Determining the change in snow microstructure during large deformation processes by the method of quantitative stereology
by Michael Quast Edens

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering Mechanics
Montana State University

Abstract:
Modern constitutive theories for snow are now using microstructural parameters in their formulation. In order to improve the theories, more advanced methods of describing the microstructural behavior are needed. This is particularly important since simplifying assumptions must be made in order that the resulting theory is manageable. To confirm understanding of microstructural behavior it is necessary to obtain experimental data pertinent to the density range, deformation range, and deformation rate being modeled. This data is also needed for the evaluation of empirical parameters.

The microstructural variables selected to characterize the behavior of snow must be able to represent dominant mechanisms such as pore collapse, bond fracture, and neck and bond growth due to pressure sintering as well as effects of pore pressure, a mechanism to account for a reduction in grain mobility, coupling of deviatoric and volumetric responses, work hardening, and local inertial effects.

In this thesis a set microstructural variables that meet these criteria and corresponding mathematical relations from quantitative stereology are reviewed along with relations and techniques required for numerical evaluation. An experimental investigation is carried out to understand the effect changes in these variables have on the behavior of snow subjected to large deformations. Measurements at several stages of deformation are used to understand microstructure changes, dominant mechanisms, and effects on bulk behavior.

Microstructure measurements of six snow samples subjected to confined compression tests are presented for precompressed and compressed states corresponding to final loads of 11.2 KN, 22.4 KN, or 44.8 KN. Order of magnitude changes in microstructural parameters are compared with corresponding magnitude changes in stress level for densities ranging from 0.5 g/cc to 0.64 g/cc. Generally, microstructural variables underwent order of 2 changes compared with 16 for the stress level.

This experimental investigation presents changes in microstructural variables that result from large deformations and high deformation rate. These results go a long-way toward filling in a gap in the presently available data. Microstructural variables which, to the present, have not been adequately measured and mathematically described are evaluated.

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ABSTRACT

Modern constitutive theories for snow are now using microstructural parameters in their formulation. In order to improve the theories, more advanced methods of describing the microstructural behavior are needed. This is particularly important since simplifying assumptions must be made in order that the resulting theory is manageable. To confirm understanding of microstructural behavior it is necessary to obtain experimental data pertinent to the density range, deformation range, and deformation rate being modeled. This data is also needed for the evaluation of empirical parameters.

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INTRODUCTION

A material that will affect nearly everyone living in polar climates at some time is snow. Its mechanical properties can exhibit a strong influence on everything from vehicle travel (Brown, 1979) and winter recreation to avalanche prediction and control. One of the features most commonly observed by those living in regions where snow ground cover occurs is its compressibility. Snow can undergo large volumetric deformations due to moderate bulk stresses. These deformations are basically irreversible. The ability to undergo large deformation is due in large part to its porosity. Brown (1979) demonstrated the ability of a constitutive law based on a pore collapse model to accurately represent the observed behavior of snow subjected to large strains and strain rates.

In its natural state snow exhibits a wide range of grain shapes and densities. Able and Gow (1975 and 1976) have demonstrated the affects initial snow density has on load bearing ability. From their experiments, it was clearly demonstrated that for undisturbed snow with densities from 0.1 to 0.27 g/cc and subjected to high rate deformation, lower initial densities resulted in lower final densities for a given final stress than that with a higher initial density. They attributed this to the very irregular shaped grains found in very low density snow and the more uniform rounded nature of the higher density material. It was found that the initial density was a significant factor for final stresses < 1 Mpa. Beyond this, the effects of higher initial density became less obvious. Their studies also showed the rate dependent nature of snow's load bearing ability.

Snow is a thermodynamically active material. This has been clearly observed in collected samples left in storage longer than two months. Volume reductions on the order of 0.5 have been observed. This reduction is a result of the snow trying to reduce its surface area to volume ratio. This same mechanism is important in neck growth due to pressure sintering. A volumetric constitutive law based on a neck growth model was developed by Brown (1980). This model included structural parameters that account for changes in neck length, grain radius, and coordination number. The resulting equation accurately described the behavior of snow subjected to a given deformation rate for a range of densities.

A constitutive law based upon nonlinear thermodynamics that accounts for the mechanical properties of snow subjected to finite strain under high rate uniaxial deformation has been developed by Hansen (1985). The microstructural properties of snow in this theory are included in the form of internal state variables. This theory was shown to predict the behavior of mechanical properties such as rate dependence, stress relaxation, and strain recovery.

Modern constitutive theories are now using microstructural parameters. In order to improve the theories, more advanced methods of describing the microstructural behavior are needed, thus this thesis.

Purpose

In all of the theories previously mentioned, accuracy is dependent on understanding the behavior of the microstructural variable included. This is particularly important since simplifying assumptions must be made in order that the resulting theory is manageable. To confirm understanding of microstructural behavior and validity of assumptions of such, it is necessary to obtain experimental data pertinent to the density range, deformation range, and deformation rate being modeled. This data is also needed

for the evaluation of empirical parameters. At the present time, little or no data on microstructural behavior exists for snow densified by large deformation to densities in the 0.5 to 0.65 g/cc range. The work of Able and Gow mentioned earlier provides a strong basis for understanding bulk behavior of snow subjected to high deformation rates and large strains, but little or no data was obtained relating the microstructure to the deformation. That obtained was for samples that had been unloaded for several hours. The resulting data may not accurately reflect the actual microstructure existing at the end of the deformation process since crystallization had time to occur in significant amounts. Gubler (1978a, 1978b) and Kry (1975a, 1975b) carried out extensive microstructural evaluation in their work, but it was generally confined to final densities below 0.5 g/cc and very low deformation rates.

It is the major purpose of this experimental investigation to evaluate the changes in microstructural variables that result from large deformations and high deformation rate. The results obtained will go a long way toward filling in a gap in the presently available data. In addition, it is intended to measure microstructural variables which to the present have not been adequately measured and mathematically described. An empirical equation relating the number of bonds per volume to the mean pore length will be given and its physical significance discussed. A new definition of bandwidth in Gubler's (1978a) work will also be given.

STEREOLOGY AND MICROSTRUCTURAL PARAMETERS

Stereology is a statistically based method by which quantities measured directly from a two dimensional plane, intersecting a three dimensional body, are used to obtain average values for the body's three dimensional structure. The relationships between two and three dimensions and the resulting numerical values can be used as a means for understanding the average microstructural behavior of the body. The task then is to select those parameters most important in characterizing the average behavior and making the pertinent section measurements. In this chapter, possible microstructural variables discussed by Hansen (1985) are reviewed along with the necessary two dimensional measurements. A new definition of the bandwidth defined by Gubler (1978a) for his method of calculating mean three dimensional coordination number is also presented.

Requirements for Microstructure Variables

The variables selected to represent the microstructure of snow under large deformations and high rate deformation must be able to characterize the average rearrangement of the microstructure. These variables must be able to represent dominant mechanisms such as pore collapse, bond fracture, and neck and bond growth due to pressure sintering. Hansen (1985) lists 5 other phenomena that these parameters must account for : (1) effects of pore pressure, (2) a locking mechanism to account for a reduction in grain mobility, (3) coupling of deviatoric and volumetric responses, (4) work hardening and (5) local inertial effects.

Two Dimensional Parameters and Measurements

Measurements from a section plane are generally based upon three counting methods: point, lineal and areal. The main assumptions necessary for validity of measurements are homogeneity and isotropy. Homogeneity requires that the material exhibits the same properties throughout the body. This may be difficult to meet in small regions, but if large enough areas are measured this requirement can be satisfied. Isotropy requires that the orientation of material structures are not predominate in any direction. For oriented structures, a series of randomly placed and oriented test lines or grids may be used.

Hansen (1985) considered the microstructure of snow by representing it as a three phase mixture. The phases were air, neck, and ice grain. The subscripts used to represent each phase and a brief description of each follow.

a: air phase — The air phase represents the pore space of the material matrix.

b: neck phase — The neck phase represents the material joining grains together.

g: ice grain phase — The grain phase represents the ice grains.

The required measurements of each phase in the section plane are represented by the following variables:

N_{gL}: number of grains intercepted per unit length of test line

N_{bA}: number of bonds per unit area

N_{gA}: number of grains per unit area

A_{aA}: area fraction of air phase.

A_{bA}: area fraction of neck phase.

A_{gA}: area fraction of ice grain phase.

The remaining surface section measurements required are

α: ratio of ice density to snow density, ρ_i / ρ_s

D_2: mean bond diameter

E: harmonic mean of bond diameter

f0: fraction of grains with 0 bonds appearing in the section plane

f1: fraction of grains with 1 bond appearing in the section plane

f2: fraction of grains with 2 bonds appearing in the section plane

f3: fraction of grains with 3 bonds appearing in the section plane.

Before proceeding further, two definitions are needed. The first is that of a grain bond. The definition used is that given by Kry (1975a). A grain bond intercepted by the section plane will appear as a line joining two opposite edges of the ice in the plane. To identify this line as a bond, three criteria must be met. First, there must be a minimum constriction of the ice in the section plane. Secondly, the constriction must be present on both edges of the ice. Finally, the constrictions must point approximately towards each other. See Figure 1. In practice, applying this definition will have some subjectivity involved. As long as consistency in applying the definition is observed, comparison of measurements should not be seriously affected.

The second definition is that of the neck region shown in Figure 1. This definition comes from Hansen (1985). The neck is considered to be that portion of the ice joining two ice grains and lying between the points where the surface goes from convex to concave with respect to the outward normal. The application of this definition also contains an element of subjectivity. These regions tend to contain less than 2% of the total section area. Errors in location of the change in curvature or identification will have only small affect on other measurements. The region enclosed by the large rectangle in Figure 2 clearly shows all phases of interest for an image enhanced surface section. Now the section variables and their measurement can be discussed. In discussing each variable, it is indicated whether the measurement is visual or automatic using computer analysis software.

Number of Grains Per Unit Length of Test Line: N_{gL}

N_{gL} is found by taking a number test lines spaced evenly across the section, counting the total number of grains intercepted by these lines and dividing this by the total length of the test lines. In the surface analysis program, a test line is displaced parallel to itself across the entire area being analyzed in steps of 0.2 scale markings. At each step, the number of intercepted grains are then counted. Grains intercepted by the left region boundary are not counted; those on the right are counted as 1 each. Refer to Figure 1 for a graphic representation of the counting procedure.

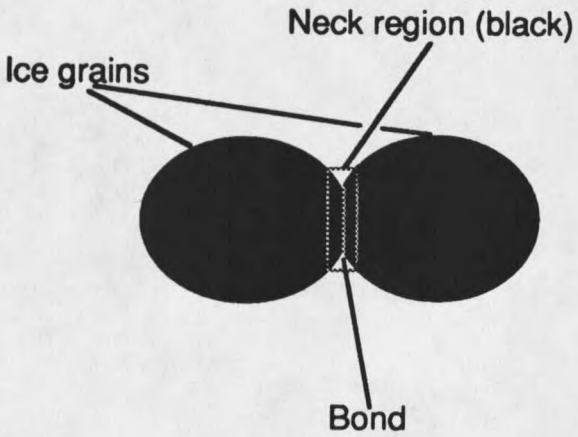

Figure 1. An idealized picture of bonded ice grains. This shows the line representing the bond in two dimensions and the neck region, the black portion within the rectangle.

Number of Bonds Per Unit Area: N_{bA}

N_{bA} is simply the number of bonds in the test area divided by the test area. Each bond is automatically counted as the bond diameter is measured. Bonds that are intercepted by the region boundary are counted as 1/2 and are not measured.

The Number of Grains
Per Unit Area: N_{gA}

N_{gA} is similar to N_{bA}. In this case the number of grains must be counted manually. The grains are actually counted in five separate categories, grains having 0, 1, 2, 3, or more than 3 bonds appearing in the surface section. Each category of grain intercepted by the region boundary is counted as 1/2. Summing all the categories gives the number of grains. Figure 1. shows an example of this for the top boundary.

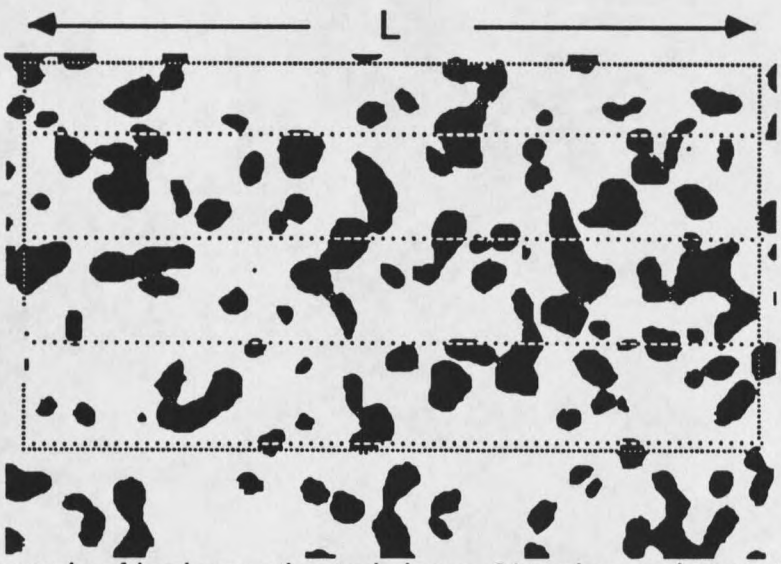

Figure 2. An example of basic counting techniques. 21 grains are intercepted by a total line length of 4*L. The resulting value of N_{gL} is 21/(4*L). Looking at the top boundary line, the count for 0 bonds is 1.5, for 1 bond 1/2+1/2 =1, 2 bonds 1/2, 3 and >3 bonds both are 0. The total number of grains along the top boundary is then 3.

Air, Bond and Ice AreaFractions:
A_{aA}, A_{bA}, A_{gA}

Area fraction measurements are simply the total area of the phase of interest divided by the total analysis area. The neck regions are delineated when the bond is measured and appear gray on the section. These measurements are an automatic feature of the analysis program.

Mean 2-D Bond Diameter and Harmonic Mean of 2-D Bond Diameters: D_2, E

The 2-D bond diameter D_2 is the average of all bonds within the analysis region. Those intercepted by the boundary are excluded from this measurement.

The Harmonic mean E, given by Fullman (1953) is

$$E=\frac{1}{N}\sum_{I=1}^{N}\frac{1}{D_{2i}} \qquad (1)$$

where N is the number of 2-D bonds in the section plane and D_{2i} is length of the i^{th} bond as it is measured in the surface section. Though E has no readily apparent physical meaning it is important as a variable in equations for polydispersed spheres and disks. In the work presented, bonds in 3-D are approximated by disks, and it is in this context that E is used.

Bond Frequency: f0,f1,f2,f3

f_0, f_1, f_2, and f_3 represent the relative frequency of grains with 0, 1, 2, and 3 bonds respectively. This measurement requires visually counting the total number of bonds in each category. This is the same counting measurement used for N_{gA}. The importance of these measurements will become more evident when the method for calculating the 3-D coordination number is discussed later in this chapter.

Density Ratio: α

The last parameter discussed in this section is the density ratio α. It is the ratio of ice density ρ_i to the snow density ρ. For ice at -10°C ρ_i has a value of 0.917 g/cc. The stereologic relationship for α is given by the following (Underwood, 1970):

$$\alpha = \frac{1}{A_{gA}+A_{bA}}. \qquad (2)$$

As well as being a variable in several of the 3-D parameters, α serves as a check on the accuracy of the image enhancement process, in particular, any interpretation required of section features. It is often necessary to determine whether pits that occur in the plane are grains that have been dislodged, grains or pore space which have been poorly polished, or air bubbles sliced by the section.

3-D Parameters

Two types of parameters will be discussed here, those that come directly from the 2-D values and those that normally require some assumption of a particle shape factor. Those which need only 2-D values are based purely on statistical assumptions and are therefore valid for particles of any shape. Before proceeding, the definition of a bond in three dimensions is needed. As given by Kry (1975a), a grain bond is defined to be the plane surface of minimum area located in the neck region of two joined grains.

Mean Bond Radius: R_3

The neck regions within the three dimensional snow structure represent locations of high stress concentration when the bulk material is subjected to external loading. These regions may have large constrictions relative to the grain structure. Since the grain bond is the location of the largest constriction, it will then have the highest stress concentration. Due to this, R_3 along with the mean three dimensional coordination number N_3 are important in determining the load bearing ability of the snow. R_3 is also important in characterizing bond growth due to pressure sintering. It was used by Brown (1979) as a parameter in a volumetric constitutive law based on a neck growth model. Kry (1975a) suggested that the bonds in three dimensions may be represented by circular disks. Based upon this assumption, the necessary relationship for the bond radius as derived by Fullman (1953) is

$$R_3 = \frac{\Pi}{4E}. \tag{3}$$

Mean Neck Length: h

Hansen (1985) gives the following relation for h:

$$h = \frac{A_{bA}}{N_{bV}\Pi r^2}. \tag{4}$$

h was used by Brown in his neck growth constitutive law. An increase in h occurs during pressure sintering and from mass transport as snow attempts to reduce its surface area to volume ratio.

Mean Pore Length: λ

The mean pore length is important in characterizing grain mobility within the material matrix. This grain mobility is critical in controlling the amount of deformation the material can undergo for a given load condition. Pressure sintering becomes more dominant as λ decreases under external loading. Another important result is an increase in the number of grain contacts. The equation for λ, derived by Fullman (1953) is valid regardless of grain shape, size, or distribution, and is given as

$$\lambda = \frac{\alpha - 1}{\alpha N_{gL}}. \tag{5}$$

Mean Number of Bonds
Per Unit Volume: N_{bV}

When a load is placed upon snow, not only bond area but also the number of bonds per grain affect the stress concentration at a bond. An increase in the number of bonds prior to an increase in load is effectively like increasing the area of a fewer number of bonds. N_{bV}'s importance is as a variable in the mean three dimensional coordination number N_3. Based on the assumption of bonds being circular disks, Fullman (1953) has

derived the following relation necessary for determining the mean number of bonds per volume:

$$N_{bV} = \frac{8EN_{bA}}{\Pi^2}. \tag{6}$$

Mean Intercept Length: L_3

The equation for the mean intercept length derived by Underwood (1970) is valid without assumptions of grain shape, size, or distribution. It is given by

$$L_3 = \frac{A_{gA}+A_{bA}}{N_{gL}}. \tag{7}$$

L_3 provides a measure of the grain length. It is useful along with the mean pore length in characterizing grain mobility.

Mean Grain Surface Area Per Volume: S_v

The mean surface area per unit volume is related to the free energy in the sample. As grain bonds break there will be an increase in surface area and, as a result, an increase in free energy. This will affect the neck and bond growth occurring during pressure sintering. The equation derived by Hansen (1985) is

$$S_V = 4N_{gL} - 2\Pi R^2 N_3 N_{gV}. \tag{8}$$

This represents the mean surface area per volume of detached grains, minus, the mean surface area of the grain bonds.

3-D Microstructure Requiring a Grain Shape Factor

The parameters that remain to be calculated are N_{gV}, the mean number of grains per unit volume; N_3, the mean number of bonds per grain; and V, the mean grain volume.

These parameters require some knowledge of grain shape. Attempting to characterize alpine snow by a grain shape factor may not be a valid generalization due to the wide range of shapes present within a given sample. Gubler (1978a) and Hansen (1985) provided a method for determining these parameters without any assumption of grain shape, other than for an initial guess.

Gubler's method for evaluating N_3 was based upon several parameters. The first was the probability p defined to be the probability that if a grain with coordination number $N_3=1$ is cut by the section plane, its bond lies in the section plane. Next, the distribution of coordination number about the mean coordination number N_3 was used. This distribution could be changed by an adjustable free parameter i. i has no apparent physical meaning.

The procedure was then to adjust N_3 and i until the theoretical values of the two dimensional distribution given by

$$f_2(l)=N_2\sum_{n=l}^{12}\binom{n}{l}p(1-p)^{n-l}f_3(n) \tag{9}$$

converged to the measured values. N_2 is a normalizing constant introduced by Hansen (1985) and is given by

$$N_2\sum_{l=0}^{6}f_2(l)=1. \tag{10}$$

$\binom{n}{l}$, the number of combinations is given by

$$\binom{n}{l}=\frac{n!}{l!(n-l)!}. \tag{11}$$

$f_3(n)$, the three dimensional probability distribution function of a grain having a coordination number n, derived by Gubler (1978a), is given by

$$f_3(n)=N3\,n^{3i-1}\exp(-\beta n^{2i}). \tag{12}$$

N3 is a normalizing constant given by

$$N3 = 4\,i\frac{\beta 2^{\frac{3}{2}}}{\sqrt{\Pi}}. \tag{13}$$

β is given by

$$\beta = \left[\frac{2}{\sqrt{\Pi}}\frac{1}{N_3}\Gamma\left(\frac{3}{2}+\frac{1}{2\,i}\right)\right]^{2i}, \tag{14}$$

where Γ is the gamma function.

The definition of p is given by

$$p = \frac{s_\beta}{s_k}, \tag{15}$$

where

$$s_\beta = \frac{\Pi N_{gL} b}{N_{gA}}, \tag{16}$$

and

$$s_k = \frac{4N_{gL}}{N_{gV}}. \tag{17}$$

s_β is the area of a band of width b whose center line is the section plane. s_k is the mean grain surface area. b is given by

$$b = 2\left[R_3^2 - \left(\frac{D_2}{2}\right)^2\right]^{\frac{1}{2}}. \tag{18}$$

Hansen adjusts p and N_3 by varying N_{gV} and using the equation

$$N_3 = \frac{2N_{bV}}{N_{gV}} \tag{19}$$

which is neglected by Gubler. It is by using equation 19 and directly varying N_{gV} and i

to force equation 9 to converge to the measured values, rather than N_3 and i, that Hansen is able to determine N_{gv} without resorting to the assumption of a shape factor. One problem with Gubler's approach is that he has fixed N_{gv} by assuming a shape factor C and using

$$N_{gv} = C\frac{N_{gA}^2}{N_{gL}}. \tag{20}$$

This fixes the value of N_3 since N_{bV} is known, yet he still allows N_3 to vary which may lead to uniqueness problems.

A slight digression will now be taken in order to introduce a new definition of the bandwidth b. In equation 18, b/2 is the height of a right triangle necessary to yield a line of length $D_2/2$ lying in the intersecting plane, when R_3, the hypotenuse is projected onto the plane. The small bonds intersected by the section plane provided the most weight in determining the magnitude of R_3, with the largest necks having much less influence. This occurs, because R_3 is based upon the sum of the reciprocal of the two dimensional bonds (equation 1 and 3). With this in mind it is then reasonable to assume that when R_3 is projected onto the section plane, one should find those values which most affect its magnitude. The definition as given does not allow for this. Only those values lying between $D_2/2$ and R_3 will be found within the resulting bandwidth. If this band is narrow, it may happen that all but a small number of measured necks will lie within the band. By replacing the term in parenthesis in equation 18 with the minimum value found in the section plane, b will then contain most of the necks found in the section plane, particularly those that most heavily influence its magnitude. The new definition for bandwidth becomes

$$b= 2\left[R_3^2 - \left(\frac{D_{2min}}{2}\right)^2\right]^{\frac{1}{2}}. \tag{21}$$

Once N_{gv} is determined using Hansen's approach along with the new definition of bandwidth, the mean grain volume can be determined. The governing equation given by Hansen (1985) is

$$V = \frac{A_{gA} + A_{bA}}{N_{gV}} . \tag{22}$$

Hansen included the neck area fraction since the neck becomes an integral part of the grain when the bond fractures. This term is important in measuring inertial effects, particularly at very high rate deformation.

The equations in this chapter have been incorporated in to two computer programs. A new program written in Forth contains code that will allow evaluation of all surface section variables except those based upon the work of Gubler. A Fortran program written by Hansen and since modified to run on a Macintosh computer and incorporating the new bandwidth, contains code for the remainder of the variables. These will be discussed further in the next chapter along with results.

EXPERIMENT AND RESULTS

In this chapter the experimental procedure, results, and conclusions will be presented. The data provided in this chapter will fill a gap in that presently available by providing extensive measurements of snow microstructure over a wide density range, resulting from a high deformation rate and large strains.

Experimental Procedure

Alpine snow was collected and stored in a cold room at -10° C. Confined compression tests were carried out within a period of two to six months after the samples were first placed in the cold room. After two months, the original sample volume had decreased by nearly 50%, and volume changes occurring after this were generally very small. The snow samples consisted of snow from two years, 1987 and 1988. The large grained samples 10k01, 5k001 and 2k501 were from February 1987, 5k002 and 2k502 were from February 1988.

The first part of the experiment involved subjecting samples to a constant deformation rate, in this case 1" per minute, until a predetermined maximum load value was reached. The possible maximums were 44.8 KN, 22.4 KN, or 11.2 KN. Once this load was attained, the test was stopped. Surface sections were then immediately prepared from the uncompressed and compressed samples.

The compressive tests were carried out using an MTS 880 materials test system capable of loadings as high as 222.5 KN. The crosshead and samples were enclosed in an environmental chamber maintained at -10° C. Samples were prepared in the cold room for testing by cutting them in square segments with edge dimensions slightly larger

than the compression ring diameter. The compression ring was then pushed onto the cubed sample and the protruding snow trimmed flush with the top of the ring. The bottom edge of the compression ring has a beveled edge which facilitates cutting through the sample without damage. The sample and compression ring were then weighed together. The weight of the ring was subtracted, and the initial density calculated using the ring volume.

The compression ring consists of four separate rings placed on top of one another and held together by three binding posts which were loosened during testing. Four strain gauges were mounted to the rings, two (dummy) on the top and two (active) on the bottom ring. These were used to measure hoop strain. A schematic of the test setup is shown in Figure 3. Three voltages from the MTS system were fed into a GW Instruments MacAdios Data Acquisition System connected to a Macintosh Plus computer. These voltages corresponded to the axial displacement, axial load, and the hoop strain. Conversion of voltages to stress and strain data and data manipulation were carried out on the Macintosh.

Surface sections were prepared from portions of each sample in the uncompressed and the compressed states. Surface preparation was based on the procedure presented by Perla (1982). The sections were prepared in 3.8X3.8X1.7 mm^3 trays. A solution of dimethylpthalate and methyl blue was allowed to cool to between -2° and -4° C, then slowly poured into the samples. After the solution had completely frozen, pedestals were attached, the sample was removed from the tray, then microtomed.

The top millimeter of the sample was removed before polishing began. The exposed surface was then allowed to sublimate for 3 minutes before it was polished with Lamp Black. Once the polishing procedure was complete, the section was photographed using a 35 mm camera with 55 mm lens and a flash ring light source. Instructions and recommendations for surface section preparation are presented in appendix A.

Photographic prints of the sections were made with the image as large as possible, while maintaining good contrast and avoiding graininess. The magnifications used ranged between 10X and 20X. Various filters were used as needed, when making the prints, to enhance images with low pore-grain contrast. Images were digitized using a Thunder Scanner image digitizing system, which works in conjunction with a Macintosh

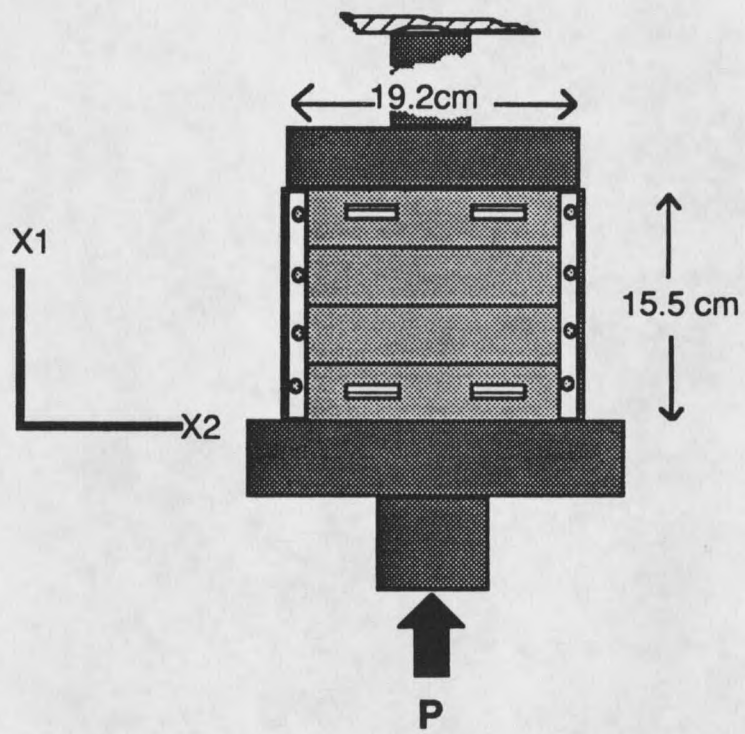

Figure 3. Schematic of Compression Ring with Upper and Lower Platens.

computer. Using the digitizing software supplied with the Thunder Scanner, it was then possible to run the image file through a filter to remove blemishes and maximize contrast. In order for the digitized images to be compatible with the image analysis software, the section contrast has to be set so that the ice grains are black and the pore space white.

To analyze an image it is brought up on the screen while the analysis program is running. The image scale is first measured. The region to be analyzed is then delineated

by marking off a rectangular region. Next, each grain bond is measured by marking a rectangular region enclosing the neck. The bond diameter, the plane, and neck area are then automatically measured. After all bonds connected to a particular grain have been measured, the number of bonds are tallied by selecting the appropriate box, either 1, 2, 3, >3, or 1/2 of one of these if the bond was intersected by the region boundary. After all bonds have been measured, the number of grains not having a bond are tallied using the box marked 0, or 1/2 if the grain is intercepted by the region boundary. At this point the calculate parameters box was selected to complete the analysis. A sample of the image window after all steps were completed for sample 2k501p is shown in Figure 4. Detailed instructions for the analysis program operation are available in appendix B along with a listing of the Forth language computer code.

Experimental Results

Two areas of each image were analyzed. The results were then averaged to obtain mean values and relative errors. The major factors limiting the size of each area analyzed were the actual photograph size and the portion of the image for which each features identity could be accurately determined. The resolution of the digitized images were better than 0.0333 mm/pixel. The resolution as well as respective areas of the section are shown in Table 1. The notation used for identifying samples was

1) the first four terms (2k50, 2500 lbs (11.2 KN)) indicated the maximum load to which a sample was be subjected,
2) p or f indicate if the sample is precompressed or in its final compressed state, and
3) 1 or 2 indicate the sample number in order tested at a given load.

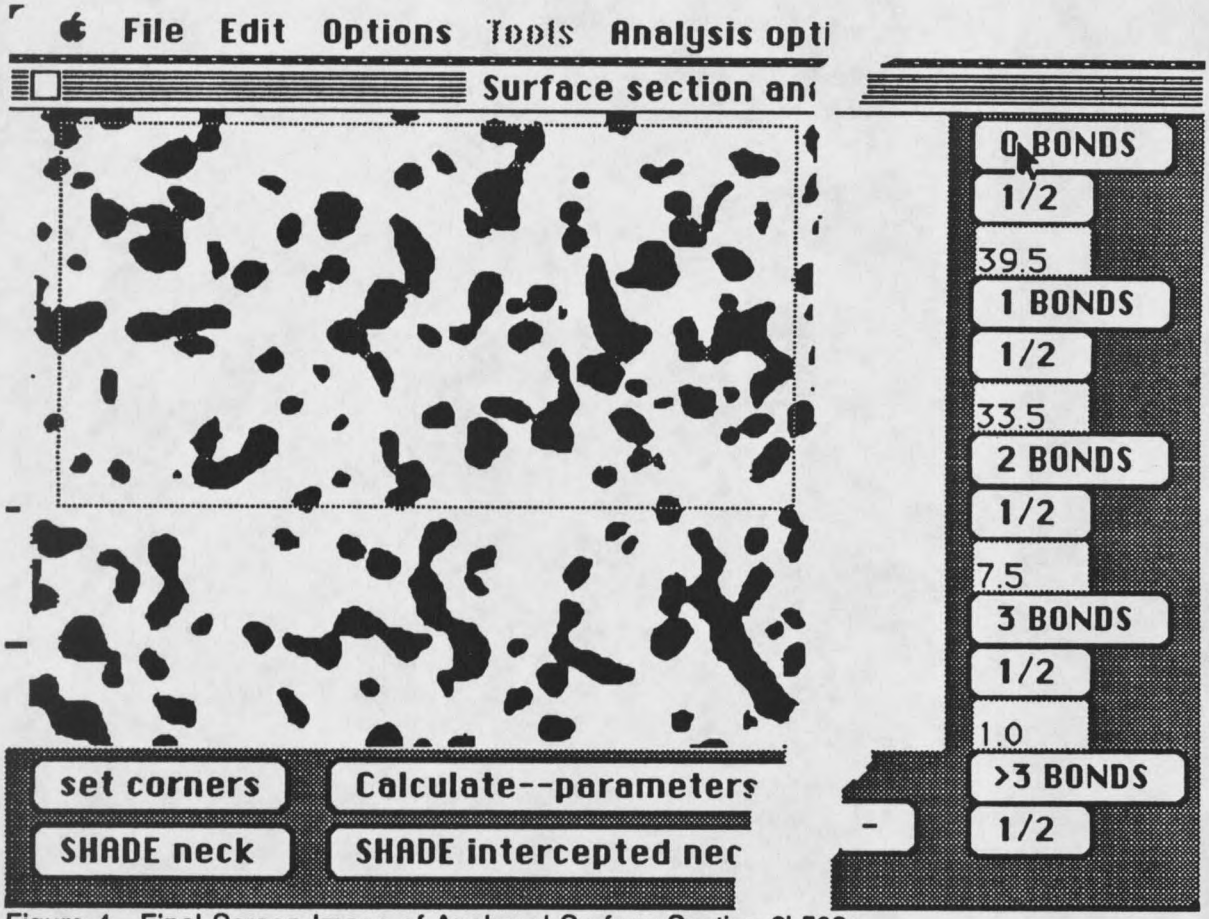

Figure 4. Final Screen Image of Analyzed Surface Section 2k502p.

The initial density for each snow sample was measured to within ±2%. The final densities were determined from readings of the crosshead displacement which is measurable to within ± 0.2 cm. Surface section values for the density were obtained from area fraction measurements. These results along with the percent difference are listed in Table 2. In all cases, measured density is larger than that obtained from surface section measurements. Two sources of error are most likely. The first is due to subjectivity. Subjectivity is evident when a section is poorly polished or when pits occur. The pits require a decision as to whether they are dislodged grains or air bubbles. This decision process is necessary to some extent in all photographs but usually the identity of these objects is obvious and so the error introduced small. The subjectivity

required in enhancing poorly polished sections may be quite significant. This may require determining the identity of an object as well as its shape, neither of which may be obvious. Most of the error in 2k501p is due to this. The other major source of error is due to the digitization process. If the digitization process produces features with areas larger than their actual size, an overestimate in density will occur. If the process produces features with areas smaller than their actual size, the resulting density measurement will be an underestimate. A slight decrease in length along the horizontal direction has been observed in the digitized images, indicating that the grain area may be on the small side. Another indicator is the much larger underestimate of density obtained for the compressed samples than that for the precompressed.

Sample	Resolution (mm/pixel)	Area 1 (mm^2)	Area 2 (mm^2)	Total (mm^2)
10k01p	0.0333	27.7026	26.6016	54.3042
10k01f	0.0270	21.1070	17.6584	38.7654
5k001p	0.0238	15.7109	14.0051	29.7160
5k001f	0.0256	11.8778	9.9599	21.8377
5k002p	0.0217	14.8966	16.7974	31.6940
5k002f	0.0178	5.9108	6.6409	12.5509
2k501p	0.0244	15.5657	13.2164	28.7821
2k501f	0.0217	10.5281	11.0685	21.5966
2k502p	0.0196	15.4865	18.2175	33.7040
2k502f	0.0185	14.4241	10.2023	24.6264

Table 1. Digitized Image Resolution and Analysis Area Sizes.

In the remaining results it is important to keep in mind that two distinct ranges of grain size exist. The large grains corresponding to the number 1 samples have values of L_3 ranging from 0.34 to 0.39 mm in the uncompressed state whereas the small grained number 2 samples range from 0.19 to 0.22 mm in the uncompressed state. The values of L3 and relative errors are presented in Table 4. The ratios of L_3/L_{3o} vs. $\Delta\rho/\rho o$ are plotted in Figure 5. The o subscript indicates initial value. $\Delta\rho$ is the change in density

resulting from the loading. Much of the decrease in the mean intercept length ratio can be accounted for by the increase in coordination number and changes in the average neck length. As indicated in Table 3, neck length tends to remain relatively constant throughout the deformation process. An increase in coordination number does occur however. The coordination number will be discussed later. Since for rounded grains, the necks incorporate part of the grain into the neck region, each new neck will then result in less material considered as ice grain, thus a net decrease in mean intercept length.

In practice, h is very difficult to measure. Location of the points where the concavity changes sign is subjective in many cases, particularly for irregular geometries. As can be seen in Table 3, there is no definite trend present in the change in neck length due to the deformation process.

At -10^{o} C and for pressureless sintering, the dominant mechanisms in neck growth will be those due to surface diffusion and vapor transport (Maeno and Ebinuma). Maeno and Ebinuma found that 35 μm ice particles in contact had normalized growth rates ($\dot{x}/r$) of 100/sec for initial bond radius (x) to grain radius (r) ratios of 0.01. One of the major driving mechanisms for mass transport, is the curvature difference (1/a-1/x)+2/r, where a is the neck curvature. This indicates that bond and neck growth may be very rapid, particularly for necks with a small radius of curvature.

If the surfaces in contact for two grains have large radii of curvature, for example flat planes, only an increase in bond radius will be observed while the bond area is less than the area of the plane surfaces since the mass is essentially trapped between the planes. If the radii of curvature for the contact regions are similar to spheres, both an increase in bond radii and neck length occurs. This increase in neck length will come at the expense of the ice grains. Mass will be transported to the neck region and as a result, the neck will migrate further onto the ice grain surface, effectively shortening the ice

grain. It should also be noted that the grain centers will maintain the same relative separation.

Sample	Measured Density (g/cc)	Section Density (g/cc)	%Difference
10k01p	0.2915	0.2736	-6.1
10k01f	0.6394	0.5636	-11.8
5k001p	0.3206	0.3087	-3.7
5k001f	0.5801	0.5062	-12.7
5k002p	0.2511	0.2421	-3.6
5k002f	0.5706	0.4931	-13.6
2k501p	0.3097	0.2785	-10.1
2k501f	0.5304	0.4798	-9.5
2k502p	0.2588	0.2527	-2.3
2k502f	0.5047	0.4612	-8.6

Table 2. Measured and Section Density.

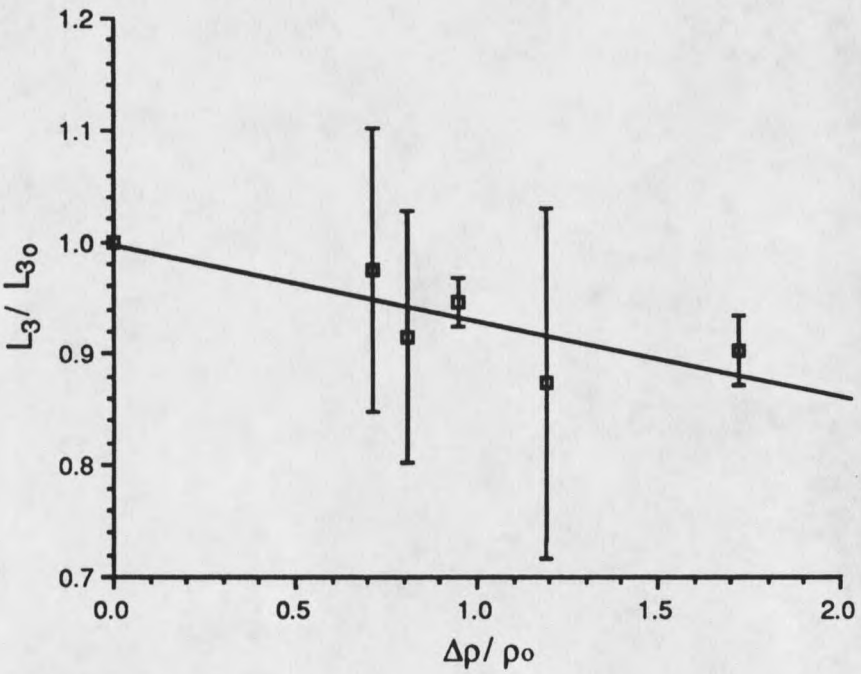

Figure 5. L/Lo VS. $\Delta\rho/\rho_o$. Ratio of mean final and initial mean intercept length versus change in density.

In pressure sintering the effects of plastic flow must also be taken into account. For an applied pressure, the separation between grain centers and opposite edges of the necks will decrease due to straining within the grains and necks. As the grains are forced together, mass will flow outward from the region of contact. For the grains with large radii of curvature at the contact region this will result in an increase in bond radii but a decrease in neck length. For the smaller radii of curvature contact regions, the neck lengths will decrease due to the strain, but will increase due to the transport of material outward from the contact regions. The outward flow of material will eventually migrate onto the grain surface, and this along with the vapor transport and surface diffusion mechanisms will cause an increase in neck length.

Other major factors determining the mean neck length will be bond breakage and bond formation. If all bonds are broken at some point prior to attaining the final load, all necks occurring will have been formed during the deformation process and tend to be short due to the time available for growth mechanisms to act. This would then give an average relative decrease in neck length. Breakage of only small bonds will increase the average neck length but formation of new bonds will tend to reduce the average nearer to its original value. For this case there may be little relative change. From this discussion it is evident that neck growth is a rather complicated process controlled by many competing mechanisms. This may explain the lack of a definite trend in change in neck length as mentioned earlier. Finally, due to the difficulties in measuring the neck length mentioned previously, the neck length values are best considered as order of magnitude estimates.

Table 3 also includes values of λ, the mean distance from grain surface to grain surface. Looking at the values of λ for the compressed samples, the ratio λ/L_3 ranges from 0.99 to 0.62 for the sample bearing the largest final load. It is evident that as the grains become more immobile, the load bearing ability increases.

Sample	L_3 (mm)	λ (mm)	h (mm)
10k01p	0.3897± 3.5%	0.8851± 7.7%	0.0958±14.6%
10k01f	0.3405±13.8	0.2114± 7.1	0.0708± 7.8
5k001p	0.3506± 8.0	0.6987± 8.5	0.0713±13.3
5k001f	0.3209± 3.2	0.2612±12.8	0.0877± 2.9
5k002p	0.2190± 1.3	0.6106± 4.1	0.0650±11.9
5k002f	0.1977± 2.2	0.1703±12.6	0.0621± 5.1
2k501p	0.3501± 8.5	0.8022± 7.6	0.0667± 6.7
2k501f	0.3416± 3.4	0.3112± 4.0	0.0607± 6.5
2k502p	0.1922± 0.8	0.5053± 3.4	0.0535± 8.9
2k502f	0.1820± 1.4	0.1796± 9.0	0.0588± 4.2

Table 3. Mean Intercept Length, Mean Pore Length and Mean Neck Length

In Table 4, two and three dimensional bond diameters and radii are given. The two dimensional diameters show a definite decrease from the precompressed to compressed state. This indicates that little or no growth in the radius of old bonds occurs, in fact, very few of these bonds appear to survive the deformation process. This can be seen in Figures 6-10 where number vs. median diameter is plotted for all necks measured. It is particularly evident in Figure 8. An increase in the number necks in the middle diameters is clear in all cases. Evidence of new bond formation can be seen in Figures 6-8.

As the grain mobility is decreased there should be some point at which the rate of bond breakage decreases and an increase in average bond radius begins. Figure 11 shows some indication of this for the largest density change. A complementary factor to the size of the grain bond is the number of bonds per volume N_{bV}. Over the range of loading conditions in this study, N_{bV} increased 2.5 fold at the 11.2 KN load and 6.9 fold at the 44.8KN load. The values of N_{bV} are shown in Table 5.

Sample	D_2(mm)	E(mm^{-1})	R_3(mm)	D_{2min}(mm)
10k01p	0.1692±2.1%	7.2841± 3.7%	0.1080± 3.7%	0.0652±14.3%
10k01f	0.1299±0.2	9.2088± 0.1	0.0853± 0.1	0.0434± 4.5
5k001p	0.1488±3.9	8.0084± 5.6	0.0984± 5.6	0.0895± 5.0
5k001f	0.1420±5.1	9.5567± 6.6	0.0826± 6.6	0.0413± 0.9
5k002p	0.0899±5.7	15.6394± 3.4	0.0503± 3.4	0.0178±21.9
5k002f	0.0693±0.4	19.5801± 3.1	0.0402± 3.1	0.0118± 2.5
2k501p	0.1485±3.1	7.8755± 0.8	0.0998± 0.9	0.0859± 0.5
2k501f	0.1315±9.5	8.5692±10.1	0.0926±10.1	0.0703± 9.8
2k502p	0.0808±1.9	15.3586± 0.8	0.0512± 0.9	0.0270± 9.1
2k502f	0.0677±0.8	18.8240± 2.0	0.0417± 2.0	0.0196±42.9

Table 4. Values Obtained Directly From Measured 2-D Bonds in The Plane

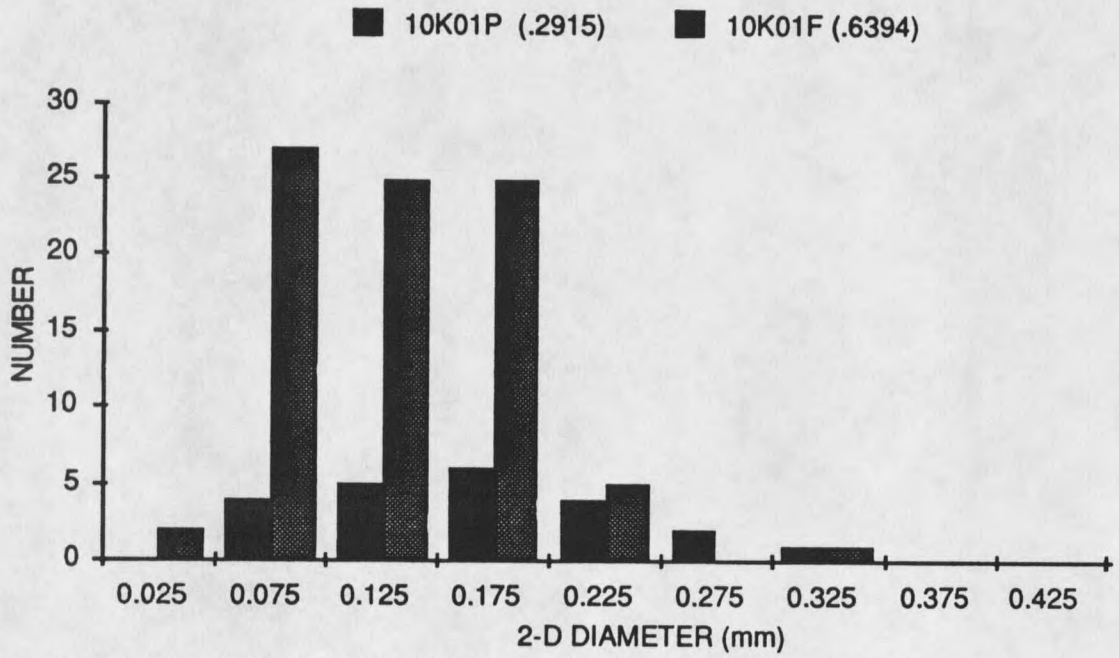

Figure 6. Number VS. 2-D Bond Diameter for Sample 10k01. Density is shown in parentheses.

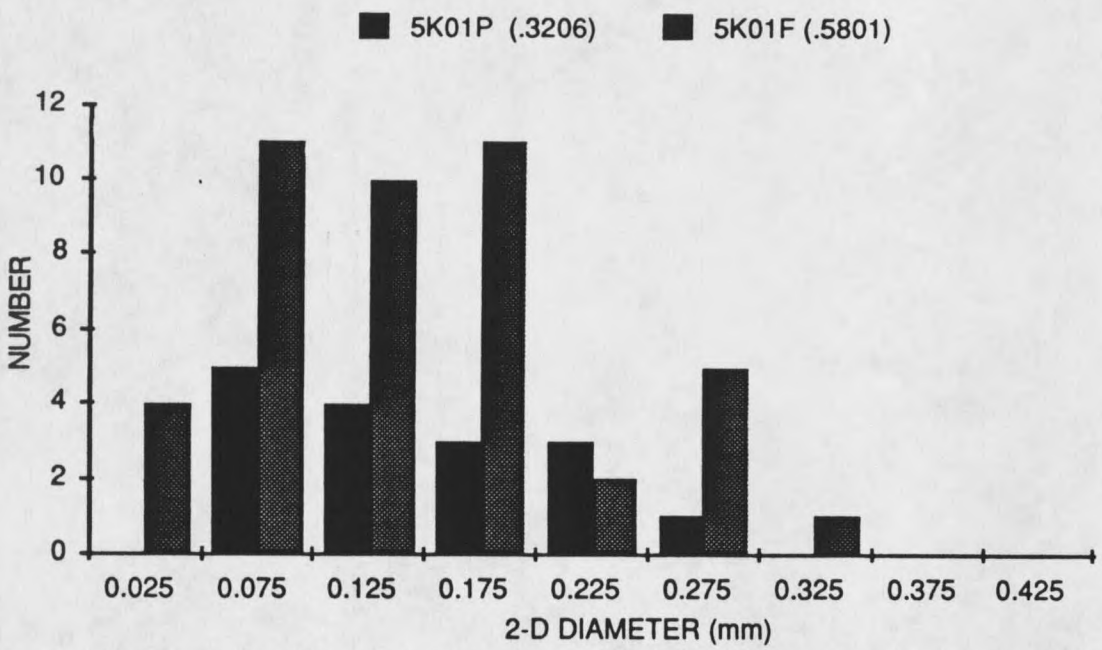

Figure 7. Number VS. 2-D Bond Diameter for Sample 5k001. Density is shown in parentheses.

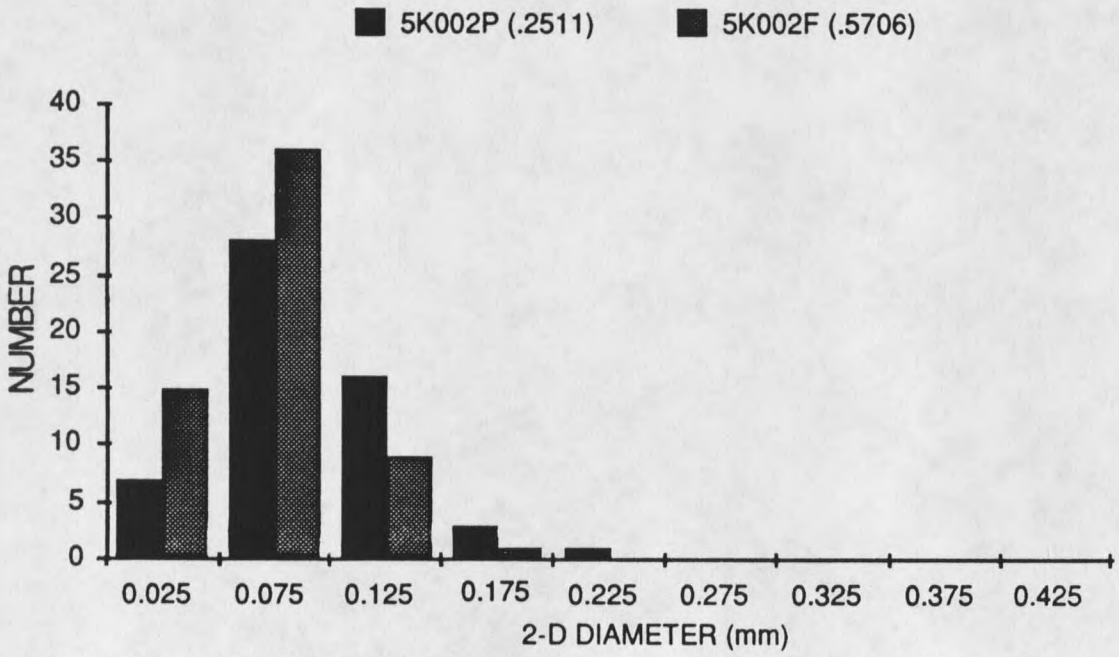

Figure 8. Number VS. 2-D Bond Diameter for Sample 5k002. Density is shown in parentheses.

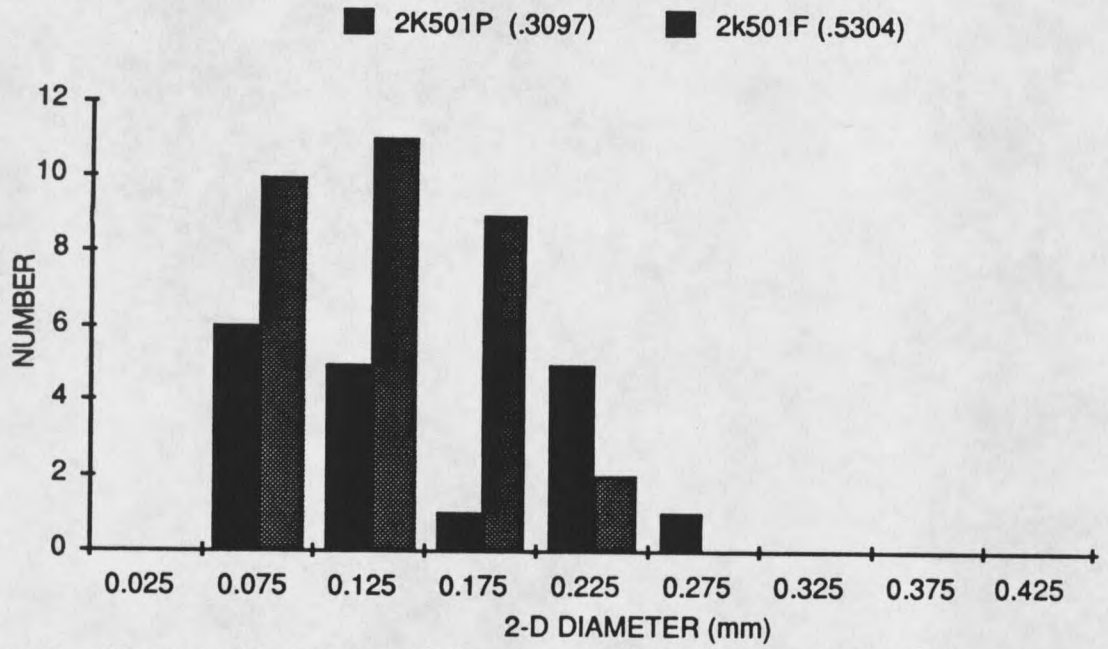

Figure 9. Number VS. 2-D Bond Diameter for Sample 2k501. Density is shown in parentheses.

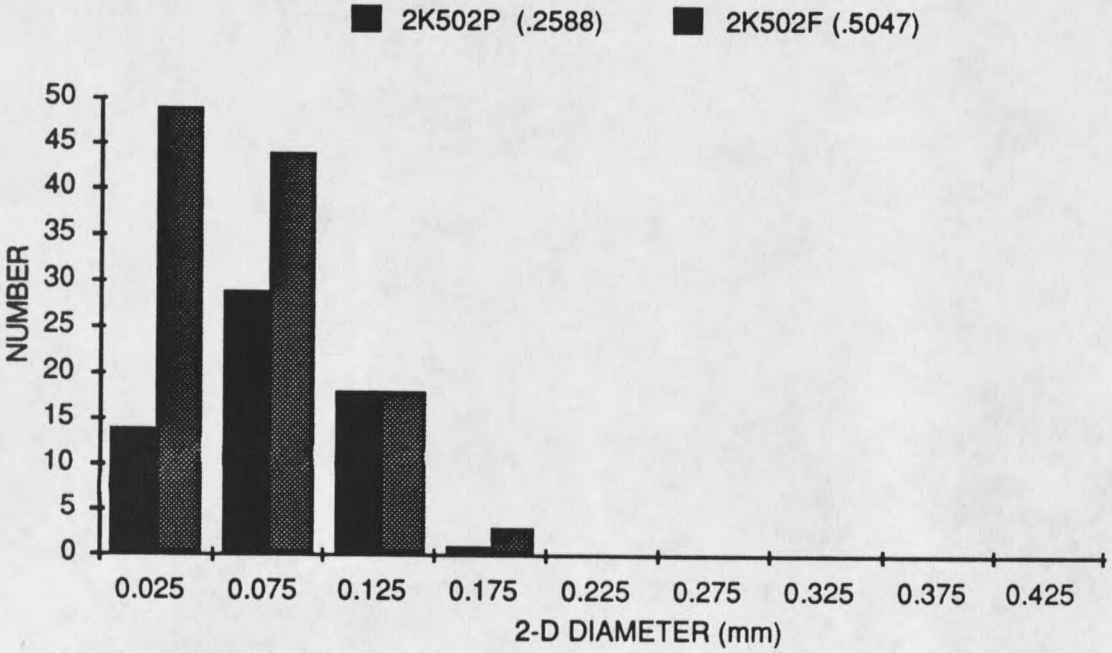

Figure 10. Number VS. 2-D Bond Diameter for Sample 2k502. Density is shown in parentheses.

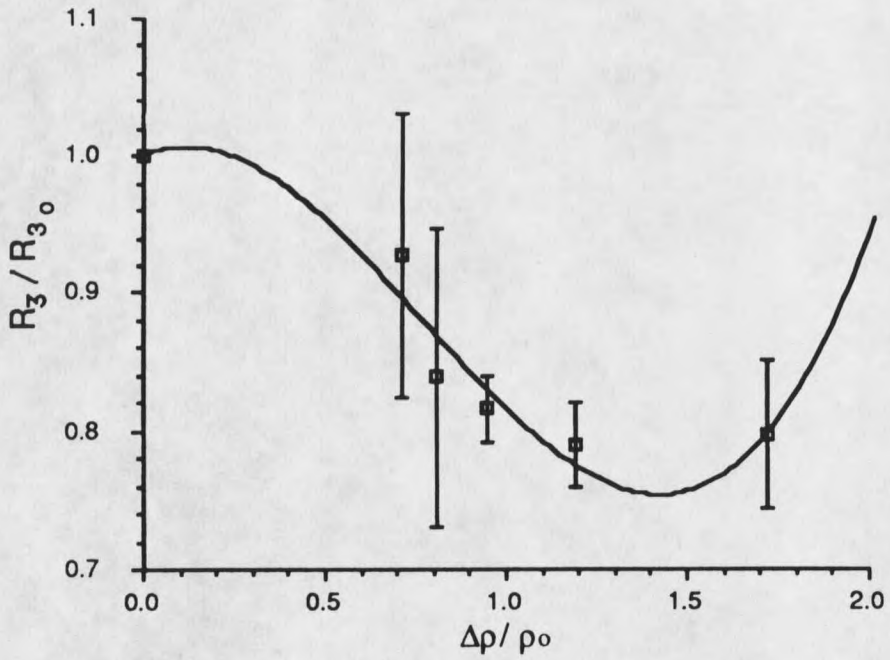

Figure 11. R/Ro VS. $\Delta\rho/\rho_o$. The ratio of final and initial 3-D bond radius versus change in density.

Sample	$N_{gL}(mm^{-1})$	$N_{gA}(mm^{-2})$	N_{bA} (mm^{-2})	$N_{bV}(mm^{-3})$
10k01p	0.7877± 6.4%	1.6359± 9.2%	0.4061±11.1%	2.4072±14.7%
10k01f	1.8350±11.2	3.8293± 7.9	2.2160± 8.0	16.5392± 7.9
5k001p	0.9597± 8.3	2.4884±10.5	0.5441±18.1	3.4961±12.6
5k001f	1.7280± 7.5	3.4117± 0.1	2.0394± 0.9	15.2510± 2.3
5k002p	1.2069± 3.4	5.1403± 6.0	1.2812±16.2	16.1515±12.8
5k002f	2.7312± 7.0	8.7129± 8.1	4.9842± 5.8	78.9629± 2.7
2k501p	0.8733± 7.9	2.2839± 4.4	0.6182±14.3	3.9414±13.5
2k501f	1.5343± 3.5	3.6731±16.4	1.5527± 4.0	10.8287±14.1
2k502p	0.4349± 2.7	5.2799± 0.3	1.7912± 2.7	22.2938± 1.9
2k502f	2.7692± 3.8	11.4273± 2.9	4.7383± 3.4	72.3471± 5.4

Table 5. Number of Features Per Length, Per Area and Per Volume.

It seems that if the effects of grain size is factored out, some correlation between the mean pore length and the number of bonds per volume should exist. To check this, $N_{bV}{}^{*}L_3^2$ was plotted versus λ. L_3^2 was used to normalize the effects of grain size. The results are shown in Figure 12. A curve was fit to the data with a confidence level of 98%. The resulting equation is

$$N_{bV}L_3^2 = 0.347\lambda^{-1.153}. \tag{22}$$

Solving 22 for N_{bV} and multiplying the right hand side by L_3/L_3 yields

$$N_{bV} = 0.347\left[\frac{1}{L_3}\right]^3\left[\frac{L_3}{\lambda}\right]\left[\frac{1}{\lambda}\right]^{0.153}. \tag{23}$$

The physical meaning of the equation becomes clearer by introducing three constants: C1, C2, and C3 in place of 0.347 . The resulting equation is

$$N_{bV} = C_1\left[\frac{1}{L_3}\right]^3 C_2\left[\frac{L_3}{\lambda}\right] C_3\left[\frac{1}{\lambda}\right]^{0.153}. \tag{24}$$

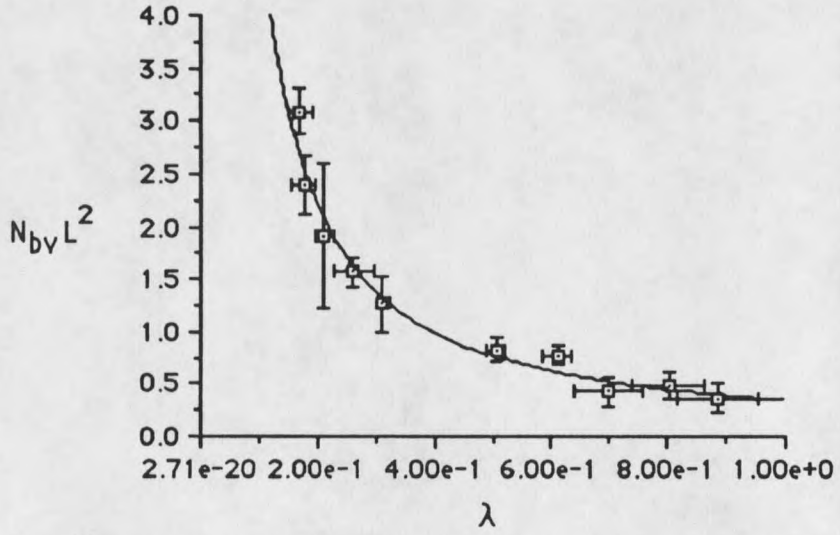

Figure 12. $N_{bV}{}^{*}L^2$ VS. λ. The product of the number of bonds per volume and the square of the mean intercept length is plotted versus the mean pore length. Relative error bars are shown for both sets of values.

It can be seen that by including the constant with the inverse of the cube of the mean intercept length, the inverse of the average grain volume will result. The inclusion of grain volume certainly makes sense as a factor influencing the number of bonds per volume. The average grain volume determines the number of grains that can fit into a given volume and as a result the number of possible grain to grain contacts. The second group of terms measures the grain mobility. This would also be expected to influence the number of bonds per volume. The less mobile the grains the more likely that bonds will survive any further deformation. As the mean pore length decreases the probability of grains coming into contact and forming bonds should increase. The rate at which the pore length decreases will also affect the formation of bonds, since bond growth is time dependent. From this point of view, the last group of terms can be considered as the probability that a bond exists or will be formed. The constant normalizes the probability to 1. It may be possible to relate the exponent to the deformation rate.

The discussion of the terms appearing in equation 23 indicate the validity of this equation on physical grounds. As a result, equation 23 should be significant in determining the mean coordination number of a sample undergoing large deformation.

Evaluation of the remaining microstructural variables, N_3, N_{gV}, and V required the use of the Fortran program mentioned at the end of the last chapter. This program was written by Andrew Hansen and incorporates his modifications of Gubler's work. It has been further modified with the introduction of the new bandwidth definition. Detailed operating instructions are provided in appendix C along with a program listing.

Only the results for N_{gV} and N_3 obtained from Hansen's computer program yielded consistent, physically reasonable results. For these samples, the relative error between theoretical 2-D bond distribution values f2(0) and f2(1) obtained from the computer program and experimental values f20 and f21, were less than 1%. Recall that the theoretical distribution is determined from

$$f_2(l) = N_2 \sum_{n=l}^{12} \binom{n}{l} p(1-p)^{n-l} f_3(n), \qquad (9)$$

and that this is varied by adjusting N_3 and i in f3(n). The error in measuring f20 ranged from 0.8% to 11% and for f21 from 0.9% to 13%. For the precompressed samples, the dominant bond distribution was for grains having 0 bonds in the section plane. The bond frequency distributions are very similar for these samples as are the resulting coordination numbers. The 1 bond distributions dominate the compressed samples. There also tends to be a significant increase in the number of two bonds counted. For these samples the bond frequency distributions don't appear to be strongly correlated with the density or the coordination numbers obtained. It's important to keep in mind that mean bond radius also influences the number of bonds per volume. As a result, frequency distribution won't be a complete indicator of the number of bonds per volume or alternatively the mean coordination number. The frequency distributions for both the precompressed and compressed samples are given in Table 6.

For the compressed samples, it was possible to get convergence between theoretical and measured frequency distributions, but the resulting values of N_{gV} were either much larger or much smaller than reasonable based upon N_{gV} for the corresponding precompressed samples. The definite decrease in intercept length tends to rule out the possibility that the net number of grains is decreasing due to grain consolidation. There doesn't appear to be sufficient grain fracture to account for an increase in the total number of grains in the sample, thus ruling out values of NgV which tended to be larger than 3 times the original value.

Sample	f20	f21	f22	f23
10k01p	0.5991±11.3%	0.3591±12.8%	0.0418±51.6%	0.0000± 0.0%
10k01f	0.3015±13.6	0.4065±11.7	0.2475± 4.9	0.0390± 2.2
5k001p	0.6539± 2.7	0.2785± 2.6	0.0675±15.4	0.0000± 0.0
5k001f	0.2755± 1.4	0.4158± 1.0	0.2670± 6.4	0.0418±40.8
5k002p	0.6158± 1.5	0.3032± 9.9	0.0685±59.4	0.0124±12.0
5k002f	0.3059± 1.4	0.4442± 0.9	0.1669± 1.9	0.0831± 3.7
2k501p	0.6121± 6.3	0.2826± 1.1	0.0906±29.9	0.0147±100
2k501f	0.4486±11.5	0.3505± 4.9	0.1752±17.6	0.0258±14.0
2k502p	0.4807± 0.8	0.4024± 2.1	0.1030±10.7	0.0114± 8.4
2k502f	0.4031± 3.9	0.4255± 3.8	0.1458± 2.8	0.0255±14.3

Table 6. Measured Bond Frequency Distributions and Relative Errors.

The values of N_3, N_{gV}, V, and S_V obtained for each of the 5 precompressed samples are shown in Table 7. For these samples the resulting mean coordination number N_3 ranged from 2.28 bonds/grain to 2.67 bonds/grain. These values appear to be independent of the mean grain size and sample density for the range studied. In obtaining values of N_{gV} for the compressed samples, the following equation was used to obtain an estimate:

$$N_{gVf} = \frac{\rho_f}{\rho_i} N_{gVi}, \tag{25}$$

where the subscripts f and i refer to initial and final states respectively; ρ is the density.

Sample	N_3(bonds/grain)	N_{gV} (mm^{-3})	V (mm^3)	S_V (mm^{-1})
10k01p	2.28±10.7%	2.09± 4.8%	0.15± 4.6%	2.80±17.5%
5k001p	2.41± 7.1	2.89± 5.6	0.12± 5.4	3.41± 6.8
5k002p	2.64± 3.4	12.19± 3.9	0.02± 6.1	4.31± 8.5
2k501p	2.58±10.6	3.05± 2.9	0.10± 2.8	3.00±13.2
2k502p	2.36± 9.3	19.22± 6.5	0.01± 6.1	5.01± 2.6

Table 7. N_3, N_{gV}, V and S_V for The Precompressed Samples.

A large decrease in the mean intercept length would indicate the possibility of significant grain fracture occurring. As discussed earlier, most of the change in mean intercept length can be accounted for by the increase in coordination number and the resulting inclusion of more of the grain into necks. Any grain fracture that the decreased intercept length might indicate will then be small. As mentioned previously, a significant decrease in the net number of grains due to consolidation is not supported by the results for L_3. These arguments would indicate no significant change in the net number of grains. Thus equation 25 should provide a good estimate for the number of grains in the compressed sample based upon the number of grains in the corresponding uncompressed sample. Results based upon the estimate for N_{gV} are listed in Table 8. Figure 13 plots coordination number vs. density for all of the values.

Values of S_V in Tables 7 and 8 indicate an increase in the surface area per unit volume. These are plotted in Figure 14. As would be expected, S_V increases steadily for that deformation in which bond breakage is frequent. It then levels out as the grain mobility decreases, the number of bonds broken decreases, and the increase in surface area resulting from the previously broken bonds is counteracted by an increase in the number of bonds. At some point beyond those available this ratio should begin to decrease as bond breakage all but ceases and bond mobility approaches 0.

Sample	N_3(bonds/grain)	N_{gV} (mm^{-3})	V (mm^3)	S_V (mm^{-1})
10k01f	7.14±19.6	4.74±13.6	0.14± 4.6	5.83±13.6
5k001f	5.85± 8.0	5.24± 6.0	0.11± 5.4	5.60± 7.2
5k002f	5.72± 6.6	27.69± 4.1	0.02± 3.7	9.32± 7.1
2k501f	4.17±16.8	5.22± 3.0	0.10± 2.8	4.99± 6.1
2k502f	3.89±11.9	37.48± 7.0	0.01± 6.1	9.49±10.2

Table 8. N_3, N_{gV}, V and S_V for The Compressed Samples.

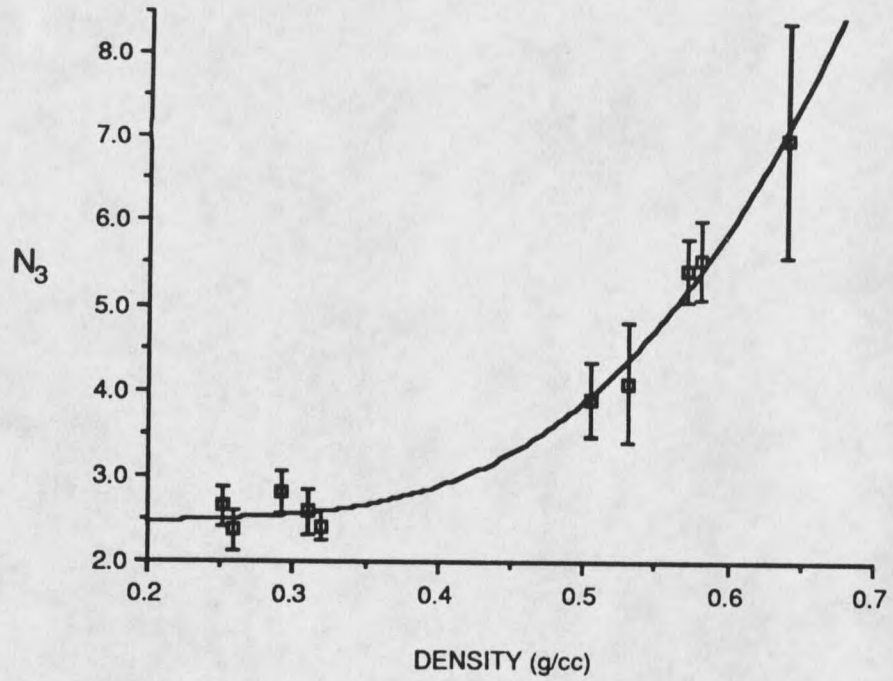

Figure 13. Three Dimensional Coordination Number VS. Density.

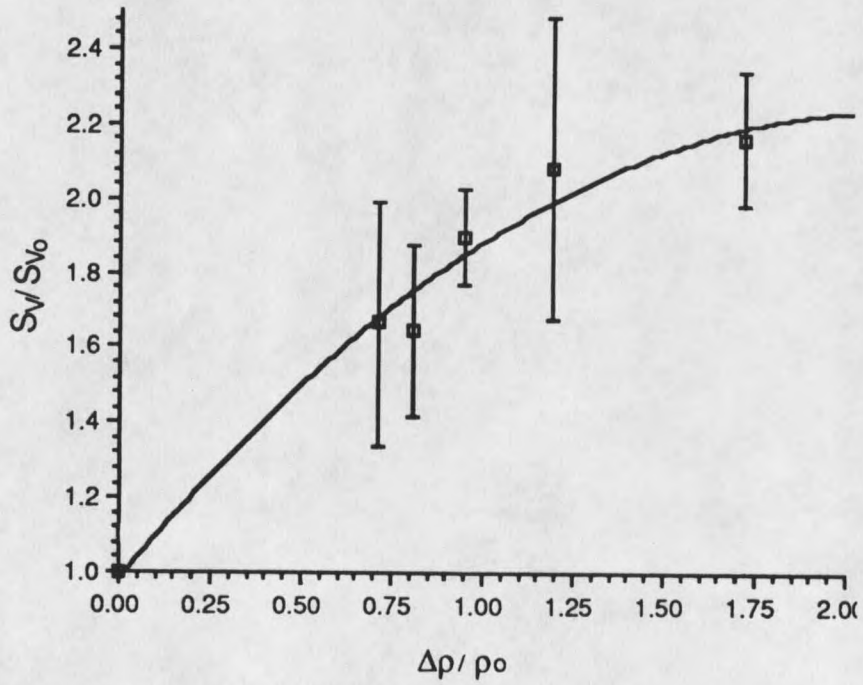

Figure 14. Ratio of Surface Area Per Unit Volume VS. $\Delta\rho/\rho_o$.

Discussion

At the maximum density attained, there is approximately a 3.5 fold increase in coordination number. The coordination number does not increase very rapidly until densities of 0.5 g/cc are reached as shown in Figure 13. This corresponds to strains of approximately 33% and occurs at a point located near the base of the sharp rise in stress on the stress strain data plotted for 10k01 in Figure 15. The stress is about 0.1 Mpa at this point. At the maximum stress, the density, 0.6394 g/cc, represents a 25% increase over that at 0.1 Mpa. Axial stress increased 16 fold and the coordination number by 2. There is approximately a 60% decrease in grain mobility as indicated by the ratios λ/L_3 occurring at densities of 0.5 g/cc and 0.6394 g/cc.

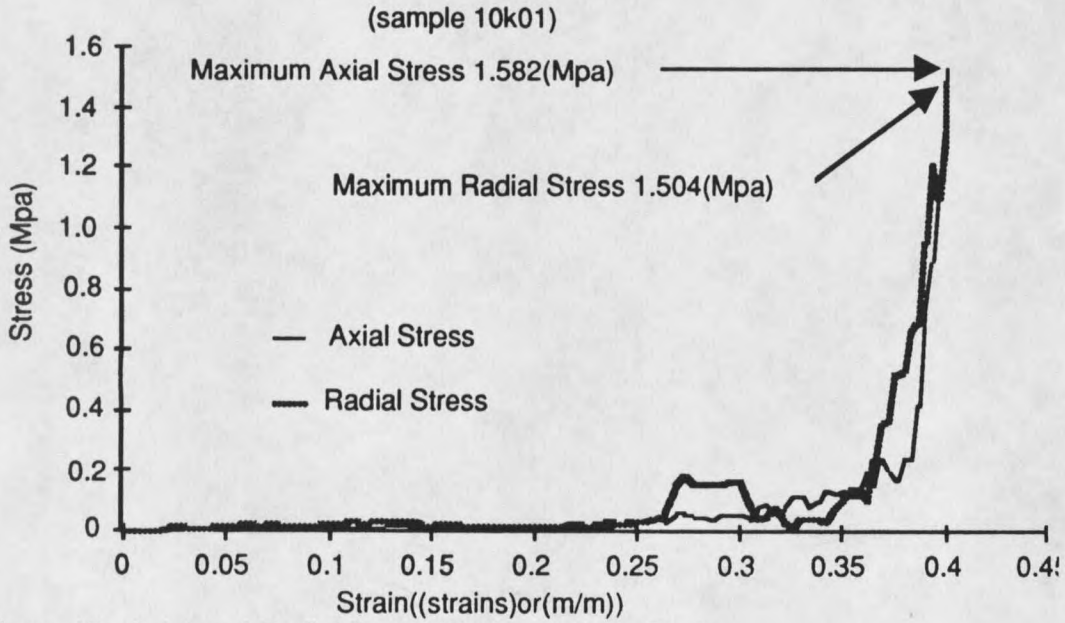

Figure 15. Axial and Radial Stress VS. Lagrangian Strain.

The largest relative decrease in bond radius, 22%, occurs at the maximum stress. This corresponds to a 38% reduction in the original bond area, but is countered by the

increased coordination number. There was a factor of 1.9 increase in the net area over the entire deformation process.

This discussion indicates the affect that small relative order of magnitude changes in microstructure have on increasing the load bearing ability of snow. The effects of reduced grain mobility due to pore collapse may account for much of this at high densities as indicated by Brown's pore collapse model constitutive equation.

RECOMMENDATIONS AND SUMMARY

Recommendations

There are several experimental results that would compliment this study. Values of L_3 occurring for density changes of 25, 50, 150, and 200% will help verify any trends in length change that are occurring. Refer to Figure 4. It appears that the average grain bond radius, shown in Figure 11, may begin to increase at a density change of 150% . Measurements of R_3 at 140, 150, and 200% are needed to confirm this. The other measurement needed is the coordination number corresponding to a final density of 0.45 g/cc. This along with measurements given in Table 4 should be sufficient to determine the location of the transition from low to high coordination numbers in Figure 13. The last significant result of any further study would be to understand the affect that deformation rate has on the microstructural changes of snow.

Summary

Microstructural parameters suggested by Hansen as being important in understanding and modeling the microstructural behavior of snow have been reviewed. Basic surface section variables needed to evaluate these parameters were also discussed. A new definition of the bandwidth, used in Gubler's work for evaluation of N_3, was also presented.

Finally, a discussion of experimental procedures and results was presented. Order of magnitude changes in the microstructural parameters were compared with the corresponding magnitude changes in stress level for densities ranging from 0.5 g/cc to

0.6394 g/cc. In general, the microstructural parameter under went order of 2 changes relative to a factor of 16 for the stress level.

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APPENDICES

APPENDIX A

SURFACE SECTION PREPARATION

In this appendix, preparation of surface sections will be discussed. The basic procedure used to prepare surface sections is based on a paper by Perla (1982). It is suggested that Perla's paper be read before proceeding with this appendix. Any deviations from his method will be pointed out.

The first step in the preparation of the snow surface sections is to make a solution of Methyl Blue and Dimethylpthalate. This is used as a pore filler in the snow samples and when frozen allows layers of the sample to be shaved off while the original grain structure is maintained. The Methyl Blue is used to enhance the pore-grain contrast. Note that Perla used Oil Blue N as the dye in solution with the Dimethylpthalate. In the tests carried out, the surface sections were prepared in aluminum trays of 1.7 X 3.8 X 3.7 mm^3 or 24.5 mm^3. The pore filler solution was also prepared in these trays. A quantity of Methyl Blue with the area of this 0 and 1 or 2 grains thick, in solution with one tray full of dimethylpthalate was sufficient to enhance the pore-grain contrast to the desired level.

The solution should be cooled to no lower than -5^{o} C. For snow densities > 0.45 g/cc, -2^{o} C is preferred since it will have a lower viscosity, thus it can flow through the sample much easier, and as a result, fewer pores are likely to be left unfilled. Unfilled pores will show up as pits when the section is polished and may be mistaken for grains dislodged during microtoming. This is important when considering the accuracy of the measurements to be made. The lower temperature is ok for samples with densities of < 0.35 g/cc. For any density snow, slowly pouring the solution into one corner of the sample tray reduced the number of air pockets in the sample. Filling the sample with the solution should take several minutes. It was found, that the solution should be prepared no more than one hour before it was to be cooled. The methyl blue tends to suppress the freezing point of the dimethylpthalate when the solution is prepared more than several hours in advance.

While the solution is cooling, samples are cut and placed into empty trays. After the solution has cooled it is poured into the sample until saturation occurs. The sample with the solution is then frozen by placing it in a foam container containing crushed dry ice. Some of the dry ice should be placed onto the exposed portion of the sample. Two hours should be allowed for the solution in the sample to freeze completely. Once this is done, a base used to hold the sample to the microtome is frozen onto the exposed portion. By immersing the tray in warm water up to its top edge the sample and attached base can be removed from the tray after the contact surfaces have thawed. The sample and base are then placed in the dry ice for 15 minutes.

Next, the sample is placed on the microtome and the first 1 mm is cut away before polishing. This removes any material that may have been affected by proximity to the bottom end of the tray. At this point Perla recommends allowing the freshly exposed surface to sublimate for 10 minutes before polishing. In the sections prepared for this thesis, the samples were allowed to sublimate for 3 minutes before the surface was polished with Lamp Black.

The section is polished using photographic lens paper. A clothespin with the clip end rounded works reasonably well as a means to hold the lens paper. Insulated gloves are also acceptable, but tend to be a bit too cumbersome. Rubber lab gloves are less cumbersome, but care must be taken not to touch the sample since these gloves are uninsulated. The main point is not to allow heat from the fingers to melt the surface while polishing.

In order to obtain good contrast between the grains and the pore filler, several applications of Lamp Black may be needed. The section has acceptable contrast when the ice grains are dark gray and the pore space is the color of the pore filler. Essentially the section is properly polished when the Lamp Black has been removed from all pore filler and is only present on the ice grains, or in pits on the surface section. The Lamp Black

must be removed from all pore filler that is in the area to be analyzed. It is very important that polishing continue until this condition is attained. Use of a hand lens is very useful in checking the condition of the surface section as polishing proceeds.

The final step in the process is to photograph the surface section. A 35 mm camera with a flash ring light source was used. The film used was Kodak Pan X 125 black and white. A slow film allows for greater enlargement of the photographic prints before graininess becomes a problem. In photographing the image the magnification should be as large as possible. This reduces the magnitude of enlargement necessary to achieve a print with details large enough to maintain accuracy in the digitized image and large enough for accurate measurements to be taken.

Two things are very critical in photographing the section : accurate focus, and good lighting. It can be difficult to focus on the surface section, but care must be taken to achieve as accurate of a focus as possible. Uneven lighting will make proper focusing difficult and will make the digitization and filtering process much harder than necessary. It will also diminish the resulting accuracy of the digitized image. A ruled grid should be included in the picture when photographing the surface section. It is preferable to lay the grid flat and along one edge of the sample.

To summarize, 3 points should be constantly kept in mind when preparing the surface section: proper temperature of the pore filler solution and careful application to the snow sample, proper polishing of the surface section resulting in lamp black only remaining on the ice grains and in the pits, and the proper focus and lighting of the image to be photographed. Care taken in preparing the surface section will save many hours and a great deal of effort during later digitization and filtering of the image.

APPENDIX B

SURFACE SECTION ANALYSIS

After the preparation of the surface section has been completed and photographic prints of the sections have been made, the prints are then digitized, filtered, and analyzed. In this appendix, prints as well as the process of digitization and filtering are briefly discussed. The main topic presented here is the analysis of the images resulting from digitization and filtering.

Preliminaries

In making photographic prints for analysis, the size of the paper used in making the print is not critical, but large prints allow for more freedom when selecting the section of the image to digitize and analyze. Eight by eleven black and white paper was found to work the best. When making prints, the surface section image should be magnified as much as possible up to a point where contrast begins to decrease significantly and graininess from the negative becomes visible. Contrast filters are useful when making the prints. It is important to have as much contrast between the grains and the pore space as possible. The prints must contain the scale markings photographed with the surface section. When making the prints, it is usually best if they are positioned along one edge.

The actual process of digitization and filtering are well covered in the instruction manuals; therefore, only several points not in the manuals will be mentioned here. The first is, if the digitizer uses the printer as part of the scanning device, it is very important to keep the timing tape clean. If this is not done, the scan lines corresponding to the dirty sections on the tape will be displaced. When actually scanning the prints, it is the information contained in the range of gray levels representing the ice grains and necks that is most important. Any contrast controls that can be set for the scanning process should be set so that these gray levels take up as wide a range of digitizer gray levels as possible. This allows for more pore-grain interface details to be distinguished,

resulting in better accuracy for the digitized ice grain images.

The best way to learn and understand this process is to make one or two trial surface sections, prints of the sections, digitizing these images, and by this process discovering the problems inherent in creating accurate digitized pictures. This will also point out what must be done during surface section preparation to make the digitization process easier and more accurate. This is certainly preferable to making many sections and then finding out that not enough care was taken in preparation. If the surface sections are prepared carefully, with the knowledge of the digitization and filtering process firmly in mind, good accuracy can be achieved and many hours of work can be saved.

The final result of the filtering process is a digitized image. In this image, the pore space is represented by white pixels and the ice grains by black. The image is then saved as a MacPaint file, or any file of type pict which can be copied into the clipboard, and that has dimensions of 415 X 240 pixels.

Analysis Procedure

Once an acceptable digitized image is made, the next step is the actual analysis. To begin this process, the MacPaint image is copied into the clipboard. The size of the image must be exactly that of the paint window, otherwise the image displayed in the analysis program will be distorted. After copying the image to the clipboard, MacPaint is quit and Surface.analysis is started. Once the blinking cursor appears, typing "go" will start the analysis program.

The process begins by selecting **display image**, the first item in the **Surface Analysis Options** menu. The image is then displayed on the screen and stored in an array called GRAYMAP as 1's and 0's. While the program is building the image map, the mouse pointer changes to a wrist watch and remains so until the map is completed.

After the image map is built, the number of pixels per unit length is calculated. First, **set scale** is selected from the **Surface Analysis Options** menu. Using the mouse, the cursor is moved to one of the scale markings photographed with the surface section. Clicking the mouse with the cursor point on the marking, sets the initial point, and draws a vertical reference line. The cursor is then moved to a point on the next marking which intersects the reference line. Clicking the mouse then sets the number of pixels per scale marking.

Next, the portion of the section image to be analyzed is determined. This begins by selecting **working window** from the **Surface Analysis Options** menu. The cursor is then moved to the upper left corner of the desired analysis area. At this point, the mouse button is pressed and held down while dragging the mouse to the lower left of the area to be analyzed. Releasing the mouse button and holding the mouse in place sets the coordinates for a rectangular analysis region. A new region replacing the one displayed can be created by starting over in the upper left of the desired area. The analysis area is not permanently set until an item from the menu is selected.

Once an acceptable analysis area is set, **color grains** is selected from the **Surface Analysis Options** menu. This is the portion of the program where 2-D bond diameters, neck areas, and number of bonds per grain are determined. The grain necks within the working rectangle can be colored in any order desired. The coloring process changes each selected neck region to a gray pattern on the screen, and sets the appropriate portion of the image map to a value of 2 where a 0 represents pore space and 1, ice. As the neck region is filled in, the area is calculated and the bond diameter is measured.

Grain bond measurements are begun by selecting a grain neck to be colored, and by visually determining 4 corner points of a polygon shaped region completely enclosing the neck. These points should be located where the curvature of the boundary changes sign.

The selection of these points will often be somewhat subjective. The control, **set corners**, at the bottom of the screen is then selected. This control box then remains highlighted until the mouse button is clicked 4 times. The cursor is moved to the first location selected on the neck region boundary, 1 in Figure 16, and the mouse is single clicked. The next point must be the one, for which a line connecting the two points would lie entirely within the ice grain as shown by number 2 in Figure 16. The cursor is then moved to this point and the mouse is single clicked. This process continues by always moving in a cyclic manner, similar to that indicated in Figure 16, until one cycle

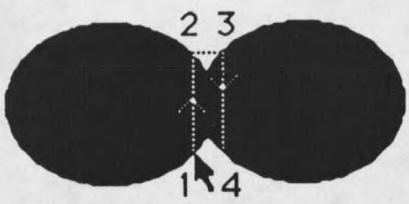

Figure 16. Cursor Movement During a Typical Grain Bond Coloring Procedure. The four cyclic movements possible are 1234, 2143, 3412, 4321. The lines shown do not actually appear.

is completed. The actual starting point is arbitrary since the direction motion is dependent on the location of the first point. If the points selected aren't correct, **set corners** can be selected again and the process repeated. Once 4 points have been marked, **shade inside outline** is selected. This becomes highlighted and remains so until the neck region is colored gray. This process is continued for each neck region lying within the working rectangle. For necks intercepted by the region boundary, two points will be on the boundary and two within the region. **Shade intercepted neck** is used in this case rather than **color grain bond**. In this situation only the portion of the neck within the region is colored and measured; no bond diameter is calculated.

The number of bonds associated with a particular grain can be counted at any time, but the best procedure is to count the number of bonds associated with a grain after each of

its necks have been colored. This is done by simply counting the number of nonintercepted necks (1 each) and intercepted necks (1/2 each) visually. The actual number is then registered by selecting the appropriate combination of bond count controls. A value of 2 in the 1 bonds counter result for the grain combo in Figure 16. It tends to be easier to count the number of grains with 0 bonds after all the necks have been colored and the bond numbers counted. A value of 1 is tallied for grains having no bonds lying wholly within the region and 1/2 for each intercepted by the region boundary. A typical screen as it appears after all of the above procedures have been completed is shown in Figure 4.

The last step after all necks have been colored is to select the **calculate parameters** at the bottom of the screen. When this is selected, the program calculates all the needed parameter values that can be obtained from direct measurements of the surface section. Once the calculations are finished, the resulting file is named by typing a name in the dialog box which appears. This then completes the analysis possible with this program. A MacForth program listing is given in Figure 17.

Figure 17. Surface Section Analysis Program

```
ANEW Surface.analysis-v2.52
130000 MINIMUM.OBJECT
1500 MINIMUM.VOCAB
variable xmouse
variable ymouse
fvariable pixel.length \ pixels per unit length
variable num.pixel
variable flg
variable ct  \ general counter
variable ct1  \ general counter
4 4 1array num.points \ save number of points in each side of box
4 2 12 2array corner \ 2 X 4 floating point array
100 8 12 2ARRAY LINE \ LINE holds  lines defining the outline
                     \ of a parallelpiped enclosing the grain bond
                     \ presently under consideration ;100x8 floating point array
100 12 1ARRAY D2radii \ first 100 2-D bond radii
415 CONSTANT xmax \ maximum x dimension of window
240 CONSTANT ymax \ maximum y dimension of window
416  241  1 2array GRAYMAP  \ 280X500 MATRIX CALLED
                           \GRAYMAP USING 8 BIT INTEGERS


NEW.WINDOW SURFACE.SECTION
      " Surface section analysis"   SURFACE.SECTION     W.TITLE
      40  0  340  512                    SURFACE.SECTION     W.BOUNDS
      CLOSE.BOX                     SURFACE.SECTION     W.ATTRIBUTES
      SURFACE.SECTION     ADD.WINDOW

NEW.CONTROL COLOR.G
      " SHADE neck "                      COLOR.G             C.TITLE
      10  270                       COLOR.G             C.POSITION
      A.PUSH.BUTTON                 COLOR.G             C.TYPE

NEW.CONTROL COLOR.BND.G
      " SHADE intercepted neck"     COLOR.BND.G         C.TITLE
      120  270                      COLOR.BND.G         C.POSITION
      A.PUSH.BUTTON                 COLOR.BND.G         C.TYPE

NEW.CONTROL setcorn
      " set corners"                setcorn             C.TITLE
      10  245                       setcorn             C.POSITION
      A.PUSH.BUTTON                 setcorn             C.TYPE

NEW.CONTROL       calculate
      " Calculate--parameters "     calculate           C.TITLE
      120  245                      calculate           C.POSITION
      A.PUSH.BUTTON                 calculate           C.TYPE
```

Figure 17. Surface Section Analysis Program Continued.

```
NEW.CONTROL NG0.BUTTON
        " 0 BONDS "          NG0.BUTTON   C.TITLE
        425  1               NG0.BUTTON   C.POSITION
        A.PUSH.BUTTON        NG0.BUTTON   C.TYPE

NEW.CONTROL BNG0.BUTTON
        " 1/2 "              BNG0.BUTTON  C.TITLE
        425  21              BNG0.BUTTON  C.POSITION
        A.PUSH.BUTTON        BNG0.BUTTON  C.TYPE

NEW.CONTROL NG1.BUTTON
        " 1 BONDS "          NG1.BUTTON   C.TITLE
        425  61              NG1.BUTTON   C.POSITION
        A.PUSH.BUTTON        NG1.BUTTON   C.TYPE

NEW.CONTROL BNG1.BUTTON
        " 1/2 "              BNG1.BUTTON  C.TITLE
        425  81              BNG1.BUTTON  C.POSITION
        A.PUSH.BUTTON        BNG1.BUTTON  C.TYPE

NEW.CONTROL NG2.BUTTON
        " 2 BONDS "          NG2.BUTTON   C.TITLE
        425  121             NG2.BUTTON   C.POSITION
        A.PUSH.BUTTON        NG2.BUTTON   C.TYPE

NEW.CONTROL BNG2.BUTTOn
        " 1/2 "              BNG2.BUTTON  C.TITLE
        425  141             BNG2.BUTTON  C.POSITION
        A.PUSH.BUTTON        BNG2.BUTTON  C.TYPE

NEW.CONTROL NG3.BUTTON
        " 3 BONDS "          NG3.BUTTON   C.TITLE
        425  181             NG3.BUTTON   C.POSITION
        A.PUSH.BUTTON        NG3.BUTTON   C.TYPE

NEW.CONTROL BNG3.BUTTON
        " 1/2 "              BNG3.BUTTON  C.TITLE
        425  201             BNG3.BUTTON  C.POSITION
        A.PUSH.BUTTON        BNG3.BUTTON  C.TYPE

NEW.CONTROL NG4.BUTTON
        " >3 BONDS "         NG4.BUTTON   C.TITLE
        425  241             NG4.BUTTON   C.POSITION
        A.PUSH.BUTTON        NG4.BUTTON   C.TYPE

NEW.CONTROL BNG4.BUTTON
        " 1/2 "              BNG4.BUTTON  C.TITLE
        425  261             BNG4.BUTTON  C.POSITION
        A.PUSH.BUTTON        BNG4.BUTTON  C.TYPE
```

Figure 17. Surface Section Analysis Program Continued.

```
NEW.CONTROL SUB.BUTTON
        " - "                   SUB.BUTTON  C.TITLE
        375  260                SUB.BUTTON  C.POSITION
        A.PUSH.BUTTON           SUB.BUTTON  C.TYPE

-102 CONSTANT WRONG.TYPE \ used when getting picture from scrap
global PICT.HANDLE \ handle for the pictur
0 0 240 415 RECT MY.RECT \ destination rectangle
ASCII MYID CREATOR !
: assign.handle 0 from.heap to pict.handle ;

: FORCE.WINDOW ( wptr -- )
        dup show.window         dup window
        dup active.window !     select.window ginit ;
variable  file.id
variable  file.size
: (SAVE.AS) ( --- )
        0 locals| file# |
        reply 74 erase
    "TEXT dflt.file.type !
        ascii MACA CREATOR !      \ MACWrite file
        75  75  xy>point
        " Save under what name?"
        " UNTITLED "
        reply
        (put.file)  \  discard.activates          front.window  (window)
        reply c@
        if
                reply 10+ delete$
                file.size @
              reply  10+  new.file  to  file#    ?file.error
        then  file#
;
: even.odd ( n\-- 0 or 1)
        2 mod 0 = if 0 else 1 then \ returns 0 if n even 1 if n odd
;
variable num.necks   \ number of grain bonds or necks (N2)
variable num.neckpix \ number of neck phase  pixels
VARIABLE FLAG
VARIABLE FIRSTCT                        \  holds cell number of first neck pixel
variable LASTCT                               \ holds cell number of last neck pixel
fvariable  Diam
:  turn.it.gray  (  ---)
        0 FLAG !
        0 ct ! begin
                        CT @ 0 LINE F@ INT  \ X
                         CT @ 1 LINE F@ INT GRAYMAP C@  \ Y
                         0 = NOT  IF FLAG @ 0 = IF CT @ FIRSTCT ! 1 flag ! 0
                        \ look for edge of neck
```

Figure 17. Surface Section Analysis Program Continued.

```
                                                    ELSE FLAG @ 1 + DUP FLAG !
                                                      2 = IF -1 ELSE 0
                                                      THEN
                                                    \ 2 consecutive
                                                     \pixels signify neck
                                                THEN
                                            ELSE 0 FLAG ! 0
                                            THEN CT @ 1 + dup ct ! 99 > if drop -1 then
                    UNTIL
    0 FLAG !
    CT1 @ 1 - ct ! begin
                    CT @ 0 LINE F@ INT \ X
                     CT @ 1 LINE F@ INT GRAYMAP C@ \ Y
                     0 = NOT IF FLAG @ 0 = IF CT @ LASTCT ! 1 flag ! 0
                                                    \ look for edge of neck
                                            ELSE FLAG @ 1 + DUP FLAG !
                                                2 = IF -1 ELSE 0 THEN
                                                  \ 2 consecutive pixels signify neck
                                                THEN
                            ELSE 0 FLAG ! 0
                            THEN CT @ 1 - dup ct ! 0 < if drop -1 then
                    UNTIL
    FIRSTCT @ CT !
            BEGIN
                    2 CT @ 0 LINE F@ INT CT @ 1 LINE F@ INT 2DUP GRAYMAP c@
                            \ has the neck pixel been set
                    1 = IF num.neckpix @ 1 + num.neckpix !
                            \ add new pixel only if prevously uncounted
                    GRAYMAP C!              \ set neck pixel in graymap to 2
                            ELSE 2drop drop    \ pixel already set
                            THEN
                    CT @ 0 LINE F@ INT even.odd   \ is x even or odd
                    CT @ 1 LINE F@ INT even.odd   \ is y even or odd
                    + float -1E fswap f** int 1 +   \ ( 1 +( -1)**(even.odd(x)
                    +even.odd(y)))
                    0 = IF  WHITE PENPAT CT @ 0 LINE F@ INT
                                    CT @ 1 LINE F@ INT
                                    DOT \ pixel white for x+y odd

                    ELSE BLACK PENPAT CT @ 0 LINE F@ INT
                                    CT @ 1 LINE F@ INT
                                    DOT  \ pixel black for x+y even
                    THEN
                    CT @ 1 + dup CT ! LASTCT @ > IF DROP -1 ELSE  0 THEN
            UNTIL
                    FIRSTCT @ 0 LINE F@ LASTCT @ 0 LINE F@ F- 2E F**
                    FIRSTCT @ 1 LINE F@ LASTCT @ 1 LINE F@ F- 2E F**
                    F+ FSQRT FDUP Diam F@ F<
                    IF Diam F! else fdrop THEN \ find shortest diameter
```

Figure 17. Surface Section Analysis Program Continued.

```
                                BLACK PENPAT
 ;
fvariable ADDER \ increment for steping along line
fvariable x  \ storage for current x value ; increment is on x
fvariable y  \ storage for current y value ; increment is on y
variable ex  \ stores number of points of shortes line being stepped on
fvariable slope \ slope of line
: vert.line  ( index\m\n- )
        locals| nv mv index |
        nv 0 corner f@ fdup
        mv 0 corner f@ fdup frot frot \ yn ym yn ym
        fmin   \ yn ym min(ym,yn)
        F= IF  .5e adder f! ELSE -.5e adder f! THEN
               fdrop
           mv 0 corner f@ y f!
           mv 1 corner f@ x f!
               0 ct1 !
               begin
                  y f@ fdup fdup ct1 @ index 2 * 1 + LINE f! \ new y
                       x f@ ct1 @ index 2 *  LINE f! \ new x
                       int nv 0 corner f@ int =  \ is y >= to the endpoint Yn
                       IF fdrop -1 \  SET EXIT VALUE
                       ELSE adder f@ f+ y f! 0 \ increment y exit flag = false
                       THEN ct1 @ 1 + ct1 ! \ increment number of points counter
               until
;

: norm.line  ( ndex\m\n- )
       locals| n m ndex |
        n 0 corner f@ m 0 corner f@ fover fover f- fabs \
        n 1 corner f@ m 1 corner f@ f- fabs fswap  \  yn ym dx dy on fstack
    fover fmin \ yn ym dx min(dy,dx)
         F= IF \ smallest delta is delx , step on y
               fswap fover  \ ym yn ym  on stack
               fmin
               F= if .5e adder f! else -.5e adder f! then
               n 0 corner f@ m 0 corner f@ f- \ dely
               n 1 corner f@ m 1 corner f@ f- \ delx
               f/ 1e fswap f/ slope f! \ 1/(dely/delx)=slope
               m 0 corner f@ y f! \ set y=y1
               0 ct1 !
               BEGIN                  \ equation: x=(y-y1)slope + x1
                 y f@ m 0 corner f@ f- slope f@ f* m 1 corner f@  f+
                 ct1 @ ndex 2 *  LINE f! \ x
                 y f@ ct1 @ ndex 2 * 1 + LINE f! \ y
                 y f@ int n 0 corner f@ int
```

Figure 17. Surface Section Analysis Program Continued.

```
                                        = IF  -1 \ set exit flag true
                                       ELSE y f@ adder f@ f+ y f! 0
                                              \ new y set exit flag false
                                       THEN   ct1 @ 1 + ct1 !
            UNTIL
        ELSE \ smallest delta is dely , step on x
            fdrop fdrop m 1 corner f@ fdup n 1 corner f@ fmin
                  \ xm  min(xm,xn)  on stack
            F= if .5e adder f! else -.5e adder f! then
            n 0 corner f@ m 0 corner f@ f- \ dely
            n 1 corner f@ m 1 corner f@ f- \ delx
            f/ slope f! \ dely/delx=slope
            m 1 corner f@ x f! \ set x=x1
           0 ct1 !
            BEGIN       \ equation: y=(x-x1)slope + y1
             x f@ m 1 corner f@ f- slope f@ f* m 0 corner f@ f+
             ct1 @ ndex 2 * 1 + LINE f! \ y
             x f@ ct1 @ ndex 2 * LINE f! \ x
             x f@ int n 1 corner f@ int
                                       =  IF -1 \ set exit flag true
                                         ELSE x f@ adder f@ f+ x f! 0 \ new x set exit flag
false
                                       THEN   ct1 @ 1 + ct1 !
            UNTIL
        THEN
;

: L1  ( I\-- )
       case
            0 of 0 0 1 vert.line  endof
            1 of 1 1 2 vert.line  endof
            2 of 2 2 3 vert.line  endof
            3 of 3 0 3 vert.line  endof
            4 of ( 2 3 2 vert.line ) endof
       endcase
;
: L2  ( I\- )
       case
            0 of 0 0 1 norm.line endof
            1 of 1 1 2 norm.line endof
            2 of 2 2 3 norm.line endof
            3 of 3 0 3 norm.line endof
            4 of ( 2 3 2 norm.line ) endof
       endcase
;

: BUILD.BOX  ( --- )
       3 1 corner f@ 0 1 corner f@ f= IF  3 L1  ELSE 3 L2  THEN
       ct1 @ 1 - 3 num.points !
```

Figure 17. Surface Section Analysis Program Continued.

```
        0 CT ! \ COUNTER
        BEGIN
           CT @ DUP 1 + 1 corner f@ CT @ 1 corner f@
                \ COUNTER COUNTER Xn X(n-1) on stack
                 int int
          = IF  L1 \ XM=XN => INFINITE SLOPE
                 ELSE  L2 \ USE NORMAL EQUATION FOR A LINE
                 THEN ct1 @ 1 - ct @ num.points !
       CT @ 1 + DUP CT ! 2 = IF  -1 ELSE 0 THEN
        UNTIL
        1 num.points @ 3 num.points @ >
        IF 4 ELSE 2 THEN \  case number
        CT @ 1 + 1 corner f@ CT @ 1 corner f@ int int \ COUNTER COUNTER
          = IF  L1 \ XM=XN => INFINITE SLOPE
                 ELSE  L2 \ USE NORMAL EQUATION FOR A LINE
                 THEN ct1 @ 1 - ct @ num.points !
;
variable ex
fvariable temp.diam
variable cts
: NEW.LINE ( delp\index1\index2- )
                        locals| ndex2 ndex1 delp |
                ndex1 num.points @ 0 DO
                        I 2 LINE f@ fdup 0 1 corner f!  \ new x1
                        I 3 LINE f@ 0 0 corner f!  \ new y1
                        I 6 LINE f@ fdup 1 1 corner f! \ new x2
                        I 7 LINE f@ 1 0 corner f! \ new y2
                           F= IF  0 L1   turn.it.gray
                           ELSE  0 L2  turn.it.gray
                          THEN
                        I cts !
                LOOP
                diam f@ temp.diam f! \ save final diam value past following routine
                ndex2 num.points @ ex ! \ store stopping criteria
                delp 0 = not IF ndex2 1 = if 1 0 corner f@ 0 0 corner f!
                         \ swap pivot corner index
                                                1 1 corner f@ 0 1 corner f!
                                then
                                = begin
                                        cts @ ndex2 2 * LINE f@ fdup 1 1 corner f! \ new x1
                                         cts @ ndex2 2 * 1 + LINE f@ 1 0 corner f! \ new y1
                                        0 1 corner f@ \ x2
                                        F= IF  0 L1 turn.it.gray
                                        ELSE  0 L2 turn.it.gray
                                        THEN
                                        cts @ ex @ = if -1 else cts @ 1 + cts ! 0 then
                                until
                THEN
```

Figure 17. Surface Section Analysis Program Continued.

```
                temp.diam f@ diam fl
;
fvariable ave.necklngth     \ mean neck length calculated
fvariable D1                \ summ of bond diameters
fvariable E                 \ summ of 1/bond diameter
: FILLER ( --- )
        10000e Diam fl \ initial diameter for comparison
        1 num.points @ dup 3 num.points @ dup rot
        = IF
          2drop 0  1 1 NEW.LINE \  delp num.points:indexs(1 1) on stack
        ELSE 2dup < IF  - abs  1 3 NEW.LINE
                        \  delp num.points:indexs(1 3) on stack : np1<np3
                ELSE  - abs 3 1 NEW.LINE
                                \ delp num.points:indexs(3 1) on stack : np3<np1
                THEN
         THEN
;
: FILL.GRAIN ( --- )
                BUILD.BOX
           FILLER
                sp! 0 \ set stack pointer to top ( clears stack ) false exit flag
  ;
: copy.corner  ( n\-- )
        1 - dup
    @mousexy  float float \ n -1 puts values in rows 0 thru 3 not 1 thru 4
        1   corner fl
        0   corner fl  ;
variable  which.corner \
: SET.CORNERS ( --- )
        SETCORN @ IN.BUTTON HILITE.CONTROL
    1 which.corner !
        begin 0
                do.events
                mouse.down = IF  which.corner @
                                        case
                                                1 of  1 copy.corner 2 which.corner !  endof
                                                2 of  2 copy.corner 3 which.corner !  endof
                                                3 of  3 copy.corner 4 which.corner !  endof
                                                4 of  4 copy.corner drop -1           endof
                                        endcase
                THEN
        until  setcorn @ 0 HILITE.CONTROL
;
: QUITS ( 0\-- -1\ ) DROP   -1 ;
VARIABLE NG0
VARIABLE NG1
VARIABLE NG2
VARIABLE NG3
VARIABLE NG4
```

Figure 17. Surface Section Analysis Program Continued.

```
VARIABLE BNG0
VARIABLE BNG1
VARIABLE BNG2
VARIABLE BNG3
VARIABLE BNG4
VARIABLE BNDFLG
: long.pen ( --- ) 1 18 pensize white penpat ;
: normal.pen ( --- ) black penpat 1 1 pensize ;
: WIPE0 ( --- )
        long.pen 427 42 469 42 VECTOR normal.pen
        427 59 MOVE.TO NG0 @ FLOAT BNG0 @ FLOAT .5E F* F+ 1 PLACES F. ;
: WIPE1 ( --- )
        long.pen 427 102 469 102 VECTOR normal.pen
        427 119 MOVE.TO NG1 @ FLOAT BNG1 @ FLOAT .5E F* F+ 1 PLACES F. ;
: WIPE2 ( --- )
        long.pen 427 162 469 162 VECTOR normal.pen
        427 179 MOVE.TO NG2 @ FLOAT BNG2 @ FLOAT .5E F* F+ 1 PLACES F. ;
: WIPE3 ( --- )
        long.pen 427 222 469 222 VECTOR normal.pen
        427 239 MOVE.TO NG3 @ FLOAT BNG3 @ FLOAT .5E F* F+ 1 PLACES F. ;
: WIPE4 ( --- )
        long.pen 427 282 469 282 VECTOR normal.pen
        427 299 MOVE.TO NG4 @ FLOAT BNG4 @ FLOAT .5E F* F+ 1 PLACES F. ;
variable intercept.neck
variable bound.pix
variable temp.pix
:ACTION         ( FLAG\CONTROL HANDLE\- FLAG )
        CASE
                COLOR.G @  OF COLOR.G @ IN.BUTTON HILITE.CONTROL
                        num.necks @ 1 + num.necks !
                        WATCH SET.CURSOR FILL.GRAIN INIT.CURSOR
                        DIAM F@ pixel.length f@ f/  num.necks @ 100 <
                                IF fdup num.necks @ 1 - D2radii f! then
                                D1 F@ F+ D1 F!  \ used to calc D2
                        1E DIAM F@ pixel.length f@  F/ f/ E F@ F+ E F!
                                \ used to calc Ebar
                        COLOR.G @ 0 HILITE.CONTROL
                        ENDOF
                COLOR.BND.G @ OF COLOR.BND.G @ IN.BUTTON HILITE.CONTROL
                        num.neckpix @ temp.pix ! watch set.cursor fill.grain
                        init.cursor intercept.neck @ 1 + intercept.neck !
                        num.neckpix @ temp.pix @ - bound.pix @ + bound.pix !
                        COLOR.BND.G @ 0 HILITE.CONTROL
                        ENDOF
                setcom @  OF  SET.CORNERS  ENDOF
               calculate @  OF   QUITS     ENDOF
                NG0.BUTTON @ OF NG0 @ 1 + NG0 ! WIPE0 ENDOF
                BNG0.BUTTON @ OF BNG0 @ 1 + BNG0 ! WIPE0 ENDOF
                NG1.BUTTON @ OF NG1 @ 1 + NG1 ! WIPE1 ENDOF
```

Figure 17. Surface Section Analysis Program Continued.

```
            BNG1.BUTTON @ OF BNG1 @ 1 + BNG1 ! WIPE1 ENDOF
            NG2.BUTTON @ OF NG2 @ 1 + NG2 ! WIPE2 ENDOF
            BNG2.BUTTON @ OF BNG2 @ 1 + BNG2 ! WIPE2 ENDOF
            NG3.BUTTON @ OF NG3 @ 1 + NG3 ! WIPE3 ENDOF
            BNG3.BUTTON @ OF BNG3 @ 1 + BNG3 ! WIPE3 ENDOF
            NG4.BUTTON @ OF NG4 @ 1 + NG4 ! WIPE4 ENDOF
            BNG4.BUTTON @ OF BNG4 @ 1 + BNG4 ! WIPE4 ENDOF
            calculate @ OF  QUITS                          ENDOF
      ENDCASE
;
: subtraction  ( --- )
      begin 0
      do.events
            case
            mouse.down of in.control?
            if follow.mouse
               if LAST.CONTROL
                 CASE
                    NG0.BUTTON @ OF NG0 @ 1 - NG0 ! WIPE0 drop -1 ENDOF
                    BNG0.BUTTON @ OF BNG0 @ 1 - BNG0 ! WIPE0 drop -1 ENDOF
                    NG1.BUTTON @ OF NG1 @ 1 - NG1 ! WIPE1 drop -1 ENDOF
                    BNG1.BUTTON @ OF BNG1 @ 1 - BNG1 ! WIPE1 drop -1 ENDOF
                    NG2.BUTTON @ OF NG2 @ 1 - NG2 ! WIPE2 drop -1 ENDOF
                    BNG2.BUTTON @ OF BNG2 @ 1 - BNG2 ! WIPE2 drop  -1 ENDOF
                    NG3.BUTTON @ OF NG3 @ 1 - NG3 ! WIPE3 drop  -1 ENDOF
                    BNG3.BUTTON @ OF BNG3 @ 1 - BNG3 ! WIPE3 drop -1 ENDOF
                    NG4.BUTTON @ OF NG4 @ 1 - NG4 ! WIPE4 drop  -1 ENDOF
                    BNG4.BUTTON @ OF BNG4 @ 1 - BNG4 ! WIPE4 drop -1 ENDOF
                 ENDCASE
                 then
            then
            endof
            endcase
      until  ;
:GETCONTROL
      LAST.CONTROL DUP SUB.BUTTON @ = IF DROP subtraction ELSE action THEN
;
:GRAY.NECK
      0E D1 F! 0E E F!
      0 num.neckpix ! 0 num.necks !
       0 intercept.neck ! 0 bound.pix !
      0 NG0 ! 0 NG1 ! 0 NG2 ! 0 NG3 ! 0 NG4 !
      0 BNG0 ! 0 BNG1 ! 0 BNG2 ! 0 BNG3 ! 0 BNG4 !
      0 BNDFLG ! \ boundary control not initially selected
      begin 0
            do.events
            case
                  mouse.down of in.control?  IF follow.mouse
                                                if getcontrol then
```

Figure 17. Surface Section Analysis Program Continued.

```
                                                        then
                                        endof
            endcase
      until
;
: GRAYMAP.ARRAY ( --- )
      241 0  DO
            416 0 DO
                  I J GET.PIXEL abs
                  I J GRAYMAP c! \ 1 for black : 0 for white
            loop ;
: checktype ( --- flag )
            PICT.HANDLE "PICT GET.SCRAP WRONG.TYPE = \ wrong type?
;
: DRAW.PICT ( -- ) \ draws picture from scrap if available
      PICT.HANDLE MY.RECT DRAW.PICTURE ;
: DRAWING ( --- ) PICT.HANDLE MY.RECT DRAW.PICTURE ;
variable place
VARIABLE XR1 \ working image initial x
VARIABLE XR2 \ working image final x
VARIABLE YR1 \ working image initial y
VARIABLE YR2 \ working image final y
:WORK.RECT  (  ---  )
      BEGIN 0
            do.events
            case
             mouse.down of @MOUSEXY  DUP YR1 ! SWAP DUP XR1 !
                  \ start of image rectangle
                  SWAP  2DUP
                  BEGIN
                        DRAW.PICT DRAWING \ update image as rectangle moves
                        @MOUSEXY  \  mouse position  xr2,yr2
                        FRAME RECTANGLE \  image rectangle
                        @MOUSEXY YR2 ! XR2 !  \ store last position
                        2DUP            \ duplicate XR1,YR1
                        STILL.DOWN NOT \ continue as long as mouse is down
                  UNTIL
                  2DROP
                  DRAW.PICT DRAWING
                  XR2 @ YR2 @ FRAME RECTANGLE
                        \ draw based on last recorded xr2,yr2
                  endof
                  IN.MENUBAR OF -1 ENDOF
            endcase
      UNTIL
      xr2 @ 415 > if 415 xr2 ! then
      xr1 @ 415 > if 415 xr1 ! then
      yr1 @ 240 > if 240 yr1 ! then
      yr2 @ 240 > if 240 yr2 ! then
```

Figure 17. Surface Section Analysis Program Continued.

```
;
: draw.coord ( x\y\- )
        locals| y x |
        gray penpat
        x 0 x ymax  vector
;
: pixel.per.length  ( --- )
        0 flg ! 0 \ set flag and put counter on stack
        begin
        do.events
                case
                        mouse.down of  wait.mouse.up drop 1 +  endof
                        mouse.up   of dup 1 = if @mousexy  ( y ! x ! ) 2dup draw.coord
                                                                rot
                                        else dup 2 = if drop @mousexy rot - dup * float
                                                        - dup * float f+ fsqrt
                                                        pixel.length f!  -1 flg !
                                                        \ set exit flag
                                                then
                                        then
                        endof
                endcase
        flg  @ until
;
fvariable  num.grains \ N1
fvariable N1A  \ number of grains per area
fvariable N2A  \ number of bonds per area
fvariable N1L  \ number of grain interceptions per unit test line length
fvariable alpha \ theoretical density
fvariable  D2                           \ mean bond radius
fvariable  Ebar                                 \ harmonic mean of bond diameter
fvariable Rbar  \ mean 3-D bond radius
fvariable N2V   \ number of bonds per unit volume
fvariable  Hbar         \ mean neck length theoretical
fvariable Lambda \ mean pore size
fvariable  L3bar          \ mean intercept length
fvariable  F20
FVARIABLE F21
FVARIABLE F22
FVARIABLE F23
VARIABLE airpixel    \ number of air phase pixels
VARIABLE icepixel    \ number of ice phase pixels
FVARIABLE airfrac    \ area fraction of air
FVARIABLE icefrac    \ area fraction of ice
FVARIABLE neckfrac  \ area fraction of necks
fvariable  area
variable step \ random test line increment
FVARIABLE total.length \ # grain intersection per unit length of test line
```

Figure 17. Surface Section Analysis Program Continued.

```
variable iceflag
variable num.lines
variable num.inter
variable ice.at.right
: PORE.COUNT ( xw1\xw2\yr1\yr2 )
                locals| yw2 yw1 xw2 xw1 |
        0 airpixel !
        yw2 yw1 1 + do
          xw2 xw1 1 + do
              I J GRAYMAP C@ 0 = IF airpixel @ 1 + airpixel ! THEN
          loop
        loop ;
: GRAIN.LENGTH ( xw1\xw2\yr1\yr2 )
    locals| yw2 yw1 xw2 xw1 |
    .5e pixel.length f@ f* int 1 + step !
    0 num.lines !
    0 num.inter !
    0 iceflag !
    0 ice.at.right !
    yw2 yw1 1 + DO
        num.lines @ 1 + num.lines !
        xw2 xw1 1 + DO
            I J GRAYMAP c@
            1 = IF I xw2 = IF ice.at.right @ 1 + ice.at.right ! THEN
                iceflag @ 0 = IF 1 iceflag ! \ moved from pore to grain
                    num.inter @ 1 + num.inter ! THEN
                ELSE iceflag @ 1 = IF 0 iceflag ! THEN
                THEN
        LOOP
    step @ +LOOP
    num.inter @ ice.at.right @ - num.inter !
        \ subtract ice grain bordering right side of window
    num.inter @ float
    xw2 xw1 - 2 - num.lines @ * float pixel.length f@ f/
    total.length f!
    num.inter @ float total.length f@ f/ N1L fl
;
: parameter.calc ( --- )
    WATCH SET.CURSOR
    xr1 @ xr2 @ 2dup > if swap then
    yr1 @ yr2 @ 2dup > if swap then
  PORE.COUNT
    xr1 @ xr2 @ 2dup > if swap then
    yr1 @ yr2 @ 2dup > if swap then
    GRAIN.LENGTH
    xr1 @ 1 - xr2 @ 1 - - abs yr1 @ 1 - yr2 @ 1 - - abs * num.pixel !
    num.pixel @ num.neckpix @ bound.pix @ + airpixel @ + - icepixel !
    airpixel @ float num.pixel @ float f/ airfrac fl \ area fraction of air (p1)
    icepixel @ float num.pixel @ float f/ icefrac fl \ area fraction of ice grain (p2)
```

Figure 17. Surface Section Analysis Program Continued.

```
        num.neckpix @ bound.pix @ + float num.pixel @ float f/  neckfrac f!
                \ area fraction of necks (p3)
        num.pixel @ float pixel.length f@ fdup f* f/ area f!
        NG0 @ NG1 @ + NG2 @ + NG3 @ + NG4 @ + float
        BNG0 @ BNG1 @ + BNG2 @ + BNG3 @ + BNG4 @ + float .5e f* f+ fdup
        num.grains f! AREA F@ F/ N1A F!
        NUM.NECKS @ float intercept.neck @ float .5e f* f+ AREA F@ F/ N2A F!
        D1 f@ num.necks @ float f/ D2 f! \ mean bond length
        E f@ num.necks @ float f/ Ebar f! \ harmonic mean of bond length
        1e icefrac f@ neckfrac f@ f+ f/ alpha f!
        PI 4E Ebar f@ f* f/ Rbar f!
        8e Ebar f@ N2A f@ f* f* PI PI f* f/ N2V f!
        neckfrac f@ Rbar f@ fdup f* N2V f@ f* PI f* f/ Hbar f!
        alpha f@ 1e f- alpha f@ N1L f@ f* f/ Lambda f!
        icefrac f@ neckfrac f@ f+ N1L f@ f/ L3bar f!
        NG0 @ float BNG0 @ FLOAT .5E F* F+ num.grains f@  f/ F20 f!
        NG1 @ float BNG1 @ FLOAT .5E F* F+ num.grains f@  f/ F21 f!
        NG2 @ float BNG2 @ FLOAT .5E F* F+ num.grains f@  f/ F22 f!
        NG3 @ float BNG3 @ FLOAT .5E F* F+ num.grains f@  f/ F23 f!
        INIT.CURSOR
;
: write.loop ( address and counts\file#\n\m )
        locals| m n file# |
        n 0 do \ numer of lines
                m 0 do \ numer of strings per line
                        file#  write.text
                loop
                CRLF 1 file# write.text \ next line in file
        loop
;
: writer
        locals| file# |
                file# write.text file# write.text crlf 1 file# write.text ;
: WRIT
        locals| file# |
                file# write.text crlf 1 file# write.text ;

: write.to.file
        1 file.size !
        (save.as) file.id !
        reply 10+ open$
         icefrac f@ 4 places (F.)  file.id @ writ
         neckfrac f@  4 places (f.)  file.id @ writ
         airfrac f@  4 places (f.)  file.id @ writ
         D2 f@  4 places (f.)  file.id @ writ
         Rbar f@  4 places (f.)  file.id @ writ
         N1L F@  4 places (f.) file.id @ writ
         N1A F@  4 places (f.)  file.id @ writ
         N2V f@ 4 places (f.)  file.id @ writ
```

Figure 17. Surface Section Analysis Program Continued.

```
        F20 f@  4 places (f.)  file.id @ writ
        F21 f@  4 places (f.)  file.id @ writ
        F22 f@  4 places (f.)  file.id @ writ
        F23 f@  4 places (f.)  file.id @ writ
        N2A F@  4 places (f.)  file.id @ writ
        alpha f@  4 places (f.)  file.id @ writ
        Hbar f@  4 places (f.)  file.id @ writ
        Lambda f@  4 places (f.)  file.id @ writ
        L3bar f@  4 places (f.)  file.id @ writ
        Ebar f@  4 places (f.)  file.id @ writ
        area f@  4 places (f.)  file.id @ writ
        pixel.length f@ 4 places (f.) file.id @ writ
        num.necks @ 0 DO I 1 + D2radii f@ 4 places (f.) file.id @ writ LOOP
      close file.id @
;
\       build and display menu
10 CONSTANT ANALYSIS.MENU       \ id number 10 assigned to analysis.menu
: Disable.menu.item
      2 0 ANALYSIS.MENU ITEM.ENABLE
      3 0 ANALYSIS.MENU ITEM.ENABLE
      4 0 ANALYSIS.MENU ITEM.ENABLE
;
:ANALYSIS.OPTIONS  ( -- )
      0 " Analysis options" ANALYSIS.MENU       NEW.MENU    \ hilite menu

      " Display Image;set scale(;working image(;color grain bonds("
   ANALYSIS.MENU  APPEND.ITEMS  \ menu items
      " new window;QUIT" ANALYSIS.MENU       APPEND.ITEMS

      DRAW.MENU.BAR     ANALYSIS.MENU       MENU.SELECTION: 0 HILITE.MENU
      CASE
            1 OF checktype IF     50 50 MOVE.TO
                                        ." no picture in scrap "
                                  ELSE WATCH SET.CURSOR DRAW.PICT DRAWING
                                        GRAYMAP.ARRAY INIT.CURSOR
                                        2 -1 analysis.menu item.enable
                                  THEN
              ENDOF \ paste image from clipboard
            2 OF pixel.per.length 3 -1 analysis.menu item.enable
              ENDOF \ set number of pixels per unit length
            3 OF 4 -1 analysis.menu item.enable WORK.RECT
              ENDOF \ put rectangle around section to be analysed
            4 OF GRAY.NECK parameter.calc write.to.file
               disable.menu.item
              ENDOF \ set grain neck to gray calc. parameter write them to file
            5 OF SURFACE.SECTION add:window 1 -1 ANALYSIS.MENU ITEM.ENABLE
             ENDOF
            6 of bye endof
      ENDCASE;
```

Figure 17. Surface Section Analysis Program Continued.

```
: BACK.COLOR ( --- )
        gray penpat
        1 100 pensize
         0 241 512 241 vector
        96 1 pensize
         416 0 416 241 vector
        black penpat
        1 1 pensize
;
: WINDOW.CONTROL ( activate flag -- )
        BACK.COLOR
        INIT.CURSOR
        SURFACE.SECTION COLOR.G       APPEND.CONTROL
        SURFACE.SECTION COLOR.BND.G APPEND.CONTROL
        SURFACE.SECTION setcom      APPEND.CONTROL
        SURFACE.SECTION calculate   APPEND.CONTROL
        SURFACE.SECTION NG0.BUTTON  APPEND.CONTROL
        SURFACE.SECTION NG1.BUTTON  APPEND.CONTROL
        SURFACE.SECTION NG2.BUTTON  APPEND.CONTROL
        SURFACE.SECTION NG3.BUTTON  APPEND.CONTROL
        SURFACE.SECTION NG4.BUTTON  APPEND.CONTROL
        SURFACE.SECTION BNG0.BUTTON APPEND.CONTROL
        SURFACE.SECTION BNG1.BUTTON APPEND.CONTROL
        SURFACE.SECTION BNG2.BUTTON APPEND.CONTROL
        SURFACE.SECTION BNG3.BUTTON APPEND.CONTROL
        SURFACE.SECTION BNG4.BUTTON APPEND.CONTROL
        SURFACE.SECTION SUB.BUTTON APPEND.CONTROL
        IF      ANALYSIS.OPTIONS

                BEGIN
                        DO.EVENTS
                                CASE
                                        IN.CLOSE.BOX OF 1 0 ANALYSIS.MENU ITEM.ENABLE
                                                                disable.menu.item
                                                        ENDOF
                                ENDCASE
                AGAIN
        ELSE
        THEN
;
 SURFACE.SECTION ON.ACTIVATE WINDOW.CONTROL
: go ( --- ) \ turnkey word
 surface.section add.window
 assign.handle
 0 0 1 1 sys.window w.bounds
 GINIT
  100 xr1 ! 200 xr2 ! 100 yr1 ! 200 yr2 !
 apple.menu
  init.cursor   show.cursor event.loop ; -floating
```

APPENDIX C

STATE VARIABLE CALCULATION

The calculation of the state variables N_{gv}, N_3, and V are done using the program State Variable. The original program was written by Andrew Hansen. It has been modified as described in the Stereology chapter. Other modifications were done merely to facilitate its use on the Macintosh and with the data files created by the image analysis program. This appendix describes the basic operation of State Variable.

The actual use of State Variable is straightforward. Upon starting the program, it prompts for the name of a data file containing the following variables, in the order listed and one variable per row: P1 (grain area fraction), P2 (neck area fraction), P3 (air area fraction), D2 (mean 2-D bond diameter), RBAR (mean 3-D bond radius), N1L (number of grains per unit length of test line), N1A (number of grains per unit area), N2V (number of bonds per volume), F20 (fraction of grains having 0 bonds), F21 (number of grains having 1 bond), F22 (fraction of grains having 2 bonds), F23 (number of grains having 3 bonds) and D2MIN (minimum measured 2-D bond diameter). After typing the data file name and then pressing return the program prompts for an output file name. The output file will contain output similar to that shown in Figures 18-21. Steps 10-30 have been removed.

After typing the output file name and pressing return, the program prompts with DO YOU HAVE N1V DETERMINED ? (TRUE/FALSE). For the first run with a data set, the response "false" followed by "return" is entered as indicated in Figure 18. The first iteration of the program will generate output as shown. At this stage the theoretical values of frequency distribution are compared with the measured values listed at the bottom of each iteration. By comparing the values, it is determined whether FP needs to be increased or decreased. For the data used to generate the figures it was necessary to increase the value of FP. Usually steps of 0.2 work well. Values of FP > 1 will cause F2(0) to decrease and F2(1) to increase. Values of FP < 1 will cause F2(0) to increase and F2(1) to decrease. The values of FP should remain in the range of $0.2 < FP < 2.0$ for

most data. If not, it may be necessary to adjust the value of N1V. This will be discussed later.

In Figure 18 the first prompt for an updated value of FP is shown along with the new value used FP=1.3. Figures 19-21 show the second form of prompt used for FP update. In this case T was typed in response to DO YOU WANT TO REFINE FP ? T/F. For the original data used here, three changes were used before acceptable theoretical values of the frequency distributions were obtained. It may happen that FP begins to get larger than 2.0. In this case when the prompt inquires about refining FP, F is entered as shown in Figure 21. Instead of responding to DO YOU WANT TO UPDATE N1VM ? T/F with F as shown, T is entered. N1VM is the middle value of N1V listed in the second column of the output. If the values of FP are getting large it will be necessary to increase N1V. This is done by entering values slightly larger than 1 when the program asks for values of DELTA. If values of FP are becoming very small, values of DELTA less than 1 are used. It is best to proceed in small increments when changing N1V. The new value of N1V is simply the product of the old value and the value of DELTA. For most surface section data it will not be necessary to change N1VM.

As can be seen in Figure 21, the value of FP=1.7 yields good convergence between the theoretical and measured frequency distributions. If this is the case, F is entered when the program prompts for refining FP and for updating N1VM. In general, several changes of FP will bracket the necessary value needed for convergence. Once the range has been bracketed the program is quit and the output file is analyzed to determine the actual range of FP needed to fine tune the convergence. The program is run again using values of FP in the new range until acceptable convergence accuracy is achieved. The data file is then analyzed again to find the resulting values of N1V and N3. The program can then be run again to calculate V, this time responding with TRUE when the program prompts with DO YOU HAVE N1V DETERMINED ? (TRUE/FALSE) or the value of V can be

calculated by hand. This completes the calculation of the state variables.

It must be kept in mind that this program did not give good results for the snow samples subjected to large deformations. For the uncompressed samples the program works quite well. The actual upper density limit for uncompressed snow is unknown, but will be obvious from difficulty in obtaining even marginal convergence with only one value of frequency distribution or when the results don't make sense physically. The complete Fortran listing of the program State Variable is given in Figure 22.

D0 YOU HAVE N1V DETERMINED ? (TRUE/FALSE)
FALSE

1IP= 1 FP= 1.00000

I	N1V	N3BAR	P	F2(0)	F2(1)	F2(2)	F2(3)	PTOT
1	7.34367	6.07157	.107805	.524943	.338333	.109070	.234358E-01	1.00000
2	7.58846	5.87571	.111398	.523234	.339295	.109628	.235850E-01	1.00000
3	7.83325	5.69209	.114992	.521783	.340223	.110055	.236645E-01	1.00000
4	8.07804	5.51961	.118585	.520538	.341132	.110376	.236855E-01	1.00000
5	8.32283	5.35726	.122179	.519451	.342031	.110615	.236583E-01	1.00000
6	8.56762	5.20420	.125772	.518485	.342927	.110789	.235920E-01	1.00000
7	8.81241	5.05964	.129366	.517609	.343826	.110915	.234950E-01	1.00000
8	9.05720	4.92289	.132959	.516798	.344729	.111005	.233738E-01	1.00000
9	9.30199	4.79334	.136553	.516035	.345638	.111067	.232341E-01	1.00000
31	14.6873	3.03578	.215610	.500618	.367487	.110676	.189413E-01	1.00000
32	14.9321	2.98602	.219203	.499885	.368568	.110616	.187244E-01	1.00000
33	15.1769	2.93785	.222797	.499148	.369658	.110552	.185061E-01	1.00000
34	15.4217	2.89122	.226390	.498406	.370755	.110484	.182863E-01	1.00000
35	15.6665	2.84605	.229984	.497661	.371862	.110411	.180651E-01	1.00000
36	15.9113	2.80226	.233577	.496911	.372978	.110334	.178424E-01	1.00000
37	16.1561	2.75980	.237171	.496157	.374103	.110253	.176184E-01	1.00000
38	16.4009	2.71861	.240764	.495399	.375237	.110167	.173930E-01	1.00000
39	16.6457	2.67863	.244358	.494635	.376380	.110077	.171663E-01	1.00000
40	16.8904	2.63981	.247951	.493868	.377533	.109982	.169382E-01	1.00000
41	17.1352	2.60210	.251545	.493095	.378695	.109882	.167089E-01	1.00000

F20=0.4807 F21=0.4024 F22=0.1030 F22=0.0113

ENTER NEW VALUE FOR FP
1.3

Figure 18. State Variable Output after First Iteration

```
1IP=  2      FP= 1.30000
```

I	N1V	N3BAR	P	F2(0)	F2(1)	F2(2)	F2(3)	PTOT
1	7.34367	6.07157	.107805	.513527	.348829	.111659	.224502E-01	1.00000
2	7.58846	5.87571	.111398	.512727	.349745	.111756	.223077E-01	1.00000
3	7.83325	5.69209	.114992	.511994	.350662	.111815	.221428E-01	1.00000
4	8.07804	5.51961	.118585	.511297	.351583	.111853	.219638E-01	1.00000
5	8.32283	5.35726	.122179	.510619	.352510	.111878	.217759E-01	1.00000
6	8.56762	5.20420	.125772	.509950	.353442	.111895	.215825E-01	1.00000
7	8.81241	5.05964	.129366	.509283	.354379	.111906	.213853E-01	1.00000
8	9.05720	4.92289	.132959	.508617	.355323	.111915	.211853E-01	1.00000
9	9.30199	4.79334	.136553	.507950	.356272	.111920	.209831E-01	1.00000
31	14.6873	3.03578	.215610	.492675	.378693	.111321	.160479E-01	1.00000
32	14.9321	2.98602	.219203	.491953	.379787	.111254	.158016E-01	1.00000
33	15.1769	2.93785	.222797	.491229	.380888	.111184	.155535E-01	1.00000
34	15.4217	2.89122	.226390	.490501	.381997	.111109	.153034E-01	1.00000
35	15.6665	2.84605	.229984	.489772	.383113	.111031	.150516E-01	1.00000
36	15.9113	2.80226	.233577	.489040	.384235	.110947	.147979E-01	1.00000
37	16.1561	2.75980	.237171	.488306	.385366	.110860	.145424E-01	1.00000
38	16.4009	2.71861	.240764	.487569	.386503	.110767	.142851E-01	1.00000
39	16.6457	2.67863	.244358	.486830	.387648	.110670	.140260E-01	1.00000
40	16.8904	2.63981	.247951	.486088	.388801	.110568	.137652E-01	1.00000
41	17.1352	2.60210	.251545	.485344	.389961	.110461	.135027E-01	1.00000

F20=0.4807 F21=0.4024 F22=0.1030 F22=0.0113

```
DO YOU WANT TO REFINE FP ? T/F
T
ENTER NEW VALUE FOR FP
1.5
```

Figure 19. State Variable Output after Second Iteration.

```
IIP=  3      FP= 1.50000
```

I	N1V	N3BAR	P	F2(0)	F2(1)	F2(2)	F2(3)	PTOT
1	7.34367	6.07157	.107805	.510203	.352945	.112044	.216674E-01	1.00000
2	7.58846	5.87571	.111398	.509527	.353872	.112072	.214752E-01	1.00000
3	7.83325	5.69209	.114992	.508860	.354804	.112092	.212777E-01	1.00000
4	8.07804	5.51961	.118585	.508193	.355741	.112108	.210769E-01	1.00000
5	8.32283	5.35726	.122179	.507526	.356684	.112121	.208738E-01	1.00000
6	8.56762	5.20420	.125772	.506857	.357633	.112132	.206685E-01	1.00000
7	8.81241	5.05964	.129366	.506185	.358588	.112141	.204613E-01	1.00000
8	9.05720	4.92289	.132959	.505511	.359548	.112147	.202521E-01	1.00000
31	14.6873	3.03578	.215610	.489266	.383367	.111548	.148855E-01	1.00000
32	14.9321	2.98602	.219203	.488526	.384484	.111483	.146275E-01	1.00000
33	15.1769	2.93785	.222797	.487785	.385607	.111414	.143674E-01	1.00000
34	15.4217	2.89122	.226390	.487040	.386738	.111341	.141053E-01	1.00000
35	15.6665	2.84605	.229984	.486293	.387877	.111264	.138411E-01	1.00000
36	15.9113	2.80226	.233577	.485543	.389023	.111183	.135748E-01	1.00000
37	16.1561	2.75980	.237171	.484790	.390177	.111098	.133065E-01	1.00000
38	16.4009	2.71861	.240764	.484033	.391338	.111008	.130361E-01	1.00000
39	16.6457	2.67863	.244358	.483274	.392507	.110915	.127639E-01	1.00000
40	16.8904	2.63981	.247951	.482512	.393684	.110817	.124897E-01	1.00000
41	17.1352	2.60210	.251545	.481747	.394868	.110714	.122137E-01	1.00000

F20=0.4807 F21=0.4024 F22=0.1030 F22=0.0113

```
DO YOU WANT TO REFINE FP ? T/F
T
ENTER NEW VALUE FOR FP
1.7
```

Figure 20. State Variable Output after Third Iteration.

```
IIP=  4      FP= 1.70000
```

I	N1V	N3BAR	P	F2(0)	F2(1)	F2(2)	F2(3)	PTOT
1	7.34367	6.07157	.107805	.508058	.355823	.112207	.210568E-01	1.00000
2	7.58846	5.87571	.111398	.507392	.356760	.112226	.208536E-01	1.00000
3	7.83325	5.69209	.114992	.506724	.357703	.112242	.206484E-01	1.00000
4	8.07804	5.51961	.118585	.506054	.358652	.112256	.204411E-01	1.00000
5	8.32283	5.35726	.122179	.505381	.359606	.112267	.202318E-01	1.00000
6	8.56762	5.20420	.125772	.504706	.360566	.112277	.200206E-01	1.00000
7	8.81241	5.05964	.129366	.504028	.361532	.112285	.198073E-01	1.00000
8	9.05720	4.92289	.132959	.503347	.362504	.112290	.195921E-01	1.00000
9	9.30199	4.79334	.136553	.502663	.363482	.112293	.193748E-01	1.00000
30	14.4426	3.08724	.212016	.487644	.385517	.111736	.143271E-01	1.00000
31	14.6873	3.03578	.215610	.486897	.386642	.111673	.140629E-01	1.00000
32	14.9321	2.98602	.219203	.486148	.387776	.111607	.137962E-01	1.00000
33	15.1769	2.93785	.222797	.485396	.388917	.111535	.135267E-01	1.00000
34	15.4217	2.89122	.226390	.484642	.390067	.111460	.132545E-01	1.00000
35	15.6665	2.84605	.229984	.483883	.391225	.111381	.129796E-01	1.00000
36	15.9113	2.80226	.233577	.483119	.392393	.111299	.127025E-01	1.00000
37	16.1561	2.75980	.237171	.482349	.393569	.111216	.124236E-01	1.00000
38	16.4009	2.71861	.240764	.481570	.394754	.111132	.121434E-01	1.00000
39	16.6457	2.67863	.244358	.480783	.395947	.111047	.118626E-01	1.00000
40	16.8904	2.63981	.247951	.479988	.397146	.110962	.115817E-01	1.00000
41	17.1352	2.60210	.251545	.479185	.398352	.110876	.113012E-01	1.00000

```
 F20=0.4807      F21=0.4024      F22=0.1030      F22=0.0113

DO YOU WANT TO REFINE FP ? T/F
F
DO YOU WANT TO UPDATE N1VM? T/F
F
```

Figure 21. State Variable Output after Fourth Iteration.

Figure 22. State Variable Program Listing

```
C       ORIGINAL PROGRAM WRITTEN BY
C       ANDREW C. HANSEN
C       MODIFICATIONS BY
C       MICHAEL Q. EDENS (1988)
        PROGRAM STVAR
C       THIS PROGRAM COMPUTES VALUES FOR THE INTERNAL STATE
C       VARIABLES DEVELOPED IN THE THEORY OF HIGH RATE DEFORMATION
C       OF SNOW.
C
        CHARACTER*1 ASK,RESP
              CHARACTER*12 NAME
         REAL  X(41),Y(41,3),RANGE(4)
         REAL  YC(123),FP(10)
        REAL*8   F3(12),F2(7),FN
        REAL*8  N1A,N1L,N1V,N1VM,N2V
        REAL*8  N3BAR,D2MIN
        EQUIVALENCE  (YC(1),Y(1,1))
        DATA  RANGE/0.0,0.0,0.0,0.0/
        DATA   PI/3.1415926/
        DATA   FP/0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0/
              OPEN(2,FILE='      ')
              REWIND 2
C       P1=ICE AREA FRACTION
C       P2=NECK AREA FRACTION
C       P3=AIR AREA FRACTION
C       D2=MEAN 2-D BOND DIAMETER
C       RBAR=MEAN 3-D BOND RADIUS
C       N1L=NUMBER OF GRAINS PER UNIT LENGTH
C       N1A=NUMBER OF GRAINS PER UNIT AREA
C       N2V=NUMBER OF BONDS PER UNIT VOLUME
C       F20=2-D 0 BOND FREQUENCY DISTRIBUTION (MEASURED)
C       F21=2-D 1 BOND FREQUENCY DISTRIBUTION (MEASURED)
C       F22=2-D 2 BOND FREQUENCY DISTRIBUTION (MEASURED)
C       F23=2-D 3 BOND FREQUENCY DISTRIBUTION (MEASURED)
C       D2MIM=MINIMUM VALUE OF SAMPLE 2-D BOND DIAMETERS
      8   READ (2,*,ERR=5) P1
              READ(2,*)  P2
              READ(2,*)  P3
         READ (2,*) D2
              READ(2,*)  RBAR
              READ(2,*)  N1L
              READ(2,*)  N1A
              READ(2,*)  N2V
           READ (2,*) F20
              READ(2,*)  F21
              READ(2,*)  F22
              READ(2,*)  F23
              READ(2,*)  D2MIN
              CLOSE(2)
```

Figure 22. State Variable Program Listing Continued.

```
            OPEN(*,FILE='      ')
C       INITIALIZE PLOT ARRAYS
        DO 10 I=1,41
        X(I)=0.0
  10            CONTINUE
        DO 30 I=1,123
        YC(I)=0.0
  30            CONTINUE
C  I    NITIALIZE PROBABILITY DISTRIBUTION ARRAYS
        DO 40 I=1,12
        F3(I)=0.0
  40    CONTINUE
        DO 60 I=1,7
        F2(I)=0.0
  60            CONTINUE
C  BEGIN COMPUTING INTERNAL STATE VARIABLES

C  DO YOU HAVE N1V DETERMINED? (T/F)
  17            WRITE  (*,932)
        READ (*,928) RESP
        IF (RESP.EQ.'T') GO TO 200
              IF (RESP.NE.'F') GO TO 17
C       THE NEXT EQUATION IS BASED ON THE NEW DEFINITION OF BANDWIDTH
        B=2.0*SQRT(RBAR**2-(D2MIN*0.5)**2)
        SBBAR=PI*N1L*B/N1A
        N1VM=0.63*N1A**2/N1L
        IFLAG=1
 100            CONTINUE
C  IP IS SIMPLY AN INDEX PARAMETER USED TO RETAIN MULTIPLE
C  VALUES OF FP
        IP=1
        FP(IP)=1.0
        IF  (IFLAG.NE.1)  WRITE  (*,956)
        IF  (IFLAG.NE.1)  READ  (*,906,ERR=3)  DELTA
        IF  (IFLAG.NE.1)  N1VM=N1VM*DELTA
 110            CONTINUE
        IF  (IFLAG.NE.1)  WRITE  (*,952)
        IF  (IFLAG.NE.1)  READ  (*,906,ERR=4)  FP(IP)
C COMPUTE THE GAMMA FUNCTION
        G=0.5+(1.0/(2.0*FP(IP)))
        COEF=SQRT(2.0*PI*G)*(G**G)*(EXP(-G))
        SERIES=1.0+1.0/(12.0*G)+1.0/(288.0*G**2)-139.0
   1    /(51840.0*G**3)
        GAMMA=COEF*SERIES
C UNIT * IS THE SCREEN AND UNIT 4 IS A FILE
        WRITE  (*,934)  IP,FP(IP)
        WRITE  (*,936)
              WRITE  (4,934)  IP,FP(IP)
              WRITE  (4,936)
```

Figure 22. State Variable Program Listing Continued.

```
            WRITE(*,938)  I,N1V,N3BAR,P,F2(1),F2(2),F2(3),F2(4),PTOT
            WRITE(4,938)  I,N1V,N3BAR,P,F2(1),F2(2),F2(3),F2(4),PTOT
C  COMPUTE F2 AS A FUNCTION OF N1V FOR A FIXED FP
      DO 120 I=1,41
      X(I)=(0.6+(I-1)*0.02)*N1VM
      N1V=X(I)
      N3BAR=2.0*N2V/N1V
      SKBAR=(4.0*N1L)/N1V
      P=SBBAR/SKBAR
      IF (P.GT.1.0) WRITE (*,930) X(I),I
      IF (P.GT.1.0) READ (*,902) KR
      IF (P.GT.1.0) P=1.0
C  COMPUTE THE COEFFICIENTS BETA AND F3N. THESE ARE GIVEN BY
C  ALPHA AND N(I) RESPECTIVELY IN GUBLER'S PAPER [6].
      BETA=((2.0*GAMMA)/(SQRT(PI)*N3BAR))**(2.0*FP(IP))
      F3N=4.0*FP(IP)*BETA**1.5/SQRT(PI)
C  COMPUTE THE 3-D THEORETICAL PROBABILITY DISTRIBUTION OF
C  COORDINATION NUMBERS REPRESENTING GRAIN BONDS (F3).
      F3T=0.0
      DO 130 J=1,12
      3(J)=F3N*(FLOAT(J)**(3.0*FP(IP)-1.0))*EXP((-BETA)
         2   *FLOAT(J)**(2.0*FP(IP)))
      F3T=F3T+F3(J)
 130        CONTINUE
      F3T1=1.0/F3T
      DO 135 J=1,12
      F3(J)=F3(J)*F3T1
 135        CONTINUE
C  COMPUTE THE 2-D THEORETICAL PROBABILITY DISTRIBUTION OF
C  COORDINATION NUMBERS REPRESENTING GRAIN BONDS (F2).
       DO 140 L=1,7
       DO 150 K=1,12
      IF (L.NE.1) GO TO 160
      PR=(1.0-P)**K*F3(K)
      F2(L)=F2(L)+PR
      GO TO 150
 160        M=L-1
      IF  (K-M)  150,170,180
 170        PR=P**M*F3(K)
      F2(L)=F2(L)+PR
      GO TO 150
 180        CALL CMBN(K,M,N)
      PR=FLOAT(N)*P**M*(1.0-P)**(K-M)*F3(K)
      F2(L)=F2(L)+PR
 150        CONTINUE
 140        CONTINUE
      F2T=0.0
      DO 145 JJ=1,7
      F2T=F2T+F2(JJ)
```

Figure 22. State Variable Program Listing Continued.

```
 145          CONTINUE
      F2T1=1.0/F2T
      DO 155 JJ=1,7
      F2(JJ)=F2(JJ)*F2T1
 155          CONTINUE
      Y(I,1)=F2(1)
      Y(I,2)=F2(2)
      Y(I,3)=F2(3)
      PTOT=F2(1)+F2(2)+F2(3)+F2(4)+F2(5)+F2(6)+F2(7)
      C   REINITIALIZE F2(I)
      DO 185 JJ=1,7
      F2(JJ)=0.0
 185          CONTINUE
 120          CONTINUE
      WRITE  (*,940)  F20,F21,F22,F23
             WRITE  (4,940)  F20,F21,F22,F23
      NIN=5
      NOUT=6
      IOPT=3
      INC=1
      N=41
      M=3
      IOPT=0
      IFLAG=2
      IP=IP+1
      IF (IP.EQ.2) GO TO 110
      WRITE  (*,962)
C  DETERMINE WHETHER OR NOT TO UPDATE FP
  32          READ  (*,928)  ASK
      IF (ASK.EQ.'T') GO TO 110
             IF (ASK.NE.'F') GO TO 32
C       DECIDE IF YOU WANT TO UPDATE N1VM
  47          WRITE  (*,960)
      READ (*,928) ASK
      IF (ASK.EQ.'T') GO TO 100
             IF (ASK.NE.'F') GO TO 47
      IF (RESP.EQ.'F') GO TO 210
 200          CONTINUE
      WRITE  (*,942)
      READ (*,906) N1V
      WRITE  (*,944)
      READ (*,906) FF
      N3BAR=2.0*N2V/N1V
      SV=4*N1L-2.0*PI*RBAR**2*N3BAR*N1V
      VBAR=(P1+P2)/N1V
      WRITE (*,972) N3BAR
      WRITE (*,972) VBAR,FF
 210          CONTINUE
```

Figure 22. State Variable Program Listing Continued.

```
C  PRINT INTERNAL STATE VARIABLES
      WRITE  (*,998)
      IF (RESP.EQ.'F') GO TO 220
      WRITE  (*,1008)  N3BAR
      WRITE  (*,1010)  N1V
      WRITE  (*,1012)  VBAR
      WRITE  (*,1013)  SV
      WRITE  (*,1014)  FF
 220          CONTINUE
              CLOSE(4)
              GOTO 2000
  3           WRITE(*,*) ' INPUT REAL NUMBER '
      GOTO 100
  4   WRITE(*,*) ' INPUT REAL NUMBER '
              GOTO 110
  5   REWIND 2
      GOTO8
 902          FORMAT  (I6)
 904          FORMAT  (G15.5,A4,G12.6)
 906          FORMAT  (G12.6)
 908          FORMAT  (3G12.6)
 910          FORMAT  (4G12.6)
 912          FORMAT  (2I6)
 916          FORMAT (A4)
 928          FORMAT (A1)
 930          FORMAT (' WARNING: P>1.0 AT N1V=',G12.6,4X,'I=',I6)
 932          FORMAT (/,' DO YOU HAVE N1V DETERMINED? (True/False)')
 934          FORMAT  ('1IP=',I3,6X,'FP=',G12.6)
 936          FORMAT(///,4X,'I',6X,'N1V',12X,'N3BAR',9X,'P',10X,
               1'F2(0)',10X,'F2(1)',10X,'F2(2)',10X,'F2(3)',10X,'PTOT',/)
 938          FORMAT  (1X,I4,3X,8(G12.6,2X))
 940          FORMAT  (///,3X,'F20=',G12.6,6X,'F21=',G12.6,6X,'F22=',G12.6,
        1'F23=',G12.6)
 942          FORMAT (/,' INPUT N1V')
 944          FORMAT (/,' INPUT FP')
 952          FORMAT (' ENTER NEW VALUE FOR FP')
 956          FORMAT (' ENTER UPDATED VALUE FOR DELTA (N1VM=N1VM*DELTA)')
 960          FORMAT ('$DO YOU WANT TO UPDATE N1VM? T/F')
 962          FORMAT ('$DO YOU WANT TO REFINE FP? T/F')
 972          FORMAT  (4X,3(G12.6,4X))
 990          FORMAT(/' PROBABILITY DENSITY OF GRAINS WITH 0 BONDS IN THE SUR
      6FACE SECTION=',G12.6)
 992          FORMAT(/' PROBABILITY DENSITY OF GRAINS WITH 1 BOND IN THE SURF
      7ACE SECTION=',G12.6)
 994          FORMAT(/' PROBABILITY DENSITY OF GRAINS WITH 2 BONDS IN THE SUR
      8FACE SECTION=',G12.6)
 996          FORMAT(/' PROBABILITY DENSITY OF GRAINS WITH 3 BONDS IN THE SUR
      9FACE SECTION=',G12.6)
 998          FORMAT  (///' *******INTERNAL  STATE  VARIABLES*******')
```

Figure 22. State Variable Program Listing Continued.

```
 1008          FORMAT (/' MEAN NUMBER OF BONDS/GRAIN=',G12.6)
 1010          FORMAT (/' MEAN NUMBER OF GRAINS/VOLUME=',G12.6)
 1012          FORMAT (/' MEAN GRAIN VOLUME=',G12.6,1X,A4,'**3')
 1013          FORMAT (/' MEAN SURFACE AREA/VOLUME=',G12.6,1X,A4,'**-1')
 1014          FORMAT (/' SHAPE PARAMETER=',G12.6)
 2000 PAUSE' RETURN TO QUIT '
      END
C
      SUBROUTINE CMBN(K,M,N)
C  THIS SUBROUTINE IS BASED ON COMBINATORIAL ANALYSIS AND
C   RETURNS THE VALUE N=K!/(M!*(K-M)!)
      KF=1
      MF=1
      KMF=1
      DO 10 I=1,K
      KF=KF*I
 10         CONTINUE
      DO 20 I=1,M
      MF=MF*I
 20         CONTINUE
      DO 30 I=1,K-M
      KMF=KMF*I
 30         CONTINUE
      N=KF/(MF*KMF)
      RETURN
      END
```