



The effect of formative testing, prescribed remediation, and retesting on student performance in calculus

by David Allen Thomas

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University

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Abstract:

During the Fall Quarter of 1982, the researcher conducted an experiment at Montana State University involving six sections of Math 121, freshman calculus. The purpose of the study was to determine the effect of formative testing, diagnostic and prescriptive remediation, and retesting on student performance in calculus in a conventional classroom instructional setting. The experimental treatments were limited to the use of weekly formative quizzes and did not intrude into lecture time or homework practices.

The students in the six sections were divided randomly into two groups for the study, an experimental group and a control group. Experimental and control group students all took the same quizzes and examinations. The control group students received conventional forms of written feedback on their quizzes and remedial assistance based on their own perceptions of need and initiative in seeking help. Experimental group students received a detailed error analysis for each formative quiz and remediation individualized to address the diagnosed errors.

At the end of the quarter, student performance was evaluated as the sum of the three one hour examination scores plus the final examination score. Of 500 possible points, the control group had a mean score of 375.9 and the experimental group had a mean of 402.3 for students finishing the course. The analysis of variance showed significant main effect differences for the experimental and control groups. The p-value for the difference in mean performance for entering freshmen was .0186. No aptitude-treatment interaction was found.

On the basis of this analysis, the researcher concluded that student performance in calculus can be significantly improved by using formative testing, prescribed remediation, and retesting, procedures which do not require extensive intrusion into the conventional lecture- discussion format of college calculus.

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AND RETESTING ON STUDENT PERFORMANCE IN CALCULUS**

by

David Allen Thomas

**A thesis submitted in partial fulfillment
of the requirements for the degree**

of

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ABSTRACT

During the Fall Quarter of 1982, the researcher conducted an experiment at Montana State University involving six sections of Math 121, freshman calculus. The purpose of the study was to determine the effect of formative testing, diagnostic and prescriptive remediation, and retesting on student performance in calculus in a conventional classroom instructional setting. The experimental treatments were limited to the use of weekly formative quizzes and did not intrude into lecture time or homework practices.

The students in the six sections were divided randomly into two groups for the study, an experimental group and a control group. Experimental and control group students all took the same quizzes and examinations. The control group students received conventional forms of written feedback on their quizzes and remedial assistance based on their own perceptions of need and initiative in seeking help. Experimental group students received a detailed error analysis for each formative quiz and remediation individualized to address the diagnosed errors.

At the end of the quarter, student performance was evaluated as the sum of the three one hour examination scores plus the final examination score. Of 500 possible points, the control group had a mean score of 375.9 and the experimental group had a mean of 402.3 for students finishing the course. The analysis of variance showed significant main effect differences for the experimental and control groups. The p-value for the difference in mean performance for entering freshmen was .0186. No aptitude-treatment interaction was found.

On the basis of this analysis, the researcher concluded that student performance in calculus can be significantly improved by using formative testing, prescribed remediation, and retesting, procedures which do not require extensive intrusion into the conventional lecture- discussion format of college calculus.

CHAPTER ONE

INTRODUCTION

Introduction

Mathematics, the physical sciences, engineering in all its forms, and many other disciplines share a characteristic orientation toward sequential, task-oriented development. In these disciplines, a student progresses from the basic to the more complex by mastering increasingly more sophisticated concepts and skills in a well defined order. Bloom (1976) argues that failure at any point in such a sequence to master the task at hand not only weakens the student's understanding at the point of failure, but also inhibits his ability to master subsequent tasks. In light of this characteristic, the curriculum must provide some way of dealing with student errors and misunderstanding.

At Montana State University, the first quarter of calculus was Math 121. The course briefly reviewed algebraic skills before beginning differential calculus. Approximately two-thirds of the quarter was spent on this topic, which deals primarily with a study of the rate of change of functional values and associated applications to science and engineering. The last topic introduced in the course was integral calculus, which is often applied to the problem of finding the area under a given curve. A thorough investigation of integration

was presented in Math 122 and subsequent course offerings. The problem of dealing with student errors and misunderstandings in Math 121 was the focus of this study.

The Mastery Learning paradigm proposed by Bloom (1976) stresses large group instruction, use of formative tasks, and remediation to achieve mastery. The classroom teacher's role is extended beyond conventional patterns of presentation and evaluation to include diagnosis of student characteristics and prescription of individualized instruction and remediation.

Another systematic approach to individualization is the Keller, or PSI, Plan. PSI (Personalized System of Instruction) programs stress the use of the written word as the primary vehicle of communication for course content. Tutors are used extensively in PSI programs to respond to student questions and administer unit quizzes, which must be repeated until mastery is achieved (Keller, 1968). The primary task of course lectures is to motivate the students and direct the course of study. Research conducted on PSI courses in psychology (Cooper and Greiner, 1971), child development (Semb, 1974), and pre-calculus mathematics (Haver, 1978) indicates that the PSI method often results in higher student performance than conventional lecture-recitation methods of instruction.

Use of PSI as an instructional format in college calculus has resulted in seemingly contradictory findings by different researchers. Pascarella (1977a, 1977b, 1978), Peluso and Baranchik (1977), and Kulik, Kulik and Smith (1976) found that students in PSI sections of

calculus scored higher on exams than students in conventionally taught sections. Klopfenstein (1977) and Thompson (1980) found no significant differences between exam scores of PSI and conventionally taught students.

In recent years, research on the nature and importance of individual learning styles (Dunn and Dunn, 1978; Gregorc, 1979) has led to the use of these concepts in the individualization of instruction (Kusler, 1979). This study makes use of learning styles research as a vehicle for identifying appropriate modes of remediation for individual students.

In introductory calculus, the problem of dealing with student misunderstanding is complicated by the wide range of abilities and degrees of preparation in the student population. In partial answer to this problem, the Montana State University Department of Mathematical Sciences operates a Learning Center every school day from 8 am to 5 pm. At the learning center, students have access to tutorial help from qualified staff members and graduate students. Students may seek assistance on homework problems, clarification of topics presented in class, or review in preparation for an examination. This service has been offered since 1977 and was originally provided to provide students with assistance when their instructors were unavailable (Tiahrt, 1982).

Statement of the Problem

Student failure in introductory calculus has created problems both for the student personally and for college departments faced with scheduling trailer sections for students who have delayed taking course work in their major area because of failure in prerequisite calculus courses. The purpose of this study was to determine the effect of formative testing, prescribed remediation, and retesting on student performance in freshman calculus when remediation is adjusted to accommodate individual learning styles.

Need for the Study

In 1981, approximately 30% of the students enrolled in introductory calculus at Montana State University received a grade of D or F. This was typical of failure rates in comparable courses across the country (Struik and Flexer, 1977). Additionally, this failure rate created of considerable personal disappointment to both students and parents. When the effect of such failures was seen in the broader perspective of the university disciplines for which mathematics is a foundation, the failures took on further significance. Student failures created a need for trailer sections of Math 121 and other courses sequenced with Math 121 in other departments. As an example, consider the impact of student failure in Math 121 on course scheduling in the Engineering Mechanics Department. With enrollments in engineering growing every year, the university was

facing shortages in qualified staffing and facilities. Ultimately, engineering students who failed calculus and had to repeat the course added to the burden of the engineering departments by creating scheduling problems when mathematics prerequisites were not completed ahead of specified engineering courses as planned (Williams, 1982). The same type of scheduling problems occurred in other departments as well. Thus, in order to deal with student failures both as a personal problem and as a university problem, there was a need for research addressing the improvement of instruction in introductory calculus.

A review of the literature revealed studies which support the use of Mastery Learning and PSI strategies as effective approaches to improving student performance in calculus (Pascarella, 1977a and Peluso and Baranchik, 1977). The review also revealed studies which concluded that the Mastery Learning and PSI strategies made no difference in student performance when compared to conventional methods of instruction (Klopfenstein, 1977). The variety of instructional formats and procedures employed in the studies cited above and in other studies discussed in Chapter Two of this paper ranged from moderate modifications of conventional classroom procedure to major diversions from traditional procedures. This study tested the effectiveness of certain Mastery Learning and PSI strategies in an instructional setting that was close to conventional classroom procedures.

Finally, the solution to the problem of failures in calculus does not lie in college pre-calculus remedial courses. Whitesitt (1980)

showed that pre-calculus courses such as college algebra and trigonometry offered at Montana State University were not effective in preparing students for calculus. Thus, the problem of student errors and misunderstanding in calculus must be dealt with as a part of the calculus curriculum.

Questions Answered

This study answered nine questions.

1. Did the random assignment of students to treatment groups result in an experimental and control group of equal ability?

2. Did the random assignment of students to instructors result in the instructors teaching groups of equal ability?

3. For students of known ability, did the experimental group students perform at a different level than the control group students?

4. Was there an aptitude-treatment interaction? In particular, when the difference in exam performance of control group and experimental group students of relatively low ability was compared to the difference in performance of control and experimental group students of relatively high ability, were the differences constant or did one treatment benefit students of one ability level more than students of another ability level?

5. For students of unknown ability, did the experimental group students perform at a different level than the control group students?

6. Was the standard deviation of the experimental group scores different than the standard deviation of the control group scores for

each of the four examinations?

7. Was there any relationship between the amount of time a student spent in remediation with his/her instructor and his/her overall performance on examinations?

8. Was there any relationship between a student's expected GPA and his/her overall examination performance?

9. Was there any relationship between a student's learning style and his/her overall examination performance?

These nine questions were restated operationally in Chapters 3 and 4.

General Procedures

Arrangements were made through the chairman of the Department of Mathematical Sciences at Montana State University for four Math 121 instructors to participate in the study. These individuals taught a total of six sections of introductory calculus. One of the instructors was a visiting professor with many years teaching experience at the university level. The other three instructors were Graduate Teaching Assistants in the Mathematics Department with varying degrees of experience at the secondary and college level.

To control for differences between instructors, each instructor taught roughly the same number of experimental and control group students. Two instructors taught only one section each of Math 121. The students in each of these sections were randomly assigned membership in either the experimental group or control group, thus

splitting the students in each section. Two instructors taught two sections each of Math 121. In both cases, these sections met at consecutive hours. For each of these instructors, one section was assigned to the experimental group and one section to the control group. The assignments were all made the second week of class after the last day to add a class had passed.

The researcher used the expected GPA, a statistic generated by the university for entering freshmen, as a measure of aptitude or ability to do college work. All the entering freshmen in the Math 121 sections in the study were ranked by the researcher on the basis of their expected GPA scores. The top third of the ranking was labeled the high ability group, the low third the low ability group, and the middle of the list was the middle ability group. This information was used in the data analysis to look for interactions of treatment with ability, but it was not made available to the instructors or students. Thus, this classification had no bearing on the treatments given.

During the third week of class, Gregorc's Learning Styles Delineator (Gregorc, 1982) was administered to the students. At the end of the third week of class, students in the six sections began a series of weekly formative tasks (Appendix A). These tasks took the form of quizzes taken in class. Instructors graded the quizzes of the experimental group students and marked a table of specifications (Appendix B) for each student characterizing the student's errors both by subject matter and level of difficulty. When the student reported to the instructor's office the next day to pick up his quiz, the

instructor discussed the student's error analysis and remediated (Appendix D) the student on the indicated objectives, taking into account the student's preferred learning style (Appendix E). Following the remediation, a retest (Appendix C) was given to the student to complete within a few days and return to the instructor. For the experimental group, the instructor directed the student's attention to the diagnosed areas of concern.

The control group students received identical quizzes at the same time as the experimental group students. The classroom teachers evaluated these papers in the conventional manner and returned them to the students with written feedback but without a systematic error analysis. Students reported to the instructor in the same manner as the experimental group students to claim their quizzes. At all times, students in the control group were free to seek help from their instructors and initiate any discussion concerning their progress or perceived problems. For the control group, the student initiated remediation by requesting help.

For both groups, the measure of performance was a series of three one hour exams and a two hour final exam (Appendix F). All students took the same exams at the same time and had their papers scored by the same committee of instructors. All students had equal access to help in the Math Learning Center. The treatments and procedures described here applied only to the use of remedial procedures relevant to the formative tasks. All questions arising from classroom discussions, homework assignments, etc. were handled by the

instructors in a conventional manner without reference to the strategies outlined in this study.

Analysis of the data was accomplished using computer facilities at Montana State University. The statistical software package MSUSTAT (Lund, 1978) was used. For each of the three hour exams and the two hour final, the mean performance levels for the control and experimental groups were compared using a 2X3 ANOVA in which the interaction of treatment with ability was examined. Mendenhall and Reinmuth (1982) state that the ANOVA is an appropriate technique for identifying important independent variables in a study and how they interact and affect the dependent variable. For identifying the strength of a relationship between variables, Linder (1979) states that the correlation coefficient is most useful. Thus, Pearson correlation coefficients were calculated for the relationships between overall performance on examinations and student ability level as measured by the expected GPA and between overall examination performance and the amount of time spent in remediation. The effect of student learning style to overall performance was examined using ANOVA.

Limitations and Delimitations

This study was limited to students enrolled in Math 121 at Montana State University during Fall quarter, 1982.

The study was delimited to addressing the instructional value of formative testing, prescribed remediation, and retesting in calculus

with regard to examination performance. No questions were asked regarding affective aspects of the instructional procedures or other possible dependent variables or outcomes.

Definitions

Terms defined for this study:

1. Ability or Ability Level referred to a student's relative position with regard to all the other students in the study in a ranking based on expected MSU grade point average. Students ranked in the top third of this listing were designated as being of "high" ability. Students ranked in the bottom third of this listing were designated as being of "low" ability. The remaining third was designated "average".

2. Formative Testing or Evaluation is a form of criterion based testing used to diagnose student errors and misunderstandings for the purpose of identifying instructional strategies appropriate to the student's needs. In this study, students receiving formative testing were remediated on the basis of their individual learning styles. Thus, two students who answered the same problems incorrectly on a formative task were remediated over the same content material. However, if the students had different learning styles, the format for the remediation was different for the two students.

3. A mathematics Learning Center was a room or group of rooms where students went for tutorial help or evaluation. At Montana

State University, students enrolled in pre-calculus classes went to a different room than students enrolled in calculus or more advanced courses. The MSU learning center was staffed by instructors, graduate teaching assistants, and advanced undergraduates with a strong background in mathematics.

4. Learning Style in this study refers to the range of cognitive behaviors by which individuals perceive and process information in a learning environment. The four orientations towards information processing used in this study are identified as concrete/sequential, abstract/random, abstract/sequential, and concrete/random (Gregorc, 1982).

5. Mastery Learning refers to a theory of instruction developed by Bloom (1976). The essential features of mastery learning require the instructor to provide the student several specific services. First, directions, demonstrations, and explanations are provided in large group instruction. Second, the student's time is utilized in such a way as to provide sufficient time on task practice of each topic to be mastered. Third, individualized corrective feedback and reinforcement are used to develop both mastery of each topic and a positive attitude toward learning.

6. In this study, Prescribed Remediation refers to a process whereby the tutor uses the diagnosis provided by the formative test to determine the content and difficulty level of the materials used for remediation.

7. PSI or Personalized System of Instruction is a theory of instruction developed by Keller (1968). The most distinctive features of the system are as follows: self-pacing, a unit perfection requirement, use of lectures as vehicles of motivation rather than sources of critical information, dependence upon written student-teacher communication, and the use of proctors in evaluation, feedback, and reinforcement.

8. Retesting refers to the practice of evaluating a student's knowledge after remediation using a parallel form of the unit test.

9. A Summative Evaluation was used to establish a grade or ranking and was not used in a formative sense.

10. A Table of Specifications is a diagnostic report form used to classify errors on a test. In this study, the table listed all the learning objectives for a unit down the left margin and the headings " notation/definition, direct application, synthesis, and abstract" across the top. Inserted in this matrix was a list of the problem numbers from the formative quiz. By inserting the problem number in the correct row and column, the content of the problem and its level of difficulty were characterized.

11. A Tutor in this study was the general term used to denote learning center personnel. These individuals were either instructors, graduate teaching assistants, or advanced undergraduates with a strong background in mathematics.

12. Unstructured Help refers to informal student-tutor interaction in which the student determines the substance of the discussion or problems to be addressed.

Summary

Student failure (grade of D or F) rates in college and university introductory calculus courses in the United States are typically in the range of 30% - 35% of course enrollment. The percentage of failures in freshman calculus at Montana State University has also typically fallen in this range. A study of the pre-calculus courses offered at MSU has shown that they are generally not effective in preparing a student for success in calculus (Whitesitt, 1980). The purpose of this study was to examine the effect of formative testing, prescribed remediation, and retesting on student performance in introductory calculus when remediation is adjusted to accommodate a student's preferred learning style.

The data for this study was collected during the Fall quarter, 1982 at Montana State University. A sample of approximately 200 students was randomly selected from the students enrolled in the course (700 - 800). These subjects were divided into a control and an experimental group for treatment. The experimental group received formative testing, prescribed remediation, and retesting with remediation individualized to accommodate the student's learning style as indicated by Gregorc's Learning Styles Delineator (Appendix F).

The control group students took the same quizzes but did not receive the diagnostic treatments offered the experimental group. Feedback was in conventional form, written comments on the quizzes, etc.

At the end of the quarter, the data was analyzed using the software statistical package MSUSTAT (Lund, 1978) on the CP6 computer at Montana State University. The analysis made use of one and two way analysis of variance to compare examination performance for the experimental and control groups, Pearson correlations to measure the strengths of relationships between variables, and F tests to compare examination variance scores.

CHAPTER TWO

REVIEW OF THE LITERATURE

Introduction

For the purpose of this study, the literature was reviewed with regard to the following major topics: the theoretical frameworks of Mastery Learning and Personalized Systems of Instruction (PSI), Learning Styles, feedback, remediation, and retesting as instructional practices, affective aspects of testing and evaluation procedures, interaction effects of various instructional practices with aptitude, preparation or readiness, and motivation, "no-difference" studies, and organizational and managerial practices in PSI and Learning Center environments.

Mastery Learning and Personalized Systems of Instruction

At the foundation of the Mastery Learning paradigm is a rejection of the notion that student achievement should be normally distributed.

Bloom (1981) states

There is nothing sacred about the normal curve. It is the distribution most appropriate to chance and random activity. Education is a purposeful activity and we seek to have the students learn what we have to teach. If we are effective in our instruction, the distribution of achievement should be very different from the normal curve (p. 155).

This statement is based in part on Carroll's (1963) view that aptitude is the amount of time required by a learner to master a learning task. Given a group of learners normally distributed with respect to aptitude for some subject, and given an instructional strategy that treats all students alike without regard for personal differences, then the end result will be a normal distribution on an appropriate measure of achievement. Bloom (1981) concludes, however, that if

... the kind and quality of instruction and the amount of time available for learning are made appropriate to the characteristics and needs of each student, the majority of students may be expected to achieve mastery of the subject (p. 156).

Bloom (1976) suggests three independent variables that effect learning: Cognitive Entry Behaviors, Affective Entry Characteristics, and Quality of Instruction. Cognitive Entry Behaviors are prerequisite learnings or behaviors necessary for the completion of a learning task. Affective Entry Characteristics are a "compound of interests and attitudes towards the subject" (p. 168). Quality of Instruction refers to the extent to which a teacher effectively communicates information to an individual student, engages the student in practice of the learning task, and reinforces (positively or negatively) the student via feedback. The use of formative evaluation over short units lasting one or two weeks is recommended (Bloom) as an effective means of providing the necessary reinforcement.

In brief, the Mastery Learning model suggests that the educational process should be highly individualized. When a teacher

faces a class having a wide range of Cognitive Entry Behaviors, the instructional process may call for several alternative introductions to the learning task, each introduction bridging the gap between some students' entry behaviors and the prerequisites for the learning task. Various methods may be used to motivate the class and prepare the students affectively to deal with the learning task. Also, feedback can be given frequently via formative testing and individualized remediation strategies. Bloom (1981) suggests that, given such a program, a much higher percentage of students will perform at a mastery level (B+ or A) than in a conventional program of instruction.

The Personalized System of Instruction (PSI) is characterized by Keller (1968) as differing from conventional teaching procedures in the following ways:

1. The go-at-your-own-pace feature, which permits a student to move through the course at a speed commensurate with his ability and other demands upon his time.
2. The unit-perfection requirement for advance, which lets the student go ahead to new material only after demonstrating mastery of that which preceded.
3. The use of lectures and demonstrations as vehicles of motivation, rather than sources of critical information.
4. The related stress upon the written word in teacher-student communications; and, finally:
5. The use of proctors, which permits repeated testing, immediate scoring, almost unavoidable tutoring, and a marked enhancement of the personal-social aspect of the educational process (p. 83).

The most essential feature in the PSI paradigm is the constant use of reinforcement. Keller (1968) states

The kind of change needed in education today is not one that will be evaluated in terms of the percentage of A's in a grade distribution or of differences at the .01 level of confidence. It is one that will produce a reinforcing state of affairs for everyone involved--- a state of affairs that has heretofore been reached so rarely as to be the subject of eulogy in the world's literature, and which, unfortunately, has led to the mystique of the "great teacher" rather than a sober analysis of the critical contingencies in operation (p. 86).

Learning Styles

Keefe (1979) defines learning style as follows:

Learning styles are characteristic cognitive, affective, and physiological behaviors that serve as relatively stable indicators of how learners perceive, interact with, and respond to the learning environment (p. 4).

Cognitive styles are habits representing the learner's preferred or typical mode of perceiving, thinking, problem solving, and remembering. Gregorc's (1979) investigations revealed that there is a duality in learning preference.

People learn both through concrete experiences and through abstraction. Further, both of these modes have two subdivisions, sequential and random preference (p. 20).

Gregorc (1979) discovered that these sets of dualities merged to form four distinct cognitive styles: concrete-sequential, concrete- random, abstract-sequential, and abstract-random. Most people exhibit all four patterns or styles to some degree. It is common for individuals to

prefer one or two of these styles and to use those preferred styles most of the time.

A concrete-sequential learner prefers step by step directions and will follow them. Such individuals like clearly ordered presentations and a quiet atmosphere.

A concrete-random learner is intuitive and often does not "show his work". They use trial and error approaches but dislike cut and dried procedures. Such individuals work well independently and may not respond well to teacher intervention.

An abstract-sequential learner is skillful with symbolic systems and makes use of diagrams or pictures extensively. Such individuals work well with their teachers and other authority figures and prefer logical, sequential presentations. Main ideas or principles are readily accepted and used.

Abstract-random learners are sensitive to human interactions and function well in group activities and busy environments. They like to receive information in an unstructured manner and dislike rules and guidelines. Such learners organize information through a process of reflection.

Dunn and Dunn (1978) have identified a set of environmental and social factors influencing an individual's ability to learn in a given situation. For instance, some people most readily learn in a well ordered room free of distractions. Other individuals would be made quite uncomfortable by such an environment and would consequently have difficulty learning.

The intent of the identification of an individual's learning style is to provide the educator with information useful in the individualization of instruction for that student. The extent to which a learning process recognizes and accommodates individual differences in learning styles may be seen as one measure of the quality of instruction offered by the instructional program.

Feedback, Remediation, and Retesting

Many questions remain unanswered, if not unaddressed, concerning the nature of feedback best suited to a given instructional task. The timing of the feedback, relative effectiveness of verbal vs. written feedback, the length of the unit being tested, and the criteria for passing each unit all have a bearing on this study.

Sturges (1978) compared the effect of immediate vs. delayed feedback on retention of learning. Various delays in feedback treatments following computer managed multiple choice tests were employed in a study of 112 students enrolled in a child psychology course. Results confirmed previous studies indicating that retention following delayed feedback is not degraded by the delay. In fact, feedback delays of 20 minutes to 24 hours had a greater positive effect on student confidence on subsequent retention tests than did immediate feedback. As a result of this study, Sturges hypothesized that with a longer delay students "engage in a more thorough semantic analysis of information presented at feedback" (1978).

In another experiment involving 98 introductory psychology students, Cooper and Greiner (1971) compared student performance in a traditionally taught lecture section with student performance in a PSI section. Feedback in the PSI section was provided in the following manner: after collecting a weekly quiz, the papers were redistributed randomly to the class, no student receiving his own paper. The quizzes were then corrected in class. This provided each student with fast feedback on the correct solutions. The quizzes were then turned in for double-checking and recording by TA's. Within two hours, these papers were available to the students for study. Tutors could then assist them in remediation and evaluation. Students failing to meet the mastery standard retested on equivalent forms of the quiz later in the week. At the conclusion of the study, it was found that the PSI students earned higher course grades and had better long-term retention than students in the conventionally taught section. Cooper and Greiner hypothesized that PSI reinforces regularly spaced practice, while conventional methods reinforce massed practice prior to exams and that this is the reason for the differences in performance.

With regard to the effectiveness of written corrective feedback on quizzes, Belanger (1976) found in a study of 51 undergraduates enrolled in a PSI psychology course that only 23% took advantage of the written feedback on their quizzes. Observations made during the study led Belanger to suggest further research into the effect of verbal interaction between instructors or TA's and students.

Semb (1974) investigated the effect of unit length on student performance in a study of 193 students enrolled in an introductory child development course. The Mastery Learning method of instruction was used. When a student completed a quiz he or she took it to the TA or proctor for immediate grading. Answers were graded either correct, partially correct or incorrect. If the student could justify or complete a partially correct answer the item was marked correct. Otherwise, it was scored incorrect. The effect of various levels of mastery on final exam performance was also studied. The results of this study indicate that use of a high mastery criterion with short assignments produces better test performance than use of a low mastery criterion on long assignments.

Summarizing research on the PSI method, Kulik, Kulik, and Smith (1976) emphasize the findings of several studies examining the reasons for the reported effectiveness of the PSI method. There appear to be 3 key features: small units of instruction, effective feedback, and a unit-mastery requirement. Other aspects of the PSI approach seem to offer less benefit than has been previously assumed. In citing research by Calhoun (1976) the authors reported that immediate verbal feedback from proctors or TA's was superior to written feedback.

In the Mastery Learning paradigm, the follow-up to feedback is remediation. Hassett and McCoy (1979) investigated the effect of post-quiz prescribed remediation on student performance in an introductory college algebra course involving 100 students. In this study, the students in the experimental group received remediation

keyed to their errors on quizzes. The controls were told to go over their mistakes, but received no additional drill, as did the experimental group students. The results of this study showed a significantly smaller variance in final exam scores for the experimental group than for the control group. Performance was higher also for the experimental group.

In a study of 235 calculus students, Struik and Flexer (1977) compared the traditional lecture method with a self-paced mastery approach. In the mastery approach students had the option of retesting up to four times, with only the best score counting. Feedback occurred within a day. The results of this study showed that students in the self-paced, mastery learning course did substantially better than those in the traditional course.

Affective Aspects of Testing and Evaluation Procedures

It is not uncommon for students to experience anxiety in anticipation of and during an exam. How does this anxiety affect the way students feel about a given course or mathematics in general? Douthitt (1978) reported in a study of 47 students enrolled in a course in finite mathematics that frequent use of informal evaluation procedures (homework, boardwork, etc.) instead of traditional unit examinations resulted in achievement levels on the final exam which were not significantly different from a control group taking the conventional unit exams. However, mean scores on the Aiken-Dreger Mathematics Attitude Test were higher for the experimental group. In

his conclusion, Douthitt recommended that instructors should give more attention to finding other less anxiety producing methods of evaluation.

In the previously cited study by Cooper and Greiner (1971) an analysis was also made of student anxiety levels. The PSI-taught students experienced essentially the same levels of anxiety over quizzes as did the conventionally taught students over unit exams. However, at the end of the course the PSI students rated the value of their course significantly higher than did the students in the conventionally taught sections.

A comparison of student attitudes was conducted by Haver (1978). In the study, 1200 students enrolled in a college intermediate algebra course were divided into various groups, each of which received a different instructional format. Of the various formats tested, the Mastery approach produced the highest grades, highest completion rate, and the highest student opinion rating. The Mastery Learning format was also significantly more successful in improving initially negative attitudes towards mathematics during the course than was any other approach in the experiment. This reshaping of affective behavior is consistent with Bloom's theoretical framework for Mastery Learning.

Interaction Effects

The application of the same treatment, x , to subjects in an experiment often produces varying results or degrees of variability in the dependent variable, y . If this variability is attributable to

some other variable, z , active in the experiment, then it is said that there is an interaction effect between variables x and z on variable y . Such is the case in a study by Born, Gledhill, and Davis (1972). Sixty students in a Psychology of Learning class were divided into two groups, one taught by PSI methods and the other by a conventional approach. At the end of the course student performance on the final exam was higher on all test item types from PSI-taught students. This effect was most pronounced for the students with "poor" to "good" academic records. The greatest difference in scores occurred on essay and fill-in item types rather than on recognition type questions. Thus, the level of student preparation or ability interacted with the type of instruction to effect the level of student performance.

Similar results are reported by Pascarella (1977a) in a study of 248 students enrolled in calculus. The results indicated that students with the lowest levels of ability and preparation were most likely to perform better in the PSI sections than in the conventionally taught sections. Students of high ability and preparation did not perform at significantly different levels when taught by different methods. In addition, it was found that level of preparation is a more significant factor in predicting student performance when the student is enrolled in a conventionally-taught section than when in a PSI section.

Pascarella (1977b) also compared the effect of motivation on student performance in calculus in a PSI section with performance in a conventionally taught course. This study involved 94 students. It

was found that the most dramatic differences in both achievement and attitude were indicated at the highest levels of motivation. This seems to indicate that the greatest benefit of PSI in mathematics instruction may go to the most highly motivated students.

A study by Peluso and Baranchik (1977) involving 395 calculus students reported similar findings. Students enrolled in a Learning Center (PSI) course did better than students taught by the traditional approach. Based on final exam performance and drop rates of the "top" students as measured by a calculus readiness test, it was also suggested that the PSI method may not be appropriate for the best students.

The study by Kulik, Kulik, and Smith (1976) places some restraints on conclusions concerning aptitude-teaching method interactions. The study points out the large differences between Bloom's theoretical relationships concerning aptitude and performance under different teaching methods and cites research in which the interaction is the opposite of what Bloom's Mastery Learning theory would predict. Clearly, more work needs to be done in the area of interaction effects in Mastery Learning or PSI environments.

"No-Difference" Studies

As an indication of the mixed results reported regarding PSI, Klopfenstein (1977) reported that in a study involving 57 students in a PSI calculus section and an unspecified number of students in a conventionally taught section, that no difference was observed either

in student performance or attitude. In this version of PSI, no lectures were given. All time was spent in the PSI section working with the study guides and proctors.

A study by Harris and Liquori (1974) comparing the PSI and conventional approaches in the teaching of a course in mathematics for business students found no significant differences in student learning between groups. The study involved 128 students.

Organizational and Managerial Practices in PSI and Learning Center Environments

The proctor's or tutors's role was discussed by Romizowski, Bajpai, and Lewis (1976). In particular, self-marking of quizzes by students followed by an analysis of student errors by the proctor and remediation based on that analysis was preferred by students over grading procedures in which the student's paper was graded by a tutor. The reasons given by the students for this preference were three. Grading was smoother and less time was lost waiting for the tutor to correct a paper. Once graded, students could receive immediate feedback and remediation, rather than wait while the tutor graded someone's paper. Quizzes could be graded and some feedback given regardless of whether or not a tutor was available. The main disadvantage seen by students was a worry that in grading their own papers a student might pass himself on a problem that a tutor would not accept and that this would lead to trouble on the summative or final exams where students do not grade their own papers. The need

for detailed marking keys was discussed as a partial remedy for inexperienced student graders.

Summary

In summarizing the literature, the following points stand out: 1) in the Mastery Learning and PSI approaches, the most significant features are the use of short units, frequent quizzes followed by undelayed feedback and remediation, and the use of a high performance criterion, 2) in comparisons of the PSI and conventional approaches, student attitudes were better in PSI courses, 3) significant interaction effects between motivation and instructional method on performance and between aptitude and instructional method on performance were noted, and 4) mixed results are reported in studies dealing with various subject areas at the college level. In light of these findings, the need for research in the area of individualized instructional methods involving Mastery Learning or PSI in college mathematics is apparent.

CHAPTER THREE

PROCEDURES

Introduction

The specific assumptions and procedures for this study are discussed under the following headings: population description and sampling procedures, treatments, content validity and departmental examinations, examination reliability, statistical hypotheses, statistical procedures, precautions taken for accuracy, and summary.

Population Description and Sampling Procedure

The population under study was Montana State University students enrolled in Math 121, introductory calculus, during Fall Quarter, 1982. Nearly 700 students enrolled in the course, requiring 21 sections of Math 121. The inferred population was students at the college level enrolled in introductory calculus courses using the Swokowski (1979) text and a large group instructional format.

Based on a sample of 100 Math 121 students' final percentage scores in 1980, the researcher estimated the 1980 Math 121 population performance standard deviation to be 20.45% and the mean to be 70.4%. Using the 1980 standard deviation as an estimate of the 1982 value, the following formula (Snedecor and Cochran, 1980, p103) yielded a recommended sample size for each group, control and experimental:

$$n = (Z_{2\alpha} + Z_{\beta})^2 \sigma_D^2 / \delta^2$$

where n = sample size

α = significance level

β = probability of type 2 error

σ_D = population standard deviation

δ = true difference to be detected by experiment

Using tables provided by Snedecor and Cochran (1980, p104) to simplify calculations, if the power of a one-tailed t-test is set at .95, the significance level at .05, and the percentage difference to be detected between the two groups' exam scores at 6%, then the recommended sample size per group was 125. Thus, if the true difference between the exam scores of the two groups was as large as 6%, there was a 95% probability that the difference would be detected statistically.

Initial class enrollments for Math 121 sections were set between 33 and 38 students. On the basis of the researcher's estimates of the sample size needed to meet statistical requirements in the interpretation of the experimental data, the researcher decided to seek the participation of a minimum of six sections of Math 121 in this study. The selection of the six sections took place in a meeting of all Math 121 instructors shortly after the start of the Fall Quarter. The researcher explained the purpose of the research and

experimental procedures to the group and solicited volunteers to participate in the study. From those instructors offering to participate, four were selected by the researcher to take part in the study. Between them, these instructors taught 6 sections of calculus. For purposes of identification, these four individuals will henceforth be referred to as instructors M1, M2, F1, and F2.

Instructor M1 was an experienced, male, college mathematics instructor with a PhD and an interest in research on instructional questions. Instructor M2 was a male graduate student working on a Masters in mathematics with a teaching assistantship. M2 participated in the study but did not manifest any personal interest in the questions addressed by the study. Instructor F1 was a female graduate student working on a Masters degree in mathematics with a teaching assistantship. F1 was a foreign student, having been born, raised and educated through college in Germany. Instructor F2 was a female graduate student working on a Masters degree in mathematics with a teaching assistantship and had a genuine professional interest in the study.

After the last day to add a class passed in the second week of the quarter, each instructor randomly assigned each of his students to one of two groups of roughly equal size, a control group and experimental group. Instructors M2 and F1 each taught two sections of Math 121. For both instructors, these two sections met at consecutive hours of the day. Each of these two instructors randomly assigned one entire section to the control group and one entire section to the

experimental group. Instructors M1 and F2 each taught one section of Math 121. Each of these two instructors assigned half of his/her section to the control group and the other half to the experimental group using a random process.

The selection procedures outlined above were used to control for the effect of teacher differences on group performance by having each teacher make equivalent contributions instructionally to each group. In this way, neither group received the benefits or liabilities of a given teacher's instruction differently than the other group. Thus, teachers, students, and treatments were randomly assigned.

Within both the experimental and control groups, three subgroups were identified by partitioning each group on the basis of general academic readiness. The university generates an expected GPA for all incoming freshmen. This study used that statistic to rank all the students in each group. Students in the top third of the list in each group were designated as being of high ability for the purpose of this study. Students in the lowest third of the list were designated as being of lowest ability. The middle third were designated as being of average ability. This partitioning of the groups lead to roughly equal subgroup sizes, a useful design element for statistical purposes. However, it did not partition each Math 121 section equally. Also, because of transfer students and returning students that were enrolled in the sections involved in the study, no expected GPA was available for some individuals. The data for these individuals was analyzed in a fourth category for students of unknown

ability. This classification information was used by the researcher in the analysis and interpretation of the data gathered in this study. The information was not supplied to the instructors nor to the students in the study.

Treatments

The treatments under investigation in this study were administered in class and at the instructors' offices beginning the third week of class. The experimental group received the following treatments:

1. During the third week of class, a Learning Styles Delineator (Gregorc, 1982) was administered to all students in the experimental group with the exception of instructor M2's students. This omission was not part of the experimental design, but came about because of instructor M2's characteristic procrastination and inspite of the researcher's promptings.
2. Beginning with the third week, a weekly formative task was assigned. These tasks (Appendix A) took the form of a quiz to be completed in class. Each task was a review of the previous week's classroom lectures.
3. Each instructor administered the formative quizzes in class and collected the papers at the end of the allotted time. These papers were graded by the

instructor outside of class and the scores entered in the grade book as part of the student's overall grade. A table of specifications (Appendix B) was then marked for each student, indicating which problems had been missed. The table of specifications displayed the problem numbers in an array that crossed the skill or concept being tested with the level of difficulty of the problem.

When a problem was missed on a formative quiz, the instructor circled the problem number on the table of specifications for that quiz. Each problem was characterized in two ways by its location in the table. The row heading identified the learning objective for the problem by citing a section number in the remediation materials (Appendix D). This section number consisted of three digits separated by periods. The first digit corresponded to the chapter number in the Swokowski (1979) text where the learning objective was presented. The second digit gave the section number in the chapter. The third digit served as a counter for objectives from the same chapter and section. For example, on quiz #1 (Appendix A), problem 11 appeared on a row with heading 2.4.3. The textbook reference was chapter 2, section 4. The remedial examples for the objective tested in problem 11 were found in the

remediation materials (Appendix D):

2.4.3 Use limits in curve sketching.

The column heading identified the level of difficulty of the problem. The four levels used in this study were Definition and Notation [Def/Not(1)], Direct Application [App(2)], Synthesis With Other Concepts [Syn(3)], and Abstraction [Abs(4)]. The parenthetical numbers associated with the levels of difficulty were printed in the remediation materials beside each example. The instructor attached a marked table of specifications to each student's graded quiz paper.

4. To get their quiz papers back, the students went to their instructor's office the day after taking the quiz. When a student from the experimental group reported to his/her instructor's office, the instructor briefly reviewed the student's table of specifications and summarized the student's performance. For example, failure to study definitions and notation showed up as a series of missed problems in the Def/Not(1) column.

Before beginning remediation on the indicated topics, with the exception of instructor M2 who failed to administer the Gregorc instrument, the instructors looked up the student's learning style profile and

identified an appropriate strategy for remediating the student in harmony with the student's learning style. The instructors in the study were trained in the identification of appropriate remediation strategies during the first two weeks of class and prior to the introduction of the experimental group treatments. The training was accomplished in two stages. First, all four instructors and the researcher met as a group to discuss the research objectives and procedures. Second, the researcher met privately with each instructor a few days later to review procedures, discuss the use of the various research instruments, and answer questions.

The profile the Gregorc instrument provided was used as a point of departure for the instructors in the individualization of remediation strategies. Based on the indications the Learning Style Delineator provided concerning a student's preferences for processing cognitive information, the instructor proceeded with one of the four following remedial formats (Appendix E). Concrete-sequential learners received a teacher-led step-by-step review of the principles and procedures relevant to the problems missed on the quiz. In this review, the emphasis was placed on use of concrete examples and an inductive approach. Abstract-sequential learners received a review based on a deductive analysis

of the principles at work in the given problems. Leading questions were used to prompt deductive thinking. For these first two types of individuals, the remediation was concluded in one session with the instructor. Concrete-random learners received a review that provided a basis of several examples and then sent the student off to consider a few leading questions away from the instructor. When these students had synthesized a solution to the problems in question, they returned to the instructor to report their conclusions and continue the remediation. Abstract-random learners received a review that included a discussion of the relevant principles involved and a few leading questions to discuss with others in the class also remediating. Following time for reflection and synthesis, these students returned to complete the remediation.

The remediation strategy relied on the Gregorc instrument to indicate a learning style preference. However, the instructors were not bound to continue with any strategy that the student disliked or found inconvenient. If a student requested a different format for the remediation, the instructor attempted to satisfy the student's present felt need for information and remediation in whatever format seemed most productive.

Once the instructor had decided which remedial format to use with a student, the substance of the remediation was addressed using a set of remedial examples prepared to address all the learning objectives for the course (Appendix D). Each quiz problem was keyed to a specific learning objective in the table of specifications (Appendix B). Thus, when a given problem was missed on a student quiz, circling the problem number on the table of specifications immediately identified which learning objective had to be reviewed and where the appropriate remedial examples could be found in the remediation materials.

5. After the student completed the remediation, the instructor verified that the student had mastered the objectives of the formative task using a written retest. This retest was in the form of a take home quiz to be completed and returned within a few days. A posted answer key served to inform the student of the correct responses to the retest problems. Further assistance was available at the request of the student.

The control group received essentially the same treatment that all students have received in class for several years, that is, written evaluation of quizzes and assistance on homework problems. In

addition, instructors M1 and F2, the split-section instructors, administered the Learning Styles Delineator to their control group students. This data was not used, however, in the treatment the control group students received. In the classroom, when the experimental group students received the weekly formative task, the control group students received an identical quiz or worksheet which was completed within the same time limitations as the task given the experimental group students. This paper was graded by the teacher in the conventional manner and returned to the students the next day at the instructor's office. No systematic diagnosis of student errors or prescribed remediation was offered the control group students. They were free to seek unlimited assistance in the conventional manner. But they did not have access to the materials used by the experimental group.

Content Validity of the Departmental Examinations

During the Spring quarter of 1982, the researcher prepared a list of course objectives for Math 121. This list was sent to the Math 121 instructors in the form of a questionnaire. The purpose of the questionnaire was to identify the course objectives that the instructors were concerned with testing on the final exam. The instructors indicated the objectives that should appear on all Math 121 finals, the objectives that should never be tested, and the objectives which could be included or left out at the discretion of the committee preparing the examination. Additions to the list were

written in by the instructors at the end of the questionnaire.

The results of this questionnaire were used as a guideline for the construction of the final exam for Fall quarter, 1982. Because a calculus examination emphasizes problem solving, and because problem solving is time consuming, it is typically not possible for the final exam to address all the course objectives. Following the procedure just outlined, the final consisted of test items covering all the topics considered necessary and as many of the optional topics as could be included given the time limitations of the exam.

As a result of this item selection process, the examination questions reflected the course objectives and the instructors' priorities. Since the function of the exam in this study was to quantify student response to the treatments under study, a high content validity made the interpretation of any differences in performance between the groups more meaningful. It should be noted here that the formative tests addressed all the course objectives, not just those which appeared on the final.

Three one hour departmental unit exams (Appendix F) were administered Fall quarter in addition to the final exam. The results of the course objectives questionnaire were used in the construction of these exams also.

The selection process for the unit exams allowed for evaluation of course objectives which were not measured on the final exam. Such a procedure broadened the scope of the overall evaluation process. This contributed to the overall content validity of the evaluations.

Examination Reliability

The reliability, or precision of measurement, of an individual's performance level on a test is often quantified by using a reliability coefficient. A perfectly reliable test would have a reliability coefficient of 1.0 and would always indicate the true level of performance of an individual. A totally unreliable test would have a reliability coefficient of 0.0 and would reveal nothing about an individual's true level of performance (Hopkins & Stanley, 1981). Davis (1964, p 24) made the following statement regarding reliability coefficients in studies of group behavior:

For measuring the average characteristics of groups of the size of many classes, say twenty-five to fifty, scores with reliability coefficients as low as .50 may often be highly serviceable.

The same conclusion is found in Kelley's (1927) analysis of the reliability requirements for group studies.

The problem of determining the reliability of scores for college level mathematics problem solving examinations was investigated by Hill (1976). Using equivalent form examinations, Hill found that the reliability coefficients for the three-hour exams in his study lay within the range .55 to .86, that error variance was independent of student ability, and that scorer reliability lay within the range .90 to .95. Thus, most of the unreliability lay with the instrument itself and not with the scoring procedures.

Other studies dealing specifically with student performance in calculus have reported similar reliability coefficients. Pascarella

reported reliabilities of .72 (1977, b) and .75 (1978). Thompson (1980) reported Chronbach reliability estimates of .85 and .80 on departmentally written calculus final exams.

This study used departmentally written examinations as instruments for measuring student achievement. Partial credit was given on test items involving problem solving. In order to estimate reliability coefficients for examinations used during the '81-'82 school year, the researcher examined two MSU departmental exams used during Fall and Spring quarters.

The 30 item final exam used Fall quarter for Math 121 was evaluated on the basis of 20 student papers selected at random from the researcher's 68 students. The examination was worth 200 points, with a sample mean of 142, variance 585, and a standard error of the mean of 5.4. In order to estimate the reliability using the Kuder-Richardson formula #20 (Sax, 1980) the papers, which had been scored giving partial credit on problem solving items were rescored giving either full credit if the item was answered 100% correct or no credit if there were any mistakes in the solution. Based on this procedure, KR-20 yielded a reliability of .69. Next, using the same scoring method, a split-half technique and Spearman-Brown formula (Sax, 1980) yielded a coefficient of .76. In as much as the item discrimination of the procedure thus defined may have been less than that of a consistently administered procedure for assigning partial credit, the reliability of the results obtained under the partial credit format

may have been higher than the coefficients obtained by rescoreing without partial credit. The significant point is that the coefficients obtained were greater than .50 and comparable to results obtained by other researchers already cited.

A sample of 30 student papers from a unit examination was photocopied prior to scoring during Spring quarter, 1982. These duplicate exams were scored twice by the same instructors approximately two weeks apart. Based on the correlation between the two sets of scores, a scorer reliability of .98 was calculated.

From these observations and from considerations of content validity of the departmental examinations, the researcher concluded that departmental examinations used in the Fall of 1982 were sufficiently reliable to use as measures of student performance in groups.

Statistical Hypotheses

Ferguson (1976) states that it is a common convention to use significance levels of .05 or .01 and that an investigator may adopt, perhaps arbitrarily, a particular level of significance. For the purposes of this study, all hypotheses were tested at the .05 level. In some situations a p-value was also reported. The numbering of the hypotheses reflected the numbering of the general questions asked in Chapter 1. For example, all questions and hypotheses dealing with general question number 1 begin with a 1 followed by a dot (.), such as hypothesis 1.2 .

1.1 There was no difference between the experimental and control group expected GPA's.

1.2 There was no difference between the variance of expected GPA scores for the experimental group and the variance of expected GPA scores for the control group.

2.1 There were four instructors, each teaching an experimental group and a control group, forming eight instructor sub-groups. There was no difference in mean expected GPA for any of the eight subgroups.

2.2 There was no difference in overall exam performance means between students assigned to different instructors.

2.3 There was no student aptitude-instructor interaction for overall examination performance.

3.1 On the first examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

3.2 On the second examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

3.3 On the third examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

3.4 On the final examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

3.5 In overall performance (total of all four examination scores), there was no difference in mean performance for the experimental and control groups for students of known ability.

4.1 On the first examination, there was no aptitude-treatment interaction for students of known ability.

4.2 On the second examination, there was no aptitude-treatment interaction for students of known ability.

4.3 On the third examination, there was no aptitude-treatment interaction for students of known ability.

4.4 On the final examination, there was no aptitude-treatment interaction for students of known ability.

4.5 In overall performance, there was no aptitude-treatment interaction for students of known ability.

5.1 On the first examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

5.2 On the second examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

5.3 On the third examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

5.4 On the final examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

5.5 In overall performance, there was no difference in mean performance for the experimental and control groups for students on unknown ability.

6.1 For the low ability group, there was no difference in standard deviations for the experimental and control groups on each of the four examinations.

6.2 For the middle ability group, there was no difference in standard deviations for the experimental and control groups on each of the four examinations.

6.3 For the high ability group, there was no difference in standard deviations for the experimental and control groups on each of the four examinations.

6.4 Neither the standard deviation of the experimental group nor the standard deviation of the control group was consistently less than the other for students of known ability.

6.5 For the unknown ability group, there was no difference in standard deviations for the experimental and control groups on each of the four examinations.

6.6 Neither the standard deviation of the experimental group nor the standard deviation of the control group was consistently less than the other for students of unknown ability.

7.1 There was no relationship between the amount of time spent in remediation and overall examination performance.

8.1 There was no relationship between expected GPA and overall examination performance for the experimental group.

8.2 There was no relationship between expected GPA and overall examination performance for the control group.

8.3 There was no difference between the strengths of the relationships obtained in the tests of hypotheses 8.1 and 8.2 .

9.1 There was no difference in overall performance for students with a sequential learning style and students with a random learning style for students of known ability.

9.2 There was no difference in overall performance for students with a concrete learning style and students with an abstract learning style for students of known ability.

9.3 There was no difference in overall performance for students with a sequential learning style and students with a random learning style for students of unknown ability.

9.4 There was no difference in overall performance for students with a concrete learning style and students with an abstract learning style for students of unknown ability.

Statistical Procedures

All the hypotheses involving crossed variables were analyzed using a 2X3 analysis of variance. Mendenhall and Reinmuth (1982) state that ANOVA is an appropriate technique to identify important independent variables and to determine their interactions and affect on the dependent variable. The MSUSTAT program AVMF was used for

these calculations. This program can be used with unequal numbers of observations per cell. For the students of unknown ability, AVMF was used as a one-way ANOVA with two levels to address the hypotheses. Least significant differences, D, for comparing cell means were calculated using the Q table discussed by Snedecor and Cochran (1980). The formula used (p. 235) was

$$D = Q_{.05} \sqrt{MSE / n_h}$$

where n_h was the harmonic mean

Cell counts are shown in the Tables found in Chapter 4. The cell count associated with each cell mean appears in parenthesis following the cell mean.

The hypotheses dealing with the strength of a relationship were tested by computing the indicated Pearson correlation coefficients using the MSUSTAT program SUMSTAT. Lindner (1979) states that correlation is the most useful measure of relationship between variables. A test of the null hypotheses that $r = 0$ were performed at the .05 level.

Precautions Taken for Accuracy

The original documents from which data was derived were the formative tests, the list of expected GPA scores generated by Montana State University, and the departmental examinations which serve as the dependent variable in this study. Student scores on the departmental examinations were obtained by photocopying each Math 121 instructor's

grade book at the end of the quarter. Following remediation, each instructor retained the top portion of each student's formative test in a file box. The contents of these boxes were collected periodically by the researcher.

All data pertaining to a given student in the study was then put on a computer file and rechecked for accuracy. All statistical hypotheses were tested using the CP6 computer.

Summary

In the Fall of 1982 at Montana State University, 6 sections of freshman calculus, Math 121, were selected at random to participate in this study. Half the students in the selected sections were assigned to the experimental group and half to the control group using a random assignment procedure. The assignment procedure used resulted in each instructor teaching roughly equal numbers of experimental and control group students. This controlled for teacher differences.

Using the expected GPA, a university generated statistic for entering freshmen, as a measure of ability or aptitude for college work, all the entering freshmen in the selected sections were ranked by the researcher on the basis of the expected GPA. The top third of this ranking was labeled the high ability group. The bottom third was labeled the low ability group. The third group was labeled the middle ability group. This information was not used as the basis of or part of any treatment in this study nor was the information made available to the instructors. It was used, however, by the researcher

in analysis and interpretation of the data gathered.

Beginning with the third week of class, the classroom teachers administered formative tasks. The instructors evaluated student performance on the tasks, prescribed remediation, and retested the experimental group students as they ^acome in to pick up their tests. Remediation was individualized to accommodate each student's learning style as determined by the Gregorc (1982) Learning Styles Delineator administered the third week of class. The control group students received the conventional written evaluation of quizzes and assistance on homework, etc. that has been the practice in the past.

At the end of the quarter, the data gathered in the study was entered into a data file on the CP6 computer at Montana State University. An analysis of the data was conducted using the software statistical package MSUSTAT (Lund, 1978). Two way analysis of variance was used to identify significant variables and interactions. Pearson correlations were used to examine the strengths of relationships.

CHAPTER FOUR

RESULTS

Introduction

The data reported in this chapter are arranged into the following categories: effectiveness of sampling procedures, main treatment effects, incomplete data sets, hypotheses suggested by examination of the collected data, and selected descriptive tabular data. Cell counts are shown in parenthesis in the tables.

Effectiveness of Sampling Procedures

An underlying assumption of the experimental design was that the students were randomly assigned to various sections of Math 121 at registration and that the distribution of ability or talent with respect to calculus would be the same from section to section and for the experimental (Expt) and control (Ctrl) groups. In assessing the strength of this assumption, the following questions were answered.

Question 1.1: Was the mean expected GPA (EGPA) the same for the experimental and control groups?

Hypothesis 1.1: There was no difference between the experimental and control group expected GPA.

Test: The mean EGPA for the experimental group was 2.783. The mean EGPA for the control group was 2.692. A t-test failed to

reject hypothesis 1.1 at the .05 level with 110 df. The p-value obtained was .2413.

Result 1.1: There was no difference between the experimental and control group expected GPA.

Question 1.2: Was the variance in EGPA scores the same for the experimental and control groups?

Hypothesis 1.2: There was no difference between the variance of expected GPA scores for the experimental group and the variance of expected GPA scores for the control group.

Test: The experimental group EGPA variance was .1570. The control group EGPA variance was .1741. An F test failed to reject hypothesis 1.2 for $\alpha = .05$. The p-value was .6968.

Result 1.2: There was no difference in the EGPA variances for the experimental and control groups.

Question 2.1: With four instructors (F2, M1, F1, and M2) each teaching an experimental and control group, there were eight treatment-instructor subgroups. Were the mean EGPA's the same for these subgroups?

Hypothesis 2.1: There were four instructors, each teaching an experimental and a control group, forming eight instructor subgroups. There was no difference in mean EGPA for any two of the eight sub-groups.

Table 1 shows the mean EGPA's for the sub-groups.

Table 1

EGPA Means for Instructor Subgroups

	F2	Instructors M1	F1	M2
Expt	2.70 (11)	2.79 (16)	2.82 (27)	2.64 (29)
Ctrl	2.73 (14)	2.66 (17)	2.63 (25)	2.70 (29)

Test: For $\alpha = .05$, the least significant difference for the means shown in Table 1 was .409. No pair of means has a difference as large as .409. Thus, the test failed to reject hypothesis 2.1.

Result 2.1: There was no difference in mean EGPA's among the eight teacher sub-groups.

Question 2.2: The experimental design relied on random assignment of students to sections of Math 121 to distribute ability equally among the sections. Was overall student performance on all four examinations the same between instructors?

Hypothesis 2.2: There was no difference in overall exam performance means between students assigned to different instructors (F2,M1,F1,M2).

Hypothesis 2.3: There was no student aptitude-instructor interaction for overall examination performance.

Overall student performance means are shown by instructor in Table 2. Table 3 shows the results of the ANOVA for the data summarized in Table 2.

Table 2

Overall Student Performance by Ability Level and Instructor

	F2	Instructors M1	F1	M2
L	337.2 (6)	365 (4)	335.3 (11)	331.8 (14)
M	394.4 (9)	371.1 (9)	390.3 (13)	421.6 (8)
H	442.3 (10)	423.1 (7)	465.6 (12)	426.3 (9)

Table 3

ANOVA for Overall Student Performance by Ability Level and Instructor

<u>Source</u>	<u>df</u>	<u>F</u>	<u>p-value</u>
Ability level	2	22.88	.000
Instructor	3	.1425	.9336
Interaction	6	1.195	.3148
Residual	100		

Test: At the .05 level, the ANOVA test failed to reject hypotheses 2.2 and 2.3.

Result 2.2: There was no difference in overall exam performance means between students assigned to different instructors.

Result 2.3: There was no student aptitude-instructor interaction for overall examination performance.

Main Treatment Effects

Every student in this study assigned an EGPA was a first-quarter freshman. The students of unknown ability did not all share this unique entry-level characteristic. Some were second, third, or fourth quarter students. Some had previously failed Math 121 and were retaking the course. Because of the many potential differences between the students of known ability and the students of unknown ability, this study analyzed data from these two groups separately. Separate hypotheses are stated and separated statistical tests carried out. Questions 3.1 - 4.5 dealt with the students of known ability as measured by their EGPA.

Question 3.1: On the first exam, did experimental group students and control group students perform at the same level?

Hypothesis 3.1: On the first examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

Hypothesis 4.1: On the first examination, there was no aptitude-treatment interaction for students of known ability.

Group means for the first exam for students of known ability are shown in Table 4. A summary of the ANOVA for the first exam for students of known ability is shown in Table 5.

Table 4

First Exam Means: Students of Known Ability

	Ability Level		
	L	M	H
Expt	71.94 (16)	76.62 (21)	92.45 (22)
Ctrl	66.26 (19)	75.72 (18)	82.69 (16)

(least significant difference = 3.02)

Table 5

ANOVA for First Exam: Students of Known Ability

<u>Source</u>	<u>df</u>	<u>F</u>	<u>p-value</u>
Expt/Ctrl	1	4.444	.03739
L/M/H	2	17.35	.000
Interaction	2	.9843	.3788
Residual	106		

Test: The ANOVA test rejected hypothesis 3.1 at the .05 level. Hypothesis 4.1 was not rejected.

Result 3.1: On the first examination, the experimental group performed at a higher level than the control group for students of known ability.

Result 4.1: There was no student aptitude-treatment interaction on the first exam for students of known ability.

Question 3.2: On the second exam, did experimental group students and control group students perform at the same level?

Hypothesis 3.2: On the second examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

Hypothesis 4.2: On the second examination, there was no aptitude-treatment interaction for students of known ability.

Group means for the second exam for students of known ability are shown in Table 6. A summary of the ANOVA for the exam for students of known ability is shown in Table 7.

Table 6

Second Exam Means: Students of Known Ability

	Ability Level		
	L	M	H
Expt	75.19 (16)	83.86 (21)	93.73 (22)
Ctrl	72.68 (19)	80.11 (18)	88.06 (16)

(least significant difference = 2.53)

Table 7
ANOVA for Second Exam: Students of Known Ability

<u>Source</u>	<u>df</u>	<u>F</u>	<u>p-value</u>
Expt/Ctrl	1	3.389	.06843
L/M/H	2	20.62	0.000
Interaction	2	.1817	.8354
Residual	106		

Test: This test failed to reject hypothesis 3.2 at the .05 level. The p-value was .06843. Hypothesis 4.2 was retained.

Result 3.2: On the second exam, there was no difference in mean performance for the experimental and control groups for students of known ability.

Result 4.2: On the second exam, there was no aptitude-treatment interaction for students of known ability.

Question 3.3: On the third exam, did experimental group students and control group students perform at the same level?

Hypothesis 3.3: On the third examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

Hypothesis 4.3: On the third examination, there was no aptitude-treatment interaction for students of known ability.

Group means for the third exam for students of known ability are shown in Table 8. A summary of the ANOVA for the exam is shown in Table 9.

Table 8

Third Exam Means: Students of Known Ability

	Ability Level		
	L	M	H
Expt	65.81 (16)	76.48 (21)	88.91 (22)
Ctrl	61.11 (19)	73.94 (18)	80.91 (16)

(least significant difference = 3.46)

Table 9

ANOVA for Third Exam: Students of Known Ability

<u>Source</u>	<u>df</u>	<u>F</u>	<u>p-value</u>
Expt/Ctrl	1	3.138	.07935
L/M/H	2	17.23	0.000
Interaction	2	.3445	.7145
Residual	106		

Test: The test failed to reject hypothesis 3.3 at the .05 level. The p-value was .07935. Hypothesis 4.3 was retained.

Result 3.3: On the third examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

Result 4.3: On the third examination, there was no aptitude-treatment interaction for students of known ability.

Question 3.4: On the final exam, did experimental group students and control group students perform at the same level?

Hypothesis 3.4: On the final examination, there was no difference in mean performance for the experimental and control groups for students of known ability.

Hypothesis 4.4: On the final examination, there was no aptitude-interaction for students of known ability.

Group means for the final exam for students of known ability are shown in Table 10. A summary of the ANOVA for the exam is shown in Table 11.

Table 10

Final Exam Means: Students of Known Ability

	Ability Level		
	L	M	H
Expt	136.8 (16)	163.8 (21)	184.0 (22)
Ctrl	127.3 (19)	154.8 (18)	168.2 (16)

(least significant difference = 6.73)

Table 11

ANOVA for Final Exam: Students of Known Ability

<u>Source</u>	<u>df</u>	<u>F</u>	<u>p-value</u>
Expt/Ctrl	1	3.947	.04955
L/M/H	2	19.81	0.000
Interaction	2	.1440	.8664
Residual	106		

Test: The test rejected hypothesis 3.4 at the .05 level.

Hypothesis 4.4 was retained.

Result 3.4: On the final examination, the experimental group performed at a higher level than the control group for students of known ability.

Result 4.4: On the final examination, there was no aptitude-treatment interaction for students of known ability.

Question 3.5: In overall points earned on all four exams, did experimental group students and control group students perform at the same level?

Hypothesis 3.5: In overall performance (total of all four examinations), there was no difference in mean overall performance for the experimental and control groups for students of known ability.

Hypothesis 4.5: In overall performance, there was no aptitude-treatment interaction for students of known ability.

Group means for overall performance for students of known ability are shown in Table 12. A summary of the ANOVA for overall performance is shown in Table 13.

Table 12

Overall Performance Means: Students of Known Ability

	Ability Level		
	L	M	H
Expt	349.7 (16)	400.7 (21)	458.9 (22)
Ctrl	327.4 (19)	384.6 (18)	419.4 (16)

(least significant difference = 12.77)

Table 13

ANOVA for Overall Performance Means: Students of Known Ability

<u>Source</u>	<u>df</u>	<u>F</u>	<u>p-value</u>
Expt/Ctrl	1	5.714	.0186
L/M/H	2	28.44	0.000
Interaction	2	.4159	.6666
Residual	106		

Test: Hypothesis 3.5 was rejected at the .05 level. The p-value was .0186. Hypothesis 4.5 was retained.

Result 3.5: In overall points earned, the experimental group performed at a higher level than the control group for students of known ability.

Result 4.5: In overall points earned, there was no aptitude-treatment interaction for students of known ability.

Questions 5.1 - 5.5 dealt with the performance of the students of unknown ability. No EGPA was available for the students addressed in the following questions.

Question 5.1: On the first exam, did the experimental and control group students perform at the same level?

Hypothesis 5.1: On the first examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

The group means for the first exam for students of unknown ability are shown in table 14.

Table 14

First Exam Means: Students of Unknown Ability

Expt	73.08	(24)
Ctrl	74.41	(32)

Test: A t-test failed to reject hypothesis 5.1 at the .05 level with 54 df. The p-value was .7422.

Result 5.1: On the first examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

Question 5.2: On the second exam, did the experimental and control groups perform at the same level?

Hypothesis 5.2: On the second examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

Group means for the second examination for students of unknown ability are shown in Table 15.

Table 15

Second Exam Means: Students of Unknown Ability

Expt	85.92 (24)
Ctrl	80.50 (32)

Test: A t-test failed to reject hypothesis 5.2 at the .05 level with 54 df. The p-value was .2604.

Result 5.2: On the second examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

Question 5.3: On the third exam, did the experimental and control groups perform at the same level?

Hypothesis 5.3: On the third examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

Group means for the third examination for students of unknown ability are shown in Table 16.

Table 16

Third Exam Means: Students of Unknown Ability

Expt	71.08 (24)
Ctrl	74.58 (32)

Test: A t-test failed to reject hypothesis 5.3 at the .05 level with 54 df. The p-value was .5383.

Result 5.3: On the third examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

Question 5.4: On the final exam, did the experimental and control groups perform at the same level?

Hypothesis 5.4: On the final examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

Group means for the final exam for students of unknown ability are shown in Table 17.

Table 17

Final Exam Means: Students of Unknown Ability

Expt	156.7 (24)
Ctrl	148.7 (32)

Test: A t-test failed to reject hypothesis 5.4 at the .05 level with 54 df. The p-value was .3548.

Result 5.4: On the final examination, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

Question 5.5: In overall points earned, did the experimental and control groups perform at the same level?

Hypothesis 5.5: In overall performance, there was no difference in mean overall performance for the experimental and control groups for students of unknown ability.

Group means for overall performance for students of unknown ability are shown in Table 18.

Table 18

Overall Student Performance Means: Students of Unknown Ability

Expt	386.8	(24)
Ctrl	378.2	(32)

Test: A t-test failed to reject hypothesis 5.5 at the .05 level with 54 df. The p-value was .6685.

Result 5.5: In overall performance, there was no difference in mean performance for the experimental and control groups for students of unknown ability.

Questions 6.1 - 6.5 dealt with the standard deviations of the mean exam scores for the low, middle, high, and unknown ability groups.

Question 6.1: For the low ability group, was there a difference in standard deviations for the experimental and control groups on the four exams?

Hypothesis 6.1: For the low ability group, there was no difference in standard deviations for the experimental and control groups on each of the four examinations.

The standard deviations for the low group are shown in Table 19.

Table 19

Examination Standard Deviations: Low Ability Group

	Expt (16)	Ctrl (19)	p-value
EXAM 1	12.61	16.65	.2825
EXAM 2	17.05	11.62	.1226
EXAM 3	17.94	20.78	.5698
EXAM 4	34.41	38.22	.6870

Test: An F test was used to compare the variances. This test failed to reject hypothesis 6.1 at the .05 level.

Result 6.1: For the low ability group, there was no difference in standard deviations for the experimental and control groups for the four examinations.

Question 6.2: For the middle ability group, was there a difference in standard deviations for the experimental and control groups on the four exams?

Hypothesis 6.2: For students in the middle ability group, there was no difference in standard deviation for the experimental and control groups on each of the four exams.

The standard deviations for the middle ability group are shown in Table 20.

Table 20

Examination Standard Deviations: Middle Ability Group

	Expt (21)	Ctrl (18)	p-value
EXAM 1	15.39	13.23	.5324
EXAM 2	7.786	11.84	.075
EXAM 3	10.67	13.00	.3962
EXAM 4	23.51	28.35	.4199

Test: An F test was used to compare the variances. This test failed to reject hypothesis 6.2 at the .05 level.

Result 6.2: For the middle ability group, there was no difference in standard deviations for the experimental and control groups on each of the four exams.

Question 6.3: For the students of high ability, was there any difference in standard deviation for the experimental and control groups on the four exams?

Hypothesis 6.3: For the high ability group, there was no difference in standard deviations for the experimental and control groups for each of the four exams.

The standard deviations for the high ability group are shown in Table 21.

Table 21

Examination Standard Deviations: High Ability Group

	Expt (22)	Ctrl (16)	p-value
EXAM 1	7.469	14.73	.00525
EXAM 2	6.504	12.47	.00717
EXAM 3	12.95	16.89	.2575
EXAM 4	20.40	35.62	.01968

Test: An F test was used to compare variances. This test rejected hypothesis 6.3 for every exam except EXAM 3 at the .05 level.

Result 6.3: With the exception of EXAM 3, the standard deviations of the experimental group were less than the standard deviations of the control group on the examinations.

Question 6.4: For the students of known ability, was the experimental group standard deviation the same as the control group standard deviation?

Hypothesis 6.4: Neither the experimental group standard deviation nor the control group standard deviation was consistently less than the other.

The greater and lesser standard deviations for the examinations are shown in Table 22.

Table 22

Signs Comparison of Examination Standard Deviation Magnitude:
Students of Known Ability

Ability Level	L				M				H			
Exam ¹	1	2	3	F	1	2	3	F	1	2	3	F
Expt	-	+	-	-	+	-	-	-	-	-	-	-
Ctrl	+	-	+	+	-	+	+	+	+	+	+	+

Test: For the twelve pairs of data shown, the signs indicate which group had the greater (+) or lesser (-) standard deviation for each exam. The signs test rejected hypothesis 6.4 at the .05 level with a p-value of .039.

Result 6.4: For students of known ability, the standard deviation of the experimental group was consistently less than that of the control group.

Question 6.5: For students of unknown ability, was the standard deviation of the experimental group the same as the standard deviation of the control group?

Hypothesis 6.5: For the unknown ability group, there was no difference in standard deviations for the experimental and control groups on each of the four exams.

The standard deviations for the unknown ability group are shown in Table 23.

Table 23

Examination Standard Deviations: Unknown Ability Group

	Expt (24)	Ctrl (32)	p-value
EXAM 1	13.66	15.62	.5095
EXAM 2	9.155	21.90	.00018
EXAM 3	18.02	22.95	.2342
EXAM 4	28.13	34.38	.3234

Test: An F test was used to compare variances. This test rejected hypothesis 6.5 for EXAM 2 at the .05 level. Hypothesis 6.5 was retained for all the other exams.

Result 6.5: For the unknown ability group, other than EXAM 2, there was no difference in standard deviations for the experimental and control groups.

Question 6.6: For the unknown ability group, was the standard deviation of the experimental group consistently less than that of the control group?

Hypothesis 6.6: Neither the standard deviation of the experimental group nor the standard deviation of the control group was consistently less than the other for students of unknown ability.

The greater and lesser standard deviations for students of unknown ability are shown in Table 24.

Table 24

Signs Comparison of Examination Standard Deviation Magnitude:
Students of Unknown Ability

Exam#	1	2	3	F
Expt	-	-	-	-
Ctrl	+	+	+	+

Test: The signs test failed to reject hypothesis 6.6 at the .05 level. The p-value is .0625 and is the lowest p-value obtainable with only four pairs of data points.

Result 6.6: For the unknown ability group, the standard deviation of the experimental group was consistently less than that of the control group, but not at the .05 level.

Incomplete Data Sets

In the original proposal for this study, the experimental design called for the instructors to keep an accurate record of the amount of time spent remediating each student. The relationship between student performance on the exams and time spent in remediation was to have been examined using correlation. With the exception of one instructor, F2, this data was not gathered as planned. The other three instructors gathered this data occasionally but not in such a manner as to be statistically valuable. Recognizing that the data from instructor F2 is limited to only one split-section and that the

number of students involved is small, the following information is presented.

Question 7.1: Was there any relationship between the amount of time spent in remediation and overall performance?

Hypothesis 7.1: There is no relationship between the amount of time spent in remediation and exam performance for experimental and control group students.

Test: A Pearson correlation coefficient was calculated for both the experimental and control groups. For the experimental group, the value $r = -.09$ was obtained. For the control group, the value $r = .006$ was obtained. Neither of these values is sufficiently large to reject hypothesis 7.1 at the .05 level.

Result 7.1: There was no relationship between the amount of time spent in remediation and overall performance for either the experimental or control groups.

Another proposed hypothesis which was not tested dealt with the relationship of formative test scores to exam performance. Although all the formative quizzes were given in each section, the relative timing between the formative tests and the summative exams differed greatly from section to section and from exam to exam. Because of this inconsistency, the researcher elected not to address this hypothesis in the data analysis.

Another hypothesis not tested dealt with the relationship between the gain in formative test points on retesting and summative exam performance. As with the remediation time hypothesis, the

instructors failed to gather the data as planned, making this test impossible.

Hypotheses Suggested By Examination Of The Collected Data

Question 8.1: For the experimental group, was the EGPA related to overall performance on the exams?

Hypothesis 8.1: There is no relationship between EGPA and overall exam performance for the experimental group.

Test: The Pearson correlation coefficient obtained was $r = .7654$. This rejected hypothesis 8.1 at the .05 level. The p-value was .01.

Result 8.1: For the experimental group, the EGPA was related to overall exam performance.

Question 8.2: For the control group, was there any relationship between EGPA and overall exam performance?

Hypothesis 8.2: There was no relationship between the EGPA and overall exam performance for the control group.

Test: The Pearson correlation coefficient obtained was $r = .4653$. This rejected hypothesis 8.2 at the .05 level. The p-value was .01.

Result 8.2: For the control group, EGPA was related to overall exam performance.

Question 8.3: Were the two correlations between EGPA and overall performance significantly different for the experimental and control groups?

Hypothesis 8.3: There is no difference between the correlations of the experimental and control groups with EGPA.

Test: A Z-transformation was used to compare the two correlations. The Z value obtained was 2.589, which is significant at the .01 level. Thus, hypothesis 8.3 was rejected.

Result 8.3: The correlation between overall performance and EGPA was stronger for the experimental group than for the control group.

Question 9.1: Among students of known ability, was a student's learning style related to his/her overall exam performance?

Hypothesis 9.1: There was no difference in overall performance for students with a sequential style and students with a random learning style for students of known ability.

Hypothesis 9.2: There was no difference in overall performance for students with a concrete learning style and students with an abstract learning style for students of known ability.

Group means for students of known ability are shown in Table 25. A summary of the ANOVA is shown in Table 26.

Table 25

Learning Styles and Overall Performance: Students of Known Ability

	Concrete	Abstract
Sequential	409.5 (31)	413.6 (8)
Random	382.9 (11)	377.3 (14)

Table 26

ANOVA for Learning Styles and Overall Performance: Students of Known Ability

<u>Source</u>	<u>df</u>	<u>F</u>	<u>p-value</u>
Sequential/Random	1	2.738	.1032
Concrete/Abstract	1	.0016	.9684
Interaction	1	.065	.7990
Residual	60		

Test: Neither hypothesis 9.1 nor 9.2 was rejected at the .05 level. There was no interaction.

Result 9.1: Sequential and random learners performed at the same level.

Result 9.2: Concrete and Abstract learners performed at the same level.

Question 9.3: Among students of unknown ability, was a student's learning style related to his/her exam performance?

Hypothesis 9.3: There was no difference in overall performance for students with a sequential learning style and students with a random learning style for students of unknown ability.

Hypothesis 9.4: There was no difference in overall performance for students with a concrete learning style and students with an abstract learning style for students of unknown ability.

Group means for students of unknown ability are shown in Table 27. A summary of the ANOVA is shown in Table 28.

Table 27

Learning Styles and Overall Performance: Students
of Unknown Ability

	Concrete	Abstract
Sequential	399.5 (11)	324.0 (1)
Random	379.3 (3)	377.0 (2)

Table 28

ANOVA for Learning Styles and Overall Performance: Unknown
Ability Students

<u>Source</u>	<u>df</u>	<u>F</u>	<u>p-value</u>
Sequential/Random	1	.0707	.7944
Concrete/Abstract	1	.3992	.5384
Interaction	1	.3528	.5627
Residual	13		

Test: The test failed to reject both hypotheses 9.3 and 9.4 at the .05 level. No interaction was present. The reader is cautioned to note the small cell sizes before accepting the following results.

Result 9.3: Sequential and random learners performed at the same level for students of unknown ability.

Result 9.4: Concrete and Abstract learners performed at the same level for students of unknown ability.

Selected Descriptive Tabular Data

The following tables are presented as incidental information. The majority of students taking Math 121 in the Fall of 1982 at Montana State University were not involved with this study. A few comparisons are therefore offered between the students in the study and those not in the study. Table 29 shows a breakdown of final letter grades. Table 30 shows the dropout rates for the course. Table 31 shows the final percent averages.

Table 29

Grade Distribution On The Final Exam

	Ctrl Group Only	Expt Group Only	Expt & Ctrl Groups	All Other Math 121 Students
A	17.4%	27.7%	22.35%	14.79%
B	32.6%	31.3%	28.82%	32.39%
C	15.1%	18.1%	21.18%	29.81%
D	17.4%	18.1%	17.06%	16.19%
F	17.4%	5%	10.59%	6.81%

Table 30

Dropout Rates

Experimental group students	10.88%
Control group students	12.79%
All other students	10.8%

Table 31

Overall Averages

Experimental group students	80.33%
Control group students	75.28%
All other students	77.53%

Summary

The results of the data analysis and statistical hypothesis testing are as follows:

Hypotheses retained at the .05 level: 1.1, 1.2, 2.1, 2.2, 2.3, 3.2, 3.3, 4.1, 4.2, 4.3, 4.4, 4.5, 5.1, 5.2, 5.3, 5.4, 5.5, 6.1, 6.2, 6.5, 7.1, 9.1, 9.2, 9.3, 9.4

Hypotheses rejected at the .05 level: 3.1, 3.4, 3.5, 6.3, 6.4, 6.6, 8.1, 8.2, 8.3

The following results and p-values were most influential in reaching the conclusions drawn in Chapter 5:

1. Result 1.1 There was no difference in the experimental and control group EGPA's. $p = .2413$

2. Result 1.2 There was no difference in the EGPA variances for the experimental and control groups.

3. Result 3.5 In overall points earned, the experimental group performed at a higher level than the control group for students of known ability. $p = .0186$

4. Result 4.5 In overall points earned, there was no aptitude-treatment interaction for students of known ability.
 $p = .6666$

5. Result 5.5 In overall points earned, there was no difference in mean performance for the experimental and control groups of unknown ability. $p = .6685$

6. Result 6.4 For students of known ability, the standard deviation of the experimental group was consistently less than that of the control group. $p = .039$. This must be seen in the context of the other tests which showed the magnitude of the differences to be non-significant at the .05 level.

7. Result 7.1 There was no relationship between the amount of time spent in remediation and overall performance.

8. Result 8.3 The correlation of EGPA with overall performance for the experimental group was significantly higher than

the correlation of EGPA with overall performance for the control group. $p = .01$

9. Result 9.1 Sequential and random learners performed at the same level for students of known ability . $p = .1032$

10. Result 9.2 Concrete and random learners performed at the same level for students of known ability. $p = .9684$

The interpretation of these results clearly showed that for first quarter freshmen, formative testing, prescribed remediation, and retesting resulted in higher levels of performance than did conventional methods of feedback. Table 12 on page 64 illustrated this fact. Recalling that there were 500 points possible, a difference in points earned of 5 points represented a grade difference of 1%. Thus, the experimental group students of low ability performed at a level approximately 4.5% higher than students of comparable ability in the control group. Similarly, the students in the experimental group of middle ability performed at a level approximately 4% higher than comparable control group students. Finally, the students of high ability in the experimental group performed at a level approximately 8% higher than comparable control group students. The variance scores for the experimental group were systematically less than those of the control group. This is in agreement with the Mastery Learning Paradigm.

The mean performance levels for students who were not entering freshmen were displayed in Table 18. The experimental group students did perform at a slightly higher level, but the difference was not

statistically significant. The reasons for the limited differences observed for this group were not addressed formally in this study other than noting that the students of unknown ability were a much more heterogeneous group in terms of academic background. Variance scores for the experimental group students were consistently less than the scores for the control group students for students of unknown ability.

As reported in result 7.1, the relationship of time spent with an instructor in remediation to performance was so weak that the explanation of the performance differences for the experimental and control groups could not be reduced to simply a case of giving the experimental group a greater quantity of help. The difference in performance rested on a quality of help.

Finally, for entering freshmen, Table 25 indicated that students with a sequential learning style performed between 5% and 7% higher than students with random learning styles, depending on whether they favored concrete or abstract learning styles.

CHAPTER FIVE

CONCLUSIONS

Introduction

The purpose of this study, the sample chosen, the experimental procedures used, and the results obtained are briefly reviewed in this chapter. The conclusions reached are then discussed. Finally, recommendations for further study and action are presented.

Summary of the Study

Bloom's (1976) Mastery Learning paradigm suggested that student performance could be increased by individualizing remediation following large group instruction and formative testing. The model also suggested that as student performance increased, variance scores would decrease. Hassett and McCoy (1979) tested the effect of prescribed remediation following class quizzes in a study of college algebra students. The results of their study showed improved performance and smaller variance scores on the final exam for students receiving the prescribed remediation.

Struik and Flexer (1977) compared performance of students in a self-paced Mastery approach calculus course to performance of students in a traditional lecture based calculus course. Repeated retesting

was a main element in the study. The results showed that the students using the Mastery approach performed at a higher level than the students using the conventional approach.

Semb (1974) investigated the effect of unit length on student performance in a child development course using the Mastery Learning approach. The results of the study showed that the use of short units with a high criterion for mastery produced greater student performance on examinations.

The recognition that individuals process information using a variety of learning styles was used by Gregorc (1982) to develop an instrument suitable for the identification of student learning styles. The intent of the identification of an individual's learning style is to provide the student and educator with data useful in the individualization of instruction. The extent to which a learning process accommodates an individual's learning style may be seen as one measure of the quality of instruction spoken of by Bloom in the Mastery Learning paradigm.

The purpose of this study was to determine the effect of formative testing, prescribed remediation, and retesting on student performance in freshman calculus when remediation is adjusted to accommodate individual learning styles. The setting for the experiment was Montana State University, located in Bozeman, Montana. The experiment took place during Fall Quarter, 1983. Of the 21 sections of Math 121, freshman calculus, offered in the Fall of 1983, six were selected as the sample for this study. At the time of registration,

this involved approximately 200 students.

The study designated the group of students receiving weekly formative testing (Appendix A), diagnostic error analysis (Appendix B), prescribed and individualized remediation (Appendix D), and retesting (Appendix C) the experimental group. Each of these students received an error analysis based on the use of a table of specifications and remediation structured to accommodate his/her individual learning style (Appendix E).

The control group students took the same weekly quizzes as the experimental group. The feedback these students received consisted of written notes on their papers and any verbal comments the instructors cared to make. This has been the conventional form of feedback in math classes for many years. Students were free to ask as many questions as they wished and had essentially unlimited help available in the Math Learning Center.

All students took the same summative examinations (Appendix F). There were three one hour exams and a two hour final exam, making a total of 500 points possible. All examination papers were scored by a committee of instructors.

The six sections selected were taught by four instructors. Two of the instructors, a male and a female, taught two sections each of Math 121. The other two instructors, a male and a female, taught one section each. To control for teacher differences, each instructor randomly assigned half of his/her students to the experimental group and half to the control group. The instructors who taught one section

of Math 121 partitioned the section into two groups. The instructors who taught two sections assigned one section to the experimental group and the other section to the control group. For both instructors, these sections met at consecutive hours of the day.

In addition to the data collected on the examinations, the students' learning style was obtained using a Gregorc Learning Styles Delineator and the expected GPA for incoming students was obtained from the university. This information was used in the statistical analysis to look for any interaction of the treatments with either learning style or ability.

The statistical analysis was carried out using the statistical software package MSUSTAT on the CP6 computer at Montana State University. Use was made of the following procedures: analysis of variance, Pearson correlations and z-transformations, F-tests, and sign tests.

For students of known ability, the main effects test comparing the overall performances of the experimental group and control group rejected the null hypothesis at the .05 level, yielding a p-value of .0186. The null hypothesis testing for interaction between ability and treatment was retained, yielding a p-value of .6666.

For students of unknown ability, students who were not entering freshmen, the null hypothesis comparing experimental and control group overall performance was retained.

The correlations between time spent in remediation and overall performance were nonsignificant. The correlations between expected

GPA and overall performance were significant.

Finally, the null hypotheses dealing with learning styles and performance were retained, although the p-value for the sequential/random learning style trait as a factor in overall performance was approximately .10 .

Conclusions

The researcher has reached the following conclusions based on an analysis of the data collected in this study.

1. The random assignment of students to treatment groups resulted in groups of equal ability.

2. The random assignment of students to Math 121 sections resulted in the four instructors teaching groups of equal ability,

3. For entering freshmen, students receiving formative testing, prescribed remediation and retesting earned more total points on examinations during the course than did the students receiving conventional forms of feedback. Thus, the experimental treatment was a superior method of instruction to the conventional approach for entering freshmen.

4. For entering freshmen, there was no aptitude-treatment interaction. That is, the experimental treatment produced consistent gains in performance over the control group for students of all abilities represented in the study.

5. For students who were not entering freshmen, students receiving formative testing, prescribed remediation, and retesting

performed at the same level as similar students receiving conventional forms of remediation.

6. For entering freshmen, the variance in examination scores for students receiving formative testing, prescribed remediation, and retesting was less than that for students receiving conventional forms of feedback. This is in agreement with the theoretical expectations of the Mastery Learning paradigm.

7. There was no relationship between the amount of time spent in remediation and overall performance

8. The relationship between expected GPA and overall performance was stronger for the experimental group than for the control group. Thus, the experimental treatment was a more effective developer of student talent than the conventional approach.

9. For entering freshmen, students with sequential learning styles performed at a higher level than students with random learning styles. The statistical test addressing this trait was not significant at the .05 level, but the trait itself is not defined as a fixed or invariant behavior. Consequently, this researcher concluded that the observed difference in overall performance between sequential and random style learners was significantly related to this trait.

It was also concluded that the concrete/abstract trait was not a factor related to performance for entering freshmen. Neither the sequential/random trait nor the concrete/abstract trait was related to performance for the non-entering freshmen.

Recommendations for Action

Based on the findings of this study, the researcher recommends that the Department of Mathematical Sciences of Montana State University take the following actions:

1. Make formative tasks, prescribed remediation, and retesting materials available to Math 121 students.
2. Train all Learning Center staff in the use of such materials so that motivated students can monitor their progress in a systematic manner.
3. Train all Math 121 instructors in the use of such materials and encourage their use as an element of the instructional process.

Recommendations for Further Study

The researcher recommends the following additional research:

1. A study should be conducted extending the treatments and hypotheses of this paper to include the second and third quarters of calculus, Math 122 and Math 123. Additional hypotheses addressing the longitudinal aspects of the main treatment effects should be included.
2. A study should be conducted to examine more closely the element of learning styles as a factor in achievement in calculus. Interaction of student learning style with instructor teaching style might be considered.
3. A study should be conducted to identify the significant differences in characteristics of the entering freshmen enrolled in

Math 121 and those labeled "unknown ability" in this study. An attempt should be made to learn why the experimental treatment was not effective in raising the performance of this "unknown" group.

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LITERATURE CITED

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APPENDICES

APPENDIX A

FORMATIVE QUIZZES

M121 Quiz #1

2.1-2.5

NAME _____

SEC # _____

DATE _____

For office use only:

Time: start _____ stop _____

Total score _____

Circle T or F

1. (5) T F All polynomials are continuous on $[0,1]$.
2. (5) T F If $f(x)=x^2-4$ is defined on $[-2,2]$, then $f'(x)=0$ somewhere in the interval.

Multiple Choice

3. (10) a b c d The $\lim_{x \rightarrow 5} \frac{x^2-25}{x-5}$ is
- a) 0
b) 5
c) 10
d) ∞
4. (10) a b c d $f(x) = 2x/(x^2-1)$ is continuous
- a) at 0,1,-1
b) at 1,-1
c) for all reals except 1,-1
d) for all reals except 0,1,-1
5. (10) a b c d The $\lim_{x \rightarrow 4} x^2-9x$ is
- a) -20
b) -1
c) -28
d) 0

Short Answer

6. (10) State the definition of a limit.

7. (10) State the three conditions for continuity at a point.

Problems: Show your work

8. (10) Show that $\lim_{x \rightarrow 1} 7x+3 = 10$

9. (10) Show that $\lim_{x \rightarrow 8} \frac{|x+8|}{x+8}$ does not exist.

10. (10) Find $\lim_{x \rightarrow 2^+} \frac{|x+2|}{x+2}$

11. (10) Sketch the graph of $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ x & \text{if } 0 < x < 3 \\ -1 & \text{if } x > 3 \end{cases}$

M121 Quiz #2

3.1 - 3.3

NAME _____ SEC # _____

DATE _____

For office use only:

Time: start _____ stop _____

Total score _____

Circle T or F

1. (6) T F If a function f is continuous at a point a , then it is differentiable at a .
2. (6) T F $D_x y = dy/dx$
3. (6) T F $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x+a) - f(x)}{x-a}$

Short Answer

4. (8) State the definition of the derivative.
5. (8) State the power rule for finding the derivative of $f(x)=x^n$.
6. (8) State the product rule for a function with two factors, f and g .
- $(f g)' =$
7. (8) State the quotient rule for a function. $(f/g)' =$

Problems

8. (10) A stone is dropped from an altitude of 1600 ft. The stone's position above the ground is given by

$$P(t) = 1600 - 16t^2$$

How long does the stone fall and with what speed does it strike the ground?

9. (10) Using the definition of the derivative, find $f'(x)$ if $f(x) = \sqrt{x}$

10. (10) $f(x) = 6x^3 - 2x^2 + 1$ Find $f'(x)$

11. (10) $f(x) = x/(x-2)$ Find $f'(x)$

12. (10) $f(x) = (x^2 + 1)(\sqrt{x})$ Find $f'(x)$

M121 Quiz #3

3.4 - 3.6

NAME _____ SEC # _____

DATE _____

For office use only:

Time: start _____ stop _____

Total score _____

Circle T or F

1. (10) T F $D_{xy} = D_{yx}$
2. (10) T F $dy = f'(x) dx$
3. (10) T F $x^2 + y^2 = 1$ determines exactly one implicit function.

Short Answer

4. (10) State the chain rule using 2 different notations.
5. (10) $f(x) = x^4 - 10x^2 + x + 2$ Using differentials to approximate the change in $f(x)$ as x changes from 2 to 2.005 you should let

dx=

df=

and set x=

6. (10) What expression should you select for $u(x)$ in differentiating

$$f(x) = \frac{4x + 12}{(x^2 - 2)^3}$$

Problems

7. (10) Find $f'(x)$ if $f(x) = (x^2 + 2x + 1)^3$ [Don't simplify]

8. (10) Estimate the change in volume of a cylinder of radius 4cm and height 5cm if the radius increases to 4.01cm and the height remains constant. Use differentials.

9. (10) Find y' if $y^4 + 3y - 4x^3 = 5x + 1$

10. (10) $xy^2 = 5y$ Find an equation of the tangent to the curve at the point (0,0)

M121 Quiz #4

3.7 - 4.2

NAME _____ SEC # _____

DATE _____

For office use only:

Time: start _____ stop _____

Total score _____

Circle T or F

1. (10) T F The slope of a position-time graph is velocity.
2. (10) T F A function can take on a maximum value exactly once.
3. (10) T F A local extrema is a function's largest or smallest value.
4. (10) T F A critical number for $f(x) = 1/(x-2)$ is 2.

Short Answer

5. (10) Give 3 different notations for the second derivative of $y = f(x)$.
6. (10) $f(x) = |x-3|$ is defined on $[2,4]$. Does the Mean Value Theorem apply? Yes or No and why.

Problems

7. (10) $f(x) = 6x^3 + 2x - 1$ Find the second derivative.
8. (10) $f(x) = 6x^2 + 4$ Find the critical numbers.

9. (10) $f(x) = x^4 - 10x^2 + x + 2$ Use differentials to estimate the change in $f'(x)$ as x changes from 2 to 2.005.

10. (10) Find the critical numbers and sketch the graph of

$$f(x) = x^2 - 36x$$

M121 Quiz #5

4.3 - 4.6

NAME _____ SEC # _____

DATE _____

For office use only:

Time: start _____ stop _____

Total score _____

Circle T or F

1. (10) T F A function is concave down at a point c if $f''(c) < 0$.
2. (10) T F The second derivatives on either side of a point of inflection have different signs.
3. (10) T F At local extrema, $f''(x) = 0$.

Multiple Choice

4. (10) a b c d Horizontal asymptotes may be found by examining a graph's characteristics at
 - a) points of inflection
 - b) relative maxima or minima
 - c) points not in the domain
 - d) infinity
5. (10) a b c d Vertical asymptotes may be found by examining a graph's characteristics at
 - a) points of inflection
 - b) relative maxima or minima
 - c) points not in the domain
 - d) infinity

6. (10) a b c d At a point a in the domain of $f(x)$
 $\lim_{x \rightarrow a} f'(x) = \infty$. At point a there is a(n)
 a) relative maxima
 b) zero of the function
 c) asymptote
 d) vertical tangent

Problems

7. (10) Find $\lim_{x \rightarrow \infty} \frac{2x^2 - 9x + 1}{7x^2 - 3}$

8. (10) $f(x) = 5/(x^2 - 4x + 4)$ Does this function have any vertical asymptotes? If so, find them.

9. (10) Sketch the graph of $f(x) = x^3 + 10x^2 + 25x - 50$

10. (10) A farmer has 900 feet of fence to enclose a rectangular pasture bounded on one side by a stream and on the other three sides by fencing. What dimensions for the pasture will enclose the most area?

M121 Quiz #6

4.7 - 4.8

NAME _____ SEC # _____

DATE _____

For office use only:

Time: start _____ stop _____

Total score _____

Circle T or F

1. (10) T F If
- $g(t)$
- is a position-time function, then

$$\frac{g(t+h) - g(t)}{h} \text{ is an expression for acceleration}$$

2. (10) T F
- $x + y = 4$
- and
- $\frac{dx}{dt} + \frac{dy}{dt} = 0$
- Then
- $\frac{dx}{dt}$
- and
- $\frac{dy}{dt}$
- are called related rates.

Problems

3. (20) The temperature of a reaction chamber is given by

$C(t) = 120t - 6t^2$. Find the average rate of change of $C(t)$ during the time interval $[1,2]$ and the instantaneous rate of change at $t = 2$.

4. (20)
- $P(t) = 8t^2 + 14t + 2$
- is a position-time function. Find expressions for velocity and acceleration.

5. (20) Show that the rate of change of volume of a sphere with respect to its radius is numerically equal to the surface area of the sphere.
6. (20) A metal plate has a square cross section and is 50 times as long on a side as it is thick. As it is heated it expands, lengthening 1% in each direction per minute. Find the rate of change of volume when the side of the square is 10cm.

M121 Quiz # 7

5.1 - 5.4

NAME _____ SEC # _____

DATE _____

For office use only:

Time: start _____ stop _____

Total score _____

Circle T or F

1. (5) T F $A = \lim_{\Delta x \rightarrow 0} \sum f(u_i) \Delta x$

2. (5) T F In a Riemann Sum, the rectangles used to estimate the area under the curve must all be the same width.

3. (5) T F $\int_a^b f(x) dx = \lim_{\|P\| \rightarrow 0} \sum f(w_i) \Delta x_i$

4. (5) T F If f is continuous on $[a, b]$, then f is integrable on $[a, b]$.Short Answer

5. (5) $\sum_{i=1}^n c =$

6. (10) $\int_a^a f(x) dx =$

7. (10) $\int_a^b k dx =$

8. (10) Write in integral form: $\lim_{\|P\| \rightarrow 0} \sum_{i=1}^n (3w_i^2 - 1)^2 \Delta x_i$, $[0, 5]$

Problems

9. (5) $\sum_{i=1}^4 i^2 + 1 =$

10. (10) Compute by graphical interpretation: $\int_{-2}^2 \sqrt{4 - x^2} \, dx$

11. (10) $\int_1^5 3 \, dx =$

12. (10) If $\int_0^1 \sqrt{x} \, dx = 2/3$, find $\int \sqrt{3x} \, dx$

13. (10) $\int_0^4 x^2 \, dx = 64/3$ Find a number z which satisfies
Mean Value Theorem for this
integral.

M121 Quiz #8

5.5 - 5.6

NAME _____ SEC # _____

DATE _____

For office use only:

Time: start _____ stop _____

Total score _____

Circle T or F1. (10) T F If F is any antiderivative of f defined on $[a, b]$,

$$\text{then } \int_a^b f(x) dx = F(a) - F(b)$$

2. (10) T F $\int_a^b kx^r dx = \frac{k}{r+1} [b^{r+1} + a^{r+1}]$ 3. (10) T F $\int 2x^3 dx = x^4/2$ Problems4. (10) $\int_0^1 x^2 dx =$ 5. (10) $\int_0^4 \frac{x+1}{\sqrt{x}} dx$ 6. (10) $\int_{-1}^4 dx =$ 7. (10) $\int_{-2}^3 |x+1| dx =$

8. (10) $\int (2x - 1)^2 dx =$

9. (10) $\int z \sqrt{4 - z^2} dz =$

10. (10) $\int \frac{1}{z^2} \left(\frac{1}{z} + 1 \right)^3 dz =$

APPENDIX B

TABLES OF SPECIFICATION

Quiz #1

Section	Def/Not(1)	Appl(2)	Syn(3)	Abs(4)
2.1.1		3		
2.2.1	6			
2.2.2		8	9	
2.3.1		5		
2.4.2		10		
2.4.3		11		
2.5.1	1,7			
2.5.2		2		
2.5.3		4		

Quiz #2

Section	Def/Not(1)	Appl(2)	Syn(3)	Abs(4)
3.1.1		8		
3.2.1	1,2,3,4			
3.2.2		9		
3.3.1	5,6,7			
3.3.2		10,11,12		

Quiz #3

Section	Def/Not(1)	Appl(2)	Syn(3)	Abs(4)
3.4.1	2			
3.5.1	1,4			
3.5.3	6			
3.5.4			7	
3.5.7	5		8	
3.6.1	3			
3.6.4		9		
3.6.5			10	

Quiz #4

Section	Def/Not(1)	App1(2)	Syn(3)	Abs(4)
3.8.1	5			
3.8.2		7		
3.8.3			9	
3.8.4	1			
4.1.1	2,3,4			
4.1.2		8,10		
4.2.1	6			

Quiz #5

Section	Def/Not(1)	App1(2)	Syn(3)	Abs(4)
4.4.1	1,2,3			
4.4.2		9		
4.5.1	4			
4.5.2	5	8		
4.5.3		7		
4.5.4	6			
4.6.1			10	

Quiz #6

Section	Def/Not (1)	App1(2)	Syn(3)	Abs(4)
4.7.1	1	3		
4.7.2		4		
4.7.3				5
4.8.1	2	6		

Quiz #7

Section	Def/Not(1)	Appl(2)	Syn(3)	Abs(4)
5.1.1		9		
5.1.2	5			
5.1.3	1			
5.2.1	2,3			
5.2.2	4,6			
5.2.3		8		
5.2.4			10	
5.3.1	7			
5.3.2		11,12		
5.4.2		13		

Quiz #8

Section	Def/Not(1)	App1(2)	Syn(3)	Abs(4)
5.5.1	1,2			
5.5.2		4,5,6,7		
5.6.1	3			
5.6.2		8,9,10		

APPENDIX C

RETESTS

Quiz #1

2.1-2.5

NAME _____ SEC _____

1. (10) Show that $\lim_{x \rightarrow 2} |4x+8| = 16$
2. (10) Give the three conditions for continuity at a point.
3. (10) Sketch the graph of $f(x) = \begin{cases} -1 & \text{if } x < 5 \\ 2x & \text{if } 5 \leq x < 8 \\ 3 & \text{if } x \geq 8 \end{cases}$
4. (10) Find $\lim_{x \rightarrow 2} x^3 - 1$
5. (10) Where is $f(x) = \frac{x^2 - 4}{x + 1}$ continuous?
6. (10) Give an example of a function that is continuous over the real numbers.
7. (10) State the Mean Value theorem in your own words.

8. (10) Find $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$.
9. (10) Show that $\lim_{x \rightarrow 0} 1/x$ does not exist.
10. (10) State the definition of a limit.

Quiz #2

3.1 - 3.3

NAME _____ SEC _____

DATE _____

For office use only:

Total score _____ Time: start _____ stop _____

1. (10) Give 2 notations for the first derivative of $f(x)$.
2. (10) State the definition of the derivative.
3. (20) Using the definition of the derivative, find $f'(x) = \sqrt{x}$
4. (20) $f(x) = (\sqrt{x} + 1)(x^2 - 9)$ Find $f'(x)$
5. (20) $f(x) = 2x/(x^2 - 1)$ Find $f'(x)$
6. (20) $f(x) = 3x^5 - 9x^2 + 2x$ Find $f'(x)$

Quiz #3

3.4 - 3.6

NAME _____ SEC # _____

DATE _____

For office use only:

Total score _____ Time: start _____ stop _____

1. (10) State the chain rule using 2 different notations.
2. (15) $f(x) = x^3 + 9x - 1$ Use differentials to estimate the change in $f(x)$ as x changes from 1 to 1.02
3. (15) $f(x) = x^3 / (x^2 + 1)^7$ Find $f'(x)$ [Don't simplify]
4. (20) $y^3 + 3xy = x^2 + 1$ Find y'
5. (20) Find an equation for the tangent to $x^2y = 5x$ at $(0,0)$.

6. (20) Give 2 examples of implicit functions derived from

$$x^2 + y^2 = 25$$

Quiz #4

3.7 - 4.2

NAME _____ SEC. # _____

DATE _____

For office use only:

Total score _____ Time: start _____ stop _____

1. (20) Find the critical numbers for $f(x) = 12x^2 - 6x$.
2. (20) $f(x) = x^3 - 9x^2 + 2x + 5$ Use differentials to estimate the change in $f'(x)$ as x changes from 1 to 1.01.
3. (20) $f(x) = x^2 + 6x - 1$ Does the Mean Value Theorem apply on the interval $[-1, 4]$? Why or why not.
4. (20) State Rolle's Theorem.
5. (20) Find the third derivative of $f(x) = x^{3/5}$.

Quiz #5

4.3 - 4.6

NAME _____ SEC # _____

DATE _____

For office use only:

Total score _____ Time: start _____ stop _____

1. (20) Find $\lim_{x \rightarrow \infty} \frac{6x^2 - 9x}{x^3 + 1}$

2. (20) Sketch the graph of $f(x) = x^3 + 3x^2 + 3x + 10$

3. (20) $f(x) = 3/(x^2 - 9)$ Find the vertical asymptotes.

4. (20) The sum of two numbers is 60. Their product is to be as large as possible. Find the two numbers.

5. (20) $f(x) = (x - 2)^{1/3} + 3$ Show that there is a vertical tangent at $(2, 3)$.

Quiz #6

4.7 - 4.8

NAME _____ SEC # _____

DATE _____

For office use only:

Total score _____ Time: start _____ stop _____

1. (25) Find the average velocity over the interval $(0, 5)$ for an object with a position-time function $P(t) = 6t^2 - 9t + 1$.

Find the object's instantaneous velocity at $t = 2$.

2. (25) $x^2 + y^3 = 1400$ if $\frac{dx}{dt} = -1$
find $\frac{dy}{dt}$ when $x = 20$.

3. (25) A ladder 13 feet long is leaning against a building. The foot of the ladder is 5 feet from the building and is slipping at 1 ft/sec away from the building. How fast is the top of the ladder slipping down the building?

4. (25) Show that the rate of change of volume of a cylinder of constant height but changing radius is directly proportional to the circumference of its base.

Quiz #7

5.1 - 5.4

1. (10) Write in integral form: For the interval $[-1, 4]$,

$$\lim_{||P|| \rightarrow 0} \sum (6w_i^2 - 2w_i + 1)^2 \Delta x_i$$

2. (15) $\sum_{i=2}^7 i^2 + 3 =$

3. (15) $\int_{10}^{10} x^2 - 7 \, dx =$

4. (20) $\int_1^7 5 \, dx =$

5. (20) $\int_1^3 x \, dx =$ [Hint: use a graphical interpretation of $f(x) = x$]

6. (20) Find a number satisfying the Mean Value Theorem for

$$\int_0^4 3x^2 \, dx = 64$$

Quiz #8

5.5 - 5.6

NAME _____ SEC # _____

DATE _____

For office use only:

Total score _____ Time: start _____ stop _____

Evaluate the following integrals:

1. (10) $\int_0^6 x \, dx =$

2. (10) $\int_7^{10} \frac{x^2 + 3}{x} \, dx$

3. (10) $\int_{-5}^6 dx =$

4. (20) $\int_{-2}^1 |x| \, dx =$

5. (20) $\int \frac{(1/x - 3)^2}{x^2} \, dx =$

6. (15) $\int x(5x^2 - 1)^4 \, dx =$

7. (15) $\int x(x^2 + 5)^3 \, dx =$

APPENDIX D

REMEDICATION MATERIALS

Remediation Materials

Each learning objective is numbered by indicating the location of the task in the Swokowski (1979) text used at Montana State University. The numbers are read as follows: first number= chapter, second number= chapter section, third number= used to count objectives within a given section of a chapter. For example, 1.2.3 indicates the learning objective comes from chapter 1, section 2, and that it is the third task in the remediation materials from that section. The level of difficulty (1 - 4) is indicated in parenthesis: (1) = definition or notation, (2) = application, (3) = synthesis, (4) = abstraction.

1.1.1 Evaluate absolute value expressions.

(2) Evaluate $|2| + |4-7| + |-9|$

Ans: 14

(2) Evaluate $|4-6| - |5+7|$

Ans: -10

1.1.2 Solve inequalities.

(2) $5x-6 > 4$ Find the solution set.

Ans: $\{x:x>2\}$

(2) $2x-8 < 8x+10$ Find the solution set.

Ans: $\{x:x>-3\}$

(3) $|2x-4| > 4$ Find the solution set.

Ans: $\{x:0>x>4\}$

- (3) $(x+1)/(x-4) < 0$ Find the solution set.

Ans: $\{x: -1 < x < 4\}$

1.2.1 Find the distance between points in the plane.

- (2) Find the distance between $(-2, 4)$ and $(5, -1)$

Ans: $7\frac{1}{2}$

- (2) Find the midpoint of the segment joining $(3, 5)$ and $(1, 46)$

Ans: $(2, 25.5)$

1.2.2 Determine the equation of a circle in standard form.

- (2) Find the center and radius of the circle given by

$$x^2 + y^2 - 10x + 2y + 22 = 0$$

Ans: center = $(5, -1)$ radius = 2

- (2) Find the equation of the circle with center $(2, 4)$ and radius $r=5$.

Ans: $(x-2)^2 + (y-4)^2 = 25$

1.2.3 Sketch a curve by plotting a table of points.

- (2) Sketch $f(x) = x^2$

1.3.1 Determine the slope of a line.

- (1) The slope of a line is found by taking the coordinates of two points on the line and dividing the change in the _____ coordinates by the change in the _____ coordinates.

Ans: y, x

- (2) Find the slope of the line joining (9,6) and (4, -3).

Ans: $9/5$

- (2) Find the slope of a line perpendicular to the segment joining (1,1) and (5,9).

Ans: $-1/2$

- (2) Find the slope of a line parallel to the line

$$2y-3x=4.$$

Ans: $2/3$

- (3) Show that (2,3), (5,-1), (0,-6), and (-6,2) are vertices of a trapezoid.

Ans: Compute all the slopes. Two will be the same.

1.3.2 Determine the equation of a line.

- (1) Give the general form for the slope-intercept equation of a line.

Ans: $y=mx+b$

- (1) Give the general form for the point-slope equation of a line.

Ans: $y-y_1 = m(x-x_1)$

- (2) Find an equation for the line passing through (1,1) & (3,7)

Ans: $y=3x-2$ or $y-1=3(x-1)$ or $y-3x+2=0$

- (3) Find an equation of the line perpendicular to $2y-4x+8=0$ that passes through the point (1,1).

Ans: $y=2x-1$

1.4.1 Determine the domain and range of a function.

- (2) Find the domain of $f(x) = \sqrt{5x-1}$

Ans: $x \geq 1/5$

- (2) Find the domain of $f(x) = x-1 / (1-x^2)^{1/2}$

Ans: $-1 < x < 1$

- (2) Find the range of $f(x) = 1/x^2$

Ans: all positive real numbers

1.4.2 State functional notation and terminology.

- (1) The x-intercepts of a function are called _____

a) asymptotes

b) zeros

c) range

d) maps

Ans: b

(1) The symbol $[x]$ means_____

- a) absolute value
- b) minimum value
- c) greatest integer
- d) none of the above

Ans: c

(2) T or F $f(x) = x^2$ is one-to-one.

Ans: F

(2) T or F $f(x) = 2x^5$ is even.

Ans: F

1.4.3 Graph linear equations.

(2) Graph $y = (2/3)x + 1$

(2) Graph $y = |x+1| + 1$

(2) Graph $y = x$

(2) Graph $y = 4$

1.5.1 Evaluate combinations and composites of functions.(1) $(f+g)(x)$ equals_____

a) $f(x) + g(x)$

b) $f(x)g(x)$

c) $f[g(x)]$

d) $f+g+x$

Ans: a

(1) $(fg)(x)$ equals_____

a) $f(x) + g(x)$

b) $f[g(x)]$

c) $f+g+x$

d) none of the above

Ans: d

(2) $f(x) = x^2$ $g(x) = x-1$ Find $(f+g)(x)$

Ans: x^2+x-1

Find $(fg)(2)$

Ans: 1

Find $(f-g)(3)$

Ans: 7

Find $\left(\frac{f}{g}\right)(-1)$

Ans: $-1/2$

2.1.1 Find limits by algebraic manipulation and substitution.

(2) Find $\lim_{x \rightarrow 7} \frac{x^2 - 49}{x - 7}$

Ans: 14

(2) Find $\lim_{x \rightarrow -1} \frac{x^3 + 1}{x + 1}$

Ans: 3

(3) Find $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$

2.1.2 Find the slope of a tangent to a curve using limits.

- (2) Find the slope of the tangent to the curve $f(x) = 5x^2$ at the point (1,5).

Ans: 10

2.2.1 State the definition of a limit.

- (1) Write the definition of a limit.

2.2.2 Use the definition of a limit to verify that a limit has a given value or does not exist.

(2) Show that $\lim_{x \rightarrow 3} 5x - 9 = 6$

(3) Show that the limit does not exist: $\lim_{x \rightarrow 5} \left| \frac{x-5}{x-5} \right|$

(3) Show that the limit does not exist: $\lim_{x \rightarrow 0} 1/x^2$

2.3.1 Apply theorems on limits to evaluate expressions.

Find the limits if they exist.

$$(2) \lim_{x \rightarrow 4} x^2 - 2x + 1$$

Ans: 9

$$(2) \lim_{x \rightarrow 3} (x+4)^{1/3}$$

Ans: $7^{1/3}$

$$(2) \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x} - 2}$$

Ans: 4

2.3.2 Use the Sandwich theorem to evaluate an expression.

- (1) If $f(x) \leq h(x) \leq g(x)$ for all x in an open interval containing a , except possibly at a itself, and if

$$\lim_{x \rightarrow a} f(x) = L = \lim_{x \rightarrow a} g(x), \text{ then } \underline{\hspace{2cm}}$$

$$\text{Ans: } \lim_{x \rightarrow a} h(x) = L$$

- (2) Use the Sandwich theorem to prove that $\lim_{x \rightarrow 0} x^\pi = 0$.

2.4.1 Use one-sided limits notation.

(1) $\lim_{x \rightarrow 1^-} -2x^2 + 1$ is a

- a) right hand limit
- b) left hand limit
- c) limit
- d) virtual point

Ans: b

(1) A function is defined on $[-3, -2]$. The limit as x approaches -2 would be written

- a) $\lim_{x \rightarrow 2^-} f(x)$
- b) $\lim_{x \rightarrow -2^+} f(x)$
- c) $\lim_{x \rightarrow -2^-} f(x)$
- d) $\lim_{x \rightarrow 2} -f(x)$

Ans: c

2.4.2 Evaluate one-sided limits.

(2) Find the limit if it exists. $\lim_{x \rightarrow 6^+} \sqrt{x+6} - x$

Ans: 6

(2) Find the limit if it exists. $\lim_{x \rightarrow 8^-} 1/(x-8)$

Ans: DNE

2.4.3 Use limits in curve sketching.

(2) Sketch the graph of $f(x) = \begin{cases} -1 & \text{if } x \leq 0 \\ 2x & \text{if } 0 < x < 3 \\ x^2 & \text{if } 3 \leq x \end{cases}$

Then find $\lim_{x \rightarrow 3^+} f(x)$, $\lim_{x \rightarrow 3^-} f(x)$, and $\lim_{x \rightarrow 3} f(x)$.

2.5.1 State the definition of continuity and recall important facts

- (1) A function is continuous at a point a if three conditions are satisfied. What are they?

Ans: 1. f is defined on an open interval containing a

2. $\lim_{x \rightarrow a} f(x)$ exists

3. $\lim_{x \rightarrow a} f(x) = f(a)$

- (1) A function is continuous on an open interval if f is continuous _____ in the interval.

Ans: at every point

- (1) T or F If $f(x)$ and $g(x)$ are continuous on (a,b) , then the product $f(x)g(x)$ is continuous on (a,b) .

Ans: T

- (1) T or F All polynomials are continuous over the entire set of real numbers.

Ans: T

- (1) T or F A rational function is continuous at every point of its domain.

Ans: T

2.5.2 Verify the Intermediate Value Theorem for a function.

- (1) T or F If a function is continuous on $[a,b]$ and if $f(a) \neq f(b)$, then f must take on every value between $f(a)$ and $f(b)$ in the interval $[a,b]$.

Ans: T

- (2) Verify the intermediate value theorem if $f(x) = x^2 - 1$ on the interval $[0,4]$.

2.5.3 Evaluate the continuity of a function.

- (2) Find all numbers for which $f(x) = 5/(x^2 - x^3)$ is continuous.

Ans. As a rational expression, f is continuous on its domain. Thus it is continuous everywhere except at $x=0$ and $x=1$.

3.1.1 Use derivatives in rectilinear motion problems.

- (1) State the definition of the slope of a tangent line to a graph.

$$\text{Ans: } m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

- (1) State the definition of instantaneous velocity of an object in rectilinear motion.

$$\text{Ans: } v(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

- (2) Find the slope of the tangent to the graph $f(x) = x - x^2$ at the point (1,0).

$$\text{Ans: } -1$$

- (2) A stone is dropped from an altitude of 400 ft. The stone's position above the ground is given by $p(t) = 400 - 16t^2$. How long does it fall before striking the ground and how fast is it falling on impact?

$$\text{Ans: } t = 5 \text{ sec} \quad v = 160 \text{ ft/sec}$$

3.2.1 Recall the definition of the derivative and related facts.

- (1) State the definition of the derivative.

$$\text{Ans: } f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

- (1) State the condition(s) necessary for a function to be differentiable on an open interval.

$$\text{Ans: } \text{must be differentiable at every point in the open interval.}$$

(1) T or F If $\lim_{h \rightarrow 0^+} \frac{f(a+h)-f(a)}{h} = \lim_{h \rightarrow 0^-} \frac{f(a+h)-f(a)}{h}$

Then the derivative at a does not exist.

Ans: T

- (1) T or F If a function f is differentiable at a , then f is continuous at a .

Ans: T

(1) T or F $D_y x = \frac{dy}{dx}$

Ans: F

(1) T or F $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x+a)-f(x)}{x-a}$

Ans: F

3.2.2 Evaluate derivatives by the definition.

(2) $f(x) = \sqrt{x}$ Find $f'(x)$ using the definition.

Ans: $1/(2\sqrt{x})$

3.3.1 State rules for finding derivatives.

- (1) State the power rule for finding the derivative of $f(x) = x^n$

Ans: $f'(x) = nx^{n-1}$

- (1) State the product rule for a function with two factors, f and g .

Ans: $(fg)' = f'g + fg'$

- (1) State the quotient rule for $(f/g)(x)$.

$$\text{Ans: } (f/g)'(x) = \frac{gf' - fg'}{g^2}$$

3.3.2 Evaluate derivatives.

- (2) $f(x) = 2 \cdot x$ Find $f'(x)$

$$\text{Ans: } 2$$

- (2) $f(x) = 3x^{1/2}$ Find $f'(x)$

$$\text{Ans: } 3 / (2 \sqrt{x})$$

- (2) $f(x) = 6x^4 - 3x^3 + x - 1$ Find $f'(x)$

$$\text{Ans: } 24x^3 - 9x^2 + 1$$

- (2) $f(x) = \frac{2x-1}{5x+2}$ Find $f'(x)$

$$\text{Ans: } 9/(5x+2)^2$$

- (2) $f(x) = (3x^2-1)(2x+4)$ Find $f'(x)$

$$\text{Ans: } 18x^2 + 24x - 2$$

- (2) $f(x) = \frac{2}{x} - \frac{1}{x^2} + \frac{3}{x^3}$

$$\text{Ans: } -2x^{-2} + 2x^{-3} - 9x^{-4}$$

- (3) $f(x) = (x+1)(2x^2-1)(x+2)$ Find $f'(x)$ [don't simplify]

$$\text{Ans: } (2x^2-1)(x+2) + (x+1)(4x)(x+2) + (x+1)(2x^2-1)(1/2 x)$$

3.4.1 Use differential notation.

- (1) T or F The differential dx of an independent variable x is given by $dx = \Delta x$.

Ans: T

- (1) T or F The differential dy of the dependent variable y is given by $dy = f'(x) dx$

Ans: F

- (1) T or F If $y=f(x)$ then $y = dy = f'(x)dx$

Ans: F

- (1) T or F average error = $\frac{\text{measured value}}{\text{error on measurement}}$

Ans: F (reversed)

3.4.2 Use differentials to estimate changes in functional values.

- (2) $f(x) = 3x^2 + 2x + 1$ Use differentials to estimate the change in $f(x)$ as x changes from 1 to 1.01 .

Ans: .08

- (2) The sides of a square are measured as 8 cm $\pm .05$ cm. Use differentials to estimate the area of the square.

Ans: $A = 64 \pm .8$ cm

3.5.1 State various forms of the chain rule.

- (1) T or F $D_x y = D_x y \cdot D_x u$

Ans: F

- (1) State the chain rule using 2 different notations.

Ans: $D_x y = D_u y \cdot D_x u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

3.5.2 State the form of the power rule.

- (1) State the power rule using appropriate notation.

Ans: If $f(x) = ax^n$, then $f'(x) = nax^{n-1}$

- (2) T or F : If $f(x) = 6x^3$, then $f'(x) = 9x^2$

Ans: F

3.5.3 When using the chain rule, select appropriate choices for $u(x)$.

- (1) Select one answer as the most appropriate expression for $u(x)$ in differentiating $f(x) = \underline{4x+9}$

$$(x^2-3)^4$$

a) $4x+9$

b) x^2-3

c) $2x$

d) x^2

Ans: b

(1) [Same instructions] $f(x) = (2x-1)^5$

a) $2x$

b) $(2x-1)^5$

c) 2

d) $2x-1$

Ans: d

3.5.4 Identify appropriate uses of the product rule in problems requiring the chain rule.

(1) T or F If $f(x) = (x-1)^2 x^{1/2}$, the product rule $(fg)' = f'g + fg'$ would set $f=x-1$ and $g=x$.

Ans: F

(2) In which problem(s) could you use the product rule in finding $f'(x)$?

a) $f(x) = (x^2+1)^{1/3}$

b) $f(x) = x(x-1)$

c) $f(x) = \frac{x-1}{x+2}$

d) $f(x) = 3$

Ans: b & c

(3) $f(x) = (x^2[x+1]^{1/2})^3$. Set up an expression for $f'(x)$. [Don't simplify]

Ans: $3(x^2[x+1]^{1/2})^2(2x[x+1]^{1/2} + x^2 \cdot 2[x+1]^{-1/2})$

- (4) $f(x) = (x^2-9)^2(3x)^4$ Set up an expression for $f'(x)$. [Don't simplify]

$$\text{Ans: } 2(x^2-9)(2)(3x)^4 + 4(3x)^3(3)(x^2-9)^2$$

3.5.6 Apply the chain rule in the solution of problems involving products and quotients of polynomials.

- (2) $h(x) = (4x-1)^3(2x^2+1)^2$. Find $h'(x)$ [Don't simplify]

$$\text{Ans: } 3(4x-1)^2(4)(2x^2+1)^2 + (4x-1)^3[2(2x^2+1)(4x)]$$

- (3) $g(x) = (3x^2-1)^2(2x+4)$. Find $g'(x)$ [Simplify]

$$\text{Ans: } 90x^4 - 144x^3 - 36x^2 - 48x + 2$$

- (2) $h(x) = (4x-1)/(2x^2+1)^2$. Find $h'(x)$ [Don't simplify]

$$\text{Ans: } 4(2x^2+1)^{-2} + (4x-1)[-2(2x^2+1)^{-3}(4x)]$$

- (3) $m(x) = (x-1)^2/(3x^2+1)$. Find $m'(x)$ [Simplify]

$$\text{Ans: } (12x^2 - 8x - 4)/(3x^2+1)^2$$

3.5.7 Apply the chain rule to problems involving differentials.

- (1) $f(x) = x^4 - 10x^2 + x + 2$. Using differentials to approximate the change in $f(x)$ as x changes from 2 to 2.005, you would use $dx =$ $df =$

and you would set $x =$

$$\text{Ans: } dx = .005, df = 4x^3 - 20x + 1, \text{ and } x = 2$$

- (2) Use differentials to approximate the change in $f(x)$ as x changes from 3 to 2.95 if $f(x) = 2x^3 - x + 1$

Ans: $df = -2.65$

- (3) Find the change in volume of a cylinder of radius 4cm and height 5cm if the radius increases to 4.01cm and the height remains constant.

Ans: $dV = 1.256$

3.6.1 State the definition of implicit functions.

- (1) A function $f(x)$ is defined implicitly by an equation in x and y if and only if substitution of $f(x)$ for _____ leads to a(n) _____.

Ans: y , identity

3.6.3 Identify implicit functions.

- (1) T or F $y^2 + x^2 = 1$ determines exactly one implicit function.

Ans: F

- (2) Circle any equations determining implicit functions.

a) $-4y^2 + 3x^2 = 12$

b) $4x^2 - y = 6$

c) $x^2 + y^2 = 1$

Ans: a, b, c

- (2) Find at least one implicit function determined by the equation $3x-2y+4=2x^2+3y-7x$

Ans: $y = (-2x^2+10x+4)/5$

3.6.4 Use implicit differentiation.

- (2) $y^4+3y-4x^3=5x+1$. Find y'

Ans: $y' = (12x^2+5)/4y^3$

- (2) $(y^2-9)^4=(4x^2+3x-1)^2$. Find y'

Ans: $y' = 2(4x^2+3x-1)(8x+3)/[8y(y^2-9)]$

- (2) $(1/x)+(3/y)=1$ Find y'

Ans: $y' = y^2/-3x^2$

3.6.5 Use implicit differentiation to determine the equation of the tangent to a graph.

- (3) $xy^2=5y$ Find an equation for the tangent to the graph at the point $(0,0)$.

Ans: $y=0$

3.6.6 Use implicit differentiation and differentials to determine approximations to functional values.

- (1) T or F $df=f'(x)dx$

Ans: T

- (2) $x^3+xy+y^4=19$ Use differentials to estimate the change in y as x changes from 1 to 1.01 at the point $(1,2)$.

Ans: $dy = -5/3300$

3.6.7 Use implicit differentiation to find n^{th} order derivatives.

(3) $y^4 + 3y - 4x^3 = 5x + 1$ Find y''

$$\text{Ans: } y'' = \frac{24x - 12y^2(y')^2}{4y^3 + 3}$$

$$\text{where } y' = (12x^2 + 5)/(4y^3 + 3)$$

3.8.1 Notations for the n^{th} order derivatives.

(1) Give 3 notations for the second derivative of $y=f(x)$.

$$\text{Ans: } f''(x)$$

$$\frac{d^2f}{dx^2}$$

$$y''$$

3.8.2 Finding n^{th} order derivatives.

(2) $f(x) = 6x^3 + 2x - 1$ Find the second derivative.

$$\text{Ans: } f''(x) = 36x$$

(2) $f(x) = (x^2 - 1)^2(3x + 2)$ Find the second derivative.

[Don't simplify]

$$\text{Ans: } (12x^2 - 4)(3x + 2) + (4x^3 - 4x)(3) + 6(x^2 - 1)(2x)$$

(2) $f(x) = 2x/(x^2 - 1)$ Find the second derivative.

[Don't simplify]

$$\text{Ans: } [(x^2 - 1)^2(-4x) - (-2x^2 - 2)(2)(x^2 - 1)(2x)] / (x^2 - 1)^4$$

- (2) $f(x)=x^{2/3}$ Find the second derivative.

Ans: $f''(x) = -2x^{-4/3}/9$

3.8.3 Use higher order derivatives to approximate changes in functional values.

- (3) $f(x) = x^4 - 10x^2 + x + 2$ Use differentials to approximate the change in $f'(x)$ as x changes from 2 to 2.005.

Ans: .140

3.8.4 Interpretation of the 2nd and 3rd derivatives of a position-time graph.

- (1) T or F The slope of a position-time graph is acceleration.

Ans: F

- (1) $P(t) = t^2$ is a position-time graph of an object. $P'(t)$ is interpreted as the _____ of the projectile. $P''(t)$ is interpreted as the projectile's _____.

Ans: velocity, acceleration

4.1.1 State definitions and use notation for local extrema.

- (1) T or F A function can take on a maximum value exactly once.

Ans: F

- (1) T or F Another name for absolute maxima and minima is local extrema.

Ans: F

- (1) T or F A local extrema is the largest or smallest functional value on a given interval.

Ans: F

- (1) T or F If a function has a local extremum at a point c , then either $f'(c) = 0$ or $f'(c)$ does not exist.

Ans: T

- (1) T or F A critical number must be in the domain of the function.

Ans: T

4.1.2 Finding critical numbers of functions.

- (2) Find the critical numbers for $f(x) = 6x^2 + 4$

Ans: $x = 0$

- (2) Find the critical numbers for $f(x) = x^2 - 36x$

Ans: $x = 18$

4.2.1 State Rolle's Theorem and the Mean Value Theorem.

- (1) State Rolle's Theorem.

Ans: If a function f is continuous on a closed interval $[a, b]$ and differentiable on (a, b) and $f(a) = f(b)$, then $f'(c) = 0$ for at least one number in the interval (a, b) .

- (2) T or F Rolle's Theorem means that if the endpoints of some portion of a continuous curve have the same functional value, then at some point in the interval the curve has a critical point.

Ans: T

- (1) State the Mean Value Theorem.

Ans: If a function is continuous on a closed interval $[a,b]$ and differentiable on (a,b) then there is a c in (a,b) such that $f(b)-f(a)=f'(c)(b-a)$.

- (2) $f(x) = |x-3|$ is defined on the interval $[2,4]$. Does the mean value theorem apply?

Ans: no, it's not differentiable at $x=3$

4.3.1 State the first derivative test

- (1) What does the first derivative reveal about the trend in functional values over an interval?

Ans: If $f'(x) > 0$ then $f(x)$ is increasing
If $f'(x) < 0$ then $f(x)$ is decreasing

(1) State the first derivative test.

Ans: If c is a critical number of a function f and if f is continuous and differentiable on the interval containing c , then $f(c)$ is a local maximum if f is increasing on the left of c and decreasing on the right, a local minimum if f is decreasing on the left of c and increasing on the right, and c is neither a local maximum or minimum if the function is either increasing or decreasing on both sides of c .

4.3.2 Use the first derivative test.

(2) $f(x) = 6x^2 - 9x + 5$ Sketch the graph.

(2) $f(x) = x^4 - 8x^2 + 1$ Sketch the graph.

4.4.1 State definitions of concavity and inflection points.

(1) T or F A function is concave-up at point c if $f''(c) < 0$.

Ans: F

- (1) T or F At a point of inflection, the second derivative on either side of the point has the same sign.

Ans: F

- (1) T or F At a local maximum, $f''(x) > 0$.

Ans: F

4.4.2 Use the second derivative test.

- (2) $f(x) = x^3 + 10x^2 + 25x - 50$ Sketch the graph.

Ans: $f''(x) = 6x + 20$: $f''(x) > 0$ when $x > -3.33$

$f''(x) < 0$ when $x < -3.33$

- (2) Sketch a graph satisfying all the following criteria:

$f(1)=4$: $f'(x) > 0$ if $x < 1$: $f'(x) < 0$ if $x > 1$: $f''(x) > 0$ for all $x \neq 1$

4.5.1 Identify horizontal asymptotes.

- (1) Horizontal asymptotes may be found by examining a graph's characteristics at

- a) points of inflection
- b) relative maxima or minima
- c) points not in the domain of the function
- d) infinity

Ans: d

- (2) Does $f(x) = (x^2 - 2x)/(4x^2 + 9)$ have any horizontal asymptotes?

If so, find them.

Ans: $f(x) = 1/4$

- (3) Determine the horizontal asymptotes and sketch the graph of $y = 2x/(1+x^2)$

Ans: horizontal asymptote $y = 0$

4.5.2 Identify vertical asymptotes.

- (1) Vertical asymptotes may be found by examining a graph's characteristics at

- a) points of inflection
- b) relative maxima or minima
- c) points not in the domain of the function
- d) infinity

Ans: c

- (2) Does $f(x) = 5/(x^2 - 4x + 4)$ have any vertical asymptotes?

If so, find them.

Ans: $x=2$

- (3) Determine the vertical asymptotes and sketch the graph of $f(x) = 2x^2/(x^2 - x - 2)$

Ans: $x=-1$, $x=2$

4.5.3 Determine the limit of an expression.

- (1) The statements $\lim_{x \rightarrow a} f(x) =$ and $\lim_{x \rightarrow a} g(x) = c$ imply that

$$\lim_{x \rightarrow a} g(x)/f(x) = ?$$

Ans: 0

- (2) Find $\lim_{x \rightarrow \infty} (3x^3 - x^2 + 1)/(6x^3 + x - 7)$

Ans: $1/2$

- (3) Find $\lim_{x \rightarrow \infty} (x^2 - 1)^{1/2 - x}$

Ans: 0

4.5.4 Identify vertical tangent lines.

- (1) At a point 'a' in the domain of $f(x)$, $\lim_{x \rightarrow a} f'(x) = \infty$

At point a there is a(n)

- a) relative maximum
- b) zero of the function
- c) asymptote
- d) vertical tangent line

Ans: d

- (2) Find the vertical tangent points, if any exist, for

$$f(x) = (x-5)^{2/3} - 4$$

Ans: $x=5$ or $(5, -4)$

4.5.5 Curve sketching involving both vertical and horizontal asymptotes.

- (2) $f(x) = [1/(x+3)^2] + 3$ Sketch the graph.

Ans:

- (3) $f(x) = 1/x(x-2)^2$ Sketch the graph.

4.6.1 Applications of the derivative to word problems.

- (3) A farmer has 900 feet of fence to enclose a rectangular pasture bounded on one side by a stream and on the other 3 sides by fencing. What dimensions for the pasture will enclose the most area?

Ans: dimensions are x by $900-2x$

$$\text{Area} = x(900-x)$$

$$A' = 900 - 4x = 0 \quad \text{so } x = 225 \text{ and length} = 450$$

- (3) The sum of two numbers is 40. Their product is to be maximized. How will you represent the two factors? What expression will you maximize? Find the two numbers.

Ans: x , $40-x$

$$P = x(40-x)$$

$$x = 20 \quad 40-x = 20$$

- (3) The product of two positive numbers is 81. Their sum is to be minimized. Find the two numbers.

Ans: $x=9$ $81/x = 9$

- (3) On the sale of a single radio, a company makes a profit of 25 dollars. If more than one radio is bought by a customer, the cost per radio is reduced by an amount equal to 25 cents for each radio purchased up to 60 radios. What size sale results in the greatest profit for the company.

Ans: 50 radios

4.7.1 Determine average and instantaneous rates of change.

- (1) Let $g(t)$ be the position function for a time interval $[t, t+h]$. Then $\frac{g(t+h)-g(t)}{h}$ is an expression for

- a) instantaneous rate of change of position
- b) average rate of change of position
- c) acceleration
- d) none of the above

Ans: b

- (2) The temperature in degrees centegrade, C , is given by $C(t) = 120t - 6t^2$. Find the average rate of change of $C(t)$ during the interval $(2, 3)$ and the instantaneous rate of change at $t=3$.

Ans: 90, 84

4.7.2 Finding acceleration and velocity as derivatives.

- (1) $f(t) = 16t^2 + 4t - 1$ is a position-time function. Find expressions for velocity and acceleration.

Ans: $V = 32t + 4$ $A = 32$

- (2) A projectile fired straight up with velocity 200ft/sec has a position given by $s(t) = -16t^2 + 200t$. Find the object's velocity and acceleration at $t=4$ sec.

Ans: $V = 72$ $A = -32$

4.7.3 Use the derivative as a rate of change in the solution of word problems.

- (4) Show that the rate of change of a sphere with respect to its radius is numerically equal to the surface area of a sphere.

Ans: take the derivative of the volume formula with respect to r .

4.8.1 Using related rates.

- (2) A metal plate has a square cross section and is 50 times as long on a side as it is thick. It expands as it is heated, lengthening in each direction 1% per min. You wish to find the rate of change of volume when the side of the square is 10 cm in length. What is dV/dt ?

$$\text{Ans: } ds/dt = .01(10) = .1 \quad V = s^3/50$$

$$dV/dt = (3/50)s^2 ds/dt = .6 \text{ cm}^3/\text{min}$$

- (3) A ladder 13 feet long is resting against a building. The foot of the ladder is 5 feet from the building and is slipping at 1 ft/sec away from the building. How fast is the top of the ladder slipping down the building?

$$\text{Ans: } -5/24 \text{ ft/sec}$$

5.1.1 Use summation notation correctly.

(1) $\sum_{i=1}^n a_n$ The letter i is called the

a) index of summation

b) summation variable

c) notation indicator

d) sigma

Ans: a

(2) $\sum_{i=1}^4 i^2 + 1 =$

Ans: $2 + 5 + 10 + 17$

5.1.2 Recall algebraic facts for summations

(1) $\sum_{i=1}^n c =$

Ans: nc

(1) $\sum_{i=1}^n (a_i + b_i) =$

Ans: $\sum_{i=1}^n a_i + \sum_{i=1}^n b_i$

$$(1) \quad \sum_{i=1}^n c a_i =$$

$$\text{Ans:} \quad c \sum_{i=1}^n a_i$$

$$(1) \quad \sum_{i=1}^n (a_i - b_i) =$$

$$\text{Ans:} \quad \sum_{i=1}^n a_i - \sum_{i=1}^n b_i$$

$$(1) \quad \sum_{i=1}^n i =$$

$$\text{Ans:} \quad n(n+1)/2$$

$$(1) \quad \sum_{i=1}^n i^2 =$$

$$\text{Ans:} \quad n(n+1)(2n+1)/6$$

$$(1) \quad \sum_{i=1}^n i^3 =$$

$$\text{Ans:} \quad (n(n+1)/2)^2$$

5.1.3 Using a limit of rectangular polygons, find the area under a curve.

$$(1) \quad T \quad F \quad A = \lim_{\Delta x \rightarrow 0} \sum_{i=1}^n f(u_i) \Delta x$$

$$\text{Ans: } T$$

- (3) Find the area under the curve $f(x) = 4 - x^2$ on the interval $[0, 2]$ by taking the limit of a sum of rectangles.

Ans: $16/3$

5.2.1 Use integral notation correctly.

- (1) The largest subdivision of a partition of a closed interval is called the _____.

Ans: norm

- (1) T F In a Riemann Sum, the rectangles used to estimate the area under the curve must all be the same width.

Ans: F

- (1) T F $\int_a^b f(x) dx = \lim_{||P|| \rightarrow 0} \sum f(w_i) \Delta x_i$

Ans: T

- (1) For $\int_a^b f(x) dx$, the symbol b is called the _____.

Ans: upper limit of integration

5.2.2 Recall facts about integral forms.

- (1) T F If $c > d$, then $\int_c^d f(x) dx = \int_d^c f(x) dx$

Ans: F

$$(1) \quad \int_a^a f(x) dx =$$

Ans: 0

(1) T F If f is continuous on $[a,b]$ then f is integrable on $[a,b]$.

Ans: T

5.2.3 Given an area expressed in limit form, write an equivalent integral form.

$$(2) \quad \lim_{\|P\| \rightarrow 0} \sum_{i=1}^n (2w_i^2 - 5w_i) \Delta x_i, \quad [0,4]$$

$$\text{Ans: } \int_0^4 2x^2 - 5x \, dx$$

$$(2) \quad \lim_{\|P\| \rightarrow 0} \sum (w_i^3 - 1)^2 \Delta x_i, \quad [-1,1]$$

$$\text{Ans: } \int_{-1}^1 (x^3 - 1)^2 dx$$

5.2.4 Compute integrals by graphical interpretation.

$$(2) \quad \int_0^4 x dx =$$

Ans: 8

$$(2) \quad \int_2^2 x^3 - 1 \, dx =$$

Ans: 0

$$(2) \quad \int \sqrt{1 - x^2} \, dx =$$

Ans: $3.14/2$

5.3.1 Recall properties of the definite integral.

$$(1) \quad \int_a^b k \, dx =$$

Ans: $k(b - a)$

$$(1) \quad \int_a^b k f(x) \, dx =$$

Ans: $k \int_a^b f(x) \, dx$

$$(1) \quad \int_a^b f(x) + g(x) \, dx =$$

Ans: $\int_a^b f(x) \, dx + \int_a^b g(x) \, dx$

$$(1) \quad \int_a^c f(x) \, dx + \int_c^b f(x) \, dx =$$

Ans: $\int_a^b f(x) \, dx$

$$(1) \quad \text{If } f(x) > g(x) \text{ for all } x \text{ in } [a, b], \text{ then } \int_a^b f(x) \, dx >$$

Ans: $\int_a^b g(x) \, dx$

5.3.2 Use these properties to evaluate integrals.

(2) $\int_{-1}^3 7dx =$

Ans: 28

(2) If $\int_1^4 \sqrt{x} \, dx = 14/3$ and $\int_1^4 x \, dx = 15/2$

then $\int_1^4 \sqrt{2x} + 3x \, dx =$

Ans: $14\sqrt{2}/3 + 45/2$

(2) If $\int_2^5 f(x) \, dx = 7$, then $\int_{-3}^5 2f(x) \, dx + \int_2^{-3} 2f(x) \, dx =$

Ans: 14

5.4.1 Recall the Mean Value Theorem for definite integrals.

(1) If f is continuous on $[a, b]$ then there is a number z in (a, b) such that $\int_a^b f(x) \, dx =$

Ans: $f(z)(b - a)$

5.4.2 Demonstrate the Mean Value Theorem.

(2) $\int_0^4 x^2 \, dx = 64/3$. Find a number which satisfies

the Mean Value Theorem for this integral.

Ans: $4/\sqrt{3}$

5.5.1 Recall the Fundamental Theorem of Calculus.

- (1) T F If
- F
- is any antiderivative of
- f
- , then

$$\int_a^b f(x) dx = F(a) - F(b)$$

Ans: F

$$(1) \quad T \quad F \quad \int_a^b kx^r dx = \frac{k}{r+1} [b^{r+1} - a^{r+1}]$$

Ans: T

5.5.2 Evaluate integrals using the power rule.

$$(2) \quad \int_0^1 x^2 dx =$$

Ans: 1/3

$$(2) \quad \int_0^4 \frac{x+1}{\sqrt{x}} dx =$$

Ans: 28/3

$$(2) \quad \int_{-1}^4 dx =$$

Ans: 5

$$(2) \quad \int_{-2}^3 |x+1| dx =$$

Ans: 17/2

5.6.1 Use notation for indefinite integrals correctly.

(1) T F $\int 2x^3 dx = x^4/2$

Ans: F

(1) In $\int f(x)dx = F(x) + C$, C is called the _____.

Ans: constant of integration

5.6.2 Evaluate integrals using a change of variable.

(2) $\int (2x - 1)^2 dx =$

Ans: $(1/6)(2x - 1)^3 + C$

(2) $\int z \sqrt{4 - z^2} dz =$

Ans: $(-1/3)(4 - z^2)^{3/2} + C$

APPENDIX E
REMEDIATION STRATEGIES

CS: Concrete-Sequential learners are logical, inductive, structured, patient, and they like lots of examples. Lead this learner through a series of examples which will illustrate the principle or process in question. You talk. Let the student do the writing and drawing, it's an important feature of his learning process.

AS: Abstract-Sequential learners are analytical and like broad principles, prefer deductive processes to inductive processes, and will accept your "expert" advice without much question. Relate the problem in question to general principles. Ask leading questions that point in the direction of the solution. You do the writing. Get the student to answer questions. The object is to use deductive reasoning, which these students prefer over inductive processes.

AR: Abstract-Random learners are emotional, reflective (not apt to accept your advice without thinking about it for a while), and activity-oriented (they work better in groups than alone). Ask this person a few leading questions that point the way to the solution. Then send him/her off to think about it and discuss the problem with other students. Have him/her report back to you for re-evaluation.

CR: Concrete-Random learners are independent, intuitive (check for unsupported conclusions), and like to work alone. Do several examples and ask leading questions, then send the student off to work for a while on his own. Recheck his thinking when he returns.

APPENDIX F
EXAMINATIONS

MATH 121

Exam 1

Name.....

Part I. Multiple Choice. Circle the appropriate answer.

1. All real numbers that satisfy $|2-3x| < 8$ are

$$\left[-\infty, -2\right] \cup \left[\frac{10}{3}, \infty\right] \quad \left(-2, \frac{10}{3}\right) \quad \left[-2, \frac{10}{3}\right)$$

$$\left(-2, \frac{10}{3}\right] \quad \text{None of these}$$

2. A line with equation $4x - 6y + 2 = 0$

a) has slope

$$4, 6, \frac{2}{3}, -\frac{3}{2}, \text{None of these}$$

b) has y-intercept

$$\frac{1}{3}, 3, 4, \frac{1}{4}, \text{None of these}$$

3. Suppose points A and B have coordinates A(-5,1) and B(0,-8).

Then

a) the midpoint of the line segment AB has coordinates

$$(1,3), \left(\frac{1}{2}, \frac{3}{2}\right), \left(2, \frac{2}{3}\right), \left(\frac{3}{2}, \frac{1}{2}\right), \text{None of these}$$

b) the length of the line segment AB is

$$15, \sqrt{175}, 13.5, \sqrt{178}, \text{None of these}$$

c) the line through the points A and B has slope

$$\frac{9}{5}, \frac{7}{5}, \frac{-7}{5}, \frac{-9}{5}, \text{ None of these}$$

4. The equation of the line passing through $P(6, -5)$ and perpendicular to the line with equation $4x - 6y + 2 = 0$ is

$$3x + 2y - 8 = 0 : 2x + 3y - 8 = 0 : -3x + 2y + 8 = 0 :$$

$$3x - 2y + 8 = 0 : \text{ None of these}$$

Part II. Short answer questions and problems. Simplify each answer as much as possible.

5. If $f(x) = \frac{1}{1+x^2}$ and $g(x) = 3x + 5$, find

a) $(f + g)(x) =$

b) $(f \cdot g)(x) =$

c) $(f/g)(x) =$

d) the domain of f is

e) the range of g is

6. Find the largest subset of $\mathbb{R}$ that can serve as the domain of the function $f(x) = \frac{\sqrt{x-4}}{\sqrt{9-x}}$

7. Suppose $f(x) =$

$$\begin{cases} -1 & \text{if } x < -1 \\ 2x + 1 & \text{if } -1 < x < 1 \\ -3 & \text{if } x = 1 \\ 3 & \text{if } 1 < x \end{cases}$$

a) Graph this function

b) Find $\lim_{x \rightarrow 1^+} f(x)$ if it exists. If not, tell why not.

c) Find $\lim_{x \rightarrow -1} f(x)$ if it exists. If not, tell why not.

d) Is f continuous at $x = -1$? Why or why not?

e) Is f continuous at $x = 1$? Why or why not?

8. By definition, a function f is said to be continuous at the point a if(complete the definition)

9. Using an intuitive approach, find the limit, if it exists, of

a) $\lim_{t \rightarrow 1} \frac{(1/t) - 1}{t - 1} =$

b) $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} =$

c) $\lim_{x \rightarrow 0} |x \sin \{1/(3\sqrt{x})\}| =$

10. Using the epsilon-delta definition of a limit, prove that

$$\lim_{x \rightarrow 3} (2 - 3x) = -7$$

MATH 121

Exam 2

Name.....

In problems 1-4, do not simplify your answers.

1. If $g(x) = 3x^4 + \frac{1}{3x^3} + \sqrt{x} + 4\pi^2$, find $g'(x)$

2. Find $f'(t)$ when $f(t) = (t + 3)(t - 4)^3$

3. For $y = \frac{x^2 + 1}{2x - 1}$, what is y' ?

4. Let $f(x) = [(x + 1)^3 + x^2]^2$. Find $f'(x)$.

5. Given the following equation $5x^2 + 2x^2y + y^2 = 8$, find

a) y' in terms of the variables x and y .

b) an equation of the tangent line to the graph at $P(1,1)$.

6. Find all of the critical numbers for the function $f(x)$ given by

$$f(x) = x^{4/3} + 4x^{1/3}$$

7. The radius of a spherical balloon is estimated to be 10 inches with a maximum error in measurement of 0.1 inches. Use differentials to estimate the maximum error in the calculated volume of the sphere.

8. The domain of the function $f(x) = (x - 1)^3$ is the set of all real numbers. For this function, $f'(x) = 3(x - 1)^2$.

a) Determine on which intervals $f(x)$ is increasing and on which intervals $f(x)$ is decreasing.

b) Identify all local maxima and all local minima of $f(x)$.

9. Find both the absolute minimum and the absolute maximum value of the function $f(x) = -x^3 + 3x - 4$ on the interval $\left[-3, \frac{4}{3}\right]$

10. In a) and b) below, use the definition of the derivative of $f(x)$ to show that the given $f'(x)$ is the derivative of $f(x)$.

a) $f(x) = x^2 - 1$ $f'(x) = 2x$

b) $f(x) = x^2 + \sqrt{x}$ $f'(x) = 2x + \frac{1}{2\sqrt{x}}$ assume $x > 0$

11. State the Mean Value Theorem.

MATH 121

Exam 3

Name.....

SHOW ALL WORK

1. Let $f(x) = 2x^3 + 9x^2 - 24x + 1$. Find the coordinates of the local maximum and minimum points of f . Describe the intervals in which f is increasing or decreasing.

2. Given $f''(x) = 6x + 2$, $f'(0) = 1$, and $f(0) = 2$, find $f(x)$.

3. A ball is thrown upward from ground level and has position equation $s(t) = -16t^2 + 144t$ (in feet).

Find a) the acceleration when $t = 3$.

b) the maximum height of the ball.

4. Given the following information, sketch an accurate graph and label all extrema and points of inflection.

$$f(-2) = f(2) = 0$$

$$f(-1) = f(1) = 1$$

$$f(0) = 2$$

$$\lim_{x \rightarrow -3} f(x) = \infty$$

$$\lim_{x \rightarrow 3^-} f(x) = \infty$$

$$f'(-2) = f'(0) = f'(2) = 0 \quad \text{only at these points}$$

$$\lim_{x \rightarrow 3^+} f(x) = -\infty$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$$

$$f''(x) > 0 \text{ on } (-\infty, -3) \cup (-3, -1) \cup (1, 3)$$

$$f''(x) < 0 \text{ on } (-1, 1) \cup (3, \infty)$$

5. Find the area under the graph of $f(x) = x^2$ from 0 to 1 using inscribed rectangles. Recall that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

and that $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

6. If a box with a square base and an open top is to have 4 cubic feet of volume, find the dimensions that require the least material.

7. Point $P(x,y)$ moves on the graph of the equation $y = x^3 + x^2 + 1$, the x -coordinate changing at a rate of 2 units per second. Find the rate at which y is changing when $x = 10$ units.

8. At 1:00 p.m., ship A is 25 miles due north of ship B. If ship A is sailing east at a rate of 16 mi/hr and ship B is sailing north at a rate of 20 mi/hr, find the rate at which the distance between the ships is changing at 1:30 p.m.

MATH 121

Final Exam Name.....

SHOW ALL WORK

1. Line L contains points (1,1) and (2,4). Write an equation of the line M which is perpendicular to line L at the point (0,-2).

2. Write an equation of the line which is tangent to the curve

$$x^2 + xy + 4y^2 = 6 \quad \text{at the point (1,1).}$$

3. Given $F(x) = \frac{-x}{\sqrt{4-x^2}}$, find:

$$\sqrt{4-x^2}$$

a) The largest subset of the real numbers that can serve as the domain of F.

b) $F(F(0))$

c) $F(3)$

Complete the following limits if they exist. If not, write π does not exist.

4. $\lim_{x \rightarrow 2} \sqrt[3]{2x^2 - 9x + 2}$

5. $\lim_{x \rightarrow 1} x(x-1)^{-1/2}$

6. $\lim_{x \rightarrow 4^+} \frac{x-4}{\sqrt{x}-2}$

7. $\lim_{h \rightarrow 0} \frac{\frac{x+h+1}{x+h} - \frac{x+1}{x}}{h}$

8. A piece of wire 80 inches long is to be bent into a rectangular frame. What dimensions should be chosen so that the area of the rectangle enclosed is a maximum ? (Show all work !)

9. Use the following graph to answer these questions. In questions a-d, find each limit if it exists.

a). $\lim_{x \rightarrow -2} f(x) =$

b) $\lim_{x \rightarrow -1^+} f(x) =$

c) $\lim_{x \rightarrow -1^-} f(x) =$

d) $\lim_{x \rightarrow 2^+} f(x) =$

e) Is f continuous at $x = -1$?

f) Is f continuous at $x = 2$?

g) Is f differentiable at $x = 2$?

h) Is f differentiable at $x = -2$?

10. $f(x) = (x^2 - 8)^{-1/3}$ Find $f'(1)$

11. Given $F(x) = x^3 + 8\sqrt{x}$, find $F'''(1)$

12. Suppose $x^2 + 4xy - y^2 = 4$ implicitly defines y as a function of x .

a) Find y' at the point $(1,1)$

b) Find y'' at the point $(1,1)$

13. Given $G(x) = ax^3 + bx + 1$, determine values for a and b such that G has a local maximum of 7 at $x = -1$.

14. Two boats, A and B, start from the same point and move away from each other along paths that are perpendicular. If boat A travels at 6 miles per hour and boat B travels at 8 miles per hour, at what rate are they separating two hours later?

Evaluate the following integrals.

15. $\int (x^2 + 1)^2 dx$

16. $\int (1 - x^{1/4} + x^{-3/4}) dx$

17. $\int_0^2 x^3 \sqrt{x^4 + 9} dx$

18. $\int_1^2 \frac{(s^3 + 2s^2 + 1)}{s^2} ds$

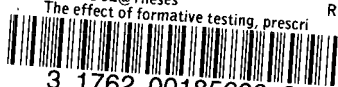
19. $\int \frac{2w + 1}{\sqrt{w^2 + w}} dw$

20. Using the formula $A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(u_i) \Delta x$

with u_i being the left end point of the i^{th} interval, show that the area under the curve $f(x) = 1 - x$ from $x = 0$ to $x = 1$ is exactly $1/2$. You may need these formulas:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

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