

TOWARDS BETTER CONSTRAINED MODELS OF THE
SOLAR MAGNETIC CYCLE

by

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in

Physics

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July, 2010

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ABSTRACT

The best tools we have for understanding the origin of solar magnetic variability are kinematic dynamo models. During the last decade, this type of models has seen a continuous evolution and has become increasingly successful at reproducing solar cycle characteristics. The basic ingredients of these models are: the solar differential rotation – which acts as the main source of energy for the system by shearing the magnetic field; the meridional circulation – which plays a crucial role in magnetic field transport; the turbulent diffusivity – which attempts to capture the effect of convective turbulence on the large scale magnetic field; and the poloidal field source – which closes the cycle by regenerating the poloidal magnetic field. However, most of these ingredients remain poorly constrained which allows one to obtain solar-like solutions by “tuning” the input parameters, leading to controversy regarding which parameter set is more appropriate. In this thesis we revisit each of those ingredients in an attempt to constrain them better by using observational data and theoretical considerations, reducing the amount of free parameters in the model.

For the meridional flow and differential rotation we use helioseismic data to constrain free parameters and find that the differential rotation is well determined, but the available data can only constrain the latitudinal dependence of the meridional flow.

For the turbulent magnetic diffusivity we show that combining mixing-length theory estimates with magnetic quenching allows us to obtain viable magnetic cycles and that the commonly used diffusivity profiles can be understood as a spatiotemporal average of this process.

For the poloidal source we introduce a more realistic way of modeling active region emergence and decay and find that this resolves existing discrepancies between kinematic dynamo models and surface flux transport simulations. We also study the physical mechanisms behind the unusually long minimum of cycle 23 and find it to be tied to changes in the meridional flow. Finally, by carefully constraining the system through surface magnetic field observations, we find that what is believed to be the primary source of poloidal field (also known as Babcock-Leighton mechanism) may not be enough to sustain the solar magnetic cycle.

1. INTRODUCTION: THE SOLAR MAGNETIC CYCLE

1.1. Discovery

Since the dawn of time the Sun has been an object of fascination for mankind. Giver of life, light and heat – its worship as a deity has been part of most of the major polytheistic religions of the world. From a practical point of view, the study of the Sun was crucial in determining a solar calendar that could be used to coordinate agricultural activities. However, development of astronomical sciences also spurred an interest on the Sun from the intellectual, scientific point of view.

The first step towards the discovery of the solar magnetic cycle was the discovery of sunspots. Although the first recorded indication of sunspot observation goes as far back as 27 B.C. by Chinese astronomers, it was not until the invention of the telescope that detailed observations were performed by Galileo Galilei, Thomas Harriot, and Christoph Scheiner, David and Johannes Frabicius. Once sunspots had been observed, it was only a matter of time before improvements of the telescope setup and tracking mechanism would allow for fairly accurate drawings of them (see Fig. 1).

The next step was the discovery of the cycle itself by Samuel Schwabe (1844). Schwabe was trying to find whether there was a planet inside Mercury's orbit (tentatively called Vulcan). Schwabe believed that by carefully observing the sun and keeping track of sunspots Vulcan could be observed while in transit. Although the search for Vulcan was not fruitful, after more than two decades of observation Schwabe

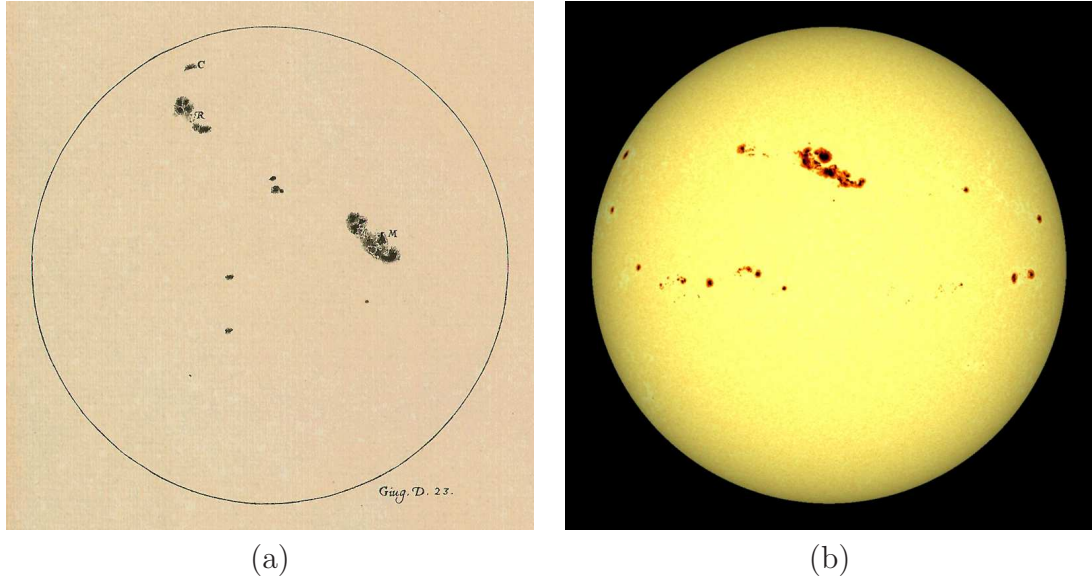


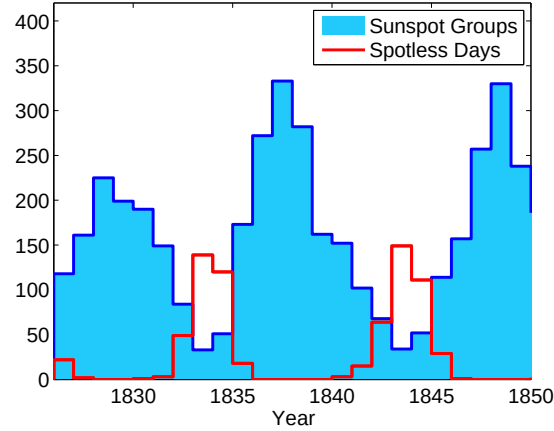
Figure 1. (a) Original sunspot drawing by Galileo Galilei. Image taken from the Galileo project. (b) SOHO/MDI white light image of the Sun. The quality of Galileo's drawings is evident when both are compared.

noted the nearly decadal periodicity of the number of sunspots present at any given time on the surface of the sun (see Fig. 2).

Soon after the discovery of the sunspot cycle, Richard Carrington (1858) noted the equatorward migration of the latitude of emergence of sunspots with the progress of the cycle, starting from mid-latitudes. This result was further refined by Gustav Spörer (1861). The best and most popular way of visualizing this characteristic was first introduced by Edward and Annie Maunder (1904) and involves plotting sunspot locations for each solar Carrington rotation and stacking such plots in time (Fig. 3), this synoptic map (also known as butterfly diagram because of the distinct pattern formed by sunspot migration) is one of the chief observational constraints for models of the solar cycle.

Jahr.	Gruppen.	Stiefenfreie Tage.	Beobachtungstage.
1826	118	22	277
1827	161	2	273
1828	225	0	282
1829	199	0	244
1830	190	1	217
1831	149	3	239
1832	81	49	270
1833	33	139	267
1834	51	120	273
1835	173	18	244
1836	272	0	200
1837	333	0	168
1838	282	0	202
1839	162	0	205
1840	152	3	263
1841	102	15	283
1842	68	64	307
1843	34	149	312
1844	52	111	321
1845	114	29	332
1846	157	1	314
1847	257	0	276
1848	330	0	278
1849	238	0	285
1850	186	2	308

(a)



(b)

Figure 2. (a) Scan of Schwabe's original sunspot observations as published on the third volume of von Humboldt's Kosmos series (1845). The columns from left to right are: year, number of groups, spotless days and number of days in which the sun was observed. (b) Graphic representation of Schwabe's data showing three sunspot cycles and their associate minima.

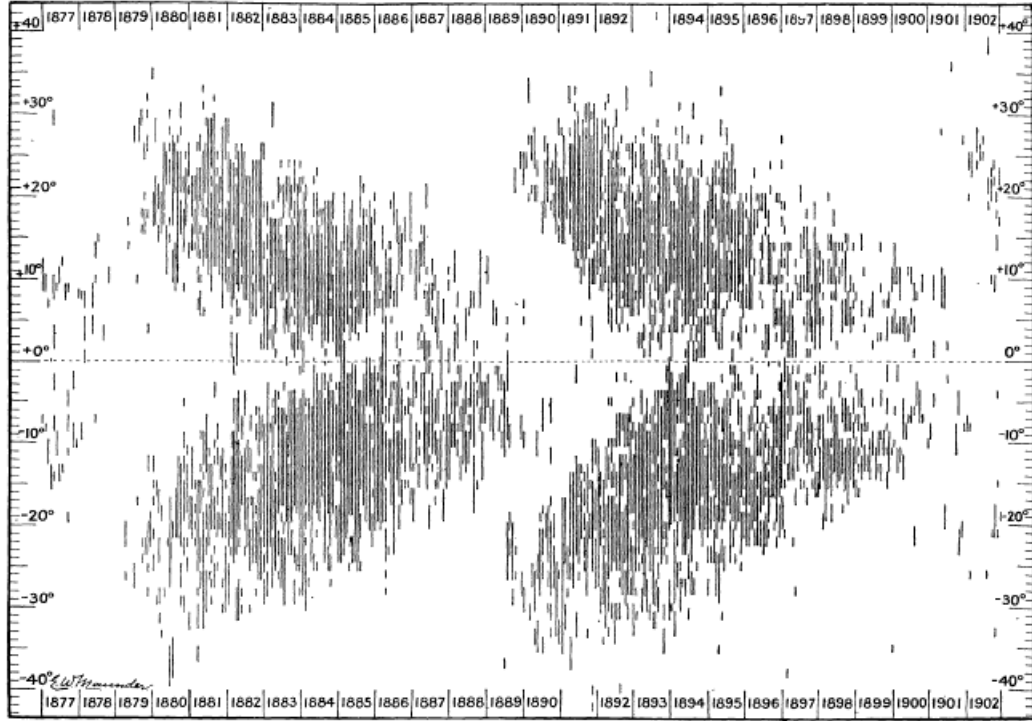


FIG. 8.—DISTRIBUTION OF SPOT-CENTRES IN LATITUDE, ROTATION BY ROTATION, 1877-1902.

Figure 3. Butterfly diagram published by Edward and Annie Maunder (1904) showing the equatorward migration of active latitudes as the cycle progresses. This process begins anew each cycle, with the first active regions of the cycle appearing at mid-latitudes.

George Hale (1908) made another breakthrough by making the first measurements of the magnetic field on the Sun using spectral line splitting due to the presence of strong magnetic fields (commonly known as Zeeman effect; Zeeman 1897). Furthermore, by comparing the line splitting measured on the Sun with laboratory experiments Hale was able to estimate the strength of the magnetic field inside sunspots (see Fig. 4). Additionally, after devising a way of measuring the line of sight component of the magnetic field, Hale observed that sunspot groups are really large scale bipolar

regions (also known as Active Regions; ARs). This typical magnetic configuration is shown in Figure 4, where we can see a white light image showing two sunspots (b) and their associated magnetic field (c) as measured by the Solar Optical Telescope aboard *Hinode*. Hale and his collaborators discovered that most AR have the following properties (Hale et al. 1919):

- The magnetic field of most active regions of a given hemisphere has the same East-West orientation and this orientation reverses across the equator (commonly known as Hale’s law).
- Active regions present a systematic tilt with respect to a line parallel to the equator such that the leading (east-most) polarity is closer to the equator (commonly known as Joy’s law).
- The polarity of active regions (and the Sun’s global magnetic field) reverses from cycle to cycle, such that two sunspot cycles correspond to a full magnetic cycle.

Today, a hundred years after Hale’s discoveries and thanks to the development of increasingly refined and sophisticated instruments, the magnetic nature of the solar cycle has been established beyond any doubt.

1.2. Evolution of our Understanding

As our observations of the Sun have become more sophisticated, so has our understanding of solar magnetic field evolution. In a nutshell, the solar magnetic cycle is

The following doublets have been measured in the spectrum of chromium:

TABLE III
CHROMIUM DOUBLETS

Wave-Length	$\Delta\lambda$, Spark	$\frac{\Delta\lambda}{4.9}$, Spark	$\Delta\lambda$, Spot	δ	$\frac{\Delta\lambda}{\Delta\lambda, \text{Spot}}$, Spark
5304.36	0.636	0.130	0.188	+0.058	3.4
5387.16	0.676	0.138	0.085	-0.043	8.0
5713.00	0.610	0.124	0.161	+0.037	3.7
5781.40	0.755	0.154	0.121	-0.033	6.2
5781.97	0.922	0.188	0.212	+0.024	4.3
5783.29	0.772	0.158	0.137	-0.021	5.6
5784.08	0.720	0.147	0.121	-0.026	6.0
5785.19	0.707	0.144	0.137	-0.007	5.1

In photographing these lines in the spark, the strength of the field was 12,500 gauss. The strength of the field in spots, as indicated by the mean separation of the chromium doublets, is therefore 2600 gauss.

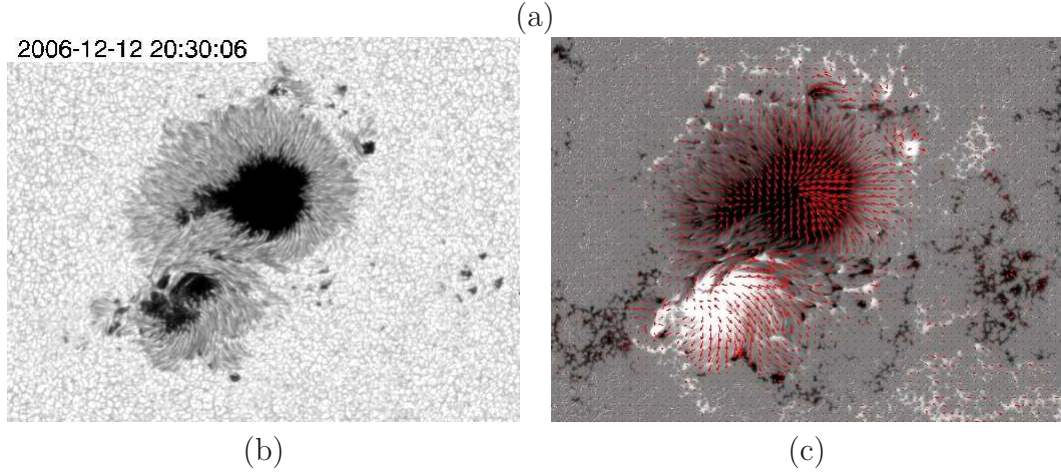


Figure 4. (a) Scan of the original paper by Hale (1908) showing his estimation of the magnetic field strength of a sunspot by comparing the spectral line splitting with that obtained in the lab. (b) White light image taken by Hinode/SOT showing two sunspots. (c) Vectormagnetogram of the same region showing the bipolar nature of the associated sunspots. White (black) corresponds to positive (negative) line of sight polarity. The little red arrows show a calculation of the field components perpendicular to the line of sight. Images taken from the main Hinode website.

a process in which the magnetic field switches from a configuration which is predominantly poloidal (confined to the meridional plane, B_r and B_θ ; see Fig. 5-a) to one

which is predominantly toroidal (normal to the meridional plane, B_ϕ ; see Fig. 5-b) and back, drawing on the available energy in solar plasma flows. This transfer of energy is possible thanks to the of the solar plasma: due to its high conductivity (low resistivity) and the size of the length-scales involved, the interaction of the magnetic field with the plasma flows is more important than the dissipation of the field due to ohmic losses. The relative importance of these processes is nicely captured in a number called the magnetic Reynolds number:

$$R_m = \frac{vL}{\lambda} \quad (1.1)$$

where for the v , L and λ are the characteristic speed, length-scale and magnetic diffusivity of the system. In the solar plasma $R_m \gg 1$, which leads to the magnetic flux being conserved or “frozen” in moving plasma. This quality first proved by Alfvén (1942) gives the magnetic-field (and its associated plasma) a very distinct identity.

The first part of the process (Poloidal $\rightarrow$ Toroidal field) was proposed in the context of the Sun by Larmor (1919) and relies on the fact that the Sun doesn’t rotate uniformly. The idea is that the shearing of a large scale poloidal field by the solar differential rotation results in the production of large scale toroidal belts of opposite sign across the equator. These toroidal belts then act as the source of active regions which, due to this antisymmetry across the equator, match Hale’s law. This process, beautifully illustrated in Figure 6, is very well established in our understanding of the solar magnetic cycle. it is the mechanism behind the second part (Toroidal $\rightarrow$ Poloidal) which has proven elusive and is still debated. The first breakthrough in this

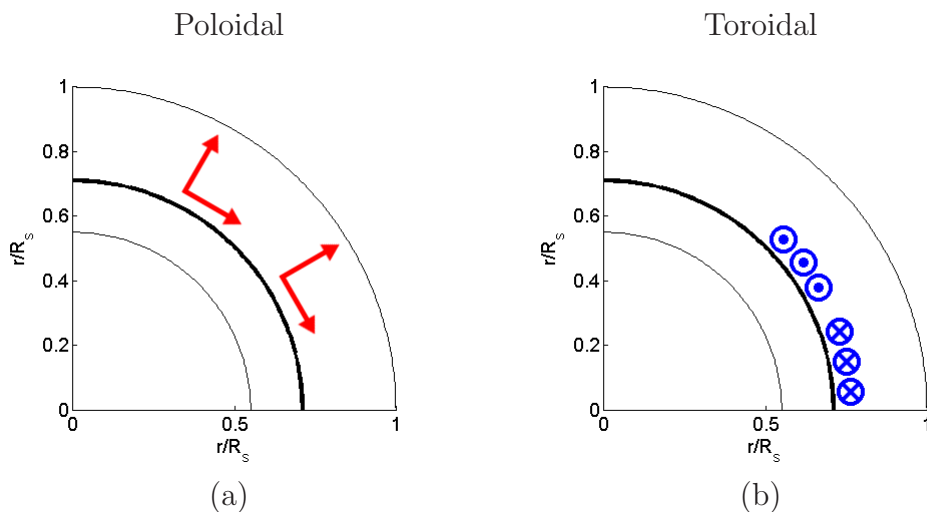


Figure 5. Important terminology: (a) Poloidal components are confined to the meridional plane, B_r and B_θ . (b) The toroidal component is normal to the meridional plane, B_ϕ . (i. e., in the direction of rotation).

direction was made by Parker (1955). His idea was that the effect of the coriolis force on turbulent convection could impart a systematic twist on toroidal fields producing a net poloidal component (Fig. 7-a); the global effect of this small scale dynamo would work together to produce a global poloidal field closing the cycle (Fig. 7-b to c). Furthermore, Parker found that solutions of such a system consisted of propagating waves which signify the migration of active latitudes with the progress of the solar cycle. However, it would take roughly a decade to put this conceptual idea into a solid mathematic formulation through mean-field electrodynamics (Steenbeck, Krause & Rädler 1966).

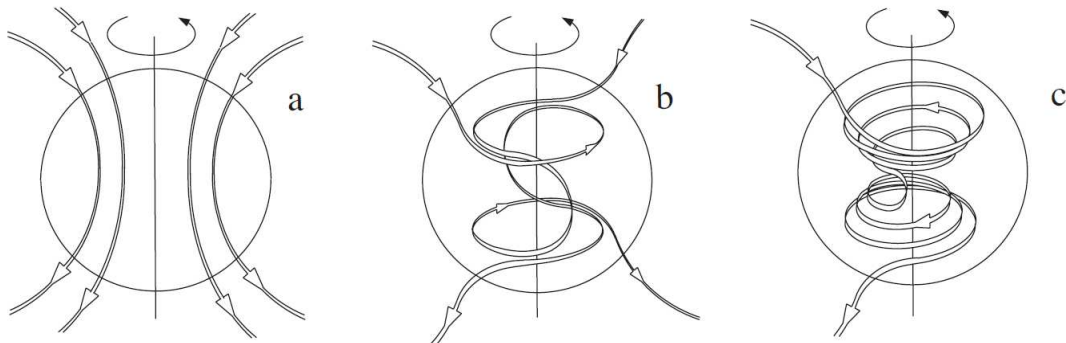


Figure 6. Poloidal $\rightarrow$ Toroidal field conversion. The interaction of a predominantly poloidal field (a) with differential rotation builds up a toroidal component (b), which given enough time produces a predominantly toroidal configuration (c). Illustrations by J. J. Love (1999).

1.2.1. Mean-Field Electrodynamics

In the solar interior fluid motions are non-relativistic, the plasma is electrically neutral and the collisional mean-free path of electrons and ions is much smaller than other relevant spatial scales. This means that Ohm's law is valid and the displacement current term in Ampère's law can be neglected. Under this conditions the evolution of the magnetic field is governed by the Magnetohydrodynamic (MHD) induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \lambda \nabla \times \mathbf{B}), \quad (1.2)$$

where $\mathbf{B}$ denotes the magnetic field, $\mathbf{v}$ the velocity field and λ the magnetic diffusivity.

Due to the turbulent nature of the system, the fields can be written in terms of their average and fluctuating parts:

$$\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}' \quad \& \quad \mathbf{v} = \overline{\mathbf{V}} + \mathbf{v}', \quad (1.3)$$

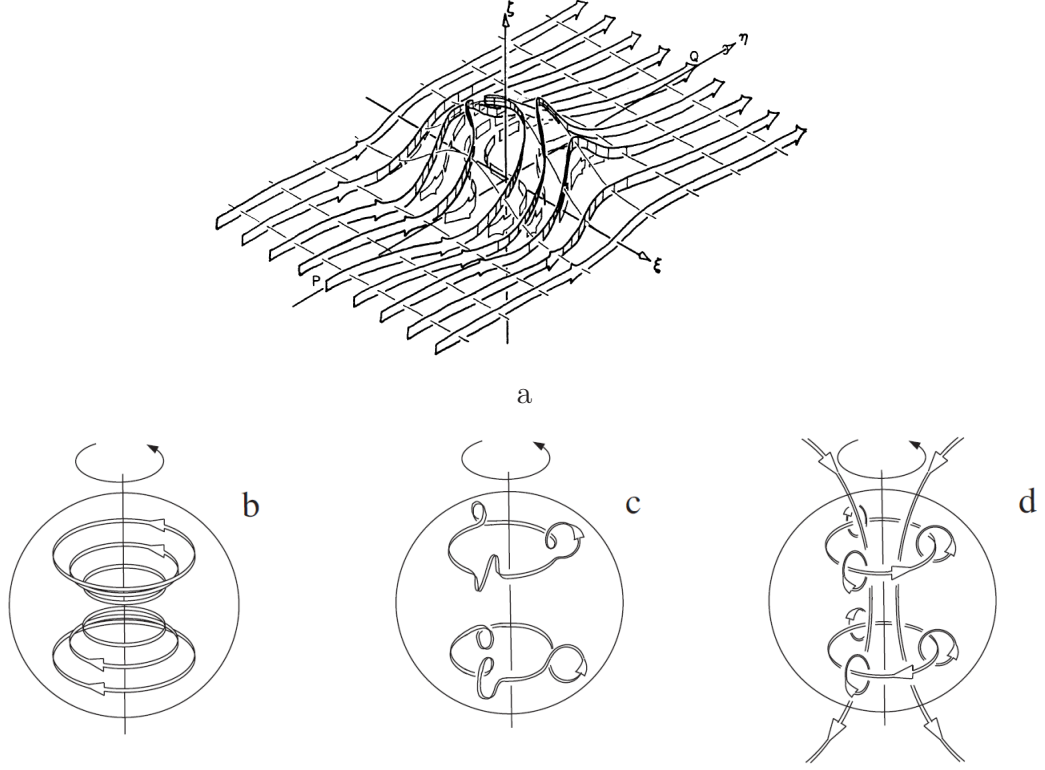


Figure 7. Toroidal $\rightarrow$ Poloidal field conversion. The interaction between the toroidal field (b) and helical turbulence can impart a poloidal component to the field (a). When this happens at a global scale (c), the resulting configuration acquires a poloidal component which closes the cycle setting up the stage for the next one (d). Illustrations by E. Parker (1955; a) and J. J. Love (1999; c-d).

where the overline (prime) indicates the average (fluctuating) part. Substituting these new variables in the induction equation (Eq. 1.2) and averaging again one obtains the equation for the evolution of the mean and fluctuating components of the magnetic field:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{V}} \times \overline{\mathbf{B}} + \overline{\mathcal{E}} - \lambda \nabla \times \overline{\mathbf{B}}) \quad (1.4)$$

$$\frac{\partial \mathbf{b}'}{\partial t} = \nabla \times (\overline{\mathbf{V}} \times \mathbf{b}' + \mathbf{v}' \times \overline{\mathbf{B}} + \mathcal{G} - \lambda \nabla \times \mathbf{b}'), \quad (1.5)$$

were $\mathcal{E} = \langle \mathbf{v}' \times \mathbf{b}' \rangle$ is the mean Electro Motive Force (EMF) and $\mathcal{G} = \mathbf{v}' \times \mathbf{b}' - \langle \mathbf{v}' \times \mathbf{b}' \rangle$.

If one assumes that at some initial time ($t = 0$), there is no fluctuating component of the magnetic field ($\mathbf{b}' = 0$), then there is a linear relationship between $\mathbf{b}'$ and $\overline{\mathbf{B}}$ (see Eq. 1.5). From that it follows that $\mathcal{E}$ and $\overline{\mathbf{B}}$ are likewise linearly related. If additionally the spatial scale of the average magnetic field is much larger than that of the fluctuating components, the EMF can be developed as a series of the form:

$$\mathcal{E}_i = \alpha_{ij} \overline{B}_j + \beta_{ijk} \frac{\overline{B}_j}{\partial x_k} + \gamma_{ijkl} \frac{\partial^2 \overline{B}_j}{\partial x_k \partial x_l} + \dots \quad (1.6)$$

Assuming that the turbulence is isotropic and that this series converges quickly (some-what questionable assumptions in the solar case), we can concentrate on the first two terms of the series and write

$$\alpha_{ij} = \alpha \delta_{ij} \quad \beta_{ijk} = \beta \varepsilon_{ijk}, \quad (1.7)$$

where α and β are pure scalars and ε_{ijk} is the purely anti-symmetric tensor. Finally, in the weak diffusion limit (which corresponds to the physical nature of the solar convection zone) one obtains that the mean EMF can be approximated by (Moffat 1974; 1978):

$$\mathcal{E} \simeq \alpha \overline{\mathbf{B}} - \beta \nabla \times \overline{\mathbf{B}}, \quad (1.8)$$

where

$$\alpha = -\frac{\tau}{3} \langle \mathbf{v}' \cdot \nabla \times \mathbf{v}' \rangle \quad (1.9)$$

and

$$\beta = -\frac{\tau}{3} \langle \mathbf{v}' \cdot \mathbf{v}' \rangle. \quad (1.10)$$

Here τ corresponds to the correlation time for the convective turbulence. We can see that α is related to the amount of helicity present in the turbulent velocity field, or in other words the cyclonic motions that impart the systematic twist proposed by Parker (this type of poloidal source is commonly known as the mean-field α -effect). On the other hand β quantifies the diffusivity of magnetic field due to convective turbulence. Substituting Eq. 1.8 in Eq. 1.4 we finally obtain the kinematic dynamo equation:

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} = \nabla \times (\bar{\mathbf{v}} \times \bar{\mathbf{B}} - \eta \nabla \times \bar{\mathbf{B}} + \alpha \bar{\mathbf{B}}), \quad (1.11)$$

where $\eta = \lambda + \beta$ is the net magnetic diffusivity (normally referred to as turbulent magnetic diffusivity because $\beta \gg \lambda$). The overall idea behind this equation is that the evolution of the magnetic field is governed by its interaction with the mean velocity field (advection, compression, rarefaction and shearing; first term on the R.H.S.), a diffusive process enhanced by turbulent convection (second term on the R.H.S) and a mean electromotive force caused by systematic helical motions in the turbulent velocity field (third term on the R.H.S).

1.2.2. Turbulent Times: The Arrival of Flux-Tube Simulations and Helioseismic Measurements of the Differential Rotation

Although mathematically well founded and of beautiful simplicity, mean-field electrodynamics and the mean-field α -effect were soon called into question by several advances on other branches of solar physics. Chief among them was the development of simulations of the buoyant toroidal flux-tubes that results in the emergence of

active regions at the surface. These simulations showed that only flux-tubes with a field strength of at least $50 - 100$ KGauss are consistent with observed latitudes of emergence (first reported in Choudhuri & Gilman 1987) and observed active region tilt (first reported in D'Silva and Choudhuri 1993). These results have been confirmed by several independent studies of increasing sophistication (Choudhuri 1989; Fan, Fisher & DeLuca 1993; Fan, Fisher & McClymont 1994; Schüssler et al. 1994; Caligari, Moreno-Insertis & Schüssler 1995; Fan & Fisher 1996; Caligari, Schüssler & Moreno-Insertis 1998; Fan & Gong, 2000).

The conflict arises when one remembers that the proposed mechanism for poloidal field regeneration (mean-field α -effect) involves the twisting of the toroidal field by helical turbulent convection. In order to do this efficiently, the plasma needs to have enough energy to dominate the dynamics of the combined system and this mechanism is expected to saturate once the energy density of the magnetic field reaches equipartition values. The problem is that the equipartition magnetic field B_e is two orders of magnitude below the values found to be necessary to form ARs ($50 - 100$ KGauss), making it quite difficult to reconcile the mean-field α -effect with flux-tube simulations.

Parker, in his landmark paper (1955), found that the linear dynamo equations support traveling wave solutions; this was found to be also true for spherical coordinates and non-linear models (Yoshimura 1975; Stix 1976). The direction of propagation of

such waves (s), was found to be:

$$s = \alpha \vec{\nabla} \Omega \times \hat{e}_\phi, \quad (1.12)$$

where Ω quantifies the solar differential rotation, which means that in order to have equatorial propagation of the dynamo wave as observed in active latitudes the following condition must be satisfied:

$$\alpha \frac{\partial \Omega}{\partial r} < 0. \quad (1.13)$$

Given that at that time the shape of the differential rotation was unknown, there was relative freedom regarding its shape inside the convection zone. However, once the differential rotation was measured accurately (Thompson et al. 1996; Kosovichev et al. 1997; Schou et al. 1998), the correct profile turned out to have a positive radial shear at low latitudes leading to poleward propagating solutions (see Eq. 1.13). This created yet another problem that needed to be addressed by the classical mean-field $\alpha\Omega$ dynamo and in combination with the other standing issues led to its fall from favor.

In spite of this setback, the dynamo community was quick to step up to the challenge and started looking for alternate sources of poloidal field regeneration. It is beyond the scope of this introduction to make a comprehensive review of all of them; for which we point the interested reader to the review by Charbonneau (2005). Instead, we will concentrate on what has become the most widely accepted mechanism for regenerating the poloidal field: the Babcock-Leighton mechanism.

1.2.3. Babcock-Leighton Dynamo Models

First proposed by Babcock (1961) and further elaborated by Leighton (1964; 1969), it was originally envisaged as a shallow dynamo operating in the near surface layers of the Sun, as opposed to the mean-field $\alpha\Omega$ dynamo which operates throughout the solar convection zone. As with the $\alpha\Omega$ dynamo, the first part of the cycle (Poloidal $\rightarrow$ Toroidal) is achieved by the shearing of the poloidal field by differential rotation. On the other hand, the second part of the cycle (Toroidal $\rightarrow$ Poloidal), is achieved by the collective effect of bipolar active region emergence and decay. At the core of this process, known as the Babcock-Leighton (BL) mechanism, resides the fact that active regions have a systematic hemispheric orientation and tilt (Hale's and Joy's laws; see Section 1.1). This means that the leading polarity of most active regions is closer to the equator than the following polarity. Given that this orientation is opposite in each hemisphere, there is a net cancellation of flux across the equator and a net accumulation of open field on the poles, which produces the cancellation and reversal of the poloidal field closing the cycle (see Fig. 8). Another way of understanding this process is in terms of the magnetic moment: due to their systematic orientation and tilt, most active regions in a cycle will carry a dipole moment of the same sign (and of opposite sign as that of the old cycle's dipole moment). After eleven years of active region emergence and diffusive action, higher order moments would have decayed leaving a new bipolar field as the starting point for the next cycle.

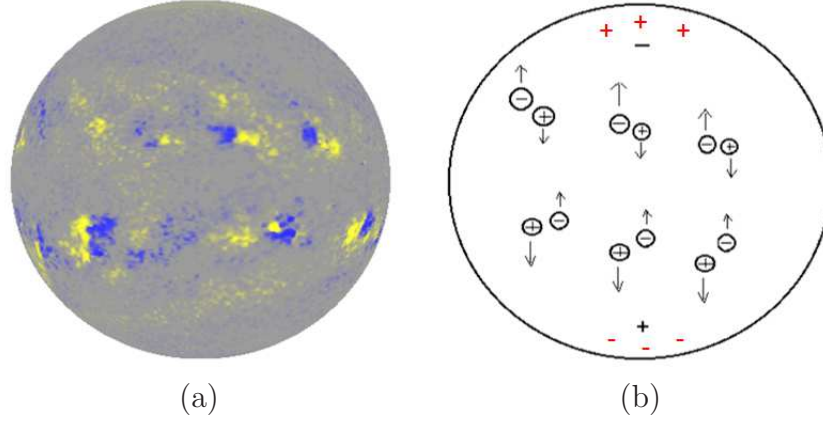


Figure 8. (a) MDI magnetogram showing a snapshot of photospheric magnetic field. The systematic orientation and tilt known as the Hale’s and Joy’s laws (see Sec. 1.1) can be seen very clearly: In the northern (southern) hemisphere the leading (eastmost) polarity is consistently positive (negative), colored in yellow (blue), and it is closer to the equator than the following (westmost) polarity of the opposite sign. (b) Simplified diagram of this typical configuration, showing the form of polarity migration that leads to flux cancelation across the equator and accumulation at the poles. This accumulation of field cancels and then reverses the old polarity (shown in red).

it is important to note that contrary to the original Babcock’s idea, modern BL dynamo models do not operate as a shallow dynamo. This is because a shallow dynamo is incompatible with our current understanding of active regions as the top part of buoyant flux-tubes, which rise from the bottom of the convection zone (see Section 1.2.2). Instead, they take the best aspects of both types of dynamos by amplifying and storing the toroidal field inside the convection zone (as in the classical $\alpha\Omega$ dynamo) and recreating the poloidal field through the BL mechanism at the surface.

1.2.4. Meridional Circulation

While flux-tube simulations were casting doubt on classical $\alpha\Omega$ dynamos, the development of high resolution magnetographs was paving the way for a discovery that would have far reaching consequences for dynamo models: the meridional circulation. Observations of small magnetic features on the surface of the Sun showed a 10 – 20 m/s flow of mass from the equator towards the poles (Komm, Howard & Harvey 1993; Latushko 1994; Snodgrass & Dailey 1996; Hathaway 1996). These observations were later confirmed by helioseismic measurements, which found that the poleward flow is present in at least the top 10% of the convection zone (Giles et al. 1997; Schou & Bogart 1998; Braun & Fan 1998; González-Hernández et al. 1999). While there are still no measurements of the meridional flow in the rest of the convection zone (and it may take a while until we have them, see Braun & Birch 2008), it is reasonable to assume that due to mass conservation there must be a return flow somewhere deep in the convection zone. Nevertheless, indirect measurements of this flow using the rate of equatorward migration of the active latitudes have estimated values of 1 m/s at the bottom of the convection zone (Hathaway et al. 2003) and have found a strong anti-correlation between its amplitude and the duration of the cycle.

From the point of view of dynamo theory, the existence of such a large scale flow has a very important implication: since the field on the Sun is frozen in the plasma (see Section 1.2; Alfvén 1942), an equatorward flow at the bottom of the convection zone will drag the field along – helping circumvent the condition required for an

equatorial propagation of the dynamo wave r obviating the requirement altogether (Eq. 1.13; Choudhuri, Schüssler & Dikpati 1995; Durney 1995). Because of this, the meridional circulation has become an integral part of modern kinematic dynamo models.

1.3. The 2.5 Kinematic Babcock-Leighton Dynamo Model

After this brief introduction to the discovery of the solar magnetic cycle and the evolution of our understanding, it is time to introduce the model which we use for the remainder of this work. We start by expressing the magnetic and velocity fields in an axisymmetric spherical coordinate system:

$$\mathbf{B} = B\hat{\mathbf{e}}_\phi + \nabla \times (A\hat{\mathbf{e}}_\phi) \quad \mathbf{v} = r \sin(\theta)\Omega\hat{\mathbf{e}}_\phi + \mathbf{v}_p, \quad (1.14)$$

where B is the toroidal component of the magnetic field, $\nabla \times (A\hat{\mathbf{e}}_\phi)$ the poloidal components obtained by taking the curl of the vector potential $A\hat{\mathbf{e}}_\phi$; Ω corresponds to the mean differential rotation profile and $\mathbf{v}_p$ the meridional circulation. Substituting these equations in the mean-field dynamo equation (1.11), one obtains the axisymmetric dynamo equations:

$$\frac{\partial A}{\partial t} + \frac{1}{s} [\mathbf{v}_p \cdot \nabla (sA)] = \eta \left(\nabla^2 - \frac{1}{s^2} \right) A + S(r, \theta, B)B \quad (1.15)$$

$$\frac{\partial B}{\partial t} + s \left[\mathbf{v}_p \cdot \nabla \left(\frac{B}{s} \right) \right] + (\nabla \cdot \mathbf{v}_p)B = \eta \left(\nabla^2 - \frac{1}{s^2} \right) B + s ([\nabla \times (A\hat{\mathbf{e}}_\phi)] \cdot \nabla \Omega) + \frac{1}{s} \frac{\partial (sB)}{\partial r} \frac{\partial \eta}{\partial r}, \quad (1.16)$$

where $s = r \sin(\theta)$. The terms on the LHS of both equations with the poloidal velocity ($\mathbf{v}_p$) correspond to the advection and deformation of the magnetic field by

the meridional flow. The first term on the RHS of both equations corresponds to the diffusion of the magnetic field. The second term on the RHS of both equations is the source of that type of magnetic field (BL mechanism for A and rotational shear for B). Finally, the third term on the RHS of Equation 1.16 corresponds to the advection of toroidal field due to a turbulent diffusivity gradient. Note that the mean-field α -effect has been substituted by a generic source $S(r, \theta, B)$, the reasons for this will be clarified in Section (1.3.4).

In Equations 1.15 and 1.16, there are four ingredients that need to be defined so that the model can be used: the differential rotation Ω , the meridional flow $\mathbf{v}_p$, the magnetic diffusivity η and the poloidal source $S(r, \theta, B)$. We will proceed to define them in the following sections. Unless specifically noted, the parameters defined here will be used in different parts of this work.

1.3.1. Meridional Flow

A crucial ingredient in modern dynamo models, meridional flow is believed to be responsible setting the speed of the migrating toroidal belts at the bottom of the convection zone, from which sunspots emerge. Additionally, it plays an important role in setting the period and amplitude of the cycle, as well as affecting the amount of flux cancelation across the equator and flux accumulation at the poles. As mentioned in Section 1.2.4, it has only been measured in the top 10% of the convection zone. However, it has been found to arise self consistently in full MHD simulations (see Miesch et al. 2008 and references therein), where it appears as a highly fluctuating

large scale flow pattern. it is commonly defined as a single cell flow in dynamo modeling, with poleward flow in the top half of the convection zone and equatorward flow in the bottom half (see Figure 9-b). We use the meridional profile of Muñoz-Jaramillo, Nandy and Martens (2009), which closely resembles the observed features present in helioseismic meridional flow data (see Chapter 2) and is defined by the following stream function:

$$\Psi(r, \theta) = \frac{v_0}{r}(r - R_p)(r - R_\odot) \sin\left(\pi \frac{r - R_p}{R_1 - R_p}\right)^a \sin^{(q+1)}(\theta) \cos(\theta), \quad (1.17)$$

where v_0 is a constant which sets the amplitude of the flow, q governs the latitudinal dependence (we use $q = 1$), $R_p = 0.675R_\odot$ the penetration depth, $a = 1.795$ and $R_1 = 1.027R_\odot$ govern the location of the peak of the poleward flow and the amplitude and location of the equatorward return flow. One can then obtain the meridional flow from the expresion:

$$\vec{\mathbf{v}}_p(r, \theta) = \frac{1}{\rho(r)} \vec{\nabla} \times (\Psi(r, \theta) \hat{\mathbf{e}}_\phi), \quad (1.18)$$

where $\rho(r)$ is the solar density profile

1.3.2. Differential Rotation

As the main source of energy for the solar magnetic cycle, the differential rotation shears poloidal field to create the strong toroidal belts from which active regions emerge. First discovered by Carrington (1859), its measurement in the bulk of the convection zone is one of the greatest achievements of helioseismology – a field which uses the propagation of acoustic waves inside the solar convection zone in combination

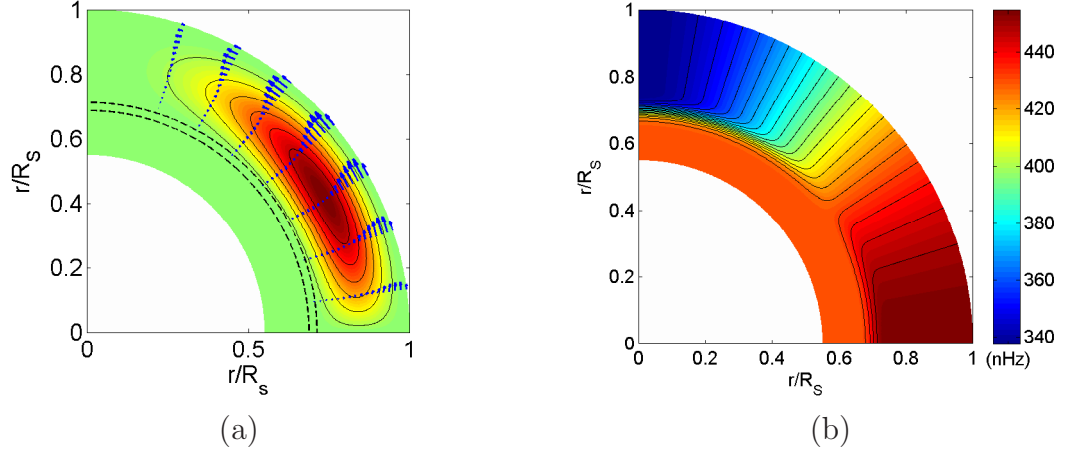


Figure 9. Global Flows: (a) Meridional flow from Muñoz-Jaramillo, Nandy & Martens (2009). (b) Differential rotation from Charbonneau et al. (1999).

with models of the solar convection zone to map internal flows. Shown in Figure 9-a, the basic characteristics are the fast rotation of the equator with respect to the poles and the solid body rotation of the radiative region at a rate somewhere between those of the equator and the poles. In this work we use the analytical form of Charbonneau et al. (1999;). It is defined as:

$$\Omega_A(r, \theta) = 2\pi \left[\Omega_c + \frac{1}{2} \left(1 - \operatorname{erf} \left(\frac{r-r_{tc}}{w_{tc}} \right) \right) (\Omega_e - \Omega_c + (\Omega_p - \Omega_e)\Omega_S(\theta)) \right] \quad (1.19)$$

$$\Omega_S(\theta) = a \cos^2(\theta) + (1 - a) \cos^4(\theta),$$

where $\Omega_c = 432$ nHz is the rotation frequency of the core, $\Omega_e = 470$ nHz is the rotation frequency of the equator, $\Omega_p = 330$ nHz is the rotation frequency of the pole, $a = 0.483$ is the strength of the $\cos^2(\theta)$ term relative to the $\cos^4(\theta)$ term, $r_{tc} = 0.7$ the location of the tachocline and $w_{tc} = 0.025$ half of its thickness. For more information about the differential rotation refer to the review by Howe (2009).

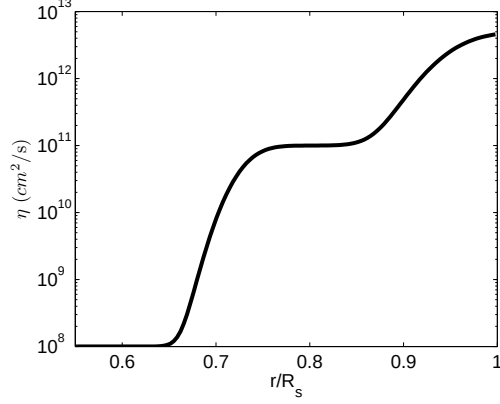


Figure 10. Turbulent magnetic diffusivity.

1.3.3. Turbulent Magnetic Diffusivity

The turbulent magnetic diffusivity is an ingredient which attempts to capture the net effect that convective turbulence has on the large scale magnetic field. It sets the properties of the transport process in combination with the meridional flow and strongly affects the memory of the dynamo (see Yeates, Nandy & Mackay 2008). It is also responsible for flux cancellation and its relative strength can lead to decaying dynamo solutions. It is commonly modeled through an ad-hoc double step profile (Dikpati et al. 2002; Chatterjee, Nandy & Choudhuri 2004, Guerrero & de Gouveia Dal Pino 2007, Jouve & Brun 2007; see Figure 10):

$$\eta(r) = \eta_{bcd} + \frac{\eta_{cz} - \eta_{bcd}}{2} \left(1 + \operatorname{erf} \left(\frac{r - r_{cz}}{d_{cz}} \right) \right) + \frac{\eta_{sg} - \eta_{cz} - \eta_{bcd}}{2} \left(1 + \operatorname{erf} \left(\frac{r - r_{sg}}{d_{sg}} \right) \right), \quad (1.20)$$

where $\eta_{bcd} = 10^8 \text{ cm}^2/\text{s}$ corresponds to the diffusivity at the bottom of the computational domain, $\eta_{cz} = 10^{11} \text{ cm}^2/\text{s}$ corresponds to the diffusivity in the convection

zone, $\eta_{sg} = 10^{12} \text{ cm}^2/\text{s}$ corresponds to the near-surface supergranular diffusivity and $r_{cz} = 0.71R_{\odot}$, $d_{cz} = 0.015R_{\odot}$, $r_{sg} = 0.95R_{\odot}$ and $d_{sg} = 0.025R_{\odot}$ characterize the transitions from one value of diffusivity to the other.

1.3.4. The Babcock-Leighton Source

This ingredient has the crucial role of closing the cycle, ensuring oscillatory solutions and preventing the dynamo from decaying. The most common way of modeling the BL mechanism is through a continuous non-local source, first introduced by Dikpati & Charbonneau (1999); it is the formulation we use in the first part of our work (see Chapter 2). This source term models the conversion of toroidal field at the bottom of the convection zone to poloidal field at the surface. Using the notation introduced in the poloidal field dynamo equation (Eq. 1.15), it is defined as:

$$S(r, \theta, B) = \alpha_0 f(r, \theta) F(B_{tc}), \quad (1.21)$$

where B_{tc} is the toroidal field at the bottom of the solar convection zone. The first element of the poloidal source (α_0), is a constant that sets the strength of the source term and it is usually used to ensure supercritical solutions.

The second element $f(r, \theta)$, attempts to capture the spatial properties of the BL mechanism: confinement to the surface, observed active latitudes and latitudinal dependence of tilt (see Figure 11-a). It is defined as:

$$f(r, \theta) = \frac{1}{16} \cos(\theta) \left(1 + \operatorname{erf} \left(\frac{\theta - (90^\circ - \beta)}{\gamma} \right) \right) \left(1 - \operatorname{erf} \left(\frac{\theta - (90^\circ + \beta)}{\gamma} \right) \right) \\ * \left(1 + \operatorname{erf} \left(\frac{r - r_{al}}{d_{al}} \right) \right) \left(1 - \operatorname{erf} \left(\frac{r - r_{ah}}{d_{ah}} \right) \right). \quad (1.22)$$

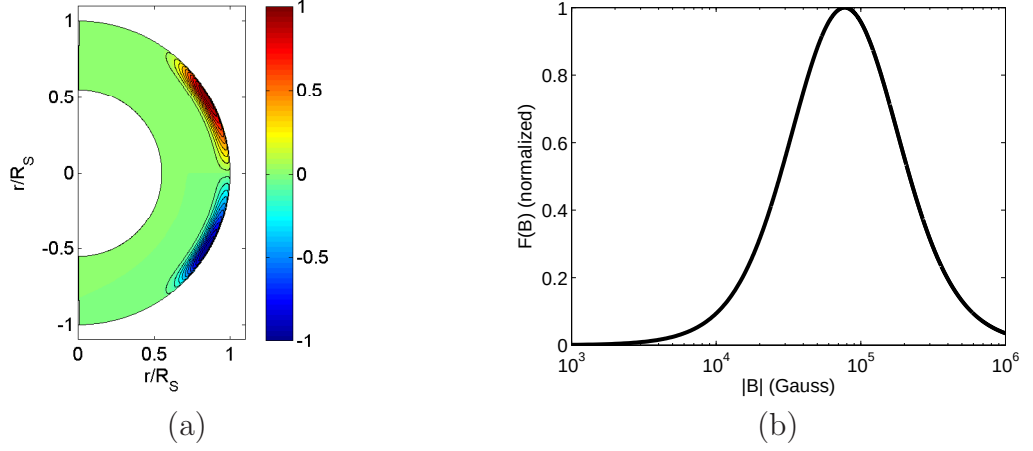


Figure 11. Poloidal Source: (a) Spatial dependence $\alpha(r, \theta)$. (b) Quenching function $F(B_{tc})$.

Here the parameters $\beta = 40^\circ$ and $\gamma = 10^\circ$ characterize the active latitudes and $r_{al} = 0.94R_\odot$, $d_{al} = 0.04R_\odot$, $r_{ah} = R_\odot$ and $d_{ah} = 0.01R_\odot$ characterize the radial extent of the region in which the poloidal field is deposited.

The final element of the source $F(B_{tc})$ adds non-linearity to the dynamo by quenching the source term for large and small values of toroidal field strength (see Figure 11-b). It is defined as:

$$F(B_{tc}) = \frac{K_{ae}}{1 + (B_{tc}/B_h)^2} \left(1 - \frac{1}{1 + (B_{tc}/B_l)^2} \right), \quad (1.23)$$

where K_{ae} is a normalization constant and $B_h = 1.5 \times 10^5 G$ and $B_l = 4 \times 10^4 G$ are the operating thresholds. The presence of a lower threshold is due to the fact that weak flux-tubes do not become unstable to buoyancy (Caligari, Moreno-Insertis & Schussler 1995) and those which manage to rise have very long rising times (Fan, Fisher and De Luca 1993). On the other hand, the higher threshold is a consequence of strong flux

tubes not being tilted enough when they reach the surface to contribute to poloidal field generation (D'Silva and Choudhuri 1993; Fan, Fisher and Deluca 1993).

1.4. A Problem of Constraints

By the beginning of 2006, when we decided to write a new dynamo code as part of this thesis (rather than use a pre-existing one), BL dynamos had achieved remarkable success in reproducing several of the characteristics of the solar magnetic cycle. The dynamo community was experiencing a general feeling of optimism and was preparing to put dynamo models to the test by making predictions for the coming sunspot cycle 24. However, as our work progressed we also realized that this type of models was far from being perfect.

In spite of their simplicity, modern kinematic dynamos have a large number of relatively free parameters, but not as many observational constraints. This has led to a heuristic exploration of the parameter space in search for solutions which would reproduce as closely as possible the different characteristics of the solar magnetic cycle. It goes without saying that this exploration has taught us a lot about the general properties of the system and the basic physics of the solar cycle. On the other hand, it has also shown us that solar-like solutions can be found in widely different physical regimes. This has led to controversy regarding which set of parameters is more appropriate (Nandy & Choudhuri 2002; Dikpati et al. 2002; Chatterjee, Nandy & Choudhuri 2004; Dikpati et al. 2005; Choudhuri, Nandy & Chatterjee 2005; Dikpati,

DeToma & Gilman 2006; Choudhuri, Chatterjee & Jiang 2007), but as of now, no consensus has been reached. However, if one analyzes carefully the different dynamo ingredients, it is clear that no consensus will ever be reached by relying exclusively on a heuristic approach. Because of this, we decided to approach the problem from a different perspective: instead of striving for the most solar-like solutions and assuming that it alone justifies our set of parameters, we take each ingredient independently and attempt to constrain it to the best of our capabilities; even if that means compromising the fidelity of our solutions.

1.4.1. The Meridional Flow

Since its discovery more than twenty years ago, the meridional flow has become a very prominent component of our understanding of the solar cycle and kinematic dynamo models. However, helioseismic measurements are still confined to a narrow layer just below the surface which unfortunately leaves a huge part of the meridional flow unobserved. This is clearly illustrated in Figure 12, where one can compare the required measurements by dynamo theory (top), versus the observed meridional circulation (bottom). However, in spite of the insufficiency of data, there has been little effort to systematically constrain meridional flow profiles used in dynamo theory by taking advantage of the available helioseismic data. Instead, the dynamo community has been content with a vague agreement between profiles used and observations, allowing for some freedom to fine tune the solutions. In the first part of our work we lay the necessary steps to constrain meridional flow profiles by making the most of

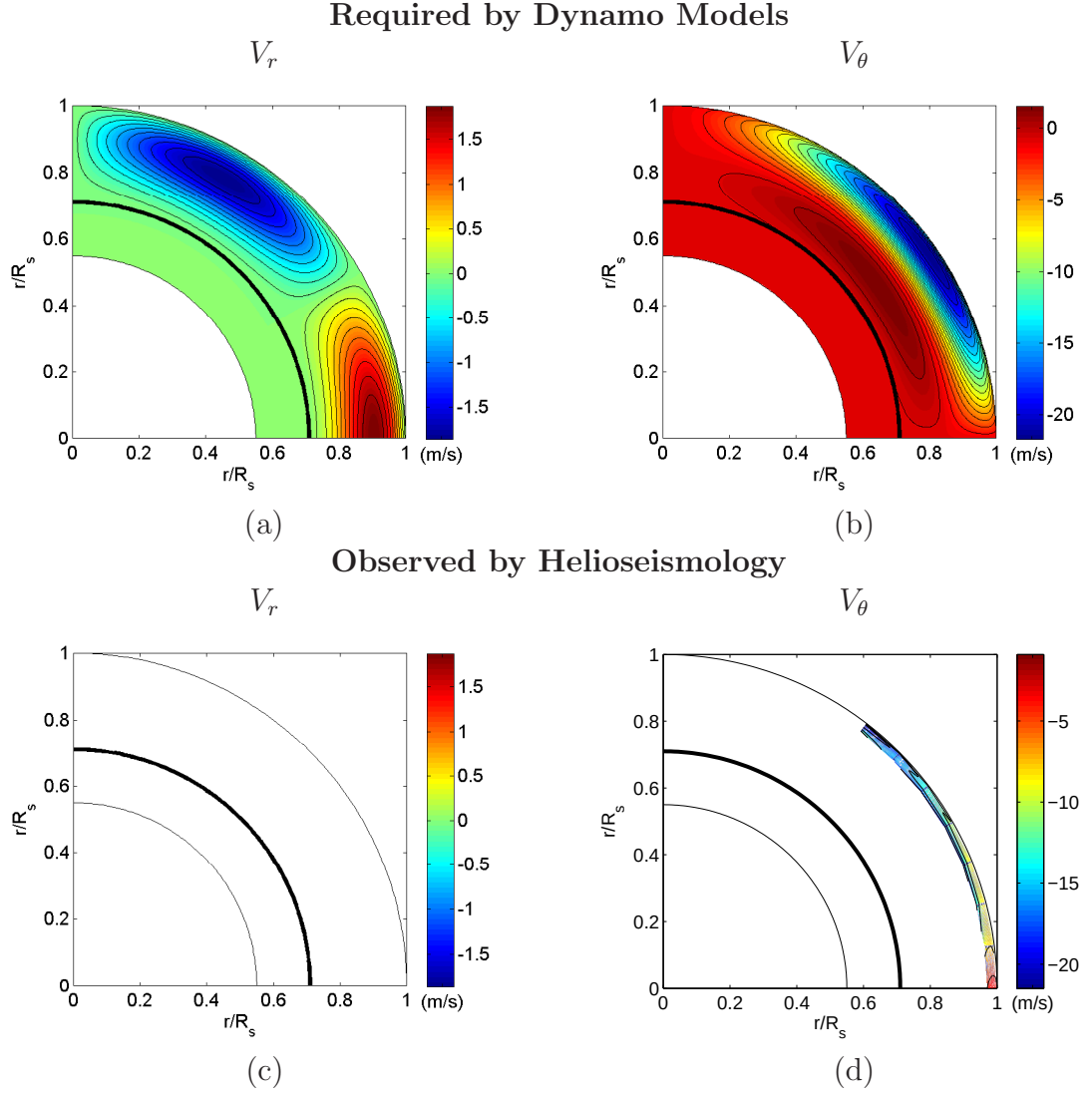


Figure 12. Radial (a) and latitudinal (b) components of a single cell meridional flow used in dynamo models. Radial (no data; c) and latitudinal (d) components of the observed meridional flow. Meridional flow profile from Muñoz-Jaramillo, Nandy & Martens (2009; a & b). Helioseismic data courtesy of Irene Gozález-Hernández (d).

the available data (see Chapter 2), paving the way for progressively improving these profiles as more data becomes available.

1.4.2. The Turbulent Magnetic Diffusivity: How to Tune Your Dynamo

Of all the different dynamo ingredients, none is treated more casually than the turbulent diffusivity when it comes to fine tuning the solutions. Fairly unconstrained with the exception of the very top of the computational domain, one of its main unofficial functions during the last decade has been to ensure that the solutions have the correct period (11.04 years to be precise). Because of this, different profiles used in the dynamo community can vary by as much as two orders of magnitude (see colored profiles in Figure 13). To add insult to injury, these profiles can be orders of magnitude below estimates based on mixing-length theory and models of the solar convection zone (see the black profile in Figure 13). We address that discrepancy and take measures to constrain the radial dependence of the turbulent diffusivity profile in a meaningful physical way (see Chapter 4).

1.4.3. The Babcock-Leighton Poloidal Source (or How I Learned to Stop Wor- rying and Love the Dynamo)

Although conceptually well defined (see Section 1.2.3), there is no solid mathematical formulation, akin to mean-field electrodynamics, from which the Babcock-Leighton poloidal source arises self-consistently. The first attempt to address this shortcoming was proposed by Durney (1995, 1997). In his approach, whenever the toroidal field at the bottom of the convection would reach a buoyant threshold, an axisymmetric double ring would be deposited at the surface. Unfortunately, although

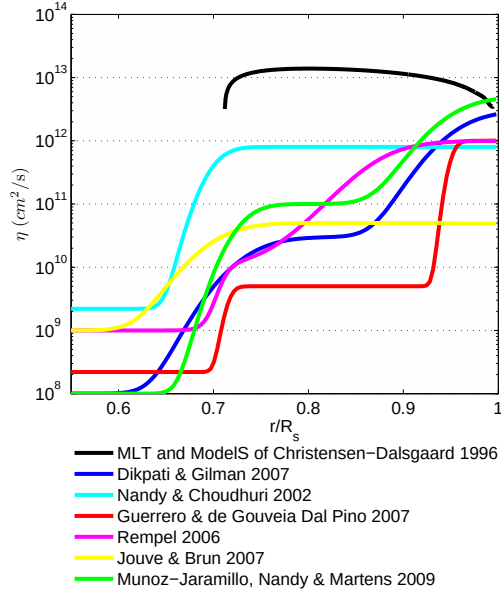


Figure 13. Turbulent magnetic diffusivity profiles used by the dynamo community (colored) in comparison with an estimate based on mixing-length theory and a model of the solar convection zone.

closest to the essence of the BL mechanism, this approach fell victim to its shortcomings (see Chapter 3) and was quickly replaced by continuous and semi-discrete formulations. In these type of approximations, dynamo modelers took advantage of the pre-existing machinery of the classical mean-field α (see Section 1.2.1), to model the BL mechanism as a “mean-field BL α -effect”.

Currently, the most commonly used formulation is the continuous non-local source explained in detail in Section 1.3.4. As an alternative, there is the local semi-discrete formulation proposed by Nandy & Choudhuri (2001; 2002). In this approach there is also a buoyancy threshold above which toroidal field is allowed to erupt to the surface (as in Durney’s double-ring), but the actual source of poloidal field involves

a term of the same nature as the one used by the non-local approach (see Eqs. 1.15 & 1.21):

$$S(r, \theta, B) = \alpha_0 f(r, \theta) F(B), \quad (1.24)$$

the main difference being that instead of creating poloidal field from the toroidal field at the bottom (B_{tc}), it uses the local surface toroidal field (B) as deposited by the buoyant eruptions. Although closer to the nature of the BL mechanism, its main problem is the same as that of the non-local approach, which is the vague quantitative connection between the global effect of active region emergence and decay and a mean-field α -effect.

Unfortunately, little work has been done to improve the BL poloidal source since its initial formulation (1999-2002); an oversight that may have resulted in a fundamental problem with the efficacy of this mechanism remaining unrecognized (see Chapter 6). Because of this, we took upon the task of dusting off Durney's double ring and improving upon it, in order to bring the truest implementation of the BL mechanism back to the forefront of dynamo theory (see Chapter 3). Finally, taking advantage of this new formulation, we also explore the causes of the unusual minimum of cycle 23 (see Chapter 5).

2. INCLUSION OF HELIOSEISMIC DATA IN SOLAR DYNAMO MODELS

At present, kinematic dynamo models incorporate the information on large-scale flows as analytic fits to the differential rotation profile and a theoretically constructed meridional circulation profile that is subject to mass conservation but matches the flow speed only at the solar surface (i.e., without incorporating the depth-dependent information that is available). However, these large-scale flows are crucial to the generation and transport of magnetic fields; the differential rotation is the primary source of the toroidal field that creates solar active regions, and the meridional flow is thought to play a crucial role in coupling the two source regions for the poloidal and toroidal field through advective flux transport. Given this, it is obvious that the next step in constructing more sophisticated dynamo models of the solar cycle is to move towards a more rigorous use of helioseismic data to constrain these models in a way such that they conform more closely to the best available observational constraints; that is the goal of this chapter.

On one hand, the differential rotation is probably the best constrained of all dynamo ingredients but the actual helioseismology data is rarely used directly in dynamo models, rather an analytical fit to it is used. We discuss below how the actual rotation data can be used directly within dynamo models through the use of a weighting function to filter out the observational data in the region where it cannot be trusted. On the other hand, the meridional circulation is one of the most loosely constrained ingredients of the dynamo. Traditionally only the peak surface

flow speed is used to constrain the analytical functions that are used to parameterize it, in conjunction with mass conservation. In this work we take advantage of the properties of such functions and make a fit to the helioseismic data on the meridional flow that constrains the location and extent of the polar downflow and equatorial upflow, as well as the radial dependence of the meridional flow near the surface – thereby taking steps towards better constrained flow profiles.

2.1. Specifics of the Model Used in this Work

There are two differences between the model used for these simulations and the one described in Section 1.3. The first one is the diffusivity profile (see Section 1.3.3) for which we use the following parameters: $\eta_{bcd} = 10^8 \text{cm}^2/\text{s}$ for the diffusivity at the bottom of the computational domain, $\eta_{cz} = 10^{11} \text{cm}^2/\text{s}$ for the diffusivity in the convection zone, $\eta_{sg} = 10^{13} \text{cm}^2/\text{s}$ for the supergranular diffusivity and $r_{cz} = 0.73R_{\odot}$, $d_{cz} = 0.03R_{\odot}$ to characterize the first step (η_{bcd} to η_{cz}), $r_{sg} = 0.95R_{\odot}$ and $d_{sg} = 0.05R_{\odot}$ to characterize second step (η_{cz} to η_{sg}).

The second difference are the flows themselves, since they are the the direct object of this chapter. In Sections 2.2 and 2.3 we present the methodologies for using the helioseismically observed solar differential rotation and constraining the meridional circulation profiles within this dynamo model and describe how they improve upon the commonly used analytic profiles.

The simulations are performed by integrating the dynamo equations (Eqs. 1.15 and 1.15) using a recently developed and novel numerical technique called exponential propagation (see Appendix). Our computational domain is a 250×250 grid covering only one hemisphere. Since we run our simulations only in one hemisphere our latitudinal boundary conditions at the equator ($\theta = \pi/2$) are $\partial A/\partial\theta = 0$ and $B = 0$. Furthermore, since the equations we are solving are axisymmetric, both the vector potential and the toroidal field need to be zero ($A = 0$ and $B = 0$) at the pole ($\theta = 0$). For the lower boundary condition ($r = 0.55R_\odot$), we assume a perfectly conducting core, such that both the radial field and the toroidal field vanish there (i.e., $A, B = 0$ at the lower boundary). For the upper boundary condition we assume that the magnetic field has only a radial component ($B = 0$ and $\partial(rA)/\partial r = 0$); this condition has been found necessary for stress balance between subsurface and coronal magnetic fields (for more details refer to van Ballegoijen and Mackay 2007). As initial conditions we set $A = 0$ throughout our computational domain and $B \propto \sin(2\theta) \times \sin[\pi((r - 0.55R_\odot)/(R_\odot - 0.55R_\odot))]$. After a few cycles, all transients related to the initial conditions typically disappear and the dynamo settles into regular oscillatory solutions whose properties are determined by the parameters in the dynamo equations.

2.2. Using the Measured Differential Rotation

As opposed to meridional circulation, there are helioseismic measurements of the differential rotation for most of the convective envelope which can be used directly in our simulations. Here we use data from the Global Oscillation Network Group (GONG) (courtesy Dr. Rachel Howe) obtained using the RLS inversion mapped onto a 51×51 grid (see Figure 14-a). However, these observations cannot be trusted fully in the region within $0.3R_{\odot}$ of the rotation axis (specifically at high latitudes), because the inversion kernels have very little amplitude there. Below we outline a method to deal with this suspect data by creating a composite rotation profile that replaces these data at high latitudes with plausible synthetic data, that smoothly matches to the observations in the region of trust.

2.2.1. Adaptation of the Data to the Model

In the first step we use a splines interpolation in order to map the data to the resolution of our simulation (a grid of 250×250 see Figure 14-a). The next step is to make a composite with the data and the analytical form of Charbonneau et al. (1999; see Figure 14-b). The analytical form is defined as:

$$\Omega_A(r, \theta) = 2\pi \left[\Omega_c + \frac{1}{2} \left(1 - \operatorname{erf} \left(\frac{r-r_{tc}}{w_{tc}} \right) \right) (\Omega_e - \Omega_c + (\Omega_p - \Omega_e) \Omega_S(\theta)) \right] \quad (2.1)$$

$$\Omega_S(\theta) = a \cos^2(\theta) + (1 - a) \cos^4(\theta),$$

where $\Omega_c = 432$ nHz is the rotation frequency of the core, $\Omega_e = 470$ nHz is the rotation frequency of the equator, $\Omega_p = 330$ nHz is the rotation frequency of the pole,

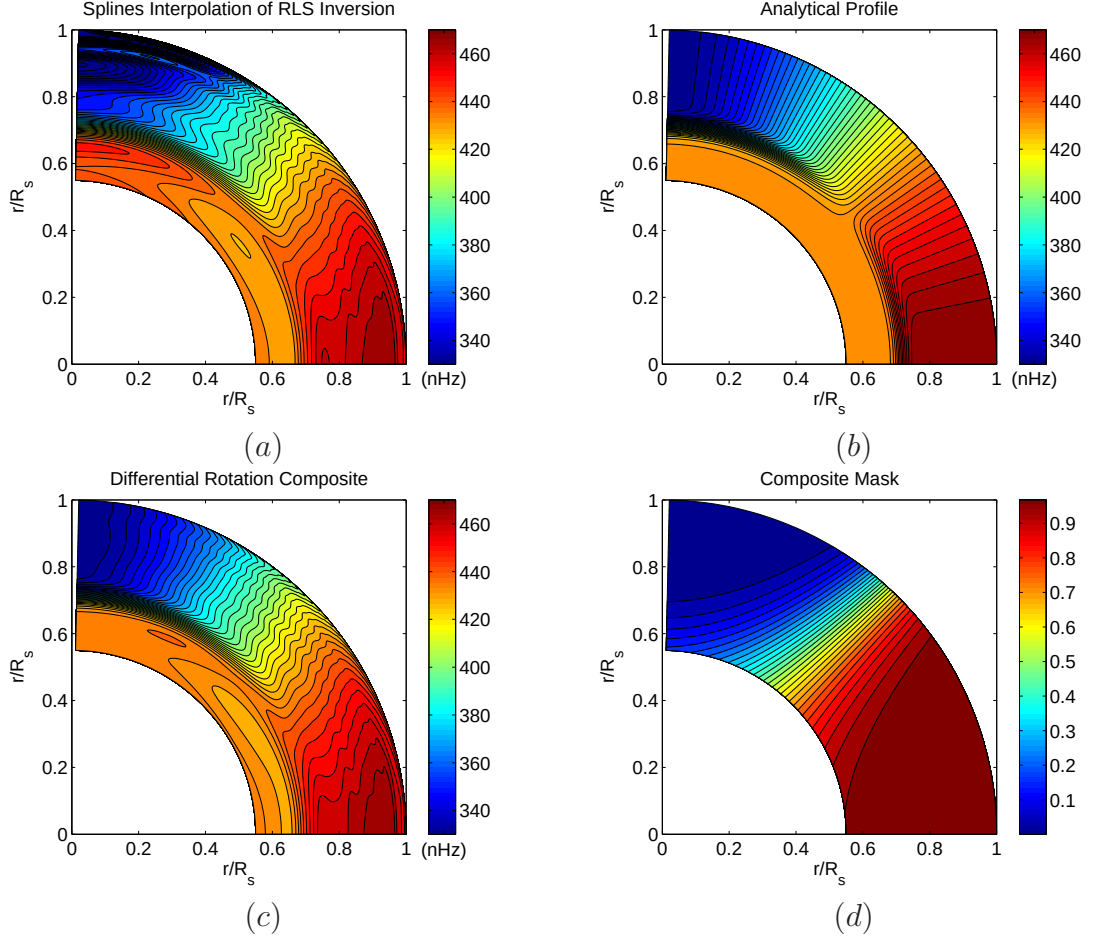


Figure 14. (a) Spline interpolation of the RLS inversion. (b) Analytical profile of Charbonneau et al. (1999). (c) Differential rotation composite used in our simulations. (d) Weighting function used to create a composite between the RLS inversion and the analytical profile of Charbonneau et al. (1999). For all figures the red denotes the highest and blue the lowest value and the units are nHz with the exception of the weighting function.

$a = 0.483$ is the strength of the $\cos^2(\theta)$ with respect to the $\cos^4(\theta)$ term, $r_{tc} = 0.716$ the location of the tachocline and $w_{tc} = 0.03$. We use the parameters defining the tachocline's location and thickness as reported by Charbonneau et al. (1999) for a latitude of 60° . This is because they match the data better at high latitudes (which is the place where the data merges with the analytical profile) than those reported

for the equator. This composite replaces the suspect data at high latitudes within $0.3R_{\odot}$ of the rotation axis with that of the analytic profile. However, it is important to note that at low latitudes, within the convection zone, the actual helioseismic data is utilized.

In order to make the composite we create a weighting function $m(r, \theta)$ with values between 0 and 1 for each grid-point defining how much information will come from the RLS data and how much from the analytical form (see Figure 14-d). We define the weighting function in the following way:

$$m(r, \theta) = 1 - \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{r^2 \cos(2\theta) - r_m^2}{d_m^2} \right) \right), \quad (2.2)$$

where $r_m = 0.5R_{\odot}$ is a parameter that controls the center of the transition and $d_m = 0.6R_{\odot}$ controls the thickness. The resultant differential rotation profile, which can be seen in Figure 14-c, is then calculated using the following expression:

$$\Omega(r, \theta) = m(r, \theta)\Omega_{RLS}(r, \theta) + [1 - m(r, \theta)]\Omega_A(r, \theta) \quad (2.3)$$

2.2.2. Differences Between the Analytical Profile and the Composite Data

It is instructive to compare the analytical and composite profiles with the actual helioseismology data. In Figure 15-a we present the residual error of subtracting the analytical profile of Charbonneau et al. (1999), with $r_{tc} = 0.7$ and $w_{tc} = 0.025$, from the RLS data. In Figure 15-b we present the residual error of subtracting our

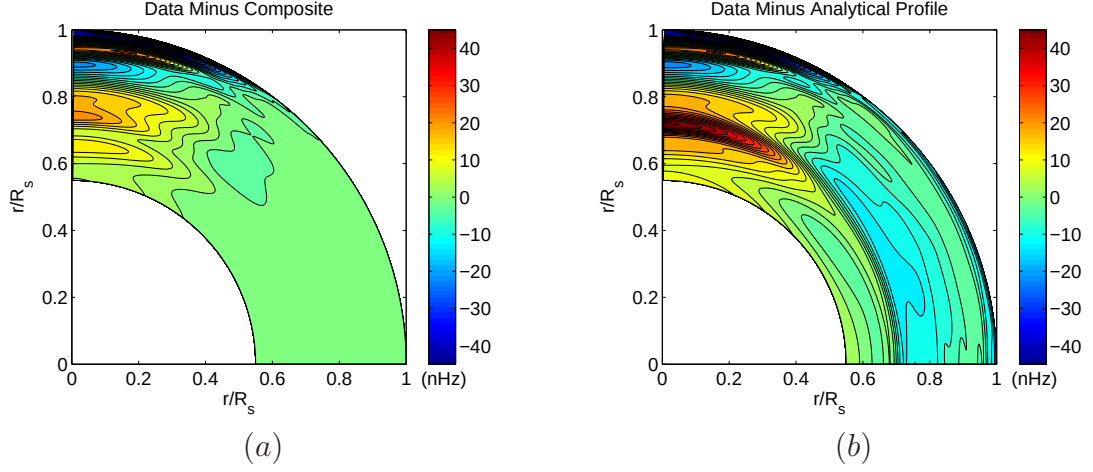


Figure 15. (a) Residual of subtracting the composite used in this work from the RLS inversion. (b) Residual of subtracting the analytical profile commonly used by the community from the RLS inversion. Red color corresponds to the highest value and blue to the lowest. Graphs in units of nHz.

composite from the RLS data. As is expected, there is no difference between the composite and raw data at low latitude, but the residual increases as we approach the rotation axis – where the RLS data cannot be trusted. For the analytical profile, the residuals errors are more significant, even at low latitudes. This demonstrates the ability of our methodology to usefully integrate the helioseismic data for differential rotation.

2.3. Using the Measured Meridional Circulation

The meridional flow profile remains rather poorly constrained in the solar interior even though the available helioseismic data can be used to constrain the analytic flow profiles that are currently in use. Here we present ways to better constrain this profile with helioseismic data. In order to do that, we use data from GONG (Courtesy Dr.

Irene González-Hernández) obtained using the ring-diagrams technique. This data, which we can see in Figure 17-a, corresponds to a time average of the meridional flow between 2001 and 2006; and it comprises 19 values of r from $R_\odot$ down to a depth of $0.97R_\odot$, and 15 different latitudes between -52.5° and 52.5° . It is important to note that our work relies heavily in the assumption that the meridional flow is adequately described by a stream function with separable variables. This is consistent with the assumption present implicit still in all work on axisymmetric solar dynamo models up to this date. Below we use this property of our stream function, along with weighted latitudinal and radial averages of the data, to completely constrain its latitudinal dependence, as well as the topmost ten percent of its radial dependence. As this data currently does not constrain the depth of penetration of the flow in the deep solar interior, we explore two different plausible penetration depths of the circulation. For reasons described later, we choose to perform simulations with two different peak meridional flow speeds, therefore exploring four plausible meridional flow profiles altogether.

2.3.1. Latitudinal Dependence of the MF

Meridional circulation has been typically implemented in these type of dynamo models by using a stream function consistent with $\nabla \cdot (\rho \mathbf{v}_p)$, i.e.:

$$\vec{\mathbf{v}}_p(r, \theta) = \frac{1}{\rho(r)} \vec{\nabla} \times (\Psi(r, \theta) \hat{\mathbf{e}}_\phi). \quad (2.4)$$

The two stream functions that are commonly used were proposed by van Ballegooijen and Choudhuri (1988) and by Dikpati and Choudhuri (1995). They have in common the separability of variables and thus can be written in the following way:

$$\Psi(r, \theta) = v_0 F(r) G(\theta), \quad (2.5)$$

where v_0 is a constant which controls the amplitude of the meridional flow.

Using such a stream function the components of the meridional flow become:

$$v_r(r, \theta) = v_0 \frac{F(r)}{r\rho(r)} \frac{1}{\sin(\theta)} \frac{\partial}{\partial \theta} (\sin(\theta) G(\theta)), \quad (2.6)$$

$$v_\theta(r, \theta) = -v_0 \frac{1}{r\rho(r)} \frac{\partial}{\partial r} (rF(r)) G(\theta), \quad (2.7)$$

which can themselves be separated into the multiplication of exclusively radially and latitudinally dependent functions. This property allows us to constrain the entire latitudinal dependence of this family of functions by using the available helioseismology data for v_θ at the surface. This can be done because the latitudinal dependence of v_θ is exactly the same as that of the stream function and only the amplitude of this functional form changes with depth, see for example Figure 16 for the latitudinal velocity at different depths used by van Ballegooijen and Choudhuri (1988). In this work we assume a latitudinal dependence like the one they used, i.e.,

$$G(\theta) = \sin^{(q+1)}(\theta) \cos(\theta). \quad (2.8)$$

In order to estimate the parameter q we first take a density weighted average of the helioseismic data using the values for solar density from the Solar Model S

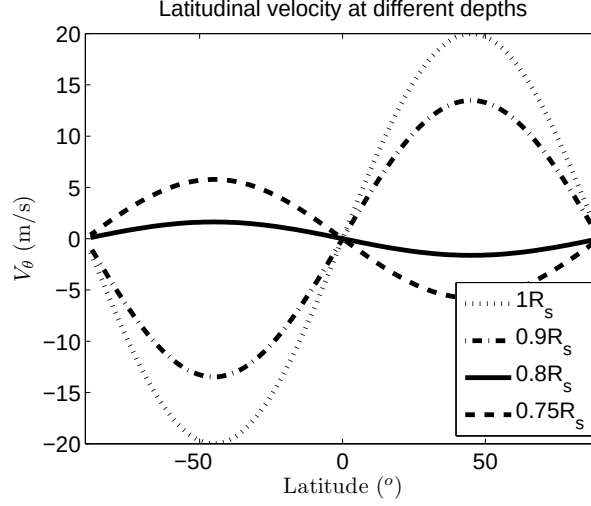


Figure 16. Latitudinal velocity as a function of θ used by van Ballegoijen and Choudhuri (1988) for different depths. Notice that the curves differ from each other only in their amplitude.

(Christensen-Dalsgaard et al. 1996), such that

$$\bar{v}_\theta(\theta_j) = \frac{\sum_i v_\theta(r_i, \theta_j) \rho(r_i)}{\sum_i \rho(r_i)}, \quad (2.9)$$

where the subindexes i and j denote the location of the helioseismic data-points in radius and latitude respectively. In Figure 17-b we plot the meridional flow at different depths weighted by density, which we add for each latitude in order to find the average. We then use this average to make a least squares fit to the analytical expression (eq 2.8), which we can see in Figure 17-c. We find that a value of $q = 1$ fits the data best. This therefore constrains the latitudinal (θ) dependence of the flow profile.

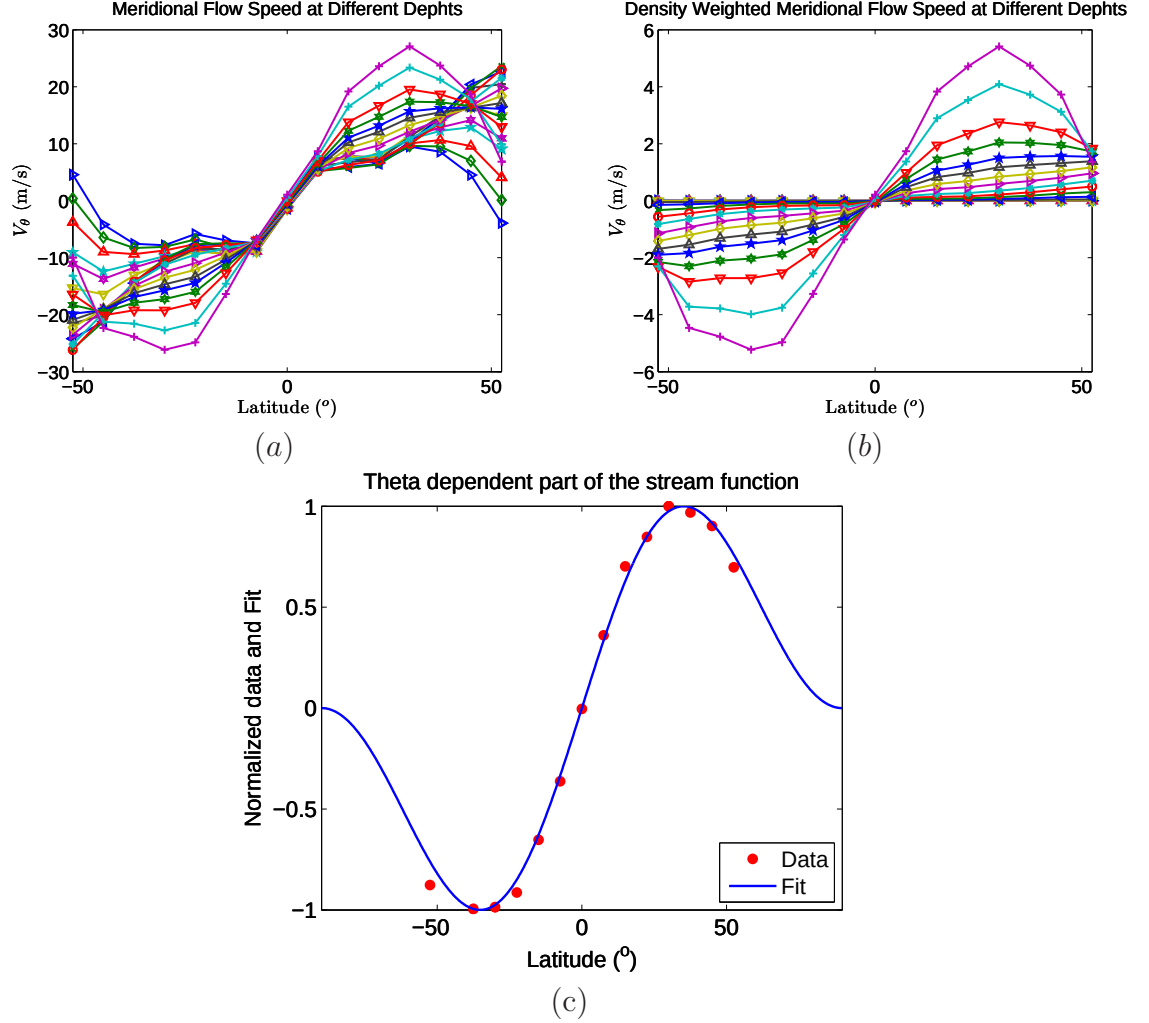


Figure 17. (a) Measured meridional flow as a function of latitude at different depths (Courtesy Dr. Irene González-Hernández), each combination of colors and markers corresponds to a different depth ranging from $0.97R_\odot$ to $R_\odot$. (b) Meridional flow after being weighted using solar density. We sum all data points at each latitude to obtain the average velocity. (c) Normalized average velocity and analytical fit.

2.3.2. Radial Dependence of the MF

As opposed to the latitudinal dependence, the radial dependence of the meridional flow is less constrained since there is no data below $0.97R_\odot$. However, at least some of the parameters can be constrained: We start with the solar density, for which we

perform a least squares fit to the Solar Model S using the following expression:

$$\rho(r) \sim \left(\frac{R_{\odot}}{r} - \gamma \right)^m \quad (2.10)$$

we find that values of $\gamma = 0.9665$ and $m = 1.911$ fit the model best (see Figure 18-c).

In the second step we constrain the radial dependence of the stream function. We begin with the function

$$F(r) = \frac{1}{r}(r - R_p)(r - R_{\odot}) \sin \left(\pi \frac{r - R_p}{R_1 - R_p} \right)^a, \quad (2.11)$$

where $R_{\odot}$ corresponds to the solar radius, R_p to the maximum penetration depth of the meridional flow and a and R_1 control the location in radius of the poleward peak and the value of the meridional flow at the surface. In order to constrain them we use the helioseismic data again, but this time we use the radial dependence (see Figure 18-a). We first remove the latitudinal dependence, which we do by dividing the data of each latitude by $G(\theta)$ using the value of $q = 1$ found in the previous Section (2.3.1). In Figure 18-b we plot the flow data after removing the latitudinal dependence, note that there is no longer any sign difference between the two hemispheres. The next step is to generate the latitudinal average which we can see as black dots in Figure 18-c. It is evident from looking at the radial dependence of the meridional flow that the velocity increases with depth for most latitudes, and that the point of maximum velocity is not within the depth up to which the data extends. Since the exact radial dependence of the data is too complex for our functional form to reproduce, the features we concentrate on reproducing are the presence of a maximum inside the

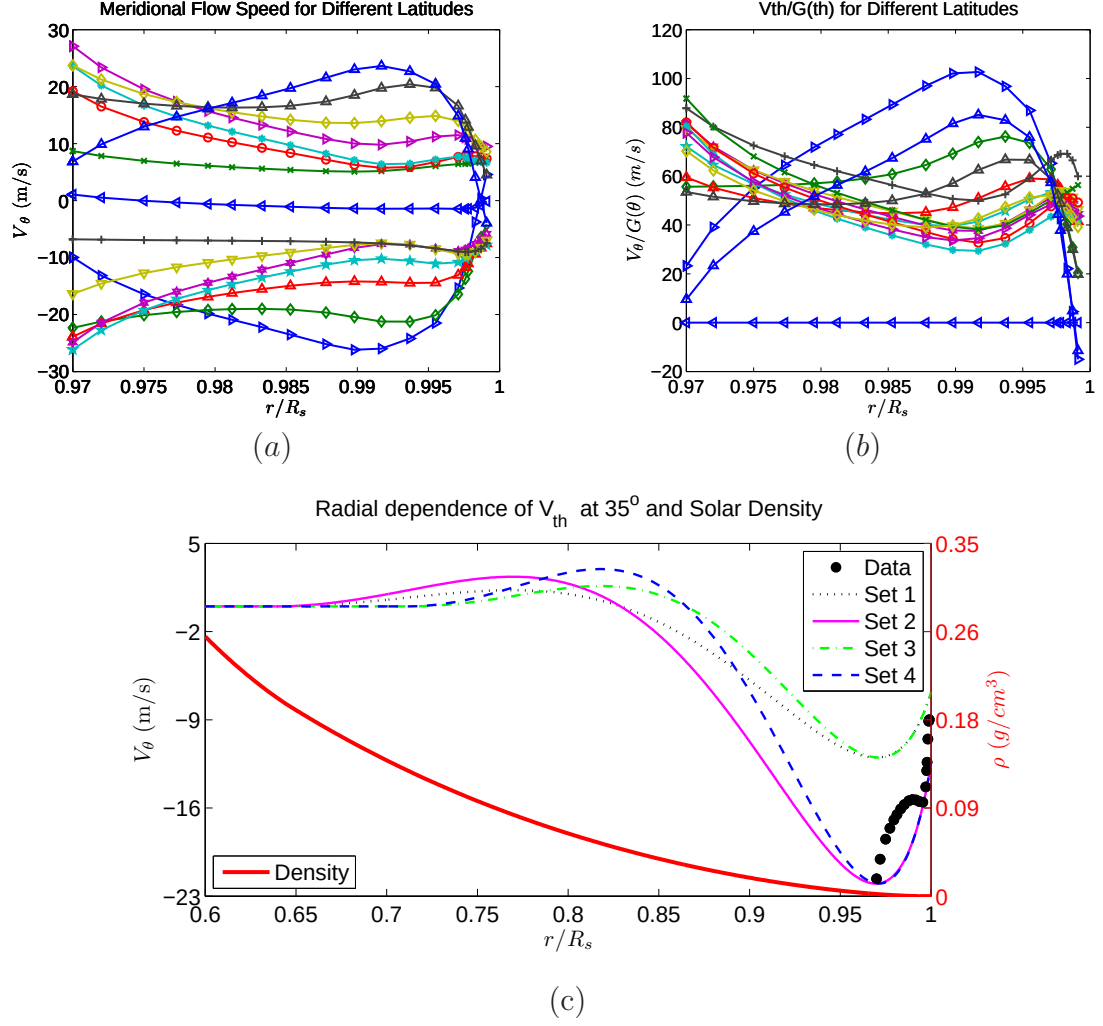


Figure 18. (a) Measured meridional flow as a function of radius at different latitudes (Courtesy Dr. Irene González-Hernández), each combination of colors and markers corresponds to a different latitude varying from -52.5° to 52.5° . (b) Meridional flow after removing the latitudinal dependence, i. e. $v_\theta/G(\theta)$. The horizontal line with zero latitudinal velocity corresponds to the equator. (c) Radial dependence of the latitudinally averaged meridional flow for our helioseismic data is depicted as large black dots. Other curves correspond to the radial dependence of the meridional flow profiles used in our simulations and solar density: Set 1 (black dotted) $R_p = 0.64R_\odot$, $v_o = 12m/s$; Set 2 (magenta solid) $R_p = 0.64R_\odot$, $v_o = 22m/s$; Set 3 (green dash-dot) $R_p = 0.71R_\odot$, $v_o = 12m/s$ and Set 4 (blue dashed line) $R_p = 0.71R_\odot$, $v_o = 22m/s$. The solar density taken from the solar Model S (Christensen-Dalsgaard et al. 1996) is depicted as a solid red line. The left-vertical axis is in units of velocity and the right-vertical in units of density.

convection zone, as well as the amplitude of the flow at the near surface-layers. The logic here is to use the fewest possible parameters and a simple, physically transparent profile that does a reasonable job of matching the data. Lacking better constraints, we assume here that the peak of the return flow is at $0.97R_{\odot}$ (which is the depth at which the current helioseismic data has its peak).

Following the procedures and steps above we construct profiles to fit the depth-dependence pointed out in the available helioseismic data; however, this does not constrain how far the meridional flow can penetrate and therefore we try two different penetration depths, one shallow $0.71R_{\odot}$ (i.e., barely beneath the base of the SCZ) and one deep $R_p = 0.64R_{\odot}$ (into the radiative interior) – both of which match the latitudinal and radial constraints as deduced from the near-surface helioseismic data. Note that observed light element abundance ratios limit the depth of penetration of the circulation to about $R_p = 0.62R_{\odot}$ (Charbonneau 2007). Now we know that the meridional flow speed is highly variable, with fluctuations that can be quite significant and the measured flow speed can change depending on the phase of the solar cycle (Hathaway 1996; Gizon & Rempel 2008). The magnetic fields are also expected to feed back on the flow (Rempel 2006). Taken together these considerations point out that the effective meridional flow speed to be used in dynamo simulations could be less than that implied from the González-Hernández et al. data, a possibility that is borne out by the work of Braun & Fan (1998) and Gizon & Rempel (2008), who find much lower peak flow speeds in the range 12–15 m/s. Keeping this in mind, we

use the same latitudinal and radial constraint as deduced earlier, but consider in our simulations two additional profiles with peak flow speeds of 12 m/s with deep and shallow penetrations. Therefore, in total we explore four plausible meridional flow profiles in our dynamo simulations (see Figure 18-c and Table 1 for an overview), the results of which are presented in the next Section (2.4).

	v_o	R_p	a	R_1
Set 1	12 m/s	$0.64R_\odot$	1.795	$1.027R_\odot$
Set 2	22 m/s			
Set 3	12 m/s	$0.71R_\odot$	2.03	$1.03R_\odot$
Set 4	22 m/s			

Table 1. Sets of parameters characterizing the different meridional flow profiles used in our dynamo simulations. v_o corresponds to the meridional flow peak speed, R_p the maximum penetration of the flow, and a and R_1 are parameters that control the location of the poleward flow as well as the surface speed.

2.4. Dynamo Simulation: Results and Discussions

2.4.1. Analytic vs. Helioseismic DR

We first compare dynamo solutions found using the helioseismology composite of the differential rotation (Section 2.2.1) and the analytical profile of Charbonneau et al. (1999). We perform dynamo simulations and generate field evolution maps for the toroidal and poloidal fields. From the simulated butterfly diagrams for the evolution of the toroidal field at the base of the SCZ and the surface-radial field evolution (Figure 19), we find that large-scale features of the simulated solar cycle are generally

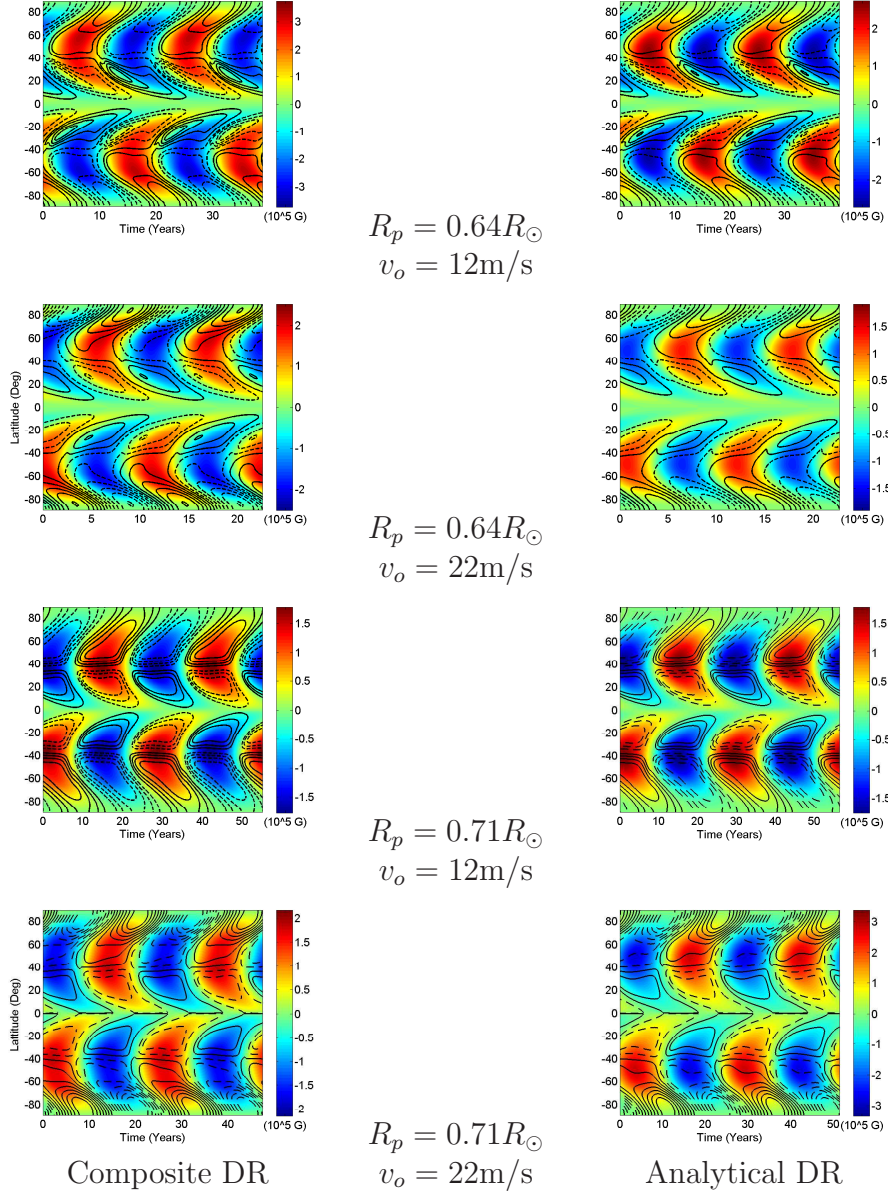


Figure 19. Butterfly diagram of the toroidal field at the bottom of the convection zone (color) with radial field at the surface (contours) superimposed. Each row corresponds to one of the different meridional circulation sets. The left column corresponds to simulations using the helioseismic composite and the right one to simulations using the analytical profile.

similar across the two different rotation profiles (even with different meridional flow penetration depths and speeds), especially for the shallowest penetration (sets 3 and 4 with $R_p = 0.71R_\odot$).

In order to understand this similarity, it is useful to look at Figures 21 and 22 corresponding to simulations using the composite differential rotation, and the meridional flow sets 1 ($R_p = 0.64R_\odot$, $v_o = 12m/s$) and 4 ($R_p = 0.71R_\odot$, $v_o = 22m/s$). The first two columns, from left to right of both figures show the evolution of the shear sources ($\mathbf{B}_r \cdot \nabla_r \Omega$) and ($\mathbf{B}_\theta \cdot \nabla_\theta \Omega$) – which contribute to toroidal field generation by stretching of the poloidal field. It is evident that the location and strength of these sources is different – the radial shear source is mainly present near the surface whereas the latitudinal shear is spread throughout the convection zone. It can also be seen that the radial shear source is roughly five times stronger than the latitudinal. However, if attention is paid to the evolution of the toroidal field (third column from the left in both figures), it is clear that this radial shear term has no significant impact on the structure and magnitude of the toroidal field (a similar result was found by Dikpati et al. 2002). The reason is that the upper boundary condition ($B = 0$ at $r = R_\odot$), in combination with the high turbulent diffusivity (and thus short diffusive time scale) there, imposes itself very quickly on the toroidal field generated by the radial shear – washing it out. This greatly reduces the relative role of the surface shear as a source of toroidal magnetic field, effectively making the surface dynamics very similar across

simulations using the analytical rotation profile (without any surface shear) or the composite helioseismic profile.

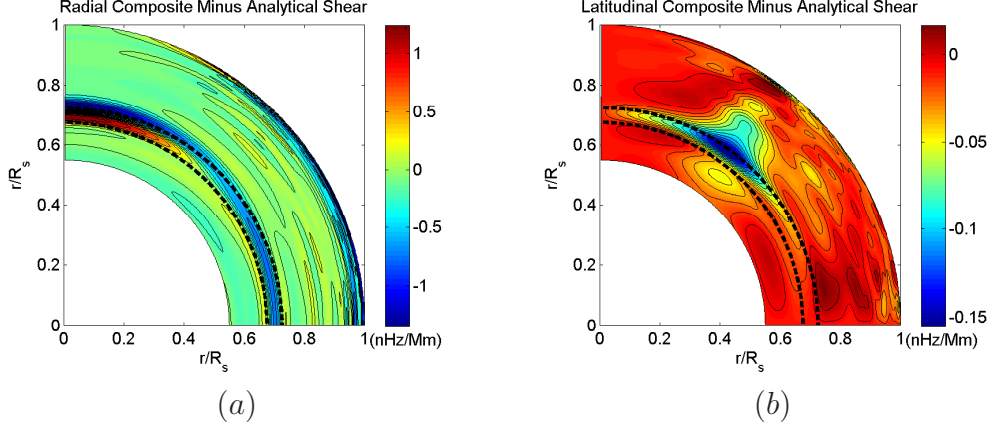


Figure 20. (a) Residual after subtracting the radial shear of the analytical profile commonly used by the community from the radial shear of our composite data. (b) Residual of subtracting the latitudinal shear of the analytical profile commonly used by the community from the latitudinal shear of our composite data.

Once the surface layers are ruled out as important sources of toroidal field generation we are faced with the fact that the strongest source of toroidal field is the latitudinal shear inside the convection zone (and not the tachocline radial shear), as is evident in Figures 21 and 22. This goes against the commonly held perception that the tachocline is where most of the toroidal field is produced. However, the importance of the latitudinal shear term in the SCZ is clearly demonstrated here where we have plotted the shear source terms, which is not normally done by dynamo modelers.

The establishment of the SCZ as an importance source region of toroidal field is of relevance when regarding the similarity of the solutions obtained using the composite data and the analytical profile. This is because the region where the shear of the

analytical profile and the helioseismic composite data differ most (both for radial and latitudinal shear) is in the tachocline. This is evident in Figure 20 where we plot the residual of subtracting the radial and latitudinal shear of the analytical profile from the shear of the composite data. The similarity between solutions is especially important for shallow meridional flow profiles with low penetration – which does not transport any poloidal field into the deeper tachocline, thereby further diminishing the role of the tachocline shear.

2.4.2. Shallow vs. Deep Penetration of the MF

In the second part of our work we compare dynamo solutions obtained for each of the four different meridional flow profiles with two different penetration depth and with two different peak flow speeds. First, from Figure 19 it is evident that the shape of the solutions changes with varying penetration depth; this is caused by the increasing role of the tachocline shear in generation and storage of the toroidal field as the penetration depth of the meridional flow increases. This is apparent when one compares Figure 21 (deep penetration) to Figure 22 (shallow penetration). If we look at the inductive shear sources it is evident that for the shallowest penetration no field is being generated inside the tachocline. In the poloidal field plots (right column) we see that no poloidal field is advected into the tachocline region for the shallowest penetration, but some is advected for the deepest.

Second we compare the periods of our solutions in Table 2: We find that most solutions have a sunspot cycle (i.e., half-dynamo cycle) period that is comparable to

that of the Sun, with the exception of the fast flow with deep penetration and the slow flow with shallow penetration, which have respectively a comparatively smaller and larger period. As the meridional flow is buried deeper, one expects the length of the advective circuit to increase, thereby resulting in larger dynamo periods. However, it is evident that as we increase the penetration, the period decreases – even if the length of the flow loop that supposedly transports magnetic flux increases; this is counterintuitive but has a simple explanation.

Our simulations and exhaustive analysis points out that it is not how deep the flow penetrates that governs the cycle period, but it is the magnitude of the meridional counterflow right at the base of the SCZ ($R_p = 0.713R_\odot$) that is most relevant. This is because most of the poloidal field creation at near-surface layers is coupled to buoyant eruptions of toroidal field from this layer of equatorward migrating toroidal field belt at the base of the SCZ (see third columns in Figures 21 and 22) and it is the flow speed at this region that governs the dynamo period. In Figure 18 it is clear that the speed of the counterflow in this convection zone–radiative interior interface increases as the flow becomes more penetrating (a consequence of the constraints set by mass conservation and the fits to the near-surface helioseismic data), thereby reducing the dynamo period.

Overall, an evaluation of the butterfly diagram (Figure 19), points out that the toroidal field belt extends to lower latitudes (where sunspots are observed) for deeper penetrating meridional flow, although there is a polar branch as well. For the shallow

flow, we find that the toroidal field belt is concentrated around mid-latitudes with almost symmetrical polar and equatorial branches – a signature of the convection zone latitudinal shear producing most of the toroidal field (as in the interface dynamo models, see, e.g., Parker 1993 and Charbonneau & MacGregor 1997).

	R_p	v_o	τ - HS data (yrs)	τ - Analytical (yrs)
Set 1	$0.64R_\odot$	12 m/s	9.67	10.00
Set 2		22 m/s	5.63	5.67
Set 3	$0.71R_\odot$	12 m/s	14.67	14.02
Set 4		22 m/s	11.85	12.85

Table 2. Simulated sunspot cycle period for the different sets of meridional flow parameters. R_p corresponds to the maximum penetration depth of the meridional flow, v_o to the peak speed in the poleward flow and τ is the period of the solutions in units of years. For the rest of the parameters in each set please refer to Table 1.

2.4.3. Dependence of the solutions on changes in the turbulent diffusivity profile

Although a detailed exploration of the turbulent diffusivity parameter space is outside the scope of this work, we study two special cases in which we vary a single parameter while leaving the rest fixed.

For the first case we lower the diffusivity in the convection zone, η_{cz} , from $10^{11}cm^2/s$ to $10^{10}cm^2/s$. As can be seen in Figure 23, this introduces two important changes in the dynamo solutions: The first one is an overall increase in magnetic field magnitude due to the reduction in diffusive decay while keeping the strength of the field sources constant. The second is a drastic increase in the dynamo period (which can be seen tabulated in Table 3) for the solutions that use a meridional flow

with a penetration of $R_p = 0.71R_\odot$. The reason behind such a change resides in the nature of the transport processes at the bottom of the convection zone, which are a combination of both advection and diffusion. In the case of flow profiles with deep penetration the velocity at the bottom is high enough for downward advection to transport flux into the tachocline. On the other hand, in the case of low penetration, the last bit of downward transport into the tachocline is done by diffusive transport and thus dominated by diffusive timescales. Because of this, by decreasing turbulent diffusivity by an order of magnitude, we drastically increase the period of the solutions.

	R_p	v_o	τ - HS data (yrs)	τ - Analytical (yrs)
Set 1	$0.64R_\odot$	12 m/s	12.46	12.86
Set 2		22 m/s	6.47	6.55
Set 3	$0.71R_\odot$	12 m/s	87.78	90.92
Set 4		22 m/s	70.48	74.41

Table 3. Simulated sunspot cycle period for the different sets of meridional flow parameters when using a low diffusivity in the convection zone ($\eta_{cz} = 10^{10} \text{cm}^2/\text{s}$). R_p corresponds to the maximum penetration depth of the meridional flow, v_o to the peak speed in the poleward flow and τ is the period of the solutions in units of years.

In the second parameter space experiment, we lower the super-granular diffusivity η_{sg} from $10^{13} \text{cm}^2/\text{s}$ to $10^{11} \text{cm}^2/\text{s}$. This was done to study the impact of the surface radial shear under low diffusivity conditions. However, as can be seen in Figure 24, there is very little difference between the two solutions. This means that even after reducing the super-granular diffusivity by two orders of magnitude, the radial shear has very little impact on the solutions and the upper boundary conditions still play

an important role in limiting the relative contribution from the near-surface shear layer.

2.5. Conclusions

In summary, we have presented here methods which can be used to better integrate helioseismic data into kinematic dynamo models. In particular, we have demonstrated that using a composite between helioseismic data and an analytical profile for the differential rotation, we can directly use the helioseismic rotation data in the region of trust and substitute the suspect data by smoothly matching it to the analytical profile where the data is noisy. This paves the way for including the helioseismically inferred rotation profile directly in dynamo simulations. We have also shown how mathematical properties of the commonly used analytic stream functions describing the meridional flow can be fit to the available near-surface helioseismic data to entirely constrain the latitudinal dependence of the meridional flow, as well as weakly constrain the radial (depth) dependence.

In our simulations, comparing the helioseismic data for the differential rotation with the analytical profile of Charbonneau et al. (1999), with four plausible meridional flow profiles, we find that there is little difference between the solutions using the helioseismic composite and the analytical differential rotation profile – specially for shallow penetrations of the meridional flow and even at reduced super-granular diffusivity. This is because the impact of the surface radial shear, which is present

in the helioseismic composite but not the analytic profile, is greatly reduced by the proximity of the upper boundary conditions. Also, for the shallow circulation, the toroidal field generation occurs in a region located above the tachocline with mainly latitudinal shear, where the difference between the composite data and the analytical profile is not significant.

The main result from this comparative analysis is that the latitudinal shear in the rotation is the most dominant source of toroidal field generation in these type of models that are characterized by high diffusivity at near surface layers, but lower diffusivity within the bulk of the SCZ – specially near the base where most of the toroidal field is being created. Since this latitudinal shear exists throughout the convection zone, an interesting question is whether toroidal fields can be stored there long enough to be amplified to high values by the shear in the rotation, without being removed by magnetic buoyancy. If this were to be the case, i.e., the latitudinal shear is indeed confirmed to be the dominant source of toroidal field induction, we anticipate then that downward flux pumping (Tobias et al. 2001; see also Guerrero & de Gouveia Dal Pino 2008) – which tends to act against buoyant removal of flux, may have an important role to play in this context. This could also call into question the widely held view that the solar tachocline is where most of the toroidal field is created and stored (see Brandenburg 2005 for arguments favoring a more distributed dynamo action throughout the SCZ).

Our attempts to integrate helioseismic meridional flow data into dynamo models and related simulations have uncovered points that are both encouraging and discouraging.

On the discouraging side, we find that the currently available observational data are inadequate to constrain the nature and exact profile of the deep meridional flow, especially the return flow. Neither do the simulation results and their comparison with observed features of the solar cycle clearly support or rule out any possibility. A recent analysis on light-element depletion due to transport by meridional circulation indicates that solar light-element abundance observations restrict the penetration to $0.62R_{\odot}$ (Charbonneau 2007); however this analysis does not necessarily suggest that the flow *does* penetrate that deep. Also vexing is the fact that different inversions, involving different helioseismic techniques such as ring-diagram or time-distance analysis recovers different profiles and widely varying peak meridional flow speeds (Giles et al. 1997; Braun & Fan 1998; González-Hernández et al. 2006; Gizon & Rempel 2008). In our analysis, we chose to use the González-Hernández et al. data because at present, this provides the (relatively) deepest full inversion of the flow within the SCZ. Chou and Ladenkov (2005) reported time-distance diagrams reaching a depth of $0.79R_{\odot}$ but have not yet reported a full inversion that could be used on our simulations.

We point out that there is an important consequence of the presence of the flow speed maximum inside the convection zone – which is related to mass conservation:

If the maximum poleward flow speed is found to be deeper inside the convection zone this would result in a stronger mass flux poleward, which needs to be balanced by a deeper counterflow subject to mass conservation; the density of the plasma increases rapidly as one goes deeper; e.g., the density at $0.97R_{\odot}$ is ten thousand times larger than at the surface. Although that is not achieved currently, our extensive efforts to fit the data point out that stronger constraints on the return flow may be achieved even with data that does not necessarily go down to where this return flow is located, a fact that may be usefully utilized when better depth-dependent helioseismic data on meridional circulation becomes available.

Although the depth of penetration of the circulation is an important constraint on the flow itself, our results indicate that the period of the dynamo cycle does not in fact depend on this depth. Rather, our simulations point out that the period of the dynamo cycle is more sensitive to changes on the speed of the counterflow than changes anywhere else in the transport circuit, as this is where the dynamo loop originates. An accurate determination of the average meridional flow speed over this loop closing at the SCZ base is very important in the context of the field transport timescales. As shown by the analysis of Yeates, Nandy & Mackay (2008), the relative timescales of circulation and turbulent diffusion determines whether the dynamo operates in the advection or diffusion dominated regime – two regimes which have profoundly different flux transport dynamics and cycle memory (the latter may lead to predictability of future cycle amplitudes). Getting a firm handle on the average

meridional flow speed is therefore very important and that is not currently achieved from the diverging helioseismic inversion results on the meridional flow.

This suggests that a concerted effort using different helioseismic techniques on data for the meridional flow over at least a complete solar cycle (over the same period of time) may be necessary to generate a more coherent picture of the observational constraint on this flow profile. It is important to note that even though we used time averaged data, nothing prevents one from using the same methods to assimilate time dependent helioseismic data at different phases of the solar circle, allowing us to study the impact of time varying velocity flows on solar cycle properties and their predictability.

On the encouraging side, our dynamo simulations show that it is relatively straightforward to use the available helioseismic data on the differential rotation (on which there is more consensus and agreement across various groups) within dynamo models. Also encouraging is the fact that the type of solar dynamo model presented here is able to handle the real helioseismic differential rotation profile and generate solar-like solutions. Moreover, as evident from our simulations, this dynamo model also generates plausible solar-like solutions over a wide range of meridional flow profiles, both deep and shallow, and with fast and slow peak flow speeds. This certainly bodes well for assimilating helioseismic data to construct better constrained solar dynamo models – building upon the techniques outlined here.

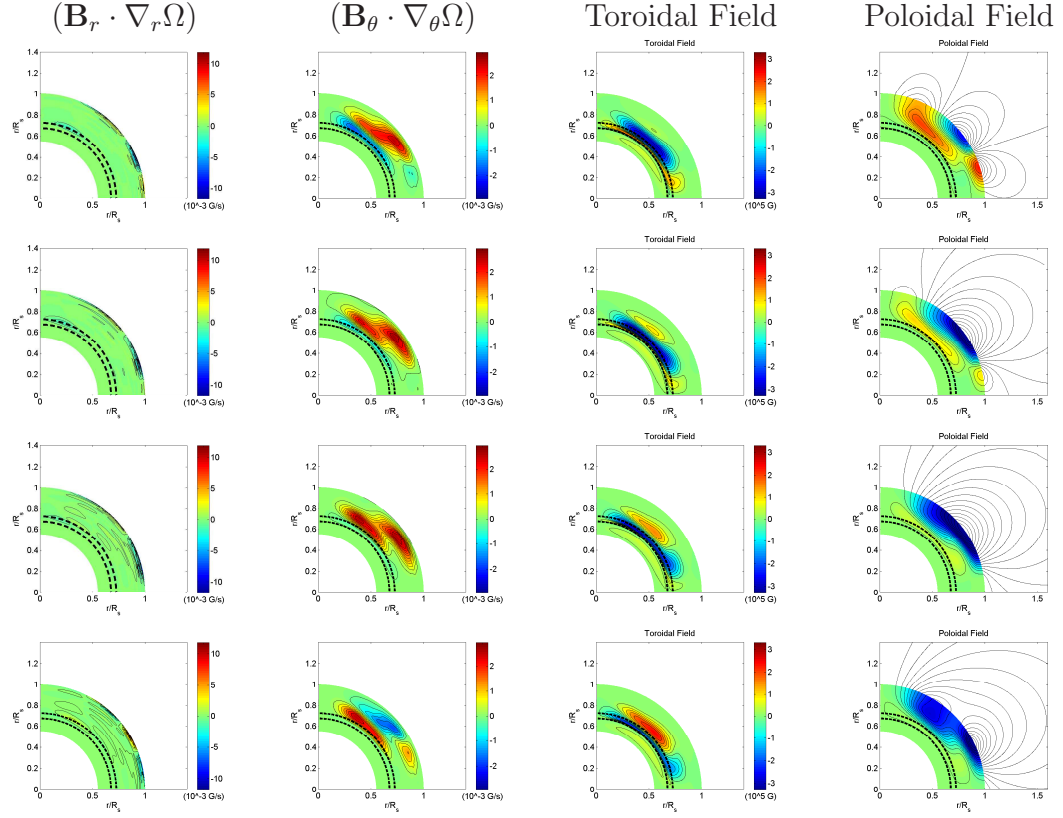


Figure 21. Snapshots of the shear source terms and the magnetic field over half a dynamo cycle (a sunspot cycle). Each row is advanced by an eighth of the dynamo cycle (a quarter of the sunspot cycle) i.e., from top to bottom $t = 0, \tau/8, \tau/4$ and $3\tau/8$. The solution corresponds to the composite differential rotation and meridional flow Set 1 (deepest penetration with a peak flow of 12 m/s).

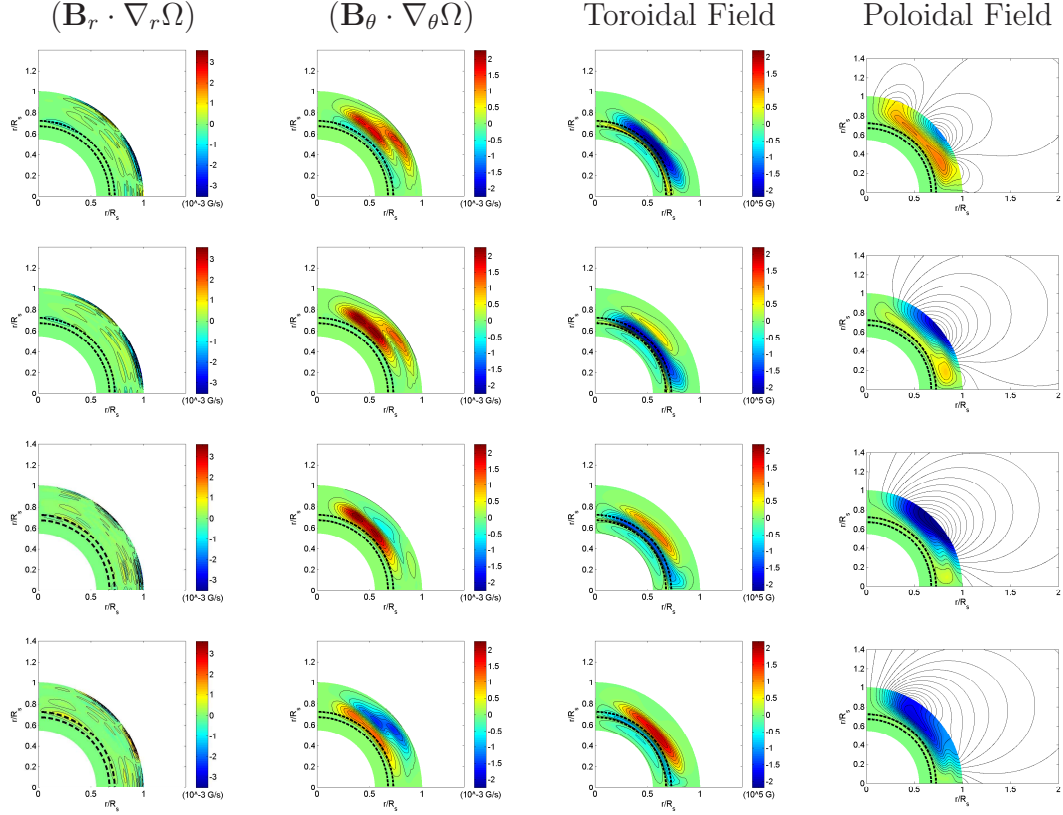


Figure 22. Snapshots of the shear source terms and the magnetic field over half a dynamo cycle (a sunspot cycle). Each row is advanced by an eighth of the dynamo cycle (a quarter of the sunspot cycle) i.e., from top to bottom $t = 0, \tau/8, \tau/4$ and $3\tau/8$. The solution corresponds to the composite differential rotation and meridional flow Set 4 (shallowest penetration with a peak flow of 22 m/s).

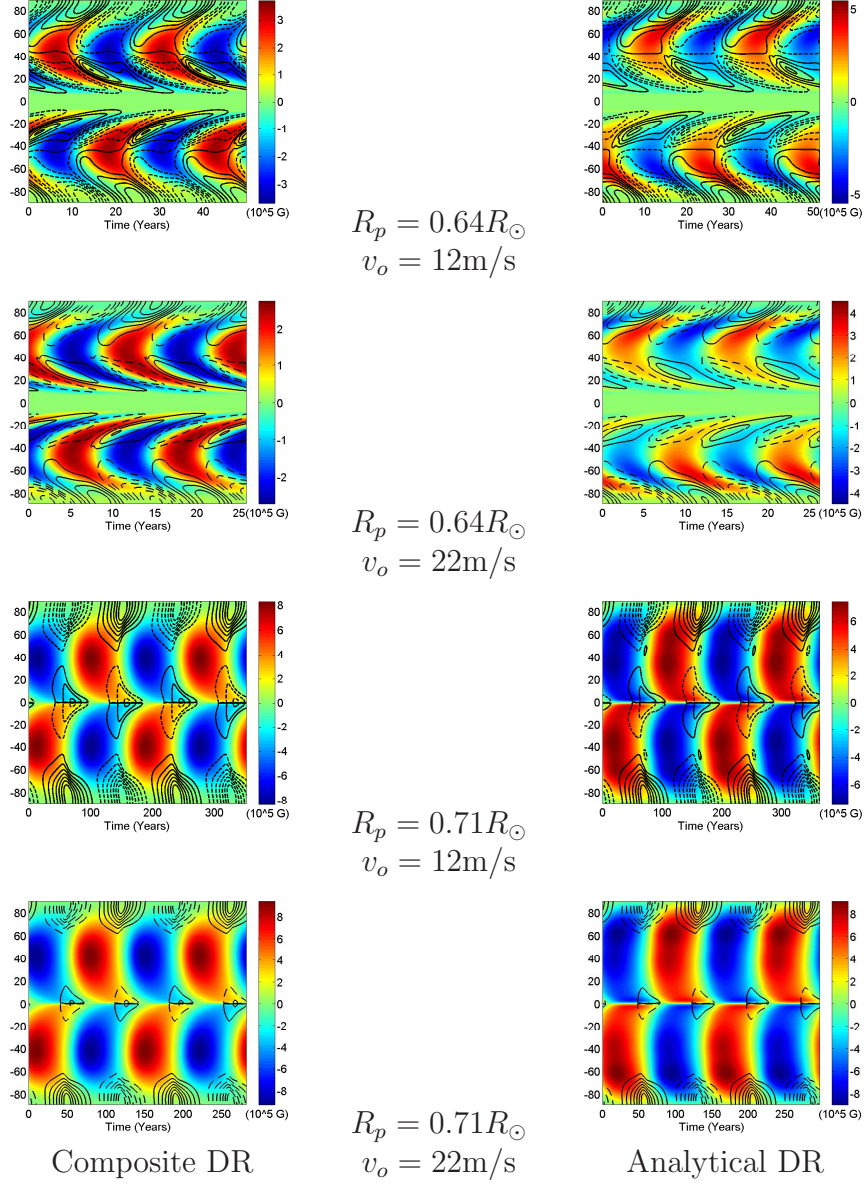


Figure 23. Butterfly diagram of the toroidal field at the bottom of the convection zone (color) with radial field at the surface (contours) superimposed using a low diffusivity in the convection zone ($\eta_{cz} = 10^{10}\text{cm}^2/\text{s}$). Each row corresponds to one of the different meridional circulation sets. The left column corresponds to simulations using the helioseismology composite and the right one to simulations using the analytical profile.

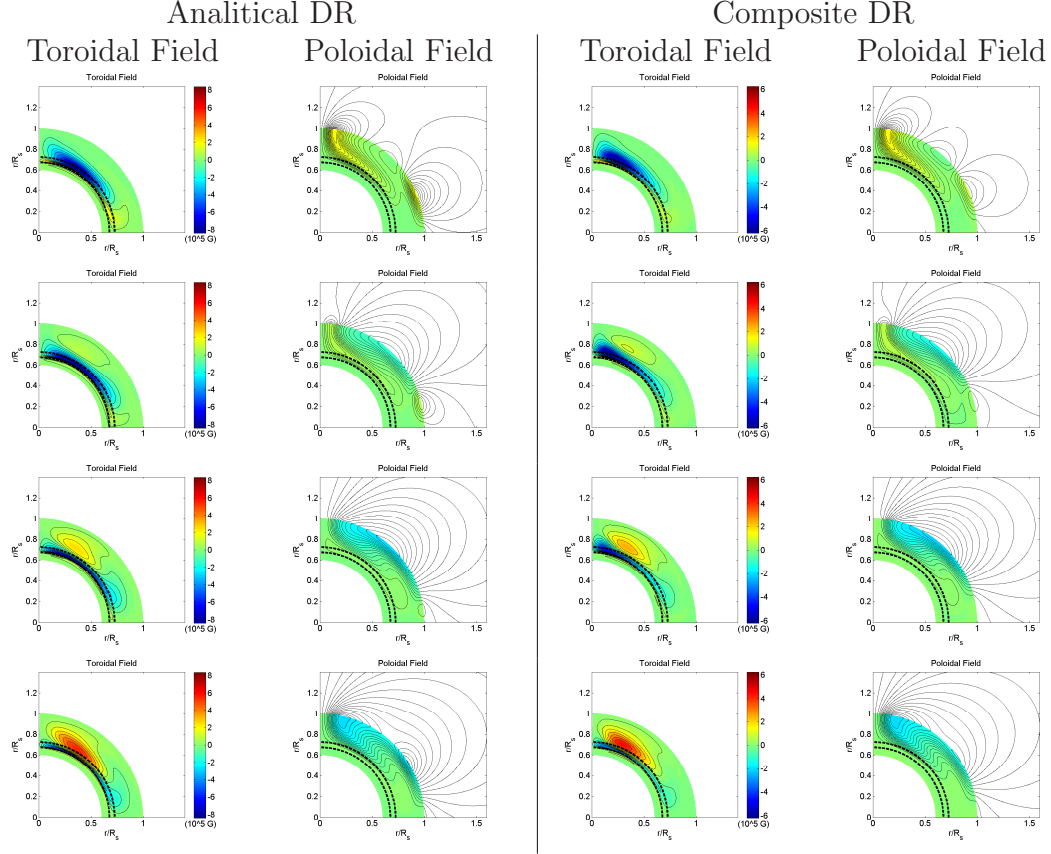


Figure 24. Snapshots of the magnetic field over half a dynamo cycle (a sunspot cycle) using a low super-granular diffusivity ($\eta_{cz} = 10^{11} \text{cm}^2/\text{s}$). Each row is advanced by an eighth of the dynamo cycle (a quarter of the sunspot cycle) i.e., from top to bottom $t = 0, \tau/8, \tau/4$ and $3\tau/8$. The solutions correspond to the meridional flow Set 2 (deepest penetration with a peak flow of 12 m/s) and analytic differential rotation (left) and composite data (right).

3. THE DOUBLE-RING ALGORITHM: A MORE ACCURATE METHOD FOR MODELING THE BABCOCK-LEIGHTON MECHANISM

As discussed earlier, the main mechanism for the recreation of poloidal field is believed to be the emergence of Active Regions (ARs), and their subsequent diffusion and transport towards the poles (Babcock 1961; Leighton 1969). However, after the introduction of the Babcock-Leighton (BL) mechanism in kinematic dynamo models (Choudhuri, Schüssler & Dikpati 1995; Dikpati & Charbonneau 1999; Nandy & Choudhuri 2001; Nandy & Choudhuri 2002), little has been done to improve upon the mean-field formulation. In this chapter we improve upon the idea proposed by Durney (1997) and further elucidated by Nandy & Choudhuri (2001) of using axisymmetric ring duplets to model individual active regions. We show how this captures the surface dynamics better than mean-field formulations and resolves previously found discrepancies between kinematic dynamo models and surface flux transport simulations.

3.1. Modeling Individual Active Regions

The initial implementation of the double-ring algorithm by Durney (1997) and Nandy & Choudhuri (2001) had two important deficiencies: strong sensitivity to changes in grid resolution and the introduction of sharp discontinuities in the ϕ component of the vector potential A . We address both of them through a careful mathematical definition of the vector potential associated with each double ring, in

combination with a substantial increase in grid resolution thanks to the increase of computational power during the last decade. We define the ϕ component vector potential A corresponding to an AR as:

$$A_{ar}(r, \theta) = K_0 A(\Phi) F(r) G(\theta), \quad (3.1)$$

where K_0 is a constant we introduce to ensure super-critical solutions and $A(\Phi)$ defines the strength of the ring-duplet and is determined by flux conservation. $F(r)$ is defined as

$$F(r) = \begin{cases} 0 & r < R_\odot - R_{ar} \\ \frac{1}{r} \sin^2 \left[\frac{\pi}{2R_{ar}} (r - (R_\odot - R_{ar})) \right] & r \geq R_\odot - R_{ar} \end{cases}, \quad (3.2)$$

where $R_\odot = 6.96 \times 10^8$ m corresponds to the radius of the Sun and R_{ar} represents the penetration depth of the AR. On the other hand, $G(\theta)$ is easier to define in integral form and in the context of the geometry of the radial component of the magnetic field on the surface. On Fig. 25-a we present a plot of the two super-imposed polarities of an AR after being projected on the r - θ plane. In order to properly describe such AR we need to define the following quantities: the co-latitude of emergence θ_{ar} , the diameter of each polarity of the duplet Λ , for which we use a fixed value of 6° (heliocentric degrees) and the latitudinal distance between the centers $\chi = \arcsin[\sin(\gamma) \sin(\Delta_{ar})]$, which in turn depends on the angular distance between polarity centers $\Delta_{ar} = 6^\circ$ and the AR tilt angle γ ; χ is calculated using the spherical law of sines. In terms of these quantities, the latitudinal dependence for each polarity is determined by the following piecewise function (use the top signs for the positive polarity and the lower

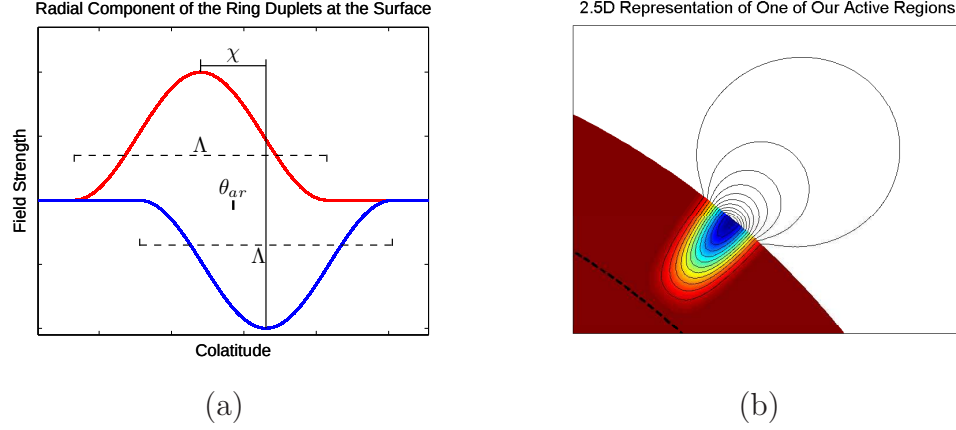


Figure 25. (a) Superimposed magnetic field of the two polarities of a modeled active region (tilted bipolar sunspot pair). The different quantities involved are: the colatitude of emergence θ_{ar} , the diameter of each polarity of the duplet Λ and the latitudinal distance between the centers χ . (b) Field lines of one of our model active regions including a potential field extrapolation for the region outside of the Sun. Contours correspond to field lines that trace the poloidal components and in this example their sense is counter-clockwise.

for negative):

$$B_{\pm}(\theta) = \begin{cases} 0 & \theta < \theta_{ar} \mp \frac{\chi}{2} - \frac{\Lambda}{2} \\ \pm \frac{1}{\sin(\theta)} \left[1 + \cos\left(\frac{2\pi}{\Lambda}(\theta - \theta_{ar} \pm \frac{\chi}{2})\right) \right] & \theta_{ar} \mp \frac{\chi}{2} - \frac{\Lambda}{2} \leq \theta < \theta_{ar} \mp \frac{\chi}{2} + \frac{\Lambda}{2} \\ 0 & \theta \geq \theta_{ar} \mp \frac{\chi}{2} + \frac{\Lambda}{2} \end{cases} \quad (3.3)$$

In terms of these piecewise functions $G(\theta)$ becomes:

$$G(\theta) = \frac{1}{\sin \theta} \int_0^{\theta} [B_{-}(\theta') + B_{+}(\theta')] \sin(\theta') d\theta'. \quad (3.4)$$

A model AR is shown in Fig. 25-b. This AR is located at a latitude of 40° and has a penetration depth of $0.85R_{\odot}$. The depth of penetration of the AR is motivated from results indicating that the disconnection of an AR flux-tube happens deep down in the CZ (Longcope & Choudhuri 2002).

3.2. Recreating the Poloidal Field

Given that the accumulated effect of all ARs is what regenerates the poloidal field, we need to specify an algorithm for AR eruption and decay in the context of the solar cycle. On each solar day of our simulation we perform the following procedure (once per hemisphere):

1. Search for magnetic fields exceeding a buoyancy threshold $B_c = 5 \times 10^4$ Gauss on a specified layer at the bottom of the CZ ($r = 0.71R_\odot$), and record their latitudes.
2. Choose randomly one of the latitudes found on Step 1 and calculate the amount of magnetic flux present within it is associated toroidal ring. The probability distribution we use is not uniform, but is restricted to observed active latitudes. We do this by making the probability function drop steadily to zero between 30° (-30°) and 40° (-40°) in the northern (southern) hemisphere:

$$P(\theta) \propto \left(1 + \operatorname{erf} \left[\frac{\theta - 0.305\pi}{0.055\pi} \right] \right) \left(1 - \operatorname{erf} \left[\frac{\theta - 0.694\pi}{0.055\pi} \right] \right). \quad (3.5)$$

3. Calculate the corresponding AR tilt, using the local field strength B_0 , the calculated flux Φ_0 and the latitude of emergence λ . For this we use the expression found by Fan, Fisher & McClymont (1994; $\gamma \propto \Phi_0^{1/4} B_0^{-5/4} \sin(\lambda)$).
4. Reduce the magnetic field of the toroidal ring from which the AR originates.

In order to do this, we first estimate how much magnetic energy is present on a

partial toroidal ring (after removing a chunk with the same angular size as the emerging AR). Given that this energy is smaller than the one calculated with a full ring, we set the value of the toroidal field such that the energy of a full toroidal ring filled with the new magnetic field strength is the same as the one calculated with the old magnitude for a partial ring.

5. Deposit an AR (as defined in Section 3.1), at the same latitude chosen on Step 2, whose strength is determined by the flux calculated in Step 2 and whose tilt was calculated on Step 3.

3.3. Evolution of Surface Magnetic Field

Given that AR emergence is strictly a non-axisymmetric process, it is important to study the amount of information lost by getting rid of the longitudinal dimension when modeling active regions through an axisymmetric ring. We do this by performing surface transport simulations in collaboration with Anthony Yeates who developed a state of the art surface transport model (see Yeates, Mackay & van Ballegooijen 2007). In a nutshell, surface transport simulations study the evolution of the photospheric magnetic field by integrating the induction equation using prescribed meridional flow, differential rotation and turbulent diffusivity. There are two main differences between them and kinematic dynamo models: the computational domain is restricted to the surface (but is not axisymmetric) and they are not self-excited, being driven by the deposition of AR bipolar pairs. This type of models has proved a successful tool for

understanding surface dynamics on long time-scales (see for example Mackay, Priest & Lockwood 2002; Wang, Lean & Sheeley 2002; Schrijver, De Rosa & Title 2002) and the evolution of coronal and interplanetary magnetic field (see for example Lean, Wang & Sheeley 2002; Yeates, Mackay & van Ballegooijen 2008).

In order to study the relevance of the non-axisymmetric component on timescales comparable with the solar cycle we perform a regular surface flux transport simulation in which the bipolar ARs are distributed all across the surface of the Sun (Case 1) and another in which the same set of ARs is deposited at the same Carrington longitude while leaving other properties (time, latitude of emergence and flux) intact (Case 2). The difference between both simulations is illustrated in the top row of Fig. 26, where we show a snapshot of the surface magnetic field at the peak of the cycle for Case 1 (Fig. 26-a) and Case 2 (Fig. 26-b). It is clear that these cases yield entirely different magnetic configurations at the time of deposition. However, the surprising result comes when the magnetic field is averaged in longitude and stacked in time to create a magnetic synoptic map (also known as butterfly diagram; Figs. 26-b & 26-c). A careful comparison shows that they are essentially the same within a margin of 1% (Figs. 26-e & f). Note that our claim is not that surface flux transport simulations are unnecessary; non-axisymmetry is essential for the evolution of the corona and interplanetary magnetic field. Instead, this result suggests that an axisymmetric representation of surface dynamics may not be far off the mark if we are concerned

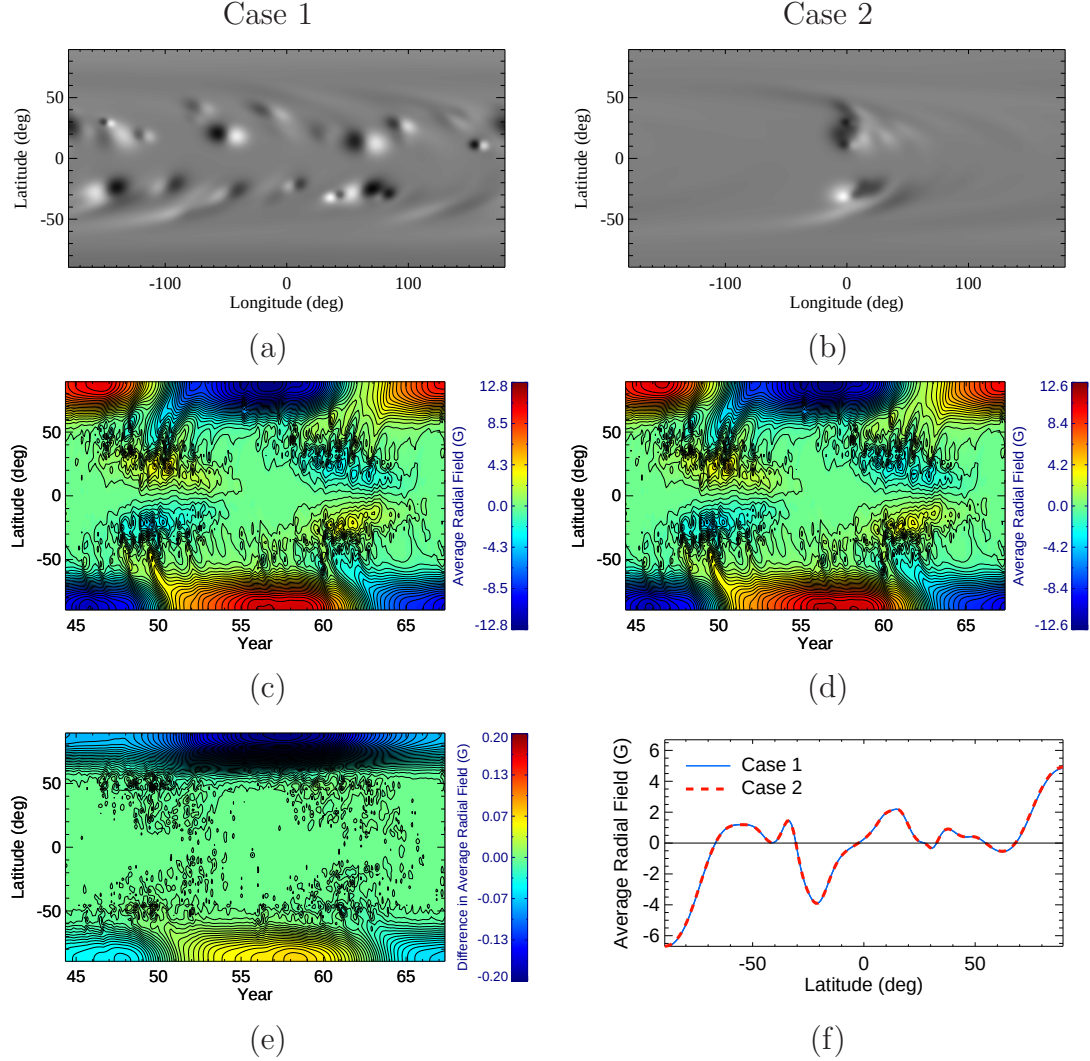


Figure 26. Long term evolution of the photospheric magnetic field in a surface flux transport simulation. The left column shows results of a simulation in which active regions are deposited across the surface of the Sun (Case 1). The right column shows results of a simulation in which the same set of active regions is deposited at the same Carrington longitude. The top row shows a snapshot of the magnetic field at the peak of the cycle for Case 1 (a) and Case 2 (b). The middle row shows the butterfly diagram for Case 1 (c) and Case 2 (d) obtained by averaging the surface magnetic field in longitude. It is clear that in spite of very different magnetic field configurations the evolution of the axisymmetric component is essentially the same. It is important to highlight that these are not the same simulation, as can be seen from their difference (e). However their butterfly diagrams are similar within a margin of 1%. To further illustrate we show the longitudinal average of the magnetic configurations shown on the top row (f), the blue solid line corresponds to the top left panel (a) and the red dashed line to the top right panel (b). It is evident that their axisymmetric component is essentially the same. Simulations performed by Anthony Yeates.

with the general properties of the magnetic field at the surface over solar cycle time-scales. In Chapter 6 we will show more evidence supporting this claim.

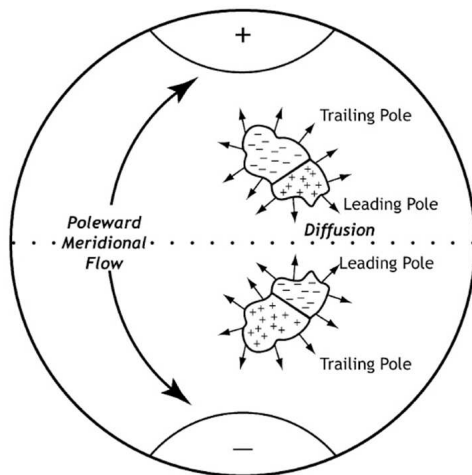


Figure 27. Diagram showing the essential role of the cancellation across the equator under the BL mechanism. Unless there is cross-equatorial cancellation there will not be much net flux available for concentration at the pole. This process of cancellation is enhanced by diffusion and opposed by meridional flow. Figure by Yi-Ming Wang (2004).

3.4. Addressing the Discrepancy Between Kinematic Dynamo Models and Surface Flux Transport Simulations

A discrepancy between kinematic dynamo models and surface flux transport simulations exists regarding the relationship between meridional flow amplitude and the strength of the polar field (Schrijver & Liu 2008; Hathaway & Rightmire 2010). On one hand kinematic dynamo models find that a stronger meridional flow results in

stronger polar field (Dikpati, de Toma & Gilman 2008), whereas surface flux transport simulations find an inverse relationship (Wang, Sheeley & Lean 2002). In order to resolve this discrepancy it is useful to go back to the basics of surface dynamics. In order to have a net accumulation of unipolar field at the poles, it is necessary to have an equal amount of flux cancelation across the equator. Since the meridional flow is poleward in the top part of the convection zone (see Fig. 27), it essentially acts as a barrier against flux cancellation by sweeping active regions towards the poles. This leads to the inverse relationship found by flux transport simulations. However, if there is already a strong separation of charges, a strong meridional flow will lead to an enhancement of the polar field due to flux concentration. This unrealistically strong separation is typical of kinematic dynamo models which use a non-local mean-field BL source (see Fig. 28-b & c). The reason is that by increasing the vector potential A proportionally to the toroidal field B at the bottom of the convection zone (see Eq. 1.21), one creates strong gradients in the vector potential above the edges of the toroidal field belt; this ends up producing poloidal field immediately which is as large in length-scale as the toroidal itself, circumventing the whole process of flux transport by circulation and diffusion. This separation is not present in a dynamo simulation using the double ring algorithm (see Fig. 28-a & d).

3.4.1. Specifics of the Model Used in this Work

In order to study the relationship between meridional flow and polar field strength, we perform simulations in which we track the solar cycle and randomly change the

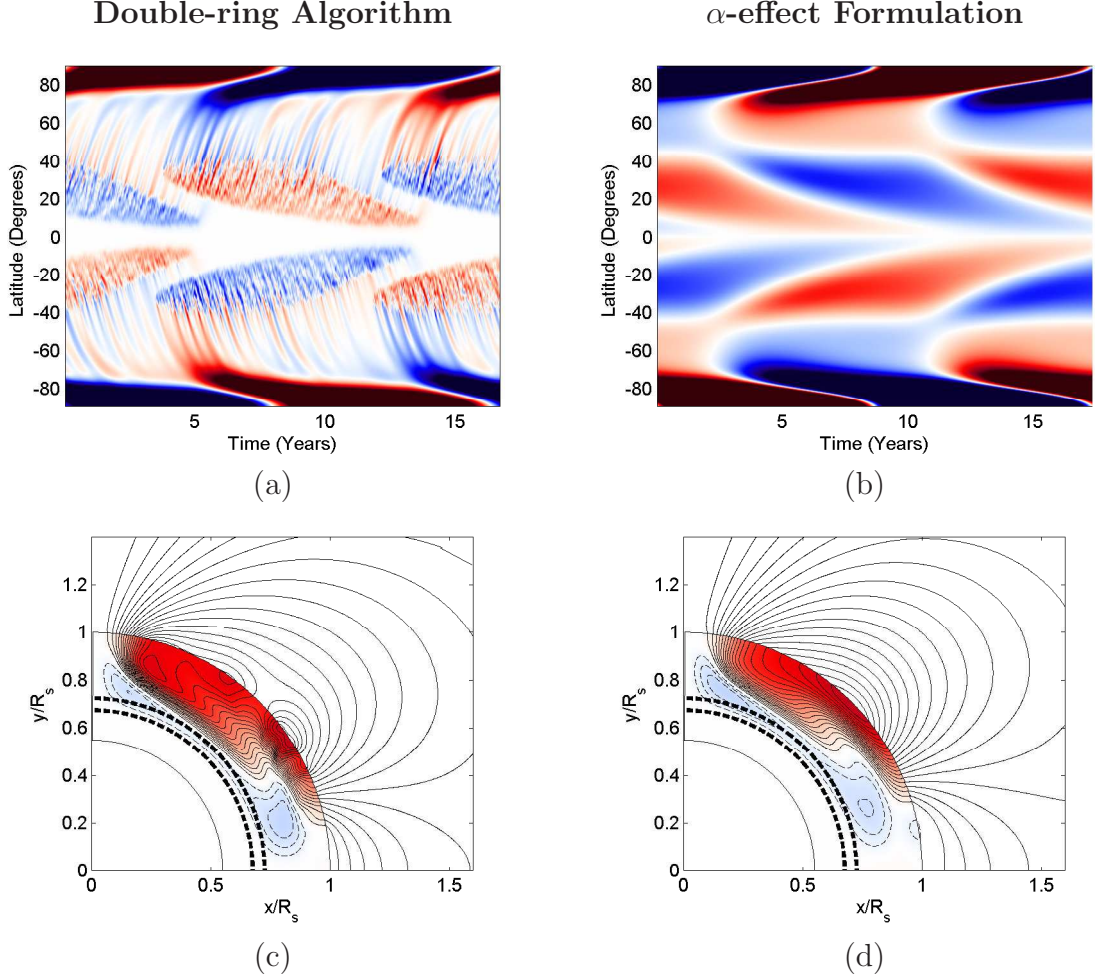


Figure 28. Comparison between surface dynamics as captured by the double-ring algorithm (left column) and the α -effect formulation (right column). The top row shows the evolution of the surface magnetic field in the form of synoptic maps – the colormap is saturated to enhance the visibility of the field at mid to low latitudes. The bottom row shows a snapshot of the poloidal components of the magnetic field taken at solar max. The solid contours corresponds to clockwise field-lines, the dashed contours correspond to counter-clockwise field-lines. The thick dashed lines mark the location of the tachocline.

meridional flow amplitude from one sunspot cycle to another (between 15 – 30 m/s).

This is illustrated in Fig. 29 where a series of sunspot cycles is plotted along with their associated meridional flow. We then evaluate the correlation between the amplitude

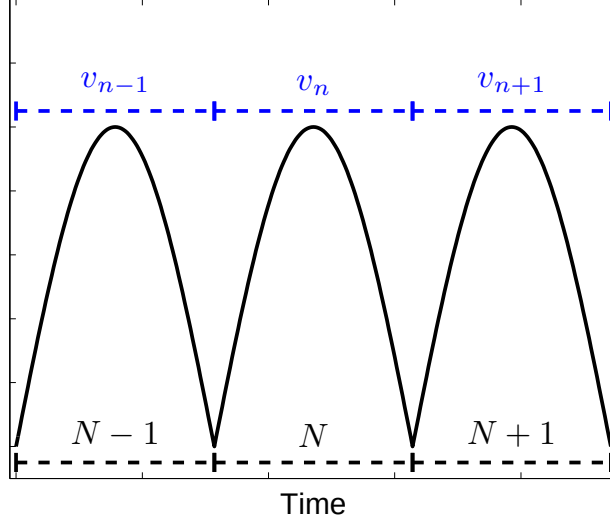


Figure 29. Diagram showing the evolution of the meridional flow amplitude with respect to the sunspot cycle: each solar cycle has a unique meridional flow strength which is randomly chosen between 15 – 30 m/s.

of the meridional flow of a given cycle and the polar field strength at the end of it. Since we want to evaluate the relative performance of the double-ring algorithm as opposed to the non-local BL source, we perform the same simulation for both types of sources. Aside from the meridional flow amplitude and the poloidal source, we use the rest of the ingredients as described in Section 1.3. It is important to note that partly due to difficulties in tracking the exact moment of solar minimum, the two hemispheres eventually get out of phase in long simulations – sometimes this phase difference leads to quadrupolar solutions and sometimes back to the observed dipolar solution. However, this parity issue only appears when the meridional flow is changed at solar minimum: if there are no variations, or if the variation takes place at solar

max, the cycle always goes back in phase. To avoid muddling the results, in this work we accumulate statistics only from cycles in which the two hemispheres are in phase. The statistics performed for both types of source contain about 200 sunspot cycles.

3.4.2. Results

Fig. 30 shows the results of both simulations. We find a weak positive correlation between meridional flow and polar field strength for the simulations using the mean-field non-local formulation (Fig. 30-top), which is in general agreement with the results of Dikpati, de Toma & Gilman (2008). On the other hand, the simulations using the double ring distinctively show a negative correlation (Fig. 30-bottom), as found by surface flux transport simulations (Wang, Sheeley & Lean 2002). This clearly shows that the discrepancy between the models is solved by introducing the double-ring algorithm and that it does a better job at capturing the observed and simulated physics of the surface dynamics. After performing this two simple tests we are now in a position in which we can use the double-ring algorithm with confidence.

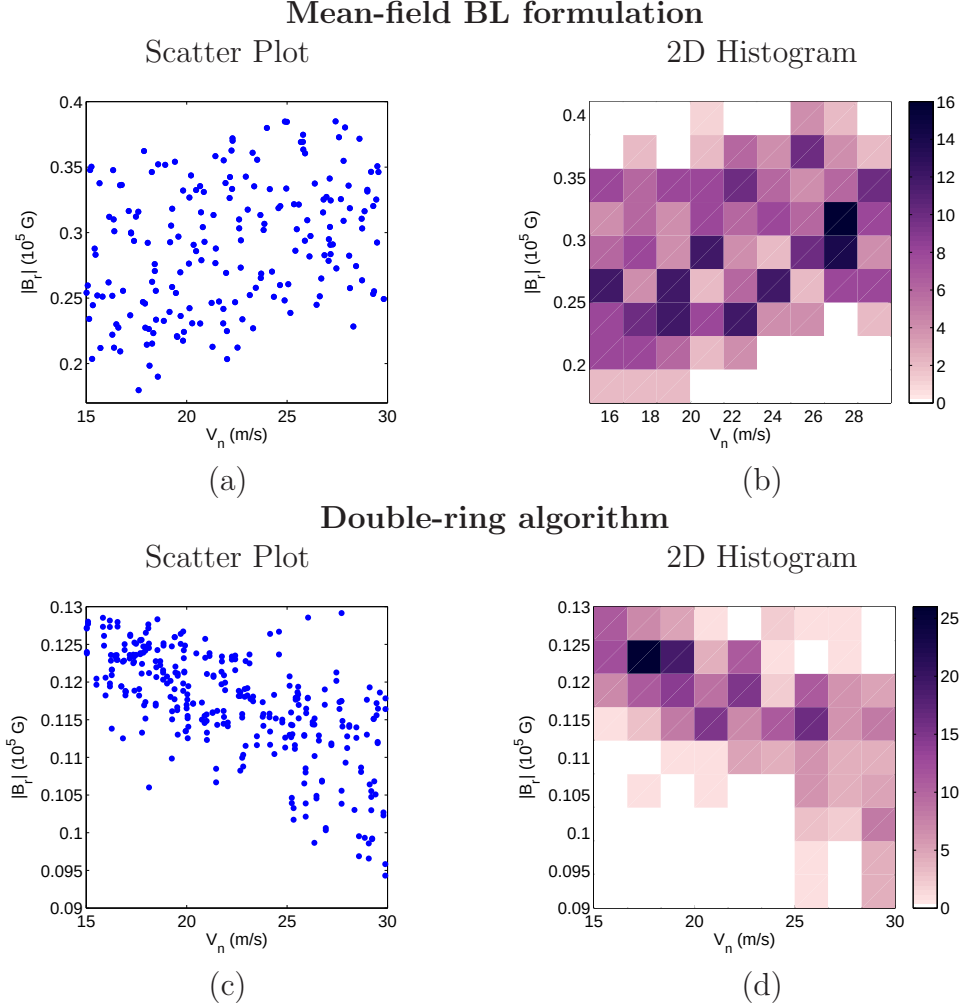


Figure 30. Relationship between randomly varying meridional flow speed and polar field strength. The polar field strength (in Gauss) is represented by the maximum amplitude of the polar radial field (Br) attained during a solar minimum. The relationship between the above parameters is determined by the Spearman's rank correlation coefficient. Top-row: (correlation coefficient, r 0.325, confidence, p 99.99%). Bottom-row: (r -0.625, p 99.99%).

4. MAGNETIC QUENCHING OF TURBULENT DIFFUSIVITY: RECONCILING MIXING-LENGTH THEORY ESTIMATES WITH KINEMATIC DYNAMO MODELS OF THE SOLAR CYCLE

The solar magnetic cycle involves the recycling of the toroidal and poloidal components of the magnetic field which are generated at spatially segregated source layers that must communicate with each-other (see e.g., Wilmot-Smith et al. 2006; Charbonneau 2005). This communication is mediated via magnetic flux-transport, which in most kinematic solar dynamo models, is achieved through diffusive and advective (i.e., by meridional circulation) transport of magnetic fields. The relative strength of turbulent diffusion and meridional circulation determines the regime in which the solar cycle operates, and this has far reaching implications for cycle memory and solar cycle predictions (Yeates, Nandy & Mackay 2008; Nandy 2010). As shown in Yeates, Nandy & Mackay (2008), different assumptions on the strength of turbulent diffusivity in the bulk of the Solar Convection Zone (SCZ) lead to different predictions of the solar cycle (Dikpati, DeToma & Gilman 2006; Choudhuri, Chatterjee & Jiang 2007). Previously this lack of constraint has led to controversy regarding what value of turbulent diffusivity is more appropriate and yields better solar like solutions (Nandy & Choudhuri 2002, Dikpati et al. 2002, Chatterjee et al. 2004, Dikpati et al. 2005, Choudhuri et al. 2005). Currently, most dynamo modelers use double-step diffusivity profiles which are somewhat ad-hoc and different from one-another (see Figure 1; Rempel 2006, Dikpati and Gilman 2007, Guerrero and de Gouveia Dal Pino 2007, Jouve and Brun 2007). There is however, a way of theoretically estimating

the radial dependence of magnetic diffusivity based on Mixing Length Theory (MLT; Prandtl 1925).

4.1. Order of Magnitude Estimation

Going back to the derivation of the mean-field dynamo equations (after using the first order smoothing approximation), we find that the turbulent diffusivity coefficient becomes (Moffat 1978; Eq. 1.10):

$$\eta = \frac{\tau}{3} \langle v^2 \rangle, \quad (4.1)$$

where τ is the eddy correlation time and v corresponds to the turbulent velocity field. In order to make an order of magnitude estimation we turn to MLT, which although not perfect, has been found to be in general agreement with numerical simulations of turbulent convection (Chan & Sofia 1987; Abbett et al. 1997). More specifically we use the Solar Model S (Chistensen-Dalsgaard et al. 1996), which is a comprehensive solar interior model used by GONG in all their helioseismic calculations. Among other quantities, this model estimates the mixing length parameter α_p , the convective velocity v for different radii and the necessary variables to calculate the pressure scale height H_p . In terms of those quantities the diffusivity becomes:

$$\eta \sim \frac{1}{3} \alpha_p H_p v, \quad (4.2)$$

which we plot in Figure 31 (solid black line) and show how it compares to commonly used diffusivity profiles.

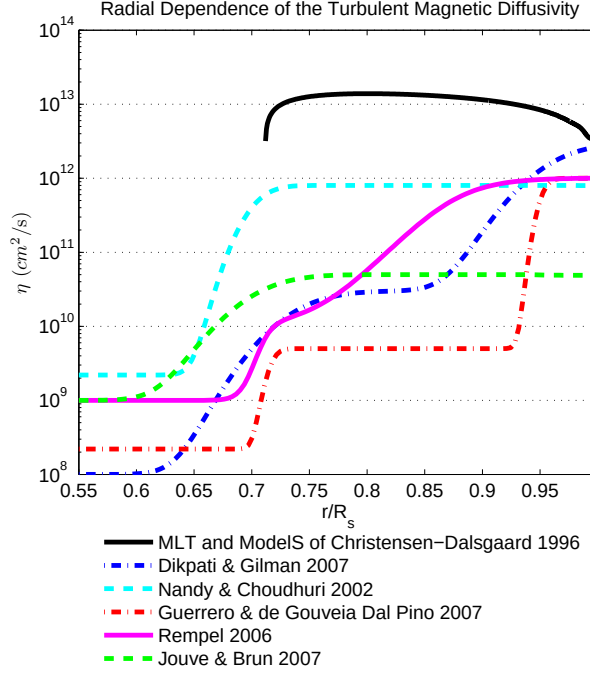


Figure 31. Different diffusivity profiles used in kinematic dynamo simulations. The solid black line corresponds to an estimate of turbulent diffusivity obtained by combining Mixing Length Theory (MLT) and the Solar Model S. The fact that viable solutions can be obtained with such a varied array of profiles has led to debates regarding which profile is more appropriate. Nevertheless, it is well known that kinematic dynamo simulations cannot yield viable solutions using the MLT estimate.

4.2. The Problem and a Possible Solution

It is evident that there is a major discrepancy between the theoretical estimate and the typical values used inside the convection zone (around two orders of magnitude difference), dynamo models simply cannot operate under such conditions.

A possible solution to this inconsistency resides in the back-reaction that strong magnetic fields have on velocity fields, which results in a suppression of turbulence and thus of turbulent magnetic diffusivity (Roberts & Soward 1975). This magnetic

“quenching” of the turbulent diffusivity has been studied before in different contexts (Rüdiger et al. 1994; Tobias 1996; Gilman & Rempel 2005; Muñoz-Jaramillo, Nandy & Martens 2008; Guerrero, Dikpati & de Gouveia Dal Pino 2009). However, although this issue has been common knowledge for more than a decade, it is only because of current improvements in computational techniques (Hochbruck & Lubich 1997; Hochbruck, Lubich & Selhofer; Muñoz-Jaramillo, Nandy & Martens 2009; MNM09 from here on), that this question can be finally addressed quantitatively. In this chapter we study whether introducing magnetic quenching of the diffusivity can solve this discrepancy and whether the shape of the currently used diffusivity profiles can be understood as a spatiotemporal average of the effective turbulent diffusivity after taking quenching into account.

4.3. Turbulent Magnetic Diffusivity and Diffusivity Quenching

In order to study the effect of magnetic quenching on dynamo models we introduce an additional state variable η_{mq} governed by the following differential equation:

$$\frac{\partial \eta_{mq}}{\partial t} = \frac{1}{\tau} \left(\frac{\eta_{MLT}(r)}{1 + \mathbf{B}^2(r, \theta, t)/B_0^2} - \eta_{mq}(r, \theta, t) \right). \quad (4.3)$$

In a steady state, η_{mq} corresponds to the MLT estimated diffusivity $\eta_{MLT}(r)$ quenched in such a way that the diffusivity is halved for a magnetic field of amplitude $B_0 = 6700$ Gauss (G). This value corresponds to the average equipartition field strength inside the SCZ calculated using the Solar Model S. The characteristic time of relaxation $\tau = 30$ days is an estimate of the average eddy turnover time.

We make a fit of $\eta_{MLT}(r)$ using the following analytical profile (see Figure 32):

$$\eta(r) = \eta_0 + \frac{\eta_1 - \eta_0}{2} \left(1 + \operatorname{erf} \left(\frac{r - r_1}{d_1} \right) \right) + \frac{\eta_2 - \eta_1 - \eta_0}{2} \left(1 + \operatorname{erf} \left(\frac{r - r_2}{d_2} \right) \right) \quad (4.4)$$

where $\eta_0 = 10^8 \text{cm}^2/\text{s}$ corresponds to the diffusivity at the bottom of the computational domain; $\eta_1 = 1.4 \times 10^{13} \text{cm}^2/\text{s}$ and $\eta_2 = 10^{10} \text{cm}^2/\text{s}$ control the diffusivity in the convection zone; $r_1 = 0.71R_\odot$, $d_1 = 0.015R_\odot$, $r_2 = 0.96R_\odot$ and $d_2 = 0.09R_\odot$ characterize the transitions from one value of diffusivity to the other. With this in mind, we define the effective diffusivity at any given point as

$$\eta_{eff}(r, \theta, t) = \eta_{min}(r) + \eta_{mq}(r, \theta, t). \quad (4.5)$$

with the minimum magnetic diffusivity $\eta_{min}(r)$ given by the following analytical profile (see Figure 32):

$$\eta_{min}(r) = \eta_0 + \frac{\eta_{cz} - \eta_0}{2} \left(1 + \operatorname{erf} \left(\frac{r - r_{cz}}{d_{cz}} \right) \right), \quad (4.6)$$

where $\eta_{cz} = 10^{10} \text{cm}^2/\text{s}$, $r_{cz} = 0.69R_\odot$, and $d_{cz} = 0.07R_\odot$. Since diffusivity is now a state variable, small errors can lead to negative values of diffusivity, which in turn leads to unbound magnetic field growth. By putting a limit on how small can the diffusivity become, we successfully avoid this type of computational instability.

4.4. Specifics of the Model Used in this Work

For this work we use the model described in Section 1.3, but instead of using the non-local poloidal source (Section 1.3.4), we use the double-ring algorithm described in Chapter 3 with a super-criticality constant of $K_0 = 3900$.

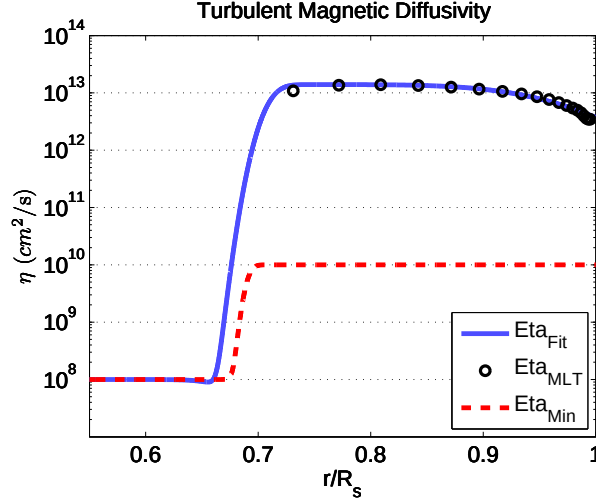


Figure 32. Fit (solid line) of diffusivity as a function of radius to the mixing-length theory estimate (circles). As part of our definition of effective diffusivity we put a limit on how much the diffusivity can be quenched. This minimum diffusivity has a radial dependence shown as a dashed line.

We use the SD-Exp4 code (see the Appendix) to solve the dynamo equations (Eqs. 1.15 and 1.15). Our computational domain comprises the SCZ and upper layer of the solar radiative zone in the northern hemisphere ($0.55R_{\odot} \leq r \leq R_{\odot}$ and $0 \leq \theta \leq \pi$). In order to approximate the spatial differential operators with finite differences we use a uniform grid (in radius and co-latitude), with a resolution of 400×400 gridpoints.

Our boundary conditions assume that the magnetic field is anti-symmetric across the equator ($\partial A / \partial \theta|_{\theta=\pi/2} = 0$; $\partial B / \partial \theta|_{\theta=\pi/2} = 0$), that the plasma below the lower boundary is a perfect conductor ($A(r = 0.55R_{\odot}, \theta) = 0$; $\partial(rB) / \partial r|_{r=0.55R_{\odot}} = 0$), that the magnetic field is axisymmetric ($A(r, \theta = 0) = 0$; $B(r, \theta = 0) = 0$), and that field at the surface is radial ($\partial(rA) / \partial r|_{r=R_{\odot}} = 0$; $B(r = R_{\odot}, \theta) = 0$). Our initial

conditions consist of a large toroidal belt and no poloidal component. After setting up the problem we let the magnetic field evolve for 200 years allowing the dynamo to reach a stable cycle.

4.5. Results and Discussion

The first important result is the existence of a uniform cycle in dynamic equilibrium. The presence of a diffusivity quenching algorithm allows the dynamo to become viable in a regime in which kinematic dynamo models cannot operate thanks to the creation of pockets of relatively low magnetic diffusivity (where long lived magnetic structures can exist). This can be clearly seen in Figure 33, which shows snapshots of the effective turbulent diffusivity and the toroidal and poloidal components of the magnetic field at different moments during the sunspot cycle (half a magnetic cycle). As expected the turbulent diffusivity is strongly suppressed by the magnetic field (especially by the toroidal component), increasing the diffusive timescale to the point where diffusion and advection become equally important for flux transport dynamics. This slow-down of the diffusive process is crucial for the survival of the magnetic cycle since it gives differential rotation more time to amplify the weak poloidal components of the magnetic field into strong toroidal belts, while providing them a measure of isolation from the top ($r = R_{\odot}$) and polar ($\theta = 0$) boundary conditions ($B=0$).

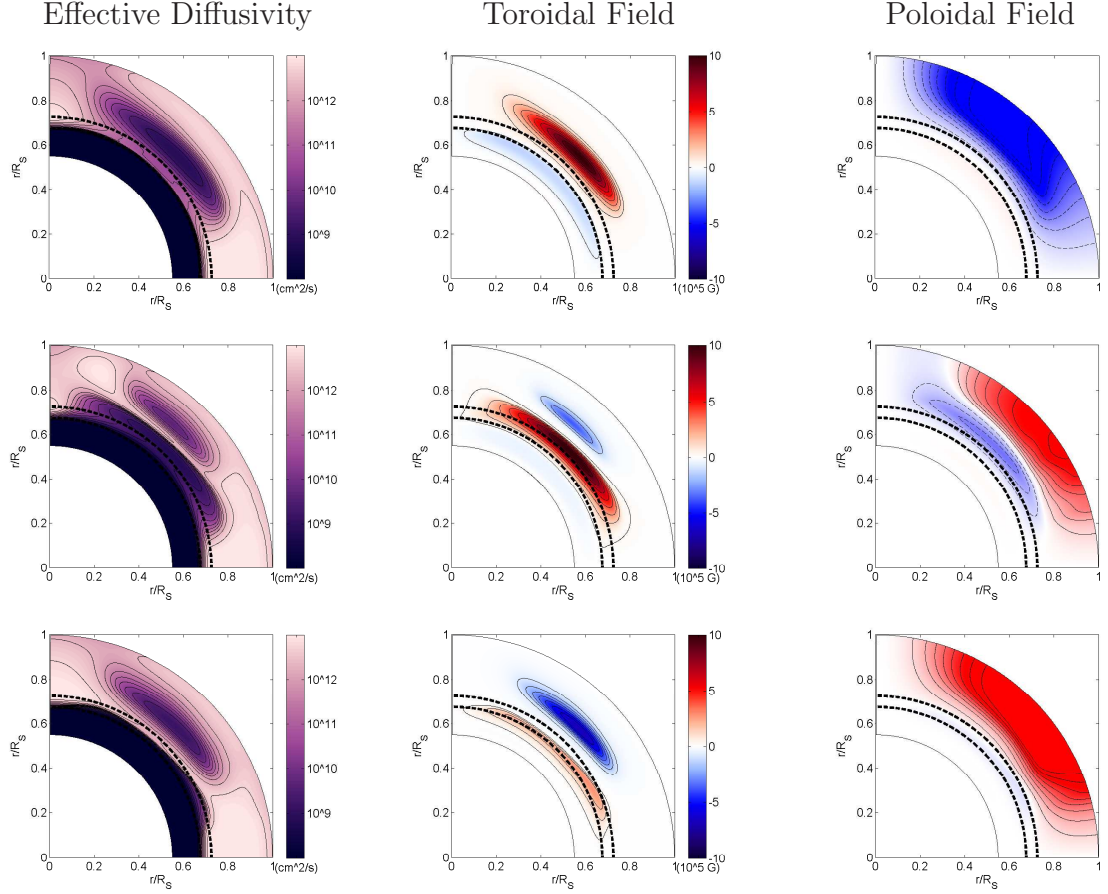


Figure 33. Snapshots of the effective diffusivity and the magnetic field over half a dynamo cycle (a sunspot cycle). For the poloidal field a solid (dashed) line corresponds to clockwise (counter-clockwise) poloidal field lines. Each row is advanced in time by a sixth of the dynamo cycle (a third of the sunspot cycle) i.e., from top to bottom $t = 0, \tau/6, \tau/3$ and $\tau/2$. As expected, the turbulent diffusivity is strongly depressed by the magnetic field (especially by the toroidal component). This reduces the diffusive time-scale to a point where the magnetic cycle becomes viable and sustainable.

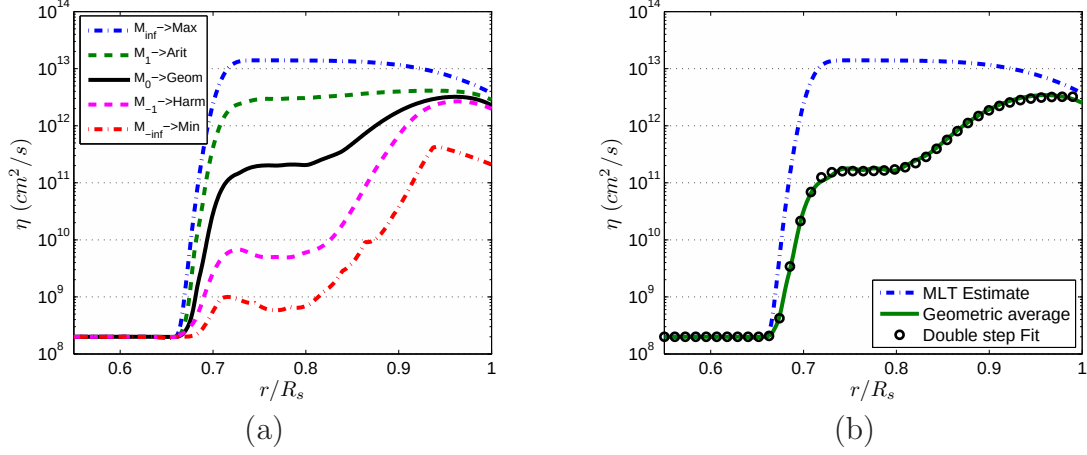


Figure 34. Spatiotemporal averages of the effective diffusivity (a). We find that the geometric time average (b) captures the essence of diffusivity quenching the best..

4.5.1. Spatiotemporal Averages of the Effective Diffusivity

Given that we ultimately want to understand how adequate kinematic diffusivity profiles are and whether they are plausible representations of physical reality consistent with MLT, we need to find a connection between kinematic profiles and the dynamically quenched diffusivity. Because of this, the next natural step is to find spatiotemporal averages of the effective diffusivity. For this purpose we use the generalized mean:

$$\eta_{av}(r_i) = M_p = \left(\frac{1}{N_\theta N_t} \sum_j \sum_n \eta_{eff}(r_i, \theta_j, t_n) \right)^{1/p} \quad (4.7)$$

where p controls the relative importance given to high values ($p > 0$) and low values ($p < 0$) of the diffusivity. From this generalized mean one can obtain the most commonly used averages: $p \rightarrow \infty$ yields the maximum value, $p = 1$ the arithmetic

average, $p \rightarrow 0$ the geometric average, $p = -1$ the harmonic average and $p \rightarrow -\infty$ the minimum value. The results of calculating these averages is shown in Figure 34.

4.5.2. Comparison with Simulations

Once we calculate the spatiotemporal averages of the effective diffusivity we obtain radial diffusivity profiles that can be used by kinematic dynamo simulations, leaving all ingredients intact, in order to compare their solutions with those of the dynamically quenched simulation. We find that the geometric average ($p \rightarrow 0$; also known as logarithmic average) shown in Figures 34-a & b as a solid lines, captures best the essence of the diffusive transport by striking a balance between high and low values of the diffusivity. Interestingly, this average can be accurately described as a double-step profile (Eq. 4.4) with the following parameters: $\eta_0 = 10^8 \text{cm}^2/\text{s}$, $\eta_1 = 1.6 \times 10^{11} \text{cm}^2/\text{s}$, $\eta_2 = 3.25 \times 10^{12} \text{cm}^2/\text{s}$, $r_1 = 0.71 R_\odot$, $d_1 = 0.017 R_\odot$, $r_2 = 0.895 R_\odot$ and $d_2 = 0.051 R_\odot$ (see circles on Fig. 34-b).

In order to compare the general properties of both simulations we cast the results in the shape of synoptic maps (also known as butterfly diagrams) as can be seen in Figure 35. The results obtained using the MLT estimate and diffusivity quenching (Fig. 35-a) and the results obtained using a kinematic simulation with the geometric average fit (Fig. 35-c) are remarkably similar given the very different nature of the two simulations. It is clear that the shape of the solutions differs mainly in the active region emergence pattern. However, the general properties of the cycle (amplitude,

period and phase) are successfully captured by the geometric average and are essentially the same. This result argues in favor of the capacity of kinematic diffusivity profiles of capturing the essence of turbulent magnetic quenching.

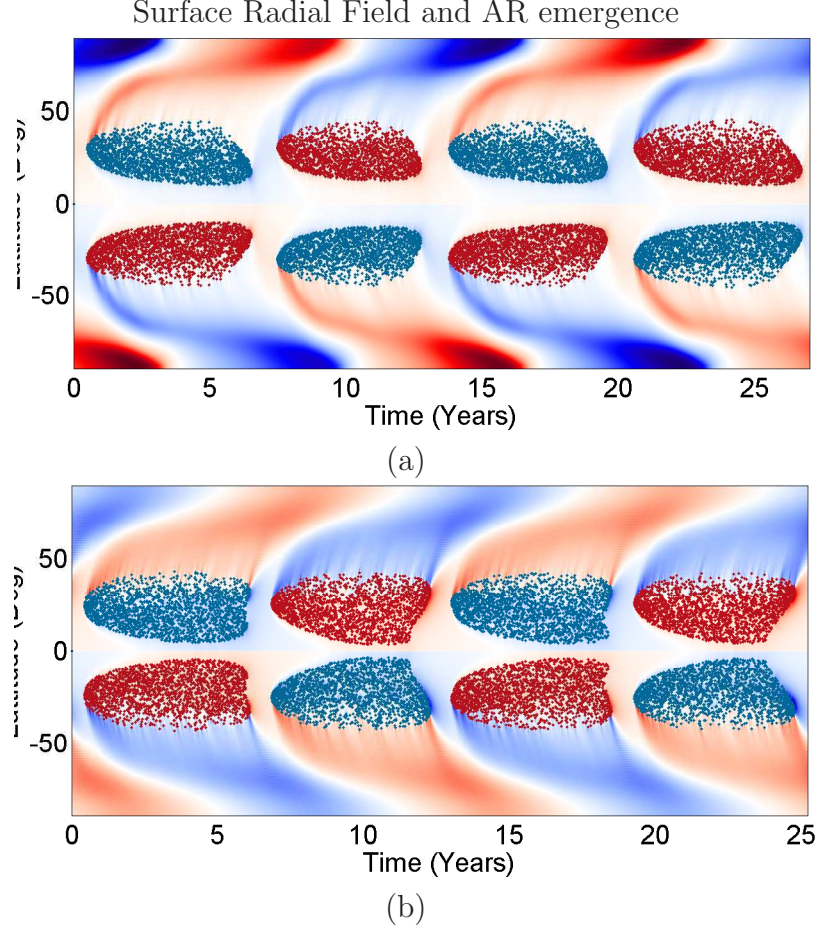


Figure 35. Synoptic maps (butterfly diagrams) showing the time evolution of the magnetic field in a simulation using the Mixing-Length Theory (MLT) estimate and diffusivity quenching (a), and a kinematic simulation using the geometric spatiotemporal average of the dynamically quenched diffusivity (b). They are obtained by combining the surface radial field and active region emergence pattern. For diffuse color, red (blue) corresponds to positive (negative) radial field at the surface. Each red (blue) dot corresponds to an active region emergence whose leading polarity has positive (negative) flux .

4.5.3. Comparison with Observations

Ultimately, the goal of dynamo models is to understand the solar magnetic cycle and reproduce and predict its main characteristics. It is therefore important to compare our results with solar observations. It is clear that the solutions are not exactly similar to those of kinematic dynamo simulations whose parameters have been finely tuned: cycle period of 7 years instead of 11, broad wings and incorrect phase. These differences point to an overestimation of the turbulent diffusivity; mainly near the surface (affecting phase and period) and at the bottom of the SCZ (which affects period and the shape of the wings). The cause of this overestimation likely resides in our definition of diffusivity quenching: in this work we use the average kinetic energy present in convection, which means that diffusivity is quenched equally through the convection zone. However, convection is less energetic near the bottom of the SCZ (due to low convective speeds) and near the surface (due to low mass density). This means that simulations taking this factor into account will probably yield more correct solutions.

4.6. Conclusions

In summary, we have shown that coupling magnetic quenching of turbulent diffusivity with the estimated profile from mixing length theory, allows kinematic dynamo simulations to produce solar-like magnetic cycles, which was not achieved before. Therefore, we have reconciled mixing length theory estimates of turbulent diffusivity

with kinematic dynamo models of the solar cycle. Additionally, we have demonstrated that kinematic simulations using a prescribed diffusivity profile based on the geometric average of the dynamically quenched turbulent diffusivity, are able to reproduce the most important cycle characteristics (amplitude, period and phase) of the non-kinematic simulations. Incidentally, this radial profile can be described by a double step profile, which has been used extensively in recent solar dynamo simulations. From the simulations reported here we provide an analytic fit to this double-step diffusivity profile that best captures the effect of magnetic quenching. A posteriori, our results strongly support the use of kinematic dynamo simulations as tools for exploring the origin and variability of solar magnetic cycles.

5. THE DEEP MINIMUM OF SUNSPOT CYCLE 23 CAUSED BY VARIATIONS IN THE SUN'S PLASMA FLOWS

Direct observations over the past four centuries (Hoyt & Schatten 1998) show that the number of sunspots observed on the Sun's surface varies periodically, going through successive maximum and minimum phases. Following sunspot cycle 23, the Sun went into an unusually prolonged minimum, from which it began to recover in mid 2010. This minimum has been characterized by a very weak polar magnetic field strength (Schrijver & Liu 2008; Wang, Robbrecht & Sheeley 2009) and a large number of days without sunspots that has been unprecedented in the space age (Data Source SIDC). Sunspots are strongly magnetized regions (Solanki 2003) and are generated by a dynamo mechanism involving complex interactions between plasma flows and magnetic fields in the Sun's interior (Charbonneau 2005). Here we report results from kinematic solar dynamo simulations which indicate that the character of the minimum in activity between solar cycles is caused by changes in the Sun's internal meridional plasma flows. Specifically, we find that a change from a faster to a slower average flow from the early half to the latter half of the cycle can explain both characteristics of the minimum of cycle 23, namely the large number of spotless days and the very weak polar field. The presence of sunspots govern the solar radiative energy flux (Krivova, Balmaceda & Solanki 2007) and radio flux, while the polar field strength modulates the solar wind, heliospheric open flux and consequently cosmic ray flux (Solanki, Schussler & Fligge 2000; Wang, Robbrecht & Sheeley 2009); our results

therefore provide a consistent link between solar internal dynamics and the atypical values of these parameters during the just concluded solar minimum.

While dynamo models self-consistently solve for both the toroidal and poloidal components of the magnetic field, solar surface flux transport models are often used to study in detail the contribution of surface flux transport processes to the solar polar field evolution (quantified as the radial-component of the poloidal field). Surface flux transport simulations indicate that the polar field strength at cycle minimum is determined by a combination of factors, including the flux and tilt angles of bipolar sunspot pairs and the amplitude and profile of meridional circulation and super-granular diffusion (Schrijver & Liu 2008, Wang, Robbrecht & Sheeley 2009). Analysis of the sunspot tilt-angle distribution of cycle 23 shows that the average tilt angle did not differ significantly from earlier cycles (Schrijver & Liu 2008). The amplitude of the super-granular diffusion coefficient is also not expected to change significantly from cycle to cycle. However, the axisymmetric meridional circulation of plasma (Giles et al. 1997), which is only observationally constrained in the top 10% of the Sun and has an average poleward speed of 20 m/s there, is known to exhibit significant intra- and inter-cycle variation (Zhao and Kosovichev 2004; Gonzalez-Hernandez et al. 2006; Švanda, Kosovichev & Zhao 2007). The equatorward counter-flow of the circulation near the base of the convection zone is coupled through mass-conservation to the poleward surface flow and therefore this return flow should also be variable. It is believed that this equatorward return flow of plasma is crucial to the solar cycle;

it drives the equatorward migration of sunspots, determines the solar cycle period and the spatio-temporal distribution of sunspots (Nandy & Choudhuri 2002, Charbonneau 2005). Motivated by this, we perform kinematic solar dynamo simulations to investigate whether internal meridional flow variations can produce deep minima between cycles in general, and in particular, explain both the defining characteristics of the minimum of cycle 23 - a weak dipolar field strength and a long period without sunspots.

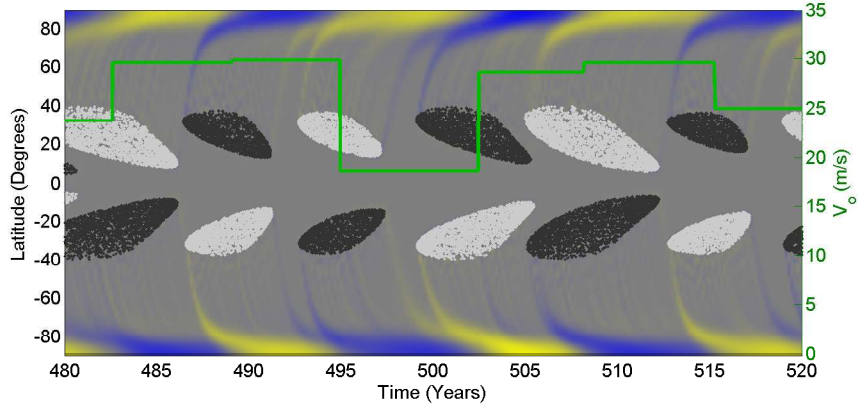


Figure 36. Simulated sunspot butterfly diagram from our solar dynamo simulations showing the time (x-axis)-latitude (left-hand y-axis) distribution of solar magnetic fields. The green line depicts the meridional flow speed which is made to vary randomly between 15-30 m/s (right-hand y-axis) at sunspot maximum, staying constant in between. The varying meridional flow induces cycle to cycle variations in both the amplitude as well as distribution of the toroidal field in the solar interior, from which bipolar sunspot pairs buoyantly erupt. This variation is reflected in the spatiotemporal distribution of sunspots shown here as shaded regions (darker shade represents sunspots that have erupted from positive toroidal field and lighter shade from negative toroidal field, respectively). The sunspot butterfly diagram shows varying degrees of cycle overlap (of the “wings” of successive cycles) at cycle minimum. The polar radial field strength (depicted in colour, yellow-positive and blue-negative) is strongest at sunspot cycle minimum and varies significantly from one cycle minimum to another.

5.1. Specifics of the Model Used in this Work

For this work we use the model described in Section 1.3, but instead of using the non-local poloidal source (Section 1.3.4), we use the double-ring algorithm described in Chapter 3 with a super-criticality constant of $K_0 = 400$.

In order to explore the effect of changing meridional flows on the nature of solar minima from one cycle to another, one needs to introduce fluctuations in the meridional flow. The large-scale meridional circulation in the solar interior is believed to be driven by Reynolds stresses and small temperature differences between the solar equator and poles; variations in the flows may be induced by changes in the driving forces, or through the feedback of magnetic fields (Rempel 2007). The feedback is expected to be highest at solar maximum (polar field minimum), when the toroidal magnetic field in the solar interior is the strongest. We therefore, perform dynamo simulations, by randomly varying the meridional flow speed at solar cycle maximum between 15-30 m/s (with the same amplitude in both the hemispheres) and study its effect on the nature of solar cycle minimum. We do this by tracking the polar field and defining solar max as the point where the polar field reverses.

Our simulations extend over 1860 years containing about 210 sunspot cycles; for each of these cycles we determine the meridional circulation speed, the cycle-overlap (which includes the information on number of sunspot-less days) and the strength of the polar radial field at cycle minimum. Fig. 36 shows the sunspot butterfly diagram and surface radial field evolution over a selected 40 year slice of simulation. Here

cycle to cycle variations (mediated by the varying meridional flows) in the structure of the sunspot butterfly diagram, including strength of the polar radial field and cycle overlap at minimum phases, are clearly apparent.

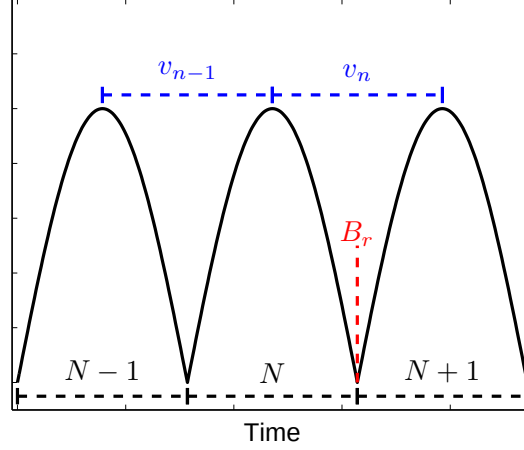


Figure 37. Diagram showing the evolution of the meridional flow amplitude with respect to the sunspot cycle: The meridional flow amplitude is changed at solar maximum such that v_n covers the second half (and minimum) and v_{n-1} the first half. The polar field at minimum of cycle n (B_r), is measured at the end of cycle n .

5.2. Understanding the 23-24 Minimum

In order to explore the relationship between the varying meridional flow, the polar field strength and cycle overlap, we need to define these quantities in relationship with any given sunspot cycle. We designate the overlap of cycle n as the amount of days in which active regions of cycle n coexist with those of cycle $n+1$. For those cycles without overlap, a negative number denotes the amount of spotless days between them. Along the same lines (illustrated in Fig. 37), B_r denotes the amplitude of the

polar field at the end of cycle n (beginning of cycle $n + 1$). Finally, we denote the meridional flow speed v_n as the amplitude which the meridional flow assumes after the random change at the solar maximum of cycle n , and which remains constant

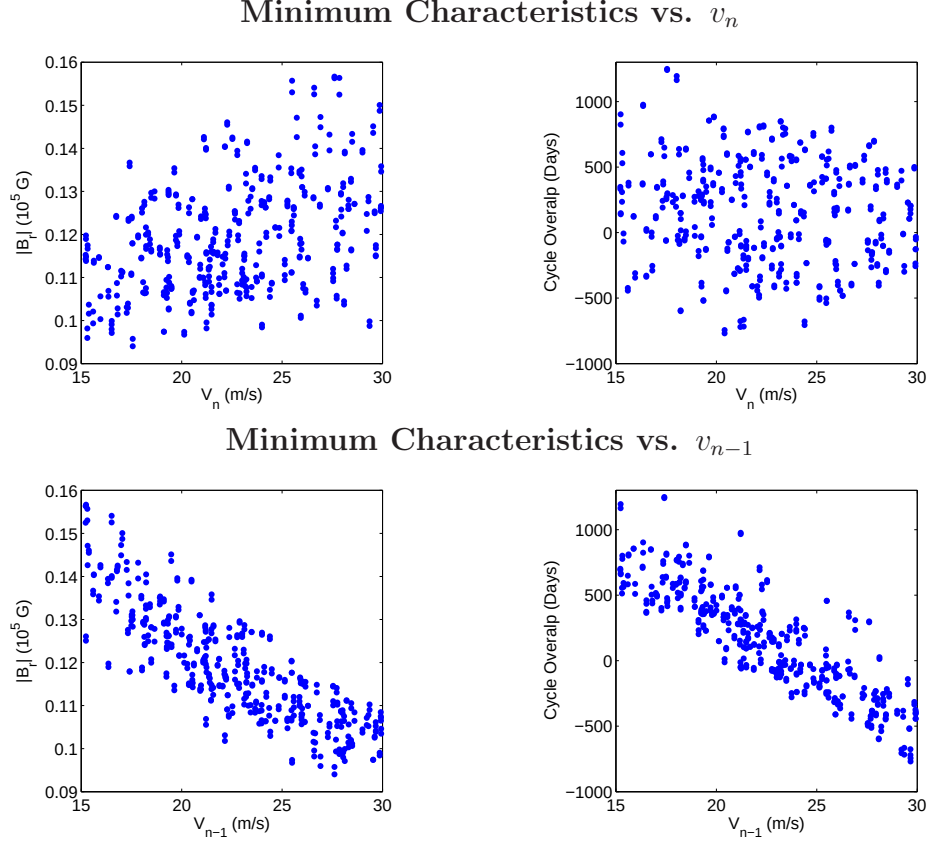


Figure 38. Relationship between randomly varying meridional flow speed and simulated solar minimum characteristics quantified by cycle overlap and solar polar field strength. Cycle overlap is measured in days. Positive cycle overlap denotes number of days where simulated sunspots from two successive cycles erupted together, while negative cycle overlap denotes number of sunspot-less days during a solar minimum (large negative overlap implies a deep minimum). The polar field strength (in Gauss) is represented by the maximum amplitude of the polar radial field (Br) attained during a solar minimum. The relationship between the above parameters is determined by the Spearman's rank correlation coefficient (420 data points, 210 data points contributing from each solar hemisphere). Top-left: (correlation coefficient, r 0.13, confidence, p 99.31%). Top-right: (r 0.44, p 99.99%). Bottom-left: (r 0.81, p 99.99%). Bottom-right: (r 0.84, p 99.99%).

until the maximum of cycle $n + 1$. Therefore, the speed during the early (rising) half of cycle n would be v_{n-1} . With these definitions in mind, we generate statistical correlations between these quantities by combining the measurements from both the solar hemispheres from our simulations over 210 sunspot cycles.

Perhaps surprisingly we find that there is no correlation between the flow speed at a given minimum (say, v_n), and cycle overlap (or the number of sunspot-less days) during that minimum, while the polar field strength at that minimum (B_r) is only moderately correlated with v_n (Fig. 38-top panel). Since transport of magnetic flux by the meridional flow involves a finite time, it is likely that the characteristics of a given minimum could depend on the flow speed at an earlier time. We find that this is indeed the case (Fig. 38-bottom panel), with cycle overlap (or number of spotless-days) and the polar field strength at a given minimum n , being strongly correlated with the flow speed v_{n-1} (i.e., meridional flow during the early, rising part of that cycle). We also find that the cycle overlap is moderately correlated and the polar field strength (B_r) is strongly correlated with the change in flow speed from the earlier to the latter half of the cycle (Fig. 39). Taken together, these results indicate that a faster flow speed during the early, rising part of the cycle, followed by a slow speed during the latter, declining part of the cycle, results in low values of the polar field combined with a large amount of days without sunspots.

The main characteristics of the minimum of solar cycle 23 are a large number of spotless days and weak polar field strength. In Fig. 40 we plot the polar field

versus cycle overlap and find that a deep minimum is in fact associated with weak polar field strength. Thus, both the defining characteristics of the deep minimum of sunspot cycle 23 are self-consistently explained in our simulations driven by changes in the changes in the Sun's meridional plasma flows.

Valuable insights to our simulation results may be gained by invoking the physics of meridional flow mediated magnetic flux transport. A faster flow (v_{n-1}) before and during the early half of a cycle n would sweep the poloidal field of the previous cycle faster through the region of differential rotation responsible for toroidal field generation; this would allow less time for toroidal field amplification and hence result

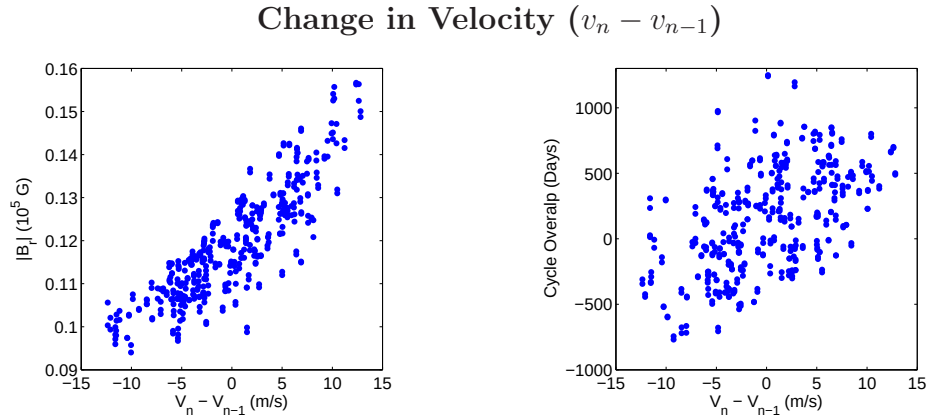


Figure 39. Relationship between change in flow speed and simulated solar minimum characteristics quantified by cycle overlap and solar polar field strength. Cycle overlap is measured in days. Positive cycle overlap denotes number of days where simulated sunspots from two successive cycles erupted together, while negative cycle overlap denotes number of sunspot-less days during a solar minimum (large negative overlap implies a deep minimum). The polar field strength (in Gauss) is represented by the maximum amplitude of the polar radial field (Br) attained during a solar minimum. The relationship between the above parameters is determined by the Spearman's rank correlation coefficient (420 data points, 210 data points contributing from each solar hemisphere). Left: (r 0.45, p 99.99%). Right: (r 0.87, p 99.99%). Evidently, a change from fast to slow meridional flow speeds result in a deep solar minimum.

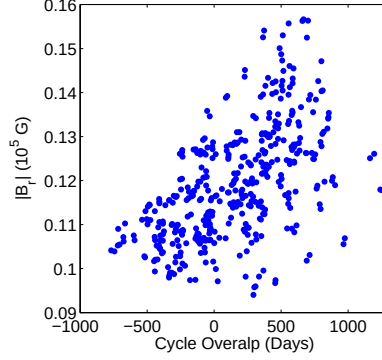


Figure 40. Simulated polar field strength (in Gauss) versus cycle overlap at sunspot cycle minimum in units of days (r 0.46, p 99.99%). The results show that a deep solar minimum with a large number of spotless days is typically associated with weak polar field strength, whereas cycles with overlap can have both weak and strong polar fields.

in a sunspot cycle (n) which ends quickly. The fast flow, followed by a slower flow during the latter half of cycle n persisting to the early part of the next cycle ($n + 1$) would also distance the two successive cycles, thereby contributing to a higher number of sunspotless days during the intervening minimum. In conjunction, a fast flow during the early half of cycle n would sweep both the positive and negative polarity sunspots of cycle n (erupting at mid-high latitudes) to the polar regions; therefore lower net flux would be available for cancelling the old cycle polar field and building up the polar field of cycle n ultimately resulting in a weak polar field strength at the minimum of cycle n . Finally, a slow down of the meridional flow in the declining phase of the cycle will allow the polar flux to weaken by allowing the field to diffuse over a larger area. We believe that a combination of these effects contribute to the occurrence of deep solar minima such as that of cycle 23.

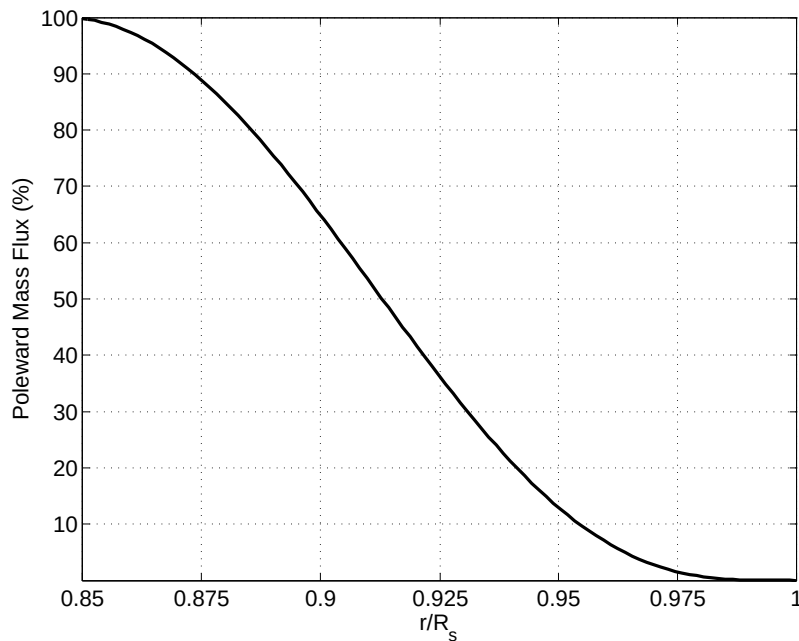


Figure 41. A plot of the cumulative meridional flow mass flux (between the radius in question and the surface; y-axis) versus depth (measured in terms of fractional solar radius $r/R_\odot$; x-axis). The mass flux is determined from the typical theoretical profile of meridional circulation used in solar dynamo simulations including the one described here. This estimate indicates that only about 2% of the poleward mass-flux is contained between the solar surface and a radius of $0.975R_\odot$, the region for which current observations of the meridional flow is limited to.

5.3. How do Our Simulations Compare to Observations Related to the Minimum of Cycle 23?

Helioseismic observations of the equatorward migration of the torsional oscillation show that the torsional oscillation pattern of the upcoming cycle 24 (which originated near the maximum of the preceding cycle) is migrating relatively slowly compared to that of cycle 23 (Howe et al. 2009). Since the torsional oscillation pattern is believed to be associated with the migration of the magnetic cycle (Rempel 2007),

this could be indirect evidence that the meridional flow driving (the toroidal field belt of) cycle 24 in the solar interior, is relatively slower compared to that of the previous cycle; this is in agreement with our theoretical simulations. On the other hand, direct surface observations (Hathaway & Rightmire 2010) and near-surface helioseismic observations (González-Hernández et al. 2010) show that the flow near the surface has increased (roughly in a sinusoidal fashion) from the maximum of cycle 23 to its minimum in apparent conflict with the earlier, indirect evidence of a slower flow and our simulations. However, the helioseismic observations (González-Hernández et al. 2010) show that this contradictory surface flow speed variation is almost wiped out at depths of $0.979R_{\odot}$. Therefore we argue that this is a near-surface phenomenon (driven by surface magnetic activity) and has no significant impact on the magnetic field dynamics in the solar interior. In support of our argument, we plot in Fig. 41 the depth-dependence of the cumulative poleward mass flux in the meridional flow (based on a standard meridional flow profile) and find that only about 2% of the poleward mass-flux is contained within the surface and a depth of $0.975R_{\odot}$. Evidently, much of the flux transport dynamics associated with meridional flow occur deeper down in the solar interior as yet inaccessible to observations. Our hypothesis is independently supported by the simulations of Jiang et al. (2010), who are unable to reproduce the weak polar field of the minimum of cycle 23 by using observed surface flows. Dynamo simulations which encompass the entire solar convection zone and a part of the radiative interior therefore remain our best bet in probing the internal

processes that govern the dynamics of the solar magnetic cycle, including the origin of deep minima such as that of cycle 23. We anticipate that the recently launched Solar Dynamics Observatory will provide more precise constraints on the structure of the plasma flows deep down in the solar interior, which will be useful for complementing these dynamo simulations.

6. ARE ACTIVE REGIONS A CRUCIAL LINK IN THE SOLAR CYCLE OR MERELY A SYMPTOM OF SOMETHING WE CAN'T SEE?

The Babcock-Leighton (BL) mechanism is now believed by many to be the main contender for poloidal field regeneration (see Section 1.2.3). The strongest point in its favor comes from surface magnetic field observations as well as simulations which have clearly shown that the surface polar field reversal is triggered by Active Region (AR) decay (Wang, Nash & Sheeley 1989; Wang & Sheeley 1991). However, whether the observed surface field dynamics (leading to polar field reversal) plays the dominant or integral role on closing the cycle remains an open question. To the extent of our knowledge, the work by Choudhuri (2003) represents the only consistent effort in that direction during the last decade and the issue has never been explored quantitatively. In this chapter we attempt to estimate the relative importance of the BL mechanism in perpetuating the cycle through both estimations and simulations.

6.1. Getting the Right Amount of Toroidal Field: A Task of Increasing Complexity

Following the original approach of Babcock (1961), which was also used by Choudhuri (2003), we start with a back of the envelope calculation. The idea is to estimate how much toroidal field (B_ϕ) can be inducted from the observed polar field (B_r) at solar max (10 Gauss) and whether this is enough to generate the necessary values for the creation of strong flux-tubes capable of forming ARs (at least 50,000 Gauss; see Section 1.2.2 and the review by Fan 2009 for information regarding this value), which

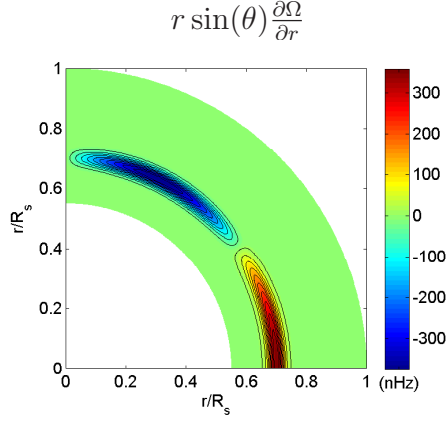


Figure 42. Radial shear of the differential rotation profile of Charbonneau et al.(1999; see Eq. 1.19), weighted by $r \sin(\theta)$. Note that this quantity is directly proportional to the amplification factor in a radial shear estimation (see Eq. 6.3).

means an amplification factor of 5000. The simplest possible calculation consists on estimating how many rotations can the equator (with a rotation period T_{eq} of 25 days) gain on the poles (with a rotation period T_{pl} of 35 days) during a solar cycle ($\tau = 11$ years). The resultant amplification factor:

$$\frac{B_\phi}{B_p} = \tau \left(\frac{1}{T_{eq}} - \frac{1}{T_{pl}} \right) \approx 50 \quad (6.1)$$

is clearly not enough. However, a more precise estimate is possible using the toroidal field induction equation. Since we are interested in an upper bound for the amplitude of the toroidal field, we make some simplifying assumptions that make the problem tractable. We assume that there is no transport of field by advection or diffusion; this essentially means that no loss occurs through cancelation or diffusion and that the poloidal field stays constant during the entire induction process. Additionally, we assume that the initial toroidal field is zero; this also leads to overestimation of

the strength of the induced field, because under normal conditions part of the newly induced toroidal field must be spent in order to cancel the old one. With these assumptions in mind, the only term remaining in the evolution of the toroidal field (Eq. 1.16) is the shearing of poloidal field ($\mathbf{B}_p$) by differential rotation (Ω):

$$\frac{\partial B}{\partial t} = r \sin(\theta) \mathbf{B}_p \cdot \nabla \Omega = r \sin(\theta) \left[B_r \frac{\partial \Omega}{\partial r} + \frac{B_\theta}{r} \frac{\partial \Omega}{\partial \theta} \right]. \quad (6.2)$$

Assuming that the poloidal field is radial in the place of maximum radial shear, which is 400nHz for the analytical profile of Charbonneau et al. 1999 (see Figure 42 and Eq. 1.19), one obtains:

$$\frac{B_\phi}{B_r} = \tau \left[r \sin(\theta) \frac{\partial \Omega}{\partial r} \right] \approx 140, \quad (6.3)$$

which although larger, falls short again. Note that there is a strong argument against using the radial shear: the BL mechanism involves the recreation of a dipolar field. This means that the latitudinal component of the field (B_θ) should be more important than the radial component (B_r). This has been proved by simulations which show that the most important source of toroidal field is actually the latitudinal shear and not the radial shear (Guerrero & de Gouveia Dal Pino 2007; Muñoz-Jaramillo, Nandy & Martens 2009).

In order to estimate the amplification due to the latitudinal shear, we start by calculating the amount of flux on a polar cap:

$$\Phi_{pc} = \int_0^{2\pi} \int_0^{\theta_{pc}} B_{pc} R_\odot^2 \sin(\theta) d\theta d\phi = 2\pi B_{pc} R_\odot^2 [1 - \cos(\theta_{pc})] \quad (6.4)$$

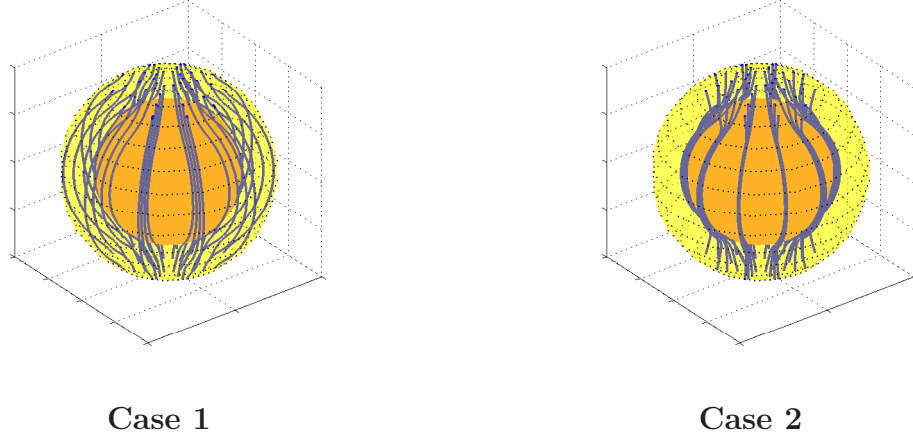


Figure 43. In our calculations we consider two extreme magnetic configurations: the poloidal flux is distributed over the entire solar convection zone (Case 1), the poloidal flux is concentrated in the tachocline (Case 2).

where B_{pc} is the average magnetic field in the polar cap, $R_{\odot} = 6.96 \times 10^{10}$ cm is the solar radius and $\theta_{pc} = \pi/6$ is the co-latitudinal extent of the cap. Given that this flux is fully contained inside the Sun (until it exits through the other polar cap), we can use flux conservation to calculate the magnitude of poloidal field crossing a conical area of constant colatitude. Since we do not know the exact distribution of flux inside the Sun, we use two extreme cases (see Fig. 43):

1. The poloidal field is distributed across the entire Solar Convection Zone (SCZ).
2. The poloidal field is concentrated in a narrow region at the bottom of the SCZ (tachocline).

With this in mind, we calculate the area where the field is contained:

$$A_{\Phi} = \int_0^{2\pi} \int_{R_b}^{R_t} r \sin(\theta) dr d\theta = \pi \sin(\theta) [R_t^2 - R_b^2], \quad (6.5)$$

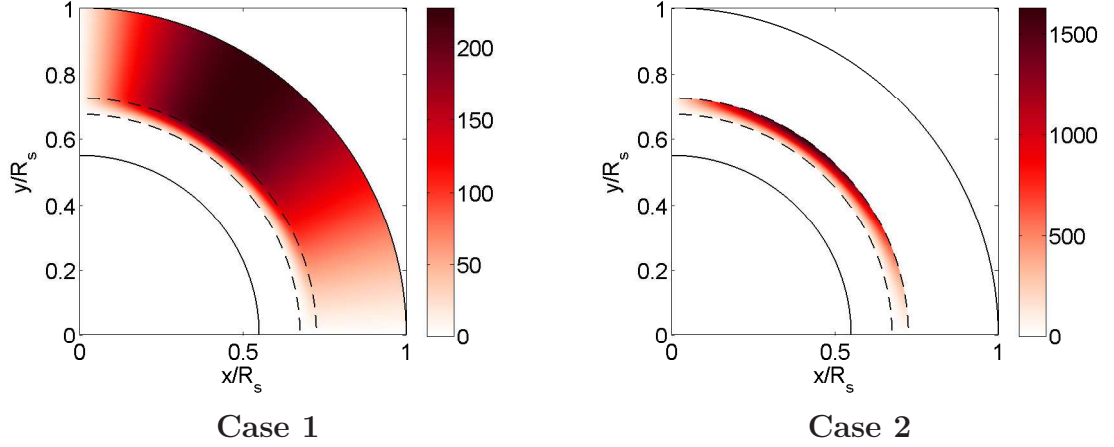


Figure 44. Estimated toroidal field amplification after 11 years of evolution. The left figure assumes that the poloidal flux is distributed over the entire solar convection zone (Case 1). The right figure assumes that the poloidal flux is concentrated in the tachocline (Case 2). The dashed lines mark the top and bottom boundaries of the tachocline.

where $R_b = .675R_\odot$ marks the bottom boundary of this area and $R_t = R_\odot$ ($R_t = 0.725R_\odot$) marks the top boundary for Case 1 (Case 2). This way Case 1 corresponds to the SCZ and Case 2 to the tachocline (which is the layer with the strongest rotational shear). Combining Eqs. 6.4 and 6.5 the magnetic field in this layer becomes:

$$B_\theta(\theta) = \frac{2B_{pc}R_\odot^2 [1 - \cos(\theta_{pc})]}{\sin(\theta) [R_t^2 - R_b^2]}, \quad (6.6)$$

Combining Eqs. 6.2 and 6.6 with the differential rotation of Charbonneau et al. (1999; see Eq. 1.19) we obtain the final expression for toroidal field amplification:

$$\frac{B_\phi}{B_{pc}}(\theta) = 4\pi\tau R_\odot^2 \frac{[1 - \cos(\theta_{pc})] (\Omega_e - \Omega_p)}{[R_t^2 - R_b^2]} F(r)G(\theta), \quad (6.7)$$

where

$$F(r) = 1 + \operatorname{erf} \left(\frac{r - r_{tc}}{w_{tc}} \right) \quad (6.8)$$

$$G(\theta) = \sin(\theta)[a \cos^2(\theta) + (1 - a) \cos^4(\theta)] \quad (6.9)$$

and $\Omega_e = 470$ nHz is the rotation frequency of the equator, $\Omega_p = 330$ nHz is the rotation frequency of the pole, $a = 0.483$ is the strength of the $\cos^2(\theta)$ term relative to the $\cos^4(\theta)$ term, $r_{tc} = 0.7$ the location of the tachocline and $w_{tc} = 0.025$ half of its thickness.

Figure 44 shows the estimated amplification factor as a function of latitude for both cases. It is evident that although closer to the required amplification factor of 5000, even the most optimistic of calculations still falls short (Case 2 with an amplification factor of 1600). This raises the question whether the BL mechanism is really the primary source of poloidal field as we believe it to be.

6.2. What Can Dynamo Simulations Tell us About this Issue?

After the introduction of the BL “mean-field” alpha (see Section 1.4.3), the dynamo community has made the tacit assumption that the BL mechanism is enough to sustain the dynamo. If one recalls the nature of the mean field BL source (Sec. 1.4.3):

$$S(r, \theta, B) = \alpha_0 \alpha(r, \theta) F(B), \quad (6.10)$$

the quantitative disconnection between the real process of active region emergence and decay and such a source becomes evident when one considers the constant α_0 .

The habitual approach is the following: If the solutions decay, the value is raised until super-criticality is achieved. Given that there is no rigorous way of quantifying this constant, due to the lack of a solid mathematical definition of the “mean-field BL α -effect”, this parameter has been treated with a high amount of freedom. However, no such freedom exists under the double-ring algorithm because each active region is perfectly quantified. Nevertheless, we were forced to introduce a super-criticality constant (K_0 ; see Chapter 3) in order to have non-decaying solutions which is much harder to justify than in the continuous and semi-discrete formulations. However, this quantitative relationship between the double-ring algorithm and active regions also presents us with the unique opportunity of directly using observed active region data in kinematic dynamos for the first time.

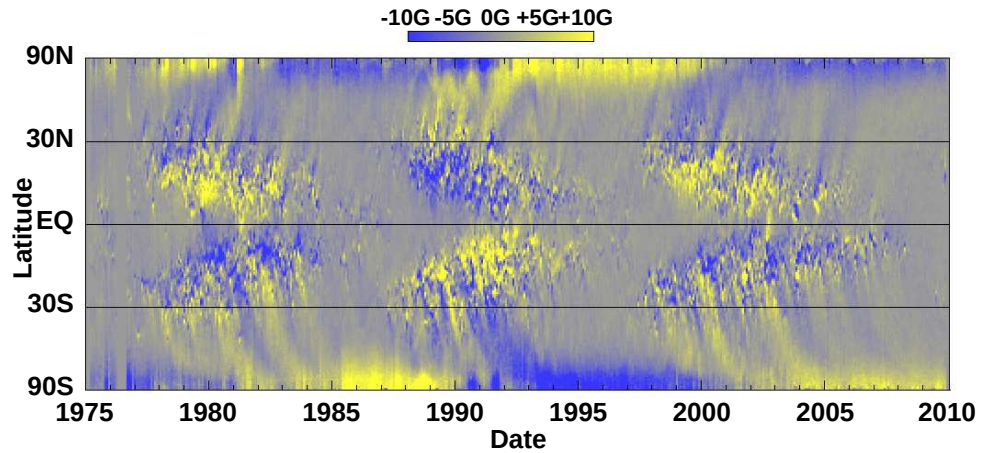


Figure 45. Longitudinally averaged magnetic field at the surface of the Sun. This map is commonly known as the ‘Butterfly Diagram’ and captures the essence of the solar magnetic cycle: Emergence of ARs that migrates towards the equator as the cycle progresses, transport of diffuse magnetic field towards the poles and polarity reversals from cycle to cycle. Image courtesy of David Hathaway.

An important step before estimating the toroidal field amplification factor from dynamo simulations is to test whether double rings are able to capture surface dynamics accurately. In Chapter 3 we took the first steps in that direction, by verifying that a dynamo model using the double-ring algorithm is in general agreement with surface flux transport simulations. In order to take this a step further we need to compare the results of our simulation with observations of the surface magnetic field (see Figure 45). The basic idea is the following: If we drive the kinematic dynamo using real active region data (without involving any super-criticality constants), do we get surface behavior which agrees with the observed evolution of the magnetic field (Figure 45)? If that turns to be the case, then we can use the simulation to estimate a more realistic toroidal field amplification factor.

6.3. Specifics of the Model Used in this Work

For this work we use the model described in Section 1.3, but instead of using the non-local poloidal source (Section 1.3.4), or allowing double-ring eruptions to arise self-consistently out of the dynamo (see Chapter 3), we force the system using the double-ring algorithm and data taken by Dr. Neil R. Sheeley, Jr. (Sheeley, DeVore & Boris 1985; Wang & Sheeley 1989) including a constant flux correction factor of 3 necessary to match the interplanetary field strength (Wang, Sheeley & Lean 2002). These data contain around 3000 active regions measured between August 1976 and April 1986, encompassing solar cycle 21 (Fig. 46). In order to be able to simulate

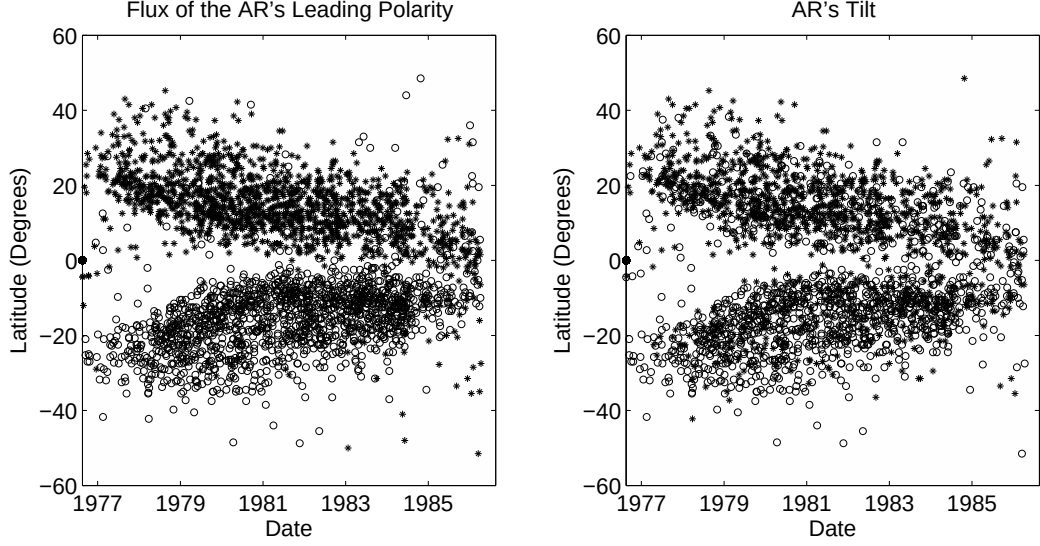


Figure 46. Active Region (AR) database of Neil R. Sheeley, Jr. (Sheeley, DeVore & Boris 1985; Wang & Sheeley 1989) comprising solar cycle 21. ARs are marked using their latitude and time of emergence. On the left figure, circles (asterisks) correspond to ARs whose leading polarity is positive (negative). On the right figure, circles (asterisks) correspond to ARs positive (negative) tilt angle with respect to a line parallel to the equator.

more than one solar cycle, we construct a series based on these data switching the polarity of the magnetic field every other cycle, to enforce polarity reversal, and switching hemispheres every two cycles in order to minimize the buildup of strong asymmetries between them. it is important to highlight that by driving the dynamo this way, we have essentially transformed a kinematic dynamo model into a 2.5D flux transport simulation (see Section 3.3 and references therein for a brief introduction to flux transport simulations). However, in contrast to traditional surface flux transport simulations, we are solving for the toroidal field in our simulation.

Since our main objective is placing an upper bound on the amplification factor between polar field and toroidal field, we choose the meridional flow and turbulent diffusivity parameters using the following criteria (in order of importance):

1. The radial field at the surface should be qualitatively as close as possible to the observational data (Fig. 45)
2. The combined effect of diffusion and advection must yield an good overlap between active region data and the evolution of the toroidal field at the bottom of the SCZ.
3. The parameters chosen should maximize the amplitude of the toroidal field at the bottom of the SCZ.

The results shown here are obtained using the set of parameters mentioned in Sections 1.3.1 and 1.3.3 with a meridional flow amplitude of 15 m/s.

6.4. Results

The results of our simulation can be seen in Fig. 47. The first thing to note is the good correspondence between our surface magnetic field (Fig. 47-A) and the observational data shown in Fig. 45 (the closest ever to have been produced by a kinematic dynamo model). Our model is able to adequately capture the mixture of polarities on active latitudes and the transport of flux towards the poles by diffusion and advection. Furthermore, the overall strength of the magnetic field is in general

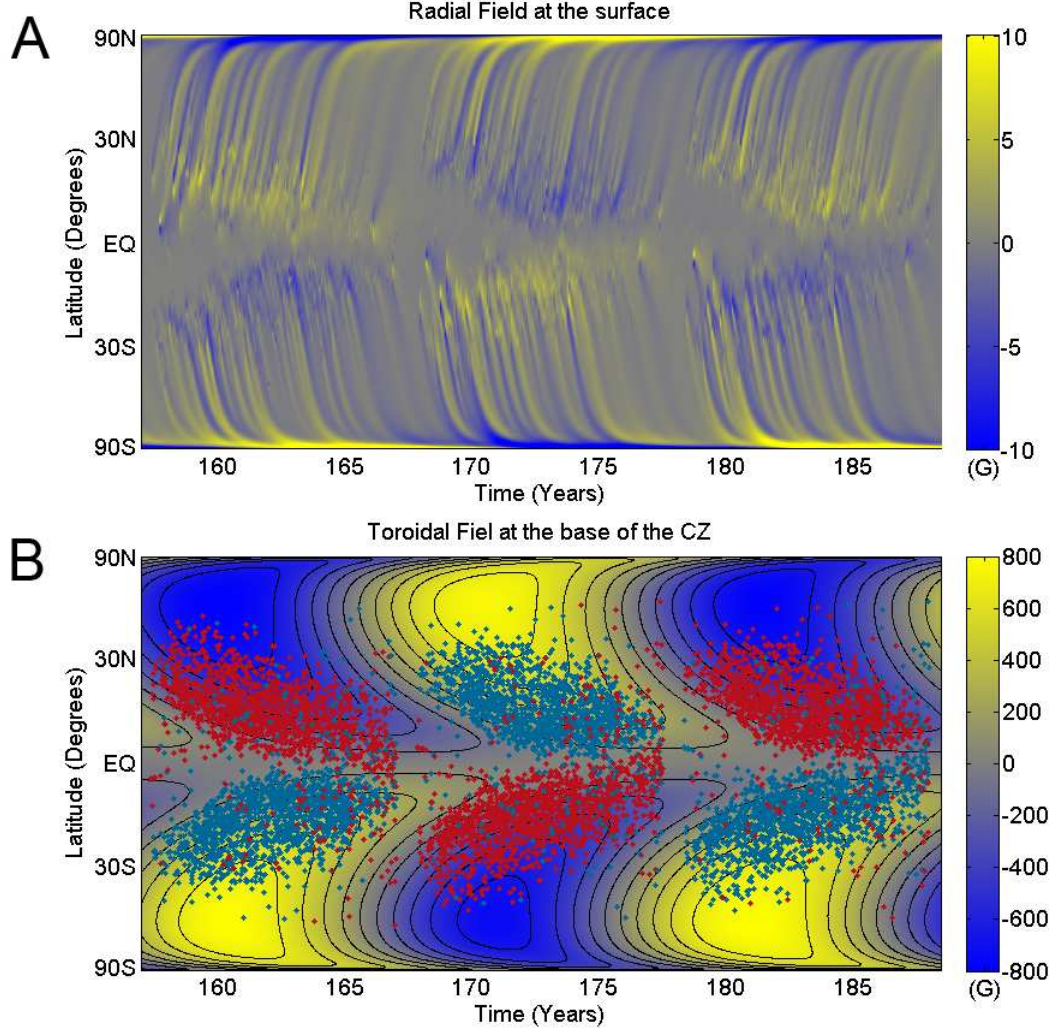


Figure 47. Results of the simulations using our kinematic dynamo model driven by a time series based on the Active Region (AR) database of Dr. Neil R. Sheeley, Jr. (Sheeley, DeVore & Boris 1985; Wang & Sheeley 1989). On the top figure (A) we can see the radial magnetic field at the surface which qualitatively agrees very well with the observational data shown in Fig. 45. On the bottom (B) we see a superposition of the AR data of our time series (dots) on the toroidal field at $0.71R_{\odot}$, which corresponds to the bottom of the solar convection zone (blue-yellow and contours). Red (blue) dots correspond to ARs whose easternmost polarity is positive (negative). Although it is possible to overlap the migration of the toroidal belts with the migration of the ARs, we obtain an amplification that is 50 to 100 times less than what is necessary for the successful recreation of next cycle.

agreement with the observations without involving any fine tuning. This gives us confidence that we are being able to capture successfully the dynamics of surface transport. However, when we look at the evolution of the magnetic field at the bottom of the convection zone: Besides the well known problem of the amplitude of the toroidal field peaking at high latitudes (Nandy & Choudhuri 2002; Charbonneau 2005), we also find that its amplitude is only of the order of 1kG – which corresponds to an amplification factor of 50 times less than necessary (5000) for the successful recreation of next cycle (see above). This agrees with the estimates obtained above. The implications of this result are of great importance, especially regarding solar cycle predictions: Current predictions of the solar cycle rely on the assumption that ARs have a causal role on determining the cycle’s properties (Dikpati, de Toma & Gilman 2006; Choudhuri, Chatterjee & Jiang 2007). If that proves not to be the case, as our results suggest, then we have to carefully reassess the understanding necessary for solar cycle prediction.

It is important to note that these estimates and results cast doubt on the crucial role that has been attributed to the BL mechanism. However, there may be additional processes (independent of the value of the poloidal field) working in tandem with the stretching and amplification due to solar differential rotation, raising the toroidal field amplification rate (an example of this is the mechanism for field amplification by flux implosion proposed by Rempel & Schüssler 2001). Nevertheless, the question remains of how the dynamo survived the long period without active region emergence

during the Maunder minimum (Eddy 1976), without an additional source of poloidal field. With this in mind, it is our opinion that the most probable situation is that the recreation of the poloidal field is not being performed exclusively by the BL mechanism, but that this is shared by other sources (for alternatives see Charbonneau 2005 and references therein). Full MHD simulations are now beginning to find large coherent toroidal belts (Browning et al. 2006; Ghizaru, Charbonneau & Smolarkiewicz 2010) including reversals (Ghizaru, Charbonneau & Smolarkiewicz 2010). Therefore we point out that the mean-field α -effect may also be playing a role in the generation of the poloidal field inside the solar interior, in addition to the BL mechanism involving active regions.

7. FINAL REMARKS

Through the research done in this thesis we have systematically reduced the amount of free parameters and improved upon each of the ingredients in solar kinematic dynamo models (see Chapters 2, 3 and 4). We illustrate our progress in Table 4 where we tabulate the status of the parameters of each of the dynamo ingredients before and after this work. The criteria used for classification denote the relative freedom a modeler has in fine tuning the dynamo through a given parameter – this means that a “well constrained” parameter is fixed for all practical purposes, whereas a “poorly constrained” parameter is essentially free.

Ingredient	Parameters Status Before	Parameters Status After
Differential Rotation	7 well constrained	Assimilated helioseismic data
Meridional Flow	2 well constrained 1 reasonably constrained 4 poorly constrained	3 well constrained 4 poorly constrained
Turbulent Diffusivity	2 reasonably constrained 5 poorly constrained	7 reasonably constrained
Poloidal Source	Mean-field formulation	Fully discrete formulation

Table 4. Status of different ingredients of the dynamo before and after this thesis.

This overall improvement has placed us in a better position to understand the underlying physics of the solar magnetic cycle. It has allowed us to resolve discrepancies between kinematic dynamos and flux transport simulations (see Chapter 3), explore the origin of the deep minimum of cycle 23 (see Chapter 5) and assess the adequacy

of the Babcock-Leighton (BL) mechanism as a source of poloidal field regeneration (see Chapter 6).

It is clear that much work remains to be done. Overall, our work lays the basis on which further refinements to dynamo models may be done when better data become available. However, the role of the BL mechanism in poloidal field regeneration needs to be studied in detail before any further applications of the model is attempted on specific problems, such as solar cycle predictions. In any case, it is clear that dynamo theory is going through a cycle of renewal where, old ideas are being revisited and cherished beliefs are being questioned.

No one knows what the future holds, but I am definitely excited to be part of it. It is my hope that contrary to what cycle 24 seems to be doing, dynamo theory will find newer and higher peaks during the next decade.

APPENDIX A

Numerical Methods

In order to use exponential propagation, we transform our system of Partial Differential Equations(PDEs) in to a system of coupled Ordinary Differential Equations(ODEs) by discretizing the spatial operators using the following finite difference operators:

For advective terms $\left(\frac{\partial A}{\partial t} = -v\frac{\partial A}{\partial x} + \chi(x)\right)$, where v is the velocity, we use a third order upwind scheme:

$$v\frac{\partial A}{\partial x} = \begin{cases} \frac{v}{6\Delta x}(-2A_{i-1} - 3A_i + 6A_{i+1} - A_{i+2}) & \text{if } v < 0 \\ \frac{v}{6\Delta x}(2A_{i+1} + 3A_i - 6A_{i-1} + A_{i-2}) & \text{if } v \geq 0 \end{cases} + \mathcal{O}(\Delta x^3) \quad (7.1)$$

For diffusive terms $\left(\frac{\partial A}{\partial t} = \eta\frac{\partial^2 A}{\partial x^2} + \chi(x)\right)$, where η is the diffusion coefficient, we use a second order space centered scheme:

$$\eta\frac{\partial^2 A}{\partial x^2} = \frac{\eta}{(\Delta x)^2}(A_{i-1} - 2A_i + A_{i+1}) + \mathcal{O}(\Delta x^2) \quad (7.2)$$

For other first derivative terms $\left(\frac{\partial A}{\partial t} = \frac{\partial B}{\partial x} + \chi(x)\right)$ we use a second order space centered scheme:

$$\frac{\partial B}{\partial x} = \frac{1}{2\Delta x}(A_{i-1} - A_{i+1}) + \mathcal{O}(\Delta x^2) \quad (7.3)$$

Here, $\chi(x)$ corresponds to all the additional terms a PDE might have on the righthand side besides the term under discussion and $A_i = A(x_0 + i\Delta x)$, $i = 1, 2, \dots, N_x$

is our variable evaluated in a uniform grid of N_x elements separated by a distance Δx .

Exponential Propagation

After discretization and inclusion of the boundary conditions we are left with an initial value problem of ordinary differential equations:

$$\frac{\partial \mathbf{U}(t)}{\partial t} = F(\mathbf{U}(t)) \quad (7.4)$$

$$\mathbf{U}(t_0) = \mathbf{U}_0 \quad (7.5)$$

where $\mathbf{U}$ is the solution vector in $\mathcal{R}^N$. Provided that the Jacobian $\partial F(\mathbf{U}(t))$ exist and is continuous in the interval $[t_0, t_0 + \Delta t]$, we can linearize $F(\mathbf{U}(t_0 + \Delta t))$ around the initial state to obtain

$$\frac{\partial \mathbf{U}(t)}{\partial t} = F(\mathbf{U}_0) + \partial F(\mathbf{U}_0)(\mathbf{U}(t_0 + \Delta t) - \mathbf{U}_0) + R(\mathbf{U}(t_0 + \Delta t)) \quad (7.6)$$

where $R(\mathbf{U}(t_0 + \Delta t))$ are the residual high order terms. The solution to this equation can be written as

$$\mathbf{U}(t_0 + \Delta t) = \mathbf{U}_0 + \frac{e^{A\Delta t} - I}{A} + \mathcal{O}(\Delta t^2) \quad (7.7)$$

where $A = \partial F(\mathbf{U}_0)$. Neglecting higher order terms leaves us with a scheme that is second order accurate in time and is an exact solution of the linear case. However there is a way of increasing the time accuracy of this method by following a generalization of Runge-Kutta methods for non-linear time-advancement operators proposed by Rosenbrock (1963). The combination of exponential propagation with Runge-Kutta

methods was first proposed by Hochbruck and Lubich (1997) and then generalized by Hochbruck, Lubich and Selhofer (1998). In this work we use a fourth order algorithm which goes to the following intermediary steps to advance the solution vector between timesteps ($\mathbf{U}^n \rightarrow \mathbf{U}^{n+1}$):

$$\begin{aligned}
k_1 &= \Phi\left(\frac{1}{2}\Delta t A\right) F(\mathbf{U}^n) \\
k_2 &= \Phi(\Delta t A) F(\mathbf{U}^n) \\
w_3 &= \frac{3}{8}(k_1 + k_2) \\
u_3 &= \mathbf{U}^n + \Delta t w_3 \\
d_3 &= F(u_3) - F(\mathbf{U}^n) - \Delta t A w_3 \\
k_3 &= \Phi\left(\frac{1}{2}\Delta t A\right) d_3 \\
\mathbf{U}^{n+1} &= \mathbf{U}^n + \Delta t \left(k_2 + \frac{16}{27}k_3\right)
\end{aligned}$$

$$A = F(\mathbf{U}^n) \quad , \quad \Phi(\Delta t A) = \frac{e^{A\Delta t} - I}{A}$$

Krylov Approximation to the Exponential Operator

Without any further approximation, this method is very expensive computationally due to the need of continuously evaluating the matrix exponential. However, it is possible to make a good approximation by projecting the operator into a finite dimensional Krylov subspace

$$S_{Kr} = \text{span}\{\mathbf{U}_0, A\mathbf{U}_0, A^2\mathbf{U}_0, \dots, A^{m-1}\mathbf{U}_0\}. \quad (7.8)$$

In order to do this we first compute an orthonormal basis for this subspace using the Arnoldi algorithm (Arnoldi 1951):

1. $v_1 = \mathbf{U}_0/|\mathbf{U}_0|^2$
2. For $j = 1, \dots, m$ do
 - for $i = 1, \dots, j$ compute $h_{i,j} = v_i^T * Av_j$
 - calculate $w = Av_j - \sum_{i=1}^j h_{i,j}v_i$
 - evaluate $h_{j+1,j} = |w|^2$
 - if $h_{j+1,j} < \epsilon$ stop, else compute the next basis vector $v_{j+1} = w/h_{j+1,j}$

ϵ is the parameter that sets the error tolerance in this approximation. Once we have finished computing the algorithm we have the relationship

$$AV_m \approx V_m H \quad (7.9)$$

where $V_m = [v_1, \dots, v_m]$ and H is a matrix whose elements are $h_{i,j} = v_i^T * Av_j$. The validity of this approximation depends on the dimension of the Krylov subspace but numerical experiments have found that 15-30 Krylov vectors are usually enough (see Hochbruck and Lubich 1997; Tokman 2001). Since $\{v_1, \dots, v_m\}$ is an orthonormal basis of the Krylov subspace S_{K_r} then $V_m^T V_m$ is a $m \times m$ identity matrix and $V_m V_m^T$ is a projector from $\mathcal{R}^N$ onto S_{K_r} . Using this projector we can find an approximation to any matrix vector multiplication by projecting them onto the Krylov subspace by calculating $Ab \approx V_m V_m^T A V_m V_m^T b$. After using Equation 7.9 this becomes

$$Ab \approx V_m H V_m^T b. \quad (7.10)$$

In fact we can approximate the action of any operator $\Phi(A)$, that can be expanded on a Taylor series, on the vector $\mathbf{U}_0$ using the Krylov subspace projection:

$$\Phi(A)\mathbf{U}_0 \approx V_m \Phi(H) V_m^T \mathbf{U}_0 = |\mathbf{U}_0|^2 V_m \Phi(H) e_1 \quad (7.11)$$

where $e_1 = [1 \ 0 \ \dots \ 0]$ and we used $v_1 = \mathbf{U}_0/|\mathbf{U}_0|^2$ and $V_m^T v_1 = e_1$. As we can see in Equation 7.11, the use of the Krylov approximation effectively reduces the size of the matrix operator; this makes the use of the exponential operator relatively inexpensive.

These two algorithms, in combination with a robust error control and an adaptive time-step mechanism strategy, form the core of the Exp4 integrator (for more details see Hochbruck and Lubich (1997); Hochbruck, Lubich and Selhofer (1998) and Tokman (2001)).

Running the Dynamo Benchmark

In order to verify the validity of the Exp4 code, as well as evaluate the code's performance in general we have compared it to the Surya code, which has been studied extensively in different contexts (e.g. Nandy & Choudhuri 2002, Chatterjee et al. 2004, Chatterjee & Choudhuri 2006), and is made available to the public on request. We have made tests concerning, runtime, resolution and overall performance and the results have been very favorable. We also ran case C' of the dynamo benchmark by Jouve et al. (2008). This case is similar in nature to the simulations done in this work with the following differences:

- The meridional flow profile has different radial and latitudinal dependence.

- The poloidal source term has no quenching term and a different radial and latitudinal dependence.
- The turbulent diffusivity profile consists of only one step and reaches a peak value of $10^{11} \text{ cm}^2/\text{s}$.
- The analytic differential rotation has no $\cos^4(\theta)$ dependence and uses a thinner tachocline.

In order to compare the performance of the different codes two quantities are used: $C_s^{crit} = \alpha_0^{crit} R_\odot \eta_{cz}$ which quantifies the maximum source strength that yields stable oscillations and $\omega = 2\pi R_\odot^2 / (T \eta_{cz})$ which quantifies the frequency of the magnetic cycle related to the diffusive timescale. The dependence of this quantities is then plotted versus resolution for all the different codes (see Figure 11 of Jouve et al. 2008). In order to compare the results of our simulation we plot this two quantities versus resolution using the same axis scale used in Jouve et al. 2008. As can be seen in Figure 48, we find lower values of C_s^{crit} than the values obtained in Jouve et al. 2008 (1.86 as opposed to an average 2.46) and we also find that this quantity is less sensitive to resolution in our code than on other codes. On the other hand, we find values of ω in perfect agreement with those found in Jouve et al. 2008 (540 as opposed to an average of 538.2). It is our belief that ω is a much more appropriate quantity for comparison between codes than C_s^{crit} since it is a solar observable quantity whereas C_s^{crit} , which is not, is more sensitive to the intrinsic difference between our numerical

scheme and the other codes used in the benchmark. On that light we consider that our code can successfully reproduce results obtained in the benchmark.

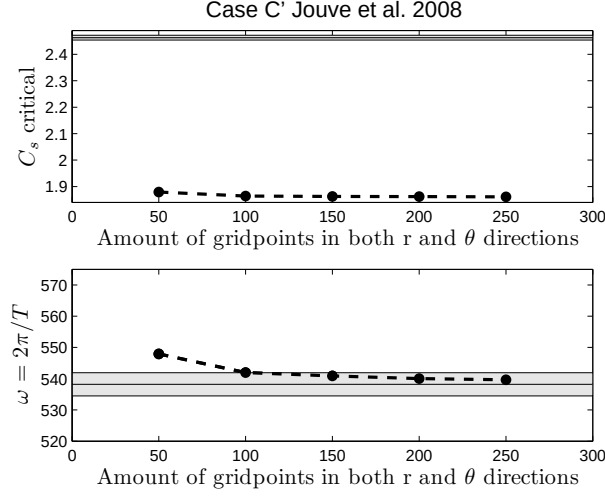


Figure 48. Results of running case C' of the dynamo benchmark by Jouve et al. (2008). In order to make the plots comparable we used the same quantities and same axis scale to make the plots. For more details refer to Jouve et al. (2008).

On a final remark, in Table 4 of Jouve et al. 2008 the different time steps that are used by the different codes is specified, ranging from values of 10^{-8} to 10^{-5} code time units. Thanks to the use of Krylov approximations we are able to use time steps as large as 10^{-2} code time units while solving Case C'. Although this doesn't mean a performance improvement of several orders of magnitude (due to the added computations in the Krylov approximation) we have found, by comparisons with the Surya code, that the Exp4 achieves a performance improvement that reduces runtime from a half to a tenth depending on the particularities of the simulation.

APPENDIX B

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REFERENCES CITED

- [1] ABBETT, W. P., BEAVER, M., DAVIDS, B., GEORGIOBIANI, D., RATHBUN, P., AND STEIN, R. F. Solar Convection: Comparison of Numerical Simulations and Mixing Length Theory. *Astrophys. J.* 480 (May 1997), 395–+.
- [2] ALFVÉN, H. Remarks on the Rotation of a Magnetized Sphere with Application to Solar Rotation. *Arkiv for Astronomi* 28 (1942), 1–9.
- [3] ARNOLDI, W. E. The principle of minimized iterations in the solution of the matrix eigenvalue problem. *Quart. Appl. Math.* 9 (1951), 17–29.
- [4] BABCOCK, H. W. The Topology of the Sun’s Magnetic Field and the 22-YEAR Cycle. *Astrophys. J.* 133 (Mar. 1961), 572–+.
- [5] BRANDENBURG, A. The Case for a Distributed Solar Dynamo Shaped by Near-Surface Shear. *Astrophys. J.* 625 (May 2005), 539–547.
- [6] BRAUN, D. C., AND BIRCH, A. C. Prospects for the Detection of the Deep Solar Meridional Circulation. *Astrophys. J.* 689 (Dec. 2008), L161–L165.
- [7] BRAUN, D. C., AND FAN, Y. Helioseismic Measurements of the Subsurface Meridional Flow. *Astrophys. J.* 508 (Nov. 1998), L105–L108.
- [8] BROWNING, M. K., MIESCH, M. S., BRUN, A. S., AND TOOMRE, J. Dynamo Action in the Solar Convection Zone and Tachocline: Pumping and Organization of Toroidal Fields. *Astrophys. J.* 648 (Sept. 2006), L157–L160.
- [9] CALIGARI, P., MORENO-INSERTIS, F., AND SCHUSSLER, M. Emerging flux tubes in the solar convection zone. 1: Asymmetry, tilt, and emergence latitude. *Astrophys. J.* 441 (Mar. 1995), 886–902.
- [10] CALIGARI, P., SCHUESSLER, M., AND MORENO-INSERTIS, F. Emerging Flux Tubes in the Solar Convection Zone. II. The Influence of Initial Conditions. *Astrophys. J.* 502 (July 1998), 481–+.
- [11] CARRINGTON, R. C. On the Distribution of the Solar Spots in Latitudes since the Beginning of the Year 1854, with a Map. *Mon. Not. R. Astron. Soc.* 19 (Nov. 1858), 1–3.
- [12] CARRINGTON, R. C. On certain Phenomena in the Motions of Solar Spots. *Mon. Not. R. Astron. Soc.* 19 (Jan. 1859), 81–84.
- [13] CHAN, K. L., AND SOFIA, S. Validity tests of the mixing-length theory of deep convection. *Science* 235 (Jan. 1987), 465–467.

- [14] CHARBONNEAU, P. Dynamo Models of the Solar Cycle. *Living Reviews in Solar Physics* 2 (June 2005), 2–+.
- [15] CHARBONNEAU, P. Babcock Leighton models of the solar cycle: Questions and issues. *Advances in Space Research* 39 (2007), 1661–1669.
- [16] CHARBONNEAU, P., CHRISTENSEN-DALSGAARD, J., HENNING, R., LARSEN, R. M., SCHOU, J., THOMPSON, M. J., AND TOMCZYK, S. Helioseismic Constraints on the Structure of the Solar Tachocline. *Astrophys. J.* 527 (Dec. 1999), 445–460.
- [17] CHARBONNEAU, P., AND MACGREGOR, K. B. Solar Interface Dynamos. II. Linear, Kinematic Models in Spherical Geometry. *Astrophys. J.* 486 (Sept. 1997), 502–+.
- [18] CHATTERJEE, P., NANDY, D., AND CHOUDHURI, A. R. Full-sphere simulations of a circulation-dominated solar dynamo: Exploring the parity issue. *Astron. Astrophys.* 427 (Dec. 2004), 1019–1030.
- [19] CHOU, D.-Y., AND LADENKOV, O. Evolution of Solar Subsurface Meridional Flows in the Declining Phase of Cycle 23. *Astrophys. J.* 630 (Sept. 2005), 1206–1212.
- [20] CHOUDHURI, A. R. The evolution of loop structures in flux rings within the solar convection zone. *Sol. Phys.* 123 (Sept. 1989), 217–239.
- [21] CHOUDHURI, A. R. On the Connection Between Mean Field Dynamo Theory and Flux Tubes. *Sol. Phys.* 215 (July 2003), 31–55.
- [22] CHOUDHURI, A. R., CHATTERJEE, P., AND JIANG, J. Predicting Solar Cycle 24 With a Solar Dynamo Model. *Physical Review Letters* 98, 13 (Mar. 2007), 131103–+.
- [23] CHOUDHURI, A. R., AND GILMAN, P. A. The influence of the Coriolis force on flux tubes rising through the solar convection zone. *Astrophys. J.* 316 (May 1987), 788–800.
- [24] CHOUDHURI, A. R., NANDY, D., AND CHATTERJEE, P. Reply to the Comments of Dikpati et al. *Astron. Astrophys.* 437 (July 2005), 703–704.
- [25] CHOUDHURI, A. R., SCHUSSLER, M., AND DIKPATI, M. The solar dynamo with meridional circulation. *Astron. Astrophys.* 303 (Nov. 1995), L29+.

- [26] CHRISTENSEN-DALSGAARD, J., DAPPEN, W., AJUKOV, S. V., ANDERSON, E. R., ANTIA, H. M., BASU, S., BATURIN, V. A., BERTHOMIEU, G., CHABOYER, B., CHITRE, S. M., COX, A. N., DEMARQUE, P., DONATOWICZ, J., DZIEMBOWSKI, W. A., GABRIEL, M., GOUGH, D. O., GUENTHER, D. B., GUZIK, J. A., HARVEY, J. W., HILL, F., HOUDEK, G., IGLESIAS, C. A., KOSOVICHEV, A. G., LEIBACHER, J. W., MOREL, P., PROFFITT, C. R., PROVOST, J., REITER, J., RHODES, JR., E. J., ROGERS, F. J., ROXBURGH, I. W., THOMPSON, M. J., AND ULRICH, R. K. The Current State of Solar Modeling. *Science* 272 (May 1996), 1286–+.
- [27] DIKPATI, M., AND CHARBONNEAU, P. A Babcock-Leighton Flux Transport Dynamo with Solar-like Differential Rotation. *Astrophys. J.* 518 (June 1999), 508–520.
- [28] DIKPATI, M., AND CHOUDHURI, A. R. On the Large-Scale Diffuse Magnetic Field of the Sun. *Solar Physics* 161 (Oct. 1995), 9–27.
- [29] DIKPATI, M., CORBARD, T., THOMPSON, M. J., AND GILMAN, P. A. Flux Transport Solar Dynamos with Near-Surface Radial Shear. *Astrophys. J.* 575 (Aug. 2002), L41–L45.
- [30] DIKPATI, M., DE TOMA, G., AND GILMAN, P. A. Predicting the strength of solar cycle 24 using a flux-transport dynamo-based tool. *Geophys. Res. Lett.* 33 (Mar. 2006), 5102–+.
- [31] DIKPATI, M., DE TOMA, G., AND GILMAN, P. A. Polar Flux, Cross-equatorial Flux, and Dynamo-generated Tachocline Toroidal Flux as Predictors of Solar Cycles. *Astrophys. J.* 675 (Mar. 2008), 920–930.
- [32] DIKPATI, M., AND GILMAN, P. A. Steps for Building a Calibrated Flux-Transport Dynamo for the Sun. *Sol. Phys.* 241 (Mar. 2007), 1–5.
- [33] DIKPATI, M., REMPEL, M., GILMAN, P. A., AND MACGREGOR, K. B. Comments on "Full-sphere simulations of circulation-dominated solar dynamo: Exploring the parity issue". *Astron. Astrophys.* 437 (July 2005), 699–702.
- [34] D'SILVA, S., AND CHOUDHURI, A. R. A theoretical model for tilts of bipolar magnetic regions. *Astron. Astrophys.* 272 (May 1993), 621–+.
- [35] DURNEY, B. R. On a Babcock-Leighton dynamo model with a deep-seated generating layer for the toroidal magnetic field. *Sol. Phys.* 160 (Sept. 1995), 213–235.

- [36] DURNEY, B. R. On a Babcock-Leighton Solar Dynamo Model with a Deep-seated Generating Layer for the Toroidal Magnetic Field. IV. *Astrophys. J.* 486 (Sept. 1997), 1065–+.
- [37] EDDY, J. A. The Maunder Minimum. *Science* 192 (June 1976), 1189–1202.
- [38] FAN, Y. Magnetic Fields in the Solar Convection Zone. *Living Reviews in Solar Physics* 6 (Dec. 2009), 4–+.
- [39] FAN, Y., AND FISHER, G. H. Radiative Heating and the Buoyant Rise of Magnetic Flux Tubes in the Solar interior. *Sol. Phys* 166 (June 1996), 17–41.
- [40] FAN, Y., FISHER, G. H., AND DELUCA, E. E. The origin of morphological asymmetries in bipolar active regions. *Astrophys. J.* 405 (Mar. 1993), 390–401.
- [41] FAN, Y., FISHER, G. H., AND MCCLYMONT, A. N. Dynamics of emerging active region flux loops. *Astrophys. J.* 436 (Dec. 1994), 907–928.
- [42] FAN, Y., AND GONG, D. On the twist of emerging flux loops in the solar convection zone. *Sol. Phys.* 192 (Mar. 2000), 141–157.
- [43] GHIZARU, M., CHARBONNEAU, P., AND SMOLARKIEWICZ, P. K. Magnetic Cycles in Global Large-eddy Simulations of Solar Convection. *Astrophys. J.* 715 (June 2010), L133–L137.
- [44] GILES, P. M., DUVAL, T. L., SCHERRER, P. H., AND BOGART, R. S. A subsurface flow of material from the Sun’s equator to its poles. *Nature* 390 (Nov. 1997), 52–54.
- [45] GILMAN, P. A., AND REMPEL, M. Concentration of Toroidal Magnetic Field in the Solar Tachocline by η -Quenching. *Astrophys. J.* 630 (Sept. 2005), 615–622.
- [46] GIZON, L., AND REMPEL, M. Observation and Modeling of the Solar-Cycle Variation of the Meridional Flow. *Sol. Phys.* 251 (Sept. 2008), 241–250.
- [47] GONZÁLEZ HERNÁNDEZ, I., HOWE, R., KOMM, R., AND HILL, F. Meridional Circulation During the Extended Solar Minimum: Another Component of the Torsional Oscillation? *Astrophys. J.* 713 (Apr. 2010), L16–L20.

- [48] GONZÁLEZ HERNÁNDEZ, I., KOMM, R., HILL, F., HOWE, R., CORBARD, T., AND HABER, D. A. Meridional Circulation Variability from Large-Aperture Ring-Diagram Analysis of Global Oscillation Network Group and Michelson Doppler Imager Data. *Astrophys. J.* 638 (Feb. 2006), 576–583.
- [49] GONZÁLEZ HERNÁNDEZ, I., PATRÓN, J., BOGART, R. S., AND THE SOI RING DIAGRAM TEAM. Meridional Flows from Ring Diagram Analysis. *Astrophys. J.* 510 (Jan. 1999), L153–L156.
- [50] GUERRERO, G., AND DE GOUVEIA DAL PINO, E. M. How does the shape and thickness of the tachocline affect the distribution of the toroidal magnetic fields in the solar dynamo? *Astron. Astrophys.* 464 (Mar. 2007), 341–349.
- [51] GUERRERO, G., AND DE GOUVEIA DAL PINO, E. M. Turbulent magnetic pumping in a Babcock-Leighton solar dynamo model. *Astron. Astrophys.* 485 (July 2008), 267–273.
- [52] GUERRERO, G., DIKPATI, M., AND DE GOUVEIA DAL PINO, E. M. The Role of Diffusivity Quenching in Flux-transport Dynamo Models. *Astrophys. J.* 701 (Aug. 2009), 725–736.
- [53] HALE, G. E. On the Probable Existence of a Magnetic Field in Sun-Spots. *Astrophys. J.* 28 (Nov. 1908), 315–+.
- [54] HALE, G. E., ELLERMAN, F., NICHOLSON, S. B., AND JOY, A. H. The Magnetic Polarity of Sun-Spots. *Astrophys. J.* 49 (Apr. 1919), 153–+.
- [55] HATHAWAY, D. H. Doppler Measurements of the Sun’s Meridional Flow. *Astrophys. J.* 460 (Apr. 1996), 1027–+.
- [56] HATHAWAY, D. H., NANDY, D., WILSON, R. M., AND REICHMANN, E. J. Evidence That a Deep Meridional Flow Sets the Sunspot Cycle Period. *Astrophys. J.* 589 (May 2003), 665–670.
- [57] HATHAWAY, D. H., AND RIGHTMIRE, L. Variations in the Sun’s Meridional Flow over a Solar Cycle. *Science* 327 (Mar. 2010), 1350–.
- [58] HOCHBRUCK, M., AND LUBICH, C. On Krylov subspace approximations to the matrix exponential operator. *SIAM Journal on Numerical Analysis* 34, 5 (1997), 1911–1925.
- [59] HOCHBRUCK, M., LUBICH, C., AND SELHOFER, H. Exponential integrators for large systems of differential equations. *SIAM Journal on Scientific Computing* 19, 5 (1998), 1552–1574.

- [60] HOWE, R. Solar Interior Rotation and its Variation. *Living Reviews in Solar Physics* 6 (Feb. 2009), 1–+.
- [61] HOWE, R., CHRISTENSEN-DALSGAARD, J., HILL, F., KOMM, R., SCHOU, J., AND THOMPSON, M. J. A Note on the Torsional Oscillation at Solar Minimum. *Astrophys. J.* 701 (Aug. 2009), L87–L90.
- [62] HOYT, D. V., AND SCHATTEN, K. H. Group Sunspot Numbers: A New Solar Activity Reconstruction. *Sol. Phys.* 181 (Aug. 1998), 491–512.
- [63] JIANG, J., IŞIK, E., CAMERON, R. H., SCHMITT, D., AND SCHÜSSLER, M. The Effect of Activity-related Meridional Flow Modulation on the Strength of the Solar Polar Magnetic Field. *Astrophys. J.* 717 (July 2010), 597–602.
- [64] JOUVE, L., AND BRUN, A. S. On the role of meridional flows in flux transport dynamo models. *Astron. Astrophys.* 474 (Oct. 2007), 239–250.
- [65] JOUVE, L., BRUN, A. S., ARLT, R., BRANDENBURG, A., DIKPATI, M., BONANNO, A., KÄPYLÄ, P. J., MOSS, D., REMPEL, M., GILMAN, P., KORPI, M. J., AND KOSOVICHEV, A. G. A solar mean field dynamo benchmark. *Astron. Astrophys.* 483 (June 2008), 949–960.
- [66] KOMM, R. W., HOWARD, R. F., AND HARVEY, J. W. Meridional Flow of Small Photospheric Magnetic Features. *Sol. Phys* 147 (Oct. 1993), 207–223.
- [67] KOSOVICHEV, A. G., SCHOU, J., SCHERRER, P. H., BOGART, R. S., BUSH, R. I., HOEKSEMA, J. T., ALOISE, J., BACON, L., BURNETTE, A., DE FOREST, C., GILES, P. M., LEIBRAND, K., NIGAM, R., RUBIN, M., SCOTT, K., WILLIAMS, S. D., BASU, S., CHRISTENSEN-DALSGAARD, J., DAPPEN, W., RHODES, JR., E. J., DUVALL, JR., T. L., HOWE, R., THOMPSON, M. J., GOUGH, D. O., SEKII, T., TOOMRE, J., TARBELL, T. D., TITLE, A. M., MATHUR, D., MORRISON, M., SABA, J. L. R., WOLFSON, C. J., ZAYER, I., AND MILFORD, P. N. Structure and Rotation of the Solar Interior: Initial Results from the MDI Medium-L Program. *Sol. Phys.* 170 (1997), 43–61.
- [68] KRIVOVA, N. A., BALMACEDA, L., AND SOLANKI, S. K. Reconstruction of solar total irradiance since 1700 from the surface magnetic flux. *Astron. Astrophys.* 467 (May 2007), 335–346.
- [69] LARMOR, J. How could a rotating body such as the sun become a magnet? *Brit. Assn. Adv. Sci. Rep.* (1919), 159–160.

- [70] LATUSHKO, S. Meridional drift in the large-scale solar magnetic field pattern. *Sol. Phys.* 149 (Feb. 1994), 231–241.
- [71] LEAN, J. L., WANG, Y., AND SHEELEY, N. R. The effect of increasing solar activity on the Sun’s total and open magnetic flux during multiple cycles: Implications for solar forcing of climate. *Geophys. Res. Lett.* 29, 24 (Dec. 2002), 240000–1.
- [72] LEIGHTON, R. B. Transport of Magnetic Fields on the Sun. *Astrophys. J.* 140 (Nov. 1964), 1547–+.
- [73] LEIGHTON, R. B. A Magneto-Kinematic Model of the Solar Cycle. *Astrophys. J.* 156 (Apr. 1969), 1–+.
- [74] LONGCOPE, D., AND CHOUDHURI, A. R. The Orientational Relaxation of Bipolar Active Regions. *Sol. Phys.* 205 (Jan. 2002), 63–92.
- [75] LOVE, J. J. Reversals and excursions of the geodynamo. *Astronomy and Geophysics* 40 (Dec. 1999), 14–+.
- [76] MACKAY, D. H., PRIEST, E. R., AND LOCKWOOD, M. The Evolution of the Sun’s Open Magnetic Flux - II. Full Solar Cycle Simulations. *Sol. Phys.* 209 (Oct. 2002), 287–309.
- [77] MAUNDER, E. W. Note on the distribution of sun-spots in heliographic latitude, 1874–1902. *Mon. Not. R. Astron. Soc.* 64 (June 1904), 747–761.
- [78] MIESCH, M. S., BRUN, A. S., DE ROSA, M. L., AND TOOMRE, J. Structure and Evolution of Giant Cells in Global Models of Solar Convection. *Astrophys. J.* 673 (Jan. 2008), 557–575.
- [79] MOFFAT, H. K. The mean electromotive force generated by turbulence in the limit of perfect conductivity. *J. Fluid Mech.* 57 (1974), 625–649.
- [80] MOFFAT, H. K. *Magnetic field generation in electrically conducting fluids*. Cambridge University Press, 1978.
- [81] MUÑOZ-JARAMILLO, A., NANDY, D., AND MARTENS, P. C. Effect of the Magnetic Quenching of the Turbulent Diffusivity in a Mean-Field Kinematic Solar Dynamo. *AGU Spring Meeting Abstracts* (May 2008), A9+.
- [82] MUÑOZ-JARAMILLO, A., NANDY, D., AND MARTENS, P. C. H. Helioseismic Data Inclusion in Solar Dynamo Models. *Astrophys. J.* 698 (June 2009), 461–478.

- [83] NANDY, D. Outstanding Issues in Solar Dynamo Theory. In *Magnetic Coupling between the Interior and Atmosphere of the Sun* (2010), S. S. Hasan & R. J. Rutten, Ed., pp. 86–95.
- [84] NANDY, D., AND CHOUDHURI, A. R. Toward a Mean Field Formulation of the Babcock-Leighton Type Solar Dynamo. I. α -Coefficient versus Durney’s Double-Ring Approach. *Astrophys. J.* 551 (Apr. 2001), 576–585.
- [85] NANDY, D., AND CHOUDHURI, A. R. Explaining the Latitudinal Distribution of Sunspots with Deep Meridional Flow. *Science* 296 (May 2002), 1671–1673.
- [86] PARKER, E. N. Hydromagnetic Dynamo Models. *Astrophys. J.* 122 (Sept. 1955), 293.
- [87] PARKER, E. N. A solar dynamo surface wave at the interface between convection and nonuniform rotation. *Astrophys. J.* 408 (May 1993), 707–719.
- [88] PRANDTL, L. *Math. Mech.* 5 (1925), 133–139.
- [89] REMPEL, M. Flux-Transport Dynamos with Lorentz Force Feedback on Differential Rotation and Meridional Flow: Saturation Mechanism and Torsional Oscillations. *Astrophys. J.* 647 (Aug. 2006), 662–675.
- [90] REMPEL, M. Origin of Solar Torsional Oscillations. *Astrophys. J.* 655 (Jan. 2007), 651–659.
- [91] REMPEL, M., AND SCHÜSSLER, M. Intensification of Magnetic Fields by Conversion of Potential Energy. *Astrophys. J.* 552 (May 2001), L171–L174.
- [92] ROBERTS, P. H., AND SOWARD, A. M. A unified approach to mean field electrodynamics. *Astronomische Nachrichten* 296 (1975), 49–64.
- [93] ROSENBROCK, H. Some general implicit processes for the numerical solution of differential equations. *Comput. J.* 5 (1963), 329–330.
- [94] RUEDIGER, G., KITCHATINOV, L. L., KUEKER, M., AND SCHULTZ, M. Dynamo models with magnetic diffusivity-quenching. *Geophysical and Astrophysical Fluid Dynamics* 78 (Dec. 1994), 247–+.
- [95] RWC BELGIUM. *Data Source: Solar Influences Data Analysis Center* (2010), <http://sidc.oma.be/sunspotdata/dailyssn.php>.

- [96] SCHOU, J., ANTIA, H. M., BASU, S., BOGART, R. S., BUSH, R. I., CHITRE, S. M., CHRISTENSEN-DALSGAARD, J., DI MAURO, M. P., DZIEMBOWSKI, W. A., EFF-DARWICH, A., GOUGH, D. O., HABER, D. A., HOEKSEMA, J. T., HOWE, R., KORZENNIK, S. G., KOSOVICHEV, A. G., LARSEN, R. M., PIJPERS, F. P., SCHERRER, P. H., SEKII, T., TARBELL, T. D., TITLE, A. M., THOMPSON, M. J., AND TOOMRE, J. Helioseismic Studies of Differential Rotation in the Solar Envelope by the Solar Oscillations Investigation Using the Michelson Doppler Imager. *Astrophys. J.* 505 (Sept. 1998), 390–417.
- [97] SCHOU, J., AND BOGART, R. S. Flow and Horizontal Displacements from Ring Diagrams. *Astrophys. J.* 504 (Sept. 1998), L131+.
- [98] SCHRIJVER, C. J., DE ROSA, M. L., AND TITLE, A. M. What Is Missing from Our Understanding of Long-Term Solar and Heliospheric Activity? *Astrophys. J.* 577 (Oct. 2002), 1006–1012.
- [99] SCHRIJVER, C. J., AND LIU, Y. The Global Solar Magnetic Field Through a Full Sunspot Cycle: Observations and Model Results. *Sol. Phys.* 252 (Oct. 2008), 19–31.
- [100] SCHÜSSLER, M., CALIGARI, P., FERRIZ-MAS, A., AND MORENO-INSERTIS, F. Instability and eruption of magnetic flux tubes in the solar convection zone. *Astron. Astrophys.* 281 (Jan. 1994), L69–L72.
- [101] SCHWABE, M. Sonnenbeobachtungen im Jahre 1843. Von Herrn Hofrath Schwabe in Dessau. *Astronomische Nachrichten* 21 (1844), 233+.
- [102] SHEELEY, JR., N. R., DEVORE, C. R., AND BORIS, J. P. Simulations of the mean solar magnetic field during sunspot cycle 21. *Sol. Phys.* 98 (Aug. 1985), 219–239.
- [103] SNODGRASS, H. B., AND DAILEY, S. B. Meridional Motions of Magnetic Features in the Solar Photosphere. *Sol. Phys.* 163 (Jan. 1996), 21–42.
- [104] SOLANKI, S. K. Sunspots: An overview. *Astron. Astrophys. Rev.* 11 (2003), 153–286.
- [105] SOLANKI, S. K., SCHÜSSLER, M., AND FLIGGE, M. Evolution of the Sun’s large-scale magnetic field since the Maunder minimum. *Nature* 408 (Nov. 2000), 445–447.

- [106] SPOERER, F. W. G. Beobachtungen von Sonnenflecken, von Herrn Dr. Spoerer in Anclam (Fortsetzung). *Astronomische Nachrichten* 55 (Aug. 1861), 379–+.
- [107] STEENBECK, M., KRAUSE, F., AND RÄDLER, K. H. A Calculation of the Mean Electromotive Force in an Electrically Conducting Fluid in Turbulent Motion Under the Influence of Coriolis Forces. *Zeitschrift Naturforschung Teil A* 21 (1966), 369–376.
- [108] STIX, M. Differential rotation and the solar dynamo. *Astronom. Astrophys.* 47 (Mar. 1976), 243–254.
- [109] THOMPSON, M. J., TOOMRE, J., ANDERSON, E. R., ANTIA, H. M., BERTHOMIEU, G., BURTONCLAY, D., CHITRE, S. M., CHRISTENSEN-DALSGAARD, J., CORBARD, T., DE ROSA, M., GENOVESE, C. R., GOUGH, D. O., HABER, D. A., HARVEY, J. W., HILL, F., HOWE, R., KORZENNIK, S. G., KOSOVICHEV, A. G., LEIBACHER, J. W., PIJPERS, F. P., PROVOST, J., RHODES, JR., E. J., SCHOU, J., SEKII, T., STARK, P. B., AND WILSON, P. R. Differential Rotation and Dynamics of the Solar Interior. *Science* 272 (May 1996), 1300–1305.
- [110] TOBIAS, S. M. Diffusivity Quenching as a Mechanism for Parker’s Surface Dynamo. *Astrophys. J.* 467 (Aug. 1996), 870–+.
- [111] TOBIAS, S. M., BRUMMELL, N. H., CLUNE, T. L., AND TOOMRE, J. Transport and Storage of Magnetic Field by Overshooting Turbulent Compressible Convection. *Astrophys. J.* 549 (Mar. 2001), 1183–1203.
- [112] TOKMAN, M. *Magnetohydrodynamic Modeling of Solar Magnetic Arcades Using Exponential Propagation Methods*. PhD thesis, California Institute of Technology, 2001.
- [113] ŠVANDA, M., KOSOVICHEV, A. G., AND ZHAO, J. Speed of Meridional Flows and Magnetic Flux Transport on the Sun. *Astrophys. J.* 670 (Nov. 2007), L69–L72.
- [114] VAN BALLEGOOIJEN, A. A., AND CHOUDHURI, A. R. The possible role of meridional flows in suppressing magnetic buoyancy. *Astrophys. J.* 333 (Oct. 1988), 965–977.
- [115] VAN BALLEGOOIJEN, A. A., AND MACKAY, D. H. Model for the Coupled Evolution of Subsurface and Coronal Magnetic Fields in Solar Active Regions. *Astrophys. J.* 659 (Apr. 2007), 1713–1725.

- [116] WANG, Y. The Sun's Large-Scale Magnetic Field and Its Long-Term Evolution. *Sol. Phys.* 224 (Oct. 2004), 21–35.
- [117] WANG, Y., LEAN, J., AND SHEELEY, JR., N. R. Role of a Variable Meridional Flow in the Secular Evolution of the Sun's Polar Fields and Open Flux. *Astrophys. J.* 577 (Sept. 2002), L53–L57.
- [118] WANG, Y., ROBBRECHT, E., AND SHEELEY, N. R. On the Weakening of the Polar Magnetic Fields during Solar Cycle 23. *Astrophys. J.* 707 (Dec. 2009), 1372–1386.
- [119] WANG, Y., AND SHEELEY, JR., N. R. Average properties of bipolar magnetic regions during sunspot cycle 21. *Sol. Phys.* 124 (Mar. 1989), 81–100.
- [120] WANG, Y., AND SHEELEY, JR., N. R. Magnetic flux transport and the sun's dipole moment - New twists to the Babcock-Leighton model. *Astrophys. J.* 375 (July 1991), 761–770.
- [121] WANG, Y., SHEELEY, JR., N. R., AND LEAN, J. Meridional Flow and the Solar Cycle Variation of the Sun's Open Magnetic Flux. *Astrophys. J.* 580 (Dec. 2002), 1188–1196.
- [122] WANG, Y.-M., NASH, A. G., AND SHEELEY, JR., N. R. Evolution of the sun's polar fields during sunspot cycle 21 - Poleward surges and long-term behavior. *Astrophys. J.* 347 (Dec. 1989), 529–539.
- [123] WILMOT-SMITH, A. L., NANDY, D., HORNIG, G., AND MARTENS, P. C. H. A Time Delay Model for Solar and Stellar Dynamos. *Astrophys. J.* 652 (Nov. 2006), 696–708.
- [124] YEATES, A. R., MACKAY, D. H., AND VAN BALLEGOOIJEN, A. A. Modelling the Global Solar Corona: Filament Chirality Observations and Surface Simulations. *Sol. Phys.* 245 (Sept. 2007), 87–107.
- [125] YEATES, A. R., MACKAY, D. H., AND VAN BALLEGOOIJEN, A. A. Modelling the Global Solar Corona II: Coronal Evolution and Filament Chirality Comparison. *Sol. Phys.* 247 (Jan. 2008), 103–121.
- [126] YEATES, A. R., NANDY, D., AND MACKAY, D. H. Exploring the Physical Basis of Solar Cycle Predictions: Flux Transport Dynamics and Persistence of Memory in Advection- versus Diffusion-dominated Solar Convection Zones. *Astrophys. J.* 673 (Jan. 2008), 544–556.

- [127] YOSHIMURA, H. Solar-cycle dynamo wave propagation. *Astrophys. J.* 201 (Nov. 1975), 740–748.
- [128] ZEEMAN, P. On the Influence of Magnetism on the Nature of the Light Emitted by a Substance. *Astrophys. J.* 5 (May 1897), 332–+.
- [129] ZHAO, J., AND KOSOVICHEV, A. G. Torsional Oscillation, Meridional Flows, and Vorticity Inferred in the Upper Convection Zone of the Sun by Time-Distance Helioseismology. *Astrophys. J.* 603 (Mar. 2004), 776–784.