The effect on mathematics achievement of teaching reading in a mathematics class at Casper College by Allan Geoffrey Skillman

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF EDUCATION
Montana State University
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Abstract:
The problem investigated in this study was the effectiveness of teaching a college mathematics class by a method that stressed instruction in reading and the effect on mathematics achievement of training mathematics teachers to teach reading.

Literature research indicates the existence of reading problems in the reading of mathematical material. Several suggested methods for overcoming these problems are given in the literature. The effectiveness of these methods has not been sufficiently researched.

A mathematics teacher was found who had taken a college course entitled "Teaching of Reading". This teacher and another member of the Casper College, Casper, Wyoming, mathematics faculty agreed to emphasize reading in their classes. Two other Casper College mathematics teachers were chosen to act as a control measure and did not stress reading in their classes.

The reading topics emphasized were technical vocabulary; symbol vocabulary; the purpose for reading; the use of the text including the color codes, author's organization, and structure keys to effective use of the text; reading of text and examples, or graphs, or pictorial aids simultaneously, A statistical analysis of the data derived from six Casper College mathematics classes gave the following results.

(1) The hypothesis that emphasizing reading in the mathematics classroom would improve the mathematical achievement of the students was supported at the 0.05 level of significance in the college algebra class but was not supported in the basic algebra class or the calculus class.

(2) The hypothesis that a mathematics teacher trained to teach reading can more effectively teach reading of mathematical material than can a mathematics teacher not trained to teach reading was not supported at the 0.05 level of significance.

(3) There was a significant interaction between the reading treatment and the course at the 0.05 level.

(4) The correlations found for IQ and ACT mathematics scores, for IQ and ACT reading scores, and for ACT reading scores and gain in mathematics achievement are not significantly different from those reported in the literature at the 0.05 level.

• lx (5) The correlation "between the ACT mathematics scores and gain in mathematics achievement as measured by teacher-made tests was 0.1708.

The following major conclusions were drawn from the statistical results. (I) It is possible to obtain better achievement in mathematics by emphasizing reading techniques. (2) The teacher's attitude or interest has an effect on the effectiveness of emphasizing reading in the mathematics classroom, (3) The ACT mathematics test is not a good predictor of success for Casper College students in mathematics classes where gain in mathematics achievement is used as a grading criterion.

# THE EFFECT ON MATHEMATICS ACHIEVEMENT OF TEACHING READING IN A MATHEMATICS CLASS AT CASPER COLLEGE

by

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A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

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## ABSTRACT

The problem investigated in this study was the effectiveness of teaching a college mathematics class by a method that stressed instruction in reading and the effect on mathematics achievement of training mathematics teachers to teach reading.

Literature research indicates the existence of reading problems in the reading of mathematical material. Several suggested methods for overcoming these problems are given in the literature. The effectiveness of these methods has not been sufficiently researched.

A mathematics teacher was found who had taken a college course entitled "Teaching of Reading". This teacher and another member of the Casper College, Casper, Wyoming, mathematics faculty agreed to emphasize reading in their classes. Two other Casper College mathematics teachers were chosen to act as a control measure and did not stress reading in their classes.

The reading topics emphasized were technical vocabulary; symbol vocabulary; the purpose for reading; the use of the text including the color codes, author's organization, and structure keys to effective use of the text; reading of text and examples, or graphs, or pictorial aids simultaneously.

A statistical analysis of the data derived from six Casper College mathematics classes gave the following results.

(1) The hypothesis that emphasizing reading in the mathematics classroom would improve the mathematical achievement of the students was supported at the 0.05 level of significance in the college algebra class but was not supported in the basic algebra class or the calculus class.

(2) The hypothesis that a mathematics teacher trained to teach reading can more effectively teach reading of mathematical material than can a mathematics teacher not trained to teach reading was not supported at the 0.05 level of significance.

(3) There was a significant interaction between the reading treatment and the course at the 0.05 level.

(4) The correlations found for IQ and ACT mathematics scores, for IQ and ACT reading scores, and for ACT reading scores and gain in mathematics achievement are not significantly different from those reported in the literature at the 0.05 level.

(5) The correlation between the ACT mathematics scores and gain in mathematics achievement as measured by teacher-made tests was 0.1708.

The following major conclusions were drawn from the statistical results. (1) It is possible to obtain better achievement in mathematics by emphasizing reading techniques. (2) The teacher's attitude or interest has an effect on the effectiveness of emphasizing reading in the mathematics classroom. (3) The ACT mathematics test is not a good predictor of success for Casper College students in mathematics classes where gain in mathematics achievement is used as a grading criterion.

# CHAPTER I

## INTRODUCTION

The changes in society over the last half century have given the ability to read and the ability to apply mathematics an ever increasing priority. The places in life are few where reading of some kind and mathematics of some kind are not necessities.

The ability to succeed very often depends on the ability to read; therefore the ability to read fluently is given a very high priority. Heilman says that "the failure of large numbers of children to learn to read at a level commensurate with their intellectual ability may well be our number one educational problem."[1]

The mathematics teacher should help students learn to read mathematical materials so that they might better learn mathematical concepts. The language of mathematics has vocabulary characteristics and a system of writing structure forms such as exponents and subscripts, which

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[1]Arthur W. Heilman, Principles and Practices of Teaching Reading, (2nd ed.; Columbus, Ohio: Charles E. Merrill Publishing Company, 1967), p. 452.

complicate the reading problem.[1]

The students should be taught to attack and solve problems that will not exist until some future time. The solution of these problems requires the ability to read mathematics and to think mathematically. This seems to require an educational system that teaches not only fundamental mathematical techniques but stresses understanding and original thinking in its mathematics classes. The efficient completion of this task seems to demand the use of every reasonable teaching tool.

## The Problem

The specific problem explored by this study was the effectiveness of teaching a college algebra class by a method that stressed instruction in reading. A second problem was to determine whether a mathematics teacher with minimal training in the teaching of reading was better able to provide this instruction than a mathematics teacher without training in the teaching of reading.

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[1]Harold H. Lerch, "Improving Reading in the Language of Mathematics--Grades 7-12," Improving Reading in Secondary Schools, ed. by Lawrence E. Hafner, (New York: The MacMillan Company, 1967), pp. 344-52.

## Purpose of the Study

The purpose of the study was to compare statistically the mathematical achievement of students receiving a reading emphasis in the mathematics class with the achievement of students not receiving a reading emphasis. The achievement was measured by teacher-made tests. A second purpose of the study was to compare the mathematics achievement of students receiving reading instruction from a mathematics teacher trained to teach reading with the mathematics achievement of students receiving reading instruction from a mathematics teacher who had no training in the teaching of reading.

## Need for the Study

It has been suggested that the failure of students to solve reasoning problems in arithmetic is in some degree caused by deficiencies in reading ability. It is logical to believe that a pupil who is unable to understand the situation described in a problem will not be able to solve the problem.[1] Balow found that both general reading

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[1]B. A. Stevens, "Problem Solving in Arithmetic," Journal of Educational Research, 25 (April-May, 1932), pp. 253-60.

ability and computational ability have a significant effect on the ability to solve verbal problems.[1]

Mathematics requires special reading skills and the language of mathematics has characteristics which complicate the reading. Technical mathematical terms must convey precise meanings in order to communicate the desired concept to the mind of the reader.[2]

Students frequently have a lack of prerequisite learnings necessary to read and understand mathematics and they may lack a knowledge of mathematical concepts. They often are easily confused and frustrated by mathematical symbolism.[3] It may be incorrectly assumed that students have already learned through previous experience the necessary vocabulary for understanding the topics.[4]

An analysis of materials and learning difficulties

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[1]I. H. Balow, "Reading and Computation Ability as Determinants of Problem Solving," Arithmetic Teacher, 11 (January, 1964), pp. 18-22.

[2]J. H. Banks, Learning and Teaching Arithmetic, Boston: Allyn and Bacon, 1959).

[3]Lerch, "Reading Mathematics," pp. 344-52.

[4]Ray F. Deck, "Vocabulary Development to Improve Reading and Achievement in Science," Improving Reading in Secondary Schools, ed. by Lawrence Hafner, (New York: The MacMillan Company, 1967), pp. 148-53.

shows that skill in reading is a necessary part of the work in algebra. For example faulty vocabulary or failure to note detail can result in faulty applications. Therefore the mathematics teacher also must be a teacher of reading in that he is concerned with the skills involved in learning.[1]

These problems were summarized by Aaron as follows: To read mathematics one needs both basic reading ability and the specialized skills unique to mathematics. Special skills used in mathematical reading are: mathematical vocabulary, both words and symbols; concept background; ability to select skills and rate for the material being read; proficiency in the special reading tasks of mathematics; and skill in the interpretation of mathematical symbols.

The development of these skills is the responsibility of the mathematics teacher. In addition to this the teacher must help the poor readers as well as the excellent readers to find appropriate reading material. These

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[1]Leo C. Fay, "The Program in the Content Areas," What Research says to the Teacher No. 11 Reading in the High School, Department of Classroom Teachers, American Educational Research Association of the National Education Association, 1956, pp. 23-26.

reading needs grow out of the specialized nature of mathematics.[1] Systematic attention to the development of the specialized reading skills in mathematics will result in better readers and greater achievement in mathematics.

Call and Wiggin performed an experiment using an English teacher to teach a ten day section of algebra II. One of the conclusions of their experiment was; "We also suggest that this experiment has some cogent implications for further testing. . . . It is time we took note of the necessity for studying relationships between disciplines."[2]

A search of the literature has revealed no listing of the reading skills that should be included in the mathematics course outlines. The teachers' manuals that accompany mathematics texts have few references to methods for teaching vocabulary and no other methods for teaching reading. It has perhaps been incorrectly assumed that the mathematics teacher knows what special reading skills are used to read mathematics. Therefore it seems reasonable to infer that there is a need for study guides in mathematics

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[1] I. E. Aaron, "Reading in Mathematics," Journal of Reading, 18 (May, 1965), pp 391-95.

[2] Russell J. Call and Neal A. Wiggin, "Reading and Mathematics," Mathematics Teacher, 59 (February, 1966) p. 57.

which include reading techniques.

## General Hypotheses

(1) Presentation of reading techniques in the mathematics classroom will improve the mathematical achievement of the students.

(2) Teachers of mathematics with minimal training in reading can more effectively provide the reading techniques than can mathematics teachers not trained to teach reading.

## Definitions of Terms

Basic Algebra, 55-097: a three credit course that meets five hours a week, designed for students who have a deficiency in high school algebra I.

College Algebra, 55-117: a three credit course, "for students in life sciences, business, and liberal arts."[1]

Calculus, 55-151: a five credit course covering single variable calculus, using an intuitive approach.

Gain Score. The score found by subtracting the number of items correct on the diagnostic test from the

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[1]Casper College, Casper College Catalog, (Casper, Wyoming: Casper College, 1971), p. 154.

number of items correct on the final examination for each student.

## Basic Assumptions

It was assumed that the assignment of students to the several sections of a mathematics course by the registration procedure used at Casper College, Casper, Wyoming, was done randomly.

## Procedures

Two of the Casper College mathematics teachers agreed to emphasize reading in their classes. One of these teachers had completed a two semester hour course entitled "Teaching of Reading." Two other Casper College teachers were carefully chosen to provide a control measure for the experiment. This choice of teachers dictated the choice of four algebra classes and two calculus classes for the study.

Teacher-made tests were revised to make them acceptable for the measurement of student achievement in the college algebra course. Test items were given for diagnostic purposes at the beginning of the semester and were repeated as part of the final examination to yield two measurements for each student included in the study. The

difference of these two measurements was the gain score.

Reading techniques were developed by the investigator and the teacher for the college algebra course. The reading emphasis was given to the experimental sections of each course in the study and statistical methods were used to analyze the resulting data.

## Limitations

The size of the sample was determined by the availability of scores for concomitant variables and by the liberal withdrawal policy of Casper College which allows a student to drop a course until one week before the final examination for that course.

The teaching assignment at Casper College severely altered the experimental design. The teacher trained to teach reading was assigned to teach calculus instead of college algebra, which necessitated the inclusion of the calculus course in the study. The teaching assignment change was made between the registration dates and the beginning of classes for the semester.

The exercises and tests used in this study were made to agree with the texts used in the particular classes at Casper College during the spring semester of 1972. No

claim is made for their use in any other situation.

The method used to obtain the IQ scores for the students at Casper College made it inadvisable to use them for any statistical inference. The method and the reasons for not using the IQ scores are explained in chapter III and chapter IV of this paper.

# CHAPTER II

## REVIEW OF LITERATURE

In this review three major topics are developed: (1) reading problems associated with learning mathematics, (2) recommended methods for teaching reading in mathematics and (3) a brief summary of investigations similar in purpose to that of this study.

### Reading Problems Associated With Learning Mathematics

An analysis of materials and learning difficulties shows that skill in reading is very much a part of the work in algebra and geometry. The mathematics teacher is a reading teacher in that he is concerned with the skills involved in learning as well as the mathematical information learned. Students can be helped to learn more if attention is given to the reading and study skills used.[1]

Miller says that mathematics requires special reading skills. Much of the difficulty with reading in mathematics stems from the student's lack of awareness of the purpose for the reading. In most instructional reading a

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[1]Fay, "The Program in the Content Areas," pp. 23-26.

seventy-five per cent comprehension level is acceptable, but in a sequentially structured subject like mathematics comprehension of all the material is necessary.[1] The purpose for reading has an important bearing on the rate at which the reading is done, as well as on the reading skills used for a particular reading task.

The language of mathematics has characteristics which complicate the problem of reading. Technical mathematical terms must convey precise meanings if they are to evoke the desired concept in the mind of the reader. It is too easy to assume that the students have learned through previous experiences the necessary vocabulary for various topics.[2]

There is a distinct vocabulary of mathematics and about one-fourth of the reading difficulty in mathematics is caused by difficult vocabulary. The technical words are most often the cause of student difficulty.[3] Students in

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[1]Lyle L. Miller, Developing Reading Efficiency, (Laramie, Wyoming: Developmental Reading Distributors, 1962), p. 12.

[2]Lerch, "Reading Mathematics," pp. 344-52.

[3]Bernice E. Leary, "Meeting Specific Reading Problems in the Content Fields," Reading in the High School and College, Forty-Seventh Yearbook of the National Society for the Study of Education, Part II (Chicago, Ill.: The University of Chicago Press, 1968), p. 158.

general have a distinct lack of prerequisite learnings necessary to read and understand the mathematical content presented; they lack the necessary understanding or knowledge of mathematical concepts; and they do not have the necessary speaking or reading vocabulary in terms of mathematical reference.[1]

Leary summarized the reading problems in mathematics, for the National Society for the Study of Education in the following manner. The vocabulary of mathematics is more specific and more descriptive than vocabulary used in other areas. It consists of many words which have different connotations in common usage. To understand the mathematical usage of these words, they must be related to a specific classification, process, or concept. Another difficulty with words is that the same word is sometimes used to convey different technical meanings. In addition, unfamiliar nontechnical words cause problems for many students. Research shows that students will not grow into an understanding of technical terms.

In addition to English words, some of which have special technical meanings, the language of mathematics

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[1]Lerch, "Reading Mathematics," pp. 344-52.

uses a large system of signs and symbols. The literal numbers, a, b, c, . . . x, y, z, must convey meanings in sentences exactly as do other nouns. There are exceptions, such as $n^a$ and $n_b$, where the a is an adverb and b acts as an adjective. The reading of these symbols requires a greater effort than most reading since the size and relative position of the symbol are as important as the symbol itself. If a student reads a as a letter of the alphabet rather than as some number, he is reading with neither meaning nor purpose beyond that of manipulating symbols. The verbs of a mathematical expression are the operational symbols +, -, =, ·, (), and others. In addition to learning to read all of the symbols, a student must learn to read the arithmetic numerals correctly, including a sense of direction when one is indicated by a plus or minus sign. The same sign is sometimes used for two different technical meanings, as x - m and $x^{-m}$. Where the first minus sign indicates the operation subtraction, and the second minus sign means take the reciprocal of $x^m$.

The formula is a universal tool. It is a combination of algebraic symbols used in sequence. The ability to understand and translate formulas is the heart of applied mathematics. Since the formula is more often a design

rather than a line of print, or a series of words, it presents special reading problems. The perception involved in reading formulas and graphs makes a greater demand on the eye than reading algebraic prose.

Reading mathematics, like reading other subject materials, demands that the student recognize the vocabulary and symbols, not only when they are isolated, but also when they are in combination. The student must be able to draw on the implied meanings of the context to get the ideas intended by the author. The extent to which the student has clear referents for the words and symbols determines the extent to which he can read them.[1]

In addition to the reading problems caused by the nature of mathematical material, there are poor reading habits that add to the reading problem in mathematics. In the following quotation is a list of poor reading habits which are likely to be crucial in reading mathematical material.

> Number attraction. Some readers come to a complete stop every time they reach a number. They seem to want to study it carefully as if it were a completely different idea in communication. . . .
> Word analysis. Stopping to think slowly and carefully about a strange word as to its origin, structure,

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[1]Leary, "Meeting Problems," pp. 150-62.

> prefixes, and suffixes may be a sound vocabulary building exercise, but it destroys the general direction of your thought in reading. . . .
>
> Monotonous plodding. Keeping the same pace of reading in all materials, from light fiction to heavy study is tiresome. One needs to develop his ability to change habits of reading to adjust to different needs. . . .
>
> . . . . . .
>
> Clue blindness. Like the driver who is too busy watching the road to see the sign posts that direct him to his destination, many readers become too involved in word reading to notice such things as headings, titles, styles of type, listings, illustrations, introductions, and summaries. . . .
>
> Back tracking. Going back to reread words or phrases is an indication that you doubt your own ability to pick out the important material. . . .
>
> Rereading. . . . If you concentrate on doing a good job of reading in the first place, a few minutes of thinking about what you have read will be far more valuable than rereading.
>
> Daydreaming. Allowing your attention to wander to other things while you are reading leaves you with the feeling of having covered pages but having no knowledge of what you have read.[1]

All of these habits can be overcome unless the student has a serious mental or physical handicap. An individual will overcome the poor habits much more quickly with guidance and controlled practice than he will if he is expected to learn to read on his own.[2]

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[1]Miller, Developing Reading, pp. 7-8.

[2]Vernon Troxel, "Effects of Purpose on the Reading of Expository Mathematical Materials in Grade Eight," Journal of Educational Research, 55 (February, 1962), pp. 221-27.

One of the marks of a good reader is his ability to adjust his reading skills to his purpose and to the type of material which he is reading. The kinds of reading skills, as defined by Dr. Vandiver, most used in reading mathematics are described briefly here.

Study reading is a type in which the reader should get seventy to eighty per cent understanding of the ideas presented and how they fit together.

Critical reading requires that the reader understand all of the ideas presented. At the same time it requires that he must consider what is known of the source of the material; he must watch for faulty logic and false comparisons; and he must be aware of what the author is trying to make him believe. Some errors in mathematical material cause the noncritical reader wasted study time while the critical reader recognizes them as obvious errors.

Analytical reading uses a questioning approach seeking complete understanding and requires careful and intense concentration.[1]

When reading skills are graded by the amount of

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[1]Dr. Willis C. Vandiver, Lecture given to his Teaching of Reading class, Montana State University, Winter Quarter, 1968.

concentration needed it is observed that the three skills previously listed demand more concentration than any of the other reading skills. Generally this need for more intense concentration makes the kinds of reading skills used in mathematics harder to learn and more difficult to apply than other kinds of reading skills. The successful solution of a story problem or the completion of a mathematical proof requires the student to combine parts of these three reading skills with good reading habits, with an understanding of the vocabulary, concepts, and symbols of mathematics, and with the necessary computational skills.

## Recommended Methods for Teaching Reading in Mathematics

Two basic assumptions concerning the teaching of reading in mathematics were held by the writers who were surveyed in this section. They were that students need specific classroom instruction in reading techniques and that teachers need a periodic reminder of this fact. Two of the emphases were teaching of vocabulary and teaching the student to read with a predetermined purpose.

Students must be taught to read for a specific purpose. Smith suggests four readings of a story problem

with the following reasons for each. (1) Read the problem to grasp the whole situation. (2) Read it a second time to concentrate on the question. (3) Use the third reading to decide on the process or the formula. (4) Last, pull out and use the given number facts.[1]

Spache claims that the approach stated in the preceding paragraph is not the same as the steps: "(1) what is given, (2) what is to be found, (3) what steps should I take (4) about what is the answer?"[2] He says also that research has shown these four steps to be unprofitable as a method for teaching problem solving.

Rereading in some situations is called a reading problem, but Maribeth Henry, says, "Rereading is often unnecessary with narrative materials, but is essential for successful reading of verbal problems."[3]

Vocabulary development is an important part of

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[1]Nila Banton Smith, "Reading in Subject Matter Fields: Reading in Mathematics," Educational Leadership, 22 (March, 1965), p. 384.

[2]George D. Spache and Evelyn B. Spache, Reading in the Elementary School, (2nd ed.; Boston: Allyn and Bacon Inc., 1969), pp. 305-307.

[3]Maribeth Henry, "Improving Mathematics Verbal Problem-solving Ability Through Reading Instruction," Arithmetic Teacher, 18 (April, 1971), p. 224.

teaching reading in mathematics. First, the teacher must identify the technical words for each lesson, use a word list and teach the technical vocabulary. Second, the teacher must teach unfamiliar words which occasionally give the key to understanding a specific topic. Third, words with multiple meanings should be explained to the student, with their mathematical meaning clearly separated from the other usages. Jo Phillips points out that we should "clarify the meaning of each term by pointing out what it does not mean."[1]

If the practice problems given to the students are to effectively aid the learning process, the teacher should adjust the problem length to the student ability. Problems with unnecessary data or with insufficient data should be used carefully and the students should be reminded to read all problems critically, looking for the useless information and expecting to find some problems that cannot be completed.[2]

Vocabulary instruction in mathematics should include an explanation of signs and symbols. The word exponent,

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[1]Leroy Barney, "Problems Associated with the Reading of Arithmetic," Arithmetic Teacher, 19 (February, 1972), p. 384.

[2]Jo McKeeby Phillips, "Reading Math Content," Instructor, 81 (November, 1971), p. 64.

for example, is not a part of the student's vocabulary until he identifies the $\underline{x}$ in $3^{\underline{x}}$ with both the word and its corresponding meaning. The teaching error occurs when the student reads exponent as the little number above the line and omits the mathematical operation which the author used as his referent for the symbol.

Children need to write and to tell number stories the same way they write and tell experience stories.[1] For example the student should practice statements such as: During the past thirty-two years a man's age has increased by $2^5$ while his net worth has increased by $10^6$ and he is now called a millionaire. Practice of this type needs to be both written and oral.

The need for vocabulary emphasis is repeated by Shepherd. He also states the need for teaching the use of the text. Students should be shown the significance of typographical differences and vocabulary aids, such as color, underlining, and shaded areas. The use of the glossary, table of contents and the index should be explained by the mathematics teacher. The use of the introductions and summaries given in the text should be

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[1]Frayda F. Cooper, "Math as a Second Language," Instructor, 81 (October, 1971), p. 76.

taught as review techniques.[1]

"An absolutely essential skill that everyone needs if he is to read a math book with understanding is that of referring to an example, or a diagram, or a graph, or a picture, that goes with an explanatory paragraph."[2] It is sometimes a help to use both hands as well as the eyes to read this kind of explanatory material, one hand to keep the place in the paragraph and the other to keep the place in the pictorial aid or example.[3]

The effectiveness of these teaching techniques and methods is determined by whether or not they are an integral part of the mathematics teacher's unit plans. If the reading is taught as a separate item the students will usually keep reading and mathematics separated and therefore will not profit from the teaching effort.

The Arithmetic Teacher, The Mathematics Teacher and The Reading Teacher contain some articles that give, in detail, specific lesson plans designed to treat items that

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[1]David L. Shepherd, "Teaching Science and Mathematics to Retarded Readers in High School," Reading Teacher, 17 (September, 1963), pp. 25-30.

[2]Phillips, "Reading Math Content," p. 65.

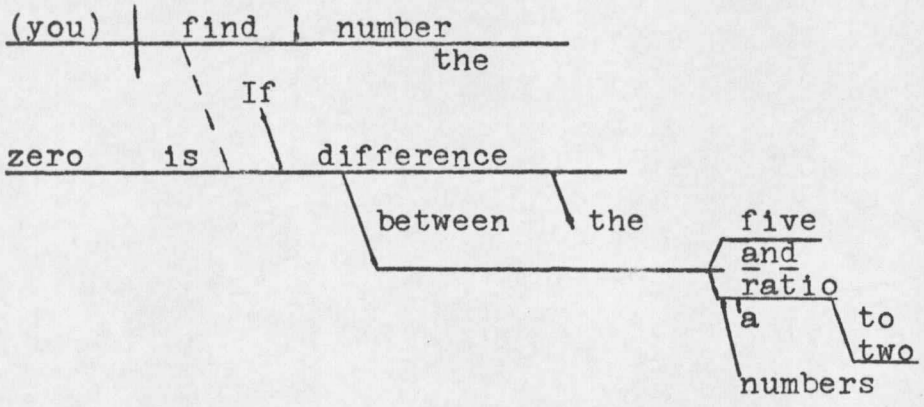
[3]Ibid.

directly relate reading to mathematics. The effective mathematics teacher will search out and use the ideas given in these and other periodicals. Some brief summaries of this kind of article are given in the following paragraphs.

> Without a close look, however, the full impact of the statement that mathematics is a language escapes most of us. The truth of the matter is that mathematics is a language and many teachers are missing an opportunity to teach it as such, or at least to show a strong correlation between a mathematical expression and an English sentence.[1]

The author gives lists of the more common mathematical nouns and their English equivalents. The following is an example of the steps suggested for changing an English sentence to a mathematical sentence.

> Example I. If zero is the difference between five and a number's ratio to two, find the number.

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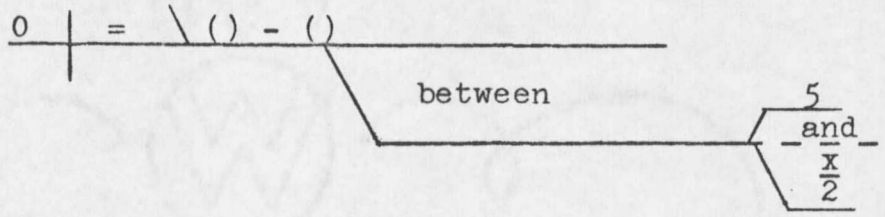
[1]King W. Jamison Jr., "Grammarmathically Speaking," Mathematics Teacher, 59 (November, 1966), p. 640.




$0 \mid = \quad \setminus (5) - (x/2)$

$0 = 5 - (x/2)$.[1]

The purpose of this lesson was to develop in the students an understanding of the idea that sentences of the forms p only if q, p is a sufficient condition for q and q is a necessary condition for p, say the same thing as the corresponding sentence if p then q. The method employed in this article to give this understanding was a dialogue expressing the same statement in its various forms.[2]

An article by Capps uses language analogies to teach mathematical concepts. He says there are some dangers in using analogies but that the competent teacher should be able to avoid them. Some examples from the article are given in the following quotation.

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[1]Jamison, "Grammarmathically Speaking," p. 641.

[2]Steven Szabo, "Several ways of Translating a Conditional Sentence," Mathematics Teacher, 61 (January, 1968), pp. 22-24.

| | |
|---|---|
| $4 + 7 = 7 + 4$ | He ate cake and ice cream. |
| | He ate ice cream and cake. |
| $(3 + 4) + 5 = 3 + (4 + 5)$ | Tom and Fred joined Mary. |
| | Tom joined Fred and Mary. |
| $16_2 \neq 16_9 \neq 16_{10}$ | Don't tear your dress. |
| | Wipe the tear from your eye.[1] |

Bamman contends that few secondary school teachers are adequately prepared to teach their students to read more effectively. He states that teachers can obtain help in discovering means to present mathematical content and reading skills simultaneously from the testing and counseling department and from the English teachers in most schools. He contends that teachers of mathematics should take training in the teaching of reading and provide in-service help to their fellow teachers. He says that mathematics teachers need this help and should seek it from every source available to them.[2]

In contrast to what Bamman says, Twining has given a list of the qualifications which content area teachers have for teaching various reading skills.

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[1]Lelon R. Capps, "Teaching Mathematical Concepts Using Language Arts Analogies," Arithmetic Teacher, 17 (April, 1970), p. 330.

[2]Henry A. Bamman, "Reading in Science and Mathematics," Perspectives in Reading No. 2, Reading Instruction in Secondary Schools, International Reading Association, (Newark, Delaware: 1964), pp. 59-71.

> (1) He is most capable of teaching the new vocabulary. (2) He is most knowledgeable in setting purposes for reading. (3) He is most able to develop and motivate student interest. (4) He is most adept in identifying important concepts. (5) He is most familiar with resources for developing background experiences. (6) He is familiar enough with the text to know best how to read and study it.[1]

The authors reviewed have stated some problems in reading mathematics and have proposed some possible solutions for some of the problems. In the next section the research related to these problems and proposed solutions has been reviewed.

## Related Research

A study of 1400 California students compared the relationship between reading ability and problem solving ability, and the relationship between computational ability and problem solving ability. The conclusions of this study which are related to the investigator's problem are given in the following quotation.

(1) General reading ability does have an effect on

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[1]Ellen E. Sargent, "Integrating Reading Skills in the Content Areas," Fusing Reading Skills, and Content, eds. H. Alan Robinson and Ellen Lamar Thomas, Newark, Delaware: International Reading Association, 1969, quoted in James E. Twining, "Content Area Reading Skills and the Community Junior College," Journal of Reading, 15 (February, 1972), p. 348.

problem solving ability.

(2) Both reading ability and problem solving ability correlate highly with IQ. When this is controlled the degree of the relationship between the two is drastically reduced.

(3) Computational ability has a significant effect on problem solving ability and this effect is not significantly reduced by controlling IQ.

(4) There is a lack of interaction. . . .

(5) Both factors are important in dealing with verbal problems.[1]

Troxel reports that the purpose for reading influences the speed with which the material is read and that speed and comprehension scores on general reading tests are not good predictors for speed and comprehension for other kinds of material. The kinds of things measured by speed and accuracy tests are factors in arithmetic achievement. Practice without instruction does not produce better scores for speed and accuracy.[2]

An experiment was performed to determine whether there is some correlation between a student's ability to solve word problems in second year algebra and the presence or absence of special reading instruction. The subjects for the experiment were two sections of algebra II, one taught by an English teacher trained in the teaching of reading and

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[1]Balow, "Reading and Computation Ability," p. 22.

[2]Troxel, "Effects of Purpose," pp. 221-27.

the other taught by the regular mathematics teacher. The time duration of this experiment was ten days. The results of the experiment seemed to indicate that the group taught by the English teacher did better, even when reading abilities and mathematical abilities were controlled. This control was achieved by pairing students on the basis of scores on the Cooperative Reading Test and by pairing them on the basis of quantitative scores from the DAT.

The authors, Call and Wiggin, claim that the following inferences can be made from the experiment.

> (1) There is some merit in teaching special reading skills for the solution of mathematical problems.
> (2) Even very good readers, . . . have difficulty in the interpretation of the kind of reading found in word problems.
> . . . . . .
> (5) If by teaching reading instead of mathematics we can get better results, it seems reasonable to infer that the competent mathematics teacher might get considerably better results if he were trained to teach reading of the kind encountered in mathematics problems.[1]

The research indicates that reading ability does have an effect on mathematical learning. From this indication, inference is made that the mathematics teacher should be able to teach appropriate reading techniques as an integral part of each mathematics course.

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[1]Call and Wiggin, "Reading and Mathematics," p. 157.

Research reported by Stevens gives correlation coefficients between 0.70 and 0.85 for IQ and problem solving ability, IQ and Reading ability, and between IQ and arithmetic fundamentals.[1] The high correlation between IQ and both mathematics and reading scores indicates that some control over IQ should be exercised in any study of these abilities.

## Summary of Chapter II

The problems encountered in reading mathematics material are caused by a lack of awareness of the purpose for reading the material, a technical vocabulary, a large vocabulary of signs and symbols, a vocabulary of numerals, a system of context clues coded by both size of the symbol and the location of the symbol. Problems are also encountered because of poor reading habits and a necessity for intense concentration.

Reading methods that can overcome many of these problems are available to the mathematics teacher through consulting reading experts and through study of the literature on reading in mathematics.

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[1]Stevens, "Problem Solving in Arithmetic," pp. 253-60.

The reported research relating reading and mathematics is extremely limited. Those studies reviewed by the investigator indicate that a gain in mathematics achievement was the result of emphasizing reading in the mathematics class.

# CHAPTER III

## PROCEDURES

A teacher was found who had training in the teaching of reading; then teacher-made examinations were written, revised and used as measures of mathematical achievement. Reading techniques were specifically designed for use in the college algebra course taught at Casper College, spring semester, 1972. The same test questions were administered both as a diagnostic test and as a final examination to supply a measure of mathematical achievement for the semester. Finally a statistical analysis was used to explain the data.

### The Search for a Teacher

The search for a mathematics teacher who had completed at least one college level course in the teaching of reading, was conducted in the fall of 1970. The Montana State Department of Public Instruction's records were searched by Mr. Alan Nicholson, State Supervisor of Mathematics. The investigator through personal contacts checked the Bozeman, Helena, Conrad, Shelby, and Cut Bank school systems for a mathematics teacher who had training in the teaching of reading. In addition Dr. Adrien Hess, of the

Montana State University, Mathematics Department used his personal knowledge and contacts to aid in this search. No suitable mathematics teacher was found.

In 1971 the investigator accepted a teaching position with Casper College and discovered a suitable teacher on the mathematics staff there. The teacher had taken a college course entitled "Teaching of Reading."

## Construction of the Tests

The topics covered in basic algebra and in college algebra were listed and reviewed for content by the Casper College mathematics faculty. A copy of each list was included in appendix A, page 75. Each faculty member was given the list and asked to mark any items that were not appropriate for final examination questions. The items marked with * were not considered for inclusion in the tests. The unmarked items in appendix A, page 75, had the unanimous approval of the Casper College mathematics faculty as reasonable topics for final examination questions. The topics for the thirty test items for each class were selected from these lists. To avoid any teacher preference for certain topics a table of random numbers was used to make the topic selection.

During the fall semester of 1971, the test items were constructed using the symbolism and vocabulary of the texts used for basic algebra[1] and college algebra.[2] The tests were submitted to the Casper College mathematics faculty for review of their content validity. The instructors were asked to approve or disapprove each item for inclusion on the final examination to be given to the sections of college algebra and basic algebra that they were teaching. The items included in the tests had the approval of all teachers teaching that particular course at Casper College spring semester, 1972.

An item analysis program was written for the IBM 1130 computer by this investigator, consulting with Miss Cynthia Grenier and Mr. Fred Wenn of the Casper College Data Processing Center. The output of this program included; (1) a count of correct responses and a count of responses and a percent for each alternative on each question, (2) a rank order of the students by number of correct

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[1]Irving Drooyan, Walter Hadel, and Frank Fleming, Elementary Algebra Structure and Skills, (2nd ed. New York: John Wiley & Sons, Inc., 1969), pp. 1-300.

[2]Elbridge P. Vance, Modern College Algebra, (2nd ed. Reading Massachusetts: Addison-Wesley Publishing Company, Inc., 1967), pp. 1-185.

responses, (3) the number of items correct for each student on the test, on the even numbered items and on the odd numbered items, (4) a difficulty index and a discrimination coefficient for each item. The difficulty index is the proportion of students answering the item correctly. The discrimination coefficient was found using the formula $(W_l - W_h)/N$, where $W_l$ represents the number of incorrect responses from students in the lower one-third of the class, $W_h$ represents the number of incorrect responses from students in the upper one-third of the class and N represents the number of students in the upper one-third of the class.

In his discussion of test item analysis Nunnally defines relatively large as necessarily over fifty and preferably over one hundred[1] and says, "pay little attention to statistical results unless they are based on relatively large numbers of students."[2] An attempt was made to use student volunteers to check the reliability, difficulty and the discriminatory power of the instrument. Only one student responded to an appeal made to the psychology

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[1]Jum C. Nunnally, Educational Measurement and Evaluation, (New York: McGraw-Hill Book Company, 1964), p. 135.

[2]Ibid., p. 137

and education classes at Casper College. The initial form of the college algebra test was then given to a college algebra class, a calculus class and to a linear algebra class.

The computer output for the difficulty and discrimination of the items on the college algebra test are shown in table 1. This data is for forty-four students. The items marked * were included in the basic algebra test.

TABLE 1

ITEM ANALYSIS FOR THE INITIAL FORM OF THE COLLEGE ALGEBRA TEST

| Ques. | Diff. | Disc. | Ques. | Diff. | Disc. |
|---|---|---|---|---|---|
| 1 | 0.5333 | -0.1333 | 16* | 0.4666 | 0.6666 |
| 2* | .8666 | .1333 | 17 | .4666 | .5333 |
| 3* | .8000 | .0000 | 18 | .4333 | .6000 |
| 4* | .6333 | .4666 | 19* | .4333 | .3333 |
| 5* | .6000 | .5333 | 20* | .5666 | .6000 |
| 6* | .6000 | .5333 | 21 | .5333 | .5333 |
| 7* | .7000 | .2000 | 22 | .3666 | .3333 |
| 8 | .6666 | .5333 | 23 | .2000 | .2666 |
| 9 | .2666 | -.2666 | 24 | .1666 | .2000 |
| 10* | .4333 | .6000 | 25 | .5000 | .4666 |
| 11 | .3333 | -.1333 | 26* | .1000 | .0666 |
| 12* | .6333 | .3333 | 27 | .4000 | .1333 |
| 13* | .3333 | .5333 | 28 | .5000 | .6000 |
| 14 | .5000 | .3333 | 29 | .0666 | .0000 |
| 15* | 0.4333 | 0.4666 | 30* | 0.4666 | 0.5333 |

To judge item difficulty, "a good rule to follow is to use few items that are above 80 per cent or below 20 per cent."[1] Items 24 and 26 of the college algebra test are difficult because they require several steps to get the solution. This leaves only items 2 and 29 outside the desirable range, therefore, none of the test items were changed on the basis of the difficulty index.

Nunnally says, "in general it is wise to be suspicious of items for which the difference is not at least 20 percentage points,"[2] when writing about item discrimination. On this basis the wording of problems 9 and 11 of the college algebra test was revised. The topics questioned by the problems were not changed. Some of the alternative answers for problems 10,12,23, and 25 were changed because no student selected them.

The difficulty index and the discrimination coefficient for each of the fifteen items included in both algebra tests, based on the final examination results of the sixty students in the four algebra classes, are given in appendix B, page 79. Three items are outside the

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[1]Nunnally, Measurement, p. 133.

[2]Ibid., p. 136.

desired range for difficulty and three items have low, positive discrimination coefficients. No item is outside the desired range on both measures. The sample size was too small to make valid judgments of the item difficulty indices or discrimination coefficients for the thirty item tests.

The set theory section was taught to the college algebra class just prior to the administration of this test. Item number one on the college algebra test was not changed because the negative discrimination coefficient was caused by the fact that the topics from set theory were familiar to the algebra students, who scored generally low in this group, but were not so familiar to the calculus and linear algebra students who generally scored high on this test.

The Kuder Richardson formula 20 gave a reliability coefficient of 0.7414 for the group of students given the initial form of the college algebra test. This was high in light of the fact that table 1 page 35, shows that the range of the item difficulty was 0.066 to 0.866, and also some of the items measured vocabulary knowledge and others measured computational skills.[1] Ferguson says that both

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[1]George A. Ferguson, Statistical Analysis in Psychology and Education, (2nd ed. New York: McGraw-Hill Book Company, 1966), p. 379.

of these characteristics will lower the reliability coefficient of the test.

The validity of the test was further confirmed by the fact that the sections of college algebra had 9.46 and 10.73 average number of items correct on final examinations, while the basic algebra classes had averages of 9.00 and 8.66 for the fifteen items which were included on both final examinations. The averages can be found in appendix F, pages 98 - 104.

## Reading Techniques for College Algebra

For convenience the sections of all classes receiving the treatment are numbered 01 in this paper. The reading treatment given to section 01 of college algebra was in the form of consultation between the investigator and the teacher. Prior to the beginning of the spring semester the investigator and the teacher carefully surveyed the text to determine the vocabulary to be covered. Specific words were selected for use in teaching the techniques of word analysis. The word list used is presented in appendix C, page 81.

Meetings to discuss the emphasis on reading were held daily for the first week of the semester and the

frequency of the meetings decreased gradually to bi-weekly toward the end of the semester. During the early meetings the reading helps in the book were discussed, including the printed keys to vocabulary, color codes, organization of sequential topics, and unusual characteristics such as the inclusion in the problems of important text material and definitions included in the footnotes.

As a review session for chapter one of the college algebra text by Vance, it was explained to the students that the vocabulary was italicized in the text. Most of the definitions and all of the axioms and theorems were printed in brown ink for easy reference. The footnotes, and colored print were to be checked for definitions and vocabulary words since Vance includes them in these locations as well as in the text material.[1] The student was reminded that the problems marked with '▷' were an important part of the course and needed to be reviewed. There was no summary section for the chapters in this book. The title page of each chapter, the table of contents and the section headings printed in brown ink at the top of each page were helpful in locating the important topics

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[1]Vance, Algebra, p. 6.

and in forming a summary of the chapters.

In later meetings the teacher and the investigator discussed topics similar to the following example. The organization of the text was usually in the form:

> Theorem 3-4. If $a \in R (a \neq 0)$ and n and m are positive integers,
>
> $$\frac{a^n}{a^m} = \begin{cases} a^{n-m} & \text{if n is larger than m} \\ \frac{1}{a^{m-n}} & \text{if m is larger than n} \\ 1 & \text{if } m = n \end{cases}$$
>
> Proof. . . .
> Illustration 1. $\frac{x^8}{x^3} = x^5$, $\frac{x^4}{x^{11}} = \frac{1}{x^7}$; $\frac{x^6}{x^6} = 1$.
>
> We are now prepared to divide any multinomial by a monomial. This is done by . . . [The text material here was often excellent review material and usually contained an explanation of the theory.]
> Example 1. Divide $12x^3y^4 + 18x^4y^2 - 36xy^3$ by $3x^2y^2$.
> Solution. . . .[1]

The text material following the examples usually explained the example and it was the place to look to find the directions for working the problems. Sometimes the steps were numbered and listed.[2]

The illustrations and examples in the text were numbered and placed in order from the easiest to the more

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[1]Vance, Algebra, p. 42.

[2]Ibid., p. 43.

difficult. To explain multiple step problems and processes the author used numbered steps.[1] There were some lists of related items as on page 96 of the text by Vance. The use of the index was explained and the students were told the location of the answers for most of the odd numbered problems.

Herber says that exercises like the one shown in appendix D, page 83, are intended to teach critical thinking.[2] The exercises in appendix D, page 83, were used to give the students practice in thinking about the ideas included in chapter one of the college algebra text.

Questions similar to the following were used to reinforce the vocabulary. These questions forced the successful student to read the text because there was not enough time for the teacher to teach each vocabulary word in class.

1. True or false: (a,b R)

a. _____ If $a < b$ then $a^2 < b^2$
b. _____ If $a < 1$ then $1/a > 1$
c. _____ If $a < b$ then $a - b < 0$
d. _____ $a < -a$ if $a < 0$

---

[1]Vance, Algebra, pp. 3, 43, 48, 49.

[2]Harold L. Herber, Teaching Reading in Content Areas, (Engelwood Cliffs, New Jersey: Prentice-Hall, Inc., 1970), p. 195.

2. Express each statement by using symbols.

a. _____ x is negative.
b. _____ x is 2 units away from 0.
c. _____ x is between -3 and 2.
d. _____ x is more than 2 times y.
e. _____ The distance from x to 2 is more than or equal to 5 units.

3. Matching

| | | |
|---|---|---|
| $\neq$ | _____ | a. subset |
| $\ni$ | _____ | b. element |
| $\emptyset$ | _____ | c. such that |
| $\in$ | _____ | d. not equal |
| $\subset$ | _____ | e. empty set |

4. Give an example of four of the basic axioms of the real number system. Label each example with the correct axiom name.

5. Matching: place the letters from column 2 in the blank after the words in column 1 if the statement in column 2 is an example of the word in column 1.

| | | | |
|---|---|---|---|
| 1. | _____ algebraic expression | a. | x in $3a^x$ |
| 2. | _____ term | b. | a in $3a^x$ |
| 3. | _____ coefficient | c. | 3 in $3a^x$ |
| 4. | _____ monomial | d. | $2x^2 + 5x + 9$ |
| 5. | _____ base | e. | $3y^2$ |
| 6. | _____ factor | f. | $36 = 2^2 \cdot 3^2$ |
| 7. | _____ exponent | g. | the 5x in d |
| 8. | _____ power | h. | the $2x^2$ in d |
| 9. | _____ polynomial | i. | $5y - 4$ |
| 10. | _____ prime factors | j. | 3xyz |

The word analysis instruction had the following form. From chapter one of the college algebra text the words, equivalent, page 4, exhaustive, page 11, and composite, page 14, were selected to be analyzed by the teacher as examples for the class. For example, the class

members were asked to give their first thought when hearing the word exhaustive. The results were listed on the blackboard. Next, the meanings of the root word, exhaust and the suffix, ive, were posed as questions. The responses, if any, were listed on the blackboard also and if no class member responded the teacher looked up the meanings in the dictionary. Next the suggested meanings for the root word and the suffix were compared with the context, " . . . set of subsets of A is said to be exhaustive if . . .,"[1] to see if one of the known uses of the words fitted this technical use of the word.

The students were given the words transitive, and distributive, from pages 20 and 24 respectively of the college algebra text, as two separate assignments to be analyzed and the analysis was compared with the technical usage of the word. For the first assignment, a written analysis was requested and, for the second, the directions were to be prepared for a class discussion of the word.

For the remainder of the semester oral analyses of vocabulary words were incorporated into the lectures and class discussions.

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[1]Vance, Algebra, p. 11.

The explanation of synthetic division on pages 45 and 46 of the text was assigned and checked in class to give the students practice in the use of text material and examples jointly. The method given by Phillips[1] was suggested to the students. The method is outlined on page 22 of this paper.

## Collection of the Data

The IQ scores and ACT standard scores for mathematics and reading given in appendix F, pages 98 - 104 were supplied by the Testing and Counseling Department of Casper College. The standard deviation for each test was reported to be seven or eight.

After the administration of the final examinations it was decided by the Casper College faculty that the item numbered 7 on the basic algebra test and 9 on the college algebra test was not a fair item because the incorrect alternative choice, x multiplied by itself five times, is used by many people. Therefore the results of this test item were not used in this study. The tests given to the basic algebra classes and to the college algebra classes

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[1]Phillips, "Reading Math Content," p. 65.

contained fifteen joint items and fourteen items each that were unique to the particular course to make two twenty-nine item tests. The calculus classes were given a different twenty-nine item test. Copies of each of the tests are given in appendix E, pages 85 - 97.

The algebra diagnostic tests were scheduled to be given on the first day of the semester but, due to severe weather conditions causing poor attendance, administration of the tests was delayed. This resulted in some of the college algebra students receiving instruction on some of the test items before the diagnostic test was given. Therefore the college algebra beginning scores are based on only the questions numbered above eight. The score on these items was multiplied by 29/21 to get an adjusted beginning score.

The teacher with training in teaching reading was not assigned an algebra class for the spring semester of 1972, therefore the calculus classes were added to the study. Due to time limitations a diagnostic test for the calculus classes was not constructed. The test scores, IQ scores, and ACT mathematics and reading scores are given in appendix F, pages 98 - 104.

## The Design of the Experiment

The control teachers were chosen from the Casper College faculty by matching them with the experimental teachers on the basis of experience, training and supervisor evaluation. The supervisors say that teachers one and two differ only in approach to the student and that there is no observable difference between teacher three and teacher four. The other comparisons made are shown in table 2. The training is given in semester hours.

TABLE 2

TEACHER COMPARISONS

| Teacher | Years of Teaching Experience | Training |
|---|---|---|
| 1 | 13 | MA |
| 2 | 15 | MS + 22 |
| 3 | 34 | MS + 22 |
| 4 | 21 | MS + 18 |

The design of the experiment was determined by the teaching assignment of these teachers. The reading treatments were given to the students in sections labeled with an 01 code in figure 1. The students in the sections labeled 02 were used as a control measure.

| Classes | Reading Emphasis | Control |
|---|---|---|
| Basic Algebra | Teacher 1<br>55-097-01 | Teacher 2<br>55-097-02 |
| College Algebra | Teacher 1<br>55-117-01 | Teacher 2<br>55-117-02 |
| Calculus | Teacher 3<br>55-151-01 | Teacher 4<br>55-151-02 |

Fig. 1.--Teaching assignment and treatment assignment.

The treatment given to the students taking college algebra was described on pages 38 - 44, of this paper. Teacher one applied the same treatment to section one of basic algebra but without consulting with the investigator concerning specific methods for the course and text. Teacher three who had the reading training met with the investigator in two meetings held before the semester and twice during the semester to discuss reading techniques. This teacher then emphasized the reading techniques in calculus section 01.

# CHAPTER IV

## ANALYSIS OF THE DATA

The literature reviewed indicated that IQ scores, measures of mathematical achievement and general reading scores all correlate with problem solving ability. Therefore measures of these items were considered as possible covariates.

### Analysis of Covariance

The IQ scores are not exact measures and therefore do not meet the assumption of exact measure necessary for a covariate. Winsor says of this case:

> Our general principle, it appears should be: if it is possible and meaningful, arrange the experiment so that the desired regression can be determined directly. . . . In those numerous situations where this is not possible, use the inverted regression.[1]

Since the adjustment of IQ scores based on the data of this experiment has no value, the inverted regression was chosen.

The bivariate frequency distributions shown in appendix G, pages 105-108, were used to justify choosing a linear regression model. An inspection of the distributions

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[1]Charles P. Winsor, "Which Regression," Biometrics, 2 (December, 1946), p. 109.

showed that a linear function would be as likely to fit the data as any of the elementary functions.

The standard deviation for IQ scores for the samples in this study was higher than the seven or eight range given by the source of the scores.[1] (See table 7, page 60 and table 8, page 61, in this paper.) This large standard deviation was evidently caused by the fact that the IQ scores were not all from the same test of mental ability but were taken from college entrance records.

The effect of this larger standard deviation was a decrease in the slope of the regression line used to arrive at the adjusted mean value for the gain scores found by subtracting the number of correct items on the diagnostic test from the number of items correct on the final examination. In figure 2, page 50, the dotted lines represent the case with a larger standard deviation and the solid lines the case where the smaller standard deviation was used.

This larger standard deviation generally tends to reduce the size of the adjustment made by regression. This can be visualized in figure 2. Let a point move along one

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[1]Means, Interoffice memo.

of the regression lines a distance fixed by an interval on the IQ axis. Repeat the process for the other regression line and observe the respective change in Y score caused by the motion along each regression line.

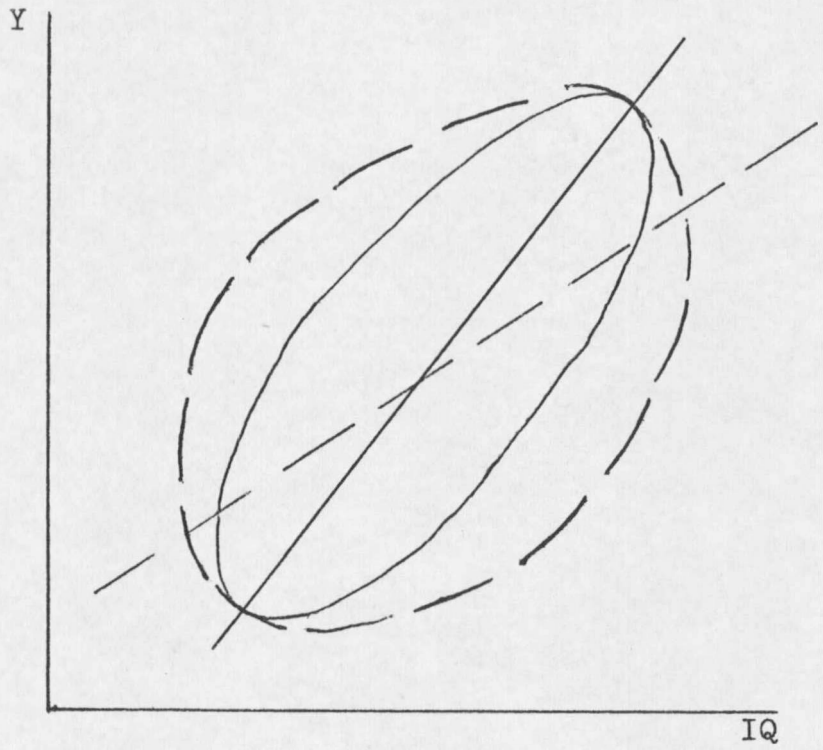


Fig. 2.--The effect of larger variance on the regression adjustment.

The data for the fifteen test items used in all of the algebra classes was analyzed by means of Dr. Lund's Multiple Linear Regression computer program. Standard output of this program included, "variable means, partial

correlations, regression coefficients, standard errors, t-values, intercept, $R^2$, and ANOV."[1] For use in the program the variables were coded as shown in table 3.

TABLE 3

CODE USED IN THE MULTIPLE LINEAR REGRESSION PROGRAM

| Variable Name | Variable No. |
|---|---|
| Gain score on 29 questions | 1 |
| IQ | 2 |
| ACT Mathematics Score | 3 |
| ACT Reading Score | 4 |
| Treatments | 5 |
| Course | 6,7 |
| Interaction | 8,9 |
| 55-097-01 | 10 |
| 55-097-02 | 11 |
| 55-151-02 | 12 |
| 55-117-02 | 13 |
| 55-151-01 | 14 |
| Gain score on 15 questions | 15 |

The code given in table 3 was used for all data from linear regression sources in this study. The regression analysis was used to test the effectiveness of IQ, mathematics scores and reading scores as covariates. The computer output is shown in table 4, page 52.

The calculated t-scores were for testing the null

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[1]Richard E. Lund, "Multiple Linear Regression Program," Department of Mathematics, Montana State University, Bozeman, Montana, 1971. (Mimeographed.)

hypothesis, $H_o$: the value of the regression coefficient for x is zero. Where x takes on the values 2,3,4,5,6,8 from table 3.

TABLE 4

COMPUTER OUTPUT FOR REGRESSION ANALYSIS OF THE EFFECTIVENESS OF THE COVARIATES

| Var. No. | Regression Coef. | Computed t-value |
|---|---|---|
| 2 | 0.0993 | 2.909 |
| 3 | -.0836 | -1.378 |
| 4 | 0.0194 | 0.271 |
| 5 | 0.8884 | 2.332 |
| 6 | -2.724 | -4.171 |
| 8 | -1.268 | -2.339 |

15 Dependent

Analysis of Variance for this Regression

| Source | df | SS | MS | F value |
|---|---|---|---|---|
| Due to Regression | 6 | 235.249 | 39.208 | 9.248 |
| Residual | 53 | 224.684 | 4.239 | |
| Total (N = 60) | 59 | 459.933 | | |

Correlation Coefficients

| | |
|---|---|
| IQ and Gain Score (15 items) | 0.5359 |
| ACT Mathematics Score and Gain (15) | 0.4043 |
| ACT Reading Score and Gain Score (15) | 0.3792 |
| IQ and ACT Mathematics Score | 0.6814 |
| IQ and ACT Reading Score | 0.6408 |
| ACT Mathematics and ACT Reading | 0.4735 |
| IQ and Gain Score (29 items) | 0.4565 |
| ACT Mathematics Score and Gain (29) | 0.1708 |
| ACT Reading Score and Gain Score | 0.2538 |

The results shown in table 4 indicate that the coefficient for the reading score was not significantly different from zero at the 0.05 level and therefore the ACT reading score would probably make no adjustment in the gain score if it were used as a covariate. The computed t-value for the test of the null hypothesis, $H_o$: the regression coefficient for the ACT mathematics scores is zero, was -1.378. The sign was ignored in the two-tailed test, giving a significance level of 0.12. This did not meet the predetermined significance level of 0.05 and therefore the ACT mathematics score was not used as a covariate.

This failure of the ACT mathematics and reading scores to explain any of the gain on the teacher-made mathematics tests supported a hypothesis stated by Mr. Richard Means of the Casper College Testing and Counseling Department that, "the ACT test is not a good predictor of success for many community college students."[1]

The regression coefficient for the IQ scores was significant at the 0.05 level, therefore an analysis of

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[1]Mr. Richard R. Means, Peronal interview, Casper, Wyoming, February, 1972.

covariance on this data would be biased when IQ was not used as a covariate.[1]

In the analysis of the data for this study, the slope of the gain on IQ regression line was reduced by the large variance, until the adjustment made in the gain scores was so small that it had no effect on the significance of the differences of gain scores. For the reasons stated in this section the use of covariance as a method of analysis was abandoned.

The correlation between the ACT reading scores and gain scores was 0.2538 and a 95 per cent confidence interval for this correlation was (0.10, 0.58). This correlation is not significantly higher than the 0.224 correlation between gain in reading and gain in arithmetic reported by Scott.[2] This low correlation supported the conclusion of Call and Wiggin that good readers as well as poor readers often have difficulty reading mathematical materials.[3]

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[1]Dr. Richard E. Lund, Personal interview, Montana State University, Bozeman, Montana, June, 1972.

[2]Carrie M. Scott, "Relationships Between Intelligence Quotients and Gain in Reading Achievement with Arithmetic Reasoning, Social Studies, and Science," Journal of Educational Research, 56 (February, 1963), pp. 322-26.

[3]Call and Wiggin, "Reading and Mathematics," p. 157.

The correlation between IQ scores and ACT mathematics scores and the correlation between IQ scores and ACT reading scores both have the 95 per cent confidence interval (0.45, 0.87). This confidence interval includes the 0.70 to 0.85 correlations found by Stevens for these same measures.[1] Therefore there is no significant difference at the 0.05 level between the correlations found here and those reported by Stevens.

## The Randomized Complete-Block Design

The randomized complete-block design could be used when it was known that grouping the experimental units would make them more uniform. From table 4, page 52, the t-value was -4.171. This t-value has a significance level below 0.001. This indicates that the blocking by class level did remove a significant amount of variation from the error term. This blocking made it possible to attribute more of the observed difference to the treatments.

For the analysis of the gain scores from the fifteen test items, selected from the algebra tests, the assignment of blocks and treatments was completed as shown in figure 3.

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[1]Stevens, "Problem Solving in Arithmetic," pp. 253-60.

| Blocks | Treatments | |
|---|---|---|
| | reading emphasis | no reading emphasis |
| 55-097 | section one | section two |
| 55-117 | section one | section two |

Fig. 3.--Assignment of experimental units to the blocks and treatments.

Since a difference was expected between the college algebra class and the basic algebra class the classes were called the blocks. The treatments were the emphasis of reading in sections numbered one and no reading emphasis in sections numbered two.

An analysis of the gain scores on the fifteen test items used the linear additive model,

$$X_{ijk} = M + T_i + B_j + E_{ij} + D_{ijk},$$

where the score of any student $X_{ijk}$ was made up of an overall mean M plus a treatment effect $T_i$, plus an effect of the class $B_j$, plus an experimental error $E_{ij}$, and the sampling error $D_{ijk}$. It was assumed that the errors were distributed normally with a mean value of zero. Under this model the analysis of variance for the randomized complete block design with subsampling is given in table 5.

In order to find out if the experimental error term

contained variation in addition to that among subsamples the following test was made. $H_o$: there was no interaction. $H_1$: there was an interaction. Under $H_o$ $F^1_{56} = 26.66/5.91 = 4.51$ which was greater than the critical value $F^1_{56,0.5} = 4.04$. This test showed that there was a significant interaction. Two factors are said to interact if there is a failure of the additivity. The presence of the interaction indicates that the data do not follow the linear model. The data used can be found in appendix F, pages 98 - 104.

TABLE 5

ANALYSIS OF VARIANCE FOR THE FIFTEEN ITEM TEST

| Source | df | SS | MS |
|---|---|---|---|
| Blocks | 1 | 147.27 | 147.27 |
| Trts. | 1 | 4.27 | 4.27 |
| Exp. Err. | 1 | 26.66 | 26.66 |
| Samp. Err. | 56 | 331.00 | 5.91 |
| Total | 59 | 509.20 | |

An examination of table 6, the class means for the fifteen question test, shows the lack of additivity. The difference between the treatment mean and the control mean in the basic algebra block was -0.80 while the difference between the treatment mean and the control mean in the college algebra block was 1.87. In an additive case the

differences in the two blocks should have been nearly equal if the same treatment had been applied to both blocks.

TABLE 6

MEANS AND STANDARD DEVIATIONS FOR THE FIFTEEN ITEM SUBTEST

| Class | df | Treatment | | Control |
|---|---|---|---|---|
| Basic Algebra | 28 | $\bar{X}$ | 2.06 | 2.86 |
| | | s | 2.74 | 2.06 |
| College Algebra | 28 | $\bar{X}$ | 6.53 | 4.66 |
| | | s | 1.73 | 2.31 |

The test of the hypothesis of equal means for the two sections of college algebra had a calculated t-statistic of 3.40, which had a significance level between 0.001 and 0.0001, which meets the predetermined level of 0.05. This significant difference, along with the significant interaction, implied that the treatment given to the basic algebra students was not the same as that given to the college algebra students. The difference could not be explained as a course effect since the blocking in the randomized complete-block design removed this variation. The teacher effect was controlled between the two experimental sections because the same teacher taught both of them. The teacher effect was minimized between experimental and

control groups by equating the teachers. The student differences were equalized by the random assignment of students to the various sections and by accounting for this variance in the subsampling error term in the model. Therefore the difference was probably caused by a factor of teacher motivation or teacher interest in the treatment.

## t-Tests of Independent Means

A comparison of the gain scores on the twenty-nine question tests was made by considering each class to be a random sample of the students registering for that course at Casper College during the spring semester of 1972. The following table shows the means and standard deviations of the four measures available, for each class.

The data in table 7, page 60 was used to calculate the F statistics which were used to test the hypothesis that the populations from which the classes were drawn had equal variances. The populations in this case referred to all students registering for basic algebra, or for college algebra, or for first semester calculus, at Casper College, spring semester, 1972. Equality of variance is a necessary condition for the standard t-test of difference of means.

TABLE 7

MEANS AND STANDARD DEVIATIONS FOR THE TWENTY-NINE QUESTION TESTS AND COVARIATES

| Class | n | | IQ | Math | Read | Gain |
|---|---|---|---|---|---|---|
| 55-097-01 | 15 | $\bar{X}$ | 104.10 | 13.06 | 17.73 | 5.20 |
| | | s | 8.99 | 5.79 | 4.99 | 3.83 |
| 55-097-02 | 17 | $\bar{X}$ | 107.24 | 14.12 | 17.71 | 5.82 |
| | | s | 14.81 | 6.80 | 5.72 | 3.41 |
| 55-117-01 | 21 | $\bar{X}$ | 113.28 | 21.52 | 20.57 | 9.62 |
| | | s | 8.69 | 4.52 | 4.47 | 3.22 |
| 55-117-02 | 21 | $\bar{X}$ | 115.52 | 21.33 | 21.48 | 6.52 |
| | | s | 13.51 | 5.13 | 4.74 | 4.06 |
| 55-151-01 | 18 | $\bar{X}$ | 113.50 | 26.70 | 22.71 | 19.05* |
| | | s | 5.20 | 2.74 | 4.63 | 4.14 |
| 55-151-02 | 29 | $\bar{X}$ | 114.50 | 26.48 | 22.90 | 18.31* |
| | | s | 9.10 | 3.26 | 4.70 | 5.57 |

* indicates final examination scores.

Using the data from table 7, the following hypothesis was tested. $H_o$: the variance of the scores on the ACT mathematics test was the same for section one and section two of basic algebra, 55-097. The alternative hypothesis was that the variances for these two classes was not the same. Under $H_o$ $s_{02}^2/s_{01}^2 = F_{14}^{16} = 1.3996$. This value was compared with the critical value, at the 0.05 level of significance, $F_{14,0.05}^{16} = 2.44$. The difference in this case

was not significant and there was no evidence at the 0.05 level that the variance for the two basic algebra classes was not the same on the ACT mathematics test. Therefore the t-test for difference of means could be used to test the difference of means for ACT mathematics scores on this pair of classes. The F statistics for making this test on each pair of classes and for each of the measurements from table 7, page 60, are given in table 8.

TABLE 8

CALCULATED F STATISTICS FOR TESTING EQUALITY OF VARIANCE FOR THE TWO SECTIONS OF EACH COURSE

| Class | df | Calculated F Statistics | | | | |
|---|---|---|---|---|---|---|
| | | IQ | Math | Read | | Gain |
| 55-097 | $F^{16}_{14}$ | 2.6773 | 1.3996 | 1.2281 | $F^{14}_{16}$ | 1.2253 |
| 55-117 | $F^{20}_{20}$ | 2.2845 | 1.2881 | 1.1246 | | 1.1589 |
| 55-151 | $F^{28}_{17}$ | 3.0627 | 1.4156 | 1.0305 | | 1.8102 |

Critical Values: $F^{16}_{14,0.05} = 2.44$; $F^{14}_{16,0.05} = 1.95$;

$F^{20}_{20,0.05} = 2.12$; $F^{28}_{17,0.05} = 2.15$.

An examination of table 8 shows, that for the ACT mathematics and reading scores and for the gain scores,

there was no evidence at the 0.05 significance level that the populations had unequal variances. The t-test for difference of means could be used to compare the respective sections on the basis of ACT mathematics scores, ACT reading scores and the gain score measured by teacher-made examinations.

The significant difference, at the 0.05 level, from section to section in each class, on IQ scores was a statistical confirmation of the fact that the variation in IQ scores available for this study was too large for use in an analysis of covariance. The fact that different IQ tests were used to obtain these scores may have been the reason for some of this difference. Other sources of the variation in IQ scores could have been the fact that some very bright students registered for basic algebra because it was the lowest numbered mathematics course at Casper College and it satisfied the school mathematics requirement. Sampling error could have accounted for some of the variation, but the ACT scores did not show the same variance differences for the same samples.

The means for the two sections of each class were compared using the t-test to check for significant differences. The t-scores used are given in table 9.

TABLE 9

t-SCORES TO TEST EQUALITY OF CLASS MEANS

| Class | df | t - Scores | | |
|---|---|---|---|---|
| | | Math | Read | Gain |
| 55-097 | 30 | -0.5751 | 0.3650 | -0.7351 |
| 55-117 | 40 | 0.1250 | -0.6276 | 2.7403 |
| 55-151 | 45 | 0.2418 | -0.1377 | 0.4836 |

Critical Values: $t_{30,0.05} = 2.042$; $t_{40,0.05} = 2.021$; $t_{45,0.05} = 2.021$.

The hypothesis tested was, $H_o$: there is no difference in the means of the two sections for each subject. The negative signs in table 9 indicate the measures where the control group was higher than the experimental group. The lack of significant differences, at the 0.05 level, between the respective sections of each of the three classes on the ACT scores indicated that the Casper College registration process did equalize the sections on the basis of mathematics ability and reading ability. This result supported the assumption that the registration procedure was a random process.

At the 0.05 level there was no significant

difference between the basic algebra control and experimental groups in mathematical achievement. This result agreed with the results from the randomized complete-block analysis of the fifteen items used in both the basic algebra and the college algebra tests. The significant interaction in the randomized complete-block model and the difference found here between the treatment and control groups for the college algebra class indicate that the treatment was applied in levels. The amount, kind, or intensity of reading emphasis was not the same in the college algebra class as it was in the basic algebra class.

The statistic in table 9, indicated that there was no significant difference in the final examination scores for the two sections of calculus. The indication was that possibly the treatment given to the calculus students was not the same as the treatment given to the college algebra class.

## Summary of Chapter IV

The analysis of covariance was attempted to control the variables for which the review of literature had indicated a correlation with mathematics achievement scores. The covariate measures available failed to make a

significant adjustment in the mathematics achievement scores. This method of analysis was then abandoned.

The results from the fifteen test items were used to compare the four algebra sections. A randomized complete block design gave no significant difference for treatments but did show a significant interaction.

The analysis of the data from the twenty-nine item tests showed a significant difference in mathematics achievement between the treatment and the control groups for college algebra, but showed no significant difference for the basic algebra classes and for the calculus classes. This result was the reason for the significant interaction found in the randomized complete-block analysis.

# CHAPTER V

## SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The implication has been made in the literature that a special reading problem exists in the reading of mathematical material. This study attempted to investigate the effect on mathematics achievement of emphasizing reading techniques in the mathematics classroom. A summary of the study, a discussion of the conclusions drawn from the research and recommendations implied by the conclusions are presented in this chapter.

### Summary

The problem investigated in this study was the effect on mathematics achievement of teaching reading of the mathematics text in the mathematics classroom. The gain in mathematics knowledge over one semester was measured by administering a set of questions as a diagnostic test near the beginning of the semester and again administering the same questions as part of the final examination.

Two teachers were chosen to emphasize reading in their classes. One of them, teacher three, had a teaching of reading course. The teachers who were used to teach the

control sections were matched with the teachers of the experimental sections on the basis of supervisor evaluation, experience, and formal training.

The two teachers of the experimental sections and the investigator applied the treatments to the samples of students in the following manner. The students were assigned to the classes by the Casper College registration process. It was assumed that the registration process made a random assignment of students to the sections of each class. Teacher three and the investigator discussed the reading techniques in four meetings, two before the semester began and two during the semester. Teacher three emphasized the reading techniques in the experimental section of calculus. Teacher one and the investigator met often during the semester to discuss the teaching methods used in college algebra, using the methods discussed, teacher one also emphasized reading in the basic algebra class in a manner similar to that used in the college algebra section. The investigator and teacher one did not discuss the specific applications given to the basic algebra class.

The review of literature indicated high correlations between IQ and arithmetic problem solving ability, IQ and reading ability, and between IQ and mathematical

achievement. An analysis of covariance was attempted using measures of these characteristics. The adjustment made by each of the covariates was not significant and the analysis by that method was terminated.

Three different twenty-nine item tests were used to measure student achievement. They were written and revised on the basis of valid content by the Casper College mathematics teachers. An item analysis computer program was used in the revision process for the algebra tests to check the difficulty level, the discriminatory power, and the rate of selection for the alternative answer choices, for each item. The revised test items were used in a diagnostic test as well as in the final examinations.

There was a fifteen item section common to both algebra tests. A randomized complete-block design was used to analyze the data from this subtest. This analysis showed no significant effect of the reading treatment at the five per cent level. The hypothesis of a significant interaction was supported at the five per cent level. The analysis of the interaction indicated that the treatment was not given uniformly to the basic algebra and college algebra classes.

The gain score was found by subtracting the number

of items correct on the diagnostic test from the number of items correct on the final examination for each student. The control group mean gain score and the experimental group mean gain score for each class were compared using the t-test. The data for this test was taken from the twenty-nine item examinations. The hypothesis that emphasizing reading would improve mathematics achievement was not supported at the 0.05 level of significance in the basic algebra and the calculus classes. This same hypothesis was supported at the five per cent level in the college algebra class. This result was further evidence that the treatment was given in levels and not uniformly to all experimental groups. The significant difference between the two college algebra classes indicated that it was possible to improve mathematics achievement by emphasizing reading. The lack of significant difference in the other two classes indicated that there was a factor of teacher motivation or teacher interest that must be overcome if the improvement was to be general.

The correlation coefficients found in this study for IQ and mathematics achievement, for IQ and ACT reading scores, and for IQ and ACT mathematics scores are not significantly lower than those reported for the same measures

by Stevens.[1] The correlation coefficient for ACT mathematics scores and mathematics achievement scores as measured by teacher-made examinations is very low, 0.1708.

## Conclusions

The conclusions made in this study are related to the group of students used in the experiment, the particular tests administered and the text on which the materials were based.

(1) It is possible to obtain better achievement in mathematics by emphasizing reading techniques. This was evidenced by the fact that the difference between the mean gain scores for the college algebra class receiving the reading treatment and the control section of college algebra was significant at the five per cent level.

(2) There was a difference between the treatment applied to the college algebra class and the treatment given to the basic algebra class. This conclusion was supported by the significant interaction in the randomized complete block analysis.

(3) There was a difference between the treatment

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[1]Stevens, "Problem Solving in Arithmetic," pp. 253-60.

applied to the calculus class and the treatments given to the two algebra classes. This is shown by the significant interaction found when the calculus scores are compared with the algebra scores.

(4) The teacher interest in the treatment may have an effect on the way the reading emphasis was given to the students.

(5) The amount of reading training may have an effect on the effectiveness of the reading emphasis. This conclusion and conclusion four are possible explanations of the significant interaction found. This study, since it was not replicated, does not effectively isolate these effects.

(6) The IQ scores from college entrance records are not precise enough measurements to be an effective covariate for this analysis of gain in mathematical achievement.

(7) The ACT mathematics and the ACT reading scores do not correlate highly enough with gain in mathematics achievement scores to use them as covariates in an analysis of this data.

(8) The ACT mathematics test is not a good predictor of success for Casper College students in mathematics classes where gain in mathematics achievement is used in the grading process.

## Recommendations

Several recommendations based on the conclusions of this study may be made. In each case the limitations and the lack of replication should be considered.

(1) Further research in this area is needed. It is evident that this study should be repeated, except that the diagnostic test be given on the first day of the semester. This experiment cannot be replicated because the diagnostic test was not given on the first day of class, spring semester, 1972, and the text for college algebra at Casper College has been changed. To implement this research it is necessary that more mathematics teachers be trained to teach reading.

A similar replicable experiment should allow the same teacher to teach both control and experimental groups, thus giving a better control over the teacher effect. The experimental units could then be blocked on the basis of both teacher and class. This arrangement would also eliminate from the error term any effect of teacher preference for a particular class.

(2) Further experimentation is needed to determine the effect of teacher interest and teacher training, in the teaching of reading, on the reading emphasis given to the

students.

(3) It is recommended that the students used in a similar study be given an IQ test to improve the adjustment made by this measure on the gain in mathematics achievement measurement.

(4) It is recommended that the Casper College Testing and Counseling Department search for an instrument that will correlate more with the results on teacher-made tests for use in mathematics placement at Casper College.

# APPENDICES

# APPENDIX A

## COURSE OUTLINES FOR BASIC ALGEBRA AND COLLEGE ALGEBRA

# BASIC ALGEBRA COURSE OUTLINE

Sets
- Set Construction and Subsets
- Set Operations
- Venn Diagrams*
- Cartesian Products

Whole numbers and Integers
- Cardinality and Ordinality*
- Equality and Order Axioms
- The Distributive Law
- Consequences of the Axioms
- Inverse Operations
- Factoring
- Open Sentences
- Absolute Value
- Sums and Differences
- Products and Quotients
- Open Sentences Involving Integers

Rational Numbers
- An extension of the Set of Integers*
- Properties of Rational Numbers
- Sums Differences Products Quotients
- Equations Involving Rational Numbers
- Decimal Notation
- Radical Notation

Real Numbers
- Irrational Numbers
- Properties of Real Numbers

Polynomials
- Definitions and Forms of Products
- Sums and Differences
- Products and Quotients of Monomials
- Integer Exponents, Scientific Notation
- Products of Polynomials
- Factoring
- Quotients

Rational Expressions
- Reducing and Building to Common Terms
- Sums Differences Products Quotients
- Complex Fractions

First Degree Equations and Inequalities--One Variable
- Equivalent Forms
- Solving for Specified Symbols

Solution of Inequalities
Mathematical Models for Word Problems
Word Problems*

Relations, Functions, Graphs
Two Equations in Two Variables
Relations
Functions
Notation
Graphs of Functions
Graphs of Inequalities in Two Variables*

Systems of Linear Equations and Inequalities
Graphical Solutions*
Algebraic Solutions
Word Problems

Roots and Radicals
Roots in Radical Form
Sums Differences Products Quotients

Second Degree Equations
Solution by Factoring
Solution by the Quadratic Formula
Graphing

* These items were not considered for inclusion on the test.

# COLLEGE ALGEBRA COURSE OUTLINE

- Sets and Numbers
  - Basic Notation and Sets
  - Operations on Sets
  - Subsets of the Real Numbers
- Logical System of the Real Numbers
  - Axioms
  - Basic Theorems
  - Further Theorems
- Operations using the Logic
  - Four Operations on Expressions
  - Special Products and Factoring
  - Four Operations on Rational Forms
  - Integral and Rational Exponents
  - Four Operations on Radicals
- Inequalities, Absolute Value, and Coordinate Systems
  - Order Axioms
  - The Completeness Property
  - The Distance Formula
- Functions
  - Relations and Functions
  - Graphing
- Linear Functions
  - Arithmetic Progressions
- Quadratic Functions
  - Solutions
  - Relations Between Zeros and Coefficients
  - Equations in Quadratic Form
  - Equations Involving Radicals
  - Variation
  - Systems of Equations
    - Linear
    - Quadratic
    - One Linear, One Quadratic
- Matrices and Determinants
  - Basic Properties
  - Products of Matrices
  - Inverse Matrix
  - Determinants of Order Two and Three
  - Expansion by Minors
  - Solution of Systems of Equations
- Polynomial Functions
  - Certain Theorems
  - Graphing

# APPENDIX B

## COMPUTER OUTPUT FOR ITEM ANALYSIS

TABLE 10

ITEM ANALYSIS FOR FIFTEEN ITEMS INCLUDED ON BOTH ALGEBRA TESTS

| Ques.* | Diff. | Disc. |
|---|---|---|
| 2 | 0.9000 | 0.2000 |
| 3 | 0.8750 | 0.2500 |
| 4 | 0.7750 | 0.2500 |
| 5 | 0.6750 | 0.0500 |
| 6 | 0.5000 | 0.6000 |
| 7 | 0.6000 | 0.3000 |
| 10 | 0.8500 | 0.2000 |
| 13 | 0.3750 | 0.0500 |
| 16 | 0.5500 | 0.6000 |
| 19 | 0.5500 | 0.1000 |
| 20 | 0.3250 | 0.4500 |
| 26 | 0.5500 | 0.3000 |
| 30 | 0.5250 | 0.4500 |
| 15 | 0.4000 | 0.6000 |
| 12 | 0.5750 | 0.8500 |

*Numbers from the college algebra final exam.

# APPENDIX C

# VOCABULARY FOR COLLEGE ALGEBRA

# VOCABULARY FOR 55-117

finite
element
equivalent
subset
union
exhaustive
factors
multiples
odd
prime
irrational
equality
symmetric
identity
associative
inverse
definition
parentheses
exponent
polynomial
coefficient
factor
algorithym
radical
trichotomy
dimension
function
range
linear
proportion
matrix
determinant
infinite
equal
complement
intersection
integers
closure
divisors
even
rational
real

$\overline{\exists}$
$\{\}$
$\in$
:
|
$\ni$
A'
$\subset$
$\neq$
I
$\not\emptyset$
$\cup$
N
$\notin$
$\Sigma$
F
$\forall$
$\exists$
R
$\cap$
$\exists!$
!
l.u.b.
n(S)
g.l.b.

transitive
identical equations
commutative
distributive
cancellation
reciprocal
power
base
binomial
term
degree
synthetic division
perfect square trinomial
complex fraction
principal root
rationalize
coordinate system
upper bound
quadrant
domain
ordered pair
parabola
arithmetic progression
quadratic equation
inequality
direct variation
scalar
minor
rule method
universal set
cartesian product
natural numbers
conditional equations
difference of squares
zero of a function
linear function
quadratic function
completing the square
one to one correspondence
composite
reflexive

# APPENDIX D

## VOCABULARY EXERCISE

# VOCABULARY HANDOUT

55-117 Extra Problems

Name_____

Directions: 1. Mark three words in each row below that have something in common.

2. Identify the common "thing".

1. element, roster method, rule method, tabulation, finite.

2. one to one correspondence, equal, equivalent, subset intersection.

3. universal, empty, disjoint, null, union.

4. cardinal, reflexive, transitive, commutative, symmetric.

5. exhaustive, associative, disjoint, Venn diagrams, ordinal.

6. positive, integers, natural, counting, composite.

7. closed, factors, multiple, even, odd.

8. integer, natural, rational, irrational, real.

9. composite, prime, even, natural, integer.

10. identity, one, equation, periodic, expression.

# APPENDIX E

# FINAL EXAMINATIONS

FINAL EXAMINATION

55-097 Name_____

DIRECTIONS: Circle the letter indicating the correct answer. All questions have only one correct answer, if more than one of the choices are correct there will be a choice allowing you to pick them.

Use this given for problems 1 - 4.

Given: $A = \{1,2,3,4\}$ $B = \{5,6,7,8\}$ $C = \{2,4\}$

$D = \{2,4,6,8\}$ $E = \{0,1,3,5,9,7\}$ $F = \{a,b\}$

1. C is a subset of a. A b. D c. A and B d. A and D e. B and D.

2. The following are equal sets: a. A and B b. B and C c. E and F d. C and F e. none of these.

3. The following are disjoint sets: a. A and E b. A and C c. B and C d. B and D e. none of these.

4. Let $U = \{0,1,2,3,4,5,6,7,8,9\}$ then, a. $A = B'$ b. $B = C'$ c. $D = E'$ d. $C = F'$ e. none of these.

5. The following is an example of the symmetric property of equality: a. $5 = 5$ b. if $a = b$ and $b = c$ then $a = c$, c. if $a + c = b + c$ then $a = b$, d. if $a = b$ then $ac = bc$, e. if $a = b$ then $b = a$.

6. The following is an example of the commutative property of addition: a. $ab = ba$ b. $(3 + x) + 5 = (x + 3) + 5$ c. if $2,3 \in N$ then $5 \in N$ d. $(4 + 7) + 8 = 4 + (7 + 8)$ e. none of these.

7. x to the fifth power means a. $5x$ b. $5^x$ c. x multiplied by itself five times. d. a base numeral x used as a factor five times. e. none of these.

8. Factor $6x^2 - 7x - 24$ a. $(2x - 3)(3x + 8)$ b. prime c. $(2x + 3)(3x - 8)$ d. $(6x - 8)(3x + 3)$ e. none of these.

9. A radical a. always represents an irrational number, b. is always in simplest form, c. cannot contain a fraction, d. cannot be used as a denominator, e. none of these.

10. The statement: If a,b and c $\in$ R such that a > b and b > c then a > c is called a. the addition axiom, b. the multiplication theorem, c. the symmetric axiom, d. the real number axiom, e. none of these.

11. If f is a function then a. f is a relation b. f(x) could be 0 for all x c. f is a set of ordered pairs d. a,b,c are all true e. none of these

12. A linear function a. can have the form $y = mx + b$ b. can have the form $y = ax^2 + bx + c$ c. can have the form $y - y_1 = m(x - x_1)$ d. a and c are both true e. none of these.

13. Find the roots of $4x^2 - 2x = 6$. a. -1 and 3/2 b. $\emptyset$ c. 1 and 3/2 d. -1 and -3/2 e. none of these.

14. Solve the pair of equations $12x - 7y = 2$ and $20x - 9y = 6$. a. (3/4,1) b. (3/4, -7/11) c. {(x,y): x = 3/4, or y = 1} d. {(x,y); x = 3/4, or y = -7/11} e. none of these.

15. The following are identity elements. a. 1 b. 0 c. $\emptyset$ d. all of these e. none of these.

16. The statement: If a,b $\in$ S then ab $\in$ S is a. the commutative axiom for multiplication on S. b. the reflexive axiom for multiplication on S. c. the closure axiom for multiplication on S. d. the inverse for multiplication on S. e. none of these.

17. The inverse of the number a, for the operation multiplication on the real numbers a. is an element a* of R such that $a + a^* = 1$. b. exists for all a $\in$ R. c. is division. d. is an element a* of R such that $a \cdot a^* = 0$. e. is an element a* of R such that $a \cdot a^* = 1$.

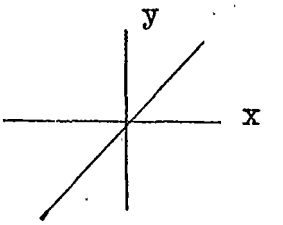
18. The following is an example of a rational number:
a. 8.4/1.5 b. 16 c. $0.24\overline{47}$ d. both a and c
e. a,b and c are all rational

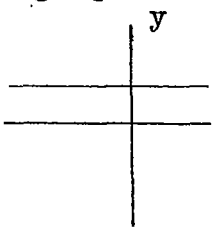
19. The following is an example of a binomial a. (2x)(6y)
b. $3x^2 + 4x + 2$ c. x - y d. both a and c
e. none of these.

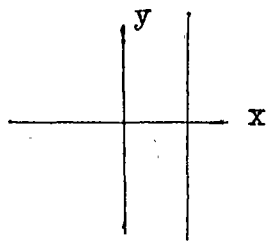
20. The following is an algebraic expression a. $3x^2 + 4y - 9 = 0$ b. $x^2 - y^2 = 4$ c. $\frac{3y}{y+5} = \frac{5}{y-2}$
d. a, b and c are all expressions e. none of these.

21. Change 3x - 5 + y = 2x - 15 + 8 to an equivalent form with 0 as the right member. a. cannot be done. b. x + 18 = 0 c. x - 28 + y = 0 d. c + 2 + y = 0
e. none of these.

22. The following is a graph of a constant function.

a. 


b. 


c. 


d. all of these e. none of these.

23. Write $\frac{\frac{x}{3}}{\frac{x^2}{51}}$ as a reduced simple fraction.

a. $\frac{x^3}{153}$ b. $\frac{51}{3x}$ c. $\frac{3x}{51}$ d. $\frac{17}{x}$ e. none of these

24. The symbol -x a. always represents a negative number. b. sometimes represents a positive number. c. is never used to mean subtract the number x. d. is ambiguous, therefore not used in mathematics. e. none of these.

25. The following are relatively prime numbers a. 5 and 18 b. 15 and 22 c. 6 and 35 d. all of these e. none of these.

26. Find the least common multiple of 36 and 28. a. 2 b. 4 c. 64 d. 1008 e. none of these.

27. A system of equations is said to be inconsistent if a. their graphs intersect. b. their graphs do not intersect. c. if one is linear and one is quadratic. d. if their graphs are the same line. e. none of these.

28. The reciprocal of x a. is the same number as the multiplicative inverse of x. b. does not exist if $x = 0$. c. is x written upside down. d. all of the above e. both a and b are true.

29. The following is an example of a composite number and its prime factors: a. 5;1,5 b. 6;1,2,3,6 c. 8; 2 d. both a and c are correct. e. none of these.

30. Solve: $3x - 2 < 8x + 4$. a. $x > -6/5$ b. $x < 2/3$ c. $x = -1\frac{1}{5}$ d. $x < -2/5$ e. none of these.

FINAL EXAM

55-117 Name_____

DIRECTIONS: Circle the letter indicating the correct answer. All questions have only one correct answer, if more than one of the choices are correct there will be a choice allowing you to pick them.

USE THIS GIVEN DATA FOR PROBLEMS 1 - 5.

Given: $A = \{1,2,3,4\}$ $B = \{5,6,7,8\}$ $C = \{2,4\}$
$D = \{2,4,6,8\}$ $E = \{0,1,3,9,7,5\}$ $F = \{a,b\}$

1. The following are equivalent sets: a. A and D b. B and E c. A and C d. B and F e. none of these.

2. C is a subset of a. A b. D c. A and B d. A and D e. B and D.

3. The following are equal sets. a. A and B b. B and C c. E and F d. C and F e. none of these.

4. The following are disjoint sets. a. A and E b. A and C c. B and C d. B and D e. A and D.

5. Let $U = \{0,1,2,3,4,5,6,7,8,9\}$ then, a. $A = B'$ b. $B = C'$ c. $D = C'$ d. $C = F'$ e. none of these.

6. The following is an example of the symmetric property of equality: a. $5 = 5$ b. if $a = b$ and $b = c$ then $a = c$ c. if $a + c = b + c$ then $a = b$ d. if $a = b$ then $ac = bc$ e. if $a = b$ then $b = a$.

7. The following is an example of the commutative property of addition: a. $ab = ba$ b. $(3 + x) + 5 = (x + 3) + 5$ c. if $2,3 \in N$ then $5 \in N$ d. $(4 + 7) + 8 = 4 + (7 + 8)$ e. none of these.

8. Simplify: $-(2x + 3[4 - (-2)] - 4)$ a. $-2x + 14$ b. $-2x - 16$ c. $2x - 14$ d. $-2x - 14$ e. none of these.

9. x to the fifth power means a. $5x$ b. $5^x$
c. x multiplied by itself five times d. a base numeral x used as a factor five times e. none of these.

10. Factor $6x^2 - 7x - 24$. a. prime b. $(2x - 3)(3x + 8)$ c. $(2x + 3)(3x - 8)$ d. $(6x - 8)(3x + 3)$ e. none of these.

11. $(x^3 + 5x - 2)/(x - 2)$ use synthetic division.

12. Write $(5x - 3)(5x + 3)$ as a difference of squares.
a. $25x^2 + 30x - 9$ b. $25x^2 - 30x + 9$ c. $25x^2 + 9$
d. $9 - 25x^2$ e. none of these.

13. A radical a. always represents an irrational number. b. is always in simplest form. c. cannot contain a fraction. d. cannot be used as a denominator.
e. none of these.

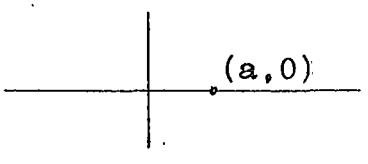
14. $(\sqrt{x} + \sqrt{y})^2 =$ a. $x + y$ b. $\sqrt{x + y}$
c. $x + \sqrt{2xy} + y$ d. $x + 2\sqrt{xy} + y$ e. none of these.

15. The statement: If a and b $\in$ R, then exactly one of the following is true: $a < b$, $a = b$, $a > b$, is called:
a. the transitive axiom b. the trichotomy axiom
c. the completeness axiom d. the remainder theorem
e. none of these.

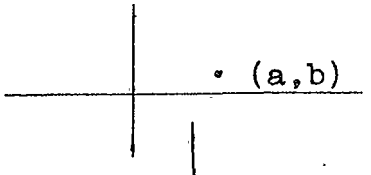
16. The statement: If a,b and c $\in$ R such that $a > b$ and $b > c$ then $a > c$, is called: a. the addition axiom
b. the multiplication theorem c. the symmetric axiom
d. the real number axiom e. none of these.

17. The following is a one dimensional graph.

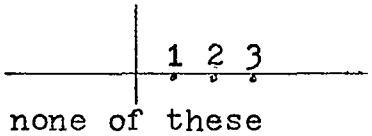
a.



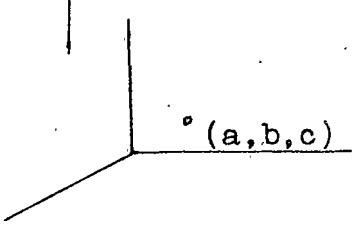

b.




c.




d.




e. none of these

18. The least upper bound for the set $F = \{x: \quad x \in R$ and $-3 < x\}$ a. does not exist b. is an element of the set F c. is not an element of the set F, but does exist. d. is -3 e. none of these.

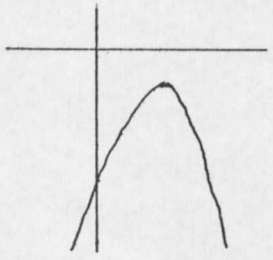
19. If f is a function then a. f is a relation b. f(x) could be 0 for all x c. f is a set of ordered pairs d. a,b,c are all true e. none of these.

20. A linear function a. can have the form $y = mx + b$ b. can have the form $y = ax^2 + bx + c$ c. can have the form $y - y_1 = m(x - x_1)$ d. a and c are both true. e. a, b and c are all true.

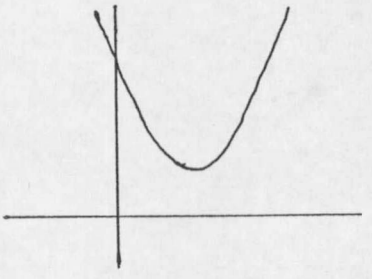
21. Which of the following is an arithmetic progression?
a. 2,4,6,16,32,. . . b. 1, -1, 1, -1, 1, . . .
c. 1,2,4,7,11, . . . d. 5,9,13,17,21, . . .
e. 1/2, 1/6, 1/18, 1/54, 1/162, . . .

22. The graph of the quadratic function $-2x^2 + 12x - 14 = f(x)$ is:

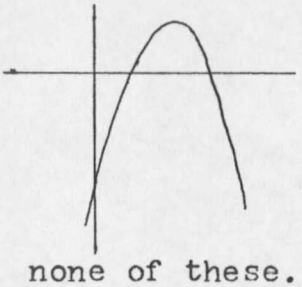
a



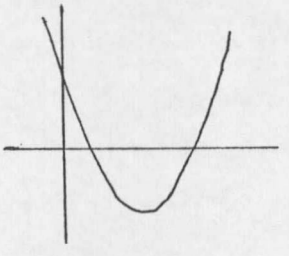
b



c.



d.



e. none of these.

23. Find the roots of $4x^2 - 2x = 7$. a. $\frac{1 \pm \sqrt{29}}{4}$

b. $\frac{-1 \pm \sqrt{29}}{4}$ c. $\frac{1 \pm 3\sqrt{3}}{4}$ d. $\frac{-1 \pm 3\sqrt{3}}{4}$

e. none of these.

24. Solve: $2x^2 + 3x < 14$. a. $\{x: x < 2 \text{ and } x < -7/2\}$ b. $\{x: -7/2 < x < 2\}$ c. $\{x: x > 2 \text{ or } x < -7/2\}$ d. $\{x: -2 < x < 7/2\}$ e. none of these.

25. If y varies directly as x and if $y = 5$ when $x = 2$ find y when $x = 6$. a. 5/2 b. 12/5 c. 15 d. 9 e. none of these.

26. Solve $12x - 7y = 2$ and $20x - 9y = 6$ as a pair of equations. a (3/4,1) b. (3/4, -7/11) c. $\{(x,y): x = 3/4 \text{ or } y = 1\}$ d. $\{(x,y): x = 3/4 \text{ or } y = -7/11\}$ e. none of these.

27. If $A \begin{bmatrix} 1, & 2 \\ -3, & 5 \end{bmatrix}$ then $-A$

a. is not defined b. is such that $A(-A) = 0$

c. is such that $A(-A) = \begin{bmatrix} 1, & 1 \\ 1, & 1 \end{bmatrix}$ d. equals $\begin{bmatrix} -1, & -2 \\ 3, & -5 \end{bmatrix}$

e. none of these

28. $3\begin{bmatrix} x, & y, & 2 \\ 0, & -1, & 3 \end{bmatrix} =$ a. $\begin{vmatrix} 3x, & 3y, & 6 \\ 0, & -3, & 9 \end{vmatrix}$

b. $\begin{vmatrix} 3x, & y, & 2 \\ 0, & -1, & 3 \end{vmatrix}$ c. $\begin{vmatrix} 3x, & 3y, & 2 \\ 3, & -1, & 9 \end{vmatrix}$ d. $\begin{vmatrix} x+3, & y+3, & 5 \\ 3, & 2, & 6 \end{vmatrix}$

e. none of these.

29. The remainder theorem states that when f(x) is divided by (x - r) the remainder a. is less than the divisor. b. can be found by synthetic division. c. can be found by evaluating f(r). d. all of the above. e. none of the above.

30. The following are identity elements. a. 1 b. $\begin{bmatrix} 1, & 0 \\ 0, & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0, & 0 \\ 0, & 0 \end{bmatrix}$ d. all of these

e. none of these.

55-151 FINAL Spring 1972

Name_____

1. Use the definition of the derivative to differentiate $y = 1/x^2$.

MULTIPLE CHOICE:

_____ 2. $\lim_{x\to 0} \frac{\sin x}{x}$ = (a) 0 (b) undefined (c) 1 (d) none of these (e) indeterminant

_____ 3. $\lim_{x\to 0} \frac{2x - 4}{x - 3}$ = (a) undefined (b) indeterminant (c) $-4/3$ (d) $4/3$ (e) none of these

_____ 4. If $y = \cos^2 x$, then dy = (a) $-2 \sin x\, dx$ (b) $2 \cos x\, dx$ (c) $-\sin 2x\, dx$ (d) $-\sin^2 x\, dx$ (e) none of these

_____ 5. $\int (5x - 4)^2 dx$ = (a) $5x - 4 + c$ (b) $\frac{(5x - 4)^3}{15} + c$ (c) $5(5x - 4) + c$ (d) none of these (e) $15(5x - 4)^3 + c$

_____ 6. If $f'(x) > 0$ over the interval (a,b), then (a) f is increasing over (a,b) (b) f is concave upward over (a,b) (c) f is decreasing over (a,b) (d) f is concave downward over (a,b) (e) none of these

_____ 7. The number of points of inflection of the curve of $y = x^5 + 5x^3 + 10x + 1$ is (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

_____ 8. $\int \tan^2 x\, dx$ = (a) $\sec x + k$ (b) $\tan x - x + k$ (c) $-\tan x + x + k$ (d) $\frac{\sec^3 x}{3} - x + k$ (e) $\frac{\tan^3 x}{3} + k$

_____ 9. $\int \frac{\sec^2 x}{1 + \tan x}\, dx$ = (a) $\tan x + \ln \tan x + k$ (b) $\ln(\sec x + \tan x) + k$ (c) $\ln(1 + \tan x) + k$ (d) $-2(1 + \tan x)^{-2} + k$ (e) none of these

____ 10. $\int e^{\cos x} \sin x \, dx =$ (a) $e^{\cos x} + C$ (b) $e^{\sin x} + C$ (c) $-e^{\cos x} + C$ (d) $\frac{e^{\cos x + 1}}{\cos x + 1} + C$ (e) none of these

____ 11. $\int \sin(3 - 2x)dx =$ (a) $\frac{\cos(3 - 2x)}{2} + k$ (b) $-2\cos(3 - 2x) + k$ (c) $\frac{\cos x(3 - 2x)}{2} + k$ (d) $2\cos(3 - 2x) + k$ (e) $-\cos(3 - 2x) + k$

____ 12. $\int x\, e^{-3x}\, dx =$ (a) $\frac{-x^2 e^{-3x}}{-3x^6} + k$ (b) $\frac{-x\, e^{-3x}}{3} - \frac{e}{9} + k$ (c) $-3xe^{-3x} - 9e^{-3x} + k$ (d) $\frac{x\, e^{-3x}}{3} - \frac{e^{-3x}}{9} + K$ (e) none of these

____ 13. $\int_0^3 e^{-2x}\, dx =$ (a) $\frac{-e^{-6}}{2}$ (b) $e^{-8}$ (c) $1 - e^{-8}$ (d) $\frac{1 - e^{-6}}{2}$ (e) none of these

____ 14. If $y = \ln x^3$, then $\frac{dy}{dx} =$ (a) $\frac{3\ln 3}{x}$ (b) $\frac{3}{x}$ (c) $\frac{3}{x\ln 3}$ (d) $3x^2$ (e) $3\ln x^2$

____ 15. If $y = e^{\ln \sqrt{x}}$, then $\frac{dy}{dx} =$ (a) $2\sqrt{x}$ (b) $\frac{\sqrt{x}}{2}$ (c) $\frac{2}{\sqrt{x}}$ (d) $\frac{1}{2\sqrt{x}}$

____ 16. The equation of the line tangent to the curve $y = \sin 2x + 3\cos 2x$ at the point on the curve where $x = 0$ is
(a) $2x - y = -3$ (b) $2x - y = -1$
(c) $6x + y = 1$ (d) $2x + y = 3$
(e) $x - 2y = -3$

_____ 17. A point moves along a line according to the law $s(t) = 3t^2 - 3t^3$. When the acceleration is 0 the velocity is (a) 0 (b) 1 (c) 2 (d) 3 (e) none of these

_____ 18. $\int_2^5 \frac{dx}{x-1} =$ (a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2 (e) 3

_____ 19. $\int_0^{\pi/4} \tan^2 x\, dx =$ (a) $\frac{1}{3}$ (b) $1 - \pi/4$ (c) $3 - \pi/4$ (d) 1 (e) 0

_____ 20. If $f''(x) > 0$ over (a,b), then (a) f is decreasing over (a,b) (b) f is concave upward over (a,b) (c) f is concave downward over (a,b) (d) f is increasing over (a,b) (e) none of these

_____ 21. A local maximum value of the function $y = x^3 - \frac{3x^2}{2} + 2$ is

(a) 0 (b) 1/2 (c) 1 (d) 3/2 (e) 2

_____ 22. A local maximum value of the function $y = x^3 - \frac{3x^2}{2} + 2$ occurs at x =

(a) 0 (b) 1/2 (c) 1 (d) 3/2 (e) 2

_____ 23. If $y = \log_7 \operatorname{cosc}$, then $\frac{dy}{dx} =$ (a) tan x (b) $\frac{-\tan x}{\ln 7}$ (c) -cot x (d) $(-\tan x)(\ln 7)$ (e) none of these

_____ 24. The graph of $y = \frac{x^2 - 5}{x}$ is symmetric

(a) about the origin (b) with respect to the x-axis (c) with respect to the y-axis (d) has no symmetry (e) has symmetry about the y-axis and the origin.

_____ 25. Which one of the following statements is false with regards to the graph of $y = \frac{x - 1}{x^2(x - 2)}$

(a) the x intercept is one (b) there is no y intercept (c) there are two vertical asymptotes (d) there is one horizontal asymptote (e) the graph is symmetric to the y-axis

_____ 26. If $y = 9^{x^2}$, then $\frac{dy}{dx} =$ (a) $x^2 \ln 9$ (b) $9^{x^2}$

(c) $9^{x^2} \ln 9$ (d) $2x \ln 9 + \frac{x^2}{9}$ (e) none of these

27. Find the area of the region bounded by $y = x^2$ and $y = 2x + 3$.

28. If $y = \frac{(4x - x^3)^{1/2}}{(x^2 - 2x)^{1/3}}$, then $\frac{dy}{dx} =$

29. Find the volume of the solid generated when the region bounded by $y = 3x^2$ and $y = 12x$ is rotated about the y-axis.

# APPENDIX F

## RAW DATA

TABLE 11

RAW DATA FOR BASIC ALGEBRA SECTION ONE

| Student Number | ACT Standard Score | | | All Questions | | | Fifteen Questions | | |
|---|---|---|---|---|---|---|---|---|---|
| | IQ | Math | Read | End | Beg | Gain | End | Beg | Gain |
| 710913 | 97 | 16 | 16 | 7 | 7 | 0 | 5 | 7 | -2 |
| 670301 | 115 | 13 | 24 | 8 | 6 | 2 | 4 | 3 | 1 |
| 711105 | 94 | 14 | 18 | 9 | 8 | 1 | 6 | 7 | -1 |
| 711087 | 100 | 15 | 15 | 12 | 13 | -1 | 6 | 10 | -4 |
| 720025 | 92 | 3 | 14 | 13 | 7 | 6 | 10 | 6 | 4 |
| 711174 | 100 | 17 | 14 | 13 | 10 | 3 | 10 | 8 | 2 |
| 720056 | 123 | 21 | 28 | 13 | 9 | 4 | 9 | 6 | 3 |
| 702041 | 95 | 16 | 14 | 14 | 10 | 4 | 9 | 7 | 2 |
| 670012 | 97 | 2 | 13 | 15 | 7 | 8 | 7 | 5 | 2 |
| 620184 | 110 | 15 | 20 | 16 | 7 | 9 | 9 | 6 | 3 |
| 720150 | 107 | 5 | 14 | 17 | 4 | 13 | 10 | 4 | 6 |
| 710980 | 114 | 13 | 21 | 17 | 11 | 6 | 10 | 8 | 2 |
| 711438 | 111 | 21 | 11 | 18 | 11 | 7 | 10 | 7 | 3 |
| 711604 | 103 | 10 | 25 | 19 | 10 | 9 | 12 | 8 | 4 |
| 661137 | 104 | 15 | 19 | 21 | 14 | 7 | 14 | 8 | 6 |
| Total | 1,562 | 196 | 266 | 212 | 134 | 78 | 130 | 100 | 31 |
| $\bar{X}$ | 104.1 | 13.1 | 17.7 | 14.1 | 8.9 | 5.2 | 8.7 | 6.7 | 2.1 |
| s | 9.0 | 5.8 | 5.0 | 4.0 | 2.7 | 3.8 | 2.6 | 1.8 | 2.7 |

TABLE 12

RAW DATA FOR BASIC ALGEBRA SECTION TWO

| Student Number | ACT Standard Score | | | All Questions | | | Fifteen Questions | | |
|---|---|---|---|---|---|---|---|---|---|
| | IQ | Math | Read | End | Beg | Gain | End | Beg | Gain |
| 720555 | 98 | 11 | 15 | 9 | 5 | 4 | 7 | 5 | 2 |
| 711578 | 80 | 7 | 11 | 9 | 8 | 1 | 7 | 6 | 1 |
| 701781 | 92 | 1 | 14 | 10 | 9 | 1 | 6 | 7 | -1 |
| 711904 | 95 | 13 | 11 | 10 | 4 | 6 | 5 | 4 | 1 |
| 700400 | 108 | 16 | 20 | 10 | 6 | 4 | 8 | 5 | 3 |
| 701712 | 108 | 15 | 22 | 11 | 6 | 5 | 8 | 5 | 3 |
| 700884 | 89 | 10 | 13 | 12 | 10 | 2 | 7 | 5 | 2 |
| 710978 | 109 | 14 | 17 | 13 | 9 | 4 | 10 | 7 | 3 |
| 671155 | 116 | 19 | 20 | 15 | 11 | 4 | 10 | 9 | 1 |
| 701457 | 96 | 7 | 16 | 16 | 6 | 10 | 8 | 5 | 3 |
| 711673 | 102 | 14 | 12 | 16 | 9 | 7 | 11 | 6 | 5 |
| 720030 | 122 | 24 | 30 | 16 | 12 | 4 | 13 | 7 | 6 |
| 660404 | 112 | 16 | 11 | 18 | 9 | 9 | 11 | 8 | 3 |
| 691649 | 128 | 22 | 19 | 18 | 9 | 9 | 12 | 8 | 4 |
| 671861 | 132 | 27 | 24 | 19 | 6 | 13 | 12 | 5 | 7 |
| Total | 1,587 | 216 | 255 | 202 | 119 | 83 | 135 | 92 | 43 |
| $\bar{X}$ | 105.8 | 14.4 | 17.0 | 13.5 | 7.9 | 5.5 | 9.0 | 6.1 | 2.9 |
| s | 14.7 | 6.8 | 5.5 | 3.6 | 2.3 | 3.5 | 2.4 | 1.5 | 2.1 |
| 711681 | 113 | 6 | 19 | 13 | 7 | 6 | | | |
| 720032 | 123 | 18 | 27 | 17 | 7 | 10 | | | |
| Total | 1,823 | 240 | 301 | 232 | 133 | 99 | | | |
| $\bar{X}$ | 107.2 | 14.1 | 17.7 | 13.6 | 7.8 | 5.8 | | | |
| s | 14.8 | 6.8 | 5.7 | 3.4 | 2.2 | 3.4 | | | |

TABLE 13

RAW DATA FOR COLLEGE ALGEBRA SECTION ONE

| Student Number | ACT Standard Score | | | All Questions | | | Fifteen Questions | | |
|---|---|---|---|---|---|---|---|---|---|
| | IQ | Math | Read | End | Beg | Gain | End | Beg | Gain |
| 690735 | 121 | 23 | 26 | 15 | 6 | 9 | 8 | 3 | 5 |
| 710401 | 110 | 23 | 18 | 15 | 4 | 11 | 9 | 2 | 7 |
| 701285 | 112 | 20 | 17 | 16 | 12 | 4 | 11 | 8 | 3 |
| 710936 | 127 | 19 | 18 | 16 | 4 | 12 | 11 | 2 | 9 |
| 700693 | 105 | 17 | 19 | 17 | 9 | 8 | 9 | 4 | 5 |
| 710760 | 118 | 22 | 15 | 18 | 7 | 11 | 11 | 4 | 7 |
| 710846 | 123 | 24 | 26 | 18 | 6 | 12 | 11 | 3 | 8 |
| 711280 | 101 | 22 | 17 | 18 | 12 | 6 | 10 | 6 | 4 |
| 711456 | 120 | 20 | 20 | 20 | 9 | 11 | 11 | 4 | 7 |
| 630505 | 115 | 17 | 23 | 20 | 6 | 14 | 10 | 3 | 7 |
| 711153 | 108 | 25 | 16 | 20 | 9 | 11 | 12 | 4 | 8 |
| 690807 | 115 | 18 | 27 | 21 | 6 | 13 | 12 | 3 | 9 |
| 711130 | 127 | 26 | 25 | 21 | 13 | 8 | 12 | 6 | 6 |
| 710392 | 108 | 12 | 21 | 22 | 10 | 12 | 11 | 5 | 6 |
| 711579 | 121 | 33 | 18 | 22 | 13 | 9 | 13 | 6 | 7 |
| Total | 1,731 | 321 | 306 | 279 | 126 | 151 | 161 | 63 | 98 |
| $\bar{X}$ | 115.4 | 21.4 | 20.4 | 18.6 | 8.4 | 10.1 | 10.7 | 4.2 | 6.5 |
| s | 7.9 | 4.8 | 4.0 | 2.4 | 3.1 | 2.7 | 1.3 | 1.7 | 1.7 |
| 661402 | 97 | 21 | 19 | 13 | 12 | 1 | | | |
| 710005 | 121 | 22 | 23 | 15 | 6 | 9 | | | |
| 711272 | 113 | 22 | 26 | 16 | 9 | 6 | | | |
| 700814 | 106 | 17 | 24 | 17 | 6 | 11 | | | |
| 690611 | 100 | 20 | 10 | 18 | 6 | 12 | | | |
| 710966 | 111 | 29 | 24 | 25 | 13 | 12 | | | |
| Total | 2,379 | 452 | 432 | 383 | 178 | 202 | | | |
| $\bar{X}$ | 113.3 | 21.5 | 20.6 | 18.2 | 8.5 | 9.6 | | | |
| s | 8.7 | 4.5 | 4.5 | 3.0 | 3.1 | 3.2 | | | |

TABLE 14

RAW DATA FOR COLLEGE ALGEBRA SECTION TWO

| Student Number | ACT Standard Score | | | All Questions | | | Fifteen Questions | | |
|---|---|---|---|---|---|---|---|---|---|
| | IQ | Math | Read | End | Beg | Gain | End | Beg | Gain |
| 701286 | 104 | 18 | 16 | 9 | 9 | 0 | 4 | 4 | 0 |
| 720012 | 122 | 21 | 24 | 14 | 7 | 7 | 7 | 4 | 3 |
| 711040 | 107 | 17 | 16 | 15 | 9 | 6 | 10 | 4 | 6 |
| 711453 | 107 | 22 | 21 | 16 | 6 | 10 | 8 | 3 | 5 |
| 711147 | 122 | 14 | 26 | 16 | 7 | 9 | 11 | 4 | 7 |
| 711432 | 115 | 21 | 18 | 16 | 9 | 7 | 13 | 5 | 8 |
| 710049 | 83 | 10 | 17 | 16 | 10 | 6 | 7 | 5 | 2 |
| 711128 | 104 | 22 | 19 | 16 | 9 | 7 | 8 | 4 | 4 |
| 720065 | 120 | 23 | 27 | 17 | 9 | 8 | 8 | 5 | 3 |
| 710744 | 128 | 26 | 16 | 18 | 13 | 5 | 9 | 6 | 3 |
| 700947 | 129 | 22 | 23 | 19 | 9 | 10 | 12 | 5 | 7 |
| 711264 | 121 | 19 | 30 | 20 | 12 | 8 | 13 | 6 | 7 |
| 712020 | 136 | 26 | 27 | 20 | 15 | 5 | 11 | 8 | 3 |
| 711875 | 113 | 18 | 20 | 20 | 7 | 13 | 9 | 4 | 5 |
| 720055 | 108 | 20 | 23 | 21 | 10 | 11 | 12 | 5 | 7 |
| Total | 1,719 | 309 | 323 | 253 | 141 | 112 | 142 | 72 | 70 |
| $\bar{X}$ | 114.6 | 20.6 | 21.5 | 16.9 | 9.4 | 7.5 | 9.5 | 4.8 | 4.7 |
| s | 13.1 | 4.8 | 4.6 | 3.0 | 2.4 | 3.1 | 2.6 | 1.2 | 2.3 |
| 710908 | 96 | 27 | 19 | 11 | 16 | -5 | | | |
| 710578 | 110 | 18 | 18 | 14 | 12 | 2 | | | |
| 700903 | 109 | 14 | 13 | 15 | 10 | 5 | | | |
| 711396 | 138 | 27 | 26 | 16 | 12 | 4 | | | |
| 711447 | 129 | 29 | 17 | 17 | 9 | 8 | | | |
| 710046 | 125 | 24 | 25 | 20 | 9 | 11 | | | |
| Total | 2,426 | 448 | 451 | 346 | 209 | 137 | | | |
| $\bar{X}$ | 115.5 | 21.3 | 21.5 | 16.5 | 9.9 | 6.5 | | | |
| s | 13.5 | 5.1 | 4.7 | 3.0 | 2.6 | 4.1 | | | |

TABLE 15

RAW DATA FOR CALCULUS SECTION ONE

| Student Number | IQ | ACT Standard Score | | Test Score |
|---|---|---|---|---|
| | | Math | Read | |
| 701138 | 113 | 23 | 24 | 12 |
| 680546 | 107 | 24 | 11 | 14 |
| 700916 | 115 | 24 | 23 | 14 |
| 701172 | 113 | 27 | 22 | 15 |
| 650541 | 112 | 27 | 25 | 16 |
| 700770 | 111 | 27 | 22 | 16 |
| 700730 | 115 | 24 | 23 | 16 |
| 700837 | 111 | 27 | 30 | 17 |
| 710758 | 118 | 21 | 27 | 17 |
| 711508 | 105 | 31 | 19 | 20 |
| 711027 | 112 | 30 | 22 | 20 |
| 710426 | 123 | 30 | 27 | 21 |
| 700158 | 117 | 23 | 25 | 21 |
| 710858 | 107 | 28 | 22 | 22 |
| 700752 | 115 | 27 | 24 | 22 |
| 720001 | 107 | 27 | 13 | 23 |
| 710859 | 114 | 29 | 27 | 24 |
| 710849 | 113 | 28 | 25 | 24 |
| 710733 | 121 | 30 | 23 | 28 |
| Total | 1,816 | 507 | 409 | 362 |
| $\bar{X}$ | 113.50 | 26.70 | 22.70 | 19.05 |
| SS | 206,524 | 13,671 | 9,663 | |
| s | 5.20 | 7.80 | 4.60 | 4.14 |

TABLE 16

RAW DATA FOR CALCULUS SECTION TWO

| Student Number | IQ | ACT Standard Score | | Test Score |
|---|---|---|---|---|
| | | Math | Read | |
| 711008 | 126 | 24 | 22 | 9 |
| 710975 | 118 | 26 | 27 | 12 |
| 711163 | 108 | 30 | 21 | 14 |
| 710802 | 122 | 25 | 27 | 14 |
| 711134 | 128 | 28 | 23 | 15 |
| 710680 | 108 | 25 | 17 | 15 |
| 710953 | 110 | 28 | 25 | 15 |
| 711461 | 110 | 27 | 26 | 17 |
| 720070 | 117 | 26 | 22 | 17 |
| 710919 | 113 | 23 | 21 | 17 |
| 700873 | 120 | 29 | 27 | 18 |
| 701140 | 100 | 25 | 10 | 18 |
| 720057 | 113 | 26 | 30 | 18 |
| 710657 | 132 | 27 | 20 | 19 |
| 711323 | 112 | 27 | 20 | 19 |
| 711148 | 116 | 29 | 26 | 19 |
| 710857 | 104 | 26 | 27 | 20 |
| 710819 | 113 | 26 | 29 | 20 |
| 710950 | 108 | 29 | 26 | 20 |
| 700664 | 101 | 17 | 19 | 21 |
| 711608 | 121 | 29 | 22 | 22 |
| 710755 | 106 | 31 | 19 | 22 |
| 661139 | 109 | 26 | 24 | 22 |
| 710674 | 117 | 35 | 25 | 22 |
| 711115 | 127 | 26 | 27 | 24 |
| 710011 | 105 | 27 | 12 | 25 |
| 661423 | 107 | 25 | 22 | 26 |
| 710883 | 133 | 28 | 28 | 26 |
| 710821 | 116 | 27 | 25 | 27 |
| Total | 3,094 | 771 | 666 | 531 |
| $\bar{X}$ | 114.50 | 26.50 | 22.90 | 18.31 |
| SS | 356,716 | 20,797 | 15,916 | 10,589 |
| s | 9.10 | 3.26 | 4.70 | 5.57 |

# APPENDIX G

## BIVARIATE FREQUENCY DISTRIBUTIONS

FIGURE 4

BIVARIATE FREQUENCY DISTRIBUTION

IQ scores and Gain Scores
x represents 55-117 scores
o represents 55-097 scores

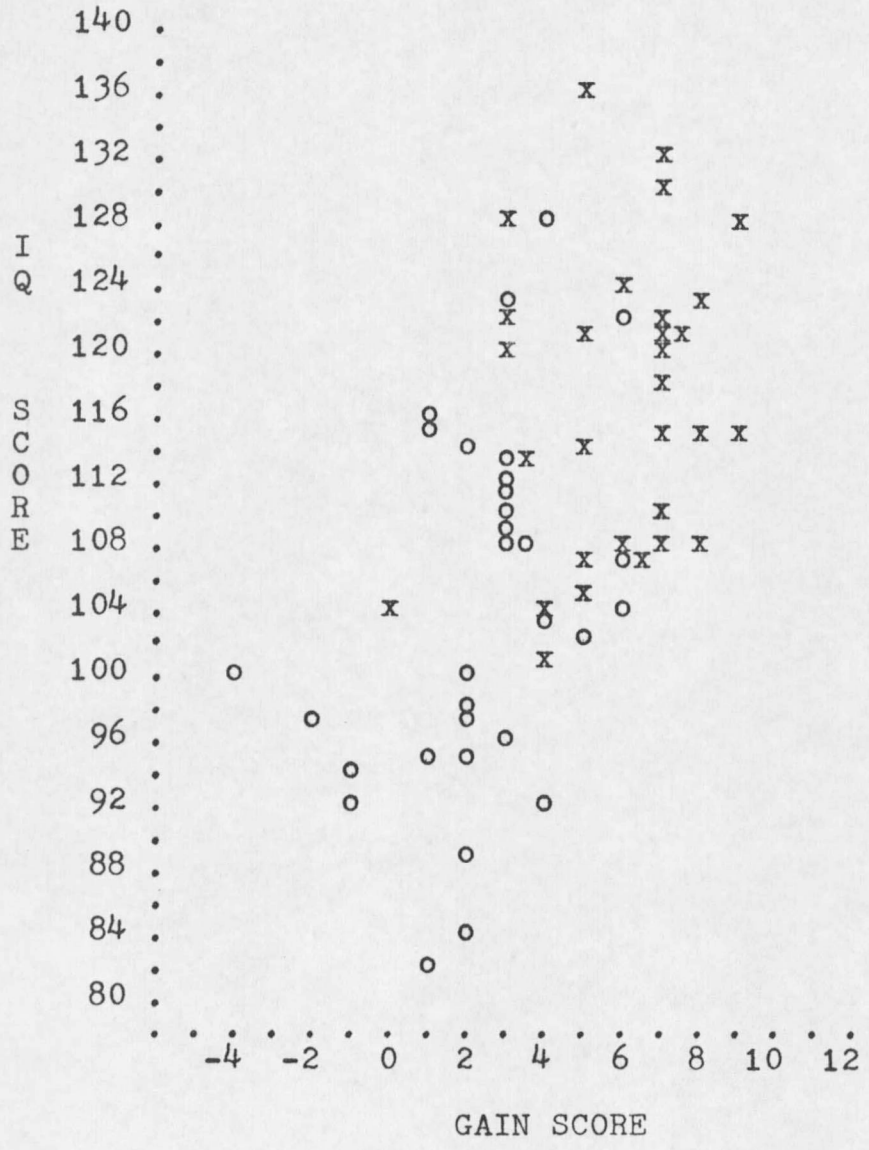


FIGURE 5

BIVARIATE FREQUENCY DISTRIBUTION

ACT Mathematics Scores and Gain Scores
x represents 55-117 scores
o represents 55-097 scores

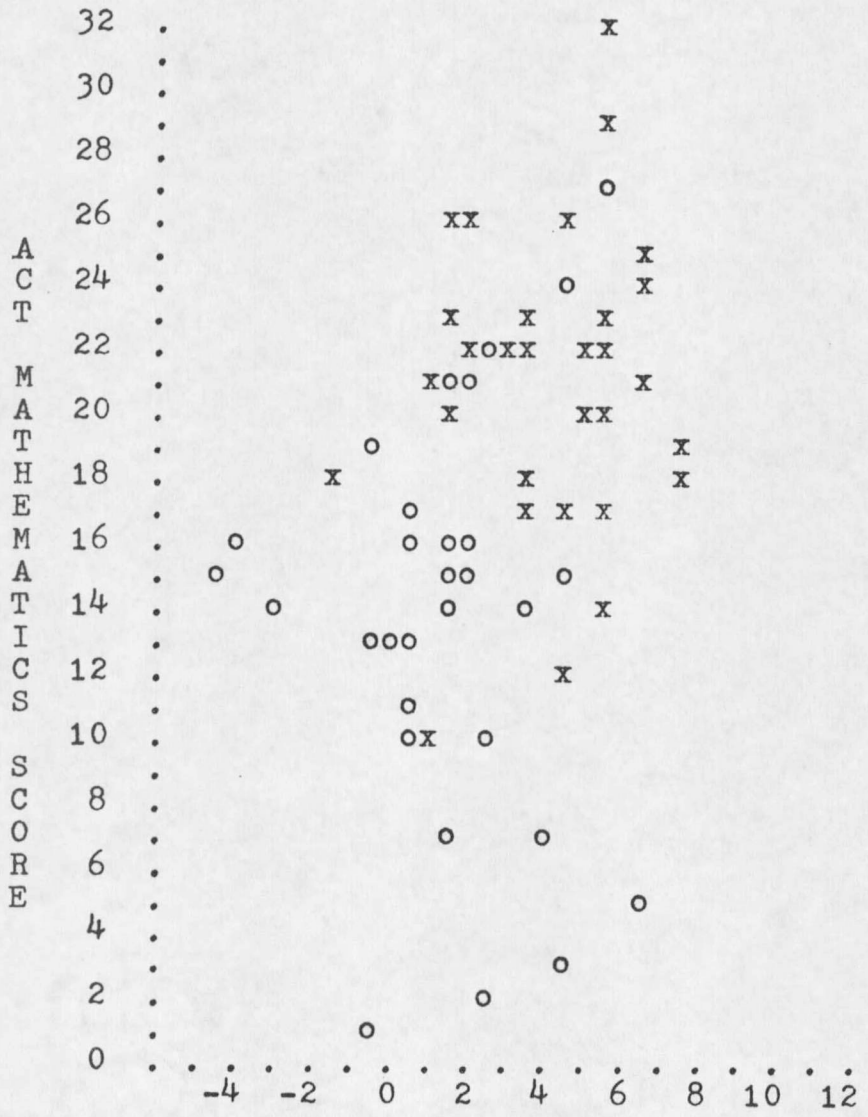
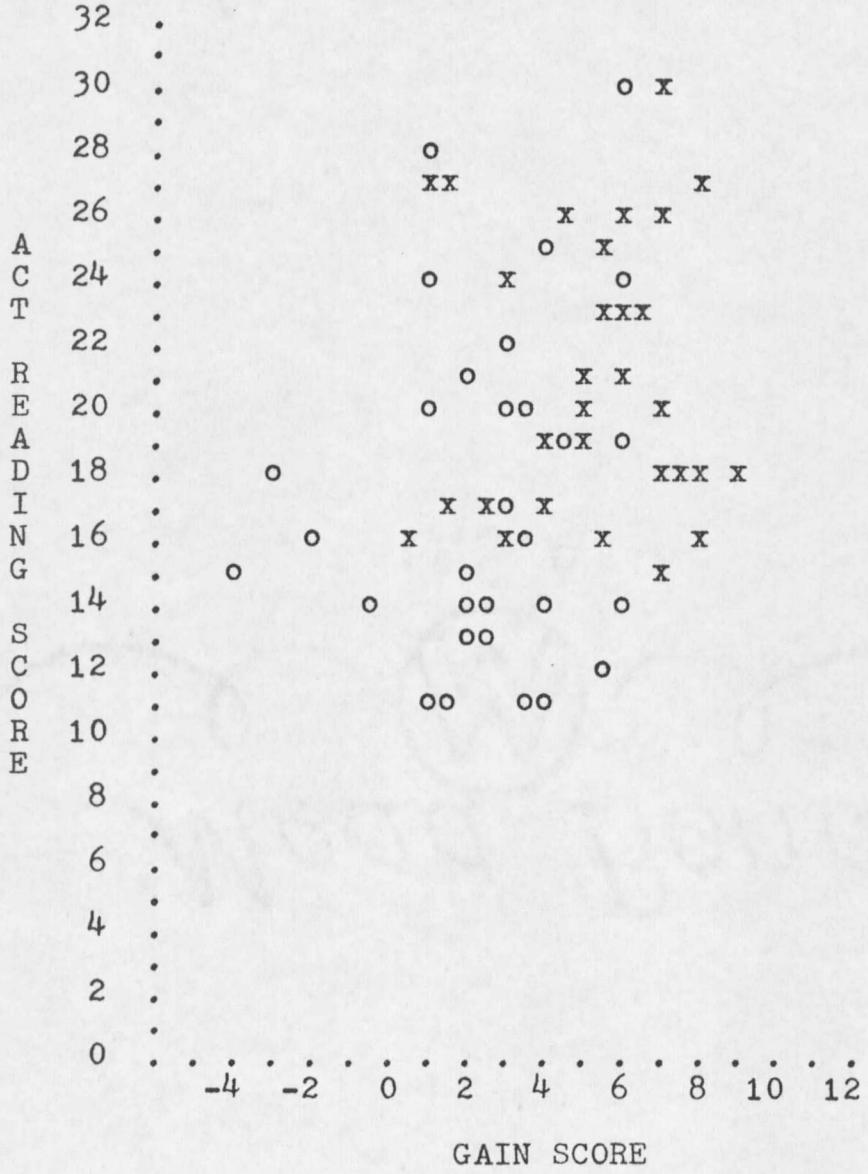


FIGURE 6

BIVARIATE FREQUENCY DISTRIBUTION

ACT Reading Scores and Gain Scores
x represents 55-117 scores
o represents 55-097 scores



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