



Identification of the failure mechanism of a welded connection
by James Harlan King

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
in Civil Engineering

Montana State University

© Copyright by James Harlan King (1981)

Abstract:

An analytical study of a statically indeterminate welded connection was performed for the purpose of identifying the failure mechanism. The reactions in the connection were determined by the direct stiffness method. The weld stiffness coefficients were derived by applying Castigliano's second theorem to a curved beam model of the weld and the contact stiffness coefficient was determined from misaligned roller bearing equations. Stresses in the weld were estimated from the curved beam formula. It was discovered that stresses at the inner radius of the weld were high.

The failure mechanism was identified as the low rotational stiffness of the root contact reaction which allowed the weld to rack. The higher rotational stiffness of the weld then carried a larger proportion of the moment which induced high stresses at the inner radius.

STATEMENT OF PERMISSION TO COPY

In presenting this thesis in partial fulfillment of the requirements for an advanced degree at Montana State University, I agree that the Library shall make it freely available for inspection. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by my major professor, or, in his absence, by the Director of Libraries. It is understood that any copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Signature James H. King

Date Feb. 19, 1981

IDENTIFICATION OF THE FAILURE MECHANISM
OF A WELDED CONNECTION

by

JAMES HARLAN KING

A thesis submitted in partial fulfillment
of the requirements for the degree

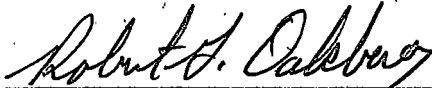
of

MASTER OF SCIENCE

in

Civil Engineering

Approved:



Chairperson, Graduate Committee



Head, Major Department



Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

February 1981

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to Dr. Robert G. Oakberg under whose guidance this investigation was made.

In addition, the author wishes to thank Wallace F. Jones, vice president of the Rapids Machinery Company, for providing the problem and support required to investigate it.

TABLE OF CONTENTS

	<u>Page</u>
VITA.....	ii
ACKNOWLEDGMENTS.....	iii
TABLE OF CONTENTS.....	iv
LIST OF TABLES.....	v
LIST OF FIGURES.....	vi
ABSTRACT	viii
INTRODUCTION.....	1
MECHANICAL PROPERTIES OF STAINLESS STEEL.....	6
APPROXIMATE LOAD ANALYSIS.....	8
THE CONTACT STRESS FAILURE MECHANISM.....	17
Derivation of Stiffness Coefficients.....	24
Stiffness Coefficient in Tension.....	28
Stiffness Coefficient in Vertical Shear.....	32
Stiffness Coefficient for Moment.....	34
Stiffness Coefficient for Horizontal Shear.....	36
Stiffness Coefficient for Contact.....	38
Order of Magnitude Study of the Stiffness Coefficients.....	41
Solution of the Equilibrium Equations.....	42
Stress Components from Mainshaft Bending.....	45
Shear Stress Components from Torsion.....	47
WELD RACKING FROM ARM ROTATION.....	50
Equivalent Forces on a Unit Length Weld.....	50
Influence of Arm Flexure.....	54
CONCLUSION	55
RECOMMENDATIONS.....	56
REFERENCES CITED.....	57

LIST OF TABLES

<u>Table</u>	<u>Page</u>
1 Mechanical Properties	7
2 Perpendicular Blade Forces for the Datum Position	12
3 Blade Forces and Bearing Reactions for All Load Positions.	13
4 Bending Moments at Station 9	16

LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
1 Interior of a typical continuous mixer.....	2
2 Cross section of a failed shaft	3
2a Beach marks on the failed cross section.....	4
3 Torque arm reducer drive	9
4 Machinist's drawing of a shaft that failed in service.....	11
5 Applied loads and bearing reactions for the 270 degree load position (pounds)	14
6 Perpendicular blade forces translated to the mainshaft (pounds)	15
7 Cross section of the arm to mainshaft welded connection.....	18
8a Plan view of the weld	19
8b Cross section of the weld.....	19
9 Planar analogy of the arm and blade force reactions	21
10 Rigid body relationships of the weld and contact reactions.....	23
11 Curved beam representation of a fillet weld	25
12 Curved beam geometry	27
13 Freebody diagrams for the derivation of weld stiffness coefficients	29
14 Displacement relationship for a toroidal weld segment	31
15 Horizontal shear relationships	37
16 Misaligned roller bearing geometry	39
17 Geometry of the drilled cross section	46

<u>Figure</u>	<u>Page</u>
18a Stress state in the weld	49
18b Principal stresses in the weld	49
19 Freebody diagram of the arm and mainshaft connection.	51
20 Maximum forces on a unit length weld	53

ABSTRACT

An analytical study of a statically indeterminate welded connection was performed for the purpose of identifying the failure mechanism. The reactions in the connection were determined by the direct stiffness method. The weld stiffness coefficients were derived by applying Castigliano's second theorem to a curved beam model of the weld and the contact stiffness coefficient was determined from misaligned roller bearing equations. Stresses in the weld were estimated from the curved beam formula. It was discovered that stresses at the inner radius of the weld were high.

The failure mechanism was identified as the low rotational stiffness of the root contact reaction which allowed the weld to rack. The higher rotational stiffness of the weld then carried a larger proportion of the moment which induced high stresses at the inner radius.

INTRODUCTION

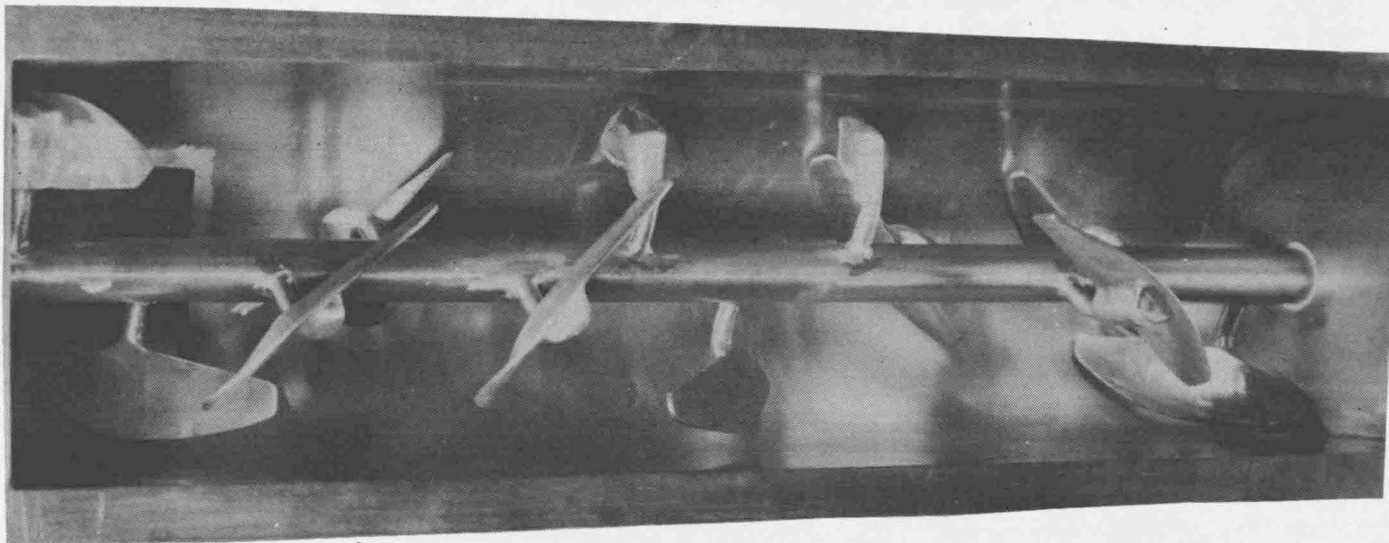
The Rapids Machinery Company of Marion, Iowa, designs and manufactures mixers for the chemical processing industries. Mixers manufactured by the company have performed satisfactorily. However, the shafts of a few mixers have failed after extended periods of service. Such failures are infrequent, but costly, since the Rapids Machinery Company must modify the mixer to accept a redesigned shaft.

A shaft that failed in service is shown in Figure 1. The shaft was mounted in a continuous mixer which operated approximately 8.5 million revolutions. An approximate analysis of the shaft revealed that the major principal stress in the shaft was 45 ksi, which exceeded the material's endurance limit of 39 ksi (Graham, 1978). The diameter of the shaft was increased from 2-7/8 inches to 3½ inches; unfortunately, the larger shaft also failed in service. Failure of the larger shaft suggested that the failure mechanism was not caused by an undersized main shaft.

The purpose of this project is to identify the failure mechanism and to suggest alternative designs that would increase the shaft life.

First, the fracture surface was inspected for clues to the failure mechanism. Figure 2 shows the top of a hole that accepts a mixer arm. The mixer arm is attached by bringing it to bear against the side of the hole and welding it to the shaft. Faint beach marks that radiate from the top of the hole are visible in Figure 2a. These marks suggested that the failure mechanism was initiated at the point of contact between the arm and the rim of the hole. The contact stress, when combined with the torsional and flexural stresses in the shaft, could have developed a maximum normal stress that exceeded the endurance limit.

The proportion of the total load supported by the contact stress depends on the relative stiffnesses of the shaft and weld. The relative stiffnesses of the weld were estimated



2

Figure 1. Interior of a typical continuous mixer.

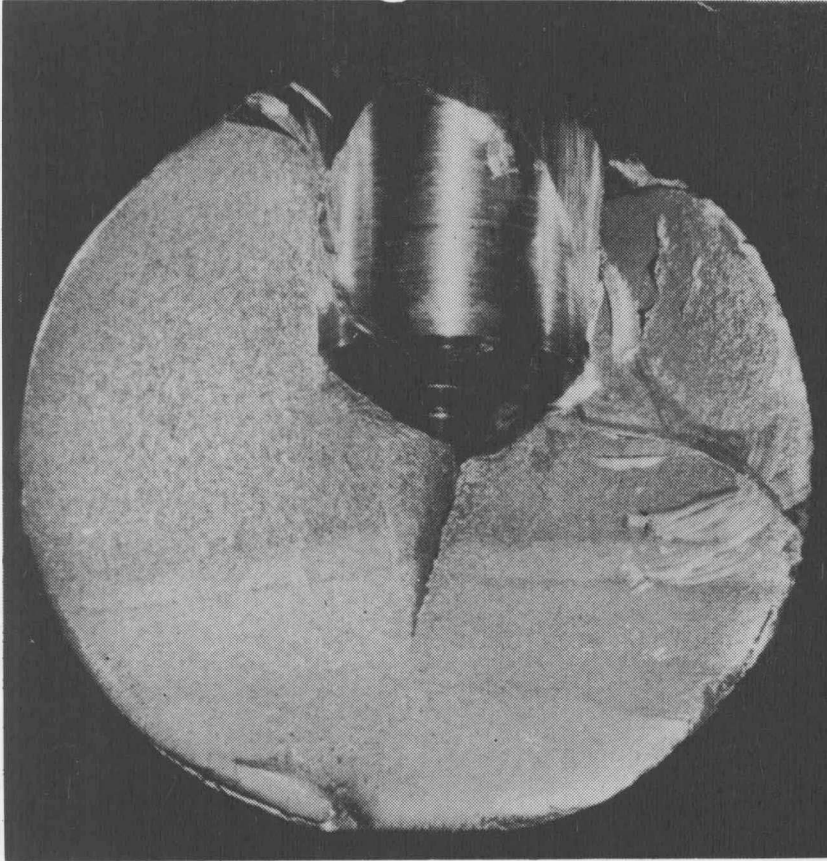


Figure 2. Cross section of a failed shaft.

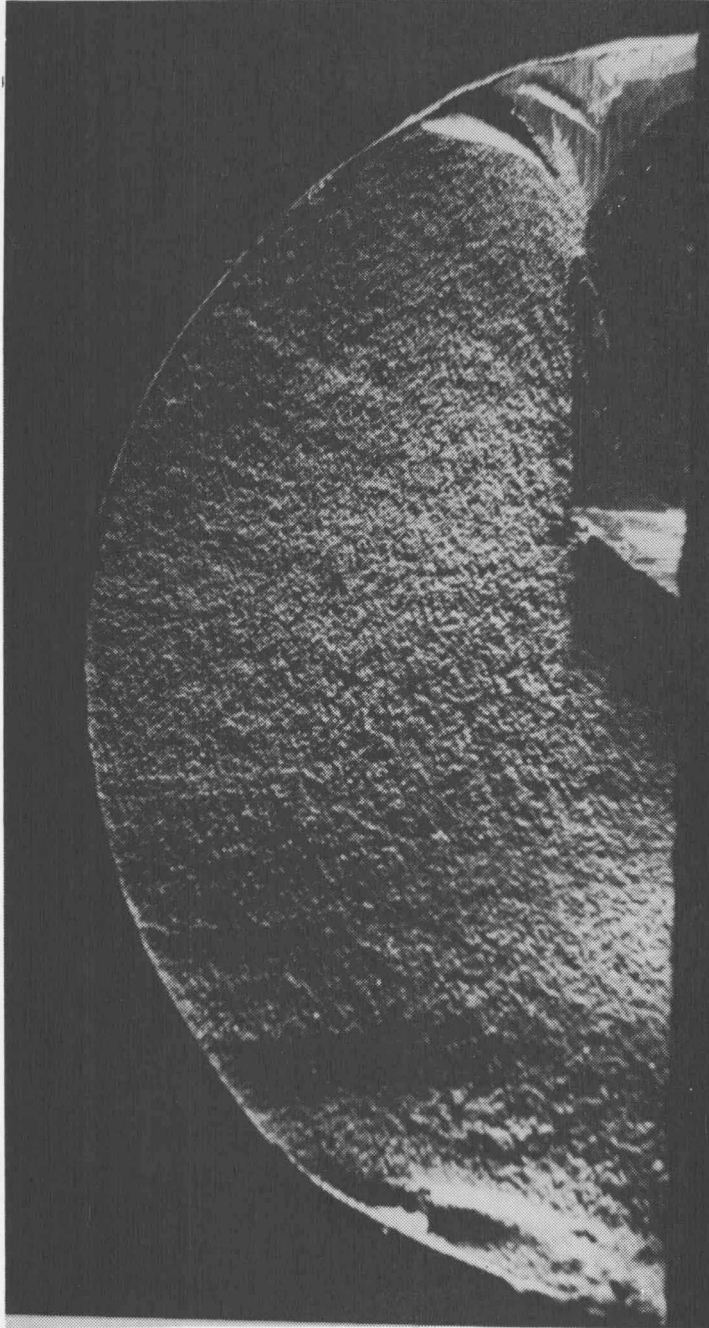


Figure 2a. Beach marks on the failed cross section.

with curved beam equations modified for the circular shape of the weld. The analogy of a roller bearing provided an estimate of the contact stiffness with the hole rim. The misaligned roller bearing analogy was also used to estimate the contact force at the hole root. Compatibility equations were derived, assuming that the arm segment within the hole rotated and translated as a rigid body.

The force and displacement equations were solved simultaneously and the contact stress calculated from the contact force. The contact stress at the hole rim was found to be insignificant.

Although the contact stress was small, it was evident that the weld stresses from arm rotation and displacement could be high enough to initiate failure. The weld stresses were determined by imposing the displacements at the critical section on a unit length curved beam. The stiffness coefficients for the unit length curved beam were calculated and multiplied by the displacements to provide forces and moments. The stress at the inner radius was calculated by the curved beam formula. It was found that weld racking does produce stresses of significant magnitude to cause yielding and ultimately failure.

The second moment area theorem was applied to the freebody diagram to determine the influence of arm flexure on weld racking. It was determined that flexure did contribute to the failure.

Two factors combined to produce the failure mechanism. First, the root contact deflection allowed the arm to rotate rigidly which induced high stresses in the weld. Second, elastic flexure of the arm rotated the arm in the weld which induced additional stresses in the weld.

MECHANICAL PROPERTIES OF STAINLESS STEEL

Types 304 and 316 austenitic stainless steel are used in chemical processing mixers. Both types are easily welded (Linnert, 1977:232). Type 308 stainless steel rod is used to join type 304 base metal and type 316 rod is used to join type 316 base metal. The submerged metal arc welding method is commonly used and provides joints that withstand heavy fatigue loads (Zapfee, 1977:9).

Research has indicated that the weld metal generally has better fatigue resistance than the base metal (Shahinian, 1972:5325). The fatigue resistance is attributed to the high tensile strength, ductility, and hardness of the metal (Zapfee, 1977:9).

The endurance limits are 35,000 pounds per square inch for type 304 stainless steel and 39,000 pounds per square inch for type 316 stainless steel (Zapfee, 1977:11). The endurance limit is defined as the stress at which 50% of the test specimens will survive ten million stress reversals (Boresi, 1978:147).

In general, the fatigue mechanism involves cyclic strain hardening until all of the ductility is expended. The brittle fracture takes place as microscopic shear failures (Sandor, 1972:142).

Table 1 provides the static mechanical properties of stainless steel used for mixer shafts (Shahinian, 1972:5285; Parker, 1977:96).

Table 1. Mechanical Properties

Material	Temperature F	0.2% Yield Strength ksi	Compression		Uniform Elongation %	Poissons Ratio
			Ultimate Strength ksi	Reduction of Area %		
304 Plate	77	37.0 ^(a)	83.0 ^(a)	65	57 ^(b)	.24
316 Plate	77	44.1 ^(a)	82.1 ^(a)	70	68 ^(b)	.26
308 Weld	77	66.0	107.4	65 plate	65	
316 Weld	77	50.9	98.1		73	

(a) Tensile data.

(b) Tensile total elongation.

APPROXIMATE LOAD ANALYSIS

The loads on the mixer blades are not known. However, a plausible load distribution can be estimated from the following parameters:

1. Motor horsepower
2. Shaft speed
3. Length, diameter and density of the mainshaft
4. Bearing locations
5. Distance from the shaft center to the blade tip
6. Number of blades
7. Blade locations
8. Blade pitch
9. Gear reducer weight

A typical torque arm reducer is shown in Figure 3. The mixer had a ten horsepower motor mounted on a 380 pound reducer drive. The entire drive weight was carried by the 2-7/8 inch diameter mainshaft into the bearings. The mainshaft was driven at 33 revolutions per minute.

The torque on the shaft was calculated from the motor horsepower and shaft rotational speed. The equation is

$$\begin{aligned}\text{Torque} &= \text{horsepower} \frac{33000 \text{ ft-lbs}}{\text{HP minute}} \frac{1}{\text{RPM}} \frac{\text{Rev}}{2\pi \text{ radians}} \\ &= 5252 \text{ horsepower/RPM} \\ &= 5252 (10/33) \\ \text{Torque} &= 1591 \text{ ft-lbs}\end{aligned}$$

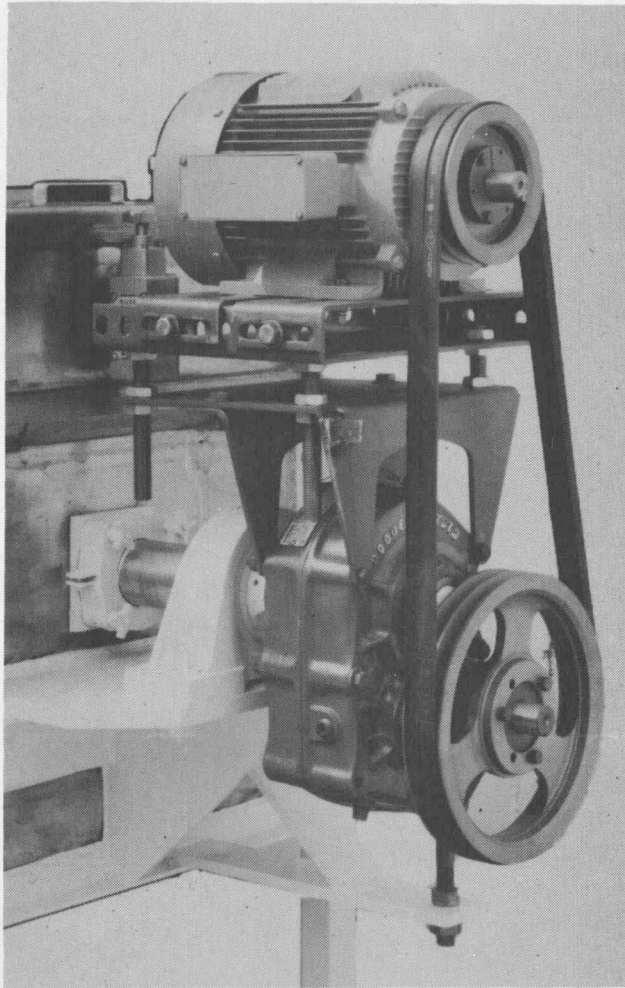


Figure 3. Torque arm reducer drive.

A plan view of the shaft that failed in service is shown in Figure 4. The position in Figure 4 shall be considered to be a datum position. The loads on each blade depend on the orientation of the shaft. Each 90° , 180° , and 270° rotation of the shaft from the datum position provides a separate load position. The shaft rotation is clockwise when one views the shaft from the keyway in Figure 4.

The material to be mixed is dropped in at station 15. Blades 15 and 14 are pitched at 45° so that the material moves from right to left. Blade 13 is pitched so that the flow is reversed. Each set of three blades continues the process until the mixed material discharges out of the bottom at station one.

It was assumed that the forces on blades sweeping downward and across the bottom balanced a major portion of the applied torque. Forces on blades sweeping upward and across the top balanced a minor portion of the torque.

The shaft was not symmetrical because there were three blades in one quadrant and four blades in the three remaining quadrants. Intuitively, one would expect that three downsweeping blades would carry more force per blade than four downsweeping blades. The following method was used in order to obtain such a load distribution.

The datum position in Figure 4 shows four blades sweeping downward. The total 1591 ft-lb torque was distributed to the four blades, which resulted in 397.7 ft-lbs per blade. There were three blades sweeping across the bottom. The total 1591 ft-lbs torque was distributed to the three blades and resulted in 530 ft-lbs per blade.

One would also expect the upward sweeping and topward sweeping blades to carry comparatively smaller loads. Those loads were estimated by dividing the total 1591 ft-lbs

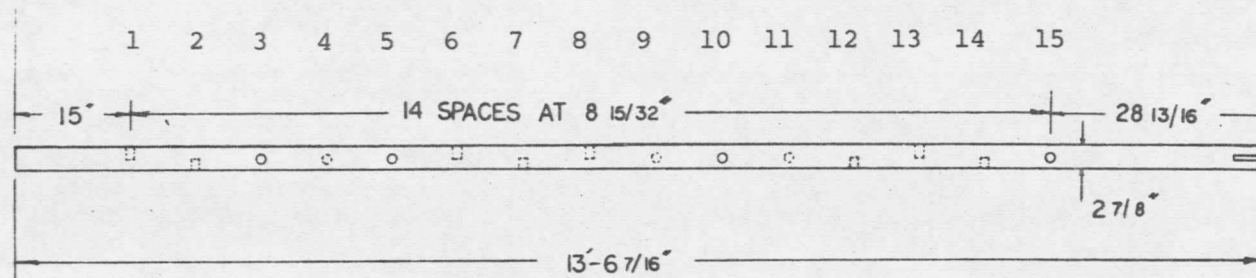


Figure 4. Machinist's drawing of a shaft that failed in service.

torque by fifteen blades, which provided 106 ft-lbs per blade. The 106 ft-lbs average was assigned to the eight upward and topward sweeping blades.

The total torque on the shaft was summed and the result was 4030 ft-lbs. Since 1591 ft-lbs was the maximum possible torque, a reduction factor was calculated by dividing 1591 by 4030 which equals .395. The 397.7, 530 and 106 ft-lbs per blade torques were multiplied by the .395 reduction factor to give the final torques for each blade. They were

157 ft-lbs downsweep

209 ft-lbs bottomsweep

41.8 ft-lbs upsweep

41.8 ft-lbs topsweep

The distance from the shaft center to the extreme blade edge was 12 inches. It was assumed the force on the blade was applied two inches from the extreme edge, which is 10 inches from the shaft center. The component of the blade force that acts perpendicular to the mainshaft was the blade torque divided by the 10/12 ft moment arm. The blade forces are given in Table 2.

Table 2. Perpendicular Blade Forces for the Datum Position

Sweep Position	Torque per Blade (ft-lbs)	Moment Arm (ft)	Blade Force (lbs)
Down	157	10/12	189
Bottom	209	10/12	250
Up	41.8	10/12	50
Top	41.8	10/12	50

The blades were pitched 45° . Therefore, there were force components parallel to the mainshaft. The magnitude of the force parallel to the main shaft was assumed equal to the force perpendicular to the mainshaft. Forces parallel to the mainshaft contribute to mainshaft bending but do not affect the total torque.

Similar calculations were carried out for the three remaining load positions so that the horizontal and vertical bearing reactions could be determined. Table 3 gives the arm loads and bearing reactions for the four load positions. The stationary shaft coordinate system, applied loads, and bearing reactions are shown in Figure 5 for the 270° load position.

Table 3. Blade Forces and Bearing Reactions for All Load Positions

Load Position	Sweep				Bearing Reactions, lbs			
	Down	Bottom	Up	Top	Station 1		Station 15	
	Force lbs				Fx	Fy	Fx	Fy
Datum	189	250	50	50	277	-65	272	480
90°	194	194	51	51	368	-54.3	204	401
180°	194	194	51	51	334	36	289	363
270°	250	289	50	50	250	-3	306	418

Consider the 270° load position. The perpendicular blade forces are translated to the mainshaft in Figure 6 with the associated equivalent couples omitted for clarity. In the YZ plane the down-sweeping blades forced the shaft into negative curvature. The weight of the reducer drive increased the negative curvature and blade forces parallel to the mainshaft applied concentrated couples to the shaft. The weight of the shaft acted to reduce the negative curvature and consequently the negative moment.

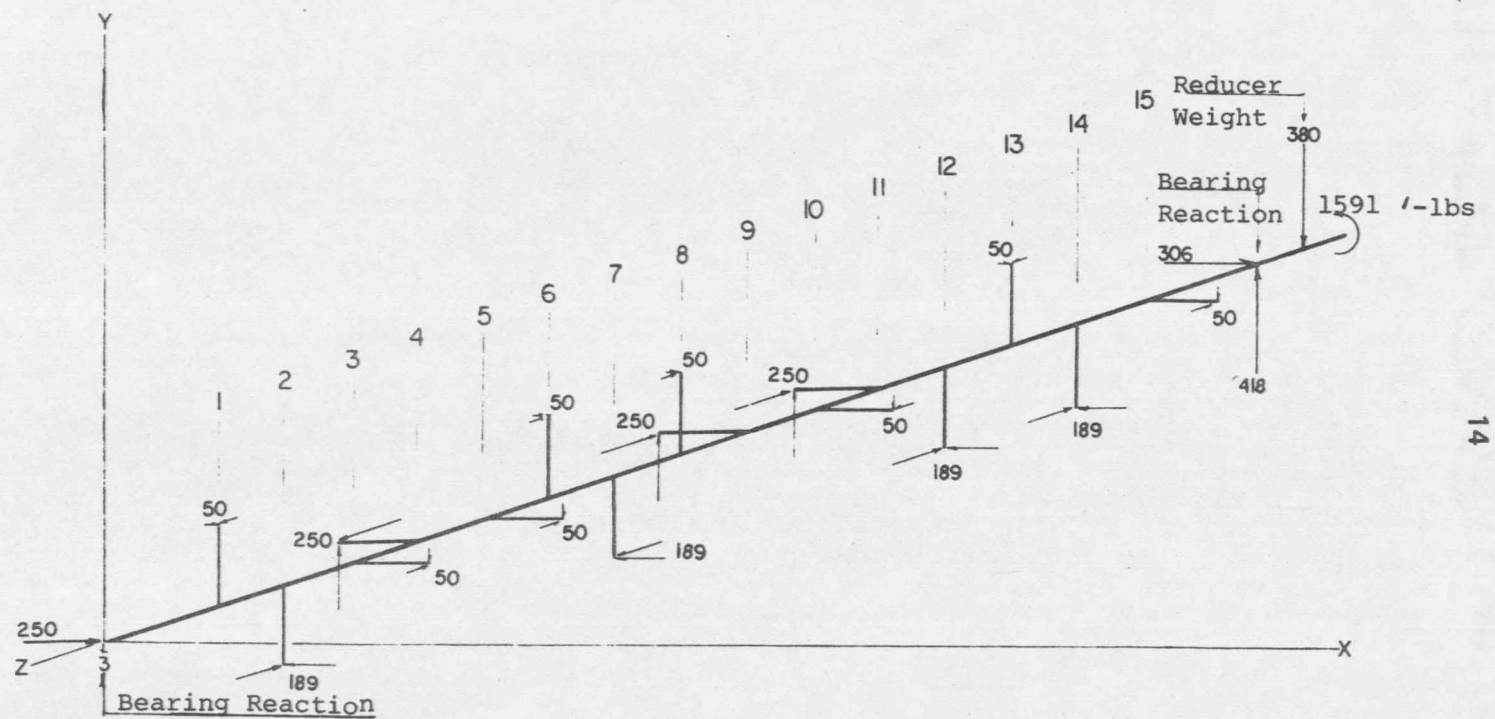


Figure 5. Applied loads and bearing reactions for the 270 degree load position (pounds).

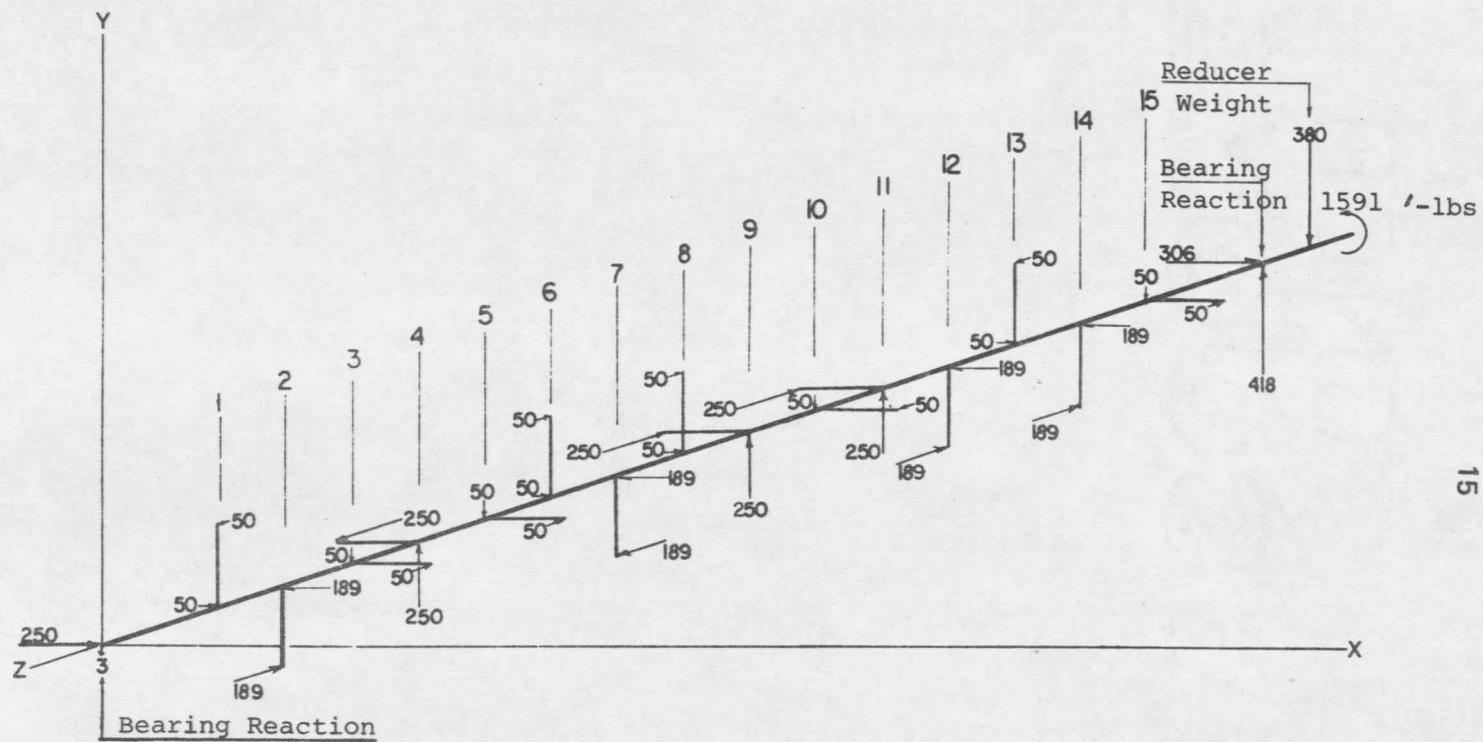


Figure 6. Perpendicular blade forces translated to the mainshaft (pounds).

In the XZ plane the bottom-sweeping blades forced the shaft into negative curvature. Concentrated couples were also present but the shaft weight and gear reducer weight were not.

Bending moments were summed at each arm for the four load positions. The maximum bending moments were, in general, at station 9. Table 4 lists the bending moments about the stationary X and Y axis for each load position.

Table 4. Bending Moments at Station 9

Load Position	Mx (in-lbs)	My (in-lbs)
Datum	2548	14200
90°	9264	9531
180°	9108	13200
270°	8278	14400

THE CONTACT STRESS FAILURE MECHANISM

The contact force and corresponding contact stress in the weld was calculated at station 9. A cross section of the welded arm-to-shaft connection is shown in Figure 7. The mainshaft was drilled one inch deep and reamed to accept a one inch diameter arm. A force was applied to the blade so that the arm would firmly contact the hole rim and hole root while the arm was welded into place. Several passes were made with the welding rod to build up a fillet approximately the size and shape of that shown in Figure 7. The weld flux was chipped off and the weld was wire brushed between passes.

The weld force and root contact force act as an indeterminate system of reactions that balance the blade force. The blade force and weld reactions perpendicular to the shaft are shown in Figure 8. The circular shape of the weld is the primary reason for the complexity of the system.

Part of the weld reaction is transverse. In Figure 8 the transverse reaction is labeled tension and compression. Due to symmetry, the force in tension and compression are equal and will be discussed collectively as tension in the weld.

Part of the weld force acts parallel to the blade force. That portion of the weld is labeled as horizontal shear in Figure 8. Horizontal shear acts primarily at the location shown in Figure 8a, which is adjacent to the critical section.

A vertical shear reaction occurs when the arm rotates under load. Vertical shear resists the rotation by forming a couple.

Arm rotation also induces a rotation of the weld faces. An analogous problem is the application of a concentrated moment to the end of a cantilevered curved beam.

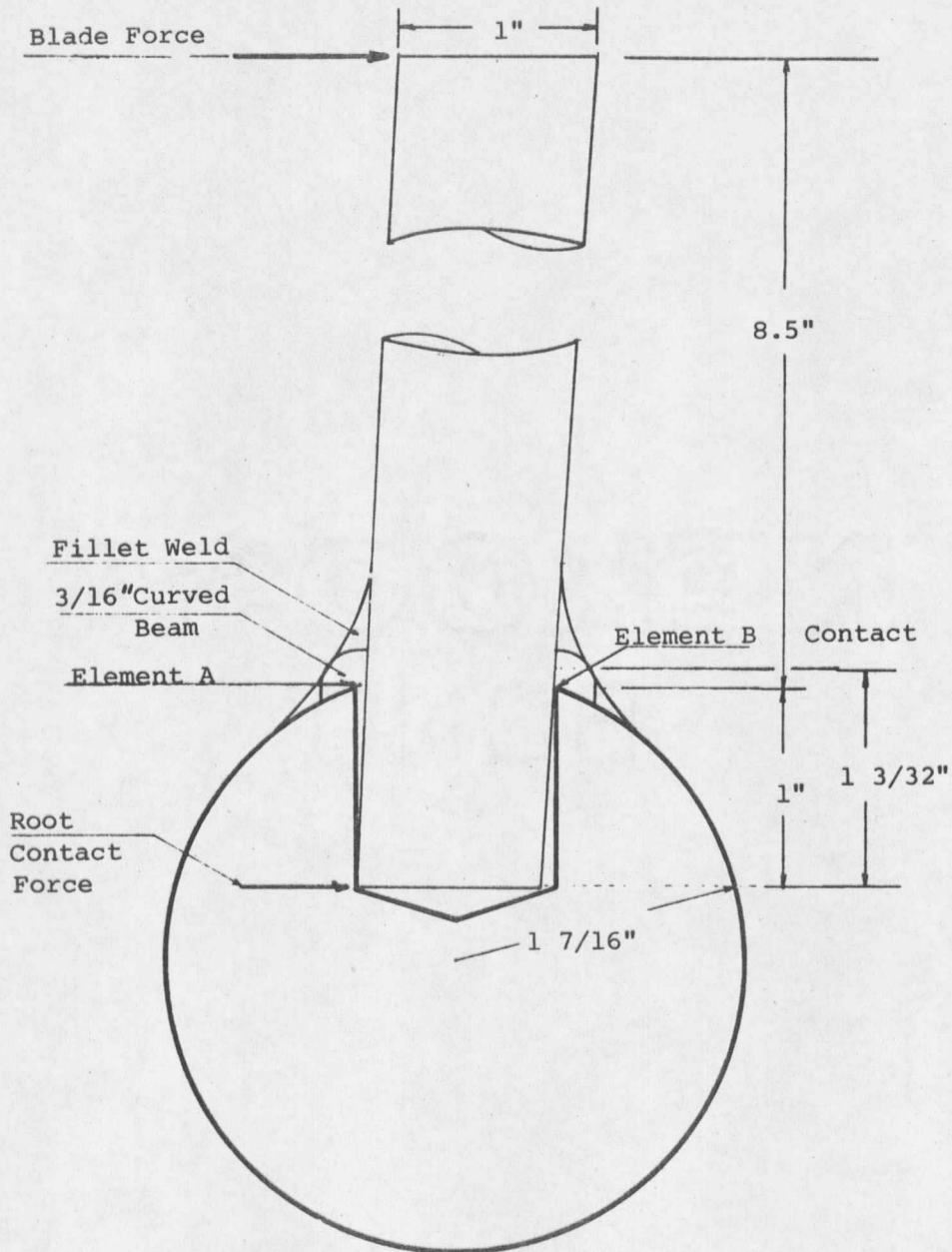


Figure 7. Cross section of the arm to mainshaft welded connection.

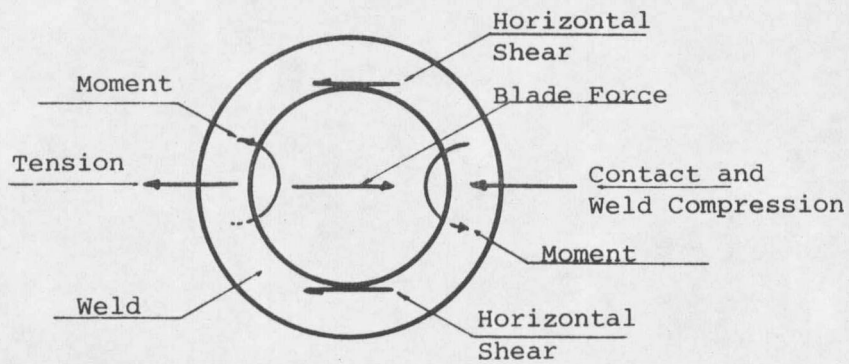


Figure 8a. Plan view of the weld.

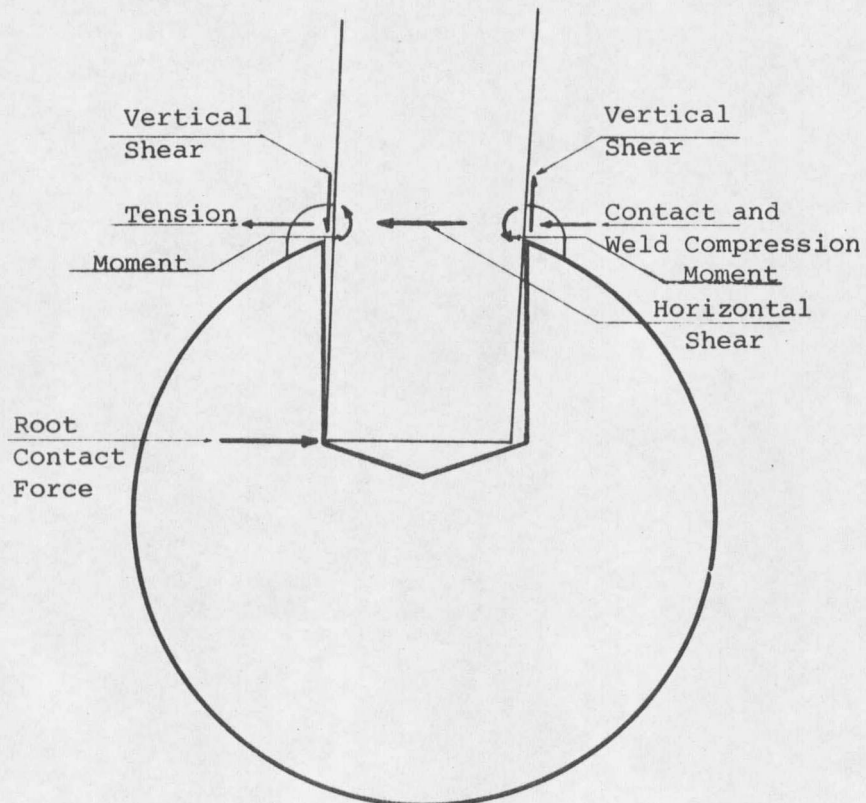


Figure 8b. Cross section of the weld.

Figure 8. Possible weld reactions.

A contact force between the arm and hole rim is induced when the blade is loaded. It acts at only one point. A contact force also acts at the hole root. Its action resists rotation of the arm by forming a moment about the weld.

The total horizontal reaction in the weld is the sum of the horizontal shear, tension, and contact reactions. All of the reactions are related by the arm displacement within the weld. The amount of force each reaction accepts is determined by its stiffness relative to the total weld stiffness. It was anticipated that the contact stress would be high, which implied that the contact force was also high. This would require that the contact force must have a stiffness coefficient greater than the tension and shear coefficients.

The stiffness coefficients of the weld were determined by applying Castigliano's second theorem to a curved beam. The contact stiffness for the weld and root contact reactions were derived from misaligned roller bearing equations.

A planar analogy of the arm reactions is given in Figure 9. Summing moments about P_c yields the first equation and summing horizontal and vertical forces the second and third equations.

$$\sum MP_c = 0 = -W\ell + P_w D + 2Vr + M \quad (1)$$

$$\sum F_x = 0 = W - P + Q \quad (2)$$

$$\sum F_y = 0 = V - V \quad (3)$$

The equilibrium equations are statically indeterminate and were rewritten in terms of the stiffness coefficients and displacements.

$$-W\ell + (K_s + K_t + K_{cw}) \delta_w D + 2rK_v \delta_v + K_m \theta_m = 0 \quad (1a)$$

$$W - (K_s + K_t + K_{cw}) \delta_w + K_c \delta_c = 0 \quad (2a)$$

Equation 3 does not provide additional information.

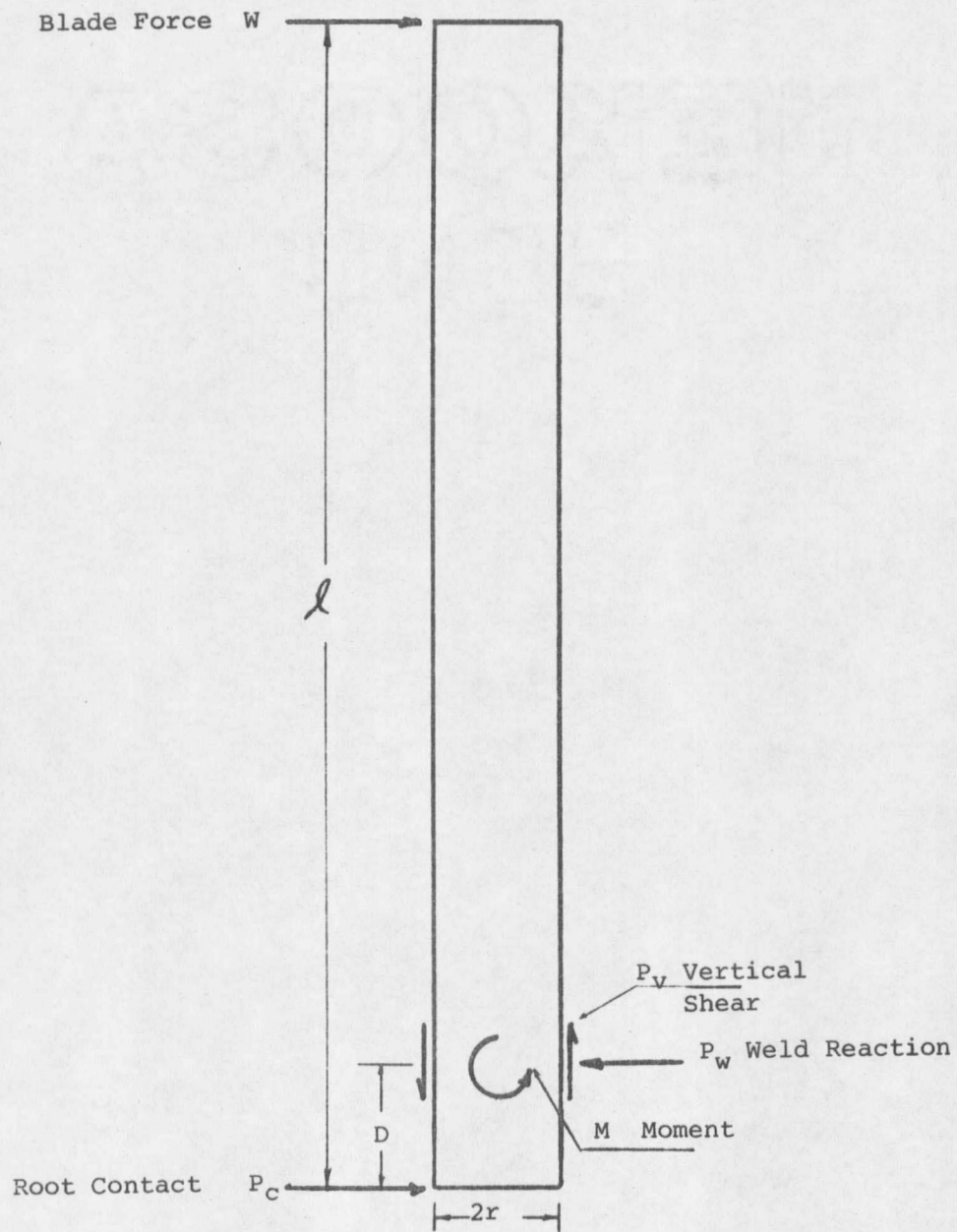


Figure 9. Planar analogy of the arm and blade force reactions.

Independent variables:

W	=	Perpendicular blade force
K_s	=	Stiffness coefficient for horizontal shear
K_t	=	" " " tension
K_{cw}	=	" " " weld contact
K_v	=	" " " vertical shear
K_m	=	" " " weld moment
K_c	=	" " " root contact

Dependent variables:

δ_w	=	Displacement of the weld-contact reaction
δ_c	=	" " " root contact reaction
δ_v	=	" " " vertical shear reaction
θ_m	=	Rotation of the arm at the weld
θ_c	=	The angle of contact between the arm and hole.

The unknown displacements were related by assuming the arm translated and rotated as a rigid body. Figure 10 illustrates the rigid body rotation and displacement of the weld and root contact force. It was assumed that the weld displacement does not contribute significantly to the rigid body rotation at the arm. This assumption was verified to be correct later in the analysis.

The variable θ_c has been defined as the angle of contact between the arm and hole. Later in the analysis it will be demonstrated that θ_c is a variable required for the stiffness coefficient in contact. The variable T is the difference of the hole and arm diameters. This analysis assumes that T is equal to .002 inch.

The variable θ_m is the rigid rotation of the arm in radians and δ_v is the vertical shear displacement. The rotation and displacement variables were derived in terms of the root contact displacement δ_c .

$$\theta_c = \frac{\delta_c + T}{D}$$

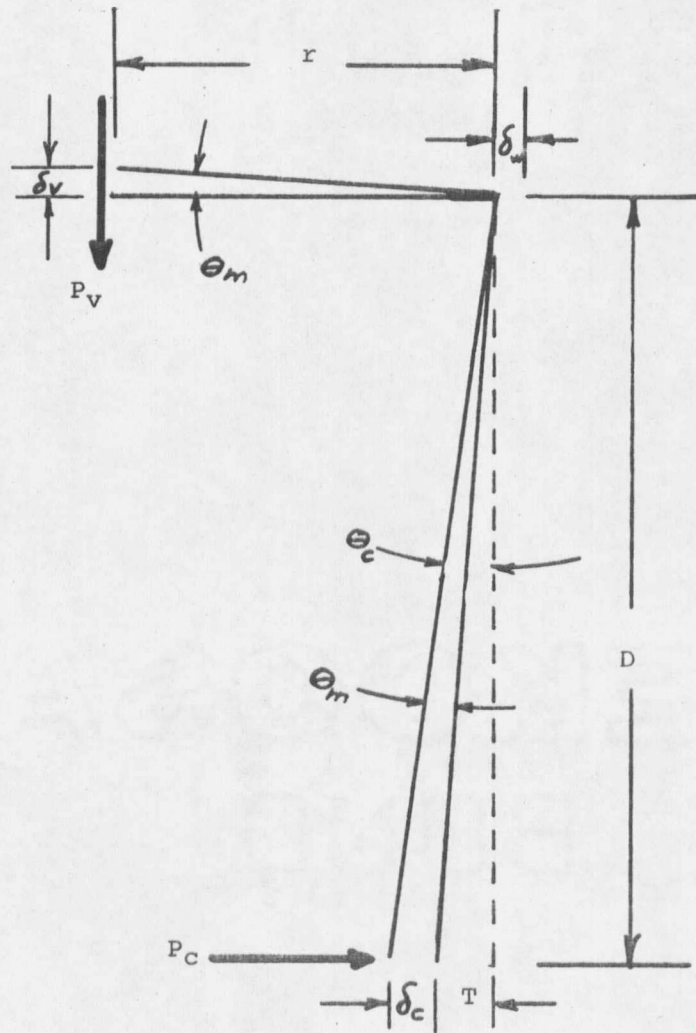


Figure 10. Rigid body relationships of the weld and contact reactions.

$$\theta_m = \frac{\delta_c}{D}$$

$$\delta_v = \frac{\delta_{cr}}{D}$$

The radius of the arm is the constant r and D is the distance from the root contact force added to the radius of the curved beam model of the weld.

The compatibility equations for θ_m and δ_v were substituted into Equation (1a) and Equation (1b), which resulted in two equations in two unknowns.

$$-W\ell + (K_s + K_t + K_{cw}) \delta_w D + 2r^2 K_v \delta_c + \frac{K_m \delta_c}{D} = 0 \quad (1c)$$

$$W - (K_s + K_t + K_{cw}) \delta_w + K_c \delta_c = 0 \quad (2c)$$

The independent variables are the weld displacement δ_w and the root contact displacement δ_c .

Derivation of the Stiffness Coefficients

Castigliano's second theorem as applied to curved beams was used to determine the stiffness coefficients (Boresi, 1978:354).

Figure 11 shows a curved beam with an outer radius of 3/16 placed within the fillet weld. The inner segment is filled, but inactive due to the tolerance so the inner radius chosen was .002 inch. Neglecting the inner segment probably underestimated the stiffness of the weld, which, in turn, underestimated the shear force and moment that the weld carried.

The total strain energy for a curved beam was derived for the condition of plane stress. An element of weld is actually a slice of a toroid which is more closely a condition of

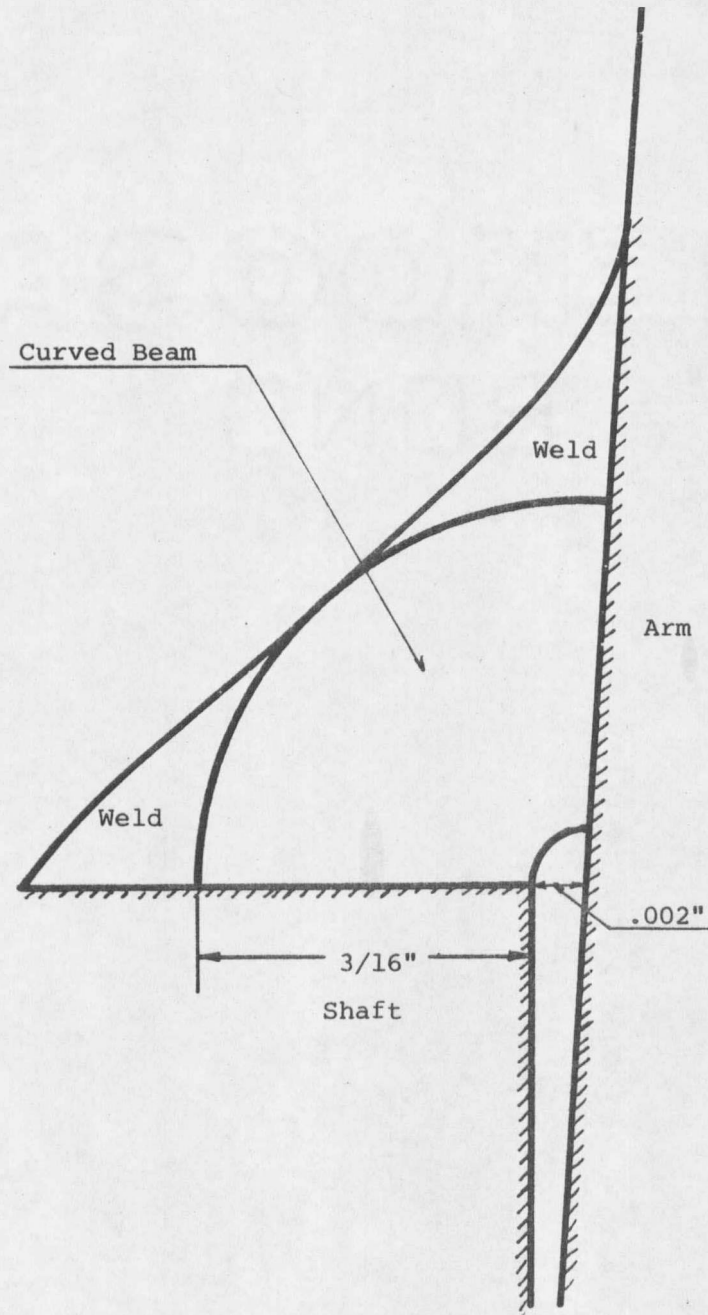


Figure 11. Curved beam representation of a fillet weld.

plain strain. The effect is that the stiffness coefficients were probably underestimated and the weld actually carried more shear force and moment than predicted.

The total strain energy U for a curved beam is given by Equation 4 (Boresi, 1978: 356).

$$U = \int \frac{KV^2 R d\theta}{2AG} + \int \frac{N^2 R d\theta}{2AE} + \int \frac{AmMx^2 d\theta}{2A(RAm - A)E} - \int \frac{MxNd\theta}{EA} \quad (4)$$

The material constants are: shear modulus, $G = 11.1 \times 10^6$ psi; Young's modulus, $E = 28 \times 10^6$ psi; shear factor for a rectangular cross section, $K = 1.5$. The shear, normal force, and bending moment are denoted to be V , N , and Mx .

The integral was taken over the arc of the beam. For this analysis the arc was zero to $\pi/2$.

The geometric constants are illustrated in Figure 12.

The deflection of the free end is equal to the strain energy differentiated with respect to the applied load at the free end.

$$\delta_v = \frac{\partial U}{\partial V}$$

$$\delta_n = \frac{\partial U}{\partial N}$$

$$\theta_m = \frac{\partial U}{\partial Mo}$$

This analysis treated each applied load separately to obtain the individual stiffness coefficients.

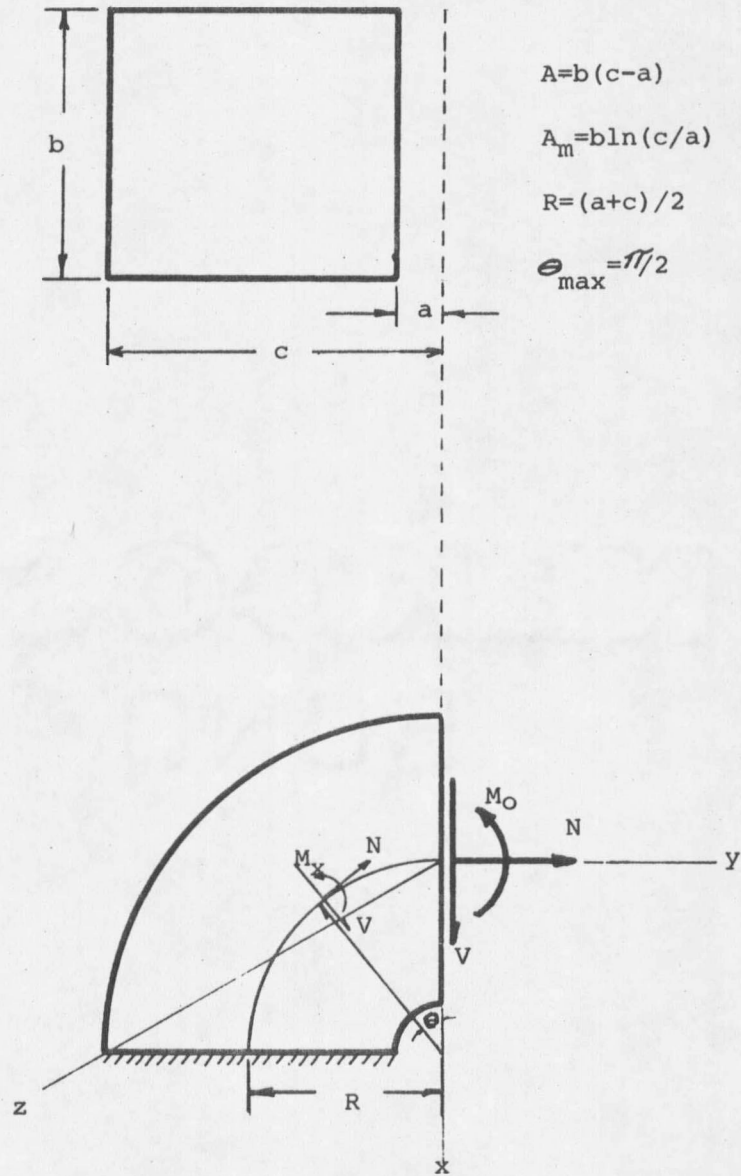


Figure 12. Curved beam geometry.

Stiffness Coefficient in Tension. The stiffness coefficient for tension on the weld was derived from the freebody diagram of Figure 13a. The equations for shear force, normal force, and bending moment at a section are

$$V = P \sin \theta$$

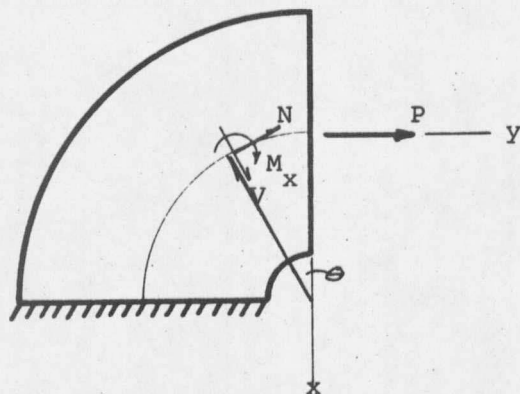
$$N = P \cos \theta$$

$$M_x = -PR(1 - \cos \theta).$$

Substituting into Equation 4 and integrating over $\pi/2$ yields the total strain energy, U . Differentiating the energy with respect to P yields the deflection, δ_p , of the free end in the direction of P .

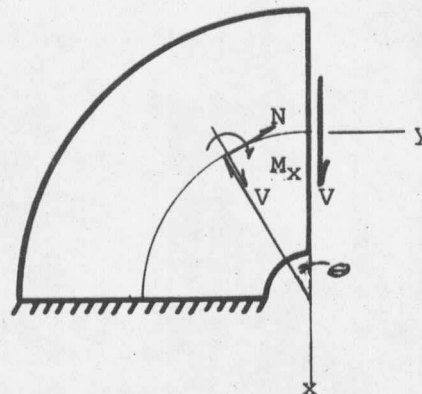
Algebraically the sequence is

$$\begin{aligned}
 U &= \int_0^{\pi/2} \frac{KRP^2 \sin^2 \theta}{2AG} d\theta + \int_0^{\pi/2} \frac{RP^2 \cos^2 \theta}{2AE} d\theta + \int_0^{\pi/2} \frac{AmP^2 R^2 (1 - \cos \theta)^2}{2AE (RAm - A)E} d\theta \\
 &\quad + \int_0^{\pi/2} \frac{P^2 R \cos \theta}{EA} d\theta \\
 \frac{\partial U}{\partial P} &= \delta_p = \int_0^{\pi/2} \frac{KRP \sin^2 \theta}{AG} d\theta + \int_0^{\pi/2} \frac{RP \cos^2 \theta}{AE} d\theta + \int_0^{\pi/2} \frac{AmPR^2 (1 - \cos \theta)^2}{AE (RAm - A)} d\theta \\
 &\quad + \int_0^{\pi/2} \frac{2PR \cos \theta (1 - \cos \theta)}{EA} d\theta \\
 \delta_p &= \frac{\pi KRP}{4AG} + \frac{1.215 RP}{AE} + \frac{3\pi AmPR^2}{2AE (RAm - A)}
 \end{aligned}$$



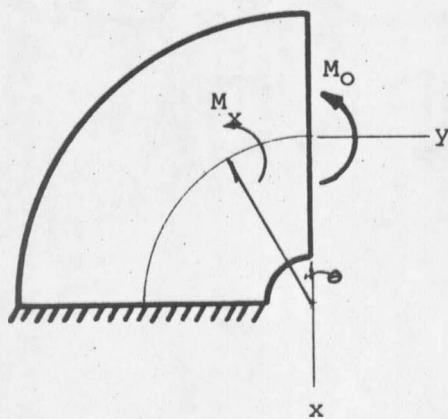
$$\begin{aligned} V &= P \sin \theta \\ N &= P \cos \theta \\ M_x &= -PR(1 - \cos \theta) \end{aligned}$$

Figure 13a. Tension



$$\begin{aligned} V &= P_v \cos \theta \\ N &= P_v \sin \theta \\ M_x &= -P_v R \sin \theta \end{aligned}$$

Figure 13b. Vertical Shear



$$\begin{aligned} V &= 0 \\ N &= 0 \\ M_x &= M_0 \end{aligned}$$

Figure 13c. Moment

Figure 13. Freebody diagrams for the derivation of weld stiffness coefficients.

$$G = \frac{E}{2(1 + \nu)}$$

$\nu = .26$ poissons ratio

$K = 1.5$ shape factor

$$\delta_p = \frac{4.183 RP}{AE} + \frac{4.712 AmPR^2}{AE(RAm - A)}$$

Solving for the form

$$\delta_p K_t = P$$

$$\delta_p \left[\frac{(RAm - A) AE}{4.183 R (RAm - A) + 4.71 AmR^2} \right] = P_t$$

Substituting Am , A and R from Figure 12 yields

$$\delta_p \left[\frac{\left(\frac{a+c}{2} b \ln \frac{c}{a} - b(c-a) \right) b(c-a) E}{4.183 \frac{a+c}{2} \left(\frac{a+c}{2} b \ln \frac{c}{a} - b(c-a) \right) + 4.712 \frac{(a+c)^2}{4} b \ln \frac{c}{a}} \right] = P_t$$

$$\delta_p = \delta_w$$

$$\delta_w \left[\frac{bE(c-a) \left(\frac{a+c}{2} \ln \frac{c}{a} - c+a \right)}{2.09(a+c) \left(\frac{a+c}{2} \ln \frac{c}{a} - c+a \right) + 1.18(a+c)^2 \ln \frac{c}{a}} \right] = P_t \quad (5a)$$

The stiffness coefficient in tension, K_t , is the quantity within the brackets. It is valid for a straight weld and was modified for a circular weld by substituting

$$b = 2\pi r$$

for the weld length. The variable r is the radius of the arm.

The displacement δ_w was for a circular weld and was also modified. Figure 14 shows a toroidal segment of weld and the rigid arm displacement in the radial direction. The

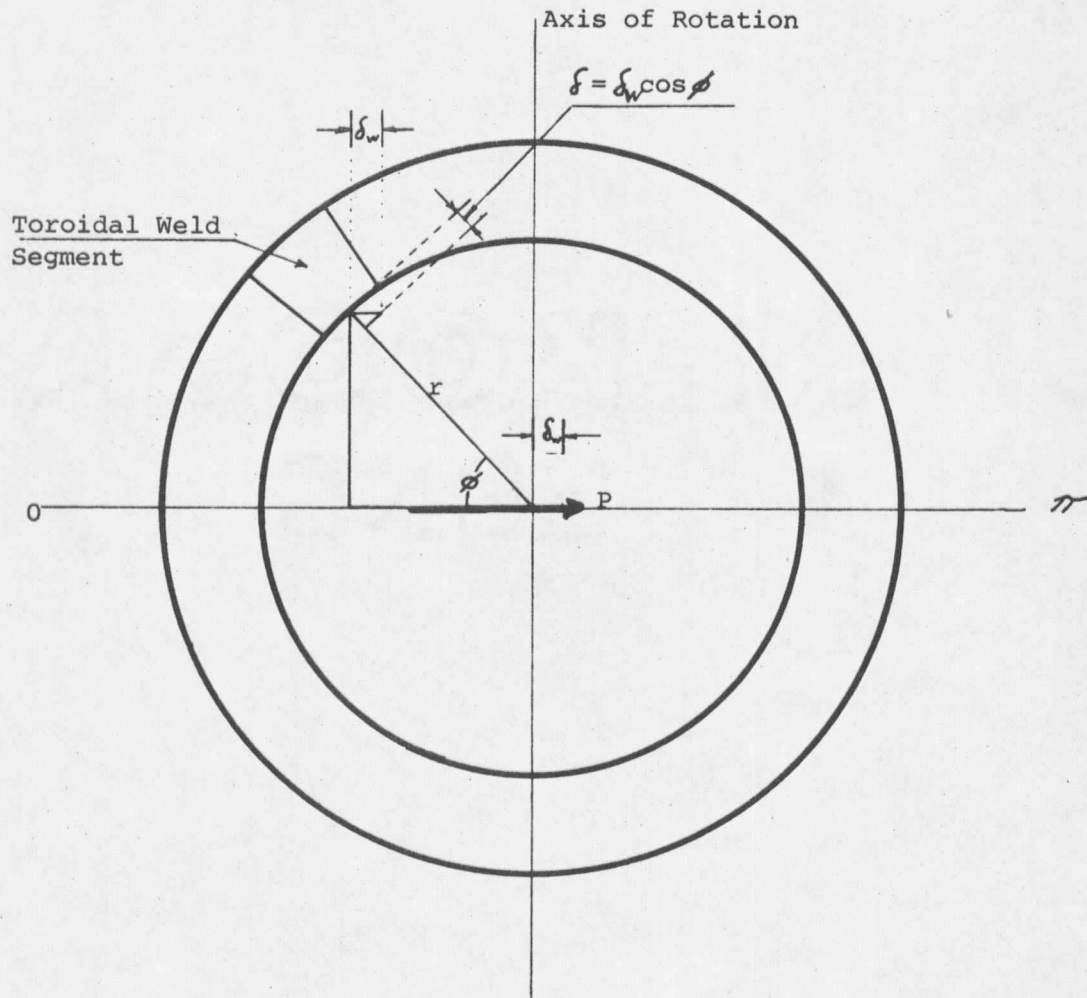


Figure 14. Displacement relationship for a toroidal weld segment.

radial displacement for the segment is

$$\delta = \delta_w \cos\phi$$

Since each segment of weld, $rd\phi$ is translated differently, an effective displacement for the entire weld length was calculated. That effective displacement was the square root of the mean of the squares, commonly referred to as the RMS value. The RMS value is

$$\delta_{\text{eff}} = \frac{\sqrt{2}}{2} \delta_w.$$

Substituting the weld length and effective weld displacement into Equation 5a yields the stiffness coefficient in tension for a circular weld.

$$\delta_w \left[\frac{4.44r(c-a) E \left(\frac{a+c}{2} \ln \frac{c}{a} - c+a \right)}{2.09(a+c) \left(\frac{a+c}{2} \ln \frac{c}{a} - c+a \right) + 1.18(a+c)^2 \ln \frac{c}{a}} \right] = P_v \quad (5b)$$

$$K_t = \left[\frac{4.44r(c-a) E \left(\frac{a+c}{2} \ln \frac{c}{a} - c+a \right)}{2.09(a+c) \left(\frac{a+c}{2} \ln \frac{c}{a} - c+a \right) + 1.18(a+c)^2 \ln \frac{c}{a}} \right] \quad (5c)$$

Stiffness Coefficient in Vertical Shear. The stiffness coefficient for vertical shear on the weld was derived from the freebody diagram of Figure 13b. The equations for shear force, normal force, and bending moment at a cross section are

$$V = P_v \cos\theta$$

$$N = P_v \sin\theta$$

$$M_x = -P_v R \sin\theta$$

Substituting into Equation 4, integrating over $\pi/2$ and differentiating with respect to P_v yields the simplified equation

$$\delta_v \left[\frac{4AE(RAm - A)}{21.3R(RAm - A) + \pi AmR^2} \right] = P_v$$

Substituting Am , A , and R

$$\delta_v \left[\frac{4b(c-a) E \left(\frac{c+a}{2} \ln \frac{c}{a} - c+a \right)}{21.3 \frac{c+a}{2} \left(\frac{c+a}{2} \ln \frac{c}{a} - c+a \right) + \pi \frac{(c+a)^2}{4} \ln \frac{c}{a}} \right] = P_v \quad (6a)$$

The stiffness coefficient K_v is the quantity within the brackets of Equation 6a. It is valid for a straight weld and was modified for a circular weld by substituting

$$b = \pi r$$

for the weld length. The length is for only half the weld because it was coupled with the remaining half through the effective moment arm.

The effective displacement is also the root-mean-square value of the maximum displacement

$$\delta_{veff} = \frac{\sqrt{2}}{2} \delta_v$$

substituting into Equation 6a the weld length and the effective displacement yields the stiffness coefficient in vertical shear for a circular weld.

$$\delta_v \left[\frac{8.88r(c-a) E \left(\frac{c+a}{2} \ln \frac{c}{a} - c+a \right)}{10.65(c+a) \left(\frac{c+a}{2} \ln \frac{c}{a} - c+a \right) + .785(c+a)^2 \ln \frac{c}{a}} \right] = P_v \quad (6b)$$

$$K_v = \left[\frac{8.88r(c-a)E\left(\frac{c+a}{2}\ln\frac{c}{a} - c+a\right)}{10.65(c+a)\left(\frac{c+a}{2}\ln\frac{c}{a} - c+a\right) + .785(c+a)^2\ln\frac{c}{a}} \right] \quad (6c)$$

The vertical shear in Figure 9 was replaced by an equivalent system of a force and a couple. In the planar analogue of Figure 9, the couple arm was assumed to equal $2r$ which is no longer valid and must be replaced with the effective radius. Figure 14 demonstrates that the distance from the axis of rotation to a toroidal segment is the cosine function

$$r' = r \cos \phi$$

The effective radius is again the RMS value.

$$r_{\text{eff}} = \frac{\sqrt{2}}{2} r$$

The vertical shear force from one-half the weld was coupled with the adjacent half through twice the effective radius.

$$d = \frac{2\sqrt{2}}{2} r$$

$$d = \sqrt{2} r$$

Equation 1c was modified later in the analysis to reflect the new vertical shear couple.

Stiffness Coefficient for Moment. The stiffness coefficient for moment on the weld was derived from the freebody diagram of Figure 13c. The equations for shear force, normal force, and bending moment at a section are

$$V = 0$$

$$N = 0$$

$$M_x = M_o.$$

Substituting into Equation 4 and integrating over $\pi/2$ yields the total strain energy, U.

Differentiating the energy with respect to M_o yields the simplified equation

$$\theta_m \left[\frac{2AE (RAm - A)}{\pi Am} \right] = M_o$$

Substituting A_m , A , and R

$$\theta_m \left[\frac{2b(c-a) E \left(\frac{c+a}{2} \ln \frac{c}{a} - c+a \right)}{\pi \ln \frac{c}{a}} \right] = M_o \quad (7a)$$

The stiffness coefficient, K_m , is valid for a straight weld and was modified by substituting

$$b = 2\pi r$$

for the weld length.

The effective rotation also follows the root-mean-square value as with the previous derivations.

$$\theta_{meff} = \frac{\sqrt{2}}{2} \theta_m$$

Substituting the weld length and effective rotation into Equation 7a yields the stiffness coefficient for moment applied to a circular weld.

$$\theta_m \left[\frac{2.83r(c-a) E \left(\frac{c+a}{2} \ln \frac{c}{a} - c+a \right)}{\ln \frac{c}{a}} \right] = M_o \quad (7b)$$

$$K_m = \left[\frac{2.83r(c-a) E \left(\frac{c+a}{2} \ln \frac{c}{a} - c+a \right)}{\ln \frac{c}{a}} \right] \quad (7c)$$

Stiffness Coefficient for Horizontal Shear. The horizontal shear stiffness coefficient was developed from Figure 15 (Oakberg, 1979). A displacement, U_z , induced by the shear force P_h is assumed to be linear with θ .

$$U_z = \delta \theta$$

The other displacement components are assumed to be zero. There is only one non-zero strain component if cylindrical coordinates are employed.

$$\epsilon_{\theta z} = \frac{1}{2} \left[\frac{1}{R} \frac{\partial U_z}{\partial \theta} + \frac{\partial U_z}{\partial z} \right]$$

$$\frac{\partial U_z}{\partial \theta} = \delta$$

The shear strain reduces to

$$\epsilon_{\theta z} = \frac{\delta}{2R}$$

The shear stress is related to the shear strain by the shear modulus, G .

$$\tau_{\theta z} = 2G\epsilon_{\theta z}$$

$$\tau_{\theta z} = G \frac{\delta}{R}$$

The force per unit length is equal to the shear integrated over the radius

$$\frac{F}{b} = \int_a^c \tau_{\theta z} dR$$

$$F = G \delta b \ln \frac{c}{a}$$

The weld displacement, U_z , is δ_w at θ equal to $\pi/2$. Dividing the force by the relative displacement yields the stiffness coefficient

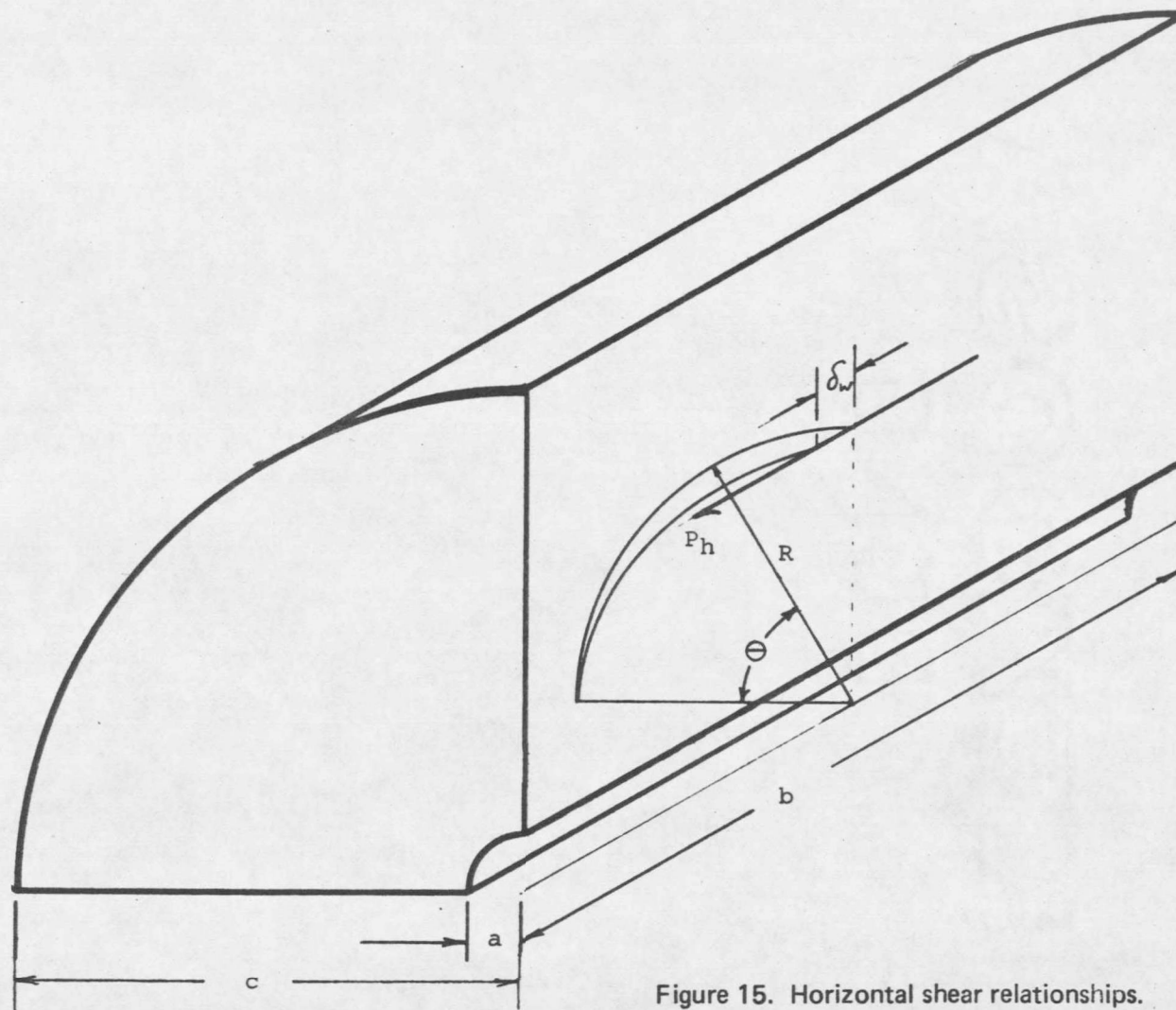


Figure 15. Horizontal shear relationships.

$$K_h = \left[\frac{G \delta b \ln \frac{c}{a}}{\delta \pi/2} \right]$$

$$\delta_w \left[\frac{2Gb \ln \frac{c}{a}}{\pi} \right] = P_h \quad (8a)$$

The adjustment for a circular weld is identical to the tension and moment coefficient adjustment.

$$b = 2\pi r$$

$$\delta_{weff} = \frac{\sqrt{2}}{2} \delta_w$$

$$\delta_w \left[2.83 Gr \ln \frac{c}{a} \right] = P_h \quad (8b)$$

$$K_s = 2.83 Gr \ln \frac{c}{a} \quad (8c)$$

Stiffness Coefficient for Contact. The stiffness coefficient for contact was developed from misaligned roller bearing equations originally derived to determine the maximum equivalent line load between the bearing and race (Daring, 1964:177). In this analysis the arm acts as the bearing and the hole acts as the race.

Figure 16 illustrates the misaligned roller bearing geometry. The first equation determines the minimum deflection of the two contacting surfaces. Since the arm and hole lose contact, the minimum deflection y_o is equal to zero.

$$y_o = \frac{AP_o}{L} - \frac{\theta_c L}{2}$$

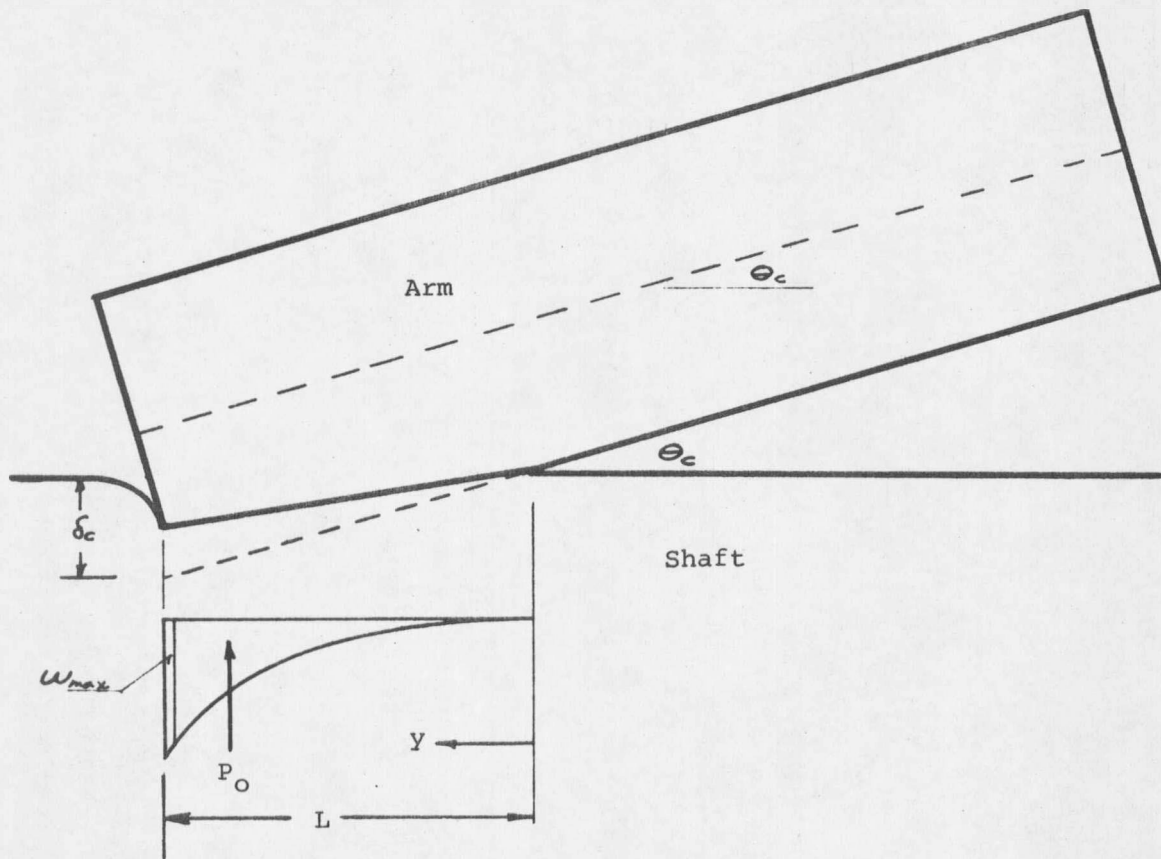


Figure 16. Misaligned roller bearing geometry.

$$A = \frac{8(1 - \nu^2)}{\pi E}$$

The equation was solved for L, the length of contact. The variables are

ν = poissons ratio, .26

θ_c = Angular misalignment between arm and hole

P_o = Resultant concentrated force

$$L = \sqrt{\frac{2AP_o}{\theta_c}} \quad (9a)$$

The second equation determines the equivalent maximum line load.

$$w_{\max} = \frac{P_c}{L} + \frac{\theta_c L}{2A} \quad (9b)$$

Substituting Equation 9a into 9b and simplifying produces

$$w_{\max} = \sqrt{\frac{2P_c \theta_c}{A}} \quad (9c)$$

The maximum deflection of the entire system is

$$\begin{aligned} \delta_c &= A w_{\max} \\ \delta_c &= A \sqrt{\frac{2P_c \theta_c}{A}} \end{aligned} \quad (9d)$$

Equation 9d was solved for the force P_c in terms of the stiffness coefficient and the maximum displacement.

$$\delta_c^2 \left[\frac{\pi E}{16(1 - \nu^2)} \right] \frac{1}{\theta_c} = P_c \quad (9e)$$

$$K_c = \left[\frac{\pi E}{16(1 - \nu^2)} \right] \quad (9f)$$

The quantity within the brackets is the contact stiffness K_c . The contact angle θ_c was substituted into Equations 1c and 2c in terms of the independent variable δ_c .

Order of Magnitude Study of the Stiffness Coefficients: Summing forces at the weld results in the equation

$$\frac{K_{cw} \delta_w^2}{\theta_c} + (K_t + K_s) \delta_w = P_w.$$

It was evident that the contact force in the weld was a higher order term of the weld deflection. Higher order terms of small numbers may often be disregarded because of their small influence. It was desirable to check if the contact force in the weld was insignificant before solving Equations 1c and 2c so that the weld contact force could be omitted. Substituting values for the stiffness coefficients of the approximate order of magnitude will provide an estimate of the force carried by contact in the weld.

All of the stiffnesses contained Young's modulus or the shear modulus so an approximate stiffness was 1×10^7 psi for K_t , K_s , and K_c . The angle, θ_c , was dominated by the tolerance term. An approximation from machining capability was 0.001 radians. A weld displacement of 10^{-5} was the only practical value for the weld displacement. A weld displacement of 10^{-4} will result in a total weld force greater than what is physically possible.

$$\frac{10^7 (10^{-5})^2}{10^{-3}} + (1 \times 10^7 + 1 \times 10^7) 10^{-5} = P_w$$

$$1 + 200 = P_w$$

The contact force was apparently insignificant and was disregarded in the solution of Equations 1c and 2c.

Solution of the Equilibrium Equations

Neglecting the contact force in the weld and substituting the compatibility equations into Equations 1c and 2c results in Equations 11 and 12.

$$-W\ell + (K_t + K_c)\delta_w D + \frac{\delta_c K_m}{D} + \frac{K_v \delta_c r^2 \sqrt{2}}{D} = 0 \quad (11)$$

$$W - (K_t + K_s)\delta_w + \frac{K_c D \delta_c^2}{\delta_c + T} = 0 \quad (12)$$

Equation 12 was solved for δ_w , the weld displacement

$$\frac{W(\delta_c + T) + K_c \delta_c^2 D}{(K_t + K_s)(\delta_c + T)} = \delta_w \quad (13)$$

Equation 13 was substituted into Equation 11 and Equation 11 was solved for δ_c , the root contact displacement.

$$\delta_c = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad (14)$$

$$A = K_c D^3 + K_m + K_v r^2 \sqrt{2}$$

$$B = DW(D - \ell) + T(K_m + K_v r^2 \sqrt{2})$$

$$C = WTD(D - \ell)$$

The positive root of Equation 14 provides the physical solution.

The weld material for the fractured shaft was type 316 stainless steel with the following material constants obtained from Table 1: $E = 28 \times 10^6$ psi; $G = 11.1 \times 10^6$ psi; $\nu = 0.26$.

The geometry of the weld was discussed previously: it has an outer radius of 3/16 inch and an inner radius of .002 inch. Substituting this information into Equations 5c, 6c, 7c, 8c, and 9f resulted in the following stiffness coefficients:

$$K_t = 9.75 \times 10^6 \text{ lbs/in;}$$

$$K_v = 1.81 \times 10^7 \text{ lbs/in;}$$

$$K_m = 3.96 \times 10^5 \text{ in-lbs/rad;}$$

$$K_s = 7.13 \times 10^7 \text{ lbs/in;}$$

$$K_c = 5.89 \times 10^6 \text{ lb-rad/in}^{-2}.$$

The independent variables have the following values:

$$r = 1/2 \text{ inch}$$

$$D = 1 \text{ } 3/32 \text{ inch}$$

$$l = 9.5 \text{ inch}$$

$$w = 250 \text{ lbs}$$

The 250 pound blade force at station 9 occurs at the 270° load position.

Solving Equations 13 and 14 resulted in a root contact displacement and weld displacement respectively of

$$\delta_c = .000295 \text{ inch}$$

$$\delta_w = .00000611 \text{ inch}$$

The weld deflection is on the order of 10^{-5} , which verified that the contact force in the weld was insignificant.

It was also assumed that the weld displacement did not affect the rigid body rotation of the arm. The rigid body rotation was found from the compatibility equation for weld rotation.

$$\theta_m = \frac{\delta_c}{D}$$

$$\theta_m = .00027 \text{ radians}$$

The actual rigid body rotation was

$$\theta_m = \frac{\delta_c + \delta_w}{D}$$

$$\theta_m = .0003011 \text{ radians}$$

The difference between the rigid body rotations was 2.03% which verifies that the assumption was correct.

Although the contact force in the weld is small, the contact stress may be large. The contact force in the weld was calculated from Equation 9e to be .105 pounds and the maximum equivalent line load, $w_{\max}$, was calculated from Equation 9c.

$$w_{\max} = \sqrt{\frac{2(.105)(.00210)}{8.48 \times 10^{-8}}}$$

$$w_{\max} = 72.11 \text{ lbs/in}$$

The equation for the maximum normal stress between two cylinders in contact was modified for this analysis into the form of Equation 15 (Radzimovsky, 1953:11).

$$p_{\max} = \sqrt{\frac{w_{\max} TE}{\pi (1 - \nu^2)}} \quad (15)$$

$$p_{\max} = 1174 \text{ psi}$$

The contact stress does not appear to be critical. However, the additional stresses due to mainshaft bending and torsion remain to be calculated.

Stress Components from Mainshaft Bending

The stress components from mainshaft bending were calculated for the 270° load position. The moments at station 9 were calculated previously and are repeated for convenience in the mainshaft coordinate system on the positive z plane.

$$M_x = 8278 \text{ in-lbs}$$

$$M_y = 14400 \text{ in-lbs}$$

The σ_{zz} normal stress was calculated from the flexure stress equation for bending multiplied by a stress concentration factor for a transverse hole in a round bar (Peterson, 1974:235). The flexure equation is

$$\sigma_{zz} = \frac{K_{tn} M c}{I}$$

in which K_{tn} represents the stress concentration factor for the net section, M denotes the bending moment about the X and Y axis, c is the distance from the neutral axis to the weld contact point, and I represents the net moment of inertia about the neutral axis.

Figure 17 illustrates the cross section geometry for computing I_x and I_y . The weld and the triangular drill point were omitted because of their small contributions. The moments of inertia about the X and Y axes are 3.27 inches⁴ and 2.43 inches⁴.

The stress concentration factor for the net section was estimated to be 1.8 about the Y axis and 2 about the X axis.

The combined stress, σ_{zz} , from bending about the X and Y axis for element A was estimated to be

$$\sigma_{zz} = \frac{1.8(14400)1.5}{2.43} - \frac{2(8278).5}{3.27}$$

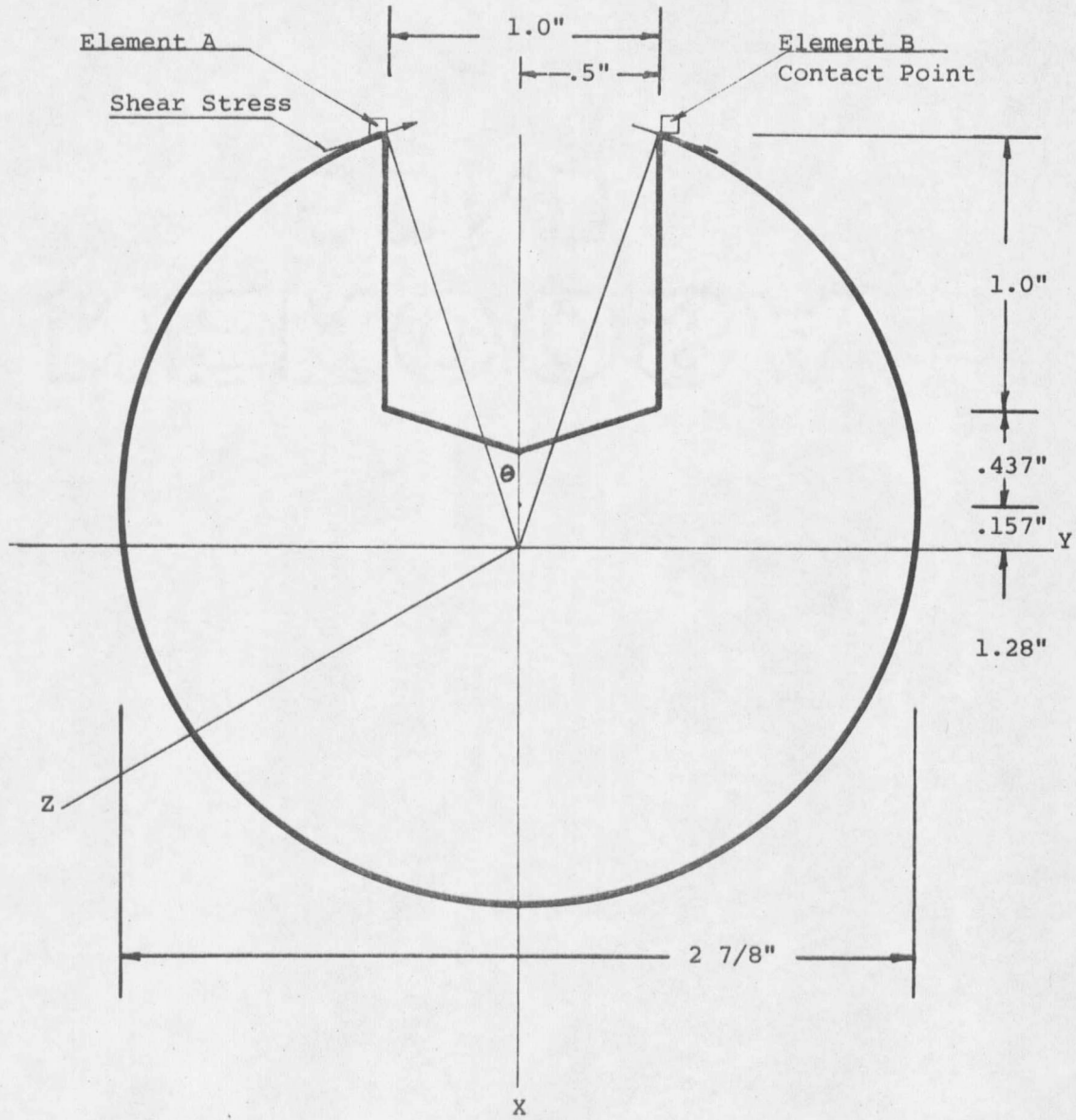


Figure 17. Geometry of the drilled cross section.

$$\sigma_{zz} = 13470 \text{ psi}$$

The stress at the weld contact point on element B was

$$\sigma_{zz} = \frac{1.8(14400)(1.5)}{2.43} + \frac{2(8278).5}{3.27}$$

$$\sigma_{zz} = 18530 \text{ psi.}$$

Shear Stress Components from Torsion

The torsional stress on the z plane was estimated by the torsion formula

$$\tau = \frac{K_{tn} T c}{J_o}$$

in which T denotes the torque, c is the radius from the neutral axis to element B, K_{tn} represents the stress concentration factor for shear (Peterson, 1974:244). J_o is the polar moment of inertia, and τ denotes the shear stress.

The moments of inertia were summed to produce the polar moment of inertia J_o . There was probably some error in this operation, because the moments of inertia were based upon an open section. The weld and arm probably close the section which would raise the polar moment of inertia and lower the shearing stress. The open cross section assumption was conservative.

The values substituted into the torsion formula were 11280 in-lbs for the torque, 1.58 in for the radius, 1.5 for the stress concentration factor, and 5.7 in⁴ for the polar moment of inertia. The shear stress was

$$\tau = 4690 \text{ psi}$$

The circumferential shear stress τ was broken into components by the trigonometric relationships

$$\tau_{zx} = \tau \sin \theta$$

$$\tau_{zy} = \tau \cos \theta$$

The angle theta, shown in Figure 17, is equal to 17.6°. Substituting into the relationships yielded component stresses of 1415 psi for τ_{zx} and 4471 psi for τ_{zy} on element B. The stress state on element B is shown in Figure 18a and the principal stresses in Figure 18b.

The maximum normal stress was 19590 psi. The maximum shearing stress was 10,880 psi which provided a safety factor of 1.8. The maximum octahedral shearing stress was 9800 psi which provided a safety factor of 1.9 (Boresi, 1976:123). It was apparent that the contact, torsional, and bending stresses did not combine to produce a critical stress state.

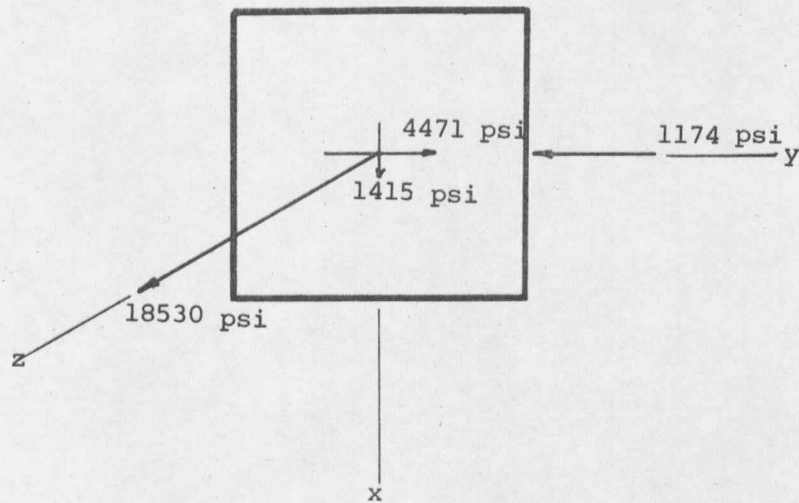


Figure 18a. Stress state in the weld.

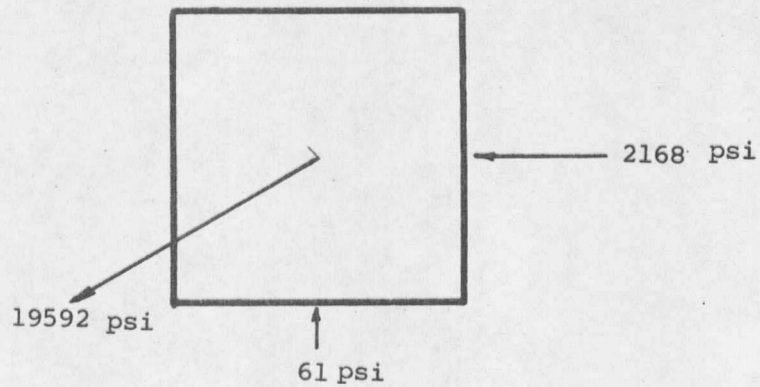


Figure 18b. Principal stresses in the weld.

Figure 18. Stress states in the weld at the contact point.

WELD RACKING FROM ARM ROTATION

A second failure mechanism became evident from the first analysis. The freebody diagram of Figure 19 shows that the weld supplied 1832 in-lbs of rotational resistance and the root contact force supplied 267 in-lbs of rotational resistance. This occurred because the weld rotational stiffness was greater than the root contact rotational stiffness. The rotational resistance of the weld was supplied mainly from vertical shear which racks the weld.

A method was required to estimate stresses in the weld at elements A and B to check if weld racking was the major failure mechanism. The method used was to impose the displacements at element A on a one inch long curved beam representation of the weld. A one inch long weld provided a convenient means to calculate the stiffness corresponding to the displacements and eventually the stresses on element A.

Equivalent Forces on a Unit Length Weld

The critical elements A and B of Figure 7 were at points where the displacements corresponding to tension, vertical shear, and moment were a maximum. Multiplying the displacements by the corresponding stiffnesses for a one inch long curved beam resulted in the forces applied to the weld. Recalling Equations 5a, 6a, 7a, and 3/16 inch for the outer radius and .002 inch for the inner radius produced the following stiffness coefficients:

$$\delta_w \quad 4.31 \times 10^6 = P_t$$

$$\delta_v \quad 8.175 \times 10^6 = P_v$$

$$\theta_m \quad 1.782 \times 10^5 = M_o$$

The vertical shear deflection δ_v and weld rotation θ_m were evaluated from the compatibility equations. The results were

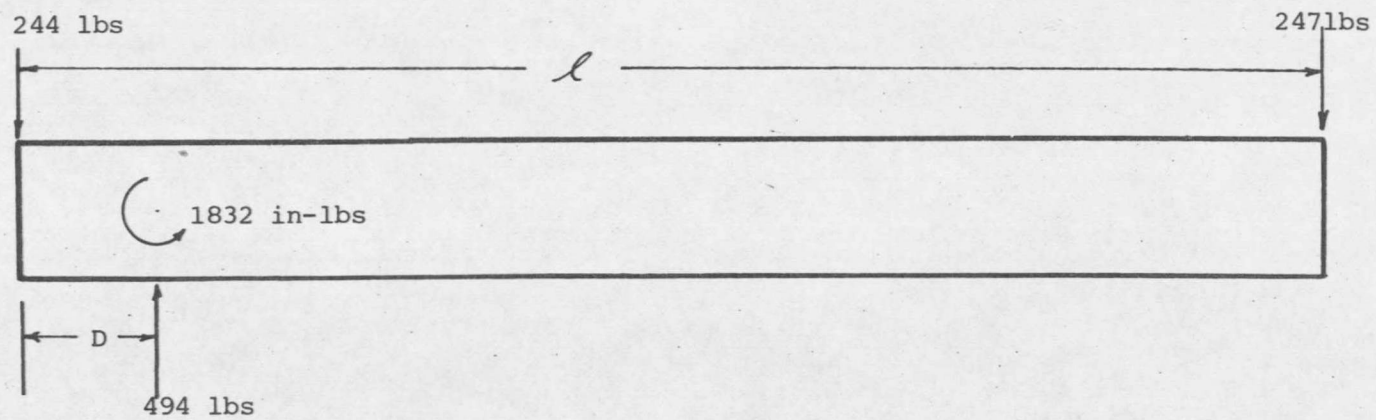


Figure 19. Freebody diagram of the arm and mainshaft connection.

$$\delta_v = .000135 \text{ inch}$$

$$\theta_m = .00027 \text{ radian}$$

The forces applied to the unit length of weld are shown in Figure 20.

The maximum circumferential stress was found from the curved beam equation (Boresi, 1978:332).

$$\sigma_{\theta\theta} = \frac{N}{A} + \frac{Mx(A - \zeta Am)}{A\zeta(RAm - A)} \quad (16a)$$

The maximum stress $\sigma_{\theta\theta}$ occurs at the inner radius ($\zeta = .002$) at the fixed base of the weld. The normal stress N is 1104 pounds and the bending moment is 52.9 inch-pounds. Substituting into Equation 16a the values of $c = 3/16$, $a = .002$, $\zeta = .002$, $b = 1$, and $Am = b \ln c/a$ produces

$$\sigma_{\theta\theta} = \frac{N}{c-a} + \frac{Mx((c-a) - \zeta \ln \frac{c}{a})}{(c-a)\zeta \left(\frac{c+a}{2} \ln \frac{c}{a} - c+a \right)}$$

$$\sigma_{\theta\theta} = 5950 + 103000$$

$$\sigma_{\theta\theta} = 109000 \text{ psi.}$$

It appears that the stress on the inner radius is greater than the yield stress. Solving Equation 16a for ζ provided a means of determining the extent of yielding.

$$\zeta = \frac{MxA}{(\sigma_{\theta\theta} - \frac{N}{A}) A(RAm - A) + MxAm} \quad (16b)$$

Substituting the yield stress of 44,000 psi into Equation 16b gives the following yield radius:

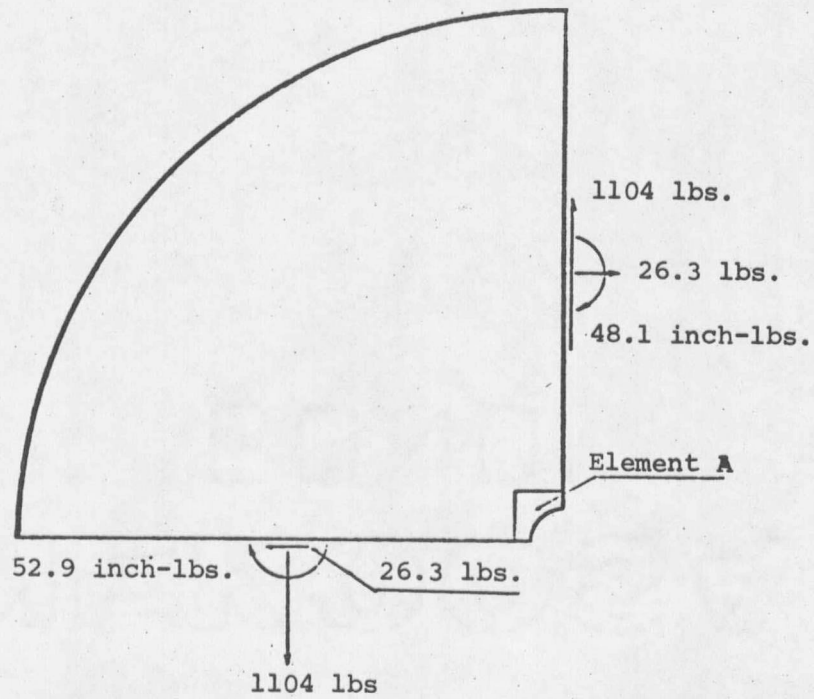


Figure 20. Maximum forces on a unit length weld.

$$\zeta = \frac{Mx(c-a)}{(\sigma_{\theta\theta} - \frac{N}{c-a})(c-a)(\frac{c+a}{2} \ln \frac{c}{a} - c+a) + Mx \ln \frac{c}{a}}$$

$$\zeta = .005 \text{ inch}$$

Equation 16b indicates that a small area from the inner radius to a .005 inch radius may have a circumferential stress greater than the yield stress. Element B on the opposite side of the arm has the same property.

Influence of Arm Flexure

It was assumed at the beginning of the analysis that the arm rotated as a rigid body. The validity of the assumption was checked by applying the second moment area theorem to the freebody diagram of Figure 19. The slope of the elastic curve at the weld was

$$\frac{P_c D^2}{6EI} = \theta_{\text{flexure}}$$

$$.0000354 \text{ radians} = \theta_{\text{flexure}}$$

The elastic flexural rotation was an order of magnitude less than the rigid body rotation of the arm. The overall effect is to raise the moment and vertical shear forces on the unit weld, which increases the yield radius.

CONCLUSION

This paper presented a planer analogy of a three dimensional problem. The analogy did not account for the stresses resulting from the blade force parallel to the mainshaft or the frictional force on the mixer blade. Including these forces complicates the analysis and overshadows the purpose of identifying the failure mechanism.

The analysis considered two failure mechanisms. The first failure mechanism assumed that the contact stress in the weld was high and contributed to the torsional and bending stresses, which may have produced a critical stress state. The contact stress in the weld was determined to be low and not the cause of failure.

The second failure mechanism was identified in the course of investigating the first failure mechanism. It was determined that the rotational stiffness of the weld was much greater than the rotational stiffness of the root contact reaction. The higher weld stiffness required the weld to carry a large preparation of moment which racked the weld and induced high stresses at the inner radius.

The prediction that stresses are high at the inner radius is supported by research conducted on fillet welds (Bissell, 1950:885). It was found that fillet welds developed approximately forty percent of the fatigue strength that a butt weld could under the same load conditions (Bissell, 1950:897).

Flexure of the arm was also identified as a possible contributor to weld racking.

RECOMMENDATIONS

It was concluded that high stresses in the weld were caused by a shift of the rotational resistance from the root contact force to the stiffer shear couple in the weld. In order to eliminate this problem, it is recommended that the mainshaft be drilled completely through.

Drilling through the mainshaft has several benefits. First, it eliminates the root contact reaction and replaces it with a weld. A stiff weld on both sides of the shaft will prevent the rigid body motion of the arm and thereby lessen flexure in the welds. Second, the double-welded joint allows the use of a rational design procedure. Once a blade load has been determined, the arm can be sized with the elastic flexure formula and a large safety factor to provide a stiff arm. The mainshaft could be sized by choosing an arm-to-mainshaft diameter ratio that will limit the stress concentration factors to practical values. Third, welding on both sides of the mainshaft may eliminate some of the warping caused by differential heating. Fourth, it would be beneficial to bevel the hole rim to provide a larger effective weld.

Most of the recommendations would lead to a larger and heavier mainshaft. The larger dead weight would counteract the gear reducer weight and reduce the uplift on the mainshaft. The result would be a reduced bending moment about the X axis, which would lower the maximum bending stress in the mainshaft.

Although the added weight lowers the bending stresses, it also raises the cost of the shaft. A thick walled tube would lower the weight while providing a large moment of inertia. Unfortunately, thick walled tubing is not available in the surface finish required for this application.

REFERENCES CITED

- Bissell, G. A. 1950. *Welding Handbook*, ed. Simon A. Greenberg. New York: American Welding Society.
- Boresi, A. P., O. M. Sidebottom, F. B. Seely, and J. O. Smith. 1978. *Advanced Mechanics of Materials*. 3rd. New York: John Wiley and Sons.
- Dareing, D. W., and E. I. Radzimovsky. 1964. "Misaligned Roller Bearings, . . . how fast will they fail?" *Machine Design*, Feb. 13, 1964.
- Graham, F. M. 1978. Personal correspondence between Dr. Frederick Graham, Iowa State University, and Wallace Jones, Rapids Machinery Co., December 26, 1978.
- Oakberg, R. G. 1979. Personal correspondence between Dr. Robert Oakberg and the writer, Montana State University.
- Parker, T. D. 1977. *Source Book on Stainless Steels*. Dir. Allen G. Gray. American Society for Metals, Metals Park, Ohio.
- Peterson, R. E. 1974. *Stress Concentration Factors*. New York: John Wiley and Sons.
- Radzimovsky, E. I. 1953. "Stress Distribution and Strength Condition of Two Rolling Cylinders Pressed Together," University of Illinois Bulletin 408, Urbana, Illinois, February 1953.
- Sandor, B. I. 1972. *Fundamentals of Cyclic Stress and Strain*. Madison, University of Wisconsin Press.
- Shahinian, P., H. H. Smith, and J. R. Hawthorne. 1972. "Fatigue Crack Propagation in Stainless Steel Weldments at High Temperature," *Welding Journal*, 51:11.
- Zapfee, C. A. 1977. Chairman, ASM Committee on Wrought Stainless Steels, *Source Book on Stainless Steels*. Dir. Allen G. Gray. American Society for Metals, Metals Park, Ohio.

STKS N310.123456
Identification of the failure mechanism

6 4362 00169405 6

[illegible]