Effect of parameter ranges upon choice between centralized and decentralized facilities
by Alfredo Roque Valderrama

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE in Industrial and Management Engineering
Montana State University
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Abstract:
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More emphasis was placed in solving a practical problem than a theoretical one. Therefore, several small businesses -in Montana were analyzed and some of them requested cooperation. At the end of this feasibility analysis, the recapping business was chosen. There are two principal reasons for solving the recapping industry problem. First, the recapping industry could be centralized or decentralized, and second, there is remarkable data and collaboration available from the small shops- in Montana.

It was concluded that the recapping industry having the State of Montana as a market area should be centralized. The Transportation cost and the unit production cost, as a function of plant size, played a great role in the solution of the problem. It also was concluded that when an optimization problem has more than one state parameter and/or more than one control variable at each stage the Dynamic Programming Approach is not practical. Therefore, only the Linear Programming Approach was worked.

The actual problem is a non-linear problem and a method was developed to adapt it to the linear case.

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ABSTRACT

The purpose of this study was to compare the Linear Programming Technique with the Dynamic Programming Technique in solving the Transportation Problem when the constraints and the objective function are non-linear functions.

More emphasis was placed in solving a practical problem than a theoretical one. Therefore, several small businesses in Montana were analyzed and some of them requested cooperation. At the end of this feasibility analysis, the recapping business was chosen. There are two principal reasons for solving the recapping industry problem. First, the recapping industry could be centralized or decentralized, and second, there is remarkable data and collaboration available from the small shops in Montana.

It was concluded that the recapping industry having the State of Montana as a market area should be centralized. The Transportation cost and the unit production cost, as a function of plant size, played a great role in the solution of the problem. It also was concluded that when an optimization problem has more than one state parameter and/or more than one control variable at each stage the Dynamic Programming Approach is not practical. Therefore, only the Linear Programming Approach was worked.

The actual problem is a non-linear problem and a method was developed to adapt it to the linear case.

CHAPTER I - INTRODUCTION

BACKGROUND AND DEVELOPMENT OF THE PROBLEM

The principal reason for the existence of the recapping industry is that the final product, i.e., the recapped (retreaded) tire, is more economical from the overall aspect of initial investment and cost per mile than a new tire (5). This fact has played a major role in the tire industry.

Until 1941 "retread tires" were a relatively insignificant part of the total tire market. Prior to this time, retread tires never achieved more than eight per cent (5) of the total tire market -- even during the sharp recession of the thirties. However, passenger retreads showed a steady growth rate averaging about ten per cent between 1930 and 1940. During World War II the proportion between new and retread tires has undergone a major but temporary change. New tire shipments dropped from 55 million units to 3 million units and retreads increased more than three times their prewar sales volume to about 10 million units per year. By 1944, however, the volume of retread units was only about 30 million per year and new tire volume was 18.5 million units per year.

After 1945, passenger tire retreading began its contemporary decline to what soon (1947) became a level almost in perfect agreement with projections based on a prewar growth trends. After World War II the output of passenger retreads grew steadily at an average rate of about 11 per cent per year until 1959. After 1959, however, the retreading industry began to change. The new tire manufacturers developed and produced the cheapest possible types of tires. These tires were shipped in car load lots to discount houses, department stores, and

chain stores. Early efforts to combat this competition consisted largely of retread price reduction. As profit margins narrowed, however, it was clear to many retreaders and equipment manufacturers that the basic cost of producing a retread also had to be reduced. One way in which to do this was to increase production volume and spread production overhead costs over a larger number of units. The profit reduction in the recapping industry forced the marginal producers out of business and the better "capitalized" retreaders augmented their share of the tire market. Edwards (10) estimated that 1,200 retread shops closed between June 1961 and December 1962 throughout the United States.

The second way to reduce cost was to cheapen the product. This process has been taking place since 1961 through such means as second line retreads, lower grades of tread rubber, and reduced carcass specifications. In addition to this, production methods and equipment have been improved.

As a consequence of these processes the daily production volume per shop is increasing while the production cost per unit is decreasing. At the same time, however, the transportation cost per unit is increasing because the actual retreader is usually unable to retail his entire output through his own store.* In a sense the retreader is assuming the same marketing functions as his major new tire producer competitor, and, as is known, similar marketing functions tend to generate similar expenses.

This paper is not going to deal with the second way of reducing

*See Appendix A

costs, but will attempt to establish a technique by which the optimum combination of plant location, size of the plant, and number of plants for a given market area can be determined to yield a minimum production and distribution cost.

Essentially two methods will be presented and compared in this study. One approach employs the Linear Programming Technique. The other uses the Dynamic Programming Technique. These techniques will be described briefly in Chapters III and IV respectively.

STATEMENT OF THE PROBLEM

In recent years, due to inherent characteristics of the industry, there has been observed a tendency toward establishment of large re-capping plants and an accompanying failure of plants. If this trend continues, there will be fewer plants and the existing ones will be larger with a greater volume output. This, of course, can be expected to increase transportation costs but, at the same time, decrease the unit production costs.

The State of Montana is the market area involved in this study. To facilitate the solution of the problem of cost minimization it can be assumed that the retail prices are essentially the same throughout this market area. The variables will be production and transportation costs, and if the retail selling prices are fixed then the profit will depend only on the production cost and transportation cost: Therefore, if cost is minimized the profits will maximize at the same time. This is the usual economic objective of a private enterprise.

Thus, the object of this study is to optimize the number of plants in Montana as well as the size and location of each plant in a given market area.

To reduce the problem to manageable proportions, several assumptions had to be made, as in any other case where the industrial engineer has to represent the real world by some model. The mathematical model had to be structured so that it would represent reality with a fair degree of approximation, and at the same time be as simple as possible in order to make the analytical approach practicable and economical.

To start with, it is assumed that production costs, fixed and variable, are dependent upon the size of the plant and not upon the relative location. It is also assumed that the transportation cost depends upon distance and not on the relative location of the plant in the state.

Attention is called to the fact that a contemporary retreading plant, in order to have a competitive production cost, must have a minimum production of about 80 tires daily. Montana has an average daily demand of 808 recapped tires.* Since 1955 (8) the number of retreading shops in the state has been about 20. This makes a daily

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* From Table 10, the annual demand for the state of Montana is 209,000 tires per day, and if we estimate the number of working days per year as 260, there are 209,000/260 = 808 tires recapped per day.

average output per shop of 40.0 tires. Most of these plants are not only in the retreading business but also sell batteries and automobile accessories and maintain automobile repair shops. The typical plant is too small to permit the hiring of technical personnel to determine where are the origins of its costs or the source of profit, if any. Therefore, it appears that in the very near future the marginal recapping shops will be forced to become only retailers or distribution centers.

Throughout this study an attempt is made to estimate the future situation of the recapping industry in Montana, given of course that the intrinsic conditions remain the same.

SUMMARY OF PAST WORK

Considerable study was given to the recapping industry in the past. The studies encountered during the research for this paper were concerned with the individual problems mentioned in previous sections and were dealt with as independent inputs. In other words the problems of transportation cost, production cost, marketing costs, etc. have been studied individually without provision for interaction one with another in a mathematical way.

Many studies have been made by companies who are in the tire business. Some of these deal with the production processes (23), some with production costs, (17), and others with the distribution and marketing processes (5). But actually none of these problems are isolated and independent problems of the business. The production costs depend upon the production process; the total cost depends upon

the production and transportation costs.

Facts reveal the necessity of a study that shows the interaction between the production costs and the distribution and transportation costs.

METHOD OF ATTACK

As previously proposed in the second section of this chapter, the object of this study is to attempt to solve production and transportation cost problems, and not to optimize separately (or minimize in this case) first the production cost and then the transportation cost as isolated systems, but to optimize both problems simultaneously by having them interact as sub-systems of the total production - distribution system.

An attempt to minimize the total cost of the production - distribution system by two completely different mathematical techniques was also made.

The first technique used was the Linear Programming approach (22). This method is very useful in optimizing a linear function in "n" variables subjected to "m" linear constraints. Although the actual case is not a linear programming problem, it has been transformed into a linear problem (Chapter III) by making several assumptions in order to facilitate solution. It is clear that any simplification of a mathematical model will be reflected in a loss of accuracy. However, this difficulty can be overcome by the method of solution employed in Chapter III, essentially by dividing the unit production costs-volume curve into straight line segments (see Fig. 5).

The second technique employed the Dynamic Programming approach. This technique was chosen because the transportation problem can usually be stated as a multi-stage problem (see Chapter 4). Another reason for the attempt to employ the Dynamic Programming technique is due to the fact that this method does not require that the constraints and the objective function be linear and, because in many problems of this sort, Dynamic Programming has shown a high computational efficiency. This study also made it possible to arrive at tentative criteria for the efficiency of Dynamic Programming in solving the transportation problem as a non-linear programming problem.

CHAPTER II - DEVELOPMENT OF A REALISTIC FORMULATION OF THE TOTAL COSTS OF PRODUCTION AND PLANT LOCATION

PARAMETERS OF THE PRODUCTION-TRANSPORTATION PROBLEM

In the production-transportation cost problem there are two principal independent variables. One of them is the unit production cost and is a function of the plant size or daily production volume. The other variable is the transportation cost and is a function of the distance. The total production-transportation cost is given by the following mathematical expression:

$$C_{T_{ij}} = C_{P_i} + C_{t_{ij}} \qquad (2\text{-}1)$$

where, $C_{T_{ij}}$ = total cost of allocating one unit from production center i to distribution center j

C_{P_i} = unit production cost for production center i

$C_{t_{ij}}$ = unit transportation cost from production center i to distribution center j

Usually the production centers will have an upper limit of desired production volume that will be denoted by the letter b_j, and at the same time the distribution centers will have a limited volume which can be made available to retailers and will be denoted by the letter a_i.

In the actual case, production-transportation of retreads, the distance between any production center to any distribution center will be fixed by the system of transportation used i.e. railroad, truck etc.

As can be seen, the parameters of the problem will be given by

the size or daily production of plant i, a_i, and by the daily demand of distribution center j, b_j. The variables will be the quantities x_{ij} to be allocated from plant i to distribution center j. The unit transportation cost from plant i to distribution center j will be constant. The restrictions will be given by the range of plant sizes that cannot produce less than 80 tires/day or more than 810 tires/day. Finally the objective function will be given by the following mathematical expression:

$$Z = \min \Sigma (C_{P_i} + C_{t_{ij}})x_{ij} \qquad (2\text{-}2)$$

COST ANALYSIS

Actually there are around thirty kinds of rubber treads for passenger and non-passenger tires. Therefore, it is not practical nor economical for a small business like a retreading shop to keep records of each one of the thirty kinds of retreaded tires produced.

From personal interviews with the manager and foreman of a medium-sized retreading shop* in Bozeman, Montana, along with available business records, the following percentages of units produced in the shop during the first half of the year, 1967, were collected. All different kinds of retreaded tires produced in the shop were summarized into five main general types as given by Table 1. The average list prices, averages selling prices, average production costs, and the average gross profit are given in the same table.**

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* Long's Tire Service, Bozeman, Montana

** Appendix B gives the dealer's data

Estimation of the Average Selling Price per Tire

From the dealer's records the following percentages of dollar sales by type of retreaded tire were obtained:

Highway Passenger Tire = 27.0%, Passenger Snow Tire = 40.0%, Truck Tire = 33.0%.

The truck tires were further divided into three main types:

Highway Truck Tire = 45.0%, Super Cross Rio Hi-Miler = 35.0%, Hi-Miler Xtra Grip = 20.0%.

TABLE 1

Average List Price, Selling Price, Production, Cost, Gross Profit and % of Production Volume per Type of Recap Tire*

COLUMN NUMBER / TYPE OF TIRE	(1) (LIST PRICE) . (%)/100	(2) ** ACTUAL AVRG. SELLING PRICE (20% DISCOUNT)	(3) AVRG. PROD. COST (50% OF THE LIST PRICE)	(4) AVRG. GROSS PROFIT PER TYPE OF TIRE	% OF PRODUCTION VOLUME
HIGHWAY PASSENGER TIRE	14.79	11.82	7.40	4.42	20
PASSENGER SNOWTIRE	16.94	13.55	8.47	5.08	30
HIGHWAY TRUCK TIRE	38.08	26.70	19.04	7.66	22.5
TRUCK TIRE	60.69	42.49	30.35	12.14	17.5
TRUCK TIRE	28.55	20.00	14.27	5.73	10
TOTAL					100%

This table is based on cost analysis approach as illustrated on the next page.

* Source: "TIRE RETREADING AND REPAIRING PRICE LIST." THE GOOD YEAR TIRE COMPANY and also from personal interview with the manager of Long's Tire Service, Bozeman, Montana.

** Truck tires have a 30% discount.

Average Selling Prices then are:

$$\text{Per Passenger Tire} = \frac{11.82 \times 27.0 + 13.55 \times 40.0}{27.0 + 40.0} = \$12.85 \qquad (2\text{-}3)$$

$$\text{Per Non-Passenger Tire} = \frac{26.78 \times 45 + 42.49 \times 35 + 20.00 \times 20}{45 + 35 + 20} = \$30.64 \qquad (2\text{-}4)$$

$$\text{Per Average Tire} = \frac{12.85 \times 140{,}000 + 30.64 \times 69{,}000}{140{,}000 + 69{,}000} = \$18.71 \qquad (2\text{-}5)$$

Estimation of the Average Production Cost per Tire.

$$\text{Per Passenger Tire} = 12.85 \times \frac{50\%}{80\%} = \$8.03 \qquad (2\text{-}6)$$

$$\text{Per Truck Tire} = 30.64 \times \frac{50\%}{70\%} = \$22.02 \qquad (2\text{-}7)$$

$$\text{Per Average Tire} = \frac{8.03 \times 140{,}000 + 22.02 \times 69{,}000}{140{,}000 + 69{,}000} = \$12.90 \qquad (2\text{-}8)$$

The weights of 140,000 and 69,000 for passenger and non-passenger tires were used in the above calculations on the assumption that the characteristics of the demand for passenger and non-passenger tires are the same for any location of Montana. The policy of the dealers is to make a 20% discount on passenger tires and 30% discount on non-passenger tires. The average profit per unit sold is computed as follows:

Avrg. Profit/Pass. Tire = (Avrg. Selling Price/Pass. Tire) - (Avrg. Prod. Cost/ Pass. Tire = 12.85 - 8.03 = \$4.82 (2.9)

Avrg. Profit/Truck Tire = (Avrg. Selling Price/Truck Tire) - (Avrg. Prod. Cost/Truck Tire) = 30.64 - 22.02 = \$10.62 (2-10)

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* The weights, 140,000 and 69,000, given in eq. (2-5) correspond to the magnitude of demand for passenger and truck tires respectively (see Table 8).

Avrg. Profit/Unit Sold = (Avrg. Selling Price/Tire) - (Avrg. Prod. Cost/Tire) = 18.71 - 12.90 = \$5.81 (2-11)

Breakdown of the Production Cost for an 80 Tire/Day Plant Size

From a study made by Goerge R. Edwards (23), the percentage per item of the production cost was taken for an 80 tire/day production plant. Edwards has made this production cost breakdown by item and by type of tire. In order to obtain the average cost per item, the weighted factors concept has been applied.

$$\text{Item Avrg. Cost} = \frac{(C_1 \times \%_1 \times fr_1) + (C_2 \times \%_2 \times fr_2) + (C_3 \times \%_3 \times fr_3)}{fr_1 + fr_2 + fr_3} \quad (2\text{-}12)$$

where:

C_1 = average production cost per highway passenger tire

C_2 = average production cost per passenger snowtire

C_3 = average production cost per truck tire

$\%_1$ = percentage of the production cost per item for highway passenger tire

$\%_2$ = percentage of the production cost per item for passenger snowtire

$\%_3$ = percentage of the production cost per item for truck tire

fr_1 = frequency of sales for highway passenger tire

fr_2 = frequency of sales for passenger snowtire

fr_3 = frequency of sales for truck tire

Then, the "average cost per tire for each item of production" was computed as follows:

Tread Rubber = (7.40 x .54 x 27) + (8.47 x .58 x .40) +
(22.02 x .55 x .33) = $7.08 (2.13)

Labor, = (7.40 x .15 x .27) + (8.47 x 13 x .40) + (22.02 x .19 x
.33) = $2.17 (2-14)

Rent, = (7.40 x .05 x .27) + (8.47 x .05 x .40) + (22.02 x .05
x .33) = $0.64 (2-15)

Depreciation = (7.40 x .05 x .27) + (8.47 x .05 x .40) +
(22.02 x .05 x .33) = $0.64 (2-16)

Supervision = (7.40 x .04 x .27) + (8.47 x .04 x .40) + (22.05
x .04 x .33) = $0.52 (2-17)

Curing Tubes = (7.40 x .03 x .27) + (8.47 x .02 x .40) + (22.02
x .02 x .33) = $0.27 (2-18)

Office = (7.40 x .03 x .23) + (8.47 x .02 x .40) + (22.02
x .01 x .33) = $0.20 (2-19)

Adjustments = (7.40 x .02 x .23) + (8.47 x .02 x .40) + (21.02 x
.02 x 33) = $0.25 (2-20)

Heat, Light, Power = (7.40 x .02 x .23) + (8.47 x .02 x .40) +
(22.02 x .02 x .33) = $0.25 (2-21)

Insurance = (7.40 x .02 x .23) + (8.47 x .02 x .40) + (22.02 x
.02 x .33) = $0.25 (2-22)

Miscellaneous = (7.40 x .05 x .23) + (8.47 x .05 x .40) + (22.02
x .03 x .33) = $0.65 (2-23)

TABLE 2

Average Costs per Item of Production Input

for an 80 Tires Daily Production Plant Size

ITEM	AVRG. PRODUCTION COST/TIRE (DOLLARS)	ANNUAL PRODUCTION %	COST/ITEM DOLLARS
Tread Rubber	$ 7.08	54.60	$141,600
Labor	2.17	16.75	43,400
Rent	.64	4.95	12,800
Depreciation	.64	4.95	12,800
Supervision	.52	4.02	10,400
Curing Tubes	.27	2.08	5,400
Office	.20	1.54	4,000
Adjustments	.25	1.93	5,000
H.L. & P.	.25	1.93	5,000
Insurance	.25	1.93	5,000
Other Items	.65	5.01	13,000
TOTAL	$12.90	100.00	$258,400

Production Costs

During the research stage allotted to this study it was possible to obtain data only for a medium size plant that is 80 tires/day or about "100 units/day".* Therefore, it was necessary to estimate the production costs for the larger size plants having as a starting point the data for the 80 tires/day size plant.

The production costs of the 80 tires/day plant have already been broken down into the following ten main items as shown in Table 2, namely:

Tread Rubber	Office
Labor	Adjustments
Rent	H.L. & P.
Depreciation	Insurance
Supervision	Other Items
Curing Tubes	

From Table 2 similar tables have been derived for larger plant sizes. As this type of industry is not highly integrated, more than 50% of the production cost is due to raw materials and is essentially constant. The remaining 50% of the production cost per unit varies with the production volume. It has been assumed that the unit variable costs for a given plant size would be at 90% of the unit variable costs of the preceding plant size.

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* Unit is the time required to produce one passenger tire. 1 unit = 20 man-minutes. One small truck tire = 2 units, and one truck tire is equivalent to 3 units.

TABLE 3

Estimated Cost per Item of Production Input

for a 300 Tires/Daily Production Plant Size

ITEM	AVRG. PRODUCTION COST PER TIRE	ANNUAL PRODUCTION %	COST/ITEM DOLLARS
Tread Rubber	$ 7.08	57.30	$551,000
Labor	1.95	15.80	154,000
Rent	.58	4.70	45,300
Depreciation	.58	4.70	45,300
Supervision	.47	3.81	36,700
Curing Tubes	.24	1.95	18,700
Office	.18	1.46	14,000
Adjustments	.23	1.86	17,400
H.L. & P.	.23	1.86	17,400
Insurance	.23	1.86	17,400
Other Items	.59	4.78	46,000
TOTAL	$12.36	100.00	$964,000

TABLE 4

Estimated Cost per Item of Production Input

for a 600 Tires/Daily Production Plant Size

ITEM	AVRG. PRODUCTION COST/ TIRES DOLLARS	ANNUAL PRODUCTION %	COST/ITEM DOLLARS
Tread Rubber	$ 7.08	59.60	$1,040,000

continuation of Table 4

ITEM	AVRG. PRODUCTION COST/ TIRES DOLLARS	ANNUAL PRODUCTION %	COST/ITEM DOLLARS
Labor	$ 1.76	14.83	$ 274,000
Rent	.53	4.57	82,500
Depreciation	.53	4.57	82,500
Supervision	.43	3.62	67,000
Curing Tubes	.22	1.85	34,300
Office	.16	1.35	24,950
Adjustments	.21	1.77	32,800
H.L. & P.	.21	1.77	32,800
Insurance	.21	1.77	32,800
Other Items	.54	4.55	84,400
TOTAL	$11.88	100.00	$1,850,050

TABLE 5

Estimated Cost per Item of Production Input

for a 850 Tires/Daily Production Plant Size

ITEM	AVRG. PRODUCTION COST/ TIRE DOLLARS	ANNUAL PRODUCTION %	COST/ITEM DOLLARS
Tread Rubber	$ 7.08	61.90	$1,565,000
Labor	1.59	13.90	307,500
Rent	.48	4.20	106,000

continuation of Table 5

ITEM	AVRG. PRODUCTION COST/ TIRES DOLLARS	ANNUAL PRODUCTION %	COST/ITEM DOLLARS
Depreciation	$ .48	4.20	$ 106,000
Supervision	.39	3.42	75,600
Curing Tubes	.20	1.75	44,200
Office	.14	1.23	39,500
Adjustments	.19	1.66	42,000
H.L. & P.	.19	1.66	42,000
Insurance	.19	1.66	42,000
Other Items	.49	4.29	108,200
TOTAL	$11.42	100.00	$2,525,000

ANALYSIS OF THE MARKET DEMAND

During the research phase of this study, several enterprises* and trade associations** were requested by letter for information or estimates of the characteristics of the tire market of Montana. Unfortunately,

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* Super Mold Corporation, O. K. Rubber Welders, etc.

** Rubber Manufacturers Association, National Tire Dealers and Retreaders Association, etc.

many did not answer and the few who answered did not supply directly applicable information. The NATIONAL TIRE DEALERS AND RETREADERS ASSOCIATION, INC. sent information on the average retread sales per car in 1966 in Montana (0.4918 retreads/car/year), (16). It can be seen from Table 8 that this is in reasonably good agreement with this study's estimate.

Estimation of the Magnitude of Market Demand

From (*) the following table was constructed:

TABLE 6

Passenger and Non-Passenger Tire Production

for Replacement for the U.S. 1964 Domestic Market

PASSENGER TIRE	New	84,000,000
	Retread	35,000,000
NON-PASSENGER TIRE	New	12,000,000
	Retread	7,700,000

Also from (*) the following table was constructed:

TABLE 7

Number of Cars, Trucks and Buses in the U.S. and Montana

Register During the Year 1966

VEHICLE	U.S.	MONTANA
CARS	71,984,000	288,000
BUSES AND TRUCKS	14,325,000	130,000

* STATISTICAL ABSTRACTS OF THE U.S. 1966

It will be assumed that the average number of new and retread replacement tires per car or truck is the same for every part of the U.S. including Montana. The ratios for the U.S. in 1967 will be assumed to be approximately equal to the 1966 ratios as given by Table 8.

TABLE 8

Calculation of the Replacement Ratios

and Magnitude of Demand for Tires in Montana, 1966

TYPE OF TIRE	REPLACEMENT RATIO R_i	MAGNITUDE OF DEMAND FOR MONTANA IN 1966, (TIRES)
New Passenger Tire	$R_1 = \frac{84,000,000 \text{ tires}}{71,984,000 \text{ cars}} = 1.17$	1.17 x 288,000 = 337,000
Retread Passenger	$R_2 = \frac{35,000,000 \text{ tires}}{71,984,000 \text{ cars}} = 0.49$	0.49 x 288,000 = 140,000
New Non-Passenger	$R_3 = \frac{12,000,000 \text{ tires}}{14,325,000 \text{ cars}} = 0.84$	0.84 x 130,000 = 109,000
Retread Non-Passenger	$R_4 = \frac{7,700,000 \text{ tires}}{14,325,000 \text{ cars}} = 0.53$	0.53 x 130,000 = 69,000

where:

$$R_1 = \frac{\text{number of new passenger tires sold in 1966}}{\text{number of cars in Montana in 1966}} \quad (2\text{-}24)$$

$$R_2 = \frac{\text{number of new non-passenger tires sold in 1966}}{\text{number of buses and trucks in Montana in 1966}} \quad (2\text{-}25)$$

$$R_3 = \frac{\text{number of retreaded passenger tires sold in 1966}}{\text{number of cars in Montana in 1966}} \quad (2\text{-}26)$$

$$R_4 = \frac{\text{number of retreaded non-passenger tires sold in 1966}}{\text{number of buses and trucks in Montana in 1966}} \quad (2\text{-}27)$$

TABLE 9

Automotive Vehicle Registrations In Montana For The Year Of 1966*

COUNTY	PASSENGER	TRUCKS	TRAILERS	EXEMPT
Beaverhead	3,298	2,062	539	5
Big Horn	3,710	2,970	854	28
Blaine	2,371	1,870	259	29
Broadwater	1,436	990	162	2
Carbon	3,452	2,210	349	112
Carter	807	784	126	-
Cascade	35,178	9,963	4,284	272
Chouteau	3,163	3,481	482	96
Custer	5,431	2,295	688	61
Daniels	1,621	1,599	185	41
Dawson	5,107	2,701	592	80
Deer Lodge	6,072	1,689	627	27
Fallon	2,094	1,577	451	16
Fergus	6,011	3,546	764	114
Flathead	15,722	7,606	3,191	79
Gallatin	12,466	5,101	1,909	18
Garfield	739	829	118	15
Glacier	3,791	2,447	657	72
Golden Valley	469	438	90	10
Granite	1,362	952	228	23
Hill	6,801	3,765	845	119
Jefferson	1,702	974	244	27

TABLE 9 (continued)

Judith Basin	1,521	1,344	202	38
Lake	5,587	3,298	901	11
Lewis & Clark	2,099	4,552	2,041	106
Liberty	1,335	1,375	189	29
Lincoln	5,587	3,081	888	79
Madison	2,099	1,517	296	53
McCone	1,335	1,526	206	23
Meagher	934	714	179	26
Mineral	1,166	796	208	29
Missoula	24,447	7,182	3,151	135
Musselshell	1,796	1,113	276	34
Park	6,085	2,543	840	79
Petroleum	333	355	57	12
Phillips	2,244	1,803	227	82
Pondera	3,911	3,125	796	88
Powder River	985	1,506	522	17
Powell	3,150	1,420	552	45
Prairie	854	796	93	53
Ravalli	5,768	3,316	814	96
Richland	4,391	2,964	408	84
Roosevelt	4,044	3,064	547	75
Rosebud	1,995	1,357	250	64
Sanders	2,676	1,765	451	56

TABLE 9 (continued)

Sheridan	2,922	2,361	334	67
Silver Bow	19,185	4,670	2,196	70
Stillwater	2,276	1,535	271	65
Sweet Grass	1,507	961	184	25
Teton	3,181	2,788	644	124
Toole	2,996	2,326	496	66
Treasure	480	530	79	1
Valley	6,326	3,432	924	62
Wheatland	1,284	689	166	1
Wibaux	778	756	146	62
Yellowstone	39,078	11,507	4,623	282
STATE EXEMPTS				2,617
TOTAL	299,378	141,916	41,801	5,975

SUMMARY

PRIVATELY OWNED VEHICLES	
Passenger	299,378
Trucks	141,916
Trailers	41,801
Total	483,095

EXEMPT PUBLICLY OWNED VEHICLES	
Passenger	870
Trucks	4,565
Trailers	534
Motorcycles	43
Total	6,012

* State Highway Commission, Planning Survey Division, Helena, Montana

TABLE 10

Number of Cars, Buses and Trucks. Estimated Passenger and Non-Passenger Tires Demand per County and Estimated Total Profit per County

COUNTY	(1)	(2) (1)x.49	(3) (2)x4.82	(4)	(5) (4)x.53	(6) (5)x10.62	(7) (3)+(6)	(8) (2)+(5)
Beaverhead	3,298	1,615	7,790	2,601	1,381	14,650	22,440	2,996
Big Horn	3,710	1,819	8,250	3,824	2,015	21,650	29,900	3,834
Blaine	2,371	1,162	5,600	2,129	1,130	12,000	17,600	2,292
Broadwater	1,436	704	3,380	1,152	611	6,500	9,880	1,315
Carbon	3,452	1,690	8,140	2,559	1,355	14,400	22,540	2,945
Carter	807	396	1,909	910	482	5,120	7,629	878
Cascade	35,178	17,200	82,900	14,247	7,560	80,300	163,200	24,760
Chouteau	3,163	1,520	7,460	3,963	2,100	22,350	29,810	3,620
Custer	5,431	2,660	12,800	2,983	1,580	16,850	29,650	4,240
Daniels	1,621	795	3,830	1,784	945	10,800	14,630	1,740
Dawson	5,107	2,505	12,050	3,293	1,745	18,530	30,580	4,350
Deer Lodge	6,072	2,980	14,350	2,316	1,265	13,100	27,450	4,245

TABLE 10 (continued)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fallon	2,094	1,025	4,930	2,028	1,075	11,430	16,360	2,100
Fergus	6,011	2,945	14,450	4,310	2,285	24,300	38,750	5,230
Flathead	15,722	7,710	37,150	10,797	5,710	60,700	97,850	13,420
Gallatin	12,466	6,100	29,390	7,010	3,720	39,600	68,990	9,820
Garfield	739	357	1,740	946	503	5,340	7,808	860
Glacier	3,791	1,859	8,940	3,104	1,645	17,500	26,440	3,504
Golden Valley	469	230	1,109	528	279	2,970	4,079	509
Granite	1,362	668	3,220	1,180	625	6,650	9,850	1,293
Hill	6,801	3,340	16,050	4,610	2,450	26,000	42,085	5,790
Jefferson	1,702	834	4,020	1,218	645	6,850	10,870	1,479
Judith Basin	1,521	745	3,590	1,546	820	8,700	12,290	1,565
Lake	5,665	2,780	13,380	4,199	2,220	23,590	36,970	5,000
Lewis & Clark	14,493	7,090	35,390	6,593	3,485	37,100	72,490	10,575
Liberty	1,083	494	2,550	1,564	830	8,800	11,350	1,324
Lincoln	5,587	2,730	1,318	3,967	2,905	22,500	23,818	5,635

TABLE 10 (continued)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Madison	2,099	1,029	4,940	1,813	961	10,220	15,160	1,990
McCone	1,335	654	3,150	1,732	919	9,750	12,900	1,573
Meagher	934	458	2,200	893	474	5,030	7,230	932
Mineral	1,166	572	2,745	1,004	531	5,650	8,395	1,103
Missoula	24,447	12,000	57,800	10,333	5,340	58,200	116,000	17,340
Musselshell	1,796	880	4,240	1,389	735	7,820	12,060	1,615
Park	6,085	2,980	14,350	3,383	1,793	19,200	33,550	4,773
Petroleum	333	163	785	412	218	2,320	3,105	381
Phillips	2,244	1,100	5,300	2,030	1,075	11,430	16,730	2,175
Pondera	3,911	1,919	9,230	3,921	2,080	22,150	31,380	3,999
Powder River	985	483	2,320	2,028	1,075	11,850	14,170	1,558
Powell	3,150	1,545	7,430	1,972	1,046	11,120	18,550	2,591
Prairie	854	419	2,015	889	471	5,000	7,015	890
Ravalli	5,768	2,825	13,600	4,130	2,185	23,220	36,820	5,010
Richland	4,391	2,150	10,350	3,372	1,788	19,000	29,350	3,938
Roosevelt	4,044	1,980	9,530	3,611	1,860	20,350	29,880	3,840

TABLE 10 (continued)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Rosebud	1,995	979	4,700	1,607	851	9,050	13,750	1,830
Sanders	2,676	1,310	6,300	2,216	1,175	12,490	18,790	2,485
Sheridan	2,922	1,465	6,900	2,695	1,425	15,190	22,090	2,890
Silver Bow	19,185	9,400	45,200	6,886	3,650	38,750	83,950	13,050
Stillwater	2,276	1,115	5,360	1,806	956	10,490	15,850	2,071
Sweet Grass	1,507	738	3,545	1,145	608	6,450	9,995	1,346
Teton	3,181	1,560	7,500	3,432	1,820	19,350	26,850	3,380
Toole	2,996	1,469	7,060	2,822	1,495	15,900	22,960	2,964
Treasure	480	235	1,135	609	322	3,430	4,565	557
Valley	6,326	3,100	14,920	4,356	2,315	24,550	39,470	5,415
Wheatland	1,284	630	3,150	855	464	4,820	7,970	1,094
Wibaux	778	380	1,835	902	479	5,090	6,925	859
Yellowstone	39,078	19,150	92,100	16,130	8,500	91,000	183,100	28,700

Explanation of Table 10:

Column (1): number of passenger cars

Column (2): column (1) x average number of recapped tires/car/year

Column (3): column (2) x average profit/recapped passenger tire

Column (4): number of buses and trucks

Column (5): column (4) x average number of recapped non-passenger tires/truck/year

Column (6): column (5) x average profit/recapped non-passenger tire

Column (7): column (3) + column (6) = total profit/area

Column (8): column (2) + column (5) = total demand in number of tires/year

LOCATION ANALYSIS

In general the design of a production system is dependent on its location because physical factors will influence plant layout and location will influence operating and capital costs.

In the case of a complex manufacturing system, careful analysis must be made of both objective and subjective factors. Objective factors include cost of land, cost of buildings, labor costs, transportation costs, raw material cost, power etc. Subjective factors include attitude of unions, attitude of the community to the industry, schools, hospital etc. When all the objective and subjective factors have been weighted,* then the final decision can be made.

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*Some mathematical approach such as (21) can be used to weigh these factors.

This process of selecting the factors for plant location requires a detailed study by management of the actual situation of the industry as well as experience and skill in order to compare respective locations. In our actual situation, that is, of comparatively small plant size and because of the particular characteristics of the recapping industry, the factors involved in the decision process are fewer. Furthermore the criteria for decision making were stated as simply as possible in order to contain the problem.* These decision criteria will be defined in the following sections.

Criteria for Selecting of Plant Location and the Corresponding Plant Size

To facilitate computation, only our representative sizes of plants have been selected: 80 tires/day, 300 tires/day, 600 tires/day and 850 tires/day, with an annual output of 20,800 tires/year, 78,000 tires/year, 156,000 tires/year and 210,000 tires/year respectively.

To form an idea of the relative sizes of these plants, one plant of the largest size would be enough to supply all the market demand in Montana, two plants of the second largest size would be enough to supply the same market demand, three shops of the third largest size would supply the same demand, and finally 10 plants of the smallest size would be needed to supply all the market demand of the State of Montana.

In any real problem like this, management first has to determine the several probable potential plant locations for each size of plant, and

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* Because of the limited resources to make this study it has been necessary sometimes to make reasonable assumptions that would permit the continuation of the problem solution.

after that, it has to make a decision among these potential locations as well as the actual size of plant for each potential location. It is therefore necessary first to establish criteria to determine potential plant locations. The proposed criteria are as follows:

(1) To allocate the largest plant size, the county has to have a population of 50,000 people or more.

(2) To allocate the second plant size, the population of the county has to be 40,000 people or more.

(3) To allocate the third plant size, the population of the county has to be 30,000 people or more.

(4) To allocate the fourth or smaller size, the population of the county has to be 20,000 people or more.

Applying these criteria it was found from the U.S. Census of 1960 that the following locations had the required markets. These potential plant locations are given by Table 11.

TABLE 11

POPULATION OF COUNTY	NUMBER OF POTENTIAL PLANT LOCATIONS	NAMES OF COUNTIES	RANGE OF PLANT SIZES TIRES/DAY
50,000	2	Cascade, Yellowstone	751 - 1000
40,000	4	Cascade, Missoula, Silver Bow, Yellowstone	501 - 750
30,000	5	Cascade, Yellowstone, Missoula, Silver Bow, Flathead	201 - 500
20,000	7	Cascade, Gallatin, Lewis & Clark	80 - 200

Criteria for Selecting Distribution Centers

In the same way that criteria were established to select potential plant locations, criteria now have to be established to select the sizes and locations of the distribution centers.

To integrate the distribution sub-system with the production sub-system is a very difficult task in which mathematical analysis has not been effective as yet because of the great complexity of the sub-systems involved. The ideal study would be to integrate inventory levels at the plant warehouse and at the distribution centers, but this subject alone is so complex that it could be a matter of an entirely separate study. Simulation (11) is the analytical technique which seems to hold the greatest promise for analyzing large-scale integrated systems of material flow.

To make the problem manageable, the criteria to be established will be based on past experience, intuition, and rudimentary logic. The criteria adopted are as follows:

(1) Each distribution point should be located in such a way that it should supply the retailers in an area no smaller than 7,000 square miles and no bigger than 25,000 square miles.

(2) Furthermore, it should be located as near as possible to the population centers of gravity in these areas.

(3) Also, these areas must have a market demand not less than 8,500 tires/year.

(4) The city in which such a distribution center is located should have a population of not less than 2,500 people.

(5) The market demand for the county in which the city is located should not be less than 2,300 tires/year.

It has been assumed that a detailed location analysis has been made and that as a result of this analysis, eleven distribution centers have been selected (see Fig. 1 and Table 12).

TABLE 12

Locations of Distribution Centers, Distribution Areas Adjacent to These Centers, and Magnitudes of Demand of These Areas per Distribution Center

DISTRIBUTION CENTER d_j	LOCATION, CITY AND COUNTY	AREA OF DISTRIBUTION CENTER IN SQ. MILES	MAGNITUDE OF* DEMAND IN TIRES/ YEAR
d_1	Browning, Glacier	16,861	30,846
d_2	Chinook, Blaine	12,421	10,257
d_3	Wolf Point, Roosevelt	24,717	26,355
d_4	Miles City, Custer	18,012	11,163
d_5	Missoula, Missoula	12,252	32,321
d_6	Helena, Helena	8,632	15,960
d_7	Great Falls, Cascade	14,673	34,257
d_8	Dillon, Beaverhead	7,010	20,291
d_9	Bozeman, Gallatin	10,520	17,929
d_{10}	Lewistown, Fergus	10,380	8,829
d_{11}	Billings, Yellowstone	11,541	37,550
TOTAL	MONTANA	145,736	245,668

* These numbers represent the magnitude of demand for the State of Montana for the year 1967. This data has been used only to estimate the market areas because similar data was not available for the year 1966.


MONTANA
LINCOLN
LIBBY
FLATHEAD
KALISPELL
GLACIER
CUT BANK
TOOLE
SHELBY
LIBERTY
CHESTER
HILL
HAVRE
CHINOOK
BLAINE
PHILLIPS
MALTA
VALLEY
GLASGOW
SCOBEY
DANIELS
PLENTYWOOD
SHERIDAN
ROOSEVELT
WOLF POINT
PONDERA
CONRAD
CHOUTEAU
FORT BENTON
TETON
CHOTEAU
SANDERS
LAKE
POLSON
THOMPSON FALLS
LEWIS AND CLARK
GREAT FALLS
CASCADE
STANFORD
JUDITH BASIN
FERGUS
LEWISTOWN
PETROLEUM
WINNETT
GARFIELD
JORDAN
MCCONE
CIRCLE
RICHLAND
SIDNEY
DAWSON
GLENDIVE
WIBAUX
PRAIRIE
TERRY
SUPERIOR
MINERAL
MISSOULA
POWELL
HELENA
MEAGHER
WHITE SULPHUR SPRINGS
HARLOWTON
WHEATLAND
GOLDEN VALLEY
MUSSELSHELL
ROUNDUP
MILES CITY
BAKER
FALLON
CUSTER
FORSYTH
HYSHAM
GRANITE
PHILIPSBURG
DEER LODGE
HAMILTON
BOULDER
TOWNSEND
BROADWATER
RYEGATE
ANACONDA
RAVALLI
BUTTE
SILVER BOW
JEFFERSON
SWEET GRASS
BIG TIMBER
YELLOWSTONE
TREASURE
ROSEBUD
EKALAKA
GALLATIN
BOZEMAN
LIVINGSTON
STILLWATER
COLUMBUS
BILLINGS
HARDIN
CARTER
POWDER RIVER
BROADUS
BEAVERHEAD
DILLON
MADISON
VIRGINIA CITY
PARK
CARBON
RED LODGE
BIG HORN
Potential Plant Locations
Distribution Centers

Figure 1

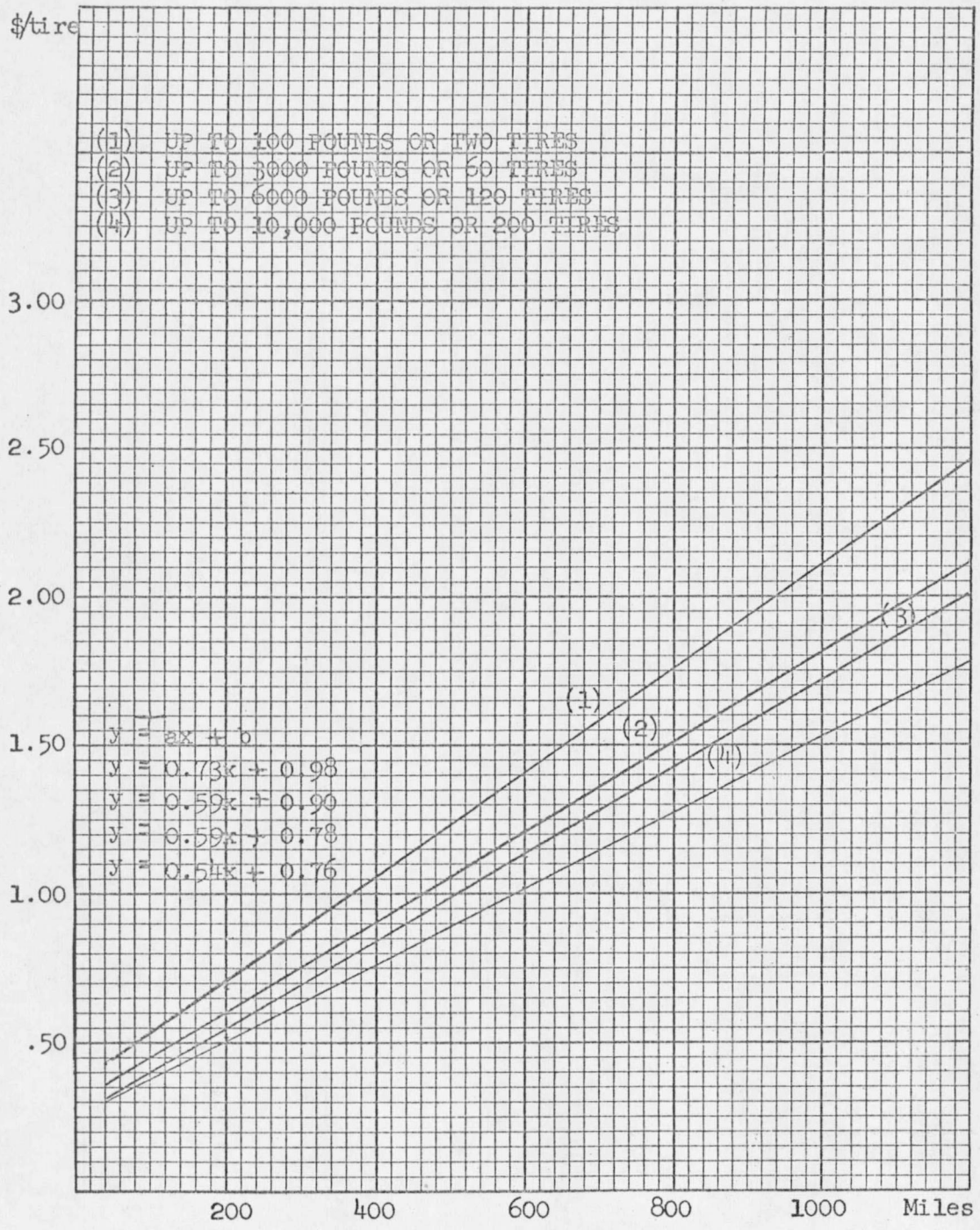


Figure 2

Transportation Cost as a Function of Distance and Weight

TRANSPORTATION ANALYSIS

To decide whether to put one plant of size A, or two plants of size B, or three plants of size C, or 10 plants of size D or any combination of these types of plants that will satisfy the market demand in Montana, one must determine what combination will minimize total costs, that is production costs C_P plus transportation cost C_T.

At this stage of the problem solution, the distances between production centers and distribution centers must be known. These distances are given by Table 14.

The transportation costs per average weight of tire produced and per distance shipped must be obtained.

The transportation cost however, is also a function of the load to be shipped. These costs are given in Table 13. These data were used to derive Fig. 2.

TABLE 13

Truck Transportation Prices, Dollars/100 Pounds*

UP TO / DISTANCE	SIZE OF SHIPMENT			
	100 POUNDS	3000 POUNDS	6000 POUNDS	10,000 POUNDS
Bozeman-Missoula (210 miles)	1.37	1.22	1.10	1.05
Bozeman-Billings (150 miles)	1.15	1.03	0.93	0.87
Bozeman-Helena (100 miles)	0.99	0.90	0.78	0.77

* Prices obtained from Northern Pacific Railway Co., Bozeman, Montana

The next step is to find the average load that will be shipped, assuming the fact that it has been selected. The calculations are as follows:

$$\text{Avrg. Weight/Tire} = 25 \text{ pounds} \times \frac{140{,}000}{209{,}000} + 100 \text{ pounds} \times \frac{69{,}000}{209{,}000}$$

$$\cong 50.0 \text{ pounds/tire} \qquad (2\text{-}28)$$

Average number of shipped tires from plant A_i to 11 distrib. centers

$$= \frac{209{,}000 \text{ tires/year}}{12 \text{ months} \times 11 \text{ distrib. centers}}$$

$$= 1585 \text{ tires/month/distrib. center} \qquad (2\text{-}29)$$

Average number of shipped tires from plant B_i to 11/2 distrib. centers

$$= \frac{104{,}500 \text{ tires/year}}{12 \text{ months} \times 5.5 \text{ distrib. centers}}$$

$$= 1585 \text{ tires/month/distrib. center} \qquad (2\text{-}30)$$

Average number of shipped tires from plant C_i to 11/3 distrib. centers

$$= \frac{69{,}000 \text{ tires/year}}{12 \text{ month} \times 11/3 \text{ distrib. centers}}$$

$$= 1585 \text{ tires/month/distrib. center} \qquad (2\text{-}31)$$

Average number of shipped tires from plant D_i to $\frac{11}{10}$ distrib. centers

$$= \frac{20{,}900 \text{ tires/year}}{12 \text{ months} \times 11/10 \text{ distrib. centers}}$$

$$= 1585 \text{ tires/month/distrib. center} \qquad (2\text{-}32)$$

From above it can be seen that on the average, each production plant is going to ship around 1500 to 1600 tires/month, given that shipping is scheduled once a month. Now it is possible to find the

transportation cost per unit as a function of distance (from Fig. 2). These costs are given in Table 14.

TABLE 14

Transportation Distance

Between Plant Locations and Distribution Centers

and Unit Shipping Cost

Distrib. Center	Browning	Chinook	Wolf Point	Miles City	Missoula	Helena	Great Falls	Dillon	Bozeman	Lewistown	Billings	Dummy City
Great Falls	126	133	320	318	166	94		225	192	106	220	10,000*
	.41	.42	.67	.67	.48	.37		.54	.50	.38	.53	**
Billings	346	229	295	153	340	214	220	261	142	127		10,000
	.70	.54	.64	.44	.69	.54	.53	.58	.43	.41		
Butte	238	291	478	382	121	64	158	67	87	241	229	10,000
	.57	.63	.87	.75	.41	.32	.45	.33	.37	.58	.54	
Missoula	207	299	486	463		116	166	174	208	272	340	10,000
	.52	.64	.88	.85		.40	.48	.48	.52	.60	.69	
Kalispell	101	283	470	545	116	232	227	290	324	333	447	10,000
	.38	.62	.86	.96	.40	.54	.54	.63	.67	.68	.83	
Bozeman	272	316	412	295	208	98	192	119		162	142	10,000
	.60	.60	.78	.64	.52	.38	.50	.41		.47	.43	
Helena	174	227	414	347	116		94	131	98	188	224	10,000
	.48	.54	.78	.69	.40		.37	.42	.38	.50	.54	

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* Distances, miles.

** Unit cost, dollars.

CHAPTER III - SOLUTION OF THE TOTAL COST PROBLEM AS A ONE STAGE PROBLEM

THE TRANSPORTATION PROBLEM

The problem to be solved here is one of allocation. Problems of allocation arise whenever there are a number of activities to perform, but limitations on either the quantity of resources and/or, the way they can be utilized, affect the performance of each separate activity. In such situations it is necessary to allot the available resources to the activities in such a way that effectiveness of the total system is optimized.

The alternatives of allocating the resources to the activities can be finite or infinite. In problems with a finite number of alternatives, it is possible, in theory, to enumerate all the alternatives. But doing this, in practice, is lengthy and sometimes impossible, relative to the time available. For example, in the case where there are 18 resources or plants and 12 activities (11 real destinations plus one dummy destination), if the problem were solved by enumeration there would be $\begin{bmatrix}18\\12\end{bmatrix}$ C combinations. In the simplest case of all the plants and all the destinations could have fixed volume of production and fixed volume of demand, respectively. But this is not the case, because at each location the plant may have a range of production from 80 to 810 tires per day. Therefore, in reality, the number of alternatives for allocating resources to activities is infinite for each combination.

Only in recent years have mathematicians realized that for many practical purposes, solutions by enumeration were inefficient. The first situations discussed were ones in which the effectiveness and the restrictions were stated in terms of linear functions. The analysis of

these situations are called Linear Programming. The techniques used can be divided into three main groups according to the methods used for solution. These techniques are known as: The Assignment Problem, The Transportation Problem, and The Simplex Problem. (2), (9), (20), (24).

The Assignment Problem is a type of allocation problem in which n items are distributed among n boxes, one item to a box, in such a way that the return resulting from the distribution is optimized. The method of solving this type of problem is the Assignment Problem Technique (19).

The Transportation Problem is a generalization of The Assignment Problem in which the matrix of effectiveness is no longer necessarily square; in this case each origin can be associated with one or more numbers of destinations in such a way that costs are minimized, or profits maximized, or any other objective function is optimized. That is to say there are m origins, with origin i possessing a_i items, and n destinations (possibly a different number from m), with destination j requiring b_j items, and with $a_j = b_j$. Given the mn costs associated with shipping one item from any origin to any destination, and given the requirement to empty the origins and fill the destinations in such a way the the total cost is minimized. The problem may be stated formally as follows:

given an m-by-n array of real number (C_{ij}), as well as two sets of positive integers $(a_1, a_2, ---, a_m)$ and $(b_1, b_2, ---, b_n)$ with

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (3\text{-}1)$$

Determine, among all m-by-n arrays (x_{ij}) of non-negative integers that satisfy:

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{for each } j \qquad (3\text{-}2)$$

as well as:

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad \text{for each } i \qquad (3\text{-}3)$$

that array (x_{ij}) for which the quantity

$$\sum_{i} \sum_{j} x_{ij} \cdot C_{ij} \qquad (3\text{-}4)$$

takes its minimum value. C_{ij} represents the cost associated with shipping one item from origin i to destination j.

There are several methods to solve the transportation problem. If the case were one where restriction equations and the objective functions were linear, then the transportation problem could be solved by the Northwest Corner Method, the Unit Penalty Method, as a Linear Programming Problem by the Simplex Method, etc.

In this particular case each element of the array C_{ij} depends on the distance between a given production center and a given distribution center as well as on the size of the plant, (i.e. production volume).

It happens that $C_{T_{ij}} = C_{t_{ij}} + C_{P_i}$, where:

$C_{T_{ij}}$ = the sum of unit transportation cost from production center i to distribution center j plus unit production cost from production center i to distribution center j

$C_{t_{ij}}$ = the unit transportation cost from production center i to distribution center j

C_{P_i} = the unit production cost at production center i

and it can be seen from Table 18 and from Fig. 6 that the unit production cost is a function of plant size. This function is given by the following equation (Appendix C):

$$C_P = .135099 \times 10^2 - .8235 \times 10^{-2}\, s + .149 \times 10^{-4}\, s^2 - .96 \times 10^{-8}\, s^3 \qquad (3\text{-}5)$$

Therefore each element in the array C_{ij} is not constant and varies with the plant size and instead of minimizing $z = \sum_i \sum_j C_{ij} \cdot x_i$ as a linear function, z has to be minimized as a non--linear function:

$$z = \sum_i \sum_j C_{T_{ij}} (C_{ij}, s) \cdot x_{ij} \qquad (3\text{-}6)$$

s is the size of the production plant and depends on the total quantity to be allocated to all the distribution centers. From equation (3-6) it can be seen that the objective function is not a linear function and therefore the problem cannot be solved by the Simplex Method of Linear Programming. Thus a Non-Linear Programming Technique (12) should be applied. But the problem is even more complicated because not only the objective function is non-linear but the constraints are not linear too.

At this point in the solution a decision must be made between two alternatives. To use a more complicated technique and get a very accurate answer which may not be in agreement with the relatively inaccurate input data; to make some assumptions and simplify the problem, obtain an acceptable solution to guide management in the decision and save time and therefore money.

From the above alternatives the second has been selected because it was more appropriate in the present case. The following assumptions are made to simplify the problem.

The production costs, instead of following a curve of the third order as a function of plant size*, will assume different constant values for each range of plant size as shown in Fig. 6. In practice this assumption is very reasonable, since if several points on the true curve are joined by straight lines as in Fig. 3, there will exist a poligonal curve which will approximate the true curve.

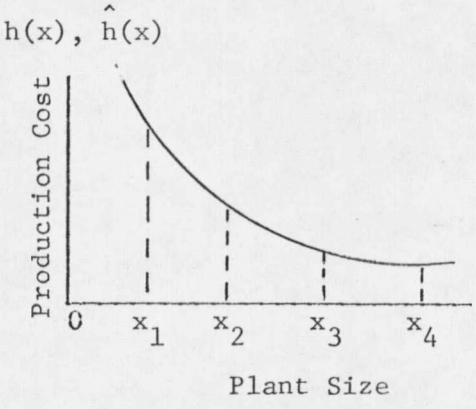


Figure 3, CONVEX CURVE OF PRODUCTION COST VS. PLANT SIZE

This assumption (12), as given by the curve in Figure 3, still doesn't simplify our problem to the degree we want because we have to approximate the following non-linear equations:

$$\sum_{j=1}^{n} g_{ij}(x_j) \{ \leq , = , \geq \} b_i, \qquad i = 1, ---, m,$$

$$x_j \geq 0, \qquad j = 1, ---, n,$$

$$\max = \sum_{j=1}^{n} f_m(x_j) \qquad (3\text{-}7)$$

---

* See Equation (3-5)

to

$$\sum_{j=1}^{n} \hat{g}_{ij}(x_j) \quad \{ \leq , = , \geq \} \; b_i,$$

$$x_j \geq 0, \qquad j=1, \text{---}, n,$$

$$\max \hat{Z} = \sum_{j=1}^{n} \hat{f}_j(x_j) \tag{3-8}$$

where:

$h(x)$ = an arbitrary continuous function of a single variable x which is defined for all x,

$\hat{h}(x)$ = the poligonal approximation to h (x)

$\hat{g}_{ij}(x_j)$ = poligonal approximation to the function $g_{ij}(x_j)$

$\hat{f}_j(x_j)$ = poligonal approximation to the function $f_j(x_j)$

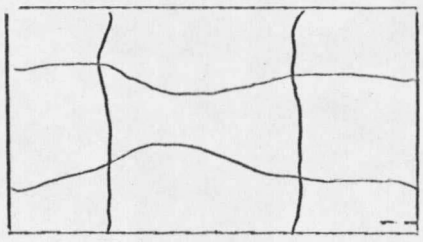
This approximation, however, still requires too many computations to adapt the problem so it can be solved by the Simplex Method.

The problem has to be simplified to an even greater extent. The third degree curve will be represented approximately by a non-continuous straight line segments as shown in Fig. 6. Each of these segments represents the production cost within a range of s, or plant size. At the same time, one or more sizes of plants will be allocated to a given potential location if this location meets the requirements of the decision criteria for potential plant location. The explanation of this plant location procedure is outlined in Chapter II.

It may appear at the outset that it is not possible or practical to allocate two different plants in the same location. This logical inconvenience is overcome by allocating a dummy destination or distribution

point that will absorb the production of these marginal plants and will leave for the actual market only the plants whose total costs (unit production cost and transportation cost) are lowest. This lower cost will allow the final product to be sold at a lower retail price in a given market. This can be better explained by the following example:

Assume a geographic area A and that this area is divided into n sub-areas (see Fig. 4). In each of these sub-areas is going to be located one distribution point that meets criteria given for its selection. Also there are going to be m plants allocated to the area A according to a previously stated criteria of selection and these are going to be the potential plants.



d_d = dummy distribution center

Fig. 4

Area A, Sub-Areas A_j, and Plant Locations L_1, L_2, L_i --- L_m, and Distribution Centers D_j

Assume m = 4 and n = 3, the distance between production centers and distribution centers are d_{ij}, the transportation costs between the same points are given by the array $C_{t_{ij}}$ and the production cost per unit per type of plants are C_{P_i}. Then the total cost of allocating one tire from P_i to D_i would be given by $C_{T_{ij}}$ where $C_{T_{ij}} = C_{t_{ij}} + C_{P_i}$.

In the next step the Transportation Problem is stated in a mathematical form:

given the following two arrays $C_{T_{ij}}$ and x_{ij}

$$\text{minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{T_{ij}} \cdot x_{ij} \quad \text{subjected to the constraints}$$

$$\sum_{i=1}^{n} x_{ij} = b_j \,, \quad j = 1, 2, \text{----}, n$$

$$\sum_{j=1}^{n} x_{ij} = a_i \,, \quad i = 1, 2, \text{----}, m \qquad (3\text{-}9)$$

and also, supply must equal demand, or

$$\sum_{j=1}^{n} b_j = \sum_{i=1}^{m} a_i \qquad (3\text{-}10)$$

The two arrays, $C_{T_{ij}}$ and x_{ij}, are given by

TABLE 15

ORIGIN \ DESTINATION	D_1	D_2	----------	D_n
P_1	C_{11}	C_{12}	----------	C_{1n}
P_2	C_{21}			.
.	.			.
.	.			.
.	.			.
P_m	C_{m1}			C_{mn}

Given: m=4 and n=3

ORIGIN \ DESTINATION	D_1	D_2	----------	D_n	Available
P_1	x_{11}	x_{12}	----------	x_{1n}	a_1
P_2	x_{21}			.	a_2
.	.			.	.
.	.			.	.
.	.			.	.
P_m	x_{m1}			x_{mn}	a_m

Required b_1 b_2 ----------b_n

The following values for a_i and b_j are assumed:

$a_i = 10$ $\qquad$ $b_1 = 8$

$a_2 = 10$ $\qquad$ $b_2 = 7$

$a_3 = 5$ $\qquad$ and $\qquad$ $b_3 = 3$

$a_4 = 3$

In the example, $\Sigma a_i > \Sigma b_j$, to satisfy the constraints it is necessary to introduce a dummy destination D_4 that will absorb the difference between $\sum_{i=1}^{5} a_i$ and $\sum_{j=i}^{4} b_j$, the D_4 will have a demand of 9 tires per day and the Transportation Problem will be stated mathematically by the following expressions:

given the arrays C_{ij} and x_{ij} (Table 16)

TABLE 16

DESTINATION

ORIGIN i \ j	D_1	D_2	D_3	D_4
P_1	C_{11}	C_{12}	C_{13}	C_{14}
P_2	C_{21}	C_{22}	C_{23}	C_{24}
P_3	C_{31}	C_{32}	C_{33}	C_{34}
P_4	C_{41}	C_{42}	C_{43}	C_{44}

DESTINATION

ORIGIN i \ j	D_1	D_2	D_3	D_4
P_1	x_{11}	x_{12}	x_{13}	x_{14}
P_2	x_{21}	x_{22}	x_{23}	x_{24}
P_3	x_{31}	x_{32}	x_{33}	x_{34}
P_4	x_{41}	x_{42}	x_{43}	x_{44}

$$\text{minimize } Z = \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} \cdot x_{ij} \quad \text{subject to the restrictions}$$

$$\sum_{i=1}^{4} x_{ij} = b_j, \; j = 1, 2, 3, \text{ and } 4$$

$$\sum_{j=1}^{4} x_{ij} = b_i, \; i = 1, 2, 3, \text{ and } 4 \qquad (3\text{-}11)$$

In addition, supply must equal demand or $\sum_{j=1}^{4} b_j = \sum_{i=1}^{4} a_i = 28$ (3-12)

Now as mentioned before this Transportation Problem is a Linear Programming problem whenever all the constraints and the objective function are linear functions and $x_{ij} \geq 0$ for all i and j. The problem can now be solved by the simplex method. To do this, coefficients of the variables of the constraint equations and of the objective function must be placed in array form. This array is given by the matrix* in Table 17.

TABLE 17

Row Name	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	x_{41}	x_{42}	x_{43}	x_{44}		
Cost	C_{11}	C_{12}	C_{13}	C_{14}	C_{21}	C_{22}	C_{23}	C_{24}	C_{31}	C_{32}	C_{33}	C_{34}	C_{41}	C_{42}	C_{43}	C_{44}		
A_1	1	1	1	1													=	10
A_2					1	1	1	1									=	10
A_3									1	1	1	1					=	5
A_4													1	1	1	1	=	3
B_1	1				1				1				1				=	8
B_2		1				1				1				1			=	7
B_3			1				1				1				1		=	3
B_4				1					1			1				1	=	10

*The inefficiency present in the Simplex approach, can be seen when cost matrix becomes a 9 x 16 simplex tableau.

APPROACH TO AN OPTIMUM REALISTIC SOLUTION OF THE TOTAL COST PROBLEM AS A ONE-STAGE PROBLEM

As was shown the total cost, C_T depends on two other costs, the production cost C_P, and the transportation cost, C_t, This latter cost (per unit) is fixed because it depends only on the distance between the origin and the destination. The distance between these two points is fixed, therefore C_t, transportation cost per unit, is fixed. The production cost per unit C_P, however, is a function of the plant size. For further simplification, it will be assumed that every plant is producing at 100% capacity and works only one shift.

Hence it can be seen that the total unit cost $C_{T_{ij}}$ is a function of C_{P_i} plus C_{P_i} as a function of the plant size. Therefore, $C_{T_{ij}} = f(s)$ must be solved for s where s is the size of the plant. But from the production cost analysis it is noted that C_{P_i} is not a linear function of s, therefore, the linear programming technique cannot be applied directly. Several assumptions must be made in order to adapt the problem.

First Assumption: There will be a fixed number of plant size.

Second Assumption: One or more plants can be allocated to a given distribution point.

Third Assumption: The excess or production volume is going to be absorbed by a dummy destination.

Fourth Assumption: The unit total cost for the dummy destination is going to be one order of magnitude higher than the retail unit total cost for the real destinations.

The first assumption implies that the unit production cost per tire is about the same for a given range of plant size s. From Fig. 6 it can be seen that as more and smaller classes are taken on the s axis the poligonal line approaches the unit production cost curve given by equation (3-5) and Fig. 5. For illustration the curve, $C_P = f(s)$, has been divided into four ranges of s, corresponding to practicable plant sizes for the Montana market.

With four possible plant sizes, one, two, three, or four plants can be set up in any potential plant location, to meet the production and market requirements. At the first glance it may not seem practical to locate several plants of different sizes, at the same time, in a given plant location because the total production would exceed the market demand (that is the demand of Montana). This logical inconvenience is overcome by introducing in the matrix a dummy destination with very high transportation costs.

Now the transportation problem can be converted to a linear programming problem by use of the simplex algorithm. Solution of this algorithm obliges the marginal plants (i.e. the plants which cannot compete because their total unit costs C_T are higher than the competitor's cost) to allocate their products to the dummy destination. The remaining plants which can allocate their products in the real market because of a lower total cost, will allocate their products in this market in such a combination as to minimize total costs, $\sum_{\text{all plants}} C_{T_{ij}} = Z$.

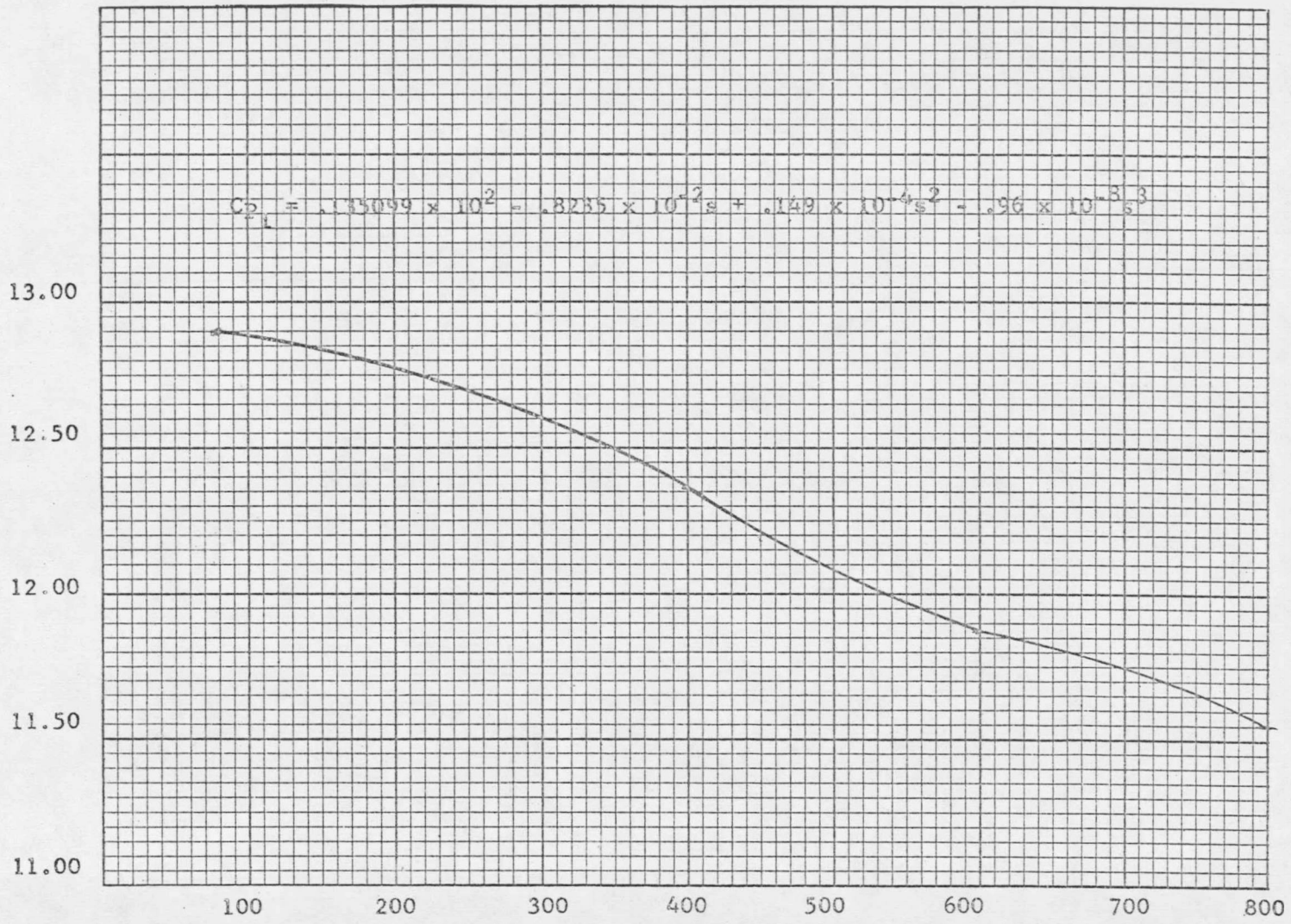


Figure 5 Unit Production Cost as a Function of Plant Size

In the actual problem four ranges of plants s_i, are given, therefore, the problem involves: four unit production costs; five possible plant locations (given by Table 12 and Fig. 1); and eleven real distribution points. Since the proposed method of solution is the Simplex Method of Linear Programming, the problem is now a transportation problem, in which the constraints and the objective functions are linear, and the variables are separable. The problem has now become linear.

Since it has been assumed that unit production cost is constant and independent of plant size, for a given small range of s (see Table 18), the mathematical statement of the problem is given by the following expressions:

$$\sum_{i=1}^{18} a_i = \sum_{j=1}^{12} b_j = 8{,}900. \qquad (3\text{-}13)$$

To determine, among all 18-by-12 arrays $||x_{ij}||$ of non-negative integers that satisfy,

$$\sum_{i=1}^{18} x_{ij} = b_j \text{ for each } j; \quad j = 1, 2, \text{-----}, 12 \qquad (3\text{-}14)$$

as well as,

$$\sum_{j=1}^{12} x_{ij} = a_i \text{ for each } i; \quad i = 1, 2, \text{----}, 18 \qquad (3\text{-}15)$$

so that array $||x_{ij}||$ for which the quantity

$$\sum_{i,j} x_{ij} \cdot C_{ij} \qquad (3\text{-}16)$$

takes its minimum value.

C_{ij} represents the cost associated with shipping one item from origin i to destination j.

TABLE 18

Production Cost C_{P_i} as a Function of The Plant Size, s

PLANT SIZE, s UNITS/DAY	PRODUCTION COST, $	RANGE OF s UNITS/DAY
80	12.90	80-200
300	12.36	201-500
600	11.88	501-750
850	11.42	751-1000

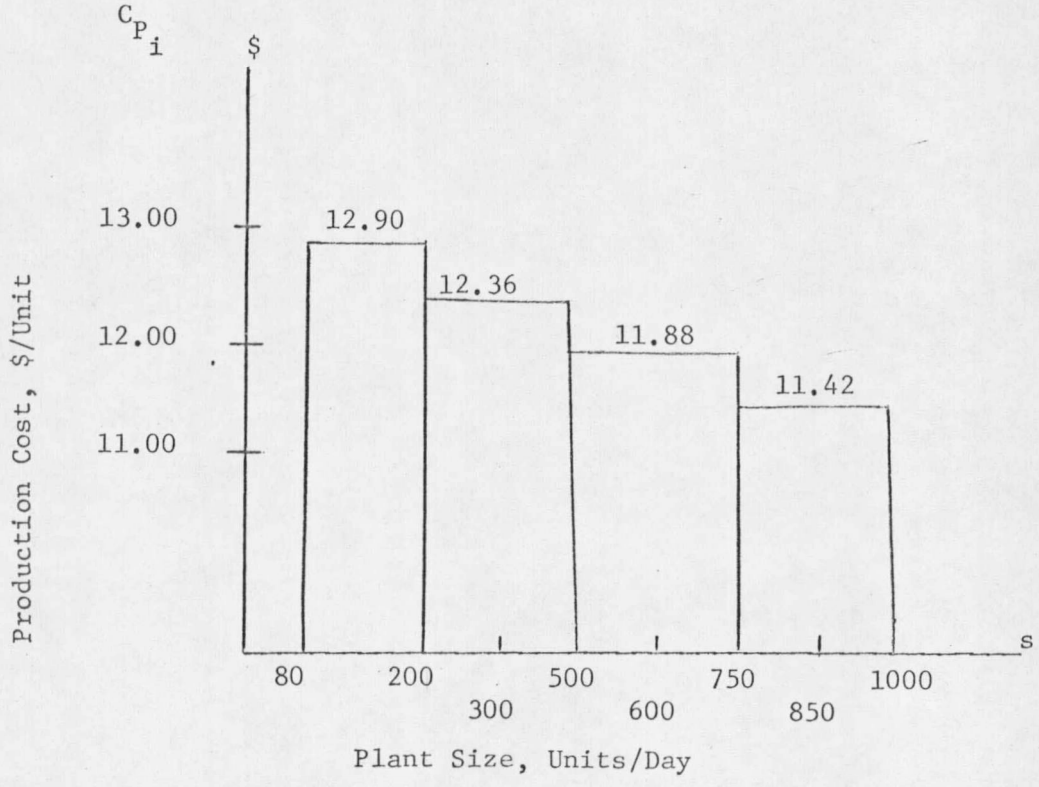


Fig. 6 Production Cost C_{P_i} as a Step Function of Plant Size, s

Because of the dimensions of the simplex tableau a computer has been used to solve the transportation problem. The program used to solve the problem is the I.B.M. 1620-1311 Linear Programming System, which is a general purpose programming system designed to provide the 1620-1311 users with a sophisticated mathematical technique to determine the most efficient use of various resources for carrying out alternative activities. The system is composed of a number of programs which are stored on the 1311. Each program routine is called into storage by procedure control cards (agendum cards) when that program routine is to be executed. The sequence of the control cards defines the solution procedure. Besides direct optimization the program can also provide an extensive post-optimal analysis to indicate the effect of changes in constraints, costs, or technology.

The program was ran in the 1620 Model 2-60 K at Montana State University. The method of solution is shown in Tables 19, 20, 21, and 22. the input data and agenda cards are given in Appendix D. Summary of the program output from Appendix E, shows the following results:

$$\min Z = 614269.850$$

and (3-17)

TABLE 19

Summary of Output

From Appendix E IBM 1620-1311 Linear Program

x11	=	118.5	x17	=	131.8
x12	=	39.5	x18	=	78.0
x16	=	61.4	x110	=	33.9

TABLE 19 (continued)

x112	=	411.9	x812	=	350.0
x23	=	101.5	x912	=	350.0
x24	=	42.9	x1012	=	350.0
x29	=	69.0	x1112	=	350.0
x211	=	148.0	x1212	=	140.0
x212	=	513.6	x1312	=	140.0
x312	=	625.0	x1412	=	140.0
x412	=	625.0	x1512	=	140.0
x512	=	625.0	x1612	=	140.0
x65	=	124.1	x1712	=	140.0
x612	=	500.9	x1812	=	140.0
x712	=	350.0			

TABLE 20

Unit Production Cost per Size of Plant, Unit Shipping Cost between Plants and Distribution Centers and Total Cost per Tire

DISTRIB. CENTER j / PLANT i		d_1		d_{11}	d_{12}	DAILY PROD. VOLUME
11.42	A1	.41 11.83		.53 11.42	100.00	$751 \leq a_1 \leq 1000$
	A17					
12.91	A18	.48 13.38				$80 \leq a_{18} \leq 200$
Prod. Cost/Tire	Avrg. Daily Demand by Distrib. Centers	b_1 = 39.5 Browning		b_{11} = 148.0 Billings	b_{12} = 7951.0 Dummy Destin.	

TABLE 21

Allocation Variables and Total Cost Matrix

DISTRIB. CENTER / PLANT		d_1	— — — — — — — —	d_{11}	d_{12}	DAILY PROD. VOLUME
11.42	A1	x11 11.83	— — — — — — — —	x111 11.42	x112 100.00	$751 \leq a_1 \leq 1000$
⋮	A2 ⋮ A17	⋮	— — — — — — — — — — — — — — — — —	⋮	⋮	⋮
12.91	A18	x181 13.38	— — — — — — — —	x1811 13.44	x1812 100.00	$80 \leq a_{18} \leq 200$
Prod. Cost/Tire	Avrg. Daily Demand by Distrib. Centers	$b_1 = 39.5$ Browning	— — — — — — — —	$b_{11} = 148.0$ Billings	$b_{12} = 7951.0$ Dummy Destin.	X

TABLE 22

Final Allocation Matrix

The Total Cost Problem Solved as a Transportation Problem

with Linear Constraints and Linear Objective Function

by the I.B.M. 1620-1311 Linear Programming Program

PLANT	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}
A_1	118.5	39.5				61.4	131.8	78.0		33.9		411.9
A_2			101.5	42.9					69.0		148.0	513.6
A_3												625.0
A_4												625.0
A_5												625.0
A_6					124.1							500.9
A_7												350.0
A_8												350.0
A_9												350.0
A_{10}												350.0
A_{11}												350.0
A_{12}												140.0
A_{13}												140.0
A_{14}												140.0
A_{15}												140.0
A_{16}												140.0
A_{17}												140.0
A_{18}												140.0
	$b_1 = 118.5$	$b_2 = 39.5$	$b_3 = 101.5$	$b_4 = 42.9$	$b_5 = 124.1$	$b_6 = 61.4$	$b_7 = 131.8$	$b_8 = 78.0$	$b_9 = 69.0$	$b_{10} = 33.9$	$b_{11} = 148.0$	$b_{12} = 7951.0$

The next step is to put the values of x_{ij} in a matrix form; see Table 22, and to interpret the results.

Interpretation of the Results

The program output, Appendix E, has been summarized in Table 22. It can be seen that only the production centers, A_1, A_2, and A_6 make allocations to real distribution centers. Analyzing in detail Table 22, it can be seen that plant A_1 allocates only 52.9% of its production volume to real distribution centers and 47.1% to the dummy distribution center d_{12}. Plant A_2 allocates 41.2% of its production volume to real distribution centers and 58.8% to the dummy distribution center d_{12}. In the case of plant A_6 the allocation to a real distribution center is only 24.8% and 75.2% is allocated to the dummy distribution center d_{12}.

It is evident that these three plants cannot operate at the same time because in realty the distribution center d_{12}, that represents a dummy market demand, does not exist, therefore, the plants will be producing at 52.9%, 47.1% and 24.8% capacity respectively. The production cost will not remain the same, but will be much higher if the plants operate at these partial capacities.

If plant A_1 or A_2 is operating alone, then 100% of their production capacity can be absorbed by the real distribution centers. The final cost of tires per unit production plus transportation cost for the preferred plant will be lower than that for any other plant producing at 100% capacity, thus the final decision has to be made only between plant A_1 and plant A_2.

Next step will be to calculate the total cost for plants A_1 and A_2.

$$\sum_{j=1}^{11} x_{1j} C_{1j} = 11{,}151.10 \quad ; \quad \sum_{j=1}^{11} x_{2j} C_{2j} = 9{,}776.39 \qquad (3\text{-}18)$$

It can be seen from (3-18) that plant A_2, located in Billings, is the plant that has the lowest total costs.

CHAPTER IV - ATTEMPTED SOLUTION OF THE TOTAL COST PROBLEM AS A MULTISTAGE PROBLEM AMENDABLE TO DYNAMIC PROGRAMMING TECHNIQUE

THE DYNAMIC PROGRAMMING APPROACH

Dynamic programming is a computational technique rather than a particular type of non-linear programming problem. The basic ideas which led to this computational technique were developed by Richard Bellman in the early 1950's (3), (4).

The method of Dynamic programming is based on the mathematical notion of recursion given by Bellman's Principle of Optimality, which states:

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with respect to the state resulting from the first decision."

Using the dynamic programming strategy, one can convert a multistage optimization problem into a sequence of equivalent one-decision problems. This technique can be illustrated by the following non-linear programming problem:

$$\sum_{j=1}^{n} a_j x_j \leq b \quad ; a_j > 0, \; j = 1, \text{------}, n$$

$$x_j \geq 0 \; , \; j = 1, \text{------}, n; \text{ all } x_j \text{ integers}$$

$$\max Z = \sum_{j=1}^{n} f_j (x_j). \qquad (4\text{-}1)$$

In this example of a non-linear problem there is only one constraint, and the objective function has separable variables which assume integral values. It is also required that the variables assume only integral values.

This kind of problem could also be solved with the use of calculus if the $f_j(x_j)$ were concave and differentiable everywhere. It might also be feasible to use the Lagrange multiplier technique to solve the problem. But if Z has more than one local maximum, then through the use of the Lagrange multiplier technique one could not be sure if he had found a local maximum or overall total maxima. And if in addition, it is required that the x_j be integers, one cannot use calculus to determine an optimum solution (12). These difficulties, mentioned above, do not present themselves in the case of Dynamic Programming technique, where the x_j has to be integers; in this case it will take submiltipliers of each unit, i.e. if cubic feet are given as a unit and x_j is a continuous variable, then the new unit for x_j will be cubic inches or a smaller unit. Therefore, in working a problem it does not matter if x_j is a continuous variable, because it is worked with a submiltiple of the original unit. Therefore, an acceptable degree of accuracy can be achieved.

The goal of the Dynamic Programming technique is to optimize the objective function:

$$Z^* = \max_{x_1,---,x_n} \{ \sum_{j=1}^{n} f_j(x_j) \}, \qquad (4\text{-}2)$$

the maximization should be made over non-negative integers, x_j satisfying:

$$\sum_{j=1}^{n} a_j x_j \leq b. \qquad (4\text{-}3)$$

The procedure of the Dynamic Programming technique is as follows:

1 - Select a value of x_n

2 - Hold x_n as a constant

3 - Maximize Z over the remaining variables, i.e., over $x_1,---,$ x_{n-1}. Then the values of $x_1,---, x_{n-1}$, which maximize Z, under these conditions will depend on the previous selected value for x_n. Therefore, Z* will be the largest Z value for each x_n selected, and the set x_1----, x_n has been determined. All this can be expressed by the following mathematical expression:

$$Z^* = \max_{x_1,----,x_{n-1}} \{\sum_{j=1}^{n} f_j(x_j)\} = f_n(x_n) + \max_{x_1,----,x_{n-1}} \sum_{j-1}^{n-1} f_j(x_j). \qquad (4\text{-}4)$$

The term $f_n(x_n)$ has been taken out of the maximization of $\Sigma f_j(x_j)$ because this term is independent of the set $x_1.----, x_{n-1}$. Once x_n has been selected, $x_1,----, x_{n-1}$ must be restricted to non-negative integers, which satisfy:

$$\sum_{j=1}^{n-1} a_j x_j \leq b - a_n x_n \qquad (4\text{-}5)$$

Then the expression,

$$\max_{x_1,----,x_{n-1}} \sum_{j=1}^{n-1} f_j(x_j) \quad ,$$

for non negative integers satisfying (4-4) depends on x_n, or even better on $b - a_n x_n$. We can express this statement by the following mathematical form:

$$\Lambda_{n-1}(b - a_n x_n) = \max_{x_1,\text{----},x_{n-1}} \sum_{j=1}^{n-1} f_j(x_j) \qquad (4\text{-}6)$$

and therefore

$$Z^* = \max_{x_n} \{ f_n(x_n) + \Lambda_{n-1}(b - a_n x_n) \} \qquad (4\text{-}7)$$

where x_n can take the values 0, 1, ----, (b/a_n).

To evaluate Z^* in (4-7), $\Omega_n(x_n)$ has to be evaluated first, where $\Lambda_n(x_n)$ is given by,

$$\Omega_n(x_n) = f_n(x_n) + \Lambda_{n-1}(b - a_n x_n) \qquad (4\text{-}8)$$

for each possible x_n, then select the largest Ω_n. This value of x_n^* that optimizes Ω_n is also an optimal value of x_n.

Now assume any arbitrary non-negative integer ξ, then

$$\Lambda_{n-1}(\xi) = \max_{x_1,\text{----},x_{n-1}} \sum_{j=1}^{n-1} f_j(x_j) \qquad (4\text{-}9)$$

subjected to the restriction

$$\sum_{j=1}^{n-1} a_j x_j \leq \xi. \qquad (4\text{-}10)$$

and it will be as above,

$$\Lambda_{n-1}(\xi) = \max_{x_{n-1}} [f_{n-1}(x_{n-1}) + \Lambda_{n-2}(\xi - a_{n-1}x_{n-1})], \qquad (4\text{-}11)$$

where

$$\Lambda_{n-2}(\theta) = \max_{x_1,\text{----}x_{n-2}} \sum_{j=1}^{n-2} f_j(x_j) \qquad (4\text{-}12)$$

and the maximization is carried out for non-negative integers x_1,----, x_{n-2} satisfying

$$\sum_{j-1}^{n-2} a_j x_j \leq \theta. \qquad (4\text{-}13)$$

In (4-11), x_{n-1} can take on the values 0, 1,----, $[\xi/a_{n-1}]$. Then if the function $\Lambda_{n-2}(\theta)$ is known, $\Lambda_{n-1}(\xi)$ can be evaluated by carrying out a maximization over the single variable x_{n-1}. For each different value of ξ, $\Lambda_{n-1}(\xi)$ has to be evaluated, then the maximization computation in (4-11) must be repeated.

To evaluate $\Lambda_{n-2}(\theta)$ the same process as above is used, until the last step where,

$$\Lambda_1(\rho) = \max_{x_1} f_1(x_1) \; ; \qquad (4\text{-}14)$$

x_1 can take the values 0, 1,----, $[\rho/a_1]$. In practice one would start by finding the values of $\Lambda_1(\rho)$, then $\Lambda_2(\xi)$, and so on until all values for $\Lambda_{n-1}(\xi)$ are found and finally Z* or $\Lambda_n(b)$.

The computational part is as follows:

1 - Compute $\Lambda_1(\xi)$ for each permissable value of ξ

$$\Lambda_1(\xi) = \max f_1(x_1)$$
$$0 \leq x_1 \leq \xi/a_1 \qquad (4\text{-}15)$$

2 Construct a table where the values of ξ, $\Lambda_1(\xi)$, and the optimum value of $x_1(\xi)$ i.e. $\hat{x}_1(\xi)$, can be shown.

TABLE 23

ξ	$\Lambda_1(\xi)$	$\hat{x}_1(\xi)$
0	$\Lambda_1(0)$	$\hat{x}_1(0)$
1	$\Lambda_1(1)$	$\hat{x}_1(1)$
2	$\Lambda_1(2)$	$\hat{x}_1(2)$
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
b	$\Lambda_1(b)$	$\hat{x}_1(b)$

3 - Now that the value (s) of $\hat{x}_1$ for each $\Lambda_1(\xi)$ have been obtained, it is possible to calculate each value (s) of $\hat{x}_2$ for each $\Lambda_2(\xi)$ using the relationship (4-11) for every $\xi = 0, 1, ----, b$, then

$$\Lambda_2(\xi) = \max_{0 \leq x_2 \leq [\xi / a_2]} [f_2(x_2) + \Lambda_1(\xi - a_2 x_2)]. \qquad (4\text{-}16)$$

4 - Substituting appropriately in equation (4-8), $\Omega_2(x_2; \xi)$ can be computed for each $x_2 = 0, 1, ----, \xi/a_2$. $\Lambda_2(\xi)$ is the largest of values.

$$\Omega_2(0 ;\xi) = f_2(0) + \Lambda_1(\xi)$$

$$\Omega_2(1 ;\xi) = f_2(0) + \Lambda_1(\xi - a_2)$$

$$\Omega_2(2 ;\xi) = f_2(0) + \Lambda_1(\xi - 2a_2)$$

.

.

$$\Omega_2\left(\left[\frac{\xi}{a_2} ; \xi\right]\right) = f_2\left(\left[\frac{\xi}{a_2}\right]\right) + \Lambda_1\left(\xi - a_2\left[\frac{\xi}{a_2}\right]\right),$$

5 - First $\Omega_2(x_2 ;0)$ has to be computed for all feasible x_2. The $\Omega_2(x_2 ; 1)$ for all feasible x_2 and so on, until finally $\Omega_2(x_2 ; b)$ has been computed. Then the second table is constructed (Table 24).

TABLE 24

ξ	$\Lambda_2(\xi)$	$\hat{x}_2(\xi)$
0	$\Lambda_2(0)$	$\hat{x}_2(0)$
1	$\Lambda_2(1)$	$\hat{x}_2(1)$
2	$\Lambda_2(2)$	$\hat{x}_2(2)$
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
b	$\Lambda_2(b)$	$\hat{x}_2(b)$

6 - Construct the following tables for each $\Lambda_k(\xi)$ until $\Lambda_{n-1}(\xi)$ has been constructed. The last table is built only for $\Lambda_n(b)$ because the other values for ξ are not required to estimate Z^* as it can be seen from equation (4-7)

If the problem is going to be solved on a computer, then the only values that have to be kept in storage are the tables of $\Lambda_{k-1}(\xi)$ and $\Lambda_k(\xi)$, the other values may be raised to save storage space.

Once $\Lambda_n(b)$ has been found, then x_n^* is known and to find the remaining values of x_k^* the following relation is used

$$x_{n-1}^* = \hat{x}_{n-1}(b - a_n x_n^*) \qquad (4\text{-}17)$$

this expression means that x^*_{n-1} is a function of $(b - a_n x^*_n)$. Then from Table $\Lambda_{n-1}(\xi)$, $\xi = b - a_n x^*_n$ can be read for the respective value (s) of x^*_{n-1}. Then using the values of Λ_{n-2}, Λ_{n-3}----Λ_1 (relation 4-17), the values for x^*_{n-i} can be found. All the above can be generalized by the following expression:

$$x^*_{n-i} = \hat{x}_{n-i}\left(b - \sum_{u=0}^{i-1} a_{n-u}\, x^*_{n-u}\right), \quad i = 1,----, n-1 \qquad (4\text{-}18)$$

ATTEMPT TO COMPUTE IDEAL MINIMUM TOTAL COST BY DYNAMIC PROGRAMMING

In the present case, variable production cost plus transportation cost, the optimum size of each plant, the location of these plants and the quantity to be shipped must be determined. The optimum combination is the one that minimizes total costs. This, as was shown in Chapter III, can be expressed by the following mathematical expressions:

$$\sum_{j=1}^{n} a_{ij} x_j = b_j \quad , \; i = 1,------,m$$

$$x_j \geq 0 \quad , \quad j = 1,------, n \quad ; \quad \text{all } x_j \text{ integers,}$$

$$\min Z = \sum_{j=1}^{n} f_j(x_j) \qquad (4\text{-}19)$$

Before entering into the computational details of the problem, one must investigate what happens to the procedure when the problem is increased in size. In the present case as given by (4-19), to define the problem, the one state parameter ξ_i is required for each constraint, but only one control variable, x_k, is required for each stage. This can be seen better if equation (4-19) is expanded:

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \text{------} + a_{1j} x_j + \text{------} + a_{1n} x_n = b_1$$

$$a_{21} x_1 + a_{22} x_2 + \text{------} + \text{------} + a_{2j} x_j + \text{------} + a_{2n} x_n = b_2$$

$$\vdots \qquad \vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \qquad \vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + \text{------} + \text{------} + a_{mj} x_j + \text{------} + a_{mn} x_n = b_m$$

$$x_j \geq 0 \ , \ j = 1,\text{------}, n \ ; \text{ all } x_j \text{ integers},$$

$$\min Z = \sum_{j=1}^{n} f_j(x_j) \qquad (4\text{-}20)$$

The optimum policy, $\hat{x}_1,\text{----}, \hat{x}_k$, will be given, for any stage, by the following expression:

$$\Lambda_k (\xi_1,\text{---- } \xi_m) = \underset{x_1,\text{----},x_k}{\text{minimum}} \sum_{j=1}^{k} f_j(x_j), \quad j = 1,\text{----}, n, \qquad (4\text{-}21)$$

where the minimization is taken over non-negative integers, $x_1,\text{----},x_k$, satisfying

$$\sum_{j=1}^{k} a_{ij} x_j \leq \xi_i \ , \text{ for } i = 1,\text{------}, m \qquad (4\text{-}22)$$

and the recurrent relation then will be given by

$$\Lambda_{k-1} (\xi_1,\text{----}, \xi_m) = \underset{0 \leq x_k \leq k}{\text{minimum}} [f_k(x_k) +$$

$$+ \Lambda_{k-1} (\xi_1 - a_{1k} x_k \ ,\text{----}, \xi_m - a_{mk} x_k)] \qquad (4\text{-}23)$$

$$k = 2,\text{----}, n,$$

and $Z^* = \Lambda_n(b_1, \text{----}, b_m)$, (4-24)

when $\delta_k = \min \{ [\frac{\xi_1}{a_{1k}}, \text{----}, \frac{\xi_m}{a_{mk}}] \}$. (4-25)

where if $\Lambda_k (\xi_1, \xi_2)$ is determined, then $\hat{x}_k(\xi_1, \text{----}, \xi_m)$ is also determined. At the last stage, i.e. n stage, $\Lambda_n(b_1, \text{----}, b_m)$ is determined and consequently x_n^* is also determined. The optimum values of the remaining x_k^* is determined from tables as for (4-18):

$$x_{n-i}^* = \hat{x}_{n-i} \left(b_1 - \sum_{u=0}^{i-1} a_{1,n-u} x^*_{n-u}, \text{----}, b_m - \sum_{u=0}^{i-1} a_{2,n-u} x^*_{n-u}\right), \quad i = 1, \text{----}, n-1 \tag{4-26}$$

The state functions, $\Lambda_k (\xi_1, \text{----}, \xi m)$ are the maximum return from the first k activities when $\xi_1, \text{----}, \xi_m$ units of resources 1,----, m, are available for allocation to these stages. The policy function $\hat{x}_k (\xi_1, \text{----}, \xi_m)$ is the optimal value from all the x_k's when quantities, $\xi_1, \text{----}, \xi_m$, from resources 1,----, m are available at stage k.

COMPUTATIONAL EFFICIENCY OF DYNAMIC PROGRAMMING IN A LARGE ALLOCATION PROBLEM

Now that the recurrent relations have been defined for a n-mulit-stage problem, that is, defined at stage k by m state parameters and one control variable, the method of computation will next be explained.

From the preceding sections of this chapter it can be seen that it is necessary to compute and tablulate Λ_k and $\hat{x}_{k1}$ for every $\xi = 0$,

1,----, b. In this case where m state parameters are given, it is necessary to compute and then tabulate $\Omega\ (x_k,\ \xi_1,\text{----},\ \xi_m)$ for every x_k subjected to the constraints of the equation (4-25), and for the values of $\Lambda_k(\ \xi_1,\text{----},\ \xi_m)$, for every combination of $\xi_1,\text{----},\ \xi_m$ where ξ_1 can assume the values of 0, 1,----, b, and ξ_2 = 0, 1,----, b_2,---- ξ_m = 0, 1,----, b_m.

In the case that a small transportation problem is given, where m = 4, n = 3, and b_1 = 10, b_2 = 8, b_3 = 7, b_4 = 5, b_5 = 11, b_6 = 10, six equations are required, instead of seven, as it was explained in Chapter 3. It is always possible to reduce by one, the number of equations in the transportation problem. The cost matrix and the allocation matrix will then be given by Tables 25, 26, and 27.

TABLE 25

Cost Matrix				Allocation Matrix			
							Available
From	C_{11}	C_{12}	C_{13}	x_{11}	x_{12}	x_{13}	b_1
	C_{21}	C_{22}	C_{23}	x_{21}	x_{22}	x_{23}	b_2
	C_{31}	C_{32}	C_{33}	x_{31}	x_{32}	x_{33}	b_3
	C_{41}	C_{42}	C_{43}	x_{41}	x_{42}	x_{43}	b_4
To	d_1	d_2	d_3	Required b_5	b_6	b_7	

TABLE 26

Restrictions for a 4 x 3 Transportation Problem

$$
\begin{aligned}
x_{11} + x_{12} + x_{13} &= b_1 \\
x_{21} + x_{22} + x_{23} &= b_2 \\
x_{31} + x_{32} + x_{33} &= b_3 \\
x_{41} + x_{42} + x_{43} &= b_4 \\
x_{11} + x_{21} + x_{31} + x_{41} &= b_5 \\
x_{12} + x_{22} + x_{32} + x_{42} &= b_6 \\
x_{13} + x_{23} + x_{33} + x_{43} &= b_7
\end{aligned}
$$

TABLE 27

Transformation of Table 26 with Variables of the Form x_{ij} to Variables of the Form x_k

$$
\begin{aligned}
x_1 + x_2 + x_3 &= b_1 \\
x_4 + x_5 + x_6 &= b_2 \\
x_7 + x_8 + x_9 &= b_3 \\
x_{10} + x_{11} + x_{12} &= b_4 \\
x_1 + x_4 + x_7 + x_{10} &= b_5 \\
x_2 + x_5 + x_8 + x_{11} &= b_6 \\
x_3 + x_6 + x_9 + x_{12} &= b_7
\end{aligned}
$$

Note: $a_{ij} = 1$ for all i and all j. i = 1,----,6 ; j = 1,----,12.

The next step is to find out how many tables must be computed to solve this small transportation problem. The first set of values are: $\xi_1 = 0$, $\xi_2 = 0$, $\xi_3 = 0$, $\xi_4 = 0$, $\xi_5 = 0$ and $\xi_6 = 0$. Then $\Lambda_1(0,0,0,0,0,0,)$ must be tabulated, and $\Omega_1(1,0,0,0,0,0,)$ must be computed. After this, $\Lambda_1(1,0,0,0,0,0)$ must be tabulated, and so on for all possible combinations of $\Lambda_1(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)$. Now the number of entries for table Λ_k are estimated. If each ξ_i can take the values 0, 1,----, b_i, then the number of entries for Λ_k will be given by $b_1 \cdot b_2 \cdot b_3 \cdot b_4 \cdot b_5 \cdot b_6$ or 10 x 8 x 7 x 5 x 11 x 10 = 308,000 entries. To arrive at table $\Lambda_k(\xi_1,----, \xi_m)$ it would be first necessary to make millions of computations for each value of $\Omega_k(x_k, \xi_1;----; x_k, \xi_m)$.

From all the above and preceding sections, the conclusion must be that it is not practical to solve linear or non-linear problems by the Dynamic Programming Approach if to define a state for each stage we need more than two state parameters or more than two control variables.

Several promising approaches are being developed for reduction in dimensionality of dynamic programming problems, such as the use of a Lagrange multiplier, but these it appears have not yet been developed to the point to make them universal or applicable to problems of "allocation" and "transportation" as found in the real world (12).

CHAPTER V – THE PROPOSED METHOD

COMPARISON OF THE TWO METHODS

When the postulated total cost problem (i.e. transportation cost plus variable production cost) was stated algebraically, a technique was devised to adapt the essentially linear problem to a linear type transportation problem. Then a solution became possible by the Simplex Method of Linear Programming.

As long as the assumptions do not infringe upon the required accuracy of the results, the technique developed in this study can be used as a fast, practical and economical method to solve the problems of simultaneous variation in plant size and plant location. However, the solution to the same problem, with the same assumptions, was attempted by the Dynamic Programming Approach, the computational efforts became so large that it became apparent at once that it was not practical, nor economical to solve problems of this type by the Dynamic Programming Technique.

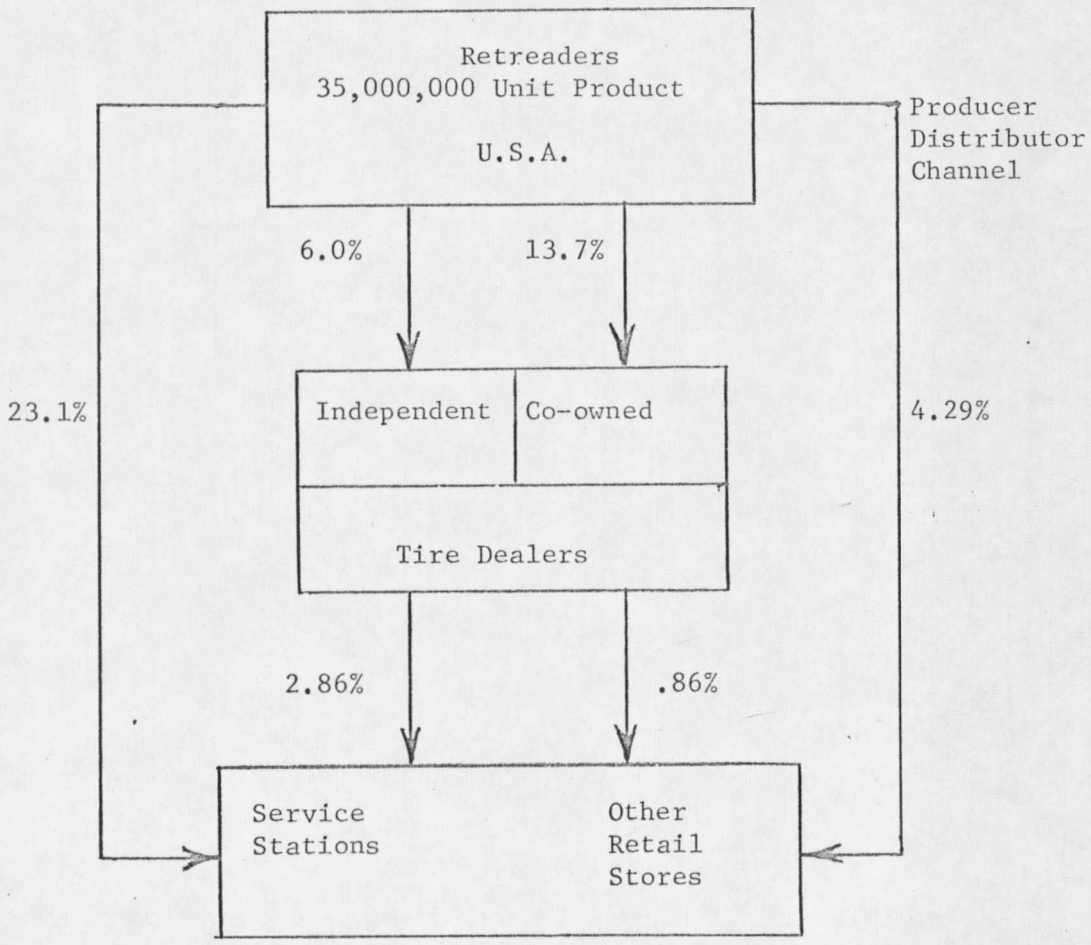
SUGGESTED TECHNIQUE FOR USE IN OPTIMIZATION OF ALLOCATION AND TRANSPORTATION PROBLEMS

From the procedures and results obtained in Chapter 3 and 4, it is evident that the Linear Programming Technique should be used to solve allocation and transportation problems whenever the problems are linear or can be transformed into a Linear Programming Problems.

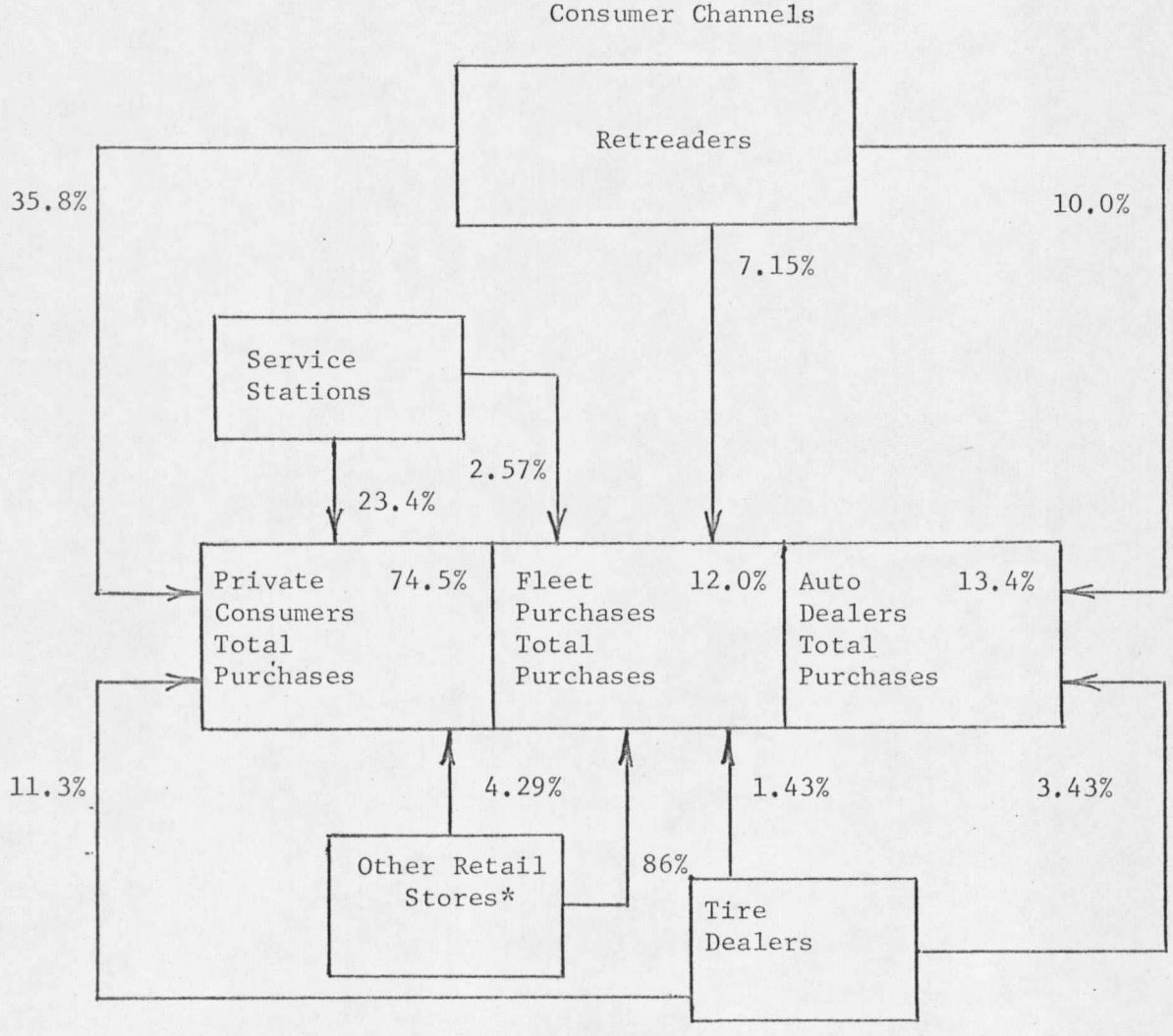
An additional generalization can be drawn: if a non-linear programming problem has more than one control variable or more than one stage parameter, then this problem should be solved by such a technique as Integer Programming, Linear Programming, Gradient Method, etc., rather than by the Dynamic Programming Technique.

APPENDIX A - CHANNELS OF DISTRIBUTION OF PASSENGER TIRE RETREADS

Channels of Distribution of Passenger Tire Retreads and Estimated Percentages Flowing through Each Channel in 1963



Channels of Distribution of Passenger Tire Retreads and Estimated Percentages Flowing through Each Channel in 1963



* Source: Retreaders Survey

APPENDIX B - AVERAGE SELLING PRICES AND PRODUCTION COST ESTIMATION

ESTIMATION OF THE AVERAGE SELLING PRICES

All passenger and non-passenger tires (new and recapped) have a List Retail Price. A policy of the retailers is to make a discount of 20% on passenger retread tires and a 30% discount on non-passenger retread tires. This is the base point of the calculations for the actual selling prices and for the production cost estimations. The next step is to estimate the average List Prices: for the Highway Passenger Tire, Passenger Snow Tire, Highway Truck Tire, and Mud and Snow Truck Tire. To calculate these average List Prices, the following statistical model was used:

Average List Price = (List Price/type of tire) (frequency of sales)

To estimate the production cost, the assumptions were based on (5) who states that the production cost is 50% of the List Selling Prices of middle size shops. The List Selling Prices used, were based on (25). From these tableaus and from the retailers policy, Table 1 was obtained.

Frequency of Sales of Highway Retreaded Passenger Tires

HIGHWAY PASSENGER TIRES

SIZE		LIST PRICE	% OF UNITS SOLD
-	6.00-13	11.80	2.78
-	6.50-13	13.05	2.78
-	7.00-13	13.45	2.78
6.45-14	6.00-14	13.05	2.78
6.95-14	6.50-14	13.60	2.78
7.35-14	7.00-14	13.60	2.78
7.75-14	7.50-14	13.80	25.00

HIGHWAY PASSENGER TIRES (continued)

SIZE		LIST PRICE	% OF UNITS SOLD
8.25-14	8.00-14	15.15	25.00
8.55-14	8.50-14	16.60	2.78
8.85-14	9.00-14	18.55	2.78
9.15-14	9.50-14	19.15	2.78
6.35-15	5.60-15	11.80	2.78
6.85-15	5.90/6.00-15	11.80	2.78
7.35-15	6.40/6.50-15	13.05	2.78
7.75-15	6.70-15	13.80	2.78
8.15-15	7.10-15	15.15	2.78
8.45-15	7.60-15	16.60	2.78
8.85-15	8.00-15	18.55	2.78
9.00-15	8.20-15	19.15	2.78
9.15-15	-	19.15	2.78

PASSENGER SNOW TIRE

SIZE	LIST PRICE	% OF UNITS SOLD
6.50-13	14.75	4.16
7.00-13	15.15	4.16
7.00-14	15.40	4.16
7.50-14	15.65	25.00
8.00-14	17.10	25.00
8.50-14	18.80	4.16
9.00-14	20.95	4.16

PASSENGER SNOW TIRE (continued)

SIZE	LIST PRICE	% OF UNITS SOLD
5.60-15	13.25	4.16
6.00-15	13.35	4.16
6.70-15	15.65	4.16
7.10-15	17.10	4.16
7.60-15	18.80	4.16
8.00-15	20.95	4.16
8.20-15	21.70	4.16

HIGHWAY TRUCK TIRE (TRACTION HI-MILER)

SIZE	LIST PRICE	% OF UNITS SOLD
6.50-16	16.10	8.59
7.00-15 7-17.5	19.35	8.59
7.00-17	19.55	8.59
8.25-20 9-22.5	34.25	8.59
9.00-20 10-22.5	41.35	8.59
10.00-20 11-22.5	45.80	8.59
10.00-22 11-24.5	46.70	40.00
11.00-2- 21-22.5	49.70	8.59

SUPER CROSS RIO HI-MILER (SYNTHETIC)

9.00-20 10-22.5	54.70	20.00
10.00-20 11-22.5	60.25	20.00
10.00-22 11-24.5	61.55	40.00
11.00-20 12-22.5	65.40	20.00

HI-MILER XTRA GRIP

6.50-16	20.00	40.00
7.00-15 7-17.5	24.00	15.00
7.00-16	24.40	15.00
8.25-20 9-22.5	44.35	30.00

APPENDIX C – SOLUTION OF THE UNIT PRODUCTION COST EQUATION

Solution of the unit production cost (C_{P_i}) equation as a function of plant size (s).

From tables 2, 3, 4, and 5, the following tableau was obtained

UNIT PRODUCTION COST				
C_P . dollars	12.90	12.36	11.88	11.42
PLANT SIZE				
s, tires/day	80	300	600	850

There are four points of the curve. Therefore, from these points, a curve of the third order, $y = a + bx + cx^2 + dx^3$ can be found. In the present case the following system of equations has been derived:

$$1.a + 80b + 6{,}400c + 512{,}000d = 42.90$$

$$1.a + 300b + 90{,}000c + 27{,}000{,}000d = 12.36$$

$$1.a + 600b + 360{,}000c + 216{,}000{,}000d = 11.88$$

$$1.a + 850b + 722{,}500c + 614{,}125{,}000d = 11.42$$

This matrix of 4 by 5 was solved in the I.B.M. 1620 computer at the computer center of Montana State University. The program used, SOLUTION OF SYSTEM OF EQUATIONS WITH OPTIONAL MATRIX INVERSION IN OUTPUT, gave the following readout results:

a = .135099E-02

b = -.8235E-02

c = .149E-04

d = -.96E-08

Therefore the unit production cost C_P, equation will be given by the following expression

$$C_P = .135099 \times 10^2 - .8235 \times 10^{-2}\, s + .149 \times 10^{-4}\, s^2 - .96 \times 10^{-8}\, s^3$$

APPENDIX D - PROGRAM INPUT

```
3400032007013600032007024902402511963611300102      COLD START
ZZJOB 5                                ALFREDO VALDERRAMA
ZZXEQ LP1620                          03
INPUT.C02000TRAN
ROW.ID
             COST
            0A1
            0A2
            0A3
            0A4
            0A5
            0A6
            0A7
            0A8
            0A9
            0A10
            0A11
            0A12
            0A13
            0A14
            0A15
            0A16
            0A17
            0B1
            0B2
            0B3
            0B4
            0B5
            0B6
            0B7
            0B8
            0B9.
            0B10
            0B11
            0B12
* COL. ID, ZERO MATRIX ENTRIES OPTIONAL
MATRIX
      X11    COST      11.83
      X11    A1         1.
      X11    B1         1.
      X12    COST      11.84
      X12    A1         1.
      X12    B2         1.
      X13    COST      12.09
      X13    A1         1.
      X13    B3         1.
      X14    COST      12.09
      X14    A1         1.
      X14    B4         1.
      X15    COST      11.90
      X15    A1         1.
      X15    B5         1.
```

X16	COST	11.79
X16	A1	1.
X16	B6	1.
X17	COST	11.42
X17	A1	1.
X17	B7	1.
X18	COST	11.76
X18	A1	1.
X18	B8	1.
X19	COST	11.92
X19	A1	1.
X19	B9	1.
X110	COST	11.80
X110	A1	1.
X110	B10	1.
X111	COST	11.42
X111	A1	1.
X111	B11	1.
X112	COST	100.
X112	A1	1.
X112	B12	1.
X21	COST	12.12
X21	A2	1.
X21	B1	1.
X22	COST	11.96
X22	A2	1.
X22	B2	1.
X23	COST	12.05
X23	A2	1.
X23	B3	1.
X24	COST	11.86
X24	A2	1.
X24	B4	1.
X25	COST	12.11
X25	A2	1.
X25	B5	1.
X26	COST	11.96
X26	A2	1.
X26	B6	1.
X27	COST	11.95
X27	A2	1.
X27	B7	1.
X28	COST	12.00
X28	A2	1.
X28	B8	1.
X29	COST	11.85
X29	A2	1.
X29	B9	1.
X210	COST	11.83
X210	A2	1.
X210	B10	1.
X211	COST	11.42

X211	A2	1.
X211	B11	1.
X212	COST	100.
X212	A2	1.
X212	B12	1.
X31	COST	12.29
X31	A3	1.
X31	B1	1.
X32	COST	12.30
X32	A3	1.
X32	B2	1.
X33	COST	12.55
X33	A3	1.
X33	B3	1.
X34	COST	12.55
X34	A3	1.
X34	B4	1.
X35	COST	12.36
X35	A3	1.
X35	B5	1.
X36	COST	12.25
X36	A3	1.
X36	B6	1.
X37	COST	11.88
X37	A3	1.
X37	B7	1.
X38	COST	12.42
X38	A3	1.
X38	B8	1.
X39	COST	12.38
X39	A3	1.
X39	B9	1.
X310	COST	12.26
X310	A3	1.
X310	B10	1.
X311	COST	12.41
X311	A3	1.
X311	B11	1.
X312	COST	100.
X312	A3	1.
X312	B12	1.
X41	COST	12.58
X41	A4	1.
X41	B1	1.
X42	COST	12.42
X42	A4	1.
X42	B2	1.
X43	COST	12.52
X43	A4	1.
X43	B3	1.
X44	COST	12.32
X44	A4	1.

X44	B4	1.
X45	COST	12.57
X45	A4	1.
X45	B5	1.
X46	COST	12.42
X46	A4	1.
X46	B6	1.
X47	COST	12.41
X47	A4	1.
X47	B7	1.
X48	COST	12.46
X48	A4	1.
X48	B8	1.
X49	COST	12.31
X49	A4	1.
X49	B9	1.
X410	COST	12.29
X410	A4	1.
X410	B10	1.
X411	COST	11.88
X411	A4	1.
X411	B11	1.
X412	COST	100.
X412	A4	1.
X412	B12	1.
X51	COST	12.45
X51	A5	1.
X51	B1	1.
X52	COST	12.51
X52	A5	1.
X52	B2	1.
X53	COST	12.75
X53	A5	1.
X53	B3	1.
X54	COST	12.63
X54	A5	1.
X54	B4	1.
X55	COST	12.29
X55	A5	1.
X55	B5	1.
X56	COST	12.20
X56	A5	1.
X56	B6	1.
X57	COST	12.33
X57	A5	1.
X57	B7	1.
X58	COST	12.21
X58	A5	1.
X58	B8	1.
X59	COST	12.25
X59	A5	1.
X59	B9	1.

X510	COST	12.46
X510	A5	1.
X510	B10	1.
X511	COST	12.42
X511	A5	1.
X511	B11	1.
X512	COST	100.
X512	A5	1.
X512	B12	1.
X61	COST	12.40
X61	A6	1.
X61	B1	1.
X62	COST	12.52
X62	A6	1.
X62	B2	1.
X63	COST	12.76
X63	A6	1.
X63	B3	1.
X64	COST	12.73
X64	A6	1.
X64	B4	1.
X65	COST	11.88
X65	A6	1.
X65	B5	1.
X66	COST	12.28
X66	A6	1.
X66	B6	1.
X67	COST	12.36
X67	A6	1.
X67	B7	1.
X68	COST	12.36
X68	A6	1.
X68	B8	1.
X69	COST	12.40
X69	A6	1.
X69	B9	1.
X610	COST	12.48
X610	A6	1.
X610	B10	1.
X611	COST	12.57
X611	A6	1.
X611	B11	1.
X612	COST	100.00
X612	A6	1.
X612	B12	1.
X71	COST	12.77
X71	A7	1.
X71	B1	1.
X72	COST	12.78
X72	A7	1.
X72	B2	1.
X73	COST	13.03

X73	A7	1.
X73	B3	1.
X74	COST	13.03
X74	A7	1.
X74	B4	1.
X75	COST	12.84
X75	A7	1.
X75	B5	1.
X76	COST	12.73
X76	A7	1.
X76	B6	1.
X77	COST	12.36
X77	A7	1.
X77	B7	1.
X78	COST	12.90
X78	A7	1.
X78	B8	1.
X79	COST	12.86
X79	A7	1.
X79	B9	1.
X710	COST	12.74
X710	A7	1.
X710	B10	1.
X711	COST	12.89
X711	A7	1.
X711	B11	1.
X712	COST	100.
X712	A7	1.
X712	B12	1.
X81	COST	13.06
X81	A8	1.
X81	B1	1.
X82	COST	12.90
X82	A8	1.
X82	B2	1.
X83	COST	12.96
X83	A8	1.
X83	B3	1.
X84	COST	12.80
X84	A8	1.
X84	B4	1.
X85	COST	13.05
X85	A8	1.
X85	B5	1.
X86	COST	12.90
X86	A8	1.
X86	B6	1.
X87	COST	12.89
X87	A8	1.
X87	B7	1.
X88	COST	12.94
X88	A8	1.

X88	B8	1.
X89	COST	12.79
X89	A8	1.
X89	B9	1.
X810	COST	12.77
X810	A8	1.
X810	B10	1.
X811	COST	13.36
X811	A8	1.
X811	B11	1.
X812	COST	100.
X812	A8	1.
X812	B12	1.
X91	COST	12.93
X91	A9	1.
X91	B1	1.
X92	COST	12.99
X92	A9	1.
X92	B2	1.
X93	COST	13.23
X93	A9	1.
X93	B3	1.
X94	COST	13.11
X94	A9	1.
X94	B4	1.
X95	COST	12.77
X95	A9	1.
X95	B5	1.
X96	COST	12.68
X96	A9	1.
X96	B6	1.
X97	COST	12.81
X97	A9	1.
X97	B7	1.
X98	COST	12.69
X98	A9	1.
X98	B8	1.
X99	COST	12.73
X99	A9	1.
X99	B9	1.
X910	COST	12.94
X910	A9	1.
X910	B10	1.
X911	COST	12.90
X911	A9	1.
X911	B11	1.
X912	COST	100.
X912	A9	1.
X912	B12	1.
X101	COST	12.88
X101	A10	1.
X101	B1	1.

```
X102    COST      13.
X102    A10        1.
X102    B2         1.
X103    COST      13.24
X103    A10        1.
X103    B3         1.
X104    COST      13.21
X104    A10        1.
X104    B4         1.
X105    COST      12.36
X105    A10        1.
X105    B5         1.
X106    COST      12.76
X106    A10        1.
X106    B6         1.
X107    COST      12.84
X107    A10        1.
X107    B7         1.
X108    COST      12.84
X108    A10        1.
X108    B8         1.
X109    COST      12.88
X109    A10        1.
X109    B9         1.
X1010   COST      12.96
X1010   A10        1.
X1010   B10        1.
X1011   COST      13.05
X1011   A10        1.
X1011   B11        1.
X1012   COST     100.
X1012   A10        1.
X1012   B12        1.
X11I    COST      12.74
X11I    A11        1.
X11I    B1         1.
X1I2    COST      12.98
X1I2    A11        1.
X1I2    B2         1.
X113    COST      13.22
X113    A11        1.
X113    B3         1.
X114    COST      13.32
X114    A11        1.
X114    B4         1.
X115    COST      12.76
X115    A11        1.
X115    B5         1.
X116    COST      12.90
X116    A11        1.
X116    B6         1.
X117    COST      12.90
```

X117	A11	1.
X117	B7	1.
X118	COST	12.49
X118	A11	1.
X118	B8	1.
X119	COST	13.04
X119	A11	1.
X119	B9	1.
X1110	COST	13.05
X1110	A11	1.
X1110	B10	1.
X1111	COST	13.19
X1111	A11	1.
X1111	B11	1.
X1112	COST	100.
X1112	A11	1.
X1112	B12	1.
X121	COST	13.31
X121	A12	1.
X121	B1	1.
X122	COST	13.32
X122	A12	1.
X122	B2	1.
X123	COST	13.57
X123	A12	1.
X123	B3	1.
X124	COST	13.57
X124	A12	1.
X124	B4	1.
X125	COST	13.38
X125	A12	1.
X125	B5	1.
X126	COST	13.27
X126	A12	1.
X126	B6	1.
X127	COST	12.90
X127	A12	1.
X127	B7	1.
X128	COST	13.44
X128	A12	1.
X128	B8	1.
X129	COST	13.40
X129	A12	1.
X129	B9	1.
X1210	COST	13.28
X1210	A12	1.
X1210	B10	1.
X1211	COST	13.43
X1211	A12	1.
X1211	B11	1.
X1212	COST	100.
X1212	A12	1.

X1212	B12	1.
X131	COST	13.60
X131	A13	1.
X131	B1	1.
X132	COST	13.44
X132	A13	1.
X132	B2	1.
X133	COST	13.54
X133	A13	1.
X133	B3	1.
X134	COST	13.34
X134	A13	1.
X134	B4	1.
X135	COST	13.59
X135	A13	1.
X135	B5	1.
X136	COST	13.44
X136	A13	1.
X136	B6	1.
X137	COST	13.43
X137	A13	1.
X137	B7	1.
X138	COST	13.48
X138	A13	1.
X138	B8	1.
X139	COST	13.33
X139	A13	1.
X139	B9	1.
X1310	COST	13.31
X1310	A13	1.
X1310	B10	1.
X1311	COST	12.90
X1311	A13	1.
X1311	B11	1.
X1312	COST	100.
X1312	A13	1.
X1312	B12	1.
X141	COST	13.47
X141	A14	1.
X141	B1	1.
X142	COST	13.53
X142	A14	1.
X142	B2	1.
X143	COST	13.77
X143	A14	1.
X143	B3	1.
X144	COST	13.65
X144	A14	1.
X144	B4	1.
X145	COST	13.31
X145	A14	1.
X145	B5	1.

```
X146   COST      13.22
X146   A14        1.
X146   B6         1.
X147   COST      13.35
X147   A14        1.
X147   B7         1.
X148   COST      13.23
X148   A14        1.
X148   B8         1.
X149   COST      13.27
X149   A14        1.
X149   B9         1.
X1410  COST      13.48
X1410  A14        1.
X1410  B10        1.
X1411  COST      13.44
X1411  A14        1.
X1411  B11        1.
X1412  COST     100.
X1412  A14        1.
X1412  B12        1.
X151   COST      13.42
X151   A15        1.
X151   B1         1.
X152   COST      13.54
X152   A15        1.
X152   B2         1.
X153   COST      13.78
X153   A15        1.
X153   B3         1.
X154   COST      13.75
X154   A15        1.
X154   B4         1.
X155   COST      12.90
X155   A15        1.
X155   B5         1.
X156   COST      13.30
X156   A15        1.
X156   B6         1.
X157   COST      13.38
X157   A15        1.
X157   B7         1.
X158   COST      13.38
X158   A15        1.
X158   B8         1.
X159   COST      13.42
X159   A15        1.
X159   B9         1.
X1510  COST      13.50
X1510  A15        1.
X1510  B10        1.
X1511  COST      13.57
```

```
X1511 A15        1.
X1511 B11        1.
X1512 COST     100.
X1512 A15        1.
X1512 B12        1.
X161  COST     13.20
X161  A16        1.
X161  B1         1.
X162  COST     13.52
X162  A16        1.
X162  B2         1.
X163  COST     13.26
X163  A16        1.
X163  B3         1.
X164  COST     13.86
X164  A16        1.
X164  B4         1.
X165  COST     13.30
X165  A16        1.
X165  B5         1.
X166  COST     13.44
X166  A16        1.
X166  B6         1.
X167  COST     13.44
X167  A16        1.
X167  B7         1.
X168  COST     13.53
X168  A16        1.
X168  B8         1.
X169  COST     13.57
X169  A16        1.
X169  B9         1.
X1610 COST     13.58
X1610 A16        1.
X1610 B10        1.
X1611 COST     13.73
X1611 A16        1.
X1611 B11        1.
X1612 COST     100.
X1612 A16        1.
X1612 B12        1.
X171  COST     13.50
X171  A17        1.
X171  B1         1.
X172  COST     13.56
X172  A17        1.
X172  B2         1.
X173  COST     13.68
X173  A17        1.
X173  B3         1.
X174  COST     13.54
X174  A17        1.
X174  B4         1.
```

```
    X175   COST      13.42
    X175   A17        1.
    X175   B5         1.
    X176   COST      13.28
    X176   A17        1.
    X176   B6         1.
    X177   COST      13.40
    X177   A17        1.
    X177   B7         1.
    X178   COST      13.31
    X178   A17        1.
    X178   B8         1.
    X179   COST      12.90
    X179   A17        1.
    X179   B9         1.
    X1710  COST      13.37
    X1710  A17        1.
    X1710  B10        1.
    X1711  COST      13.33
    X1711  A17        1.
    X1711  B11        1.
    X1712  COST     100.
    X1712  A17        1.
    X1712  B12        1.
    X181   COST      13.38
    X181   B1         1.
    X182   COST      13.44
    X182   B2         1.
    X183   COST      13.68
    X183   B3         1.
    X184   COST      13.59
    X184   B4         1.
    X185   COST      13.30
    X185   B5         1.
    X186   COST      12.90
    X186   B6         1.
    X187   COST      13.77
    X187   B7         1.
    X188   COST      13.32
    X188   B8         1.
    X189   COST      13.28
    X189   B9         1.
    X1810  COST      13.40
    X1810  B10        1.
    X1811  COST      13.44
    X1811  B11        1.
    X1812  COST     100.
    X1812  B12        1.
FIRST.B
```

```
A1        875.
A2        875.
A3        625.
A4        625.
A5        625.
A6        625.
A7        350.
A8        350.
A9        350.
A10       350.
A11       350.
A12       140.
A13       140.
A14       140.
A15       140.
A16       140.
A17       140.
B1        118.5
B2         39.5
B3        101.5
B4         42.9
B5        124.1
B6         61.4
B7        131.8
B8         78.0
B9         69.0
B10        33.9
B11       148.0
B12      6031.40

ENDATA
ASSIGN      444
MIN....   COST
OUTPUT
CHECK.
ENDJOB
```

APPENDIX E - PROGRAM OUTPUT

```
MIN....      COST
 ITER  VARBL.  EXIT  VARBL. ENTR  CNT  OBJ. FUNCTION
 0001  B12     U.B.  X1812        029    603140.000
 0002  A2      U.B.  X212         028    603140.000
 0003  A1      U.B.  X112         027    603140.000
 0004  A6      U.B.  X612         026    603140.000
 0009  A10     U.B.  X1012        021    603140.000
 0010  A9      U.B.  X912         020    603140.000
 0011  A8      U.B.  X812         019    603140.000
 0012  A7      U.B.  X712         018    603140.000
 0013  B11     U.B.  X211         017    604830.160
 0014  A17     U.B.  X1712        016    604830.160
 0015  A16     U.B.  X1612        015    604830.160
* MAX ERROR =          .00000000
 0016  A15     U.B.  X1512        015    604830.160
 0017  A14     U.B.  X1412        014    604830.160
 0018  A13     U.B.  X1312        013    604830.160
 0019  A12     U.B.  X1212        012    604830.160
 0020  B7      U.B.  X17          011    606335.310
 0021  B5      U.B.  X65          010    607809.610
 0022  B1      U.B.  X11          009    609211.460
 0023  B3      U.B.  X23          008    610434.530
 0024  B8      U.B.  X18          007    611351.810
 0025  B9      U.B.  X29          006    612169.460
 0026  B6      U.B.  X16          005    612893.360
 0027  B4      U.B.  X24          003    613402.150
 0028  B2      U.B.  X12          002    613869.830
 0029  B10     U.B.  X110         001    614269.850
OUTPUT
BASIS.                MANTISSA 08    TOLERANCES  05 03 03 03
VARBLS  TYPE  NAME  ACTIVITY LEVEL
            FX11          118.500
            FX12           39.500
            FX16           61.400
            FX17          131.800
            FX18           78.000
            FX110          33.900
            FX112         411.900
            FX23          101.500
            FX24           42.900
            FX29           69.000
            FX211         148.000
            FX212         513.600
            FX312         625.000
            FX412         625.000
            FX512         625.000
            FX65          124.100
            FX612         500.900
            FX712         350.000
            FX812         350.000
            FX912         350.000
```

	FX1012	350.000		00056
	FX1112	350.000		00057
	FX1212	140.000		00058
	FX1312	140.000		00059
	FX1412	140.000		00060
	FX1512	140.000		00061
	FX1612	140.000		00062
	FX1712	140.000		00063
	FX1812	140.000		00064
SLACKS	TYPE NAME	ACTIVITY LEVEL	SIMPLEX MULT.	00065
	FCOST	614269.850		00066
	OGA1	.000	.000	00067
	OGA2	.000	.000	00068
	OGA3	.000	.000	00069
	OGA4	.000	.000	00070
	OGA5	.000	.000	00071
	OGA6	.000	.000	00072
	OGA7	.000	.000	00073
	OGA8	.000	.000	00074
	OGA9	.000	.000	00075
	OGA10	.000	.000	00076
	OGA11	.000	.000	00077
	OGA12	.000	.000	00078
	OGA13	.000	.000	00079
	OGA14	.000	.000	00080
	OGA15	.000	.000	00081
	OGA16	.000	.000	00082
	OGA17	.000	.000	00083
	OGB1	.000	11.830	00084
	OGB2	.000	11.840	00085
	OGB3	.000	12.050	00086
	OGB4	.000	11.860	00087
	OGB5	.000	11.880	00088
	OGB6	.000	11.790	00089
	OGB7	.000	11.420	00090
	OGB8	.000	11.760	00091
	OGB9	.000	11.850	00092
	OGB10	.000	11.800	00093
	OGB11	.000	11.420	00094
	OGB12	.000	100.000	00095
CHECK.				00096
CHECK.	ROW NAME	UPPER LIMIT	SOL. VALUE	ROW ERROR 00097
	COST		614269.870	.02000000- 00098
	A1	875.000	875.000	.00000000 00099
	A2	875.000	875.000	.00000000 00100
	A3	625.000	625.000	.00000000 00101
	A4	625.000	625.000	.00000000 00102
	A5	625.000	625.000	.00000000 00103
	A6	625.000	625.000	.00000000 00104
	A7	350.000	350.000	.00000000 00105
	A8	350.000	350.000	.00000000 00106

A9	350.000	350.000	.00000000	00107
A10	350.000	350.000	.00000000	00108
A11	350.000	350.000	.00000000	00109
A12	140.000	140.000	.00000000	00110
A13	140.000	140.000	.00000000	00111
A14	140.000	140.000	.00000000	00112
A15	140.000	140.000	.00000000	00113
A16	140.000	140.000	.00000000	00114
A17	140.000	140.000	.00000000	00115
B1	118.500	118.500	.00000000	00116
B2	39.500	39.500	.00000000	00117
B3	101.500	101.500	.00000000	00118
B4	42.900	42.900	.00000000	00119
B5	124.100	124.100	.00000000	00120
B6	61.400	61.400	.00000000	00121
B7	131.800	131.800	.00000000	00122
B8	78.000	78.000	.00000000	00123
B9	69.000	69.000	.00000000	00124
B10	33.900	33.900	.00000000	00125
B11	148.000	148.000	.00000000	00126
B12	6031.400	6031.400	.00000000	00127
* MAX ERROR =	.02000000-			00128
ENDJOB				00129

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