

An economic analysis of nitrogen fertilization of livestock pastureland in a semi-arid region
by Frederick Vincent Linse

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
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Abstract:
Physical experimentation has shown nitrogen carryover in livestock pastureland fertilization to be of,considerable importance. Little attempt has been made to evaluate the economic significance of nitrogen carryover. The present paper utilizes an implicit method of estimating nitrogen carryover to provide quantitative information for a dynamic programming analysis of the economic decision-making problem. The results of the study demonstrate the potential for increases in economic efficiency through use of non-zero nitrogen fertilization policies.

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Date December 6, 1971

AN ECONOMIC ANALYSIS OF NITROGEN FERTILIZATION OF LIVESTOCK PASTURELAND IN A SEMI-ARID REGION

by

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ABSTRACT

Physical experimentation has shown nitrogen carryover in livestock pastureland fertilization to be of considerable importance. Little attempt has been made to evaluate the economic significance of nitrogen carryover. The present paper utilizes an implicit method of estimating nitrogen carryover to provide quantitative information for a dynamic programming analysis of the economic decision-making problem. The results of the study demonstrate the potential for increases in economic efficiency through use of non-zero nitrogen fertilization policies.

INTRODUCTION

Returns to the agricultural production activity are low relative to some other forms of productive endeavor in our society. In response, ambitious agricultural research programs have been initiated to search out potential return-increasing technology. One such area of research interest has been nitrogen fertilization of livestock pasture lands in semi-arid areas as a means of increasing livestock output.

Physical research has indicated that grasses growing in the northern great plans usually respond to applications of nitrogen fertilizer [1,2,3]. In addition, such grasses have shown a residual response to nitrogen applied in previous years [3,4,5]. The latter result implies that nitrogen fertilizer in the soil is carried over from production period to production period in a plant-available form. The observed possibility of nitrogen carryover brings forth significant implications for economic decision-making.

If the carryover effect is important, optimal decisions pertaining to management of plant available nitrogen in the soil will depend heavily upon levels of nitrogen carried over from past production periods. Also, when consideration is given to initiating a nitrogen fertilization program, knowledge of expected nitrogen carryover levels are required to weigh the total discounted value marginal products against the total current costs for each alternative rate of nitrogen application.

A complicating factor to the decision process is the stochastic character of nitrogen carried over into succeeding production years. This stems from the established fact that nitrogen fertilizer recovery and plant growth are directly related to precipitation levels [2].

Prior economic analyses have failed to evaluate the stochastic and dynamic propensities of the problem. Black [1] emphasized comparisons of expected returns under alternative rates of nitrogen application, but did not consider the carryover effect of nitrogen fertilization. Stauber and Burt [6] provided a more sophisticated approach to the problem using an implicit estimate of nitrogen carryover in a deterministic framework to determine an optimal level of nitrogen to maintain in the soil.

It was the intent of this study to provide a meaningful economic analysis of the nitrogen fertilization of pasture land. The approach taken incorporates the stochastic and dynamic characteristics of the problem into an appropriately formulated economic decision model. The major objective is to provide a set of policy guidelines which livestock producers in the general study area can utilize in determining the proper decisions for their individual situation.

OPTIMAL APPLICATION OF NITROGEN FERTILIZER WHEN CARRYOVER IS SIGNIFICANT

Dynamic programming possesses features which lend themselves readily to problems similar to the one currently under consideration. Typically, dynamic programming is concerned with multistage decision processes where a given set of decisions recurrs at each stage of the process. The objective is to find the sequence of decisions which maximizes (or minimizes) some appropriately defined criterion function for all possible combinations of states and stages. The selected decision sequence is then termed the optimal policy for the multistage process.

The optimal policy depends upon two primary factors: 1) the process length and 2) the initial state of the system. In regard to the length of the process, a grassland livestock-producing firm with a one stage or a single production year planning horizon would likely pursue a different fertilization policy than if the expected process length were infinite. This conclusion follows from the apparent inability of the firm, under a one stage planning constraint, to capture all of the expected benefits attributable to a decision optimal under an infinite planning horizon. In addition, the optimal policy is dependent upon the initial state of the system, where a state is defined as the sum total of all relevant information pertaining to the process of concern. For example, decisions relative to the amount of nitrogen to apply will likely depend upon the current levels of plant-available nitrogen in the soil.

A decision made at a given stage will generally change the state of the process in the succeeding stage. The nature of the change brought about by a specific decision will either be deterministic or stochastic depending upon the inherent characteristics of the process considered. If decisions made at a given initial state enable one to predict exactly the subsequent new state, the nature of the process change is deterministic. Obviously, the inclusion of weather, as a relevant variable in the determination of nitrogen carryover requires a stochastic approach to the problem at hand.

In view of the foregoing discussion, it would seem appropriate to formally define the essential dynamic programming components in terms of the decision problem currently under consideration. Accordingly, define the state of the process to be the current level of plant-available nitrogen in the present stage, the decision variable to be the alternative rates of nitrogen which can be applied, and each stage in the process to be equivalent to one production year.

Note that our state variable definition must satisfy the Markovian requirement of dynamic programming [7]. Specifically, it requires the optimal policy beginning in a particular state to be dependent on that state and not upon how the state was attained. The state variable as defined above, complies quite readily with the requirements.

In order to construct the appropriate dynamic programming model, or recurrence relation, some justification is required with regard to

linkage of the n-stage process to the (n-1) stage process of a given multistage activity. To determine the optimal policy for an n-stage process, we consider the sum of the immediate expected return in the nth stage at some initial state "i" and the optimal expected return from the (n-1) stage process, which begins at some new state which may or may not be different from "i". The above procedure is firmly supported by the dynamic programming principle of optimality. Said principle states, "an optimal policy has the property that whatever the initial state and the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision [7].

The discrete stochastic recurrence relation, below, (1) was selected for use in the dynamic programming analysis of the nitrogen fertilization decision problem.

$$f_n(Y) = \underset{k}{\text{Max}} \left[R_Y^X + \beta \sum_{j=1}^{m} P_{YV}^X f_{n-1}(V)\right], \quad Y = 1, 2, \ldots, m. \tag{1}$$

where

$f_n(Y)$ = total discounted expected returns under an optimal policy from an n-stage process when the initial state if "Y".

R_Y^X = expected immediate return under decision X when the initial state is "Y".

β = the appropriate discount rate.

R_{YV}^{X} = the probability of moving from state "Y" in stage n to state "V" in stage n-1, given that decision X is selected; this is commonly termed a transition probability.

$f_{n-1}(V)$ = total discounted expected return under an optimal policy from an (n-1) stage process when the initial state is "V".

Clearly, (1) has been formulated in a manner consistent with the principle of optimality.

It will be noted that n, in (1), denotes the number of stages remaining in the decision process. This is the result of the unique dynamic programming computational technique of driving optimal policies by beginning at the last stage of the process and, thence, working back, stage by stage, to the present. Note further that optimal policies, resulting from (1), tend to converge to a constant decision rule for large n. Although a systematic proof of convergence will not be offered here, reference can be made to several published sources for detailed presentations of the proof [8,9].

Physical Data

As indicated within the recurrence relation, two primary types of information are required to determine the optimal policies of dynamic programming. Specifically, knowledge is necessary relative to the expected immediate returns and the transition probabilities.

The data requirements for computation of these vital components include determination of a production-response relationship which adequately describes grass hay yield as a function of plant available nitrogen and quantitative information pertaining to the stochastic carryover effect of nitrogen fertilizer.

The yield and carryover relationships used in this paper were estimated by Stauber and Burt [6]. Basically, the estimated yield response relationship attempts to explain the relationship between grass yield and the two independent variables, plant-available nitrogen and weather. Equally as important, the model provides an implicit estimate of carryover nitrogen. The data set used in their analysis was derived from an experiment site in western South Dakota near Newell, approximately 30 miles southeast of the Montana border [10]. The study evaluated the interrelationships of nitrogen, phosphorus and seasonal precipitation and their relative influences on the yield and composition of bromegrass-crested wheatgrass hay. The effects of residual nitrogen were also examined.

The yield model was hypothesized to be

$$Y_t = f(U_t, W_t)$$

where

Y_t = hay yield in pounds per acre in year "t". (2)

$U_t = N_t + R_t$ = total plant available nitrogen in year "t".

N_t = pounds of nitrogen applied in year "t".

R_t = residual nitrogen, where R_t is assumed to be a measure of nitrogen carried over from applications in previous years.

R_t was estimated in first order difference equation form as

$$R_t = p(N_{t-1} + R_{t-1})/W_{t-1},\ t = 1, 2, 3, \ldots \quad (3)$$

where

p = the proportion of nitrogen available in year t carried over to year t+1.

W_j = a standardized measure of seasonal precipitation adjusted for temperature in the jth year. Or,

$$W_j = [\frac{P_j}{T_j}]/W^*$$

where

P_j = total precipitation in the jth growing season.

T_j = mean daily maximum temperature in the jth growing season.

W^* = the sample mean of the ratio of growing season precipitation to growing season mean daily maximum temperature.

The logic of the weather variable's formulation is based on the adverse effect of high temperatures on plant growth. Division of adjusted seasonal precipitation by the sample mean of adjusted seasonal precipitation results in a standardized weather variable which reflects weather deviations from the sample mean. Specifically, W_j values greater than unity depict situations of above average adjusted seasonal precipitation while values less than unity are indicative of the converse.

Returning to residual nitrogen as hypothesized earlier (3), assuming $R_o = N_o = 0$ and iterating (3), the following sequence of equations results

$$R_1 = 0$$

$$R_2 = pN_1/W_1$$

$$R_3 = p[N_2 + R_2]W_2 = pN_2/W_2 + p^2N_1/W_1W_2$$

$$R_4 = p[N_3 + R_3]W = pN_3/W_3 + p^2N_2/W_2W_3 + p^3N_1/W_1W_2W_3$$

$$\vdots$$

$$R_t = p[N_{t-1} + R_{t-1}] = pN_{t-1}/W_{t-1} + p^2N_{t=2}/W_{t-1}W_{t-2} +$$

$$p^3N_{t-3}/W_{t-1}W_{t-2}W_{t-3} + \cdots$$

It follows that total plant available nitrogen can be expressed as

$$U_t = N_t + R_t = N_t + pN_{t-1}/W_{t-1} + p^2N_{t-2}/W_{t-1}W_{t-2} + \qquad (4)$$

$$p^3N_{t-3}/W_{t-1}W_{t-2}W_{t-3} + \cdots$$

or total plant available nitrogen is the sum of nitrogen applied in year t and a weighted sum of nitrogen applied in years prior to "t".

Previously, in (2) yield (Y_t) was defined as a function of total plant available nitrogen (U_t) and adjusted seasonal precipitation (W_t). In explicit form, yield was hypothesized to be a polynomial considering both the individual and interactive influences of nitrogen and weather. Expressed mathematically

$$Y_t = b_1U_t + b_2W_t + b_3U_t^2 + b_4W_t^2 + b_5U_tW_t + b_6U_t^2W_t + b_7U_tW_t^2 + \varepsilon_t \qquad (5)$$

ε_t = random error term assumed to be normally distributed.

b_i = the contribution of the ith variable to yield.

Estimates of the forementioned parameter, including p, were derived by nonlinear regression techniques. These coefficients plus the values of other statistical indicators are listed in Table I.

As emphasized earlier, p (the proportion of U_t carried over to year t+1 under average weather conditions) is of particular interest from a decisionmaker's standpoint. The estimated value of p, approximately 62 percent, points out the important influence of residual nitrogen on hay yields in years subsequent to the year of application.

The R^2 coefficient of .906 indicates the model is capable of fairly accurate hay yield predictions. Also, it will be noted the model possesses diminishing marginal returns of hay yields with respect to additions of either precipitation or nitrogen. This is evidenced by the second order mixed derivative of the yield equation.

$$\frac{\partial^2 Y_t}{\partial U_t \partial W_t} = 18.04 - .0364U_t - 5.70W_t$$

To quote Stauber and Burt [6], "the marginal effect of precipitation on the marginal yield response of nitrogen or the marginal effect of nitrogen on the marginal yield response of precipitation is seen to diminish with increases in usage of either variable."

TABLE I. STATISTICAL RESULTS OF THE REGRESSION ANALYSIS.

Parameter	Variable	Estimate	Standard Error
P		0.6194	0.0389
b_1	U_t	-6.3136	2.2124
$*b_3$	U_t^2	0.0029	.0084
$*b_4$	W_t^2	-19.7000	126.3052
b_5	$U_t W_t$	17.0422	3.3473
b_6	$U_t^2 W_t$	-0.0173	0.0068
$*b_7$	$U_t W_t^2$	-2.8495	1.6496

Standard error of the estimate 214.43

Coefficient of determination 0.906

* Variable insignificant at the 5% probability level.

DERIVATION OF IMMEDIATE EXPECTED RETURNS AND TRANSITIONAL PROBABILITIES

To derive the expected immediate returns, it was necessary to reduce the yield equation to a function of plant available nitrogen independent of the stochastic weather effects. Taking the expectation of the yield equation with respect to weather gave the desired result. In functional form, we have

$$E(Y_t|U_t) = -6.3136\ U_t + 614.0656\ E(W_t) + .0029\ U_t^2 - 19.700\ E(W_t^2) +$$

$$17.0422\ U_t E(W_t) - .0173\ U_t^2 E(W_t) - 2.8495\ U_t E(W_t^2)$$

Estimates for $E(W_t)$ and $E(W_t^2)$ were obtained through use of available weather history data for Newell, South Dakota. These values, 1.1418 and 1.5761 for $E(W_t)$ and $E(W_t^2)$, respectively, were then substituted into the yield equation giving the expected yield response function

$$E(Y_t\ U_t) = 670 + 8.65\ U_t - 0.0167\ U_t^2$$

As noted earlier, U_t is the sum of nitrogen applied in production year t (N_t) and nitrogen carried over into year t from applications made in prior productionmyears (R_t). Similarly, these components of U_t correspond to "Y" and "X" within the dynamic programming framework.

Computation of the expected immediate returns, in a manner consistent with the discrete dynamic programming approach, required the assignment of incremental values for "Y" and "X". In order to provide increased clarity and interpretative value in the dynamic programming

results, the state variable was divided into five-pound intervals while the decision variable was assumed to take on discrete values of five pounds per acre. Consideration was given to a 0 to 300 pound range of total nitrogen (U_t). This range was felt to be more than adequate in view of the fact that the yield equation under expected weather conditions attains a maximum at 259 pounds of total nitrogen per acre. Consequently, 61 possible decisions (X = 0, 5, 10 ... , 300) and 60 potential initial states (Y = 0-5, 5-10, 10-15, ... , 295-300) were defined.

In equation form the expected immediate returns are expressed as

$$R_X^Y = P_H \cdot E(Y_t | U_t) - P_N \cdot X_k - \phi$$

$$= P_H[670 + 8.65(y_i + x_k) - 0.0167(y_i + x_k)^2 - P_N \cdot x_k - \phi$$

where "Y" and "X" are defined above and,

y_i = Midpoint corresponding to Yth initial state for i = 1, 2, ... , 60.

x_k = kth amount of nitrogen applied for k = 1, 2, ..., 61.

P_H = Price per pound for hay

P_N = Price per pound for nitrogen

ϕ = Costs of spreading fertilizer

A set of immediate expected returns was subsequently determined for all meaningful combinations of states and decisions using a range of cost and price parameters.

The primary function of transition probabilities is to explain changes in the level of plant available soil-nitrogen as movement occurs

from production stage to production stage. Specifically, their basic concern is definition of the probability distribution associated with future levels of nitrogen carried over from the current production period. Since it is within the decisionmaker's power to influence the amount of nitrogen carried over by varying the quantities of nitrogen fertilizer applied in the present production stage, the probabilistic statement is necessarily conditional in form. In terms of the current problem, transition probabilities can, therefore, be represented as

$$P^{X}_{YV} = \text{Prob}[v_j < V^{n-1} \leq v_{j+1} \mid X^n = x_k \; Y^n = y_i] \qquad (5)$$

Where

V^{n-1} = Carryover nitrogen in stage n-1.

Y^n = Carryover nitrogen in stage n.

X^n = Amount of nitrogen applied in stage n.

v_j = an endpoint corresponding to the Vth state interval for stage n-1, (j = 1, 2, ..., 61).

y_i = ith midpoint corresponding to state interval for stage n, (i=1, 2, ..., 61).

x_k = kth level of nitrogen applied in stage n (k=1, 2, ..., 61).

Quantitative estimates of the transition probabilities were derived through use of the carryover equation (3). Specifically, (3) was reformulated in terms of R_{t+1} so as to reflect nitrogen carryover one production year into the future. Restatement of $R_t + 1$ within the context of dynamic programming gave

$$R_{n-1} = P\left(\frac{N_n + R_n}{W_n}\right) \tag{6}$$

Redefining (6) according to the notation set forth in (5) and substituting the resulting expression into (5) yields

$$P_{YV}^{X} = \text{Prob}\left[v_j < P\left(\frac{x_k + y_i}{W_n}\right) < v_{j+1}\right] \text{ for } X^n = x_k \text{ and} \tag{7}$$

$Y^n = y_i$, where $k = 1$ corresponds to zero pounds of nitrogen applied per acre, $j = 1$ is equivalent to zero pounds per acre of carryover nitrogen and $i = 1$ is associated with 2.5 pounds per acre of carryover nitrogen. The remaining integer-state and decision variable correspondences follow in ascending numerical order.

Computation of the transition probabilities was achieved through use of a range of W_n values derived from a 32-year weather record for Newell, South Dakota. Coefficients for the individual probability elements were computed by determining the proportion of years in which W_n satisfied the above inequality for every meaningful combination of initial state, decision and state of carryover nitrogen in stage n-1.

To clarify the above discussion, let us consider a numerical example. Assume we entered stage n with plant available nitrogen between 0 and 5 pounds per acre. Also, assume 20 pounds of nitrogen fertilizer was immediately applied in response to this low level of soil nitrogen. It is our desire to determine the probability of carryover nitrogen being between 5 and 10 pounds per acre in the next succeeding production year.

To begin, we approximate the level of nitrogen carried over from past years by selecting the midpoint of our initial state interval, or 2.5 pounds per acre. Substituting this value plus the given magnitudes of the decision and new state variable into (7) yields

P_{YV}^{X} = Prob $[5 < p(20 + 2.5) \leq 10]$ for $X = x_5 = 20$ and $Y^n = y_1 = 2.5$. Utilizing our estimate for p(.6194) and solving the inequality for W_n gives P_{YV}^{X} = Prob $[2.7873 > W_n \geq 1.3937]$.

Scanning Table II, we see that in ten out of the past thirty-two years, weather would have been favorable enough to induce the stated level of nitrogen carryover. Hence, on the basis of past weather experience, we say that the probability of nitrogen carryover falling within the interval of 5 and 10 pounds per acre is 10/32, given we have an initial state of zero to five pounds of nitrogen per acre and that we apply 20 pounds per acre in the current production year.

In several cases, non-zero probability elements were associated with nitrogen carryover levels of greater than 100 percent. This peculiarity occurred whenever the computed W_t values were less than the estimate for the carryover parameter "p". Since, for every positive value an even smaller W_t is possible, greater than 100 percent carryover predictions could occur regardless of our estimate for "p".

To prevent carryover estimates greater than 100 percent, all W_t values less than "p" were assigned magnitudes equal to "p". In Table II,

TABLE II. COMPUTED W_n VALUES FOR THE PERIOD 1938-1969.

Year	W_n	Year	W_n
1938	.7745	1954	1.1461
1939	.5986	1955	.6151
1940	1.0334	1956	.7669
1941	2.1381	1957	1.2724
1942	1.6932	1958	1.4245
1943	.7292	1959	.6202
1944	1.4303	1960	.4426
1945	1.2570	1961	.6567
1946	2.4922	1962	2.0834
1947	1.2019	1963	1.4273
1948	1.2593	1964	1.3479
1949	.6339	1965	1.7780
1950	.7422	1966	.5476
1951	.9458	1967	1.5362
1952	.4540	1968	.7764
1953	1.7321	1969	.9791

we see that five years fall into this category hence, it was necessary to adjust these values to .6194 (the estimate for the carryover parameter "p").

Numerical Results

To begin our discussion of the dynamic programming solutions, let us consider the optimal policies pertaining to the parameter set where the price of hay equals $15 per ton, the price of nitrogen is $.10 per pound and the interest rate appropriate for discounting is ten percent. Within this basic situation two variations related to spreading costs are examined. In the first case the fertilizer is assumed to be custom spread at a cost of $1.50 per acre. In the second case it is assumed the operator owns the necessary equipment to spread the fertilizer and a charge of $.25 is made which represents the operating costs for a typical farm situation. These policies and their associated expected returns are listed in Table III for stages prior to policy convergence and a fifty stage planning horizon.

From Table III, the optimal application rates and the lowest state level at which nitrogen is applied are noted to increase as the length of the planning horizon increases. This result can be attributed to the enhanced ability of the process to capture a greater share of the distributed value marginal products associated with each application decision. In addition, the lower ownership costs of spreading encourage nitrogen applications earlier in the process than custom spreading costs.

To illustrate the decision rule, examine the optimal solutions corresponding to stage fifty under spreading costs of $1.50 per acre. Here, we see that the level of total nitrogen (nitrogen applied plus the initial state of nitrogen) is maintained between 105 and 110 pounds per acre for the five lowest initial states of nitrogen. For higher initial state levels, the optimal fertilization policies recommend that nitrogen not be applied. The decision rule implied by this behavior indicates that if the level of plant available nitrogen in the soil falls below 25 pounds, the optimal response is to apply enough nitrogen to bring soil nitrogen up to at least 105 pounds per acre. On the other hand, if carryover nitrogen is greater than 25 pounds, expected returns will be maximized by not applying any nitrogen.

The decision rule described is identical to the structure of (S,s) policies of inventory systems analysis [15].

Using inventory theory terminology, s is defined as the reorder level of the item currently in stock where as "S" represents the reorder level of said item. The decision rule proposed by (s,S) policies states that if the amount on hand of the given item falls below a specified level (s), the appropriate policy response is to restock the inventory of that item until a previously determined upper level is reached (S). On the other hand, if the inventory level falls somewhere between s and "S", the proper optimizing decision is to make no further additions to the inventory in question. In terms of current problem, s represents

TABLE III. DYNAMIC PROGRAMMING OPTIMAL POLICIES FOR STAGES N* = 3, 4, 5, 6, and 50 WHEN THE PRICE OF HAY EQUALS $15/TON, THE PRICE OF NITROGEN IS $.10/LB AND APPROPRIATE INTEREST RATE EQUALS TEN PERCENT.

Initial State of Carryover Nitrogen lb/A	Spreading Costs = $.25/A		Spreading Costs = $1.50/A	
	Optimal Policy lb/A	Expected Returns $	Optimal Policy lb/A	Expected Returns $
	N=3			
0 - 5	70	14.51	0	14.18
5 - 10	65	15.01	0	14.66
10 - 15	60	15.51	0	15.30
15 - 20	55	16.01	0	15.85
20 - 25	50	16.51	0	16.42
25 - 30	45	17.01	0	17.01
30 - 35	0	17.59	0	17.59
45 - 50	0	19.21	0	19.21
95 - 100	0	23.96	0	23.95
145 - 150	0	27.71	0	27.71
195 - 200	0	30.48	0	30.47
245 - 250	0	32.21	0	32.21
295 - 300	0	32.89	0	32.88
	N=4			
0 - 5	85	19.23	0	18.08
5 - 10	80	19.23	0	18.58
10 - 15	75	20.23	0	19.26
15 - 20	70	20.73	0	19.85
20 - 25	65	21.23	0	20.48
25 - 30	60	21.73	0	21.13
30 - 35	55	22.23	0	21.78
35 - 40	50	22.73	0	22.38
40 - 45	45	23.23	0	23.01
45 - 50	40	23.73	0	23.57
50 - 55	35	24.23	0	24.15
55 - 60	0	24.76	0	24.72
95 - 100	0	28.89	0	28.88
145 - 150	0	33.16	0	33.15
195 - 200	0	36.40	0	36.40
245 - 250	0	38.55	0	38.55
295 - 300	0	39.60	0	39.59
	N=5			
0 - 5	90	23.47	100	22.08
5 - 10	85	23.97	95	22.58
10 - 15	80	24.47	90	23.08
15 - 20	75	24.97	85	23.58
20 - 25	70	25.47	0	24.10
25 - 30	65	25.97	0	24.78
30 - 35	60	26.47	0	25.46
35 - 40	55	26.97	0	26.09
40 - 45	50	27.47	0	26.76
45 - 50	45	27.97	0	27.35
50 - 55	40	28.47	0	27.97
55 - 60	35	28.97	0	28.57
60 - 65	0	29.52	0	29.19
95 - 100	0	33.15	0	32.56
145 - 150	0	37.63	0	27.59
195 - 200	0	41.15	0	41.14
245 - 250	0	43.58	0	43.57
295 - 300	0	44.88	0	44.87
	N=6			
0 - 5	90	27.32	105	25.65
5 - 10	85	27.83	100	26.15
10 - 15	80	28.32	95	26.65
15 - 20	75	28.82	90	27.15
20 - 25	70	29.32	85	27.65
25 - 30	65	29.82	0	28.20
30 - 35	60	30.32	0	28.87
35 - 40	55	30.82	0	29.48
40 - 45	50	31.32	0	30.14
45 - 50	45	31.82	0	30.73
50 - 55	40	32.32	0	31.35
55 - 60	35	32.82	0	31.96
95 - 100	0	37.00	0	36.53
145 - 150	0	41.54	0	41.31
195 - 200	0	45.18	0	45.05
245 - 250	0	47.74	0	47.67
295 - 300	0	49.17	0	49.12

Initial State of Carryover Nitrogen lb/A	Spreading Costs = $.25/A		Spreading Costs = $1.50/A	
	Optimal Policy lb/A	Expected Returns $	Optimal Policy lb/A	Expected Returns $
	N=50			
0 - 5	90	65.38	105	59.86
5 - 10	85	65.88	100	60.36
10 - 15	80	66.38	95	60.86
15 - 20	75	66.88	90	61.36
20 - 25	70	67.38	85	61.86
25 - 30	65	67.88	0	62.48
30 - 35	60	68.38	0	63.12
35 - 40	55	68.88	0	63.73
40 - 45	50	69.38	0	64.37
45 - 50	45	69.88	0	64.95
50 - 55	40	70.38	0	65.57
55 - 60	35	70.88	0	66.16
60 - 65	0	71.43	0	66.79
65 - 70	0	71.01	0	67.31
70 - 75	0	72.54	0	68.01
75 - 80	0	73.02	0	68.52
80 - 85	0	73.48	0	69.13
85 - 90	0	74.11	0	69.71
90 - 95	0	74.63	0	70.26
95 - 100	0	75.06	0	70.62
100 - 105	0	75.62	0	71.33
105 - 110	0	76.11	0	71.86
110 - 115	0	76.55	0	72.33
115 - 120	0	76.99	0	72.80
120 - 125	0	77.49	0	73.33
125 - 130	0	77.97	0	73.85
130 - 135	0	78.40	0	74.29
135 - 140	0	78.79	0	74.71
140 - 145	0	79.29	0	75.25
145 - 150	0	79.64	0	75.61
150 - 155	0	80.10	0	76.10
155 - 160	0	80.48	0	76.50
160 - 165	0	80.87	0	76.92
165 - 170	0	81.25	0	77.31
170 - 175	0	81.58	0	77.66
175 - 180	0	82.03	0	78.13
180 - 185	0	82.30	0	78.41
185 - 190	0	82.71	0	78.85
190 - 195	0	82.99	0	79.15
195 - 200	0	83.34	0	79.51
200 - 205	0	83.64	0	79.83
205 - 210	0	84.01	0	80.22
210 - 215	0	84.21	0	80.43
215 - 220	0	84.48	0	80.71
220 - 225	0	84.78	0	81.02
225 - 230	0	85.01	0	81.27
230 - 235	0	85.29	0	81.57
235 - 240	0	85.53	0	81.82
240 - 245	0	85.77	0	82.08
245 - 250	0	85.99	0	82.30
250 - 255	0	86.18	0	82.51
255 - 260	0	86.36	0	82.70
260 - 265	0	86.56	0	82.92
265 - 270	0	86.74	0	83.10
270 - 275	0	83.24	0	86.87
275 - 280	0	83.47	0	87.08
280 - 285	0	83.57	0	87.17
285 - 290	0	83.75	0	87.34
290 - 295	0	83.88	0	87.46
295 - 300	0	83.95	0	87.92

* Non-zero application rates did not appear until stage 3.

the lowest level to which plant available nitrogen will be allowed to fall before non-zero applications are required to maintain optimality. Similarly, "S" would denote the amount of nitrogen which must be attained when non-zero application decisions are implemented. Consequently, in reference to the results listed in Table III, the (s,S) policies are (57.5, 87.5) and (22.5, 107.5) for spreading costs of $0.25 and $1.50 per acre, respectively.

Since this approach allows a much more concise expression of the dynamic programming results, further discussion of the optimal decision rules will be confined to (s,S) definition.

Sensitivity Analysis

The changes in the optimal inventory of plant available nitrogen with regard to variations in the economic environment are summarized in Table IV.

Movements in the relationship between the price of hay and the price of nitrogen are seen to exert a strong influence on the relative magnitudes of s and "S". Specifically, more favorable price relationships tend to increase both the reorder point, s, and the reorder level, S, for plant available nitrogen. The increased returns to nitrogen fertilization indicate the desirability of maintaining higher overall levels of nitrogen in the soil.

Increases in the fertilizer spreading costs resulted is higher levels for "S" and lower levels for s. It is clear that increases in spreading costs penalize applications throughout a given planning horizon.

Hence, with the prices of hay and nitrogen remaining constant, fewer applications in conjunction with heavier amounts of nitrogen applied, at each application point is a logical consequence.

Fluctuations in the rate of interest influence the relative postponement of non-zero nitrogen application decisions. This conclusion is consistent with the human behavioral assumption that future returns are worth less in the present period than their realizable values at future production stages.

As the interest rate rises the value of future returns decreases; therefore investment for future benefits (and hence both "S" and s) would be expected to diminish. As shown in Table IV, this result was observed.

Comparison of Dynamic-Stochastic and Non-Stochastic Solutions

A non-stochastic evaluation of the nitrogen fertilization decision problem when nitrogen carryover is significant was given by Burt and Stauber [6]. Their approach begins by noting that under static single period assumptions, marginal economic analysis ensures the optimal employment of a given variable by equating value marginal product of that resource with the resource's unit cost. The condition of optimality in terms of the problem at hand is represented as

$$[P_N = P_H \, MPP_{N_t}]$$

TABLE IV. OPTIMAL (S,s) POLICIES UNDER VARIED PARAMETER AND PLANNING HORIZON ASSUMPTIONS.

Parameter Values				Stages of Production						Expected Returns for Initial State (0-5 lb/A) at Stage 50
Interest	Price/Cost			1	2	3	4	5	50	
Rate	Hay	Nitrogen	Spreading	S s	S s	S s	S s	S s	S s	
%	$/T	$/LB	$/A	LB/A	LB/A	LB/A	LB/A	LB/A	LB/A	$/A
10	10	0.10	1.50	0,0	0,0	0,0	0,0	0,0	0,0	37.71
10	10	0.10	.25	0,0	0,0	0,0	0,0	0,0	0,0	37.71
10	10	0.05	1.50	0,0	0,0	127.5,22.5	142.5,47.5	152.5,47.5	177.5,42.5 a/	
10	10	0.05	.25	0,0	72.5,32.5	127.5,82.5	127.5,87.5	142.5,92.5	142.5,92.5	41.59
10	15	0.05	1.50	0,0	142.5,57.5	187.5,92.5	187.5,92.5	187.5,92.5	187.5,92.5	87.68
10	15	0.05	.25	57.5,17.5	142.5,112.5	117.5,137.5	177.5,142.5	177.5,142,5	177.5,142.5	95.89
10	15	0.10	1.50	0,0	0,0	0,0	0,0	147.5,22.5	107.5,27.5 b/	59.82
10	15	0.10	.25	0,0	0,0	72.5,32.5	87.5,57.5	92.5,57.5	92.5,62.5	65.28
5	15	0.10	1.50	0,0	0,0	0,0	102.5,22.5	127.5,37.5	127.5,32.5 c/	110.10
15	15	0.10	1.50	0,0	0,0	0,0	0,0	0,0	102.5,12.5 d/	40.65

a/ Optimal policy converges in Stage 9.

b/ Optimal policy converges in Stage 16.

c/ Optimal policy converges in Stage 7.

d/ Optimal policy converges in Stage 8.

where

P_N = Price per pound of nitrogen.

P_H = Price per pound of hay.

MPP_{N_t} = Marginal physical product of nitrogen applied in year t.

However, this criterion assumes all returns attributable to the factor employed are received within the immediate production period. A more suitable condition for the optimal employment of nitrogen would be to increase the rate of nitrogen application in year t until the sum of the discounted value marginal products equals the marginal cost of nitrogen in year t.

In order to secure an optimal solution to the problem, knowledge of future marginal products and, hence, future soil nitrogen levels are required. This impossible task can be avoided somewhat be deriving solutions based on the assumption of a constant rate of nitrogen application.

In modified form, the condition of optimality is redefined as

$$P_N = \sum_{t=0}^{\infty} P_H \left[\frac{MPP_N \, t}{(1+r)^t}\right] \qquad (8)$$

where

t = Time in discrete units.

r = The appropriate interest rate for discounting.

The marginal physical product of nitrogen applied is given by the partial derivative of yield in year t with respect to nitrogen applied in year zero. It follows from (4) and (5) that

$$\frac{\partial Y_t}{\partial N_o} = \frac{\partial Y_t}{\partial U_t} \frac{\partial U_t}{\partial N_o} = \frac{\partial Y_t}{\partial U_t} \quad p^t \tag{9}$$

where all variables including "p", the carryover proportion, are defined as previously.

Substituting (9) into (8) yields

$$P_N = \sum_{t=0}^{\infty} P_H \frac{\partial Y_t}{\partial U_t} \; p^t \; \frac{1}{(1+r)^t} \tag{10}$$

Under the assumption that $N_o = N_1 = N_2 = \ldots$, and its direct implication that $U_o = U_1 = U_2 = \ldots$, the optimality condition resolves to one variable. Further, given $U_t = U_o$, $\frac{\partial Y_t}{\partial U_t}$ is a constant independent of t. With these modifications, (10) becomes

$$P_N = P_H \frac{\partial Y_o}{\partial U_o} \sum_{t=0}^{\infty} \left(\frac{p}{1+r}\right)^t \tag{11}$$

Since $\sum_{t=0}^{\infty} \left(\frac{p}{1+r}\right)^t$ is the sum of the terms of a geometric progression, (11) can be written as

$$P_N = P_H \frac{\partial Y_o}{\partial U_o} / (1 - p/1+r)$$

Substituting in the necessary informational items including the first derivative of the yield equation under expected weather conditions

and solving for U_o gives the profit maximizing level of total plant available nitrogen. Table V lists the economic results for total nitrogen levels ranging from 0 to 110 pounds per acre, under pricing and interest rate assumptions of \$.0075 per pound for hay, \$.10 per pound for nitrogen, and a ten percent rate of interest.

As readily apparent, returns are maximized at approximately 100 pounds per acre. The associated expected optimal application rate and the expected amount of nitrogen carryover at this level are 33.1 and 66.9 pounds per acre, respectively.

The decision rule, advocated by this rather simple-minded decision model, recommends maintenance of 100 pounds of nitrogen in the soil each growing season. Since nitrogen consumed in any production year is replaced in the immediately succeeding production year, the corresponding (s,S) solution is (100,100).

The state of equivalence between "s" and "S" is attributable to the omission of spreading costs from the non-stochastic decision model. Upon reflection we observe that costs of spreading fertilizer are attached directly to the application of nitrogen. Conventional economic theory dictates that spreading costs will inhibit the applications of nitrogen in any given growing season. Subsequently, we would expect to apply nitrogen less and less frequently over time as spreading costs become greater. Further, given non-stochastic nitrogen carryover, such expectations would imply existence of a well defined time interval between applications. It

TABLE V. DETAILED ECONOMIC DATA FOR NON-STOCHASTIC ANALYSIS ASSUMING THE PRICE OF HAY IS $15/TON, THE PRICE OF NITROGEN EQUALS $.10/LB, AND THE INTEREST RATE (r) IS TEN PERCENT.

Total Nitrogen	Nitrogen Carryover a/	Nitrogen Applied b/	Average Profits c/
lb/A	lb/A		$
0	0	0	5.025
5	3.34	1.66	5.147
10	6.69	3.31	5.263
15	10.03	4.97	5.373
20	13.37	6.63	5.476
25	16.72	8.28	5.573
30	20.07	9.93	5.664
35	23.41	11.59	5.749
40	26.76	13.24	5.827
45	30.10	14.90	5.900
50	33.45	16.55	5.965
55	36.79	18.21	6.025
60	40.13	19.87	6.078
65	43.48	21.52	6.126
70	46.83	23.17	6.166
75	50.17	24.83	6.201
80	53.52	26.48	6.229
85	56.86	28.14	6.251
90	60.21	29.79	6.267
95	63.55	31.45	6.277
100	66.90	33.10	6.280
105	70.24	34.76	6.277
110	73.58	36.42	6.268

a/ Carryover Nitrogen = $P/W_t[U_t]$

b/ Nitrogen applied = $1 - P/W_t\ [U_t] \equiv 1 - [.6194\ (1.08)\ (U_t)]$

c/ Computed by the functional relationship, average profits = $P_H \cdot E(Y_t/U_t) - P_N\ [N_t + r.R_t]$ where all variables are as defined earlier.

follows that the appropriate decision rule, when spreading costs are non-zero, must consider two distinct levels of nitrogen or, in other words, a (s,S) solution structure.

To derive an optimization model which provides adequate evaluation of the impact of spreading costs upon the optimal non-stochastic fertilization policies, consideration must be given to application cycle length. Because this factor could be expected to vary with spreading costs, it is of sufficient importance to be included as important variable within the appropriate optimizing relationship. The resulting relation would be optimized with respect to nitrogen level and the application cycle length.

A mathematical model endowed with these desirable qualities has been formulated [11]. However, the work has yet to be published and, therefore, is unavailable for use in the current paper by reason of professional ethics. In substitution, we will utilize a method which yields approximate solutions to the present problem.

To begin, we note that when spreading costs are non-zero, the individual decision-maker must decide whether average returns can be increased by applying nitrogen, given he has an accurate estimate of carryover nitrogen. Specifically, application of nitrogen must yield a large enough increase in average returns to cover the costs of spreading in order to allow fertilization in any given production year. If increased average returns exceed spreading costs, the appropriate policy response is to apply enough nitrogen to reach the maximum average return level.

Alternatively, if spreading costs cannot be covered by increased average returns, no attempt should be made to apply nitrogen.

Returning to Table V, let us assume spreading costs are $0.25 per acre. To determine "s", we deduct $0.25 from maximum average profits and relate the resulting average profits to the corresponding level of nitrogen. Using this procedure, we find the (s,S) decision rule to be (55,100) when spreading costs are $0.25 per acre.

To provide a basis on which to evaluate the relative merits of the non-stochastic and dynamic programming analyses, we need to determine the effect of spreading costs upon maximum average profits. In order to achieve this result, it is necessary to ascertain average spreading costs by obtaining an estimate of the proportion of years in which we would expect to apply nitrogen.

Recall in our discussion of the derivation of transition probabilities, we redefined nitrogen carryover in year t+1 as

$$R_{t+1} = p/W_t \, [N_t + R_t] = p/W_t \, [U_t]$$

By setting R_{t+1} equal to 55 pounds and letting U_t equal the average profit maximizing level of total nitrogen and solving for W_t, the relative value of W_t which would be expected to induce a carryover amount of 55 pounds can be determined. Completing this operation, we obtain W_t equal to 1.126.

Since we are concerned with years in which weather is relatively favorable, we would be most interested in years which have weather values

greater than 1.126. Analysis of the 32-year weather record (Table III) for the experimental site indicates that the value of the weather variable was greater than 1.126 in 16 years. Thus, we would expect to apply nitrogen in fifty percent of the years constituting a given planning horizon. Consequently, the average spreading costs for any given year would be in the neighborhood of $0.125.

Subtracting average spreading costs from maximum average profits, we have average profits under $0.25 spreading costs equal to $6.155. By discounting this value over the appropriate planning horizon and summing the result, we obtain the desired basis for comparison with dynamic programming.

Before proceeding, we might point out two major inadequacies of the preceding analysis. First, although emphasis was given to the determination of "s", "S" was assumed to remain constant (100 pounds). Prior experience with dynamic programming suggests that both "S" and "s" are readjusted when spreading costs are non-zero. Specifically, "s" tends downward while "S" inclines upward. Second, when spreading costs exceed $1.25 per acre, the approximately method proposes that nitrogen fertilization is not feasible. In as much as the non-stochastic approach closely approximates the stochastic consideration, we would not expect substantial differences in their ultimate solutions. Examination of Table IV will reveal significant disagreement between the approximate non-stochastic solutions and the stochastic dynamic programming results.

In general, both descrepancies can be attributed to lack of effective consideration of the application cycle length as determined by the relative magnitudes of spreading costs.

Table VI presents the dynamic programming and approximate non-stochastic solutions under listed price/cost assumptions and a twenty year planning horizon.

Contrasting the solutions derived under $.25 spreading costs, we observe the (s,S) decision rule recommended by the approximate non-stochastic analysis advocates fewer applications over planning horizons of specified lengths than dynamic programming. In present value terms, pursuance of the latter policy would yield $4.69 greater discounted expected returns than the latter.

Similarly, if we make the same comparison using discounted expected returns from the non-stochastic analysis under zero spreading costs, we note greater returns are obtainable through use of dynamic programming.

Hence, it is apparent that consideration of the problem's dynamic and stochastic characteristics does offer an economic advantage over the simpler non-stochastic method.

Practical Implementation of Dynamic Programming

To demonstrate how an individual livestock producer might utilize the dynamic programming decision rules, let us begin by assuming the typical parametric situation, with spreading costs of $1.50 per acre. That is, we assume the price of hay to be $15 per ton, the price of

TABLE VI. COMPARISON OF APPROXIMATE NON-STOCHASTIC AND STOCHASTIC DYNAMIC PROGRAMMING SOLUTIONS UNDER A TWENTY-YEAR PLANNING HORIZON ASSUMPTION. a/

Method of Analysis	(s,S) Policy lbs/A	Spreading Costs $	Discounted Expected Returns $
Non-Stochastic Approx.	(55,100)	0.25	57.58
Stochastic-Dynamic	(62.5, 92.5)	0.25	62.27 b/
Non-Stochastic Optimal	(100,100)	0	58.75

a/ Parameter assumptions: P_H = \$15/ton, P_N = \$.10/lb, and r = 10%.

b/ Discounted expected returns when initial state is 0-5 pounds of nitrogen per acre.

nitrogen to be $.10 per pound and the appropriate rate of interest for discounting to be ten percent. In addition, assume the nitrogen fertilization program was initiated in 1965 and that the producers expects to conduct firm operations for at least thirty more years. The latter assumption allows the use of a constant decision rule which in turn simplifies the following discussion.

Since the corresponding decision rule in (s,S) terms is (22.5, 107.5), we apply 107.5 pounds of nitrogen in 1965. To determine the proper policy response for 1966, we need to ascertain the level of nitrogen carried over from 1965.

Recall from Chapter 2, the proportion of carryover under average weather conditions was represented by the parameter "p" with the expected value of the weather variable equal to 1.0. Estimation of nitrogen carryover for any specific year can be accomplished through use of the formula

$$R_t = p\left(\frac{N_{t-1} + R_{t-1}}{W_{t-1}}\right)$$

where all variables are defined as in Chapter II. The variable which distinguishes one year's carryover from the next is weather. Computation of W_{t-1} requires knowledge of the precipitation and average daily maximum temperature for the year t-1. In 1965, for example, these readings were 10.48 inches and 63.42 fahrenheit. Dividing this quotient by the average precipitation and temperature quotient for the sample period yields

$$W_{t-1} = \frac{63.42}{.0920} = \frac{.1640}{.0920} = 1.7780$$

Applying the ratio of p/W_{t-1} to total plant available nitrogen in 1965 $(N_{t-1} + R_{t-1})$ we have

$$R_t = p/W_{t-1}\,[N_{t-1} + R_{t-1}] = \frac{(.6194)}{(1.778)}\,[107.5] = 37.44$$

our estimate of nitrogen carried into 1966.

As this value is above 22.5 pounds per acre, the optimal decision is not to apply fertilizer in 1966.

The weather in 1966 was considerably below normal such that a carryover proportion of 131 percent was estimated. Since the probability of the event is very near zero, we adjust predicted carryover downward to 100%. The optimal policy response for 1967 is, consequently, the same as in 1966.

Favorable climatic conditions in 1967 reduced nitrogen carryover to 40.3 percent of plant available nitrogen in 1967, or 15 pounds per acre. The appropriate decision, then, for 1968 is to apply the difference between 107.5 and 15 pounds per acre as the initial state of nitrogen has fallen below 22.5 pounds.

Hence, we apply 92.5 pounds of nitrogen fertilizer at the beginning of the 1968 growing season.

In this manner, optimal policy responses can be derived for any production year. However, each individual livestock producer would likely need to modify the decision rules in accordance with his own specific situation.

SUMMARY AND CONCLUSIONS

The foregoing analysis has indicated potential for increases in economic efficiency obtainable under non-zero nitrogen fertilization policies. However, it must be re-emphasized that a multiproduction period viewpoint is a virtual necessity for the individual decision-maker considering a nitrogen fertilization program. A very short run approach to the problem will generally not be a fair assessment of nitrogen application decisions due to the significance of nitrogen carryover from production period to production period. Another consideration, which has negative implications for the latter point, is the importance of return variability. Given the unstable nature of seasonal precipitation in semiarid continental type climates, the variation of returns can become quite pronounced over the span of a few short years. This is especially true when contemplation is given to maintenance of high nitrogen levels in the soil [12]. In light of the narrow return margins faced by the agricultural production sector, the extent of return variability is a decisive factor influencing the choice among alternative decisions.

Extrapolation of agriculture research findings, to areas other than that in which the study was specifically located, has been and will continue to be a very contestable issue among the different concerned research disciplines. The criticisms relating to such extrapolations are not unfounded as significant fluctuations in results are apparent over even the smallest of geographical regions. However, the value of

agricultural research is not contained within its ability to make very specific recommendations to each and every decision-maker, but rather to provide some general information which the individual farmer or rancher can utilize in making decisions most appropriate for his particular circumstance. It is upon this pretext that I base justifications for broader geographical generalizations of research conclusions.

Clearly, as revealed in Chapter II, effective seasonal precipitation is of paramount importance in determining pasture grass yields. Consequently, for any region to qualify as having extrapolation potential, similar precipitation patterns must be present. In addition, like soil types and grass species are necessary requisites.

Of the regions considered, those areas included in the extreme southeastern portion of Montana appeared to hold the most promising similiarities. As well as comparable precipitation levels, the general soil type of the area is closely related to that existing at Newell [13]. Also, both crested wheat grass and bromegrass possess traits compatible with their establishment in the area [14].

As a final note, I would like to suggest some possibilities for future research in the area of range land fertilization.

First, and foremostly, more extensive research following experimental designs similar to that used by Thomas and Osenbrug is badly needed to effectively evaluate the economic feasibility of range land fertilization over the northern great plains as a whole. Such

experimentation should consider not only nitrogen, but also other plant growth enhancing chemicals.

Secondly, economic research concerned with the integration of range land fertilization into the overall program of range management is worth future consideration.

And third, both physical and economic research might do well to consider potential externalities associated with livestock pastureland fertilization. Nitrate poisoning and long-run effects of fertilization on productivity are two possible areas for future exploration.

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