



A computer model of the coupled dynamic fluid and electrical interactions within an MHD duct
by Donald James Hammerstrom

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Electrical Engineering

Montana State University

© Copyright by Donald James Hammerstrom (1991)

Abstract:

The introduction of commercial magnetohydrodynamic (MHD) power generation ducts to existing power grids and the associated control problems arising from MHD power take off schemes necessitate an accurate, dynamic model of the coupled gas fluid and electrical behavior of an MHD duct. This thesis presents such a working model and lays the groundwork for further improvements.

A system of differential and algebraic equations is developed to model the duct. A Lax-Wendroff fluid flow integration scheme for the entire duct is coupled with a circuit representation of the generator channel. FORTRAN computer programs define the finite difference approximations to the three gas dynamic, one dimensional flow conservation equations, integrate these differential equations, and calculate all related algebraic fluid flow and electrical variables.

A number of modeled transient conditions are presented and discussed. Pulse amplitude synthesis and control (PASC), a low harmonic, DC power consolidation and conversion scheme, is then applied to the model to show that the model may be used with somewhat complex switching and control schemes. The model is shown to model the coupled, dynamic gas fluid and electrical behaviors of an MHD channel during a variety of transient conditions, and it may prove very useful for planning future MHD applications. Some future improvements are suggested.

A COMPUTER MODEL OF THE COUPLED DYNAMIC FLUID AND
ELECTRICAL INTERACTIONS WITHIN AN MHD DUCT

by

Donald James Hammerstrom

A thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Electrical Engineering

MONTANA STATE UNIVERSITY
Bozeman, Montana

November 1991

N378
H1832

ii

APPROVAL

of a thesis submitted by

Donald James Hammerstrom

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

Dec. 2, 1991
Date

Roy M. Johnson
Chairperson, Graduate Committee

Approved for the Major Department

Dec. 2, 1991
Date

V. Leroy D. A. Pierre
Head, Major Department

Approved for the College of Graduate Studies

December 17, 1991
Date

Harry L. Parsons
Graduate Dean

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a master's degree at Montana State University, I agree that the Library shall make it available to borrowers under rules of the Library. Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of source is made.

Permission for extensive quotation from or reproduction of this thesis may be granted by my major professor, or in his absence, by the Dean of Libraries when, in the opinion of either, the proposed use of the material is for scholarly purposes. Any copying or use of the material in this thesis for financial gain shall not be allowed without my written permission.

Signature Donald J. Hammerstrom
Date November 27, 1991

TABLE OF CONTENTS

	Page
LIST OF FIGURES	v
LIST OF SYMBOLS	vii
ABSTRACT	x
INTRODUCTION	1
Background on MHD Power Generation	1
Present State of MHD Research	3
Purpose of this Thesis	4
COMPUTER MODEL FORMULATION	6
Duct Fluid Dynamic Model Formulation	6
Channel Electrical Model Formulation	14
FORTRAN Implementation of the Duct Model	19
DEMONSTRATION OF DYNAMIC MODEL RESULTS	23
Description of Hypothetical MHD Duct	23
Steady State Behavior under Variable Load	23
Dynamic Validity Testing of the MHD Model	29
Transient Model Behavior	31
Flow Shut Off Transient	31
Electrical Load Loss Transient	35
Shorted Electrode Pair Transient	39
Open Electrode Pair Transient	41
A Switching Transient Demonstration	43
SUGGESTIONS FOR FUTURE WORK	46
CONCLUSIONS	48
LITERATURE CITED	50
APPENDIX	52

LIST OF FIGURES

Figure		Page
1.	MHD Channel Interactions.....	3
2.	Basic Equations of Continuity.....	7
3.	One Dimensional Equations of Continuity.....	9
4.	Schematic Viscosity Terms.....	11
5.	Circuit Model for a Single Electrode pair.....	18
6.	Many Circuit Models Connected in Series for a Hall Configuration.....	19
7.	Matrix Formulation of State Equations in STATE.FOR.....	20
8.	Overview of FORTRAN Model Implementation.....	22
9.	The MHD Duct Model Measurements Used in this Thesis.....	24
10.	Axial Profile of Fluid and Electrical Variables under Constant Resistive Faraday Loading.....	25
11.	Load Curve for a Faraday Configuration MHD Channel.....	26
12.	Power Curve for a Faraday Configuration MHD Channel.....	26
13.	Axial Profile for Fluid Variables in an Unloaded MHD Channel.....	27
14.	Axial Profile for Fluid Variables Showing a Shock at Node 28 as Input Pressure is Decreased.....	28
15.	Comparison of Modeled Shock Tube Pressure Behavior to Theoretical.....	30
16.	Flow Shut Off Transient Pressure Profile in Time and Axial Distance.....	33
17.	Flow Shut Off Transient Velocity Profile in Time and Axial Distance.....	34

LIST OF FIGURES (continued)

Figure	Page
18. Flow Shut Off Transient Faraday Current Profile in Time and Axial Distance.	35
19. Load Loss Transient Faraday Current Profile in Time and Axial Distance.	36
20. Load Loss Transient Pressure Profile in Time and Axial Distance.	37
21. Load Loss transient Velocity Profile in Time and Axial Distance.	38
22. Node 18 Short Circuit Transient Faraday Current Profile.	40
23. Node 18 Short Circuit Transient Pressure Profile.	40
24. Node 18 Short Circuit Transient Velocity Profile.	41
25. Node 18 Open Circuit Transient Faraday Current Profile.	42
26. Node 18 Open Circuit Transient Pressure Profile.	42
27. Node 18 Open Circuit Transient Velocity Profile.	43
28. PASC Electrode Consolidation Switching for a Single Phase Triangle Wave Generation.	44
29. Single Phase Switching Transients Faraday Current Profile for PASC Power Takeoff.	45
30. Single Phase Switching Transients Velocity Profile for PASC Power Takeoff.	45
31. FORTRAN Program Listing.	52

LIST OF SYMBOLS

Symbol	Meaning
A	duct cross sectional area (sq. m)
AC	alternating current
B	scalar magnetic flux density (Webers / sq. m)
$\vec{B}$	vector magnetic flux density (Webers / sq. m)
B_z	$\vec{z}$ component of vector magnetic flux density (Webers / sq. m)
C_v	constant volume specific heat (J / K Kg)
CDIF	U. S. component development and integration facility
DC	direct current
E	scalar electric field (V / m)
$\vec{E}$	vector total electric field (V / m)
E_x	$\vec{x}$ component of vector electric field (V / m)
E_y	$\vec{y}$ component of vector electric field (V / m)
f	function multipliers within a state matrix
F	friction term
g	additional functions used in the state formulation
h	channel height (m)
i	index for axial position
I_x	filamentary Hall current (amperes)
I_y	filamentary Faraday current (amperes)

$\vec{j}$	vector total current density (amperes / sq. m)
j_x	Hall component to vector current density (amperes / sq. m)
j_y	Faraday component to vector current density (amperes / sq. m)
k	second index for axial position
M	Mach number
MHD	magnetohydrodynamic
MSE	MHD test facility in Butte, Montana
P	static pressure (Pa)
PASC	pulse amplitude synthesis and control consolidation scheme
Q	heat loss term
R	gas constant (cu. m Pa / K Kg)
R_x	equivalent axial resistance in electrical circuit model (Ohms)
R_y	equivalent resistance to Faraday current in circuit model (Ohms)
t	time (s)
T	static temperature (K)
U	internal energy (sq. m / s ²)
v	scalar velocity--usually the $\vec{x}$ component (m / s)
$\vec{v}$	vector total velocity (m / s)
V_v	equivalent potential due to magnetomotive interactions (V)
V_x	Hall potential across an axial step Δx (V)
V_y	Faraday potential across channel height h_i (V)
V_{Hallx}	axial potential due to Faraday current (V)
V_{Hally}	potential in $\vec{y}$ direction due to Hall current (V)
w	channel width (m)

W	mass flow rate (Kg / s)
x	axial variable of differentiation
$\vec{x}$	x vector direction
Δx	discrete axial step size (m)
$\vec{y}$	y vector direction
z	schematic viscosity term (sq. m / s)
$\vec{z}$	z vector direction
β	Hall parameter
γ	specific heat ratio
ϵ	permittivity (F / m)
ϵ	stagnation energy state variable (sq. m / s ²)
μ	permeability (H / m)
ρ	density state variable (Kg / cu. m)
$\rho \epsilon$	density-energy product state variable (Kg / s ² m)
σ	fluid conductivity (Mhos)

ABSTRACT

The introduction of commercial magnetohydrodynamic (MHD) power generation ducts to existing power grids and the associated control problems arising from MHD power take off schemes necessitate an accurate, dynamic model of the coupled gas fluid and electrical behavior of an MHD duct. This thesis presents such a working model and lays the groundwork for further improvements.

A system of differential and algebraic equations is developed to model the duct. A Lax-Wendroff fluid flow integration scheme for the entire duct is coupled with a circuit representation of the generator channel. FORTRAN computer programs define the finite difference approximations to the three gas dynamic, one dimensional flow conservation equations, integrate these differential equations, and calculate all related algebraic fluid flow and electrical variables.

A number of modeled transient conditions are presented and discussed. Pulse amplitude synthesis and control (PASC), a low harmonic, DC power consolidation and conversion scheme, is then applied to the model to show that the model may be used with somewhat complex switching and control schemes. The model is shown to model the coupled, dynamic gas fluid and electrical behaviors of an MHD channel during a variety of transient conditions, and it may prove very useful for planning future MHD applications. Some future improvements are suggested.

INTRODUCTION

Background on MHD Power Generation

Magnetohydrodynamic (MHD) power generation is the extraction of power from conducting fluids as they flow through a magnetic field. Two types of interactions occur within an MHD duct. The first type of interaction is fluid dynamic, and the second type of interaction is that due to electromagnetic power extraction in a portion of the duct called the channel. The two interactions intimately coexist such that a perturbation of one of the interactions must also affect the other.

An MHD generator cycle may be described as closed or open. Molten metals like sodium are perpetually circulated in a closed cycle; combustion gases from fossil fuel combustion are expelled from an open cycle. One chooses open or closed systems to meet economic, environmental, or system constraints.

MHD generators may use any conducting fluid. Some fluids like molten metal naturally conduct, but most open cycle systems require supplementary conducting metallic seed, which may be recovered after use. The word "compressible" describes those fluids for which density is a function of pressure. Compressible fluids propagate sound at finite speed. This consideration of sound speed exacerbates compressible fluid behavior and analysis. Regardless of fluid properties, one investigates MHD fluid behavior as he would investigate many other similar fluid systems like jet propulsions or steam flows.

Generally, MHD generation takes place at high temperature. There are two important benefits to operating gas fluid, open cycle generators at high temperature: The seeded gas fluid conducts much better the higher the temperature, and hot conducting gas fluid--plasma--transports denser internal energy at high temperatures. Therefore, greater energy extraction and system efficiency can occur for hotter combustion processes. Of course, the high temperatures also bring about detrimental corrosion, containment and control problems.

Electromagnetic interactions can be designed to extract power from the fluid within an MHD channel. By the Lorentz force equation, conducting particles deflect from their trajectories in a direction perpendicular to the magnetic field through which they flow. This charge separation is completed on sets of electrodes on the walls of an MHD channel, and the electrodes may be connected in various external circuit configurations. Faraday generator electrode configurations are those for which the majority of power extraction is perpendicular to fluid flow, and Hall configurations are those for which the majority of power extraction is parallel to fluid flow. Diagonal electrode configurations utilize both Hall and Faraday currents. Generally, MHD generators produce direct current (DC) electrical power.

Throughout this thesis the fluid flow and electromagnetic interactions shown in Figure 1 will prevail. Figure 1 shows the conducting fluid flow in the $\vec{x}$ direction and magnetic flux density $\vec{B}$ in the positive $\vec{z}$ direction. Conducting fluid flow in the $\vec{x}$ direction interacts with the given magnetic flux density $\vec{B}$ to produce a motional emf in the negative $\vec{y}$ direction. The resulting electric field is called the Faraday field. If electrodes separated in $\vec{y}$ within this generator are attached to an external load, current flows in the external load, and a current density flows in the negative $\vec{y}$ direction within the channel. This current in the negative $\vec{y}$ direction also interacts with the magnetic flux to produce an electromotive force field in the $\vec{x}$ direction called the Hall field, and

electrodes separated in $\vec{x}$ may supply Hall current to external circuitry. Although both Hall and Faraday fields may be present in either the $\vec{x}$ or $\vec{y}$ directions, the $\vec{y}$ fields in MHD literature are called Faraday fields, and those in the axial channel direction are called Hall fields because these are the directions of major Faraday and Hall interaction, respectively.¹⁰

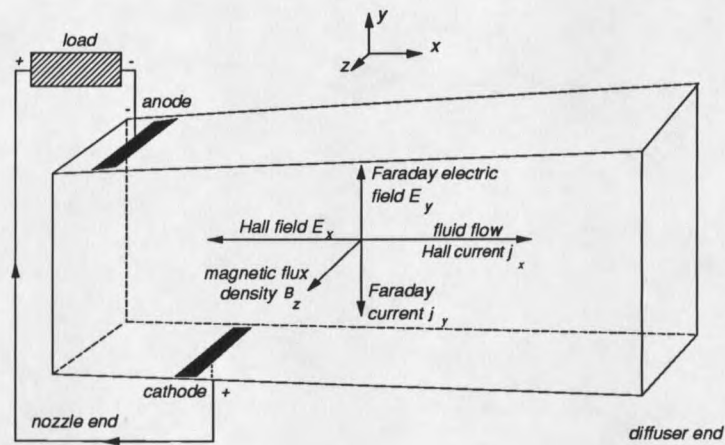


Figure 1
MHD Channel Interactions

Those readers desiring more complete technical background may refer to a book by Rosa¹⁰ or any comprehensive book on plasma dynamics or compressible fluid flows.^{9,11,12}

Present State of MHD Research

A number of MHD test facilities have come to operation. However, the addition of an MHD topping cycle to a conventional fossil fuel steam plant is of most interest to introduce MHD into commercial power generation. This design retrofit can fairly easily augment existing power plants, and one may predict a significant increase in overall plant efficiency due to the efficient, high combustion temperatures necessary for an MHD topping cycle.

Presently, the United States Department of Energy's Component Development Integration Facility (CDIF), which is operated by MSE Inc. in Butte, Montana is investigating the technology for coal fired MHD commercial power generation. Considerable success has been demonstrated. However, problems of short electrode life and unpredictable transient behavior still plague the operation. Other countries have shown the viability of clean fuel MHD generation plants, but the retrofitted coal fired plant would be novel in its prolonged use of coal. A retrofit of an MHD coal fired topping cycle to a conventional steam plant is now being planned for possible implementation in the middle 1990s.¹⁴

Purpose of this Thesis

Most research in MHD power generation and generator design focuses on steady state DC generation. Even DC generation under controlled conditions, however, may encounter unexpected transients, and MHD alternating current (AC) generation may yet have a resurgence of interest. Switching algorithms for power take off and channel optimization also introduce transients. Therefore, the dynamic interaction of the coupled fluid and electrical behavior within an MHD generator should be studied. This thesis presents the formulation for a working dynamic model of the coupled fluid and electromagnetic behavior of an MHD power generation duct. Suggestions for further improvements will be proposed as well.

A good MHD channel model, which correctly models both the steady state and the transient behaviors of the MHD generator from combustor to diffuser end, should greatly enhance the study of control for MHD generation. First, the model would allow fairly accurate extrapolation of small test plant behavior to larger power facilities. Some costly scale-up errors consequently may be avoided. Second, transient behaviors may be investigated to help operators create contingency plans for unplanned transients. One

could prepare proper control procedures for electrical transients due to sudden failures and other less severe load fluctuations. Thorough contingency planning can extend component life, maximize operation time, and protect a system from catastrophic failure. Third, switching and control algorithms could be carefully studied before application to a costly test or retrofit facility. Novel control schemes and inversion schemes like pulse amplitude synthesis and control (PASC),^{3,4} a source consolidation and inversion scheme, warrant continued investigation.

The pending implementation of a coal fired MHD topping cycle retrofit and the practicality of such an endeavor have biased the formulation and demonstration of the model developed in this thesis. Specifically, the model used in the transient examples represents an open cycle, compressible combustion gas, Faraday power take off system, but the formulation procedure is not inherently limited to MHD generation with any of these attributes. This thesis is intended to emphasize the dynamic capabilities of the developed model, not the abilities of the model to accurately represent any specific MHD duct.

The next section presents the model formulation and the mathematical justification for the formulation. The fluid dynamic and electromagnetic model formulations will be discussed separately. Then the program FORTRAN computer model which models the coupled fluid dynamic and electromagnetic behavior will be presented. The last major section of the thesis presents the results of some dynamic transient experiments as simulated by computer model. The model will be shown to be applicable to fluid transients, electromagnetic transients, and switching transients.

COMPUTER MODEL FORMULATION

Duct Fluid Dynamic Model Formulation

Although the fluid and electromagnetic interactions within an MHD duct are intertwined, it is beneficial to discuss the fluid dynamics apart from the electromagnetic dynamics. Therefore, this section will discuss the mathematical formulation of the fluid MHD behavior only. The model is formulated for compressible, combustion gas behavior, but simple parameter changes would permit the model to be used for general fluids in subsonic or supersonic MHD ducts.

The starting point for the fluid dynamic modeling is a set of three partial differential conservation equations. The three conservation principles are conservation of mass, momentum and energy. Each equation is derived by considering conservation principles applied to an infinitesimal volume. For example, the equation of mass conservation states that the rate of mass flow per volume across an infinitesimal volume's boundaries must be equal to the rate of change in the fluid's density. Similar statements may be made for each of the conservation equations, although the interpretations may not be as obvious. The equation forms shown in Figure 2 come from Rosa.¹⁰

One makes assumptions from this point to make the given continuity equations tractable. In this formulation it is assumed that the field terms $\frac{eE^2}{2}$ and $\frac{B^2}{2\mu}$, the energy storage terms of the system fields, are small or constant. This assumption simplifies the calculation of energy partial differentials. The assumption is reasonable because the applied magnetic field is usually constant in time, and the product of electric field

Mass:

$$\frac{\partial \rho}{\partial t} = -(\rho \nabla \cdot \vec{v} + \vec{v} \cdot \nabla \rho)$$

Momentum:

$$\rho \frac{\partial \vec{v}}{\partial t} = -\rho \vec{v} \cdot \nabla \vec{v} - \nabla P + \vec{j} \times \vec{B}$$

Energy:

$$\rho \frac{\partial}{\partial t} \left(\frac{v^2}{2} + U + \frac{\epsilon E^2}{2} + \frac{B^2}{2\mu} \right) = -\rho \vec{v} \cdot \nabla \left(\frac{v^2}{2} + U \right) - \nabla \cdot (\vec{v} P) + \vec{j} \cdot \vec{E}$$

Figure 2
Basic Equations of Continuity

strength and space permittivity ϵ is quite small. The change of magnetic field in the vicinity of fluctuating conduction currents and current densities may indeed be significant, but this problem is reserved for later models.

The biggest assumption made is that a one dimensional flow model adequately models the behavior of an MHD duct for the studied phenomena. The resulting mathematical simplification is noteworthy in that all gradient operations may then be represented as simple partial derivatives in one variable. The mass and momentum

equations, in fact, then contain only partials in respect to $\vec{x}$, the direction of fluid flow. The energy conservation equation will still contain partials in respect to $\vec{z}$ and $\vec{y}$ since energy is indeed exchanged with duct walls!

The consequences of the one dimensional flow assumption are as follows: First, fluid flow occurs primarily in the axial direction down the duct. This assumption somewhat limits duct architectures for which the model is valid. Rapid cross sectional area changes along a duct will prevent accurate solution because at these places expansion causes non-axial fluid velocities, which are not modeled. Inclusion of area gradients in later model formulations may improve model accuracy in this respect. Second, the one dimensional model assumes that physical conditions of pressure, velocity, *etc.* are homogeneous in any duct cross section. This statement is, of course, absurd. Friction slows gases near walls, temperatures differ near walls, and greater compressible fluid densities and pressures occur near the channel anodes. The best one can hope for is a representative example--an average of sorts--for the fluid behavior at any axial position. Other research has successfully developed higher dimension models,² but the one dimensional model should provide a good first order representation of dynamic MHD duct behavior for fast control algorithms. The two and three dimensional models are left for later research.

The described simplifications are completed to give a set of equations like those in Figure 3. The formulations shown are those of Ostling,⁷ who chose density ρ , mass flow rate W , and the product of density ρ and stagnation energy $\epsilon = U + \frac{v^2}{2}$ as his state variables. He also includes terms for friction and heat losses. The choice of state variables is a natural consequence of the described formulation, but the choice has significant advantage over others because it allows calculation across flow discontinuities like shocks. Regardless, conservation must be maintained. The choice of mass flow rate

state variable W also helps assure that mass flow rate will become constant down the duct as steady state is approached. The problem of shocks will be addressed in more detail later.

Mass:

$$\frac{\partial \rho}{\partial t} = -\frac{1}{A} \frac{\partial W}{\partial x}$$

Momentum:

$$\frac{\partial W}{\partial t} = -\frac{\partial W v}{\partial x} - A \frac{\partial P}{\partial x} - F + A(j_y B_z)$$

Energy:

$$\frac{\partial \rho \epsilon}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial x} \left[W \left(\epsilon + \frac{P}{\rho} \right) \right] + Q + \vec{j} \cdot \vec{E}$$

Figure 3
One Dimensional Equations of Continuity

The partial differential equations are then made discrete in axial position by finite difference approximation. The effect of this approximation is that the set of partial differential equations in two independent variables becomes a much larger set of approximated ordinary differential equations in the single independent variable time. The resulting set of equations is fully time dynamic, and the number of equations is three times the number of discrete axial position steps. One's choice of position step size will be determined by his desired accuracy, the importance of the observed region of the duct, and the expected rates of change along the duct due to energy exchange and duct geometry. The axial position steps used in initial examples within this thesis may be considered excessive to some in MHD research.

As an example of the finite difference approximation, the first partial derivative

$$\frac{\partial W}{\partial x}$$

may be approximated by

$$\frac{W_{i+1} - W_{i-1}}{2\Delta x}$$

where Δx is the axial position step size. The second partial derivative

$$\frac{\partial^2 \rho}{\partial x^2}$$

will be approximated by

$$\frac{\rho_{i+1} - 2\rho_i + \rho_{i-1}}{(\Delta x)^2}$$

Usually the choice of small step size will improve the accuracy of the finite difference approximation.

Following the procedure outlined in Ostling's paper,⁷ schematic viscosity terms were then included in the three sets of differential equations. Schematic viscosity is an artificial viscosity which eases mathematical computer integration. Unlike a true fluid viscosity, it is independent of channel surface area and is instead represented by second order combinations of the fluid dynamic variables. The evaluation of a set of schematic viscosity terms may be positive or negative, but it nearly always moderates the rate of change. It may be thought of as a penalty for any nonlinear parameter variation over short axial distances. The conservation equations, equations of first order, then react to the schematic viscosity terms to force compliance with the laws of conservation at each

axial position. The net effect is a reduction in oscillation and an increase in stability. The three schematic viscosity additions to the three conservation equations are shown in Figure 4. These additional terms were also made discrete by the finite difference method as described above.

Mass:	$+\frac{\partial z}{\partial x} \frac{\partial \rho}{\partial x} + z \frac{\partial^2 \rho}{\partial x^2}$
Momentum:	$+\frac{\partial z}{\partial x} \frac{\partial W}{\partial x} + z \frac{\partial^2 W}{\partial x^2} + \rho z A \frac{\partial^2 v}{\partial x^2}$
Energy:	$+z \frac{\partial^2 \rho \epsilon}{\partial x^2} + \frac{\partial z}{\partial x} \frac{\partial \rho \epsilon}{\partial x}$
where	$z = \left v \right \frac{\Delta x}{2},$
the schematic viscosity, and all other variables are described elsewhere.	

Figure 4
Schematic Viscosity Terms

The state equations could have been formulated to eliminate all non-state variables, but such a formulation would require that inherent assumptions be imbedded within the state equations and would hide some of the more intuitively helpful physical variables like pressure and temperature. Therefore, a two step Lax-Wendroff^{5,9} approach is taken to alternately integrate the state equations, then calculate the set of related algebraic equations. The related algebraic equations are briefly listed below:

The most important algebraic equation is the specifying equation, or equation of state. This equation specifies the relationship between density and pressure. For this initial formulation, an ideal gas assumption was made. No gas will behave ideally, but the accuracy provided by the assumption allows good initial dynamic study. Later models must include gas nonlinearities.

$$P = \rho RT$$

Mass flow rate, density, cross sectional duct area and gas velocity are related by

$$W = \rho A v.$$

This equation allows the calculation of duct velocities.

It is assumed that internal energy is a function of temperature only. Consequently, temperatures may be calculated

$$TC_v = \epsilon - \frac{v^2}{2}.$$

And Mach number can be calculated

$$M^2 = \frac{v^2}{R\gamma T}.$$

Here, γ is the ratio of specific heats taken arbitrarily to be 1.4, the specific heat ratio for air at room temperatures. This assumption may be improved in later models, and the ratio may even be calculated dynamically for each axial position.

Only the first and last nodes pose problems to the integration scheme since no upstream or downstream variables exist at these nodes to formulate correct finite

difference approximations. Ostling⁷ presented a interesting formulation of the input conditions at the back wall of a combustor using the concept of reflection, and his results may be helpful for future models, but the problem is not as well defined at an open input to a nozzle. Therefore, one must define boundary conditions at the first and last nodes, the input to the nozzle and the exit from the diffuser, respectively.

Two possible input boundary conditions were posed. First, since no combustor is included in the model, one may choose to provide the input combustor pressure and stagnation temperature, then calculate mass flow rate. Second, he may provide mass flow rate and stagnation temperature for the combustor and calculate input pressure. The equations for these boundary conditions are those used by Matair.⁵ Once either of these two input boundary conditions is accepted, one need only extrapolate the velocity back to the input boundary node and calculate the remaining variables. The MHD operator will most likely have control of mass flow rate rather than combustor pressure. But one should choose the input variable which is least susceptible to change during the transient of interest. Should a combustor be added to the model, the first boundary condition formulation must be chosen and no choice remains.

At the diffuser exit, the most important boundary condition is the requirement that the exit pressure is only slightly higher than atmospheric. This condition is imposed to assure that flow will continue through the bottoming portion of a power plant. Density and mass flow rate are then extrapolated from near nodes. Further, the time differential of the energy-density product state variable is extrapolated to allow its calculation through integration.

These boundary conditions were formulated to allow the greatest possible degree of freedom at the boundaries. Insufficient description, however, gives instability. These boundary conditions may continue to evolve with the computer model. For example, should a combustor be included in the model, the first node will become a reflective wall,

and the boundary condition will become relatively well defined as described by Ostling.⁷ Such a formulation is especially applicable to current tangential slag and coal input designs.

Before this section is concluded, more must be said about the model's fluid behavior near shocks. A shock front is the boundary between a region which is supersonic and a region which is subsonic. The shock front may be either transient like that which must occur when a supersonic duct is shut off, or the shock front may be static. The speed of sound is primarily dependent upon temperature and the compressible fluid's relationship between pressure and density. Consequently, a shock may result in rather severe discontinuities for temperature, velocity, pressure and other variables on the two sides of any shock. The fortunate thing about the method of formulation used in this thesis is that because the formulation is based on conservation principles, a shock poses no exception. The Rankine-Hugoniot relations still hold at a shock location in the duct. This type of formulation is sometimes called a *through, smoothing* or Lax-Wendroff method for the way it treats shocks. The alternative to this formulation would be to shock fit, a method which usually requires that the shock be located in the duct before an accurate curve fit can be made at that location. The shock fit method is quite accurate, but it is not as flexible in dynamic situations as the smoothing method.

The inclusion of schematic viscosity smooths shock fronts slightly, but one gains a startling increase in integration step size, and therefore, one also decreases his program run time.

Channel Electrical Model Formulation

Neither the fluid dynamic model formulation nor the electric model formulation of this thesis are novel, but the combination of fluid dynamic model and circuit representation of the channel interactions is original. The advantage to this union is that

the complete model taps both the voluminous experience in fluid dynamics and the extensive available circuit theory of the electrical engineers. This section shall describe the circuit model of the electrical interactions within the MHD channel. The formulation is very similar to that posed by Trung.¹³

The formulation is based upon a fairly complete equation for Ohm's law.¹⁰

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) - \frac{\beta}{B}(\vec{j} \times \vec{B})$$

The equation states that the current density within the MHD channel is equal to the net electromotive field multiplied by conductivity, less a component due to Hall interaction. The Hall parameter β represents the magnitude of influence perpendicular current density components have on the vector total current density and increases with magnet strength and plasma ion mobilities.¹⁰ The presence of current density on both sides of the equation complicates interpretation, but a few simple cases can be addressed to acquire some intuition about MHD interactions: First, assume that only an axial component to fluid flow is significant. If no electric current flows, it is obvious that the electric field and magnetomotive field vectors are equal in magnitude but opposite in direction. Second, if only Faraday current flows, the magnitude of the magnetomotive field must be somewhat greater than the electric field in the $\vec{y}$ direction, and current flows in the direction $\frac{\vec{v} \times \vec{B}}{|\nu B|}$. The electric field in the $\vec{x}$ direction divided by conductivity must be approximately equal in magnitude to the Hall interaction term. Similar arguments may be made for the case of conduction in the Hall direction only. These simple cases should provide some insight into the behavior and complexity of the general MHD channel interactions.

The vector components of Ohm's law are separated to begin the formulation of a circuit model for electrical interactions. After initial separation one finds

$$j_x = \sigma E_x - \beta j_y$$

and

$$j_y = \sigma(E_y - vB) + \beta j_x.$$

One can immediately rearrange to get

$$E_x = \frac{j_x + \beta j_y}{\sigma}$$

and

$$E_y = \frac{j_y - \beta j_x}{\sigma} + vB.$$

Assume homogeneity throughout each discrete channel region in the one dimensional, finite difference approximation to obtain current density representations in the filamentary currents I_x and I_y .

$$j_x = \frac{I_x}{wh}$$

$$j_y = \frac{I_y}{w\Delta x}$$

One may then integrate the field equations over Δx , the discrete axial position step, and h , the height of the channel in the $\vec{y}$ direction at an axial position, respectively. The result will be the negative of the potential across the integrated distance.

$$V_x = -\frac{I_x \Delta x}{\sigma w h} - \frac{\beta I_y}{\sigma w}$$

$$V_y = -\frac{I_y h}{\sigma w \Delta x} + \frac{\beta I_x}{\sigma w} - v B h$$

One now begins to see the relationship between the expressions derived and the circuit diagram for a single electrode pair shown in Figure 5. The first terms in the above equations resemble products of currents and resistances; the remaining terms resemble voltage sources. Some notation changes are made to simplify the model.

Let

$$V_x = -I_x R_x + V_{Hallx}$$

and

$$V_y = -I_y R_y + V_{Hally} - V_v,$$

where

$$R_x = \frac{\Delta x}{\sigma w h},$$

$$R_y = \frac{h}{\sigma w \Delta x},$$

$$V_{Hallx} = -\frac{\beta I_y}{\sigma w},$$

$$V_{Hally} = \frac{\beta I_x}{\sigma w},$$

and

$$V_v = vBh.$$

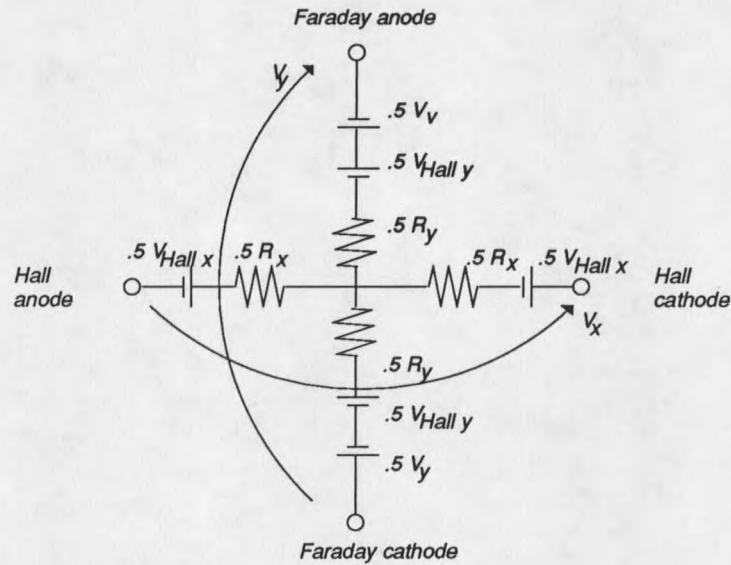


Figure 5
Circuit Model for a Single Electrode Pair

One might notice that the sign of the Hall source $V_{Hall x}$ is carried into its definition. This is done to reflect the fact that the Faraday current is usually in the negative $\vec{y}$ direction, and therefore positive Hall current should flow in the positive $\vec{x}$ direction. The model then avoids negative voltage sources, and the model is quickly and correctly interpreted. Keep in mind that all shown sources and resistances will vary with currents, conductivities, and channel dimensions.

Numerous models may be combined as shown in Figure 6--here shown in a Hall configuration. Each electrode pair should be represented by an integer multiple of models. This suggestion maintains a correspondence between filamentary currents and meaningful channel outputs. It is assumed that no energy storage elements are required

in the model, but later studies may show the need for capacitors and inductors for high frequency transients. Although the examples given later in this thesis will be for Faraday connections, the model is not limited to Faraday interactions. Faraday interactions are simply the easiest to model for these first demonstrations. Faraday connections are implemented by loading each individual circuit model in the $\vec{y}$ direction; diagonal and Hall connections would be implemented by shorting the channel across desired equipotentials and extracting currents diagonally or axially.

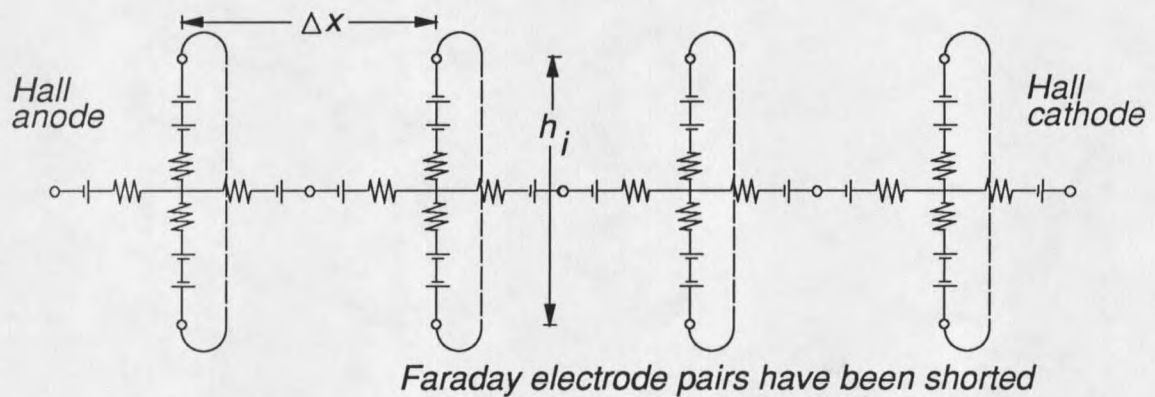


Figure 6
Many Circuit Models Connected in Series for a Hall Configuration

Finally, the external circuit may be defined to model inverters, loads, or control scheme behavior. Any external circuit which can be mathematically defined in the time domain may be modeled, but the examples given in this thesis largely assume resistive load and ideal switching for simplicity.

FORTTRAN Implementation of the Duct Model

Two FORTRAN subroutines were written to implement the formulations described above. The first subroutine, SIDE.FOR (see appendix), defines a state matrix containing all the multipliers for the three state variables ρ_i , W_i , and $\rho \in_i$ used in the three sets of

partial differential conservation equations which were approximated by the finite difference scheme described earlier. The multipliers are prepared from the most recently available fluid variables. A matrix multiplication is performed to multiply the matrix just described by the vector of state variables. The resulting vector, the incomplete vector of time differentials, is three times as long as the number of axial nodes in the model. Therefore, the order of the differential equation set is three times the number of discrete position steps. The matrix multiplication is carried out in parts to take advantage of the matrix sparsity.

Two of the three differential equation sets have additional terms including electromagnetic interactions, pressure terms, and thermal loss terms. These terms are included to complete the calculation of the time differential. Control is then returned to the integration routine to project the next set of independent variables for the next time interval.

$$\begin{bmatrix} \frac{d\rho_i}{dt} \\ \frac{dW_i}{dt} \\ \frac{d\rho_{\in i}}{dt} \end{bmatrix} = \begin{bmatrix} f_{\rho, \rho_k}(v_i) & f_{\rho, W_k}(A_i) & 0 \\ f_{W, \rho_k}(v_i, F, A_i) & f_{W, W_k}(v_i) & 0 \\ 0 & f_{\rho \in i, W_k}(\in_i, P_i, A_i, \rho_i) & f_{\rho \in i, \rho \in k}(v_i) \end{bmatrix} \begin{bmatrix} \rho_k \\ W_k \\ \rho \in k \end{bmatrix} + \begin{bmatrix} 0 \\ g_{W_i}(B, v_i, A_i, I_{iy}, P_i) \\ g_{\rho \in i}(Q, T, A_i, I_{ix}, I_{iy}, E_{ix}, E_{iy}) \end{bmatrix}$$

Figure 7
Matrix Formulation of State Equations in STATE.FOR

The new state variable set for the next time interval is then passed to subroutine SIDE.FOR (see appendix), where related gas fluid and electrical algebraic variables are updated. The user has the choice of two boundary conditions: In the first formulation, stagnation pressures and temperatures are given to the program for combustor conditions. The velocity in the nozzle is projected back to the input node, and all dependent variables

may be calculated for the first node. In the second formulation, one supplies the mass flow rate and stagnation temperature for the first node of the nozzle. Again the velocity is extrapolated to the first node, and the other dependent variables and pressure may be calculated. The user should choose one input boundary condition option and comment out the other.

Any integration scheme may be used to solve the differential equation set. The differential equations for this thesis were integrated by a fourth order Runge-Kutta integration scheme. The numerical stability to date has been very good for time steps up to one tenth millisecond. Success was also met with a very simple Euler integration scheme, and at time intervals of 50 microseconds, the solution by Runge-Kutta is indistinguishable from that obtained by the simple Euler integration method. Rules like the Courant number^{5,9} exist for determining maximum allowable time step for integration of fluid flows, but position steps in this initial formulation were large enough that time step optimization was unnecessary. The Runge-Kutta integration procedure accuracy and stability may be improved by automatically decreasing the integration time step upon observation of severe transients or significant errors. The time step could again be increased as transients or estimated errors abate. These improvements should be incorporated into later models.

The FORTRAN model implementation is summarized in Figure 8.

The complete set of states and related fluid and electrical variables was archived for numerous integration steps. Samples were collected every 0.1 millisecond for durations of two hundred milliseconds in most of the transient studies given later in the thesis. The resulting files were very storage intensive, but one must weigh the trade off between computer processing unit time and computer storage. Once a particular transient study was performed, numerous analyses may then be performed consuming minimal additional processor time.

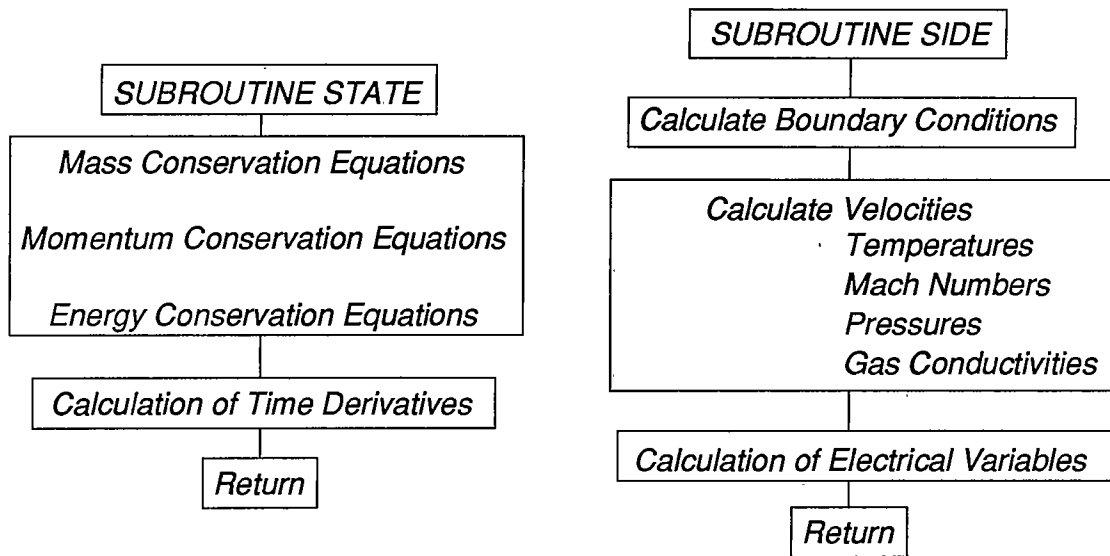


Figure 8
Overview of FORTRAN Model Implementation

These storage files were then scanned by short FORTRAN programs to extract important data into much smaller files. The short files were manipulated with the software package MATLAB⁶ to graphically portray data. Of particular note is the MATLAB program "mesh," which generates a three dimensional relief graph of matrix values. By use of this program, individual magnitudes (pressures, for example) may be graphed as functions of two independent variables--time and axial position.

DEMONSTRATION OF DYNAMIC MODEL RESULTS

Description of the Hypothetical MHD Duct

MHD duct dimensions were chosen to resemble the U-25 Soviet test channel.¹ This resemblance allowed comparison of steady state fluid values to a designed channel for which test results were available. The model is not, however, intended to model the electrical behavior of the U-25 channel, which is a diagonal, low interaction MHD test channel. The modeled duct is indeed low interaction with a magnetic flux density of 2.0 Tesla, but the model is strictly Faraday. A Faraday configuration was chosen for the simplicity of its internal and external circuit and for its flexibility over varying load and transient conditions. The diagonal duct is designed for only one steady state operating condition, which determines the desired electrode pitch. The measurements for the duct model used in the following studies is shown in Figure 9.

The duct is designed for 3.2 atmospheres combustor pressure at 2800 K, and these input conditions are maintained unless otherwise stated.

Steady State Behavior Under Variable Load

First, steady state channel profiles were investigated with a constant resistive load on each Faraday configuration electrode pair. Fifty nodes were used to model the duct as previously described. A number of different constant resistive loads were investigated to roughly demonstrate load curves for the modeled generator. The general fluid and electrical behaviors of the channel were observed and related to the cross sectional areas of the columns at each node. As expected for subsonic channels, density and pressure

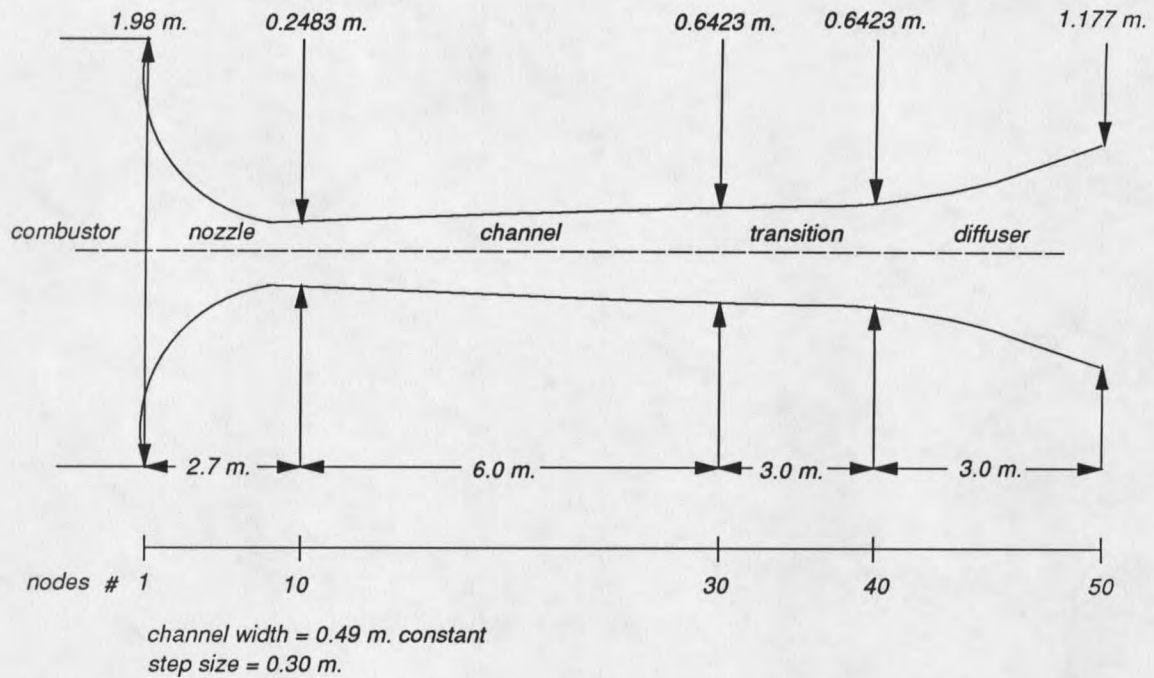


Figure 9
The MHD Duct Model Measurements Used in this Thesis

have a direct relationship to cross sectional area while velocity and temperature have an inverse relationship to cross sectional area. The channel operated near sonic with a loading of 0.5 ohms across each Faraday electrode pair, so this loading was chosen as a baseline. Observe the profile of fluid and electrical variables upon constant 0.5 ohm loading shown in Figure 10.

Variation of the Faraday load should cause a change in Hall interaction. Specifically, an increase in Faraday load (smaller resistance and greater currents) should increase the Hall interaction. The increased Hall interaction must slow the plasma velocity, increase the Hall field, and consequently decrease the power available to the strictly Faraday channel. These results are observed for constant loading resistances of 0.1, 0.25, 0.5, 0.75, 1.0 and 1.5 ohms. Figure 11 demonstrates the trade off between load

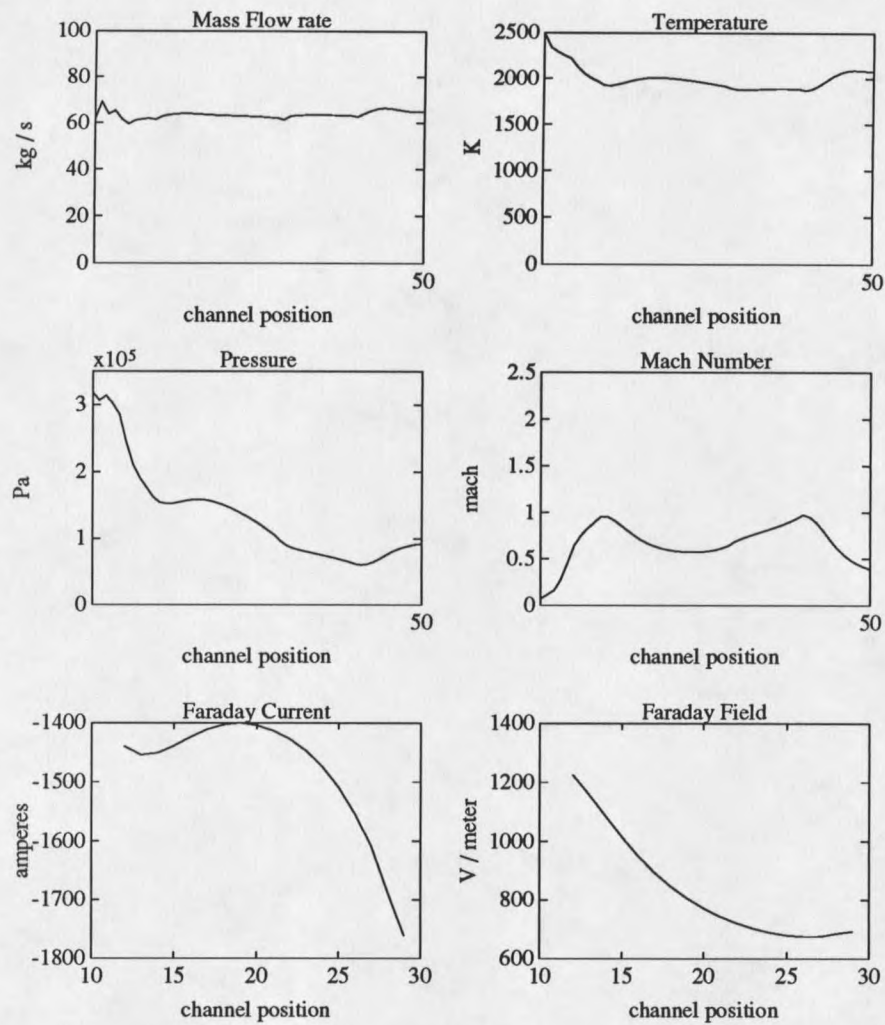


Figure 10
Axial Profile of Fluid and Electrical Variables under Constant Resistive Faraday Loading

current and terminal voltage--a load curve--for the Faraday MHD channel. The crosses mark averages across the channel length for each loading resistance. The power output of the channel in the Faraday mode is maximum near a loading of 1.0 ohms decreasing to either side as shown in Figure 12.

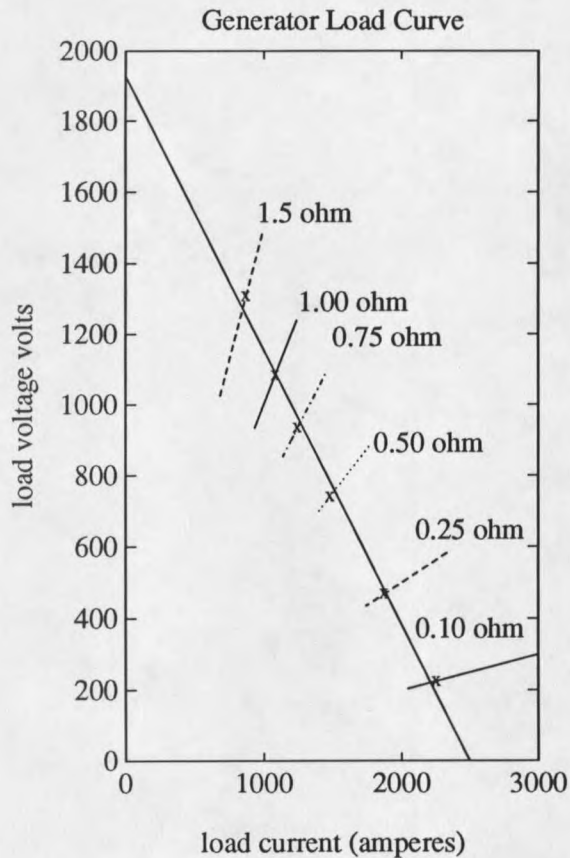


Figure 11
Load Curve per Electrode Pair for a Faraday
Configuration MHD Channel

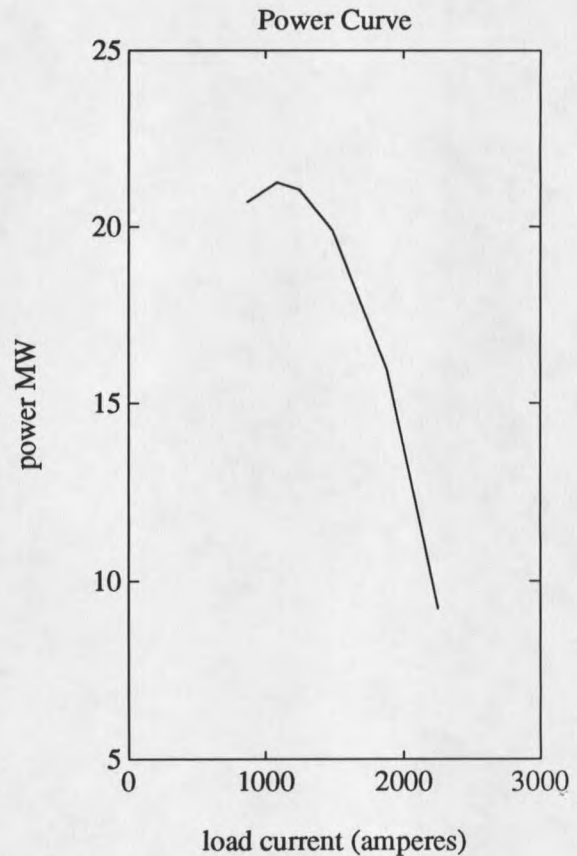


Figure 12
Power Curve for a Faraday
Configuration MHD Channel

One of the extremes to the loadability of a Faraday channel is the unloaded condition. This particular channel is supersonic when no load is applied. Nothing different need be done, however, to model the new supersonic condition because the

conservation equation problem formulation readily models both subsonic and supersonic behaviors. The unloaded channel profile is shown in Figure 13. Notice that all physical channel behaviors change under supersonic conditions. Whereas velocity decreased in the expanding diffuser for the subsonic case, the velocity now increases in the expanding diffuser.

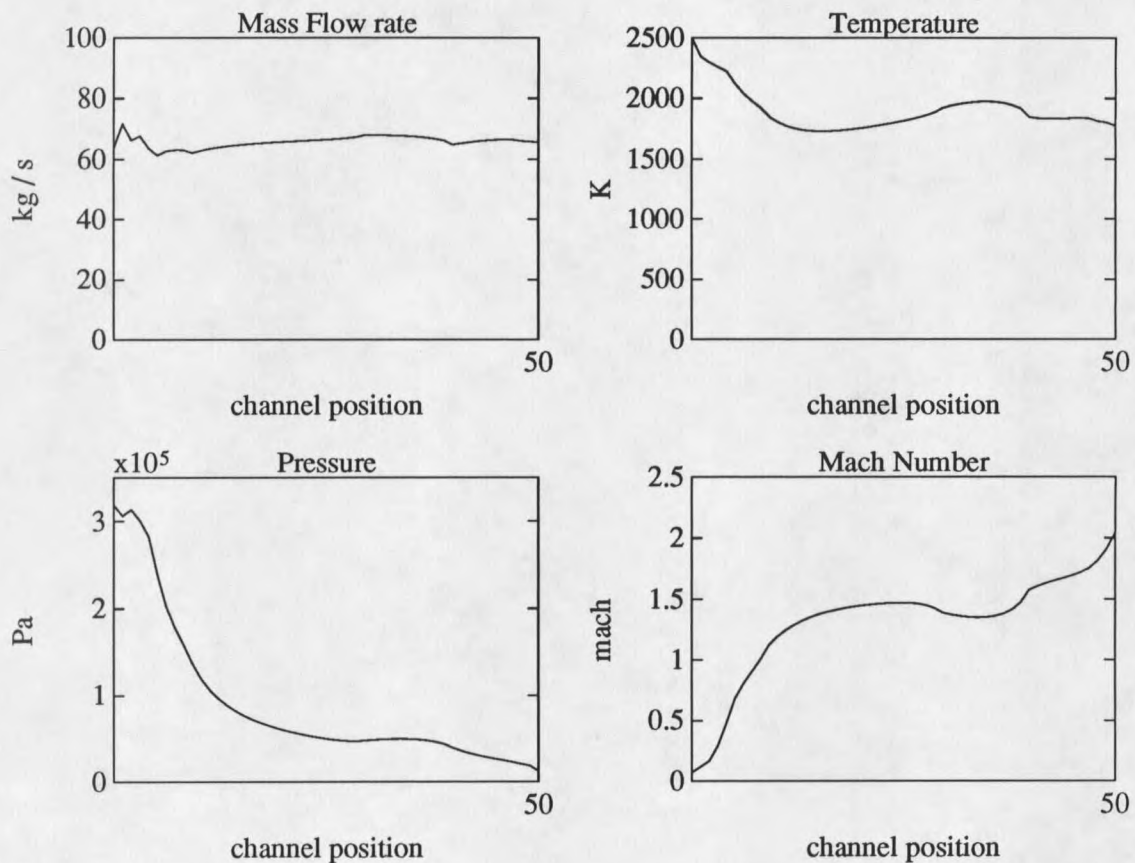


Figure 13
Axial Profile for Fluid Variables in an Unloaded MHD Channel

If the combustor pressure is decreased for the unloaded duct to two atmospheres, a shock develops within the generator portion of the model. At this input pressure, the system is no longer able to support a shock at the diffuser output, and the shock moves

into the diffuser and generator channel until it comes to rest halfway into the generator. Although the shock may be very narrow in the axial direction, and variation may be quite severe across the shock, this model will tend to spread the shock over a few nodes. It is hypothesized that an increase in the number of nodes will narrow the width of the shock. A shock may be observed in Figure 14 at about node 28 in the variables of density, velocity, pressure, and possibly temperature. This shock is not extreme in magnitude, but it is demonstrative.

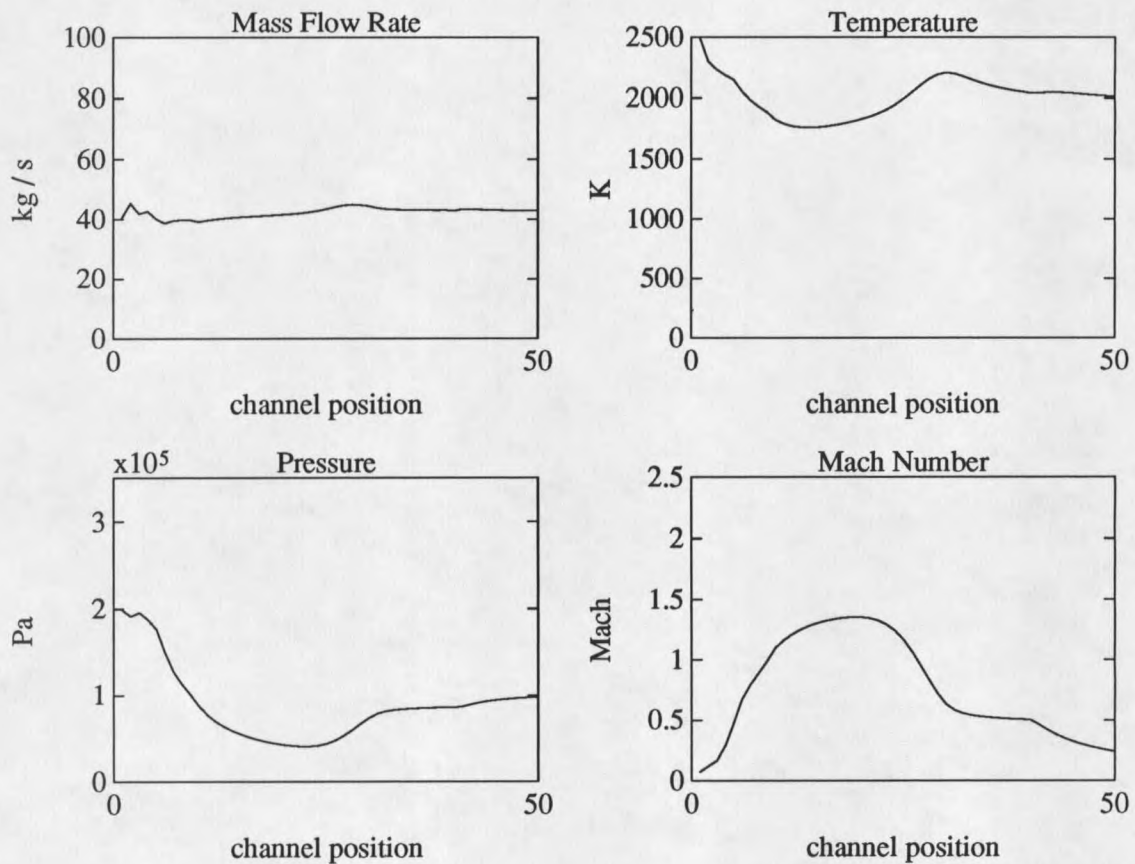


Figure 14
Axial Profile for Fluid Variables Showing a Shock at Node 28 as Input Pressure is Decreased

Dynamic Validity Testing of the MHD Model

Once the steady state MHD duct behavior was shown to be reasonable, the dynamic behavior was investigated. One of the first validity tests done on the fluid dynamic portion of the model was a replication of a classic shock tube experiment as given by Shapiro¹² and described by Ostling.⁷ A shock tube is nothing more than a length of tube in which a septum separates two different pressure regions. When the septum is pierced, a complex history of shock and rarefaction waves occurs.

A theoretical pressure history was prepared by Shapiro from gas dynamic theory for a shock tube experiment. The same shock tube was simulated using fifty nodes and the computer simulation of this thesis. The ends of the tube were modeled to be reflective wall boundaries such that vectored values like velocity become zero at the wall boundaries and other values are conserved. Any correlation between the theoretical and modeled fluid behavior should lend credence to the fluid dynamic computer model. The electrical portion of the model was not tested by this trial.

A septum separating a 2 atmosphere pressure region from a 1 atmosphere region was then ruptured. The simulated septum was between nodes twenty and twenty-one. The subsequent modeled gas pressure behavior at node 35 is compared to the theoretical pressure history in Figure 15. Notice that the model tends to smear shock waves over a number of nodes. This smoothing is due to discretization and the schematic viscosity. The smoothing due to discretization may decrease with finer mesh size, but the smoothing due to schematic viscosity may not be circumvented. Interestingly enough, this lossless model may still model the true pressure behavior better than the theoretical model! It is intuitively impossible for a real system to have infinitesimal shock width and no overshoot--properties exhibited in the theoretical curve.

The given results match those given by Ostling.

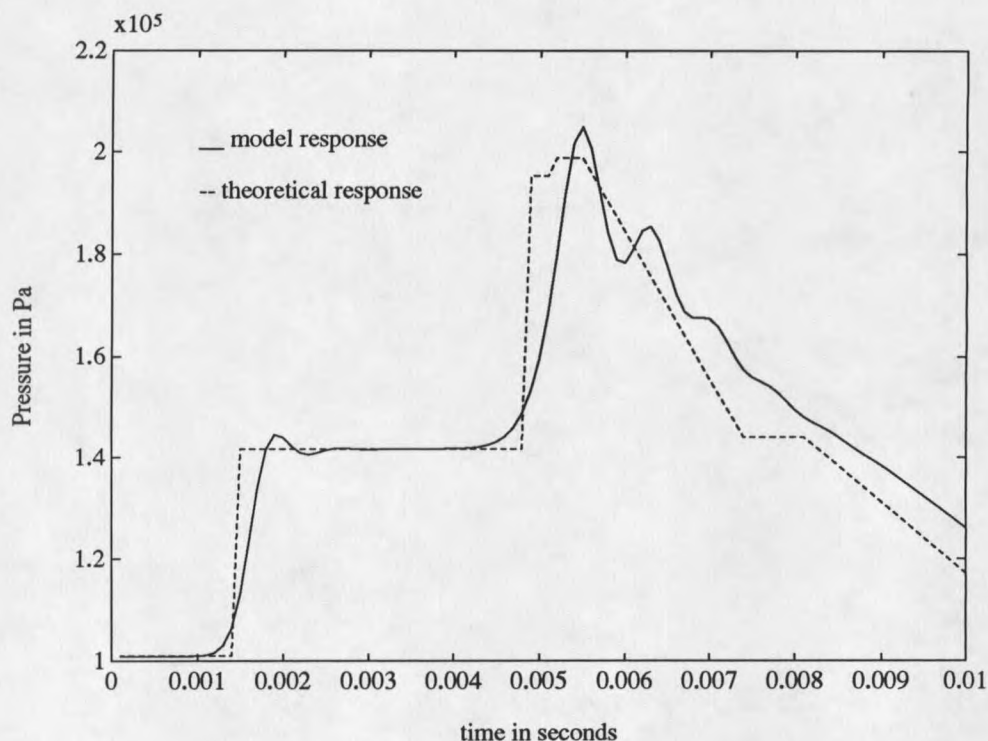


Figure 15
Comparison of Modeled Shock Tube Pressure Behavior to Theoretical

The electrical behavior of the MHD model is more difficult to verify for a number of reasons: The modeling of plasma behavior is at best an art, and the number of controllable parameters is nearly uncountable. The behavior of plasma is highly nonlinear so that minor inaccuracies in modeling may bring about considerable discrepancies with experimental results despite correctness of dynamic modeling. Very little electrode by electrode profile data exists for an entire MHD channel length and even less transient data exists for MHD behavior.

Nonetheless, the electrical and fluid dynamic behavior of the duct will be supported by a fair number of transient examples. In every example, the model behavior mimics the expected duct behavior in gross behavior, if not accurately. In other words,

an increase in one variable will correctly bring about an increase or decrease in another. The absolute accuracy of the model may be easily improved once the dynamic behavior is shown to be acceptable by inclusion of polynomial fits or look up tables, especially for the equation of state. The remainder of this thesis details examples of dynamic model behavior and explains how the results correspond to expected MHD channel behavior.

Transient Model Behavior

Flow Shut Off Transient

Several transient behaviors were then investigated. First, a flow shut off is presented to show the effect of a fluid dynamic transient on the fluid dynamic and electrical variables under constant 0.5 ohm loading. Then a temporary loss of the resistive load is allowed to take place to demonstrate the effect of a transient starting in the electrical portion of the channel model. Third, other less severe electrical transients of interest like single electrode shorts and single electrode open circuits are presented. The Faraday configuration model is maintained throughout.

A short section was added to the FORTRAN code of SIDE.FOR to ramp up the input pressure over 25 milliseconds from an initial ambient pressure of about one atmosphere, hold the pressure for another 25 milliseconds, and quickly ramp the pressure back to one atmosphere in 5 milliseconds. The model was robust to this severe transient with a step size of 50 microseconds. A complete set of data was collected and stored every tenth millisecond. The complete run of 200 milliseconds took about 25 minutes on a VAXstation 3100/76. The resulting output data file required some 30K blocks of memory. Very short FORTRAN programs were then run to extract the data from this large file; the data was combined as a large matrix in MATLAB⁶ and printed with the three-dimensional plotting routine of MATLAB called "mesh". Note that the plots

shown represent only one fortieth of the collected data because the mesh plots of this time duration were cleaner with a time step of 4 milliseconds. Consequently, five milliseconds of this file could be viewed with the same visual graph resolution!

Figure 16 shows the pressure profile of this transient as the combustor pressure is ramped up, held, then ramped down. The user of this program has the choice of using the independent variable pressure and letting flow rate vary or using mass flow rate as the independent variable and letting combustor pressure vary. Each method has some advantages, but pressure was chosen as the independent variable for this example. The plane $\vec{x}=0$ shows how pressure is varied to cause a pressure transient at the duct input. The combustor pressure begins at about one atmosphere. After some 25 ms the input reaches the operating pressure of 3.2 atmospheres, and this pressure is held for another 25 ms. before being ramped back down. The variation in the $\vec{x}$ direction shows the pressure profile along the entire duct length. The pressure is seen to decrease to the narrowest part of the nozzle, increase slightly upon reaching the electrical generation portion of the duct and either increase or decrease through the transition and diffuser sections to match the boundary condition at the exit of the diffuser.

In time, the pressure along the channel overshoots the steady state pressure profile, then approaches the steady state profile before being ramped off again. The combustor pressure is ramped off so suddenly that the channel pressure exceeds the pressure in the combustor side of the nozzle for a short period. A brief reverse of plasma flow occurs at this point in time. Damped oscillations in pressure take place as the system settles to a constant ambient pressure condition. It is interesting to note that the period of this oscillation is about 36 ms, roughly twice the transition period for this modeled MHD duct.

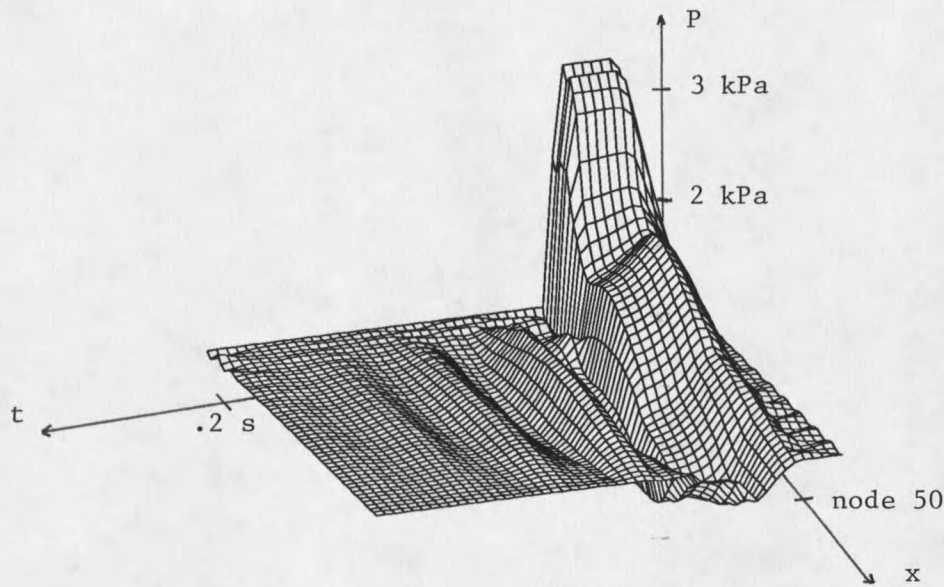


Figure 16
Flow Shut Off Transient Pressure Profile in Time and Axial Distance

The velocity profile corresponding to the flow shut off transient is shown in Figure 17. In axial position along the MHD channel, one can see that as the channel is turned on, velocity in the nozzle becomes great, decreases somewhat through the generation portion of the duct where power is being extracted, again increases throughout the transition section of the channel, and finally decreases toward the end of the diffuser. As the steady state profile is approached, the velocity in the nozzle decreases and the velocity in the transition section decreases. When the channel is rapidly ramped off, the plasma flows toward both the diffuser and combustor for a short time. This is shown by the negative trough in the profile immediately after the ramping begins. The oscillations which occur as the system again approaches constant zero velocity represent periods of time for which gas flows into the channel from both ends, then out both ends, etc. This graph helps explain the reason that the oscillation period is two duct transition intervals--the wavelength of each variable's variation seems to be two channel lengths.

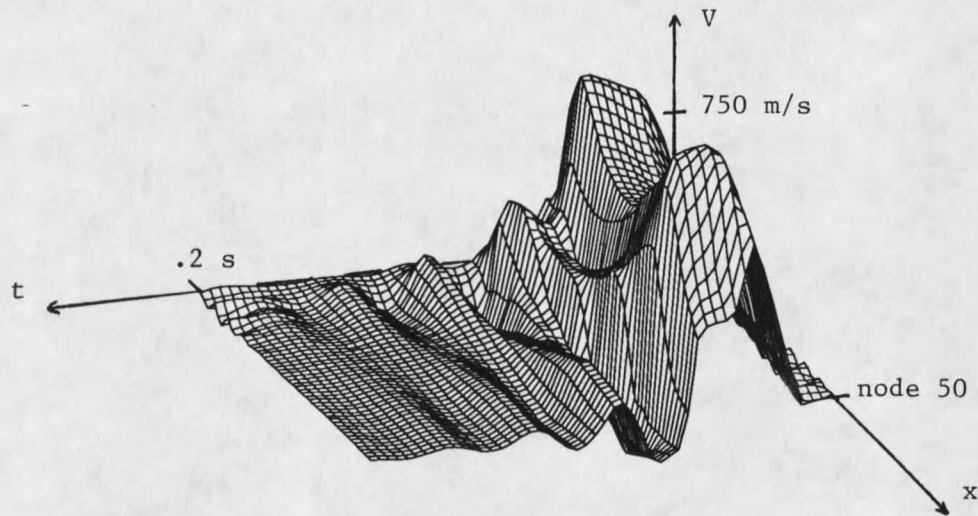


Figure 17
Flow Shut Off Transient Velocity Profile in Time and Axial Distance

Figure 18 shows the effect of the flow shut off transient upon the electrical portion of the duct. Only the Faraday current is shown, but for a constant resistive loading in a strictly Faraday channel, Hall field is proportional to the Faraday current, and Hall current is zero. Therefore, only Faraday current is needed to see most of the electrical interactions in the channel model even though a number of additional electrical variables are available if desired. Interactions will be more complex for other electrode configurations. Only the generator channel portion of the duct, nodes 12 through 29 in this example, are shown. Notice that the load current profile corresponds to the velocity profile in Figure 17. As the velocity increases, load current increases. As velocity decreases or goes negative, load current also decreases or changes sign.

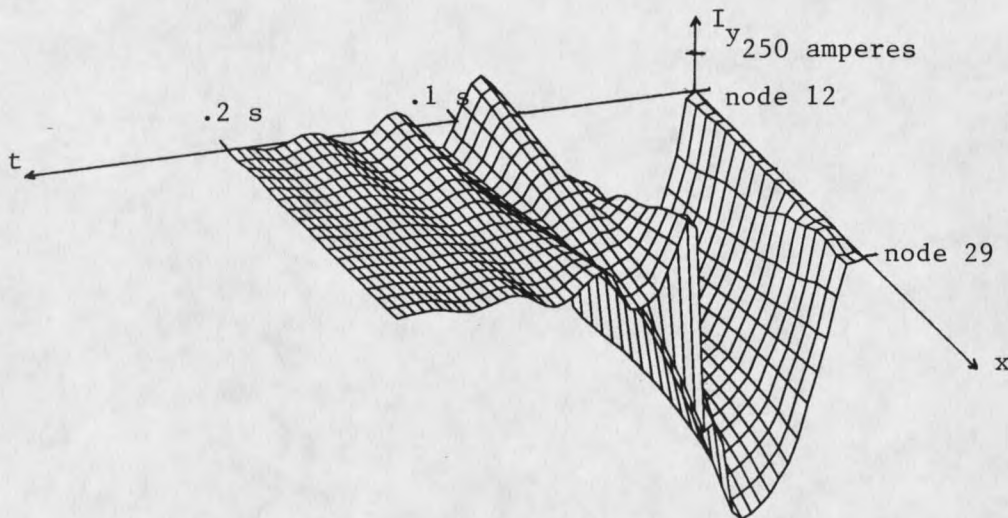


Figure 18
Flow Shut Off Transient Faraday Current Profile in Time and Axial Distance

Electrical Load Loss transient

The electrical power takeoff from the MHD channel can bring about transients in the entire MHD duct just as fluid transients can affect the electrical power takeoff from the channel. The electrical portion of the duct model was ramped off and on as the input pressure had been in the previous example. Starting from the active steady state for constant 0.5 ohm resistive load across each Faraday electrode pair, the electrical portion of the duct was ramped completely off in 25 ms, held off for 25 additional ms, and ramped quickly back on in 5 ms. Since the model is still a simple, uniform load Faraday channel, the electrical load loss could have been equivalently modeled by either ramping to infinite impedance or by turning off the magnetic interaction. Each method should give about the same fluid dynamic behavior. The first method is truer to the physical system, but the second method was chosen because of its inherent simplicity.

The Faraday current transient is shown in Figure 19. Again the figure is limited to the nodes in the active electrical generation portion of the MHD duct, and the Faraday current is chosen as a representative example of electrical interactions for the simple Faraday generator. The electrical response is shown to be immediate because of the intimate relation between load current and magnetic field. One might think of this example as a simplified version of a recloser action near the MHD facility. The current reaches exactly zero after being ramped off from its active steady state (a current in the negative $\vec{y}$ direction). It is interesting that the end of the generator channel near the diffuser reacts more quickly to the return of electrical generation than does the front (toward the nozzle). This statement may be made from the evidence of the diagonal time line on the figure face as electrical generation is being ramped back on. The diffuser end of the channel very quickly reaches its steady state value, then overshoots it. The reason for this behavior will be obvious after looking at the velocity profile.

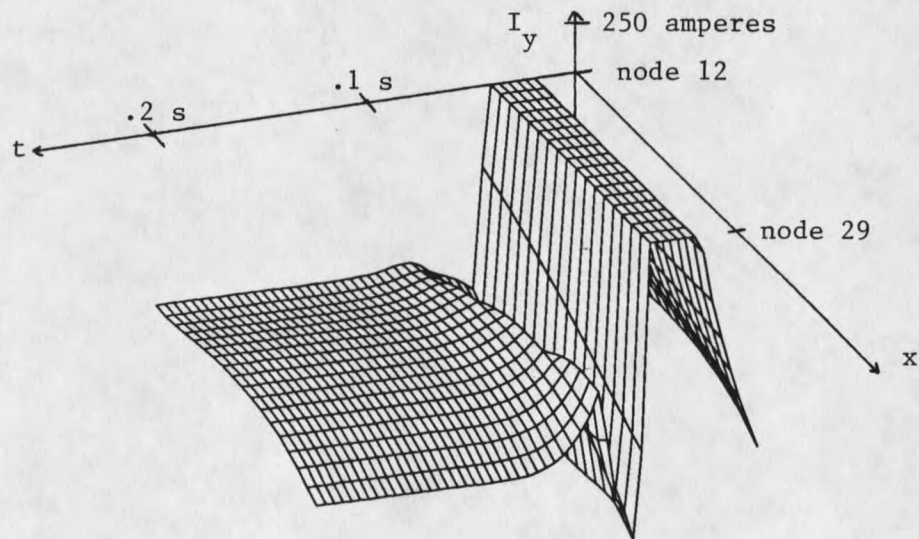


Figure 19
Load Loss Transient Faraday Current Profile in Time and Axial Distance

Figure 20 shows part of the fluid response to the electrical transient. The pressure profile decreases as the electrical portion of the model is turned off. Every node beyond the nozzle opening seems to be affected, but the effect upstream into the nozzle appears to be minimal. This decrease is an expected result since the presence of a Faraday current in the generator brings about a back pressure in the negative axial direction. Upon loss of load, therefore, the channel fails to provide this back pressure and the pressure profile decreases within the generator portion of the duct, especially toward the nozzle end of the generator channel. The great decrease in pressure at the diffuser exit is a result of temporary supersonic duct behavior, which will be better demonstrated in the velocity profile. Some overshoot occurs as the load is switched back on, but oscillations are not as obvious for the generating channel as they were for the nongenerating channel.

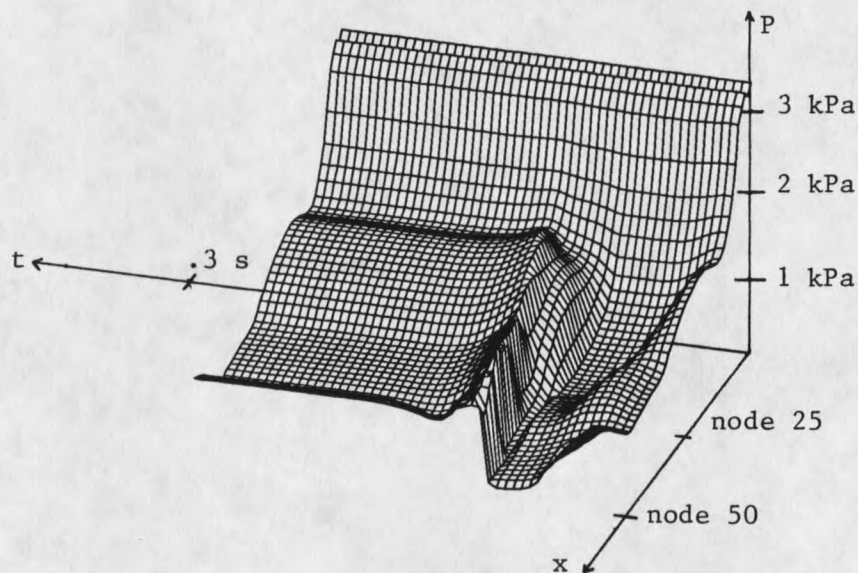


Figure 20
Load Loss Transient Pressure Profile in Time and Axial Distance

Perhaps the velocity profile for the load loss transient is most interesting. Figure 21 shows that the loss of electrical load causes the channel to become supersonic from the nozzle to the diffuser exit. As expected for a supersonic fluid, the plasma accelerates despite the expansion of the duct through the generation and diffuser portions. The highest velocity exceeds Mach two. This increase in velocity down the channel length helps explain why the end of the generator responds so quickly to load recovery--the gases at the end of the generator have considerably greater momentum than those at the front.

As the electrical load is once again established, a shock front develops and retreats from the diffuser to the nozzle. These shock fronts can be seen as the set of wave-shaped velocity lines in the latter third of the channel as the load is ramped back on. One can also see the quick velocity decrease resulting from the return of the Faraday current back force in the active generator portion of the model.

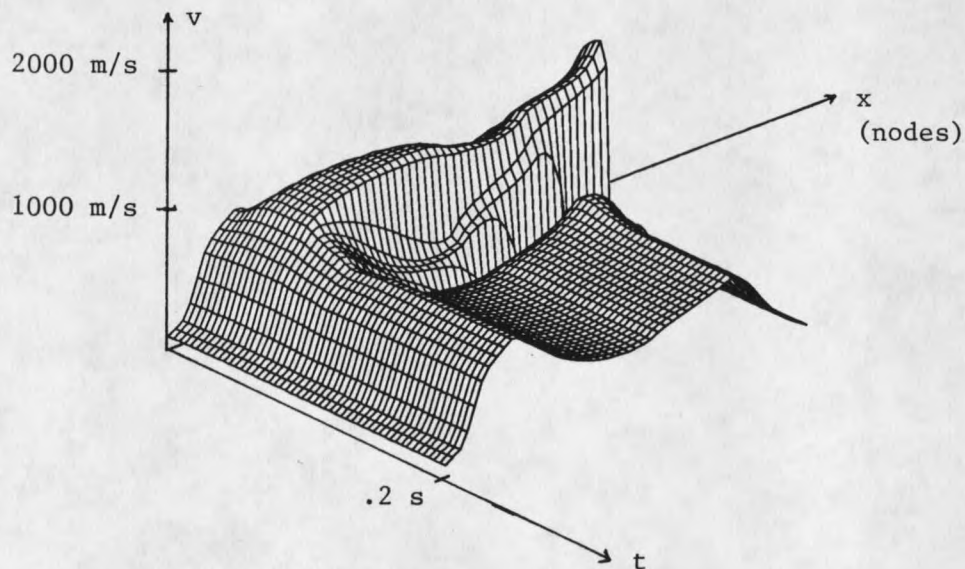


Figure 21
Load Loss Transient Velocity Profile in Time and Axial Distance

Shorted Electrode Pair Transient

One hopes that electrical transients will not be as severe as the load loss transient just presented. Many electrical transients may affect only one electrode pair. Therefore, a single Faraday electrode pair short circuit was investigated by shorting out the Faraday electrode pair at node eighteen of the model, a node about one third of the distance down the generator channel. The short circuit was ramped on and off in the same time scale of the previous two transients.

The immediate result of a short circuit is very large current flows for the shorted electrode pair. Figure 22 shows how the node eighteen current becomes very large when the short circuit occurs. The only impedance to current flow is the internal Faraday resistance of the generator channel node. This internal resistance is relatively small. The upstream electrodes seem to have their currents decreased, but there is surprisingly little effect on the downstream nodes.

Figures 23 and 24 show the pressure and velocity profiles respectively due to the single electrode pair short circuit transient. In Figure 23 a sharp pressure gradient is shown at the shorted node. This increased pressure results from the great Hall back pressure from the excessive Faraday short circuit current. Once again it is observed that the major interaction occurs upstream to the short, and little variation is seen downstream. Figure 24 shows an analogous result in the velocity. The back pressure slows fluid flow upstream to the short, and the short affects the gas velocity even into the nozzle. Some variation is seen downstream of the short, but this deviation is relatively small and seems to occur only immediately after the transient switching has occurred.

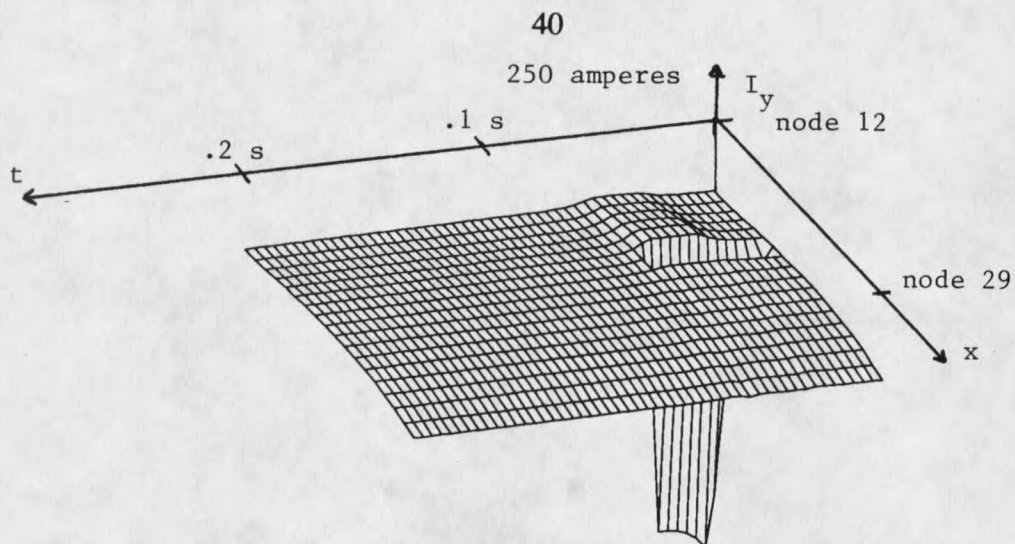


Figure 22
Node 18 Short Circuit Transient Faraday Current Profile

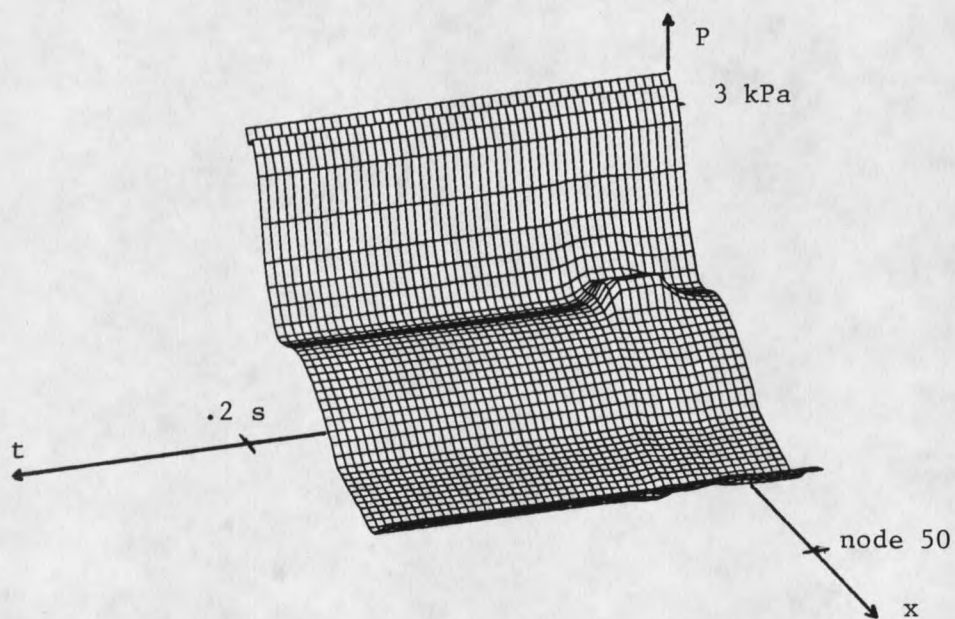


Figure 23
Node 18 Short Circuit Transient Pressure Profile

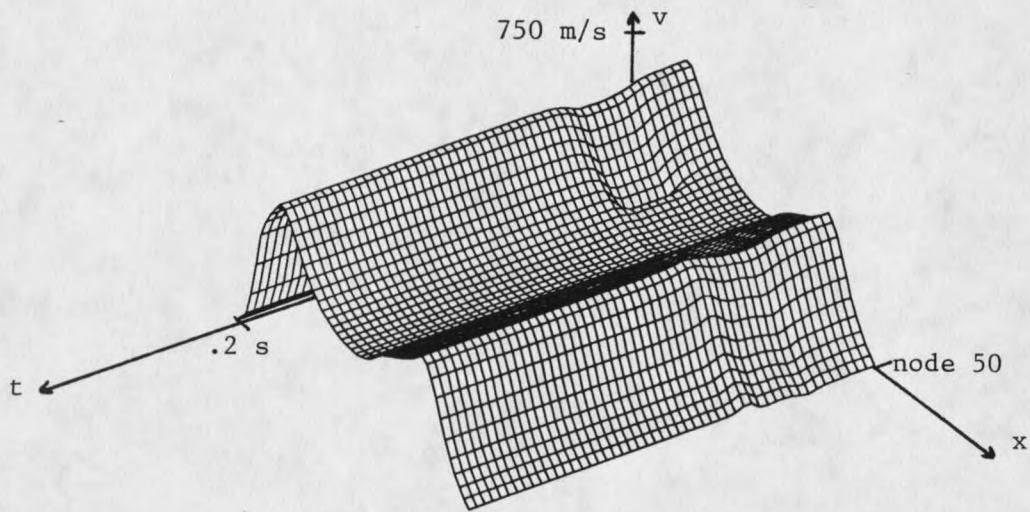


Figure 24
Node 18 Short Circuit Transient Velocity Profile

Open Electrode Pair Transient

The open circuit electrode pair is of great interest because an opened Faraday electrode pair is likely to induce arcing between adjacent electrodes. Figures 25-27 show the response of the MHD channel to an opened electrode pair on node eighteen. The results may be readily compared to those of the short circuit on node eighteen.

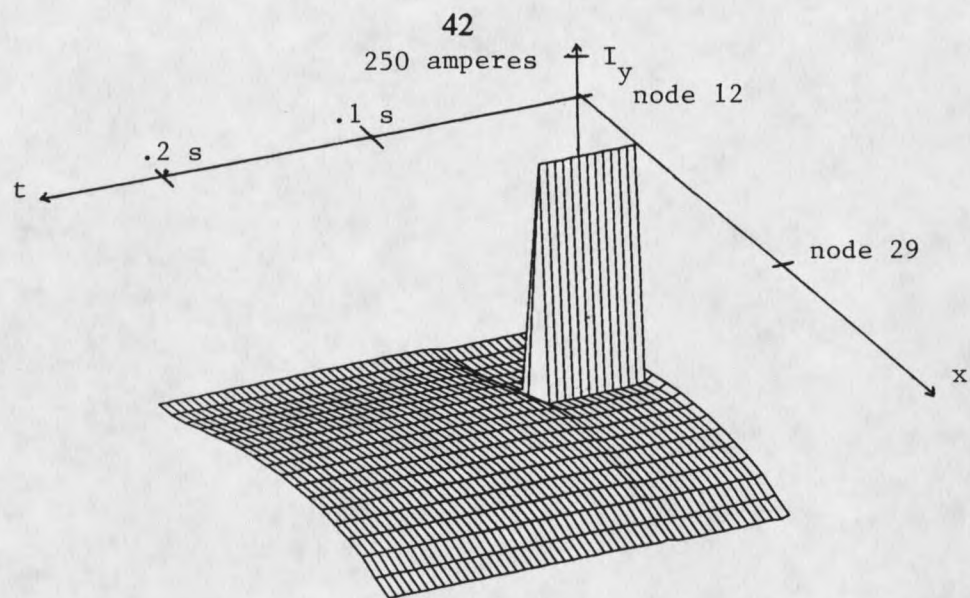


Figure 25
Node 18 Open Circuit Transient Faraday Current Profile

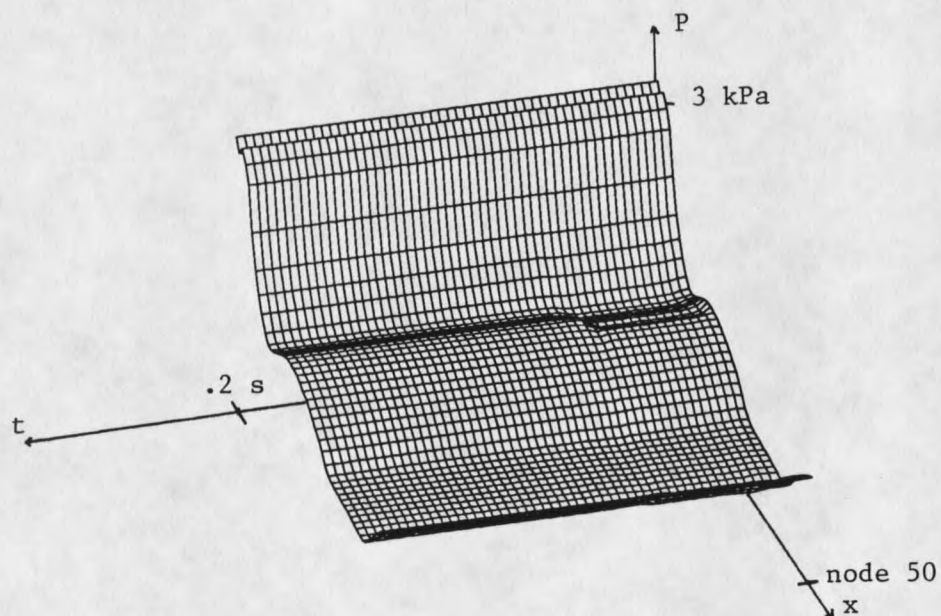


Figure 26
Node 18 Open Circuit Transient Pressure Profile

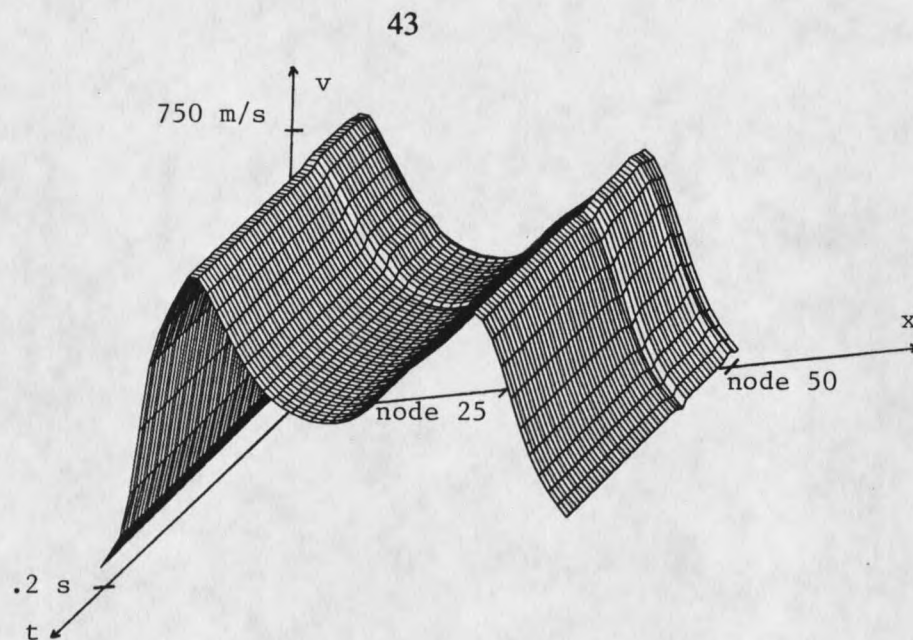


Figure 27
Node 18 Open Circuit Transient Velocity Profile

A Switching Transient Demonstration

Pulse Amplitude Synthesis and Control (PASC)^{3,4} is a power consolidation scheme by which unlike DC sources may be synthesized into any reasonable wave shape. This method was originally investigated at Montana State University for high frequency strategic defense initiative (SDI) applications, but it may be useful in battery array storage and commercial MHD power generation. It is static, dense and requires minimal power conditioning. A simple single phase application was investigated to exemplify extreme switching transients. A multiphase system would not have the extreme switching transients present in the single phase system because electrode pairs would have a nearly continuous duty cycle for three phase applications.

The PASC scheme requires that the pulses from each electrode pair be added inductively through a set of multiple primary transformers. The switching sequence determines the final wave form. The triangle wave form of Figure 28 was generated by

the switching of 18 electrode pairs as shown. The switching was modeled by simple on-off logic through a transformer. At this time resolution, the transformer model is of little consequence. The resulting load current is shown in Figure 29. The deep chasms in the load current profile represent the periods during which the electrodes are on. Each electrode is on about one fourth of the total period. The triangle wave is slightly evident in the figure, but the time resolution of the graph is insufficient to show the exact on-off switching sequence.

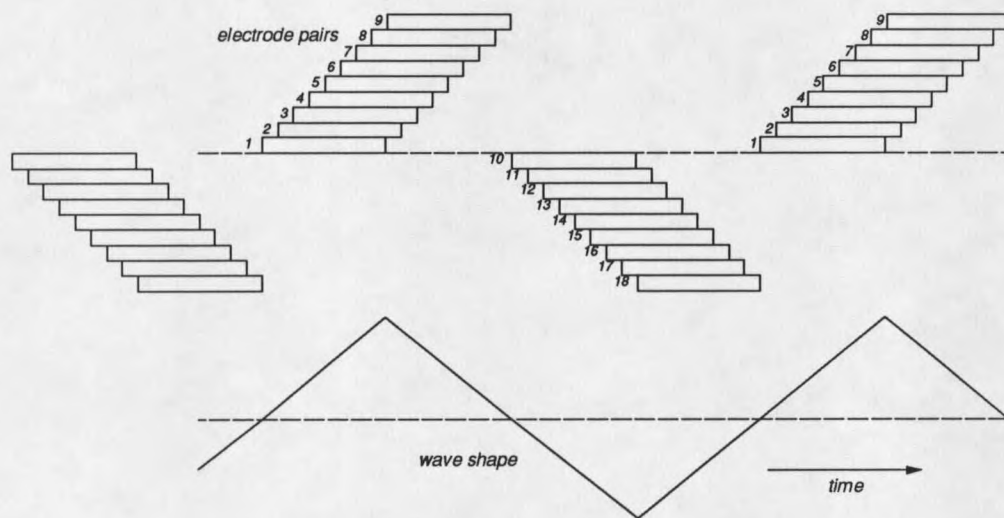


Figure 28
PASC Electrode Consolidation Switching Scheme for Single Phase Triangle Wave Generation

The loading from the single phase PASC power takeoff system is insufficient to prevent the channel from becoming transonic, and the velocity profile of Figure 30 shows the diffuser of the channel wagging back and forth between subsonic and supersonic behavior. Such extreme variations would most likely be unacceptable for any practical MHD channel, but the example demonstrates the flexibility of the simulation to model

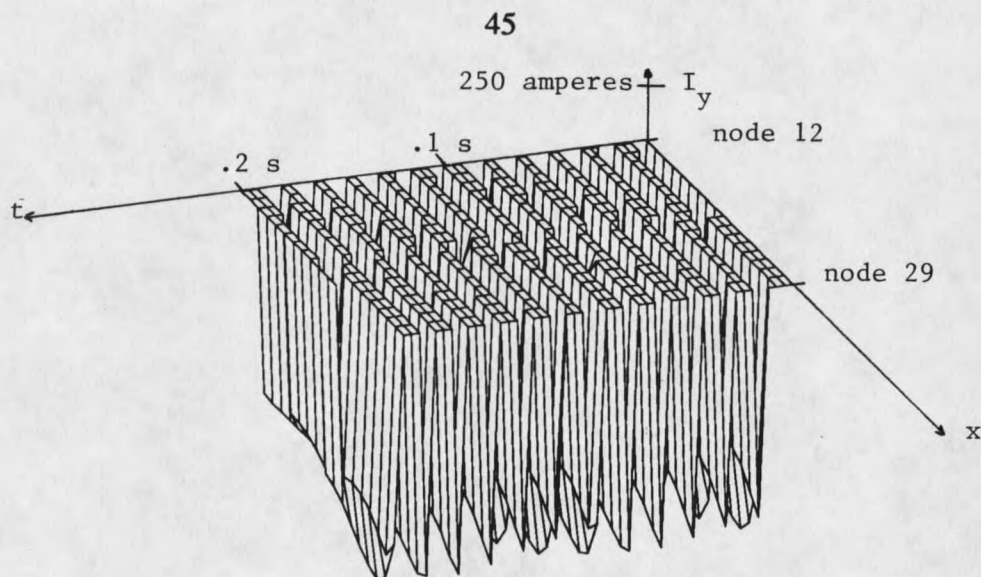


Figure 29
Single Phase Switching Transients Faraday Current Profile for PASC Power Takeoff

fairly complex switching transients. The next step in this example would be to model a three phase power takeoff, a more likely loading configuration for commercial power frequencies.

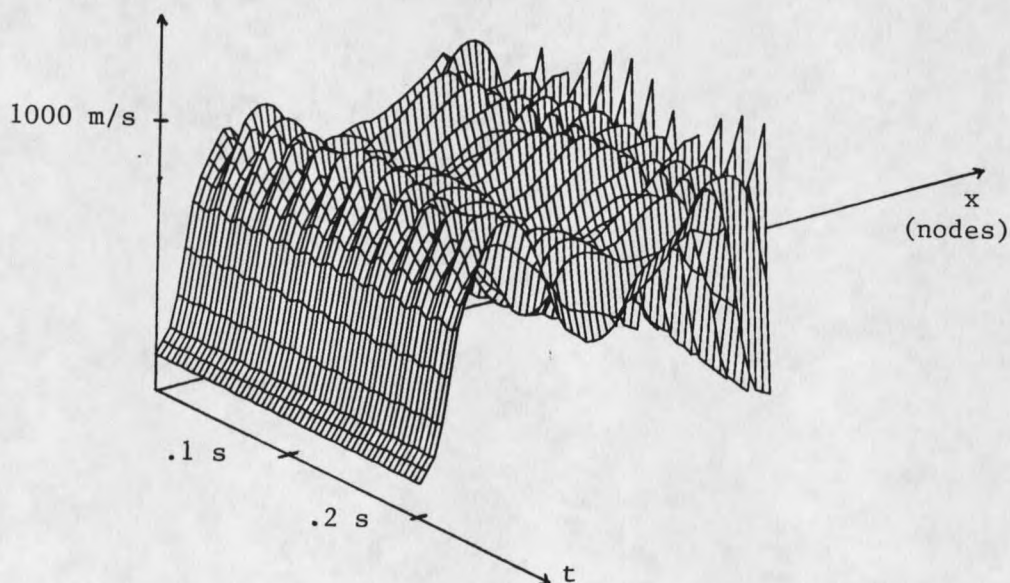


Figure 30
Single Phase Switching Transients Velocity Profile for PASC Power Takeoff

SUGGESTIONS FOR FUTURE WORK

(1) Hall interactions have not yet been adequately investigated. Faraday connections were chosen in this first battery of demonstrations due to the simplicity of Faraday configurations. There is no reason to doubt that Hall and diagonal interactions may also be investigated with this model.

(2) The fluid and electrical dynamic data obtained from this model should eventually be compared to similar data collected for test channel's entire length during transient behavior. This comparison is an essential step in confirming the model's accuracy and limitations.

(3) An ideal gas assumption is inadequate for accurate plasma modeling. The model formulation and computer implementation provides the flexibility, however, for one to improve model accuracy by including curve fits, standard look up tables, or any other acceptable property definitions.⁸

(4) The model must be made more flexible for, and friendly to the user. The number of spatial steps, the channel dimensions, the magnetic field distribution, and perhaps the electrode connections should be made easily changeable.

(5) The model has shown some ability to model switching transients like those needed for PASC power takeoff or load sharing algorithms. There is a plethora of control investigations which warrant investigation including, but not limited to PASC, optimization algorithms and protection or contingency control schemes.

(6) Novel loading configurations--perhaps continued interest in direct AC generation--may be cheaply investigated using this model.

(7) The model may be made more complete and accurate by modeling of the combustor and combustor inputs, slag layers, seeding effects, *etc.*

(8) The model may be augmented to a two or three dimensional model. Two and three dimensional models permit investigation of Faraday pressure gradients, generated eddy currents, plasma induced magnetic fluctuations, oblique shocks, and more. Note that the presentation of dynamic data becomes significantly more difficult as the model dimension is increased. Presentation of data from even this one dimensional model has required three-dimensional graphics.

CONCLUSIONS

(1) The Lax-Wendroff MHD fluid model with schematic viscosity stabilization has been effectively paired with a circuit description of the generator portion of an MHD generator. The main advantages to the formulation appear to be comparatively fast integration speed, avoidance of shock fitting techniques, and flexibility in electrical modeling. The circuit model provides insight into electrical behavior and allows one to use the wealth of knowledge available in circuit theory.

(2) The circuit models for individual electrode pairs may be combined to simulate Faraday, Hall, or diagonal configurations by appropriate interconnection of electrode terminals. Electrode pairs may be modeled by any integer multiple of circuit models. This thesis has considered only simple Faraday electrode configurations, but the extension to other configurations should be straightforward.

(3) The FORTRAN programs used for this simulation model the nozzle, channel, and diffuser sections of an MHD generator duct. The computer code may be readily augmented to include combustor, slag layers, and other components. Model accuracy may be improved by curve fitting techniques and choice of a more accurate specifying equation. At this stage, dynamic behavior has been stressed more than absolute accuracy.

(4) The dynamic fluid behavior of the model has been verified by replication of a shock tube simulation first carried out by Ostling. The dynamic electrical behavior of the model is not so easily verified and is only supported by a number of reasonable transient examples.

(5) Examples of several transients have been presented. Such transients may originate in either the fluid or the electrical portions of the model; the demonstrated interactions between fluid dynamics and electrical generation are dynamic and reasonable.

LITERATURE CITED

- [1] Brogan, T., J. Hill and A. Aframe. "A Preliminary Design of a Single Output Variable Wall Angle Windowframe Diagonal MHD Channel for the U-25 Power Plant." Sixth International Conference on Magnetohydrodynamic Power Generation, Washington, DC: 9-13 June 1975, pp. 267-85.
- [2] Fritzer, P., L. Lengyel, and K. Witte. "Self Consistent Calculation of the Gasdynamic and Electrical Properties of a Two Dimensional MHD Flow." Argonne, IL: 12th Symposium Engineering Aspects of Magnetohydrodynamics, 27-29 March 1972, III.7.1-8.
- [3] Johnson, R., K. Marcotte, and M. Donnelly. "A Feasibility Demonstration Experimental Facility Simulation of a PASC Consolidation/Inversion System for Faraday Connected MHD Generators." Pittsburgh, PA: Final Technical Progress Report, Contract DE-AC22-87PC79680, U.S. DOE/PETC, 1990.
- [4] Johnson, R., And V. Skurdal. "Pulse Amplitude Synthesis and Control (PASC) of MHD Commercial Power Output." Proceedings of the 24th Symposium on Engineering Aspects of Magnetohydrodynamics (SEAM). Butte, MT: 24-27 June 1986.
- [5] Matair, S. Interaction of an MHD Generator with a Line-commutated Inverter System. Sydney, Australia: Diss. Univ. Of Sydney, March 1985.
- [6] MATLAB. The Math Works, Inc., South Natick, MA.
- [7] Ostling, A. R. "Stable Formulation and Solution of the Transient MHD Flow Equations." Butte, MT: Montana Energy and MHD Research and Development Institute, c. 1980.
- [8] Pierre, D., C. Dudding, and D. Miller. "MHD Combustor/Nozzle/Channel/Diffuser Simulation and Input-Output Modeling." Bozeman, MT: Montana State Univ. ERL Report No. 1276, 1976.
- [9] Roache P. J. Computational Fluid Dynamics. Albuquerque, NM: Hermosa Publishers, 1972, Chapter V.
- [10] Rosa, R. J. Magnetohydrodynamic Energy Conversion. New York, NY: McGraw Hill, 1968.
- [11] Saad, M. A. Compressible Fluid Flow. Englewood Cliffs, NJ: Prentice-Hall, 1985.

- [12] Shapiro, A. H. The Dynamics and Thermodynamics of Compressible Fluid Flow. New York, NY: Ronald Press, 1954, p. 1027.
- [13] Trung, D. T. System Analysis of the Integration of a Segmented MHD Generator into an AC Power Network. Sydney, Australia: Diss. Univ. Of Sydney, January 1981.
- [14] VanBibber, L., D. Wiseman, H. Muenchow, R. Holman, and F. Bernard.
"Conceptual Design of a Coal-fired MHD Retrofit Plant." Pittsburgh, PA:
Quarterly Report, Contract DE-AC22-87PC79668, U. S. DOE/PETC,
February 1989.

Figure 31
FORTRAN Program Listing

```

C EQUIVALENT CIRCUIT AND LOAD PARAMETERS:
C
C FARADAY LOAD CURRENTS: (AMP)
C X(401) TO X(450)
C HALL GAS CURRENTS: (AMP)
C X(451) TO X(500)
C LOAD RESISTANCES: (OHM)
C X(501) TO X(550)
C CONDUCTIVITIES (SIEMENS)
C X(551) TO X(600)
C HALL VOLTAGES: (VOLT)
C X(601) TO X(650)
C COMPLETE ELECTRIC FIELD IN Y DIRECTION: (V / M)
C X(651) TO X(700)
C COMPLETE ELECTRIC FIELD IN THE X DIRECTION: (V / M)
C X(701) TO X(750)
C
C
C SUBROUTINE STATE(X,DX,T,DT,N)
C REAL*8 X(750),DX(750),T,DT,AIM,UNUSED1,UNUSED2,C,B(150,150),
C 2 S,BMAG,KF,KQ,W,BETA,TW,CV,R,MAGNET,MACH,GAMMA
C 3 DIFFPRESSURE,INTEMPERATURE,INFLOWRATE,INPRESSURE
C INTEGER I,J
C DATA
C 2 KF/5.D-3/,KQ/5.D4/,W/0.49D0/,BETA/2.D0/,TW/2000.D0/,
C 3 R/287.D0/,CV/716.5D0/,DIFFPRESSURE/1.02D5/,BMAG/2.D0/
C 4 ,S/0.30D0/,INFLOWRATE/10.D0/,
C 5 INAREA/34519D0/,GAMMA/1.4D0/
C SOME OF THE DATA IN THE ABOVE LIST CONSTITUTE BOUNDARY CONDITIONS, AND
C OTHERS CONSTITUTE COMPRESSIBLE PLASMA CHARACTERISTICS. THE BOUNDARY
C CONDITIONS AND PLASMA CHARACTERISTICS ARE ASSUMED CONSTANT FOR THE INITIAL
C CAPABILITIES TESTING; HOWEVER, TIME DEPENDENCE AND LOOK-UP TABLES MAY BE
C LATER INCLUDED TO BETTER MODEL SPECIFIC CHANNEL AND PLASMA BEHAVIORS.
C THE FOLLOWING COMMON STATEMENT IS PROVIDED TO SATISFY THE "INTEG"
C INTEGRATION PROGRAM. GENERALLY, THE COMMON STATEMENT IS UNUSED.
C COMMON/ABC/UNUSED1(1,1),AIM(1,1),UNUSED2(1,1),C(1,1)
C INITIALIZE THE TIME DERIVATIVES. THE FIRST AND LAST NODE TIME
C DERIVATIVES CANNOT BE CALCULATED BECAUSE NO KNOWN DOWNSTREAM/UPSTREAM
C CONDITIONS EXIST.
C DO 5,I=1,150
C 5 DX(I)=0.D0
C ASSIGN ALL B MATRIX VALUES FOR DENSITY CONSERVATION VARIABLES. THE
C FIRST AND LAST NODES ARE NOT CALCULATED. THE "B" MATRIX IS MULTIPLIED BY THE
C CONSERVATION VARIABLE VECTOR TO DETERMINE THE TIME DERIVATIVE VECTOR.
C
C DO 10,I=2,49
C DENSITY CONSERVATION CALCULATIONS.
C B(I,I-1)=(DABS(X(150+I-1))-DABS(X(150+I+1)))+
C 2 4.D0*DABS(X(150+I)))/(8.D0*S)
C B(I,I)=(-DABS(X(150+I)))/S
C B(I,I+1)=(DABS(X(150+I+1))-DABS(X(150+I-1)))+
C 2 4.D0*DABS(X(150+I)))/(8.D0*S)
C B(I,I+50-1)=0.5D0/(S*X(300+I))
C B(I,I+50+1)=-0.5D0/(S*X(300+I))
C MASS FLOWRATE CONSERVATION CALCULATIONS.
C B(I+50,I)=-KF*X(150+I)**2
C 2 +(X(300+I)*DABS(X(150+I))*(X(150+I+1)+X(150+I-1)
C 3 -2.D0*X(150+I)))/(2.D0*S)
C B(I+50,I+50-1)=(DABS(X(150+I-1))-DABS(X(150+I+1))
C 2 +4.D0*DABS(X(150+I))+4.D0*X(150+I-1))/(8.D0*S)
C B(I+50,I+50)=-DABS(X(150+I))/S
C B(I+50,I+50+1)=(DABS(X(150+I+1))-DABS(X(150+I-1))
C 2 +4.D0*DABS(X(150+I))-4.D0*X(150+I-1))/(8.D0*S)
C ASSIGN ALL B MATRIX VALUES FOR DENSITY*ENERGY CONSERVATION VARIABLES. . .
C OR ALTERNATE FORMULATION.
C B(I+100,I+50+1)=-(X(100+I+1)+X(250+I+1))/
C 2 (2.D0*S*X(300+I)*X(I+1))
C B(I+100,I+50-1)=(X(100+I-1)+X(250+I-1))/
C 2 (2.D0*S*X(300+I)*X(I-1))

```

Figure 31
FORTRAN Program Listing (continued)


```

      B(I+100,I+100-1)=(DABS(X(150+I-1))-DABS(X(150+I+1))
2      +4.D0*DABS(X(150+I)))/(8.D0*S)
      B(I+100,I+100)=DABS(X(150+I))/S
      B(I+100,I+100+1)=(DABS(X(150+I+1))-DABS(X(150+I-1))
2      +4.D0*DABS(X(150+I)))/(8.D0*S)
10     CONTINUE
C   THE B MATRIX IS USED TO CALCULATE TIME PARTIALS AS SHOWN HERE. THE B
C   MATRIX IS VERY SPARSE, SO THE MULTIPLICATION IS DONE IN PARTS TO REDUCE CPU
C   TIME.
      DO 13,J=2,49
      DO 13,I=J-1,J+1
      DX(J)=DX(J)+B(J,I)*X(I)
      DX(J)=DX(J)+B(J,50+I)*X(50+I)
      DX(J+50)=DX(J+50)+B(J+50,I)*X(I)
      DX(J+50)=DX(J+50)+B(J+50,50+I)*X(50+I)
      DX(J+100)=DX(J+100)+B(J+100,50+I)*X(50+I)
      DX(J+100)=DX(J+100)+B(J+100,100+I)*X(100+I)
13     CONTINUE
C   TAKE CARE OF THE SOURCE FUNCTION VECTORS FOR THE STATE EQUATIONS.
C   NOTE THAT THE DENSITY CONSERVATION EQUATIONS HAVE NO SOURCE VECTORS.
C   THIS MEANS THAT THE EQUATIONS TWO THROUGH FORTY-NINE ARE FULLY DESCRIBED
C   AT THIS TIME. THE MASS FLOWRATE CONSERVATION EQUATIONS DO HAVE SOURCE
C   VECTORS. LET THE CURRENT DENSITY BE DEFINED IN TERMS OF THE FARADAY AND
C   HALL CURRENT. THE HALL CURRENT WILL BE ZERO FOR A FARADAY CONFIGURATION, BUT
C   IT IS INCLUDED SO DIAGONAL AND HALL CONFIGURATIONS MAY BE LATER INVESTIGATED.
C   CARE IS TAKEN THAT THE MAGNETIC FIELD DOES NOT APPLY TO ANY NODES BEYOND THE
C   GENERATOR. THE MAGNETIC FIELD MAY BE BETTER MODELLED LATER TO REPRESENT A
C   TRUE DISTRIBUTION.
      DO 15,I=2,49
      IF(I.LT.12.OR.I.GT.29) THEN
      MAGNET=0.D0
      ELSE
      MAGNET=BMAG
      END IF
      DX(I+50)=DX(I+50)+(X(300+I)*MAGNET*X(400+I)/S)/W
2      -X(300+I)*(X(250+I+1)-X(250+I-1))/(2.D0*S)
C   THE DENSITY*ENERGY PRODUCT CONSERVATION EQUATIONS ALSO HAVE SOURCE
C   VECTORS. BETA IS DEFINED AS THE HALL PARAMETER. TW IS DEFINED AS THE WALL
C   TEMPERATURE.
      DX(I+100)=DX(I+100)
2      +X(450+I)*X(700+I)/X(300+I)+X(400+I)*X(650+I)/(S*W)
3      -KQ*(X(200+I)-TW)
15     CONTINUE
C   THIS IS SUFFICIENT TO COMPLETELY DETERMINE THE CONSERVATION VARIABLES
C   AND THEIR TIME DERIVATIVES FOR THE NOZZLE, CHANNEL AND THE DIFFUSER.
C   THE FIRST AND LAST NODE CONSERVATION VARIABLES OR TIME DERIVATIVES ARE TAKEN
C   CARE OF BY THE BOUNDARY CONDITIONS IMPOSED IN THE "SIDE" SUBROUTINE.
C   AS PART OF THE EXIT BOUNDARY CONDITIONS, DENSITY*ENERGY PRODUCT IS ALLOWED
C   TO VARY AS ITS TIME DIFFERENTIAL IS EXTRAPOLATED FROM THE LAST NODES.
      DX(150)=2.D0*DX(149)-DX(148)
C   RETURN TO THE INTEGRATION PROGRAM TO CALCULATE THE CONSERVATION VARIABLES
C   AFTER THIS TIME STEP.
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C   SUBROUTINE: "SIDE"
C
C   DATE: AUGUST 1991
C
C   PROGRAMMER: DONALD HAMMERSTROM, GRADUATE STUDENT
C   DEPARTMENT OF ELECTRICAL ENGINEERING
C   MONTANA STATE UNIVERSITY
C   BOZEMAN, MONTANA
C
C   SUPERVISOR: WORK COMPLETED UNDER THE DIRECTION OF DR. ROY JOHNSON.
C
C   PURPOSE: THIS SUBROUTINE IS TO BE USED IN CONJUNCTION WITH "STATE"
C   TO CALCULATE THE RELATED PHYSICAL AND ELECTRICAL STATES.
C
C   DEFINITIONS: CP AND CV ARE THE HEAT CAPACITIES OF THE GAS, R IS THE
C   GAS CONSTANT, KG1 AND KG2 ARE THE CONSTANTS USED IN
C   DETERMINING CONDUCTIVITY, W IS THE CHANNEL WIDTH, S IS THE

```

Figure 31
FORTRAN Program Listing (continued)

```
C      DISTANCE BETWEEN ELECTRODE PAIRS. ADDITIONAL VARIABLE
C      DEFINITIONS MAY BE FOUND IN "STATE."
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

SUBROUTINE SIDE(X,DX,T,DT,N)
REAL*8X(1),DX(1),T,DT,AIM,UNUSED1,UNUSED2,C,DENOM
CP,CV,R,KG1,KG2,DIFFPRESSURE,BMAG,BETA,W,S,
INTEMPERATURE,INFLOWRATE,MAGNET,G(56),MACH,GAMMA,
INPRESSURE
INTEGER I,J
DATA CV/716.5D0/,CP/1003.5D0/,R/287.D0/,
KG1/11581.D0/,KG2/601300.D0/,
DIFFPRESSURE/1.02D5/BMAG/2.D0/,BETA/2.D0/,W/0.49D0/,
S/0.30D0/,INFLOWRATE/20.D0/,
INAREA/34519D0/GAMMA/1.4D0/
C THE COMMON STATEMENT IS REQUIRED BY THE "INTEG" INTEGRATION PROGRAM.
C GENERALLY, THE COMMON STATEMENT IS NOT USED.
COMMON/ABC/UNUSED1(1,1),AIM(1,1),UNUSE2(1,1),C(1,1)
C THERE ARE TWO ACCEPTABLE INPUT BOUNDARY CONDITION SETS FOR THE FIRST NODE:
C COMMENT OUT ONE OF THE FORMULATIONS.
C FORMULATION #1: STAGNATION COMBUSTOR TEMPERATURE AND
C PRESSURE IS ASSIGNED. VELOCITY IS EXTRAPOLATED TO FIRST NODE. THE THREE
C CONSERVATION VARIABLES, DENSITY, MASS FLOWRATE AND ENERGY*DENSITY PRODUCT,
C ARE CALCULATED.
    INTemperature=2500.D0
    INPressure=320.D3
C   EXTRAPOLATE VELOCITY TO THE FIRST NODE (FIRST OR SECOND ORDER)
    X(151)=2.D0*X(152)-X(153)
C   X(151)=(10.D0*X(152)-5.D0*X(153)+X(154))/6.D0
C   CALCULATE THE STATIC INPUT TEMPERATURE FROM THE GIVEN STAGNATION INPUT
C TEMPERATURE
    X(201)=INTemperature-X(151)**2/(2.D0*CP)
C CALCULATE THE STATIC INPUT PRESSURE FROM THE GIVEN STAGNATION INPUT PRESSURE
    MACH=DABS(X(151)**2/(GAMMA*R*X(201)))
    X(251)=DEXP(DLOG(INPressure)-DLOG(1.D0+(GAMMA-1)*MACH**2))
C   *MACH/2.D0)/(GAMMA/(GAMMA-1.D0)))
C   CALCULATE THE INPUT DENSITY FROM KNOWN PRESSURE AND TEMPERATURE
    X(1)=X(251)/(R*X(201))
C   CALCULATE INPUT MASS FLOWRATE FROM DENSITY AND VELOCITY
    X(51)=X(1)*X(151)*X(301)
C   CALCULATE THE INPUT ENERGY*DENSITY FROM TEMPERATURE AND VELOCITY AND
C DENSITY
    X(101)=X(1)*(X(201)*CV+X(151)**2/2.D0)
C ALTERNATE FORMULATION #2: FLOWRATE AND COMBUSTOR STAGNATION TEMPERATURE
C ARE ASSIGNED. VELOCITY IS EXTRAPOLATED FROM NEAR NODES. SOLVE FOR DENSITY,
C PRESSURE AND ENERGY*DENSITY PRODUCT RESPECTIVELY.
C ASSIGN MASS FLOWRATE AND STAGNATION TEMPERATURE.
    X(51)=INFLOWRATE
    INTemperature=2800.D0
C   EXTRAPOLATE THE VELOCITY OF THE FIRST NODE. (FIRST OR SECOND ORDER)
    X(151)=2.D0*X(152)-X(153)
C   X(151)=(10.D0*X(152)-5.D0*X(153)+X(154))/6.D0
C   CALCULATE THE INPUT DENSITY FROM THE MASS FLOWRATE AND VELOCITY.
    IF(X(151).LE.0.D0) X(151)=1.E-6
    X(1)=X(51)/(X(301)*X(151))
C   CALCULATE THE STATIC INPUT TEMPERATURE FROM THE GIVEN STAGNATION INPUT
C TEMPERATURE
    X(201)=INTemperature-X(151)**2/(2.D0*CP)
C   CALCULATE THE INPUT PRESSURE FROM THE TEMPERATURE AND DENSITY.
    X(251)=X(1)*R*X(201)
C   CALCULATE THE INPUT ENERGY*DENSITY FROM TEMPERATURE AND VELOCITY AND
C DENSITY
    X(101)=X(1)*(X(201)*CV+X(151)**2/2.D0)
C THE EXIT BOUNDARY CONDITIONS ARE CALCULATED AS FOLLOWS:
C THE DENSITY AND MASS FLOWRATE AT THE EXIT ARE EXTRAPOLATED FROM THE NEAR
C NODES. (FIRST OR SECOND ORDER)
    X(50)=2.D0*X(49)-X(48)
    X(100)=2.D0*X(99)-X(98)
C   X(50)=(10.D0*X(49)-5.D0*X(48)+X(47))/6.D0
C   X(100)=(10.D0*X(99)-5.D0*X(98)+X(97))/6.D0
```

Figure 31
FORTRAN Program Listing (continued)

```

C THE ENERGY*DENSITY PRODUCT IS CALCULATED IN "STATE" BY EXTRAPOLATION OF
C THE TIME PARTIAL.
C VELOCITY IS THEN CALCULATED AND CHECKED TO MAKE SURE THE MAXIMUM VELOCITY
C POSSIBLE IS NOT EXCEEDED.
      X(200)=X(100)/(X(50)*X(350))
C IF(X(200).GT.DSQR(2.D0*GAMMA*(X(250)+X(200)**2/
C 2 (2.D0*CP))*R/(GAMMA-1.D0)))
C 3 X(200)=DSQR(2.D0*GAMMA*(X(250)+X(200)**2/
C 4 (2.D0*CP))*R/(GAMMA-1.D0))
C ASSIGN THE OUTPUT STATIC PRESSURE AS A CONSTANT AT THE DIFFUSER.
      IF(X(250).LT.1.D0) X(250)=1.D0
      MACH=DABS(X(200)**2/(GAMMA*R*DABS(X(250))))
      X(300)=DEXP(DLOG(DIFFPRESSURE)-DLOG(1.D0+(GAMMA-1)
      2 *MACH/2.D0)*(GAMMA/(GAMMA-1.D0)))
C LET EXIT TEMPERATURE BE CALCULATED JUST AS ALL OTHER CHANNEL TEMPERATURES
C ARE CALCULATED.
C SET UP A DO LOOP TO CALCULATE REMAINING STATES ALONG CHANNEL LENGTH.
      DO 20,I=1,50
C VELOCITIES ALONG THE CHANNEL MAY NOW BE DETERMINED FROM THE NODE
C DENSITIES AND FLOWRATE AND CHANNEL AREAS.
      X(150+I)=X(50+I)/(X(I)*X(300+I))
C STATIC TEMPERATURES MAY THEN BE DETERMINED ALONG THE CHANNEL FROM
C INPUT NODE TO THE LAST NODE OF THE CHANNEL FROM THE ENERGY-DENSITY
C CONSERVATION VARIABLE AND CALCULATED VELOCITIES.
      X(200+I)=(X(100+I)/X(I)-X(150+I)**2/2.D0)/CV
C MACH NUMBERS ARE CALCULATED FOR EACH NODE.
      X(350+I)=X(150+I)/DSQR(R*GAMMA*DABS(X(200+I)))
C STATIC PRESSURES MAY BE DETERMINED FROM THE DENSITIES AND
C TEMPERATURES AT ANY GIVEN NODE.
      X(250+I)=X(I)*R*X(200+I)
C LET THE GAS CONDUCTIVITY VARY WITH TEMPERATURE AND PRESSURE. THE TWO
C CONSTANTS USED ARE SPECIFIC FOR A GIVEN GAS AND CAN BE DETERMINED FROM
C TWO POINTS AT THE EXTREMES OF THE OPERATING RANGE FOR THE GAS. CARE MUST
C BE TAKEN TO PREVENT ZERO CONDUCTIVITY FOR MATH REASONS.
      X(550+I)=KG2*DSQR(DSQR(DABS(X(200+I))))
      2 *DEXP(-KG1/DABS(X(200+I)))/DSQR(DABS(X(250+I)))
      IF(X(550+I).LE.0.D0) X(550+I)=1.D-6
      20 CONTINUE
C RESET THE EXIT AND INPUT PRESSURES SINCE THEY MAY HAVE BEEN CHANGED
C IN THE SET OF ASSIGNMENTS ABOVE. THIS IS IMPORTANT SINCE PRESSURE IS VERY
C INFLUENTIAL IN TIME BEHAVIOR NEAR BOUNDARIES.
      MACH=DABS(X(200)**2/(GAMMA*R*X(250)))
      X(300)=DEXP(DLOG(DIFFPRESSURE)-DLOG(1.D0+(GAMMA-1)
      2 *MACH/2.D0)*(GAMMA/(GAMMA-1.D0)))
      MACH=DABS(X(151)**2/(GAMMA*R*X(201)))
      X(251)=DEXP(DLOG(INPRESSURE)-DLOG(1.D0+(GAMMA-1)
      2 *MACH/2.D0)*(GAMMA/(GAMMA-1.D0)))
C THE FARADAY CONFIGURATION GIVES SOME VERY SIMPLE FIRST ORDER CALCULATIONS
C OF FIELDS AND CUURENTS. THE RELATIONSHIP IS GREATLY SIMPLIFIED IF ONE
C ASSUMES THAT THE SEGMENTED GENERATOR CAN HAVE NO AXIAL CURRENTS. HIGHER
C ORDER CALCULATIONS WOULD REQUIRE EXTENSIVE MESH CIRCUITS AND POSSIBLY WOULD
C REQUIRE FINE MODELLING OF THE ELECTRODE SHAPES THEMSELVES.
C CALCULATE THE FARADAY CURRENTS.
      DO 30,I=1,50
      IF(I.LT.12.OR.I.GT.29) THEN
        MAGNET=0.D0
      ELSE
        MAGNET=BMAG
      END IF
      30 X(400+I)=-X(150+I)*MAGNET*X(300+I)*X(550+I)*S*W
      2 /(X(300+I)+X(550+I)*W**2*S*X(500+I))
C ALL THE HALL CURRENTS REMAIN ZERO TO THE FIRST APPROXIMATION.
      DO 40,I=1,50
      X(450+I)=0.D0
      40
C CALCULATE THE ELECTRIC FIELD FOR THE FARADAY DIRECTION
      DO 50,I=1,50
      IF(I.LT.12.OR.I.GT.29) THEN
        MAGNET=0.D0
      ELSE
        MAGNET=BMAG
      END IF
      50 X(650+I)=+X(150+I)*MAGNET+X(400+I)/(X(550+I)*S*W)
C CALCULATE THE ELECTRIC FIELD IN THE HALL DIRECTION

```

Figure 31
FORTRAN Program Listing (continued)


```
      DO 60, I=1,50
60     X(700+I)=BETA*X(400+I)/(X(550+I)*S*W)
C     CALCULATE THE HALL VOLTAGES. THIS IS HELPFUL FOR DETERMINING THE
C     LIKELIHOOD OF ELECTRODE ARCING.
      DO 70, I=1,50
70     X(600+I)=-X(700+I)*S
C     SET THE NODE 50 FIELDS EQUAL TO THOSE AT NODE 49. THIS IS A MINOR
C     POINT, BUT IT CLEANS UP GRAPHS. THE NODE 50 FIELDS AND CURRENTS CANNOT BE
C     CALCULATED BY THIS SCHEME.
      X(700)=X(699)
      X(750)=X(749)
      RETURN
      END
```

Figure 31
FORTRAN Program Listing (continued)

MONTANA STATE UNIVERSITY LIBRARIES



3 1762 10182623 6

HOUCHEN
BINDERY LTD
UTICA/OMAHA
NE.