

HISPANIC ELL MIDDLE-SCHOOL STUDENTS AND
THEIR TEACHERS: PERCEPTIONS OF THREE
MATHEMATICS TEACHING PRACTICES

By

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ABSTRACT

Even though Hispanic English Language Learners (ELL) are one of the fastest growing student demographics in the United States, limited research exists exploring how Hispanic ELLs benefit from mathematics teaching practices that have been determined to be highly effective. The purpose of this qualitative case study was to examine the perceptions of Hispanic ELL middle-school students regarding the efficacy of research-informed instructional practices, as embodied in the Mathematics Teaching Practices described by the National Council of Teachers of Mathematics (NCTM, 2012).

Capturing Hispanic ELL student perceptions is of great importance due to the challenging situations that many Hispanic ELL students and their families face (Schneider et al., 2006) in the United States. Additionally, the concerning differences have been observed in achievement data for Hispanic and non-Hispanic students and for ELL and non-ELL students. To address the layered challenges that Hispanic ELL students face daily in school, teachers, school leaders, and researchers can turn to student consultation as one way to gather information to inform teaching practices and develop positive mathematical identities.

Findings from this multi-phased qualitative case study suggest that both students and teachers value and find benefit in the use of mathematical discourse, posing of purposeful questions, and attention to multiple representations. Additional insights were uncovered as alignments and divergences between perceptions were analyzed. Recommendations for practice include promoting mathematical discourse, posing purposeful questions for multiple purposes, emphasizing multiple representations, including student voice, building mathematical identity through instruction, and leveraging alignments and divergences in perceptions.

CHAPTER ONE

INTRODUCTION

In 2001, Robert Moses announced the status of mathematics literacy to be an urgent issue in the United States (Moses, 2001). He expanded on his claim in 2018 when he added that the ability to do math is a civil right as it contributes to a person's opportunity to fully participate in society (Curry, 2018). Moses argued that students with low mathematical skills often end up with the lowest paying jobs (Moses 2001; Hanushek et al., 2013). This relationship occurs because high-level coursework in high school mathematics often serves as a gateway to higher education, especially in the fields of engineering and technology (Schoenfeld, 2002; Hanson et al., 2017). In support of Moses's observations, researchers at Harvard University found that even after controlling for socioeconomic factors, rigorous high school mathematics coursework was the most important predictor of success in introductory college science courses like biology and chemistry, even more than high school science coursework itself (Harvard, 2007). Furthermore, literacy in mathematics is critical in the consumption of data, information, and technology, which is an integral component of everyday life regardless of education or career pathways (Aguirre et al., 2013; Curry, 2018). Thus, successful completion of high school mathematics coursework can have lifelong implications for educational access, career decisions, and financial success (Aguirre et al., 2013; Schoenfeld, 2002; Tyson, 2007).

Success in high school mathematics coursework has been linked to a higher likelihood of attending college (Rodriguez, 2017), success in a variety of disciplines at the post-secondary level, and degree attainment leading to higher paying employment (NCES, 2019). However, to

be accepted into most universities or colleges, one must have completed 2–3 years of high school mathematics. For more selective post-secondary options, students are often required to have completed four years of high school mathematics including pre-calculus (Rodriguez, 2018; Clotfelter, Hemelt, & Ladd, 2018). As mathematics courses often occur in a linear progression in school systems, students must take more than the minimal level or remedial course options in middle school to meet this target. The trajectory to higher level high school mathematics classes starts with the successful completion of middle school mathematics courses.

Unfortunately, racial and socioeconomic disparities in academic opportunity at the secondary level remain a barrier to higher education for minority students (Rodriguez, 2018). A particular demographic of concern is English Language Learners (ELL) of Hispanic descent. In 2019, the National Center for Educational Statistics (NCES) reported that 10.1% of all school age students were identified as ELLs, and this segment of the population is the fastest growing student demographic (NCES, 2020). Furthermore, the most recent available data from the Office of Civil Rights (OCR) reported that 76% of the ELL population are of Hispanic descent (U.S. Department of Education, Office for Civil Rights, 2017-18). Taken together, these statistics indicate that Hispanic ELL students are a substantial and growing population in U.S. schools. However, both Hispanic and ELL students are not faring as well as their counterparts on national achievement data. Eighth-grade Hispanic students scored 23 points lower than white students in mathematics on the 2019 National Assessment for Educational Progress (NAEP, 2020). Even more concerning, the difference between ELL and non-ELL students in eighth-grade mathematics was a staggering 41 points (NAEP, 2020). The alarm is clear: students in the fastest growing demographic group in the United States may be struggling to attain mathematics

proficiency. However, Gutierrez (2008) does warn against “gap-gazing” in which a focus on achievement gaps excludes other issues surrounding equity. She suggests that supporting mathematical identities and mathematical literacy be used as levers for informing gaps as they are areas that can be addressed within schools (Gutierrez, 2008).

The conditions described above illuminate the layered challenges faced by many Hispanic ELL students in mathematics. Addressing the first challenge in the scenario—success in middle grade mathematics—may be the first step in smoothing the academic pathway for these students. The National Council of Teachers of Mathematics suggests in the 2014 publication *Principles to Actions* that “effective teaching is the non-negotiable core that ensures all students learn mathematics at high levels” (p. 4).

An abundance of literature exists describing educational barriers that ELL students face. These barriers include parents’ immigrant and socioeconomic status (Schneider, Martinez, & Owens, 2006), a lack of preschool learning opportunities (Peters & Engerrand, 2016), a lack of English proficiency (Reardon & Galindo, 2009), and high mobility rates (Hanson et al., 2017). Schneider, Martinez, and Owens (2006) refer to the Hispanic educational experience as one of “accumulated disadvantage” (p. 179), listing weak relationships with teachers, inadequate school resources, and a lack of guidance around course selection as factors contributing to this challenging environment. Furthermore, the growing population of ELL students is outpacing the availability of professional development for teachers aimed at meeting the needs of these students (Vladu, 2016). Melhuish et al. (2022) recently concluded that professional learning that attends to the needs of minority groups will need to incorporate bias reduction as a major component. Their study found that while their professional-development model made gains in

teacher knowledge and the implementation of quality instruction, it did not increase standardized test scores for students from minority backgrounds. They describe their findings as a “cautionary tale” for educators that a “race neutral approach to equity may account for an inequitable result for otherwise successful professional development” (p. 308). Thus, studies involving minority student groups and instructional practices will need to attend to the experiences of the students involved specifically including multiple types of equity-based pedagogy.

Research has demonstrated that highly effective instructional practices have a positive impact on student learning (Hiebert & Grouws, 2006; Hattie, 2009; Marzano, 2007). In recent decades, scholars have called for an in-depth examination of instructional practice that would uncover effective approaches to accommodate differences among students, especially those from marginalized groups based on low socioeconomic status (SES), immigrant status, minority populations, and ELLs (Battey, 2013; Lubienski, 2002; Gutierrez, 2013). In response, the National Council of Teachers of Mathematics (NCTM) produced *Principles to Actions* (2014) to support the adoption of the Common Core State Standards for Mathematics (CCSSM) in over 40 states. While the CCSSM provides guidance regarding what mathematics to teach and when, the Mathematics Teaching Practices (MTPs) described in *Principles to Actions* recommend how best to deliver this content. These recommendations stem from cognitive science and mathematics education research identifying specific types of experiences that benefit learners in a mathematics classroom. They include engagement with challenging tasks, connection with prior knowledge, development of conceptual and procedural knowledge, a construction of knowledge in a social environment, access to timely feedback, and development of metacognitive awareness (NCTM, 2014). NCTM drew on research supporting the effectiveness of these student

experiences in developing the eight MTPs into a framework for teaching and learning in mathematics.

However, little research to date investigates student perceptions regarding the classroom instructional practices that encourage their success in mathematics (De Araujo et al., 2018; Sanchez-Gutierrez et al., 2015). Including students as stakeholders who have knowledge to shape education can improve current practice and point to additional areas of improvement (Cook-Sather, 2002). Sanchez-Gutierrez et al. (2011) state, “Latina/o students have seldom been asked for their perspectives on their classroom mathematics experiences or what insights they might provide about the possibilities of enhanced mathematics learning opportunities” (p. 27).

However, the limited, available research does suggest that students’ perceptions of their learning influence their academic achievement (Thompson, 2009). Sands (2007) notes, “Despite intense endeavors to promote educational change to affect student achievement, one voice, perhaps the most critical voice that could inform the debate on how to increase student achievement, is sorely lacking: that of students themselves” (p. 324). This is especially true for the Hispanic ELL perspective in the mathematics classroom (Garza, 2009).

Purpose of Study

In recent years, effective teaching practices founded on research have become more prevalent in mathematics classrooms (Hattie, 2009; Smith et al., 2017). However, there remain many questions about whether these practices are equally effective between Hispanic ELL students and their non-Hispanic, non-ELL counterparts. Limited research exists exploring how Hispanic ELLs benefit from practices that have been determined as highly effective. The purpose

of this qualitative case study is to examine the perceptions of Hispanic ELL middle-school students regarding the efficacy of research-informed instructional practices, as embodied in the Mathematics Teaching Practices. The goal is to determine if Hispanic ELL students perceive the practices as contributing to their success in mathematics and if those perceptions align to those of their teachers. This comparison is important because Huang and Barlow (2013) found that, when there was consistency between what the teacher intended and the students' perceived importance of the learning, it was more likely the students would engage with the mathematical task at the cognitive level the teacher intended.

Attention to student perspectives is relatively unique in education literature. Sands et al. (2007) posit, "As schools, parents, and districts coalesce around improving schools, many voices echo concerns about the future of America's children. One voice falls eerily silent—the voice of the child." (p. 341). This voice can enrich current understandings of how mathematical learning occurs (Anthony et al., 2013). It often seems as if educational systems are designed without consulting those they serve (Cook-Sather, 2002). This study will contribute to the literature about mathematics teaching and learning by including Hispanic ELL middle-school students as contributors in the educational process.

Research Questions

This study examines the perceptions of Hispanic ELL middle school mathematics students and their teachers regarding the effectiveness of specific mathematics teaching practices. It is guided by the following research questions:

1. How do Hispanic ELL middle-school students perceive the effectiveness of three research-informed mathematics teaching practices in supporting their mathematical understanding?
2. How do teachers of Hispanic ELL middle-school students perceive the effectiveness of the mathematics teaching practices in supporting their students' mathematical understanding?
3. In what ways do these students' and teachers' perceptions align and diverge?

Significance of Study

Examining the perceptions of Hispanic ELL middle-school students regarding the efficacy of certain mathematics teaching practices provides insight about how research-informed teaching practices support mathematical understanding and learning for this population. This effort holds significance for middle school teachers, school leaders, students, and researchers as they work to improve current practice and shape further efforts to be taken in school systems toward improving outcomes for Hispanic ELL students (Cook-Sather, 2002).

Within the mathematics classroom, findings from this study offer practitioners a better understanding of how implementing research-informed mathematics teaching practices can support the learning of Hispanic ELL students. Emphasizing the student perspective provides insight into the effectiveness of various mathematics teaching practices so that teachers can successfully utilize them in the classroom setting. Constructivist theorists argue that teachers can improve their practice by listening to what students have to say (Cook-Sather, 2002).

For school leaders, this study identifies ways to guide a collaborative approach to teaching and learning between teachers and Hispanic ELL students in several ways (Cook-Sather, 2002). First, it contributes to the conversation around the role of student perceptions in better understanding which mathematics teaching practices are effective and in which ways (Cook-Sather, 2002). Second, it brings to light issues from a Hispanic ELL perspective that may have been omitted from the conversation in the past (Davila, 2010). Last, it validates students as stakeholders in the educational process (Cook-Sather, 2002). Inviting students into educational conversations will support school leaders' efforts in making their schools places where students want to learn.

Being considered as contributors to the educational process is beneficial for learners, especially Hispanic ELL students whose perspective has been found to be lacking in the literature (Garza, 2009; Halx & Ortiz, 2011). When students see themselves as key players in their educational experience, they feel more empowered and motivated to participate in the process (Sands et al., 2007; Cook-Sather, 2002; Thompson, 2009). Thus, this study has the potential to provide information that will improve mathematics learning for Hispanic ELL students.

The findings from this study support educational research by contributing to the sparse literature base on Hispanic ELL mathematics students (de Arujo et al., 2019; Roberson & Summerlin, 2005) and research that incorporates student perceptions, especially from the Hispanic ELL viewpoint (Sands, 2007; Garza, 2009; Halx & Ortiz, 2011). Thus far, research has focused on broad classroom practices that support Hispanic ELL students (Echevarria et al., 2008; Goldenberg, 2014) rather than pinpointing practices that specifically support mathematical

understanding and learning (de Arujo et al., 2019; Roberson & Summerlin, 2005). Additionally, by examining student perceptions, this study enhances the literature with a data source that is often overlooked. Davila (2010) found a similar gap in the literature when working in Chicago schools to gather insights from Hispanic high school students and highlighted how pedagogical shifts like using student input when making curricular decisions can impact practices to better serve the needs of Hispanic students.

Theoretical and Conceptual Framework

The theoretical framework for this study is based on social constructivist theory, which suggests that learning is constructed through social interactions, reflection, and internalization (Fox, 2001; Schunk, 2016). Grounded in the work of theorists such as Lev Vygotsky (DeVries, 2000), the social constructivist paradigm views learning as an active rather than passive process and as being socially constructed (Fox, 2001; Liu & Matthews, 2005). In mathematics education, the movement toward social constructivism as a key theoretical lens for practice, management, and policy in the 1980s provided researchers with a new window for exploring and understanding mathematics learning. Assuming that the nature of cognition is inherently social (De Freitas & Walshaw, 2016; Gutierrez, 2013), a social constructivist paradigm provides a lens for how individual students create meaning and knowledge to better interpret the world around them (Vladu, 2016). The social constructivist perspective considers participants' views, values, beliefs, feelings, assumptions, and ideologies, lending itself as a useful framework for incorporating student perspective (Creswell & Creswell, 2018).

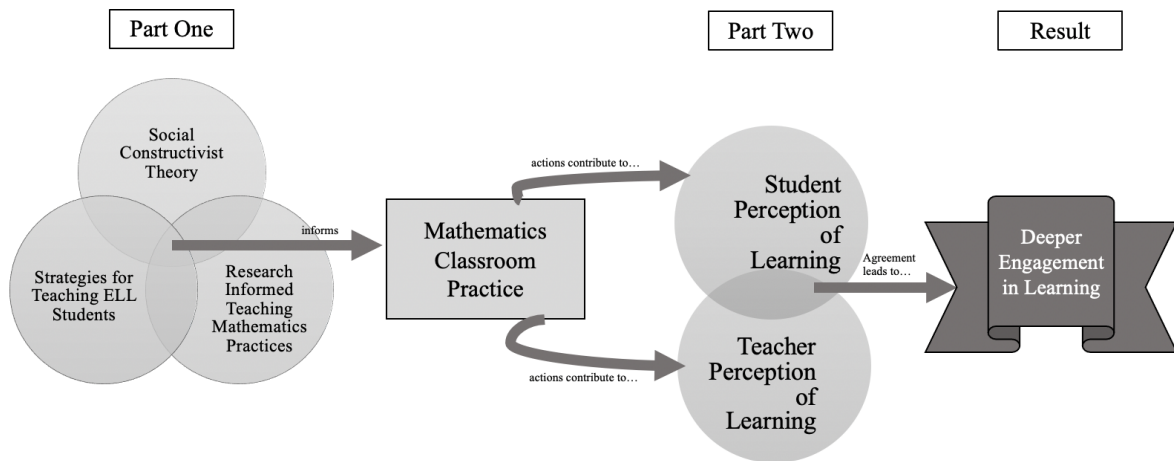
Many of the general classroom practices that have been reported in the literature as supportive of ELL student learning, such as revoicing of student statements (Moschkovich, 1999), asking questions that promote critical thinking (Echevarria et al., 2008), promoting and privileging meaning regardless of language (Moschkovich, 2012), treating everyday language as a resource (Moll, 2019; Moschkovich, 2012), using hands-on activities, and supported discussion (Echevarria et al., 2008), are considered social constructivist practices. These practices allow students to create meaning through social interaction (Vladu, 2016).

General practices that support ELL students closely align with the NCTM Mathematics Teaching Practices. For example, revoicing student statements, elevating everyday language, and supported discussion are all elements of the MTP “facilitating meaningful mathematical discourse” (Smith et al., 2017). Privileging meaning regardless of language and the use of hands-on activities align with the MTP “using and connecting mathematical representations” (Smith et al., 2017). Privileging meaning regardless of language means that teachers are seeking to ensure that students are making sense of the mathematics with whichever communication means possible (Moschkovich, 2012). Asking questions that promote critical thinking aligns with the MTP “posing purposeful questions.” In fact, the NCTM vision for students as mathematics learners states that “students are active learners, constructing their knowledge of mathematics through exploration, discussion, and reflection” (Steele, et al., 2017, p. 2), thus highlighting this critical aspect of social constructivist theory in mathematics learning.

Conceptual frameworks guide research and provide a model of the researcher’s thinking by synthesizing what is already known with new or under-researched elements in an empirical study (Savin-Baden & Howell Major, 2013). The conceptual framework for this study draws on

the theoretical stance that student learning occurs through social interaction. This relationship is illustrated in Figure 1. Part One of the diagram depicts how social constructivist theory, known strategies for teaching ELL students, and research-informed mathematics teaching practices inform classroom practice. The actions involved in classroom practice generate both teacher and student perceptions of the learning as seen in Part Two of the diagram. It is posited that areas in which these perceptions align have the potential to result in a deeper engagement with the learning.

Figure 1.1. Conceptual Framework



Research Methodology

This study takes a pragmatic design approach in investigating perceptions of middle school Hispanic ELL students and their teachers regarding the efficacy of specific mathematics teaching practices. When utilizing a pragmatic methodology, the researcher is concerned with the outcome and application of the findings in a desire to find a solution to a problem, and the research occurs in a specific context (Creswell & Poth, 2018); both conditions that are well

suited to this study. Additionally, a pragmatic approach emphasizes actionable knowledge (Kelly & Cordeiro, 2020). The results from this study are immediately actionable for educators working with Hispanic ELL students in mathematics. Finally, pragmatism treats inquiry as an experiential process. This means that the process is embedded in everyday situations, and the inquiry focuses on the human experience (Dewey, 1920/2008; Morgan, 2014). In this study, the research questions are based on the experiences of Hispanic ELL students and their teachers in a middle school mathematics classroom.

Given the nature of a pragmatic approach, case study is an appropriate methodology to investigate the research questions (Savin-Baden & Howell Major, 2013). Case studies have many advantages for the researcher. They allow flexibility and the opportunity for in-depth investigation (Savin-Baden & Howell Major, 2013). The in-depth investigation required of a case study can produce a wide variety of data, such as interviews, observations, documents, and media that are likely to provide insight in answering the research questions (Creswell & Poth, 2018). Greig (2013) suggests a case study approach when exploring strategies that do and do not work for children, especially when working with a small group of individuals in a bounded situation. Thus, a case study methodology is best suited to examine mathematics teaching practices through the perspectives of teachers and Hispanic ELL students.

An important feature of a case study is that it can be defined within several parameters (Johnson & Christensen, 2017). In this research design, the case study is bound by three parameters. First, the case is situated in a rural Western resort town, which is of interest as resort towns support the service industries in which many Hispanic immigrants find employment. The findings may be transferable to other similar resort locations in the Intermountain West. Second,

purposive sampling intentionally constrains the study population to Hispanic ELL students in one middle school to focus on the perspectives and shared experiences of a specific demographic group in a specific location. Last, the design is bound by the timeframe of one semester. These boundaries limit the object of study, which is the “single most defining characteristic of a case study” (Merriam, 1998, Location number 395).

This study was conducted in two phases. Phase One: Preparation and Pilot Study included site selection, rating scale development, teacher and classroom selection, and an alignment of operational terms between the researcher and teacher participants. Phase One also involved a pilot study to test the interview protocol. Yin (2014) recommends a pilot study to refine relevant interview questions. A pilot group of Hispanic ELL students (N=2) were interviewed to refine the questions and logistics that were used with the study group (Sands et al., 2007). Phase Two: Data Collection occurred during the second semester of the school year and employed a video stimulated response (VSR) and semi-structured interview process. In three different middle school mathematics classrooms, the researcher made a series of video recordings of distinct mathematics lessons that incorporated at least one of the MTPs. The videos were reviewed during the interview process to support an accurate recall of the teacher’s implementation and the students’ response to the MTP. Teachers and Hispanic ELL mathematics students from the three classrooms were interviewed three times each to ensure adequate data for answering the research questions.

Operational Definitions

Emergent Bilingual: Emergent bilinguals are students who are in the “early stages of developing an additional language” (Freeman et al., 2021, p. xvii).

English Language Learner (ELL): The National Center for Educational Statistics (NCES) defines English Language Learners as students who participate in language assistance programs that guide them to English proficiency to meet academic standards required by all students (NCES, 2020). The U.S. Department of Education defines an ELL student as “a national-origin-minority student who is limited-English-proficient” (U.S. Dept. of Education, 2021, Developing programs for ELLs section). According to federal policy, ELLs in public schools are entitled to language support until they demonstrate sufficient proficiency on state-approved English language proficiency tests (Title III, No Child Left Behind, 2005). Other terms commonly found in the literature include English Learners (ELs), Limited English Proficient (LEP) students, English as a second Language (ESL) students, and Multilingual Learners (MLL).

Hispanic: The U.S. Office of Management and Budget, which conducts the U.S. Census, defines Hispanic as a “person of Cuban, Mexican, Puerto Rican, South or Central American, or other Spanish culture or origin regardless of race” (Bureau U.S.C, 2020, About Hispanic Origin section). The majority of students involved in this study were from Mexico, or their immediate family (mother, father) were from Mexico.

Language Partners: This term was used at the study site to describe a partnership between students in which one student was bilingual and the other was monolingual. The bilingual student was able to provide translation for the monolingual student. The partnerships shifted

throughout the semester. Bilingual students would rotate the language partner role throughout the semester to support the monolingual students.

Mathematics Teaching Practices (MTPs): The National Council of Teachers of Mathematics developed the Mathematics Teaching Practices serving as a framework for teaching and learning in mathematics. The eight practices are: (1) establish mathematics goals to focus learning, (2) implement tasks that promote reasoning and problem-solving, (3) build procedural fluency from conceptual understanding, (4) pose purposeful questions, (5) use and connect mathematical representations, (6) facilitate meaningful mathematical discourse, (7) elicit and use evidence of student thinking, and (8) support productive struggle in learning mathematics (Smith et al., 2017). The three chosen for this study were: pose purposeful questions, use and connect mathematical representations, and facilitate meaningful mathematical discourse because of their connection with social constructivist theory and literature on general classroom strategies that support Hispanic ELL students in all subject areas.

Monitor status: The definition of monitor status varies from state to state. At the study site, monitor status indicated that an ELL student had a composite score of 4.7 or higher on the WIDA ACCESS assessment for ELLs. The student no longer received direct support services, however, school staff continued to “monitor” the student’s achievement and success.

Student Consultation: Student consultation involves seeking student views about existing programs (Bragg, 2010). It can be collaborative and participatory in nature or collected through data gathering instruments such as surveys. Rudduck and McIntyre (2007) define student consultation as a conversation between teachers and students that allows students to know that their views are being sought because they have something to contribute.

Student Perception: Perception is a way in which one attends to reality and experience through the senses. “Individual perception influences opinion, judgment, understanding of a situation or person, meaning of an experience, and how one responds to a situation. Qualitative researchers are most interested in individual perception to gain access to understanding the meaning of experience for an individual, a culture, and or social groups” (Given, 2008, Perception Section).

Student Perspective: The direct articulation from a student about events and circumstances that influence the student’s world (Wilson & Corbett, 2001).

WIDA ACCESS Assessment: “ACCESS for ELLs (ACCESS) is the collective name for WIDA's suite of summative English language proficiency assessments. ACCESS is taken annually by English language learners in Kindergarten through Grade 12 in WIDA Consortium member states” (WIDA, 2022, ACCESS section).

Assumptions, Limitations, and Delimitations

Assumptions

In this study, the researcher assumed that the students interviewed are representative of Hispanic ELL students at the study site.

Limitations and Delimitations

All studies are limited by a variety of factors. This study was no exception, with limitations that existed both within and outside of the researcher’s control. These limitations must be stated and considered in the context of the study and its findings. There are two limiting factors and four delimiting factors to consider in this study.

Limitations. First, the experience and skill set of the teachers chosen for the study is a limitation. Teachers have varying levels of understanding of the NCTM Mathematics Teaching Practices or practices that are supportive of Hispanic ELL students and varying levels of comfort and experience in a classroom. These varying levels of skill and experience may have affected the results of this study.

Second, a small number of students from a single school district were invited to participate in this study and, thus, may not be representative of all Hispanic ELL students in the region or state. This small number of students represents a unique population of Hispanic ELL students that live in a rural, affluent community with strong school supports. This may limit the applicability of the findings to other contexts.

Delimitations. This study is subject to several delimitations. The first delimiting factor of this study is the selection of only three Mathematics Teaching Practices. While there are eight MTPs that could be studied, this study focused on three MTPs that align closely with a social constructivist paradigm. The additional MTPs may or may not be more effective in supporting learning for Hispanic ELL middle-school students.

Second, the choice to conduct video stimulated recall interviews could be considered a delimiting factor. Students may or may not react well to seeing themselves on video. While VSR has been shown to produce positive interview situations for students, (Meier & Vogt, 2015; Hunter, 2017), this may not be the case in all situations.

Third, student perception can be considered a delimiting factor. Some researchers contend that students cannot provide accurate accounts of a situation (Thompson, 2009). In this study, interviews were supported by video recordings of the MTPs being used in a classroom

setting. The videos provided memory stimulation and accuracy in responding to interview questions.

Last, data were collected over a fixed timeframe of one semester. This choice was made because seventh- and eighth-grade students at the chosen site only had math for one semester of the 2021–2022 school year. There was not an opportunity to interview students during different times of the year or over the course of several years. This is a snapshot of students’ perspectives from a particular class with a particular teacher. This snapshot may or may not reflect their experience with other teachers or levels of mathematics.

Chapter Summary

There is an observable disparity between the achievement of white, English-speaking students and Hispanic ELL students in mathematics (NCES, 2019). The consequences resulting from disparities in academic opportunity are far reaching. These include inequitable mathematics programming, a growing excellence gap, and the economic future of families (Tyson, 2007; Plucker, 2010). Students with low mathematical skills tend to end up with the lowest paying jobs, thus repeating the cycle of poverty (Moses, 2001; Hanushek et al., 2013). Our educational systems have an “obligation to provide equitable high-level mathematics to all students,” (Schoenfeld, 2002, p. 13).

While research has identified several teaching practices that generally support students’ mathematical learning, less is known about how to best meet the needs of the growing Hispanic ELL population, especially in mathematics instruction (Vladu, 2016; De Araujo et al., 2018; Baker, 2013). By examining middle-school, Hispanic ELL students’ perceptions related to

specific Mathematics Teaching Practices, the results of this study contribute to the literature base on teaching mathematics to Hispanic ELL students. Findings may provide school practitioners valuable insight regarding which research-informed instructional practices are effective in supporting mathematical learning for Hispanic ELL middle-school students. Additionally, they may also benefit other audiences: school leaders, middle-school Hispanic ELL students, and researchers.

CHAPTER TWO

LITERATURE REVIEW

This chapter synthesizes the relevant theoretical and empirical literature regarding how Hispanic English Language Learners (ELLs) learn mathematics and the practices that support them. Additionally, literature that incorporates student perceptions as data in answering research questions will be part of this review. This review is organized into four topics: learning theory as it connects to teaching and learning for middle school mathematics, strategies for teaching ELL students, the Mathematics Teaching Practices as described by the National Council of Teachers of Mathematics in their *Principals to Actions* (NCTM, 2014) publication, and how student perception and consultation inform research.

Learning Theory

The complex nature of learning and how it occurs has been a topic explored by theorists for generations. Some of the earliest theories are represented in the behaviorist ideas of Ivan Pavlov and John Watson and constructivist theories of Jean Piaget and Lev Vygotsky. The social cognitive theories of Albert Bandura and, most recently, the expanding field of neuroscience (Schunk, 2016) also contribute to the vast literature base about the nature of learning. Within mathematics education, two distinct and somewhat contradictory learning theories have guided educational researchers: behaviorist theory that proposes learning occurs outside of the mind and constructivist theory claiming that learning is actively built inside the mind (Bozkurt, 2017).

However, beginning in the 1980s, constructivism rose to the forefront as the dominant paradigm of mathematics education research and practice.

The underlying premise of constructivist theory is that knowledge is built or actively constructed, (Fox, 2001). From that premise, two seemingly opposing theories have evolved: cognitive constructivism and social constructivism. Cognitive constructivism views the individual as the unit of analysis in respect to how learning occurs. Conversely, social constructivism sees the unit of analysis in the learning process as the individual in a social interaction (Cobb, 1994). However, Cobb (1994) counters that the two theories should be considered complementary within mathematical learning. He states, “mathematical learning should be viewed as both a process of active individual construction and a process of enculturation into mathematical practices of wider society” (p.13). In recent years, social constructivism has been more prevalent in the field of mathematics education, emphasizing that learning happens through social interaction both inside and outside of school contexts (Bozkurt, 2017). After defining learning, both cognitive and social constructivist theories will be included in this literature review as both provide a foundation for understanding how students learn mathematics.

Learning Defined

Over the years, the construct of student learning has been defined in a variety of ways. For example, Schunk (2016) defines learning as a change of behavior due to practice or experience. However, Hattie and Donoghue (2016) view learning as an outcome of processes that lead to enhanced skills, dispositions, and motivations. While these definitions have distinct emphases, common to both definitions is the idea that an enduring change occurs within the

learner. This study will use Schunk's (2016) definition that learning results in enduring change which evolves from experiences.

Cognitive Constructivism and the Learning of Mathematics

Piaget's theory of learning (1963) grew out of his observations of children, with a focus on what occurs within an individual during the knowledge acquisition process (Ojose, 2008; Loc, 2019). Even though social interaction is present in learning, Piaget identified the individual cognitive process as the point of learning (Bozkurt, 2017). Loc (2019) explains this by stating that in a Piagetian view, "people cannot be provided with information for immediate understanding and use. They must build their own knowledge through a cognitive process" (p.719).

Piaget's theory of cognitive constructivism involves a process in which assimilation, defined as the "application of existing cognitive schema without changing that schema into a new cognitive object" (Loc, 2019, p. 719) causes an imbalance in which new cognitive schema is in contrast with past cognitive schema. Cognitive schema is defined as a "cognitive structure that organizes large amounts of information into a meaningful system" (Schunk, 2016, p. 496). Cognitive schema is often described by educators as background knowledge (Cho & Ma, 2020). The resulting imbalance requires an accommodation to return to balance (Loc, 2019). Accommodation as defined by Piaget (1953) is a replacement of current cognitive schema because of an acquisition of new knowledge. We accommodate to make sense of the new schema being taken in (Schunk, 2016). Once the accommodation is made, the learning process returns to a balanced state and there is a new cognitive schema (Loc, 2019).

Loc (2019) tested the effectiveness of Piaget's approach in a mathematics classroom while teaching 39 Grade 10 and 11 geometry students in Vietnam to define the directional vector of a straight line. Teachers in the study guided students through six phases: recalling relevant knowledge, inputting information and assimilating, creating cognitive conflict, accommodating, testing and evaluation, and connecting the learning. Analysis found that to ensure that students learn new knowledge, all six phases had to be effectively implemented by their teachers to avoid misconceptions in the cognitive schema.

Piaget also identified four stages of cognitive development that all students pass through, which shape what and how they learn. In the sensorimotor stage between birth and the appearance of language, elements relevant to mathematics learning that manifest are object permanence, counting, and quantity comparison such as "who has more?" (Ojose, 2008). During the preoperational stage (between 18 months and 7 years), students begin characterizing concepts and determining which items belong together in groups. The concrete operational stage (ages 7–11) represents an important period of cognitive growth. In this stage, students begin to make connections between mathematics activities and concepts to make abstract ideas concrete. It is in the final stage, formal operations (over 12 years old), that students begin to think abstractly, hypothesize, and create inductive and deductive inferences (Ojose, 2008).

In the field of mathematics education, Piagetian theory is often criticized by researchers who claim that the stages underestimate the abilities of younger learners and overestimate the abilities of older learners (Ojose, 2008). Additionally, this focus on the individual has been a foundational debate among mathematics education researchers who seek to choose between cognitive constructivism and social constructivism (Cobb, 1994). Confrey (1995) claims that the

individual perspective is crucial. If social constructivist researchers overlook the individual, they risk under investigating the strength and diversity of individual constructive processes.

Social Constructivism and the Learning of Mathematics

Contrary to classical, behaviorist views in which learning resides in the mind rather than with interaction (Schunk, 2016), social constructivists posit that both social interaction and individual meaning are vital within the construct of learning mathematics (Bozkurt, 2017).

Walshaw (2016) states, “The work of Lev Vygotsky has provided researchers with the theoretical tools to interpret the social origins of thinking and learning” (Walshaw, 2016, p. 293). From a Vygotskian perspective, mathematical development is a process involving participation, communication, and collaboration (Walshaw, 2016), thus, Vygotsky’s theories will provide a framework for this discussion.

The Vygotskian theory of learning prioritizes the role of social and linguistic influences in the learning process. His studies suggested that a child’s learning was influenced by their interactions with their social environment as well as their cultural and historical background (De Freitas & Walshaw, 2016). In Vygotskian theory, all learning is social through the semiotics of language mediated by symbols, art, road signs, writing, drawings, and other cultural tools (De Freitas & Walshaw, 2016). The Vygotskian view suggests that, in order to learn, the student must become aware of their thoughts and engage in critical reflection resulting in self and social transformation. This process is called internalization (De Freitas & Walshaw, 2016). Vygotsky’s critics have questioned the claim that all higher mental functions originate in the social environment, citing examples showing that children may be biologically predisposed to learning (Fox, 2001).

Vygotsky's theory also contends that learning and cognitive functions occur within the Zone of Proximal Development (ZPD). When students are in the ZPD, the tasks they are attempting and the concepts they are learning are beyond their immediate capabilities (Walshaw, 2016). This means that a teacher, another adult, or a more knowledgeable peer must assist the learner in reaching a new understanding (Albert, 2000). For example, in a study involving seventh-grade mathematics students, Albert (2000) investigated the relationship between oral and written thought processes. The study was grounded in a Vygotskian framework and thus included collaborative, individual, and whole class reflections about their mathematical problem-solving and was designed to explore the ways in which the ZPD supports middle school students' oral and written ideas about problem-solving.

In her research, Albert concluded that understanding mathematics through written processes was supported by being situated in the ZPD. Additionally, Albert stated in her discussion that “the social nature of mathematics was an integral part of the learning process” (Albert, 2000, p. 138). Based on the findings from this study, the author concluded that the social nature of mathematics and positioning students within their ZPD supported using Vygotskian principles in the mathematics classroom.

Learning in the Mathematics Classroom

Social constructivist learning theory proposes that students develop higher functions through social and collaborative activities (Walshaw, 2016). In classrooms that include social and collaborative interactions, students actively construct their own knowledge rather than passively taking in knowledge. The social constructivist theory can be observed in mathematics classrooms in which students are part of a mathematical community. Within these communities,

students share their mathematical thinking through discussion and mathematical representations (Moschkovich, 2002). Mathematics education researchers have seen evidence of successful social constructivism strategies in several studies (e.g., Cobb et al., 1993; Taylor & Cox, 1997) in which students co-create knowledge through collaboration and meaningful tasks (Harkness, 2009). For example, after studying a group of second graders for a full year, Cobb et al. (1993) concluded that “mathematical activity can be viewed as intrinsically social in that what counts as problem and resolution has normative aspects” (p. 93).

Learning (enduring change which evolves from experiences) of mathematics is influenced by both instructional practices and the social norms of the classroom community. For example, Taylor and Cox (1997) conducted a study focused on the social aspect of mathematics with a group of fourth-graders. Using a control group and two study groups incorporating peer interaction, the researchers investigated peer interactions. One group used modeling techniques and the other group used socially assisted (reflective questioning and shared ownership) techniques. Both peer interaction groups showed superior quiz results when compared to the control group, and the socially assisted group outperformed the modeling group as well as the control group. This example illustrates how social constructivist theory influences practices and social interactions in a mathematics classroom.

Conclusion

Some educational researchers have turned to constructivist theories to guide their inquiry related to teaching and learning (Cobb et al., 1993). Mathematics educators have specifically been drawn to social constructivist paradigms to provide insight regarding how students learn and construct meaning in mathematics (Albert, 2000). Boaler and Staples (2008) found that

classrooms with social constructivist practices promoted more active intellectual roles for high school students and led to increased achievement and narrowing of the opportunity gap. The social constructivist paradigm provides support when working with ELL students (Boaler, 2008) as they navigate educational systems that are still working to meet their needs (Moschkovich, 2013). The research evidence suggests that mathematics students, especially ELL students, benefit from the use of social-constructivist practices providing students the opportunity to co-create mathematics knowledge with a community of learners (Goos et al., 2004; Moschkovich, 1999).

Teaching Mathematics to English Language Learners

As presented in Chapter One, the most recent data from the National Center for Educational Statistics (NCES, 2019) reported that 10.1% of all school-age students were identified as ELL, thus representing the fastest-growing student demographic (NCES, 2020). Of those students, 76% are of Hispanic descent (U.S. Department of Education, Office for Civil Rights, 2015–16). Hispanic ELL students are a substantial and growing population in U.S. schools. National achievement paints a comparative picture between the success of Hispanic and ELL students and their non-Hispanic, non-ELL counterparts in the area of mathematics. In the most recently released National Assessment of Educational Progress (NAEP) data, Hispanic eighth-grade mathematics students scored 23 points lower than white eighth-grade mathematics students (NAEP, 2020).

The available research about why this gap in achievement exists has identified several out-of-school and within-school factors that affect the Hispanic ELL students’ “opportunity to

learn” mathematics (Gandara, 2006). The construct of “opportunity to learn” has been defined as “not only students’ access to mathematical content and discourse practices, but also their access to (positional) identities as knowers and doers of mathematics” (Herbel-Eisenmann et al., 2013). Out-of-school barriers that may impede a student’s learning of mathematics include healthcare, nutrition, housing stability, neighborhood environment (Gandara, 2006), socioeconomic status and family background, and their linguistic environment (Reardon & Galindo, 2009). In-school barriers include an unclear message about the purpose of mathematics (Gutstein, 2003; Shoenfeld, 2002), negative teacher beliefs about student ability (Rousseau & Tate, 2003), lack of student participation in the learning process (Gutierrez, Willey, & Khisty, 2001), limited English proficiency, and poor school quality (Reardon & Galindo, 2009).

Research has also identified several school-based solutions that can support Hispanic ELL students’ learning of mathematics. The literature recommends four areas of focus when improving the teaching of mathematics to Hispanic ELL students: (1) provide an increased opportunity to learn (OTL) (Abedi et al., 2006), (2) encourage classroom structures that support constructivist theory (Boaler & Staples, 2008; NCTM, 2014), (3) promote the language of mathematics (Moschkovich, 2012), and (4) develop positive relationships (Cobb, 1994). In all four areas, high-quality instruction for ELL students treats language as a resource and emphasizes academic achievement, not just English language development (Moschkovich, 2013; Echevarria et al., 2008).

Increasing Opportunity to Learn

The National Research Council (2001) claimed that a vital predictor of student achievement is the opportunity to learn (OTL), which accounts for differences in mathematics

learning across contexts and links teaching to learning (Hiebert & Grouws, 2007; Harris & Anderson, 2012). As immigrant students and the children of immigrant parents, Hispanic ELL students face a variety of challenges such as high poverty, attendance in schools with fewer educational resources (Hanson et al., 2017), lower levels of parental spending (Kornrich & Furstenberg, 2013; Peters & Engerrand, 2016), and limited early literacy (Peters & Engerrand, 2016). These have resulted in unequal access to resources inside and outside the classroom and have created a lack of equal opportunity to learn (OTL) (Harris & Anderson, 2012; Sanchez-Gutierrez, 2015). OTL describes access to learning that goes beyond curriculum and addresses issues such as teacher content knowledge, pedagogical awareness, resources, and preschool learning (Gutierrez, 2008; Peters & Engerrand, 2016). A student's prior knowledge and experience are also an essential component of their OTL (Hiebert & Grouws, 2007). Understanding the challenges facing Hispanic ELL students and their limited OTL helps educators position these students in learning environments that increase the likelihood of engaging in the learning (Hiebert & Grouws, 2007; De Freitas & Walshaw, 2016).

Previous research has documented a reduced OTL for low-income and minority students (Hiebert & Grouws, 2007; De Freitas & Walshaw, 2016). For example, in a study involving 314 kindergarten and first-grade students in 48 districts in 3 states, Stipek (2004) found that the quality of schools and observed classroom instruction were associated with the student body's demographics. High proportions of Hispanic and African American students in these classrooms were correlated with high levels of didactic instruction, low levels of constructive teaching practices, and negative social climates (Stipek, 2004). Another study (Abedi et al., 2006) produced a technical report that highlighted the impact of OTL on eighth-grade ELL students.

The study measured three class-level components of OTL: (1) a student report of content coverage, (2) teacher content knowledge, and (3) prior mathematics ability. Using hierarchical linear modeling, the team found that all three OTL components were significantly related to mathematics performance. Of particular interest was the finding that the ability level of a class had almost twice the effect on the performance of mathematical outcomes when compared to teacher content knowledge and content coverage. This is interesting because previous research (Wang & Goldschmidt, 1999) had shown that ELL students are traditionally placed in lower-performing classrooms. To increase the OTL for Hispanic ELL students, the discussion may need to move beyond the general awareness of changing demographics to the meaningful inclusion of strategies that address the coverage of content, teacher content knowledge, and the appropriate placement of students in mathematics courses (Abedi et al., 2006; Aguirre et al., 2013).

Constructivist Classroom Structures

The National Council of Teachers of Mathematics (NCTM) has identified five classroom structures that can contribute to lower levels of conceptual understanding among students (NCTM, 2014). These five structures are: (1) a focus on procedures without connection to meaning, (2) low expectations for students, especially ELLs, (3) over-assessment of skills, (4) isolated teachers that do not have the opportunity for collaboration or professional development, and (5) a limited access to tools and technology (NCTM, 2014). Examples of how schools can address these barriers were explored in a study by Boaler and Staples (2008) in California. In a study of three secondary schools in California, Boaler and Staples (2008) compared the outcomes of over 700 students across three high schools. One school was in an urban setting

with an ethnically, linguistically, and economically diverse student body. One of the schools was on the coast with a predominately white community, and the third school was rural with mostly white and Hispanic students. Two of the schools were traditional in the presentation of mathematics, meaning that a procedure-oriented curriculum was taught. The third school relied on a “reform” approach. In the Boaler and Staple (2008) study, conceptually oriented mathematics materials, taught well and with consistency broadly defined a reform approach. The third school was also highly committed to utilizing challenging conceptualized problems, student presentation, heterogeneous grouping, and teacher collaboration (Boaler & Staples, 2008). The case study found, through interviews and questionnaires, that the students at the reform school enjoyed mathematics more and believed that they had the authority and capacity to validate mathematical methods. After the three-year study was complete, the reform school no longer saw significant gaps between white and Hispanic students on achievement data, even though the reform school students were performing at significantly lower levels than the other two schools prior to the start of the study (Boaler & Staples, 2008).

The researchers concluded that the positive outcomes were due to the fact that the teachers at the reform school were able to create classes in which there was a high level of cognitive demand while placing emphasis on effort over ability (Boaler & Staples, 2008). This was achieved using a social constructivist approach in which teachers incorporated projects that encouraged students to create mathematical beliefs and practices consistent with the type of cognitive demands required of mathematics in real-world contexts (Boaler & Staples, 2008). This structure allowed students to present a variety of methods and solutions, all of which were valued within the classroom setting. This process, known as multidimensionality, was thought to

build student efficacy in their thinking and allowed teachers to raise student competence through student contributions to discussions and examples (Boaler & Staples, 2008).

A similar study in Australia by Merrillyn Goos (2004) highlighted a different reform approach to teaching in a mathematics classroom. In this study, the researcher explored how a culture of inquiry in a secondary mathematics classroom could develop mathematics sense-making, justification of mathematical thinking, and mathematical learning practices within students (Goos, 2004). The study was supported by a framework of sociocultural themes that link teaching practices to learning outcomes through mathematical communication.

The researcher observed one classroom over the course of two years. The classroom was chosen because of how the teacher structured the learning and the teacher's beliefs about how mathematical learning should occur. In this classroom, the explanation and justification of ideas were strongly emphasized, self-directed and personal sense-making were a key component, and there was a high incidence of mathematical discussion among students. The teacher stated that he was highly committed to these principles in his teaching and the development of his classroom community.

After observing the classroom, the researcher conducted video stimulated recall interviews with the teacher and students to uncover if a reform approach could increase learning by increasing participation in a mathematical community. This brought the researcher to question "in what kinds of practices students should participate and what specific actions should a teacher take to improve student participation" (p. 281). The teacher used three principles in this classroom, which the researcher suggested could be a template for other inquiry-based classrooms. First, he modeled positive mathematical attitudes through his personal sense-making

and willingness to work with justification and proof. Second, he modeled the inquiry process by presenting a problem to students while withholding his own judgment or thought process to allow students an authentic approach to problem-solving. Last, he facilitated mathematical discussion and required students to justify their thinking to others. Ultimately, the researcher concluded the use of the Zone of Proximal Development as a framework, especially in peer collaborations, explicitly informed teaching practices.

Over the years, the literature has coalesced around a few key insights regarding classroom structures that support higher levels of conceptual understanding in mathematics. These practices include high levels of student discourse (Cobb, 1993), challenging problem-solving (Lubienski, 2000), student presentation (Boaler & Staples, 2008), student explanation and justification (Martino & Maher, 1999), teacher collaboration (Boaler & Staples, 2008), and the development of a mathematical community (Freedman et al., 2005). Schools that employ these structures often see an improvement in achievement gaps between minority and non-minority students (e.g., Boaler & Staples, 2008).

The Language of Mathematics

Previous research suggests that educators may assume that mathematics classrooms are a “safe harbor” for ELL students as numbers and symbols are common across contexts (Moschkovich, 2012; De Araujo et al., 2018). However, studies have shown that this is a misconception (Moschkovich, 2012). Mathematics classrooms are steeped in complex language, multiple representations such as objects, pictures, words, symbols, tables, and graphs, and a wide variety of text such as word problems, student explanations, and teacher explanations (Moschkovich, 2012). The way in which students talk about mathematics often differs from

colloquial language, thus refuting the assumption that mathematics classrooms are “safe harbors” for ELL students.

Language is inherently a social activity, and the language of mathematics is necessary for participation in mathematics discourse practices (Moschkovich, 2015). Researchers have identified several characteristics of learning environments that support language development in conjunction with content learning (Echevarria et al., 2008). For Hispanic ELL students, classrooms that offer abundant and diverse speaking, listening, reading, and writing opportunities are especially supportive (Garcia & Gonzalez, 1995; Echevarria et al., 2008; Barton & Griffin, 2009). These environments align with social constructive classrooms in which student-to-student interactions and discourse are the norm (Moschkovich, 2015). Empirical studies by Brenner (1998), Castellón et al. (2011), and Vidot (2011) highlight how collaborative tasks (Brenner, 1998), interactive interview procedures (Castellón et al., 2011), and the Sheltered Instruction Observation Protocol (SIOP) (Vidot, 2011) increase speaking, listening, reading, and writing through social constructivist approaches in mathematics classrooms (Moschkovich, 2015).

Collaborative Tasks. As far back as 1998, researchers and educators explored classroom practices that would support ELL students in mathematics. In her study of ELL mathematics students in California, Mary Brenner (1998) stated,

There is an urgent need for researchers and teachers to work together to find effective ways of including LEP [ELL] students in the new mathematics programs, using the resources that are currently available, so that linguistic minority students are not left behind as mathematics education is transformed (p. 149).

In her study, Brenner compared two first-year algebra teachers that were both utilizing a new resource. The resource focused on small-group problem-solving sessions and cooperative task

structures. She found that the teacher that used the suggested small-group structures as prescribed by the resource more successfully engaged ELL students in large-group discussions. The small-group structure enabled ELL students to have a higher level of competency when the mathematical discussion returned to the whole group. They had ample opportunity to discuss their thinking in a small group, often in their native language, so they could bring that thinking back to the larger group environment.

SIOP. Vidot (2011) examined the effectiveness of the Sheltered Instruction Observation Protocol (SIOP) recommended by Echevarria et al. (2008) in a mixed-methods case study. Vidot's study included three classrooms within a high school that had a student population that was 93% Hispanic. Instruction, teacher attitude, and student learning were themes developed from the qualitative data gathered from high school mathematics teachers in the case study as they examined the efficacy of the SIOP. The theme of instruction meant that the teachers believed that SIOP was based on sound instructional practices. Through the teacher attitude theme, teachers expressed that they needed to be cognizant of choosing the best strategies to use so that the utilization of SIOP was manageable and consistent. In the final theme of student learning, teachers stated they felt that SIOP did indeed help student comprehension of the content they were teaching.

Interactive Interviews. In the 2011 publication *Latinos/as and Mathematics Education*, Castellón et al. (2011) examined four sixth-grade English language learners' conceptual understandings of fractions through a four-stage interactive interview approach. The researchers found that written responses were limited in nature, providing only simple solutions. However,

when using an interactive interview approach in which ELL students could ask questions, use their own knowledge base rather than prescribed algorithms, get language support, and were allotted ample time for problem-solving, the researchers were able to gather a better understanding of the students' conceptual understanding of fractions. The researchers suggested that the interactive interview using social interactions and purposeful questioning could be utilized as a formative assessment structure within classrooms (Castellón et al., 2011).

In a review of studies of classroom practices that support ELL students, Moschkovich (2013) prepared four recommendations specifically around the language of mathematics designed to support ELL students in mathematics classrooms. The recommendations suggest that educators shift their focus from language accuracy to mathematical reasoning and mathematical practice. This recommendation means that educators should focus on the mathematical reasoning students are trying to convey rather than whether their English is accurate. The recommendations also suggest that educators treat home language as a resource and recognize the complexity of language acquisition. The findings concluded that, in order to assist with language acquisition, teachers need to use scaffolds that include multiple representations (Moschkovich, 2013), use visual tools and realia (real-world objects), and preview and review methods (Nguyen & Cortes, 2013). The researcher also recommended that a focus should be placed on mathematical practice as language is encouraged for reasoning, explanation, and constructing mathematical arguments (Moschkovich, 2013).

Building Positive Relationships

Teacher-student relationships have been identified as a barrier for ELL students for various reasons. Often cited is a lack of professional development for teachers (Greenfield, 2013)

and deficit viewpoints of student ability (Walker, Shafer, & Liams, 2004). To address these barriers, research suggests that, in addition to supportive environments that view language as a resource rather than a deficiency (Moschkovich, 2013), Hispanic ELL mathematics students also need teachers who provide student-to-home communication, hold high expectations for all students, and not see ELL students as intellectually disadvantaged (Lubienski, 2002; Garcia & Gonzalez, 1995). Relationship-building between three groups (teacher and other teachers, teachers and students, and students and other students) at the reform school in the Boaler and Staples (2008) study contributed to the level of responsibility that all participants felt for each other's learning and increased the quality of discourse that took place in the classrooms.

In a brief prepared for the National Council of Teachers of Mathematics, Civil and Menendez (2010) summarized the research that underscores the need for parental involvement from Hispanic families in the United States. A positive parent-school relationship is important as research evidence indicates a positive association between parent involvement and student achievement. Henderson et al. (2002), examined nine characteristics shared by high-performing high schools in the state of Washington. The characteristics were determined after the Office of the Superintendent of Public Schools had published a literature review of over 20 studies concerning common attributes of high-performing high schools. Henderson et al. (2002) then extended the work by examining the results of an additional 51 studies regarding high levels of parent and community involvement. In their synthesis, they uncovered that schools that focus on (1) building trust between families and schools, (2) attending to the needs of families and cultural differences, and (3) embracing a "philosophy of partnership" (p. 7) were able to engage families and communities in the work of schools and saw higher levels of achievement. They determined

that the studies identified a “positive and convincing relationship between family involvement and benefits for students” (p. 24). While this finding was consistent for families from all economic and racial backgrounds, the researchers recommended that supporting involvement for minority parents could close achievement gaps.

Aspiazu, Bauer, and Spillett (1998) examined the impact of a community center designed to support Hispanic families by creating leadership roles for parents and an after-school tutoring center for students. They found that parents were willing to participate actively in educational programming if the opportunity presented was seen as an educational benefit to their family. The families in the study recognized that homework contributed to many factors in school success. Through after-school attendance at the tutoring center, parents stated that their students’ reading and mathematics skills had improved, and they were successfully completing homework checks. They also saw less fighting in the areas where they lived as many students were occupied at the center. The parents in the study “embraced the opportunity to become personally involved in a mechanism that could improve their children’s performance and achievement at school.” (Aspiazu, et al.,1998, p. 144).

Civil and Mendendez (2010) found that teacher-parent and school-parent relationships should be culturally responsive and avoid a deficit perspective, suggesting home visits, workshops in mathematics, and classroom visits as pathways to relationship-building. In a study by Valencia and Black (2002), they examined the deficit perspective myth that Mexican-American parents do not value education. A deficit perspective in education is highlighted by a “language of lacking” (Ladson-Billings, 1995, p. 479) rather than assigning levels of competence

to capabilities and assets. Valencia and Black (2002) debunk this myth by uncovering that Mexican-American parents have a history of transgenerational parental involvement.

Conclusion

There are many factors to consider when addressing the instructional needs of ELL mathematics students. Moschkovich (2013) posits that three generalizations should be considered. First, educators need to have a knowledge of their students' experiences with mathematical instruction by highlighting teaching frameworks and classroom structures that support ELL students. Second, teachers should understand their students' language histories by addressing language needs within the classroom and building relationships with students and families. Understanding OTL can also assist in the overall picture of an ELL student's language history. Last, teachers should understand their students' overall educational background by understanding OTL and developing family relationships.

Mathematics Teaching Practices

While theories on learning are prolific and robust, some experts have suggested that theories on specific practices in classroom teaching are sparse (Hiebert & Grouws, 2007). According to Hiebert & Grouws (2007), "teaching consists of classroom interactions among teachers and students around content directed toward facilitating students' achievement and goals" (p. 372). This focus on interactions lends itself to a social constructivist paradigm. Of all school-based factors, teaching practices have been consistently shown to have the most significant impact on student learning (Hiebert & Grouws, 2007). While that may seem an obvious correlation, there is little empirical research connecting specific teaching practices to

learning, especially in the field of mathematics (Hiebert & Grouws, 2007). Conducting empirical studies identifying teaching practices that consistently lead to effective student learning is one of the most significant challenges in educational research (Hiebert & Grouws, 2007).

In a review of empirical studies prior to 2007, Hiebert and Grouws (2007) identified two features of mathematics instruction that facilitate conceptual understanding in mathematics: (1) explicit attention to connections among mathematical ideas and (2) engagement of students as they struggle with essential mathematics. However, in a nationally representative sample of eighth-grade mathematics classrooms in the United States, a video study by the Trends in International Mathematics and Science Study (TIMSS) showed so few examples of work in mathematics classrooms that promoted conceptual connections between math ideas that the percentage was rounded to 0% (Heibert, 2003).

Wilson et al. (2005) gathered the perspectives of nine mathematics teachers in a series of three interviews to determine what actions could be classified as “good teaching” in mathematics. After a rigorous coding process of over 135 pages of qualitative data, the researchers identified seven common themes. The first two themes were: (1) good teaching requires prerequisite knowledge and (2) good teaching promotes mathematical understanding. These two themes support attention to both conceptual understandings and procedural fluency (Wilson et al., 2005). All the teachers in the study were concerned with student understanding beyond procedural fluency. The following two themes represented in the data were: (3) connecting mathematics and (4) visualizing mathematics. This includes making connections between mathematical representation and having resources that support students in seeing the mathematics in various formats such as graphs, manipulatives, and images. These two themes

were tied to the themes of: (5) “refrain from telling,” in which teachers described the need for modeling, developing, and showing mathematics rather than telling students how to perform mathematics and (6) frequent formative assessment as a hallmark of good teaching, stating that that formative assessment gave them information about whether to move on to new learning with their students. Last, teachers in this study claimed that: (7) good teaching motivates and engages students. They suggested utilizing a variety of modalities when teaching, like technology-assisted programs, applications to real-life situations, and laboratory activities (Wilson et al., 2005).

Moschkovich (2013) extended this research around instructional practices to practices that could specifically support ELL students in *Principles and Guidelines for Equitable Mathematics Teaching Practices and Materials for English Language Learners*. She based her recommendations on Gutierrez’s (2013) four dimensions of equity: access, achievement, identity, and power. Her recommendations support all students in mathematical discussion but focus on students who are in the process of learning English. First, there is a focus on mathematical reasoning, not accuracy in language. Second, there should be a focus on mathematical practices, not single words or vocabulary. Third, it is essential to recognize the complexity of language in a mathematics classroom, and last, home languages should be treated as an asset, not a deficit (Moschkovich, 2013).

Since research has shown that teaching practices impact student learning (Hiebert & Grouws, 2007) and theories of learning suggest the benefits of a social constructivist approach to learning mathematics (Cobb, 1994), implementing these teaching practices that specifically address the needs of Hispanic ELL students could benefit their learning and achievement. Thus,

the NCTM has created the *Principles to Actions* document to guide recommendations for highly effective teaching practices (NCTM, 2014).

Overview of Principles to Actions

In 2014, the National Council of Teachers of Mathematics (NCTM) produced *Principles to Actions* in response to the overwhelming success and adoption of the Common Core State Standards for Mathematics (CCSSM). The adoption of the CCSSM in over 40 U.S. states provides guidance to the “what” of teaching mathematics and provides opportunity for systemic improvement. In response to national and international achievement data that identify several areas of concern, the NCTM has identified “realities” that may be at the root cause of static and declining scores in assessments like the National Assessment for Educational Progress (NAEP) and the Programme for International Student Assessment (PISA) (NCTM, 2014). Conclusions suggested there may be too much focus on procedure without connection to meaning, low expectations, remedial tracks, too much skill assessment, and isolated teachers, which are all areas within mathematics education that can be addressed for systemic improvement by the NCTM (NCTM, 2014).

The NCTM (2014) identified effective teaching as the core variable that will ensure high level mathematics. It organized its guiding principles around teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism (NCTM, 2014). From the guiding principles, the NCTM has outlined eight high-leverage Mathematics Teaching Practices (Hattie, 2009; Haystead & Marzano, 2009). These practices provide educators with the “how” of teaching mathematics. Practices refer to actions that teachers do habitually to support student learning, and high-leverage teaching practices show the most significant impact (Hlas &

Hlas, 2012). While the Common Core State Standards in Mathematics (CCSSM) provide guidance regarding which mathematics to teach and when, the Mathematics Teaching Practices (MTPs), as described in *Principles to Actions* (NCTM, 2014), recommend how best to deliver this content.

The MTPs stem from cognitive science and mathematics education research (Mayer, 2002; Bransford et al., 2000) that identify specific types of experiences that learners should have in a mathematics classroom. Bransford (2000) lists engagement with challenging tasks, connection with prior knowledge, development of conceptual and procedural knowledge, construction of knowledge in a social environment, access to timely feedback, and development of metacognitive awareness as the principles of learning that provide a foundation for mathematics education (Bransford, 2000). The findings from these studies guided the NCTM in designing the MTPs, which serve as a framework for teaching and learning in mathematics. High-leverage practices are a “core set of practices that have the greatest impact on student learning” (Hlas & Hlas, 2012, p. 77). The eight high-leverage practices promoted by NCTM are: (1) establish mathematics goals to focus on learning, (2) implement tasks that promote reasoning and problem-solving, (3) use and connect mathematical representations, (4) facilitate meaningful mathematical discourse, (5) pose purposeful questions, (6) build procedural fluency from conceptual understanding, (7) promote productive struggle in learning mathematics, and (8) elicit and use evidence of student thinking.

Trends in Data. With these experiences and practices articulated, research has explored areas of mathematics education that continue to show unsettling trends. For example, in 2018, the National Center for Educational Statistics (NCES, 2019) reported that eighth-grade Hispanic

students lagged behind white students by 24 points on the 2018 NAEP average scale score in mathematics. Even more concerning, the difference between ELL and non-ELL students on the 2018 NAEP in mathematics was a staggering 39 points (NCES, 2019). NCES defines English Language Learners as students who participate in language assistance programs that guide them to English proficiency to meet academic standards required by all students (NCES, 2019). In 2018, NCES reported that 9.7% of all school-age students were identified as English Language Learners (ELLs) (NCES, 2019).

The NCTM states in *Principles to Actions* that there is a need to eliminate racial, ethnic, and income achievement gaps and ensure that all students have access to high-level mathematics (NCTM, 2014). Considering the recent data that illustrate lagging achievement for Hispanic and ELL students, access and equity continue to be areas of particular concern. Research from Lubienski (2002) shows that there has been a lack of attention to ethnicity and class when examining opportunity gaps. Lubienski (2002) found significant differences between lower and higher socioeconomic status (SES) students in their ability to participate in whole-class discussions and open-ended mathematics problems. ELL students often reside in households with a low SES (Reardon & Galindo, 2009). Murphy, Madill, and Guzman (2017) also noted that close to half of all Hispanic kindergarteners lived in poverty. Equity and access are at the forefront of concern in mathematics education as educational systems work to address inequitable out-of-school factors (Gutierrez, 2013; NCTM, 2014).

Mathematics Teaching Practices. With equity and access at the forefront, three of the eight Mathematics Teaching Practices (MTP) were chosen for use in this study: pose purposeful questions, use and connect mathematical representations, and facilitate meaningful discourse.

These three were selected due to their strong connection with social constructivism (NCTM, 2014; Smith et al., 2017) and best practices for teaching ELL students (Echevarria et al., 2008). Equitable mathematics teaching aims to progress mathematical learning and understanding while developing a solid mathematics identity (Aguirre et al., 2013). The chosen MTPs support engagement and success for all students (NCTM, 2014).

Pose Purposeful Questions

When posing purposeful questions, teachers can shape the context of the mathematics they teach and how students engage in learning (NCTM, 2014). Purposeful questioning has been linked to gains in student achievement (Redfield & Rousseau, 1981) as teachers use questions to probe thinking, make mathematical connections evident and visible, and encourage explanation and justification. Redfield and Rousseau (1981) analyzed 14 studies to produce a meta-analysis that investigated the relationship between the level of teacher questioning and student achievement. Their findings suggest that students exposed to higher cognitive levels of questioning experience higher achievement results. Additionally, Boaler and Staples (2008) found that high cognitive demand could be maintained and supported through teacher questioning strategies. A study by Stigler & Hiebert (1999) found a shift from a traditional I-R-E (initiate-respond-evaluate) questioning stance to more open-ended questioning provided an opportunity for students to explain and justify their thinking. Herbel-Eisenmann & Breyfogle (2005) build upon the study by Stigler and Hiebert by examining interaction patterns that occur after a purposeful question is posed. They uncovered that a funneling pattern in which teachers guide students through a series of questions to steer them to the desired end was less limiting than the I-R-E pattern. They then explored a different pattern referred to as focusing, in which

the teacher listens to the student's response and guides the students based on their thinking rather than how they would solve the problem. In the examination, Herbel-Eisenmann & Breyfogle (2013) suggest teachers work to convert I-R-E and funneling questions into focusing questions to encourage students to articulate their thinking.

Posing purposeful questions aligns with a social constructivist paradigm and supports ELL learners in two ways. First, being able to answer questions positions ELL students as competent members of the mathematical community (Gresfali & Cobb, 2006). As teachers make instructional decisions, they need to be aware “of whom they are asking which types of questions and whom they are positioning as competent” (Smith et al., 2017, p. XX). Facilitating participation for ELL students is a crucial component in their mathematical learning, as it provides language development (Gibbons, 2015) and requires scaffolding techniques by teachers (Echevarria et al., 2008). Scaffolding for ELLs includes actions such as reducing linguistic demand, sentence stems, and using graphic organizers (Echevarria et al., 2008). Second, answering purposeful questions promotes speaking and writing in mathematics which are two of the four domains of language encouraged in the literature (Martino & Maher, 1999). Developing a culture that encourages speaking and writing to answer questions supports both mathematics and second language learning when integrated into the content learning (Chval & Khisty, 2009).

Use and Connect Mathematical Representations

Mathematics is an abstract system that people understand through physical representations (NCTM, 2014). Studies have found that using physical and visual representations supports the development of mathematical reasoning by enabling students to make connections between the concrete and abstract (Suh & Moyer, 2007). For example, in a study with third-

grade students, Suh and Moyer (2007) showed that physical and virtual manipulative models significantly supported gains in achievement between pre- and post-test assessments. The students were also more able to translate flexibly between multiple representations of the algebra concepts they were learning. The extent to which students see and connect multiple representations of a mathematics concept informs teachers about the depth of their understanding of the concept. However, while visuals and math manipulatives are often plentiful in elementary mathematics programs, middle school programs often see them fade away as students progress through grade levels. This may be a serious oversight as such representations have been shown to support ELL students and promote equitable classrooms (Aguirre et al., 2013).

Allowing students to gain access to mathematical understandings by building, drawing, creating tables, or working with digital manipulatives encourages them to engage in mathematics in ways that help them to make sense of the concepts. Promoting a wide variety of mathematical representations supports the equity-based practice of drawing on multiple sources of knowledge (Aguirre et al., 2013) and positions learners as mathematically competent. Additionally, the social interactions of sharing and co-creating representations align with the social constructivist paradigm found supportive in the mathematics classroom (Tripathi, 2008).

Facilitate Mathematical Discourse

Incorporating discourse in mathematics classrooms has become a staple for building and honoring student thinking, providing opportunities for idea sharing and comparing, clarifying understandings, and developing convincing arguments (NCTM, 2014). Gee (1996) defines discourse as “ways of being in the world or forms of life that integrate words, acts, values,

beliefs, attitudes, social identities, as well as gestures, glances, body positions, and clothes” (p. 127).

Participating in discourse in a mathematics classroom asks students to utilize words, gestures, and attitudes that are specific to mathematics. This requires knowing more than just specific vocabulary words. In a study of professional development on teacher discourse moves, Herbel-Eisenmann et al. (2011) determined that discourse practices were productive and powerful for students’ learning. They define productive as the “way in which participants can support student access to mathematical content” (p. 181) and powerful as “the ways in which participant’s discourse practices can support student identities as knowers and doers of mathematics” (p. 181).

The MTP of facilitating meaningful discourse requires a shift from a traditional warm-up, homework review, lecture, and practice problem cycle that had been typical in mathematics classrooms for many decades (NCTM, 2014). To create a classroom that maintains meaningful mathematical talk (Moschkovich, 1999), NCTM (2014) suggests utilizing the Five Practices for Orchestrating Productive Discourse: anticipating, monitoring, selecting, sequencing, and connecting. Anticipating asks the teacher to consider possible student responses to mathematical tasks. Through the use of monitoring, teachers listen to students talk in small groups. Teachers then select responses to be discussed by the whole group and connect those responses to critical mathematical learning. The connecting component allows teachers to make additional thinking explicit through discourse and mathematical representations.

Mathematical discourse provides an ideal opportunity for ELL students to practice speaking while actively engaging in mathematical reasoning (Moschkovich, 2013). It also offers

the opportunity to continue to build academic vocabulary. Creating the learning environment to support this MTP is critical. For ELL students, classrooms that view language as a resource, offer abundant speaking, listening, reading, and writing opportunities, and encourage students to take risks while learning mathematics will be most supportive (Garcia & Gonzalez, 1995). Classrooms with high levels of discourse promote equitable practices by providing access to peer thinking and including all student voices (Herbel-Eisenmann et al., 2013; Aguirre et al., 2013). Teachers must also carefully consider who is chosen to share their thinking to continue to position all learners as people who know and do math (NCTM, 2014).

Student Voice and Perception

In a recent review of the literature regarding ELL students and mathematics, De Araujo, Roberts, Willey, and Zahner (2018) located 891 articles pertaining to the topic. However, only 75 were empirically based and dealt with teaching, learning, or teacher education. Furthermore, researchers have found the voice of Hispanic ELL students to be almost nonexistent in the literature (Garza, 2009; Vladu, 2016; Halx & Ortiz, 2011). Student voice is defined as the “ways in which students share input into their education, their school, or the decisions that impact on their lives” (Kahne et al., 2022, p. 392). Cook-Sather extends this definition and understanding in her 2006 review of literature on student voice by adding “that young people have unique perspectives on learning, teaching, and schooling; that their insights warrant not only the attention but also the responses of adults; and that they should be afforded opportunities to actively shape their education” (Cook-Sather, 2006, p. 359).

Cook-Sather explains why the voice of the student is vital in making significant progress and change within educational systems:

As long as we exclude student perspectives from our conversations about schooling and how it needs to change, our efforts at reform will be based on an incomplete picture of life in classrooms and schools and how that life could be improved (Cook- Sather, 2002, p. 3).

Despite intense endeavors to meet the needs of all students through high-leverage instructional practices, the voice of the student is glaringly absent from the literature (Halx & Ortiz, 2011; Sands, 2007). Halx & Ortiz (2011) suggest that this absence of student voice can cause students to believe that their life chances are limited, therefore giving up on school prematurely. De Los Reyes and Gozemba (2002) state, “the inequalities facing the US Latino population cannot be understood apart from the wider socio-economic structures of society in which they are embedded” (p. 147).

For these reasons, there is a compelling argument for including student voice, perspective, and perception in research. The definitions of perspective and perception are unique in the literature and require attention. First, perspective is the direct articulation from a student about events and circumstances that influence the student’s world (Wilson & Corbett, 2001). Perception takes into account the ways in which one attends to reality and experience through the senses (Given, 2008, Perception Section).

Scholarly attention to understanding the experiences of Hispanic ELL students can provide steps of action and an understanding of how ELL students create their meaning within classrooms (Halx & Ortiz, 2011). By gathering student perception and perspective regarding the actions within a classroom, researchers can better understand the experience of the studied population (Cook-Sather, 2006). Additionally, Gutierrez (2013) contends that, by examining how

marginalized groups such as ELL students participate in classrooms, a systemic understanding of underachievement can open the door for systemic improvement.

Student Consultation

Theorists like Giroux (1986) encourage student consultation in matters concerning their experiences and engagement to develop a fuller understanding of life in classrooms and schools (Rudduck & McIntyre, 2007). Student consultation involves seeking student views about existing programs (Bragg, 2010). It can be collaborative and participatory in nature or collected through data gathering instruments such as surveys. Ruddick and McIntyre (2007) define student consultation as a conversation between teachers and students that allows students to know that their views are being sought because they have something to contribute.

Additionally, social constructivist theorists would contend that student consultation plays a role in a teacher's ability to determine a student's current state of understanding to make instructional decisions (Rudduck & McIntyre, 2007). This presents the need to engage in dialogue with students around teaching and learning regarding classroom practice (Morris, 2019). Rudduck (2005) reports that student consultation may result in students contributing to the wellbeing of their school and classroom.

Improving Practice. A longstanding body of scholarship outside of the United States would argue that student consultation is vital in improving teacher practice and promoting agency in learning. For example, Morris (2019) conducted a "teach the teacher" project with 16 students and 8 teachers in a school on the outskirts of London. Students were asked to provide professional learning to their teachers about technology topics. Through observations, one-on-

one interviews, and focus groups, Morris found that the use of student voice and student consultation in the project resulted in a sense of empowerment and confidence in the students and a broader appreciation of the work of teachers. She defined empowerment as times in which students are in the position to determine the agenda of the event, whether it be teaching teachers or classroom learning. She states, “In its purest form, student voice affords the opportunity to build the capacity for students to lead student voice initiatives themselves and become active in the decision-making process.” (p. 113).

In a large-scale project by Rudduck and McIntyre (2007), researchers in the United Kingdom interviewed Year 8 students and their teachers about student consultation practices. They found that students responded positively about being consulted, but thought that the success of the consultation was dependent on how seriously the teachers took the recommendations and whether or not they implemented them. In this study, students claimed they felt that student consultation also contributed to their learning. They felt that they knew, more so than their teachers, which strategies would be most beneficial to their learning.

In 2007, Sands et al. collected data about factors that Hispanic students considered valuable in supporting their efforts in school. The researchers determined that listening to the Hispanic student voice could considerably impact educational reform initiatives. While the results of the study are of interest, the methods also contributed to the overall implications of the study. Sands et al. (2007) state, “Helping students to voice their views not only makes sense for their cognitive, affective, and behavioral development, but it promotes the underlying social and political goals we hold as a society” (p. 337).

As this general body of literature continues to grow, especially within the United States, the integration of student perspective into workable teaching practice increases (Sands et al., 2007). In the study by Sands et al. (2007), student perspective was gathered around variables contributing to student effort. Sands et al. (2007) concluded that, as early as elementary school and continuing through high school, students were able to articulate the variables that affected their performance, thus providing valuable insight for school reform.

Alignment. Seeing the classroom from a student's perspective can result in the ability of a teacher to make content more accessible and can be especially useful in informing local contexts (Anthony et al., 2013). As students are asked to seriously contribute to the conceptualization of teaching and learning, they become more motivated to participate in their learning processes and provide crucial information for an understanding of learning and improvement of instruction (Cook-Sather, 2002; Huang & Barlow, 2013).

Previous research suggests that, as students enter critical conversations around their learning, they are providing missing and often differing perspectives from other stakeholders such as teachers and administrators (Sands, 2007; Cook-Sather, 2006). Researchers have found that matching what teachers and students value can result in higher performance levels (Anthony et al., 2013). For example, as part of the Learner's Perspective Study, an international research collaboration in mathematics classrooms, Huang and Barlow (2013) found that consistency between a teacher's intention and the students' perceived importance of a teaching event meant that students were more likely to engage in the learning task at the cognitive level the teacher intended. The study by Rudduck and McIntyre (2007) in the United Kingdom found that when teachers recognized the value of perspectives other than their own when making instructional

decisions in their classrooms, students were able to provide information that made teachers more aware of the effects of their practice. In this same study, students were able to provide information regarding which practices their teacher utilized that were more or less helpful in their learning (Rudduck. & McIntyre, 2007). This growing body of literature confirms that students can be a rich source of information about the relationships between teaching and learning, especially when they provide a perspective different from the current authority (Cook-Sather, 2002; Rudduck & McIntyre, 2007).

Student Agency. Increased student agency, or the ability to intentionally influence one's environment (Bandura, 2006), has been identified as an essential outcome in studies examining the impact of student consultation. Rudduck and McIntyre (2007) found that student consultation promoted student agency by (1) strengthening student self-esteem, (2) enhancing attitudes toward school and learning, and (3) developing new skills for learning. When students are consulted and their perspective is sought, they are in a position in which they contribute to their own learning (Rudduck, 2005). This is especially valuable when working with minority groups whose perspectives and perceptions are often omitted.

Garza (2009) used student consultation to uncover how Hispanic students perceived teacher behaviors that conveyed caring. Garza (2009) argued that caring may be perceived differently by students than common practice currently suggests. They found that Hispanic students valued scaffolding during teaching as a sign of caring from teachers. This valuable insight could only be gathered by consulting with students about the topic. Davila (2010) used student voice in a study about the lived experience of Latino students to explore how to best serve the population's academic and social needs. Through conversations with students, he found

that Latino students often felt that school systems worked against their needs. Without student consultation, this insight may not have been unearthed (Davila, 2010). When the perspectives of minority groups are sought, they can provide rich examples of their unique experience and the intricacies of learning in a structure that was not necessarily designed for them (Davila, 2010), thereby informing future instruction.

The Hispanic ELL experience is uniquely different from that of their white, English-speaking peers and, as such, their perceptions and perspectives may differ significantly from their peers and teachers. Access to those differences is difficult unless students are heard and conditions for dialogue are supported (Fielding, 2010). Fielding's *Patterns of Partnerships* (2011) suggests that researchers need to start with students as a data source and build to a fellowship dimension in which students and adults have a shared commitment to the common good.

While the literature utilizing student voice and consultation in research is vast and ever-growing in countries such as Australia and the United Kingdom (Rudduck & McIntyre, 2007; Morris, 2019; Cook-Sather, 2006), the Hispanic, ELL voice in the United States is almost non-existent (Garza, 2009). Halx and Ortiz (2011) posit that the Hispanic ELL voice has been socialized into silence. Including student voice through student consultation can provide schools with valuable insights that cannot be gathered in any other format (Morgan, 2011). As schools and districts continue to seek opportunities for improvement in practice and policy, understanding and responding to the primary stakeholders is essential (Sands et al., 2007).

Chapter Summary

In exploring the Mathematics Teaching Practices that support Hispanic middle-school ELL mathematics students, this literature review explored findings established in prior research relative to learning theory in mathematics, teaching mathematics to ELL students, and the role of Mathematics Teaching Practices, as well as the role of student perspective and voice in understanding how best to support Hispanic ELL mathematics students.

CHAPTER THREE

METHODS

This chapter describes the methodology, context, methods, procedures, and data analysis used to explore Hispanic ELL middle-school students' perceptions and their teachers' perceptions of the efficacy of a set of research-informed mathematics teaching practices. The first section of the chapter outlines the rationale for the methodology chosen to best answer the research questions. The second section provides an overview of the community where the study took place and includes a description of the researcher's positionality and relationship to the study context. The third section describes the data collection methods used to discover the students' and teachers' perceptions. The final section outlines the approaches used for data coding and analysis.

Research Design and Rationale

The purpose of this study was to examine middle-school Hispanic ELL students' perceptions and their teachers' perceptions about the level of effectiveness found in the research-informed National Council of Teachers of Mathematics' (NCTM) Mathematics Teaching Practices (MTPs). Very little research exists that includes the perceptions of Hispanic ELL students (De Araujo et al., 2018; Sanchez-Gutierrez et al., 2015). Documenting and analyzing students' views may provide more understanding of their learning experience and possible solutions to the challenge of teaching mathematics to ELL students. Thus, to best address the

research questions of this study that incorporate student and teacher perceptions, a qualitative pragmatic research design approach was selected to frame the study.

The foci of a pragmatic research design are (1) to describe an experience or event to better understand the perspective of the people involved (Savin-Baden & Howell Major, 2013) and (2) to find solutions to a contextualized problem. In a pragmatic approach, individuals, structures, and processes are the phenomenon of interest, and data are collected using interviews, documents, and observations (Savin-Baden & Howell Major, 2013). Thus, this study included interviews and observations of mathematics teachers and middle-school Hispanic ELL students using and experiencing NCTM Mathematics Teaching Practices.

Pragmatic methodology is anchored by three underlying principles: a focus on highlighting actionable knowledge, connecting experience to knowing and acting, and using an inquiry-based, experiential process (Kelly & Cordeiro, 2020). These three tenets were present in this study, thus aligning it to a pragmatic methodology. First, research that emphasizes actionable knowledge allows researchers to act on a problem (Kelly & Cordeiro, 2020) and participate in an iterative inquiry process as the phenomena change and develop (Morgan, 2014). Given that this study's results could be used to inform mathematics instruction for Hispanic ELL students, the results can be seen as highly actionable.

The second tenet of a pragmatic approach recognizes the interconnectedness between the experience, the knowledge, and the action of the study or inquiry (Kelly & Cordeiro, 2020). This means that a focus on knowledge alone is not enough; there must be resulting action attached to the knowledge. Mathematics teachers are often familiar with or know strategies that support ELL students, and these often overlap with effective practices for teaching and learning mathematics.

This study compares mathematics teachers' knowledge and use of effective practices in their classrooms to their students' experiences and perceptions of the practices. The results of this study may lead teachers in this context or similar contexts to take different actions based on the data gathered from their Hispanic ELL students. For example, if the participants state that a particular practice was less helpful in their learning, teachers can adjust that lesson to include an alternative approach in the future.

Last, a pragmatic approach is an inquiry-based, experiential process. This means that the process is embedded in everyday situations, and the inquiry focuses on the human experience (Dewey, 1920/2008; Morgan, 2014). Additionally, the participants involved in the inquiry engage in an experience that allows the researcher to better understand the phenomena (Kelly & Cordeiro, 2020). In the current study, the focus of the inquiry relied on the experience and perception of the participants and the experiential inquiry process of data collection was designed to identify participants' perceptions of the effectiveness of mathematical practices for learning.

Pragmatic research designs can be embedded into practical, everyday situations relevant to researchers and practitioners (Morgan, 2014). This is important when working in an educational context with teachers and students. By demonstrating the applicability of the inquiry, researchers can demonstrate to teachers and students how the study is relevant to them. Additionally, pragmatic research allows for a choice of methods most appropriate for answering the research questions (Morgan, 2014). Pragmatism encourages research actions that are useful within a particular context (Morgan, 2014). Data collection techniques can be chosen that are most likely to provide practice-based knowledge (Kelly & Cordeiro, 2020).

Case Study Methodology

A case study methodology was an appropriate choice of methodology for this study because it offered the flexibility and opportunity for the in-depth investigation (Savin-Baden & Howell Major, 2013) necessary for understanding whether specific MTPs effectively support Hispanic ELL students when learning mathematics.

The design of the case study used in this research project included the five essential features recommended by experts (Yin, 2014; Merriam, 1998). First, a case study must be bounded, meaning that it is narrow in scope. For example, a case study may be defined within the parameters of location, time, or group (Johnson & Christensen, 2017). Second, a case study is holistic (Savin-Baden & Howell Major, 2013), describing the case in its entirety, including the relationships between the parts and the whole of the case. A case study is particularistic, meaning that it is specific in nature rather than general. A case study is contextual as the context is vital in understanding the case. Last, a case study is concrete relying on in-depth descriptions to help the reader understand the meaning of the case (Savin-Baden & Howell Major, 2013; Merriam, 1998). This research study adhered to these essential features to maximize the quality of data collection, analysis, and interpretation regarding ELL students' perceptions of mathematics instruction.

Attending to the first essential feature, this study was bounded by three parameters: location, participants, and a finite number of observations. First, the location was a rural resort town in the Intermountain West, which is of interest as resort towns support the service industries in which many Hispanic immigrants find employment. Second, purposive sampling with maximum variation was selected because it would constrain the participant group to

middle-school Hispanic ELL students and allow the case to show variations in perceptions (Palinkas et al., 2015). Last, data collection was time-bound, taking place over one semester with a limited number of classroom observations and student interviews. Limiting the number of interviews was viewed as essential when working with middle school students to help prevent interview fatigue (Savin-Baden & Howell Major, 2013; Wilson & Corbett, 2001).

This case study was also designed to be holistic, particularistic, contextual, and concrete in the following ways. The study was holistic because it examined not only the perceptions of Hispanic ELL students but also the perceptions of their teachers. The study was particularistic as the focus was on three specific MTPs. It did not attempt to investigate all research-informed teaching practices or other aspects of a mathematics classroom that may support Hispanic ELL students, such as sentence stems or learning supports such as anchor charts (Echevarria et al., 2008; Goldenberg, 2014). The study was contextual because the geographic location, a resort town with a large population of Hispanic service workers, was important, as was the school and classroom environment.

The middle school in the study maintains a Hispanic ELL student population of roughly 26%; a pragmatic interest in supporting this group contributed to the decision to explore the perceptions of this population. Last, this study provided thick descriptions of its participants and their perceptions, allowing deep understanding and contributing to the concrete aspect of the case study (Gay, Mills, & Airasian, 2009). Thick description is a term borrowed from anthropological research. Researchers provide concrete, literal descriptions of an event of interest to give the reader a realistic understanding of the experience (Merriam, 1998; Creswell & Creswell, 2018).

In summary, this study aimed to understand an educational action and its impact on students through narrative with thick description (Savin-Baden & Howell Major, 2013). The investigation was pragmatic in nature as it took an inquiry approach to exploring experience with the goal of actionable results. Since this study aimed to examine MTPs through the perceptions of those most closely involved, a case study design was the best choice for gathering the perceptions of teachers and Hispanic ELL students (Yin, 2014; Greig, 2013).

Research Context

Site Selection

The site chosen for this study was a middle school in a medium-sized district located in a resort community in the Intermountain West. The district is classified as rural fringe, meaning that the 2019 U.S. Census defined it as territory that is less than or equal to 5 miles from an urbanized area, as well as rural territory that is less than or equal to 2.5 miles from an urban cluster (NCES, 2020). The county population that feeds the middle school was estimated at 23,464 residents in the 2019 U.S. Census, and the town population was estimated at 10,559 residents. Approximately 81.5% of the population in the surrounding county is considered white, not of Hispanic descent, and 14.9% are Hispanic (U.S. Census Bureau, 2019).

The community is considered a gateway to major recreation and parklands and is home to a large ski resort. As such, the accommodation and food services industries are the most significant economic contributors, constituting about 20% of the total industry in the area (datausa.io). Many Hispanic families have relocated to the area because of the wide availability of service jobs. These community characteristics, economic factors, and demographic shifts are

not unique to this study site. Several similar resort communities across the Intermountain West support a large population of Hispanic families (*The Quiet Force*, 2019).

The school district where this study took place is comprised of one high school (student population of 729), one middle school (student population of 697), one dual immersion elementary school (student population 457), three larger elementary schools (student populations of 345, 263, and 203), and three small, remote elementary schools (student populations of 55, 17, 38). The overall student population in the district during the 2020–21 school year was 2,869 students, supported by 244 classroom teachers with a student-teacher ratio of approximately 12:1. The ELL population was 418 students across all schools.

The student population of the middle school of interest, Mountain West Middle School (pseudonym), was 697 students with a student/teacher ratio of 13:1 (NCES, 2021). Minority enrollment made up 35% of the population and mainly represented Hispanic students. This was higher than the state minority enrollment of 23%, placing it as one of the most diverse schools in the state (NCES, 2021).

As of October 2021, the ELL population of this middle school was 26% of the total enrollment, or roughly 180 ELL students. Of these, 62 had a composite English Language Proficiency (ELP) level between 1 and 4.6 on the WIDA ACCESS for ELLs. WIDA ACCESS for ELLs is a large-scale, summative language proficiency assessment given to kindergarten through twelfth-grade students in the United States (WIDA, 2021). This means that these students were receiving ELL instructional services during the study. The remaining 118 identified ELLs were on a four-year monitor schedule. This means that they were categorized as ELL students, but had achieved a composite ELP score of 4.7 or greater on the WIDA ACCESS

for ELLs. The monitor years were labeled as M1, M2, M3, and M4, indicating the first year of monitor status through the fourth year of monitor status. Two ELL students did not participate in WIDA testing in the spring of 2021 and, thus, were not represented in the data. There were 18 former ELL students enrolled who had exited ELL instructional services completely. Table 3.1 illustrates the composition of current and former ELLs attending Mountain West Middle School during Fall 2021.

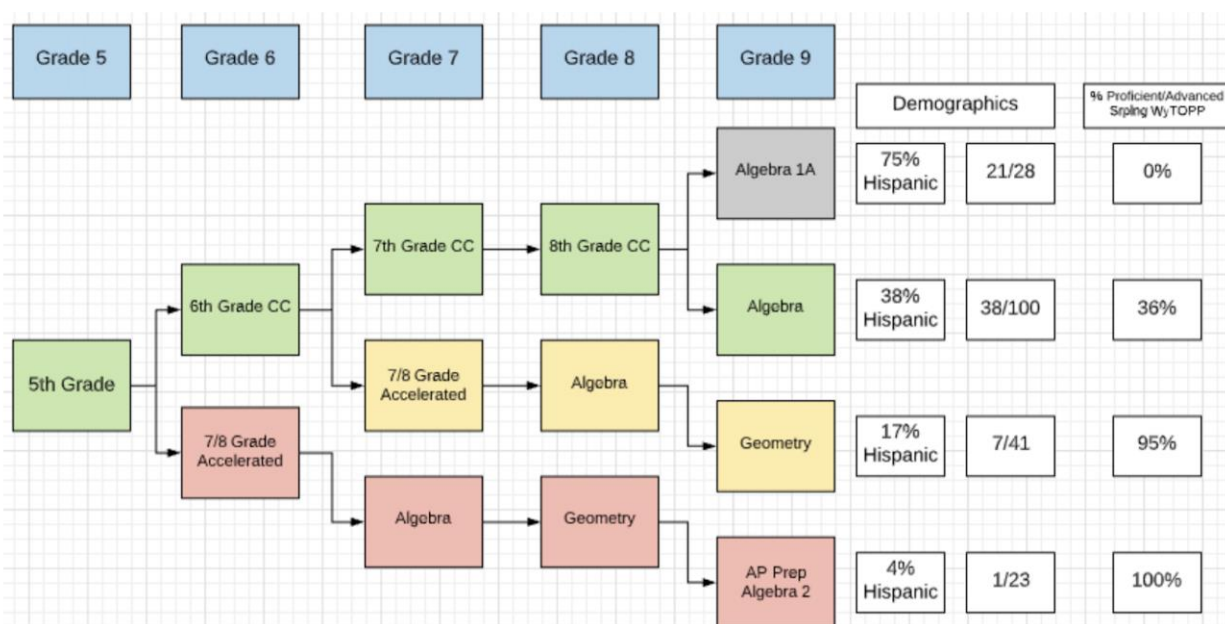
Table 3.1. 2021 WIDA ACCESS Level

<i>Composite score</i>	<i>Number of students</i>
1.0 – 1.9	5
2.0 – 2.9	10
3.0 – 3.9	29
4.0 – 4.6	16
M1	9
M2	25
M3	34
M4	50
Exited (Former ELL)	18

Data marking students' progression through the district's mathematics curriculum from fifth grade through ninth grade for the 2019–2020 school year show that Algebra 1a, the lowest available level of mathematics available in ninth grade, was comprised of 75% Hispanic students, 25% of which were ELL (Figure 3.1). Based on state testing information from the previous spring, 0% of the students enrolled in Algebra 1a were grade-level proficient in mathematics. Algebra 1, the course considered “on grade level” for ninth grade, was comprised of 100 students, 38 of whom were Hispanic. In the Algebra 1 group, only 36% of the total population were proficient on the state exam. In the “above grade level” classes of Geometry, 95% of the students were proficient on the state exam. Similarly, 100% of the students in

Algebra 2 were proficient on the state exam. Students in the “above grade level classes” averaged 12.5% Hispanic and 87.5% White. The school district built the model in Figure 3.1 utilizing Spring 2019 testing data as 2020 testing did not occur due to COVID-19 restrictions. This model was intended to illustrate the demographic composition of the mathematics classes in order to examine mathematics course pathways and issues of equity in class make-up.

Figure 3.1. Mountain West Middle School Mathematics Course Series



Participants

Teachers. Teacher participants for this study were identified through classroom observations as described in the Data Collection section of this chapter. Four teachers demonstrated a high occurrence of the Mathematics Teaching Practices. Three of the four participated in the full study as a result of the required qualifications of the student participants.

Students. Student participant selection was based on purposive sampling techniques with three specific characteristics required of the student to be part of the study (Johnson & Christensen, 2017). First, they had to be enrolled in a middle school mathematics class. Second, they had to be of Hispanic descent. Last, participants had to be identified as an ELL student at the selected site. In this study, the goal was to obtain 6 to 10 participants to allow the researcher access to a breadth of experience while keeping data collection manageable. To increase the maximum variation, Maxwell (2013) suggests systematically selecting participants that represent the most important variations. In this study, the goal was to have at least two students from each grade level with differing language levels.

A review of the school's mathematics class lists revealed that many of the identified eighth-grade ELL students were either on monitor status or had another identification, such as special education, that kept them from being considered for the study. This resulted in 11 students from sixth and seventh grade who had composite WIDA ACCESS scores ranging from untested (too new to the district) to 1.5 through 4.1. The demographics of the group are shown in Table 3.2. While gender and country of origin were not factors in this study, this demographic information is included in the table to provide a thorough depiction of the participants.

The researcher met with all ELL students who met the selection criteria from each participating classroom to determine their interest and to describe the study in detail. A translator was present to support any students who needed translation of the information. The students were informed of the nature and purpose of the research, including any possible risks, their right to refuse to answer a question or withdraw from the project at any time, and the timing, length, and number of interviews. Students who volunteered to participate were given the parental

consent form to take home for signatures. The parental consent form and the student assent form were translated into Spanish.

Table 3.2. Student Demographics

	Grade Level	Gender	Country of Origin	Years in the United States	WIDA ACCESS composite
<i>Pilot Study</i>					
Zelda	6	F	Mexico	less than one	2.1
Xena	6	F	Mexico	three years	3.3
<i>Classroom #1</i>					
Nayeli	6	F	US	born in US	4.1
Lupe	6	F	US	born in US	4.1
Elena	6	F	Mexico	four years	3.7
Julian	6	M	Mexico	two years	1.7
<i>Classroom #2</i>					
Noemi	6	F	Puerto Rico	one year	1.7
Jimena	6	F	Mexico	one year	1.5
<i>Classroom #3</i>					
Abril	7	F	Colombia	one year	3.2
Carlos	7	M	Mexico	7 months	NA
Daniel	7	M	Mexico	3 months	NA

The school secretary and the researcher made phone calls to the parents to describe the study and to let them know that permission slips were being sent home. The secretary spoke fluent Spanish and thus called all the parents who spoke only Spanish. The researcher sat with her while calls were made in case the parents had additional questions. The researcher called parents who indicated on school records that they spoke English at home. The students also signed the student assent form for themselves. No research was conducted until all documents were signed and returned.

To ensure confidentiality, pseudonyms were used for all participants and the research site. Access to data was controlled in a locked file cabinet at the researcher's home office to

protect confidentiality; this includes audio recordings, transcripts, field notes, video recordings, and district evidence used for documentation. Digital data were stored on a password-protected computer.

Positionality

Due to the descriptive nature of qualitative research, especially the in-depth methods used in case study, the positionality of the researcher can influence data collection, analysis, and interpretation. The researcher's past experiences, biases, and views in relationship to the studied experience may shape the study in a way that is impactful (Creswell & Poth, 2018).

As a former elementary and middle school teacher, I taught Hispanic ELL students for over 20 years in the same district where the study was conducted. Teaching ELL students in various contexts and over many years may have created biases (both positive and negative), which I need to notice and address in my work on this project. I thoroughly enjoyed teaching ELL students and continually took opportunities to learn more about strategies for working with ELL students. I also participated in a year-long, district-level, professional development strand that focused on providing sheltered instruction for ELL students. I then went on to complete a series of university courses that resulted in a state-approved endorsement in teaching ELL students. Two of my earlier research projects focused on the inclusion of ELL students in gifted programming. These experiences have influenced my views about ways to teach ELL students.

I also have a passion for mathematics education. I taught mathematics every year of my 20-year career and taught mathematics solely for six years at the middle school level. I also completed a Master of Science degree in Mathematics Education at the start of my career. I

currently work as an educational consultant, often guiding school districts in designing mathematics programs, and I also coach mathematics teachers. My beliefs about how mathematics should be taught informed my choices for methodology and the interpretation of the data.

I consider myself an advocate for immigrant families and ELL students. In my professional career, I have advocated for and conducted action research around supporting ELL students. I worked to produce avenues for a more equitable representation of Hispanic and ELL students in the gifted program in my past school district. In my personal life, I have supported families in the legal processes of immigration and acknowledge personal biases about these processes in the United States.

Last, I have chosen to conduct research in the school district where I spent much of my career. This is a logical choice as the research proposed here stems from questions that developed while I was teaching in the district. However, this proximity can be a benefit or a hindrance. I have a good working rapport with many administrators and teachers in the district. I am also very familiar with the context of the school district and the middle school. I recognize the need to continue to be aware of my preconceived understandings about the context, so they do not inhibit the collection and interpretation of data.

Data Collection

In order to collect interview data about the perceptions of Hispanic ELL students and their teachers regarding three research-informed Mathematics Teaching Practices, this study employed a two-phase sequence of preparation and data collection. Phase One included (1) the

identification of classrooms where specific MTPs were consistently utilized, (2) the calibration of operational terms, and (3) a collaborative rating scale development. This sequence included a pilot study of the Student Interview Protocol (Appendix D). Phase Two was informed by the results generated in Phase One so that data collection from video stimulated recall (VSR), semi-structured interviews with student and teacher participants could occur. The data collection protocol is described in the following section and outlined in Table 3.3.

Phase One: Design

This study examined the perceptions of Hispanic ELL students' and their teachers about the efficacy of three specific and research-informed MTPs promoted by the National Council of Teachers of Mathematics (NCTM) to support students' mathematical learning. The three MTPs included in the study were: (1) use and connect mathematical representations, (2) facilitate mathematical discourse, and (3) pose purposeful questions (NCTM, 2014). These three practices were selected from a potential pool of eight MTPs because they closely align with a social constructivist framework and literature regarding classroom practices that support ELL students (Moschkovich, 1991, 2012; Echevarria et al., 2008).

A series of collaborative, qualitative research steps were developed to answer the research questions effectively. Table 3.3 outlines these steps, the participants involved in each action, the action's purpose, and the tool used or created in the research action.

Table 3.3. Research Action Steps

Research Action	Participants	Purpose	Tool
Classroom Walk-Throughs	Researcher All middle school math teachers at selected site	To determine consistent use of the MTPs	<i>Mathematics Teaching Practices Classroom Observation Tool</i>
Classroom Selection	Researcher	To determine the participating teachers and class periods	<i>Mathematics Teaching Practices Classroom Observation Tool</i>
Proxy term list creation	Researcher Selected middle school math teachers	To create a list of commonly used classroom terms for the examined MTPs	<i>Common Terminology for Mathematical Practices Reference Page</i>
Helpfulness scale creation	Researcher Selected middle school math teachers	To create a rubric with commonly used classroom terminology for levels of helpfulness	<i>The Mathematical Practices Helpfulness Rating Scale</i>
Student participant invitation	Researcher Selected middle school math teachers	To invite qualifying students to participate in the study	Description of the study <i>Parental Consent Form</i> <i>Student Assent Form</i>
Classroom visit	Researcher Selected middle school classrooms	To inform all students about the study and how the videos will be used	Description of the study
Pilot study	Researcher Two Hispanic, ELL mathematics students from participating classrooms Translator	To refine interview protocol and questions	<i>Student Interview Protocol</i>
Group Orientation Session	Researcher Selected middle school math teachers Selected student participants Translator	To introduce student participants to the interview protocol	<i>Student Interview Protocol</i> <i>The Mathematical Practices Helpfulness Rating Scale</i>

Table 3.3. Research Action Steps Continued

Classroom videos (3 per classroom)	Researcher Participating classrooms	To capture the MTPs in the classroom with student participants	Video equipment
Student interviews (3 interviews per student pair)	Researcher Student participants Translator	To collect data regarding the efficacy of the selected MTPs	<i>Student Interview Protocol</i>
Teacher Interviews (3 interviews per teacher)	Researcher Selected middle school math teachers	To collect data regarding the efficacy of the selected MTPs	<i>Teacher Interview Protocol</i>
Data analysis & preparation	Researcher Colleague	To analyze and represent the collected data in an understandable and readable format	Open, axial, selective coding methods Cohen's Kappa
Teacher member checking session (one per teacher)	Researcher Selected middle school math teachers	To ensure that the researcher accurately captured the teacher perception	Completed findings

Classroom Walk-throughs. To determine the extent of teachers' skillful and consistent use of the three selected MTPs, the researcher conducted a series of classroom observation walk-through sessions using the Mathematics Teaching Practices Classroom Observation Tool (Appendix A). The researcher developed the tool utilizing the Teacher and Student Actions for Effective Mathematics Learning components from NCTM's *Principles to Actions* (NCTM, 2014). The "teacher actions" in the *Principles to Actions* document described what could be observed in a classroom setting when a teacher was utilizing the specified MTP. During a two-

week observation period, the researcher visited all the mathematics classrooms in Mountain West Middle School. There were six math teachers at the selected site, two at each grade level (sixth, seventh, and eighth).

The researcher collected observation data in each classroom three times for a duration of 20 minutes each, spending a total of one hour in each of the teachers' classrooms to collect observational data about the MTPs. Classroom walk-throughs are a widely used strategy for collecting data snapshots around the use of a particular strategy in a school. Research suggests using an observational guide referred to as "look-fors" (Rouleau & Corner, 2020; Bloom, 2007) related to the purpose of the data collection. The "look-fors" in this study were the teacher actions described in Mathematics Teaching Practices Classroom Observation Tool. For each of the three MTPs, several teacher actions were listed. For example, when observing a teacher using the MTP, use and connect mathematical representations, one might see the teacher selecting tasks that allow students to use the mathematical representation that makes the most sense to them. If that action was observed, a tally mark was made in the "observed" column of the tool. Field notes were added during the observation as well. The teachers with the most consistent use of the teacher actions associated with the MTPs were invited to participate in the study.

Four teachers were identified as frequently using the MTPs and were thus invited to participate in the study. All the teachers agreed to participate. However, it was determined that one of the teachers did not have enough student participants who fit the parameters of the case study, leaving three participants. Mr. CL, Mr. BW, and Ms. NG. All names are pseudonyms to maintain confidentiality. All three teacher participants had been teaching for more than five years.

Proxy Terms. Once participating classrooms were selected, the teachers and the researcher collaboratively identified commonly used proxy terms for elements of the investigated three MTPs. For example, if a teacher used “turn and talk” to facilitate mathematical discourse, the researcher used that phrase when interviewing the participating students. It was essential to gather this information from each participating teacher as they used different terminology for the same practice.

Using common terminology is supportive when working with ELL students and promoted clarity in the interview process (Echevarria et al., 2008; Powell & Harvill, 1990). When developing the interview questions, the word “helpful” was used in place of effectiveness, efficacy, or support. This purposeful decision was made to provide language that would be comprehensible to ELL students. During the proxy term alignment, teachers related that they would identify an MTP as “helpful” if it assisted in moving students to mathematical independence. In other words, students could better complete the mathematical task on their own because of the use of the MTP. This meaning was added into the interview protocol for students (Appendix F) so that the definition of helpful was clear to them.

Using familiar classroom language in which students feel comfortable helped to build rapport with students (Savin-Baden & Howell Major, 2013; Wilson & Corbett, 2001). The proxy terms identified for each teacher are documented in the Common Terminology for Mathematical Practices Reference Page (Appendix B). The Reference Page has categories for each of the three MTPs, organized by teacher, and lists the terminology they each individually use in their mathematics classrooms.

Rating Scale. Next, the teachers and researcher co-created a rating scale for students that aligns with commonly used language in the classroom (Echevarria et al., 2008; Powell & Harvill, 1990). The researcher and teachers chose to incorporate thumbs-up symbols that signify different levels of rating. Using images and gestures is a way to support ELL students with new or unfamiliar terms and procedures (Echevarria et al., 2008). The Mathematical Practices Helpfulness Rating Scale (Appendix C) provided students with a way to rate the degree to which a particular practice was more or less supportive in their mathematical learning and success. The student participants used The Mathematical Practices Helpfulness Rating Scale to evaluate the effectiveness of each MTP as they observed them in video segments during the interview process. To answer research question #2, teachers also rated each practice on the scale from not helpful to helpful in supporting ELL students' mathematical understanding during the teacher interview component of the data collection. Comparison of the ratings identified if students and teachers have similar or disparate perceptions of the MTPs as supportive for learning mathematical concepts and to what level, and these findings were used to answer research question #3. Comparing student and teacher perceptions arises within the literature surrounding student consultation (Huang & Barlow, 2013; Anthony et al., 2013; Rudduck & McIntyre, 2007). Huang and Barlow (2013) found that, when there was consistency between what the teacher intended and the students' perceived importance of the learning, it was more likely that the students would engage with the mathematical task at the cognitive level the teacher intended.

Student Selection. At this point in the study, Hispanic middle school ELL students from the selected classrooms were invited to participate (N = 9 students and N=2 pilot study students). Those who volunteered completed the parental consent and participant assent forms. Following

the collection of consent and assent forms, the researcher visited each participating classroom to explain the study and the actions the students would see happening in their classroom. For example, the researcher asked all students to go about their classroom routines like they normally would, even though she would be collecting video data. She also let the class know that she may bring the video camera physically close to them to be able to record what they were saying. During these class visits, students asked a lot of questions, including how long the researcher had been in school and why she was asking students about mathematics.

Pilot Study. Phase One concluded with a pilot study of the semi-structured interview protocol to be used with students. The pilot study helped clarify the researcher's understanding of the logistics of the study (Maxwell, 2013). The interview protocol for this study included a welcome, an introduction in which the selected MTP was named, a viewing of the video clip, a summary statement, and interview questions. Yin (2014) explicitly recommends a pilot test to refine relevant interview questions. In this study, the interview questions needed to be clear and accessible for Hispanic ELL students in middle school mathematics classrooms. The interview questions were purposefully designed to constrain the focus of the interview to factors within the school environment. This constraint was chosen for the interview for the purpose of generating findings that would be relevant and valuable for improving mathematics instructional practices. The questions did not ask broadly what factors enabled Hispanic ELL students to accomplish mathematics goals, but whether the three specific MTPs did so.. If the interview questions had been more open-ended, students may have given responses that were not within the reach of classroom teachers. For example, teachers cannot control if a student had a brother or sister who

enjoyed helping them with mathematics homework. Teachers can improve their implementation of the MTPs.

The interview was pilot tested with two middle school Hispanic ELL students taught by one of the participating teachers to refine the questions used with later groups (Sands et al., 2007). While using students for the pilot study from a participating teacher reduced the number of potential students available for the study, doing so helped to increase the fidelity of the data collection. Furthermore, the two students included in the pilot study had varying language abilities to ensure ELL students' ability to clearly understand and respond to the interview questions. The two students in the pilot study had composite scores of 2.1 and 3.3 on the district-administered WIDA ACCESS, which determines language ability for ELL assessment.

The Mathematical Practices Helpfulness Rating Scale was also examined and refined. The researcher asked the pilot student participants if the scale was clear, if it was easy to use, and if they had suggestions for improvement. All of these steps were audio and video recorded so that the researcher could provide an audit trail of all the decisions made throughout the research action steps.

The interview protocol employed video stimulated recall (VSR) to provoke memory and prompt discussion during interviews (Meier & Vogt, 2015; Hunter, 2017). VSR methods allow participants to relive a situation with vividness and accuracy (Bloom, 1953). In this study, a semi-structured interview supplemented by VSR allowed students to view a video clip from a recent lesson that included elements of the selected MTPs. Previous research recommends allowing a short period between the video and the interviews, from 2 to 48 hours (Meier & Vogt, 2015; Morgan, 2007). In this study, the VSR interviews were conducted within 24 hours of the

recorded lesson. The pilot study mirrored the actual research study in these methods; however, the focus of the pilot study was on the wording of the questions and the ease with which students could respond. The interview questions and the helpfulness scale were edited based on the feedback from the students. Additionally, a Spanish language translator was present in the pilot session to facilitate clear communication if necessary.

Pilot study participants watched a video clip from their classroom in which instruction featured one of the MTPs. The video clip was selected based on its ability to highlight the student participant actively engaging with the MTP. The pilot study participants then responded to the draft interview questions in the same way that the actual study was conducted. Both ELL students were interviewed at the same time during the pilot study. This allowed the students to clarify with one another and compare interpretations of the questions.

For the pilot study lesson, the researcher asked one participating teacher to focus on one of the MTPs. Mr. BW selected facilitating mathematical discourse. Three procedural changes were brought to light in the pilot study that the researcher needed to address. First, the researcher needed to slow down her speech and allow time for translation to occur. Second, she needed to ensure that each student's name was introduced at the beginning of the recording and used each time the student was asked a question. This added clarity to the transcribed interviews. It was often difficult to differentiate the students' voices when listening to the audio recording, especially because they both wore medical masks on and off during the interview due to the COVID 19 pandemic. Last, the researcher needed to repeatedly remind the students to say as much as possible in response to each of the questions. In addition to the logistical shifts, the pilot

study participants helped refine the group orientation protocol (Appendix E) and interview questions (Appendix F).

The group orientation protocol was adjusted to include the logistical operations mentioned above. Based on a suggestion from the pilot participants, the researcher also added several reminders about being able to use “thumbs up and thumbs down” to indicate the level of helpfulness of the MTP. This was especially helpful for Hispanic students who were familiar with the side-to-side hand gesture that means “más o menos” in Spanish or “so-so/somewhat” in English. The pilot participants suggested to only read two or three sentences at a time and repeatedly ask if students understood during the group orientation protocol. They also suggested that the researchers ask, “Can you tell me what that means? Does anyone have any questions? Does it make sense?”

The pilot participants also had valuable feedback about the interview protocol. They recommended that the researcher explain more about the Mathematics Teaching Practices and give examples using the terminology they used in class, like “turn and talk” for mathematical discourse. At the end of the interview protocol, the students suggested asking a question more like “What helps you learn math?” rather than “Is there anything else you would like to add?” They stated that the researcher could use the original question, but not without first asking what else helps you learn math.

Both pilot participants were very helpful in the pilot study process, offering many valuable suggestions. They both said that they would help more if needed. During the actual study, the two pilot participants would often stop the researcher in the middle school hallway to talk.

After the pilot, teachers were asked to incorporate the MTPs into three class periods. Three class periods were chosen to limit the number of interviews and interview fatigue for students (Savin-Baden & Howell Major, 2013). While there are many teaching practices to select when planning a lesson, for this study, the participating teachers purposefully incorporated one of the MTPs each time the researcher videoed the classroom. This ensured that the researcher could document instructional episodes that helped the student participants rate the MTP's efficacy.

Phase Two: Data Collection

Orientation. The first step in the data collection phase was to hold the Group Orientation Session. The purpose of this session was to introduce students to the study procedures, clarify any unknown terminology, and build rapport and comfort with the students. The Orientation Session also introduced students to the researcher so that more natural discourse could occur during the interview process (Bragg, 2010).

During the Group Orientation Session, students viewed the same video clip as the pilot students so they could be introduced to the language the teachers and researcher used when naming classroom practices. Different teachers used different terminology and facilitation, which was highlighted in the Group Orientation Session. The students were also introduced to the Mathematical Practices Helpfulness Rating Scale, which was co-constructed by the researcher and pilot study teacher and reviewed in the pilot study. This introduction allowed the researcher to explain the differences between the levels on the rating scale. This was an essential component of the Group Orientation Session because students often feel less intimidated when presented

with new information as a group (Hill, 2006; Sanchez-Gutierrez, 2015; Savin-Baden & Howell Major, 2013). Questions from students were fielded during this session. Direct interview questions were not asked during this session to avoid some of the pitfalls of group interviews, such as conversations dominated by more assertive speakers or conversations derailed by other topics (Bragg, 2010).

The researcher, with a translator present, facilitated the Group Orientation Session. Having the translator available during the Group Orientation Session allowed student participants another opportunity to clarify any remaining questions. The translator provided additional familiarity for the students since she was the school secretary and translated for their families often. All these steps made input from the orientation comprehensible for ELL students (Echevarria et al., 2008; Hill, 2006).

Documenting Real-Time Practices. After the Group Orientation Session was complete, the researcher video recorded each participating classroom three times so that all the selected MTPs were examined. Teachers preplanned to utilize MTPs in each of three class periods. Selecting one MTP for each lesson placed a focus on the specific MTP and allowed the researcher several choices of video segments to use in the student and teacher interviews. The researcher recorded the entire lesson to maximize the number of usable segments. During the video recording, the researcher took time-stamped notes to be able to locate usable video segments quickly. A segment was usable if it highlighted the student participants utilizing the MTP. Following the classroom visits, VSR-enhanced semi-structured student interview sessions were conducted within 24 hours of the actual class period. Teacher interviews were conducted within 36 hours of the class period.

VSR Interviews. The student participants viewed a video clip from the most recent recorded lesson that included the selected MTP. The choice was made to conduct the interviews in classroom groups (N=4, N=2, N=3) to increase the level of comfort for the students (Morgan, 2007) and as a measure of language support for ELL students (Echevarria et al., 2008). These sessions happened during lunch so that the students did not miss any class time. The worry of missing class time could negatively affect the students' ability to complete the process (Greig, 2013). A special lunch was provided to participating students so that they had an enjoyable experience and as a thank you for missing a lunch period. This process was repeated three times to examine all the selected MTPs.

Students viewed preselected segments of the lesson rather than the entire lesson. Preselecting segments helped students focus on the selected MTPs rather than being distracted or overwhelmed by the entirety of the lesson. The chosen segments were the segments that best demonstrated the student participants utilizing the selected MTPs. The student and teacher interviews were video recorded for later transcription and reference.

The researcher followed the Student Interview Protocol (Appendix F) during the interview. Each question on the Student Interview Protocol was asked aloud by the researcher and translated by the translator. After a welcome statement, the interview started by introducing the student names clearly for the recording and naming the mathematical goal of the lesson. For example, inequalities were named, followed by the MTP incorporated into the lesson. The researcher used the Common Terminology for Mathematical Practices Reference Page when discussing MTPs.

The selected video segment was then shown to the participating students. They were each individually asked to recall what they were thinking at the moment that the identified MTP was being used in the classroom. There was often more than one participating student in the selected clip, so they were each asked to recall their thinking separately. This sometimes meant that the small group observed several video clips so that the researcher could gather feedback from each participant. This is referred to as Summarizing the Math in the Student Interview Protocol. This initial question was purposefully designed to encourage the students to recall and describe their experiences and thinking. This laid a foundation for the evaluative questions later in the interview process (Savin-Baden & Howell Major, 2013).

After summarizing the math, the students were asked to rate the selected MTP using The Mathematical Practices Helpfulness Rating Scale, indicating if the MTP was helpful (Sands, 2007) in learning mathematics. A high rating indicated that the student found the MTP highly supportive of their mathematical learning. The Helpfulness Rating Scale was co-created with teachers and introduced during the Group Orientation Session to facilitate ease of use. The researcher read the rating descriptions aloud. The translator translated and was available for clarification. The students responded orally, used hand gestures (thumbs up or down), or pointed to the corresponding rating on the Helpfulness Rating Scale.

After rating the efficacy of the MTP, additional questions asked in what ways the MTP was supportive in their mathematical learning, or not, and if the MTP was helpful at other times in mathematics class. Additional open-ended questions followed these questions. First, students were asked what their teachers did that helped them learn mathematics or what they wished their teacher did, as suggested by the pilot study. The interview closed with questions that asked if

there was anything the student would like to add, if they had any questions, and if there was anything else they remembered about learning mathematics in the selected lesson. These questions were designed to help the researcher get a broader understanding of practices that support Hispanic ELL middle school mathematics students. At the end of the interview, students were reminded that they would repeat this process, so as they thought of ideas about practices that would help them learn mathematics, they could share them in the following interview.

Enhancing Understanding. Reading and writing can add a layer of difficulty when working with ELL students (WIDA, 2020; Baker, 2013; De Oliveira, 2019), and steps must be taken to mitigate these challenges. In this study about student perceptions, interviews provided the opportunity to gather abundant information, probe deeper into topics, bring up new issues, and allow the participants a personal forum for their voice (Bragg, 2010). The semi-structured interview was considered more accessible to ELL students as there is no questionnaire to read and interpret or writing involved. Additionally, VSR methods allowed open-ended questioning without describing the event to the student. This reduced the risk of the researcher influencing what occurred in the event (Meier & Vogt, 2015). A translator was present for all student interviews to translate any oral communication and provide clarification if necessary.

Teacher Interviews. The Teacher Interview Protocol mirrored the experience of the Student Interview Protocol; however, the interview questions were worded to address teachers. The teacher interviews were conducted within 36 hours of the lesson, as suggested in research utilizing VSR methods (Meier & Vogt, 2015; Morgan, 2007). The interview session opened with a welcome and focus statement reminding the teacher about the math goal and MTP that was

incorporated into the lesson. As with the student interviews, teachers viewed a preselected video segment at the beginning of their interview. This was the same video segment that their students viewed.

After viewing the video segment, teachers were also asked to Summarize the Math, followed by questions about the helpfulness of the incorporated MTP. The teacher version of the interview questions asked the teachers to consider how the MTP may or may not have been helpful to the mathematical understanding of their Hispanic ELL students. Open-ended questions asked if the MTP was helpful to Hispanic ELL students in different mathematical lessons and if other practices had been perceived as helpful. Teachers were encouraged to ask questions and add additional thoughts about the MTPs and other practices. The teacher interviews took place before school, after school, during planning time, or at another time within the 36-hour timeframe that was convenient for the teacher. Each participating teacher completed three interviews (some using Zoom), one following each lesson incorporating the selected MTP.

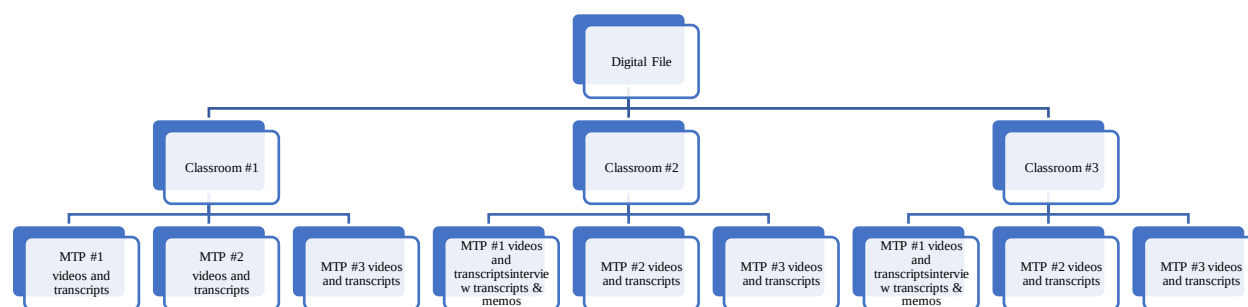
Data Analysis

To conduct a valid, comprehensive analysis, qualitative researchers must immerse themselves in their data through an iterative process (Savin-Baden & Howell Major, 2013; Creswell & Poth, 2018). Creswell and Poth (2018) refer to the data analysis spiral in which researchers move in “analytic circles” (p. 185), entering with data from transcripts, video, and text, and exiting with a narrative describing the case. Further supporting the spiraling idea, Merriam (1998) suggests that data collection and analysis should be simultaneous in qualitative research. This study's data analysis process occurred in four spiraling sets of activities as

suggested by Creswell and Poth (2018). First, the researcher prepared, transcribed, and organized the interview data. Memos were then recorded that helped produce an initial set of codes to identify key ideas in the text. Next, the data was broken into sentence unit segments, and preliminary codes were created. Finally, the codes were described and classified into themes. These activities will be described in detail in the following paragraphs.

During step one, preparation, transcription, and organization, it was essential to have a solid organizational system in place (Creswell & Poth, 2018). After classrooms had been videoed, participants had been interviewed, and interviews had been transcribed, a digital file system was prepared. Each file housed the video segments that illustrate each MTP and the corresponding interview transcripts (Figure 3.2) for each classroom. Figure 3.2 illustrates this organization.

Figure 3.2. File Organization



After transcription, the researcher followed Agar's (1980) suggestion to read the transcripts entirely in order to be immersed in the details before breaking the data into segments. This initial reading was followed by open coding, a process of developing significant categories within the data. As categories were identified, codes were assigned to the data segments. As the

reading and memoing process spiraled, axial codes were assigned. The constant comparison analytical method, initially described by Glaser and Strauss (1967), was used to determine which categories were the central focus of the data (Savin-Baden & Howell Major, 2013). Finally, during selective coding, the researcher described the interrelation between the categories through a visual model.

During the open coding process, all interview data were given a “first read,” in which the researcher highlighted words and phrases that stood out as compelling. Then the data were read a second time. During the “second read,” the researcher color-coded similar words and phrases or words and phrases that presented a similar concept within the text. Memos were created during this process of reading and rereading as they facilitated analytical thinking and synthesis of the data (Maxwell, 2013; Creswell & Poth, 2018). Additionally, memoing allowed the researcher to create an audit trail to serve as a validation strategy by documenting the thinking process (Creswell & Poth, 2018).

After the open coding process, the researcher color-coded the data to fit the preliminary codes. These were then entered into an Excel spreadsheet that the researcher used to synthesize the codes through an axial coding procedure. Like ideas were grouped and then reduced to a limited number of final codes. The number of times each final code was observed in each classroom was recorded in the spreadsheet to construct a logical pathway justifying why the final codes were assigned. Participant quotes from the transcribed interviews were also included to support the coding process. Last, definitions (Appendix L) for each code were created by analyzing the open coding to axial coding process. Definitions reflected what students and teachers were stating that supported the overall meaning of the code.

To assess the dependability of the codes and categories developed in the analysis, a colleague used the established codes to review three transcripts. Creswell and Poth (2018) suggest that an external check be completed by someone familiar with the research and phenomena. The colleague in this case was a doctoral candidate in educational leadership with an extensive background in ELL education. The colleague used the axial codes and definitions (Appendix L) to review the transcribed interview data. Cohen's kappa coefficient was calculated at 0.785 to measure inter-rater reliability. Sun (2011) stated that “A well-defined measure of agreement describes how well one rater’s rating agrees with what another rater would have reported and indicates the generalizability of a rating beyond the specific rater” (Sun, 2011, p. 147). The 0.785 level of agreement indicates substantial agreement between the two raters (McHugh, 2012). Cohen’s kappa is considered a more robust measure than other agreement calculations (Sun, 2011). As a further measure of dependability, the researcher represented the data visually through color-coding, spreadsheets, and diagrams (Savin-Baden & Howell Major, 2013) to enhance interpretation. Colleagues were then asked to review initial data displays and provide feedback about which displays most clearly represented the data.

It should be noted that data interpretation actually began early in the research process (Merriam, 1998). For example, in this study, interpretation of critical ideas started to develop during conversations with teachers as the Common Terminology Resource was created and during the interview process. Marshall and Rossman (2015) suggest that, throughout the entire interpretative process, researchers keep an open mind to alternative understandings that may challenge their own.

Validity and Trustworthiness

Valid and trustworthy research is plausible, credible, and defensible (Johnson & Christensen, 2017). In a qualitative study, the “gold standard” of validity and trustworthiness (Savin-Baden & Howell Major, 2013, p. 475) is outlined by Lincoln and Guba (1985). They suggest that a qualitative study provides credibility through convincing and believable results, transferability to show that the results have applications in similar situations, dependability indicating that the results will endure over time, and confirmability ensuring that the researcher has remained neutral. A wide variety of strategies can maximize a qualitative study’s overall quality regarding the previously mentioned criteria (Savin-Baden & Howell Major, 2013).

This study used an audit trail, member checking, peer examination, and thick descriptions to meet this criterion. Creating an audit trail allows the researcher to trace a path to their final findings (Creswell & Poth, 2018), contributing to credibility and confirmability. An audit trail (Merriam, 1998) depicts the entire research process in detail, including how data was collected, how categories were coded and created, and how decisions were made. Video and audio recordings, researcher memos, and transcription records are part of the audit trail in this study. Additionally, inter-rater agreement was tested to ensure the appropriateness of the selected codes and the consistency with which they were assigned to the data.

Member checking was used in this study, which also contributes to confirmability by allowing participants to verify the researcher’s interpretation of their words (Savin-Baden & Howell Major, 2013). In this study, teachers reviewed the findings to ensure that their ideas were accurately presented. For two reasons, students did not review the findings. First, the school year had ended before all the data were analyzed, and it was not feasible to locate the student

participants during the summer months. Second, the data findings were presented in highly academic English. While they could have been translated into Spanish, the high level of academic language utilized in the data summaries and findings did not match the language abilities of the student participants.

Peer examination and inter-rater agreement were used to ensure the appropriateness of the codes and the consistency with which they were assigned to the data. This served as an additional layer of confirmability and credibility for this study. The researcher attempted to generate thick descriptions (Creswell & Poth, 2018) of the student and teacher perceptions that allow the results to be convincing, to be used in multiple situations, to endure over time, and to be confirmed by others. Finally, the researcher's long-term relationship with the study site supported all four of the criteria for trustworthiness.

Chapter Summary

This qualitative case study aimed to examine the perceptions of Hispanic ELL middle-school students and their teachers regarding the effectiveness of research-informed, high-leverage Mathematics Teaching Practices (MTPs). The goal was to determine how Hispanic ELL students perceive these practices as contributing to their mathematics success and to compare students' perceptions to those of their teachers. Since the problem presented in this study is pragmatic in nature, the results are intended to inform instruction in a practical manner. A two-phase approach was utilized in the research design. Phase One involved preparation for the study and included the creation of instruments and resources such as The Helpfulness Rating Scale and Common Terminology Reference Page. Classroom observations were conducted during this

phase to select participating teachers. Phase One also included a pilot study used to refine the interview protocol and address logistics. Phase Two focused on data collection through thorough student and teacher interviews coupled with video-stimulated recall of classroom episodes.

Data analysis was conducted through an iterative process that included organization, preliminary coding, and cycles of open and axial coding in search of themes that help to explain students' perceptions of the effectiveness of selected MTPs. The data were arranged first in multiple spreadsheets and later in visual displays to help the researcher ascertain and interpret meaning in the results. Rigorous memoing, tests for inter-rater reliability, and thick descriptions supported the validity and trustworthiness of the findings.

CHAPTER FOUR

RESULTS

This study examined the perceptions of Hispanic ELL middle-school students and their teachers regarding the efficacy of three research-informed, high-leverage instructional practices known as Mathematical Teaching Practices (NCTM, 2014; Smith et al., 2017). The data for this study were generated and collected primarily through a series of student group interviews and individual teacher interviews that all utilized video stimulated response (VSR) to provoke memory and prompt reflection during the interviews. Nine middle school students and three middle school teachers participated in this study in a medium-sized district located in a resort community in the Intermountain West.

The findings presented in this chapter are the result of analyzing and interpreting the collected data to answer the following research questions:

1. How do Hispanic ELL middle-school students perceive the effectiveness of research-informed mathematics teaching practices in supporting their mathematical understanding?
2. How do teachers of Hispanic ELL middle-school students perceive the effectiveness of mathematics teaching practices in supporting their students' mathematical understanding?
3. In what ways do these students' and teachers' perceptions align and diverge?

As described in Chapter Three, Methodology, this study employed a two-phase sequence of preparation and data collection. The preparation phase included (1) identifying classrooms

where specific MTPs are consistently employed, (2) calibrating operational terms, and (3) developing a collaborative rating scale. The data collection phase involved using video stimulated recall to collect interview data. The interviews were recorded, transcribed verbatim, and analyzed through an iterative process of open coding and axial coding techniques to identify key concepts and common threads. The results of this analysis are presented in this chapter.

Description of Participants

Within this study, there were two groups of participants. The first group included three teacher participants, and the second group included nine students from those teachers' classrooms.

Teachers

As described in depth in Chapter Three, the classroom practices of all middle school mathematics teachers at the study site were observed to determine their suitability as potential participants in the study. A researcher-developed Classroom Observation Tool (Appendix A) was used to document occurrences in which each teacher utilized the three Mathematics Teaching Practices (MTPs) of interest in this study. Four teachers were identified as frequently using these MTPs and were thus invited to participate in the study. All the teachers agreed to participate. However, it was determined that one of the teachers did not have enough student participants who fit the parameters of the case study, leaving three participants, Mr. CL, Mr. BW, and Ms. NG. All names are pseudonyms to maintain confidentiality.

All three teacher participants had been teaching for more than five years. Mr. CL and Ms. NG had taught for more than five years at the selected school site, and Mr. BW had transferred

from a private school in the area. It was his first year at the selected site. Mr. CL and Ms. NG both had their own classrooms, and Mr. BW moved from classroom to classroom. All the teachers at the selected site taught at least one section of sixth-grade math. Mr. CL also taught eighth-grade math. Ms. NG and Mr. BW both taught seventh-grade math as well as sixth grade. The entire mathematics department had one hour per day as a common planning time, and they participated in a Professional Learning Community (PLC) once per week. The teachers in this study stated an interest in the study because they felt that learning more about how to support ELL students in mathematics was appropriate for personal and school-wide growth.

Students

One of the essential features of a case study is that it is bound by specific parameters to keep the study narrow in focus. One set of parameters for this study defined student characteristics. To qualify as a participant in this study, students were required to be classified as a Hispanic, ELL, middle school student currently enrolled in a mathematics class. After identifying the participating teachers, the list of Hispanic ELL students within each classroom was examined to ensure that at least two students were eligible to participate within a classroom before selecting that classroom for observation and interviews. This allowed students to be interviewed with a peer, which is recommended in the literature when conducting interviews with students (Hill, 2006).

Other disqualifying factors further limited the choices for student participants. One of the disqualifying factors was a dual identification of special education and ELL. In many of the eighth-grade classrooms, the students identified as ELL were either on monitor status (exited from direct ELL services) or receiving special education services. Being provided an additional

service could have influenced students' responses to the interviews, which had been designed to determine the helpfulness of MTPs for ELL students in traditional instruction. These parameters left only sixth- and seventh-grade students as participants in the study.

Two sixth-grade classrooms and one seventh-grade classroom were identified with an adequate number of ELL students who could participate. The corresponding student groups are labeled Classroom #1, Classroom #2, and Classroom #3 for confidentiality. Table 4.1 shows the demographic data of the groups including grade level, gender, country of origin, years in the United States, and WIDA ACCESS composite language level scores. While gender, country of origin, and number of years living in the United States were not factors analyzed in this study, they are included in the demographic information to build a contextual background for the students interviewed. The WIDA ACCESS score is relevant to the demographic data because it gives information about the types of input and output communication a language learner is capable of utilizing. The WIDA ACCESS score measures a student's progress in learning English by assessing four domains: reading, writing, speaking, and listening. Students at the study site exit ELL programming when their composite score is 4.7 or higher. They are then on a monitor status where they no longer receive direct ELL services. The students who do not have WIDA ACCESS scores have not been tested yet. All names appearing in the table and throughout the study are pseudonyms to maintain confidentiality.

Table 4.1. Student Demographics

	Grade Level	Gender	Country of Origin	Years in the United States	WIDA ACCESS composite
<i>Classroom #1</i>					
Nayeli	6	F	US	born in US	4.1
Lupe	6	F	US	born in US	4.1
Elena	6	F	Mexico	four years	3.7
Julian	6	M	Mexico	two years	1.7
<i>Classroom #2</i>					
Noemi	6	F	Puerto Rico	one year	1.7
Jimena	6	F	Mexico	one year	1.5
<i>Classroom #3</i>					
Abril	7	F	Colombia	one year	3.2
Carlos	7	M	Mexico	7 months	NA
Daniel	7	M	Mexico	3 months	NA

Classroom #1

Classroom #1 consisted of four students from a sixth-grade classroom, three girls and one boy. Nayeli sat in the front of the class and appeared to enjoy working independently or one-on-one with her teacher. She also seemed to like to attempt challenging problems. Lupe sat with her

best friend, who was not an ELL student. They would frequently talk about mathematics together in class. Both Nayeli and Lupe had high WIDA ACCESS scores close to the exit cut score. Elena was an outgoing, talkative student who had been living in the United States for four years. During the classroom observations, Elena sat with another Hispanic student who spoke Spanish but was not designated as an ELL student. While Elena's WIDA ACCESS score was not as high as Nayeli and Lupe's, she contributed the most to the interview conversations. Julian had been in the United States for the least amount of time and had the lowest WIDA ACCESS score. He did not contribute as often to the interview conversations as the other three students in this classroom. Julian was the only student in this group who actively relied on a translator during the interview sessions.

Classroom #2

Classroom #2 contained two sixth-grade students who were fairly new to the United States; both girls had been in the country for one year. Their WIDA ACCESS scores were typical for the range of students often described as "newcomers." Noemi and Jimena sat with a third Hispanic girl who spoke fluent Spanish but was not classified as an ELL student. The trio was purposely seated together to be language partners during class. At the study site, the term "language partner" was used to describe a partnership between students in which one student was bilingual and the other was monolingual. The bilingual student was able to provide translation for the monolingual student. While this role usually occurred at the request of the teacher, students were allowed to "opt out" if were not comfortable taking on the role of a language partner. Both Noemi and Jimena were less talkative than some of the other student participants. They held hands during the first interview, stating that they were nervous. After the first

interview, they did not hold hands again. Both Noemi and Jimena relied on the translator to understand and respond to the interview questions.

Classroom #3

Classroom #3 had three seventh-grade students; Abril, Carlos, and Daniel. Abril had only been in the United States for one year but was scoring higher on the WIDA ACCESS than some of the other students. Both Carlos and Daniel had been in the United States for less than a year; therefore, they did not have WIDA ACCESS scores. Daniel had been in the United States when he was younger but had not returned for over three years. In this classroom, Abril, Carlos, and Daniel were seated close together in the front of the room. An additional Spanish speaker was seated with Daniel so that he would have a language partner. This position was rotated through available Spanish speakers in the class. This class had one other native Spanish speaker who could translate and a dual immersion student who could translate. At the time of the study, Daniel was seated with the dual immersion student. Dual immersion is a program model in which students are immersed in two languages each day. In this school district, dual immersion students spent half of the day immersed in Spanish and half immersed in English. The district had one dual immersion elementary school. At the middle school level, dual immersion students took a combination of language arts and either science or history in Spanish and the rest of their courses in English.

All the participants, both teachers and students, were very supportive and helpful throughout the data collection process. The teachers made time in their schedules to participate in the interviews, and the students met with the researcher during their lunch periods. The students smiled often, spoke openly, and asked to stay until their lunchtime finished rather than

rejoining the lunchroom crowd. At the end of the interview process, several student participants offered hugs. Everyone was cooperative during the research process.

Research Question #1: Student Perceptions of
Effectiveness of Mathematical Teaching Practices

Students were asked about the effectiveness of Mathematics Teaching Practices (MTPs) within their classroom settings. During the interviews, effectiveness was defined and articulated as helpfulness in understanding the mathematics students were learning that day. To draw out the perceptions of Hispanic ELL middle-school students, video stimulated recall (VSR) was incorporated into the interviews. Students were shown a video clip of themselves engaging in a given MTP during a mathematics lesson, then asked about the helpfulness of the practice they observed within the video clip. All the video clips shown during the interviews included the participants engaged in the MTP. Students could respond with words or a thumbs up, thumbs down rating system. The rating system had been previously discussed in the group orientation protocol so that students were familiar with the language or gestures they could use to indicate the helpfulness of each MTP. After reporting the overall helpfulness of each MTP, students were then asked how the MTP contributed, or not, to their learning that day and on other occasions. Additionally, students were asked what other practices their teacher engaged in that helped them learn mathematics. They were also asked if there was anything they wished that their teacher would do to help them learn mathematics. The Student Interview Protocol can be found in Appendix F.

To organize the findings for this research question, each of the MTPs was examined separately. For each MTP, the different ways in which the three teachers facilitated the MTP

within their classrooms is described first. Data about facilitation of MTPs were obtained from the researchers observations while in the classroom and while observing the classroom actions on the video recordings. Then the helpfulness of the MTP as perceived by the students is reported. This is followed by a description of student perceptions regarding how the MTP supported their learning of mathematics. Finally, there is a discussion of themes and common threads that emerged around the three MTPs during the analysis of the data from the student interviews.

Findings for Mathematical Discourse

Teachers' Approaches to Mathematical Discourse. Mathematical discourse involves using words, gestures, and attitudes specific to mathematics in oral and written formats (NTCM, 2014). Mathematical discourse can be facilitated in many ways, including structures like “turn and talk,” small-group and whole-class debriefs and discussion, and written explanations. Each of the classroom teachers in this study facilitated mathematical discourse in different ways. For example, in Mr. CL’s classroom, the students were not explicitly asked by their teacher to turn and talk or to engage in conversation. However, mathematical discourse appeared to be the norm. For example, the students felt comfortable turning to their partners and talking at any time during the class when the teacher was not instructing. They were also encouraged to walk to another student to ask questions or to talk about mathematics. Mr. CL stated that he chose this arrangement because the students are “going to get a lot of questions answered working together before coming to me. And I think it's a really important skill that they have.” A student from this class claimed, “[Talking to peers when needed is] much easier than your teacher giving you permission to talk with [other students].”

Mr. BW facilitated mathematical discourse mainly through small-group and partnership situations. The mathematical discourse often occurred inside those groupings and was then debriefed in a whole-class discussion. Mathematical discourse in this classroom occurred when the teacher instructed it to happen using phrases like “turn and talk” or “talk with your partner.” Ms. NG had yet another way of facilitating mathematical discourse. The teacher encouraged mathematical discourse in various formats, including partnerships, small groups, and whole groups. She also incorporated movement into the class period. For example, when students were discussing opposite numbers on a number line, the teacher assigned an integer to each student. They were required to talk to one another to decide how to order themselves in a human number line.

While there were differences in the ways the teachers facilitated mathematical discourse, there were also commonalities. In all classrooms, students were seated next to other students to be able to converse with one another easily. They were not set up in singular desks arranged in rows but rather at tables with at least two students. Another commonality was that, in all classrooms, teachers had seated ELL students with other students who could speak Spanish to provide a language partner.

Student Ratings of Mathematical Discourse. To rate the practice of mathematical discourse, students viewed a video clip of the MTP being utilized within their classroom. They were asked to rate the practice’s helpfulness in that situation using the researcher-provided rating scale. The rating scale was illustrated with a thumbs-up symbol to indicate that the practice was helpful. A side-to-side hand symbol meant that the practice was somewhat helpful, and a thumbs down suggested that the practice was not helpful. Students could use hand gestures, point to the

illustration, or articulate their ratings. Of the nine students interviewed, six rated mathematical discourse as helpful, really helpful, a lot helpful, or with a thumbs up. While “really helpful” and “a lot helpful” were not on the rating scale, the students often used these words when rating the practices. Two students stated that mathematical discourse was somewhat helpful, and one said mathematical discourse was a little bit helpful. Two of the students noted that mathematical discourse was only helpful when it was conducted in Spanish.

Student Perceptions of How Mathematical Discourse Supported Learning. After assessing the helpfulness of each MTP, students were asked to describe their perceptions of how the MTP was supportive in their learning that day and on other occasions. The analysis of interview data revealed two common themes: (1) the importance of explanations from peers and self and (2) the differences between written and oral discourse. The theme of explanations from peers and self was characterized by opportunities for students to considering other viewpoints and strategies about the mathematics they were attempting. The second theme emerging from the student interviews around mathematical discourse was that written and oral discourse are perceived differently in terms of how helpful they are in understanding mathematics. This could be seen as especially important for ELL students experimenting with forms of output (Krashen & Terrell, 1983). Depending on their level and type of English development, as well as comfort in a classroom, students may prefer one modality over another. Nayeli articulated this preference.

I think [writing] helps me a little bit because I get to out all my thoughts that are inside. I can just write [my answers and explanations] out instead of explaining it because it's a bit hard when I'm explaining [how I solved the problem and my thinking].

Both modalities are considered “output,” which is often developed later in second language acquisition (Freeman et al., 2021).

Explanations from Peers and Self. When engaging in mathematical discourse, students felt that gathering opinions from one another, learning together, and seeing an alternative way to solve a problem helped support their understanding of the mathematics that they were learning. One student felt that mathematical discourse was helpful in her learning because “I saw both sides, my thinking, and my friend’s thinking. How does she think about [the mathematics]? If she understands [the mathematics] more than me and how she can help me.” This is an example of how some students relied on each other for an alternative way to think about the problems they were attempting. Students viewed their peers as collaborators in their mathematical understanding. Carlos described this type of interaction. “So I could give him my opinion. He could give me his opinion. And through this, we could determine what was the correct answer.”

Additionally, articulating thinking out loud to oneself is a form of mathematical discourse and this action supports the development of mathematical identities and understanding. Students claimed they were able use the strategy to independently support their mathematics thinking. Lupe described that it was helpful when she articulated her thinking out loud because “you walk yourself through [the problem] and you take on your pace instead of going really fast.” Several students agreed that self-talk was an essential way for them to understand the problems they were attempting in mathematics class. In one of the observed videos, Elena was shown reading the question out loud to herself and talking herself through the steps she would take to complete the problem. Elena summarized the process she utilized: “Well, sometimes I [self-talk] to learn the words, but sometimes I [self-talk] because when I read [the problem] in my head, [the

problem] doesn't make sense for me. I'm used to just reading [the problem] out loud to myself." Lupe added the importance of being able to self-regulate and go at your own pace when she shared, "Yeah, it's really helpful because you walk yourself through [the problem] and you take on your pace instead of going really fast. You take [the problem] at your own pace and it's really helpful." For both Elena and Lupe, engaging in self-discourse helped these students develop a mathematical independence in which they support themselves and their mathematical understandings.

Written Versus Oral. Both written and oral forms of mathematical discourse are considered output in the process of second language acquisition (Freeman et al., 2021). Krashen and Terrel (1983) identified stages that emergent bilinguals pass through as they develop language. Emergent bilinguals are students who are in the "early stages of developing an additional language" (Freeman et al., 2021, p. xvii). Students start in a preproduction or silent language stage before moving into an early production stage. After early production, speech emergence occurs. Finally, students pass into intermediate fluency where they can complete complex sentences. These stages of speech align with writing output abilities (Krashen & Terrell, 1983). Four of the students in this study were in the preproduction stage. They could understand much of what was said but were not producing language. The role of the translator was vitally important in the interviews so that data could be accurately collected.

Participants in this study had differing perceptions about whether written and oral discourse were helpful in their understanding of mathematical concepts. Students had various opportunities to speak out loud to language partners, peers, teachers, and the whole class. When engaging with the whole class, teachers would often have a language partner translate the words

of the ELL students so that their voices could be included in the discussion. Most of the written mathematical discourse took place in the software platform the middle school teachers used in their mathematics classrooms. The software had the capability of translating questions into Spanish with one “click.” Answers could be typed in Spanish; however, they could not be translated from Spanish to English so that the teacher could read them. One of the teachers stated that he had the students write their answers in Spanish. Then he would copy and paste it into a translating platform to see what the students were trying to explain in their native language.

One student, Nayeli, explained that written discourse was more accessible for her.

I think [writing] helps me a little bit because I get to out all my thoughts that are inside. I can just write [my answers and explanations] out instead of explaining it because it's a bit hard when I'm explaining [how I solved the problem and my thinking].

Lupe felt the same way: “I agree with Nayeli. It's much easier to type than explain yourself because you can use a bunch of the different stuff that [the software] offers you and it just gives you more ideas.” These two students saw writing as supportive in their understanding. They could use their resources and time to better explain themselves in written format. However, other students in the study felt differently. For example, Julian and Elena felt they could communicate better by speaking. Elena said “I'm better at explaining and it's easier for me to say [my explanations and thinking] than to write it [thinking].”

Interestingly, the analysis also revealed different perceptions of the impact of grouping arrangements in mathematical discourse. For example, three student participants stated that grouping could serve as both a support and a distraction within mathematical discourse. In the video these students viewed, the teacher had grouped the students first with partners and then in a grouping of four to enhance the mathematical discourse and idea-sharing. Even though all

members of the foursome spoke Spanish, the students interviewed stated that they preferred the partnership and that it was more helpful. Carlos claimed, “when it was group of two [it was more helpful] because we were almost hiding more in the group of four. We were hidden, and it was harder to understand what was happening with what everybody was saying.” Daniel added that he felt that “we were making more errors when it was a group of four.” Grouping may be a factor in oral discourse that makes a difference in how students perceive the helpfulness of mathematical discourse.

Findings for Posing Purposeful Questions

Teachers’ Approaches to Posing Purposeful Questions. As with mathematical discourse, posing purposeful questions was also implemented differently in the three classrooms that were observed in the study. In two classrooms, Mr. CL and Ms. NG posed purposeful questions in either small groups or individualized settings. The questions were designed to advance student thinking as well as assess where they were in their thinking. For example, Ms. NG had a straightforward set of questions to determine what students knew and how. “The next question was so simple, it was just, ‘How did you know that?’ Sometimes you think a question needs to be this big, I don’t know, very monumental question. Instead, you just need to ask them, ‘How do you know that? What’s your reasoning?’”

In one classroom observation, the teacher made a very specific choice about which mathematical question/problem to pose to the entire class based on the depth of mathematical understanding that would potentially be uncovered while students explored the problem. While the facilitation move of posing purposeful questions was enacted differently in the classrooms,

the teachers were asking questions with the same goals in mind. They supported high-level thinking, prompting for explanations and connections, and encouraging students to explore mathematics deeply. Ms. NG explained how questioning helped her see what her students were thinking. “Without that next question, I wouldn’t have known how they knew that.” Mr. CL reflected that his students were exploring mathematics deeply by sharing, “I think they respond really well. It’s just a really hard question. Trying to open up minds and get people to think about things that they’ve never thought or seen before [requires questioning].”

Student Ratings of Posing Purposeful Questions. As with mathematical discourse, students were asked to rate the helpfulness of purposeful questioning after viewing video clips from their mathematics lesson. One student was absent for this round of interviews. Of the eight others interviewed, five rated purposeful questioning as helpful, two said it was between somewhat helpful and helpful, and one rated purposeful questioning as “a little” helpful. Elena, Lupe, and Nayeli all stated that their teacher always asked a lot of questions. Elena provided an example when she said, “[the practice] helped me because he asked us questions, and while I answer them, he’s like, “Yeah. But...” And then he keeps on asking questions, and I keep on thinking.”

Student Perceptions of How Posing Purposeful Questions Supported Learning. The interview data about purposeful questioning uncovered two critical student perceptions. First, students described how questions helped to organize and articulate thinking. Posing purposeful questions can provide students with a prompt that encourages them to organize their thinking. Other questions require students to respond, meaning they must articulate their thinking and

understanding. Second, students described how questions helped to shape their mathematical identity. The theme of mathematical identity emerged within the other MTPs as a common thread, but it was clearly articulated by students when discussing purposeful questioning. Choosing problems that require more than computation positions students to believe they can tackle challenging mathematical questions. Being able to answer questions that require thought positions ELL students as competent members of the mathematical community (Gresfali & Cobb, 2006). Gresfali and Cobb (2006) describe positioning in a mathematical classroom as “the way that students are entitled, expected, and obligated to interact with one another as they work on content together” (p. 51).

Organizing and Articulating Thinking. Throughout the interviews about purposeful questioning, students related times in which intentional questioning techniques helped them organize and articulate their thinking. Lupe explained the questioning process as follows:

He doesn't say right away what the answer is, he explains carefully what you should do, like he asked, “What could you do instead of this? Why should you do this? Can you explain to me what is this and that?” So, [questioning] helps you understand in reality what's in your mind.

Teachers also used probing questions to continue to advance student thinking. Elena stated, “[the practice] helped me because he asked us questions and while I answer them he's like, “Yeah. But...” And then he keeps on asking questions, and I keep on thinking.” Nayeli supported the idea that questioning helps students articulate their mathematical thinking. “[Questioning] helps me more. I can't pull my own thoughts out of my mind like that.” This type of questioning most often occurred when the students were having one-on-one or small-group conversations with the

teacher. This environment allowed teachers to pose questions to specific students and support them individually as they synthesized their thinking about mathematics.

Shaping Mathematical Identity. During the classroom observations, teachers were observed asking purposeful questions of their students. In some situations, teachers had pre-selected questions and, in others, they used a string of questions that achieved their purpose for questioning. In one-on-one and small-group settings, the teachers would use probing questions, assessing questions, and advancing questions to draw out student thinking so that students could synthesize the mathematical process they were exploring. An example of both probing and assessing questions was observed when Ms. NG asked her students, “How do you know that? What's your reasoning?” She used this line of questioning frequently when assessing the depth of her students’ understanding. An advancing question observed in Mr. CL’s class was “How do the three representations we’ve worked with (weights/hangers, equations, graphs) connect to this real-life example?” His question was met with a question from students, “What happens if we do ____ in this situation?” which exemplifies that the students were engaged in applying the representations to the real-world example.

Teachers again made purposeful choices when choosing questions for the class to explore. For example, Mr. BW chose a question for the whole class that was highlighted by the grade-level resource because it required students to look at multiple ways to represent the same mathematical situation. This multiple-choice question was not a straightforward computation problem; it asked the students to explore all the options available to decide which were correct (there were more than one correct option). This facilitation method was seen again in Mr. CL’s class where the teacher conducted a whole-group discussion focused on one purposeful question.

In this instance, Mr. CL had chosen a “level-four” question. A “level-four” question is a question that requires students to think above the grade-level minimum requirements. He also told the students that the question was a level-four question, thus above grade level. It would require some serious thinking. These choices illustrate how the teachers did not choose different questions for their ELL students but instead provided supports so that the students could access the above grade-level content. Both teachers said they often used the district resource when selecting these questions. These decisions supported mathematical identity by showing students that they are capable of grade-level (and above) questions. Nayeli shared the example, “There was a day when you were recording, [Mr. CL said] it was tenth-grade math, and so I was like, “Wow, I already know [can do] this,” and I’m so speechless because I’m barely in sixth grade!” Nayeli’s teacher’s choice to have all students engage in challenging mathematical questions led to an increased confidence level for Nayeli.

Findings for Using and Connecting Mathematical Representations

Teachers’ Approaches to Using and Connecting Mathematical Representations.

Mathematical representations can appear in many different forms within a mathematics classroom. For example, visual representations such as diagrams and graphs, symbolic representations such as algebraic equations, verbal and contextual representations, and physical representations such as manipulatives and models may be used daily in a mathematics classroom (Smith et al., 2017). For these classroom observations, the teachers purposefully chose to highlight using and connecting mathematical representations within their teaching. In Mr. BW’s classroom, he connected the relationship between a double number line, a table, and an equation

to solve a contextualized problem about turtles. Ms. NG demonstrated various number lines to help students understand positive and negative values and their opposites. She drew vertical and horizontal number lines on the board, had the students create a human number line, and demonstrated a student representation of a number line that looked like a rainbow. Mr. CL demonstrated inequalities using weights on a hanger, equations, graphs, and real-life contextualized situations. All three teachers explicitly discussed the connections between the different representations and the mathematical understanding under consideration that day.

Student Ratings of Using and Connecting Mathematical Representations. All nine students from all three of the mathematics classrooms rated using and connecting mathematical representations as helpful, except for one student who said it was sometimes helpful. The reasons the students found mathematical representations supportive in their mathematical learning varied across two themes.

Student Perceptions of How Using and Connecting Mathematical Representations Supported Learning. The analysis of the interview data regarding mathematical representations revealed two critical student perceptions: the importance of options and the value of seeing the math. When using mathematical representations, students stated that they have many options for finding a solution. Students also talked about the value of seeing the math, which the researcher interpreted as referring to how a concrete representation helped them understand abstract mathematical ideas. These themes were consistently evident when the students spoke about mathematical representations.

Options. Students were able to clearly articulate how mathematical representations provided them with choices and options when solving problems inside their mathematics classroom. For example, Elena stated, “Well, I think it's helpful because you have three different options to [solve the problem]. If you don't understand one, then you could do it the other way. And you have other options.” Elena viewed mathematical representations as a way to have choices in the pathway to understanding mathematics. Daniel supported this idea by stating, “[Multiple representations] teach us another process to solve [the problem].” Students also viewed mathematical representations as a way to confirm their solutions. For example, if they got a solution using a table, they would then ensure they got the same answer when using a graph. Having choices and opportunities for confirmation provides a wide range of access to the content and multiple entry points to the content, and draws on multiple resources of knowledge, (Moschkovich, 2013).

Seeing the Math. Seeing the math was articulated multiple times throughout not just the mathematical representations interview but also in the mathematical discourse and purposeful questions interviews. Students referred to many ways in which they can “see the math.” Elena summarized, “I need to see [the mathematical idea] to be able to understand it.” Noemi and Jimena found that drawings that supported mathematical ideas were especially supportive and stated, “Because [the problem is] referring to the drawings, but you see math,” and “For me, it helps me on the iPad when we are seeing animals or persons.”

Seeing the math included physical representations as well as drawings. Mr. CL used many hand gestures when explaining mathematics to students. His students stated, “He uses hand techniques to see which way. And he really knows how to get you to understand.” Ms. NG used

a physical number line to help students understand the concept of positive and negative numbers and opposites. Her students described this as helpful because “we could see which numbers came before and after.” One of the students explained that she often helped her brother with his mathematics at home and used drawings to help him understand elementary-level concepts. Seeing the math was very supportive for these students and they desired even more visual representations. For example, Elena said, “Well, I think one of the things that they could do to help me would be kind of having the actual thing. More hands-on, we could say. So that when you're right there, and then you can actually show the teacher what you are thinking.” The student was describing her desire for realia or objects that she could use to represent what was going on inside the mathematical situation.

Common Threads

In Chapter Three, it was recommended that, throughout the entire interpretative process, researchers keep an open mind to alternative understandings that may challenge their own (Marshall & Rossman, 2015). Keeping an open mind allows for unexpected findings to emerge. During the analysis of student interview data, two additional findings emerged woven through the data about the helpfulness of the MTPs. Students often spoke about the need for translation. They referred to the translation of materials as well as the translation of the content that the teacher was delivering orally. Thus, a common thread related to Research Question #1 is translation.

Translation. While examining the data across all three MTPs, evidence was found supporting translation as a common thread. Almost all the students mentioned translation as a

supportive structure during the interview process. For example, the need for translation was made evident when the students were asked what they were thinking during the math lesson they observed via the video clips. Daniel stated, “I wasn't really thinking about anything because he's speaking in English, and it's hard to understand him.” Every time Carlos and Daniel were asked what other things helped them learn mathematics, they mentioned translation. The translator who was present at the interviews stated that Daniel said, “His partner helping him translate” is helpful. Carlos added that the Spanish translations needed to be done well to be helpful, noting that “Sometimes it's more confusing if they don't speak Spanish very well.” The translator further clarified, “So we assume just because it's in broken Spanish that it might help, but sometimes it's more confusing.” Julian talked about the helpfulness of the online resource that his teacher used, claiming that it was easier for him because it would translate the questions to Spanish for him. His teacher also encouraged him to type his responses in Spanish. Then the teacher would use translating software to read his work. Julian viewed this as helpful. To access the mathematical content, the students in this study found translated material very helpful and articulated the need for more translated material.

Conclusion

Research question #1 explored student perceptions of the helpfulness of three research-informed Mathematical Teaching Practices. Data analysis uncovered six themes within the student data as well as one common thread, identified as such because it was woven through the discussions of all three MTPs.

Teachers' Perceptions of the Effectiveness of MTPs

Interviews with students about the helpfulness of three MTPs resulted in data that provided new insight into students' perceptions of these practices. It is informative to contrast these findings with the teachers' perceptions to identify any areas in which student and teacher perceptions align and diverge. As discussed in Chapter Two, findings from research about student consultation indicate that consistency between a teacher's intention for a teaching event and the students' perceived importance of that event makes students more likely to engage in the learning task at the cognitive level the teacher intended (Huang and Barlow, 2013). This section explores teachers' perceptions of the helpfulness of the three MTPs; alignments and divergences will then be examined in discussing the third research question.

Once participating classrooms for this study were identified, each teacher was given a copy of *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6–8* (Smith et al., 2017). The intent of this was to clarify what the three Mathematics Teaching Practices (MTP) selected for this study looked and sounded like in a classroom setting. The text was provided to address questions about how the practices were defined or could be implemented. Two teachers stated that they reviewed the sections on the three practices being investigated, and one teacher had not had a chance to look at the text. Therefore, while the text served as an optional support, knowledge and proficient implementation of the three MTPs was validated primarily through the researcher's prior knowledge and classroom observations. Teachers were, however, purposeful in the incorporation of the selected MTPs into their lesson plans so that data collection could occur.

To draw out teachers' perceptions about the helpfulness of the selected MTPs, they also

participated in video stimulated recall interviews. During the interviews, teachers were asked about the effectiveness of the MTPs within their classroom settings. Effectiveness was defined and articulated as the MTP's level of helpfulness in assisting students in the doing of mathematics. During Phase One, when proxy terms for the MTPs were being developed, teachers stated that an MTP was "helpful" if it assisted students in being able to do the mathematics independently.

As with the student participants, teachers were shown a video clip of their students engaging in a given MTP during a mathematics lesson. Then teachers were asked to assess the helpfulness of the practice they observed within the video clip. After evaluating the helpfulness of each MTP, teachers were asked how the MTP supported student learning that day and on other occasions. Like the students, teachers were additionally asked what other practices they perceived as helpful to Hispanic ELL middle-school mathematics students. Throughout the interviews, teachers' responses often addressed both the first and second interview questions. For example, after rating an MTP, the teacher would explain how and why the MTP was helpful before being directly asked. Analysis of interview data revealed multiple ways that each of the three Mathematical Teaching Practices effectively supported student learning.

Teacher Ratings and Perceptions of Mathematical Discourse

All three participating teachers rated mathematical discourse as "very helpful" or "very, very helpful." Similar to the interviews with students, teachers used terms outside of the rating scale to suggest an even greater level of helpfulness. The teachers had a wide variety of reasons they perceived mathematical discourse to be supportive of mathematical learning for Hispanic ELL students. In analyzing the transcripts from teachers' mathematical discourse interviews, five

themes emerged from the data: environment, independence, processing, evidence, and access. The theme of environment refers to how mathematical discourse practices contribute to positive classroom culture and increase the comfort level of students. The theme of independence focuses on how students utilize the MTP to help themselves. The processing theme highlights the importance of creating the opportunity to work through mathematical thinking and revise solutions. The evidence theme captures the ways teachers are able to gather information about student thinking when engaged in mathematical discourse, and the theme of access reveals the teachers' perceptions regarding how mathematical discourse provides ELL students an entry point to the content. Each of these is discussed separately in the following sections.

Positive Learning Environment. The teacher participants frequently spoke about how mathematical discourse encouraged a positive learning environment for their Hispanic ELL students. They spoke about the importance of all voices being heard within their classrooms and recognized that this may not always be the case for Hispanic ELL students. Ms. NG stated, "I don't know if Hispanic ELL students always get the opportunity to have a voice, or feel comfortable enough having a voice, even if it's in their native language in a large group setting." Ms. NG's statement connected to what Mr. BW said about small-group conversations encouraging engagement:

So I think that, for them, understanding that during turn-and-talk they have a comfortable small group space in order to do it in, provides them with an opportunity to have a voice in a math classroom in which they might just be passive participants.

In both examples, teachers illustrate how mathematical discourse encourages a classroom environment in which students feel comfortable enough to have a voice in the mathematical

discussion. The teachers in this study did this by explicitly facilitating “turn and talk” opportunities, classroom activities that included discourse, and classroom situations in which students were allowed to seek feedback from their peers at any time.

Building Independence. When a supportive learning environment is fostered, teachers found that students were able to build independence by utilizing mathematical discourse. They saw that, when discourse routines were facilitated within the math classroom, the students were soon using the routines independently without being prompted by teachers. Mr. BW mentioned:

I think by embedding the routine of turn-and-talk, it actually is facilitating more of a long-term skill development so that they know that the people around them are supports for them. So that even without a teacher asking them to turn and talk, they know that turning and talking will support their learning.

The students were finding ways to help themselves and their mathematical understanding. Mr. CL added to this thinking by stating, “Taking discourse into account, they can help each other. And then if they both get stuck, then you can ask for help and get another point of view.”

Mathematical Processing. Teachers in the study viewed mathematical discourse as a way in which Hispanic ELL students could process their mathematical thinking, identify misconceptions, and make corrections. The opportunity to think and speak with another person allowed them to consider multiple viewpoints and reconsider their own lines of thinking. Mr. BW summarized the opportunity to process during mathematical discourse by stating, “I just think of it as different forms of processing, first giving yourself the opportunity to think of it on your own, and then the turn and talk processing with one person.” Ms. NG observed students using mathematical discourse to clarify misconceptions and shared, “I think [mathematical discourse] was really helpful because they were able to identify a misconception that they had

from the beginning. It gave them the opportunity to discuss a misconception and then rectify it.” Mr. CL also noticed this synergy in his classroom: “You’re not just thinking to yourself, you’re thinking with someone else. And then they can catch each other’s mistakes and catch each other at how they work.” All three teachers viewed mathematical discourse as a way for students to process their mathematical thinking.

Evidence of Learning. Teachers in this study saw mathematical discourse as evidence of a student’s understanding of the mathematics being studied. Ms. NG stated, “I wonder sometimes if a student isn’t able to verbalize their understanding, how embedded is it in them? If you can’t say [your mathematical thinking] out loud, how well do you know [the mathematical concept]?” In her classroom, students were encouraged to talk about mathematics in their native language, “And while Noemi had to say [her thinking] in Spanish, it was still proof of her understanding. That’s like doing a second layer. I know this. I know this in Spanish, and now I can tell you why.” Mr. CL agreed with Ms. NG when he stated, “Because you have to understand [the mathematical concept] at a deep level to be able to explain [your mathematical thinking].” He also saw discourse as supportive because “it is really helpful that they can explain because then it gives you a window into what they believe.” The teachers in this study saw discourse as a method in which ELL students were able to show evidence of their understanding.

Access to Content. The final theme uncovered in the discussions about mathematical discourse was increasing students’ access to content. None of the teachers in the study were fluent in Spanish. All teachers relied on pairing ELLs with language partners to translate for their Spanish-speaking students. Most of the Spanish-speaking students sat with another student who

spoke Spanish fluently but was not an identified ELL student. This was sometimes a native speaker or a student in the Spanish dual immersion program. The students in this study from Classrooms #2 and #3 relied on their language partners for oral translation of the spoken content. In Classroom #1, only Julian was very reliant on his language partner. The other students in Classroom #1 had enough English language to access much of the mathematics content. However, they frequently relied on the Spanish translation of their electronic resource.

Mathematical discourse was perceived as a way in which the ELL students could have more access to mathematics content through translations from their language partners. Mr. BW summarized this situation:

With ELL students in particular, that conversation's going on in Spanish. And so I'm excited that that's occurring, because then that's in the language that they can access the information. So to me although I can't speak Spanish, that seemed to be a fruitful conversation.

Mr. BW did comment that, due to his limited ability to speak Spanish, he was not always sure that the mathematical discourse was productive:

I was hearing from my English students, so many great conversations. So I think when I saw the two ELL students with two language partners sitting together, my assumption was the same fruitful conversation was occurring and the fact that they got two correct out of three, to me only confirmed that assumption.

This is important because Mr. BW had to assume that the math was happening until he could confirm it with the correctness of their answers. This indicates that mathematical discourse may need to be combined with additional checks for understanding.

In this study, mathematical discourse was perceived as very helpful to Hispanic ELL students by all three teacher participants. The teachers created encouraging environments in which students were using mathematical discourse to guide their learning and gain greater

independence. Teachers witnessed mathematical processing and used mathematical discourse as evidence of understanding. Mathematical discourse provided a layer of access to mathematical content that was not presented in the native language.

Teacher Ratings and Perceptions of Posing Purposeful Questions

Based on the video segments they viewed, two teachers rated posing purposeful questions as helpful to Hispanic ELL students in their mathematical learning. One of the teachers stated that, in the video clip he viewed, posing purposeful questions was probably not as helpful. This was partly due to how long the question took to facilitate within the class period. This teacher stated that, when questions were translated, they could be very helpful.

Two themes emerged from the interviews about posing purposeful questions. First, the data revealed that purposeful questioning was used to assess and advance student thinking toward mathematical understandings. Second, purposeful questioning was also used to encourage students' participation and ownership of their learning. Participation in the mathematics classroom is a crucial element in learning for ELL students who need peer and teacher interactions to increase content knowledge and language development (Chval et al., 2021).

Assessing and Advancing Student Thinking. Teachers in the study found purposeful questioning to be highly effective when working with students one-on-one to assess their understanding and advance their thinking about mathematics. Mr. BW summarized this by stating, "Being able to sit down with one student, and just get them to think, can just have this amazing effect. Just finding the time." When using this type of questioning, Ms. NG found that a simple line of questioning was helpful for her students:

Literally the next question was so simple, it was just, “How did you know that?” Sometimes you think a question needs to be this big, very monumental question. Instead, you just need to ask them, “How do you know that? What's your reasoning?”

Utilizing assessing questions like: “How did you know that?”, “What’s your reasoning?”, and “Where is your confusion happening?” helped teachers evaluate where students were positioned in their learning.

Purposeful questioning also helped students to develop effective thinking habits like applying what they know to new situations. Mr. CL saw that challenging questions could be used to extend his students’ understanding. He observed, “They seem to be able to apply their understanding that they got from that struggle into more questions. I think she responded really well to being pushed, to think about it in a different way.” Advancing questions move students to extend what they know to a new situation and encourage them to consider a line of thinking they had not considered before (Smith et al., 2017).

Ownership of Learning. The environments and relationships these teachers had established with their students assisted them in posing challenging questions that led students to take ownership of their learning. Teachers in this study viewed ownership as students being able to develop their own answers to problems rather than being told how to solve them step by step. Mr. CL saw this as an important element in learning mathematics. He claimed:

Using purposeful questions allows them to take ownership of their learning and they feel like they’re the one that got there, as opposed to being told the answer. I think just giving somebody an answer or not getting them to think about something, leads to them developing bad habits and bad ways of thinking. By posing purposeful questions, you start leading them in the direction, but they take ownership of [their thinking and learning].

The teacher participants saw their role not just as deliverers of knowledge but also as developers of thinking habits. They also saw their role as builders of student efficacy. They were consistently working to build mathematical confidence in their students. Ms. NG stated:

Me believing in them, other students believing in them, them being paired with all levels of students, and giving them voice, and giving them purpose in the classroom helps with them feeling like they belong. That helps build their confidence and that helps lower their affective filter and that helps them to feel like engaged participants in the community. We can't believe that because they don't know English, that they can't understand what's going on in the classroom.

Ownership of learning is developed when students are actively involved and have a personal investment in the learning taking place (Voltz & Damiano-Lantz, 1993). The teachers in this study appeared to value having students create their own answers and responses rather than being told how to perform the mathematics. By asking challenging questions to ELL students, these teachers were working to ensure mathematical equity and the development of a mathematical identity (Aguirre, Mayfield-Ingram, & Martin, 2013).

Teacher Ratings and Perceptions of Connecting and Using Mathematical Representations

Two participating teachers rated connecting and using mathematical representations as “very helpful” to Hispanic ELL students. One teacher felt that the way he facilitated connecting and using mathematical representations was “not very helpful this time.” Two themes emerged from this portion of the interview data: tools for independence and connections between representations. Teachers in this study felt that the more options and connections between those options they could present to students when solving problems, the more independent the students could become in their mathematical thinking. It is worth noting that this topic had been part of a broader discussion in the mathematics department at the participating school. The team was

exploring, “Is it better to teach students to master one way to solve problems or to provide multiple ways to solve problems?” In addition, teachers in this study saw that other types of connections—to background knowledge, real-world situations, and student lives—also strengthened their students’ mathematical understandings.

Tools for Independence. Regarding connecting and using mathematical representations, independence means that students can choose and utilize mathematical representations without relying on the teacher to provide or model them. This skill allows students to identify various options in how they solve problems. As mentioned, the mathematics department at the school was exploring beliefs around teaching students multiple representations (many options) and mathematical algorithms (one option). Mr. CL observed that the debate centered on whether faculty were teaching students “too much.” He summarized the conversations:

[They are saying] we're teaching them too much. We're exposing too much. And I don't believe that's the case. I don't believe that you can expose too much. I don't think that you can show them too many different ways to do [the mathematics], because they're going to find what works for them. We have to do this because, otherwise, you're forcing kids into a box that they don't fit in.

By teaching multiple representations of mathematical concepts and processes, Mr. CL stated, “It's opening up that door so that it's not one way. Giving them multiple ways allows them to actually choose what they like to do. What makes sense to them, so that they can put sense into their own math.”

Presenting multiple representations is similar to providing numerous tools to complete a job. Each individual can choose the tool that helps them best complete the job. Ms. NG stated, “This is showing me that they then have a tool. And so they don't need me as much and that's the

whole point, right?” She saw that students could be more mathematically independent by exposure to a wide variety of mathematical representations.

Connections Between Representations. Teachers in the study also viewed mathematical representations as a way to make connections between the representations and previous learning, real-world situations, and the students’ own lives. Mathematical representations were presented by teachers, pulled from the classroom resource, or created by the students themselves. An example of a student-created representation was when a student in Classroom #2 used a rainbow to illustrate opposite numbers on a number line. The teacher highlighted this representation so that other students could use it. The teacher stated:

But it was so powerful because I've never looked at it as this idea of a rainbow, like something that all students have likely seen. And so I think that really helped to build or just connect back to their own lives and build that background knowledge and make connections to previous nonacademic experiences that they've had.

Common Threads

The previous sections discussed themes from the teacher data that are linked to each of the three MTPs. Three additional findings emerged from the teacher interviews. In addition to the themes that developed from the data from the teacher interviews surrounding each of the MTPs, additional findings were also uncovered during the data analysis process. Teachers in this study spoke about the importance of translation through the use of language partners and the shaping of positive mathematical identities within their students.

Language Partners. Language partners were relied upon frequently within the mathematics classrooms in this study. The study site followed the practice of placing emergent

bilinguals in a classroom with students who are fluent or close to fluent in English as well as the native language of the emergent bilingual. Mr. BW described this situation: “I put Carlos who does not speak much English at all, next to Abril, who is fluent in both languages. Abril understands what's going on, and then I have her engage in the content with Carlos in Spanish.” Mr. BW found this arrangement very helpful for mathematical discourse and stated, “To have mathematical discourse occur in Spanish as a result of a language partner seemed very helpful.” Mr. CL noticed that the emergent bilingual student in his class was more successful when paired with a language partner claiming, “His scores and his ability were a thousand times better. Every time he was working with Elena. Now we're on a more simple concept. And he's struggling with it because he is not sitting with Elena anymore.” Ms. NG saw language partners as effective when encouraging emergent bilinguals to share their thinking with the whole class and stated, “They will say it aloud to the class, and then one of their language partners will then repeat the translated version.”

Even though the three teacher participants found language partners to be supportive for Hispanic ELL students, they articulated two primary concerns about the practice. The first concern was introduced in a previous section when Mr. BW stated that he was not always sure if the language partners were having productive mathematical conversations due to his lack of fluency in Spanish. Mr. BW also expressed concern about the cognitive and social burdens placed on a language partner. By definition, when a student is functioning as a language partner, they must grapple with the mathematics content as well as how to explain the content in another language. Mr. BW described the situation he saw developing in his class when he said, “One of my worries is that it's a burden on the language partners in that they're not only doing math but

then also translating it.” At the start of the semester, Mr. BW had thought that he would identify two students to be language partners for the whole semester. He soon saw that he would need to rotate the position after “Daniel's partner had requested to not be a language partner anymore.” He shared, “I loved it that Angela advocated for herself, stating ‘I've done my role and now I'm ready to move on from this role.’ My guess is she was not excited about only having two partners for the entire semester.”

Mr. CL also saw social implications in being a language partner. In his class, Julian and Elena had been dating. They enjoyed working together until they broke up, and Julian's scores started declining. Thus, Mr. CL worked to pair him up with another language partner. He stated that Nayeli would do it sometimes, but she also liked to work alone. Issues related to the use of language partners appear to intersect with and influence efficacy of the Mathematical Teaching Practices.

Shaping Mathematical Identities. Equitable teaching in mathematics aims to move mathematical learning and understanding forward while at the same time helping students develop a solid mathematics identity (Aguirre, Mayfield-Ingram, & Martin, 2013). Mathematical identities are defined as “how students see themselves and how they are seen by others, including teachers, parents, and peers as doers of mathematics” (Aguirre, Mayfield-Ingram, & Martin, 2013, p. 14). While mathematical identity was not an explicit focus of this study, elements that contribute to mathematical identities, such as classroom culture and belief in student ability, were discussed by teachers.

Teacher participants consistently mentioned the importance of student comfort levels within their classrooms to break down affective filters. Colorin Colorado, a bilingual site for

educators and families of English Language Learners, defines affective filters as “a metaphor that describes a learner's attitudes that affect the relative success of second language acquisition. Negative feelings, such as lack of motivation, lack of self-confidence, and learning anxiety, act as filters that hinder and obstruct language learning” (Colorin Colorado, 2022). Mr. CL summarized this importance. “For them to be comfortable and communicate is the most important thing.” Ms. NG elaborated on the concept of comfort when discussing mathematical discourse:

I don't always see my ELL Hispanic students engaging in as much conversation unless they know that routine of turn-and-talk and they really understand it. So I think that, for them, understanding that the turn-and-talk, that they have a comfortable small group space in order to do it in, provides them with an opportunity to have a voice in a math classroom in which they might just be passive participants.

Ms. NG later shared her views about students being able to share their thinking in the native language as a way to break down affective filters:

And while Natalia had to say [her thinking] in Spanish, it was still proof of her understanding whereas I don't know if Hispanic ELL students always get the opportunity to have a voice, or feel comfortable enough having a voice, even if it's in their native language in a large group setting.

For these teachers, developing a classroom environment where students could express their thinking was advantageous in developing positive mathematical identities.

Teachers in this study also spoke about shaping positive mathematical identities through their belief in their students’ abilities. Ms. NG illustrated this idea when she said:

Me believing in them, other students believing in them, them being paired with all levels of students, and giving them voice, and giving them purpose in the classroom helps with them feeling like they belong. That helps build their confidence and that helps lower their affective filter and that helps them to feel like engaged participants in the community. We can't believe that because they don't know English, that they can't understand what's going on in the classroom.

Ms. NG also spoke about the connection that independence has with mathematical identities: “This is showing me that they then have a tool. And so they don't need me as much, and that's the whole point, right?” Mr. CL saw student independence as a way to shape a positive mathematical identity through purposeful questioning. “Using purposeful questions allows them to take ownership of their learning, and they feel like they're the one that got there, as opposed to being told the answer.” The teachers in this study provided data demonstrating how their belief that students could accomplish grade-level mathematics contributed to the development of a positive mathematical identity for ELL students.

Conclusion

Research question #2 explored teacher perceptions of the helpfulness of three research-informed Mathematical Teaching Practices. Rigorous and iterative use of open and axial coding techniques resulted in the identification of nine themes and two common threads. The common threads were identified as such because they were woven through the discussions of all three MTPs.

Alignment and Divergence Between Student and Teacher Perceptions

In this study, the first research question examined student perceptions regarding the helpfulness of three Mathematical Teaching Practices (MTPs). The second research question examined teacher perceptions about the helpfulness of the same MTPs. Two distinct sets of codes evolved in answering Research Questions #1 and #2. One reason this occurred was that different interview questions were asked of students and teachers. A second reason this occurred was that two sets of participants interpreted the questions about MTPs through very different sets

of perceptions. Delpit (1988) suggests that these differences may also occur as reflection of the culture of the selected site. Students and teachers may have differing perceptions of “how things work” within the school, especially considering that the students and teacher in this study also had differing cultural backgrounds.

The final research question explored areas of alignment and divergence between the perceptions of the students and the teachers. The purpose of the third research question in this study was to unearth areas of alignment and divergence between the teacher and student codes. Identifying alignments and divergences is valuable because previous research has found that, when students and teachers align in their perceptions of the helpfulness of the MTPs, students are more likely to engage in learning tasks at the cognitive level the teacher intended (Huang and Barlow, 2013).

The data from research question #1 and research question #2 were coded as aligned when there was agreement in shared ideas between what students and teachers said about the purpose of the MTPs. The data were coded as divergent when students and teachers had some degree of variance in how they perceived the MTPs or disagreed to some extent about the MTPs’ purpose. Aligned codes were identified across all three MTPs, whereas only mathematical discourse demonstrated considerable areas of divergence. Major areas of alignment are first discussed individually, then summarized in Figure 4.1, which provides an illustrated view of the alignments to clarify the connections between the MTPs. The discussion of divergences follows Figure 4.1.

Alignment of Perceptions around Mathematical Discourse

Based on an analysis of data across the two participant groups, mathematical discourse was the MTP with the highest level of alignment between teachers and students. Students and teachers agreed that mathematical discourse was helpful in learning mathematics. The student themes of explanations from peers and self connected to four of the teacher themes in this category: environment, independence, processing, and access. The student theme of written versus oral connected with one additional teacher theme of evidence. Each alignment is presented below, identified by the corresponding teacher theme and the way in which the themes connect.

A Place and Method for Testing Thinking. Alignment between explanations from peers and self and environment was exemplified when Carlos mentioned that he felt able to give his opinion and seek feedback from his peers. He stated, “I could give him my opinion. He could give me his opinion. And through this, we could determine what was the correct answer.” This aligned with a comment from Ms. NG. She stated, “Turn and talk is helpful in that it provides a safe environment for students to test out their thinking with one other person. So, they get feedback on their thinking.” These comments illustrate how both teachers and students saw mathematical discourse as creating an environment where students felt comfortable seeking feedback from their peers. Mathematical discourse facilitated a method and place in which students could test their thinking with their peers.

Seeking Support Outside of the Teacher. An alignment was interpreted in the data about mathematical discourse as explanations from peers and self helped to develop mathematical

independence. Mathematical independence suggests that students are able to seek help from each other before having to rely on a teacher for help. For example, Elena reported that she often turned to peers for mathematical guidance. She explained, “I saw both sides, my thinking, and my friend's thinking. How does she think about [the mathematics]? If she understands [the mathematics] more than me and how she can help me.” This thinking aligned with what Mr. CL was observing in his class when he said, “Taking discourse into account, they can help each other. And then if they both get stuck, then they can ask for help and get another point of view.” The teachers and students aligned in their comments about how explanations from peers and self facilitated mathematical independence because students were positioned to seek help outside of the teacher.

Methods for Thinking Through the Mathematics. Data reinforcing the student theme explanations from peers and self also aligned with data around the teacher theme of processing. When students described the ways that mathematical discourse supported their learning, they highlighted the value of explanations from their peers as well as the explanations they created for themselves. When describing the purpose of talking out loud to herself, Elena stated, “I [self talk] because when I read [the problem] in my head, [the problem] doesn't make sense for me.” Lupe added, “[Self talk is] really helpful because you walk yourself through [the problem] and you take on your pace instead of going really fast.” Students also spoke about how being asked to explain something aloud helped them think through the mathematics. Noemi stated, “[Mathematical discourse] helped us a lot because we had to explain what we were thinking.” These findings aligned with the teachers’ perceptions about how mathematical discourse helps to improve students’ cognitive processing. Mr. CL stated, “You have to understand [the

mathematical concept] at a deep level to be able to explain it.” Mr. BW’s statements supported an alignment between explanations between peers and self and processing when he explained, “I just think of it as different forms of processing, first giving yourself the opportunity to think of [how to solve the problem] on your own, and then the turn and talk processing with one person.” These student and teacher statements align in the sense that mathematical discourse aids mathematical processing by providing a method to think through mathematics orally, whether with a peer or with oneself.

Providing Comprehensible Input. An alignment was identified connecting explanations from peers and self and increasing access to content for ELL students. Access to content, referred to as comprehensible input in the literature (Krashen, 1983) about ELLs, is when “the teacher makes verbal communication more understandable by consciously attending to students’ linguistic needs” (Echevarria et al., 2008, p. 79). In this alignment, explanations from peers provided access to content through an increased level of comprehensible input.

Mr. BW explained the importance of facilitating mathematical discourse because he could not speak Spanish to his students. He stated, “Having students who cannot speak English has really made me push mathematical discourse more. I just see the students are way more engaged when they're working on their own, as opposed to listening to me tell them about [the mathematics].” Daniel’s thinking aligned with Mr. BW’s when he stated, “I wasn't really thinking about anything because he's speaking in English, and it's hard to understand him.” Daniel said this when asked what he was thinking after viewing the video that showed him listening to a whole-group mathematical explanation from his teacher. Based on the definition of

access to content, Daniel was not able to access the content that was delivered; this aligned with Mr. BWs perceptions of the event.

Another example of this alignment was when Elena stated, “I saw both sides, my thinking and my friend’s thinking. How does she think about [the mathematics]? If she understands [the mathematics] more how than me, how can she help me?” This was a time when content was more accessible for Elena because of the open use of mathematical discourse in the classroom. Ms. NG described this type of access as “I think by embedding the routine of turn-and-talk, actually is facilitating more of a long-term skill development so that they know that the people around them are supports for them.”

Ways to Share Thinking. The researcher found alignment between written and oral mathematical discourse as a way in which students could show evidence of their mathematical thinking. Nayeli explained that written discourse helped her show her thinking when she stated,

I think [writing] helps me a little bit because I get to out all my thoughts that are inside. I can just write [my answers and explanations] out instead of explaining it because it's a bit hard when I'm explaining [how I solved the problem and my thinking].

Mr. CL backed up this statement when he said, “you actually get to see [the writing] and go back and explain to them anything that you see that might not be [correct].” Both Nayeli and Mr. CL seem to agree that discourse in written form serves as a way for students’ to share thinking. However, there was also some divergence in this area that will be discussed in a later section.

In summary, the researcher identified alignment between five pairs of themes within the category of facilitating mathematical discourse. In these five areas, teachers and students gave overlapping reasons to justify why this MTP was helpful in understanding mathematics. While

student and teacher data demonstrated alignment on several points regarding the helpfulness of mathematical discourse, this MTP also produced the most evidence of divergences.

Alignment of Perceptions around Posing Purposeful Questions

Alignment of student and teacher perspectives about the purpose of the MTP of posing purposeful questions was identified in two distinct areas. First, the data indicated alignment between the student theme of organizing and articulating thoughts and the teacher theme of assessing and advancing thinking. Second, data from student ideas around shaping identity aligned with teacher views about ownership of learning. These alignments are discussed below, identified by the corresponding teacher theme and connecting thread.

Clarifying Thinking for Advancement. Both student and teacher comments pointed out that, when a teacher was able to sit with one student and ask purposeful questions, the teacher was able to assess and advance the thinking of the student. In other words, as students started to organize and articulate their mathematical thinking, teachers saw opportunities to assess their thinking and advance them to the next level of understanding. Teachers found one-on-one interaction to be highly effective in this process. For example, one teacher stated, “Being able to sit down with one student, just be able to get that and just get them to think, can have this amazing effect.” Students corroborated the teacher perspective. Elena provided an example of when a teacher used purposeful questioning to help her organize her thinking and advance them to the next mathematical understanding:

How [questioning] helps me understand more better is that he doesn't say right away what is the answer, he explains carefully what you should do, like he asked, "What could you do instead of this? Why should you do this? Can you explain to me what is this and that?" So, it helps you understand in reality what's in your mind.

When Elena spoke about what was in her mind, she highlighted the experience of organizing her thinking. The teacher's "What can you do instead of this?" is an example of a question that advances student thinking to consider alternative pathways and to justify their mathematical thinking. Purposeful questioning provided a pathway for clarifying and advancing student thinking.

Mathematical Efficacy. The second alignment between student and teacher perspectives dealing with purposeful questions occurred between the themes of shaping identity and ownership of learning. The example of Elena presented above shows how her math identity, as a student who can justify her thinking, contributed to her taking ownership of her learning. She was not discouraged that her teacher had not simply given her the correct answer or a set of steps to take. Instead, she felt encouraged as the line of questioning helped her bring the mathematical thinking in her mind out in words. She saw herself capable of mathematical ideas but needed help organizing them. Similarly, Nayeli shared, "I can't pull my own thoughts out of my mind like that."

Teachers saw this alignment while they viewed the classroom videos. Mr. CL stated, "Using purposeful questions allows them to take ownership of their learning and feel like they're the one that got there, as opposed to being told the answer." After having a language partner translate a response from an ELL student for a whole-class discussion, Ms. NG recognized an alignment between ownership and identity when she said,

Giving them voice, and giving them purpose in the classroom helps with them feeling like they belong. That helps build their confidence, and that helps lower their affective filter, and that helps them to feel like engaged participants in the community.

The connection found here between mathematical identity and ownership of learning can be described as mathematical efficacy, or the belief that you can solve mathematical problems and complete mathematical tasks (Watson, 2015).

Alignment of Perceptions around Connecting and Using Mathematical Representations

Analysis revealed two areas of alignment between the student and teacher perspectives for the MTP of connecting and using mathematical representations. First, teachers and students offered similar thoughts represented by the student theme of options and the teacher theme of independence. Second, the student and teacher comments aligned between the student theme of seeing the math and the teacher theme of connections.

Utilize Math Tools. The alignment between options and independence can be seen in Mr. CL's claim that giving students opportunities to use multiple mathematical representations supported their learning. He stated:

[Connecting representations is] opening up that door so that [solving a mathematical problem is] not one way. So, giving them multiple ways allows them to actually choose what they like to do. What makes sense to them so that they can put sense into their own math.

One of his students similarly stated, "Well, I think [multiple mathematical representations are] helpful because you have three different options to do [the problem]. If you don't understand one, then you could do [the problem] the other way. And you have other options." Furthermore, when the students have options about the mathematical representations that work best for them, they can independently uncover and understand the mathematics they are learning. Ms. NG stated this relationship by describing how her provided tools led to independence. She explained,

“This is showing me that they then have a tool. And so they don't need me as much, and that's the whole point, right?” As students utilized their many options for mathematical representation, teachers saw these tools as a pathway to mathematical independence.

Visual Schema. The data also highlighted an alignment between student perceptions about seeing the math and teacher perceptions about connections. Several students described how visual representations and actions led them to connect mathematical ideas and real-world situations. In the following examples, students are using and adding on to their visual schema, or background knowledge. For example, Elena and Jimena talked about how visual representations helped them make mathematical connections. Jimena shared her thoughts after her teacher showed a mathematical drawing of a number line and then supported it with a drawing from a real-world situation. Jimena explained, “[The problem is] referring to the drawings, but you see math.” Elena also said, “I need to see [the mathematical idea] to be able to understand it.” Lupe described how her teacher used gestures to develop mathematical connections. She explained that “He uses hand techniques to see which way. And he really knows how to get you to understand.”

The teachers were able to name times in which being able to see the math helped their students. Sometimes they would experience a new representation that a student presented that was supportive of the learning. After a class period in which a student used a drawing similar to a rainbow that connected opposite numbers on a number line, Ms. NG shared,

I forget which student brought this up. But [the explanation] was so powerful because I've never looked at [the number line and opposites] as this idea of a rainbow, like something that all students have likely seen. And so I think that, that really helped to build or like just connect back to their own lives and like build that

background knowledge and make connections to previous nonacademic experiences that they've had.

In this example, the teacher could recognize that the student's drawing connected to a visual they understood and that she could leverage that to help with mathematical understandings.

In closing, nine alignments were identified between the student and teacher themes. For example, the broad student theme of “explanations from peers and self” aligns with five distinct teacher themes. Figure 4.1 provides a visual representation of the alignments that were identified within each MTP, representing where student and teacher perceptions were in alignment regarding the purpose of the MTPs. Figure 4.1 also illustrates the ideas from the data that connected the themes to one another thus creating the alignment.

Figure 4.1. Alignment of Student and Teacher Perceptions

Mathematical Discourse

<i>Student Code</i>	<i>Alignment</i>	<i>Teacher Code</i>
Explanations from Peers and Self	A Place and Method for Testing Thinking	Environment
Explanations from Peers and Self	Seeking Support Outside of the Teacher	Independence
Explanations from Peers and Self	Methods for Thinking Through the Mathematics	Processing
Explanations from Peers and Self	Providing Comprehensible Input	Access
Written Versus Oral	Ways to Share Thinking	Evidence

Posing Purposeful Questions

<i>Student Code</i>	<i>Alignment</i>	<i>Teacher Code</i>
Organize and Articulate Thinking	Clarifying Thinking for Advancement	Assess and Advance Thinking
Mathematical Identity	Mathematical Efficacy	Ownership of Learning

Use and Connect Mathematical Representations

<i>Student Code</i>	<i>Alignment</i>	<i>Teacher Code</i>
Options	Utilize Math Tools	Independence
See the Math	Visual Schema	Connections

Divergences in Perceptions

Four areas were noted where student and teacher perceptions about aspects of the Mathematical Teaching Practices diverged. A divergence indicates when students and teachers had some degree of variance in the ways in which they were perceiving the MTP.

Mathematical Discourse. Multiple divergences were identified within data related to the MTP of facilitating mathematical discourse. These divergences occurred between the student theme of written and oral discourse and the teacher theme of evidence as well as between the students' theme of explanations from peers and self and the teacher theme of evidence. The first divergence occurred between the student theme of written versus oral and the teacher theme of evidence. Students had different preferences about whether this MTP was helpful based on whether they could present their thinking in written or oral formats. Students were able to articulate that written discourse was helpful, especially when the classroom resource was translated into Spanish on their iPads. When the resource was translated, students were able to provide written discourse in response to the questions asked in the resource. Nayeli described this by stating, "I can just write [my answers and explanations] out instead of explaining it because it's a bit hard when I'm explaining [how I solved the problem and my thinking]." This differs from what was articulated by teachers. Ms. NG wondered, "I wonder sometimes if a student isn't able to verbalize their understanding, how embedded is [the mathematical concept in them]? If you can't say [your mathematical thinking] out loud, how well do you know [the mathematical concept]?" The data highlights a divergence between how students and teachers were perceiving the purpose of mathematical discourse.

The second divergence within mathematical discourse was highlighted in the data within the teacher theme of evidence and the student theme of explanations from peers and self.

Teachers saw oral discourse as a way to show evidence of mathematical thinking and understanding. Mr. CL stated, “it is really helpful that they can explain because then [what they say] gives you a window into what they believe [about mathematics].” In contrast, students did not talk about discourse as a way to show what they know, but more as a way to get help and confirmation. Noemi stated, “It can help me understand more when I hear people talking.” She expanded on this thinking by saying, “[Mathematical discourse] helped me because talking to other people that explain it differently or better.” While these divergences are slight, they exemplify a diverging perception about the purpose of mathematical discourse.

Other Divergences. Additional divergences were found within the MTPs of posing purposeful questions and connecting and using mathematical representations. The themes identified within posing purposeful questions were intertwined, and clear divergences were not observed across the themes. However, there was one individual instance in the data where the students and teacher were not in agreement about when and in which ways purposeful questioning was helpful. In this example, Mr. BW had mentioned that he felt the discussion resulting from a purposeful question he had chosen was “too long and in English.” His students said that “the longer time was helpful because at first we didn't understand [what the problem was asking].” Essentially, the longer the discussion continued, the more time the ELL students had to process what was being said and explored mathematically. While this individual instance does not support a finding, the purpose of the study is to highlight student voice and consultation

in the use of MTPs with Hispanic ELL students. This example illustrates that when we ask students about their experiences, we may find insightful information that can guide teaching.

Analyses revealed that there were no divergences between the students' and teachers' perceptions of connecting and using mathematical representations. There were, however, instances in the interviews where students and teachers offered differing suggestions that could be useful to continue to build the purpose of the practice. When students were asked for suggestions on how their teachers could guide their learning in mathematics, the most common response was a request for more visual representations, more realia or real-life items, more gestures, and more manipulatives to help them understand the mathematics. This was not articulated by the teachers as a need within the MTP of using and connecting mathematical representations. However, Elena stated, "Well, I think one of the things that they could do to help me would be kind of having the actual thing. More hands-on, we could say. So that when you're right there, and then you can actually show the teacher what you are thinking." Thus, in this example, the student found visual representation so helpful that she was requesting even more. Overall, teachers and students were in alignment in their perceptions about the helpfulness of mathematical representations.

Relationships Between Common Threads Across MTPs

The discussion of findings for research questions #1 and #2 highlighted three concepts that were evident across all three MTPs: translation, language partners, and shaping mathematical identities. Within these common threads were indications of alignment and

divergence similar to those identified within the MTPs. First, an interesting relationship was noted between the student thread of translation and the teacher thread of language partners.

Alignment and Divergence in Perceptions about Translation and Language Partners

When considering the common threads that are woven throughout all three MTPs, alignment can be seen between the student common thread about translation and the teacher common thread about language partners. This is a reasonable connection as language partners provide translation for students. Students expressed the need to have both verbal and written content translated into their native languages. Teachers saw language partners as one way to accomplish this outcome. For example, when students were asked what strategies were helpful in learning mathematics, Daniel and Carlos both responded that having Abril translate for them was helpful. Both Noemi and Jimena stated that they found their language partners helpful as well. Jimena mentioned that her language partner was helpful because “other students explain [the mathematical problem] differently [than the teacher].” Ms. NG described the benefits of language partners when she stated:

I was very explicit that [language partners] sit together so that they can work in their native language, and then also start to learn some of the English terms, but that their math learning can be supported as well. So it can be both an English Language Learning class and then also a math class versus it being a math class without ELL support or only an ELL class without understanding any of the math.

While teachers and students agree that more translation is necessary, the facilitation can be challenging. The divergence in perceptions about translation and language partners was found in the amount and types of translation options provided for students. Teachers perceived that the

translation that occurred from a language partner was helpful, and students agreed. However, students perceived the need for more translation support differently.

Teachers spoke frequently about the helpfulness of language partners, while students expressed a need for a translation of the classroom lectures, debriefs, and resources. When asked about his mathematical thinking during a problem-solving session in class, Daniel stated, “I wasn't really thinking about anything because he's speaking in English, and it's hard to understand him.” Daniel was seated with a language partner, but during the facilitation of this whole-group discussion, Daniel was not able to utilize his language partner because the teacher was talking to the whole class and the language partner would have had to talk over the teacher. Julian also confirmed that he found whole-group lectures and discussions challenging because, “I don't understand the teacher because he doesn't speak Spanish.” Mr. BW had attempted to have language partners translate while he was speaking but found the timing to be challenging. When asked about a whole-group discussion, Mr. BW shared,

I think it was supportive for English speaking students. And so I would say the same for ELL students, but I didn't probably leave enough time for translation. And that's a dynamic that I want to work on. I'm talking a lot and now I want to let the translation occur, but I also don't know how to deal with the middle school [student feeling awkward].

In summary, ELL students found translation to be supportive and felt that even more translation would be helpful. Teachers saw a need to have whole-group lectures and discussions translated, but had yet to find a system or platform that could accomplish that smoothly within the classroom. Language partners were not a satisfying solution to translation of whole-class discussions, lectures, and debriefs.

Perceptions Around Shaping Mathematical Identities

A number of student perceptions and comments intersected with the teachers' common thread of shaping positive mathematical identities. These points of intersection fell within the student themes of explanations from peers and self, organizing and articulating thinking, and options. Aguirre et al. (2013) defines mathematics identity in the following way:

We define mathematics identity as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical context and use mathematics in powerful ways across the context of their lives. Depending on the context, a mathematics identity may reflect a sense of oneself as a competent performer who is able to do mathematics or as the kind of person who is unable to do mathematics (Aguirre et al., 2013, p. 14).

In this study, students were able to see that seeking explanations from their peers was a way in which they could learn mathematics and become more independent mathematical learners, thus shaping a positive mathematical identity. For example, Carlos stated, "I could give him my opinion. He could give me his opinion. And through this, we could determine what was the correct answer." This exemplifies how Carlos saw the discussion with his partner as supportive in confirming his solution, thus shaping his belief that he can get mathematical problems correct without direct teacher support.

Organizing her thinking helped Nayeli to develop a positive math identity because the solution was not just given to her; she had to arrive at it herself. She explained:

How [purposeful questioning] helps me understand more better is that he doesn't say right away what is the answer, he explains carefully what you should do, like he asked, "What could you do instead of this? Why should you do this? Can you explain to me what is this and that?" So it helps you understand in reality what's in your mind.

Her teacher reinforced this perception when he stated:

I think just giving somebody an answer or not getting them to think about something, leads into them developing bad habits and bad ways of thinking. By posing personal questions, you start leading them in the direction, but they take ownership of [their learning and thinking].

Last, providing students with options in their problem-solving helped to shape a positive mathematical identity. Lupe said, “Well, I think [multiple representations are] helpful because you have three different options to do [the problem]. If you don't understand one, then you could do [the problem] the other way. And you have other options.” The teachers supported this alignment by stating, “So giving them multiple ways allows them to actually choose what they like to do. What makes sense to them, so that they can put sense into their own math.”

In summary, the shaping of positive mathematical identities was a common thread for teachers across all three MTPs and was present in a number of student themes that emerged within each MTP. Although not directly addressed in the teacher and student interview protocols, features of mathematical identity arise naturally in discussions about the effectiveness of the Mathematical Teaching Practices.

The third research question in this study sought to explore alignments and divergences between student and teacher perceptions about the helpfulness of three Mathematical Teaching Practices as identified in the first two research questions. This section presented several cases of alignment between students and teachers in their perceptions about the helpfulness of facilitating mathematical discourse, posing purposeful questions, and using and connecting mathematical representations. Several nuanced examples of divergence among perceptions were also discussed.

Chapter Summary

The purpose of this qualitative case study was to examine the perceptions of Hispanic ELL middle-school students regarding the efficacy of the research-informed Mathematics Teaching Practices (NCTM, 2014; Smith et al., 2017). The goal was to determine if Hispanic ELL students perceived the practices as helpful in learning mathematics and to compare their perceptions to those of their teachers. Nine Hispanic ELL students and three teachers from a single school participated in the study. A total of eighteen interviews featuring video stimulated recall provided data to answer the research questions.

A thorough analysis of the data revealed that the student participants of the study found mathematical discourse, posing purposeful questions, and using and connecting mathematical representations helpful when learning mathematics. From within the student data surrounding these MTPs, six themes emerged. The students shared that, when engaging in mathematical discourse, explanations from peers and self were helpful in their learning. Additionally, students saw written and oral discourse as helpful. When teachers asked purposeful questions, students stated that they could organize and articulate their thinking in ways they could not without the questioning strategies. They also saw purposeful questioning as helpful in building their confidence and identities as learners of mathematics. Students noted that, by using and connecting mathematical representations, they could have options among how they were able to solve problems, and they could see the math so that it was easier to understand.

The teacher interviews produced eight themes from the data surrounding the MTPs. Regarding mathematical discourse within their classrooms, teachers found that it led to a positive learning environment, helped when building independence in students, assisted in mathematical

processing, and gave access to content for ELL students. Posing purposeful questions helped teachers to assess and advance the thinking of their students and helped students take greater ownership of their learning. As students were using and connecting mathematical representations, teachers found that those representations were tools for independence in the mathematics classroom, and their students could make connections between representations and real life.

Three common threads could be identified across data related to all three of the Mathematical Teaching Practices. The common theme of translation wove itself through the students' perceptions about all three MTPs; they requested support in this area during all of the interview sessions. Within the teacher data, themes of language partners and mathematical identities were threaded throughout multiple interviews. Alignments and divergences were also found within these common threads.

In Chapter Five, the results of the data analysis will be further discussed in terms of important findings and implications. Chapter Five will discuss contributions to literature, contributions to practice, and recommendations for further research.

CHAPTER FIVE

DISCUSSION

This study provided an in-depth exploration of the perceptions of nine Hispanic ELL middle-school students and their teachers regarding the efficacy of three research-informed Mathematics Teaching Practices (NCTM, 2014). Existing literature describes the educational barriers that ELL students face and gaps in achievement levels between Hispanic and non-Hispanic students and ELL and non-ELL students (Schneider et al., 2006). However, fewer studies have sought to include perceptions from Hispanic ELL middle-school students about learning mathematics (Sanchez-Gutierrez et al., 2011). This is a serious gap in the literature because student perceptions can offer valuable insight into the effectiveness of teaching practices (de Araujo et al., 2018). This study attempts to bring those student perceptions into the discussion about the effectiveness of mathematics teaching practices.

Thus, this study investigated the efficacy of three research-informed Mathematics Teaching Practices (MTPs) using data centered around the perceptions of Hispanic ELL middle-school students and their teachers about the practices. Research-informed practices have become more prevalent in the literature about teaching mathematics (Hattie, 2009; Smith et al., 2017). However, limited research exists exploring how Hispanic ELL students perceive these practices.

Summary of Findings

To examine the perceptions of Hispanic ELL middle-school mathematics students and their teachers regarding the helpfulness of specific mathematics teaching practices, three research questions guided this study:

- 1) How do Hispanic ELL middle-school students perceive the effectiveness of three research-informed mathematics teaching practices in supporting their mathematical understanding?
- 2) How do teachers of Hispanic ELL middle-school students perceive the effectiveness of three mathematics teaching practices and supporting their students' mathematical understanding?
- 3) In what ways do these students' and teachers' perceptions align and diverge?

The purpose of this chapter is to summarize and interpret the research findings, relate findings to the literature, discuss implications for mathematics learning and teaching, and offer recommendations for both practice and research.

Summary of Research Question #1: Perceptions of Hispanic ELL Students

The first research question of this study aimed to explore how Hispanic ELL middle-school students perceived the level of effectiveness of three specific Mathematics Teaching Practices (MTPs) in supporting their mathematics learning. The three MTPs examined in this study include: facilitating mathematical discourse, posing purposeful questions, and using and connecting mathematical representations. These three practices were chosen from eight practices as described by the National Council of Teachers of Mathematics (NCTM) (NCTM, 2014). They

were chosen for their close connection to social constructivist theory (NCTM, 2014) and their connection to previous research discussing general strategies that were found to be effective for ELL students (Echevarria et al., 2008). Findings for each practice are presented separately in the following sections.

Mathematical Discourse. Of the nine Hispanic ELL middle-school students interviewed, six rated mathematical discourse as helpful, two somewhat helpful, and one said “a little bit” helpful. When students were asked about the perceived helpfulness of mathematical discourse, two themes were identified: explanations from peers and self and written versus oral discourse. Students identified mathematical discourse as providing the opportunity to gather opinions from each other, learn together in a supportive environment, and hear alternative ways to understand or solve mathematical problems. Student participants also found self-talk helpful in understanding mathematics. The second theme revealed that students had different preferences in mathematical discourse modalities. Several participating students stated that writing was easier for them to express their mathematical understandings, while other students could better explain their thinking orally. Hispanic ELL students also indicated that group size during oral discourse made a difference.

Purposeful Questions. Of the eight students interviewed regarding posing purposeful questions, five stated that the MTP was helpful, two said somewhat helpful, and one remarked “a little bit” helpful. The students offered two key perspectives regarding posing purposeful questions: organizing and articulating thinking and shaping mathematical identity. The Hispanic ELL students in this study stated that, when their teachers posed purposeful questions, they could

“pull my thoughts out of my mind.” This indicated that the questioning was helping them to organize their thinking and articulate it better. Most of the participating students said that purposeful questioning was helpful in whole-group, small-group, and one-on-one settings. Additionally, participant students reported that the questions supported them to feel more successful in their ability to do mathematics. This led to the designation of a second theme, shaping mathematical identities.

Mathematical Representations. All nine students in this study rated using and connecting mathematical representations as helpful in supporting their understanding of mathematics. Two themes about this MTP emerged from the data collected from students: the importance of options and the value of seeing the math. The students noted that using and connecting mathematical representations allowed them choices in their problem-solving methods and provided a way to confirm their solutions. They also spoke about the value of “seeing the math.” They described multiple ways this happened in their classrooms ranging from hand gestures and student drawings to pictures within software programs.

Common Threads. One common thread was woven throughout the students’ conversations about all three MTPs. Of particular concern from the student perspective was the need to have more class resources translated into Spanish. Students stated that translation needs were especially important during whole-group classroom lectures and discussions. It was during these times that the students stated that they were not thinking about anything mathematical because they did not understand what the teacher and other students were saying. Students

reflected that very little learning was taking place when whole-group discussions were happening.

Summary of Research Question #2: Teacher Perceptions

The second research question in this study explored teachers' perceptions of the helpfulness of the same three MTPs to support mathematical understanding for their Hispanic ELL students. As with research question #1, findings for each practice are presented separately.

Mathematical Discourse. All the teachers in this study rated mathematical discourse as helpful. Five themes related to facilitating mathematical discourse were identified within the data collected during teacher interviews: a positive learning environment, building independence, mathematical processing, evidence of learning, and access to content. Teachers stated that mathematical discourse provided a positive learning environment because students could have a voice in the mathematics classroom. Additionally, teachers said that an increased use of mathematical discourse contributed to the comfort level of Hispanic ELL students because they were able to discuss mathematics with other students. This led to increased student independence. For example, Hispanic ELL students used mathematical discourse routines to seek support without being prompted by a teacher. Mathematical processing was observed as students faced misconceptions and used discourse to clarify misunderstandings. The teachers also mentioned that the process of thinking, sharing ideas, and completing a task contributed to mathematical understanding. Teachers were able to witness evidence of student learning through mathematical discourse, stating that when students could explain their mathematical thinking, the

students understood the mathematics on a deeper level. Last, teachers saw mathematical discourse as increasing access to content through language partners.

Posing Purposeful Questions. Two of the teachers rated purposeful questioning as helpful, and one teacher rated it as not as helpful in the lesson that he taught. He stated that this was reflective of his delivery of the MTP rather than the MTP itself. After analyzing the interview data surrounding posing purposeful questions, two themes emerged from the teacher data. The two themes from this MTP were assessing and advancing student thinking and ownership of learning. In whole-group, small-group, and one-on-one situations, teachers perceived that they could assess and extend what students understood about a mathematical topic and encourage them to consider alternative approaches by asking questions. They also perceived that students furthered their ownership of learning through responding to purposeful questions rather than being told how to solve mathematical problems.

Mathematical Representations. Two of the three teachers rated mathematical representations as helpful to the students. One teacher said that they were not helpful at this time due to the way in which he presented the representations rather than the MTP itself. Two themes emerged from the data collected about using and connecting mathematical representations. Teachers saw this MTP as providing tools for independence and connections between representations. By introducing multiple representations for solving mathematical problems, teachers found that students were able to make choices for themselves about which representations were most helpful and made the most sense. They saw this as a pathway to student independence. Additionally, teachers related that when they could connect past learning

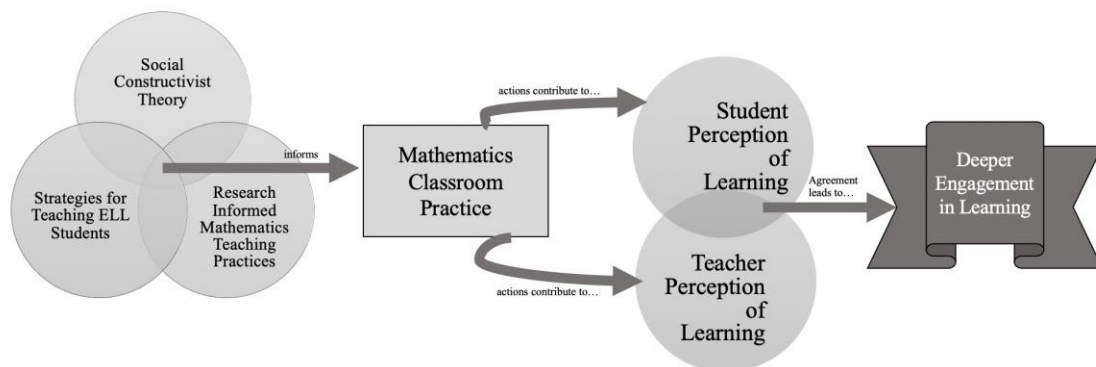
and real-life situations to the mathematical representations, students had a more comprehensive understanding of the concept. This was illustrated when a student compared opposites on the number line to a rainbow because she had drawn connecting arches between opposites.

Common Threads. As with research question #1, common threads were identified that were woven through the findings in research question #2. In this study, these common threads represented concerns that teachers were sharing about how they were accommodating the needs of ELL students. Throughout their discussion of each of the MTPs, teachers spoke about language partners and shaping mathematical identities. Teachers articulated concern about how to facilitate language partners effectively and whether or not their classroom environment led to the shaping of positive mathematical identities for Hispanic ELL students. The structure of using language partners had been introduced by the English language development teachers in the school as a way to provide translation for students in class. It was a relatively new structure for the mathematics teachers, and they were finding that while language partners were able to provide language translation during partner and small-group work, there were additional considerations to be made. For example, not every student was comfortable in the role of language partner and language partners were not able to translate during whole-group instruction. Additionally, the common thread of shaping mathematical identities was identified as teachers discussed their personal beliefs about Hispanic ELL student ability and classroom actions that led to the shaping of mathematical identities. These actions included opportunities to communicate within the classroom and to share thinking as well as ways in which teachers worked to make their classroom comfortable for all learners.

Summary of Research Question #3

The third research question in this study examined alignment and divergence between the findings for students and their teachers. Alignments occurred when there was agreement in shared ideas between students and teachers about the purpose of the MTPs. In contrast, a divergence indicates some degree of variance in the ways in which students and teachers were perceiving the MTP. Alignment in the results is of interest because it is within the alignments that teachers have the greatest opportunity to have the agreed-upon practices support deeper learning. Researchers have found that matching what teachers and students value can result in higher performance levels (Anthony et al., 2013) and would contend that student consultation plays a role in unearthing where these alignments exist (Rudduck & McIntyre, 2007). This relationship is illustrated in the study's conceptual framework found in Figure 5.1. Divergences present an important opportunity to help teachers recognize areas where nuanced differences may exist. This area of disconnect illuminates an opportunity for a shift in practice that could bring about alignment.

Figure 5.1. Conceptual Framework



Mathematical Discourse. Analysis of perceptions of mathematical discourse revealed multiple alignments between student and teacher themes. Connections were identified between the student theme of explanations from peers and self and the teacher themes of environment, independence, and processing, and access. The analysis showed that environment and explanations from peers and self were connected by the idea that teachers had created a place and method in which students could test their mathematical thinking. The connecting thread between explanations from peers and self and independence was that students were seeking help outside of the teacher to confirm their mathematical thinking. Explanations from peers and self connected to processing because students were using mathematical discourse to think through the mathematics. Access and explanations from peers and self were connected by the notion that comprehensible input (understandable verbal communication) was being provided to students through mathematical discourse. Alignment was also seen between the student theme of written versus oral discourse and the teacher theme of evidence as students shared their thinking through multiple modalities.

Additionally, two areas of divergence were found within mathematical discourse. An example of a divergence is when teachers stated that, for a student to know math deeply, they had to be able to talk about it. This differed from what the students reported. Their perception was that writing about their thinking was sometimes more conducive for sharing their thinking than speaking was. The second divergence occurred between the student theme of explanations from peers and self and the teacher theme of evidence. Students viewed mathematical discourse as an opportunity to seek help and confirmation from peers while teachers saw mathematical discourse as an opportunity to seek evidence of student learning. If teachers and students were in

alignment on these two points, there may be an opportunity for deeper engagement in the learning.

Posing Purposeful Questions. There was also alignment in perceptions about posing purposeful questions between students and teachers. Students' and teachers' perceptions aligned when discussing organizing and articulating thinking from the student perspective and assessing and advancing thinking from the teacher perspective. The analysis showed that the connecting idea between these two themes was that mathematical thinking was being clarified so that it could be advanced. An additional alignment was seen between ownership of learning from the teacher perception and mathematical identity from the student perception. These two themes are related through the connecting idea of mathematical efficacy, or the belief that you can complete mathematical problems and tasks.

Only one divergence was found between the themes associated with posing purposeful questions. Teachers and students offered different perceptions about the amount of time taken with problems. One teacher felt he had stretched the problem out too long. The students, however, appreciated the time to process their thinking. This was one example of how, without student consultation, we would not be aware of these differing perceptions. During the interview with the teacher in this example, he stated disappointment in the way in which he had facilitated the problem, but the students stated that they needed the extra time.

Using and Connecting Mathematical Representations. Most teachers and students interviewed in this study agreed that using and connecting mathematical representations was helpful for understanding mathematics. Alignment was determined to exist between the student

theme of options and the teacher theme of tools for independence. Another alignment was seen between the theme of seeing the math and the theme of connections. The analysis identified an alignment between teachers and students because when students had options for solving problems by using mathematical tools [representations] they could take ownership of their learning. Second, when students could see visual representations of the mathematics, they could use and add to their visual schema as they connected real-world situations.

Common Threads. Students and teachers were alike in their perceptions about the need for translation of resources. At this site, teachers focused on language partners for that purpose. The main purpose of a language partner is to provide opportunities for student to discuss their thinking in their native language. Having content translated is also a product of two native language speakers working together. At the same time, students were concerned with the translation of lectures and whole-group discussions when using a language partner was inappropriate. The teachers perceived that they were making efforts to build positive identities, but this contradicted the students' feelings of being unsuccessful during whole-group interactions. This was exemplified when Daniel stated that he was not thinking about anything mathematical during whole-group interactions because he did not understand the teacher.

Limitations of the Findings

There are three primary limitations in the design of this case study that affect the potential application of these findings. First, only nine Hispanic ELL middle-school students and their three teachers provided data through interviews. This small number of students represents a unique population of Hispanic ELL students that live in a rural, affluent community. As a result,

generalizability of the findings may be limited due to the relatively small size of the sample and the study's unique location. Second, the language levels of the student participants may have affected the findings. Even with a translator present, some students found it difficult to talk about how they perceived their mathematical learning. Last, this study took place over one semester and involved nine teaching episodes; therefore, it represents only a brief snapshot of student perception. The students may have different experiences and perceptions of the MTPs with varying teachers of mathematics or mathematics courses.

Evaluation of Research Methods

The methodology and procedures for data collection and analysis involved in the study were conducted in two phases. Phase 1 included the identification of classrooms, the calibration of operational terms, and the development of a collaborative rating scale. Phase 1 also included a pilot study with two students to refine the student interview protocol. Phase 2 was informed by Phase 1 and included data collection utilizing video stimulated recall and semi-structured interviews with student and teacher participants. The strategies and tools used in these two phases were very supportive of the research process. There were also areas where modification might increase the effectiveness of the methodology in future research applications.

Effective elements of the research design included the calibration of operational terms and the development of the collaborative rating scale. These two processes were carried out collaboratively with the classroom teachers. This increased the alignment of language used when working with Hispanic ELL students and reduced potential misinterpretation during the interview process. Additionally, the pilot study was very informative in adjusting the student

interview protocol to improve clarity for Hispanic ELL middle-school students. The pilot study was highly effective at obtaining Hispanic ELL students' perceptions of the questions that were going to be asked of their peers. The pilot students were able to provide specific insights about ways in which to ask the questions to gather the most information possible. For example, the pilot students suggested that the researcher read only two or three sentences at a time, then allow time for translation and thinking. This suggestion was particularly useful during the interviews because it allowed everything to move at a slower pace. The slower pace made it so everyone felt more comfortable and also provided wait time for students to consider what they would like to say.

There are two elements of the research methods to consider revising if this study were to be replicated. While the teachers selected were viewed by the researcher as most often utilizing the Mathematics Teaching Practices in their school, this study may have benefited from some opportunity to instruct teachers on the workings of each MTPs. Each teacher was given *Taking Actions: Implementing Effective Mathematics Teaching Practices in Grades 6-8* from NCTM (Smith et al., 2017); however, they were not required to read it. Professional learning might have ensured greater fidelity to the research-informed features of each practice and greater consistency across classrooms.

Second, different methods may be available to further activate and capture student perceptions. For example, studies utilizing photographs to elicit student input have also been successful with ELL students (Baker, 2013). Another possibility for the future would be to incorporate student work samples during the interview process. Using more visual

representations was a request from students during the interviews and could be incorporated into further studies to elicit even deeper student conversation.

Last, this study was conducted over the course of one semester due to the school's course scheduling structure. If there had been extended time to collect additional data in differing ways, there may have been additional insights into the helpfulness of the MTPs. For example, if there had been an opportunity to collect data from these and additional participants over the course of several years, then it might be possible to see whether different teachers and different mathematics courses affected the perceptions of the Hispanic ELL students.

Contributions to Literature

This study was designed to address a specific gap in the body of literature regarding teaching and learning in mathematics—in particular, to add to the knowledge base about the effectiveness of NCTM's Mathematics Teaching Practices for Hispanic ELL middle-school students. Additionally, this study contributes to knowledge and research practice regarding student consultation and the role of student perceptions in research specifically involving alignments and divergences in perceptions. This section discusses how findings related to Hispanic ELL students' perceptions about Mathematics Teaching Practices might inform the literature base.

Effectiveness of Mathematics Teaching Practices

As Hispanic ELL students in this study provided insight into the helpfulness of the MTPs, they also contributed their broader perceptions about mathematics teaching and learning. Drawing from the work of Cook-Sather (2002), including students as contributors in the

educational process can improve current practice and point to additional areas of improvement. In this study, consultation with Hispanic ELL students extended what we already know in the literature about effective mathematics teaching practices in two important ways. First, the analysis from mathematical discourse contributed to the knowledge base about opportunity to learn (OTL). Second, the data indicated an additional way in which posing purposeful questions can support ELL students.

Students appreciated when mathematical discourse was incorporated into their classroom structure. Building on the work of Abedi et al. (2006) in which they found that content coverage, teacher knowledge base, and student prior ability contributed to an increased OTL, this study would contend that a supportive classroom environment built through the inclusion of mathematical discourse would also increase the OTL for Hispanic ELL students. Heibert and Grouws (2007) have suggested that educators position students in learning environments that increase the likelihood of engaging in the learning, thus increasing OTL for students. The purposeful inclusion of the mathematical discourse into the classroom environment contributed to a positive classroom environment in this study, therefore contributing to the OTL for these Hispanic ELL students. Additionally, students in this study had more access to mathematical content because of the high levels of mathematical discourse that were incorporated into their classroom environment. This access also increases OTL for Hispanic ELL students.

Students in this study highlighted how teachers could assist them in organizing and articulating their thinking by posing purposeful questions. Research clearly shows that maintaining a high cognitive demand when questioning raises student confidence (Boaler & Staples, 2008) and positions ELL students as contributors in a mathematical community

(Gresfali & Cobb, 2006). This study found that, in addition to these benefits of purposeful questioning, teachers can also support ELL students in organizing their mathematical thinking through questioning. As ELL students grapple with the cognitive demand of both learning English and mathematical content, purposeful questioning can help students organize the complexities of learning a language and content. Nayeli said it best when she stated, “I can't pull my own thoughts out of my mind like that” when referring to how purposeful questioning was helpful to her.

Student Consultation

Attending to the inclusion of the Hispanic ELL student perception is unique in the literature (Sands, 2007) and has been sparse in scope (de Arujo et al., 2019). Sanchez-Gutierrez et al. (2011) state, “Latina/o students have seldom been asked for their perspectives on their classroom mathematics experiences or what insights they might provide about the possibilities of enhanced mathematics learning opportunities” (p. 27). This study contributes to the body of knowledge about the value of including student consultation in the classroom and in research as well as the specific inclusion of perceptions from Hispanic ELL students.

While student consultation in and of itself is not unique in the literature, including the perceptions of Hispanic ELL students is both timely and valuable. In a recent study conducted by Melhuish et al. (2022), researchers concluded that race-neutral professional development (PD) did not increase achievement for students of minority groups. The study found that, as a result of standards-based PD in which teacher content knowledge and implementation of quality instruction increased, student achievement scores saw some impact. However, minoritized groups did not see the same gains as their counterparts. Recommendations from this study about

the successful use of mathematics teaching practices for Hispanic ELL students will add to this body of knowledge and be useful in strengthening race-conscious, standards-based teacher professional learning.

Additionally, the consultation of students could assist in the development of more accessible large-scale assessments. As described in Chapter One, there are notable discrepancies between Hispanic and non-Hispanic students and ELL and non-ELL students on large-scale mathematics assessments (NAEP, 2019). These achievement gaps have contributed to a “gap-gazing fetish” as describe by Gutierrez (2008). However, student consultation may uncover mathematical understandings in ways that standardized assessments cannot measure. In addition to using consultation to gain a more comprehensive picture of student understanding, Hispanic ELL students may have insights about how to create assessments that can more accurately measure their knowledge base. Recommendations from this study could not have been articulated without student consultation. To know which practices Hispanic ELL students perceive as supportive in their learning, we must ask them.

Alignments and Divergences

Extant literature discusses alignments as areas to leverage for increased learning (Anthony et al., 2013); thus, adjusting teaching practice to create alignments is beneficial to learners. Expanding on this theme, research question #3 focused on identifying both alignment and divergence between teacher and student perceptions. While previous research suggests that points of alignment can result in deeper learning (Huang & Barlow, 2013), this study extended that reasoning by analyzing so-called “points of divergence.” Divergence occurred when students and teachers had some degree of variance in the ways they were perceiving the MTPs. Several

points of divergence were observed between the teachers' and students' perceptions in this study that could inform future practice.

One example from the study that highlights this phenomenon is the finding about divergent perceptions between the student common thread of translation and the teacher common thread of language partners. At the outset, these two themes appeared to represent the same idea: language partners provide translation. Additionally, and perhaps more importantly, teachers believed that language partners met the need for native language translation. However, after consulting with students, it became apparent that students wanted and needed more translation support than could be provided by their language partners. For example, students wanted translation to be available to them during whole-class discussions and lectures.

Identifying divergences between teacher and student perceptions adds depth to the research on instructional practice by illuminating where teachers' perceptions are not in alignment with those of their students. In the example given above, teachers assumed they were serving their Hispanic ELL students in their use of native language translation, when in fact they were only partially meeting students' needs. Similarly, teachers perceived discourse as a way to evaluate student understanding. While students valued discourse, they were not perceiving it as a way to evaluate their understanding; rather, they perceived it as a way to seek help and guidance from peers and teachers. This divergence is a critical component of understanding both the purpose and value of the MTP. Divergences in name seem to have a negative connotation; however, they are one of the hidden gems revealed by student consultation. When a divergence or disconnect is recognized, teachers and students then have the opportunity to engage in a conversation about how to come into alignment.

Recommendations for Practice

Besides contributing to the work of researchers who seek to find ways to improve teaching and learning, the findings of this study are significant for practitioners, school leaders, and Hispanic ELL students. The results can be translated into several recommendations for educational practice, some specific to the three Mathematical Teaching Practices explored in the study and others that are broader in scope. Acknowledging the limitations of this study, it still offers several principles that may be useful to improve mathematics teaching and learning for Hispanic ELL students.

Recommendations Related to Mathematical Teaching Practices

Promote Mathematical Discourse. This study suggests that facilitating mathematical discourse within the classroom can provide Hispanic ELL students with an environment in which they feel comfortable seeking support from their peers. Additionally, mathematical discourse can allow students a way in which to process their mathematical thinking either by explaining their thinking to another student or to themselves. This phenomenon was observed in the study, with students stating that they could get other students' perspectives about the mathematical learning they were experiencing. Students also valued talking aloud as they could grapple with language and meaning while controlling the pace of their thinking. Teachers of Hispanic ELL students could explicitly teach these highlighted components of mathematical discourse to their students. For example, a teacher could suggest that students "talk themselves through" a problem in their native language.

Pose Purposeful Questions for Multiple Purposes. When teachers pose purposeful questions, Hispanic ELL students can sort through their mathematical thinking and see themselves as students of mathematics who can answer questions successfully. As one teacher stated, sometimes a simple question like “Why do you think that?” can help students articulate their thinking and understanding. Teachers of mathematics could plan more purposeful questioning in their lessons. They may also consider incorporating resources that teach students to pose purposeful questions to one another.

In addition to assessing and advancing student thinking as described in *Principles to Actions* (NCTM, 2014), students in this context claimed that purposeful questioning helped them to organize thinking. When teachers are cognizant of this additional layer of scaffolding that purposeful questioning provides, they can begin to shape their questions to accomplish multiple goals. Discussions can follow that guide students to replicate this type of questioning for themselves so that they are able to “pull the thinking out of [their] minds” as described by Nayeli in this study. Thus, the recommendation is to preplan not only assessing and advancing questions, but also questions that assist students in organizing their thinking like “Why did you make that decision?” and “What will be your next step?”

Emphasize Hands-On Representations. Using and connecting mathematical representations was also seen as helpful by all Hispanic ELL students in this study. As one student stated, “I need to see it to be able to understand it.” The suggestion made by students was that mathematics teachers of ELL students include more visual representations, make connections between mathematical representations and real-world visuals, and utilize more gestures when teaching. It is important to note that, while teachers in this study utilized multiple

visual representations and presented connections between visuals and real-life examples, students still felt the need for more visual representations, especially concrete objects which they could handle or manipulate.

Value Student Perception. As seen in each of the examples with the MTPs, attending to student perceptions and applying consultation in this study allowed students to engage in the process of teaching and learning. Student consultation may be considered a strategy to increase student engagement, rather than solely a method for data collection. This practice can be implemented at the individual classroom or whole-school levels. This recommendation need not be limited to mathematics education.

Additionally, student perception can be a leverage point for addressing “gap-gazing” issues as described by Gutierrez (2008). Large-scale assessments on student achievement provide one picture of a student’s mathematical understanding. Consulting with students about their mathematical understandings and how we can better support those understandings can provide a more comprehensive picture of a student’s achievement in mathematics. An extension of this is described in the contributions to literature section as it pertains to both extending what we know about student consultation in the literature and how it can be enacted in professional practice.

Implications Beyond the MTPs

Elements of three common threads were woven throughout all of the interviews, although they do not directly reference one of the MTPs. A common thread for students was translation, and common threads for teachers were language partners and shaping mathematical identity. These findings provide implications for professional practice in several ways.

Expand Resources for Translation. Students repeatedly spoke about issues surrounding translation. Students claimed that they were not doing any mathematical thinking when they could not understand the language. Thus, evidence from this study suggests that classrooms could benefit from increased use of translation, especially during lectures and whole-group conversations. For example, several presentation platforms can translate spoken language in the moment. The translations look like closed captioning for the speaker. This type of translation software could be used during whole-group lectures or debriefs when much of the talking is done by the teacher.

Be Sensitive to Language Partner Dynamics. Teachers spoke frequently about the use of language partners who were able to translate in partnerships and small groups and share a partner student's thinking with the whole class. While language partners are useful in a classroom that supports ELL students, there are several considerations to be made. First, teachers need to be aware that language partners cannot be the only resource for translation. There are technology-enhanced options for "in the moment" translation such as applications on tablets that teachers and schools could consider. Second, there are social implications for language partners that must be considered. The evidence from this study suggests that teachers need to monitor how long a language partner supports an ELL student and assess if the relationship is a comfortable one. Relationships between students at the middle school level are ever-changing. Thus, teachers should look for ways in which a language partner can exit the relationship easily when needed. Last, teachers need to be mindful of the fact that the work of translation carries an increased cognitive load for the language partner, who is simultaneously working to learn the mathematics for themselves.

There are several logistical adjustments that may be effective in the facilitation of language partners. Where possible, identify a rotating group of students who are both bilingual and willing to be language partners. Then the onus of translation could shift throughout the year. In this same vein, it may be more effective to assign two or three students to one emerging bilingual as a small group, thus sharing the responsibility. Last, it may be beneficial to provide some training for language partners around how, when, and how much to translate as well as how to advocate for a break if they need it. With the many advances in translation technology, there may be an app or software that could fill any translation gaps that language partners cannot fill.

Build Identity through Instruction. Teachers also spoke about how the MTPs shaped mathematical identities. This is a vital conversation for mathematics teachers who are trying to develop positive mathematical identities in their students. High-quality facilitation of the MTPs can support this positive development. First, creating a comfortable environment that fosters mathematical discourse is suggested in developing positive mathematical identities. Second, posing high-level questions with scaffolded supports also contributes to fostering productive student beliefs about their abilities. Last, being able to connect mathematical representations to students' lives helps them to see themselves as doers of mathematics.

Most importantly in the discussion of mathematical identity is the inclusion of student consultation about teaching practices. For example, in this study both pilot participants were very helpful in the pilot study process, offering many valuable suggestions. They both said that they would help more if needed. During the actual study, the two pilot participants would often stop

the researcher in the middle school hallway to talk to the researcher. They viewed themselves as students who had something to offer to the educational process.

Leverage Alignments and Divergences. As alignments are identified, teachers can continue to use them to more effectively meet the needs of Hispanic ELL students. As divergences are identified, teachers and students provide adjustments to come to a closer alignment. In the translation/language partner divergence mentioned above, teachers can find ways in addition to language partners to provide translation. For example, teachers could record themselves teaching the lesson that students can then view later with Spanish closed captioning. With that adjustment, Hispanic ELL student perceptions will more closely align with their teacher perception that their language needs are being met. This brings teachers and students to closer alignment. Then, the next iteration may be using a presentation software to provide closed captioning of teacher lectures “in the moment.” Each new iteration brings the teacher and student perceptions closer to alignment.

Recommendations for Future Research

The conclusions from this study call for future research in several areas regarding teaching mathematics to Hispanic ELL students in the middle-school setting. These areas include (1) expanding to additional MTPs, (2) expanding data collection methods, (3) increasing length of study, and (4) exploring additional contexts.

First, the most obvious extension of this research would be to replicate the methodology with the remaining five Mathematics Teaching Practices: establishing mathematics goals to focus learning, implementing tasks that promote reasoning and problem-solving, building

procedural fluency from conceptual understanding, eliciting and using evidence of student thinking, and supporting productive struggle in learning mathematics. It is unknown if these MTPs may be perceived as beneficial in learning mathematics by Hispanic ELL students and their teachers. The MTPs chosen for this study were specifically selected for their connection with social constructivist theory and research-informed practices that support ELL students. The remaining MTPs may require alternative methods to elicit student consultation. For example, building procedural fluency from conceptual understanding may require a series of video recordings and a different line of questioning to unearth if the practices support mathematical learning for Hispanic ELL students.

Second, this study utilized video stimulated recall and semi-structured interviews to gather data about the perceptions of Hispanic ELL students. While interviews are the mainstay of qualitative case studies (Savin-Baden & Howell Major, 2013), there may be additional ways to gather student perceptions. Other studies that have collected information about lived experiences from the Hispanic ELL perspective have used photo-elicitation interviews (Baker, 2013) and a phenomenological narrative approach (Bohensky, 2014). The collection of photos and student work samples may present an even more complete picture of the MTP in the classroom setting.

Third, a longitudinal study could confirm and strengthen findings about the perceptions of Hispanic ELL mathematics students. This study was conducted over one semester, which was appropriate since the seventh-grade participants only took math for one semester. It would be of interest to continue gathering these nine students' perceptions longitudinally and document the ways in which these nine students continue to see these MTPs in future mathematics courses with different teachers. Data collection could also explore the ways in which teacher knowledge,

pedagogical awareness, and facilitation of the MTPs affect the students' perceptions of helpfulness?

Fourth, the study could also be expanded to include more than nine students, more than three teachers and with more than three lessons. Professional learning around the MTPs could be included so that a more consistent facilitation of the MTPs could be analyzed. This could offer a comprehensive look at the effectiveness of the MTPs across a wider variety of classrooms in which teachers had a deeper knowledge base around the MTPs. In this setting, a teacher focus group may also contribute to the discussion in a different way than individual interviews might.

Last, this study was conducted with nine Hispanic ELL students in a rural resort town in the Intermountain West. It would be of interest to conduct similar studies in different contexts. For example, studies could explore if the findings would be consistent in a school with a lower proportion of Hispanic ELL students or with a school that had fewer educational resources than the one in this study? Conducting the study in an urban setting with different student demographics may provide additional insights about the needs of minoritized student groups and their teachers.

Chapter Summary

This study examined the perceptions of nine Hispanic ELL middle-school students and their teachers regarding a subset of research-informed Mathematics Teaching Practices. Capturing Hispanic ELL student perceptions is of great importance due to the challenging situations that many Hispanic ELL students and their families face (Schneider et al., 2006) in the United States and the concerning differences seen in achievement data for Hispanic and non-

Hispanic students and for ELL and non-ELL students. To address the layered challenges that Hispanic ELL students face daily in school, teachers, school leaders, and researchers can turn to student consultation as one way to gather information to inform teaching practices and develop positive mathematical identities.

This study found that both students and teachers value and find benefit in the use of mathematical discourse, posing of purposeful questions, and attention to multiple representations. While the three MTPs examined in this study were found to be effective in supporting mathematical learning, some divergences were noted between teachers' and students' perceptions. Among these were translation support, the relative importance of written versus oral discourse, and the use of discourse as an evaluation of learning. Additionally, an important common thread across the practices was the development of students' mathematical identities.

The findings of this study translate to implementable strategies for both teachers of Hispanic ELL students and other researchers. The ability to do mathematics is a civil right—it contributes to a person's opportunity to fully participate in society (Curry, 2018)—thus, the outcomes of this study contribute to our understanding of how we can better support the growing population of Hispanic ELL students in our country.

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APPENDICES

APPENDIX A

MATHEMATICAL TEACHING PRACTICES

CLASSROOM OBSERVATION TOOL

Appendix A
Mathematical Teaching Practices Classroom Observation Tool

Teacher _____

Observation Date _____

Teacher Behavior	Observed	Field Notes
Use and connect mathematical representations: Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.		
Selecting tasks that allow student to decide which representations to use in making sense of problems.		
Allocating substantial instructional time for students to use, discuss, and make connections among representations.		
Introducing forms of representations that can be useful to students.		
Asking students to make math drawings or use other visual supports to explain and justify their reasoning.		
Focusing students' attention on the structure or essential feature of mathematical ideas that appear, regardless of the representation.		
Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.		

Appendix A continued
Mathematical Teaching Practices Classroom Observation Tool

Teacher Behavior	Observed	Field Notes
Pose purposeful questions: Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.		
Advancing students understanding by asking questions that build on, but do not take over or funnel, student thinking.		
Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.		
Allowing sufficient wait time so that more students can formulate and offer responses.		
Facilitate mathematical discourse: Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.		
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches using varied representations.		
Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.		
Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.		
Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.		

APPENDIX B

PROXY TERMS FOR MATHEMATICAL
TEACHING PRACTICES REFERENCE PAGE

Appendix B
Proxy Terms for Mathematical Teaching Practices Reference Page

Teacher Name	MTP: Pose Purposeful Questions	MTP: Facilitate Meaningful Mathematical Discourse	MTP: Use and Connect Mathematical Representations
	Proxy/Classroom Terms Used	Proxy/Classroom Terms Used	Proxy/Classrooms Terms Used
1. CL	1. gathering information questions 2. probing thinking questions What will you do next? 3. making mathematics visible questions 4. encouraging reflection questions	- Opportunities - Ask a partner before me	- Options - Choices - opportunities
3. NG	1. gathering information questions - How do you know? 2. probing thinking questions - What is your reason? 3. making mathematics visible questions - Can you show me? 4. encouraging reflection questions	- Turn and talk - Consult with a friend - Small group - Music mingle	You must use a model

Appendix B Continued
Proxy Terms for Mathematical Teaching Practices Reference Page

4. BW	1. gathering information questions - Cool down 2. probing thinking questions 3. making mathematics visible questions 5. encouraging reflection questions - Question, comment, concern	- Turn to a seat partner 2 people one I pad - Student teachers - Don't. leave your partner behind	- Ratio tables, double number lines - Which representation will you use - Tool - Strategy - model
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APPENDIX C

THE MATHEMATICAL PRACTICES
HELPFULNESS RATING SCALE

Appendix C
The Mathematical Practices Helpfulness Rating Scale
Did this help you to be able to do your math on your own?

Practice	0 – no 	1 – somewhat 	2 – yes 
Pose Purposeful Questions	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.
Facilitate Meaningful Mathematical Discourse	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.
Use and Connect Mathematical Representations	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.

APPENDIX D

THE MATHEMATICS TEACHING
PRACTICES HELPFULNESS STUDY

Appendix D
The Mathematics Teaching Practices Helpfulness Study
Phase One Student Pilot Study Protocol
 Part One

Project:

Time of Interview:

Date:

Place:

Interviewee:

Introduction:

Say:

Thank you for participating in this project with me. This part of the project is set up to help me make sure that I am asking questions that make sense and can be answered by students your age. Today we are going to go through an interview process, but we are not going to answer any of the questions. I am going to stop often to find out if each of the questions make sense to you. I will also ask you if there is a way to make the wording or steps more understandable.

This session will be divided into five sections. During four of the sections, we will look at the wording of the introductions and questions. During one of the sections, we will look at the wording in a rating scale. After I fix up any wording from the suggestions you make today, we will visit one more time to try out the interview.

In all this work, you are the expert helping me out. I value your opinion and appreciate the time you are taking to help me make this project the best it can be. Everything you say today is confidential and I will not share your thoughts with anyone else. I might write about what you say, but I will not use your name or any other information that might identify you.

Do you have any questions?

Interview Protocol:

Section One

Say:

This is the first section of the interview process. I will read it out loud then ask you a couple questions about whether the section makes sense. I can repeat anything that you need repeated, and our translator can also read it in Spanish if that will make it clearer.

Read aloud:

Welcome:

Thank you for participating in this project with me. Remember that this interview is confidential. That means I will not share your thoughts with anyone else. Anything you tell me will be private, and you won't get in trouble for anything you say. I might write about what you say, but I will not use your name or any other information that might identify you. You can tell me you don't want to answer any question you don't feel like answering, and you can stop the interview at any point.

Ask:

Can you tell me what you think this section is about? It's okay if you don't know. That helps me know that I need to change some things.

Say:

Now we will go through 2-3 sentences at a time. If you think that the sentences make sense, just give me a thumbs up. If you think they don't make sense, just give me a thumbs down. This will help us figure out exactly which parts need fixing up. If a sentence doesn't make sense, we will stop to rewrite it so that it does make sense.

Ask:

Now, can you tell me what you think this section is about?

Say:

Now we will go through the same steps with section two.

Section Two

During this interview, you will be asked to tell what you think about things that happen in your mathematics classrooms. We will be watching a short video clip from a recent lesson in your mathematics classroom. The video clip will show your teacher using a Mathematical Teaching Practice or strategy that educators believe to be helpful in learning mathematics. Your job is to tell me what you were thinking and feeling when you were doing the mathematics and if the practice or strategy was helpful to you as you were learning this math concept. I am very interested in hearing about what you think. This study is all about learning about what students really think about the ways their teachers try to help them understand mathematics, so you honestly tell me if the strategy helped or not.

Ask:

Can you tell me what you think this section is about? It's okay if you don't know. That helps me to know that I need to change some things.

Say:

Now we will go through 2-3 sentences at a time. If you think that the sentences make sense, just give me a thumbs up. If you think they don't make sense, just give me a thumbs down. This will help us figure out exactly which parts need fixing up. If a sentence doesn't make sense, we will stop to rewrite it so that it does make sense.

Ask:

Now, can you tell me what you think this section is about?

Say:

Section Three is an introduction that will be read before watching a video clip of your classroom.

*Section Three***Introduction:**

In this video clip, your class was working on _____ (name the math goal; for example, students will be able to simplify fractions).

The Mathematical Teaching Practice or strategy that your teacher was using was _____ (pose purposeful questions, facilitate mathematical discourse, or connect mathematical representations). Your teacher often calls this strategy _____ (proxy term).

We will watch the video clip of this practice together. Please signal me anytime you'd like to stop the video to tell me about what you were thinking or feeling. I'll do the same thing.

While watching the clip, stop the video to ask:

1. At this moment, what do you remember about the math you were doing? What were you thinking and feeling about the way you were learning math?

Ask:

Can you tell me what you think this section is about? It's okay if you don't know. That helps me to know that I need to change some things.

Say:

Now we will go through 2-3 sentences at a time. If you think that the sentences make sense, just give me a thumbs up. If you think they don't make sense, just give me a thumbs down. This will help us figure out exactly which parts need fixing up. If a sentence doesn't make sense, we will stop to rewrite it so that it does make sense.

Ask:

Now, can you tell me what you think this section is about?

Section Four

Say:

Next, we are going to look at a rating scale that your teacher and I came up with together. It is supposed to help students in your class tell me if the strategy that their teacher is using is helpful or not. If you think there is a better way to describe helpfulness, I would like to hear about that.

Your teacher has described a helpful strategy as one in which _____ (teacher description). Do you agree with that? What would you change?

Now we will look at each section of the rating scale (show document). If you think there is a better way to describe if a strategy is helpful, please let me know.

Read each section of the scale aloud. Then ask:

Did that description make sense to you?

Is there another way to say this that would make more sense?

Do you think this scale would help you describe if the strategy that your teacher was using was helpful or not?

Section Five

Say:

In this section, we are going to go through each of the questions that I am going to ask the students during their interview. Remember, that they will have seen a video clip of themselves in math class. Today, I am just trying to find out if the questions make sense. You don't have to answer the question.

Ask each question one at a time. After each question, ask:

Did the questions make sense?

Is there a way to say the question that would make more sense?

Can you tell me what the question is asking you to do or talk about?

Questions Part 1: related to episode

1. How much did _____ (Mathematical Teaching Practice) help you understand _____ (math goal) during this lesson? You can use the rating scale to help you describe how much it helped.
2. Why do you think _____ (Mathematics Teaching Practice) was or was not helpful in learning this math idea?
3. Is there anything you would like to add that you remember about learning mathematics during this math lesson?

Questions Part 2: general

4. Does _____ (Mathematical Teaching Practice) help you at other times in math class? Please share an example.
5. Is there anything that you would like to add that you think might help me understand which strategies help you learn mathematics?
6. Do you have any questions?

Closing:

Thank you so much for answering my questions. We will watch a video clip and have a conversation like this ____ more times.

Pilot Study Part One Closing:

Say:

Thank you so much for helping with this part of my project. The next time we get together, we will go through the interview process using some of your suggestions.

The Mathematical Practices Helpfulness Phase One

Student Pilot Study Protocol

Part Two

Project:

Time of Interview:

Date:

Place:

Interviewee:

Introduction:

Say:

Thank you for participating in this project with me. This part of the project is set up to help me make sure that the interview portion of my project makes sense and works within the time limits we have. During our last session together, you helped me make sure that the interview introductions and questions made sense. I really appreciate your ideas and help on that part. You may see some of your suggestions in the work we do today.

For this session, we will go through the interview process just like I would during the actual study. However, I will stop after each section to ask you several questions.

In all this work, you are the expert helping me out. I value your opinion and appreciate the time you are taking to help me make this project the best it can be. Everything you say today is confidential and I will not share your thoughts with anyone else. I might write about what you say, but I will not use your name or any other information that might identify you.

Do you have any questions?

Interview Protocol:

Say:

This is the first section of the interview process. I will read it out loud then ask you a couple questions about whether the section makes sense. I can repeat anything that you need repeated, and our translator can also read it in Spanish if that will make it clearer.

Read aloud:

Welcome:

Thank you for participating in this project with me. Remember that this interview is confidential. That means I will not share your thoughts with anyone else. Anything you tell me will be private, and you won't get in trouble for anything you say. I might write about what you say, but I will not use your name or any other information that might identify you. You can tell me you don't want to answer any question you don't feel like answering, and you can stop the interview at any point.

During this interview, you will be asked to tell what you think about things that happen in your mathematics classrooms. We will be watching a short video clip from a recent lesson in your mathematics classroom. The video clip will show your teacher using a Mathematical Teaching Practice or strategy that educators believe to be helpful in learning mathematics. Your job is to tell me what you were thinking and feeling when you were doing the mathematics and if the practice or strategy was helpful to you as you were learning this math concept. I am very interested in hearing about what you think. This study is all about learning about what students really think about the ways their teachers try to help them understand mathematics, so you honestly tell me if the strategy helped or not.

Ask:

How did you feel about that section?

Did it make sense?

Do I need to read faster or slower?

Do you have any other suggestions for that section that would help other students understand what they are supposed to do?

Say:

Now we will start the interview.

Introduction:

In this video clip, your class was working on _____ (name the math goal; for example, students will be able to simplify fractions).

The Mathematical Teaching Practice or strategy that your teacher was using was _____ (pose purposeful questions, facilitate mathematical discourse, or connect mathematical representations). Your teacher often calls this strategy _____ (proxy term).

We will watch the video clip of this practice together. Please signal me anytime you'd like to stop the video to tell me about what you were thinking or feeling. I'll do the same.

While watching the clip, stop the video to ask:

1. At this moment, what do you remember about the math you were doing? What were you thinking and feeling about the way you were learning math?

Ask:

How did you feel about that section?

Did it make sense?

Do you have any other suggestions for that section that would help other students understand what they are supposed to do?

Questions Part 1: related to episode

7. How much did _____ (Mathematical Teaching Practice) help you understand _____ (math goal) during this lesson? You can use the rating scale to help you describe how much it helped.
8. Why do you think _____ (Mathematics Teaching Practice) was or was not helpful in learning this math idea?
9. Is there anything you would like to add that you remember about learning mathematics during this math lesson?

Questions Part 2: general

10. Does _____ (Mathematical Teaching Practice) help you at other times in math class? Please share an example.
11. Is there anything that you would like to add that you think might help me understand which strategies help you learn mathematics?
12. Do you have any questions?

Ask:

How did you feel about that section?

Did it make sense?

Do you have any other suggestions for that section that would help other students understand what they are supposed to do?

Closing:

Thank you so much for answering my questions. We will watch a video clip and have a conversation like this ____ more times.

Pilot Study Part One Closing:

Say:

Thank you so much for helping with this part of my project. All your suggestions will help my project be the best it can be.

APPENDIX E

GROUP ORIENTATION PROTOCOL

Appendix E

Group Orientation Protocol

Project:
Time of Interview:
Date:
Place:
Participants:

Introduction:

Say:

Thank you for participating in this project with me. This project is important because it will help teachers understand how to teach mathematics in a way that is helpful to you. During this project, I am going to video record some of your math classes and then ask you some questions about the class. This part of the project is set up to show you how the interviews will work and to let you ask any questions you might have.

In all this work, you are the expert helping me out. I value your opinion and appreciate the time you are taking to help me make this project the best it can be. I have already asked two students to go through this process with me once before as a practice/ trial run. I can share their responses as examples.

Do you have any questions?

Interview Protocol:

Say:

This is the first section of the interview process. I will read it out loud then ask you a couple questions about whether the section makes sense. I can repeat anything that you need repeated, and our translator can also read it in Spanish if that will make it clearer.

Read aloud:

Welcome:

Thank you for participating in this project with me. Remember that this interview is confidential. That means I will not share your thoughts with anyone else. Anything you tell me will be private, and you won't get in trouble for anything you say. I might write about what you say, but I will not use your name or any other information that might identify you. You can tell me you don't want to answer any question you don't feel like answering, and you can stop the interview at any point.

Ask:

Can someone tell me what you think that section is all about?

If there are not any responses, say:

Another student said that this section was about _____ (response from pilot).

Does anyone have any questions about this section?

Read out loud:

During this interview, you will be asked to tell what you think about things that happen in your mathematics classrooms. We will be watching a short video clip from a recent lesson in your mathematics classroom. The video clip will show your teacher using a Mathematical Teaching Practice or strategy that teachers believe to be helpful in learning mathematics. Your job is to tell me what you were thinking and feeling when you were doing the mathematics and if the practice or strategy was helpful to you as you were learning this math concept. I am very interested in hearing about what you think. This study is all about learning about what students really think about the ways their teachers try to help them understand mathematics, so you honestly tell me if the strategy helped or not.

Ask:

Can someone tell me what you think that section is all about?

If there are not any responses, say:

Another student said that this section was about _____ (response from pilot).

Now, turn to the person sitting next to you and tell them what you think this section is about.

Does anyone have any questions about this section?

Introduction:

Read out loud:

In this video clip, your class was working on _____ (name the math goal; for example, students will be able to simplify fractions).

The Mathematical Teaching Practice or strategy that your teacher was using was _____ (pose purposeful questions, facilitate mathematical discourse, or connect mathematical representations). Your teacher often calls this strategy _____ (proxy term).

We will watch the video clip of this practice together. Please signal me anytime you'd like to stop the video to tell me about what you were thinking or feeling. I'll do the same thing.

Ask:

Can someone tell me what you think we will be doing in this part?

If there are not any responses, say:

Another student said that this section was about _____ (response from pilot).

Say:

Imagine that we just saw a video clip from your math class. I will tell you the responses that the previous students gave to the questions I asked.

Read out loud:

I asked:

2. At this moment, what do you remember about the math you were doing? What were you thinking and feeling about the way you were learning math?

Say:

Other students responded by saying _____ (response from pilot) for these questions.

Ask:

Does anyone have a question about that part of the interview?

Say:

Before I ask any questions, I want to share a rating scale with you. Your teacher and two students helped to make the rating scale and helped me come up with a definition for the word helpful. The definition of helpful is _____ (state co-created definition). When you use the rating scale, you will decide if the strategy that your teacher was trying was helpful or not.

Do:

Read each section of the rating scale out loud for the MTP that was used in the pilot study.

Ask:

Do you have any questions about how to use the rating scale?

Say:

Now I will go through each of the questions that I asked, and I will share a sample response from the practice study.

Questions Part 1: related to episode

13. How much did _____ (Mathematical Teaching Practice) help you understand _____ (math goal) during this lesson? You can use the rating scale to help you describe how much it helped.
 - a. **Say:** Another student said _____ (response from pilot).
 - b. **Ask:** Are there any questions about this question?
14. Why do you think _____ (Mathematics Teaching Practice) was or was not helpful in learning this math idea?
 - a. **Say:** Another student said _____ (response from pilot).
 - b. **Ask:** Are there any questions about this question?
15. Is there anything you would like to add that you remember about learning mathematics during this math lesson?
 - a. **Say:** Another student said _____ (response from pilot).
 - b. **Ask:** Are there any questions about this question?

Questions Part 2: general

16. Does _____ (Mathematical Teaching Practice) help you at other times in math class? Please share an example.
- a. **Say:** Another student said _____ (response from pilot).
 - b. **Ask:** Are there any questions about this question?
17. Is there anything that you would like to add that you think might help me understand which strategies help you learn mathematics?
- a. **Say:** Another student said _____ (response from pilot).
 - b. **Ask:** Are there any questions about this question?
18. Do you have any questions?
- a. **Say:** Another student said _____ (response from pilot).
 - b. **Ask:** Are there any questions about this question?

Closing:

Say:

Thank you so much for being a part of my project. Your time and opinion matters very much to me, and you are helping teachers understand how to teach math even better!

APPENDIX F

THE MATHEMATICS PRACTICES HELPFULNESS
STUDENT INTERVIEW PROTOCOL

Appendix F

The Mathematics Practices Helpfulness Student Interview Protocol

Project:

Time of Interview:

Date:

Place:

Interviewee:

Interview Protocol:

Welcome:

Thank you for participating in this project with me. Remember that this interview is confidential. That means I will not share your thoughts with anyone else. Anything you tell me will be private, and you won't get in trouble for anything you say. I might write about what you say, but I will not use your name or any other information that might identify you. You can tell me you don't want to answer any question you don't feel like answering, and you can stop the interview at any point. The interview will be very much like what we practiced during the Group Orientation Session.

During this interview, you will be asked to tell what you think about things that happen in your mathematics classrooms. We will be watching a short video clip from a recent lesson in your mathematics classroom. The video clip will show your teacher using a Mathematical Teaching Practice or strategy that educators believe to be helpful in learning mathematics. Your job is to tell me what you were thinking and feeling when you were doing the mathematics and if the practice or strategy was helpful to you as you were learning this math concept. I am very interested in hearing about what you think. This study is all about learning about what students really think about the ways their teachers try to help them understand mathematics, so you honestly tell me if the strategy helped or not.

Introduction:

In this video clip, your class was working on _____ (name the math goal; for example, students will be able to simplify fractions).

The Mathematical Teaching Practice or strategy that your teacher was using was _____ (pose purposeful questions, facilitate mathematical discourse, or connect mathematical representations). Your teacher often calls this strategy _____ (proxy term).

We will watch the video clip of this practice together. Please signal me anytime you'd like to stop the video to tell me about what you were thinking or feeling. I'll do the same thing.

While watching the clip, stop the video to ask:

3. At this moment, what do you remember about the math you were doing? What were you thinking and feeling about the way you were learning math?

Questions Part 1: related to episode

1. How much did _____ (Mathematical Teaching Practice) help you understand _____ (math goal) during this lesson? You can use the rating scale to help you

Practice	Not Very Helpful	Somewhat Helpful	Very Helpful
Pose Purposeful Questions	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.
Facilitate Meaningful Mathematical Discourse	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.
Use and Connect Mathematical Representations	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.

describe how much it helped.

2. Why do you think _____ (Mathematics Teaching Practice) was or was not helpful in learning this math idea?
3. Is there anything you would like to add that you remember about learning mathematics during this math lesson?

Questions Part 2: general

4. Do you remember a time when _____ (Mathematical Teaching Practice) helped you in math class? Please share an example.
5. Is there anything that you would like to add that you think might help me understand which strategies help you learn math? Basically, what helps you learn math?

Closing:

Thank you so much for answering my questions. We will watch a video clip and have a conversation like this ____ more times.

MTP Helpfulness Rating Scale

(the exact terminology used in this scale will be determined during the teacher interview protocol)

APPENDIX G

THE MATHEMATICS PRACTICES HELPFULNESS
TEACHER INTERVIEW PROTOCOL

Appendix G
The Mathematics Practices Helpfulness Teacher Interview Protocol
 Phase One

Date:

Place:

Interviewee:

Introduction:

Thank you for participating in this project with me. The teacher interview protocol will occur in two phases. During the first phase, we will develop documents that will support both students and teachers during the interview process. The second phase will occur after I have video recorded a National Council of Teachers of Mathematics (NCTM) Mathematics Teaching Practice (MTP) being utilized in your classroom. The focus of the recording will be the previously selected Hispanic ELL students.

Phase One:

During Phase One, we will co-create several documents that will help student and teacher interviews be as clear as possible. We will create a proxy term reference page as well as a rating scale.

Common Terminology for Mathematical Practices Reference Page

During this part of Phase One, we will collaboratively identify commonly used proxy terms for elements of the three MTPs to be investigated. For example, if you use “turn and talk” as a means of facilitating mathematical discourse, I will use that phrase when interviewing the participating students. It will be important to gather this information from each participating teacher as you all may use different terminology for the same practice. Using common terminology is supportive when working with ELL students and promotes clarity in the interview process. Additionally, using familiar classroom language in which students feel comfortable helps build rapport with the students.

We will record the proxy terms in the *Common Terminology for Mathematical Practices Reference Page* (Appendix B).

Questions:

1. Which terms do you commonly use when you pose purposeful questions? NCTM categorizes questions into four types:
 - gathering information questions in which students recall facts, definitions, or procedures,
 - probing questions in which you ask students to explain their thinking
 - making the mathematics visible questions in which you ask about connections and relationships
 - encouraging reflection and justification where students might make an agreement for their work.
 We will record terms for each of these types separately in our document.
2. Which terms do you commonly use when facilitating mathematical discourse? Some common ones might be “turn and talk”, “does anyone have questions for (student name)”, “explain your thinking”, and “do you agree or disagree”.

3. Which terms do you commonly use when using and connecting mathematical representations?
Ways that teachers might encourage this could be to ask if anyone found another way to solve the problem or to ask how different solution pathways are similar or different.

The Mathematics Practices Helpfulness Rating Scale

Next, we will co-create a rating scale for students that aligns with commonly used language in the classroom. We can also incorporate emojis and other symbols, like a thumbs up, that signify different levels of rating. *The Mathematics Practices Helpfulness Rating Scale* (Appendix C) will provide students with a way to rate the degree to which a particular practice is more or less helpful in their mathematical learning. The student participants will use *The Mathematics Practices Helpfulness Rating Scale* to evaluate the helpfulness of each MTP as they observe them in video segments during the interview process. Teachers will also rate each practice from not at all helpful to very helpful in supporting ELL students' mathematical understanding during the teacher interview component of the data collection.

Questions:

1. How might we define the term “helpful” for this project? What does it mean for a strategy to be helpful?
2. How might you and your students decide if a MTP is helpful or not?
3. What might we see or hear from students if a MTP is helpful? Not helpful?
4. What words might you and your students use when describing if a MTP is helpful?
5. How might you and your students differentiate between levels of helpfulness? For example, would you say, “very helpful, somewhat helpful, and not at all helpful”?

The Mathematics Practices Helpfulness Teacher Interview Protocol Phase Two

Introduction:

Thank you for participating in this project with me. To remind you, this interview is confidential. I might write about what you say, but I will not use your name or any other information that might identify you. You can tell me you don't want to answer any question you don't feel like answering, and you can stop the interview at any point.

During this interview, you will be asked talk about your perceptions of the MTPs that were being utilized in your class during this lesson. Remember, I am specifically wondering about the perceived helpfulness of the selected MTP.

We will be watching a short video clip from a recent lesson that you taught. The video clip will show you using a MTP that educators believe to be helpful in learning mathematics. Your job is to reflect on the video segment and discuss if you perceived the MTP as helpful to your Hispanic, ELL students in learning this mathematics concept. This perception should be based on how we defined “helpful” in Phase One of the interview protocol. I am very interested in hearing what you think.

Introduction:

We are going to watch and discuss 3 video clips from your classroom. In this (first, second, third) video clip, your class was working on _____ (name the math goal; for example, students will be able to simplify fractions). The Mathematical Teaching Practice that you used was _____ (pose purposeful questions, facilitate mathematical discourse, or connect mathematical representations).

We will watch the video clip of this (first, second, third) practice together. Please signal me anytime you'd like to stop the video to tell me about what you were thinking or feeling. I'll do the same thing.

Summarizing the Mathematics

While watching the clip, stop the video to ask:

4. At this moment, tell me what you remember about how Hispanic, ELL students were responding to the selected MTP?

Questions:

6. Based on our definition of helpful, how helpful was _____ (Mathematical Teaching Practice) in supporting your Hispanic, ELL students' understanding of _____ (math goal) during this lesson? You can use the rating scale to help you describe how much it helped.
7. Why do you think _____ (MTP) was or was not helpful in learning this math idea?
8. What evidence leads you to believe that the MTP was helpful or not? Did you hear, observe, or sense something that caused you to make this conclusion?
9. Does this MTP help your Hispanic, ELL students at other times in math class? Please share an example.
10. Is there anything that you would like to add that you think might help me understand which teaching practices help your Hispanic ELL students learn mathematics?
11. Is there anything you would like to add that you remember about teaching mathematics during this mathematics lesson?
12. Do you have any questions?

Closing:

Thank you so much for answering my questions about (first, second, third) practice.

MTP Helpfulness Rating Scale

Practice	Not Very Helpful	Somewhat Helpful	Very Helpful
Pose Purposeful Questions	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.
Facilitate Meaningful Mathematical Discourse	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.
Use and Connect Mathematical Representations	This practice was not very helpful when I was learning mathematics in class today.	This practice was somewhat helpful when I was learning mathematics in class today.	This practice was very helpful when I was learning mathematics in class today.

APPENDIX H

PARENTAL CONSENT FORM FOR PARTICIPATION IN
HUMAN RESEARCH AT MONTANA STATE UNIVERSITY

Appendix H**PARENTAL CONSENT FORM FOR PARTICIPATION IN HUMAN RESEARCH AT
MONTANA STATE UNIVERSITY**

Title: Hispanic ELL Student Perceptions of Mathematics Teaching Practices

Dear Parent/Guardian,

My name is Christi Roberts, and I am a graduate student at Montana State University and a certified teacher in the state of Wyoming. I am interested in understanding whether or not Hispanic ELL students see Mathematics Teaching Practices as helpful to them when learning mathematics. Mathematics Teaching Practices are specific strategies that math teachers use to help students learn math. I think some teaching practices such as posing purposeful questions, facilitating mathematical discourse, and connecting mathematical representations could help to increase Hispanic, ELL students' mathematical understanding. I am conducting a study at your child's middle school to help teachers understand if Hispanic ELL students see these practices as helpful in learning mathematics.

Your child is being asked to take part in this study because he/she is a Hispanic middle school student enrolled in mathematics coursework and receives ELL (English Language Learner) services in school. Taking part in this study is completely voluntary, meaning that you can choose for your child to participate or not. Your child has the right to end their participation at any time without consequence.

Your child's participation in this study is confidential. I will not record your child's name on any document. Instead, I will assign each child a unique pseudonym, which will be recorded in a master list. All information including video recordings and interview transcripts will be kept for at least six years on a secure server in a password-protected database. Your child will not be identified in any report about this study. The school will not receive information about your student's interview responses.

If you agree to allow your child to be part of this study, your child will participate in three thirty-minute informal interview sessions. There will be two students, a translator, and myself present at each interview session. In each session, the students will view a video recording of their mathematics class. They will then respond to several questions about how helpful they found the Mathematical Teaching Practices to be in their learning of the mathematics. The interviews will take place during lunch, and lunch will be provided for your student on those days. Your student will not miss any class time.

Participation in this study carries no foreseen risks, no monetary benefit, and no cost to you or your child. Please know that I will conduct my project in an ethical manner, and your child's information will be kept confidential.

If you have any questions about the research, please contact me, Christi Roberts at christi729@hotmail.com; 208-520-7303. If you have additional questions about the rights of human subjects you can contact the Chair of the Institutional Review Board, Mark Quinn, (406) 994-4707 [mquinn@montana.edu].

If you consent to allow your child to participate, please sign and return this form to your child's teacher.

Parent/Guardian Consent

AUTHORIZATION: I have read the above and understand the discomforts, inconvenience and risk of this study. I, _____ (*name of parent or guardian*), related to the subject as _____ (*relationship*), agree to the participation of _____ (*name of subject*) in this research. I understand that the subject or I may later refuse participation in this research and that the subject, through his/her own action or mine, may withdraw from the research study at any time. I have received a copy of this consent form for my own records.

Parent or Guardian Signature: _____

Child's Assent Signature: _____ (if applicable)

Investigator: _____

Date: _____

Appendix H Continued**FORMULARIO DE CONSENTIMIENTO DE LOS PADRES PARA PARTICIPAR EN INVESTIGACIONES HUMANAS EN LA UNIVERSIDAD ESTATAL DE MONTANA**

Título: Las percepciones de los estudiantes hispanos ELL (Aprendices del idioma inglés) sobre las prácticas de enseñanza de las matemáticas

Estimado padre/tutor legal,

Me llamo Christi Roberts y soy un estudiante graduado en la Universidad Estatal de Montana y una maestra certificada en el estado de Wyoming. Estoy interesada en comprender si los estudiantes perciben o no las prácticas de enseñanza de las matemáticas como útiles para ellos cuando aprenden matemáticas. Creo que algunas prácticas de enseñanza, como formular preguntas con propósito, facilitar el discurso matemático y conectar representaciones matemáticas, podrían ayudar a aumentar la comprensión matemática de los estudiantes. Estoy realizando una investigación en la escuela secundaria de su hijo para ayudar a los maestros a comprender si los estudiantes hispanos de desarrollo del idioma inglés perciben estas prácticas como útiles para su aprendizaje de las matemáticas.

Se le pide a su hijo que participe en este estudio porque es un estudiante hispano de la escuela secundaria inscrito en cursos de matemáticas y recibe servicios de ELL (Aprendiz del idioma inglés) en la escuela. La participación en este estudio es completamente voluntaria, lo que significa que puede elegir si su hijo participa o no. Su hijo tiene derecho a finalizar su participación en cualquier momento sin consecuencias.

La participación de su hijo en este estudio es confidencial. No grabaré el nombre de su hijo en ningún documento. En cambio, asignaré a cada niño un seudónimo único, que se registrará en una lista maestra. Toda la información, incluidas las grabaciones de video y las transcripciones de entrevistas, se mantendrá durante al menos seis años en un servidor seguro en una base de datos protegida con contraseña. Su hijo no será identificado en ningún informe sobre este estudio. La escuela no recibirá información sobre las respuestas de la entrevista de su estudiante.

Si acepta permitir que su hijo sea parte de este estudio, su hijo participará en tres sesiones informales de entrevistas de treinta minutos. Habrá dos estudiantes, un traductor y yo presente en cada sesión de entrevistas. En cada sesión, los estudiantes verán una grabación de video de su clase de matemáticas. Luego responderán a varias preguntas sobre cuán útiles les parecieron las prácticas de enseñanza de las matemáticas en su aprendizaje de las matemáticas. Las entrevistas se llevarán a cabo durante el almuerzo y el almuerzo se le proporcionará a su estudiante en esos días. Su estudiante no perderá ningún tiempo de clase.

La participación en este estudio no conlleva riesgos previstos, ningún beneficio monetario o ningún costo para usted o su hijo. Tenga en cuenta que llevaré a cabo mi proyecto de manera ética y que la información de su hijo se mantendrá confidencial.

Si tiene alguna pregunta sobre la investigación, comuníquese conmigo, Christi Roberts, en christi729@hotmail.com; 208-520-7303. Si tiene preguntas adicionales sobre los derechos de los sujetos humanos, puede comunicarse con el presidente de la Junta de Revisión Institucional, Mark Quinn, (406) 994-4707 [mquinn@montana.edu].

Si da su consentimiento para que su hijo participe, firme y devuelva este formulario al maestro de su hijo.

Consentimiento del padre / tutor legal

AUTORIZACIÓN: He leído lo anterior y entiendo las molestias, inconvenientes y riesgos de este estudio. Yo, _____ (*nombre de padre o tutor legal*), relacionado con el sujeto como _____ (*parentesco*), aceptar la participación de _____ (*nombre del sujeto*) en esta investigación. Entiendo que el sujeto o yo podemos rechazar la participación en esta investigación y que el sujeto, a través de su propia acción o la mía, puede retirarse del estudio de investigación en cualquier momento. He recibido una copia de este formulario de consentimiento para mis propios registros.

Firma del padre o tutor legal : _____

Investigador: _____

Fecha: _____

APPENDIX I

STUDENT CONSENT FORM FOR PARTICIPATION IN
HUMAN RESEARCH AT MONTANA STATE UNIVERSITY

Appendix I

STUDENT CONSENT FORM FOR PARTICIPATION IN HUMAN RESEARCH AT MONTANA STATE UNIVERSITY

You are invited to take part in a study on the helpfulness of Mathematics Teaching Practices.

What are Mathematics Teaching Practices (MTPs)?

MTPs are specific strategies that your math teachers use to help students learn math. These are ways of teaching that your teachers use because that they think will help students understand math better.

What's our study about?

I am trying to learn if the MTPs are helpful to you when you are trying to learn math. I'm also interested to know how much you think they help you.

If you agree to be part of this project, you will be asked to watch a few video clips from your math class at a time when your teacher has chosen to use a particular MTP. You will then be asked some questions about what you were thinking during that time, how helpful the MTP was, and if the MTP has been helpful at other times in math class. All of your responses will be given orally, and you will not need to do any writing.

There are no right or wrong answers. We will not share your answers with the school or your parents/legal guardian. Your grades in school are not affected by your decision to agree to take part in the study.

Your parent/legal guardian says it's okay for you to be in our study. But if you don't want to be in the study, you don't have to be. If you want to be in our study now but change your mind later, that's okay. You can stop at any time. If there is anything you don't understand you can tell us and we can explain it to you.

Do you AGREE to participate in the study?

☐ **I agree to participate in the study**

Signature _____

Date: _____

Appendix I

FORMULARIO DE CONSENTIMIENTO DEL ESTUDIANTE PARA PARTICIPAR EN INVESTIGACIÓN HUMANA EN LA UNIVERSIDAD ESTATAL DE MONTANA

Estás invitado a participar en un estudio sobre la utilidad de las prácticas de enseñanza de las matemáticas.

¿Qué son las prácticas de enseñanza de las matemáticas (MTP)?

Los MTP son estrategias específicas que sus profesores de matemáticas utilizan para ayudar a los estudiantes a aprender matemáticas. Estas son formas de enseñanza que usan sus maestros porque creen que ayudarán a los estudiantes a comprender mejor las matemáticas.

¿De qué trata nuestro estudio?

Estoy tratando de averiguar si los MTP son útiles para usted cuando está tratando de aprender matemáticas. También me interesa saber cuánto crees que te ayudan.

Si aceptas ser parte de este proyecto, te le pedirá que veas algunos videoclips de tu clase de matemáticas en un momento en que tu maestro haya elegido usar un MTP en particular. Luego te le harán algunas preguntas sobre lo que estabas pensando durante ese tiempo, qué tan útil fue el MTP y si el MTP ha sido útil en otros momentos de la clase de matemáticas. Todas tus respuestas te darás oralmente y no necesitarás escribir nada.

No hay respuestas correctas o incorrectas. No compartiremos tus respuestas con la escuela o tus padres/tutor legal. Tus calificaciones en la escuela no se ven afectadas por tu decisión de aceptar participar en el estudio.

Tu padre/tutor legal dice que está bien que participes en nuestro estudio. Sin embargo, si no deseas participar en el estudio, no es necesario. Si quieres estar en nuestro estudio ahora pero cambias de opinión más tarde, está bien. Puedes detenerte en cualquier momento. Si hay algo que no comprendes, puedes informarnos y te lo explicaremos.

¿ACEPTAS participar en el estudio?

☐ **Acepto participar en el estudio**

Firma _____

Fecha: _____

APPENDIX J

PARTICIPANT CONSENT FORM FOR PARTICIPATION IN
HUMAN RESEARCH AT MONTANA STATE UNIVERSITY

Appendix J

PARTICIPANT CONSENT FORM FOR PARTICIPATION IN HUMAN RESEARCH AT MONTANA STATE UNIVERSITY

Dear Educator,

My name is Christi Roberts, and I am a graduate student at Montana State University and a certified teacher in the state of Wyoming. I am interested in understanding whether students see Mathematics Teaching Practices as helpful to them when learning mathematics. I think some teaching practices such as posing purposeful questions, facilitating mathematical discourse, and connecting mathematical representations could help to increase students' mathematical understanding. I am conducting a study at your middle school to help teachers understand if Hispanic English Language Learners (ELL) see these practices as helpful in their learning of mathematics.

You are being asked to take part in this study because you are a mathematics teacher of Hispanic middle school students who receive ELL services in your school. Taking part in this study is completely voluntary, meaning that you can choose to participate or not. You have the right to end your participation at any time without consequence.

Your participation in this study is confidential. I will not record your name on any document. Instead, I will assign a unique pseudonym, which will be recorded in a master list. All information including video recordings and interview transcripts will be kept for at least six years on a secure server in a password-protected database. Your will not be identified in any report about this study. The school will not receive information about your interview responses.

If you agree to a to be part of this study, you will participate in three thirty-minute informal interview sessions. In each session, you will view a video recording of your mathematics class. You will then respond to several questions about how helpful you found the Mathematics Teaching Practices to be for Hispanic ELL students in learning mathematics. The interviews will take place within 48 hours of the video recording at a time that is convenient for you.

Participation in this study carries no foreseen risks, no monetary benefit, and no cost to you. Please know that I will conduct my project in an ethical manner, and your information will be kept confidential.

AUTHORIZATION: I have read the above and understand the discomforts, inconvenience, and risk of this study. I, _____ (*name of subject*), agree to participate in this research. I understand that I may later refuse to participate and that I may withdraw from the study at any time. I have received a copy of this consent form for my own records.

Signed: _____

Participant Name: _____

Investigator: Christi Roberts

Date: _____

APPENDIX K

PROJECT INFORMATION STATEMENT/LETTER
OF INVITATION TO SCHOOL PRINCIPAL

Appendix K

Hispanic ELL Student Perceptions of Mathematics Teaching Practices Project Information Statement/ Letter of Invitation to School Principal

My name is Christi Roberts, and I am a doctoral student at Montana State University. I am conducting research on the helpfulness of Mathematics Teaching Practices for Hispanic English Language Learners (ELL) under the supervision of Doctor Jayne Downey. I invite you to consider taking part in this research.

Research Questions:

4. How do Hispanic ELL middle school students perceive the effectiveness of research informed mathematics teaching practices in supporting their mathematical understanding?
5. How do teachers of Hispanic ELL middle school students perceive the effectiveness of mathematics teaching practices in supporting their students' mathematical understanding?
6. In what ways do these students' and teachers' perceptions align and diverge?

Significance of the Research Project

The research is significant in three ways.

1. Findings from this study will guide practitioners in better understanding which mathematics teaching practices could best support Hispanic ELL students.
2. This study will open the conversation around using student perception as a method for making content more accessible thus validating students as stakeholders in the educational process.
3. This study will contribute to the sparse literature base on Hispanic ELL mathematics students.

Confidentiality

All information collected will be treated in strictest confidence and neither the school nor individual students will be identifiable in any reports that are written. Participants may withdraw from the study at any time without penalty. The role of the school is voluntary, and the school principal may decide to withdraw the school's participation at any time without penalty.

School Involvement

Once I have received your consent to approach learners and teachers to participate in this study, I will

- arrange for informed consent to be obtained from participants' parents
- arrange a time for data collection to take place
- obtain informed consent from all participants.

Attached for your information are copies of the parent consent form, the student assent form, and the teacher consent form.

If you would like your school to participate in this research, please complete and return the attached form. Thank you for taking the time to read this information.

School Principal Consent Form

I give consent for you to approach middle school mathematics students to participate in this project.

I have read the project information statement explaining the purpose of the research project and understand that:

- The role of the school is voluntary.
- I may decide to withdraw the school's participation at any time without penalty.
- Middle school mathematics students will be invited to participate, and permission will be sought from them and their parents.
- Only students who consent and whose parents consent will participate in the project.
- All information obtained will be treated in the strictest confidence.
- The students' names will not be used, and individual students will not be identifiable in any written records about the study.
- The school will not be identifiable in any written reports about the study.
- Participants may withdraw from the study at any time without penalty.
- A report of the findings will be made available to the school.
- I may seek further information on the project from Christi Roberts at 208-520-7303.

Principal

Signature

Date

Please return to Christi Roberts.

APPENDIX L

DEFINITIONS OF AXIAL CODES

Appendix L
Definitions of Axial Codes

Mathematics Teaching Practice	Code	Definition	Supporting Quotes
Facilitate Mathematical Discourse			
Student	Explanations from peers and self	Students confirm their own thinking by comparing it to the viewpoints of their peers. Students gain an understanding of mathematics by talking with peers. Student “talk themselves” through a problem.	“I could give him my opinion. He could give me his opinion. And through this, we could determine what was the correct answer.” “I saw both sides, my thinking and my friend's thinking. How does she think about it? If she understands it more than me and how she can help me.” “It's really helpful because you walk yourself through [the problem] and you take on your pace instead of going really fast or going really fast.”
Student	Written versus oral	Varying student preference of mathematical discourse modality.	“I get to out all my thoughts that are inside. I can just write [my thinking] out instead of explaining it because it's a bit hard when it's, when I'm explaining it.”
Teacher	Environment	A level of comfort within a classroom in which students feel at ease when talking with one another and with the whole group.	“I think that, for them, understanding that with turn-and-talk, that they have a comfortable small group space in order to do it in, provides them with an opportunity to have a voice in a math classroom in which they might just be passive participants.”
Teacher	Processing	Students’ cognitive work through the mathematical task.	“I just think of it as its different forms of processing, first giving yourself the opportunity to think of it on your own, and then the turn and talk processing with one person.”
Teacher	Independence	Students have options for support beyond the teacher: peers, themselves, representations.	“I think by embedding the routine of turn-and-talk, that when the teacher requires them or asks them to do it, it actually is facilitating more of a long-term skill development so that they know that the people around them are supports for them.”
Teacher	Evidence	Teachers actively listen to student discourse to check their understanding of mathematical concepts.	“[Mathematical discourse] is really helpful that they can explain because then it gives you a window into what they believe as well.”
Teacher	Access	Opportunities to talk about mathematics in their native language provides access to the	“With ELL students in particular that conversation's going on in Spanish. And so I'm excited that that's

		words, ideas, and methods that are being presented.	occurring, because then that's in the language that they can access the information."
Pose Purposeful Questions			
Student	Organize and articulate thinking	Putting mathematical thinking in order and relating that thinking aloud.	"I can't pull my own thoughts out of my mind like that."
Student	Mathematical identities	A student's mathematical confidence about solving problems and completing mathematical tasks.	"It was a 10th grade math, and so I was like, "Wow, I already know this," and I'm so speechless because I'm barely in sixth grade so it's four years away from here."
Teacher	Assess and advance thinking	Directing students toward understanding the learning goal. Assessing questions evaluate what students know. Advancing questions move students to extend what they know.	"Without that next question, I wouldn't have known how they knew that." "I think they respond really well. Trying to open up minds and get people to think about things that they've never thought or seen before."
Teacher	Ownership of learning	Student participation in and efficacy around problem solving.	"Using purposeful questions allows them to take ownership of their learning and them feel like they're the one that got there, as opposed to being told the answer."
Use and Connect Mathematical Representations			
Student	Options	Choices when solving problems.	"I think it's helpful because you have three different options to do it. If you don't understand one, then you could do it the other way. And you have other options."
Student	See the math	A concrete representation the helps understand the abstract mathematical ideas.	"I think one of the things that they could do to help me would be kind of having the actual thing. More hands-on, we could say."
Teacher	Independence	Students choose and utilize mathematical representations without relying on the teacher.	"This is showing me is that they then have a tool. And so they don't need me as much and that's the whole point, right?"
Teacher	Connections	Relationships between representations are explicit. Mathematical representations are related to real-life events.	"It's like lowering the affective filter for them, right? Because then they can see and they don't necessarily have to make the connection to the language piece. They can just use their eyes. They don't have to use their ears in terms of input."