Risk and uncertainty in the appraisal of sunken investments in range development
by Richard J McConnen

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Agricultural Economics
Montana State University

Abstract:
Northern Great Plains ranchers contemplating a sunken investment in range development face the problem of arriving at and properly discounting expected increments to net ranch income. Methods of determining mean expected increments to net ranch income are made in terms of long run averages rather than determining the increments as they occur over time.

The discounting process for time preference is quite well developed. However, the problem of selecting the proper discount rate is"somewhat obscure. The method of valuing such-sunken investments does not presently have adequate criteria to value the effect of risk and uncertainty.

The approach to the problem is in four phases. First, the method of extrapolating expectations was based on Hart's three principles plus a fourth principle added by the author. It was established that the time preference discount process should account for time preference only, using !Keynes' three factors to determine the cost to hold money. An Uncertainty Correction factor (HG) which is a function of the Coefficient of Variation (CV) was developed from the indifference analysis of variation and effective income used by Lange and the Lutzes. The above mentioned approaches to the three main problems were then combined in an attempt to apply them in valuing a sunken investment.

The method of valuing which was developed represented a framework for valuing rather than an operational method of valuing. The importance of properly valuing the effect of risk and uncertainty on both the individual rancher and society is considered. The study reaches conclusions which are general rather than specific, inasmuch as the solutions to the three main problems mentioned are of. a largely psychological origin.

RISK AND UNCERTAINTY IN THE APPRAISAL OF SUNKEN INVESTMENTS IN RANGE DEVELOPMENT

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While this work is by no means highly original, the author reserves the right to take full credit for any deficiencies and/or inconsistencies which are present.

ABSTRACT

Northern Great Plains ranchers contemplating a sunken investment in range development face the problem of arriving at and properly discounting expected increments to net ranch income. Methods of determining mean expected increments to net ranch income are made in terms of long run averages rather than determining the increments as they occur over time. The discounting process for time preference is quite well developed. However, the problem of selecting the proper discount rate is somewhat obscure. The method of valuing such sunken investments does not presently have adequate criteria to value the effect of risk and uncertainty.

The approach to the problem is in four phases. First, the method of extrapolating expectations was based on Hart's three principles plus a fourth principle added by the author. It was established that the time preference discount process should account for time preference only, using Keynes' three factors to determine the cost to hold money. An Uncertainty Correction factor (UC) which is a function of the Coefficient of Variation (CV) was developed from the indifference analysis of variation and effective income used by Lange and the Lutzes. The above mentioned approaches to the three main problems were then combined in an attempt to apply them in valuing a sunken investment.

The method of valuing which was developed represented a framework for valuing rather than an operational method of valuing. The importance of properly valuing the effect of risk and uncertainty on both the individual rancher and society is considered. The study reaches conclusions which are general rather than specific, inasmuch as the solutions to the three main problems mentioned are of a largely psychological origin.

I. The Problem, and Associated Definitions and Assumptions

The formulation of a problem consists of establishing the boundaries of the problem and identifying the contents within these boundaries. The clarification of the problem will not come about until the solution, or attempted solution, is presented.

This problem is identified as "the difficulty which arises for the planning agent in the area of decision making with regards to the valuation of a sunken investment". In this particular case, the planning agent is a livestock rancher in the Northern Great Plains. The task at hand is that of either finding or developing the method of valuation which is best suited for the geographic area and the type of enterprise in question. The problem approach will consist of three phases. First, the method of valuing which it is assumed ranchers are prone to use in practice will be developed. Secondly, a method of valuing a sunken investment will be presented which is based on pure theory. Thirdly, a model of valuing will be presented which will be an attempt to utilize the usable portions from both the first and second methods.

A "sunken investment" is an investment which when once made becomes an intrinsic part of the land. However, certain qualifications of the above definition should be made.

The mobility or immobility of an investment is a relative, not an absolute concept. Some investments may be moved or sold apart from the land with various degrees of ease (or difficulty). Immobility itself is not the sole criterion of a sunken investment. For this paper, a sunken investment will be held to exist when the following qualifications are met.

1. The investment has to do with acquisition of new land, or the development of old land. The investment is not expected to have a scrap value. The investment is calculated to effect the slope and/or shape of the ranch's total product function.

2. The investment is made with the intent of utilizing it for more than one production cycle.

To analyze the intentions of a man is a difficult procedure. Even the man committing the act may find it impossible to know his true intent. In the case of an investment on the part of a rancher, he must answer this question himself, for he alone can know his intent. In answering, he must be realistic enough to give himself a relevant answer as to his intentions with regard to the expected life of his investment. 1/

A sunken investment is such because of both purpose and intent. The classification of investments as to sunken or not is based on a discreet decision of the rancher. Opinion may vary on border line cases, but a clarification of the intent of the investor should determine whether or not the investment is sunken.

1/ To explain: A rancher spends say, $550 in digging a well. The well is literally, as well as physically, a sunken investment. Economically speaking, such a well may be anything but a sunken investment. If such a well were dug to enable a rancher to sell a half section of land, and the well was dug with this purpose in mind, the investment is anything but sunken to the primary investor.

The internal costs and revenues are assumed to be the causal factors which will be the basis upon which the entrepreneur will base his decision. Undoubtedly, the causal factors are of an almost infinite number. Many of the causal factors are of an intangible nature and many enjoy a complex inter-relationship with other causal factors.

This multiple hodgepodge of causal factors seem to present a Gordian knot to the investigator. For certainly, an unraveling would entitle the doer to recognition, master of a wide scope of knowledge. No such noble goal will be the purpose of this paper. In order to understand or analyze a problem of such intricacy, it is often found necessary to examine only those most relevant causal factors. Such a practice will be followed in this paper. However, it will not be presupposed that those less relevant causal factors may be ignored completely, for oft times the aggregate effect of the less relevant causal factors will exercise a governing effect on the decision in question.

Since this paper will be concerned with the problem of evaluating fixed investments on range land (range development) in the Northern Great Plains region, it is only appropriate that the nature of range land, with special reference to the area in question, be defined and examined.

The formal definitions of range may be said to number only slightly fewer than the definers. The Congress of the United States saw fit to define western range as follows: 2/

2/ The Western Range, Senate Document 199, 74th Congress, 2nd Session, 1936.

"The Western range is largely open and unfenced with control of stock by herding; when fenced, relatively large areas are enclosed. It supports with few exceptions, only native grasses and other forage crops, is never fertilized or cultivated, and can in the main be restored and maintained only through control of grazing. It consists almost exclusively of lands, which because of relatively meager precipitation or other adverse climatic conditions, or rough topography, or lack of water for irrigation, cannot successfully be used for any other agriculture."

The completeness of the above definition may be questioned, especially in light of the purpose of this paper. Yet, it is felt the definition is generally good.

The first questionable passage is the "never fertilized or cultivated." If "seldom", "practically never" or some such meaning phrase replaced the word never, the definition would be much more correct. Brush eradication, range reseeding, and other such range development programs involve varying degrees of cultivation. The mere act of cultivation need not change the basic properties of range land. Reseeded range land is used in practically the same manner as it was before the reseeding. Usually, the intensity of use increases directly with the intensity of cultivation. Fertilization is practiced to a lesser degree than cultivation, but fertilizing may still be profitable. Trace mineral fertilization of range land in Australia has proven this.

The mentioning of "adverse climatic conditions" in addition to inadequate rainfall as a limiting factor in land use, merits some additional attention for this particular area. Prevailing winds, length of growing season, etc., may place limitations on the land use that precipitation may not entail. Such limitations are, however, usually quite unimportant when compared to the limitations caused by precipitation. Variance of annual precipitation and variance of seasonal distribution will, if the variance dips below the "critical level" for dryland farming often enough, restrict land use to that of range. It is noteworthy to mention that the "average year" seldom prevails. Saunderson mentions the "mythical" character of the average year in this area in his discussion of fluctuating grazing capacities. 3/ Also of particular interest is the type of distribution the average rainfall pattern makes for the Northern Great Plains. There seem to be a non-cyclical groupings of like averages. Clawson discusses this at some length and develops a means of statistical classification. 4/

The uncertainty of annual precipitation may not only prescribe the intensity of use to which a land should be put, but may also determine the flexibility this intensity should enjoy. This will be discussed later.

3/ M. H. Saunderson, Western Stock Ranching (Univ. of Minn. Press, 1950), pp. 103-104.

4/ Marion Clawson, The Western Range Livestock Industry (McGraw-Hill Book Co., 1950), pp. 41-44.

II. A Description of the Method of Valuing Commonly Used by Commercial Livestock Ranchers in the Northern Great Plains.

The purpose of this section will be the presentation of a formalized expression of the method of valuing commonly used by livestock ranchers in the Northern Great Plains. It is not held that all ranchers act in the following manner, but it is felt that the principles presented are in wide acceptance.

Consciously or unconsciously, the rancher bases his valuing method on the principles of the theory of the firm. However, the valuing method for the rancher is quite apt to be in much more informal terms than those used by the economist. For the purpose of clarity, economic expressions will be used to identify the various principles of the theory of the firm.

Quenemoen, while working on an actual problem of this type, defined the theory of the firm as "a general set of principles relating factors of production into an economic unit capable of producing economic goods".1/ Such a definition seems acceptable for this paper.

The next task will be that of exposing the content of the theory of the firm as it is applied by the ranchers to the problem and the area in question. The examination of the content of the theory of the firm will be restrictive in the sense that any presentation will be an intrinsic part of the explanation of what ranchers are assumed to do.

1/ M. E. Quenemoen, Economic Aspects of Water Spreader Developments on Southeastern Montana Ranches (Mimeo Circular 69, Dec., 1952, Ag. Exp. Sta., Montana State College, Bozeman, Montana) p. 17.

Where profit maximization is held to be the goal of a rancher, as is the case for a "pure economic rancher", the point of profit maximization is determined by specific relationships of cost and revenue factors. 2/ While ranchers are not "purely economic", it is felt that the primary valuation of a sunken investment is made by the rancher with economic responses in mind.

A. <u>A Consideration of Revenue Factors.</u> The realistic assumption of a purely competitive market for the seller will greatly reduce the scope of a rancher's investigation of revenue factors. A firm with a purely competitive market for its goods is one in which every firm sells such a small percentage of any commodity that its effect on the commodity price is insignificant, and the price is the same to every seller.

The above assumption seems realistic as far as the Northern Great Plains livestock rancher is concerned. The prices paid to each rancher may in effect vary depending on the rancher's bargaining powers since livestock products are not perfectly homogeneous products. However, for theoretical purposes, these small differences may be neglected.

Under pure competition, the price received for a unit of commodity will be constant with respect to any level of production for a single firm.

2/ If losses are considered as negative profits, maximization of profits will also include the minimization of losses.

Both the marginal revenue and average revenue curves will coincide with the price line.

In the above discussion, it was implied that price is a single valued expectation held with certainty. Such is not the case, unless the rancher either made his analysis ex post, or he possessed perfect knowledge as to future prices at the time of analysis. If made ex post, it would have been a fact, not an expectation. An equilibrium position for a sunken investment involves one or more forward positions in time. The distance of these forward points from the present may vary from the very next moment in time to some point in the far distant future. The greater the length of this projection period, the greater the uncertainty of predicting an expected price.

Two types of outcomes are possible in future expectations. They are risk and uncertainty. "Risk refers to variability of outcomes which are measurable in an empirical or quantative measure". 3/ In contrast, "Uncertainty is always present when knowledge of the future is less than perfect in the sense that the parameters of the probability distribution cannot be determined". 4/ The type of outcome of expectations accompanying a sunken investment will be both uncertainty and risk, with uncertainty in expectations predominating. Expectations of the future may be

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3/ E. O. Heady, Economics of Agricultural Production and Resource Use, (Prentice-Hall, New York, 1952), p. 440.

4/ Ibid., p. 443.

determined, but it is not possible to assemble enough observations to predict the exact parameters of the probability distribution for expectations in the case of a commercial rancher. While the exact parameters cannot be established for many of the probability distributions of the rancher's expectations, the parameters may be known with enough certainty to treat them as known. It is on this basis that the treatment of uncertainty will be more rigorous than Heady's definition would allow. Risk and uncertainty will receive common treatment and will be jointly referred to as uncertainty.

The measurement of uncertainty may be determined by the magnitude of variation. The results are therefore subjective in nature. The degree of uncertainty is indicated by the characteristics of the probability distribution which surround the expectations in question. The unweighted consideration of the subjective probability distribution is in terms of the variation present. Variance, standard deviation, range of expected values, the degree of skewness, and the degree of kurtosis are the measurements of variation.

A skewed probability distribution is asymmetrical. Kurtosis indicates the degree of peakedness or flat topness which exists in a probability distribution. 5/

5/ Croxton and Cowden, Applied General Statistics (Prentice-Hall, New York, 1939), pp. 234-235.

The magnitude and character of variation is not the sole indicator of uncertainty. Another indicator is that of the possibility of loss. This will be discussed later. The magnitudinal measures of variation are, however, important indicators of the possibility of loss.

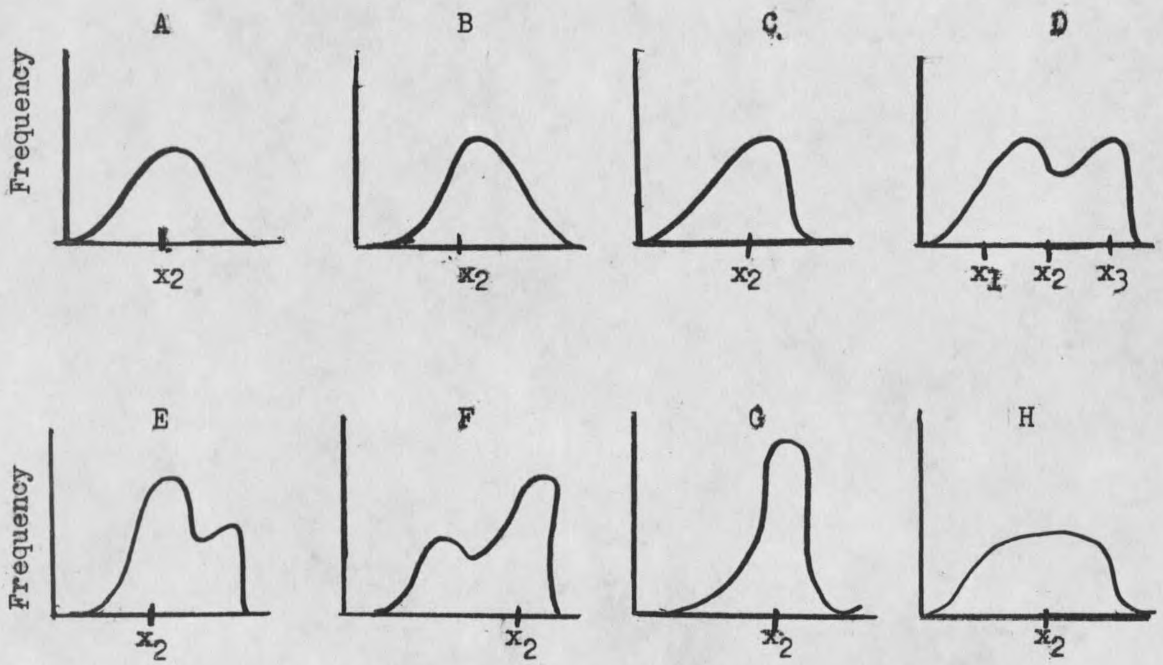

Figure 1.

Relation of Subjective Probability (frequency) Distributions and Uncertainty.

Heady discusses some of the more prominent type of probability distributions which may represent a rancher's expectations for a certain variable; in this case, price. Figure 1 is the basis on which the discussion is made. 6/

Figure 1A represents a normal distribution of expectations. A normal curve exhibiting a greater standard deviation would entail greater uncertainty. 1B is a right-skewed probability distribution which represents a lesser chance of loss than is represented by 1A. 1C is a left-skewed probability distribution which represents a greater possibility of loss than either 1A or 1B. The U shaped probability distribution in 1D indicates that there is about equal chance of an extremely high or low price, with little chance of an intermediate price. Figure 1E is a mirrored J distribution which has connotation similar to those of 1B. The J distribution in 1F has connotations similar to those in 1C. The "chance" of a higher or lower price than the mode are about equal in 1G and 1H, but the probability attached to the mode in 1G is much greater.

The price used in reaching a static equilibrium is an expected price. The amount the expected price will vary from the present price is measured by the elasticity of expectation. 7/ But inasmuch as the

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6/ Heady, op. cit., pp. 451-453.

7/ Lange, Price Flexibility and Employment, (Cowles Comm., 1944), pp. 20-21.

elasticity of expectations is never known with certainty, price expectations cannot be determined with complete certainty. An entrepreneur cannot rationally act on some definite unique value chosen indiscriminately. At best, he can only choose a range of possible values. 8/

Some particular price out of all possible prices may appear to the entrepreneur as the most probable value, and this most probable value may become the basis of his expectations. The preciseness with which the most probable price is expected to occur will depend on the range of the probability distribution of possible prices around the modal price.

An alterative basis of establishing the expected price is the use of the mean of the probability distribution for expected prices. As in the case of the modal expectation, the preciseness with which the average price is expected to occur will depend on the range of the probability distribution of possible prices around the average price.

If the distribution is truly normal, the mode, arithmetic mean, and the median will be equal. However, since many probability distributions are not likely to be normal, there are some economists who favor the use of the arithmetic mean as a basis to determine the most probable expected price. 9/

8/ Ibid, p. 29.

9/ Some of the advocates of the mean as a tool for dealing with uncertainty, are A. Pigou, The Economics of Welfare, pp. 773-774, J. R. Hicks, A Suggestion for Simplifying the Theory of Money, Economics Vol. 6, February, 35, pp. 1-19.

Either a single expected price, or a very narrow range of expected prices, must be chosen, but now the choice may be more rational than if a definite unique price had been chosen with no formal recognition of its probability of occurrence. Lange states, "The greater the range, the less definite the expected price. The range can thus be taken as a measure of the degree of uncertainty of the expectations. In most cases, the entrepreneur or consumer does not consider the whole range of possible values of the expected price, but disregards the extreme values at both tails of the probability distribution. He does so because the joint probabilities of these extreme values is too small to bother about. The range with the tail values thus cut off will be called the practical range and will serve as our measure of the degree of uncertainty of price expectations." 10/

The rancher, in making a decision with regard to a future price, will think in terms of a range of expectations. There are an almost infinite number of possible (though not probable) prices which might be expected, and these possible prices may be distributed over a great range. Certainly, the entrepreneur cannot consider the complete range of possible prices, but, as Lange suggests, will deal with a practical range of expected prices. The upper and lower limits of the practical range may be chosen common sensical or with some degree of mathematical precision at a predetermined level of probability. Seldom does the commercial entre-

10/ Ibid., pp. 29, 30.

preneur find it possible to compute mathematically the limits of the practical range for a certain level of probability, but informally, he will attempt to decide upon a practical range which has some unknown, but high, level of probability. The more definite the value of the practical range is, the better. However, such definiteness may be obtained only at a cost.

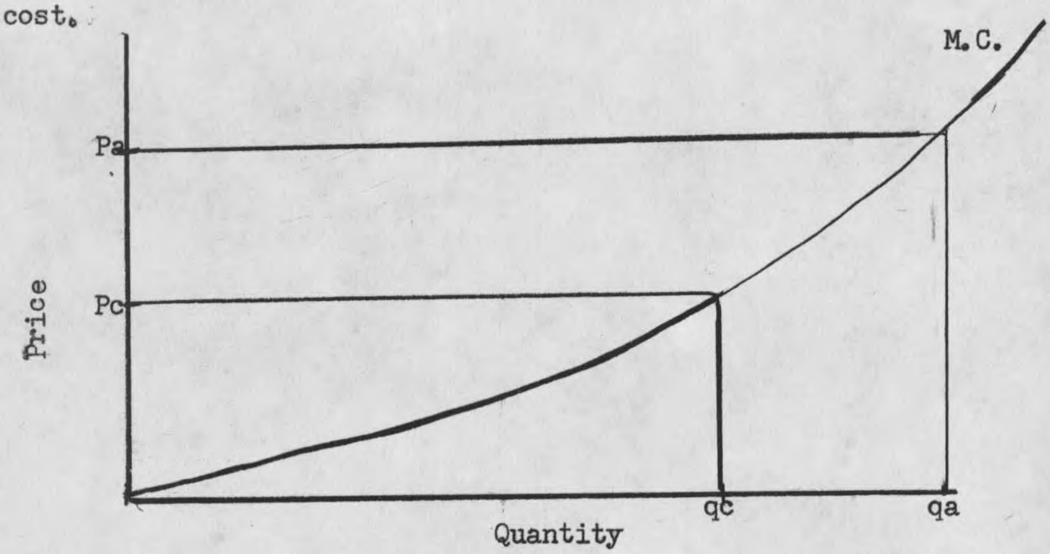


Figure 2.

Range of Production Based on Expected Prices Where the Marginal Cost Curve Is Assumed to be Known.

The rancher cannot bargain, he has but to accept a market price. He must plan his production on one price or a narrow range of prices. As a hypothetical case, an entrepreneur expects at the 95% probability level, a P_x between Pa and Pc as depicted in Figure 2. For the purpose of illustration, it is assumed his marginal cost curve is known. If the upper limit price of the practical range, Pa, occurred, he would produce qa. If the lower limit price, Pc, occurred, he would produce qc. Since production is not an instantaneous process, the entrepreneur must plan to produce some quantity of X between qa and qc, the exact quantity being dependent on some one price in the practical range which the entrepreneur decides to act upon.

Some data have been collected by Williams at the University of Illinois with regard to farmers' willingness to assume various degrees of uncertainty. 11/ Williams found a pattern of preference for various levels of income and the certainty of these incomes. Jensen found a like result while engaged in a study of irrigated pastures in Montana. Jensen discussed this concept in terms of income vulnerability. 12/

Examination of a hypothetical ranch investment with regard to uncertainty of income for the investment is given below. Figure 3 gives the practical range of expected prices Pa - Pc, the most probable P, Pb, all prices for the product of the investment, and an assumed marginal cost curve for the investment.

11/ Williams, "Price Expectations and Reactions to Uncertainty by Farmers in Illinois," J.F.E., February, 51, Vol. XXXII, pp. 20-39.

12/ C. W. Jensen, The Economics of Pasture Integration and Irrigated Farms. (Mimeo Circular 67, July, 52, Ag. Exp. Sta., Bozeman, Montana, pp. 74-78.)

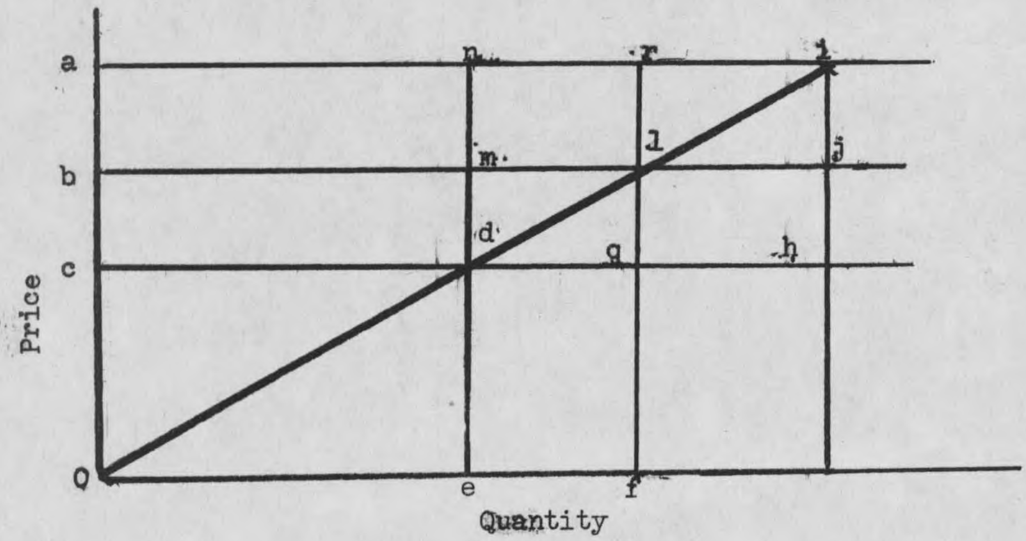


Figure 3.

Possible Incomes Based on Expected Prices Where the Marginal Cost Curve Is Assumed to be Known.

The problem is that of choosing some quantity of X as a production goal. Suppose this rancher has an aversion to risk. The effective expected price will lay somewhere below b. The greater the aversion to risk, the nearer the expected price will be to c. It is extremely unlikely that the P will fall below price c. Operating in light of Pc will insure a net of variable costs income of at least cdo. The planned production will be oe. It is very unlikely that a P greater than Pa will occur. Therefore, the greatest net income which the firm could expect producing oe will be ando. Therefore, a firm producing oe in light of Pc may expect a range of incomes from cdo to ando, with a most probable net income of bmdo.

If this firm were to operate in light of the most probable P, Pb, of would be produced. With a range of incomes running from (cdo-dlg) to a high of arlo with an average net income of blo. While the average net income is greater in the second case (blo-cdo), the range of expected income is also greater. In addition, any entrepreneur operating in light of price Pb increases the possibility of realizing a negative income. (Range (cdo-dlg) to arlo cdo to ando). 13/ This firm's aversion to risk may make the lower average net incomes acceptable because of the lesser degree of variance. 14/ The introduction of fixed costs into the analysis would further complicate the question of uncertainty.

13/ (cdo - dlg) - [arlo - (cdo - dlg)] > (ando - cdo)

[(arlb + blo) - (cdo - dlg)] > (andc + cdo - cdo)

(arlb + blo - cdo + dlg) > andc

(anmb + nmlr + bmdc + dlm + cdo - cdo + dlg) > andc

argc > andc by ndgr

also, at Oe, min. profit is cdo, at Of, min. profit is cdo - dlg. Hence, the price Oc involves not only less variability, but also greater reward for the risk involved.

14/ This situation is explained in Lange's Price Flexibility and Expectations, pp. 31, footnote No. 7.

B. <u>A Consideration of the Cost Factors.</u> To reach a meaningful decision, a rancher will accumulate knowledge of cost factors as well as revenue factors. Those cost factors which the rancher is ultimately concerned with are the marginal costs of the physical product. Marginal cost of output (MC) = the price of the aggregate of variables (P_V) divided by marginal physical product of the variable (MPP_V). The marginal cost for the physical product is determined by (1) the price of the production factor and (2) the rate at which this production factor is converted into the physical product.

The cost of the production factor may be viewed as (1) the unit factor price, and (2) cost in terms of the amount of the production factor needed to produce a unit of physical product. The unit factor price is of no value unless the rate at which the production factor is converted into the end product is known. It may be assumed that the rancher anticipates paying a price for the factors used equal to the prices paid by others. The price of the production factor may vary with the quantity purchased. But the unit factor price is of importance only in so far as it affects the cost per unit of produced product. Because the cost in terms of marginal cost of produced product is determined by the production function and factor prices, the discussion of change of unit factor prices is abandoned.

Analysis of cost in terms of cost per unit of product produced must be tied to the rates at which the production factor are converted to end products. The production function is the relationship from which the relevant rates of conversion are derived. 15/

The production function is a physical relationship and of a technical nature. It is not the purpose of this paper to engage in a discussion of the exact nature of the detailed technical determinants of a rancher's various production functions, for such information is seldom available to the rancher. Rather, the purpose will be that of examining the general nature of the production functions for sunken investments. The primary means of obtaining this end will be the employment of seemingly realistic assumptions. While these assumptions will not hold true for all livestock ranches and/or ranchers, their chance of occurrence is great enough so as to lend some insight into the nature of the planning agent's means of making a decision with regards to sunken investments. These assumptions may later be relaxed singularly or in groups, but unless such a relaxation is specifically designated these assumptions shall be held for the remainder of the paper. These assumptions shall be the following:

1. The ranch will be owned by the manager, debt free. However, as is the practice in this area, various amounts of land may be leased or rented. With this exception, all physical resources are held to be owned in full.

15/ G. J. Stigler, The Theory of Price, (The Macmillan Co., New York, Revised Edition, 1952) p. 106.

2. The type of product will be held as fixed. The quantity may vary. A single enterprise firm will be held to be the case. While this assumption will seem unrealistic, it will materially aid in the adaption of the theory of the firm to the area in question, as the problem of shifting enterprise (cattle to sheep, etc.) is arbitrarily eliminated.

3. The ranch or ranches will be fixed as to size. This assumption will prescribe the size within limits. The limits will be narrow, allowing only for the adjustments needed for a closer approximation of a least cost combination for the particular level of resource ownership in question. However, the size may vary from ranch to ranch.

For the purpose of explaining the nature of a rancher's production function, the quantity of product previous to the advent of a sunken investment will be some average quantity, say OQ_9. For the purpose of discussion, it will be assumed the only varying factors, with the exception of the sunken investment will be those production factors needed to utilize any added primary production which may result from the particular sunken investment in question. For instance, a certain investment may result in a possible greater forage yield. Certain amounts of livestock, labor, and management are needed to utilize this possible added production. Since a rancher measures his production in terms of beef or mutton and wool, a conversion of the added forage into these commodities is necessary. Under like conditions of climate, soil etc., a fixed investment should yield each rancher an equal quantity of the primary product, in this case forage. However, the above need not hold true when the conversion of this primary resource (forage) into the secondary resource (beef or mutton and wool) takes place. It is quite likely, within certain limits that livestock and labor of approximately the same quality are available to all ranchers in an area. Such is not true for the remaining variable, that of management. The nature of the production function for a sunken investment will be considered initially without the variable of management. Later, the management factor will be included. With these considerations in mind, a production function for a sunken investment is presented below in Figure 4.

Inasmuch as a sunken investment entails production over more than one production period, it is extremely important to fully explain the labeling of the axes of a diagramatic presentation of a relevant production function. This is especially true in the case of a static analysis since the consideration of the element of time is somewhat obscured. The X axis will represent the quantity of livestock product per year. The Y axis will measure the various levels of annual investment.

As stated before, (by assumption), OQ will be produced providing there is no disinvestment. The production function was constructed on the basis of the law of eventually diminishing marginal physical productivity. Any level of investment up to Od will result in an increased production of livestock product. Any investment beyond this level will result in a loss in total production. Somewhere between the limits a and d is the economically relevant area of production. It is not

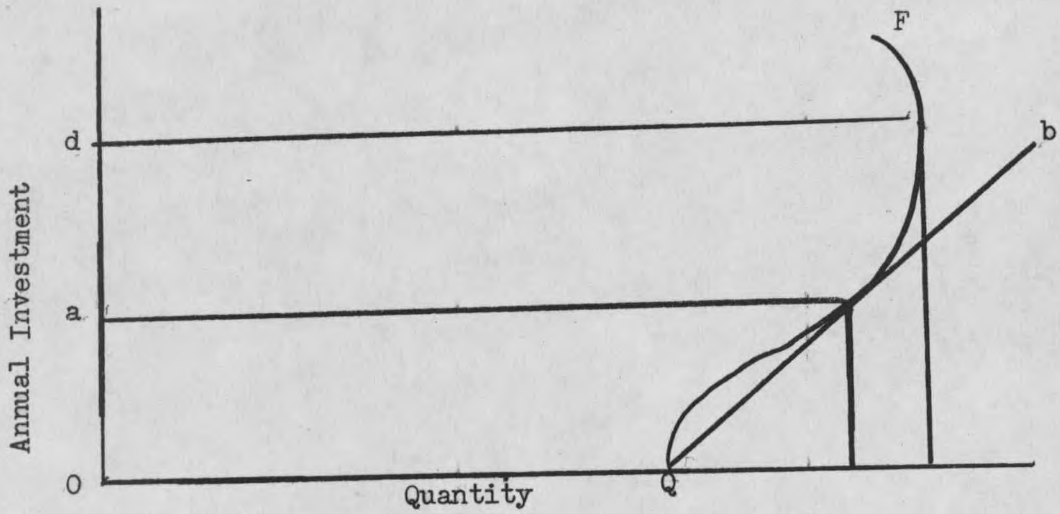


Figure 4.

Range of Feasible Annual Investments.

readily evident that it would be economically feasible to invest at a level higher than Od. Line Qb is drawn tangent to the production function. At the point of tangency, marginal product = average product. From O to a, the average productivity of the annual investment is increasing. It is not readily evident that it would be economically feasible to limit the annual investment to an area where the average productivity is increasing.

A marginal cost curve for the investment ($M\ C_I$, in Figure 5), may be constructed from the production function by plotting a series of points denoting the added average level of investment per unit of product increase. However, an addition must be made to this $M\ C_I$ curve, the marginal cost involved in applying other variables needed to utilize the additional primary resource provided by the investment. The variables needed to utilize the primary resource in this case are labor, livestock, tax payments, and all such obvious costs. The marginal cost curve for such variables (MC_V) is usually assumed to increase with an increase in the quantity of product produced. This is especially true when the rancher omits the variable of management. Total marginal cost is derived by summing the vertical values for both marginal cost curves (MC_V and MC_I) for each value on the horizontal axis.

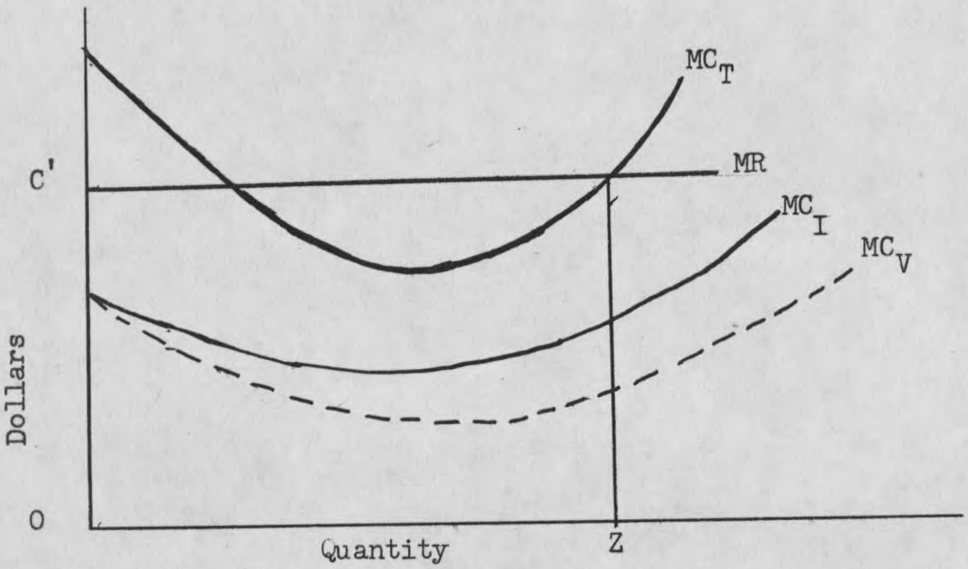


Figure 5.

Marginal Costs Curves for an Investment.

Since the valuing procedure is ex ante in nature, it holds that if the MR is above any portion of the MC_T, it would be possible to maximize profit or minimize loss by equating MC_T to MR. To determine the level of average investment most profitable, a line should be drawn perpendicular to the X axis and passing through the point where M C_T equals M R. The point of intersection on the X axis, oz, will indicate the net quantity of livestock product which is desirable.

Such an analysis is highly over simplified, but essentially this is the process a rancher goes through, but at a more informal level. It is seemingly quite possible that the rancher is apt to use an economic horizon of short duration and attempt to cover his total costs of investment in this duration, assuming all the product produced after this horizon to be "gravy" or profit on the investment. However, the admission of a gravy period extends the horizon in question, but this extension horizon is of a more indefinite type than the original horizon.

The rancher usually does not stop here. Once he has determined the direct benefits received from the investment, he is apt to temper his decision by considering the seemingly indirect benefits and costs which may occur as the result of the investments. Such a consideration will be made outside of the body of the formal analysis and after the formal analysis has been completed. It is now that the effect the investment will have on the elements of time preference, risk and uncertainty, available leisure time, the chance of success or failure, etc., will be considered. Discussion of these subjects will be presented later, for it is in these areas that suggestions might possibly be made as a means of improving the rancher's decision making analysis. It may be possible for the commercial rancher to include these less obvious benefits and costs as an intrinsic part of his analysis.

In discussing a production function, the two important considerations are (1) the probable shape of the production function and (2) the degree of certainty with which the shape of this function may be predicted. The rancher may not recognize the production function as such, but rather deal directly with the cost factors. However, it is felt that the production function is the best means of explaining any peculiarities which may arise in the cost factors.

Even the best, or perhaps, only the best technicians in livestock production would hesitate to describe the exact shape of the production function for a rancher's fixed investment.

It seems quite feasible, however, that the law of eventually diminishing physical productivity operates in the case of fixed investments. The question of when this "eventually diminishing" takes place is a horse of another color. The determination of exact shapes and slopes is too much to expect, but by examining like investments in the local area and referring to technical literature released by both private and public institutions, the chance of gross false prediction is materially reduced. Also, it is quite possible to obtain valuable information from local soil conservationists, bankers, and county agents. By utilizing such information it should be possible to determine the general type of production function which might be expected. Because there is a degree of uncertainty in determining the production function, the rancher may elect to deal directly with the cost factors, realizing the existence of a production function, but ignoring it as such, and making an instantaneous transfer from production to cost factors.

The continuous smooth line production function assumes that the production factor in question is infinitely divisible. For many of a rancher's possible fixed investments, this might not be so. Real estate is a prime example. Usually a sale and hence the purchase is made on the basis of all or none, and seldom is the rancher able to sell any of the "all" which he feels he may not want. If the possible investment is of one level only, the production function becomes one point. For instance, if a level of investment of only Od_1 were available, (as shown in Figure 6) the net quantity Oq_1 would be produced.

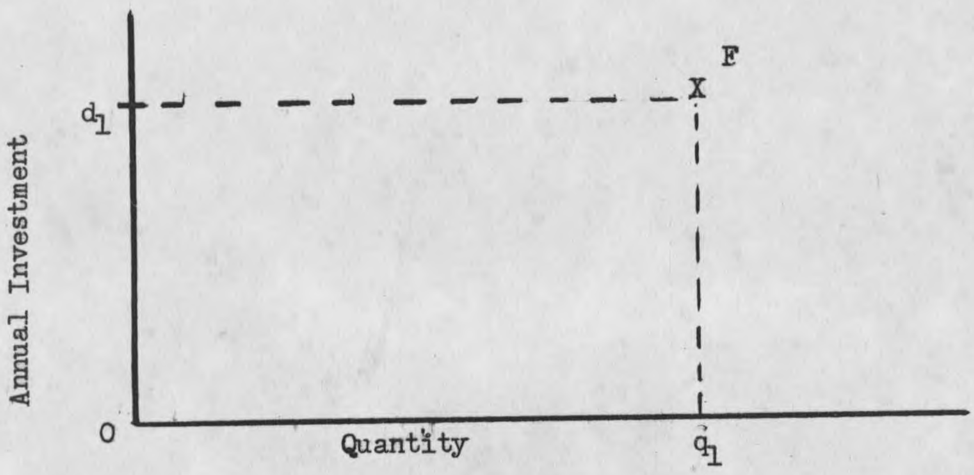


Figure 6.

The Production Function When Only One Level of Annual Investment Is Possible.

A more likely occurrence will be a stair-step production function where the production factor is available in different quantities, but the quantities available are in fairly large parcels. The size and arrangement of the parcels themselves may vary in size. In the immediate vicinity of a rancher, he may find three parcels of land offered for sale which could be incorporated into his present operation. They are 80 acres, 320 acres and 640 acres. It is assumed they are offered at the same price per acre and are of approximately the same productivity. A production function somewhat akin to that in Figure 7 will result.

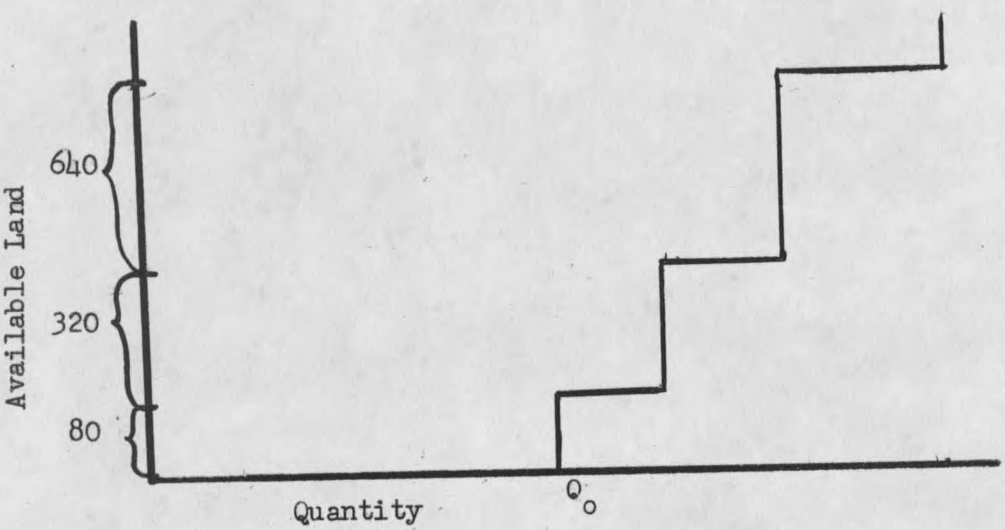


Figure 7.

The Production Function Where Set Levels of Annual Investments Arise.

The resulting M C from the two production factors mentioned above are depicted in Figure 8 where 8a gives the M C when only one level of investment is available (Figure 6), and 8b the M C where a stair-stepped production function is in evidence (Figure 7).

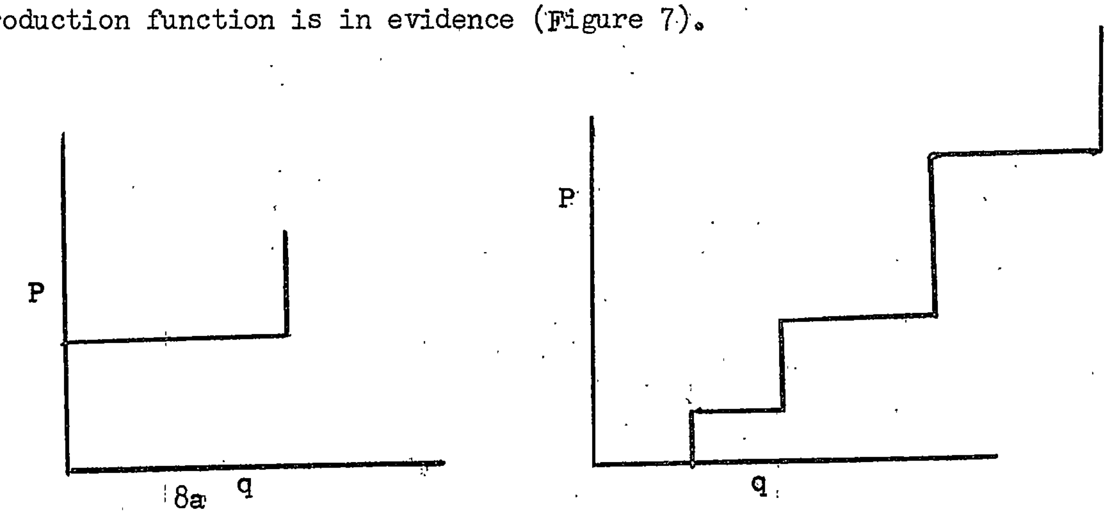
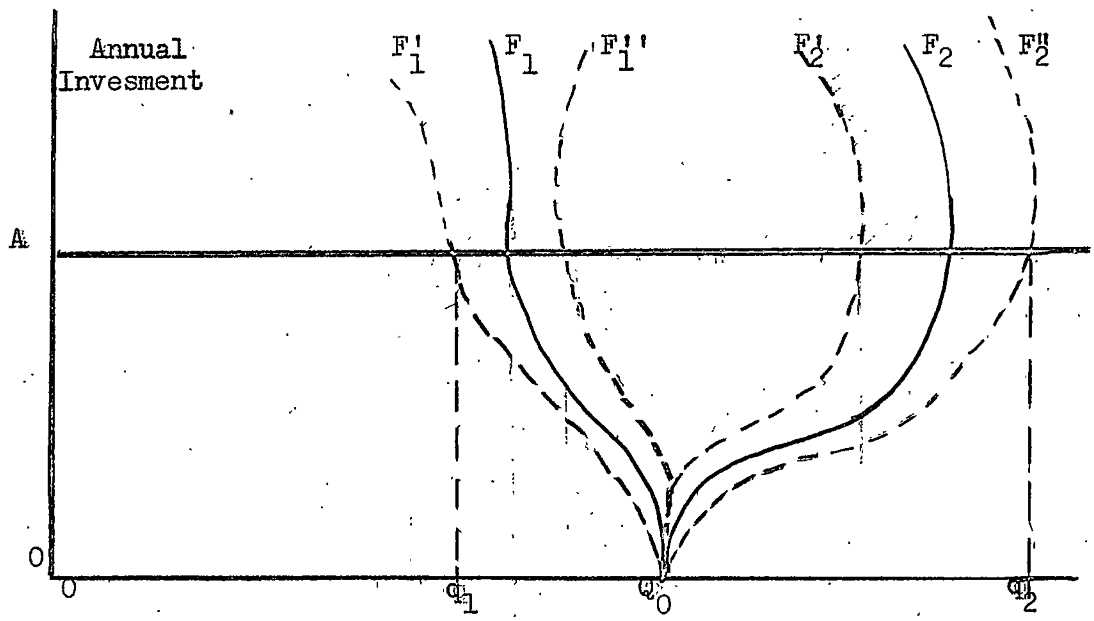


Figure 8.

The Marginal Cost Curves for
Set Levels of Investment.

Even if a rancher could know the shape and position of a production function from technical material, this production function is likely computed for the average rancher. While the manager in question might be an average rancher, it is likely that he differs considerably from the theoretical average in many respects. Therefore, the production functions will be apt to vary for each individual rancher on the same type of investment depending on the ability of the rancher as a manager. What confronts the rancher, then, is a possible range of production response and he must rationally choose the production response which he considers to be the most probable for himself.

The management ability of the rancher may affect both the shape and the position of the production function. Figure 9 gives the relevant range of production functions which may occur as the result of some investment, say a water spreader on a cattle ranch.



Quantity of Product

Figure 9.

Range of Possible Production Functions.

If the investment resulted in a poor physical structure and/or poor management of the physical structure, a negative production function such as F_1 might result. However, if the rancher exercises the utmost care in constructing the physical structure involved and exercises the best possible management practices, a positive production function such as F_2 might be expected. These production factors would result assuming average climatic conditions.

As can be seen, one of the most important determinants of the shape and slope of a production function besides management, has been partially omitted. This determination is the expected climatic condition. These climatic factors are of extreme importance over which the rancher exercises no control in this area other than that of adapting his operation to the expected climatic conditions. Both the lower and upper limits have a range of possible occurrence dependent upon climatic conditions. The curves F_1^{1}, F_1^{11}, F_2^{1} and F_2^{11} are then added to Figure 9. F_1^{1} and F_2^{1} are those production functions which might exist if climatic conditions were extremely bad for the production period in question with management held as under average conditions. Conversely F_1^{11} and F_2^{11} are those production functions which might be expected to exist if extremely favorable climatic conditions occurred. It is assumed above that Q_o is produced with certainty and will not vary with climatic conditions. The Q_o is really an average production for the ranch.

It is, therefore, possible for an investment of O A to result in net additional beef production from a minus Q_oq_1 to a plus Q_oq_2. It is impossible to make a decision when such a large range of possible production is considered as such. However, the rancher needs to know what the possible results of his course of action are. If he is to determine his course of action from such a group of production functions, he will select some one F, or a narrow range of F curves which he deduces are most probable to occur.

In deciding upon which production function to adopt, the rancher must first relate his abilities to the investment in question. A rancher may find it helpful to discuss this problem with an interested but frank and unbiased party (a Utopian conception). The rancher must in some way, decide on the nature of his ability to manage the investment.

The above discussion is applied as it relates to the rancher considering the water spreader in Figure 9. The rancher in question is assumed to be a careful manager who seems to have a knack for achieving a good deal with little material which may be attested from the manner in which he operates his present ranch setup. He is not seemingly overwhelmed by decision making, and is actively engaged in making his ranch a better producer. He also sees the possibility of some of the intangible values of the investment; for example, reduced uncertainty for the ranch as a complete unit. After careful consideration, he feels he is capable of operating the water spreader in question at some production function in the shaded area F_R of Figure 10. Between F_3^{1} and F_3^{11} are the production factors likely to occur at both climatic extremes.

His degree of aversion to risk and his interpretation of the likely climatic conditions will determine the ultimate production function he expects to materialize. If he anticipates a dry year and/or has an aversion to risk, he may well choose a production function near F_3^{1}.

Such a discussion may seem unrealistic, but it is quite likely that the better managers go through a process such as that above, but on a more informal level. It is equally unlikely that the poorer managers go

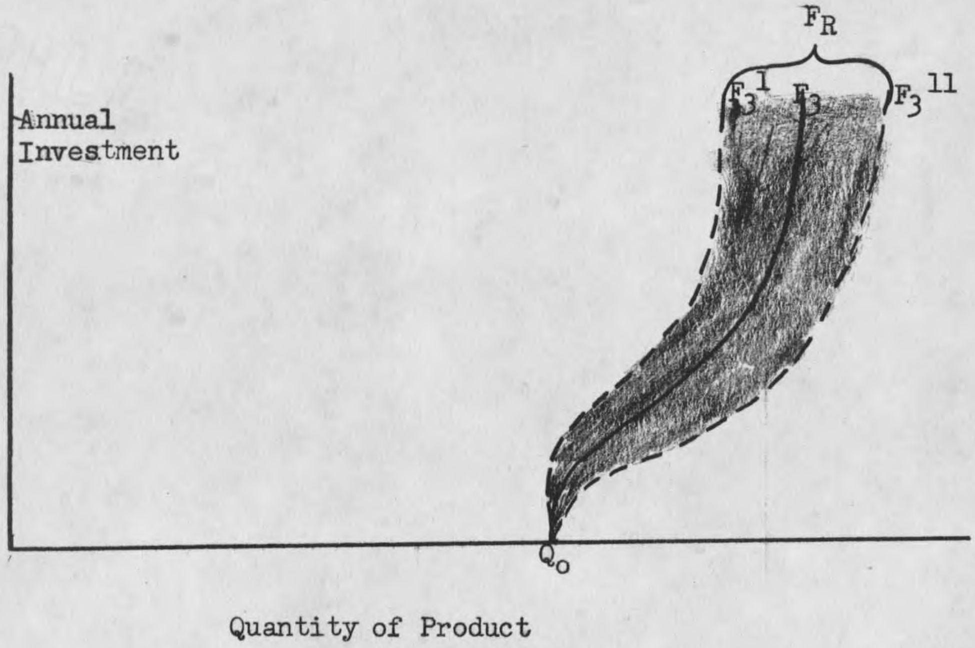


Figure 10.

The Effect of Climate on a Chosen Production Function.

through any such process, but replace logic with luck. It may seem paradoxical that a "good manager" with poor luck may be a heavy loser and a "poor manager" with good luck might be an equally heavy winner. The degree of "goodness" or "poorness" of a manager is difficult to know, for management is an art, not a science. But it must be remembered that even an art has principles which must be adhered to. While there may be many heretics, only a few are successful, and the remaining unsuccessful heretics are likely to be artists far under par.

Up to now the discussion has been in terms of the formalized method a rancher would use in valuing one particular investment. How many possible alternatives are there which compete for a rancher's limited capital? Certainly they must be many, and it is doubtful that a rancher considers each in the manner prescribed previously. He will narrow down the alternative courses of investment to those he considers economically feasible. He will accomplish this by three means.

First, he will deem unworthy of valuing those investments which require a capital outlay greater than he is willing to bear, either because of his capital position or the added risk which may be involved in large scale borrowing.

Secondly, he will eliminate those alternative investments whose "horizon" is such that he is unwilling to consider them. The horizon is that period of time over which the revenue from the investment is not less than the variable costs of production. First, the horizon for the investment may be of a greater duration than the entrepreneur's horizon. That is, the investment may be profitable, but only over a longer period of time than the rancher is willing or able to expose his capital. The length of an investment's horizon may be acceptable, but the nature of the returns during this horizon may be considered unfavorable. For instance, the investment may yield no returns for the first three or four production periods. It is at this point that the rancher's liquid capital balance is apt to be at a low ebb.

Thirdly, he will not consider these investments which might possibly be profitable, but where there are remaining alternatives which will obviously yield a greater profit.

The resultant is a lesser number of possible alternative investments which the rancher deems worthy of a formalized valuing process. These alternatives are then valued by utilizing the principles set forth under Revenue Factors and Cost Factor Analysis. Once the formal valuing has been completed, a comparison of the results for each alternative will be made.

C. "Additional Considerations". The final choice or decision cannot be reached as yet, for there are additional considerations not included in the formal valuation. The valuation was made in terms of the investment itself. Its impacts on the remainder of the firm were not fully considered. This, then, is the next step. In what ways may it affect the firm other than the increase in one physical product?

Most likely the first consideration will be the status of the demands made upon the entrepreneur and his immediate family. Some ranchers would consider unfavorably an investment which would tie the rancher or his family down, or would require exacting labor and management which the rancher feels he is capable of, but which he considers very laborious or perhaps humiliating. Consequently, a highly profitable investment might be deemed undesirable because of the heretofore unconsidered non-monetary costs to the rancher and his family. Conversely, an unusually pleasant endeavor associated with an investment may place the investment in a more favorable light.

Next is a consideration of the investment's effect on the firm's economic stability (risk and uncertainty). It is quite possible that production flexibility might be considered the key to economic stability.

An investment with only one possible use might be considered unfavorable when compared with an investment with several possible uses even though the single use investment is seemingly more profitable. The investment might strengthen other phases of the ranching organization other than that phase with which it is directly concerned. For instance, a crested wheat pasture may result in forage production which is valuable for reasons other than the quantity and the quality produced. The time of production, early spring, might give grazing at a very crucial time of the early lactation period of ewes and cows, hence reduce the uncertainty of a poor lamb or calf turnoff.

A particular investment, while not unduly profitable in a dollars and cents way, may eliminate an aggravating problem. A water spreader system may be more profitable than a new section of fence. The rancher, however, considers the cost of hunting strays and continually patching fence to be greater than the actual monetary cost involved.

Some tribute to time preference, however inadequate, will be given. The rancher may realize the profitability of an investment, but not be willing to give up his liquid capital for a greater amount at a series of points in the future.

A rancher lives in a community and supposedly is part of that community. As such, he is not immune to the wishes of that community. If he feels the community will be greatly displeased over a particular investment, he may refrain from making the investment regardless of its profitability.

The above mentioned are but a few, although it is likely they are the important ones, of the additional considerations given after a formal valuation for each investment and a comparison of the results is complete.

It would be difficult to say with any degree of truth that a rancher is irrational in valuing a possible investment when all the above have been considered. However, there seems to be a possibility of incorrect weighing of three important factors. They are, production over time, time preference, and risk and uncertainty (all of these considered as they affect the firm as a unit). It is the purpose of the next section of this paper to construct a model which will have as an intrinsic part of the evaluation method these three factors.

III. Development of an Economic Model for Valuing

The first object will be that of defining the goal of the rancher. As previously, the rancher will be assumed to be an "economic man" (one who seeks profit maximization). The Lutzes list four possible criteria for profit maximization, all four of which have been used in literature.1/ Lutz lists them as, "First, the entrepreneur may find the present value of the future gross revenue stream (V) and the present value of the future cost stream (C) by capitalizing at the interest rate ruling the market, and maximize the difference (V - C) between these present values. Secondly, he may maximize the present value of the future revenue stream (again found by capitalizing at the given market rate of interest) divided by the present value, similarly calculated of the future cost stream, i.e. he may maximize V/C. Thirdly, he may maximize the internal rate of return on the total capital sum invested. Fourthly, he may maximize the rate of return on his own capital, which is assumed to be a given amount and may be smaller than the total sum invested whenever part of the latter is financed out of borrowed funds." 2/

It may be assumed that at all times a rancher will be interested in maximizing the rate of return on his own investment. Lutz argues that the maximization of the rate of return on the entrepreneur's investment will coincide with the maximization of V - C in practically all important cases. The two conditions which will cause the entrepreneur's maximum

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1/ Friedrich and Vera Lutz, The Theory of Investment of the Firm (Princeton University Press, 1951) p. 16.

2/ Ibid, p. 16.

rate of return on his capital not to coincide with maximum V - C are (1) where the entrepreneur does not have unrestricted access to the market for borrowed funds, and (2) where both the funds horizon and the internal investment horizon are of the dependent type and coincide as to length. (T).3/

It will be remembered, that it was assumed the rancher owned his ranch in full, and the investment would be made for the purpose of achieving a better balance of production factor use at approximately the same scale as was present before the investment. Therefore, in all but the most peculiar cases, it will be assumed that the rancher has access to the market for borrowed funds, as the magnitude of the investment will not be so great as to render the present assets of the ranch inadequate security for a loan (if a loan be needed) to cover the capital outlay for the investment.

If the maximum rate of return on the entrepreneur's capital is equal to the rate of interest on the loan or borrowed capital and both are equal to the internal rate of return, V - C will be maximized but will not coincide with maximization of the entrepreneur's rate of return on his own capital. If it is possible to sell the investment capitalized at the market rate of interest, even though the entrepreneur does not elect this alternative, the investment will be most profitable when V - C is maximized even though the rate of return on the entrepreneur's own capital will not be maximized. Even if either of the above exceptions to the exception may not hold, maximization of the rate of return on the entrepreneur's

3/ _Ibid_, p. 42.

capital will not coincide with the maximized V - C only when the funds horizon and the internal investment horizon are dependent and of equal length. The chances of this occurring are slight indeed, for it is highly unlikely that a rancher will base the life of his firm on the length of time an investment will economically produce unless that investment is of such a nature that it is crucial to the existence of the firm. 4/

If the rancher earns other than the going rate of interest on his investment, each criterion gives different results though the maximization of the rate of return on the entrepreneur's capital and the maximization of V - C will coincide in all but the above mentioned cases. 5/ In accor-

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4/ Ibid, pp. 27-31.
5/ Where p_m equals the marginal internal rate of return and p_a equals the average internal rate of return, V - C is maximized where the marginal internal rate of return is equal to the rate of interest at C_2. Maximization of V/C requires that the average internal rate of return be equal to the marginal internal rate of return as at C_1. Maximization of the internal rate of return requires that the average internal rate of return should equal the marginal internal rate of return, which is identical with the maximization of V/C. The fourth criterion, maximization of k, the rate of return on the entrepreneur's own capital, where the entrepreneur's own capital is a fixed amount, is identical with the maximization of V - C. This will be true because any additional funds above the entrepreneur's will be borrowed funds, and he will increase the use of borrowed funds until the marginal internal rate of return equals the rate of interest on the last unit of borrowed funds. The above will hold true where the rate of interest, r, is held constant.

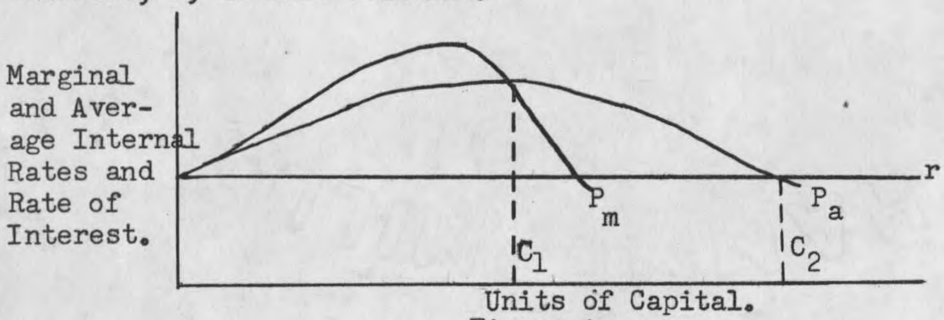

Figure A.
Different Results For Different Criteria of Maximizing Profit.

dance with Lutz, the criteria of maximization of V - C will be the basis of the valuing analysis used. While maximization of the rate of return on the entrepreneur's capital (k) is the desired end, Lutz feels the manipulating difficulties which are present when dealing directly with k do not compensate for the small chance of a situation where the maximization of k and V - C do not coincide. 6/

By definition, a sunken investment involves production over time. It is therefore necessary that the element of time involved (T) be singled out and clarified as to its meaning. The period of time over which economic production "takes place" is referred to as the "economic horizon". 7/ It is over this period that the rancher plans his sales of product or optimum time length of operation. There exists what might be called two sub-horizons which in turn determine the nature of the economic horizon. These two sub-horizons are called the "funds horizon" and the "internal investment horizon". The funds horizon is that period of time over which the entrepreneur anticipates investment possibilities for the funds in question either within or outside his firm. The internal investment horizon is that period of time over which he may expect to realize a positive quasi rent in the last time period (t_n) for a particular investment within his firm. 8/ Each sub-horizon may be of either the independent or dependent type. A dependent horizon is one

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6/ Ibid, p. 43.

7/ Lange, op. cit., p. 32.

8/ Lutz, op. cit., p. 25.

which is dependent on the value given to the variable time, a series of t's (T). The independent horizon is one which is not dependent on the value given to the various t's. There are three possible combinations of the fund horizon and the internal investment horizon.

The first possibility is that where both the fund horizon and the internal investment horizon are of the dependent type. In other words, there is a known period of time over which the funds will be available and over which a positive quasi rent will result. These two sub-horizons may or may not coincide. If they do not coincide, the shorter of the two horizons will determine the length of the economic horizon.

The second possibility is that where one of the two sub-horizons is of the independent type, the remaining sub-horizon being of the dependent type. In this case, the dependent sub-horizon will determine the length of the economic horizon.

The third and last possibility is where both sub-horizons are of the independent type. In this case, the economic horizon is also of the independent kind (thus has no definite value).

The economic horizon is that period of time over which a rancher will discount the optimum economic operation of his investment and his flow of a stream of discounted quasi rents. Because the economic horizon determines the time length over which profit is calculated, it is extremely important to determine the length of the relevant economic horizon.

Since the criteria of the maximization of V - C has been designated as the method of valuing a sunken investment, it is appropriate to examine the component parts first separately and then together.

A. <u>Considerations of V - C.</u> V has been described as the present value of the gross revenue stream. It may also be defined as a series of prospective quasi rents discounted at the market rate of interest that gives the entrepreneur the estimated capital value of the investment. Lutz gives the formula for approximating the value of V where r = the rate of interest, T = the life of the investment in terms of years, $(t_1)\ Q_t$ = the quasi rent on the machine in any unit period t, and S = the present value of the scrap value of the investment (which might conceivably be a negative value). 9/

$$V = \frac{Q_1}{(1 + r)} + \frac{Q_2}{(1 + r)^2} \cdots\cdots \frac{Q_T}{(1 + r)^T} + \frac{S_T}{(1 + r)^T}$$

Quasi rent is the excess of receipts over what Marshall refers to as the "supplementary costs", or "--- --- the money cost of the raw materials used in making the commodity and the wages of that part of the labor spent on it which is paid by the hour or the piece and the extra wear and tear on the plant." 10/ The supplementary costs are the operating costs, or those costs which are incurred during each production cycle only if the investment is utilized or operated. Operating costs are those costs which must be paid each production period if the investment is to be

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9/ <u>Ibid</u>, p. 13. Lutz derives this formula from the calculus

$$V = \int_0^T Q(t)e^{-rt}\,dt + Se^{-rt}$$

10/ Alfred Marshall, <u>Principles of Economics</u>, (Macmillan and Co., Ltd., St. Martins Street, London, 1936), pp. 359-360.

operated, unless future expected (discounted to present value) quasi rents will compensate for any incurred. It is possible that the quasi rent may be negative for one or more unit periods (production cycle) t. Q_1 is the quasi rent resulting from the first production cycle, Q_2 the quasi rent for the second production cycle, etc. According to Lutz, 11/

$$Q_t = \left[p_t\ (q_t)\quad q_t\right] - \left[E_t\ (q_t)\right]$$

and Q_t is maximized when

$$p_t'(q_t)\ q_t + p_t\ q_t = E_t\ '(q_t) \text{ or (when MC = MR)}$$

Where p_t (q_t) is the average price of the product, where q_t is the number of units of output produced in a production cycle: where E_t (q_t) are the "variable" or operating costs (exclusive of depreciation and of interest charges on the cost of the fixed equipment) also as a function of the number of units of output produced in the production cycle.

Again, the rancher will find it necessary to determine the price which he expects to receive for his product, only now it is necessary to determine a price for each separate production cycle instead of deciding on only one price for the life of the investment. It is also likely that the p_t may vary within the production cycle and hence p_t (q_t) might be dependent on several prices. However, in the name of simplification, it will be assumed that the total production during a production cycle (q_t) will be sold or valued at one price (p_t) (q_t). With few exceptions, the rancher finds it necessary to extrapolate his price predictions. Hart, in discussing the nature of forecasting by extrapolation, states, "There is no golden rule. Instead, we have three basic assumptions, which may be applied in an infinite number of combinations: (a) that the

11/ Op. cit., p. 12.

recent level of the variable under study will continue, (b) that the recent rate of change of the variable will continue -- an assumption which admits of variants, as we may consider the significant rate of change to be either the rate of growth or the rate of acceleration, or some higher derivative, and (c) that the variable will tend toward some level thought of as 'normal'." 12/ The inadequacies of the above assumptions becomes more apparent when the element of dynamics is considered. However, if a fourth assumption (d) is added, that of the effect of exogenous change, a deeper understanding of the nature of expected prices results. Exogenous changes are those changes which occur because of probable future developments rather than the result of past or present conditions. The future price for a commodity (say X) for a particular period in the future is the function of the supply and demand which exist in that period, i.e., $P_x = f(S_x, D_x)$. Hence, any future development in either supply or demand may have some effect upon the price for that particular period which is not considered in either the past trend, past norm, or present price. A graphic illustration might better present this. (Figure 11).

Product X has a present price of OA, a normal price of OB, and a recent trend line of CA. A synthetic product is replacing X, and researchers are on the threshold of rapid improvement of this substitute. On the x axis is measured time. The curve AD' is the actual line which will exist for product X in the future because of the technological developments which are about to take place in the competing synthetic

12/ A. G. Hart, "Anticipations, Uncertainty, and Dynamic Planning", (Augustus M. Kelly, Inc., New York, 1951), p. 78.

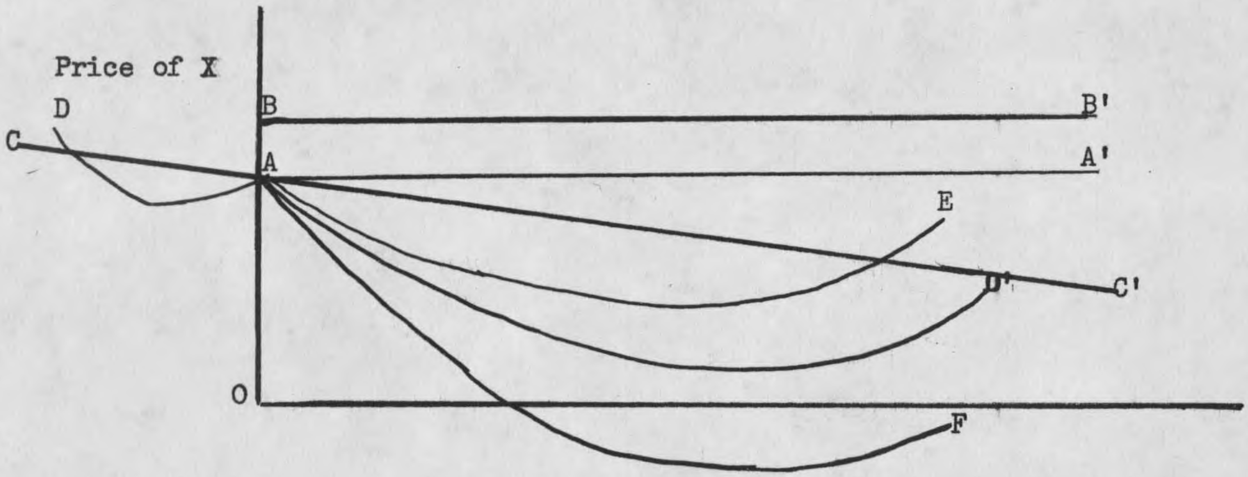

Figure 11.

Predictions for the Price of Commodity X.

industry. It therefore becomes apparent that the fourth assumption may be applicable in some cases.

It is extremely unlikely that a curve such as AD' could be predicted with any high degree of accuracy. Therefore, the expected future price for commodity X is likely to have several values at any one period (t) in time. It might be stated that AD' is the mean or average price line with a probability distribution scattered at the 95% probability level between lines AE and AF. The standard deviation for any one mean value for one period of time (assuming a normal distribution) will be the vertical distance between line AD' and either AE or AF divided by 1.96. It will be noticed that the σ increases with an increased period of projection.

The same may be said for the other projections into the future; BB', AA'; and AC'. These lines are in a sense an average price expected with dispersion around the mean. The σ for these projections will also increase as the period of projection increases.

Quantity (q) like price must be determined for each production period in the future. Since the future production is not known with certainty, the rancher finds it necessary, as in the case of price expectations, to extrapolate his production expectations. In the livestock industry, barring any revolutionary developments such as some unheard of disease, or an overcrowding from oil derricks, the techniques of production and the climatic conditions determine the quantity of a product produced in any future production cycle. The principle of Hart's three assumptions plus the principle of consideration of exogenous change used in determining price expectations could seemingly be applied in considering the possible effects of techniques of production on future production. Climatic conditions for future production periods are of a somewhat different nature, as they are not greatly affected by present conditions. It may be possible to place a cardinal value on the probability distribution of TDN's actually produced in the area by studying the past history of the area in question. The result will be an average climatic condition in terms of production of TDN's or acres needed per animal unit. The arithmetic mean would be determined for a long term record of production and the standard deviation calculated.

Climatic conditions seem to follow no easily recognized pattern. If the acres needed per AU were correlated with tree ring growth from 1784

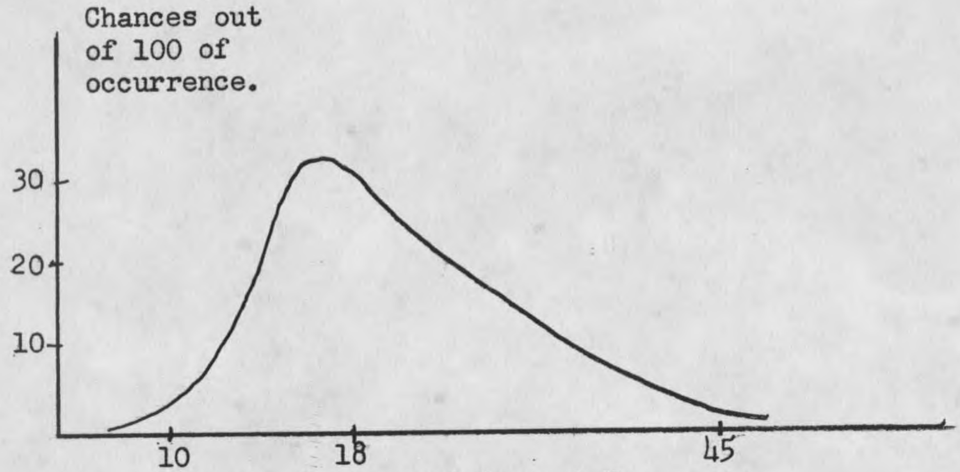

Figure 12.

Frequency Distribution for Acres Needed per AU near Havre, Montana.

to 1949 near Havre, Montana, it would be found that an average of 18 acres were required per AU. 13/ The standard deviation would be in the neighborhood of 3.8 acres. The distribution is definitely skewed to the right, and the resulting flatness indicates a degree of kurtosis. Figure 12 is a diagramatic display of the distribution. The data and calculations were rough, and the results are not necessarily empirically sound.

Without further refinements and computations, it would be difficult indeed to place a significant cardinal probability value on the probability of the occurrence of a good, average, or poor year. Added research and refinements of correlating the acres needed per animal unit with either tree ring growth, or some other long time record, might give a more valuable relationship than that which the rough approximation

13/ Source: M. A. Bell, Montana Experiment Station.

provided. It is not apparent that any valuable information might be obtained by considering the expected climatic conditions to adhere to the present conditions, the short run trend, or rely on some normal or average conditions. With refined statistical treatment, it might be possible to develop a reliable probability distribution of climatic conditions.

By combining any registered trend in technique of production and the probabilities of production, a future expectation of production could have a cardinal value and the possibility of its occurrence determined. The entrepreneur's attitude towards risk could be considered and a fairly reasonable production expectation arrived at.

The total variable costs for a production period, (E_t) (q_t), is determined by the technique of production, and the price of the variables. The technique of some future production period has been previously considered, and it may be possible to utilize this information to determine the quantity of the product which will be expected for some particular period in the future.

Consideration of the price of the variables used will be somewhat dependent on the previously considered price of the final product. It is not inferred that the price of the variables used is a function of price of the product. However, there is certainly some correlation between the prices paid and the prices received. One would not expect an extremely high price for beef and also expect to hire the needed labor at an extremely low price. Certainly, the relationship of the prices paid and the prices received are important. Again, it might be possible to construct a probability distribution of the ratios of prices paid to prices received and place a cardinal probability on the prices expected to be paid based on the expected price received.

By taking the difference between the index of prices received by farmers and the index of prices paid by farmers 14/, and assuming a normal distribution for such differences, it was found that there was a standard deviation of about 19. At the 95% level of probability ($1.96 \times \sigma$), it was found that the index of prices paid would not vary more than ± 38% of the change in the index of prices received. If for instance, a price increase of 10% in prices received was expected, 95% of the time the prices paid would be between 62% and 138% of the present prices paid. If, however, the indexes of prices paid and prices received for the specific area in question were computed, a more reliable figure could be obtained. 15/ For instance, if the index of prices received consisted only of the indexes of the prices of products sold in the area, and the indexes of prices paid were only for the variables used to produce the product in mind. If the first mentioned figures were to be taken as a sufficiently reliable basis for action, and beef were expected to go up in price about 10% at some date in the future, and where the present average variable cost per cwt. of beef sold was known to be about $12.00, it could be expected that 95% of the time the rancher could expect his variable costs to run between $7.44/cwt. and $17.76/cwt. The rancher is now in a posi-

14/ "1952 Agricultural Outlook Charts", (Bureau of Agricultural Economics, U.S.D.A., Washington, D.C., October 1951), pp. 12-13.

15/ This was done by Gray and Baker in Commercial Family-Operated Sheep Ranches, Range Livestock Area, Northern Great Plains 1930-50, Bul. 478, Nov. 1951, M.S.C. Agri. Exp. Station, Bozeman, Montana, p. 90.

tion to make a better decision on a likely average variable cost for that period in the future.

S_T is the residual value of the investment. As previously mentioned, this may either be a positive, zero or negative value depending on the nature of the investment. If the investment were usable for one purpose for a certain length of time (T), but was no longer efficient enough for that purpose after the occurrence of t_n (where the revenue no longer covers variable costs) it might be used for another purpose which demands a lesser degree of efficiency (the variable costs may be covered in the new use). As such, the value of the investment area may be greater than if the investment had not taken place. The positive net value of this residual of the original investment (the worth of the investment over that worth which would have existed if the investment had never been made) is then the residual value of the investment. If, however, in order to prepare the area for another investment, a capital outlay would be required that would not have been necessary if the investment (before t_n) had never been made, a negative residual value would result and the previous summation of the discounted quasi rent would be lessened by this amount discounted at the discount rate.

1. Choice of an interest rate or discount rate. Tarshis describes the interest rate as the cost of holding money. 16/ What then determines the cost to hold money? The interest rate for society is determined by the supply and demand curves for money. The resultant is an interest

16/ Tarshis, Lorie, The Elements of Economics, (Houghton Mifflin Company, Boston 1947), p. 321.

rate for all investments to all of society.

The interest rate may vary for individual ranchers and, certainly, it will vary with the type of investment. The motives for holding money as money, according to Keynes are three fold. They are the transactions motive, the precautionary motive, and the speculative motive. The reason for this is that money is the most liquid of assets. The rancher must either keep some of his own capital or have a source to borrow this capital from, to carry on the transactions of ordinary business endeavors. The rancher also finds a certain emergency relief or insurance in liquid capital, or a liquid source of capital. This then is the precautionary motive. The real value of capital changes, and when such changes occur there is the possibility of altering one capital position by speculating in the change of price levels. The speculative motive will up the interest rate if it is felt the "real" value of the capital is about to increase and vice versa if it is felt the real value of the capital is about to decline. 17/

Black states the appropriate interest rate to use is dependent on the above plus the amount of risk involved and the servicing required. 18/ For our purpose it may be assumed that no servicing is required, hence no cost will be charged to servicing. For the present, no consideration will be given to the risk involved for the particular investment, and hence, the going rate of interest for the area, or the rate of interest charged

17/ Ibid, pp. 325-327.

18/ John D. Black, et al., Farm Management, (The Macmillan Company, New York, 1949), p. 740.

for the average risk, will be used. The choice of a time discount rate cannot be dismissed so lightly, however, for Black points out the effect a wrong choice of the rate of time discount may have.

The effects given are summarized in Table I below. 19/

Table I. The Effect of Changes In r On The Present Value

Change in r		Change in Present Value
From	To	
6%	4%	+50%
5%	4%	+25%
4%	3%	+33%

In light of the above, the rancher may find it advisable to take the going rate of interest as a base in establishing the rate of time discount which he finds most suitable for his purpose, if he elects to adjust for risk in the discount rate. He will then adjust the interest upwards if he feels his need for transaction capital is especially tight, if he foresees some future happening that may require considerable liquid capital, or if he feels the economy is about to enter a depression. If he expects an inflationary period, he will lower the interest rate, as he feels the value of his capital will increase more rapidly as an investment. After considering these three things, he may then arrive at an adjusted rate of time discount which will better fit his purpose.

2. The effects of a change in the rate at which the entrepreneur discounts his profits. A fall (rise) in the interest rate will cause a

19/ Black, op. cit., p. 740.

shift to a more (less) capital intensive process; i.e., the average period of investment, if the price of variables remain constant. The present value of V will adjust upward (downward) as the result in a fall (rise) in the discount rate used. A fall in the time discount rate will also make it more profitable to substitute initial cost (a better physical plant) for future variable costs. 20/ Likewise, a fall in the interest rate or time discount rate will (1) cause the investment period, or funds horizon, to lengthen, and (2) cause an increase in scale. These two causes may cancel one another, or at least modify the effect of one another depending on the shapes of successive revenue growth curves and the degree of rise in the interest rate. 21/

After consideration of the variables discussed above, the rancher finds himself in a position to determine the value of his expected discounted flow of quasi rents (V).

Where C is the original cost of the investment, the formula V - C will then indicate the difference which must be maximized. The rancher would value all feasible investments in the above manner. It is understandable that each investment might have a series of V's. The rancher will choose that combination which he thinks most likely to occur because of his management ability, etc. The choice of the investment with the maximum value of V - C does not necessarily follow. If the rancher had four feasible investments on his ranch, with original costs (O.C.),

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20/ Lutz, op. cit., p. 127.

21/ Ibid, p. 81.

economic horizons, (T) and V - C's as listed below in Table II, it does not follow that he should choose Investment I.

Table II. Returns on Hypothetical Investments With Various Economic Horizons.

Investment No.	Original Costs	V - C	Economic Horizon
I	$10,000	$1,800	10 yrs.
II	$ 1,500	$ 500	5 yrs.
III	$ 5,000	$ 800	7 yrs.
IV	$ 3,500	$ 700	6 yrs.

Investment profit maximization (maximization of V - C) depends both on the original cost (capital invested) and the period of time over which the investment is exposed (economic horizon). It stands to reason that the rancher would rather realize the 33 1/3% profit over the period of five years in Investment II, than the 18% profit over a period of ten years on Investment I. The rancher is interested in total profit, and hence it might be such that if the rancher had $10,000 of free capital his total profit would be more in Investment I than if he invested $1500 in Investment II and deposited the remaining $8,500 in the bank at no profit, in the sense that profit is used here. Below is a table giving the total percent profit for each investment, the percent profit per year per investment. The weighted profit per year is determined by multiplying the % profit per year by $10,000 divided by each investment. The aggregate weighted profit for investment II, III, and IV is a summation of the weighted profit per year for those investments.

Table III. Total, Annual, and Weighted Profits For the Investments in Table I.

Investment No.	Total % Profit	% Profit per year	(% of $10,000) Wt. of Invest.	Weighted Profit per year with $10,000 to invest
I	18%	1.8%	100%	1.8%
II	33.3%	6.7%	15%)	
III	16%	2.3%	50%)	3.3%
IV	20%	3.3%	35%)	

If the rancher chooses Investments II, III, and IV, the percent profit per year would be an effective 3.3%, as compared to 1.8% for Investment No. I. It is not to prove that the maximization of V - C is not a true criterion for profit maximization that this has been done, but to point out the possible necessity of considering several smaller investments as one investment when a certain amount of capital is available for investment, and some investments may be made with a lesser amount of capital than that available for investment. The V - C must be maximized for the whole fund of liquidity available to the rancher.

B. Considerations of Risk and Uncertainty. Expected values, whether for price, quantity, labor costs or interest rates, have been treated as single valued expectations. The probability distribution of expected values have been discussed, but their significance in terms of risk and uncertainty has not been explored. The presence of a probability distribution for expected values may be presented in a rather concrete way with the aid of statistics. Terms such as skewness, kurtosis, range, standard deviation, mode, median, and mean describe the nature of the probability distribution. The attitude of the entrepreneur towards this probability distribution is in part a psychological phenomenon. For the present, it will be assumed that the probability distributions in question assume the shape of a normal curve. In this case, the risk preference function may be described in terms of the mean (M) and the standard deviation. 22/ Inasmuch as this paper deals mainly with the uncertainty aspect rather than risk, and since the uncertainty has been described in terms of unknown probability distributions, and since in a sunken investment more than one production period is in question, the uncertainty preference function will be considered to be much the same in nature as the risk preference function and will include both risk and uncertainty. For the present, it will be assumed the uncertainty preference function for an entrepreneur is of a negative nature.

In effect, the entrepreneur must correct his single valued expectations for uncertainty. This corrected value will be referred to as N. Lutz states that $N = f(\sigma, M)$, (N is a function of the standard deviation and the mean of the probability distribution in question). If U.C. were used to signify the "Uncertainty Correction Factor", N would equal (M)(UC). The correction factor is not a function of σ alone, but the comparative magnitude of the variation of the distribution. The comparative magnitudinal variation may be measured by the coefficient of variability (CV). $C.V. = \frac{\sigma}{M}$. Therefore, U.C. = F (CV).

22/ Lutz, op. cit., p. 189.

With a negative uncertainty preference function, it is conceivable that an entrepreneur might prefer a low mean with small dispersion to a greater mean with greater dispersion. Both Lange and Lutz give geometrical presentation similar to that in Figure 13 below: 23/ 24/

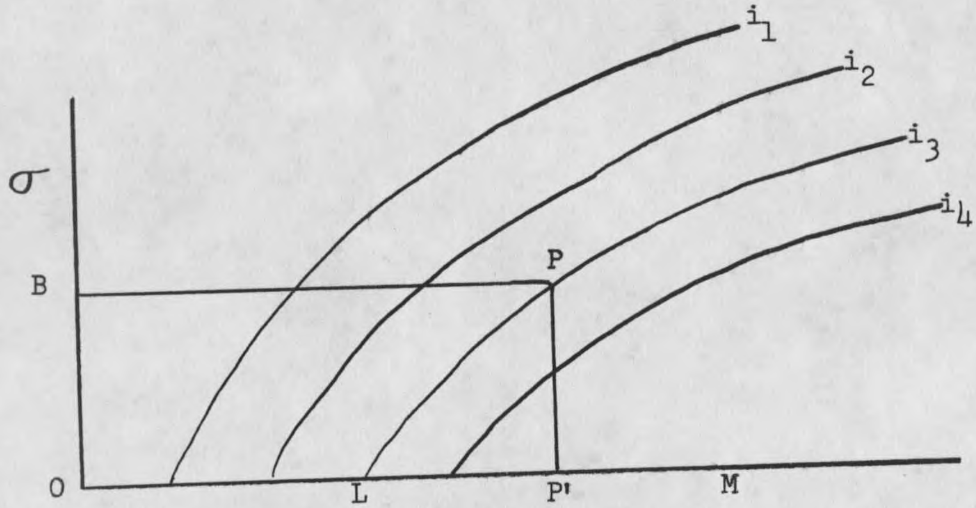

Figure 13.

Variation and Effect Average Indifference Curves.

The curves i_1, i_2, etc., are indifference curves with i_4 signifying a high level of satisfaction, and i_1 a lower level of satisfaction. If some particular investment results in a V - C of BP (assuming the V - C = M of the probability distribution of possible values of V - C) with a σ of B, the corrected value of V - C (N) would equal OL. (M = OL, σ = 0). OL would be the "effective average", and LP the "uncertainty equivalent". In this case, the correction factor for the uncertainty involved would =

23/ Lutz, op. cit., p. 190.

24/ Lange, op. cit., p. 31.

OL/BP = UC. If several investment alternatives are possible, the entrepreneur will choose that investment which will place him on the highest possible indifference curve. Or, in effect he will maximize N which is the value read on the x axis along line OM. 25/

Again, if the V - C is thought of in terms of the rate of profit per year, it is then possible to determine the optimum of the various alternatives. Each investment has several possible V - C's, and it is assumed that the V - C used for each alternative investment is that combination of M and σ which places the entrepreneur on the highest possible indifference curve. Each investment has some point which signifies this point, such as point P in Figure 14. The resultant is a scattering of such points, one for each alternative investment. The point which touches the highest indifference curve (the greatest N) represents that investment which gives the maximum corrected V - C or N. Since it might well be difficult to examine each of the points in question, a line connecting the outermost points, (those points furthest to the right at each level of σ) called the Boundary Line, is drawn. That point at which the Boundary Line touches the highest indifference will indicate that investment which is most profitable. 26/

Figure 14 below gives the boundary line for numerous alternative investments, each of which is represented by one of the dots to the left of the boundary line. Point P is that dot which represents that invest-

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25/ Lutz, op. cit., p. 190.

26/ Lutz, op. cit., p. 191.

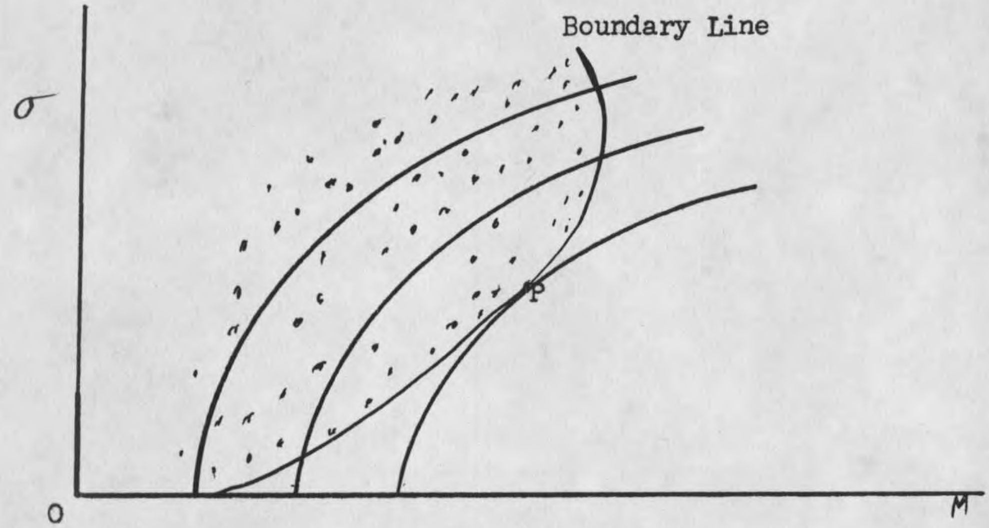

Figure 14.

Use of the Boundary Line as a Means of Determining the Maximum Corrected V - C.

ment with the maximum V - C.

The discussion so far has been in terms of the most profitable investment of several alternative investments. The rancher also has the problem of choosing maximum V - C for each investment. The maximum V - C for each investment depends on the technique of use of the investment and the scale of the investment. Therefore, the rancher is not only confronted with the valuing of alternative investments, but also the valuing of alternative scales and techniques for each alternative investment. This problem may be treated in the same manner as the valuing of alternative investments. However, by assumption, the problem is somewhat narrowed when problems of scale for the firm as a whole are ruled out.

Therefore, the problem of proper scale of an investment is limited to that scale which would not materially alter the scale of the firm. In many cases, the investment may be fixed as to scale because of physical limitations of the investment in question. The possible techniques of use are also limited somewhat (in some cases there might be only one possible technique or scale for an investment) by the nature of the investment and the abilities of the entrepreneur.

The use of an uncertainty correction factor (UC) has been discussed previously, where UC = f (CV) (a function of the coefficient of variability). It was also previously assumed that the entrepreneur had a negative uncertainty preference function. It is not to be inferred, however, that the negative value is of the same magnitude for all types of uncertainty. In other words, a negative uncertainty preference function indicates a UC with a value less than one, but the value of UC may vary for various types of uncertainty. However, it is highly likely that the UC value will be of the same functional relationship for rather broad categories of uncertainty for each entrepreneur. Figure 4 is a representation of the functional relationship which might exist for some particular categories (f_1 & f_2) of functional relationship between UC and CV. 27/

27/ The choice of elements to be included in a category represented by one functional relationship between UC and CV is dependent upon the nature of the uncertainty premium for each element to be included. To be completely accurate, each element will have a functional relationship between UC and CV which is unique. However, it is likely that a number of these unique relationships will bear enough resemblance that they may in effect be considered as one relationship.

An extreme case would be where the probability distributions for expected prices of two like products, A and B, have the same standard deviation, but the mean expected price of B is greater than the mean expected price of A. In this case, the CV value would be greater for product A than product B, as CV is a measure of relative dispersion. The set of indifference curves describing the uncertainty premiums would have to be the same for each product if the same functional relationship between UC and CV were to be used for both products. The slope of these indifference curves would have to increase as the height of the indifference curve, or utility value, increased. This relationship is shown in Figure B.

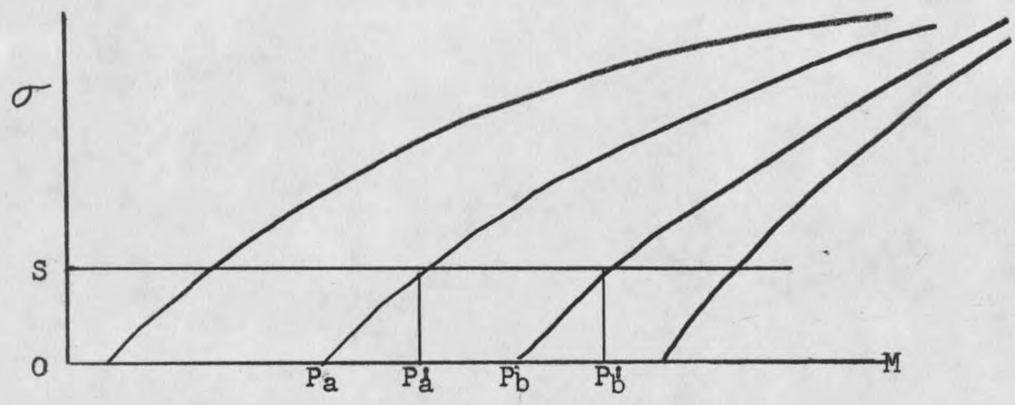

Figure B.

Nature of the Indifference Curves for Categories Which Have Like Functional Relationships Between UC and CV.

Where OS is the standard deviation for the price expectation probability distribution for both product A and B, OP_a the mean price for A, OP'_a the effective price for A, OP_b the mean price for B, OP'_b the effective price for B, the ratio on the uncertainty equivalents for A and B ($P_aP'_a$ / $P_bP'_b$) must be equal to the ratio of the CV's for A and B (CV_a/CV_b), if the functional relationship between A and B is the same for both products.

From Figure B it will be noted that the importance of the standard deviation in determining the effective mean value increases disproportionately with its increase in actual value. While the above principle may seldom exist to this magnitude, it is likely that the disproportionate increase in the importance of the standard deviation with an increase in actual value will hold for many situations in real life. If the above principle is true, it is seemingly more realistic to talk of a category of elements having nearly the same functional relationship between UC and CV.

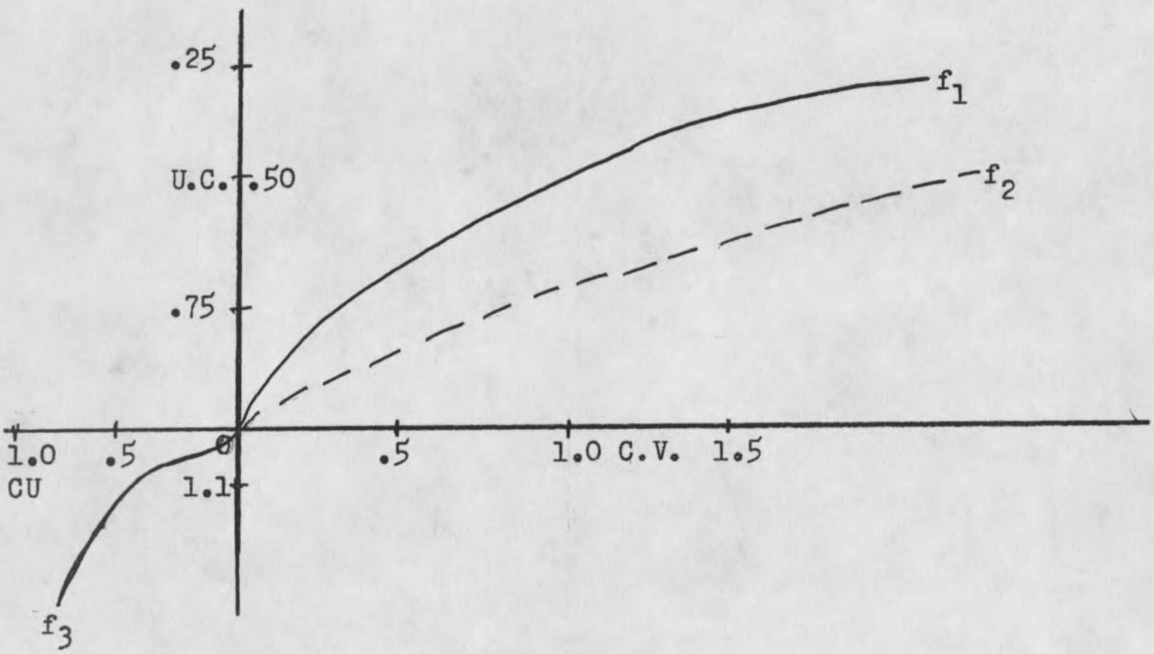

Figure 15.

Three Functional Relationships Between UC and CV.

The UC value is used to correct the value of the mean value which is felt to exist. In a sense, the UC value is not the correction factor, or shrinkage factor, but rather a yield factor which gives the corrected net value (N) of the mean.

It might be possible that a rancher would have one functional relationship (f_3) which is to the left of the y axis and below the x axis for a category such as gambling at small stakes. Or a functional relationship coinciding with the x axis for some category of which a certain limit is required, such as insulin for a diabetic rancher where a certain amount is necessary to continue life, but any amount beyond that is of no value. The shape of these functional relationships are variable for each rancher.

The use of the assumption of normal distributions has greatly simplified the above discussion. However, empirical observation of expected values, yields, prices, etc. for the area considered seems to indicate that normal curves or near normal curves for future expectations are likely to be the exception rather than the rule. In reality, there is likely to be some degree of skewness or kurtosis in the probability distributions for expected values.

The adjustment for uncertainty (use of UC) may take place at several levels. For an example, the following discussion will be in terms of the adjustment of the V to uncertainty. It will be assumed that the values for UC are known for the various probability distributions of expected values.

If the mean quantity for each time period (q_t) were adjusted for uncertainty (M multiplied by the UC), the same done for the price in each time period (p_t) and the variable costs for each time period (E_t), the resultant quasi rent would be corrected for uncertainty. The resultant discounted V would then be a discounted value which would give the N value when the original cost (C) is subtracted.

The correction for uncertainty may also be made on a basis of the probability distribution of expected quasi rents. The quasi rents will be multiplied by the UC signifying the adjustment for uncertainty. The discounting of the V value (discounted if a negative uncertainty preference is assumed) may also be used. Here, the mean V which might result from an investment is discounted on the basis of the probability distribution's standard deviation, or rather the resulting CV which is converted to a UC value. The discounted or effective V = (V)(UC).

C. Abnormal Probability Distributions, And Their Effect On Uncertainty Correction. As previously mentioned, three general types of abnormal curves exist. They are: a skewed curve, a curve showing kurtosis, and a bi-modal curve. They and their effect on the uncertainty correction factor (UC) will be discussed in the above order.

1. The effect of a skewed distribution on UC. A skewed curve is an asymmetrical curve. Figure 16 gives two examples of skewed curves. The solid line curve is a probability distribution which is skewed to the right (line AA'). The dotted line curve is a probability distribution which is skewed to the left (line BB').

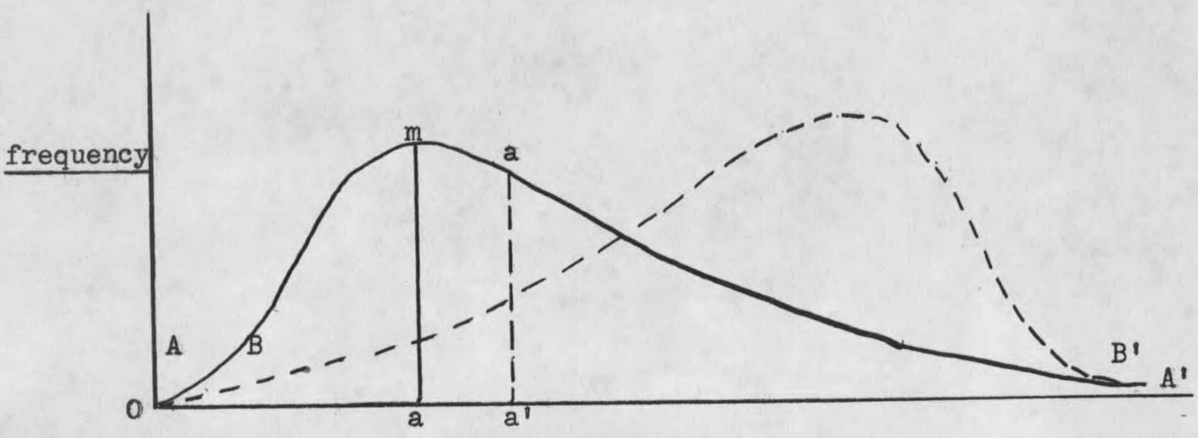

Figure 16.

Types of Skewed Distributions.

UC was stated to be a function of CV. This is strictly true only for a normal distribution. The standard deviation measures the dispersion around the mean. In the case of a probability distribution such as AA', the mean will be in the approximate vicinity of aa', to the right of the mode (Ma). If the standard deviation (σ) were calculated, and the 95% limits of probability were calculated, it is possible that the limit less than the mean might be to the left of the zero axis, which would be irrational in many cases. In any case, this limit will be to the left of that point which cuts off the extreme 2-1/2% of the lower limits. The use of UC at face value, therefore, gives undue weight to the values less than the mean. In effect, the upper limits of the curve are slighted. The mean plus 1.96 (σ) establishes the right limit of the 95% fiducial limits. The area to the right of this point (under the curve) is greater than the 2-1/2% which should theoretically be there.

Where the probability distribution is skewed to the right, and UC = f, CV, too much weight is placed on the values less than the mean, too little weight is placed on the values greater than the mean. Where the probability distribution is skewed to the left, and UC = f (CV), the reverse is true. The skewed distribution seems to reduce the importance of CV. While CV is indicative of the comparative dispersion for various probability distributions, there are certain limitations to its use. 28/

The amount of skewness may be measured by several methods. For the purpose of this paper, the Pearsonian method will be used. While this

28/ For a further discussion of the limitations of CV, see JFE Vol. 24, 1942, p. 892, "Some Limitations to Use of Coefficient of Variation", Joe J. Livers.

method may not be as adequate as the measures based on quartiles, percentiles, or the third moment, its simplicity of calculation is attractive. For the remainder of the paper, skewness will be signified by the notation Sk, when a formula is used. The Pearsonian measure of skewness is a relative measure of dispersion. $Sk = \frac{\overline{X} - Mo}{\sigma}$, but since it is sometimes difficult to locate the mode in some frequency distributions, the formula following may be substituted: $Sk = \frac{3(\overline{X} - Med)}{\sigma}$. When the skewness is to the right, the algebraic sign of the Sk will be positive. When the distribution is skewed to the left, the algebraic sign will be negative. The value of Sk will never be greater than ± 3, and values of greater than ± 1 are unusual. 29/

The UC will now be designated as UC_1. This correction factor must itself be corrected. The UC_1 corrected for skewness is denoted as UC_2.

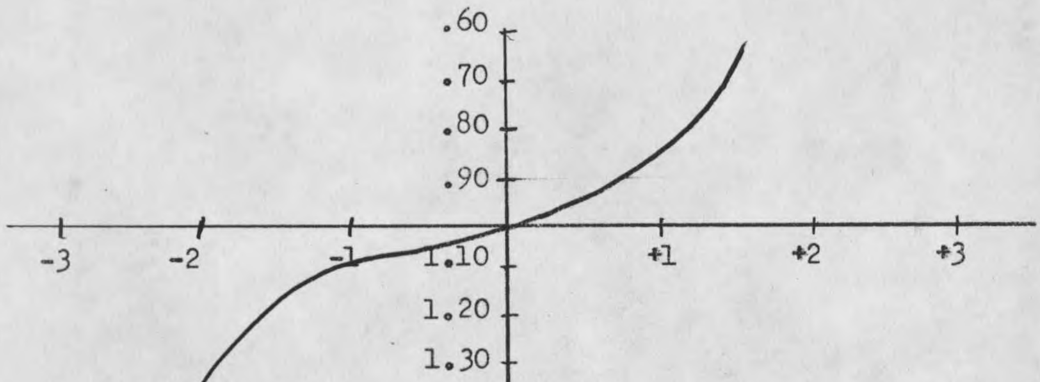

Figure 17.

Where the x Axis Measures Value of Sk and the y Axis the Percentage UC_2/UC_1.

29/ Croxton and Cowden, op. cit., pp. 251-257.

UC_2 is a percent of UC_1 and a function of the amount of skewness. Figure 17 shows the type of functional relationship which might exist between the amount of skewness and $\frac{UC_2}{UC_1}$ (100).

The UC_2 may then be used in place of the UC_1 where a distribution is skewed. It would be much easier, however, if the distributions were not skewed and UC_1 could be used. A distribution which is skewed to the right (the Sk value is positive) may approach a normal distribution if the logarithmic (log to the base 10) value of the observation is plotted or used in the calculations rather than the value itself. Where the resultant distribution is still markedly skewed, it may be possible to use a log with a base of less than ten. The σ and the mean will be calculated from the logarithmic values, and the resultant CV used to determine the UC value. 30/

2. <u>The effect of kurtosis on the UC.</u> Kurtosis is present where the distribution is more peaked or less peaked than normal. A distribution which is normal is referred to as mesokurtic (kurtic = humpbacked, meso = in the middle). A distribution which is more peaked than normal is referred to as leptokurtic. (lepto = slender, narrow). A distribution which is less peaked than normal is referred to as platykurtic. (platy = broad, wide, flat). The degree of kurtosis may be measured by using the fourth moment where π_4 = the fourth moment, and $\pi_4 = \frac{\Sigma x^4}{N}$. π_4 is an absolute measure of kurtosis. π_4 may be converted to a relative measure of kurtosis by dividing by σ^4 (i.e. π_2^2). This measure is referred

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30/ Ibid., p. 225.

to as the α_4 or β_2. $\pi_4 = V_2 - V_1^2$, where $V_1 = \frac{\Sigma x}{N}$, and where $V_2 = \frac{\Sigma x^2}{N}$. $\alpha_4 = \beta_2 = \frac{\pi_4}{\pi_2^2}$. The value of alpha_4 will have a value of 3 when the distribution is normal, a value of less than 3 when the distribution is leptokurtic. 31/ CC' in Figure 18 is a platykurtic distribution, while DD' is a leptokurtic distribution.

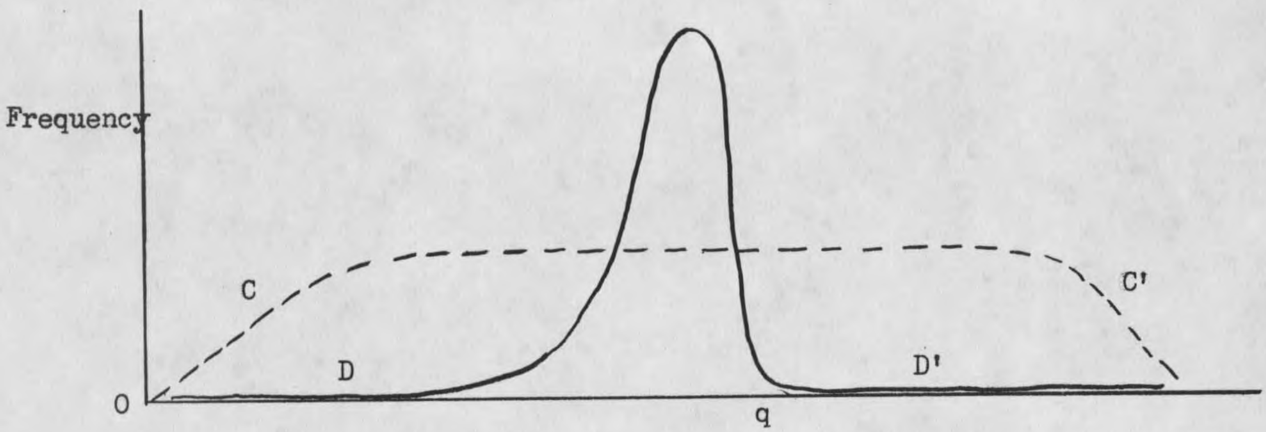

Figure 18.

Distributions Showing Two Types of Kurtosis.

Where kurtosis exists, the UC_1 based on the CV is no longer an adequate picture of the y uncertainty involved. Where the curve is leptokurtic, (α_4 is greater than 3), the UC_1 is too great for the uncertainty involved and this value must be corrected downward. Conversely, where the curve is platykurtic, the UC_1 value must be corrected upward. The functional relationship between the value of the α_4 and the corrected UC_3, (UC_3 is the UC_1 corrected for kurtosis) is shown in

31/ See Croxton and Cowden, op. cit., pp. 258-262.

Figure 19. The UC_1 value will be multiplied by the value read on the y axis to arrive at the UC_3 value. The value for α_4 is read on the x axis.

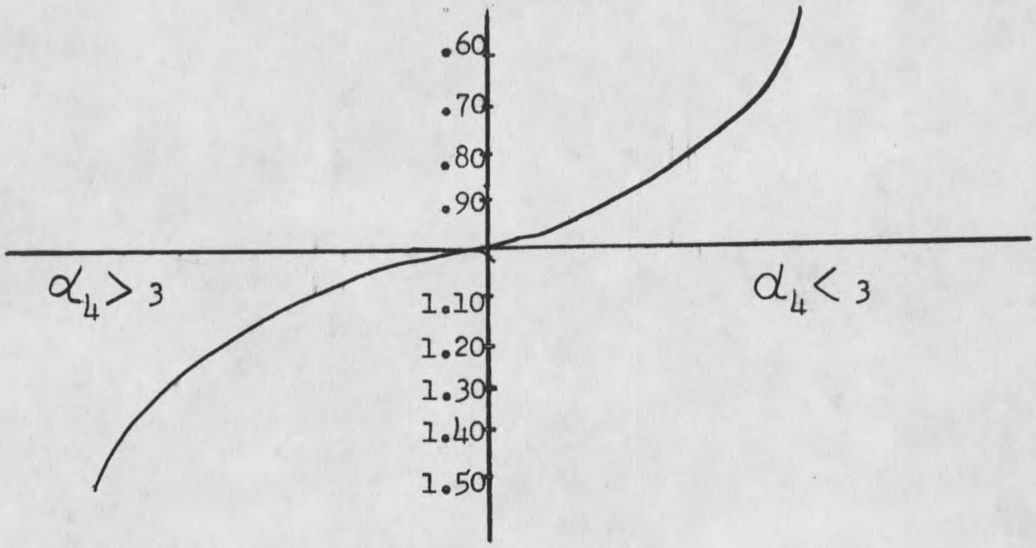

Figure 19.

Where the x Axis Measure Value of α_4 And the y Axis Measures the Percentage UC_3/UC_1.

3. The effect of bi-modal probability distributions on the UC value. A bi-modal probability distribution, in contrast to a unimodal distribution, is defined as "Having two modes; having a distinct tendency toward concentration (of observations or scores) at two different points or regions; having two peaks". 32/ As previously noted in Section II, Heady depicts three types of bi-modal distributions as U-shaped, J-shaped, and mirrored J-shaped. 33/ With two exceptions, a bi-modal

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32/ Albert K. Kurtz and Harold A. Edgerton, Statistical Dictionary of Terms and Symbols, (John Wiley & Sons, Inc., New York, 1939), p. 18.

33/ Heady, op. cit., p. 450.

probability distribution should be treated as a corollary of a normal distribution. Kurtosis may exist in all three types of bi-modal distributions and the kurtosis should be treated accordingly. Both J type distributions show skewedness, the J distribution is skewed to the left, the mirrored J distribution is skewed to the right. These skewed distributions should be afforded the treatment used previously in adjusting the UC value.

The first exception where a bi-modal distribution should receive special consideration with regard to the UC value is where the x values of either peak seem to approximate some type of grouping, but observe no cyclical pattern. If the economic horizon is of great enough length to include many of these groupings, the effect may be negligible. However, if the economic horizon is of such nature so as to make it possible for the expectations to be concentrated in one mode, with equal chance of either mode predominating, a material adjustment in the UC value is warranted. For, in this case, the average is of little use, and it cannot be asserted with any degree of probability of correctness which of the modes will materialize. Decisions made in view of the higher expectations expose the rancher to great possible loss, while decisions based on the lower mode of expectations eliminate the possibility of highly favorable gains. It is quite likely that the lower mode of expectations is the reasonable choice to make, if the firm could be liquidated by decisions based on the higher mode of expectations when the lower mode of expectations materialized. The UC value should be adjusted downward according to the degree of domination which will be exercised by one of the modes, and by the effect the materialization of the lower mode of expectations will have on the firm.

The second exception where a bi-modal distribution should receive special consideration with regard to the UC value is where the x value of one of the modes occurs below or above some crucial point. If, for instance, a bi-modal distribution of yield expectations is evidenced, and the lower mode is to the left of a yield which is necessary to cover variable costs, the frequency with which yields below this crucial point occur is greater than that indicated by the CV. In this case, the UC value should be corrected downward to account for this fact.

D. <u>An Investment's Effect on the Uncertainty of the Firm as a Unit.</u> Other things remaining equal, any increase in the average production of a firm after the advent of an investment is credited to the investment. The uncertainty involved in the increase is considered in determining the UC value as a means of correcting the average increase for uncertainty. While no change is admitted for the firm's previous averages, the standard deviations around these averages may evidence some change. If, for instance, the investment not only increases net income, but also reduces the vulnerability of the average net income, the value of the investment is greater than indicated by the value of the corrected net increase. It is also possible that a particular investment may bring about a negative net increase in income, yet reduce the value of the variation around the average net income (also the CV) enough to make the investment feasible. The average net income previous to the investment may be assumed to be a gross value, not corrected to consider the variability around the average. The correction of this average net income is dependent upon a correction factor, which, as before, will be signified by the notation UC. Since the value of UC is a function of the coefficient of variation (CV), the corrected value of the average may be greater with an equal or smaller uncorrected average if the CV value is changed to a smaller value.

With the above in mind, it is apparent that an investment's effect on the dispersion around the firm's average net income is of importance, as well as the investment's effect on the level of the average net income for the firm. This is especially noticeable when the average net income is considered. An investment may increase the average net income a certain corrected figure; i.e., the (V-C X UC) for an investment. This same investment may have an effect on the CV for the average net income without the investment. Likewise, the investment may affect the degree of kurtosis or skewness of the distribution around the average uncorrected net income. An increase in skewness $(S_k)n$, an increase in the α_4 value, and a decrease in the CV value, all will cause an increase in the value of UC. Therefore, a result of the type mentioned above will result in an increase in the effective value of an investment (V). It is possible that any change in the UC value due to a change in the CV value may be obliterated by an opposite change in the UC value caused by a change in the degree of skewedness or kurtosis existing.

If the functional relationship between CV and UC existed for a firm as illustrated in Figure 20, and an investment would in no way affect either the degree of skewedness or kurtosis of a probability distribution around the average net income for the firm, any change in the CV value should be reflected in the investment's worth.

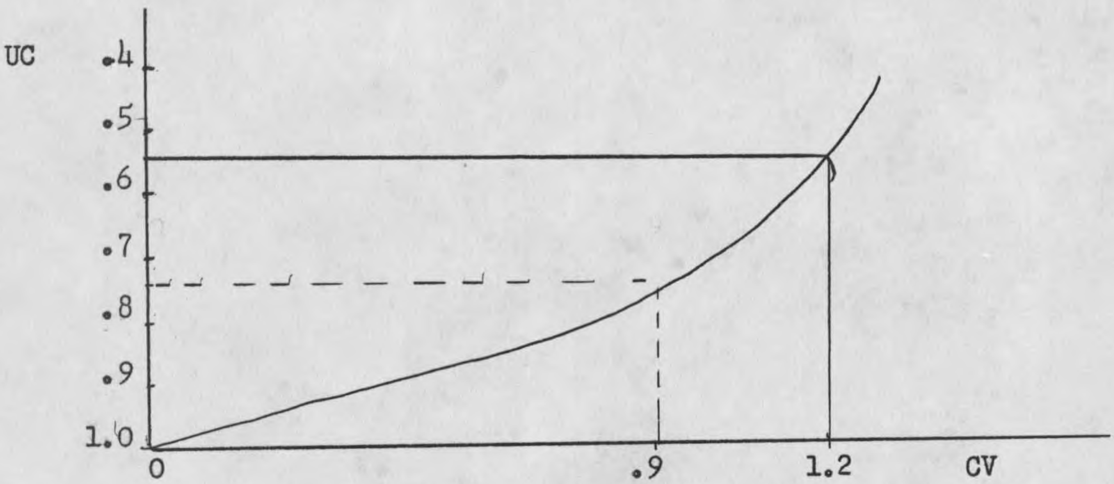

Figure 20.

Hypothetical Relationship Between UC and CV.

Assume that the average net income (uncorrected) for a ranch previous to an investment is $5,000 with a resultant CV of 1.2. The effective or corrected net income (n) is $2,750. (The $5,000 x .55, the UC value read off the y axis for a CV of 1.2). The investment in question, we will assume, in no way affects the average net income, but it does lower the CV value to .9. The investment may not be condemned because it in no way affected the average income (uncorrected). The effective net income or N is affected by a decrease in the value of the CV value. With a CV of .9, the UC value becomes .75. Hence, the effective income N increases from the previous $2,750 to $3,750. The investment, therefore, results in an increase of effective or corrected income of $1,000 for each year the CV is .75. It is likely that the CV may change each year. This thousand dollars may be treated as the positive quasi rent resulting for each period of the life of the investment if the investment's rate of efficiency does not decrease throughout the life of the investment. This quasi rent will be discounted over time and the resulting V computed by summation. The value of V - C will then give the true profit resulting from the investment, where C = the initial cost of the investment.

If the efficiency for the investment increases first at an increasing and then at a decreasing rate, the CV will change accordingly. The change in the effective average net income will then change from year to year. If such an efficiency pattern existed, as is likely in reality, the CV would then change over time, and any change in effective income would have to be computed for each year, and discounted accordingly to the resultant quasi rent for each period. In Figure 21, EE' is the efficiency of an investment over time as reflected in the value of the CV.

The treatment of skewedness and kurtosis may be conducted in the manner used above on Pages 72-77. Where the investment affects the average net income, any change in the CV should be computed on the basis of the old average net income, as the addition to the net has already been corrected for uncertainty by calculating the effective value of V - C.

Below in Table IV is a breakdown of the discounting process for an investment. The initial investment was $5,000, with an internal invest-

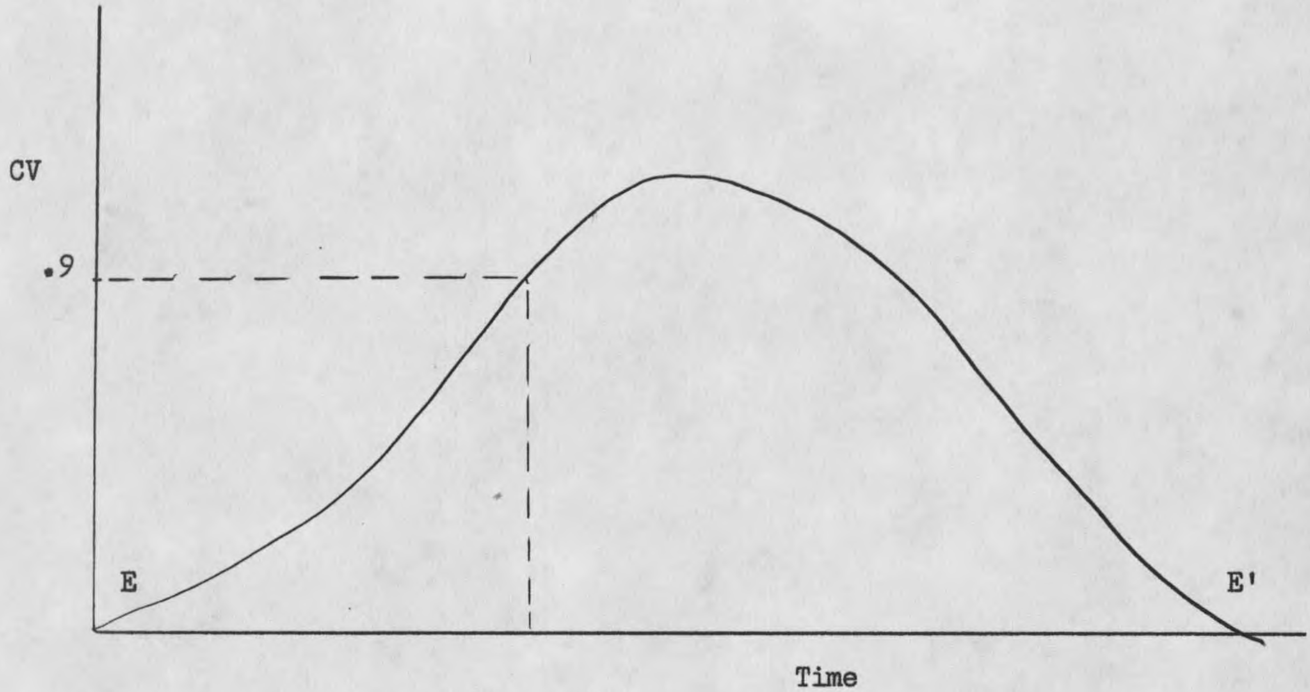

Figure 21.

Change In CV Over Time As A Reflection
Of Changing Efficiency Of An Investment.

Table IV. Use of the Time Discount Rate As A Means of Correcting For Uncertainty

$i=4.5\%$, $T=10$ yrs., Investment=$5,000, UC=f(CV), Q'=correct discounted Q

1. Year	2. Q	3. UC	4. N	5. $(1+i)^{-n}$	6. Q' $(N)(1+i)^{-n}$	7. Q/Q' $(1+i')^n$	8. $\sqrt[n]{(1+i)^n \, (L+i')}$	9. i' in %
1	$1,000	.90	$900	.9569	$861	1.16	1.16	16
2	1,000	.85	850	.9157	778	1.29	1.135	13.5
3	1,000	.80	800	.8763	701	1.43	1.127	12.7
4	1,000	.75	750	.8386	629	1.59	1.123	12.3
5	1,000	.70	700	.8025	562	1.78	1.122	12.2
6	1,000	.65	650	.7678	499	2.00	1.122	12.2
7	1,000	.60	600	.7348	441	2.27	1.124	12.4
8	1,000	.55	550	.7032	387	2.58	1.126	12.6
9	1,000	.50	500	.6729	336	2.98	1.129	12.9
10	1,000	.45	450	.6439	290	3.45	1.142	14.2
					Total=$5,483			*

*Rates of discount which must be used in each time period to correct for both uncertainty and time preference.

ment horizon of ten years. It will be assumed for the present that the economic horizon is determined by the internal investment horizon, with equal positive rents of $1,000 occurring in each production period (one year). It will be assumed that the CV increases with the length of projection in the future; hence, the UC also increases with the length of projection. This UC increases at an increasing rate. The rate of time preference discount is 4.5%. Q' is the discounted value of N, the Q corrected for uncertainty. The discounted value of the quasi rent is found by using the multiplier $(1 + i)^{-n}$, which is the reciprocal of $(1 + i)^n$. The value of the $(1 + i')^n$ which must be used in each period if the discount rate is to correct for uncertainty is computed by dividing Q by Q'. By finding the N'th root of $(1 + i')^n$, the value of $(1 + i')$ for each production period is determined. The value of i', the rate of discount which must be used in each production period to account for both uncertainty correction and time preference discount, is determined by subtracting one from the value in column 8 for each production period and multiplying by 100.

The i' for each production period is a function of the length of time the production period is from the time of investment and the CV for that period. In turn, the increase in the CV value is a function of that same time. This double functional relationship is difficult to analyze, and it would in practice be much simpler to deal directly with a UC value for each production period. It is possible to deal with a discount rate (i') which will account for time preference and correction for uncertainty, but the difficulties involved seem to outweigh the advantages.

Klein notes what he calls the "bearish" attitude on the part of modern day business men. This is evidenced by the fact that they "require that a capital asset must pay for itself in one to five years, although they know that the useful life of the asset is likely to be much longer than five years." 32/ One may argue with the one to five years, but in the case of ranchers, casual observation leads one to believe that the statement is fairly accurate. This is an attempt to consider the uncertainty involved in a very simple way. No attention is paid to discount for time preference.

In the investment presented in Table IV, it will be noted that the investment is "paid back" in five years, and the entrepreneur would consider any returns after five years to be "gravy". Using the above procedure of valuing, the rancher would not be likely to hesitate in making the investment. It is interesting to note that while the uncorrected rate of return to the investment is "20%", the V value is only $5,483 when the elements of time preference and correction for uncertainty are considered. The V - C value, where C = $5,000, would be $483. The investment would be only moderately profitable due to uncertainty and time preference, even though the investment would be "paid back" in five years. It is quite likely that some alternative investment may be more profitable even though the alternative may not be "paid back" in five years.

32/ Lawrence R. Klein, "The Keynesian Revolution", (The Macmillan Co., New York, 1949), p. 64.

IV. A Model for Valuing Sunken Investments for Northern Great Plains Livestock Ranchers

The title of this section is something of a misnomer inasmuch as the model presented below is meant to be a means of valuing a rancher's method used in making a decision with regards to a fixed investment. It is not meant as an investigating tool for all possible alternatives, but rather as a means of valuing a tentative decision of a rancher who anticipates making the investment.

A hypothetical sheep ranch in eastern Montana will be examined. It will be assumed that the rancher in question has tentatively planned to develop a 100 acre water spreader on his ranch. The ensuing budgets are taken directly from a Masters Thesis written by Merle E. Quenemoen. 1/ All cost and production figures will be taken from this paper. Additional data are also gathered from secondary sources.

It will be assumed that present values would continue in the future if no water spreader were developed. Therefore, any ex-ante change in the ranch organization will be credited to the water spreader.

The following data were developed by Quenemoen from a study made by Gray and Baker of Northern Great Plains sheep ranches; 2/ and Montana Experiment Station Circular 51. 3/

For the purpose of illustration, it will be assumed that the rancher

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1/ Quenemoen, op. cit.

2/ Gray and Baker, op. cit.

3/ Montana Farm and Ranch Prices, Mimeograph Cir. 51, Montana State College, Bozeman, Montana, April, 1949.

will use the water spreader to produce hay. Quenemoen found that the hypothetical or average sheep ranch must, on the average, purchase forty tons of hay. 4/ It will be assumed that the ranch uses forty tons of the hay produced on the spread. This hay will be valued at the market price. The remaining hay will be regarded as sold, and/or valued at the market price. For the present, no attention will be paid to the convenience and security of producing the hay on the ranch. The value of the water spreader shall be valued in terms of the market value for the hay produced.

A. Cost of Developing a 100 Acre Water Spreader. The cost of developing the 100 acre water spreader on a ranch in Southeastern Montana is given below in Table V.. It is assumed that all work with the exception of the cost of sowing the alfalfa hay is done custom rates. The prices are based on 1952 averages, the year in which the rancher tentatively planned to build the spreader. It will be assumed that costs similar to the ones below were confronting the rancher. 5/

B. Consideration of Expected Yields. Quenemoen reported that annual alfalfa yields in the area range from 3/4 to 2 tons per acre, with an arithmetic annual average of 1.3 tons. This range of yield will be used as the range of expected yields at the 95% level of probability.

It will be assumed, as Quenemoen suggests, that no yield occurs the first year, but after that, the average yield is constant. While it is highly likely that the stand of hay may need re-establishing about every

4/ Quenemoen, op. cit., p. 47.

5/ Ibid, p. 51.

Table V. Cost of Developing Water Spreader and Alfalfa Hay Meadow (100 Acres)

Item	Units	Price*	Cost
Dam	13,000 yds.	$ .15	$ 1950
Ditch	6,840 yds.	.15	1026
Land Leveling & Diking	1,000 yds.	.17	170
Head Gate & Pipe			35
Land Preparation			
Plowing	100 a.	5.00	500
Double Disking	100 a.	1.25	125
Alfalfa Seed	800 lbs.	.55	440
Sowing	100 a.	1.25	125
Fence	$\frac{1}{4}$ mi.	800.00	200
Total	XXX	XXX	$ 4571

*Prices are based on 1952 averages.

eight years, it will be assumed that the stand need not be re-established in the 15 year discount period which will be used. The rancher, on the basis of his present knowledge, feels the spreader system will only last 15 years.

C. Consideration of Expected Prices for Hay. It will be assumed that lines OA and OC in Figure 22 represent the limits which will include 95% of the expected prices for hay over the next fifteen years. Point C is roughly based on the predicted hay price for 1960. 6/ The choice of point A is somewhat more arbitrarily chosen. Some attempt was made to extend a modified recent trend in the price of hay into the future. The

6/ Long-Range Price Factors in Farm Land Appraisal Policy, a report prepared by the Bureau of Agricultural Economics at the request of the Farmers Home Administration, May, 1951, for administrative use.

method used to determine these boundaries by the rancher are apt to be subjective. The method used in this paper was only an attempt to make these boundaries seem realistic. It will be noticed however, that the dispersions of probable expected prices increase with time. Line OB is taken to be the average price per ton of hay over the next fifteen years. In this case, the average expected price per ton of hay is equal to the present price, the average price in 1952. The boundaries by no means need to be straight lines. This procedure was instituted for computational ease.

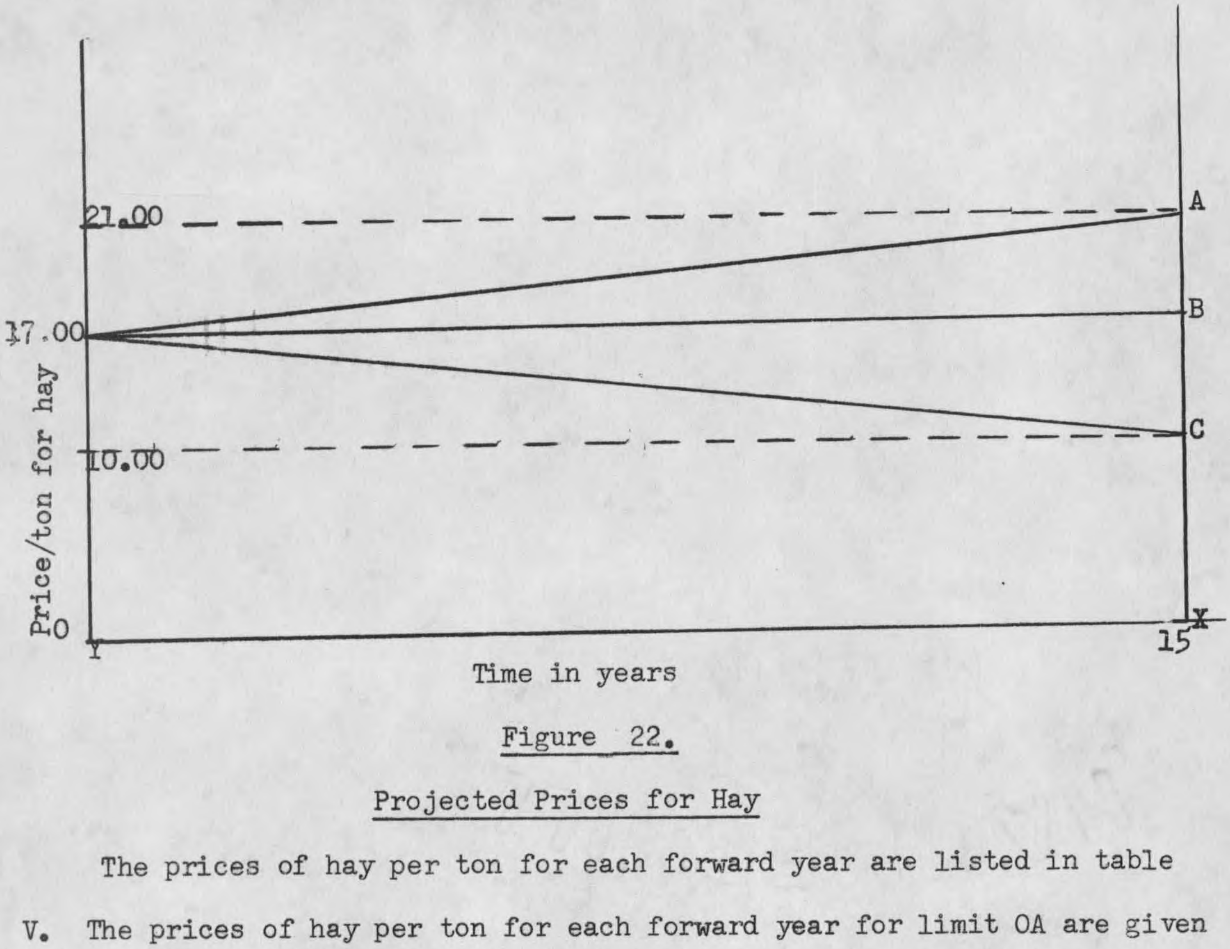


Figure 22.

Projected Prices for Hay

The prices of hay per ton for each forward year are listed in table V. The prices of hay per ton for each forward year for limit OA are given in column A, the prices of line OB in column B, etc. The values are to the nearest fifty cents.

Table VI. Prices for Hay at Three Levels of Expectations

YEAR	Price/ton of Hay		
	OA.	OB.	OC.
1	$17.00	$17.00	$16.50
2	17.50	17.00	16.00
3	18.00	17.00	15.50
4	18.00	17.00	15.00
5	18.50	17.00	14.50
6	18.50	17.00	14.00
7	19.00	17.00	13.50
8	19.00	17.00	13.00
9	19.50	17.00	12.50
10	19.50	17.00	12.00
11	20.00	17.00	11.50
12	20.00	17.00	11.00
13	20.50	17.00	10.50
14	20.50	17.00	10.00
15	21.00	17.00	10.00

D. Consideration of Changes in Variable Costs. Quenemoen found that consistant with a price level at which hay would sell for $17.00 per ton, the total variable costs for a hay operation on a spreader system of 100 acres to be approximately $1170. It will be assumed that the price level of variable inputs is positively correlated with the price of hay. The regression equation index of prices paid = 29 + 0.6 index of prices received will be used to determine the variable costs for each of the three price levels used. 7/ The year 1952 will be used as a base of 100 for prices received. Using the above regression equation, the index of variable costs for 1952 equal 89. By converting the calculated indexes of

7/ This regression equation is the result of some informal work done by C. B. Baker of the Dept. of Ag. Econ. and Rural Sociology, Montana State College, Bozeman, Montana based on a 1935-1939 base period.

prices paid to the basis of 89 = 100, and multiplying the results over 100 times $1170, the variable costs for each production period and at each price level are calculated. The method and result are shown in Table VIIa, VIIb, and VIIc.

E. Methods of Determining the Various Levels of Quasi Rents. Nine alternative streams of quasi rents will be calculated. Table VIIa represents the resulting streams of quasi rents which would result on the basis of a .75 ton per acre yield of hay. Q'_a is the stream of quasi rents which would result if price level OA materialized for both the price of hay and total variable costs. Q'_b and Q'_c are those streams of quasi rents which would exist if price levels OB and OC materialized. The subscript indicates the price level used to calculate each stream of quasi rents; subscript "a" is used for price and variable cost level OA, etc. The single prime (Q') signifies that the lower average yield has been used (.75 ton per acre). The double prime (Q'') signifies that the average yield (1.3 ton per acre) has been used. The triple prime (Q''') signifies that the upper yield (2 ton per acre) was used. The resulting streams of quasi rents are shown in Table VIIIa, Table VIIIb, and Table VIIIc.

It will be noticed that in all nine streams of quasi rents the first quasi rent is a minus $160. It is assumed that no yield is realized the first year. As the result, no positive gross income results. The usual variable costs of haying, etc., are not evidenced. However, a certain amount of variable costs will exist because of the labor needed for flooding, minor repair, and miscellaneous work which amounts to about $160.

Table VIIa. Variable costs of a haying operation for price level OA (fig. 22) where index PP = 29 + 0.6 index PR

Year	Price Received Per Ton of Hay in Dollars	Index of Price Received Where Year 1 = 100	Index of Variable Costs Where PP = 29 + .6 PR	Index of Variable Costs Where Year 1 = 100	Variable Costs for Each Period When Variable Costs for Year 0 = 1170 1170 x Index of Variable Costs
0	17.00	100	89	100	1170
1	17.00	100	89	100	1170
2	17.50	103	91	102	1193
3	18.00	106	93	104	1217
4	18.00	106	93	104	1217
5	18.50	109	94	106	1240
6	18.50	109	94	106	1240
7	19.00	112	96	108	1264
8	19.00	112	96	108	1264
9	19.50	115	98	110	1287
10	19.50	115	98	110	1287
11	20.00	118	100	112	1310
12	20.00	118	100	112	1310
13	20.50	121	102	115	1346
14	20.50	121	102	115	1346
15	21.00	124	103	116	1357

Table VIIb. Variable costs of a haying operation for price level OB (fig. 22) where index of PP = 29 + 0.6 index of PR

Year	Price Received Per Ton of Hay in Dollars	Index of Price Received Where Year 1 = 100	Index of Variable Costs Where PP = 29 + .6 PR	Index of Variable Costs Where Year 1 = 100	Variable Costs for Each Period When Variable Costs for Year 0 = 1170 1170 x Index of Variable Costs
0	17.00	100	89	100	1170
1	17.00	100	89	100	1170
2	17.00	100	89	100	1170
3	17.00	100	89	100	1170
4	17.00	100	89	100	1170
5	17.00	100	89	100	1170
6	17.00	100	89	100	1170
7	17.00	100	89	100	1170
8	17.00	100	89	100	1170
9	17.00	100	89	100	1170
10	17.00	100	89	100	1170
11	17.00	100	89	100	1170
12	17.00	100	89	100	1170
13	17.00	100	89	100	1170
14	17.00	100	89	100	1170
15	17.00	100	89	100	1170

Table VIIc. Variable costs of a haying operation for price level OC (fig. 22) where index of PP = 29 + 0.6 index of PR

Year	Price Received Per Ton of Hay in Dollars	Index of Price Received Where Year 1 = 100	Index of Variable Costs Where PP = 29 + .6 PR	Index of Variable Costs Where Year 1 = 100	Variable Costs for Each Period When Variable Costs for Year 0 = 1170 1170 x Index of Variable Costs
0	17.00	100	89	100	1170
1	16.50	97	87	98	1147
2	16.00	94	85	96	1123
3	15.50	91	84	94	1100
4	15.00	88	82	92	1076
5	14.50	85	80	90	1053
6	14.00	83	79	89	1041
7	13.50	79	76	85	995
8	13.00	76	75	84	983
9	12.50	74	73	82	959
10	12.00	71	72	81	948
11	11.50	68	70	79	924
12	11.00	65	68	76	889
13	10.50	62	66	74	866
14	10.00	59	64	72	842
15	10.00	59	64	72	842

F. Choice of an Interest Rate for Capitalizing and Discounting. The capitalizing process will account for the time preference only. The interest rate used is that paid on government bonds. It was felt that this is the most favorable return a rancher may receive on his investment where practically no risk is involved. There is a certain inconvenience in converting government bonds into cash, but for all practical purposes, government bonds will be considered liquid capital. Therefore, it is felt that a 3% interest rate is fair indicator of pure time preference.

G. Initial Cost. The original cost of the investment in a water spreader is $4571. In fifteen years time, it is assumed that the spreader system is of no further value as hay land, and is of value for range land at the same rate as previous to the investment.

H. Calculation of the Discounted Quasi Rents. Where the discounted quasi rents equal $\frac{Q}{(1-i)n}$, the discounted quasi rent equal $(Q)\ (1+i)^{-n}$. The value of $(1+i)^{-n}$ is given in the first column of Table IX for each year from one to fifteen, where i equal 3%. The discounted quasi rents for all possible combination of averages and limits for both price and yield are shown in Table IX. They are summed and the original cost (C) subtracted. The residual value is V-C.

Quenemoen found that such a spreader would not be flooded 2 out of 10 years. Assuming that the yields when no flooding occurs are materially reduced, it is reasonable to assume that the V-C values are only about

VIIIa. Quasi rents where the lower fiducial limit of yield averages is used with all three levels of price lines

Year	$(P_a)(Y.75)$	VC_a	Q'_a	$(P_b)(Y.75)$	VC_b	Q'_b	$(P_c)(Y.75)$	VC_c	Q'_c
1	--	160	-160	--	160	-160	--	160	-160
2	1313	1193	120	1275	1170	105	1200	1123	77
3	1350	1217	133	1275	1170	105	1163	1100	63
4	1350	1217	133	1275	1170	105	1125	1076	49
5	1388	1240	148	1275	1170	105	1088	1053	35
6	1388	1240	148	1275	1170	105	1050	1041	9
7	1425	1264	161	1275	1170	105	1013	995	18
8	1425	1264	161	1275	1170	105	975	983	-8
9	1463	1287	176	1275	1170	105	938	959	-21
10	1463	1287	176	1275	1170	105	900	948	-48
11	1500	1310	190	1275	1170	105	863	924	-61
12	1500	1310	190	1275	1170	105	825	889	-64
13	1538	1346	192	1275	1170	105	778	866	-88
14	1538	1346	192	1275	1170	105	750	842	-92
15	1575	1357	218	1275	1170	105	750	842	-92

VIIIb. Quasi rents where the average fiducial limit of yield averages is used with all three levels of price lines

Year	$(P_a)(Yl.3)$	VC_a	Q''_a	$(P_b)(Yl.3)$	VC_b	Q''_b	$(P_c)(Yl.3)$	VC_c	Q''_c
1	--	160	-160	--	160	-160	--	160	-160
2	2275	1193	1082	2210	1170	1040	2080	1123	957
3	2340	1217	1123	2210	1170	1040	2015	1100	915
4	2340	1217	1123	2210	1170	1040	1950	1076	874
5	2405	1240	1165	2210	1170	1040	1885	1053	832
6	2405	1240	1165	2210	1170	1040	1820	1041	779
7	2470	1264	1206	2210	1170	1040	1755	995	760
8	2470	1264	1206	2210	1170	1040	1690	983	707
9	2535	1287	1248	2210	1170	1040	1625	959	666
10	2535	1287	1248	2210	1170	1040	1560	948	612
11	2600	1310	1290	2210	1170	1040	1495	924	571
12	2600	1310	1290	2210	1170	1040	1430	889	541
13	2655	1346	1309	2210	1170	1040	1365	866	499
14	2655	1346	1309	2210	1170	1040	1300	842	458
15	2730	1357	1373	2210	1170	1040	1300	842	458

VIIIc. Quasi rents where the upper fiducial limit of yield averages is used with all three levels of price lines

Year	$(P_a)(Y_2)$	VC_a	Q'''_a	$(P_b)(Y_2)$	VC_b	Q'''_b	$(P_c)(Y_2)$	VC_c	Q'''_c
1	--	160	-160	--	160	-160	--	160	-160
2	3500	1193	2307	3400	1170	2230	3200	1123	2077
3	3600	1217	2383	3400	1170	2230	3100	1100	2000
4	3600	1217	2383	3400	1170	2230	3000	1076	1924
5	3700	1240	2460	3400	1170	2230	2900	1053	1847
6	3700	1240	2460	3400	1170	2230	2800	1041	1759
7	3800	1264	2536	3400	1170	2230	2700	995	1705
8	3800	1264	2536	3400	1170	2230	2600	983	1617
9	3900	1287	2613	3400	1170	2230	2500	959	1541
10	3900	1287	2613	3400	1170	2230	2400	948	1452
11	4000	1310	2690	3400	1170	2230	2300	924	1376
12	4000	1310	2690	3400	1170	2230	2200	889	1311
13	4100	1346	2754	3400	1170	2230	2100	866	1234
14	4100	1346	2754	3400	1170	2230	2000	842	1158
15	4200	1357	2843	3400	1170	2230	2000	842	1158

85% effective. 8/

I. Determination of the Frequency Distribution and Associated Values. As assumed previously, the limits set for price and yield expectations are the 95% fiducial limits. The mid value for both price and yield are the average expectations. In this case, the data listed in Table IX would hold.

Expected Yield	Percent of time expectation will occur
.75 or lower	2.5
1.3 or lower	50.0
1.3 or higher	50.0
2.0 or higher	2.5
Expected Price Level	
Price level OC or lower	2.5
Price level OB or lower	50.0
Price level OA or lower	97.5
Price level OC or higher	97.5
Price level OB or higher	50.0
Price level OA or higher	2.5

To establish the type of frequency distribution which might occur around the mean of a V'-C' of $5,850, or the 5th alternative V'-C' since

8/ Since the variable costs of haying will not materialize when no hay crop is produced, the effective V-C values previous to any discounting for uncertainty are somewhat more effective than would be reflected by mere occurrence of yield. On this basis it is roughly approximated that the effective V-C value because of failure to flood would be 85% of the original calculation.

Table IX. Determination of V'-C', where i = 3%

Year	$(1+i)^{-n}$	.75 ton/acre			1.3 ton/acre			2 ton/acre		
		Q'_a	Q'_b	Q'_c	$Q'_a{}'$	$Q'_b{}'$	$Q'_c{}'$	$Q''_a{}'$	$Q''_b{}'$	$Q''_c{}'$
1	.97	-155	-155	-155	-155	-155	-155	-155	-155	-155
2	.94	113	99	72	1017	978	900	2169	2096	1952
3	.92	122	97	58	1033	975	842	2192	2052	1840
4	.89	118	93	44	999	926	778	2121	1985	1712
5	.86	127	90	30	1002	894	716	2116	1918	1588
6	.84	124	88	8	979	874	654	2066	1873	1478
7	.81	130	85	15	977	842	616	2054	1806	1381
8	.79	127	83	-6	953	822	559	2003	1762	1277
9	.77	136	81	-16	961	801	513	2012	1717	1187
10	.74	130	78	-35	924	770	453	1934	1650	1074
11	.72	137	76	-44	929	749	411	1937	1606	991
12	.70	133	74	-45	903	728	379	1883	1561	918
13	.68	131	71	-60	890	707	339	1873	1516	839
14	.66	127	69	-61	864	689	302	1818	1472	764
15	.64	140	67	-59	879	666	293	1820	1427	741
V = Summation		1640	996	-254	13155	11453	7600	27843	24286	17587
C =		4571	4571	4571	4571	4571	4571	4571	4571	4571
V-C =		-2931	-3575	-4825	8584	6882	3029	23272	19715	13016
V'-C' =		-3371	-4111	-5549	7266	5850	2575	19781	16758	11064
No. of V'-C'		3	2	1	6	5	4	9	8	7

average price and average yield will give the average expected V'-C', it is desirable to establish the ordinate of the nine alternative V'-C' 's. This will be done by assuming the mean for the distribution is equivalent to the mode. In this case, the model V'-C' will be given an arbitrary ordinate value of 1. The remaining eight V'-C's will be given ordinates of less than one, the exact value of which is dependent on the ratio of the joint probability of occurrence for the remaining V'-C' with the joint probability of occurrence of the mean or average V'-C'. It is noted that it is likely that the distribution would probably show a mode to the left of the mean, but for purposes of computational ease, it is assumed that the mean and mode coincide.

The values resulting from the joint probabilities of occurrence and the resulting ordinal value calculated on the basis of the mean V'-C' having a ordinal value of one are presented below.

V'-C'	Corresponding Number	Joint probability of occurrence	Ordinate, where model ordinate equal 1
$(V'-C')'_c$	1	00.06%	.0024
$(V'-C')'_b$	2	1.25%	.05
$(V'-C')'_a$	3	2.40%	.10
$(V'-C')'_c{}'$	4	12.50%	.50
$(V'-C')'_b{}'$	5	25.00%	1.00
$(V'-C')'_a{}'$	6	12.50%	.50
$(V'-C')''_c{}'$	7	2.4%	.10
$(V'-C')''_b{}'$	8	1.25%	.05
$(V'-C')''_a{}'$	9	00.06%	.0024

These values are plotted on Figure 23. The probability distribution represents an attempt to fit a curve on the basis of the nine points on the curve which were calculated. The effective value, V'-C' is found. The resultant V'-C' values are plotted on the x axis of Figure 23. The means of determining the shape of the distribution are described later.

The range of expected V'-C' is \$25,330 at the 98% fiducial limits. (A maximum V'-C' of \$19,781 and a minimum of-\$5,549). Where the mean $\pm$ 2.4σ includes 98% of the observations, 4.8σ should approximately equal the range. Hence, \$25,330/4.8 equals \$5,277 which in turn is equal

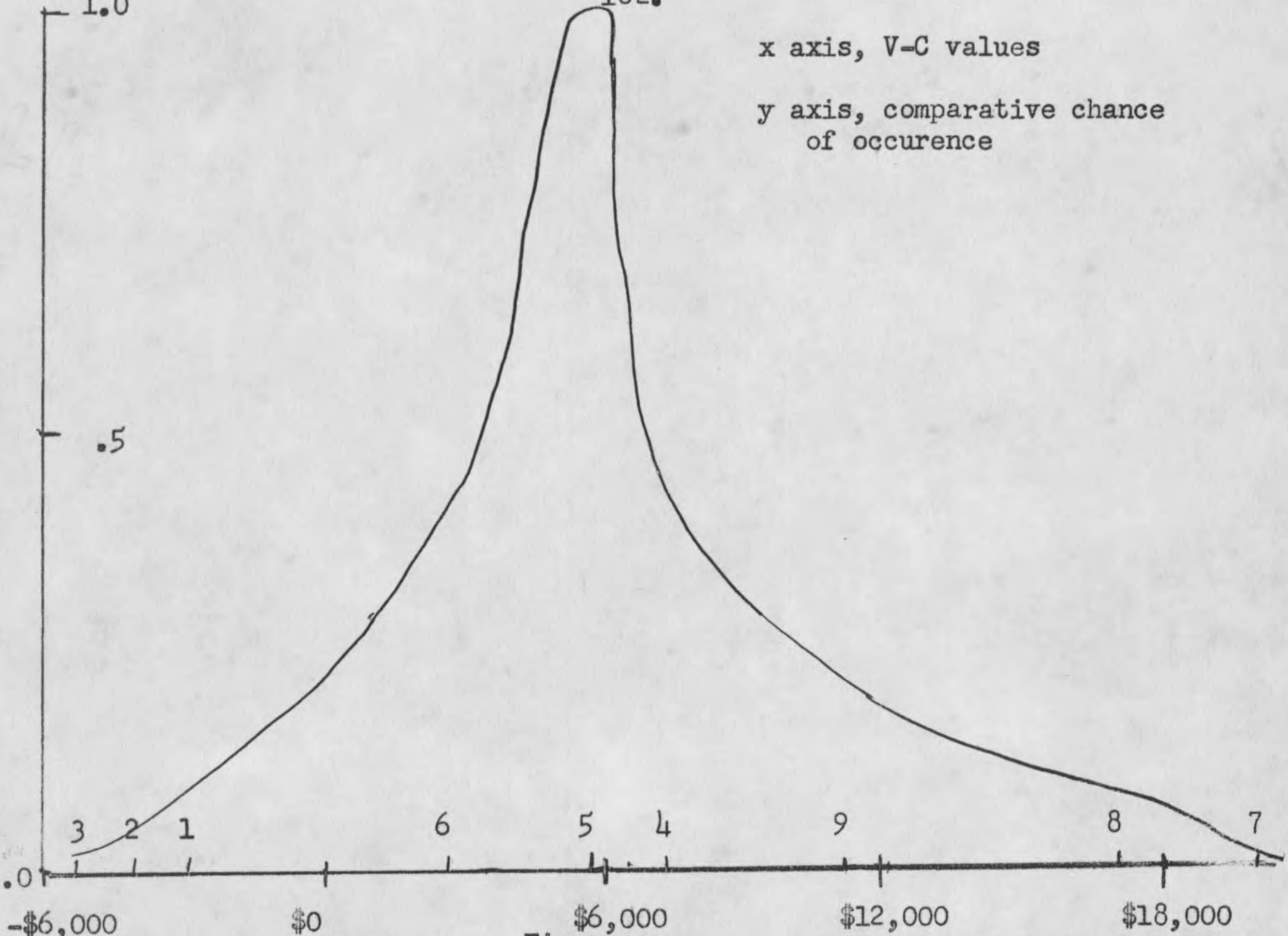
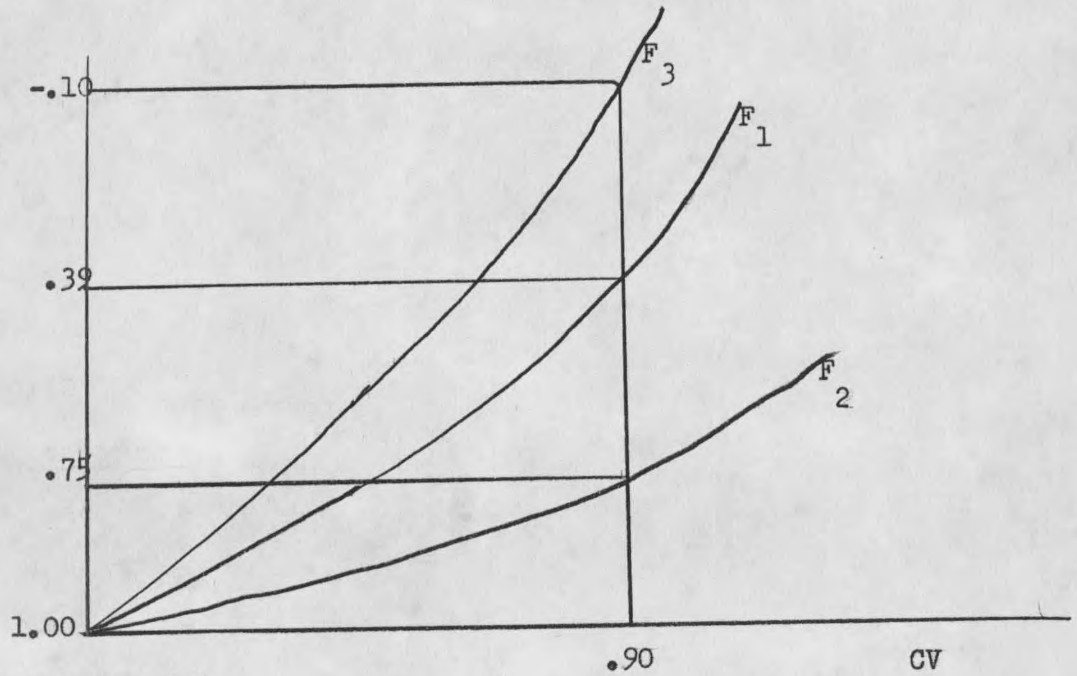


Figure 23.

Frequency Distribution For V'-C' Values.

to the σ for the distribution. Where the mean was assumed to be $5,850, the CV value is .90, ($5,277/$5850). The distribution shows signs of skewness and perhaps some kurtosis. However, for this illustration, it will be assumed that the distribution is normal.

J. Determination and Application of the UC. Where the functional relationship between the CV and UC for this category of rewards for the rancher in question is like that depicted by F_1 in Figure 24, the UC factor will be .39 where CV = .90. While the average profit will be $5,850 for the investment, the effective average income will be $2,282, or the mean, $5,850, times the UC factor, .39. In this case, it was found expedient to correct for uncertainty at the V-C level.



UC-CV Relationships Figure 24.

Not even all hypothetical ranchers can be expected to have the same functional relationship between UC and CV, for the same category of rewards. The functional relationship F_1 might be representative of a man whose financial condition is good, but not indissoluble. The possible loss of over $4,000 during a period of 15 years will hurt, but the rancher does not expect the possibility of such gains to be greatly over-shadowed by the possibility of rather material gains.

Rancher X is in somewhat a different fix than the rancher represented above. His ranch is encumbered with debt and/or his attitude are such that he would look on the possible losses amounting to better than $4,000 as a death warrant to his enterprise. If such is the case, he may discount the mean V'-C' to a negative value. Say the functional relationship between UC and CV for Rancher X is represented by F_3. In this case, where the CV value is .90, the UC factor would be a -.10. The average expected V'-C' would for this rancher have an effective value of a negative $585.00. Certainly, this rancher would not choose to make the investment.

Rancher Y as opposed to Rancher X, is in such a position that the possible loss of $4,000 plus over a period of fifteen years would not materially alter his financial condition, but the possibility of gains amounting to almost $20,000 would have a material effect. This condition might exist because of one of two extreme conditions. The rancher may be well enough fixed financially so that a possible loss of this nature would not seriously alter either his standard of living or his financial condition. This rancher can in effect afford to gamble. The other extreme is where the rancher might be so far in debt that only a material gain can preserve his ranch liquidity. He is going broke anyway so the possibility of a $4,000 plus loss can't hurt, and the possible $20,000 gain may save him. This rancher is placed in such a position that he can't afford not to gamble if he wishes to continue as a rancher. The functional relationship between UC and CV for a rancher in either of the positions mentioned above might be more nearly that relationship which is represented by F_2. In this case, where CV = .90, the UC value would be .75, and the mean V'-C' of $5,850 would have an effective value of $4,387. In this case, it is extremely likely that the rancher would choose to make the investment.

In the final decision as to whether or not to make the investment in the water spreader, he will compare the effective profit from the water spreader with the effective income of alternative investments. If the alternative investments are such that they require a different level of capital of investment over a different economic horizon, the rancher would compare the percent profit per year as was done above on pp. 62-63, Section III. That investment showing the greatest percent profit per year would be the investment with greatest chance of maximum net ranch income.

Additional value may be placed on the water spreader which was not apparent in the valuation process used above. The rancher may place high value on the convenience of having the hay produced on the ranch. However, it must be remembered that the production is only 80% efficient and the rancher may have to buy hay two years out of ten. If an alternative investment, such as developing watering places, shows a materially greater rate of profit, than does the water spreader, the rancher should carefully scrutinize any added profit he may place on the water spreader because of convenience and security. It is likely that he would find the buying of hay much more efficient than 80%. In any case, there is little if any sense of performing any valuation process to determine the soundness of a rancher's decision if the result of the test of soundness is paid no heed.

V. Critique, Appraisal, and Suggestions for Further Consideration.

A. Critique and Appraisal. The method of valuing which was developed is not, nor was it intended to be, an empirically precise and exact method of valuing. Rather, the method was meant to be a framework, or atmosphere, within which a valuing process could take place. The nine alternatives which were calculated in Section IV are only a few of the almost infinite number of alternatives which could be calculated. As assumptions are dropped, the number of alternatives increases. However, so that it might be possible to reach some conclusion in the short span of this paper, limiting assumptions were introduced.

The method used was cumbersome, but the very nature of the subject seems to have this cumbersomeness as an intrinsic part of its being. While the future is never known with certainty, complete uncertainty must be converted to some degree of certainty for the purpose of action. The planning agent necessarily takes some order of probabilities as a basis for his projections. This was the attempt made in this paper. While probabilities need not be handled in the awkward and often obscure ways that they were handled in this paper, there are difficulties inherent in the pondering of probabilities. This difficulty increases as the number of interdependent probabilities increases.

With some exceptions, the method developed is not likely to be more definitive than the method of valuing which is presently used by ranchers in practice. The method presently used by the rancher is apt to be less complete than the method developed herein because of his ignorance and/or the difficulty in handling many variables. But, it is pointless to belittle the rancher for some of the weaknesses in his method of valuing. The fact remains that ranchers do reach a decision with regard to a sunken investment. This they accomplish, and this, a systematic review of a method of valuing did not accomplish. For, when one attempts to view a complex problem such as valuing a sunken investment in the light of complete rational behavior, it is doubtful that a final and absolute value can be reached. After an attempt to accomplish the above, one is rather amazed that the commercial rancher has enough faith in his convictions to reach a decision, and is further amazed that the decision is often a wise one.

The critique of the method is apologetic. Yet, in a sense, it need not be. For if the method of valuing which was developed resulted in a "formula" into which a rancher plunked prescribed values and ground out the answer, it is quite likely that two very important facts would have been ignored. Each rancher will view like situations in a different light, and each rancher is likely to be a special case because of the physical and institutional environment in which he lives.

The method developed is not an operational method. It is a purely academic method. It is not meant to be an aid to the rancher in any direct sense. It is felt, however, that certain important problems in risk and uncertainty were explored. Some attempt was made to point out the weakness in basing investment judgments on arithmetic averages. Because of the variations around this arithmetic average and the impact of uncertainty, the "decision parameters" are somewhat different from those based on the arithmetic average. Seemingly, this is a problem that is often either ignored or dealt with inadequately. It is not proposed that this paper dealt with the problem adequately, but the problem was not ignored. Consciously or unconsciously, this is the central theme of the paper. It is this problem which is often paramount to the rancher in an area such as the Great Plains.

The study was not entirely unfruitful, however. Uncertainty was discussed in terms of uncertainty equivalents and effective incomes. From these two was derived an Uncertainty Correction Factor (UC) which is a function of the coefficient of variation (CV). This seems to be one approach to the problem of risk and uncertainty. The use of an adjusted discount rate to reflect correction for uncertainty is a convenient rather than an accurate method of discounting for risk and uncertainty. No attempt was made to distinguish risk and uncertainty as separate entities; rather, they were treated as one. While this fact weakened the paper, it provided some concise means of evaluating uncertainty.

B. <u>Suggestions for Further Consideration</u>. While it is not posited that it will ever be possible to determine the exact nature of the functional relationship between the Uncertainty Correction Factor (UC) and the coefficient of variation (CV) for each and every rancher, it may be possible to better determine the nature of this relationship for the aggregate of ranchers.

One method of approach to this problem may be an examination of past historical data to determine the type and degree of uncertainty which is apt to confront the rancher in various fields of endeavor and measure past reactions of ranchers under these conditions. Once this is done, it might be possible to build a questionnaire and/or interview on the basis of this information which would determine the attitude of the rancher toward risks and uncertainties which are apt to occur again. The attitude of a rancher, or for that matter anyone else, towards risk and uncertainty is of a psychological origin given other data; e.g., capital position, social status, etc. It might well be questioned if it is possible to put a cardinal value on a subjective attitude. By analyzing the results of the questionnaire and/or interviews, it might be possible to establish the amount and range of discounting the mean expected results that ranchers practice. The nature of the functional relationship between the Uncertainty Correction Factor (UC) and the coefficient of variation (CV) could then be found. Once this is done, it would be materially easier to put a cardinal value on the cost of risk and uncertainty for both the individual rancher and society.

Where the purpose of our agricultural policy and technical agricultural research is to increase the welfare of the individual farmer and rancher, there are three possible alternative methods of accomplishing the desired end. They are (1) to increase expected values of net income, (2) to reduce variation around expected values of net income, and (3) a combination of one and two.

The above is true not only for sunken investments, but for all the various types of endeavor a farmer or rancher may engage in. In the past, it is felt that both agricultural policy makers and technical agricultural researchers concentrated too much of their effort on the problem of increasing expected mean values and too little on the problem of reducing variation around an expected mean value. Often, while acting under the auspices of the first alternative, the third alternative resulted. This was, however, coincidental. When the social cost of risk and uncertainty is known, the agricultural policy makers and technical agricultural researchers may be better able to judge the value of a conscious effort to reduce variations. This is certainly no revelation to those men who are actively engaged in increasing the welfare position of the individual farmer and rancher in society. What is needed are some criteria with which to judge the effect of variation on the planning agent. At present there seem to be no workable criteria by which to judge the effect of variation on the planning agent. It is not proposed that this paper establishes proper criteria for this purpose, but it should point out the need and possible value of some criteria which could evaluate the effect of variation. Reduction of uncertainty need not consist of pure and simple reduction of variation around some mean. The reduction of variation may be "lopsided" in such a way as to place some critical expectation in a favorable position. Such a reduction of uncertainty may be of great value. Reducing uncertainty is therefore by no means perfectly correlated with reduction of variation.

Reduction of uncertainty of net income expectations may be beneficial for the rancher, but it does not follow that the social gain will be an aggregate of the benefits received by ranchers. While no distinction was made between risk and uncertainty for the individual rancher, it is convenient and seemingly expedient to make a distinction between the effects of uncertainty to an individual rancher and the effect of all like grouped uncertainties (i.e., risk) to society. The aggregate uncertainty for all the ranchers concerned may form a rather insignificant risk for society.

While it may not be possible to develop a specific set of criteria to value the effect of variation on the planning agent usable by each and every rancher, it may be possible to develop a general set of criteria which would be usable by technical and policy researchers in the field of agriculture. The information on the effect of variation on the planning agent could be used by these researchers to (1) develop data and results which are usable as bench marks or guides to the farmer and rancher in correcting his management decisions for risk and uncertainty; and (2) place a value on the acceptance by society of some portion of the aggregate risks and uncertainties of the individual farmers and ranchers.

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