



Quality box charts : a modern process monitoring tool
by Eric Damon Mott

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Industrial and Management Engineering
Montana State University
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Abstract:

Modern industry monitors its processes to guarantee quality of products and services. Control Charts are one of the most effective methods for process monitoring. Yet traditional control charts can be difficult to implement and interpret. Quality Box Charts, a generalized class of process monitors was developed to address these concerns. .

The present research builds upon previous Quality Box Chart results. Using statistical resampling techniques, assumptions that hampered previous research were no longer necessary. Control limit calculations were possible for a wide variety of applications.

Quality Box Charts offer increased flexibility along with applications that were not previously practical. In addition, Quality Box Charts are usually more sensitive in detecting shifts in the underlying process than traditional $\bar{x}/R$ charts. In-control run properties are also greater for the Quality Box Chart. Furthermore, Quality Box Charts are significantly easier to interpret than $\bar{x}/R$ charts for all levels of users.

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APPROVAL

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ABSTRACT

Modern industry monitors its processes to guarantee quality of products and services. Control Charts are one of the most effective methods for process monitoring. Yet traditional control charts can be difficult to implement and interpret. Quality Box Charts, a generalized class of process monitors was developed to address these concerns.

The present research builds upon previous Quality Box Chart results. Using statistical resampling techniques, assumptions that hampered previous research were no longer necessary. Control limit calculations were possible for a wide variety of applications.

Quality Box Charts offer increased flexibility along with applications that were not previously practical. In addition, Quality Box Charts are usually more sensitive in detecting shifts in the underlying process than traditional $\bar{x}/R$ charts. In-control run properties are also greater for the Quality Box Chart. Furthermore, Quality Box Charts are significantly easier to interpret than $\bar{x}/R$ charts for all levels of users.

CHAPTER 1

BUSINESS PROCESSES AND PROCESS MONITORING

Today's business environment is becoming less and less forgiving. Companies often competed on cost alone, but this is rarely possible now. World markets are integrating at higher and higher levels all the time. With the evolution of global markets, the world has witnessed the evolution of competitive position. Markets are now being fought with the weapons of cost, quality, and timing. If a company does not possess all three of these weapons in its arsenal, it may not survive. A central philosophy of this global, business evolution is quality. Quality profoundly affects cost and timing. It was believed for most of the industrial revolution that cost and quality were mutually exclusive concepts. Yet, low cost and high quality can and often do exist together. Nearly every industry has shown that proper attention to quality lowers cost and aids in the design and introduction of new products [Deming, 3]. One of the breakthrough ideas of quality and the modern industrial age is the Process Mentality which establishes every business function as a process.

A process is defined as a set of interrelated inputs that produce an output. A process is simply a set of actions and materials that join together to produce an outcome. Therefore one sees that every action taken in a business setting is a step in some process. No longer is it sufficient to consider singular actions being disjointed or unimportant because all functions are processes in a larger system. Processes follow one another to a final destination.

When an organization makes the fundamental step of recognizing its activities as a series of processes, several questions naturally begin to occur.

- What is the first step of a process?
- What is the last step of a process?
- Is the process operating in the optimum way and at the proper level?
- How can the process be monitored to provide the information necessary to answer questions about its performance?

The natural outcome of process definition is process improvement. Process improvement leads to higher quality which, when done properly, will reduce costs and improve the timeliness of operations.

In 1950, Dr. W. Edwards Deming presented the Process Mentality to the Japanese industrial leaders [Deming, 3]. He showed the leaders that raw material suppliers were just an earlier activity in the same process and the customer was the final step in the process. Every action from the earliest raw material source to the final customer is simply a series of processes that meld together. If any one step or process is neglected, all others suffer.

Deming reported, "Management learned about their responsibilities for improvement at every stage. Engineers learned theirs, and learned simple but powerful statistical methods by which to detect the existence of special (assignable) causes of variation, and that continual improvement of processes is essential . . . Once management in Japan adopted the chain reaction, everyone then from 1950 onward had one common aim, namely, quality." [Deming

Process Monitoring

Accepting the Process Mentality creates a paradigm shift. The model of disjoint activities is replaced by related processes. It is a natural reaction to define the process and to want to monitor it—the action of defining a process leads to improvement [Goetsch 8]. To understand how a process operates, it must be monitored. As it is monitored through time, it may be possible to discover trends or actions that greatly influence the performance of the process:

Motorola's Six Sigma Quality approach has been one of the most successful arguments for process monitoring. It claims that everything known about a process should be expressed as numbers. If it is not able to be expressed as numbers, then really not much is known about the process [Hahn, 9].

Fundamentally, there are two types of process monitoring. The first method monitors all inputs to the process. Inputs may be raw materials, people, actions, machinery, or similar categories. Deming believed that the important inputs were: people, materials, methods, equipment, and the environment [Roehm, 15]. Usually monitoring inputs is very complicated. Often, there are no metrics to measure the necessary qualities. Furthermore, there is a great deal of variation in each input and it is difficult to know what is appropriate and what is not. In addition, the process may be altered by the monitoring process itself as in the Hawthorne effect. In general, it is difficult and expensive to monitor the inputs to a process.

The second method of process monitoring monitors the outputs. In general, the process outputs are easier to measure. In fact, measuring can often be automated or process operators can perform the monitoring themselves. Relatively little cost is usually associated with monitoring the output. Most process monitoring involves go/no-go actions, visual go/no-go, or caliper measurements which are usually not time consuming. If output is monitored, then only limited measuring tasks are involved which should be simpler than monitoring multiple inputs to a system.

Furthermore, output monitoring is efficient for uncommon occurrences. For example, incoming raw materials are frequently sampled to guard against receiving a poor batch. Receiving inspection can often be replaced by proper process output monitoring. Output monitoring is sensitive to raw material failure. Even if value has been added to the product before it is discovered to be defective, the value of the lost product will probably be less than the cost of the continuous receiving inspection.

In addition, process monitoring is useful for detecting when and where problems occur in the process. For example, if a piece of equipment begins to malfunction or if an employee is doing something to affect the output quality, process monitoring of the output will indicate a problem.

The Politics of Variation

When variation is against business goals of an organization, the politics of variation become a factor. If quality is indeed a business priority, then the language of variation must become a common language of the organization.

Every process is variable. Process variation is universally categorized as one of two types. The first type is often called special or removable variation. Special cause variation is the variation in the process that is attributable to things not working or being done properly. Therefore, improper machine settings, power spikes, operator error, poor raw materials, machine malfunction, all are considered special causes of variations. Special causes of variation are considered acts of nature or uninformed actions. Special causes are often singular or cyclical occurrences and are not always present. Special causes generally cause large spikes in the quality of the process output.

The second type of variation is common cause variation. Common cause variation is a result of uncontrollable, natural deviations in the environment, machinery or personnel. Common cause variation is always present in a system and is not a singular or spike occurrence.

A key tenant of Dr. Deming's work is special causes do not occur regularly and when they do occur, they can usually be eliminated by supervisory level personnel [Moen, 12]. Yet, if the variation is caused by a common cause source of variation, the fault is inherent in the system itself. Only management has the ability to change the system. Therefore, it is directly the responsibility of management to improve the system and systematically eliminate the variation of a process—everyone “trying harder” will not affect the variation of the system. In his career, Deming estimated that 94% of all variation he saw was attributable to common cause variation, that is only 6% of the variation is attributable to special causes [Deming, 3]. There is a subtle undercurrent here that the workers who are usually blamed for poor quality really have no control over it and managers who have the control do not believe it is their

responsibility. Neither workers or management can distinguish between the types of variation if the process is not being monitored properly.

Attributes of a Process Monitor

The development of a process mentality, output monitoring, and the politics of variation leads one to question how a process monitor should function. The ideal process monitor indicates a special cause of variation as soon as the special cause influences the process. For example, if a batch of defective raw material was introduced into a manufacturing process, the ideal process monitor would detect the first unit affected by the defective raw material. In addition, the ideal process monitor would never make a mistake, that is, every time it indicated a problem there would indeed be a problem.

Yet, since a process contains variation, a monitor can be fooled. If the process produces output such that common cause variation looks like special cause variation, the monitor will be fooled. The fooling of a process monitor in this way is known as a *false alarm*. A false alarm occurs when the process monitor signals the influence of a special cause of variation when one is not present. In scientific terminology this is a false positive. The ideal process monitor would never suffer from a false alarm; it would have the power of perfect discernment.

A second problem faced in practice is where special cause variation affects the process and the monitor fails to detect the problem. This can be called a *late alarm* or in scientific terminology, false negative. An ideal process monitor would not have late alarms.

A third issue is that an ideal process monitor would be easy enough to use that all of the necessary personnel could use it with little or no training. In today's world, there are many capable process monitors available, yet many of these are so complicated that only professionals can gain relevant information from them which is not usually the optimum situation. Therefore, every process monitor must be evaluated on at least three characteristics: False alarm rate, sensitivity or late alarm rate, and ease of use.

Therefore, the goal of this research was to develop an accurate process monitoring tool which is more accurately stated:

Research Objective

To develop a generalized class of distribution free, variable control charts that are easy to construct and simple to interpret.

The present research develops a new generalized family of Quality Box Charts (QBC) from the singular case of White and Schroeder [White, 22]. Quality Box Charts are process monitoring tools that are easy to use and simple to interpret. QBC's also have better false alarm discriminate power than standard process monitors. In addition, QBC's are generally more sensitive with fewer late alarms than more traditional process monitors. Most important, QBC's are a general family that can be adapted to specialized needs of users. The present research greatly extends the ideas previously presented and uses Monte Carlo simulation to overcome problems encountered by previous researchers of Quality Box Charts.

Quality Box Charts are able to be used by all members of an organization in monitoring important aspects of the business process in an effort to gain an advantage with quality, productivity, cost, and timeliness. Quality Box Charts can be an important tool in today's business environment.

CHAPTER 2

QUALITY BOX CHARTS

Quality Box Charts are process monitors which are simple to implement and use. Quality Box Charts (QBC) are output monitors based upon the most common type of process monitors known as control charts. QBC's are developed by considering the general form of all control charts followed by an introduction to the boxplot. The generalized QBC family is also developed in this chapter.

Control Charts

In the 1920's Walter Shewhart of Bell Laboratories developed a method of process monitoring known as control charting [Shewhart, 17]. Control charting allows operators to distinguish common cause variation from special cause variation by using statistics and probability models. Since it helps to identify when special causes of variation are present, control charting generally provides superior process monitoring compared to other methods. Traditional methods such as operator experience or intuition usually increase the process variation.

Many types of control charts exist, yet all of the charts have basic properties relating back to the first charts developed by Shewhart. Control charts are constructed by measuring a quantity of interest and plotting the value on a time plot. The natural variation the process

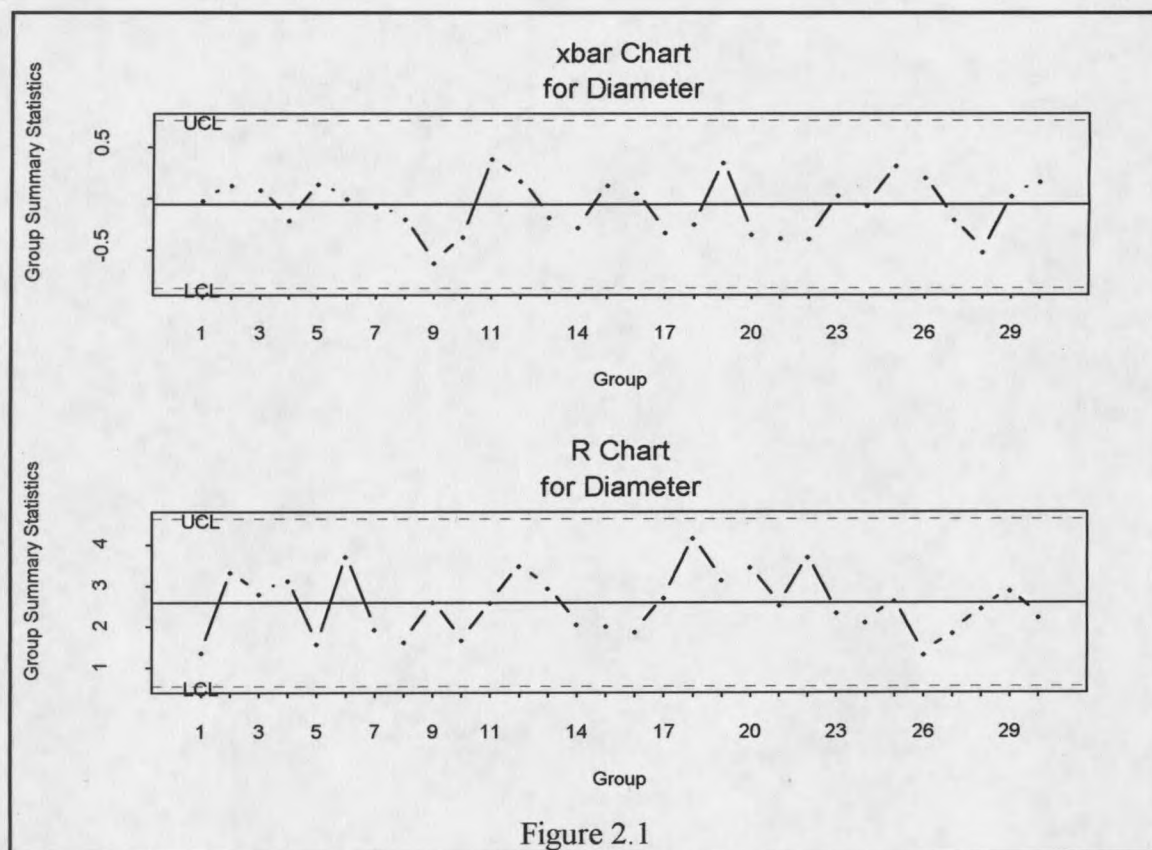


Figure 2.1

is expected to exhibit is indicated by upper and lower lines called control limits and the quantity of interest is expected to lie between these control limit values. If the measured quantity goes outside of the control limit values, a special cause of variation may be present.

The original control chart developed by Walter Shewhart is called an $\bar{x}/R$ ($\bar{x}$ -bar/range) chart. It consists of two plots used simultaneously—one for the sample mean and another for the sample range. The example shown in Figure 2.1 of an $\bar{x}/R$ chart for needle diameters from a medical device manufacturer. Each point of the top plot is the average needle diameter from a random sample of needles while the range of each sample is plotted in the bottom chart. The random variation due to common cause variation is shown by the up and down bounce of the points. The control limits are also present to show the presence of special causes of variation.

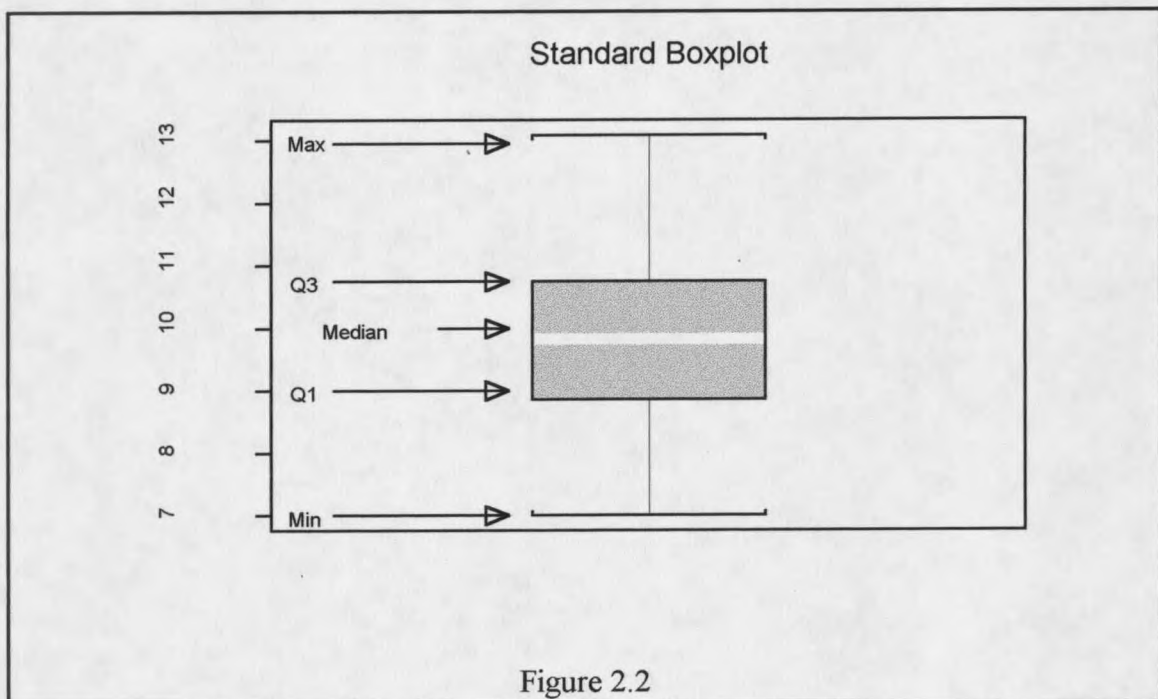
To construct an $\bar{x}/R$ chart, it is necessary to calculate the mean and range of each sample and plot each one. More important, the control limits required knowing the statistical distribution of the sample average. The control limits are key to understanding the difference between common and special cause variation—their placement is important. Control limits cannot be estimated. How would one know where to set them? Experience may help but is not sufficient. Probability theory is used to set control limits.

QBC's are Shewhart control charts where a boxplot is used to characterize the sample output rather than the mean or range. QBC's require only one graph rather than two as with the $\bar{x}/R$ chart because the sample center, range, and other information are conveyed simply in the boxplots. In addition, QBC's do not need to make assumptions about the process distribution or the distributions of the statistics. Because of this, QBC's use different methods to calculate the control limits (Chapter 3). Quality Box Charts are based on the box plot developed by [Tukey, 20].

The Boxplot

Boxplots are a standard method of examining data. Since their introduction [Tukey, 19], they have been incorporated into nearly all statistical software packages and most textbooks. Several modifications have been suggested, but few offer the ease of use given by the original design [McGill, 11; Benjamini, 1]. Boxplots are most informative when placed side-by-side in a single graph. Side-by-side orientation illustrates both within sample information as well as between sample information.

A boxplot is a graphical representation of a data set's minimum, maximum, and middle values along with its first and third quartiles. The quartiles are found by simply finding the middle value of both the upper and lower halves of the data set. The five numbers plotted by the boxplot are collectively called the statistical five number summary of a data set.



One advantage of using boxplots is their ease of interpretation. Because quartiles calculated by finding the middle of each sample half, they represent the quarter points of the ordered data ($1/2 * 1/2 = 1/4$). In addition, the median is the middle value of the data. The median can also be called Q_2 since it is the second quarter. The boxplot has the property that one-half of the data is represented by the box and 25% of the data is on each side of the box. A boxplot divides every sample into quarters. The distance that each quarter segment fills

indicates its density, that is, each segment always contains 25% of the data but may be spread out or tightly packed.

When used side-by-side, boxplots provide a large amount of information. Other than providing the five number summary values: minimum, maximum, median, and quartiles, boxplots present the range, the inter-quartile range and the mean, if desired. Boxplots also track skewness, kurtosis, trend, and within and between sample variation. Simple modifications to the boxplot allow confidence intervals and the tracking of any percentile of interest.

Even though boxplots experienced such widespread use, it took a long time for researchers to apply them to control charts. In standard control charting, all of the information in a sample is collapsed down to a single number such as the mean. Boxplots allow users to retain the original structure of the data as well as having statistically viable summaries.

Quality Box Charts

Quality Box Charts were first examined by White and Schroeder [White, 22]. White and Schroeder introduced the topic of QBC's by using the five number summary boxplot with normally distributed data. Their QBC was simply a Shewhart median chart using a boxplot median instead of a singular point. Other than using resistant measures, White and Schroeder did not show the QBC was robust to distributional assumptions. The first serious consideration of the QBC's was given by Iglewicz and Hoaglin [10]. They developed the traditional statistical theory justifying the box chart as a viable method of control charting; even though their presentation dealt with "process evaluation." Nagasawa and Ootaki [15]

consider box plot charts with independent upper and lower control limits based upon a normally distributed process. The present research builds on these researchers' work, yet it varies in its scope, monitoring approach, implications, and calculation methods. All previous research was based on the explicit assumption of a normally distributed process which is often infeasible [Shainin, 18].

Quality Box Chart Interpretation

The interpretation of QBC's is very simple. If the quantity of interest (e.g., the median) crosses the control limit, the process may be out of control, that is, special causes of variation may be present. A process is said to be in statistical control when only common cause sources of variation are present. More formally,

Quality Box Chart Run Rule: Let θ_x be the statistic of interest for which the Lower Control Limit (LCL) or Upper Control Limit (UCL) was calculated. The process is not in statistical control whenever

$$\theta_x > UCL \quad \text{or}$$

$$\theta_x < LCL$$

otherwise, the process is in statistical control.

Interpretation of the QBC is easy with this single rule. As long as the statistic(s) that is(are) being monitored stay(s) within the control limit, the process is in statistical control.

Quality Box Chart Construction

Two issues are involved in the construction of QBC's. The first issue is the construction of the boxplot itself. The boxplot can be constructed using either available

software, or it can be quickly created and sketched by hand. The sample values must only be placed in order and the appropriate five numbers (minimum, maximum, median, quartile₁, and quartile₃) chosen from the list. The five numbers are simply graphed as horizontal lines with the vertical lines added to the middle three vertical lines to construct a box. It is customary to connect the upper and lower points by a horizontal line.

Example: Consider the following process sample: 67, 56, 42, 69, 56, 66, 61, 64, 64

Step 1: Order the sample 42, 56, 56, 61, 64, 64, 66, 67, 69

Step 2: Find min and max values 42, 69

Step 3: Find the median 64

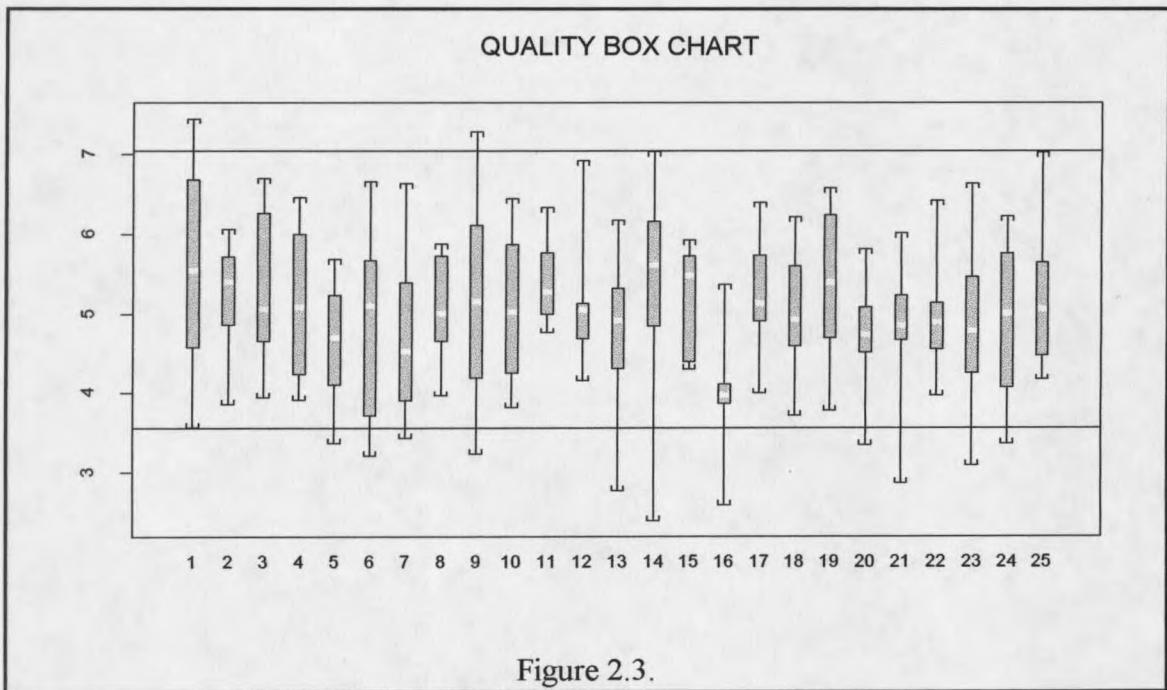
Step 4: Find the quartiles 56, 66

Now the boxplot can be drawn from these values

The second issue involved in QBC construction is the calculation of the probability control limits. The calculation method is discussed in Chapter 3. Once the control limits are calculated, they are added to the side-by-side boxplot graph to make a Shewhart style Quality Box Chart.

The Generalized Family of Quality Box Charts

The present form of the Quality Box Chart is a definite extension of the charts discussed by previous researchers. Two distinct differences exist between the previous methods and the present QBC method. First, the present method does not make distributional assumptions concerning the underlying process (discussed in Chapter 3). Secondly, the present method develops a generalized family of control charts, which are not limited by only considering the



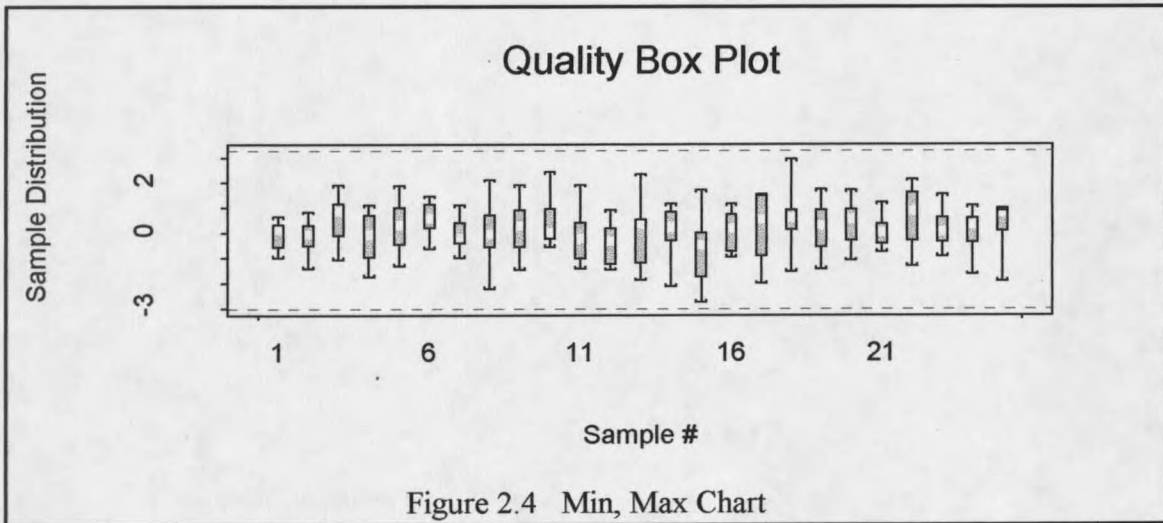
measure of location like means, median, and trimmed means as previous research has.

The present research extends previous treatments by developing methods where attention is not focused only upon the measure of location, but rather upon the variation of each point in the sample. If one wishes to concentrate upon the central tendency of the process then the generalized class of QBC's simplifies to the previous research. Yet, if more refined or more specialized applications are needed, the present research presents the generalized family of control charts to accommodate the need. The interpretation of the chart is still the same for the present research: If the quantity of interest crosses the control limit, special cause variation may be present.

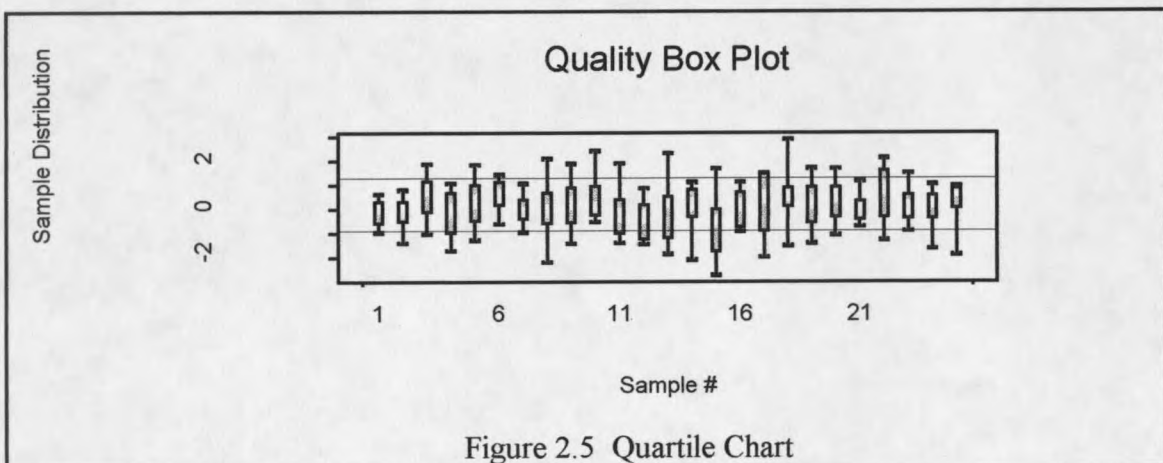
Furthermore, the generalized family is able to be applied in situations where the process is known to have serious distributional challenges such as skewness or kurtosis. The previous research attempts to show that the earlier methods are robust due to the normal distribution

assumptions. The present method is empirically accurate regardless of the continuous distribution used.

As specified before, a control chart is composed of a statistic measuring a quantity of



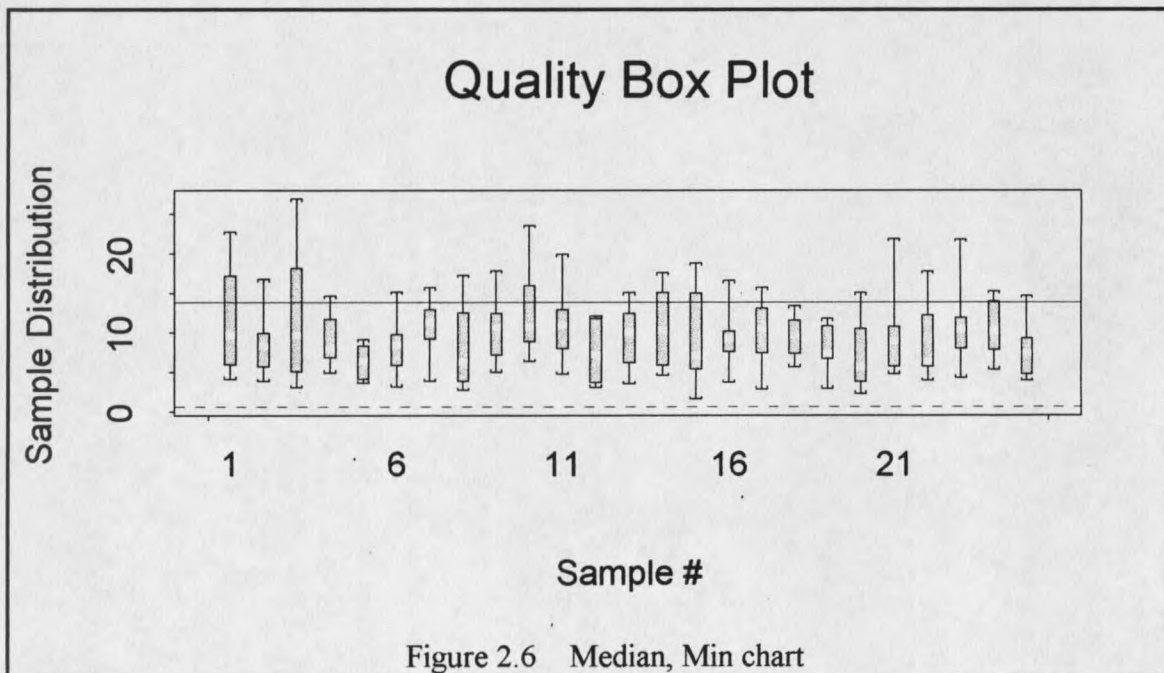
interest and control limits for the quantity of interest determined probabilistically. In the generalized family of QBC's, one-sided control limits are used. The one-sided control limit is a logical choice for boxplot quantiles since their direction is always relative to another



quantity. For example, the median is always above (or equal to) the Q1 value and the maximum value is always equal to or greater than the minimum value.

QBC's are able to be constructed for any combination of boxplot quantities. The combination that possibly has the most promise for modern quality applications is the (min, max) QBC. The (min, max) QBC indicates the total performance of the process (Figure 2.5). It sets control limits on the extreme values. $\bar{x}/R$ charts can have one or more extreme values and still not indicate problems. The (min, max) QBC does not allow this—it is very sensitive to extreme values while still tracking the mean, median, and range.

The (median, median) QBC is the simple case used in previous studies. The (median, median) control chart closely tracks the central tendency while not being sensitive to extreme values. The (median, median) chart behaves exactly like a traditional control chart, except it uses a boxplot median in place of a single point median. Sometimes, non-symmetric charts



may be of interest. For example, a gasket manufacturer is monitoring a new manufacturing process. The process manufactures gaskets for high temperature applications such as found in a submarine nuclear reactor core. The thickness of the gasket is critical for proper fit, but it is known that there is a substantially greater risk of failure due to thin gaskets rather than thick gaskets. In this case, the (min, median) QBC might be appropriate. It allows the engineers to monitor the gasket process with emphasis upon the thinner products as well as the average behavior of the process. The non-symmetric charting is something that is not available in previous methods, yet it is straightforward with the generalized family of QBC's.

The generalized QBC's can be extended even one step further. Not only is it possible to consider any combination of the boxplot quantities, but it is even possible to consider any *percentile*. In general, the p^{th} percentile of a distribution is the value with p percent of the observations falling at or below it. Thus, the median is the 50th percentile while Q_1 is the 25th percentile and Q_3 is the 75th. Boxplots use these values, but QBC's are able to capitalize on a generalized boxplot which is based upon percentiles rather than quartiles [Esty, 7]. Therefore, it is possible to monitor any level of the distribution that is of particular interest. For example, Queko Industries is a subcontractor on military contracts. The military will not accept any product that has more than 5% of the units outside of specified dimensions. Therefore, Queko Industries is very interested in monitoring the performance of the 5th and 95th percentile of its production. A 5th and 95th percentile QBC could be used to monitor the process. Traditional process monitors do not allow this level of flexibility. The example raises other issues related to process capability that are beyond the scope of the discussion here. The example is used simply as an illustration of the flexibility of the generalized family of QBC's.

CHAPTER 3

STATISTICS, BOOTSTRAPPING AND CONTROL LIMITS

The development of the Quality Box Charts involves two fundamental differences from traditional Shewhart control charts. The first difference is the use of Boxplots to display sample information instead of single summary points. The second way in which QBC's differ from traditional control charts is how the control limits are calculated. The generalized family of QBC's uses control limits calculated from empirically bootstrapped sampling distributions. Control limit calculation and related statistical concepts will be discussed here.

In general, Shewhart control limits are probability statements that are direct corollaries of statistical hypothesis tests and confidence intervals. Because of their close relationship, it is necessary to first discuss fundamental statistical concepts

Statistics

In situations where random outcomes are possible, the mathematical theory of probability is useful. The theory of statistics uses probability theory to infer information based upon limited information. In general, the random quantity of any activity is called a Random

Variable. A random variable can be defined as a variable whose value is a numerical outcome of a random phenomenon. [Moore, 14]. More formally:

DEFINITION 3.1: A *random variable* is a function from a sample space S into the real numbers where S is the set of all possible outcomes of a particular experiment.

For example, the number of heads appearing in five flips of a coin is a random variable. Similarly, the number of green M&M candies one randomly picks out of a handful is a random variable. It is not known what the number will be before the handful is chosen. The number of green M&M's may vary with every handful. The simple idea of a random variable is extended to many situations.

Every random variable has an associated distribution. The distribution of a random variable is (1) the set of all possible values that the random variable can assume and (2) the associated probabilities related to each point. Although a definition of probability is not developed here, it is often helpful to consider probability as synonymous with frequency. The frequency idea of probability can be expressed as:

$$Probability(event) = \frac{\text{Number of Successful Outcomes}}{\text{Possible Outcomes}}$$

Therefore, using the frequency illustration, the probability of a flipped coin landing with the head side up is = Number of Successful Outcomes/Possible Outcomes = $\frac{1}{2}$ = 50%. In general, with random variables that assume continuous real numbers, one does not consider the probability of a point, but rather the probability for a set of points. The distribution of every

random variable has two forms: the cumulative distribution function (cdf) and the probability distribution function (pdf).

DEFINITION 3.2: The cumulative distribution function (cdf) of a random variable X , $F_X(x)$, is defined by

$$F_X(x) = P(X \leq x), \text{ for all } x \text{ and where } P(\cdot) \text{ is a probability function}$$

The cdf of a random variable is simply the summation of the probabilities of a random variable for each possible value from the region of interest.

The second form of a random variable's distribution is the probability density function.

It is the form that is most often graphed to show the behavior of the random variables.

DEFINITION 3.3: The probability density function (pdf), $f_X(x)$ of a continuous random variable X is the function that satisfies

$$F_X(x) = \int f_X(t)dt \text{ for all } x, \text{ where the integration is from } -\infty \text{ to } x$$

The cdf is the integral of the pdf or rather, the pdf is the derivative of the cdf. A pdf for a continuous variable is plotted in Figure 3.1.

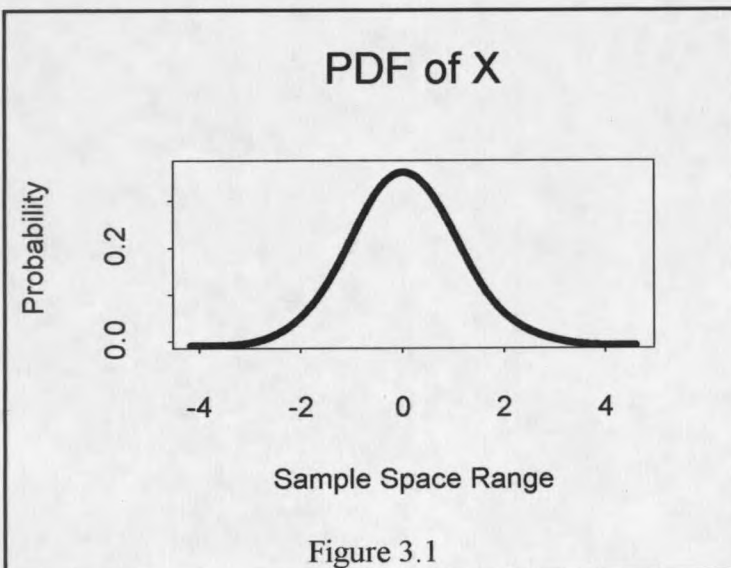
Using the fundamental theorem of calculus, the area (a,b) under a pdf curve is the integral of the pdf function evaluated at the limits of integration, $F(b) - F(a)$. Therefore by definition 3.2, the probability of an event x such that $(a < x < b)$ is the area under the pdf curve of x , $f(x)$ can be written in terms of the cumulative density function of X ,

$$P(a < x < b) = \int f(x)dx = F(b) - F(a)$$

For example, let the random variable, X be the height of a region of an injection molded part, then the probability that the surface smoothness is between ± 2 thousandths of an inch is the area under the appropriate density curve between -2 and 2 . Mathematically, this can be defined as the integral of the pdf from -2 to 2 . For the most common types of distributions, tables, calculators, and computers can be used to make calculations. The probability of a set of actions is simply the area under the proper region of the appropriate curve.

Random variables, distributions and probabilities introduce statistics. In their most basic sense, statistics are functions of data where data come from experiments that involve randomness. Calculations are made from the data such as the mean, range, minimum value, maximum value, mode, standard deviation, or proportion. The calculations made from the data are called statistics.

DEFINITION 3.4: Let $X_1, \dots, X_n$ be a random sample of size n from a population and let $T(x_1, \dots, x_n)$ be a function whose domain includes the sample space of $(X_1, \dots, X_n)$. Then $Y = T(X_1, \dots, X_n)$ is called a **statistic** and is Y is a random variable.



First, notice that any function of sample data is a statistic. Second, every statistic is itself a random variable. If a statistic is a random variable, then it has its own distribution with a

suitable range of values and associated probabilities. The distribution of a statistic is given a special name to distinguish it from distributions of other random variables, it is known as a *Sampling Distribution*. The implication is every time an experiment is performed and a statistic is calculated a different result is possible. Consider a simple example concerning heights of human subjects: A sample of people are measured for stature and if the average height is the quantity of interest, then the sample mean is an appropriate statistic. Now, if the experiment is repeated, it is very possible (even expected) that a different sample average mean would be calculated. The sample mean, $\bar{x} = 1/n * \sum x_i$ is a random variable and has variation. The variation gives $\bar{x}$ its own distribution. It is the variation of $\bar{x}$ that Shewhart took advantage of to develop the original control charts.

Control Limits as Probability Statements

In process monitoring, a statistic is chosen to summarize a characteristic of interest. The statistic is a random variable and will therefore indicate the variability in the underlying process. If the process is assumed constant, then the statistic will behave in a prescribed way based upon its sampling distribution. The true characteristic of interest, the process average for example, can only be determined if the entire population of data values are collected and the statistic of interest is calculated for the entire data population. Unfortunately, this is rarely possible. Therefore, the statistic varies because each time it is calculated, the statistic is implemented for a different subset of the data population.

Once, the distribution of the statistic is known, the behavior of the statistic is well defined. Shewhart showed that it is possible to determine if the variation in a control chart is

due simply to the variation of the statistic or whether it was due to a non-stable underlying process [Shewhart, 17]. Shewhart was able to distinguish between common and special causes of variation because variation due to the distribution of the statistic is common cause variation while special cause variation is due to a non-stable underlying process.

The key to the difference between common and special cause sources of variation is the amount of variation present. The statistic of interest has a well-defined behavior. Whenever a statistic's variation increases beyond the control limits, it signals an unstable process because the control limits are specifically chosen such that the probability of seeing a statistic more extreme than the control limit is nearly zero. Therefore, if a statistic is exhibiting behavior more extreme than the control limit, it is either a very rare event or the underlying process has become unstable. The control limits are directly related to the variation of the distribution of the statistic creating a tolerance for the statistic.

Shewhart indicated that control limits should be set in the following way:

$$\theta \pm k\sigma_{\theta}$$

where

θ is the statistic of interest

σ_{θ} is the standard deviation for the distribution of θ .

k is a factor (usually chosen to be 3)

The k value in the control limit calculation makes control limits probability statements causing the upper and lower limits to become a range in the distribution of θ . Because the

control limits define a range in the statistics sampling distribution, they define an area of the statistic's pdf curve. The area under the pdf curve is defined to be the probability of a range of events. Therefore control limits are probability statements.

It is noted that the control limit formula is of the same form as statistical confidence intervals. Confidence intervals can be used to test the hypothesis of certain events [Moore, 14]. Since confidence intervals are probability based, the null hypothesis is rejected if the calculated statistic falls outside the confidence interval. Based upon the assumption of independent samples in process monitoring, control limits can be used to test hypotheses about the statistic of interest, θ .

Example: It is believed that a certain process produces less than 5% defects. A sample from the process is taken and a 95% confidence interval of defect percentage is calculated to be (5.6, 9.9). The null hypothesis $H_0: \mu = 5\%$ must be rejected. The mean number of defects is thought to be significantly higher than 5%. The hypothesis must be rejected because the "best" estimate of μ available is $\bar{x}$. Even though $\bar{x}$ has variation, one can feel confident that the variation exhibited here is too great to be due to chance alone. Since the confidence interval for $\bar{x}$, (5.6, 9.9) did not include 5, the hypothesis is rejected.

In an analogous manner, every time a sample statistic is plotted inside of the control lines of a control chart, one can believe that the hypothesis of a stable underlying process cannot be rejected.

The Statistic of Interest

Process monitoring is concerned with a variety of characteristics. Some processes are concerned with the minimum value of a sample, the maximum value of a sample, or perhaps the average behavior of a sample. It is even common to be concerned with the range of each sample. Each of these characteristics is estimated by a statistic or in some cases one of many statistics.

Every process is concerned about its accuracy and its precision. Accuracy relates to the ability of a process to operate at a specific level. Precision is the ability of a process to be repeatable. Accuracy and precision have been likened to a hunter. The hunter may hit the center of the target--he or she is accurate. Also, the hunter may hit the exact same spot repeatedly, even though it may not be the spot that he or she is aiming for. The hunter is precise, but may not be accurate. Although two separate qualities, accuracy and precision are not mutually exclusive. It is the goal of process monitoring to help establish both simultaneously.

Because of the need to focus on both precision and accuracy, it has historically been common to use the sample mean to monitor accuracy and the sample range to monitor the process precision. This combination is called the $\bar{x}/R$ chart and was the original monitoring method proposed by Dr. Shewhart.

Even though the sample mean and range were the first statistics used to track process characteristics, they are the only ones. Statisticians have spent many years developing

alternatives. One family of alternatives is called the order statistics and will be mentioned later.

Regardless of the statistics used, it is necessary to know the statistic's sampling distribution. Recall that the distribution completely defines the behavior of the statistic. The distribution indicates what values of the statistic are possible and how likely it is to see each one of those values. If the distribution of a statistic is unknown, that statistic is not able to be used for process monitoring. The behavior of the statistic must be known in order to separate its variation from that of the process—a necessary step in identifying common cause variation.

Statistical theory often helps in identifying a statistic's distribution. Statistical theory says that the distribution of the underlying process must be known to determine the behavior of the statistic itself. Therefore, it is necessary to know one distribution to determine the other. Even still, if the distribution of the process can be determined, it is can be complicated or even impossible to determine the distribution of the statistic. The sample mean, $\bar{x}$ is one exception to this rule.

The statistic $\bar{x}$ has a useful relationship known as the Central Limit Theorem. The central limit theorem allows the distribution of $\bar{x}$ to be determined (approximated) without knowing the underlying process distribution.

Theorem 3.1: If $X_1, X_2, \dots, X_n$ comprise a random sample arising from the random variable X possessing mean μ and finite variance σ^2 , then as n tends to infinity, the probability distribution function (pdf) of $\bar{x}$ approaches the probability distribution of a normally distributed random variable.

The Central Limit Theorem indicates that the distribution of any $\bar{x}$ is approximately normally distributed and the approximation improves as the sample size used to calculate $\bar{x}$ increases.

Quality Box Charts are a general class of process monitors and do not necessarily use the sample mean, $\bar{x}$. QBC's are based upon the boxplot. Because statistics are any function of the sample data, the maximum, minimum, median, and quartiles are statistics. What then are the distributions of boxplot statistics? To answer this question, it is necessary to consider the theory of order statistics.

Order Statistics

A special category of statistics called *Order Statistics* has been very helpful in data analysis. Order statistics provide the basis for the boxplot and therefore the Quality Box Charts. Order statistics are simply the sample data sorted from smallest to largest. The various ordered sample points are used to estimate the parallel population points. For example, if the data population minimum was of interest, it is intuitive to use the minimum sample value.

DEFINITION 3.4: The order statistics of a random sample $X_1, X_2, \dots, X_n$ are the sample values placed in ascending order. The order statistics are denoted by $X_{(1)}, X_{(2)}, \dots, X_{(n)}$. In particular the order statistics are random variables that satisfy $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$.

Thus the order statistics are estimated in the following way from a data sample:

$$X_{(1)} = \min X_i$$

$$X_{(2)} = 2^{\text{nd}} \text{ smallest } X_i$$

$$\vdots$$

$$X_{(n)} = \max X_i$$

Using the order statistics as defined, the distribution of any order statistic is given by the theorem 3.2 [Casella, 2]. Furthermore, the joint distribution of order statistics is needed to monitor the quantity of interest. Casella and Berger give results for the joint distribution for any *two* order statistics. Also, they include a generalized expression to calculate the joint distribution for *any* number of order statistics.

THEOREM 3.2: Let $X_{(1)}, \dots, X_{(n)}$ denote the order statistics of a random sample, $X_1, \dots, X_n$, from a continuous population with cdf $F_X(x)$ and pdf $f_X(x)$. Then the pdf of $X_{(j)}$ is

$$f_{X_{(j)}}(x) = \frac{n!}{(j-1)!(n-j)!} f_X(x) [F_X(x)]^{j-1} [1-F_X(x)]^{n-j}$$

Notice that Theorem 3.2 depends completely upon the existence of the distribution of the underlying process. The pdf and cdf must be known to calculate the distribution of the statistic and without the distribution of a statistic, it cannot be effectively used for process monitoring.

Determining the Process Distribution

The underlying process distribution must be known in order to determine the distribution of a statistic used in monitoring the process. Traditionally two approaches have been taken to determine the underlying process distribution.

One approach has been to use statistical techniques to estimate the distribution. Once a suitable distribution could be approximated, mathematical theory can often be used to establish the distribution of the statistic. This method is the most accurate and mathematically defensible approach when it works. A problem with this approach is it often is so difficult that results become impossible for many to determine. A second problem with this method is the underlying process often changes as a function of time—especially with an imprecise process. Therefore, the tedious and complicated analysis may be repeated. In practice, the approach is accurate, but challenging and possibly expensive.

The second traditional approach to finding the underlying process distribution has been to assume that the process is normally or near normally distributed and to focus attention upon $\bar{x}$ as a statistic. In this way, even if the distribution was not normally distributed, the central limit theorem states that given enough data, the distribution of $\bar{x}$ will itself be (approximately) normally distributed. The second approach has been effective in practice, but does have the serious drawback that attention is limited to the mean, $\bar{x}$. Casalla and Berger (1990) comment about the use of $\bar{x}$ and the central limit theorem, “While reveling in the wonder of the [Central Limit Theorem], it is also useful to reflect on its limitations. Although it gives us a useful

general approximation, we have no way of knowing how good this approximation is. In fact, the goodness of the approximation is a function of the original distribution..."

The history of process monitoring has been shaped by these two constraining approaches. The mathematical method is accurate, but is difficult enough that it does not always yield results. On the other hand, the central limit theorem is helpful, but forces the sample mean to be used. The three previous attempts at Quality Box Charts took the traditional mathematical approach. Each research group made the assumption that the underlying process behaved as a normal distribution. The assumption of a normally distributed process gave them various mathematical properties to allow the research to continue, yet each result depends upon the initial assumption. Even though Iglewicz and Hoaglin spend time examining the robustness of their method to violations of the normality assumption, it would be better if no prior assumptions or limitations were necessary. The present research is unique because results were obtained without making any distributional assumptions about the underlying process. The present research introduces the statistical theory of bootstrapping to Quality Box Charts. Bootstrapping is a modern approach that does not depend upon distributional assumptions. It is applied in this research to determine the control limits as probability statements without distributional assumptions.

The Theory of Bootstrapping

The previous discussion indicated that in process monitoring, it is necessary to choose a statistic to estimate a characteristic of interest. Having chosen a particular statistic, it is of

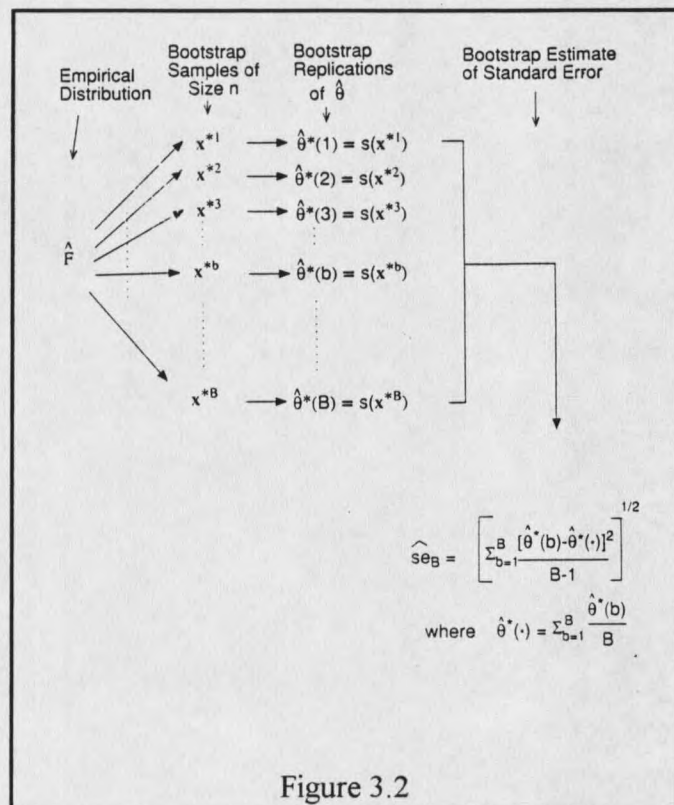
interest to know how the statistic behaves. The behavior of a statistic is completely determined by its sampling distribution.

Bootstrapping is a modern computer-based technique to empirically determine the distribution of any statistic. **Bootstrapping is a generalized central limit theorem which works for all statistics.** It does not specify the form of the distribution (e.g. normal, poison, exponential); yet it does create the distribution allowing probability calculations.

Bootstrapping is an intuitive approximation approach. What makes bootstrapping novel is that it requires a large amount of computer power to make accurate approximations. Because of this, bootstrapping's development has paralleled the development of modern computers and has therefore only recently become practical.

Bootstrapping functions as follows: Consider a characteristic of

interest, call it θ . θ is unknown and unable to be known unless the entire population of data $\mathbf{X}=(X_1, X_2, \dots, X_n)$ from $f_X(X)$ is sampled, where f_X is the probability density function (pdf) of $\mathbf{X}$. Therefore, θ is estimated by calculating the statistic $\theta^{\text{hat}} = s(\mathbf{x})$; $\mathbf{x}=(x_1, x_2, \dots, x_n)$. An approximate empirical



distribution, f_x^{hat} which puts probability of $1/n$ on each sample point is formed from the sample.

A bootstrap sample, $\mathbf{x}^*$ is a random sample of size n drawn with replacement from f_x^{hat} , therefore $\mathbf{x}^* = (x_1^*, x_2^*, \dots, x_n^*)$ where x_1^* is simply the first data point chosen at random from $\mathbf{x}$; x_2^* is the second point randomly chosen from $\mathbf{x}$ and so on. It is important to note that $\mathbf{x}^*$ may consist of the identical point from $\mathbf{x}$ which has been randomly chosen (and rechosen) several times. $\mathbf{x}^*$ may also completely lack other points from $\mathbf{x}$. The bootstrap sampling is repeated B times giving $\mathbf{x}_1^*, \mathbf{x}_2^*, \dots, \mathbf{x}_B^*$.

For each of the B bootstrap samples, the statistic is calculated, $\theta^{\text{hat}*} = s(\mathbf{x}^*)$, where $s(\mathbf{x}^*)$ is the same function applied to $\mathbf{x}^*$ that was applied to $\mathbf{x}$. Each time the statistic is calculated from a

bootstrap sample, a realization of the statistic's behavior is given. With enough realizations, the behavior becomes well defined, i.e., the distribution is

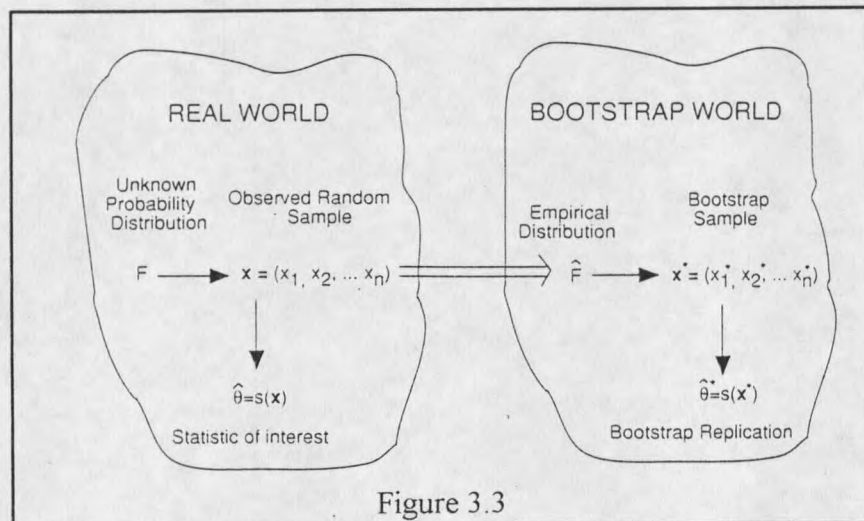


Figure 3.3

accurately approximated. It can be shown that there is a total of $\binom{2n-1}{n} = (2n-1)!/(n!(n-1)!)$ possible bootstrap samples. The number of bootstrap samples is very large even for small samples. For example a sample size $n=10$, yields 92,378 possible bootstrap samples. Because

of the large number of possible samples, the empirical distribution is usually not fully estimated.

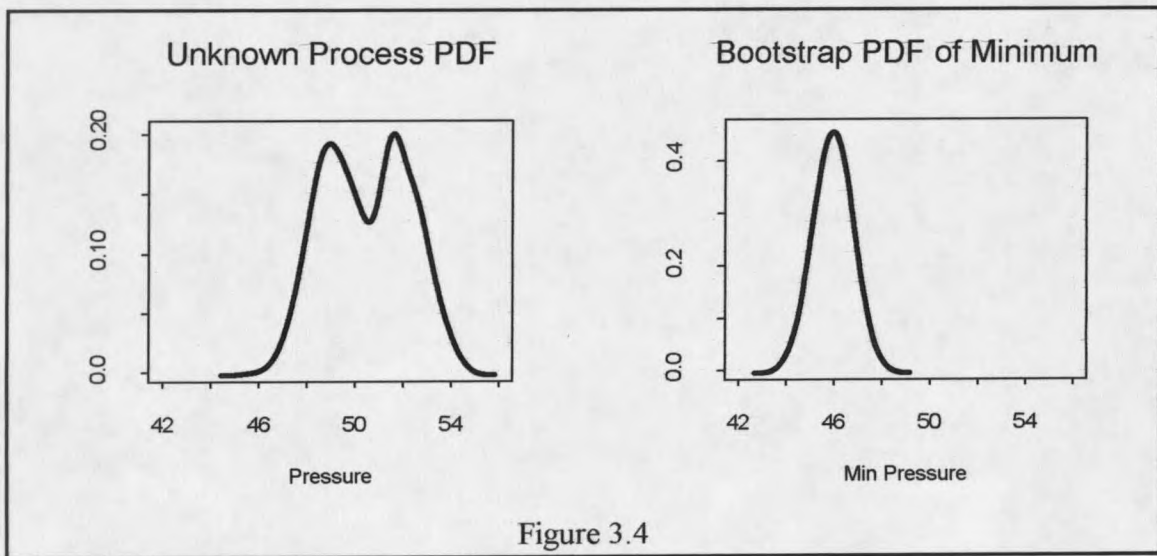
Bootstrapping with the Quality Box Chart

With the power of bootstrapping available, it is possible to generalize the Quality Box Chart to a full family of process monitors which are not restricted by distributional assumptions. Any statistic of interest may be chosen to calculate an appropriate control limit(s) for that statistic by bootstrapping.

The actual implementation of bootstrapping for the QBC's was written in the Splus language. Splus is an object-oriented language that has very powerful statistical and graphical abilities. The code that was used is included in the Appendix B. In general, the code consists of four modules (functions). The first function is a general driver and user-interface. The second module uses bootstrapping to calculate the control limits. Module three provides the graphical functions to actually draw the QBC's. The forth function calculates average run lengths and probabilities.

To illustrate the power of bootstrapping, consider the following situation:

A manufacturer is concerned about the minimum flow rate through their pumps. It is desired to monitor the process to determine if the pumps are being manufactured properly. Data have been collected for some time, but it is not known how to calculate the control limits for the minimum pump flow. Two hundred fifty bootstrap samples are taken from the available data. The minimum value $x^*_{(1)}$ is taken for each sample. The distribution of the $x^*_{(1)}$ values provides an approximation to the real



distribution $f_{x(1)}$ from the unknown process distribution f_X . The empirical distribution can be used to calculate probabilities related to the minimum value from the process and can therefore be used to calculate control limits (Figure 3.2).

Control Limit Calculation Using Bootstrapping

Control limits are lines on a control chart that help determine if the variation present is common cause variation or special cause variation. The control limits are based upon a statistic of interest. Statistics are random variables and therefore exhibit natural variation. Whenever a statistic crosses a control limit, the statistic is signaling that the process contains too much variation because “extra” variation was required to force the statistic across the control line, therefore a special cause may have entered the system.

Recall that control limit has the form:

$$\theta \pm k\sigma_{\theta}$$

where θ is the parameter of interest
 σ_{θ} is the standard deviation of θ .
 k is a factor (usually chosen to be 3)

The control limit is calculated by estimating θ with θ^{hat} and σ with σ^{hat} . For example, the exact probability limit for the mean

$$\mu \pm k\sigma_{\mu}$$

is estimated by

$$\bar{x} \pm ks_{\bar{x}}$$

and in general

$$\hat{\theta} \pm k\hat{\sigma}_{\hat{\theta}}$$

Statistically calculating the standard error is challenging and frequently impossible. Many statistics have not been used in control charting because their standard error is intractable. Estimating the standard error is the primary use of the bootstrap. The bootstrap estimate of standard error, is simply the standard deviation of bootstrap samples, that is, it is the standard deviation of the bootstrap distribution. In this way, control limits for virtually any statistic can be calculated.

Bootstrapping allows control limits to be calculated when the distribution of the statistic is not known. It also allows control limits to be calculated when the underlying process distribution is not known. Not only does it provide a method of graphically examining the statistic's distribution, it provides an estimate of the standard error of the statistic. The estimated standard error can be used to make probability statements about the behavior of the statistics itself. Bootstrapping provides a method of calculation in situations where no other method is known. Although it varies case by case, bootstrap estimates have been shown to be quite accurate [Efron, 4]. In situations where well developed statistical theory exists, bootstrap estimates give very similar results, but when the assumptions of traditional statistics are violated, bootstrapping is superior [Efron, 4].

Bootstrapping is only dependent upon the availability of a computer and a very mild assumption that the underlying process is independent and identically distributed. The independence property implies that knowing one data value does not influence what the next data value will be. The identical distributed property implies that all of the data come from the same process or data population. Both of these properties are exceptionally common in most processes, furthermore this assumption is also needed in other charting procedures. Standard methods exist to test for both conditions if necessary.

CHAPTER 4

SITUATION AWARENESS

Having overcome previous difficulties in calculating Quality Box Charts, it was desired to test if Quality Box Charts were easier to learn and use than the more traditional charts. First of all, the researcher needed to define what "Easier to use" meant. It was believed that a subject should be able to gather more and better information from the "easier" chart.

The "more and better" information was measured using a cognitive human factors measure called Situation Awareness. Situation Awareness (SA) has become a highly researched topic in the last few years. Situation Awareness is defined as a person's perception of the elements of the environment within a volume time and space, the comprehension of their meaning and their projection of the status in the near future. The majority of SA research has focused on dynamic situations such as airline flight, airplane combat, and operating rooms. It is believed that as automation increases workers will be forced out of the information loop and SA will decrease [Endsley, 5].

SA is often difficult to measure. Endsley lists and evaluates the major methods of measuring SA that are in use now and presents the SAGAT method of measurement with two studies verifying its effectiveness [6].

The present study implemented an industrial approach of the Situation Awareness Global Assessment (SAGAT) methodology. The goal was to examine the situational

awareness for a process monitoring task where quality control charts were used to simulate the process monitoring activities. It was desired to test whether the type of chart used and the level of operator involvement had significant effect on the situation awareness of subjects who had not been exposed to this monitoring process. This study considered three skill levels of participants and showed that SAGAT is also applicable with subjects of varying skills.

The experiment was performed to test two different research hypotheses. The first hypothesis focused on whether there was a difference in information gathering, assimilation, and accuracy between Quality Box Charts and traditional $\bar{x}/R$ charts.

The second experimental hypothesis tested addressed subject complacency. The field of industrial quality has always been affected by complacency and the impact is growing. Complacency is a worker's lack of attention and vigilance to the process. Endsley and Kiris [5] discuss complacency and vigilance problems in terms of a loss of situation awareness. The loss of SA appears to be increasing as the level of automation increases because as workers lose direct contact with the process, out-of-the-loop performance problems increase [Endsley and Kiris, 5]. Therefore, the project experiment considered involvement and no involvement in the activities which required two hypotheses:

I. To test the ease of use (information assimilation) of the chart types.

H_0 : No significant difference exists between the Situation Awareness of the two chart types

H_1 : A significant difference exists between the Situation Awareness of the two chart types

Statistically:

$H_0: \mu_{QBC} = \mu_{\bar{x}/R}$

$H_1: \mu_{QBC} \neq \mu_{\bar{x}/R}$

II. To test whether involvement affected the complacency and therefore the Situation Awareness.

H₀: Involvement has no significant effect on the Situation Awareness

H₁: Involvement increases the Situation Awareness

Statistically:

$$H_0: \mu_{\text{Involvement}} = \mu_{\text{No Involvement}}$$

$$H_1: \mu_{\text{Involvement}} \geq \mu_{\text{No Involvement}}$$

Testing

Subjects were asked to read a job description. After reading the job description, each subject was asked if they had questions about the task. The SAGAT procedure was explained and every subject was given sample computer output. If subjects desired, time was spent practicing the SAGAT procedure with each subject. The practice time was continued until each subject felt he or she had gained adequate confidence with the procedure. Questions related to the control charts and their interpretations were not directly answered during this period. Each time a question was asked, the researcher referred the subjects to the instructions previously given. The subjects were all initially told they were helping with an engineering related activity. Once the SAGAT procedure had begun, questions that were asked were answered by saying that the researcher would only be able to repeat the question. If after further reading, the subject was still unable to answer the questions, they were told to guess as best they could. Each test consisted of three stops with ten queries. The stopping time and the queries asked were randomly determined as suggested by [Endsley, 6].

Design

The sample population consisted of students from Montana State University. Three distinct groups of subjects performed the experiment. Each group had different educational levels. The first group (group 1) consisted of freshman students who reported never having seen control process monitoring charts before and had therefore obviously never received any training related to process monitoring. The second group (group 2) consisted of upperclass engineering students who were enrolled in an introductory quality control class. Subjects from group two had recently received training on $\bar{x}/R$ charts. No one in the group had ever been exposed to Quality Box Charts. The third group (group 3) was composed of engineering graduate students and one faculty member who all had extensive control chart experience. Most of the members of group three had previously been exposed to QBC's. Therefore, the three groups represented three distinct levels of knowledge and experience relative to interpreting process monitoring charts.

The experiment consisted of three factors: Type of chart, Level of involvement, and Group of subjects. Two types of charts were used, standard $\bar{x}/R$ Shewhart charts and Quality Box Charts. The second factor, Involvement, was simulated by having some subjects add the control limits to their charts, therefore, involvement and no involvement were the two levels of this factor. The third factor was the group. It was expected a priori that the results of each group would be significantly different. For this study, it was not of interest to test the difference between groups—only the difference between chart types and level of involvement.

Analysis

The experiment was conducted among three distinct groups making a simple factorial analysis inappropriate. The groups represented blocks or restrictions on the randomization structure. Within each group, subjects were randomly assigned to every combination of chart and involvement level. The design structure created a complete block factorial design which is generalized in the split-plot design structure. The data for error rates is shown in Table 1. Error rate is a complimentary measure of Situation Awareness, that is, $SA=1-\text{Error Rate}$.

TABLE 4.1 Error Rate by Category

	$\bar{x}/R$ chart		Quality Box Charts	
Involvement Level	No	Yes	No	Yes
Group 1	63.2	56.5	51.5	35.65
	80.2	43	57.15	30.5
	73.55	51.3	49.32	14.5
Group 2	46	33	11	20
	26.5	50	22	7.5
	31	33.75	30	12.75
Group 3	14	33.5	15	8
	25.5	15.5	10.5	8.5
	19.68	26.3	6.5	8.25

The values were analyzed using Analysis of Variance techniques using the following model:

$$Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \delta_k + \epsilon_{ijk}$$

where τ is the effect of the chart type
 β is the effect of involvement
 $\tau\beta$ is the interaction effect
 δ is the effect of the groups
 ϵ is the random error component
 μ is the overall mean

Results

The analysis is summarized in the following table.

TABLE 4.2 ANOVA TABLE FOR SA FACTORS

	Df	Sum of Sq	Mean Sq	F Value	Pr (F)
Involvement	1	630.010	630.010	7.11750	0.012189
Chart	1	2797.705	2797.705	31.60692	0.000004
Group	2	7291.073	3645.537	41.18525	0.000000
Involvement:Chart	1	200.883	200.883	2.26947	0.142403
Residuals	30	2655.468	88.516		

The analysis indicated several things. First of all, that there was a significant difference in SA between the QBC and $\bar{x}/R$ charts. The plots of the data indicate that the QBC's had a significantly lower error rate and therefore higher SA. Second, the level of involvement did have a significant effect while the interaction between the level of Involvement and the chart type was not significant. Because Groups were considered a blocking variable in the analysis, it is possible to calculate a sum-of-squares error for the variable and calculate an F-statistic,

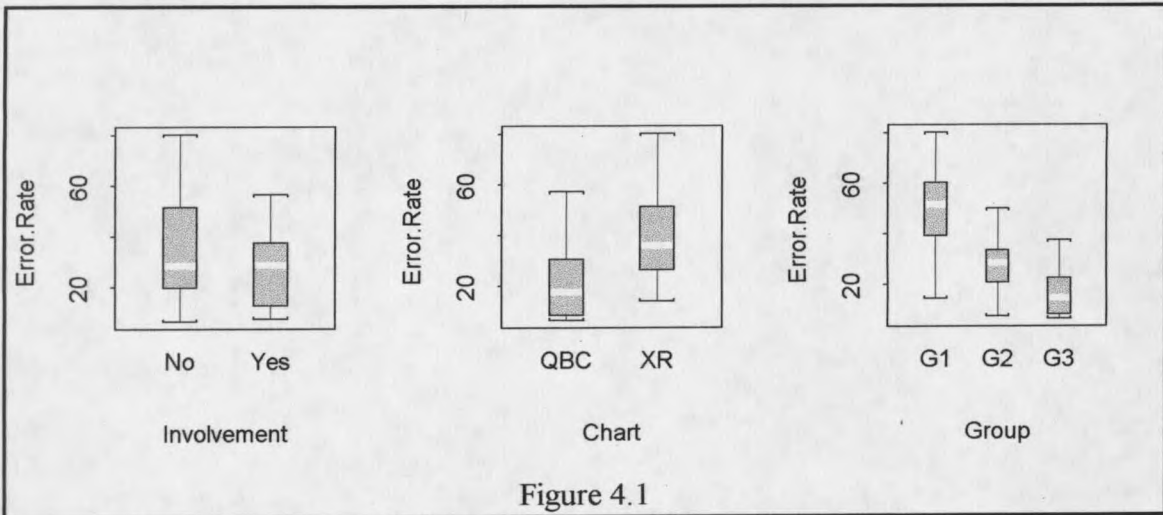


Figure 4.1

no formal statistical testing procedure exists for testing blocks. Furthermore, it was assumed from the start, that a difference existed among the group therefore multiple groups were used in the analysis. Diagnostic plots on the accuracy of the analysis (model assumptions) are given in Appendix B.

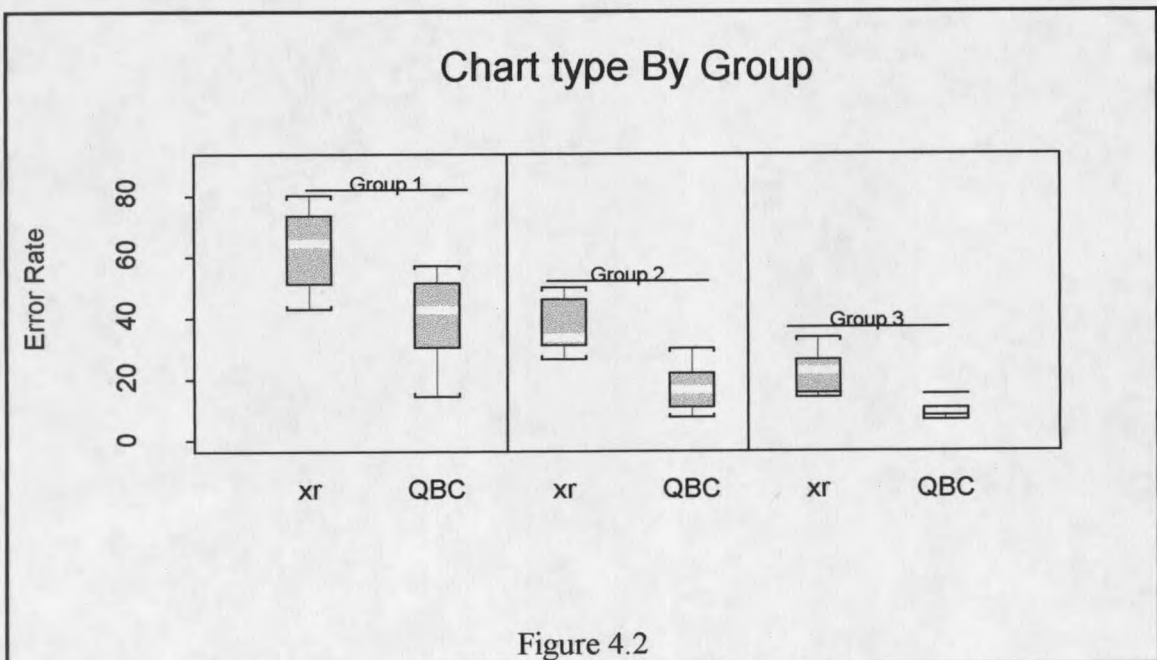


Figure 4.2

The SAGAT procedure showed that the QBC outperformed the $\bar{x}/R$ charts for unskilled workers with no training, advanced students with some training (in $\bar{x}/R$ charts), and even graduate students with extensive training. Although Group 1 and Group 2 had never seen QBC's and Group 3 had only a brief introduction months before, each group still performed better using the QBC. The monitoring task chosen for this study could be considered complex because $\bar{x}/R$ charts are known to be very complicated for workers to master even with extensive training. Before the study, Quality Box Charts were believed to be easier to interpret and the results support this.

Critics may say that it was simply too much for these subjects to interpret the process charts without any guidance--especially since nearly every subject asked fundamental questions related to the charts they were seeing and two of the three groups had never seen QBC's. Yet, in spite of the fact that no questions were answered relative to the charts, better responses were consistently recorded for the QBC chart with subject involvement. Subjects were often forced to guess because they did not "know" the proper response. Even though many guesses were made, these guesses were not made in a vacuum. The QBC chart's were superior in conveying information and were easier to interpret in the face of uncertainty.

CHAPTER 5

AVERAGE RUN LENGTH

The Quality Box Chart was introduced in Chapter 2 and its control limits analytically presented in Chapter 3. The QBC is an attempt to design a simple process monitoring tool that does not depend upon assumptions of the underlying process. Computationally, the QBC does exactly this, but it must be judged on at least two additional criteria. The first criterion is whether the QBC is truly easier to use. Chapter 4 showed that the QBC's convey more information for all levels of users. The second criterion is whether the Quality Box Chart's technical performance is comparable to other types of charts, that is, is it really an effective tool for monitoring a process.

Chapter 5 presents the technical aspect for which all charts must be compared. Furthermore, the performance of QBC's will be compared to traditional $\bar{x}$ charts. In general, control charts used as process monitoring tools are evaluated on two fundamental properties. The first property is false alarm probabilities. A false alarm occurs when the monitoring process indicates a special cause of variation is present when only common cause variation is present. The second property by which a control chart must be evaluated is the chart's sensitivity to changes in the mean and variance of the underlying process. Both of the criteria were introduced in Chapter 3 relative to an ideal process monitor.

Each of the two fundamental properties by which control charts are judged is measured by a quantity known as average run length (ARL). Each control chart has a unique average run length. The ARL is the theoretical average time between signals for a given set of operating conditions. Therefore, when a process is in control and no special causes of variation are present, it is desired to have a very large ARL. A large ARL implies long process runs between false alarms. The fewer the false alarms the better the process monitor with respect to the first criterion.

When a process begins to shift in either its operating level or variation, a good process monitor will rapidly indicate the change in conditions. In terms of average run length, a good process monitor will have a very short average run length when the process changes. Therefore, with a shift in the mean or variance of a process, a small ARL is desired from a superior process monitoring tool. Therefore, ARL can be used to measure both false-alarms and late-alarms.

- *Criterion One: Process monitors should have a large ARL for in-control processes*
- *Criterion Two: Process monitors should have a short ARL for out-of-control processes*

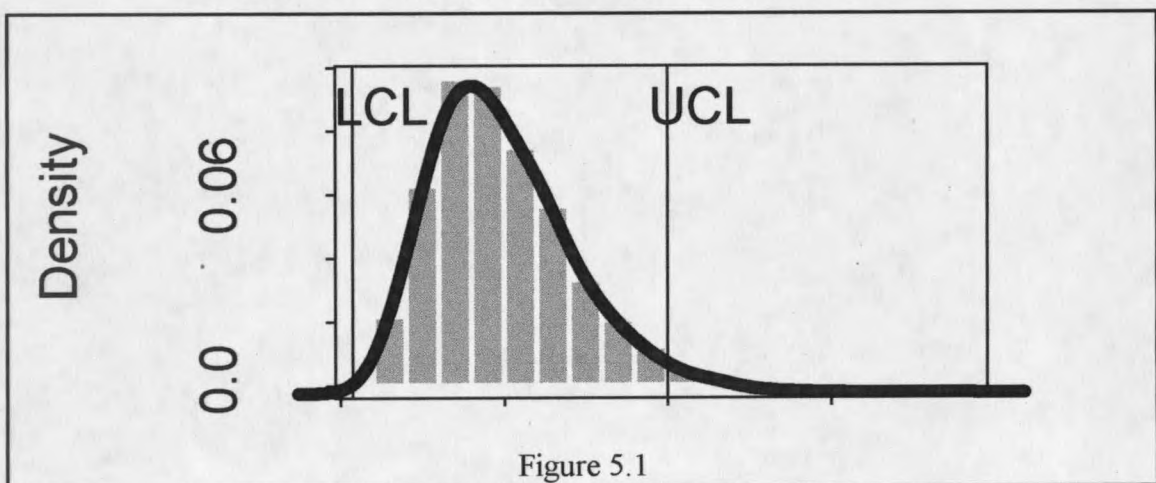
False Alarms

Of central importance to any control method is the method's responsiveness to conditions that warrant a signal. Control methods can usually be made arbitrarily sensitive, but as the sensitivity is increased the false alarms also increase. False Alarms in quality control charts are very expensive. When a control chart signals an alarm, the process must

usually be stopped, leaving idle workers. The engineering staff must investigate the process, develop a cause theory, generate a solution, and implement the solution. Often this process is very fast and possibly even done on-line, yet other times it is time consuming and therefore expensive. If the alarm that generated all of the preceding activity was false, the situation is compounded. Now the process is in a worse state than before the alarm. The engineering staff must again stop the line, leaving workers with more idle time while the engineering staff reverses their previous adjustments. Furthermore, all products that have been made during the interim, must be inspected and reworked or scrapped.

False alarms must be balanced by the cost of not noticing a poorly performing process (late alarms). The balance between false alarms and responsiveness provided the motivation for Walter Shewhart's groundbreaking work with control charts. In essence, Shewhart showed that the probability of having a false alarm was statistically related to the sensitivity of a control chart. The basic relationships are [Montgomery, 13] :

$$P(\text{False-Alarm}) = P(\text{Out-of-Control}) = 1 - P(\text{In-Control})$$



$$ARL = \frac{1}{P(\text{False-Alarm})}$$

The first relationship indicates that a chart's sensitivity to false alarms is related to the control limits chosen.

Late Alarms

The second relationship shows that the Average Run Length (ARL) is also a function of the control limits. For an *in-control process*, the ARL is the mean time between false alarms. If a chart has restrictive control limits, it will give more false alarms whereas a chart with wide limits will have, on average, more samples between false alarms. For an *out-of-control* process, the ARL is the mean time to signal for a process shift. Thus, when the process starts to drift, the ARL becomes a measure of the control chart's sensitivity. Recall the general form of the control limit:

$$\theta \pm k\sigma_{\theta}$$

where

θ is the statistic of interest

σ_{θ} is the standard deviation for the distribution of θ .

k is a factor (usually chosen to be 3)

The factor k is chosen to balance the false-alarm and late alarms. Small values of k cause more false alarms while large values of k make the control chart insensitive to changes in the process.

Implicit in the previous discussion is that the probabilities are obtainable. Probabilities are calculated directly from distributions. Yet, one of the fundamental objectives of this study is to develop control charts that require no distributional assumptions. Therefore it may seem impossible to calculate the average run length or the probability of a false alarm for QBC's.

Again using the bootstrap sampling distribution, the ARL and false alarm probabilities can be empirically estimated for any distribution. The empirical density curve is compared against the control limits. The area of the density curve outside the control limits is computed and its inverse is the empirical ARL.

Not only are false alarm probabilities are important, but it is of interest how a monitoring method performs when the data experience shifts, that is, how does the monitoring method perform relative to late alarms. Traditionally both mean and variance shifts and their combinations are considered in examining the late alarm performance of a monitoring method. This study calculated the empirical ARL for both the QBC's and the $\bar{x}/R$ chart.

Relative Performance

The relative performance of the two charts depends upon a number of factors. The $\bar{x}/R$ chart is traditionally not very sensitive to small shifts in the mean and variance. The sensitivity of the QBC depends upon which member of the generalized family is chosen. Furthermore, the behavior of the underlying distribution affects the sensitivity of all charts. One example is give

below to show the relative performance. In general, several basic principles about the relative performance can be stated:

- The QBC family has superior false alarm properties
- The QBC family has superior sensitivity for mean shifts
- The QBC family is more sensitive for ill-behaved processes
- The QBC provides more information

Table 5.1

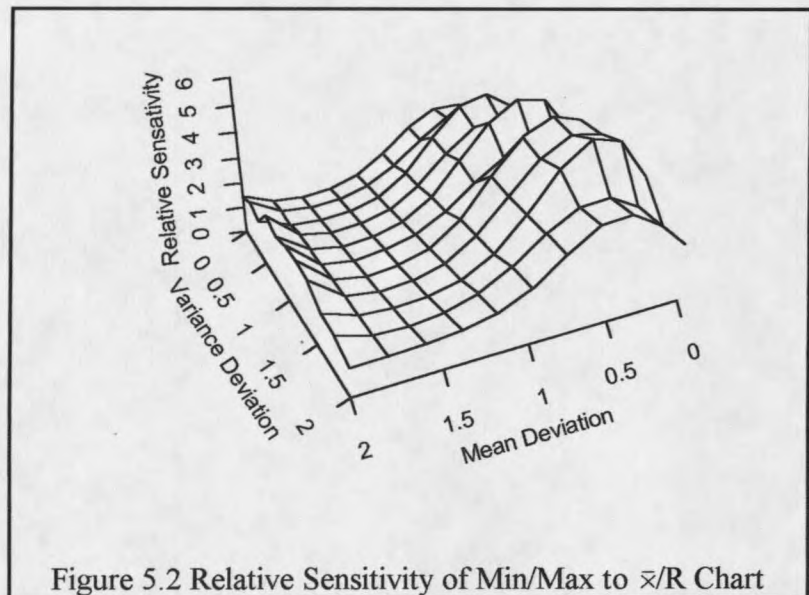
RELATIVE SENSITIVITY OF THE QBC Min/Max CHART v. $\bar{x}$ /R CHART

		Variance Shift (σ)										
		0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Mean Shift	0	1.4	2.4	3.0	2.2	4.0	3.9	6.0	8.1	3.7	2.3	2.1
	0.2	3.1	5.5	4.0	3.5	5.8	3.8	4.2	4.4	6.4	3.0	3.0
	0.4	4.4	3.4	5.5	8.2	5.4	4.0	6.1	6.0	6.6	4.9	6.7
	0.6	4.1	4.8	4.9	4.6	4.5	4.5	6.1	4.7	4.4	4.0	4.3
	0.8	3.1	3.3	3.1	2.9	3.3	2.9	2.8	3.2	3.1	2.6	2.6
	1.0	1.9	1.8	1.8	1.9	1.8	1.8	1.9	1.9	2.0	1.9	1.8
	1.2	1.5	1.4	1.3	1.3	1.3	1.4	1.3	1.3	1.3	1.4	1.3
	1.4	1.1	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1
	1.6	1.1	1.1	1.0	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	1.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

The response plot shows the relative sensitivity (with respect to ARL) of the QBC for various shifts of mean and variance. It is of special interest that all points are near or above one. The Z level of one indicates that the QBC is at least as sensitive as the $\bar{x}$ chart. Notice that when the variance shifts are small the QBC is as sensitive as the $\bar{x}$ chart and when the mean shifts are small, the QBC is much more sensitive. Graphically, this is indicated by the high ridge running along the response plot. It implies that the QBC is almost always more sensitive than the $\bar{x}$ charts. The increased sensitivity indicates the

ARL is smaller for shifts in the mean and/or variance. Generally, the QBC will give an alarm faster once the process begins to drift.

It is difficult to give global statements about the performance of



QBC's derived from empirical tools, yet the previous results have consistently been upheld through numerous simulations. The format of QBC's makes outlier detection and extreme observations more easily present. Furthermore, it is possible to still track outliers and not have the control limits affected using QBC's. Empirically, it is shown that QBC's generally have superior false alarm properties. Superior false alarm properties imply that the QBC is stable for in-control processes. Improved false alarm performance can aid in cost reduction. Since fewer false alarms are present, costly mistakes are minimized.

In addition, QBC's often have superior detection properties. Once a process begins to shift in its mean or variance structure, the QBC is often able to detect it faster than an $\bar{x}$ chart. The performance of the QBC depends in part on the behavior of the process and the type of QBC used. The QBC is usually superior to the $\bar{x}$ chart for variance shifts smaller than the mean shift.

CHAPTER 6

DISCUSSION AND CONCLUSION

The research goal of this thesis was to develop a process monitoring tool that performed well, yet was easy to use. The goal was stated specifically as:

Research Objective

To develop a generalized class of distribution free, variable control charts that are easy to construct and simple to interpret.

The Quality Box Chart was presented as the fulfillment of this objective. This research started from the simple QBC presented by previous researchers and developed a generalized family of Quality Box Charts. The generalized family of QBC's has the flexibility to apply in many situations that were not practical with previous control chart procedures.

The generalized QBC family is a family of distribution free control charts. The form of the process distribution or the sampling distribution of the control measure is not needed. Using bootstrap resampling procedures, empirical distributions are created. All necessary calculations are performed with empirical sampling distributions.

Using Situation Awareness as a measure of a person's understanding of a task, the

Quality Box Chart was compared to traditional $\bar{x}/R$ control charts. The Quality Box Chart afforded significantly better Situation Awareness for beginner, intermediate, and advanced users. QBC's afforded significantly higher Situation Awareness even when subjects had no previous exposure to QBC's.

The false alarm and late alarm properties of the generalized QBC family were compared to traditional $\bar{x}/R$ control charts. Using statistical simulation, the average run length of an in-control process was compared for both types of charts. Regardless of the particular QBC used, the average run length was always greater for the QBC. The relative factor of the average run length generally increased as the underlying distribution became increasingly non-normal.

The late alarm properties of the chart types were compared for various combinations of process mean and variance shifts. In general, the QBC family had late alarm properties equal to or greater than the $\bar{x}/R$ charts. The QBC family performed much better when shifts in the process mean were present, but still performed as well or better for variance shifts.

Future Research

At least three clear areas of research are still available relative to Quality Box Charts. First, the Splus functions that were used to calculate the control limits and perform the average run length simulations could be made more compact and efficient. In addition, the functions could be made more portable to increase their applicability. The author

would like to add several optional features to the present functions to broaden their application.

Second, during the calculation of the control limits, the tolerance factor, k , is used. The present research followed standard practices and used $k = 3$ for all calculations. Since the generalized family of QBC's has such superior false alarm properties, it should be beneficial to examine other values of k to improve the QBC's sensitivity. At this time, it is believed that $k = 3$ is NOT the optimum value for the generalized family of QBC's.

Third, because the QBC family uses the same scale as the original data, process capability can be directly examined. Whether tolerance limits are used or not, it is possible to visualize the capability. Simple graphical enhancements are also possible to assist in performing capability studies.

Conclusion

Quality Box Charts are able to be used by all members of an organization in monitoring important aspects of the business process in an effort to gain an advantage with quality, productivity, cost and timeliness. Quality Box Charts can be an important tool in today's business environment. QBC's work better for all levels of employees. Quality Box Charts have one simple rule and therefore are easier to monitor and more intuitive to learn. Quality Box Charts provide a large, flexible family of charts which can be calculated whether the process distribution is known or not. QBC's satisfy the objective of the thesis.

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APPENDIX A

Split Functions

```

.Box1_function(x, a = 1, b = nrow(x), ucl = 2, lcl = -2)
{
#####
##                                                                    ##
## S-plus function to create a Quality Box Chart. It consists      ##
## of four basic sections:                                         ##
##     I. Data acquisition and formatting                          ##
##     II. Plot area formatting                                    ##
##     III. Plotting each rational subgroup by fcts               ##
##     IV. Control chart use or calculation functions              ##
##                                                                    ##
## 14 September 1996                                              ##
## Rev. # 8                                                         ##
## Rev. Date 2 Jan . 97                                           ##
#####
    if(missing(x)) {
        stop("Data Not Given")
    }
    if(!is.matrix(x)) {
        stop("Data Must Be In MATRIX Form")
    }
    if((missing(a)) & (!missing(b)) || (missing(b)) & (!missing(a)))
    {
        stop("Both Start and End Data Points Needed")
    }
    if((a <= 0) || (b <= 0)) {
        stop("Starting Points must be Positive")
    }
    xnew <- x[a:b, ]
#####
## Menu for User choice of statistic calculation method          ##
#####
    cat("Chose the statistic for the LOWER control Limit")
    choices <- c("Minimum", "First Quartile (Q1)", + "Median",
        "Third Quartile (Q3)", "Maximum")
    pick <- menu(choices)
    .theta1 <- switch(pick,
        theta1 <- 1,
        theta1 <- 2,
        theta1 <- 3,
        theta1 <- 4,
        theta1 <- 5)
#####

```

```

## Menu for User choice of statistic calculation method    ##
#####
cat("chose the statistic for the UPPER control Limit")
choices <- c("Minimum", "First Quartile (Q1)", + "Median",
            "Third Quartile (Q3)", "Maximum")
pick <- menu(choices)
.theta2 <- switch(pick,
    theta2 <- 1,
    theta2 <- 2,
    theta2 <- 3,
    theta2 <- 4,
    theta2 <- 5)
assign("theta1", theta1, frame = 1)
assign("theta2", theta2, frame = 1)
#####
## Menu for User choice of control limit calculation method    ##
#####
cat("Chose the control limit calculation Method")
choices <- c("Empirical Sub-Group Method",
            "Bootstrap Sub-Group Method", + "Full Data Bootstrap",
            "Previously Given")
pick <- menu(choices)
.Boxlim <- switch(pick,
    .box1.a(xnew),
    .box1a.a(xnew),
    .box2.a(xnew),
    .box3.a(xnew, ucl, lcl))
#####
##### PLOTTING PORTION OF FUNCTION #####
#####
cat("Chose the Desired Output")
choices <- c("QBC Only", "QBC with Emperical Dist's",
            "QBC with X-bar Chart", + "QBC, X-bar, R Charts",
            "QBC with Shape Data", "QBC, Shape, Dist's",
            "Fit with BS Dist's")
pick <- menu(choices)
.Boxlim1 <- switch(pick,
    .plot1(xnew, ucl, lcl),
    .plot2(xnew),
    .plot3(xnew, lcl, ucl),
    .plot4(xnew, lcl, ucl),
    .plot5(xnew, lcl, ucl),
    .plot6(xnew, lcl, ucl),

```

```

        .plot7(xnew, lcl, ucl))
        cat(lcl, ucl)
    }

#####
#                                                                    #
# Control limit calculation functions--Bootstrap functions          #
#                                                                    #
#####

.box1.a_function(xnew)
{
#Generalized Function
#This function is called by function Boxchart and .Arli() to calculate control limits
# Method 1: Sub-Group Calculation (calculates statistics for each sub-group and creates
# emperical sampling distribution)

    xsub <- matrix(t(apply(xnew, 1, quantile)), ncol = 5)
    bs.5ns <- xsub #bs.5ns is used in general in other fcts
    .q1b <- xsub[, theta1]
    .q3b <- xsub[, theta2]
    q1 <- mean(xsub[, theta1])
    q3 <- mean(xsub[, theta2])
    q1.std <- sqrt(var(xsub[, theta1]))
    q3.std <- sqrt(var(xsub[, theta2]))
    lcl1 <- q1 - 3 * q1.std
    ucl1 <- q3 + 3 * q3.std      #cat(lcl1, ucl1)
    assign("ucl", ucl1, frame = 1)
    assign("lcl", lcl1, frame = 1)
    assign(".q3b", .q3b, frame = 1)
    assign(".q1b", .q1b, frame = 1)
    assign("bs.5ns", bs.5ns, frame = 1)
    cat(lcl1, ucl1)
}

.box1a.a_function(xnew)
{
#This function is called by function Boxchart to calculate control limits
# Method 1a: Boobstrap Sub-Group Calculation (calculates statistics for each
# sub-group and creates bootstrap sampling distribution)

    xsub <- matrix(t(apply(xnew, 1, quantile)), ncol = 5)

```

```

q1a <- matrix(sample(xsub[, theta1], 5 * nrow(xnew), replace = T),
              nrow = 5 * nrow(xnew))
q3a <- matrix(sample(xsub[, theta2], 5 * nrow(xnew), replace = T),
              nrow = 5 * nrow(xnew))
.q1b <- t(apply(q1a, 1, quantile))
.q3b <- t(apply(q3a, 1, quantile))
q1 <- mean(.q1b[, theta1, ])
q3 <- mean(.q3b[, theta2, ])
q1.std <- sqrt(var(xsub[, theta1]))
q3.std <- sqrt(var(xsub[, theta2]))
lcl1 <- q1 - 3 * q1.std
ucl1 <- q3 + 3 * q3.std
assign("ucl", ucl1, frame = 1)
assign("lcl", lcl1, frame = 1)
assign(".q1b", .q1b, frame = 1)
assign(".q3b", .q3b, frame = 1)
cat(lcl1, ucl1)
}

```

```
.box2.a_function(xnew)
```

```

{
#This function is called by function Boxchart to calculate control limits
#Method 2: Boxplot control limits by bootstrapping the total data set

```

```

size <- 50
qsample <- matrix(apply(xnew, 1, quantile), byrow = T, ncol = 5)
bsdata <- as.vector(xnew)
i <- 1
bs.sample <- matrix(0, nrow = nrow(xnew), ncol = size)
while(i < nrow(xnew)) {
  bs.sample[i, ] <- sample(bsdata, size, replace = T)
  i <- i + 1
}
bs.5ns <- matrix(apply(bs.sample, 1, quantile), ncol = 5, byrow =
T)
lcl <- mean(bs.5ns[, theta1]) - 3 * sqrt(var(bs.5ns[, theta1]))
ucl <- mean(bs.5ns[, theta2]) + 3 * sqrt(var(bs.5ns[, theta2]))
.q1b <- bs.5ns[, theta1]
.q3b <- bs.5ns[, theta2]
cat(lcl, ucl, a3)
assign("lcl", lcl, frame = 1)
assign("ucl", ucl, frame = 1)
assign(".q1b", .q1b, frame = 1)

```

```

    assign(".q3b", .q3b, frame = 3)
}

```

```

.box3.a_function(xnew, ucl, lcl)
{

```

```

    .q1b <- lcl
    .q3b <- ucl
    cat(ucl, lcl)
    assign("lcl", lcl, frame = 1)
    assign("ucl", ucl, frame = 1)
    assign(".q1b", .q1b, frame = 1)
    assign(".q3b", .q3b, frame = 1)
}

```

```

#####
#                                     #
#           Graphics functions       #
#                                     #
#####

```

```

.plot1_function(xnew, ucl, lcl)
{
    .plt(xnew, axdensity = 5)
}

```

```

.plot2_function(xnew)
{
    frame()
    par(fig = c(0, 1, 0.3, 1), mar = c(5, 4, 2, 2) + 0.1)
    .plt(xnew)
    par(fig = c(0, 0.5, 0, 0.3))
    plot(density(.q1b, width = 0.55, n = 100), xlab =
        "Q1 Distribution", ylab = "frequency")
    abline(v = lcl, lty = 2)
    par(fig = c(0.5, 1, 0, 0.3))
    plot(density(.q3b, width = 0.55, n = 100), xlab =
        "Q3 Distribution", ylab = "frequency")
    abline(v = ucl, lty = 2)
    par(fig = c(0, 1, 0, 1))
}

```

```

.plot3_function(xnew, ucl, lcl)
{

```

```
#####
#### Plot for QBC with X-bar #####
#####

    par(mfrow = c(2, 1))
    .plt(xnew)      #abline(h = lcl)
#abline(h = ucl)
    ydata <- qcc(xnew, type = "xbar")
    shewhart(ydata, add.stats = F, nsigmas = 3, highlight = list(pch
        = 1, col = 2))
}

.plot3_function(xnew, ucl, lcl)
{
#####
#### Plot for QBC with X-bar #####
#####

    par(mfrow = c(2, 1))
    .plt(xnew)      #abline(h = lcl)
#abline(h = ucl)
    ydata <- qcc(xnew, type = "xbar")
    shewhart(ydata, add.stats = F, nsigmas = 3, highlight = list(pch
        = 1, col = 2))
}

.plot4_function(xnew, ucl, lcl)
{
#####
#### Plot for QBC with X-bar and R #####
#####

    par(mfrow = c(3, 1))
    .plt(xnew)
    ydata <- qcc(xnew, type = "xbar")
    rdat <- qcc(xnew, type = "R")
    r2 <- shewhart(ydata, add.stats = F, nsigmas = 3, highlight = list(
        pch = 1, col = 2))
    r1 <- shewhart(rdat, add.stats = F, highlight = list(pch = 1, col
        = 2))
    .pointsr <- shewhart.rules(r1)
    .pointsx <- shewhart.rules(r2)
}

.plot5_function(xnew, ucl, lcl)
```

```
{
#####
#### Plot for QBC with X-bar and R      ###
#####
    par(mfrow = c(2, 1))
    .plt(xnew)
    Shape(xnew)
}
```

```
.plot6_function(xnew; ucl, lcl)
{
#####
#### Plot for QBC with Empirical Dist's    ###
#####
    frame()
    par(fig = c(0, 0.7, 0.3, 1), mar = c(5, 4, 2, 2) + 0.1)
    .plt(xnew)
    par(fig = c(0.7, 1, 0.3, 1))
    Shape(xnew)
    par(fig = c(0, 0.5, 0, 0.3))
    plot(density(.q1b, width = 0.55, n = 100), xlab =
          "Q1 Distribution", ylab = "frequency")
    abline(v = lcl, lty = 2)
    par(fig = c(0.5, 1, 0, 0.3))
    plot(density(.q3b, width = 0.55, n = 100), xlab =
          "Q3 Distribution", ylab = "frequency")
    abline(v = ucl, lty = 2)
    par(fig = c(0, 1, 0, 1))
}
}
```

```
#####
#### Diagnostic Plots This fuction plots the emperical data and the ##
#### bootstrap sampling distibutions for the five number summary    ##
#####
```

```
p <- 1.5 * min(xnew)
q <- 1.5 * max(xnew)
par(new = F)
plot(density(xnew, width = 0.3, n = 10000), yaxt = "n", xlim = c(p,
  q), col = 3, xlab = " ", ylab = " ")
abline(v = c(quantile(xnew)))
par(new = T)
plot(density(bs.5ns[, theta1], width = 0.2, n = 1000), yaxt = "n",
```

```

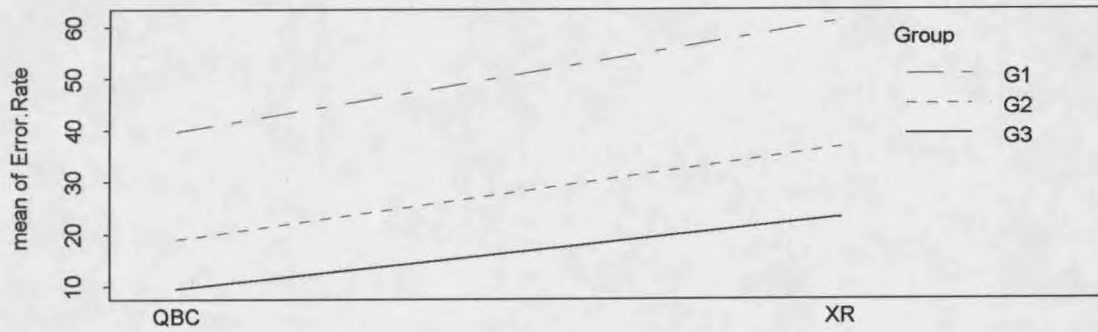
        xlim = c(p, q), col = 2, xlab = " ", ylab = " ")
    par(new = T)
    plot(density(bs.5ns[, theta2], width = 0.2, n = 1000), yaxt = "n",
        xlim = c(p, q), col = 1, xlab = " ", ylab = " ")
    #par(new=T)
    #plot(density(bs.5ns[, 1], width = 0.5,
    #n = 1000), yaxt = "n", xlim =
    #c(-3.5, 3.5), col = 5, xlab =
    #" ", ylab = " ")
    #par(new = T)
    #plot(density(bs.5ns[, 5], width = 0.5,
    #n = 1000), yaxt = "n", xlim =
    #c(-3.5, 3.5), col = 1, xlab =
    #" ", ylab = " ")
    title(main = "Empirical Sampling Distributions")
}

plt_function(object, axdensity = 4)
{
#temy plotting fct to play
    indices <- 1:dim(object)[1]
    indcs <- c(indices - 1/2, max(indices) + 1/2)
    a <- " "
    nams <- rep(a, nrow(object))
    m <- trunc(length(indices)/axdensity)
    p <- numeric()
    j <- 1
    i <- 1
    while(j < length(indices)) {
        p[i] <- (j)
        i <- i + 1
        j <- j + m
    }
    nm.grp <- as.character(p)
    nams[p] <- nm.grp
    boxplot(split(object, rep(1:dim(object)[1], dim(object)[2])), range = 0, whisklty =
        1,
        names = nams, ylim = range(max(object), min(object), lcl, ucl), xlim =
        range(indcs), ylab = "Sample Distribution", xlab = "Sample #", axes = F)
    abline(h = c(ucl, lcl), lty = 2)
    title(main = "Quality Box Plot")
    axis(2)
    axis(1, labels = F, ticks = T)
    box()}

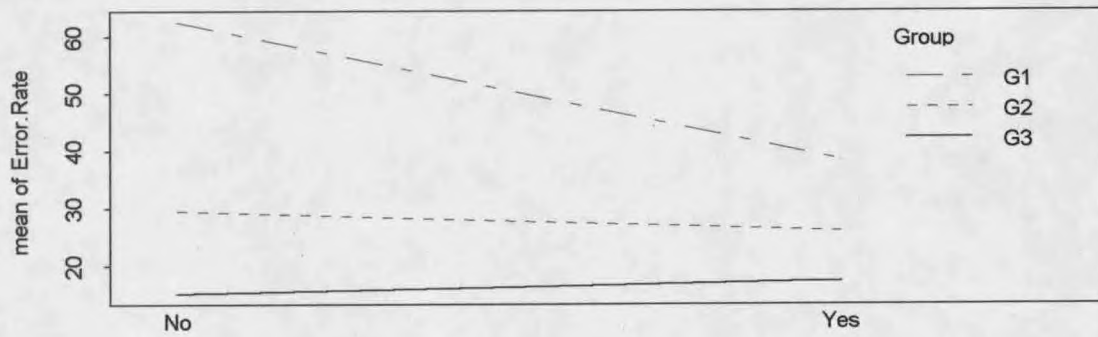
```

APPENDIX B**Model Diagnostic Plots**

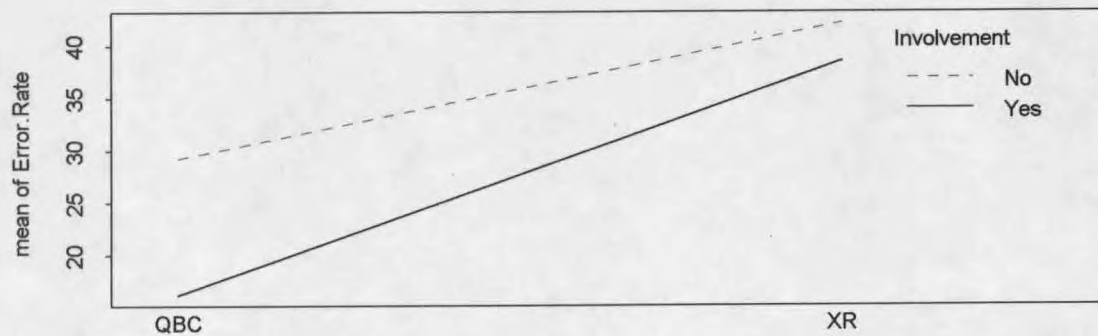
Interaction Plot



Chart

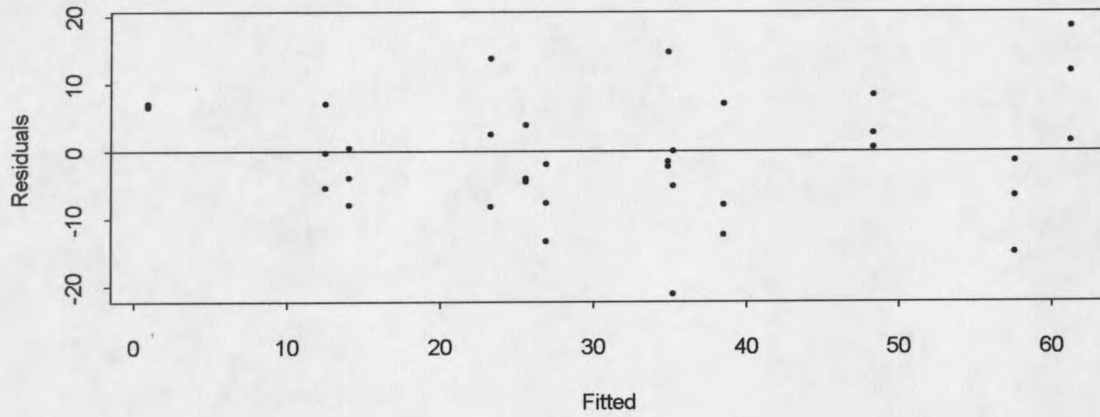


Involvement

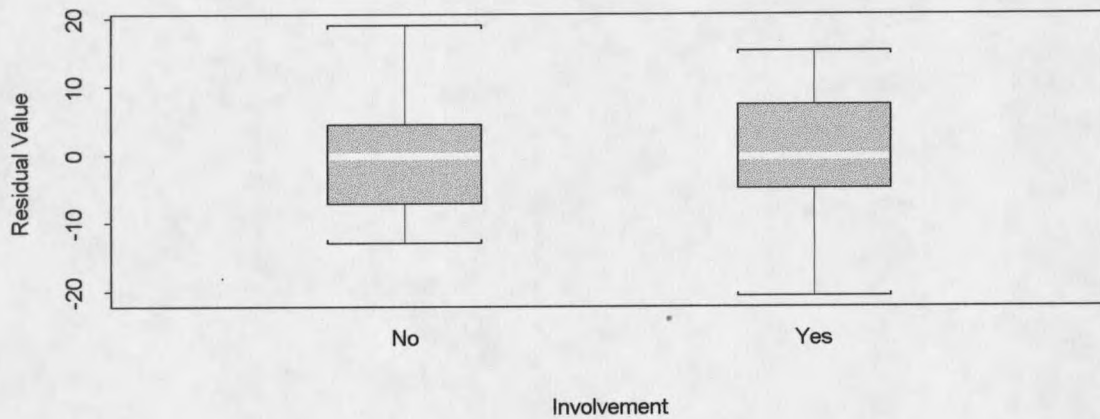


Chart

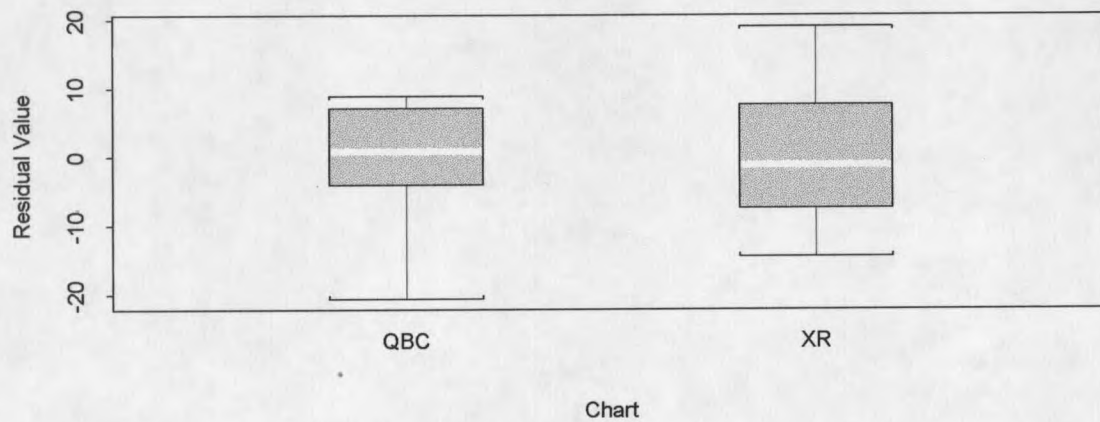
Residuals v. Fitted values



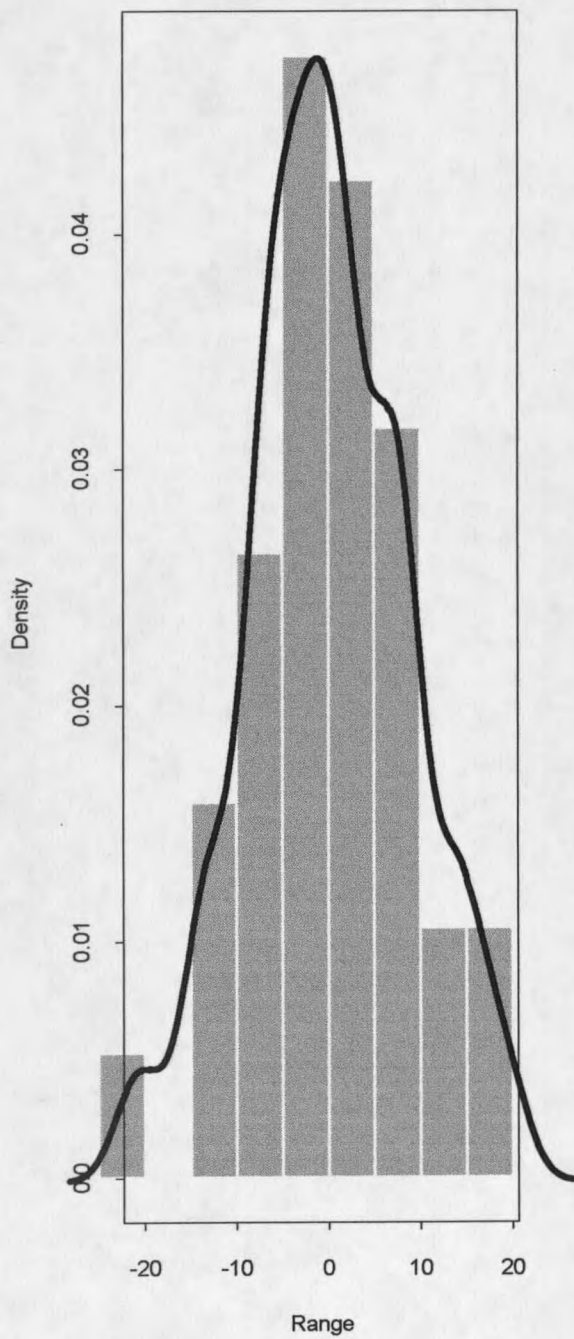
Residuals v. Involvement



Residuals v. Charts



Residual Distribution



Normal QQ Plot of Residuals

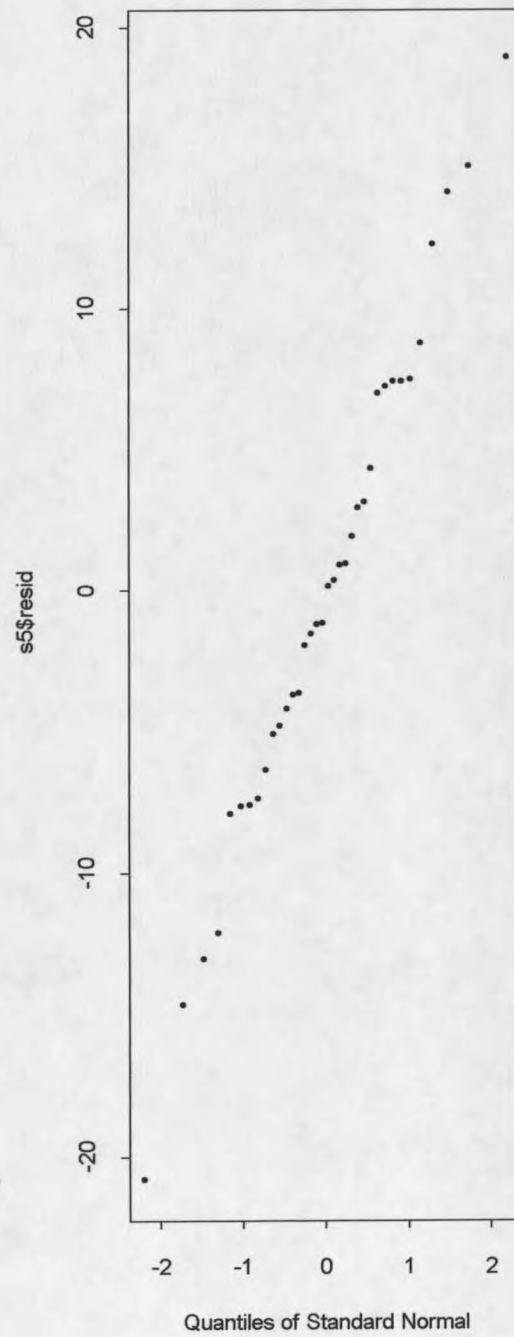
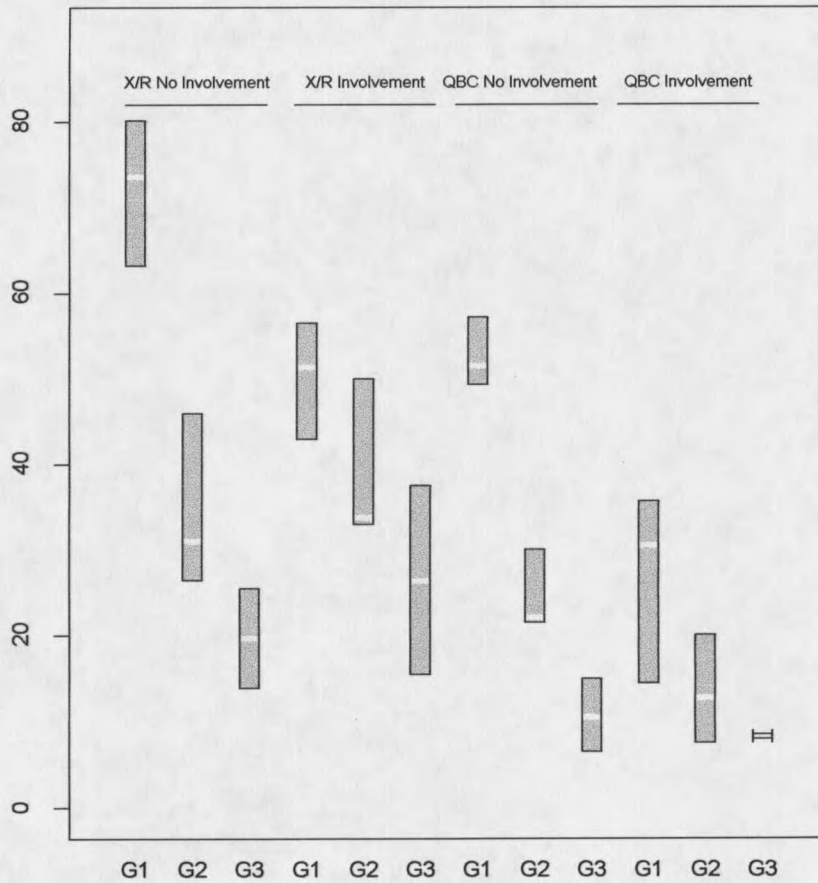


Chart-Involvement Error by Group



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