

USING THE PENALTY IMMERSSED BOUNDARY METHOD TO MODEL THE
INTERACTION BETWEEN FILIFORM HAIRS OF CRICKETS

by

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ABSTRACT

Fluid-structure interactions are important in a wide range of applications and, due to their complexity, need extensive experimental and computational research. One such example comes from crickets, which have evolutionarily developed an excellent micro-air-flow sensory system. Understanding principles of the cricket's micro-air-flow sensor will help design and manufacture artificial sensors. This thesis focuses on improving and validating a Penalty Immersed Boundary (PIB) model of the cricket sensory system, which consists of hundreds of filiform hairs. Previous efforts by others have modeled the filiform hair as a rigid inverted pendulum. Advantages to the PIB approach over previous models include a flexible fluid solver (previous models used an idealized, analytical flow field), the filiform hairs are not required to be completely rigid, and, most importantly, the entire cerci and all the filiform hairs can be modeled. The first goal was to improve the precision and accuracy of modeling a single filiform hair by adjusting model parameters so that the model predictions more accurately fit experimental data. A second goal was to model a portion of a full cercus based on filiform hair data from a real cricket and use the model to determine the interactions occurring between multiple hairs and identify any evolutionary optimization of the cercal system.

CHAPTER 1

INTRODUCTION

Fluid-structure interactions occur when a gas or liquid is in contact with a solid and each can have an effect on the other. There can be both chemical and mechanical interactions between the fluid and solid. However, this research is only concerned with mechanical interactions. While fluid-structure interactions are ubiquitous in the world, they are often computationally complex to model and, as a result, various methods and techniques have been developed to try to capture the physical phenomena. This thesis will focus specifically on a fluid-structure interaction involving a cricket's cercal system, which functions as an excellent micro-air-flow sensory system.

Current research to understand the cricket's micro-air-flow sensor uses a two-pronged approach: collecting experimental data under a wide range of air flow conditions and developing mathematical models of the fluid-structure interaction. The experimental part is focused on applying different wind stimuli to the cricket's filiform hairs to probe the interactions between the air and various filiform hairs (1-11). Much of the simulation work is focused on the development of computationally fast methods to accurately capture the physical movement of filiform hairs (12). Earlier models of filiform hairs only focused on a single, isolated hair. Since the part of the cercus proximal to the body is densely populated with filiform hairs, we are particularly interested in the impact of neighboring hairs on a specific sensory hair. The work presented here is focused on a method to capture the interactions occurring between multiple filiform hairs (13).

There were two main steps in the development of the current model. The first was to improve the precision and accuracy of modeling a single filiform hair. This is accomplished by adjusting model parameters so that the predictions more accurately fit the experimental data. The other goal was to model the response of a collection of hairs covering a significant portion of a cercus. These results could then be directly compared to recently obtained experimental data.

Mechanoreceptor Systems

The cercal system of the common house cricket, *Acheta domestica*, consists of two cone shaped appendages called the cerci, as seen in figure 1. The cercal system has been extensively studied by other groups (14-19). The cerci are approximately 1.0-1.5 cm in length, 400 microns in diameter and found at the abdominal end of the cricket. Both cerci are covered with sensory hairs, called filiform hairs, completely spread out over the surface. Each cercus has approximately 800 filiform hairs on a matured adult cricket. Every filiform hair is roughly 300-1800 microns long and 10 microns in diameter. Also, each filiform hair has a single plane of motion. Given all the filiform hairs on the cerci and their various planes of motion, the micro-air-flow sensor is able to detect air flow from all directions. The comparison of a filiform hair at rest and then set in motion by an oscillating external air flow can be seen in figure 2. Besides the filiform hairs, the cerci are also covered by a dense collection of much smaller bristle hairs. The cercal system plays an important role in the detection of approaching predators, as well as potential mates. Interestingly, all the neural processing of the cercal sensory system output takes place in the terminal ganglion, consisting of about 100 cells, and the output

to the higher centers controlling motor response is carried out by only about 20 interneurons. The cercal sensory system and the underlying neural responses have been studied in detail (3, 4, 20).

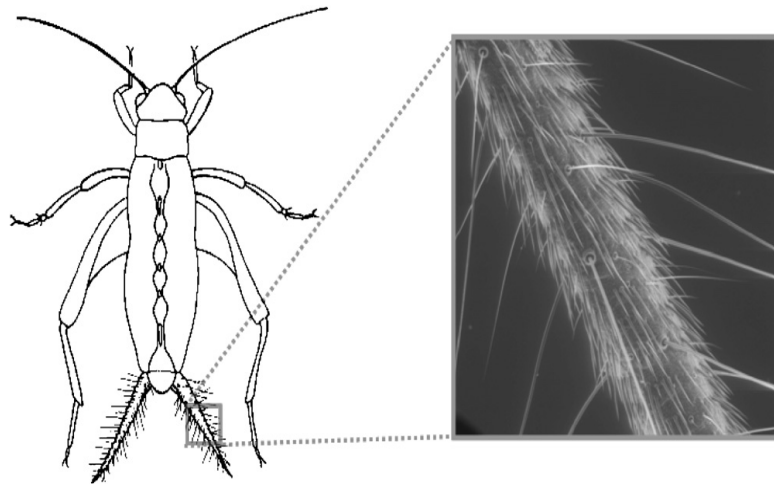


Figure 1. The cricket's two cerci, located at the abdominal end, contain about 800 filiform hairs each (courtesy of J.P. Miller, Center for Computational Biology, Montana State University).

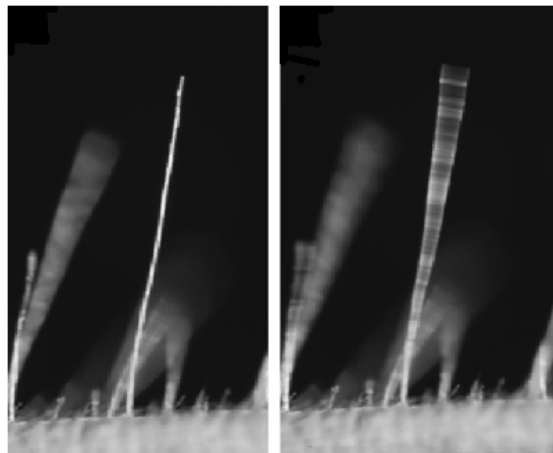


Figure 2. Image of the same filiform hair when it is at rest or in an unstimulated state and also when it is being stimulated by an air flow field (courtesy of J.P. Miller, Center for Computational Biology, Montana State University).

Besides crickets, other arthropods, such as cockroaches, caterpillars, scorpions and spiders, also have similar micro-air-flow sensory hairs. One particular type of spider, *Cupiennius salei*, has been studied much like the cricket, *Acheta domestica* (21-28). Spiders have sensory hairs, called trichobothria, on their walking legs and pedipalps. The trichobothria hairs are structurally different and arranged different spatially than filiform hairs, but they essentially provide the same information. Structurally, trichobothria hair lengths are typically between 200 and 800 μm and have a short perpendicular bend at the top. Trichobothria are also spatially arranged more in patches versus a relatively uniform distribution for filiform hairs. The comparison of a trichobothria hair at rest and one in motion due to an oscillatory external air flow can be seen in figure 3. Besides the physical differences, trichobothria are still used for acts such as mating and prey detection like the cricket.

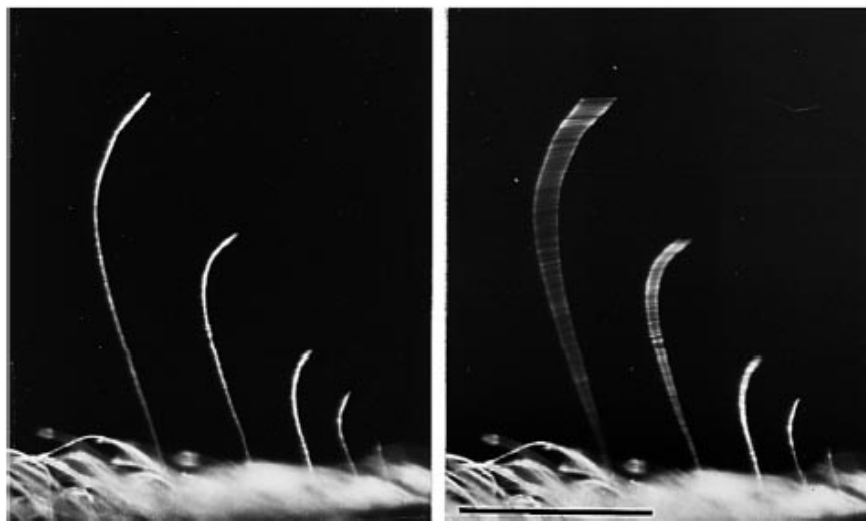


Figure 3. Image of the same trichobothria hairs when they are at rest or in an unstimulated state and also when they are being stimulated by an air flow field (24).

Inverted Pendulum Models

Most previous modeling efforts have treated each filiform hair as a rigid inverted pendulum. This is due to the fact that previous research showed that the filiform hair was nearly rigid and only elastically bent in its hair socket (29, 30). Older generation models only dealt with a single isolated hair (14, 15, 26, 31). This made sense for many reasons, including computational complexity and minimizing the number of model parameters. Once the first generation models adequately predicted the motion of a single filiform hair, there was a desire to also capture the viscous interactions occurring between multiple filiform hairs. Newer models used the foundations and theory learned from the first generation models, and combined them with much improved computational capabilities and mathematical techniques to start looking at filiform hairs having one or more neighbors in the immediate vicinity (12, 32, 33). For the inverted pendulum models, the motion of the hair is typically given by equation 1 with I as the moment of inertia, R as the torsional resistance or damping coefficient, S as the torsional restoring or spring constant, θ as the angle of displacement, and τ as the applied torque from the fluid field.

$$\tau = I\ddot{\theta} + R\dot{\theta} + S\theta \quad (1)$$

The moment of inertia can be calculated for a rod using physiological parameters. The coefficients R and S are used to match the model predictions with experimental data given a similar fluid field. A simple representation of an inverted pendulum hair can be seen in figure 4.

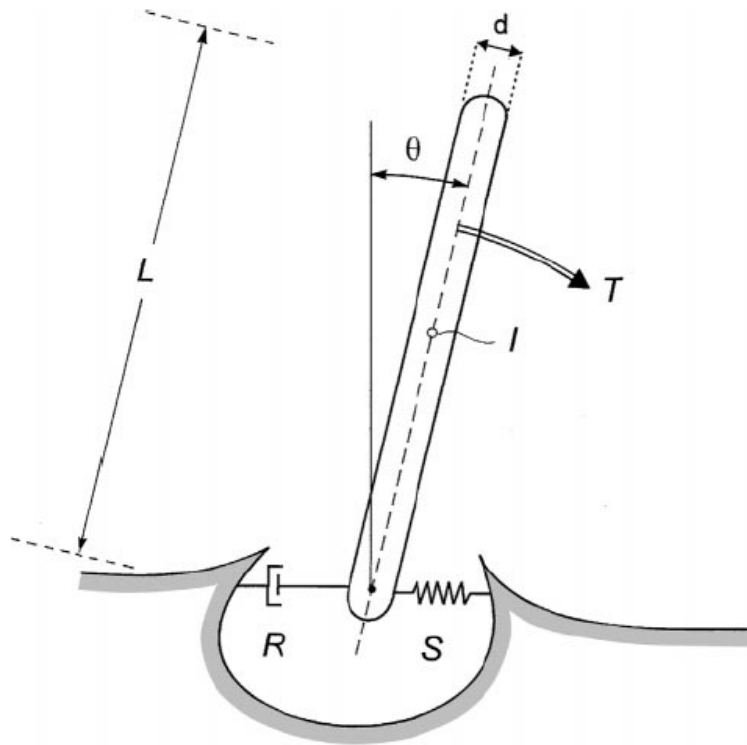


Figure 4. Depiction of an inverted pendulum mechanics (24).

Typically, the bulk fluid field is an oscillating sinusoidal wave with peak velocities anywhere from 1 millimeter per second to 1 meter per second. Previous models used a specific analytical solution to the Navier-Stokes equation in order to calculate the amount of torque applied to each filiform hair. The fluid solution is obtained by solving the Stokes equation, which is used as a low Reynolds number approximation to the Navier-Stokes equation.

Viscous Coupling Models

To more accurately represent the interactions occurring between multiple filiform hairs, two viscous coupling models were previously created. One model was developed

by Bathelier (32). To calculate the near field velocity flow, the theory of Stokes was used. This is valid for a long infinite cylinder and a Reynolds numbers much less than one. The viscous coupling between hairs comes out of the boundary layers created by the theory of Stokes. The far field air flow was taken as the sinusoidal air velocity. This air flow velocity provided the amount of torque applied to a hair. The model then took the torque value and used the hair mechanics as seen in Humphrey (23) to calculate its corresponding angular deflection. In order to calculate the viscous coupling occurring between hairs, the interaction coefficient, κ , is given in equation 2.

$$\kappa = \frac{\theta(single) - \theta(interaction)}{\theta(single)} \quad (2)$$

A κ value of zero implies no interaction has occurred, a value of 1 means that the surrounding hairs have completely damped the motion of the observed hair and a negative value indicates positive reinforcement of motion by surrounding hairs. The viscous coupling in the model between two hairs, while apparent, was very small and can be seen in figure 5.

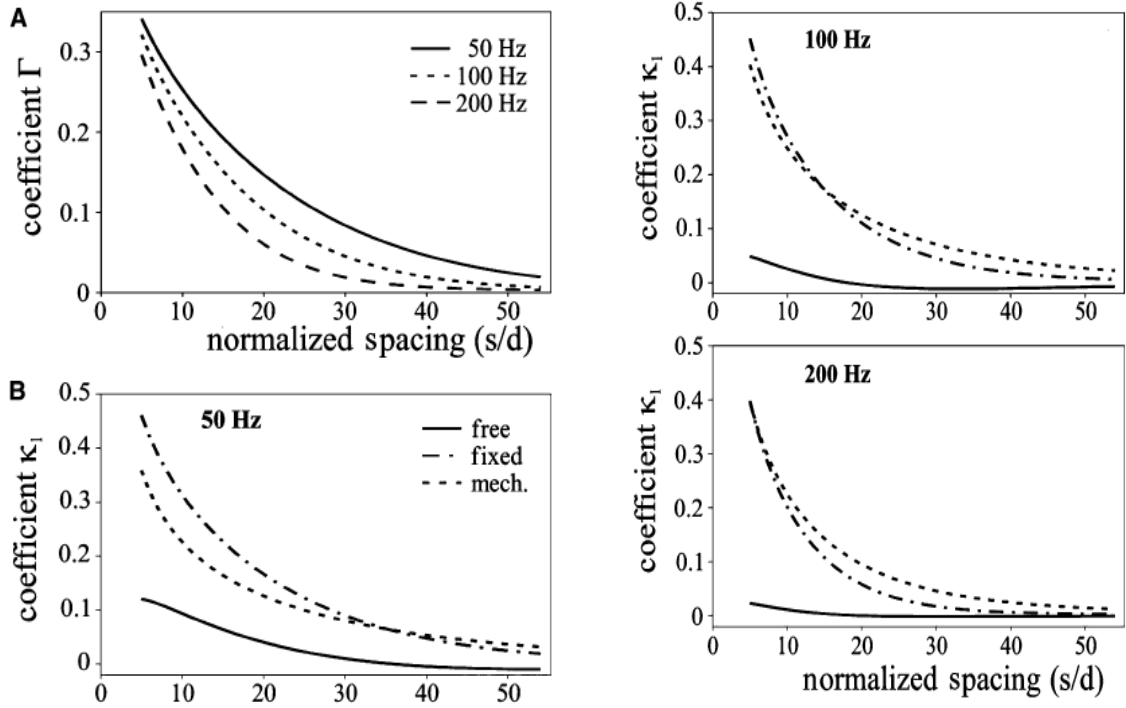


Figure 5. Results from Bathelier of interacting hairs (32). Plot A comes from two immobile cylinders placed at varied normalized distances ($d = 10 \mu\text{m}$) in varied oscillatory fluid flows. The coefficient Γ , is equivalent to κ except that the values of θ are replaced by forces applied to the cylinders from the fluid flow. Three plots of B consist of two free movement cylinders (free), a fixed and free movement cylinder (fixed) and a fixed and mechanically driven cylinder (mech) in 50 Hz, 100 Hz and 200 Hz oscillatory fluid flows, respectively. The varied normalized distances hold their same value.

The other viscous coupling model came from Gedeon and Cummins (12). This model uses a steady Stokes flow approximation for periodically driven far field air flow. The total air velocity, $\mathbf{v}$, is then given by equation 3.

$$\mathbf{v} = \mathbf{u} + \mathbf{u}_b \quad (3)$$

It is comprised of the boundary layer velocity occurring from the no slip boundary condition at the cercal surface, $\mathbf{u}_b$, and a perturbation velocity, $\mathbf{u}$. In order to obtain the perturbation velocity, the Stokes equations, given below, are used.

$$\Delta \mathbf{u} = \nabla p - \mathbf{G} \quad (4)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (5)$$

The sum of applied forces, $\mathbf{G}$, comes from whatever force equations are used to model the hair. In this case, the hair forces are based on the inverted pendulum model and coupling terms. A Stokeslet is used to model a concentrated point force, and a distribution of point forces can represent a one dimensional hair. Since a concentrated point force cannot be modeled discretely, a radial or radially symmetric blob function is used to distribute the force. Conservation of momentum and mass are then used to calculate the motion of the hairs and the surrounding fluid. The viscous coupling effects for multiple hairs can be seen in figure 6. The model has been extended using the unsteady Stokes flow as an approximation to the Navier-Stokes equations. This allowed the model to have any arbitrary periodic driving flow. The main advantage to this model is that it is computationally fast for a relatively small number of hairs (less than 20).

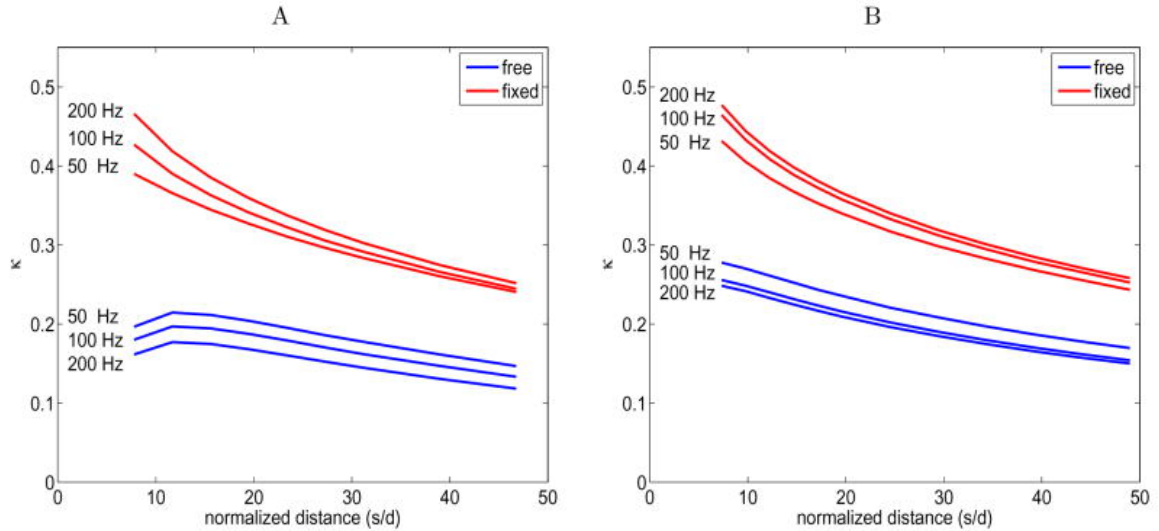


Figure 6. Results from Gedeon and Cummins of two interacting hairs (12). The two hairs were either fixed in position or allowed to freely move in oscillatory fluid flow of 50 Hz, 100 Hz and 200 Hz with a peak velocity at 5 cm/s. Plot A is for two hairs of length 700 μm and plot B is for two hairs of length 1400 μm .

Immersed Boundary Method

The modeling portion of this investigation focuses on using the Penalty Immersed Boundary (PIB) method to model a large segment of the cricket sensory system (13). The PIB method is a modification of the traditional Immersed Boundary (IB) method, which was originally developed by Peskin and others to model blood flow in the heart (34-37). It treats the entire domain as a fluid and includes ‘immersed’ boundaries to capture the physical response of solids mechanically coupled to the fluid. The immersed boundary nodes are typically connected to one another by springs, and these springs give the immersed boundary its mechanical properties. The IB method has also been used to model filament type interactions and blood vessels (38-40). A depiction of an immersed

boundary heart simulation can be seen in figure 7. The main details of the IB method along with its modification as the PIB method are in the following chapter.

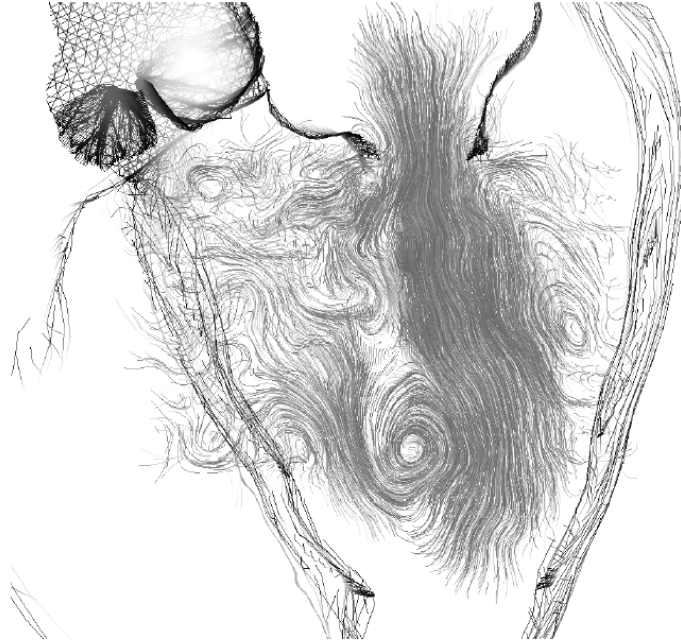


Figure 7. Depiction of blood flow in the heart simulated by the IB method (41).

Background on the PIB Method

Unlike the original IB method, the PIB method can accurately model solids with higher densities than a surrounding fluid, which is the case for many fluid-structure interactions (42-45). It does so by attaching a new mass component structural node to each of the fluid interactive structural nodes. The springs connecting the mass component structural node to the fluid interactive structural node are stiff enough so that these two immersed boundaries work as one cohesive unit. The mass component structural nodes do not interact directly with the fluid domain. With mass component

immersed boundaries, objects such as parachutes and flags have been modeled with the PIB method as seen in figures 8 and 9.

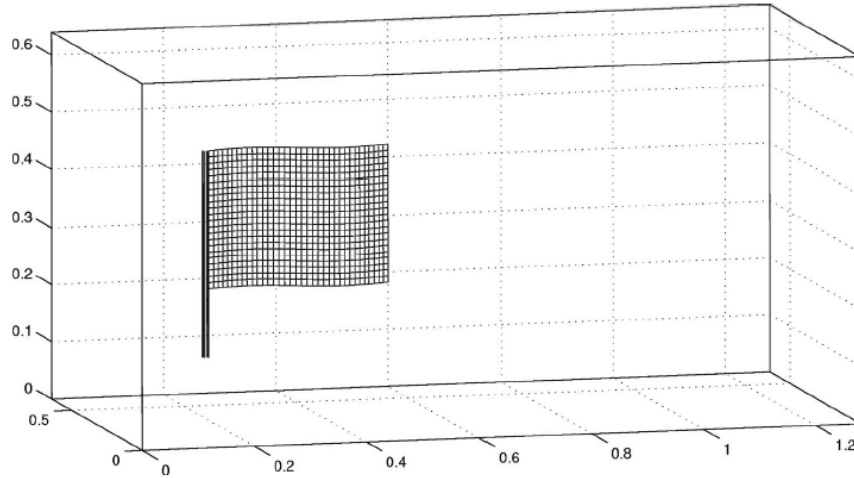


Figure 8. Rectangular fluid domain box with immersed boundary of a flag (43).

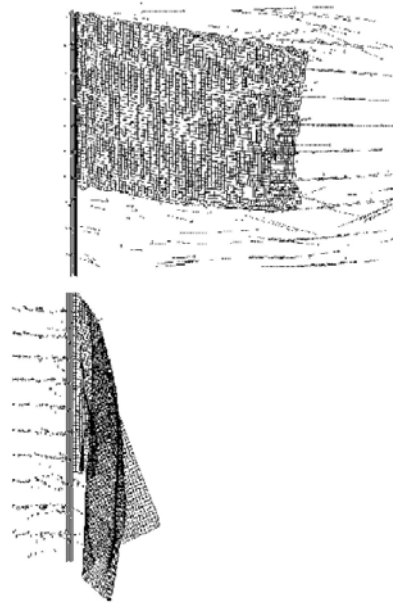


Figure 9. Depiction of the immersed boundary flag receiving both air flow velocity to make it flap as well as insignificant air flow resulting in a limp flag (43).

Related Areas

With the desire to produce micro- and nano-scale mechanical devices comes the need to have a proper understanding of the physical and mechanical phenomena occurring at that scale. Understanding the principles of the cricket's micro-air-flow sensor will help design and manufacture artificial sensors. MEMS (micro-electro-mechanical systems) are in current development and study. Most air flow sensors give velocity information and work via thermal resistors. A MEMS device could add extra information for the fluid field such as the type of input stimuli and where it is originating, if those properties are desired. Hair-to-hair interactions are currently under intense investigation for possible key roles they may play (46, 47). Particle image velocity data of fixed artificial rods can be seen in the figures 10 and 11 from those studies.

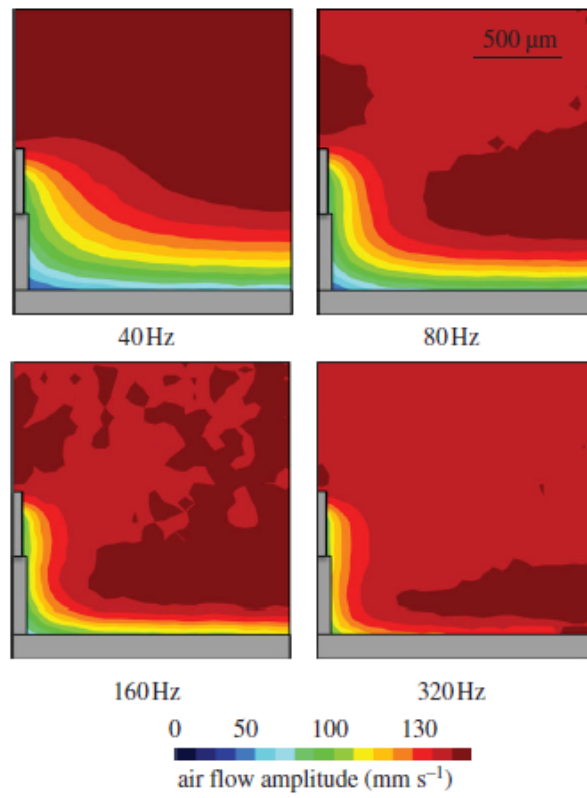


Figure 10. Particle image velocity plot of a single fixed artificial rod in varying oscillatory air flow fields (47).

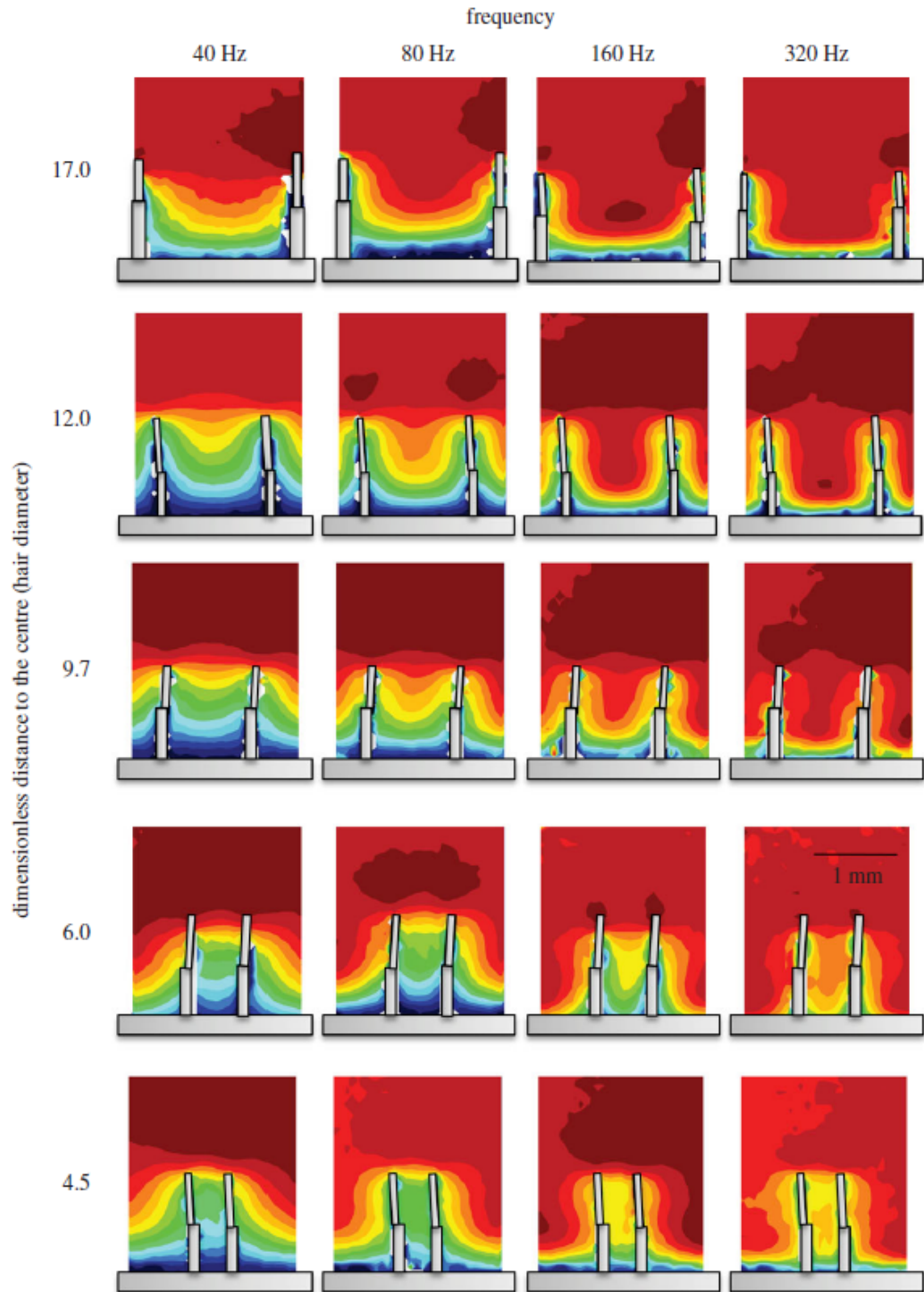


Figure 11. Particle image velocity plots of two fixed artificial rods in varying oscillatory air flow fields and spaced at varying distances (47). The dimensionless distance is given by the distance between the two artificial rods over their diameters.

CHAPTER 2

PENALTY IMMERSED BOUNDARY METHOD

There are several advantages to the PIB method over previous models for modeling the cricket cercus. The first advantage is a flexible, numerical fluid solver where previous models used an idealized, analytical flow field. Second, this new model allows filiform hairs that are not completely rigid. Most importantly, the computational scalability of the model allows nearly the entire cerci and a large portion of the filiform hairs to be modeled without prohibitive computational costs.

PIB Method Equations

The mathematical formulation of the PIB method consists of eight equations. The first two equations below are the Navier-Stokes and continuity equations for a three-dimensional, Newtonian, incompressible fluid where p and $\mathbf{u}$ are the pressure of the fluid and the velocity, respectively.

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \Delta \mathbf{u} + \mathbf{f} \quad (6)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (7)$$

The body force, $\mathbf{f}$, exerted by the hairs on the fluid is calculated from the force density, $\mathbf{F}$, using a three dimensional Dirac delta function, which will be discussed later on. It should be noted that the fluid is calculated under Eulerian coordinates and the immersed

boundary, or the filiform hairs, are in Lagrangian coordinates. $\mathbf{X}(r,s,t)$ represents the location in time for the fluid interactive structural nodes.

$$\mathbf{f}(\mathbf{x}, t) = \iint \mathbf{F}(r, s, t) \delta(\mathbf{x} - \mathbf{X}(r, s, t)) dr ds \quad (8)$$

The force density, $\mathbf{F}(r,s,t)$, can be split into two forces, $\mathbf{F}_E$ and $\mathbf{F}_K$, to represent the elastic stress and inertial gravitational force, respectively. The elastic energy related to the elastic stress force comprises two terms that include stretching or compression, c_s , and bending, c_b , of the solid. $\mathbf{Y}(r,s,t)$ represents the location in time for the mass structural nodes, and K is a spring coefficient linking the mass structural nodes with the fluid interactive structural nodes. While K should technically be infinite to force both types of structural nodes to act as one cohesive unit, computationally it is not practical to set K too large, and it has to only be set large enough to maintain the two node types in close proximity (1000 g/s^2).

$$\mathbf{F}_E = -\frac{\partial E}{\partial \mathbf{X}} \quad (9)$$

$$E(\mathbf{X}) = \frac{1}{2} c_s \int \left(\left| \frac{\partial \mathbf{X}}{\partial s} \right| - 1 \right)^2 ds + \frac{1}{2} c_b \int \left(\frac{\partial^2 \mathbf{X}}{\partial s^2} \right)^2 ds \quad (10)$$

$$M(r, s) \frac{\partial^2 \mathbf{Y}}{\partial t^2} = -\mathbf{F}_K(r, s, t) - M(r, s) \mathbf{g} \quad (11)$$

$$\mathbf{F}_K = K[\mathbf{Y}(r, s, t) - \mathbf{X}(r, s, t)] \quad (12)$$

The final equation matches the rate of displacement of the immersed solid with the fluid velocity via a discrete Dirac delta function.

$$\frac{\partial \mathbf{X}(r, s, t)}{\partial t} = \int \mathbf{u}(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{X}(r, s, t)) d\mathbf{x} \quad (13)$$

The discrete Dirac delta function, given in equation 14, allows the Lagrangian force density of the immersed boundaries to be transformed to the Eulerian fluid grid and vice versa. Ideally, the fluid would be represented as a complete continuum, but the discrete nature of the fluid grid cannot be completely masked using the discrete Dirac delta function. However, the delta function was derived such that it could be as invisible as possible and, as a result, minimizes the grid effects. At the same time, it was derived such that each fluid interactive structural node would not have to interact with every fluid grid node, only those in the immediate vicinity. Otherwise, exponential functions would be used to link every fluid interactive structural node to every fluid grid node.

$$\delta(x) = \begin{cases} 0, & x \leq -2 \\ \frac{1}{8} \left(5 + 2x - \sqrt{-7 - 12x - 4x^2} \right), & -2 \leq x \leq -1 \\ \frac{1}{8} \left(3 + 2x + \sqrt{1 - 4x - 4x^2} \right), & -1 \leq x \leq 0 \\ \frac{1}{8} \left(3 - 2x + \sqrt{1 + 4x - 4x^2} \right), & 0 \leq x \leq 1 \\ \frac{1}{8} \left(5 - 2x - \sqrt{-7 + 12x - 4x^2} \right), & 1 \leq x \leq 2 \\ 0, & 2 \leq x \end{cases} \quad (14)$$

The known parameters used in the simulations are given in table 1. The filiform hair mass depends on its length. It is given the value 1.0×10^{-6} grams/cm unless otherwise noted.

Table 1. Parameters and associated values.

Parameter	Value
Filiform Hair Mass	1.0×10^{-6} g/cm
Air Viscosity, μ	2.0×10^{-4} g/(cm·s)
Air Density, ρ	1.0×10^{-3} g/cm ³
Gravity	980 cm/s ²

When it comes to modeling the cercus and filiform hairs, the only unknown parameter in the set of model equations is the value for the bending coefficient, c_b , found in equation 10. The bending coefficient gives each filiform hair its structural integrity and is used to match the response of experimental hairs to model hairs.

To accomplish finding the bending coefficient, the experimental and model response of the hairs is typically quantified using the radial gain. The radial gain is defined as the deflection of the hair (in radians) divided by the oscillatory air velocities amplitude or its maximum value. A typical gain value for a filiform hair is roughly 1.0 θ /m/s, but this value changes depending on frequency and hair length. We expect that the gain is maximized at a certain frequency and is also a function of the hair length. However, when the bending coefficient in the model is set too small, the radial gain is a purely decreasing function of the frequency for all hair lengths. When the bending

coefficient is set too large, radial gain is almost independent of frequency and its amplitude is very small. When the bending coefficient is properly tuned, the radial gain has a unique maximum as a function of frequency and hair length. Each filiform hair is approximated using a specified number of structural nodes depending on the length of the hair. For a given fluid grid, each hair needs approximately nine structural nodes per fluid grid point interval. Also, for a given fluid grid, about sixteen nodes at the bottom of each filiform hair are fixed in place to model the attachment of the hair to the cercus. The bending coefficient for the next about sixteen nodes up the filiform hair are a constant, α , which depends on the angle of orientation, $\hat{\theta}$, of the filiform hair with respect to the fluid grid as well as the hair length, l . The rest of the filiform hair nodes are given values based on a normalized inverse of a decaying exponential multiplied by a constant, β , that also depends on the angle of orientation of the filiform hair with respect to the fluid grid as well as its length. Coefficients for filiform hair lengths of 700 μm to 1800 μm (in intervals of 100 μm) as well as every angle around the cercus in intervals of five degrees can be seen in tables 2 and 3 located in the following chapter.

$$c_b = \begin{cases} \alpha(\hat{\theta}, l) \\ \alpha(\hat{\theta}, l) + \beta(\hat{\theta}, l) \left(1 - e^{-\frac{x}{l}}\right) \end{cases} \quad (15)$$

We should note that the choice of bending coefficients depends on the fluid grid chosen for the computation. Since the fluid grid and filiform hair node forces are equated with a discrete Dirac delta function, the effective “volume” of the filiform hair decreases as the fluid grid is refined because the volume of the discrete Dirac delta function decreases.

The consequence of this is that the appropriate value for the bending coefficient, c_b , depends on the refinement of the fluid grid. Typically, we begin by selecting a properly resolved fluid grid, and then we determine the appropriate value for c_b .

General Solving of PIB Method

The numerical implementation of the PIB method has been described in detail in (43) and is summarized here. Every time step is split into two half steps and the overall process has formal second-order accuracy. First, the locations of the two immersed boundaries need to be updated to a half of a time step. The fluid interactive structural node locations, $\mathbf{X}(r,s,t)$, are updated using equation 13 and the mass component structural node locations, $\mathbf{Y}(r,s,t)$, are updated according to their previous velocity. Second, the positions of both structural nodes at the half time step are used to calculate the Lagrangian force density, which is the sum of $\mathbf{F}_E$ and $\mathbf{F}_K$. Third, the Lagrangian force density is transformed into the Eulerian force density and applied to the fluid with equation 8. Fourth, for the half time step, solve for the fluid velocity and pressure using equations 6 and 7 and solve for the velocity of mass component structural nodes using equation 11. Fifth, the two immersed boundary locations need to be updated to the full time step. Again, this is done by the fluid interactive structural nodes being updated using equation 13, and the mass component structural nodes updated according to their previous velocity (this time it's from the half time step). Finally, the fluid variables are updated to the full time step using equations 6 and 7, as well as the velocity of mass component structural nodes using equation 11. For the boundary conditions, each surface

of the rectangular fluid domain is set to periodic boundary conditions with one of the surfaces specifying the inflow velocity at each moment in time.

All computations were run on a Dell Precision T5400 with Intel Xeon n-series processor and 16 Gb of RAM. The FORTRAN 90 code has been modified from the group's previous efforts (13). The dimensions of the fluid volume being modeled are 0.6 cm by 0.3 cm by 0.3 cm as seen in figure 12. The fluid grid consists of 128 by 64 by 64 regularly spaced fluid nodes. The immersed nodes of the cercus surface are sufficiently dense to ensure that fluid does not leak through the cercus. In order to measure the total angle of deflection for a hair, the top node of each hair is tracked.

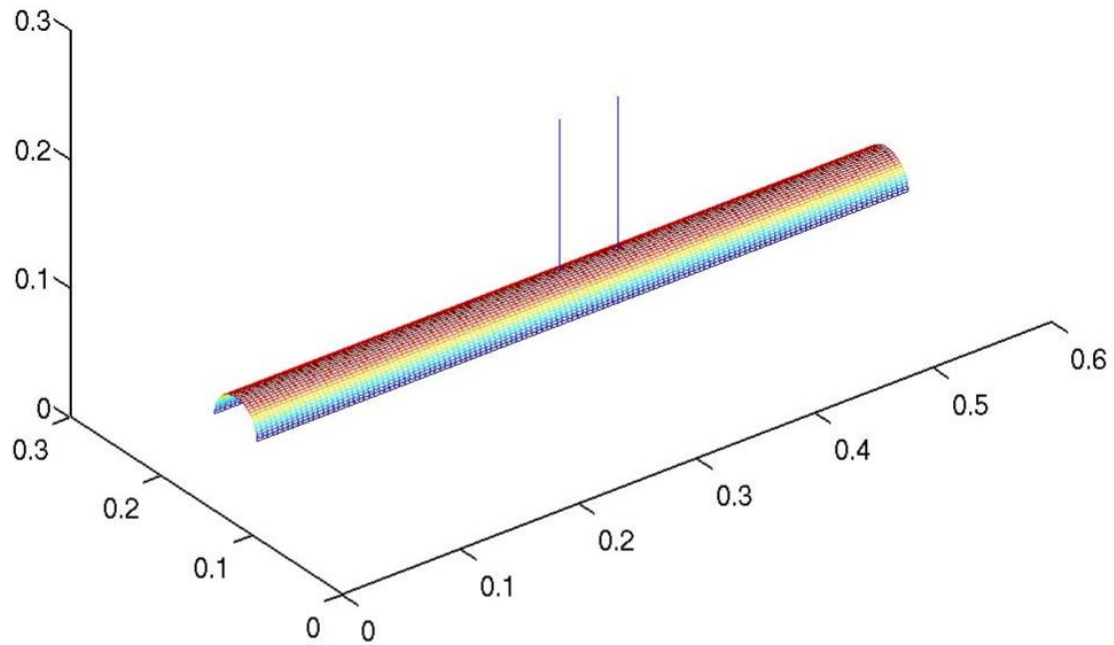


Figure 12. Half of cercus modeled with two filiform hairs on the top of a cercus with hair heights of 1000 μm each. The length scale is in centimeters.

CHAPTER 3

RESULTS AND DISCUSSION

Model Coefficients

The first task to accomplish in developing the PIB model was finding the coefficients of α and β in equation 15. Since the discrete Dirac delta function cannot completely mask the Eulerian fluid grid, there are minute differences in force densities applied depending on how the hair is angled through the fluid grid. The coefficients for α and β were found at hair length at intervals of 100 μm and angled through the fluid grid every five degrees from a perpendicular point with respect to the top of the cercus. The coefficients were found to try and best match the experimentally measured deflection of a single, isolated filiform hair. Initially, a search for a clear pattern of the coefficient values depending on both hair length and hair angle was pursued. However, that turned out not to be the case as seen in tables 2 and 3. Currently, some coefficient values at certain hair lengths and angles are not accurate enough to report. These are given by X's in tables 2 and 3. The coefficient values left blank were simply not attempted. It should be noted that even though only the angle values from 0 to 45 degrees are given, due to the symmetry of the cercus within the fluid grid, these values represent the entire 360 degrees around the cercus.

Table 2. $\beta(\hat{\theta}, l)$ coefficients for every 5 degrees and 100 μm .

cm	Degrees									
	0	5	10	15	20	25	30	35	40	45
0.07	0.0004	0.0007	0.00085	0.0012	X	X	0.002	0.0019	0.0015	0.0016
0.08	0.0005	0.0016	0.0016	0.0017	X	0.0022	X	0.0028	0.0025	0.0025
0.09	0.0006	0.0028	0.003	0.003	0.00375	0.0032	X	0.0034	0.0034	0.003
0.1	0.001	0.004	0.0047	0.003	0.00575	0.00525	X	0.0075	0.0055	0.0055
0.11	0.0012	0.0056	0.0055	0.0051	0.00833	0.007	0.008	0.0085	0.0083	0.012
0.12	0.0015	0.0085	0.0085	0.008	0.012	0.0115	0.011	0.013	0.012	0.015
0.13	0.0025	0.014	0.011	0.012	0.016	0.016	0.016	0.022	0.015	0.017
0.14	0.003	0.02	0.017	0.012	0.0225	0.022	0.023	0.023	0.023	0.034
0.15	0.004	0.0205	0.023	0.024	0.03	0.035	0.031	0.031	0.031	0.031
0.16	0.005	0.027	0.03	0.032	0.044	0.044	0.04	0.044	0.047	0.055
0.17	0.025	0.052	0.04	0.045	0.05	0.05	0.0525	0.0515	0.053	
0.18	0.045	0.0675	0.051	0.055	0.065					

Table 3. $\alpha(\hat{\theta}, l)$ coefficients for every 5 degrees and 100 μm .

cm	Degrees									
	0	5	10	15	20	25	30	35	40	45
0.07	0.0023	0.0016	0.0012	0.00075	X	X	0.008	0.0011	0.0016	0.002
0.08	0.0044	0.0022	0.00213	0.0018	X	0.0032	X	0.0033	0.0034	0.0034
0.09	0.0085	0.0048	0.0033	0.0033	0.0075	0.0064	X	0.0071	0.007	0.0075
0.1	0.013	0.0079	0.0053	0.006	0.0115	0.0105	X	0.0078	0.0105	0.011
0.11	0.0203	0.0113	0.011	0.0103	0.0166	0.018	0.016	0.017	0.0166	0.01
0.12	0.029	0.016	0.01733	0.016	0.023	0.023	0.025	0.024	0.024	0.018
0.13	0.0405	0.018	0.026	0.024	0.033	0.032	0.035	0.022	0.037	0.034
0.14	0.0555	0.022	0.034	0.037	0.045	0.044	0.0475	0.046	0.046	0.023
0.15	0.0743	0.041	0.046	0.048	0.058	0.048	0.062	0.062	0.062	0.062
0.16	0.097	0.054	0.06	0.064	0.065	0.069	0.08	0.078	0.068	0.048
0.17	0.098	0.045	0.08	0.0805	0.1	0.1	0.102	0.103	0.106	
0.18	0.093	0.052	0.102	0.105	0.126					

A model filiform hair with a height of 700 μm was compared to experimental data from Cummins and Kumagai et al. with filiform hair heights at 500 μm and 600 μm , respectively, as seen in figure 13 (15, 48). The experimental results probably come from

looking at a specific hair on a cercus with a full set of hairs. It should be noted that the model filiform deflection peak occurs at a slightly lower frequency as well as a slightly lower magnitude compared to the experiment results. This causes the general shape of the inverted parabolic response to be flatter.

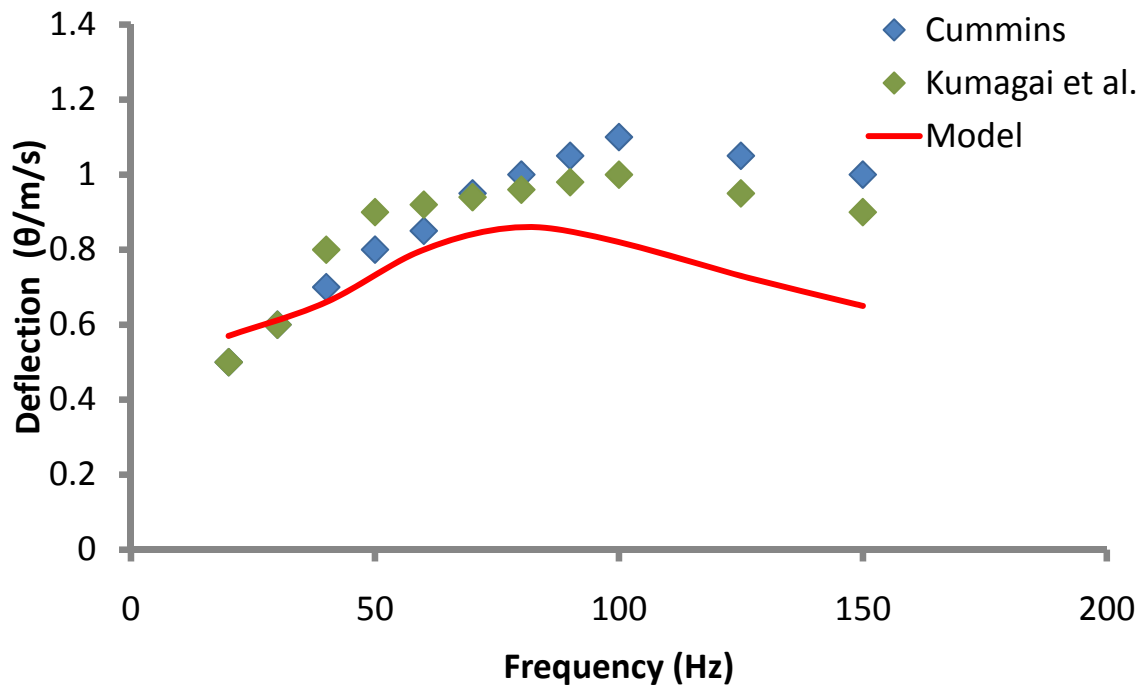


Figure 13. Normalized deflection of a single filiform hair with a height of 700 μm from the model compared with two experimental data sets with hair heights at 500 μm and 600 μm , respectively (15, 48).

A model filiform hair with a height of 1000 μm was also compared to experimental data from Cummins and Kumagai et al. with filiform hair heights at 900 μm each, as seen in figure 14 (15, 48). Similar to the previous figure, the deflection peak is not as high as those seen experimentally. Also, the general deflection profile of the inverted parabolic response is fairly flat.

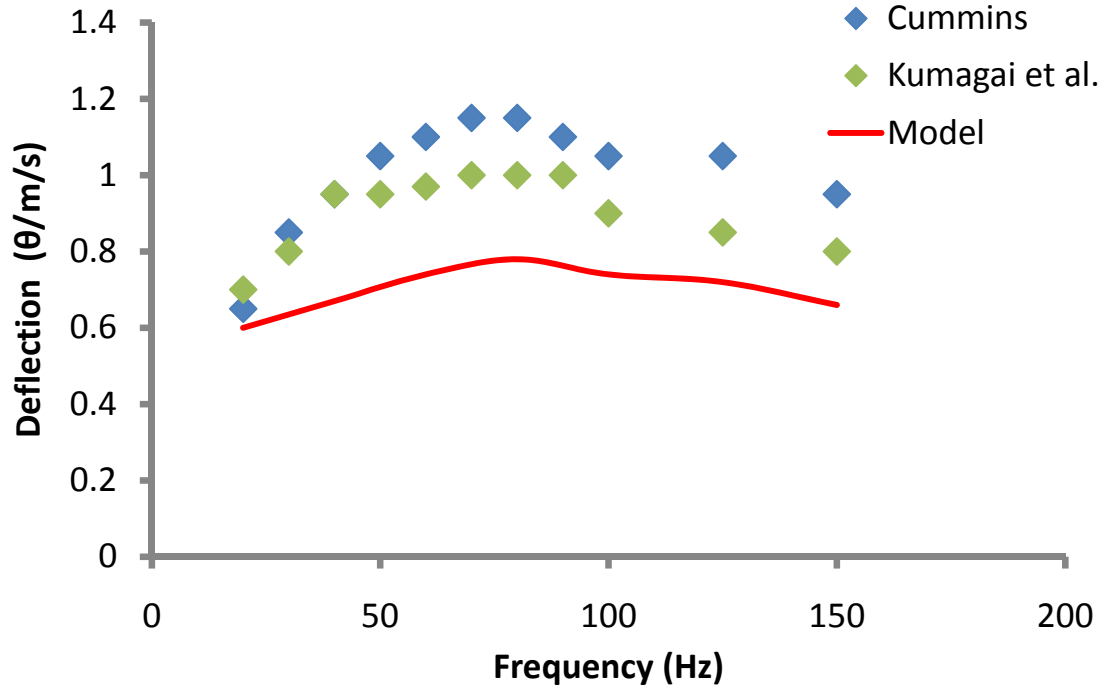


Figure 14. Normalized deflection of a single filiform hair with a height of $1000\ \mu\text{m}$ from the model compared with two experimental data sets with hair heights at both $900\ \mu\text{m}$ (15, 48).

Pair of Interacting Filiform Hairs

Once the optimal model parameters were found, we investigated the interaction between two hairs using the PIB model. To compare the original deflection (i.e., a single hair without neighbors) to the deflection with neighbor interaction, the value of κ is calculated for two filiform hairs using equation 2. In the model, both filiform hairs were allowed to move freely. As seen in figure 15, κ converges towards 0.5 as the distance between the filiform hairs is reduced. Alternatively, as the distance between the filiform hairs is increased, the interaction, as seen by the κ values, decreases. At higher frequencies, κ decays more rapidly with the distance between the hairs. Comparing these

results to similar data of the two viscous coupling models of Gedeon and Cummins and Bathelier shows their models have much less variance in κ values over different frequencies. There is a flatter response in κ values over increased distance separation.

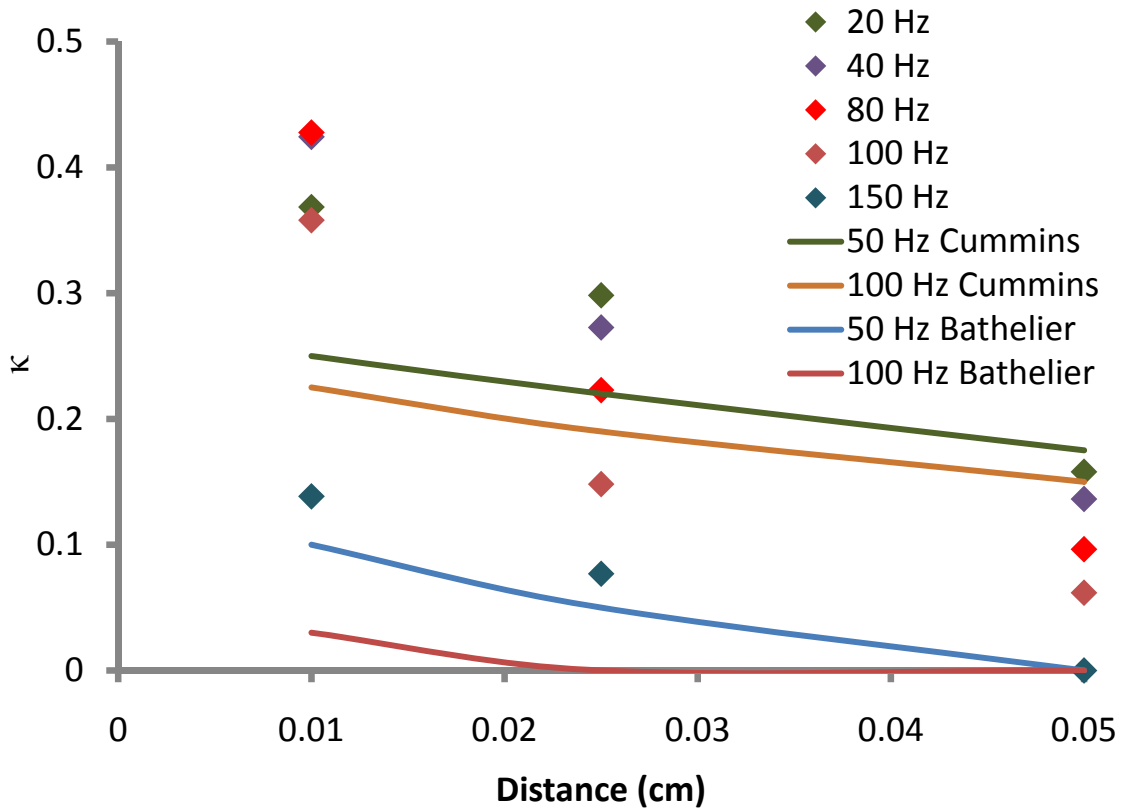


Figure 15. The interaction between two filiform hairs with the setup seen in figure 12 and the distance being between the filiform hairs. Both filiform hairs were allowed to move and their normalized deflection was averaged when κ values were calculated. Similar model results can be compared (32, 48).

Three Partial Cerci

Experimental trials, currently unpublished, were run by Andrew Larson from the Center for Computational Biology at Montana State University to obtain the deflection of

a specific filiform hair to various air flow stimuli both in the presence and absence of neighboring filiform hairs. The absence of neighboring filiform hairs occurs for a full millimeter around the specific hair of interest as the rest of the hairs remained intact. Using video and photographs of those particular experimental cerci, the physical representation of filiform hair lengths, locations and angle of orientation were replicated as an input into the PIB model. Three sets of trials are given below in figures 16 to 19, figures 20 to 23 and figures 24 to 27. Each trail set has a picture taken before the experimental trial of the entire cercus and a modeled depiction in the first two figures. Only the neighboring filiform hairs in the photo plane of the particular filiform hair of interest could be accurately represented. Neighboring filiform hairs wrapping around the cercus at different angles of orientation could not be viewed due to being out of focus. If, however, a data set was rerun, and afterwards the cercal surface was viewed for the leftover filiform hair sockets, measured socket dimensions and location could tease out their related filiform hair lengths. Then, all of the surrounding neighbors of the particular filiform hair of interest could be obtained. As such, for this trial only four filiform neighbors could be used.



Figure 16. Picture of cricket cercus used in the experiment and taken during trial 1.

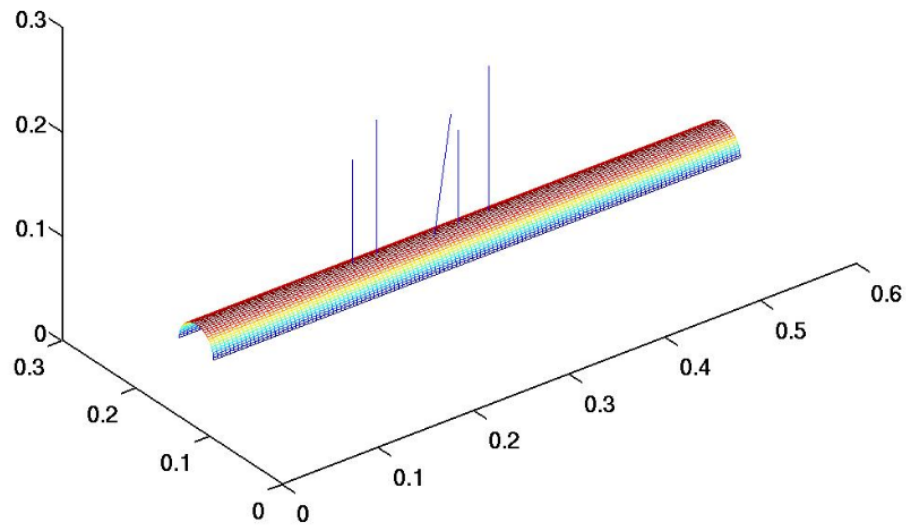


Figure 17. Model representation of the cricket cercus from experimental trial 1.

The experimental and model results for the first trial can be compared in figure 18. For a filiform hair with a mass of 1×10^{-6} grams/cm the full filiform hair results match well. However, the single filiform hair results still suffer from a lack of magnitude in deflection with respect to frequency. The exact opposite is found for a filiform hair with a mass of 1×10^{-5} grams/cm. The single filiform hair has more deflection except for it quickly decaying at higher frequencies, while the full filiform hair results are further off. A filiform hair mass near the middle of those two values, 5×10^{-6} grams/cm, strikes somewhat of a compromise. For a single filiform hair it matches the peak deflection, but still suffers from being too narrow to match surrounding frequencies. For a full set of filiform hairs its deflection is too high at its peak and not flat enough at higher frequencies.

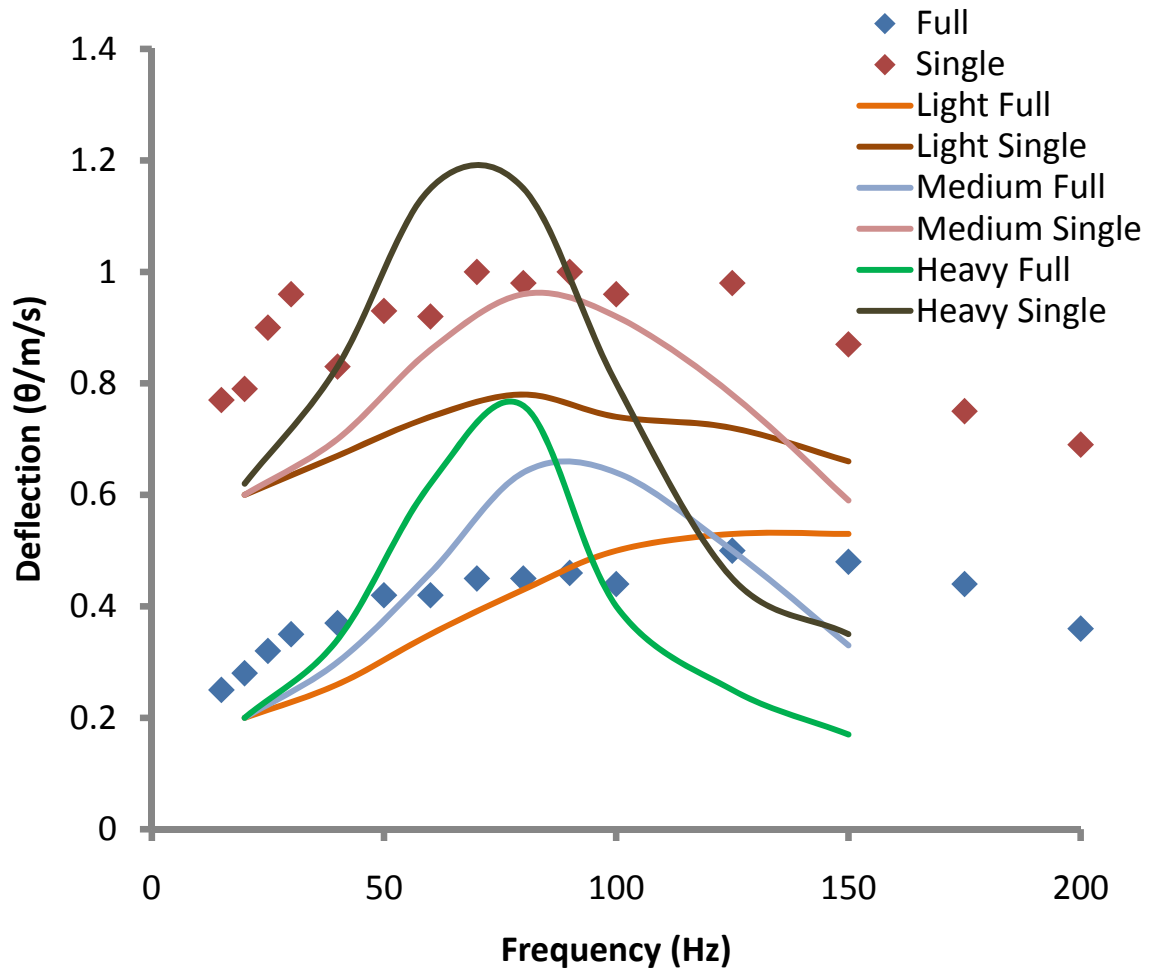


Figure 18. A 900 μm long filiform hair with full near field surrounding filiform hairs and also without any surrounding filiform hairs. The experimental results are shown in diamonds, model results with a hair weighing 1×10^{-6} grams/cm is labeled light, a hair weighing 5×10^{-6} grams/cm is labeled medium and a hair weighing 1×10^{-5} grams/cm is labeled heavy.

Comparing the hair interaction κ values between the experimental and model shows decent results. The filiform hair with a mass of 1×10^{-6} grams/cm matched about half of the κ values while the filiform hair with a mass of 1×10^{-5} grams/cm matched nearly all of the κ values relatively well. In general, κ values from filiform hairs with a mass of 1×10^{-6} grams/cm usually show a decreasing linear relationship with respect to

increasing frequency. Also, this same decreasing linear relationship with respect to increasing frequency can be seen in filiform hairs with a mass of 1×10^{-5} grams/cm at frequencies below 100 Hz. Above 100 Hz, there is instability in the model filiform hair deflection. A filiform hair with a mass of 5×10^{-6} grams/cm acted similar to the heavier hair mass, but did not destabilize until slightly higher frequencies.

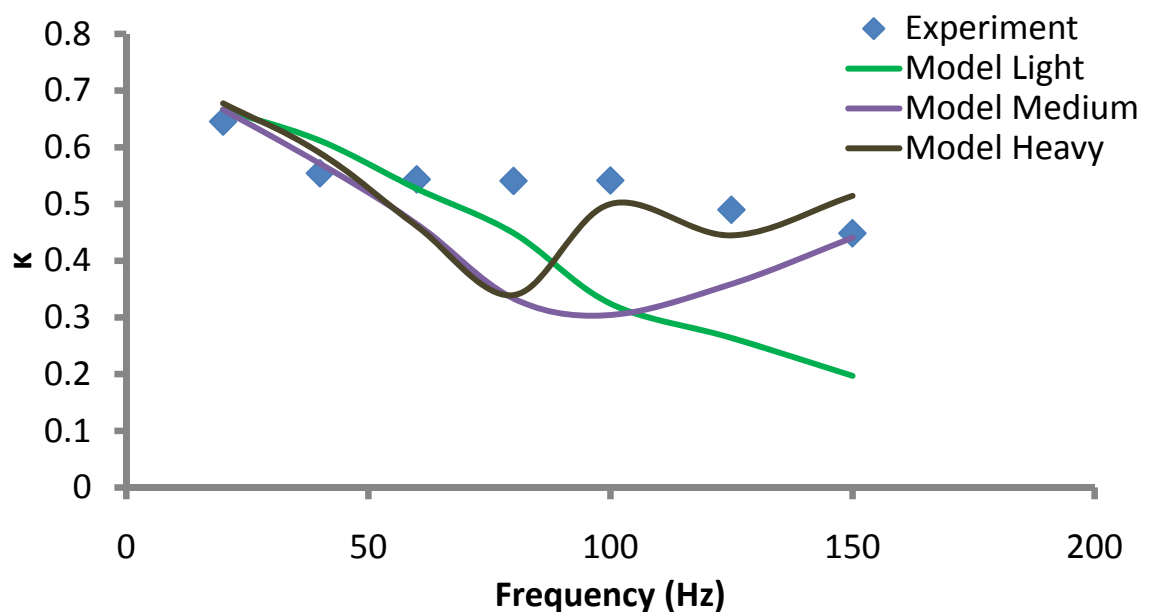


Figure 19. Comparison between κ values obtained from both the experimental and model results of a 900 μm long hair that weighs 1×10^{-6} grams/cm labeled light, one that weighs 5×10^{-6} grams/cm labeled medium and another one that weighs 1×10^{-5} grams/cm labeled heavy.

The second trial allowed for a few more neighboring filiform hairs to be placed in the model. A picture of the cricket's cercus and its representation as a model with a filiform hair surrounded by six neighbors can be seen in the following two figures.

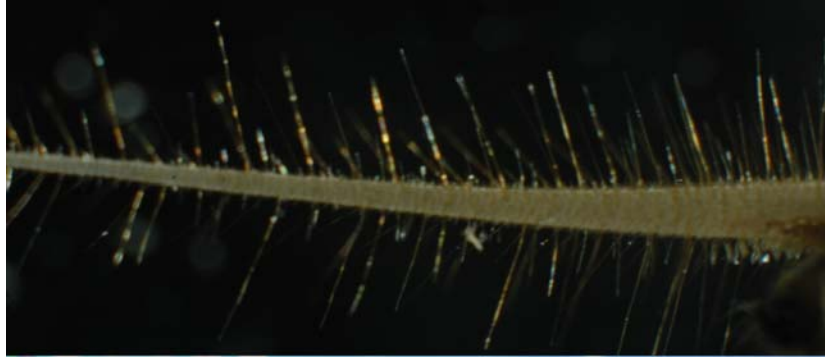


Figure 20. Picture of cricket cercus used in the experiment and taken during trial 2.

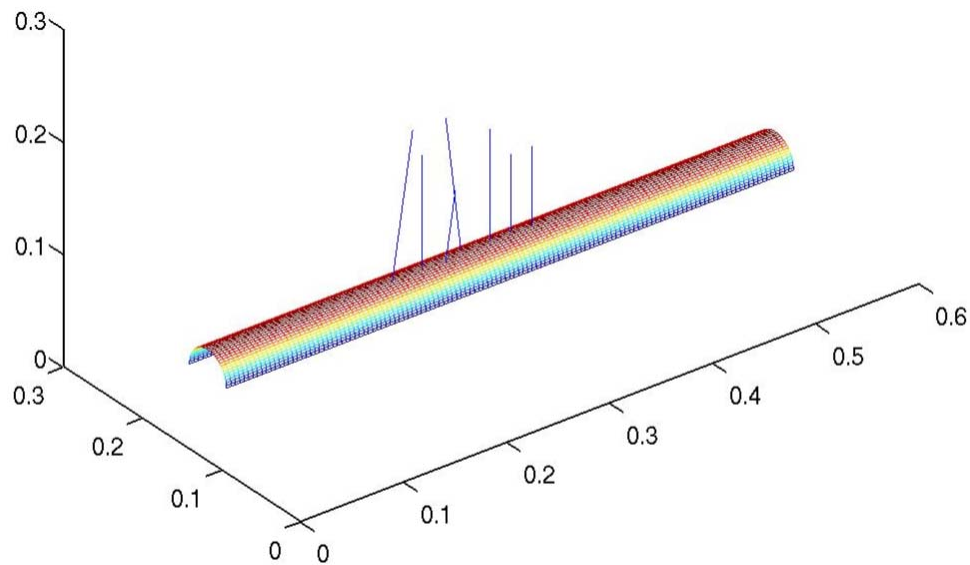


Figure 21. Model representation of the cricket cercus from experimental trial 2.

The model results for the second trial do not differ much from the first trial and indeed they were not expected to as the model yields relatively similar results. The experimental results were surprising and their validity needs to be taken into consideration. First, the experimental results show a very high deflection at lower frequencies for a single filiform hair. This tends not to be seen elsewhere and the cause is

unknown. Second, in the experimental results for a cercus full of hairs, there is a large difference in between 100 Hz and 125 Hz and a somewhat small difference between 35 Hz and 40 Hz. The cause of this can be partially attributed to the experimental methods. Frequencies are applied in groups that range of 25-35 Hz, 40-100 Hz and 125-200 Hz. Besides a smooth transition, a small jump in deflection between those frequencies may be expected and does not raise too much concern. However, a large jump in deflection does raise concern. It is therefore impossible tell which range of frequencies is more accurate.

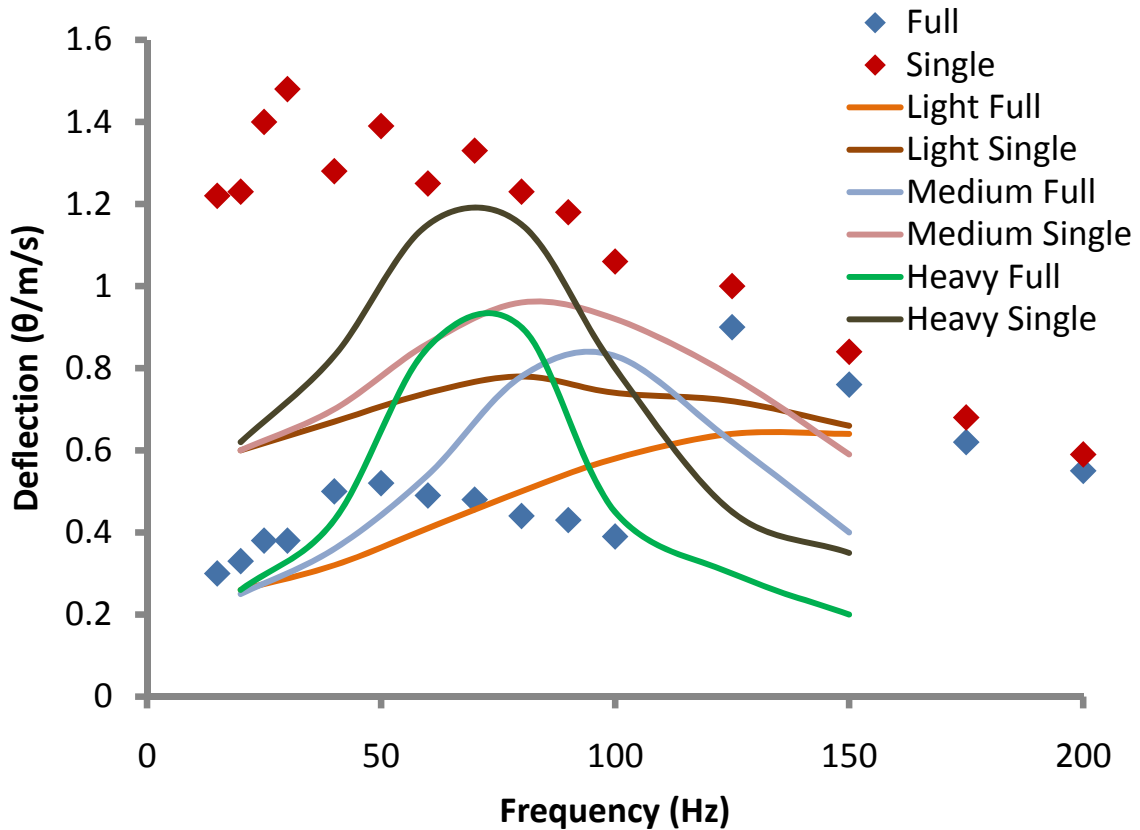


Figure 22. A 1000 μm long filiform hair with full near field surrounding filiform hairs and also without any surrounding filiform hairs. The experimental results are shown in diamonds, model results with a hair weighing 1×10^{-6} grams/cm is labeled light, a hair weighing 5×10^{-6} grams/cm is labeled medium and a hair weighing 1×10^{-5} grams/cm is labeled heavy.

As a consequence of the high deflections at lower frequencies for the experimental results, the κ values are higher at those same frequencies as well. The model κ values lie in an area between the two extremes of the experimental κ values.

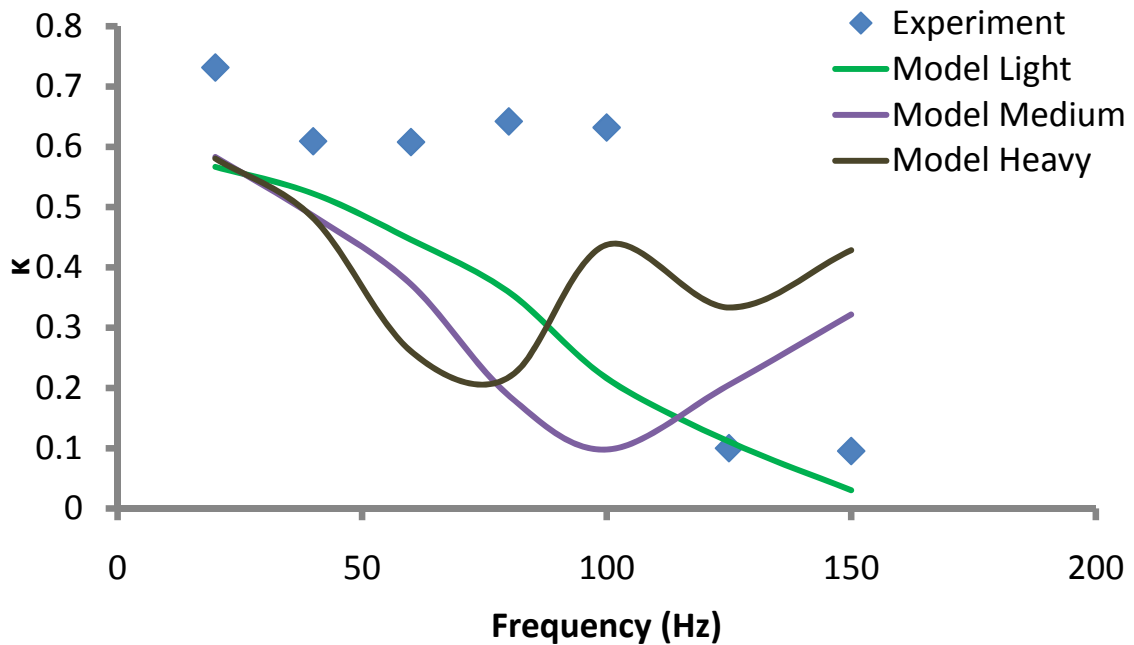


Figure 23. Comparison between κ values obtained from both the experimental and model results of a 1000 μm long hair that weighs 1×10^{-6} grams/cm labeled light, one that weighs 5×10^{-6} grams/cm labeled medium and another one that weighs 1×10^{-5} grams/cm labeled heavy.

The third trial is similar to the first. It has the same number of neighboring filiform hairs. A picture of the cricket's cercus and its representation as a model with a filiform hair surrounded by four neighbors can be seen in the following two figures.

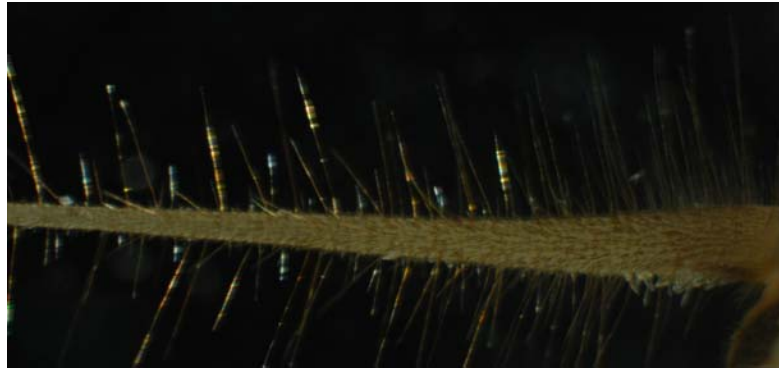


Figure 24. Picture of cricket cercus used in the experiment and taken during trial 3.

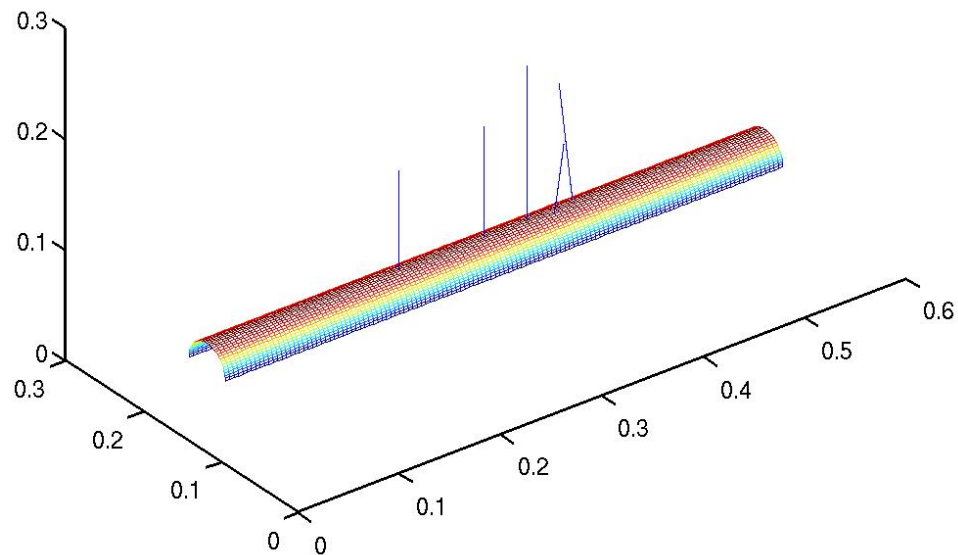


Figure 25. Model representation of the cricket cercus from experimental trial 3.

As expected, the model results for trial three were very similar to the two previous trials. Like the second trial, the experimental results had a peculiar effect. The deflections for the particular filiform hair of interest while surrounded with neighboring hairs were very high at low frequencies. It is unknown what might cause this. There was no noticeable jump between frequency ranges either, although the lowest frequencies

were not obtained. The heavier model filiform hair matches up well with the experimental results except at higher frequencies. Again, at higher frequencies the heavier model filiform hair has instability in deflection above 100 Hz.

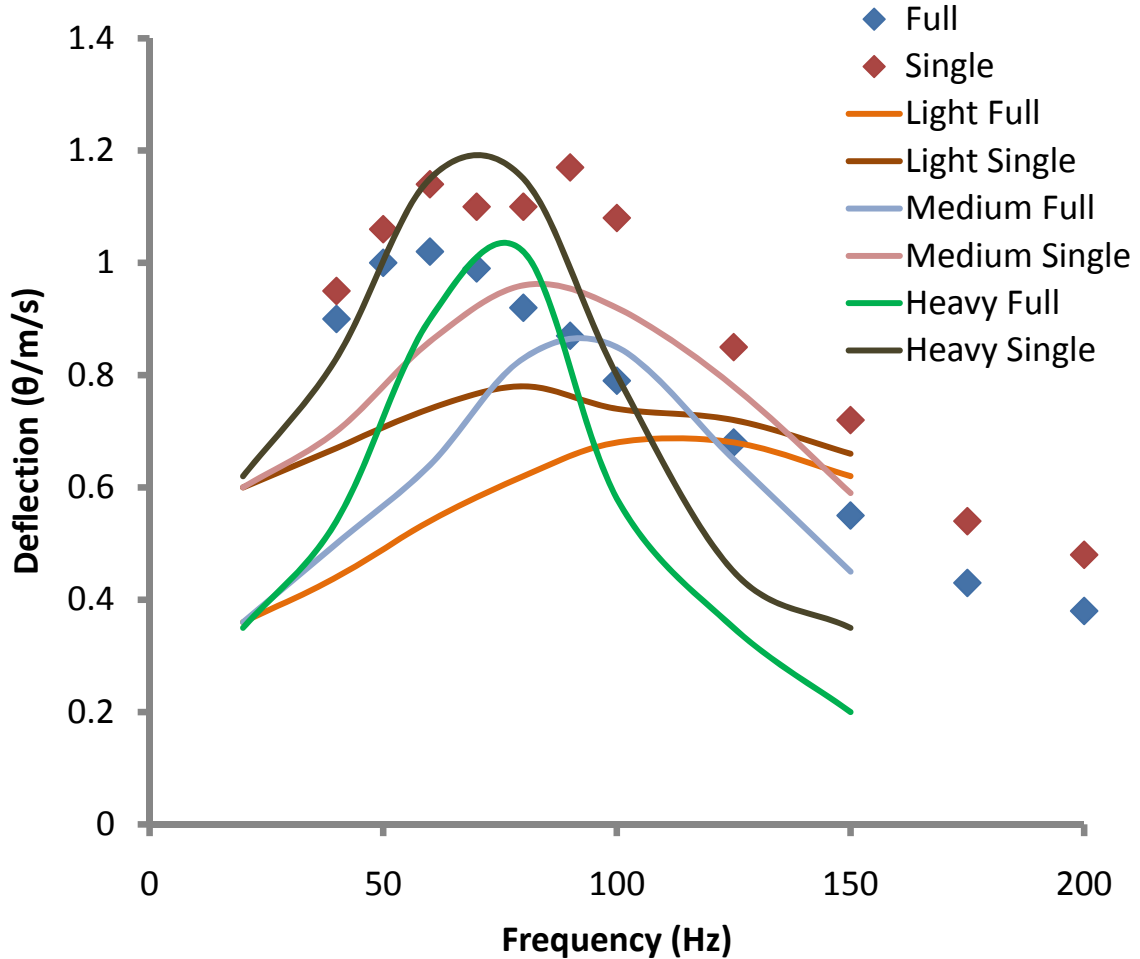


Figure 26. A 1000 μm long filiform hair with full near field surrounding filiform hairs and also without any surrounding filiform hairs. The experimental results are shown in diamonds, model results with a hair weighing 1×10^{-6} grams/cm is labeled light, a hair weighing 5×10^{-6} grams/cm is labeled medium and a hair weighing 1×10^{-5} grams/cm is labeled heavy.

The κ values were once more affected by the deflection peculiarities. The experimental κ values start low and increase with an increase in frequency. The model κ values show similar behavior as the previous trials.

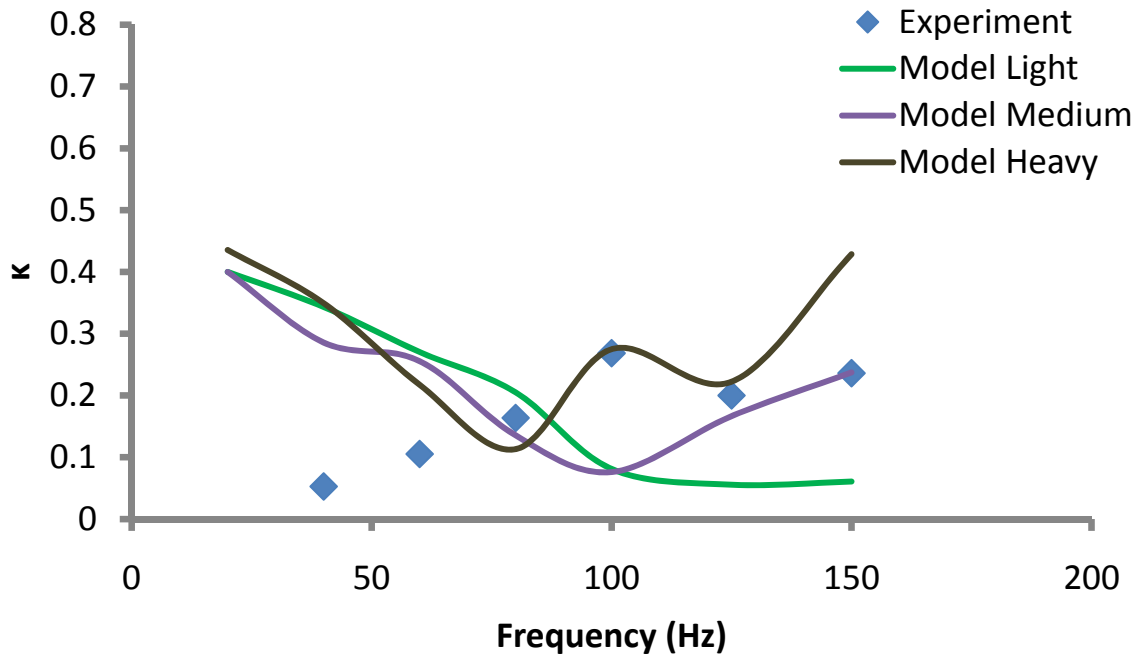


Figure 27. Comparison between κ values obtained from both the experimental and model results of a 1000 μm long hair that weighs 1×10^{-6} grams/cm labeled light, one that weighs 5×10^{-6} grams/cm labeled medium and another one that weighs 1×10^{-5} grams/cm labeled heavy.

While some of the experimental data are still being processed and examined, the results so far clearly show that neighboring hairs have a great impact on the motion of individual filiform hairs. This interaction depends on the frequency of the oscillatory fluid flow. At low frequencies, the filiform hairs have a strong interaction with each other, but the interaction decreases as the frequency of the oscillatory fluid flow is increased. This appears to be a common motif seen in other results as well. For example,

this same trend can be seen in experiments on the fixed artificial hairs in figures 10 and 11. The particle velocity field at lower frequencies has farther reaching velocity changes whereas higher frequencies cause velocity effects over a short distance. This would lead to larger κ values at the smaller frequencies and smaller κ values at the larger frequencies.

In our modeling effort, we first had to carefully determine the correct bending coefficients for every orientation the filiform hair could occupy. This proved more difficult than originally anticipated. Since the fluid grid is Eulerian and the filiform hair nodes can occupy any orientation in the fluid grid, this meant the approximated discrete Dirac delta function equating the fluid and filiform hair forces depended on the relative positional relationship. It was found relatively late that the mass of a filiform hair in the model impacted the deflection profile for a filiform hair. It appears the model best matches experimental results with a filiform hair mass in between 1×10^{-5} and 1×10^{-6} grams/cm while using the bending coefficients in tables 2 and 3. While using the same bending coefficients, the deflection profiles seem constant depending on the filiform hair mass. A heavier mass makes the deflection peak at higher magnitudes, but the deflection profile is narrow so at frequencies both lower and higher than the peak frequency, the magnitude is smaller. A lighter mass achieves a broader deflection profile and is stable at higher frequencies, but does not provide the deflection magnitude seen in experimental data. Filiform hair mass in between these is a mix of the two. Whether the model is matching the deflection or catching the interaction occurring between filiform hairs, the model seems to predict the most important aspects of the experimental data.

CHAPTER 4

CONCLUSION AND FUTURE WORK

We have developed an experimental and mathematical modeling platform for examining sensory hair interactions on the cricket cercus. Once appropriate values for bending coefficients in the model were determined, the agreement between the experimental measurements and model predictions was acceptable. Both experimental measurements and model predict that the interaction distance between hairs decreases as function of frequency. While this trend is obviously seen in the data, more consistent experimental data sets are required to confidently show deflection values and their related κ values. The model shows promise in capturing multiple filiform hairs interacting together, but it remains unknown how much parameter changes could increase the accuracy of capturing both single and multiple filiform hairs. The results suggest that filiform hair interactions are biologically significant, and they raise the obvious question of what is the function of the dense patch of sensory hairs.

Future directions for this research can include many changes to provide a better match in deflection to experimental results. While there may never be a way for the current form of the model to predict the filiform hair deflection perfectly, it can come close. First, the code should be parallelized for speed. Currently, the largest computational cost comes from the fluid solving portion. With parallelization accomplished, it could cut normal computational time down from a few days. Second, coefficient parameters set in the model could be improved. It is important to capture the

deflection profile of a single filiform hair because without this attribute capturing multiple filiform hair interactions can seem irrelevant. This can be accomplished by using tables 2 and 3 as a starting point and fine tuning the values. Third, determine an average of actual filiform hair masses. The bending coefficients and filiform hair mass couple together in the model code to provide a typical deflection profile. It would be much more convenient to find a ballpark region of filiform hair mass and acquire one set of bending coefficients for the filiform hairs. Currently, all filiform hairs in the model scale linearly to a mass dependant on its length, but this may not be completely accurate. Fourth, the effective filiform hair diameter could be broadened to increase the magnitude of the deflection profile and counteract the effect of a shrinking effective diameter as the fluid grid is refined. The fluid grid is refined in order to keep the interaction occurring between filiform hairs at an appropriate scale. However, this shrinks the effective diameter of the filiform hair. Changing the smear radius of the discrete Dirac delta function could impart more interaction occurring between the fluid interactive structural nodes and the fluid, but deriving a new discrete Dirac delta function could be very difficult. Fifth, find the interaction distance profile from experimental data and use that to scale the model. Currently, the model may be predicting interactions for unnecessarily long distances between hairs. For instance, a filiform hair 750 μm away might still experience interaction effects. An interesting event in the model occurs when many filiform hairs are put on a cercus in a row. A filiform hair at one end of the partial cercus will impact a filiform hair a much further distance away until it reaches a minimum deflection value. This would cause hairs in the middle of the partial cercus to always be

at a minimum deflection value or a κ value around 0.5 no matter where the neighboring filiform hairs are placed. Finally, more filiform hairs can be added to the cercus to make it full of neighbors with many interactions occurring. Experimentally, cercus surfaces plucked of filiform hairs show the layout of filiform hairs, and the diameters of the leftover sockets provide the information for their former filiform hair lengths. All of this can be input into the model to have a complete partial cercus. New experimental data would be needed to compare filiform hair deflection profiles though. With a number of improvements, this model could capture a partial or full cercus.

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APPENDIX A

PENALTY IMMERSED BOUNDARY METHOD CODE

The following appendix is a summarization of the FORTRAN 90 code used to implement the PIB method for a partial cricket cercus and filiform hairs. Only selected code sections are included here, and the full source code is available upon request. The first section is from the commonval.f90 file which implements any common variables used throughout the entire code. The particular selection from the commonval.f90 file shown below sets up the four point delta function as seen in equation 14. The subroutine Delta4 is called whenever a force density switch occurs between Eulerian and Lagrangian coordinates or vice versa occurs. The input to Delta4, R, is simply the distance between the respective fluid interactive structural node and the fluid grid node.

```
!Now We have to make a 4-point delta function
CONTAINS
SUBROUTINE Delta4(R,Del)
  REAL(DP), DIMENSION(:,:,:), INTENT(IN) :: R
  REAL(DP), DIMENSION(:,:,:), INTENT(OUT) :: Del
  INTEGER(I4B) :: i, j, k, r_n, c_n, h_n
  r_n=size(R,1); c_n=size(R,2); h_n=size(R,3)
  do i=1,r_n
    do j=1,c_n
      do k=1,h_n
        if (ABS(R(i,j,k))<=1.0_dp) then
          Del(i,j,k)=(3.0_dp-2.0_dp*ABS(R(i,j,k))+ &
            sqrt(1.0_dp+4.0_dp*ABS(R(i,j,k))-
4.0_dp*ABS(R(i,j,k))**2))/8.0_dp
        else if (1.0_dp<ABS(R(i,j,k)) .and. ABS(R(i,j,k))<=2.0_dp)
then
          Del(i,j,k)=(5.0_dp-2.0_dp*ABS(R(i,j,k))- &
            sqrt(-7.0_dp+12.0_dp*ABS(R(i,j,k))-
4.0_dp*ABS(R(i,j,k))**2))/8.0_dp
        else
          Del(i,j,k)=0.0_dp
        end if
      end do
    end do
  end do
END SUBROUTINE Delta4
```

The next section comes from the main programming running the code. Prior to this particular section of code, the physical location of the cercus nodes and filiform hair node locations along with their appropriate coefficients are set. The following is the main section of the time loop. The calling of subroutine order matches directly with the previous thesis section that describes the implementation of the code for the half time step, but varies a little in the calculation of variables for the full time step. The subroutines HairL_X1st and getEarth update the locations of fluid interactive immersed boundary in both the filiform hairs and cercus, respectively, at a half time step. The variable L_X holds the fluid interactive immersed boundary information while L_Xn holds the same information for the half time step. Any variable with the same name with an added “n” at the end represents that variable at the half time step ahead. The variables U, V and W hold the fluid flow information. The time step, dt, is denoted by its half time step, dt/2, as the input for the called subroutine.

```

do i=1,Iter_N
  Call HairL_X1st(U,V,W,dt/2.0_dp,L_X,L_Xn,Bot,Botn)
  Call getEarth(U,V,W,dt/2.0_dp,Earth,Earthn,1_I4B)
  Mass_Xn=Mass_X+dt*Mass_U/2.0_dp
  Call hairForce(L_Xn,Mass_Xn,Mass_F,E_Fx,E_Fy,E_Fz,PSF_old, dt)
  Call
Earth_Force(Earthn,FixEarth,Botn,Fixbot,c_earth,E_Fx,E_Fy,E_Fz)

  drag_V(1)=inflow*(sin(pi*dt*i*freq))
  Call
getUV1st_02(E_Fx,E_Fy,E_Fz,dt/2.0_dp,drag_V,U,V,W,Un,Vn,Wn,P)
  Call getUV2nd_02(E_Fx,E_Fy,E_Fz,dt,drag_V,U,V,W,Un,Vn,Wn,P)
  Call HairL_X2nd(Un,Vn,Wn,dt,L_X,L_Xn,Bot,Botn)
  Call getEarth(Un,Vn,Wn,dt,Earth,Earthn,2_I4B)

  do j=1,Hairnum; do k=1,M1_Actual(j)+1; do m=1,radpts
    Mass_F(k,m,3,j)=Mass_F(k,m,3,j)-
mass*ds_length(k,m,1,j)*A_grav

```

```

      Mass_X(k,m,:,j)=Mass_X(k,m,:,j)+dt*Mass_U(k,m,:,j)+dt**2*Mass_
      F(k,m,:,j)/(2.0_dp*mass*ds_length(k,m,1,j))

      Mass_U(k,m,:,j)=Mass_U(k,m,:,j)+dt*Mass_F(k,m,:,j)/(mass*ds_le
      ngth(k,m,1,j))
    end do; end do; end do

    if (mod(i,Interval) == 1) then
      write (*,*) i
      write (1,20) L_X
      write (2,20) Earth
      write (3,20) U
      write (4,20) V
      write (5,20) W
    end if

  end do

```

The following is the actual subroutine HairL_X1st, which updates the fluid interactive immersed boundary location. First, Delta4 is called in order to match the filiform hair velocity with the fluid velocity. The variables E_x, E_y and E_z hold the information for the location of the Eulerian fluid grid nodes. Second, the projection of the fluid velocity is determined in every direction. Finally, the fluid interactive immersed boundary location is updated according to equation 13.

```

do k=1,Hairnum; do i=1,M1_Actual(k)+1; do m=1,radpts

  i_x=floor(L_X(i,m,1,k)/H);i_y=floor(L_X(i,m,2,k)/H);i_z=floor(L_X(i,
  m,3,k)/H)
  call Delta4((E_x(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
  L_X(i,m,1,k))/H,Val_X)
  call Delta4((E_y(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
  L_X(i,m,2,k))/H,Val_Y)
  call Delta4((E_z(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
  L_X(i,m,3,k))/H,Val_Z)
  G=Val_X*Val_Y*Val_Z
  Proj_X=U(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)
  Proj_Y=V(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)
  Proj_Z=W(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)
  L_Xn(i,m,1,k)=L_X(i,m,1,k)+dt*sum(Proj_X*G)
  L_Xn(i,m,2,k)=L_X(i,m,2,k)+dt*sum(Proj_Y*G)
  L_Xn(i,m,3,k)=L_X(i,m,3,k)+dt*sum(Proj_Z*G)

```

```

end do;end do;end do
do k=1,Hairnum; do i=1,Mfix; do m=1,radpts

i_x=floor(Bot(i,m,1,k)/H);i_y=floor(Bot(i,m,2,k)/H);i_z=floor(Bot(i,
m,3,k)/H)
    call Delta4((E_x(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
Bot(i,m,1,k))/H,Val_X)
    call Delta4((E_y(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
Bot(i,m,2,k))/H,Val_Y)
    call Delta4((E_z(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
Bot(i,m,3,k))/H,Val_Z)
    G=Val_X*Val_Y*Val_Z

Botn(i,m,1,k)=Bot(i,m,1,k)+dt*sum(U(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)*G
)

Botn(i,m,2,k)=Bot(i,m,2,k)+dt*sum(V(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)*G
)

Botn(i,m,3,k)=Bot(i,m,3,k)+dt*sum(W(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)*G
)
end do;end do;end do

```

The next step in the process is to use the positions of both structural nodes at the half time step to calculate the Lagrangian force density, which is the sum of $\mathbf{F}_E$ and $\mathbf{F}_K$. Also, the Lagrangian force density is transformed into the Eulerian force density, $\mathbf{f}(\mathbf{x},t)$. The following section of the code comes from the subroutine `hairForce` and the following section of code is specifically dealing with the force from the bending resistance of each filiform hair. A central difference is used to approximate the second derivative of $\mathbf{X}(r,s,t)$ with respect to s , which holds the information of the distance between fluid interactive structural nodes. The Eulerian force density is held in the variables `E_Fx`, `E_Fy` and `E_Fz`.

```

!! Force from the bending resistance
=====
do j=1,Hairnum; L_F1=0.0_dp
    do i=2,M1_Actual(j); do m=1,radpts
        PSF=L_X(i+1,m,:,j)+L_X(i-1,m,:,j)-2.0_dp*L_X(i,m,:,j)

```

```

      L_F1(:,i-1,m,j)=L_F1(:,i-1,m,j) -c_bend(i-
1,m,j)*PSF/ds_length(i,m,1,j)**3
      L_F1(:,i,m,j)=L_F1(:,i,m,j) +c_bend(i
,m,j)*2.0_dp*PSF/ds_length(i,m,1,j)**3
      L_F1(:,i+1,m,j)=L_F1(:,i+1,m,j) -
c_bend(i+1,m,j)*PSF/ds_length(i,m,1,j)**3
      end do;end do
      do i=1,M1_Actual(j)+1; do m=1,radpts

i_x=floor(L_X(i,m,1,j)/H);i_y=floor(L_X(i,m,2,j)/H);i_z=floor(L_X(i,
m,3,j)/H)
      call Delta4((E_x(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
L_X(i,m,1,j))/H,Val_X)
      call Delta4((E_y(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
L_X(i,m,2,j))/H,Val_Y)
      call Delta4((E_z(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)-
L_X(i,m,3,j))/H,Val_Z)

E_Fx(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)=E_Fx(i_x:i_x+3,i_y:i_y+3,i_z:i_z
+3)&

+ds_length(i,m,1,j)*L_F1(1,i,m,j)*Val_X*Val_Y*Val_Z/(H**3)

E_Fy(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)=E_Fy(i_x:i_x+3,i_y:i_y+3,i_z:i_z
+3)&

+ds_length(i,m,1,j)*L_F1(2,i,m,j)*Val_X*Val_Y*Val_Z/(H**3)

E_Fz(i_x:i_x+3,i_y:i_y+3,i_z:i_z+3)=E_Fz(i_x:i_x+3,i_y:i_y+3,i_z:i_z
+3)&

+ds_length(i,m,1,j)*L_F1(3,i,m,j)*Val_X*Val_Y*Val_Z/(H**3)
      end do; end do; end do

```

To complete the half time step, the Eulerian force density from the previous step along with the boundary conditions is input into the subroutine `getUV1st_O2` to find the fluid velocity and pressure. The fluid velocity and pressure are solved using a fast Fourier transform called by `rdft3d`. This can be seen in the following section of code. To update the velocity of the mass component structural nodes held by the variable `Mass_X`, equation 11 is used and the coding can be seen at the bottom of the main time loop shown previously.

```

do k=1,N3;do j=1,N2;do i=1,N1
  PU(i+1,j+1,k+1)=U1(i,j,k)
  PV(i+1,j+1,k+1)=V1(i,j,k)
  PW(i+1,j+1,k+1)=W1(i,j,k)
end do; end do;end do
do j=1,N2;do i=1,N1
  PU(i+1,j+1,1)=U1(i,j,N3) ;PU(i+1,j+1,N3+2)=U1(i,j,1)
  PV(i+1,j+1,1)=V1(i,j,N3) ;PV(i+1,j+1,N3+2)=V1(i,j,1)
  PW(i+1,j+1,1)=W1(i,j,N3) ;PW(i+1,j+1,N3+2)=W1(i,j,1)
end do; end do
do k=1,N3;do j=1,N2
  PU(1,j+1,k+1)=U1(N1,j,k) ;PU(N1+2,j+1,k+1)=U1(1,j,k)
  PV(1,j+1,k+1)=V1(N1,j,k) ;PV(N1+2,j+1,k+1)=V1(1,j,k)
  PW(1,j+1,k+1)=W1(N1,j,k) ;PW(N1+2,j+1,k+1)=W1(1,j,k)
end do; end do
do k=1,N3;do i=1,N1
  PU(i+1,1,k+1)=U1(i,N2,k) ;PU(i+1,N2+2,k+1)=U1(i,1,k)
  PV(i+1,1,k+1)=V1(i,N2,k) ;PV(i+1,N2+2,k+1)=V1(i,1,k)
  PW(i+1,1,k+1)=W1(i,N2,k) ;PW(i+1,N2+2,k+1)=W1(i,1,k)
end do;end do
do k=1,N3;do j=1,N2;do i=1,N1
  W_1(i,j,k)=U1(i,j,k)-(dt/2.0_dp)*&
    (U1(i,j,k)*(PU(i+2,j+1,k+1)-PU(i,j+1,k+1))&
    +V1(i,j,k)*(PU(i+1,j+2,k+1)-PU(i+1,j,k+1))&
    +W1(i,j,k)*(PU(i+1,j+1,k+2)-PU(i+1,j+1,k))&
    +PU(i+2,j+1,k+1)*PU(i+2,j+1,k+1)-
  PU(i,j+1,k+1)*PU(i,j+1,k+1)&
    +PU(i+1,j+2,k+1)*PV(i+1,j+2,k+1)-
  PU(i+1,j,k+1)*PV(i+1,j,k+1)&
    +PU(i+1,j+1,k+2)*PW(i+1,j+1,k+2)-
  PU(i+1,j+1,k)*PW(i+1,j+1,k))/&
    (2.0_dp*H)+(dt/den)*E_Fx(i,j,k)
  W_2(i,j,k)=V1(i,j,k)-(dt/2.0_dp)*&
    (U1(i,j,k)*(PV(i+2,j+1,k+1)-PV(i,j+1,k+1))&
    +V1(i,j,k)*(PV(i+1,j+2,k+1)-PV(i+1,j,k+1))&
    +W1(i,j,k)*(PV(i+1,j+1,k+2)-PV(i+1,j+1,k))&
    +PV(i+2,j+1,k+1)*PU(i+2,j+1,k+1)-
  PV(i,j+1,k+1)*PU(i,j+1,k+1)&
    +PV(i+1,j+2,k+1)*PV(i+1,j+2,k+1)-
  PV(i+1,j,k+1)*PV(i+1,j,k+1)&
    +PV(i+1,j+1,k+2)*PW(i+1,j+1,k+2)-
  PV(i+1,j+1,k)*PW(i+1,j+1,k))/&
    (2.0_dp*H)+(dt/den)*E_Fy(i,j,k)
  W_3(i,j,k)=W1(i,j,k)-(dt/2.0_dp)*&
    (U1(i,j,k)*(PW(i+2,j+1,k+1)-PW(i,j+1,k+1))&
    +V1(i,j,k)*(PW(i+1,j+2,k+1)-PW(i+1,j,k+1))&
    +W1(i,j,k)*(PW(i+1,j+1,k+2)-PW(i+1,j+1,k))&
    +PW(i+2,j+1,k+1)*PU(i+2,j+1,k+1)-
  PW(i,j+1,k+1)*PU(i,j+1,k+1)&
    +PW(i+1,j+2,k+1)*PV(i+1,j+2,k+1)-
  PW(i+1,j,k+1)*PV(i+1,j,k+1)&

```

```

      +PW(i+1,j+1,k+2)*PW(i+1,j+1,k+2)-
PW(i+1,j+1,k)*PW(i+1,j+1,k))/&
      (2.0_dp*H)+(dt/den)*E_Fz(i,j,k)
end do ;end do; end do
W_1(1:Dline,1:N2,1:N3)=W_1(1:Dline,1:N2,1:N3)+(dt*fric_alpha/den)*(d
rag_V(1)-U1(1:Dline,,:))
W_2(1:Dline,1:N2,1:N3)=W_2(1:Dline,1:N2,1:N3)+(dt*fric_alpha/den)*(d
rag_V(2)-V1(1:Dline,,:))
W_3(1:Dline,1:N2,1:N3)=W_3(1:Dline,1:N2,1:N3)+(dt*fric_alpha/den)*(d
rag_V(3)-W1(1:Dline,,:))

call rdft3d( ldim1, ldim2, N1, N2, N3, 1, W_1, work, ip, wsave)
call rdft3dsort(ldim1, ldim2, N1, N2, N3, 1, W_1)
call rdft3d( ldim1, ldim2, N1, N2, N3, 1, W_2, work, ip, wsave);
call rdft3dsort(ldim1, ldim2, N1, N2, N3, 1, W_2)
call rdft3d( ldim1, ldim2, N1, N2, N3, 1, W_3, work, ip, wsave);
call rdft3dsort(ldim1, ldim2, N1, N2, N3, 1, W_3)

do k=1,N3; do j=1,N2; do i=1,N1/2+1
  k1=2*i-1;k2=2*i
  pff(k1,j,k)=-(den*H/dt)*((sin(2.0_dp*pi*(i-
1.0_dp)/N1))*W_1(k2,j,k)+ &
      (sin(2.0_dp*pi*(j-1.0_dp)/N2))*W_2(k2,j,k)+
&
      (sin(2.0_dp*pi*(k-
1.0_dp)/N3))*W_3(k2,j,k))/DDPP(i,j,k)
  pff(k2,j,k)= (den*H/dt)*((sin(2.0_dp*pi*(i-
1.0_dp)/N1))*W_1(k1,j,k)+ &
      (sin(2.0_dp*pi*(j-1.0_dp)/N2))*W_2(k1,j,k)+
&
      (sin(2.0_dp*pi*(k-
1.0_dp)/N3))*W_3(k1,j,k))/DDPP(i,j,k)
  Uff(k1,j,k)=(W_1(k1,j,k)-pff(k2,j,k)*&
      (dt/(den*H))*sin(2.0_dp*pi*(i-1.0_dp)/N1))/AA(i,j,k)
  Uff(k2,j,k)=(W_1(k2,j,k)+pff(k1,j,k)*&
      (dt/(den*H))*sin(2.0_dp*pi*(i-1.0_dp)/N1))/AA(i,j,k)
  Vff(k1,j,k)=(W_2(k1,j,k)-pff(k2,j,k)*&
      (dt/(den*H))*sin(2.0_dp*pi*(j-1.0_dp)/N2))/AA(i,j,k)
  Vff(k2,j,k)=(W_2(k2,j,k)+pff(k1,j,k)*&
      (dt/(den*H))*sin(2.0_dp*pi*(j-1.0_dp)/N2))/AA(i,j,k)
  Wff(k1,j,k)=(W_3(k1,j,k)-pff(k2,j,k)*&
      (dt/(den*H))*sin(2.0_dp*pi*(k-1.0_dp)/N3))/AA(i,j,k)
  Wff(k2,j,k)=(W_3(k2,j,k)+pff(k1,j,k)*&
      (dt/(den*H))*sin(2.0_dp*pi*(k-1.0_dp)/N3))/AA(i,j,k)
end do ;end do; end do

! do k=1,N3; do j=1,N2/2+1; do i=1,N1
!   k1=2*j-1;k2=2*j
!   pff(i,k1,k)=-(den*H/dt)*((sin(2.0_dp*pi*(i-
1.0_dp)/N1))*W_1(i,k2,k)+ &
!       (sin(2.0_dp*pi*(j-1.0_dp)/N2))*W_2(i,k2,k)+
&

```

```

!           (sin(2.0_dp*pi*(k-
1.0_dp)/N3))*W_3(i,k2,k))/DDPP(i,j,k)
!   pff(i,k2,k)= (den*H/dt)*((sin(2.0_dp*pi*(i-
1.0_dp)/N1))*W_1(i,k1,k)+ &
!           (sin(2.0_dp*pi*(j-1.0_dp)/N2))*W_2(i,k1,k)+
&
!           (sin(2.0_dp*pi*(k-
1.0_dp)/N3))*W_3(i,k1,k))/DDPP(i,j,k)
!   Uff(i,k1,k)=(W_1(i,k1,k)-pff(i,k2,k)*&
!           (dt/(den*H))*sin(2.0_dp*pi*(i-1.0_dp)/N1))/AA(i,j,k)
!   Uff(i,k2,k)=(W_1(i,k2,k)+pff(i,k1,k)*&
!           (dt/(den*H))*sin(2.0_dp*pi*(i-1.0_dp)/N1))/AA(i,j,k)
!   Vff(i,k1,k)=(W_2(i,k1,k)-pff(i,k2,k)*&
!           (dt/(den*H))*sin(2.0_dp*pi*(j-1.0_dp)/N2))/AA(i,j,k)
!   Vff(i,k2,k)=(W_2(i,k2,k)+pff(i,k1,k)*&
!           (dt/(den*H))*sin(2.0_dp*pi*(j-1.0_dp)/N2))/AA(i,j,k)
!   Wff(i,k1,k)=(W_3(i,k1,k)-pff(i,k2,k)*&
!           (dt/(den*H))*sin(2.0_dp*pi*(k-1.0_dp)/N3))/AA(i,j,k)
!   Wff(i,k2,k)=(W_3(i,k2,k)+pff(i,k1,k)*&
!           (dt/(den*H))*sin(2.0_dp*pi*(k-1.0_dp)/N3))/AA(i,j,k)
!end do ;end do; end do
call rdft3dsort(ldim1, ldim2, N1, N2, N3, -1, Uff)
call rdft3d( ldim1, ldim2, N1, N2, N3, -1, Uff, work, ip, wsave)
U2=2.0_dp*Uff(1:N1,1:N2,1:N3)/(N1*N2*N3)
call rdft3dsort(ldim1, ldim2, N1, N2, N3, -1, Vff)
call rdft3d( ldim1, ldim2, N1, N2, N3, -1, Vff, work, ip, wsave);
V2=2.0_dp*Vff(1:N1,1:N2,1:N3)/(N1*N2*N3)
call rdft3dsort(ldim1, ldim2, N1, N2, N3, -1, Wff)
call rdft3d( ldim1, ldim2, N1, N2, N3, -1, Wff, work, ip, wsave);
W2=2.0_dp*Wff(1:N1,1:N2,1:N3)/(N1*N2*N3)
call rdft3dsort(ldim1, ldim2, N1, N2, N3, -1, pff)
call rdft3d( ldim1, ldim2, N1, N2, N3, -1, pff, work, ip, wsave);
Pressure=2.0_dp*pff(1:N1,1:N2,1:N3)/(N1*N2*N3)

```

Now the variables for the full time step can be calculated much the same way as the half time steps were calculated. This can be seen in the main time loop when the subroutines getUV2nd_O2 and HairL_X2nd are called as well as the bottom of the main time loop where the mass structural nodes are updated.