

DESIGN, METHODOLOGIES, AND CHARACTERIZATION
OF MAGNETO-GRAVITATIONAL TRAPS FOR USE IN
MEASURING NEWTONIAN GRAVITATIONAL CONSTANT G

by

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of the requirements for the degree

of

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in

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NOMENCLATURE

G	Newton's constant of gravitation
g	local acceleration due to gravity (estimated for Bozeman, MT) – 9.80 m/s^2 [136]
χ	volume magnetic susceptibility, SI units if not specified
m	particle mass (mass of test mass) – about 1 mg for Edwin (HB05-2.4-lapped)
ρ	mass density
$\vec{B}$	magnetic flux density (though often referred to as magnetic field)
B	magnitude of magnetic flux density (though often referred to as magnetic field)
μ_0	permeability of free space – $4\pi \times 10^{-7} \text{ N/A}^2$
U	potential energy
F_i	short-hand for the i th component of the force $\vec{F}$, indicating that the relationship holds for each $i \in (1, 2, 3)$, such that $F_1 = \vec{F} \cdot \hat{x}$, $F_2 = \vec{F} \cdot \hat{y}$, $F_3 = \vec{F} \cdot \hat{z}$
x_i	short-hand for i th coordinate dimension, indicating that the relationship holds for each $i \in (1, 2, 3)$, such that $x_1 = x$ (transverse trap axis), $x_2 = y$ (vertical trap axis), $x_3 = z$ (axial trap axis)
ρ_{SiC}	mass density of SiC – $3.21 \times 10^3 \text{ kg/m}^3$ [75]
ρ_C	mass density of highly-ordered pyrolytic graphite (HOPG) – $2.26 \times 10^3 \text{ kg/m}^3$ [94]
χ_{SiC}	volume magnetic susceptibility of SiC at room temperature – -1.29×10^{-5} [49]
$\chi_{C\parallel}$	volume magnetic susceptibility of highly-ordered pyrolytic graphite (HOPG) parallel to its c-axis at room temperature – -6.2×10^{-4} [52]
$\chi_{C\perp}$	volume magnetic susceptibility of highly-ordered pyrolytic graphite (HOPG) perpendicular to its c-axis at room temperature – -1.4×10^{-5} [52]
B_{test}	set-point of magnetic flux density inside metal inductor core for measuring eddy-current power loss in an inductor – $8.0 \times 10^{-4} \text{ T}$
f	frequency, typically of oscillator
V	volume
Φ_M	magnetic scalar potential
$Y_l^m(\theta, \phi)$	spherical harmonic
b_0	multipole-fit coefficient describing strength of linear quadrupole in the trap
c_4	multipole-fit coefficient describing difference between x - and y -field gradients in the trap
α_z	multipole-fit coefficient describing strength of axial asymmetry in the trap
d_4	multipole-fit coefficient describing the axial weakening of the quadrupole field in the trap
y_{eq}	effective y -equilibrium of particle, defined not by center of mass but by how it appears as a scaling factor in the multipole fit – -0.222 mm for the Long MGT, on the order of tens of microns for the Short MGT

NOMENCLATURE – CONTINUED

ω_i	natural oscillation frequency of trapped particle along the x_i trap axis – for the Long MGT , $\omega_x/2\pi \approx 30$ Hz, $\omega_y/2\pi \approx 40$ Hz, and $\omega_z/2\pi \approx 0.3$ Hz; for the Short MGT , $\omega_x/2\pi \approx 80$ Hz, $\omega_y/2\pi \approx 90$ Hz, and $\omega_z/2\pi \approx 8$ Hz
c_0	coefficient containing physical parameters of the particle that affect its behavior in the trap
α	dimensionless multipole-fit coefficient describing strength of axial asymmetry in the trap
β	dimensionless multipole-fit coefficient describing the axial weakening of the quadrupole field in the trap
γ	dimensionless multipole-fit coefficient describing difference between x - and y -field gradients in the trap
y_{COM}	the distance from the estimated geometric center of the trap to the particle's center of mass – about -0.1 mm for Edwin (HB05-2.4-lapped) , assumed to be similar to y_{eq} for small particles in the Short MGT
y_{COF}	the distance from the estimated geometric center of the trap to the center of magnetic force on the particle when offset by a distance equal to y_{COM}
z_0	amplitude of particle's axial motion in trap – anywhere from a few microns to 1-2 millimeters
z_{eq}	axial equilibrium position for $\beta > \alpha$
r_0	radius of a cylindrical shape, where r is an internal radius for integration
χ_{HB}	volume magnetic susceptibility of Pentel 0.5mm HB pencil lead perpendicular to long axis of particle – $-2.0^{+0.4}_{-0.1}$
g_e	electron gyromagnetic ratio – 2.002319
D	energy of zero-field splitting in SiC – $(70 \text{ MHz})\frac{\hbar}{2}$
E_R	magnetic-field dependent resonant energy splitting for spin states of V_{Si}^- (V2) defect centers
μ_B	Bohr magneton -9.274×10^{-24} J/T
S	total spin of defect center – $\frac{3}{2}$
m_S	z-component of defect center spin – $\pm\frac{1}{2}, \pm\frac{3}{2}$
B_0	magnitude of external static field for ODMR/MDMR – $2.5 \text{ mT} \rightarrow 70 \text{ MHz}$ resonant frequency shift
θ	used as spherical coordinate, but also as angle between c-axis of SiC and external static field B_0
ν	frequency of microwave signal – typically 20-2000 MHz
ν_0	central microwave drive sweep frequency – 1050-1950 MHz

NOMENCLATURE – CONTINUED

$\Delta\nu$	deviation ($\pm$) of microwave drive sweep frequency about $\nu_0 - 90$ MHz
A_x	effective cross-sectional area of particle when treated as loop of wire – on the order of $1 \times 10^{-6} \text{ m}^2$
R	effective resistance of particle when treated as a loop of wire
$\varepsilon(t)$	time-varying voltage induced in loop of wire
$i(t)$	time-varying current induced in loop of wire
b_y	characteristic vertical field gradient in trap
b_z	scaling of characteristic axial field gradient in trap
$\vec{\mu}$	magnetic dipole moment
Γ_i	damping rate of particle motion defined by $\Gamma_i = \frac{1}{m} \frac{\partial F_i}{\partial \dot{x}_i}$
$\omega_{i,0}$	undamped frequency of harmonic oscillator
z_{rms}	root-mean-squared of $z(t)$, the axial position of the particle
k_B	Boltzmann constant – $1.38 \times 10^{-23} \text{ J/K}$
m_0	mass of gas particles for damping model – $4.65 \times 10^{-23} \text{ kg}$
T	room temperature – 24°C
ρ_{HB}	measured density of Pentel 0.5mm HB pencil lead – 1.70 g/cm^3
M	mass of field mass (source mass) for G -measurement apparatus – 1.3 kg
U_G	gravitational potential energy
d	distance between center of masses of field masses
k_{Gz}	gravitational contribution to trap- z spring constant from field masses – $-3.5 \times 10^{-7} \text{ N/m}$
δx_i	particle misalignment from center along x_i axis
$\text{d}x_i$	field mass misalignment from center along x_i axis
M'	other mass nearby contributing perturbing potential to trap
a, b	minor, major axis of ellipse
$\text{d}s$	difference between major and minor axes of ellipse
Φ_G	gravitational potential
ρ_{water}	density of water – 1 g/cm^3
$\vec{\mu}_F$	magnetic dipole moment of magnetized field mass

NOMENCLATURE – CONTINUED

$\chi_F^{(V)}$	volume magnetic susceptibility of field mass – -1×10^{-5}
$\chi_T^{(V)}$	volume magnetic susceptibility of test mass – -2×10^{-4}
$\vec{B}_{ext}$	external field responsible for magnetizing field masses – about 50 μ T
R	radius of field masses – 4 inches
ρ_T	density of field masses – 2.51 g/cm ³
ρ_P	density of particle – 1.70 g/cm ³
$\vec{B}_F$	magnetic field produced by both magnetized field masses
$k_{ext,z}$	magnetic contribution to trap- z spring constant from field masses alone – 3×10^{-13} N/m
$k_{trap,z}$	magnetic contribution to trap- z spring constant from field masses interacting with trap field – 5×10^{-3} N/m
$B_{EM,x}$	transverse magnetic field produced by end magnets at field masses – 190 μ T
G_0	actual value of gravitational constant, as contrasted against measured value of G
σ_{HB}	measured electrical conductivity of Pentel 0.5mm HB pencil lead – $4.63(15) \times 10^5 \Omega^{-1}\text{m}^{-1}$

NOMENCLATURE – CONTINUED

LOMS	levitated optomechanical system
SiC	silicon carbide
NIST	national institute of standards and technology
TOS	time of swing
DTM	single point diamond turning machine
ppm	part(s) per million
MGT	magneto-gravitational trap
DC	direct current
Q	quality factor
GCP	graphite composite particle
FFT	fast Fourier transform
HOPG	highly-oriented pyrolytic graphite
NA	numerical aperture
PEEK	polyetheretherketone
SAR	sublimation-activated release
COM	center of mass
PETG	polyethylene terephthalate glycol
ESR	electron spin resonance
ODMR	optically detected magnetic resonance
MDMR	mechanically detected magnetic resonance
ZFS	zero-field splitting
LED	light-emitting diode
ND	neutral density
PL	photoluminescence
SPCM	single-photon counting module
SMF	single-mode fiber
MMF	multi-mode fiber
MD	microwave drive
DAC	digital to analog converter
SNR	signal-to-noise ratio
RF	radio frequency
ABS	acrylonitrile butadiene styrene
TPU	thermoplastic Polyurethane
UHV	ultra-high vacuum
ASD	amplitude spectral density
SS	stainless steel

ABSTRACT

There is poor consensus to date on the accepted value of the Newtonian constant of gravitation G , despite multiple recent studies attempting high-precision measurements of the parameter. In this work we discuss the design of a levitated optomechanical system, constructed using a magneto-gravitational trap under ultra-high vacuum with a graphite composite test mass, as a system for performing such a measurement. Through tilt-stabilization, the use of a single point diamond turning machine, and careful control of the trapping potential, an oscillator with a frequency of about 0.28 Hz with a damping rate insignificant to other noise sources in the system has been constructed. Additionally, work on this system resulted in the development of a selective loading technique with potential application in a multitude of levitated optomechanical systems and a possible measurement scheme for building a spin-mechanical coupled system using V_{Si}^- (V2) vacancy defect centers in levitated silicon carbide. The results are a significant step towards the construction of a high-precision levitated optomechanical system for measuring G .

INTRODUCTION

One of the most basic concepts in Physics is the principle that applying a net force to a massive object effects a change in its position in space that can be observed empirically. This idea, quantified in Newton's Second Law, implies that any effect that creates a force can, in theory, be measured and characterized experimentally.

The practical difficulty in performing such experiments lies in the details. First, the force created by the effect being characterized must be of a magnitude large enough compared to other forces acting on a test mass so as to significantly alter the net force. Second, the acceleration of the test mass produced by a change in the net force must result in a displacement large enough to be reliably measured. These two principles drive the design of many fundamental physics experiments.

Techniques used to perform sensitive experiments while addressing the former principle tend to involve controlling other forces acting on the test mass, such as by reducing mechanical coupling of the test mass to the outside world and reducing ambient pressure and temperature. The latter principle is predominantly one of precise and unambiguous measurement, and is accomplished by coupling mechanical motion of a test mass to more directly measurable quantities such as electrical current or light, which can then be measured by a photodiode.

As an interesting side note, even some experiments not relying on directly detecting forces applied to a test mass often must meet similar requirements, such as the Michelson-Morley experiment of 1887 aimed at measuring a shift in the speed of light due to motion through a hypothesized "aether". In this experiment, mechanical isolation was accomplished by setting the experiment on a sandstone slab and floating it in a bath of mercury, while

precise detection was accomplished using light in a Michelson interferometer. This example also serves to illustrate another important observation: experimentally probing fundamental physics often leads to the development of highly useful techniques (in this case the Michelson interferometer) that may become widely applied across disciplines. [18, 19, 90, 119, 151]

A relatively recent solution to these underlying requirements on experimental physics is the technique of levitated optomechanics, pioneered by Ashkin in 1977 [13] with optically levitated dielectric particles. In this case, the test mass is isolated from its environment by levitating it to remove contact forces, reducing the pressure in the chamber by placing it under vacuum, and feedback-cooling its motion using radiation pressure to further reduce background noise. The position of the test mass is then detected optically by projecting an image of the particle onto a split photodiode detector.

In this manuscript we discuss the development of a specific levitated optomechanical system (LOMS) with the intended application of performing a precise measurement of G . This work builds off of results already published on selective loading of micron-scale particles into magneto-gravitational traps [106] and achieving ultra-low damping using a graphite composite particle levitated in a magneto-gravitational trap. [107]

First, in the remainder of this chapter, we will discuss previous measurements of G and their challenges, then we will revisit the topic of levitated optomechanics and focus down on the system at hand.

Next, in Chapter 2, we will describe the design of the apparatuses used for most of the work described in this manuscript, report measured material properties for custom components, and present a theoretical model for the trapping mechanism of the LOMS under development.

In Chapter 3 we discuss work related to performing spin-resonance measurements on V_{Si}^- (V2) vacancy defect centers in 4H-SiC, its possible applications as a quantum magnetometer to a measurement of G , and some of the merits of the work apart from its application in

magnetometry.

Chapter 4 will present the results of the LOMS under study, specifically its extremely low damping rate and other factor affecting its suitability for measuring G .

Then, in Chapter 5, we will go through several calculations estimated the tolerances on experimental design that must be accounted for to reduce systematic error in a measurement of G that is good to 10 ppm or 1 ppm, emphasizing tolerances for our system but with some general application to any measurement of G .

Finally, Chapter 6 will present a brief summary of the results of the previous sections as well as notes on future research and improvements that may be beneficial.

Measuring G

The Newtonian constant of gravitation, G , is a difficult parameter to measure due to the relative weakness of its effect. We can see the effect of magnetic fields easily in refrigerator magnets, electric forces in static electricity, but to see the effect of gravity in every-day life we typically need something roughly the size of a planet.

Recent Measurements and Accepted Value

Nevertheless, G is a critical parameter in astronomical simulations and data calibration, and in any field of gravity research. The current accepted value of $6.67430(15) \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ [112] is only specified to 22 parts per million, with several recently measured values falling well outside that range.

A chart comparing recent measurements [8, 14, 47, 65, 77, 85, 92, 110, 114, 120, 121, 129, 132, 149] of G is shown in Figure 1, compared against the value recommended by the national standard of institutes and technology (NIST).

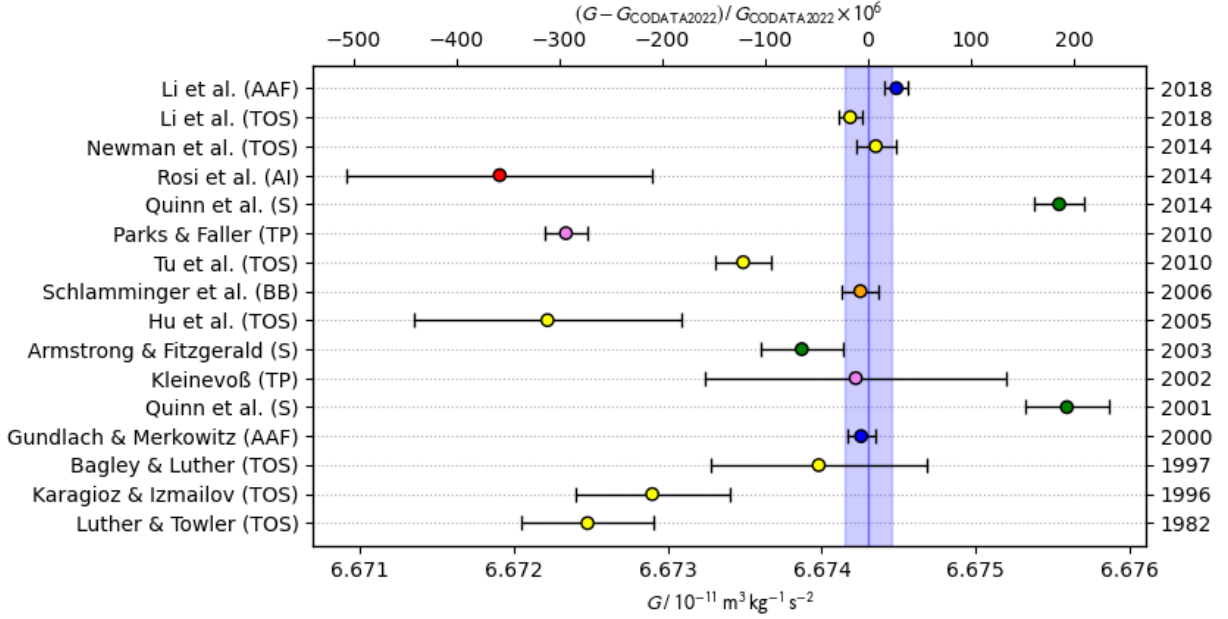


Figure 1: This figure was generated by `G_measurements_plot.py`. A chart displaying the values and uncertainties of recent measurements of G . TOS = Time of Swing, BB = Balance Beam, S=Servo-control, AI=Atom Interferometry, TP = Two-Pendulum, AAF=Angular Acceleration Feedback.

These recent measurements of G , using methods ranging from torsion pendulums and simple pendulums to cold atom interferometry, do not reach consensus around a reproducible measurement of the quantity. Even more disturbingly, measurements are often not even close to agreeing to within listed uncertainties. These data imply that unaccounted-for sources of systematic error are present in some or all of the measurements, leaving doubt as to the true accuracy (not just precision) of any given measurement, regardless of its claimed uncertainty.

What Makes it Challenging

As mentioned previously, the small amplitude of gravitational forces produced by laboratory-scale masses compared to other forces makes any measurement of G quite challenging. In general, seismic activity, [77, 85, 110, 149] the magnetic field of the Earth [121] or in some cases of the experimental setup, [85, 110, 114, 132, 149] and other sources of

vibration must be carefully considered. Additional difficulty often arises from the long-term change in the properties of the torsion fibers typically used in time-of-swing (TOS) measurements, [65, 85, 92, 149] although this effect seems to be well-characterized enough to account for in the data analysis.

In every measurement, precisely determining the precise properties and locations of experimental objects proves to be quite a challenge. The precise position of the field masses, in particular, is commonly a leading source of uncertainty [8, 85, 114] and requires very careful measurement and control. Many different field mass materials have been used, ranging from 500 g copper cylinders [8] to 576 kg brass masses [77] and even tanks filled with mercury. [132] Other field mass materials include 316 stainless steel [85, 149] and tungsten. [114] This last is an especially attractive option due to its high density, low-amplitude magnetic susceptibility, and hardness allowing for precise machining. It is, however, more costly and more difficult to work with than materials such as copper or stainless steel.

While larger field masses, such as bodies of water or granite slabs, could in principle be used to increase the gravitational force on a test mass, the homogeneity and precise shape and mass of such source masses is typically difficult to control or measure carefully enough to justify the increase in signal strength.

In addition to the weakness of the effect making precise force isolation difficult, we currently have no way to shield from gravitational fields (there is no known material with a relative gravitational permeability or permittivity that differs from one as with electric and magnetic forces). This makes determinations of G subject to perturbations from other nearby masses, an effect which is potentially always present but rarely specifically addressed in most descriptions of experimental procedures reported in the literature.

Possible Solutions

A LOMS is a natural choice for performing a measurement of G due to the stability of the trapping potentials that can be produced and their potential for extreme isolation of a test mass from its environment. In addition, the rapidly growing nature of the field aids in timely development of new techniques that may become useful for improving precision measurements of all varieties. In particular, a stable potential such as that formed by a magneto-gravitational trap will not be subject to the same anelasticity and gradual material decay as a torsion pendulum.

One major source of uncertainty in many measurements, the location and positioning of the field masses, can be dealt with in a robust way using a single point diamond turning machine (DTM). The experiment is mounted on a Moore Nanotech 250UPL Ultra-Precision Diamond Turning Lathe, modified by removing the spindle and covers, and outfitted with custom-machined metal plates. The glass slides on this machine allow high-resolution feedback (0.034 nm) and programming resolution of about 1 nm over 200 mm of travel. The straightness of the slides is specified to be within 200 nm over that range, and the granite slab the slides are mounted on can be floated on four pillows of air, allowing for active tilt-stabilization of the table.

As for the effects of nearby masses causing systematic error in a way that cannot be shielded, this can only be addressed by paying very careful attention to nearby masses and documenting any change that takes place between measurements taken with different arrangements of field masses. Stray laboratory equipment must be controlled, but the nature of a 10 ppm measurement or better will likely also require consideration of factors outside the lab such as equipment in neighboring labs in the same building or even cars parked outside. Such considerations are difficult to control, but must be considered to truly eliminate these possible sources of systematic error.

Levitated Optomechanical Systems

A full review of levitated optomechanical systems (LOMSs) is beyond the scope of this manuscript. However, a brief description of some common techniques employed and of some recent applications is presented here to illustrate some of the depth of this rich field. [70]

Due to the isolated nature of levitated systems, extreme force sensitivity can be achieved, especially by taking advantage of techniques such as optical feedback cooling. [44, 51, 86, 87, 103, 123] Indeed, asymmetrical particles can also be used in such systems to measure rotations and torques acting on trapped particles. [4, 5, 54, 150].

This extreme sensitivity has been used to build detectors to probe many different kinds of quantities, including electric and magnetic fields [6, 66, 68, 104, 166] and quantum properties of materials. [32, 53, 69, 93, 159] These systems have potential application in pushing the boundaries of our understanding of physics in areas such as dark matter searches, [1, 23] constraining non-Newtonian gravity, [26, 43] the construction of gravity wave detectors, [3, 10, 158] and creating superposition states of macroscopic objects. [16, 42, 118, 127, 153] Finally, LOMSs can be engineered with the capability to characterize the properties and behavior of small objects and exotic materials in many different fields including stochastic thermodynamics, [2, 35, 45, 55, 98, 124, 128] thermal properties and control, [36, 50, 122] and biological sensing. [7]

Trapping Mechanism

There are two major requirements to create a levitated system near the surface of the Earth. First, a vertical force strong enough to oppose gravity must be created to keep a test mass elevated in free space. Second, a trapping potential must be formed in all three spatial dimensions such that the test mass (typically referred to as a particle) is confined in a stable potential minimum. In many cases, such as in optical traps, a single trapping mechanism

may satisfy both these conditions if the spring constant is high enough.

Optical Traps In an optical trap, sharply focused light exerts a restoring force on a dielectric particle. [21, 101] This technique, first achieved by Ashkin in 1971 [12] when he levitated $\sim 20\text{ }\mu\text{m}$ glass particles, has since been used to levitate silica nanoparticles as small as 43 nm [73] and even single atoms. [72, 117]

These traps generally have high frequencies up to hundreds of kHz, [93] and work with neutral or charged particles, but can be affected by heating of the particles [9, 154] due to the high powers often used.

Paul Traps Paul traps levitate charge particles by creating pseudo-potentials using high-frequency alternating electric fields to levitate particles between electric poles. [116] These traps have been used to trap particles as small as 100 nm [22] and as large as $10\text{ }\mu\text{m}$. [37]

This trapping technique is highly versatile in what materials can be trapped, since the particles need only to be charged. It also has the advantage that the potential strength can be adjusted dynamically, making it especially useful in creating hybrid traps. [20, 99] However, the necessity of using a charged particle and the dynamic trapping potential can lead to coupling mechanisms between the particle's motion and external noise sources.

Magneto-Gravitational Traps Magneto-gravitational traps (MGTs) offer static trapping potentials for diamagnetic particles, with frequencies ranging from less than 1 Hz [107] to over 1 kHz [17] and particle sizes ranging from about $1\text{ }\mu\text{m}$ [140] to several millimeters. [28]

Such traps are typically used to levitate diamagnetic materials (although diamagnetic materials can be used to aid in the levitation of magnets in some cases, [41, 67, 137, 139, 164]) somewhat limiting the scope of materials that can be used. The static trapping potential, however, offers greater stability compared to more dynamic traps.

Position Tracking

While some experiments, like Ashkin’s original feedback-cooled system, use a split photodetector for position tracking, [13, 17, 60] other experiments use different arrangements of photodiodes, [11] area-scan [60, 97, 99, 140] or line-scan [39] cameras, a 3D split detection system, [86] interferometric detection. [31, 44, 97]

Analysis of the position data is usually done in real-time to allow active feedback cooling, which can pose an additional challenge when the imaging device used for feedback cooling is an area-scan camera. [140] In the work presented in this manuscript, we use cross-correlation based image analysis developed in collaboration with Leonardo R. Werneck and Zachariah B. Etienne. [156]

In some cases a quicker image analysis technique can be used for feedback, while a slower detector capable of collecting more information is used to collect position data for analysis. [60]

Feedback Cooling

Feedback cooling can be accomplished in several ways, including the use of electrostatic forces if the particle is charged [60, 99], radiation pressure via cooling lasers, [97, 140] magnetic forces in the case of a MGT, [31, 91] or cavity cooling for cavity-based optical traps. [11, 57]

This feedback cooling is critical for experiments attempting to reach the quantum ground state of an oscillator, but can also be useful when trying to isolate information from a single degree of freedom of an oscillator moving in three dimensions. Specifically, if a LOMS is to be used to measure G along a lower-frequency axis of motion, the other two degrees of freedom should be cooled in order to reduce the effect of higher-frequency motion coupling into the axial data.

Trap Loading

One problem that must be solved for any LOMS is how to load a particle into a trap. Many techniques, such as using an atomizer, [87] tapping the particles off of a rigid object, [37, 58] electrospray injection, [34, 71, 80] sublimation activated release, [106] and piezo/optical selective loading, [12, 113] are typically performed with the trap under atmospheric pressure. In addition to the difficulty of getting a loading apparatus into an evacuated chamber, the other difficulty in non-atmospheric loading is that a trap with a conservative potential in general cannot be loaded without some dissipative mechanism to reduce the particle's energy.

Nevertheless, there are still several methods that can be used to accomplish loading under vacuum, such as Piezo-based loading (only accomplished up to rough vacuum), [60, 73, 140?] light-induced acoustic desorption (LIAD), [11, 22, 100] Load-Lock, [97] and hollow-core loading. [46, 88, 133, 134] However, these methods (with the exception of Load-Lock and hollow core techniques, which rely on careful placement of the particle inside the potential) rely on some additional dissipative mechanism, such as optical feedback cooling, to remove energy from the particle as it is being loaded.

Our Magneto-Gravitational Trap

The designs and methodologies described in this manuscript focus on the use of a specific type of MGT based on using ferromagnetic pole pieces to create slightly perturbed linear quadrupole magnetic fields that form trapping potentials in conjunction with Earth's gravitational acceleration g .

Trapping Potential

It is beneficial at this point to discuss the physics of MGTs in more detail.

Diamagnetism in materials is caused by the orbital spin of electrons in materials without an unpaired electron (paramagnetism, resulting from the spin of the electron itself, tends to

dominate in materials with unpaired electrons). The result is that diamagnetic materials, where the magnetic susceptibility χ is less than 1, form magnetic fields that oppose any external magnetic field applied.

A particle of mass m , mass density ρ , and magnetic susceptibility χ (where $|\chi| \ll 1$) in a magnetic field with magnitude B is subject to the potential

$$U = -\frac{\chi m}{2\mu_0\rho}B^2. \quad (1)$$

The force on such a particle can be readily calculated by taking a gradient of this potential:

$$F_i = -\frac{\partial U}{\partial x_i} = \frac{\chi m}{\mu_0\rho}\vec{B} \cdot \frac{\partial \vec{B}}{\partial x_i}. \quad (2)$$

It is easy to see that χ must be negative in order for this force to create a stable trap. Additionally, in order to levitate a particle near the surface of the earth, F_y must be able to overcome the acceleration due to gravity of the earth g with the particle inside the trapping region. Via Eq. (2), it can be seen that this necessitates both a high B -field strength and a high B -field gradient.

For the work described in this manuscript, this was accomplished by creating a strong field resembling a linear quadrupole using high-strength permanent magnets (producing a high B -field) in conjunction with sharp pole pieces (producing a high B -field gradient) made from a material with high magnetic saturation (Hiperco 50A). The specific physics of the MGTs used will be discussed further in Chapter 2, but a schematic of the basic design of the MGTs used in this work is shown in Figure 2.

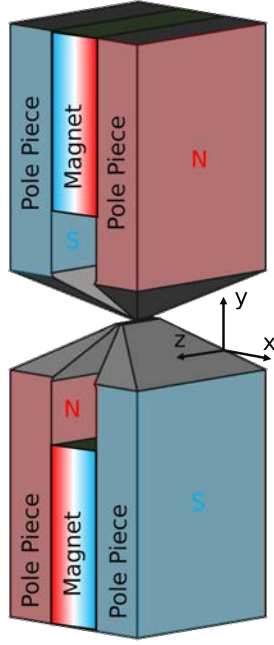


Figure 2: A schematic of a magneto-gravitational trap showing the orientation of the magnets and pole pieces. The tightly-spaced pole piece tips make a strong quadrupole field in the x - y plane, which produces a strong transverse (x) and vertical (y) trapping force, and the relatively long edges in the axial (z) direction create a weak field gradient along the trap axis. The asymmetry between the top and bottom sets of pole pieces causes the magnetic potential to curve up at the ends of the trap, and gravity keeps the particle from falling out the ends.

One consequence of this type of geometry is a decoupling between the three principal axes of the trap – the vertical offset of the particle due to gravity introduces a change in frequency between x - and y -motion, which combines with the fundamentally weaker z -motion to prevent coupling of energy between the normal modes of the oscillator. This allows cooling to be done separately on each degree of translational freedom of a particle trapped in such a MGT.

Test Masses

Several different possibilities for test masses were considered and used for different experiments discussed in this manuscript. Here we briefly mention each one and the motivations for using it.

Silica Microspheres Silica microspheres have been used for many experiments in this lab and others due to their favorable properties for trapping. Silica microspheres can be readily found in many different sizes ranging almost continuously from 10s of nanometers to millimeters in diameter (although finding borosilicate spheres between 50 μm and 300 μm did pose something of a challenge, motivating the work mentioned in this manuscript to develop a technique for flame spheroidization). Silica is fairly transparent at many wavelengths commonly used for optical trapping, which also allows them to be feedback-cooled optically without excessive heating. They also are insulating, which is important for avoiding eddy current damping in MGTs.

However, while silica microspheres behave well in strong MGT geometries (8 μm spheres were used to demonstrate the sublimation-activated release technique developed in this lab [106]), the weaker geometry used for creating a low oscillation-frequency trap for measuring G necessitated that the particles be charged. This charge, while it allowed a DC bias voltage to be applied to overcome gravity sufficiently to stably levitate the particle, ultimately led to excessive charge damping.

4H-SiC Another material considered for use in measuring G was 4H-SiC. This crystalline material can form silicon vacancy centers when bombarded with high-energy electrons, [75] making it a very interesting material for consideration in a MGT. One particularly useful feature of the V_{Si}^- (V2) centers formed in 4H-SiC is their potential usefulness for vector magnetometry [33, 81, 82, 105, 111, 138]. This would not only allow potential characterization of the magnetic field within the small length scale of a MGT, but also give a measurement of the particle's orientation in the trap, which can otherwise be difficult to determine for nearly spherical particles. For measuring G in particular, unmeasured coupling of rotational energy into the translational motion of an oscillating particle could lead to systematic error.

Another interesting application of levitating a particle of 4H-SiC in a MGT is the potential for coupling the spin of the particle's V_{Si}^- (V2) centers to its mechanical motion in the trap. [130, 131, 163] While not related directly to measuring G , this result on its own would be quite significant.

Graphite Composite Particles Graphite at first seems like a clear choice as a test mass for use in a MGT due to the relative strength of its diamagnetism, particularly along its c-axis. However, there are two major issues that threaten to make it a poor choice compared to options like silica spheres. First, it is opaque to visible light, making it difficult to apply laser-based feedback cooling. Second, the conductivity of graphite suggests that it should be highly susceptible to eddy current damping.

Recent advancements in the use of highly oriented pyrolytic graphite (HOPG) as a test mass in MGTs [27–31, 48, 56, 74, 89, 91, 126, 148, 155, 161, 165] has already revealed several methods for reducing this eddy current damping problem. Some groups have noted the improvement in Q accomplished by mixing the graphite with an insulating material such as wax [148] or epoxy. [29, 31] In other cases, damping rates were reduced by engineering certain geometries in the particles. [56, 126, 160] Finally, damping rates were also reduced significantly in a rotating graphite or graphite-composite plate by leveraging the axial symmetry of a magnetic potential formed by a cylindrical permanent magnet. [31, 74]

A recent result from this lab is the achievement of unmeasurably low damping rates through the use of both a graphite-composite particle (GCP) and a highly anisotropic magnetic field with no linear field gradient along the axial direction. [107] Another consequence of this result is that the eddy current damping is quite strong in the two strong trapping directions, resulting in passive cooling of the other degrees of freedom in the system. This long MGT with low axial oscillation frequency and low axial damping rate used with a GCP as a test-mass shows significant promise as the oscillator in a system for performing a

measurement of G .

Analysis

There are several analysis techniques commonly employed throughout this manuscript that are worth mentioning here.

Many measurements in this experiment rely on extracting values of displacements from images taken by a camera (Basler acA2440-75um). This analysis is done using cross-correlation code [84, 156] developed in collaboration with Leonardo R. Werneck and Zachariah B. Etienne.

Any time a frequency is extracted from displacement data, it is done using code from `find_freq_avg.py` or a derivation of this code, which does the following.

1. Reads data from a file, taking into account the frame rate at which it was collected.
2. Splits that data into many segments, and possibly pads the data with zeros to interpolate between frequency bins in Fourier space.
3. Uses fast-fourier transforms (FFTs) to compute the power spectrum for each segment, then averages together the values at each frequency.
4. Plots the square root of the power spectrum average (average amplitude spectrum), and finds the highest value in a certain range around a user-specified frequency guess.
5. Interpolates a parabola to the maximum value and the two points to either side.
6. Returns the vertex of the interpolated parabola.

This results in a much higher resolution in resolving frequency values than the frequency bin size, a convenient feature in data analysis.

EXPERIMENTAL DEVICES AND DESIGN

This chapter is devoted to describing and analyzing the basic experimental setups used to do conduct the bulk of the research discussed in this manuscript. Most prominently, two different styles of magneto-gravitational trap (MGT) are described: one with sharply tapered pole pieces resulting in a short axial trapping region which will be referred to as “Short MGT ”, and one with only slightly tapered top pole pieces resulting in a long axial trapping region which will be referred to as “Long MGT ”.

Trap

The basic design of the MGTs used in this research has already been introduced in Chapter 1. There are two specific designs used for different purposes in this work. The first design, which will be referred to as the Short MGT , is shown in Figure 3. This design was made to prioritize maximizing the flux through the Hiperco 50A pole pieces in order to make them as saturated as possible, so as to maximize trapping strength.

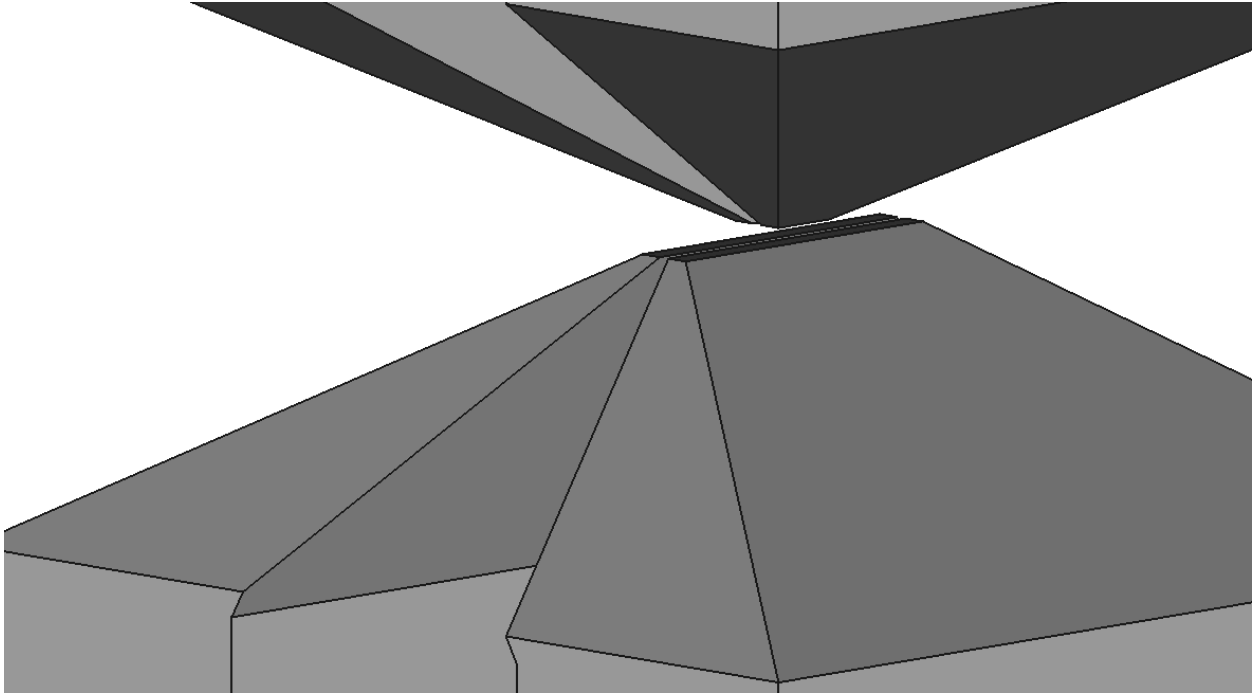


Figure 3: This figure was generated from `murphy_pole_pieces.FCStd`. Generally the tips are lapped such that the desired vertical/transverse gap is achieved when the unlapped tips would all touch in the center of the trap. In practice the gaps are aligned by hand depending on the application – narrower gaps are used for stronger traps, wider gaps are used if the particle dimension begins to approach the size of the trap gap.

The actual vertical gap of the assembled trap changed slightly each time it was taken apart and reassembled, but generally the transverse gap was set to about $100\mu\text{m}$ and the vertical gap was set to about $150\mu\text{m}$. These dimensions are reflected in Figures 4 and 5, which show schematics of the particle imaged down its transverse and axial axes, respectively.

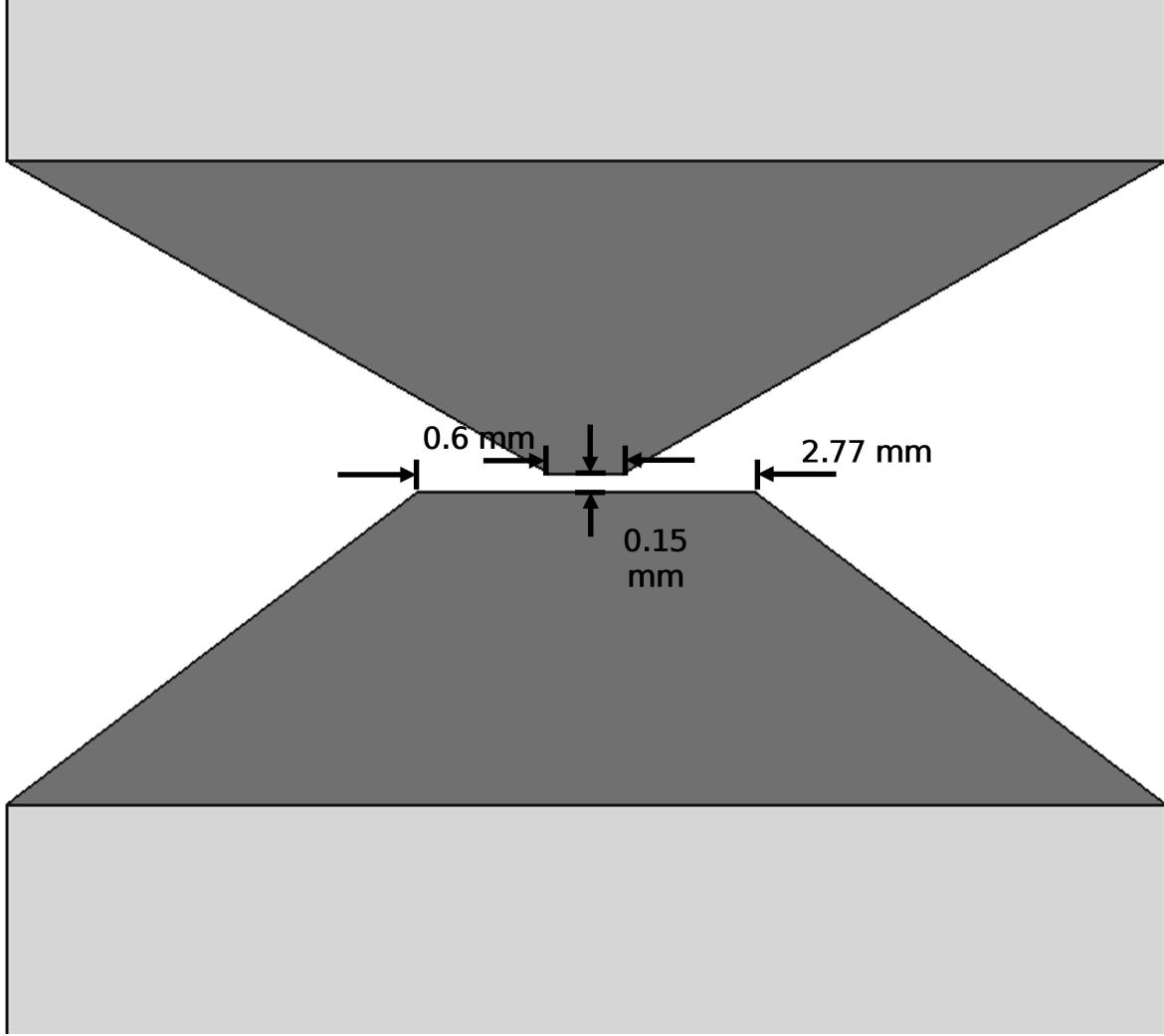


Figure 4: This figure was generated from `murphy_pole_pieces.FCStd`. Although the axial length of the trap (2.77 mm on the bottom, 0.6 mm on the top) is long compared to the vertical ($\sim 150\ \mu\text{m}$) or transverse ($\sim 100\ \mu\text{m}$) gaps, the strong tapering of the pole pieces still encourages high saturation of the pole pieces. This is essential to keep the trapping B -field dominated by geometry and not by the magnetic limitations of the Hiperco 50A .

Optical access is available down both the transverse and axial directions, but the numerical aperture (NA) for imaging a particle at the center is significantly higher when viewed down the transverse axis ($NA \sim 0.3$) as in Figure 4 than the axial axis ($NA \lesssim 0.2$). For this reason, primary detection is typically done along this axis, while the axial view (shown in Figure 5) is used predominantly for cooling the motion of the particle in the

transverse direction.

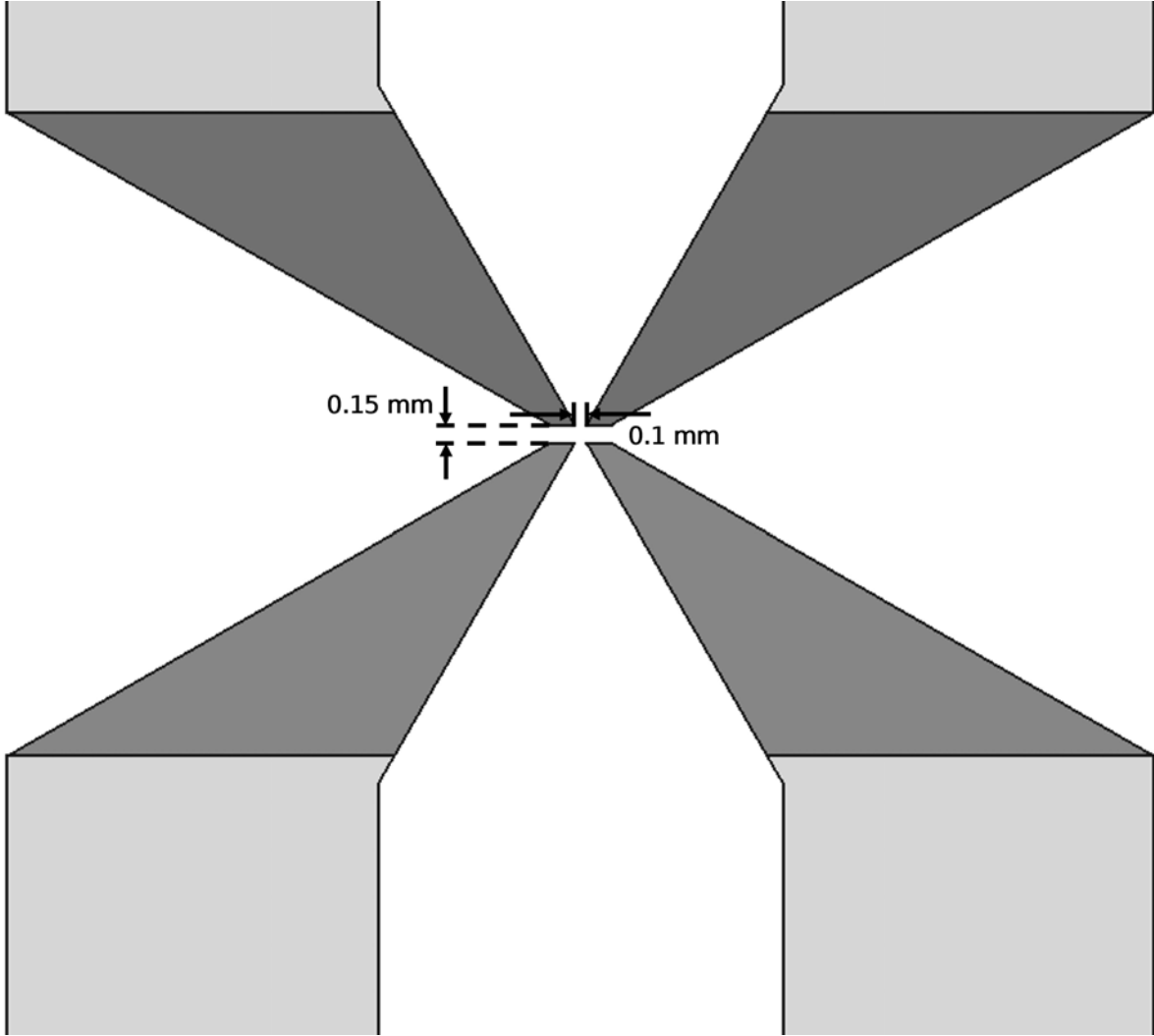


Figure 5: This figure was generated from `murphy_pole_pieces.FCStd`. The transverse and vertical gaps in this figure are set to $100\text{ }\mu\text{m}$ and $150\text{ }\mu\text{m}$, respectively. In practice, the transverse gap is set by placing PEEK or Kapton films between the pole pieces and the magnets. This also allows each pole piece to be electrically insulated from each other pole piece, a useful feature that allows the possibility of generating an electric field near the center of the trap in nearly any direction in the x - y plane by applying a different offset voltage to each pole piece. The vertical gap is adjusted by hand and checked under a microscope.

The MGT is housed in a vacuum chamber with $4\frac{1}{2}$ -inch flanges and viewed through an anti-reflection coated window (Viewport VPZL-450DUC3) as shown in Figure 6.

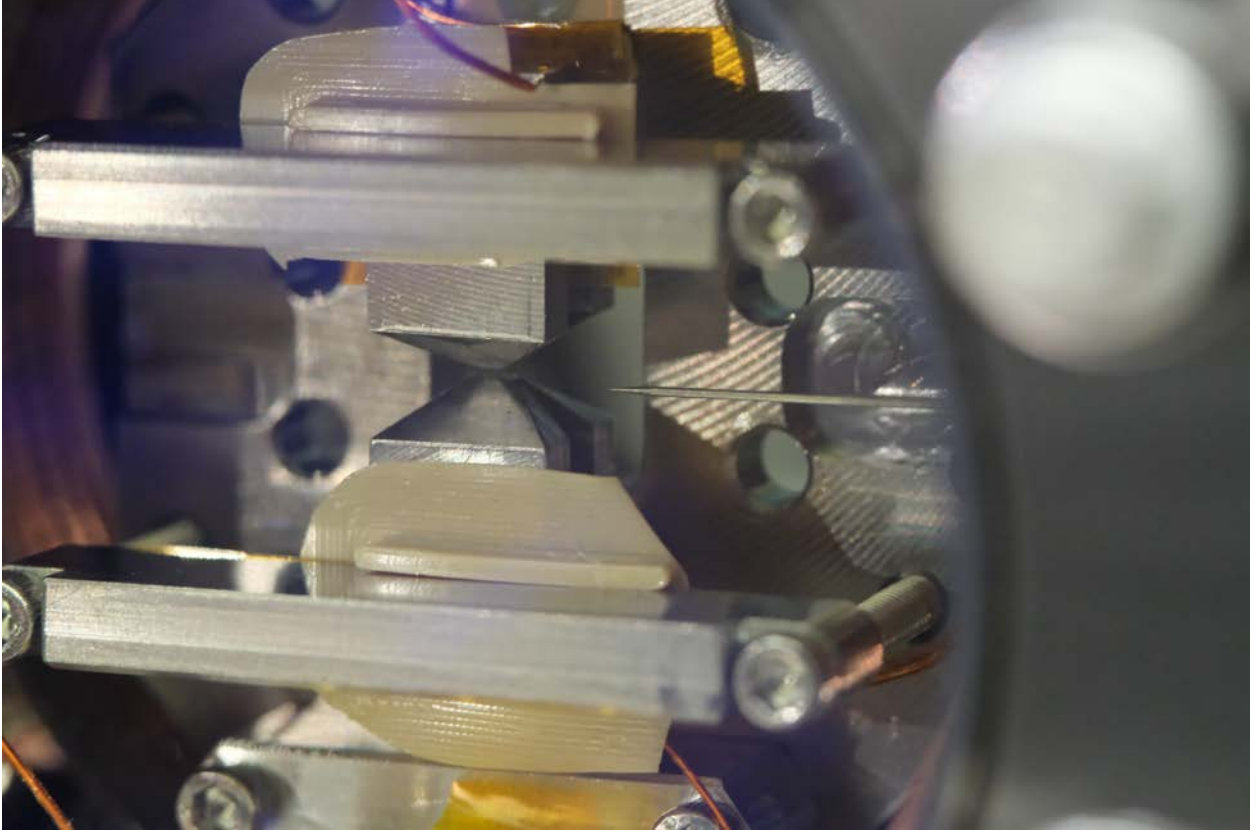


Figure 6: Photo of the assembled Short MGT housed in a vacuum chamber. On the right can be seen a tungsten probe tip used for sublimation-activated release (SAR) loading (discussed in Section 2). The wide tan pieces are 3D-printed PEEK used to electrically insulate the pole pieces from the metal mount, allowing independent control of the pole piece voltages.

The second type of trap, which will be called the Long MGT, is designed to minimize the axial oscillation frequency of a trapped particle while still reliably levitating it. A schematic for this type of trap is shown in Figure 7. This design was used to trap charged $\sim 50\text{ }\mu\text{m}$ borosilicate microspheres with the aid of a DC bias voltage applied from the bottom set of pole pieces to the top set of pole pieces.

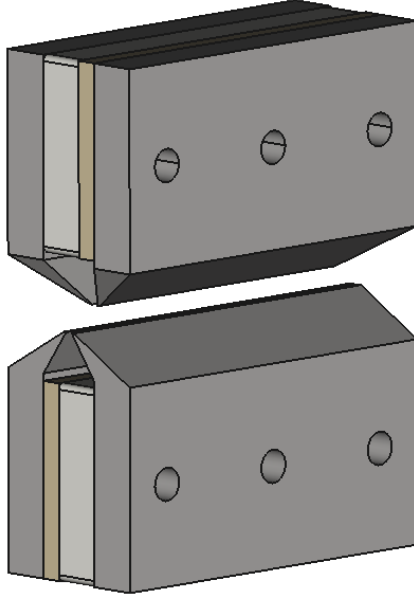


Figure 7: This figure was generated from `longtrap.FCStd`. The gap sizes shown here are about 0.5 mm for the transverse gap and about 1.5 mm for the vertical gap. These values were much smaller (0.25 mm transverse and 0.5 mm vertical) for the version used to levitate borosilicate spheres.

The main difference of this trap can be clearly seen from a front view of the trap as shown in Figure 8; the bottom pole pieces do not taper at all, and the top pole pieces only taper slightly. This only weakly breaks the y -symmetry with respect to z , forming a weak restoring force axially which corresponds to low oscillation frequencies in the corresponding normal mode.

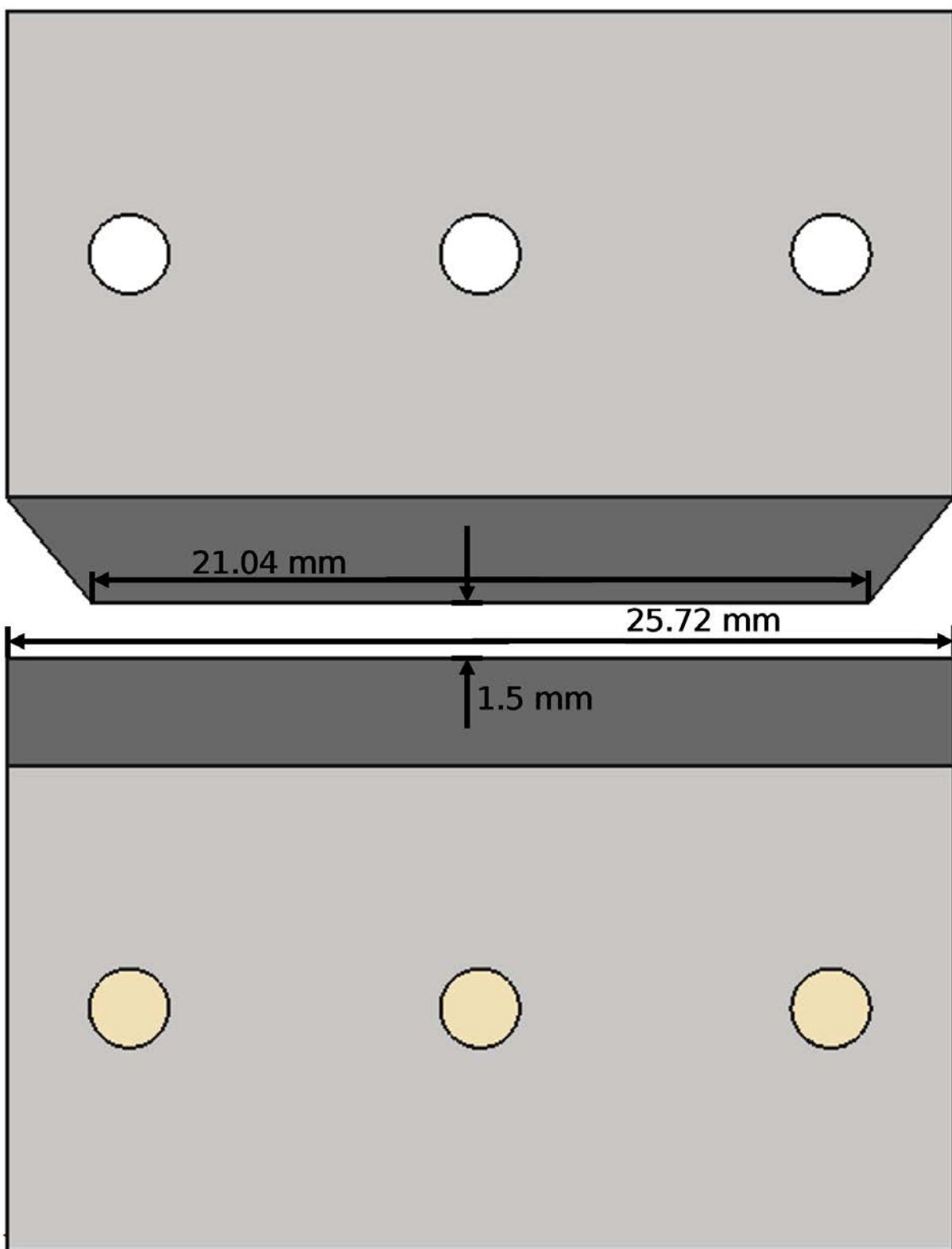


Figure 8: This figure was generated from `longtrap.FCStd`. We note that the difference in top and bottom axial trap lengths is only about 15% in this design, intended to produce a significantly lower axial oscillation frequency for trapped particles. Bottom pole piece length is 25.72 mm and top pole piece length is 21.04 mm.

The basic quadrupole field is still very similar, as can be inferred from the view shown in Figure 9. There is a somewhat greater asymmetry between the x - and y -gap dimensions than in the Short MGT , but for the most part the geometry favoring the formation of a linear quadrupole field can be clearly seen.

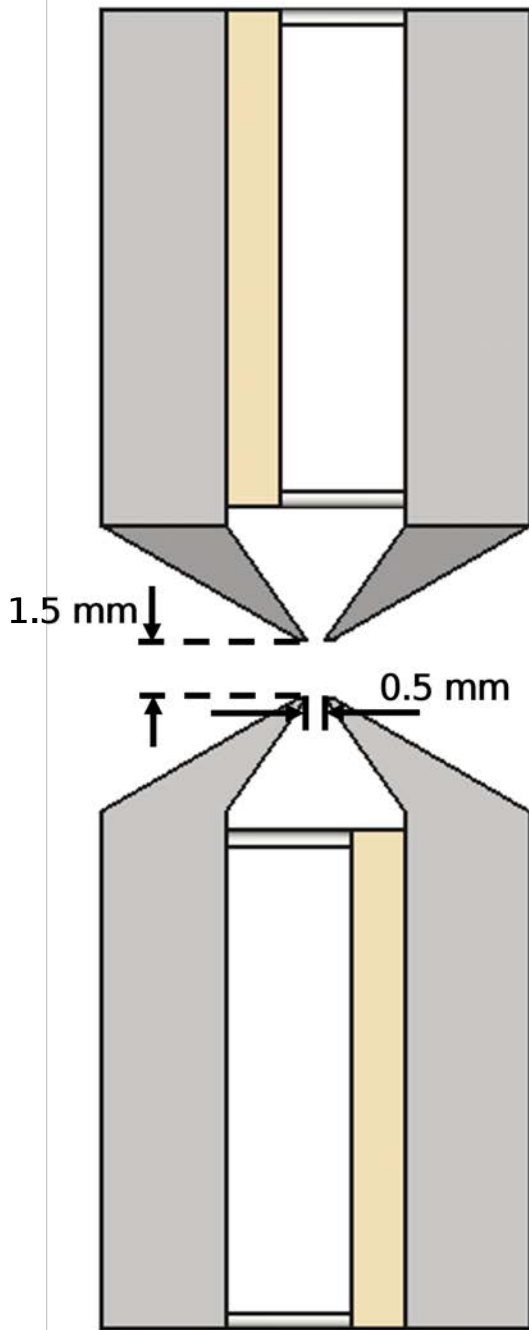


Figure 9: This figure was generated from `longtrap.FCStd`. A very similar image to that shown in Figure 5, but with heavier lapping on the tips and significantly larger (about a factor of 10) gap sizes – the gap sizes shown here are about 0.5 mm for the transverse gap and about 1.5 mm for the vertical gap.

This long-pole-piece trap design, while originally intended for use trapping borosilicate spheres (or perhaps even SiC particles eventually), ultimately was used to levitate particles made of a graphite-composite material. During the transition of this trap design to be better suited to that purpose, several discoveries were made.

First, experimentation with new materials for manufacturing pole pieces (discussed further in Section 2) revealed that annealing Hiperco 50A powder before mixing it with epoxy to form composite pole pieces significantly increased the trapping ability of the pole pieces to the point that pole pieces constructed with un-annealed Hiperco 50A powder did not successfully levitate particles. For this reason, the solid Hiperco 50A pole pieces shown in Figure 11 were annealed as described in Appendix A.

Second, it was found that some traps did not adequately confine levitated graphite composite particles (GCPs) on their own, and needed additional magnets to be attached to the ends of the top primary magnets. This arrangement is shown in Figure 10; the end magnets must be placed so that their orientation is the same as the bottom primary magnet and opposite the top primary magnet (this is their naturally preferred orientation), and in some cases need to be pushed down so that their lower edge is roughly level with the center of the trap.

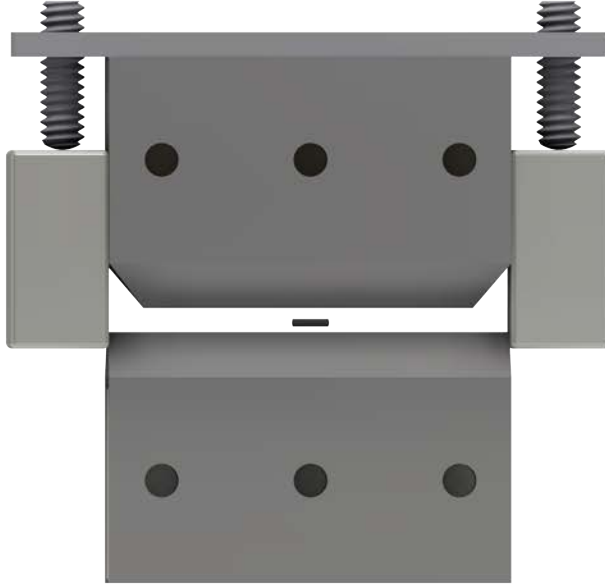


Figure 10: Front schematic of the Long MGT with end magnets added and a black rod representing a GCP shown for scale. The end magnets tend to slide up in order to sit level with the top primary magnet, so the set screws applying a downward force are sufficient to hold them in place. The beveling of the top pole pieces in this case is not strictly necessary, and may even weaken the quadrupole field in this case. However, these are the solid Hiperco 50A pole pieces used for the bulk of the tests discussed in this manuscript.

Of the pole pieces tested, Xflux pole pieces (an iron-silicon powder material used to reduce eddy current loss), annealed composite pole pieces, and annealed solid Hiperco 50A pole pieces all required end magnets to trap. The only set of pole pieces that was able to trap without end magnets, relying solely on the beveling of the top pole pieces, was the set of un-annealed solid Hiperco 50A pole pieces. This likely indicates a higher degree of magnetic saturation in these pole pieces, resulting in a less-pronounced quadrupole weakening effect. For the Xflux and composite pole pieces this lower saturation is likely due to the other materials mixed in with the magnetic material. For the annealed Hiperco 50A pole pieces, it is possible that the higher saturation magnetization results in a greater disparity between the strength of the quadrupole field at the edges of the trap versus the middle of the trap.

Third, it was found that if the transverse gap was too small ($\lesssim$ the particle diameter) it was difficult to stably trap a particle. This is one of the reasons for the large transverse and vertical gaps seen in the Long MGT .

Finally, to reduce the effects of patch charges on particle motion, thin copper sheets were used to wrap the pole pieces. The copper was cleaned with acetic acid (commercial vinegar) prior to use to remove its oxide layer.

The particle shown in Figure 10 represents a piece of 0.5mm (really 0.56 mm) Pentel HB pencil lead used in our MGTs. The photo of the setup in Figure 11 shows a particular particle used in several experiments, which will be called Edwin (HB05-2.4-lapped) . The pole pieces in this image are annealed solid Hiperco 50A .

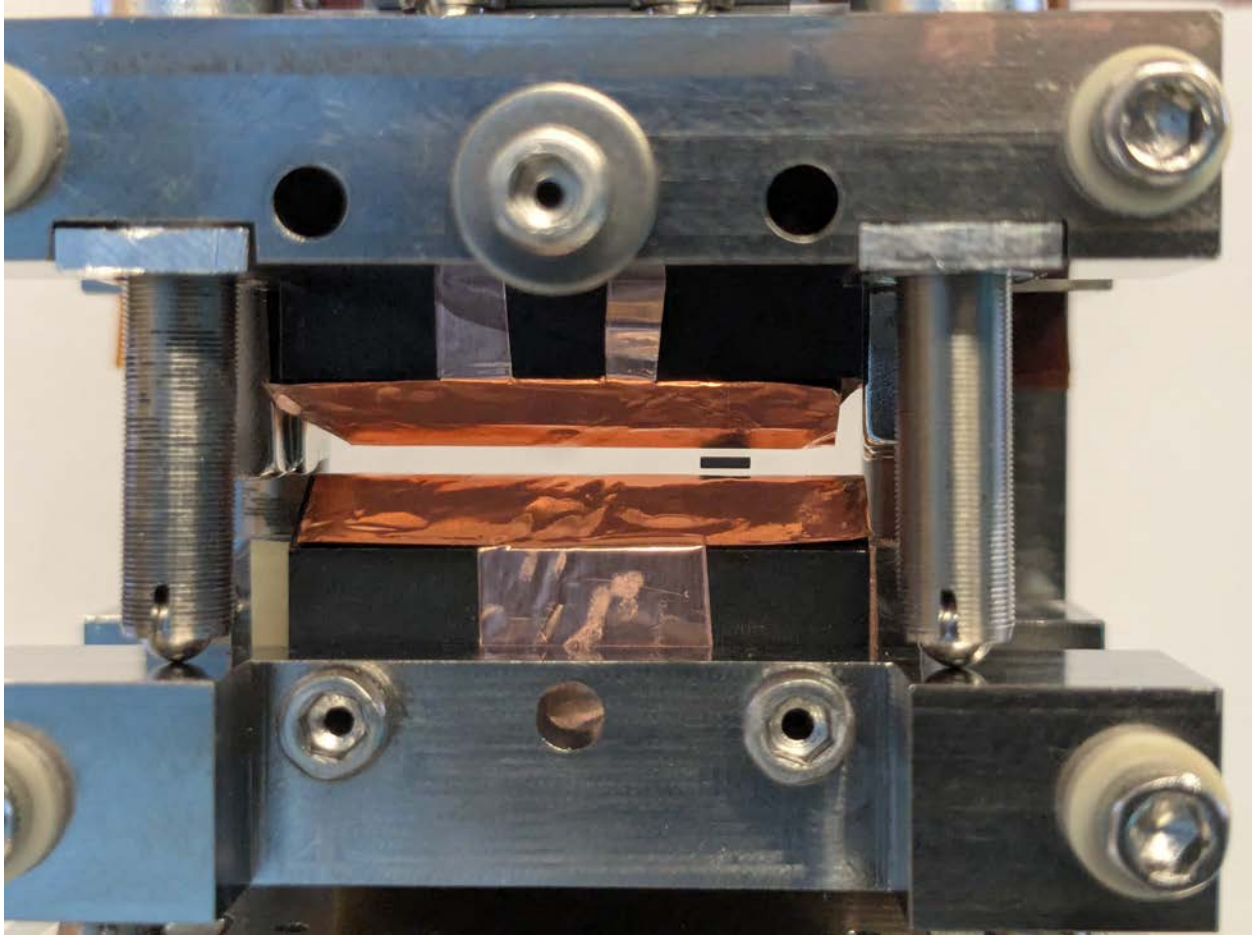


Figure 11: Photo of Edwin (HB05-2.4-lapped) , a 2.4 mm-long piece of pencil lead, sitting near the right side of the trapping region of the Long MGT . The top and bottom pole pieces are covered in copper foil to shield the particle from electric fields generated by patch charges on the pole pieces. The large screws running vertically on either side of the trap are used to adjust the tilt of the pole pieces about the transverse axis and the vertical gap of the trap. Three magnets (K&J N42SH) are stacked together and held in place on either side of the top magnets to provide an axial trapping force. The trap assembly is not well-leveled in this photo, resulting in the visible offset of the particle from the center of the trap.

A photo of the foil piece used to electrically shield the pole pieces is shown in Figure 12.

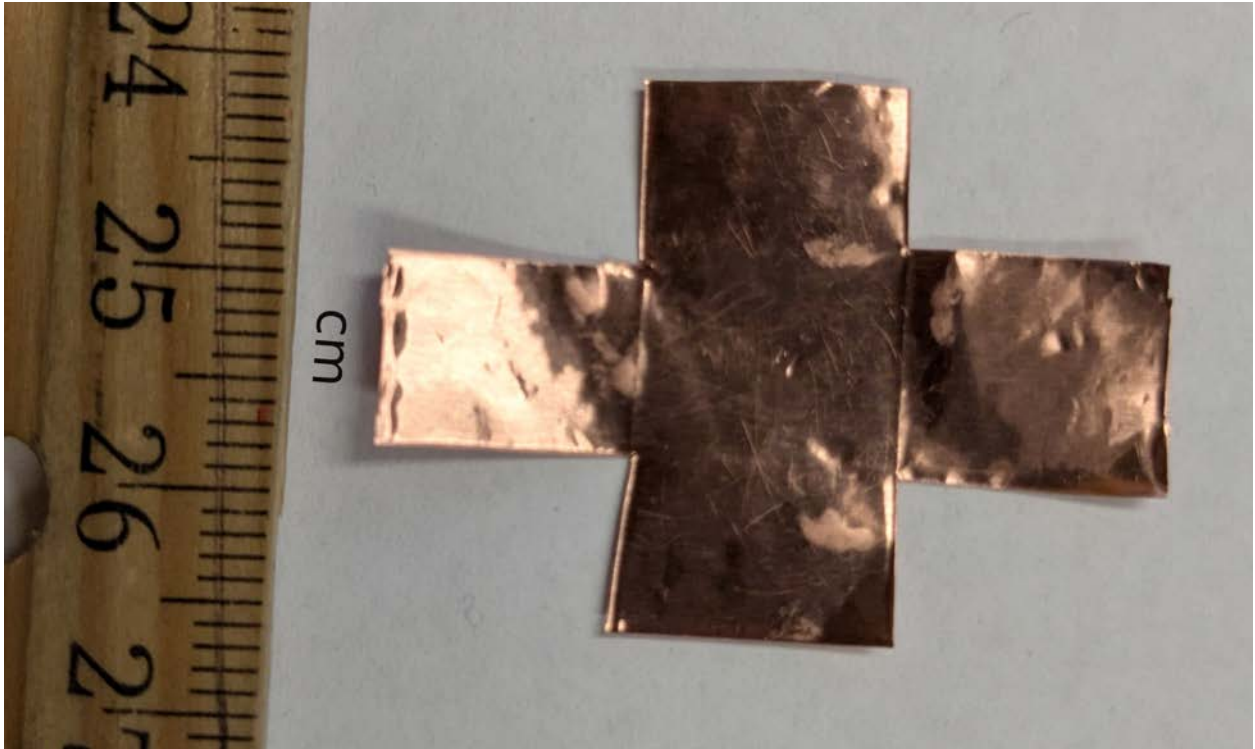


Figure 12: A piece of copper foil used to cover the pole pieces in the Long MGT . The tabs on right and left are shaped to fit between the screws attaching to the pole pieces, but can be cut in the middle to accomodate more screws as needed.

Vacuum Chamber

Two slightly different vacuum systems are used for the two systems discussed. In both systems, a rough pump is used to bring the pressure in the chamber down to about 10^{-2} Torr, then a turbomolecular pump is used to bring the vacuum down to about 10^{-7} Torr, and finally an ion pump is used in conjunction with a long bake of the chamber to bring the pressure down to about 10^{-9} or 10^{-8} Torr.

The rough pump Short MGT (Kurt J. Lesker AV-V6SATSFAMF-R Alcatel ACP28, S/N 611692 AJ) and the turbo pump (TMU 071 P) for the Short MGT were mounted on the same metal table using custom-cut mounting holes as shown in Figure 13. Aluminum foil was placed over the large windows for baking, which was accomplished using two variacs and some heating tape. In some cases a small hole was cut in the aluminum-foil window-cover

on the illumination side of the chamber to allow transverse imaging of the particle.



Figure 13: Left: Photo of the rough pump/turbo pump setup for the Short MGT vacuum chamber. Right: Photo of the top of the table containing the vacuum gauges and bellows connection to the chamber.

Similarly, photos of the vacuum pumps for the Long MGT can be seen in Figure 14. The rough pump (Edwards XDS35i) was on the bottom of a large rack, with the turbo pump (Leybold TURBOVAC 1000C) connected via bellows at the top of the rack. Bricks were set against the bellows connecting the turbo pump to the vacuum chamber to reduce vibration coupling into the chamber.

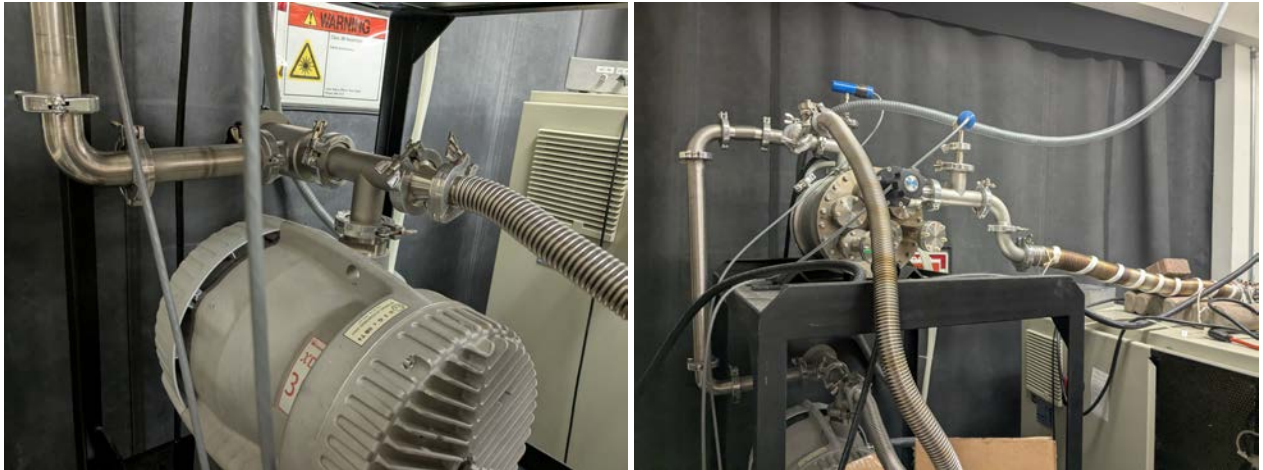


Figure 14: Left: Photo of the rough pump setup for the Long MGT vacuum chamber. Right: Photo of the turbo pump setup for the Long MGT vacuum chamber.

Figure 15 shows images of the two vacuum chambers as they were setup for the primary measurements described in this document.

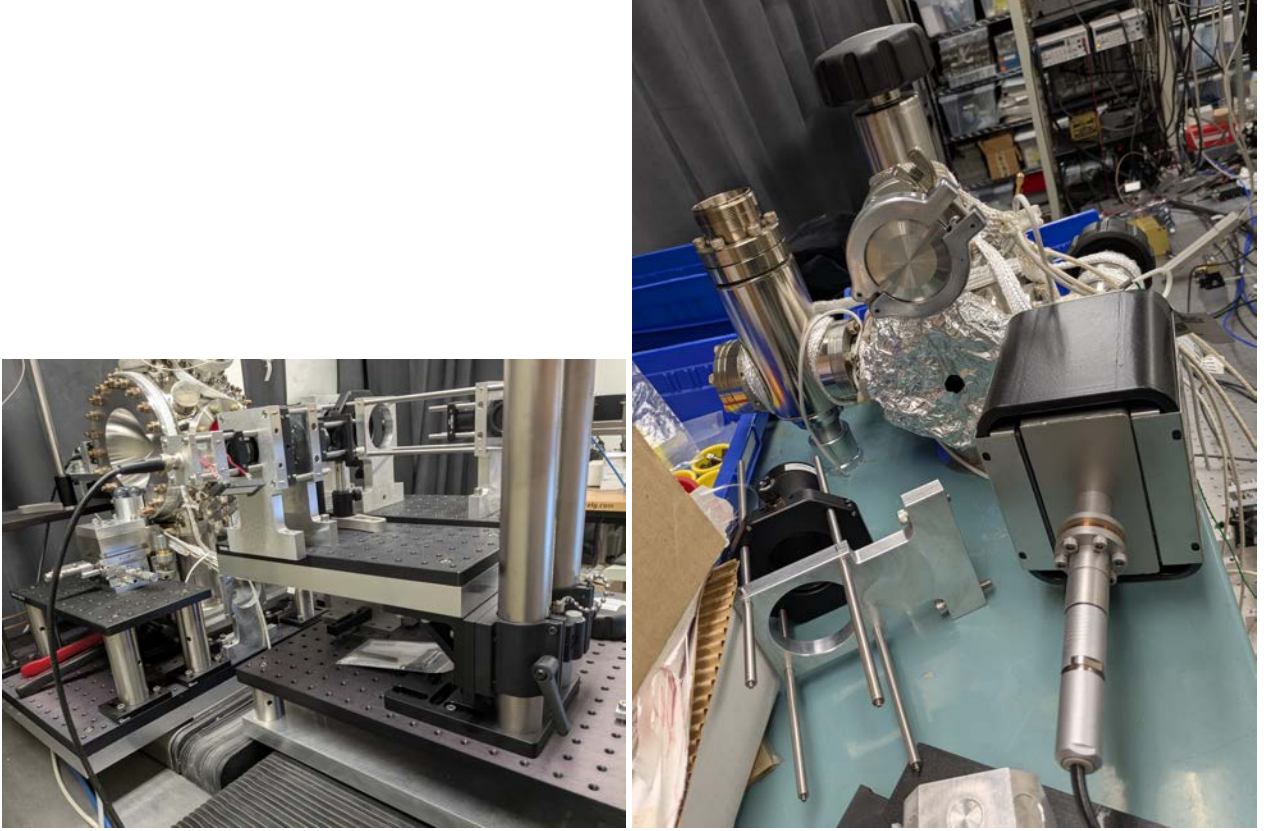


Figure 15: Left: Photo of the Long MGT chamber set up on the diamond-turning machine (DTM). Right: Photo of the Short MGT vacuum chamber while not in use.

The ion pump – Duniway 50 L/s Galaxy ion pump w/ Magnets (DGD-050-5125-M) – was connected to the controller (Duniway T-751A-1F110) via a VIP/Fischer connector (Duniway FS-110N-FR). An ion gauge (Duniway I-Nude-F UHV Ion Gauge Dual Iridium Filament, Type I) was used to measure the vacuum pressure some distance from the chamber when the turbo pump was running, and convectron gauges (Kurt J. Lesker KJL275185 convectron enhanced Pirani gauge and Granville-Phillips 275 Convectron Gauge) were used to measure roughing-pump vacuum and turbo-pump vacuum at higher pressures. A Varian-style titanium sublimation pump with three filaments was used to temporarily achieve even

lower pressures.

Pole Pieces

The perturbation of the trap magnetic field caused by its interaction with a diamagnetic particle generates a changing magnetic field experienced by the nearby pole pieces as the particle moves in the trap. This changing magnetic field is likely to induce eddy currents in the pole pieces, an effect that might damp the motion of the particle significantly if not addressed.

It was hypothesized that mixing Hiperco 50A powder with high-temperature epoxy (Epo-Tek 353ND) might produce usable pole pieces significantly less prone to hosting eddy currents. Some tests of this were done using a solid rod of Hiperco 50A and a rod of Hiperco-epoxy composite made from an annealed mix of Hiperco 50A powder and epoxy mixed at a 2:1 volume ratio and then cured at 80°C. These rods are shown in Figure 16, alongside the inductor coil used to test the losses in the rods.



Figure 16: Left: Inductor used to measure the loss of the metal rods – 260 turns, $A_{cc} = 1.1 \text{ cm}^2$ core cross-sectional area, length 8.6 cm. Middle: Hiperco-epoxy composite rod – broken pieces were fit back together using high-temperature epoxy (Epo-Tek 353ND) . A small current could still be passed through the length of the rod, indicating at least a partial electrical connection throughout its length. Right: A solid rod of Hiperco 50A prior to in-house annealing.

Sierra Holleman made use of a documented procedure [40] to measure the loss induced in an inductor under set flux density $B_{test} = 8.0 \times 10^{-4} \text{ T}$ with these two materials as cores of the inductor shown in Figure 16. Cylindrical neodymium magnets were added to the ends of the cores for some tests to see if saturating portions of the rods affected the loss in the cores.

The composite core had no discernable loss associated with it compared to the air-core

measurements, as seen in Figure 17.

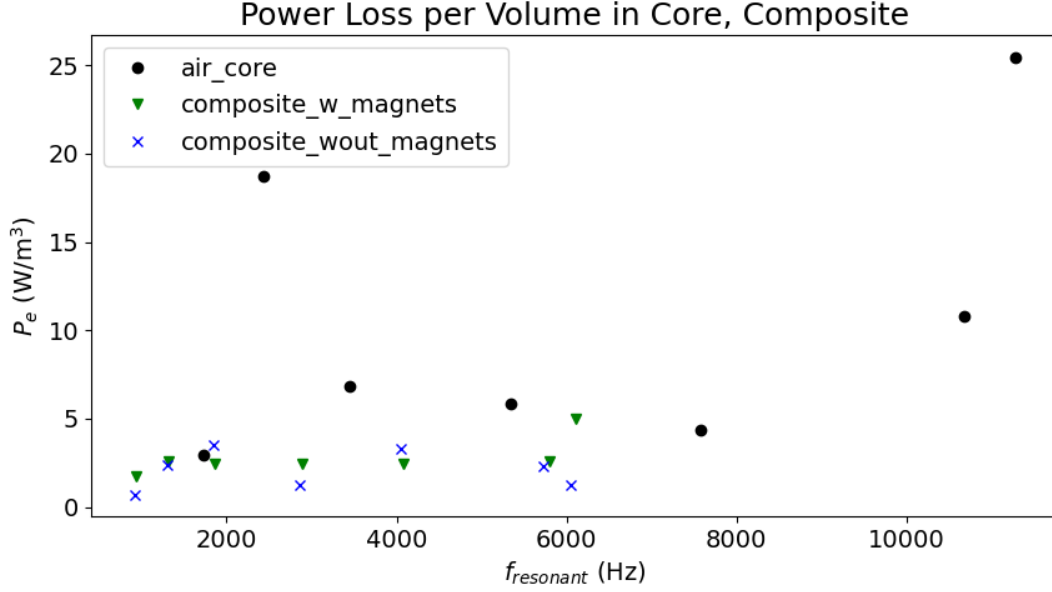


Figure 17: This figure was generated by `core_loss/plot_data.py` from the file `core_loss`. Plot of the power loss in the core of an inductor, where the magnetic field B in the core is held constant and the frequency is varied by changing the capacitors in the circuit, as described in. [40] Resonant frequency was varied by using different values of capacitance in the LC circuit. B set to 8.0×10^{-4} T.

The solid Hiperco 50A rod was tested before and after annealing, with measurable loss compared to the air core in both cases, as shown in Figures 18 and 19.

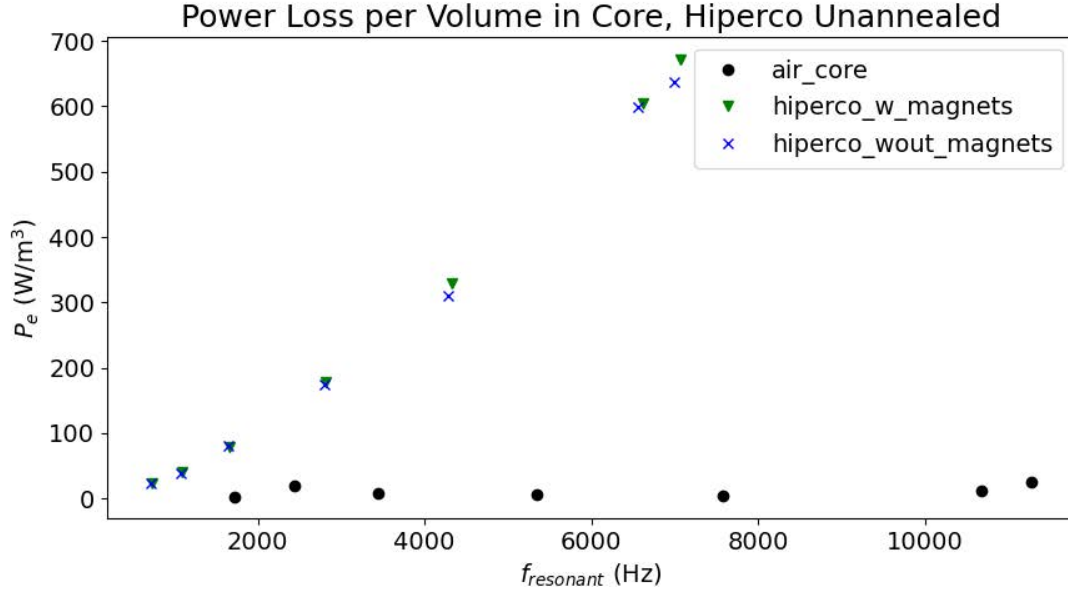


Figure 18: This figure was generated by `core_loss/plot_data.py` from the file `core_loss`. Plot of the power loss in the core of an inductor, where the magnetic field B in the core is held constant and the frequency is varied by changing the capacitors in the circuit, as described in. [40] Resonant frequency was varied by using different values of capacitance in the LC circuit. B_{test} set to $8.0 \times 10^{-4} \text{ T}$.

The presence of added magnets to the ends, meant to induce magnetic saturation in the Hipercó 50A rod, did not have a very noticeable effect in either case, perhaps due to insufficient field strength to penetrate deeply into the rods. Annealing the sample, by contrast, decreased the losses in the core by more than a factor of 2 (see Figure 19), a significant effect.

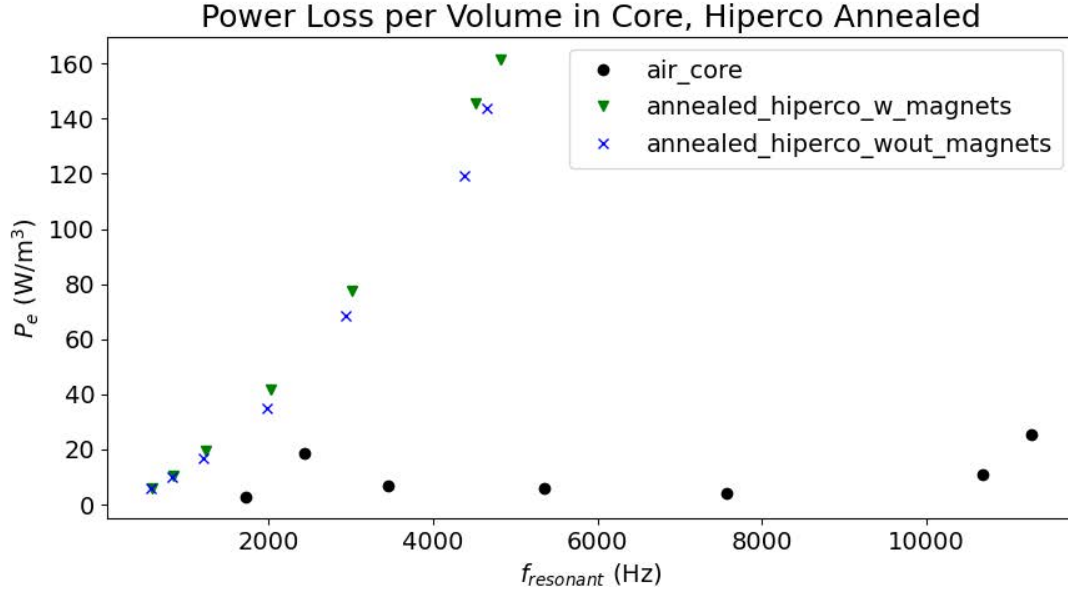


Figure 19: This figure was generated by `core_loss/plot_data.py` from the file `core_loss`. Plot of the power loss in the core of an inductor, where the magnetic field B in the core is held constant and the frequency is varied by changing the capacitors in the circuit, as described in. [40] Resonant frequency was varied by using different values of capacitance in the LC circuit. B_{test} set to 8.0×10^{-4} T.

There are two likely sources of loss in a material such as Hiperco 50A . The first is hysteresis loss, which can be estimated from [59]

$$P_h(f) = P_{ref} \left(\frac{f}{f_{ref}} \right)^\alpha \left(\frac{B_{test}}{B_{ref}} \right)^2 \rho_{Hiperco}, \quad (3)$$

where $P_{ref} = 1.67$ W/kg, $f_{ref} = 50$ Hz, $\alpha = 1.31$, $B_{ref} = 1.5$ T, and $\rho_{Hiperco} = 8.11 \times 10^3$ kg/m³. This yields P_h as power per volume.

The second loss mechanism to consider is eddy current losses, which can be estimated considering a constant conductivity for Hiperco 50A of $\sigma_{Hiperco} = 2.44 \times 10^6 \Omega^{-1}\text{m}^{-1}$ as follows.

First, we describe the volume of the rod by

$$V = \pi r_0^2 h, \quad (4)$$

where r_0 is the radius of the rod and h is its height. The skin depth of a material is given by

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}, \quad (5)$$

where μ is the permeability of the material, σ is its conductivity, and ω is the frequency of the magnetic field, which we will approximate by $\vec{B}(t) = B_{test} \cos \omega t \hat{z}$. For a completely saturated rod of pure Hiperco ($\mu \approx \mu_0$), $\delta \approx 10$ mm. For an unsaturated rod ($\mu \approx 10000\mu_0$), however, $\delta \approx 0.1$ mm. This means that for a rod with radius $r_0 = 5.9$ mm, the saturation of the rod has a significant effect on the power dissipation due to eddy currents.

Next, we can use the Kelvin-Stokes theorem to write

$$\oint_0^{2\pi} \vec{E} \cdot \hat{\phi} r \, d\phi = - \int_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}, \text{ so} \quad (6)$$

$$\delta \gtrsim r_0 \implies E_\phi(r) \approx \frac{1}{2} r \omega B_{test} \sin \omega t \text{ and} \quad (7)$$

$$\delta \ll r_0 \implies E_\phi(r \approx r_0) \approx \delta \omega B_{test} \sin \omega t. \quad (8)$$

We can then use $J_\phi = \sigma E_\phi$ and Eqs. (4), (5), (7), and (8) to write

$$\begin{aligned} P_{\delta \gtrsim r_0}(f) &\approx \left\langle \frac{1}{V} \int_V E_\phi J_\phi \, dV \right\rangle_t \\ &= \frac{1}{4r_0^2} \sigma B_{test}^2 \omega^2 \int_0^{r_0} r^3 \, dr \\ &= \frac{1}{16} \sigma \omega^2 B_{test}^2 r_0^2 \end{aligned} \quad (9)$$

and

$$\begin{aligned}
 P_{\delta \ll r_0}(f) &\approx \left\langle \frac{1}{V} \int_V E_\phi J_\phi \, dV \right\rangle_t \\
 &= \frac{2\omega}{\mu r_0^2} B_{test}^2 \int_{r_0-\delta}^{r_0} r \, dr \\
 &= \sqrt{\frac{8\omega}{\sigma \mu^3}} \frac{B_{test}^2}{r_0}.
 \end{aligned} \tag{10}$$

These two approximations give very different estimates for core loss, as can be seen in Figure 20. It is also clear that neither approach adequately matches either the scale or the frequency-dependence of the data, indicating the need for a better model to fully explain the results.

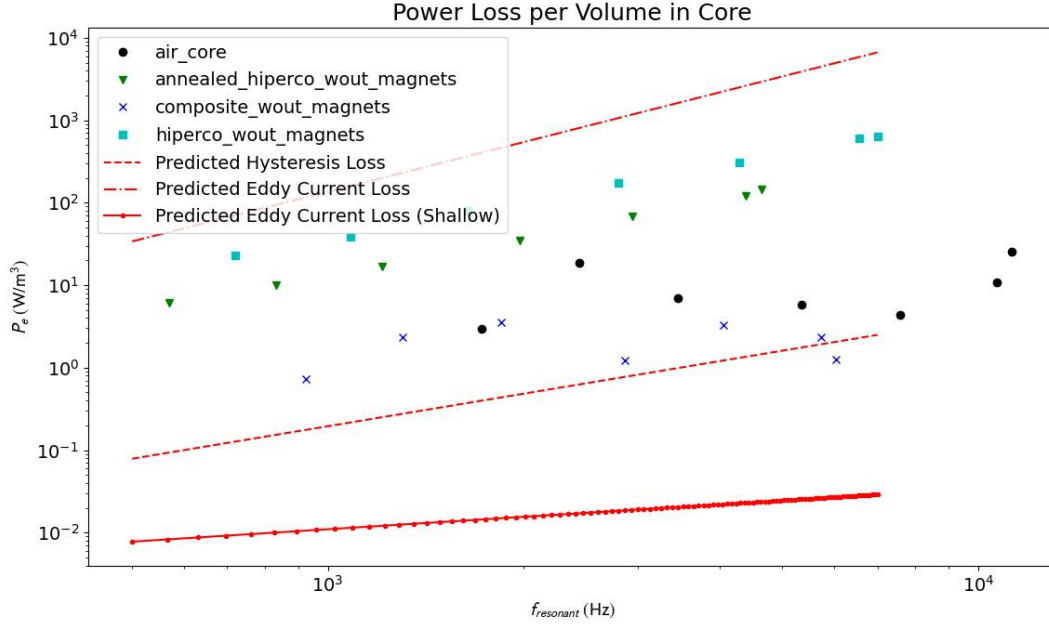


Figure 20: This figure was generated by `core_loss/plot_data.py` from the file `core_loss`. Plot of the power loss in the core of an inductor, where the magnetic field B in the core is held constant and the frequency is varied by changing the capacitors in the circuit, as described in. [40] Resonant frequency was varied by using different values of capacitance in the LC circuit. B_{test} set to 8.0×10^{-4} T.

One reason it is a difficult system to model is that both the magnetic permeability and the conductivity of Hiperco 50A vary with field strength, making an accurate estimate of the skin depth of an applied alternating B -field extremely difficult. With all this said, it is important to take the prediction of Eq. (9) as only a rudimentary approximation to indicate the power dependence.

What can be said with little ambiguity is that annealing the Hiperco 50A decreased its eddy current losses, the composite material had significantly lower losses than the solid Hiperco 50A, and the actual loss in a piece of Hiperco 50A likely depends heavily on the degree of its saturation in a magnetic field.

This Hiperco-epoxy composite material was used to make composite pole pieces using the method described in Appendix A. The resulting pole pieces were found to be quite effective at trapping (although the annealed solid Hiperco 50A pole pieces still trapped more strongly, as indicated by higher equilibrium positions and more robust containment), and considerably easier to fabricate than the solid Hiperco 50A pole pieces due to the ability to cast the pole pieces directly rather than needing to use conventional machining to shape the pieces.

The ratio of powder used was based on an estimate of random packing density for variably-sized spheres by Yamada et al. [162] This paper informed both the large-grain to small-grain ratio used and the ratio of powder to epoxy used.

Tahereh Naderishahab was primarily responsible for the development and testing of these pole pieces; the procedure for fabricating them is included here primarily for completeness.

Magnetic Field and Potential

It is useful to be able to estimate the behavior of the magnetic field in the trap. One way to do this is to express the magnetic field using a multipole expansion, including only

terms that match the symmetry of the geometrical arrangement. In this way a theoretical model can be fit to actual data, so long as the number of spherical harmonics used does not exceed the number of experimentally observable parameters.

This is harder than it may sound; many quantities are difficult to directly observe, either because certain simplifying approximations in the math disallow a precise measurement, or because the quantities appear as differences between much larger terms. Nevertheless, what follows presents the B -field as a linear combination of four spherical harmonics which can be linked to directly observable quantities.

The magnetic scalar potential, Φ_M , can be expressed in terms of spherical harmonics as follows:

$$\begin{aligned} \Phi_M = & A_2^2 i \sqrt{\frac{1}{2}} (Y_2^{-2}(\theta, \phi) - Y_2^2(\theta, \phi)) r^2 + A_3^1 \sqrt{\frac{1}{2}} (Y_3^{-1}(\theta, \phi) - Y_3^1(\theta, \phi)) r^3 \\ & + A_4^4 i \sqrt{\frac{1}{2}} (Y_4^{-4}(\theta, \phi) - Y_4^4(\theta, \phi)) r^4 + A_4^2 i \sqrt{\frac{1}{2}} (Y_4^{-2}(\theta, \phi) - Y_4^2(\theta, \phi)) r^4, \end{aligned} \quad (11)$$

where each term can be associated with a different expression of symmetry in the trap.

The A_2^2 -term describes a linear quadrupole field, the dominant trapping field in the trap that is responsible for the high transverse and vertical trapping frequencies. It is critical that this term be large enough to overcome the gravitational force on a particle in order for levitation to be possible. The A_3^1 -term describes a breaking of symmetry of the quadrupole field along the axial direction. This term contributes a stable trapping force in the z -direction. The A_4^2 -term, by contrast, describes the weakening of the linear quadrupole field along the axial direction, and generally opposes the contribution from the A_3^1 -term, contributing an unstable trapping force in the z -direction. Finally, the A_4^4 -term contributes an asymmetry to the x - and y -quadrupole fields, accounting for the difference in x - and y -frequencies observed in the trap.

It is worth noting here that a similar analysis of the magnetic field of a MGT has

already been done. [60] In that analysis, a scaling factor, y_0 , was used to define constants a_2 , a_3 , and a_4 such that $a_2 = \mu_0 y_0 A_2^2$, $a_3 = \mu_0 y_0^2 A_3^1$, and $a_4 = \mu_0 y_0^3 A_4^4$ (no term comparable to A_4^2 was included in that analysis).

The magnetic potential can be expressed in Cartesian coordinates as

$$\begin{aligned}\Phi_M = & \frac{1}{4} \sqrt{\frac{15}{\pi}} A_2^2 xy \\ & + \frac{1}{4} \sqrt{\frac{7}{6\pi}} A_3^1 x (4z^2 - x^2 - y^2) \\ & + \sqrt{\frac{315}{4\pi}} A_4^4 xy (x^2 - y^2) \\ & + \sqrt{\frac{45}{4\pi}} A_4^2 xy (6z^2 - x^2 - y^2),\end{aligned}\tag{12}$$

We can then compute the magnetic flux density, $\vec{B}$, from $\vec{B} = -\mu_0 \vec{\nabla} \Phi_M$ as follows:

$$\vec{B} = \begin{bmatrix} b_0 y + \frac{1}{8} \alpha_z (4z^2 - y^2 - 3x^2) + c_4 y^3 + \frac{1}{2} (d_4 - 6c_4) x^2 y - \frac{1}{2} d_4 y z^2 \\ b_0 x - \frac{1}{4} \alpha_z xy + 3c_4 xy^2 + \frac{1}{2} (d_4 - 6c_4) x^3 - \frac{1}{2} d_4 x z^2 \\ \alpha_z x z - d_4 x y z \end{bmatrix},\tag{13}$$

where $b_0 = -2\mu_0 \frac{1}{4} \sqrt{\frac{15}{\pi}} A_2^2$, $\alpha_z = -8\mu_0 \frac{1}{4} \sqrt{\frac{7}{6\pi}} A_3^1$, $c_4 = \mu_0 \left(\sqrt{\frac{315}{4\pi}} A_4^4 + \sqrt{\frac{45}{4\pi}} A_4^2 \right)$, and $d_4 = 12\mu_0 \sqrt{\frac{45}{4\pi}} A_4^2$. These new terms, b_0 , α_z , c_4 , and d_4 , can be used to describe the field independently of the specifics of the trap (such as magnetic susceptibility of the particle and gravity).

The field gradients are then:

$$\frac{\partial \vec{B}}{\partial x} = \begin{bmatrix} -\frac{3}{4} \alpha_z x + (d_4 - 6c_4) xy \\ b_0 - \frac{1}{4} \alpha_z y + 3c_4 y^2 + \frac{3}{2} (d_4 - 6c_4) x^2 - \frac{1}{2} d_4 z^2 \\ \alpha_z z - d_4 y z \end{bmatrix},\tag{14}$$

$$\frac{\partial \vec{B}}{\partial y} = \begin{bmatrix} b_0 - \frac{1}{4}\alpha_z y + 3c_4 y^2 + \frac{1}{2}(d_4 - 6c_4)x^2 - \frac{1}{2}d_4 z^2 \\ -\frac{1}{4}\alpha_z x + 6c_4 xy \\ -d_4 xz \end{bmatrix}, \quad (15)$$

and

$$\frac{\partial \vec{B}}{\partial z} = \begin{bmatrix} \alpha_z z - d_4 yz \\ -d_4 xz \\ \alpha_z x - d_4 xy \end{bmatrix}. \quad (16)$$

The constants can now be used to describe different aspects of the trap. The value of b_0 functions as a scaling factor to describe the dominant field gradient along the trap x - and y -axes. The value of α_z determines the strength of the dominant field gradient in the z -direction, which we note is not constant. The value of c_4 , which is determined by a combination of both the strength of A_4^4 and A_4^2 , determines the degree of asymmetry between x - and y -motion. And finally, the value of the d_4 term determines the degree of anharmonicity in the trap caused by high-amplitude axial oscillations.

Before continuing, it is important to have an idea of the relative sizes of various parameters in these equations, so that small terms can be dropped in order to simplify the analysis. In the MGTs discussed here, the x and y -motion of a particle is typically much smaller than the offset to its y -equilibrium position (y_{eq}). We therefore take $x \ll y_{eq}$ and $y \rightarrow y_{eq} + y$ so that $y \ll y_{eq}$. It is useful also to note that this implies $y^2 \rightarrow y_{eq}^2 + 2y_{eq}y$ and $y^3 \rightarrow y_{eq}^3 + 3y_{eq}^2y$.

The z -motion can be small if cooled actively as in the Short MGT, but especially in the case of the Long MGT cannot generally be considered small compared to other length scales. Even so, the motion is assumed to be harmonic with only small perturbations contributing anharmonicity, so terms of order z^3 and higher (z^4 and higher for z -motion) are neglected in this analysis.

Terms of the form $\frac{\alpha_z y_{eq}}{b_0}$ and $\frac{d_4 y_{eq}^2}{b_0}$ are related to the symmetry-breaking in the axial direction (responsible for determining the axial frequency), and can generally be considered small since it is empirically observed that $\omega_z^2/\omega_{x,y}^2 \ll 1$ in both trapping setups (ω_i is the natural trap oscillation frequency along the x_i -axis). Terms on the order of $\frac{c_4 y_{eq}^2}{b_0}$ are almost small enough to be considered much less than 1, but not quite. Therefore we consider terms of this order to be much less than 1 if and only if they are multiplied by another term that is $\lesssim 1$.

With these approximations in hand, we can now express the equations of motion in three dimensions for diamagnetic particles levitated in such a magnetic field by combining Eq. (2) with Eq. (13) and Eqs. (14), (15), and (16) to get

$$\ddot{x} = -c_0 b_0^2 \left[\left(1 - \frac{5}{4} \frac{\alpha_z y_{eq}}{b_0} + \frac{d_4 y_{eq}^2}{b_0} + 3 \frac{c_4^2 y_{eq}^4}{b_0^2} \right) - \left(\frac{d_4}{b_0} \right) z^2 \right] x, \quad (17)$$

$$\begin{aligned} \ddot{y} = & -c_0 b_0^2 \left[\left(1 - \frac{3}{4} \frac{\alpha_z y_{eq}}{b_0} + 12 \frac{c_4 y_{eq}^2}{b_0} + 15 \frac{c_4^2 y_{eq}^4}{b_0^2} \right) - \left(\frac{d_4}{b_0} \right) z^2 \right] y \\ & - g - c_0 b_0^2 y_{eq} \left[\left(1 - \frac{3}{8} \frac{\alpha_z y_{eq}}{b_0} + 4 \frac{c_4 y_{eq}^2}{b_0} + 3 \frac{c_4^2 y_{eq}^4}{b_0^2} \right) + \left(-\frac{d_4}{b_0} + \frac{1}{2} \frac{\alpha_z}{y_{eq} b_0} \right) z^2 \right], \end{aligned} \quad (18)$$

and

$$\begin{aligned} \ddot{z} = & -c_0 b_0^2 \left[\left(\frac{\alpha_z y_{eq}}{b_0} - \frac{d_4 y_{eq}^2}{b_0} \right) \left(1 + \left(\frac{c_4 y_{eq}^2}{b_0} - \frac{\alpha_z y_{eq}}{8 b_0} \right) \right) z + \left(\frac{1}{2 y_{eq}^2} \left(\frac{\alpha_z y_{eq}}{b_0} - \frac{d_4 y_{eq}^2}{b_0} \right)^2 \right) z^3 \right] \\ \approx & -c_0 b_0^2 \left[\left(\frac{\alpha_z y_{eq}}{b_0} - \frac{d_4 y_{eq}^2}{b_0} \right) z + \left(\frac{1}{2 y_{eq}^2} \left(\frac{\alpha_z y_{eq}}{b_0} - \frac{d_4 y_{eq}^2}{b_0} \right)^2 \right) z^3 \right], \end{aligned} \quad (19)$$

where $c_0 = -\frac{\chi}{\rho \mu_0}$.

In each equation for $\ddot{x}_i$ above, the terms proportional to x_i describe the harmonic motion of the particle in that direction. The second line in Eq. 18 describes the static force on a

levitated particle in the vertical direction, and thus must be equal to 0 at equilibrium.

It is convenient now to define several dimensionless parameters: $\alpha = \frac{\alpha_z y_{eq}}{b_0}$, $\beta = \frac{d_4 y_{eq}^2}{b_0}$, $\gamma = \frac{c_4 y_{eq}^2}{b_0}$. Note that these are primarily trap parameters rather than directly magnetic field parameters. We can then use the approximations $(1 + \alpha) \approx 1$, $(1 + \beta) \approx 1$, $(1 + \gamma^2) \approx 1$ to write four descriptive equations for the system:

$$\omega_x^2(z) \approx c_0 b_0^2 \left(1 - \beta \frac{z^2}{y_{eq}^2} \right) \quad (20)$$

$$\omega_y^2(z) \approx c_0 b_0^2 \left[(1 + 12\gamma) - \beta \frac{z^2}{y_{eq}^2} \right] \quad (21)$$

$$\omega_z^2(z) \approx c_0 b_0^2 \left[(\alpha - \beta)(1 + \gamma) + \frac{1}{2}(\alpha - \beta)^2 \frac{z^2}{y_{eq}^2} \right] \quad (22)$$

$$\frac{g}{y_{eq}(z)} = -c_0 b_0^2 \left[(1 + 4\gamma) - \left(\beta - \frac{1}{2}\alpha \right) \frac{z^2}{y_{eq}^2} \right], \quad (23)$$

where we again treat the motion as primarily harmonic with anharmonic perturbations.

While in theory the value of c_0 can be calculated independently of the trap, in practice it is convenient to include it in these fits since the value of χ is an experimentally difficult value to measure precisely. By fitting $c_0 b_0^2$ instead of calculating c_0 directly and fitting b_0 , we are in effect using empirically measured values to fit the actual system under test, regardless of whether we know the precise density and magnetic susceptibility of the test mass.

B-Field: Short MGT At this point we are ready to discuss the Short MGT in greater detail, since for most of its applications the z -motion is relatively small. In such a case, the

anharmonicity disappears and we are left with

$$\omega_x^2(z) \approx c_0 b_0^2 \quad (24)$$

$$\omega_y^2(z) \approx c_0 b_0^2 (1 + 12\gamma) \quad (25)$$

$$\omega_z^2(z) \approx c_0 b_0^2 (\alpha - \beta) (1 + \gamma) \quad (26)$$

$$\frac{g - E_y \frac{q}{m}}{y_{eq}} \approx -c_0 b_0^2 (1 + 4\gamma), \quad (27)$$

where the term $E_y \frac{q}{m}$ was added to account for the fact that the particle's height in the trap was typically altered by applying a voltage across the top and bottom pole pieces.

We can use typical empirically measured values of $\omega_x/2\pi = 79$ Hz, $\omega_y/2\pi = 84$ Hz, $\omega_z/2\pi = 7.8$ Hz (measured from Cinderella (HY0667-21B-2A) , **collection-C-190**), and $y_{eq} = -4.7$ μm (measured from Cinderella (HY0667-21B-2A) , Figure 132) to estimate these parameters. For the interested reader, the value E_y can be estimated by

$$E_y \approx \frac{V_{tb}}{y_{gap}} = \frac{19.0 \text{ V}}{170 \mu\text{m}} = 112 \text{ kV/m}. \quad (28)$$

The estimates for the trap parameters are then

$$c_0 b_0^2 \approx \omega_x^2 = 2.5 \times 10^5 \text{ s}^{-2} \quad (29)$$

$$\gamma \approx \frac{1}{12} \left(\frac{\omega_y^2}{\omega_x^2} - 1 \right) = 0.011 \quad (30)$$

$$(\alpha - \beta) \approx \frac{\omega_z^2}{\omega_x^2 (1 + \gamma)} = 9.6 \times 10^{-3} \quad (31)$$

$$\frac{q}{m} \approx \frac{c_0 b_0^2 y_{eq}}{g E_y} (1 + 4\gamma) = 0.23 \text{ C/kg} \quad (32)$$

(the last value listed, q/m , is included more for completeness as it does not play a major

role in any other calculations).

In the case of the Short MGT , the usual cooling of the axial motion [140] means that the terms of higher order in z can be neglected. In this case, the resulting multipole expansion of the field is essentially identical to that described by Hsu et al. [60] In fact, it is likely that the value of β here is small compared to α , since the geometry of the Short MGT is specifically designed to saturate the Hiperco 50A pole pieces, reducing any weakening of the linear quadrupole field in the z -direction.

There is one more point worth noting here. At one point, as shown in Figure 21, many $8\mu\text{m}$ silica microspheres were loaded at once using an ultrasonic horn. Although it is not easy to see from the figure, by panning the objective back and forth it was clear that the vertical equilibrium positions of the particles increased towards the edges of the trap. From Eq. (27), this tells us that $\alpha > 2\beta$ for this trap, allowing a somewhat quantitative statement about the magnitude of α vs. β .

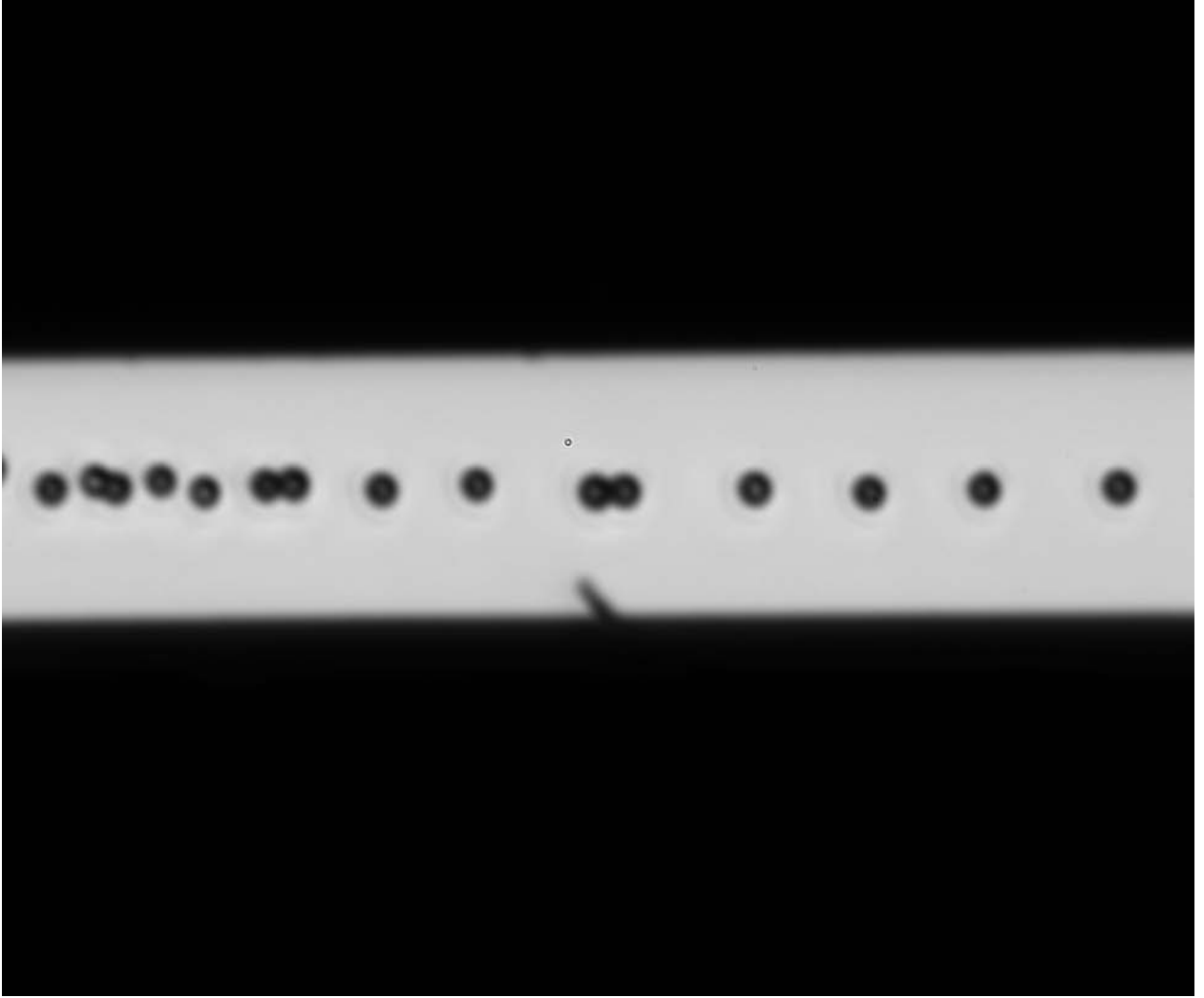


Figure 21: Several levitated $8\,\mu\text{m}$ borosilicate spheres, separated due to charge build-up of the same sign on each particle. The top and bottom borders shown are the outline of the slit used for illumination, but previous observation of the particles' positions relative to the pole pieces revealed an upward curving of the y -equilibria of the particles.

B-Field: Long MGT There are two significant differences when we consider the Long MGT compared to the Short MGT . First, the axial motion can be fairly large (within an order of magnitude of the axial length scale of the trap), so the anharmonic perturbations of order z^2 need to be considered. Second, as will be discussed further in Section 2, the particles being levitated are large cylinders of graphite-composite – so large that it is no longer clear precisely what the parameter y_{eq} represents.

For a small diamagnetic particle levitated in a magnetic field gradient, the center of mass (COM) of the particle aligns with the center of magnetic force (COF) on the particle. This is not generally true if the field strength can change significantly over the length of the particle; in this case the lower parts of the particle rest in a region of significantly higher field strength (particularly the x -component of the B -field) than the upper parts.

Consider a cylindrical particle of radius r_0 and non-negligible size resting at position y_{COM} in a MGT. For simplicity, assume the vertical force on a slice of the particle with thickness dy is harmonic so that the force can be expressed as $dF = -kL x(y) y dy$, where k is the harmonic spring constant per volume, L is the length of the cylinder, and $x(y)$ is the transverse extent of the particle at vertical location y in the trap. Note that all this is what we should expect for a vertical force on a diamagnetic particle in a linear field gradient.

We now consider the center of force of such a particle, the point where the average magnetic restoring force on the particle is centered:

$$\begin{aligned}
 y_{COF} &= \frac{\int y dF}{\int dF} = \frac{\int_{y_{COM}-r}^{y_{COM}+r} y^2 \sqrt{r^2 - (y - y_{COM})^2} dy}{\int_{y_{COM}-r}^{y_{COM}+r} y \sqrt{r^2 - (y - y_{COM})^2} dy} \\
 &= \frac{\int_{-r}^r (u + y_{COM})^2 \sqrt{r^2 - u^2} du}{\int_{-r}^r (u + y_{COM}) \sqrt{r^2 - u^2} du} \\
 &= y_{COM} \left(1 + \frac{1}{4} \frac{r^2}{y_{COM}^2} \right),
 \end{aligned} \tag{33}$$

where the integrals were taken from Schaum's 17.11.9 and 17.11.10. [145] The result for the values $y_{COM} = -0.10$ mm and $r = 0.28$ mm (measured from Figure 155) is $y_{COF} = -0.30$ mm. This number is drastically different from the position of the center of mass.

With this result in mind, we will treat y_{eq} as a quantity that needs to be determined, not as a measurable value. Data taken of Edwin (HB05-2.4-lapped) in the Long MGT, found in 0_amplitude_blue/displacements_offline_04.zia, was used to extract the frequencies of the particle's oscillation for low-amplitude motion $z_{rms} \approx 6$ μ m. These values

are $\omega_x/2\pi = 29.19$ Hz, $\omega_y/2\pi = 40.61$ Hz, and $\omega_z/2\pi = 0.2493$ Hz.

We can use these empirically determined frequencies to estimate three of the fit parameters – $c_0 b_0^2$, γ , and $(\alpha - \beta)$ – and y_{eq} :

$$c_0 b_0^2 \approx \omega_x^2 = 3.4 \times 10^4 \text{ s}^{-2} \quad (34)$$

$$\gamma \approx \frac{1}{12} \left(\frac{\omega_y^2}{\omega_x^2} - 1 \right) = 0.0780 \quad (35)$$

$$(\alpha - \beta) \approx \frac{\omega_z^2}{\omega_x^2(1 + \gamma)} = 6.8 \times 10^{-5} \quad (36)$$

$$y_{eq} \approx -\frac{g}{\omega_x^2(1 + 4\gamma)} = -0.222 \text{ mm} \quad (37)$$

It is interesting to note that this value for y_{eq} falls between what we would expect for the center-of-mass and center-of-force estimates. This discrepancy will be discussed in greater detail in Section 2.

Up to this point in our analysis we have only been able to determine the value of $\alpha - \beta$, not α or β independently. Looking at Eq. (22) it is clear that even if we measure the anharmonicity in the axial motion, which is the primary behavior impacted by the value of β that we are interested in, this is also only sensitive to the term $\alpha - \beta$. As we have seen, Eq. (23) is not very useful since the value y_{eq} cannot be easily measured empirically.

Another solution is presented by the forms of Eqs. (20) and (21); if we can measure the frequency of the transverse and vertical oscillations of the particle at large amplitudes, it should be possible to extract the value of β from the difference between the high- and low-amplitude values of ω_x^2 and ω_y^2 .

This information can be extracted from particle motion with high z -amplitude by the following method:

1. From axial motion data with a large amplitude, identify all times at which $z \geq 0.95z_0$,

where z_0 is the maximum amplitude of the axial motion.

2. For each set of vertical motion data collected during each of these “turn-around” time intervals, take an FFT of the data.
3. Use the analysis technique described in Section 1 to extract the x - and y -frequencies.

This technique is employed in the file `coefficient_extraction.py` to obtain the values $\omega_x(z = 1.475 \text{ mm})/2\pi = 28.76 \text{ Hz}$ and $\omega_y(z = 1.475 \text{ mm})/2\pi = 40.00 \text{ Hz}$ by averaging together results from the three files:

- `textttz_ringdown_blue/displacements_offline_01.zia`
- `z_ringdown_blue/displacements_offline_02.zia`
- `z_ringdown_blue/displacements_offline_03.zia`.

The value listed of $z = 1.475 \text{ mm}$ is the average value of z for all the “turn-around” time intervals, which we will call $z_{0,avg}$.

These can each be used independently with the values measured for low-amplitude motion to estimate the value of β from

$$\beta \approx y_{eq}^2 \frac{\omega_x^2(0) - \omega_x^2(z_{0,avg})}{z_{0,avg}^2 \omega_x^2(0)} \approx y_{eq}^2 \frac{\omega_y^2(0) - \omega_y^2(z_{0,avg})}{z_{0,avg}^2 \omega_x^2(0)} \stackrel{avg}{=} 6.7 \times 10^{-4} \pm 3 \times 10^{-4}. \quad (38)$$

It is then trivial to find $\alpha = 7.4 \times 10^{-4}$ from Eq. (36), completing the multipole fitting to the Long MGT .

We can express the potential energy per mass of a particle levitated in such a system as

$$U/m = \frac{1}{2} (c_0 b_0^2) \left(\frac{B}{b_0} \right)^2 - gy, \quad (39)$$

where

$$\frac{\vec{B}}{b_0} = \begin{bmatrix} y + \frac{1}{8} \frac{\alpha}{y_{eq}} (4z^2 - y^2 - 3x^2) + \frac{\gamma}{y_{eq}^2} y^3 + \frac{1}{2} \left(\frac{\beta}{y_{eq}^2} - 6 \frac{\gamma}{y_{eq}^2} \right) x^2 y - \frac{1}{2} \frac{\beta}{y_{eq}^2} y z^2 \\ x - \frac{1}{4} \frac{\alpha}{y_{eq}} x y + 3 \frac{\gamma}{y_{eq}^2} x y^2 + \frac{1}{2} \left(\frac{\beta}{y_{eq}^2} - 6 \frac{\gamma}{y_{eq}^2} \right) x^3 - \frac{1}{2} \frac{\beta}{y_{eq}^2} x z^2 \\ \frac{\alpha}{y_{eq}} x z - \frac{\beta}{y_{eq}^2} x y z \end{bmatrix}. \quad (40)$$

This way of expressing the potential allows it to be computed if only the value of $c_0 b_0^2$ is known, making it agnostic as to how much of the trapping strength is due to particle composition vs. the strength of the B -field and its gradient. The potential energy per unit mass is plotted in Figures 22 and 23 using Eqs. (39) and (40).

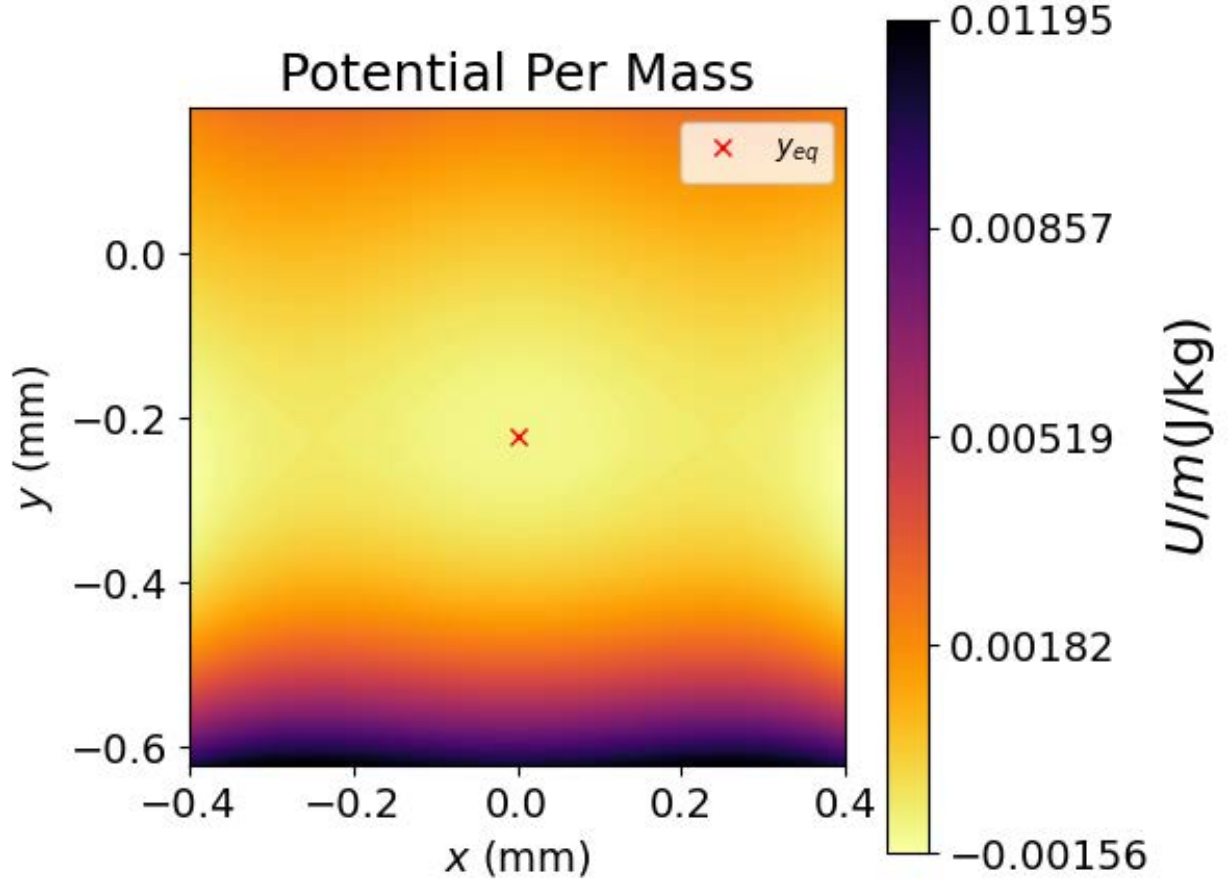


Figure 22: This figure was generated by `plot_field_potential.py` from the files `coefficient_extraction.py` and `z_ringdown_blue/coeffs.txt`. The potential is plotted by fitting the frequencies of oscillation at low amplitude, and the average over three data sets of the x - and y -frequencies at high amplitudes, to the parameters $c_0 b_0^2$, α , β , and γ in Eqs. (34), (35), (36), and (38). The value displayed for y_{eq} is computed from Eq. (37). Slice shown for $z = 0$.

Notice in Figure 22 that the potential is highly anisotropic in its vertical/transverse strength about the equilibrium position due to the offset in the particle's vertical displacement as shown by the red "x" at y_{eq} . This means that, for the sake of most estimates of force on the particle, the dominant B -field is in the x -direction.

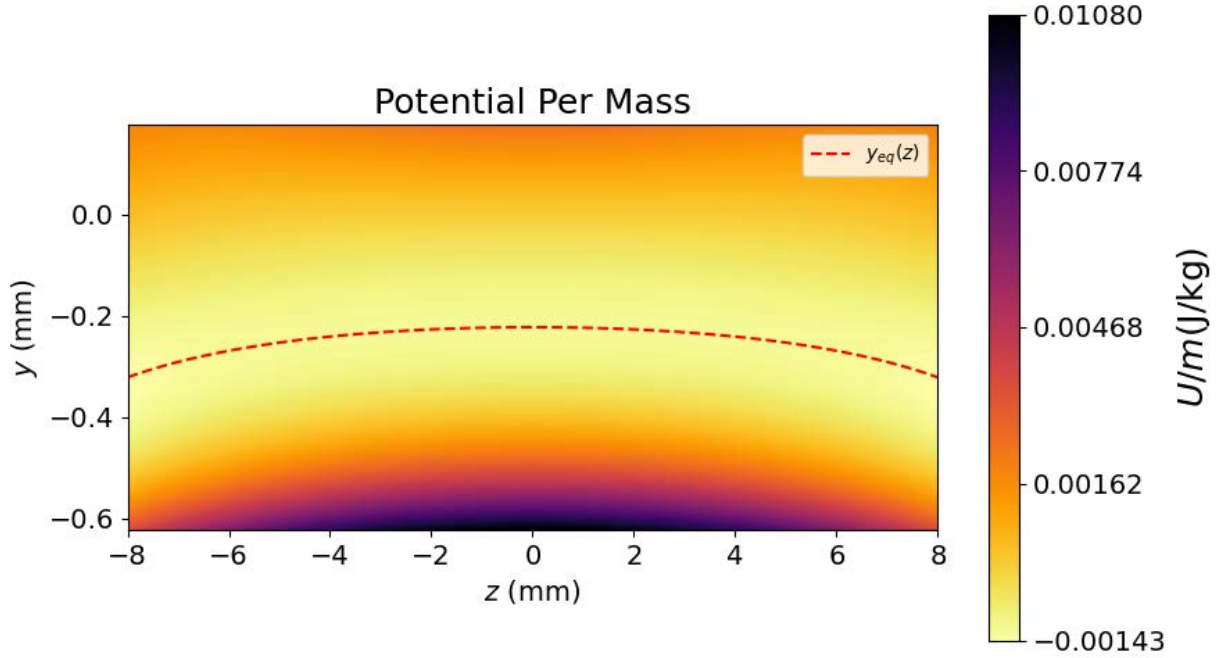


Figure 23: This figure was generated by `plot_field_potetential.py` from the files `coefficient_extraction.py` and `z_ringdown_blue/coeffs.txt`. The potential is plotted by fitting the frequencies of oscillation at low amplitude, and the average over three data sets of the x - and y -frequencies at high amplitudes, to the parameters $c_0 b_0^2$, α , β , and γ in Eqs. (34), (35), (36), and (38). The values displayed for $y_{eq}(z)$ are computed from Eq. (23). Slice shown for $x = 0$.

Note also that, due to the form of Eq. (23), the actual y -offset in the trap is a function of z , as can be seen by the dashed red line in Figure 23. It is important to note that, while this effect has been observed (especially in weaker versions of the Long MGT using Xflux pole pieces), it is not typically nearly so dramatic as what is calculated in theory. This is likely due to the fact that, at high amplitudes, the potential in the axial direction becomes dominated by highly nonlinear terms due to proximity to the magnets placed at the ends. So while a slight dipping of one end of the particle near the edges of the trap can sometimes be observed, the z -dependence of y_{eq} actually observed is considerably smaller than in theory.

There is one more tool that can be used to evaluate the axial potential of the particle. The analysis performed by `potential_analysis_Fourier_data2.py` works using

the following process:

1. A set of z -displacement data (calibrated by known pixel size and framerate) is Fourier transformed and filtered using a single-sided Gaussian filter (see Figure 24) to remove high-frequency noise. This is done because it is assumed that for low oscillation frequencies, any high-frequency signal is due to either process or measurement noise.
2. The data is inverse-FFTed and a polynomial is subtracted from the time-domain data to force the values, derivatives, and second derivatives at the endpoints to 0.
3. The data is again Fourier transformed and differentiated twice in Fourier space to obtain velocities and accelerations.
4. The data is again inverse-FFTed back to the time domain as velocities and accelerations, where the derivative and second-derivative of the polynomial evaluated analytically are added back on to the data. This polynomial addition/subtraction helps avoid troubles that might be caused by assuming periodicity at the endpoints of data without periodic boundary conditions.
5. A polynomial is then fit to a as a function of z and v (see Figures 26 and 25.).
6. A fit to the potential is then performed by analytically integrating the polynomial fit to the acceleration.

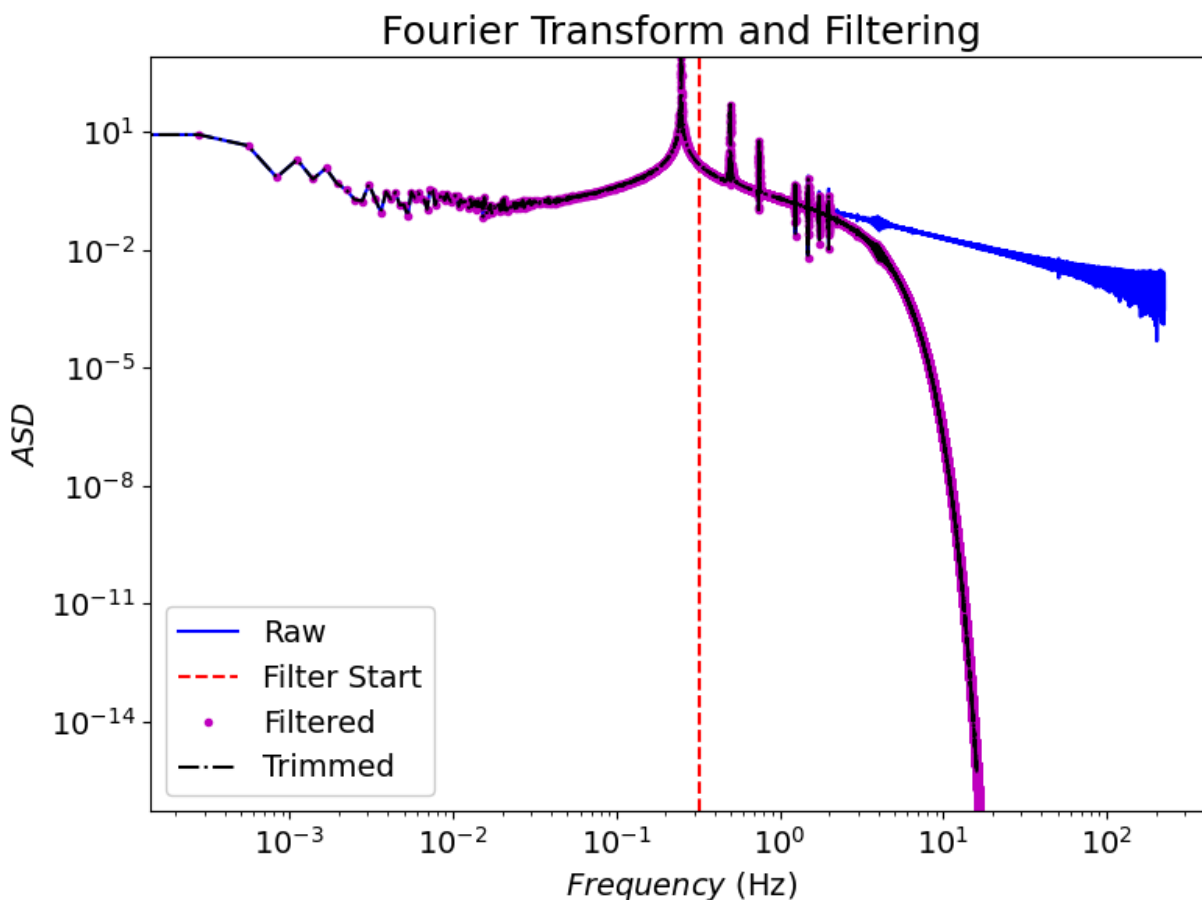


Figure 24: This figure was generated by `plot_field_potential.py` from the file `z_ringdown_blue/displacements_offline_02.zia`. The Gaussian filter is applied starting at $\mu = 0.32$ Hz and $\sigma = 2$ Hz. Note how the high-frequency data is effectively removed while having little impact on the low-frequency information likely to be the primary contributor to the potential.

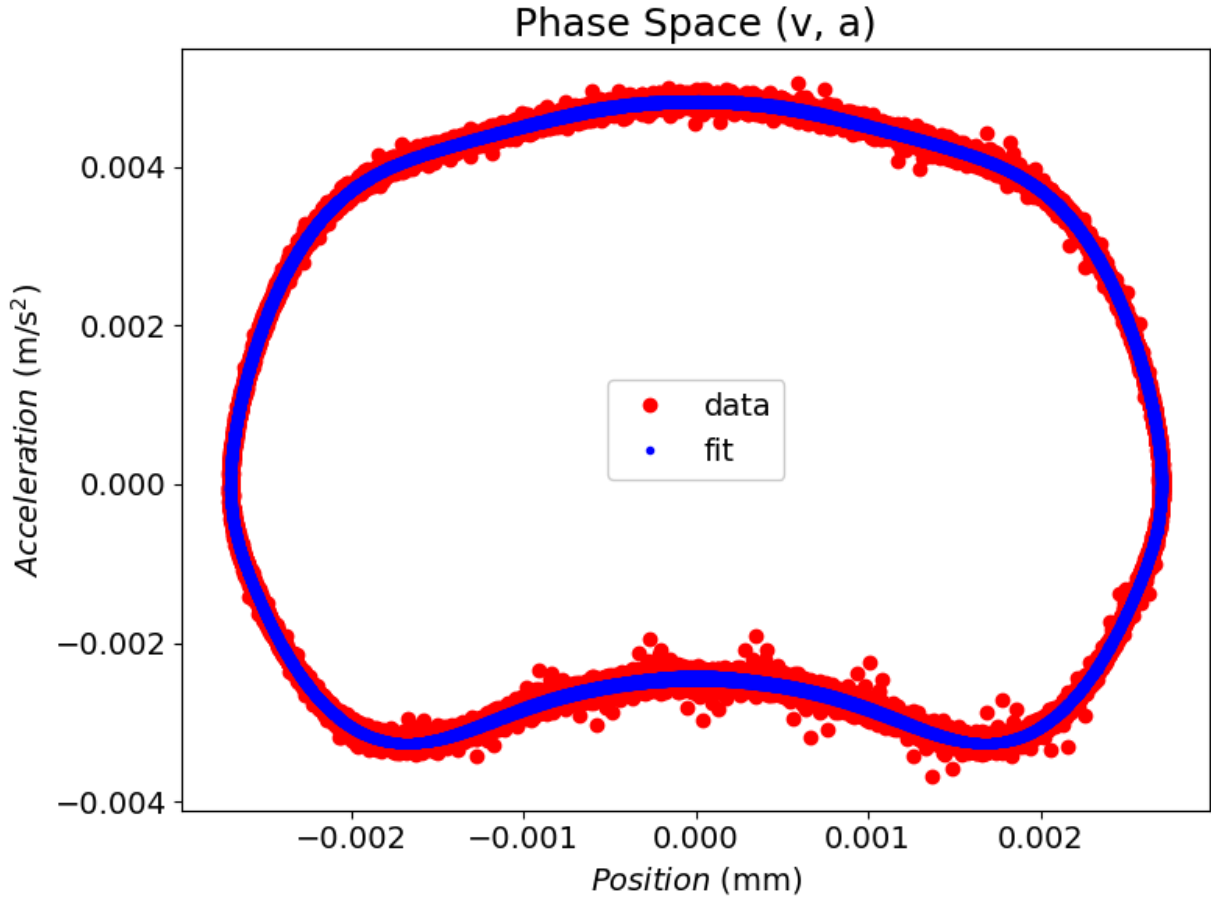


Figure 25: This figure was generated by `plot_field_potential.py` from the file `z_ringdown_blue/displacements_offline_02.zia`. The Gaussian filter is applied starting at $\mu = 0.32$ Hz and $\sigma = 2$ Hz. The phase space of the data after being differentiated, along with the 21-degree polynomial fit. Note that a harmonic oscillator would have a circular phase space.

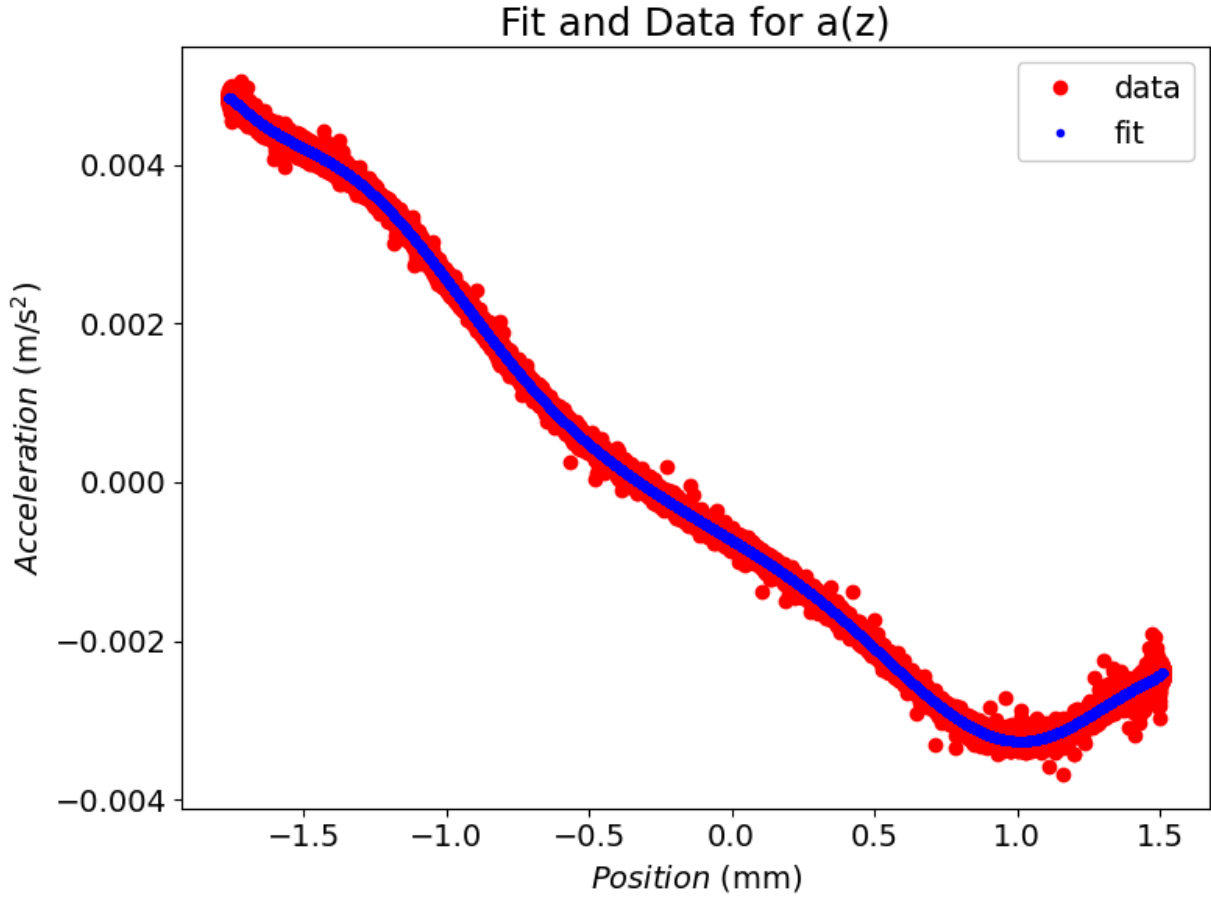


Figure 26: This figure was generated by `plot_field_potential.py` from the file `z_ringdown_blue/displacements_offline_02.zia`. The acceleration calculated from the displacement data by Fourier-domain differentiation, along with the 21-degree polynomial fit. Note that a harmonic oscillator plotted in this way would have a straight line with a negative slope in keeping with Hooke's Law.

The potential fit using this method is shown in Figure 27 plotted next to the axial potential estimated from the multipole fit discussed previously.

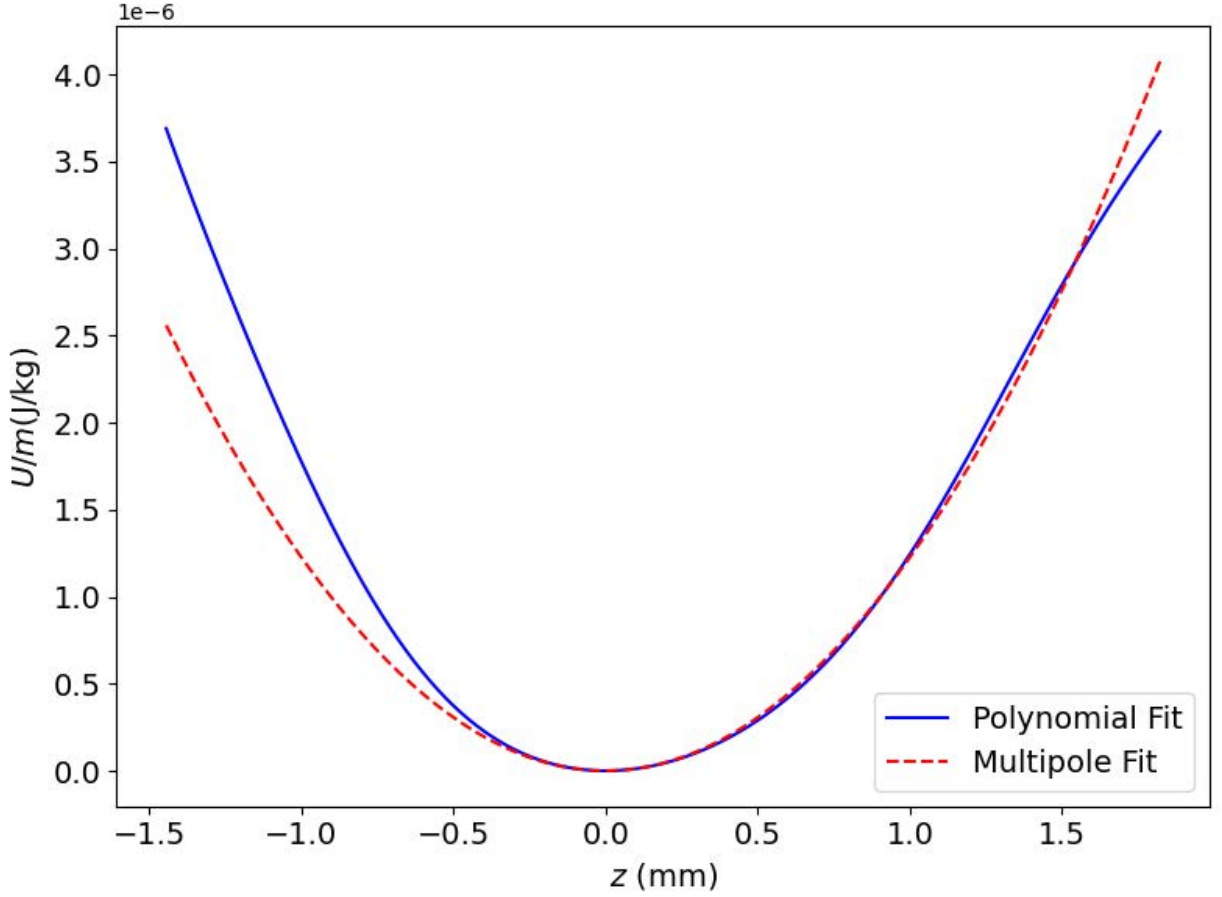


Figure 27: This figure was generated by `plot_field_potential.py` from the files `z_ringdown_blue/displacements_offline_02.zia` and `z_ringdown_blue/coeffs.txt`. Polynomial fit performed using $\sigma = 2$ Hz and a 21-degree polynomial. Multipole fit performed by averaging over the values obtained from three different data sets.

There is one more piece of analysis that is interesting to consider. So far we have only fit parameters to measured systems. The parameter α may be varied while holding β constant, allowing an insight into the effect of changing the magnitude of the two terms competing to affect the axial trapping strength. It should be noted that varying α without adjusting β is not the simple effect of adjusting the positions of the end magnets; in general, one would expect that bringing the end magnets closer to the trapping region should both increase α (the asymmetry term) and β (the quadrupole-weakening term).

Examining Eq. (19), we can set $\ddot{z} = 0$ to find the equilibria of the motion:

$$0 = (\alpha - \beta)(1 + \gamma)z + \frac{1}{2y_{eq}^2} (\alpha - \beta)^2 z^3. \quad (41)$$

There is clearly an equilibrium position at $z = 0$. The derivative of the acceleration at this location, $-c_0 b_0^2 (1 + \gamma) \left(\alpha - \beta + \frac{3}{2y_{eq}^2 (1 + \gamma)} (\alpha - \beta)^2 z^2 \right)$, is negative (stable equilibrium) if $\alpha > \beta$ and positive (unstable equilibrium) if $\alpha < \beta$, indicating the behavior of the primary trapping well.

There are two additional equilibria in some cases, located at $z_{eq}^2 = -\frac{2y_{eq}^2(1+\gamma)}{\alpha-\beta}$. If $\alpha > \beta$, these are imaginary and there are no additional equilibrium positions. If, however, $\alpha < \beta$, we have

$$z_{eq} = \pm y_{eq} \sqrt{\frac{2(1 + \gamma)}{\beta - \alpha}}. \quad (42)$$

At either of these locations the derivative yields $-2c_0 b_0^2 (\beta - \alpha)$, which is always negative for $\beta > \alpha$, indicating stable trapping wells. This behavior is demonstrated by plotting the potential $U(0, y_{eq}, z)/m$ in Figure 28.

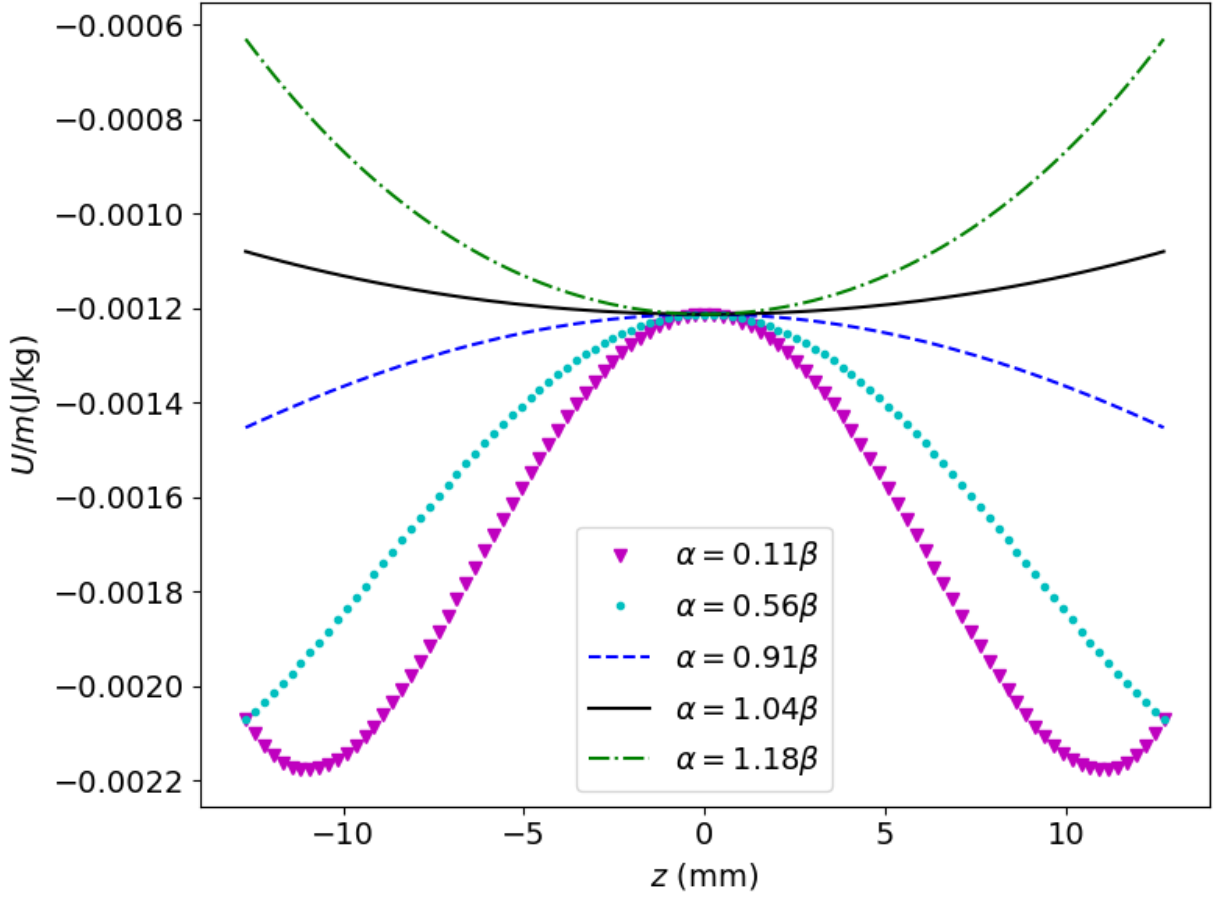


Figure 28: This figure was generated by `plot_field_potetential.py` from the files `coefficient_extraction.py` and `z_ringdown_blue/coeffs.txt`. Here the fitted parameters found earlier for $c_0 b_0^2$, β , and γ are used, but α is allowed to vary.

We see that, indeed, stable equilibria form in a double-well potential for appropriate choices of α . This behavior has been observed in actual trap behavior; however, the equilibrium positions actually observed are typically closer to the center than expected, likely owing to higher-order terms as the particle approaches the end-magnets.

Particles

There is a variety of diamagnetic materials that could theoretically be used for magneto-gravitational levitation. Silica microspheres have commonly been levitated in these types of MGTs, ranging from $\sim 1 \mu\text{m}$ spheres[140] to $50 \mu\text{m}$ borosilicate spheres. [84] Other materials,

such as nanodiamonds, [60] have also been used. Several different material options considered for levitation are discussed here as possibilities explored for viability.

The particles used for initial trials towards a measurement of G were $50\text{ }\mu\text{m}$ borosilicate spheres, loaded using an ultrasonic horn assembly. However, these could not be levitated without the aid of a DC bias across the top and bottom pole pieces, used to help offset gravity. This necessitated using charged particles, which lead to high damping rates. Additionally, the pole pieces of the Long MGT were placed further apart than in the Short MGT to account for the larger particle size while still allowing transverse optical access.

The possibility of using a large glass sphere (somewhere between $50\text{ }\mu\text{m}$ and $300\text{ }\mu\text{m}$ in diameter) in a modified Short MGT was briefly pursued. Figure 29 shows a schematic of the modified trap, including a sample depiction of a $300\text{ }\mu\text{m}$ polished sphere, a product commercially available from Swiss Jewel. This design sacrificed optimal optical access to see if, by allowing the particle to rest lower in the trap, levitation could be achieved without the application of a voltage bias across the pole pieces. It also made use of shaved samarium cobalt magnets in order to strengthen the trapping field.

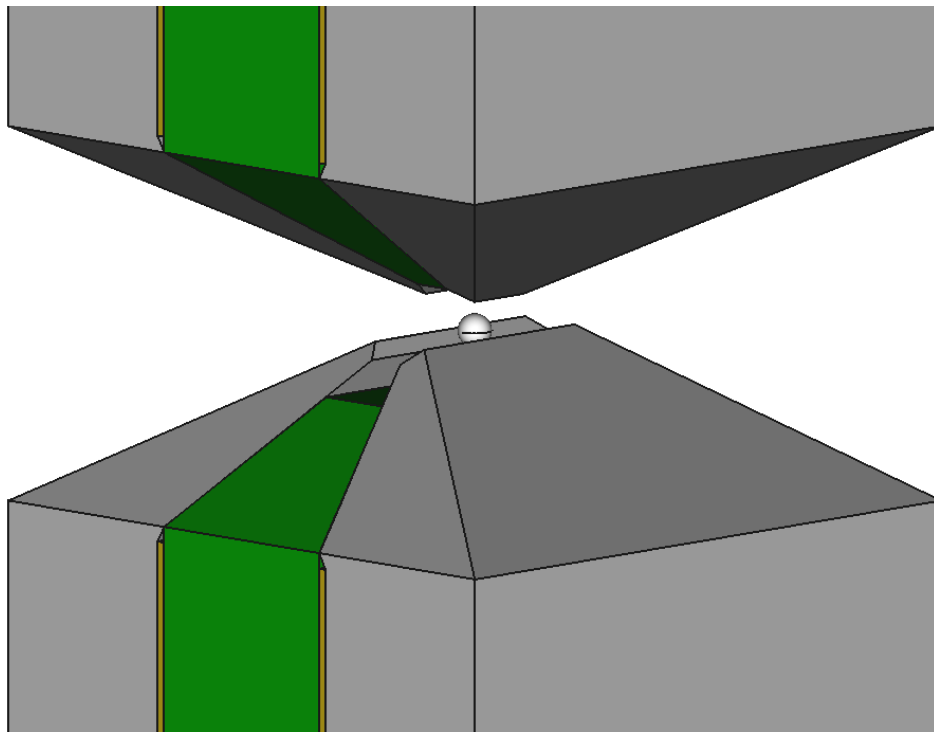


Figure 29: This figure was generated by `murphy_pole_pieces_large.FCStd`. Pole Pieces lapped at an angle relative to horizontal and magnets shaved to fit all the way up inside the pole piece tips form a trap optimized for strength and particle size by potentially sacrificing transverse optical access and allowing proximity to nearby surfaces.

In addition to shaving these solid Hipercro 50A pole pieces to allow the particle to rest lower in the trap, the magnets between the pole pieces were shaved to increase the field strength in the trapping region. The Hipercro 50A pole pieces were not annealed, as that technique had not become common practice in the lab yet. Loading of $300\text{ }\mu\text{m}$ glass spheres was attempted using tweezers for some time, but it was found that forming a small trough for the spherical particles from paper allowed the particle to be rolled into the trapping region. A tungsten probe tip was positioned in the trap using a Signatone Motion Stage (Signatone S-931) to stop the particle from rolling out the end (see Figure 30).



Figure 30: Image of a 300 μm sphere being loaded into a modified Short MGT . The paper could be gently jostled by hand to release the sphere, while the tungsten probe tip was placed to block its exit from the trap on the far side. The relatively large size of the sphere made it possible for small vibrations to overcome its contact force to other surfaces, allowing for this kind of release.

This approach was tricky to execute, but even with some attempts that seemed well-executed, the glass sphere was never trapped, typically instead falling between the pole pieces. Attempts made to load chunks of bismuth of a similar size also failed to levitate any particles. Loading by SAR was not rigorously attempted. Finally, pieces of Pentel 0.5mm HB pencil lead were loaded using a similar method shown in Figure 31.

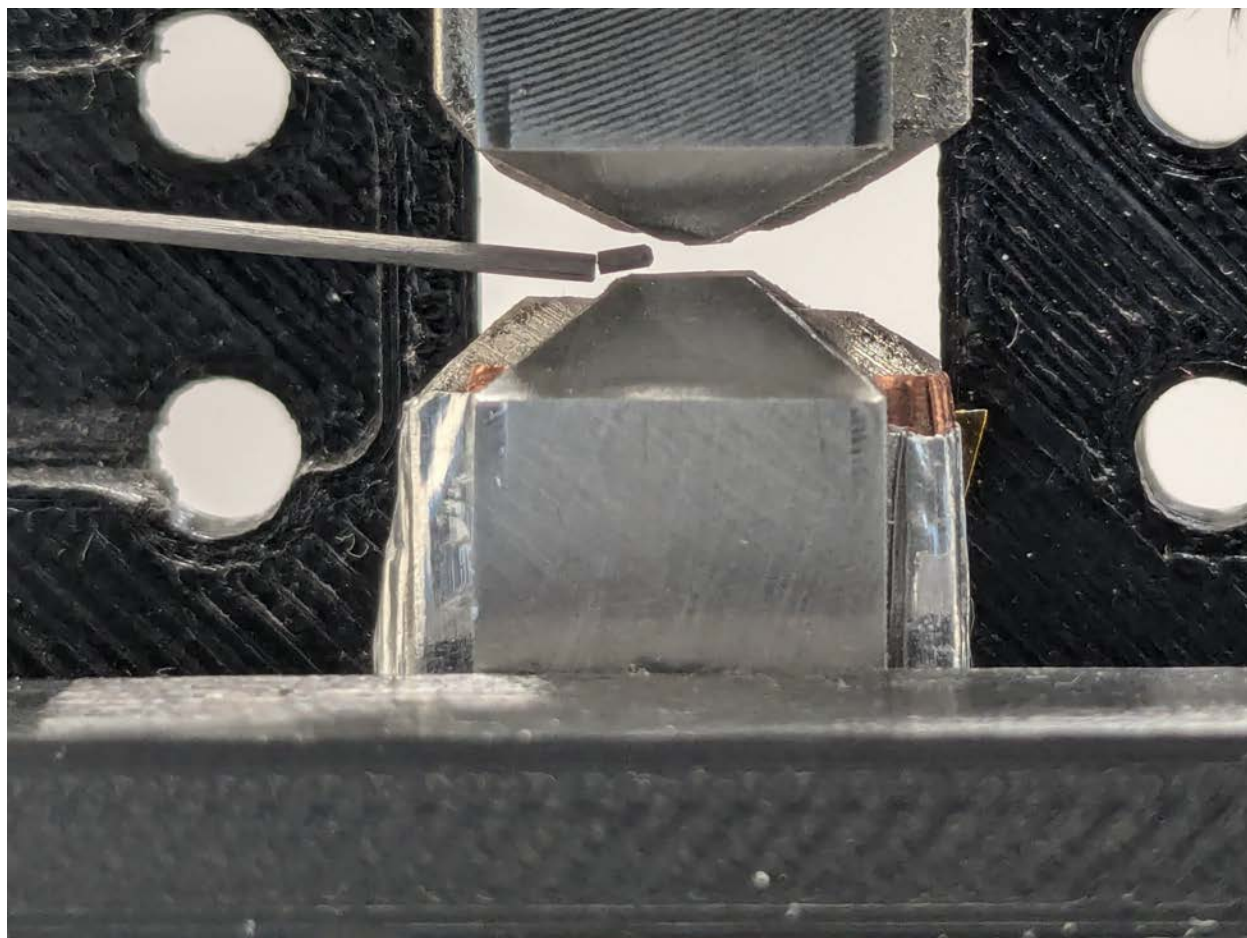


Figure 31: Pentel 0.5mm HB pencil lead loaded using the paper trough method shown in Figure 30, with a piece of 0.7mm pencil lead mounted to a Signatone probe station used as a stopper for the particle. Note that, while the particle is not truly levitated, it is magnetically confined in the transverse and vertical trap directions and only mechanically confined along the axial direction.

The graphite was magnetically trapped along the x - and y -axes of the trap, but still needed a blocker along the z -direction to keep it contained; gradually pushing the blocking piece of lead towards the center of the trap consistently resulted in the particle falling out the other end. Even tapping the blocking rod of pencil lead to release the particle always resulted in either the particle falling back against the blocking piece or going out the far end of the trap, never a true magneto-gravitational trapping result.

Although these tests did not achieve true levitation, they did clearly demonstrate the

immense trapping strength of graphite, even when mixed with a polymer as in pencil lead, compared to other diamagnetic alternatives.

SiC One other material considered as a possible candidate for a G -measurement test mass was SiC. It was already known that SiC grain could be readily levitated in a version of a Short MGT, and the potential applications of the material were highly attractive. 4H-SiC can be bombarded with high-energy electrons to form V_{Si}^- (V2) vacancy defect centers inside, where a Si atom has been knocked out of the crystal lattice. These defect centers behave like spin-3/2 systems, [102] and can be used for vector magnetometry. [81]

The ability to measure the magnetic field experienced by a particle was attractive for measuring G for two reasons. First, it opened up the possibility of directly measuring B -field values at different locations in the trap, allowing the possibility of truly characterizing the magnetic field through direct measurement. Second, the measured signal would be dependent on the orientation of the particle in the trap, possibly giving the ability to measure particle rotations or librations that could couple into the translational motion.

The methodology used to fabricate rounded SiC particle for use in MGTs is described in Appendix A. An image of the jet-milling apparatus used and the resulting SiC particles obtained is shown in Figure 32.

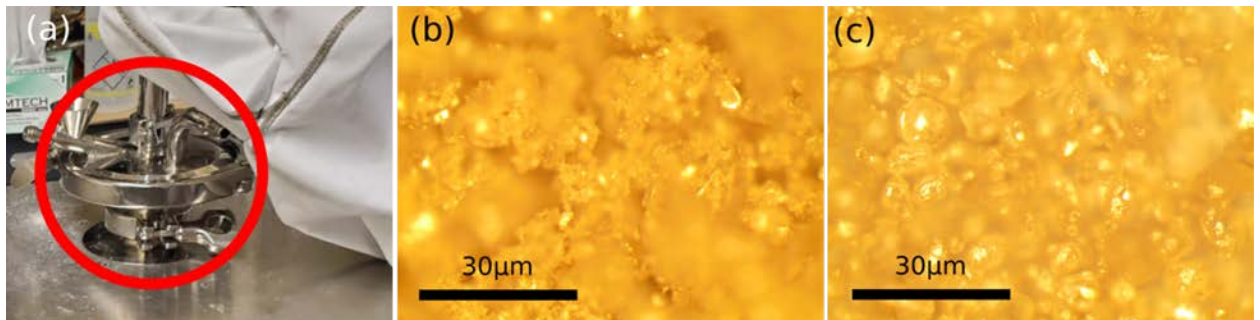


Figure 32: (a) Head of the jet mill used to tumble SiC fragments to smooth them. (b) Image of the rough-edged SiC after processing in a tool steel mortar and pestle (used so that HCl etching would remove metal contaminants). (c) Rounded particles after tumbling in the jet mill.

Because of the nature of the fabrication process such that particles of many different sizes and shapes were all mixed together, and due to the short supply of properly irradiated 4H-SiC , a highly selective and low-waste method was needed for loading SiC into MGTs. This method of loading, which makes use of the room-temperature sublimation of camphor to release particles into a trapping region at low velocity, is detailed in Section 2.

SiO₂ There was interest in learning the maximum size of particle that could be levitated in one of the MGTs in the lab without the necessity of a DC bias voltage and a charged particle. The 50µm spheres typically loaded into the Long MGT with a DC voltage bias were loaded into a Short MGT (without the additional angling inwards to increase trapping volume) as shown in Figure 33.

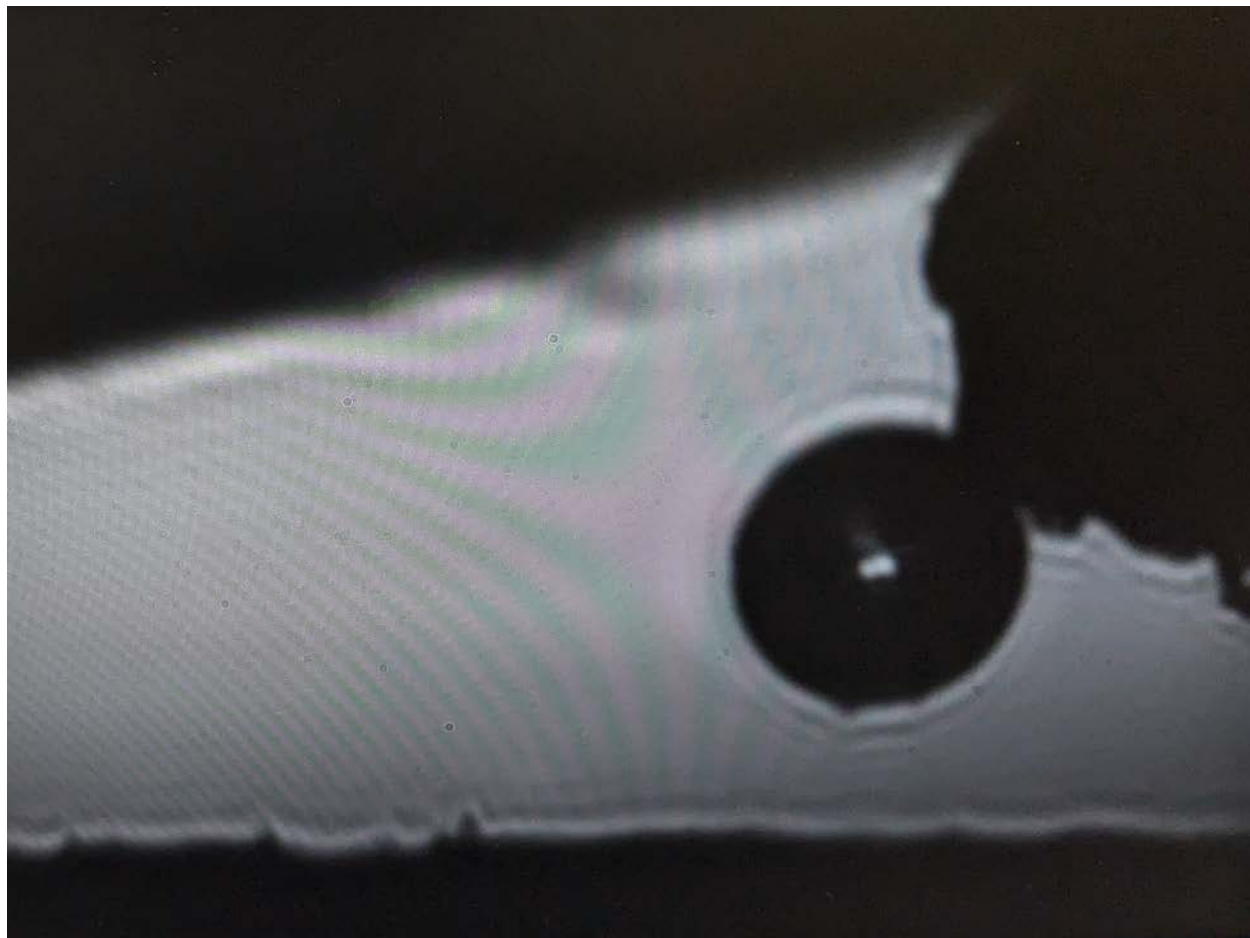


Figure 33: Example image of an attempt to load a 50 μm borosilicate sphere into a Short MGT . The particle is shown attached to a camphor-coated tip as the particle is held in the trapping region.

After many failed loading attempts it was hypothesized that perhaps the borosilicate spheres are low enough in purity that either only a small percentage could be trapped (allowing them to be reasonably trapped only by high-volume, high-waste loading techniques such as an ultrasonic horn), or that they were impure enough that they were only barely able to be levitated. In order to try to trap large glass spheres of potentially higher purity, the possibility of in-house fabrication of glass spheres was pursued.

Flame spheroidization is a process in which glass powder is passed through a high-temperature flame, causing the glass to melt and form a spherical shape due to surface tension

before cooling back down and being re-collected. The most recent procedure (still needing significant refinement) for this process of flame spheroidization is detailed in Appendix A.

Images of particles successfully produced in this way are shown in Figures 34 and 35.

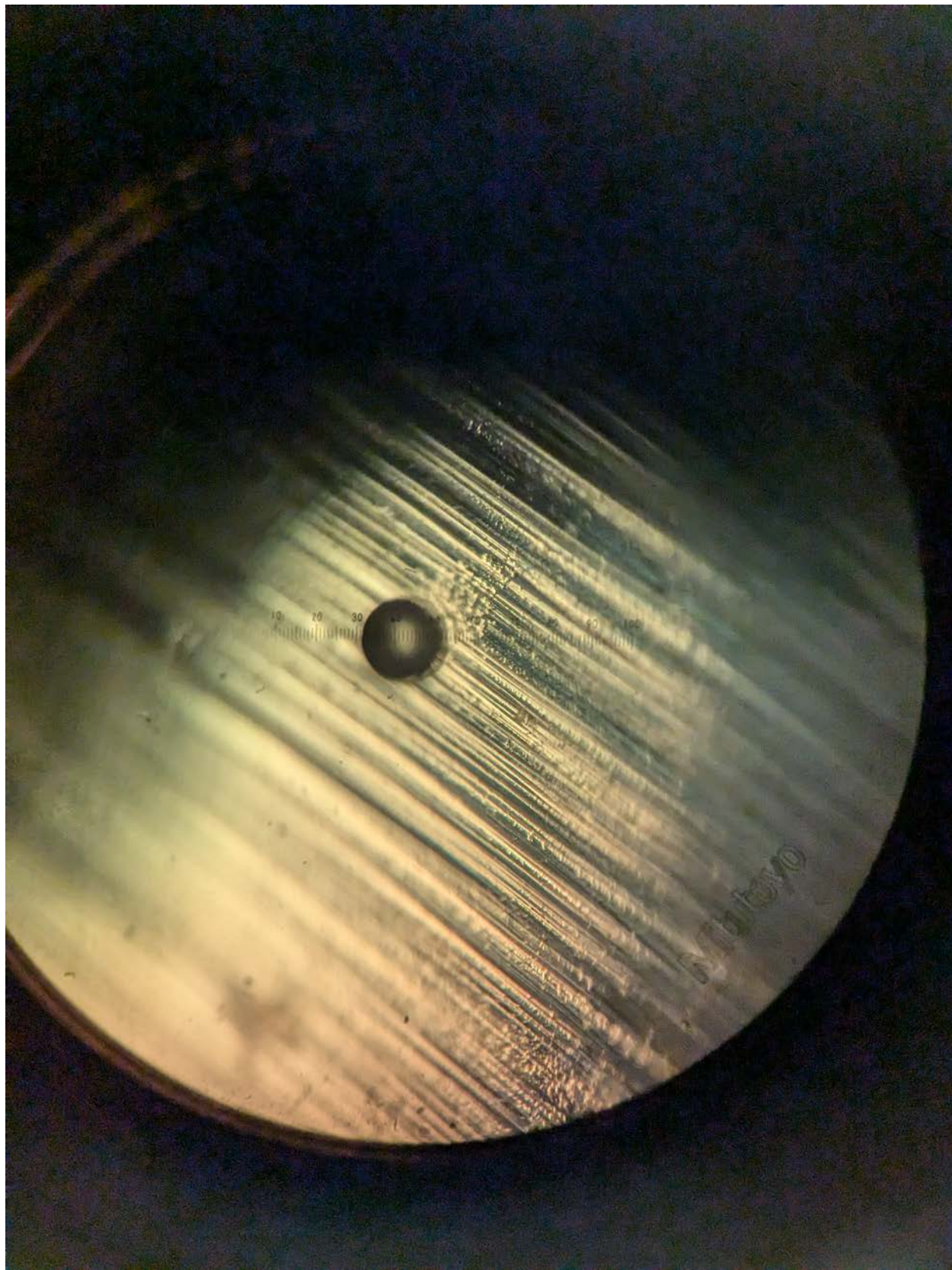


Figure 34: The image is taken using a 20x microscope objective, with each tick shown equaling about $5\mu\text{m}$. It is uncertain whether the dark shade of the particle is due to impurities or a trick of the illumination.

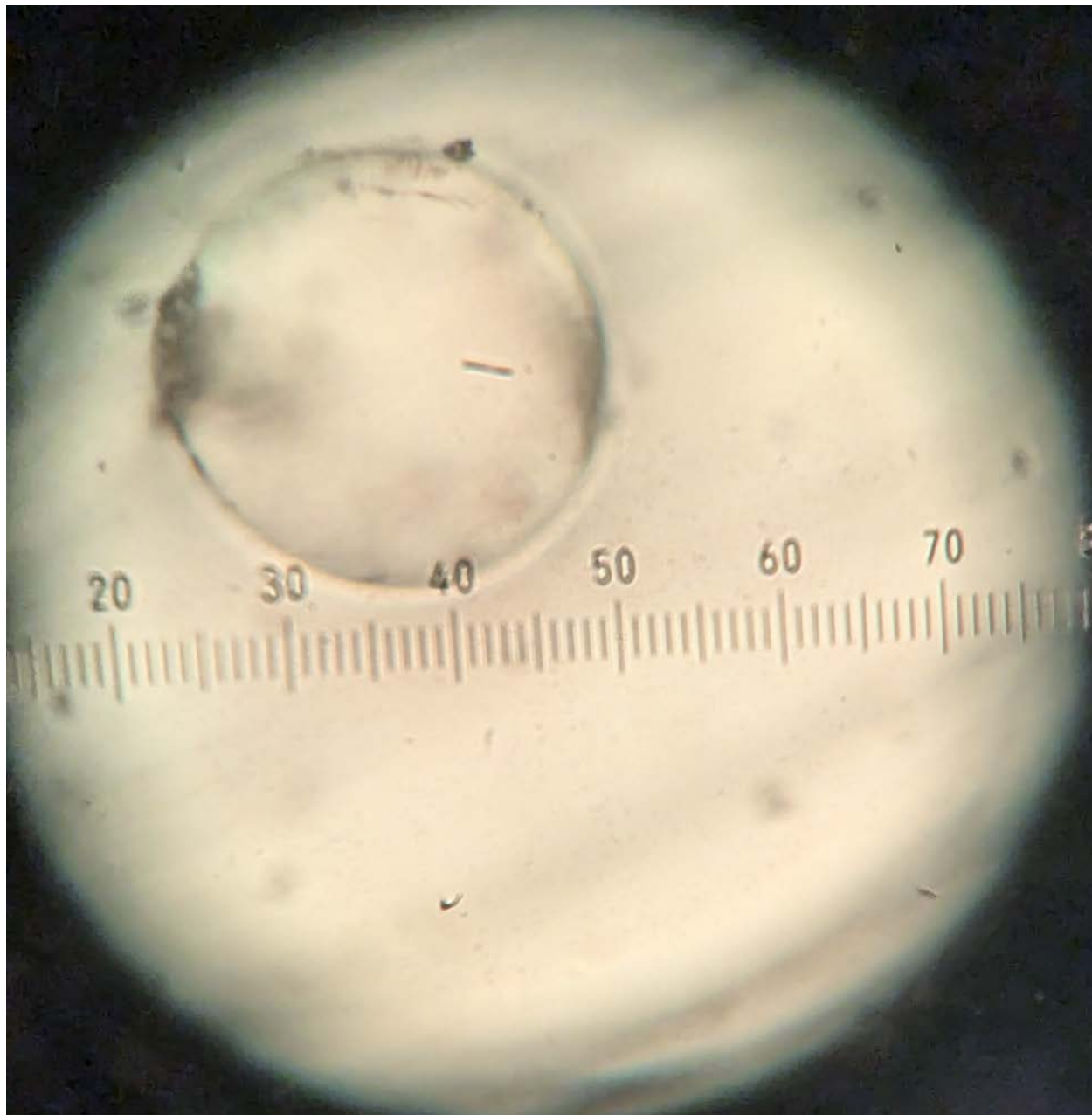


Figure 35: The image is taken using a 20x microscope objective, with each tick shown equaling about $5\mu\text{m}$. The rough dark patches indicate surface contaminates on the glass particle.

Graphite Composite The initial test shown in Figure 31 suggested the possibility of using a graphite composite material for levitation. There were two primary concerns with using graphite as a levitated test mass which had previously discouraged such attempts.

First, graphite is opaque to most light, making it likely to absorb significant heat due to the high-intensity laser beams typically used to cool non-axial motion of particles levitated in the Long MGT . This heating could be highly problematic in a vacuum chamber where the only dissipation method is through radiation. Second, The conductivity of graphite can lead to eddy current damping, [74] making it a potentially poor choice due to the high dissipation. However, the presence of an insulating material could be enough to offset this somewhat, so the direction was pursued further.

Initial tests in a Long MGT constructed from Xflux pole pieces (a material with lower conductivity and less favorable magnetic properties compared to Hiperco 50A) demonstrated that Pentel pencil lead tended to levitate vertically, but slide out the ends of the trap. It was found that adding magnets to the axial ends of the trap as in Figure 10 produced a confining field such that the particle was trapped in all three directions. It is worth noting that the Xflux pole pieces used for this initial case were weak enough magnetically that no borosilicate spheres had ever been trapped in them, even with the aid of a DC bias voltage across the top and bottom sets of pole pieces.

Commercially available pencil lead is usually described using a hardness (such as HB) and a diameter (such as 0.5). Information received on pencil lead from manufacturers[144] indicates that pencil lead is typically composed of 5% wax with the other 95% being a mix of graphite and polymer/clay (by mass). Higher values of H (e.g2H) in the lead specification indicate Harder lead with more polymer, while higher values of B (e.g4B) indicate Bolder lead with more graphite. Of particular interest in this work, HB pencil lead is about 68% graphite by mass, while 4B pencil lead is about 79% graphite by mass.

One more note for Pentel lead in particular is that the specified lead diameters are not precise. The diameters of available leads were measured directly using calipers: 0.2mm lead is 0.27 mm in diameter, 0.3mm lead is 0.37 mm in diameter, and 0.5mm lead is 0.56 mm in diameter. Other leads measured – 0.7mm, 0.9mm, and 1.3mm – all matched the reported

value to within to significant figures.

Figure 11 shows an image taken of a piece of levitated pencil lead, called Edwin (HB05-2.4-lapped) , in the Long MGT used for the work discussed in Chapter 4.

The particle was prepared by breaking off a piece of Pentel 0.5mm HB pencil lead in a lapping plate printed from polyethylene terephthalate glycol (PETG) and drilled using the Mini Milling Machine (KLUTCH 49657) and a 0.55 mm (504021 McMaster-Carr) drill bit (gripped in the chuck by wrapping it in aluminum foil). The lapping plate (shown in Figure 36) was then placed between two sides of a folded piece of printer paper and translated back and forth to lap the ends of the particle smooth.

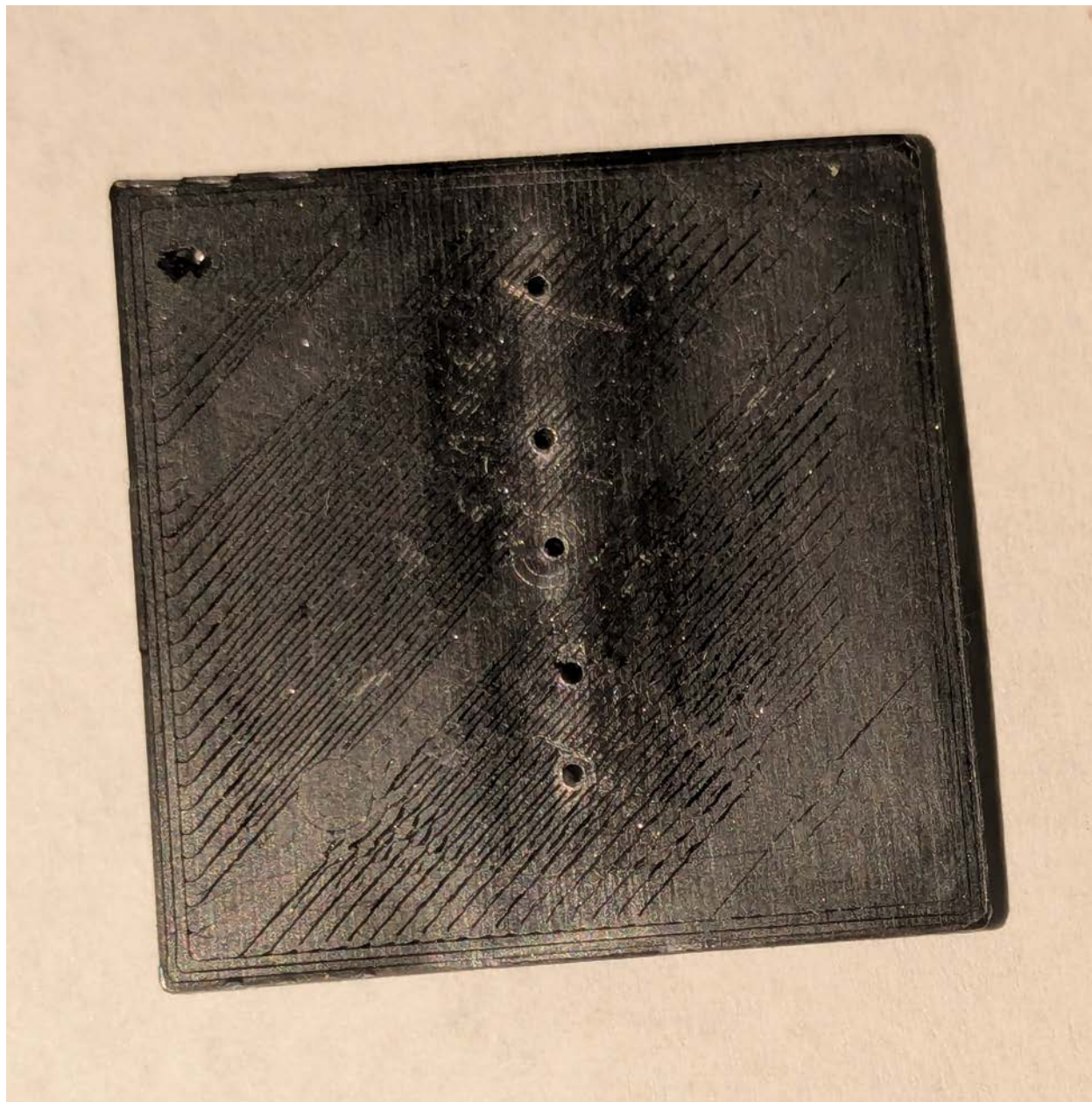


Figure 36: The plate used to lap pencil lead so that its ends are smooth and its length is somewhat controlled. 0.5mm Pentel pencil lead is placed in the holes, broken off jaggedly, and then lapped smooth on both ends simultaneously using printer paper. The plate is 2 mm thick, and the holes are drilled with a 0.55 mm drill bit.

The resulting product had slightly rounded ends, and was about 2.4 mm long with a diameter of about 0.56 mm. An image of this particle was taken under the microscope using a 2x objective, as shown in Figure 37.

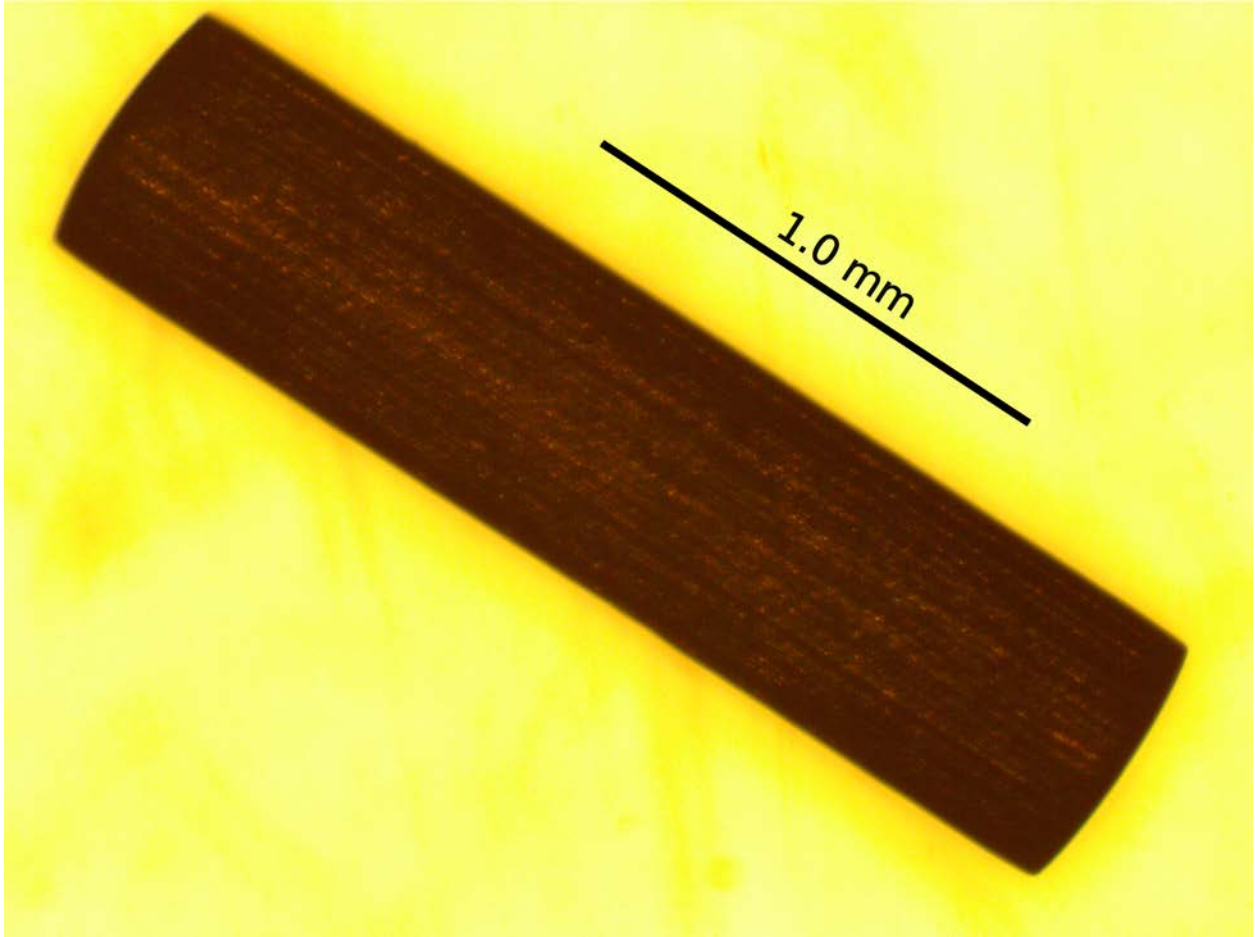


Figure 37: Image of Edwin (HB05-2.4-lapped) taken with the microscope , 2x objective, intermediate zoom. Note the uniformity of the ends produced using the lapper shown in Figure 36, and the ridges running down its length.

Levitation Trials Several experimental trials were performed to characterize the levitation properties of Pentel pencil lead and other graphite composites. One of these trials was to levitate several different hardnesses of 0.5mm Pentel pencil lead, as shown in Figure 38.

The results, though imprecise, show the expected trend of a weaker trapping force for particles with lower concentrations of graphite. Qualitative observation also indicated that higher-polymer pencil leads tended to be more weakly constrained in the transverse direction, falling out easily especially near the axial edges of the trap. This effect was far more noticeable to the eye than the change in y_{COM} demonstrated in Figure 38.

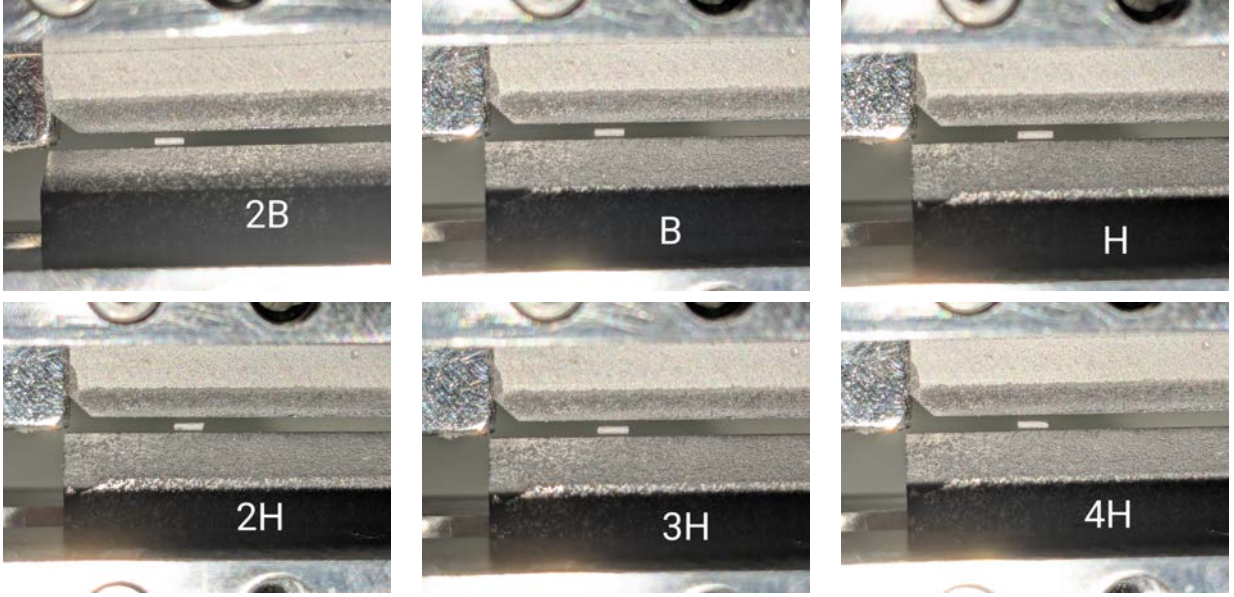


Figure 38: Photos taken of several similar pieces of pencil lead of different hardnesses levitating in the same trap (Xflux pole pieces). The phone was held by a tripod for these images, allowing a rough estimate of the value of y_{COM} to be measured for each hardness: $y_{COM}^{2B} \approx -0.08$ mm, $y_{COM}^B \approx -0.09$ mm, $y_{COM}^H \approx -0.10$ mm, $y_{COM}^{2H} \approx -0.13$ mm, $y_{COM}^{3H} \approx -0.11$ mm, $y_{COM}^{4H} \approx -0.13$ mm. An attempt was made at loading a piece of $5H$ lead, but it could not be trapped.

Because of the proprietary nature of the Pentel pencil lead, some effort was taken to see if similar results could be achieved using homemade composite particles from high-temperature epoxy (Epo-Tek 353ND) and high-purity graphite powder (Nanografi NG04EO072379). Some of the results of these and other tests follow.

Initial attempts at producing composite graphite particles used high-purity carbon tape (no adhesive, just a roll of flattened graphite flakes) mixed with epoxy to extrude cylindrical particles using mechanical pencils. It was found, however, that this process was highly sensitive to contamination by ferromagnetic materials; blending the tape in a coffee grinder with stainless steel blades contaminated the tape enough that the resulting powder was ferromagnetic.

Starting with $1\mu\text{m}$ - $5\mu\text{m}$ graphite powder proved effective, and cylinders of graphite

were created as described in Appendix A. Some plates of graphite-epoxy composite were also created by pressing the powder-epoxy mixture between two flat plates before curing. These flat particles could be levitated over a checkerboard magnet pattern as shown in Figure 39.

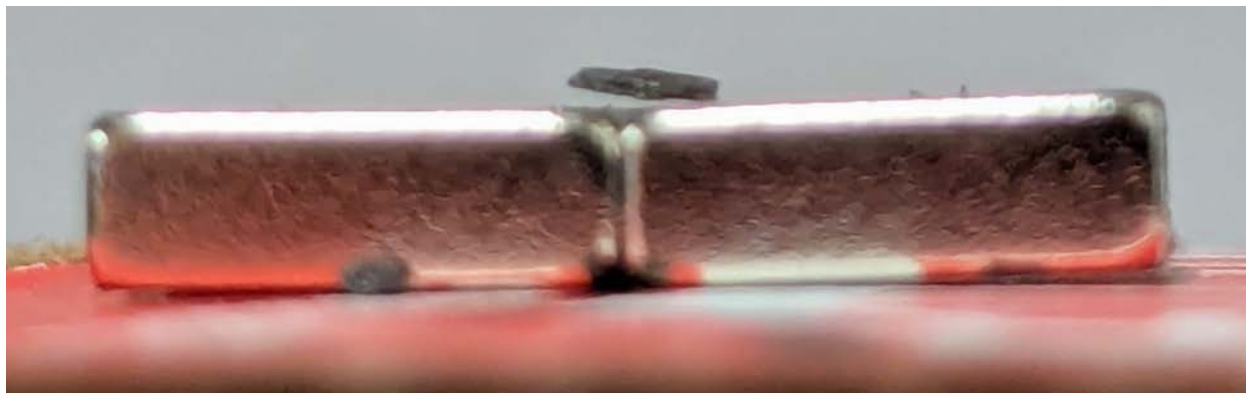


Figure 39: A piece of $\sim 67\%$ graphite, $\sim 33\%$ high-temperature epoxy (Epo-Tek 353ND) (by volume) graphite composite levitated over a checkerboard arrangement of neodymium magnets.

The cylindrical particles had mixed results, with some levitating well and others not. The determining factors seemed to have something to do with length (longer particles were more likely to levitate) and uniformity (particles with burs on the edges tended to catch or not levitate well). Some levitated fairly well as shown in Figure 40, but not nearly so well as the Pentel lead.



Figure 40: A piece of $\sim 67\%$ graphite, $\sim 33\%$ high-temperature epoxy (Epo-Tek 353ND) (by volume) graphite composite levitated in a Long MGT with end magnets, annealed Hiperco 50A pole pieces, and unshaved magnets. The graphite volume ratio of this particle slightly exceeds that of Pentel HB pencil lead, which has a volume ratio of about 51%, but levitates less well.

The results also showed that the composite particles were likely to exhibit different trapping behavior across their lengths. This could be due to contamination, inhomogeneity in the material, or both.

A few other brands of pencil lead were tested as well. BIC and Papermate pencil lead did not exhibit clear paramagnetic or diamagnetic behavior and could not be trapped, but Faber Castell trapped about as well as the homemade composite pieces tested. Koh-I-Noor and Pacific Arc exhibit paramagnetic (or perhaps ferromagnetic) behavior, showing macroscopic attraction to magnets and resisting attempts to trap them.

Tumbling Process Some effort was made to control the shape of graphite composite particles, with the intent being to develop methods useful for shaping both commercial pencil lead and homemade composite using high-temperature epoxy (Epo-Tek 353ND). A tumbler was designed for this purpose (schematic shown in Figure 41), which uses compressed air to tumble particles along a circular path such that collisions with the walls of the tumbler

eventually wear down its edges.

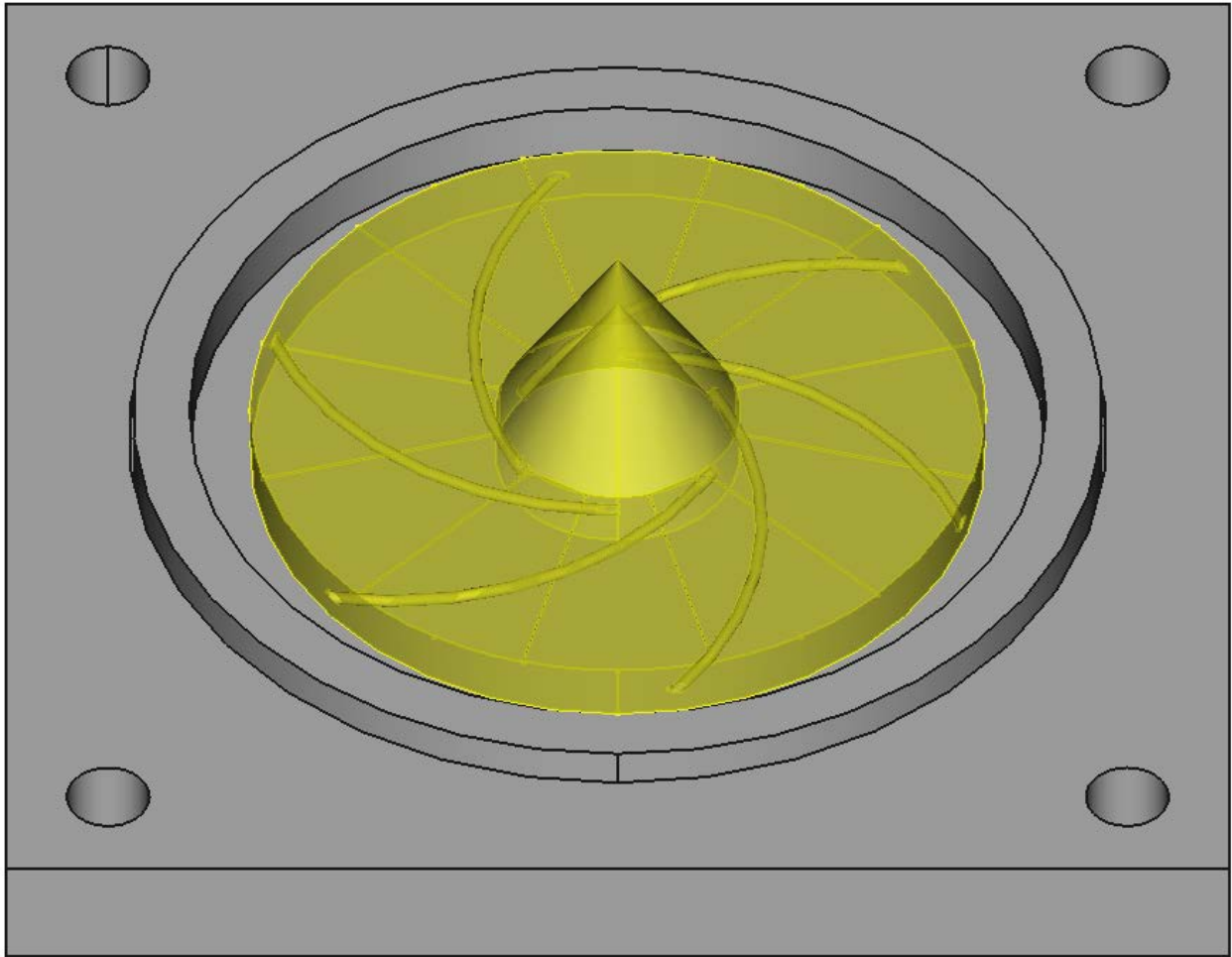


Figure 41: This figure was generated by `sphere_tumbler.FCStd`. Partial schematic of the tumbler used for smoothing pieces of graphite composite. The bottom plate (square piece) has a hole in the bottom for 1/4-inch tubing as the input for the compressed air. The inner wall (highlighted piece in yellow) redirects the input air out through 6 spiraling holes so that the air pushes the particle in a clockwise direction. The air then escapes between small slits in the top of the inner wall (outlined by the 12 radial lines extending outward on the inner wall piece) and the top plate (not shown). The outer wall (the ring piece) sets the size of the tumbling track by its height (shared by the inner wall) and its inner diameter. An abrasive paper with a hole cut in the center is placed below and above the two wall pieces, clamped in place by 1/4-20 screws at the corners of the top and bottom plates.

An image of the tumbler is shown in Figure 42 with a particle placed inside. In this case paper disks were used as abrasives.



Figure 42: An image of the particle tumbler (printed from PETG with a 3 mm tumbling track) with a particle inside ready to be tumbled. The top plate is not shown. The conical shape at the center was designed to make 3D-printing the part possible without additional supports.

The Mini Milling Machine (KLUTCH 49657) can be used to shape pencil lead by

mounting a lead into the chuck, setting it rotating at several hundred rpm, and then lathing it with a tungsten carbide blade. This process was used to shape and cut pieces of pencil lead in preparation for being used in the tumbler. Figure 43 shows several pieces of 6H 0.5mm pencil lead shaped to be roughly spherical using this method.



Figure 43: Pieces of 0.5mm Pentel 6H pencil lead, lathed on the Mini Milling Machine (KLUTCH 49657) by use of a tungsten carbide blade. Rough edges can be seen where the pieces broke off at their thinnest point during the lathing process.

These particles were then placed in the tumbler with sandpaper used as an abrasive (this was eventually replaced with paper to avoid contamination and to improve surface finish); the results can be seen in Figure 44.



Figure 44: Pieces of 0.5mm Pentel 6H pencil lead tumbled in the particle tumbler for 40 minutes with 2000 grit silicon carbide sandpaper followed by 70 minutes using 3000 grit sandpaper (flow rate not well-controlled or documented). Note the slightly oblong shape and smooth surface finish.

Attempting to trap spherical particles of pencil lead did not work. In fact, trapping pencil lead below a certain length (nominally about 1 mm, though it is likely trap-dependent and should be tested further) also consistently failed. The reasons for this phenomenon are discussed in Section 2. An image taken of a tumbled 0.5mm Pentel HB tumbled sphere is shown in Figure 45, where levitation was not achievable.

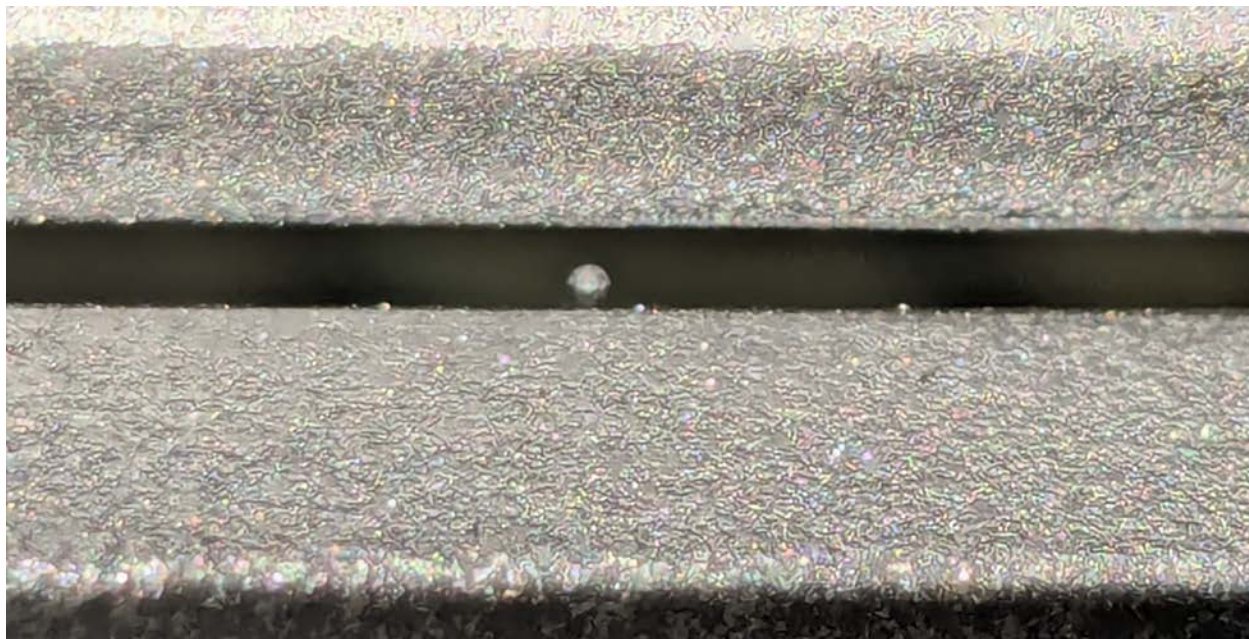


Figure 45: Image taken of Pentel 0.9mm HB pencil lead sphere resting on the lapped tip of one of the Xflux pole pieces after many failed loading attempts.

Because of this, the tumbler was tested as a device for smoothing the ends of particles in the interest of possibly creating ellipsoidal particles with high eccentricity. The results favored the formation of spherocylindrical shapes rather than ellipsoids, as shown in Figure 46. This is likely due to the geometry of tumbling favoring collisions with edges rather than faces such that flat surfaces have a near-zero chance of experiencing a collision compared to edges, causing the ends to tumble before the center.

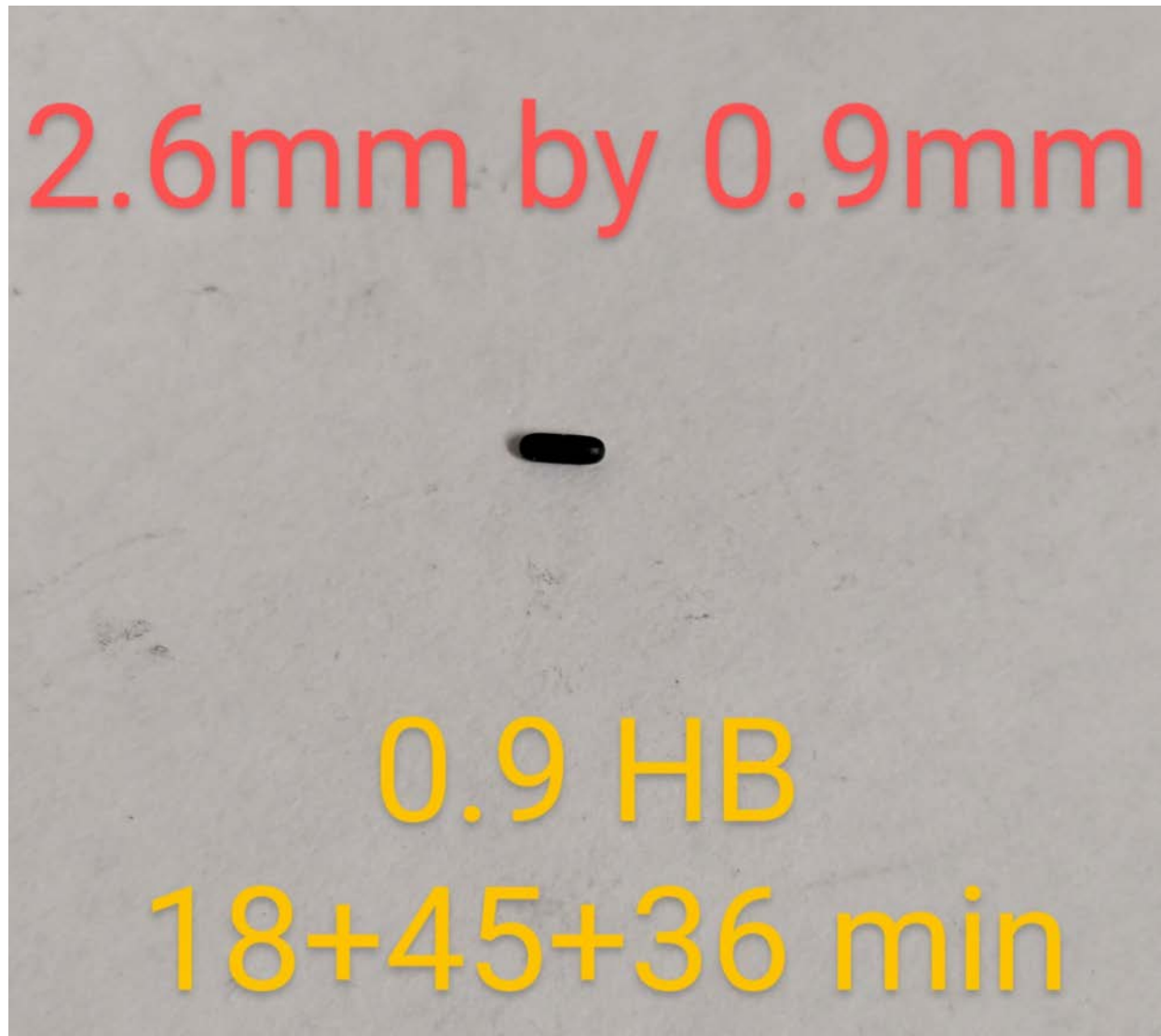


Figure 46: A spherocylindrical (capsule-shaped) piece of Pentel pencil lead tumbled using printer paper as the abrasive for a total of about 100 minutes. The black color is due to the build up of oil on the particle; subsequent tumbling trials were performed with a filter on the building's air line. Note that the end diameter of the particle was the same as at the start, owing to the preferred tumbling of particle ends over centers.

This shape is actually quite favorable for several reasons. First, it allows consistent shaping of a long particle that can be trapped in our MGTs. Second, the smooth and uniform polishing results in fairly good approximations of the center of mass of a particle by its center of brightness, a straightforward parameter to measure for a back-illuminated

particle imaged onto an area camera. Finally, the hemispherical ends allow unambiguous focusing on the center of the particle; for the lapped particle shown in Figure ??, the focal plane is somewhat unclear, since focusing on the front edge of the end of the particle leaves the back end blurry and vice versa. Hemispherical ends mean that if the furthest point on the end of the particle is in focus, the imaging optics are centered on the central plane of the whole particle.

In the Long MGT , the dominant motion of the particle is in the axial direction. This means that, for the most part, only the pixels at the ends of the image of the particle convey information about its location; all the pixels along its length are essentially informationless. One can imagine that the information per frame could be greatly improved by increasing the number of pixels that change each frame for axial translation. One possible way to do this is to cut grooves into a piece of pencil lead by lathing it on the Mini Milling Machine (KLUTCH 49657) . Such a particle is shown in Figure 47, shown next to two other particles for comparison.

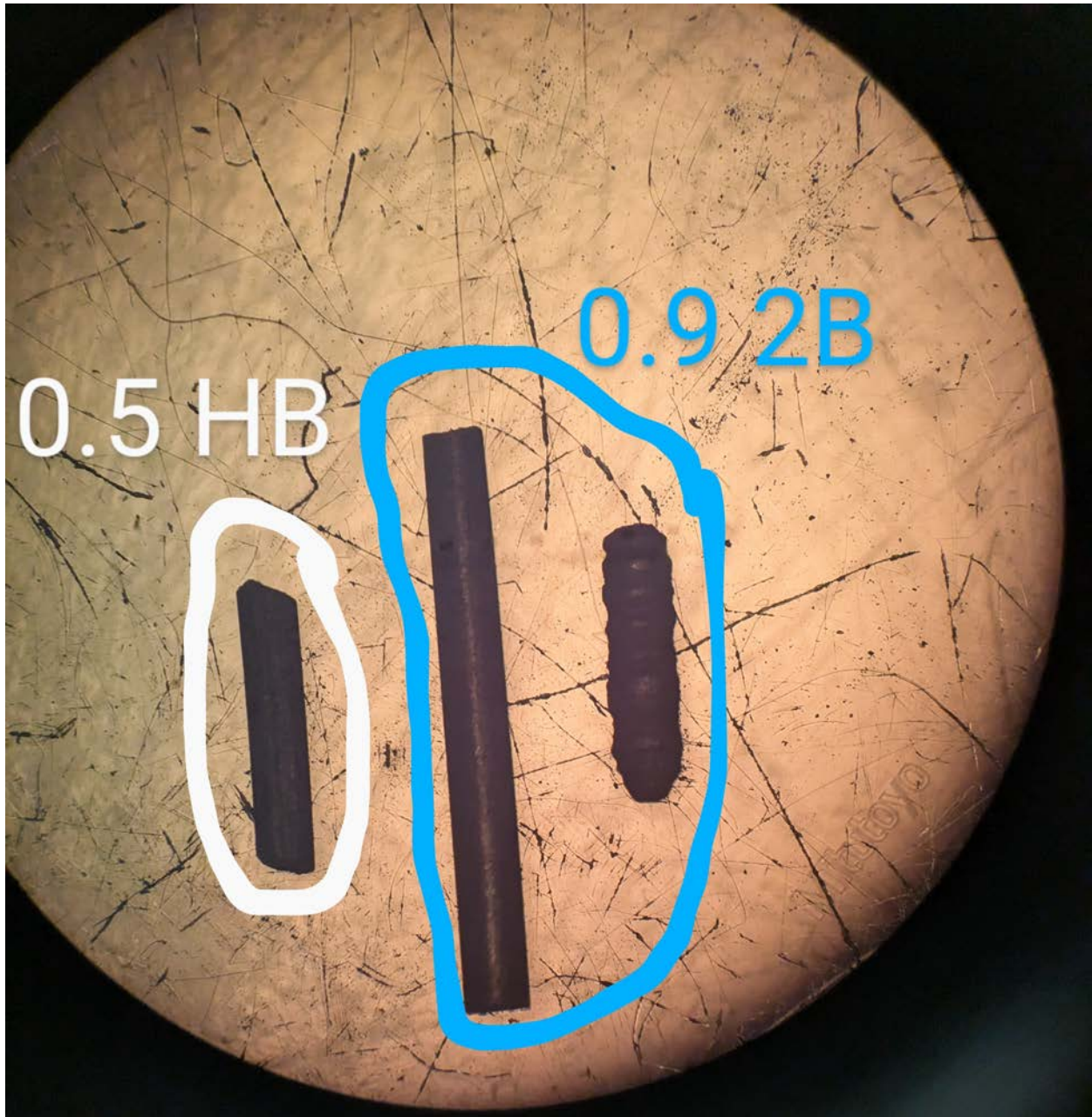


Figure 47: Three pieces of Pentel pencil lead shown side-by-side for comparison. The left-most particle is un-modified 0.5mm HB lead. The middle particle is a piece of 0.9mm 2B lead lathed to form a smooth cylinder using the Mini Milling Machine (KLUTCH 49657) . The furthest right particle is also a piece of lathed 0.9mm 2B lead, this time milled using the Mini Milling Machine (KLUTCH 49657) to create a grooved surface. Note that the smooth lathed surface appears smoother than the un-lathed surface of the 0.5mm lead.

We can see that larger-diameter pieces of lead can be fairly reliably lathed down to

smaller diameters, a useful technique for obtaining specific shapes of certain sizes. Figure 48 shows a close-up view of the same particles.

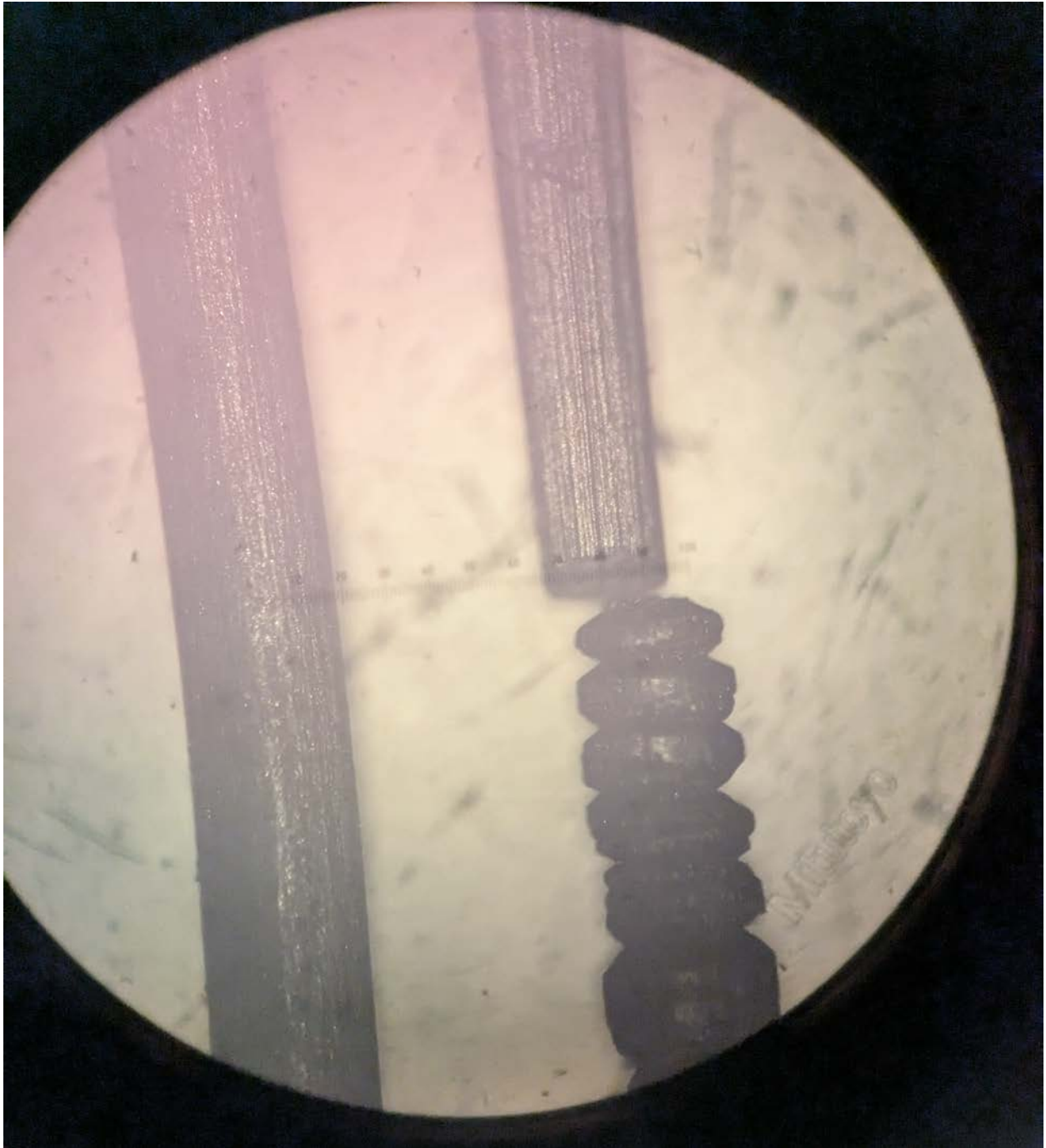


Figure 48: Three pieces of Pentel pencil lead shown side-by-side for comparison. The left-most particle is a piece of 0.9mm 2B lead lathed to form a smooth cylinder using the Mini Milling Machine (KLUTCH 49657) . The upper right particle is un-modified 0.5mm HB lead. The lower right particle is also a piece of lathed 0.9mm 2B lead, this time milled using the Mini Milling Machine (KLUTCH 49657) to create a grooved surface. Note the asymmetrical grooves of the lower right particle, likely due to wobble in the milling machine and the hand holding the cutting tool.

The asymmetry in the groovy particle is concerning for the sake of center-of-mass calculations and general homogeneity of the particle. However, the proof of concept motivates the possibility of using a more precise lathe to shape future particles to be optimized for different qualities, such as information efficiency. Figures 49 and 50 demonstrate the fact that particles lathed in this way can still be readily trapped in the Long MGT .

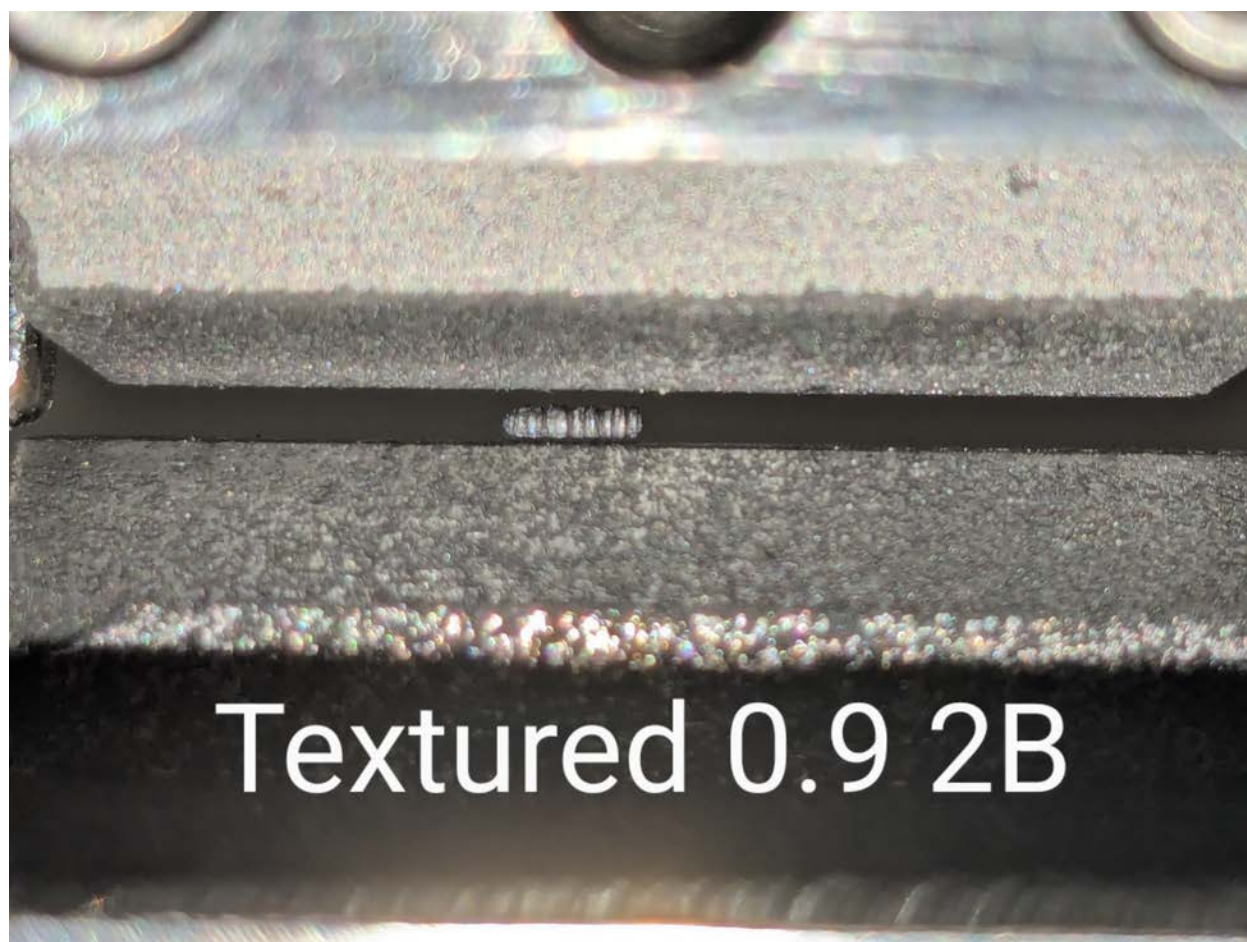


Figure 49: A piece of 0.9mm Pentel 2B pencil lead, lathed down to be about the same diameter as a piece of 0.5mm lead and levitated in a Long MGT with Xflux pole pieces. Note the grooved surface, a potentially useful feature for information efficiency in tracking the location of the particle using image analysis.

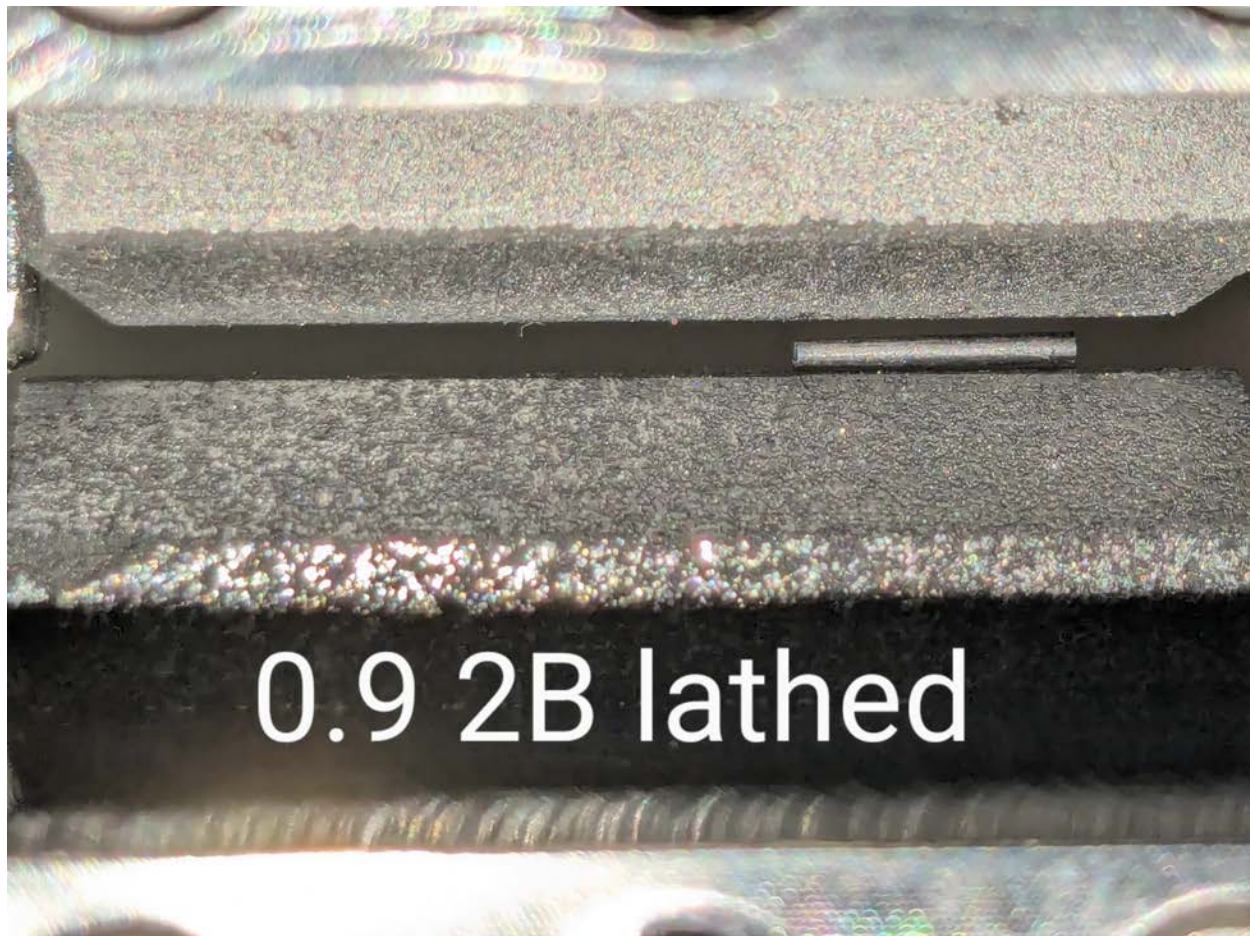


Figure 50: A piece of 0.9mm Pentel 2B pencil lead, lathed down to be about the same diameter as a piece of 0.5mm lead and levitated in a Long MGT with Xflux pole pieces. Note the smoothness of the surface.

Characterization Several hardnesses of pencil lead were tested for density and resistivity using a pair of calipers with resolution to $10\mu\text{m}$, a scale sensitive out to 0.1mg , and a Keithley 2400 sourcemeter combined with a custom holder for performing four-wire resistance measurements (see Figures 51, 52, and 53). Each measurement was taken on 12 separate pieces of the same kind of pencil lead from the same case. The results of these measurements are displayed in Table 1.

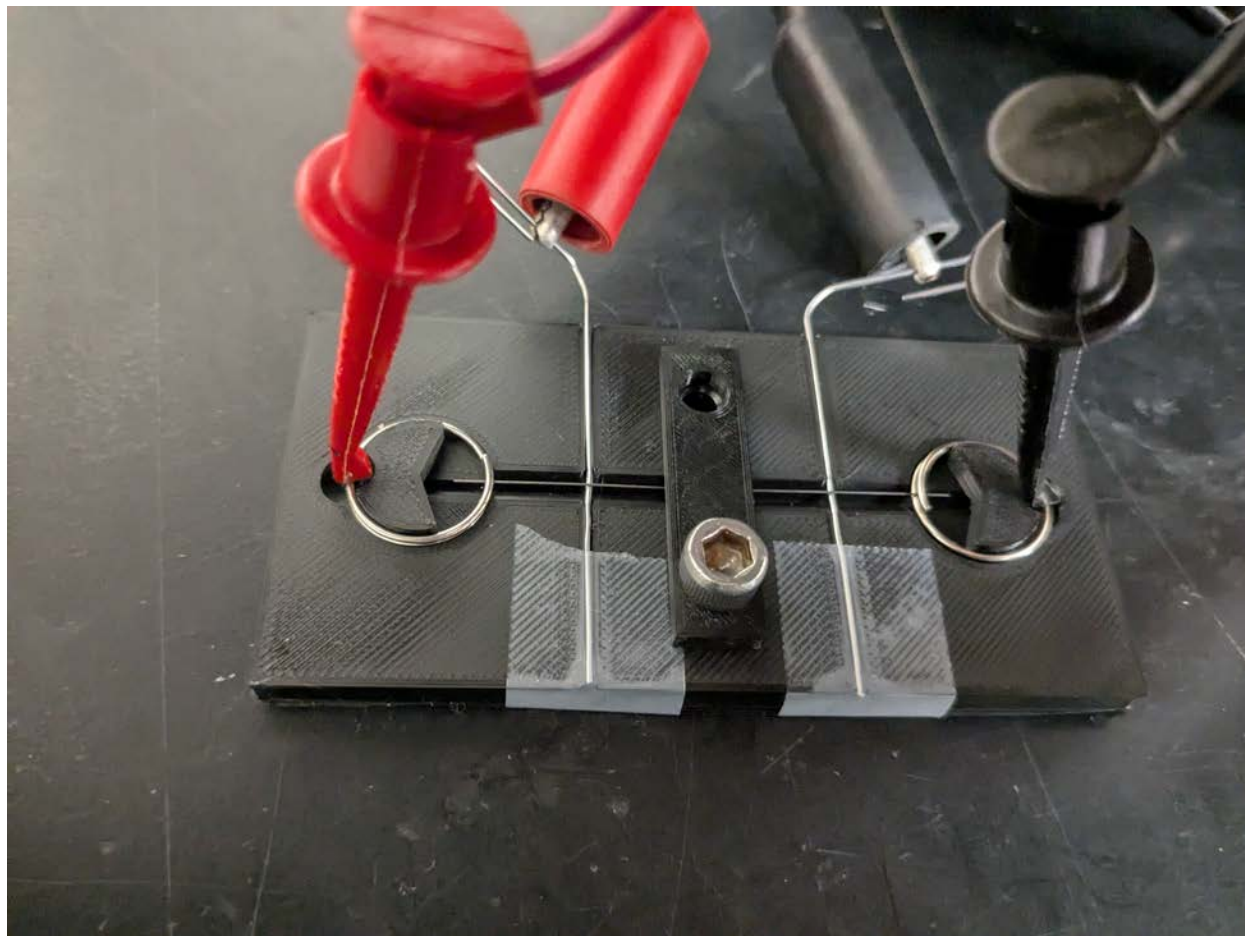


Figure 51: The setup used to measure the resistivity of pencil lead using a four-wire resistance measurement. The pencil lead is clamped between two split rings, then placed on top of the straightened paper clips. the small plastic beam is used to press down on the pencil lead, flexing it to ensure that it makes good electrical contact with the paper clips.

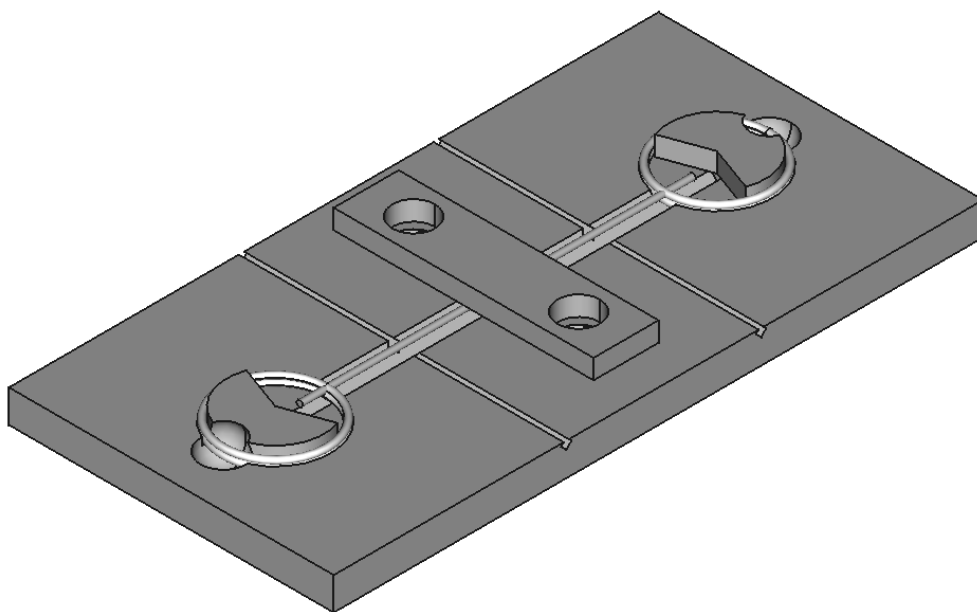


Figure 52: A schematic of the 4-wire resistance measurement setup, including the actual design of the 3D-printed plastic parts. The divots on the outer edges of the circular protrusions are placed to make it easier to make electrical contact with the split rings.

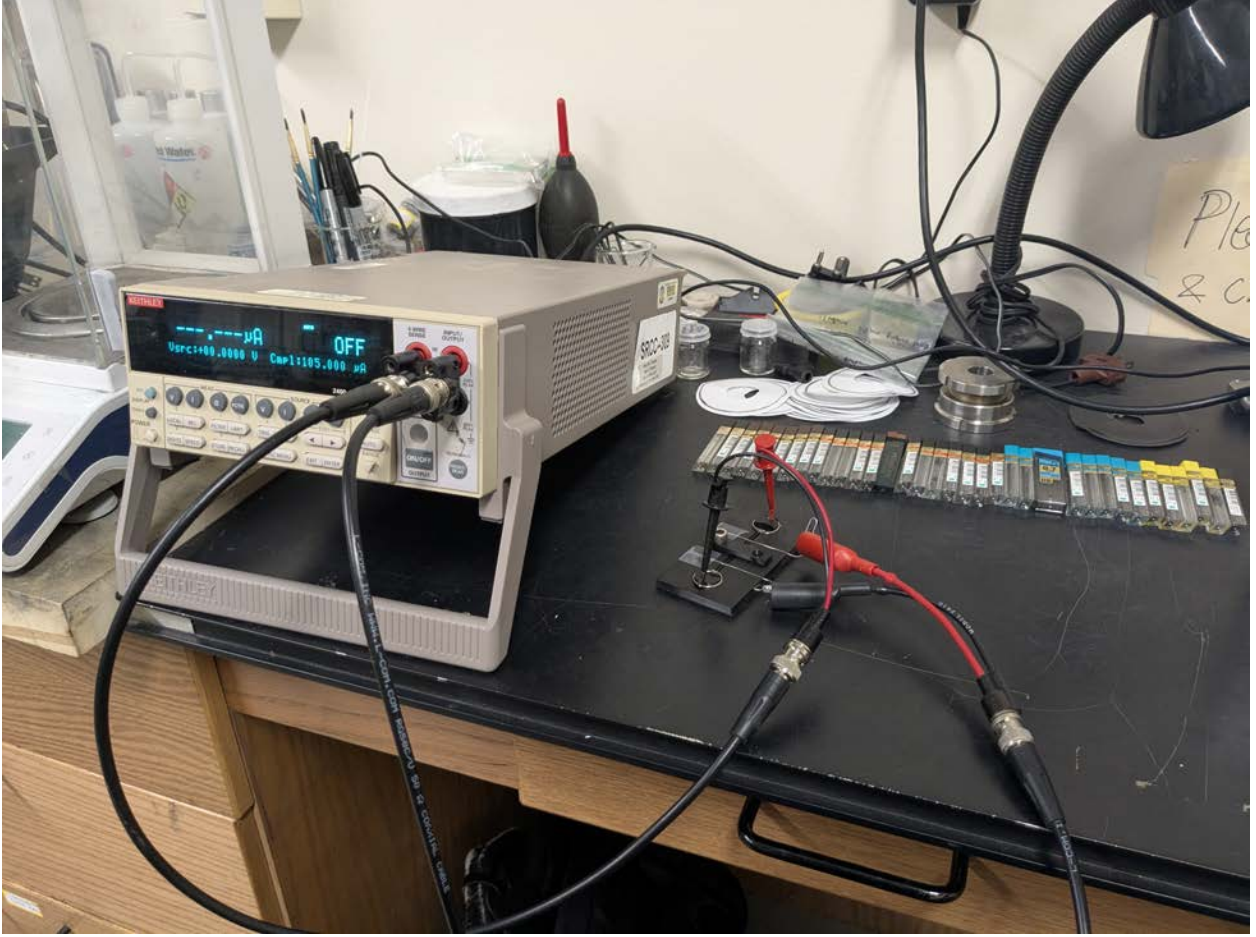


Figure 53: The sourcemeter used to perform resistivity measurements of pencil lead. A known current is run through the two leads connected to the ends of the pencil lead, and the voltage across the two leads in the center is measured. By measuring the distance between these central leads, a value for the resistivity of the rod between the two central contact points can be calculated with minimal dependence on the electrical connection at the contact points.

Table 1: Intrinsic Properties of 0.5mm Pentel Pencil Lead

Hardness	Density	Density Uncertainty	Resistivity	Resistivity Uncertainty
4B	1.67 g/cm ³	0.02 g/cm ³	$5.38 \times 10^{-6} \Omega \text{ m}$	$1.0 \times 10^{-7} \Omega \text{ m}$
2B	1.74 g/cm ³	0.02 g/cm ³	$3.46 \times 10^{-6} \Omega \text{ m}$	$5 \times 10^{-8} \Omega \text{ m}$
HB	1.70 g/cm ³	0.02 g/cm ³	$2.16 \times 10^{-6} \Omega \text{ m}$	$7 \times 10^{-8} \Omega \text{ m}$
2H	1.74 g/cm ³	0.05 g/cm ³	$2.10 \times 10^{-6} \Omega \text{ m}$	$3 \times 10^{-8} \Omega \text{ m}$
4H	1.70 g/cm ³	0.01 g/cm ³	$2.13 \times 10^{-6} \Omega \text{ m}$	$1 \times 10^{-8} \Omega \text{ m}$
6H	1.80 g/cm ³	0.02 g/cm ³	$2.80 \times 10^{-6} \Omega \text{ m}$	$4 \times 10^{-8} \Omega \text{ m}$

In order to measure the magnetic susceptibility of HB pencil lead along both its long axis and its cross-sectional axis, approximately 540 pieces of roughly equal length 0.5mm-HB lead were packed in rows in the PETG holder shown in Figure 54.



Figure 54: Pentel 0.5mm HB pencil lead packed by hand into a 9 slots for use in the TeachSpin magnetic susceptibility measuring kit. After packing, the ends of the lead were lapped smooth to try to keep the lengths as close to uniform as possible. Measurements of the susceptibility of the holder in both orientations were taken separately and subtracted off the results for the lead-packed holder.

The value of χ obtained from the measurement was approximately 0 with the magnetic field oriented along the long axis of the pencil lead, and $\chi_{HB} = -2.0^{+0.4}_{-0.1}$ across its cross-section. This indicates a likely extrusion-induced orientation of graphite flakes such that their c-axes are primarily perpendicular to the long axis of the lead pieces.

Also of note, the magnetic susceptibility of 0.5mm Pentel 4B pencil lead was measured using the same method. The value measured was $\chi_{4B} = -2.1 \pm 0.1$. This slightly higher value matches with the expected behavior, given that the mass concentration of graphite in 4B pencil is higher than in HB pencil lead – about 79% as opposed to 68%. [144]

If the plates of graphite are indeed ordered in concentric shells within the pencil lead

cylinder such that the effective susceptibility of the lead for fields parallel to the plates is near 0, we should expect χ_x inside the particle to be proportional to $|\hat{\rho} \cdot \hat{x}| = |\cos \phi|$, where ϕ is the polar angle from the particle's x -axis. Furthermore, if χ_0 is the maximum magnetic susceptibility of the pencil lead with radius r_0 , length L , and coordinates x, y, z centered on the particle's center of mass, we can write the average value of χ_x in a uniform field as

$$\begin{aligned}
\chi_x &= \frac{1}{\pi r_0^2 L} \int_z \int_y \int_x \chi_0 |\cos \phi| \, dx \, dy \, dz \\
&= \frac{2}{\pi r_0^2} \int_{-r_0}^{r_0} \int_0^{\sqrt{r_0^2 - y^2}} \chi_0 \frac{x}{\sqrt{x^2 + y^2}} \, dx \, dy \\
&= \int_{-r_0}^{r_0} \frac{2\chi_0}{\pi r_0^2} (r_0 - |y|) \, dy \\
&= \frac{2}{\pi} \chi_0.
\end{aligned} \tag{43}$$

There are two interesting conclusions to draw from this. First, we can estimate the maximum average magnetic susceptibility achievable for highly-ordered graphite arranged in concentric shells. The graphite concentration by mass for HB pencil lead is 68%. [144] The volume concentration can be estimated from

$$\frac{V_C}{V_{HB}} = 0.68 \frac{\rho_{HB}}{\rho_C} = 0.51. \tag{44}$$

The maximum expected value for χ_{avg} , then, is

$$\chi_{HB} = 0.51 \frac{2}{\pi} \chi_{C\parallel} = -2.0 \times 10^{-4}. \tag{45}$$

This is within the uncertainty for the measurement performed here of χ_{HB} , indicating that this is indeed likely the arrangement of the graphite in Pentel HB pencil lead.

Second, if we define $\chi'_x(y)$ according to line 3 of Eq. (45) such that $\int \chi'_x(y) dy = \chi_x$, we can adjust our estimate of y_{COF} obtained from Eq. (33) by taking into account the variability of the magnetic susceptibility with height:

$$\begin{aligned}
 y_{COF} &= \frac{\int y dF}{\int dF} = \frac{\int_{y_{COM}-r}^{y_{COM}+r} \chi'_x(y) y^2 \sqrt{r^2 - (y - y_{COM})^2} dy}{\int_{y_{COM}-r}^{y_{COM}+r} \chi'_x(y) y \sqrt{r^2 - (y - y_{COM})^2} dy} \\
 &= \frac{\int_r^r (r - y) (u + y_{COM})^2 \sqrt{r^2 - u^2} du}{\int_{-r}^r (r - y) (u + y_{COM}) \sqrt{r^2 - u^2} du} \\
 &= \frac{y_{COM}^2 + \frac{1}{8}(r - 2y_{COM})r}{y_{COM} - \frac{1}{8}r}.
 \end{aligned} \tag{46}$$

Plugging in values of $y_{COM} = -0.1$ mm and $r = 0.28$ mm now yields $y_{COF} = -0.199$ mm – a value within about 10% of the value for y_{eq} of -0.222 mm obtained using the multipole fitting technique. This difference can now easily be accounted for by measurement error (recall that the fact that the pole pieces are covered in foil makes it difficult to precisely estimate the geometric center of the trap), an actual difference in the equilibrium position caused by the particle being off-center axially, or a combination of these or similar effects.

Loading

Because of the interest in levitating irregularly shaped particles of irradiated 4H-SiC in relatively limited supply, a method for selectively loading particles with high efficiency was needed.

In general, small particles can be readily attached to surfaces due to Van der Waals forces, capillary forces, and Coulombic attraction; in fact, these forces dominate inertial forces on small particles to such an extent that releasing them from a surface is far more of a challenge. A common technique in this lab is to load particles by launching them from an ultrasonic horn – a device that leverages mechanical advantage from the narrowing of a

titanium metal rod to vibrate the tip of the launcher at a high amplitude using a piezo driven by a frequency-swept signal that includes the resonant frequency of the titanium tip. This method, however, is neither selective nor efficient, as it involves picking up many particles and launching them in indiscriminate directions hoping that one will become trapped.

The new technique developed works by first sticking the particle to a probe tip using camphor, then placing it in the trapping region and allowing the particle to fall into the trap as the camphor sublimates. The procedure for using this sublimation-activated release (SAR) loading technique is outlined in Appendix A. Figure 55 illustrates the steps of the process for the loading of a large SiC particle.

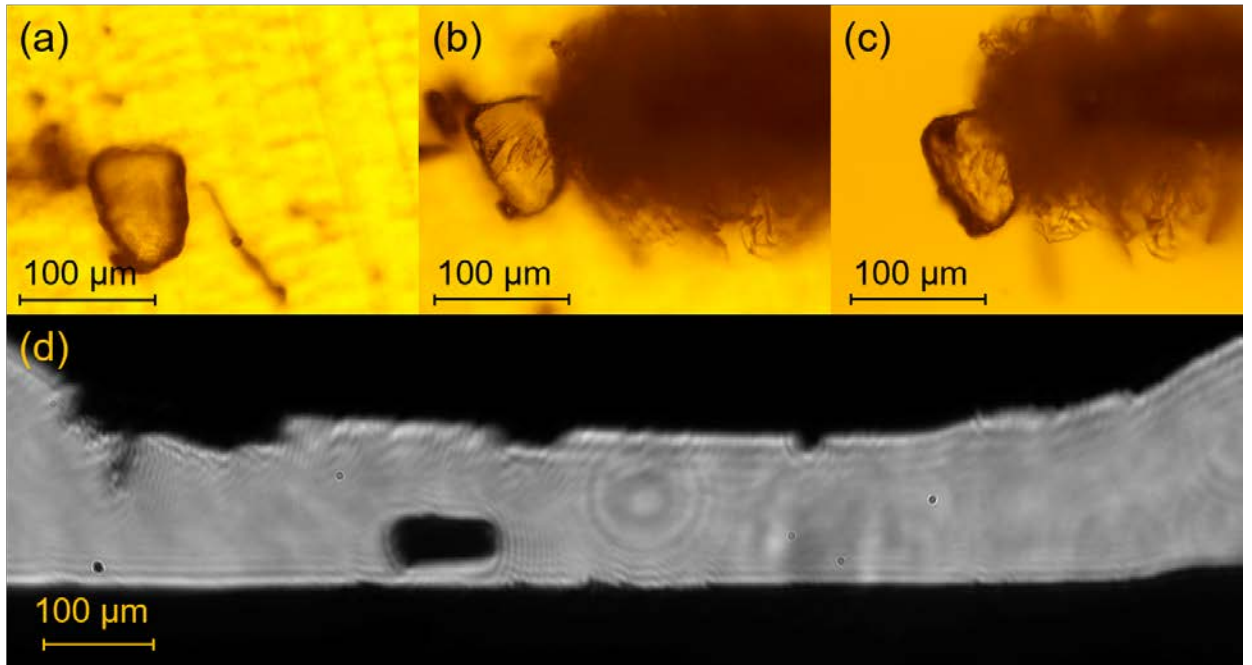


Figure 55: (a) A loading candidate is identified on an aluminum-foil covered microscope slide. (b) A camphor-coated probe tip is pressed against the selected particle. (c) The probe tip is lifted from the microscope slide, particle attached to the sticky camphor coating. (d) An image of the particle is captured using a camera and an LED for back-illumination after it has been successfully loaded into the trap. The variety of viewing images obtained for this particle allows the rough dimensions of the particle along each axis to be measured, giving the total dimensions of: $95\text{ }\mu\text{m}$ by $69\text{ }\mu\text{m}$ by $44\text{ }\mu\text{m}$ ($\pm 5\text{ }\mu\text{m}$).

The SAR loading technique relies on the fact that, at the point of release, the contact

point between the particle and the camphor it is attached to vanishes. This phenomenon can be seen somewhat conceptually in the images shown in Figure 56.

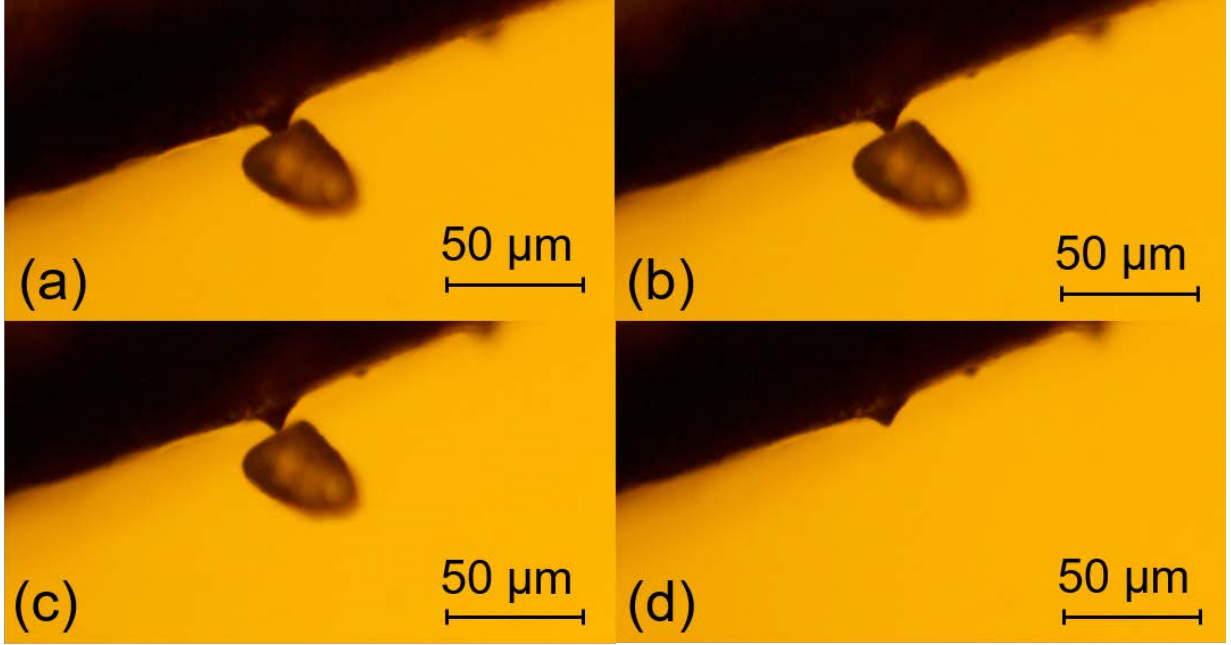


Figure 56: (a) A small SiC particle has been picked up by a camphor-coated tungsten probe tip and is now attached only by a thin cantilever of camphor extending from the metal tip. (b) The camphor continues to sublime, narrowing the connecting bridge of material. (c) The contact point between the particle and the camphor becomes extremely small, resulting in very low capillary and Van der Waals forces on the particle. (d) The particle releases from the camphor, falling with essentially negligible initial velocity.

A schematic using actual trap and probe tip dimensions is shown in Figure 57. The probe tip is typically slowly extended further into the trapping region than shown in the Figure.

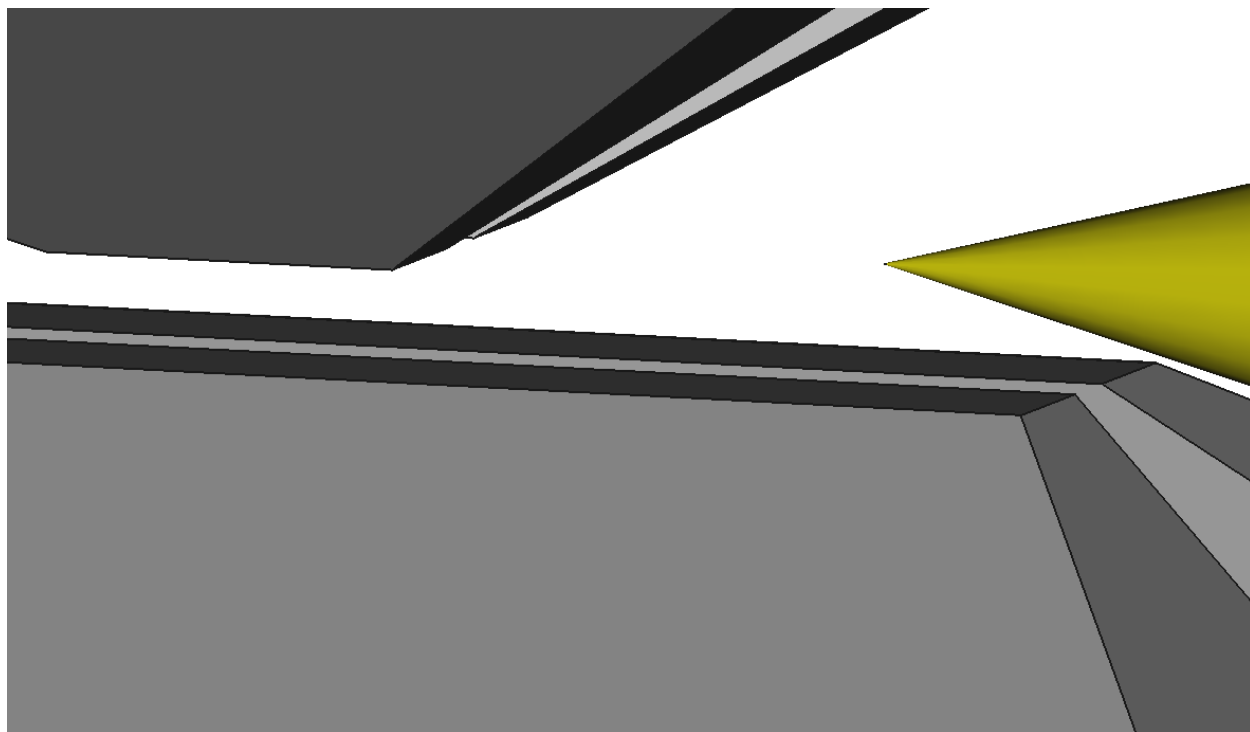


Figure 57: To-scale schematic showing the insertion of a SE-20T Signatone tungsten probe tip into a Short MGT . A camera imaging down the transverse trap axis is typically used to position the tip optimally in y and z . The transverse positioning is trickier, as discussed in Appendix A.

The SAR-loading method was tested extensively with many different particles. Over 50 trials using $\sim 20\text{ }\mu\text{m}$ borosilicate spheres were performed with approximately 30-40% success rates. Dozens of trials using $8\text{ }\mu\text{m}$ glass spheres were performed with slightly lower rates of success due to residual charge causing the lower-mass particles to stick more to the loading tip. And dozens of SiC-loading trials successfully loaded SiC particles of varying shapes and sizes, as illustrated in Figure 58.

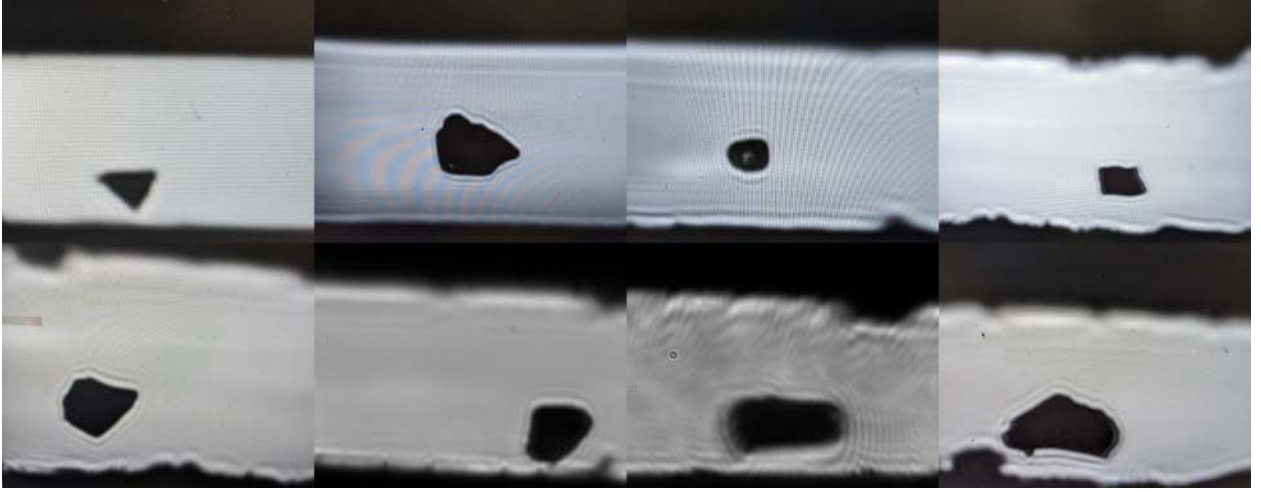


Figure 58: A collection of images of trapped SiC particles. The images are only roughly scaled, and were taken using different optics and across multiple different trap assemblies (though all with the same set of pole pieces). Each particle was loaded via SAR.

The technique, while somewhat time-consuming to learn and practice, provided unparalleled control over exactly which particles were loaded, and prevented significant build-up of untrapped particles, a consequence in other vacuum chambers in the lab of loading using an ultrasonic horn. Most critically, it allowed selective loading of individual 4H-SiC particles, a necessity for the spin-resonance measurements detailed in Chapter 3.

Loading GCPs into the Long MGT is significantly simpler. The particle is picked up with nonmagnetic tweezers and placed carefully into the trapping region. This is typically done with the particle entering along the transverse axis, and relies on the vertical gap being larger than the net thickness of the tweezers plus the particle diameter, but has a high rate of success. Particles can easily be unloaded in this manner as well.

Control

Much of the control code for various experiments discussed in this manuscript was operated with the aid of Pythics, a software written by Brian R. D’Urso. However, several pieces of software were written by the author of this manuscript for various aspects of

experiment control.

List of Control Code written by Connor E. Murphy for Experiment Control

- `analyze_collectionsA-D.py`. Analyzes unprocessed data from background-subtraction MDMR data for data folders A,B,C, and D.
- `analyze_collectionsE.py`. Analyzes unprocessed data from background-subtraction MDMR data for data folder E.
- `analyze_collectionsF.py`. Analyzes unprocessed data from background-subtraction MDMR data for data folders F.
- `analyze_collectionsI.py`. Analyzes unprocessed data from background-subtraction MDMR data for data folders I. Version for use with background-subtraction MDMR data can be found under `Data/trapped_data/HY0667-21B-2A`. Version for lock-in MDMR data can be found in `pulsed_mdmr_tests`.
- `analyzeA-D.py`. Analyzes processed data from background-subtraction MDMR data for data folders A,B,C, and D.
- `analyzeE.py`. Analyzes processed data from background-subtraction MDMR data for data folder E.
- `analyzeF.py`. Analyzes processed data from background-subtraction MDMR data for data folders F and I.
- `find_freq_avg.py`. Splits time data up into segments and averages to find frequency of averaged data.
- `find_freq_coherent.py`. Finds frequency of time domain data for a coherent signal without splitting into segments and averaging.

- `find_freq_mdmr.py`. Finds averaged frequency for time domain MDMR data that is split into bright and dark segments. Version for use with background-subtraction MDMR data can be found under `Data/trapped_data/HY0667-21B-2A`. Version for lock-in MDMR data can be found in `pulsed_mdmr_tests`.
- `first_look.py`. Gives a preliminary look at a single data set of time-domain background-subtracted MDMR data. Version for use with background-subtraction MDMR data can be found under `Data/trapped_data/HY0667-21B-2A`. Version for lock-in MDMR data can be found in `pulsed_mdmr_tests`, and produces processed data for `analyze_collectionsI.py`.
- `galvo_controller.py`. Used for controlling the THORLABSgalvo scanning mirror.
- `ODMR_control.py`. Used to control the ODMR measurement setup.
- `record_control_microwave.py`. Used to control the microwave drive for analysis separately from ODMR or MDMR setups.
- `mdmr_instrument_control.py`. Used to control the MDMR experimental setup.

SPIN-RESONANCE DETECTION IN SILICON CARBIDE

In this chapter a novel method for performing magnetic spin-resonance detection of V_{Si}^- (V2) centers in 4H-SiC is proposed, the steps taken to attempt the measurement are described, and the results from the attempt are reported.

Well-established methods such as optically detected magnetic resonance (ODMR) [15, 24, 83, 108, 142, 143, 152, 157] and electron spin resonance (ESR) [62–64, 102, 141] have already been shown to be effective for probing the nature of the V_{Si}^- (V2) defect centers, but this new method has the potential advantage of coupling the quantum states of individual centers to the mechanical motion of a trapped particle.

Point defect centers in SiC are a common subject for ESR measurements [15, 24, 62–64, 79, 81, 102, 108, 125, 135, 142, 143, 152, 157]. While various defects in SiC have been studied and characterized [64, 135], the defects of interest here are V_{Si}^- (V2) vacancy defect centers in 4H-SiC. The valance electrons in these defect centers can be treated as spin-3/2 systems which undergo Zeeman splitting under static magnetic fields on the order of a few milliteslas when driven on resonance by a microwave-frequency magnetic field.

Another common and effective way to characterize these vacancy centers is through optically detected magnetic resonance (ODMR) [24, 79, 142, 143, 157], where a change in the photoluminescence intensity of optically excited SiC can be observed in the presence of a resonant microwave drive. In ODMR, the measurement relies on the fact that the photoluminescence intensity of the bulk material is dependent on the spin states of the individual vacancy centers inside. Similarly, the magnetic moment of a sample of irradiated SiC material also depends on the spin state of individual defect centers. It is interesting, then, to consider the possibility of using a magnetic force on a piece of 4H-SiC to mechanically detect the spin-states of defect centers inside.

A detailed analysis of how to use spin-3/2 systems for vector magnetometry has been

published by Lee et al. [81], but a brief summary of the relevant physics starts with the expression for the Hamiltonian of a general defect center,

$$\mathcal{H} = g_e \mu_B \mathbf{S} \cdot \mathbf{B} + D \left[S_z^2 - \frac{1}{3} S(S+1) \right], \quad (47)$$

where $g_e \approx 2.002319$ is the electron gyromagnetic ratio, B_0 is an applied static magnetic field, $D \approx (70 \text{ MHz}) \frac{\hbar}{2}$ is a constant describing the strength of the zero-field splitting (ZFS), and S is the spin of the defect center. Here we have neglected the term describing the hyperfine splitting (often represented by the parameter E), since for the V_{Si}^- (V2) centers in SiC, $E \ll D$.

The real value of using V_{Si}^- (V2) centers for vector magnetometry can be seen from the dependence of the resonant energy E_R on both field strength B_0 and polar angle θ ,

$$E_R(\theta) = g_e \mu_B B_0 m_S + \frac{D}{6} (3 \cos^2 \theta - 1) (3m_S^2 - S(S+1)), \quad (48)$$

which allows for the magnetic field strength to be determined from the central point of the two measured resonant frequencies, and the angle to be determined from the difference between them.

It is important to note that Eq. (48) is only valid for relatively high field strengths; measurements made with $g_e \mu_B B_0 \sim D$ tend to be complicated by the mixing of eigenstates [81]. This effect is shown in Figures 59, 60, 61, and 62, where the eigenmodes for just two crossing are used to compare numerical solutions to Eq. (47) considering only two spin transitions (there are really four to be considered at low field strengths [24]) against approximations from Eq. (48).

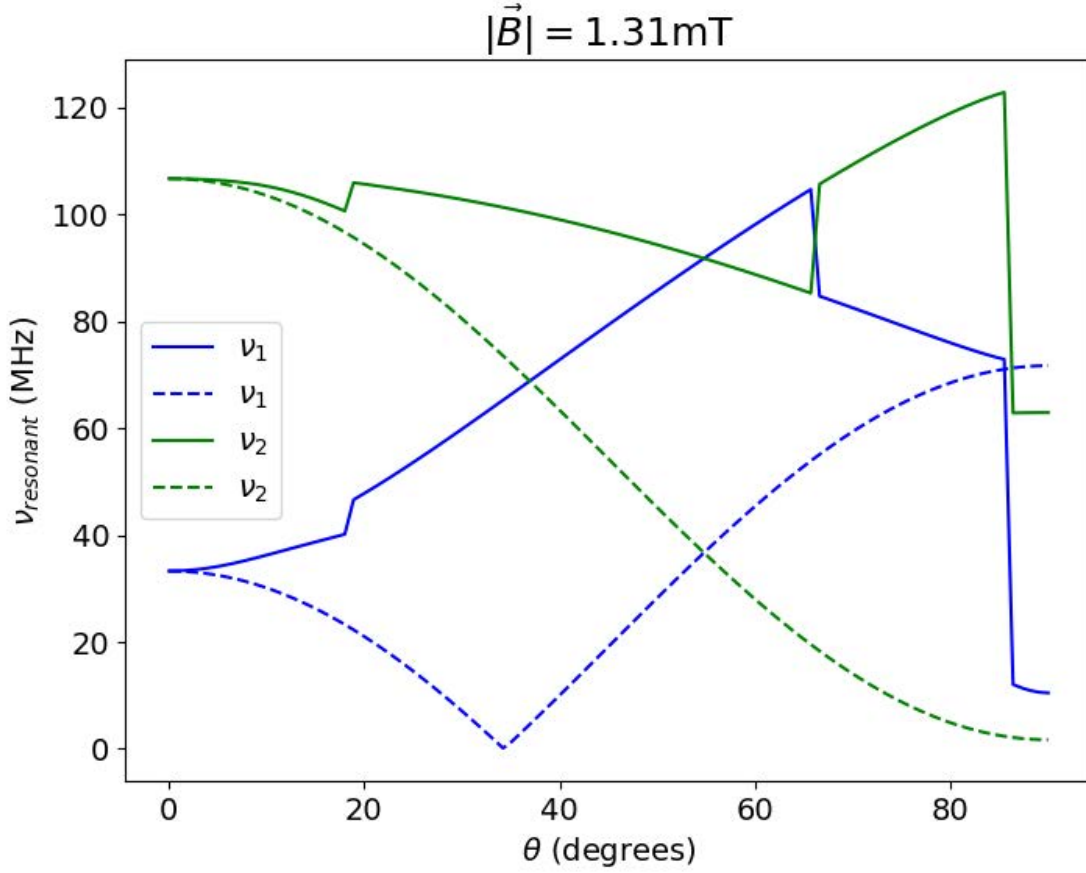


Figure 59: This figure was generated by `resonance_splitting_sic.py`. Theoretical resonant frequencies for $+\frac{3}{2} \rightarrow +\frac{1}{2}$ - and $-\frac{1}{2} \rightarrow -\frac{3}{2}$ spin transitions in V_{Si}^- (V2) centers found in 4H-SiC at fixed background field B_0 . The dashed lines are approximations from perturbation theory (taking $D \lesssim g_e \mu_B B_0$), while the solid lines are computed numerically from the eigenvalues of the Hamiltonian in Eq. (47). The choppy behavior in the numerical computations comes from eigenstates mixing.

Mixed eigenstates form a signal that would be hopelessly difficult to decipher without high SNR and some information about the c-axis of the 4H-SiC, neither of which would be available in measurements performed on a levitated particle.

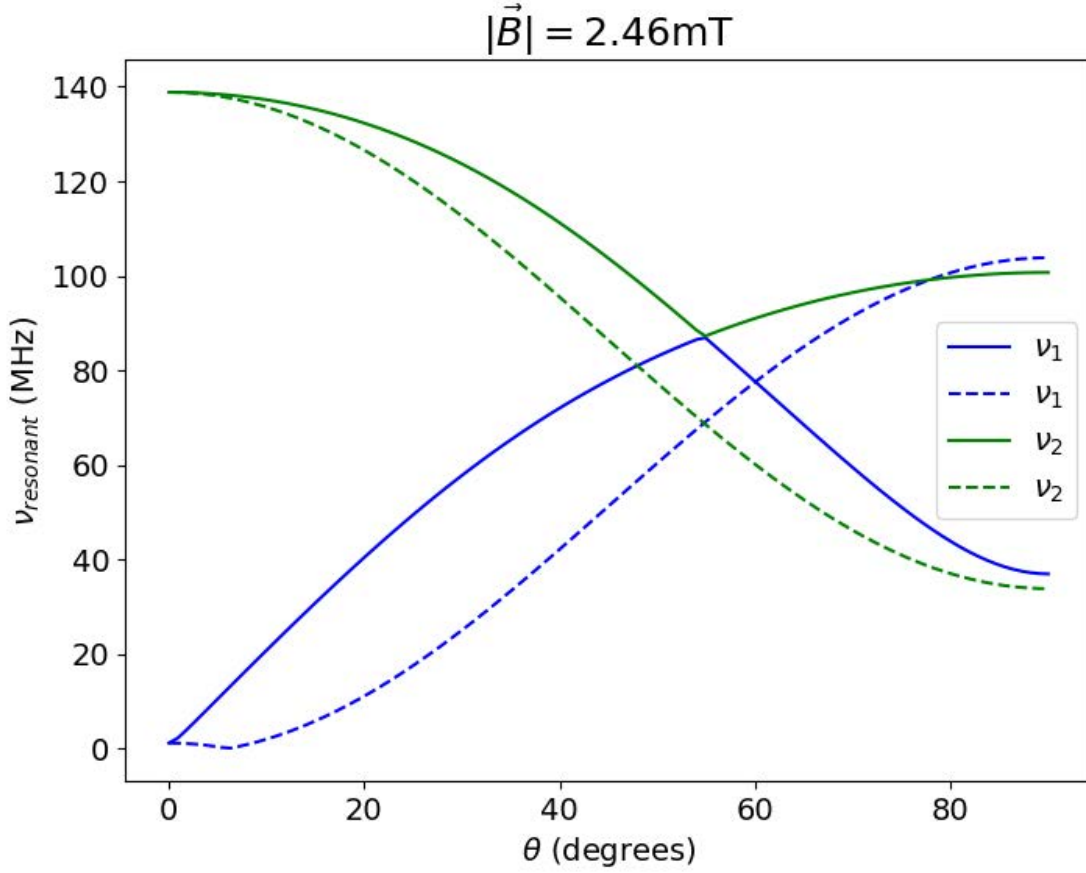


Figure 60: This figure was generated by `resonance_splitting_sic.py`. Theoretical resonant frequencies for $+\frac{3}{2} \rightarrow +\frac{1}{2}$ - and $-\frac{1}{2} \rightarrow -\frac{3}{2}$ spin transitions in V_{Si}^- (V2) centers found in 4H-SiC at fixed background field B_0 . The dashed lines are approximations from perturbation theory (taking $D \lesssim g_e \mu_B B_0$), while the solid lines are computed numerically from the eigenvalues of the Hamiltonian in Eq. (47).

As the field strength increases, mixed eigenstates become less of an issue as Eq. (48) becomes a better approximation to the resonant energies.

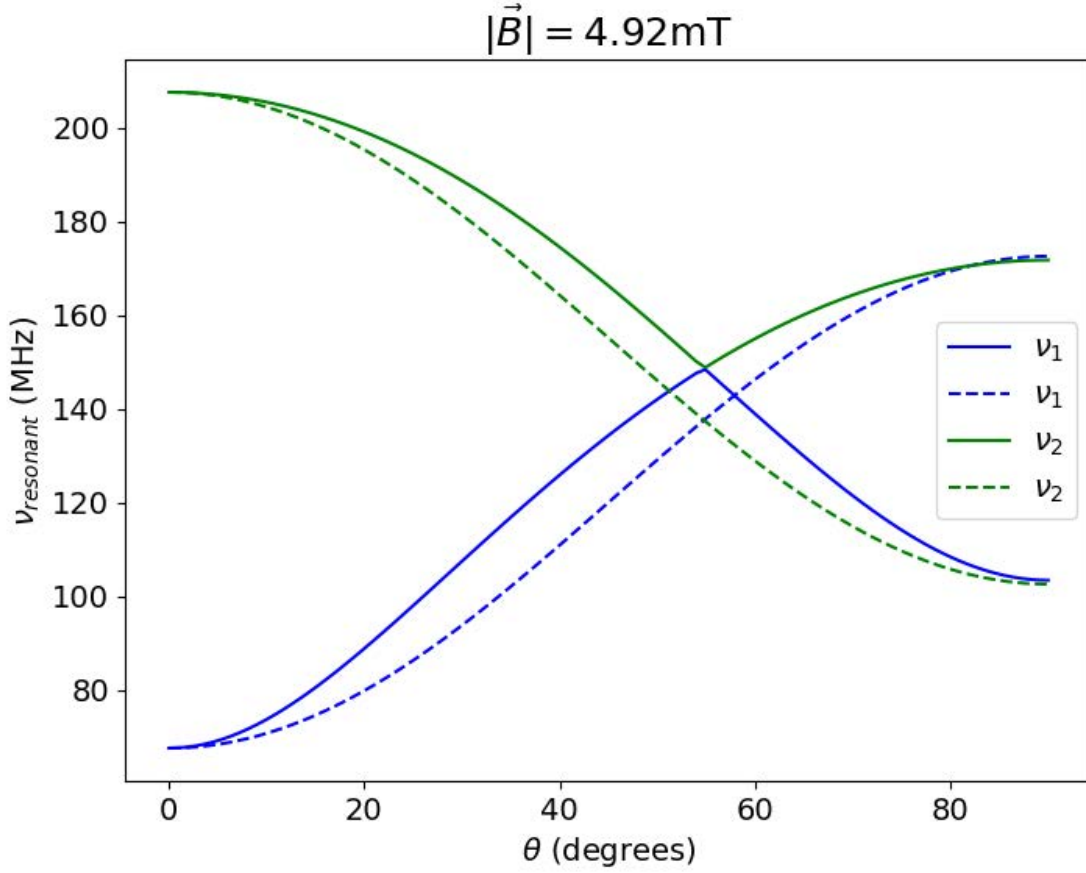


Figure 61: This figure was generated by `resonance_splitting_sic.py`. Theoretical resonant frequencies for $+\frac{3}{2} \rightarrow +\frac{1}{2}$ - and $-\frac{1}{2} \rightarrow -\frac{3}{2}$ spin transitions in V_{Si}^- (V2) centers found in 4H-SiC at fixed background field B_0 . The dashed lines are approximations from perturbation theory (taking $D \lesssim g_e \mu_B B_0$), while the solid lines are computed numerically from the eigenvalues of the Hamiltonian in Eq. (47).

We see that for field strengths B_0 such that $g_e \mu_B B_0 \gtrsim D$ the perturbation theory already starts to fit the numerical solution fairly closely.

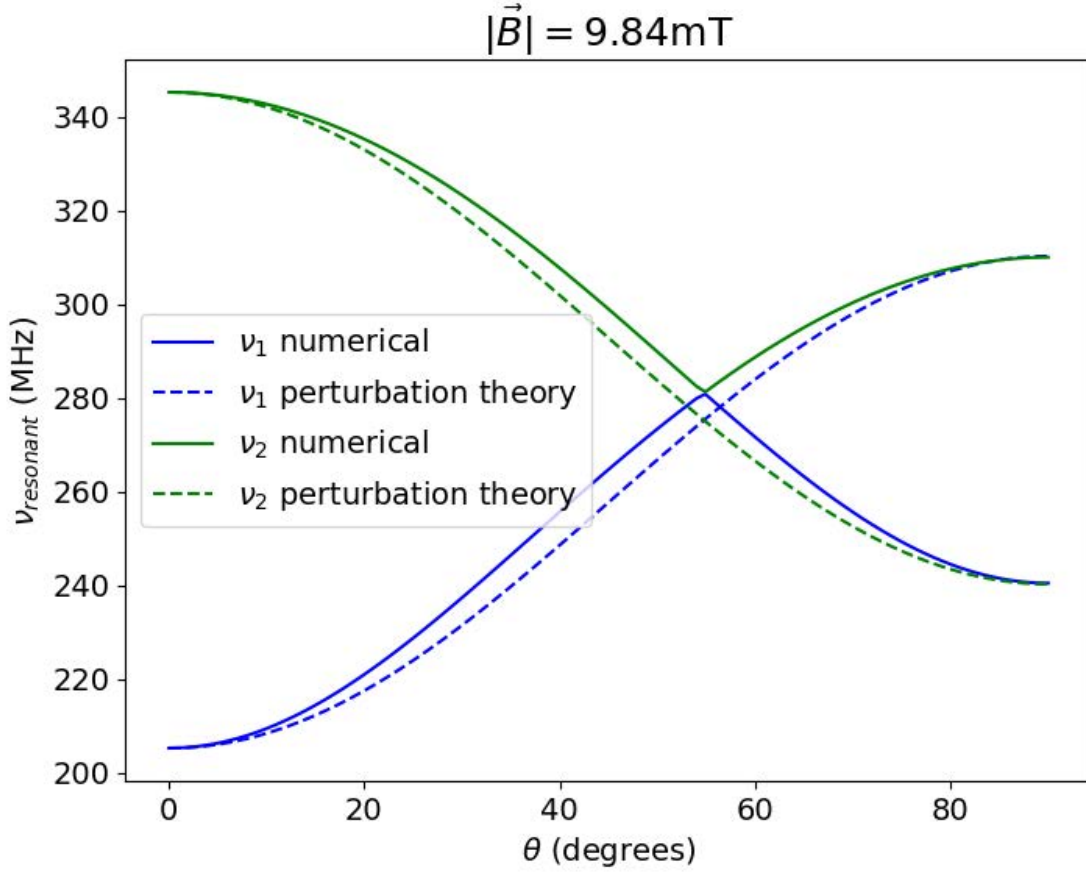


Figure 62: This figure was generated by `resonance_splitting_sic.py`. Theoretical resonant frequencies for $+\frac{3}{2} \rightarrow +\frac{1}{2}$ - and $-\frac{1}{2} \rightarrow -\frac{3}{2}$ spin transitions in V_{Si}^- (V2) centers found in 4H-SiC at fixed background field B_0 . The dashed lines are approximations from perturbation theory (taking $D \lesssim g_e \mu_B B_0$), while the solid lines are computed numerically from the eigenvalues of the Hamiltonian in Eq. (47).

If the external field is oriented parallel to the c-axis of the crystal, the signal is considerably simpler and can be represented accurately by Eq. (48) as shown in Figure 62.

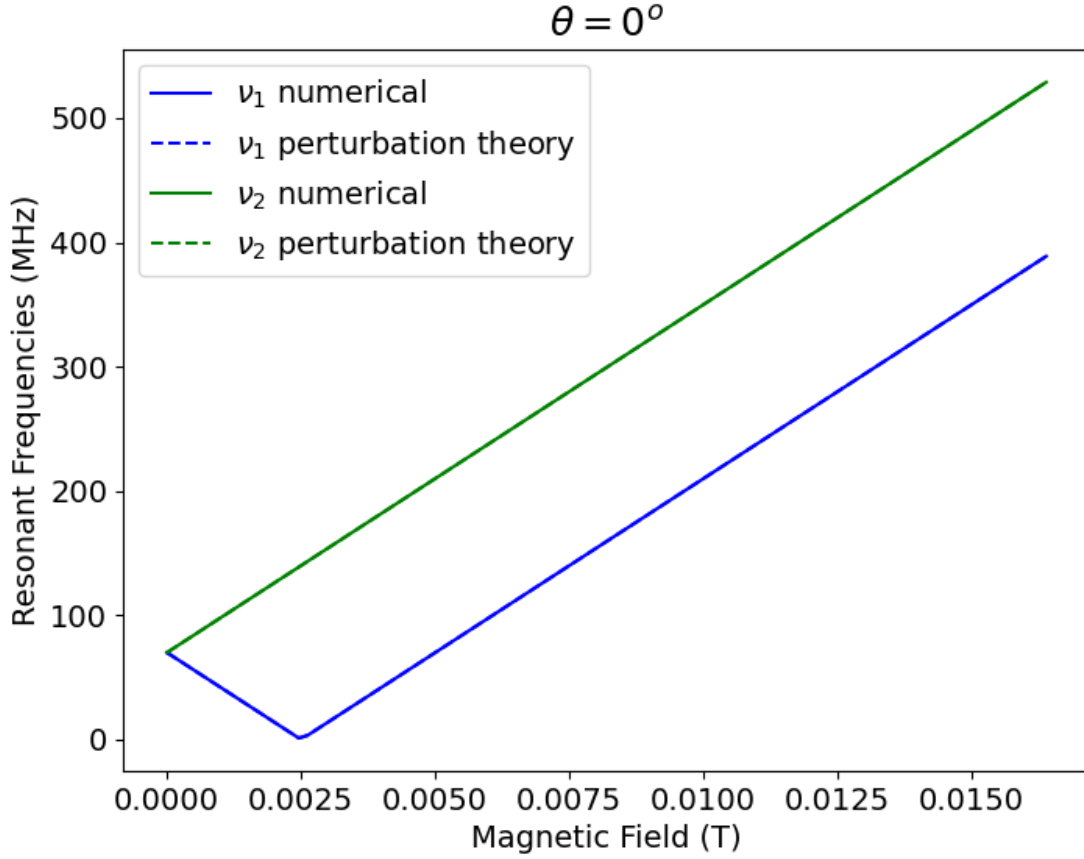


Figure 63: This figure was generated by `resonance_splitting_sic.py`. Theoretical resonant frequencies for $+\frac{3}{2} \rightarrow +\frac{1}{2}$ - and $-\frac{1}{2} \rightarrow -\frac{3}{2}$ spin transitions in V_{Si}^- (V2) centers found in 4H-SiC at fixed B -field polar angle θ . The dashed lines are approximations from perturbation theory (taking $D \lesssim g_e \mu_B B_0$), while the solid lines are computed numerically from the eigenvalues of the Hamiltonian in Eq. (47). Notice that the theory and the numerical calculation in this case give the same result.

At other angles, however, the fit is clearly less good unless B_0 is sufficiently strong, as can be seen in Figure 64.

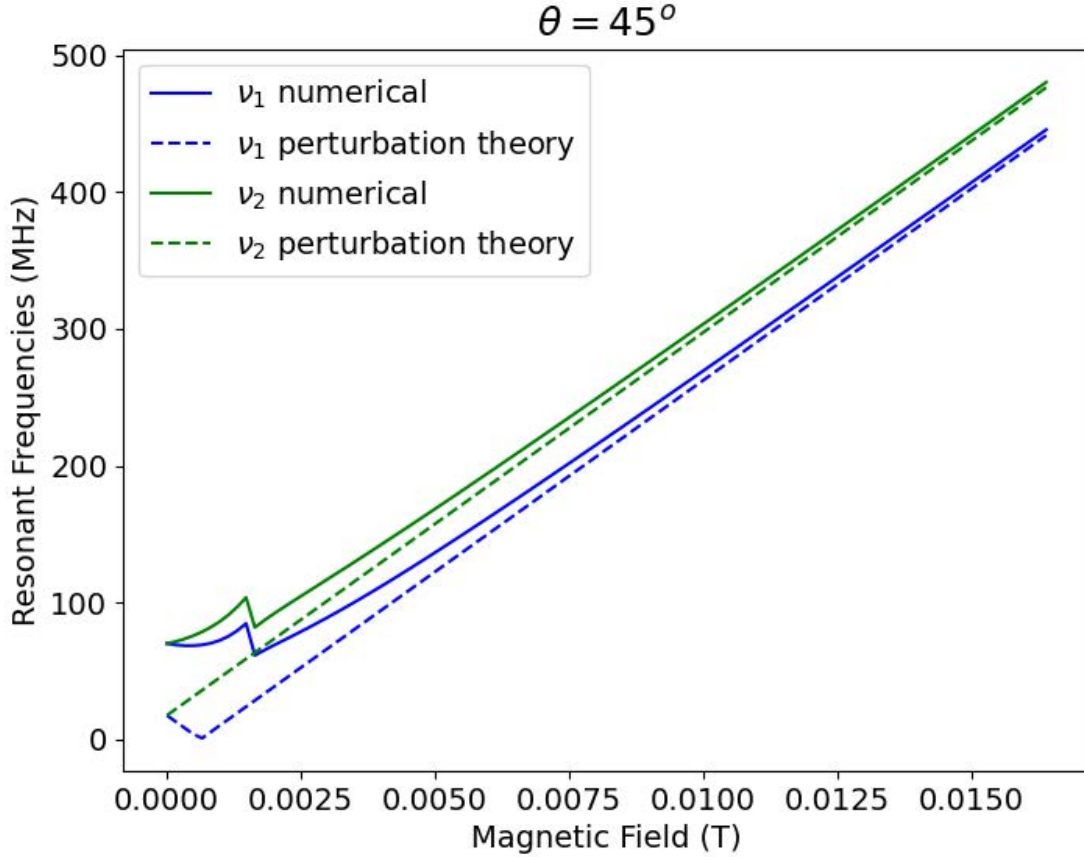


Figure 64: This figure was generated by `resonance_splitting_sic.py`. Theoretical resonant frequencies for $+\frac{3}{2} \rightarrow +\frac{1}{2}$ - and $-\frac{1}{2} \rightarrow -\frac{3}{2}$ spin transitions in V_{Si}^- (V2) centers found in 4H-SiC at fixed B -field polar angle θ . The dashed lines are approximations from perturbation theory (taking $D \lesssim g_e \mu_B B_0$), while the solid lines are computed numerically from the eigenvalues of the Hamiltonian in Eq. (47).

In general, since the goal is to perform spin-resonance measurements on levitated particles, the orientation of the c -axis will not be known. This leads to the conclusion that levitated trials should be performed with $g_e \mu_B B_0 \gtrsim D$ where possible to simplify the measurements.

A magneto-gravitational trap (MGT) can act as a low-loss, high-precision position detector for small diamagnetic particles [60, 109, 115, 140], allowing a way to directly measure forces acting on levitated particles. Because they rely on strong magnetic fields and magnetic

field gradients to create a trapping potential, the force acting on a particle containing defect centers in a MGT can be made to be dependent on the spin-states of the electrons in those defect centers. This leads to an interesting idea for an alternative measurement of phenomena in V_{Si}^- (V2) vacancy centers in 4H-SiC .

The measurement proposed in this chapter is mechanically detected magnetic resonance (MDMR). The basic idea is to manipulate the spin-states within the V_{Si}^- (V2) vacancy centers of 4H-SiC to apply a force to a piece of 4H-SiC levitated in a MGT under ultra-high vacuum on resonance with the particle's motion. One application of this is then to use the Zeeman splitting of the resonant frequencies to create a precise magnetometer capable of measuring magnetic field strengths and directions [79, 81] inside a MGT. A more ambitious direction would be to attempt to trap a 4H-SiC particle containing only one defect center, allowing the possibility to couple the motion of a macroscopic object to the spin-state of a single vacancy defect center.

This proposed technique works by tracking the motion of a magnetically levitated piece of 4H-SiC containing V_{Si}^- (V2) centers as it moves through a field gradient while manipulating the spin-centers as in ODMR using a microwave-frequency magnetic field and optical pumping at 785 nm.

It may also be possible to perform ODMR measurements on a trapped particle. This is not such a fundamentally different measurement than what has been done, but may be a reasonable step towards MDMR, and would allow the implementation of an in-trap magnetometer due to the splitting of spin resonances in V_{Si}^- (V2) centers. This has its own difficulties, however, since the static field gradient in our MGTs is high compared to the length scale of a trapped particle, or even of a diffraction-limited spot given the NA restraints imposed on the collection and excitation optics by the geometry of the trap and the vacuum chamber in which it is housed. This would likely cause any ODMR signal collected to be very broad, making it potentially difficult to observe.

Nevertheless, the initial direction pursued was to aim for performing ODMR on a trapped particle of 4H-SiC. Several samples of both irradiated and non-irradiated SiC were obtained from the Choyke/Devaty research group in Pittsburgh, of which a few were used for performing measurements. They are listed below with labels given to refer to them in the text to follow.

- Unknown sample C4 for making particle batches A18 and A20.
 - Camphor loading trials
 - Albert (A18-4)
 - Billy Joe (A18 Stuck)
- **03021903T4 #29.** Epi sample for ODMR. Electron fluence $1 \times 10^{16} \text{ cm}^{-2}$ @ 1 MeV, annealed in Argon for 1 hour at 550°C . Additional details can be found in `SiC_Sample_List.pdf`.
- **04052911T1 #19.** Epi sample for ODMR. Electron fluence $1 \times 10^{15} \text{ cm}^{-2}$ @ 1 MeV, annealed in Argon for 1 hour at 550°C . Additional details can be found in `SiC_Sample_List.pdf`.
- **HY0667-21B #1.** Boule sample for ODMR. Electron fluence $1 \times 10^{16} \text{ cm}^{-2}$ @ 1 MeV, un-annealed. Additional details can be found in `SiC_Sample_List.pdf`.
- **HY0667-21B #2.** Boule sample for ODMR and MDMR. Electron fluence $1 \times 10^{16} \text{ cm}^{-2}$ @ 1 MeV, annealed in Argon for 1 hour at 550°C . Additional details can be found in `SiC_Sample_List.pdf`.
 - Cinderella (HY0667-21B-2A)
 - Dorito (HY0667-21B-2B)

Photoluminescence of Trapped Particle

There were two preliminary results to demonstrate before attempting an ODMR measurement on a levitated particle. The first was to demonstrate that we could measure ODMR on a piece of bulk 4H-SiC , both to demonstrate the viability of the sample and to develop expertise in how to do it. The second was to demonstrate that a photoluminescence spectrum could be obtained from a levitated particle of SiC. For the former, I inherited an incomplete setup originally designed by Charlie Lewandowski. For the latter, a Fast Fastie spectrometer (Jarrell Ash 75-000, S/N 55261) was used.

The experimental setup used to take a photoluminescence spectrum of SiC levitated in a MGT is shown in Figure 65. The same setup was used to take a spectrum of a bulk sample of SiC of the same material as the levitated particle, just with the trap replaced by a sample held in an adjustable mount (THORLABS LH160CA).

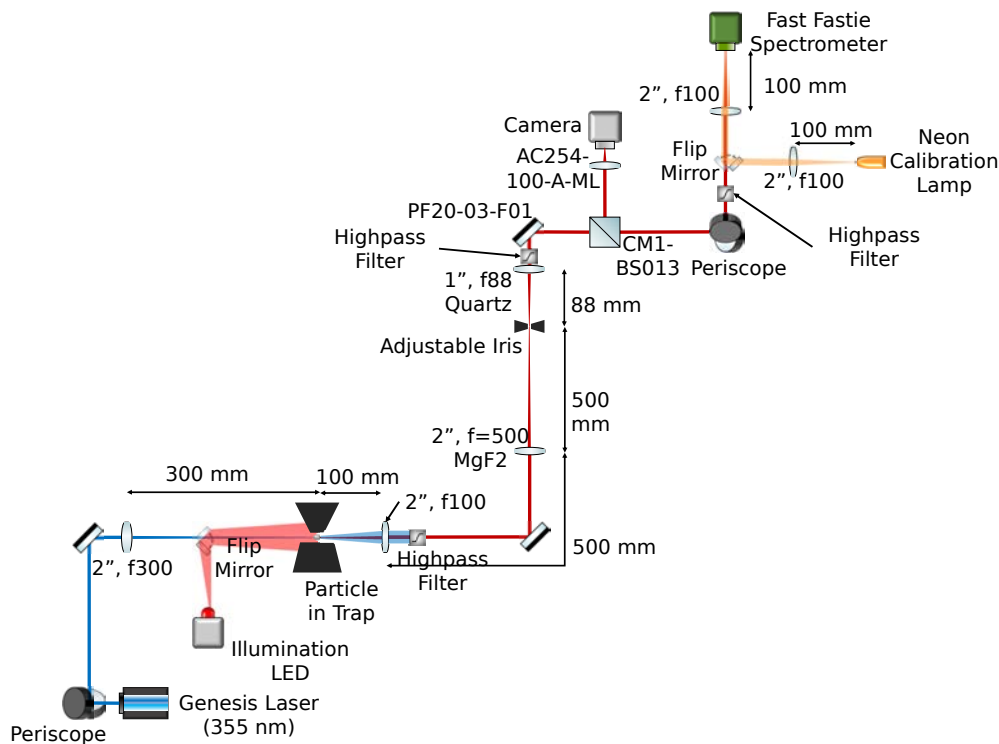
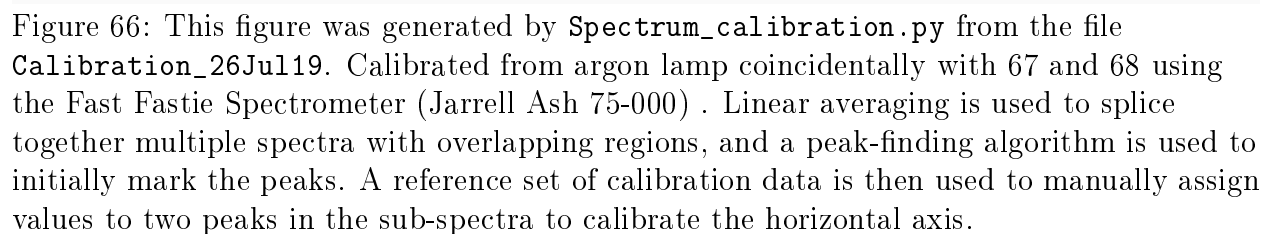


Figure 65: Many optical elements for this setup were not well-documented. However, the exact elements used are specified where available, and the focal lengths of all the lenses are still specified. The LED used was labeled LUXDRIVE RHEA 8018, and was used for experiments throughout all spectroscopy for illumination of trapped particles and back-illumination of bulk samples for aligning optics for ODMR trials.

The Fast Fastie Spectrometer (Jarrell Ash 75-000) was set up with a 500 nm diffraction grating with a 590 lines/mm blaze. Additionally, the corners were taped up with reflective tape to prevent excess light from entering. It was always operated with its dewar filled with liquid nitrogen.

Several spectra were collected of this sample using the Fast Fastie Spectrometer (Jarrell Ash 75-000) , but only the most useful are shown here. Initial calibrations were done



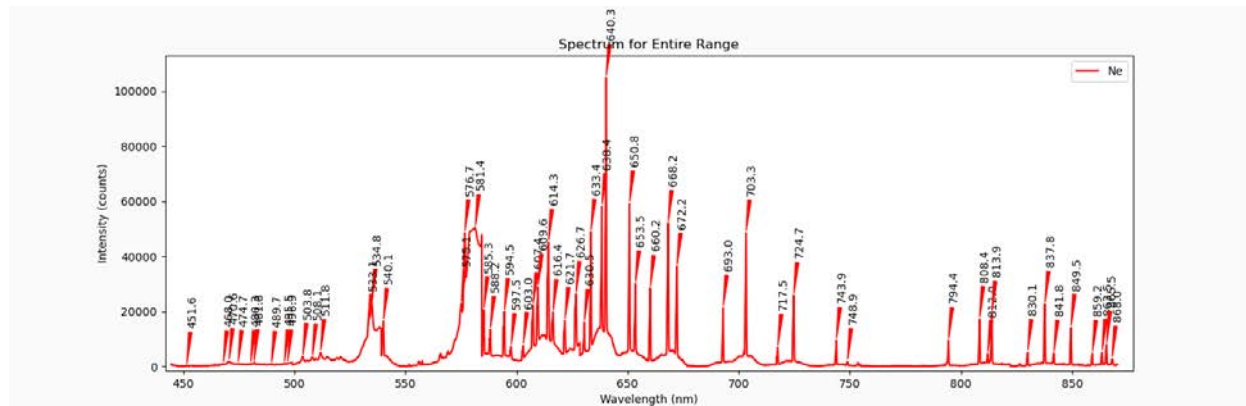


Figure 67: This figure was generated by `Spectrum_calibration.py` from the file `Calibration_26Jul19`. Calibrated from neon lamp coincidentally with 66 and 68 using the Fast Fastie Spectrometer (Jarrell Ash 75-000) . Linear averaging is used to splice together multiple spectra with overlapping regions, and a peak-finding algorithm is used to initially mark the peaks. A reference set of calibration data is then used to manually assign values to two peaks in the sub-spectra to calibrate the horizontal axis.

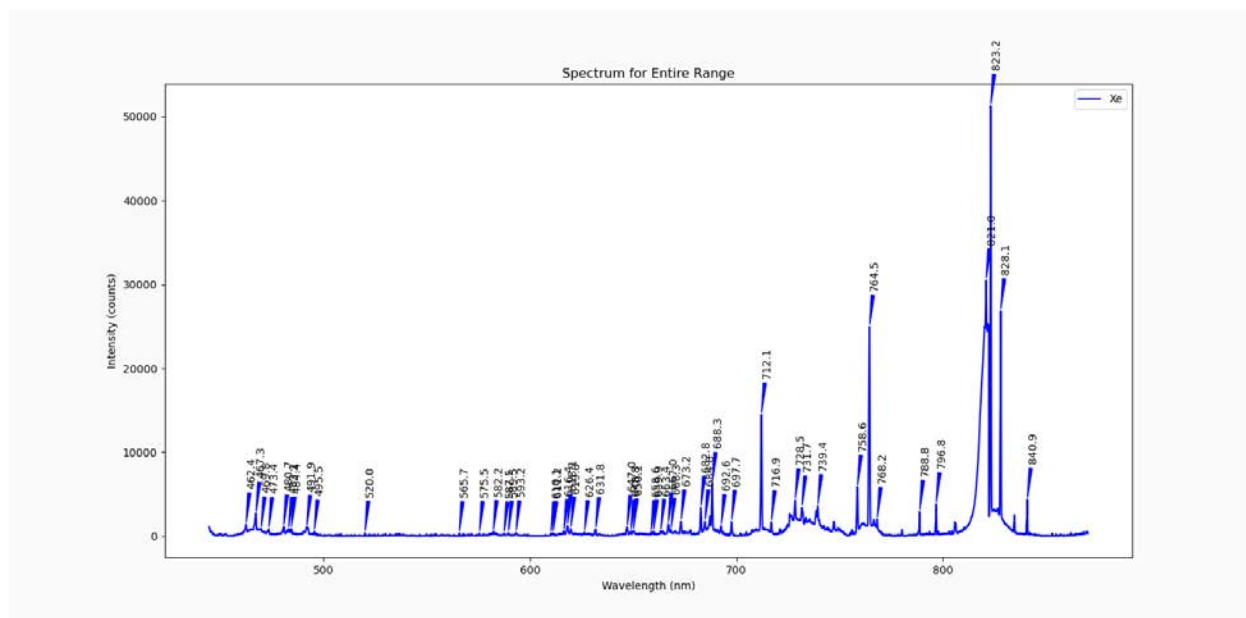


Figure 68: This figure was generated by `Spectrum_calibration.py` from the file `Calibration_26Jul19`. Calibrated from xenon lamp coincidentally with 66 and 67 using the Fast Fastie Spectrometer (Jarrell Ash 75-000) . Linear averaging is used to splice together multiple spectra with overlapping regions, and a peak-finding algorithm is used to initially mark the peaks. A reference set of calibration data is then used to manually assign values to two peaks in the sub-spectra to calibrate the horizontal axis.

A sample calibration spectrum plotting spectra from all three calibration lamps over a single range can be seen in Figure 69. Using these spectra in conjunction with calibration data from NIST [78], spectra could be calibrated over wavelengths from about 450 nm to 720 nm.

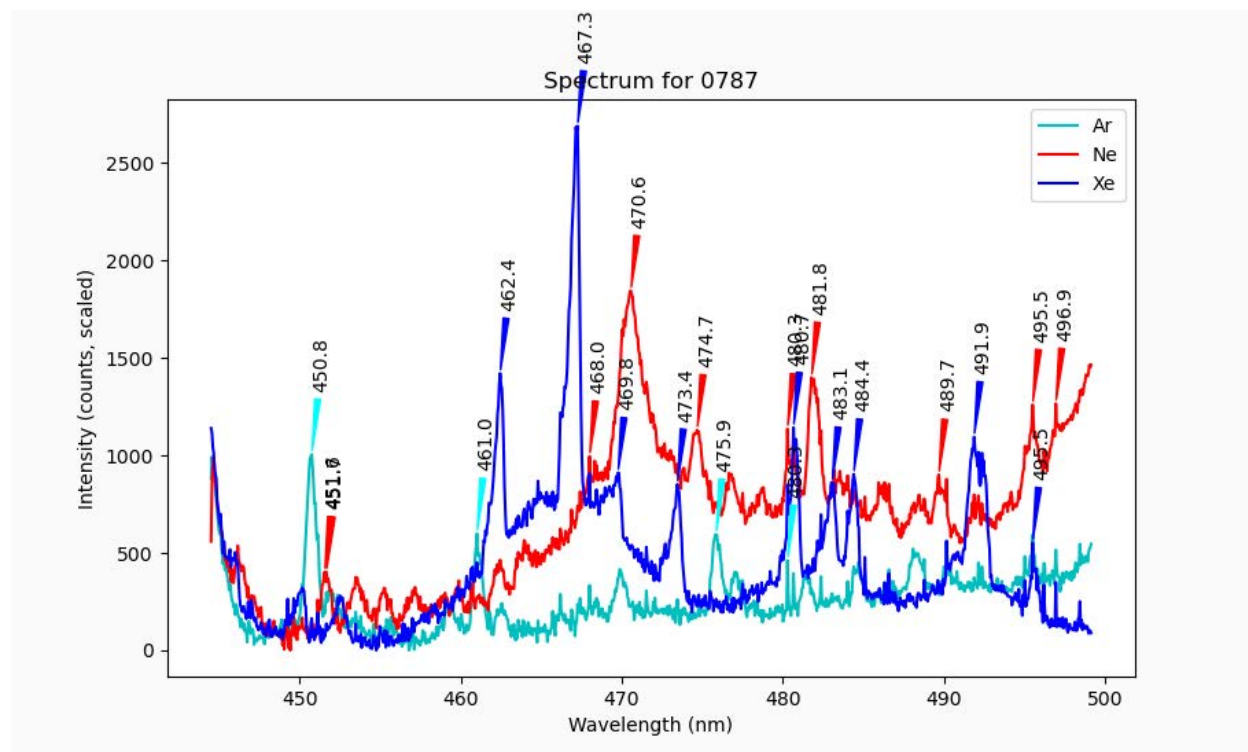


Figure 69: This figure was generated by `Spectrum_calibration.py` from the file `Calibration_26Jul19`. Spectrum obtained using three different calibration lamps without adjusting the dial on the spectrometer in between. Peaks automatically found using a peak-finding algorithm, and labeled by hand with by comparing to reference spectra for each lamp.

Once all three lamps had been used to get a calibration over the desired wavelength range, only the neon lamp was used for calibration of the data. Because the grating on the spectrometer was so fine, multiple spectra were taken and spliced together, which led to a method of moving the grating in such a way that every two consecutive spectra had at least one neon calibration peak in common. A neutral density (ND) filter was used for light collection from the calibration lamp, but not for actual measurement data.

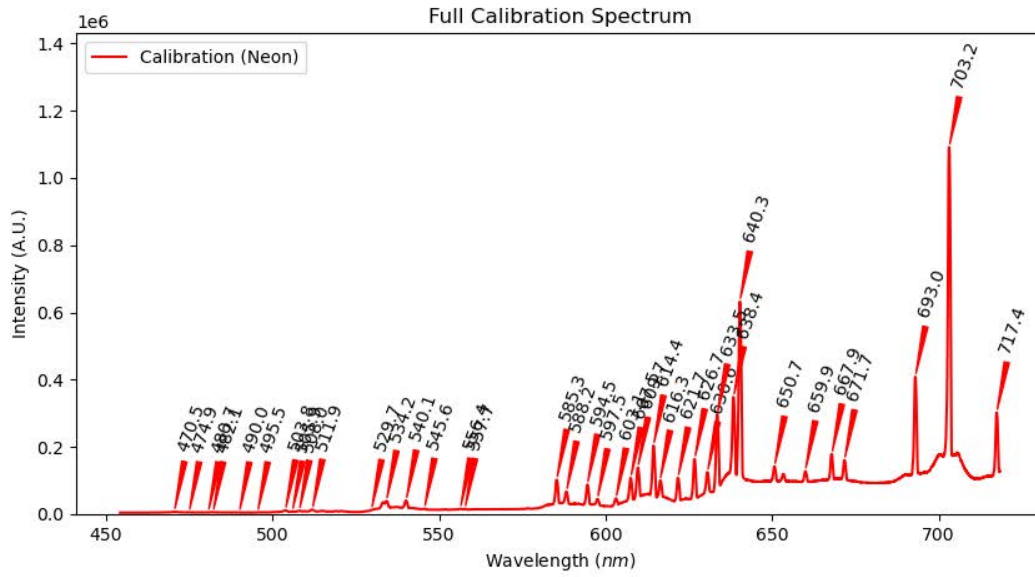


Figure 70: This figure was generated by `Plot_spectrum.py` from the file `A18_Bulk`, with calibration specifications obtained from `Calibrate_Spectra.py`. Linear averaging is used to splice together multiple spectra with overlapping regions, and a peak-finding algorithm is used to initially mark the peaks. A reference set of calibration data is then used to manually assign values to two peaks in the sub-spectra to calibrate the horizontal axis.

Two additional techniques were employed to analyze data from the actual SiC samples. First, a background collection was taken with the sample light blocked. These data were then subtracted off the measured data. Second, it was found that several very fine spikes made it difficult to make out the signal we saw. These were located using an algorithm to find high, narrow spikes, and set equal to the average value of the surrounding points. This can be seen in the Figure 71.

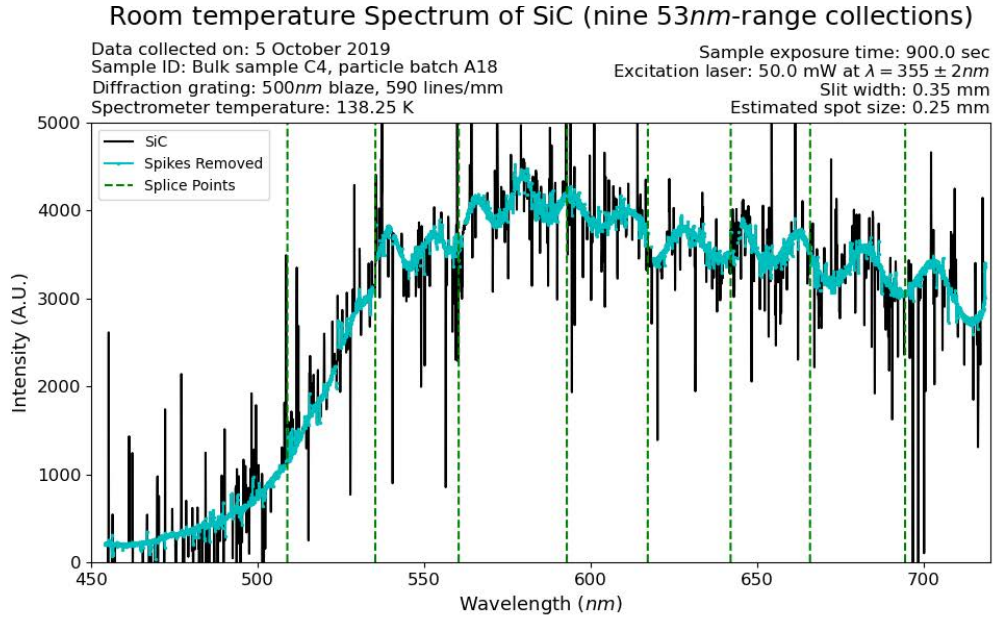


Figure 71: This figure was generated by `Plot_spectrum.py` from the file `A18_Bulk`, with calibration specifications obtained from `Calibrate_Spectra.py`. The background signal was measured and subtracted off from the sample signal, a linear averaging is used to splice together multiple spectra with overlapping regions, and high narrow spikes are removed using an algorithm.

Note the characteristic broad hump for room-temperature SiC starting around 525 nm. Because of the high-temperature, none of the peaks from defect centers can be clearly seen.

The SAR loading technique was used to load Albert (C4-A18-4) into the short MGT. Another particle from the same batch was loaded previously and spectra were collected from it, but here only data collected from Albert (C4-A18-4) are shown since we were able to acquire images of him at every stage of the SAR loading process (see Figure 55), making for a neat demonstration of the technique. An image of Albert (C4-A18-4) without the SAR loading cartoon is shown in Figure 72.

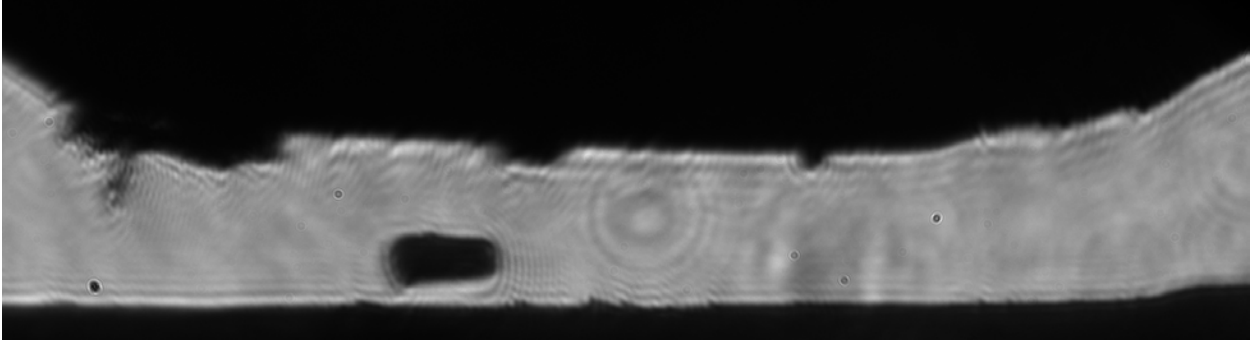


Figure 72: Image of the particle denoted as Albert (C4-A18-4) taken both inside and outside the trap. This particle was used to do spectroscopy on levitated SiC at room temperature. This is also the largest measured particle ever trapped in this lab using the SAR loading technique, measuring $95\text{ }\mu\text{m}$ by $69\text{ }\mu\text{m}$ by $44\text{ }\mu\text{m}$ ($\pm 5\text{ }\mu\text{m}$).

The calibration spectrum for the trapped particle was taken coincidentally with the sample spectrum, and is shown in Figure 73. The data was taken with the trap shut off from the outside in a vacuum chamber left at atmospheric pressure.

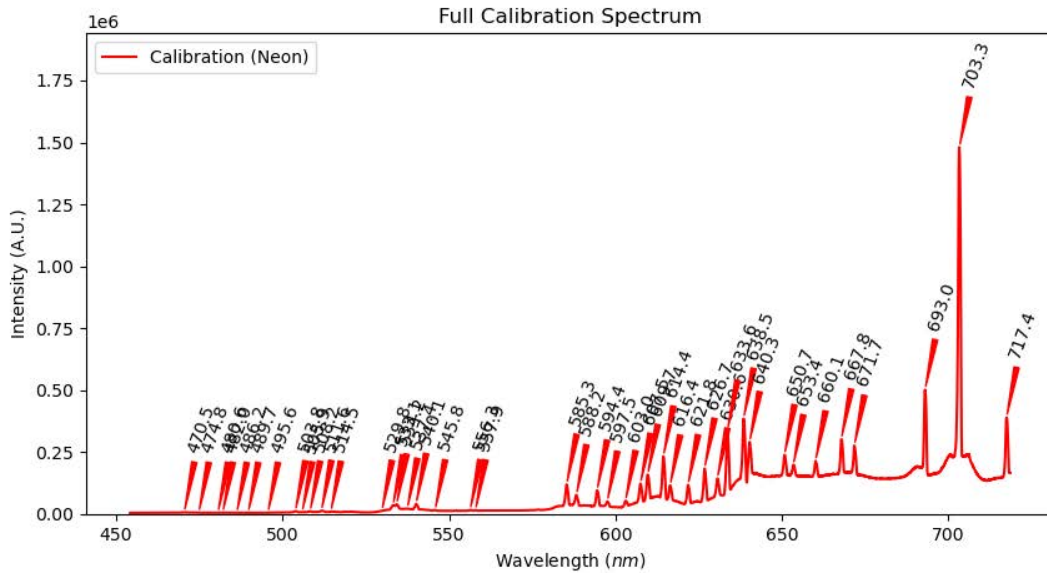


Figure 73: This figure was generated by `Plot_spectrum.py` from the file `A18_Particle_4`, with calibration specifications obtained from `Calibrate_Spectra.py`. Linear averaging is used to splice together multiple spectra with overlapping regions, and a peak-finding algorithm is used to initially mark the peaks. A reference set of calibration data is then used to manually assign values to two peaks in the sub-spectra to calibrate the horizontal axis.

The photoluminescence (PL) spectrum for magnetically levitated SiC is shown in Figure 74. Note that the spikes in this data are much more prevalent due to the longer exposure time and lower signal strength. Nevertheless, the broad glow of SiC can still be clearly seen.

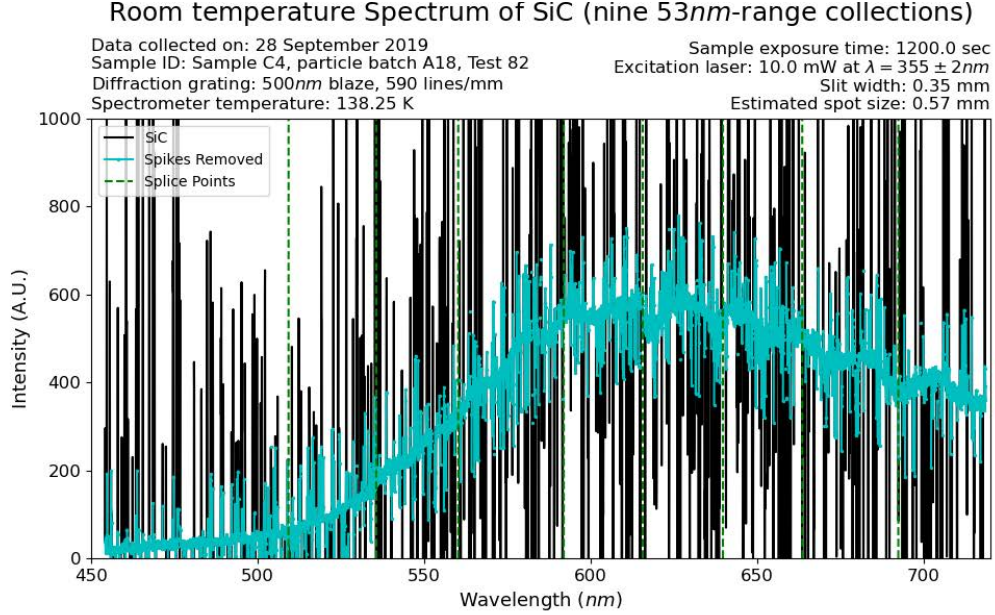


Figure 74: This figure was generated by `Plot_spectrum.py` from the file `A18_Particle_4`, with calibration specifications obtained from `Calibrate_Spectra.py`. The background signal was measured and subtracted off from the sample signal, a linear averaging is used to splice together multiple spectra with overlapping regions, and high narrow spikes are removed using an algorithm.

Figure 75 shows the PL spectra from the bulk sample and the trapped particle plotted next to each other. Aside from the lower SNR for the trapped particle, we also see that the broad peak has shifted down slightly in energy. This may be due to a difference in heat-dissipation or some other effect. Either way, the data demonstrate the feasibility of performing spectroscopy on levitated particles.

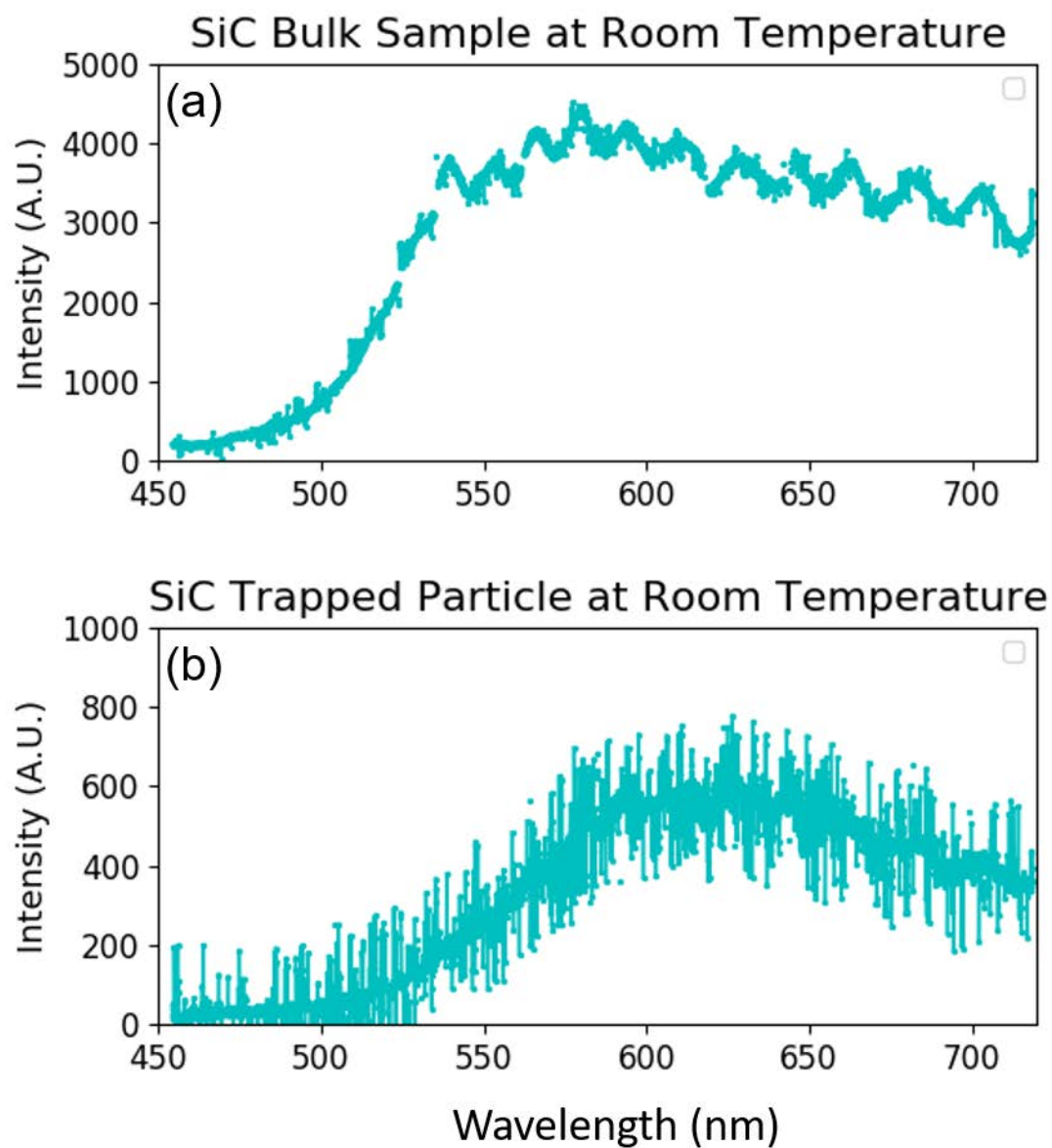


Figure 75: This figure was generated by `Plot_spectrum_stack.py` from the files `A18_Bulk` and `A18_Particle_4`, with calibration specifications obtained from `Calibrate_Spectra.py`.

The SNR of the two sets of data also indicates a measure of the relative difficulty associated with an ODMR measurement of a trapped particle.

ODMR

The basic setup employed to perform ODMR was as follows: A 785 nm power-stabilized laser was aimed at a sample of irradiated 4H-SiC . PL light was collected back through the same optics and split off from the laser light using a dichroic mirror. This light was then coupled into a SPCM connected to a gated photon counting module (SR400). Simultaneously, a microwave-frequency current was run through a 30 μm gold-plated tungsten wire glued in such a way that its length ran across the surface of the sample. In this way the V_{Si}^- (V2) centers were optically pumped by the laser and excited using the microwave drive (MD).

Setup

A full diagram of the optical setup used for ODMR is shown in Figure 76. Several changes were made to the setup from its initial version in order to get a measurement of ODMR.

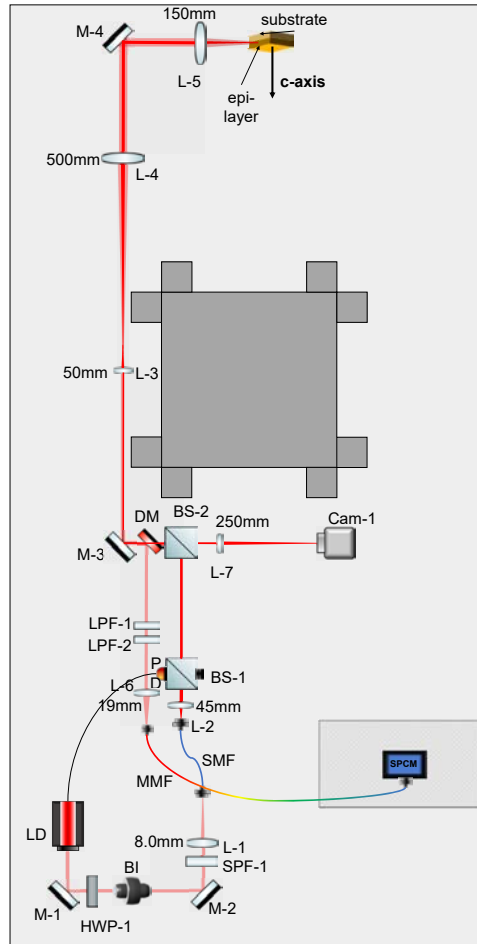


Figure 76: The index of part lists for this figure can be found in Section 3. The diagram is meant to give a detailed parts list for the optics used, as well as a rough outline of the placement of the optics.

First, the objective was changed to a 100 mm achromat (AC508-100-B-ML). Second, the laser diode was changed to a frequency-stabilized diode (LD785-SEV300). After making these changes, the Galvo-scanning mirror (THORLABSGVS012 TSH32411-X) was used to make several scans of the sample surface, such as the one shown in Figure 79. Despite power-stabilizing the laser, the intensity of the PL coming from the sample from any given point fluctuated greatly. Moving the photodiode used to provide feedback to the laser diode after the single-mode fiber (SMF) coupling helped somewhat, but still did not resolve the issue.

The issue was finally resolved when the beam splitter adjacent to the dichroic mirror (BS-2) was replaced with the one shown in Figure 76 (BS-014); it had originally been PBS202. Once this was resolved, adjusting the thresholding levels on the SR400 allowed the first measurements of ODMR.

When coupling light into the multimode fiber, a chopper wheel was placed between BI and M-2, connected to a lock-in amplifier (SRS Model SR810 DSP). This allowed the very small PL signal from the sample to be found even with significant background light present.

Optical Component List for Figure 76 For optical component lists, items marked with a “*” are new additions from the previous setup displayed. The notation “[C]” indicates that the components were fixed rigidly to cage rods, usually via custom aluminum mounts, and “[Al]” indicates that the optics were covered in aluminum foil to prevent stray light interfering with the signal.

- **LD.** 785 nm, 300 mW, Ø9 mm TO Can, E Pin Code, VHG Wavelength-Stabilized Single-Frequency Laser Diode (LD785-SEV300). Mounting: Newport Model700C-42Lens.
- **M-1.** Ø1" Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: POLARIS-K1E.
- **HWP-1.** 780 nm half-wave plate (WPH05M-780). Mounting: RSP1. [C]
- **BI.** Free-Space Isolator, 780 nm, Ø4.7 mm Max Beam, 1.7 W Max (IO-5-780-VLP). Mounting: SM1RC.
- **M-2.** Ø1" Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: POLARIS-K1E.
- **SPF-1.** 800 nm shortpass filter (FESH0800). Mounting: KC1-T. [C]

- **L-1.** $f = 8.0$ mm, $NA = 0.5$, Mounted Geltech Aspheric Lens, AR: 600 - 1050 nm (C171TMD-B). Mounting: CP1XY. [C]
- **SMF.** $NA = 0.12$, Single Mode Fiber (P5-780PM-FC-2). Mounting: SM1FC/A, SM1Z. [C]
- **L-2.** $f = 45$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-045-B-ML). Mounting: CXY1A. [C]
- **BS-1.** 10:90 (R:T) Non-Polarizing Beamsplitter Cube, 700-1100 nm, 20mm (BS044). Mounting: CCM1-4ER. [C]
- **PD.** Large Area Mounted Silicon Photodiode, 350-1100 nm, Anode Grounded (SM1PD1B, NE20B-B). Mounting: CCM1-4ER. [C]
- **BS-2.** 50:50 Non-Polarizing Beamsplitter Cube, 700-1100 nm, 20mm (BS014). Mounting: B6C. [C]
- **DM.** 25 mm x 36 mm Shortpass Dichroic Mirror, 805 nm Cutoff (DMSP805R). Mounting: FFM1, B4CRP. [C]
- **M-3.** Optotune Galvo-scanning mirror (MR-E-2). Mounting: TR50/M.
- **L-3.** $f = 50$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-050-B-ML). Mounting: CP02. [C]
- **L-4.** $f = 500$ mm, Ø2" Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC508-500-B-ML). Mounting: LCP34. [C]
- **M-4.** 2" Broadband Dielectric Elliptical Mirror, 750 - 1100 nm (BBE2-EO3). Mounting: KCB2C. [C]

- **L-5.** $f = 100$ mm, $\varnothing 2''$ Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC508-100-B-ML). Mounting: CXY2, SM2V05. [C]
- **LPF-1.** Semrock 830 nm longpass (LP02-830RU-25). Mounting: B4CRP. [C]
- **LPF-2.** $\varnothing 1''$ Longpass Filter, Cut-Off Wavelength: 900 nm (FEL0900). Mounting: CP02. [C]
- **L-6.** $f=19$ mm, $\varnothing 1/2''$ Achromatic Doublet, SM05-Threaded Mount, ARC: 650-1050 nm (AC127-019-B-ML). Mounting: CXY1A. [C]
- **MMF.** $NA = 0.10$, Multimode Fiber (FG105LVA). Mounting: SM1FC/A, SM1Z. [A]
- **SPCM.** Single Photon Counting Module (SPCM-AQRH-14-FC). Mounting: $-$.
- **L-7.** $f = 250$ mm, $\varnothing 1''$ Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-250-B-ML). Mounting: SM1Lxx. [C]
- **Cam-1.** Basler Camera (acA2440-75um). Mounting: CP12, C-MOUNT EXT. [C]

Microwave Drive The MD was generated by an SRS SG384 and then amplified using a Mini Circuits ZHL-25W-272+ RF amplifier. The system is described in greater detail in Figure 129, but for now it is sufficient to state that the power at the wire was amplified to be +28 dB higher than at the signal generator. For the ODMR tests, no bias tees were used to isolate the signal, and no explicit DC bias voltage was set.

Epi Samples Describe the setup used and the samples available, as well as the annealing process used.

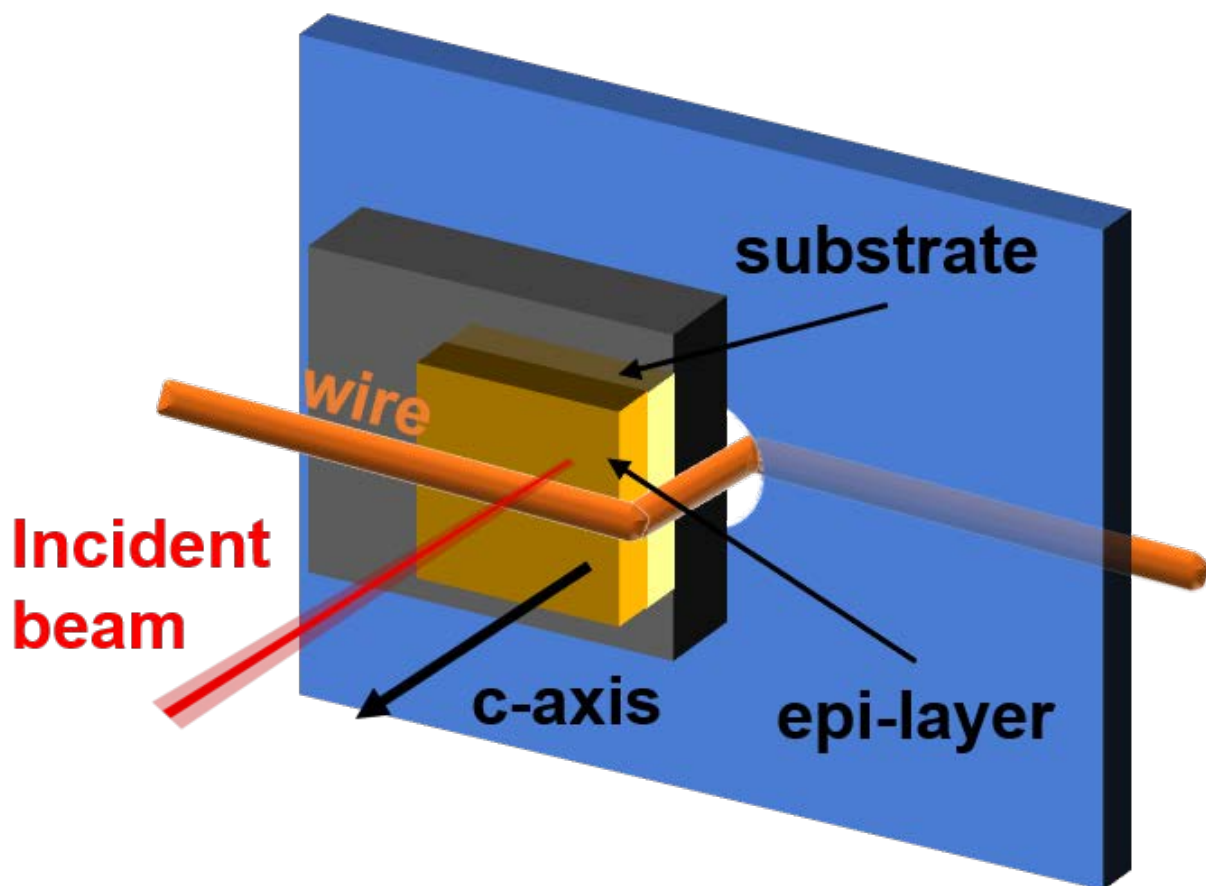


Figure 77: The setup used to mount the SiC for measurements. The blue plate was a piece of acrylic mounted on an optical post, and the hole shown allowed rough alignment of the optics by back-illuminating the sample using a red LED. The thickness of the epi layer is $75\text{ }\mu\text{m}$ and the thickness of the wire is $30\text{ }\mu\text{m}$.

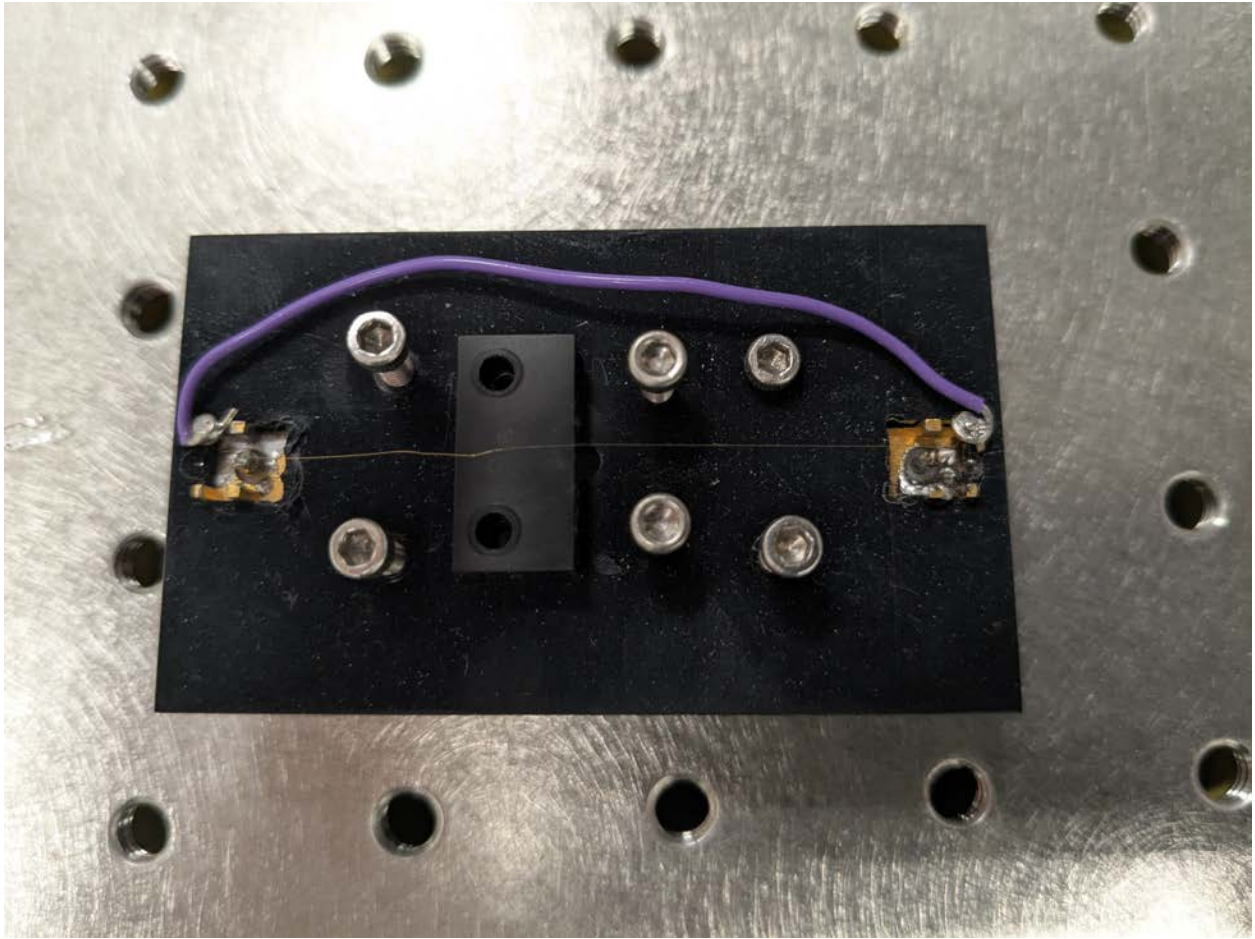


Figure 78: A photo of the mount used to hold the SiC samples in place for ODMR (with sample removed). The purple wire on top connects the shields of the two SMA connectors on the reverse side of the plate for running a microwave signal through the gold-plated tungsten wire (30 μm). The wire would typically be held in tension over the surface of the sample to get as strong a signal as possible near the optical collection point on the sample.

Galvo Scanning Mirror The Galvo-scanning mirror (THORLABSGVS012 TSH32411-X) was used to scan across the sample via a computer interface (`galvo_scanner.py`). It was helpful for determining the precise location of the wire, and was used to try to estimate regions of high PL intensity and low intensity gradient to mitigate effects from beam drift. Figure 79 shows a sample plot obtained by use of the Galvo-scanning mirror (THORLABSGVS012 TSH32411-X) .

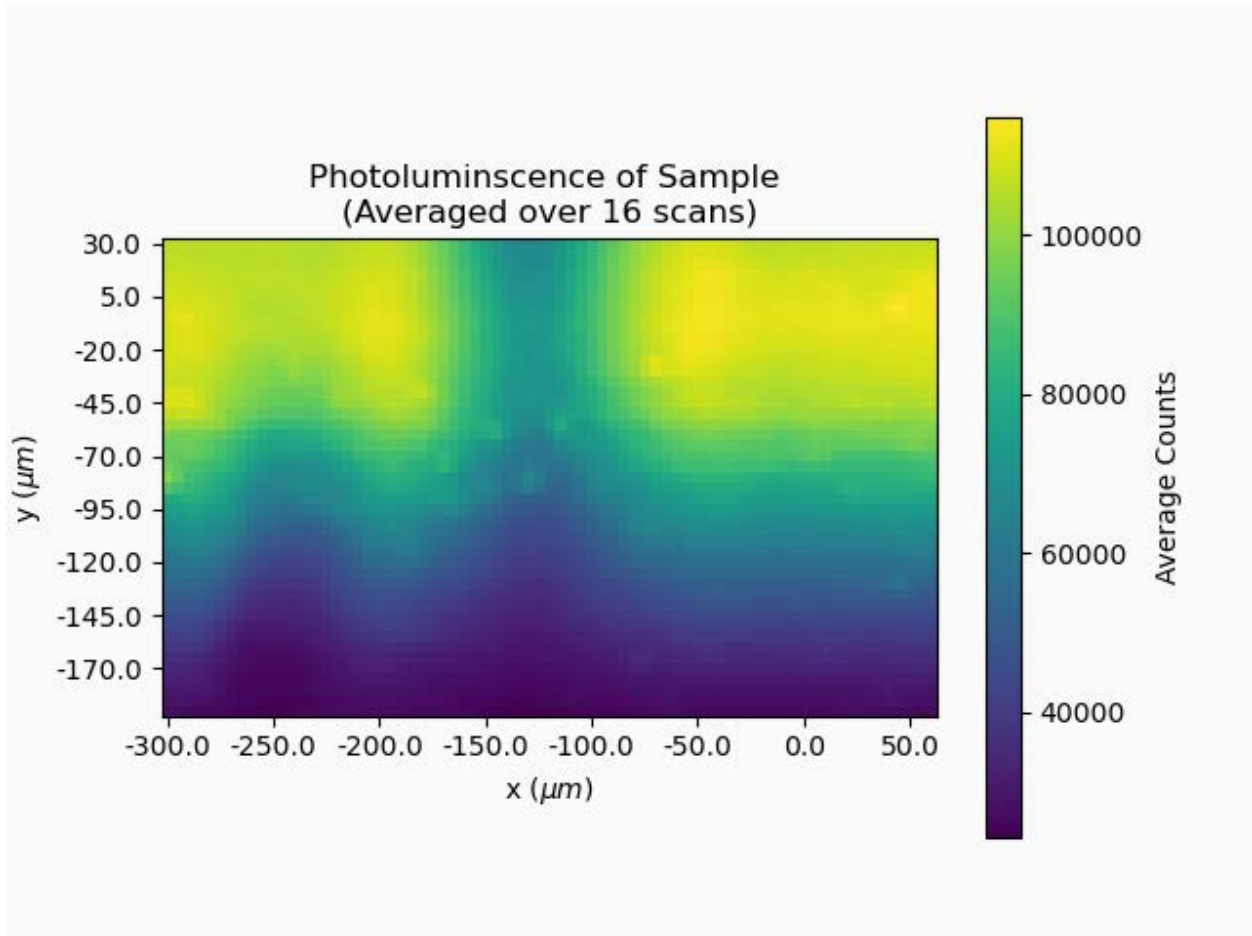


Figure 79: This figure was generated by `galvo_odmr_processing_3.py` from the file `galvo_l300_d190_i5_n16_16_p8.csv`. Scan oriented 90° from subsequent scans, showing the edge of the sample at the bottom and the $30\mu\text{m}$ wire running vertically near the center.

Results

With all the above tools in place, it was possible to measure ODMR in the bulk sample of 03021903T4 #29 (10^{16} fluence) .

Primary ODMR Results The first step in taking a set of ODMR data was to choose a suitable location on the sample. This was done by using the Galvo-scanning mirror (THORLABSGVS012 TSH32411-X) to collect PL data from near the edge of the sample, as shown in Figure 80.

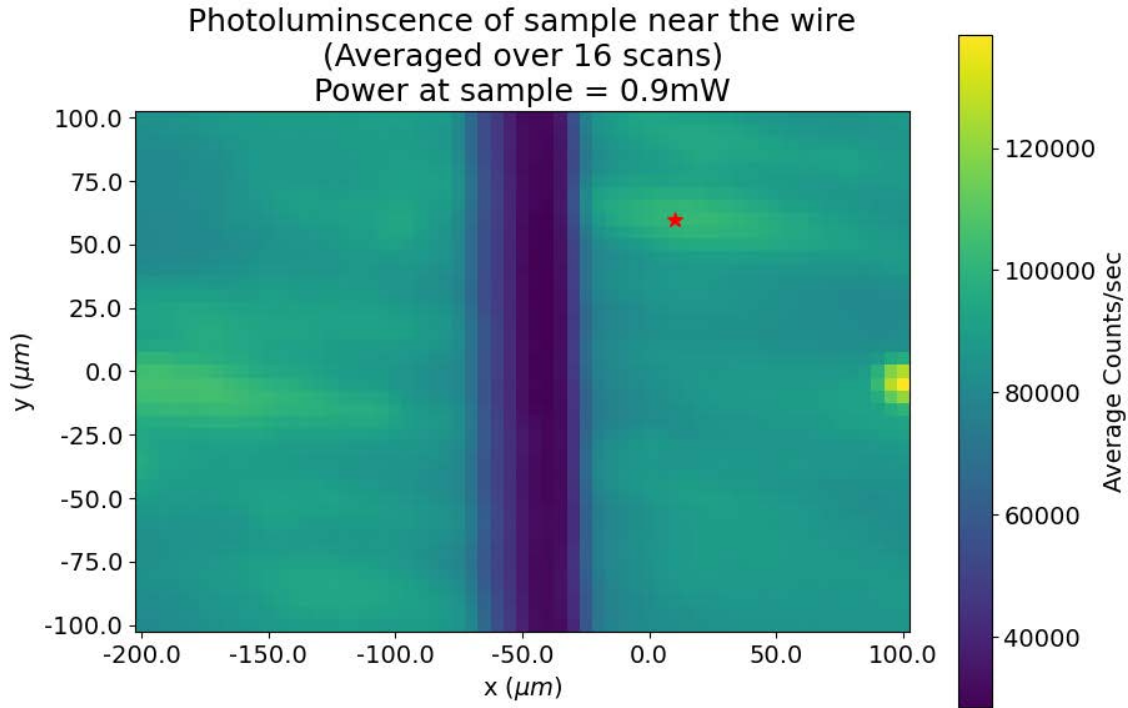


Figure 80: This figure was generated by `galvo_odmr_processing_new.py` from the files `galvo_l200_d100_i5_n16_16_p9.csv` and `galvo_x10_y60_n25.csv`. PL scan of sample used to collect ODMR data. The red star indicates the location used for power and ODMR measurements, selected for its location in a spot of high PL intensity and low gradient (see Figure 81) not far from the wire.

The data from the galvo scan was differentiated (linearly, turing every 4 points into 1 to avoid any shift from discrete differentiation), and a point with low PL intensity gradient and high PL intensity was selected as shown in Figure 81.

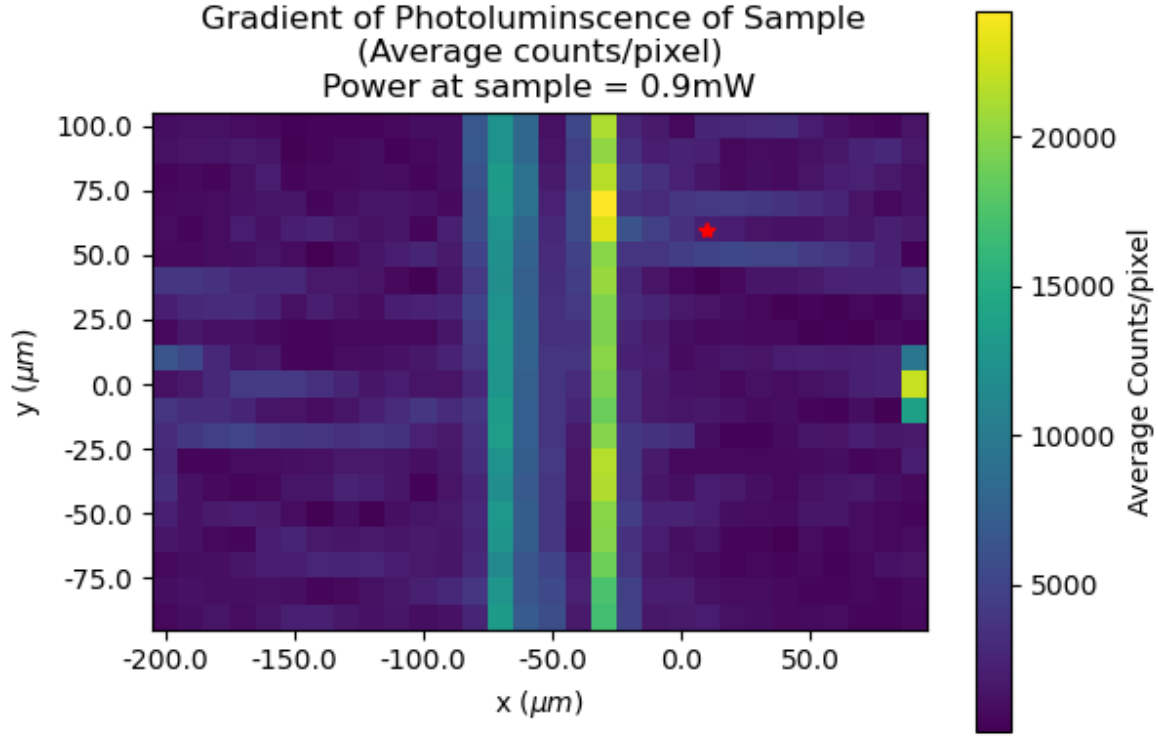


Figure 81: This figure was generated by `galvo_odmr_processing_new.py` from the files `galvo_l200_d100_i5_n16_16_p9.csv` and `galvo_x10_y60_n25.csv`. Gradient of PL scan of sample used to collect ODMR data. The red star indicates the location used for power and ODMR measurements, selected for its location in a spot of high PL intensity and low gradient (see Figure 80) not far from the wire. Note the anisotropy in the gradient around the wire, likely indicating a shadow caused by optical misalignment.

Once the location was selected, the PL intensity from the sample at that point was characterized as a function of input laser power. No PL saturation limit can readily be seen in the data.

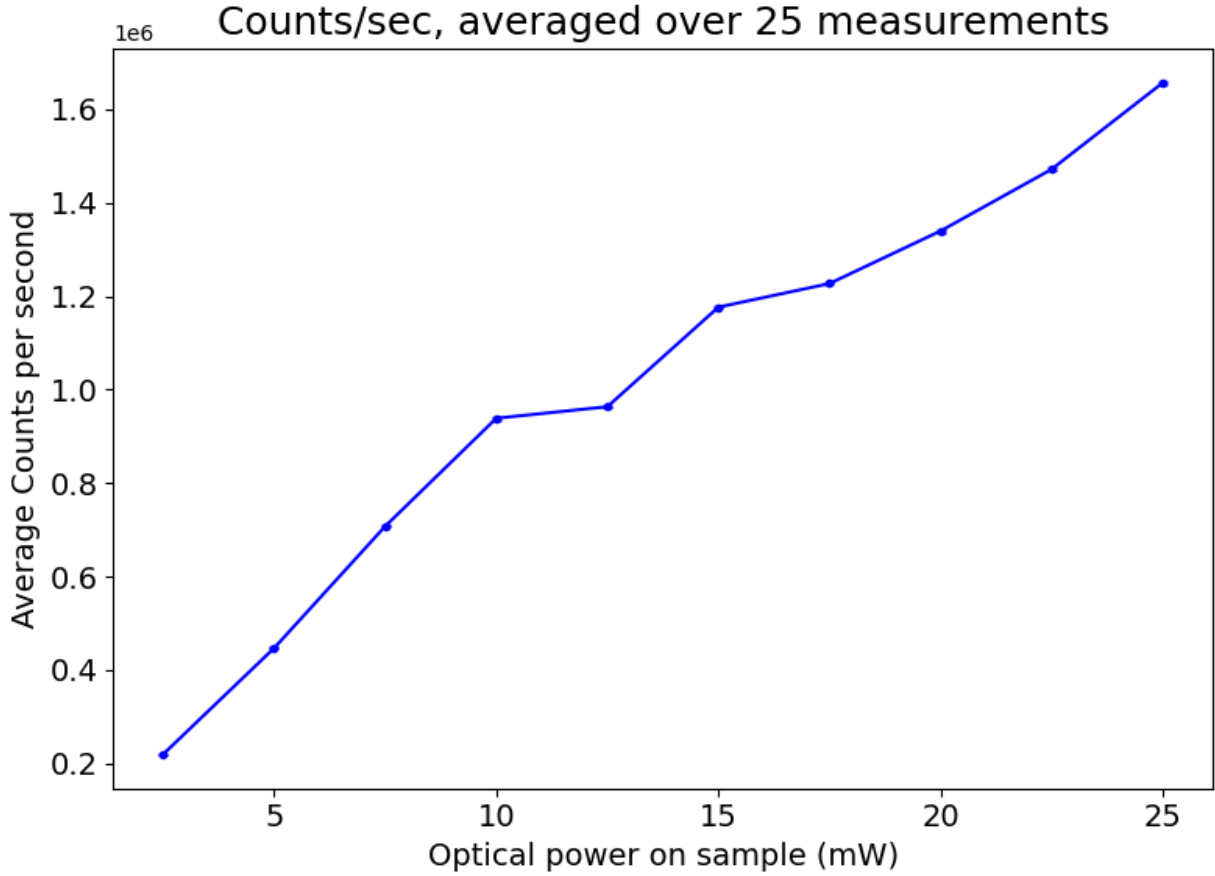


Figure 82: This figure was generated by `galvo_odmr_processing_new.py` from the files `galvo_l200_d100_i5_n16_16_p9.csv` and `galvo_x10_y60_n25.csv`. Plot of the PL intensity in counts collected by the SPCM at the location indicated by a red star in Figures 80 and 81. The error bars for this plot are too small to be seen, thanks to the successful power stabilization of the laser.

An ODMR scan was taken at the location shown by the red star in Figures 80 and 81. It is worth noting here that this geometrical arrangement is not ideal. Only the component of the microwave-frequency B-field perpendicular to the c-axis of the 4H-SiC sample will drive spin transitions, and at this placement from the wire the magnetic field should be primarily into and out-of the page, aligned with the c-axis. However, for a wire of finite thickness some distance above the surface of the sample, there should also be a non-negligible B-field component running along the x -axis of Figure 80.

Several reproducible scans were taken at this same location, with different MD power levels and optical pumping power levels using the file `galvo_scanner_m.py`. One such scan is shown in Figure 83, where a clear ODMR peak can be seen at the expected frequency of 70 MHz.

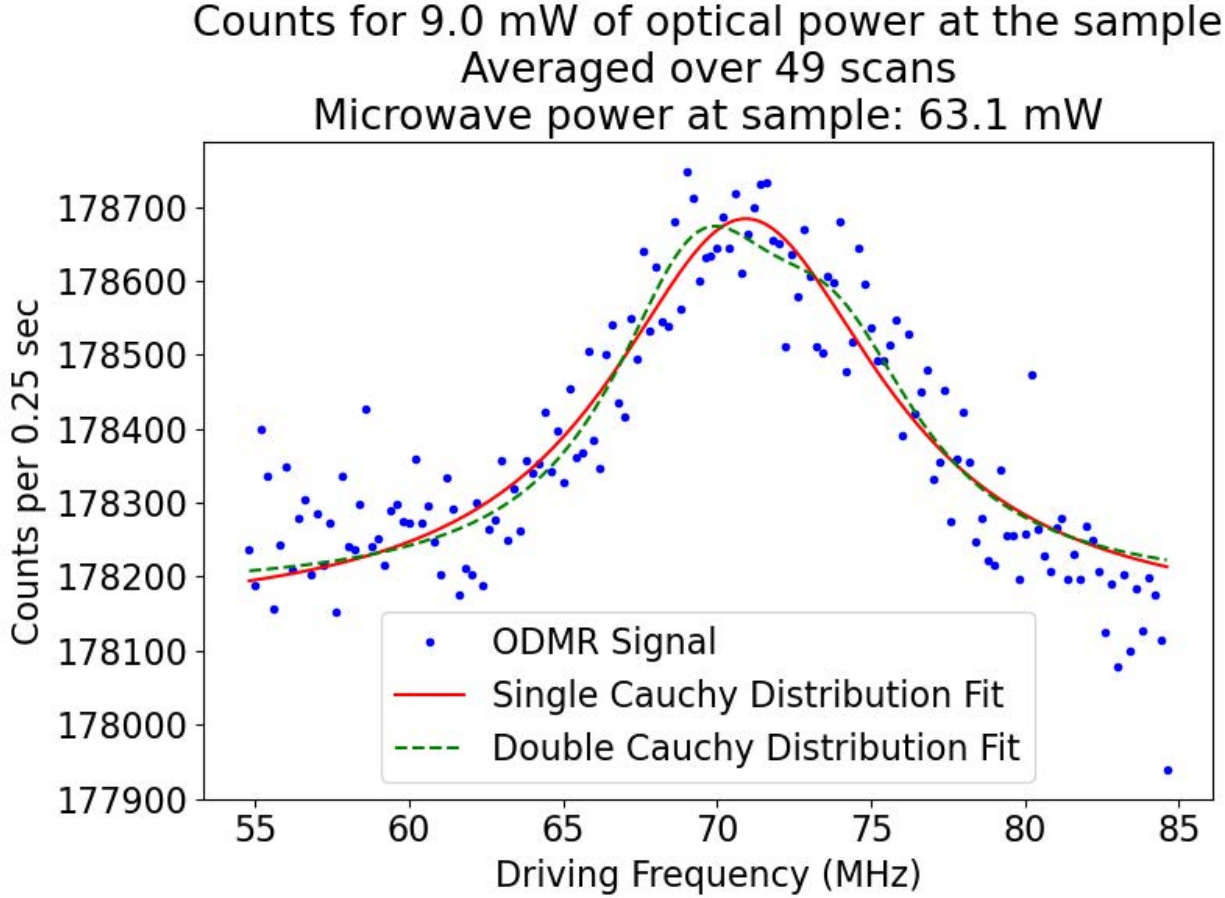


Figure 83: This figure was generated by `odmr_plot_epi_good.py` from the file `odmr_x10_y60_p90_t250_s200_d-10`. Measurement taken with an exposure time of 250 ms and drive frequency step size of 0.2 MHz. The double Cauchy fit (see Figure 84) was used to better fit the shape of the resonance peak, and to look for any splitting of the peak due to the presence of a background magnetic field.

The peak observed in the ODMR signal is consistent with what is expected for 4H-SiC in the presence of negligible static magnetic field. In the absence of Zeeman splitting, a single Cauchy fit (Figure 84) should sufficiently describe the signal shape. However, a

double-Cauchy fit is better in the presence of a static magnetic field.

For reference, Earth's magnetic field at our location is around 50 μT (measured using a BELL gaussmeter (Model 620) with 10x probe Model HTL4-0608, Ser. 150284 and BELL Zero Gauss Chamber: Model YA-111), which would correspond to a maximum frequency shift (if aligned parallel to c-axis) of about 1.4 MHz. Such a shift is too small to clearly identify from the data recorded, but may help to explain why the double-Cauchy fit has such a consistent shape throughout most of the data taken.

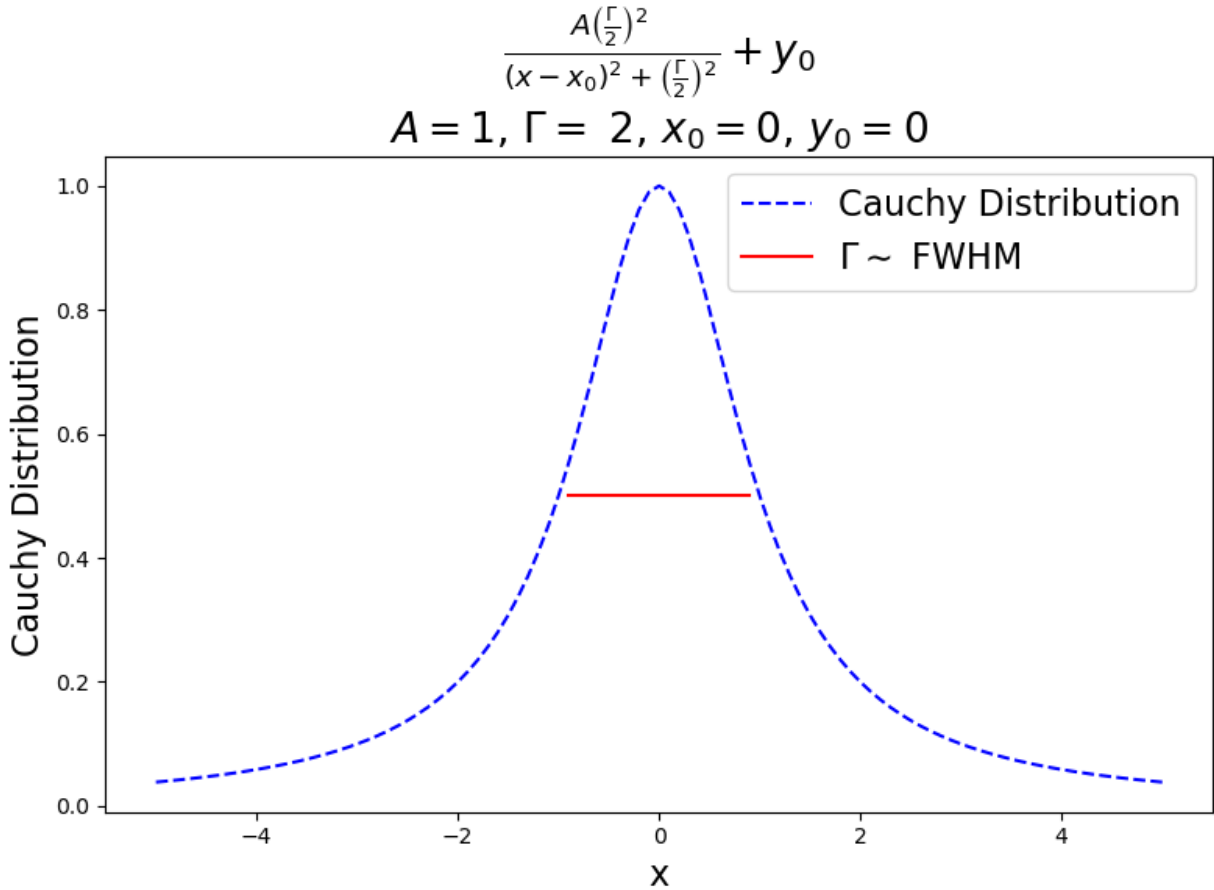


Figure 84: This figure was generated by `odmr_plot_epi_good.py`. It shows a sample Cauchy function for use in fitting ODMR signals to find peak widths and amplitudes.

The cauchy fit allowed the extraction of peak height and width for several different MD and optical pumping power levels. A second sample data set with slightly different values is

shown in Figure 85.

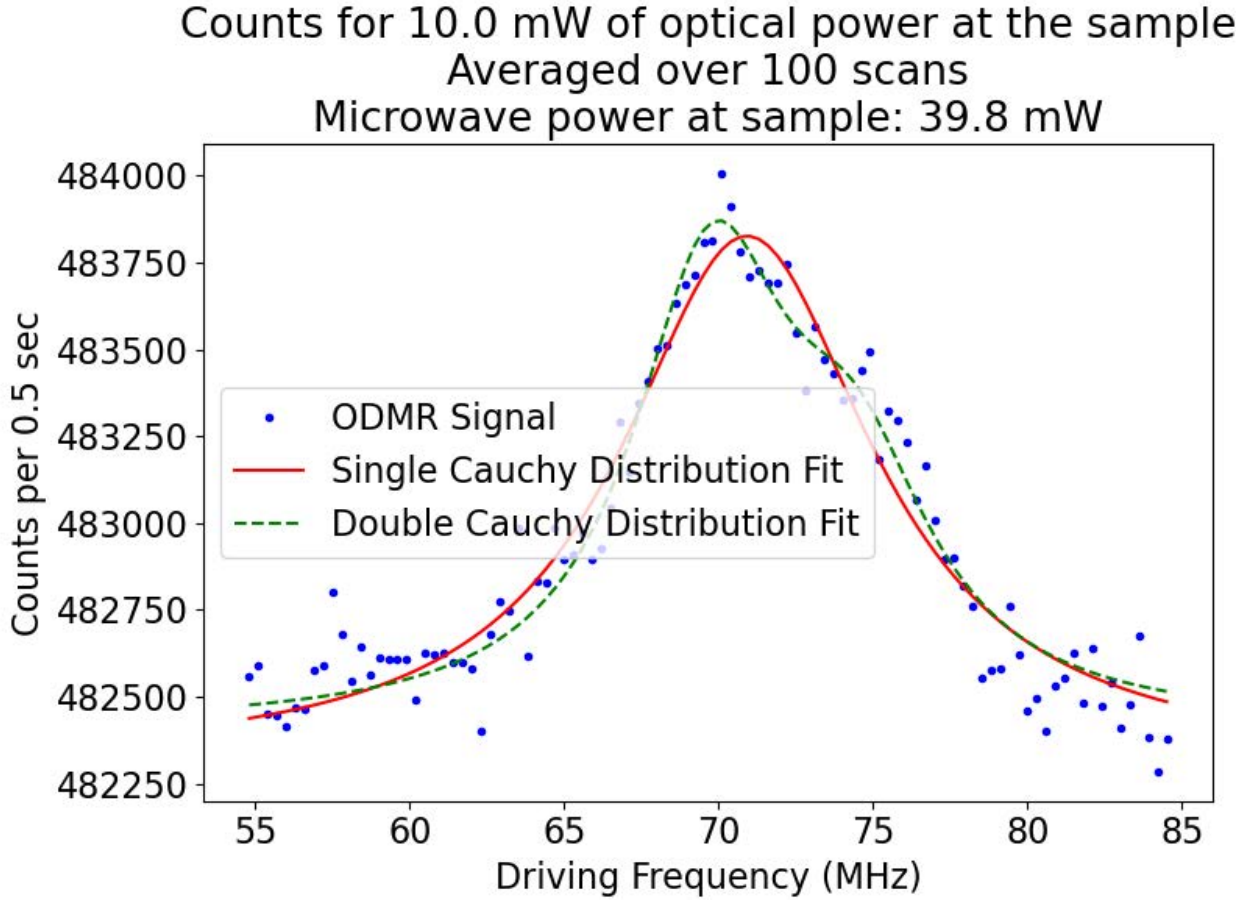


Figure 85: This figure was generated by `odmr_plot_epi_good.py` from the file `odmr_x10_y60_p100_t500_s300_d-12`. Measurement taken with an exposure time of 500 ms and drive frequency step size of 0.3 MHz. The double Cauchy fit (see Figure 84) was used to better fit the shape of the resonance peak, and to look for any splitting of the peak due to the presence of a background magnetic field.

Two series of measurements were taken, one with the MD power at the sample set to 16 dBm while the optical power was varied (Figure 86), and the other taken with the optical power held constant at 9 mW while the MD power was allowed to vary (Figure 87).

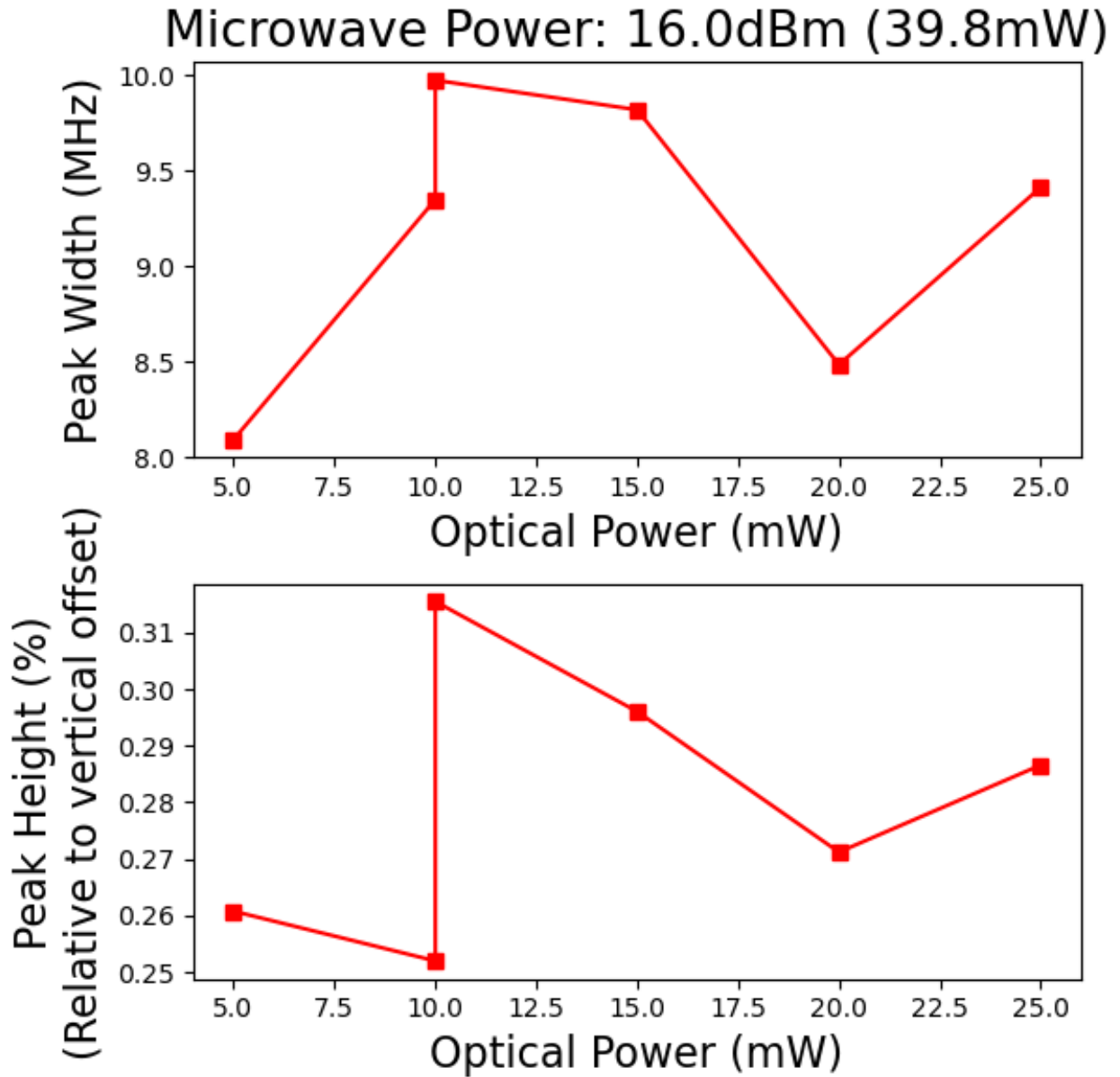


Figure 86: This figure was generated by `odmr_plot_epi_good.py` from the files `odmr_x10_y60_p50_t500_s300_d-12`, `odmr_x10_y60_p100_t500_s250_d-12`, `odmr_x10_y60_p100_t500_s300_d-12`, `odmr_x10_y60_p150_t500_s300_d-12`, `odmr_x10_y60_p200_t500_s300_d-12`, and `odmr_x10_y60_p250_t500_s300_d-12`. Peak height and width determined from single-Cauchy fit (see Figure 84).

It can be seen that neither the bandwidth of the signal nor the strength of the signal show any clear dependence on optical pumping power. In both cases, the signal is about

0.25%-0.3%. Literature results often cite a signal strength of about 0.5% [24], which is within a factor of 2 of the results presented here.

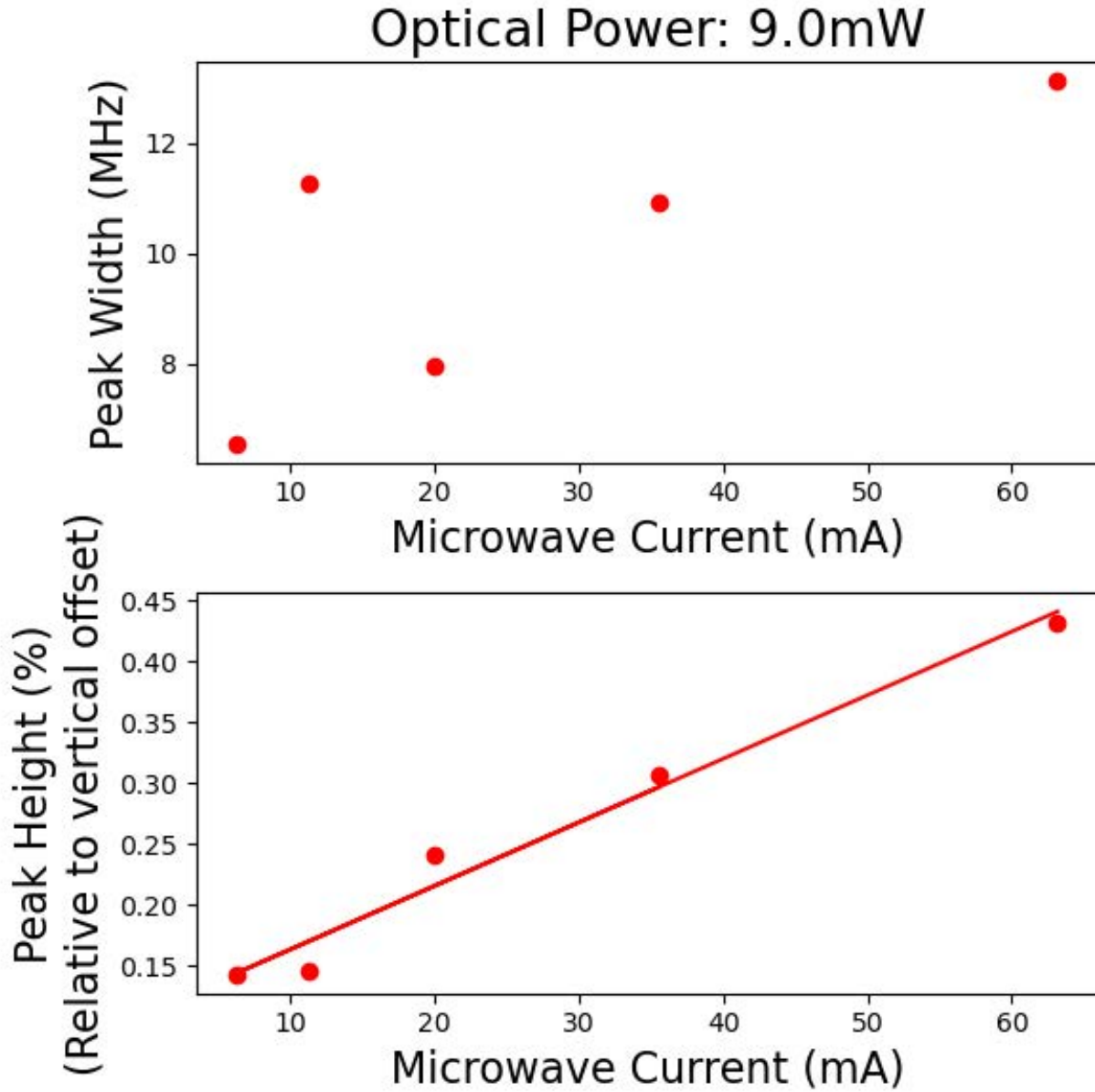


Figure 87: This figure was generated by `odmr_plot_epi_good.py` from the files `odmr_x10_y60_p90_t500_s300_d-5`, `odmr_x10_y60_p90_t250_s200_d-10`, `odmr_x10_y60_p90_t250_s400_d-15`, `odmr_x10_y60_p90_t250_s300_d-20`, and `odmr_x10_y60_p90_t250_s300_d-25`. Peak height and width determined from single-Cauchy fit (see Figure 84). The line shown in the lower plot is a linear fit to the data.

By contrast to the optical power dependence, a clear trend can be seen in the peak height shown in Figure 87. Here increasing the MD power showed a linear increase in signal strength with no sign of saturation, indicating that some signal improvement could likely be achieved by further optimizing the drive wire geometry. However, the signal strength is unlikely to improve by very much, since 0.5% is not significantly higher than what was achieved at the highest MD power tested.

Further Tests Some brief attempts were made to measure the resonant-frequency splitting of the signal due to a static magnetic field applied parallel to the crystal c-axis. A field was generated by stacking 5 neodymium magnets together and mounting them on an optical post about 51(2) mm (73(2) mm) from the sample oriented parallel to the c-axis for Figure 89 (88). The approximate magnetic field strength at the sample was then measured to be 1.41(8) mT (0.48(4) mT) using the BELL gaussmeter (620) .

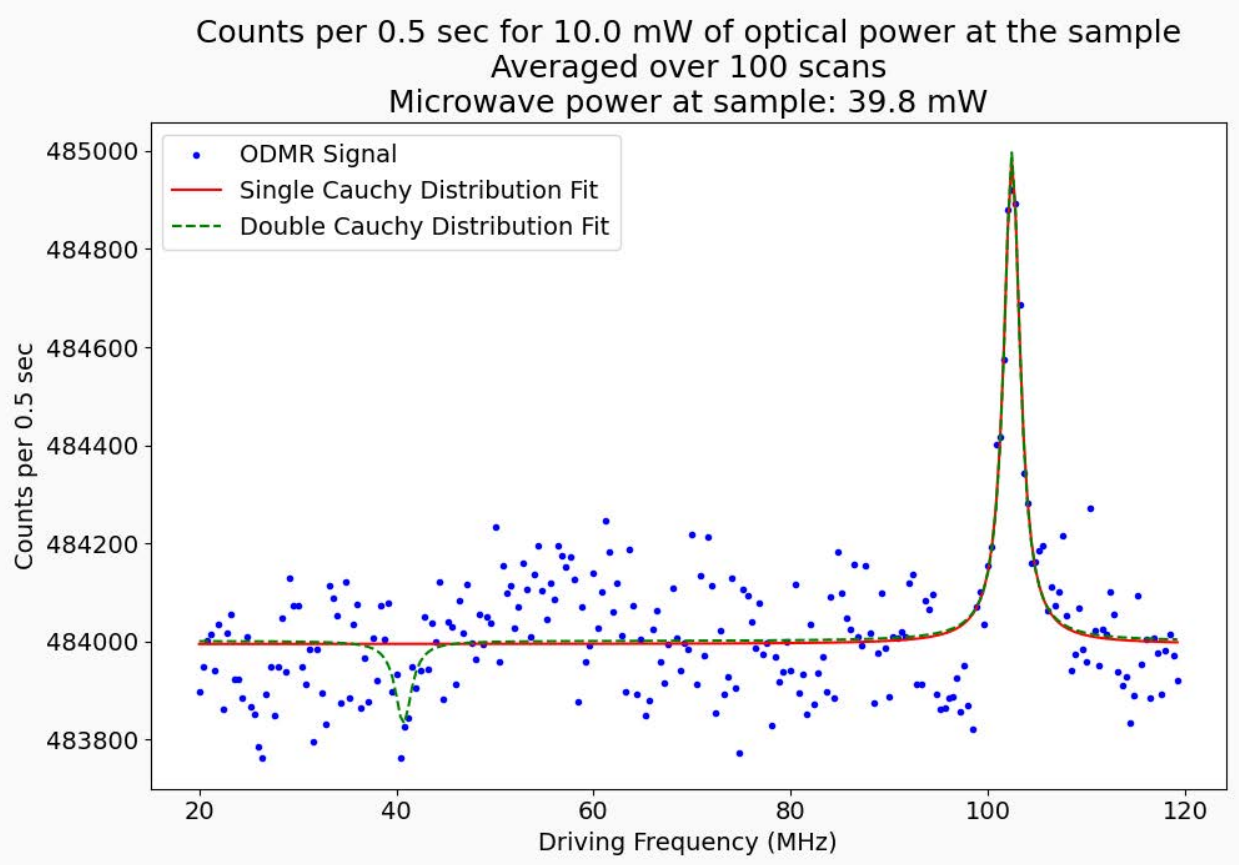


Figure 88: This figure was generated by `odmr_plot_epi_static_field.py` from the file `odmr_x10_y60_p100_t500_s300_d-12_b51`. Measurement taken with an exposure time of 500 ms and drive frequency step size of 0.3 MHz. The double Cauchy fit (see Figure 84) was used to better fit the shape of the resonance peak, and to look for any splitting of the peak due to the presence of a background magnetic field, estimated to be about 1.41(8) mT.

While there is a clear peak at about 102 MHz (expected 109 MHz), the result is largely inconclusive since the expected peak at 31 MHz is not to be seen (a quick scan from 220 to 260 MHz also revealed no peak at 240 MHz). The magnets were moved farther away to reduce the B-field by about a factor of 3 and the measurement was repeated, as shown in Figure 88.

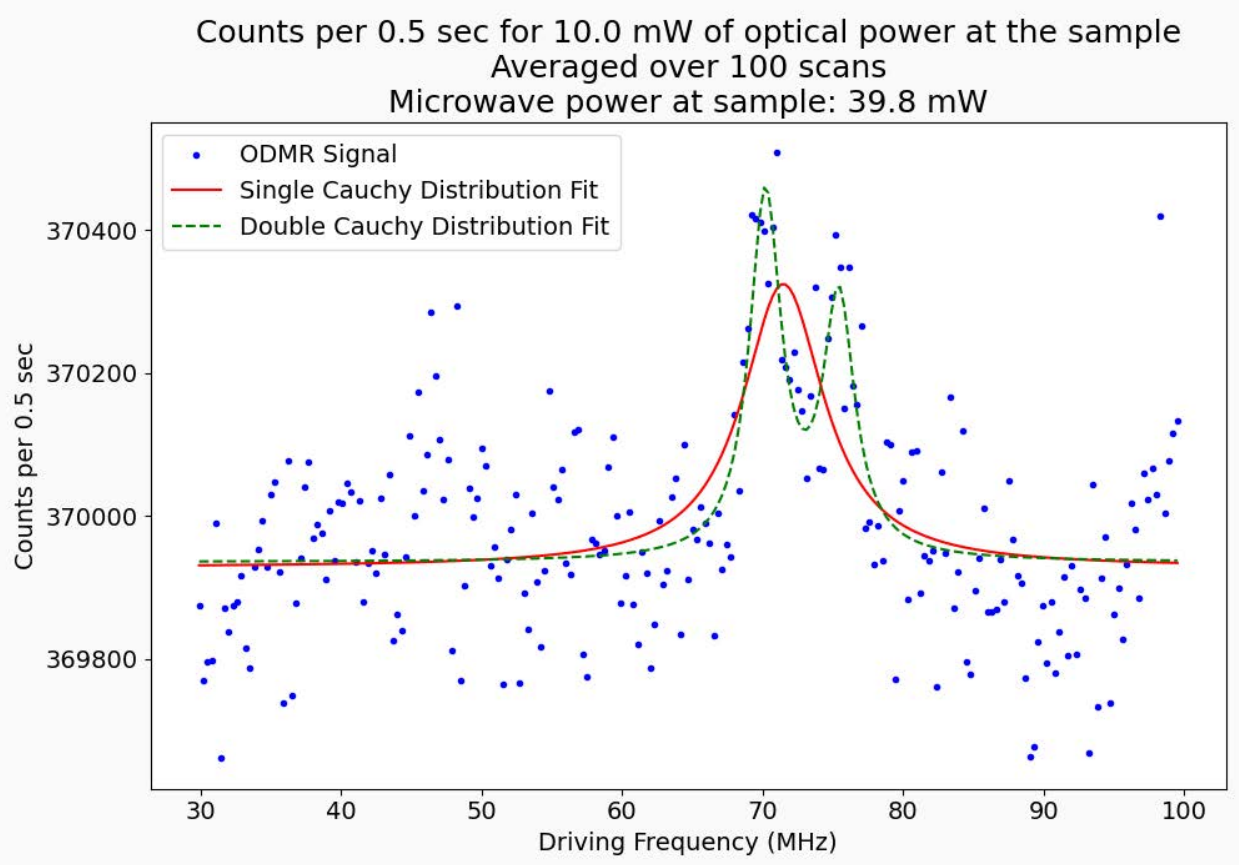


Figure 89: This figure was generated by `odmr_plot_epi_static_field.py` from the file `odmr_x10_y60_p100_t500_s300_d-12_b73`. Measurement taken with an exposure time of 500 ms and drive frequency step size of 0.3 MHz. The double Cauchy fit (see Figure 84) was used to better fit the shape of the resonance peak, and to look for any splitting of the peak due to the presence of a background magnetic field (estimated to be about 0.48(4) mT

Once again, the result was largely inconclusive, since no peaks can be seen at the expected frequencies of 57 MHz and 83 MHz. The peaks at 70 MHz and 75 MHz obtained from the double-Cauchy fit may or may not be significant.

Either way, two conclusions may be drawn from these results. First, applying a static field clearly altered the resonance peaks. Second, to clearly understand what is going on, more data would need to be collected. One likely issue is the non-uniformity of the magnetic field at the sample, and the ambiguous nature of its strength. It is important to note that the angle of the field relative to the c-axis also affects the splitting of the resonance frequencies,

and that cleaner results might be obtained by using a stronger field.

Due to time constraints and the fact that the results demonstrated the viability of the sample for ODMR, we moved on to other trials.

There was some question as to whether a lower fluence (10^{15} rather than 10^{16}) might be preferable for ODMR trials. For this reason data was collected from a new sample (04052911T1 #19). The old sample was removed by placing the sample and holder in an acetone bath in an ultrasonic agitator (VWR B2500A-DTH) to remove the glue, and the new sample was glued on using Norland Optical Adhesive 63 . A plot of the galvo-scan used to choose a collection spot is shown in Figure 90.

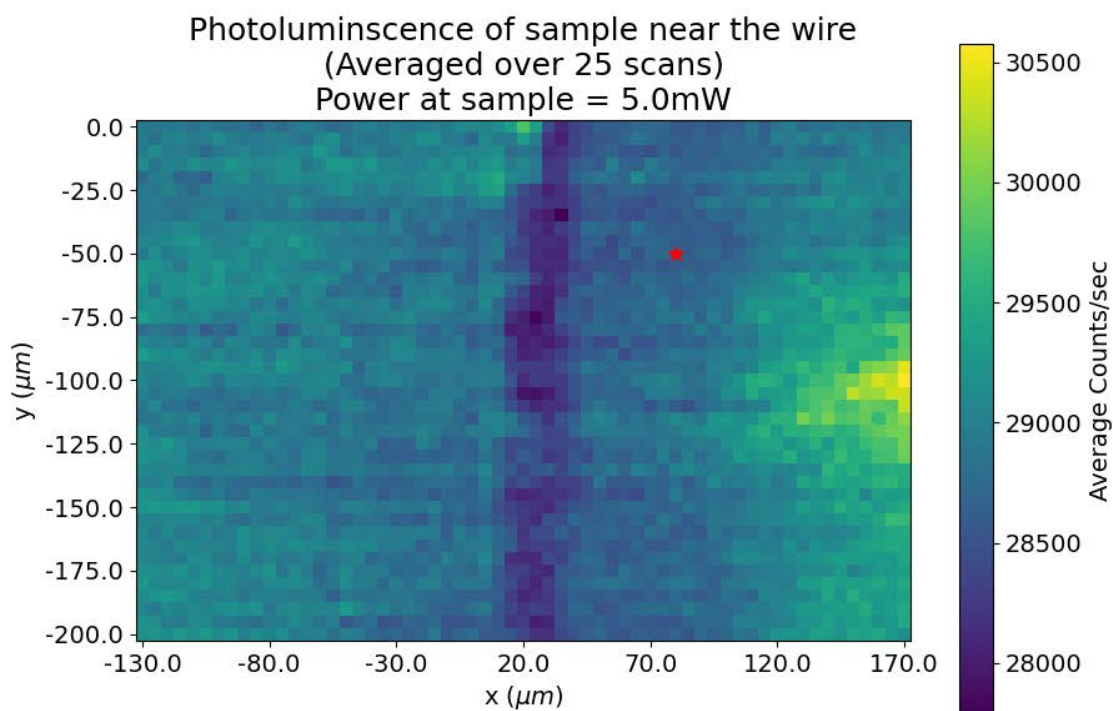


Figure 90: This figure was generated by `galvo_odmr_processing_new.py` from the files `galvo_l1130_d200_i5_n25_25_p50.csv` and `galvo_x80_y-50_n100.csv`. PL scan of sample used to collect ODMR data. The red star indicates the location used for power and ODMR measurements, selected for its location in a spot of high PL intensity and low gradient (see Figure 91 not far from the wire. Note how much lower contrast there is in contrast to Figure 80.

It is immediately obvious that the PL intensity from this sample is lower than for the previous sample. The gradient of the data (Figure 91 is similarly less clear, making good spot selection difficult.

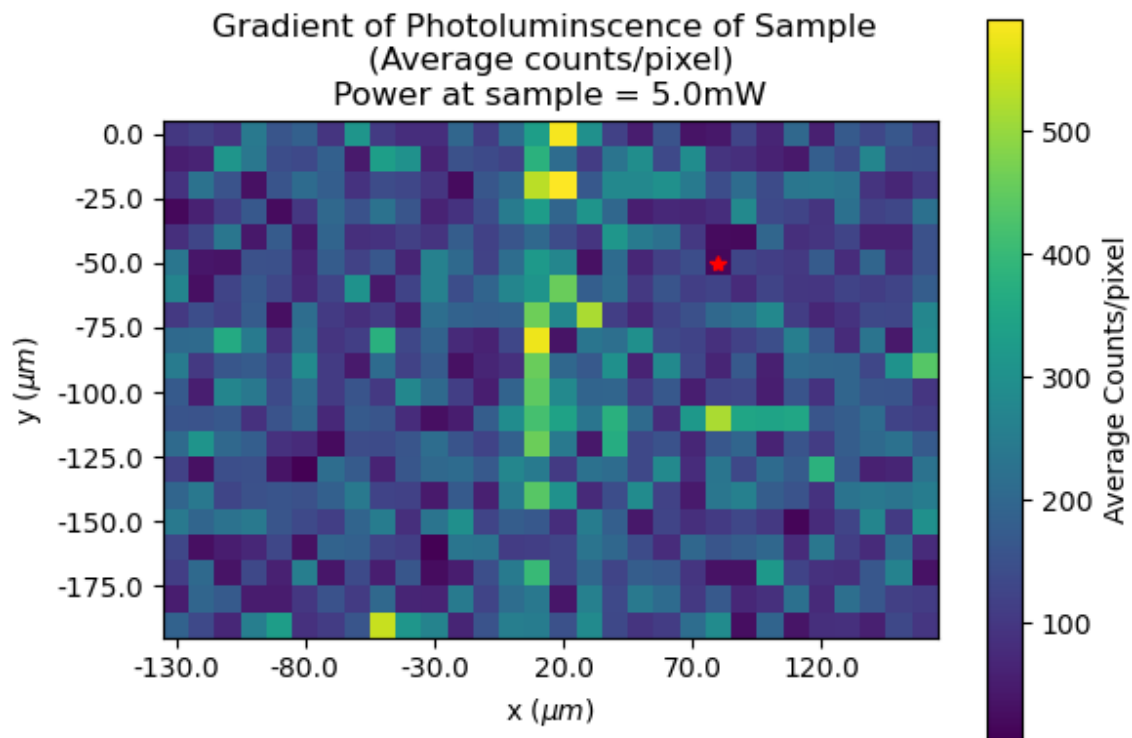


Figure 91: This figure was generated by `galvo_odmr_processing_new.py` from the files `galvo_l1130_d200_i5_n25_25_p50.csv` and `galvo_x80_y-50_n100.csv`. Gradient of PL scan of sample used to collect ODMR data. The red star indicates the location used for power and ODMR measurements, selected for its location in a spot of high PL intensity and low gradient (see Figure 90 not far from the wire).

The PL intensity as a function of laser power (Figure 92) similarly shows that the sample does not photoluminesce as brightly as the previous one.

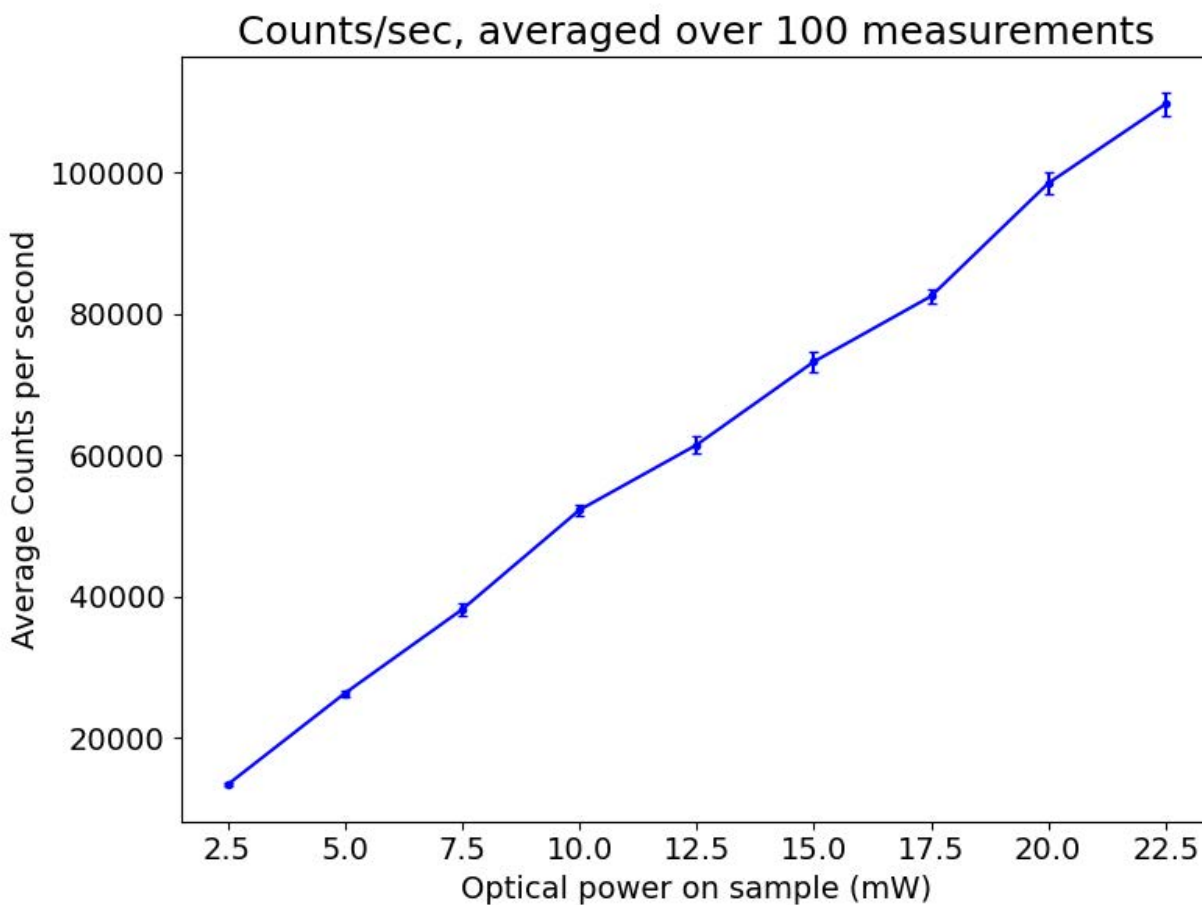


Figure 92: This figure was generated by `galvo_odmr_processing_new.py` from the files `galvo_l1130_d200_i5_n25_25_p50.csv` and `galvo_x80_y-50_n100.csv`. Plot of the PL intensity in counts collected by the SPCM at the location indicated by a red star in Figures 80 and 81. The error bars for this plot are mostly too small to be seen, thanks to the successful power stabilization of the laser.

It is no surprise, then, that the signal from the sample is not clear in Figure 93.

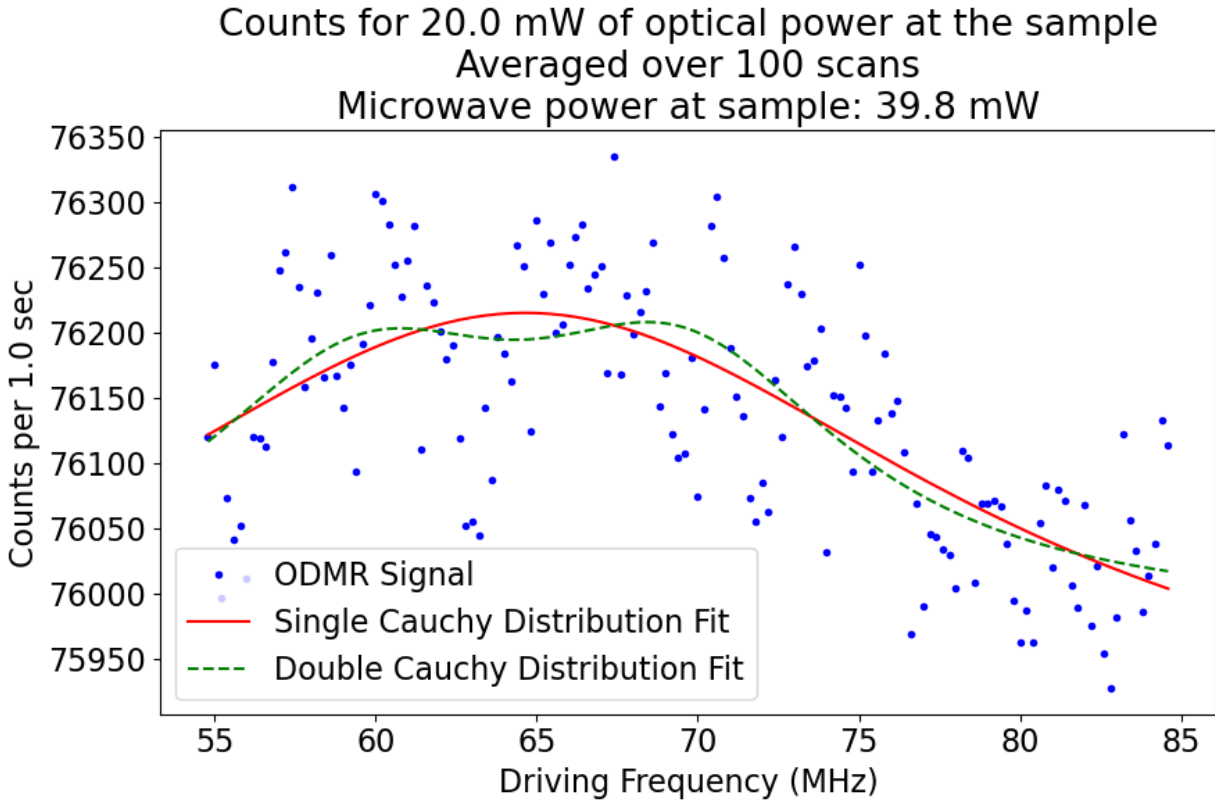


Figure 93: This figure was generated by `odmr_plot_epi_miscellaneous.py` from the file `odmr_x80_y-50_p200_t1000_s200_d-12_lower`. Measurement taken with an exposure time of 1000 ms and drive frequency step size of 0.2 MHz. The double Cauchy fit (see Figure 84) was used to better fit the shape of the resonance peak, and to look for any splitting of the peak due to the presence of a background magnetic field.

While it is possible that improvements could be made to the measurement scheme (such as an objective with less aberration, a longer integration time, or improved wire geometry), it is clear that a higher fluence is preferable for ODMR in this case.

Boule Samples

In considering what would need to be accomplished to perform spin-resonance measurements (such as ODMR) on trapped particles of 4H-SiC, there was one significant challenge to using the epi-grown material. Because the epi-material is grown as a thin (75 μm) layer on a thick substrate, crushing it into pieces and then levitating a piece of the

required epi-material might be challenging, since most of the material would be substrate, and the two are not easy to tell apart under a microscope.

For this reason, the setup was modified to attempt ODMR on irradiated boule-grown 4H-SiC that was made available by the Devaty/Choyke group in Pittsburgh.

Initial Experiment Setup The experiment setup for performing ODMR in boule-grown 4H-SiC did not at first differ greatly from the setup used for the epi-grown material. This time the sample orientation was adjusted to be more favorable as shown in Figure 94.

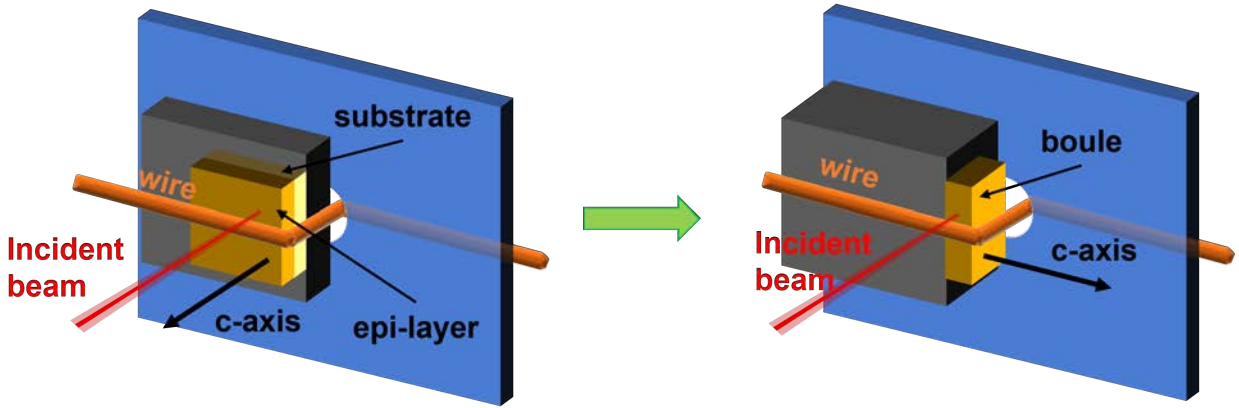


Figure 94: Indicates the change in the setup from the epi-grown sample to the boule-grown sample. The drive wire geometry here favors the formation of a B -field oriented 90° relative to the crystal c -axis at the collection point.

The boule sample used (HY0667-21B-2), was polished on one edge to form a smooth surface for performing ODMR, and was glued on the side of the acrylic plate in order to facilitate a more favorable drive wire geometry.

After a few unsuccessful attempts at measuring ODMR in this sample, it became clear that there was a new challenge here in collecting PL from the same location in the sample as was being optically pumped by the 785 nm laser. After several iterations of optical setup designs, the one shown in Figure 95 was ultimately used.

Early iterations used a higher-NA objective (Mitutoyo WA58086718, M Plan App NIR, 5x / 0.14, $f=200$), but it eventually became clear that illuminating a broad region with

the laser (achieved by use of the AC508-150-B-ML) and collecting from a narrow region for the PL was more effective for obtaining a clear signal. At first this narrow collection was attempted by trying to couple the PL into a single-mode fiber with a core diameter of $5\mu\text{m}$, but this proved too difficult. Instead, a $30\mu\text{m}$ pinhole was used to restrict the sampling point. This diameter was found to reduce the light collected without being prohibitively difficult to couple into.

Additionally, the optics leading from the dichroic mirror to the SPCM tended to allow a great deal of background light to be collected, even with lights turned off and the optical curtains closed. Data collection was so sensitive that the monitor had to be turned off to start any kind of data collection. For this reason different sections of the optics were encased in aluminum foil to reduce the effects of background light. The results of this aluminum-foil covering are presented in Table 2.

Table 2: Effect of Using Aluminum Foil to Suppress Background Light (0.2 sec exposure time)

Location	Cts Before (Off)	Cts After (Off)	Cts Before (On)	Cts After (On)
SM to MM fiber coupling	3643	547	12742	2515
fiber-SPCM connection	547	435	2515	2329
SPCM fiber	435	225	2329	1209
Illumination Side	221	226	1139	1137
Collection Side	226	228	1137	1155
Copper Sheaths on SM fibers	228	142	1155	598
First SM coupling+sheath	142	142	598	177
Removed Illumination+Collection	142	145	177	193

The last two columns in Table 2 indicate the result of applying aluminum foil to all previous locations in addition to the presently examined one. All counts are the average of 4 measurements.

These tests were done with the laser turned on at 98.0mA with 4.5mW of power at the input of the initial fiber coupling. The monitor previously used for background light was

turned off. Tests were run with and without the laser on.

An optotune scanning mirror was temporarily used in the M-3 position. The Optotune Scanning Mirror (MR-E-2) was connected to a DUE board using the pin layout shown in Table 3. The file `OptotuneGalvo.ino` was used to communicate with the mirror.

Table 3: DUE to MR-E-2 Pin Layout

Due	Terminal	MR-E-2
MOSI	4	5
SCK	5	6
10	6	7
MISO	7	8
-	GND	9
8	3	10

This mirror was not ultimately used due to stability issues. Instead, a homemade galvo-scanning mirror[61] was used. It had a much smaller range but considerably higher stability.

Final Setup The optical setup shown in Figure 95 is ultimately what was used to perform ODMR on the HY0667-21B-2 sample. Laser light was coupled into the pinhole initially by removing the longpass filters, but there was always a small adjustment that needed to be made due to chromatic aberration in the optics to get any signal.

The other major change was the use of the Homemade galvo-scanning mirror [61] in place of the Galvo-scanning mirror (THORLABSGVS012 TSH32411-X) for the mirror M-3. This mirror was setup using the 3-channel Arduino Due based digital to analog converter (DAC) in conjunction with two A18H1400 Acopian power supplies.

This galvo allowed for a small scanning range while still providing good stabilization for the mirror. It also seemed like a more suitable choice for the eventual possibility of ODMR on levitated SiC particles due to its smaller scanning range.

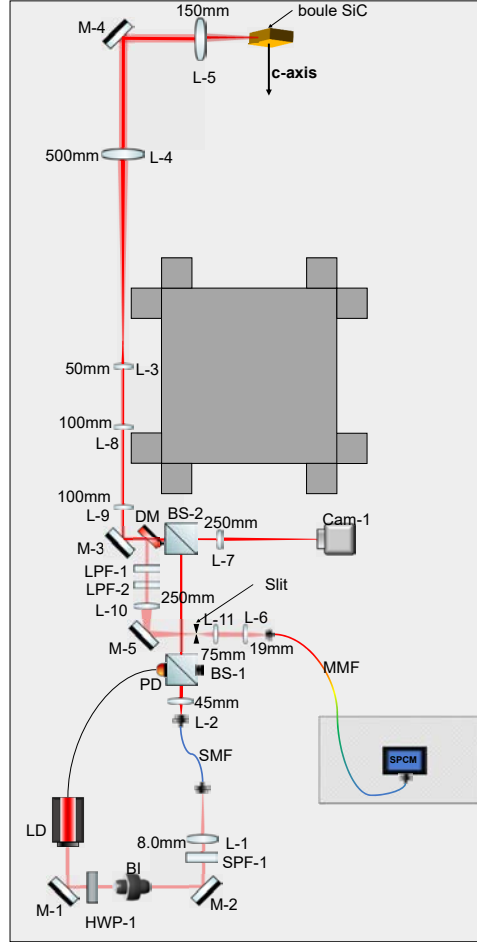


Figure 95: The optical setup used for performing ODMR on bulk samples of boule-grown 4H-SiC . Note in particular the changes: A lower-NA objective to illuminate a deeper section of material (L-5), a relay system between the galvo-scanning mirror and the sample, and a pinhole in the collection optics to select the depth of focus for PL collection.

Optical Component List for Figure 95 For optical component lists, items marked with a “*” are new additions from the previous setup displayed. The notation “[C]” indicates that the components were fixed rigidly to cage rods, usually via custom aluminum mounts, and “[Al]” indicates that the optics were covered in aluminum foil to prevent stray light interfering with the signal.

- **LD.** 785 nm, 300 mW, Ø9 mm TO Can, E Pin Code, VHG Wavelength-Stabilized Single-Frequency Laser Diode (LD785-SEV300). Mounting: Newport Model700C-

42Lens.

- **M-1.** Ø1" Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: POLARIS-K1E.
- **HWP-1.** 780 nm half-wave plate (WPH05M-780). Mounting: RSP1. [C]
- **BI.** Free-Space Isolator, 780 nm, Ø4.7 mm Max Beam, 1.7 W Max (IO-5-780-VLP). Mounting: SM1RC.
- **M-2.** Ø1" Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: POLARIS-K1E.
- **SPF-1.** 800 nm shortpass filter (FESH0800). Mounting: KC1-T. [C]
- **L-1.** $f = 8.0$ mm, $NA = 0.5$, Mounted Geltech Aspheric Lens, AR: 600 - 1050 nm (C171TMD-B). Mounting: CP1XY. [C]
- **SMF.** $NA = 0.12$, Single Mode Fiber (P5-780PM-FC-2). Mounting: SM1FC/A, SM1Z. [C]
- **L-2.** $f = 45$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-045-B-ML). Mounting: CXY1A. [C]
- **BS-1.** 10:90 (R:T) Non-Polarizing Beamsplitter Cube, 700-1100 nm, 20mm (BS044). Mounting: CCM1-4ER. [C]
- **PD.** Large Area Mounted Silicon Photodiode, 350-1100 nm, Anode Grounded (SM1PD1B, NE20B-B). Mounting: CCM1-4ER. [C]
- **BS-2.** 50:50 Non-Polarizing Beamsplitter Cube, 700-1100 nm, 20mm (BS014). Mounting: B6C. [C]

- **DM.** 25 mm x 36 mm Shortpass Dichroic Mirror, 805 nm Cutoff (DMSP805R).
Mounting: FFM1, B4CRP. [C]
- **M-3*.** Homemade galvo mirror (-). Mounting: -.
- **L-3.** $f = 50$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-050-B-ML). Mounting: CP02. [C]
- **L-4.** $f = 500$ mm, $\varnothing 2$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC508-500-B-ML). Mounting: LCP34. [C]
- **M-4.** 2" Broadband Dielectric Elliptical Mirror, 750 - 1100 nm (BBE2-EO3).
Mounting: KCB2C. [C]
- **L-5*.** $f = 150$ mm, $\varnothing 2$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC508-150-B-ML). Mounting: CXY2, SM2V05. [C]
- **LPF-1.** Semrock 830 nm longpass (LP02-830RU-25). Mounting: B4CRP. [C]
- **LPF-2.** $\varnothing 1$ " Longpass Filter, Cut-Off Wavelength: 900 nm (FEL0900). Mounting: CP02. [C]
- **L-6.** $f=19$ mm, $\varnothing 1/2$ " Achromatic Doublet, SM05-Threaded Mount, ARC: 650-1050 nm (AC127-019-B-ML). Mounting: CXY1A. [C]
- **MMF.** NA = 0.10, Multimode Fiber (FG105LVA). Mounting: SM1FC/A, SM1Z. [Al]
- **SPCM.** Single Photon Counting Module (SPCM-AQRH-14-FC). Mounting: -.
- **L-7.** $f = 250$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-250-B-ML). Mounting: SM1Lxx. [C]
- **Cam-1.** Basler Camera (acA2440-75um). Mounting: CP12, C-MOUNT EXT. [C]

- **L-8***. $f = 100$ mm, $\varnothing 1"$ Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-100-B-ML). Mounting: CP02. [C]
- **L-9***. $f = 100$ mm, $\varnothing 1"$ Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-100-B-ML). Mounting: CP02. [C]
- **L-10***. $f = 250$ mm, $\varnothing 1"$ Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-250-B-ML). Mounting: CXY1A. [C,Al]
- **M-5***. $\varnothing 1"$ Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: KCB1C. [C,Al]
- **PH***. 30 μ m Pinhole, Stainless Steel Foil, $\varnothing 1"$ Housing (P30K). Mounting: CP360R. [C,Al]
- **L-11***. $f = 75$ mm, $\varnothing 1"$ Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-075-B-ML). Mounting: CP02. [C,Al]

Results Once all the optics were adjusted as shown in Figure 95), a scan was taken that revealed an ODMR signal in HY0667-21B-2 (Figure 96. The signal was small (about 1%), but gave a starting point for future measurement attempts. For this measurement, the pinhole was placed at an axial position that maximized the number of counts recorded using the lock-in amplifier.

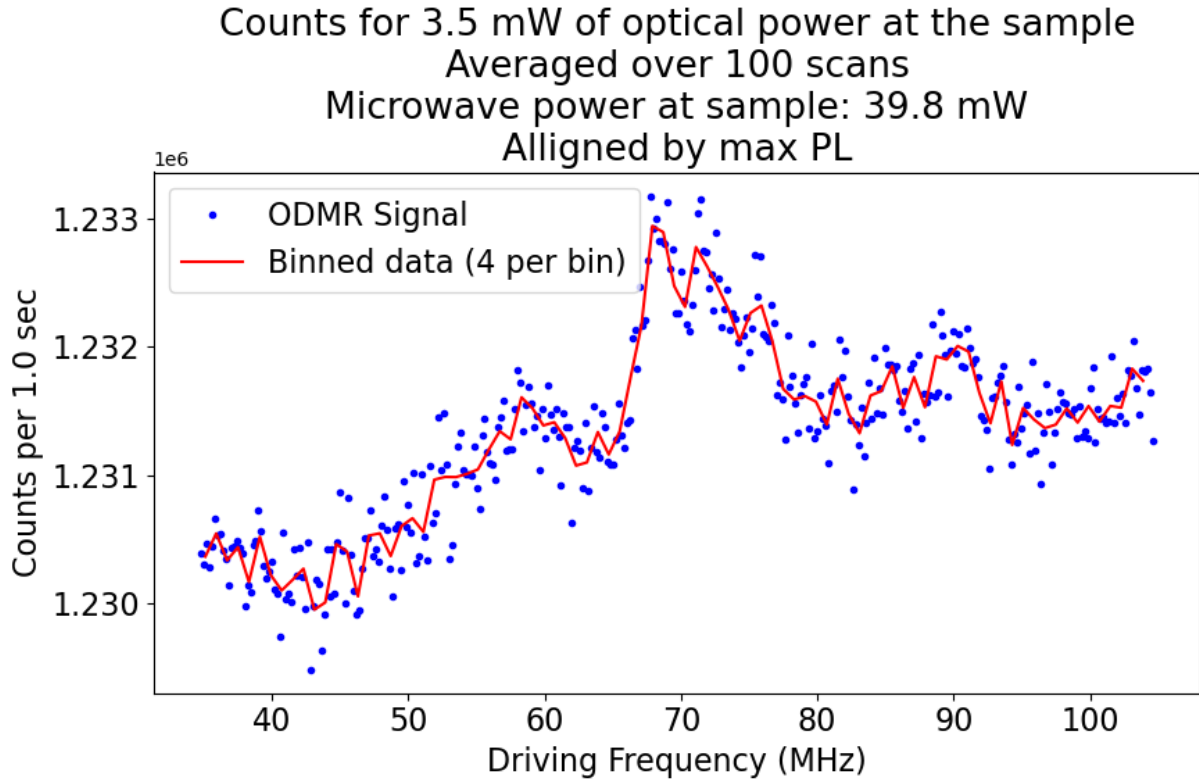


Figure 96: This figure was generated by `odmr_plot_boule.py` from the file `odmr_x40_y30_p35_t1000_s200_d-12`. Measurement taken with an exposure time of 1000 ms and drive frequency step size of 0.2 MHz. A running average is shown in red to guide the eye.

Further measurements were performed, and it became clear that the system was highly sensitive to misalignment along the optical axis. Figure 97 shows data collected by aligning the pinhole to maximize the ODMR signal using the lock-in amplifier, then shifting the pinhole to collect light from deeper into the sample. No signal can be seen in this plot.

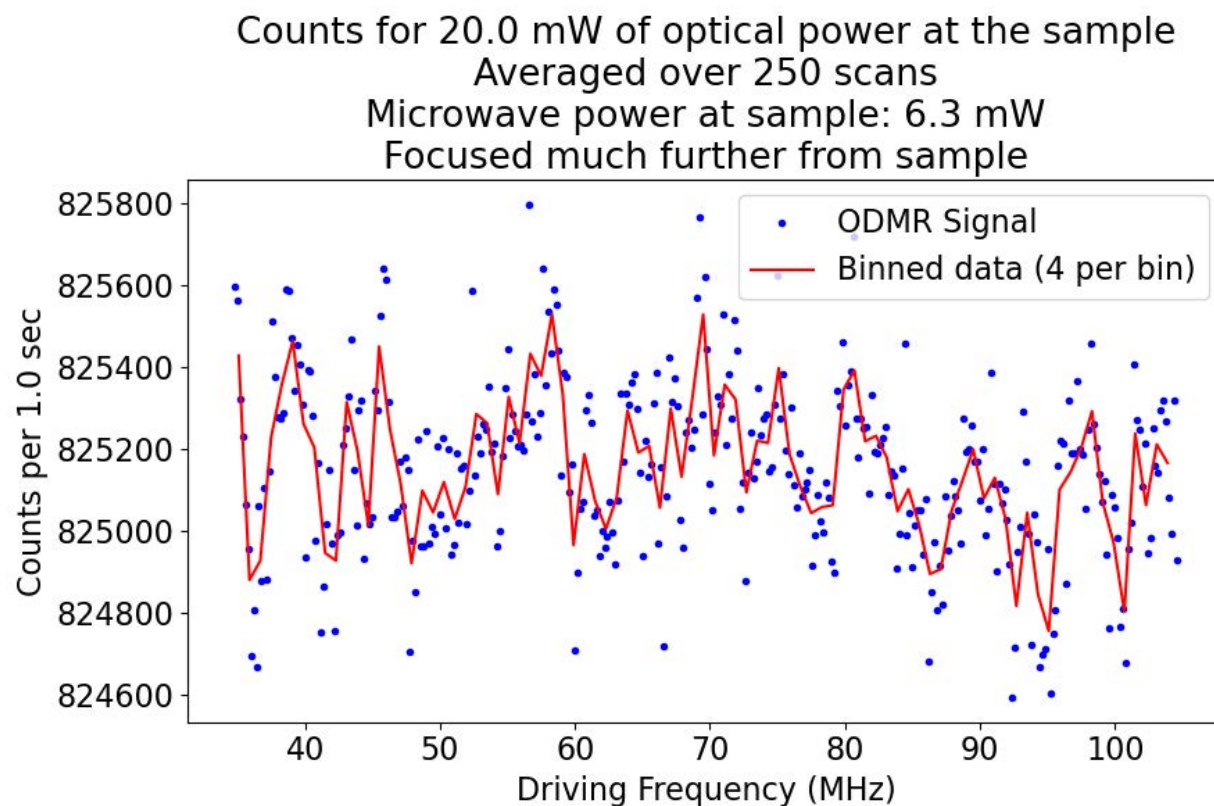


Figure 97: This figure was generated by `odmr_plot_boule.py` from the file `odmr_x40_y30_p200_t1000_s200_d-20`. Measurement taken with an exposure time of 1000 ms and drive frequency step size of 0.2 MHz. A running average is shown in red to guide the eye.

By adjusting the pinhole again, the beginning of a signal could just be seen in the next set of data, shown in Figure 98. Note the slightly wider step size to aid in finding the signal without a prohibitively long scan time.

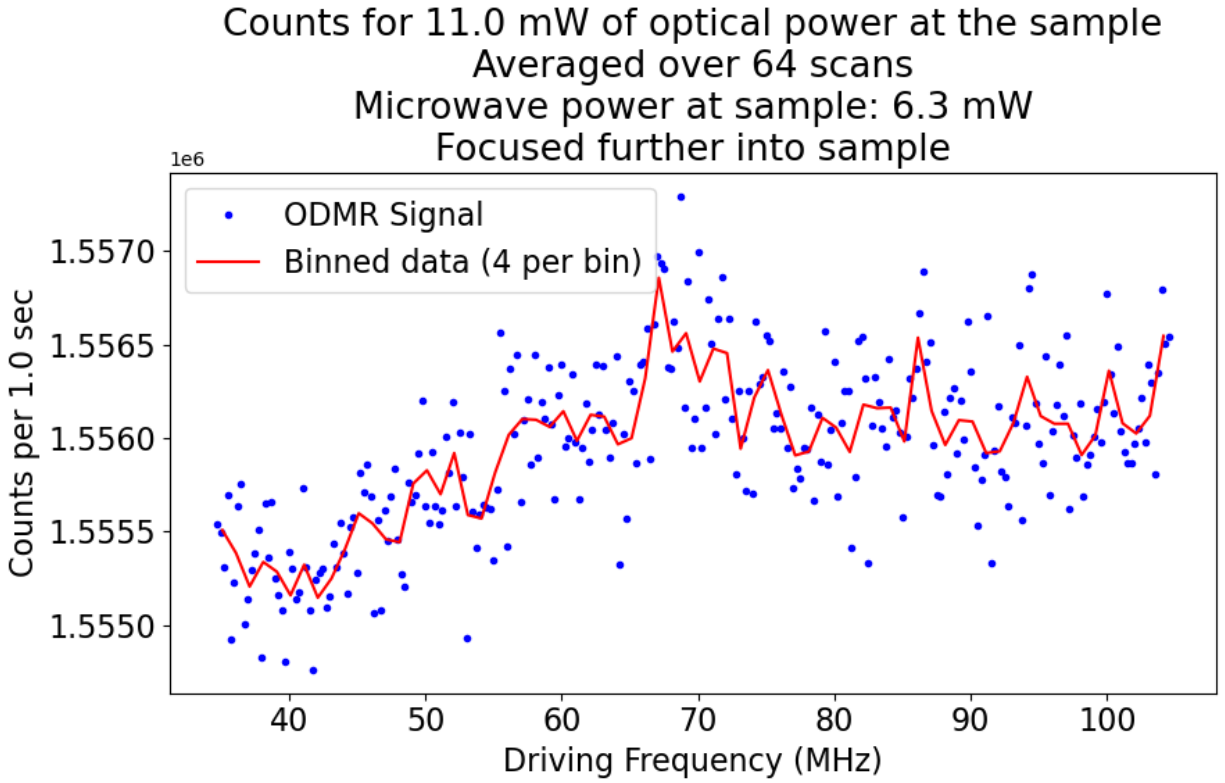


Figure 98: This figure was generated by `odmr_plot_boule.py` from the file `odmr_x40_y30_p110_t1000_s250_d-20`. Measurement taken with an exposure time of 1000 ms and drive frequency step size of 0.25 MHz. A running average is shown in red to guide the eye.

By increasing the step size significantly and increasing the number of scans averaged over, a scan was collected with the pinhole at a slightly adjusted location from the position corresponding to maximum PL intensity that showed a clear signal (Figure 99). The signal is still small, less than about 0.1%. This pinhole location was used for all subsequent measurements.

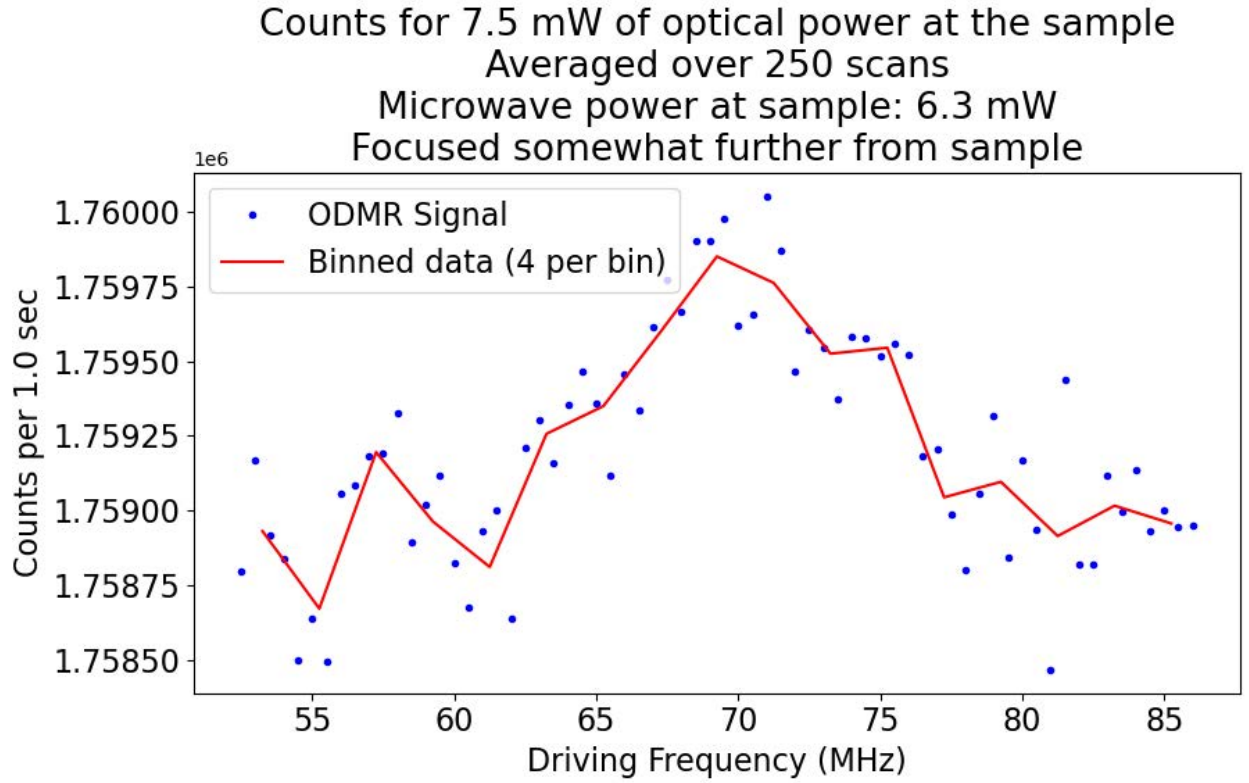


Figure 99: This figure was generated by `odmr_plot_boule.py` from the file `odmr_x40_y30_p75_t1000_s500_d-20`. Measurement taken with an exposure time of 1000 ms and drive frequency step size of 0.5 MHz. A running average is shown in red to guide the eye.

A linear polarizer was placed after the dichroic mirror to pass only horizontally polarized light through. This resulted in a PL-intensity reduction of about 30%, and the signal shown in Figure 100.

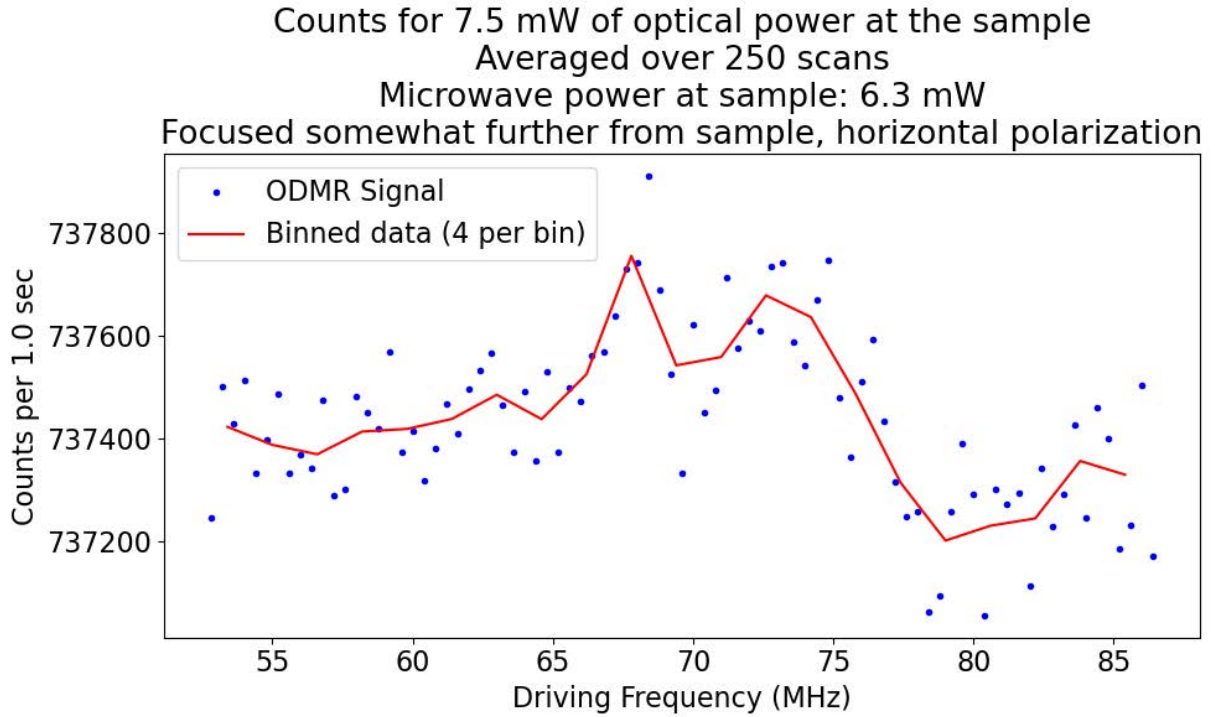


Figure 100: This figure was generated by `odmr_plot_boule.py` from the file `odmr_x40_y30_p75_t1000_s400_d-20`. Measurement taken with an exposure time of 1000 ms and drive frequency step size of 0.4 MHz. A running average is shown in red to guide the eye.

The signal is not particularly clearer with the polarization filtering, indicating that this is not a likely method to significantly increase signal-to-noise ratio (SNR).

Similarly to before, 4 neodymium magnets (B662-N52) were placed on optical posts at different distances to try to cause a splitting of the resonance frequencies. With the magnets placed closer to the sample, a field of about 1.0 mT (28 MHz shift) was measured near the sample.

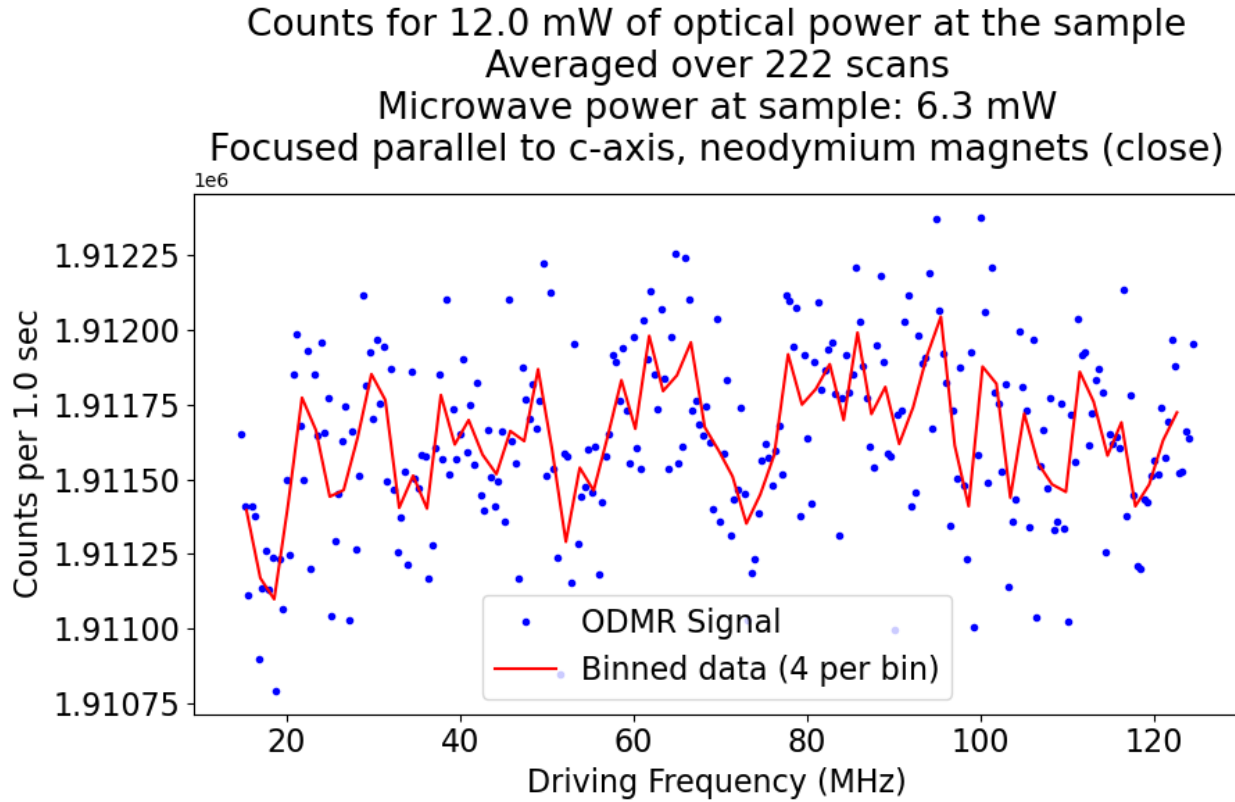


Figure 101: This figure was generated by `odmr_x40_y30_p120_t1000_s400_d-20_b100.py` from the file . Measurement taken with an exposure time of 1000ms and drive frequency step size of 0.4MHz. A running average is shown in red to guide the eye. Neodymium magnets placed to generate a B -field aligned with the c-axis of about 1.0mT at the sample.

No clear signal was seen here. It is possible that the actual field experienced by the sample was larger than the measurement indicated, or that the SNR was simply not high enough to resolve. It is also possible that a smaller step size in driving frequency might have allowed for the signal to be measured.

With the magnets placed closer to the sample, a field of about 0.3mT (8MHz shift) was measured near the sample.

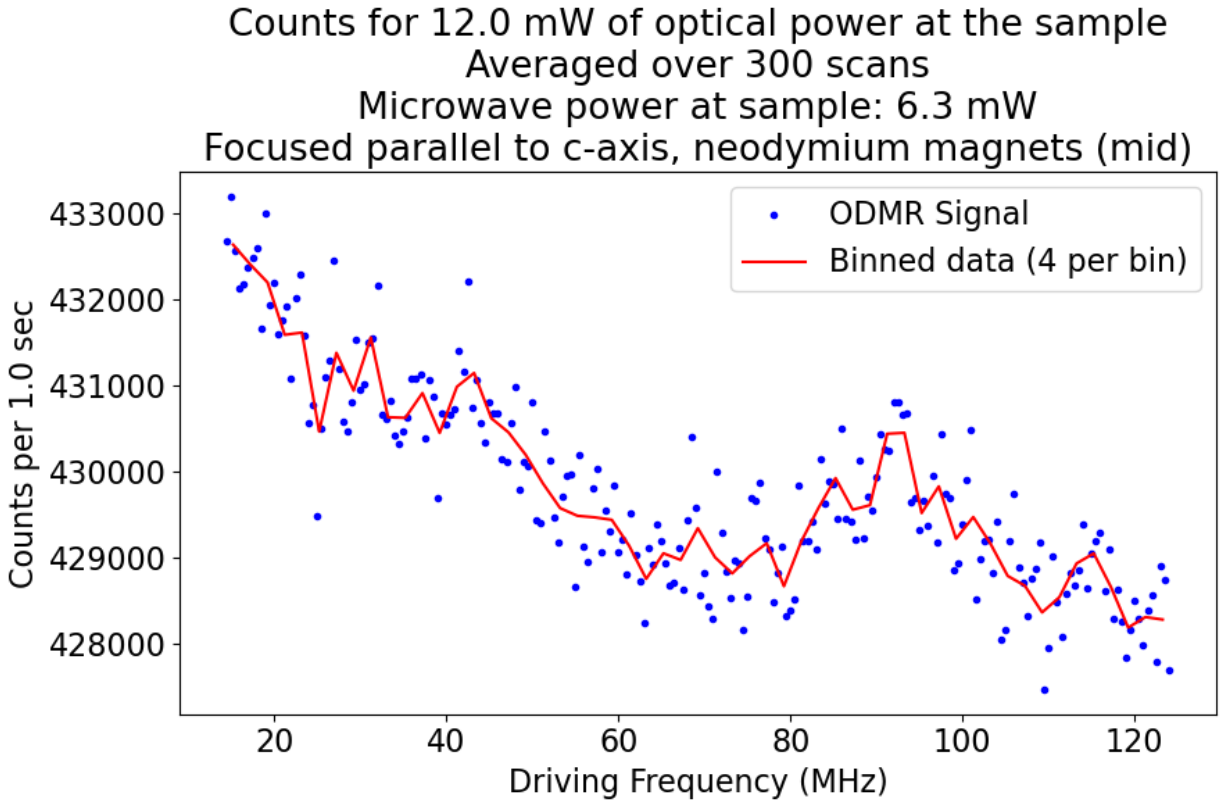


Figure 102: This figure was generated by `odmr_plot_boule.py` from the file `odmr_x40_y30_p120_t1000_s500_d-20_b30`. Measurement taken with an exposure time of 1000 ms and drive frequency step size of 0.5 MHz. A running average is shown in red to guide the eye. A running average is shown in red to guide the eye. Neodymium magnets placed to generate a B -field aligned with the c-axis of about 0.3 mT at the sample.

It looks like there may be a peak around 45 MHz and 95 MHz, indicating a magnetic field stronger than expected by a factor of 3. However, it is difficult to say anything with certainty more than that the signal seems to be affected by the external field.

With the magnets placed closer to the sample, a field of about 0.15 mT (4 MHz shift) was measured near the sample.

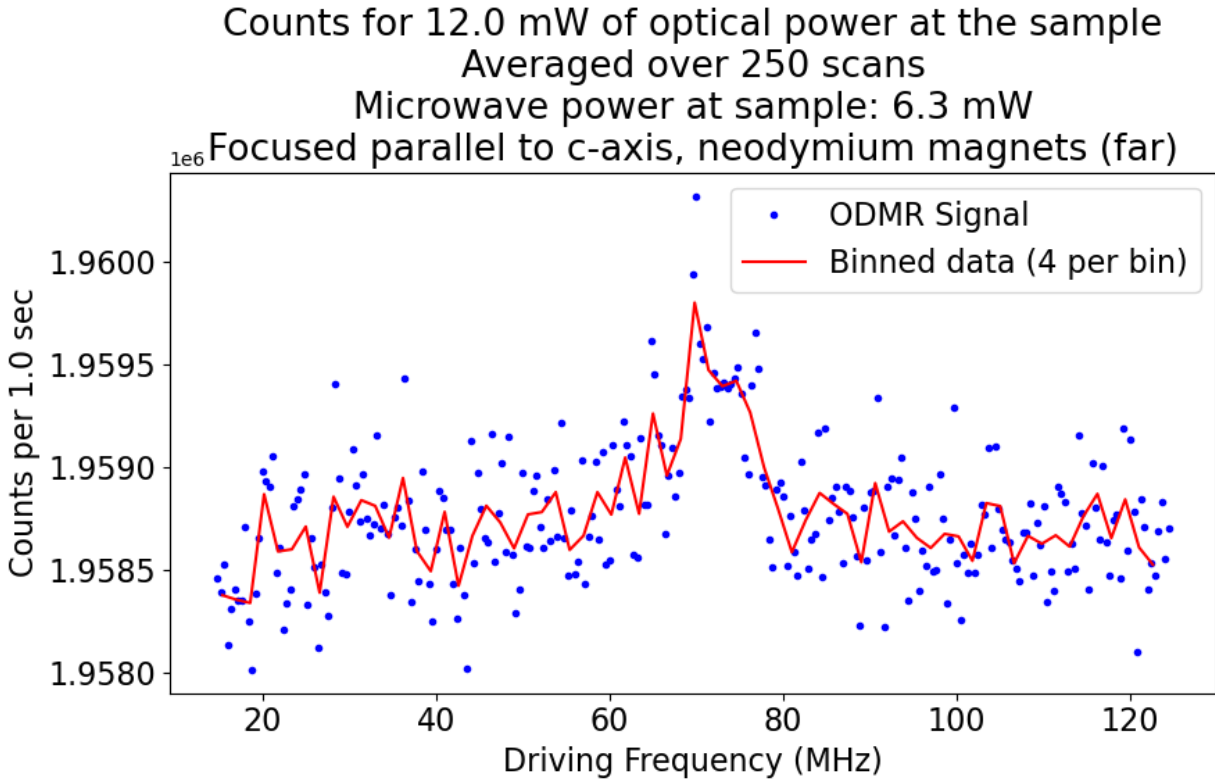


Figure 103: This figure was generated by `odmr_plot_boule.py` from the file `odmr_x40_y30_p120_t1000_s400_d-20_b15`. Measurement taken with an exposure time of 1000 ms and drive frequency step size of 0.4 MHz. A running average is shown in red to guide the eye. A running average is shown in red to guide the eye. Neodymium magnets placed to generate a B -field aligned with the c-axis of about 0.15 mT at the sample.

This signal appears to show a single peak at the usual 70 MHz frequency. This may indicate that the magnetic field was too weak (or not aligned well-enough) to produce a noticeable effect.

These results are all difficult to make out owing to the low signal strength and the difficulty in obtaining the measurements (long integration times were used, and lights in adjacent labs had to be turned off to sufficiently reduce background light). However, it seems clear that an ODMR signal was measured, both by the consistent location of the signal at 70 MHz and the variability when subject to an external static magnetic field.

MDMR

The basic idea of MDMR is to use the magnetic moments of the V_{Si}^- (V2) centers in 4H-SiC to alter the bulk magnetic properties of the material by populating some spin states more than others. If there is a net change in the sum of the spin states of the electrons in V_{Si}^- (V2) vacancy centers, the result will be an overall change in the magnetic moment of the particle. This change in the magnetic moment, in turn, causes a change in the force applied to the particle by the trapping magnetic field, which results in a change in the position of the particle. If such a force is applied on resonance with the mechanical oscillation frequency of the particle, it can result in a measurable increase in the on-resonance amplitude of motion of the particle. In essence, MDMR uses the same effect as ODMR to produce a mechanical, rather than an optical, signal.

Models and Theory

The vertical equation of motion for a diamagnetic particle in a simple quadrupole magnetic field can be rewritten from Eq. (18) as

$$\ddot{y} = \frac{\chi}{\rho\mu_0} \left(\frac{\partial B_x}{\partial y} \right)^2 y. \quad (49)$$

The vertical oscillation frequency (ν_y) in our system with no DC voltage bias across the pole pieces was measured to be about 94.8 Hz, and can be used to estimate the field gradient in the trap by rearranging equation (49) to get

$$\frac{\partial B_x}{\partial y} = 2\pi \sqrt{\frac{\rho_{SiC}\mu_0}{-\chi_{SiC}}} \nu_y. \quad (50)$$

Using values of $\rho_{SiC} = 3.21 \text{ g/cm}^3$ and $\chi_{SiC} = -1.28 \times 10^{-5}$, we estimate the field gradient to be about $10 \text{ mT}/\mu\text{m}$.

The particle sits below the center of the magnetic quadrupole field generated by the pole pieces due to the offset in its equilibrium position caused by gravity, which results in a steep field gradient across the whole length of the particle. For a particle approximately $20 \mu\text{m}$ in diameter, the resonant frequency of V_{Si}^- (V2) vacancy centers within the particle can be estimated to change by about 5000 MHz from one end of the particle to the other.

A model of the force applied to a particle containing many vacancy defect centers is shown in Figure 104. The B -field shown is estimated from the observed frequencies in the trap, and the red and green dots are used to represent vacancy centers activated by a MD sweep. The width of the active band is estimated from the data obtained by Carter et al. [24].

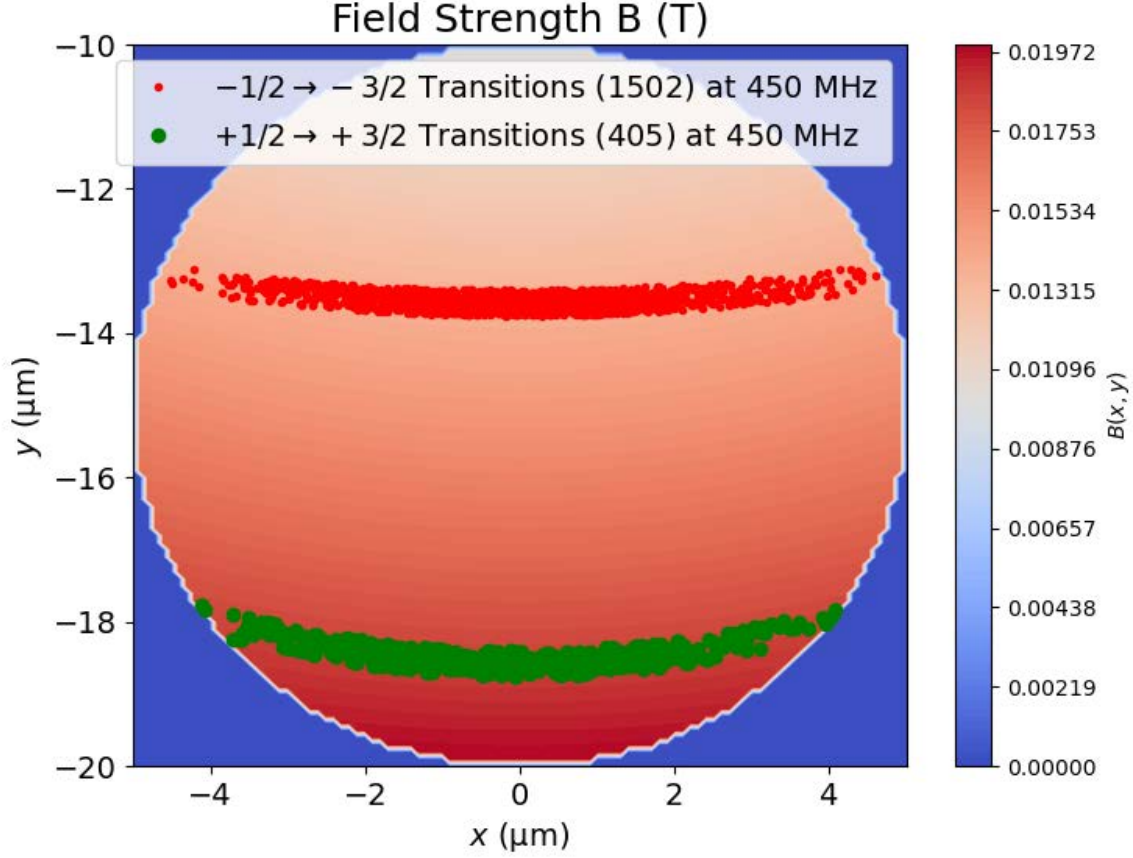


Figure 104: This figure was generated by `microwave_drive_7.py`. Model of the cross section of a particle sitting at equilibrium in a MGT, where the dominant field B_x is approximately linear in y . The bands of colored dots are meant to represent active vacancy centers triggered by a MD sweep.

Figure 105 shows an estimate for the force on the particle modeled in Figure 104 calculated using

$$\vec{F} = (\gamma_f - \gamma_0) \sum_{i=1}^{n_{\text{res}}} \nabla (\mu_{\pm 3/2, i} - \mu_{\pm 1/2, i}) \cdot \vec{B} \approx \pm g \mu_B \left(\frac{\partial B_x}{\partial x} \Big|_{x=0} \right) (\gamma_f - \gamma_0), \quad (51)$$

where $\mu_{ms, i}$ is the magnetic moment of spin- m_s vacancy center and $\gamma_f = 0.99$ and $\gamma_0 = 0.55$ were used to represent the population density of targeted spin states with and without a driving microwave field. Note that this can only be taken as a very primitive estimate, since

the exact population densities γ_f , γ_0 and total defect density are not known. Nevertheless, efforts were made to get the order of magnitude of the estimate as close as possible.

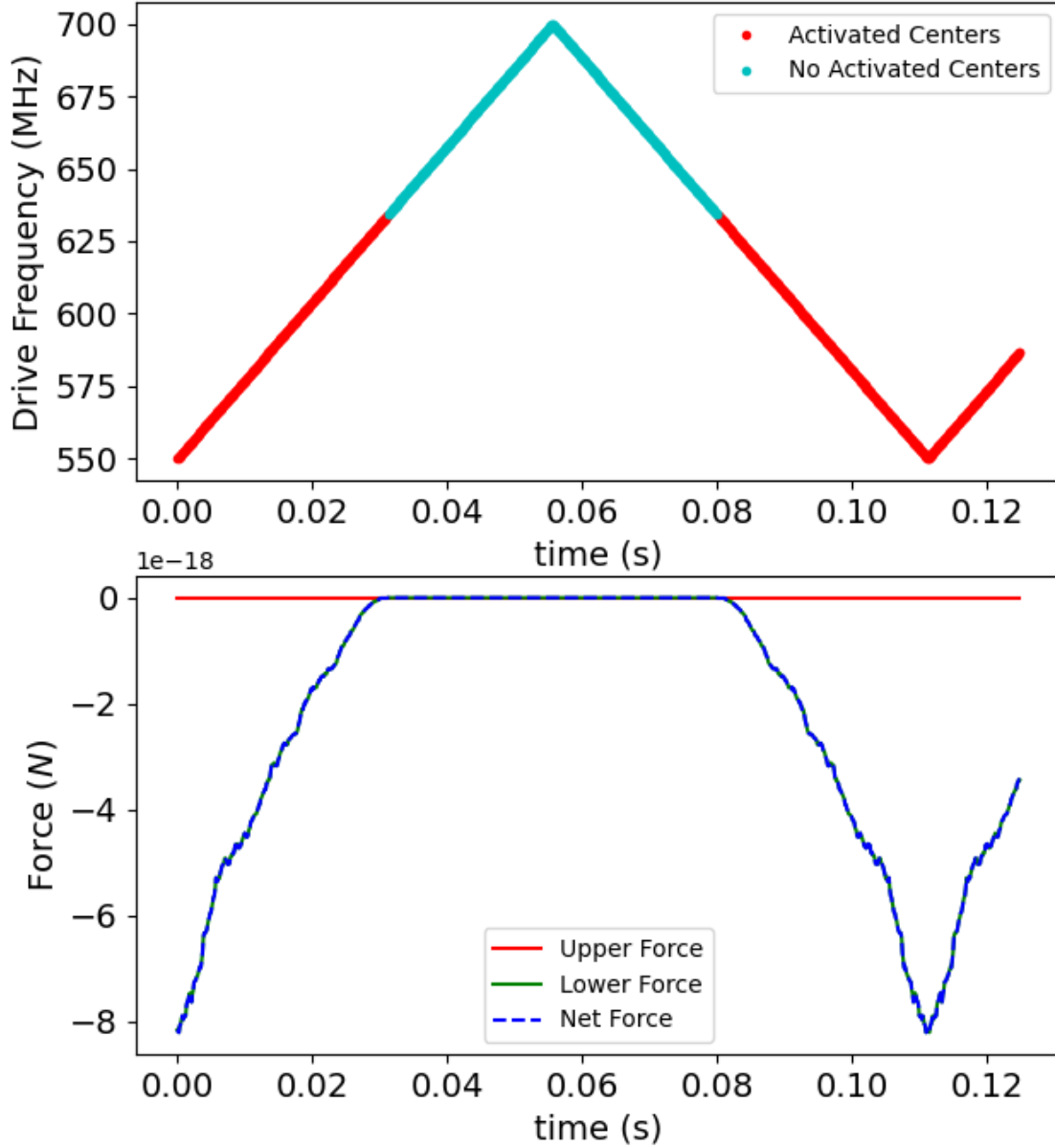


Figure 105: This figure was generated by `microwave_drive_7.py`. Top: The microwave sweep applied to the particle, color-coded by whether or not it is activating any frequencies in the particle shown in Figure 104 at a given time. Bottom: Plot of the force applied to a particle by the microwave signal shown in the top plot as a function of time, calculated from Eq. (51). The lower force refers to the force caused by driving $+\frac{1}{2} \rightarrow +\frac{3}{2}$ transitions, while the upper force is caused by driving $-\frac{1}{2} \rightarrow -\frac{3}{2}$ transitions.

In the case of Figure 105, the force is simply harmonic and would likely provide a good efficiency for on-resonance detection. A similar effect can be achieved in a different frequency range nearer the top of the particle, as seen in Figure 106.

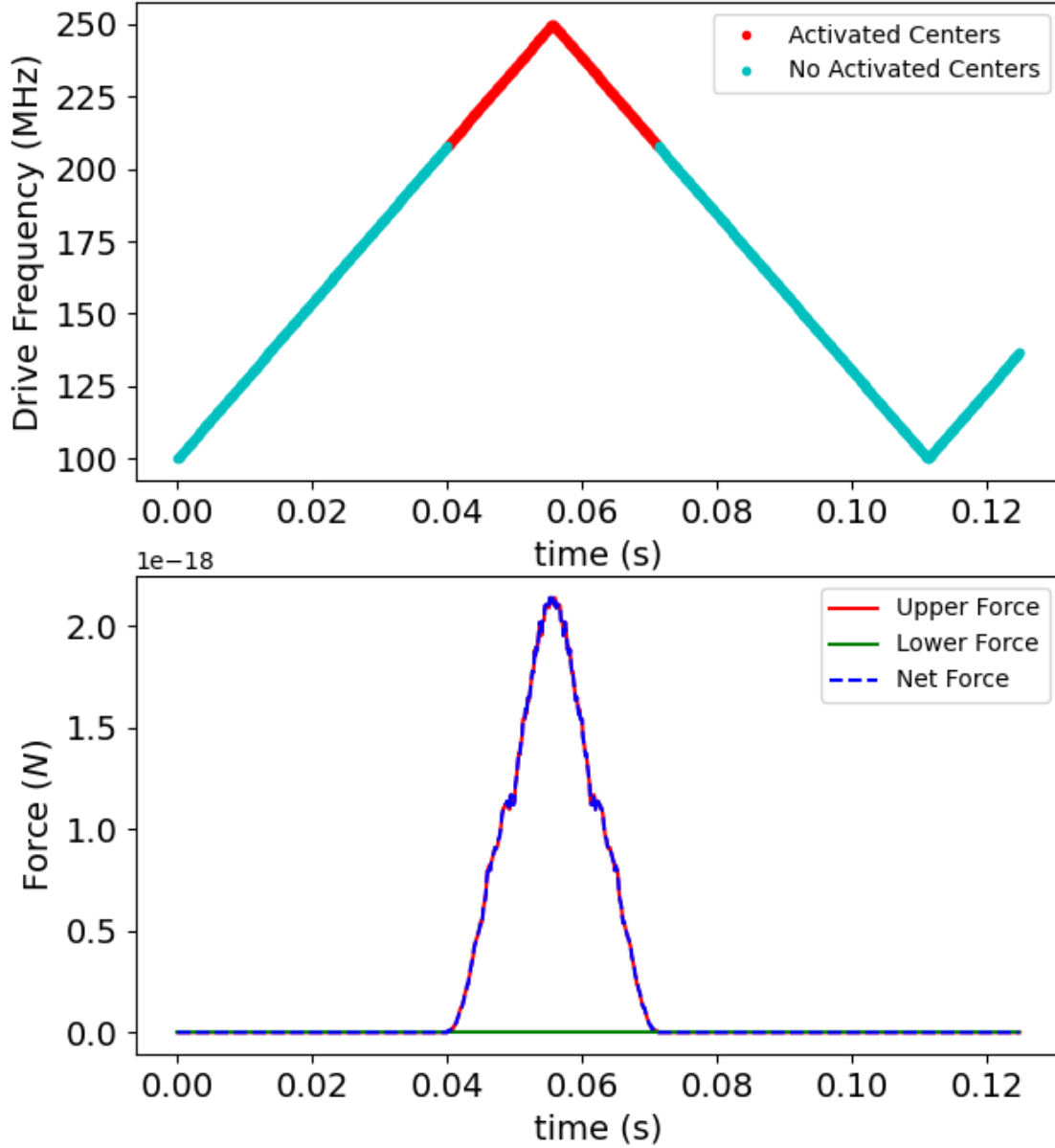


Figure 106: This figure was generated by `microwave_drive_7.py`. Top: The microwave sweep applied to the particle, color-coded by whether or not it is activating any frequencies in the particle shown in Figure 104 at a given time. Bottom: Plot of the force applied to a particle by the microwave signal shown in the top plot as a function of time, calculated from Eq. (51). The lower force refers to the force caused by driving $+\frac{1}{2} \rightarrow +\frac{3}{2}$ transitions, while the upper force is caused by driving $-\frac{1}{2} \rightarrow -\frac{3}{2}$ transitions.

Due to the geometry of the particle, even sweeps that trigger spin transitions with opposing forces can still be useful for creating a resonant force, as seen in Figure 107. This is because the bands of activated centers near the center of a sphere contain many more defect centers than bands near the edges of the sphere.

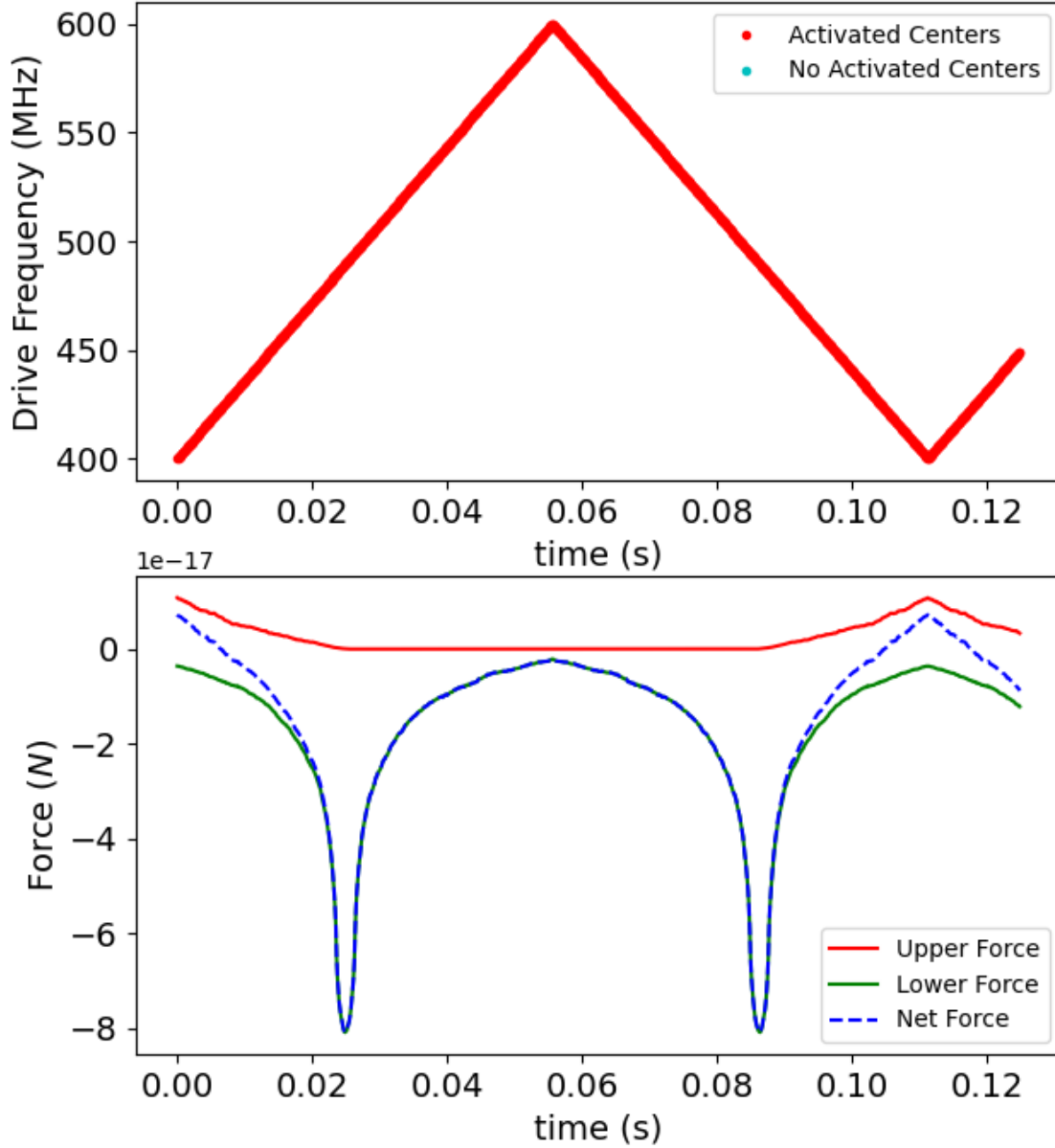


Figure 107: This figure was generated by `microwave_drive_7.py`. Top: The microwave sweep applied to the particle, color-coded by whether or not it is activating any frequencies in the particle shown in Figure 104 at a given time. Bottom: Plot of the force applied to a particle by the microwave signal shown in the top plot as a function of time, calculated from Eq. (51). The lower force refers to the force caused by driving $+\frac{1}{2} \rightarrow +\frac{3}{2}$ transitions, while the upper force is caused by driving $-\frac{1}{2} \rightarrow -\frac{3}{2}$ transitions.

However, sweeps spanning wider sections of the particle (or as in the case of Figure 108, the whole particle) produce somewhat more complicated force profiles that may not provide good resonant force detection.

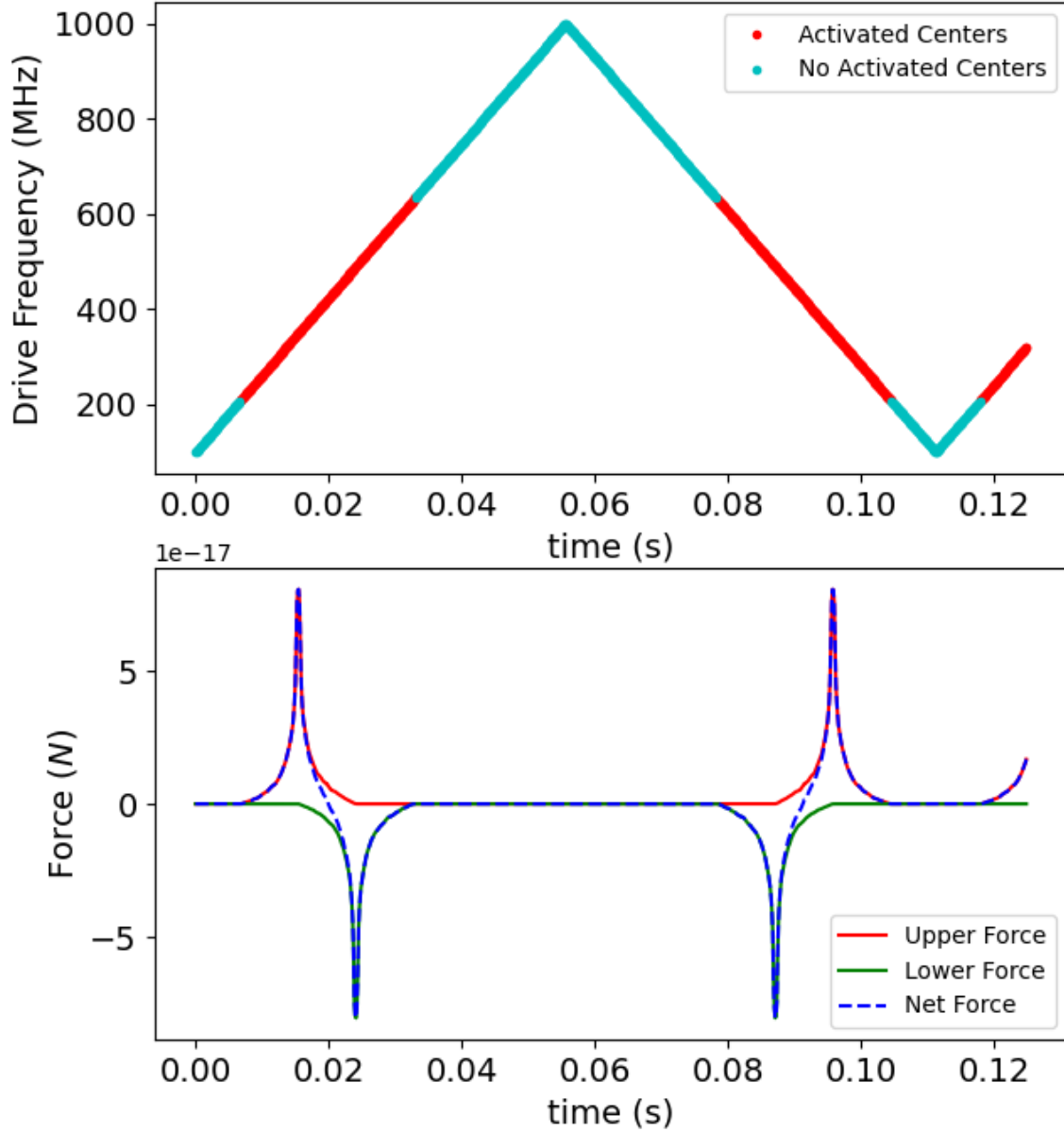


Figure 108: This figure was generated by `microwave_drive_7.py`. Top: The microwave sweep applied to the particle, color-coded by whether or not it is activating any frequencies in the particle shown in Figure 104 at a given time. Bottom: Plot of the force applied to a particle by the microwave signal shown in the top plot as a function of time, calculated from Eq. (51). The lower force refers to the force caused by driving $+\frac{1}{2} \rightarrow +\frac{3}{2}$ transitions, while the upper force is caused by driving $-\frac{1}{2} \rightarrow -\frac{3}{2}$ transitions.

It is therefore advantageous to choose MD sweep ranges that primarily target one edge of the particle, rather than sweeping over the entire volume or the middle section of a particle.

The shape of the particle has a significant impact on the shape of the signal observed. Figures 109 and 110 show some estimates of what signal shapes might look like for a central drive frequency ranging from 1050-1950 MHz (a frequency band that can be continuously modulated over in a phase-continuous manner by our microwave signal generator (SG384) with a modulation of ± 90 MHz. These plots are estimations based on integrating the density of point defects in space $\rho_{pd}(y \propto \nu)$ for all frequencies ν around each central driving frequency ν_0 with a harmonic weighting function:

$$F_{resonant}^{\pm}(\nu_0) \propto \int_{\nu_0 - \Delta\nu}^{\nu_0 + \Delta\nu} \rho_{pd} \left(\nu \pm \frac{1}{2} \nu_{V2} \right) \sin \left(\frac{\pi}{2} \frac{\nu - \nu_0}{\Delta\nu} \right) d\nu. \quad (52)$$

Here ν_{V2} is the frequency associated with the V_{Si}^- (V2) centers in 4H-SiC, approximately $(70 \text{ Hz}) \times (3 \cos^2 \theta - 1)$ [24, 79], where θ is the angle between the c-axis and the static magnetic field (B_x). The resulting net force is then $F_{resonant}^+ - F_{resonant}^-$.

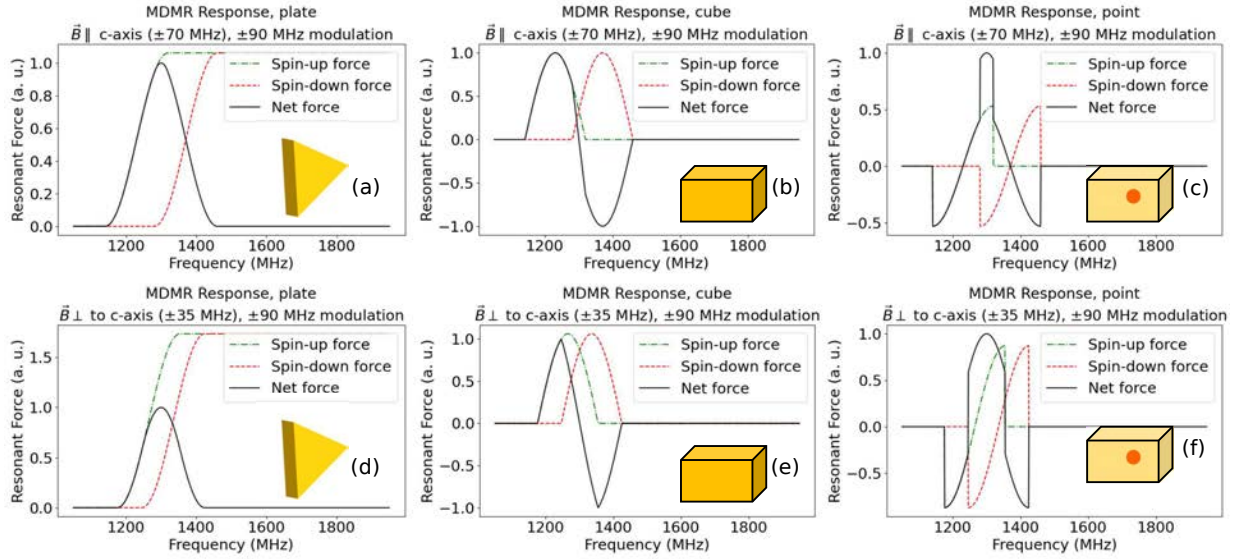


Figure 109: This figure was generated by `signal_shape.py`. This figure shows the expected response of a vacancy-rich 4H-SiC particle to modulation by a microwave-frequency magnetic field applied perpendicular to the c-axis of the crystal in the presence of a static magnetic field applied either parallel (top row) or perpendicular (bottom row) to the c-axis. Panels a, b, d, and e assume a homogenous distribution of vacancy centers throughout the particle's geometry, where the upper-most edge of the particle is located at a static field strength corresponding to a central resonance frequency of 1300 MHz (~ 46 mT). Panels c and f show the expected signal shape when a single vacancy center (represented as a red dot inside a particle of arbitrary shape) is subject to the same modulated signal.

One interesting feature of these signals is the fact that the signal vanishes after the edge of the particle is passed. Therefore, the magnetic field at the edge of the particle would need to be known fairly precisely to judge whether or not the frequency range of the sweep

would even generate a signal. This is not quite true for the signals shown in panels c and f; in that case, one might expect the signal to appear several times throughout the volume of the particle. However, for a clear signal to be distinguished in this case, these signals would need to be far enough apart so as not to overlap significantly. Other particle shapes, by contrast, yield a continuous signal throughout the entire volume of the particle, as shown in Figure 110.

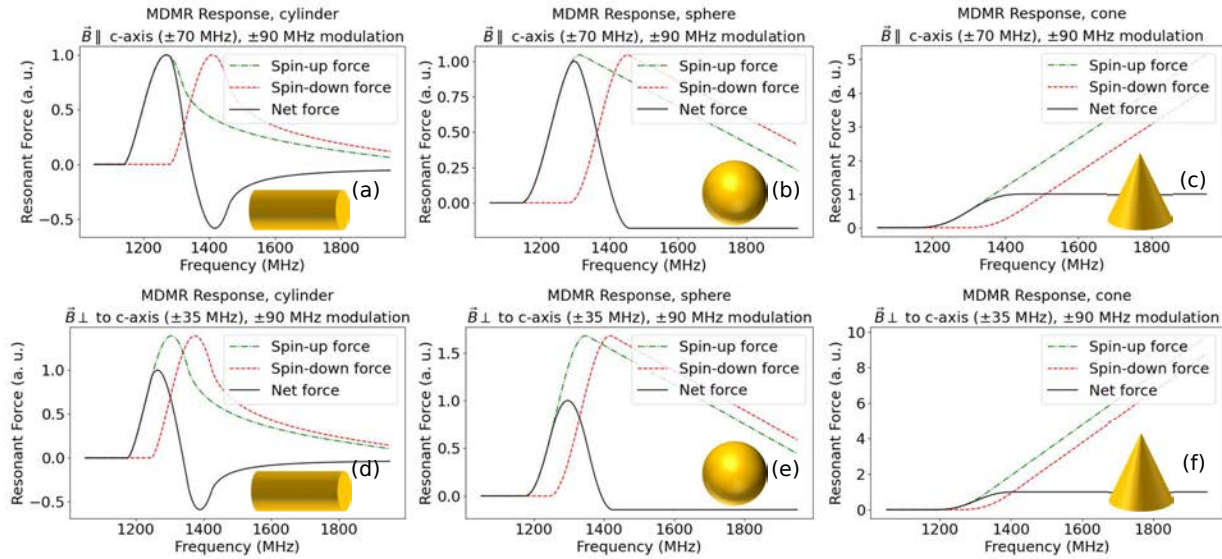


Figure 110: This figure was generated by `signal_shape.py`. This figure shows the expected response of a vacancy-rich 4H-SiC particle to modulation by a microwave-frequency magnetic field applied perpendicular to the c-axis of the crystal in the presence of a static magnetic field applied either parallel (top row) or perpendicular (bottom row) to the c-axis. All panels assume a homogenous distribution of vacancy centers throughout the particle's geometry, where the upper-most edge of the particle is located at a static field strength corresponding to a central resonance frequency of 1300 MHz (~ 46 mT).

These results were not well-realized at the time data was collected. Had this result been known, extra care would have been taken to ensure that the range of positions where the particle rested in the trap was broader, increasing the likelihood of a successful measurement. Increasing the modulation amplitude might also have increased the range where a signal could be detected, however, this would also result in a decreased range for the available central frequencies, due to the limitations on the phase-continuous modulation capabilities of the SG384.

Setup

The HY0667-21B-2 sample of 4H-SiC used to obtain ODMR data was broken into pieces, and one of those pieces was used to create particles of SiC for levitation. There was some concern that the full process of particle milling would damage the defect centers, so some modifications were made to minimize surface fracturing.

The particles were processed according to the procedure detailed in A, with the following changes. First, the jet-milling step was skipped entirely. Second, the particle was crushed more carefully with the mortar and pestle: it was broken initially by placing it in the mortar and pressing down firmly to fracture it. It was then ground by rotating the pestle exactly 10 times around the mortar. Third, only 2 mL of HCl were used during the etching process due to the small sample size (the initial sample size was 6.5 mg and the final was 2.9 mg). Once these particles were made, the remainder of the boule sample was saved for reference.

The pole pieces were electrically isolated from each other (except for the first set of data, where the bottom two pole pieces were shorted together by the drive wire) so that they could be used to shape a static electric potential in the trapping region, allowing a force to be applied to a charged particle. A schematic of the electronics used to set the pole piece voltages is shown in Figure 111.

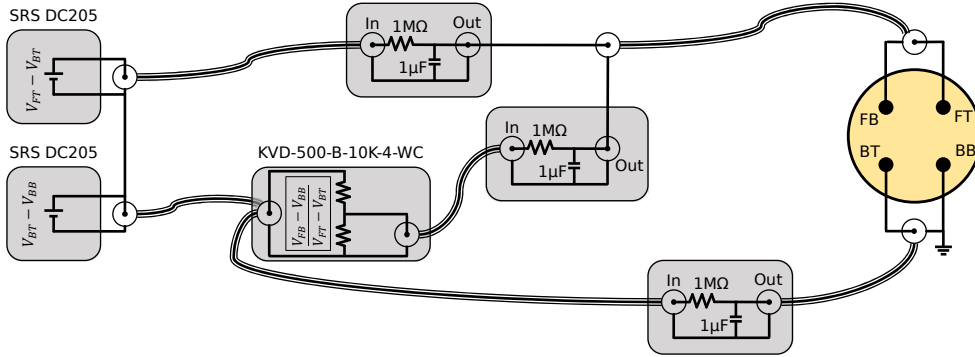


Figure 111: Schematic of the circuits used to set the DC bias voltages on the pole pieces for adjusting particle equilibrium position. A voltage divider was used to gain an additional degree of freedom, so that if one pole piece is grounded, each other can in theory be set to a different voltage. Several 1-second low-pass filters were placed to reduce the impact of high-frequency noise in the power supplies and to smooth out the effect of adjusting voltage levels. Ground was tied directly to the optical table, and the power supplies were operated with a floating ground.

In practice, adjusting the voltage across the transverse pole piece gap did not seem to affect the transverse location of the particle very significantly. Since adjusting the position of the particle relative to the drive wire was a desirable capability, an alternative wiring system, shown in Figure 112, was used for the second set of data.

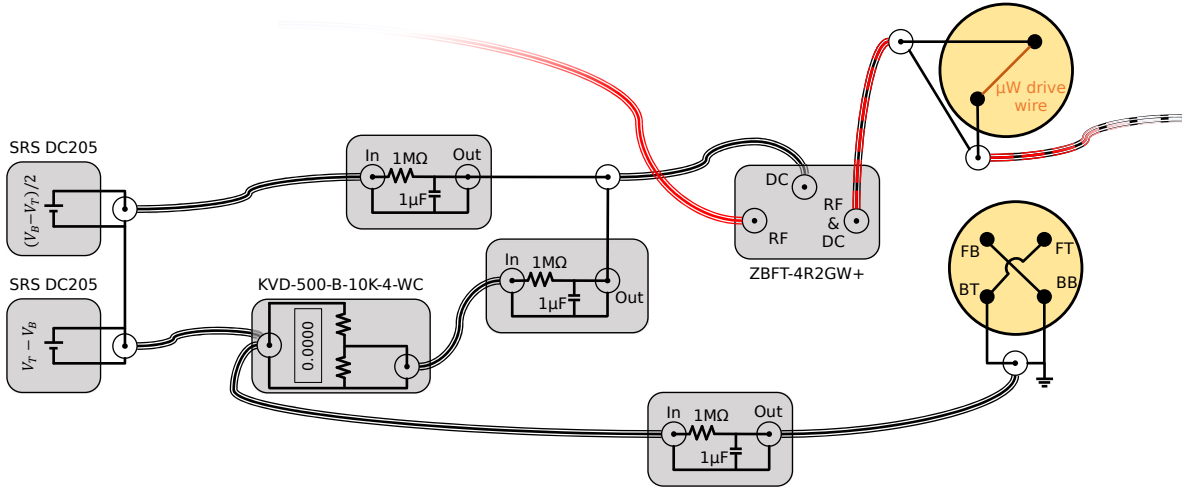


Figure 112: Schematic of the circuits used to set the DC bias voltages on the pole pieces for adjusting particle equilibrium position. Several 1-second low-pass filters were placed to reduce the impact of high-frequency noise in the power supplies and to smooth out the effect of adjusting voltage levels. Ground was tied directly to the optical table, and the power supplies were operated with a floating ground. In this case the top pole pieces were shorted together and the bottom set were shorted together, leaving two degrees of freedom; the difference in voltage between the top and bottom sets of pole pieces and the voltage on the wire. This setup was operated with the voltage divider essentially unused and the wire voltage set halfway between the top and bottom pole piece voltages.

The set voltages for all measurements taken are recorded in the collection parameters file in each data folder. In both cases, the voltage divider was usually set to 0.0000; the setup was engineered, however, to leave open the possibility of adjusting the value at some point.

In some cases (most notably Billy Joe (2023-06-02-D)), the particle did not respond significantly to 10s of volts across the pole pieces. In such cases, charge was deposited on the particle by shining a UV lamp through the trap window in order to induce a charge to form on the pole pieces. The SiC tended to become negatively charged, as indicated by the sign of the voltage required to move it upward in the trap.

As before, the $30\text{ }\mu\text{m}$ gold-plated tungsten wire needed to be placed near the sample of 4H-SiC (in this case a trapped particle) in order to manipulate the spins via the MD signal. This proved to be quite challenging, but the task was made easier by 3D-printing PEEK shims, shown in Figure ?? to hold the wire in place as it was clamped between the pole pieces and the mounting plates.

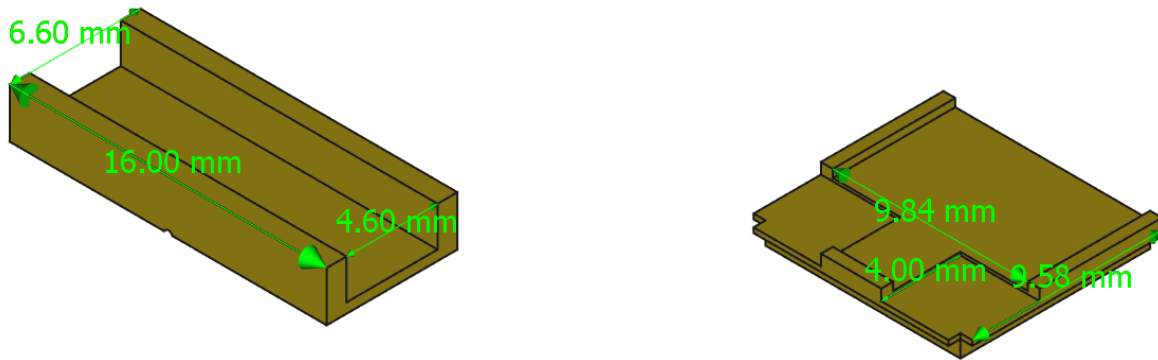


Figure 113: A FreeCAD design of shims for the Short MGT generated from `trap_plate_shims.FCStd`. The ridges were meant to keep the clamping plates (left piece) and pole pieces (right piece) from sliding around. The cutouts in the right piece allowed a thicker wire loop tied to the thin tungsten wire to be clamped firmly between the shim and the pole piece without offsetting the position of the pole piece in the process.

The connections to the wire were ported out through the chamber using a four-wire feed-through (Kurt J. Lesker EFT0143052) which was connected to the SMA inputs from the microwave signal generator. As the trap was being assembled, the microscope was used to take images of the microwave drive wire. These images are shown in Figures 114 and 115.

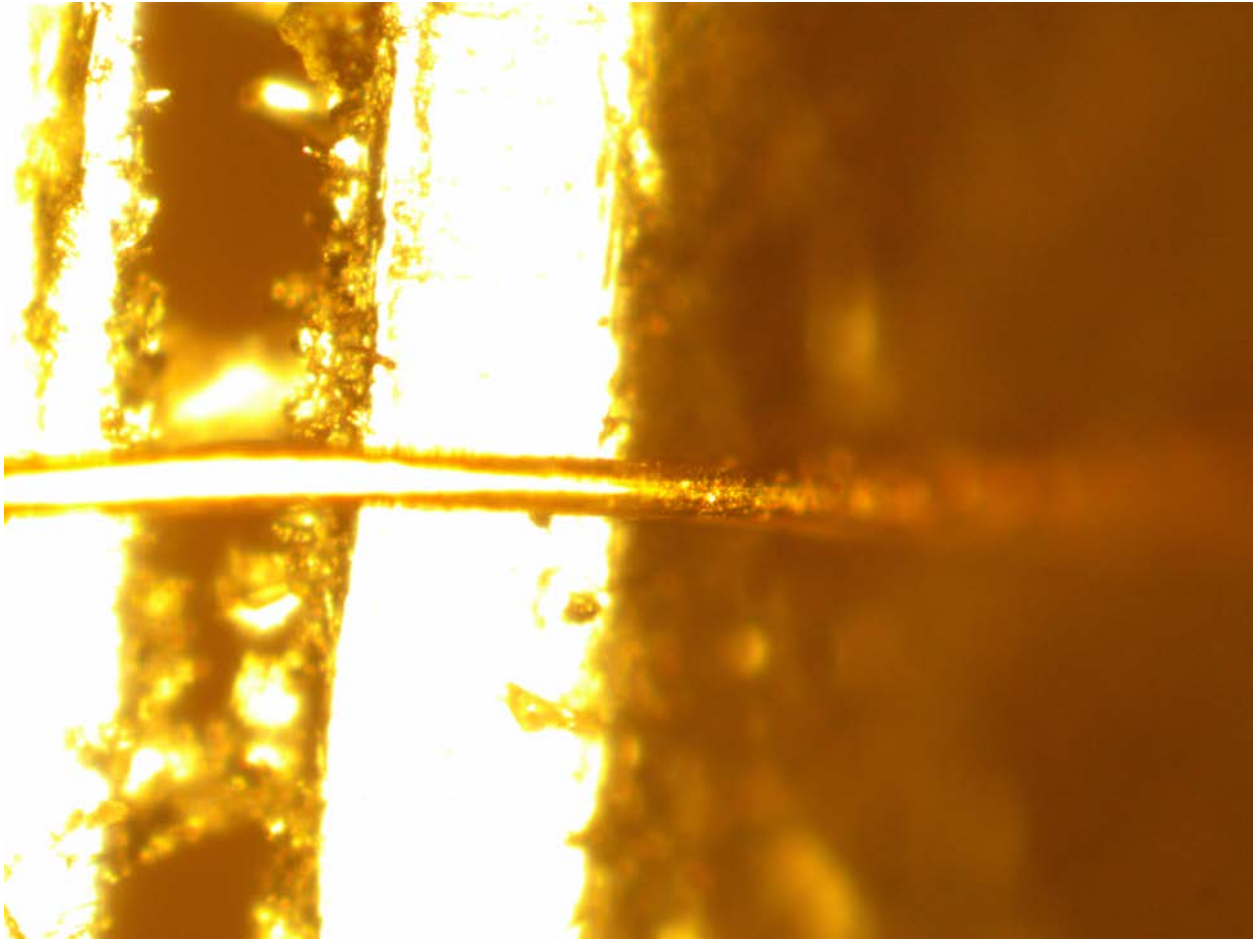


Figure 114: A microscope image (looking down the trap vertical axis) of the 30 μm gold-plated tungsten wire strung across the bottom pole pieces. Generally, this wire shorted the bottom two pole pieces together.

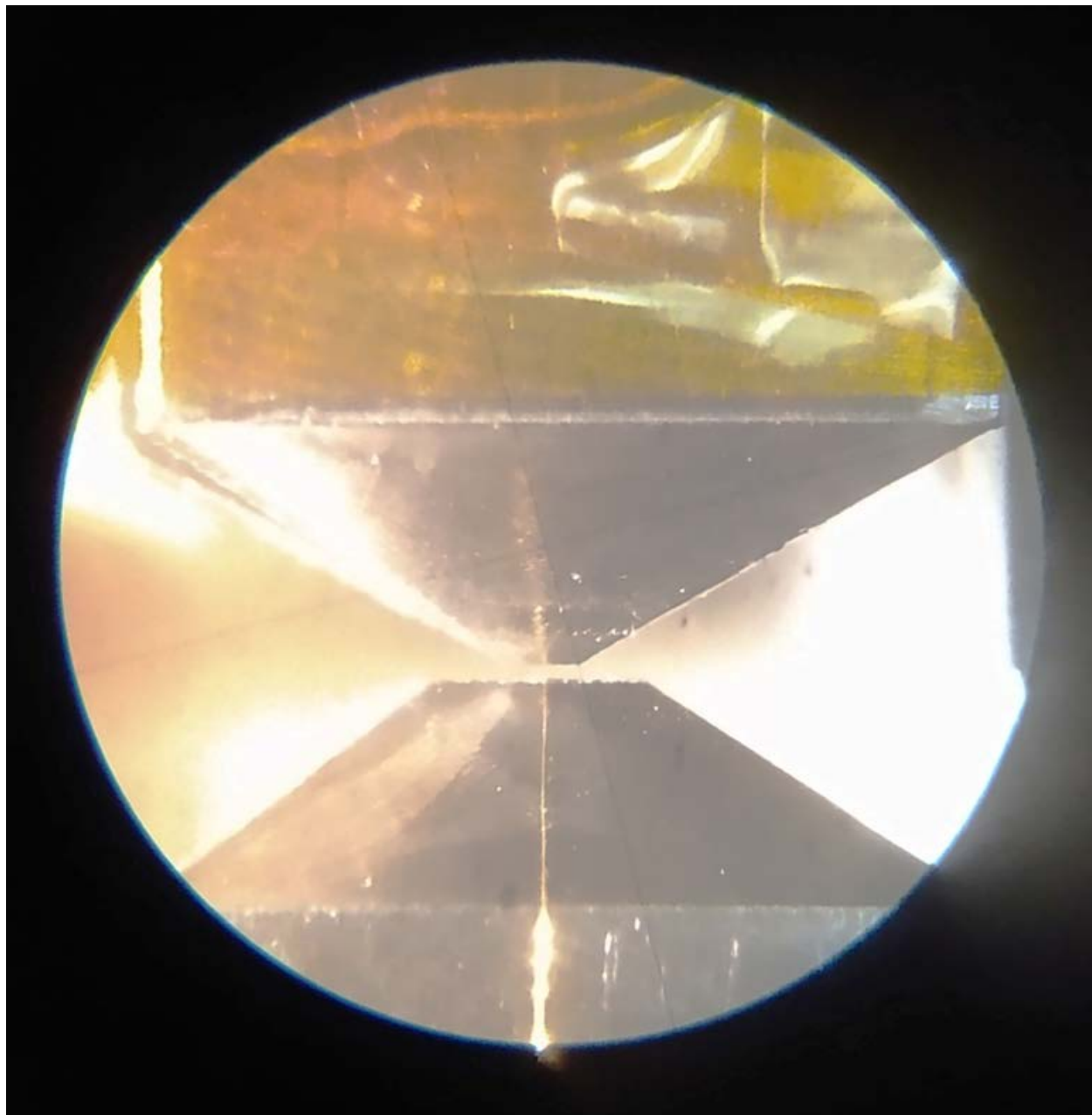


Figure 115: A microscope image (looking down the trap transverse axis) of the assembled pole pieces with the $30\text{ }\mu\text{m}$ gold-plated tungsten wire strung across the bottom pole pieces. When an RF signal is run through the wire, it will generate a field at the particle's location that is roughly in the vertical direction.

In order to try to keep DC to low-frequency noise from leaking into the signal experienced by the trapped particle, bias tees (see Figure 116) were placed at the input

and output wires on the trap.

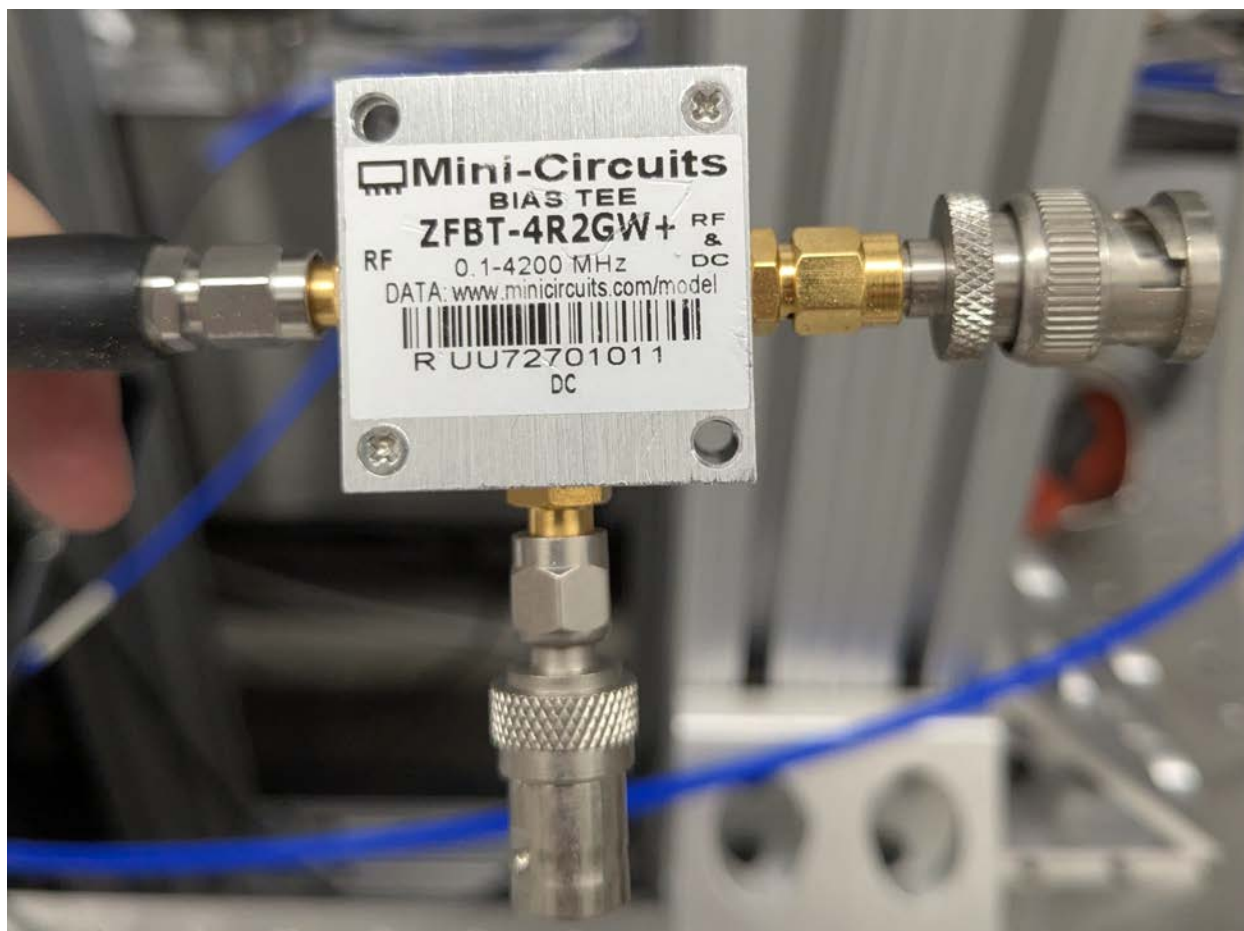


Figure 116: A bias tee used to separate low-frequency signals originating outside the chamber from the high-frequency microwave drive being transferred through the wire.

Data was collected by taking the output after the radio frequency (RF) signal was passed through the trap and putting it into a Keithley 2000 multimeter through a crystal detector (see Figure 129). The results are shown in Figure 117 alongside the output signal from the SG384.

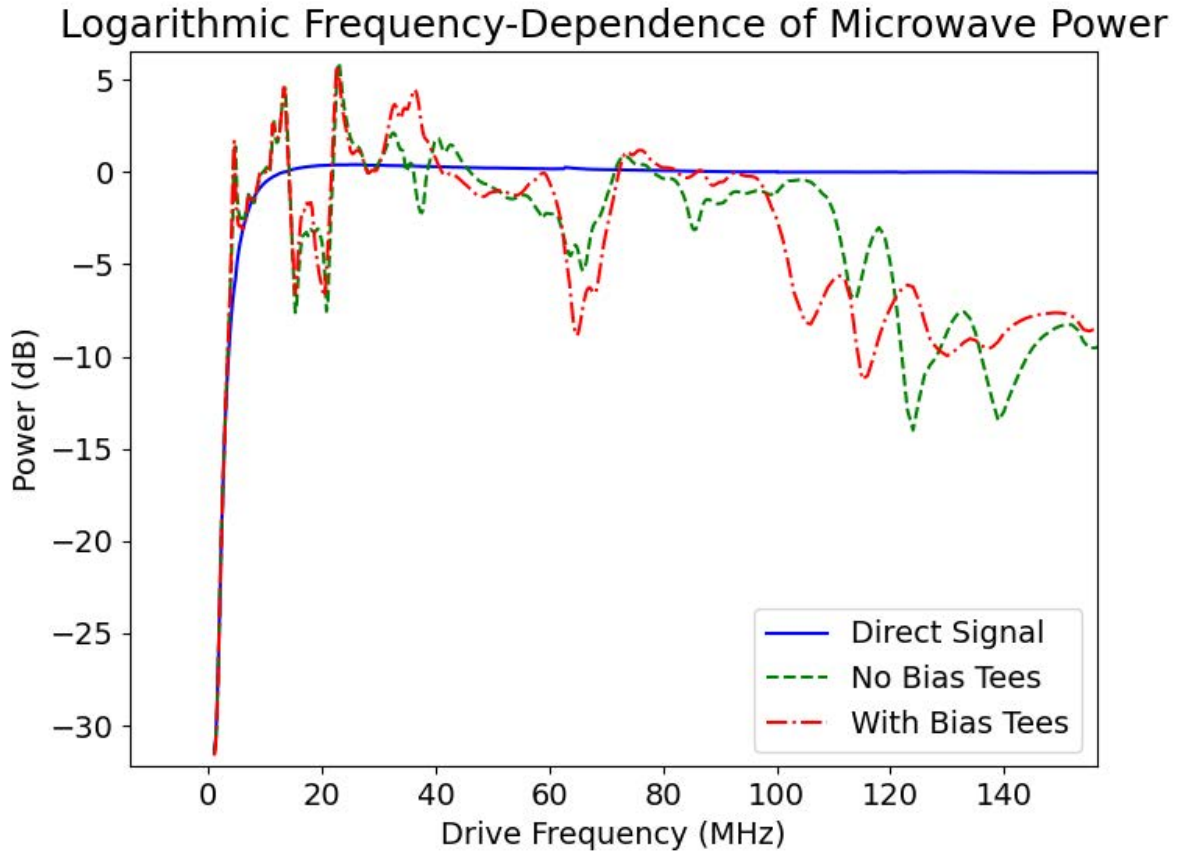


Figure 117: This figure was generated by `plot_ma_data1.py` from the files `signal_vs_frequency_250_direct`, `signal_vs_frequency_250_wbt`, and `signal_vs_frequency_250_nbt`. Output power and measured power after running an RF signal through the trap with and without bias tees.

While the bias tees clearly have an effect, it is clear that there is a great deal of frequency-dependent modulation of the signal. This is likely to make MDMR measurements quite difficult; if the current running through the drive wire varies with frequency, it may cause a resonant force on a trapped particle under frequency modulation even in the absence of any spin-resonance effect.

In order to modulate the RF signal in a phase-continuous manner, it may be helpful to have an idea of how the signal behaves over a much broader frequency range, as shown in Figure 118.

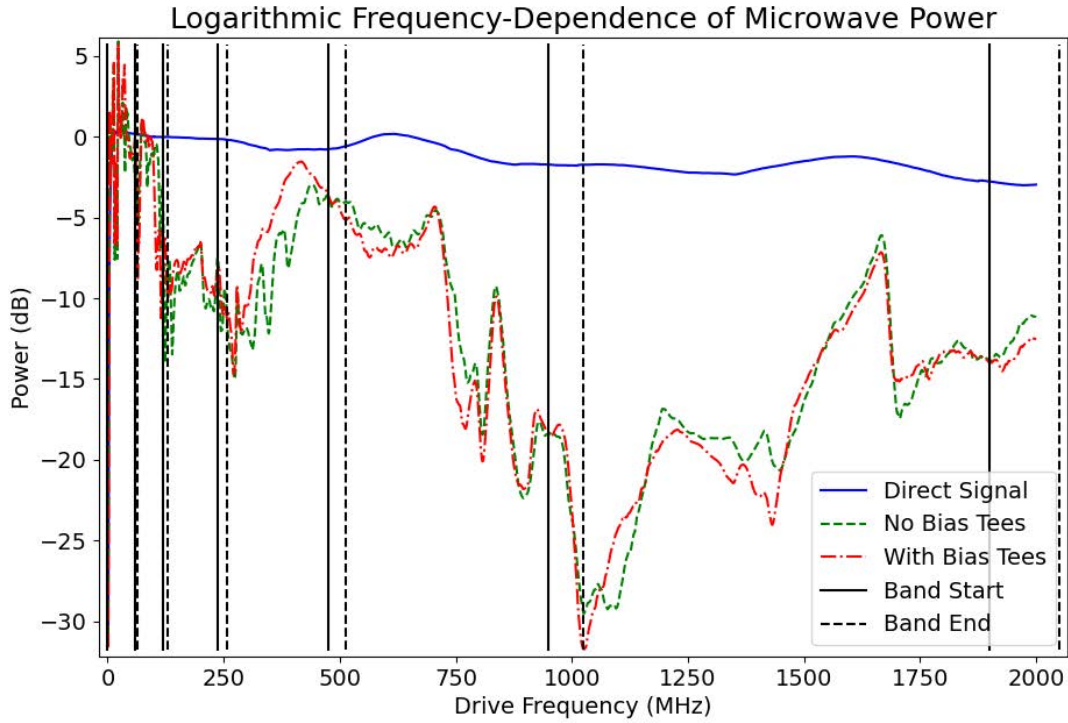


Figure 118: This figure was generated by `plot_ma_data1.py` from the files `signal_vs_frequency_1000_direct`, `signal_vs_frequency_1000_wbt`, and `signal_vs_frequency_1000_nbt`. Output power and measured power after running an RF signal through the trap with and without bias tees. Bands shown correspond to the frequency ranges over which the SG384 can provide phase-continuous frequency modulation of the RF signal.

The largest frequency range we can use goes from just below 1000 MHz to just over 2000 MHz; this is the range that was used for all MDMR measurements to follow.

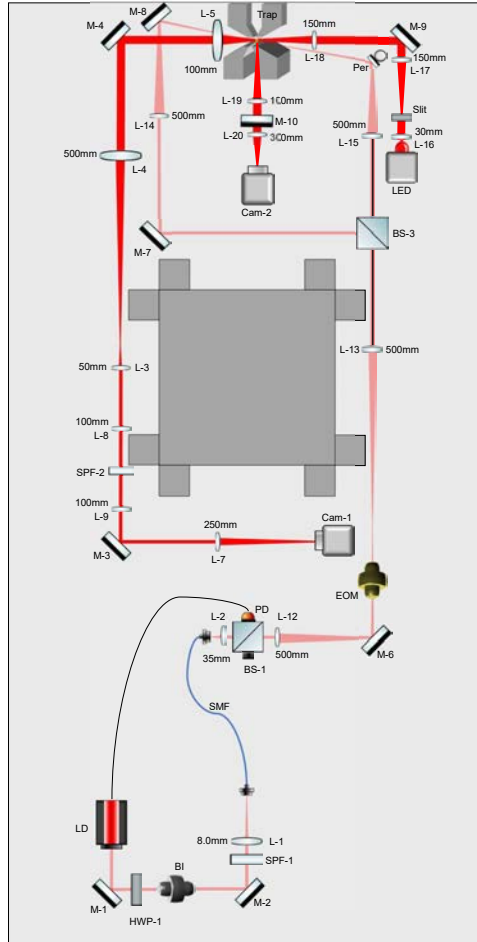


Figure 119: The optical setup used for attempting MDMR on trapped pieces of boule-grown 4H-SiC . An illumination arm is added with a slit to illuminate the trapping region uniformly, and the laser is split into two cooling beams after passing through an EOM. Note that the dichroic mirror and SPCM arm have been removed; these could be placed in again if an ODMR-like measurement was desired.

Optical Component List for Figure 119 For optical component lists, items marked with a "*" are new additions from the previous setup displayed. The notation "[C]" indicates that the components were fixed rigidly to cage rods, usually via custom aluminum mounts, and "[Al]" indicates that the optics were covered in aluminum foil to prevent stray light interfering with the signal.

- **LD.** 785 nm, 300 mW, Ø9 mm TO Can, E Pin Code, VHG Wavelength-Stabilized Single-Frequency Laser Diode (LD785-SEV300). Mounting: Newport Model700C-42Lens.
- **M-1.** Ø1" Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: POLARIS-K1E.
- **HWP-1.** 780 nm half-wave plate (WPH05M-780). Mounting: RSP1. [C]
- **BI.** Free-Space Isolator, 780 nm, Ø4.7 mm Max Beam, 1.7 W Max (IO-5-780-VLP). Mounting: SM1RC.
- **M-2.** Ø1" Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: POLARIS-K1E.
- **SPF-1.** 800 nm shortpass filter (FESH0800). Mounting: KC1-T. [C]
- **L-1.** $f = 8.0$ mm, $NA = 0.5$, Mounted Geltech Aspheric Lens, AR: 600 - 1050 nm (C171TMD-B). Mounting: CP1XY. [C]
- **SMF.** $NA = 0.12$, Single Mode Fiber (P5-780PM-FC-2). Mounting: SM1FC/A, SM1Z. [C]
- **L-2*.** $f = 35$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-035-B-ML). Mounting: CXY1A. [C]
- **BS-1.** 10:90 (R:T) Non-Polarizing Beamsplitter Cube, 700-1100 nm, 20mm (BS044). Mounting: CCM1-4ER. [C]
- **PD.** Large Area Mounted Silicon Photodiode, 350-1100 nm, Anode Grounded (SM1PD1B, NE20B-B). Mounting: CCM1-4ER. [C]
- **M-3*.** Round Protected Silver Mirror (PF10-03-P01). Mounting: POLARIS-K1E.

- **L-3.** $f = 50$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-050-B-ML). Mounting: CP02. [C]
- **L-4.** $f = 500$ mm, $\varnothing 2$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC508-500-B-ML). Mounting: LCP34. [C]
- **M-4.** 2" Broadband Dielectric Elliptical Mirror, 750 - 1100 nm (BBE2-EO3). Mounting: KCB2C. [C]
- **L-5*.** $f = 100$ mm, $\varnothing 2$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC508-100-B-ML). Mounting: CXY2, SM2L20. [C]
- **Cam-1.** Basler Camera (acA2440-75um). Mounting: CP12, C-MOUNT EXT. [C]
- **L-8.** $f = 100$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-100-B-ML). Mounting: CP02. [C]
- **L-9.** $f = 100$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-100-B-ML). Mounting: CP02. [C]
- **SPF-2*.** 750 nm shortpass filter (EO 64332). Mounting: CP02. [C]
- **L-12*.** $f = 500$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-500-B-ML). Mounting: CP02. [C]
- **M-6*.** $\varnothing 1$ " Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: POLARIS-K1E.
- **EOM*.** Electro-optic modulator, New Focus (4102). Mounting: 9071.
- **HWP-2*.** 780 nm half-wave plate (WPH05M-780). Mounting: RSP1. [C]
- **L-13*.** $f = 500$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-500-B-ML). Mounting: CP02. [C]

- **BS-3***. 20mm Polarizing Beamsplitter Cube, 620 - 1000 nm (PBS202). Mounting: CCM1-4ER. [C]
- **M-7***. Ø1" Broadband Dielectric Mirror, 750 - 1100 nm (BB1-E03). Mounting: KS1.
- **L-14***. $f = 500$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-500-B-ML). Mounting: CP02. [C]
- **M-8***. Ø1/2" Broadband Dielectric Mirror, 750 - 1100 nm (BB05-E03). Mounting: KS05.
- **L-15***. $f = 500$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 650-1050 nm (AC254-500-B-ML). Mounting: CP02. [C]
- **Per***. Periscope Assembly (BB1-E03 ($\times 2$)). Mounting: RS99.
- **LED***. Red XLamp XP-E LED (CREEXPE2-Red-1). Mounting: custom. [C]
- **L-16***. $f = 30$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 400 - 700 nm (AC254-030-A-ML). Mounting: CP02. [C]
- **Slit***. Ø1" Mounted Slit, 200 ± 4 μ m Wide, 3 mm Long (S200RD). Mounting: CRM1. [C]
- **L-17***. $f = 150$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 400 - 700 nm (AC254-150-A-ML). Mounting: CP02. [C]
- **M-9***. Ø1" Broadband Dielectric Mirror, 400 - 750 nm (BB1-E02). Mounting: KCB1C. [C]
- **L-18***. $f = 150$ mm, Ø1" Achromatic Doublet, SM1-Threaded Mount, ARC: 400 - 700 nm (AC254-150-A-ML). Mounting: CXY1A. [C]

- **L-19***. $f = 100$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 400 - 700 nm (AC254-100-A-ML). Mounting: SM1Lxx. [C]
- **M-10***. $\varnothing 1$ " Broadband Dielectric Mirror, 400 - 750 nm (BB1-E02). Mounting: KCB1C. [C]
- **L-20***. $f = 300$ mm, $\varnothing 1$ " Achromatic Doublet, SM1-Threaded Mount, ARC: 400 - 700 nm (AC254-300-A-ML). Mounting: SM1Lxx. [C]
- **Cam-2***. Basler Camera (acA2440-75um). Mounting: C-MOUNT EXT. [C]

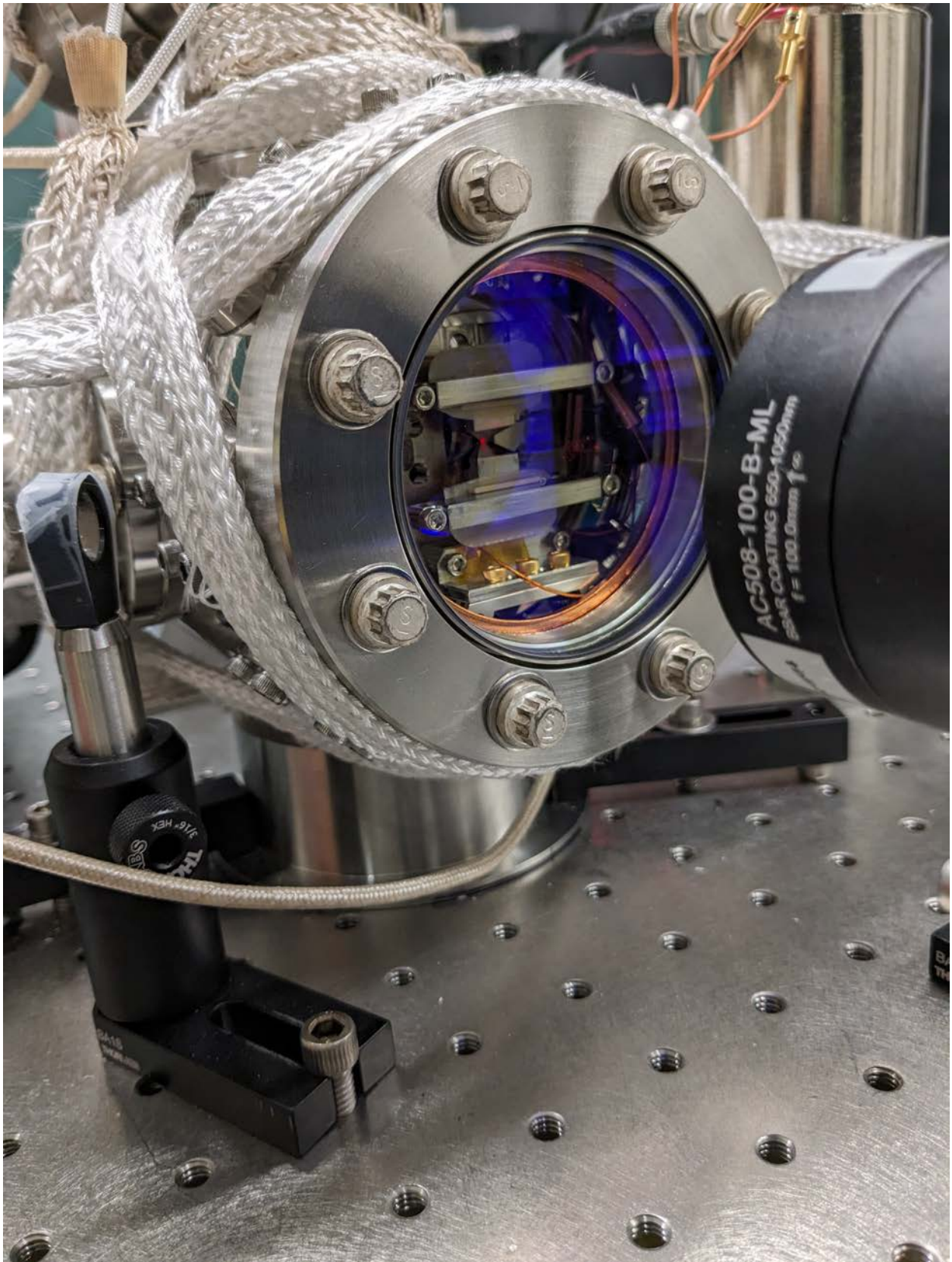


Figure 120: An image of the chamber with the objective placed for measurement. The cooling beams only just cleared the lens tube holding the objective, which was placed as far forward in its mount as possible (the retaining ring extends out slightly past the edges of the mount).

The electro-optic modulator was used to apply optical feedback cooling to the motion of the particle using two counter-propagating beams aligned with none of the principle axes in the trap, using previously developed cooling code. [140] Due to the separation of normal mode frequencies, cooling of all three degrees of freedom can be achieved using only a single beam; in this case two were used to mitigate the impulse on the particle from radiation pressure due to switching the beams on or off.

Microwave Drive Wire While the first set of MDMR data was taken using the setup shown in Figure 115, the results shown in Figure 118 motivated the development of a new wire-mounting system.

The idea was to 3D print a wire holder that passed the wire near the trapping region twice with current running in the same direction; that way, the region below the two current-carrying wires would be subject to a more uniform field.

Michael Testagrossa designed and printed the test plates out of PEEK, and was able to successfully mount the 30 μm gold-plated tungsten wire to them as shown in Figure 121.

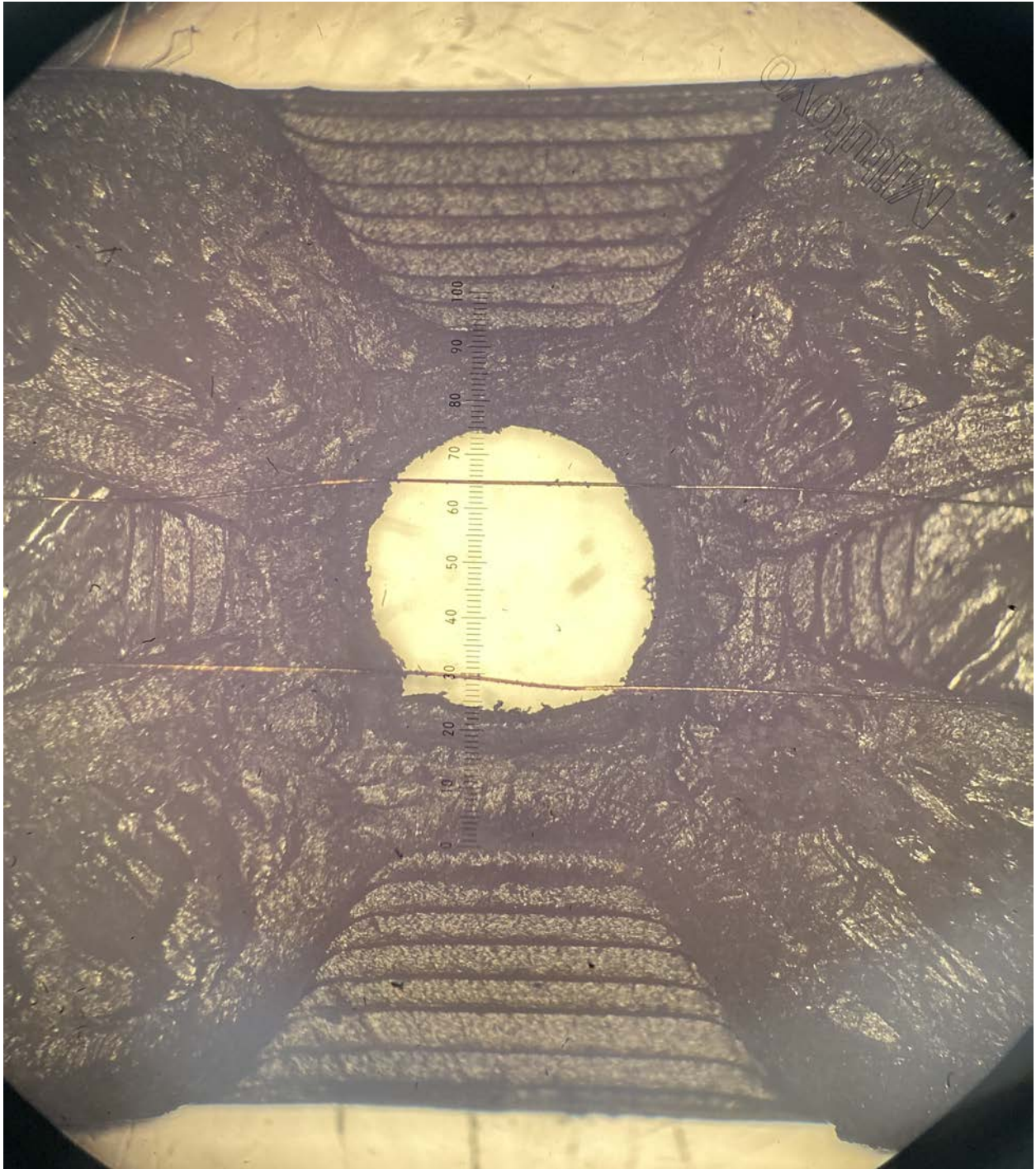


Figure 121: These wires are about 1.75 mm apart, with each tick mark shown being equal to about 50 μm .

Three plates were fitted to the pole pieces and mounted in an assembled trap using

small set screws in tapped holes in the plates. With a great deal of fine adjustment, it was possible to leave a clear pathway for optical access down the transverse axis, as shown in Figure 122.

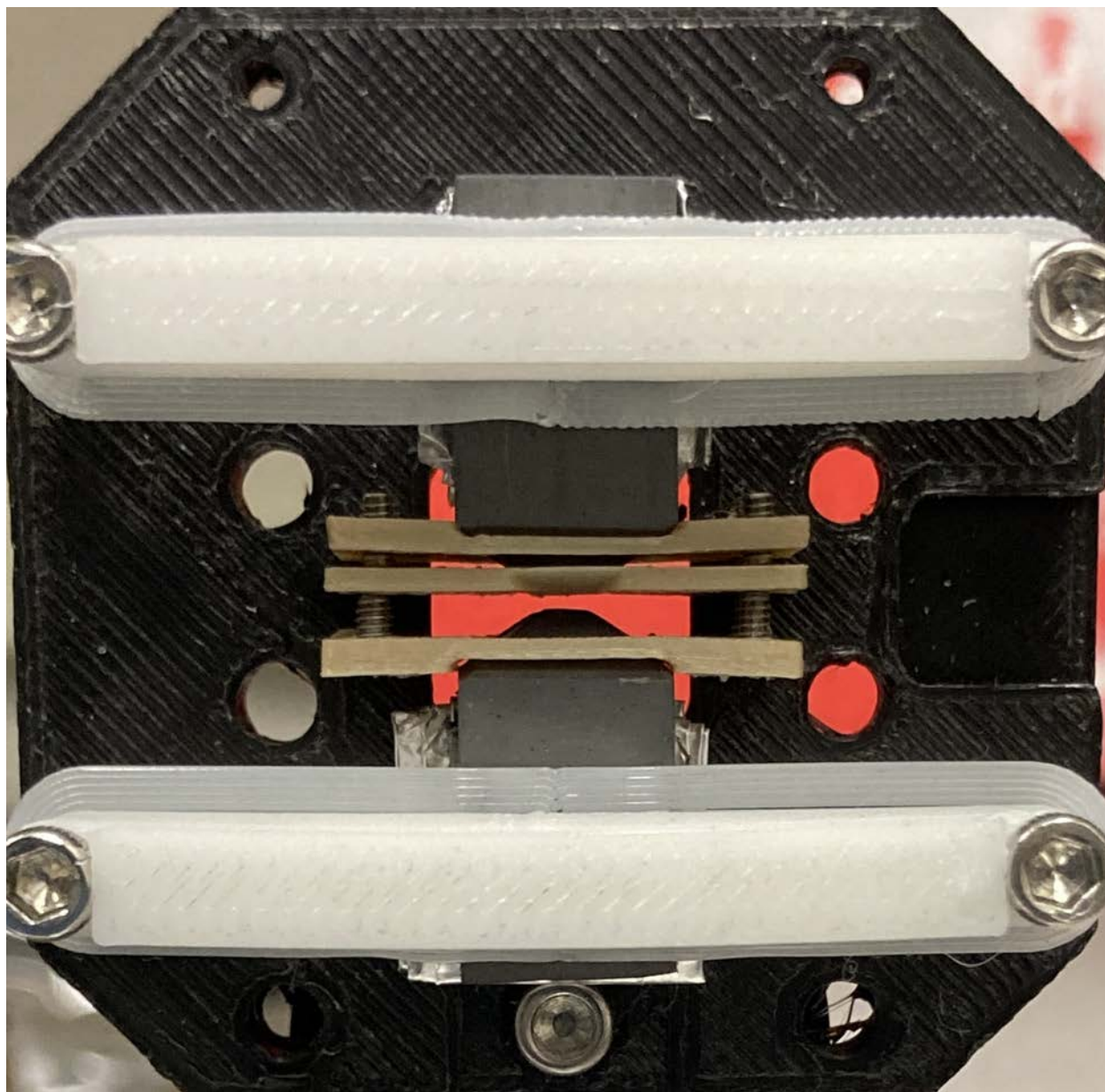


Figure 122: The two-wire PEEK mount for creating a more uniform RF signal in the trap. Assembled without the wire in an ABS/TPU test mount for the trap.

However, one unforeseen consequence of this arrangement was that there was no longer

sufficient room to maneuver a tungsten probe into the trap for SAR loading of SiC particles. Due to time constraints, this effort was abandoned and a simpler approach for reducing electrical contact between the wire and the pole pieces was employed.

A piece of 25 μm Kapton was cut into a particular shape as shown in Figure 123.



Figure 123: Kapton piece used to keep the 30 μm gold-plated tungsten wire electrically isolated from the pole pieces. The thin strip where the wire rests is offset to one side so that the wire sits at one end of the trap, so as not to interfere with the primary trapping region.

This piece was then clamped in place with the pole pieces and the wire simultaneously (this was a very time-consuming and difficult process to get right – the Kapton and the wire were both prone to breaking, and tended to slip around in a manner that was difficult to control).

A microscope image of the bottom pole pieces with the wire attached in this way is shown in Figure 124.

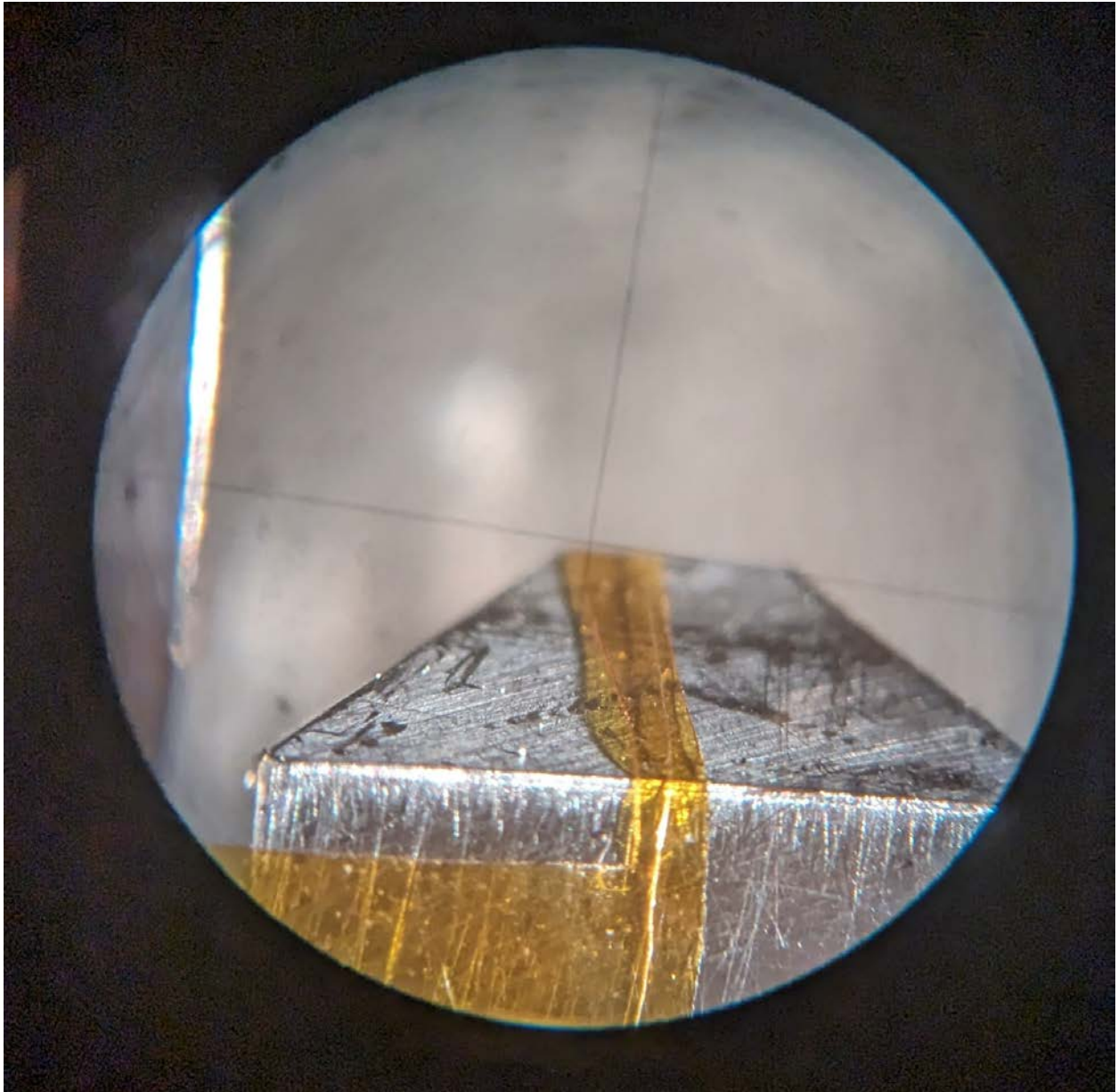


Figure 124: Side view of the Kapton piece and wire clamped in place.

Figure 125 shows a closer view of the point of contact at the edge of the pole piece before it tapers.

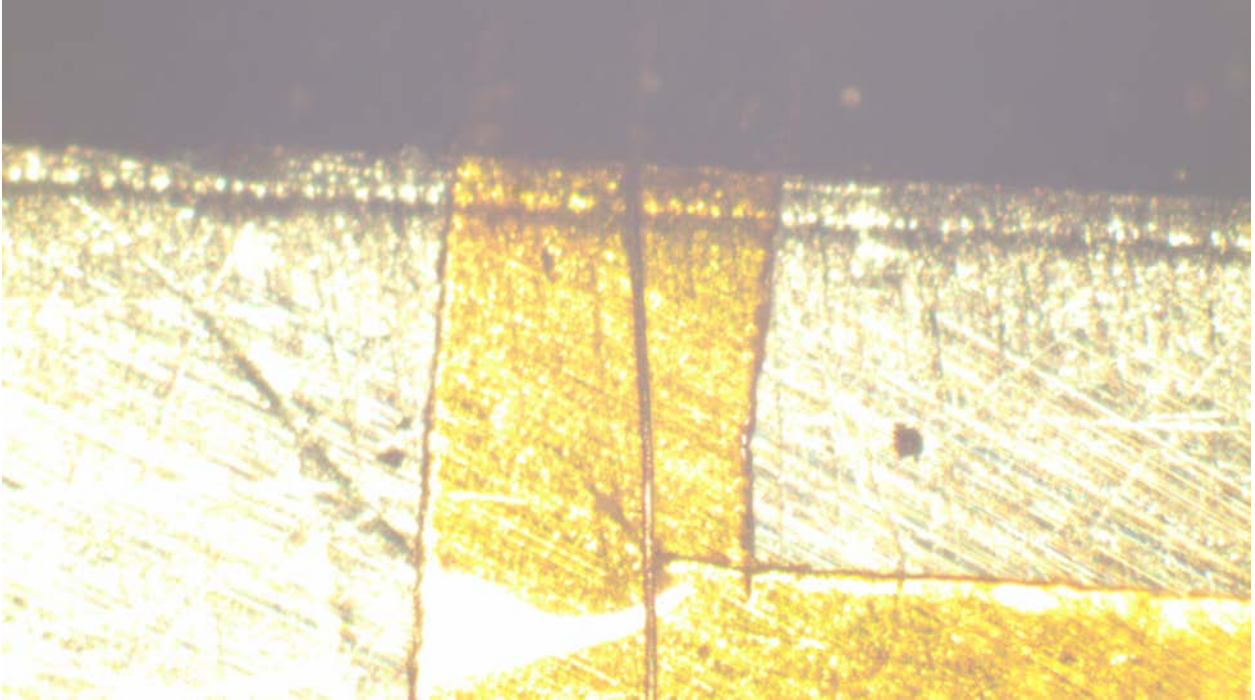


Figure 125: Zoomed-in side view of the Kapton piece and wire clamped in place.

A silhouette of the Kapton piece and the wire is shown in Figure 126; this image is helpful for seeing how much space is actually taken up by the Kapton film in the trap.



Figure 126: Side view of the Kapton piece and wire clamped in place. Note how close the wire sits to the tips of the pole-pieces; this is essential so as to still leave room for a particle to be trapped close enough to the wire to feel its signal.

The arrangement is not nearly so optimal as the 2-wire arrangement had the potential to be. It also has the downside that the signal strength at the particle may be small not only due to the distance from the wire to the particle's equilibrium position, but also due to the possibility electromagnetic shielding from the pole pieces. Nevertheless, it was what

could be achieved in the time available. An image demonstrating the final wire placement for the second MDMR measurement attempt is shown in Figure 127.

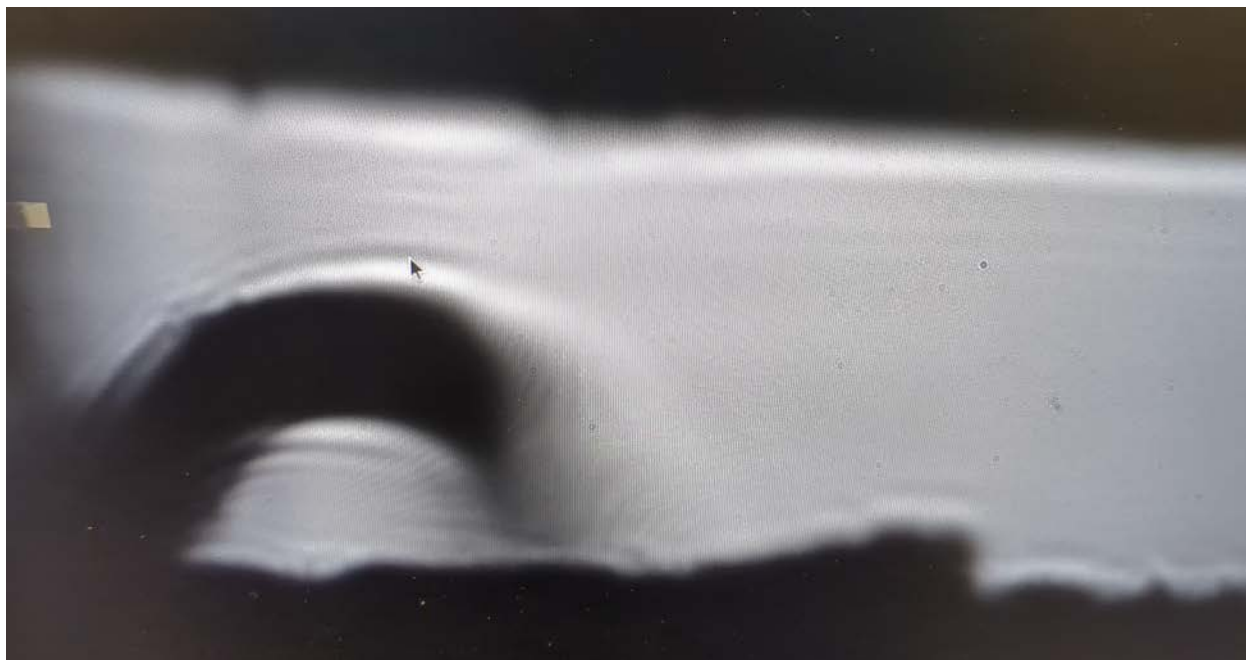


Figure 127: The location of the wire in the trap for the second MDMR measurement attempt. Note that the wire is not touching any of the pole pieces due to the insulating piece of Kapton film shown in Figure 123.

A schematic of the arrangement is shown in Figure 128.

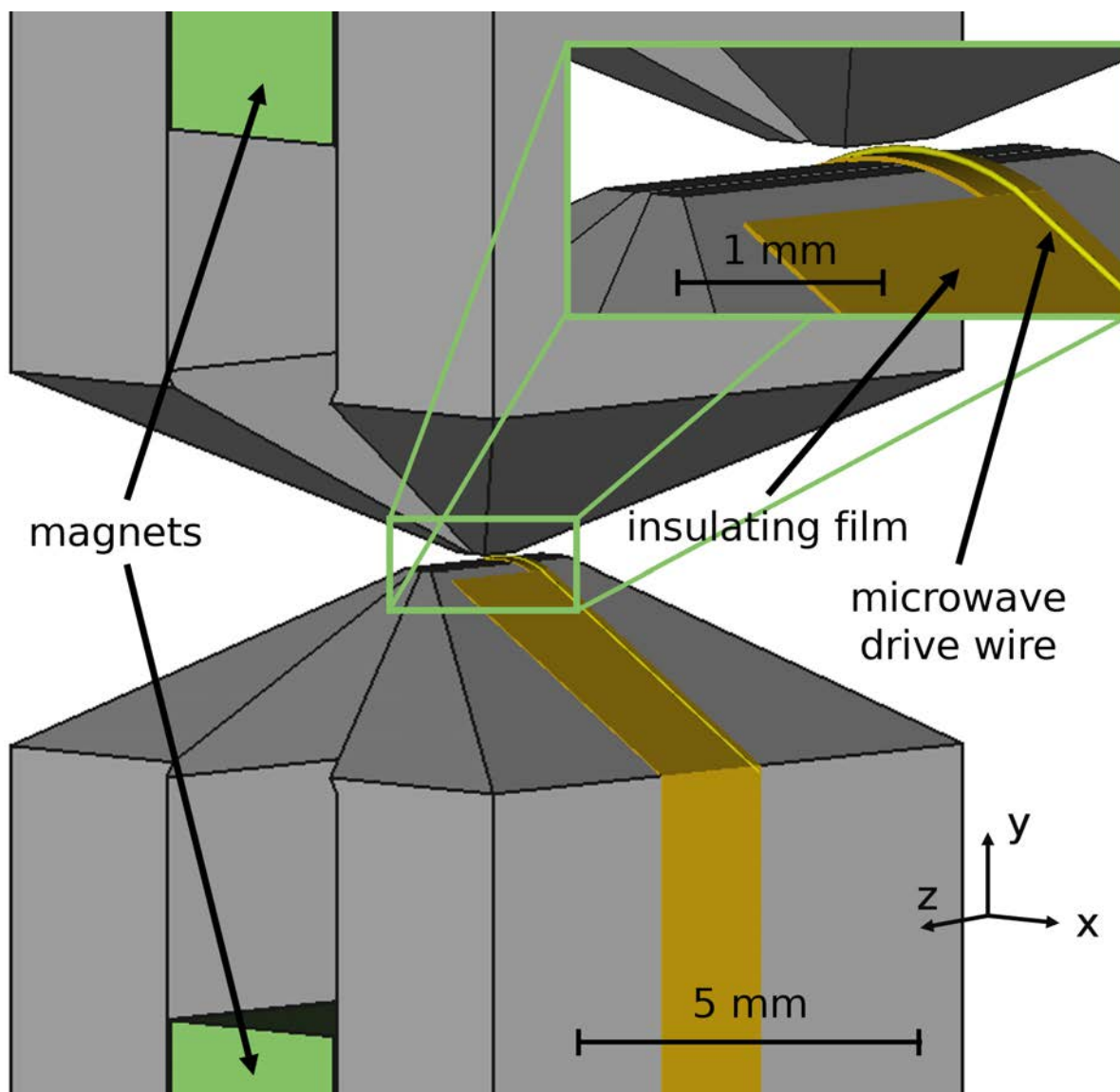


Figure 128: A diagram of the MGT used to levitate a 4H-SiC particle containing V_{Si}^- (V_2) vacancy defect centers. The wire shown generates the microwave signal used to drive spin transitions in the vacancy centers, and the insulating film helps reduce extraneous signals resulting from contact between the wire and the conducting pole pieces.

A schematic of the microwave system used to generate a signal in the wire (modified only slightly from that used for ODMR measurements) is shown in Figure 129.

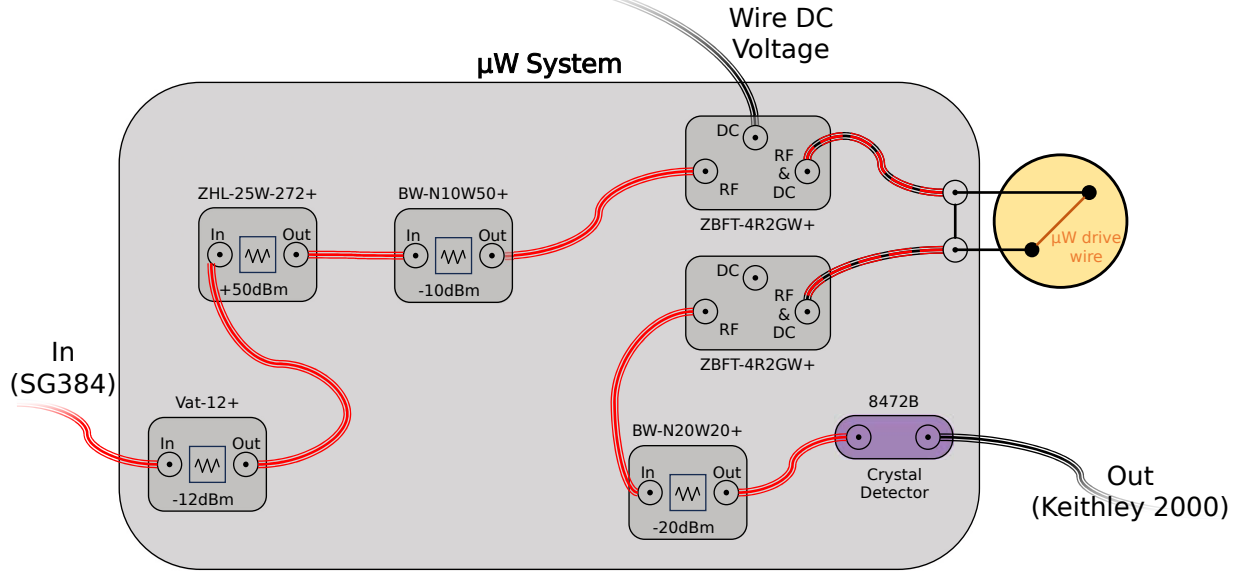


Figure 129: A schematic of the system used for generating a microwave signal at the drive wire for ODMR and MDMR. All the red lines shown are CBL-10FT-SMSM+ cables connecting to SMA ports. The black wires shown are BNC cables connecting to BNC ports. Note that the power at the drive wire is +28 dB higher than the power generated by the SG384.

A large piece of non-irradiated SiC (Billy Joe (2023-06-02-D)) was trapped to test the system parameters and see if a clear effect from the microwave drive could be seen on its motion under resonant frequency modulation. Figure 130 shows an image of this particle.

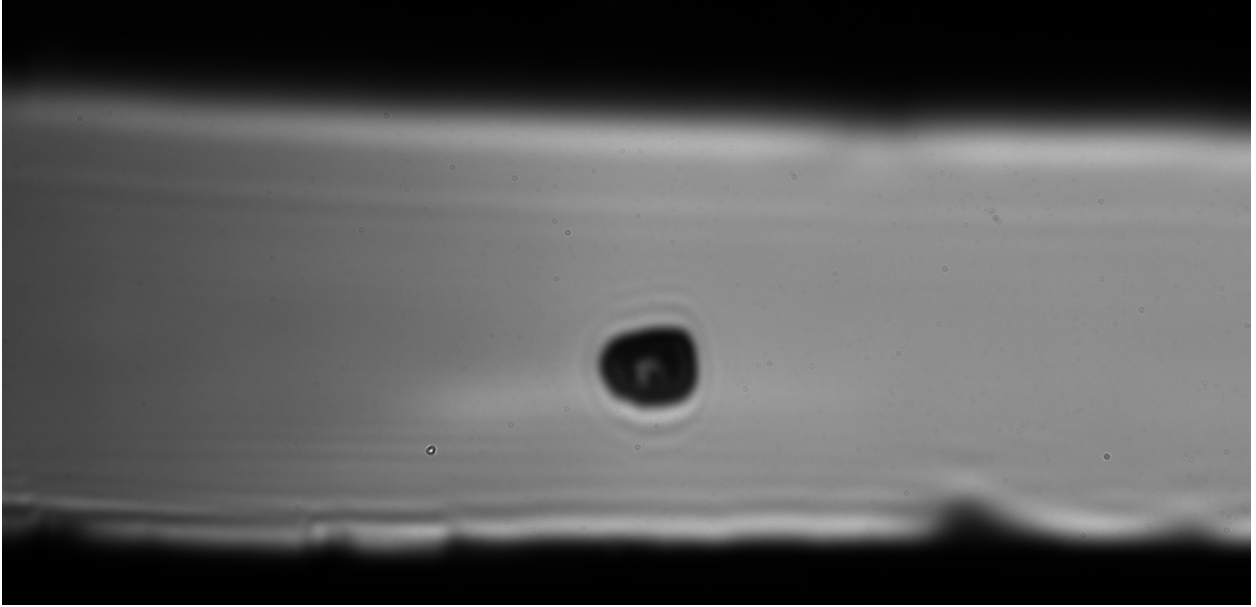
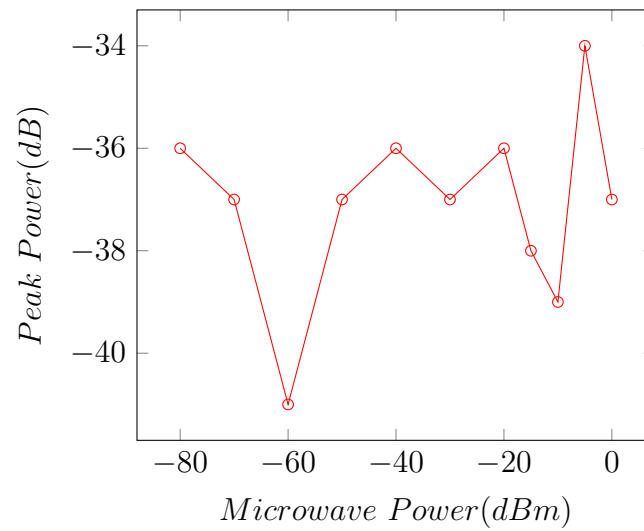


Figure 130: Image of Billy Joe (2023-06-02-D) , levitated in the MGT used for MDMR with the drive wire to the left of the particle (not shown). The top and bottom pole pieces can be seen at the top and bottom of the image.

We set the sweep frequency to 120 Hz, corresponding to the vertical motion of Billy Joe (2023-06-02-D) . The following plot shows the approximate peak height of the particle motion at that frequency in the vertical-motion PSA for varying power levels of the microwave signal.



The data seem to suggest only noise in the resonant amplitude, which is what was

expected for Billy Joe (2023-06-02-D) , who contained no V_{Si}^- (V2) centers. However, Billy Joe (2023-06-02-D) was quite large compared to the other SiC particles used for MDMR, so this may have been partly due to the mass of the particle.

Results

What follows is a description of the MDMR measurements recorded, as well as notes on the changes that took place between measurements and how they were performed.

Background-Subtraction MDMR Measurement Attempt A particle obtained from a sample of irradiated boule material, Cinderella (HY0667-21B-2A) , was loaded into the trap using the SAR loading technique. She was tile-shaped, and tended to sit with her thin axis aligned axially (as shown in Figure 131) when low in the trap, and with her short axis aligned transversely (as shown in Figure 132) when higher in the trap.

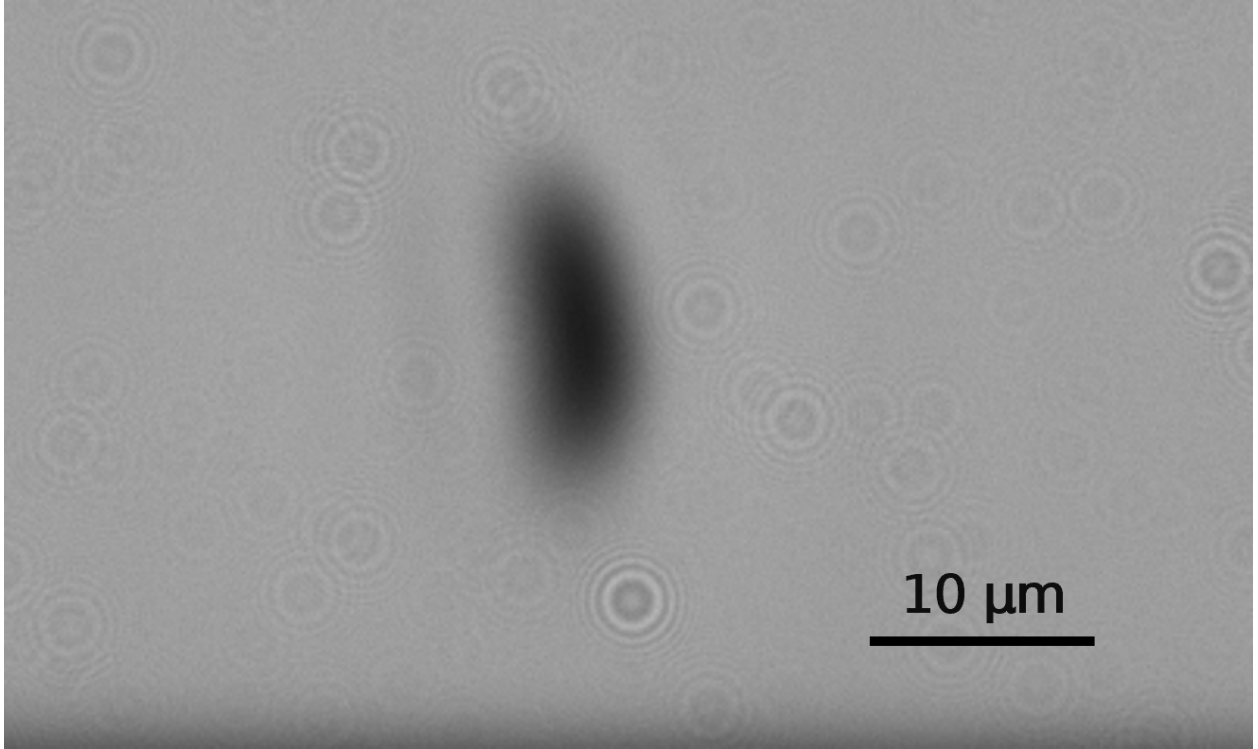


Figure 131: Image of Cinderella (HY0667-21B-2A) taken down the transverse axis of the trap. Height was controlled by adjusting the voltage between the top and bottom pole piece, V_{tb} . In this case, $V_{tb} = 9.0$ V.

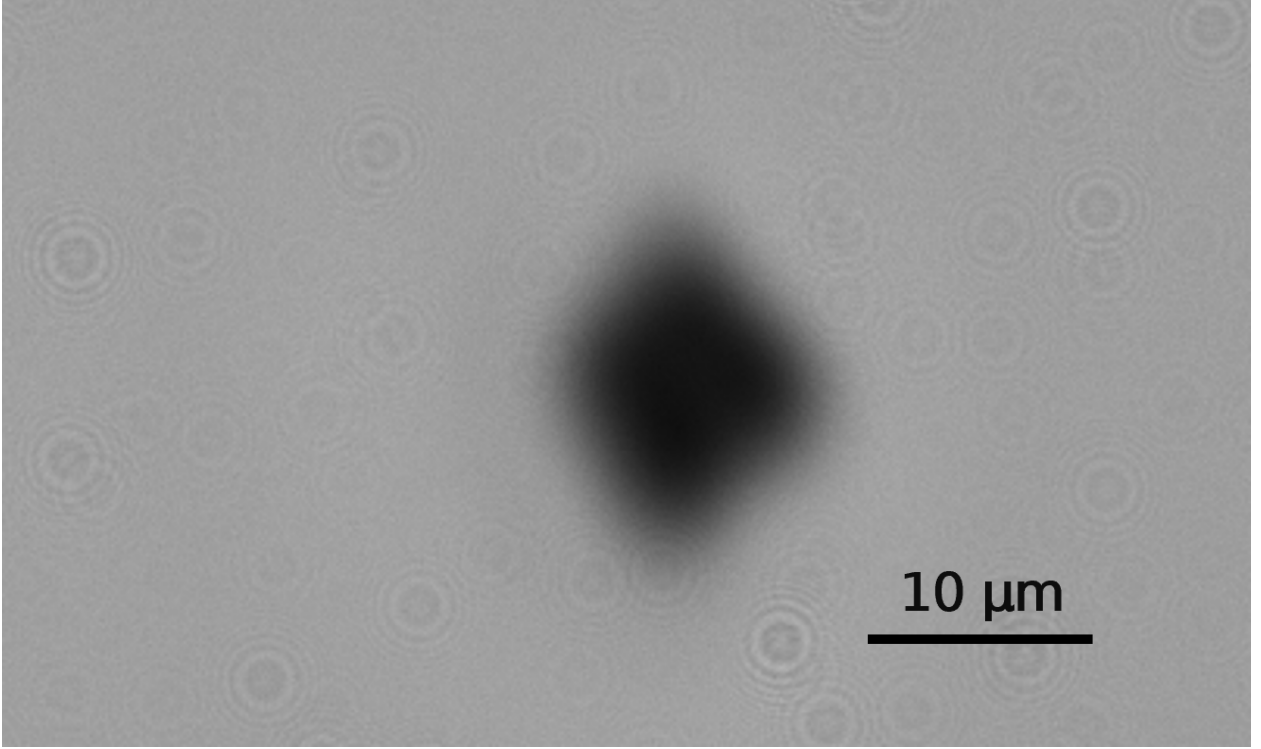


Figure 132: Image of Cinderella (HY0667-21B-2A) taken down the transverse axis of the trap. Height was controlled by adjusting the voltage between the top and bottom pole piece, V_{tb} . In this case, $V_{tb} = 19.0$ V.

The measured vertical oscillation frequency for this particle at 10.0 V was $\nu_y = 82$ Hz (from `collection-F-1500`). Using this with equation (50), the field gradient experienced by Cinderella (HY0667-21B-2A) can be estimated to be about 9.1 mT/ μ m. The top of the particle was estimated to rest around 3.8 μ m below the geometric center of the trap (located at about 899 px). This corresponds to a resonant frequency of about 960 MHz at the tip, meaning that most of the particle's vacancy centers should be resonant for drive frequencies in the tested frequency range from about 960 MHz to 2040 MHz. Figure 133 shows a schematic of the particle's position and expected resonant frequency range.

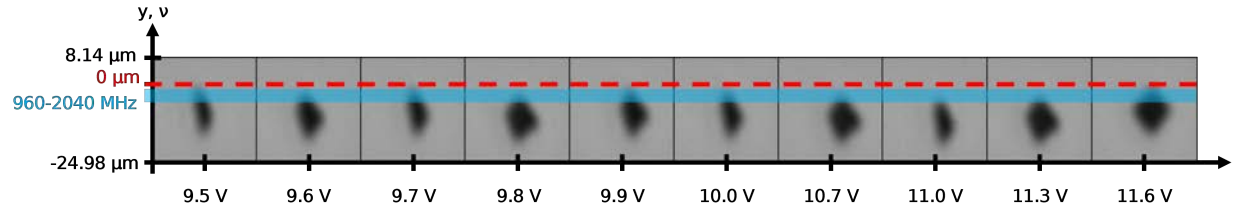


Figure 133: Sample images of Cinderella (HY0667-21B-2A) taken at several different DC offset voltages. The red dashed line shows the estimated geometric center of the trap, and the blue transparent line shows the estimated band of resonant frequencies (based field gradient approximated from the measured vertical frequency) in the particle. Note that these sample images are representative of single moments, not necessarily the average position of the particle over the course of a measurement.

The first MDMR measurement was taken with the hardware control system shown in Figure 134.

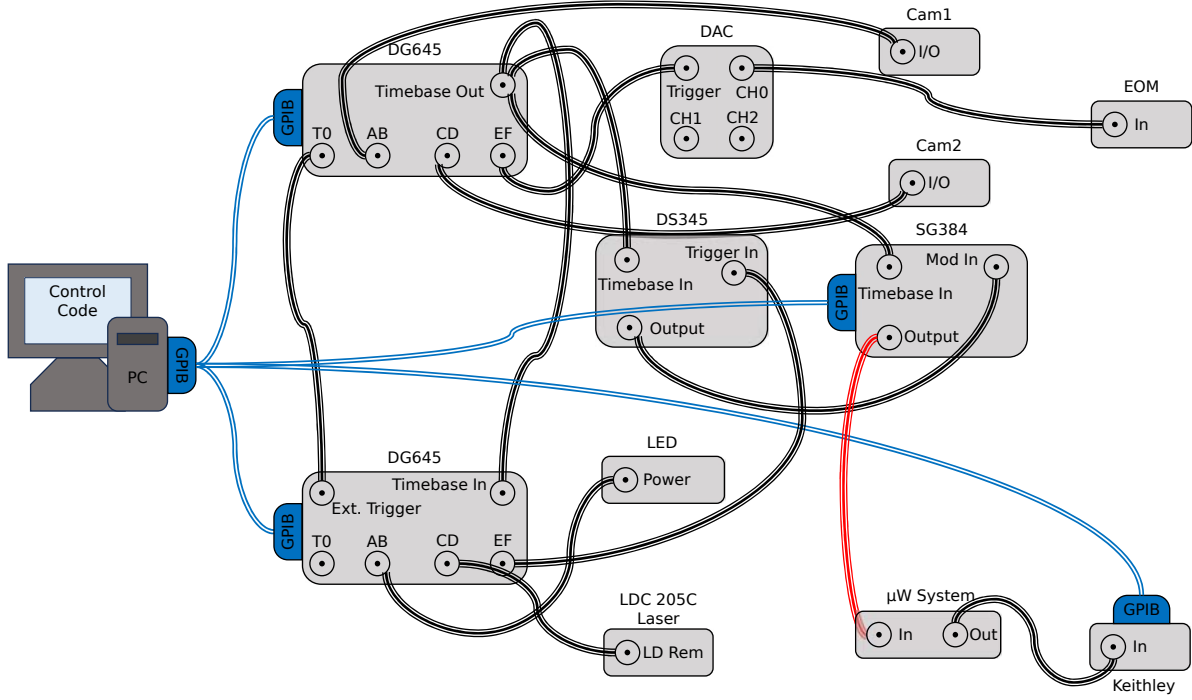


Figure 134: The settings for the pulse generators can be found in the camera cooling folder in `settings_cc.py` under `SiC` $\rightarrow$ `PULSE_GENERATOR_COOLING_AND_uW_DICT`. The red wires shown are CBL-10FT-SMSM+ cables connecting to SMA ports. The black wires shown are BNC cables connecting to BNC ports. The blue wires are GPIB serial bus cables used for interfacing with the PC used to control the experiment. See Figure 129 for details about the microwave drive.

In this measurement scheme, the idea was to first cool the motion of the particle using the camera cooling code and the 785nm laser, then record the uncooled motion of the particle. This was done in alternating segments, one where the laser was kept off and one where it was left on, but the cooling was not active. This way, the bright data would show an MDMR signal if present, while the dark data would show only background effects. A sample of time-domain data using this measurement scheme is shown in Figure 135.

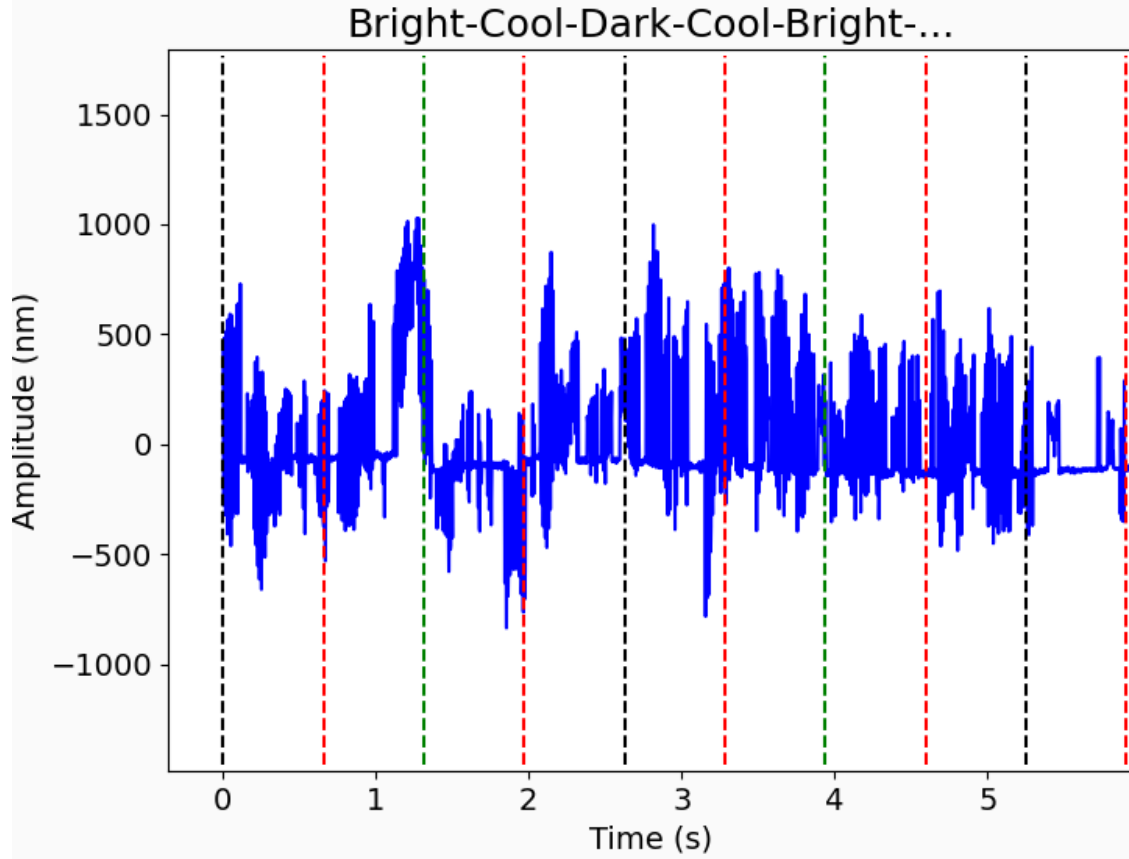


Figure 135: This figure was generated by `first_look.py` from the file `collection-E-1080`. It shows the particle motion recorded with and without the cooling laser on and with and without the cooling active. The laser is off during the dark sections and the cooling is off during the bright uncooled sections.

The amplitude of the motion at microwave sweep modulation rate (matched to the vertical oscillation frequency of the particle) is measured for the uncooled motion, both bright and dark, as depicted in Figure 136.

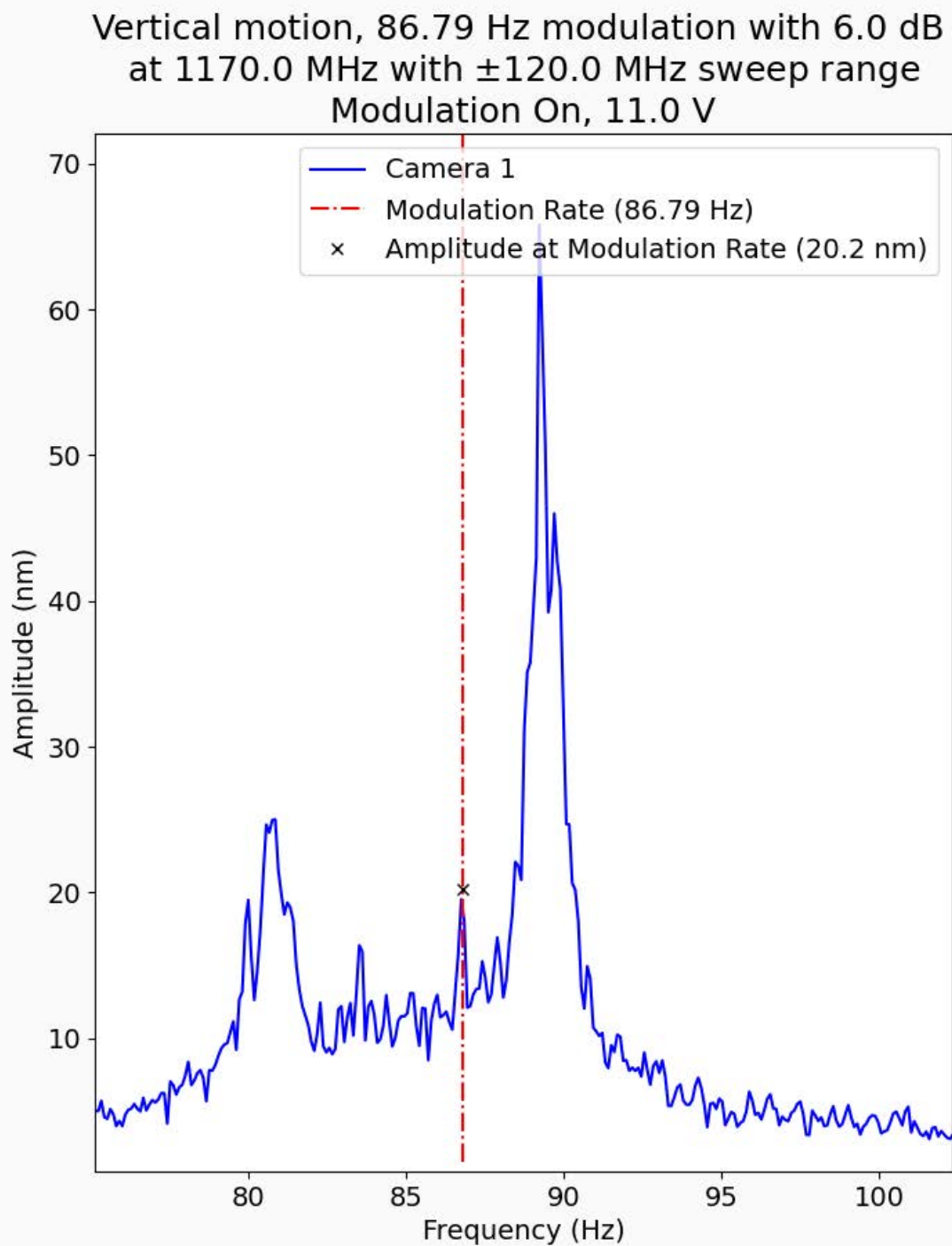


Figure 136: This figure was generated by `analyzeE.py` from the file `collection-E`. Sample data set showing the peak-locating for a set of data with the MD on.

The signal is then calculated by subtracting the dark resonant amplitude from the bright resonant amplitude, as shown in Figure 137.

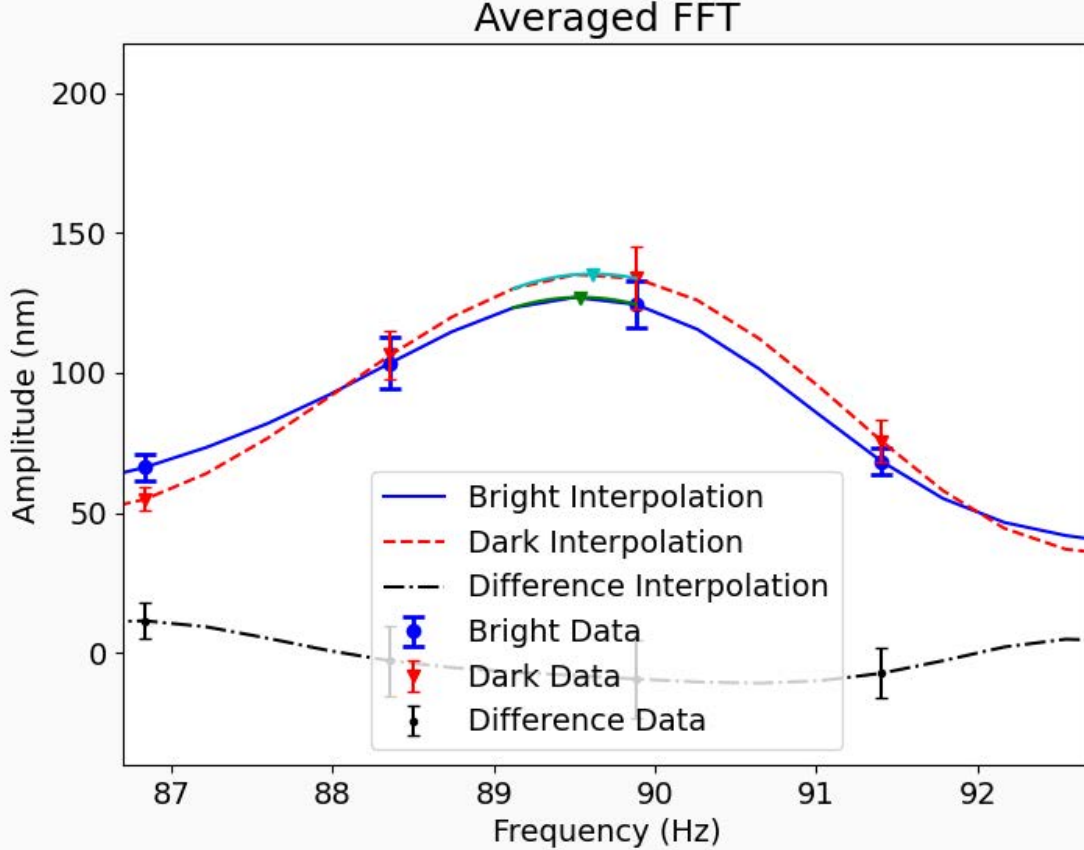


Figure 137: This figure was generated by `first_look.py` from the file `collection-E-1080`. Sample data set showing the dark-signal subtraction.

This process is repeated twice: once with the modulation turned on, and one with it turned off (but the unmodulated RF signal still on). In this way any MDMR signal should appear as a distinct signal in the modulation-on data that is not present with the modulation off. In addition, this signal should change with the B -field at the edge of the particle; therefore, a change in the particle's vertical position (induced by changing the voltage difference between the top and bottom sets of pole pieces) ought to result in the signal shifting to be centered on a different central microwave drive frequency.

It is with this in mind that we consider Figure 138. The laser power was set to 8.0 mW after the fiber coupling for all of these measurements.

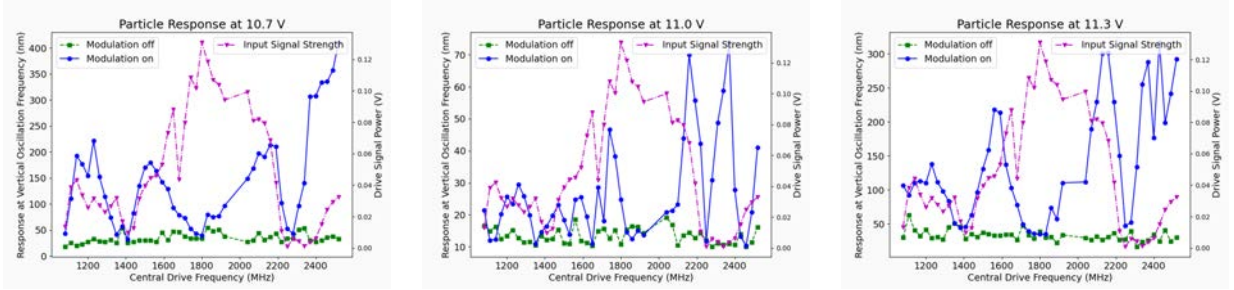


Figure 138: This figure was generated by `analyzeE.py` from the file `collection-E`. Laser power: 8.0 mW, MD modulation amplitude: ± 90 MHz, microwave power: 6.0 dBm (34.0 dBm in the trap), 3.3×10^{-8} Torr

It is very clear from looking at these results that the modulation of the microwave drive exerts a clear force on the particle, as seen by the much higher resonant amplitudes in the modulated signal. There is not, however, a clear signal that shifts in central MD frequency, as we would expect to observe for MDMR.

A more complete set of data was taken spanning a similar voltage range in finer steps (corresponding to smaller differences in particle location, and hence more sensitive dependence on the central microwave frequency corresponding to the expected signal). These data are displayed in Figure 139.

Once again, although the modulation has a clear impact on the amplitude of the particle's vertical motion on resonance, there is no clear signal shifting in frequency indicative of an MDMR result.

Lock-In MDMR Measurement Attempt A new particle (Dorito (HY0667-21B-2B)) was obtained from the same sample of boule material as Cinderella (HY0667-21B-2A) and was loaded into the trap using the SAR loading technique. He was also tile-shaped, resembling

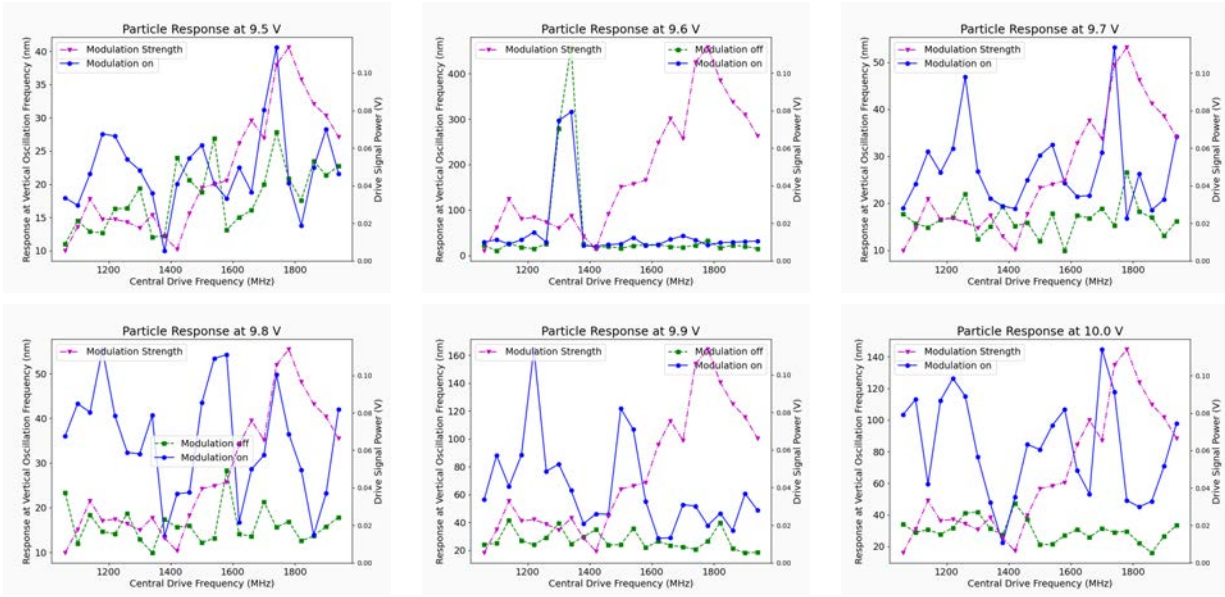


Figure 139: This figure was generated by `analyzeF.py` from the file `collection-F`. Laser power: 8.0 mW, MD modulation amplitude: ± 90 MHz, microwave power: 6.0 dBm (34.0 dBm in the trap), $\sim 4 \times 10^{-8}$ Torr

an equilateral triangle with side lengths of about $23 \mu\text{m}$, and normally sat with his thin axis aligned with the trap's transverse direction.

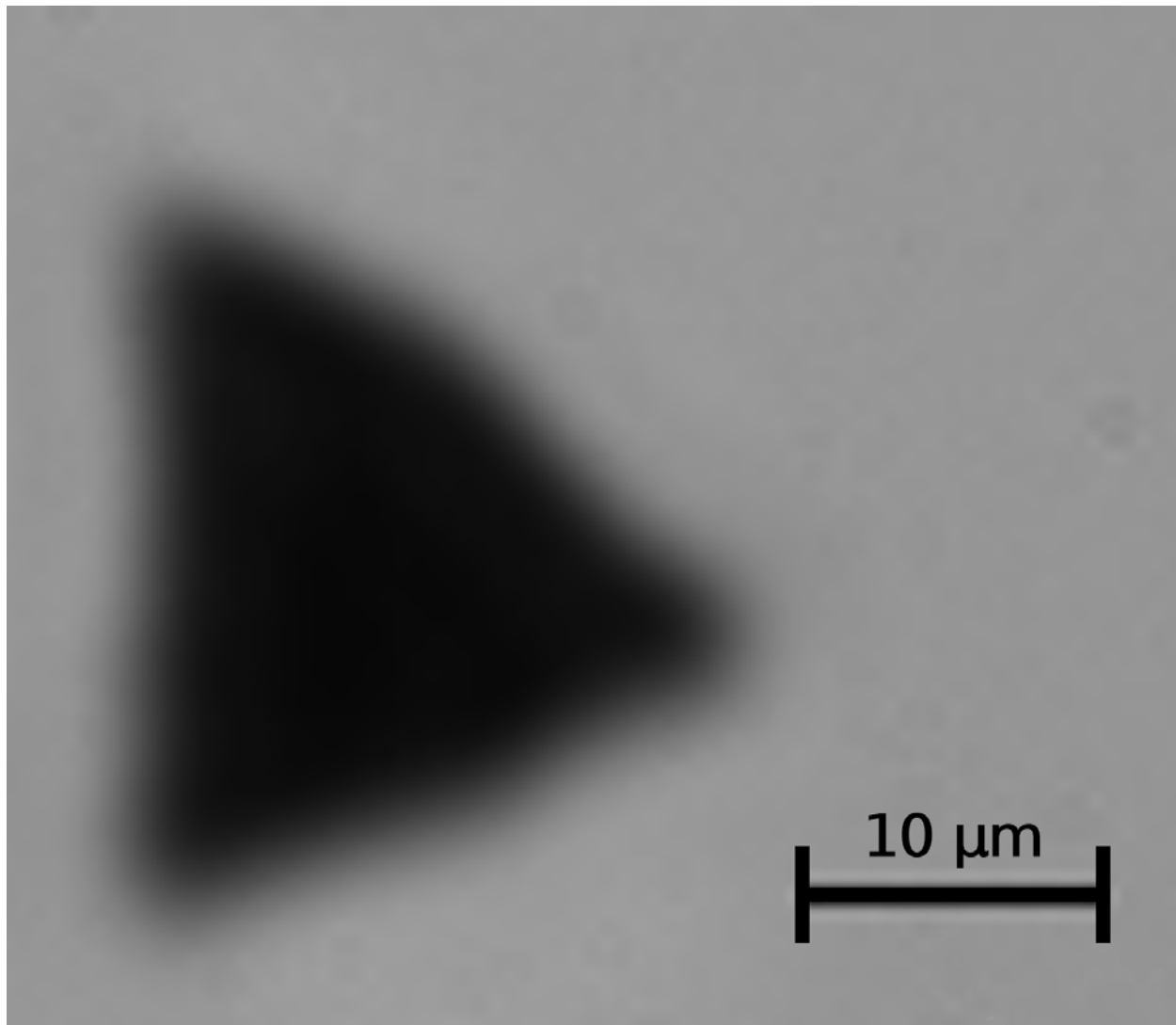


Figure 140: Image of Dorito (HY0667-21B-2B) taken down the transverse axis of the trap. Height was controlled by adjusting the voltage between the top and bottom pole piece, V_{tb} . In this case, $V_{tb} = 3.3 \text{ V}$.

The measured vertical oscillation frequency for this particle at 3.3 V was $\nu_y = 98.2 \text{ Hz}$ (from `collection-C-1050`). Using this with equation (50), the field gradient experienced by Dorito (HY0667-21B-2B) can be estimated to be about $10.9 \text{ mT}/\mu\text{m}$. In this case the top of the particle was near or slightly above the geometric center of the trap (located at about 1388 px), as can be seen in Figure 133.

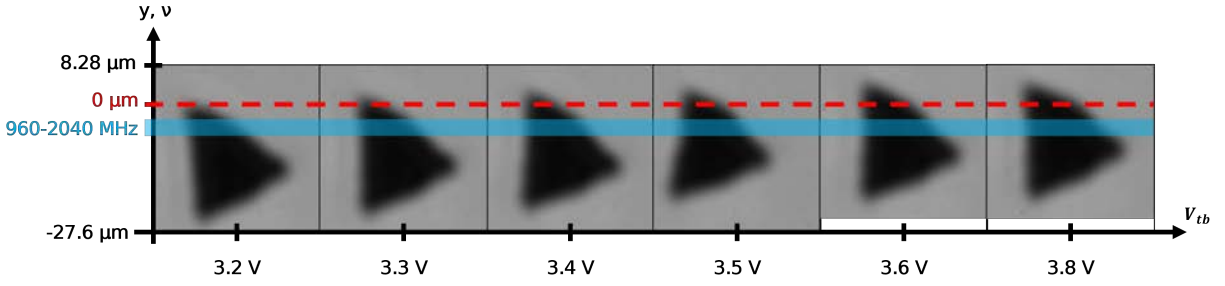


Figure 141: Sample images of Dorito (HY0667-21B-2B) taken at several different DC offset voltages. The red dashed line shows the estimated geometric center of the trap, and the blue transparent line shows the estimated band of resonant frequencies (based field gradient approximated from the measured vertical frequency) in the particle. Note that these sample images are representative of single moments, not necessarily the average position of the particle over the course of a measurement.

The second MDMR measurement was taken with the hardware control system shown in Figure 142.

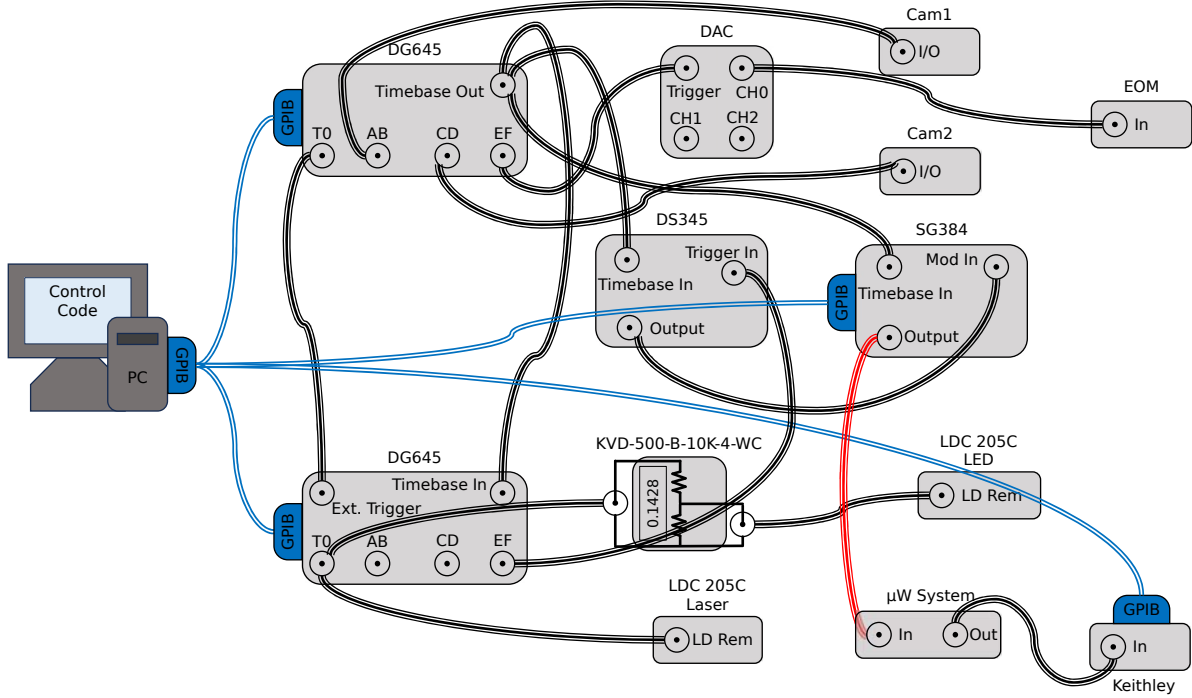


Figure 142: The settings for the pulse generators can be found in the camera cooling folder in `settings_cc.py` under `SiC` → `PULSE_GENERATOR_COOLING_AND_uW_LOCKIN_DICT`. The red wires shown are CBL-10FT-SMSM+ cables connecting to SMA ports. The black wires shown are BNC cables connecting to BNC ports. The blue wires are GPIB serial bus cables used for interfacing with the PC used to control the experiment. The 50Ω terminator expected at the LDC 205C controlling the red LED was not connected in order to get a higher driving voltage for the LED. See Figure 129 for details about the microwave drive.

In this measurement scheme, the approach was modified from the previous attempt to try to cancel out the effect of the MD in the absence of the optical vacancy-center pumping via the 785nm light and the LED.

This MD force can be mitigated by taking data in bursts such that every other burst is 180° out of phase from the previous one, causing any excitation to cancel out every two burst

cycles. The MDMR signal is only present with the illumination on, so it can be conserved by switching the light off for every other burst cycle.

This introduces the second resonant force we would not like to be sensitive to, which is the radiation-pressure-induced kick the particle receives each time the illumination is turned on or off. This force also cancels due to the half-wavelength cycle between bursts. This timing diagram is shown in Figure 143 for a 4-burst cycle and Figure 144 for a 1-burst cycle.

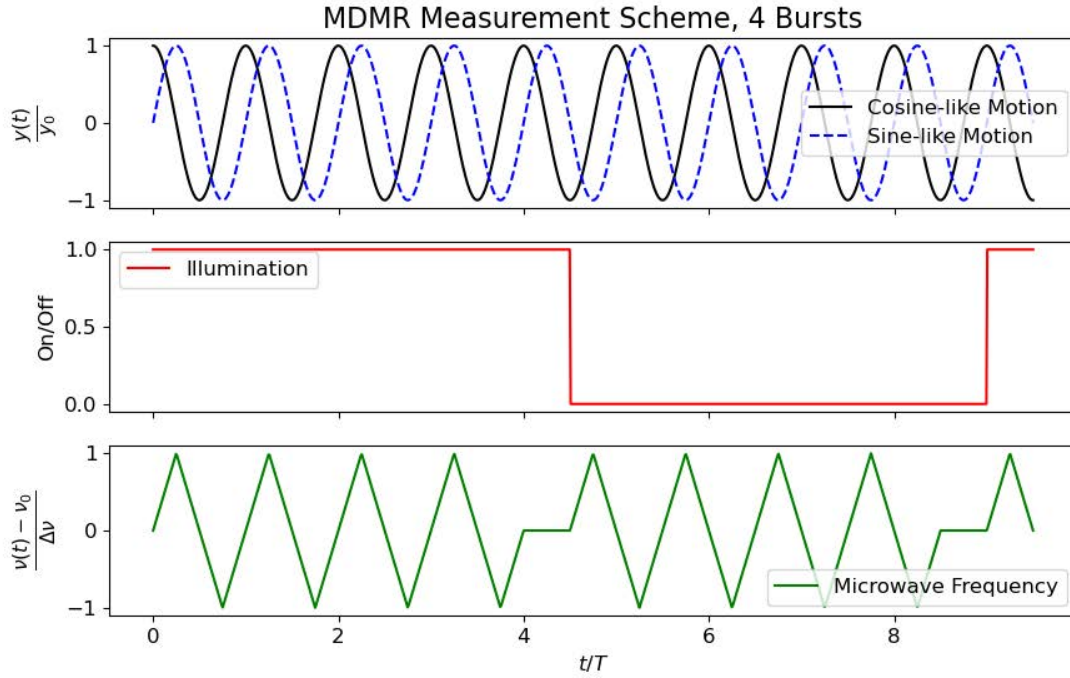


Figure 143: This figure was generated by `measurement_scheme.py`. The proposed measurement scheme for measuring MDMR. The top plot represents the two quadratures of particle motion in the vertical direction, the middle plot shows when the illumination (788 nm laser and 660 nm LED) is on or off, and the lower plot shows the time dependence of the frequency of the modulated microwave signal for a 4-burst cycle. Here y_0 is the amplitude of vertical motion and $\nu(t)$ is the frequency of the microwave drive, with ν_0 and $\Delta\nu$ being the center frequency and the modulation amplitude, respectively.

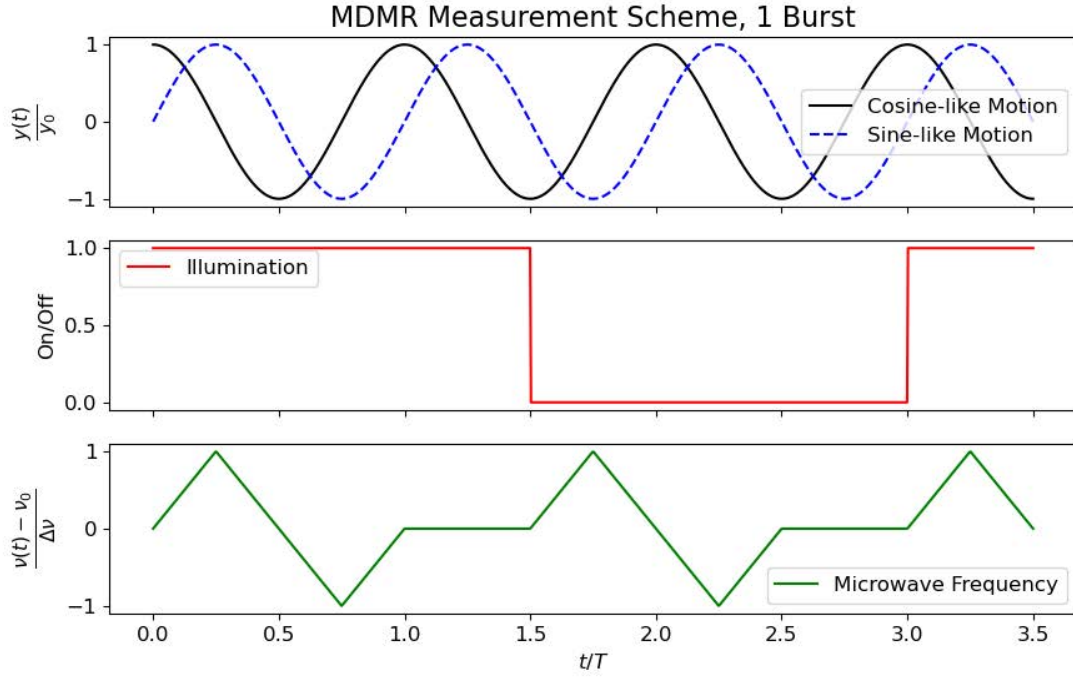


Figure 144: This figure was generated by `measurement_scheme.py`. The proposed measurement scheme for measuring MDMR. The top plot represents the two quadratures of particle motion in the vertical direction, the middle plot shows when the illumination (788 nm laser and 660 nm LED) is on or off, and the lower plot shows the time dependence of the frequency of the modulated microwave signal for a 1-burst cycle. Here y_0 is the amplitude of vertical motion and $\nu(t)$ is the frequency of the microwave drive, with ν_0 and $\Delta\nu$ being the center frequency and the modulation amplitude, respectively.

A sample of time-domain data using this method is shown in Figure 145. The motion of the particle can only be collected when the illumination is on, during the first half of each segment. Note that in this measurement scheme, the particle is cooled semi-continuously (cooling is on any time illumination is on).

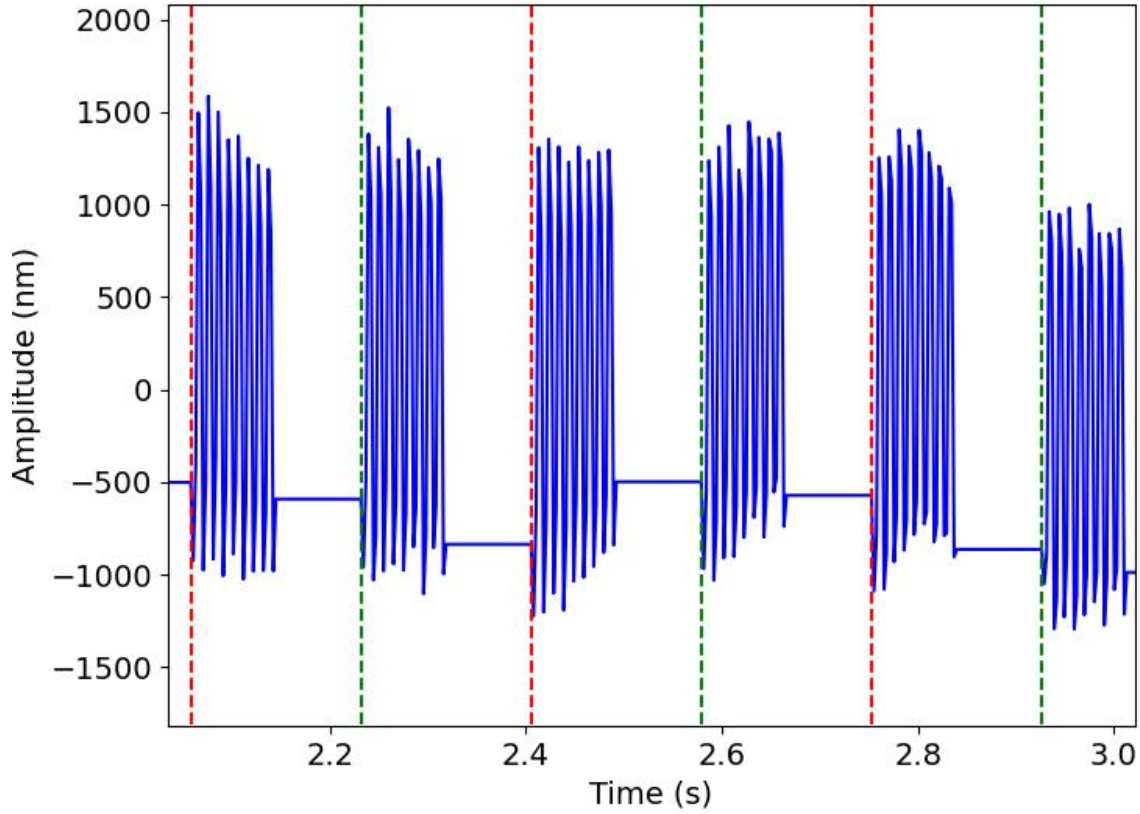


Figure 145: This figure was generated by `first_lookA.py` from the file `collection-C-1950`. $V_{tb} = 3.2 \text{ V}$, $\nu_y = 97.9 \text{ Hz}$. Sample time-domain data for lock-in MDMR measurement.

The data is then Fourier transformed and the power spectra are averaged together. In this case no frequency-finding algorithm is needed, since the framerate is set to exactly 4 times the measured oscillation frequency, which is used to set the microwave modulation frequency. This means that the desired resonance signal always falls exactly into one bin, as shown by the peak in Figure 146.

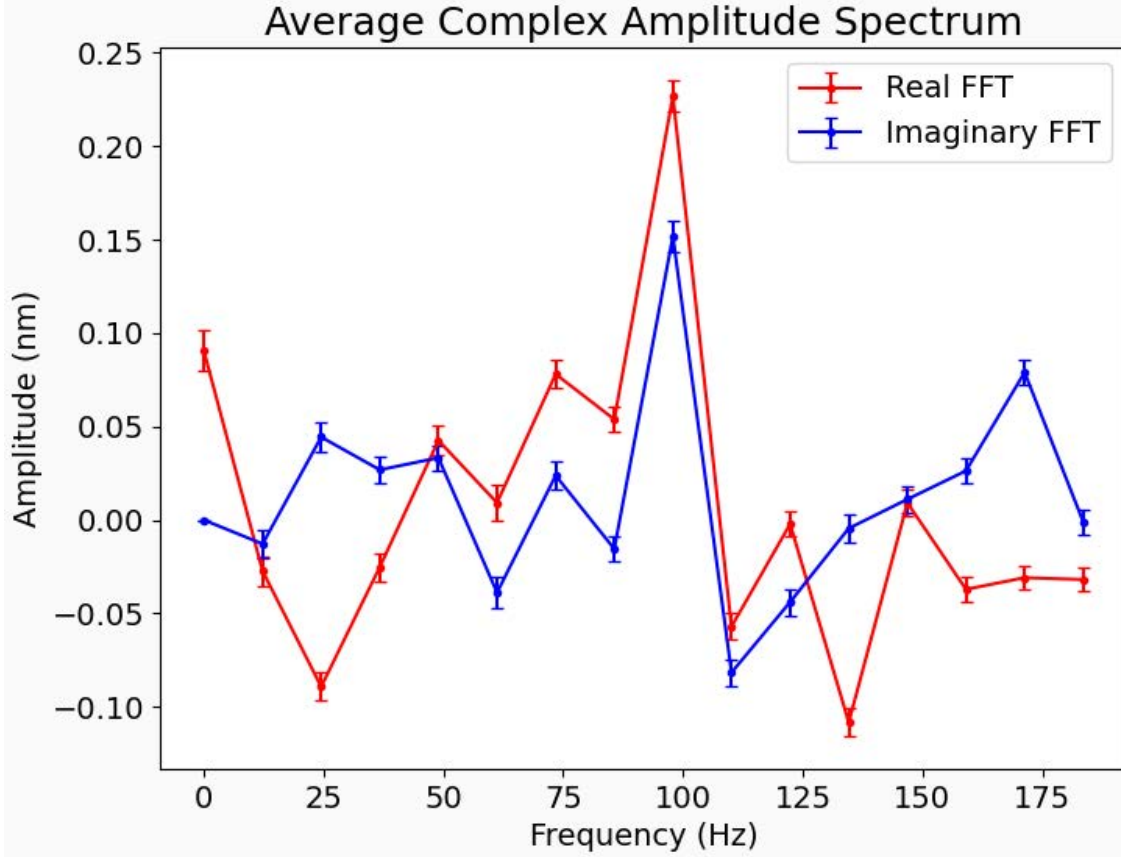


Figure 146: This figure was generated by `first_lookA.py` from the file `collection-C-1950`. $V_{tb} = 3.2$ V, $\nu_y = 97.9$ Hz. Sample averaged ASD for lock-in MDMR measurement.

One final thing to note about this measurement setup is how it interacts differently depending on which quadrature of the particle's motion is considered. Consider first the force acting on the sine-like quadrature, shown as a blue-dashed line in Figure 144. Any force caused directly by the accidental modulation of the amplitude of the microwave drive will add energy to this quadrature, while any force that occurs only at a particular frequency (i.e., the MDMR signal we are looking for) will cancel out. In the cosine-like quadrature, by contrast, any amplitude modulation cancels out, while forces occurring at a particular frequency add together.

Thus, in each illuminated burst, the imaginary part of the Fourier transform contains information about the effect of the accidental modulation of the microwave drive amplitude, while the real part contains information about the MDMR force caused by the optical pumping of the spin states in the V_{Si}^- (V2) vacancy defect centers.

The first set of data was taken over a fairly wide range of frequencies, shown in Figure 147. One major complication in cooling the particle's motion that persisted throughout all measurements was that it librated significantly about the transverse trap axis. One cooling channel was set to the frequency corresponding to these librations since the transverse motion of the particle was negligible. However, this cooling had only limited success, as the libration frequency sometimes shifted around mid measurement, and the cooling lasers are designed to exert uniform forces on particles, not torques.

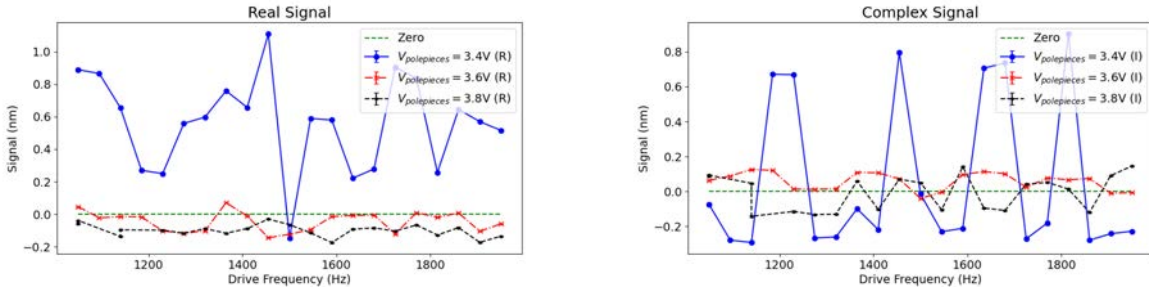


Figure 147: This figure was generated by `analyze_collectionsA.py` from the file `collection-A`. Laser power: 5.6 mW, MD modulation amplitude: ± 90 MHz, microwave power: 6.0 dBm (34.0 dBm in the trap), 4 burst cycles, $\sim 3 \times 10^{-8}$ Torr. These data shows the in-phase (left) and out-of-phase (right) response of the particle's vertical motion on resonance with the modulation of the microwave signal. Each change in voltage of 0.1 V is expected to change the resonant frequency for MDMR by about 200 MHz.

These librations were especially concerning for the fact that, with the particle well-cooled by the cooling beams, the librations shifted the location of the particle tip by significantly more than the vertical motion did. Thus, this allowed for the possibility of a resonant response to variations in microwave drive power at a frequency other than microwave-sweep frequency.

The next set of data, displayed in Figure 148, was taken at two more voltages to fill out the range of the data shown in Figure 147.

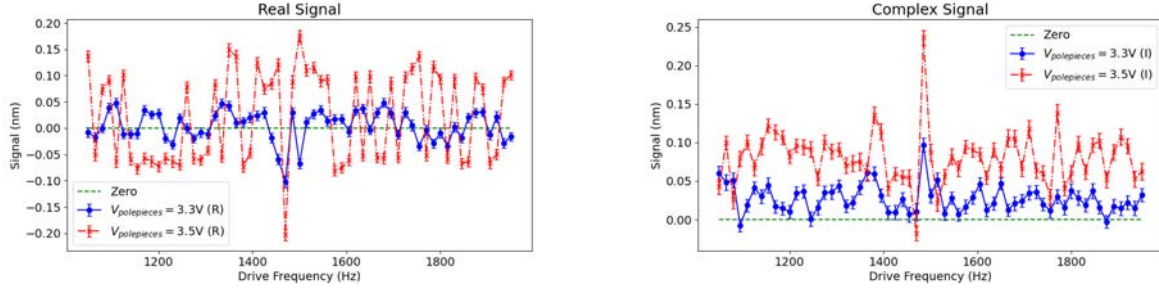


Figure 148: This figure was generated by `analyze_collectionsB.py` from the file `collection-B`. Laser power: 5.6 mW, MD modulation amplitude: ± 90 MHz, microwave power: 6.0 dBm (34.0 dBm in the trap), 4 burst cycles, 3.5×10^{-8} Torr. These data shows the in-phase (left) and out-of-phase (right) response of the particle's vertical motion on resonance with the modulation of the microwave signal. Each change in voltage of 0.1 V is expected to change the resonant frequency for MDMR by about 200 MHz.

The data shown in Figure 149 was taken over a set of three close voltages. This was done partially to try to avoid the particle shifting modalities; it had a tendency to get apparently spontaneously excited, after which the libration frequency might be significantly changed and its motion noticeably different. Nevertheless, some change still occurred.

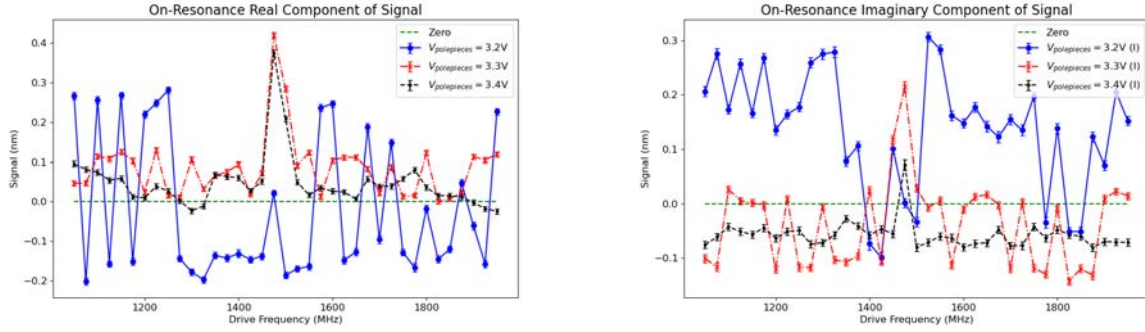


Figure 149: This figure was generated by `analyze_collectionsC.py` from the file `collection-C`. Laser power: 5.6 mW, MD modulation amplitude: ± 90 MHz, microwave power: 6.0 dBm (34.0 dBm in the trap), 4 burst cycles, 3.7×10^{-8} Torr. These data shows the in-phase (left) and out-of-phase (right) response of the particle's vertical motion on resonance with the modulation of the microwave signal. Each change in voltage of 0.1 V is expected to change the resonant frequency for MDMR by about 200 MHz.

Next a set of data was taken to test if altering the number of bursts significantly altered the signal if the particle was held at the same voltage (see Figure 150). This time no major shift in behavior was observed over the three measurements, largely due to the absence of any voltage changes.

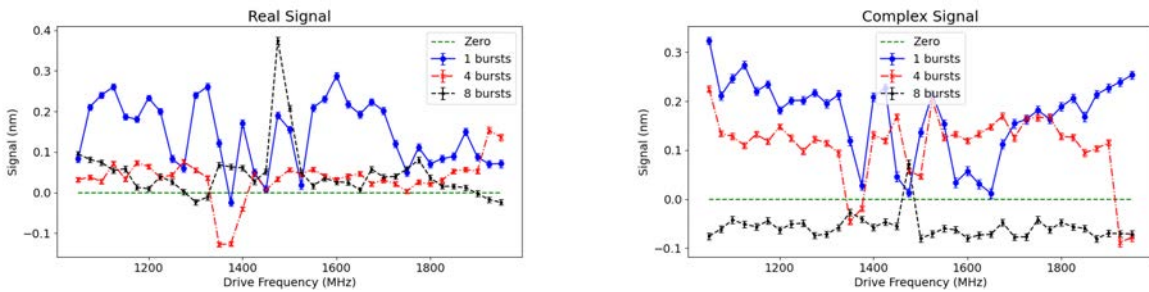


Figure 150: This figure was generated by `analyze_collectionsD.py` from the file `collection-D`. Laser power: 5.6 mW, MD modulation amplitude: ± 90 MHz, microwave power: 6.0 dBm (34.0 dBm in the trap), 3.7×10^{-8} Torr. These data shows the in-phase (left) and out-of-phase (right) response of the particle's vertical motion on resonance with the modulation of the microwave signal. These data were collected to see how much variation in the signal there was due to varied microwave burst durations. The resonant frequency was measured once and then assumed to be fixed.

Since varying the burst count seemed to have some impact on the signal, but it was not clear which was better, a burst count of 1 was chosen for the next set of data to increase how many values could be averaged over for the same collection time. This data (shown in Figure 152) was taken with a stack of three permanent magnets placed on top of the trap, aligned in order to primarily alter the transverse magnetic field.

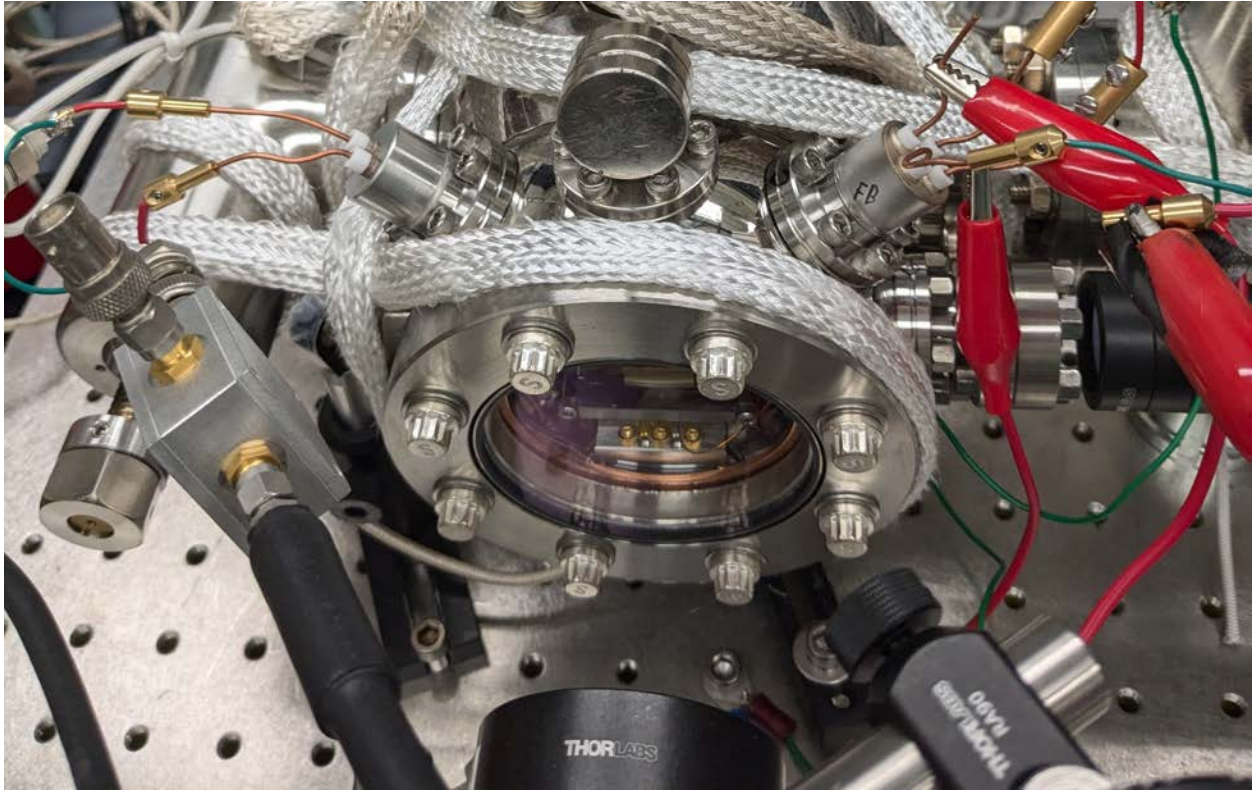


Figure 151: Arrangement of the magnets used to try to shift the magnetic field experienced by the trapped particle without changing the pole piece voltages. Measurements were taken with magnets absent and in two orientations, both like the one shown in this figure, but with the poles reversed.

While there are intriguing signals in each set of data shown, none has the clear frequency-shifted peak expected of MDMR.

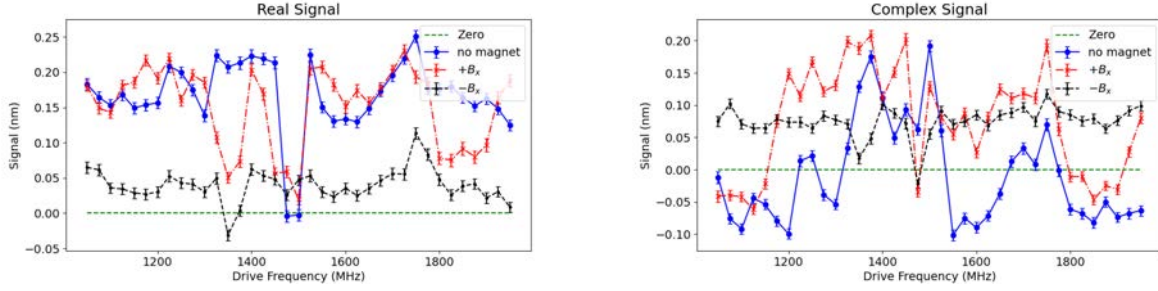


Figure 152: This figure was generated by `analyze_collectionsE.py` from the file `collection-E`. Laser power: 5.6 mW, MD modulation amplitude: ± 90 MHz, microwave power: 6.0 dBm (34.0 dBm in the trap), 1 burst cycle, 3.8×10^{-8} Torr. These data shows the in-phase (left) and out-of-phase (right) response of the particle's vertical motion on resonance with the modulation of the microwave signal. The resonant frequency was measured once and then assumed to be fixed, despite the fact that (in hindsight) the oscillation frequency likely does change somewhat to the additional B -field present.

Conclusions

We can make several observations from these data. First, we do not see a clear peak that shifts in frequency as expected. Second, despite the chaotic appearance of the data, it is not actually dominated by noise sources, as can be seen by the relative sizes of the error bars. However, there are still statistically significant features that seem consistent across runs, and some that seem to shift around in ways that are difficult to characterize. This indicates that our experiment is likely not limited by sensitivity or noise, but rather by sources of systematic error.

There are many possible sources of error here: confused signal due to the coupling of the particle's rotational instability into the vertical tracking of the particle, potentially random placement of vacancy centers within the particle, a poor choice of material (the epi-material had a far stronger ODMR signal than the boule material), sub-optimal particle placement for detecting a signal with many vacancies present (see Figures 109 and 141), and a complete lack of knowledge concerning the orientation of the SiC c -axis.

The precision and low noise of the measurement combined with the inconsistent

structure of the data suggest that if an MDMR signal is present, it is confused by whatever else it is that causes the observed structure. A significant difficulty arose in reproducing measurements due to the inconsistency of state preparation of the particle. Specifically, its librations increased or decreased in amplitude and frequency over time scales of minutes, days, and hours, and could not be well controlled or characterized.

The results do not demonstrate a clear detection of an MDMR signal. However, the methodology shows theoretical promise and experimental precision, if the obstacles preventing clear signal detection can be overcome.

In summary, there are several possible factors that may contribute to the difficulty of obtaining a clear MDMR signal.

First, while the translational motion of the particle was cooled effectively, the particle librated about the transverse (x) axis of the trap. This motion could not be cooled, was large compared to vertical translational motion, occurred at a different frequency than the microwave drive modulation linked to vertical motion, and almost certainly would have changed the resonance frequency of V_{Si}^- (V_2) vacancy centers present in the trapped particle.

Second, the results from the ODMR trials suggest that the boule-material, which was used to create the particle used, may be a worse candidate for MDMR than the epi-material. With a small number of vacancy centers present, the signal shape would be dominated not by the overall shape of the particle, but by the specific locations of individual vacancy centers scattered throughout the particle. This would make interpreting any observed signal very difficult.

Finally, if the particle's orientation changes significantly during detection, it causes two additional problems. One is that the strongest signal occurs when the resonant magnetic field is oriented perpendicular to the c -axis of the crystal, so the signal strength may depend on particle orientation. The second is that the expected frequency shift also depends on the angle between the c -axis and the static magnetic field, which is also dependent on particle

orientation.

Thus the following steps are clear choices for future attempts to measure MDMR in a magnetically levitated particle. First, use a material with potentially higher concentration of V_{Si}^- (V2) vacancy centers, such as the epi-material, for the test particle. Second, use a test particle that is rotationally stable enough that any librations are small enough to be neglected. This can likely be accomplished by using a particle that more closely resembles a prolate ellipsoid than an oblate one. Care must be taken, however, since smooth rotation about any axis could result in a time-varying c-axis orientation. Thus the ideal particle would be stable enough such that its libration is small compared to its vertical motion while still maintaining a consistent orientation in the trap.

A sphere is a tempting choice due to the existence of straightforward methods for shaping them and their favorable signal shape (see Figure 110), but may be subject to a changing c-axis orientation that could not be readily measured.

CHARACTERIZATION AND REDUCTION OF DAMPING RATES IN MAGNETO-GRAVITATIONAL TRAPS (MGTS)

When charged borosilicate spheres were levitated in the Long MGT , they damped at a rate of about $1.3 \times 10^{-4} \text{ s}^{-1}$. For a measurement of G lasting several hours, this is clearly unacceptable. It was observed that this damping rate depended on the amount of charge on the microspheres, a result of image-charge damping.

Trapping graphite composite particles (GCPs) has the clear advantage of eliminating the need for charged particles due to its strong diamagnetism, but due to graphite's higher conductivity it was hypothesized that eddy current damping might become the new dominant source of damping. However, some careful consideration of the specific MGT being used indicates that this may not be so great an issue after all.

Damping Rate Estimates

What follows is a rough characterization eddy current damping in GCPs in a magnetic trap with field magnetic field approximated by the multipole expansion discussed in depth in Chapter 2. It assumes a particle small relative to the trap dimensions (which is only true in this case to a very approximate extent), and models the particle as a loop of wire for estimating the effects of eddy currents rather than a full three-dimensional object.

As such the parameters such as the resistance R and cross-sectional area A_x of the particle must be understood in more of an effective sense; that is, they are unlikely to match any specific observable or directly measurable parameters of the GCP, but should scale roughly with its resistivity and area, respectively. In this sense, these parameters are not dissimilar to the vertical equilibrium scaling factor y_{eq} , which can be computed but not directly measured.

Particle Eddy Current Damping

The eddy current damping in a GCP may be estimated by first considering the forces acting on the particle to derive the equation of motion (EOM), which in this case resembles a damped harmonic oscillator, then using that EOM to compute the power loss of the system over time.

Equations of Motion From Eq. (18), we estimate the vertical and axial equations of motion to be

$$\ddot{y} = -c_0 b_0^2 (1 + 12\gamma) y \quad (53)$$

$$\ddot{z} = -c_0 b_0^2 (\alpha - \beta) z, \quad (54)$$

where we have considered $\alpha, \beta, \gamma^2 \ll 1$ and have neglected anharmonic terms proportional to z^2 or z^3 .

It is also convenient to define a parameter b_y to denote the characteristic vertical field gradient as $\frac{\partial B_x}{\partial y} = b_y = b_0(1 + 3\gamma)$. We similarly define b_z to denote the characteristic axial field gradient as $\frac{\partial B_x}{\partial z} = b_z z = \frac{b_0(\alpha - \beta)}{y_{eq}} z$. Note that b_z is not constant in z and does not have the same units as b_0 or b_y .

For the sake of estimating eddy current damping, we model this particle as a loop of wire with cross-sectional area A_x and resistance R . It should be noted that only A_x really matters for the purpose of these calculations, since Eq. (40) tells us that for $x = 0$, the primary field experienced by the particle is in the x -direction.

We are now ready to consider the damping force exerted on the particle in the y -direction due to eddy current losses. Treating the particle like a loop of wire with cross-sectional area A_x and resistance R , the current running through the wire can be estimated using Eq. (40)

as

$$i(t) = \frac{\varepsilon(t)}{R} = -\frac{A_x}{R} \frac{dB_x}{dt} \approx -\frac{A_x b_y}{R} \dot{y}, \quad (55)$$

where we have neglected terms of order x_i^2 and higher.

Using the fact that the dipole moment of a current-carrying wire is given by $\vec{\mu} = i\vec{A}$, and that the energy of a dipole in a magnetic field is $U = -\vec{\mu} \cdot \vec{B}$, we can take the gradient of the potential to obtain the following expression for force:

$$F_{\text{eddy},y} = \vec{\nabla} (\vec{\mu} \cdot \vec{B}) \cdot \hat{y} = -\frac{\partial}{\partial y} \left[\left(\frac{A_x^2 b_y}{R} \dot{y} \right) (b_y y) \right] = -\frac{A_x^2 b_y^2}{R} \dot{y}. \quad (56)$$

We can use this to see that the damped equation of motion (EOM) in the y -direction is

$$\ddot{y} = -\omega_y^2 y - \Gamma_y \dot{y}, \quad (57)$$

where $\Gamma_y = \frac{A_x^2 b_y^2}{mR}$ is the damping rate of the oscillator, and $\omega_{y,0}^2 = c_0 b_y^2 (1 + 12\gamma)$. Note that in our system $\omega_{y,0} \gg \Gamma_y$, so we can make the approximation that $\omega_y \approx \omega_{y,0}$, where ω_y is the actual angular frequency of the damped oscillator.

Before we move on to the EOM for the axial motion of the particle, let us take a moment to consider the force that acts on a loop of wire as it moves through a field gradient. We consider the same loop of wire considered earlier, this time in a transverse magnetic field, $B_x(z)$, with an arbitrary gradient in z .

In order to calculate the force on the loop, we break it up into individual rectangular sub-loops with height Δy and width Δz . If we consider a current I moving clockwise ($-\hat{x}$ direction) through this sub-loop centered at axial position z , we can calculate the force from

the Lorentz force law:

$$\Delta \vec{F}(z) = -I (B_x(z + \Delta z/2) - B_x(z - \Delta z/2)) \Delta y \hat{z}. \quad (58)$$

We can now imagine building a loop of arbitrary shape by combining all these sub-loops together. So long as the current I in each loop is held constant, all interior currents cancel out, leaving only the exterior currents circulating around the perimeter of the total loop. We therefore take $\Delta \rightarrow d$ to turn Eq. (58) into

$$dF_z = -I dy \, dz \frac{dB_x}{dz} = -I dA_x \frac{\partial B_x}{\partial z}. \quad (59)$$

Note that for $\frac{\partial B_x}{\partial z} = \text{const.}$, the total force becomes simply $F_z = -I A_x \frac{\partial B_x}{\partial z} = \frac{\partial}{\partial z}(\mu_x B_x)$, matching the form of eqn. (56). Thus we see that the force on a loop of current-carrying wire due to motion through a linear field gradient is independent of the shape of the wire.

For more complicated fields, more analysis is required. At this point it should be noted that, in order to keep I constant for each sub-loop, the area of each sub-loop must vary in order to keep the same change in flux for a varying field gradient. Specifically,

$$I = \frac{\varepsilon}{dR} = -\frac{dA_x \frac{\partial B_x}{\partial z} \dot{z}}{dR} = \text{const.}, \quad (60)$$

where dR is the resistance of a given sub-loop. Note that this resistance is not the same for each loop, but depends instead on dA_x .

If we solve eqn. (60) for $\frac{\partial B_x}{\partial z}$ and substitute into eqn. (59), we obtain

$$dF_z = \frac{I^2 dR}{\dot{z}}. \quad (61)$$

We can further observe that if the total resistance of the loop is $R = \int dR$, then the instantaneous power dissipation is given by

$$P = \dot{z} \int dF_z = I^2 R. \quad (62)$$

Because dR is not trivial to calculate, Eq. (61) is not a very useful formula for computation, but it does serve as a nice check on the result.

At this point there are two simple cases we can consider. The first case is for a rectangular loop of width l_y and height l_z momentarily centered at position $(0, y, z)$. In this case, eqn. (59) yields

$$\begin{aligned} F_z &= \int dF_z = -I \int_{y-l_y/2}^{y+l_y/2} \int_{-l_z/2}^{l_z/2} \left. \frac{\partial B_x(z')}{\partial z'} \right|_{z+z'} dz' dy' \\ &= -Il_y (B_x(z + l_z/2) - B_x(z - l_z/2)) \\ &= -IA_x \frac{B_x(z + l_z/2) - B_x(z - l_z/2)}{l_z}. \end{aligned} \quad (63)$$

This gives a way to directly calculate the force on a large rectangular loop of wire in an arbitrary field gradient. Note that in the limit that $l_z \ll z$, this reduces to

$$F_z = -IA_x \frac{\partial B_x}{\partial z}, \quad (64)$$

regardless of whether $\frac{\partial B_x}{\partial z}$ is constant or not.

Even when we are not in that limit, however, eqn. (63) is useful for calculating the force on a rectangular loop of wire in an arbitrary field gradient.

The second case to consider is the case of a quadratic B_x -field in the axial direction,

where for the trap being considered $B_x(z) = \frac{1}{2}b_z z^2$ per Eq. (40). In this case, eqn. (59) for a loop of wire with shape specified by functions $y_1(z)$ and $y_2(z)$ defining the bottom and top boundaries of the loop, respectively, yields

$$\begin{aligned}
 F_z &= -I \int_{A_x} b_z(z + z') \, dA'_x \\
 &= -IA_x \frac{\partial B_x}{\partial z} - Ib_z \int_{-l_z/2}^{l_z/2} \int_{y_1(z')}^{y_2(z')} z' dy' \, dz' \\
 &= -IA_x \frac{\partial B_x}{\partial z} - Ib_z \int_{-l_z/2}^{l_z/2} (y_2(z') - y_1(z')) z' \, dz',
 \end{aligned} \tag{65}$$

or equivalently in cylindrical coordinates,

$$\begin{aligned}
 F_z &= -IA_x \frac{\partial B_x}{\partial z} - Ib_z \int_0^{2\pi} \int_0^{r(\phi'+\pi)} r'^2 \cos \phi' \, dr' \, d\phi' \\
 &= -IA_x \frac{\partial B_x}{\partial z} + \frac{1}{3} Ib_z \int_{-\pi}^{\pi} r(\phi')^3 \cos \phi' \, d\phi',
 \end{aligned}$$

where $z = r \cos \phi$ and $y = r \sin \phi$.

We can see three things from this. First, if $z' \ll z$ for all $|z'| < l_z$, then this reduces once again to eqn. (64) as with eqn. (63), except this time independently of the shape of the loop. Second, if the function $r(\phi)$ is odd or constant (as would be the case for, e.g., a circular loop of wire), then we once again get eqn. (64). Finally, if the function $(y_2(z) - y_1(z))$ is even or constant (as would be the case for, e.g., a rectangular loop of wire), then we also get eqn. (64).

In conclusion, for a quadratic B -field gradient in z , we can approximate the force on the particle using eqn. (64) if the particle can be considered small relative to the scale of its motion, or if it is modeled as a circular or rectangular loop of wire even if it is not small. If none of these conditions is met, however, eqn. (64) may not be sufficient to accurately estimate the force on a loop of wire moving in such a field.

We are now ready to combine eqns. (40), (55), and (64) to obtain the eddy-current damping force on the particle in the z -direction (for a particle modeled as a rectangular or

circular loop of wire):

$$\begin{aligned}
 F_{eddy,z}(t) &= i(t)A_x \frac{\partial B_x}{\partial z} = -\frac{A_x^2 b_z z}{R} \frac{dB_x}{dt} \\
 &\approx -\frac{A_x^2 b_z z}{R} \left(b_y \dot{y} + b_z z \dot{z} - \frac{1}{4} \alpha_z y_{eq} \dot{y} \right),
 \end{aligned} \tag{66}$$

where we have once again ignored terms of order x_i^2 and higher.

We now take the average in time, recalling that the amplitude of oscillation in x and y is small compared to z and that $\omega_x, \omega_y \gg \omega_z$:

$$\begin{aligned}
 \langle F_{eddy,z} \rangle_z &\approx -\frac{A_x^2 \alpha_z}{R} \left\langle b_y z \dot{y} + b_z z^2 \dot{z} - \frac{1}{4} \alpha_z y_{eq} z \dot{y} \right\rangle_z \\
 &\approx -\frac{A_x^2 b_z^2}{R} \langle z^2 \dot{z} \rangle_z,
 \end{aligned} \tag{67}$$

where $\langle f(t) \rangle_z = \frac{\omega_z}{2\pi} \int_0^{2\pi/\omega_z} f(t) dt$.

We are interested here in attempting to discover the long-term behavior of the particle, specifically how it is damped over many oscillation periods. Therefore, when considering how to estimate the long-term equation of motion, we will neglect the terms in the damping force that average out to 0 over time. The equation of motion, then, can be written for our purposes as

$$\ddot{z} = -\omega_z^2 z - \Gamma_z(z) \dot{z}, \tag{68}$$

where in this case the damping force is amplitude-dependent, unlike for the y -motion, and given by $\Gamma_z(z) = \frac{A_x^2 b_z^2 z^2}{mR}$.

Average Power Loss It is interesting to observe that, similarly to eqn. (62), we can also use eqn. (55) to calculate the power-loss related force on a loop of current carrying wire as

$$F_y = -\frac{I^2 R}{\dot{y}} = -\frac{A_x^2 b_y^2}{R} \dot{y} \quad (69)$$

$$F_z = -\frac{I^2 R}{\dot{z}} = -\frac{A_x^2 b_z^2 z^2}{R} \dot{z}, \quad (70)$$

which match nicely with eqns. (68) and (57).

In fact, we can get information about the long term behavior of the particle directly without computing the force acting on it by calculating the power dissipation. For the vertical motion we see that the solution to (57) is

$$\begin{aligned} y(t) &= y_{eq} + y_0 e^{-\Gamma_y t/2} \cos \omega_y t \\ \dot{y}(t) &= -\frac{1}{2} \Gamma_y y_0 e^{-\Gamma_y t/2} \cos \omega_y t - \omega_y y_0 e^{-\Gamma_y t/2} \sin \omega_y t \end{aligned} \quad (71)$$

where we will take Γ_y to be an unknown quantity. Then,

$$-\langle I^2 R \rangle_y = -\frac{A_x^2 b_y^2}{R} \langle \dot{y}^2 \rangle = -\frac{A_x^2 b_y^2 y_0^2 \omega_y^2}{2R} e^{-\Gamma_y t} \quad (72)$$

$$\left\langle \frac{d}{dt} \left(\frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \omega_y^2 y^2 \right) \right\rangle_z = m \langle \dot{y} \ddot{y} + y \dot{y} \rangle \quad (73)$$

$$= m \langle \dot{y} (-\omega_y^2 y - \Gamma_y \dot{y} + \omega_y^2 y) \rangle_z \quad (74)$$

$$= -\frac{1}{2} m \Gamma_y y_0^2 \omega_y^2 e^{-\Gamma_y t} \quad (75)$$

where we recall again that $\Gamma_y \ll \omega_y$.

Combining eqns. (72) and (75) yields a familiar expression for Γ_y :

$$\Gamma_y = \frac{A_x^2 b_y^2}{mR}. \quad (76)$$

We can similarly estimate an equivalent Γ_z if we treat the amplitude-dependent damping force as a perturbation so that the EOM is approximately that of a damped harmonic oscillator, following the general form of Eq. (71). Specifically, we consider the power loss due to the particle's motion on time scales small such that $\Gamma_z t \ll 1$. Consider again a comparison between the following two ways to compute the power loss:

$$-\langle I^2 R \rangle_z = - \left\langle \frac{A_x^2 b_z^2 z^2 \dot{z}^2}{R} \right\rangle_z = - \frac{A_x^2 b_z^2 \omega_z^2 z_0^4}{8R} e^{-2\Gamma_z t} \text{ and} \quad (77)$$

$$\left\langle \frac{d}{dt} \left(\frac{1}{2} m \dot{z}^2 + \frac{1}{2} m \omega_z^2 z^2 \right) \right\rangle_z = - \frac{1}{2} m \Gamma_z z_0^2 \omega_z^2 e^{-\Gamma_z t}. \quad (78)$$

For $\Gamma_z t \ll 1$, setting eqns. (77) and (78) equal to each other yields an effective Γ_z of

$$\Gamma_z = \frac{A_x^2 b_z^2 z_0^2}{4mR}. \quad (79)$$

We can now examine the relative damping rates between the axial and transverse motion, which can be expressed in terms of the fit parameters discussed in Chapter 2 as

$$\frac{\Gamma_z}{\Gamma_y} = \frac{b_z^2 z_0^2}{4b_y^2} = \frac{1}{2} \left[\left(\frac{z_{rms}}{y_{eq}} \right) \left(\frac{\alpha - \beta}{1 + 3\gamma} \right) \right]^2, \quad (80)$$

where we have made the assumption that $z_{rms}^2 \approx \frac{1}{2} z_0^2$.

If we plug in the values listed in Section 2 and the value of $z_{rms} = 1.16$ mm (taken from `z_ringdown_blue/displacements_offline_01.zia`), we can approximate this ratio to be about $\frac{\Gamma_z}{\Gamma_y} = 4.1 \times 10^{-8}$.

Note that this can be approximated by

$$\frac{\Gamma_z}{\Gamma_y} \approx \frac{1}{2} \left[\left(\frac{z_{rms}}{y_{eq}} \right) \left(\frac{\omega_z^2}{\omega_y^2} \right) \right]^2 \quad (81)$$

if $9\gamma \ll 1$, which is not a good approximation in our system. Nevertheless, an order-of-magnitude estimate of the ratio of the damping rates can still be obtained from the frequencies alone without performing a multipole fit. [107]

Estimated Gas Damping

The motion of the particle is also subject to damping from residual gas pressure in the chamber, which can be estimated for cylinder moving in a finite volume of gas by [25]

$$\Gamma_{gas,cylinder} = \frac{1}{\rho_{HB}h} \sqrt{\frac{8m_0}{\pi k_B T}} \left(1 + \frac{h}{2R} + \frac{\pi}{4} \right), \quad (82)$$

where m_0 was assumed to be equal to the mass of a N_2 molecule, T was measured to be approximately 24 °C at the time of the measurement, and $h = 2.4$ mm was used to match the length of Edwin (HB05-2.4-lapped) . A plot of the theoretical gas damping rate for several diameters of pencil lead is shown in Figure 153.

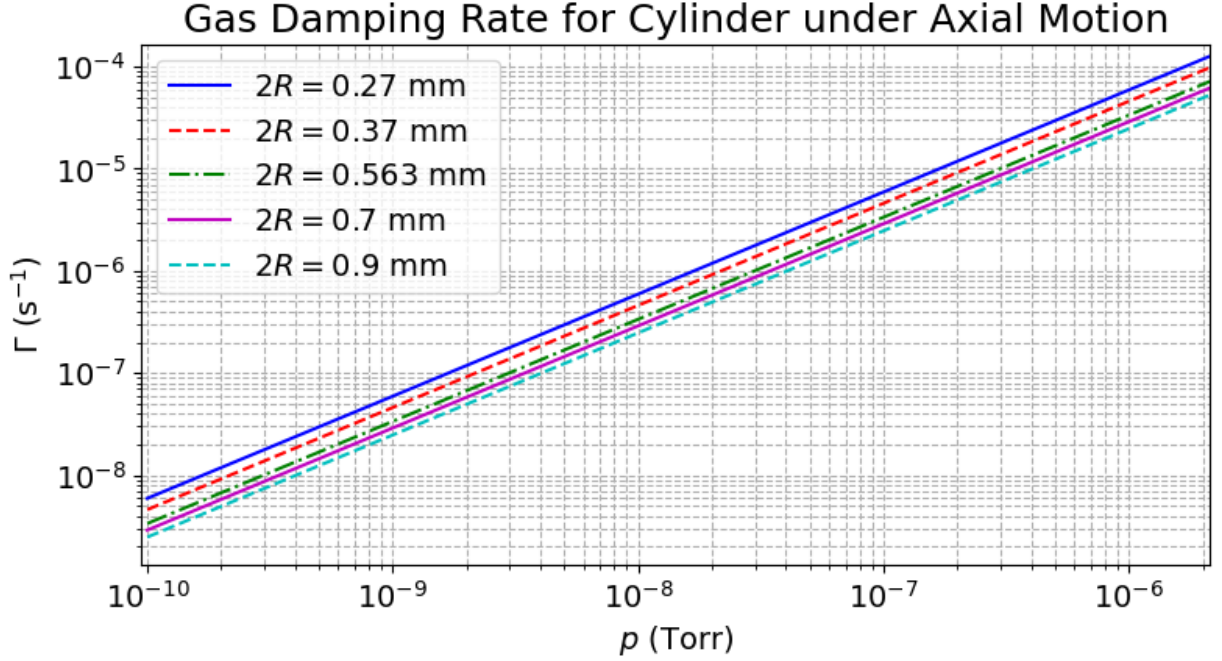


Figure 153: This figure was generated by `gas_damping_estimates.py`. A plot estimating the damping rate for a cylinder moving axially through a volume of gas, estimated using the analysis by Cavalleri et al. [25] and the parameters measured in Chapter 2 for Pentel HB pencil lead. Length held constant at 2.4 mm. A value of $\rho_{HB} = 1.7 \text{ g/cm}^3$ was used for the density of HB pencil lead (assumed to be equal for all lead diameters, which is only approximately true). Radii were chosen to match the diameters of commercially available Pentel pencil lead.

The damping rate is proportional to chamber pressure and, due to the relatively large length of the particle, roughly inversely proportional to the radius of the particle.

A similar result can be obtained for a sphere under the same conditions from [25]

$$\Gamma_{gas,sphere} = \frac{3}{4\rho_{HB}R} \sqrt{\frac{128m_0}{9\pi k_B T}} \left(1 + \frac{\pi}{8}\right). \quad (83)$$

A plot of the theoretical damping rate for several different particle sizes is shown in Figure 154.

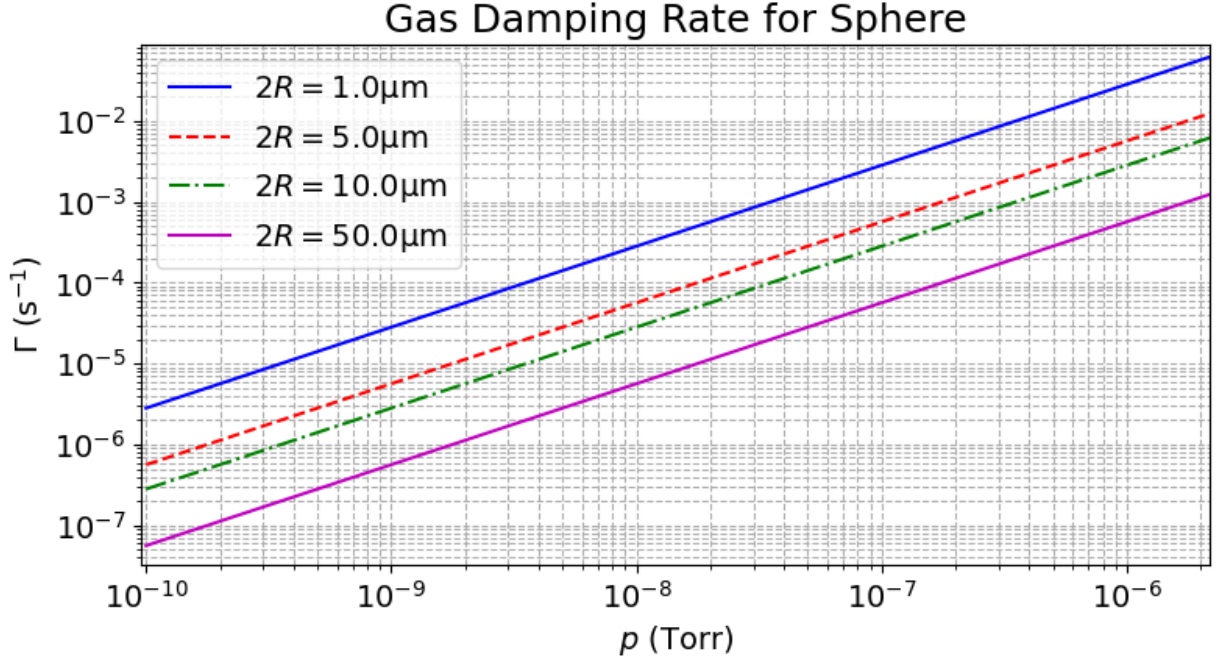


Figure 154: This figure was generated by `gas_damping_estimates.py`. A plot estimating the damping rate for a sphere moving through a volume of gas, estimated using the analysis by Cavalleri et al. [25] and the parameters of a borosilicate sphere. A value of $\rho_{\text{SiO}_2} = 2.23 \text{ g/cm}^3$ was used for the density of borosilicate glass.

Low-Damping Rates Achieved in the Long MGT

The particle denoted as Edwin (HB05-2.4-lapped) was levitated in the Long MGT and placed under ultra-high vacuum (UHV) by use of the roughing pump, turbo pump, chamber baking, and occasional firing of the Ti-sublimation pump. The chamber pressure was not the same for every measurement taken, and is recorded in each figure caption in this section.

It was found that if the chamber pressure stayed at too high of a pressure for very long (e.g. 10×10^{-7} Torr for a few hours), charge build-up on the GCP became a dominant source of damping in the system.



Figure 155: An image of Edwin (HB05-2.4-lapped) taken under UHV. The diameter of the particle is about 0.56 mm and the length is about 2.4 mm. The bottom and top black parts of the image are the silhouettes of the bottom and top pole pieces, respectively.

Setup

An image of the setup used is shown in Figure 156. The slides and vacuum chamber shown were mounted in such a way as to allow active tilt-stabilization of the system as mentioned in Chapter 2. [84, 84]

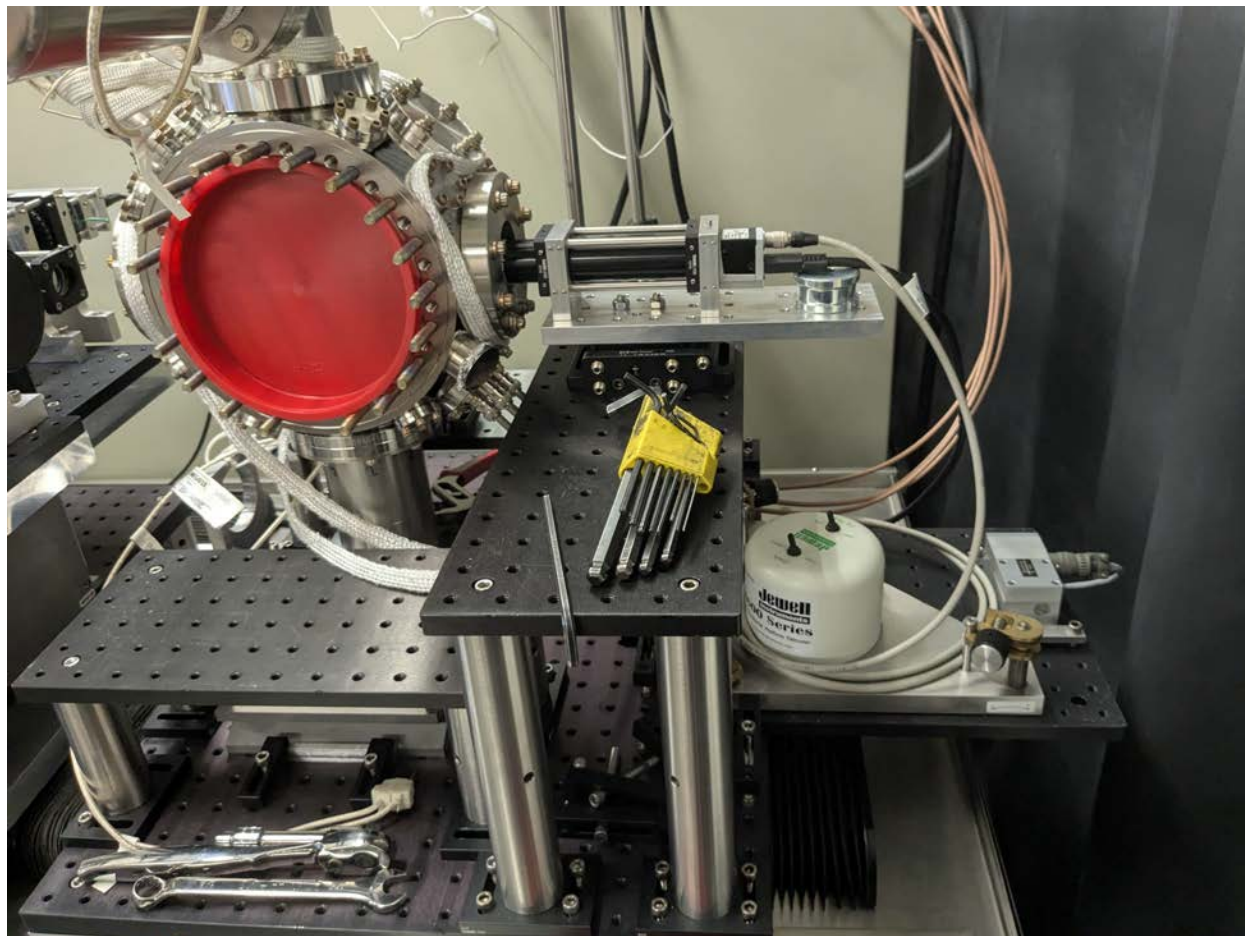


Figure 156: A photo of the setup used with the large vacuum blanks removed. The camera (right) is setup to image the particle down the transverse trap axis.

A diagram of the optics is shown in Figure 157, provided by Cody Jessup. The basic idea is to approximately collimate light from an LED onto a slit, then image that slit into the same plane as the trapped particle. Focusing can be tricky in this way, since the left and right edges of the particle do not focus easily in the same plane as themselves.

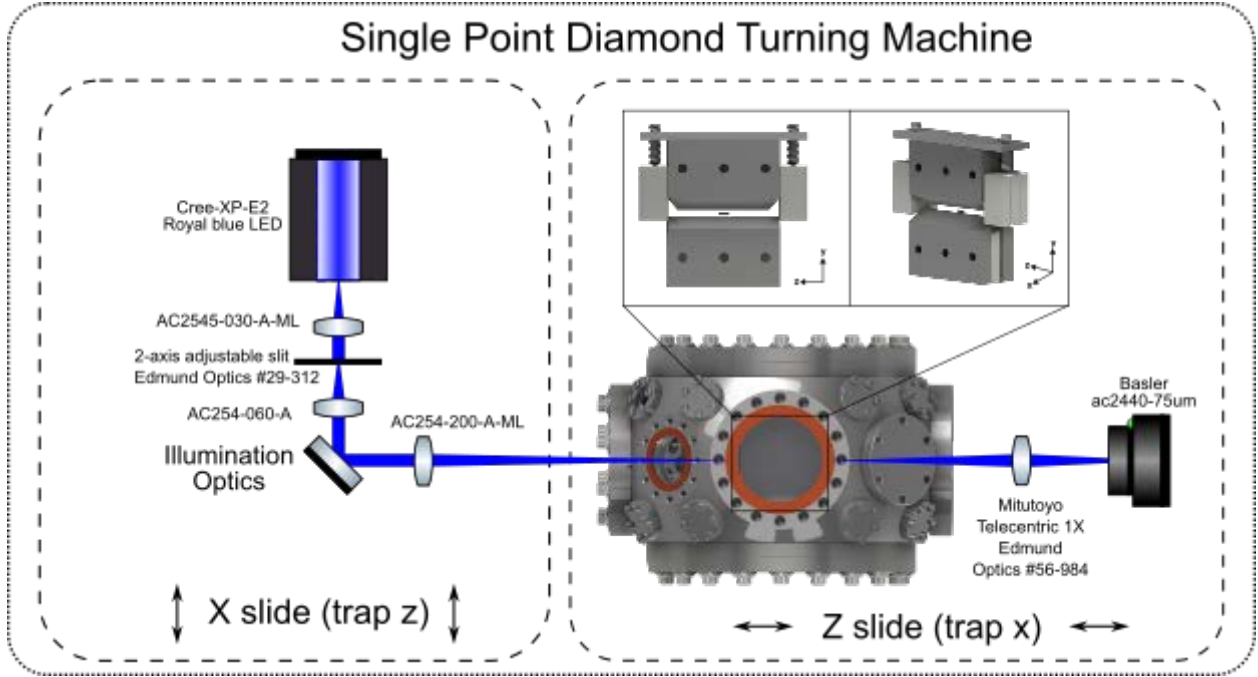


Figure 157: A diagram of the basic setup used for collecting the data discussed in this section. Notice that the x - and z -axes for the diamond turning machine are defined oppositely to those for the trap x - and z -axes.

The slides on the DTM were held fixed in place for all measurements reported, but are useful to have available for an eventual measurement of G , where the precise locations of the field masses could be measured on the apparatus just prior to each measurement.

Data

The camera exposure time was $25\text{ }\mu\text{s}$ throughout the measurements described in this section.

Motion data of Edwin (HB05-2.4-lapped) were collected with the DTM servos off. The vacuum level varied over the course of the measurements, but stayed close to $3\text{--}6 \times 10^{-8}$ Torr, and the ambient temperature in the lab was measured to be about 24°C .

The image analysis separates the vertical and transverse motion of the particle by the camera axes. As such, any tilt in the camera results in coupling between the recorded y - and

z -motion. If the angle of the camera is known, this can be mitigated by applying a digital rotation to the data.

This angle was estimated from the first harmonic of the axial motion as follows:

First, the data were split into several overlapping segments in the time domain using a Welch Segmenter. They were then Fourier transformed and averaged to obtain an amplitude spectral density (ASD) for both the y and z motion. This ASD was then interpolated with a parabola at its peak to obtain the axial frequency ($f_z = 0.249$ Hz) (shown in Figures 158, 168, and 174).

Second, each individual time segment was Fourier transformed, and the complex amplitudes of the vertical and axial motion at that frequency ($\tilde{y}(f_z)$ and $\tilde{z}(f_z)$, respectively) were determined via interpolation in each time segment.

Finally, the optimal θ required to minimize the coupling of the axial oscillation into the vertical data was found by minimizing

$$\tilde{y}'(f_z, \theta) = \sum_i |\tilde{y}_i(f_z) \cos \theta - \tilde{z}_i(f_z) \sin \theta|^2, \quad (84)$$

with respect to θ .

This value of θ was determined separately for each data set containing significant axial motion. Since no consistent angle was found that matched across all measurements, the data containing primarily vertical motion, where it was difficult to measure a clear angle due to short time segments, were not rotated by any correction angle.

Data of the particle's low-amplitude behavior were collected by pushing on the chamber by hand to de-excite the particle's axial motion until it was barely visible on the camera. The axial tilt stabilization was allowed to equilibrate before collecting data. The ASD of these data is shown in Figure 158.

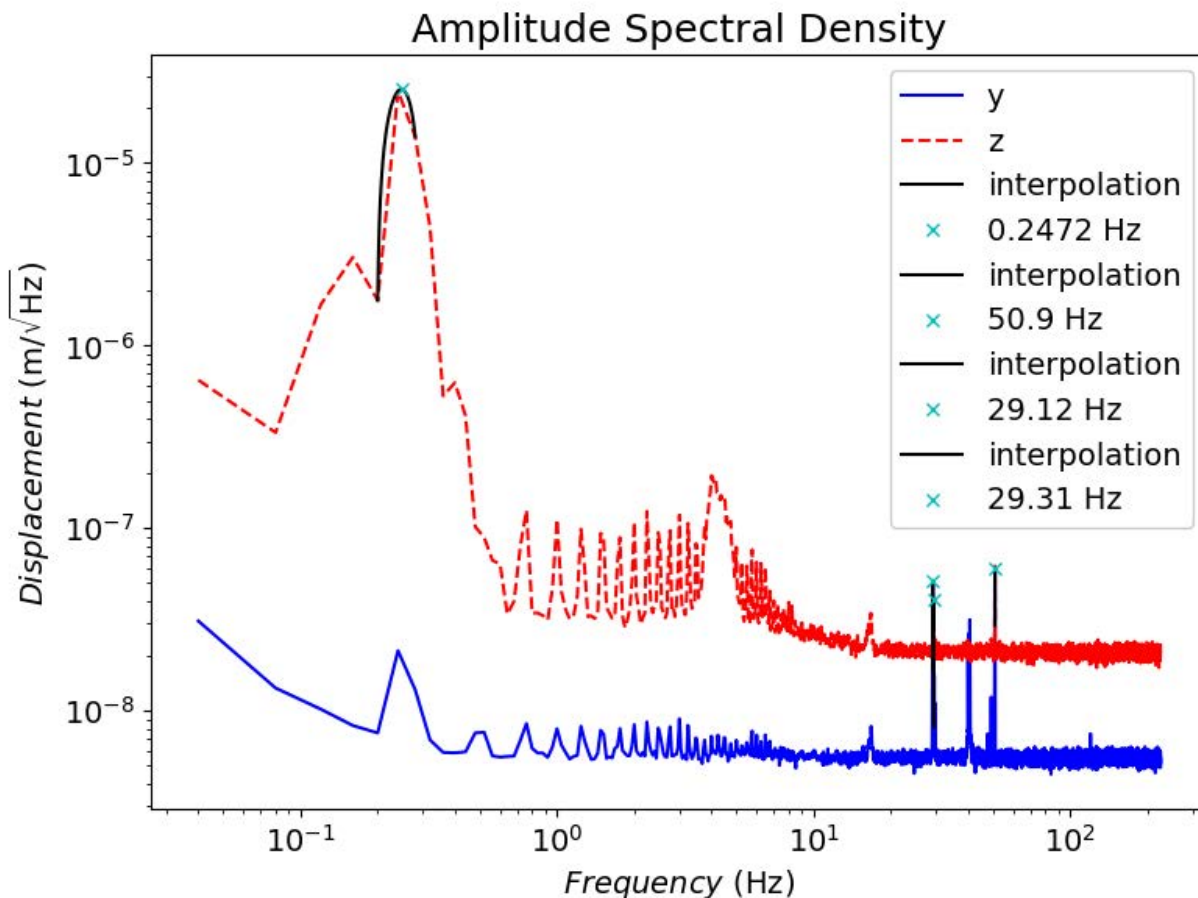


Figure 158: This figure was generated by `damping_processing_blue.py` from the file `0_amplitude_blue/displacements_offline_04.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr. Unrotated amplitude spectral density of the displacements of a particle with low amplitude ($z_{rms} = 5.9 \mu\text{m}$, $y_{rms} = 0.13 \mu\text{m}$) oscillations. Parabolic interpolation used to find peak frequencies (see Figure 159 for a closer look at the transverse frequencies).

There are several features of this plot that are worth noting. First, there is a broad peak in the axial data around 4-5 Hz. This may be due to particle libration, or it may be due to some other effect. There are three higher-frequency peak clusters at about 29 Hz, 40 Hz, and 51 Hz. The peaks at 29 Hz and 40 Hz correspond to the transverse and vertical oscillation frequencies, respectively. The origin of the 51 Hz peak, however, is unknown. The transverse frequency has sidebands, as can be seen in Figure 159.

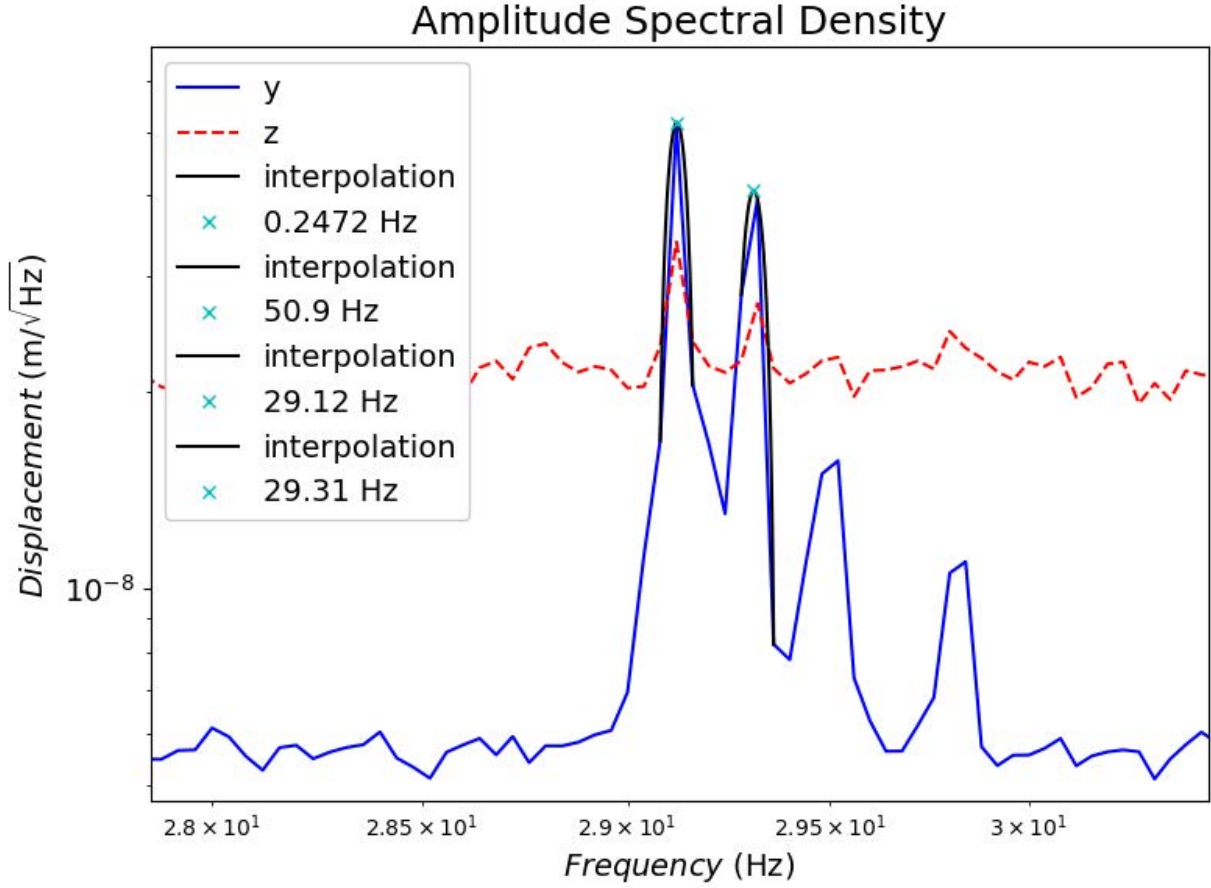


Figure 159: This figure was generated by `damping_processing_blue.py` from the file `0_amplitude_blue/displacements_offline_04.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr. Unrotated amplitude spectral density of the displacements of a particle with low amplitude ($z_{rms} = 5.9 \mu\text{m}$, $y_{rms} = 0.13 \mu\text{m}$) oscillations. Parabolic interpolation used to find peak frequencies. Note that the precise transverse frequency is difficult to estimate accurately due to the presence of significant sidebands in the data.

The transverse frequency sidebands make estimating the true transverse frequency difficult, since two of the peaks have similar amplitudes. The estimation of this value obtained in Chapter 2 was found by averaging the FFT over more segments to get wider frequency bins and avoid the issue. This however, means that the approximate for the absolute value of β given in Eq. (38) should be taken as only an estimate.

The effect of rotating the data using Eq. (84) in this case yields roughly a factor of 2

decrease in the amplitude of the vertical motion at the axial frequency, as seen by comparing Figure 160 to Figure 158.

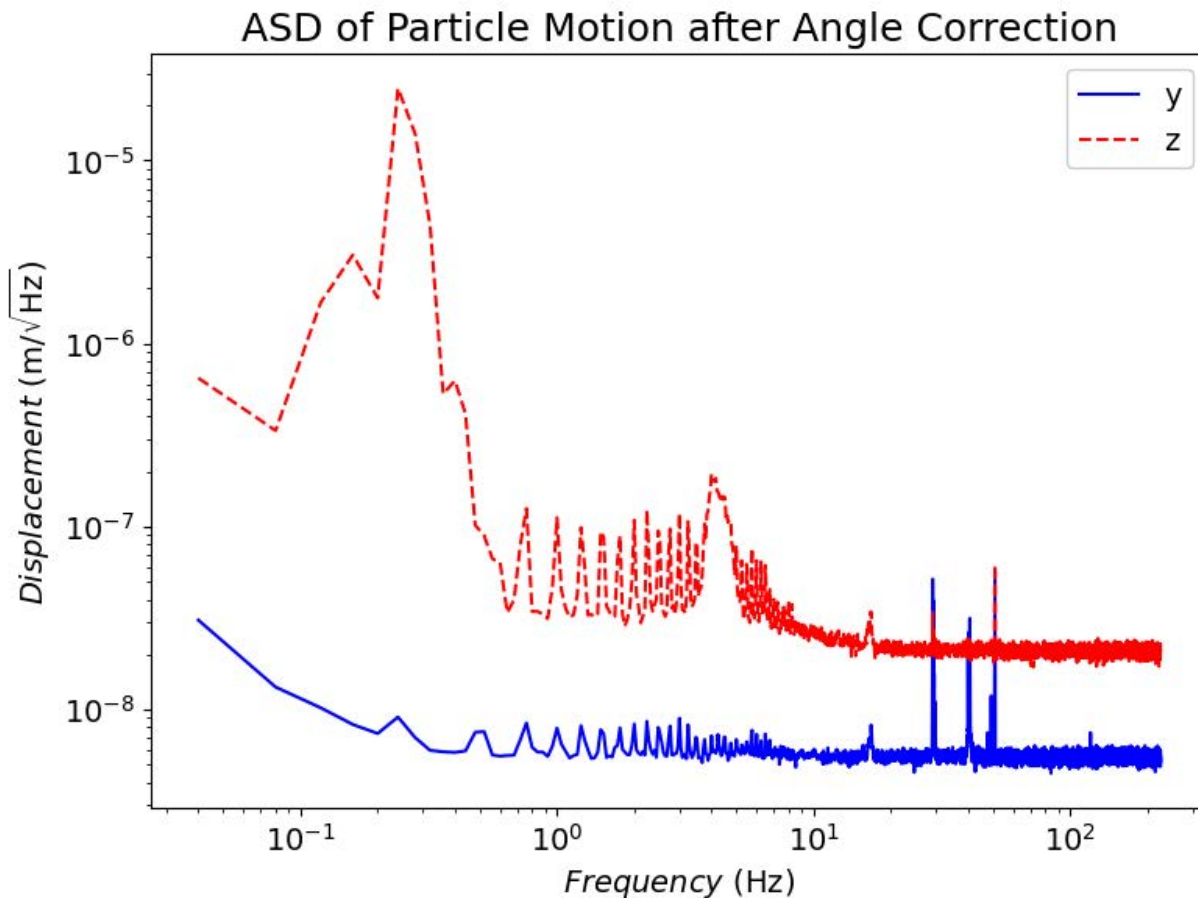


Figure 160: This figure was generated by `damping_processing_blue.py` from the file `0_amplitude_blue/displacements_offline_04.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.044^\circ$. Rotated amplitude spectral density of the displacements of a particle with low amplitude ($z_{rms} = 5.9 \mu\text{m}$, $y_{rms} = 0.13 \mu\text{m}$) oscillations. Note how much lower the amplitude of the vertical noise is at the axial oscillation frequency after rotation.

One useful feature of this plot is that we can estimate the measurement noise by examining the noise floor at high frequencies, where the noise is frequency independent (notice that force noise is proportional to $1/f^2$, producing the negative slope seen in the data at low frequencies). In this case, the axial (vertical) position uncertainty is about $20 \text{ nm}/\sqrt{\text{Hz}}$ ($6 \text{ nm}/\sqrt{\text{Hz}}$), or about 420 nm/Image (130 nm/Image).

The ratio of the particle's length to its width predicts that the uncertainty in the z position should be higher than the uncertainty in the y motion by about a factor of 2.1 (ratio of the square root of the number of pixels on the top/bottom edges); however, we observe a difference of about a factor of 3.3. This discrepancy is likely due to the fact that the top and bottom edges are better focused than the left and right edges, resulting in higher precision per pixel.

The maximum kinetic energy of the particle (equivalent to its total energy) can be estimated by fitting the displacements of the particle as it crosses equilibrium to a cubic, then analytically differentiating the fit to obtain a velocity curve and finding the vertex of that velocity curve. The cubic fit for this process is illustrated in Figure 161.

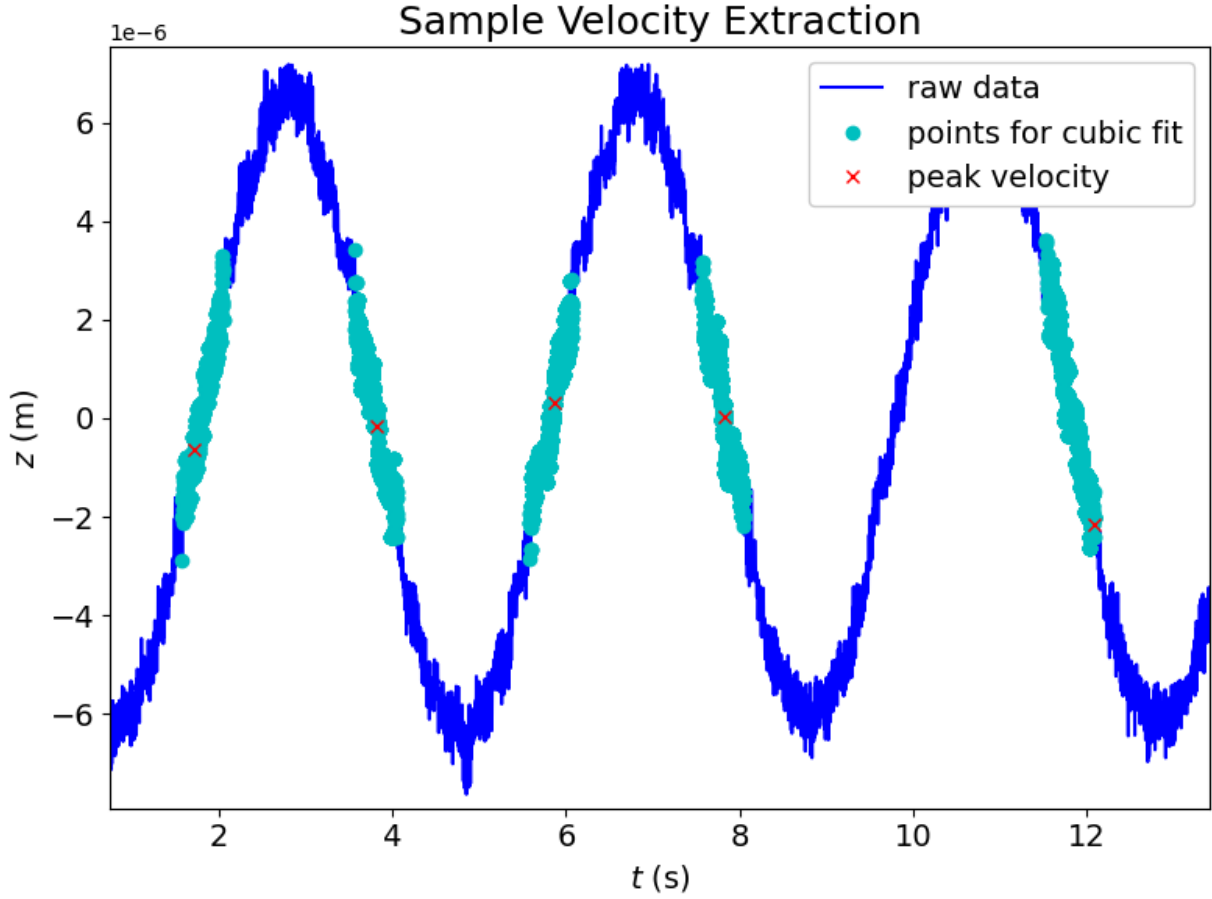


Figure 161: This figure was generated by `damping_processing_blue.py` from the file `0_amplitude_blue/displacements_offline_04.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.044^\circ$. Peak velocity estimation for low-amplitude ($z_{rms} = 5.9 \mu\text{m}$, $y_{rms} = 0.13 \mu\text{m}$) axial oscillations. The high-frequency component of the signal leads a poor fit to the slope of the curve at its crossing points.

Note that, in this case, the choppiness of the data resulted in multiple “zero-crossings” of the displacement. Only one set of data points was used to produce a fit at each zero-crossing, but this set of data had a relatively poor fit and was not carefully selected, leading to the potential for somewhat inaccurate energy estimates.

The damping rate can be estimated from the data by fitting an exponential to the kinetic energies computed from the velocities fit, as shown in Figure 162.

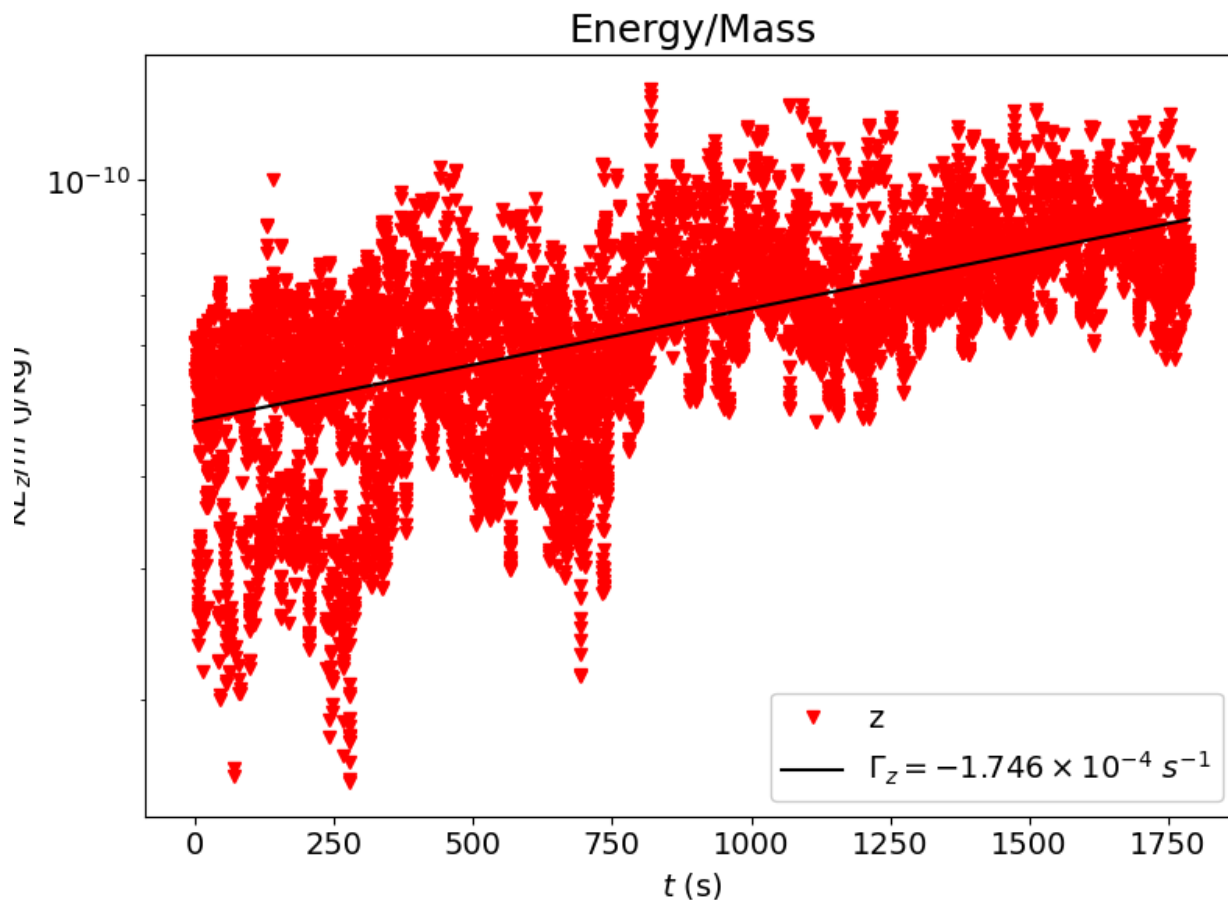


Figure 162: This figure was generated by `damping_processing_blue.py` from the file `0_amplitude_blue/displacements_offline_04.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.044^\circ$. Translational kinetic energy calculated from the peak velocities for low-amplitude ($z_{rms} = 5.9 \mu\text{m}$, $y_{rms} = 0.13 \mu\text{m}$) axial oscillations. The negative damping rate indicates that the particle was gaining energy over the course of the measurement.

The amplitude of the axial motion of the particle can actually be seen to increase slightly in time, indicating the presence of external sources of noise adding energy to the system. These sources may include vibrations, since only the axial tilt control was activated without any transverse tilt stabilization.

The time-domain data is plotted in Figure 163 for this data set, along with the estimated energy change (scaled by assuming that the maximum potential energy was given by $\frac{1}{2}m\omega_z^2 z^2$

and equal to the maximum kinetic energy.

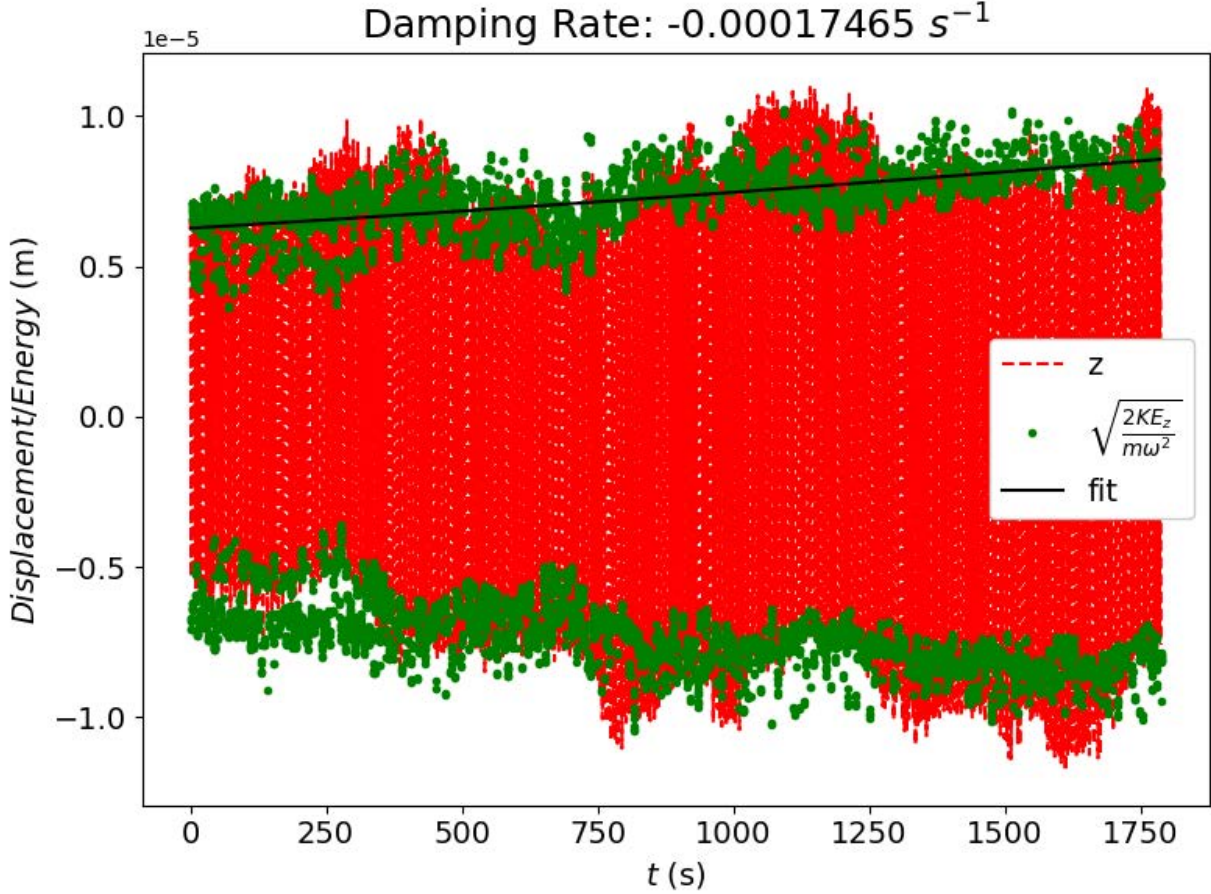


Figure 163: This figure was generated by `damping_processing_blue.py` from the file `0_amplitude_blue/displacements_offline_04.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.044^\circ$. A plot of the time-domain displacements for low-amplitude ($z_{rms} = 5.9 \mu\text{m}$, $y_{rms} = 0.13 \mu\text{m}$) axial oscillations with an estimate of the trend for the amplitude of the motion calculated from the kinetic energy of the particle over time. Discrepancies between the estimated amplitudes and the actual amplitudes are likely due to the rough fit to the particle velocities shown in Figure 161.

The next set of data was primarily collected to analyze the particle's vertical motion. With the axial motion cooled, the vertical motion was excited by striking the top of the chamber with a rubber mallet. This excited the vertical motion of the particle enough to observe it ringdown (Figure 164 shows the ASD of these data). The tilt stabilization was not allowed to equilibrate in this case due to the rapid damping time of the vertical motion.

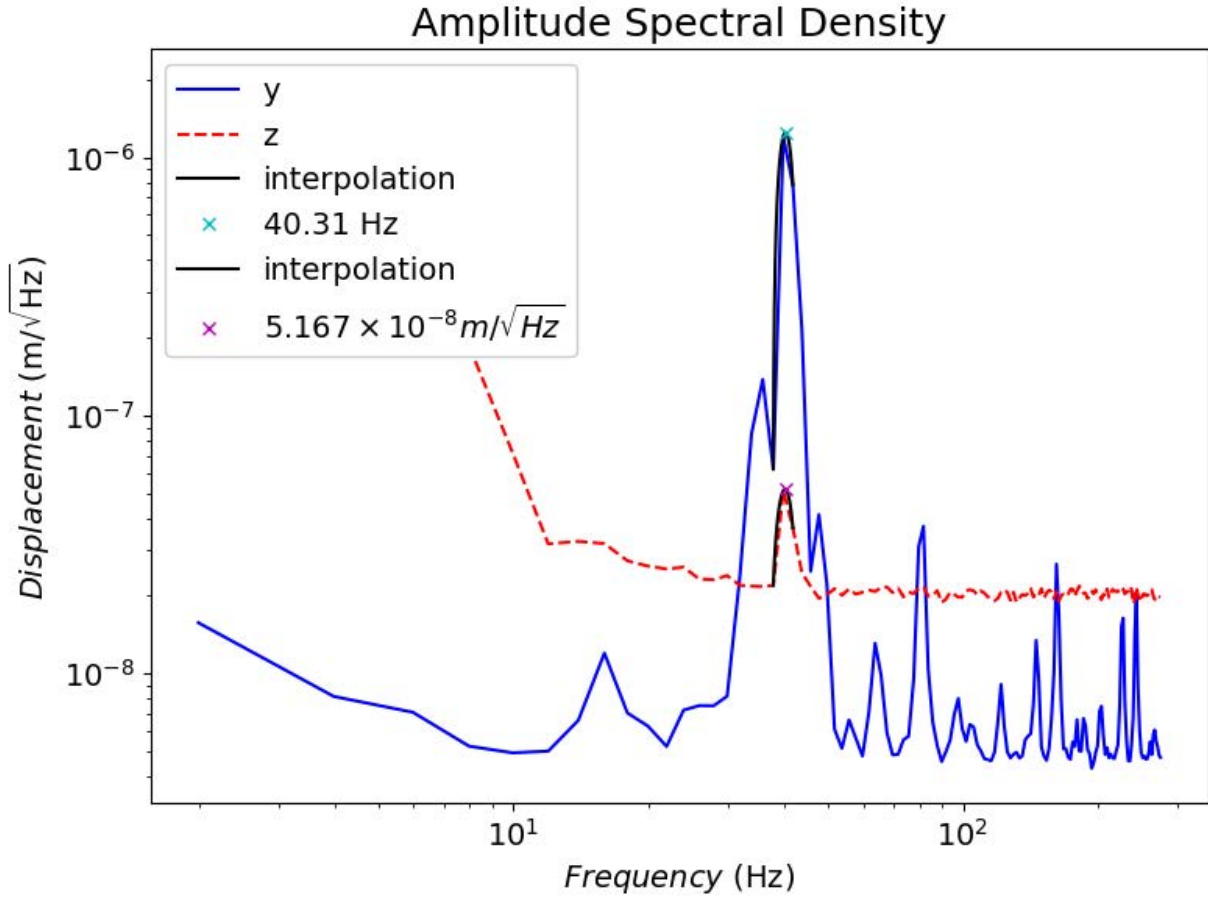


Figure 164: This figure was generated by `damping_processing_blue.py` from the file `y_ringdown_blue/displacements_offline_07.zia`. Frame rate: 550 fps, Pressure: 3.6×10^{-8} Torr. Amplitude spectral density of the displacements of a particle after the chamber was struck by a rubber mallet. Parabolic interpolation used to find peak frequencies.

The frequency measured in Figure 164 is the primary indicator used to determine what each frequency appearing in Figure 158 corresponds to; since the transverse frequency is expected to be lower than the vertical frequency and the vertical frequency is clearly around 40 Hz, it follows that the transverse frequency corresponds to the peaks near 29 Hz.

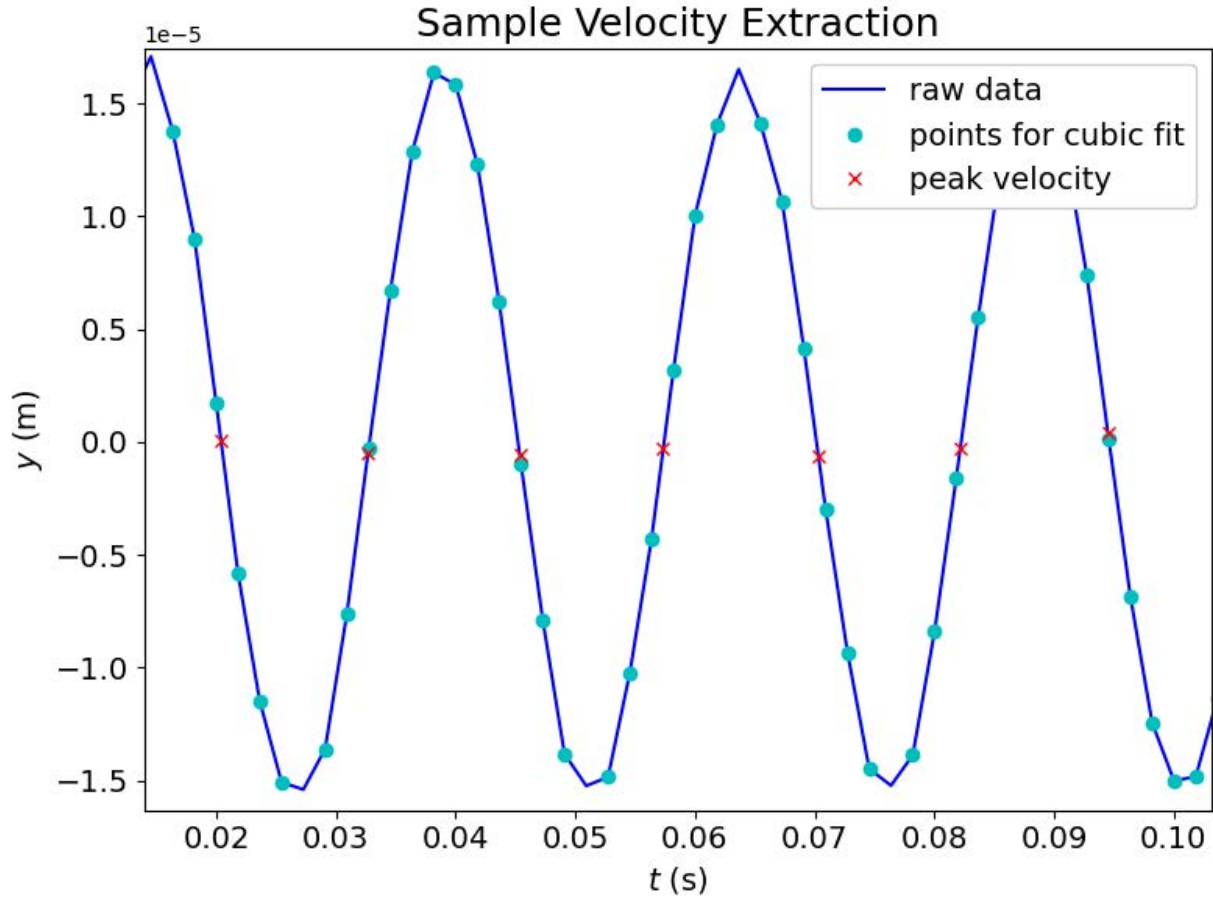


Figure 165: This figure was generated by `damping_processing_blue.py` from the file `y_ringdown_blue/displacements_offline_07.zia`. Frame rate: 550 fps, Pressure: 3.6×10^{-8} Torr. Peak velocity estimation for the vertical motion of a particle after the chamber was struck by a rubber mallet. Exactly 6 points were used in the fit at each zero-crossing.

The same cubic fit to differentiation technique was used to extract peak velocities for this data, except that due to the higher oscillation frequency, only 6 data points could be used in each fit, as shown in Figure 166.

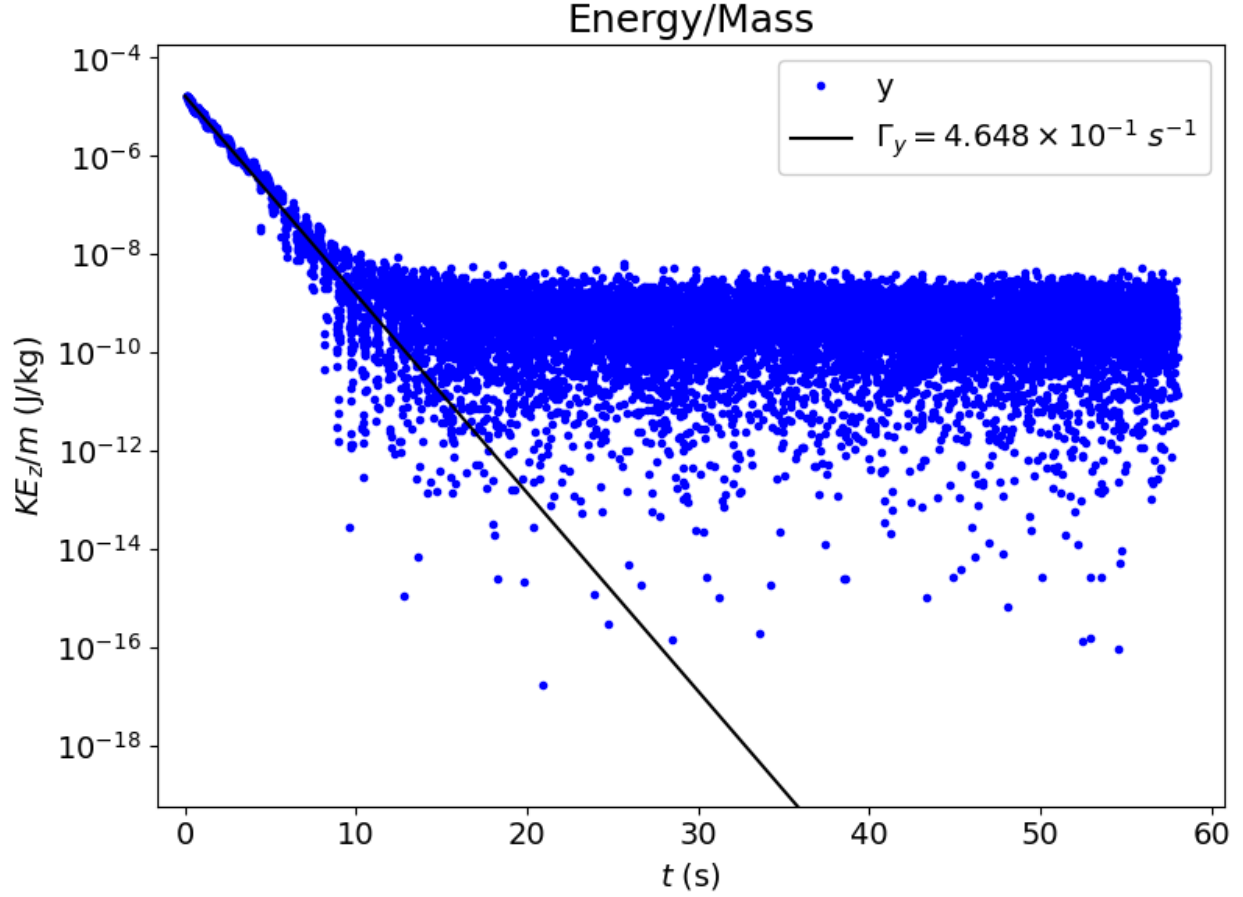


Figure 166: This figure was generated by `damping_processing_blue.py` from the file `y_ringdown_blue/displacements_offline_07.zia`. Frame rate: 550 fps, Pressure: 3.6×10^{-8} Torr. Translational kinetic energy calculated from the peak velocities for the vertical motion of a particle after the chamber was struck by a rubber mallet. Note that after about 20 seconds the kinetic energy of the particle can no longer be measured, indicating vertical motion that is effectively completely damped.

The peak velocities were once again used to estimate the damping rate, in this case about 0.46 s^{-1} . Using Eq. (80), if we assume the vertical damping is due to eddy currents (a reasonable assumption given that no other effect is likely to have such a strong damping effect), we can estimate that the axial eddy current damping rate should be about $1.9 \times 10^{-8} \text{ s}^{-1}$.

The time-domain displacement data are once again displayed in Figure 167. In this

case, the enveloping function fits the data quite well, due to the smoother velocity fit and straightforward ringdown motion.

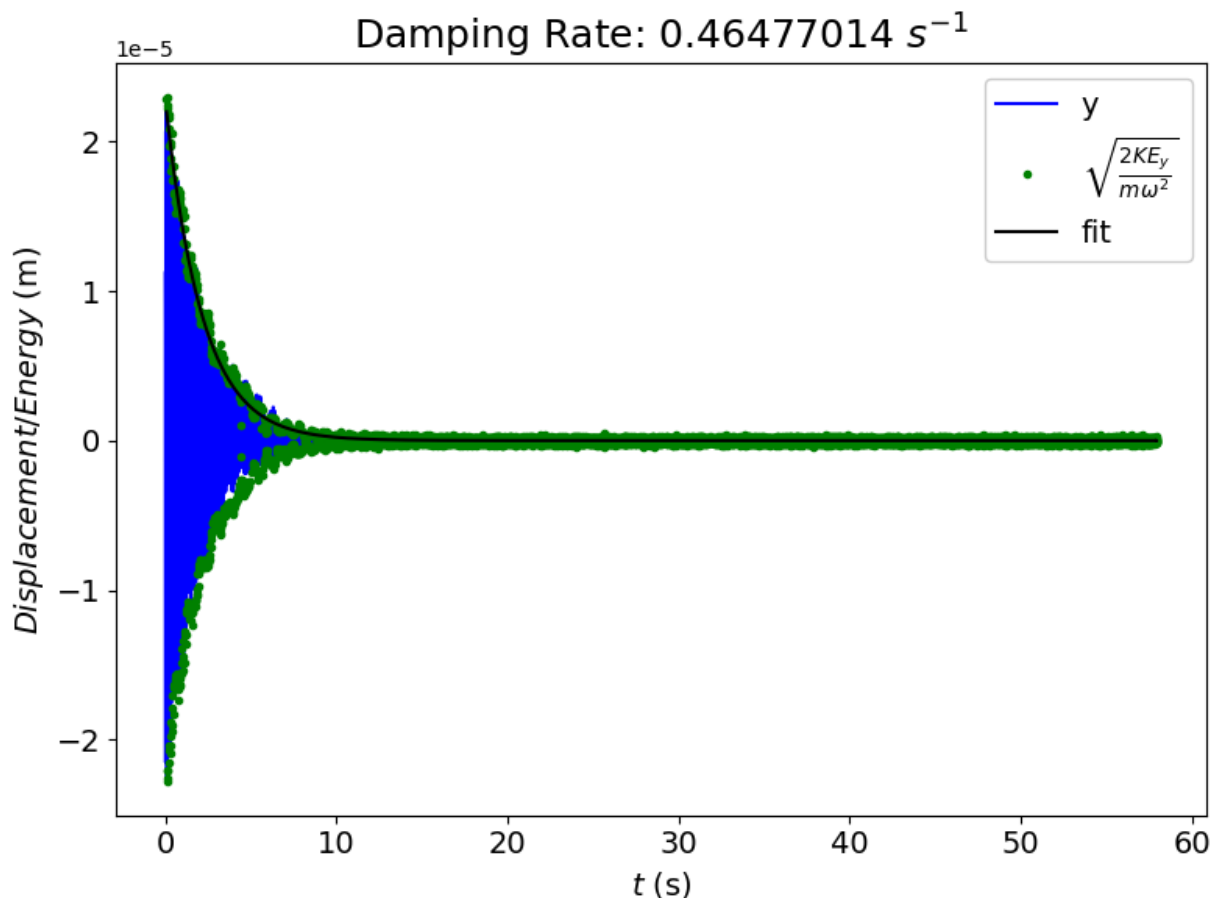


Figure 167: This figure was generated by `damping_processing_blue.py` from the file `y_ringdown_blue/displacements_offline_07.zia`. Frame rate: 550 fps, Pressure: 3.6×10^{-8} Torr. A plot of the time-domain displacements for the vertical motion of a particle after the chamber was struck by a rubber mallet, with an estimate of the trend for the amplitude of the motion calculated from the kinetic energy of the particle over time.

The next sets of data considered are motion with a high axial amplitude, excited by pushing on the chamber by hand. Once again the axial tilt stabilization was allowed to equilibrate after the excitation was complete. Unsurprisingly, the ASD shown in Figure 168 has higher peaks with more harmonics than for the low-amplitude data.

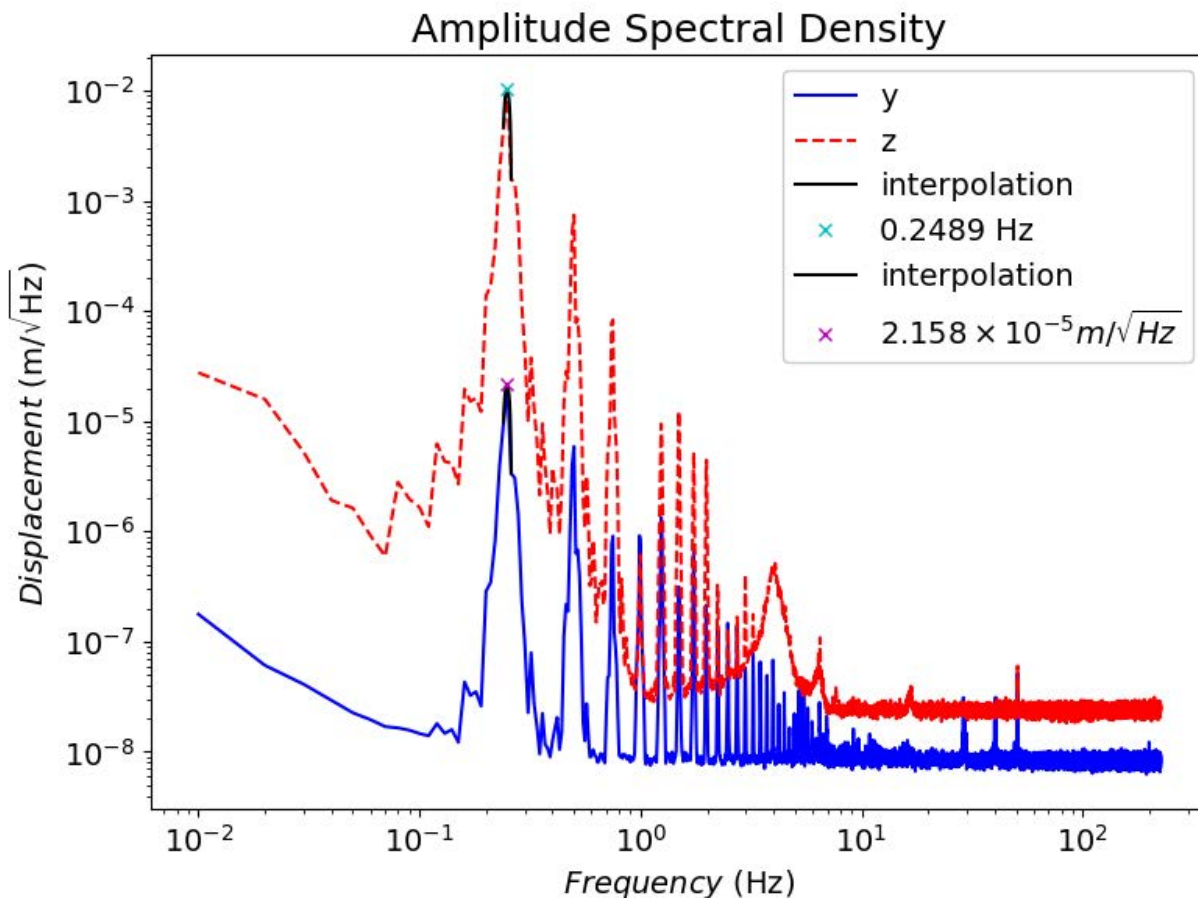


Figure 168: This figure was generated by `damping_processing_blue.py` from the file `z_ringdown_blue/displacements_offline_01.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.120^\circ$. Unrotated amplitude spectral density of the displacements of a particle with high amplitude ($z_{rms} = 1.16$ mm, $y_{rms} = 2.59$ μm) oscillations. Parabolic interpolation used to find peak frequencies.

We also note that there are now two broad peaks between the axial and transverse oscillation frequencies: one large peak at about 4 Hz and one smaller peak around 6 Hz. Once again the camera tilt angle was estimated and used to rotate the data, resulting in the data shown in Figure 169.

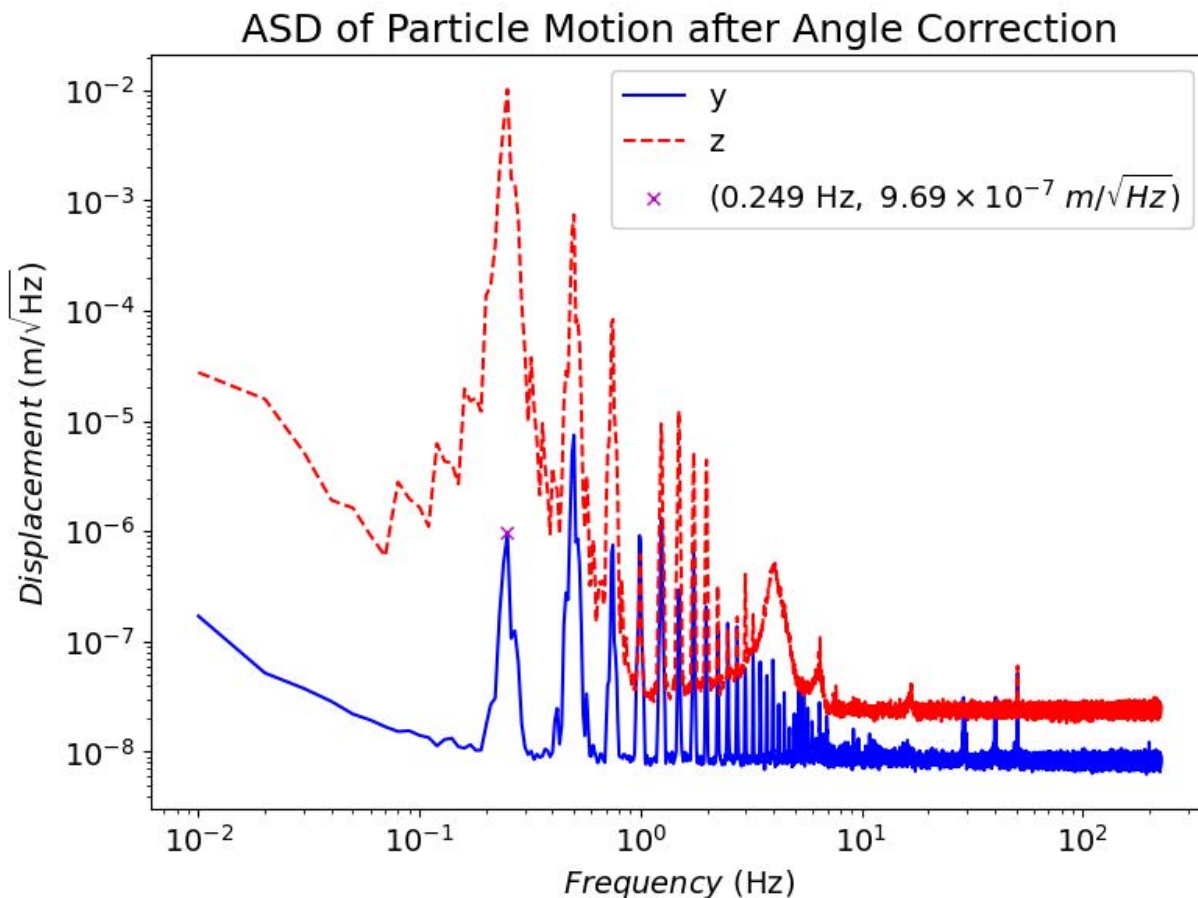


Figure 169: This figure was generated by `damping_processing_blue.py` from the file `z_ringdown_blue/displacements_offline_01.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.120^\circ$. Rotated amplitude spectral density of the displacements of a particle with high amplitude ($z_{rms} = 1.16$ mm, $y_{rms} = 2.59$ μ m) oscillations. Note how much lower the amplitude of the vertical noise is at the axial oscillation frequency after rotation.

In this case the effect of rotating the data by the specified angle is dramatic, resulting in a factor of 22 difference between the amplitude of the particle's vertical motion at the axial frequency with and without angle adjustment. These data strongly suggest that it is worth using this or a similar technique to mechanically adjust the chamber-camera tilt to minimize this effect in order to properly level the system.

The same method described previously was used to extract peak velocities from the data, as depicted in Figure 170, and this time the fit was quite robust, using many points

and containing unambiguous zero-crossings.

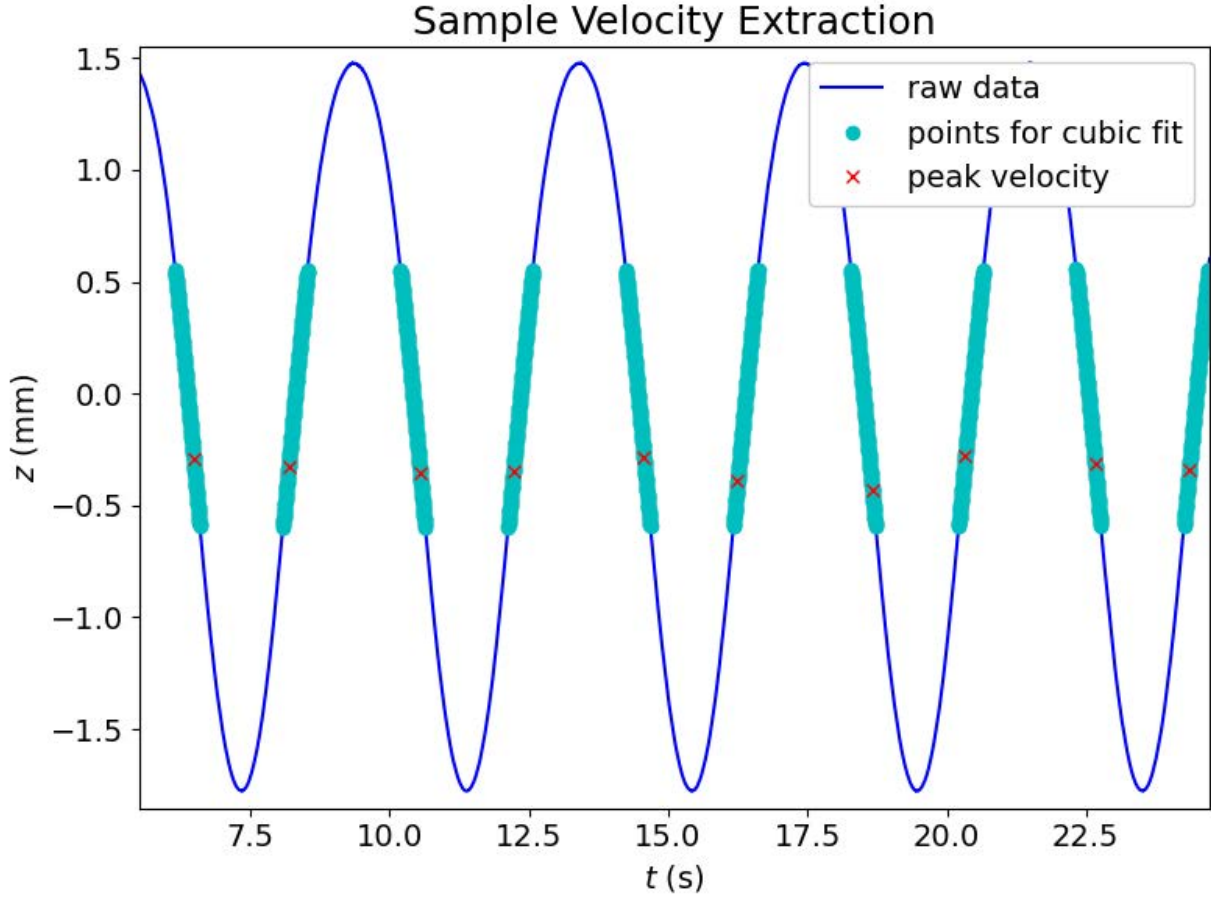


Figure 170: This figure was generated by `damping_processing_blue.py` from the file `z_ringdown_blue/displacements_offline_01.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.120^\circ$. Peak velocity estimation for high-amplitude ($z_{rms} = 1.16$ mm, $y_{rms} = 2.59$ μ m) axial oscillations.

The resultant velocities are used to estimate the exponential decay of the particle's energy, as shown in Figure 171.

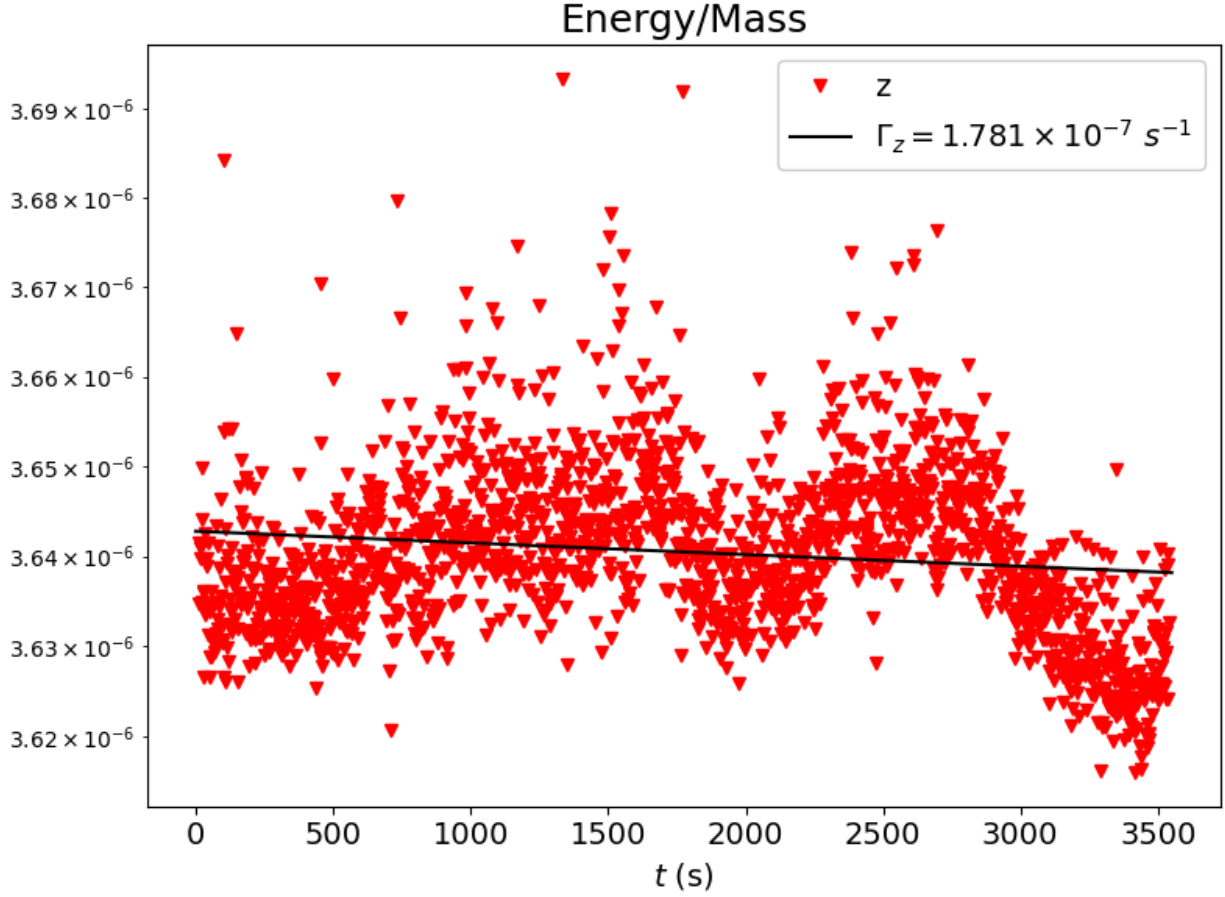


Figure 171: This figure was generated by `damping_processing_blue.py` from the file `z_ringdown_blue/displacements_offline_01.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.120^\circ$. Translational kinetic energy calculated from the peak velocities for high-amplitude ($z_{rms} = 1.16$ mm, $y_{rms} = 2.59$ μ m) axial oscillations.

The energy of the particle appears to be more dependent on variations on the scale of many minutes rather than being characterized by steady damping over the course of the measurement.

Once again the time-domain data is plotted next to the energy enveloping function in Figure 172. In this case, the extreme anharmonicity of the potential well makes potential energy estimated for a harmonic oscillator a poor fit to the data, where the time-averaged equilibrium position does not align with the center-point between the axial displacement minimum and maximum.

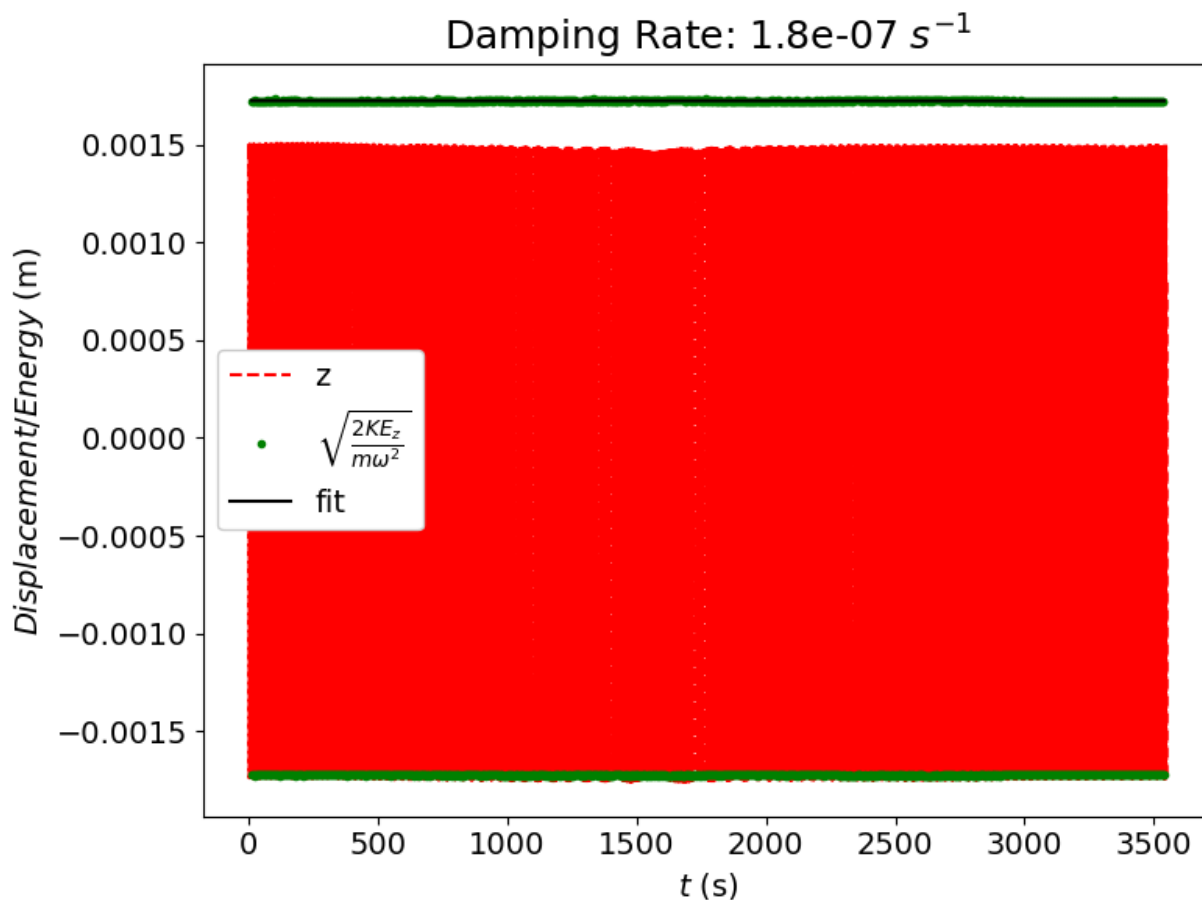


Figure 172: This figure was generated by `damping_processing_blue.py` from the file `z_ringdown_blue/displacements_offline_01.zia`. Frame rate: 450 fps, Pressure: 3.3×10^{-8} Torr, $\theta = -0.120^\circ$. A plot of the time-domain displacements for high-amplitude ($z_{rms} = 1.16$ mm, $y_{rms} = 2.59$ μ m) axial oscillations with an estimate of the trend for the amplitude of the motion calculated from the kinetic energy of the particle over time. Discrepancies between the estimated amplitudes and the actual amplitudes are likely due to the anharmonicity of the potential.

It was noted that the particle did not always exhibit the same behavior between data collections. As can be seen by examining two additional data sets taken under very similar conditions (shown in Figure 173, the sign and magnitude of the particle's effective damping vary greatly from measurement to measurement.

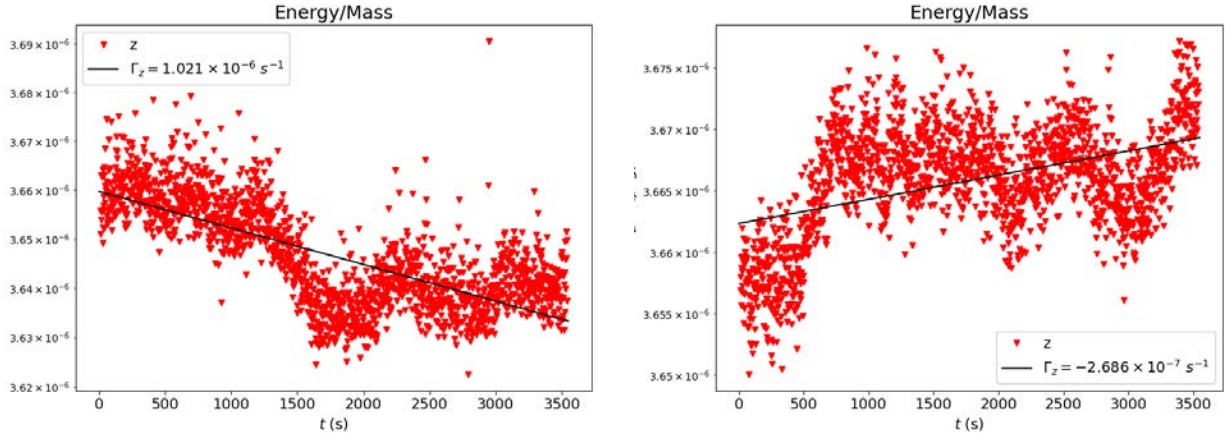


Figure 173: This figure was generated by `damping_processing_blue.py` from the files `z_ringdown_blue/displacements_offline_02.zia` and `z_ringdown_blue/displacements_offline_03.zia`. Frame rate: 450 fps, Pressure: 4.3×10^{-8} Torr (02) and 3.3×10^{-8} Torr (03), $\theta = -0.118^\circ$ (02) and -0.125° (03). Translational kinetic energy calculated from the peak velocities for high-amplitude axial oscillations.

Similar data, taken at a lower frame rate but otherwise under much the same conditions, were collected over a 24 h time period. The raw ASD for these data is shown in Figure 174.

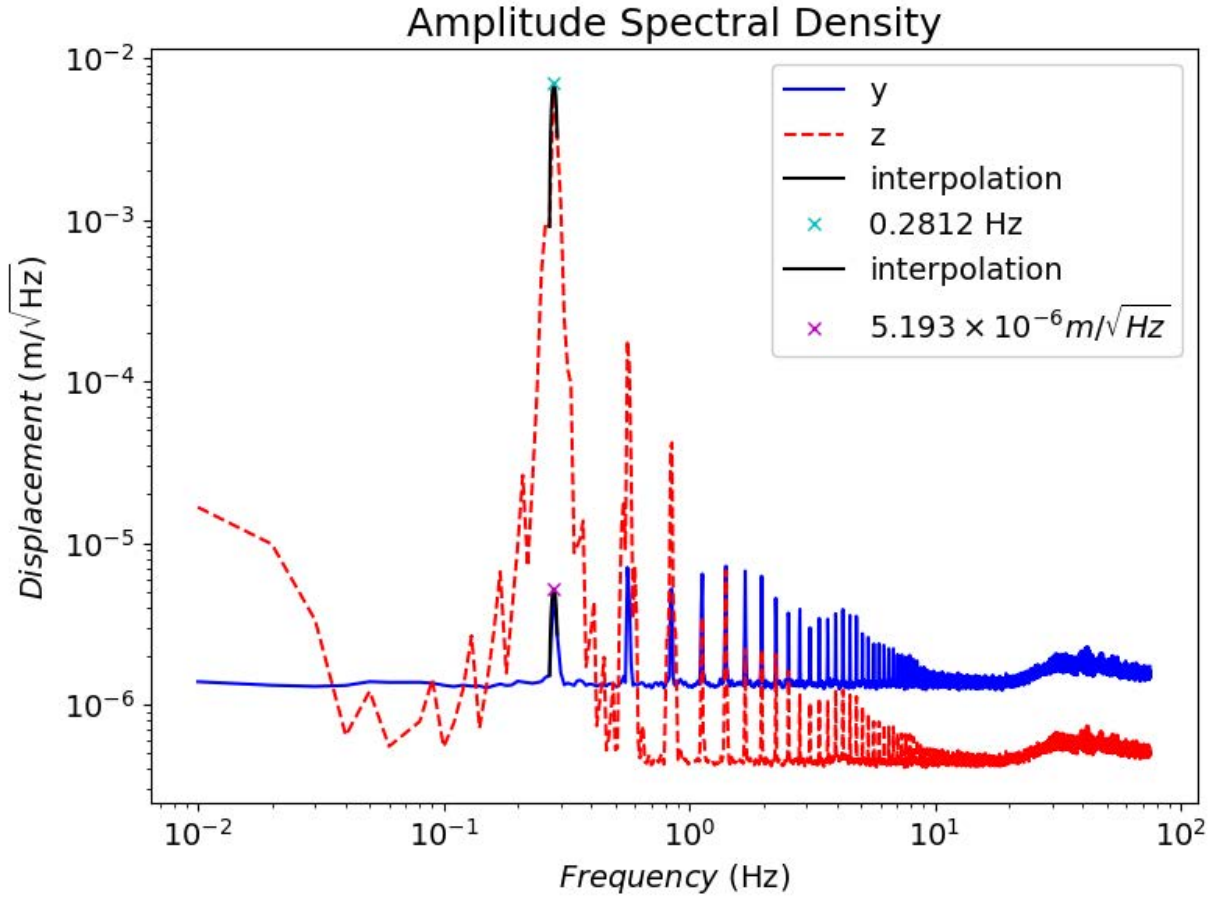


Figure 174: This figure was generated by `damping_processing_blue.py` from the file `24hr_ringdown_blue/displacements_offline_01.zia`. Frame rate: 150 fps, Pressure: 5.2×10^{-8} Torr, $\theta = -0.005^\circ$. Unrotated amplitude spectral density of the displacements of a particle with high amplitude ($z_{rms} = 0.79$ mm) oscillations. Parabolic interpolation used to find peak frequencies.

At such a low framerate it is difficult to distinguish the higher oscillation frequencies. Additionally, the effect of rotating the data by the optimized angle θ was very small in this case, as can be seen in Figure 175. This may indicate a change in the experimental setup that corrected for previous perturbations.

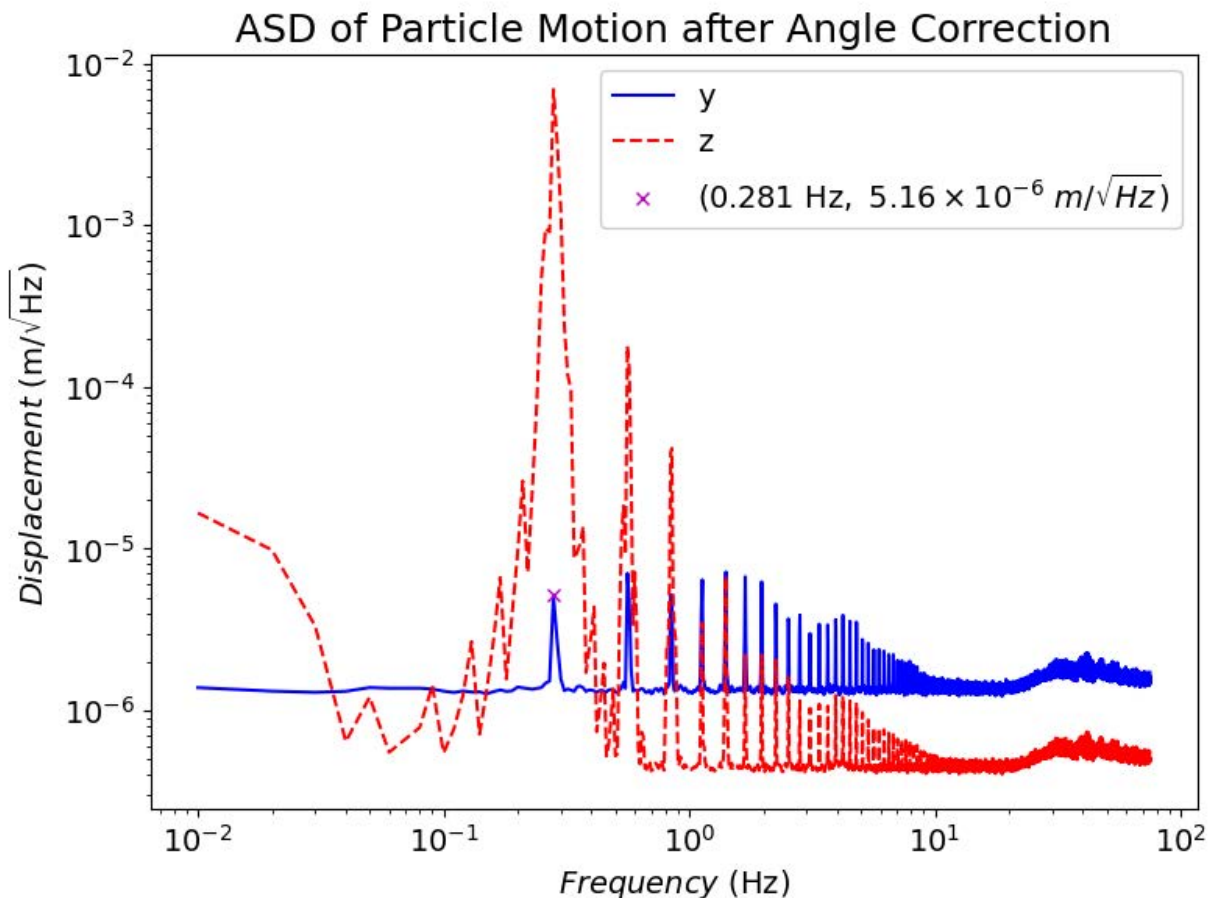


Figure 175: This figure was generated by `damping_processing_blue.py` from the file `24hr_ringdown_blue/displacements_offline_01.zia`. Frame rate: 150 fps, Pressure: 5.2×10^{-8} Torr, $\theta = -0.005^\circ$. Rotated amplitude spectral density of the displacements of a particle with high amplitude ($z_{rms} = 0.79$ mm) oscillations. Note how much lower the amplitude of the vertical noise is at the axial oscillation frequency after rotation.

The peak velocities were computed in the same manner as before, as shown in Figure 176.

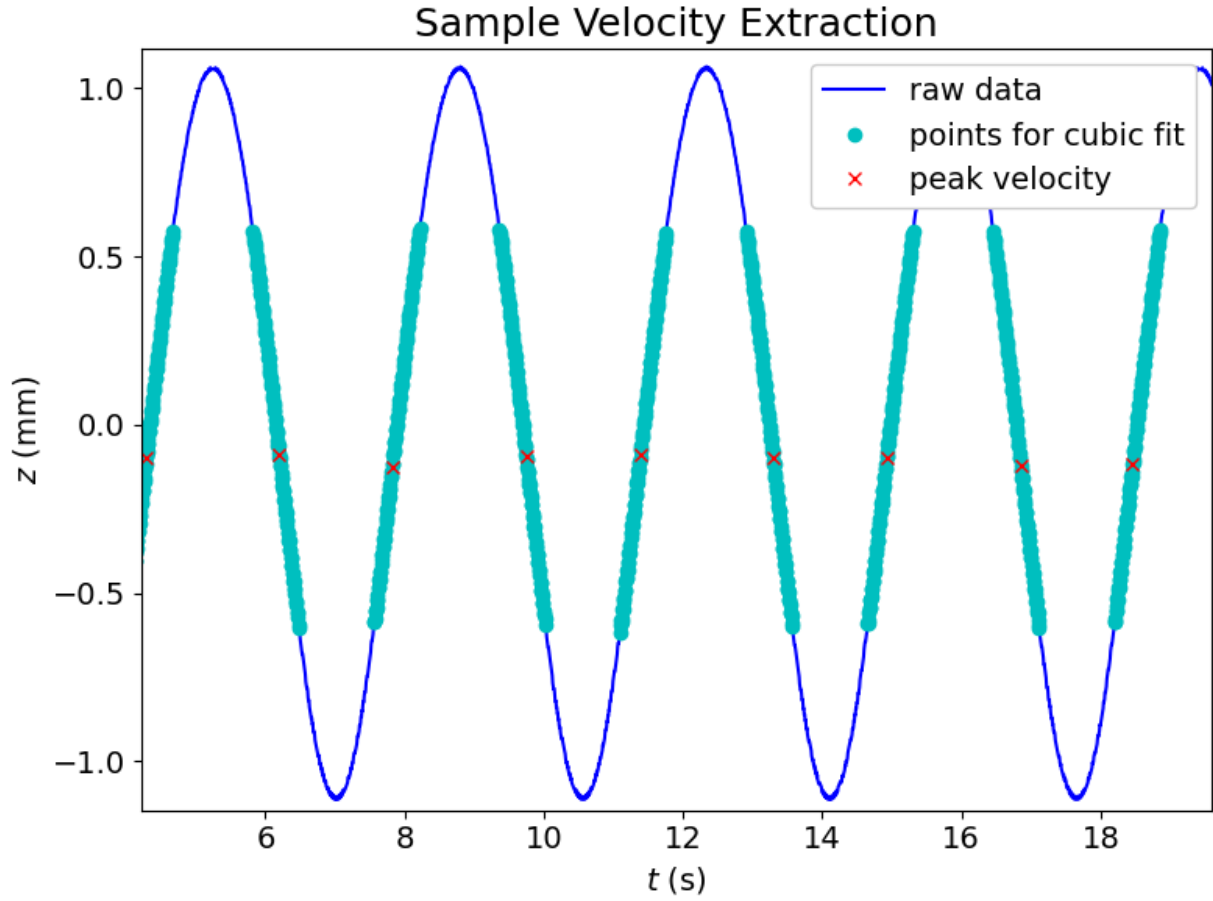


Figure 176: This figure was generated by `damping_processing_blue.py` from the file `24hr_ringdown_blue/displacements_offline_01.zia`. Frame rate: 150 fps, Pressure: 5.2×10^{-8} Torr, $\theta = -0.005^\circ$. Peak velocity estimation for high-amplitude ($z_{rms} = 0.79$ mm) axial oscillations.

In this case the estimated damping rate over 24 hours was seen to increase on average, although a few longer time segments with $\Gamma_z > 0$ can be seen in Figure 177.

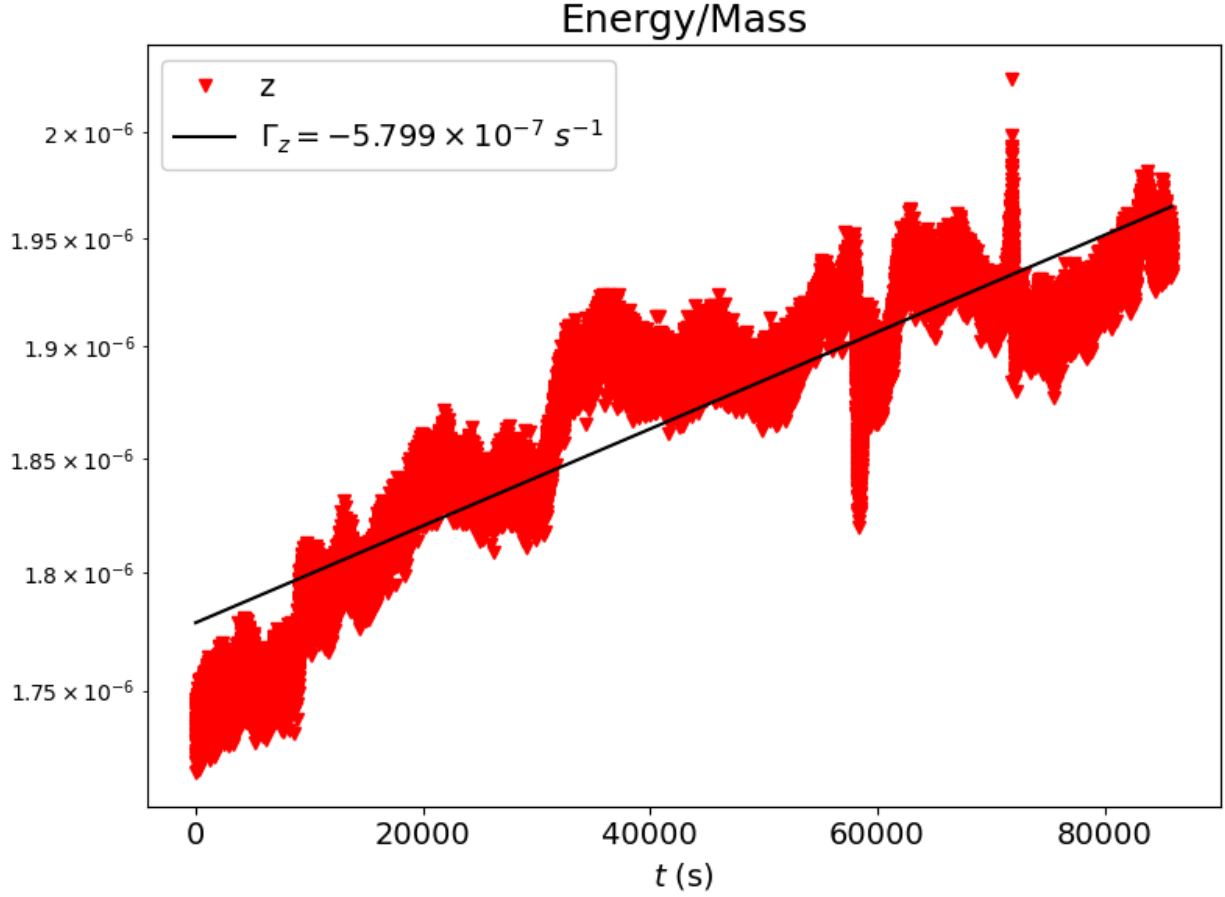


Figure 177: This figure was generated by `damping_processing_blue.py` from the file `24hr_ringdown_blue/displacements_offline_01.zia`. Frame rate: 150 fps, Pressure: 5.2×10^{-8} Torr, $\theta = -0.005^\circ$. Translational kinetic energy calculated from the peak velocities for high-amplitude ($z_{rms} = 0.79$ mm) axial oscillations.

In this case the enveloping function also fit the displacement data better (Figure 178), likely due to the slightly lower amplitude of oscillation.

The estimate of Γ_z obtained using Eq. (80) in this case yields $\Gamma_z \approx 8.8 \times 10^{-9} \text{ s}^{-1}$ due to the lower amplitude of oscillation compared to the data used to produce Figure 171.

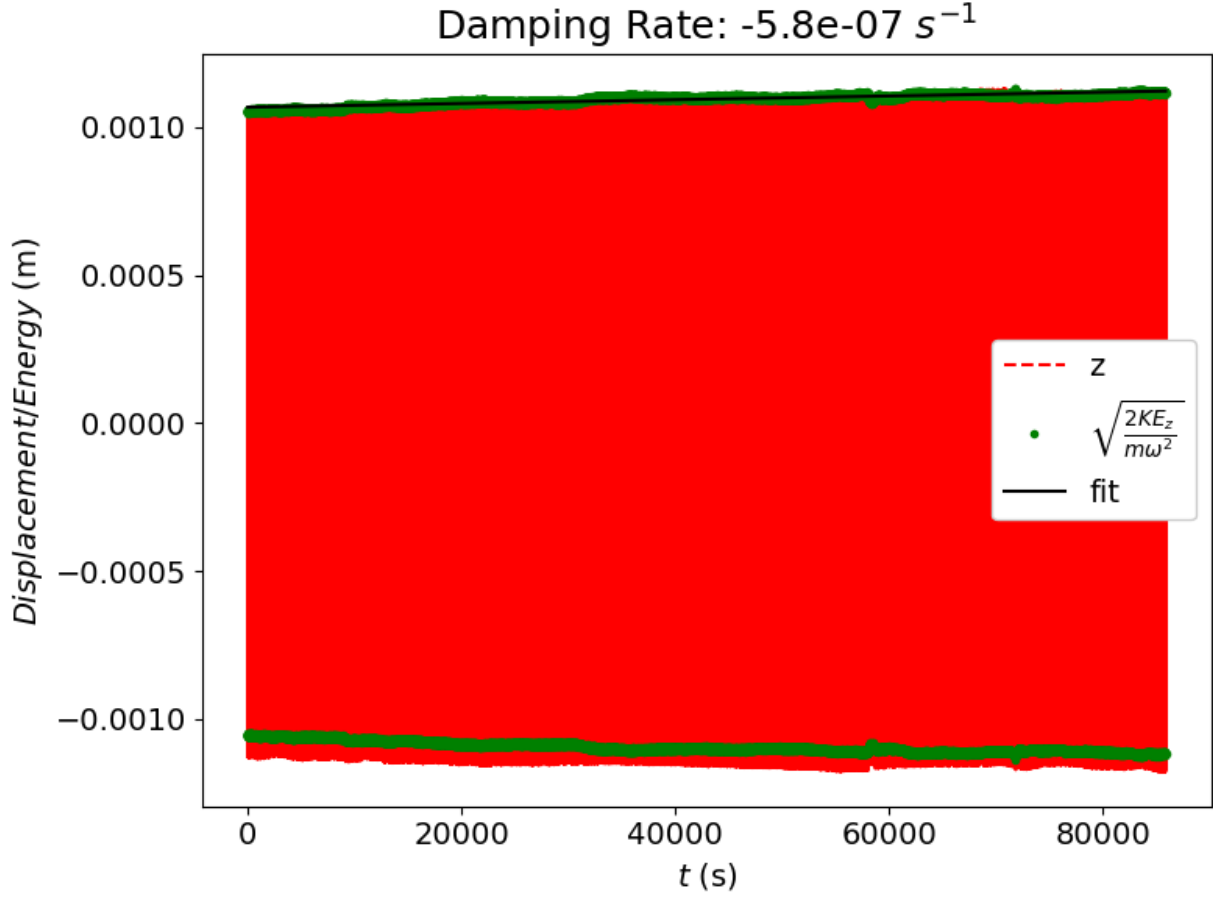


Figure 178: This figure was generated by `damping_processing_blue.py` from the file `24hr_ringdown_blue/displacements_offline_01.zia`. Frame rate: 150 fps, Pressure: 5.2×10^{-8} Torr, $\theta = -0.005^\circ$. A plot of the time-domain displacements for high-amplitude ($z_{rms} = 0.79$ mm) axial oscillations with an estimate of the trend for the amplitude of the motion calculated from the kinetic energy of the particle over time. Discrepancies between the estimated amplitudes and the actual amplitudes are likely due to the anharmonicity of the potential.

These results strongly imply two conclusions about the system. First, the system is not well-isolated from the environment, allowing for energy to be added to the system over a timespan of several hours to days. Second, any damping that is present in the system is too small to be directly measured at present.

The results shown in Figure 153 combined with the estimated eddy current damping rate might suggest that, for the pressures used for the measurements discussed, gas damping

is likely to dominate over eddy-current damping. However, it is possible that actual eddy current damping rates may exceed what is predicted in theory due to coupling between either transverse/vertical translational motion or librations about the transverse or vertical axes.

TOLERANCE CALCULATIONS FOR A MEASUREMENT OF G

There are several important factors to consider when determining what is required to take an accurate measurement of G . Specifically, there are two primary sources of concern when it comes to taking an accurate measurement.

First, any parameters used in the calculation of G , such as the mass of the field masses M , must be known to within a certain tolerance depending on the acceptable uncertainty in the measurement. When considering these effects, we will focus on what is required to match the current lowest reported uncertainty of about 10 ppm [85], and what would be required to beat it by a factor of 10 to get about 1 ppm.

Second, the time-of-swing method relies on the assumption that the only difference to the oscillation frequency of the test mass that occurs due to the field masses being removed is due to the gravitational force they exert on the particle. Therefore, any effect besides the gravitational attraction of the field masses that is present when they are nearby but is not present when they are removed (or vice versa) must be considered a potential source of systematic error.

Figure 179 shows the basic dimensions of the system as it is being constructed in this lab. Field masses of two materials are considered: K9 glass spheres due to the readily available supply and low cost, and tungsten spheres for their high density.

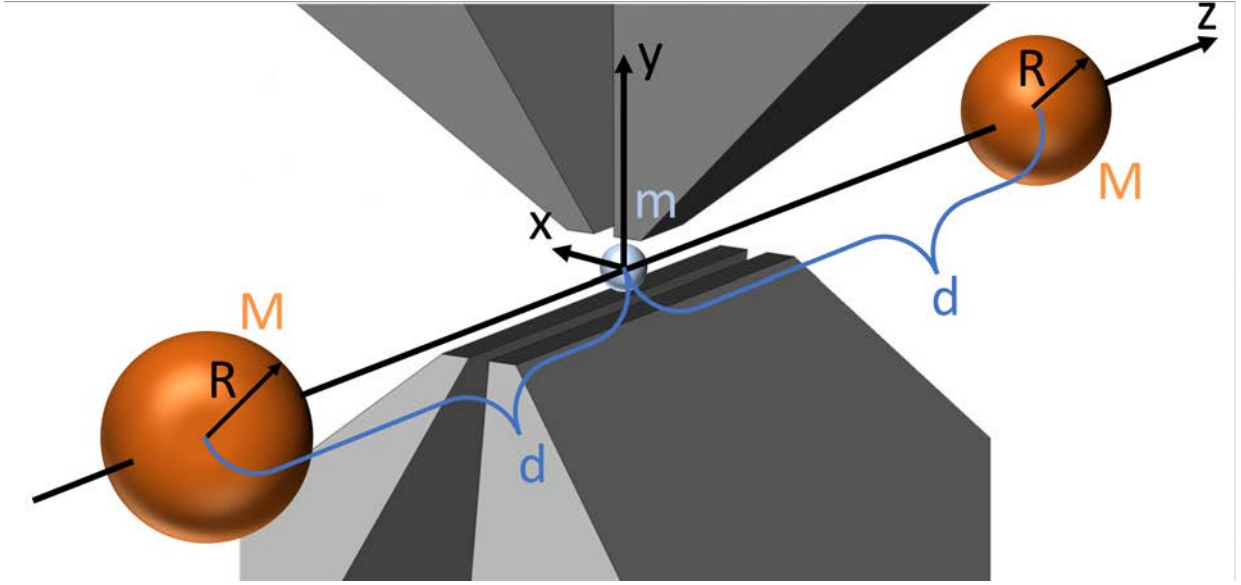


Figure 179: Illustration of the trap geometry and orientation relative to the two spherical field masses (orange). The pole pieces shown here are exaggerated in their dimensions.

Calculations

Any two forces acting on a test mass that obey Hooke's law can be expressed by a single force with a spring constant equal to the sum of the spring constants of the individual forces. It is therefore convenient to express different effects as effective spring constants, so that they can be summed arithmetically.

Particle Offset

Considering test masses alone, the gravitational potential experienced by a trapped particle is given by

$$U_G(x, y, z) = -\frac{GMm}{(x^2 + y^2 + (z - d)^2)^{1/2}} - \frac{GMm}{(x^2 + y^2 + (z + d)^2)^{1/2}}. \quad (85)$$

We use $F_z = -\frac{\partial U}{\partial z}$ to find the force in the z -direction,

$$F_z(x, y, z) = -\frac{GMm(z-d)}{(x^2 + y^2 + (z-d)^2)^{3/2}} - \frac{GMm(z+d)}{(x^2 + y^2 + (z+d)^2)^{3/2}}, \quad (86)$$

and $k_z = -\frac{\partial F_z}{\partial z}$ to find the effect of the test masses on the spring constant. Let $r_- = (x^2 + y^2 + (z-d)^2)^{1/2}$ and $r_+ = (x^2 + y^2 + (z+d)^2)^{1/2}$. Then we have

$$k_{Gz}(x, y, z) = GMm \left(\frac{1}{r_-^3} + \frac{1}{r_+^3} - 3\frac{(z-d)^2}{r_-^5} - 3\frac{(z+d)^2}{r_+^5} \right), \quad (87)$$

which we can simplify in the limits where $z \ll d$ or $x \ll d$.

For small displacements of the field masses δx and δz we have

$$\begin{aligned} k_{Gz}(0, 0, \delta z) &= -2GMm \left(\frac{1}{r_-(0, 0, \delta z)^3} + \frac{1}{r_+(0, 0, \delta z)^3} \right) \\ &= -\frac{2GMm}{d^3} \left[\left(1 - \frac{\delta z}{d} \right)^{-3} + \left(1 + \frac{\delta z}{d} \right)^{-3} \right] \\ &\approx -\frac{4GMm}{d^3} \left(1 + 6 \left(\frac{\delta z}{d} \right)^2 \right) \end{aligned} \quad (88)$$

and

$$\begin{aligned} k_{Gz}(\delta x, 0, 0) &= GMm [2(\delta x^2 + d^2)^{-3/2} - 6d^2(\delta x^2 + d^2)^{-5/2}] \\ &\approx -\frac{4GMm}{d^3} \left(1 - 3 \left(\frac{\delta x}{d} \right)^2 \right). \end{aligned} \quad (89)$$

Note that the case where $\delta x \ll d$ is identical at lowest order to the case when the trap is misaligned by an angle $\theta = \frac{\delta x}{d}$.

Field Mass Offset

If we only offset one mass along the x -axis by a distance dx , we instead come up with

$$k_{Gz}(0, 0, 0)_{dx \ll d} \approx -\frac{4GMm}{d^3} \left(1 - \frac{3}{2} \left(\frac{dx}{d} \right)^2 \right). \quad (90)$$

If, however, the masses are not spaced a distance $2d$ apart, but rather $2d + dz$, then we have:

$$k_{Gz}(0, 0, 0)_{dz \ll d} \approx -\frac{4GMm}{d^3} \left(1 - \frac{3}{2} \frac{dz}{d} \right). \quad (91)$$

Other masses

For a spherical mass located at position $(x_0, 0, 0)$ or $(0, 0, z_0)$ with mass M' , we have the following effective spring constants:

$$k_{Gz}(0, 0, 0)_{M' \rightarrow x_0} = -\frac{4GMm}{d^3} + \frac{GM'm}{x_0^3} \quad (92)$$

and

$$k_{Gz}(0, 0, 0)_{M' \rightarrow z_0} = -\frac{4GMm}{d^3} - \frac{2GM'm}{z_0^3}. \quad (93)$$

A Single Ellipsoidal Field Mass

If one field mass is ellipsoidal such that

$$x = as \sin \theta \cos \phi \quad (94)$$

$$y = as \sin \theta \sin \phi \quad (95)$$

$$z = bs \cos \theta \quad (96)$$

$$(97)$$

with $b = a + ds$ and $a^2b = R^3$, then the quadrupole term of the potential is given by

$$U_{quad}(x=0, y=0, z) = -\frac{Gm(\hat{r} \cdot \mathbb{Q} \cdot \hat{r})}{r^3} = -\frac{2GMm(b^2 - a^2)}{5(z-d)^3} \approx -\frac{4GMmRds}{5(z-d)^3}, \quad (98)$$

where the quadrupole moment of the ellipsoid is given by

$$\mathbb{Q} = \frac{M}{5} \begin{bmatrix} a^2 - b^2 & 0 & 0 \\ 0 & a^2 - b^2 & 0 \\ 0 & 0 & 2b^2 - 2a^2 \end{bmatrix}. \quad (99)$$

We can then write the spring constant due to field mass ellipticity as

$$k_{Gz}(0, 0, 0)_{e \rightarrow \frac{ds}{R}} = -\frac{4GMm}{d^3} - \frac{48GMmRds}{5d^5}. \quad (100)$$

Note that the eccentricity of the ellipsoid, defined as $e = \sqrt{1 - \frac{a^2}{b^2}}$, reduces to $e = \frac{ds}{R}$ in the limit that $ds \ll R$.

Tidal Deformation

When the sun and the moon align to pull on the earth from the same direction, this is known as a Spring Tide. When this occurs with the moon and sun at Perigee (closest approach), it is known as a King Tide. Not only does this effect the water, it also deforms the Earth, thereby impacting its gravitational field.

To represent the most extreme case of this, we consider two different arrangements which result in the greatest difference to the spring constant of the trap, shown in Figure 180.

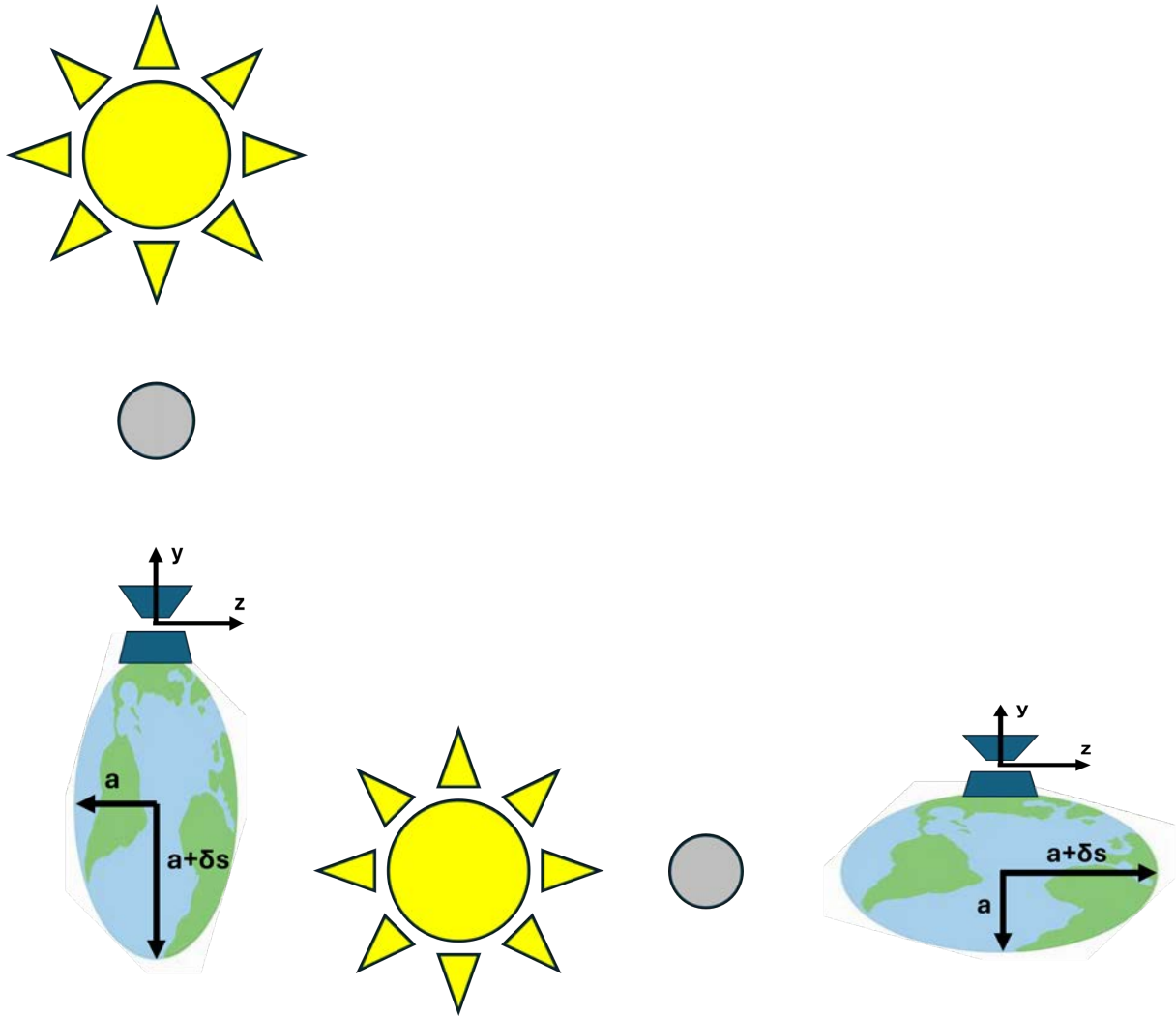


Figure 180: Illustration of the two contrasting positions of the moon and sun relative to the trap geometry that result in the greatest difference in the spring constant of the trap.

The effect of the tidal deformation of the Earth can be estimated using the Love number, $k_2 \approx 0.308$, associated with the correction to an external tidal potential applied to the Earth by the Sun and the Moon. That is, the external potential is approximately given by

$$U_{tidal} = (1 + k_2) \left(\frac{GM_{\odot}}{R_{\oplus,\odot}} + \frac{GM_{\zeta}}{R_{\oplus,\zeta}} \right), \quad (101)$$

where $M_{\odot} = 1.988 \times 10^{30}$ kg, $M_{\zeta} = 7.347 \times 10^{22}$ kg, $R_{\oplus,\odot} = 1.47 \times 10^{11}$ m, $R_{\oplus,\zeta} = 3.57 \times 10^8$ m.

In one case this leads to the new spring constant

$$k_{Gz}(0, 0, 0)_{z,tidal} = -\frac{4GMm}{d^3} - 2(1 + k_2)Gm \left(\frac{M_{\odot}}{R_{\oplus,\odot}^3} + \frac{M_{\zeta}}{R_{\oplus,\zeta}^3} \right), \quad (102)$$

and in the other case produces

$$k_{Gz}(0, 0, 0)_{x,tidal} = -\frac{4GMm}{d^3} + (1 + k_2)Gm \left(\frac{M_{\odot}}{R_{\oplus,\odot}^3} + \frac{M_{\zeta}}{R_{\oplus,\zeta}^3} \right). \quad (103)$$

The worst case scenario occurs when data is collected on two different days where the Sun and the Moon have competing effects. Therefore we consider the worst possible measurement error from this effect to be:

$$\varepsilon_{tidal} = \frac{k_{Gz}(0, 0, 0)_{z,tidal} - k_{Gz}(0, 0, 0)_{x,tidal}}{k_{Gz}} = -\frac{3(1 + k_2)d^3}{4M} \left(\frac{M_{\odot}}{R_{\oplus,\odot}^3} + \frac{M_{\zeta}}{R_{\oplus,\zeta}^3} \right) = -2.2 \times 10^{-7}. \quad (104)$$

Therefore, we can safely ignore this effect for any measurement attempting no better than 1 ppm accuracy.

Local Tides

Because the precise gravitational field of local tides can be, in principle, quite complicated, a simplified model is used to estimate the impact of local tides on the spring constant of the test mass. To simplify the calculations, local tides are modeled as an area of uniformly perturbed mass bounded by two arcs and two radii as shown in Figure 181, with thickness Δt . The gravitational potential generated by such a distribution of mass in a region $\mathfrak{R}$ is given by

$$\Phi_G(0, 0, z) = Gm \int_{\mathfrak{R}} \frac{\rho(\vec{x}')}{\sqrt{x'^2 + y'^2 + (z - z')^2}} d^3x'. \quad (105)$$

We can solve for k_z by taking two derivatives with respect to z and then setting $z = 0$:

$$\begin{aligned} k_z(0, 0, 0) &= Gm \int_{\mathfrak{R}} \rho(\vec{x}') \left(\frac{1}{(x'^2 + y'^2 + z'^2)^{3/2}} - \frac{3z'^2}{(x'^2 + y'^2 + z'^2)^{5/2}} \right) d^3x' \\ &= Gm\rho_{water}\Delta t \int_R^{\Delta R} \int_{\phi}^{\pi-\phi} \frac{1}{r'^2} (1 - 3\sin^2 \phi') dr' d\phi'. \end{aligned} \quad (106)$$

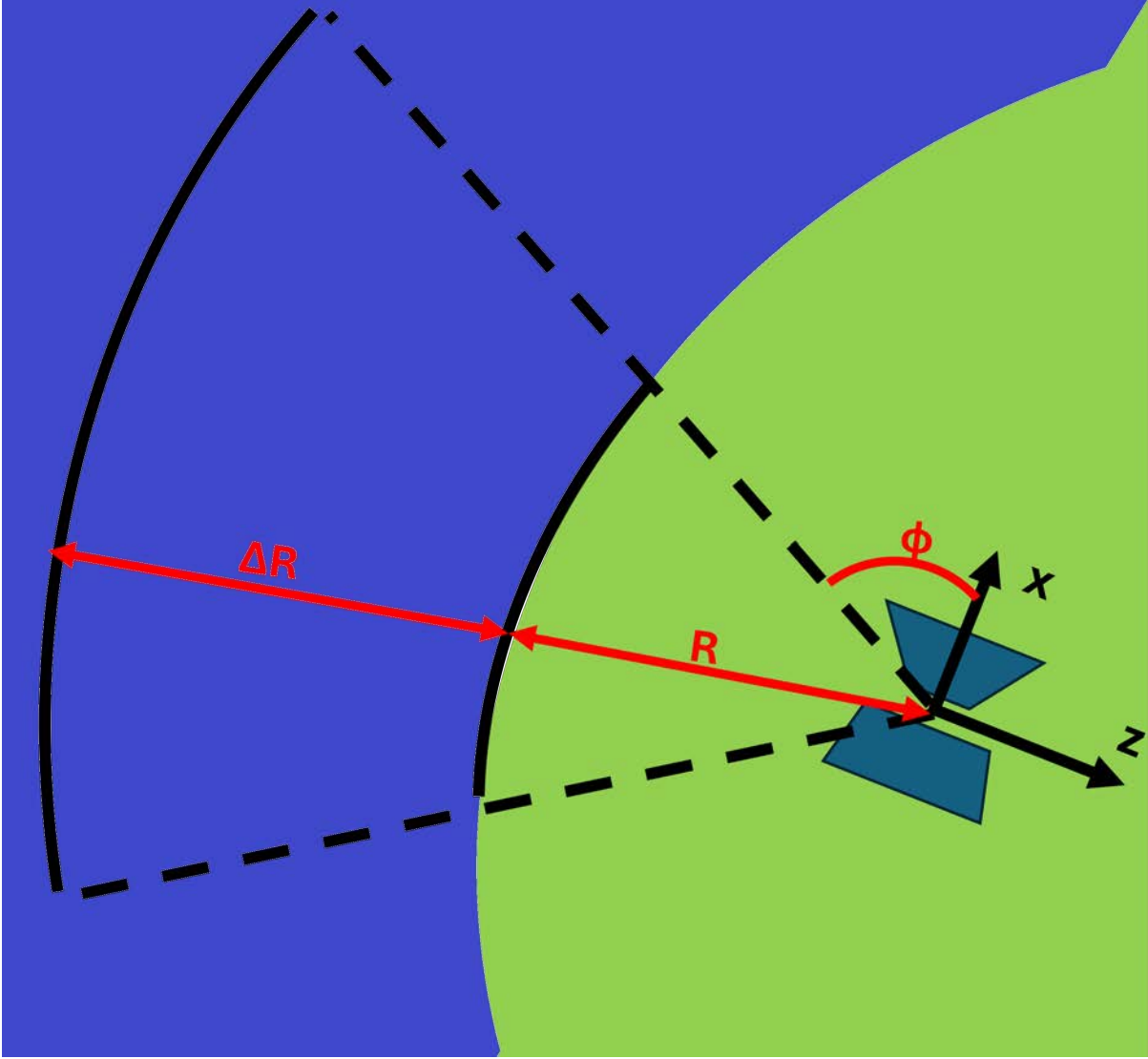


Figure 181: Diagram of the model used to estimate local tides. $\Delta R = 1.0 \times 10^6$ m was chosen to be as large as possible for coastal calculations while still being able to neglect the curvature of the Earth. R and ϕ were considered to be variable, and were used in making the plot shown in Figure 182.

It should be noted that if the trap z -axis is instead oriented parallel to the coast, we instead obtain

$$k_z(0, 0, 0) = Gm\rho_{water}\Delta t \int_R^{\Delta R} \int_{\phi-\frac{\pi}{2}}^{\frac{\pi}{2}-\phi} \frac{1}{r'^2} (1 - 3 \sin^2 \phi') dr' d\phi'. \quad (107)$$

With a little bit of algebra, it can be shown that for the two cases, k_z simplifies down to:

$$k_{z,tides} = -Gm\rho_{water}\Delta t \frac{\Delta R}{R(R + \Delta R)} \left(\frac{\pi}{2} - \phi \pm \frac{3}{2} \sin 2\phi \right), \quad (108)$$

where the $+$ corresponds to the trap orientation shown in Figure 181 with the z -axis perpendicular to the coastline, and the $-$ corresponds to the case when the z -axis of the trap is parallel to the coastline. It is instructive at this point to consider the case when $\frac{k_{z,tides}}{k_{Gz}} < \varepsilon_{0,1}$.

A plot of the minimum required distance from the coast R as a function of ϕ is shown in Figure 182 for all combinations of orientation and tolerance, where $\Delta t = 2.0$ m was used to estimate the typical effect from King Tides on the Pacific coast.

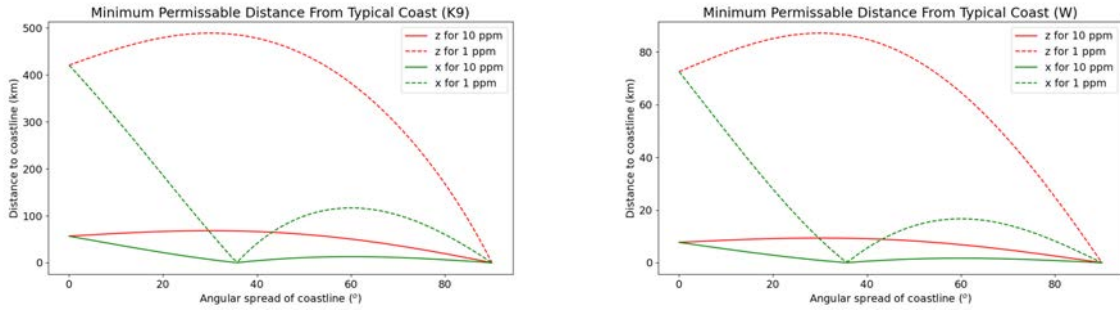


Figure 182: Minimum distance from the Pacific coast during a King tide before tidal considerations become significant to the measurement of G . Tide heights taken to be 2 m, tide extent taken to be 1000 km. Calculations performed considering K9 glass (left) and tungsten (right) field masses.

Figure 182 demonstrates the possible effect of King tides from a typical coastline. It is also worth noting, however, that specific locations (such as the Bay of Fundy in Canada) can have much larger tides, as high as 16 m. Figure 183 illustrates the potential impact of such an effect, using $\Delta R = 100$ km and $\Delta t = 16.0$ m.

Finally, we can model the effect of additional snow banks outside the building using a similar model. For relatively dense settled snow, we can take $\rho_{snow} = 300$ kg/m³. Considering

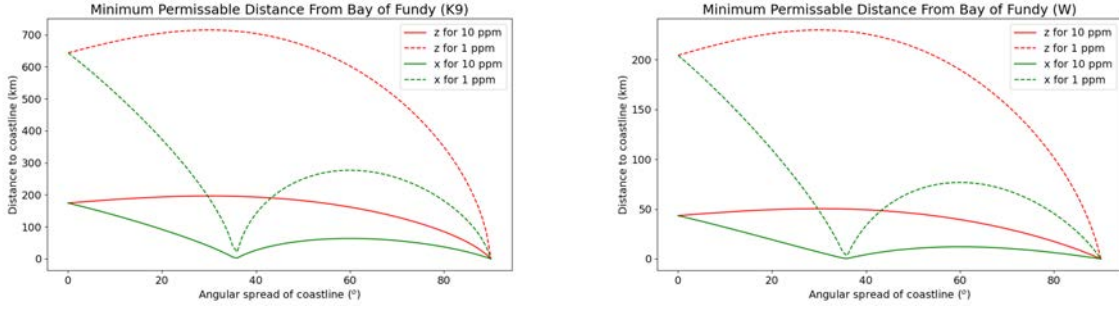


Figure 183: Minimum distance from a bay with extreme tides such as the Bay of Fundy in Canada. Tide height considered is 16 m, tide extent is the approximate average width of the Bay of Fundy, estimated here as 100 km. Calculations performed considering K9 glass (left) and tungsten (right) field masses.

50 m of snow with a depth of 0.5 m, we obtain the plot shown in Figure 184.

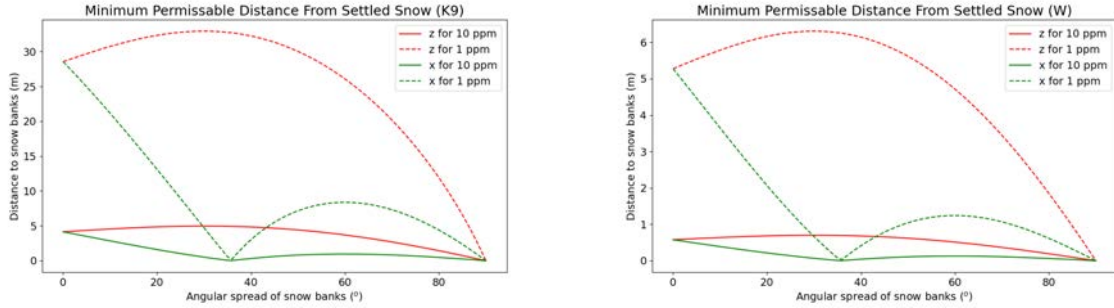


Figure 184: Minimum distance from snow banks that have changed from one measurement to the next before they begin to affect the measurement. The snow was modeled to have the same geometry as the oceanic tides (see Figure 181), but with a density of $\rho_{\text{snow}} = 300 \text{ kg/m}^3$, a patch extending out 50 m from the edge of the building, and a depth of 0.5 m. Calculations performed considering K9 glass (left) and tungsten (right) field masses.

Field Mass Magnetization

If there is a static magnetic field present at the locations of the field masses (such as Earth's magnetic field), they can become magnetized and perturb the trap's magnetic field, giving rise to a potential source of systematic error. In fact, this effect is present to a lesser extent even in any measurement of G , regardless of whether or not the oscillation frequency of the test mass is determined predominantly by magnetic forces[121].

First, let us consider the magnetic field generated by two spherical field masses becoming magnetized by an external magnetic field B_{ext} with no field gradient. The magnetic dipole moment of a single sphere, $\vec{\mu}_F$, is given by

$$\vec{\mu}_F = \frac{4\pi R^3 \chi_F^{(V)} \vec{B}_{ext}}{3\mu_0}. \quad (109)$$

We can then express the field generated halfway between two such spheres lying separated along the z -axis by $2d$ to be

$$\vec{B}_{\pm}(0, 0, z) = \frac{\mu_0}{4\pi(d \pm z)^3} (3(\hat{z} \cdot \vec{\mu}_F) \hat{z} - \vec{\mu}_F) \quad (110)$$

$$= \frac{\chi_F^{(V)} R^3}{(d \pm z)^3} \left(B_{ext,z} \hat{z} - \frac{1}{3} \vec{B}_{ext} \right), \quad (111)$$

where the $+$ indicates the field mass located on the negative part of the z -axis and the $-$ indicates the field mass located on the positive part of the z -axis. It is informative first to consider the magnitude of this effect by itself. The correction to the potential experienced by a particle in the trap from this effect is given by

$$U_{ext} = -\frac{\chi_T^{(V)} B_F^2}{2\mu_0 \rho_T}, \quad (112)$$

where $\vec{B}_F = \vec{B}_+ + \vec{B}_-$.

It is useful at this point to write down $\vec{B}_F$ and its partial derivatives at $(0, 0, 0)$:

$$\begin{aligned} \vec{B}_F(0, 0, 0) &= \frac{2\chi_F^{(V)} R^3}{3d^3} \left(3B_{ext,z} \hat{z} - \vec{B}_{ext} \right) \\ \frac{\partial \vec{B}_F}{\partial z}(0, 0, 0) &= 0 \\ \frac{\partial^2 \vec{B}_F}{\partial z^2}(0, 0, 0) &= \frac{8\chi_F^{(V)} R^3}{d^5} \left(3B_{ext,z} \hat{z} - \vec{B}_{ext} \right). \end{aligned} \quad (113)$$

The effective spring constant of this effect alone can be computed from $\frac{\partial^2 U_B}{\partial z^2}$ to be

$$\begin{aligned} k_{ext,z}(0,0,0) &= -\frac{\chi_T^{(V)}}{2\mu_0\rho_T} \left(\vec{B}_F \cdot \frac{\partial \vec{B}_F}{\partial z^2} \right) \\ &= -\frac{8\chi_T^{(V)} \left(\chi_F^{(V)} \right)^2 R^6}{3\mu_0\rho_T d^8} (B_{ext,x}^2 + B_{ext,y}^2 + 4B_{ext,z}^2). \end{aligned} \quad (114)$$

We can similarly consider the effect caused by $\vec{B}_F$ interacting with the trap field $\vec{B}_T$ by combining Eqs. (13) and (113) to get

$$\begin{aligned} k_{trap,z}(0,0,0) &= -\frac{\chi_T^{(V)}}{2\mu_0\rho_T} \left(B_{ext,x} \frac{\partial^2 B_x}{\partial z^2} + \frac{\partial^2 B_{ext,x}}{\partial z^2} B_x \right) \\ &= \frac{\chi_T^{(V)} \chi_F^{(V)} R^3}{\mu_0\rho_T d^3} \left(\frac{1}{3} (\alpha_z - d_4 y_{eq}) + \frac{b_0 y_{eq}}{d^2} \right) B_{ext,x}. \end{aligned} \quad (115)$$

A Pixel 6a cellphone magnetometer app was used to estimate the B -field from the Earth with no nearby magnets present near the location of the trap to be $\vec{B}_E = (16 \mu\text{T})\hat{x} - (35 \mu\text{T})\hat{y} + (18 \mu\text{T})\hat{z}$. These values will be used for calculations for the remaining calculations unless otherwise noted.

The end magnets in the Long MGT also create a nonvanishing dipole moment that will magnetize the field masses. As a very rough approximation of this effect, let us treat the dipole field from these magnets as being constant across the diameter of the field masses and equal to the value of B at their center (this will in general underestimate the magnitude of this effect).

The end magnets (N42SH) from K&J are specified to have $B_{rmax} = 1.33 \text{ T}$, which according to their website[76] can be used to calculate the magnetic dipole moment produced by the magnets as $\mu_{EM} = \frac{B_r V}{\mu_0}$, where V is the volume of the magnets. For three

magnets stacked together, $V_{3EM} \approx 7.68 \times 10^{-7} \text{ m}^3$. The centers of the magnets are placed approximately $z_{EM} = 0.016 \text{ m}$ from the center of the trap.

In this case, we find that the external field from the end magnets, B_{EM} , can be written as

$$\begin{aligned} B_{EM,x} &= \left(\frac{\mu_0}{4\pi (d + z_{EM})^3} + \frac{\mu_0}{4\pi (d - z_{EM})^3} \right) (-\mu_{EM,x}) \\ &= -\frac{B_r V_{3EM}}{4\pi} \left(\frac{1}{(d + z_{EM})^3} + \frac{1}{(d - z_{EM})^3} \right) \\ &\approx -189 \mu\text{T}, \end{aligned} \tag{116}$$

where we have considered only the transverse portion of the field for simplicity. Notice that this field is roughly an order of magnitude larger than Earth's magnetic field even in its worst orientation, making this the dominant effect.

We can now estimate the relative error in a measurement of G due to these effects by considering

$$\frac{k_{B,z}}{k_{G,z}}(\text{Earth/Earth}) = \frac{\chi_T^{(V)} \left(\chi_F^{(V)} \right)^2 R^3}{2\pi\mu_0 G \rho_T \rho_F d^5} (B_{Earth,x}^2 + B_{Earth,y}^2 + 4B_{Earth,z}^2) \tag{117}$$

$$\frac{k_{B,z}}{k_{G,z}}(\text{Trap/External}) = -\frac{3\chi_F^{(V)} \chi_T^{(V)}}{16\pi\mu_0 G \rho_T \rho_F} \left((\alpha_z - d_4 y_{eq}) + \frac{b_0 y_{eq}}{d^2} \right) B_{ext,x}. \tag{118}$$

Several estimates of the contributed error in measuring G are shown in Table 4. Values for α_z and d_4 were estimated from the measured values for 0.5mm HB Pentel pencil lead of $\chi_T^{(V)} = -1.9 \times 10^{-4}$ and $\rho_T = 1.70 \text{ g/cm}^3$ from Section 2 and the fit parameters in Eqs. (34), (35), (36), and (37).

Table 4: Magnetic Field Considerations for Various Materials

Material	ρ_F	$\chi_F^{(V)}$	$k_{B,z}/k_{G,z}$ $\oplus/\oplus$	$k_{B,z}/k_{G,z}$ Trap/ $\oplus$	$k_{B,z}/k_{G,z}$ Trap/EM
K9 Glass[49, 146]	2.51 g/cm ³	-1.01×10^{-5}	-3.0×10^{-13}	3.9×10^{-4}	-4.6×10^{-3}
Tungsten	19.3 g/cm ³	6.99×10^{-5}	-1.9×10^{-12}	-3.5×10^{-4}	4.1×10^{-3}
316 Steel[95]	8.03 g/cm ³	0.02	-3.7×10^{-7}	-0.24	2.8
410 Steel[96]	7.8 g/cm ³	750	-4.6×10^2	-8.7×10^3	1.0×10^5

Trap TiltSummary of Correction Terms

We can solve for G as follows:

$$G = -\frac{k_{Gz}d^3}{4Mm}, \text{ with } G_0 = -\frac{k_{Gz}(0,0,0)d^3}{4Mm}. \quad (119)$$

Summarizing the results from previous sections , we have

$$\text{Particle } z\text{-offset: } G = G_0 \left(1 - 6 \left(\frac{\delta z}{d} \right)^2 \right) \quad (120)$$

$$\text{Particle } x\text{-offset: } G = G_0 \left(1 + 3 \left(\frac{\delta x}{d} \right)^2 \right) \quad (121)$$

$$\text{Field Mass } x\text{-offset: } G = G_0 \left(1 + \frac{3}{2} \left(\frac{dx}{d} \right)^2 \right) \quad (122)$$

$$\text{Field Mass off by } \delta z: G = G_0 \left(1 + \frac{3}{2} \frac{dz}{d} \right) \quad (123)$$

$$\text{Other Mass } (M') \text{ at } x_0: G = G_0 \left[1 + \frac{1}{4} \left(\frac{M'}{M} \right) \left(\frac{d}{x_0} \right)^3 \right] \quad (124)$$

$$\text{Other Mass } (M') \text{ at } z_0: G = G_0 \left[1 - \frac{1}{2} \left(\frac{M'}{M} \right) \left(\frac{d}{x_0} \right)^3 \right] \quad (125)$$

$$\text{Ellipsoid: } G = G_0 \left[1 - \frac{12}{5} \left(\frac{R}{d} \right)^2 \left(\frac{ds}{R} \right) \right] \quad (126)$$

It is helpful to consider what this means in terms of nearby masses by running some numbers. Table 5 describes the required tolerances for parameters that have the same requirements regardless of what field masses are used.

Table 5: Acceptable Tolerances (Mass-Independent)

Description	Symbol	dG/G	$\leq \varepsilon_0$	$\leq \varepsilon_1$
Particle z -offset	δz	$-6 \left(\frac{\delta z}{d} \right)^2$	129 μm	41 μm
Particle x -offset	δx	$3 \left(\frac{\delta x}{d} \right)^2$	183 μm	58 μm
Single Field-mass x -offset	dx	$\frac{3}{2} \left(\frac{dx}{d} \right)^2$	258 μm	82 μm
Single Field-mass z -offset	dz	$3 \frac{dz}{d}$	0.67 μm	0.067 μm
Trap Angular Offset x -offset	θ	$3\theta^2$	0.105°	0.033°
Ellipticity of Field-Mass	ds	$-\frac{12}{5} \left(\frac{R}{d} \right) \left(\frac{ds}{d} \right)$	0.83 μm	0.083 μm

Table 6 describes the tolerances for nearby masses that might affect the measurement. Recall that these masses are acceptable so long as they remain stationary over the course

of a whole measurement; they only cause error if they are present for one measurement and absent (or moved to a very different location) for the other.

Table 6: Acceptable Tolerances (K9)

Description	Symbol	dG/G	$\leq \varepsilon_0$	$\leq \varepsilon_1$
Deviation of Mass	dM	$\frac{dM}{M}$	13.1 μg	1.3 μg
Air Bubble Inside Sphere	dr	$\frac{1}{2} \left(\frac{dr}{R} \right)^3 \left(\frac{d}{d-R} \right)^3$	679 μm , 3.29 μg	315 μm , 0.33 μg
Other Mass at 1 m	M'_{1x}	$\frac{1}{4} \left(\frac{M'_{1x}}{M} \right) \left(\frac{d}{x} \right)^3$	52.6 g	5.3 g
Other Mass at 5 m (x)	M'_{5x}	$\frac{1}{4} \left(\frac{M'_{5x}}{M} \right) \left(\frac{d}{x} \right)^3$	6.57 kg	0.66 kg
Other Mass at 15 m (x)	M'_{15x}	$\frac{1}{4} \left(\frac{M'_{15x}}{M} \right) \left(\frac{d}{x} \right)^3$	177 kg	18 kg
Other Mass at 50 m (x)	M'_{50x}	$\frac{1}{4} \left(\frac{M'_{50x}}{M} \right) \left(\frac{d}{x} \right)^3$	6571 kg	657 kg
Other Mass at 200 m (x)	M'_{200x}	$\frac{1}{4} \left(\frac{M'_{200x}}{M} \right) \left(\frac{d}{x} \right)^3$	4.21×10^5 kg	4.21×10^4 kg
Other Mass at 1 m (z)	M'_{1z}	$-\frac{1}{2} \left(\frac{M'_{1z}}{M} \right) \left(\frac{d}{z} \right)^3$	26.3 g	2.6 g
Other Mass at 5 m (z)	M'_{5z}	$-\frac{1}{2} \left(\frac{M'_{5z}}{M} \right) \left(\frac{d}{z} \right)^3$	3.29 kg	0.33 kg
Other Mass at 15 m (z)	M'_{15z}	$-\frac{1}{2} \left(\frac{M'_{15z}}{M} \right) \left(\frac{d}{z} \right)^3$	88.7 kg	8.9 kg
Other Mass at 50 m (z)	M'_{50z}	$-\frac{1}{2} \left(\frac{M'_{50z}}{M} \right) \left(\frac{d}{z} \right)^3$	3286 kg	329 kg
Other Mass at 200 m (z)	M'_{200z}	$-\frac{1}{2} \left(\frac{M'_{200z}}{M} \right) \left(\frac{d}{z} \right)^3$	2.10×10^5 kg	2.10×10^4 kg

It is not difficult to see some challenges associated with using field masses with such a low density as K9 glass – even for a desired 10 ppm measurement, nothing inside a typical lab (roughly a 5 m radius) the size of a computer monitor could be moved very far between measurements without introducing enough systematic error to invalidate the measurement.

In Table 7 the same calculations are performed again, this time using tungsten field masses. The results are a little more manageable, but still highlight the difficulty of the measurement.

These results are also summarized visually in Figure 185.

For an air bubble located a distance R' from the center of a test mass, the contribution

Table 7: Acceptable Tolerances (W)

Description	Symbol	dG/G	$\leq \varepsilon_0$	$\leq \varepsilon_1$
Deviation of Mass	dM	$\frac{dM}{M}$	101 μg	10.1 μg
Air Bubble Inside Sphere	dr	$\frac{1}{2} \left(\frac{dr}{R} \right)^3 \left(\frac{d}{d-R} \right)^3$	679 μm , 25.3 μg	315 μm , 2.5 μg
Other Mass at 1 m	M'_{1x}	$\frac{1}{4} \left(\frac{M'_{1x}}{M} \right) \left(\frac{d}{x} \right)^3$	404 g	40 g
Other Mass at 5 m (x)	M'_{5x}	$\frac{1}{4} \left(\frac{M'_{5x}}{M} \right) \left(\frac{d}{x} \right)^3$	50.5 kg	5.1 kg
Other Mass at 15 m (x)	M'_{15x}	$\frac{1}{4} \left(\frac{M'_{15x}}{M} \right) \left(\frac{d}{x} \right)^3$	1364 kg	136 kg
Other Mass at 50 m (x)	M'_{50x}	$\frac{1}{4} \left(\frac{M'_{50x}}{M} \right) \left(\frac{d}{x} \right)^3$	5053 kg	505 kg
Other Mass at 200 m (x)	M'_{200x}	$\frac{1}{4} \left(\frac{M'_{200x}}{M} \right) \left(\frac{d}{x} \right)^3$	3.23×10^6 kg	3.23×10^5 kg
Other Mass at 1 m (z)	M'_{1z}	$-\frac{1}{2} \left(\frac{M'_{1z}}{M} \right) \left(\frac{d}{z} \right)^3$	202 g	20 g
Other Mass at 5 m (z)	M'_{5z}	$-\frac{1}{2} \left(\frac{M'_{5z}}{M} \right) \left(\frac{d}{z} \right)^3$	25.3 kg	2.5 kg
Other Mass at 15 m (z)	M'_{15z}	$-\frac{1}{2} \left(\frac{M'_{15z}}{M} \right) \left(\frac{d}{z} \right)^3$	682 kg	68 kg
Other Mass at 50 m (z)	M'_{50z}	$-\frac{1}{2} \left(\frac{M'_{50z}}{M} \right) \left(\frac{d}{z} \right)^3$	25 264 kg	2526 kg
Other Mass at 200 m (z)	M'_{200z}	$-\frac{1}{2} \left(\frac{M'_{200z}}{M} \right) \left(\frac{d}{z} \right)^3$	1.62×10^6 kg	1.62×10^5 kg

from its dipole moment and its mass will be equal when the following condition is met:

$$R' = d \left(1 - 2^{-1/3} \right) = 0.2063d. \quad (127)$$

The results of these calculations are clear; to claim a measurement of G that is both precise and accurate, these factors MUST be taken into account. It is difficult to say how much such systematic errors may contribute to the varied measurements of G reported in the literature, since most do not include a description of how the presence and/or absence of nearby masses (such as people, desks, water bottles, and even cars) was controlled.

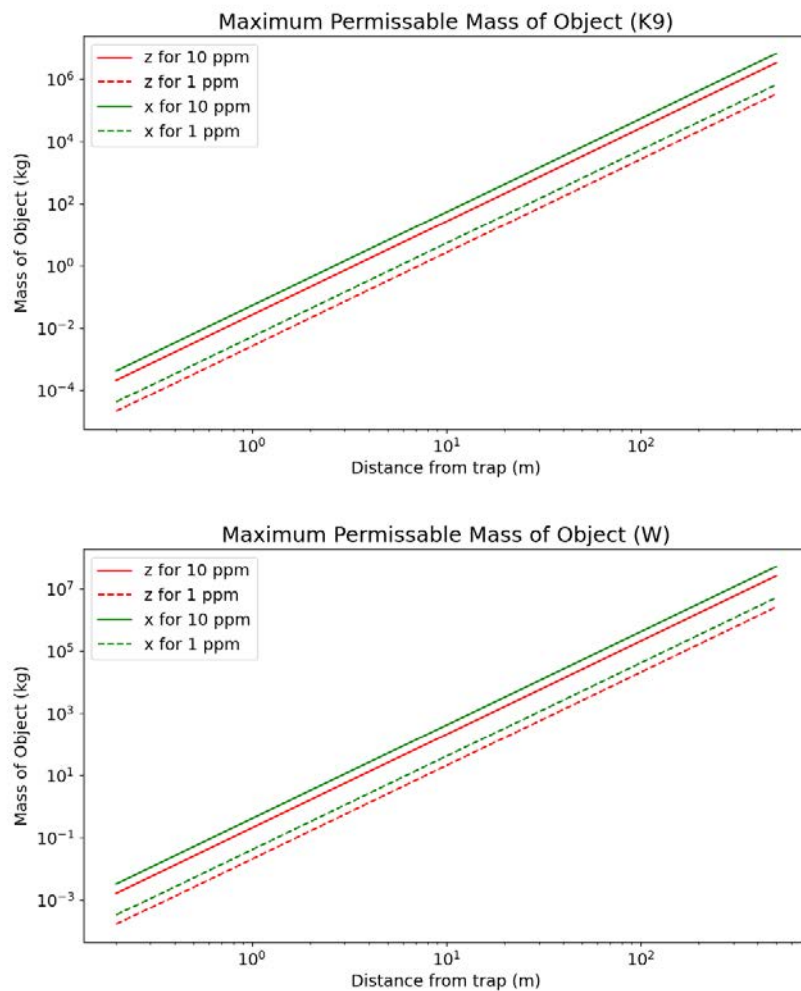


Figure 185: Minimum distance a mass must be from the test mass before its presence or absence can be safely ignored for different levels of tolerance. Calculations performed considering K9 glass (left) and tungsten (right) field masses.

CONCLUSIONS AND FUTURE WORK

In Chapter 1 we discussed the precision and versatility of levitated optomechanical systems for performing sensitive force measurements. In particular, we motivated the use of a magneto-gravitational trap (MGT) as an instrument for measuring the Newtonian constant of gravitation G ; the stable trapping potential, active tilt-stabilization on a single point diamond turning machine (DTM), cross-correlation-based image analysis, and chamber under ultra-high vacuum (UHV) all work together to form a precise, well-isolated system.

The use of a graphite composite particle (GCP) as a test mass allows stable charge-free trapping with ultra-low damping rates and passive cooling (via eddy current damping) in other motional degrees of freedom, allowing long (easily 24 hours or longer) coherent collection times needed for time-of-swing (TOS) measurements. Although the trapping frequencies are still fairly high ($\omega_z/2\pi \approx 0.3\text{Hz}$), this can be tuned in a number of ways. In addition, the partially asymmetrical shape of the particles allows direct measurement of the rotation of the particle about two of its principle axes, allowing for the possibility of directly tracking rotational motion.

This manuscript has documented progress towards building a system for measuring G ; however, several major tasks remain before a high-precision measurement of G can be obtained. In what follows, we will summarize some of these and other results in more detail.

Trap Design and Modeling

The MGT design described in Chapter 2 robustly levitates GCPs (specifically Pentel 0.5mm HB pencil lead) in a stable potential designed with a single weak axis such that one degree of freedom of the particle's motion oscillates at a low frequency with very little dissipation.

Multipole Field Expansion and Potential Characterization

In this manuscript we developed a multipole fit solution to the MGTs used. While finite-element analysis (FEA) solutions (such as using OpenFOAM, Comsol, and even Python [17]) can provide much more detail about the magnetic field of a given arrangement of magnets and magnetic materials, this multipole fit has two major advantages.

First, matching the fit parameters to empirically measured quantities instead of theoretical geometries and material properties in a simulation results in a model where the accuracy is completely independent of misalignments or defects that make the actual system differ from the theoretical system.

Second, the way the model is constructed, the fit parameters are calculated to fit the trapping potential experienced by the particle, not the magnetic field produced by the magnetic trap. This makes the result agnostic to both the absolute magnitude of the magnetic field and the magnetic susceptibility of the particle (although either quantity can be estimated if the other quantity is known), a useful feature when testing poorly understood materials.

Still, a direct simulation of the field produced by the trap could have several advantages over the multipole fit. One benefit would be that the effect on the potential from moving the end magnets up and down could be characterized, allowing much more quantitative control over the potential. Another useful result might be to simulate the effect of reshaping the pole pieces to engineer a trapping potential with a very low axial frequency formed by pole piece geometry rather than the end magnets.

This latter result might be especially useful for measurements of G , where the presence of the end magnets produces an additional dipole field that complicates a measurement of G by magnetizing the field masses, and allows for an additional coupling mechanism between mechanical effects (vibration or thermal shift of the end magnets) and the trapping potential.

The fit parameters for the trap with potential given by Eq. (39), for the trapped particle

Edwin (HB05-2.4-lapped) in the Long MGT are:

$$c_0 b_0^2 = 3.4 \times 10^4 \text{ s}^{-2}$$

$$\alpha - \beta = 6.8 \times 10^{-5}$$

$$\alpha = 7.4 \times 10^{-4}$$

$$\beta = 6.7 \times 10^{-4}$$

$$\gamma = 0.0780$$

$$y_{eq} = -0.222 \text{ mm}$$

It should be noted that, while $c_0 b_0^2$, γ , and $\alpha - \beta$ have little uncertainty, the way that α and β are calculated from $\alpha - \beta$ using the difference between transverse (vertical) oscillation frequencies at low and high amplitude (where we recall that the sidebands in these frequencies confuses the value, as seen in Figure 159) lends itself to significant uncertainty.

Nevertheless, the construction of a multipole fit containing the lowest order spherical harmonics with the appropriate symmetries provides a useful tool for measuring and predicting trap properties.

Properties of Materials Used

The Hiperco 50A and Hiperco-composite pole pieces used for experiments in this manuscript are difficult to characterize well for several reasons. First, while the material properties of Hiperco 50A are well-known, the actual magnetic saturation of the pole pieces, which affects many of its other physical characteristics, is not known for the trap assembly and likely has non-negligible geometrical dependence. Second, direct trapping properties (such as damping rates or magnetic field strengths) are nontrivial to predict from the behavior of

trapped particles, because differences in magnets used, precise pole piece geometries, and assembly make it difficult to build two analogous traps where the only difference is pole piece material.

Nevertheless, tests performed on rods of solid Hiperco 50A and Hiperco-composite indicate two principles clearly. First, eddy current dissipation in the Hiperco-composite is much lower (not measurable with the apparatus used) than in the solid Hiperco 50A . Second, annealing the Hiperco 50A rod reduced eddy current dissipation in the material. The former result is expected due to the presence of an insulating material to break up conductive pieces into small regions, while the reason for the latter result is unknown.

The magnetic susceptibility of Pentel 0.5mm HB pencil lead was measured directly using the Gouy method, with a resulting value near 0 for χ_{HB} through its long axis and a value of $\chi_{HB} = -2.0^{+0.4}_{-0.1}$ perpendicular to its long axis. These results indicate a strong anisotropy in the pencil lead, consistent with graphite flakes oriented in concentric shells down its length.

The average conductivity and density of Pentel 0.5mm HB pencil lead were also measured to be $\sigma_{HB} = 4.63(15) \times 10^5 \Omega^{-1}\text{m}^{-1}$ and $\rho_{HB} = 1.70(2) \text{g/cm}^3$, respectively.

Possible Design Improvements

One possible design improvement already mentioned would be to achieve potential control via better-stabilized methods than the end magnets. While the adjustability may be a great advantage in applications where a dynamic potential is desired [91], in a measurement of G it poses a significant concern for the stability of the trap's oscillation frequency. One way to do this is to shape the geometries of the pole pieces to induce asymmetry such that the effective α -value for the trap increases. This may be achievable with refined pole piece casting techniques such as heat or chemical smoothing of the molds used for casting the pole pieces, eliminating the need for post-cast polishing and allowing for more arbitrary

(e.g. curved) geometries.

The high resistivity of 4B Pentel lead, coupled with its higher graphite concentration of 79% by mass [144] ($\sim 58\%$ by volume), make it an attractive solution for future tests due to potentially lower eddy current damping and higher graphite concentration. Additionally, the use of tumbled particles is likely to increase position-tracking by improving the focus of the particle's profile.

Further test-mass refinements are also possible. Using a particle with a rough surface like the groovy one shown in Figure 48 is likely to reduce the measurement uncertainty per frame significantly by increasing the number of pixels carrying information about the z -displacement of the particle.

The vacuum-compatibility of Pentel 0.5mm HB pencil lead is not well-known, partially due to the proprietary nature of its fabrication. Therefore, it may be worthwhile to pursue in-house fabrication of GCPs using high-temperature epoxy (Epo-Tek 353ND) and high-purity graphite. Several challenges arise down this path. First, it is likely that a specific extrusion process is the cause of the favorable anisotropy observed in the magnetic susceptibility of Pentel 0.5mm HB pencil lead. Second, composite particles made to date have produced very crumbly particles, implying that further refinement of the packing or mixing process may be beneficial. Finally, achieving homogenous and geometrically uniform particles is not trivial.

The tumbling process suggests the possibility of creating spherical GCPs, but tests have shown that such spheres made from magnetically anisotropic pencil lead do not levitate stably. Thus, spherical GCPs would likely trap more weakly, potentially requiring a somewhat higher concentration of graphite (likely to make the particle more crumbly and difficult to tumble effectively) than cylindrical particles.

SiC Measurements

Several measurements of optically detected magnetic resonance (ODMR) in 4H-SiC samples were taken, as discussed in Chapter 3. While these measurements contribute little to the rich collection of work characterizing V_{Si}^- (V2) centers in SiC, [135, 147] they serve as a starting point for spin-manipulation experiments in levitated optomechanics. And while the attempted mechanically detected magnetic resonance (MDMR) results showed no clear signal, the techniques developed there are novel and potentially useful for future projects.

Discussion of Techniques and Results

While both the background subtraction and the lock-in approaches detailed in Chapter 3 should be able to yield resonant detection of an MDMR signal in a levitated piece of 4H-SiC containing V_{Si}^- (V2) centers, the lock-in technique shows greater promise for future measurements.

There are several factors that may have contributed to the null result of the MDMR trials attempted. First, the use of boule-grown instead of epi-grown SiC indicates the possibility of a low concentration of V_{Si}^- (V2) centers, resulting in low SNR. Second, the asymmetrical shape of the particle resulted in librations playing a major role in confusing amplitude detection and preventing narrow-bandwidth resonant excitation of the particle's vertical motion. Third, the rotational instability of the particle likely confused any signal present further. Fourth, the unknown geometry of the particle (especially along the transverse direction used for imaging) makes expected signal characterization difficult. Fifth, the placement and proximity of the drive wire, combined with the heavily frequency-dependent microwave signal at the trap, make precise driving of the spin transitions difficult.

Possible Future Directions

One clear choice is to select a better bulk sample of SiC for particle selection, such as – in this case – the epi-grown material instead of the boule-grown. It may even be beneficial to fabricate a batch of particles first, then irradiate second, in order to avoid damaging vacancy centers through the mechanical stress of fabrication, although such a process is likely to present challenges of its own.

The particle shape must also be better-controlled. A spherical particle will not have the same troubles with librations causing the magnetic field experienced by the particle to change drastically, but presents new difficulties with regard to the extreme uncertainty in particle orientation. There are at least four possible routes forward which show promise in different areas.

The first is to obtain a vacancy-rich particle with a clearly defined geometry that is rotationally stable enough or uniform enough in shape that librations do not cause parts of the particle to move through the trap's field gradient. One such option would be a long elliptical cylinder where the c-axis is well-known.

A more tricky way to accomplish a similar result might be to use a sphere. As mentioned before, this does not at first appear ideal due to the lack of orientation locking and unknown c-axis. However, the anisotropy of the magnetic susceptibility of SiC, combined with an optical technique for tracking the particle's orientation (perhaps by use of polarization), might allow such a shape to be effective.

The second approach would be to intentionally create a levitated system where the translational motion is well-cooled but the rotation is used to perform resonant detection. Such an approach would require an oblong particle, likely with cylindrical symmetry.

The third method would be to perform ODMR on a particle levitated in a trap with a weaker field gradient. This could perhaps be accomplished using a flat piece of material with a known c-axis orientation attached to a GCP to allow levitation in a trap with a weaker

field gradient. This has the advantage of a weaker field gradient for better magnetic field control and a highly stable levitated orientation. The downsides to this are that MDMR would be difficult due to the much lower effective V_{Si}^- (V2) center density and there may be additional internal mechanical modes that could complicate data analysis.

A fourth option is to obtain a stably-oriented particle of 4H-SiC containing just a single vacancy defect center. This is likely to be quite challenging, as such a particle would need to be very small. The benefit, however, would be immense. Not only would the MDMR signal be greatly simplified, but the center-of-mass motion of the particle would be directly coupled to the spin state of a single defect center.

Sublimation-Activated Release

One result that came out of the SiC experiments was the development of the sublimation-activated (SAR) loading method. [106] While this technique is not needed for the current GCPs used (which can be loaded directly with tweezers), it is immensely useful for loading smaller particles on the order of 10-100 μm when efficiency and selectivity is required.

This new technique could likely be improved in a number of ways, but one clear choice for further refinement would be to mount a small thermoelectric cooling/heating device near the tip. This would allow cooling of the camphor to slow its sublimation (particularly useful for smaller particles which can fall off in about a minute under typical ambient conditions), and heating of the camphor (particularly helpful for large particles, release times without heating can be close to an hour).

Damping Rates of Levitated Pencil Lead

In Chapter 4, data was obtained of an effectively one-dimensional harmonic oscillator with no discernable damping. [107] This result demonstrates an important capability for

constructing a system for measuring G , where long data collection times are ideal.

Methods and Results

The low damping rate achieved indicates that the combination of dissipative forces acting on the particle is small compared to external noise sources in the system such as vibrations. In particular, the eddy current damping was kept low by creating a trap with only a quadratic magnetic field gradient along the trap's z -axis. The pressure-damping and charge-damping were kept low by maintaining a pressure in the range of about $3\text{--}6 \times 10^{-8}$ Torr throughout the measurements, by neutralizing the charge on the particle using an ionizing radiation source prior to pumping down the chamber, and by coating the pole pieces in copper foil.

It is not known whether the consistently low pressure (accomplished by firing the Ti-sublimation pump every few days) or the foil coverings played a greater role in reducing charge damping; what is known is that without both of these procedures and without initial neutralization, the particle damped at a measurable rate faster than would be expected from pressure-damping alone.

Next Improvements

While the results are impressive, better characterization of the system is clearly needed. A method for vibration sensing, such as a seismometer, would help to characterize the noise in the system and perhaps identify how energy is leaking into the particle's motion. On a similar note, implementing tilt-stabilization in the transverse as well as the axial direction is an obvious next step.

It is possible that using 4B instead of HB pencil lead would further reduce eddy current damping (surprisingly, the lead with the higher graphite concentration had a lower conductivity when measured); however, eddy current damping does not appear to be the

leading source of damping anyway. Using a larger-diameter GCP would somewhat reduce the pressure-damping, but not by very much.

Improving the shape of the particle by rounding the ends and perhaps by cutting grooves into the particle could significantly reduce measurement noise, and would likely also reduce eddy-current damping. However, the grooves might be difficult to do well without interfering with center-of-mass estimations.

System Tolerances for a Measurement of G

An analysis of several potential sources of systematic uncertainty, primarily relating to the G -measurement system being designed in this lab, are discussed in Chapter 5. While several of the tolerance calculations have been done elsewhere – such as magnetic field considerations and geometrical specifications of field masses and their positions – others are not well-discussed in the literature, particularly the gravitational impact of unconsidered nearby masses. Here we summarize some of the most interesting requirements for 10 ppm and 1 ppm measurements.

Summary of Contributing Sources of Systematic Uncertainty

Certain effects discussed are unlikely to pose a problem to most experiments. Even in the worst cases, the gravitational effect of the moon and the sun, combined with the tidal deformation of the Earth, on the quadratic potential experienced by a test mass is negligible even for most 1 ppm measurements of G .

Other effects may or may not be a concern, depending on the details of the laboratory setting and the design of the experiment. Experiments attempting 10 ppm measurements within tens of kilometers of the coast will need to consider the gravitational effect of oceanic tides. For 1 ppm measurements, the experiment may need to be located hundreds of kilometers from the coast depending on field mass density and placement.

Similarly, most 10 ppm measurements should be relatively unaffected by snow banks forming or melting overnight, but many experiments in areas with heavy snowfall and laboratories located near an external wall of the building will need to consider this effect for 1 ppm measurements.

One source of uncertainty that poses a challenge in virtually every measurement of G is the precise positioning of the field masses, the contribution to oscillation frequency scales linearly with displacement. In our case, the distance between the masses must be known to within 670 nm for a 10 ppm measurement, or 67 nm for a 1 ppm measurement. The position of the particle or the transverse/vertical placement of the field masses, by contrast, need only be known to within a few hundred micrometers (several tens of micrometers for 1 ppm).

One possible source for systematic error that is especially hard to control is the presence or absence of nearby masses that perturb the gravitational potential of the particle between two sets of measurements. Apart from obvious masses that could be ignored to the experiment's detriment such as apparatuses used to hold field masses in place, more subtle effects are also possible.

For a 10 ppm measurement, some examples of objects large enough to potentially cause a problem are: a water bottle or wrench placed on a table next to the measurement apparatus, a desk or person anywhere inside the lab, a particularly large piece of equipment or elevator in the building housing the lab, or a car parked in a parking lot just outside the building.

For a 1 ppm measurement, the examples could be even more difficult to identify: a nut or bolt placed on a table next to the measurement apparatus, a stack of books or large liquid-filled container anywhere inside the lab, a desk or person in the building housing the lab, or a car parked in a parking lot across the street from the building.

It is important to note that such objects only pose a problem if they are moved substantially from one measurement to the next. Therefore, such effects can potentially be accounted for by taking the average of many measurements over many different days.

However, the best approach would be to combine this averaging with careful documentation of both the lab environment and, to the extent possible, the location of other large objects in and around the building.

In most experiments subject only to Earth's magnetic field, the magnetization of field masses made of paramagnetic or diamagnetic materials can be safely ignored, even for 1 ppm measurements. The use of martensitic stainless steel (SS) such as 410 as a field mass is completely unreasonable, possibly producing changes to a test mass oscillation frequency in greater than the change due to their gravitational force. Using an austenitic stainless steel such as SS 316 produces an effect that should be small enough to ignore, but only if cold-working of the steel has not produced areas of the field masses that are no longer austenitic. As such, even SS 316 field masses should be considered a possible source of systematic error to be considered.

In our system, the magnetization of the field masses (tungsten or K9 glass) by Earth's magnetic field contributes systematic errors on the order of 400 ppm due to the strong magnetic fields used to trap the particle. Thus local fields will need to be carefully characterized and controlled. If the end magnets are used as discussed in Chapter 2, this error becomes closer to 4000 ppm, an enormous effect. In addition, the quadrupole field generated by the trap, the impact of which was not analyzed in this work, may also cause non-negligible errors even without the dipole field produced by the end magnets. [38]

Possible Mitigation Techniques

The use of a MGT to measure G is far more susceptible to small changes in the local magnetic field of the laboratory environment than other techniques. This means that this system will need to take great care in reducing and characterizing the effects of magnetic fields on the oscillation of the test mass.

The local magnetic field at the field masses can be canceled to an extent using Helmholtz

coils; however, this is not trivial for magnetic fields with steep gradients such as those produced by MGTs. Another method for mitigation is mu-metal shielding. This is also tricky, however, since optical access must still be available and placing it too close to the trap may saturate the material, lessening its effectiveness as a magnetic field shield. Finally, carefully characterizing the field allows for its impact to be calculated and subtracted off measured data. This approach, however, necessarily introduces more measurements and material properties that may contribute uncertainties of their own.

Therefore the best path forward likely involves a combination of techniques, and perhaps the removal of the end magnets in favor of a less dynamic potential shaping that does not produce such a large dipole field at the field masses.

Summary

The system under development described in this manuscript is a tilt-stabilized magneto-gravitational trap operated under UHV with particle motion viewed using an area-scan camera and positions computed from a cross-correlation based image analysis technique. This system shows great promise as a potential apparatus for performing a measurement of G . Furthermore, the techniques and components designed in this work, such as the selective loading of micron-scale particles into traps and the design of one-dimensional high-Q harmonic oscillators, show promise for integration into other experiments as well.

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APPENDIX

PROCEDURES

Here are outlined several standard lab procedures for reference.

Tube Furnace Operation

Operation

1. First, determine what gas is to be used in the furnace. If you simply want to avoid oxidation, use pure argon. To also strip away an already-present oxidation layer, use 5% hydrogen, 95% argon. Connect the tube running to the input of the furnace to the appropriate tank.
2. Place the sample in the quartz boat and place it in the center of the furnace. Make sure that the tube is centered in the furnace shielding.
3. Place the quartz plugs in the tube, about even with where the shielding begins. Close the tube with the O-ring clamps, tightening them as tightly as you can without the use of tools. Make sure the exhaust-tube coming from the furnace is set below the surface of the liquid in the beaker, and that the exhaust coming from the beaker is vented into the fume hood.
4. Ensure the flow-control valve on the tank is shut. Open the main tank valve, then open the flow-control valve until you see a slight increase in pressure. Open the valve on the flow-meter (with a small black plastic ball) until the ball rests at about 82 (400 sccm). Let the furnace run like this for about 10-20 minutes to flush out the oxygen from inside.
5. While the furnace is being flushed, turn on the display using the green switch on the front. Check that all the settings are as desired. Several common programs are listed below. Press the page button (looks like a page) until **ProG LiSt** is in view. Then press the scroll button (circle with arrow) to cycle through the parameters.
6. After the tube has been flushed, turn the flow down to about 41 (200 sccm), then press the run button (top right) to start the program.
7. Once the furnace has cooled sufficiently, REMOVE THE EXHAUST TUBE FROM THE WATER. This is to avoid getting water sucked up inside. You may then shut off all the valves from the gas tank and open the furnace.
8. Turn off the furnace using the green switch.

SiC Annealing

Use pure argon.

- `SEG.n 1 - tYPE → rmP.t - tGt → 550 - dur → 105`

OR

- SEG.n 1 - tYPE → rmP.t - tGt → 550 - rAtE → 5
- SEG.n 2 - tYPE → dwEll - dur → 60
- SEG.n 3 - tYPE → rmP.t - tGt → 24 - dur → 105

OR

- SEG.n 3 - tYPE → rmP.t - tGt → 24 - rAtE → 5
- SEG.n 4 - tYPE → StEP - tGt → 24
- SEG.n 5 - tYPE → dwEll - dur → 0
- SEG.n 6 - tYPE → End - End → dwEll

Hiperco 50A Annealing

Use 5% hydrogen, 95% argon.

- SEG.n 1 - tYPE → rmP.t - tGt → 825 - dur → 160

OR

- SEG.n 1 - tYPE → rmP.t - tGt → 825 - rAtE → 5
- SEG.n 2 - tYPE → dwEll - dur → 220
- SEG.n 3 - tYPE → rmP.t - tGt → 200 - dur → 250

OR

- SEG.n 3 - tYPE → rmP.t - tGt → 200 - rAtE → 2.5
- SEG.n 4 - tYPE → StEP - tGt → 24
- SEG.n 5 - tYPE → dwEll - dur → 0
- SEG.n 6 - tYPE → End - End → dwEll

Composite Pole Piece Curing

Use pure argon.

- SEG.n 1 - tYPE → rmP.t - tGt → 150 - dur → 25

OR

- SEG.n 1 - tYPE → rmP.t - tGt → 150 - rAtE → 5

- SEG.n 2 - tYPE $\rightarrow$ dwEll - dur $\rightarrow$ 720
- SEG.n 3 - tYPE $\rightarrow$ rmP.t - tGt $\rightarrow$ 200 - dur $\rightarrow$ 10

OR

- SEG.n 3 - tYPE $\rightarrow$ rmP.t - tGt $\rightarrow$ 200 - rAtE $\rightarrow$ 5
- SEG.n 4 - tYPE $\rightarrow$ dwEll - dur $\rightarrow$ 720
- SEG.n 5 - tYPE $\rightarrow$ rmP.t - tGt $\rightarrow$ 25 - dur $\rightarrow$ 70

OR

- SEG.n 5 - tYPE $\rightarrow$ rmP.t - tGt $\rightarrow$ 25 - rAtE $\rightarrow$ 2.5
- SEG.n 6 - tYPE $\rightarrow$ StEP - tGt $\rightarrow$ 25
- SEG.n 7 - tYPE $\rightarrow$ dwEll - dur $\rightarrow$ 0
- SEG.n 8 - tYPE $\rightarrow$ End - End $\rightarrow$ dwEll

Additional Notes

- The actual time the furnace takes to reach a temperature you tell it to ramp to may be as much as an hour longer than you expect. It will still, however, have the appropriate dwEll duration you specify.
- The orange light below the green switch will flash while ramping, presumably because it is heating up but not doing so constantly. Supposedly the “HOLD” light will flash during SEG.n 1 and SEG.n 3, while “RUN” will be lit during SEG.n 2, but I have noticed that “RUN” may be lit continuously for sections of step 1.
- For EMERGENCY INTERRUPT of furnace operation, PRESS and HOLD the 2nd button in the 1st row and allow furnace to cool.
- Be wary when setting up a different program structure. Attempting to End a dwEll step if the last step was not a dwEll may result in the furnace continuing at the last temperature command it received. If you are unsure of the command structure you have provided, monitor the furnace operation when it transitions from one step to the next to ensure it does what you expect.

Sublimation-Activated Release (SAR) Selective Loading

General Procedure, $\lesssim 10\ \mu\text{m}$

For loading small particles via camphor sublimation, the following procedure is used. [106]

1. A loading apparatus like the one shown in Figure 186 is constructed. It must be placed at a height such that the tungsten probe (Signatone SE-20T, mounted on 10-centimeter S3-E-1 shaft) can be inserted into the trapping region while leaving the micropositioner (S-931) with room to travel up and down.
2. The particles to be loaded are placed on a conductive surface, such as an Al-foil coated microscope slide, then placed under a microscope at 10X or 20X magnification. An ionizing radiation source is brought near to reduce charge on the particle, as shown in Figure 187. This step is critical for smaller particles to prevent sticking to the probe tip.
3. About 18 grams of crushed camphor (roughly the texture of snow, which can be accomplished using a mortar and pestle) is placed in the bottom of a copper crucible like the one shown in Figures 188. and 189. The aluminum foil at the top (sometimes double- or quadruple-layered for insulation must have a hole in the top to allow the probe tip to be inserted. The crucible is heated on a hot plate to 225 °C for about 20 minutes, covered by a glass lid while heating. Note: over time, camphor builds up on the foil cover as shown in Figure ??, and needs to be scraped off and crushed up again.
4. A particle is selected by centering the microscope over it. Then one probe station is used to pick up the particle on its tip, lifting it above the surface of the slide.
5. The loading tip is inserted into the hole in the top of the crucible for about 10 seconds (the precise time required is not fixed – sometimes more or less time is required depending on camphor build-up in the crucible – and then mounted on a second micropositioner by the microscope.
6. The camphor is then monitored under the microscope until the large crystal structures have sublimated. It is now ready for loading.
7. The loading tip is then maneuvered over to the selected particle to transfer it from the uncoated tip to the coated one. At this point a timer begins (roughly 1 minute) – the loading tip must be positioned in the trap before the particle falls off the camphor-coated loading tip.
8. The loading tip is removed from the microscope micropositioner and placed on the one mounted on the track. This track is then manually inserted into roughly the correct position, then the position of the particle is fine-tuned using the trap camera to put it in focus in the trapping region.

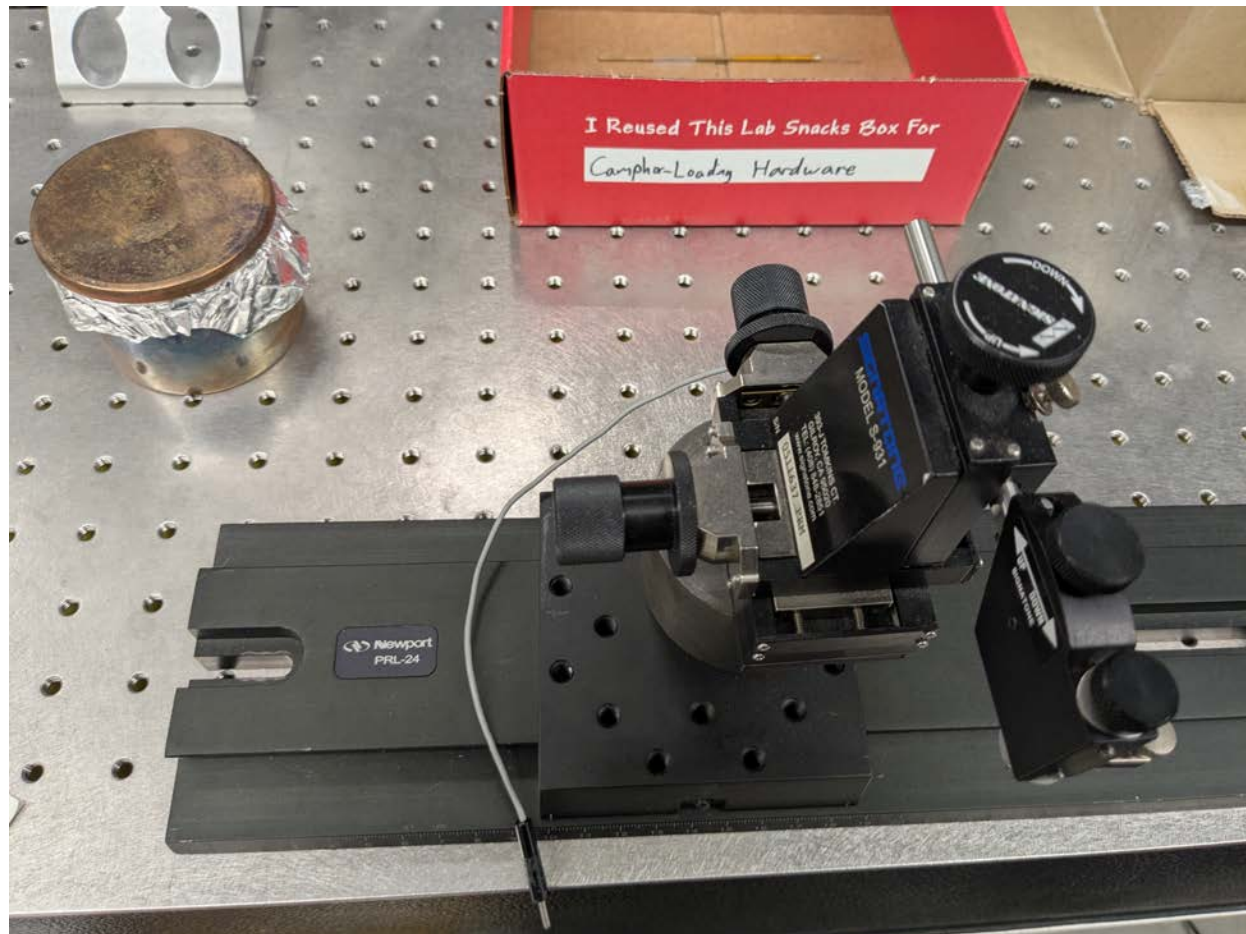


Figure 186: The track used for coarse adjustment of the probe location in the trap along its axial axis.

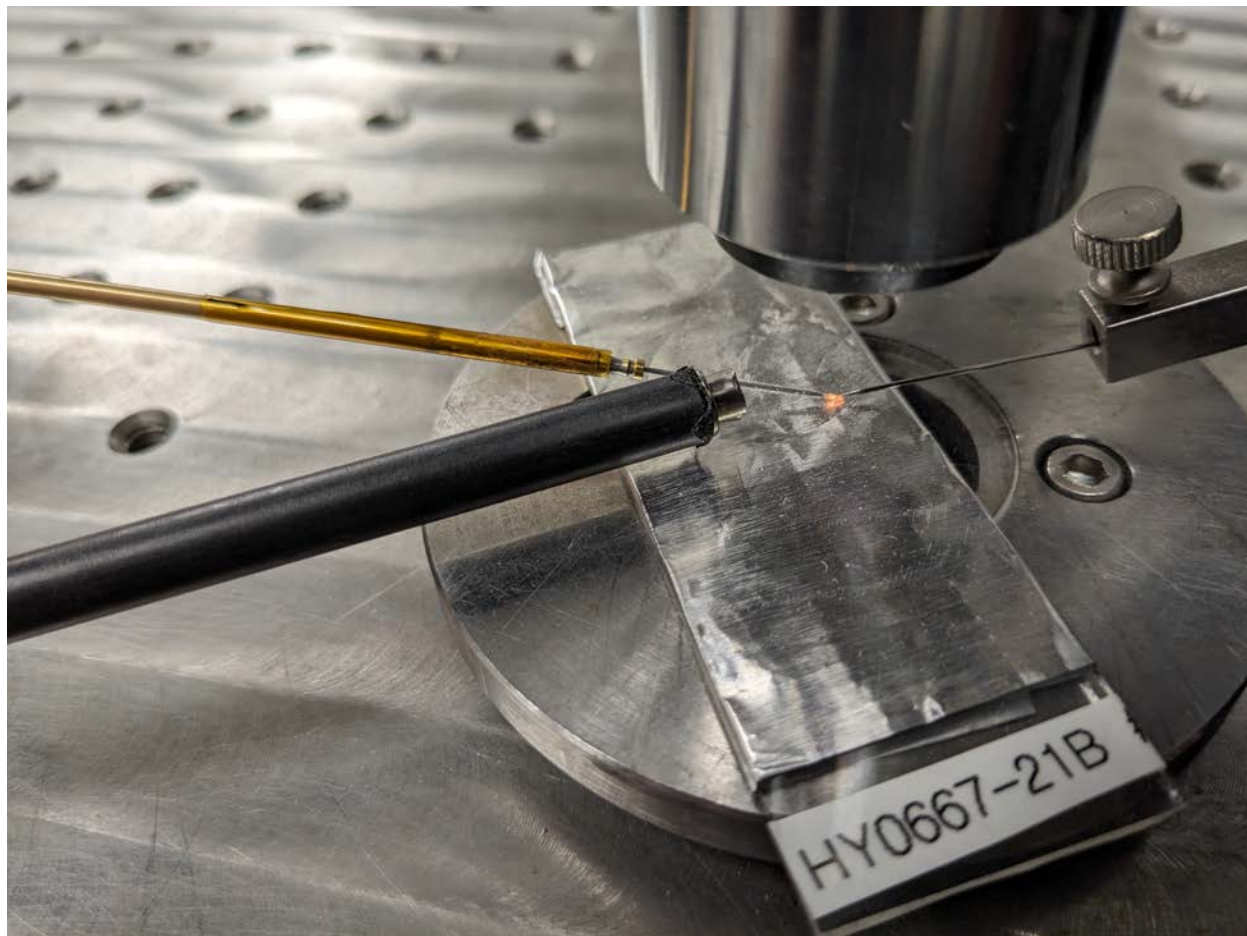


Figure 187: Radiation source and microscope objective used for bleeding charge off of particles prior to loading via sublimation-activated release.

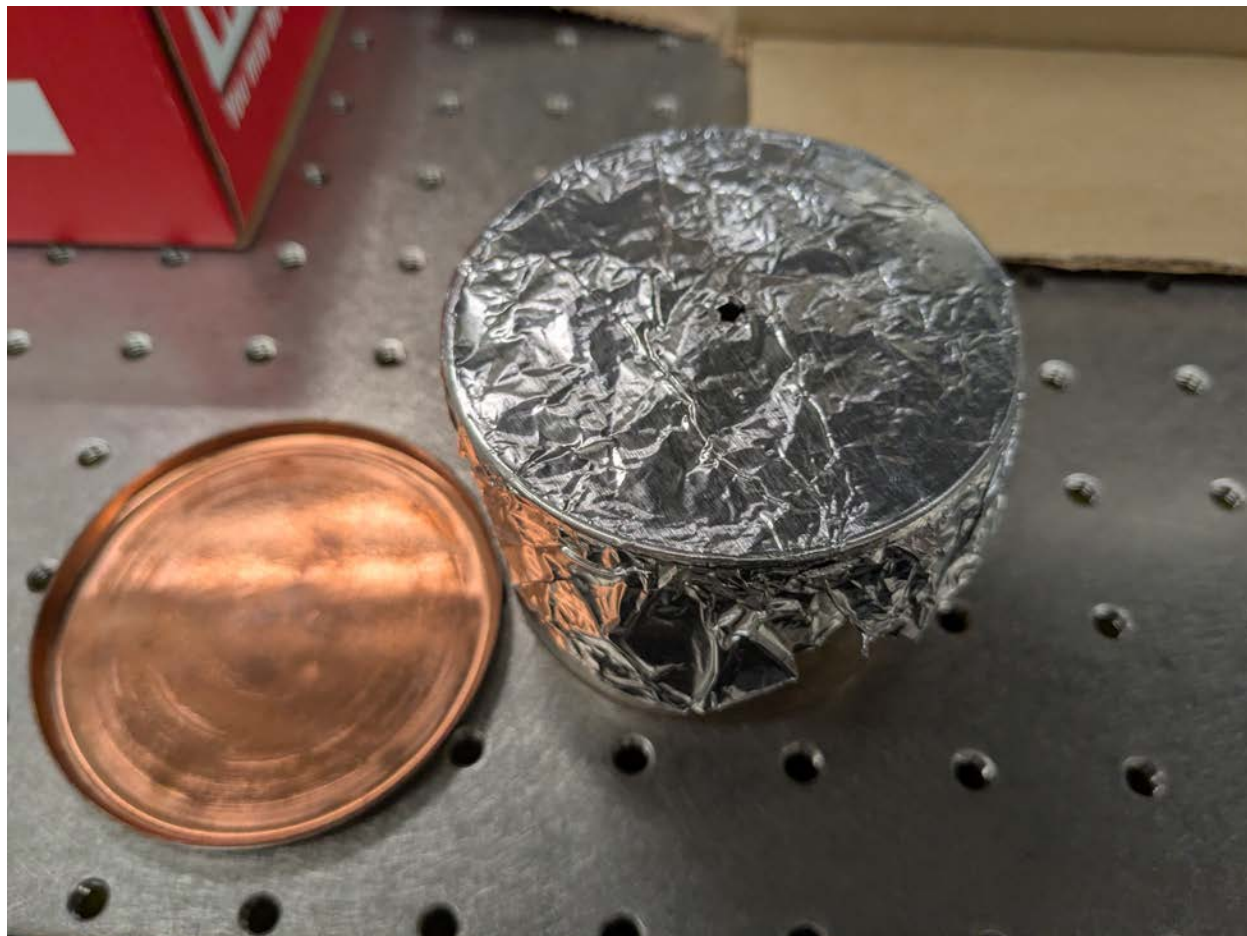


Figure 188: Copper crucible used for heating camphor with Al-foil lid. The loading tip is inserted into the hole in the top when the camphor is warm.

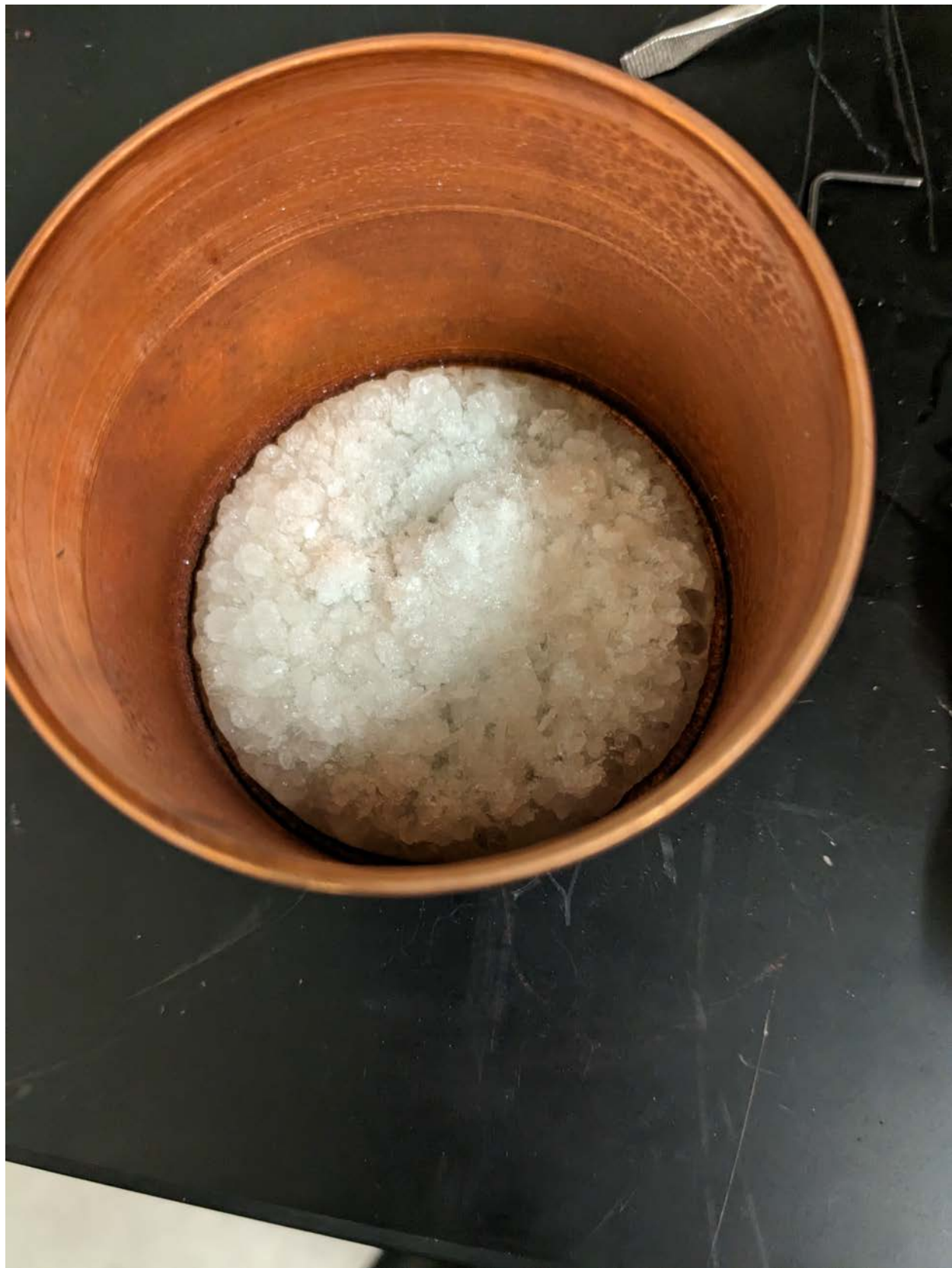


Figure 189: Crushed camphor in the bottom of the beaker after it has been used for a few trials and clumped up somewhat in the bottom.



Figure 190: A thick layer of camphor that has formed on the inside of the Al-foil cover to the copper crucible used for camphor heating after many loading trials. It must be scraped off and re-crushed to produce a high-enough concentration of camphor in the beaker when heated.

Additional Techniques, $\gtrsim 10\ \mu\text{m}$

For large particles, some things can be done differently.

- Consider attaching a wire to the tungsten probe tip using an ultrasonic solderer. Running a DC current through this connection results in Ohmic heating of the probe tip, expediting the sublimation of the camphor. An early version of this setup involving two probe tips made to touch one another, is shown in Figure 191.
- A thicker coating of camphor is desirable for larger particles, and the secondary probe station for initially isolating and picking up the particle is often unnecessary. Instead, simply hover the camphor-coated tip over the desired particle, then bring it straight down on top of the particle, pressing it into the foil-covered slide.
- If too much camphor is used, sometimes it can be lodged in between the pole pieces with the particle still attached. If the tip is then heated, the probe tip can be removed and the camphor can be left to sublime on its own – this method has occasionally succeeded in trapping particles, though the limited positioning makes it inefficient.

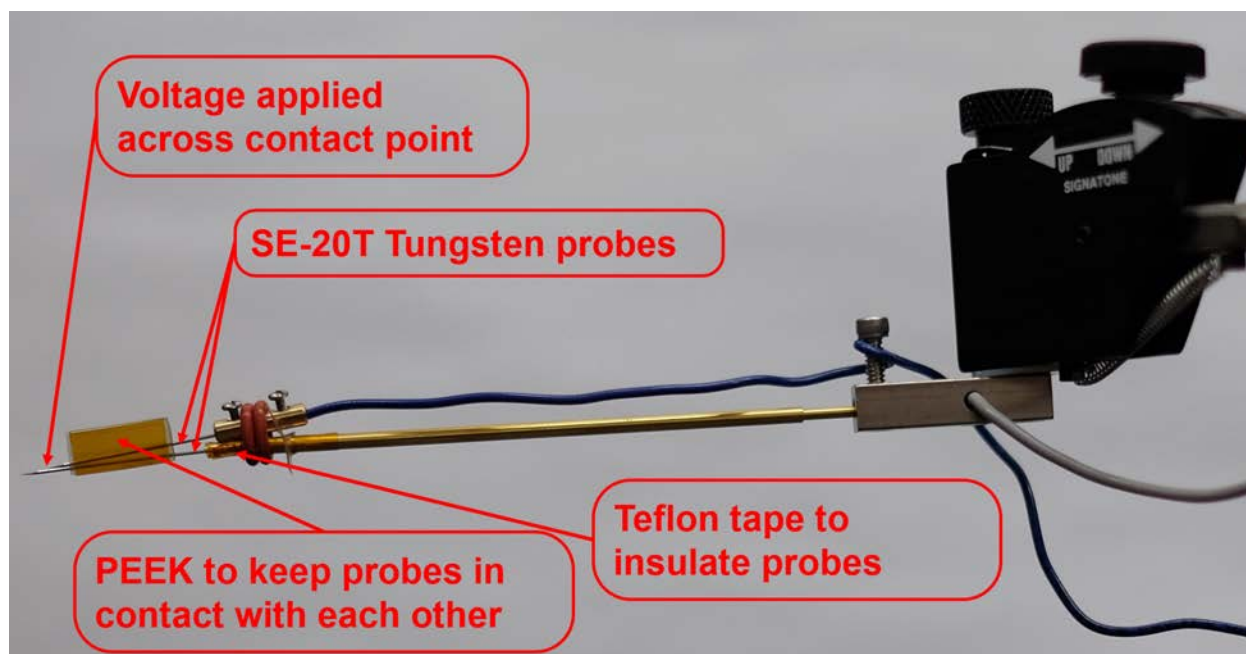


Figure 191: Loading apparatus used for heated SAR trials. The two tungsten probe tips can be replaced by a soldered wire if desired.

Additional Notes

- If the particle sticks to the tip, one of several things could be wrong:
 - The particle has too much charge.

- The camphor coating is too thin.
- The tip is not grounded.
- Consider gently tapping the micropositioner until the particle falls off. On occasion, the particle will trap when it does fall off, though this is not a high success rate.
- If the particle falls off too soon, consider pressing it in more deeply into the camphor, and note that you may simply need to practice your insertion technique to get faster. Be warned! Going too hastily may irreversibly damage a probe tip from a single collision with a hard object.
- If the pole pieces become contaminated, the micropositioner system can be outfitted with a bent and curved tungsten tip for hooking obstructions, or with a tip like the one shown in Figure 192 to be used like a pipe cleaner. This allows cleaning debris off the pole pieces without removing the trap from the vacuum chamber.



Figure 192: A piece of Kimwipe skewered on a SE-20T Signatone tungsten probe tip, used for cleaning debris off of pole pieces in magneto-gravitational traps.

Flame spheroidization

This procedure has been shown to work, but is highly time-consuming and irregular in its results. Revision is recommended upon further testing of the technique.



Figure 193: A photo of the type of flame used for flame spheroidization. Glass powder was dropped through the tip of the smaller, brighter blue flame at the center.



Figure 194: A photo of a piece of aluminum foil used to catch glass after it fell through the flame. Karger catching pieces may be beneficial, partly because many of the rounded particles tended to be found further from the flame, possibly due to a longer amount of time spent in the jet of hot air.

1. Fold up the edges of some printer paper and place a tube of glass inside. Cover with second piece, then smash with pestle. Once relatively small chunks are obtained, place in tool-steel mortar.
2. Grind the glass with pestle until it seems to be making little more of a difference, then pick out large chunks and place the remaining powder in a vile (we used a funnel made of paper to do this). Place large chunks back in and repeat. At the end, rinse the last bits out with a small amount of DI water.
3. Place the glass in a bath of HCl.
 - (a) Use roughly 1 mL of HCl per 5 mg of SiC.
 - (b) Agitate HCl acid and SiC in a glass vial.
 - (c) Place the solution on a hotplate at 100 °C without a cap for 30 minutes.
 - (d) Remove HCl.
 - (e) Repeat steps 4a-4d 2 times or until the HCl acid no longer changes color.
 - (f) Rinse the particles with DI water 2 times.
 - (g) Remove isopropanol until left with a saturated cohesive mixture.
 - (h) Place the vial on the hot plate at 50 °C without a lid for 30 minutes.
 - (i) Place the capped vial in a sonic agitator for 20 minutes to separate the particles.
4. Make a large sheet of aluminum foil to catch particles (as long as space will allow, and about a foot wide. Round up the edges. Place this below and in front of the torch to catch particles.
5. Setup a metal funnel with a squashed end over the flame location on a ringstand.
6. Pour particles in funnel slowly, adjusting position of funnel as needed to get the particles to pass through the hottest part of the flame. It is likely that the further particles will be most favorable, since they stayed in the flame the longest.
7. Carefully collect the aluminum foil and sort through the particles to try to find spheres.

SiC Particle Processing

The following procedure was largely developed and recorded by Dakota Chapman. Bulk samples of SiC can be prepared for

1. Select silicon carbide (SiC) wafer and thoroughly clean with isopropanol.
2. Crush the SiC wafer piece into particles $\sim 50\ \mu\text{m}$ in diameter using a mortar and pestle.

3. Rinse any excess SiC particles from the mortar and pestle using distilled water.
4. Run a HCl acid etch on the particles.
 - (a) Use roughly 1 mL of HCl per 5 mg of SiC.
 - (b) Agitate HCl acid and SiC in a glass vial.
 - (c) Place the solution on a hotplate at 100 °C without a cap for 30 minutes.
 - (d) Remove HCl.
 - (e) Repeat steps 4a-4d 2 times or until the HCl acid no longer changes color.
 - (f) Rinse the particles with DI water 2 times.
 - (g) Remove isopropanol until left with a saturated cohesive mixture.
 - (h) Place the vial on the hot plate at 50 °C without a lid for 30 minutes.
 - (i) Place the capped vial in a sonic agitator for 20 minutes to separate the particles.
5. Run particles through the jet mill.
 - (a) Open the check valves.
 - i. Open the feed valve (valve A) to 40 psi, then open the grind valve (valve B) to 15 psi.
 - ii. Readjust the pressures (feed pressure will drop when the grind valve is opened).
 - (b) Pour in the particles. For rounder particles, pour in the particles quickly. To decrease particle size, pour in the particles slowly.)
 - (c) Pour in one packet of white sugar and wait about 2.5 minutes.
 - (d) Close the grind valve (valve B) and then close the feed valve (valve A).
 - (e) Rinse all material from the jet mill into the collection container before removing it.
 - (f) Use a pipet to transfer the particles to a new glass
 - (g) Remove DI water.
 - (h) Rinse the particles with DI water to remove the sugar at least 2 times until water is clear.
 - i. Fill the vial with water and agitate the particles.
 - ii. Allow time for the particles to settle at the bottom of the vial, about 5 minutes.
 - iii. Pipet out the water.
6. Run another HCl Acid Etch on the particles to remove metal contaminants. (Repeat step 4).

7. Repeat Jet Mill (Step 5) and then HCl etch (Step 4) until the particles are satisfactory (the particles are the desired size or roundness). On average, it takes 2-3 jet millings to achieve satisfactory roundness.

Additional Notes

Graphite Particle Fabrication

For initial graphite-composite fabrication trials, a roll of high-purity carbon tape which was known to be diamagnetic was ground up using a coffee grinder to produce a fine powder. However, the steel blades of the coffee grinder appeared to contaminate the powder too much for it to be usable, as it became paramagnetic (if not ferromagnetic) after this form of processing.

Subsequent trials used a fine graphite powder (Nanografi NG04EO072379) with a grain size of 1-5 μm . The powder was mixed with high-temperature epoxy (Epo-Tek 353ND) , then shaped and cured, to produce graphite-composite objects. The rough procedure for this process, developed with the assistance of Tahereh Naderishahab, is outlined below:

1. The epoxy is mixed and placed in a wide dish (a few inches across) that has been tared so as to measure the mass of the epoxy.
2. Sufficient powder is measured out carefully (so as to avoid contamination) to match the measured weight of the epoxy. Successful mixtures of both 1:1 graphite:epoxy and 2:1 graphite to epoxy mixtures (by volume) were both performed.
3. The graphite is slowly added to the epoxy until it starts to behave more like a wet powder than a liquid (this was typically after significantly less than half the powder was added. At this point the remainder of the powder is added.
4. The graphite is cut into the epoxy using a method similar to cutting butter into pie dough. For 2:1 graphite:epoxy, it was necessary to wet the powder with isopropanol during this step. This is done until the powder looks homogeneous. In addition to cutting the mixture, it is repeatedly packed with a spoon then cut again to break up the chunks that are formed due to light packing. This helps to mix in clumps of epoxy.
5. The resultant mixture at this point is very powdery until packed, at which point it self-adheres somewhat. It is now ready to be packed and/or extruded.
6. The powder can be packed into a metal-sleeved mechanical pencil (or plastic-sleeved with somewhat less regular results) using the following method:
 - (a) Depress the clicker to release the lead in the pencil, then press down into the powder. Tap about 10 times to press the powder up into the sleeve, pushing the lead back into the pencil. Tap more times for a longer particle.

- (b) Release the clicker and tap about 10 more times to pack the powder in further and more densely. Tap more times to get denser particles, at the risk of jamming the pencil.
 - (c) Hold the pencil tip over a collection unit (such as a glass vial) and click the clicker until the pencil lead forces the particle out. Gently brush with finger to release from lead if needed.
7. The particles are then placed in an oven (Cole-Parmer OVF-400-80-120) for 1 hour at 80 °C. Ideally this should be done no later than two hours after the epoxy is mixed.
 8. It is NOT recommended that the particles be cleaned in the ultrasonic bath after curing, as they may disintegrate.
 9. Sheets of graphite composite may be formed by pressing the wet powder between two hard flat surfaces. A reliable method for high-pressure extrusion of cylinders is desirable but has not yet been achieved.

Pole Piece Fabrication

Tahereh Naderishahab is primarily responsible for the development of the pole piece fabricating process. However, a brief description of the current process is included here for completeness.

1. Two Hipercos powders (FUSHEL Alloy Cobalt Powder Co 46% Fe 54% and NANOHEMA-ZONE Cobalt Iron Alloy Powder – 7440-48-4) with different grain sizes are mixed together.
2. The Hipercos powder mixture is annealed as described elsewhere in this appendix (this step is necessary – without it, the pole pieces are much weaker).
3. The powder is mixed together with high-temperature epoxy (Epo-Tek 353ND) in a 2:1 Hipercos:epoxy ratio (by volume). The difference in grain sizes for the two powders allows a random closest packing ratio similar to this value. [162].
4. The resultant powder is denser and somewhat wetter than the graphite/epoxy powder, but still more like wet sand than liquid. It also is typically mixed in more gradually, and the last half of the mixing is more akin to kneading dough than cutting pie dough. IT is done with gloved fingers.
5. The TPU-molds are coated with a non-stick spray, then held together loosely with C-clamps (contact Tahereh Naderishahab for details). The Hipercos/epoxy powder is then scooped into the molds and packed in with small plastic rods fitted to the mold openings. It has been found that packing the powder with many small taps of a small hammer works better than a few harder taps.

6. The pole pieces are then placed in an oven (Cole-Parmer OVF-400-80-120) for 1 hour at 80 °C. Ideally this should be done no later than two hours after the epoxy is mixed.
7. The pole pieces are then lapped to the desired final shape.
8. Aquedag (2:1 water:Aquedag mixture) can be painted onto the pole pieces to increase surface conductivity in order to mitigate the effects of patch charges.
9. The pole pieces can then be cured in the furnace for about 24 hours as detailed elsewhere in this appendix to prevent outgassing.

Pencil Lead Tumbling

This process is still under development. However, what follows is a description of the current techniques used.

1. Use a 3" hole puncher to cut a circle out of a piece of paper (cardstock tends to be less noisy and behave better). Fold in half, then center by eye and cut out a hole with a 1" puncher. Do this twice, then place in top and bottom pieces of the tumbler.
2. Break the pencil lead into the desired length, about 3-4 mm. Place wall and center piece into tumbler, then place 1-2 particles in the trough.
3. Clamp the tumbler shut with the four bolts on the corners. Consider applying pressure to the middle of the tumbler using a C-clamp.
4. Connect the air to the bottom, making sure that the tumbler is upright.
5. Open the main air valve and other valves all the way except for the valve on the flow meter. Slowly adjust that valve until the flow reaches the desired rate (needs to be refined – currently flow rate is increased until just below the point when the tumbler starts screeching).
6. The duration for good tumbling results is typically on the order of 10-60 minutes, but still needs significant testing and refinement.

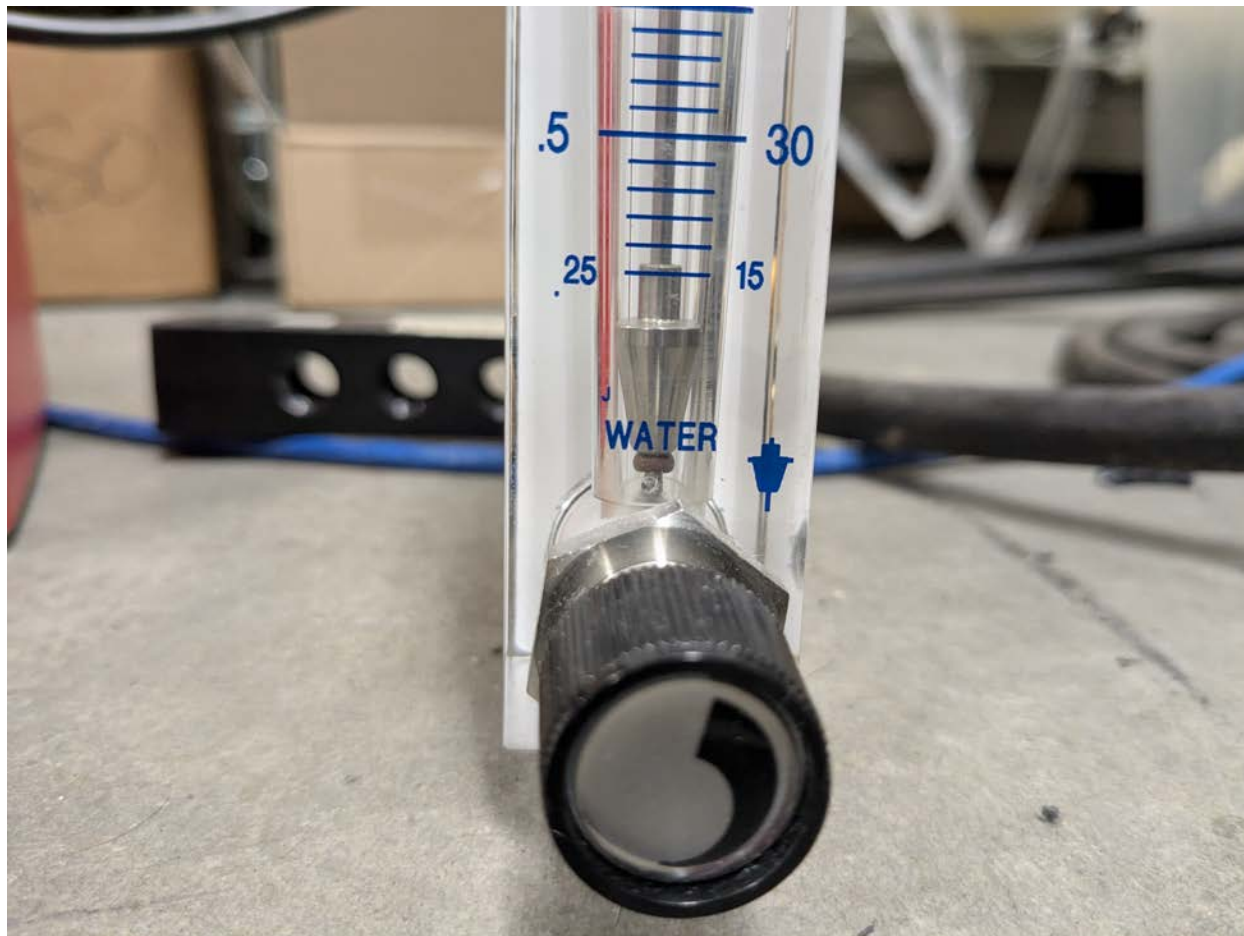


Figure 195: Zoomed in photo of the flowmeter indicator. When flow rates are reported, care must be taken to specify which part of the indicator is aligned with the reported value.



Figure 196: The flow meter used to determine the flow of air for the tumbler. It is designed to be used with water, but could in principle be calibrated to get an absolute measurement of the flow rate for air.



Figure 197: In image of the tumbler components disassembled. The taped-together Thorlabs boxes on the left hold the tumbler upright while the black tube is inserted through the hole in the bottom plate. The filters on the right help regulate pressure and keep the tumbler clean.



Figure 198: Photo of the hole punchers used to cutout pieces of printer paper for use in the pencil lead tumbler. The large green cutter is used first, then the piece is folded over and the gray cutter is used to cut out a half circle from the folded piece, which can then be unfolded.