



The preparation of perspective teachers in the geometry content of elementary school mathematics texts

by Margaret Neoma Botkin Banning

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
DOCTOR OF EDUCATION

Montana State University

© Copyright by Margaret Neoma Botkin Banning (1971)

Abstract:

The problem investigated in this study has been the assessment of the adequacy of preparation of Montana State University (MSU) majors in elementary education in selected concepts of geometry and the estimation of the effectiveness of certain mathematics courses in such preparation.

Mathematics course requirements in the curriculum were Math 107 and, since 1970, Math 108, a sequence taught using Webber and Brown's Number Concepts and Geometry. Annually since spring quarter, 1969, an informal geometry course using Smart's Introductory Geometry has been offered by the Department of Mathematics.

Literature research failed to reveal an instrument adequate for testing knowledge of geometric concepts common to modern mathematics programs in the elementary school. Consequently, a test was constructed based upon a detailed analysis of the geometry of four widely recognized text series: Houghton Mifflin, Laidlaw, SMSG, and GCMP. Construction of the twenty-nine item test and confirmation of its validity, reliability, and discrimination formed an important part of the study.

Test score data were gathered from the following groups with the indicated mean scores on the test instrument: 1971 seniors in elementary education, 13.60 [with 109, 23.50; with 108 but not 109, 10.17; with 107 only, 9.80]; students who completed Math 109 spring, '71, 23.88; students who completed Math 108 winter, '71, 13.83; and students with no college mathematics, 8.69.

Results were as follows: 1. The effect, on scores of seniors of the most advanced course in the mathematics sequence taken was significant at the .05 level. No evidence was found to support the hypothesis that the grade point average or the time lapsed since the last mathematics course was taken had an effect on test scores. The multiple linear regression model explained over fifty-six per cent of the- variation among the test scores of seniors.

2. The hypothesis that completion of Math 109 had the greatest positive effect upon knowledge of the selected geometric concepts was supported. This has been based upon (a) the Mann-Whitney U test for equality of scores for students completing Math 109 and those completing Math 108 being rejected at the .01 level of significance; (b) the rejection (at the .05 level) of the null hypothesis that the scores for Seniors with Math 108 as the highest course and those with Math 109 were the same using the Wald-Wolfowitz runs test; and (c) the significance of the effect of the highest level mathematics course taken by seniors. Direction was indicated by the group means.

3. The hypothesis that seniors with both Math 107 and Math 108 had better knowledge of geometry concepts than did those with Math 107 was not supported at the .05 level using the Wald-Wolfowitz runs test.

Major conclusions were that (1) many elementary teachers prepared at MSU have insufficient

background in the geometry taught in elementary school; (2) Math 109 should be strongly recommended as an elective by advisors or should be made a required course; (3) the year in which the mathematics sequence is scheduled and the academic excellence of the student may not affect proficiency in geometric concepts; (4) the content of Math 109 has been effectively taught at an appropriate level; and (5) the Math 108 requirement cannot be justified on the basis of geometric content.

THE PREPARATION OF PROSPECTIVE TEACHERS IN THE GEOMETRY
CONTENT OF ELEMENTARY SCHOOL MATHEMATICS TEXTS

by

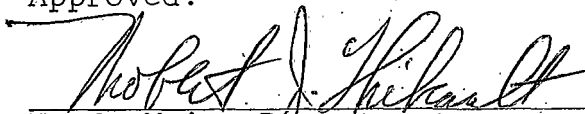
MARGARET NEOMA BOTKIN BANNING

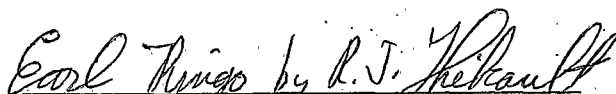
A thesis submitted to the Graduate Faculty in partial
fulfillment of the requirements for the degree

of

DOCTOR OF EDUCATION

Approved:


Head, Major Department


Chairman, Examining Committee


Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

August, 1971

ACKNOWLEDGEMENTS

Thanks are due to the members of the Doctoral Committee, especially Dean Earl N. Ringo of the College of Education who served as Chairman. The reading committee composed of Dean Ringo, Dr. Robert A. VanWoert, and Dr. J. Eldon Whitesitt is thanked for their generous donation of time and energy to that work. I wish to express my gratitude to the consultants in test revision or in statistical design: Dr. J. Eldon Whitesitt, Dr. Adrien L. Hess, Dr. Richard E. Lund, and Dr. Roy E. Byrd of the Department of Mathematics, and Dr. Albert Suvak, Head, Testing and Counseling Services at Montana State University. Special note is given to Dr. Willis C. Vandiver, Professor of Education, who aided in the plan of the study and offered his advice, his time, and the services of his office, and to Dr. Robert J. Thibeault, Head, Educational Services, for his help in enabling the completion of the thesis on schedule.

Appreciation is expressed to the College of Education and to the Department of Mathematics for providing computer time on the IBM Sigma 7. Finally, I wish to thank Marian Kennedy, Mary Ann McCusker, and my son, Charles, for their help in preparation of the manuscript.

TABLE OF CONTENTS

	Page
LIST OF TABLES	vii
LIST OF FIGURES	ix
ABSTRACT	x
Chapter	
1. INTRODUCTION	1
THE PROBLEM	1
Statement of the Problem	1
Purpose of the Study	2
Importance of the Problem	3
ANALYSIS OF THE STUDY	7
The Hypotheses	7
THEORETICAL FRAMEWORK	8
Definition of Terms	8
Elementary education seniors.	8
Math 107, Math 108, Math 109.	8
Geometric concepts of elementary school mathematics texts.	9
Basic Assumptions	9
Limitations	12
2. REVIEW OF RELATED LITERATURE	14

Chapter	Page
3. DESIGN OF THE STUDY	23
CONSTRUCTION OF THE TEST	23
Selection of Elementary Mathematics Text Series	24
Analysis of Geometry Content	25
The Test--Construction, Item Discrimi- nation, Validity, Reliability	26
THE DATA	35
Sources of the Data	35
Methods of Gathering Data	37
Elementary education seniors.	38
Students completing Math 108.	39
Students with no college mathematics.	39
THE TEST INSTRUMENT	41
The Test	43
4. PRESENTATION AND ANALYSIS OF DATA	52
THE POPULATION	52
Group A	53
THE STATISTICAL ANALYSIS	57
The Statistical Models	57
Multiple linear regression.	58
The Mann-Whitney U test.	60

Chapter	Page
The Wald-Wolfowitz runs test.	61
Analysis of the Data	63
Analysis of selected factors affecting Group A scores.	66
Analysis of the effectiveness of Math 108 and Math 109 as prepar- ation for the selected concepts of geometry tested using Group B and Group C.	74
Analysis of the effectiveness of Math 108 and Math 109 as prepar- ation in geometric concepts for elementary education seniors.	80
Analysis of test scores for Group A ₂ and Group A ₃	81
SUMMARY	83
5. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS . . .	88
SUMMARY	88
CONCLUSIONS	93
RECOMMENDATIONS	96
BIBLIOGRAPHY	99
APPENDIXES	103
A. The Test--Preliminary Form and First Revision	104
B. Computer Print-outs: Item Analyses, Correlations	119
C. Letters of Invitation	124

LIST OF TABLES

Table	Page
1. Total Grade Point Averages of Elementary Education Majors	10
2. Preparation and Practice Time of Intermediate Teachers	15
3. Computer Print-out: Item Analysis, Correlations for Preliminary Test Form	31
4. Performance on Test Items by Students with No College Mathematics and Students Completing Math 109	32
5. Kuder Richardson Correlations on the Final Form of the Test for Various Samples	33
6. Grade Levels at Which Test Item Concept Is Introduced in Teacher's Editions of the Four Mathematics Text Series	51
7. Code Symbol and Number of Students for Participating Groups	53
8. The Composition of Group A and of Forty-five Elementary Education Seniors	56
9. Means and Standard Deviations of Scores for Participating Groups	65
10. Scores, Most Advanced Mathematics Course Taken GPA, and Year Course Taken for Group A, 15 Elementary Education Seniors	68
11. Computer Program Print-out: Multiple Linear Regression, Effects of Three Variables on Group A Test Scores	70
12. Computer Program Print-out: Multiple Linear Regression, Effects of X_2 and X_3 Variables on Group A Test Scores	75

Table		Page
13.	Test Scores and Ranks for Group B and Group C	79
14.	Computer Print-out: Item Analysis, Correlations; Group A--Elementary Education Seniors	120
15.	Computer Print-out: Item Analysis, Correlations; Group B--Students Completing Math 109	121
16.	Computer Print-out: Item Analysis, Correlations; Group C--Students Completing Math 108	122
17.	Computer Print-out: Item Analysis, Correlations; Group D--Students with No College Mathematics	123

LIST OF FIGURES

Figure	Page
1. Scatter Diagram of X_{ij} Levels and Test Scores for Group A	71

ABSTRACT

The problem investigated in this study has been the assessment of the adequacy of preparation of Montana State University (MSU) majors in elementary education in selected concepts of geometry and the estimation of the effectiveness of certain mathematics courses in such preparation.

Mathematics course requirements in the curriculum were Math 107 and, since 1970, Math 108, a sequence taught using Webber and Brown's Number Concepts and Geometry. Annually since spring quarter, 1969, an informal geometry course using Smart's Introductory Geometry has been offered by the Department of Mathematics.

Literature research failed to reveal an instrument adequate for testing knowledge of geometric concepts common to modern mathematics programs in the elementary school. Consequently, a test was constructed based upon a detailed analysis of the geometry of four widely recognized text series: Houghton Mifflin, Laidlaw, SMSG, and GCMP. Construction of the twenty-nine item test and confirmation of its validity, reliability, and discrimination formed an important part of the study.

Test score data were gathered from the following groups with the indicated mean scores on the test instrument: 1971 seniors in elementary education, 13.60 [with 109, 23.50; with 108 but not 109, 10.17; with 107 only, 9.80]; students who completed Math 109 spring, '71, 23.88; students who completed Math 108 winter, '71, 13.83; and students with no college mathematics, 8.69.

Results were as follows:

1. The effect on scores of seniors of the most advanced course in the mathematics sequence taken was significant at the .05 level. No evidence was found to support the hypothesis that the grade point average or the time lapsed since the last mathematics course was taken had an effect on test scores. The multiple linear regression model explained over fifty-six per cent of the variation among the test scores of seniors.

2. The hypothesis that completion of Math 109 had the greatest positive effect upon knowledge of the selected geometric concepts was supported. This has been based upon (a) the Mann-Whitney U test for equality of scores for students completing Math 109 and those completing Math 108 being rejected at the .01 level of significance; (b) the rejection (at the .05 level) of the null hypothesis that the

scores for seniors with Math 108 as the highest course and those with Math 109 were the same using the Wald-Wolfowitz runs test; and (c) the significance of the effect of the highest level mathematics course taken by seniors. Direction was indicated by the group means.

3. The hypothesis that seniors with both Math 107 and Math 108 had better knowledge of geometry concepts than did those with Math 107 only was not supported at the .05 level using the Wald-Wolfowitz runs test.

Major conclusions were that (1) many elementary teachers prepared at MSU have insufficient background in the geometry taught in elementary school; (2) Math 109 should be strongly recommended as an elective by advisors or should be made a required course; (3) the year in which the mathematics sequence is scheduled and the academic excellence of the student may not affect proficiency in geometric concepts; (4) the content of Math 109 has been effectively taught at an appropriate level; and (5) the Math 108 requirement cannot be justified on the basis of geometric content.

Chapter 1

INTRODUCTION

Speaking before the Oklahoma Conference October 1, 1962, Bernard Jacobson, Associate Director of the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America, stated:

. . . the level of the high school program will depend upon the mathematics taught at the elementary school level and upon how it is taught. More important is the fact that a student's attitude toward the subject may be fixed for life by the time he finishes the elementary grades. . . . Probably nowhere else in the educational world do teachers understand as little of the material that they are teaching. In most cases this is not the fault of the elementary school teacher, but is due to the fact that in the past little or no attention was paid by college people to their mathematical training.¹

The preparation of elementary school teachers of mathematics, then, is a crucial factor in mathematics education.

THE PROBLEM

Statement of the Problem

The problem was to ascertain the adequacy of the

¹Bernard Jacobson, "CUPM--Its History, Activities, and Elementary School Teacher Training Recommendations," Ten Conferences on the Training of Teachers of Elementary School Mathematics, CUPM Report Number 7 (Berkeley: Committee on the Undergraduate Program in Mathematics, February, 1963), p. 23.

preparation of graduating Montana State University (MSU) majors in elementary education in the geometry of modern elementary school texts. A subsidiary problem was the determination of the effectiveness of certain courses in mathematics offered by the Department of Mathematics at MSU.

Purpose of the Study

The purpose of the study was threefold:

1. the development of an instrument to test knowledge of selected geometric concepts common to elementary school mathematics text series
2. the use of statistical analysis of test scores on this instrument to estimate (a) the competency of MSU elementary education seniors relative to these selected geometric concepts, (b) the effectiveness of courses in the Math 107, 108, 109 sequence in contributing to such knowledge, and (c) the effect of other factors such as total grade point average (GPA) and the lapse of time since the most advanced mathematics course was taken
3. the determination of possible improvements in course content or course requirements of the curriculum of the MSU Department of Elementary Education based upon the inferences resulting from the statistical analysis.

Importance of the Problem

CUPM Level I recommendations deal with the preparation of teachers of mathematics for the elementary grades and are recognized as important and realistic criteria.² These recommendations, based upon a prerequisite of two years of college preparatory mathematics in high school, suggest as minimal requirements (1) a two course sequence devoted to the structure of the real number system and its subsystems, (2) a course devoted to the basic concepts of algebra, and (3) a course in informal geometry.³ The texts used in the Math 107, 108, 109 sequence are claimed to satisfy CUPM Level I recommendations.⁴ The importance of a course in informal geometry for MSU elementary education majors has been investigated in the present study.

²Committee on the Undergraduate Program in Mathematics, Forty-one Conferences on the Training of Teachers of Elementary School Mathematics, CUPM Report Number 15 (Berkeley: Committee on the Undergraduate Program in Mathematics, June, 1966). (The Committee on the Undergraduate Program in Mathematics is hereafter referred to as CUPM.)

³CUPM, Ten Conferences on the Training of Teachers of Elementary School Mathematics, CUPM Report Number 7 (Berkeley: CUPM, February, 1963), p. 1.

⁴G. Cuthbert Webber and John A. Brown, Number Concepts and Geometry (Reading: Addison-Wesley Publishing Company, 1969); James R. Smart, Introductory Geometry: An Informal Approach (Belmont: Brooks/Cole Publishing Company, 1968).

The mathematics required for the elementary education curriculum at MSU has increased during recent years. Prior to the 1970-1972 Montana State University Bulletin the requirement was a one quarter mathematics content course, Math 107, and a course in methods of arithmetic.⁵ As of July, 1970, an additional mathematics course, Math 108, has been required.⁶ At the present time completion of Math 107 and Math 108 presumably prepares MSU's prospective elementary teachers in the background needed to teach modern mathematics programs in the elementary schools.

A course in informal geometry, Math 109, was initiated by the MSU Department of Mathematics spring quarter, 1969, and has been offered annually since then.⁷ In practice, some geometry has been taught in Math 108.⁸ Although the three courses are listed as a sequence, the practice has been not to demand Math 107 and Math 108 as

⁵Montana State University, Montana State University Bulletin, Biennial Catalog Issue for 1968-1970 (Bozeman: Montana State University, February, 1968), p. 207. (Montana State University is hereafter referred to as MSU.)

⁶MSU, Biennial Catalog Issue for 1970-71 and 1971-72 (Bozeman: MSU, July, 1970), p. 109.

⁷Ibid., p. 257.

⁸Webber and Brown, op. cit., pp. 220-253, 307-334.

prerequisites for Math 109.⁹ However, completion of Math 107 has been required for registration in Math 108.

If the CUPM recommendations involving two years of mathematics content were used as the criteria, MSU graduates who completed only the two quarters of required mathematics courses would be deemed poorly prepared to teach the mathematics of kindergarten through sixth grade (K-6). Certainly the brief introduction to geometric concepts in Math 108 would not approach the recommended level of a semester course in informal geometry. Such a geometry course appears desirable in view of the fact that a survey of mathematics text series for K-6 revealed that as much as twenty-four per cent of the total pages in a student text may be devoted to informal geometry, not including coordinate geometry or the extensive use of geometry in the development of the number line, fractions, and graphing.¹⁰

A need for a definitive study of the mathematics preparation of MSU elementary education majors has existed. Geometry is an important part of the mathematics taught in

⁹ Comment by Dr. Adrien L. Hess, March 3, 1971.

¹⁰ School Mathematics Study Group, Mathematics for the Elementary School, Grade 4 (Stanford: The Board of the Leland Stanford Junior University, 1962). (The School Mathematics Study Group is hereafter referred to as SMSG.)

the elementary grades that has been neglected in the MSU elementary education curriculum. A course in informal geometry recommended for preparation of elementary teachers has been made available by the MSU Department of Mathematics. If Math 108 adequately prepares the students in needed geometric concepts, Math 109 would not be considered a justifiable additional requirement. The significance of Math 109 in preparing students specifically in the geometry of modern elementary mathematics texts is of concern to both the Department of Elementary Education and the Department of Mathematics.

How well the elementary education majors performed on tests of Math 107, 108, 109 content was not the question. The question was whether the graduating senior was knowledgeable in the mathematics of modern programs for grades K-6. The construction of a test with items related directly to geometric concepts common to selected elementary school mathematics texts constituted an important preliminary project. A search of the literature has failed to reveal an adequate instrument to measure such understandings.

ANALYSIS OF THE STUDY

The research design involved (1) the construction of a testing instrument, (2) the administration of the test to appropriate subjects, (3) the choice of statistical models to elicit information about the significance of the data, and (4) drawing valid inferences. Literature research has been an integral method used in the present study at each stage. Accepted standard methods have been used to construct an instrument of suitable validity and reliability. The opinions of experts have been sought, and their suggestions have been implemented. Data have been analyzed using computer programs for the IBM Sigma 7 computer.

The Hypotheses

The hypotheses to be tested were:

1. Test performance by elementary education seniors is a function of several factors: (a) the mathematics courses completed in the Math 107, 108, 109 sequence; (b) the general academic ability as evidenced by total GPA; and (c) the time lapsed since the most advanced mathematics course taken was completed.
2. Completion of Math 109 has the greatest positive effect on knowledge of geometry contained in the mathematics

texts of elementary school.

3. Seniors who completed both Math 107 and Math 108 have significantly better knowledge of the selected geometric concepts than do seniors who completed only Math 107.

4. The course content of Math 109 specifically prepares students in the concepts of geometry being tested.

THEORETICAL FRAMEWORK

For the purposes of the present study certain definitions, certain basic assumptions, and certain limitations have been specified.

Definitions of Terms

Elementary education seniors. The term "elementary education seniors" has referred to those MSU students who are in the fourth and final year of the regular elementary education curriculum. The objective of the curriculum is the preparation of graduates to teach in grades K-6.

Math 107, Math 108, Math 109. Math 107, Math 108, and Math 109 are courses offered by the MSU Department of Mathematics. The catalog descriptions indicate that Math 107 and Math 108 are quarter courses to be taken in sequence

and are intended to be "an introduction to basic mathematical concepts and structure."¹¹ The text used for this sequence has been that authored by Webber and Brown. Math 109 is titled "Geometry from an Intuitive Approach" and has used the text by Smart. The three course sequence "gives an understanding of modern mathematics as it is found in the elementary grades."¹²

Geometric concepts of elementary school mathematics texts. "Geometric concepts of elementary school mathematics texts" has been used to indicate those concepts presented by the four elementary mathematics text series for grades K-6 that were used in this study. The text series have been widely recognized and used in numerous school systems.

Basic Assumptions

In this study the assumption has been made that elementary education seniors have been prepared for the teaching of elementary school mathematics through the Math 107, 108, 109 sequence and the methods of arithmetic course. Other courses in mathematics may have been completed by

¹¹MSU, Montana State University Bulletin, Biennial Issue for 1970-71 and 1971-72 (Bozeman: MSU, 1970), p. 109.

¹²Ibid.

individual seniors, but the emphasis in such courses has not been the mathematics background for elementary education.

The elementary education seniors of the 1971 class were assumed to be representative of such classes in all instances of generalization from the data of a sample of the 1971 seniors to more inclusive groups. From the standpoint of the overall GPA means shown in Table 1 the assumption seemend reasonable. Attention is directed to the fact that the group of all elementary education students included transfer students, all college classes, and some students who would not complete the curriculum.

Table 1

Total Grade Point Averages of Elementary Education Majors

Description of Group	Mean	Range
All students, elementary teaching option, fall, 1969 ^a	2.85	2.03-3.73
1971 seniors ^b	2.75	2.20-3.87
Sample used in study ^c	2.93	2.20-3.87

^aData from Dr. Albert Suvak, Head, Testing and Counseling, MSU, Bozeman, Montana. N = 78.

^bThe seniors whose mathematics courses were taken at MSU from the Math 107, 108, 109 sequence. N = 45.

^cN = 15.

A further assumption has been that the experience received in a given course has not differed significantly from year to year nor from instructor to instructor. Math 107 and Math 108 have been multi-sectioned courses with as many as six different instructors teaching the various sections. However, Dr. Adrien L. Hess, Professor of Mathematics, has been the supervisor of the sequence and has arranged the class schedules identically in the sections. Uniformity of course content has been insured, also, by common testing and evaluation procedures.

The geometry concepts contained in the four selected elementary school mathematics text series have been assumed not to differ significantly from the geometric concepts of other contemporary programs in mathematics for the elementary school. Since the School Mathematics Study Group (SMSG) texts have been used as models for revision of many commercially published texts and since the SMSG text series for K-6 has been one of the four series used in the present study the assumption seemed justifiable.¹³

¹³Phillip S. Jones and Arthur F. Coxford, Jr., "Reform, 'Revolution,' Reaction: 1945-Present," A History of Mathematics Education in the United States and Canada, Thirty-second Yearbook of the National Council of Teachers of Mathematics (Washington: NCTM, 1970), p. 76.

Limitations

No attempt has been made in this study to investigate effects on elementary education seniors' knowledge of the selected geometric concepts except for the following:

(1) the courses in the Math 107, 108, 109 sequence experienced; (2) the total GPA; and (3) the time lapsed since the most advanced mathematics course was completed. In particular, an analysis of the effect of the required course in methods of arithmetic has not been a part of this study, nor has the high school mathematics preparation been researched.

The population involved in the present study has consisted of those elementary education seniors whose college mathematics courses were from the Math 107, 108, 109 sequence. Seniors who transferred from another major into elementary education, who transferred mathematics credits substituting for Math 107 or Math 108 from another institution, or who completed more advanced courses offered by the MSU Department of Mathematics have not been a part of the present study.

In addition to the seniors, certain groups of students who had just completed Math 108, who had just completed Math 109, or who had experienced no college mathematics beyond intermediate algebra have been involved in the study. In

some cases and for some subsets of these groups, the major and the class rank of students have not been considered.

No claim has been made that the instrument constructed to determine knowledge of selected geometric concepts assesses all the geometric competencies that should be possessed by elementary education seniors. All statements related to the achievement of elementary education majors and others on the test instrument refer only to the geometry contained in the test items.

Chapter 2

REVIEW OF RELATED LITERATURE

Much has been written about the elementary school teacher's lack of preparation in mathematics. The evidence has supported such a conclusion and has resulted in the announcement of goals and recommendations for teacher education in mathematics. The CUPM Level I recommendations have been discussed. Other significant groups have proposed plans for improving the situation--the Cambridge Conference on Teacher Training of 1965¹ and the joint committee of the National Association of State Directors of Teacher Education and Certification and the American Association for the Advancement of Science.²

In 1964 Roush presented results that demonstrated that of the four main areas of subjects taught in elementary school, mathematics was the subject that received the least

¹Cambridge Conference on Teacher Training, Goals for Mathematical Education of Elementary School Teachers (Boston: Houghton Mifflin Company, 1967).

²National Association of State Directors of Teacher Education and Certification and the AAAS, Guidelines for Science and Mathematics in the Preparation Program of Elementary School Teachers (Washington: AAAS, 1961).

attention relative to the time spent on the subject area in the classroom. For the intermediate grades the comparison of preparation and practice time was as shown in Table 2. Thus, although seventeen per cent of an intermediate teacher's classroom time might be spent in the teaching of arithmetic, only four and one half per cent of the semester credit hours of the teacher's college curriculum were devoted to preparation in the subject.³

Table 2

Preparation and Practice Time of Intermediate Teachers

Subject	Preparation (% of 132 semester hours)	Practice (% of classroom time)
Social studies	18.2	12.5
Language arts	16.7	28.0
Science	14.4	12.5
Mathematics	4.5	17.0

Source: Donald C. Roush, "Implementation of CUPM Recommendations," Forty-one Conferences on the Training of Teachers of Elementary School Mathematics, A Summary, CUPM Report Number 15 (Berkeley: CUPM, June, 1966), Chart I, p. 31.

³ Donald C. Roush, "Implementation of CUPM Recommendations," Forty-one Conferences on the Training of Teachers of Elementary School Mathematics, A Summary, CUPM Report Number 15 (Berkeley: CUPM, June, 1966), pp. 30-31.

Other research has indicated the consequences of such inadequate mathematics training. Reys found that fifty-five per cent of the preservice elementary education majors in his study scored below the median for eighth or ninth grade students on the Contemporary Mathematics Test, Algebra Level.⁴ Wozencraft in a study at a highly selective state university had shown earlier that although the elementary education students were in the sixty-fifth percentile on the American Council on Education Psychological Test they only achieved at a seventh grade median on a standardized mathematics test.⁵ In a similar vein, Carroll concluded that education students had only about half the mathematical understandings needed to competently teach arithmetic.⁶

The critical need for improving the mathematics preparation of elementary school teachers has been recognized nationwide. Fisher found that in almost every state

⁴Robert E. Reys, "Mathematical Competencies of Preservice Elementary School Teachers," School Science and Mathematics, April, 1968, pp. 302-308.

⁵Marian Wozencraft, "Even the Teacher Can't Do It!," Journal of Teacher Education, September, 1960, pp. 387-390.

⁶Emma C. Carroll, "A Study of Mathematical Understandings Possessed by Undergraduate Students Majoring in Elementary Education" (unpublished Doctor's dissertation, Wayne State University, 1961).

resolutions were passed in support of the CUPM Level I recommendations at the various regional conferences. There the delegate teachers agreed that the present requirements for elementary school teachers were inadequate.⁷ Research reported by Kenney in 1965 showed that teacher experience did not increase understanding of mathematical concepts or processes. Kenney constructed an instrument to test understandings of number systems, measurements, graphs, scales, whole numbers, common fractions, decimal fractions, and percent. Kenney was not testing geometric concepts, yet he found no adequate instrument in the literature to measure the understandings being tested.⁸

Research findings have supported the notion that completion of mathematics courses by preservice or inservice elementary teachers serves to increase conceptual understandings of mathematics. Garnet found that scores on a mathematical concepts test increased as the number of high school and college mathematics courses increased. Here,

⁷ John J. Fisher, "Extent of Implementation of CUPM Level I Recommendations," The Arithmetic Teacher, March, 1967, pp. 194-197.

⁸ Russel A. Kenney, "Mathematical Understandings of Elementary School Teachers," The Arithmetic Teacher, October, 1965, pp. 431-442.

again, Garnet found it necessary to construct the evaluation instrument.⁹ A 1970 research report concluded that elementary teachers did progressively better on a test of mathematical understandings with more years of high school mathematics, with more college mathematics courses, and with more exposure to "modern" mathematics.¹⁰

The relationship between a teacher's mathematics preparation and his pupils' achievement has interested researchers. Hunkler¹¹ investigated the achievement of sixth-grade pupils in modern mathematics related to their teachers' mathematics preparation and concluded that (1) completion of one college mathematics course had no effect, (2) completion of two or more college mathematics courses had a positive effect, and (3) three years of college preparatory

⁹Emma Whitlock Garnet, "A Study of the Relationship Between the Mathematics Knowledge and the Mathematics Preparation of Undergraduate Elementary Education Majors," (unpublished Doctor's dissertation, George Peabody College for Teachers, 1968).

¹⁰Thomas C. Gibney, John L. Ginther, and Fred L. Pigge, "A Comparison of the Number of Mathematical Courses taken by Elementary Teachers and Their Mathematical Understandings," School Science and Mathematics, May, 1970, pp. 377-381.

¹¹Richard Frederic Hunkler, "Achievement of Sixth-Grade Pupils in Modern Mathematics as Related to Their Teachers' Mathematics Preparation," (unpublished Doctor's dissertation, Texas A&M University, 1968).

mathematics in high school had a positive effect only if the courses were in a modern program. An interesting study by Moody involved the relationship between fifth-grade student and teacher performance on selected tasks in nonmetric geometry. This investigator found support for the hypothesis that the level of success of the teacher on those selected conceptual tasks and the level of success of pupils in the class following instruction tended to be the same.¹² The interrelationship between teacher mathematical competency and pupil understanding was supported further by the work of Lampela.¹³

In a study more closely related to the present study, Kipps investigated elementary teachers' ability to understand concepts used in new mathematics curricula. A section on geometry was included in the test instrument and the test was based upon mathematics concepts in sixteen textbook

¹²William Braun Moody, "An Investigation of the Relationship between Fifth-Grade Student and Teacher Performance on Selected Tasks Involving Nonmetric Geometry," (unpublished Doctor's dissertation, University of Maryland, 1968).

¹³Roland Mitchell Lampela, "An Investigation of the Relationship between Teacher Understanding of Selected Concepts and Change in Pupil Understanding of Selected Concepts in Elementary School Mathematics," (unpublished Doctor's dissertation, University of California, Los Angeles, 1966).

series (K-6) published since 1962. The geometry section of the instrument included the following subsections: names and properties of common shapes; boundaries and regions; congruence, similarity and symmetry; measurement and estimation of lengths, areas and volumes; and graphs (circle, bar, line); construction and interpretation of tables. The test of forty-two questions covered twenty-eight subsections of which geometry involved the five given above. The results indicated that the mean scores on the geometry section were lower than on any other part of the test, that less than half the teachers involved in the study could find areas of common geometric figures (given a picture on graph paper or a formula), that half knew the names of different kinds of triangles, and that one fourth knew criteria for determining a plane figure. One of the conclusions of the study was that an appropriate course in informal geometry needed to be offered or required in more teacher training institutions.¹⁴

Neatrou researched the geometric content in the mathematics curriculum of the middle school. He found that while the geometric content varied greatly among the sixteen

¹⁴ Carol Kipps, "Elementary Teachers' Ability to Understand Concepts Used in New Mathematics Curricula," The Arithmetic Teacher, April, 1968, pp. 367-370.

mathematics texts used in 156 middle schools in a nineteen state region, that the geometric content was roughly three times as much as that of 1900 and emphasized informal geometry.¹⁵ Such results would be expected since mathematics educators have been concerned about the geometry content of elementary school mathematics. All the experimental programs have stressed the need for geometry to be introduced at an earlier grade and to be integrated in the curriculum. Hawley and Suppes published Geometry for Primary Grades in an attempt to improve the geometry of the early grades.

The decisions of major study groups who developed mathematics text series have been put into practice by the commercial publishers of textbooks. Hughes¹⁷ did a study to determine the impact of SMSG and of the Greater Cleveland Mathematics Program (GCMP) on contemporary texts. An analysis of ninety-four textbooks involved readability,

¹⁵ Charles Raymond Neatrour, "Geometric Content in Mathematics Curriculum of the Middle School" (unpublished Doctor's dissertation, Indiana University, 1968).

¹⁶ Newton Hawley and Patric Suppes, Geometry for Primary Grades (San Francisco: Publishers Book Service, 1960).

¹⁷ Eugene Morgan Hughes, "The Impact of Selected Experimental Curriculum Projects on Commercially Published Elementary School Mathematics Textbooks: (unpublished Doctor's dissertation, George Peabody College for Teachers, 1968).

vocabulary, scope and sequence, and teaching strategies. The conclusion reached was that SMSG had a greater overall impact on post 1960 textbooks than GCMP had. Hughes speculated that this was due to the fact that SMSG was developed to serve as a model and a source of suggestions for authors of commercially published textbooks.

In summary, a diligent search of the literature has revealed no study comparable to the present research design. No studies of a nature similar to the present one have been found among the research papers and dissertations at MSU. No instrument to test the knowledge of the geometry of elementary school mathematics texts has been located. One test was based upon the mathematics of elementary school texts, but this test did not involve geometry as the main subject matter nor was it of the comprehensive nature necessary for the purposes of this study. The literature has provided ample research findings to confirm the importance of geometry in elementary school mathematics programs, the importance of SMSG and GCMP text series in the field, and the lack of elementary school teacher preparation in mathematics.

Chapter 3

DESIGN OF THE STUDY

The research study has involved (1) the construction of a valid, reliable instrument to test knowledge of the geometry content of selected contemporary elementary education texts, (2) the administration of the test to subjects whose scores would supply the desired information, (3) the choice of suitable statistical models for eliciting information about the significance of the resulting data, (4) the use of such models to determine the various statistics, and (5) the interpretation of the statistical analysis and dissemination of the conclusions and recommendations.

CONSTRUCTION OF THE TEST

The construction of the test instrument has been based upon the geometry taught in modern elementary school programs. Since the study has been designed specifically to relate to the MSU elementary education curriculum, texts likely to be used by these graduates in the field have been considered. Construction of the instrument has involved (1) the selection of elementary mathematics text series to

be used as a basis for test content, (2) the analysis of the geometry content of these selected series and determination of common concepts, (3) the construction of a test form from items testing these concepts, and (4) the revision of the test on the basis of accepted indications of validity, reliability, and discrimination.

Selection of Elementary Mathematics Text Series

A survey of MSU elementary education graduates of the years 1968 through 1970 revealed that, in Montana, the public school systems hiring the greatest numbers (per single system) of MSU graduates in elementary education were Billings, Great Falls, and Bozeman.¹ Therefore, the text series used in those systems were chosen as two of the series: the Laidlaw series, used in most of the Billings schools, and Houghton Mifflin, used in the Bozeman and Great Falls systems.² The other K-6 mathematics text series chosen as a basis for geometry content selection were the nationally well-accepted SMSG and the version of the GCMP³

¹Information received from Mr. John Breeden, Assistant Director, MSU Career Placement Office, October, 1970.

²See page 25.

³The GCMP originated as a project of the Educational Research Council of Greater Cleveland.

published by Science Research Associates. The elementary school mathematics text series analyzed for geometry content along with the symbol or abbreviation used throughout this paper in referring to the chosen text series were as follows:

Gundlach, Bernard H. and others. The New Laidlaw Mathematics Program. River Forest, Illinois: Laidlaw Brothers Publishers, 1968. (LAIDLAW)

Duncan, Ernest R. and others. Modern School Mathematics, Structure and Use. Boston: Houghton Mifflin Company, 1967. (H-MIFFLIN)

SMSG. Mathematics for the Elementary School. New Haven: Yale University Press. Publication dates range from 1962 to 1965 for various grades. (SMSG)

Educational Research Council of America. Greater Cleveland Mathematics Program. Chicago: Science Research Associates, Inc., 1968. (GCMP)

In each case both the student text and the teacher's edition have been utilized for analysis of geometry content.

Analysis of Geometry Content

The geometry content of the four text series was subjected to careful analysis. From detailed, annotated lists of geometric content which omitted coordinate geometry and use of geometry related to development of the number line, fractions, and graphing, the concepts common to at least two of the series were determined. After careful consideration of exact terminology and differences in definitions and usage,

a selection of concepts from the master list was made. A thirty item test was constructed based upon this list of common concepts.⁴

The Test--Item Discrimination, Validity, Reliability

The test was constructed with detailed attention to the vocabulary and to the symbolism used in the text series. Familiarity with elementary set theory and operations on sets has been assumed since each of the four series used a point-set approach to geometry. Nevertheless, caution has been taken not to use a symbol when a written word would be understood and not be too cumbersome. For example, the symbol for intersection of sets has not been employed.

No item was included in the test that used a definition or notation about which the text series differed. The terminology related to area differed. H-MIFFLIN defined "area" for a rectangular region with region defined as the interior of a simple closed curve. SMSG defined "area" of a rectangular region, but in this case the region was defined to be the union of a simple closed curve and its interior. GCMP, on the other hand, discussed the technical idea of a region in the Teacher's Edition but used the phrase "area of

⁴See Appendix A, pp. 105-111.

a rectangle." Therefore, no item on the test has sought the definition of either "area" or "region," but instances for calculation of areas have been provided. Another consideration was the term "sup set" which was peculiar to the H-MIFFLIN series. That term has not been used in any item of the test.

The thirty-item test was administered to ninety-seven students on the final day of classes for Math 108, winter quarter, 1971. An item analysis was made for these scores which has been reproduced in Table 3. The biserial correlation was used as a measure of the discriminatory power of each item. The number of students who answered a question incorrectly and who chose each of the four incorrect choices indicated the drawing power of that choice. In cases in which no student selected a choice, revision of that particular completion was indicated.⁵

Based upon the item analysis, the initial revision of the test had altered forms for the following questions

⁵George A. Ferguson, Statistical Analysis in Psychology and Education (New York: McGraw-Hill Book Company, 1966), pp. 242-244. The program used for the IBM Sigma 7 computer was originated by Dr. Albert Suvak, Head, MSU Testing and Counseling Service, who acted as a consultant in test construction and designated significance levels for the biserial correlation.

or choices: 1(b); 2(a), 2(b); 4(c); 7(b), 7(c); 8(c); 9(c), 9(d); 11(b); 12(c); 15(b); 18(d), 18(e); 22; 25(d). The exact changes may be seen by comparing the preliminary test form and the first revision of the test. Both of these test forms have been reproduced in Appendix A.

The revised form was then submitted for validation by expert opinion to Dr. J. Eldon Whitesitt, Professor and former Head, MSU Department of Mathematics, and to Dr. Adrien L. Hess, Professor of Mathematics and Supervisor of the Math 107, 108, 109 sequence. The final form of the test instrument incorporated the suggestions of these two experts in the field mathematics education.

The final form of the test was composed of twenty-nine items. Item number 28 of the original form was omitted from the final form because of a negative biserial correlation. The test question was as follows:

The union of two distinct angles in the same plane

- (a) may be the empty set, $\emptyset$.
- (b) may be a set with just one point as an element.
- (c) may be a ray.
- (d) may be an angle.
- (e) None of the above choices is possible.

The opinion of the author and of the expert consultants was that this question should not have been difficult for students who had completed Math 108. Dr. Hess suggested

that there may have been some reluctance to choose the correct response, choice (e), because of its nature or because Item 28 was the only completion of that type. Another possibility was that the student thought "intersection" rather than "union." The intersection of two distinct angles may be the empty set, a singleton set, or a ray among the choices offered. Such an error in thinking would not account for the twenty students who chose "may be an angle," however. The decision was made to eliminate this question from the test.

Further confirmation of the validity of the test was possible after other groups of students had taken the test. A comparison of performances of students with no college mathematics and those who had just completed Math 109 has been made relative to percentage of each group correctly answering a given problem. The per cents have been given in Table 4. If the item were valid a higher per cent of students completing Math 109, the course in informal geometry, should complete the item correctly. Two items, number 1 and number 27, have not been considered since the concepts involved have not been stressed in Math 109. For every item the per cent for students who had completed Math 109 was greater than for students with no

college mathematics. All of the group who had Math 109 answered items 4 (definition of angle), 9 (lines of symmetry of a plane figure), 13 (relationship of squares and rectangles), 17 (area of a circular region), 18 (definition of scalene triangle), 22 (definition of octagon), and 29 (the intersection of a line and a plane) correctly. The per cents for the group with no college mathematics ranged from twelve to sixty-four for these same problems. Other instances were Item 11 (concept of a prism) and Item 12 (lengths of sides of a right triangle) which less than ten per cent of the group with no mathematics in college answered correctly while over seventy per cent of the group who had taken Math 109 responded correctly on these items. Since the significance of the differences between the per cents for the two groups was obvious, no statistical tests have been necessary.

The Kuder Richardson correlation has been used as an indicator of test reliability. The fact that the test instrument was constructed on the basis of material not all a part of Math 108 content influenced the value for that group.⁶ The Kuder Richardson formula 20 statistic for the

⁶Ferguson, op. cit., pp. 379-380. 385-386.

Table 3

Computer Print-out: Item Analysis, Correlations
for Preliminary Test Form

Item	Per cent correct	Biserial R	Choice				
			A	B	C	D	E
1	36	.28598	20	7	25	35*	9
2	55	.13902	1	54*	2	10	29
3	39	.30069	25	2	38*	4	28
4	83	.21674	81*	6	0	6	4
5	71	.37678	69*	16	4	1	7
6	18	.33486	19	18*	8	40	11
7	59	.30543	19	1	1	17	58*
8	20	.20938	20*	51	1	9	14
9	35	.06999	18	34*	0	6	38
10	39	.20581	10	11	10	38*	25
11	5	.26827	26	0	5*	5	60
12	32	.46904	10	32*	1	22	25
13	69	.34400	67*	4	10	6	9
14	40	.06685	10	14	19	15	39*
15	61	.18665	5	1	3	60*	25
16	68	.36927	3	3	15	7	66*
17	49	.48726	3	48*	18	3	25
18	80	.46721	11	78*	2	0	5
19	16	.26774	7	24	16*	27	21
20	43	.32162	13	3	32	42*	6
21	27	.34147	27*	12	15	5	31
22	89	.09366	1	87*	3	3	2
23	29	.34803	47	29*	3	2	10
24	61	.28760	18	3	4	12	60*
25	52	.32138	22	5	51*	0	18
26	59	.36655	4	2	24	58*	7
27	55	.29806	54*	6	15	2	16
28	4	-.04633	32	12	12	20	17*
29	15	.25106	1	15*	45	13	22
30	61	.34930	17	8	60*	7	3

*Correct choice for the given item.

N = 97. Mean = 13.84536. Sigma = 3.78413.

Kuder Richardson Correlation = .59201. Variance = 14.31962.

Table 4

Performance on Test Items by Students
with No College Mathematics and
Students Completing Math 109

Item	Per cent correct response	
	No math course ^a	Math 109 ^b
1 ^c	38	50
2	20	81
3	30	75
4	12	100
5	48	93
6	12	50
7	25	93
8	25	81
9	64	100
10	15	81
11	7	75
12	5	81
13	41	100
14	30	81
15	33	81
16	38	93
17	20	100
18	33	100
19	23	43
20	30	87
21	12	87
22	87	100
23	20	93
24	43	75
25	28	68
26	43	81
27 ^c	25	37
28	15	93
29	33	100

^aN = 39.^bN = 16.^cConcept not stressed in Math 109.

Source: Computer print-outs. See Appendix B.

participating groups has been given in Table 5. The correlation for elementary education seniors, .9397, denoted a sufficiently high reliability for the test relative to this major group of students.

Table 5

Kuder Richardson Correlations on the Final Form
of the Test for Various Samples

Group of students	Kuder Richardson correlation
Students with no college math	.5991 ^a
Students completing Math 108	.5993 ^a
Students completing Math 109	.5522 ^a
Elementary education seniors	.9397 ^a

^aSee Appendix B. Figures rounded to the nearest ten-thousandth.

The test instrument used in the present study has been shown to be a valid and reliable test. Validity has been determined by means of an item analysis and the opinion of experts, and later validity was checked by comparison of per cents of two groups, students with no college mathematics and students completing Math 109, selecting the correct answer for each of the twenty-nine items.

The Kuder Richardson correlation for the scores of

elementary education seniors, .9397, was close enough to the perfect correlation, one, to indicate high level of reliability. Ferguson has explained:

. . . . It may be shown that if a test is split in all possible ways, the average of all the split-half reliability coefficients with the Spearman-Brown correction is the Kuder-Richardson formula 20. . . .

The Kuder-Richardson formula 20 is a measure of the internal consistency, or homogeneity, or scalability, of the test material. In this context these three terms may be considered synonymous. If the items on a test have high intercorrelations with each other and are measures of much the same attribute, then the reliability coefficient will be high. . . .⁷

Item discrimination has been checked using the biserial correlation statistic which correlates an item with the total test score. These correlations served to indicate those items to be revised after administration of the preliminary form of the test. A variation for a given item has been noted. For example, Item 27 had biserial correlations .29801, .49678, -.01978, and .18408 for the various participating groups. However, because the .29801 value was for a group of size ninety-seven and the .49678 value was for the group of size thirty-nine these are the values to be considered since the significant levels are lower as the number in the group increases. During the

⁷Ibid., pp. 379-380.

construction of the test instrument only the data for students completing Math 108 were available. For the final test form the discrimination of the items was at an acceptable level.⁸

THE DATA

Data pertinent to the present study have been gathered as scores on the test instrument from four distinct samples of MSU students. In some cases the test was administered in a classroom situation. Other groups were volunteers.

Sources of the Data

Maximum information about the preparation of MSU elementary education graduates entering the teaching profession has been obtained from test data from a sample of the 1971 seniors in elementary education. Participants have been selected with the following restrictions:

1. The senior had experienced his college mathematical preparation at MSU from among those courses

⁸George W. Snedecor and William G. Cochran, Statistical Methods (6th ed., Ames, Iowa: The Iowa State University Press, 1967), p. 557.

designated for training in elementary school mathematics: Math 107, Math 108, and Math 109. Seniors whose mathematical background did not fit this specification have not been a part of the study.

2. The senior volunteered as a participant by attending one of the meetings scheduled for administration of the test. Since the nature of the study was not given to the seniors in the announcements, the volunteers were not seniors who felt well prepared in mathematics.

These restrictions did not cause the sample to be atypical as far as the overall GPA was concerned. The GPA of the seniors has been compared with other groups in Table 1, page 10. Other evidence supporting the similarity of the sample and other elementary education seniors who met the given restrictions has been found in the statistical analysis and the score means.

In order to determine the relevance and effectiveness of Math 108 and Math 109 in instilling knowledge of the selected geometric concepts of elementary school mathematics, the test has been administered to samples of the following students:

1. students completing Math 108 winter quarter, 1971 (those students in attendance the final day of classes)

2. students completing Math 109 spring quarter, 1971

3. students who had taken none of the Math 107, 108, 109 sequence and had not more than an intermediate algebra course during college.

Students with no college mathematics were selected from two populations. Students registered for Psychology 103 were chosen since that course is required for the elementary education major as a lower level course, prior to the scheduled mathematics curriculum. This sample consisted of volunteers. Another group was composed of those students taking Math 107 summer quarter, 1971, who had no previous college mathematics courses.

Methods of Gathering Data

Methods used in gathering the test scores for the selected groups of students varied in the location and time of administration of the test. Answers were recorded using IBM Form 1230 Document No. 512, a standard form, since the mark sense forms were to be processed using the IBM Sigma 7. No time limit was imposed on the test. Scores were guaranteed to be treated confidentially. Directions for taking the test were the same for each group and were

given either orally or both orally and written. The instructions were as follows:

DIRECTIONS:

1. On the answer sheet enter your name only. Do not enter your Social Security number.
2. READ each multiple choice question CAREFULLY and COMPLETELY.
3. Record the letter of the ONE MOST CORRECT AND COMPLETE CHOICE FOR EACH COMPLETION.
4. If you are uncertain about the correct choice:
 - a. Consider each possibility.
 - b. Enter the letter of your best educated guess.
5. Now take the test. Scratch paper is provided. There are five (5) pages. There are twenty-nine (29) questions. Do not write on the test.
6. This test is scored by the number of correct responses.

Elementary education seniors. During May, 1971, two times were designated for administration of the test instrument. Seniors who met the specifications previously stated were invited to the meeting by a letter mailed to each senior from the Department of Elementary Education. The letter was composed jointly by Dr. Willis Vandiver, then Head, Department of Elementary Education at MSU, and the author.⁹ Purposely, the letter did not mention the nature of the meeting since it was felt that a prior

⁹See Appendix C, p. 125.

announcement of the mathematics test might (1) encourage the student to review his mathematical experience or (2) discourage the student from attending the meeting.

Eleven elementary education seniors attended the first scheduled meeting, and, after a follow-up communication,¹⁰ four more seniors attended the second meeting. There were fifteen elementary education seniors participating in the study.

Students completing Math 108. Four sections of Math 108 were offered winter quarter, 1971. The test was given in each of the four sections on the last class day. The sample included every student who attended class on the given day, irrespective of the student's major, GPA, or other factors. No differentiation has been made among the sections taught by the three instructors of the various sections.

Students with no college mathematics. An effort was made to administer the test to a satisfactory group of elementary education majors who had taken none of the required mathematics courses of the elementary education

¹⁰See Appendix C, 126.

curriculum. When this plan met with failure through no fault of the author, a substitute plan was implemented. A group of volunteers were obtained from among students in an introductory course in psychology, Psych 103. Dr. Truman M. Mast, Associate Professor of Psychology, assisted by making announcements of the administration of the test to his classes. The Psych 103 course has been required by elementary education majors, usually during the first year of the curriculum.¹¹ The test was administered for this group in combination with the second group of elementary education seniors in May, 1971.

Additional subjects in this category took the test during the first week of summer quarter, 1971. The test was given during a regularly scheduled Math 107 class by Dr. J. Eldon Whitesitt, the instructor. Those students in the class who had taken no other mathematics, except possibly a course in intermediate algebra, and who had no experience teaching mathematics in elementary school were included in the sample.

¹¹MSU, Montana State University Bulletin, Biennial Catalog Issue for 1970-1971 and 1971-1972 (Bozeman: MSU, 1970), p. 109

The sample employed in the study, then, has been composed of all students with no college mathematics who either volunteered or who took Math 107 during summer, 1971.

THE TEST INSTRUMENT

The test used to elicit the data has been constructed as an integral part of the present study. The purpose of the test has been to ascertain the knowledge of geometric concepts of elementary school mathematics attained by selected students at MSU. Content of the test items has been determined by an analysis of the geometry of four selected elementary school mathematics text series. The final form of the instrument and Table 6 which is a tabulation of the test items and the grade level or levels at which the given concepts appear in each of the text series have been presented here.

The data in Table 6 disclosed that in some instances a given concept was introduced in the Teacher's Edition notes or in the student text at approximately the same level in most of the text series. On the other hand, some concepts may have been omitted from a given series for grades K-6. The choice of the grade level listed in Table 6 has not been intended to convey the impression that the given

level represents the entire development of the concept. The given notion may have been developed at more elementary grades and usually has been extended in subsequent grade levels. The actual term involved in the test item may not have been used in a text in order to have that level entered in the table. The attempt has been made to test the concept at the teacher's level. For example, once the sum of measures of angles has been introduced and the right angle has been defined in terms of degrees, the notion of "complementary angles" naturally arises whether or not the phrase has been used in a text. Another instance was the term "scalene triangle." Three of the text series introduced the term in the Teacher's Edition, two introduced the term in the student text, and one did not introduce the term. However, since all terms used in test item No. 16 other than "scalene" had been defined and developed in each of the four text series, the question is appropriate for prospective teachers of each of the elementary school mathematics programs.

As references for item construction and for defense of the multiple-choice question type two publications of the Educational Testing Service have been used: Making the classroom test: A guide for teachers and Multiple-choice

Questions: A close look.¹²

The Test

For convenience, the final form of the test instrument has been reproduced here.

1.

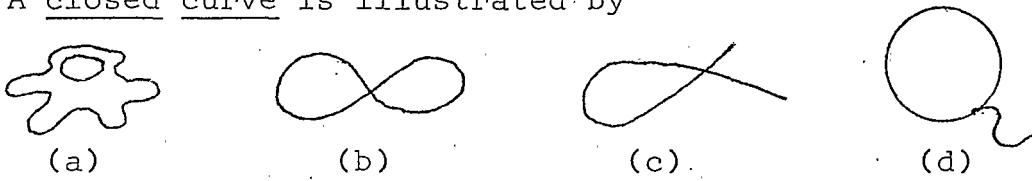


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

A geometric figure is illustrated in

- (a) Figures 5 and 4 only.
- (b) Figures 2, 3, and 4 only.
- (c) Figures 2, 3, 4, and 5 only.
- (d) All of the figures; 1, 2, 3, 4, and 5.
- (e) None of the above choices is correct.

2. A closed curve is illustrated by



- (a)
- (b)
- (c)
- (d)
- (e) More than one of the figures shown.

¹²Educational Testing Service, Making the classroom test: A guide for teachers (2nd ed.; Princeton, N. J.: Educational Testing Service); Educational Testing Service, Multiple-choice questions: A close look (Princeton, N. Y.: Educational Testing Service, 1963).

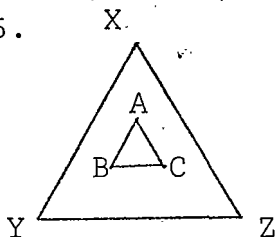
3. By definition, two lines are said to be parallel if and only if they are

- (a) any two lines whose intersection is the empty set.
- (b) opposite sides of a parallelogram.
- (c) lines in the same plane whose intersection is the empty set.
- (d) any two lines in different parallel planes.
- (e) Both choice (a) and (d) are correct.

4. By definition an angle is

- (a) the union of two noncollinear rays with a common endpoint.
- (b) the union of two line segments with one endpoint in common.
- (c) the union of any two rays in the same plane.
- (d) the union of two rays with a common endpoint and all segments that have one endpoint in one of the rays and the other endpoint in the other ray.
- (e) the union of two intersecting lines.

5.



Consider the triangles shown at the left. The intersection of $\triangle ABC$ and $\triangle XYZ$ is

- (a) the empty set, $\emptyset$.
- (b) $\triangle ABC$.
- (c) $\triangle XYZ$.
- (d) points A, B, and C (i.e., $\{A, B, C\}$)
- (e) None of the above choices is correct.

6. The relationship between angle ABC ($\angle ABC$) and the triangle ABC ($\triangle ABC$) is that

- (a) $\angle ABC$ is a subset of $\triangle ABC$.
- (b) the intersection of $\angle ABC$ and $\triangle ABC$ is the union of $\overline{AB}$ and $\overline{BC}$.
- (c) line segment $\overline{AC}$ is a subset of $\angle ABC$.
- (d) More than one of (a), (b), and (c) are correct.
- (e) None of the above statements is correct.

7. If the sum of the measures, in degrees, of two angles is 90 then the angles are said to be
- (a) right.
 - (b) adjacent.
 - (c) congruent.
 - (d) supplementary.
 - (e) complementary.
8. A triangular pyramid is also called a
- (a) tetrahedron.
 - (b) triangular prism.
 - (c) triangular solid.
 - (d) solid with five faces.
 - (e) None of the above choices is correct.
9. A figure that has four different lines of symmetry is
- 

(a)

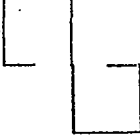


(b)



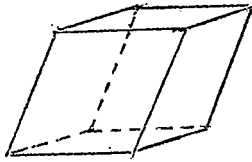
(c)
- 

(d)

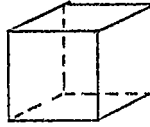


(e)
10. A plane is separated uniquely into exactly three sets of points which are disjoint in pairs by
- (a) any curve.
 - (b) any closed curve.
 - (c) any simple curve.
 - (d) any simple closed curve.
 - (e) none of the above.

11.



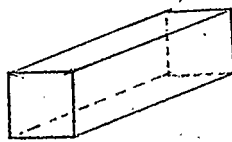
(1)



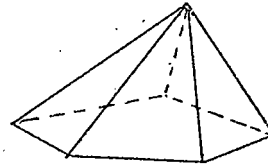
(2)



(3)



(4)



(5)

Of the figures above the following show prisms:

- (a) figure (5) only.
 - (b) figure (3) only.
 - (c) figures (1), (2), (3), and (4) only.
 - (d) figures (1), (2), (3), (4), and (5).
 - (e) None of the above choices is correct.
12. A right triangle (one angle has measure 90°) may have sides with lengths
- (a) 8, 9, and 12 units.
 - (b) 5, 12, and 13 units.
 - (c) 12, 13, and 25 units.
 - (d) More than one of (a), (b), and (c).
 - (e) None of the above choices is correct.
13. Which of the following is a correct relationship?
- (a) All squares are rectangles.
 - (b) All rectangles are squares.
 - (c) All parallelograms are squares or rectangles.
 - (d) No rectangle is a square.
 - (e) Every quadrilateral is a parallelogram.
14. The intersection of three distinct planes in space is always
- (a) the empty set, $\emptyset$.
 - (b) a single point.
 - (c) a line.
 - (d) a line or a point.
 - (e) the empty set or a point or a line.

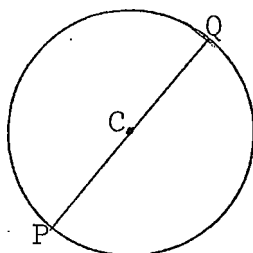
15. A plane is determined by

- (a) any three distinct points.
- (b) any two distinct rays.
- (c) any two lines.
- (d) any three distinct points not all in the same line.
- (e) More than one of the above choices are correct.

16. A triangle that has no two of its angles congruent is called

- (a) equiangular.
- (b) equilateral.
- (c) isosceles.
- (d) right.
- (e) scalene.

17.



Point C denotes the center of the circle shown at left. The length of $\overline{PQ}$ is 8 units. Then the area of the circular region, in square units, is

- (a) 8π .
- (b) 16π .
- (c) 64π .
- (d) 4π .
- (e) $4\pi^2$.

18. A triangle with two of its sides congruent is called

- (a) an equilateral triangle.
- (b) an isosceles triangle.
- (c) a scalene triangle.
- (d) a right triangle.
- (e) None of the above choices is correct.

19. If the intersection of a sphere and a plane is non-empty then the intersection is

- (a) always a circle.
- (b) always a circular region, i.e., the union of a circle and its interior.
- (c) always either a circle or a point.
- (d) always either a circular region or a point.
- (e) always a point, a line segment, or a circle.

20. The surface area of a cube measuring 4 feet on each edge is

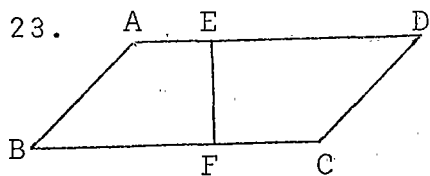
- (a) 16 square feet.
- (b) 24 square feet.
- (c) 64 square feet.
- (d) 96 square feet.
- (e) 256 square feet.

21. Let k denote a counting number greater than one. If there are k distinct points, no three of which are collinear, then the number of lines determined is

- (a) $\frac{k(k-1)}{2}$.
- (b) $\frac{k(k+1)}{2}$.
- (c) k .
- (d) $2k$.
- (e) $2k+1$.

22. A polygon with eight sides is called

- (a) a decagon.
- (b) an octagon.
- (c) a hexagon.
- (d) a pentagon.
- (e) None of the above choices is correct.

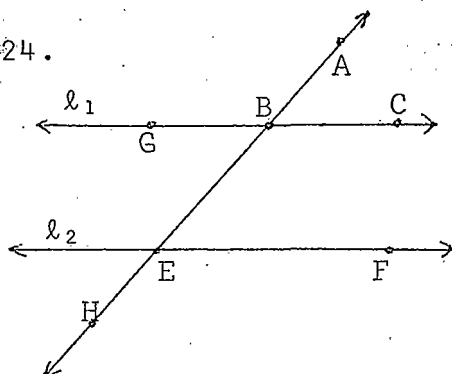


The figure ABCD is a parallelogram.
 $\overline{EF}$ is perpendicular to $\overline{BC}$.
 The measure of $\overline{AB}$ is 5, in inches.
 The measure of $\overline{BC}$ is 10, in inches.
 The measure of $\overline{EF}$ is 4, in inches.

Then the area of the simple closed region ABCD is

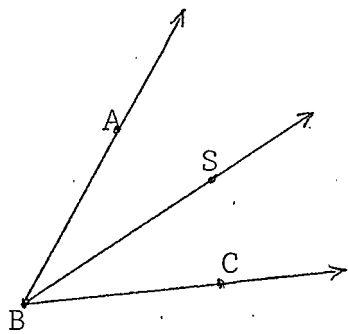
- (a) 50 square inches.
- (b) 40 square inches.
- (c) 32 square inches.
- (d) 25 square inches.
- (e) 20 square inches.

24.

If l_1 and l_2 are parallel lines then

- (a) $\angle BEF$ and $\angle ABC$ are congruent angles.
- (b) $\angle BEF$ and $\angle EBG$ are congruent angles.
- (c) $\angle GBA$ and $\angle HEF$ are congruent angles.
- (d) Choices (a) and (b) only are correct.
- (e) All of the choices (a), (b), and (c) are correct.

25.

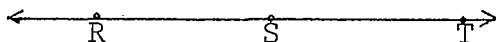
In the diagram at left suppose $\angle SBC$ and $\angle SBA$ are congruent. Then we say $\angle ABC$ is bisected. The bisector is

- (a) the line $\overleftrightarrow{BS}$.
- (b) the angular region SBC .
- (c) the half line determined by point B and containing point S, $\overrightarrow{BS}$.
- (d) the line segment $\overline{BS}$.
- (e) None of the above choices is correct.

26. The faces of a pyramid are

- (a) always triangles or rectangles.
- (b) always closed triangular regions except for two faces.
- (c) always closed triangular regions.
- (d) always closed triangular regions except perhaps one face, the base.
- (e) parallel in pairs.

27.



Using the points R, S, and T as shown above, the line can be named in

- (a) exactly 6 different ways.
- (b) exactly 4 different ways.
- (c) exactly 3 different ways.
- (d) exactly 2 different ways.
- (e) more than 6 different ways.

28. If two triangles have corresponding angles congruent then

- (a) the triangles are equal and congruent.
- (b) the triangles are similar.
- (c) the triangles are congruent.
- (d) the triangles are similar and congruent.
- (e) the triangles are equal, similar, and congruent.

29. The intersection of a line and a plane

- (a) must be a unique point.
- (b) must be either a unique point or the empty set.
- (c) must be a unique point, the empty set, or the line itself.
- (d) must be the line itself.
- (e) None of the above choices is correct.

Table 6

Grade Levels at Which Test Item Concept Is Introduced
in Teacher's Editions of the Four
Mathematics Text Series

Test item	Grade level at which concept introduced			
	H-MIFFLIN*	LAIDLAW*	SMSG*	GCMP*
1	K	1	1	K
2	K	1	1	
3	3	4	3	3
4	2	2	2	5
5	3	3	2	5
6	5	3	4	
7	5		6	
8	5	6		
9	1		6	2
10	4	3	3	
11	5	5	4	
12	5	5		
13	2	1	1	4
14	4	4		4
15	4	4	4	4
16	6	2	6	
17	6	6		6
18	6	5	4	
19	5		4	
20	5		6	
21	3	2		4
22	5	5		
23	3	6		5
24	6	6		
25	6	5		
26	4		4	
27	3	4		
28	5	6		
29	4		4	

*See page 25 for code.

Chapter 4

PRESENTATION AND ANALYSIS OF DATA

The data, test scores, have been gathered from students with four distinct types of mathematical preparation. Some discussion of the sample of elementary education seniors has been included. The various groups have been coded for convenience of reference. A presentation of the reasoning behind the choice of appropriate statistical models has been given. Then the statistical tests and analyses have been recorded in this section of the thesis.

THE POPULATION

The four samples to whom the test was administered were (1) elementary education seniors, 1971, (2) students who completed Math 108 winter quarter, 1971, (3) students who completed Math 109 spring quarter, 1971, and (4) students who had completed no college mathematics courses other than perhaps Math 114, a course in intermediate algebra at MSU. The fourth group, students with no college mathematics, was administered the test mainly to provide a validity check for the test instrument. However, the mean and the range of scores for this group has also been of

interest in this study.

For convenience the several groups have been coded. The samples, the number of students in each, and the coding symbol have been given in Table 7.

Table 7
Code Symbol and Number of Students
for Participating Groups

Group	Symbol	Number in group
Elementary education seniors	A	15
with Math 109*	A ₁	4
with Math 108*	A ₂	6
with Math 107*	A ₃	5
Students completing Math 109 spring, 1971	B	16
Students completing Math 108 winter, 1971	C	97
Students with no college mathematics	D	39

*Highest level course completed.

Group A

How representative Group A has been of the elementary education seniors whose mathematics was taken at MSU has

been of interest because any generalization from the data depends upon randomness of the sample. Group A consisted of volunteers and was not a random sample. The sample has been examined with respect to the highest level mathematics course in the Math 107, 108, 109 sequence taken, the overall GPA, and the academic year in which the highest level mathematics course was taken. These have been the factors whose effects have been analyzed in the study for Group A. No other factors have been investigated. The number of students and the per cent of the total group for each factor level combination have been compiled in Table 8.

The voluntary participants, Group A, constituted roughly one third of the 1971 seniors in elementary education at MSU who had taken their mathematics courses at MSU and who were available at their listed addresses at the times the test was administered. A few seniors were student teaching in other parts of the state. Forty-five seniors qualified and were invited to attend the meetings. In all, fifteen seniors attended one of the two meetings and became participants in the study. The figures shown in Table 8 indicated that only six of those forty-five seniors elected to complete Math 109. Of these, four responded and took the test. The six who had taken Math 109 were 13.3 per

cent of the eligible seniors whereas the four seniors in Group A₁ were 26.7 per cent of Group A. Another difference was that 40.0 per cent of Group A belonged to Group A₂ while 58.7 per cent of the forty-five seniors had Math 108 as the highest level mathematics course.

The conclusion has been that Group A, the participating seniors, was of a slightly higher level of mathematical experience than was the group of all elementary education seniors who had completed their mathematics at MSU. Additional differences have been noted. For example, differences have been observed in the per cents of the two groups taking the highest level mathematics course prior to the 1969-1970 academic year. Some of the factor combinations have no representatives in Group A but do have as many as 8.8 per cent of the forty-five seniors as representatives. Necessarily caution has been taken whenever data from Group A has been used to make inferences.

In spite of the many differences some qualified statements have been possible. The importance of lack of a factor combination representative in Group A decreases when only one or two representatives were available in the entire group of forty-five seniors. A study of Table 8 revealed that in only two cases, both involving the academic

Table 8

The Composition of Group A and of Forty-five Elementary Education Seniors

Group A

GPA	Math 107*				Math 108*				Math 109*			
	Prior to 68-69	68-69	69-70	70-71	Prior to 68-69	68-69	69-70	70-71	Prior to 68-69	68-69	69-70	70-71
2.00- 2.62	1 6.7%						2 13.3%					6.7%
2.63- 3.05	3 20.0%						1 6.7%					1 6.7%
3.06- 4.00	1 6.7%				1 6.7%	1 6.7%	1 6.7%			2 13.3%		
Totals	5 (33.3%)				6 (40%)				4 (26.7%)			

Forty-five Elementary Education Seniors

2.00- 2.62	2 4.4%	1 2.2%		1 2.2%		4 8.8%	7 15.5%					2 4.4%
2.63- 3.05	7 15.5%				1 2.2%	3 6.6%	4 8.8%	1 2.2%				1 2.2%
3.06- 4.00	2 4.4%				2 4.4%	2 4.4%	2 4.4%			3 6.6%		
Totals	13 (28.9%)				26 (57.8%)				6 (13.3%)			

*Highest numbered mathematics course taken by student.

year 1968-1969, were more than two representatives available in the larger group of seniors and no representative in Group A. These findings together with the use of nonparametric tests for some analyses have allowed inferences under such conditions.

THE STATISTICAL ANALYSIS

The statistical analysis has involved several mathematical models because of the particular problems involving assumptions, small numbers, and lack of randomness among the samples. Both parametric and nonparametric tests have been employed in order to elicit as valid inferences as possible in the present study.

The Statistical Models

Multiple linear regression has been utilized to estimate the selected effects on test performances for Group A and its subgroups. Two nonparametric statistical tests have been used: (1) the Mann-Whitney U test for equality of means and medians and (2) the Wald-Wolfowitz runs test which tests the null hypothesis (H_0) that two samples have been drawn from the same population against the alternative that the two samples differ in any way whatever. More detailed description of the models follows.

Multiple linear regression. A given test score may be thought of as a function of the selected factors and others whose effects would be included in experimental error. Multiple linear regression appeared to be a suitable and adequate manner in which to analyze test data from Group A in order to estimate the effects of the various selected factors.¹

The mathematical model expressed a given test score in the following manner:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \epsilon_i$$

where

$i = 1, 2, 3, \dots, 15$ indicates the i -th senior;

Y_i is the test score of the i -th senior;

X_{1i} denotes the most advanced course of the Math 107, 108, 109 sequence taken by the i -th senior;

X_{2i} is the grade point average of the i -th senior;

X_{3i} denotes the academic year the most advanced mathematics course was taken, i.e., prior to 1968-1969, 1968-1969, 1969-1970, or 1970-1971, by the i -th senior.

¹Roy N. Byrd, Ph. D., served as a consultant for the choice of statistical designs appropriate for the particular data of the present study.

ϵ_i represents the experimental error for the i -th score;

the β 's represent the unknown parameters which are to be established from the test score data.

The assumptions are that the X 's are measured without error and that the ϵ_i 's are independently and normally distributed with zero mean and with variances independent of the X 's.

Dr. R. L. Lund's Multiple Linear Regression Program for the IBM Sigma 7 computer was used. The output of this program has been described as follows:

Normal output includes program and regression titles, data formats used, check values for the first observation both before and after transformation, and for the regression selection: means, standard deviations, regression coefficients, t values, intercept, R^2 , standard error and ANOVA.²

The grade point averages were entered on the data cards exactly as received from the Office of the Registrar without coding or interval assignment. The other two variables were given codes to indicate intervals for the GPA and the academic years.

²Richard E. Lund, "Multiple Linear Regression Program" (Bozeman: MSU Department of Mathematics, February 17, 1970), p. 1. (Mimeographed.)

The Mann-Whitney U test. According to Siegel,³ the Mann-Whitney U test may be used to test whether two independent groups have been drawn from the same population. The test has been referred to as one of the most powerful of the nonparametric tests and a most useful alternative to the parametric t test when the measurement in the research is weaker than interval scaling. The test employs a method of ranking combined scores and calculating the statistic U which gives the number of times a score of one group precedes a score of the other group.

Siegel has discussed the power-efficiency of the Mann-Whitney U test:

If the Mann-Whitney test is applied to data which might properly be analyzed by the more powerful parametric test, the t test, its power-efficiency approaches $\frac{3}{\pi} = 95.5$ per cent as N increases and is close to 95 per cent even for moderate sized samples. It is therefore an excellent alternative to the t test, and of course it does not have the restrictive assumptions and requirements associated with the t test.⁴

³Sidney Siegel, Nonparametric Statistics for the Behavioral Sciences (New York: McGraw-Hill Book Company, 1956), p. 116.

⁴Ibid., p. 126.

Tables are available for experiments involving groups of size less than nine each and for groups in which the larger group has size not greater than twenty but greater than eight.⁵ For groups in which the larger group has over twenty members, a z value is calculated and tested in the regular manner.⁶ No provision for tied scores has been made for the first two cases; a correction for ties is available for the normal curve approximation employed for large samples.⁷ In the present study, then, because of tied scores and because of group sizes, only the Mann-Whitney U test for large samples has been used. The actual formulas have been given with the statistical analysis.

The Wald-Wolfowitz runs test. The Wald-Wolfowitz runs test is a nonparametric test used to test the null hypothesis that two independent samples have been drawn from the same population against the alternative hypothesis that the two samples differ in any respect whatsoever. The Wald-Wolfowitz runs test makes the assumption that the variable under consideration has an underlying distribution

⁵Ibid., pp. 117-120.

⁶Ibid., pp. 120-123.

⁷Ibid., pp. 123-125.

that is continuous and requires that the measurement of that variable be in at least an ordinal scale.⁸ These conditions can be assumed satisfied in the case of data used in the present study.

A run is any sequence of scores from the same group when the combined scores have been ranked. Calculation of the statistic r has been simplified by determining the absolute minimum value. Since the critical values for groups with sizes n_1 and n_2 are given in a table⁹ and the significance of the calculated r (at the .05 level) depends upon the number of runs being less than or equal to the critical value, the least possible r value allows the test to be made. Any other r value would necessarily be greater than or equal to the minimum.

In order to understand the rationale of the Wald-Wolfowitz runs test suppose two samples are from the same population. Then the scores of the two samples would be well mixed. In such a case r , the number of runs, would be relatively high and H_0 , the hypothesis that the two samples are from the same population would fail to be rejected. That null hypothesis would be rejected when r is small

⁸Ibid., p. 136.

⁹Ibid., p. 252.

enough. It can be shown that r would be small whenever there was a sufficient difference in central tendency, in variability, in skewness, or in medians.

In discussing the power-efficiency of the Wald-Wolfowitz runs test, Siegel has stated:

Little is known about the power-efficiency of the Wald-Wolfowitz test. Moses points out that statistical tests which test H_0 against many alternatives simultaneously--and the runs test is such a test--are not very good at guarding against accepting H_0 erroneously with respect to any one particular alternative.

For instance, if we are interested simply in testing whether two samples come from populations with the same location, the Mann-Whitney U test would be a more powerful test...¹⁰

The Mann-Whitney U test would have been a preferable test in the present study where the Wald-Wolfowitz has been employed. However, the occurrence of tied scores with small numbers prevented use of the Mann-Whitney U in two instances because of the lack of a correction figure or because no tables of critical U values were available.

Analysis of the Data

The means and standard deviations have been listed in Table 9 for the various groups participating in the

¹⁰Ibid., p. 144.

study. Some statistics were taken from computer print-outs¹¹ and some were calculated by hand. As would be expected, Group A had the greatest standard deviation since the mathematical backgrounds for members of this group were less homogeneous than in other groups. Interesting also were the standard deviations for the subgroups of Group A: 2.646 for Group A₁; 8.785 for Group A₂; and 8.174 for Group A₃. Obviously, Group A₁, those seniors who took Math 109, had much less variation among scores on the test. Whereas the small number in Group A₁, four, the size of Group A₂, six, and that of Group A₃, five, were not much greater. Further, the means and standard deviations for Group A₁ and for Group B, students completing Math 109 winter quarter, 1971, were in close agreement as shown by the entries in Table 9.

An inspection of the means given in Table 9 revealed that students with no college math, Group D, performed least well on the instrument, students with Math 109 whether in Group A or Group B scored highest, and students with Math 108 as the most advanced mathematics course achieved a mean between those for Group D and students who had taken

¹¹See Appendix B.

Table 9

Means and Standard Deviations of Scores
for Participating Groups

Group ^a	n	Mean ^b	Standard deviation ^c
A	15	13.60	7.149
A ₁	4	23.50	2.646
A ₂	6	10.17	8.785
A ₃	5	9.80	8.174
B	16	23.88	2.630
C	97	13.83	3.805
D	39	8.69	3.555

^aGroup descriptions:

A --Elementary education seniors

A₁ --Seniors whose most advanced mathematics course was Math 109

A₂ --Seniors whose most advanced mathematics course was Math 108

A₃ --Seniors whose most advanced mathematics course was Math 107

B --Students completing Math 109 spring quarter, 1971

C --Students completing Math 108 winter quarter, 1971

D --Students with no college courses in mathematics

^bRounded to the nearest hundredth.

^cRounded to the nearest thousandth.

Math 109. Certainly the difference between the mean scores for Group B and Group C as well as for Group B and Group D indicated that a significant difference existed. However, no tests for the significance of differences of means have been made here because of failure to meet the necessary assumptions and restrictions. Nonparametric tests have been used for this purpose.

Analysis of selected factors affecting Group A scores. The effects of (1) the highest level mathematics course completed, (2) the overall GPA, and (3) the time lapsed since the most advanced mathematics course was taken have been investigated for Group A by means of a multiple linear regression analysis. The mathematical model has been discussed.¹² The program was run on the Sigma 7 computer at the MSU Computer Center. The effects being tested, their codes and their levels were as follows:

$Y = X_1$ = the dependent variable, the test score.

X_2 = the most advanced mathematics course taken:

1: Math 107

2: Math 108

3: Math 109

¹²See pp. 58-59.

X_3 = the GPA, rounded to two decimal places, as of the end of winter quarter, 1971.

X_4 = the academic year during which the most advanced mathematics course was completed:

1: prior to autumn quarter, 1968

2: 1968-1969

3: 1969-1970

4: 1970-1971

Information entered on the data cards has been compiled in Table 10. The print-out for the original run of the multiple linear regression has been reproduced in Table 11.

The value of R squared, .5655, for the regression analysis with three independent variables indicated that approximately 56.55 per cent of the variation had been explained by the model. This result was satisfactory since a number of possible effects have not been investigated and these have been reflected in the error term. Among these would be the high school mathematics experience of the participants and the arithmetic methods course required in the curriculum.

The t value for testing the effect of X_4 , the academic year during which the highest level mathematics course was taken, was $-.2672$. The negative value has

Table 10

Scores, Most Advanced Mathematics Course Taken,
GPA, and Year Course Taken for Group A
15 Elementary Education Seniors

Student	$Y = X_1^a$	X_2^b	X_3^c	X_4^d
1	17.0	1.0	2.78	1.0
2	6.0	2.0	2.20	3.0
3	9.0	2.0	2.58	3.0
4	8.0	1.0	2.59	1.0
5	8.0	2.0	2.10	3.0
6	7.0	1.0	2.78	1.0
7	8.0	1.0	3.35	1.0
8	18.0	2.0	2.10	2.0
9	21.0	3.0	3.87	2.0
10	12.0	2.0	2.97	3.0
11	8.0	2.0	3.08	1.0
12	9.0	1.0	2.84	1.0
13	24.0	3.0	3.28	2.0
14	22.0	3.0	2.46	4.0
15	27.0	3.0	3.01	4.0

$^aY = X_1$ = the test score.

bX_2 = the most advanced mathematics course:

1--Math 107

2--Math 108

3--Math 109

cX_3 = the GPA.

dX_4 = the academic year in which X_2 was taken:

1--prior to 1968-1969

2--1968-1969

3--1969-1970

4--1970-1971

resulted from the assignment of code values and was not important. The absolute value of t was 0.2672. From a table of t values with eleven degrees of freedom, 0.2672 indicated a probability slightly less than .80 that the null hypothesis that X_4 , the time lapsed since the most advanced mathematics course was taken, had no effect would be accepted.¹² This result led to a decision to run the multiple linear regression analysis with only effects X_2 and X_3 being considered. The decision was supported by the lack of correlation between test scores and academic year of the most advanced mathematics course observed in the scatter diagram shown in Figure 1, page 71.

Since the t value, .4425, indicated that the probability that H_0 , X_3 has no effect, would be accepted was greater than .60 but less than .70, the decision was to keep this factor, the GPA, in the model.

The computer print-out for the multiple linear regression program with the two independent factors, most advanced mathematics course and GPA, has been reproduced in Table 12. The R squared for this analysis, .5626, indicated

¹²William G. Cochran and Gertrude M. Cox, Experimental Designs (2d ed.; New York: John Wiley & Sons, Inc., 1957), p. 613.

Table 11

Computer Program Print-out: Multiple Linear Regression,
Effects of Three Variables on Group A Test Scores

Variable number*	Mean	Standard deviation	Correlation X vs. Y	Regression coefficient	Std. error of reg. coef.	Computed t value
2	1.933	.7988	.7329	6.930	3.408	2.0333
3	2.933	.4067	.3492	2.096	4.737	.4425
4	2.133	1.1250	.4332	-.6407	2.398	-.2672
Dependent						
1	13.600	7.149				

*See footnotes, Table 10, p. 56.

Intercept: -4.579. R squared: .5655. Std. error- $S_{Y \cdot X}$: 5.317

70

Analysis of Variance for the Regression

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F value
Attributable to regression	3	404.642	134.881	4.77134
Deviation from regression	11	310.958	28.2689	
Total (N = 15)	14	715.600		

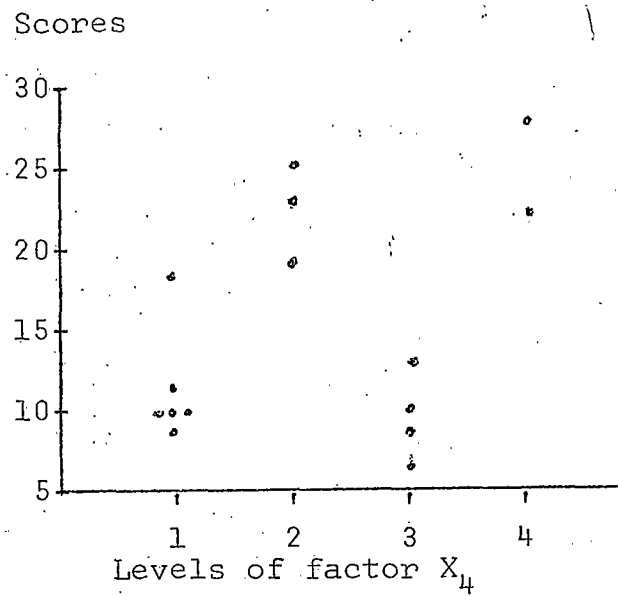


Figure 1

Scatter Diagram of X_4 Levels
and Test Scores for Group A

Source: Table 10.

approximately 56.26 per cent of the variation was accounted for by the model. Again, this finding has supported the deletion of the X_4 factor since with the X_4 effect included the model explained 56.55 per cent of the independent variables' effect on the test scores.

Tests were made concerning the significance of the t values for effects X_2 and X_3 at the .05 level. In all discussions H_0 has been used to represent the null hypothesis and H_1 the alternative hypothesis. The regression coefficient for X_2 has been denoted by β_2 and that for X_3 has been denoted by β_3 for convenience.

The test for significance of effect X_2 , the most advanced mathematics course taken, at the .05 level was as follows:

i) H_0 : $\beta_2 = 0$, i.e., the most advanced mathematics course taken by a senior had no effect upon his test score.

H_1 : $\beta_2 \neq 0$, i.e., the most advanced mathematics course taken by a senior had an effect on the test score.

ii) If H_0 is true, then t_{12} is the ratio:

$$\frac{\text{regression coefficient}}{\text{standard error of the regression coefficient}}$$

for the regression coefficient β_2 .

iii) Reject H_0 if $|t_{12}| \geq t_{0.975;12} = 2.179$. This

is for a two-tailed test at the .05 level.

iv) From Table 12, $t_{12} = \frac{6.165}{1.733} \doteq 3.477.$

Now $3.477 > 2.179$. Therefore, reject H_0 .

v) The effect of the most advanced course in mathematics taken is not zero unless a five per cent sampling error has been made.

If the result of the t test just given is considered in light of the means for the different subgroups of Group A the conclusion may be given that completion of Math 109 had a positive effect upon test performance.

The test for the significance (at the .05 level) of factor X_3 was exactly as the test just completed for steps ii and iii. The other three steps were as follows:

i) $H_0: \beta_3 = 0$, i.e., the GPA has no effect on the test scores for Group A.

$H_1: \beta_3 \neq 0$, i.e., the GPA has an effect on the test scores for Group A.

iv) From the computer print-out given in Table 12, using the coefficients for β_3 , $t_{12} = \frac{2.911}{3.482} \doteq .8359.$

Since $.8359 < 2.179$, H_0 was not rejected.

v) There is no evidence at the .05 level that the GPA of students in Group A had an effect on test scores.

The test for significance of factor X_3 has indicated that the GPA of the seniors in Group A has not been a significant effect on test performance. Here the conclusion has been drawn that the highest level mathematics course completed by a senior has the greatest effect of the effects tested upon his test score on the instrument constructed for the present study.

Because of the results of these two tests, a test of the hypothesis $H_0, \beta_2 = \beta_3 = 0$, has not been warranted.

Analysis of the effectiveness of Math 108 and Math 109 as preparation for the selected concepts of geometry tested using Group B and Group C. The Mann-Whitney U test seemed the most appropriate and powerful nonparametric test for estimation of the effectiveness of Math 108 and Math 109 in preparing students in the selected concepts of geometry tested in the present study. Because of the large size of Group C the calculations for large samples (one group with size greater than twenty) has been used. Since there were several instances of tied scores as shown in Table 13, the correction for ties has been employed to make the statistic as accurate as possible. The notation and formulas given by Siegel have been used throughout the discussion of the Mann-Whitney U test, as have been the form for presentation

Table 12

Computer Program Print-out: Multiple Linear Regression,
Effects of X_2 and X_3 Variables on Group A Test Scores

Variable number*	Mean	Standard deviation	Correlation X vs. Y	Regression coefficient	Std. error of reg. coef.	Computed t value
2	1.933	.7988	.7329	6.165	1.773	3.477
3	2.933	.4067	.3492	2.911	3.482	.8359
Dependent						
1	13.600	7.149				

*See footnotes, Table 10, p. 55.

Intercept: -6.854. R squared: .5626. Std. error- $S_{Y.X}$ = 5.107

75

Analysis of Variance for the Regression

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F value
Attributable to regression	2	402.624	201.312	7.31860
Deviation from regression	12	312.977	26.0814	
Total (N = 15)	14	715.600		

of the analysis.¹³ The ranking of the combined scores has been given in Table 13. In the case of tied scores, the mean of the positions occupied has been assigned as the rank for each of the tied scores. This technique was not demanded whenever all the ties were from the same group. Table 13 entries that are tied scores have all been treated in the same manner for the sake of consistency.

Using the data given in Table 13, the Mann-Whitney U test resulted in the following analysis:

i) Null hypothesis. H_0 : Group B, the students who were taking Math 109 spring quarter, 1971, and Group C, the students completing Math 108 winter quarter, 1971, are equally well prepared in the geometric concepts tested.

H_1 : Group B and Group C have not been drawn from the same population, i.e., the two groups are not equally well prepared in the geometric concepts.

ii) Statistical test. Group B and Group C constitute independent groups insofar as the highest level mathematics course completed and the test used to measure knowledge of geometric concepts is ordinal at best. For these reasons the Mann-Whitney U test for large numbers is appropriate for

¹³Siegel, op. cit., pp. 120-126.

analyzing these data.

iii) Significance level. Let $\alpha = .01$. $n_1 = 16$ = the number of students in Group B. $n_2 = 97$ = the number of students in Group C.

iv) Sampling distribution. For $n_2 > 20$ and since there are tied scores,

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\left(\frac{n_1 n_2}{N(N-1)}\right) \left(\frac{N^3 - N}{12} - \Sigma T\right)}} \quad \text{where}$$

ΣT is found by summing over all groups of tied observations and $T = \frac{t^3 - t}{12}$, t the number of observations tied for a given rank;

$N = n_1 + n_2$, the total number of scores;

$$U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2,$$

where R_2 is the sum of the ranks assigned to Group C and R_1 is the sum of the ranks assigned to Group B, respectively.

v) Rejection region. Since H_1 predicts the direction of difference, the rejection region is one-tailed. It is all values of z which are so extreme that their associated probability is equal to or less than $\alpha = .01$.

vi) Decision. The ranking assigned to each of the 113 scores has been shown in Table 13. For these data,

$R_2 = 4768.5$ and $R_1 = 1672.5$. Then the calculation of the

ΣT was as follows:

2	scores of 5;	T =	.5
3	scores of 8;	T =	2.0
5	scores of 9;	T =	10.0
13	scores of 10;	T =	182.0
2	scores of 11;	T =	.5
10	scores of 12;	T =	82.5
15	scores of 13;	T =	280.0
7	scores of 14;	T =	28.0
11	scores of 15;	T =	110.0
10	scores of 16;	T =	82.5
6	scores of 17;	T =	17.5
2	scores of 18;	T =	.5
5	scores of 19;	T =	10.0
3	scores of 20;	T =	2.0
3	scores of 21;	T =	2.0
2	scores of 22;	T =	.5
3	scores of 23;	T =	2.0
2	scores of 24;	T =	.5
2	scores of 25;	T =	.5
2	scores of 26;	T =	.5
2	scores of 27;	T =	.5

$$\Sigma T = 814.5$$

Then

$$U = (16)(97) + \frac{(16)(17)}{2} - 1672.5 = 15.5$$

$$\text{and } z = \frac{15.5 - \frac{(97)(16)}{2}}{\sqrt{\frac{((97)(16))}{((113)(112))} \left(\frac{113^3 - 113}{12} - 814.5 \right)}} = \frac{-760.5}{89.13} < -8.45.$$

Under H_0 , $p(z \leq -3.43)$ is less than .0003. Hence, for $\alpha = .01$, reject H_0 in favor of H_1 . That is, students just completing Math 109 are better prepared in geometry concepts of elementary school mathematics than are students just completing Math 108.

Table 13

Test Scores and Ranks for Group B and Group C

C scores	B scores	Rank	C scores	B scores	Rank	C scores	B scores	Rank
5		1.5	13		45	16		75.5
5		1.5	13		45	16		75.5
6		3	13		45	16		75.5
7		4	13		45	16		75.5
8		6	13		45	17		84.5
8		6	13		45	17		84.5
8		6	13		45	17		84.5
9		10	13		45	17		84.5
9		10	13		45	17		84.5
9		10	13		45	17		84.5
9		10	13		45	18		87.5
9		10	13		45	18		87.5
10		19	13		45	19		91
10		19	13		45	19		91
10		19	14		56	19		91
10		19	14		56	19		91
10		19	14		56		19	91
10		19	14		56	20		95
10		19	14		56	20		95
10		19	14		56		20	95
10		19	14		56	21		98
10		19	15		65	21		98
10		19	15		65		21	98
10		19	15		65		22	100.5
10		19	15		65		22	100.5
11		26.5	15		65	23		103
11		26.5	15		65		23	103
12		32.5	15		65		23	103
12		32.5	15		65		24	105.5
12		32.5	15		65		24	105.5
12		32.5	15		65		25	107.5
12		32.5	15		65		25	107.5
12		32.5	16		75.5		26	109.5
12		32.5	16		75.5		26	109.5
12		32.5	16		75.5		27	111.5
12		32.5	16		75.5		27	111.5
12		32.5	16		75.5		28	113
13		45	16		75.5			

Analysis of the effectiveness of Math 108 and Math 109 as preparation in geometric concepts for elementary education seniors. No member of Group A_2 was a member of Group C. Therefore, in determining the effectiveness of Math 108 and Math 109 as preparation in the geometric concepts of elementary school mathematics Group A_1 and Group A_2 were the reference samples. The Mann-Whitney U test was not suitable in this case because of the lack of appropriate tables for the U value calculated. The Wald-Wolfowitz runs test was chosen for this analysis. The notation and form recommended by Siegel¹⁴ have been followed.

i) Null hypothesis. H_0 : the test scores of Group A_1 and Group A_2 are the same. H_1 : the test scores of Group A_1 and Group A_2 differ.

ii) Statistical test. Since the data are in an ordinal scale and since the hypothesis relates to differences of any kind between the test scores, the Wald-Wolfowitz runs test is appropriate.

iii) Significance level. Let $\alpha = .05$, the significance level for which a table of critical values of r is available.¹⁵ $n_1 = 4$ = number of students in Group A_1 and

¹⁴Ibid., pp. 136-140.

¹⁵Ibid., p. 252.

$n_2 = 6$ = number of students in Group A_2 . There were no tied scores for these two groups.

iv) Rejection region. The region of rejection consists of all values of r where $n_1 = 4$ and $n_2 = 6$ that are so small that the probability associated with their occurrence under H_0 is equal to or is less than $\alpha = .05$. Thus, in this instance, H_0 would fail to be accepted if $r \leq 2$.

v) Decision. Each test score has been tabulated for Group A_1 and Group A_2 .

Group A_1 :	21	22	24	27	
Group A_2 :	6	8	8	9	12 18

Combined and arranged in increasing order, this gave

Score	6	8	8	9	12	18	21	22	24	27
Group	A_2	A_2	A_2	A_2	A_2	A_2	A_1	A_1	A_1	A_1
Run				1					2	

Hence, $r = 2$ which allows rejection of H_0 at the .05 level. The decision was that the scores of Group A_1 and of Group A_2 differ.

Analysis of test scores for Group A_2 and Group A_3 .

The Wald-Wolfowitz runs test was utilized again to test any differences between the test scores of Group A_2 and Group A_3 . The difficulty of tied scores was solved by calculation of the minimum number of runs possible with

under any permutation of the ties. If this value was greater than the critical value for r then all other numbers of runs would also be greater.

i) Null hypothesis. H_0 : the test scores for Group A_2 are the same as those for Group A_3 . H_1 : the test scores for Group A_2 and for Group A_3 differ.

ii) Statistical test. Since the data are in an ordinal scale and since the hypothesis relates to differences of any kind among test scores for the two groups, the Wald-Wolfowitz runs test appeared appropriate.

iii) Significance level. Let $\alpha = .05$; $n_1 = 5$ = the number of students in Group A_3 ; $n_2 = 6$ = the number of students in Group A_2 .

iv) Rejection region. The region of rejection consists of all values for r where $n_1 = 5$ and $n_2 = 6$ that are so small that the probability associated with their occurrence under H_0 is equal to or less than $\alpha = .05$. For these group sizes, H_0 would fail to be accepted if $r \leq 3$.

v) Decision. The test scores for Group A_2 and Group A_3 have been given below.

Group A_3 :	7	8	8	9	17	
Group A_2 :	6	8	8	9	12	18

The scores were then combined and arranged to give the least

r value. The following sequence resulted, with the indicated number of runs:

Score	6	7	8	8	8	8	9	9	12	17	18
Group	A ₂	A ₃	A ₃	A ₃	A ₂	A ₂	A ₂	A ₃	A ₂	A ₃	A ₂
Run	<u>1</u>	<u>2</u>			<u>3</u>			<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>

The value of r in this case was seven and this was the least possible number of runs. Seven is greater than the critical value $r = 3$. Therefore, H_0 was not rejected. There was insufficient evidence at the .05 level to reject the hypothesis that the scores made by seniors with Math 107 only were the same as scores made by seniors with both Math 107 and Math 108. In other words, for the geometric concepts being tested there is no evidence that Math 108 has increased the knowledge of members of Group A₂ related to the geometry of elementary school mathematics programs.

SUMMARY

An instrument testing selected concepts of geometry found in selected elementary school mathematics text series was constructed. The concepts of geometry were chosen after an analysis of the content of four text series likely to be used by MSU elementary education graduates upon entering the profession. The outline format, the characteristics of the instrument were as follows:

1. a high level of reliability as evidenced by the Kuder Richardson Formula 20 value for elementary education seniors on the final form of the test, .9397

2. suitable validity as verified by experts in the field and by a comparison of per cent selecting the correct choice for a given item of students with no college mathematics courses and of students with exposure to the given concept

3. appropriate item discrimination as shown by the biserial correlation coefficients and item analyses

The test instrument constructed for use in this study has been demonstrated to be suitable for use for evaluation of knowledge of the selected concepts of geometry involved.

The preliminary form of the test was administered to students who were completing Math 108 winter quarter, 1971. After test revision, the answer sheets for this group of ninety-seven students were revised so that the results were for an equivalent form of the test in length and in concepts tested. The item analysis for test results from this group and from other participating groups using the final test form have been made available in Appendix B.

Other groups involved in the present study have been seniors who were in the regular elementary education

curriculum, class of 1971, students completing Math 109, spring quarter, 1971, and students with no college mathematics other than perhaps a course in intermediate algebra.

The data gathered was subjected to statistical analyses. Both parametric and nonparametric statistics appropriate for the samples and for the test data were utilized.

For the factors influencing test performance of elementary education seniors the following results were obtained from the multiple linear regression analysis:

1. the time lapsed since completion of the most advanced mathematics course taken had no effect (at the .05 level) upon test scores;
2. the overall GPA had no effect (at the .05 level) upon test scores for this group of fifteen seniors;
3. the most advanced mathematics course in the Math 107, 108, 109 sequence taken by a senior had an effect significant at the .05 level.

These findings considered in view of the mean scores for the various subgroups of participating seniors showed that Math 109 completion had the greatest positive effect on test scores. For the sample of seniors Math 108 completion had no effect on the geometry test scores.

Use of nonparametric tests produced the following findings:

1. students completing Math 109 spring quarter, 1971, and students completing Math 108 winter quarter, 1971, do not come from the same population, i.e., there is a significant difference (at the .01 level) between test scores of the two groups of students;

2. there is a significant difference (at the .05 level) between scores on the geometry test of participating seniors who completed Math 109 and those who completed only Math 107 and Math 108;

3. there is no significant difference (at the .05 level) between test scores of seniors who completed only Math 107 and those who completed both Math 107 and Math 108..

In summary, then, a valid and reliable instrument has been developed to test the knowledge of selected concepts of geometry found in modern mathematics programs for the elementary school. Analyses of data in the form of test scores for various participating groups have resulted in findings about the significance of several effects: the overall GPA, the time lapse since a mathematics course was taken, and the most advanced course in the Math 107

108, 109 sequence taken. Other findings have related to the extent to which Math 108 and Math 109 content prepares students in the geometric concepts being tested.

Chapter 5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The CUPM Level I recommendations include a semester course in informal geometry. The geometry content of contemporary elementary mathematics programs has changed greatly during the past fifteen years by an increase in the amount of geometry introduced and by adoption of an informal or intuitive approach. The present study has investigated the preparation of majors in elementary education at Montana State University in geometric concepts common to selected elementary school mathematics texts. A summary of the study, a discussion of conclusions supported by the research findings, and recommendations suggested by these conclusions for the elementary education curriculum, for the content and teaching of certain mathematics courses, and for further research have been presented here.

SUMMARY

The problem investigated in this study has been the estimation of the adequacy of preparation of Montana State University seniors in elementary education in selected concepts of geometry. Estimation of the effectiveness of

certain courses in mathematics for such preparation was a second important problem.

Literature research revealed no test instrument adequate for the purposes of the study. Consequently, the construction of a reliable and valid instrument based upon concepts of geometry necessary for understanding the content of the selected text series was a major part of the study. The Kuder Richardson correlation, biserial correlations and item analyses, expert opinion, and a comparison of performance on items in the test by students completing Math 109 and by students with no college mathematics were employed in arriving at the final form of the instrument.

The test was administered to groups of students in order to gather data suitable for statistical analysis. These groups were: (1) elementary education seniors who responded to an invitation to one of the two scheduled meetings, (2) students completing Math 108 winter quarter, 1971, (3) students completing Math 109 spring quarter, 1971, and (4) students who had experienced no college mathematics beyond intermediate algebra.

The statistical treatment involved three types of tests: (1) tests for the significance of the student t statistics resulting from a multiple linear regression

analysis of test scores for seniors, (2) the Mann-Whitney U test, and (3) the Wald-Wolfowitz runs test. The multiple linear regression model involves parameters. The Mann-Whitney U test and the Wald-Wolfowitz runs test are non-parametric in nature. The statistical tests were chosen so that a minimum number of assumptions had to be met and so that unequal and small numbers were permissible.

Any statement about the research findings has meaning only in relation to the selected concepts of geometry tested by the instrument constructed for this study. The findings have not been claimed to hold for populations other than the samples used in the present study except where specifically stated.

The multiple linear regression model was found to account for over fifty-six per cent of the variation of test scores whether three factors (most advanced mathematics course taken, overall GPA, and the academic year the course was taken) were used as independent variables or whether only the first two factors listed were used. Thus the multiple linear regression model was satisfactory for analysis of data from participating seniors in elementary education although the number, fifteen, was small. Decisions considered with sample means yielded important

relationships. Subsamples of the sample of seniors were also treated nonparametrically.

The findings of this research study, based upon statistical treatment of data, have been as follows:

1. The hypothesis that the most advanced course in the Math 107, 108, 109 sequence completed by a senior in elementary education had a significant effect upon the test score (knowledge of tested geometric concepts) was supported at the .05 level.

2. There was insufficient evidence at the .05 level to support the hypothesis that the overall GPA had an effect upon the test scores of seniors.

3. The hypothesis that the lapse of time since a senior experienced the most advanced course in the sequence had a significant effect upon the test scores was not supported at the .05 level.

4. The hypothesis that completion of Math 109 had the greatest positive effect upon knowledge of the selected geometric concepts has been supported. This hypothesis has been accepted because of the results of three distinct statistical tests and because of the means for the various groups. Regardless of their curricula, students completing Math 109 performed at a higher level on the test than did

students with only Math 107 and Math 108 when the Mann-Whitney U test for large samples was employed and the mean scores were considered. A Wald-Wolfowitz runs test resulted in rejection of the null hypothesis that seniors who completed Math 108 and those who completed Math 109 were drawn from the same population. The third result has been given as the first finding listed. The mean for students who completed Math 109 was greater than twenty-three. The mean for students with Math 108 but not Math 109 was less than fourteen. The fact that both the sample of seniors and the groups finishing Math 109 and Math 108, for which the sample was essentially the population, have been involved made support for the hypothesis quite strong.

5. The hypothesis that Math 109 content specifically prepares students in the concepts of geometry tested in the study has been supported. The support has been seen in the means for the scores of the groups and in the consistent finding of significant differences when students with Math 109 have been tested against other groups.

With the limitations and restrictions imposed by the nature of the groups involved well in mind, the results of the research may be stated in a more abbreviated manner:

1. There was no difference in the scores obtained

on the test of geometric concepts between students with Math 107 and students with Math 108 as the most advanced course completed.

2. There was a difference in the scores obtained on the test between students with Math 107 and Math 108 and students with Math 109 as the most advanced course.

3. The GPA of the senior and the time lapse since completion of highest level mathematics course were not significant effects on test scores.

4. Math 109 specifically prepares in the geometric concepts of elementary school mathematics.

5. Of all the factors investigated Math 109 had the greatest positive effect on test scores.

CONCLUSIONS

Conclusions based upon the present study are related to the test instrument, to the research design, and to the statistics and the analyses. These conclusions have been given in itemized form.

1. The test of knowledge and understanding of selected geometric concepts of elementary school mathematics constructed as a part of the study is a reliable, valid instrument with good discriminatory power among the

items. Since the test was based upon content of nationally recognized text series and tests only concepts introduced in at least two of the series, the test would be of value (1) in future research at Montana State University, (2) in assessing the effectiveness of geometry preparation of majors in elementary education at other institutions, and (3) as a diagnostic instrument for practicing elementary school teachers. Since no test of this nature has been located through research in the literature, this instrument may serve as a model.

2. The design of the study allowed satisfactory testing of the stated hypotheses. Improvements could be made in using random samples and in securing larger numbers. However, at the time of this study, sixteen students constituted the total registration in Math 109. Only six seniors would have completed Math 109 by graduation. Since joint research involving the Department of Mathematics and the Department of Elementary Education at Montana State University has been sadly neglected, perhaps this study may set a precedent for similar research into student preparation in other mathematical topics.

3. With the mean score for every group of students who had not taken Math 109 under fifty per cent of the

twenty-nine questions a qualified conclusion is that many elementary teachers prepared at Montana State University have insufficient mathematical background in the geometry taught in the elementary schools.

4. The geometry taught in Math 108 is ineffective as far as the concepts tested in this study. Therefore, the addition of Math 108 as a requirement in the elementary education curriculum is not justified from the standpoint of geometry content alone.

5. Math 109 should be strongly recommended as an elective to every elementary education major or should be required in the curriculum.

6. The year in college when the elementary education major takes the Math 107, 108, 109 sequence appears to have no relation to that student's knowledge of geometric concepts of elementary school mathematics at the time of graduation.

7. The total grade point average of an elementary education senior has no demonstrated relationship to his understanding of the geometric concepts for K-6 tested. The informal geometry of Math 109 has been taught at an appropriate level and manner for successful learning by the general run of students.

RECOMMENDATIONS

Several recommendations based upon the conclusions of the study may be made. In each case, however, the limitations and sometimes the small numbers of the research data should be considered.

A recommendation is made that a long-range study be initiated to investigate the geometry preparation of Montana State University majors in elementary education. Research findings of this study emphasize the importance of the problem and the need for careful research on a continuing basis. The suggestion is made that a longitudinal study be implemented.

Further, the study could be extended to other topics in mathematics or to mathematics in general in the curriculum for elementary education majors. Not only should course content be examined but also the type of instructor assigned to such courses and the teaching approach.

Regardless of the limitations of the present study, sufficient evidence of the positive effect of Math 109 upon knowledge of geometric concepts of elementary school mathematics has been demonstrated to warrant consideration of Math 109 as a required course or as a strongly recommended

elective. The Department of Mathematics has made an appropriate course in informal geometry available. The Department of Elementary Education would take another step toward satisfying the CUPM Level I recommendations if the course were a part of the curriculum. Therefore, the recommendation is made that Math 109 be made a required course for elementary education majors.

Further, a recommendation is made that elementary education majors be made aware of the research findings presented here. More of these students might be encouraged to register for Math 107 voluntarily.

The results of the study appear to indicate that the geometry content of Math 108 has no significant effect upon the knowledge of geometry for the samples involved. The recommendation is made that the geometry content of the course be examined in detail. Perhaps time spent on geometry in Math 108 could be utilized to greater advantage with other mathematical topics. On the other hand, with a change of text or with additional time devoted to the present geometry content the entire situation might improve relative to knowledge of geometry.

The extension of the present close cooperation between the Department of Elementary Education and the

Department of Mathematics at Montana State University to a combined effort toward productive research is urgently recommended. The major objective should be improvement of the mathematics preparation of prospective teachers of elementary school arithmetic.

BIBLIOGRAPHY

Breeden, John. Personal interview. Various dates, October, 1970.

Cambridge Conference on Teacher Training. Goals for Mathematical Education of Elementary School Teachers. Boston: Houghton Mifflin Company, 1967.

Carroll, Emma C. "A Study of Mathematical Understandings Possessed by Undergraduate Students Majoring in Elementary Education." Unpublished Doctor's dissertation, Wayne State University, 1961.

Cochran, William G. and Gertrude M. Cox. Experimental Design. 2nd. ed. New York: John Wiley & Sons, Inc., 1957.

Committee on the Undergraduate Program in Mathematics. Ten Conferences on the Training of Teachers of Elementary School Mathematics. CUPM Report Number 7. Berkeley: Committee on the Undergraduate Program in Mathematics, June, 1966.

_____. Forty-one Conferences on the Training of Teachers of Elementary School Mathematics, A Summary. CUPM Report Number 15. Berkeley: Committee on the Undergraduate Program in Mathematics, June, 1966.

Educational Research Council of America. Greater Cleveland Mathematics Program. Teacher's Edition and Student Text. 7 vols. Chicago: Science Research Associates, Inc., 1968.

Educational Testing Service. Making the classroom test: A guide for teachers. 2d ed. Princeton: Educational Testing Service, 1961.

_____. Multiple-choice questions: A close look. Princeton: Educational Testing Service, 1963.

Duncan, Ernest R. and others. Modern School Mathematics, Structure and Use. 7 vols. Boston: Houghton Mifflin Company, 1967.

Ferguson, George A. Statistical Analysis in Psychology and Education. New York: McGraw-Hill Book Company, 1966.

Fisher, John J. "Extent of Implementation of CUPM Level I Recommendations," The Arithmetic Teacher, March, 1967, pp. 194-197.

Garnet, Emma Whitlock. "A Study of the Relationship Between the Mathematics Knowledge and the Mathematics Preparation of Undergraduate Elementary Education Majors." Unpublished Doctor's dissertation, George Peabody College for Teachers, 1968.

Gundlach, Bernard H. and others. The New Laidlaw Mathematics Program. 7 vols. River Forest, Illinois: Laidlaw Brothers Publishers, 1968.

Gibney, Thomas C., John L. Ginther, and Fred L. Pigge. "A Comparison of the Number of Mathematics Courses Taken by Elementary Teachers and Their Mathematical Understandings," School Science and Mathematics, May, 1970, pp. 377-381.

Hawley, Newton, and Patric Suppes. Geometry for Primary Grades. San Francisco: Publishers Book Service, 1960.

Hess, Adrien L. Personal interview. March 3, 1971.

Hughes, Eugene Morgan. "The Impact of Selected Experimental Curriculum Projects on Commercially Published Elementary School Mathematics Textbooks." Unpublished Doctor's dissertation, George Peabody College for Teachers, 1968.

Hunkler, Richard Frederic. "Achievement of Sixth-Grade Pupils in Modern Mathematics as Related to Their Teacher's Mathematics Preparation." Unpublished Doctor's dissertation, Texas A & M University, 1968.

Kenney, Russel A. "Mathematical Understandings of Elementary School Teachers," The Arithmetic Teacher, October, 1965, pp. 431-442.

Kipps, Carol. "Elementary Teachers' Ability to Understand Concepts Used in New Mathematics Curricula," The Arithmetic Teacher, April, 1968, pp. 367-370.

Lampela, Roland Mitchell. "An Investigation of the Relationship between Teacher Understanding of Selected Concepts and Change in Pupil Understanding of Selected Concepts in Elementary School Mathematics." Unpublished Doctor's dissertation, University of California, Los Angeles, 1966.

Lund, Richard E. "Multiple Linear Regression Program," Bozeman: The Department of Mathematics, Montana State University, July 17, 1970. (Mimeographed.)

Montana State University. Montana State University Bulletin. Biennial Catalog Issue for 1968-1970. Bozeman: Montana State University, February, 1968.

_____. Biennial Catalog Issue for 1970-1972. Bozeman: Montana State University, July, 1970.

Moody, William Braun. "An Investigation of the Relationship between Fifth-Grade Student and Teacher Performance on Selected Tasks Involving Nonmetric Geometry." Unpublished Doctor's dissertation, University of Maryland, 1968.

National Association of State Directors of Teacher Education and Certification and the AAAS. Guidelines for Science and Mathematics in the Preparation Program of Elementary School Teachers. Washington: AAAS, 1961.

National Council of Teachers of Mathematics. A History of Mathematics Education in the United States and Canada. Thirty-second Yearbook. Washington: National Council of Teachers of Mathematics, 1970.

Neatrou, Charles Raymond. "Geometric Content in the Mathematics Curriculum of the Middle School." Unpublished Doctor's dissertation, Indiana University, 1968.

Reys, Robert E. "Mathematical Competencies of Preservice Elementary School Teachers," School Science and Mathematics, April, 1968, pp. 302-308.

* * * School Mathematics Study Group. Mathematics for the Elementary School. 7 vols. Stanford: The Board of the Leland Stanford Junior University, 1962.

Siegel, Sidney. Nonparametric Statistics for the Behavioral Sciences. New York: McGraw-Hill Book Company, 1956.

Smart, James R. Introductory Geometry: An Informal Approach. Belmont: Brooks/Cole Publishing Company, 1968.

Snedecor, George W., and William G. Cochran. Statistical Methods. 6th ed. Ames: The Iowa State University Press, 1967.

Suvak, Albert. Personal interviews. March, 1971.

Webber, G. Cuthbert, and John A. Brown. Number Concepts and Geometry. Reading: Addison-Wesley Publishing Company, 1969.

Wozencraft, Marian. "Even the Teacher Can't Do It!," Journal of Teacher Education, September, 1960, pp. 387-390.

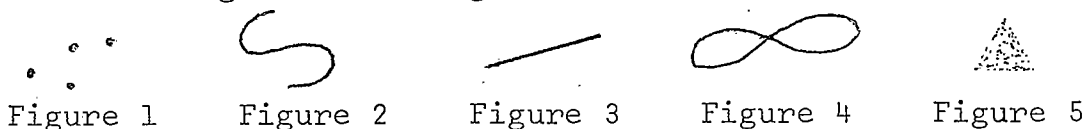
APPENDIXES

APPENDIX A

The Test--Preliminary Form and First Revision

THE TEST--PRELIMINARY FORM

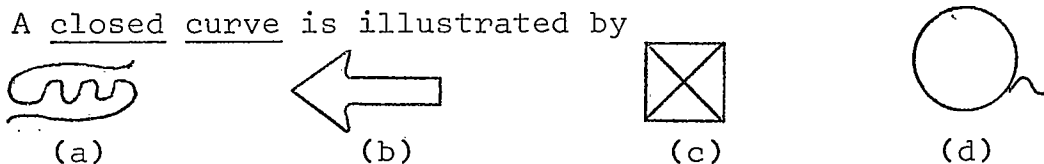
1. Consider figures 1 through 5 shown below.



A geometric figure is illustrated in

- (a) Figures 5 and 4 only.
- (b) Figures 2, 4, and 5 only.
- (c) Figures 2, 3, 4, and 5 only.
- (d) All of the figures: 1, 2, 3, 4, and 5.
- (e) None of the above choices is correct.

2. A closed curve is illustrated by



- (e) More than one of the figures shown.

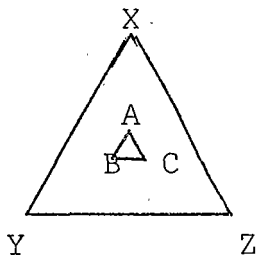
3. Two lines are said to be parallel if and only if they are

- (a) any two lines whose intersection is the empty set.
- (b) opposite sides of a rectangle.
- (c) lines in the same plane that do not intersect.
- (d) any two lines in different parallel planes.
- (e) Either choice (a) or choice (d).

4. By definition an angle is

- (a) the union of two noncollinear rays with a common endpoint.
- (b) the union of two line segments with one endpoint in common.
- (c) a vertex of a triangle.
- (d) the union of two rays with a common endpoint and all segments that have one endpoint in one of the rays and the other endpoint in the other ray.
- (e) the union of two intersecting lines.

5.



Consider the triangles shown at left.

The intersection of $\triangle ABC$ and $\triangle XYZ$ is

- (a) the empty set, $\emptyset$.
- (b) $\triangle ABC$.
- (c) $\triangle XYZ$.
- (d) points A, B, and C (i.e., $\{A, B, C\}$).
- (e) None of the above choices is correct.

6. The relationship between angle ABC ($\angle ABC$) and triangle ABC ($\triangle ABC$) is that

- (a) $\angle ABC$ is a subset of $\triangle ABC$.
- (b) the intersection of $\angle ABC$ and $\triangle ABC$ is the union of $\overline{AB}$ and $\overline{BC}$.
- (c) line segment $\overline{AC}$ is a subset of $\angle ABC$.
- (d) More than one of (a), (b), and (c) are correct.
- (e) None of the above statements is correct.

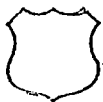
7. If the sum of the measures, in degrees, of two angles is 90 then the angles are said to be

- (a) right.
- (b) reflexive.
- (c) straight.
- (d) supplementary.
- (e) complementary.

8. A triangular pyramid is also called a

- (a) tetrahedron.
- (b) triangular prism.
- (c) truncated cone.
- (d) solid with five faces.
- (e) None of the above choices is correct.

9. A figure that has four different lines of symmetry is



(a)



(b)



(c)



(d)

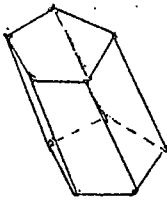


(e)

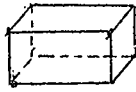
10. A plane is separated into exactly three sets of points which are disjoint in pairs by

(a) any curve.
 (b) any closed curve
 (c) any simple curve.
 (d) any simple closed curve.
 (e) None of the above.

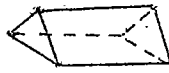
11.



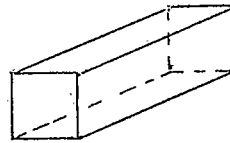
(1)



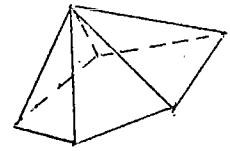
(2)



(3)



(4)



(5)

Of the illustrations above the following show prisms

- (a) figure (5) only.
 (b) figures (2), (3), and (4) only.
 (c) figures (1), (2), (3), and (4) only.
 (d) figures (3) and (4) only.
 (e) figures (1), (2), (3), (4), and (5).
12. A right triangle (one angle has measure 90°) may have sides with lengths
- (a) 8, 9, and 12 units.
 (b) 5, 12, and 13 units.
 (c) 13, 25, and 28 units.
 (d) More than one of (a), (b) and (c).
 (e) None of the above choices is correct.

13. Which of the following is a correct relationship?

(a) All squares are rectangles.
 (b) All rectangles are squares.
 (c) All parallelograms are squares or rectangles.
 (d) No rectangle is a square.
 (e) Every quadrilateral is also a parallelogram.

14. The intersection of three distinct planes in space is always

- (a) the empty set, $\emptyset$.
- (b) a single point.
- (c) a line.
- (d) a line or a point.
- (e) the empty set, a point, or a line.

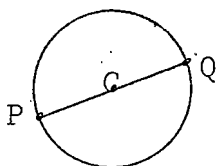
15. A plane is determined by

- (a) any three distinct points.
- (b) any point and ray.
- (c) any two lines, not necessarily intersecting.
- (d) any three distinct points not all in the same line.
- (e) More than one of the above choices are correct.

16. A triangle that has no two of its angles congruent is called

- (a) equiangular.
- (b) equilateral.
- (c) isosceles.
- (d) right.
- (e) scalene.

17.



Point C denotes the center of the circle shown at left. The length of $\overline{PQ}$ is 8 units. Then the area of the circular region, in square units, is

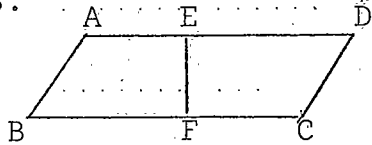
- (a) 8π
- (b) 16π
- (c) 64π
- (d) 4π
- (e) $4\pi^2$

18. A triangle with two of its sides congruent is called

- (a) an equilateral triangle.
- (b) an isosceles triangle.
- (c) a scalene triangle.
- (d) an obtuse triangle.
- (e) a right triangle.

19. If a sphere and a plane intersect the intersection is
- (a) always a circle.
 - (b) always a circular region, i.e., the union of a circle and its interior.
 - (c) either a point or a circle.
 - (d) either a point or a circular region.
 - (e) a point, a line segment, or a circle.
20. The surface area of a cube measuring 4 feet on each edge is
- (a) 16 square feet.
 - (b) 24 square feet.
 - (c) 64 square feet.
 - (d) 96 square feet.
 - (e) 256 square feet.
21. Let k denote a counting number greater than one. If there are k distinct points, no three of which are collinear, then the number of lines determined is
- (a) $\frac{k(k-1)}{2}$.
 - (b) $\frac{k(k+1)}{2}$.
 - (c) k .
 - (d) $2k$.
 - (e) $2k + 1$.
22. A polygon with six sides is called
- (a) a heptagon.
 - (b) a hexagon.
 - (c) a pentagon.
 - (d) an octagon.
 - (e) a quadrilateral.

23.

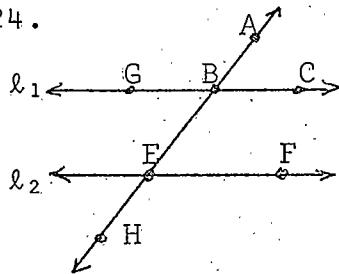


The figure ABCD is a parallelogram.
 $\overline{EF}$ is perpendicular to $\overline{BC}$.
 The measure of $\overline{AE}$ is 5, in inches.
 The measure of $\overline{BC}$ is 10, in inches.
 The measure of $\overline{EF}$ is 4, in inches.

Then the area of the simple closed region is

- (a) 50 square inches.
- (b) 40 square inches.
- (c) 32 square inches.
- (d) 25 square inches.
- (e) 20 square inches.

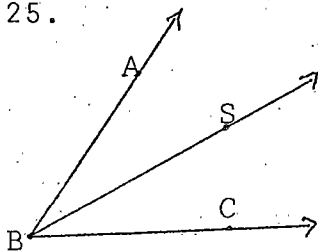
24.



If l_1 and l_2 are parallel lines then

- (a) $\angle BEF$ and $\angle ABC$ are congruent angles.
- (b) $\angle BEF$ and $\angle EBG$ are congruent angles.
- (c) $\angle GBA$ and $\angle HEF$ are congruent angles.
- (d) Choices (a) and (b) only are correct.
- (e) All of the choices (a), (b) and (c) are correct.

25.



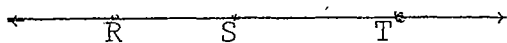
In the diagram at left suppose $\angle SBC$ and $\angle SBA$ are congruent. Then we say $\angle ABC$ is bisected. The bisector is

- (a) the line $\overleftrightarrow{BS}$.
- (b) the angular region SBC.
- (c) the half line determined by point B and containing point S.
- (d) $\triangle ABC$.
- (e) None of the choices is correct.

26. The faces of a pyramid are

- (a) always closed rectangular regions.
- (b) always closed triangular regions except for two faces, the bases.
- (c) always closed triangular regions.
- (d) always closed triangular regions except perhaps one face, the base.
- (e) parallel in pairs.

27.



Using the points R, S, and T as shown above, the line shown above can be named in

- (a) 6 different ways only.
- (b) 4 different ways only.
- (c) 3 different ways only.
- (d) 2 different ways only.
- (e) More than 6 different ways.

28. The union of two distinct angles in the same plane

- (a) may be the empty set, $\emptyset$.
- (b) may be a set with just one point as an element.
- (c) may be a ray.
- (d) may be an angle.
- (e) Non- of the above choices is possible.

29. If two triangles have corresponding angles congruent

Then

- (a) the triangles are equal.
- (b) the triangles are similar.
- (c) the triangles are congruent.
- (d) Both choice (a) and (c).
- (e) Both choice (b) and (c).

30. The intersection of a line and a plane

- (a) must be a unique point.
- (b) must be either a unique point or the empty set.
- (c) must be a unique point, the empty set, or the line itself.
- (d) must be the line itself.
- (e) None of the choices above is correct.

THE TEST--FIRST REVISION

1.

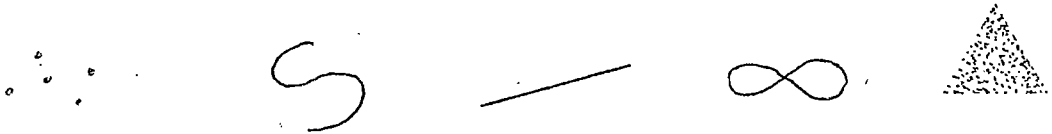
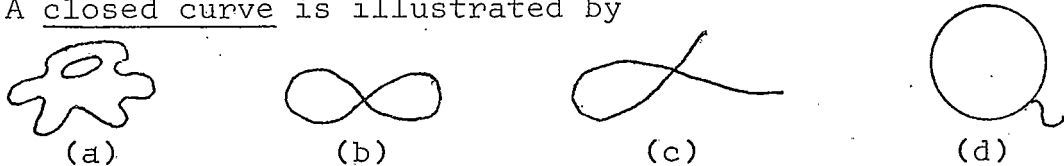


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5
 A geometric figure is illustrated in

- (a) Figures 5 and 4 only.
- (b) Figures 2, 3, and 4 only.
- (c) Figures 2, 3, 4, and 5 only.
- (d) All of the figures: 1, 2, 3, 4, and 5.
- (e) None of the above choices is correct.

2. A closed curve is illustrated by

- (e) More than one of the figures shown.

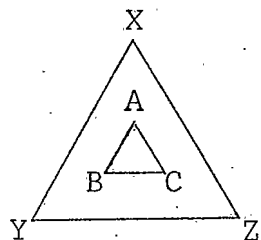
3. Two lines are said to be parallel if and only if they are

- (a) any two lines whose intersection is the empty set.
- (b) opposite sides of a parrallelogram.
- (c) lines in the same plane that do not intersect.
- (d) any two lines in different parallel planes.
- (e) Both choice (a) and choice (d) are correct.

4. By definition an angle is

- (a) the union of two noncollinear rays with a common endpoint.
- (b) the union of two line segments with one endpoint in common.
- (c) the union of any two rays in the same plane.
- (d) the union of two rays with a common endpoint and all segments that have one endpoint in one of the rays and the other endpoint in the other ray.
- (e) the union of two intersecting lines.

5.



Consider the triangles shown at left.
The intersection of $\triangle ABC$ and $\triangle XYZ$ is

- (a) the empty set, $\emptyset$.
- (b) $\triangle ABC$.
- (c) $\triangle XYZ$.
- (d) points A, B, and C (i.e., $\{A, B, C\}$).
- (e) None of the above choices is correct.

6. The relationship between angle ABC ($\angle ABC$) and triangle ABC ($\triangle ABC$) is that

- (a) $\angle ABC$ is a subset of $\triangle ABC$.
- (b) the intersection of $\angle ABC$ and $\triangle ABC$ is the union of $\overline{AB}$ and $\overline{BC}$.
- (c) line segment $\overline{AC}$ is a subset of $\angle ABC$.
- (d) More than one of (a), (b), and (c) are correct.
- (e) None of the above statements is correct.

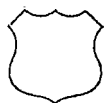
7. If the sum of the measures, in degrees, of two angles is 90 then the angles are said to be

- (a) right.
- (b) adjacent.
- (c) congruent.
- (d) supplementary.
- (e) complementary.

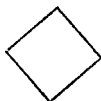
8. A triangular pyramid is also called a

- (a) tetrahedron.
- (b) triangular prism.
- (c) triangular solid.
- (d) solid with five faces.
- (e) None of the above choices is correct.

9. A figure that has four different lines of symmetry is



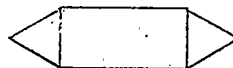
(a)



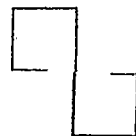
(b)



(c)



(d)

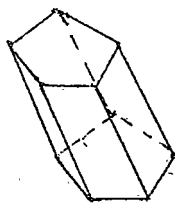


(e)

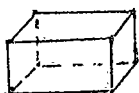
10. A plane is separated into exactly three sets of points which are disjoint in pairs by

- (a) any curve.
- (b) any closed curve.
- (c) any simple curve.
- (d) any simple closed curve.
- (e) None of the above.

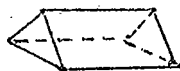
11.



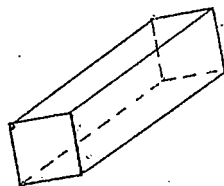
(1)



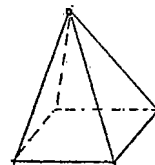
(2)



(3)



(4)



(5)

Of the figures above the following show prisms:

- (a) figure (5) only.
 - (b) figure (3) only.
 - (c) figures (1), (2), (3), and (4) only.
 - (d) figures (1), (2), (3), (4), and (5).
 - (e) None of the above choices is correct.
12. A right triangle (one angle has measure 90°) may have sides with lengths
- (a) 8, 9, and 12 units.
 - (b) 5, 12, and 13 units.
 - (c) 12, 13, and 25 units.
 - (d) More than one of (a), (b), and (c).
 - (e) None of the above choices is correct.
13. Which of the following is a correct relationship?
- (a) All squares are rectangles.
 - (b) All rectangles are squares.
 - (c) All parallelograms are squares or rectangles.
 - (d) No rectangle is a square.
 - (e) Every quadrilateral is a parallelogram.

14. The intersection of three distinct planes in space is always

- (a) the empty set, $\emptyset$.
- (b) a single point.
- (c) a line.
- (d) a line or a point.
- (e) the empty set, a point, or a line.

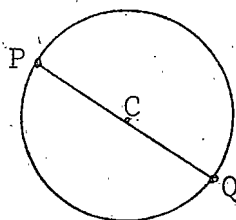
15. A plane is determined by

- (a) any three distinct points.
- (b) any two distinct rays.
- (c) any two lines, not necessarily intersecting.
- (d) any three distinct points not all in the same line.
- (e) More than one of the above choices are correct.

16. A triangle that has no two of its angles congruent is called

- (a) equiangular.
- (b) equilateral.
- (c) isosceles.
- (d) right.
- (e) scalene.

17.



Point C denotes the center of the circle shown at left. The length of $\overline{PQ}$ is 8 units. Then the area of the circular region, in square units, is

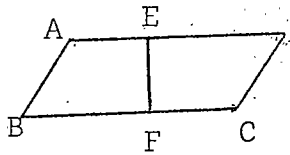
- (a) 8π .
- (b) 16π .
- (c) 64π .
- (d) 4π .
- (e) $4\pi^2$.

18. A triangle with two of its sides congruent is called

- (a) an equilateral triangle.
- (b) an isosceles triangle.
- (c) a scalene triangle.
- (d) a right triangle.
- (e) None of the above choices is correct.

19. If a sphere and a plane intersect, the intersection is
- (a) always a circle.
 - (b) always a circular region, i.e., the union of a circle and its interior.
 - (c) always either a circle or a point.
 - (d) always either a circular region or a point.
 - (e) always a point, a circle, or a line segment.
20. The surface area of a cube measuring 4 feet on each edge is
- (a) 16 square feet.
 - (b) 24 square feet.
 - (c) 64 square feet.
 - (d) 96 square feet.
 - (e) 256 square feet.
21. Let k denote a counting number greater than one. If there are k distinct points, no three of which are collinear, then the number of points determined is
- (a) $\frac{k(k-1)}{2}$.
 - (b) $\frac{k(k+1)}{2}$.
 - (c) k .
 - (d) $2k$.
 - (e) $2k + 1$.
22. A polygon with eight sides is called
- (a) a decagon.
 - (b) an octagon.
 - (c) a hexagon.
 - (d) a pentagon.
 - (e) a quadrilateral.

23.



The figure ABCD is a parallelogram.
 $\overline{EF}$ is perpendicular to $\overline{BC}$.

The measure of $\overline{AB}$ is 5, in inches.

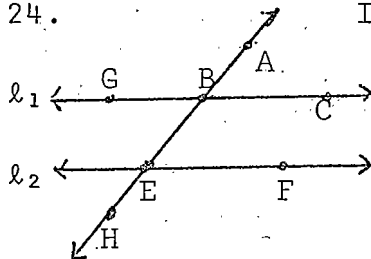
The measure of $\overline{BC}$ is 10, in inches.

The measure of $\overline{EF}$ is 4, in inches.

Then the area of the simple closed region is

- (a) 50 square inches.
- (b) 40 square inches.
- (c) 32 square inches.
- (d) 25 square inches.
- (e) 20 square inches.

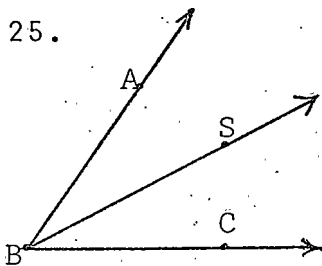
24.



If l_1 and l_2 are parallel lines then

- (a) $\angle BEF$ and $\angle ABC$ are congruent angles.
- (b) $\angle BEF$ and $\angle EBG$ are congruent angles.
- (c) $\angle LGBA$ and $\angle HEF$ are congruent angles.
- (d) Choices (a) and (b) only are correct.
- (e) All of the choices (a), (b), and (c) are correct.

25.



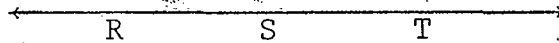
In the diagram at left suppose $\angle SBC$ and $\angle SBA$ are congruent. Then we say $\angle ABC$ is bisected. The bisector is

- (a) the line $\overleftrightarrow{BS}$.
- (b) the angular region SBC.
- (c) the half line determined by point B and containing point S, $\overrightarrow{BS}$.
- (d) the line segment $\overline{BS}$.
- (e) None of the choices is correct.

26. The faces of a pyramid are

- (a) always triangles or rectangles.
- (b) always closed triangular regions except for two faces.
- (c) always closed triangular regions.
- (d) always closed triangular regions except perhaps one face, the base.
- (e) parallel in pairs.

27.



Using the points R, S, and T as shown above, the line can be named in

- (a) 6 different ways only.
 - (b) 4 different ways only.
 - (c) 3 different ways only.
 - (d) 2 different ways only.
 - (e) more than 6 different ways.
28. The union of two distinct angles in the same plane
- (a) may be the empty set, $\emptyset$.
 - (b) may be a set with just one point as an element.
 - (c) may be a ray.
 - (d) may be an angle.
 - (e) None of the above choices is possible.
29. If two triangles have corresponding angles congruent then
- (a) the triangles are equal and congruent.
 - (b) the triangles are similar.
 - (c) the triangles are congruent.
 - (d) the triangles are similar and congruent.
 - (e) the triangles are equal, similar and congruent.
30. The intersection of a line and a plane
- (a) must be a unique point.
 - (b) must be either a unique point or the empty set.
 - (c) must be a unique point, the empty set, or the line itself.
 - (d) must be the line itself.
 - (e) None of the above choices is correct.

APPENDIX B

Computer Print-outs: Item Analyses, Correlations

Table 14

Computer Print-out: Item Analysis, Correlations
Group A--Elementary Education Seniors

Item	Per cent correct	Biserial R	Choice				
			A	B	C	D	E
1	33	.65277	1	0	5	5*	4
2	13	.18653	0	2*	1	0	12
3	53	.17196	0	1	8*	1	5
4	40	.71187	6*	0	1	4	4
5	66	.57364	10*	0	0	3	2
6	13	.21396	4	2*	0	7	2
7	60	.60147	2	0	2	2	9*
8	33	.59342	5*	2	4	2	2
9	80	.32170	1	12*	0	0	2
10	53	.13458	0	0	6	8*	1
11	33	.85057	2	3	5*	4	1
12	53	.75138	2	8*	2	1	2
13	60	.65858	9*	1	3	2	0
14	33	.53408	1	6	0	3	5*
15	46	.25794	3	0	1	7*	4
16	66	.59342	0	1	4	0	10*
17	53	.77007	0	8*	2	0	5
18	60	.67761	2	9*	0	3	1
19	33	.27693	0	4	5*	1	5
20	33	.73189	3	1	6	5*	0
21	40	.71187	6*	4	1	0	3
22	93	.20934	0	14*	1	0	0
23	33	.61320	7	5*	3	0	0
24	66	.29671	2	0	0	3	10*
25	33	.17803	5	0	5*	5	0
26	40	.73091	2	0	5	6*	1
27	33	-.01978	5*	3	7	0	0
28	60	.67761	3	9*	0	2	1
29	40	.42636	7	0	6*	1	1

*Correct choice for the given item.

N = 15. Mean = 13.60000. Sigma = 7.14973.

Kuder Richardson Correlation = .93970. Variance = 51.11429.

Table 15

Computer Print-out: Item Analysis, Correlations
Group B--Students Completing Math 109

Item	Per cent correct	Biserial R	Choice				
			A	B	C	D	E
1	50	.04753	0	4	2	8*	2
2	81	.46425	0	13*	0	0	3
3	75	.46649	1	1	12*	0	2
4	100	.00000	16*	0	0	0	0
5	93	-.11047	15*	0	0	0	1
6	50	.57035	1	8*	0	4	3
7	93	-.01229	0	0	1	0	15*
8	81	.15982	13*	0	0	0	2
9	100	.00000	0	16*	0	0	0
10	81	.34247	0	0	0	13*	3
11	75	.30185	0	0	12*	1	3
12	81	.58602	1	13*	0	1	1
13	100	.00000	16*	0	0	0	0
14	81	.34247	0	2	1	0	13*
15	81	.22071	1	0	0	13*	2
16	93	-.01229	0	0	1	0	15*
17	100	.00000	0	16*	0	0	0
18	100	.00000	0	16*	0	0	0
19	43	.42516	3	0	7*	3	3
20	87	.62875	0	0	2	14*	0
21	87	.62875	14*	1	1	0	0
22	100	.00000	0	16*	0	0	0
23	93	.18408	1	15*	0	0	0
24	75	.24697	0	1	0	3	12*
25	68	.01922	4	0	11*	0	1
26	81	.22071	0	0	2	13*	1
27	37	.18408	6*	2	7	0	1
28	93	.28225	0	15*	0	0	1
29	100	.00000	0	0	16*	0	0

*Correct choice for the given item.

N = 16. Mean = 23.87500. Sigma = 2.62995.

Kuder Richardson correlation = .55221. Variance = 6.91667.

Table 16

Computer Print-out: Item Analysis, Correlations
Group C--Students Completing Math 108

Item	Per cent correct	Biserial R	Choice				
			A	B	C	D	E
1	36	.28914	21	7	24	35*	9
2	56	.14349	1	54*	3	9	29
3	39	.30417	25	2	38*	4	27
4	83	.20814	80*	6	0	6	4
5	71	.37448	69*	15	4	1	7
6	18	.33668	18	18*	8	40	11
7	59	.30937	19	1	1	18	57*
8	20	.21796	20*	50	1	9	14
9	35	.07251	18	34*	0	6	37
10	38	.20344	10	11	10	37*	25
11	5	.26901	25	0	5*	5	60
12	32	.47522	10	31*	1	22	25
13	68	.34848	66*	4	10	6	9
14	39	.06345	10	14	19	15	38*
15	62	.16399	5	1	3	60*	26
16	67	.37374	3	3	17	7	68*
17	48	.48650	3	47*	18	3	25
18	80	.45923	11	77*	2	0	5
19	15	.27521	7	24	15*	27	22
20	43	.32560	13	3	32	42*	6
21	29	.33529	28*	12	15	5	30
22	89	.08365	1	86*	3	3	3
23	30	.35079	46	29*	3	2	10
24	61	.29157	18	3	4	12	59*
25	53	.32642	21	5	51*	0	18
26	60	.36762	4	2	23	58*	7
27	56	.29801	54*	6	16	2	16
28	15	.26767	1	15*	45	13	22
29	62	.35061	17	8	60*	7	4

*Correct choice for the given item.

N = 97. Mean = 13.83333 Sigma = 3.80489.

Kuder Richardson correlation = .59928. Variance = 14.4772.

Table 17

Computer Print-out: Item Analysis, Correlations
Group D--Students with No College Mathematics

Item	Per cent correct	Biserial R	Choice				
			A	B	C	D	E
1	38	.11289	9	2	8	15*	5
2	20	-.02748	1	8*	2	4	24
3	30	.32335	6	3	12*	7	11
4	12	.37835	5*	10	2	3	19
5	48	.43065	19*	3	0	6	11
6	12	.03319	17	5*	5	10	2
7	25	.19948	16	1	6	6	10*
8	25	.18296	10*	13	12	3	1
9	64	-.01966	2	25*	0	6	6
10	15	.27678	9	3	4	6*	17
11	7	-.02915	9	7	3*	12	8
12	5	.08552	5	2*	5	14	13
13	41	.58537	16*	1	6	13	3
14	30	-.16107	1	11	7	8	12*
15	33	.06120	10	2	2	13*	12
16	38	.33526	3	4	17	0	15*
17	20	.66910	12	8*	4	2	13
18	33	.41308	12	13*	3	7	4
19	23	.45823	4	11	9*	6	8
20	30	.22959	6	2	11	12*	7
21	12	.31364	5*	13	8	0	10
22	87	.25726	2	34*	0	2	1
23	20	.43691	18	8*	4	3	4
24	43	.22152	3	5	2	9	17*
25	28	.50301	17	2	11*	5	1
26	43	.35242	3	1	13	17*	2
27	25	.49678	10*	6	4	8	8
28	15	.33674	5	6*	8	8	8
29	33	.56608	9	4	13*	1	6

*Correct choice for the given item.

N = 39. Mean = 8.69231. Sigma = 3.55523.

Kuder Richardson correlation = .59913. Variance = 12.63968.

APPENDIX C

Letters of Invitation

LETTER OF INVITATION

Montana State University

Bozeman, Montana 59715

Tel. 406-587-3121

May 10, 1971

As you probably realize, the Department of Elementary Education conducts a continuing analysis of the curriculum in order to initiate frequent improvements. As a senior in Elementary Education, you could be of great help to us if you will lend your support to a research study now in progress.

I am asking your cooperation as a participant in a current study. This research involves data from 1971 seniors. We anticipate the findings of this study to permit inferences that will affect future changes in the curriculum you are now completing.

We need only about 35 minutes of your time and effort at a general meeting set up as follows:

Date: Tuesday, May 18, 1971

Time: 7 p.m.

Place: 201 Reid Hall

Please return the section below with appropriate information.

Cordially,

Dr. Willis C. Vandiver
Head, Elementary Education

WV/keh

I am willing to participate in the research study. I will be at 201 Reid Hall, Tuesday, May 18, at 7 p.m.

I will not be at the meeting because _____ . However, I am willing to cooperate at another time to be specified.

Signed: _____

FOLLOW-UP COMMUNICATION

Dear Senior:

A copy of the letter you received earlier urging your participation in a research study is attached. Again, the importance of your participation is being stressed.

Since you were unable to attend the Tuesday meeting, another meeting has been set as follows:

DATE: Wednesday, May 26, 1971

TIME: 7:00 p.m.

PLACE: 201 Reid Hall

Your returning the form at the bottom of the letter would be helpful. However, in any case, we look forward to seeing you this Wednesday!

3 1762 10005134 9

D378 Banning, Margaret N.
B227 The preparation of
cop.2 prospective teachers
 in geometry content of
 elementary school
 mathematics texts

NAME AND ADDRESS	
	Yale University 1218 Coward Rodgey Kennedy 401 Meisen Street Galt Taylor © DMP 522 W Main
R	
R	
R	
JAN 22 1974	Galt Taylor
R	
R	
R	
JAN 22 1974	

D378
B227
Cap. 2