



A mathematical model of laser ablation applied to ultrasonics in liquid mercury
by Stephen Eugene Garwick

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Mechanical Engineering
Montana State University
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Abstract:

When a material is irradiated by a pulsed laser, an ultrasonic wave is generated in the material. This ultrasonic wave is the culmination of two processes, rapid thermal expansion and ablation. Rapid thermal expansion occurs due to the material absorbing a portion of the laser's intensity, thereby heating a small surface layer of the material rapidly. This, in turn causes large thermal gradients which generate an ultrasonic impulse. Ablation occurs at high laser intensities when the surface layer of the material is given enough energy to cause vaporization. The recoil momentum from the escaping vapor is the cause of ultrasound generation in this case.

Ablation is able to produce a much stronger ultrasonic signal than rapid thermal expansion. However, the testing of solids using ablation is not a non-destructive method of testing. Since ablation is generally not a problem with liquids, it has applications in molten metal processes. Since such a large increase in signal strength is obtained through ablation, a greater understanding of the ablation process is desired.

This work examines the ablation process and develops a quasi-two-dimensional model. The model is compared with experimental results for liquid mercury.

Comparison between the theory and results indicate that the model qualitatively agrees with the experimental data.

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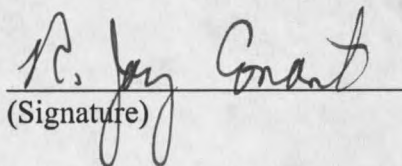
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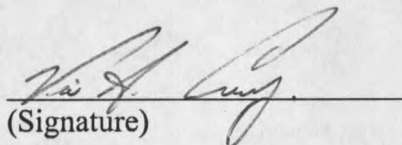
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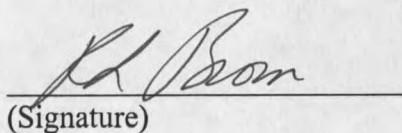
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ABSTRACT

When a material is irradiated by a pulsed laser, an ultrasonic wave is generated in the material. This ultrasonic wave is the culmination of two processes, rapid thermal expansion and ablation. Rapid thermal expansion occurs due to the material absorbing a portion of the laser's intensity, thereby heating a small surface layer of the material rapidly. This, in turn causes large thermal gradients which generate an ultrasonic impulse. Ablation occurs at high laser intensities when the surface layer of the material is given enough energy to cause vaporization. The recoil momentum from the escaping vapor is the cause of ultrasound generation in this case.

Ablation is able to produce a much stronger ultrasonic signal than rapid thermal expansion. However, the testing of solids using ablation is not a non-destructive method of testing. Since ablation is generally not a problem with liquids, it has applications in molten metal processes. Since such a large increase in signal strength is obtained through ablation, a greater understanding of the ablation process is desired.

This work examines the ablation process and develops a quasi-two-dimensional model. The model is compared with experimental results for liquid mercury. Comparison between the theory and results indicate that the model qualitatively agrees with the experimental data.

CHAPTER 1

INTRODUCTION

Non-destructive evaluation of materials using ultrasonic detection has been a successful reality for the past 3 decades. Ultrasonic testing provides information about a material's properties, such as grain size and porosity. It also detects small cracks or other flaws in a material in a non-destructive manner. This gives it advantages over other testing methods such as tension tests, which destroy the specimen.

Ultrasound is generated in a material when the material is subjected to an impulse force. The impulse force creates a molecular disturbance in the material, which creates an ultrasonic wave. The ultrasonic wave is altered by the material characteristics as it travels through the material. These characteristics include flaws, such as impurities and cracks, grain boundaries and grain size. By analyzing the wave after it passes through the material, one can obtain information about the material through non-destructive means.

The most common way of generating ultrasound in a material is with a piezoelectric transducer. The piezoelectric transducer, or PZT, undergoes strain when subjected to an electrical current and, conversely, creates an electric current when it is strained. By placing the PZT in contact with the material, the strain can be transferred to the material. One can think of the PZT as being a small hammer which hits the surface of the material, creating an impulse force. To improve the contact between molecules of the

the PZT and the material, the transducer is often coupled to the material via a liquid couplant or gel couplant, which provides a better medium for the small strain to transfer from the PZT to the material. Piezo-electric transducers provide a relatively good signal-to-noise ratio since the PZT is in contact with the material.

Another method of ultrasound generation is to use pulsed lasers. Lasers have several advantages over PZT techniques. Lasers provide a non-contacting method of ultrasonic testing. Therefore, there is no need for a couplant between the material and the ultrasonic source generator. This allows one to make ultrasonic measurements from a distance. This is advantageous in environments which would prove too hazardous for PZT techniques. For example, molten metal processes cannot be monitored by contacting techniques, but a non-contacting method using lasers makes ultrasonic measurements possible.

There are two mechanisms for creating an ultrasonic wave with lasers. At low laser intensities, the incident laser light is absorbed by a thin layer of the material at the surface. The absorbed energy causes the layer to undergo rapid thermal expansion, which creates an ultrasonic pulse. Rapid thermal expansion causes no surface damage in solids, but it produces a relatively weak signal.

The weak signal strength associated with rapid thermal expansion is one of the disadvantages of laser ultrasonics. In comparison with PZT techniques, laser ultrasonics produce a much weaker signal and thus a low signal to noise ratio. There are several ways to increase the signal to noise ratio. Some methods include using multiple lasers to

create multiple signals in the material. This creates a phased array of signals which have the effect of producing a much larger net input signal. However, this method is very costly and difficult to perform. Other methods use a single laser source modulated in time to produce a series of ultrasonic pulses. The detection device can be tuned to a narrower range of frequencies corresponding to the time modulation of the laser source. Still other ways focus not on the generation or detection of the ultrasound, but on advanced signal processing techniques.

Another way to increase the signal strength is to simply increase the laser intensity. Higher laser intensities cause some of the material to vaporize and be ejected from the surface. The ultrasonic wave is generated as the ablated vapor transfers some of its momentum to the material. The ablation mechanism creates a much stronger ultrasonic pulse than rapid thermal expansion, but it does damage the surface of the material. Consequently, ablation is not an attractive method of ultrasound generation in solids. However, ablation is generally not a problem in liquids. Therefore, ultrasound generation through ablation is an attractive technique in liquid processes such as molten metal applications. These include casting processes and weld monitoring.

There is no extra cost involved with ultrasound generation through ablation as opposed to other laser-based methods. Since the signal can be increased simply by increasing the laser intensity, a greater understanding of the ablation process is desired.

Background

The creation of ultrasonic waves using pulsed lasers was first demonstrated by White [1] in 1963. Several years later, Aindow *et al.* [2] showed the ablative effects of high intensity pulsed lasers on elastic waves. Using an Nd:YAG pulsed laser, Aindow *et al.* showed that a plasma forms in the ablation regime which leads to an enhancement of the longitudinal ultrasonic pulse. This enhancement was thought to be caused by a momentum transfer due to the ablated particles, which were ejected from the surface of the material.

Several scientists have modeled the ablative process in recent decades. Von Allmen [3] developed a one-dimensional, steady-state model for the heat transfer in materials which took into account liquid expulsion and vaporization. Later, Knight [4] formulated a one-dimensional model for steady-state flow with back pressure, taking into account the presence of a Knudsen layer adjacent to the material. Since thermodynamic equilibrium does not occur in this layer, Knight used a Mott-Smith type solution developed by Anisimov [5] to obtain jump conditions from which ratios of temperature and density could be found. Knight's model calculated the conditions for each region in the flow. His primary interest was in metallic surfaces.

Zweig [6] further developed Knight's model by concentrating on the liquid layer as it is ablated. Zweig also had an interest in the laser drilling velocity. In addition to

calculating the conditions in each flow region, Zweig also formulated an expression for the pressure at the interface of the liquid in terms of a laser intensity and flow conditions.

Recently, Telschow, Walters, and Conant [7] performed a series of experiments on liquid mercury. Mercury is a substance which is liquid at room temperature. Furthermore, its properties are well known. The scientists measured the ultrasonic pulse amplitude given a particular output laser intensity. They found that the signal strength was increased approximately two orders of magnitude in the ablation regime over that of the thermoelastic regime. Furthermore, a simple threshold model was developed to correspond with the experimental data. This model had two free parameters which were chosen by matching two points on the experimental curve.

Objective

Since such a large increase in signal strength was obtained simply by increasing the laser intensity, a greater understanding of the process of ablation as an ultrasonic generating mechanism was desired. Although the simple model developed by Telschow, Walters, and Conant [7] fit the data reasonably well, a model developed from fundamental principles and describing the ablation process in two dimensions would give a better description of what happens physically as the material is ablated. This thesis develops a model which predicts the average acoustic pressure for a given laser intensity. The results are then compared to the experiments carried out at the Idaho National Engineering Laboratory.

CHAPTER 2

EXPERIMENTAL SETUP

The experimental setup for detecting ultrasound in liquid mercury is shown in Figure 1. Mercury was enclosed in a glass container and subjected to a pulse on the order of 10 ns from a 1064 nm Nd-YAG pulsed laser. At the bottom of the container, a 10 MHz pinducer detected the resulting ultrasonic pulse. A series of experiments was performed, each with an increased laser intensity. The resulting signal amplitude from the pinducer was recorded for each laser intensity. This series of experiments was performed twice, once with a laser spot diameter of 2.3 mm and once with a laser spot diameter of 1.4 mm.

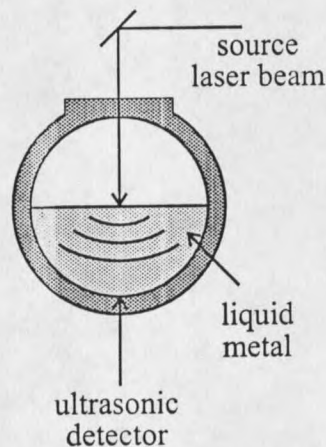


Figure 1. Experimental setup for liquid mercury ultrasonic experiments.

The results of the series of experiments are shown in Figure 2. The signal amplitude, normalized with respect to the spot area, is plotted against the output laser

intensity, or power density. At low laser intensities, rapid thermal expansion is the mechanism responsible for the resulting signal. As the laser intensity is increased, there is a transition from a thermal response to an ablative response. For mercury, this happens between about 6 MW/cm^2 and 20 MW/cm^2 . Ablation is the dominant mechanism at high laser intensities. Note that there is approximately a 2 order of magnitude increase in signal strength in the ablative regime over that of the thermoelastic regime.

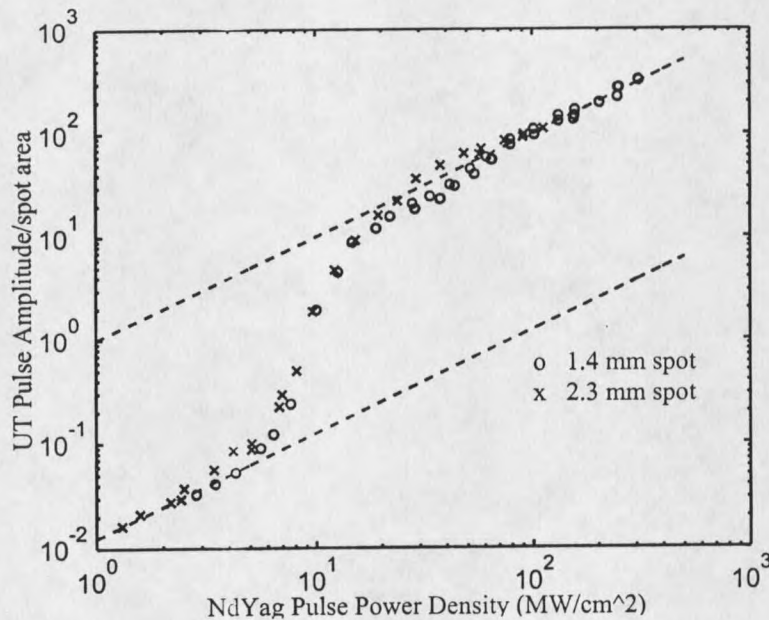


Figure 2. Experimental results for ultrasound in liquid mercury.

In short, an increase in signal strength of approximately two orders of magnitude was obtained simply by increasing the laser intensity. Operating in the ablation regime therefore warrants further investigation as a simple way to obtain a greater signal-to-noise ratio for ultrasonic testing in liquids.

CHAPTER 3

PROBLEM FORMULATION

Previous researchers modeling gasdynamic flows generally developed one-dimensional models as a simple method of describing the physics of the flow. A one-dimensional flow is a flow in which the rate of change of the fluid properties normal to the streamline direction is negligible compared to the rate of change along the streamlines. In the case of laser ablation, the velocity of the expanding vapor is much greater in the direction normal to the surface than the velocity of the vapor parallel to the surface. Although the true flow expands outward in three dimensions, it can be treated as one-dimensional for this reason. This work will use the one-dimensional model formulated by Knight and further developed by Zweig in order to gain an understanding of the physics behind laser ablation in liquid mercury.

When the laser energy reaches the surface of the material, it is assumed that the saturation temperature is immediately reached and that evaporation occurs. In other words, a steady state condition exists, with no transient portion. Furthermore, the vapor ablated from the surface of the liquid is assumed to be an ideal, monatomic gas. Indeed, when applying the ideal gas law to published saturated vapor data for mercury [8, p. 680], the absolute error between the calculated value of the gas constant and the actual value of the gas constant is less than 1.7%. Since the saturated vapor condition is the condition

when the material acts least like an ideal gas, the error can be expected to decrease for the expanding mercury vapor. Thus, the ideal gas assumption is acceptable.

As the material is ablated, some of the momentum from the expanding vapor is transferred to the surface of the material, resulting in a pressure on the surface. Since the pulse duration of the laser is very short, the pressure creates an impulse on the surface of the material. This impulse results in the creation of an ultrasonic wave which propagates through the material.

The expanding vapor pushes outward on the surrounding air, creating a shock wave in the air which moves outward with time. Since convective evaporation dominates diffusive evaporation for laser intensities which cause ablation [4], there is no mixing between the expanding vapor and the expanding air behind the shock wave. Therefore, a contact discontinuity separates the expanding vapor and the compressed air behind the shock wave. Across this discontinuity, the temperature and density are discontinuous because the flow transitions from the expanding vapor to expanding air with no mixing of the atoms. However, the pressure and flow speeds are continuous across the contact discontinuity.

As the pressure of the expanding vapor becomes larger than ambient, i.e. strong evaporation occurs, there are relatively few back-scattered particles in a thin region at the surface of the liquid. In this region, thermodynamic equilibrium does not exist due to the random motion of the particles. However, thermodynamic equilibrium is obtained within a few molecular mean free paths by collisions of the particles. This region has been

modeled as a Knudsen layer and treated as a discontinuity across which jumps in temperature and density can be found.

In the derivations that follow, the subscripts are denoted as follows: 0 refers to the ambient air, a refers to the air behind the shock wave, v refers to the vapor region, k refers to the Knudsen layer exit plane, l refers to the liquid, and s refers to the saturated vapor plane.

General Formulation

The flow regions for Knight's model are shown in Figure 3. A portion of the laser intensity which is absorbed by the material raises the temperature of the liquid to the saturation temperature. This portion is denoted I_{therm} . The remaining portion causes ablation of the material and is denoted I_{evap} . The absorbed intensity, I_{abs} , is then

$$I_{\text{abs}} = I_{\text{therm}} + I_{\text{evap}} \quad (1)$$

The output laser intensity, I_{laser} , is actually greater than the absorbed intensity because losses occur as the light passes through glass and the vapor itself, and because some light is reflected by the sample surface. Thus,

$$I_{\text{laser}} = \frac{I_{\text{abs}}}{1 - R} + I_{\text{loss}} \quad (2)$$

where R is the reflection coefficient for the material, defined as [9]

$$R = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}. \quad (3)$$

In the above equation, n is the index of refraction, and k is the extinction coefficient.

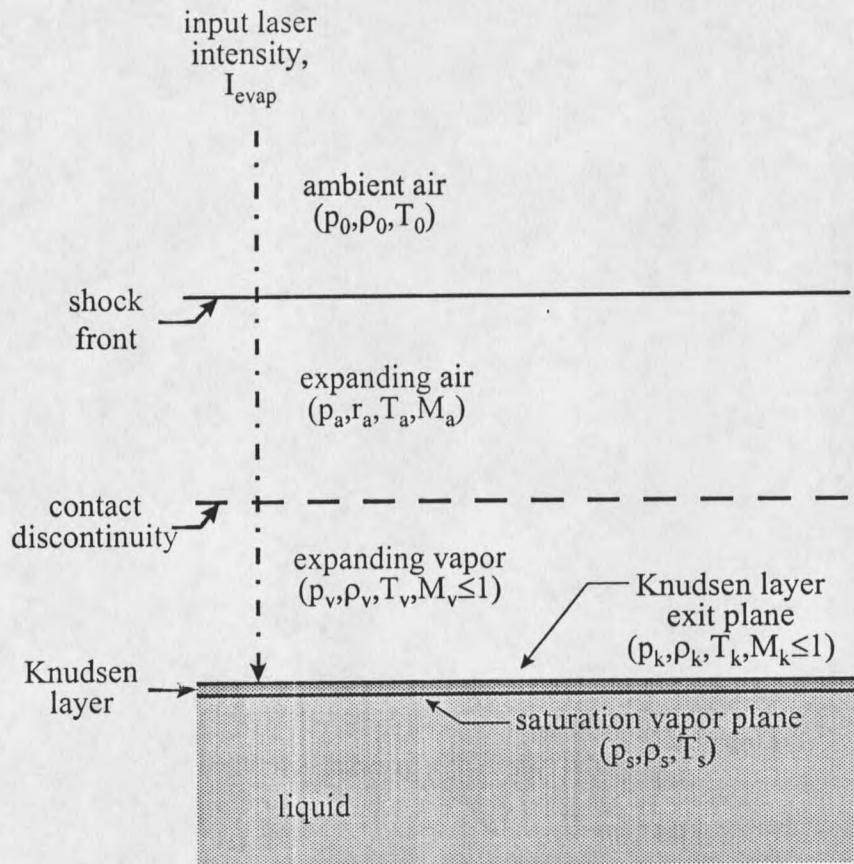


Figure 3. Flow regions for $M_v < 1$.

When the evaporation rate at the liquid surface is large, as is the case when a laser is causing ablation, the gas near the phase interface is not in translational equilibrium. However, translational equilibrium is achieved in a few molecular mean free paths by

collision of particles in a Knudsen layer region [4]. The Knudsen layer is modeled as a gasdynamic discontinuity across which jump conditions are derived from conservation of mass, momentum, and energy. Knight treated the flow as locally one-dimensional, which is usual for a gasdynamic discontinuity. Since translational equilibrium had to be reached at the outer edge of the Knudsen layer, Knight assumed a Maxwellian distribution function for the flow at the Knudsen layer exit plane. After proceeding in a Mott-Smith approach for rapidly changing flow states, he obtained the following jump conditions [4]:

$$\frac{T_k}{T_s} = \left[\sqrt{1 + \pi \left(\frac{\gamma_v - 1}{\gamma_v + 1} \frac{m}{2} \right)^2} - \sqrt{\pi} \frac{\gamma_v - 1}{\gamma_v + 1} \frac{m}{2} \right]^2 \quad (4)$$

$$\frac{\rho_k}{\rho_s} = \sqrt{\frac{T_s}{T_k}} \left[\left(m^2 + \frac{1}{2} \right) e^{m^2} \operatorname{erfc}(m) - \frac{m}{\sqrt{\pi}} \right] + \frac{1}{2} \frac{T_s}{T_k} \left[1 - \sqrt{\pi} m e^{m^2} \operatorname{erfc}(m) \right]. \quad (5)$$

T_k is the temperature at the Knudsen layer exit plane, T_s is the temperature in the saturation plane, ρ_k is the density of the vapor at the Knudsen layer exit plane, ρ_s is the density of the saturated vapor, and m is a parameter defined as

$$m = \sqrt{\frac{\gamma_v}{2}} M_k. \quad (6)$$

M_k is the Mach number at the Knudsen layer exit plane, and γ_v is the ratio of specific heats in the vapor phase. The Mach number, M , is defined as

$$M = \frac{u}{a} \quad (7)$$

where u is the fluid velocity, and a is the local sound speed given by

$$a = \sqrt{\gamma \mathcal{R}T}. \quad (8)$$

γ is the ratio of specific heats, T is the temperature, and R is the gas constant for an ideal gas.

Eqs. (4) and (5) can be rewritten by replacing the temperatures and pressures with ratios. Let

$$r_\rho = \frac{\rho_k}{\rho_s}, \text{ and } r_T = \frac{T_k}{T_s}. \quad (9)$$

The Knudsen layer equations then become

$$r_T = \left[\sqrt{1 + \pi \left(\frac{\gamma_v - 1}{\gamma_v + 1} \frac{m}{2} \right)^2} - \sqrt{\pi} \frac{\gamma_v - 1}{\gamma_v + 1} \frac{m}{2} \right]^2 \quad (10)$$

$$r_\rho = \sqrt{\frac{1}{r_T}} \left[\left(m^2 + \frac{1}{2} \right) e^{m^2} \operatorname{erfc}(m) - \frac{m}{\sqrt{\pi}} \right] + \frac{1}{2} \frac{1}{r_T} \left[1 - \sqrt{\pi} m e^{m^2} \operatorname{erfc}(m) \right] \quad (11)$$

The recoil momentum from the expanding vapor creates a pressure pulse which travels through the liquid. The pressure on the liquid surface can be found by taking a momentum balance across a control volume that spans the liquid surface and the Knudsen layer (Figure 4), and moves with the liquid surface as the material is ablated. The position of the liquid surface from a fixed reference is denoted D_l . Therefore, the velocity with which the evaporation front moves is $\dot{D}_l$. Conservation of momentum gives

$$p_l + \rho_l u_l^2 = p_k + \rho_k u_k^2, \quad (12)$$

where p is the pressure, the subscript l refers to the liquid state, and the subscript k refers

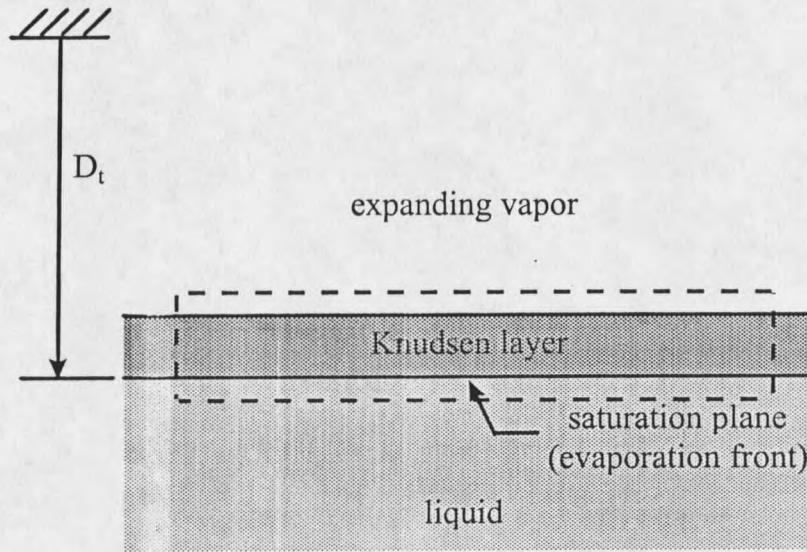


Figure 4. Control volume used for calculating the liquid pressure.

to the Knudsen layer exit plane. The liquid velocity entering the control volume, u_l , is $\dot{D}_t$, and the velocity of the vapor leaving the Knudsen layer, u_k , can be expressed in terms of the Mach number with Eq. (7). Thus, Eq. (12) can be written as

$$p_l + \rho_l \dot{D}_t^2 = p_k + \rho_k M_k^2 a_k^2. \quad (13)$$

The mass balance across the control volume can be written as

$$\rho_l \dot{D}_t = \rho_k M_k a_k. \quad (14)$$

Eq. (13) can be solved for $\dot{D}_t$ and the result used in Eq. (12) to give

$$p_l + \rho_l \left(\frac{\rho_k}{\rho_l} M_k a_k \right)^2 = p_k + \rho_k M_k^2 a_k^2.$$

This can be rearranged by combining terms, and Eq. (7) can be used to eliminate a_k .

Thus,

$$p_l = p_k + \rho_k M_k^2 \gamma_k \mathcal{R}_k T_k \left(1 - \frac{\rho_k}{\rho_l} \right).$$

Typically, $\rho_k/\rho_l \ll 1$, since the density of the vapor phase is much less than that of the liquid phase. Therefore, the density ratio can be neglected, and the following equation results:

$$p_l = p_k + \rho_k M_k^2 \gamma_k \mathcal{R}_k T_k. \quad (15)$$

Since each region can be treated as an ideal gas,

$$p_k = \rho_k \mathcal{R}_k T_k. \quad (16)$$

Eqs. (15) and (16) can be combined to obtain

$$p_l = p_k (1 + \gamma_v M_k^2) \quad (17)$$

Therefore, once the conditions in the Knudsen layer exit plane are known, the pressure exerted on the liquid can be found.

The expanding vapor from the Knudsen layer exit plane pushes the air above the surface outward. This sudden expansion of air creates a shock wave which moves outward with time. A flow diagram, in which x is the distance from the surface of the liquid and t represents time, is shown in Figure 5. Separating the expanding vapor and the air behind the shock wave is the vapor-air interface. Across the interface, the pressure and velocity are continuous but the temperature and density are discontinuous [10, p340]. The interface is therefore referred to as a contact discontinuity.

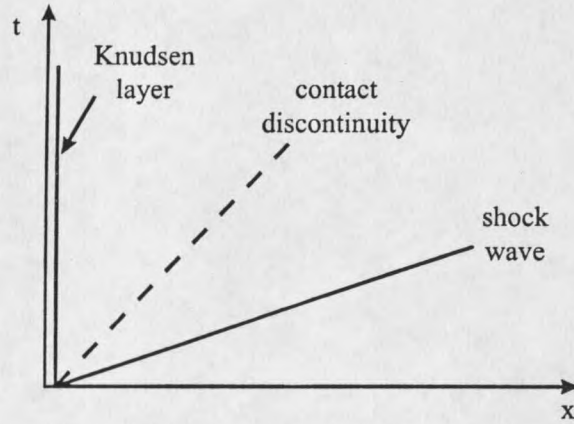


Figure 5. Subsonic x-t flow diagram.

The velocity of the airflow behind the shock wave can be expressed by a Rankine-Hugoniot relation [11, p64] as

$$u_a = \frac{a_0 \left(\frac{p_a}{p_0} - 1 \right)}{\gamma_0 \sqrt{1 + \frac{\gamma_0 + 1}{2\gamma_0} \left(\frac{p_a}{p_0} - 1 \right)}}. \quad (18)$$

Since velocity and pressure across the vapor-air interface are continuous,

$$u_a = u_v \text{ and } p_a = p_v.$$

After substitution of the above into the Rankine-Hugoniot relation, Eq. (18) becomes

$$u_v = \frac{a_0 \left(\frac{p_v}{p_0} - 1 \right)}{\gamma_0 \sqrt{1 + \frac{\gamma_0 + 1}{2\gamma_0} \left(\frac{p_v}{p_0} - 1 \right)}}.$$

The vapor Mach number is introduced on dividing by a_v and using Eq. (7):

$$M_v = \frac{u_v}{a_v} = \frac{\frac{a_0}{a_v} \left(\frac{p_v}{p_0} - 1 \right)}{\gamma_0 \sqrt{1 + \frac{\gamma_0 + 1}{2\gamma_0} \left(\frac{p_v}{p_0} - 1 \right)}}. \quad (19)$$

Eq. (18) can be rewritten upon substituting Eqs. (8) and (16) for the vapor region:

$$M_v = \frac{\frac{a_0}{\gamma_0} \left(\frac{\rho_v \mathcal{R}_v T_v}{p_0} - 1 \right)}{\sqrt{\gamma_v \mathcal{R}_v T_v} \sqrt{1 + \frac{\gamma_0 + 1}{2\gamma_0} \left(\frac{\rho_v \mathcal{R}_v T_v}{p_0} - 1 \right)}}.$$

This equation is now expanded to read:

$$M_v = \frac{\frac{a_0}{\gamma_0} \left(\frac{\rho_v}{\rho_k} \frac{\rho_k}{\rho_s} \frac{T_v}{T_k} \frac{T_k}{T_s} \frac{\rho_s \mathcal{R}_v T_s}{p_0} - 1 \right)}{\sqrt{\gamma_v \mathcal{R}_v \frac{T_v}{T_k} \frac{T_k}{T_s} T_s} \sqrt{1 + \frac{\gamma_0 + 1}{2\gamma_0} \left(\frac{\rho_v}{\rho_k} \frac{\rho_k}{\rho_s} \frac{T_v}{T_k} \frac{T_k}{T_s} \frac{\rho_s \mathcal{R}_v T_s}{p_0} - 1 \right)}}. \quad (20)$$

With the ratios

$$R_\rho = \frac{\rho_k}{\rho_v} \text{ and } R_T = \frac{T_k}{T_v}, \quad (21)$$

Eq. (20) can be written as

$$M_v = \frac{\frac{a_0}{\gamma_0} \left(\frac{r_\rho}{R_\rho} \frac{r_T}{R_T} \frac{\rho_s \mathcal{R}_v T_s}{p_0} - 1 \right)}{\sqrt{\gamma_v \mathcal{R}_v \frac{r_T}{R_T} T_s} \sqrt{1 + \frac{\gamma_0 + 1}{2\gamma_0} \left(\frac{r_\rho}{R_\rho} \frac{r_T}{R_T} \frac{\rho_s \mathcal{R}_v T_s}{p_0} - 1 \right)}}. \quad (22)$$

The gas constant $\mathcal{R}$ for a material is the same whether the material is in a vapor state or a saturated vapor state. Therefore, $\mathcal{R}_v = \mathcal{R}_s$. The ideal gas law can then be used in Eq. (22) to replace $\rho_s \mathcal{R}_s T_s$ with p_s . Eq. (22) then takes the form

$$M_v = \frac{\frac{a_0}{\gamma_0} \left(\frac{1}{p_0} \frac{r_\rho}{R_\rho} \frac{r_T}{R_T} p_s - 1 \right)}{\sqrt{\gamma_v \mathcal{R}_v \frac{r_T}{R_T} T_s} \sqrt{1 + \frac{\gamma_0 + 1}{2\gamma_0} \left(\frac{1}{p_0} \frac{r_\rho}{R_\rho} \frac{r_T}{R_T} p_s - 1 \right)}} \quad (23)$$

An equation relating the saturation pressure to the evaporation intensity can be found by considering a control volume fixed at the evaporation front and spanning the Knudsen layer as shown in Figure 4. An energy balance for the control volume results in

$$I_{evap} + \rho_l \dot{D}_l \left(H_l + \frac{1}{2} \dot{D}_l^2 \right) = \rho_k M_k a_k \left(H_v + \frac{1}{2} (M_k a_k)^2 \right), \quad (24)$$

where H_l and H_v are the specific enthalpies of the saturated liquid and saturated vapor phase of the material, respectively. Since the control volume is assumed to be very thin, the potential energy terms have been neglected. Conduction can also be neglected because laser pulses are on the order of nanoseconds, leaving no time for conduction to carry heat throughout the material.

When Eq. (14) is used to substitute for $\dot{D}_l$, Eq. (24) becomes

$$I_{evap} + \rho_k M_k a_k \left(H_l + \frac{1}{2} \left(\frac{\rho_k M_k a_k}{\rho_l} \right)^2 \right) = \rho_k M_k a_k \left(H_v + \frac{1}{2} (M_k a_k)^2 \right).$$

After simplification, the above equation reduces to

$$I_{evap} = \rho_k M_k a_k \left(H_v - H_l + \frac{1}{2} (M_k a_k)^2 \left(1 - \left(\frac{\rho_k}{\rho_l} \right)^2 \right) \right)$$

Once again, the density ratio can be neglected because $\rho_k/\rho_l \ll 1$. I_{evap} is then expressed as

$$I_{evap} = \rho_k M_k a_k H_{lv}, \quad (25)$$

where

$$H_{lv} = H_v - H_l + \frac{1}{2} (M_k a_k)^2 \quad (26)$$

The term $\rho_k M_k a_k$ can be expanded by substituting Eq. (8) for a_k and multiplying and dividing by T_s and ρ_s :

$$\rho_k M_k a_k = \frac{\rho_k}{\rho_s} \rho_s M_k \sqrt{\gamma_k \mathcal{R}_k \frac{T_k}{T_s} T_s}$$

The ideal gas law is substituted for ρ_s , and Eqs. (9) are also applied. This results in

$$\rho_k M_k a_k = r_\rho \frac{p_s}{\mathcal{R}_s T_s} M_k \sqrt{\gamma_k \mathcal{R}_k r_T T_s}. \quad (27)$$

The specific heat ratio γ_k can be replaced by γ_v . Eq. (27) can then be written as

$$\rho_k M_k a_k = r_\rho p_s M_k \gamma_v \sqrt{\frac{r_T}{\gamma_v \mathcal{R}_v T_s}}. \quad (28)$$

After substitution of Eq. (28) into Eq. (25), the saturation pressure can be expressed in terms of the laser intensity I_{evap} and saturation temperature as

$$p_s = \frac{I_{evap}}{\gamma_v M_k H_{lv}} \frac{1}{r_\rho} \sqrt{\frac{1}{r_T}} \sqrt{\gamma_v \mathcal{R}_v T_s}. \quad (29)$$

Another equation relating the saturation pressure to the saturation temperature is found by using the Clasius-Clapeyron equation, which gives

$$p_s = P^* e^{\frac{r^*}{T_s}}, \quad (30)$$

where P^* and T^* are constants. The values of these constants are found in the same manner as Zweig [6] by fitting published saturation data for the material with the Clasius-Clapeyron equation. With values of $P^* = 5.486 \times 10^4$ atm and $T^* = 6915.96$ K, Eq. (30) predicts the published saturation pressure values for mercury [8, p. 680] to within 1.2%.

Eqs. (6), (10), (11), (23), (29), and (30) contain 9 unknown quantities: r_T , r_p , R_T , R_p , p_s , T_s , M_k , M_v , and m . Three more equations are therefore needed to determine all of the unknowns.

Subsonic Flow

For relatively low laser intensities, the Mach number of the vapor coming off the Knudsen layer is less than one. In this case, the state throughout the vapor region is uniform and therefore conditions at the Knudsen layer exit plane are the same as the conditions in the vapor. Thus,

$$R_T = 1, R_p = 1, M_k = M_v \quad (31)$$

We now have a set of nonlinear, simultaneous equations that can be solved with a given I_{evap} and known ambient conditions. Once the equations are solved, the interface pressure can be obtained from Eq. (17).

Supersonic Flow

As the laser intensity is increased, the material is ablated from the liquid surface at a higher rate. Consequently, the velocity in the vapor region increases. However, at a certain laser intensity, the vapor at the Knudsen layer exit plane becomes thermally choked [4]. Extra heat addition will not increase the Mach number of the flow beyond $M_k=1$ because heat addition increases the Mach number for a subsonic flow and decreases the Mach number for a supersonic flow [12, p.192]. Therefore, at the Knudsen layer exit plane,

$$M_k = 1 \quad (32)$$

Downstream of the Knudsen layer, the flow streamlines diverge, allowing a transition from sonic speed to supersonic speed. This takes place via a centered Q-rarefaction fan [13, p. 266]. The supersonic model incorporating the Q-rarefaction fan can be visualized in Figure 6, and the corresponding x-t diagram in Figure 7.

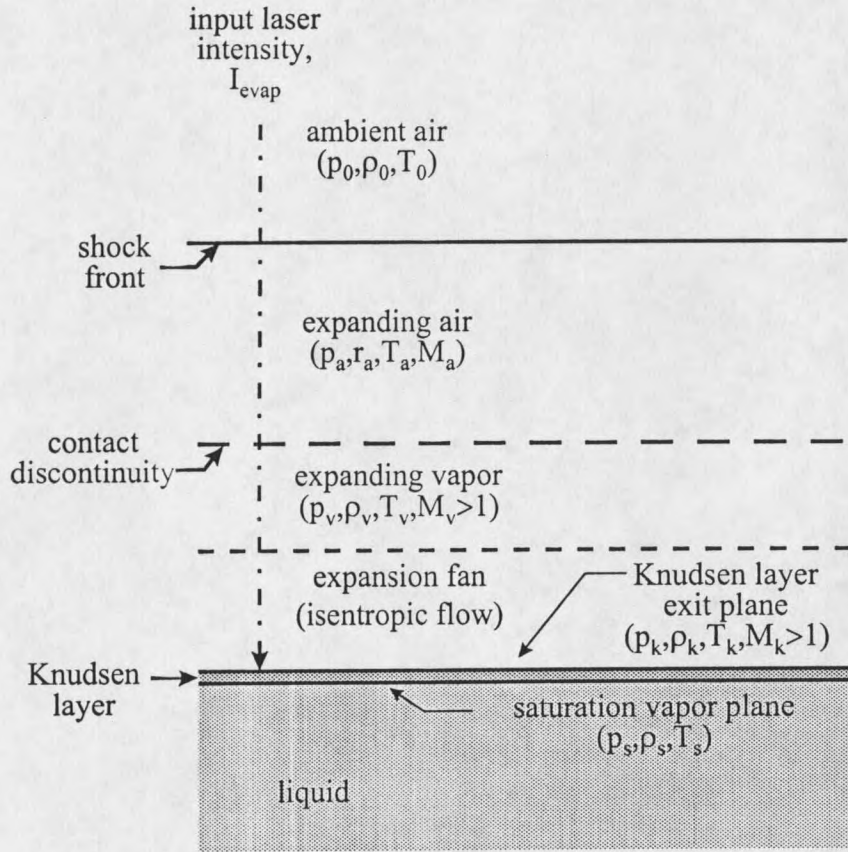


Figure 6. Flow regions for $M_v > 1$.

The x - t diagram shows that $dx/dt = 0$ at $x=0$. The characteristic lines for a flow are defined as [13, p.308]

$$\frac{dx}{dt} = u + a \text{ for a } C^+ \text{ curve} \quad (33a)$$

$$\text{and } \frac{dx}{dt} = u - a \text{ for a } C^- \text{ curve.} \quad (33b)$$

By definition, the sound speed is positive. Since the velocity of the vapor is positive as the vapor is ablated from the material, dx/dt can only equal zero if the minus

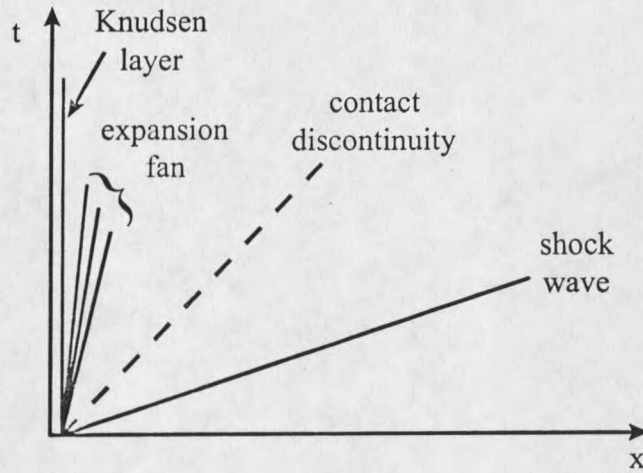


Figure 7. Supersonic x-t flow diagram.

sign is chosen in Eq. (33). Thus the straight lines shown in the expansion fan in Figure 7 are C^- characteristics, and hence the Riemann invariant

$$P = u + \frac{2a}{\gamma_v - 1}$$

is constant across the expansion fan. Therefore, the value of P can be calculated at the exit of the fan and equated to the value at the Knudsen layer exit plane:

$$u_k + \frac{2a_k}{\gamma_v - 1} = u_v + \frac{2a_v}{\gamma_v - 1}.$$

With Eq. (7) this becomes

$$M_k a_k + \frac{2a_k}{\gamma_v - 1} = M_v a_v + \frac{2a_v}{\gamma_v - 1}.$$

After the sound speed is factored from each side, the equation can be written as

$$[(\gamma_v - 1)M_k + 2]a_k = [(\gamma_v - 1)M_v + 2]a_v,$$

or,

$$\frac{a_k}{a_v} = \frac{[(\gamma_v - 1)M_v + 2]}{[(\gamma_v - 1)M_k + 2]}.$$

If Eq. (8) is substituted into the above equation, it becomes

$$\sqrt{\frac{T_k}{T_v}} = \frac{[(\gamma_v - 1)M_v + 2]}{[(\gamma_v - 1)M_k + 2]}$$

Substitution of Eq. (20) and Eq. (32) yields

$$\sqrt{R_T} = \frac{(\gamma_v - 1)M_v + 2}{\gamma_v + 1}. \quad (34)$$

Across the Q-rarefaction fan, the flow is isentropic [11, p. 79]. Therefore,

$$\frac{T_v}{T_k} = \left(\frac{p_v}{p_k} \right)^{\frac{\gamma_v - 1}{\gamma_v}}.$$

Upon substitution of the ideal gas law for the pressures, the equation becomes

$$\frac{T_v}{T_k} = \left(\frac{\rho_v T_v}{\rho_k T_k} \right)^{\frac{\gamma_v - 1}{\gamma_v}}.$$

The above equation can be rearranged, and Eqs. (20) substituted for the ratios of temperature and density, respectively, to obtain

$$R_T = (R_\rho)^{\gamma_v - 1}. \quad (35)$$

Eqs. (32), (34), and (35) can be combined with the six general equations to solve for the conditions in each region. As in the subsonic model, once the conditions are known, the interface pressure can then be found.

When working with ultrasonics, it is the acoustic pressure, not the absolute pressure, that is of interest. The acoustic pressure can be obtained simply by subtracting the back pressure of one atmosphere from the interface pressure. The acoustic pressure is shown as a function of I_{evap} in Figure 8. The acoustic pressure appears to be very nearly a linear function of the evaporation intensity except for very high laser intensities. The subsonic model is used for evaporation intensities which result in a Mach flow number at the Knudsen layer exit plane of less than one. The evaporation intensity at which this occurs according to the model is about .067 MW/cm². Above this intensity, the supersonic model is used to determine the acoustic pressure.

Quasi-two-dimensional formulation

In reality, the actual laser profile and resulting pressure are two-dimensional. The average pressure is found by integrating the pressure distribution over the area illuminated by the laser and dividing by the illuminated area. For a laser spot of radius r_0 and area A_{spot} , the average acoustic pressure is

$$\langle p_{\text{acoustic}} \rangle = \frac{2\pi}{A_{\text{spot}}} \int_0^{r_0} [p(r) - p_0] r dr \quad (36)$$

where $p(r)$ is the absolute pressure at the liquid surface and p_0 is the ambient atmospheric pressure.

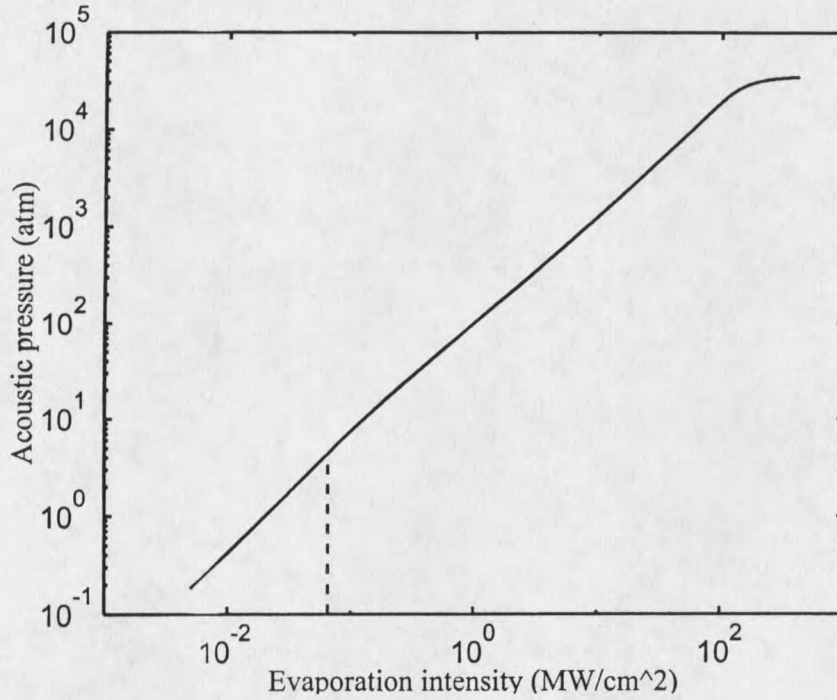


Figure 8. Acoustic pressure as a function of evaporation intensity.

The one-dimensional acoustic pressures can be used for a quasi-two-dimensional formulation by assuming that Eq. (36) can be approximated by replacing the actual pressure distribution, $p(r)$, with its one-dimensional counterpart, p_1 :

$$\langle p_{acoustic} \rangle \approx \frac{2\pi}{A_{spot}} \int_0^{r_0} [p_1 - p_0] r dr . \quad (37)$$

The liquid pressure p_1 obtained from Eq. (17) is a function of I_{evap} , which is in turn a function of radius. Therefore, p_1 is radially dependent as well. This assumption yields a quasi-two-dimensional formulation for the acoustic pressure for a given laser intensity.

The signal strength can be found by assuming that linear acoustics holds in the material. The signal strength is therefore proportional to the average acoustic pressure exerted on the liquid. Since, for a given output laser intensity, I_{laser} , the resulting average acoustic pressure can be obtained from Eq. (37), a curve similar to that shown in Fig. (2) can be obtained by using Eq. (37) for a range of output laser intensities.

CHAPTER 4

PROBLEM SOLUTION

In order to compare with experimental data, mercury was the material used in the model. Data for mercury is obtainable from a variety of sources (see, for example, [14]). The enthalpy values H_v and H_l were obtained by taking an average of published values [8, p680]. For a temperature range of 109.2 °C to 752.7 °C, the average value for the specific enthalpy in the vapor phase was found to be $H_v = 348.68$ J/gm-K. The average value for H_l was 57.49 J/gm-K for the same temperature range.

For a laser wavelength of 1064 nm, the index of refraction, n , and extinction coefficient, k , were interpolated from [9] to be

$$n = 4.187$$

$$k = 7.166.$$

Using these values, the reflectance coefficient was calculated to be $R = .786$ using Eq.

(3). Thus, for a given value of I_{laser} , the corresponding absorbed intensity, I_{abs} , is found from Eq. (2).

Mathcad 5.0 Plus [15] was used to solve the system of nonlinear equations to obtain the acoustic pressure given the output laser intensity. Mathcad was chosen over other programs to solve the nonlinear equations because it has a built-in nonlinear equation solver. The solver uses a Levenberg-Marquardt method, which requires an

initial estimate of the unknown variables. The initial values should be reasonably close to the solution values for a highly nonlinear system. Therefore, care must be taken when selecting initial values for a specific I_{evap} .

In addition to the equation solver, Mathcad performs unit conversions. It also has a graphical interface, so that the equations input into the program appear exactly as they would if written on paper. This reduced the amount of time needed to debug the program, since unit conversions were automatic and typographical errors in the equations were reduced. Mathcad 5.0 Plus does not, however, have the ability to loop through a range of input variables. Therefore, it was not possible to obtain acoustic pressures given a range of input evaporation intensities using Mathcad alone.

In order to find the average acoustic pressure over a laser spot and a range of average acoustic pressures for a range of output laser intensities, an Excel 5.0 macro was written. Excel [16] has the ability to communicate directly with Mathcad via dynamic data exchange (DDE) functions, and it passes data values back and forth using these functions. For this reason, Excel was used to loop through the range of output laser intensities. If, for a given output laser intensity, the resulting absorbed intensity was greater than the thermal intensity, Excel computed the evaporation intensity from Eq. (2) and called Mathcad for the solution to the evaporation intensity.

A flowchart illustrating how Excel and Mathcad work together is shown in Figure 9. Input values are entered into the Excel spreadsheet. These values include the lowest and highest output laser intensities, the properties for mercury, and the ambient

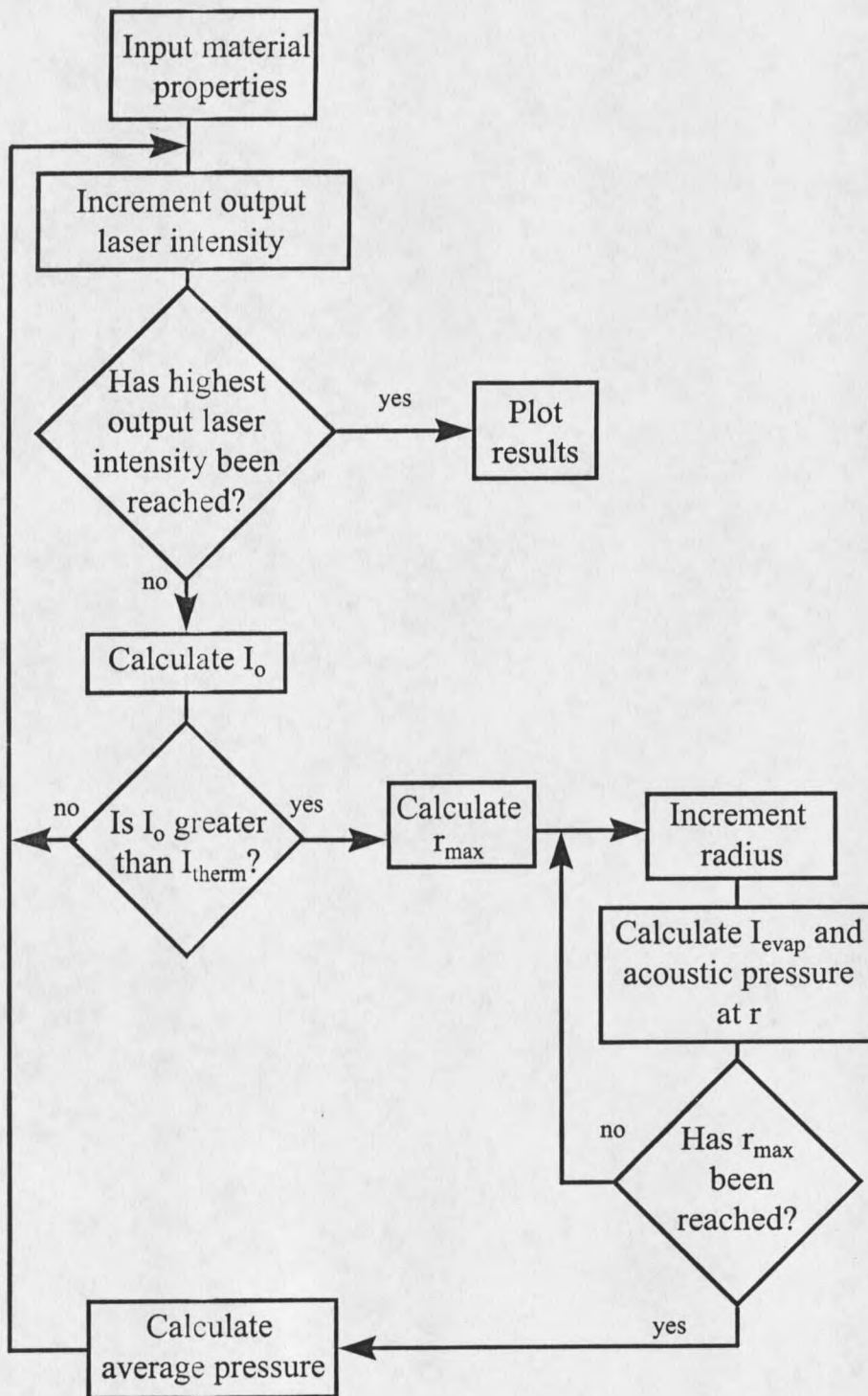


Figure 9. Programming logic for theoretical model.

conditions. The spreadsheet then loops from the lowest output laser intensity to the highest output laser intensity.

For each output laser intensity, the absorbed intensity is calculated. This absorbed intensity is equal to the maximum intensity, I_0 , on a Gaussian profile for the laser spot. I_0 is compared to I_{therm} , the intensity at which ablation begins. If I_0 is less than I_{therm} , then no ablation occurs (Fig. 10). The acoustic pressure is set to zero, and the program loops to the next higher intensity.

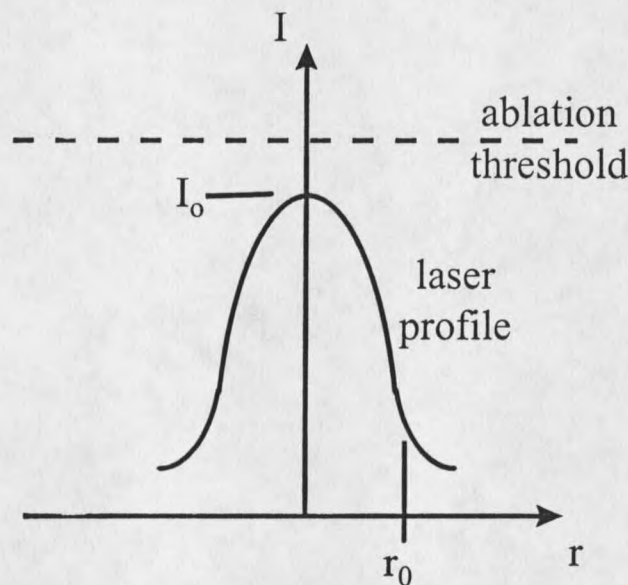


Figure 10. Laser profile below threshold, no ablation

However, if I_0 is greater than I_{therm} , then ablation occurs in the liquid. Figure 11 shows a laser profile undergoing partial ablation, while Figure 12 shows a fully-ablated

laser spot. The maximum radius at which ablation occurs, $r_{\max}$, is calculated. The evaporation intensity, I_{evap} , is calculated using Eq. (2) for each value of r less than $r_{\max}$, and Matchcad is called to find the corresponding acoustic pressure. This is done for all of the laser profile which is above the ablation threshold, or $I_{\text{abs}} > I_{\text{therm}}$.

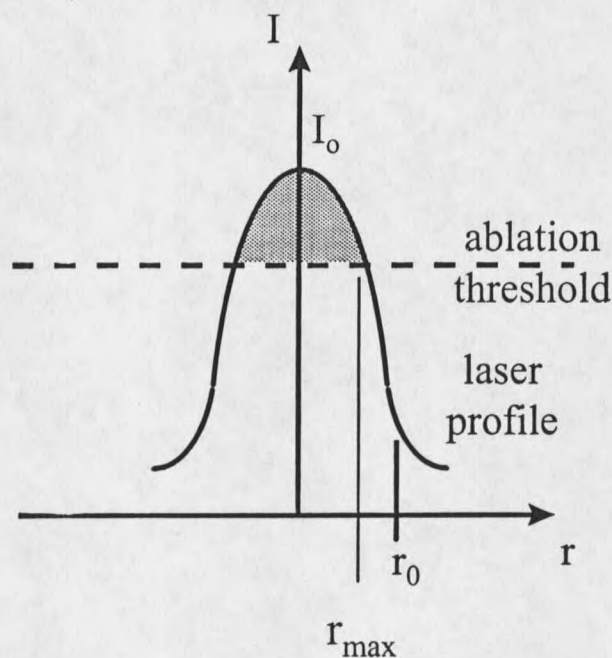


Figure 11. Partly-ablated laser spot.

Once the acoustic pressure distribution for the laser profile is found, Eq. (37) is used to find the average acoustic pressure. Numerically, this is accomplished by finding the total force on the laser spot using Simpson's rule [17] and then dividing by the spot area.

The output laser intensity is incremented, and the process begins again.

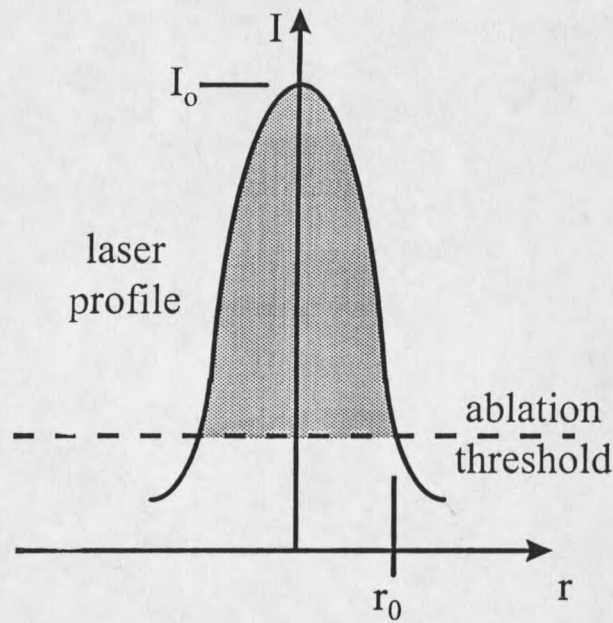


Figure 12. Fully-ablated laser spot.

The maximum amount of laser energy that can be absorbed without causing ablation can be approximated as the energy which raises the temperature of the liquid to the saturation temperature:

$$Q \approx mc(T_s - T_0).$$

T_s is taken to be the saturation temperature at one atmosphere, T_0 is the ambient temperature, c is the specific heat, and m is the mass of the liquid into which the energy flows. The mass is equal to the product of the density and the heated volume of liquid. This volume is approximated as the area of the laser spot times the optical penetration depth of the liquid, δ_p . The above equation then becomes

$$Q \approx \rho_l A_{spot} \delta_p c (T_s - T_0). \quad (38)$$

Experimentally, the amount of energy thermally absorbed by the liquid is

$$Q = I_{therm} A_{spot} t_p. \quad (39)$$

where t_p is the pulse duration of the laser. Eqs. (38) and (39) can be combined to obtain

$$I_{therm} \approx \frac{\rho_l \delta_p c (T_s - T_0)}{t_p}. \quad (40)$$

In actuality, the value of I_{therm} will vary, since the saturation temperature in the liquid varies as the laser intensity is increased. For the purposes of this work, however, it is assumed to have a constant value.

Although Excel was able to obtain a solution, there were several problems associated with using the macro. For example, the Excel macro did not reliably pass data values back and forth from the macro to the spreadsheet. Due to these idiosyncrasies, a better programming method was desired to call Mathcad. Matlab [18] fit this need very well.

Matlab also has DDE functions which enable it to communicate with Mathcad. Following the same logic as in Figure 9, a Matlab program was written to loop through the laser intensities and call Mathcad to find the acoustic pressure for a certain evaporation intensity. In addition, Matlab has a vast array of mathematical functions built into the programming, including a square root function. Therefore, the only data that needed to be passed was between the Matlab program and the Mathcad programs. In

contrast, Excel needed to pass values from the macro to the spreadsheet, and also from the spreadsheet to Mathcad.

Two types of laser profiles were used in the theoretical model (Figure 13). The first was a flat-top profile which represented the one-dimensional case. The second profile was a Gaussian profile, which is the generally accepted profile for a laser beam.

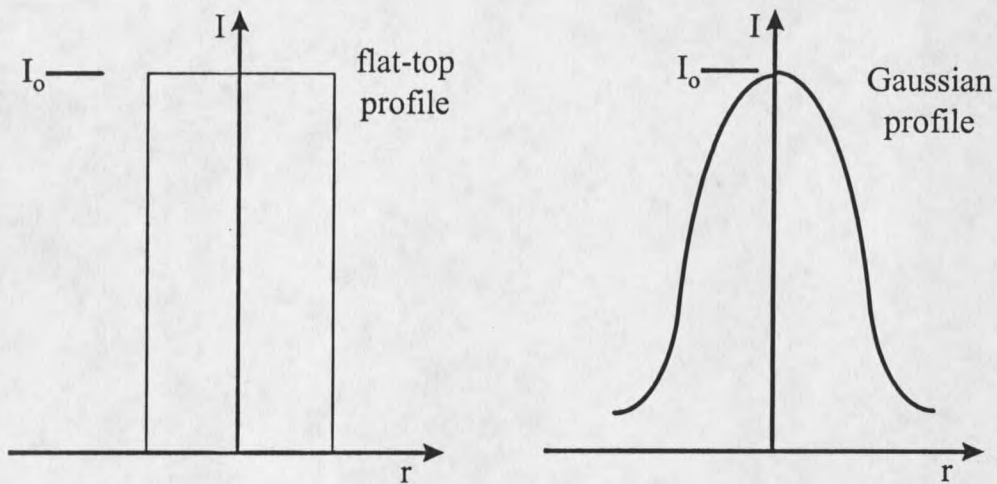


Figure 13. Laser profiles used in the theoretical model.

The average acoustic pressure is shown as a function of the output laser intensity in Figure 14. The curve starts at the onset of ablation and rises rapidly. As the output laser intensity increases, ablation dominates. At these laser intensities, the acoustic pressure is approximately a linear function of the output laser intensity.

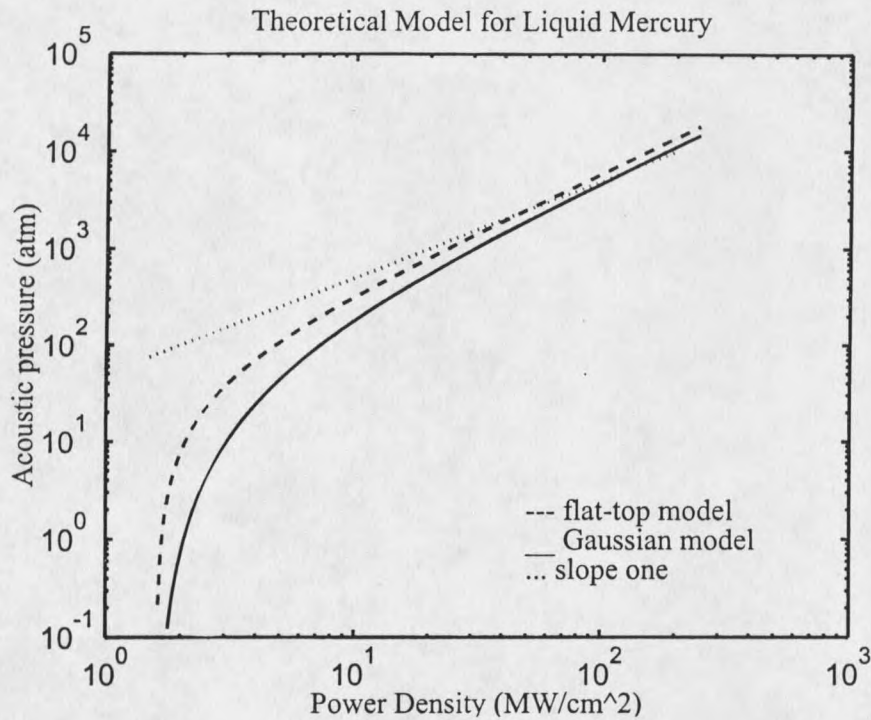


Figure 14. Average pressure as a function of output laser intensity.

The acoustic pressure is proportional to the signal amplitude if we assume that linear acoustics holds in the liquid, hence the signal amplitude = $K \langle P_{\text{acoustic}} \rangle$. The theoretical model can then be compared to experimental data. The theoretical data can be scaled up or down by the proportionality constant, which simply shifts the data vertically on a log-log plot. The comparison between theory and experiment is shown in Figure 15. The experimental data was obtained at two spot sizes, 1.4 mm and 2.3 mm. Therefore, the theoretical model used the same spot sizes for comparison. The scale factor was chosen so that the linear portions of the theoretical curves and the experimental curves coincide.

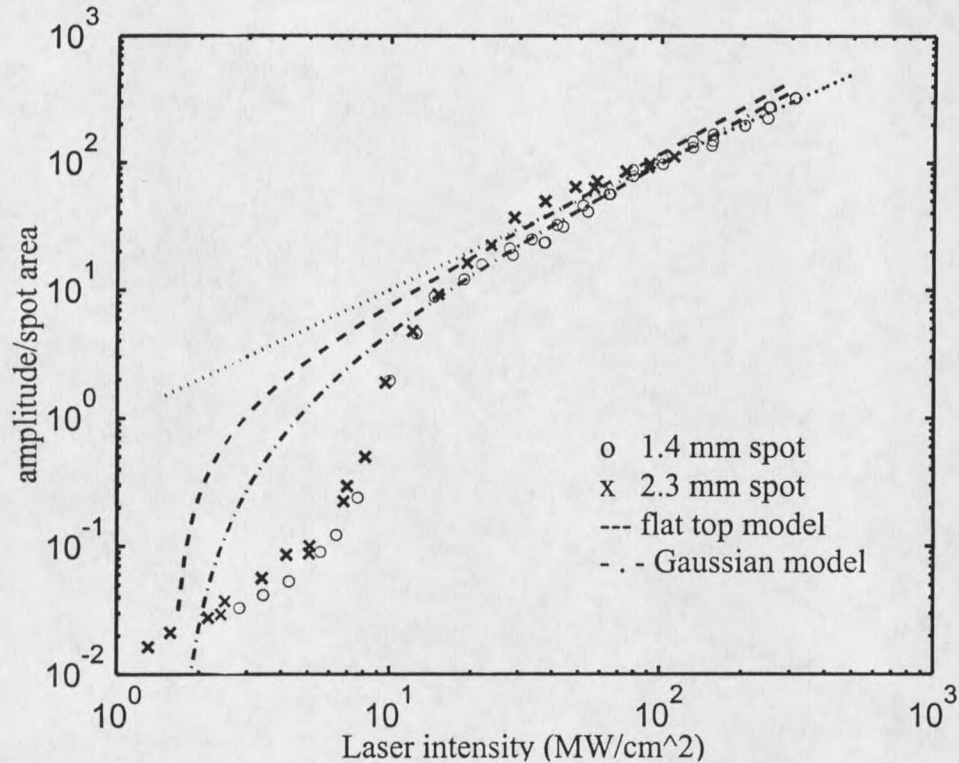


Figure 15. Comparison between theory and experiment.

The theory predicts that ablation begins at approximately 2 MW/cm^2 for the Gaussian model. This is lower than the 6 MW/cm^2 seen experimentally. This is most likely caused by the loss term in Eq. (2), which was chosen as zero in the theoretical model. The loss term will likely have a large effect on the amount of laser intensity absorbed by the material, since the expanding vapor very likely shields the mercury surface from a portion of the laser intensity. If the theoretical curves are translated along the upper ablation line so that the onset of ablation for the theoretical curves matches the

onset of ablation for the experimental data, as shown in Figure 16, the agreement between the theoretical curves and the experimental data is very good.

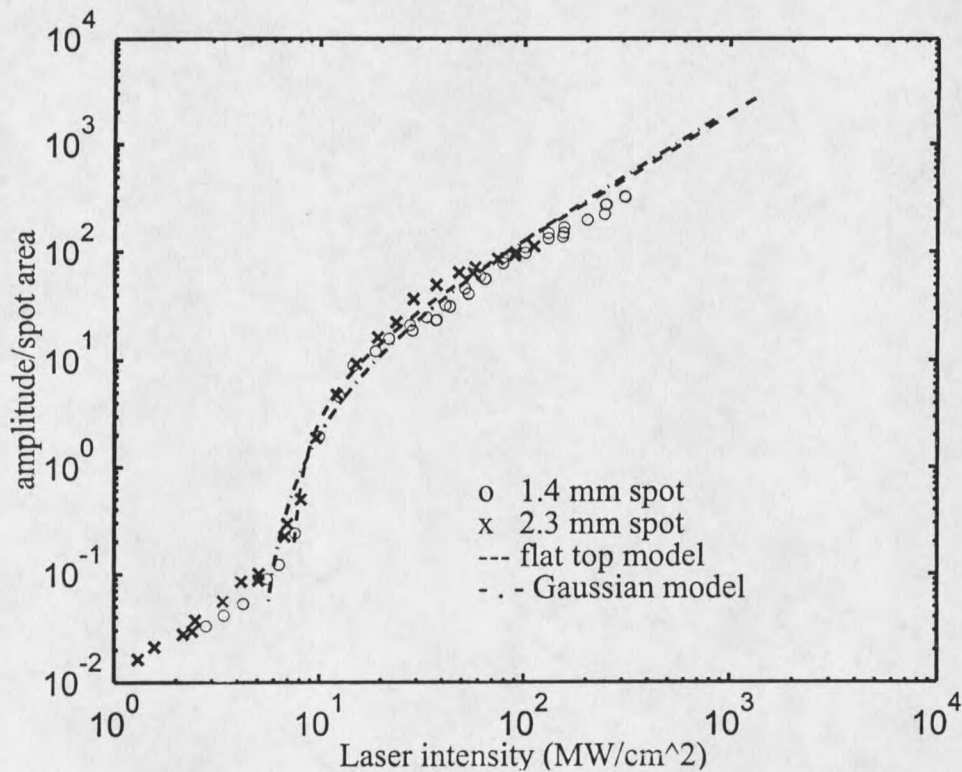


Figure 16. Qualitative comparison of theory and experiment.

Conclusion

The general shape of the curve indicates that the model is a good physical representation of what is seen experimentally. The model predicts that there is a rapid rise in the acoustic pressure at the onset of ablation, in agreement with experimental data.

Furthermore, both the theory and the experiments show that, at very high laser intensities, the acoustic pressure is a linear function of the output laser intensity.

While this quasi-two-dimensional formulation reasonably describes the physical process of ablation, a better description of ablation is needed. This description should include a full two-dimensional model that also takes into account the transient characteristics of the laser pulse.

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APPENDICES

APPENDIX A
SUBSONIC MATHCAD MODEL

SUB-ML.MCD

Units Definition:

$$\text{gmol} := \frac{1\text{M}}{1000}$$

$$\text{bar} = 10^5 \cdot \text{Pa}$$

$$\text{C} := 1 \cdot \text{K}$$

$$\text{kgmol} := 1\text{M}$$

$$\text{MW} := 10^6 \cdot \text{watt}$$

$$\text{KJ} := 1000 \cdot \text{joule}$$

Unknowns:

$$p_s \quad T_s \quad r_t \quad R_t \quad r_p \quad R_p$$

$$M_k \quad M_v \quad m$$

Input values from Excel:

$$I_{\text{evap}} := .068$$

$$M_k := .968$$

$$R_t := 1.0$$

$$r_t := .678$$

$$p_s := 9.519$$

$$M_v := .968$$

$$R_{\text{dens}} := 1.0$$

$$r_{\text{dens}} := .317$$

$$T_s := 787.082$$

Given values:

Universal gas constant

$$R := 8.31434 \frac{\text{joule}}{\text{gmol} \cdot \text{K}}$$

Specific heat ratio for mercury

$$\gamma := \frac{5}{3}$$

Ambient conditions for air

$$p_a := 1 \cdot \text{atm}$$

$$\gamma_a := 1.4$$

Molecular weight of air

$$\text{MW}_a := 29 \frac{\text{kg}}{\text{kgmol}}$$

Clausius-Clapeyron constants

$$A := 6486 \cdot 10^6 \cdot \text{Pa}$$

$$T_0 := 6937 \cdot \text{K}$$

$$T_a := 293.15 \cdot \text{K}$$

Laser input intensity

$$I_{\text{evap}} := I_{\text{evap}} \cdot \frac{\text{MW}}{\text{cm}^2}$$

Molecular weight of mercury

$$\text{MW}_{\text{hg}} := 200.56 \frac{\text{kg}}{\text{kgmol}}$$

Enthalpy variables

$$H_v := 348.68 \frac{\text{joule}}{\text{gm}}$$

$$H_l := 3.942 \frac{\text{joule}}{\text{gm}}$$

Primary Calculations:

$$R_a := \frac{R}{\text{MW}_a}$$

$$R_a = 286.70 \text{ m}^2 \cdot \text{sec}^{-2} \cdot \text{K}^{-1}$$

$$a_a := \sqrt{\gamma_a \cdot R_a \cdot T_a}$$

$$a_a = 343.023 \text{ m} \cdot \text{sec}^{-1}$$

$$R_{\text{hg}} := \frac{R}{\text{MW}_{\text{hg}}}$$

$$R_{\text{hg}} = 41.456 \text{ m}^2 \cdot \text{sec}^{-2} \cdot \text{K}^{-1}$$

Guess values:

$$p_s := p_s \cdot \text{atm} \quad T_s := T_s \cdot K \quad m := M_k \cdot \sqrt{\frac{\gamma}{2}} \quad r_p := r_{\text{dens}}$$

$$H_{lv} := (H_v - H_l) + \frac{1}{2} \cdot \left(M_k \cdot \sqrt{\gamma \cdot R_{hg} \cdot r_t \cdot T_s} \right)^2 \quad R_p := R_{\text{dens}}$$

Solve block with the simultaneous equations

Given

Knudsen layer jump conditions

$$r_t = \left[\sqrt{1 + \pi \cdot \left(\frac{\gamma - 1}{\gamma + 1} \cdot \frac{m}{2} \right)^2} - \sqrt{\pi} \cdot \frac{\gamma - 1}{\gamma + 1} \cdot \frac{m}{2} \right]^2$$

$$r_p = \sqrt{\frac{1}{r_t}} \cdot \left[\left(m^2 + \frac{1}{2} \right) \cdot e^{m^2} \cdot (1 - \text{erf}(m)) - \frac{m}{\sqrt{\pi}} \right] + \frac{1}{2 \cdot r_t} \cdot \left[1 - \sqrt{\pi} \cdot m \cdot e^{m^2} \cdot (1 - \text{erf}(m)) \right]$$

$$m = M_k \cdot \sqrt{\frac{\gamma}{2}}$$

Rankine-Huginiot relation

$$M_v = \frac{\frac{a_a}{\gamma_a} \cdot \left(\frac{1}{p_a} \cdot \frac{r_p}{R_p} \cdot \frac{r_t}{R_t} \cdot p_s - 1 \right)}{\sqrt{\gamma \cdot R_{hg} \cdot \frac{r_t}{R_t} \cdot T_s} \cdot \sqrt{1 + \frac{\gamma_a + 1}{2 \cdot \gamma_a} \cdot \left(\frac{1}{p_a} \cdot \frac{r_p}{R_p} \cdot \frac{r_t}{R_t} \cdot p_s - 1 \right)}}$$

Energy balance across evaporation front

$$p_s = \frac{I_{\text{evap}}}{\gamma \cdot M_k \cdot H_{lv}} \cdot \frac{1}{r_p} \cdot \sqrt{\frac{1}{r_t}} \cdot \sqrt{\gamma \cdot R_{hg} \cdot T_s}$$

$$H_{lv} = (H_v - H_l) + \frac{1}{2} \cdot M_k^2 \cdot \gamma \cdot R_{hg} \cdot r_t \cdot T_s$$

Clausius-Clapeyron equation

$$p_s = A \cdot e^{\frac{-T_0}{T_s}}$$

Subsonic conditions

$$R_t = 1$$

$$R_\rho = 1$$

$$M_k = M_v$$

Solve the nonlinear system of equations

$$\begin{bmatrix} M_{\text{knudsen}} \\ M_{\text{vap}} \\ m_{\text{knudsen}} \\ r_{\text{temp}} \\ r_{\text{dens}} \\ R_{\text{temp}} \\ R_{\text{dens}} \\ p_{\text{sat}} \\ T_{\text{sat}} \\ H_{\text{fg}} \end{bmatrix} := \text{find}(M_k, M_v, m, r_t, r_\rho, R_t, R_\rho, p_s, T_s, H_{lv}) \quad \text{ERR} = 8.232 \cdot 10^{-10}$$

Results:

$$\begin{aligned} M_{\text{knudsen}} &= 1.01 & r_{\text{temp}} &= 0.666 & R_{\text{temp}} &= 1 & T_{\text{sat}} &= 794.243 \text{ K} & H_{\text{fg}} &= 363.39 \frac{\text{joule}}{\text{gm}} \\ M_{\text{vap}} &= 1.01 & r_{\text{dens}} &= 0.305 & R_{\text{dens}} &= 1 & p_{\text{sat}} &= 10.306 \text{ atm} \\ m_{\text{knudsen}} &= 0.922 \end{aligned}$$

Calculate other state variables

saturation region:

$$\rho_{\text{sat}} := \frac{p_{\text{sat}}}{R_{\text{hg}} \cdot T_{\text{sat}}}$$

$$\rho_{\text{sat}} = 31.715 \text{ kg} \cdot \text{m}^{-3}$$

vapor region:

$$T_{\text{vap}} := \frac{r_{\text{temp}}}{R_{\text{temp}}} \cdot T_s$$

$$T_{\text{vap}} = 524.584 \text{ K}$$

$$a_{\text{vap}} := \sqrt{\gamma \cdot R_{\text{hg}} \cdot T_{\text{vap}}}$$

$$a_{\text{vap}} = 190.38 \text{ m} \cdot \text{sec}^{-1}$$

$$\rho_{\text{vap}} := \frac{r_{\text{dens}}}{R_{\text{dens}}} \cdot \rho_{\text{sat}}$$

$$\rho_{\text{vap}} = 9.689 \text{ kg} \cdot \text{m}^{-3}$$

$$p_{\text{vap}} := \rho_{\text{vap}} \cdot R_{\text{hg}} \cdot T_{\text{vap}}$$

$$p_{\text{vap}} = 2.079 \cdot \text{atm}$$

Speed of vapor leaving Knudsen layer

$$u_{\text{vap}} := M_{\text{vap}} \cdot a_{\text{vap}}$$

$$u_{\text{vap}} = 192.268 \text{ m} \cdot \text{sec}^{-1}$$

This also equals the speed of air after the contact discontinuity and before the shock wave

$$\rho_a := \frac{p_a}{R_a \cdot T_a}$$

$$\rho_a = 1.206 \text{ kg} \cdot \text{m}^{-3}$$

$$p_{\text{air}} := p_{\text{vap}}$$

$$p_{\text{air}} = 2.079 \cdot \text{atm}$$

$$\rho_{\text{air}} := \rho_a \cdot \frac{1 + \frac{\gamma_a + 1}{\gamma_a - 1} \frac{p_{\text{air}}}{p_a}}{\left(\frac{\gamma_a + 1}{\gamma_a - 1} + \frac{p_{\text{air}}}{p_a} \right)}$$

$$\rho_{\text{air}} = 2.011 \cdot \text{kg} \cdot \text{m}^{-3}$$

$$T_{\text{air}} := \frac{p_{\text{air}}}{\rho_{\text{air}} \cdot R_a}$$

$$T_{\text{air}} = 365.450 \text{ K}$$

$$a_{\text{air}} := \sqrt{\gamma_a \cdot R_a \cdot T_{\text{air}}}$$

$$a_{\text{air}} = 382.998 \text{ m} \cdot \text{sec}^{-1}$$

Mach number of air after the contact discontinuity

$$M_{\text{air}} := \frac{u_{\text{vap}}}{a_{\text{air}}}$$

$$M_{\text{air}} = 0.502$$

Solve for shock mach number and velocity

We can use the piston/shock model from ELEMENTS OF GASDYNAMICS, using the velocity of the vapor as the velocity of the piston.

$$M_{\text{shock}} := \sqrt{\frac{\gamma - 1}{2 \cdot \gamma} + \frac{\gamma + 1}{2 \cdot \gamma} \frac{p_{\text{air}}}{p_a}}$$

$$M_{\text{shock}} = 1.365$$

$$u_{\text{shock}} := M_{\text{shock}} \cdot a_a$$

$$u_{\text{shock}} = 468.267 \text{ m} \cdot \text{sec}^{-1}$$

Interface pressure

$$p_{\text{int}} := \frac{I_{\text{evap}}}{H_{\text{fg}}} \cdot \sqrt{\gamma \cdot R_{\text{hg}} \cdot T_{\text{sat}}} \cdot \sqrt{\frac{T_{\text{vap}}}{T_{\text{sat}}} \cdot \frac{1}{M_{\text{vap}}} \cdot \left(M_{\text{vap}}^2 + \frac{1}{\gamma} \right)}$$

$$p_{\text{int}} = 5.64 \cdot \text{atm}$$

$$f_{\text{ev}} := \frac{p_{\text{int}}}{I_{\text{evap}}}$$

$$f_{\text{ev}} = 84.035 \cdot \text{bar} \cdot \frac{\text{cm}^2}{\text{MW}}$$

$$T_{\text{satC}} := T_{\text{sat}} - 273 \cdot \text{K}$$

$$T_{\text{satC}} = 521.243 \cdot \text{C}$$

Acoustic pressure

$$P_{\text{acoust}} := P_{\text{int}} - P_a$$

$$P_{\text{acoust}} = 4.64 \cdot \text{atm}$$

Convert values to pass back to Excel:

$$M_k := M_{\text{knudsen}}$$

$$M_k = 1.01$$

$$M_v := M_{\text{vap}}$$

$$M_v = 1.01$$

$$m := m_{\text{knudsen}}$$

$$m = 0.922$$

$$R_t := R_{\text{temp}}$$

$$R_t = 1$$

$$r_t := r_{\text{temp}}$$

$$r_t = 0.666$$

$$p_s := \frac{p_{\text{sat}}}{\text{atm}}$$

$$p_s = 10.306$$

$$T_s := \frac{T_{\text{sat}}}{\text{K}}$$

$$T_s = 794.243$$

$$P_{\text{acoustic}} := \frac{P_{\text{acoust}}}{\text{atm}}$$

$$P_{\text{acoustic}} = 4.64$$

APPENDIX B
SUPERSONIC MATHCAD MODEL

SUPER-ML.MCD

Units Definition:

$$\text{gmol} := \frac{1\text{M}}{1000}$$

$$\text{bar} := 10^5 \cdot \text{Pa}$$

$$\text{C} := 1 \cdot \text{K}$$

$$\text{kgmol} := 1\text{M}$$

$$\text{MW} := 10^6 \cdot \text{watt}$$

$$\text{KJ} := 1000 \cdot \text{joule}$$

Unknowns:

$$p_s$$

$$T_s$$

$$r_t$$

$$R_t$$

$$r_p$$

$$R_p$$

$$M_k$$

$$M_v$$

$$m$$

Input values from Excel:

$$I_{\text{evap}} := .87$$

$$M_k := 1.0$$

$$R_t := 2.126$$

$$r_t := .669$$

$$p_s := 151.877$$

$$M_v := 2.833$$

$$R_{\text{dens}} := 3.101$$

$$r_{\text{dens}} := .308$$

$$T_s := 1148.0$$

Given values:

Universal gas constant

$$R := 8.31434 \frac{\text{joule}}{\text{gmol} \cdot \text{K}}$$

Specific heat ratio for mercury

$$\gamma := \frac{5}{3}$$

Ambient conditions for air

$$p_a := 1 \cdot \text{atm}$$

$$\gamma_a := 1.4$$

Molecular weight of air

$$\text{MW}_a := 29 \frac{\text{kg}}{\text{kgmol}}$$

Clausius-Clapeyron constants

$$A := 6486 \cdot 10^6 \cdot \text{Pa}$$

$$T_0 := 6937 \cdot \text{K}$$

$$T_a := 293.15 \text{K}$$

Laser input intensity

$$I_{\text{evap}} := I_{\text{evap}} \frac{\text{MW}}{\text{cm}^2}$$

Molecular weight of mercury

$$\text{MW}_{\text{hg}} := 200.56 \frac{\text{kg}}{\text{kgmol}}$$

Enthalpy variables

$$H_v := 348.68 \frac{\text{joule}}{\text{gm}}$$

$$H_l := 3.942 \frac{\text{joule}}{\text{gm}}$$

Primary Calculations:

$$R_a := \frac{R}{\text{MW}_a}$$

$$R_a = 286.701 \text{m}^2 \cdot \text{sec}^{-2} \cdot \text{K}^{-1}$$

$$a_a := \sqrt{\gamma_a \cdot R_a \cdot T_a}$$

$$a_a = 343.023 \text{m} \cdot \text{sec}^{-1}$$

$$R_{\text{hg}} := \frac{R}{\text{MW}_{\text{hg}}}$$

$$R_{\text{hg}} = 41.456 \text{m}^2 \cdot \text{sec}^{-2} \cdot \text{K}^{-1}$$

Guess values:

$$p_s := p_s \cdot \text{atm} \quad T_s := T_s \cdot K \quad m := M_k \sqrt{\frac{\gamma}{2}} \quad r_\rho := r_{\text{dens}}$$

$$H_{lv} := (H_v - H_l) + \frac{1}{2} \left(M_k \sqrt{\gamma \cdot R_{hg} \cdot r_t \cdot T_s} \right)^2 \quad R_\rho := R_{\text{dens}}$$

Solve block with the simultaneous equations

Given

Knudsen layer jump conditions

$$r_t = \left[\sqrt{1 - \pi \cdot \left(\frac{\gamma - 1}{\gamma + 1} \cdot \frac{m}{2} \right)^2} - \sqrt{\pi} \cdot \frac{\gamma - 1}{\gamma + 1} \cdot \frac{m}{2} \right]^2$$

$$r_\rho = \sqrt{\frac{1}{r_t}} \cdot \left[\left(m^2 + \frac{1}{2} \right) \cdot e^{m^2} \cdot (1 - \text{erf}(m)) - \frac{m}{\sqrt{\pi}} \right] + \frac{1}{2 \cdot r_t} \cdot \left[1 - \sqrt{\pi} \cdot m \cdot e^{m^2} \cdot (1 - \text{erf}(m)) \right]$$

$$m = M_k \sqrt{\frac{\gamma}{2}}$$

Rankine-Huginiot relation

$$M_v = \frac{\frac{\gamma_a}{\gamma_a} \cdot \left(\frac{1}{p_a} \cdot \frac{r_\rho}{R_\rho} \cdot \frac{r_t}{R_t} \cdot p_s - 1 \right)}{\sqrt{\gamma \cdot R_{hg} \cdot \frac{r_t}{R_t} \cdot T_s} \cdot \sqrt{1 + \frac{\gamma_a + 1}{2 \cdot \gamma_a} \cdot \left(\frac{1}{p_a} \cdot \frac{r_\rho}{R_\rho} \cdot \frac{r_t}{R_t} \cdot p_s - 1 \right)}}$$

Energy balance across evaporation front

$$p_s = \frac{I_{\text{evap}}}{\gamma \cdot M_k \cdot H_{lv}} \cdot \frac{1}{r_\rho} \cdot \sqrt{\frac{1}{r_t} \cdot \gamma \cdot R_{hg} \cdot T_s}$$

$$H_{lv} = (H_v - H_l) + \frac{1}{2} \cdot M_k^2 \cdot \gamma \cdot R_{hg} \cdot r_t \cdot T_s$$

Clausius-Clapeyron equation

$$p_s = A \cdot e^{\frac{-T_0}{T_s}}$$

Supersonic conditions

Riemann invariant P constant across the Q-rarefaction fan

$$\sqrt{R_t} = \frac{(\gamma - 1) \cdot M_v + 2}{\gamma + 1}$$

Isentropic flow across the expansion fan

$$R_t = R_\rho^{\gamma - 1}$$

Choked flow at outside edge of Knudsen layer

$$M_k = 1$$

Solve the nonlinear system of equations

$$\begin{bmatrix} M_{\text{knudsen}} \\ M_{\text{vap}} \\ m_{\text{knudsen}} \\ r_{\text{temp}} \\ r_{\text{dens}} \\ R_{\text{temp}} \\ R_{\text{dens}} \\ p_{\text{sat}} \\ T_{\text{sat}} \\ H_{\text{fg}} \end{bmatrix} = \text{find}(M_k, M_v, m, r_t, r_\rho, R_t, R_\rho, p_s, T_s, H_{lv}) \quad \text{ERR} = 2.634 \cdot 10^{-9}$$

Results:

$$M_{\text{knudsen}} = 1 \quad r_{\text{temp}} = 0.669 \quad R_{\text{temp}} = 2.14 \quad T_{\text{sat}} = 1.152 \cdot 10^3 \cdot \text{K} \quad H_{\text{fg}} = 371.367 \frac{\text{joule}}{\text{gm}}$$

$$M_{\text{vap}} = 2.851 \quad r_{\text{dens}} = 0.308 \quad R_{\text{dens}} = 3.13 \quad p_{\text{sat}} = 155.27 \cdot \text{atm}$$

$$m_{\text{knudsen}} = 0.913$$

Calculate other state variables

saturation region:

$$\rho_{\text{sat}} = \frac{p_{\text{sat}}}{R_{\text{hg}} \cdot T_{\text{sat}}} \quad \rho_{\text{sat}} = 329.432 \text{ kg} \cdot \text{m}^{-3}$$

vapor region:

$$T_{\text{vap}} = \frac{r_{\text{temp}}}{R_{\text{temp}}} \cdot T_{\text{s}} \quad T_{\text{vap}} = 358.996 \text{ K}$$

$$a_{\text{vap}} := \sqrt{\gamma \cdot R_{\text{hg}} \cdot T_{\text{vap}}} \quad a_{\text{vap}} = 157.493 \text{ m} \cdot \text{sec}^{-1}$$

$$\rho_{\text{vap}} := \frac{r_{\text{dens}}}{R_{\text{dens}}} \cdot \rho_{\text{sat}} \quad \rho_{\text{vap}} = 32.433 \text{ kg} \cdot \text{m}^{-3}$$

$$p_{\text{vap}} := \rho_{\text{vap}} \cdot R_{\text{hg}} \cdot T_{\text{vap}} \quad p_{\text{vap}} = 4.764 \cdot \text{atm}$$

Speed of vapor leaving Knudsen layer

$$u_{\text{vap}} := M_{\text{vap}} \cdot a_{\text{vap}} \quad u_{\text{vap}} = 449.027 \text{ m} \cdot \text{sec}^{-1}$$

This also equals the speed of air after the contact discontinuity and before the shock wave

$$\rho_a := \frac{p_a}{R_a \cdot T_a}$$

$$\rho_a = 1.206 \text{ kg} \cdot \text{m}^{-3}$$

$$p_{\text{air}} = p_{\text{vap}}$$

$$p_{\text{air}} = 4.764 \cdot \text{atm}$$

$$\rho_{\text{air}} = \rho_a \cdot \frac{1 + \frac{\gamma_a + 1}{\gamma_a - 1} \frac{p_{\text{air}}}{p_a}}{\left(\frac{\gamma_a + 1}{\gamma_a - 1} + \frac{p_{\text{air}}}{p_a} \right)}$$

$$\rho_{\text{air}} = 3.313 \text{ kg} \cdot \text{m}^{-3}$$

$$T_{\text{air}} := \frac{p_{\text{air}}}{\rho_{\text{air}} \cdot R_a}$$

$$T_{\text{air}} = 508.12 \text{ K}$$

$$a_{\text{air}} := \sqrt{\gamma_a \cdot R_a \cdot T_{\text{air}}}$$

$$a_{\text{air}} = 451.608 \text{ m} \cdot \text{sec}^{-1}$$

Mach number of air after the contact discontinuity

$$M_{\text{air}} := \frac{u_{\text{vap}}}{a_{\text{air}}}$$

$$M_{\text{air}} = 0.994$$

Solve for shock mach number and velocity

We can use the piston/shock model from ELEMENTS OF GASDYNAMICS, using the velocity of the vapor as the velocity of the piston.

$$M_{\text{shock}} := \sqrt{\frac{\gamma - 1}{2 \cdot \gamma} + \frac{\gamma + 1}{2 \cdot \gamma} \frac{p_{\text{air}}}{p_a}}$$

$$M_{\text{shock}} = 2.003$$

$$u_{\text{shock}} := M_{\text{shock}} \cdot a_a$$

$$u_{\text{shock}} = 686.987 \text{ m} \cdot \text{sec}^{-1}$$

Interface pressure ..

$$P_{\text{int}} := \frac{I_{\text{evap}}}{H_{\text{fg}}} \cdot \sqrt{\gamma \cdot R_{\text{hg}} \cdot T_{\text{sat}}} \cdot \sqrt{\frac{T_{\text{vap}}}{T_{\text{sat}}}} \cdot \frac{1}{M_{\text{vap}}} \cdot \left(M_{\text{vap}}^2 + \frac{1}{\gamma} \right) \quad P_{\text{int}} = 111.481 \cdot \text{atm}$$

$$f_{\text{ev}} := \frac{P_{\text{int}}}{I_{\text{evap}}} \quad f_{\text{ev}} = 129.837 \cdot \text{bar} \cdot \frac{\text{cm}^2}{\text{MW}}$$

$$T_{\text{satC}} := T_{\text{sat}} - 273 \cdot \text{K} \quad T_{\text{satC}} = 879.009 \cdot \text{C}$$

Acoustic pressure

$$P_{\text{acoust}} := P_{\text{int}} - P_{\text{a}} \quad P_{\text{acoust}} = 110.481 \cdot \text{atm}$$

Convert values to pass back to Excel:

$$M_{\text{k}} := M_{\text{knudsen}}$$

$$M_{\text{k}} = 1$$

$$M_{\text{v}} := M_{\text{vap}}$$

$$M_{\text{v}} = 2.851$$

$$m := m_{\text{knudsen}}$$

$$m = 0.913$$

$$R_{\text{t}} := R_{\text{temp}}$$

$$R_{\text{t}} = 2.14$$

$$r_{\text{t}} := r_{\text{temp}}$$

$$r_{\text{t}} = 0.669$$

$$p_{\text{s}} := \frac{P_{\text{sat}}}{\text{atm}}$$

$$p_{\text{s}} = 155.27$$

$$T_{\text{s}} := \frac{T_{\text{sat}}}{\text{K}}$$

$$T_{\text{s}} = 1.152 \cdot 10^3$$

$$P_{\text{acoustic}} := \frac{P_{\text{acoust}}}{\text{atm}}$$

$$P_{\text{acoustic}} = 110.481$$

APPENDIX C
EXCEL MACRO

```

'
'twoD Macro
'
'
'

Const pi = 3.14159265
Option Base 1
Public count As Integer
Public Ievap, Mk, Mv, RRt, RRdens, rt, rdens, ps, Ts, rmax, Pave, Fave As Double
Public radius(), pacoustic(), superflag(), lambda, Itherm As Double
Public title As String
Public bob, fred As Double

```

```
Sub twoD()
```

```
'Define variables
```

```

'laser wavelength (m)
lambda = Worksheets(1).Cells(8, 2) * 10 ^ -9
'pulsewidth of laser (s)
pulsewidth = Worksheets(1).Cells(9, 2) * 10 ^ -9
'spotwidth (m) is the radius
spotwidth = Worksheets(1).Cells(10, 2) * 10 ^ -3 / 2
'maximum intensity (MW/cm^2)
maxintens = Worksheets(1).Cells(11, 2)
'n and k, the optical material properties
n = Worksheets(1).Cells(14, 2)
k = Worksheets(1).Cells(15, 2)
'Specific heat (J/kg-K)
c = Worksheets(1).Cells(16, 2)
'liquid density (kg/m^3)
dens = Worksheets(1).Cells(17, 2)
'boiling temp at 1 atm (C)
tboil = Worksheets(1).Cells(18, 2)
'ambient temp at 1 atm (C)
tambient = Worksheets(1).Cells(19, 2)

```

```
'Primary calculations
```

```
'Penetration depth of laser into material (calculated on paper 6/10 p. 1)
```

```

pendepth = -Log(0.01) * lambda / (4 * pi * k)
Worksheets(1).Cells(25, 2) = pendepth
'Thermal intensity (calculated on paper 6/10 p. 2)
'Units are MW/cm^2
Itherm = dens * pendepth * c * (tboil - tambient) / (pulsewidth * 10 ^ 10)
Worksheets(1).Cells(26, 2) = Itherm
'Reflection coeffiecient
Crefl = ((n - 1) ^ 2 + k ^ 2) / ((n + 1) ^ 2 + k ^ 2)
Worksheets(1).Cells(27, 2) = Crefl
'Area of laser spot
area = pi * spotwidth ^ 2
Worksheets(1).Cells(28, 2) = area * 10 ^ 6

```

```

'number of steps
numsteps = 6
ReDim radius(numsteps + 1)
ReDim pacoustic(numsteps + 1)
ReDim superflag(numsteps + 1)

```

```

'Gaussian Model

```

```

'Set up initial guesses
Worksheets(1).Range("d7:d" & (numsteps + 7)) = 0.1
Worksheets(1).Range("e7:e" & (numsteps + 7)) = 0.1
Worksheets(1).Range("f7:f" & (numsteps + 7)) = 1
Worksheets(1).Range("g7:g" & (numsteps + 7)) = 1
Worksheets(1).Range("h7:h" & (numsteps + 7)) = 1
Worksheets(1).Range("i7:i" & (numsteps + 7)) = 1
Worksheets(1).Range("j7:j" & (numsteps + 7)) = 1
Worksheets(1).Range("k7:k" & (numsteps + 7)) = 600

```

```

For i = 1 To numsteps + 1
    superflag(i) = 0
Next i
count2 = 0
Ilaser = 1

```

```

'Timestep loop

```

```

Do While Ilaser < maxintens
    If Ilaser < 4.9 Then

```

Ilaser = Ilaser + 0.1

Else

Ilaser = Ilaser + 2

End If

count2 = count2 + 1

'Place Ilaser value in worksheet

Worksheets(1).Cells(20 + count2, 4) = Ilaser

'find Iabs, the laser intensity which is absorbed by the material

Iabs = Ilaser * (1 - Crefl)

'Iabs equals the value of Io, the peak of a Gaussian curve

Io = Iabs

'Now that we have the peak of the Gaussian curve, we can express

'laser as a radial function. We can follow the curve radially

'outward and compare the intensity at a point to the maximum

'thermal intensity. If the intensity is greater than Itherm,

'then we can calculate Ievap and find the pressure at that point.

'After all the pressures are found, we can integrate over the

'Gaussian curve to find the average pressure for the average

'laser intensity level.

'check to see if the peak intensity is greater than Itherm. If it isn't,

'then there is no ablation occurring anywhere on the spot.

If Io < Itherm Then

Pave = 0

Worksheets(1).Cells(20 + count2, 5) = Pave

Fave = 0

Worksheets(1).Cells(20 + count2, 6) = Fave

Else

'Calculate the radius at which Ievap=0, i.e. the edge of the

'ablation spot

rmax = Sqr(-spotwidth ^ 2 * Log(Itherm / Io))

'cycle through the laser profile to determine the pressures due to ablation

stepsize = rmax / numsteps

Worksheets(1).Cells(1, 1) = stepsize

count = 1

For rad = 0 To rmax Step stepsize

radius(count) = rad

'calculate local intensity at a point on the profile

```

intens = Io * Exp(-rad ^ 2 / (spotwidth ^ 2))
'calculate evaporation intensity
Ievap = intens - Itherm
Worksheets(1).Cells(count + 6, 3) = Ievap
'Get guess values
Mk = Worksheets(1).Cells(count + 6, 4)
Mv = Worksheets(1).Cells(count + 6, 5)
RRt = Worksheets(1).Cells(count + 6, 6)
RRdens = Worksheets(1).Cells(count + 6, 7)
rt = Worksheets(1).Cells(count + 6, 8)
rdens = Worksheets(1).Cells(count + 6, 9)
ps = Worksheets(1).Cells(count + 6, 10)
Ts = Worksheets(1).Cells(count + 6, 11)

'Calculate acoustic pressure from Mathcad models

If superflag(count) = 1 Then
    title = "c:\school\thesis\super-ml.mcd"
    'look up pacoustic using supersonic model
    findsoln
Else
    title = "c:\school\thesis\sub-ml.mcd"
    'look up pacoustic using subsonic model
    findsoln
    'check to see if M>1
    Mach = Worksheets(1).Cells(count + 6, 5)
    If Mach > 1 Then
        superflag(count) = 1
        title = "c:\school\thesis\super-ml.mcd"
        'look up pacoustic using supersonic model
        findsoln
    End If
End If
Worksheets(1).Cells(count + 6, 12) = superflag(count)
count = count + 1
Next rad
'
'calculate average pressure using Simpson's rule
Pave = 0
For i = 1 To numsteps + 1
    pacoustic(i) = Worksheets(1).Cells(i + 6, 13)

```

```

Worksheets(1).Cells(i + 6, 15) = pacoustic(i)
Next i
For i = 2 To count - 1 Step 2
    Paye = Pave + 2 * stepsize / (3 * spotwidth ^ 2) * 4 * radius(i) * pacoustic(i)
Next i
For i = 3 To count - 2 Step 2
    Pave = Pave + 2 * stepsize / (3 * spotwidth ^ 2) * 4 * radius(i) * pacoustic(i)
Next i
'Place Pave into worksheet
Worksheets(1).Cells(20 + count2, 5) = Pave
'Find Fave and place it into the worksheet
Fave = Pave * area
Worksheets(1).Cells(20 + count2, 6) = Fave
End If
Loop
End Sub

```

```

Sub findsoln()

```

```

'Open channel to Mathcad
ChannelNumber = DDEInitiate("MCad", title)

```

```

'Send data to Mathcad
DDEPoke ChannelNumber, "I.evap", Ievap
DDEPoke ChannelNumber, "Mk", Mk
DDEPoke ChannelNumber, "Mv", Mv
DDEPoke ChannelNumber, "R.t", RRt
DDEPoke ChannelNumber, "R.dens", RRdens
DDEPoke ChannelNumber, "r.t", rt
DDEPoke ChannelNumber, "r.dens", rdens
DDEPoke ChannelNumber, "p.s", ps
DDEPoke ChannelNumber, "T.s", Ts

```

```

'Retrieve data from Mathcad
Worksheets(1).Cells(count + 6, 4) = DDERequest(ChannelNumber, "M.k")
Worksheets(1).Cells(count + 6, 5) = DDERequest(ChannelNumber, "M.v")
Worksheets(1).Cells(count + 6, 6) = DDERequest(ChannelNumber, "R.t")
Worksheets(1).Cells(count + 6, 7) = DDERequest(ChannelNumber, "R.dens")
Worksheets(1).Cells(count + 6, 8) = DDERequest(ChannelNumber, "r.t")
Worksheets(1).Cells(count + 6, 9) = DDERequest(ChannelNumber, "r.dens")
Worksheets(1).Cells(count + 6, 10) = DDERequest(ChannelNumber, "p.s")
Worksheets(1).Cells(count + 6, 11) = DDERequest(ChannelNumber, "T.s")

```

```
Worksheets(1).Cells(count + 6, 13) = DDERequest(ChannelNumber, "p.acoustic")  
Worksheets(1).Cells(count + 6, 14) = title
```

```
DDETerminate ChannelNumber  
End Sub
```

APPENDIX D
MATLAB PROGRAM: PRESSURE.M

pressure.m

% 7/10/96

%

% This file calculates the acoustic pressure for a liquid
% material for a range of laser intensities.

%

clear

% Input parameters

numsteps=6;

Imax=100;

lambda=1064/10^9;

n=4.187;

k=7.166;

pulswidth=10/10^9;

spot(1)=2.3/2;

spot(2)=1.4/2;

dens=13546;

c=138.14;

Tboil=629.73;

Ta=293.15;

% Penetration depth of laser into material (calculated on paper 6/10 p. 1)

pendepth=-log(.01)*lambda/(4*pi*k);

% Thermal intensity (calculated on paper 6/10 p. 2)

% Units of this are MW/cm^2

Itherm=dens*pendepth*c*(Tboil-Ta)/pulswidth/10^10;

% Reflection coefficient

Crefl=((n-1)^2+k^2)/((n+1)^2+k^2);

% Area of laser spotwidths

area(1)=pi*spot(1)^2;

area(2)=pi*spot(2)^2;

% Initial guess values to be passed into Mathcad

% Initial subsonic guesses

Mk=.1;

Mv=.1;

Rt=1;

Rd=1;

rt=1;

rd=1;

ps=1;

Ts=600;

```

for count=1:1:numsteps
    Mkgauss(count)=Mk;
    Mvgauss(count)=Mv;
    Rtgauss(count)=Rt;
    Rdgauss(count)=Rd;
    rtgauss(count)=rt;
    rdgauss(count)=rd;
    psgauss(count)=ps;
    Tsgauss(count)=Ts;
end
% Initial supersonic guesses
sMk=1;
sMv=1;
sRt=1;
sRd=1;
srt=.6;
srd=.3;
sps=33;
sTs=800;

Ievap=logspace(-2,log10(Imax),200);

%
% Flat top model
%
loopstop=0;
% If flag = 0, then use subsonic solution
subflag = 0;

for count1=1:1:max(size(Ievap))
    % Call Mathcad to find solution to nonlinear system of equations
    if subflag == 0
        title = 'c:\school\thesis\sub-ml.mcd';
        [Mk,Mv,Rt,Rd,rt,rd,ps,Ts,Pac(count1),loopstop] ...
            =ablsolv(Ievap(count1),Mk,Mv,Rt,Rd,rt,rd,ps,Ts,title);
        if Mv>=1
            subflag = 1;
            Mk=sMk;
            Mv=sMv;
            Rt=sRt;
            Rd=sRd;

```

```

rt=srt;
rd=srd;
ps=sps;
Ts=sTs;
title = 'c:\school\thesis\super-ml.mcd';
[Mk,Mv,Rt,Rd,rt,rd,ps,Ts,Pac(count1),loopstop] ...
    =abltsovl(Ievap(count1),Mk,Mv,Rt,Rd,rt,rd,ps,Ts,title);
end
else
    [Mk,Mv,Rt,Rd,rt,rd,ps,Ts,Pac(count1),loopstop] ...
        =abltsovl(Ievap(count1),Mk,Mv,Rt,Rd,rt,rd,ps,Ts,title);
end
if loopstop==1
    count1=max(size(Ievap));
    msg='bad solution'
end

```

APPENDIX E
MATLAB PROGRAM: GAUSS.M

```

% gauss.m
%
%
%
% This file calculates the average interface pressure for a liquid
% material for a range of laser intensities.
% The program obtains the average pressure using both a
% one dimensional flat-top profile and a quasi-two-dimensional
% Gaussian profile for the laser intensity.
%
clear
% Input parameters

% Number of steps to separate the laser profile into. It must be an
% even integer in order to use Simpson's Rule
numsteps=6;
% Maximum laser intensity (MW/cm^2)
Imax=250;
% Wavelength of laser light (m)
lambda=1064/10^9;
% Material reflection parameters
n=4.187;
k=7.166;
% Laser pulse width, i.e. time of pulse (s)
pulswidth=10/10^9;
% Spotwidth (m). Two are defined, 2.3mm and 1.4mm
spot(1)=2.3*10^-3/2;
spot(2)=1.4*10^-3/2;
% Density of mercury
dens=13546;
% Specific heat of mercury
c=138.14;
% Saturation temperature at one atmosphere (K)
Tboil=629.73;
% Ambient temperature (K)
Ta=293.15;

% Penetration depth of laser into material (calculated on paper 6/10 p. 1)
pendepth=-log(.01)*lambda/(4*pi*k);
% Thermal intensity (calculated on paper 6/10 p. 2)
% Units of this are MW/cm^2
Itherm=dens*pendepth*c*(Tboil-Ta)/(pulswidth*10^10);

```

```

% Reflection coefficient
Crefl=((n-1)^2+k^2)/((n+1)^2+k^2);
% Area of laser spotwidths
area(1)=pi*spot(1)^2;
area(2)=pi*spot(2)^2;

% Initial guess values to be passed into Mathcad
% Initial subsonic guesses
Mk=.01;
Mv=.01;
Rt=1;
Rd=1;
rt=1;
rd=1;
ps=1;
Ts=600;

% Flat-top profile

% Begin the loop to calculate acoustic pressures
% Iexp is the output laser intensity, also the intensity incident on the surface
% of the material

Iexp=logspace(0,log10(Imax),250);

% Calculate Iabs, the total absorbed intensity
Iabs=Iexp.*(1-Crefl);

%
% Flat top model
%
loopstop=0;
% If flag = 0, then use subsonic solution
subflag = 0;

for count1=1:1:max(size(Iabs))
    if Iabs(count1)<Itherm % No ablation is occurring
        Pave_T(count1)=0;
    else
        % Calculate Iabl, the laser intensity which goes into ablation
        Iabl=round((Iabs(count1)-Itherm)*1000)/1000
    end
end

```

```

model
    % Pave_T is the average pressure, equal to Pacoustic for the flat-top

    % Call Mathcad to find solution to nonlinear system of equations
    if subflag == 0
        title = 'c:\school\thesis\sub-ml.mcd';
        [Mk,Mv,Rt,Rd,rt,rd,ps,Ts,Pave_T(count1),loopstop] ...
            =abltsov(Iabl,Mk,Mv,Rt,Rd,rt,rd,ps,Ts,title);
        if Mv>=1
            subflag = 1;
            title = 'c:\school\thesis\super-ml.mcd';
            [Mk,Mv,Rt,Rd,rt,rd,ps,Ts,Pave_T(count1),loopstop] ...
                =abltsov(Iabl,Mk,Mv,Rt,Rd,rt,rd,ps,Ts,title);
        end
    else
        [Mk,Mv,Rt,Rd,rt,rd,ps,Ts,Pave_T(count1),loopstop] ...
            =abltsov(Iabl,Mk,Mv,Rt,Rd,rt,rd,ps,Ts,title);
    end
    % Check to make sure solution is valid
    if loopstop==1
        msg='Flat top - bad solution'
        break
    end

    end
    % The forces corresponding to the 2 spotwidths are calculated
    Fave_T(1,count1)=Pave_T(count1)*area(1);
    Fave_T(2,count1)=Pave_T(count1)*area(2);
    if loopstop==1
        break
    end
end

%
% Gaussian model
%
%

% Iabs equals the value of Io, the peak of a Gaussian curve
Io=Iabs;
%
% Now that we have the peak of the Gaussian curve, we can express
% Ilaser as a radial function. We can follow the curve radially outward
% and compare the Intensity at a point to the maximum thermal intensity.

```

```

% If the intensity is greater than Itherm, then we can calculate an
% Ievap and find the pressure at that point. After all the pressures are
% found, we can integrate over the Gaussian curve to find the average pressure
% for the average laser intensity level.
% Following this procedure, we can increase the average laser intensity and
% find the average pressure as a function of average laser intensity.
%

% Initial guess values to be passed into Mathcad
% Initial subsonic guesses
Mkg(1:numsteps+1)=ones(1,numsteps+1)*.01;
Mvg(1:numsteps+1)=ones(1,numsteps+1)*.01;
Rtg(1:numsteps+1)=ones(1,numsteps+1)*1;
Rdg(1:numsteps+1)=ones(1,numsteps+1)*1;
rtg(1:numsteps+1)=ones(1,numsteps+1)*1;
rdg(1:numsteps+1)=ones(1,numsteps+1)*1;
psg(1:numsteps+1)=ones(1,numsteps+1)*1;
Tsg(1:numsteps+1)=ones(1,numsteps+1)*600;

subflagG=zeros(1,numsteps+1);
for count1=1:1:max(size(Io))
count1
    % Check to see if the peak intensity is greater than Itherm. If it isn't, then
    % there is no ablation occurring anywhere on the spot.
    if Io(count1)<Itherm
        Pave_G(count1)=0;
    else
        % Calculate the radius at which Ievap=0, i.e. the edge of the ablation spot
        rmax=sqrt(-spot(1)^2*log(Itherm/Io(count1)));
        % Cycle through the laser profile to determine the pressures due to
ablation
        %
        % Determine the size of steps to take
        step=rmax/numsteps;
        count2=1;
        Io(count1)
        for rad=step:step:(rmax-step)
            count2=count2+1;
            radius(count2)=rad;
            % Calculate local intensity (at a point on the profile)
            Intens=Io(count1)*exp(-rad^2/(spot(1)^2));
            % Calculate ablation intensity

```

```

Iabl=Intens-Itherm;
% Pave_G is the average pressure, equal to Pacoustic for the flat-
top model

% Call Mathcad to find solution to nonlinear system of equations
if subflagG(count2) == 0
    % Find subsonic solution
    progtile = 'c:\school\thesis\sub-ml.mcd';

[Mkg(count2),Mvg(count2),Rtg(count2),Rdg(count2),rtg(count2)...

,rdg(count2),psg(count2),Tsg(count2),Pliq_G(count2),loopstop] ...
    =ablsolv(Iabl,Mkg(count2),Mvg(count2),Rtg(count2)...

,Rdg(count2),rtg(count2),rdg(count2),psg(count2),Tsg(count2),progtile);
    % If Mv>1, then we need to use supersonic formulation
    if Mvg(count2)>=1
        subflagG(count2) = 1;
        progtile = 'c:\school\thesis\super-ml.mcd';

[Mkg(count2),Mvg(count2),Rtg(count2),Rdg(count2),rtg(count2)...

,rdg(count2),psg(count2),Tsg(count2),Pliq_G(count2),loopstop] ...
    =ablsolv(Iabl,Mkg(count2),Mvg(count2),Rtg(count2)...

,Rdg(count2),rtg(count2),rdg(count2),psg(count2),Tsg(count2),progtile);
    end
else
    % Use supersonic formulation
    progtile = 'c:\school\thesis\super-ml.mcd';

[Mkg(count2),Mvg(count2),Rtg(count2),Rdg(count2),rtg(count2)...

,rdg(count2),psg(count2),Tsg(count2),Pliq_G(count2),loopstop] ...
    =ablsolv(Iabl,Mkg(count2),Mvg(count2),Rtg(count2)...

,Rdg(count2),rtg(count2),rdg(count2),psg(count2),Tsg(count2),progtile);
    end
    % Check to make sure solution is valid
    if loopstop==1
        msg='Gauss - bad solution'
        Iabl
        break

```

```

        end
    end
    if loopstop==1
        break
    end
    % Calculate average pressure using Simpson's Rule
    Pave_G(count1)=2/spot(1)^2*...
        (step/3*(4*sum(radius([2:2:count2]).*Pliq_G([2:2:count2]))...
            +2*sum(radius([3:2:count2-1]).*Pliq_G([3:2:count2-1]))));
    Fave_G(1,count1)=Pave_G(count1)*area(1);
    Fave_G(2,count1)=Pave_G(count1)*area(2);
end
end
end

```

APPENDIX F
MATLAB PROGRAM: ABLTSOLV.M

```
' abltsovl.m
```

```
function [Mk, Mv, Rt, Rd, rt, rd, ps, Ts, pac, loopstop] = ...
    abltsovl(I,Mk,Mv,Rt,Rd,rt,rd,ps,Ts,title)
```

```
chan=ddeinit('MCad', title);
```

```
%Send data to Mathcad
```

```
ddepoke(chan , 'I.evap', I);
```

```
ddepoke(chan , 'M.k', Mk);
```

```
ddepoke(chan , 'M.v', Mv);
```

```
ddepoke(chan , 'R.t', Rt);
```

```
ddepoke(chan , 'R.dens', Rd);
```

```
ddepoke(chan , 'r.t', rt);
```

```
ddepoke(chan , 'r.dens', rd);
```

```
ddepoke(chan , 'p.s', ps);
```

```
ddepoke(chan , 'T.s', Ts);
```

```
pause(1)
```

```
%Retrieve data from Mathcad
```

```
Mk = ddereq(chan , 'M.k');
```

```
Mv = ddereq(chan , 'M.v');
```

```
Rt = ddereq(chan , 'R.t');
```

```
Rd = ddereq(chan , 'R.dens');
```

```
rt = ddereq(chan , 'r.t');
```

```
rd = ddereq(chan , 'r.dens');
```

```
ps = ddereq(chan , 'p.s');
```

```
Ts = ddereq(chan , 'T.s');
```

```
pac = ddereq(chan , 'p.acoustic');
```

```
if pac>=1000000
```

```
    loopstop=1;
```

```
    rc = ddeterm(chan);
```

```
elseif pac<=0
```

```
    loopstop=1;
```

```
    rc = ddeterm(chan);
```

```
else
```

```
    loopstop=0;
```

```
end
```

```
rc = ddeterm(chan);
```

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