



Natural convective flow between an isothermal spherical body and its isothermal cubical enclosure
by Thomas Keith Larson

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
in Mechanical Engineering

Montana State University

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Abstract:

Qualitative descriptions of the natural convective flow patterns between isothermal heated spherical bodies and an isothermal cooled cubical enclosure are presented. Inner bodies used had diameters of 4.5, 7.0, and 9.0 inches', and the cubical enclosure measured 9.9 inches on a side. Test fluids used in the gap between the spheres and the cube included water, 20 cs silicone oil, and 350 cs silicone oil. The silicone oils are Dow Corning 200 fluids with 20 and 350 representing the kinematic viscosity in centistokes at 25°C. All investigations were conducted with the inner body concentrically located within the enclosure. Motion pictures and still photographs of the flow patterns observed were taken to aid in the descriptions of the patterns.

For most of the fluids and geometries tested, the flow for small temperature differences was found to maintain a peripheral pattern in which high-speed fluid layers followed the solid boundaries of the system. Larger temperature differences usually resulted in separation of the high-speed layer moving over the surface of the inner body and unsteady vortex structures in the upper portion of the gap. Unsteadiness occurring with the silicone oils as test fluids was usually confined to small regions of the gap since these oils were quite viscous and acted as unsteady flow dampers. Unsteadiness occurring with water as the test fluid was generally noted to affect the whole upper central region of the gap. Some three-dimensional activity was noted for nearly all cases investigated. This is to be expected because of the lack of symmetry resulting from having a spherical body located within a cubical enclosure. Summaries of the experimental results obtained are given in Tables 4.1-4.3 and should help to categorize the flows observed.

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Date August 6, 1974

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ISOTHERMAL CUBICAL ENCLOSURE

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THOMAS KEITH LARSON

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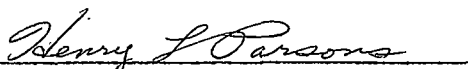
in

Mechanical Engineering

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Bozeman, Montana

August, 1974

ACKNOWLEDGEMENT

The author wishes to express his thanks and appreciation to all those who have aided him in the completion of this work. Special thanks go to Dr. J. A. Scanlan, Dr. E. H. Bishop, and Dr. R. E. Powe for their helpful suggestions and guidance. Advice and assistance received from the Mechanical Engineering machine shop was also greatly appreciated. A special note of thanks goes to his wife, Sherry, for her patience and understanding during the completion of this work.

The work reported in this paper was supported by the National Science Foundation under Grant Number GK-31908, the Atomic Energy Commission under AEC Contract Number AT(45-1)-2214, and by the Mechanical Engineering Department at Montana State University.

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ABSTRACT

Qualitative descriptions of the natural convective flow patterns between isothermal heated spherical bodies and an isothermal cooled cubical enclosure are presented. Inner bodies used had diameters of 4.5, 7.0, and 9.0 inches; and the cubical enclosure measured 9.9 inches on a side. Test fluids used in the gap between the spheres and the cube included water, 20 cs silicone oil, and 350 cs silicone oil. The silicone oils are Dow Corning 200 fluids with 20 and 350 representing the kinematic viscosity in centistokes at 25°C. All investigations were conducted with the inner body concentrically located within the enclosure. Motion pictures and still photographs of the flow patterns observed were taken to aid in the descriptions of the patterns.

For most of the fluids and geometries tested, the flow for small temperature differences was found to maintain a peripheral pattern in which high-speed fluid layers followed the solid boundaries of the system. Larger temperature differences usually resulted in separation of the high-speed layer moving over the surface of the inner body and unsteady vortex structures in the upper portion of the gap. Unsteadiness occurring with the silicone oils as test fluids was usually confined to small regions of the gap since these oils were quite viscous and acted as unsteady flow dampers. Unsteadiness occurring with water as the test fluid was generally noted to affect the whole upper central region of the gap. Some three-dimensional activity was noted for nearly all cases investigated. This is to be expected because of the lack of symmetry resulting from having a spherical body located within a cubical enclosure. Summaries of the experimental results obtained are given in Tables 4.1-4.3 and should help to categorize the flows observed.

NOMENCLATURE

Symbol	Description
a, b	Characteristic dimensions of system
C_p	Specific heat
D	Characteristic dimension
Dev	Percent deviation (equation 4.2)
g	Acceleration of gravity
k	Thermal conductivity
L	Length ratio
N_{Gr}	Grashof number (equation 2.1)
N_{Pr}	Prandtl number (equation 2.2)
N_{Ra}	Rayleigh number (equation 2.4)
r_i	Inner body radius
r_o	Distance from center of inner body to enclosure
T_{am}	Arithmetic mean temperature (equation 4.1)
$T_{i,av}$	Average inner body temperature
$T_{o,av}$	Average outer body temperature
T_o	Local outer body temperature
β	Coefficient of thermal expansion
ΔT	Temperature difference ($T_{i,av} - T_{o,av}$)
θ	Angular position measured from upper vertical axis of inner body

Symbol	Description
ρ	Density
μ	Absolute viscosity

CHAPTER I

INTRODUCTION

Until the last decade, the bulk of the literature related to natural convection concerned the convective process from a body to an infinite surrounding medium. Increased interest in the area of natural convection heat transfer from a body to a finite enclosure has been generated by the need for more accurate heat transfer rate predictions from a body to its enclosure. Projected applications may concern nuclear reactor core design, room heating, instrument packaging, and fire fighting techniques.

The need for experimental work in the area of natural convection in finite enclosures arises from the inherent complexity of the governing equations. The defining partial differential equations are nonlinear and coupled; thus no general solution is available. Additional difficulties complicating the situation are (1) the usual simplifying assumptions concerning boundary layers are not valid, (2) boundary conditions become unknown as flow unsteadiness develops, and (3) pressure distributions in the finite enclosure are unknown.

Experimental work done by Bishop (1)*, McCoy (2), Weber (3), and Scanlan et al (4) has shown that heat transfer data can be correlated to yield empirical relations predicting the convective heat transfer rate from a body to a spherical enclosure. Since natural convection is

* Underlined numbers in parentheses refer to references in the bibliography

a combination of fluid flow and thermal phenomena, it is desirable to visualize the flow patterns existing in the finite enclosure. Knowledge of the flow patterns in conjunction with the heat transfer data and temperature profiles, provides a deeper understanding of the convective process.

The objective of this experimental investigation was to photograph and qualitatively describe the fluid flow patterns resulting from the convective interaction between an isothermal spherical body and its isothermal cubical enclosure. All investigations were conducted with the spherical body located concentrically within the cubical enclosure. Test fluids used in the gap between the sphere and its enclosure included water and two different silicone oils. Similar work has been reported by Baughman (5) and Yin (6). Their experiments were conducted with a spherical rather than a cubical enclosure. Directly related work, reported by Eyler (7), was done utilizing air as the medium between the sphere and the cubical enclosure.

This work is part of a continuing project at Montana State University. The results obtained add to the currently available information and hopefully will supplement the heat transfer experiments now in progress.

CHAPTER II

LITERATURE REVIEW

Natural convection is a term used to describe transport processes in fluids wherein the motion is driven by the interaction of a difference in density with an acceleration field (such as gravity or rotation). Natural convection heat transfer is then a form of energy exchange caused by the motion of a fluid. Motion of the fluid is created by a bouyancy force resulting from the interaction between the density differences within the fluid and the acceleration field.

Review of the literature reveals that some analytical solutions are available for natural convection from a body to an infinite surroundings and that some solutions exist for a few simple enclosures. Examples can be found in Gebhart (8), Jakob (9), Holman (10), Eckert and Carlson (11), and Batchelor (12). For more complex geometries, however, experimental methods must be relied upon because of the complexity of the governing equations. For this reason, a number of experimental studies concerning natural convective flow in an enclosure have been reported.

Jakob (9) derived some basic relations on heat transfer in natural convection using the principle of similarity. He used both the differential and the dimensional methods to determine the important non-dimensional parameters. The resulting dimensionless groups are

$$N_{Gr} = \frac{g\beta\rho^2 D^3 \Delta T}{\mu^2} \quad (2.1)$$

$$N_{Pr} = \frac{\mu C_p}{k} , \quad (2.2)$$

and

$$L = \frac{a}{b} . \quad (2.3)$$

In the above equations, N_{Gr} is termed the Grashof number and N_{Pr} is the Prandtl number. The parameter L is the ratio of a and b , the characteristic dimensions of the system under investigation. Another nondimensional group, the Rayleigh number (N_{Ra}), is also useful. The Rayleigh number is defined as the product of the Grashof number and the Prandtl number, or

$$N_{Ra} = N_{Gr} \cdot N_{Pr} . \quad (2.4)$$

Initial investigations of the natural convective phenomena in a spherical annulus were carried out by Bishop (1). All of his investigations were done using air as the working fluid and with the inner sphere concentrically located within the outer sphere. Heat transfer results, temperature profiles, and qualitative information about the fluid-flow behavior are reported. Visualization studies were conducted for diameter ratios of 1.19, 1.37, 1.72, 2.53, and 3.14 and for temperature differences from 5°F to 60°F. Bishop reported the occurrence of three basic flow patterns, (1) the "crescent-eddy" type, (2) the "kidney-shaped-eddy" type, and (3) the "falling-vortices" type.

The most common flow pattern, the "crescent-eddy", was noted to occur at all temperature-differences studied for diameter ratios of 1.37

and 1.72. For diameter ratios of 1.19, 2.53, and 3.14 the "crescent-eddy" pattern occurred only at small temperature differences. The "kidney-shaped-eddy" pattern, a modification of the crescent-eddy type, was noted to occur for diameter ratios of 2.53 and 3.14 at moderate to large temperature differences. Both the "crescent-eddy" and "kidney-eddy" type patterns were noted to be steady with time. Three basic regions were apparent in both patterns. These were (1) a thin layer of high-speed fluid in the close vicinity of each sphere, (2) a central region containing slowly moving fluid, and (3) a region of transition between each high speed layer and the central region.

The only unsteady pattern observed by Bishop, the "falling-vortices" type, occurred at moderate to high temperature differences for a diameter ratio of 1.19. This unsteady pattern was characterized by the formation of counter-rotating pairs of vortices in the region near the upper vertical axis of the inner sphere. The vortices would then coalesce and merge into the outer sphere boundary flow, momentarily disrupting the central region.

Further natural convective studies utilizing a spherical annulus were conducted by Yin (6). Diameter ratios of 1.40, 1.78, and 2.17 were used with air as the working fluid. Studies with water as the working fluid were conducted with diameter ratios of 1.09, 1.40, 1.78, and 2.17. Investigations with air extended the data of Bishop (1) to a larger temperature difference. Differences observed by Yin at larger temper-

ature differences were (1) violent interior contractions, (2) slight sideways oscillations of the chimney, and (3) a three-dimensional spiral motion in the upper portion of the annulus for the smallest diameter ratio. With water as the working fluid, four flow patterns, two steady and two unsteady, were observed. These patterns were (1) the steady "dog-face" type, (2) the unsteady "dog-face" type, (3) the steady "interior tertiary" type, and (4) the unsteady three-dimensional spiral flow. The most common pattern was the steady "dog-face" flow. It occurred at small temperature differences for all diameter ratios investigated except the smallest. The pattern was characterized by three distinct regions, namely (1) regions of high-speed flow adjacent to each sphere, (2) a low speed interior region, and (3) a central stagnant region. The "interior tertiary" flow pattern occurred at large temperature differences for a diameter ratio of 1.78. This pattern was noted to be steady. The "unsteady dog-face" pattern was observed at large temperature differences for a diameter ratio of 2.17. Characteristic of the pattern was the formation and shedding of cells in the interior region. The three-dimensional spiral was an unsteady pattern that occurred at large temperature differences for diameter ratios of 1.09 and 1.40. For this case the upper portion of the annulus was dominated by a three-dimensional spiral flow.

Baughman (5) investigated the case of eccentric spheres with air and a silicone oil as the test fluids. He also used water and the

silicone oil as test fluids in concentric hemispherically ended cylinder studies. Results obtained with the eccentric spheres and air as the gap medium were similar to the concentric sphere results discussed above. Some new type flow characteristics were observed with the silicone oil as the test medium in the eccentric sphere studies. A diameter ratio of 2.17 was noted to generally yield a basic crescent shape. Negative eccentricities (inner body below the center of the outer body) tended to display a "climbing vortices" type pattern. Characteristic of this pattern was the formation of cells, one on each side of the thermal plume, and their subsequent rotation upward along the plume. Falling vortices and three-dimensional vortices were postulated to have occurred for positive eccentricities with a diameter ratio of 1.40. Negative eccentricities for this diameter ratio yielded (1) vortex drifting, (2) formation of multiple cells, and (3) undulating tendencies of the high-speed fluid layer on the surface of the inner body. Flow patterns resulting with the cylinders as the inner body and the silicone oil as the gap medium were similar to the eccentric sphere studies. Cylinder studies with water as the test fluid yielded patterns similar to the concentric sphere results obtained by Yin (6). A detailed review of the particular patterns can be found in Baughman (5).

Fluid flow patterns between a hemispherically ended cylinder and a spherical enclosure have also been investigated by Teng (13). His

studies were conducted with the cylinder concentrically located within the enclosure. Air and a silicone oil of different absolute viscosity than that used by Baughman were used as test fluids.

Additional work, primarily concerning heat transfer data and temperature profiles, has been done utilizing a spherical enclosure and various inner bodies and several different test fluids. Examples are Weber, Powe, Bishop, and Scanlan (14), McCoy, Powe, Bishop, Weber, and Scanlan (15), and Scanlan, Bishop, and Powe (4).

Natural convective flow patterns between an isothermal sphere concentrically located within a cubical enclosure have been investigated by Eyler (7). Spheres of 3.50, 4.50, 5.50, 7.00, and 9.00 inches diameter were investigated, and air was used as the gap medium. The 3.50 inch and 4.50 inch spheres yielded steady flows characterized by three distinct regions for all temperature differences investigated. These regions were (1) a high-speed boundary region, (2) an upper interior region characterized by at least one eddy formation, and (3) a slow moving lower region. For small temperature differences, the 5.50 inch sphere exhibited flow characteristics similar to the 3.50 inch and 4.50 inch bodies. At high temperature differences, unsteady flow developed and was characterized by a periodic pulsating eddy formation in the upper central region. Both the 7.00 inch and 9.00 inch bodies yielded nonperiodic unsteady flows at all temperature differences. For these two bodies, slugs of fluid were noted to randomly "fall" from the upper vertical axial region into

the upper central region.

CHAPTER III

EXPERIMENTAL APPARATUS AND PROCEDURE

EXPERIMENTAL APPARATUS

An existing apparatus used by Eyler (7) in his investigations with air as the test fluid was partially redesigned to permit the use of test fluids other than air in the study of natural convective flows and to allow a water cooling system to be employed. Figures 3.1 and 3.2 show the experimental apparatus and a schematic representing the functions of the peripheral equipment. The cubical enclosure seen in the center of Figure 3.1 contains the isothermal cubical test space and its inner body. Both the test space and inner body will be described in detail later. Peripheral equipment shown in the photograph includes (1) the water cooling system on the left, (2) inner body temperature controlling instruments to left and below, (3) temperature monitoring instruments and light source on the right, and (4) the test fluid reservoir in the upper left corner.

The main purpose of the approximately 14.0 inch cubical enclosure shown in Figure 3.1 was to allow a closed cooling system to be employed. The enclosure functioned as a water jacket surrounding the inner cubical test space. The water jacket was equipped with individual channels so that cooling water could be circulated independently over the four sides, the top, and the bottom of the cubical test space. The water cooling system was constructed so that the coolant flow rate in each channel could be controlled by means of valves. The network of plumbing required

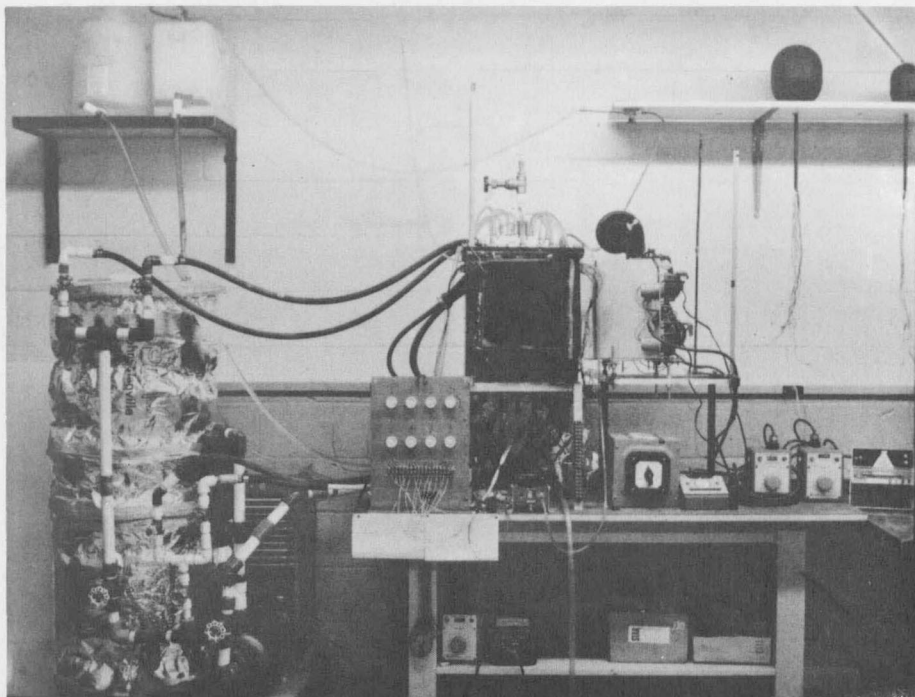


Figure 3.1 Apparatus Assembly

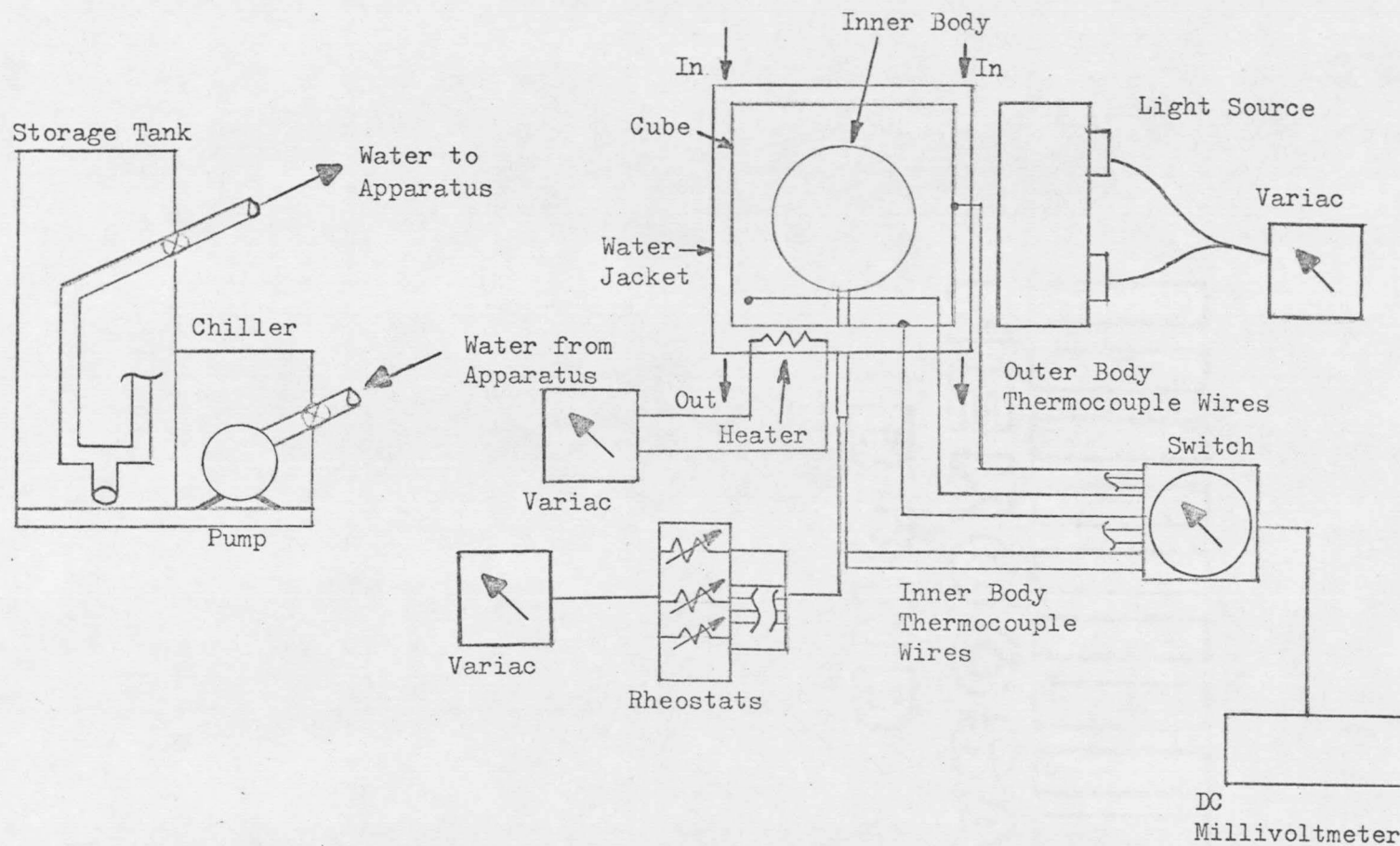


Figure 3.2 Schematic Drawing of Support Equipment

to achieve this control can be seen in Figure 3.1.

Since the apparatus could not withstand high pressures, the coolant water was actually drawn through the channels rather than forced through under pressure. Coolant water entered the side channels at the top of the water jacket and was removed at the bottom. The coolant flow entered the top channel via inlet ports on the front face of the water jacket and exited via ports on the rear face. The inlet and outlet ports for the bottom channel were located in the bottom of the jacket. The bottom channel was also fitted with a tubular resistance heater to be used in the event that heating, rather than cooling of the bottom of the cubical test space, was required to maintain isothermal conditions.

The front, right side, and top of the jacket were constructed of 0.500 inch thick transparent plexiglass so that the flow patterns could be illuminated and photographed. The remaining sides and bottom were constructed of sheet aluminum. The sides and bottom of the jacket were held together with machine screws and a leakproof silicone sealant.

Studs were fitted in the tops of the sides of the jacket. The top could then be secured with wing nuts when it was in place. This arrangement allowed for easy access to the cubical test space.

The cubical test space was constructed from sheet plexiglass approximately 0.25 inches thick. The test space measured 9.90 inches on an inside edge. The four sides and bottom were held together with machine screws and silicone sealant. Studs were fitted in the top edges of the

test space so that the top could be easily removed to allow insertion of the inner test body. Wing nuts were used to secure the top once it was in place. A centrally located 0.525 inch hole was bored in the bottom of the test space to allow passage of the inner body support stem. A vent hole was located in one corner of the top of the test space. This vent allowed entrapped air to escape as test fluid was introduced into the test space through fill holes located in the bottom.

Support for the cubical test space was provided by a frame attached to the base of the test space and to the enclosing water jacket. A solid rectangular support with a 0.525 inch hole bored through it was located concentrically with the test body stem hole in the bottom of the cubical test space. An O-ring located in the rectangular support provided a seal for the test body stem. A 1/2" Conax packing gland, located on the outside of the enclosing water jacket, was also employed to seal the inner test body stem and to secure the body at a desired location.

Temperatures of the surfaces of the cubical test space were monitored using copper-constantan thermocouples. Nineteen thermocouples were positioned at different locations on the cube. Small holes were drilled through the plexiglass, and the thermocouples were epoxied in place flush with the inner surface. All thermocouple leads exited through holes bored in the water jacket. The holes were sealed tight with silicone sealant. The temperature variation on any given side of the cubical enclosure was usually found to be less than 2°F. Thus, all thermocouples from a

common side were connected in parallel so that an average temperature reading was indicated.

Three of the five spherical inner test bodies used by Eyler (7) were used in this study. The bodies were constructed by soldering two hemispheres together. Heating tapes attached to the inner surface of the 0.030 inch thick copper spheres allowed the bodies to be kept essentially isothermal. The tapes, 0.020 inches thick and 0.125 inches wide, were constructed of a resistance wire sandwiched between an adhesive backed foil and an insulative foil. These tapes were spirally affixed to the inner surface of each half of the body under construction. Figure 3.3 illustrates the tape arrangement for the two halves of the 7.0 inch sphere. Silicone sealant was also spread over the tapes after installation to insure adhesion to the sphere surface.

Temperatures of the test body were monitored using copper-constantan thermocouples. Holes were drilled through the walls of each hemisphere, and the thermocouples were soldered in place from the inside so that they were flush with the outer surface.

The thermocouple leads and heater tape power leads exited the test body through the support stem previously mentioned. The stem, a 0.500 inch diameter stainless steel tube, was soldered to the base of one of the hemispheres.

Assembly of the spherical body involved stuffing each hemisphere with an insulative material to minimize internal convective activity

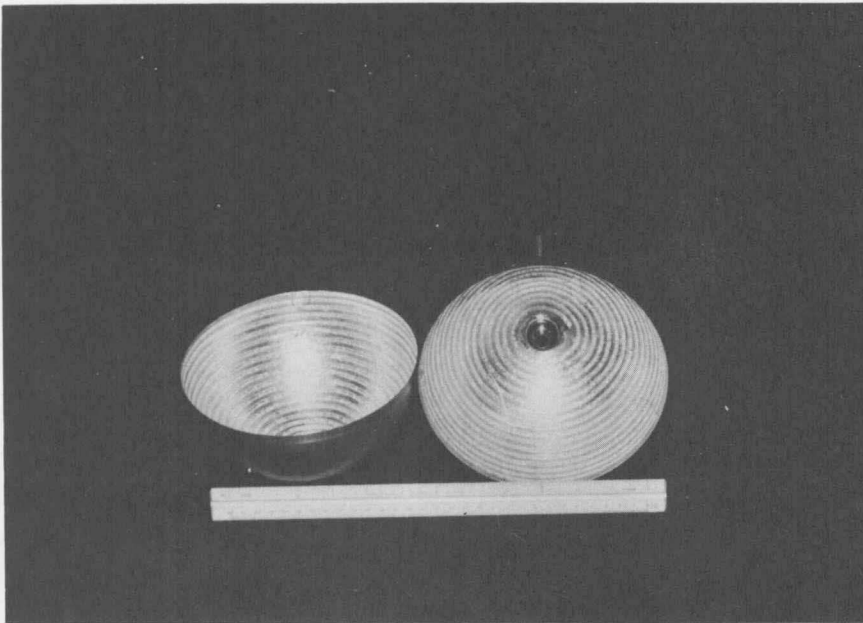


Figure 3.3 Interior of Inner Sphere

and then soldering the two hemispheres together. The assembly joint and the thermocouple locations were then filed smooth. The heating tapes and thermocouples were then checked to insure proper operation and the assembled body was checked for leaks. The final step involved painting the spherical body and its stem with flat black paint to reduce unwanted reflections when conducting visualization experiments.

A lighting system, equipped with two 650 watt movie lamps, provided a collimated beam of light necessary for photographic purposes. The light was passed through 0.250 inch wide view slits in the enclosure. The slits were located on a side of the water jacket adjacent to the frontal viewing window.

Nonsymmetry caused by having a spherical body concentrically located within a cubical enclosure required that the flow be viewed in two different vertical planes perpendicular to a cube face. The flow was viewed in (1) a plane through the sphere's vertical axis and (2) a plane parallel to but shifted 1.65 inches away from the plane through the vertical axis of the sphere. These planes were necessary in order to investigate any three-dimensional flow and the extent of the unsteadiness of the flow.

All thermocouple leads were connected to a 24 point thermocouple switch which in turn was connected to a DC millivoltmeter. A reference junction was provided by an ice bath. Temperatures were recorded as millivolt readings and later converted to degrees.

A water cooling system was used to provide chilled water for the purpose of maintaining the cubical test body in an isothermal condition. The cooling system consisted of an insulated storage tank, water chiller, and a pump.

Power was supplied to the inner body tapes by an AC Variac autotransformer. The power supply was connected to a panel of rheostats. Each tape on the inner body was then connected in series with a rheostat. This arrangement allowed positive control of the power consumed by each tape and thus allowed the inner body to be maintained in an essentially isothermal condition.

Still photographs were taken with a Calumet 4" x 5" professional camera using Kodak Tri-X Pan professional film. Moving pictures were recorded with a Beaulieu R 16 "Automatic" movie camera and Kodak 4-X Reversal film.

EXPERIMENTAL PROCEDURE

In order to visualize and photograph the flow pattern between the spherical body and its cubical enclosure, it was necessary to introduce suitable tracer particles into the test medium. Particles found in liquid "Ajax" detergent made good tracer particles in water. The solution was made by boiling the necessary quantity of distilled water, allowing it to cool to approximately 80°F, and then adding five drops of the detergent per gallon of water. The solution was then gently mixed and allowed to sit for several hours. The resulting tracer particles that appeared in the water were both highly reflective and neutrally bouyant. Fluorescent orange spray paint produced excellent tracer particles for the 20 cs and 350 cs silicone oils. The oils are Dow Corning 200 fluids with 20 and 350 representing the kinematic viscosity at 25°C. Tracer particles were made by spraying the surface of an open container of the silicone oil with the paint and allowing the atomized paint particles to fall on the surface of the fluid. Stirring the mixture then produced a homogeneous solution. Best results were obtained when the surface of the oil was sprayed from a height of approximately 18 inches.

After the test fluid was prepared for experimentation, the desired inner body was selected and prepared. The body was first painted flat black and then fitted with shrink tubing on its support stem to minimize lateral conduction along the stem. The body was then placed in the

apparatus, and the cover of the cubical test space was secured.

The central location of the body in the cube was next determined. This was done by raising and lowering the body to its maximum upper and lower positions, marking the stem at these points, and then positioning the inner body at the midpoint between the marks. The Conax packing gland was then tightened, thus securing the stem. Final assembly involved connecting the inner body power and thermocouple leads, securing the top of the water jacket, and connecting the plumbing associated with the chilling system.

Once the system was assembled, the following procedures were carried out:

- (1) The test medium was introduced into the cubical test space by gravity feed, and the water jacket was filled.
- (2) Appropriate valves were opened, and the pump and water chiller systems were turned on. Water was added to the water jacket until all of the air was forced out of the cooling system.
- (3) The AC Variac autotransformer was turned on and set to give a desired temperature difference between the cube and the inner body. After allowing sufficient time for thermal equilibrium to be reached, the rheostats were adjusted to achieve an isothermal condition on the inner body, and time was allowed to reach a new equilibrium.

- (4) Once isothermal conditions existed and the system was in equilibrium, the investigation of the flow field was carried out. Still photographs and/or moving pictures were taken, and the flow patterns were described and sketched.

After the pattern had been adequately photographed and described, the autotransformer was reset, and time was allowed to reach a new equilibrium. The process of describing the flow pattern was then repeated. This procedure was done as many times as necessary to describe the flow for the particular inner geometry and test fluid. Information recorded for each run comprised:

- (1) The date and time,
- (2) Inner body size,
- (3) The test fluid,
- (4) Run number
- (5) Wattage to the inner body,
- (6) Inner body and cube thermocouple readings,
- (7) Camera settings and photograph exposure time,
- (8) Written descriptions and sketches of the flow.

The amount of data collected required the use of a computer program to reduce the data to a desired form. A copy of the program, written for the XDS Sigma 7 computer, is listed in the Appendix.

For clarification purposes, a listing of the inner bodies and test fluids used is shown in Table 3.1.

Table 3.1

FLUID AND GEOMETRIES INVESTIGATED

Inner Body Diameter In Inches	Silicone 20	Silicone 350	Water
4.5	x	x	x
7.0	x	x	x
9.0	x	x	x

CHAPTER IV

EXPERIMENTAL RESULTS

This chapter is divided into two sections. The first section deals with the flow patterns observed and is divided into three subsections. Each subsection contains qualitative descriptions and discussion about the flow patterns observed for each of the inner bodies investigated and a particular working fluid. The second section contains a summary of the experimental results.

A few remarks are in order before proceeding to the flow pattern descriptions. A value for D in equation 2.1 had to be selected so that the Grashof number could be calculated. This dimension, hereafter referred to as the characteristic dimension, was defined as one-half the difference between the length of one side of the cube and the inner body diameter. The temperature difference upon which the Grashof number was based was defined as the difference between the average inner body temperature and the average outer body temperature. All of the test fluid properties were based on the arithmetic mean temperature of the inner and outer bodies. This temperature was defined as

$$T_{am} = \frac{(T_{i,av} + T_{o,av})}{2} \quad (4.1)$$

The computer program previously mentioned was used to calculate thermocouple temperatures, test fluid properties, and the appropriate dimensionless parameters.

In most of the cases investigated it was found that sufficient control could be maintained on the inner body to keep it in an isothermal

state. It was not possible, however, to always keep the outer body exactly isothermal. Therefore it was necessary to set some limits on the error involved. Upper and lower limits on the temperature difference imposed were chosen such that the maximum percent deviation was less than 10 percent. The percent deviation was defined as

$$\text{Dev} = \frac{|T_o - T_{o,av}|}{T_{i,av} - T_{o,av}} \times 100. \quad (4.2)$$

To keep within the limits set by the above definition, the minimum temperature difference observed was usually between 5°F and 15°F. The maximum temperature differences imposed were limited by the error limits on the percent deviation or by the fact that for the water studies the "Ajax" tracer particles disappeared when the arithmetic mean temperature exceeded approximately 120°F.

The qualitative descriptions and discussions about the flow patterns are substantiated by photographs of the actual flows. Pertinent information such as the Grashof number, imposed temperature difference, working fluid, and inner body size is listed below the photograph. A note should be said about the photographs taken on the offset perpendicular plane. The majority of the flow in this case was either directly toward the camera or approaching the corner of the cube at an angle to the camera. Information obtained from these photographs was usually limited to a speculation about the flow in other vertical planes.

FLOW PATTERN DESCRIPTIONS

SILICONE 20 AS THE GAP WORKING FLUID

The first geometric configuration investigated was that of a 4.5 inch diameter sphere located concentrically in a 9.9 inch cube. This arrangement resulted in a characteristic dimension of 2.7 inches. The minimum and maximum values of the Grashof number were 22,800 ($\Delta T = 13^\circ\text{F}$) and 534,400 ($\Delta T = 94^\circ\text{F}$) respectively.

Shown in Figure 4.1 is the flow pattern observed for a Grashof number of 22,800 ($\Delta T = 13^\circ\text{F}$). Several basic regions in this pattern can be identified. These regions consist of (1) a layer of high-speed fluid that follows the geometric boundaries of the system, (2) an upper interior region, and (3) a lower stagnant region. As seen from Figure 4.1, the high-speed fluid flows upward over the surface of the sphere and separates from the sphere surface forming a chimney around the upper vertical axis. Near the top of the cube, a portion of the fluid is turned downward toward the surface of the inner body. The fluid that is turned downward gives rise to a pair of vortices, one on each side of the chimney. The center of each cell was located approximately where the fluid flowing over the sphere surface turned to form the chimney. This pattern is quite similar to the "dog-face" flow first reported by Yin (6). The major difference is that in the "dog-face" pattern the cells were located at the top of the chimney much closer to the outer body. The portion of fluid that flowed across the top entrained a

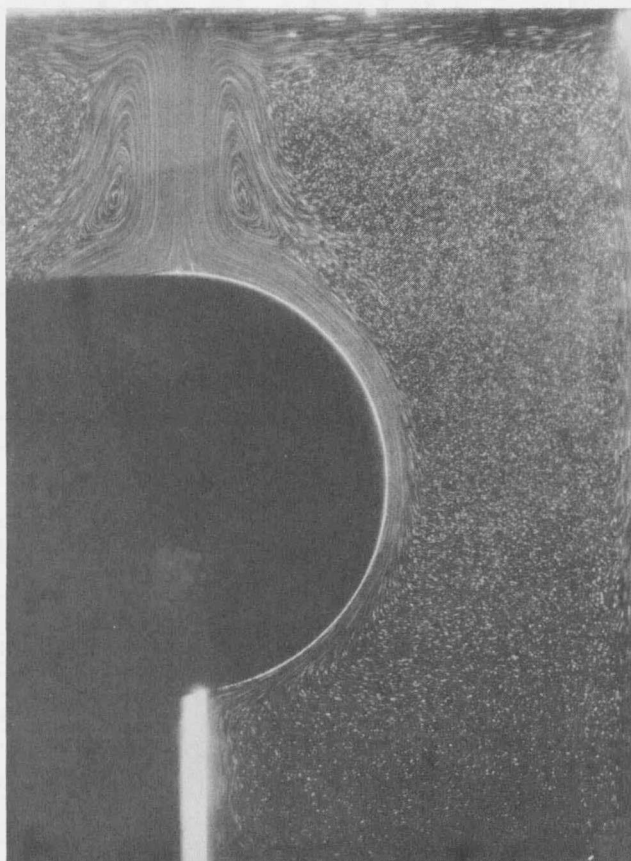


Figure 4.1 Silicone 20 Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 22,800$

$\Delta T = 13^{\circ}F$

small amount of fluid from the central region and then proceeded down the side of the enclosing cube. At a point on the side of the cube at the same elevation as the bottom of the inner body, a portion of the fluid flowing down the side of the cube separated and slowly moved inward toward the spherical body. The remaining fluid continued down the cube side into the lower stagnant region. The point of separation on the side of the cubical enclosure was more pronounced at larger temperature differences.

Examination of Figure 4.1 indicates that there is a substantial flow up the support stem of the inner body but only a relatively weak boundary layer type flow across the bottom of the cube toward the stem. This indicates the possibility of three-dimensional flow in the lower region near the stem or else strong flows in the lower region in different planes than that shown in Figure 4.1.

The flow pattern just described was steady with time and existed up to a Grashof number of 39,000. Increasing the Grashof number above this value resulted in an unsteady periodic pattern. The basic characteristics of this flow were the same as before with the exception of the two cells adjacent to the chimney. These cells remained stationary for a short period and then proceeded to move upward along the sides of the chimney. As the cells moved upward, they lost fluid to the central region and finally lost their vortex identity as they were engulfed by the high-speed fluid layer along the top of the cube. The upward

movement of the cells was noted to cause a very slight tangential oscillation of the chimney. As previously stated, the process appeared to be periodic with time. For a Grashof number of 39,900, the cells would form, rise, and disperse in about 15 seconds. New cells would form about 10 seconds after the process had been completed. As the Grashof number was increased to its maximum value of 534,400, the time required for the process to be completed had decreased to about 10 seconds, and new cells formed almost immediately. Also characteristic to this pattern was the formation of a small cell in the corner of the cubical enclosure. The cell appeared to be three-dimensional in nature and resulted, in part, from the abrupt change in the enclosing boundary. Figure 4.2 illustrates the flow pattern just discussed.

Increasing the Grashof number from 39,900 to larger values had some effect on the flow patterns observed. The basic unsteady periodic pattern remained unaltered (except for the period); but changes were noted in the upper central region which had previously been relatively stagnant. A weak return flow in the upper central region was first noticed for a Grashof number of 65,700 ($\Delta T = 29^\circ\text{F}$). For this case a small amount of the fluid turned downward alongside the chimney was noted to migrate toward the side of the enclosure. The strength of this return flow was noted to increase as the Grashof number increased. Other minor changes noticed at larger Grashof numbers were (1) the fluid in the high-speed layers adjacent to the solid boundaries of the system had increased in

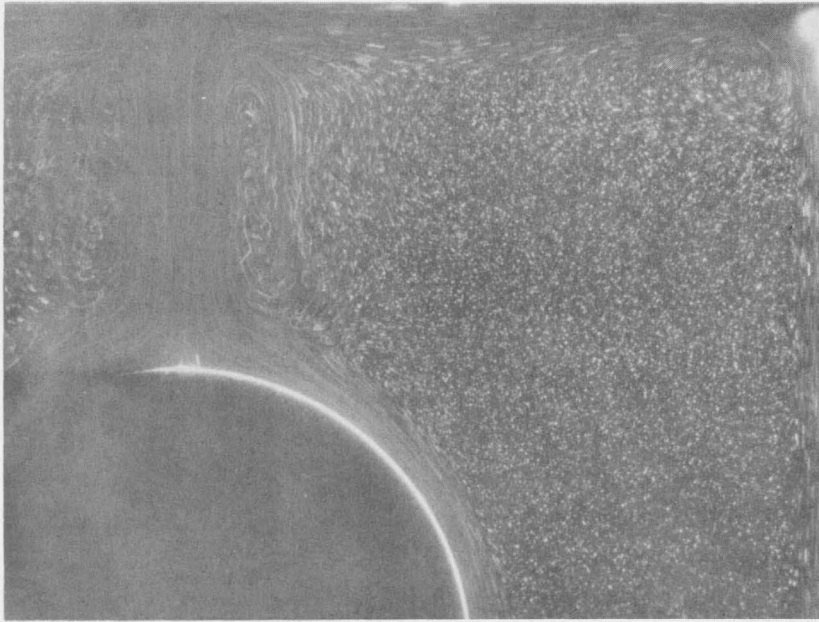


Figure 4.2 Silicone 20 Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 39,900$

$\Delta T = 20^{\circ}F$

velocity, (2) the thickness of the high-speed layers had decreased slightly, (3) the width of the chimney had decreased slightly, and (4) the point of separation of the cool layer of fluid flowing down the side of the enclosure had moved upward.

When the Grashof number reached approximately 315,700, additional flow in the form of a recirculation movement was observed in the central region. Fluid from the central region was entrained by the high-speed layer flowing over the surface of the inner body. This resulted in a movement of some of the fluid from the layer flowing down the side of the cubical enclosure toward the sphere. Figure 4.3 depicts the return and recirculation flows. Also evident from Figure 4.3 is the existence of a thin layer of stagnant fluid between the return flow and the recirculation flow. This indicates that there is no shear between the two flows. The pattern discussed above existed up to the maximum observed Grashof number of 534,400 ($\Delta T = 94^{\circ}\text{F}$).

The next inner geometry investigated was the 7.0 inch diameter sphere. This body was concentrically located within the 9.9 inch cube and resulted in a characteristic dimension of 1.45 inches. The fluid flow was investigated over a range of Grashof numbers from a minimum of 3,900 to a maximum of 123,100. This corresponds to minimum and maximum temperature differences of 15°F and 111°F respectively.

Figure 4.4 illustrates the flow pattern observed in the upper portion of the gap for Grashof numbers between the minimum value of 3,900 ($\Delta T =$

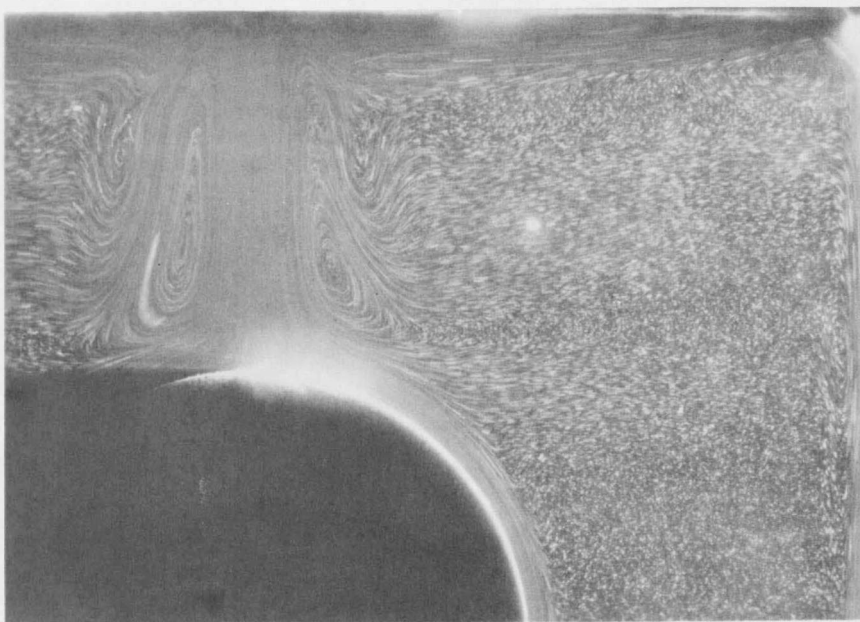


Figure 4.3 Silicone 20 Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 385,000$

$\Delta T = 80^{\circ}F$

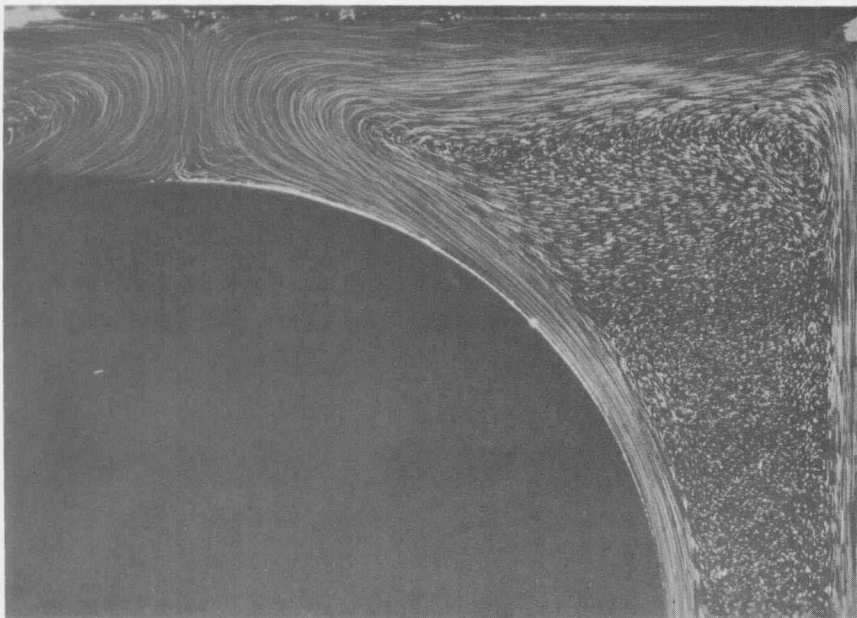


Figure 4.4 Silicone 20 Flow Pattern for

ID = 7.0 inches

$N_{Gr} = 7,400$

$\Delta T = 23^{\circ}F$

15°F) and a value of 7,400 ($\Delta T = 23^\circ\text{F}$). A high-speed fluid layer, resembling a boundary layer, basically follows the solid boundaries of the system. This fluid proceeds over the surface of the inner body and separates at a point near the upper vertical axis of the sphere forming a thermal plume around the vertical axis of the inner body. The pattern, at least for small Grashof numbers, was symmetric in the plane of observation about the vertical axis. The fluid then moves across the top and down the side of the cubical enclosure to a point of separation (Figures 4.5 and 4.6 illustrate the point of separation). The point of separation was located slightly above a horizontal plane through the bottom of the inner body for the whole range of Grashof numbers investigated. The region below the point of separation was essentially stagnant for all of the cases investigated.

Examination of Figure 4.4 indicates that recirculation of flow in the upper interior region, noted at larger Grashof numbers for the 4.5 inch diameter inner body, was also present for investigations with the 7.0 inch diameter body. The characteristics of the recirculation were as previously noted in that fluid was apparently entrained from the central region as the flow over the surface of the inner body turned upward into the thermal plume. This resulted in a general movement of some of the fluid from the cool layer flowing down the side of the cube toward the inner body. A small cell of three-dimensional nature was also noted to occur in the upper corner of the cube between the high-speed

layer across the top of the cube and the recirculation flow. This corner eddy was not steady and had no definite period. Slight three-dimensional characteristics were exhibited by the layer of fluid flowing across the top of the cube. The fluid in the layer appeared to display a slight rotational motion as it moved across the upper surface.

For Grashof numbers between approximately 7,400 ($\Delta T = 23^\circ F$) and 32,00 ($\Delta T = 55^\circ F$), flow similar to that shown in Figure 4.5 was observed. Two large rotating cells exist, one on each side of the thermal plume, where the flow over the inner body surface turns into the chimney and across the top of the cube. These cells appeared to be steady with time. The flow in the chimney area for this Grashof number range exhibited some unsteady characteristics. Figure 4.5 shows two very small counter rotating cells on the surface of the inner body near the base of the chimney. These cells would form as some fluid from the boundary flow over the sphere drifted into the vertical axis region near the top of the inner body. The main boundary flow then detoured around this small region in which the fluid moved about randomly. The random motion continued for only a few seconds, and then the two small cells formed. The formation of the cells was followed by their upward movement until they were absorbed by the boundary flow across the top. A well developed flow in the chimney region (as in Figure 4.4) would then form for a short period after which the sequence of events just described would again occur. This process was noted to occur with a period of approximately 110 seconds. The

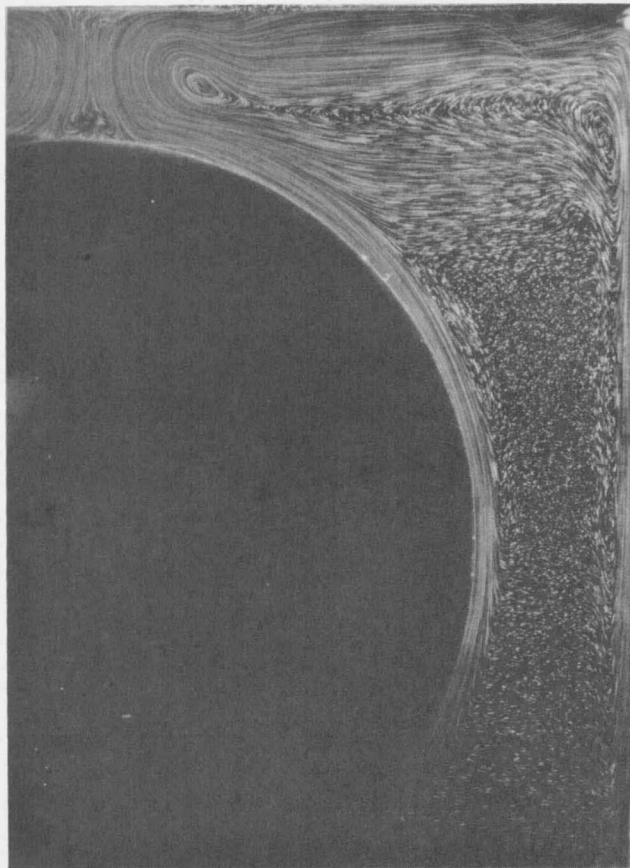


Figure 4.5 Silicone 20 Flow Pattern for

ID = 7.0 inches

$N_{Gr} = 32,000$

$\Delta T = 55^{\circ}F$

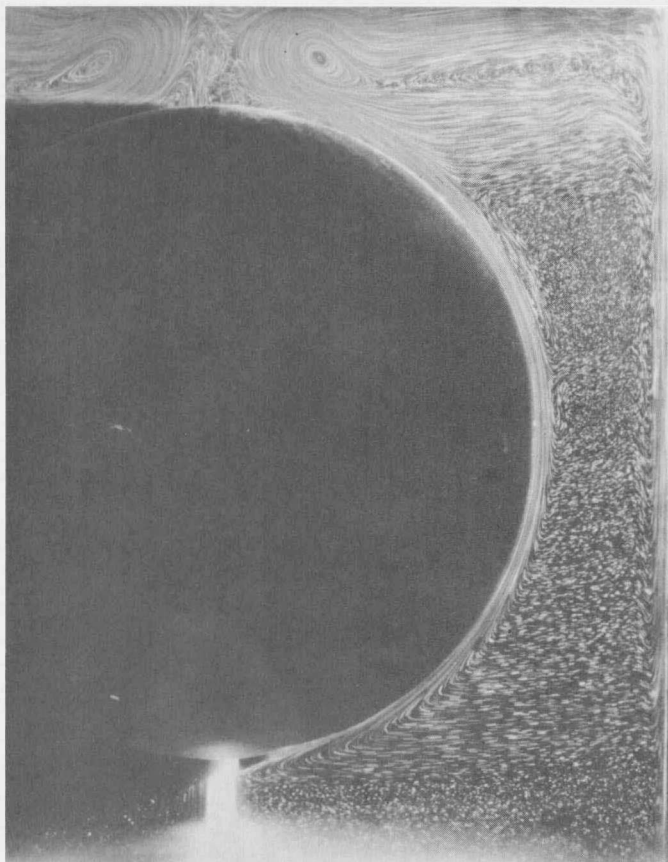


Figure 4.6 Silicone 20 Flow Pattern for
ID = 7.0 inches

$$N_{Gr} = 57,100$$

$$\Delta T = 75^{\circ}F$$

presence of the small cells was noted to have almost no affect on the rest of the flow field.

The three-dimensional corner eddy was somewhat stronger at larger Grashof numbers. This can be seen by comparing Figures 4.4 and 4.5. Also noted at larger Grashof numbers was the presence of a small steady cell attached close to the surface of the inner body. This cell was located at a point about 60 degrees below the vertical axis of the sphere. This cell was observed for the rest of the cases investigated.

The basic flow pattern observed for Grashof numbers greater than 32,000 ($\Delta T = 55^\circ F$) is shown in Figure 4.6. The primary flow over the inner body surface separates from the surface at a point located approximately 10 degrees below the vertical axis of the sphere. The separation point was noted to be dependent on the Grashof number. For the maximum Grashof number observed ($N_{Gr} = 123,100$, $\Delta T = 111^\circ F$), the point of separation had moved to about 25 degrees below the vertical axis of the sphere. The portion of the gap where the thermal plume had previously existed was now dominated by random motion. Slugs of fluid from the boundary flow over the sphere surface were ejected into the upper vertical axis region. These slugs would then rise in an erratic fashion to the top of the enclosure where the fluid was absorbed by the primary flow across the top. This activity caused the main cells in the upper region to exhibit an expansion and contraction type motion. The centers of the cells did not display any lateral translation.

The last inner geometry investigated with the 20 cs fluid was the 9.0 inch diameter sphere. This body was concentrically located in the cube and yielded a characteristic dimension of 0.45 inches. The minimum and maximum Grashof numbers for which investigations were conducted were 62 ($\Delta T = 7^\circ F$) and 2,300 ($\Delta T = 84^\circ F$) respectively.

The flow observed in the gap for Grashof numbers between 62 and 185 is shown in Figure 4.7. The high-speed primary flow over the inner body does not flow into the narrow gap in the upper vertical axis region. This flow was noted to separate at a point about 25 degrees below the vertical axis of the inner body and then proceed across the top and down the side of the cubical enclosure into the lower region.

The pattern appeared to be divided into two separate regions by the narrow spacing between the inner body and the cube at the midplane of the sphere. This division did not affect the majority of the primary high-speed flow. The upper central region was bounded by the narrow gap at the midplane of the sphere, by the flow over the sphere surface, and by the top and side of the enclosing cube. Some fluid was lost to the central region by the cool layer flowing down the side of the cube as the gap became narrower. This fluid was, in turn, absorbed by the primary flow over the sphere surface thus creating the separation between the upper and lower regions. A similar occurrence was observed in the lower region. In the narrow gap below the midplane of the inner body, small amounts of fluid from the flow over the sphere were entrained by the

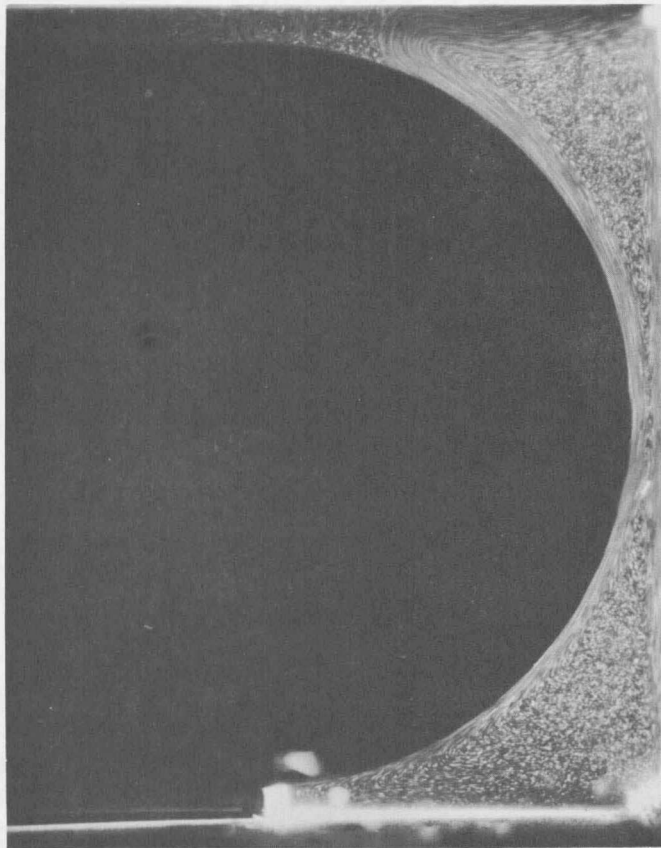


Figure 4.7 Silicone 20 Flow Pattern for

ID = 9.0 inches

$N_{Gr} = 185$

$\Delta T = 19^{\circ}F$

layer flowing down the side of the cube. The fluid in the lower region was no longer stagnant as it was for the 7.0 inch diameter inner body. Fluid in this portion of the gap slowly migrated toward the inner body.

The upper vertical axis region of the gap was dominated by unsteady activity for all the cases investigated with the 9.0 inch diameter sphere. This activity was characterized by the nonperiodic formation of multiple rotating cells in this portion of the gap and by random motion of the fluid for the time period between dissipation and formation of the cells. The cells usually did not encompass the whole of the upper vertical axis region. Fluid in the spaces between the cells exhibited random motion. Figure 4.8 shows a close view of the gap containing two cells. As many as four cells were at times noted.

Increases in the Grashof number to values larger than 184 resulted in changes in the flow pattern for the 9.0 inch diameter sphere that were similar to those observed with the 7.0 inch body. For a Grashof number in the neighborhood of 328 ($\Delta T = 29^\circ F$), a recirculation of flow in the upper central region of the gap commenced. Accompanying the recirculation were two cells. Both were located between the high-speed flow across the top of the cube and the recirculation flow. A pulsating motion was associated with the cell located closer to the inner body. Further increases in the Grashof number resulted in more intense unsteadiness in the upper vertical axis region, a stronger recirculation flow, and the occurrence of several small cells in the narrow gap region above and

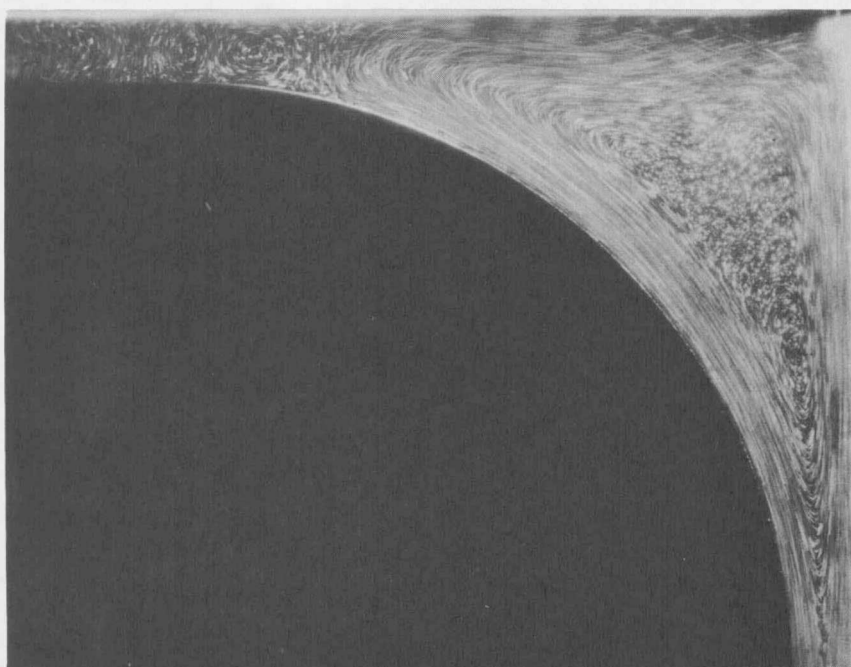


Figure 4.8 Silicone 20 Flow Pattern for

ID = 9.0 inches

$N_{Gr} = 62$

$\Delta T = 7^{\circ}F$

below the midplane of the inner body. The basic pattern observed for Grashof numbers between 328 ($\Delta T = 29^\circ\text{F}$) and 2,300 ($\Delta T = 84^\circ\text{F}$) is illustrated in Figure 4.9.

SILICONE 350 AS THE GAP WORKING FLUID

The same three inner bodies used in the 20 cs investigations were also used with the 350 cs fluid as the gap medium. All investigations were conducted with the inner body located concentrically within the 9.9 inch cube.

The fluid flow patterns for the 4.5 inch diameter inner body were investigated for Grashof numbers ranging between a minimum of 73 ($\Delta T = 10^\circ\text{F}$) and a maximum of 1,390 ($\Delta T = 78^\circ\text{F}$). For Grashof numbers less than approximately 170 ($\Delta T = 22^\circ\text{F}$), a steady flow pattern as shown in Figure 4.10 was observed. As was noted previously, a layer of high-speed fluid followed the contour of the inner body and then separated forming a thermal plume around the upper vertical axis of the sphere. The fluid then proceeded across the top and down the side of the enclosing cube. No point of separation of the high-speed fluid layer from the cube side was evident. Figure 4.10 reveals that the upper central and lower portions of the gap were almost completely stagnant. Some fluid from the thermal plume and the layer moving down the side of the cube was lost to these stagnant regions. A weak flow up the support stem of the inner body is also evident from Figure 4.10. This indicates the possibility of some three-dimensional activity in the lower portion of the

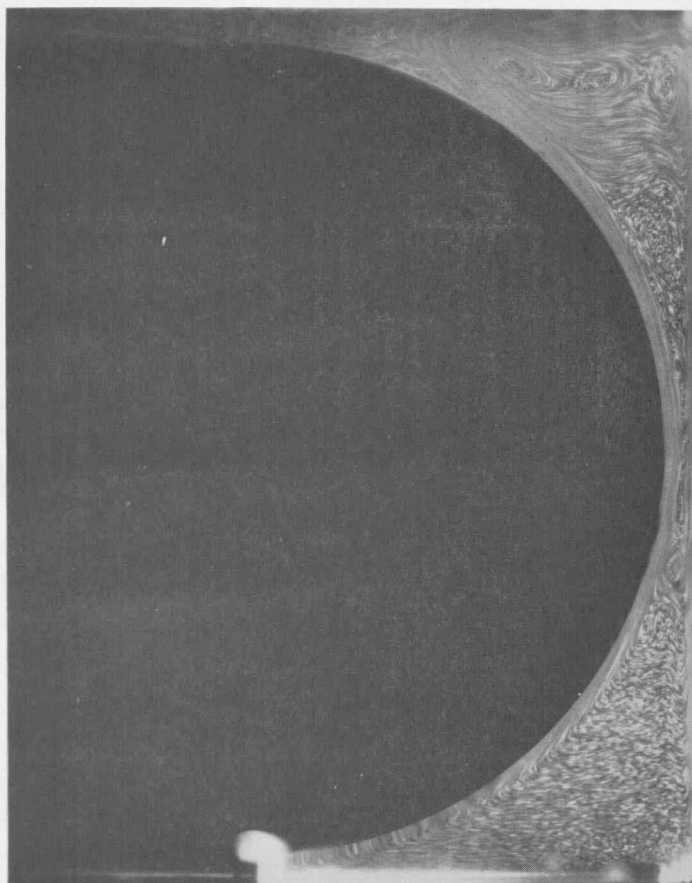


Figure 4.9 Silicone 20 Flow Pattern for

ID = 9.0 inches

$N_{Gr} = 1,548$

$\Delta T = 70^{\circ}F$

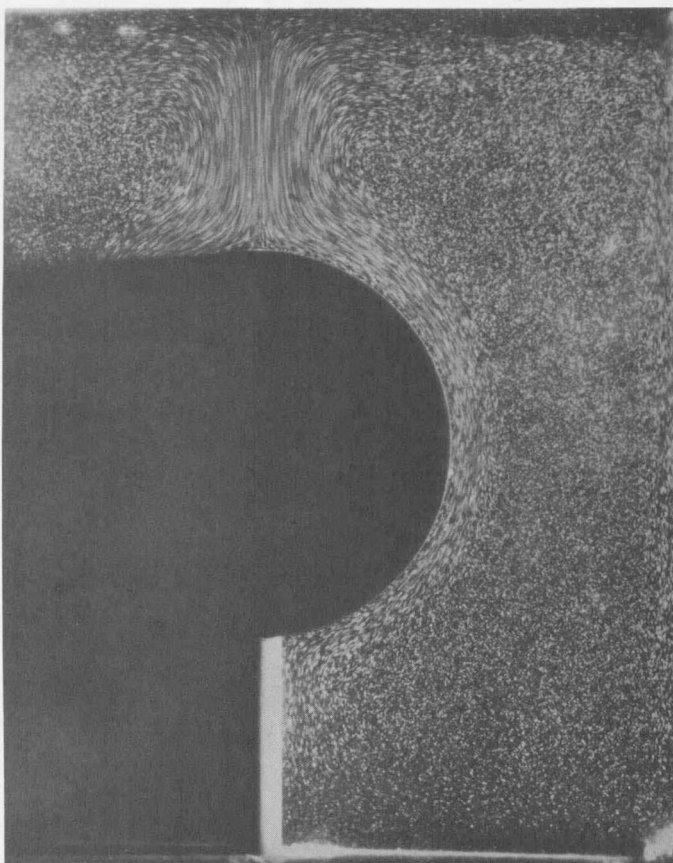


Figure 4.10 Silicone 350 Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 73$

$\Delta T = 10^{\circ}F$

gap or else increased flow in the lower region in some plane other than that investigated.

Increasing the Grashof number to a value larger than 160 resulted in the formation of steady cells, one on each side of the thermal plume, as shown in Figure 4.11. The cells were located in the neighborhood of the chimney where the flow over the sphere surface turned into the chimney. These cells first appeared for a Grashof number of approximately 170 ($\Delta T = 22^\circ F$). They remained steady and grew in size with increases in Grashof number until a value of $N_{Gr} = 560$ ($\Delta T = 48^\circ F$) was reached. This pattern (Figure 4.11) was very similar to the pattern observed at small Grashof numbers with the 20 cs fluid (Figure 4.1). Figure 4.12 illustrates the flow observed in the offset plane for $N_{Gr} = 560$. This photograph helps to substantiate past investigators' hypotheses about the uniformity of the boundary flow over the inner body surface. From Figure 4.12 it is seen that the boundary flow moves over the sphere surface in a two-dimensional manner, and the flow lines tend to converge at the top of the body. This is what was expected and had been postulated in the past, but no photographs were available to verify the occurrence. The fluid activity in the center of the upper region in Figure 4.12 is the steady cell (one on each side of the chimney) shown also in Figure 4.11. This seems to indicate the symmetrical formation of the cells about the upper vertical axis of the sphere. The primary flow along the top of the cube can also be seen in Figure 4.12. Comparison between

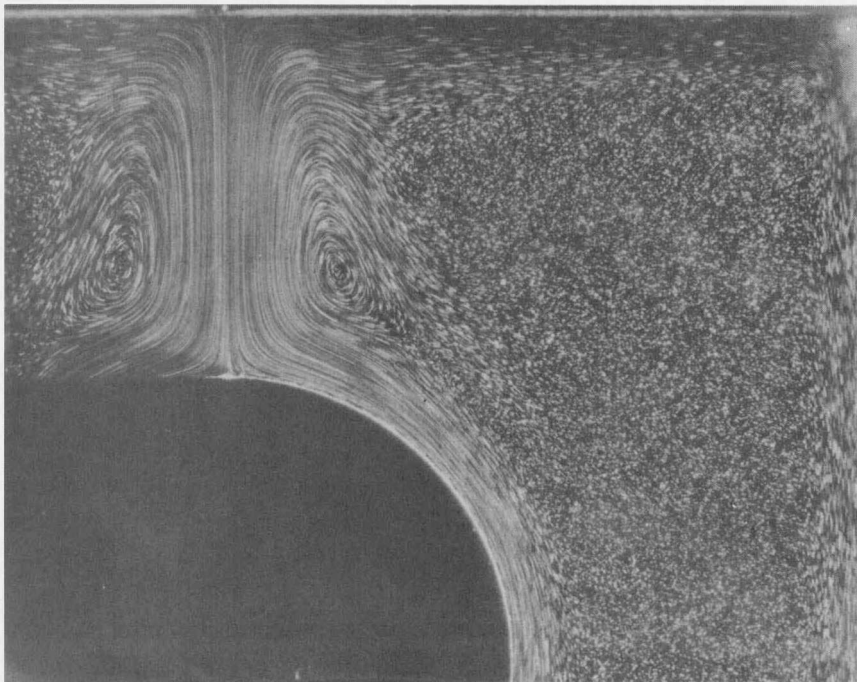


Figure 4.11 Silicone 350 Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 560$

$\Delta T = 48^{\circ}F$

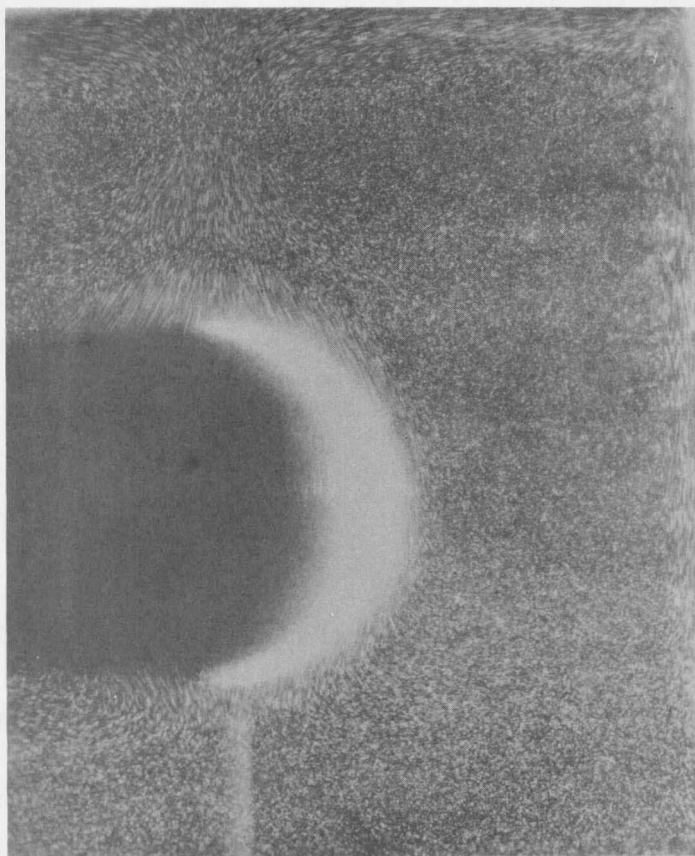


Figure 4.12 Silicone 350 Flow Pattern in the
Offset Plane for

ID = 4.5 inches

$N_{Gr} = 560$

$\Delta T = 48^{\circ}F$

Figures 4.11 and 4.12 of the boundary flows along the top and side of the cube indicates that these flows are slightly more pronounced in the offset plane. This may also be evidence that the boundary flows are more pronounced in the vertical diagonal plane of the cube than in the vertical perpendicular plane.

An increase in the Grashof number ($N_{Gr} > 560$) resulted in a very interesting unsteady pattern. The pattern was noted to be periodic, and the period was Grashof number dependent. This particular flow was first observed for a Grashof number of 650 ($\Delta T \approx 55^\circ F$). The sequence of events associated with this pattern is illustrated in Figures 4.13 and 4.14.

A cell formed periodically near the inner body surface at a point located about 20 degrees below the upper vertical axis. Another cell which was counter rotating and was somewhat removed from the chimney formed directly above the clockwise rotating cell near the inner body surface. These two cells are shown in Figure 4.13. After remaining stationary for a short period of time, the lower cell started to climb up the chimney growing in size and increasing in velocity as it did so. This caused the entire flow field adjacent to the chimney to become rotational as shown in Figure 4.14. Associated with this activity was a very pronounced upward acceleration of fluid in and adjacent to the chimney. Moving pictures taken of this pattern clearly indicate this acceleration of fluid and also indicate that the pattern is symmetric about the upper vertical axis of the inner body (filming speed was 2 frames per second). As

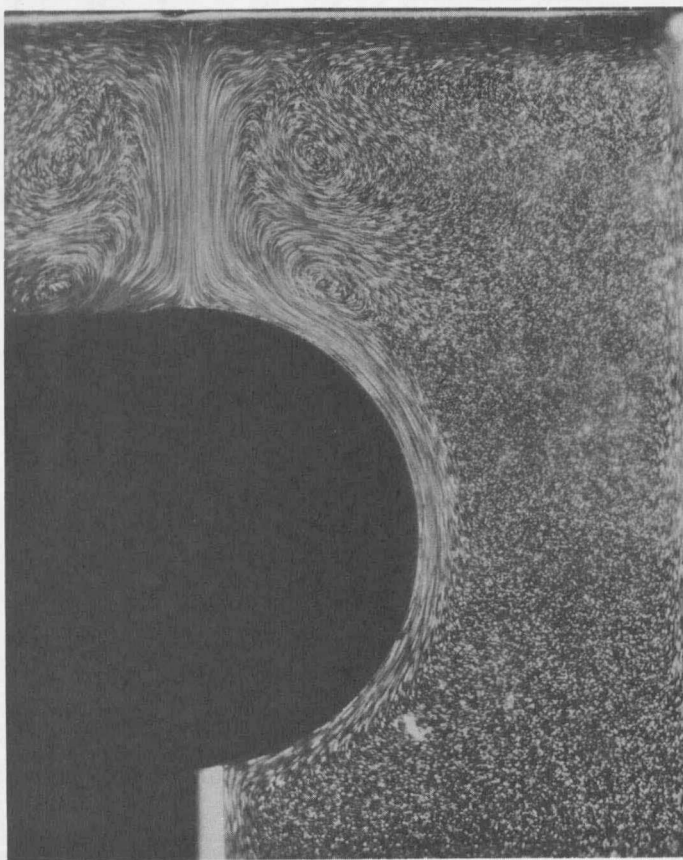


Figure 4.13 Silicone 350 Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 800$

$\Delta T = 62^{\circ}F$

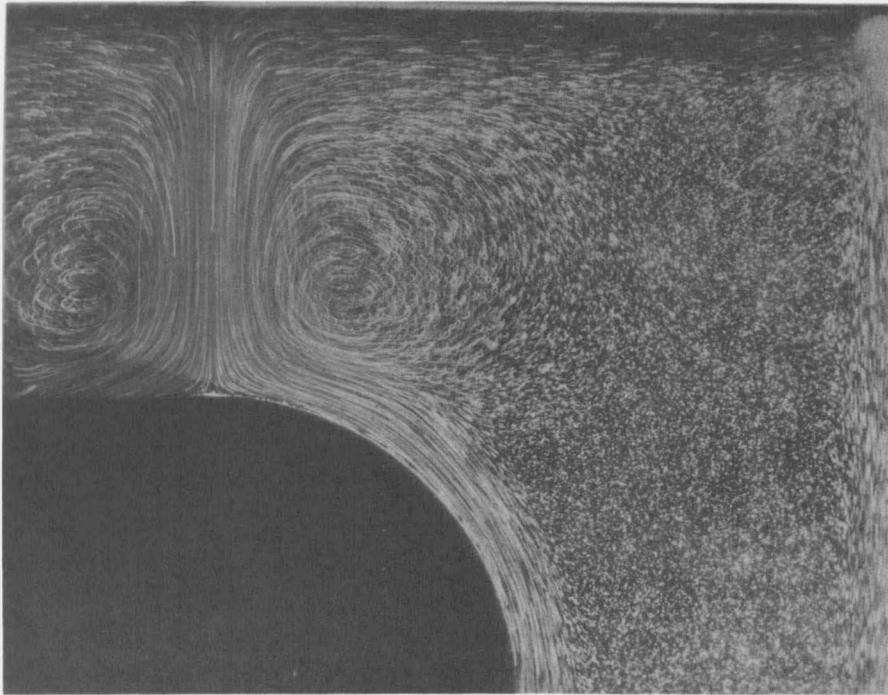


Figure 4.14 Silicone 350 Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 800$

$\Delta T = 62^{\circ}F$

stated earlier, this pattern was periodic. For $N_{Gr} = 650$, the lower cell remained stationary for about 11 seconds and the cycle repeated itself at approximately 33 second intervals. For the largest Grashof number investigated of 1,390 ($\Delta T = 78^\circ F$), the cell was stationary for approximately 6 seconds, and the cycle repeated every 15 or 20 seconds.

At the present time no definite explanation for the occurrence of this pattern is known; however, some postulations can be given. The upward acceleration of fluid adjacent to the chimney may be due to a thermal instability. Complete mixing may occur in the upper central region adjacent to the chimney, resulting in an almost uniform temperature of the fluid in this region. Fluid nearer to the inner body would then be heated, resulting in warmer fluid being trapped under cooler layers. The warmer fluid would then accelerate upward because of the larger bouyancy force acting on it. Acceleration of fluid in the chimney may be plausibly explained by viscous interaction between this fluid and the fluid adjacent to the chimney.

Another possible explanation of this pattern can be drawn from boundary layer theory. The pattern observed bears some resemblance to flow over a horizontal cylinder. Under certain conditions in cylinder flow, vortices are formed near the downstream side of the body and shed at periodic intervals into the wake. The vortices in the wake of the cylinder form a regular pattern of cells rotating alternately clockwise and counter clockwise. This pattern, known as a Karman vortex street, is

somewhat similar to the flow observed between the spherical body and the cubical enclosure. As was stated previously, the exact mechanism of this pattern is not presently known. It exhibited similarities to both of the above postulations and may be a complex combination of both thermal and hydrodynamic instabilities.

The 7.0 inch diameter inner body was utilized next in the investigations with the 350 cs fluid as the gap medium. Fluid flow observations were conducted for Grashof numbers between 25 ($\Delta T = 23^\circ\text{F}$) and 500 ($\Delta T = 111^\circ\text{F}$). For Grashof numbers less than 93 ($\Delta T = 52^\circ\text{F}$), the flow in the gap basically followed the solid boundaries of the system as shown in Figure 4.15. The primary flow moved over the surface of the sphere and turned at the upper vertical axis of the sphere forming a very wide thermal plume. The fluid then followed the contour of the outer geometry and proceeded into the lower region. No definite point of separation of fluid from the cube wall was noted. Fluid seemingly was gradually peeled off the primary flow moving down the cube side by the fluid in the lower region. This fluid was then noted to begin a very slow migration toward the inner body. It is noted by comparing Figure 4.15 with 4.4 that the boundary flow was thicker and somewhat slower with the 350 cs fluid as the gap medium than it was with the 20 cs fluid in the gap. This is partially due to the fact that the 350 fluid is an order of magnitude more viscous than the 20 fluid. A slight increase in Grashof number resulted in the formation of rotating cells adjacent to the chimney where the flow direction turned. This steady pattern persisted up to a

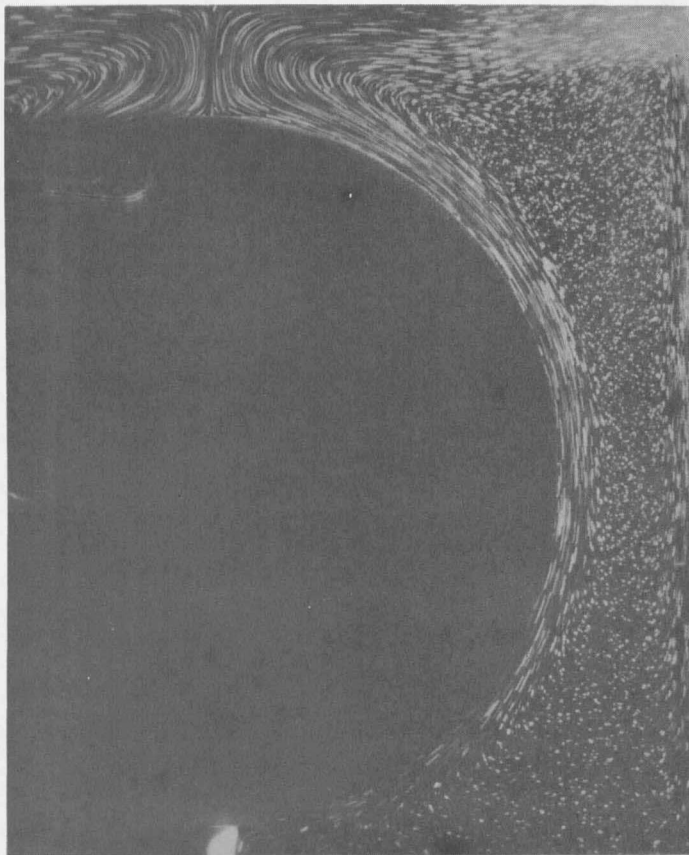


Figure 4.15 Silicone 350 Flow Pattern for

ID = 7.0 inches

$N_{Gr} = 57$

$\Delta T = 39^{\circ}F$

Grashof number of approximately 102 ($\Delta T = 59^\circ\text{F}$). Further increases in the Grashof number resulted in a basic unsteady pattern that prevailed throughout the remainder of the investigations. The unsteady pattern was characterized by the following occurrences. At periodic intervals, a portion of the upward moving fluid flowing over the sphere surface would separate before reaching the chimney. For $N_{Gr} = 170$ ($\Delta T = 71^\circ\text{F}$) the point of separation was about 15 degrees below the upper vertical axis of the sphere. At the largest Grashof number investigated ($N_{Gr} = 500$, $\Delta T = 111^\circ\text{F}$), the separation point was about 20 degrees below the sphere axis. Rotating cells, one on each side of the chimney, were observed where the separated flow turned to flow across the top of the cube. These cells appeared to form as the primary flow separated, and they dispersed shortly after forming. The thermal plume seemed to remain intact and was unaffected at small Grashof numbers ($170 < N_{Gr} < 280$) by the flow separation. After the above sequence of events had taken place, the primary flow would then flow all the way into the upper vertical axis region forming a very wide chimney. The period of the flow pattern discussed above was noted to be about 20 seconds. Figure 4.16 illustrates the basic pattern when a portion of the primary flow is separated. For Grashof numbers larger than 280, the same basic flow was observed except for the thermal plume area. This region was now characterized by slight random activity although the chimney was still evident. This pattern is shown in Figure 4.17.

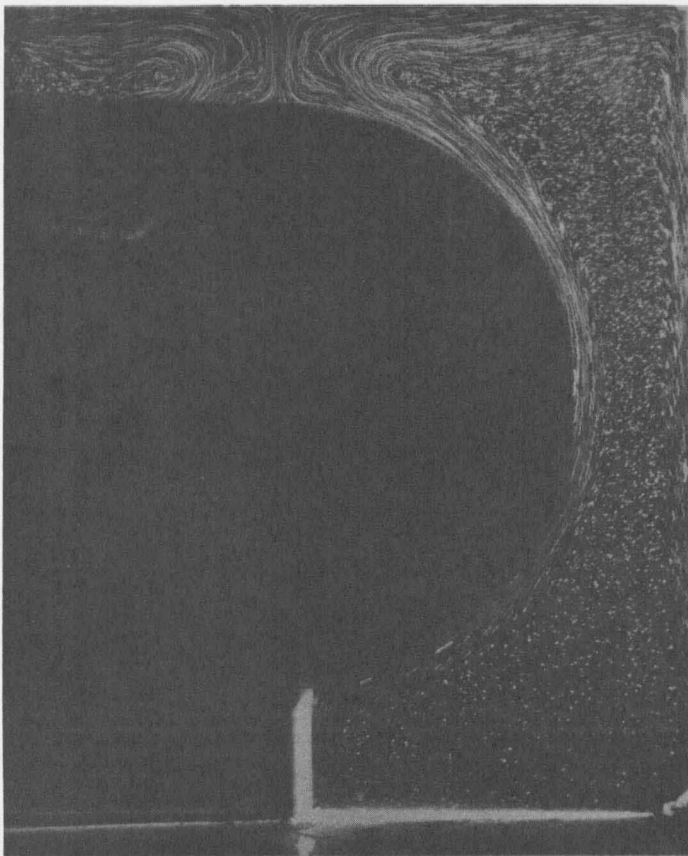


Figure 4.16 Silicone 350 Flow Pattern for

ID = 7.0 inches

$$N_{Gr} = 170$$

$$\Delta T = 71^{\circ}F$$

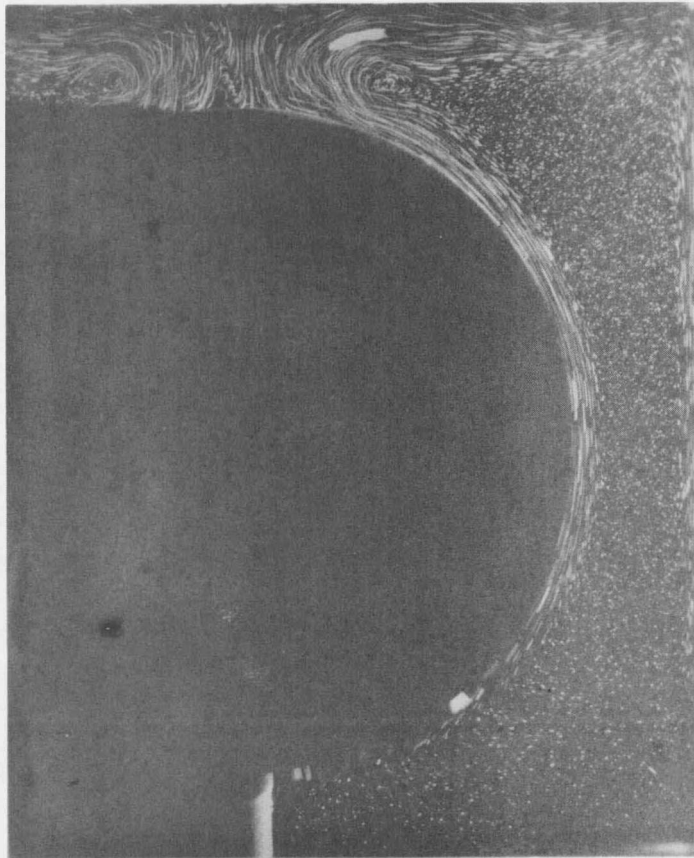


Figure 4.17 Silicone 350 Flow Pattern for

ID = 7.0 inches

$N_{Gr} = 500$

$\Delta T = 111^{\circ}F$

Flow patterns between the 9.0 inch diameter sphere and the 9.9 inch cube with silicone 350 as the gap fluid were next investigated. Investigations were conducted for Grashof numbers ranging between 0.5 ($\Delta T = 14^{\circ}\text{F}$), and 12 ($\Delta T = 103^{\circ}\text{F}$). The fluid flow pattern observed for the smallest Grashof number investigated is shown in Figure 4.18. As was the case with the 20 cs fluid in the gap, the primary flow separates from the inner body surface at a point approximately 35 degrees below the upper vertical axis of the sphere. The narrow spacing between the sphere and the enclosing cube near the midplane of the inner body provides a division between the upper and lower portions of the gap. This caused the flow in the gap to appear as two large rotating cells. In the upper region, the cell was bounded by the midplane, the inner body surface, and the top and side of the cube. The midplane, the inner body surface, and the bottom and side of the cube bounded the cell in the lower region. The division of the upper and lower region was much more pronounced at larger Grashof numbers. The distinct division between upper and lower regions was caused by interference between the upward and downward moving boundary flows. Interaction between the boundary flows in the narrow gap region has been substantiated by moving pictures.

With reference to Figure 4.18, it is seen that the extreme upper region near the vertical axis of the inner body is almost stagnant for this particular small Grashof number. This apparently is a region of almost pure conduction. Even though the midplane constituted a dividing

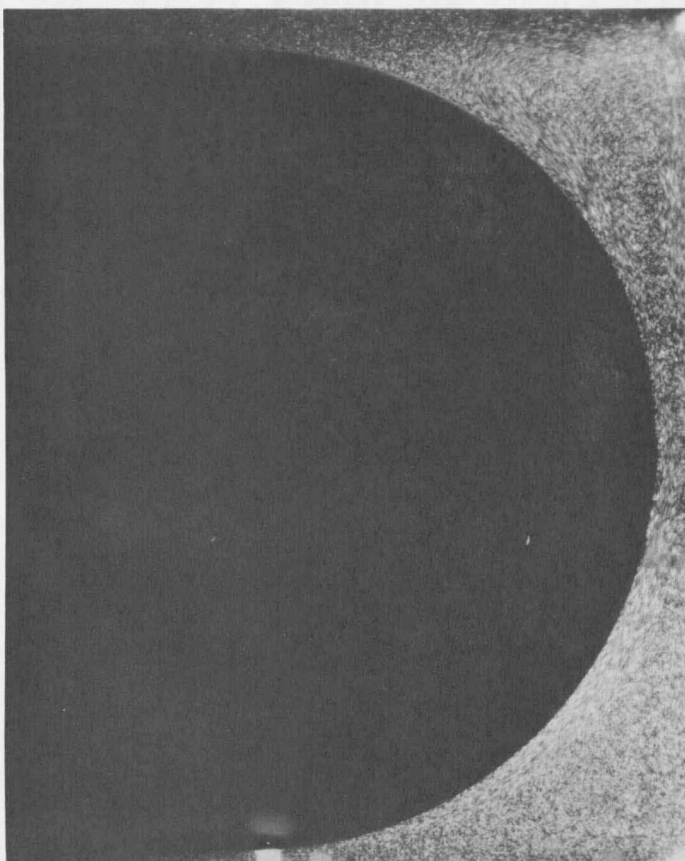


Figure 4.18 Silicone 350 Flow Pattern for

ID = 9.0 inches

$N_{Gr} = 0.5$

$\Delta T = 14^{\circ}F$

line between the upper and lower regions, it was noted that the primary flow moving down the cube side extended all the way into the lower region. The same was true of the fluid flowing over the sphere surface. Motion of the fluid in the upper vertical axis region in the gap was induced as the Grashof number was increased from a value of 0.5. At first the fluid in this region seemed to be just slowly wandering about in a random fashion. Further increases in the Grashof number resulted in the formation of multiple cells in the upper vertical axis region as shown in Figure 4.19. At times these cells completely filled the gap between points of separation of the primary flow from the sphere surface. At other times only a few cells existed, and the majority of the upper vertical axis region was characterized by random activity. The multiple cells were presumably of a three-dimensional nature. The size and therefore strength of the multiple cells was limited by the narrow gap in the upper region. As a consequence, the cells did not seem to have much effect on the primary flow. The geometry of the upper region of the gap resembles the gap between horizontal parallel flat plates. The multiple cells may therefore be similar to the ordered, cellular convection process, known as Benard Cells, which occurs in the parallel plate case for Rayleigh numbers greater than about 1700.

For Grashof numbers larger than 3 ($\Delta T = 53^\circ F$), additional activity was induced in the upper central region. A smaller secondary cell formed within the large cell encompassing the upper region. The primary flow

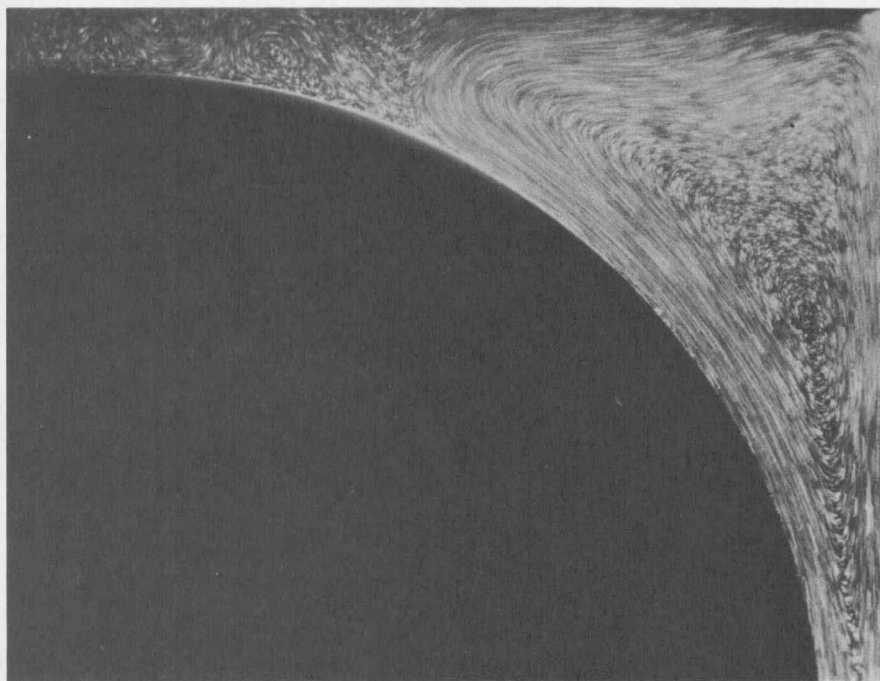


Figure 4.19 Silicone 350 Flow Pattern for

ID = 9.0 inches

$$N_{Gr} = 2$$

$$\Delta T = 41^{\circ}F$$

across the top of the cubical enclosure also was noted to display a slight rotational characteristic which indicated the presence of three-dimensional flow. This pattern can be seen in Figure 4.20. As the Grashof number was increased to its maximum value, the rotational characteristic of the boundary flow became more pronounced, and an eddy formed in the corner of the cube. Small cells in the narrow region above and below the midplane of the inner body were also noted at larger Grashof numbers.

WATER AS THE GAP WORKING FLUID

The same inner bodies used with the silicone oils as test fluids were used with water as the gap medium. All investigations were conducted with the inner bodies located concentrically within the cubical enclosure.

Fluid flow investigations were conducted with the 4.5 inch diameter sphere for Grashof numbers between 993,300 ($\Delta T = 7^\circ F$) and 48,079,000 ($\Delta T = 50^\circ F$). The steady flow observed in the gap for Grashof numbers less than 2,649,000 ($\Delta T = 14^\circ F$) is shown in Figure 4.21. The same high-speed peripheral flow observed with the silicone oils is also evident in this photograph. This flow moves over the surface of the sphere, turns upward near the top of the body to form a thermal plume, and then proceeds across the top and down the side of the enclosure to a region of separation. Two large cells form a binary pattern in the upper portion of the gap adjacent to the chimney. One of the cells is located about halfway up the chimney, and the other is near the top of the enclosure. Some

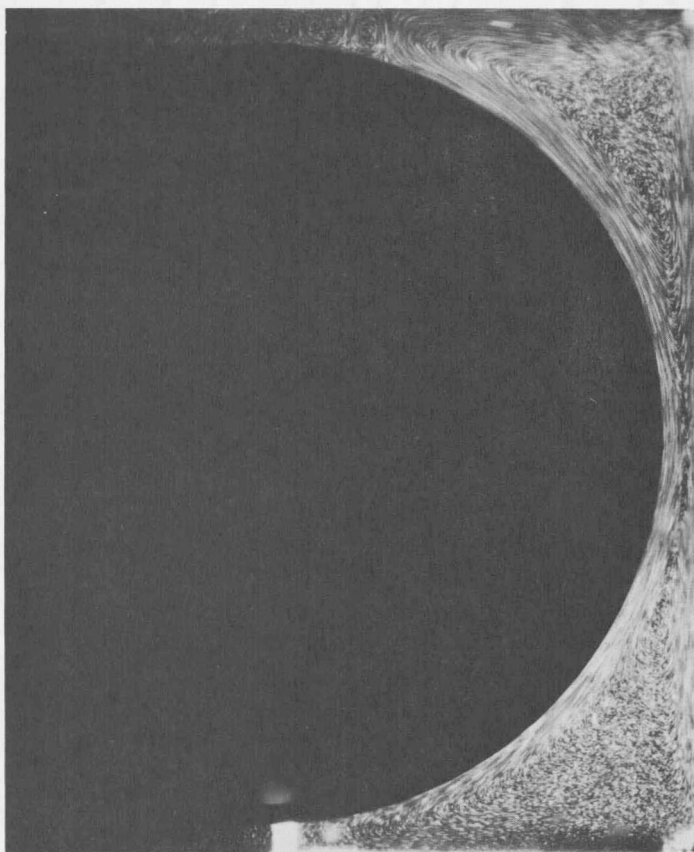


Figure 4.20 Silicone 350 Flow Pattern for

ID = 9.0 inches

$N_{Gr} = 5$

$\Delta T = 64^{\circ}\text{F}$

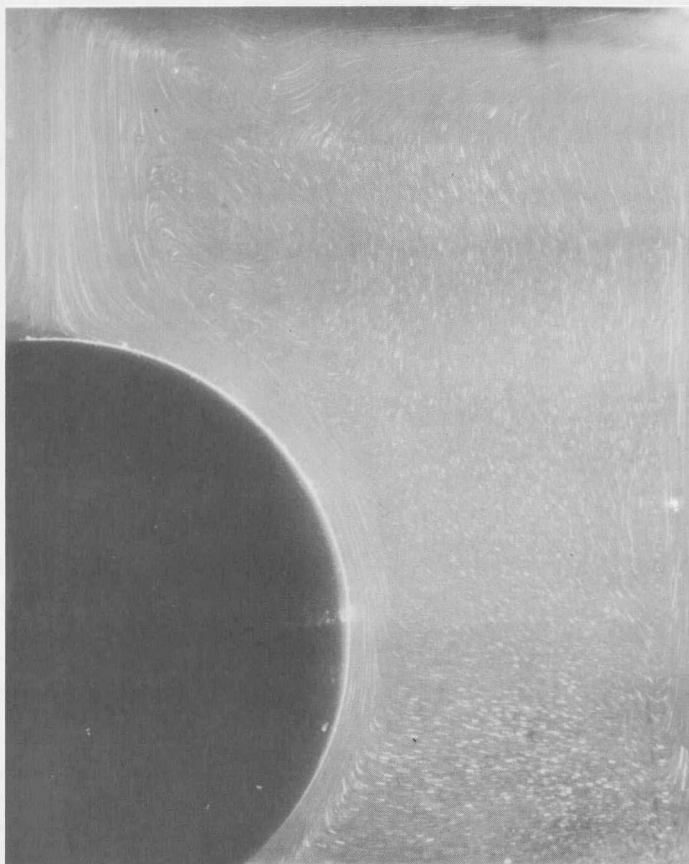


Figure 4.21 Water Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 993,300$

$\Delta T = 7^{\circ}\text{F}$

flow from the high-speed layer moving over the inner body appeared to separate just as the flow turned into the chimney. This flow then started to return toward the side of the enclosure but turned upward before reaching the side. This caused the fluid adjacent to the chimney to move around the two cells mentioned above and form a goose neck shape. The majority of the fluid in the upper central region was noted to flow upward toward the corner of the enclosure. A corner eddy and several small eddies along the side of the cubical enclosure were also noted. Considerably more motion was observed in the lower region for this case than for the silicone oils as test fluids. As stated previously, the flow down the side of the cube separated over a wide region between the bottom of the enclosure and a point just below the midpoint of the cube side. This resulted in a migration of fluid in the extreme lower region of the gap toward the inner body.

For Grashof numbers between approximately 2,649,000 ($\Delta T = 14^{\circ}\text{F}$), and 6,611,000 ($\Delta T = 22^{\circ}\text{F}$), a tertiary cellular motion occurred in the upper portion of the gap. The two cells adjacent to the chimney still existed as they did for small Grashof numbers, and a third cell was located near the surface of the inner body. Most of the return flow now migrated to the side of the enclosure rather than moving upward as it did before. This pattern is very similar to the tertiary pattern observed by Yin (6) in his concentric sphere studies. Figure 4.22 illustrates the tertiary flow.

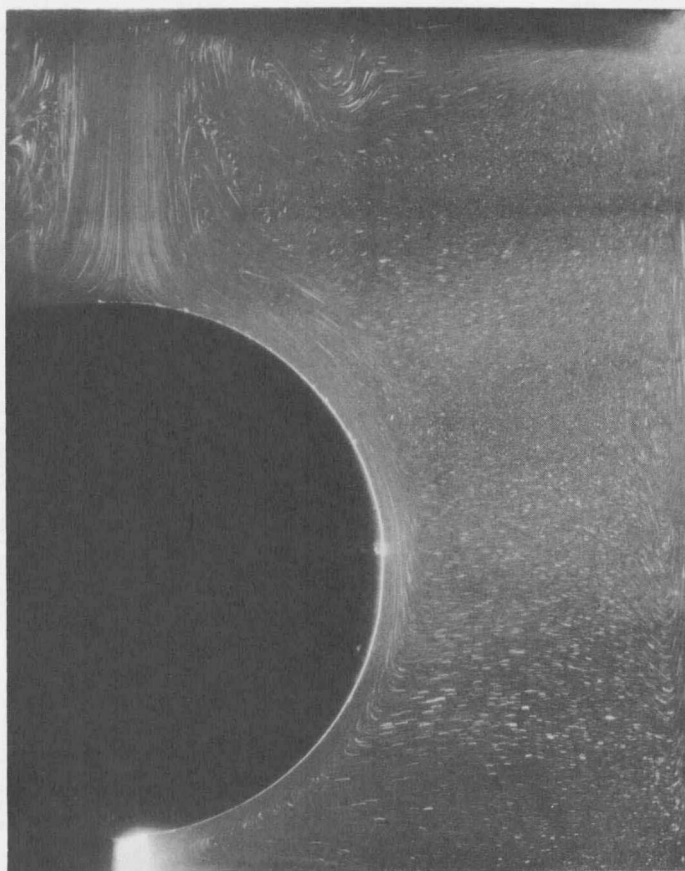


Figure 4.22 Water Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 2,649,000$

$\Delta T = 14^{\circ}F$

Increasing the Grashof number above 6,611,000 resulted in an unsteady pattern in which the upper central region was dominated by three-dimensional activity. The cell near the top of the cube was no longer steady for this case, and the tertiary pattern no longer existed. The cell, which appeared to be three-dimensional in nature, was noted to form near the top of the enclosure and then move out into the upper interior region, violently disrupting this portion of the gap. Figure 4.23 indicates this cell and the rather unpredictable activity in the upper interior region. No definite period could be established for this occurrence.

The next spherical body investigated was the 7.0 inch diameter sphere. Minimum and maximum Grashof numbers observed were 38,400 ($\Delta T = 8^\circ\text{F}$) and 2,833,000 ($\Delta T = 36^\circ\text{F}$) respectively.

For Grashof numbers below approximately 177,600 ($\Delta T = 11^\circ\text{F}$), the pattern shown in Figure 4.24 was observed in the gap. This pattern is very similar to the "dog-face" flow observed by Yin (6) in his concentric sphere investigations. Slight differences between this flow pattern and the "dog-face" pattern are (1) the small cell in the center of the interior region, (2) the upper vertical axis region, and (3) the corner eddy. The vertical axis region for small Grashof numbers was dominated by unpredictable activity.

A slight increase in Grashof number resulted in the formation of two counter rotating cells on the sphere surface at the base of the chimney.

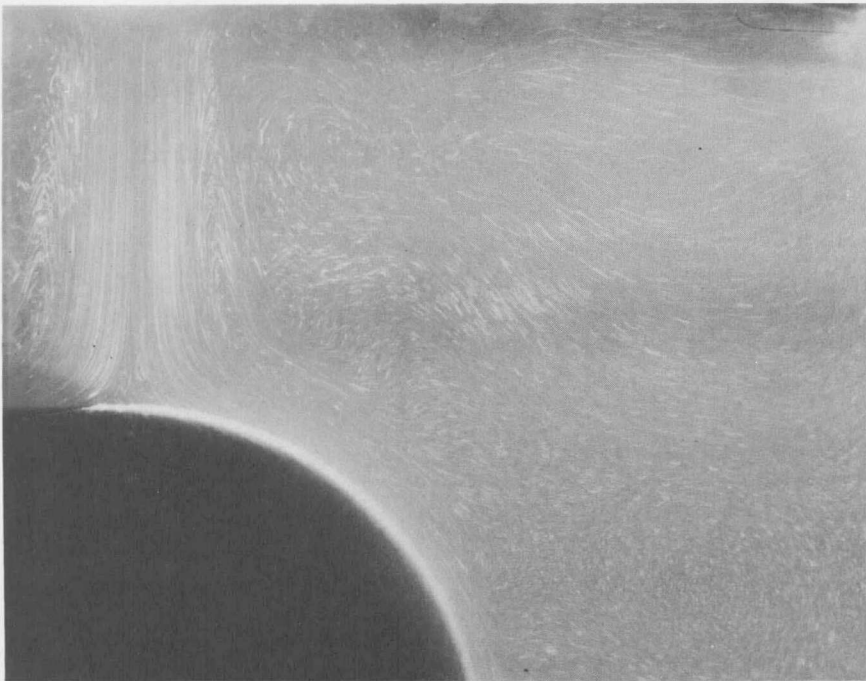


Figure 4.23 Water Flow Pattern for

ID = 4.5 inches

$N_{Gr} = 16,782,000$

$\Delta T = 32^{\circ}F$

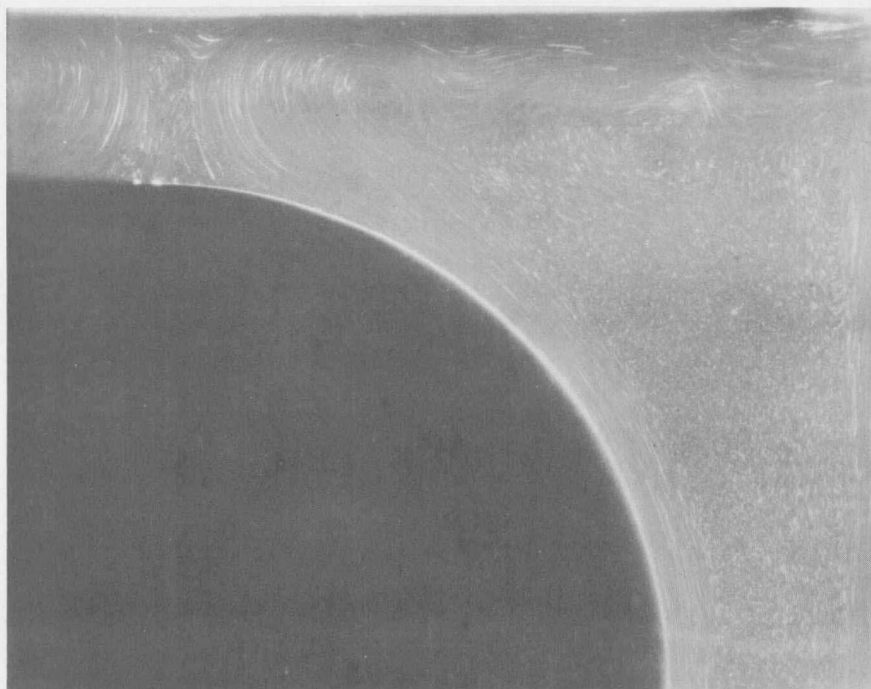


Figure 4.24 Water Flow Pattern for

ID = 7.0 inches

$N_{Gr} = 177,600$

$\Delta T = 11^{\circ}F$

These small cells circulated in opposite directions to the primary flow on each side of the vertical axis. Also noted at this point was a recirculation of flow in the upper interior region. Several small eddies were located between the primary flow across the top of the cube and the recirculating flow. Three-dimensional characteristics were also exhibited by the high-speed layer moving across the cube top.

Further increases in Grashof number resulted in a widening of the vertical axis region where the counter rotating cells had existed. When a value of $N_{Gr} = 494,000$ ($\Delta T = 18^\circ F$) was reached, the cells no longer appeared, and this portion of the gap was again dominated by random activity as shown in Figure 4.25. Unpredictable activity, highly three-dimensional in nature, was also observed in the upper central region.

The last inner body investigated with water as the test medium was the 9.0 inch diameter sphere. The flow was observed for Grashof numbers between 4,667 ($\Delta T = 6^\circ F$) and 59,700 ($\Delta T = 26^\circ F$).

Unsteady flow existed in the gap for all Grashof numbers investigated. Grashof numbers smaller than 24,000 ($\Delta T = 18^\circ F$) resulted in a flow pattern in the upper region of the gap that was very similar to the pattern observed with the 7.0 inch inner body. The region of unsteady activity in the upper vertical axis region was somewhat wider for the 9.0 inch body than for the 7.0 inch body. No counter rotating cells were observed in this portion of the gap. For most of the flows investigated, the motion in this region appeared to be unpredictable and random in

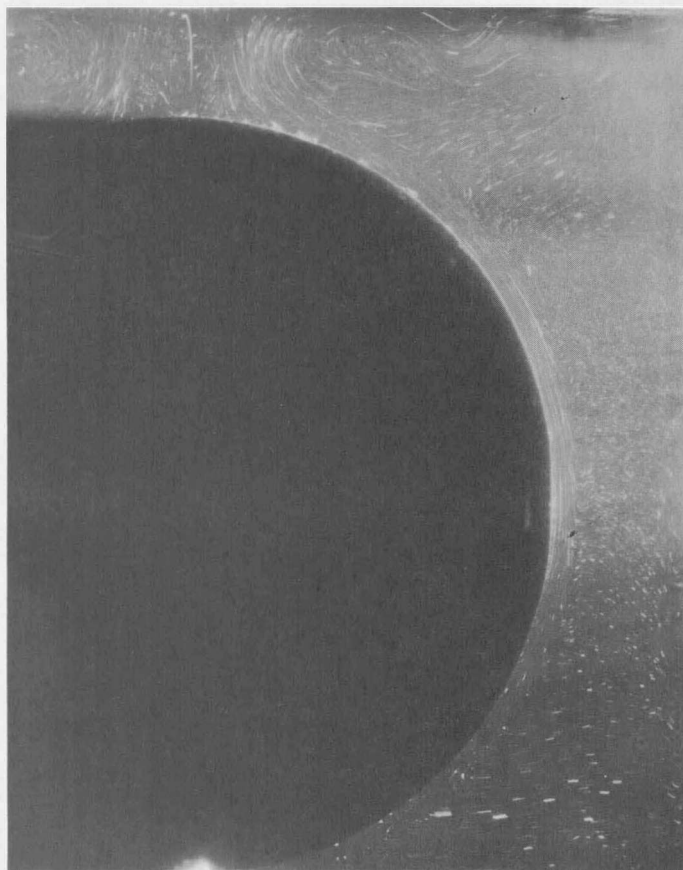


Figure 4.25 Water Flow Pattern for

ID = 7.0 inches

$N_{Gr} = 1,050,000$

$\Delta T = 24^{\circ}F$

nature. Interference between the upward and downward moving boundary flows in the narrow region between the sphere and cube at the midplane of the sphere again divided the flow into distinct upper and lower regions. Flow in the lower region basically followed the solid boundaries, and fluid in the central lower region slowly migrated toward the inner body. Figure 4.26 illustrates the basic pattern occurring for $N_{Gr} = 24,000$.

Increases in the Grashof number ($N_{Gr} > 24,000$) resulted in some of the same trends that developed for the 7.0 inch sphere. A strong recirculation flow in the upper central region developed and associated with this was the formation of cells between the primary flow and the recirculating flow. Three-dimensional flow in the upper central portion of the gap also became more apparent as the Grashof number was increased. The flow in the gap for the largest Grashof number investigated ($N_{Gr} = 59,700$, $\Delta T = 26^\circ F$) is shown in Figure 4.27.

SUMMARY OF RESULTS

The first section of this chapter contained qualitative descriptions of the fluid flow patterns observed for each spherical body and test fluid investigated. This section contains a brief summary of the results obtained. Tables 4.1-4.3 provide a categorization of all the flows observed and also indicate transition values of the important parameters involved. The transition values of Grashof numbers listed in the tables should not be interpreted as definite divisions between steady and unsteady flows. These values are suggested to be in the

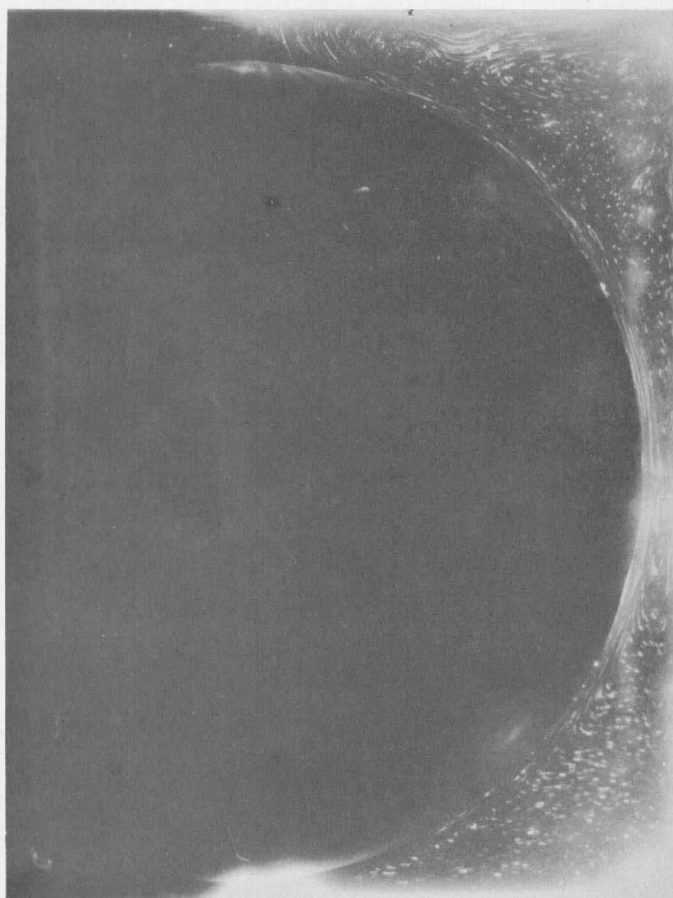


Figure 4.26 Water Flow Pattern for

ID = 9.0 inches

$N_{Gr} = 16,500$

$\Delta T = 15^{\circ}F$

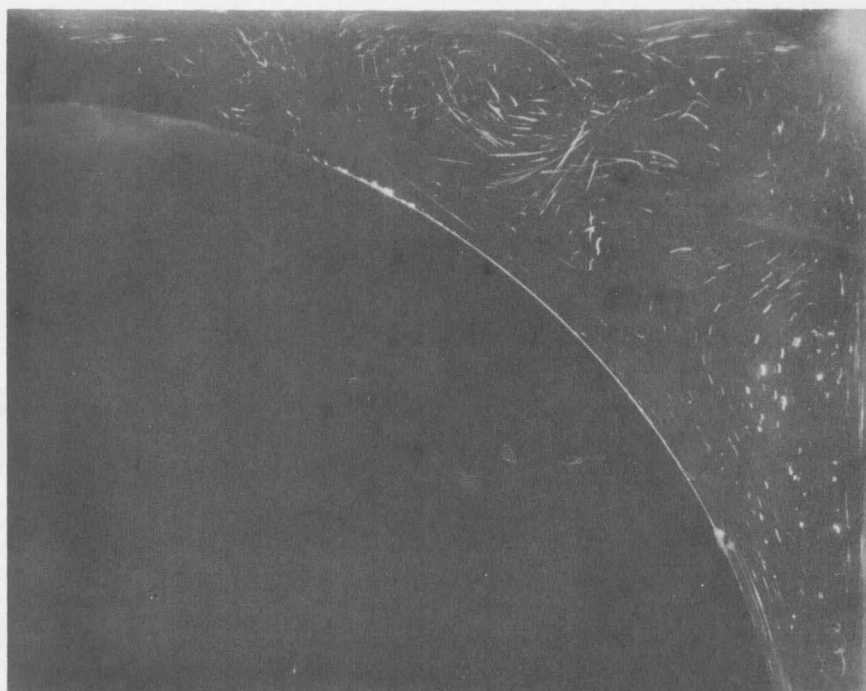


Figure 4.27 Water Flow Pattern for

ID = 9.0 inches

$N_{Gr} = 59,700$

$\Delta T = 26^{\circ}F$

Table 4.1
Summary of Silicone 20 Results

Inner Body Diameter in Inches	Steady Flow	Unsteady Flow
4.5	$22,800 < N_{Gr} < 39,900$ $7,158,000 < N_{Ra} < 11,925,000$ Peripheral Flow	$39,900 < N_{Gr} < 534,000$ $11,925,000 < N_{Ra} < 93,650,000$ Climbing-vortices Oscillating Chimney
7.0	$3,900 < N_{Gr} < 7,400$ $1,238,400 < N_{Ra} < 2,139,900$ Peripheral Flow and Recirculation	$7,400 < N_{Gr} < 123,100$ $2,139,900 < N_{Ra} < 19,215,600$ Counter-cells at Chimney Base
9.0	None	$62 < N_{Gr} < 2,300$ $18,800 < N_{Ra} < 398,200$ Multiple Cells in Upper Vertical Axis Region

Table 4.2
Summary of Silicone 350 Results

Inner Body Diameters in Inches	Steady Flow	Unsteady Flow
4.5	$73 < N_{Gr} < 650$ $295,800 < N_{Ra} < 2,100,000$ Peripheral Flow	$650 < N_{Gr} < 1,390$ $2,100,000 < N_{Ra} < 3,611,700$ Accelerating Chimney
7.0	$25 < N_{Gr} < 102$ $101,000 < N_{Ra} < 333,000$ Peripheral Flow	$102 < N_{Gr} < 500$ $333,000 < N_{Ra} < 1,019,000$ Random Motion In Chimney Region
9.0	$.5 < N_{Gr} < 1$ $1,960 < N_{Ra} < 4,500$ Peripheral Flow Separation	$1 < N_{Gr} < 12$ $4,500 < N_{Ra} < 26,000$ Multiple Cells

Table 4.3
Summary of Water Results

Inner Body Diameter In Inches	Steady Flow	Unsteady Flow
4.5	$993,000 < N_{Gr} < 6,611,000$ $8,833,000 < N_{Ra} < 48,100,000$ Binary and Tertiary Pattern	$6,611,000 < N_{Gr} < 48,079,000$ $48,100,000 < N_{Ra} < 246,726,000$ Vortex-shedding and Three-dimensional Activity
7.0	$38,400 < N_{Gr} < 177,600$ $477,000 < N_{Ra} < 1,647,000$ Similar to Dog-Face	$177,600 < N_{Gr} < 2,833,000$ $1,647,000 < N_{Ra} < 17,723,000$ Three-dimensional Activity
9.0	None	$4,667 < N_{Gr} < 59,700$ $39,300 < N_{Ra} < 377,500$ Random in Chimney and Three-dimensional Activity

neighborhood of the transition from steady to unsteady flow since in many cases the exact beginning of unsteadiness could not be determined. The general types of flow patterns observed are also indicated on the tables.

The flow in the gap between the 4.5 inch sphere and the cube with silicone 20 as the working fluid could be characterized by three distinct regions. These regions were (1) a high-speed layer that followed the solid-boundaries of the inner and outer geometries, (2) an upper interior region, and (3) a lower relatively stagnant region. For small Grashof numbers, the upper interior region contained two steady cells, one on each side of the chimney. These secondary cells were located inside the main peripheral flow and rotated in the same direction. The resulting pattern was similar to the "dog-face" pattern observed by Yin (6) with the exception of the location of the cells. Grashof numbers larger than 39,000 ($\Delta T > 20^\circ\text{F}$) resulted in an unsteady periodic pattern in which the aforementioned cells climbed up the sides of the chimney and lost their vortex identity as they were engulfed in the high-speed peripheral flow across the top of the enclosure. Slight tangential oscillations of the chimney were also associated with the climbing vortices. Additional changes in the upper central region accompanied increasing values of the Grashof number although the basic unsteady pattern remained unchanged (except for the period which decreased as the Grashof number increased). Grashof numbers larger than 65,700 ($\Delta T > 29^\circ\text{F}$) resulted in a slight

return flow and a recirculation of flow in the upper central region. A small corner eddy was also noted at large Grashof numbers.

A recirculation of flow in the upper central portion of the gap was observed throughout the range of Grashof numbers investigated for the 7.0 inch diameter sphere and the silicone 20 fluid. For Grashof numbers less than 7,400 ($\Delta T < 23^\circ\text{F}$), the flow pattern was steady. The high-speed peripheral flow basically followed the contours of the inner and outer geometries. The high-speed flow down the enclosure's side separated at a point below the inner body midplane. Axisymmetry was indicated by the thermal plume that formed around the vertical axis of the inner body. A small corner eddy, similar to that observed with the 4.5 inch sphere, was also present. An unsteady pattern occurred as the Grashof number was increased above 7,400. This pattern was characterized by the periodic formation of very small counter rotating cells at the base of the thermal plume and also by the presence of secondary cells inside the primary flow. Large Grashof numbers resulted in a separation of the upward moving high-speed primary flow from the inner body. For this case, the upper vertical axis region was dominated by random motion, and the secondary cells inside the primary flow exhibited an expansion-contraction motion although they did not translate laterally.

Separation of the boundary flow from the inner body occurred throughout the range of Grashof numbers observed ($62 < N_{Gr} < 2,300$) for the 9.0 inch body and the silicone 20 fluid. The flow separated at a point about

25 degrees below the vertical axis of the sphere. The close proximity of the inner body and the enclosure at the midplane of the sphere appeared to divide the flow in the gap into two separate regions. This occurrence was caused by the interference between the upward and downward moving boundary flows in the midplane region. The upper vertical axis portion of the gap was dominated by unsteady activity. Multiple rotating cells were noted to form nonperiodically in this portion of the gap. The geometry in the upper vertical axis region suggests that these cells may be similar to Benard Cells which are normally associated with the flow between horizontal parallel plates heated from below. Increasing the Grashof number above 184 resulted in changes in the flow pattern that were similar to those observed with the 7.0 inch inner body. Secondary cells and a recirculation of flow occurred in the upper central region.

Grashof numbers smaller than 160 with the 4.5 inch diameter body and the silicone 350 fluid resulted in a steady pattern in which the flow followed the solid boundaries of the system. Slight increases above this value ($170 < N_{Gr} < 560$) resulted in the formation of steady vortices, one on each side of the chimney, in the region where the primary flow turned to form the chimney. Grashof numbers larger than 560 produced an unsteady pattern. The cells adjacent to the chimney now formed periodically, remained stationary for a short time, and then rotated up the side of the chimney accelerating as they went. An acceleration of fluid in the chimney was associated with the upward movement of the cells. The

exact mechanism for this occurrence is unknown, but it is postulated to be due to thermal or hydrodynamic instabilities or a combination of the two.

Utilizing silicone 350 as the test fluid and the 7.0 inch inner body resulted in a steady pattern for $N_{Gr} < 102$. Fluid flow followed the contours of the inner and outer geometries. Grashof numbers near the upper range of the steady flow regime resulted in the formation of rotating cells adjacent to the thermal plume that were very similar to those observed with this body and the 20 cs fluid. A basic unsteady periodic pattern occurred for $N_{Gr} > 102$. This pattern was characterized by slugs of fluid being ejected into the chimney region and a consequent separation of the primary flow over the inner body surface. The rotating cells adjacent to the chimney were noted to form and disperse in a periodic fashion for this case.

Fluid flow patterns with the 9.0 inch body and the silicone 350 fluid were basically the same as those observed with the 20 cs fluid. Separation of the upward boundary flow from the inner body surface occurred for all the cases investigated. The upper vertical axis region was characterized by relatively stagnant fluid for very small Grashof numbers. Moderate and large Grashof numbers resulted in the formation of multiple cells in this region. The narrow space between the sphere and cube at the midplane divided the gap into distinct upper and lower regions as was the case with the 20 cs silicone.

For $N_{Gr} < 2,649,000$, a binary pattern was observed with the 4.5 inch inner body and water as the working fluid. In this pattern two steady cells existed on each side of the thermal plume. One was near the top of the enclosure, and the other was about halfway up the chimney. Flow adjacent to the chimney moved around these cells to form a gooseneck shape. The upper range of Grashof numbers in the steady flow regime led to a tertiary type pattern in the upper central region of the gap. This pattern was similar to the tertiary pattern observed by Yin (6). Grashof numbers larger than 6,611,000 resulted in unsteady flow in which the cell near the top of the cube was observed to migrate away from the chimney into the central region. This action caused violent disruptions of the flow in the central portion of the gap. Observations in the offset plane revealed that this pattern was highly three-dimensional in nature.

The 7.0 inch body with water as the gap medium had flow characteristics very similar to the same body with silicone 20 as the gap medium. Small Grashof numbers resulted in a flow similar to the "dog-face" pattern with the secondary cells shifted laterally away from the chimney region. Small counter rotating cells near the base of the chimney were observed for small and moderate Grashof numbers. Flow recirculation in the upper central region was noted until large Grashof numbers were reached. At this time, random activity dominated the chimney area, and the fluid in the upper region was in an unpredictable three-dimensional state of motion.

Unsteady flow existed for the whole range of Grashof numbers investigated with the 9.0 inch sphere and water as the gap fluid. The upper vertical axis region appeared random and unpredictable in nature for all cases investigated although the upper part of the gap flow was similar to that observed for the 7.0 inch body with water. Interference between the upward and downward boundary flows in the narrow region at the midplane again distinctly divided the flow into upper and lower regions. Three-dimensional activity became more predominate in the upper central region as the Grashof number was increased to its maximum value.

The Prandtl number in this investigation ranged from 5.1 to 4,106. Throughout this range, the basic peripheral flow pattern always existed. In general, the only apparent effect of the higher viscosity fluids was to confine any flow unsteadiness to small regions. Thus it seems there is only a relatively small Prandtl number effect on the flows observed.

A temperature profile for the 7.0 inch sphere and silicone 20 as the gap fluid is shown in Figure 4.28. The flow pattern corresponding to this particular temperature difference ($\Delta T = 76^\circ\text{F}$) is shown in Figure 4.6. Past investigators (1, 4, 14, 15) have interpreted the temperature profiles in terms of five basic regions. These regions are (1) two areas of steep gradient adjacent to the solid boundaries, (2) a relatively flat portion, and (3) transition regions between the areas of steep gradient and the flat portion of the curve. Comparison of Figures 4.28 and 4.6 indicates that the profiles agree reasonably

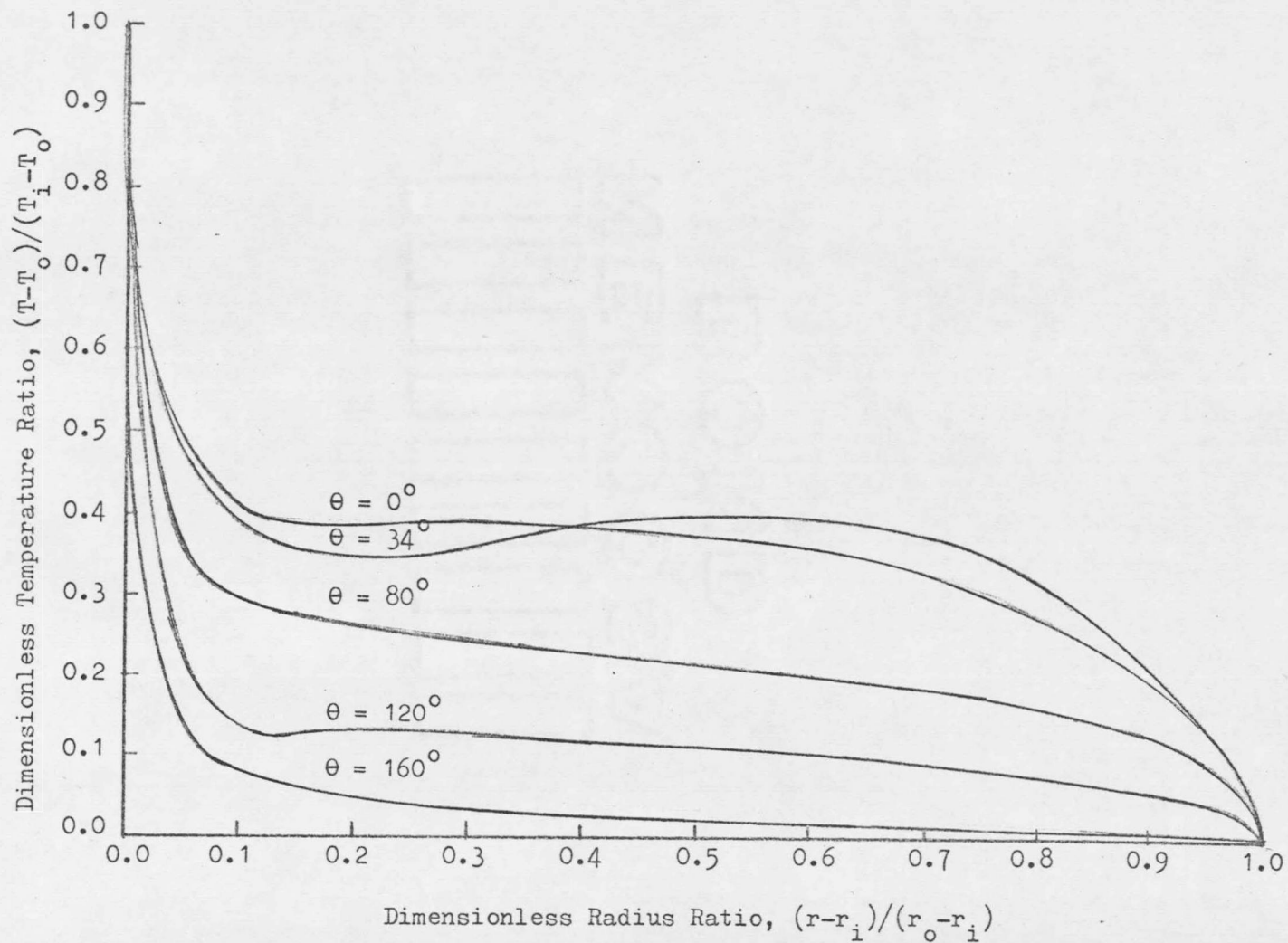


Figure 4.28 Temperature Profiles for 7.0 inch Sphere (Silicone 20), $\Delta T = 76^\circ\text{F}$ (16)

well with the flow pattern observed. The regions of steep slope on the profiles correspond to the thin high-speed peripheral flows that follow the solid boundaries of the system. In these regions hot fluid is rapidly convected away resulting in a large temperature drop in a small radial distance. As seen from Figure 4.28, the 80, 120 and 160 degree profiles (the angles signify the locations measured from the upper vertical axis of the inner body at which temperature profiles were taken) are relatively flat in the central part of the figure. These areas correspond to the relatively stagnant central and lower regions shown in Figure 4.6. In these portions of the gap, conduction is the dominant mode of heat transfer. The transition regions on the temperature profiles correspond to portions of the gap in which both conduction and convection heat transfer occur. A temperature inversion is noted at the 34 degree location on Figure 4.28. This is due to the strong recirculation of flow in the upper central region shown in Figure 4.6. Temperature variations were indicated by the temperature probe at the 0 degree location. The variations were caused by the random activity observed in the upper vertical axis region.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

An existing apparatus used to investigate the natural convective flow between a sphere and a cubical enclosure with air as the gap medium was extended to incorporate water and two silicone oils as test fluids. Fluid flow phenomena were investigated in (1) a vertical plane passing through the inner body's vertical axis and perpendicular to the cubical enclosure's side and (2) a plane parallel to but shifted 1.65 inches away from the plane through the vertical axis of the sphere.

For the variety of inner bodies and fluids used, a high-speed peripheral flow generally followed the contours of the inner and outer geometries. Unsteadiness usually occurred in the upper portion of the gap when the imposed temperature difference between the inner and outer bodies was sufficiently large. Unsteadiness was usually confined to smaller areas of the gap as the Prandtl number increased. Nonsymmetry caused by having a spherical body located in a cubical enclosure usually resulted in some three-dimensional activity. This activity was much more pronounced at large temperature differences than at small temperature differences.

Several recommendations are in order if future work is to be done with the apparatus. Modifications should be made so that visual observations can be made on a vertical diagonal plane of the cube. Poor lighting and optical distortions rendered it impossible to take photographs on this plane in this investigation. Changes in the top channel

of the water jacket should also be made so that visual observations on a horizontal plane can be conducted. The continual presence of air bubbles in the top channel prohibited horizontal plane investigations. The effects of eccentric locations of the inner body on the convective activity in the gap should also provide useful and interesting work for the future.

APPENDIX

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C:.... PROGRAM BY T.K. LARSON FOR DATA REDUCTION OF NATCON PROJECT
C:.... SPHERICAL BODY TO A CUBICAL ENCLOSURE
C:.... I = FLUID NUMBER      AIR           = 1
C:                           WATER          = 2
C:                           SILICONE 350   = 3
C:                           SILICONE 20    = 4
C:.... CP(I,1) = SPECIFIC HEAT AT CONSTANT PRESSURE OF A SPECIFIED TEST FLUID
C:.... VIS(I,1) = ABSOLUTE VISCOSITY OF A SPECIFIED TEST FLUID
C:.... CONO(I,1) = THERMAL CONDUCTIVITY OF A SPECIFIED TEST FLUID
C:.... RHO(I,1) = DENSITY OF A SPECIFIED TEST FLUID
C:.... BETA(I,1) = EXPANSION COEF. OF A SPECIFIED TEST FLUID
C:.... OD = CUBE LENGTH (IN)
C:.... ID = SPHERE DIAMETER (IN)
C:.... BARO = BAROMETRIC PRESSURE (IN HG) .... INCLUDE ONLY IF USING
C:.... AIR AS TEST FLUID
C:.... AP = ATMOSPHERIC PRESSURE (LBF/SQ IN)
C:.... GAP = DISTANCE BETWEEN THE INNER AND OUTER CONFIGURATIONS (IN)
C:.... GR = GRASHOF NUMBER
C:.... RA = RAYLEIGH NUMBER
C:.... MN = NUMBER OF DATA SETS
C:.... DATA REQUIRED
C:.... CARD 1) FLUID #, RUN #, # INNER TC, # OUTER+#INNER...4I3
C:.... CARD 2) OUTER CUBE LEN, INNER BODY DIA...2F10.4
C:.... CARD 3) INNER BODY TC READINGS... 6F10.4
C:.... CARD 4) OUTER CUBE TC READINGS... 6F10.4= 6AVERAGES
C:.... REAL ID, IR, IT, ITAVG, MAXDEVOT, MAXDEVIT
C:.... INTEGER RUN
C:.... DIMENSION OT(20), IT(20), E(20), DEV(20)
C:.... MN=7
C:.... DO 500 JJ=1, MN
C:.... READ(105,10) I, RUN, MM, MMM
C:.... READ(105,11) OD, ID
10  FORMAT(4I3)
11  FORMAT(2F10.4)
C:.... WRITE(108,50) RUN
50  FORMAT('1', //, 'RUN NO = ', I3)

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      IF(I.EQ.1) GO TO 100
      IF(I.EQ.2) GO TO 101
      IF(I.EQ.4) GO TO 111
      WRITE(108,51)
51    FORMAT(' ',///,'FLUID .... SILICONE 350 CS')
      GO TO 102
100   WRITE(108,52)
52    FORMAT(' ',///,'FLUID .... AIR')
      GO TO 102
101   WRITE(108,53)
53    FORMAT(' ',///,'FLUID .... WATER')
      GO TO 102
111   WRITE(108,82)
82    FORMAT(' ',///,'FLUID....SILICONE 200, 20CS')
102   CONTINUE
      DIAR = OD/ID
      GAP = (OD-ID)/2.
      WRITE(108,54) OD, ID, DIAR
54    FORMAT('0', 'CUBE LENG. = ', F10.4, ' INCHES',/, 'INNER BODY
C/ )
      CDIA = ' ', F10.4, ' INCHES',/, 'DIA. RATIO = ', F10.4,
C/ )
C.... READ IN THE INNER TC'S TEMPS IN MILLIVOLTS
      READ(105,12) (E(J), J=1,MM)
12    FORMAT(6F10.4)
C.... READ IN THE OUTER TC'S TEMPS IN MILLIVOLTS
      READ(105,13) (E(J), J=MM+1,MMM)
13    FORMAT(6F10.4)
C.... CONVERSION TO R FROM MV OF INNER TMPS
      DO 200 N=1,MM
      IT(N) = T(E(N))
200   CONTINUE
C.... CONVERSION TO R FROM MV OF OUTER TMPS
      DO 201 NN=MM+1,MMM
      OT(NN) = T(E(NN))
201   CONTINUE
C.... ADD UP TEMPS OF INNER TC'S

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SUM = 0.
DO 103 II=1,MM
SUM = SUM+IT(II)
103 CONTINUE
C.... FIND AVERAGE INNER TEMP
ITAVG = SUM/MM
C.... ADD UP TEMPS OF OUTER TC'S
SUM=OT(MM+1)+OT(MM+2)+4.*OT(MM+3)+4.*OT(MM+4)+4.*OT(MM+5)+
C5.*OT(MM+6)
C.... FIND AVERAGE OUTER TEMPS
OTAVG=SUM/19.
WRITE(108,59)
59 FORMAT('01', 'OUTER CUBE   TEMP:',10X, 'DEVIATION',15X, '% DEVIATION',
$//)
MAXDEVOT = 0.
DO 105 J=MM+1,MMM
DEV(J) = OTAVG-OT(J)
DUM2 = ABS(DEV(J))
IF(DUM2 .GT. MAXDEVOT) MAXDEVOT = DUM2
ERROR = DEV(J)/OTAVG*100.
WRITE(108,55) OT(J), DEV(J), ERROR
55 FORMAT('1',4X,F10.4,11X,F10.4,16X,F10.4)
105 CONTINUE
WRITE(108,60)
60 FORMAT('01', 'INNER BODY TEMP:',//)
MAXDEVIT = 0.
DO 106 J=1,MM
DEV(J) = ITAVG-IT(J)
DUM2 = ABS(DEV(J))
IF(DUM2 .GT. MAXDEVIT) MAXDEVIT = DUM2
ERROR = DEV(J)/ITAVG*100.
WRITE(108,55) IT(J), DEV(J), ERROR
106 CONTINUE
DT = ITAVG-OTAVG
PERDEVIT = MAXDEVIT/DT*100.
PERDEVOT = MAXDEVOT/DT*100.

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WRITE(108,56) ITAVG,MAXDEVIT, OTAVG,MAXDEVOT,DT,
C'PERDEVIT,PERDEVOT
56 FORMAT('11',//,'MEAN INNER BODY TEMP. = ',F10.4,' R',//,
C'MAX. INNER BODY DEV. = ',F10.4,' R',//,'MEAN OUTER
C'CUBE TEMP. = ',F10.4,' R',//,'MAX OUTER CUBE DEV. = ',
C',F10.4,' R',//,'MEAN TEMP. DROP ACROSS GAP = ',F10.4,
C',R',//,'MAX. INNER BODY DEV. = ',F10.4,' % MEAN TEMP.
C'DROP',//,'MAX OUTER CUBE DEV. = ',F10.4,' % MEAN TEMP.
C'DROP',//)
IR = ID/2.
OR = OD/2.
RAVG = (OR+IR)/2.
G = 32.174
TM = (ITAVG+OTAVG)/2.
WRITE(108,61) TM
61 FORMAT('01',20X,'PROPERTIES BASED ON THE ARITHMETIC MEAN TEMPERATUR
$E',//,'ARITHMETIC MEAN TEMP. = ',F10.4,' R')
109 VIS = U(TM,I)
SH = CP(TM,I)
COND = CON(TM,I)
DEN = RHO(TM,I)
B = BETA(TM,I)
PR = VIS*SH/COND
C.... GRASHOF NUMBER BASED ON THE GAP THICKNESS
GR = G*B*DEN**2.*GAP**3.*DT*3600.**2./(1728.0*VIS**2.)
RA = PR*GR
WRITE(108,57) VIS, SH, COND, DEN, B
57 FORMAT('11',//,'VISCOSITY = ',F10.4,' LBM/FT HR',//,'S
$SPECIFIC HEAT = ',F10.4,' BTU/LBM R',//,'THERMAL CONDUCTIV
$ITY = ',F10.4,' BTU/LBM FT R',//,'DENSITY = ',F10.
$4,' LBM/CU FT',//,'EXPANSION COEF. = ',F10.4,' 1/R')
WRITE(108,58) PR, GR, RA
58 FORMAT('11',//,'PRANDTL NO. = ',F15.4,'//,'GRASHOF NO.
$ = ',F15.4,'//,'RAYLEIGH NO. = ',F15.4)
500 CONTINUE
END

```

```

FUNCTION T(E)
DIMENSION C(8)
DOUBLE PRECISION TOT
C(1)=491.96562
C(2)=46.381884
C(3)=1.3918864
C(4)=0.15260798
C(5)=0.020201612
C(6)=0.0016456956
C(7)=6.6287090/(10.**5)
C(8)=1.0241343/(10.**6)
TOT=0.00
DO 1 I=1,8
1 TOT=TOT+C(I)*(E**(I=1))
T=TOT
RETURN
END
FUNCTION CP(T,I)
GO TO (1,2,3,4),I
1 CONTINUE
C.... SPECIFIC HEAT OF AIR
C0 = 2.236775/(10.0**5)
C1 = 0.22797749
CP = C1+C0*T
GO TO 50
2 CONTINUE
C.... SPECIFIC HEAT OF WATER
C1 = 1.3757095
C2 = .0012968965
C3 = 1.1110533/(10.0**6)
CP = C1+(C2+C3*T)*T
GO TO 50
3 CONTINUE
C.... SPECIFIC HEAT OF 350 CS DOW 200 SILICONE
TP = 5.0*(T-491.69)/9.
C1 = 0.3259583

```

```

C2 = 2.425/(10.0**4)
C3 = 5.41667/(10.0**7)
CP = C1+(C2-C3*TP)*TP
GO TO 50
4 CONTINUE
C... SPECIFIC HEAT OF 200S DOW 200 SILICONE
TP = 5*(T-491.69)/9.
C1 = .3448334
C2 = 7.7499/(10.**5)
C3 = 4.167/(10.**8)
CP = C1+(C2-C3*TP)*TP
GO TO 50
50 RETURN
END
FUNCTION U(T,I)
GO TO (1,2,3,4),I
C... 1 ABSOLUTE VISCOSITY OF AIR
CO = 134.375
C1 = .0133834
C2 = 1.8432299
C3 = 1.3347050
U = T**C2/(EXP(C1)*(T+CO)**C3)
GO TO 50
C... 2 ABSOLUTE VISCOSITY OF WATER
CONTINUE
TP = T-593.33203
C1 = .0071695149
C2 = .011751302
C3 = .0087791942
C4 = .81654704
VIS = C1*TP+C2*(1.+C3*(TP**2))**.5+C4
U = 1./VIS
GO TO 50
C... 3 CONTINUE
ABSOLUTE VISCOSITY OF 350 CS DOW 200 SILICONE
V = .03875*(5.495*10**9)/(T-259.69)**2.943

```

```

C1 = 52.754684
C2 = .045437533
C3 = 5.1832336/(10.**5)
RHO = C1+(C2=C3*T)*T
RHO = RHO*.970
U = RHO*V
GO TO 50
4 CONTINUE
C.... ABSOLUTE VISCOSITY OF 20CS DOW 200 FLUID
V = .03875*(4.*6*10**5)/(T=359.69)**1.912
C1 = 52.754684
C2 = .045437533
C3 = 5.1832336/(10.**5)
RHO = C1+(C2=C3*T)*T
U = RHO*.949*V
GO TO 50
50 RETURN
END
FUNCTION CON(T,I)
GO TO (1,2,3,4),I
1 CONTINUE
C.... THERMAL CONDUCTIVITY OF AIR
XP = 0.1
CO = 8.5964965
C1 = 34490.89
C2 = 868.23837
C3 = 8056583.8
10 X = XP
F = CO+C1*X+C2*X*X+C3*X*X*X*T
FP = C1+2.*C2*X+3.*X*X*C3
XP = X=F/FP
IF (ABS((XP=X)/X)=0.0001)20,20,10
20 CON = XP
GO TO 50
2 CONTINUE
C.... THERMAL CONDUCTIVITY OF WATER

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```

C1 = .23705417
C2 = .0017156797
C3 = 1.1563770/(10.0**6)
CON = C1+(C2=C3*T)*T
GO TO 50
3 CONTINUE
C.... THERMAL CONDUCTIVITY OF 350 CS DOW 200 SILICONE
CON = 0.00038/0.004134
GO TO 50
4 CONTINUE
C.... THERMAL CONDUCTIVITY OF 20CS DOW 200 SILICONE
CON = 0.00034/0.004134
GO TO 50
50 RETURN
END
FUNCTION RHO(T,I)
GO TO (1,2,3,4),I
1 CONTINUE
C.... DENSITY OF AIR AT LOCAL ATMOSPHERIC PRESSURE
READ (105,100) BARO
100 FORMAT(F10.4)
P = BARO*.491
WRITE(108,101) P
101 FORMAT(' ',//,'ATMOSPHERIC PRESSURE = ',F10.4,' PSI')
RHO = BARO*.491*144./(53.34*T)
GO TO 50
2 CONTINUE
C.... DENSITY OF WATER
C1 = 52.754684
C2 = .045437533
C3 = 5.1832336/(10.**5)
RHO = C1+(C2=C3*T)*T
GO TO 50
3 CONTINUE
C.... DENSITY OF 350 CS DOW 200 SILICONE
C1 = 52.754684

```

```

C2 = 0.045437533
C3 = 5.1832336/(10.0**5)
RHOW = C1+(C2=C3*T)*T
RHO = RHOW*.970
GO TO 50
4 CONTINUE
C.... DENSITY OF 20CS DOW 200 SILICONE
C1=52.754684
C2=.045437533
C3=5.1832336/(10.**5)
RHO=.949*(C1+(C2=C3*T)*T)
GO TO 50
50 RETURN
END
FUNCTION BETA(T,I)
GO TO (1,2,3,4),I
1 CONTINUE
EXPANSION COEF. OF AIR
BETA = 1.0/T
GO TO 50
2 CONTINUE
C.... EXPANSION COEF. OF WATER
TP = 1/100.
IF (T=549.59), 10,10,20
10 C1 = 603.11841
C2 = 353.03882
C3 = 68.297012
C4 = 4.3611460
BP = C1+(C2+(C3+C4*TP)*TP)*TP
GO TO 30
20 C1 = 128.44920
C2 = 68.827927
C3 = 13.858489
C4 = 1.2608585
C5 = .042495236
BP = C1+(C2+(C3+(C4+C5*TP)*TP)*TP)*TP

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30 BETA = BP/(10.0**4)
GO TO 50
3 CONTINUE
C.... EXPANSION COEF. OF 350 CS DOW 200 SILICONE
BETA = 0.00096/1.8
GO TO 50
4 CONTINUE
C.... EXPANSION COEF. OF 20CS DOW 200 SILICONE
BETA = .00107/1.8
GO TO 50
50 RETURN
END

```

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