ALGORITHMIC ASPECTS OF RESOURCE ALLOCATION IN COGNITIVE RADIO WIRELESS NETWORKS

by

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DEDICATION

To Ann Michele Snowberger, Sarah Michele Bieber, Timothy Chester Bieber, Hannah Michele Judson, and Isaiah John Michael Judson.

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ABSTRACT

Wireless networking is a critical component of today's internet infrastructure. Two examples of important wireless internet infrastructure are long distance network backbone links and last-mile solutions to remote areas. Wireless technology already supplies a wide variety of consumer solutions including analog television channels (TVWS), cellular infrastructure for massive scale real-time communication, and computer networking for seamless global connectivity. Worldwide, there are an estimated 2.5B internet users and 6B cellular phone subscribers- and those numbers are steadily growing. Sufficient capacity for divergent wireless applications, along with their growing users, calls for a more efficient use of bandwidth.

We present multiple resource allocation algorithms to address this challenge in various aspects of wireless networking. Each algorithm focuses on a single resource of wireless networking: antenna beam sector activation, directional antenna beam bearings and duration, joint routing and channel selection, and link-channel allocation. In terms of computation and memory, our topology control algorithms provide near optimal performance with significantly lower cost. For each algorithm, a rich set of simulation scenarios is presented that compare our novel algorithms performance to the optimal solution.

Ultimately, we present a topology control algorithm that provides an efficient solution to the channel rental problem: finding the most cost-effective set of communication channels (for a wireless mesh network) at a minimum performance guarantee. This problem occurs in high-density traditional wireless networking, cellular networking, and rural sparse networking with last mile internet connectivity; topology control algorithms are well suited for all applications of wireless technology. These algorithms are shown to be robust against various network challenges including topology, frequency availability, and interference.

CHAPTER 1

INTRODUCTION

Broadband internet connectivity has become an assumed resource in First World (highly developed) countries. In second world nations, populations access broadband through modern cellular networks. Population density is the largest factor affecting the quality and coverage of robust broadband infrastructure. Population density, profitability, drives private and commercial investment interests in expensive terrestrial broadband infrastructure. Environment further exacerbates the deployment of robust broadband infrastructure to underpopulated areas. Difficult terrain and certain weather factors require a more costly model of infrastructure and maintenance.

Regions of highly varying terrain and low population densities can be found all over the world. Typically, residents within these areas do not have access to broadband Internet. When they do, it is a cost prohibitive service characterized by low performance. Examples include WildBlue [1] and HughesNet [2] satellite internet which cost approximately $80 per month or $960 per year, and provide bandwidths of approximately 1-2Mbps down, and 200-400kbps up- bandwidths that are not classified as broadband according to the Federal Communications Commission [3]. Historically, when no commercial opportunity for technology deployment and maintenance exists, one of two solutions - or a combination of both - have been used to solve the problem: a rural cooperative model, or commercial interest stimulated through federal funding.

Wireless networking provides a strong basic solution for providing broadband to areas without sufficient cost justification for wired infrastructure. Commercial wireless broadband involves high cost infrastructure. Wireless towers may cost in excess of $1M, requiring deployment within areas that have enough subscribers to justify installation and maintenance costs. Because expensive infrastructures, either terrestrial (Fiber, Co-location spaces) or wireless, requires a significant investment, they also require a minimum population density to make them commercially viable. As a result, areas without sufficient population to justify broadband access lack ability to access digital resources worldwide.

20% of the worlds population has no broadband access. How can these valuable citizens gain access to digital resources? Two solutions emerge: 1. rely on commercial providers, who through tax-based incentives and subsidies may invest in infrastructure to reach remote and underpopulated areas with broadband, or 2. Enable citizens to cost-effectively provide broadband infrastructure for themselves. The latter solution engages a cooperative model that has been used in rural areas for hundreds of years. Cooperatives have been used in rural areas for hundreds of years. Historically, cooperatives have been goods-based organizations, sharing agricultural products, crafts, and other product resources. Increasingly, cooperatives have become organizations of scale: engaging in the buying and selling of goods as well as procuring services for its members. Electrification of Rural America brought electricity to rural America through federally funded electrical cooperatives. The same legislation is now being used to justify fiber based broadband infrastructure deployments.

1.1. Motivation

In order to provide equal access to digital resources worldwide, work must be done to create cost effective wireless technology that is: robust in its delivery, simple to setup, and easy to maintain. Multiple vendors already provide low cost wireless devices with high-bandwidth, a variety of frequencies, and ruggedized hardware. Some of these vendors also provide open platforms for research and development. Vendor supplied tools include setup and maintenance support to enable non-technical end-user setup, deployment and maintenance.

Still lacking are robust planning tools, better architectures, and algorithms to provide the best possible robust infrastructure. My ultimate goal, is to enable remote populations to participate in global digital communication through the internet. My goal is to build a toolkit of hardware, software, and documentation that provides everything necessary for the smallest and least technologically savvy population to implement remote affordable broadband infrastructure.

In order to build this toolkit, a set of fundamental technologies must be developed. In particular, solutions are needed for hardware, software, and social challenges. Fundamental research must engage robust wireless solutions, open hardware platforms, and claim locations where test networks can be deployed and studied over long periods of time. As more and more resources are identified, such as unlicensed access to TV white space, research can incorporate them into proposed solutions.

1.2. Contents

My research places an emphasis on developing resource allocation algorithms that can provide effective solutions to deliver broadband efficiently to sparsely populated areas with highly varying terrain. My dissertation presents the results of developming multiple algorithms focused on a variety of algorithmic techniques, addressing a set of resource allocation issues relevant to existing and emerging cognitive radio wireless networks.

The background section includes an overview of relevant wireless networking fundamentals, a review of relevant literature, and a summary of my previous work.

In each of four chapters, work is presented on resource allocation algorithms for antenna beam scheduling, multi-beam smart antennas, joint routing and channel selection, and ultimately the channel rental problem. Each of these chapters provides novel, efficient solutions to an individual resource allocation problem in wireless networks today.

Following these chapters are a chapter on the applications of my research on wireless networking and a chapter on the simulation toolkit constructed to perform this research.

CHAPTER 2

BACKGROUND

Cognitive radio wireless networks (CRNs) have nodes that scan for available wireless frequencies and then use those empty frequencies for communication. Originally, CRNs were modeled to have primary users (PU)- those with the expectation that their network demands will be met- and secondary users (SU)- those who leverage unused portions of the CRN to satisfy their demands, but whose demand does not interfere with primary users. This model has been relaxed in recent years.

In general, CRN research falls into three categories: routing; channel (or spectrum) selection; joint routing and channel/spectrum selection. Routing in CRNs is difficult to research without considering the channel selection problem. Current research organizes CRN routing into groups based on either strategy or technique. Channel selection research is limited, focusing on decision techniques that are primarily concerned with cost models. CRNs joint routing and channel selection research is the most robust area, and represents the largest subset of CRN literature.

2.1. Wireless Networking Fundamentals

Unlike wired technology that utilizes electrical signals conducted through wires, wireless networking uses radios to convert bits into radio frequencies. From an application point of view, any differences in networking protocols are hidden in the operating system and driver software, presenting a uniform network interface. The radios used in wireless networking come in a variety of different configurations, supporting a growing number of different wireless networking standards. The configuration variations include radio frequency, antenna types, radio capabilities, and radio power output.

A typical wireless network consists of one or more gateways and provides the user with access to resources outside of the wireless network. These gateways act similarly to consumer Internet routers like cable, DSL, or satellite modems. The most common consumer network has modems connected directly to computers, however, more and more, consumers are inserting wireless routers into the modem so that more than one computer can share the Internet connection. Similarly, in a wireless network, Internet gateways are connected to a set of relay nodes that provide the core routing functionality of the network. These relay nodes track each other and reconfigure the network as needed to provide users with the highest possible performance. Users in a wireless network are traditionally called Subscriber Nodes, since they are subscribing to wireless services where there may be multiple choices for services.

The most common wireless network architectures are: a tree, where the root is the Internet gateway, the internal nodes are the relay nodes, and the leaves are the subscribers; a mesh, consisting of one or more Internet gateways, a set of relay nodes, and a set of subscribers; and a ladder, consisting of two Internet gateways, relay nodes configured in two parallel lines, and where subscribers follow the lines. These architectures correspond to the most common functions of wireless networking: enterprise networking, cellular/sensor/environmental networking, and transportation networking.

Traditionally, consumer wireless networking has been done in 2.4GHz and 5GHz frequencies. Recently, devices using the 900MHz range have become available and offer more choices for consumers. Power output (and thus consumption) is not a typical factor for most wireless network implementations, but when installed in remote locations, where power is not available, it becomes a significant influence on design. Similarly, most radios have an integrated antenna, providing basic coverage for a typical usage scenario. More and more, radios provide external antenna connectors to enable augmenting the radio with higher-powered antennas to improve range and performance. Most consumer wireless products do not have advanced capabilities - like the ability to sense signals and adapt their configuration to adjust for performance. However, products are emerging with an open software platform to allow aftermarket modification that enable advanced functions.

2.2. Frequency Selection

Frequencies that are available for unlicensed use represent a fairly small part of the overall radio spectrum- typically in the 700-900MHz, 2.4GHz, and 5.8GHz regions. These frequencies provide very different transmission characteristics; 5.8GHz provides higher bandwidth, but at a significantly shorter distance than the 700-900MHz range. Lower frequencies typically provide lower bandwidth, although recent developments with multiple frequency, multiple antenna, and MIMO-type radios ensure that solutions at almost any frequency provide robust, high bandwidth, for end users.

Two more aspects to consider about frequencies are their propagation characteristics, and performance ability in the presence of interference (from weather, foliage, or other intermittent obstruction). The higher the frequency (e.g. 5.8GHz), the more sensitive it becomes to obstructions; it is less able to penetrate solid objects. The lower the frequency, the better it is able to penetrate solid objects; it goes farther and is less vulnerable to signal obstruction.

2.2.1. Whitespace Frequency Allocation

For the same reason that 700-900MHz perform differently than 2.4 and 5.8GHz frequencies, TV White space - which is approximately between 50MHz and 700MHz - is even more robust in terms of propagation distances and interference. As users

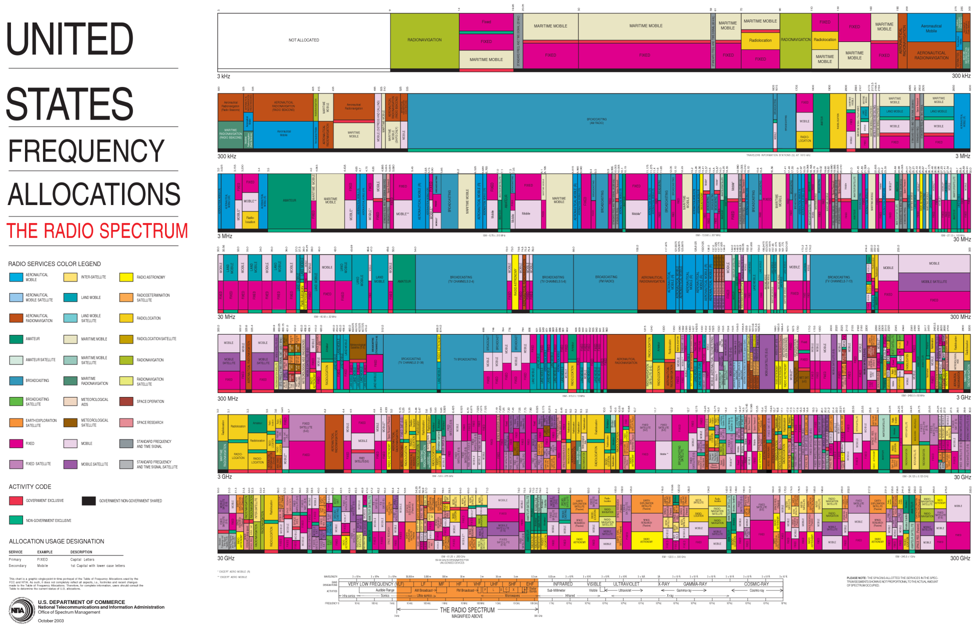

Figure 2.1: Frequency allocation chart for frequencies in use in the United States of America.

switch to digital TV, and thereby free up TV White space, proposals emerge [4–6] to use TV White space to accommodate better rural broadband. Current discussions center on how such use might impact wireless microphone technologies that cooperate at very specific frequencies. It is fairly obvious, however, that TV White space has more than enough frequency space to accommodate all user requirements.

2.3. Radios

Radios are electronic components that convert analog radio waves into meaningful data. The data is then transmitted or received through the radios antenna. The radio is the interface in both directions between the antenna and the device producing or consuming the signal ((e.g. analog sounds (as in radio stations), or digital communications protocols (as in WiMAX)). A variety of radios exist to segment unlicensed operation of equipment, and most are locked to a specific frequency or range. A number of multi-frequency scanning devices allow the user to receive signals on any frequency they can tune in, but typically radio and antenna pairs are only able to receive (and/or transmit) on a narrow range of frequencies.

Historically, radios only parameters have been transmission power, receive gain, and transmission gain. Transmitter power is a regulated control that defines an optimal maximum transmission distance. Receive and transmission gain were factors of design, including internal antennas, wire paths, routing, and electrical design.

2.3.1. Cognitive Radios

Cognitive radios are a relatively recent development. They augment traditional radios with the ability to scan and sense other traffic, and they modify their own settings to avoid congestion and interference. Because frequency space is limited, and radio transmission power is regulated, cognitive radios intelligently cooperate to provide shared frequency space between as many users as possible. This technique shows significant promise in terms of frequency resource allocation and resource management where dynamically responsive radios may be deployed in areas with cyclic or constantly changing conditions. Cognitive radios may be purposed to optimize both connectivity and performance as: a service monitor that provides different qualities of services within fixed bandwidth; as a smart power manager enabling radios to switch into low-performance mode during periods of low use; or as a management resource that learns when users are active or inactive to allocate resources respectively.

2.4. Antennas

Antennas transfer signals that are being emitted through the air into electrical signals that are then interpreted. Antennas come in a variety of designs, with wildly different performance characteristics. In recent years, an active "build-your-own-antenna'" community has been primarily building custom antennas for 2.4GHz wireless networking. Additionally, antenna research has pursued various smart antennas to break down monolithic antennas into component system that can be independently controlled, configured, and used. Beam forming antennas were one of the first solutions that allowed multiple components to work in concert to produce a higher quality signal than any single component could produce by itself. Since then, numerous improvements and inventions have been made to beam forming antennas.

2.5. Power

Electrical power management is often overlooked when considering terrestrial wireless network solutions because electricity is so pervasive and available. Many wireless systems, however, are primarily power management platforms – cellular phones, radios, and sensor network (both terrestrial and satellite) and therefore require careful management. The challenge of power management is to maintain a balance between using as little electrical power as possible while maximizing transmission power to ensure that communications are robust and consistent. This trade-off is exacerbated for satellite, ocean, and wireless systems that are not able to connect to the electrical grid. In these cases, the systems are very carefully designed with the power use carefully accounted for to ensure that the power supplying subsystem can last as long as necessary before regeneration or replacement. Solar and wind power generation systems are available to provide a low-cost, robust power generation system that can be buffered with off the shelf battery components to provide continuous power when there is no sunlight or wind.

2.6. Challenges

Sparse networks have many challenges that emerge from the low-density of nodes and large distances between them. The basic challenges for sparse networks are the same as dense networks: connectivity, throughput, and reliability. The overlap and redundancy of nodes in a dense network provide alternative solutions to problems of throughput and reliability. In a very dense network, connectivity is dependent upon the ration of relay nodes to subscriber stations because it's assumed that 100% of the space is covered by the network. As network density decreases, the challenge of connectivity becomes more important. For instance, when the network covers less than 100% of the subscriber station area, the subscriber station moves to the uncovered area, and connectivity fails for that subscriber.

Sparse networks are designed differently. Instead of being designed to pervasively saturate an entire area with wireless signals, sparse networks seek to saturate only relevant areas with signal. This choice is determined by the complexity and cost of pervasive coverage versus selective coverage. Commercial providers favor sparse networks, carefully selecting coverage because their commercial resource allocation priorities demand profitability. Saturating areas with enough equipment to provide pervasive coverage diminishes profitability. It creates: areas of high profit (where the costs are far less than the number of customers paying for service); areas where costs break even (where the costs are roughly equivalent to the customers paying for service); and areas where they lose money (where the costs are far greater than the customers paying for service).

2.6.1. Population Density

Rural broadband connectivity continues to be a challenge even as technology continues to improve networking-related products and services. The driving factor is population density, which relates directly to the recovery of infrastructure costs. In areas of low population density, there are not enough customers to cover the necessary infrastructure that can enable cost-effective infrastructure investment. The challenges of developing rural broadband are similar to the challenges this nation faced when rural electrification was an issue. As recently as the 1930s, 90% of rural homes and farms were without electricity. After the federal government enabled rural electric cooperatives in 1935, the installation of electrical systems spread quickly. By 1953, more than 90% of rural homes and farms had electricity.

Delivering broadband networking to sparsely populated areas of America is an ongoing challenge. Terrestrial broadband, using fiber or copper networking, requires the same investment in rural areas as it does in suburban or urban areas. Rural areas, however, have fewer customers to share the cost of infrastructure and makes the cost per customer unattractive to commercial providers. As a result, rural residents have few options for Internet access; often only two alternatives exist – satellite Internet or cellular broadband.

The federal government and several non-government research and public policy organizations conducted in-depth examinations of the extent of broadband infrastructure in the US. Independent results point to the same conclusion: a significant gap remains between the availability of high-speed Internet services in rural areas relative to metropolitan and suburban areas [7–9]. These reports further identify major differences between network speeds in rural areas relative to metro areas, and that rural users are paying higher prices for lower quality services. While some argue that rural demographics do not generate demand for broadband network investment, evidence shows that the availability of broadband infrastructure leads to economic growth, higher quality of education and healthcare services, and more citizen engagement in community, state, and federal services.

Internet access has three tiers: 1) Locations with broadband (cable-modem and/or digital subscriber line) Internet; 2) Locations with satellite and/or cellular broadband; and 3) Areas where no services of any kind are available. Cable-modem and DSL customers are privileged to have significantly higher bandwidths than satellite and cellular customers, and they also benefit from significantly lower costs. Satellite and cellular broadband are differentiable from cable-modem or digital subscriber lines (DSL) because of bandwidth limits. The bandwidth limits throttle the network connection, or charge additional fees, after a certain amount of bandwidth is used.

While satellite and cellular solutions can provide relatively high throughput, they do not provide low latency, and cost significantly more than cable-modem and DSL. Satellite and cellular solutions often have bandwidth usage policies that are enforced by limiting or disabling the Internet connection, and that charge significant fees for overages. While these bandwidth usage/pricing models allow providers to maintain competitiveness while still providing sufficient Internet access to rural areas, there are better and more cost-effective models that dont suffer from the same constraints.

For instance, rural broadband can be enabled using wireless technology. It is possible to build necessary infrastructure with significantly reduced costs; there are no long-haul networking connections to put in place. Wireless networking simply requires wireless nodes deployed in proximity of other wireless nodes with electricity supplied to each wireless node [10, 11]. Wireless Internet service providers (WISPS) are already providing access to many communities, but their reach is limited by what is economically viable for their business. Technology cooperatives can fill the gap. Technology cooperatives, like rural electrical cooperatives, can operate without the same profit constraints as other providers and deploy a wireless network solution by sharing the cost among its members.

With the recent conversion of television from analog to digital transmission, an additional portion of radio spectrum has been made available to unlicensed wireless use. The availability of more radio spectrum, and the potential to use lower frequencies via TV white space, currently used by WiFi networks (e.g., 2.4 GHz), offers enormous potential for rural area networks. Radio signal range D is governed by a frequency-dependent power law relationship. In areas where there is a clear, line of sight path, where f is the frequency, and in areas where he antenna height is low, or where there are significant obstructions, the range is considerably less. Furthermore, lower frequency radio signals propagate around obstructions where higher frequency signals are blocked. Hence the use of TV white space spectrum at 500 MHz could lead to an extension in range by a factor of 25, or more relative to one that operates in the current 2.4 GHz band used by most WiFi systems.

For example, Montana's Gallatin County has an average population density of 25 people per square mile. According to the 2000 U.S. Census Bureau, Montana Department of Commerce, and the Gallatin County Planning Department, Gallatin Countys population is expected to reach approximately 45 people per square mile in 2030. If a single radio, antenna, and power system can be constructed for $250 to cover one square mile, the entire county could provide a bare minimum of wireless broadband to everyone for $10 per person per square mile startup costs, plus approximately $5 per person per month for ongoing maintenance and service fees of. Quadrupling the cost to $40 per person for startup expenses, plus $20 per month for operations, would enable construction of a moderately robust system with significant capabilities. These costs are less than typical $250 startups that often charge $75 per month or more for alternative solutions [12]. A recent examination of life cycle costs, for deploying and operating wireless fixed Internet access in Gallatin County, in areas not covered by current wire line systems, yielded similar results [10].

Technical know-how inhibits the average citizen from building a wireless cloud; choices related to hardware, antennas, electricity, ongoing maintenance, and operation of the wireless network. In fact, it takes expert network operators and technicians to design, install, deploy, and support WISP installations. Fortunately, many complicated technical points can be simplified through robust software and testing infrastructure, enabling end users to deploy wireless with minimal effort and at a high success rate.

Efforts like MIT's RoofNet [13] – which was commercialized as Meraki, Inc. [14], FreiFunk [15], OpenWRT [16], and Open-Mesh are providing hardware and software that solve many of the associated issues for high-density urban and suburban populations. Much of the existing work is focused on communities where population density is high and electricity is relatively easy to access. Ubiquiti Networks, Inc. has been incorporating knowledge discovered from RoofNet and other wireless research projects into the software they develop to drive comprehensive hardware solutions and present a commercially viable product line that is robust, flexible, and that can be extended and enhanced by our proposed work.

Some projects address distances typical of rural area networks. WILDNet, for example, was built by a group at the University of California, Berkeley, using conventional WiFi technology operating at 2.4GHz [17]. Their demonstration network yielded high throughput over distances of up to 50-100km, obtained by making minor adjustments to the radio system protocols. Other work, using WiMAX (IEEE

802.16d) shows comparable results [18], and this technology is now in place for fixed wireless access in several networks in developing countries where conventional wireline infrastructure is poor or nonexistent. These projects provide excellent solutions for point-to-point, line-of-sight long distance links.

Projects in high population density areas make commercial sense because their impact, the ratio of affected users to cost, is high. Defining impact with this ratio, however, immediately prejudices areas of low population density. It is not until digital division produces significant disparity between high impact and low impact opportunities that viability will be equal between areas of varying population. Our aim is to find an efficient, cost effective, solution for areas of low population density to avoid the widening of the digital divide and enable rural communities to participate in the opportunities afforded by broadband Internet access.

For example, Buffalo Jump Technology Cooperative, Three Forks, Montana, the area of the network coverage is proposed to be 20 x 5 = 100^2 miles. In this area there are approximately $2,500$ residents, $1,500$ of whom are located within 1^2 mile in Three Forks. The other $1,000$ residents are distributed non-uniformly over 99^2 miles. Assuming a simplified uniform distribution that's 10 people per square mile.

With large distances between non-uniformly distributed residents, the challenge of designing a reliable, well-connected network significantly drives costs up. Other issues emerge as long term challenges: maintenance of the equipment and the skills necessary to deploy and maintain the network. Unless these challenges can be mitigated, by reducing the costs and skills required, and by increasing the value of the network to residents, there is not much utility in deploying a network that can't be maintained.

2.6.2. Topography

Topography can also be a major challenge in designing a sparse network. People tend to cluster around natural resources that in turn make wireless communications difficult, such as rivers, confluences of rivers, valleys, and areas with enough water to support agriculture, large trees, and other vegetation. The challenge is reliability of wireless signal delivery. In the case of Buffalo Jump Technology Cooperative, the main challenges are leaf and terrain interference. Trees surrounding homes, and other buildings, tend to scatter point-to-point wireless signals, and sharp elevation changes create shadow areas for wireless signals.

As a result, sparse networks in these areas are naturally segmented by terrain-based boundaries such as cliffs, rivers, and valleys. These terrain-based boundaries create natural dividers between high-density and sub-networks, or wireless clouds, and challenge ongoing maintenance of network equipment that is commonly located in places that are difficult to reach. Buffalo Jump Technology Cooperative has one location that is only accessible for part of the year. Design of equipment at that location must take into account that it can't be repaired or replaced in winter months.

2.6.3. Environment

Environmental factors can significantly impact wireless signal propagation. Rugged terrain, that doesn't gracefully propagate signals, and weather patterns, that cause temperature differentials and inversions, are two primary challenges affecting sparse wireless networks. Networks are particularly vulnerable when deployed in highly variable terrain, have network locations near the tops of ridges and mountains. Natural weather systems occur in the valley floors below these high places. Cold air settles into the valleys and warmer air rises, causing temperature differentials. When temperature changes occur, such as morning sun warming the air in the valley, inversions can occur. Inversions change signal propagation through the air, causing less predictable behavior and intermittent outages. Beyond the challenges of maintaining connectivity and throughput, in the presence of varying environmental conditions, environmental conditions increase wear and tear on networking components causing them to have a shorter lifespan.

2.6.4. Power

Electrical power is not pervasive in remote areas. Homes in remote areas may have electricity, but in most cases service sites, where wireless network equipment is needed and deployed, do not have electric service available. The cost to bring electricity to remote locations is prohibitive, nearly $10,000 per pole from an existing location to the desired location. Designing network to co-locate relays and subscribers near existing electrical installations, is a cost-reducing choice that also increases the likelihood of year-round accessibility.

In some cases, equipment must be deployed in locations without power using solar, wind, or generator supplied power systems. Buffalo Jump Technology Cooperatives most efficient solution is solar followed by wind. Many solar powered homes and devices exist within its service area and do not require substantial solar cell size to efficiently charge a large capacity battery. With relatively low power requirements for networking equipment, a large capacity battery is able to power equipment for 5-7 days, a usually sufficient time for recharging.

2.6.5. Economics

Because cost is directly tied to population density, there are no economies of scale for sparse networks; each participant must pay their full share of the systems cost.

Some costs are exclusive to individual users. Other costs are shared between multiple users. Exclusive and shared costs raise issues of fairness and resource management. Generally, rural American's have a rich history of using cooperatives to solve shared cost problems. Rural cooperatives form community groups that seek mutual economic, housing, agricultural, electrical, and other benefits for its members. Rural America, and western states in particular, have a history of developing cooperatives that solve problems that are too large for individuals, but not economically viable for commercial interests.

For wireless clouds, rural cooperatives can address both initial and ongoing costs with a tiered cost model: an initial cost to participate plus an ongoing cost. The initial cost is: the cost to extend the sparse network to the user; cover the users required space; and stock enough hardware to replace or repair equipment in a reasonable timeframe. Ongoing costs relate to: Internet connectivity; hardware replacement; upgrades. Ongoing costs are much lower than initial costs. A portion of initial costs may be pro-rated and combined into the ongoing costs, as long as enough resources exist to cover Internet connectivity at the gateways, and replace or repair existing hardware.

2.7. Literature Review

A significant amount of literature exists on subtopics that are combined within this work. This section presents a review of relevant literature: the overall topic of rural wireless networking under which all of the other topics fall; whitespace literature, including historic analysis of TV bandwidth and propagation data; smart antennas; topology control; cognitive radios; routing and channel selection; game theory in wireless networking.

2.7.1. Rural Wireless Networking

Rural wireless networking has only been studied [19–21] in detail over the last five years. The focus of research has been limited to three primary areas: long distance WIFI links, network protocol level analysis, and end-user access systems coupled with network equipment. Long distance WIFI-based links have been studied extensively starting with [22] and [23] and more recently WiLDnet by [17], a South African deployment by [24], two deployments, one in Venezuela and one in Italy [25], and a deployment in the village of Wray [21]. Much of the work involving long distance wifi, to enable Internet connectivity for rural regions, and mostly in underdeveloped parts of the world, has been based on analysis of the various layers of the network protocol stack including the MAC layer [17,26–31], various multi-path routing challenges [32–35], and also radio based interference [36–40].

2.7.2. Whitespace Frequency Usage

Now that the FCC has allowed the use of specific TV whitespace [41], research is being conducted to determine how to use TV whitespace for wireless networking. Whitespace research is not new, and many of the relevant sources of information are not necessarily in digital form. However, recent literature studies the possibility of using TV whitespace for wireless networking and some of that literature both reviews and re-examines historical research to produce better estimates of the bandwidths capable using modern hardware [42,43]. Work is already being pursued in algorithms to efficiently share TV whitespace [44], although devices have not yet come out commercially. This is an area of intense research and activity.

2.7.3. Beam Scheduling

Resource allocation in wireless relay networks has received much research attention. In [45], Sundaresan *et al.* showed that the scheduling problem to exploit diversity gains alone in 2-hop WiMAX relay networks is NP-hard, and provided polynomial time approximation algorithms to solve it. They also proposed a heuristic algorithm to exploit both spatial reuse and diversity gains. In [46], a similar scheduling problem was studied for OFDMA-based WiMAX relay networks. The authors provided an easy-to-compute upper bound. They presented three heuristic algorithms that were shown to provide close-to-optimal solutions and outperform other existing algorithms in simulations. Other recent works on this topic include [47, 48].

Smart antennas are being studied and improved. MAC protocols were proposed in [28, 29] for 802.11-based ad-hoc networks with switched beam antennas. The authors of these papers modified the original 802.11 MAC protocol to explore the benefits of directional antennas. In [49], Sundaresan *et al.* presented a constant factor approximation algorithm for Degree-Of-Freedom (DOF) assignment and a distributed algorithm for joint DOF assignment and scheduling in ad-hoc networks with Digital Adaptive Array (DAA) antennas. A unified representation of the physical layer capabilities of different types of smart antennas, and unified medium access algorithms are presented in [30]. Another important type of smart antennas are Multiple Input Multiple Output (MIMO) antennas that are able to support multiple concurrent streams over a single link. Resource allocation with MIMO links has been studied in [50–52].

Another approach to exploit the benefits of smart antennas for mesh networking is topology control that pre-computes an antenna pattern for each node such that a certain network topology can be formed for future communications. Using this approach, antenna beam switching is conducted in a slower time scale (in the order of seconds or more) at the potential expense of performance.

In one of the first works on this topic [53], Kumar *et al.* presents a topology control approach to effectively use directional antennas with legacy MAC protocols, that uses multiple directional antennas on each node and orients them appropriately to create low-interference topologies while maintaining network connectivity.

In [54], the authors consider the problem of power-efficient topology control with switched beam directional antennas, taking into account their non-uniform radiation pattern within the beam width. In [55], Huang and Shen present several heuristic algorithms for topology control with multi-beam directional antennas, and show that compared to Omni directional topology control approach, proposed algorithms can reduce hop count, save power and provide symmetric links.

In a recent paper [56], the authors present a distributed measurement protocol to measure the RSS of antenna patterns and a greedy distributed topology control protocol that uses this information to achieve topologies with minimal interference.

We summarize the differences between our work and this related work as follows: (1) Most related work on wireless relay networks [45–48] deal with resource allocation problems with Omni directional antennas, which are mathematically different from the optimization problems studied here; (2) Relay assignment is a special problem for wireless relay networks, and was not a concern for previous work on directional antennas; (3) Some related work on directional antennas focused on switched beam sectorized antennas [28, 29, 53, 54, 56], that can only form main beams towards a few pre-defined directions. We consider a smart adaptive antenna with an adjustable beam orientation and beam-width that make corresponding optimization problems much harder; (4) We present fast and effective algorithms to determine antenna patterns and relay assignments in a real-time manner, which can be applied to networks with mobile nodes. Topology control algorithms [53–56] may only be used in relatively static networks; (5) Generally, directional antenna related resource allocation problems are NP-hard. Most related work, including [28–30, 50, 52], presents heuristic algorithms that cannot provide any performance guarantees. Our work, however, presents a constant factor approximation algorithm for the joint beam scheduling and relay assignment problem and a polynomial-time optimal algorithm for the relay beam-scheduling problem.

2.7.4. Smart Antennas

The cross-layer approach has been studied for multihop wireless networks with directional antennas. MAC protocols were in [28,29] for 802.11-based ad-hoc networks with switched beam antennas. The authors of these papers modified the original 802.11 MAC protocol to explore the benefits of directional antennas.

In [49], Sundaresan *et al.* presented a constant factor approximation algorithm for Degree-Of-Freedom (DOF) assignment and a distributed algorithm for joint DOF assignment and scheduling in ad-hoc networks with Digital Adaptive Array (DAA) antennas. A unified representation of the physical layer capabilities of different types of smart antennas, and unified medium access algorithms are presented in [30]. Another important type of smart antennas is Multiple Input Multiple Output (MIMO) antenna that is able to support multiple concurrent streams over a single link. The authors of [52] present a centralized algorithm as well as a distributed protocol for stream control and medium access in ad-hoc networks with MIMO links. A constant factor approximation algorithm is proposed for a similar problem in [51]. In [57], Hu and Zhang devise a MIMO-based MAC protocol. They also study its impact on routing and characterize the optimal hop distance that minimizes end-to-end delay. In [58], Bhatia and Li present a centralized algorithm to solve the joint routing, scheduling, and stream control problem subject to fairness constraints for multiple wireless networks with MIMO links.

2.7.5. Topology Control

Smart antennas have received tremendous attention by researchers. Topology control with directional antennas are studied in [53–56, 59]. In one of the first works on this topic [53], Kumar *et al.* presents a topology control approach to effectively using directional antennas with legacy MAC protocols, which uses multiple directional antennas on each node and orients them appropriately to create low interference topologies while maintaining network connectivity. They show, via empirical studies, that this approach can reduce interference significantly and improve network throughput without increasing stretch factors to any appreciable extent. In [54], the authors consider the problem of power-efficient topology control with switched beam directional antennas, taking into account their non-uniform radiation pattern within the beam-width. Two cases were considered: one where the antenna orientation is assumed given, and another where the antenna orientation needs to be derived. For the first case, they present optimal and approximation algorithms for constructing power-efficient topologies. For the second case, they prove the problem to be NP-complete and present heuristic algorithms. In [55], Huang and Shen present several heuristic algorithms for topology control with multibeam directional antennas, and show that compared to the Omni directional topology control approach, the proposed algorithms provide equivalent performance in terms of the probability distribution of the number of symmetric neighbors in their resulting topologies, but can reduce hop count, save power and provide symmetric links.

The authors of [59] present a bandwidth-guaranteed topology control algorithm for TDMA-based ad hoc networks with sectorized antennas. In a recent paper [56], the authors introduce a measurement-based optimization framework for topology control in dense 802.11 networks using sectorized antennas. They present a distributed measurement protocol to measure the RSS of these antenna patterns and a greedy distributed topology control protocol that uses this information to achieve topologies of minimal interference. Extensive measurements show that the protocols operate very close to optimal and yield significant increase in network throughput compared to Omni directional antennas. Topology control with Omni directional antennas have been extensively studied in the literature [60, 61].

2.7.6. Cognitive Radios

Cognitive radio networks receive extensive attention. Spectrum allocation and access are the most important problems in such networks. In [62], the authors derive optimal and suboptimal distributed strategies for the secondary users to decide which channels to sense and access with the objective of throughput maximization under a Partially Observable Markov Decision Process (POMDP-JRCS) framework. In [63], Zheng *et al.* develope a graph-theoretic model to characterize the spectrum access problem and devised multiple heuristic algorithms to find high throughput and fair solutions. In [44], the concept of a time-spectrum block is introduced to model spectrum reservation, and a centralized and a distributed protocol is presented to allocate such blocks for cognitive radio users. Tang *et al.* introduces a graph model to characterize the impact of interference and proposed joint scheduling and spectrum allocation algorithms for fair spectrum sharing based on it in [64]. In [65], a distributed spectrum allocation scheme based on local bargaining is presented.

2.7.7. Cognitive Radio Channel Selection

Cognitive radios enabled with multi-frequency, multi-channel capabilities provide the opportunity to support a wider set of channels for the radio to choose from. While more expensive, these cognitive radios provide more powerful capabilities because the overall capacity of the CRN is directly proportional to the number of available channels. Other benefits of multi-frequency cognitive radios are the different transmission characteristics of the different frequencies. TVWS frequencies can transmit at up to 19 Mbps over a range of 30 km, while 2.4GHz and 5.8Ghz wireless signals can transmit at much higher bandwidths over a shorter range. Cognitive radios that can combine these features are very versatile nodes in a CRN.

In [66], a game theoretical approach provides a distributed channel allocation algorithm that balances selfish and cooperative needs of the users. Most channel allocation algorithms attempt to maximize certain utility characteristics of the network. The most common are fairness and throughput. Fair channel allocation algorithms are presented in [67] and [68]. In [67], both admission and power control are considered in the channel selection algorithm to ensure minimal interference to primary users while still guaranteeing reasonable performance to secondary users. In [68] an algorithm which maximizes system throughput while minimizing interference is presented. Both [67] and [68] only consider single hop cognitive radio networks.

[69] studies the channel selection problem constrained by quality and fairness. They show that by taking only quality into account, the problem can be solved in polynomial time, but when fairness is considered the problem is NP complete. The authors present a tree pruning based algorithm to solve the distance constrained channel selection problem, but they do not take into account relay stations.

Channel selection techniques that maximize system throughput are presented in [70] and [71]. In [70] algorithms that consider total transmit power of secondary users and maximum interference to primary users are presented. The algorithms attempt to meet some fairness criteria by allocating transmission opportunities to users that receive less service. A Markhov chain formulation is used to estimate the number of packets that can be transmitted by each secondary users over each channel is presented in [71]. Based on this estimation a scheduler is proposed that maximizes the aggregated system throughput.

More recent work in channel selection appears in [72–74]. In [74], the authors present the channel selection problem with the goal of proportional fair scheduling. They show that this problem is NP-hard when interference, channel quality and the usable spectrum are changing, but they present two heuristic algorithms that can meet real-time scheduling demands and perform close to optimal. [73] presents a resource minimized channel assignment algorithm that supports baseline network connectivity that provides a constant control channel and adapts to changing conditions in the network while maximizing end-to-end flow rate. A more novel approach to channel selection is presented in [72], where the authors use historical data from the network to construct a predictive model of channel use. This model is compared against random channel selection, which seems naive, but the idea of using historical data for future predictions does not appear to have been applied elsewhere in CRN channel selection literature.

2.7.8. Cognitive Radio Routing

Solving the routing problem in CRNs is difficult because of the entwined nature of routing and channel selection. In some cases, however, tackling the routing problem is achievable under the correct set of constraints. In [75, 76] opportunistic routing algorithms are presented. In [75] the authors present an opportunistic routing algorithm that uses bandwidth approximation and branch and bound searching to minimize the global knowledge required to find the optimal route. In [76] the authors leverage network coding, a technique that allows a single transmission of data for multiple nodes where the nodes can extract the information intended for them from the bundle of data. This allows co-transmission of data, reducing contention in the network. [77] combines network coding and opportunistic routing as well, however the authors provide an improved link availability prediction algorithm that helps predict both availability and duration of links, further improving the performance. Combining this with the network code significantly enhances this cross-layer routing protocol.

Gymkhana is a connectivity based routing scheme that takes node connectivity into consideration leveraging alternate routes to avoid congestion and low performance [78]. In this algorithm, global information about candidate paths is collected, then a Laplacian graph spectra is leveraged to find routes that avoid congestion caused by primary users in the network. In [79] the notion of a weighted cumulative estimation of transmission time is used to enhance a fairly typical Ad-Hoc On-demand Distance Vector (AODV) routing protocol. The authors show that this metric significantly improves the performance of the standard AODV method.

2.7.9. Joint Routing and Channel Selection

Routing and channel selection have been studied for cognitive radio networks. In [80], a novel layered graph was proposed to model spectrum access opportunities, and was used to develop joint spectrum allocation and routing algorithms. In [81], the authors present distributed algorithms for joint spectrum allocation, power control, routing and congestion control. A mixed integer non-linear programming based algorithm is presented to solve a joint spectrum allocation, scheduling and routing problem in [82]. A distributed algorithm is presented in [83] to solve a joint power control, scheduling and routing problem with the objective of maximizing data rates for a set of communication sessions.

The Spectrum Aware Mesh Routing (SAMER) [84] is a routing protocol that accounts for long term and short term spectral availability, which seeks to utilize available time-spectrum blocks by routing data traffic over paths with higher spectrum availability, without ignoring instantaneous spectral conditions. SPEctrum-Aware Routing (SPEAR) presented in [85] aimed at maximizing throughput by combining end-to-end optimization with the flexibility of link based approaches to address spectrum heterogeneity.

In [86], Hincapie *et al.* proposes a novel distributed routing protocol which can select a route and allocate channels and timeslots for a connection request to satisfy its end-to-end bandwidth requirement. The proposed protocol is based on Dynamic Source Routing (DSR) and selects time-spectrum blocks for links using a novel metric to obtain high capacity and low interference blocks for links during the route discovery procedure. In [87], Mumey *et al.* considers the problem of finding a transmission schedule and a channel selection solution for a given path, and presents a constant factor approximation algorithm based on graph coloring.

In addition, routing and channel selection have also been studied in the context of traditional WMNs with homogeneous channels [36, 88, 89]. A constant-bound approximation algorithm is proposed in [88] to jointly compute channel assignment, routing and scheduling solutions for fair rate allocation. The authors of [89] study a similar problem and derive upper bounds on the achievable throughput using a fast primal-dual algorithm. In [36], Tang *et al.* proposes an interference-aware channel assignment algorithm along with an optimal routing scheme for end-to-end bandwidth guarantees.

More recently, [90] proposes ROSA, a cross-layer joint routing and spectrum allocation algorithm that also does scheduling and transmit power control. The algorithm minimizes interference and guarantees a bounded bit error rate for receivers. [91] takes a different approach, using a Markao Decision Process to learn from the network the best spectrum utilization, then based on that information constructs routes that conform to the learned model. This approach appears robust in the case of CRNs with regular traffic patterns, but might be of limited use in a CRN with less regular traffic. In [92], channel selection is done after a collision-minimized route is selected, trading off channel costs for less interference. The proposed heuristic algorithm is tested in different scenarios, but never compared with an optimal solution.

CHAPTER 3

BEAM SCHEDULING

A wireless relay network consists of a Base Station (BS), multiple Relay Stations (RSs) and a large number of Subscriber Stations (SSs), which is illustrated in Fig. 3.1. The BS serves as a gateway connecting the network to external networks such as the Internet. If an SS is out of the transmission range of the BS, it can communicate with the BS via one or multiple RSs in a multihop manner. Such a network architecture has been adopted by emerging wireless networking standards such as IEEE 802.16j. The IEEE 802.16j [93] was proposed to extend the scope of IEEE 802.16e [94] to support multihop relay. Compared to a single-hop wireless network in which each SS directly communicates with the BS, a relay network can significantly extend the coverage range, improve network capacity and reduce dead spots [93]. Therefore, such relay networks are considered as a promising solution to provide low-cost, high-speed and long-range wireless communications for various applications such as broadband Internet access and emergency communications.

Compared to a conventional omni-directional antenna, a smart (directional) antenna offers a longer transmission range and lower power consumption by forming one or multiple beams only toward intended receivers without wasting energy in other directions. Therefore, smart antennas can enhance the functionalities of RSs and help a

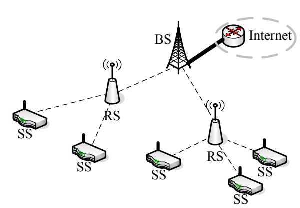

Figure 3.1: A wireless relay network

wireless relay network better achieve its goal. We focus on a smart adaptive antenna with an adjustable beamwidth and beam orientation.

Wireless relay networks have attracted extensive attention from the research community recently and various resource allocation problems have been studied in recent works [45–48]. However, most of them focused on relay networks with omnidirectional antennas. We exploit the benefits of using smart antennas in wireless relay networks by jointly considering two fundamental problems: Beam Scheduling (selecting a beamwidth and direction for the smart antenna at each RS in each scheduling period) and Relay Assignment (determining how the RSs should be assigned to serve SSs in each scheduling period). Our objective is to maximize a utility function that can lead to a stable and high-throughput system. To the best of our knowledge, we are the first to study such a joint beam scheduling and relay assignment problem in the context of wireless relay networks, and present theoretically well founded and practically useful algorithms to solve it. Specifically, we summarize our contributions in the following:

- We define the Beam Scheduling and Relay Assignment Problem (BS-RAP), show it is NP-hard and present a Mixed Integer Linear Programming (MILP) formulation to provide optimal solutions, which can serve as a benchmark for performance evaluation.

- We present two polynomial-time greedy algorithms for the BS-RAP and show that one of them has a constant approximation ratio (i.e., if the problem is a maximization problem, then the objective value of a solution given by the algorithm is guaranteed to be no smaller than the optimal value multiplied by a constant less than 1).

- We also consider a related problem in which a RS assignment schedule has been provided for some time interval and the problem is to select the best beam pattern for each RS to use in each scheduling period within the interval. Some low-cost antenna hardware may require some minimum time period between two consecutive beam pattern reconfigurations; this imposes an additional constraint on beam scheduling which we include in the problem. We term this the Beam Scheduling Problem (BSP) and provide a polynomial-time algorithm to find an optimal solution.

The rest of this chapter is organized as follows. We describe the system model and present the problem formulation in Sections 3.1 and 3.2, respectively. The proposed algorithms are presented in Section 3.3. The simulation results are presented in Section 3.4 and with a summary presented in Section 3.5.

3.1. System Model

We consider a 2-hop wireless relay network with a BS, m RSs $\{R_1, \ldots, R_i, \ldots, R_m\}$ and n SSs $\{M_1, \ldots, M_j, \ldots, M_n\}$. Each SS can communicate with the BS through an RS. The BS has an omni-directional antenna and transmits at a fixed high power level such that it can reach every RSs with a high data rate. Each RS R_i is equipped with an adaptive directional antenna that can form a main beam in any direction with a beamwidth chosen from a set of angles $\Theta = \{\theta_1 < \ldots < \theta_W\}$. We do not make any assumption on antennas at SSs, i.e., an SS can have either an omni-directional antenna or a directional antenna. Both RSs and SSs transmit at fixed power levels. For uplink communications (i.e, from an SS to an RS or from an RS to the BS), the transmitting node can simply point its main beam towards the receiving node. Hence, we focus on determining antenna orientations of RSs for downlink communications from RSs to SSs.

Let $r_{ijk} \geq 0$ be the maximum data rate that can be supported by assigning M_j to R_i and adjusting the directional antenna at R_i to cover M_j with a beam of width θ_k, i.e. the capacity of the wireless link from R_i to M_j with beamwidth θ_k. Typically, r_{ijk} depends on the transmit power at R_i, the beamwidth, the distance between M_j and R_i, the operating frequency, and maybe other factors. If the free space path loss model [95] is considered, then the Signal to Noise Ratio (SNR) at node M_j for a transmission over link (R_i, M_j) is $\mathrm{SNR}_{ij} = \frac{P_t G_t^{\theta} G_r \lambda^2}{(4\pi)^2 d_{ij}^{\alpha} N_0}$, where G_t^{θ} is the transmitter gain assuming that R_i forms a beam of width θ, d_{ij} is the distance between R_i and M_j, λ is the wavelength and N_0 is the background noise power. α is the path loss exponent and is usually between 2 and 4. Note that since we assume a fixed transmit power, operating frequency and given beamwidth θ, G_t^{θ} and G_r are considered as constants within the main beam. Note that the optimization schemes proposed in this work is independent of the propagation model. Practically, if a radio is capable of Adaptive Modulation and Coding (AMC), the maximum link data rate r_{ijk} is given by a discrete step increasing function of SNR at the receiver (instead of the continuous Shannon's function). A set of SNR thresholds, and the corresponding modulation indices and maximum data rates (link capacities) specified by IEEE 802.16e [94] is given in Table 3.3, which was used for our simulations. We can easily compute the transmission range R_i^T for RS R_i by $R_i^T = (\frac{P_t G_t^{\theta} G_r \lambda^2}{(4\pi)^2 \mathrm{SNR}_{\min} N_0})^{\frac{1}{\alpha}}$, where $\mathrm{SNR}_{\min}$ is the minimum SNR threshold. The main beam of a directional antenna at RS R_i is modeled as a sector with an angle of $\theta_k \in \Theta$ and a radius of R_i^T. We summarize the major notations in Table 3.1.

Table 3.1: Major Notations

B_{ikl}	The lth beam set of R_i using beamwidth θ_k
R_i/M_j	The ith RS / jth SS
q_j	The queue length of M_j
r_{ijk}	The data rate of link (R_i, M_j) with beamwidth θ_k
R_i^T	The transmission range of R_i
$\mathcal{S}_{ijk}$	The collection of beam sets of R_i of width θ_k that contain M_j
K_i	The maximum number of SSs that can be assigned to R_i

3.2. Problem Formulations

In this section we formulate the Beam Scheduling and Relay Assignment Problem (BS-RAP) and the Beam Scheduling Problem (BSP). We provide an MILP formulation for the BS-RAP and prove that this problem is NP-hard.

3.2.1. The Beam Scheduling and Relay Assignment Problem

For each RS R_i, we imagine rotating the main beam direction through 360 degrees (recall that the width of the beam is some angle $\theta_k \in \Theta$ degrees). As the direction changes, SSs will enter and leave the beam sector. For any fixed direction α and beamwidth θ_k there will be a set of SSs that are currently covered by the beam. We refer to this set of SSs as a *beam set* for α and θ_k. We note that there will be finite collection of distinct beams sets for all $\alpha \in [0, 360]$ degrees and $\theta_i \in \Theta$. We further assume that any beam set that is a proper subset of another is removed from the collection. Let these *beam sets* be $\mathcal{B}_{ik} = \{B_{ik1}, B_{ik2}, \ldots, B_{ikn_{ik}}\}$. For each SS M_j, let $\mathcal{S}_{ijk}$ be the collection of beam sets for R_i with angle θ_k that contain M_j, i.e. $\mathcal{S}_{ijk} = \{B_{ikl} \in \mathcal{B}_{ik} : j \in B_{ikl}\}$. Even thought the main beam of a directional antenna

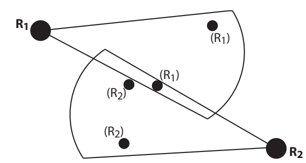

Figure 3.2: An illustration of a BS-RAP instance.

can be pointed in any direction, as explained above, we only need to consider a finite number of directions in terms of coverage for SSs.

We are interested in the problem of choosing the beam direction of the smart antenna at each RS as well as which RS should serve each SS in each scheduling period. Note that if a scheduling-based MAC protocol (such as WiMAX) is used, then a scheduling period consists of several consecutive frames. The beam direction problem is equivalent to the problem of selecting a beam set for each RS since once a beam set is chosen, the RS can point its main beam towards any direction whose corresponding sector can cover all the SSs belonging to that beam set. Once a beamwidth θ_{k_i} and beam set B_{i,k_i,l_i} is selected for each RS R_i, all SSs will know which RSs (if any) can cover it. Because each RS R_i has a limited number of channels to use simultaneously, we assume it can serve some maximum number K_i of SSs during the scheduling period. Deciding which RS (if any) should serve each SS is thus a joint problem to be solved along with selecting the relay beams. Let $r(j)$ be the RS that is assigned to SS M_j. We will use the convention that $r(j) = -1$ indicates that no RS is assigned to M_j. To summarize, in order for the RS assignment to be *valid*, we require that $|j : r(j) = i| \leq K_i$ for all $i = 1, \ldots, m$ and $r(j) = i > 0 \Rightarrow j \in B_{i,k_i,l_i}$.

The beam scheduling problem becomes the problem of creating a beam set schedule for each relay in order to best serve the SSs. We assume that at each scheduling period, a RS is able to select one of its beam sets to be active. Following [96], we assume that each SS M_j has a capacity demand represented as a queue length q_j. We formally define the optimization problem as follows.

Definition 1. *For a given scheduling period where each SS M_j has a queue length q_j, the* **Beam Scheduling and Relay Assignment Problem (BS-RAP)** *is to select for each RS R_i, a beamwidth θ_{k_i} and beam set B_{i,k_i,l_i} and jointly determine a valid RS assignment, $\langle r(j) \rangle$, such that the utility function $\sum_{\{j:r(j)\neq -1\}} q_j \min(q_j, r_{r(j),j,k_i})$ is maximized.*

Figure 3.2 provides an illustration of a simple instance of the BS-RAP problem in the case where each RS can serve at most two SSs ($K_1 = K_2 = 2$). It is known that if a scheduling algorithm can maximize the above utility function in each scheduling period, then it can keep the system stable, i.e., keep the length of each queue finite [96]. Such a stable scheduling algorithm is also considered to achieve 100% throughput [97].

3.2.2. MILP Formulation for BS-RAP

In this section, we present an MILP formulation for the BS-RAP, which can be used to provide optimal solutions and is also the basis for our LP rounding algorithm. We define the following decision variables:

$x_{ijk} \in \{0, 1\}$: indicates M_j is assigned to R_i and R_i uses beamwidth θ_k.

$s_{ikl} \in \{0, 1\}$: indicates R_i uses beam set B_{ikl}.

$y_j \geq 0$: the useful capacity supplied to M_j.

MILP: BS-RAP

$$\max \sum_j q_j y_j \tag{3.1}$$

Subject to:

$$y_j \leq q_j, \quad j \in [1, n] \tag{3.2}$$

$$y_j \leq \sum_{i,k} r_{ijk} x_{ijk}, \quad j \in [1, n] \tag{3.3}$$

$$\sum_{i,k} x_{ijk} \leq 1, \quad j \in [1, n] \tag{3.4}$$

$$\sum_{k,l} s_{ikl} = 1, \quad i \in [1, m] \tag{3.5}$$

$$\sum_{l \in \mathcal{S}_{ijk}} s_{ikl} \geq x_{ijk}, \quad i \in [1, m], j \in [1, n], k \in [1, t] \tag{3.6}$$

$$\sum_{j,k} x_{ijk} \leq K_i, \quad i \in [1, m] \tag{3.7}$$

In this formulation, constraint (3.4) ensures that each SS M_j is assigned at most one relay station R_i and beamwidth θ_k. Constraint (3.5) ensures that each relay station R_i selects exactly one beamwidth θ_k and beam set B_{ikl}. Constraint (3.6) ensures that if M_j is assigned R_i with beamwidth θ_k, then R_i must select a beam set from $\mathcal{S}_{ijk}$. Constraint (3.7) ensures that each RS R_i is assigned to at most K_i SSs (using its available independent channels).

3.2.3. The Beam Scheduling Problem

We also consider the case where a relay assignment schedule has already been pre-computed for each relay over some time interval consisting of T scheduling periods. The problem becomes how best to choose the beams used by each RS in order to improve the efficiency of the transmissions. A key consideration is that for some low-cost antennas, it may not be possible to reconfigure the beam pattern used by a given RS in each scheduling period. We assume that some minimum number of scheduling periods $0 \leq \Delta \leq T$ must elapse between two consecutive beam pattern reconfigurations. One feature of BSP which makes it easier to solve is that since the relay assignment is given; this means that set of SSs that each RS will serve during each scheduling period is known. With this in mind, we will define the beam scheduling problem with respect to a single relay $R = R_i$ and let S_t be the set of SSs scheduled to use R during the scheduling period at time t . The beams available to this RS for a given beam angle θ_k are given by the beam set $\mathcal{B}_{ik}$. For any time t, only some of these beams will provide coverage to all of the SSs in S_t. Let

$$B_t = \{(k, l) : S_t \subset B_{ikl}\}, \tag{3.8}$$

be the set of specific angles and beams that can cover all the SSs within S_t during time t. We are interested in choosing a beam angle k_t and beam l_t such that $(k_t, l_t) \in B_t$ for all $0 \leq t \leq T$. We consider $\langle(k_t, l_t)\rangle$ to be a *beam schedule* for the given scheduling duration. We define the *reconfiguration times* of the schedule as $C = \{t > 0 : (k_{t-1}, l_{t-1}) \neq (k_t, l_t)\} \cup \{0, T\}$. A beam schedule is Δ-*valid* if $t, \hat{t} \in C$ and $t \neq \hat{t}$ implies that $|t - \hat{t}| \geq \Delta$. We measure the quality of a beam schedule $\langle(k_t, l_t)\rangle$ by the worst case data rate R provides to any SS from any set S_t, for $t \in [0, T]$,

$$w(\langle(k_t, l_t)\rangle) = \min_{t \in [0,T], j \in S_t} r_{i,j,k_t}. \tag{3.9}$$

The computational problem is thus to find a valid beam schedule $\langle(k_t, l_t)\rangle$ that maximizes (3.9):

Definition 2. *Given a set of SSs S_t for each scheduling period time $t \in [0, T]$ that have been assigned RS R and a minimum same-beam period of Δ, the* **Beam Schedul-**

ing Problem (BSP) *is to find a* Δ*-valid beam schedule* $\langle(k_t, l_t)\rangle$ *that maximizes* $w(\langle(k_t, l_t)\rangle)$.

In section 3.3.5 we show that an optimal beam schedule can be computed quickly using dynamic programming.

3.2.4. Computational Complexity

In this section, we show that the BS-RAP is NP-hard. The proof is a reduction from the PARTITION problem and follows a similar approach to [98]. An instance of the PARTITION is a set of integers $a_1, a_2, \ldots a_n$ and the problem is to decide if there exists an index set $A \subset N = \{1, 2, \ldots, n\}$ such that $\sum_{i \in A} a_i = \sum_{i \in N-A} a_i$.

Theorem 1. *BS-RAP is NP-hard.*

Proof: Let $I_P : a_1, a_2, \ldots a_n$ be the given instance of PARTITION. We show how to create an instance I_B of BS-RAP (in polynomial time) such that I_P can be partitioned if and only if I_B can achieve a given utility. Let $b = \sum_i a_i$ (note b must be even, otherwise the answer to I_P is trivially no). We will create n RSs $R_1, R_2, \ldots R_n$ with beamwidths $\theta_i = (a_i/b)\pi$. For the purposes of the reduction, we let $r_{ijk} = 0$ if $k \neq i$; this means that it only useful for R_i to use beamwidth θ_i. All RSs will be located at the origin $(0, 0)$ and it will suffice to let $K_i = a_i + 1$ be the number of SSs that R_i can be assigned to. Next, we create $b + 2$ SSs at points on the unit circle. Specifically, we place $b/2+1$ SSs on the arc of the unit circle $(\pi/4, -\pi/4)$ and $b/2+1$ SSs on the arc $(-3\pi/4, 3\pi/4)$. Let $x = \pi/b$ and $\epsilon = \pi/b^2$. On the right hand side, we place SS M_j at location $\pi/4 - (j - 1)(x + \epsilon)$ (in radians) for $j = 1, 2, \ldots, b/2$. This places M_1 at $\pi/4$ and M_j at $x + \epsilon$ radians clockwise from M_{j-1} for $j = 2, \ldots, b/2$. Place $M_{b/2+1}$ at $-\pi/4$ radians. For the left hand side, we can simply add π to the right hand side locations, i.e. $M_j = M_{j-b/2}$ for $j = b/2+2, b/2+3, \ldots, b+2$. Fig. 3.3

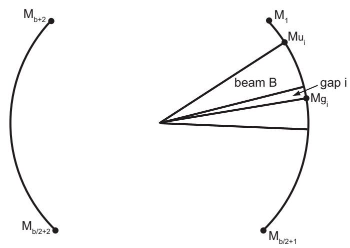

Figure 3.3: An NP-hard instance of the BS-RAP.

illustrates the scenario. Let all data rates and queue lengths be identically one: $r_{ij} = 1$ for all $1 \leq i \leq n$ and $1 \leq j \leq b+2$ and $q_j = 1$ for all $1 \leq j \leq b+2$. We claim that I_P can be partitioned if and only if I_B can achieve a total utility value of $b+2$ (each SS is served by some RS). We argue each direction of this statement as follows:

$\Rightarrow$: Let $A \subset N$ be such that $\sum_{i \in A} a_i = \sum_{i \in N-A} a_i$. We will use the relays $\{R_i | i \in A\}$ to serve the right hand side SSs and the relays $\{R_i | i \in N \setminus A\}$ to serve the left hand side. The sum of the beam angles available on each side is $\sum_{i \in A}(a_i/b)\pi = \sum_{i \in N \setminus A}(a_i/b)\pi = \pi/2$. Thus, using an non-overlapping adjacent beam pattern on each side will just cover all of the SSs and achieve the desired utility value.

$\Leftarrow$: Suppose that I_B can achieve a total utility value of $b+2$. This implies that each SS M_j is in the beam B_i of at least one RS R_i. Let A be the index set of RSs that serve SSs on the right hand side. Let $B_A = \{B_i | i \in A\}$ be the set of beams serving the right hand side. For each beam $B_i \in B_A$, let M_{iu} be the uppermost (most counter-clockwise) SS covered by B_i and let M_{il} be the lowermost (most clockwise). Rotate each B_i clockwise until the top edge of beam just touches M_{iu}. Note that all SSs remain covered after these rotations. There may be some gaps in beam coverage along the arc $(\pi/4, -\pi/4)$. Assume there are k such gaps. We will argue that the total size of these gaps must be small. Because all SSs are covered, these gaps must occur between consecutive SSs. Furthermore, the lower edge of gap i must intersect an SS, M_{g_i}, since the beams were rotated. The upper edge of gap i is formed by the lower edge of a beam B whose uppermost RS M_{u_i} must fall in between $M_{g_{i-1}}$ and M_{g_i}, so $u_i \in (g_{i-1}, g_i)$ (let $g_0 = 1$); see Fig. 3.3. The upper edge of B must intersect M_{u_i} (because of the previous rotation). The beamwidth of B is $(g_i - u_i)x$ radians, while the angle between M_{u_i} and M_{g_i} is $(g_i - u_i)(x + \epsilon)$ radians, so the size of gap i is $(g_i - u_i)\epsilon$ radians. The total size of all the gaps (in radians) is

$$\begin{aligned}\sum_{i=1}^{k}(g_i - u_i)\epsilon &\leq \sum_{i=1}^{k}(g_i - g_{i-1})\epsilon \\ &= (g_k - g_0)\epsilon \leq (b/2)\epsilon.\end{aligned}$$

It follows that the sum of the beam angles on the right hand side is

$$\begin{aligned}\sum_{i \in A}(a_i/b)\pi &\geq \pi/2 - (b/2)\epsilon \\ &= \pi/2(1 - 1/b).\end{aligned}$$

Multiplying both sides by b/π yields

$$\sum_{i \in A} a_i \geq b/2 - 1/2.$$

Since $\sum_{i \in A} a_i$ is an integer and b is even, $\sum_{i \in A} a_i \geq b/2$. A symmetric argument for the left hand side shows that $\sum_{i \in N-A} a_i \geq b/2$. It follows that $\sum_{i \in A} a_i = \sum_{i \in N-A} a_i = b/2$. ■

We remark that while pseudo-polynomial and Polynomial Time Approximation Schemes (PTASs) exist for the PARTITION problem [99], they cannot be directly applied to solve the BS-RAP in its general form.

3.3. Proposed Algorithms

We present two algorithms to solve the BS-RAP. Both are based on greedy strategies. The first algorithm has a constant factor approximation guarantee and the second is shown to be somewhat more effective in practice. Finally, we present an polynomial-time optimal algorithm for the BSP.

3.3.1. BS-RAP: A Basic Greedy Algorithm

The idea of the *BS-RAP-Greedy1* algorithm is to first get a tentative assignment of RSs to SSs and then use that assignment to guide selecting a beam set of reach RS to use. Steps 1 and 2 of the algorithm assigns RSs to SSs optimistically assuming that any SS M_j can communicate with any RS R_i using the best narrow-beam transmission rate available r_{ij1}. Thus, beam directions are ignored for the time being. This simplifies the problem to a form of the *generalized assignment problem (GAP)*; this problem considers that there are some number of activities (subscriber stations) and agents (relay stations) with various capabilities. Assigning an agent to an activity provides some value (v^r_{ij1}) and requires some cost (always 1). The objective is to assign agents to activities in order to maximize total value, with agents constrained to given budgets (in this case K_i, the maximum number of SSs that RS R_i can be assigned to). Once this tentative RS assignment is found, each RS choses a beam set that maximizes its reward given this assignment (Step 3). Any remaining SSs that have not yet been assigned a RS are then assigned RSs if possible (Step 4). Pseudocode for BS-RAP-Greedy1 is given in Algorithm 3.1.

Step 1 Let $v_{ijk} = q_j \min(q_j, r_{ijk})$ for all i, j.
Let $r(j) = -1$ for all j.
We define (and keep updated)

$$v^r_{ijk} = \begin{cases} v_{ijk} & \text{if } r(j) = -1 \\ v_{ijk} - v_{r(j)jk} & \text{otherwise.} \end{cases}$$

Step 2 **for** $i = 1$ **to** m :
  Let π^i sort $\{v^r_{ij1}\}_{j=1}^n$ into decreasing order.
  for $p = 1$ **to** K_i :
    if $v^r_{i\pi^i(p)1} > 0$
      Set $r(\pi^i(p)) = i$.
    endif
  endfor
endfor

Step 3 **for** $i = 1$ **to** m :
  Calculate the *reward* w_{ikl} of each
  beam set B_{ikl}, defined as follows:

$$w_{ikl} = \sum_{j \in B_{ikl}, r(j)=i} v_{ijk}$$

  Compute $(k_i, l_i) = \text{argmax}_{k,l}\, w_{ikl}$.
  Set $s_{ik_il_i} = 1$ and $s_{ikl} = 0$ for $k, l \neq k_i, l_i$.
  For $j \notin B_{ik_il_i}$ s.t. $r(j) = i$, set $r(j) = -1$.
endfor

Step 4 **for** $j = 1$ **to** n :
  Set $x_{ijk} = 0$ for all i, k.
  if $r(j) = -1$
    Let $\mathcal{R}_j = \{i : j \in B_{ik_il_i}, |\{j : r(j) = i\}| < K_i\}$
    if $\mathcal{R}_j \neq \varnothing$
      Set $r(j) = \text{argmax}_{i \in \mathcal{R}_j}\, r_{ijk_i}$
      Set $x_{r(j)jk_{r(j)}} = 1$.
    endif
  else
    Set $x_{r(j)jk_{r(j)}} = 1$.
  endif
endfor

Algorithm 3.1: BS-RAP-Greedy1

3.3.2. Time Complexity of BS-RAP-Greedy1

Next we examine the running time of the BS-RAP-Greedy1 algorithm. Steps 1 and 2 implement a standard GAP algorithm that can be seen to run in $O(mn \log n)$ time. We note that an upper bound on the number of beam sets for each RS is $2n$ since a new beam set is formed only when an SS enters or leaves the beam set (as the RS's antenna sweeps its beam in a circle). This implies that Step 3 can be computed in $O(mn)$ time (we assume that W, the number of possible beamwidth angles is a constant). Finally, Step 4 can be done in $O(mn)$ time so the overall running time is $O(mn \log n)$.

Theorem 2. *The BS-RAP-Greedy1 algorithm runs in $O(mn \log n)$ time and provides a solution that is within a factor $\frac{1}{2\lceil 2\pi/\theta_1 \rceil}$ of the optimal value.*

Proof. We establish the approximation ratio; the running time analysis was given above. Let $\{x^*_{ijk}, s^*_{ikl}\}$ be an optimal solution to the original MILP with utility value V^*. Let $\{x_{ijk}, s_{ikl}\}$ be the solution produced by the greedy algorithm, with utility value V. Let V^*_{gap} be the optimal solution value to the simplified GAP instance considered and let V^g_{gap} be the value of the greedy solution found by Steps 1 and 2. A relatively recent result [100] shows that the greedy solution found in this case is 2-optimal, $V^g_{gap} \geq \frac{1}{2} V^*_{gap}$. It is also clear that $V^* \leq V^*_{gap}$, since in the GAP problem the RSs are not forced to chose a beam set. Hence,

$$V^* \leq 2V^g_{bs-gap} \tag{3.10}$$

After Step 3, each RS has chosen the beam set $B_{ik_il_i}$ and assuming each M_j still uses RS $R_{r(i)}$ the value of the utility function (3.1) is $\sum_i w_{ik_il_i}$. In Step 4, SSs that have not yet been assigned an RS are free to be assigned an RS if it is has available channels; hence the utility value can only increase and so $V \geq \sum_i w_{ik_il_i}$.

For a RS R_i, we define a *beam set cover* to be any subset $C \subset \mathcal{B}_{i1}$ such that $\forall j \exists B_{i1l} \in C$ such that $j \in B_{i1l}$. Note that the narrowest beam angle θ_1 is chosen to define beam set covers. Let C_i be a minimum-sized beam set cover for R_i. Observe that $|C_i| \leq \lceil \frac{2\pi}{\theta_1} \rceil$. We have,

$$
\begin{aligned}
V^g_{gap} &= \sum_j v_{r(j)j1} \\
&= \sum_i \sum_{j:r(j)=i} v_{r(j)j1} \\
&\leq \sum_i \sum_{B_{i1l} \in C_i} \sum_{j \in B_{i1l}, r(j)=i} v_{r(j)j1} \\
&= \sum_i \sum_{B_{i1l} \in C_i} w_{i1l} \\
&\leq \sum_i \sum_{B_{i1l} \in C_i} w_{ik_il_i} = \sum_i |C_i| w_{ik_il_i} \\
&\leq \lceil \frac{2\pi}{\theta_1} \rceil \sum_i w_{ik_il_i} = \lceil \frac{2\pi}{\theta_1} \rceil V. \qquad (3.11)
\end{aligned}
$$

Combining (3.10) and (3.11), yields $V \geq \frac{1}{2\lceil 2\pi/\theta_1 \rceil} V^*$ as claimed. □

3.3.3. BS-RAP: A Joint Greedy Algorithm

The second greedy algorithm is a variation on the first. The main difference is that beam set selection and relay assignment are done jointly. This is done in Step 2 of the algorithm, which loops through all the RSs and for each, chooses the best beam set to use given the previously made SS assignments to that RS. Once a beam set is chosen for a RS R_i then up to K_i SSs are assigned that RS and the loop continues. We refer to the algorithm as the BS-RAP-Greedy2, pseudocode is given in Algorithm 3.2.

3.3.4. Time Complexity of BS-RAP-Greedy2

It is easy to see that Step 2 of the BS-RAP-Greedy2 is also $O(mn \log n)$ time, and so this algorithm has the same asymptotic time complexity as the BS-RAP-Greedy1 algorithm.

Step 1 Follow Step 1 of BS-RAP-Greedy1.

Step 2 **for** $i = 1$ **to** m :
  Calculate the *reward* w_{ikl} of each
  beam set B_{ikl}, defined as follows:
    (a) Let π^{ikl} sort B_{ikl} by decreasing v^r_{ijk}.
    (b) Let $W_{ikl} = \{p \leq K_i : v^r_{i\pi^{ikl}(p)k} > 0\}$.
    (c) Let $w_{ikl} = \sum_{p \in W_{ikl}} v^r_{i\pi^{ikl}(p)k}$.
  Compute $k_i l_i = \mathrm{argmax}_{kl}\, w_{ikl}$.
  Set $s_{ik_i l_i} = 1$ and $s_{ikl} = 0$ for $kl \neq k_i l_i$.
  for $p \in W_{ik_i l_i}$:
    Set $r(\pi^{ikl}(p)) = i$.
  endfor
endfor

Step 3 Follow Step 4 of BS-RAP-Greedy1.

Algorithm 3.2: BS-RAP-Greedy2

3.3.5. BSP: A Polynomial-Time Optimal Algorithm

In this section we present a polynomial-time optimal algorithm for computing an optimal beam schedule $w(\langle(k_t, l_t)\rangle)$ based on dynamic programming. As is the hallmark of dynamic programming, we show that an optimal solution can be constructed using optimal solutions of smaller subproblems. In this case, the subproblems are: for each t, determine the best Δ-valid beam schedule $\langle(k_t, l_t)\rangle^{*t}$ for the subinterval $[0, t]$ for which $t + 1$ could be a reconfiguration time. This means that $\langle(k_t, l_t)\rangle^{*t}$ cannot have a beam pattern reconfiguration in the interval $[t + 1 - \Delta, t]$. To simplify the presentation, we assume that R_i may use a trivial omni-direction beam to serve the scheduled SSs (perhaps with data rate 0). Note that $\langle(k_t, l_t)\rangle^{*t}$ must use some beam angle k_t^* and beam set l_t^* at time t and the reconfiguration to this beam must have been made at some earlier time $\hat{t}$ such that $\hat{t} = 0$ or $\Delta \leq \hat{t} \leq t + 1 - \Delta$. Observe that $\langle(k_t, l_t)\rangle^{*t}$ can be assumed to agree with an optimal solution $\langle(k_t, l_t)\rangle^{*\hat{t}}$ for $t \in [0, \hat{t} - 1]$, since this cannot decrease the worst-case quality of the beam schedule. This means that we can construct $\langle(k_t, l_t)\rangle^{*t}$ by simply trying all viable beams for its final beam, with last switching time $\hat{t} = 0$ or $\Delta \leq \hat{t} \leq t+1-\Delta$, checking what the minimum rate to any scheduled SS by that beam in interval $[\hat{t}, t]$ and extending it to full schedule using the previously computed solution $\langle(k_t, l_t)\rangle^{*\hat{t}-1}$. The optimal solution of the original problem will be given by $\langle(k_t, l_t)\rangle^{*T}$. The full dynamic programming algorithm is given in Algorithm 3.3.

3.3.6. Time Complexity of BSP

We next analyze the computational complexity of Algorithm 3.3. We note that there are at most $O(K_i W)$ possible beams to consider for any scheduling period time t and a given RS R_i (K_i is the maximum number of SSs that can use R_i simultaneously and W is the number of beam angles available). Thus Step 1 can be performed in $O(K_i W T)$ time. Step 2 can be performed in $O(K_i W T^2)$ time using a similar argument. If the partial solutions $\langle(k_t, l_t)\rangle^{*t}$ are represented efficiently using lists of beam intervals, they can be extended in $O(1)$ time and Step 3 can be implemented in $O(T^2)$ time. Thus the time complexity of the entire algorithm is $O(K_i W T^2)$.

Step 1 Compute B_t for $0 \leq t \leq T$ using (3.8).

Step 2 Compute $B_{\hat{t},t} = B_{\hat{t}} \cap \ldots \cap B_t$ for $0 \leq \hat{t} < t \leq T$.
Compute
$$(k^*, l^*)^{\hat{t},t} = \underset{B_{\hat{t},t}}{\text{argmax}} \min_{\tilde{t} \in [\hat{t},t], j \in S_{\tilde{t}}} r_{i,j,k},$$
for $0 \leq \hat{t} < t \leq T$.

Step 3 **for** $t = \Delta$ **to** T :
  Initialize $\langle (k_t, l_t) \rangle^{*t}$ by setting all
  $(k_t, l_t) = (k^*, l^*)^{0,t}$.
  for $\hat{t} = \Delta$ **to** $t + 1 - \Delta$:
    Let $\langle (k_{\tilde{t}}, l_{\tilde{t}}) \rangle^{test}$ be defined by:
    $\langle (k_t, l_t) \rangle^{*\hat{t}-1}$ for $\tilde{t} < \hat{t}$ and
    $(k_{\tilde{t}}, l_{\tilde{t}}) = (k^*, l^*)^{\hat{t},t}$ for $\hat{t} \leq \tilde{t} \leq t$.
    if $w(\langle (k_{\tilde{t}}, l_{\tilde{t}}) \rangle^{test}) > w(\langle (k_t, l_t) \rangle^{*t})$
      Let $\langle (k_t, l_t) \rangle^{*t} = \langle (k_{\tilde{t}}, l_{\tilde{t}}) \rangle^{test}$
    endif
  endfor
endfor

Step 4 **return** $\langle (k_t, l_t) \rangle^{*T}$

Algorithm 3.3: BSP-DP

3.4. Numerical Results

In this section, we present simulation results to show the performance of the proposed algorithms. The ILOG CPLEX [101] optimization software was used to solve all the MILP and LP problems. In the simulation, n SSs were randomly deployed within a square $l \times l$ km region, with a single BS placed at the center of the square. Then, m RSs were deployed radially from the center of the region with uniform angular spacing and random radii between zero and the maximum distance. In the simulation, we calculated SNRs using the free space path loss model [95] as described in Section 3.1. The transmitter antenna gain of each node was set to $\frac{360}{\theta}G_o$, where G_o is the gain of an antenna working in the omni-directional mode. In addition, each node was assumed to be able to receive signals from all directions, i.e., the receiver antenna gain of each node was set to G_o. The values of those parameters relevant to the propagation model and other related parameters were set according to Table 3.2.

As described previously, the link capacity is given by a discrete step increasing function. A set of SNR thresholds, and the corresponding modulation indices and link capacities specified by IEEE 802.16e [94] are given in Table 3.3.

Table 3.2: Common Simulation Settings

Omni-directional antenna gain G_o	2dB;
Operating frequency	5.8GHz;
Path loss exponent	2;
Transmit power of each RS (P_t)	1W;
Noise power	-174dBm/Hz;
Channel bandwidth	10MHz;

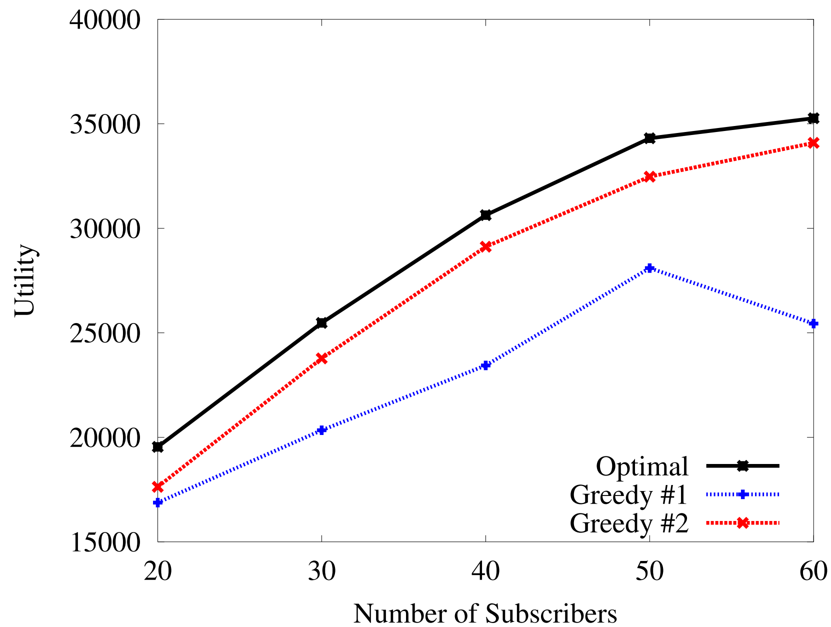

Figure 3.4: Scenario 1: Performance VS. the number of SSs (n)

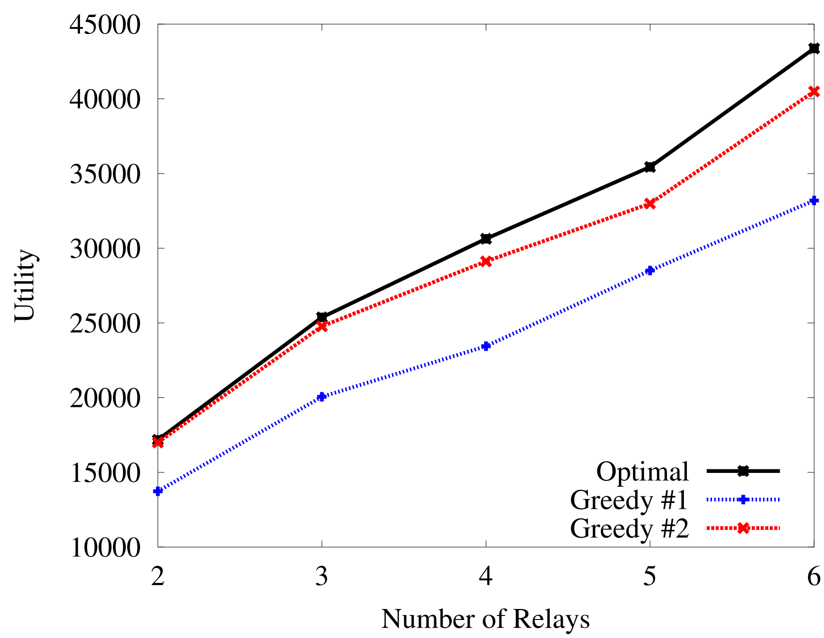

Figure 3.5: Scenario 2: Performance VS. the number of RSs (m)

Table 3.3: SNR VS. Link Capacity

SNR Threshold (dB)	Modulation Index	Link Capacity (Mbps)
10	QPSK 1/2	10
14.5	16QAM1/2	20
17.25	16QAM 3/4	30
21.75	64QAM 2/3	40
23	64QAM 3/4	45

To simplify the scenarios, we just considered that each RS could use either the beamwidth θ or the beamwidth 2θ. In each simulation scenario, we changed the value of one parameter and fixed the values of the others. We first evaluated the performance of the proposed BS-RAP algorithms, i.e., the two greedy algorithms (labeled as "Greedy #1" and "Greedy #2"), in terms of the *utility* function (3.1). In all scenarios, we also computed optimal solutions by solving the MILP for the BS-RAP (labeled as "Optimal"). We summarize our BS-RAP simulation scenarios below and present the corresponding simulation results in Figs. 3.4–3.9. Each plotted value in these figures is an average over 10 runs per parameter combination, each with a different randomly generated network. We assume that each RS has angles θ and 2θ available and that each SS queue length is drawn from the uniform distribution on $[0, 2\mu]$, where μ is the mean queue length.

Since our BSP-DP algorithm is optimal, we do not provide simulation results for the BSP problem.

We make the following observations from the simulation results, noting that the BS-RAP-Greedy2 algorithm is close to optimal in almost all cases, while the BS-RAP-Greedy1 algorithm performs less well in all cases:

1) From Fig. 3.4–3.9, we see the BS-RAP-Greedy2 algorithm closely tracks the optimal solution; it is always within 15% of the optimal solution. The BS-RAP-

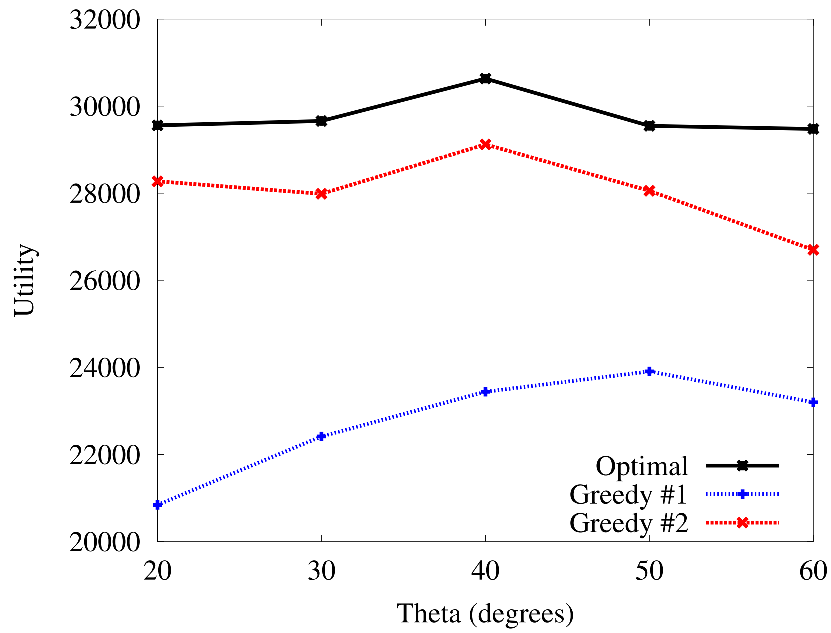

Figure 3.6: Scenario 3: Performance VS. beamwidth (θ)

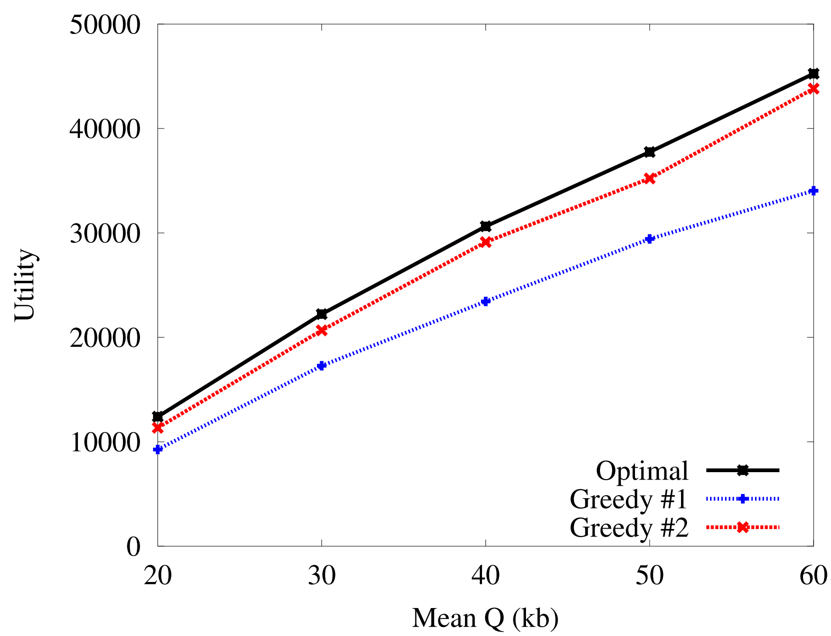

Figure 3.7: Scenario 4: Performance VS. mean queue length (μ)

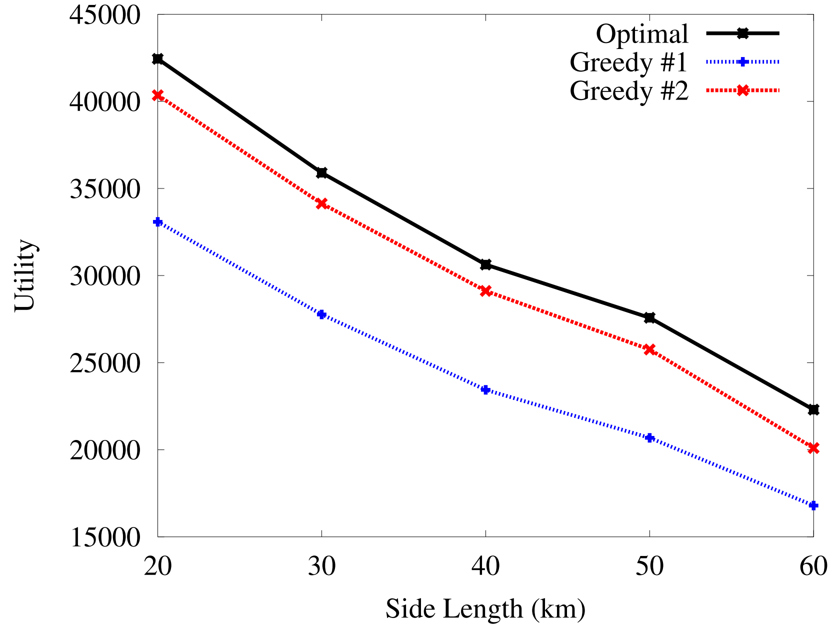

Figure 3.8: Scenario 5: Performance VS. region side length (l)

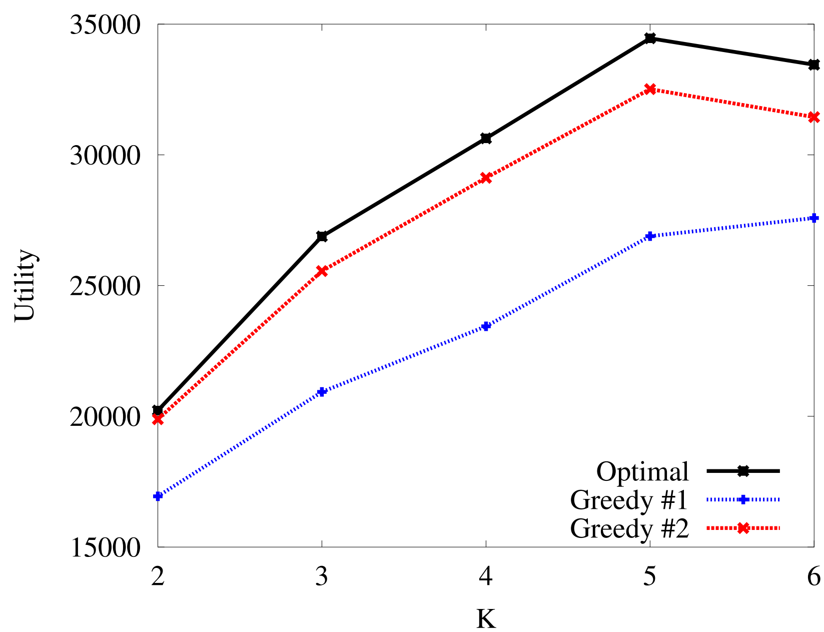

Figure 3.9: Scenario 6: Performance VS. maximum SSs per RS (K)

Greedy1 algorithm degrades to less than 75% of optimal as the number of SSs increases in Scenario 1.

2) Fig. 3.5 illustrates that a near linear increase in performance results as the number of RSs is increased in Scenario 2. Even as the number of RSs increases to the maximum value, both greedy algorithms do a relatively good job at allocating these additional resources. Eventually, there must be diminishing returns as more RSs are added but this did not occur in the range considered ($m \in [2, 6]$).

3) Fig. 3.6 suggests that θ in the range 40-50 degrees is optimal for the simulation parameters examined. Recall that each RS could form beams of width θ or 2θ. Across the simulations performed, there were somewhat more beams of width 2θ chosen versus θ. Using a narrower beam can increase the potential throughput from an RS to an SS assigned to it, but a wider beam potentially allows the RS to serve more SSs during the scheduling period.

4) From Fig. 3.7 we see that the BS-RAP utility increases close to linearly with the average queue length of each SS. BS-RAP-Greedy2 tracks the optimal solution closely, whereas the relative performance of BS-RAP-Greedy1 falls off as the queue lengths are increased.

5) Fig. 3.8 indicates that the utility function decreases approximately linearly with the side length of the simulation region. As distances increase a RS must either use a narrower beam to reach more distant SSs it is assigned to (and so potentially serve a smaller number of SSs), or accept a lower transmission rate to those SSs. In either case, the utility function will decrease.

6) In Fig. 3.9 we see that there are decreasing gains as the number of independent channels available to each RS is increased. This number determines how many SSs that can be assigned to each RS. In this case ($n = 40$ SSs, $m = 4$ RSs), it appears that around 5 channels per RS is sufficient to obtain the maximum utility value.

3.5. Conclusions

We have explored how to leverage smart antennas for efficient communications in wireless relay networks. A corresponding optimization problem was formally defined as the BS-RAP and was proven to be NP-hard. We first presented an MILP formulation to provide optimal solutions and then presented two two greedy approaches for the BS-RAP, one of which, BS-RAP-Greedy1, was shown to have an approximation ratio of $\frac{1}{2\lceil 2\pi/\theta_1 \rceil}$. These algorithms are simple, easy to implement and scale well to larger network instances. We also considered the related problem BSP and presented the BS-DP algorithm to solve it optimally in polynomial time. It has been shown by extensive simulation results that both proposed BS-RAP algorithms provide good performance, with the BS-RAP-Greedy2 algorithm better than 85% of optimal in all cases.

CHAPTER 4

MULTI-BEAM SMART ANTENNAS

Compared to a conventional omni-directional antenna, which wastes most of its energy in directions where there is no intended receiver, a smart (directional) antenna offers a longer transmission range and lower power consumption by forming one or multiple beams only toward intended receivers. This paper was co-authored with Drs. Mumey and Tang and Yun Xing, it was submitted to IEEE GlobeCom 2011, but was rejected. It has been revised and resubmitted to the IEEE International Conference on Computing, Networking and Communications 2012.

There are primarily two approaches to exploit the benefits of smart antennas for mesh networking: the cross-layer approach [52] and the topology control approach [53]. With the cross-layer approach, a joint antenna pattern assignment and scheduling solution is provided to switch beams to communicate to different neighbors at a fast time scale (e.g., on a per-time slot basis). However, the topology control approach pre-computes an antenna pattern for each node such that a certain network topology can be formed for future communications. Using this approach, antenna beam switching is conducted in a slower time scale (in the order of seconds or more) at the potential expense of performance. Compared to the cross-layer approach, the major advantage to using the topology control approach is that it is purely a link layer solution that does not require any modifications to a standard MAC protocol. Hence, it can be easily implemented in a system using Commercial-Off-The-Shelf (COTS) and standard protocols. The topology control approach is the focus of this paper, which may lead to performance comparable to the cross-layer approach with a carefully designed algorithm and full consideration for link capacity.

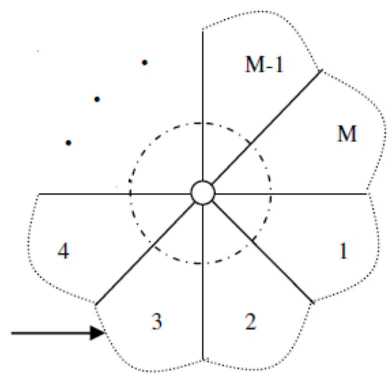

Figure 4.1: A multi-beam antenna.

Our contributions are summarized as follows:

1) We formally define the corresponding optimization problem as the Sector Selection Problem (SSP). 2) We present a Mixed Integer Linear Programming (MILP) formulation to provide optimal solutions. 3) We present an effective Linear Programming (LP) rounding based algorithm for the SSP. 4) We present extensive simulation results to show that the proposed algorithm provides close-to-optimal performance and yields good solutions in terms of both capacity and fairness compared to alternative approaches including a Minimum Spanning Tree (MST) based algorithm and the k nearest neighbors algorithm.

4.1. System Model

We consider a multihop wireless network composed of n nodes. Each node v_i is equipped with an adaptive directional antenna that can form beams in any of M different sectors (see Figure 4.1). Each sector that is activated (turned on) creates a beam of width $\frac{360}{M}$ degrees.

We assume that the data rates available to a node depends on the number of sectors it has activated. Let $c_{i,j}^a \geq 0$ be the maximum data rate that can be supported

$$\mathrm{SNR}_{ij} = \frac{P_t G_t^a G_r \lambda^2}{(4\pi)^2 d_{ij}^{\alpha} N_0} \tag{4.1}$$

by the link (v_i, v_j) assuming v_i has activated a sectors in total, including the sector in which v_j falls in. Typically, $c_{i,j}^a$ depends on the transmit power at v_i, the distance between v_i and v_j, the operating frequency, and maybe other factors. If the free space path loss model [95] is considered, then the Signal to Noise Ratio (SNR) at node v_j for a transmission over link (v_i, v_j) is
where G_t^a is the transmitter gain assuming that v_i has activated exactly a sectors, G_r is the receiver antenna gain (omni-directional reception), d_{ij} is the distance between v_i and v_j, λ is the wavelength and N_0 is the background noise power. α is the path loss exponent and is usually between 2 and 4. We assume a fixed transmit power P_t, operating frequency and receiver gain G_r, so SNR_{ij} varies only with the number of sectors activated by v_i and the transmission distance.

Practically, if a radio is capable of Adaptive Modulation and Coding (AMC), the maximum link data rate c_{ij}^a is given by a discrete step increasing function of SNR at the receiver (instead of the continuous Shannon's function). A set of SNR thresholds, and the corresponding modulation indices and maximum data rates (link capacities) specified by IEEE 802.16e [94] is given in Table 3.3, which was used for our simulations. For each $1 \leq a \leq M$, we compute the transmission range for a node using a sectors as

$$\left(\frac{P_t G_t^a G_r \lambda^2}{(4\pi)^2 \mathrm{SNR}_{\min} N_0}\right)^{\frac{1}{\alpha}},$$

where $\mathrm{SNR}_{\min}$ is the minimum SNR threshold. Each active sector beam of node v_i is as a sector with an angle of $\frac{360}{M}$ and a radius of $R^{T,a}$. We summarize the major notations in Table 4.1.

Table 4.1: Major Notations.

n	The number of nodes in the network.
v_i	The ith node in the network.
M	The number of antenna sectors.
$s(u,v)$	The sector of u that node v falls in.
SNR_{ij}	The SNR at node v_j for transmission over link (v_i, v_j)
R_i^T	The transmission range of v_i
$c_{u,v}^a$	The transmission rate of link (u,v) assuming u has activated exactly $1 \leq a \leq M$ sectors.

4.2. Problem Formulation

We are interested in the problem of choosing which antenna sectors each node should activate. We will assume that the power available to each activated sector for transmission depends on the number of sectors activated. Thus, there is a trade-off between activating additional sectors to increase the number of directions that a node can use to reach other nodes and the transmission rates achievable to those nodes. We formally define the optimization problem as follows.

In this problem, our objective is to maximize the summation of link capacities since this summation gives the *maximum (possible) capacity* (note that the actual network capacity may depend on many other factors such as the MAC protocol). It may be argued that maximizing the total link capacity may lead to unfairness, however we will show that our SSP algorithm offers good performance in terms of both maximum capacity and fairness via simulation results.

4.2.1. MILP Formulation

In this section, we present the following MILP formulation of the SSP, that can be used to provide optimal solutions.

Variables:

$s_{u,k} \in \{0, 1\}$: indicates whether u activates sector k

$x^a_{u,v} \in \{0, 1\}$: indicates whether link (u, v) is active and u uses a sectors

$z^a_u \geq 0$: used for constraining the $x^a_{u,v}$ variables (real)

$f_{u,v} \geq 0$: the amount of flow on the edge (u, v) (real)

Objective:

$$\max\ T_{total} = \sum_{u,v,a} c^a_{u,v} x^a_{u,v} \tag{4.2}$$

subject to the following constraints (indices vary over their entire domains, unless otherwise noted). We also choose one node arbitrarily to be the *source* vertex s. The purpose of s is explained below.

$$\sum_a x^a_{u,v} \leq 1 \tag{4.3}$$

$$x^a_{u,v} = 0, \quad \text{if } c^a_{u,v} = 0 \tag{4.4}$$

$$\sum_a x^a_{u,v} = \sum_a x^a_{v,u} \tag{4.5}$$

$$x^a_{u,v} \leq s_{u,s(u,v)} \tag{4.6}$$

$$z_u^{|A|-1} \leq |A| - \sum_{k\in A} s_{u,k}, \quad A \subset [1, M], |A| > 1 \tag{4.7}$$

$$x^a_{u,v} \leq z^a_u \tag{4.8}$$

$$f_{u,v} \leq 1 \quad (u,v) \in E \tag{4.9}$$

$$f_{u,v} \leq n \sum_a x^a_{u,v}, \quad (u,v) \in E \tag{4.10}$$

$$\sum_{\{(u,s)\in E\}} f_{u,s} + 1 = \frac{1}{n} + \sum_{\{v:(s,v)\in E\}} f_{s,v} \tag{4.11}$$

$$\sum_{\{u:(u,v)\in E\}} f_{u,v} = \frac{1}{n} + \sum_{\{w:(v,w)\in E\}} f_{v,w}, \quad v \in V \setminus \{s\} \tag{4.12}$$

In this formulation, constraint (4.3) ensures that for each link (u, v), the transmitting node u must fix the number of sectors, a, it is using. Constraint (4.4) ensures that links (u, v) cannot be used if u, when using a sectors, cannot provide any capacity to v. Constraint (4.5) guarantees that each link will be bidirectional in the solution (if (u, v) is operational then (v, u) must also be operational). The constraint (4.6) requires that if the link (u, v) is used, then u must activate the sector containing the node v. Constraints (4.7) and (4.8) work together to ensure that if $x^a_{u,v} = 1$ then the node u must not have more than a sectors activated. This is done by enumerating all subsets of u's sectors of size two or greater, e.g. if any two sectors are activated then u cannot use a link of the form $x^1_{u,v}$, etc. We note that there are $2^M - M - 1$

such constraints. The flow constraints are used to ensure network connectivity. In particular, one node is designated as the source node s and there must be connectivity from s to all other nodes. We view each node $v \in V$ as able to implicitly absorb $\frac{1}{n}$ units of flow. We assume that there is 1 unit of flow originating at the source node and that the source node is able to absorb $\frac{1}{n}$ of this flow and must ship the remainder to the other $n-1$ nodes to be absorbed. In particular, (4.11) stipulates that the net flow out of the source node is $1-\frac{1}{n}$ and (4.12) says that the flow into any other node equals the flow absorbed at the node plus the outgoing flow from the node. Constraint (4.9) ensures that the flow on each link is at most 1. Constraint (4.10) requires that if a link (u, v) has positive flow, then it must be active since $\sum_a x^a_{u,v} \geq \frac{1}{n} f_{u,v}$. The $\frac{1}{n}$ factor is used to reduce the influence of which node was chosen to be the source vertex s. Together, the transmission and flow constraints guarantee that the MILP solution meets both the network connectivity and bidirectional links requirements of the SSP and the objective function ensures the solution has maximum total capacity.

4.2.2. Computational Complexity

The problem is clearly NP-hard since it is related to the bounded degree spanning tree problem. In particular, we can easily reduce the problem of testing whether an undirected planar graph contains a Hamiltonian path to SSP by setting the sectors and capacities such that there is only positive capacity through a node if it enables at most two edges. This problem is known to be NP-complete [102].

4.3. Proposed Algorithms

We present two effective heuristic algorithms to solve the SSP. The first algorithm is based on LP rounding while the second is based on finding minimum spanning tree

(MST) and then greedily augmenting the solution until no further improvement is found.

4.3.1. The LP Rounding Algorithm

We begin with an observation that the $x^a_{u,v}$ variables can be relaxed to be real-valued in the MILP from the previous section, provided the $c^a_{u,v}$ constants are non-increasing with a:

Lemma 1. *If $c^1_{u,v} \geq \ldots \geq c^M_{u,v}$ for all $(u, v) \in E$, then relaxing the $\{x^a_{u,v}\}$ to be real-valued and adding additional constraints of the form $0 \leq x^a_{u,v} \leq 1$ does not change the optimal objective value of (4.2).*

Proof. The objective value of the original MILP cannot exceed that of the relaxed version. Suppose $\{x^a_{u,v}\}$ are part of an optimal relaxed solution and suppose (u, v) is some edge such that $\sum_a x^a_{u,v} > 0$. We observe that constraint (4.3) must be tight, since the right hand sides of constraints (4.6) and (4.8) are integer values. In particular, let $a' = \text{argmin}_a\, z^a_u > 0$. If we set $x^{a'}_{u,v} = 1$ and $x^a_{u,v} = 0$ for $a \neq a'$, this cannot decrease the objective value and remains feasible. It follows that an optimal solution to the relaxed LP can be found with integral $\{x^a_{u,v}\}$ values and so the original and relaxed MILPs have the same optimal objective value. □

By Lemma 1, we can solve the relaxed version of the MILP instead of the original. The idea of our algorithm is to further relax the MILP to a LP formulation in which the sector usage variables $\{s_{u,k}\}$ are also relaxed to be real-valued, with additional constraints of the form $0 \leq s_{u,k} \leq 1$. The algorithm proceeds in two phases. In the first phase, the network connectivity constraints are satisfied by ensuring that a solution contains a spanning tree of the network. The approach used is similar to Prim's algorithm for finding a minimum spanning tree in a graph: an edge (u, v)

is added to the spanning tree if nodes u and v are currently disconnected and the addition of the link (u, v) most improves the objective. If link (u, v) is selected, u and v's components are merged and we add the explicit constraint $s_{u,s(u,v)} = 1$ to the LP. In this way, sectors in the network are gradually forced to be fully activated. After phase one, if the LP remains feasible, the network will be connected. At this point, all sectors that were not explicitly turned on in phase one, are set to be off. We consider this as a baseline integer solution. In phase two, we greedily try to improve the baseline solution by checking to see if any facing sector pairs, with at least one sector of pair unactivated, can be both activated to realize a gain in the overall network capacity. If so, we choose the pair with the greatest capacity improvement and activate these sectors. We repeat this until no further improvements are found.

4.3.2. Time Complexity of SSP-LPR

We note that the time complexity of the SSP-LPR algorithm is polynomial. This because the LPs that are solved are polynomial in size and can be solved in polynomial time. Step 2 requires solving $n-1$ such LPs. A simple upper bound on the length of L in Step 4 is $n(n-1)/2$ since each pair of nodes may or may not yield an unused facing pair of sectors in L. L shrinks by 1 pair each iteration. So, in the worst case Step 4 must evaluate the effect on the objective function of turning up to $O(n^4)$ facing pairs. We note that computing the new objective value can be done in $O(n)$ time since only links using one of the endpoints of the considered facing pair will potentially be effected by turning on the facing pair and there are at most $O(n)$ such links. This implies Step 4 has a worst case complexity of $O(n^5)$ but in practice instances up to $n = 30$ can be run in a few minutes on a fast workstation.

Step 1 Let P be the fully relaxed version of the MILP
with the $\{x^a_{u,v}\}$ and $\{s_{u,k}\}$ real-valued;
for $i = 1$ **to** n: set $\mathrm{cpnt}(v_i) = i$; **endfor**

Step 2 **for** $i = 1$ **to** n-1
Solve P;
Compute

$$(u, v) = \underset{\{(u,v)|\mathrm{cpnt}(u)\neq\mathrm{cpnt}(v)\}}{\mathrm{argmax}} \sum_a c^a_{u,v} x^a_{u,v}$$

Add $\{s_{u,s(u,v)} = 1, s_{v,s(v,u)} = 1\}$ to P;
Merge $\mathrm{cpnt}(u)$ with $\mathrm{cpnt}(v)$;
endfor

Step 3 **for** $s_{u,k} \in P$ unconstrained by Step 2
Add $\{s_{u,k} = 0\}$ to P;
endfor
Let $L = \{((u_l, k_l), (u_r, k_r))\}$ be all facing
sector pairs such that $s_{u_l,k_l} = 0$ or $s_{u_r,k_r} = 0$;

Step 4 **do**
improvement = **FALSE**;
Compute $((u_l, k_l), (u_r, k_r)) =$
$\mathrm{argmax}_L\, \mathrm{obj}(P + \{s_{u_l,k_l} = 1, s_{u_r,k_r} = 1\})$;
if $\mathrm{obj}(P + \{s_{u_l,k_l} = 1, s_{u_r,k_r} = 1\}) > \mathrm{obj}(P)$
Let $P = P + \{s_{u_l,k_l} = 1, s_{u_r,k_r} = 1\}$;
Update L;
improvement = **TRUE**;
endif
while (improvement);

Algorithm 4.4: SSP-LPR

4.3.3. The MST-Greedy Algorithm

The second algorithm that we propose is based on first finding a minimum spanning tree for the network, then turning only those node sectors so as to realize the MST. Similar to the SSP-LRP algorithm, this provides a baseline solution that we then try to greedily improve by checking to see if any facing sector pairs, with at least one sector of pair unactivated, can be both activated to realize a gain in the overall network capacity. The sector pair with the greatest capacity improvement is chosen and activated. This is repeated until no further improvements are found. The complete algorithm is shown in Algorithm 4.5.

Step 1 Compute a MST T, where each edge weight $w(u, v)$ is given by the Euclidean distance $d(u, v)$.

Step 2 Let P be the corresponding MILP instance.
for $(u, v) \in T$
Add $\{s_{u,s(u,v)} = 1, s_{v,s(v,u)} = 1\}$ to P;
endfor

Step 3 Follow Steps 3 and 4 identically from the SSP-LPR algorithm.

Algorithm 4.5: MST-Greedy

4.3.4. Time Complexity of MST-Greedy

We note that the time complexity of the MST-Greedy algorithm is also polynomial. Performing the MST computation can be done in $O(n^2)$ time using a standard method such as Prim's algorithm [103]. Step 3 has a worst case of $O(n^5)$, so this is the overall time complexity of the algorithm.

4.4. Simulation Results

In our simulation scenarios we change either the value of n or M while keeping the other value fixed. We evaluate the performance of the proposed algorithm, i.e., the SSP-LP rounding algorithm (labeled as SSP-LPR), a Minimum Spanning Tree Approach (labeled, SSP-MST), and a K-Nearest Neighbors based algorithm (labeled as SSP-kNN) in terms of the summation of link capacities (i.e. maximum capacity) and the well-known Jain's fairness index,

$$f(r_1, r_2, \cdots, r_n) = \frac{\left(\sum_{i=1}^{n} r_i\right)^2}{n \sum_{i=1}^{n} (r_i)^2},$$

where r_i is the capacity of each edge in the network. Further we compute the fairness into and out of each node to verify the results. Jain's fairness index is the most commonly used metric for evaluating the performance of resource allocation algorithms in terms of fairness. In the first scenario, we compared the proposed algorithms against the optimal solutions given by solving the MILP in small cases. In the other scenarios, we compared the proposed algorithm in terms of both metrics on large input cases. We summarize our simulation scenarios in the following and present the corresponding simulation results in Figs. 4.2–4.4. Each number in these figures is an average over 10 runs, each with a different randomly generated network.

- Scenario 1: Change n from 8 nodes to 16 nodes with a step size of 2. Fix $M = 8$.
- Scenario 2: Change n from 10 nodes to 30 nodes with a step size of 5. Fix $M = 8$.
- Scenario 3: Change M from 4 sectors to 12 sectors with a step size of 2. Fix $n = 20$.

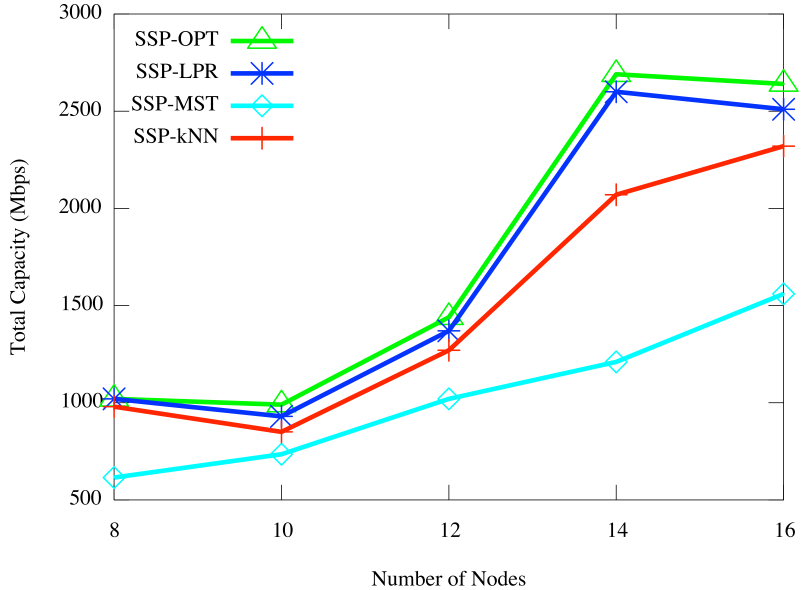

Figure 4.2: Multi-Beam Antenna Scenario 1: The proposed algorithm VS. optimal

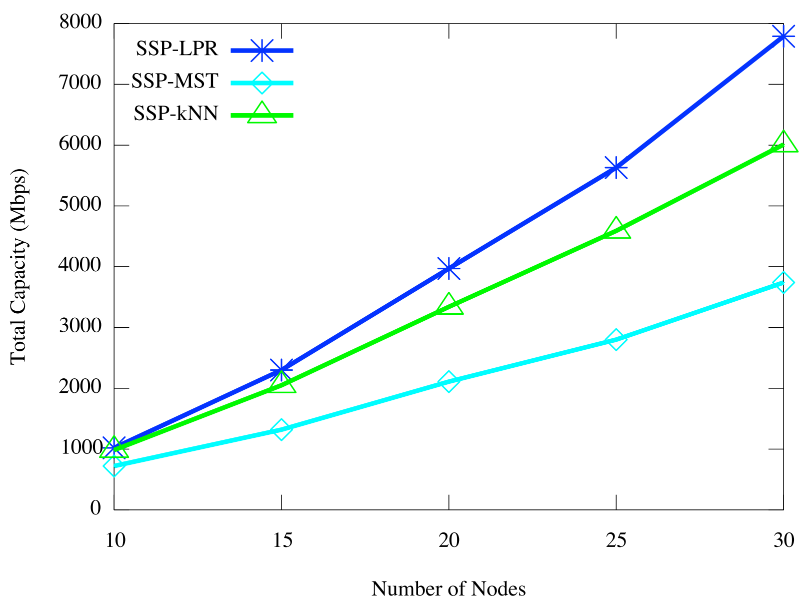

Figure 4.3: Multi-Beam Scenario 2: Performance VS. the number of nodes in the network.

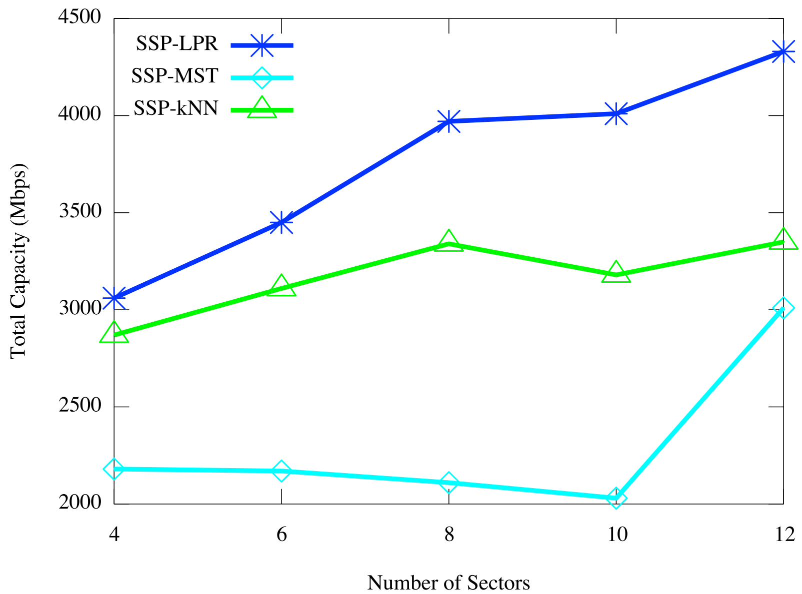

Figure 4.4: Multi-Beam Scenario 3: Performance VS. the number of sectors in each node.

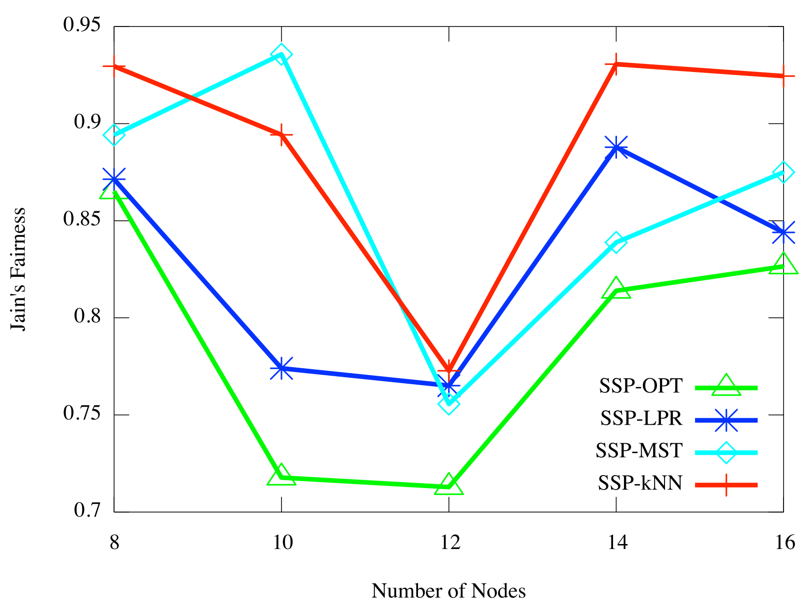

Figure 4.5: Average Jain's Fairness for each of the multi-beam antenna algorithms.

The following observations can be made from the simulation results:

1) From Fig. 4.2, we can see that the maximum capacity values given by the proposed algorithm closely track the optimal values. The average difference is only 3.61% for the SSP-LPR algorithm.

2) From Figs. 4.3–4.4, we see that when the number of nodes in the network varies, the SSP-LPR algorithm outperforms the SSP-MST by approximately a factor of two and it outperforms the SSP-kNN algorithm by an average of 30%. As the number of sectors per antenna is increased the SSP-LPR outperforms the SSP-MST algorithm by an average of 30% and the SSP-kNN by an average of 23%, however this performance shows increasing gains for the SSP-LPR algorithm as the number of sectors grows.

3) Although the goal of the proposed algorithm is to maximize the network capacity, maximizing capacity can lead to poor fairness. In Scenario 1, we computed the fairness of both incoming and outgoing node capacities. The average of these values for for each algorithm is shown in Fig. 4.5. All algorithms are relatively fair (fairness > 0.7), with SSP-kNN and SSP-MST providing the best average fairness.

4.5. Conclusions

We have studied the topology control approach for efficient communications in wireless relay networks with smart antennas. The corresponding optimization problem was formally defined as the SSP. We first presented an MILP formulation to provide optimal solutions. Then we presented a new LP rounding algorithm. It has been shown by extensive simulation results that the proposed algorithm provides close-to-optimal performance and is superior to several alternative approaches in terms of both network capacity and fairness.

CHAPTER 5

JOINT ROUTING AND CHANNEL SELECTION

Wireless Mesh Networks (WMNs) are considered an economical method of providing robust, high-speed backbone infrastructure and broadband Internet access in large areas [104]. Mesh topology offers the advantages of alternative route selection to assure throughput and Quality of Service (QoS) requirements under dynamic load conditions. As aggregate traffic volume can be substantial on backbone links converging on gateways and mesh routers, considerations of transmission path routing and how to select channels along the path are essential to assure that a WMN can meet the QoS and throughput requirements of end-users' applications, especially real-time multimedia applications. Furthermore, range considerations and propagation characteristics demand careful attention to interference. Cognitive radios are desirable for a WMN in which a large volume of traffic is expected to be delivered since they are able to utilize available spectrum more efficiently than conventional, static channel assignment methods and therefore improve network capacity significantly [105]. However, they introduce additional complexities to resource allocation. With cognitive radios, each node can access a set of available spectrum bands which may span a wide range of frequencies. Each spectrum band may be divided into channels, and the channel bandwidths may vary from band to band. Different channels may be able to support quite different transmission ranges and data rates, both of which have a significant impact on resource allocation and interference effects. Each network link has some subset of channels available due to the activities of primary users and other traffic in the network.

We study two hard resource allocation problems in cognitive radio mesh networks: the *Channel Selection (CS)* problem which is to choose a set of available channels on each link in a given routing path so as to maximize end-to-end throughput of the path, and the *Joint Routing and Channel Selection (JRCS)* problem where the routing path is not provided as part of the input and must be found along with a channel selection for each link on the path. We use a general and accurate approach to estimate end-to-end throughput of a path, in which channel capacities and availabilities are link specific. Our objective is to design efficient approaches to support emerging wireless applications demanding long-standing connections and high end-to-end throughput, such as real-time streaming video or bulk data transfer.

This work is different from some previous works on scheduling and spectrum allocation [37, 44, 62–65] which usually dealt with the problem of scheduling and allocating channels to links for link-layer throughput maximization. Here, we focus on end-to-end performance, and consider the problem of allocating channels along a multi-hop routing path, which is a much harder problem due to the constraints related to intra-flow interference [106] (links on a common path interfere with each other if assigned the same channels) and due to the fact that in general there are an exponential number of potential paths in a mesh network connecting a pair of nodes as well as an exponential number of ways to assign channels along a path. In addition, the algorithms proposed for traditional WMNs with homogeneous channels [36,88,89] cannot be applied to solve our problems here which target at a large number of heterogeneous channels that can support different data rates and transmission ranges. In short, routing and channel selection in cognitive radio mesh networks are very challenging problems, which is why most existing works [80–86] on this topic presented heuristic algorithms that cannot provide any performance guarantees. In this work, we study the CS problem and the JRCS problem from a theoretical perspective and aim at developing theoretically well-founded and practically useful algorithms to solve them. Our major contributions are summarized as follows:

- We present a new characterization of optimal CS solutions, which leads to an efficient dynamic programming algorithm that can optimally solve the CS problem, and if the path satisfies a certain natural *self-avoiding* criteria (defined in Section 5.1), in time linear in the length (hop-count) of the path. In addition, the algorithm can be easily implemented in a distributed fashion and an optimal solution can be computed in a single pass by the nodes along the path with only local information sharing.

- We also examine the much harder problem of JRCS. We show that obtaining a $(2/3 + \epsilon)$ approximation to the JRCS problem is NP-hard for any $\epsilon > 0$, which places the JRCS problem in the complexity class of APX-hard. Despite the theoretical hardness, we present two heuristic algorithms to solve the joint problem.

- Extensive simulation results show that the proposed joint algorithms outperform the approach using our optimal CS algorithm on shortest (minimum hop-count) paths .

The differences between this work and related works are summarized as follows: (1) In both of our problems, the objective is to maximize end-to-end throughput, which is different from those works addressing link layer (single-hop) throughput such as [37, 44, 62–65]. (2) We obtain an optimal algorithm for the CS problem, that runs in time linear in the length of the given routing path, provided the path satisfies a natural condition. Many related works (such as [80–86]) only presented heuristic algorithms that cannot provide any performance guarantees. (3) Our optimization problems are different from those studied in [80–87]. (4) As described above, the algorithms proposed for traditional WMNs with homogeneous channels [36, 88, 89] cannot be applied to solve our problems here due to channel heterogeneity. (5) To the best of our knowledge, we are the first to establish a bound on the complexity of the JRCS problem. We show that it is NP-hard to approximate to within a factor $(2/3+\epsilon)$ and provide effective heuristic algorithms to solve it.

5.1. Problem Formulation

We consider a wireless mesh backbone network $G = (V, E)$ with static mesh routers, where V is the set of nodes and E is the set of available communications links between the nodes. Each node is equipped with a cognitive radio. Similar as in [80, 82, 86], a spectrum occupancy map is assumed to be available to network nodes from a centrally-maintained spectrum database. This scenario has recently been promoted by the FCC to indicate over time and space the channel availabilities in the spectrum below 900 MHz and around 3 GHz [41]. In this case, spectrum (channel) availability between any given node pair is known. We study the problem of determining the optimal route and channel assignment for a communication session between two nodes in the network. Spectrum sensing is out of scope of this work.

We define our assumptions about the parameters of the cognitive radio network: Let m be the number of channels available in the network. In general, each link e will have only a subset of these channels available at any given time. This can be due to interference, the link distance being greater than the transmission range, or that channel being already in use on that link. We will also assume that each available channel j on link e has an associated bit rate $b_{e,j} \geq 0$. This bit rate can depend on the link distance and other factors. We assume that communication in the network is done using synchronized transmission frames. Let A_e be the set of channels available to link e during the current frame.

We adopt the following simple interference model: We assume that there is an interference distance R_j for each channel j such that a link $e = (u, v)$ *interferes* with another link $e' = (u', v')$ on channel j if and only if $|u - v'| \leq R_j$ or $|u' - v| \leq R_j$. We will also consider that the nodes in question are half-duplex. This means that nodes cannot simultaneously transmit and receive. The duplexing and interference constraints impose conditions on which link flows can be active at the same time. We will summarize these conditions in a well-known *conflict graph*, $G_c = (V_c, E_c)$, where the vertices V_c are the link-channel pairs (e, j) and the edges (undirected) indicate those link-channel pairs which cannot be simultaneously operational due to interference or duplexing constraints.

Suppose we have a routing path p from s to t and a set of active channels $J_e \subset A_e$ has been chosen to be used for each link $e \in p$. Let $G_{c,p}^{\langle J_e \rangle}$ be the conflict graph restricted to the link-channel pairs of the form (e, j) where $e \in p$ and $j \in J_e$. Let $t_{e,j}$ be the total amount of time allocated to the link-channel pair (e, j) in the transmission frame (assumed to be of length 1). Each clique C in $G_{c,p}^{\langle J_e \rangle}$ imposes the constraint

$$\sum_{(e,j) \in C} t_{e,j} \leq 1. \tag{5.1}$$

Let $m_{e,j}^{\langle J_e \rangle}$ be the size of the largest clique containing (e, j) in $G_{c,p}^{\langle J_e \rangle}$. We make a simplifying assumption that the scheduling mechanism creates a *uniform schedule* such that

$$t_{e,j} = 1/m_{e,j}^{\langle J_e \rangle}. \tag{5.2}$$

Observe that each constraint of the form (5.1) is satisfied by (5.2). We note that a uniform schedule may not be optimal but we adopt this assumption for algorithmic convenience and because it may not be possible to alter the scheduling mechanism used in a real network.

Let $G^j_{c,p}$ be the subgraph of $G_{c,p}$ consisting of the link-channel pairs that use channel j.

Definition 3. *We say that the routing path p is* **self-avoiding** *if all of the subgraphs $G^j_{c,p}$ are interval graphs such that the intervals occur in order of p.*

This means that the link-pairs in p involving channel j can be placed in order on the real number line R such that two links-pairs (e, j) and (e', j) conflict if and only if their corresponding intervals overlap. Interval graphs have a useful property that their cliques can be easily enumerated since a cliques will be represented by a set of consecutive intervals that all mutually overlap. In fact, all maximal cliques on an interval graph with n vertices can be enumerated in $O(n)$ time. We will focus on self-avoiding routing paths in this work because it is easy to find the maximal cliques in their interference graphs.

We define the *end-to-end throughput* τ of the path p and selected active channels $\langle J_e\rangle_{e\in p}$, as the minimum over all the links $e \in p$ of the effective throughput on link e. The effective throughput of an link e is sum of the bit rates on each active channel times the amount of transmission time allocated to each channel,

$$\tau(p, \langle J_e\rangle) = \min_{e\in p} \sum_{j\in J_e} b_{e,j}/m^{\langle J_e\rangle}_{e,j}. \tag{5.3}$$

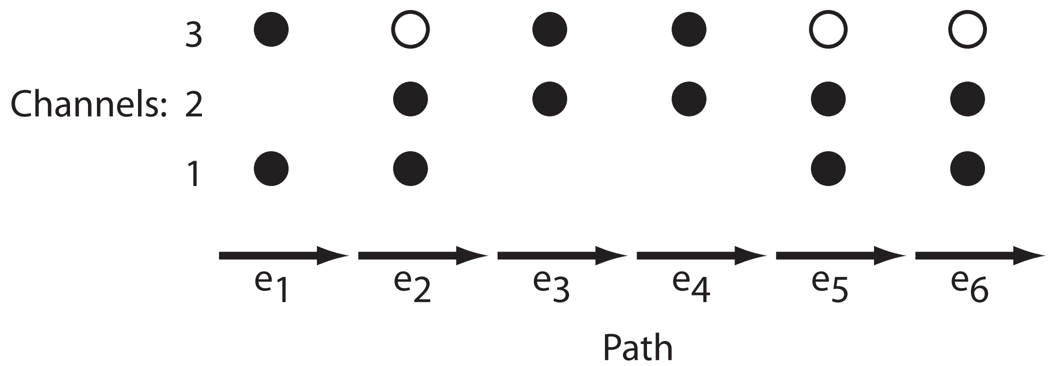

Figure 5.1: Example channel selection problem.

We can now formalize the two computational problems considered. In addition to the source node s and destination node t, we assume the available channel sets A_e and bit rates $b_{e,j}$ are provided in the input.

Channel Selection (CS): Given a routing path p from s to t, determine active channels sets $J_e \subset A_e$ for all $e \in p$ that maximize the end-to-end throughput $\tau(p, \langle J_e \rangle)$.

Joint Routing and Channel Selection (JRCS): Find a routing path p from s to t and active channels sets $J_e \subset A_e$ for all $e \in p$ that maximizes the end-to-end throughput $\tau(p, \langle J_e \rangle)$.

5.1.1. Optimal Channel Selection

In this section we present an optimal algorithm for the problem of choosing the best set of channels to use for a given routing path (CS). Our proposed CS algorithm is based upon a dynamic programming approach in which the solutions to partial problems are used to assemble a solution to the full problem. To help motivate the algorithm, we provide a simple example of channel selection in Figure 5.1.

In the example shown in Figure 5.1 we assume that the channel availability is as shown and that each channel has a capacity of 1. We further assume that two link pairs (e_i, j) and (e_k, j) interfere if and only if $|i-k| \leq 2$. The solid circles indicate an optimal channel selection. Each selected channel participates in a clique of size 3 (due to interference and duplexing constraints). This means each selected channel provides capacity $\frac{1}{3}$ and the end-to-end capacity of the path is $\frac{2}{3}$.

Let $p = (v_0, v_1, \ldots, v_n)$ be the given routing path from the source node s to the destination node t, where $v_0 = s$ and $v_n = t$. Let $e_i = (v_{i-1}, v_i)$ be the i-th link on the path $(1 \leq i \leq n)$ and let A_i be the set of available channels on e_i. The objective is to find active channel sets $J_i \subset A_i$ for $i = 1, \ldots, n$ that maximize the end-to-end throughput $\tau(p, \langle J_i \rangle)$. For $0 < i \leq n$, we define $X_i = \{(e_i, j) | j \in A_i\}$ as the available link-pairs involving e_i. For $0 \leq l \leq n$, let $p_l = (v_0, v_1, \ldots, v_l)$ be the subpath consisting of the first l links of p. For $0 < l < n$, we define the *bridging set*,

$$\begin{aligned} B_l = \{(e_i, j), (e_{i'}, j') \mid i < l, i' > l, \\ ((e_i, j), (e_{i'}, j')) \in G_{c,p}\}. \end{aligned} \tag{5.4}$$

We also let $B_0 = X_1$, the set of available link-channel pairs for the first link e_1 in the path. Observe that $X_{l+1} \subset B_l$ for all $0 \leq l < n$, due to the half-duplex constraint at each node. For $0 \leq l < n$, let $\langle J_i \rangle_{i=1}^{l}$ be a channel selection for the subpath p_l and let $B \subset B_l$.

Definition 4. *We say that $\langle J_i \rangle_{i=1}^{l}$ is B-**compatible**, written $\langle J_i \rangle_{i=1}^{l} \sim B$, if, for all $1 \leq i \leq l$, $(e_i, j) \in B \Rightarrow j \in J_i$ and $(e_i, j) \in B_l \setminus B \Rightarrow j \notin J_i$.*

Suppose $\langle J_i \rangle_{i=1}^{l} \sim B$. We are interested in calculating the throughput of this channel assignment for the partial path p_l. As B may contain link-channel pairs from further along in p, we will include those in the calculation, since these pairs may effect clique sizes. We will refer to this throughput as $\tau^B(p_l, \langle J_i \rangle)$. Next, let

$$\langle J_i^{l,B} \rangle_{i=1}^{l} = \operatorname*{argmax}_{\langle J_i \rangle_{i=1}^{l} \sim B} \tau^B(p_l, \langle J_i \rangle),$$

be a channel selection for the subpath p_l that is B-compatible and has the maximum end-to-end throughput along the subpath p_l. The main idea of the algorithm is that the $\langle J_i^{l,B} \rangle$ can be computed by dynamic programming. In particular, suppose that the $\langle J_i^{l,B} \rangle$ are known for all $B \subset B_l$, for some $0 \leq l < n-1$. We will use these channel selections to compute the $\langle J_i^{l+1,B'} \rangle$ for all $B' \subset B_{l+1}$. Let $B' \subset B_{l+1}$ and suppose that $\langle J_i^{l+1,B'} \rangle_{i=1}^{l+1}$ be an optimal channel selection for the subpath p_{l+1} that is compatible with B'. We define

$$B_{l+1}^{prev} = B_{l+1} \cap B_l \tag{5.5}$$

and

$$B_{l+1}^{new} = B_{l+1} \setminus B_l. \tag{5.6}$$

Note that $B_{l+1} = B_{l+1}^{new} \cup B_{l+1}^{prev}$. Let

$$B = (\bigcup_{i=1}^{l+1} J_i^{l+1,B'} \cup B') \cap B_l.$$

We observe that $B \subset B_l$ and that B agrees with B' on B_{l+1}^{prev} (this means $B \cap B_{l+1}^{prev} = B' \cap B_{l+1}^{prev}$). Thus,

$$\begin{aligned} \tau^{B'}(p_l, \langle J_i^{l+1,B'} \rangle_{i=1}^{l}) &= \tau^B(p_l, \langle J_i^{l+1,B'} \rangle_{i=1}^{l}) \\ &\leq \tau^B(p_l, \langle J_i^{l,B} \rangle_{i=1}^{l}), \end{aligned} \tag{5.7}$$

since the throughput on all of the links in the subpath p_l is unchanged. We use the notation, $m_{e_{l+1},j}^{\langle J^{l+1,B'}\rangle,B'}$ to refer to the size of the largest clique containing (e_{l+1}, j) present among the link-channel pairs from $\langle J^{l+1,B'}\rangle$ and B' together. Let

$$< J^{opt} >= \langle J^{l,B}\rangle, J_{l+1}^{l+1,B'};$$

the channel assignments given by $\langle J^{l,B}\rangle$ for the subpath p_l and by $J_{l+1}^{l+1,B'}$ for link e_{l+1}. A key observation is that $m_{e_{l+1},j}^{\langle J^{l+1,B'}\rangle,B'} = m_{e_{l+1},j}^{\langle J^{opt}\rangle,B'}$ for any $(e_{l+1}, j) \in J_{l+1}^{l+1,B'}$ since both B_i and B_{i+1} will contain any link-channel pairs from p_{l+1} that conflict with it. We have,

$$\begin{aligned}
&\tau^{B'}(p_{l+1}, \langle J^{l+1,B'}\rangle) \\
&\quad = \min(\tau^{B}(p_l, \langle J^{l+1,B'}\rangle_{i=1}^{l}), \\
&\qquad \sum_{j\in J_{l+1}^{l+1,B'}} b_{e_{l+1},j}/m_{e_{l+1},j}^{\langle J^{l+1,B'}\rangle,B'}) \\
&\quad \leq \min(\tau^{B}(p_l, \langle J^{l,B}\rangle_{i=1}^{l}), \\
&\qquad \sum_{j\in J_{l+1}^{l+1,B'}} b_{e_{l+1},j}/m_{e_{l+1},j}^{\langle J^{opt}\rangle,B'}) \\
&\quad = \tau^{B'}(p_{l+1}, \langle J^{opt}\rangle). \qquad (5.8)
\end{aligned}$$

Since $\langle J^{l+1,B'}\rangle$ was assumed optimal, we must have equality in (5.8), so we can take

$$\langle J^{l+1,B'}\rangle = \langle J^{opt}\rangle. \qquad (5.9)$$

Equation (5.8) shows that the optimal channel selection for p_{l+1} and B' can be expressed in terms of an optimal channel selection for p_l and B, a smaller problem. This means that we can use dynamic programming to compute $\langle J^{l+1,B'}\rangle$. The idea is to take each $\langle J^{l,B} \rangle$ found for p_l and extend to channel assignment for p_{l+1} for all $B' \subset B_{l+1}$ such that B and B' agree on B_{l+1}^{prev}. Since B fixes which link-pairs from B_{l+1}^{prev} are included in B', we can simply enumerate all subsets $B'' \subset B_{l+1}^{new}$ and for each, form $B' = (B \cap B_{l+1}^{prev}) \cup B''$. Also, since $X_{i+1} \subset B_i$, we let B also determine all channel assignments for link e_{l+1}:

$$\langle J^{test} \rangle = \langle J^{l,B} \rangle, (B \cap X_{i+1}). \tag{5.10}$$

We then evaluate $\tau^{B'}(p_{l+1}, \langle J^{test} \rangle)$ to see if J^{test} provides a better channel assignment for p_{n+1} compatible with B'; if yes, we keep it. This evaluation is done in the same fashion as (5.8):

$$\begin{aligned} \tau^{B'}(p_{l+1}, \langle J^{test} \rangle) = & \min(\tau^{B}(p_l, \langle J^{l,B} \rangle_{i=1}^{l})), \\ & \sum_{j \in B \cap X_{i+1}} b_{e_{l+1},j} / m_{e_{l+1},j}^{\langle J^{test} \rangle, B'}) \end{aligned} \tag{5.11}$$

In order to evaluate (5.11), we need to calculate the second min term since the first is already known. This is done by determining the largest clique that (e_{l+1}, j) participates in among link-channel pairs from $\langle J^{test} \rangle, B'$. If we make the assumption that the routing path p is self-avoiding, then any clique in involving (e_{l+1}, j) is either a consecutive list of link-channel pairs in $G_{c,p}^{j}$ (that all mutually overlap in the interval graph representation), or a clique involving (e_{l+1}, j) and a link-channel pair from X_l or X_{l+2} or both, using a channel other than j. Let Δ be the maximum degree of any node in any of the $G_{c,p}^{j}$ and suppose there are at most m_p channels available on any link in p. All of the clique possibilities can be checked in $O(\Delta m_p)$ time, so this is the time required to evaluate (5.11). We can now state the entire DP-ChannelSelect algorithm (Algorithm 5.6).

5.2. Proposed Algorithms

5.2.1. The DP-ChannelSelect Algorithm

INPUT: a self-avoiding path $p = (v_0, v_1, \ldots v_n)$ from s to t

Step 1 **for** $l = 0$ to $n-1$
  Compute B_l, B_l^{prev} and B_l^{new} using (5.4), (5.5) and (5.6).
endfor

Step 2 **for** $l = 1$ to $n-1$
  forall $B \subset B_l$
    forall $B'' \subset B_{l+1}^{new}$
      Construct $B' = (B \cap B_{l+1}^{prev}) \cup B''$ and $\langle J^{test} \rangle$ using (5.10).
      Calculate $\tau^{test} = \tau^{B'}(p_{l+1}, \langle J^{test} \rangle)$ using (5.11).
      if $\tau^{test} > \tau^{B'}(p_{l+1}, \langle J^{l+1,B'} \rangle)$
        set $\langle J^{l+1,B'} \rangle = \langle J^{test} \rangle$
      endif
    endforall
  endforall
endfor

Step 3 Compute
$B^* = \underset{B \subset B_{n-1}}{\operatorname{argmax}} \tau^B(p_{n-1}, \langle J^{n-1,B} \rangle, B \cap X_n)$

Step 4 **return** $\langle J^{n-1,B^*} \rangle, B^* \cap X_n$

Algorithm 5.6: DP-ChannelSelect

5.2.2. Time Complexity of DP-ChannelSelect

Next, we analyze the time complexity of the DP-ChannelSelect algorithm. The running time is dominated by Step 2 which must examine every subset of B_{l+1}^{new} for every subset of B_l for $l = 1, \ldots, n-1$. The number of subset combinations examined for each l is $2^{|B_l||B_{l+1}^{new}|}$. If p is self-avoiding, then for any l, B_l will be comprised of consecutive lists of link-channel pairs in $G^j_{c,p}$ for each channel j. For each j, the list will be at most $\Delta + 1$ in length; since every other link-channel pair in the list will conflict with the $(e_{l'}, j)$ in the list with maximum $l' \leq l$. Let m_p be the total number of channels available along p. There can be at most $m_p(\Delta + 1)$ link-channel pairs in B_l. Since $B^{new}_{l+1} \subset B_{l+1}$ the same bound applies to it. Thus the number of subset combinations examined is at most $2^{m_p^2(\Delta+1)^2}$. The time required for evaluating each subset combination is $O(\Delta m_p)$ time, as explained earlier. This implies the overall running time of the algorithm is $O(2^{m_p^2(\Delta+1)^2}\Delta m_p n)$ time, provided the input routing path is self-avoiding. In practice, the running time can be much faster, as typically each B_l will be smaller than $m_p(\Delta + 1)$ in size and B^{new}_{l+1} will be smaller still. We remark that this bound places CS in the class of *fixed-parameter tractable* problems (FPT) [107], where the parameters are Δ and m_p. The above discussion leads to the following result:

Theorem 3. *The DP-ChannelSelect computes an optimal end-to-end channel assignment in $O(2^{m_p^2(\Delta+1)^2}\Delta m_p n)$ time, where p is a self-avoiding routing path of length n, m_p is maximum number of available channels in any link in p and Δ is the maximum degree of any node in the path conflict graph $G_{c,p}$.*

We observe that the effectiveness of Algorithm 5.6 depends heavily the parameters m_p and Δ; providing these values are not too large, the algorithm runs quickly in practice; most of the optimal channel selections found in our experiments were computed in a few seconds on a laptop computer. However, the performance of the algorithm will degrade and perhaps become impractical if these parameters are too large. We observe that the DP-ChannelSelect algorithm can be implemented in a distributed fashion and carried out by the nodes themselves along the routing path p, since the algorithm makes a single pass over the length of p and the sets of intermediate path/channel assignments computed at each node along the path depend only on those from the previous node and relatively local interference information (determined by the B_l sets).

5.2.3. Channel Aware Routing

In this section we present two heuristic algorithms for the joint problem of routing and channel selection.

5.2.3.1. The RCS-PathExtend Algorithm. The first proposed algorithm is based on the idea of simultaneously finding good path/channel assignments from the source node s to all other nodes in the network, including the destination node t. The approach is similar to the Bellman-Ford shortest-path algorithm in that new path/channel assignments are generated by "relaxing" all the links in the network in a series of phases that continues until no better paths are discovered. Each node $u \neq s$ will store a list P_u of path/channel assignments of the form $(p_u, \langle J_e \rangle_{e \in p_u})$, where p_u is a path from s to u. When the link (u, v) is relaxed, the current path/channel assignments stored at u will be augmented by the link (u, v); for each subset $J' \subset A_{(u,v)}$ we will create a new path/channel assignment $(p'_v, \langle J_e, J' \rangle)$, where p'_v is the path p_u followed by the link (u, v). To limit the search, each node keeps a maximum of d best path/channel assignments (we chose $d = 100$). We also note that it is only necessary to augment those path/channel assignments that were created in the previous phase; we will refer to these as *new* in the algorithm. Clearly, a loop in a path can never improve the end-to-end throughput of the path, so we restrict attention to *simple* paths that never repeat a vertex. We also require that the path be self-avoiding. The complete algorithm is given in Algorithm 5.7. (Algorithm 5.8 is a subroutine used by Algorithm 5.7.)

INPUT: a connection request from s to t, $G = (V, E)$

Step 1 **repeat**
  forall $e \in E$
    Link-relax(e)
  endforall
until no list P_u changes

Step 2 **return** $\text{argmax}_{(p,\langle J\rangle)\in P_t}\, \tau(p, \langle J\rangle)$

Algorithm 5.7: RCS-PathExtend

INPUT: a link $e = (u, v)$

forall new $(p_u, \langle J_e\rangle_{e\in p_u}) \in P_u$
  Let $p_v = p_u + (u, v)$
  if p_v is a simple, self-avoiding path
    forall $J_{(u,v)} \subset A_{(u,v)}$
      Construct $\langle J'\rangle = \langle J_e, J_{(u,v)}\rangle$
      Compute $\tau' = \tau(p_v, \langle J'\rangle)$
      if $(|P_v| < d$ **or** $\tau' > \min_{(p,\langle J\rangle)\in P_v} \tau(p, \langle J\rangle))$
        Insert $(p_v, \langle J'\rangle)$ into P_v
        if $|P_v| > d$
          remove $\text{argmin}_{(p,\langle J\rangle)\in P_v}\, \tau(p, \langle J\rangle)$ from P_v
        endif
      endif
    endforall
  endif
endforall

Algorithm 5.8: Link-relax

5.2.3.2. Time Complexity of RCS-PathExtend. We next analyze the computational requirements of RCS-PathExtend.

Lemma 2. *All path/channel assignments created during i-th phase, have paths of length at least i.*

Proof. The proof is by induction on the number of phases completed. The statement is trivially true for $i = 1$. By the inductive hypothesis, the path/channel assignments created during phase $i - 1$, have path length at least $i - 1$. These will be the only path/channel assignments considered new in phase i and so it follows that all path/channel assignments created during phase i, have path length at least i. $\square$

Theorem 4. *RCS-PathExtend runs in $O(d2^{m_p}\Delta m_p|E||V|^2)$ time, where d is the maximum number of path/channel assignments kept at each node, m_p is maximum number of available channels in any link in G and Δ is the maximum degree of any node in the conflict graph G_c.*

Proof. The maximum length of a simple path is $|V| - 1$, so the maximum number of phases (iterations of the outer loop in Algorithm 5.7) is $|V| - 1$ by Lemma 2. Each phase calls Link-relax $|E|$ times, and each call to Link-relax creates at most $d2^{m_p}$ new path/channel assignments. The end-to-end throughput of each of these can be evaluated using (5.3) in $O(|V|\Delta m_p)$ time, since we restrict focus to self-avoiding paths. The total running time is thus $O(d2^{m_p}\Delta m_p|E||V|^2)$ time. $\square$

5.2.3.3. The Bottleneck-Route Algorithm. The second approach attempts to find a single path whose links all have a high useful capacity. We make this precise as follows. We define the link capacity $c(e)$ of a link e as, $c(e) = \sum_{j \in A_e} b_{e,j}$. The link capacity provides an upper bound on the bit flow rate achievable by link e ignoring intra-path interference. In addition to link capacity, we also take into account how close the link $e = (u, v)$ to the source s and destination t.

For each link $e = (u, v)$, we define the source-destination distance as $d(e) = ||u-s||+||u-t||+||v-s||+||v-t||$. Let $d_{max} = \max_{e \in E} d(e)$ and $d_{min} = \min_{e \in E} d(e)$, be the maximum and minimum source-destination distances, respectively. A heuristic additional weighting factor on the link capacity to better estimate the *usefulness* of a link e for an s-t path. We define

$$u(e) = (1 + \frac{d_{max} - d(e)}{d_{max} - d_{min}})c(e), \tag{5.12}$$

Note that $1 \leq u(e) \leq 2$, with $u(e) = 1$ when $d(e) = d_{max}$ and $u(e) = 2$ when $d(e) = d_{min}$. Let the *useful bottleneck capacity* of a path p be defined as

$$c(p) = \min_{e \in p} u(e).$$

Our goal is to find a path p that maximizes $c(p)$. This is a well-known problem that can be efficiently solved by computing a minimum spanning tree T on the network graph using an link weight function $w(e) = -u(e)$. The unique path in T from s to t will have maximum useful bottleneck capacity. We term this the *Bottleneck route.* It can easily computed using Algorithm 5.9. Once the bottleneck route has been been found then an optimal channel section for the route can be computed using Algorithm 5.6.

5.3. Computational Complexity of JRCS

In this section, we show that the JRCS problem is NP-hard to approximate to within a factor of $2/3+\epsilon$, for any $\epsilon > 0$. This is done via a reduction from the EXACT

INPUT: a connection request from s to t, $G = (V, E)$

Step 1 Assign edge weights $-u(e)$ to each link $e = (u, v)$ using (5.12).

Step 2 Compute a minimum weight spanning tree T.

Step 3 Compute the route p from s to t in T.

Step 4 **return** p.

Algorithm 5.9: Bottleneck-Route

COVER problem [108]. An instance of this problem consists of a set $U = \{u_1, \ldots u_m\}$ and a collection of subsets of U, $F = \{S_1, \ldots S_n\}$. The problem is to determine if there exists a sub-collection $F' \subset F$, such that for each $u_j \in U$, there is a unique $S_i \in F'$ such that $u_j \in S_i$. Figure 5.2 provides a sketch of the network for which we show that it is computational difficult (NP-hard) to find the best routing and channel selection. In each of the shaded sub-blocks in the figure, there a several options for the path to go (as shown in Figure 5.3). Because of interference, the choice of paths (and channels) in one sub-block the choices for the neighboring sub-blocks directly above and below. This leads to the following theorem:

Theorem 5. *The JRCS problem is NP-hard to approximate to within a factor of* $2/3 + \epsilon$*, for any* $\epsilon > 0$.

Proof. We can reduce an instance of the EXACT COVER problem to an instance of the JRCS problem as follows: The JRCS instance will consist of a layout of n rows by m columns of *blocks* $B_{i,j}$ as shown in Figure 5.2 (the layout for odd n is shown; for even n the final path to t will go from right-to-left instead). There are two basic types of blocks, depending on whether $u_j \in S_i$ and they can be oriented left-to-right or right-to-left (Figure 5.3). The blocks in the first (last) row are modified by removing the upper (lower) path in the construction diagram. We also use a specific sequence of four links called a *C-chain* that is also shown in the figure. We assume the following interference structure: two links both using channels 1 or 2 will interfere with each other if there is at most one link separating them on the selected path; the links (on channel 2) of the bottom path of block $B_{i,j}$ interfere with the links on the top path of block $B_{i+1,j}$ for $i = 1, \ldots n-1$; links using channel 0 don't interfere with each other; there is separate channel c_j available for each $j = 1, \ldots m$ and all links using channel c_j interfere with each other. Furthermore, we will assume that $b_{e,j} = 1$ for all e and j. We claim that a solution to the EXACT COVER instance exists if and only if there exists a routing path p from s to t in the JRCS instance with an end-to-end throughput of 1/2. For the links e with a single channel available j, this implies that the size of the largest cliques that the link-channel pair (e, j) can belong to is 2. We observe that this forces the links with two channels available in each C-chain to only select one channel, otherwise the third link will belong to a clique of size 3. This also requires that if the selected path traverses the bottom path of block $B_{i,j}$, then the path cannot go through the top path of block $B_{i+1,j}$ (otherwise a clique of size 4 is formed). In order to prevent this from happening, observe that in each column j at least on path must use the center path in the block corresponding to $u_j \in S_i$. Furthermore, no more than one center path can be used in column j, since

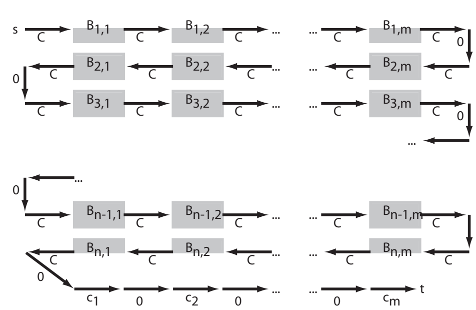

Figure 5.2: Layout of the JRCS instance corresponding to an instance of EXACT COVER.

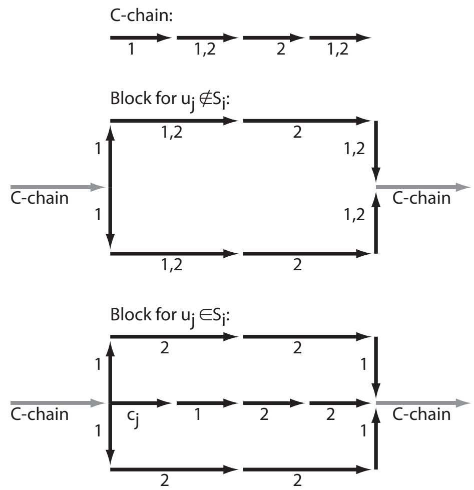

Figure 5.3: C-chain construction and block types depending on whether $u_j \in S_i$. Left-to-right versions shown.

c_j is used along each such path as well as the final portion of the path to t. These links all mutually interfere and the maximal clique size cannot exceed 2. Finally, we observe that the channel that is chosen for the second link of the first C-chain traversed in given row determines that this channel must be chosen for all C-chains in the row. We also note that if channel 2 is chosen for the second link, then the fourth link most choose channel 1; this means that the center path in any block in the row cannot be entered since this will create a clique of size 3 with the c_j link in the block. Conversely, if channel 1 is chosen for the second link in the first C-chain, then the fourth link must use channel 2 and the center path must always be chosen if a $u_j \in S_i$ block is encountered (since taking the top or bottom path creates a clique of size 3). It follows that choosing channel 1 for the second link in the first C-chain encountered in row i corresponds to adding S_i to the cover F' and choosing channel 2 corresponds to omitting S_i from F'. Observe that each $u_j \in U$ must be covered (there are n rows and only $n - 1$ top/bottom block paths so some row must use center paths) exactly once (if two center paths are used for some column j this creates a clique of size 3 on links using channel j). Hence, an end-to-end throughput of 1/2 is achievable if and only if an exact cover F' exists. The next lowest potential end-to-end throughput for this JRCS instance is 1/3 (at least one single-channel link is involved in a clique of size 3). If an approximation algorithm for JRCS was able to find a solution at least $2/3+\epsilon$ of optimal, it would find a solution with end-to-end throughput at least $(\frac{2}{3}+\epsilon)\frac{1}{2} > \frac{1}{3}$, so it would necessarily find the optimal solution and so could be used to solve EXACT COVER. It follows that joint routing and channel selection is NP-hard to approximate to within a factor of $2/3+\epsilon$. We remark this also places JRCS in the complexity class APX-hard. □

5.4. Simulation Results

To test our routing and channel selection algorithms we compared it against simple shortest path routing using Dijkstra's algorithm (the edge weights in this case were the physical link distances) followed by the DP-ChannelSelect algorithm to find the optimal channel selection for the shortest path. In all cases tried, the DP-ChannelSelect algorithm itself runs in under a second on a laptop computer.

Table 5.1: Maximum transmission distances by frequency and data rate

Transmission rate	700 Mhz	2400 Mhz	5800 Mhz
45 Mbps	15.4 km	4.5 km	1.8 km
40 Mbps	18.4 km	5.3 km	2.2 km
30 Mbps	30 km	8.6 km	3.6 km
20 Mbps	41 km	11.8 km	4.9 km
10 Mbps	68 km	20 km	8.2 km

For our experiments, we assumed there were three widely spaced frequency bands available for licensed and unlicensed operation and that the link throughput for each channel was the maximum available given the link distance and frequency used. The bands exhibit widely ranging propagation, transmission range and usage characteristics, highlighting the potential value of cognition in transmission scheduling. Tables 1 and 2 summarize our assumptions about the transmission rates and interference ranges of each frequency. These values are based on a scenario where each node transmits at 1W with a 2dBi antenna and the receiving antenna has a gain of 2dBi. The channel bandwidth is 10 MHz and the receiver noise figure is 5dB, and implementation losses of 3dB are assumed for each link. Path loss is calculated using line of sight and free space characteristics. Typical IEEE 802.16 adaptive modulation and coding parameters performance parameters were used to estimate the throughput achievable as a function of CNR (carrier to noise ratio), and were then translated into the allowable path loss threshold. The maximum channel transmission rate is a function of distance and frequency (at lower frequency, the maximum distance for a given transmission rate will be greater). We assumed that each channel was available on each link with independent probability 0.5. Primary users were placed at random locations and assigned a random channel. This channel was then made unavailable to any links of cognitive radios within the interference range of the primary user.

Table 5.2: Interference ranges by frequency

Frequency	Interference range
700 Mhz	30.8 km
2400 Mhz	9 km
5800 Mhz	3.6 km

In this scenario, the number of channels available to secondary users was varied from 3 to 15 with a step size of 3 (chosen equally from each frequency band). Ten random source-destination pairs were generated on a $50 \times 50\text{km}^2$ network with 25 nodes and 5 primary users. These pairs and the node locations were held constant

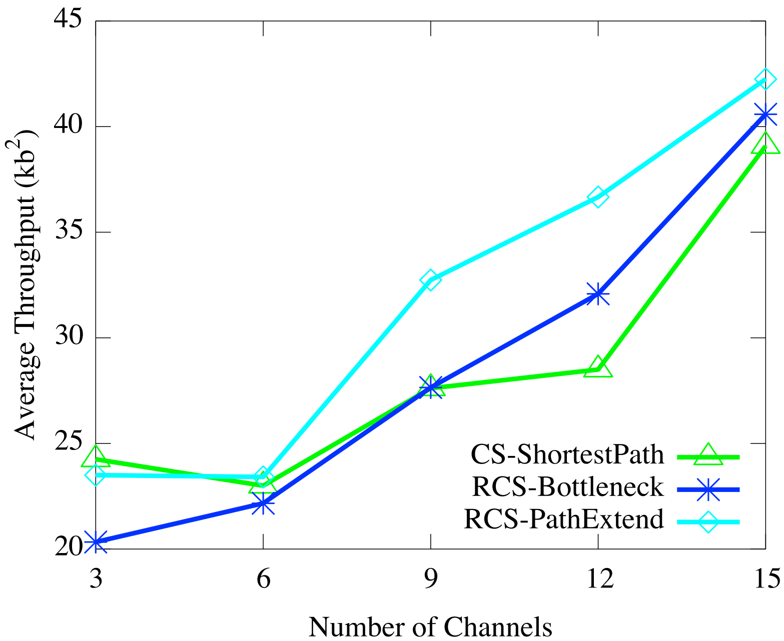

Figure 5.4: Joint Routing and Channel Selection: Average path end-to-end throughput versus the number of available channels.

for all of the experiments in this scenario. The average end-to-end transmission rate (throughput) for all routing path and channel selections is reported. The results, shown in Figure 5.4, indicate an almost linear improvement is gained by adding additional channels to the network in terms of additional throughput. The path and channel assignments found by the RCS-PathExtend were, on average, 13.23% better than those found by CS-ShortestPath, and 8.67% better than those found by RCS-Bottleneck.

In the second scenario, the physical region size was increased, but node density was held constant. Ten random source-destination pairs were generated on a network with 3 channels per frequency band, 5 primary users, and a constant density of 0.01nodes/km^2. The average end-to-end transmission rate for all routing path and channel selections is reported. As the region size grows, paths tend to get longer. The results, shown in Figure 5.5, indicate that the RCS-PathExtend algorithm continues

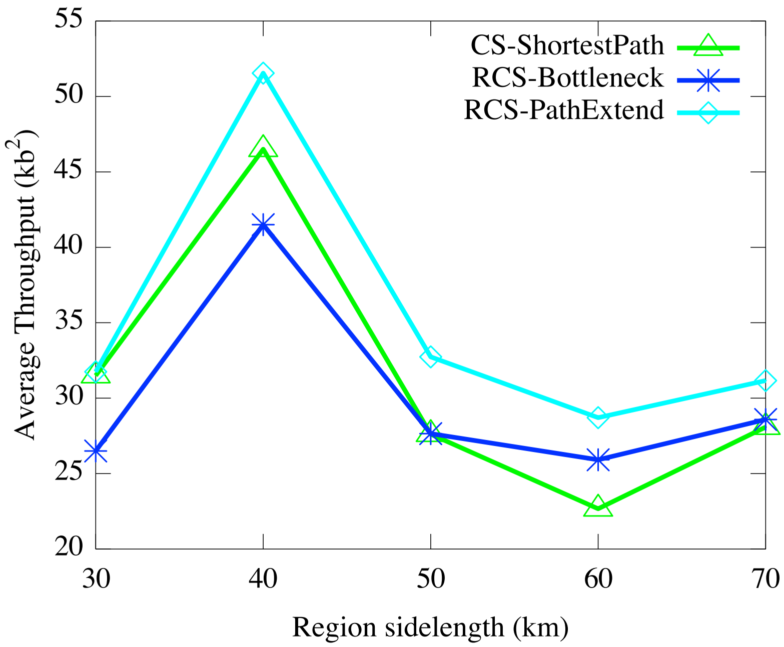

Figure 5.5: Joint Routing and Channel Selection: Average path end-to-end throughput versus network size. Node density was held constant at 0.01nodes/km^2.

to outperform the two other routing and channel assignment methods and that gap increases slightly as the region size increases. The path and channels assignments found by the RCS-PathExtend algorithm were, on average, 9.97% better than those found by CS-ShortestPath, and 20.46% better than those found by RCS-Bottleneck.

In our final scenario, the physical region size was held constant at $50 \times 50\text{km}^2$ and the number of nodes was increased. Ten random source-destination pairs were generated on a network with 3 channels per frequency band, 5 primary users. The average end-to-end transmission rate for all routing path and channel assignments is reported. As the network size grows, node density increases and the average number of links that a given link interferes with on a given channel increases. The effect is to increase the average vertex degree in the conflict graph G_c. The results, shown in Figure 5.6, indicate that again, the RCS-PathExtend algorithm outperforms the two other approaches across the range of node densities considered. That path and channel

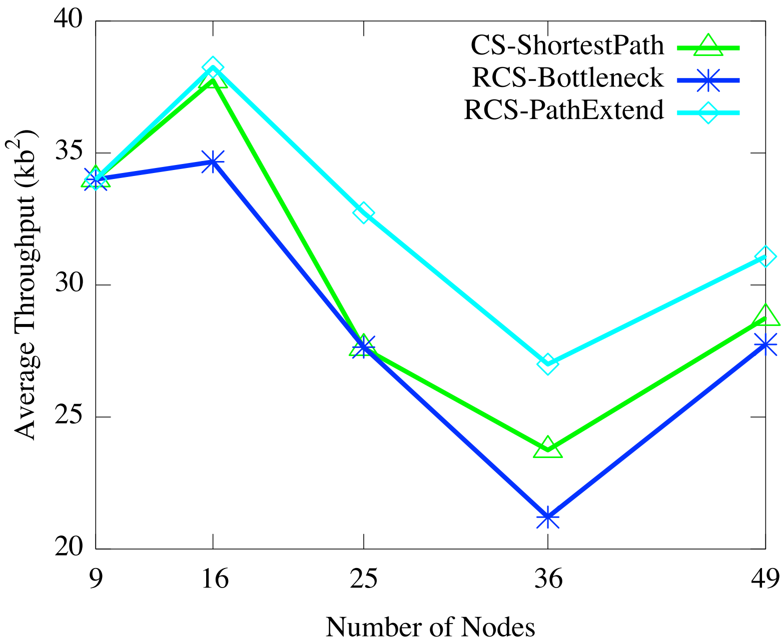

Figure 5.6: Joint Routing and Channel Selection: Average path end-to-end throughput versus node density. Region size fixed to $50 \times 50\text{km}^2$.

assignments found by the RCS-PathExtend algorithm were, on average, 8.26% better than those found by CS-ShortestPath, and 11.52% better than those found by RCS-Bottleneck.

5.5. Conclusions

We have examined two important problems for maximizing the end-to-end throughput for communication flows in cognitive radio mesh networks. For the channel selection problem on a known routing path, we developed an optimal algorithm and showed that for self-avoiding paths it runs in time linear in the length of the path. Furthermore, the algorithms only needs to propagate local information from source to destination and can be implemented in a distributed fashion. We also considered the joint problem of routing and channel selection and showed that it was NP-hard to approximate within a factor of $(2/3 + \epsilon)$. In addition we presented two novel heuristic algorithms for this problem and demonstrated the universal superiority of one of them, RCS-PathExtend, to simply using shortest path routing followed by optimal channel selection.

CHAPTER 6

THE CHANNEL RENTAL PROBLEM

Wireless Mesh Networks (WMNs) are considered an economical method of providing robust, high-speed backbone infrastructure and broadband Internet access in large areas [104]. Mesh topologies offer the advantages of alternative route selection to assure throughput and Quality of Service (QoS) requirements under dynamic load conditions. As aggregate traffic volume can be substantial on backbone links converging on gateways and mesh routers, considerations of transmission path routing and how to select channels along the path are essential to assure that a WMN can meet the throughput requirements of end-users' applications, especially real-time multimedia applications. Furthermore, range considerations and propagation characteristics demand careful attention to interference. Cognitive radios are desirable for a WMN in which a large volume of traffic is expected to be delivered since they are able to utilize available spectrum more efficiently than conventional, static channel assignment methods and therefore improve network capacity significantly [105]. However, they introduce additional complexities for resource allocation.

With cognitive radios, each node can access a set of available spectrum bands that may span a wide range of frequencies. Each spectrum band may be divided into channels, and the channel bandwidths may vary from band to band. Different channels may be able to support quite different transmission ranges and data rates, both of which have a significant impact on resource allocation and interference effects. Each network link has some subset of channels available due to the activities of primary users and other traffic in the network. Additionally, network operators may need to lease the channels from other spectrum owners. For a specific network, each channel used may have some rental cost. For this reason it is useful to determine network topologies that minimize the number of channels used.

This closely models real-world situations in both dense wireless mesh networks, such as cellular networks [109], where increasing demands for capacity are being handled by increasing the density of the network and utilizing multi-spectrum architectures, but it also models conditions in sparse rural networks, where environmental conditions can affect the availability of channels over the period of hours or days until repairs are made. Currently Ubiquiti Networks [110] provides wireless hardware in 900MHz, 2.4GHz and 5.8GHz. The cost of 900MHz and 5.8GHz hardware are 2 and 3 times the cost of 5.8GHz hardware, respectively. These costs (2x and 3x) show why channel costs should influence a topology control solution to not only find an efficient topology but also a cost-effective one.

We study the problem of how to minimize channel rental costs for topology control. We formulate the problem in general terms, provide a Mixed Integer Linear Program in order to find the optimal solutions, and propose effective algorithms that are close to optimal in practice.

6.1. System Model

We consider a multihop wireless network $G = (V, E)$ with static mesh routers, where V is the set of nodes and E is the set of available communications links between the nodes. Each node is is equipped with a cognitive radio that can transmit on multiple channels in different frequencies of the spectrum.

We assume that there is an interference distance R_j for each channel j such that a link $e = (u, v)$ interferes with another link $e' = (u', v')$ on channel j if and only if $|u - v'| \leq R_j$ or $|u' - v| \leq R_j$.

We assume each link $e \in E$ has a set of available channels A_e. Suppose that a subset of channels $J_e \subset A_e$ has been chosen to be active for each link $e \in E$. Let $G_c^{\langle J_e \rangle}$ be the *conflict graph* restricted to the link-channel pairs of the form (e, j) where $e \in E$ and $j \in J_e$. Let $t_{e,j}$ be the total amount of time allocated to the link-channel pair (e, j) in the transmission frame (assumed to be of length 1). Each clique C in $G_c^{\langle J_e \rangle}$ imposes the constraint

$$\sum_{(e,j) \in C} t_{e,j} \leq 1. \tag{6.1}$$

Let $m_{e,j}^{\langle J_e \rangle}$ be the size of the largest clique containing (e, j) in $G_{c,p}^{\langle J_e \rangle}$.

We make a simplifying assumption that the scheduling mechanism creates a *uniform schedule* such that

$$t_{e,j} = 1/m_{e,j}^{\langle J_e \rangle}. \tag{6.2}$$

Observe that each constraint of the form (6.1) is satisfied by (6.2). We note that a uniform schedule may not be optimal but we adopt this assumption for algorithmic convenience and because it may not be possible to alter the scheduling mechanism used in a real network.

Given the selected active channels $\langle J_e \rangle$, the *effective capacity* of a link e is the sum of the bit rates on each active channel times the amount of transmission time allocated to each channel,

$$c(e, \langle J_e \rangle) = \sum_{j \in J_e} b_{e,j}/m_{e,j}^{\langle J_e \rangle}. \tag{6.3}$$

We note that this link-channel model assumes that a uniform transmission schedule will be created so that equation (6.2) is satisfied and interfering link-channel pairs are not scheduled at the same time. This allows us to accurately calculate the effective capacity of each link-channel used.

6.2. Problem Formulation

The generic system model described above lets us formulate a general topology control problem: we assume that there are a set of L source and destination node pairs, each with some requested capacity demand. We formulate each of these connection requests as a tuple (s_l, t_l, d_l), where s_l is the source, t_l is the destination and d_l is the capacity demand. We further assume that for each connection request (s_l, t_l, d_l) is given some path set P_l of available routing paths from s_l and t_l that the connection may use (in any combination) to meet its capacity demand. The objective is to minimize the total number of distinct channels used in the network (or minimize the total cost of the channels used). As discussed in the introduction, it is natural to consider this objective; a network operator may need to rent channels where there is a fixed rental cost C_k that must be paid to use channel k regardless of the number of links that use the channel.

Thus, the *Channel Rental for Topology Control* (CRTC) problem is to determine:

1. *the path sets* P_l, for each connection l,
2. *the set of active channels* J_e, for each edge $e \in E$ and
3. *the capacities* $d_{l,m} \geq 0$, that should be provided by the mth routing path in P_l between s_l and t_l,

so as to minimize the total number of distinct active channels used in the network such that $\sum_m d_{l,m} = d_l$.

We note that once the P_l sets and $\langle J_e \rangle$ and $\langle d_{l,m} \rangle$ values are chosen, it is easy to check whether the solution is feasible: for each edge e compute the total demand requested by all paths using that edge and check that it is less than or equal to $c(e, \langle J_e \rangle)$.

We note that this problem formulation is quite general; for example in an access point network, we may have a single gateway node s that must connect to multiple access points (APs) $\{t_i\}$ and each AP t_i must be provisioned with some capacity d_i.

In order to simplify the problem and more effectively find solutions, we will solve CRTC in a two-step process: First, we will compute paths sets P_l and second, we will determine the active channels $\langle J_e \rangle$ and path capacities $\langle d_{l,m} \rangle$. We refer to the second problem as CRTC-P, indicating the paths sets are given as part of the input. We will use a heuristic approach to determine the path sets (Algorithm 6.10) and state a MILP and a second heuristic method for CRTC-P (Algorithm 6.11).

6.2.1. MILP Formulation for CRTC-P

In this section, we present the following MILP formulation for the CRTC-P problem that can be used to provide optimal solutions.

Constants:

C_k: Cost to rent channel k in the network.

D_{ikc}: The capacity provided on edge e_i, using channel k, with clique size c.

d_l: capacity required by connection request (s_l, t_l, d_l).

P_l: the set of $s_l t_l$-paths available for connection request (s_l, t_l, d_l).

p_{lm}: the mth path in P_l.

Variables:

$c_k \in \{0,1\}$: 1, if any edge uses channel k, 0, otherwise.

$y_{ik} \in \{0,1\}$: 1, if edge e_i uses channel k, 0, otherwise.

$x_{ikc} \in \{0,1\}$: 1, if edge e_i uses channel k and the largest clique involving (e_i, k) has size c, 0, otherwise.

$d_{lm} \geq 0$: the capacity provisioned for $p_{l,m}$.

Objective:

$$\min\ T_{total} = \sum_k C_k c_k \tag{6.4}$$

subject to the following constraints:

$$y_{ik} = \sum_c x_{ikc} \leq c_k,\ \forall i, k \tag{6.5}$$

$$y_{ik} \leq c_k,\ \forall i, k \tag{6.6}$$

$$\sum_c x_{ikc} \leq 1,\ \forall i, k \tag{6.7}$$

$$x_{ikc} \leq |Q| - \sum_{j \in Q} y_{jk},\ \forall i, k, Q, c < |Q| \tag{6.8}$$

$$\sum_m d_{l,m} = d_l,\ \forall l \tag{6.9}$$

$$\sum_{k,c} D_{ikc} x_{ikc} \geq \sum_{l,m: e_i \in p_{lm}} d_{lm}, \forall i \tag{6.10}$$

Constraint (6.5) requires that the $y_{i,k}$ variables are set in accordance with the $x_{i,k,c}$ variables, while (6.6) ensures that the c_k variables reflect which channels are in use. Constraint (6.7) states that the maximum clique size that each edge, channel pair

(i, k) participates in is unique. Constraint (6.8) enumerates all cliques Q that the pair (i, k) participates and if all pairs within Q are turned on, then $x_{i,k,c} = 0$ for all $c < |Q|$. This ensures the size of the maximum clique that (i, k) participates in is correctly set. (In practice, we assume that there is an upper bound K on the maximum clique size, so we only enumerate cliques Q up to size K.) Constraint (6.9) requires that the full capacity demand d_l for connection request (s_l, t_l, d_l) is met (using paths in P_l). Finally, (6.10) ensures that each edge is provisioned with sufficient capacity to support all of the connection paths using that edge.

6.2.2. Computational Complexity

The channel rental problem can be broken down into a sequence of two-node problems, one for each source-destination pair. The number of times the two node problem is solved is a minor component in the complexity of the overall problem when compared with the complexity of solving each instance of the two-node problem. Since the two-node problem reduces to the JRCS problem, the channel rental problem is exactly as hard to approximate.

6.3. Proposed Algorithms

We first present a simple heuristic algorithm, Algorithm 6.10, for determining the path sets P_l available to each connection request (s_l, t_l, d_l). This algorithm works by simply finding $M \geq 1$ edge-disjoint minimum spanning trees (MSTs) and adding the path from s_l to t_l in each tree to P_l.

The second algorithm we present is also a heuristic approach to solving the CRTC-P problem; pseudocode is shown in Algorithm 6.11. The idea of the algorithm is to provide each edge to have a parallel virtual overflow edge that can be used to supplement the edge's capacity. We also modify the the MILP formulation given in section 6.2.1 so that the objective is just to minimize the maximum overflow amount in the network. Initially we set all link/channel pairs (i, k) to be inactive. This corresponds to setting all of the $y_{i,k}$ variables in the MILP to 0. We observe that if the $\{y_{i,k}\}$ variables are fixed, then the MILP reduces to real-valued linear program (LP), since the $\{y_{i,k}\}$ variables determine the values of the $\{x_{i,k,c}\}$ and $\{c_k\}$ variables. Then the algorithm looks for inactive link/channel pairs (i, k) that can be turned on in order to reduce the usage of the overflow links. We only consider channels that belong to an activated channel set, S (initially empty). We accept a pair (i, k) if either (1), it reduces the maximum overflow amount, or (2) the maximum overflow amount remains the same but the number of edges with the maximum overflow is reduced. This is easily determined by solving the above LP. If no pair (i, k) is found, then S is augmented with a new channel; we will choose the least cost unused channel that is available on one of the links with maximum overflow. The search for new (i, k) pairs to activate is repeated until all overflow is reduced to 0, or there are no remaining useful channels available (the algorithm reports "No solution" in this case).

6.3.1. Time Complexity of JRCS Algorithms

We note that the time complexity of Algorithm 6.10 is dominated by the time required to find M minimum spanning trees, which is $O(M|E| + M|V| \log |V|)$ using a standard MST algorithm such as Prim's algorithm.

The time complexity of Algorithm 6.11 is also polynomial, since the modified LPs it solve are now just linear programs (as noted above in the algorithm description). We note that in each iteration of the **while** loop, either the maximum overflow amount is reduced (by some discrete amount since there only a fixed number of edge capacities), or it will be in at most $|E|$ more iterations (in the next iteration, either a new channel

Input: the network $G = (V, E)$ with a list of connection requests $\langle(s_l, t_l, d_l)\rangle$, and M, the desired path set size.

Step 1 Find M non-overlapping MSTs.

Step 2 **forall** connection requests (s_l, t_l, d_l):
$P_l = \varnothing$.
for $k = 1$ **to** M:
Add the path from s_l to t_l in the kth MST to P_l.
endfor
endforall

Step 3 **return** $\langle P_l \rangle$.

Algorithm 6.10: FindPathSets

is added or one less edge is at maximum overflow). Thus the number of iterations will be $O(|E| \sum_l d_l)$ and each iteration just solves a real-valued LP, so the total running time is polynomial. In practice, the algorithm runs in a few minutes for instances up to 60 nodes.

6.4. Simulation Results

We tested our proposed algorithms on a variety of experimental scenarios. We assumed there were three widely spaced frequency bands available and that the link throughput for each channel was the maximum available given the link distance and frequency used. The bands exhibit widely ranging propagation, transmission range and usage characteristics, highlighting the potential value of cognition in transmission scheduling. Each node v_i is equipped with a cognitive radio that can activate k channels across each of three frequency bands: 700Mhz, 2.4GHz, and 5.8GHz.

Input: the network $G = (V, E)$ with a list of connection requests $\langle(s_l, t_l, d_l)\rangle$, associate path sets $\langle P_l \rangle$, and a set of available channels A.

Step 1 Modify LP formulation: For each link $e_i \in E$, create a new variable o_i; the *overflow* needed for link e_i. Also, introduce a new variable $O \geq 0$ (max overflow), and add constraints of the form $o_i \leq O$ for all i. New LP objective: $\min O$. Also, we define: tight(LP) $= \{i : o_i = O\}$.

Step 2 Set all $y_{i,k} = 0$
Set $S = \varnothing$ (selected channels)

Step 3 **while** $O = \text{solve(LP)} > 0$:
  foundImprovement = **FALSE**.
  forall (i, k) s.t. $y_{i,k} = 0, o_i = O$:
    Let testLP = LP + $\{y_{i,k} = 1\}$.
    if (solve(testLP) $< O$ **or**
     (solve(testLP) $= O$) **and**
      $|$tight(testLP)$| < |$tight(LP)$|$))
      Set $y_{i,k} = 1$.
      foundImprovement = **TRUE**.
    endif
  endforall
  if (**not** foundImprovement **and**
   $\exists (i, k) : i \in \text{tight(testLP)}, k \in A \setminus S$)
    $k = \text{argmin}_{i \in \text{tight(testLP)}, k \in A \setminus S} \text{cost}(k)$ $S = S \cup \{k\}$.
  else
    return "No solution."
  endif
endwhile

Step 4 **return** $\langle S, y_{i,k} \rangle$.

Algorithm 6.11: CRTC-P-OverflowReduce

Table 6.1: Maximum transmission distances by frequency and data rate

Transmission rate	700 Mhz	2400 Mhz	5800 Mhz
45 Mbps	15.4 km	4.5 km	1.8 km
40 Mbps	18.4 km	5.3 km	2.2 km
30 Mbps	30 km	8.6 km	3.6 km
20 Mbps	41 km	11.8 km	4.9 km
10 Mbps	68 km	20 km	8.2 km

Table 6.2: Interference ranges by frequency

Frequency	Interference range
700 Mhz	30.8 km
2400 Mhz	9 km
5800 Mhz	3.6 km

Tables 1 and 2 summarize our assumptions about the transmission rates and interference ranges of each frequency. The table values reflect a combination of manufacturer data sheets and our own experience in using these radios in the field. The values in Table 1 are based on a scenario where each node transmits at 1W with a 2dBi antenna and the receiving antenna has a gain of 2dBi. The channel bandwidth is 10MHz and the receiver noise figure is 5dB, and implementation losses of 3dB are assumed for each link. The values in Table 2 are based on a widely used rule of thumb that the interference range is about a factor of two greater than the transmission range. Path loss is calculated using line of sight and free space characteristics. Typical IEEE 802.16 adaptive modulation and coding parameters performance parameters were used to estimate the throughput achievable as a function of CNR (carrier to noise ratio), and were then translated into the allowable path loss threshold. The maximum channel transmission rate is a function of distance and frequency (at lower frequency, the maximum distance for a given transmission rate will be greater).

The experimental scenarios considered are as follows:

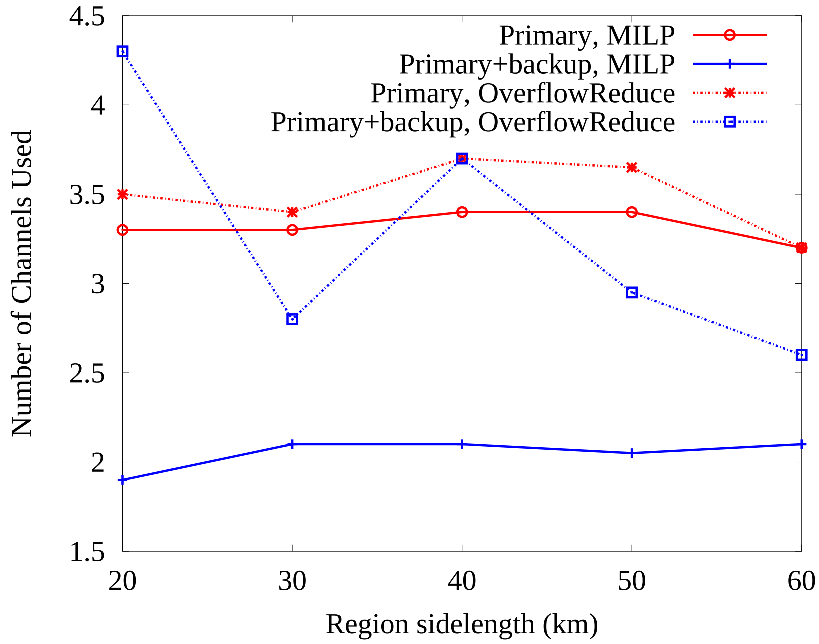

Figure 6.1: Channel Rental Scenario 1: Numbers of Channels Used vs Region Size.

Scenario 1: Vary s from 20 to 60 km with a step size of 10 km.

Scenario 2: Vary n from 10 to 60 nodes with a step size of 10.

Scenario 3: Vary a from 5 to 9 access points with a step size of 1.

In all cases, non-varying parameters were held at their median values. We also considered two cases for path sets; in the first case, a single *Primary* path from each s_l to t_l was found using Algorithm 6.10 by setting $M = 1$ and in the second case, an alternate *Backup* path was also found (by setting $M = 2$).

The results of the experiments are plotted in Figs. 6.1–6.3. Each point represents the average of 10 independent simulation runs.

The following observations can be made from the simulation results:

1) In all figures, we can see that the OverflowReduce algorithm closely tracks the optimal solution when only primary routing paths are available. In fact on average the OverflowReduce algorithm differs from the optimal solution by at most 1 channel on average.

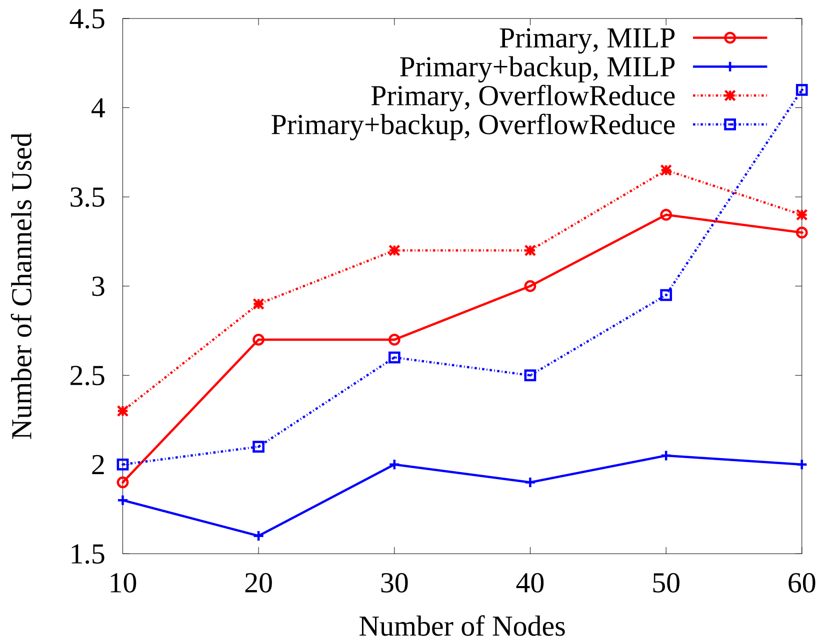

Figure 6.2: Channel Rental Scenario 2: Number of Channels Used vs Number of Nodes.

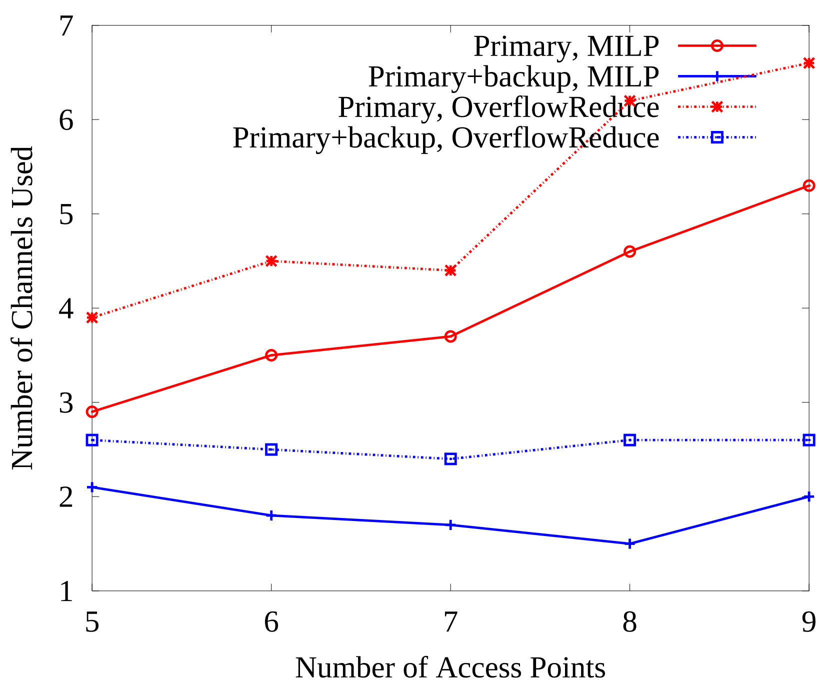

Figure 6.3: Channel Rental Scenario 3: Number of Channels Used vs Number of Access Points.

2) When backup paths are available, from Figs. 6.1 and 6.2 we see that as the density of nodes decreases, the OverflowReduce solution performs better: In Fig. 6.1, where the number of nodes is held constant, as the region size increases (thus the density decreases), the performance of OverflowReduce approaches the optimal solution. Similarly, in Fig. 6.2, where the region size is held constant, as the number of nodes increases (thus the density increases), the performance of OverflowReduce diverges from optimal.

3) From Fig. 6.3 we see that when the gateway-access point architecture is implemented the OverflowReduce algorithm performs consistently across the varying number of access points. In the primary path only case, the OverflowReduce algorithm uses only a single channel more than the optimal case and when backup paths are available, that reduces to only 0.5 channels more than the optimal.

6.5. Conclusions

We have studied the channel rental problem – a topology control approach for cost-effective spectrum management in wireless relay networks using multi-channel cognitive radios. The simulation results indicate that the proposed algorithms provides close to optimal solutions in multiple scenarios. The algorithms presented are robust to different channel cost models.

CHAPTER 7

APPLICATIONS OF RESOURCE ALLOCATION RESEARCH

Resource allocation in wireless networks is a broad research area. We focused on the algorithmic aspects of resource allocation in cognitive radio enabled wireless networks to narrow the scope. Over the course of this research, we applied scheduling and topology control techniques to develop algorithms to improve beam scheduling, multi-beam directional antennas, routing and channel selection, and ultimately the most economical end-to-end throughput for a set of demands on a wireless mesh network. In most cases, the algorithms developed address a specific aspect of the wireless network: antenna sector activation, antenna directionality, and routing and channel selection. Similarly the application of these algorithms can be applied to various problems facing wireless mesh networks today including rural broadband, high-density cellular networks, and network economics.

7.1. Real World Networks

Multiple aspects of a wireless network affect overall performance. Real world wireless networks are affected by network density, uniformity, shape, demand patterns, environmental conditions, and economics. Network density, uniformity, shape, and economics are aspects that generally change over longer periods of time (e.g. months and years). Demand patterns and environmental condition, however, can change instantly or in a matter of a few minutes. Each of these aspects can be studied in isolation, but they inevitably work together to create a complex problem that needs a solution that addresses end-to-end performance guarantees.

7.1.1. Network Density

Networks contain internet gateways, wireless relays and subscribers. Sometimes nodes in a wireless network perform more than one of these basic functions. Network density specifies how far apart wireless network nodes are located. Unlike modern switch-based wired networks, wireless networks use communication channels that can cause interference with other communication signals that are using the same frequency and channel. Interference is directly related to the signal strength that is being transmitting; signal strength is proportional to node distance. Therefore, network density has a direct effect on interference for wireless network.

In the case of network density, the right choice of frequency, channel, and power is critical to ensure that overall network demand can be satisfied. If infinite independent wireless frequency/channel sets were available it would be possible to allocate a separate frequency/channel pair to each communication link, and there would be no interference. However, since each frequency/channel has an associated cost, and the number of frequencies (and channels) is limited, infinite channels are not a solution that can satisfy the demand in a wireless network. Our work in joint routing and channel selection, and the channel rental problem, is directly applicable to network density challenges. Our work is different from previous works on scheduling and spectrum allocation [37, 44, 62–65] that deals with the problem of scheduling and allocating channels to links for link-layer throughput maximization. We focus on end-to-end performance and consider the problem of allocating channels along a multihop routing path – a much harder problem due to intra-flow interference constraints [106] (links on a common path interfere with each other if assigned the same channels) and due to the fact that there are an exponential number of potential paths in a mesh network that connect node pairs, and an exponential number of ways to assign channels along a path. Additionally, the algorithms proposed for traditional WMNs with homogeneous channels [36,88,89] cannot be applied to wireless networks with a large number of heterogeneous channels that have different data rates and transmission ranges.

7.1.1.1. High Density Networks. High density networks are becoming common. Wireless routers are inexpensive enough that consumers can buy and deploy multiple routers in 2.4GHz and 5.8GHz frequency space. These frequencies have a fixed number of communication channels available. As the power of these products increase, device coverage also expands. Increased power and expanded coverage, combined with increased population density and demand, means the amount of interference is also growing. This is evident from the emergence of features like "channel hopping" and "find the best channel" which are designed to allow consumer wireless to sense interference and reconfigure to avoid lost throughput.

In cellular networks, growing capacity requirements have been addressed by the introduction of smaller and smaller "cell" hardware, that can be deployed to supplement existing installations. All of these smaller cells need to integrate into existing infrastructure by offloading traffic from the existing infrastructure. In order to do that, these cells need to operate on different frequencies than the original infrastructure to avoid interference that would result in lower overall performance. The introduction of these multi-frequency cellular networks are what makes it possible to have both 3G, 4G and LTE operate simultaneously, and it is what allows cellular infrastructure to provide reliable throughput to a growing number of users.

By allocating and utilizing all available frequencies, cellular infrastructure addresses the reality of growing consumer demand for bandwidth, and provides a midterm solution until new frequencies and protocols can be deployed to augment existing infrastructure. The algorithms that we developed for joint routing, channel selection, and the channel rental problem can significantly reduce the cost of infrastructure required to meet issues of consumer demand. The multi-frequency cellular network problem is so new, there is no published research – our work on the channel rental problem may be considered one of the earliest works that addresses this issue.

7.1.1.2. Low Density Networks. Official and unofficial research, regarding low density network issues, rural networking, sub-Saharan Africa networking, and wireless distance challenges, have directly and indirectly sought to send a single wireless signal as far as possible to reach a very remote endpoint. These research efforts do not take into account multi-level wireless architectures, cognitive radios, or directional multi-sector antennas. In extremely low density cases, antennas are selected to specifically deliver a signal over a certain distance and in a certain direction. Our work in beam scheduling, and multi-beam directional antennas, directly addresses these goals by identifying the best direction, size, and timing for beams at the endpoints to be active. Our work is different from most research in low-density networking because we attempt to solve a robust problem that improves the overall performance of the wireless network. Other research is focused on providing long distance, high-throughput network links, without regard to overall network density.

Most work on wireless relay networks [45–48] deals with resource allocation problems involving Omni directional antennas, which are mathematically different from the optimization problems studied in our research. Relay assignment is a special problem for wireless relay networks and was not a concern in previous work on directional antennas. Some related work on directional antennas, focused on switched beam (sectorized) antennas [29, 53, 54, 56, 111], that can only form main beams toward limited predefined directions. We consider a smart adaptive antenna, with an adjustable beam orientation and beamwidth, that makes the corresponding optimization problems much harder. We present fast and effective algorithms to determine antenna patterns and relay assignments in a real-time manner, that can be applied to networks with mobile nodes. These differences make our work unique in the area of low-density networks.

7.1.1.3. Variable Density Networks. Variable density networks represent an interesting set of simulation scenarios ranging from vehicular networks to cellular networks, and even remote sensor networks. In these cases, the density of the network changes over time, some density changes occur quickly- in cellular networks during emergencies or large events- some occur more slowly much like traffic congestion during rush hour. Some of these density changes are random, in the case of traffic accidents, and some occur on a schedule, such as intentional powering down of nodes in a sensor network to extend battery life. In all cases, changes in network density cause the network to reconsider configuration in order to maintain provided throughput provided to end users over the duration of the change.

Enhancing scheduling algorithms to be efficient enough to respond quickly to network changes provides a robust and effective way to adapt network configurations under variable network density. Most of the relevant literature shows that the topology control approach [53–56] is most effective in relatively static networks. Generally, directional antenna related resource allocation problems are NP-hard. Most related work, including [29,58,111–113], present heuristic algorithms that cannot provide any performance guarantees. Our work, however, presents a constant factor approximation algorithm for the joint beam scheduling and relay assignment problem and a polynomial-time optimal algorithm for the relay beam scheduling problem.

7.1.2. Uniformity

Network uniformity describes the amount of variation in different aspects of the network. Variations in link length, channel availability, beamwidth, connectivity, and whether beams can be directionally controlled are all aspects of the network can that be regular, semi-regular or irregular. The more realistic the situation the more non-uniform these characteristics appear to be; ideally, they would be uniform and provide the simplest problem to solve – not trivial – but simple. Our algorithms and our toolkit provide for entirely non-uniform network scenarios and we find the best possible solutions for those scenarios.

The way our toolkit is designed, every node and link can have independent characteristics, providing the ability to create completely uniform and completely non-uniform networks – and every combination in between. It is these real-world networks that provide a robust platform to do algorithm development that will prove robust reliability in the real world. Simplifying the simulations provides a nice platform for developing algorithms that work in simulations, but we integrate real-world dynamics to increase success.

Our algorithms were developed in highly dynamic simulation space. We thoroughly tested each algorithm multiple times, and took an average for the solutions. The result is statistically sound evidence that our algorithms solve the problems we set out to resolve. We interpreted the problems correctly, implemented realistic network scenarios, and implemented algorithms that are directly applicable to a wide variety of networks that vary in uniformity.

7.1.3. Topology

Network topologies are widely variable and our algorithms are robust in the face of these challenges. In high-density networks, with many interconnections, the main challenge was selecting link/channel combinations that avoid interference. We realistically simulate both static interference (based on the network topology and frequency/channel availability), and dynamic interference (caused by subscriber dynamics). Our toolkit and algorithms are robust in widely varying network topologies, and prove a wide range of applicability when it comes to topological variation.

One improvement we can make, is to run all of our algorithms against a standard set of network topologies. Similar to testing pathological cases, there are a set of standard networks that we could use. This could help us identify edge cases that hinder the performance of our algorithms.

7.1.4. Demands

Network demand is the amount of throughput required to satisfy all users needs at any given point in time. Obviously, these demands change rapidly. An average demand model is used in our simulation. We adopt the same technique by assigning an average demand per subscriber in our simulations. Since our algorithms are designed to perform less frequently, and find a global solution for network challenges. it is hard to determine if the speed of changing network demands is a characteristic that can be addressed any better than the approach we have adopted.

We might construct our simulations to occur over a longer period of time, where we change the network demands, but this approach is not significantly different than running multiple simulation scenarios with different demand characteristics. Therefore, it is likely that our approach, while not continuous in the demand space, is an accurate discretization of the demand characteristics of real-world networks over time.

Our work is different from previous works on scheduling and spectrum allocation (e.g. [64]) that mainly deal with the problem of scheduling and allocating channels to links to maximize link-layer throughput. We focus on end-to-end performance to satisfy the demand, and consider the problem of allocating channels along a multi-hop routing path, a much harder problem due to intra-flow interference constraints [106] (links on a common path interfere with each other if assigned the same channels) and due to the fact that there are an exponential number of potential paths in a mesh network connecting node pairs, as well as an exponential number of ways to assign channels along a path. Additionally, the algorithms proposed for traditional WMNs with homogeneous channels [36, 88, 114] cannot be applied to solve our problems that target networks with a large number of heterogeneous channels which that have different data rates and transmission ranges. In short, routing and channel selection in cognitive radio mesh networks are very challenging problems. Most existing works [83–86, 115–118] on this topic presented heuristic algorithms that cannot provide performance guarantees. We study the CS problem and the JRCS problem from a theoretical perspective and aim at developing theoretically well-founded and practically useful algorithms to solve them.

7.1.5. Economics

Network economics cover a wide variety of topics that generally represent the processes of buying and selling network resources, understanding the underlying economic models, and understanding demand, security, and anonymity in those models. Our work is most relevant in that it provides details regarding need and/or use of the network by a current set of users with specific demands. For a supplier, our work can help identify the most efficient use of existing resources; give information about resource quantities that are available to be sold. For a buyer, our work can help identify exactly what resources need to be added in order to meet usage demands.

CHAPTER 8

WIRELESS NETWORKING TOPOLOGY CONTROL TOOLKIT

The goal of this research is to find solutions that are less computationally expensive than the optimal solution, and provide reliable (deterministic performance and accurate) results that are comparable with the optimal solution. In order to investigate the questions presented as part of this research, tools were developed that allowed the creation of a rich set of network conditions, implementation of optimal solutions, implementation of proposed solutions, and then comparison of the proposed solutions against the optimal to find quantitative proof that the proposed solution was sufficiently close to the optimal solution to be practical. While the original toolkit was done in Java, with optimal solutions implemented as linear programs using IBM's ILOG Cplex software, it was reimplemented as python. This reimplementation improved both the quality of the software and usability for other researchers.

The structure of the software is simple. It has a basic network object, implemented as an undirected graph, with edges and nodes allowing user extensible rich data. This allows each project, investigating a specific research question, to decorate the basic graph with research specific data. Each of the projects completed during the research and development of each topic, has extended the basic network with data necessary to support implemented algorithms. Each of the project implementations are encapsulated in a python module, and named to correspond to the project. Each project has a script that enables running the algorithms associated with the project to see the results. Additionally, data collection is done using scripts that run simulations across a parameterized simulation space and output data, statistics, and graphs for use in publications.

8.1. Overview

The Bobcat Wireless Networking Topology Control Toolkit is a freely available set of tools. The code is available in Java, or more completely in Python. Both implementations are available from Github. Once the git tools have been installed for your platform (instructions can be found here: https://help.github.com/articles/set-up-git),

8.1.1. Prerequisites

In order to use the Bobcat toolkit, two pieces of software must be installed, Networkx – a network simulation toolkit from Los Alamos National Laboratory, and CPLEX a solver for linear programs. Installing Networkx is straightforward for most platforms, usually consisting of only:

```
> pip install networkx
```

CPLEX installation is not trivial, but instructions are provided by IBM.

8.1.2. Installation

Once Networkx and CPLEX are installed and working properly, the installation of Bobcat is trivial:

```
> pip install git@github.com:irjudson/bobcat.git
```

This provides everything necessary to run a couple of interactive tests which show things are working:

```
$ python
```

```
Python 2.7.3 (default, Dec 18 2012, 13:50:09)
[GCC 4.5.3] on cygwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import networkx
>>> K5=networkx.complete_graph(5)
>>> K5.edges()
[(0,1),(0,2),(0,3),(0,4),(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
>>> K5.nodes()
[0, 1, 2, 3, 4]
```

8.2. Basic Network Implementation

The basic network implementation provided by the Bobcat toolkit is a network built as an undirected graph. This basic network provides capabilities to generate random networks, layout networks in a user defined space, calculate interference and throughput for nodes and edges, and visualize the network.

8.3. Beam Scheduling Tools

In order to support beam scheduling algorithms, Bobcat Wireless Networking Simulation Toolkit implemented a small number of critical computational elements that provide the framework for the algorithms. Adding antenna sectors to the basic graph, computing the bearing between nodes, and determining which nodes are covered by a specific sector of a node antenna complete what is necessary to implement beam scheduling simulations. These routines have been implemented as additions to the basic networking class.

```
$ python
```

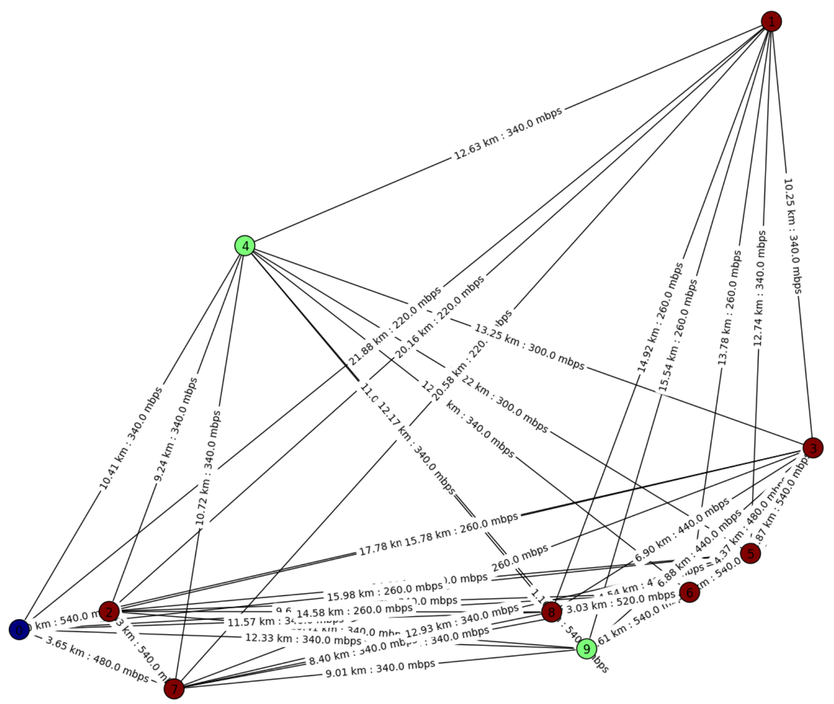

Figure 8.1: Example random network generated by the Bobcat Wireless Networking Simulation Toolkit.

```
Python 2.7.3 (default, Dec 18 2012, 13:50:09)
[GCC 4.5.3] on cygwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import networkx
>>> K5=networkx.complete_graph(5)
>>> K5.edges()
[(0,1),(0,2),(0,3),(0,4),(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
>>> K5.nodes()
[0, 1, 2, 3, 4]
```

8.4. Directional Antenna Tools

The only addition to the basic network object that is necessary to investigate directional antennas is a method to update the throughput for each node. Computing the throughput for each edge is provided as a fundamental network method. To aggregate information and provide a node throughput measure, it is necessary to extend the node with in, out, and total throughput attributes. At that point is is possible to implement a simple method that iterates through the edges and aggregate the throughput for the node.

```
$ python
Python 2.7.3 (default, Dec 18 2012, 13:50:09)
[GCC 4.5.3] on cygwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import networkx
>>> K5=networkx.complete_graph(5)
>>> K5.edges()
```

```
[(0,1),(0,2),(0,3),(0,4),(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
>>> K5.nodes()
[0, 1, 2, 3, 4]
```

8.5. Joint Routing and Channel Section Tools

The joint routing and channel selection algorithms required no additional extensions to the basic toolkit. The way these algorithms work is to take a network and find three paths through the network, a shortest path, a path in the minimum spanning tree, and the path we calculate using the channel selection model. Then for the the shortest path, and the minimum spanning tree path, we run the channel selection process that provides us with all three paths- each with channels selected to provide maximum throughput. The code for this set of simulations is in the toolkit. Both the simple and greedy algorithms are available in the Joint Routing and Channel Selection simulation code, which extends the main toolkit.

```
$ python
Python 2.7.3 (default, Dec 18 2012, 13:50:09)
[GCC 4.5.3] on cygwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import networkx
>>> K5=networkx.complete_graph(5)
>>> K5.edges()
[(0,1),(0,2),(0,3),(0,4),(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
>>> K5.nodes()
[0, 1, 2, 3, 4]
```

8.6. Channel Rental Problem Tools

The channel rental problem code is encapsulated in a new python module. This code implements two algorithms: 1. A simple heuristic that finds disjoint minimum spanning trees and combines paths from source to destination from each tree, and 2. A heuristic algorithm that simulates an overflow problem by allowing each edge to have a virtual overflow edge and modifying the Mixed Integer Linear Program so that it becomes a real valued Linear Program. The algorithm finds edge/channels that reduce overflow. This iterates until there is no overflow in the virtual network, and all the flow is in the actual network. Finally, we provide a Mixed Integer Linear Program that finds the optimal solution to the channel rental problem. Each of these implementations is available in the channel rental simulation code, not the main toolkit.

```
$ python
Python 2.7.3 (default, Dec 18 2012, 13:50:09)
[GCC 4.5.3] on cygwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import networkx
>>> K5=networkx.complete_graph(5)
>>> K5.edges()
[(0,1),(0,2),(0,3),(0,4),(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
>>> K5.nodes()
[0, 1, 2, 3, 4]
```

8.7. Future Extensions

The Bobcat Network Simulation Toolkit is easily extended to support any number of research inquiries. Because the data structures are primitive and flexible python dictionaries, it is extremely easy to add new data to edges and nodes, as well as add new methods to compute the data stored on the edges and nodes. Extensions already under consideration include adding additional frequencies, allowing for three dimensional networks (adding height), and supporting topographical data to support realistic interference models. Additionally, the ability to model complex economic scenarios would be a significant improvement.

The current implementation of the toolkit, easily allows for the addition of new frequencies and channels within those frequencies. The model assumes that there are a number of channels per frequency that are configurable as simulation parameters. Pathloss, throughput, and interference take into account the channel frequency to provide accurate values for the simulation.

Adding node height, and updating the distance calculation to include the height, is all that is needed to support three dimensional networks. Additional three dimensional parameters could include antenna downtilt, which would require additional modifications to the computations for node antennas including beam direction, pathloss, throughput, and bearing.

Extending the three dimensional model to know about topography, would allow more realistic interference models resulting in more accurate network simulations. This will require a new data structure in the toolkit, a map/topography structure, related to the network through the existing node location data. This will need to include new calculations for object interference, which is fairly complex and will entail significant effort.

To simulate even more realistic scenarios, building models could be added to provide a rich urban simulation capability. Weather and foliage models would allow richer rural simulations. Buildings provide physical interference, and the presence of other networks (inside the buildings) that can bleed out and cause complex interference situations. The fact that many cellular installations are located on top of urban buildings, is another reason to include this data in the simulation (both for accurate physical modelling and for accurate network interference models). Weather and foliage, while very difficult to model, provide realistic simulation conditions for rural sparse networks where environmental factors can significantly impact the long network links used to maintain connectivity.

CHAPTER 9

CONCLUSION

Wireless networks are pervasive infrastructures in our lives. They provide television, radio, computer and cellular phone communication, as well as CB, satellite and other mission critical communication components in our daily lives. The growing adoption of wireless networks for computer and communication infrastructure, is creating demand faster than infrastructure is growing. With 2.5B internet users and over 6B cellular subscribers, the demand on wireless infrastructure is accelerating, not slowing down. In order to meet growing demand we can deploy more infrastructure and find more efficient ways to use existing infrastructure.

My research explores the use of Topology Control Algorithms to find efficient solutions to problems within wireless networking: Beam Scheduling, Multi-Beam Directional Antennas, Joint Routing and Channel Selection, and ultimately the Channel Rental problem. The goal of my research is to develop efficient topology control algorithms that provide cost efficient solutions and guaranteed throughput for wireless networks.

We studied Beam Scheduling for efficient communications in wireless relay networks with smart antennas. The corresponding optimization problem was formally defined as the BSchP and was shown to be NP-hard. We first present a MILP formulation to provide optimal solutions. We then present two simple and fast localized greedy approaches for BSchP, one of which is shown to have an approximation ratio of $\frac{1}{2\lceil 2\pi\theta_1 \rceil}$. We show by extensive simulation results, that our proposed algorithms provide performance better than 80% of optimal.

For Multi-Beam Directional Antennas, we study the topology control approach for efficient communications in wireless relay networks with smart antennas. The corresponding optimization problem was formally defined as the SSP. We first present a MILP formulation to provide optimal solutions. We then present a new LP rounding algorithm. We show by extensive simulation results, that the proposed algorithm provides close-to-optimal performance and is superior to several alternative approaches in terms of both network capacity and fairness.

We examine two important problems for maximizing end-to-end throughput for communication flows in cognitive radio mesh networks in our work on Joint Routing and Channel Selection. For the channel selection problem on a known routing path, we develop an optimal algorithm and show that for self-avoiding paths it runs in time linear in the length of the path. Furthermore, the algorithms only need to propagate local information from source to destination and can be implemented in a distributed fashion. We also consider the joint problem of routing and channel selection, and show that it is NP-hard to approximate within a factor of $(2/3 + \epsilon)$. Additionally, we present two novel heuristic algorithms, and demonstrate the superiority of RCS-PathExtend to using shortest path routing combined with optimal channel selection.

To solve the the Channel Rental Problem, we again used a topology control approach for cost-effective spectrum management in wireless relay networks by using multi-channel cognitive radios. Simulation results indicate that proposed algorithms provide close to optimal solutions in multiple scenarios. The algorithms presented are robust to different channel cost models like those found in current Ubiquiti Networks hardware. We anticipate that the channel rental problem is an important problem for future network operators.

Bobcat Network Simulation Toolkit is a set of python simulation tools that allow researchers to develop algorithms and test them against wireless networks that can be randomly generated. The toolkit includes tools to create, visualize, and annotate networks with simulation dependent data, as well as create linear programs using IBM's ILOG CPLEX solver. The toolkit includes our algorithms as examples for teaching others how to use the toolkit in order to develop their own algorithms. The toolkit is simple, well-documented, and small in scope to so that researchers may quickly leverage it as a resource to answer their own questions about algorithms and networks. The toolkit is freely available under Apache 2.0 licensing in source form from Github, a popular source code publishing site.

9.1. Future Work

Bobcat Network Simulation Toolkit can be enhanced and extended in various ways, primarily through the addition of more frequencies and channels available for simulation, but also through the addition of complex interference models. Finally, implementing more complex simulation scenarios, to reproduce more realistic real-world examples, would benefit the toolkit significantly.

Two interference models are implemented in the toolkit: a simple two dimensional range based interference, and a randomized interference that simulates subscriber activity in the wireless network. Both of these models are accurate with respect to their goals, but limited to two dimensions. The first step in extending these interference models, would be to extend them to three-dimensions. This is moderately difficult because it involves extending the entire toolkit to three dimensions. The basic addition of a third dimension is not difficult, but all distance and interference calculations, beam bearings, and sector shape calculations, will also need to be extended to three dimensions. This change would, however, significantly move the toolkit toward more realistic simulations.

In addition to the logical extension of the toolkit to three dimensions, adding topographically aware interference would allow the toolkit to simulate wireless networks and the performance of proposed algorithms in more realistic world scenarios. The ultimate goal is to run network simulations on Google Earth models, including three dimensional models of buildings and cities. At that point, scenarios would be as close to real-world as possible without additional details of building construction and materials. Another direction this extension might take, is to integrate both weather and foliage data. These effects significantly impact the performance of wireless networks. They have been studied in isolation, but have not been integrated into a complete simulation toolkit.

Finally, two more improvements that might be made to the toolkit include the ability to run integrated simulations, and the ability to apply complex economic models. In each of these cases, the goal is to more accurately simulate complex simulations that integrate beam scheduling, joint routing, and channel selection, and apply a complex economic model to the channel rental problem. Integrating complex simulation scenarios investigates the interactions of simulation solutions to find interesting outcomes.

REFERENCES CITED

[1] WildBlue, "Wildblue satellite internet service," http://www.wildblue.net.

[2] HughesWay, "Hughesnet satellite internet service," http://www.hughesnet.com.

[3] "FCC Report to the US Congress on Advanced Telecommunications."

[4] C. Stevenson, G. Chouinard, Z. Lei, W. Hu, S. Shellhammer, and W. Caldwell, "Ieee 802.22: The first cognitive radio wireless regional area networks (wrans) standard," *IEEE Communications Magazine*, pp. 130–138, 1 2009, doi:10.1109/MCOM.2009.4752688.

[5] IEEE, "Ieee 802 lan/man standards committee 802.22 wg on wrans (wireless regional area networks)," http://www.ieee802.org/22/.

[6] E. Bangeman, "The white spaces coalition's plans for fast wireless broadband: The technology," http://arstechnica.com/articles/paedia/hardware/white-space.ars/2, 2007, ars Technica, 2007-4-17.

[7] NTIA, "Digital nation expanding internet usage," http://www.ntia.doc.gov/files/ntia/publications/ntia_internet_use_report_february_2011.pdf, February 2011.

[8] FCC, "Fcc's sixth broadband deployment report," http://transition.fcc.gov/Daily_Releases/Daily_Business/2010/db0720/FCC-10-129A1.pdf.

[9] J. Horrigan, "Home broadband adoption 2009," *Pew Internet & American Life, Washington, DC, USA*, 2009.

[10] Z. Mingliu and R. Wolff, "Crossing the digital divide: cost-effective broadband wireless access for rural and remote areas," *Communications Magazine*, vol. 42, 2004.

[11] S. Surana, R. Patra, S. Nedevschi, M. Ramos, L. Subramanian, Y. Ben-David, and E. Brewer, "Beyond pilots: keeping rural wireless networks alive," in *Proceedings of the 5th USENIX symposium on networked systems design and implementation.* USENIX Association, 2008, pp. 119–132.

[12] R. Wolff, "Citizen political enfranchisement and information access: Telecommunications services in rural and remote areas," in *Communications Research in Action.* Fordham University Press, January 2011.

[13] J. Bicket, D. Aguayo, S. Biswas, and R. Morris, "Architecture and evaluation of an unplanned 802.11 b mesh network," in *Proceedings of the 11th annual international conference on Mobile computing and networking.* ACM, 2005, pp. 31–42.

[14] Meraki, "Meraki cloud managed wireless networks," http://meraki.com.

[15] FrieFunk, "Freifunk: Free open source radio firmware," http://start.freifunk.net.

[16] C. Palazzi, M. Brunati, and M. Roccetti, "An OpenWRT solution for future wireless homes," in *Multimedia and Expo (ICME), 2010 IEEE International Conference on*, 2010, pp. 1701–1706.

[17] R. K. Patra, S. Nedevschi, S. Surana, A. Sheth, L. Subramanian, and E. A. Brewer, "Wildnet: Design and implementation of high performance wifi based long distance networks." in *NSDI*, vol. 1, no. 1, 2007, p. 1.

[18] K. Sibanda, H. Muyingi, and N. Mabanza, "Building Wireless Community Networks with 802.16 Standard," in *Broadband Communications, Information Technology & Biomedical Applications, 2008 Third International Conference on*, 2008, pp. 384–388.

[19] D. Benyamina and Hafid, "Wireless Mesh Networks Design — A Survey," *Communications Surveys & Tutorials, IEEE*, no. 99, pp. 1–12, 2011.

[20] H. Moustafa, U. Javaid, T. M. Rasheed, S.-M. Senouci, and D. Meddour, "A panorama on wireless mesh networks: Architectures, applications and technical challenges," in *First International Workshop on Wireless Mesh: Moving Towards Applications,(WIMESHNETs)*, vol. 6, 2006.

[21] J. Ishmael, S. Bury, D. Pezaros, and N. Race, "Deploying Rural Community Wireless Mesh Networks," *Internet Computing, IEEE*, vol. 12, no. 4, pp. 22–29, 2008.

[22] M. Zhang and R. Wolff, "Using Wi-Fi for cost-effective broadband wireless access in rural and remote areas," in *Wireless Communications and Networking Conference, 2004. WCNC. 2004 IEEE*, 2004, pp. 1347–1352.

[23] W. Almuhtadi, "Rural/Remote WiFi Wireless Broadband System," in *Wireless On-demand Network Systems and Services, 2005. WONS 2005. Second Annual Conference on*, 2005, pp. 180–188.

[24] D. Johnson, "Evaluation of a single radio rural mesh network in south africa," in *Information and Communication Technologies and Development, 2007. ICTD 2007. International Conference on.* IEEE, 2007, pp. 1–9.

[25] R. Flickenger, S. Okay, E. Pietrosemoli, M. Zennaro, and C. Fonda, "Very long distance wi-fi networks," in *NSDR '08: Proceedings of the second ACM SIGCOMM workshop on Networked systems for developing regions*, 2008.

[26] A. Sharma, "FreeMAC: framework for multi-channel mac development on 802.11 hardware," in *Proceedings of the ACM workshop on . . .*, 2008.

[27] R. Mahajan, M. Rodrig, D. Wetherall, and J. Zahorjan, "Analyzing the MAC-level behavior of wireless networks in the wild," in *SIGCOMM '06: Proceedings of the 2006 conference on Applications, technologies, architectures, and protocols for computer communications.* ACM Request Permissions, Aug. 2006.

[28] R. R. Choudhury and N. H. Vaidya, "Deafness: A MAC Problem in Ad Hoc Networks when using Directional Antennas," in *ICNP '04: Proceedings of the 12th IEEE International Conference on Network Protocols.* IEEE Computer Society, Oct. 2004.

[29] T. Korakis, G. Jakllari, and L. Tassiulas, "A mac protocol for full exploitation of directional antennas in ad-hoc wireless networks," in *Proceedings of the 4th ACM international symposium on Mobile ad hoc networking & computing.* ACM, 2003, pp. 98–107.

[30] K. Sundaresan and R. Sivakumar, "A unified mac layer framework for ad-hoc networks with smart antennas," in *Proceedings of the 5th ACM international symposium on Mobile ad hoc networking and computing.* ACM, 2004, pp. 244–255.

[31] Y. Ben-David, M. Vallentin, S. Fowler, and E. Brewer, "JaldiMAC: taking the distance further," in *NSDR '10: Proceedings of the 4th ACM Workshop on Networked Systems for Developing Regions*, 2010.

[32] K. Ramachandran, I. Sheriff, and E. Belding, "A multi-radio 802.11 mesh network architecture," *Mobile Networks and . . .*, 2008.

[33] A. Dhananjay, H. Zhang, and J. Li, "Practical, distributed channel assignment and routing in dual-radio mesh networks," *ACM SIGCOMM . . .*, 2009.

[34] M. Gerla and K. Tang, "TCP performance in wireless multi-hop networks," *Mobile Computing Systems . . .*, 1999.

[35] J.-H. Huang, L.-C. Wang, and C.-J. Chang, "Throughput-coverage tradeoff in a scalable wireless mesh network," *Journal of Parallel and Distributed Computing*, vol. 68, no. 3, Mar. 2008.

[36] J. Tang, G. Xue, and W. Zhang, "Interference-aware topology control and qos routing in multi-channel wireless mesh networks," in *Proceedings of the 6th ACM international symposium on Mobile ad hoc networking and computing.* ACM, 2005, pp. 68–77.

[37] J. Huang, R. Berry, and M. Honig, "Spectrum sharing with distributed interference compensation," *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on*, pp. 88–93, 2005.

[38] K. Moaveni-Nejad, "Low-interference topology control for wireless ad hoc networks," *Ad Hoc & Sensor Wireless Networks*, 2005.

[39] R. M. Lotun, "wypy: an extensible, online interference detection tool for wireless networks," Ph.D. dissertation, The University Of British Columbia, 2008.

[40] D. Gokhale, S. Sen, and K. Chebrolu, "On the feasibility of the link abstraction in (rural) mesh networks," *INFOCOM 2008 The . . .*, 2008.

[41] F. FCC, *FCC 08-260.* Unlicensed Operation in the TV Broadcast Bands, 2008. [Online]. Available: http://scholar.google.com/scholar?q=related:5KyINcAbjPsJ:scholar.google.com/&hl=en&num=30&as_sdt=0,5

[42] P. Bahl, R. Chandra, T. Moscibroda, R. Murty, and M. Welsh, "White space networking with wi-fi like connectivity," *ACM SIGCOMM Computer Communication Review*, vol. 39, no. 4, pp. 27–38, 2009.

[43] S. Deb, V. Srinivasan, and R. Maheshwari, "Dynamic spectrum access in DTV whitespaces: design rules, architecture and algorithms," in *MobiCom '09: Proceedings of the 15th annual international conference on Mobile computing and networking.* ACM Request Permissions, Sep. 2009.

[44] Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda, and Y. Wu, "Allocating dynamic time-spectrum blocks in cognitive radio networks," in *MobiHoc '07: Proceedings of the 8th ACM international symposium on Mobile ad hoc networking and computing.* ACM Request Permissions, Sep. 2007.

[45] K. Sundaresan and S. Rangarajan, "On exploiting diversity and spatial reuse in relay-enabled wireless networks," in *Proceedings of the 9th ACM international symposium on Mobile ad hoc networking and computing.* ACM, 2008, pp. 13–22.

[46] S. Deb, V. Mhatre, and V. Ramaiyan, "Wimax relay networks: opportunistic scheduling to exploit multiuser diversity and frequency selectivity," in *Proceedings of the 14th ACM international conference on Mobile computing and networking.* ACM, 2008, pp. 163–174.

[47] G. Li and H. Liu, "Resource allocation for ofdma relay networks with fairness constraints," *Selected Areas in communications, IEEE Journal on*, vol. 24, no. 11, pp. 2061–2069, 2006.

[48] K. Sundaresan and S. Rangarajan, "Efficient algorithms for leveraging spatial reuse in ofdma relay networks," in *INFOCOM 2009, IEEE*. IEEE, 2009, pp. 1539–1547.

[49] K. Sundaresan, W. Wang, and S. Eidenbenz, "Algorithmic aspects of communication in ad-hoc networks with smart antennas," in *Proceedings of the 7th ACM international symposium on Mobile ad hoc networking and computing*. ACM, 2006, pp. 298–309.

[50] R. Bhatia, "Throughput optimization of wireless mesh networks with MIMO links," *... 2007 26th IEEE International Conference on ...*, 2007.

[51] B. Mumey, J. Tang, and T. Hahn, "2008 IEEE International Conference on Communications," in *2008 IEEE International Conference on Communications*. IEEE, 8, pp. 2921–2925.

[52] K. Sundaresan, R. Sivakumar, M. Ingram, and T. Chang, "Medium access control in ad hoc networks with mimo links: optimization considerations and algorithms," *IEEE Transactions on Mobile Computing*, pp. 350–365, 2004.

[53] U. Kumar and H. Gupta, "A topology control approach to using directional antennas in wireless mesh networks," *Communications*, 2006.

[54] V. Namboodiri, L. Gao, and R. Janaswamy, "Power-efficient topology control for static wireless networks with switched beam directional antennas," *Ad Hoc Networks*, vol. 6, no. 2, Apr. 2008.

[55] Z. Huang and C.-C. Shen, "Multibeam Antenna-Based Topology Control with Directional Power Intensity for Ad Hoc Networks," *IEEE Transactions on Mobile Computing*, vol. 5, no. 5, May 2006.

[56] A. Subramanian and H. Lundgren, "Topology control protocol using sectorized antennas in dense 802.11 wireless networks," *Network Protocols*, 2009.

[57] M. Hu, "MIMO ad hoc networks: Medium access control, saturation throughput, and optimal hop distance," *Networks*, 2004.

[58] R. Bhatia and L. Li, "Throughput optimization of wireless mesh networks with mimo links," in *INFOCOM 2007. 26th IEEE International Conference on Computer Communications. IEEE*. Ieee, 2007, pp. 2326–2330.

[59] C. Ang, "iMST: A bandwidth-guaranteed topology control algorithm for TDMA-based ad hoc networks with sectorized antennas," *Computer networks*, 2008.

[60] N. Li and J. Hou, "Flss: a fault-tolerant topology control algorithm for wireless networks," in *Proceedings of the 10th annual international conference on Mobile computing and networking*. ACM, 2004, pp. 275–286.

[61] L. Li, J. Halpern, P. Bahl, Y. Wang, and R. Wattenhofer, "A cone-based distributed topology-control algorithm for wireless multi-hop networks," *Networking, IEEE/ACM Transactions on*, vol. 13, no. 1, pp. 147–159, 2005.

[62] Q. Zhao, L. Tong, and A. Swami, "Decentralized cognitive mac for dynamic spectrum access," *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on*, pp. 224–232, 2005.

[63] H. Zheng and C. Peng, "Collaboration and fairness in opportunistic spectrum access," in *Communications, 2005. ICC 2005. 2005 IEEE International Conference on*, 2005, pp. 3132–3136.

[64] J. Tang and S. Misra, "Joint spectrum allocation and scheduling for fair spectrum sharing in cognitive radio wireless networks," *Computer networks*, 2008.

[65] L. Cao and H. Zheng, "Distributed spectrum allocation via local bargaining," in *Sensor and Ad Hoc Communications and Networks, 2005. IEEE SECON 2005. 2005 Second Annual IEEE Communications Society Conference on*, 2005, pp. 475–486.

[66] N. Nie and C. Comaniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks," in *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on*. IEEE, 2005, pp. 269–278.

[67] L. B. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 12, pp. 5306–5315, 2008.

[68] A. Attar, O. Holland, M. Nakhai, and A. Aghvami, "Interference-limited resource allocation for cognitive radio in orthogonal frequency-division multiplexing networks," *Communications, IET*, vol. 2, no. 6, pp. 806–814, 2008.

[69] C. N. Mathur, M. Haleem, R. Chandramouli, and K. Subbalakshmi, "Impact of constraints on the complexity of dynamic spectrum assignment," in *Global Telecommunications Conference, 2008. IEEE GLOBECOM 2008. IEEE*. IEEE, 2008, pp. 1–6.

[70] T. Qin and C. Cyril Leung, "Fair adaptive resource allocation for multiuser ofdm cognitive radio systems," in *Communications and Networking in China,*

2007. CHINACOM'07. Second International Conference on. IEEE, 2007, pp. 115–119.

[71] V. K. Tumuluru, P. Wang, and D. Niyato, "A novel spectrum-scheduling scheme for multichannel cognitive radio network and performance analysis," *Vehicular Technology, IEEE Transactions on*, vol. 60, no. 4, pp. 1849–1858, 2011.

[72] X. Tan, H. Zhang, Q. Chen, and J. Hu, "Opportunistic channel selection based on time series prediction in cognitive radio networks," *Transactions on Emerging Telecommunications Technologies*, 2013.

[73] R. E. Irwin, A. B. MacKenzie, and L. A. DaSilva, "Resource-minimized channel assignment for multi-transceiver cognitive radio networks," *Selected Areas in Communications, IEEE Journal on*, vol. 31, no. 3, pp. 442–450, 2013.

[74] J.-C. Liang and J.-C. Chen, "Resource allocation in cognitive radio relay networks," *Selected Areas in Communications, IEEE Journal on*, vol. 31, no. 3, pp. 476–488, 2013.

[75] T. T. Le, P. T. A. Quang, S. Y. Shin, and D.-S. Kim, "Bar: Bandwidth-aware opportunistic localized-routing for cognitive radio networks," in *Globecom Workshops (GC Wkshps), 2012 IEEE.* IEEE, 2012, pp. 937–942.

[76] X. Tang and Q. Liu, "Network coding based geographical opportunistic routing for ad hoc cognitive radio networks," in *Globecom Workshops (GC Wkshps), 2012 IEEE.* IEEE, 2012, pp. 503–507.

[77] Y. Zhang, J. Guan, C. Xu, and H. Zhang, "The stable routing protocol for the cognitive network," in *Globecom Workshops (GC Wkshps), 2012 IEEE.* IEEE, 2012, pp. 1090–1095.

[78] A. Abbagnale and F. Cuomo, "Gymkhana: a connectivity-based routing scheme for cognitive radio ad hoc networks," in *INFOCOM IEEE Conference on Computer Communications Workshops, 2010.* IEEE, 2010, pp. 1–5.

[79] S. Patel, "An on-demand routing technique for cognitive radio ad hoc network," *International Journal of Engineering*, vol. 2, no. 5, 2013.

[80] C. Xin, B. Xie, and C. Shen, "A novel layered graph model for topology formation and routing in dynamic spectrum access networks," in *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on.* IEEE, 2005, pp. 308–317.

[81] Y. Xi, "Distributed algorithms for spectrum allocation, power control, routing, and congestion control in wireless networks," in *Proceedings of the 8th ACM international . . .*, 2007.

[82] Y. Hou, Y. Shi, and H. Sherali, "Optimal Spectrum Sharing for Multi-Hop Software Defined Radio Networks," in *INFOCOM 2007. 26th IEEE International Conference on Computer Communications. IEEE*, 2007, pp. 1–9.

[83] Y. Shi and Y. Hou, "A Distributed Optimization Algorithm for Multi-Hop Cognitive Radio Networks," in *INFOCOM 2008. The 27th Conference on Computer Communications. IEEE*, 2008, pp. 1292–1300.

[84] I. Pefkianakis and S. Wong, "SAMER: Spectrum aware mesh routing in cognitive radio networks," *... Spectrum Access Networks*, 2008.

[85] A. Sampath, L. Yang, L. Cao, and H. Zheng, "High throughput spectrum-aware routing for cognitive radio networks," *Proc of IEEE Crowncom*, 2008.

[86] R. Hincapie, J. Tang, G. Xue, and R. Bustamante, "QoS Routing in Wireless Mesh Networks with Cognitive Radios," in *Global Telecommunications Conference, 2008. IEEE GLOBECOM 2008. IEEE*, 2008, pp. 1–5.

[87] B. Mumey, X. Zhao, J. Tang, and R. Wolff, "Transmission Scheduling for Routing Paths in Cognitive Radio Mesh Networks," in *Sensor Mesh and Ad Hoc Communications and Networks (SECON), 2010 7th Annual IEEE Communications Society Conference on*, 2010, pp. 1–8.

[88] R. B. M. Alicherry and L. Li, "Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks," in *Proceedings of ACM MobiCom'2005*, 2005, pp. 58–72.

[89] M. Kodialam and T. Nandagopal, "Characterizing the capacity region in multi-radio multi-channel wireless mesh networks," in *Proceedings of ACM MobiCom'2005*, 2005, pp. 73–87.

[90] L. Ding, T. Melodia, S. N. Batalama, J. D. Matyjas, and M. J. Medley, "Cross-layer routing and dynamic spectrum allocation in cognitive radio¡ emphasis emphasistype=," *Vehicular Technology, IEEE Transactions on*, vol. 59, no. 4, pp. 1969–1979, 2010.

[91] S. S. Barve and P. Kulkarni, "Dynamic channel selection and routing through reinforcement learning in cognitive radio networks," in *Computational Intelligence & Computing Research (ICCIC), 2012 IEEE International Conference on.* IEEE, 2012, pp. 1–7.

[92] L. Lai, J. Wang, A. Huang, and H. Shan, "Routing and resource allocation with collision constraint in multi-hop cognitive radio networks," in *Globecom Workshops (GC Wkshps), 2012 IEEE.* IEEE, 2012, pp. 974–979.

[93] I. D. Standard, "Ieee 802.16j working group, part 16: Air interface for fixed and subscriber broadband wireless access systems—multihop relay specification," Institute of Electrical and Electronics Engineers, Tech. Rep., 2007.

[94] I. Standard, "Ieee 802.16e working group, part 16: Air interface for fixed and subscriber broadband wireless access systems amendment 2: Physical and medium access control layers for combined fixed and subscriber operation in licensed bands," Institute of Electrical and Electronics Engineers, Tech. Rep., 2005.

[95] T. S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Prentice Hall, 2002.

[96] M. Andrews and L. Zhang, "Scheduling algorithms for multi-carrier wireless data systems," in *Proceedings of the 13th annual ACM international conference on Mobile computing and networking*. ACM, 2007, pp. 3–14.

[97] G. Z. A. Brzezinski and E. Modiano, "Enabling distributed throughput maximization in wireless mesh networks—a partitioning approach," in *Proceedings of ACM MobiCom'06*, 2006, pp. 26–37.

[98] H. Ito, H. Uehara, and M. Yokoyama, "Np-completeness of stage illumination problems," in *Discrete and Computational Geometry*. Springer, 2000, pp. 158–165.

[99] T. Cormen, "Introduction to algorithms," The MIT Press, 2001.

[100] R. Cohen and L. Katzir, "An efficient approximation for the generalized assignment problem," *Information Processing Letters*, 2006.

[101] I. ILOG Software, "Cplex 12.4," http://www.ilog.com.

[102] M. R. Garey, D. S. Johnson, and L. Stockmeyer, "Some simplified np-complete problems," in *Proceedings of the sixth annual ACM symposium on Theory of computing*. ACM, 1974, pp. 47–63.

[103] T. Cormen, C. Leiserson, R. Rivest, and C. Stein, *Introduction to algorithms, 3rd edition*. The MIT Press, 2009.

[104] I. Akyildiz, "A survey on wireless mesh networks," *Communications Magazine*, 2005.

[105] M. C. V. I. F. Akyildiz, W-Y Lee and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Journal of Computer Networks*, vol. 50, no. 13, pp. 2127–2159, 2007.

[106] Y. Yang, "Contention-aware admission control for ad hoc networks," *IEEE Transactions on Mobile Computing*, 2005.

[107] R. Downey and M. R. Fellows, "Fundamentals of parameterized complexity," *Undegraduate Texts in Computer Science, Springer-Verlag*, 2012.

[108] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness (Series of Books in the Mathematical Sciences)*. W. H. Freeman, 1979.

[109] T. Nakamura, S. Nagata, A. Benjebbour, Y. Kishiyama, T. Hai, S. Xiaodong, Y. Ning, and L. Nan, "Trends in small cell enhancements in lte advanced," *Communications Magazine, IEEE*, vol. 51, no. 2, pp. 98–105, 2013.

[110] (2012, Jun.) Ubiquiti networks. [Online]. Available: http://www.ubnt.com

[111] R. R. Choudhury and N. H. Vaidya, "Deafness: A mac problem in ad hoc networks when using directional antennas," in *Network Protocols, 2004. ICNP 2004. Proceedings of the 12th IEEE International Conference on.* IEEE, 2004, pp. 283–292.

[112] K. Sundaresan, R. Sivakumar, M. A. Ingram, and T.-Y. Chang, "Medium access control in ad hoc networks with mimo links: optimization considerations and algorithms," *Mobile Computing, IEEE Transactions on*, vol. 3, no. 4, pp. 350–365, 2004.

[113] K. Sundaresan and R. Sivakumar, "A unified mac layer framework for ad-hoc networks with smart antennas," *Networking, IEEE/ACM Transactions on*, vol. 15, no. 3, pp. 546–559, 2007.

[114] M. Kodialam and T. Nandagopal, "Characterizing the capacity region in multi-radio multi-channel wireless mesh networks," in *Proceedings of the 11th annual international conference on Mobile computing and networking.* ACM, 2005, pp. 73–87.

[115] N. Kumar, M. Kumar, and R. Patel, "Capacity and interference aware link scheduling with channel assignment in wireless mesh networks," *Journal of Network and Computer Applications*, vol. 34, no. 1, pp. 30–38, 2011.

[116] V. Ramamurthi, A. Reaz, D. Ghosal, S. Dixit, and B. Mukherjee, "Channel, capacity, and flow assignment in wireless mesh networks," *Computer Networks*, vol. 55, no. 9, pp. 2241–2258, 2011.

[117] Y. Ding, K. Pongaliur, and L. Xiao, "Channel allocation and routing in hybrid multi-channel multi-radio wireless mesh networks," 2013.

[118] H. Khalife, S. Ahuja, N. Malouch, and M. Krunz, "Probabilistic Path Selection in Opportunistic Cognitive Radio Networks," in *Global Telecommunications Conference, 2008. IEEE GLOBECOM 2008. IEEE*, 2008, pp. 1–5.