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Abstract
Culturally responsive mathematical modeling empowers teachers to build on the outofclass resources that students bring to the classroom and empowers students to draw on their identities and experiences to inform mathematical work and take action. While professional development can support teachers' learning of culturally responsive mathematics modeling, research on classroom enactments is limited. The aim of this study is to understand teachers' practices for enacting culturally responsive mathematical modeling, including the opportunities and challenges they face. Data sources include 31 videotaped modeling lessons from 13 elementary grade teachers (from across grades K-4) who participated in a yearlong professional development program. Lessons were coded using an observation tool that attended to varied dimensions of teaching modeling, and analyzed for patterns and alignment with culturally responsive teaching practice. Findings included areas of strength which suggest entry points for teachers who are new to culturally responsive mathematical modeling, and areas of challenge where teachers likely require more support. Findings also identify practices for teaching modeling that align with culturally responsive mathematics teaching, charting paths for strong practice in dimensions that are both challenging and generative for culturally responsive teaching. Our findings can inform professional development and teacher education efforts focused on mathematical modeling.

Keywords Mathematical modeling · Culturally responsive mathematics teaching · Equity · Professional development · Elementary education

Introduction

Mathematical modeling is an iterative process that includes phases such as problem posing, making assumptions and identifying important quantities and variables, and building, testing, and revising mathematical models to inform decisions in relevant, real-world situations (Anhalt et al., 2018; Kaiser, 2017; Lesh & Zawojewski, 2007; Pollak, 2011). In K-8 classrooms, mathematical modeling involves students in finding and defining problems, under-

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standing and constructing mathematical models to address those problems, and evaluating whether solutions are useful in the real world. For example, in response to water contamination concerns, elementary students might pose problems about the school's drinking water needs, using equations and visual models to project the number of water bottles needed to provide safe water (e.g. Aguirre et al., 2019; Plumb et al., 2017).

International and U.S. scholars have argued for the importance of mathematics instruction that connects to students' lived experiences, elicits student thinking, and empowers them to take action in their communities and the world (e.g. Averill et al., 2009; Hunter & Miller, 2022; Parker et al., 2017; Planas & Civil, 2009). Often referred to as "culturally responsive teaching" this instructional approach affirms students' cultural identities and supports them to develop critical perspectives and challenge inequities (Ladson-Billings, 1995). In mathematics, culturally responsive instruction "empower[s] learners to see the multiple purposes for learning mathematics, helping learners appreciate why mathematics is important in their lives, and allowing learners to believe they can succeed in mathematics" (Thomas & Berry, 2019, p. 21).

As the safe water example illustrates, modeling has the potential to advance culturally responsive mathematics instruction because it can foster understanding of socially significant contexts (Turner et al. 2024), encourage critical analysis of complex situations (Cirillo et al., 2016), and give teachers opportunities to "recognize and reward a broader range of mathematical abilities than those traditionally emphasized" (Lesh et al., 2003, p. 23). When modeling is culturally responsive, teachers intentionally elicit diverse resources from students, drawing on their identities, languages, and experiences to inform mathematical work and take action (Anhalt et al., 2018; Bartell et al., 2017; Suh et al., 2017; Aguirre et al., 2019).

While professional development has begun to support teachers' learning of culturally responsive mathematics modeling instruction (Carlson et al., 2023; Jung & Magiera, 2023; Ramos-Rodríguez et al., 2022), research on teachers' *practices* during elementary classroom enactments is limited (Stohlman & Albarracín, 2016). This study builds on prior work (Turner et al., 2024) that examined teachers' reflections on the potential of mathematical modeling to support culturally responsive mathematics teaching practice. Here we draw on classroom level data from 13 teachers and 31 modeling lessons to understand teachers' *practices* for enacting culturally responsive mathematical modeling. The following Research Questions (RQ) guide our study:

1. What are strengths and challenges in teachers' practices during modeling lessons?
2. How do teachers' practices for modeling lessons support culturally responsive mathematics teaching?

Mathematical modeling in K-8 classrooms

We grounded our approach to mathematical modeling in Anhalt et al.'s (2018) modeling cycle (Fig. 1), which drew on similar representations including Blum and Leiß (2005) and the Common Core State Standards for School Mathematics (National Governers Association [NGA] & Council of ChiefState School Officers [CCSSO] (2010)). Internationally, most K-12 research on mathematical modeling has focused on secondary (high school) versus elementary or middle grade settings (Preciado Babb et al., 2023; Zbiek et al., 2024).

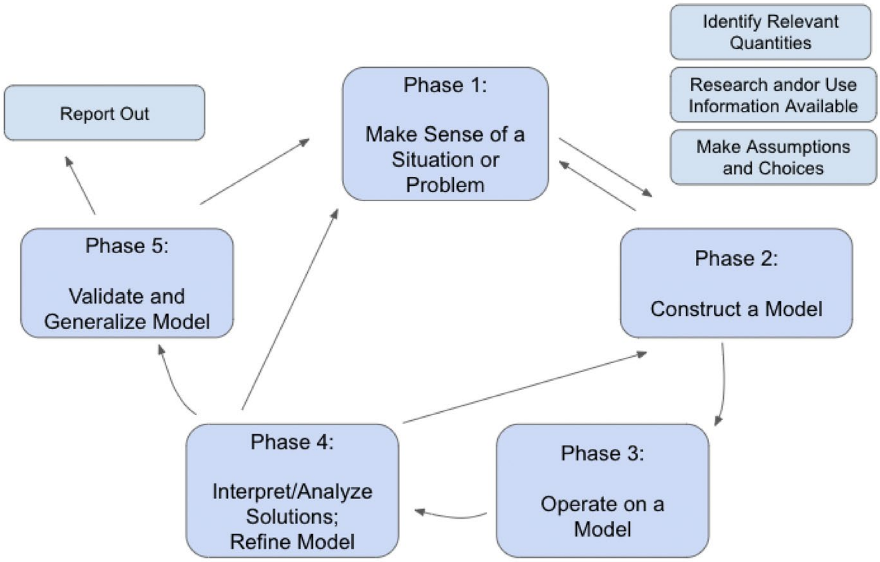

Fig. 1 The mathematical modeling cycle

Despite this trend, researchers in the U.S. (Turner et al., 2021; Chamberlin et al., 2020; Flevares & Schiff, 2013; Lehrer & Schauble, 2013; Wickstrom & Yates, 2021) and internationally (Asempapa, 2015; Bahmaei, 2011; Osana & Foster, 2021; Watters et al., 2004) have argued that elementary and middle grade students benefit from learning mathematical modeling. In contexts including Germany (Hankeln, 2020) and Australia (Parish, 2023), modeling is an essential concept and competency in national curricula. In the U.S., modeling is one of eight K-12 standards for mathematical practice (NGA & CCSSO, 2010).

Modeling gives "children a sense of agency through recognizing the potential of mathematics as a critical tool for analysis of issues important in their lives, communities, or society in general" (Greer et al., 2007, p. 89; see also Asempapa, 2015). In the elementary grades, modeling instruction tends to emphasize making sense of authentic problems and opportunities for students to connect to their experiences as they pose problems; identify relevant quantities and make assumptions; and construct models that represent their thinking (Bahmaei, 2011; Flevares & Schiff, 2013). For example, English and Watters (2005) describe modeling problems as "authentic situations that need to be interpreted and described in mathematical ways" that may include "incomplete, ambiguous, and undefined" information (p. 60). Others focus on elementary students' agency in modeling as they translate their mathematical results back to the real world through analysis and interpretation (e.g. Arnold et al., 2021). Some scholars emphasize the need for engagement in specific components of modeling like revision (e.g. Lesh & Yoon, 2007). Others acknowledge that iteration and revision may not happen in classroom settings, especially when students are satisfied with their model results (Arnold et al., 2021; Levy et al., 2016). In such cases, teachers may use age-appropriate activities like facilitating discussions about model improvements or situations that would prompt model revision, thereby giving students opportunities to see that modeling solutions are rarely perfect or static without going through a full modeling cycle.

Although there is overlap between mathematical modeling and other high cognitive demand tasks (e.g. requiring non-algorithmic thinking), there are also important differences. Modeling tasks are *authentic* because they are based in real-world situations and/or students' experiences, *complex* because they involve making decisions about how to represent and explore real-world situations with mathematics, and *open* because they allow multiple valid solutions and solution processes (Fulton, 2021; Greer et al., 2007; Tran & Dougherty, 2014). English (2013) described how 4th grade students in Australia used authentic and complex data to develop different models to rank and select swimmers for a national competition. Students made choices regarding what data to include and exclude as they built their models. Classroom contexts can also give rise to modeling tasks. In Wickstrom and Yates (2018), second graders in the U.S. posed and answered modeling questions related to sharing crackers during school. Similar to English's study, students made choices about what was important to include and exclude in their models (e.g. the cracker's color; what to do with leftovers) and developed varied strategies for distributing crackers that met their interpretation and experiences of fairness. In both studies, the tasks maximized opportunities for students to make decisions about quantities and assumptions, build varying models, and revise solutions. However, the degree to which students engaged in these modeling practices depended on teacher decisions as they implemented the modeling tasks.

Incorporating modeling in classrooms "requires a new set of teaching and learning skills" (Herget & Torres-Skoumal, 2007, p. 385). Although conceptual papers (Doerr & Lesh, 2011) and small-scale classroom cases (Chang et al., 2019; Zapata & Roth McDuffie, 2024) have acknowledged the complexity of teaching modeling in the elementary grades, as Stohlman and Albarracín (2016) note, most studies specific to the elementary grades have focused on student learning of modeling (e.g., English, 2006; Mehraein et al., 2014; Verschaffel & De Corte, 1997), rather than teacher practice. Our study addresses the critical need for research that analyzes *teacher practices* for mathematical modeling *across multiple classrooms* to better understand the opportunities and challenges teachers face.

Teacher challenges and practices for teaching modeling in K-8 classrooms

The affordances of mathematical modeling are not realized without teachers' skillful instruction. Here, we review studies that mention teacher practice for modeling lessons, highlighting challenges that arise. Many challenges around teaching modeling relate to teachers learning to cultivate environments where students' ideas and experience drive the modeling process. Although modeling creates unique opportunities for students to connect mathematics to their lived experiences (Bahmaei, 2011), teachers have reported that eliciting ideas from a broad range of students can be difficult (Turner et al., 2024) and incorporating those ideas into modeling questions and solutions makes modeling instruction unpredictable (Blum & Borromeo Ferri, 2009; Cai et al., 2014; de Oliveira & Barbosa, 2010). Centering students' ideas throughout the modeling process may be particularly challenging for teachers used to instruction that relies on teacher demonstration or other traditional approaches (Asempapa, 2015; Gould & Wasserman, 2014). Practices such as offering familiar images and artifacts (Brown, 2008) and intentionally eliciting students' real-world understandings about problem contexts (Wickstrom & Yates, 2018) before posing modeling questions may help teachers navigate this challenge.

Supporting students as they develop multiple valid solutions and use varied solution methods can also be challenging. In Jung and Brady's (2016) study, eighth grade teacher Kate expressed concern over supporting students on multi-step, open-ended modeling problems, especially as they worked in groups. Teachers navigating these challenges may direct students toward teacher-preferred methods and preconceived solutions, rather than aligning instructional scaffolds with students' conceptions of the task (Tropper et al., 2015; Warner et al., 2010). Productive teacher practices include offering prompts that redirect students toward the modeling question (Jung & Brady), eliciting student explanations (Tropper et al.) and building on student ideas about the context (Warner et al., 2010; Zapata & Roth McDuffie, 2024). Studies also suggest teachers promote social interactions around the modeling task during small-group work (Albarracín, 2021) and select and sequence student models for whole-class discussions (Fulton, 2021) to support students as they discuss, justify, and compare models.

Synergy between mathematical modeling and CRMT

Our approach to mathematical modeling draws on (Zavala et al., 2024) framework for culturally responsive mathematics teaching (CRMT). The framework consists of three interconnected strands (Knowledge and Identities, Rigor and Support, and Power and Participation) which each include three components that inform teacher practice (see Figure 2). Below we introduce each strand and discuss synergy with mathematical modeling.

The *Knowledge and Identities* strand focuses on how students' identities, out-of-school experiences, and mathematical knowledge are intertwined (Carlson et al., 2023). Modeling has the potential to align with this strand because it can be rooted in real-world situations that are relevant to students and their communities (Brown, 2008; Bonotto, 2007; Stankiewicz-Van der Zanden, 2021). Furthermore, modeling may create opportunities for teachers to de-emphasize correctness and computation, focusing instead on ways students interpret situations and the potential for more than one "right" answer (Lesh & Yoon, 2007; Wickstrom & Yates, 2021). In doing so, modeling has the potential to *rehumanize* mathematics classrooms by sustaining focus on students' curiosity and creativity and the various ways they express mathematical ideas. For example, Stankiewicz-Van der Zanden et al.'s (2021) study of a kindergarten modeling lesson recounted ways students drew on their experiences attending parties, eating too much, and dealing with leftovers as they interpreted "enough" when planning to serve cupcakes at a party. They concluded that children can be creative during modeling, "as students draw on their funds of knowledge and wonder about their world" (p. 308).

Rigor and Support focuses on the extent to which mathematics teaching gives all students, including multilingual learners, access to complex mathematical ideas. The potential for synergy with modeling lies in the ways mathematical modeling provides access to high cognitive demand tasks, sustains focus on mathematical practices like comparing and evaluating solutions, and creates opportunities for students to develop understanding of core mathematical ideas (Albarracín, 2021; English, 2006; Fulton, 2021). Additionally, the real-world contexts that inspire modeling tasks, coupled with the varied strategies modeling tasks elicit, may create teaching opportunities to *scaffold up,* supporting all students' sustained engagement in cognitively demanding modeling tasks (Anhalt, 2014). For example, an elementary school task that involved predicting the number of plastic bags needed to

	Dimension	Definition	Potential Teaching Practices in Modeling
Knowledge & Identities	Cultural & Community Funds of Knowledge	Helping students connect mathematics with relevant/authentic issues or situations in their lives	Inviting students to reflect on family or community experiences when making sense of task contexts, making assumptions and creating models
	(Re)Humanizing Mathematics	Supporting creativity, broadening what counts as mathematical knowledge, & affirming positive math identities for all students	Inviting and affirming varying conceptions of "fairness" when students make assumptions in tasks related to distributing resources
	Honoring Student Thinking & Ideas	Creating opportunities to elicit, express, & build on student mathematical thinking in multiple ways	Inviting students to use words, pictures, gestures, and realia as they construct models and explain their mathematical thinking
Rigor & Support	Sustaining Cognitive Demand	Enabling students to closely analyze math concepts, procedures, & problem-solving/reasoning strategies	Providing Venn diagrams to support students in comparing and contrasting models
	Scaffolding Up	Maintaining high rigor with high support for all students	Providing realia so that students can reference concrete objects as they explain and justify their models
	Affirming Multilingualism	Making space for multilingual learners to be central participants in mathematics activities	Inviting students to use their first language along with pictures and gestures when explaining and justifying their models
Power & Participation	Distributing Intellectual Authority	Distributing mathematics authority & making space for multiple forms of knowledge & communication	Inviting multiple students to pose and refine modeling questions
	Disrupting Status & Power	Disrupting status differences, entrenched stereotypes, & inequitable power relationships present in all mathematics classrooms	Using discourse structures that ensure all small group members have opportunities to contribute ideas and raise questions about models
	Analyzing & Taking Action	Supporting student use of mathematics to analyze, critique, & address power relationships & injustice in their lives	Inviting students to make recommendations to community stakeholders based on their model results

Fig. 2 The CRMT framework

make a school set of jump ropes elicited multiple approaches to multiplication as well as strategies ranging from reasoning with physical models to working with smaller sets of ropes and scaling up. The flexibility of the task and familiarity of the context created opportunities for the teacher to invite and showcase students' varied approaches which scaffolded their participation in modeling (Turner et al., 2021).

Power and Participation focuses on leveraging the power of mathematics instruction to confront injustice and challenge authority structures in and out of classrooms. Mathematical modeling aligns with this strand because it can disrupt traditional classroom structures that position the teacher or textbook as the source of all knowledge and because modeling tasks can create opportunities for students to develop critical consciousness as they engage with issues of access, fairness, and justice (Anhalt et al., 2018). In modeling lessons, teachers may *distribute intellectual authority* when students' questions and conceptions drive problem posing and model evaluation (Lesh et al., 2013). Modeling may also support students' *analyzing and taking action* when students reflect on how models illuminate and help communities respond to injustice. For example, in a modeling lesson focused on designing a homeless shelter Brady et al. (2023) found that students considered how those affected by problems might experience modeling solutions, "empathizing with the inhabitants and showing concern for how the interior space would be experienced" (p. 5).

Figure 2 shows (Zavala et al., 2024) framework, along with teaching practices in modeling lessons that may support CRMT. Although research suggests a potential synergy between mathematical modeling and CRMT (Anhalt et al., 2018), studies of teachers' *practices* in culturally responsive mathematical modeling lessons are needed. Specifically, the field needs to understand strengths and challenges in teachers' practices, and the ways teachers' practices support CRMT.

Methods

Research context

This study is part of a research program focused on culturally responsive mathematical modeling in elementary grades. Twenty teachers participated in year-long professional development that included monthly in-person sessions and asynchronous activities. Sessions introduced CRMT frameworks (Zavala et al., 2024) and mathematical modeling, and discussed teacher moves for supporting students' engagement in modeling practices such as posing problems and analyzing models. During sessions teachers discussed videos of modeling activities, planned lessons, and reflected on classroom enactments. Teachers also had access to digital materials (readings, modeling tasks, student work samples) to support their teaching.

Participants

This analysis focused on the 13 teachers listed in Table 1. Teachers were selected because we videotaped at least two different types of modeling tasks in their classrooms; we excluded 7 teachers that did not meet these criteria. Teachers taught kindergarten (age 5–6) through fourth grade (age 9–10), with classes of 15 to 25 students. Schools at the Southwest, Northwest and MidAtlantic sites served racially and linguistically diverse students, with significant numbers of multilingual learners. The Mountain West site schools served predominantly white students with a small but growing population of multilingual learners. All classrooms included students designated for special education services, and the racial and linguistic background of students mirrored those of their school population.

Data sources

Data sources included two or three videotaped modeling lessons from each teacher's classroom (31 lessons total). Lessons averaged 100 min and often occurred over two or three

Table 1 Participants

Teacher (pseudonym)	Site	Grade	Number of observed Lessons	Types of modeling tasks in observed lessons	Years Taught
Niya	Southwest	K	2	Snack, Making	17
Patricia	Southwest	1	2	Snack, Making	25
Ines	Southwest	3	2	Snack, Making	27
Alexa	Northwest	K	2	Snack, Making	3
Talia	Northwest	1	2	Snack, Community	6
Elizabeth	Northwest	1	2	Snack, Community	8
Isabella	Northwest	2	3	Snack, Making, Community	1
Danielle	Mountain West	1	3	Snack, Making, Community	7
Sara	Mountain West	1	3	Snack, Making, Community	7
Jessica	Mountain West	4	3	Snack, Making, Community	17
Diana	Mountain West	4	3	Snack, Making, Community	25
Melinda	MidAtlantic	3	2	Snack, Making	2
Allie	MidAtlantic	4	2	Snack, Making	6

Grade Level Variations in Modeling Questions	**Grade Level Variations in Identifying Important Quantities and Making Assumptions**	**Sample Math Connections**
First Grade: Are these egg cartons enough to make bird feeders for our class? (Ms. Danielle) Figure A3 shows sample student model	*Identify Important Quantities:* •Number of feeders the class needs to make •Number of egg cartons available *Make Assumptions About:* •Number of birds that may visit feeder at same time •Reasonable size for bird feeder (e.g., egg "cups" per feeder)	•Reasoning about array representations •Repeated addition and subtraction •Forming and counting equal groups
Fourth Grade: What materials do we need so our class can make paper flower mother's day bouquets? (Ms. Diana) Figure A4 shows sample student model	*Identify Important Quantities:* •Number of bouquets needed •Number of petals / leaves per sheet of paper *Make Assumptions About:* •Variability in number of flowers per bouquet •Variability in number of petals per flower •Whether flowers will have leaves or just petals •Acceptable amount of leftover or extra materials	•Multi-digit multiplication and division •Informal reasoning about "average"

Fig. 3 Snack sharing modeling tasks across grade levels

days. All teachers taught different modeling tasks across the year. In *snack sharing tasks*, some teachers began with a large container of snacks, and students planned how to fairly distribute the snack. Other teachers brought a small package of snacks, and students determined how many more packages were needed. In *making tasks*, some teachers started with a known amount of materials. Others started with a finished project, and students determined the quantities of materials needed to make varying numbers of items. *Community-based tasks* often involved students collecting data to make decisions about important issues in their schools or communities. Figures 3, 4 and 5 show samples modeling tasks across grade levels.

Modeling tasks varied in complexity across grade levels, and we encouraged teachers to make adaptations that aligned with their contexts. For example, tasks for fourth and fifth graders included opportunities to work with more challenging quantities (i.e., unit rates,

Grade Level Variations in Modeling Questions	Grade Level Variations in Identifying Important Quantities and Making Assumptions	Sample Math Connections
First Grade: How many boxes of hot chocolate do we need for our winter party? (Ms. Jessica) Figure A1 shows sample student model	*Identify Important Quantities:* •Number of students and adults for party •Number of hot chocolate packets in a box *Make Assumptions About:* •Reasonable serving size •Whether extras are needed	•Modeling quantities in 10s and 1s •Repeated addition or subtraction •Addition within 100 using place value understanding •Comparing quantities
Third Grade: How long will this giant container of pretzels last our class? (Ms. Inés) Figure A2 shows sample student model	*Identify Important Quantities:* •Total number of pretzels (e.g., using visual estimation, using serving size and number of servings) *Make Assumptions About:* •Number of students who will want snack •Reasonable serving size •Snack schedule •Which quantities may vary	•Visual estimation strategies •Multiplication of 1 digit by 2 digit numbers •Addition of multi-digit numbers •Comparing quantities

Fig. 4 Making modeling tasks across grade levels

Grade Level Variations in Modeling Questions	Grade Level Variations in Identifying Important Quantities and Making Assumptions	Sample Math Connections
First Grade: What should we include in a sensory space for our classroom? (Ms. Sara) Figure A5 shows sample student model	*Identify Important Quantities:* •Set of sensory tools •Values for rating scale (1 to 5) *Make Assumptions About:* •Which categories to prioritize in rating sensory items •Whether all categories are equally important	•Addition of 5 single digit numbers •Comparing quantities
Fourth Grade: How many rocks do we need to make a friendship river rock path for our school? (Ms. Jessica) Figure A6 shows sample student model	*Identify Important Quantities:* •Number of rocks currently available •Area to cover with river rock path *Make Assumptions About:* •Variability in size of rocks •Number of rocks needed to cover a given unit of area, including variability across different sections of the path •Reasonable number of rocks for students to collect •Need for extra rocks	•Linear measurement, area measurement •Reasoning about rates (rocks per square unit) •Multi-digit multiplication and division •Informal reasoning about "average"

Fig. 5 Community modeling tasks across grade levels

fractional units), construct multi-step models with different operations, and make several assumptions. Tasks for the youngest students typically involved whole number quantities and fewer assumptions and decisions. At all grade levels students made assumptions about contexts that informed decisions about quantities and specific values (see Figs. 3, 4 and 5). Students also used different representations (i.e., physical models, equations, written steps) as they constructed models. The student models in Appendix Figs. 9, 10, 11, 12, 13 and 14 illustrate this variation. Across grades, differences in students' assumptions, quantities, and models resulted in various solutions. Teachers guided students to evaluate their solutions in relation to the problem context, and when solutions were deemed unsatisfactory, teachers invited students to consider revisions.

Observational tool development

We followed Bostic et al's (2019) multi-step process to develop an observation tool that attends to mathematical modeling and culturally responsive teaching (Turner et al., 2022). *Stage 1* involved a review of observation protocols for mathematics teaching (e.g., Walkowiak et al., 2014) and CRMT (Aguirre et al., 2013), and research on effective teaching practices for mathematical modeling. In *Stage 2* we used this review to draft an initial version of the tool. This version included 6 dimensions which focused on culturally responsive teaching practices for specific phases of the modeling process (e.g., posing problems; identifying quantities and making assumptions). Dimensions were scored on a 5-point scale, with anchor descriptors to capture low (1), average (3) and high (5) levels of practice. In *Stage 3*, a review panel of 15 scholars with expertise in mathematical modeling, CRMT, and classroom observation tools provided feedback on the tool's alignment with the modeling process and the clarity of dimensions and scoring rubrics.

In *Stage 4,* we used expert feedback to revise the tool. This revision (Fig. 6) included splitting three of the initial dimensions to allow a closer focus on teachers' culturally responsive practices in each phase of the modeling process. For example, the initial tool had a single dimension for teacher practices for identifying quantities and making assumptions. The revised tool split this into two separate dimensions. In the final tool, Dimension 1 focuses on connections to students' experiences and cultural/community contexts across the modeling process, aligned with the knowledge and identities strand of the CRMT. This dimension is scored based on teacher practice across the lesson, as connections to experiences can inform different aspects of modeling. Dimensions 2 through 9 focus on specific phases of the modeling process and are scored based on teacher practices in that portion of the lesson.

We also revised the 5-point scale for each dimension to a 4-point scale to be consistent with commonly used classroom observation tools (Piburn & Sawada, 2000), and to create clearer distinctions between levels. The 4-point scale measures different levels of teacher practice: Not Present (0), Emerging (1), Proficient (2) and Advanced (3). Levels are distinguished by the extent to which teachers use culturally responsive practices in that phase of the modeling cycle, such as sustaining high cognitive demand while providing supports that increase access, and allowing student ideas to drive decisions. Figure 7 shows scoring descriptors for Dimension 7: Analyzing or Interpreting Models and Solutions. The distinction between Proficient (2) and Advanced (3) reflects the consistency of the supports provided, how teachers elicit student justification, and whether teachers allow student ideas to influence the analysis.

S*tage 5* focused on testing the revised tool with videos of modeling lessons from prior projects (Bostic et al., 2019). These lessons spanned different grade levels and included teachers with various levels of experience teaching modeling. Four researchers independently scored each video and met to discuss scores. We used these discussions to clarify score descriptors and develop exemplars of teacher practice aligned with a given score. We repeated this process for six lesson videos, at which point no additional clarifications were needed.

Dimension	Focus of Dimension	Variation in Levels of Practice
1. Connections to Out-of-Class Experiences and Cultural/Community Contexts (C)	Teachers support students to make connections to out-of-class experiences and/or cultural and community contexts. Connections inform modeling work - influencing decisions made or actions taken in any phase of modeling process.	•Connections present through the modeling cycle •Student ownership of connections •Connections inform modeling
2: Making Sense of the Context / Situation (MS)	Teachers support students to make sense of the context, solicit students' ideas or questions about the task context, and focus students on key considerations related to context.	•Presence of supports •Intensity of teacher solicitation •Focus on key considerations
3. Posing Problems (PP)	Teachers build on student ideas to pose the modeling problem. Teachers support students to ask and analyze mathematical questions.	•Student ownership of problem posing •Support asking math questions
4. Identifying Important Quantities (IQ)	Teachers support students to identify key quantities and to decide on a specific value for one or more quantities. Teacher asks students to explain the relevance of key quantities.	•Student ownership of quantities •Allowing variation in quantities •Support for explanations
5. Making Assumptions (AS)	Teachers support students to make /state assumptions, and to justify the relevance and reasonableness of assumptions.	•Student ownership of assumptions •Support for explanations
6. Building and Operating on Models (BM)	Teachers facilitate student work as students create and operate on models, soliciting student ideas and supporting students to justify work.	•Student ownership of models •Support for justification
7. Analyzing or Interpreting Models and Solutions (AI)	Teachers provide structures to support analyzing models or solutions and support participation so student ideas influence discussion.	•Presence of analysis supports •Student ownership of analysis •Support for justification
8. Revising Models (RV)	Teachers support revision, ensuring that student contributions play a central role in model revisions, and supporting students to justify.	•Student ownership of revision •Support for justification
9. Reporting Out (RO)	Teachers support students to report and explain their work. Teachers provide students with options for reporting out their results.	•Student ownership of report out •Support for student choice

Fig. 6 [Project] observation tool

Analysis

Using the observation tool, each of the 31 lessons was scored independently by randomized pairs of research team members using a process adapted from prior projects (Foote et al., 2020; Walkowiak et al., 2014). To capture details of teacher practice in the context of broader lesson activities, scorers watched videos in eight-to-ten-minute segments, pausing at transition points such as the end of an activity to script teacher and student talk and

(0) Not Present	(1) Emerging	(2) Proficient	(3) Advanced
The teacher **does not support the class** in analyzing models or solutions.	The teacher provides **limited support** for students in analyzing models or solutions. The teacher offers **overly broad prompts** that fail to direct attention to important ideas; OR **overly directed prompts** that focus on clarifying specific model components but not how specific components impact the model/solution. - AND - The **majority of talk is directed from the teacher to the students,** and the teacher acts as the ultimate authority.	The teacher provides **some support** for students in analyzing models or solutions. The teacher provides **at least one structure to support analyzing or interpreting** models such as probing questions and invitations to compare and contrast models. - AND - The teacher **supports student participation in analysis** but the *teacher still acts as final authority.* The teacher prompts students to explain their analysis, but prompts lead to a **brief and/or partial discussion** of relationships among the problem posed, quantities, assumptions, models, and solutions.	The teacher **consistently supports** students in analyzing models or solutions. The teacher provides **multiple structures to support** analyzing or interpreting models or solutions, such as probing questions and invitations to compare and contrast models. - AND - The teacher **supports student participation in analysis** so that *student ideas influence the analysis discussion.* The teacher prompts students to explain their analysis, and prompts lead to a **more substantive discussion** of relationships among the problem posed, quantities and assumptions, models, and solutions.

Fig. 7 Scoring descriptors for D7: analyzing/interpreting models and solutions

actions. This resulted in detailed lesson logs which included evidence related to observation tool dimensions. At the end of a lesson video, scorers compared evidence to the dimension descriptors to determine which descriptor (and associated score) best aligned with the evidence. Final scores, with evidence, were noted on a coding sheet. Pairs met to resolve score differences through discussion of relevant evidence and establish final scores. We calculated agreement between each scorer's independent dimension scores and the final scores. All scorers met a 80% threshold (Miles & Huberman, 1994), with an average agreement with the final scores of 83%.

Analysis for **Research Question 1** focused on strengths and challenges in teachers' practices for teaching mathematical modeling. *First***,** we investigated mean teacher scores by dimension across teachers and for each type of modeling lesson. Our goal was to determine how strengths and challenges were consistent or varied across the different modeling tasks. *Second*, for each dimension, we explored mean scores by grade level to determine whether specific dimensions of teaching culturally responsive mathematical modeling were more or less accessible to teachers at different grade levels. We clustered teachers into two grade-level bands (K-1, 7 teachers; 2–4, 6 teachers) that reflected the distribution in our data set.

Analysis for **Research Question 2** focused on how teacher practices for modeling lessons supported CRMT. ***First***, for each dimension, we analyzed the evidence of teacher practices to understand the range of ways that teachers supported particular dimensions of culturally responsive math modeling. For example, for Dimension 1: *Connections to Students' Out-of-Class Experiences*, we selected all lessons (n = 10) that evidenced teacher practices for connecting to student experiences, including lessons in which teachers scored at the emerging (n = 7), proficient (n = 0), or advanced (n = 3) levels. We then reviewed the coding sheet evidence to identify different teacher practices for making these connections. Similarly, for Dimension 8: *Revising Models*, we selected all lessons (n = 10) for which teachers scored at the emerging (n = 7), proficient (n = 3) or advanced (n = 0) levels, and used coding sheet evidence to identify teachers' practices for supporting model revision. We repeated this process for all dimensions. Our focus was not on comparing practices across levels (i.e., emerging versus proficient or advanced) but rather to understand variations and patterns in teacher practice for each dimension. For this reason, we included all lessons that included at least emerging evidence of teacher practices for a given dimension and excluded lessons for which teachers scored a 0.

Second, we considered how teachers' practices for each dimension supported different strands and components of the CRMT framework. We used analysis memos to document specific practices and their potential connections to CRMT. For example, teacher practices for *Dimension 6: Building and Operating on Models*, included inviting students to use varied tools and representations to build models (realia, pictures, words, equations), which supported the *honoring student thinking and ideas* component of CRMT. Similarly, teacher practice for *Dimension 8: Reporting Out*, included supporting students to share models and solutions in different forms (orally, through pictures and physical representations), which supported the *scaffolding up* component of CRMT. We repeated this process across dimensions and used patterns across memos to develop themes.

Findings

The first section of our findings summarizes areas of strength and challenge in teachers' practices for teaching culturally responsive mathematical modeling. Next, we present themes related to how teacher practices supported different strands and components of CRMT.

Strengths and challenges in teachers' practices for culturally responsive math modeling

We first review trends and average dimensions scores across grade levels and modeling tasks and then summarize strengths and challenges in teachers' practices.

Strengths in teachers' practices

Table 2 displays average teacher scores by dimension, modeling lesson type and grade level band. Scores ranged from 0 (not present) to 3 (advanced practice). We observed several areas of strength across teachers, including supporting students to (a) make sense of the context/situation (Dimension 2, average score of 2.78 (K-1) and 2.50 (2–4); (b) identify important quantities (Dimension 4, average score of 1.94 (K-1) and 2.00 (2–4); and (c) build and operate on models (Dimension 6, average score of 2.13 (K-1) and 1.97 (2–4). These strengths, highlighted in green in Table 2, are elaborated below.

Making sense of modeling contexts

Teachers enacted multiple strategies for helping students make sense of the modeling tasks' real-world contexts, including sharing images, realia, or videos related to the context, and inviting students to share relevant experiences (e.g., "When you have this snack, how much

Table 2 Average teacher scores by dimension, type of modeling lesson and grade level

Dimension	All Lessons			Snack Sharing			Making			Community		
	All n=31	K-1 n=16	2-4 n=15	All n=13	K-1 n=7	2-4 N=6	All n=11	K-1 n=5	2-4 n=6	All n=7	K-1 n=4	2-4 n=3
1:C	0.47	0.72	0.20	0.15	0.14	0.17	0.27	0.40	0.17	1.35	2.13	0.33
2:MS	2.65	2.78	2.50	2.69	2.71	2.67	2.41	2.80	2.08	2.93	2.88	3.00
3:PP	1.10	1.25	0.93	1.08	1.29	0.83	1.05	1.10	1.00	1.21	1.38	1.00
4:IQ	1.97	1.94	2.00	1.85	1.57	2.17	2.00	2.40	1.67	2.14	2.00	2.33
5:AS	1.03	1.03	1.03	1.08	0.71	1.50	0.82	1.10	0.58	1.29	1.50	1.00
6:BM	2.05	2.13	1.97	2.08	2.14	2.00	2.09	2.30	1.92	1.93	1.88	2.00
7:AI	1.11	1.31	0.90	1.00	1.29	0.67	1.36	1.50	1.25	0.93	1.13	0.67
8:RV	0.63	0.84	0.40	0.69	1.00	0.33	0.32	0.70	0.00	1.00	0.75	1.33
9:RO	1.69	2.03	1.33	1.46	1.71	1.17	1.95	2.30	1.67	1.71	2.25	1.00

Rows highlighted in green reflect areas of strength (proficient to advanced practice); rows highlighted in red reflect challenging practices for teachers; unhighlighted rows reflect areas of emerging practice for teachers

do you like to eat?" "How do we share snack at school?"). Individual think time followed by partner and group discussion were common strategies to promote broad participation in making sense of contexts. All thirteen teachers evidenced this strength, with all but two scoring at the proficient (2) or advanced (3) level on all three lessons. Notably, in the community modeling tasks (average score of 2.93), teachers led rich discussions of images or topics (e.g., "What do you notice/wonder in this image?" "What does it mean for our classroom library to be fair?").

Identifying important quantities

Teachers also evidenced strong practices for supporting students to identify important quantities across the three types of modeling lessons. Twelve of thirteen teachers scored a 2 or above (proficient practice) on at least one modeling lesson, and teachers' scores on this dimension increased slightly across the year (average score by lesson of 1.85, 2.00, and 2.14). This shift towards more proficient practice was most notable among the K-1 teachers, four of whom scored at the emerging level (1) on the first modeling lesson, snack sharing, while all scored at the proficient (2) or advanced levels (3) on the second modeling lesson, making. These teachers became more adept at focusing students' observations on relevant quantities and on how quantities might vary through strategically selected images and prompts that drew students' attention to variation (e.g., "How are the materials in these picture frames similar/different?). Emerging scores on this dimension reflected instances when teachers constrained choices about quantities. For example, in the making modeling task several grades 2–4 teachers identified quantities for students and then provided students with options for values, rather than leaving these decisions open.

Building and operating on models

A third area of strength consistent across grade levels and modeling lessons was supporting students to build and operate on models (average scores of 2.08, 2.09, and 1.93 on the snack sharing, making and community-based modeling lessons). All 13 teachers scored a two or above (proficient practice) on at least one modeling lesson, and 6 of the 13 teachers demonstrated proficient practice on all lessons. Teachers' strategies to support model building included encouraging students to use visuals, concrete tools, tables, and equations. Emerging practices for supporting model building were particularly prevalent among the third and fourth grade teachers (5 of the 7), who were inconsistent in fostering student ownership (e.g., they parsed model building into specific steps to follow) and eliciting student justifications. Advanced practice included teacher actions to maintain student ownership over models (i.e., affirming diverse model representations), as well as prompts to support students to justify their models.

Challenges in teachers' practices

The most challenging practices (in red on Table 2) were supporting students to (a) connect to out-of-class experiences (Dimension 1, average teacher score of 0.72 (K-1 teachers) and 0.20 (2–4 teachers); and (b) revise models (Dimension 8, average teacher score of 0.84 (K-1 teachers) and 0.40 (2–4 teachers).

Connections to out-of-class-experiences

The most challenging dimension was Dimension 1: making connections to students' experiences in out-of-class, cultural/community-based contexts to support their work in modeling. Five of the 13 teachers scored a 0 (practices not evident) on all modeling lessons, and only 3 of the 31 observed lessons (from 3 first grade teachers) reflected advanced practice. These three lessons featured community-based modeling tasks related to diverse representation in library books and the benefits of sensory spaces. Five teachers evidenced emerging practices for making connections to students' out-of-class experiences, most often via isolated invitations for students to share relevant experiences as part of a task launch.

Revising models

Another area of challenge for both K-1 and 2–4 teachers was Dimension 8: Revising Models. There were no instances of advanced practice in this dimension. Six lessons, taught by 4 teachers, included proficient practice, and five teachers evidenced emerging practices for supporting model revision in one or more lessons. Emerging practices often included overly broad revision prompts (e.g., Is there anything you want to change?), or demonstrating model revisions without creating opportunities for students to engage in revising models. Supporting revision was particularly challenging for teachers during the making modeling lessons, wherein only 3 teachers scored higher than 0. While teachers' average scores for revision were slightly higher on the last (community) modeling lesson, this remained a challenging practice across grade levels.

Summary of research question 1 findings

In modeling lessons, teachers evidenced *strengths* in supporting students to make sense of real-world contexts, identify important quantities, and build and operate on models. Teachers' practices also reflected areas of *challenge*, including connecting to students' out-of-class experiences, and supporting model revision, and areas of *emerging practice*, including supporting problem posing, assumptions and model analysis. These challenges and areas of emerging practice are not surprising as these practices are less common outside of modeling lessons and ones teachers may not employ in daily mathematics instruction. In the next section, we elaborate on teachers' practices for these areas of challenge and emerging practice.

How teachers' practices for modeling lessons support CRMT

While all dimensions evidenced teacher practices that aligned with CRMT, given space limitations and our focus on understanding the unique potential of mathematical modeling to support culturally responsive instruction, in this section, we highlight teacher practices related to four dimensions (1, 5, 6, 7) that may receive minimal attention outside of modeling lessons (i.e., Connections to Students' Out-of-School Experiences, often as part of Posing Problems; Making Assumptions; Analyzing and Interpreting Models and Solutions; and Revising Models). Interestingly, as noted above, these dimensions were also among the more challenging for teachers. Understanding how teachers' practices in these dimensions support CRMT and at the same time help teachers to navigate some of the more challenging dimensions of teaching modeling is important as findings may provide empirical support for the potential of mathematical modeling as a lever for equity in elementary mathematics classrooms.

Connections to students' out-of-class experiences and cultural/community contexts

Teachers' practices for connecting to students' out-of-class experiences supported the *Knowledge and Identities* and *Power and Participation* strands of CRMT.

Practice: eliciting and marking students' relevant knowledge and experiences

Teachers who asked students to share out-of-class experiences often invited connections during the problem posing phase of modeling lessons. Some teachers sought to *rehumanize mathematics,* sharing their own experiences or asking students to imagine what teachers might think about as they answer real-world questions, positioning these everyday practices as contexts for mathematics. For example, in a task about sharing sleeves of crackers with tablemates, Niya, a kindergarten teacher, described her experiences sharing snacks with siblings, including what it meant to share fairly. Similarly, first-grade teacher Jessica asked students to "put yourself in my shoes," and imagine, "what will I have to consider when I am planning this hot chocolate party?" as she posed a modeling problem about planning hot chocolate supplies for a first-grade celebration (Fig. 9 shows student solutions). Teachers also *elevated students' cultural and community funds of knowledge*, explicitly marking students' connections with phrases like, "I hear you connecting to your experiences with birds in the neighborhood." Some teachers extended this practice across the modeling cycle, asking students to consider out-of-class experiences as they identified quantities and made assumptions to inform models. For example, in a modeling task about whether a set of egg cartons of varying sizes was sufficient to make bird feeders for the school, first-grade teacher Danielle invited students to use what they knew about birds in their community to think about how many birds might visit their feeders and the space the birds would need, as they made decisions about their models (Fig. 11 shows student solutions).

Practice: facilitating students' connections to their feelings and priorities

Teachers also *rehumanized mathematics* by asking students to share feelings and priorities related to the problem context asking, "What does it feel like when…", "Why is this important to you?" For example, first-grade teacher Sara connected to students' social-emotional needs as she posed a modeling problem about designing a classroom sensory space. She asked students what it feels like to "flip their lid" and what might help them calm down, normalizing the need to regulate emotions during school. Sara provided examples of tools that address different sensory needs including touch, sound, and movement and invited students to share personal experiences using the tools to calm down (e.g., "Have you ever used the pedals in Ms. D's office?"). Later, students generated criteria to score different sensory tools and used each tool's total score to inform their selections (see Fig. 13). During this rating process, Sara noticed that one student, Milo, was left out of his group's discussion. Sara knew that Milo used kinetic sand at home and built on this connection to help Milo share his ideas (i.e., "Milo, what would you rate kinetic sand, for the touch criteria? … Let's let Milo come up with his idea first"). Sara used Milo's experience to *disrupt a potential status hierarchy* and position Milo's knowledge as important to the group.

Practice: facilitating connections to students' racial/cultural identities and sense of fairness

Some teachers asked students to connect to their racial and cultural identities and perspectives on fairness as they discussed problem contexts. These connections guided students' critical analysis of models and helped them *analyze situations and take action* in light of their mathematical work. For example, Elizabeth's first-grade community modeling lesson investigated concepts of fairness and racial/cultural representation in the class library. Elizabeth launched the task by asking students what it means for something to be "fair". Students described fairness in terms of racial equality (i.e., "black people and white people get the same things"), drawing on their racialized experiences and referencing prior discussions about civil rights. Next, Elizabeth presented an infographic (Huyck & Dahlen, 2019) that showed the percent of main characters in published children's books from different racial groups (1% American Indians/First Nations; 5% Latinx, 7% Asian/Pacific Islanders; 10% African American; 27% Animals or Objects; 50% White). Elizabeth asked students what the data meant to them, which prompted students to consider how the books represented their own racial identities. Raymon asked, "Ms. Elizabeth, I am both black and white, so how many (books) would that be about me?" As students realized that the infographic lacked a category for multi-racial characters and therefore did not reflect the identities of some students, Elizabeth affirmed the importance of representation, honoring students' critical analysis.

She then asked whether the main character distribution "looked fair" and whether their own classroom library might reflect similar patterns.

Hector:	It is not fair. Because 10% [African American], that [white] literally has 40 more…
Ms. Elizabeth:	You're right, 40 [percent] more than 10. What do you think, Aliyah?
Aliyah:	Why would they only make books about white people? Even though we are in America? Why don't they make equal books?
Ms. Elizabeth:	Sari …Why do you think it is fair or not fair?
Sari:	This is not fair. This one is like much bigger [50%], because the boy gets the bigger one, and this one is little
Ms. Elizabeth:	If we look at our classroom library …… What could we do with our books to see if it is fair?

Following this discussion, students decided to include a broader range of categories (e.g. mixed race) to sort their classroom books by the racial background of the main character. Next, they compared the results of their sort to the racial background of students in their classroom to determine whether the library fairly represented their diverse identities.

Making assumptions

Teachers used several practices to support students to make and justify assumptions which supported the *Knowledge and Identities* and *Rigor and Support* strands of CRMT.

Practice: focusing students on the problem context through targeted prompts

Teachers often introduced making assumptions by asking students what they needed to consider about the problem context (e.g. "What is something important to keep in mind as you make your plans?"). These questions *honored student thinking and ideas* as they supported students to draw on experiences to inform their mathematical work. Teachers then used targeted prompts to guide students' attention to aspects of the context that might require assumptions, such as how the number of people or snack days might vary. These prompts *scaffolded up,* sustaining students' engagement in this modeling practice. For example, in a lesson focused on sharing a container of snacks across the week, Inés used targeted prompts to support her third-grade students.

Ms. Inés:	What *assumptions* do we need to make for our math problem? That's kind of like, what do we need to remember when we're fair sharing?
Octavia:	The teacher's not going to eat all the extra
Nina:	Everybody has the same amount
Ms. Inés:	What else do we need to remember as we share? It sounds like you want it to be fair. But what can happen from today to Tuesday? With the numbers, with our bodies?
Saul:	There might be more or less people here

Inés recorded students' ideas and reminded them to continue to consider how assumptions about fairness and attendance might impact their models (Fig. 10 shows student solutions). Other teachers used similar targeted prompts to *scaffold* students' access to making assumptions. In a modeling task about the quantities of beads and string needed to make bracelets for community newcomers, kindergarten teacher Alexa used questions like, “Does everyone’s bracelet have to have the same number of beads?” to focus attention on quantities that might vary. These prompts supported students to express assumptions such as “some people want one bracelet, and some want two” and “bracelets can have different numbers of beads.” When students’ responses reflected implicit assumptions about the context, teachers marked students’ ideas as assumptions. For example, in a modeling task about estimating the number of rocks needed to create a friendship rock path at the school, Jessica asked her fourth-grade students what they should consider as they build models. When one student commented on the number of rocks available, Jessica framed this idea as an assumption.

Jacob:	We have 320 rocks, because all the students in each class is 80, and then 80 times 4 is 320, because everyone was supposed to bring in 4 rocks
Ms. Jessica:	Ok, so you are *assuming* that everyone brought in 4 rocks

Explicitly marking students’ ideas as assumptions was another way that teachers *scaffolded up* to support making assumptions relevant to modeling problems (Fig. 14 shows student solutions).

Practice: ensuring that aspects of the context remain open to assumptions and decisions

Teachers also supported assumptions by ensuring that aspects of the problem context remained open to variation and decisions. These practices *sustained* a *high cognitive demand* as students used understandings about the context to inform assumptions about quantities and relationships in their models. In a lesson about modeling how many granola bar boxes were needed to provide snacks for the grade level, two fourth grade teachers, Allie and Melinda, ensured that specific quantities (e.g., number of students, snacks per student) remained open, which prompted students to make and justify assumptions. When students suggested visiting each class to find out how many students were present and would like snacks, the teachers responded that visits were not possible, but instead, students could generate “a good enough guess” with their groups. This prompted students to estimate the number of students using their knowledge of classroom spaces. One group assumed that larger classrooms have more students, and another used the number of tables in particular classrooms to decide how many students to include. We observed similar patterns in other lessons; teachers left problem components open and repeatedly reminded students to work with peers to make assumptions and decisions, which *sustained a high cognitive demand*.

Practice: prompting students to explain the reasonableness of assumptions

Teachers also *sustained cognitive demand* by prompting students to explain the reasonableness of their assumptions. During a modeling lesson about predicting how many days it would take the class to read 100 books, Elizabeth *honored student thinking and ideas* by encouraging her first graders to use their experiences to justify assumptions.

Ms. Elizabeth:	Do you think we will read the same number of books every day?
Students:	No
Ms. Elizabeth:	Why not?
Gia:	Because the same people won’t be here every day

Liam:	We go to library on Tuesdays
Ms. Elizabeth:	On Tuesdays you go to library, and then do you read more books or less books?
Class:	More!
Ms. Elizabeth:	Why?
Aliyah:	Because library is where we get books and mostly where we read them

As the discussion continued, students offered additional experiences to explain their assumption that the number of books read would vary from day to day (e.g., “Saturdays I go to the library with my family”). Elizabeth *honored student thinking and ideas* by revoicing students’ reasoning and affirming that their experiences should inform their assumptions.

Analyzing and interpreting models and solutions

Teachers’ practices for supporting students to analyze models aligned with the *Rigor and Support* and *Power and Participation* strands of CRMT.

Practice: supporting students to compare models through prompts, realia and graphic organizers

Teachers used repeated analysis prompts (e.g., “How are these two models the same/different?”) along with graphic organizers, images, or realia to *scaffold up,* supporting students to notice similarities and differences among models. In a modeling task about estimating the number of popsicle sticks needed to make picture frames for a class display, Patricia, a first-grade teacher, recorded groups’ solutions and key model quantities in a table. Patricia drew students’ attention to variation in solutions asking, “Which group uses the least number of sticks? Why do you think that is?”.

Teachers also invited students to compare their decisions to those of other groups. This practice *distributed intellectual authority* among students because analysis discussions remained grounded in students’ model interpretations. For example, in a task focused on sharing a box of goldfish cracker packets across a week, Alexa, a kindergarten teacher, projected a visual model of each group’s solution (Fig. 8). As Ruby reported her group’s model, Alexa asked her to compare her group’s “bags of goldfish per day” quantity to that of other groups.

Fig. 8 Visual models of snack-sharing solutions

Ms. Alexa:	Ruby, how did you decide to do two bags of goldfish? Julio did four bags of goldfish and Saray did three bags of goldfish, but you did two bags of goldfish, why did you decide to do two?
Ruby:	Because there's me and Sol at my table

Alexa then invited other students to reason about the relationship between the number of leftovers and the number of packets distributed each day.

Ms. Alexa:	Why do you think Ruby had more extras than the other groups?
Saray:	Because she only put two
Ms. Alexa:	Because she only put two bags for each day. What about the other people, like Julio or you, Saray? Did you put two?
Saray:	[shakes head]
Ms. Alexa:	No, different numbers

Alexa's practice of alternating prompts focused on interpreting specific model components ("What did you decide for day 2?") with prompts about how model components impact solutions ("Why do you think Ruby has more extras than other groups?") *scaffolded up* supporting students' engagement in the rigorous work of analyzing and comparing models.

Practice: offering "what if" scenarios and prompts about variation and optimization

Some teachers presented "what if" scenarios that asked students to consider how changes in the situation might affect models or solutions to support analysis. These practices *sustained* a *high cognitive demand* by encouraging students to attend to model structures as quantities changed. For example, after students reported their models for estimating how long it would take their class to read 100 books, first-grade teacher Elizabeth posed a what-if scenario, asking how models or solutions would change, "if we joined together with [Ms. T's] class to read 100 books." Students recognized that they would either need less time to read the books ("less days") or would read more books in the same amount of time.

A few teachers asked students to consider variation of different model components. In a task about the materials needed to make paper flower bouquets for a Mother's Day celebration, fourth grade teacher Diana asked students to consider which model quantities could vary. Students noted that while some groups created bouquets with the same number of flowers, their total number of petals needed varied (Fig. 12 shows student solutions). Diana prompted students to consider how different model components might explain this variation.

Ms. Diana:	Why is that [number of petals] so different a number? The first team came up with a cool reason why … Maybe that's another variable that changes. Luka, why is that so different?
Luka:	Everybody has different flower [designs] and [a] different number of flowers, like the actual [number of] flowers and different number of petals
Ms. Diana:	Cool, that's really something to consider. Anya, is there anything else to consider?
Anya:	How much people in each group
Ms. Diana:	How many people could be the maximum amount of people in each group?
Class:	5

By offering diverse analysis prompts, including questions about variables and maximum values, Diana *sustained cognitive demand* in analysis discussions. Diana offered these prompts in whole and small group discussions, providing students with multiple opportunities to analyze how variation in specific model components affected their solutions. Diana's practices also *distributed intellectual authority* to students through prompts that publicly connected student-generated ideas to mathematically significant concepts (e.g. "maybe that's a variable").

Revising models

Teachers used several practices for supporting students to revise models; these practices supported the *Rigor and Support* and *Power and Participation* strands of CRMT.

Practice: asking students if models are fair or reasonable

Teachers who asked students to consider the reasonableness of models used prompts such as "Is that reasonable?" or "Do you think that would be fair?" as students shared initial models and solutions. Often, prompts were directed at particular model components, such as assumptions about quantities. For example, when one group of kindergarteners presented a model that reflected an inequitable distribution of crackers, Niya asked students to consider the fairness of their decisions.

Ms. Niya:	Would it be fair if you got all the crackers, and your friends just got one? … When I share with my siblings, we all want the same. Samuel, when you share candy with your brothers, what would you say if your brothers wanted all of them, but you just got 1?
Samuel:	It's not fair for anyone to eat all the candy

Following this exchange, students reconsidered and revised their model in a way they viewed as more fair. Prompting revision in this way *sustained cognitive demand* because it supported students to reconsider their mathematical work in light of their own experiences with fairness. These prompts also *distributed intellectual authority* because students' conceptions of reasonableness and fairness motivated the revision.

Practice: adjusting information related to the modeling context

Teachers also supported revision by adjusting information related to the context after students constructed initial models. For instance, Jessica's fourth grade students created models that predicted the amount of hot chocolate and marshmallows needed for a class party. Students initially included the marshmallow bag's suggested serving size (2/3 cup of marshmallows per person) in their models. The next day, Jessica *scaffolded up* students' efforts to revise by sharing pictures of the suggested serving size (⅔ of a cup of marshmallows in a mug) so students could "see how much was inside." Students decided the image showed too many marshmallows for one person. Jessica then *distributed intellectual* authority, allowing students to determine if or how they wanted to adjust model quantities by asking "Does this help you figure out how many marshmallows you need?".

Practice: inviting students to play out the results of their models

While a less common practice, a few teachers supported revision by inviting students to implement their ideas for specific model components and then evaluate the viability of their decisions. This practice was particularly effective in *distributing intellectual authority* among students because revisions were driven by students rather than teachers. For example, in Danielle's first-grade bird feeder modeling task students first planned the dimensions of individual bird feeders and then considered how the feeder designs impacted the total number of egg cartons needed. After several minutes of planning, the class regrouped and decided that each person would make a 6-cup (6 egg carton sections) feeder. Danielle encouraged students to "test out" their models and several groups realized that a 6-cup feeder was too large to allow all students to make a feeder with the given materials. One group used a diagram of the available egg cartons to explain the need for revision.

Max:	I have an idea, count how much is on here [egg carton diagrams], and then do circles right there [around egg cups] to tell us. …
Kyla:	Guys, we should try 4 by 2, 3 by—no try 2 by 2 so 4 [cups]!
Max:	[counting out groups of four] Guys, I'm not done
Kyla:	This actually might work!
Fiona:	[counting the number of groups of 4] 7, 8, 9, we're going to have more than enough
Kyla:	[counting groups of four] We're going to have 18 [bird feeders]

Students explained that they "tried fours" and had enough (18) for all students and teachers to make a feeder (see Fig. 11). Danielle then asked if they wanted to revise their model ("So, you're thinking we should revise our thinking about the six?"). When the class regrouped, students shared their reasoning and decided to revise their 6-cup feeder to a 4-cup feeder. Danielle's practices *distributed intellectual authority* by relying on students' own model enactments to demonstrate the need for revision. Her practices *sustained high cognitive demand* by asking students to justify their proposed revisions.

Summary of research question 2 findings

Teachers' practices for several dimensions of teaching modeling were generative for supporting strands of CRMT. Aligned with the *Knowledge and Identities* strand, teachers *rehumanized mathematics* as they modeled connections between out-of-class experiences and the problem context and helped students connect modeling tasks to their feelings and sense of fairness. Because tasks were situated in familiar contexts, teachers had opportunities to *elevate students' cultural and community funds of knowledge* by marking students' connections and reminding students that out-of-class experiences could influence their modeling decisions. This *honored student thinking and ideas* because students' experiences became resources to inform their mathematical ideas.

Aligned with the *Rigor and Support* CRMT strand, teachers *scaffolded up*, using targeted prompts to focus students' attention on aspects of the situation that might require assumptions, and then *sustained a high cognitive demand* by asking students to explain their assumptions. Teachers used diverse analysis prompts and graphic organizers to help students compare models, supporting sustained engagement in the *cognitively demanding* practice of analysis. Teacher practices for supporting revision *sustained high cognitive demand* through *scaffolds* that helped students see connections between their modeling decisions and the real world and consider how hypothetical or actual scenarios might require changes to models.

Teaching practices that supported the *Power and Participation* strand were especially present in tasks that investigated fairness and/or connected to students' identities. These tasks created opportunities to show how modeling supports *analyzing and taking action* (e.g., analyzing representation in books) and *disrupting status hierarchies* by centering student needs that may be overlooked (e.g., sensory needs). Teachers *distributed intellectual authority* amongst students, ensuring that multiple voices drove analysis through targeted prompts that focused students' attention on specific components of their own and classmates' models, and by giving students agency in deciding if and how to revise.

Discussion

Our study is among the first to examine teacher practices for culturally responsive mathematical modeling in elementary classrooms. Understanding strengths and challenges in teachers' practice contributes to the research base by revealing productive entry points for teachers new to culturally responsive mathematical modeling and areas where teachers likely require more support.

The areas of strength we identified (supporting students to make sense of contexts, identify quantities and build models) are relevant to other areas of mathematics teaching and teachers likely brought skills for these dimensions into their modeling instruction. For example, heuristics for solving word problems often include sense-making and identifying and relating quantities (Depaepe et al., 2009). Additionally, understanding operations to solve contextualized problems is a central focus of elementary mathematics (Carpenter et al., 1996, 1999). Moreover, our professional development included time for teachers to adapt task contexts to relevant situations in their school communities, and to discuss students' models, including how students represented and related key quantities. These experiences likely built on teachers' pre-existing practices to support these areas of strength.

Our findings about challenging practices reveal areas where teachers may need additional support. *Connecting to students' out-of-class experiences* is facilitated when teachers have deep knowledge of how students' cultural and community contexts intersect with mathematics learning (Land et al., 2018; Moll et al., 2005; Turner et al., 2016). Teachers likely needed more support to learn about students' out-of-class experiences, and how they can support students to draw on these connections throughout the modeling process. We suspect that our professional development provided limited opportunities for teachers to practice these highly responsive moves. The fact that the only instances of advanced practice occurred during the community-based modeling lessons also suggests a link between the task's focus and opportunities for connections to students' out-of-class experiences.

Revising Models was a second area of challenge. Revising models is likely less familiar to teachers as it departs from typical problem-solving instruction (Garcia et al., 2019). Teachers may not have had enough experience considering revision as it applies to modeling to score at the proficient or advanced levels. Although we created opportunities for teachers to engage in the entire modeling cycle in our first professional development, revision was not an explicit focus until late in the school year. Revising requires moving beyond a right-wrong dichotomy to support students to revise decisions made earlier in the modeling process, and to consider how changes to quantities and assumptions impact solutions. Another explanation is that most revising occurred at the end of the modeling cycle when instructional time was limited, an issue common in classroom settings (Levy et al., 2016).

While these dimensions of teaching modeling represented areas of challenge in the aggregate, some teachers demonstrated practices at the proficient and advanced levels. As all teachers received comparable exposure to the topics addressed in these dimensions during professional development, future research should investigate what might have contributed to some teacher's ability to engage in these practices at higher levels than others (e.g. teacher identities or prior experiences; school structures; alignment of modeling with curricular resources).

Our findings also offer detailed accounts of ways teachers' practices took advantage of unique attributes of modeling to create opportunities for culturally responsive teaching, particularly practices for connecting to students' out-of-class experiences, and for supporting students to make assumptions, analyze and revise models. For example, making assumptions rarely receives attention outside of modeling and requires modelers to draw on their own understanding of situation contexts. Teachers who ensured tasks remained open to assumptions and decisions and pressed students to explain the reasonableness of their assumptions engaged in culturally responsive teaching by *honoring student thinking and ideas* and *sustaining high cognitive demand.* Similarly, revising requires modelers to draw on their own sensibilities as they consider model results in the real world. Teachers who supported revision *sustained cognitive demand* by inviting students to verify the results of their models and consider whether their proposed solutions were satisfactory. In doing so, teachers also *distributed intellectual authority* by letting students, rather than teachers, drive revision.

Teachers' practices for connecting to students' out-of-class experiences repeatedly centered students' identities by elevating students' knowledge, honoring students' feelings and priorities, and by affirming students' racial or cultural identities and sense of fairness—practices that aligned with the *Knowledge and Identities* and *Power and Participation* CRMT strands. These findings echo other studies that point out the potential of modeling to connect to students' experiences and cultural and community contexts (Brown, 2008; Bonotto, 2007; Stankiewicz-Van der Zanden, 2021). Given that this dimension of teaching modeling was among the most challenging for teachers, the practices we observed are important because they may help more teachers facilitate and sustain these connections throughout the modeling cycle.

Notably, teachers' practices when supporting students to make assumptions, and to analyze and revise models all connected to the *Rigor and Support* strand of the CRMT framework. This suggests these dimensions of teaching modeling may be generative for culturally responsive teaching because they create opportunities for students to engage in challenging mathematics (see also Stohlmann & Albarracín, 2016). However, research suggests that a focus on rigor is insufficient for equity-oriented teaching as it can reinforce existing conceptions of who can and cannot engage in challenging mathematics (Jackson et al., 2017). Thus, it is important that all but one of the teachers' practices we observed within these dimensions also aligned with the *Power and Participation* or *Knowledge and Identities* CRMT strands. For example, teachers *honored student thinking and ideas* by reminding students to draw on their own experiences to inform mathematical work, and they *distributed intellectual authority* through targeted prompts that grounded model analysis and interpretation in student conceptions.

By describing how specific teacher practices in modeling lessons were generative for culturally responsive teaching we offer empirical evidence that culturally responsive modeling instruction contributes to equity-oriented mathematics teaching and learning.

Implications and conclusions

While our study contributes important insights about key practices related to culturally responsive mathematical modeling, it has limitations. First, our study only includes 13 teachers. Including more teachers, especially at the same grade level and/or in the same school might help us better identify sources of variation in teacher practice. Second, our sample is composed of teachers who agreed to participate in our professional development, teach multiple modeling lessons, and to be videotaped during classroom instruction. Thus, our sample may be biased toward teachers who are inclined to innovate.

Working within this sample, we identify teaching strengths and challenges encountered by K-4 teachers. Our analysis demonstrated that even teachers new to modeling can take up practices that align with CRMT. Professional development designers can leverage our findings as they consider entry points for culturally responsive mathematical modeling and dimensions that need more support. Our findings also have implications for elementary mathematics teaching. For example, presenting students with "what if" scenarios holds promise for maintaining high cognitive demand by helping students focus on important mathematical ideas during analysis; a practice that could extend to other contextualized problems. Similarly, identifying contexts for mathematical work that connect to students' identities and sense of fairness may encourage students to draw on out-of-class experiences and help them see how mathematics informs relevant issues in their lives. Future studies should examine how teachers' practices for navigating challenging aspects of culturally responsive modeling could support other areas of mathematics teaching. In conclusion, our study contributes new empirical evidence across K-4 classrooms of the unique potential of mathematical modeling to advance CRMT.

Appendix: Sample student solutions to different modeling tasks

See Figs. 9, 10, 11, 12, 13 and 14.

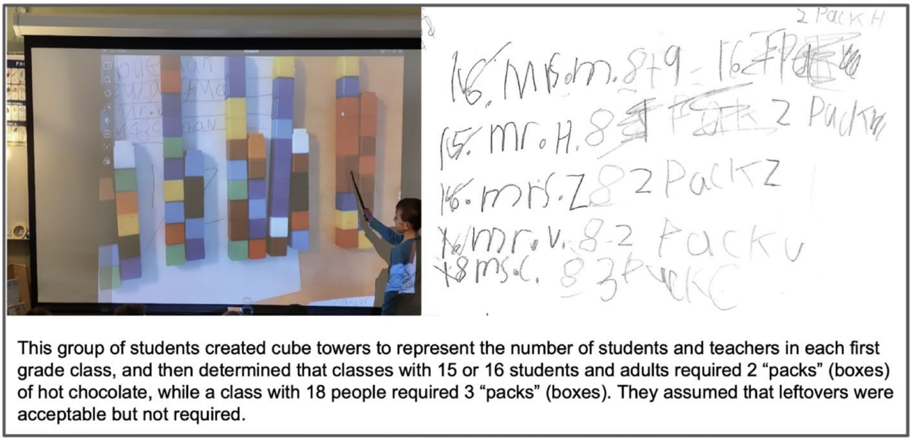

Fig. 9 1st grade student model for number of hot chocolate boxes needed

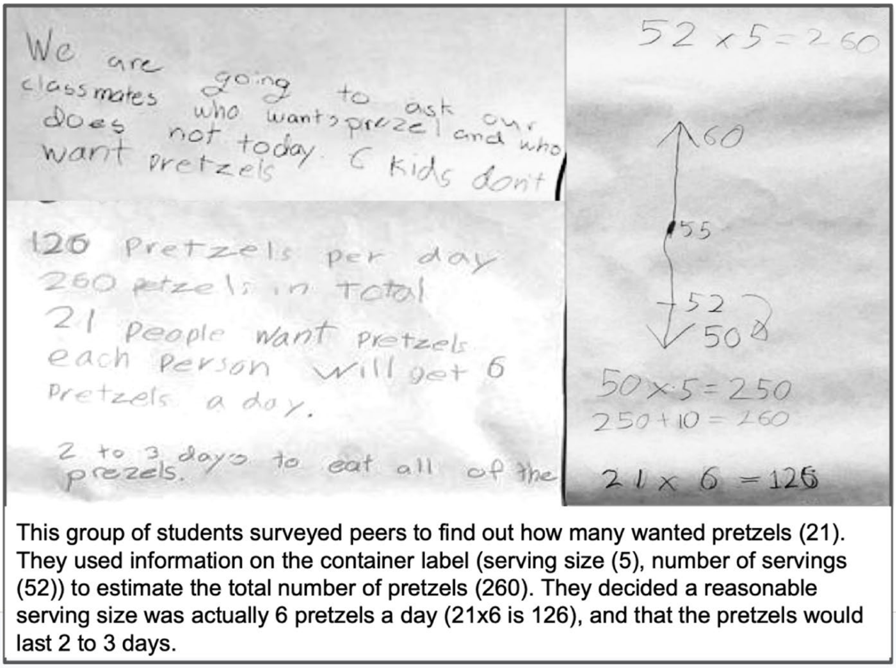

Fig. 10 3rd grade student model for sharing pretzel container

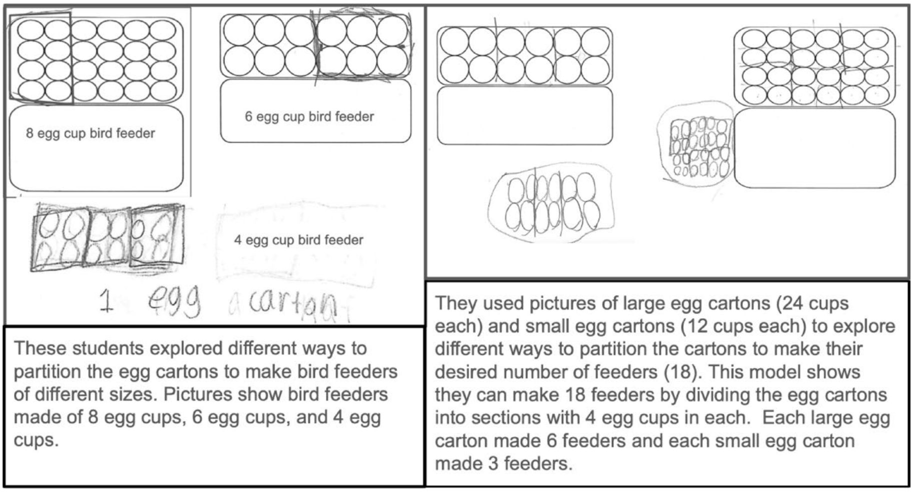

Fig. 11 1st grade student model for making bird feeders

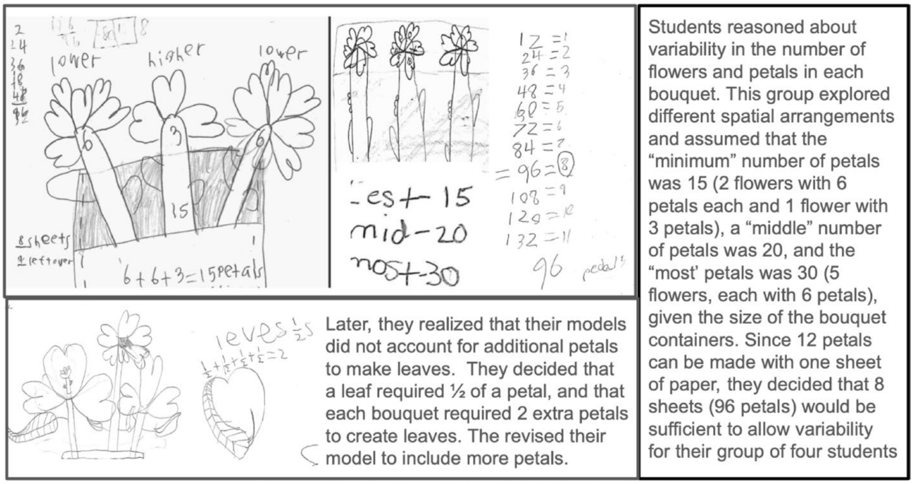

Fig. 12 4th grade student model for making paper flower bouquets

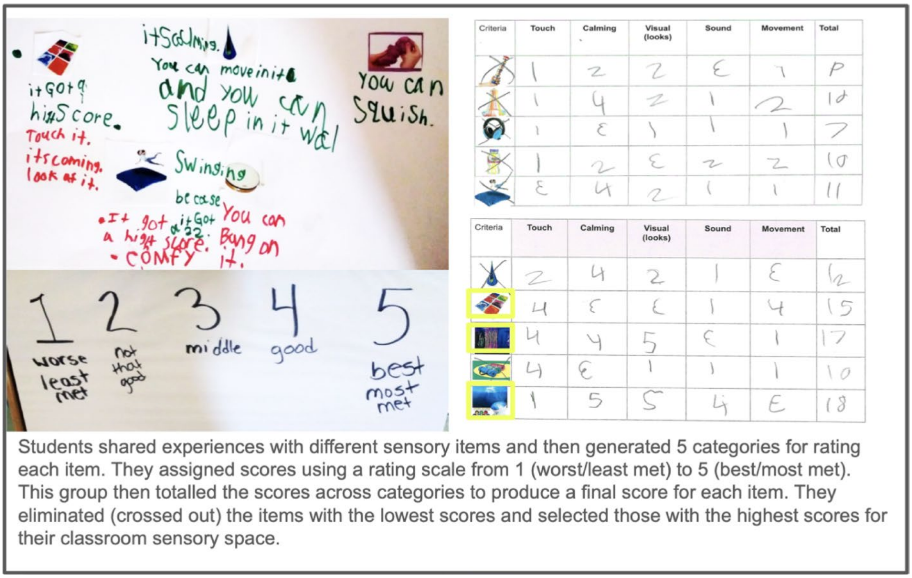

Fig. 13 1st grade student model for classroom sensory space

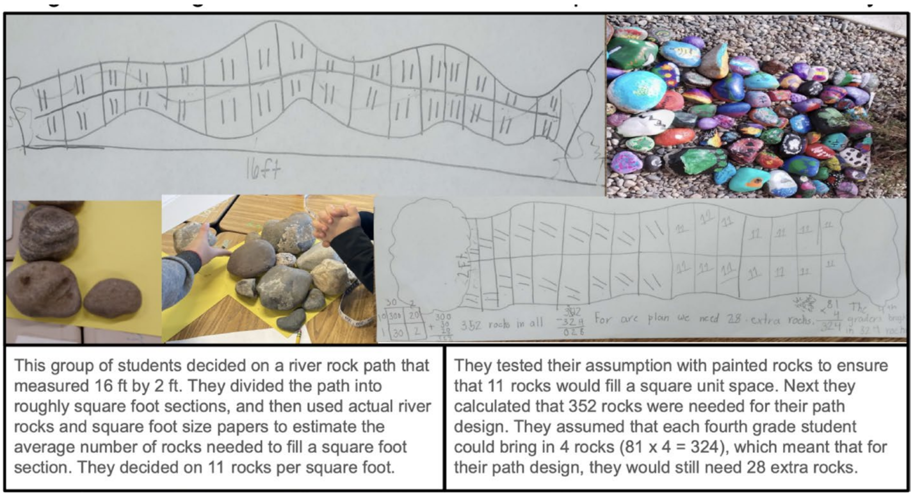

Fig. 14 4th grade student model for river rock path for school community

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Declarations

Conflict of interest On behalf of authors, the corresponding author states that there is no conflict of interest.

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