POSITIONING THEORY AS A FRAMEWORK FOR UNDERSTANDING STUDENT ENGAGEMENT IN MATHEMATICAL MODELING COMPETENCIES.

by

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DEDICATION

For my grandparents, Leslie and Kimberly Simkins. Thank you for your loving presence throughout my life. I am, and will always be, incredibly grateful for your unconditional love and support. You both meant the world to me.

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TABLE OF CONTENTS

1. THE PROBLEM OF INTEREST 1

Background of the Problem 1
Mathematical Literacy 1
Mathematical Modeling Supports Mathematical Literacy 2
Process of Mathematical Modeling 3
Background: Mathematical Modeling in Elementary Grades 4
Modeling Competencies in Elementary Grades. 5
Overview of Group Work During Modeling. 6
Positioning Theory as the Theoretical Framework 7
Statement of the Problem 8
Research Questions 9
Significance of the Study 10

2. THE LITERATURE AND THEORY 12

Mathematical Modeling in Educational Settings 12
Perspectives on Mathematical Modeling in Education 12
Purposes of Modeling Perspectives. 13
Types of Modeling in Each Perspective. 14
Research Foci of Perspectives. 15
Understanding the Modeling Process. 15
Realistic Perspective in this Study. 20
Literature on Mathematical Modeling Competencies 21
Modeling Competencies of Interest in this Study. 24
Literature on Mathematical Modeling in the Elementary Grades 25
Characteristics of Modeling in Elementary Grades. 26
Illustrative Example of Elementary Modeling in Practice. 26
Benefits of Modeling for Elementary Students. 28
Limitations of Traditional Elementary Mathematics. 28
Illustrative Example of Traditional Problem-Solving. 29
Modeling as a Holistic Alternative to Traditional Mathematics Problems. 29
Cognitive and Social Benefits of Modeling. 30
Conclusion to Modeling Benefits for Elementary Learners. 31
Areas Underexplored in Mathematical Modeling Research. 31
Group Work during Mathematical Modeling. 32
Conclusion to the Relevant Literature 34
Positioning Theory as the Theoretical Framework 35

TABLE OF CONTENTS CONTINUED

Rationale for Using Positioning Theory 35
Central Constructs of Positioning Theory 36
Overview of Central Constructs 36
Speech Acts in Conversation. 36
Positions in Conversation. 37
Story Lines in Conversation 37
Summary of the Central Constructs. 37
Positioning in Relation to Mathematical Modeling Competencies 38
Relevance of Positioning Theory for This Study 39
Closing Discussion: Linking Literature and Theory 39

3. METHODS AND ANALYSIS 41

Researcher Positionality 41
Research Methodology 43
Research Methods 44
Sampling Methods 44
Case Boundaries 45
Context of the Study 46
Participant Demographics. 46
Teacher Mathematical Modeling Experience. 46
Student Mathematical Modeling Experience. 47
Description of the Jump Rope Task. 48
The Day of the Jump Rope Task. 49
Data Collection 50
Researcher Participation. 52
Data Types 53
First-Order Data: Raw Materials from the Field 53
First-Order Data: Audiovisual Recordings. 54
First-Order Data: Student Work. 54
First-Order Data: Field Notes. 54
Second-Order Data: Products of Interpretation and Analysis 55
Second-Order Data: Jottings and Analytic Memos. 55
Second-Order Data: Content Logs. 56
Second-Order Data: Transcripts. 60
Data Management and Confidentiality 60
Analytical Approach to Data Analysis 61
Overview of the Analytical Approach 61
Holistic Coding: Preparing the Data. 62
Transcript Selection Process: Overview. 63
Preliminary Group Conversation Transcripts. 67
Final Group Conversation Transcripts. 69

TABLE OF CONTENTS CONTINUED

Ethical Considerations 71
Analysis of the Data 72
Overview of Data Analysis 72
First and Second Cycle Coding. 73
Codebook Development. 74
First Cycle Coding 74
Developing First Cycle Codes. 77
Applying First Cycle Codes. 85
Conclusion to First Cycle Coding. 89
Second Cycle Coding 90
Generating Category Codes from First-Cycle Codes. 91
Second Cycle Coding: Modeling Competency Codes 91
Second Cycle Coding: Student Interaction Codes. 92
Second Cycle Coding: Student Position Codes. 93
Using Category Codes to Examine First Cycle Coding. 94
Layered Analysis Using Category Codes. 95
Layer 1: Competency Categories. 95
Layer 2: Positional and Competency Categories. 96
Layer 3: Interactional, Positional, and Competency Categories. 96
Cross-Layer Analysis to Identify Findings. 97
Analytic Approach and Development of Findings. 97
Conclusion to Second Cycle Coding. 98
Conclusion to Analysis of the Data 98

4. FINDINGS FROM ANALYSIS 101

Introduction 101
Contextual Overview 101
Enactment of the Jump Rope Task 101
Finding 1: Contextually Grounded Mathematical Modeling 105
Modeling Competencies during Group Work Conversations 106
Contextual Grounding During Group Work Conversations 107
Group 4: Sustained Contextual Grounding of Mathematical Modeling. 109
Group 4: Development of Model and Solution. 109
Group 4: Generalized Representation of the Model. 117
Conclusion: Group 4's Sustained Contextual Grounding in Modeling. 118
Group 1: Inconsistent Contextual Grounding of Mathematical Modeling. 119
Group 1: Development of Model and Solution. 119

TABLE OF CONTENTS CONTINUED

Group 1: Generalized Representation of the Model. 131
Conclusion: Group 1's Inconsistent Contextual Grounding in Modeling..... 132
Synthesis of Contextually Grounded Mathematical Modeling..... 133
Finding 2: Positional Authority in Modeling Conversations 133
Positions as Sources of Positional Authority 135
Higher-Authority Positions in Modeling. 135
Moderate to Lower-Authority Positions in Modeling..... 138
Conclusion: Positions as Sources of Positional Authority. 141
Interruptions and Positional Authority in Modeling 142
Interruptions as Authority-Sharing Mechanisms. 143
Interruptions as Authority-Reinforcing Mechanism. 144
Conclusion: Interruptions and Positional Authority in Modeling. 152
Questions and Positional Authority in Modeling..... 154
Questions as Authority-Sharing Mechanisms. 154
Questions as Authority-Reinforcing Mechanisms. 157
Conclusion: Questions and Positional Authority in Modeling. 158
Distribution of Positional Authority Among Students 159
Students Holding Higher Positional Authority. 160
Students Holding Moderate or Lower Positional Authority. 161
Conclusion to Distribution of Positional Authority Among Students. 165
Conclusion: Positional Authority and Student Voice in Modeling 165

5. DISCUSSION AND CONCLUSION 169

Summary and Discussion of Findings 170
Summary of the Findings in Relation to the Research Questions 170
Finding 1: Contextually Grounded Modeling. 171
Finding 2: Positional Authority in Modeling Conversations. 171
Connecting the Findings: Nature of Student Engagement in Modeling Competencies. 173
Discussion of the Findings in Relation to the Relevant Literature 174
Discussion of Contextually Grounded Modeling. 174
Supporting Functions of the Task Context. 175
Student Enactment of the Modeling Process. 178
Conclusion to Discussion of Contextually Grounded Modeling. 179
Discussion of Positional Authority in Modeling. 180
Authority-Shaped Modeling Engagement. 181
Student Development of Identity and Self. 183

TABLE OF CONTENTS CONTINUED

Conclusion to Discussion of Positional Authority in Modeling. 185
Conclusion to the Discussion of the Findings 186
Implications and Significance of the Findings 187
Implications of the Findings 187
Implications for Theory. 187
Implications for Practice. 188
Implications for Research. 189
Significance of the Findings 190
Limitations of the Research 191
Contextual and Methodological Boundaries 192
Interpretive and Analytical Constraints 192
Quality of the Findings 193
Concluding Remarks 194

REFERENCES CITED 196

APPENDICES 205

UNIVERSITY CONSENT FORMS 206

FIELD NOTES AND CONTENT LOGS 213

TRANSCRIPTS 254

LIST OF TABLES

Table Page

1. Table 1. Maaß (2006) Modeling Competencies and Subcompetencies 22
2. Table 2. Modeling Competencies, Subcompetencies, and Subprocesses 24
3. Table 3. Sample Analytic Memo for Whole Class Engagement 56
4. Table 4. Sample Lesson-Level Content Log 57
5. Table 5. Named Sections of the Modeling Lesson 58
6. Table 6. Classifying the Sections of the Lesson 59
7. Table 7. Excerpt from the Group-Level Content Log 60
8. Table 8. Lesson Segments Eliminated from Transcription 64
9. Table 9. Excerpt from Group 1 Conversation Content Log 66
10. Table 10. Lesson Segments where Conversations were Transcribed 67
11. Table 11. Preliminary Transcript Sample Conversation 68
12. Table 12. Transcript Codebook 70
13. Table 13. Example of Final Transcript 71
14. Table 14. A Priori Concept Codes and Operationalized Definitions 76
15. Table 15. Competency Codes, Operational Definitions, and Examples 78
16. Table 16. Interaction Codes, Operational Definitions, and Examples 80
17. Table 17. Position Codes, Operational Definitions, and Examples 82
18. Table 18. Example Coding Competencies – Making Assumptions 86
19. Table 19. Example of Coding Interactions – Countering 87
20. Table 20. Example of Coding Positions – Instructor 88

LIST OF TABLES CONTINUED

21. Table 21. Competency Categories, Operational Definitions, and Included First-Cycle Codes 92

22. Table 22. Interactional Categories, Operational Definitions, and Included First-Cycle Codes 93

23. Table 23. Positional Categories, Operational Definitions, and Included First-Cycle Codes 94

24. Table 24. Alignment between Research Questions and Analysis 97

25. Table 25. Group 4 Sample Dialogue – Engaging in the Context 110

26. Table 26. Group 4 Sample Dialogue – Interpreting Mathematics within the Context 111

27. Table 27. Group 4 Sample Dialogue – Adjusting Assumptions 113

28. Table 28. Group 4 Sample Dialogue – Building the Model 114

29. Table 29. Group 4 Sample Dialogue – Recontextualizing the Mathematical Solution 115

30. Table 30. Group 4 Sample Dialogue – Analyzing and Estimating the Solution 116

31. Table 31. Group 1 Sample Dialogue – Connecting Mathematics to the Context 119

32. Table 32. Group 1 Sample Dialogue – Context Supported Model Development 121

33. Table 33. Group 1 Sample Dialogue – Countering Model Development with the Context 122

34. Table 34. Group 1 Sample Dialogue – Developing Model Solutions 123

35. Table 35. Group 1 Sample Dialogue – Disconnecting Mathematics from the Context 125

36. Table 36. Group 1 Sample Dialogue – Evaluating Mathematical Relationships 126

LIST OF TABLES CONTINUED

37. Table 37. Group 1 Sample Dialogue – Reconnecting Model Development with the Context 129

38. Table 38. Sample Dialogue – Learner and Instructor Positional Authority 139

39. Table 39. Sample Dialogue – Rejecting Positioning 140

40. Table 40. Sample Dialogue – Sharing Positional Authority by Interrupting 144

41. Table 41. Sample Dialogue – Exercising Positional Authority with Interrupting 146

42. Table 42. Sample Dialogue – Interruptions Granting Limited Access to the Conversation 149

43. Table 43. Sample Dialogue – Interruptions Reinforcing the Distribution of Positional Authority 150

44. Table 44. Sample Dialogue 1 – Questions Sharing Positional Authority 155

45. Table 46. Sample Dialogue 2 – Questions Sharing Positional Authority 156

46. Table 46. Sample Dialogue – Interruptions as Silencing Mechanisms 163

47. Table A1. Teacher Consent Form 207

48. Table A2. Student Consent Form 210

LIST OF FIGURES

Figure Page

1. Figure 1. Process of Translating between the Real and Mathematical Worlds (Cirillo et al., 2016) 16
2. Figure 2. Example Diagram of the Modeling Process (Maaß, 2006) 18
3. Figure 3. Synthesized Structure of the Modeling Process (Kaiser and Stender, 2013) 18
4. Figure 4 Traditional Mathematics Problems vs Modeling Problems (Muller & Burkhardt, 2007) 30
5. Figure 5. EQ-STEMM Modeling Process (Turner et al., 2025) 47
6. Figure 6. Sketch of the Fourth Grade Classroom 51
7. Figure 7. Placement of the Swivl Audio and Video Recording Devices 52
8. Figure 8. Overview of Selecting Transcription Segments 63
9. Figure 9. Group 4's Mathematical Modeling Work 117
10. Figure 10. Group 1's Mathematical Modeling Work – Sample 1 130
11. Figure 11. Group 1's Mathematical Modeling Work – Sample 2 131

ABSTRACT

While mathematical modeling is more prominent in secondary grades, research has demonstrated that elementary learners are also capable of mathematical modeling – the process of translating open-ended, real-world situations into mathematical representations, developing models, generating solutions, and interpreting solutions within the context. Student facility with modeling can be viewed through their engagement in *mathematical modeling competencies*, the set of skills students use to carry out the modeling process. Although research on mathematical modeling in the elementary grades is still in its infancy, scholars increasingly advocate for providing these learners with greater access to modeling experiences. Elementary students are both capable of and benefit from experiences with mathematical modeling. Situated within the theoretical framework of positioning theory, this study investigated how fourth-grade students engaged in mathematical modeling competencies during group-work conversations. Positioning theory illuminates how students' social interactions shape the positions they enact and, in turn, influence their engagement in modeling competencies. Using two groups of three and four students, this qualitative case study examined how students' use of the task context and their enacted positions influenced their engagement in modeling competencies during group conversations. Findings indicate that student attention to the context of the situation supported their access to and engagement in modeling competencies, particularly when developing mathematical models and generating solutions. Additionally, student positions were linked to differing levels of positional authority that influenced their access to and engagement in modeling competencies. These findings contribute to the limited literature on elementary students' modeling work by illustrating how engagement in modeling competencies is interactionally produced and mediated through positioning and contextual engagement. The study offers implications for theory, practice, and research by extending positioning theory into modeling conversations, supporting teacher facilitation of modeling tasks, and suggesting directions for future research on learning and assessment in elementary mathematical modeling.

CHAPTER ONE

THE PROBLEM OF INTEREST

Background of the Problem

Mathematical modeling (or modelling) in educational settings is broadly understood as the process where students translate between the real and the mathematical worlds to address a real-world problem. The modeling process supports students in their ability to apply mathematics to the world around them, which in turn fosters mathematical literacy skills. This chapter begins with an argument for mathematical modeling as a way to support mathematical literacy in students and concludes with the research questions, purpose, and significance of this research project. To begin, first consider mathematical literacy.

Mathematical Literacy

The term *mathematical literacy* first appeared in the United States in 1944 but was not defined until 1999 by the Organisation for Economic Co-operation and Development in the Programme for International Student Assessment (PISA) guidelines (Niss & Jablonka, 2014). Today's definition remains essentially the same. The *PISA 2022: Mathematics Framework* (2022) defines mathematical literacy as a person's ability to reason mathematically about real-world problems by using mathematics to interpret the problem and develop a solution to the problem. Consequently, mathematical literacy is not only concerned with student understanding and application of mathematical concepts but also with students' facility to reason mathematically in their everyday contexts.

The importance of using mathematics in everyday contexts is apparent by the use of international assessments. Across the globe, PISA measures how well countries equip their learners to use mathematics in their everyday lives (*PISA 2022: Mathematics Framework*, 2022). Addressing the issue, Bolstad (2021) explains that if mathematics education:

> fails to involve students in posing and answering questions, making inquiries and solving open-ended problems, students will continue to view mathematics only as a school activity, and the contexts to which students relate the use of mathematics will continue to be limited to basic everyday activities. The social justification of mathematics depends on its potential use in real-life situations. For individuals to develop their [mathematical literacy] learning and becoming, they need to encounter the use of mathematics in real-life situations a sufficient number of times, and the situations need to be significant to the students. (p. 250)

One way for students to experience this "mathematics in real-life situations" is through *mathematical modeling* – which is closely connected to mathematical literacy (Stohlman & Albarracín, 2016; Wickstrom & Yates, 2021). In fact, a mathematically literate individual is one who is able to understand, analyze, interpret, and assimilate information from a real-world problem, create and validate a mathematical model and solution through the efficient use of mathematical concepts – "in short, modelling" (Henning & Keune, 2007, p. 228; Kolar, 2021, p. 468). Thus, a mathematically literate individual has the ability to successfully model with mathematics.

Mathematical Modeling Supports Mathematical Literacy

Modeling with mathematics is listed as the fourth standard for mathematical practice in the Common Core State Standard for Mathematics (CCSSM). This standard describes what it means for students to be proficient with respect to mathematical modeling by detailing what modeling practices students are expected to develop. Students who can model with mathematics are able to: reinterpret real-world problems in the context of mathematics by identifying the

important quantities, make assumptions to define the problem, construct mathematical relationships between the selected quantities to develop a mathematical *model*, recontextualize the model to ascertain how well the model addresses the real-world problem, and revise the model if it requires further refinement to effectively address the original situation (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

The CCSSM's standard for mathematical modeling aligns with PISA's requirements for a mathematically literate student (Cevikbas et al., 2021; Stohlmann & Albarracín, 2016; Wickstrom & Yates, 2021). Both mathematical modeling and mathematical literacy are concerned with student proficiency in recognizing, applying, and interpreting mathematics in real-world contexts, and also with student ability to make decisions and assumptions related to the situation. Consequently, proficiency in mathematical literacy is connected to facility with mathematical modeling.

Process of Mathematical Modeling

Although past years have seen numerous definitions of modeling (English, 2021), mathematical modeling is widely conceptualized as the process where students translate between the real and the mathematical worlds in order to address real-world problems (Kaiser, 2017). Even with this general conceptualization, there is no universally agreed upon definition for modeling (Abassian et al., 2020). The process of modeling has been described by several researchers including Blum and Ferri (2016), Ferri (2006), Greer et al. (2007), Kaiser (2017), Maaß (2006), Pollak (2007), Stender and Kaiser (2016), and Suh (2017). The modeling process begins when students identify a problem within the real world and formulate a real-world question (Pollak, 2007). Students structure and define the bounds of the modeling problem by

making necessary assumptions and by identifying the important quantities present in the situation. Students *mathematize* their work by mathematically representing the problem and its components. In constructing relationships between problem components students develop mathematical models and use their models to determine solutions to the problem situation. Models and solutions are validated by analyzing them in relation to the mathematical decisions made and in context of the original real-world scenario. As a result of analysis, modelers may revise their work, or they may decide the model effectively addresses the situation and then conclude the modeling process by communicating their results with others. In short, mathematical modeling can be understood as the process of translating an authentic, open-ended, real-world scenario into the world of mathematics to generate a mathematical model and solution which address the original scenario.

Background: Mathematical Modeling in Elementary Grades

Traditionally, mathematical modeling experiences have been limited to students in their secondary years of school with little attention being given to the development of mathematical modeling at the elementary level (English, 2006; English, 2021; English & Watters, 2005; Greer et al., 2007; Jung & Brand, 2021; Stohlmann & Albarracín, 2016; Usiskin, 2007). In 2007 Greer et al. argued that researchers should consider mathematical modeling with elementary learners. At the time of their work they found "investigations of, discussions about, and instructional efforts for, mathematical modelling take place, almost exclusively, at the higher secondary and tertiary level" (Greer et al., 2007, p. 89). In 2016, Stohlmann and Albarracín conducted a review of the literature on mathematical modeling in the elementary grades and found that research in this area is still in its infancy. Along the same vein, Jung and Brand (2021) found few research

studies occurring between 2000 and 2017 concerning the nature of mathematical modeling in the elementary grades. Accordingly, Stohlmann and Albarracín (2016) and Jung and Brand (2021) call for further research focused on young learners' modeling experiences – a sentiment echoed throughout the literature on elementary students and mathematical modeling. Numerous researchers advocate for greater access to mathematical modeling experiences in the elementary school years and have found that these young learners have the ability to engage in mathematical modeling (Carlson et al., 2016; English, 2006; English, 2021; English & Watters, 2005; Stankiewicz-Van Der Zanden et al., 2021; Suh et al., 2021; Wickstrom & Yates, 2021). Some of the benefits elementary learners experience from modeling are enhanced critical thinking skills and a deeper understanding of mathematics (Suh et al., 2017; Wei et al., 2022). However, extending mathematical modeling from secondary into elementary grades requires consideration for potential differences between modeling in secondary grades and modeling in elementary grades.

Modeling Competencies in Elementary Grades. One of the ways researchers can investigate mathematical modeling within elementary grades is through *mathematical modeling competencies*. These competencies are detailed in the following chapter. For now, they can be thought of as the skills students use to work through the modeling process. Kaiser (2017) highlights the importance of modeling competencies stating the "relevance of modeling competencies, that is, the importance of the ability and the willingness to carry out a modeling process, is not in question" (p. 278) as the "promotion of modeling competencies … is accepted as a central goal of mathematics education worldwide" (p. 267). While recent years in mathematics education have witnessed more attention being given to elementary student

mathematical modeling competencies, there still exists a demand for research in this area (English, 2021; Stankiewicz-Van Der Zanden et al., 2021; Stohlmann & Albarracín, 2016). However, in considering elementary learners' mathematical modeling competencies, Brady and Lesh (2021) caution against thinking "that secondary-level versions of modeling can be reverse engineered to guide understandings of modeling among younger learners" (p. 100). Because of the dearth of information on elementary students' mathematical modeling competencies, this research project emphasizes how modeling competencies are realized at the elementary level within the context of a modeling problem.

Overview of Group Work During Modeling. When engaged in mathematical modeling, elementary teachers typically organize students into small working groups as these kinds of modeling problems are designed for small-group work (English, 2006, p. 306; English & Watters, 2005, p. 61; Flevares & Schiff, 2013, p. 42). Additionally, Flevares and Schiff (2013) explain that working through mathematical modeling problems in small groups creates space for student collaboration. However, this collaboration hinges on student *conversations* carried out through verbal and nonverbal *interactions*. These interactions are influential to the process and content of modeling because children's "interactions with others affect the choices of tools and creations of models" (Flevares & Schiff, 2013, p. 43). That is, interactions affect mathematical modeling. In their commentary on measuring and fostering mathematical modeling competencies, Frejd and Vos (2021) explain that current research finds:

> that the classroom culture is a matter of negotiations between the participants involved in the activity, which means that the fostering of mathematical modelling competencies does not depend solely on well-designed tasks and competent teachers. Researchers also need to take into consideration social interactions. (p.

460)The impact of these social interactions can be considered through the framework of *positioning theory*, a neo – Vygotskyan learning theory.

Positioning Theory as the Theoretical Framework

Neo-Vygotskyan learning theory "proposes that social interaction (including the use of language and other cultural tools) is the fundamental medium within which learning takes place" (Erickson, F., 2006, p. 181). Specifically, positioning theory attends to the *positions* brought about through social interactions and enacted by conversants. For instance, during group work one student may position themselves as "teacher" while positioning another student as "learner." This positioning influences the course of the conversation and the ways the two students interact with each other. Positioning theorists Davies and Harré (1990) explain that positions may "involve shifts in power, access, or blocking of access, to certain features of claimed or desired identity, and so on" (p. 49). Thus, positioning and access to mathematics affect how students see and understand themselves and others as mathematical people. But being positioned as a type of person may affect how a learner thinks they can act and how other people view the learner (Anderson, 2009). A position may afford or constrain the opportunity a learner has for participation. In short, conversants position themselves and others through interactions during conversation and the ensuing positions support or inhibit participation in the conversation.

Consequently, positions (a product of social interactions) can impact a student's access to and participation in a group's mathematical conversation, particularly during mathematical modeling. Group conversations during mathematical modeling hinge on students interacting with each other – i.e., a group's conversation evolves through the positions adopted by each of the group members. If social interaction is the "fundamental medium" through which students

engage with one another in order to learn (Erickson, F., 2006), then social interactions can be thought of as the "fundamental medium" for student engagement in mathematical modeling competencies during group work. Positioning theory provides a lens for considering the impact of positions on student access to the substance of the conversation. Thus, positioning theory is the lens through which to understand how positions influence student engagement in mathematical modeling competencies during group work.

Following Frejd and Vos's (2021) suggestion for "developing a social turn in mathematical modelling research" (p. 465) this research project adopts positioning theory as its theoretical framework. Positioning theory is an appropriate framework because it concerns the social interactions and resulting positions enacted by individuals during conversation as well as the impact positions have on the conversants and the substance of conversation. More to the point of this research project, if positions affect the opportunity a student has to participate in their group's conversation, it is likely that these positions affect the opportunities students have to engage in mathematical modeling competencies during group work conversations.

Statement of the Problem

While interest in elementary mathematical modeling is increasing, there remains a need for research concerning elementary students and mathematical modeling, and in particular, elementary student engagement with modeling competencies. This study concerns elementary student engagement in mathematical modeling competencies during group work with the understanding that current literature regarding modeling competencies is heavily influenced by research in the secondary grades. Thus, this study emphasizes modeling competencies as they are carried out in the elementary grades. As described, teachers typically support students to

collaborate in their working groups during modeling tasks. However, group work highlights student interactions and the impact those interactions have on the conversants and the substance of their conversation. The influence of these interactions is viewed via positioning theory. Positioning theory holds that through social interactions conversants position themselves and one another. The positions created through interaction can support or limit the opportunity students have to engage in mathematical conversations – and in particular, the opportunity students have to engage in mathematical modeling competencies. In short, when considering elementary students and mathematical modeling competencies, an important element for researchers to consider is student interactions and the resulting positions. This study investigates the influence of positions on elementary student engagement in mathematical modeling competencies during group work.

Research Questions

Given the considerations of the problem described, this research asks the central research question (CRQ): What is the nature of student engagement in mathematical modeling competencies during group work? To answer this question, I focus on the following sub-research questions (SRQs):

SRQ1: How does student attention to the task context influence their engagement in mathematical modeling competencies?

SRQ2: How do student positions influence their engagement in mathematical modeling competencies?

The purpose of this qualitative case study is to contribute to understanding how *fourth-grade elementary students* engage with mathematical modeling competencies during group work, and

the positions they enact through social interactions, and how those positions influence their engagement in modeling competencies.

Significance of the Study

The research presented in this work enriches the field of mathematics education because it attends to the often invisible – but always impactful – positions occurring during mathematical conversations, particularly those that arise during mathematical modeling conversations. The mathematics education community benefits from an awareness of student positions during modeling because, as Goleman (1997) reminds us:

> The range of what we think and do
> is limited by what we fail to notice.
> And because we fail to notice
> *that* we fail to notice,
> there is little we can do
> to change
> until we notice
> how failing to notice
> shapes our thoughts and deeds. (p. 24)

Mathematics educators must notice the positions students take up during modeling, as these positions shape the opportunities students have to engage in the mathematics conversation. Mathematics is not a spectator sport; students must engage in the work to grow as mathematical thinkers. The same holds true for mathematical modeling.

Bringing awareness to the impact of positions during mathematical modeling work is a step toward better understanding one of the universal factors of student engagement in any conversation: positioning – how students both position themselves and are positioned by others. More specifically, this research examines how positioning affects elementary students' engagement in modeling competencies.

This study contributes to the existing body of modeling literature by providing in-depth descriptions of seven fourth-grade elementary students working through a mathematical modeling lesson. In particular, the findings detail the positions students enact during group work conversations and the influence those positions have on their engagement in mathematical modeling competencies.

Thus, this research addresses the demand for greater understanding of elementary students and mathematical modeling, particularly with respect to modeling competencies. As such, the contribution to elementary modeling literature is significant because it extends the knowledge base on how elementary students engage in modeling competencies. Additionally, the findings concerning student positioning and its impact on engagement in modeling competencies deepen our understanding of how social interactions shape young modelers. Ultimately, this research informs both researchers' and teachers' understanding of elementary student engagement in modeling competencies during group work.

CHAPTER TWO

THE LITERATURE AND THEORY

Given the background of the problem and the stated research questions discussed in the previous chapter, this chapter builds a case for researching elementary student engagement with mathematical modeling competencies during group work. This case is accomplished through an argument from the literature related to mathematical modeling in education with attention to modeling competencies and elementary modelers. The chapter then presents positioning theory as the theoretical framework and demonstrates its appropriateness for this research study. The chapter closes with a final word for positioning theory in mathematical modeling classrooms.

Mathematical Modeling in Educational Settings

The background of the problem illustrates the need for more research concerning elementary students and mathematical modeling – particularly their engagement in mathematical modeling competencies. This literature review describes the landscape of mathematical modeling and modeling competencies with attention towards elementary learners collaborating during group work.

Perspectives on Mathematical Modeling in Education

While mathematical modeling was part of European and North American education during the 19th century, it did not become an official part of the United States curriculum until the start of the 20th century (Kaiser, 2017). Initially, mathematical modeling was loosely structured with the general goal of supporting students in applying mathematics to real-world contexts. But

"[b]eyond this general description, the process of teaching and learning mathematical modeling consist[ed] of a wide range of goals and backgrounds" (Abassian et al., 2020, p. 53). These various goals and backgrounds resulted in different conceptualizations or perspectives for mathematical modeling which were eventually formalized over the course of the 20th century (Abassian et al., 2020). The remainder of this section outlines the five perspectives on mathematical modeling, including their modeling goals, research focus, and commonalities and differences between them regarding the modeling process, and modeling competencies. Particular attention is given to the realistic perspective as this is the perspective adopted in this research.

Purposes of Modeling Perspectives. In their 2020 review of the literature on mathematical modeling, Abassian et al. consider five perspectives for mathematical modeling as organized by Kaiser and Sriraman (2006, p. 55). These perspectives are realistic, educational, models and modeling, socio-critical, and epistemological (Abassian et al., 2020; Kaiser & Sriraman, 2006). The differing goals for modeling, types of modeling problems, and foci in research contribute to these distinct modeling perspectives (Abassian et al., 2020). For instance, Abassian et al. (2020) explain that the purpose for modeling in the realistic perspective is to support student competence with modeling through real-world, open-ended scenarios. But in the models and modeling perspective the primary goal is to support student understanding of mathematical concepts within a modeling context. Modeling from an educational perspective has the dual goal of developing student modeling skills and also specific mathematical concepts. The socio-critical perspective aims to support students as they develop and apply their modeling skills to societal situations, while the epistemological perspective is concerned primarily with

student development of mathematical reasoning (Abassian et al., 2020). All five perspectives are concerned with student development of modeling skills and/or student development of mathematics.

Types of Modeling in Each Perspective. Within each of the five perspectives, a distinction can be made with respect to the type of modeling goal. One goal is student learning of mathematics. The other goal is student facility with the process of modeling – that is, the purpose here is for students to engage in the modeling process and develop skills with modeling. In the former, modeling is viewed as a *vehicle* to support student learning of mathematical concepts, while the latter view perceives modeling as *content* (Julie & Mudaly, 2007, p. 504). With modeling as a vehicle, modeling problems are designed so that students work with predetermined mathematical concepts – as in the case of the models and modeling perspective. But with modeling as content, the mathematics is secondary to the emphasis of student engagement with the process of modeling.

To address their modeling goals, the different perspectives use specific types of mathematical modeling problems. On one hand, perspectives emphasizing mathematical content over the process of modeling are more concerned with problems designed for students to engage with targeted mathematical concepts, as in the case of the socio-critical and the models and modeling perspectives. On the other hand, perspectives highlighting the modeling process use real-world problems designed so that students use the mathematical modeling process to address the real-world scenario (Wickstrom & Jung, 2024) – as in the case of the realistic perspective. The educational perspective emphasizes modeling problems that require students to engage in the modeling process and also learn mathematics (Abassian et al., 2020). Attendant to these

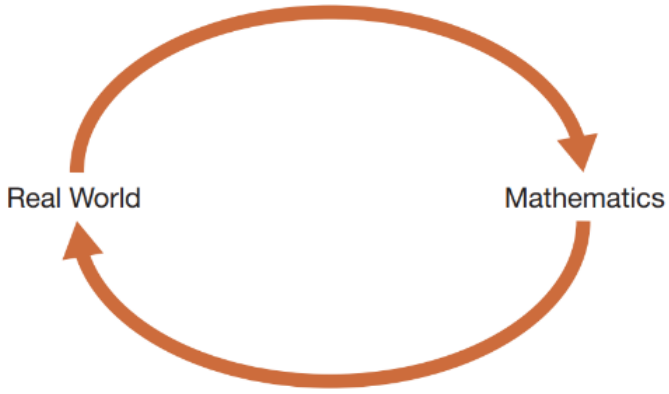
various modeling goals and types of modeling problems is the research focus from each perspective.

Research Foci of Perspectives. The research focus also varies with the intended goal of each perspective. Most relevant to this study, research from the realistic perspective focuses on mathematical modeling competencies (Abassian et al., 2020). *Mathematical modeling competencies* are the specific skills or practices students engage with as they work through the process of modeling. Research in the other perspectives emphasizes mathematical concepts – mathematical concepts in modeling or in the curricula (educational perspective), teaching mathematical concepts (epistemological and models and modeling perspectives), and applying mathematical concepts to society (socio-critical perspective) (Abassian et al., 2020). Concerning how students engage in modeling competencies, two conceptualizations can be articulated: the atomistic and holistic approaches (Blomhøj & Højgaard Jensen, 2003). The *atomistic approach* considers student modeling competencies in a targeted manner, i.e., rather than working through the entire modeling process, emphasis is placed on parts (or phases) of the modeling process that target a particular competency (Blomhøj & Højgaard Jensen, 2003; Cevikbas et al., 2021). On the other hand, the *holistic approach* "assumes that the development of modeling competencies should be fostered by performing complete processes of mathematical modeling" (Kaiser, 2017, p. 282). It is the holistic approach that is adopted for this research study because it aligns with the realistic perspective on the process of mathematical modeling.

Understanding the Modeling Process. While there exist five distinct perspectives on modeling, they do share some commonalities (Blomhøj 2009; Vorhölter, 2021). In particular, these perspectives share the understanding that the process of modeling is *cyclic*, modeling

contexts are grounded in the real world, a mathematical model needs to be developed, a mathematical solution attained which is validated in the context of the problem and then shared (Vorhölter, 2021). Consequently, mathematical modeling in schools is widely recognized as a cyclic process involving the *translation* of an authentic, realistic scenario into the world of mathematics where a model is constructed and then *translated* back into the real-world context (Carlson, 2021; Cevikbas et al., 2021; Flevares & Schiff, 2013; English, 2006; Kaiser, 2017; Stohlmann & Albarracín, 2016). In other words, translating between the real-world context and the representative mathematics is the core of mathematical modeling and the reason for conceptualizing modeling as a cyclic process, see Figure 1 (Carlson et al., 2016; Cirillo et al., 2016).

Figure 1. Process of Translating between the Real and Mathematical Worlds (Cirillo et al., 2016)



This conceptualization of the modeling process as a cyclic translation between the real and mathematical worlds is a component of the realistic perspective on mathematical modeling – the perspective adopted in this research project. In summary, the goal of the realistic perspective is to support student competence with the process of modeling by using modeling to address real-world problems that are complex, open-ended scenarios (Wickstrom & Jung, 2024). These types

of modeling problems require students to engage with the modeling process in order to develop their modeling competencies.

Research motivated by the realistic perspective focuses on student mathematical modeling competencies while students work through the entirety of a modeling problem (Abassian et al., 2020; Kaiser, 2017). It is because of the realistic perspective's research focus, type of modeling problems, and modeling goal that this research project operates from the realistic perspective. Recall from the previous chapter that this study investigates student engagement in mathematical modeling competencies as they work through the process of modeling – consequently, the use of the realistic perspective aligns with the purpose of this study.

Because modeling contexts are grounded in realistic, authentic scenarios which are open and complex, not everything in the given situation is defined (Wickstrom & Jung, 2024). Students do not have all the information needed to address the problem directly. They must make at least one assumption to productively engage in model building (Arnold et al., 2021; English, 2006). As a result, various solutions to the modeling problem are possible (Wickstrom & Yates, 2021). Working through the modeling process students form interpretations of the real-life situation; they make choices and assumptions to simplify the context and then represent and address the situation using mathematics (Arnold et al., 2021; Jung & Brand, 2021). Students can make different assumptions or decisions as they build their mathematical representations of the problem – these differences affect the outcome (the solution) of the model developed.

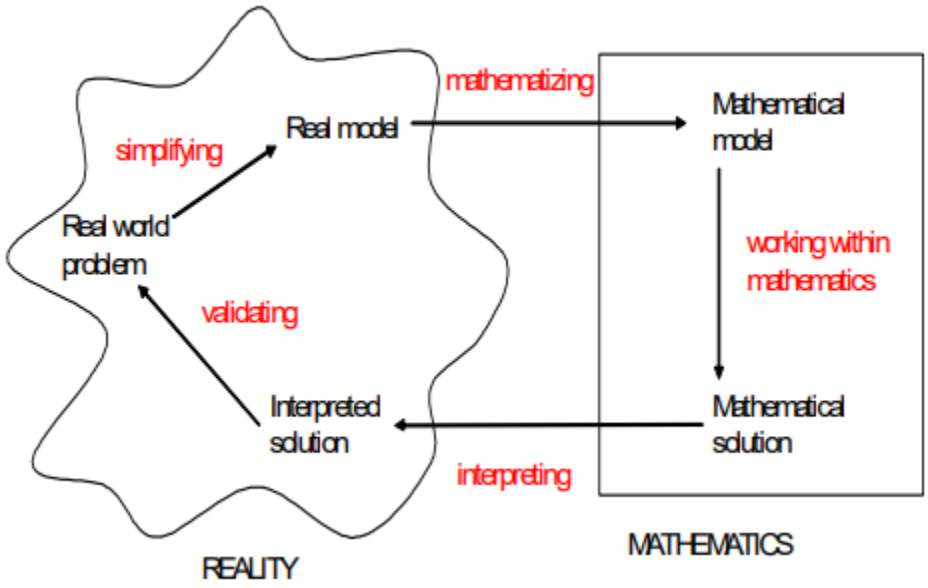
In concert with the different perspectives on modeling there exist various diagrams depicting the modeling process. Despite their differences, Kaiser (2017) explains that these

representations share the foundational idea of translating between the real and the mathematical worlds:

> All these modeling cycles share the distinction of the real-world situation from the real model and the mathematical model, and they all emphasize the necessity of interpreting the mathematical results to obtain real world results, which then need to be validated. (p. 276)

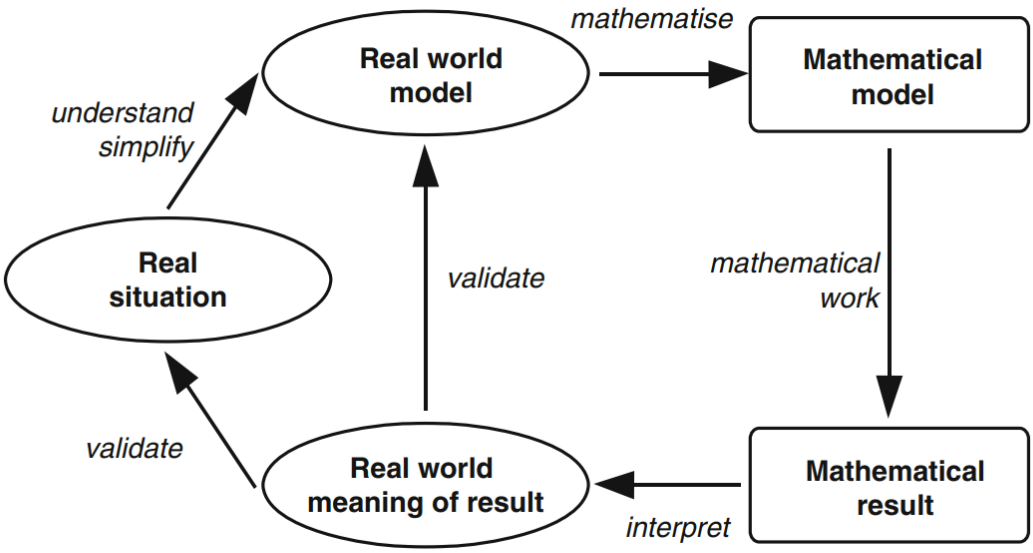
For example, Maaß (2006) emphasizes that modeling involves transitioning between the real and the mathematical worlds through the processes of mathematizing and interpreting, as illustrated in Figure 2.

Figure 2. Example Diagram of the Modeling Process (Maaß, 2006)



Considering these shared features, Kaiser and Stender (2013) analyzed commonalities among various depictions of the modeling process and synthesized them into a unified structure, shown in Figure 3.

Figure 3. Synthesized Structure of the Modeling Process (Kaiser and Stender, 2013)



Kaiser and Stender's (2013) modeling cycle depicts *modeling phases* which include understanding and simplifying a real-world, open-ended problem by formulating a modeling question, identifying quantities, and making assumptions related to the context. Modelers then take the elements created in the real-world model and represent them mathematically – a process often referred to as *mathematizing* – and develop mathematical relationships between these elements to construct their mathematical models. Modelers use the mathematical model to determine a mathematical result which they *interpret* within the real-world context. Validating the results of the model occurs as students consider their results in the context of the original problem and their real-world model. The process is complete when students communicate their models to others (Turner et al., 2021; Wickstrom & Yates, 2021).

However, students may not work through each phase of modeling through a predetermined path. While working through the process of modeling, students engage in "mini-cycles" by revisiting previous phases (Kaiser, 2017). Zbiek and Conner (2006) describe these

mini-cycles as subprocesses whereby a student moves through the phases of modeling in a non-linear manner (p. 91). These mini-cycles or subprocesses are particularly important with elementary students. In their work with primary and early elementary student modelers, Flevares and Schiff (2013) explain, "[m]odeling should be experienced in an ongoing, iterative cycle of modeling and interpreting" (p. 39). Moreover, young children's "model-building considerations tightly connect to the context, so as students construct a model, they often continue to make sense of the situation or problem" (Turner et al., 2021, p. 240). The continual interpretation of mathematical work in light of the real-world context is an important aspect of the realistic perspective on modeling as the realistic perspective focuses on student ability to move between the real and the mathematical worlds through mathematizing and interpreting – the core of modeling.

Realistic Perspective in this Study. The realistic perspective considers mathematical modeling as a cyclic process where modeling problems are grounded in real-world scenarios and are such that students engage with the entirety of the modeling process. The modeling process being thought of as the translation between the real and mathematical worlds. The overarching goal of the realistic perspective is student facility with mathematical modeling (modeling as content), with an emphasis on modeling competencies during modeling (holistic approach). Turner et al. (2021) explain, the realistic perspective "calls for a focus on the modeling competencies that support students as they make sense of real-world situations and apply phases of the modeling process." (p. 1391). Consequently, the present research study, which emphasizes student engagement with modeling competencies over the course of the modeling process, is motivated by the holistic approach within the realistic perspective.

Literature on Mathematical Modeling Competencies

Mathematical modeling requires a broad range of abilities, including social and communication competencies (Cevikbas et al., 2021, Vorhölter, 2021). Kaiser and Stender (2013) note the importance of these social competencies as they are prerequisite for working with the complexity of the modeling problems used in the realistic perspective. Another component of modeling is mathematical modeling competencies. In fact, determining student facility with mathematical modeling can be considered through the lens of modeling competencies (Anhalt et al., 2021). As a result, mathematical modeling competencies receive significant attention in mathematics education (Cevikbas et al., 2021; Kaiser, 2017). In particular, Kaiser (2017) explains that in "many national curricula, modeling competencies play a central role" (p. 267). It is through engagement with these modeling competencies that students become proficient in mathematical modeling.

In Maaß's (2006) seminal work addressing the question "What are modelling competencies?" Maaß explains, the "exact understanding of modelling competencies and skills is closely related to the definition of the modelling process" (p. 116). Maaß (2006) then enumerates modeling competencies and their related, but more specific subcompetencies as described in Blum and Kaiser's 1997 work, see Table 1.

Table 1. Maaß (2006) Modeling Competencies and Subcompetencies

	Competencies	Subcompetencies
Maaß 2006, p. 116 *Competencies and Subcompetencies*	Competencies to understand the real problem and to set up a model based on reality	▫ to make assumptions for the problem and simplify the situation; ▫ to recognize quantities that influence the situation, to name them and to identify key variables; ▫ to construct relations between the variables; ▫ to look for available information and to differentiate between relevant and irrelevant information
	Competencies to set up a mathematical model from the real model	▫ to mathematize relevant quantities and their relations ▫ to simplify relevant quantities and their relations if necessary and to reduce their number and complexity; ▫ to choose appropriate mathematical notations and to represent situations graphically
	Competencies to address mathematical questions within this mathematical model	▫ to use heuristic strategies such as division of the problem into part problems, establishing relations to similar or analog problems, rephrasing the problem, viewing the problem in a different form, varying the quantities or the available data etc.; ▫ to use mathematical knowledge to address the problem
	Competencies to interpret mathematical results in a real situation	▫ to interpret mathematical results in extra – mathematical contexts; ▫ to generalize solutions that were developed for a special situation; ▫ to view solutions to a problem by using appropriate mathematical language and/or to communicate about the solutions
	Competencies to validate the solution	▫ to critically check and reflect on found solutions; ▫ to review some parts of the model or again go through the modelling process if solutions do not fit the situation; ▫ to reflect on other ways of addressing the problem or if solutions can be developed differently; to generally question the model

Note. From (Blum & Kaiser, 1997, p. 9).

However, what Maaß terms “competencies,” Hirsch and McDuffie (2016) call “subcompetencies” because they are “indicated by the components in the modeling cycle” (p. 108). Other researchers call these competencies or subcompetencies “subprocesses,” and they consider these subprocesses to be in alignment with the phases or “stages” of modeling (Blomhøj & Højgaard Jensen, 2003, p. 125). Anhalt et al.’s, (2021) modeling competency framework is a summary of Maaß’s (2006) modeling competencies and subcompetencies. Table 2 shows these researchers and their descriptions of these modeling competencies, subcompetencies, and subprocesses.

Table 2. Modeling Competencies, Subcompetencies, and Subprocesses

Modeling Competencies, Subcompetencies, and Subprocesses	
Maaß 2006, p. 116 *Competencies*	▫ understand the problem and set up a real model ▫ set up a mathematical model from the real model ▫ address mathematical questions within this mathematical model ▫ interpret mathematical results in a real situation ▫ validate the model solution
Hirsch & McDuffie 2016, p. 108 *Subcompetencies*	▫ understand and simplify the problem by making assumptions and setting up a related real-world model ▫ translate the real-world model into a mathematical model by mathematization ▫ use the mathematical model to obtain a mathematical result ▫ translate the mathematical result back into the real-world context ▫ validate the real-world result by asking whether the result adequately answers the original problem
Blomhøj & Højgaard Jensen 2003, p. 125 *Subprocesses*	▫ formulate a task and understand the context of the problem to guide the model ▫ identify key elements from this understanding to generate a mathematical model ▫ translate these elements to a mathematical model ▫ use mathematics to generate a solution from the model ▫ interpret the mathematical results within the problem context ▫ validate the model within the problem context
Anhalt et al., 2021, p. 47 *Competencies & Subcompetencies*	▫ make assumptions, identify key variables, and construct relation between variables ▫ mathematize and simplify quantities and their relations and choose appropriate representations ▫ use mathematical knowledge to address the problem and use heuristics, such as rephrasing, dividing, and varying ▫ interpret results in context and generalize solutions ▫ critically check and reflect on solution, review model and iterate, reflect on other solution approaches, and question the model ▫ report out (communicate)

Modeling Competencies of Interest in this Study. Alignment exists between the modeling competencies, subcompetencies, and subprocesses described by Maaß (2006), Hirsch & McDuffie (2016), Blomhøj & Højgaard Jensen (2003), and Anhalt et al., (2021) (Table 2). Due to this alignment, this research study calls the modeling competencies, subcompetencies, and

subprocesses “modeling competencies” and considers them as described in the following competency framework:

(1) simplify the context by formulating the modeling problem, identifying relevant quantities, and making needed assumptions
(2) build a model by constructing relationships between quantities and using the model to generate a solution to the problem context
(3) interpret the model in the context of the situation and compare model and solution with other models and solutions
(4) check that the model and solution appropriately address the problem context and carry out identified model revisions
(5) communicate the model and solution to others and consider other contexts the model may extend to

These are the five modeling competencies this research project considers. However, recall from the first chapter that there is little research regarding elementary learners engaged in mathematical modeling competencies.

Literature on Mathematical Modeling in the Elementary Grades

Thus far, this chapter has considered mathematical modeling by outlining the modeling process, reviewing the various perspectives on modeling, and identifying the competencies associated with it. The next sections turn to the literature on elementary students’ engagement in modeling, focusing on four areas: the characteristics of modeling in the elementary grades, the

benefits of modeling, the need for a deeper understanding of how elementary learners engage in modeling competencies, and the role of group work during the modeling process.

Characteristics of Modeling in Elementary Grades. While the process of mathematical modeling is often depicted as a cyclic progression through modeling phases, the heart of modeling lies in students moving between the real and the mathematical worlds to address a complex, real-world problem. Importantly, modeling is not reducible to a checklist of phases. Even if students do not engage in every single phase, they are still modeling when they use their knowledge of mathematics to understand and make decisions regarding a real-world scenario (Arnold et al., 2021; Jung, H. & Brand, S., 2021).

Arnold et al. (2021) describe four defining characteristics of mathematical modeling: it "begins and ends outside the mathematical world," "deals with situations that are open and complex," "modelers make decisions and exercise judgement," and "modelers decide when a solution is good enough" (p. 4). Modeling tasks then, are such that students engage in decision-making, find meaning in the context of the problem, and allow for multiple mathematical approaches (p. v). In the elementary grades, students engage in modeling by posing mathematical problems, using mathematics to investigate and represent situations (where they make assumptions), and interpreting and revising their solutions (Arnold et al., 2021; Jung, H. & Brand, S., 2021).

Illustrative Example of Elementary Modeling in Practice. While there is a growing body of evidence showing that elementary learners are capable of mathematical modeling, relatively few research studies have examined the "nature and scope" of modeling with these learners (Jung, H. & Brand, S., 2021). One such study involved the *Upcycling Jump Rope* task, enacted in

four elementary grade classrooms, consisting of two third-grade and two fifth-grade classrooms of students (Turner et al., 2021, p. 242).

In this task, students drew upon their background knowledge and experiences as they engaged with the modeling process. When making sense of the situation, students built on their understanding of plastics, recycling, and upcycling, and discussed the environmental impacts of plastic use. These conversations helped them "begin to identify important quantities and assumptions" (Turner et al., 2021, p. 248) that they would use in constructing their models.

During model construction, students drew on their experience with "jumping rope to inform assumptions and decisions about appropriate jump rope lengths for different students and activities" (Turner et al., 2021, p. 249). They then used prior mathematical knowledge to relate relevant quantities through mathematical relationships and propose solutions, supported by physical tools and visual illustrations to engage in repeated addition, skip counting, and multiplication.

Finally, students interpreted their models in the context of the situation and refined them by adjusting their proposed jump rope lengths based on their lived experiences with jump roping. While only one fifth-grade class developed more generalized models, Turner et al. (2021) attribute this to both the level of complexity required for generalization and the time constraints of the classroom setting.

The *Upcycling Jump Rope* task illustrates mathematical modeling in the elementary grades as a process characterized by students moving between the real and mathematical worlds, making assumptions and decisions, and using mathematics to address authentic problems. Importantly, although three of the four student groups did not engage in generalizing their

models, they were nonetheless actively engaged in modeling as they meaningfully addressed the problem scenario. This example demonstrates that elementary learners can effectively engage with the modeling process without completing the "checklist" of phases. The following section addresses the benefits of modeling for elementary students.

Benefits of Modeling for Elementary Students. Although research on elementary learners' engagement in mathematical modeling is still emerging, a growing body of evidence highlights its benefits – especially when compared with traditional approaches to mathematics instruction. This section first addresses the limitations of traditional elementary mathematics, provides an illustrative example of how these limitations play out in practice, and then presents mathematical modeling as a holistic alternative.

Limitations of Traditional Elementary Mathematics. English (2021) explains that "[m]odeling is different to 'traditional' mathematics children complete in school, in which speed and accuracy are often considered all important, where the mathematics is of a cognitively low level and where children's thinking is not challenged" (p. 4). Frequently, elementary mathematics emphasizes procedural or computational skills, often at the expense of mathematical reasoning skills grounded in real-world scenarios. Researchers have noted that mathematics at the elementary level is often restricted to "traditional word problems where teachers may instruct students on finding key words" (Stohlmann & Albarracín, 2016, p. 1). Traditional types of problems tend to focus on computational (Flevares & Schiff, 2013; Greer et al., 2007) and procedural fluency, with a single correct method and answer (Suh et al., 2017). Such problems do little to support student recognition and understanding of the mathematics in their everyday lives (English & Watters, 2005).

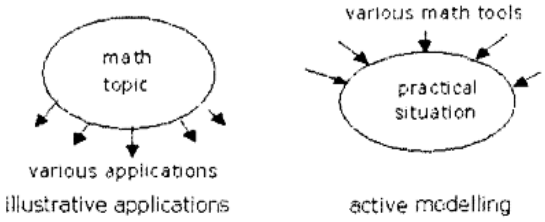
Illustrative Example of Traditional Problem-Solving. Standard problems in the elementary grades often focus on basic arithmetic competencies (English, 2006) where "the interpretation process for the child has been minimalised or eliminated" (English & Watters, 2005, p. 60). Greer et al. (2007) explain that international elementary education typically emphasizes computational proficiency in learning arithmetic, and that word problems are frequently dressed up examples of basic operations – addition, subtraction, multiplication, and division. Usiskin (2007) cautions against teaching elementary learners the operations of addition, subtraction, multiplication, and division through the models of $x + y, x - y, xy,$ and x/y arguing that such presentations lead to shallow conceptual understanding of the operations and the various contexts in which they apply.

For example, Verschaffel et al. (2000) describe the following problem: "Carl has 5 friends and Georges has 6 friends. Carl and Georges decide to give a party together. They invite all their friends. All friends are present. How many friends are there at the party?" (p. 102). In this scenario, students often default to adding 5 + 6 without considering the possibility of overlapping friendships, illustrating how students may apply arithmetic operations indiscriminately. More broadly, Biembengut (2007) finds that students in grade school learn to divorce mathematics from their real-world knowledge when solving problems, fostering a disassociation between mathematics and students' lived realities (Biembengut, 2007). As a result, students develop mental practices in direct opposition to those of mathematical modeling, i.e., effectively translating between the real and mathematical worlds.

Modeling as a Holistic Alternative to Traditional Mathematics Problems. In contrast, mathematical modeling positions mathematics as a powerful tool for making sense of real-world

situations. Muller and Burkhardt (2007) describe how traditional curricula emphasize teaching and learning mathematics as a "succession of separate topics, chapters, and formulas," whereas mathematical expertise requires that mathematics "be approached holistically rather than as an accumulation of bits and pieces of de-contextualized knowledge" (p. 268). They offer a demonstrative visual in Figure 4, showing that traditional curricula begin with a mathematical topic which is then applied in different situations, while mathematical modeling begins with a real-world situation that requires diverse mathematical tools. Muller and Burkhardt (2007) argue that mathematical modeling supports students' mathematical expertise by fostering a robust understanding of mathematics through the use of authentic, real-life scenarios.

Figure 4 Traditional Mathematics Problems vs Modeling Problems (Muller & Burkhardt, 2007)



Along the same vein, Wickstrom and Yates (2021) explain that modeling tasks require students to use many different aspects of mathematics while addressing these real-world problems.

Cognitive and Social Benefits of Modeling. The available research on mathematical modeling reports that elementary learners experience cognitive and social benefits through their modeling experiences. In a literature review, Wei et al. (2022) found that modeling fosters primary students' development in areas such as mathematical thinking, independence in learning mathematics, critical thinking, and communication skills. Similarly, Suh et al. (2017) reported that the process of mathematical modeling supported elementary learners with critical thinking,

problem solving, and constructing mathematical meaning independently by engaging deeply with the problem context (p. 18). English and Watters (2005) stressed the importance of mathematical modeling for young learners because modeling experiences build foundational skills such as mathematical argumentation, effective communication, and teamwork. In addition to these cognitive and collaborative benefits, modeling is accessible to students with diverse levels of learning (English, 2021; English & Watters, 2005; Wei et al., 2022). Singer (2007) argued that "one of the most important outcomes of a model-building learning environment is the self-development of competencies" (p. 239). He emphasized that these competencies are grade-level specific and essential for building deeper mathematical understanding.

Conclusion to Modeling Benefits for Elementary Learners. Mathematical modeling has been recognized as a valuable instructional approach for elementary learners. It is through modeling tasks that children develop "a grasp of the relationship between aspects of reality and mathematical structures [...] and a belief in the power of mathematics as a sense-making and critical tool" (Greer et al., 2007, p. 97). Overall, the available literature shows that mathematical modeling not only deepens elementary learners' mathematical understanding but also fosters collaboration and their ability to use mathematics as a meaningful tool in their world.

Areas Underexplored in Mathematical Modeling Research. While modeling tasks have been commonly reserved for secondary and higher education students, researchers over the past two decades have increasingly emphasized that young children (elementary and also primary aged learners) have the ability to successfully work through complex modeling problems (Albarracín, 2020; Biembengut, 2007; Brady & Lesh, 2021; Carlson et al., 2016; English, 2006; English, 2021; English & Watters, 2005; Flevares and Schiff, 2013; Greer et al., 2007;

Stankiewicz-Van Der Zanden et al., 2021; Suh et al., 2021). Despite this growing body of evidence, the literature consistently calls for research to increase awareness and understanding of mathematical modeling at the elementary level. For example, in their 2016 review of elementary mathematical modeling, Stohlmann and Albarracín note that while some research exists, mathematical modeling "has mainly been emphasized at the secondary level, but for students to become more adept modellers the elementary grades need to be given more attention" (p. 1). Similarly, English (2021) argues that "the foundations of mathematical modeling do not begin in the high school years – they commence long before then during children's earliest school experiences and even before this time, in their home and community environments" and concludes that the research community must "recognize these children's modeling capabilities and capitalize on them" (p. 18). Thus, a gap exists in the literature regarding mathematical modeling in the elementary grades – particularly in relation to the modeling competencies of these young learners.

In their 2021 review of the literature on mathematical modeling competencies, Cevikbas et al. found that "two studies (3%) dealt with elementary school students (one group focusing on first graders and the other on fifth graders)" (p. 219). Given this lack of research on modeling competencies (English, 2021; Jung & Brand, 2021; Stohlmann & Albarracín, 2016) and the benefits elementary learners experience from modeling, the present study seeks to address this gap by exploring elementary students' engagement in mathematical modeling competencies during a modeling lesson.

Group Work during Mathematical Modeling. With respect to mathematical modeling, many researchers argue that mathematical modeling tasks are, by design, especially appropriate

for small groups (Antonius et al., 2007; English, 2006; English & Watters, 2005; Flevares & Schiff, 2013,). The social dynamics of group work influence the modeling process by increasing students' opportunities for active participation (Featherstone et al., 2011) and by shaping the choices students make regarding tools and the models they construct (Flevares & Schiff, 2013). In elementary classrooms, teachers frequently organize students into small groups when facilitating modeling lessons, as group work can be an effective pedagogical tool for fostering increased student participation and learning in mathematics (Featherstone et al., 2011; Webb et al., 2021). Group work creates opportunities for students to expand and elaborate on the ideas of their group mates and so experience greater learning together than if they work independently (Duckworth, 2001). This collaborative advantage is especially true when students engage in mathematical modeling. As Zawojewski et al., (2003) note, the "skills of the group are potentially broader and more accurate than those of the individuals. The information brought to the [modeling] task is potentially greater, because each individual is tapped for his or her relevant knowledge" (p. 342). In this way, a group of students can bring more mathematical strength to a modeling task than could any one individual.

The benefits of group work are not guaranteed by virtue of organizing students into groups. Teachers must deliberately establish productive classroom norms and expectations to facilitate effective working groups (Flevares & Schiff, 2013; Horn & National Council of Teachers of Mathematics, 2012). Otherwise, "[s]ome students' contributions are sought out and heard, whereas others' contributions are ignored" (Horn & National Council of Teachers of Mathematics, 2012, p. 19). Featherstone et al. (2011) describe this as the "paradox of groupwork": although group work has great capacity to support every student's learning, in

practice it often benefits only some. Without intentional teacher moves, groups may unintentionally develop patterns of inequitable participation, interpersonal conflict, or status hierarchies. Horn (2012) notes that "[w]ho speaks, who stays silent, who is excluded, and who dominates class discussions are all indicators of status" (p. 21). Antonius et al. (2007) also highlight practical challenges with working groups, such as relying on group members who are later absent and unable to contribute to the work, and issues of fairness and cooperation. These dynamics require teacher attention to establish equitable and productive classroom norms and expectations for productive group work (Flevares & Schiff, 2013).

Social interactions are imbedded in students' engagement in modeling competencies and are therefore a necessary factor in examining how students engage in modeling competencies (Brady & Jung, 2021). This emphasis on social interactions aligns with Frejd and Vos (2021), who "propose further developing a social turn in mathematical modelling research" (p. 464). Similarly, Webb et al., (2021) call for greater attention to social interactions during modeling as there is a need to better understand how different patterns of interaction result in more or less productive participation for each group member. In sum, while group work is a frequently used tool during mathematical modeling, its benefits depend on the concomitant social dynamics. For this reason, research on elementary student modeling competencies must account for the ways in which group interactions support or constrain engagement.

Conclusion to the Relevant Literature

Working in groups has the potential to make student models more robust as students hear from one another and perhaps build on one another's ideas. Additionally, in any elementary classroom, there is a limit to how much support a teacher can provide for each individual student.

By organizing students into groups, the teacher sets students up as influencers to each other's thinking and mathematical work – and by extension, the way students engage in mathematical modeling competencies during a modeling task. The literature shows that elementary students frequently engage in mathematical modeling competencies by working through modeling tasks with their groupmates. But working in groups brings attention to the inherent social dimension of group work – and the impact interactions have on student access to practicing modeling competencies. It is with these social interactions that the following theoretical framework of positioning theory is concerned.

Positioning Theory as the Theoretical Framework

Rationale for Using Positioning Theory

As stated in the central and sub-research questions, this research examines the nature of elementary student engagement in mathematical modeling competencies through their interactions and positions during group work conversations. Positioning theory is well-suited as a theoretical framework for this research because student interactions can support or inhibit student opportunities to engage in the group's conversations (Horn & National Council of Teachers of Mathematics, 2012) – and consequently, modeling competencies.

Positioning theory, as described by Davies and Harré (1990), has been increasingly used as a theoretical framework in researching classroom conversation (Kayi-Aydar & Miller, 2018) because it acknowledges that social interactions form the fabric of social reality. Conversation "is the essence of the social world" as it is through conversation that social interactions take place (Van Langenhove, 2021, p. 69). The primary reason to use positioning theory in a social setting is that it attends to how chains of social interactions unfold during conversation by accounting

for the positioning of the conversants (Davies & Harré, 1990). Using positioning theory to understand student engagement in mathematical modeling competencies directly addresses the aims of this research. By tracing the chains of consequences from social interactions during modeling conversations, positioning theory provides a lens for describing and understanding student engagement in modeling competencies.

Central Constructs of Positioning Theory

Overview of Central Constructs.

Conversation unfolds through social interactions (verbal and nonverbal) as interlocutors interpret and give meaning to the interactions occurring between conversants. (Davies & Harré, 1990). Within these interactionally produced conversations, the three central constructs of positioning theory – speech acts, positions, and story lines – are enacted. These three constructs function as a "mutually determining triad" (Harré & Van Langenhove, 1991, p. 406; Van Langenhove, 2021, p. 70). Each is fundamentally related to the other two, making it impossible to discuss one without reference to another (Kayi-Aydar & Miller, 2018; McVee, 2011). Conversations begin when speakers use social interactions to conversationally situate themselves and others through *speech acts*. These speech acts result in the *positioning* of self and others, which may be accepted, adjusted, or resisted. As interlocutors adopt and adapt positions, *story lines* emerge and develop (Harré & Van Langenhove, 1991).

Speech Acts in Conversation.

Speech acts occur through social interactions (verbal and nonverbal) and are "sayings and doings of types defined by reference to their social (illocutionary) force" (Davies & Harré, 1990, p. 45). They are the meanings ascribed to a speaker's verbal and nonverbal forms of communication (Herbel-Eisenmann et al., 2015).

Positions in Conversation. A *position* is a location created within the unfolding conversation, and it arises from the process of *positioning* – how interlocutors situate themselves and one another in the developing conversation (Davies and Harré, 1990). Positioning is observable through social interactions (and subsequent speech acts) occurring between individuals participating in conversation. By adopting a position, individuals take up its accompanying rights and duties and enact it according to their understanding of its expectations (Davies and Harré, 1990). However, the individual "is not inevitably caught" in the expectations, rights, and duties associated with the position (Davies and Harré, 1990, p. 48). Positions may be available, but participants can choose to adopt, reject, or redefine them.

Story Lines in Conversation. The third construct in positioning theory, *story lines,* "are the ongoing repertoires that are already shared culturally, or they can be invented as participants interact" (Herbel-Eisenmann et al., 2015, p. 188). A story line is a living, evolving narrative shaped by the speech acts and positions of the participant which are grounded in social interactions. The positions taken up, negotiated, and discarded by conversants shape the development of the story lines (Harré & Van Langenhove, 1991).

Summary of the Central Constructs. Positioning theory emphasizes social interactions in conversation through the interrelated concepts of speech acts, positions, and storylines. Speech acts are the means by which positions are opened, constrained, taken up, negotiated, or revised. These positions, in turn, both shape and are shaped by the developing story lines. Within a given story line, individuals carry out speech acts in relation to positions, and the availability of positions is interpreted in light of the storylines in play. In this way, speech acts, positions, and

story lines function as mutually constitutive elements of a conversation grounded in social interaction.

Positioning in Relation to Mathematical Modeling Competencies

Positions are fluid "[i]n so far as the content of a position is defined in terms of rights, duties and obligations of speaking, and … are locally and momentarily specified, positions will be unstable" (Harré & Van Langenhove 1991, pp. 404-405). Not only are positions unstable, but positions may give some interlocutors greater access to rights, privileges, power, authority, aspects of identity, engagement in mathematical work – while limiting access for others.

Taking up a position and its associated rights through social interaction is critical. To have access to a conversation, to have the opportunity to develop as a person, one must "have a footing in a social episode" through the rights associated with a position (Harré et al., 2009, p. 8). Access and opportunity to engage in conversation, however, are affected by the position taken up or imposed. Positions may "involve shifts in power, access, or blocking of access, to certain features of claimed or desired identity, and so on" (Davies and Harré,1990, p. 49). This facet of positions is especially salient to this study on student engage in mathematical modeling competencies, because the various positions students adopt during group work can either support or constrain their access to engage in mathematical modeling competencies.

A primary purpose of positioning theory is to highlight social interactions that effectively support or constrain a person, or groups of people, from contribution to the discussion of interest (Kayi-Aydar & Miller, 2018). In the context of modeling competencies, positioning theory is a tool for explaining how interlocutors shift from one position to another, how these positions

affect student opportunity to engage in modeling, and ultimately, student engagement in modeling competencies during conversation.

Relevance of Positioning Theory for This Study

Positioning theory is particularly well-suited to examining elementary student engagement in mathematical modeling competencies during group work. Group work requires the use of social interactions; yet in the process of interacting, students position themselves and one other in ways that influence their access to the modeling conversation – and thus, to modeling competencies. Positions are embedded in social interaction, carry meaning for participants, impact the opportunity for engagement, influence conversational power and authority, and shape students' understanding of themselves and others. Because much of the process of modeling is carried out through group work conversations, positioning theory provides a productive framework for examining student interactions, positions, and their influence on student engagement in modeling competencies.

Closing Discussion: Linking Literature and Theory

This chapter concludes by connecting the reviewed literature and the theoretical framework. As discussed, elementary learners engage with mathematical modeling competencies as they work through the modeling process, often in groups. Group work requires conversation through verbal and nonverbal social interactions, and as students converse, they engage in speech acts, create positions, and develop story lines. In a modeling classroom, speech acts are grounded in mathematical conversation and the meanings students intend to convey. Yet these acts do more

than communicate content; they also signal how students relate to one another. Social positioning and its attendant positions may stem from, and build upon, previously established storylines.

The literature on elementary mathematical modeling underscores the need to examine student engagement in modeling competencies with particular attention to the influence of social interactions during group work. Addressing this need is the central purpose of the present study.

CHAPTER THREE

METHODS AND ANALYSIS

As described in Chapter Two, the central aim of this research is to examine student engagement in modeling competencies with particular attention to the influence of social interactions and positions during group work. Specifically, the purpose of this study is to contribute to understanding how fourth-grade students engage with mathematical modeling competencies during group work, the positions they enact through social interactions, and how those positions influence their engagement in modeling competencies. To address this purpose, this study asks the central research question (CRQ): What is the nature of student engagement in mathematical modeling competencies during group work? To answer this question, I focus on the following sub-research questions (SRQs):

SRQ1: How does student attention to the task context influence their engagement in mathematical modeling competencies?

SRQ2: How do student positions influence their engagement in mathematical modeling competencies?

The balance of this chapter addresses my positionality as a researcher and describes the manner in which the present study addressed these sub-research questions.

Researcher Positionality

The discussion of researcher positionality is especially relevant in a study that applies positioning theory as a theoretical lens. Harré and Van Langenhove (1991) note that a social science use of positioning theory to interpret meaning in social interactions is itself an act of

positioning. As such, the researcher inevitably engages in the practice of positioning through the research process. In particular, I recognize that as a researcher I position myself as observer while simultaneously positioning the study participants as observed. As a researcher, I need to acknowledge the assumptions I carry with me in this study.

My beliefs and assumptions as a researcher shape my research approach. Among these beliefs are philosophical assumptions. *Ontologically*, I acknowledge there exist multiple realities of the same event as perceived by those engaged in the event. This assumption is consistent with positioning theory's focus on individual perspectives of social interactions. *Epistemologically*, I assume that students develop mathematical modeling competencies by practicing them through collaboration during modeling. This assumption is grounded in literature on mathematical modeling competencies and on student engagement in modeling during group work. The assumption also aligns with the use of positioning theory as the theoretical framework because this theory explains how individuals interact with one another. *Axiologically*, I acknowledge that my perception of classroom events is influenced by my own story lines and background knowledge which inevitably influences what I see, understand, and interpret in the data.

My educational background positions me as a learner with limited direct experience with group work during the elementary grades. I experienced mathematics in the form of traditional instruction where students were arranged in straight rows with the teacher lecturing at the blackboard. From these experiences, I am positioned as an outsider to the group-based activities I now observe as a researcher. However, my time in graduate school provided me with relevant experience. During an elementary internship in the Fall of 2022, I spent time interacting with fourth and fifth grade students. I interned with a local mathematics teacher who taught several

classes each day. I observed lessons, interacted with the students while they worked in groups, answered questions, and tried to support their learning. Additionally, part of my training as a graduate research assistant involved watching and analyzing numerous audio-video data files of elementary mathematical modeling lessons. In particular, the opportunities I had to observe fourth grade students working through modeling lessons informed my expectations for how a modeling task might play out in a fourth-grade classroom.

Throughout this research, I am an instrument for data analysis. Student verbal and nonverbal forms of communication that I choose to record and analyze are influenced by my understanding of the literature, the theoretical lens, and by my personal and professional experiences. As a learner of mathematics, I am inherently interested in students' engagement with the subject which certainly influences my decision to pursue research questions in the field of mathematics education.

Research Methodology

This study employs a *qualitative research methodology*, which aligns with my ontological assumption of multiple realities and is motivated by the theoretical framework and also by the literature gap concerning elementary student engagement in mathematical modeling competencies. Specifically, the lens for viewing student interactions during group work, positioning theory, motivates the use of a qualitative methodology as this study aims to describe and interpret student interactions during group work. This methodology provides the structure within which to explore, describe, and better understand the ways in which student interactions shape enacted positions which influence the practice of mathematical modeling competencies. In particular, a *case study approach* is used to address the sub-research questions.

I used a case study approach first, because of the sub-research questions. Each question is explanatory which can be addressed using a case study (Mills & Gay, 2019). The second reason I used a case study approach was because the case in this research was an ideal case for developing an understanding of elementary mathematical modeling competencies as the teacher who provided the case for this study was trained in mathematical modeling and knew how to implement it in her classroom. Finally, there is still a great deal to learn about elementary students and mathematical modeling competencies. A case study approach supports in-depth descriptions in order to build an understanding of elementary students engaged in modeling competencies (Yazan, 2015). Mills and Gay (2019) explain, "case study research is a narrative account that provides the researcher (and reader of the case study) with new insights into the way things are and into the kinds of relationships that exist among participants in the study" (p. 406). Consequently, a case study approach aligned with the theoretical framework allowed for consideration of the interactions (i.e., "the kinds of relationships that exist") between the study participants.

Research Methods

Sampling Methods

In this study, I used a combination of *purposive* and *convenience* sampling to select a case with the ability to generate data through which student engagement in mathematical modeling competencies during group work could be observed. I began with *purposive sampling* by contacting four teachers I knew who had prior experience facilitating mathematical modeling lessons, specifically asking whether they would be willing and able to implement such a task

with their students. From this group of teachers, I used *convenience sampling* by working with the teacher who agreed to participate in the study.

Case Boundaries

The type of case study is *instrumental* as the goal is to better understand the topic of research through the use of a *single case*. Sfard (2008) explains that "a close and detailed picture of one little sample may sometimes be more revealing than a lengthy study with hundreds of participants" (p. 70). The boundaries of the case are delineated by one fourth grade teacher, one fourth grade classroom, one modeling lesson, and group work time throughout the lesson. The *case* is that of students interacting with each other in groups during a modeling lesson. The embedded units of analysis are the student group work conversations. I take group work conversation as the unit of analysis because modeling problems are intended for small-group work (English, 2006), and it is through conversation, the medium of social interaction, that students collaboratively address these problems. The six groups of students who participated in the modeling lesson were not predetermined or organized for the sake of this research. They were organized into their table groups in the manner of any other lesson.

It is important to note that the purpose of this research is to understand student engagement in mathematical modeling competencies and the positions they enact through interactions using the case of group work. Consequently, student groups need to generate data through which the sub-research questions can be addressed. To that end, no student groups were preselected for data collection and analysis as it was impossible to know in advance which student groups would generate the most appropriate data. I collected data for each of the six student groups in the classroom and the decision to analyze two student working groups was

determined during data analysis. Any participant names appearing throughout this research are pseudonyms. The teacher's pseudonym is Ms. Kedrik.

Context of the Study

Participant Demographics.

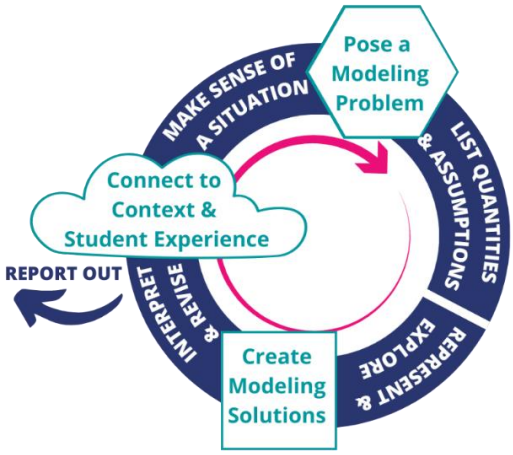
Situated in the Rocky Mountain West, the participants in this study attended an elementary school that was built in 2020. This 68,435 sq. ft. building had a 14.3 student to teacher ratio and encompassed pre-kindergarten through fourth grade students. According to the National Center for Education Statistics the enrolled elementary student population during the most recent 2023-2024 academic year was 431 students – 224 boys and 207 girls. Of the 431 students, 367 were White, 38 were Hispanic, 22 were two or more races, two were American Indian/Alaska Native, one student was Asian, and one student was Black (*National Center for Education Statistics,* 2023). The demographics of this study's participating fourth grade class were consistent with the racial demographics of the school as all but one student was White. Of the 22 students in the fourth-grade class, three were in the gifted and talented program, two students were in special education for reading, and none were English Language Learners. Additionally, Ms. Kedrik shared that the mathematics skills of this group of fourth-grade students resembled those of her classes prior to COVID-19.

Teacher Mathematical Modeling Experience.

Ms. Kedrik was in her 19th year of teaching at the time of this research study. Previously, she had participated in the grant for which I was a graduate research assistant, Advancing Equity and Strengthening Teaching through Elementary Mathematical Modeling (EQ-STEMM). EQ-STEMM was funded by the National Science Foundation and was a design and development project focused on professional development for

K-5 elementary teachers in mathematical modeling. The mathematical modeling process used during EQ-STEMM is shown in Figure 5.

Figure 5. EQ-STEMM Modeling Process (Turner et al., 2025)



Much of my role in EQ-STEMM involved helping facilitate professional development sessions, collecting data from classroom observations, and interviewing teachers. Ms. Kedrik and I met each other in the fall of 2022 during EQ-STEMM professional development. As a graduate research assistant, I interviewed Ms. Kedrik, observed her classroom, and collected data from two of her modeling lessons – which are not the lessons used in this study. Through her participation in the professional development project, Ms. Kedrik learned about and implemented a series of modeling lessons over the 2022-2023 academic year. At the time of this research study she was still involved with EQ-STEMM and was continuing to develop her ability to facilitate mathematical modeling lessons with her fourth-grade students.

Student Mathematical Modeling Experience. Prior to the day of data collection, Ms. Kedrik facilitated a complete modeling lesson with her students – their first experience with mathematical modeling. Aside from this lesson, Ms. Kedrik shared that these students had no

experience posing mathematical questions and working with the ambiguity in mathematical modeling problems. The students' second experience with mathematical modeling occurred during data collection for this study and is the context of this research.

Description of the Jump Rope Task. For this research study, Ms. Kedrik selected a jump rope mathematical modeling task – an upcycling project where students predicted the number of plastic bags they would need to make enough jump ropes for the entire fourth grade at their school. This lesson was Ms. Kedrik's second time leading this particular modeling task. The first time she facilitated the jump rope task was in early March of 2023 with her 2022-2023 fourth grade class. The 2024-2025 academic year was the second time Ms. Kedrik facilitated the jump rope modeling task with her students.

About three weeks prior to the day of the modeling lesson, the students began collecting plastic bags which I then prepared for the lesson. To prepare a plastic bag, I cut off the handles and the bottom of the bag which resulted in a connected loop. While there were a few bags that were smaller or significantly larger, the majority of the bags were approximately one foot by one foot excluding the length of the handles.

In the jump rope task, the overarching modeling question was "How many plastic bags do we need for everyone in the fourth grade to jump rope?" Each jump rope was constructed by braiding three prepared plastic bags together. The handles of the braided jump rope were made by duct taping the ends of the braided rope. The length of a jump rope depended primarily on the size of the plastic bags and how tightly or loosely the bags were braided together. The required length of the jump rope varied depending on the height of the jumper – which then affected the number of plastic bags needed for different sizes of jump ropes.

To answer the modeling question, modelers needed to know the different jump rope sizes or lengths and also the corresponding human heights for each length of jump rope. Modelers needed to determine the number of plastic bags required for each jump rope size; they needed to know the number of students in the fourth grade; they needed to decide how many students for each jump rope length. Finally, the modelers answered the question posed by determining the total number of plastic bags required for everyone in the fourth grade to jump rope.

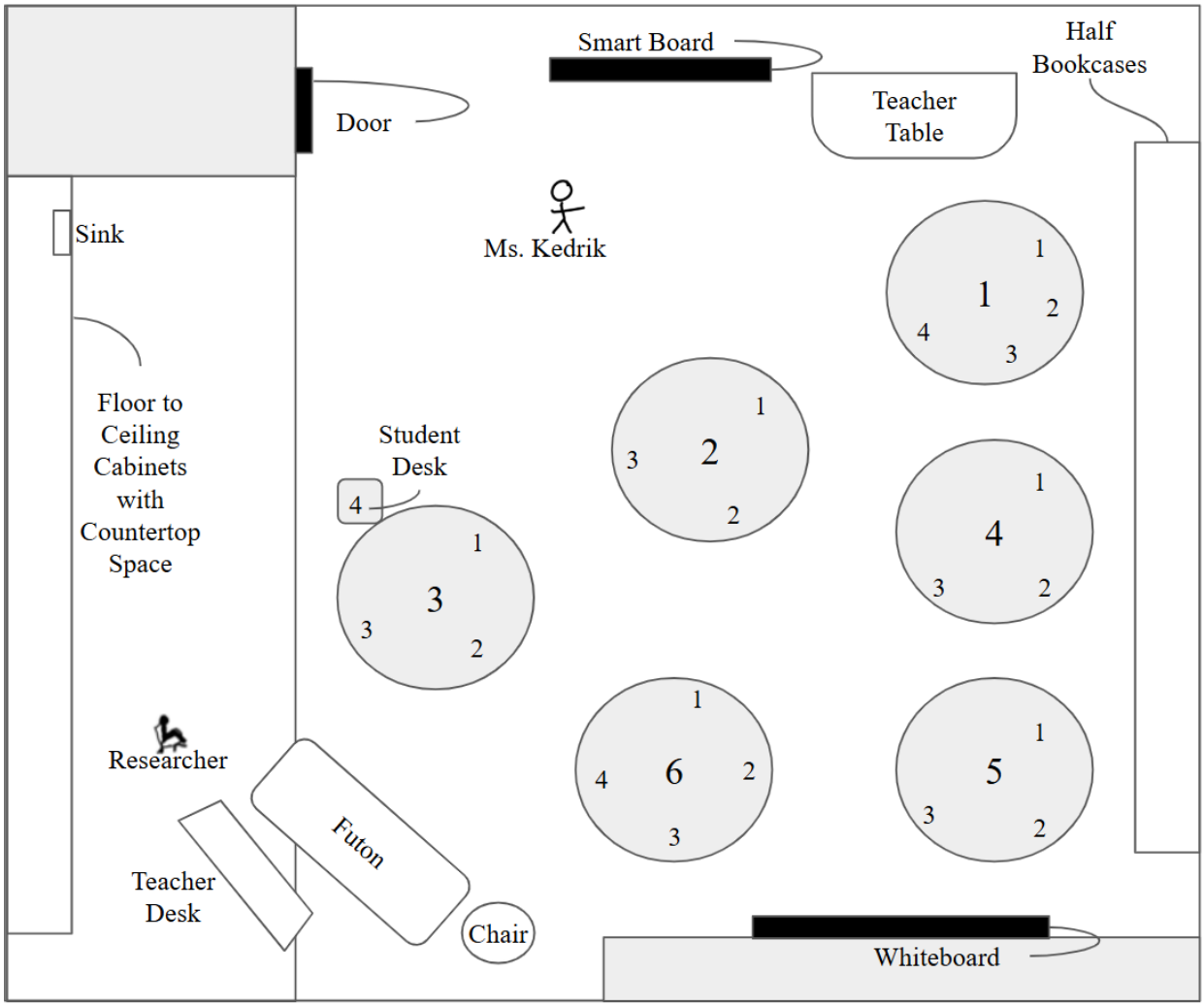
The Day of the Jump Rope Task. The Jump Rope task occurred in mid-October of 2024 from 8:30 am to 10:10 am. I arrived at 8:05 am, just in time for breakfast. Ms. Kedrik was sitting at her desk in the back of the classroom reviewing her slides. She wanted to go over a few things with me regarding the jump rope task as she was a little worried about how it would go. When she facilitated the task in March of 2023 it had been late in the school year and her students had learned multidigit multiplication. She was concerned that it was now only mid-October and her fourth-grade students had very little exposure to multidigit multiplication. I asked if they could do repeated addition. She confirmed that they could. Ms. Kedrik's second concern was that the students would not have enough background knowledge to work through the task. The 2023 fourth grade students had previously completed a jump rope unit in their Physical Education class and knew that various lengths of rope were needed for different heights of children. This 2024 class had not yet done a jump rope unit in Physical Education. We discussed that they might bring up various jump rope lengths with each other in their groups. Even if she only heard one of them say something about jump rope lengths or different heights of students, she could ask them to share with the class. Alternatively, if no student mentioned different heights or

lengths, she could point out different heights of her students and ask the class if those students could use the same rope. She felt better and was ready for the lesson.

Data Collection

On the day of the modeling lesson there were 21 of the 22 students in attendance. The attending students were organized into six table groups of three or four students. The sketch of the fourth-grade classroom in Figure 6 depicts the organization of the students into six table groups. The student at the desk adjacent to Group 3 worked as a member of Group 3 during the lesson. Student locations in their table groups are indicated by the placement of numbers along the circumference of each table and are numbered one through three or four corresponding with the number of students at each table group. My location during the modeling lesson was towards the back of the room by Ms. Kedrik's desk and is depicted and described in Figure 6 as "Researcher." Absent in Figure 6 is the wall of windows at the back of the room behind Ms. Kedrik's desk.

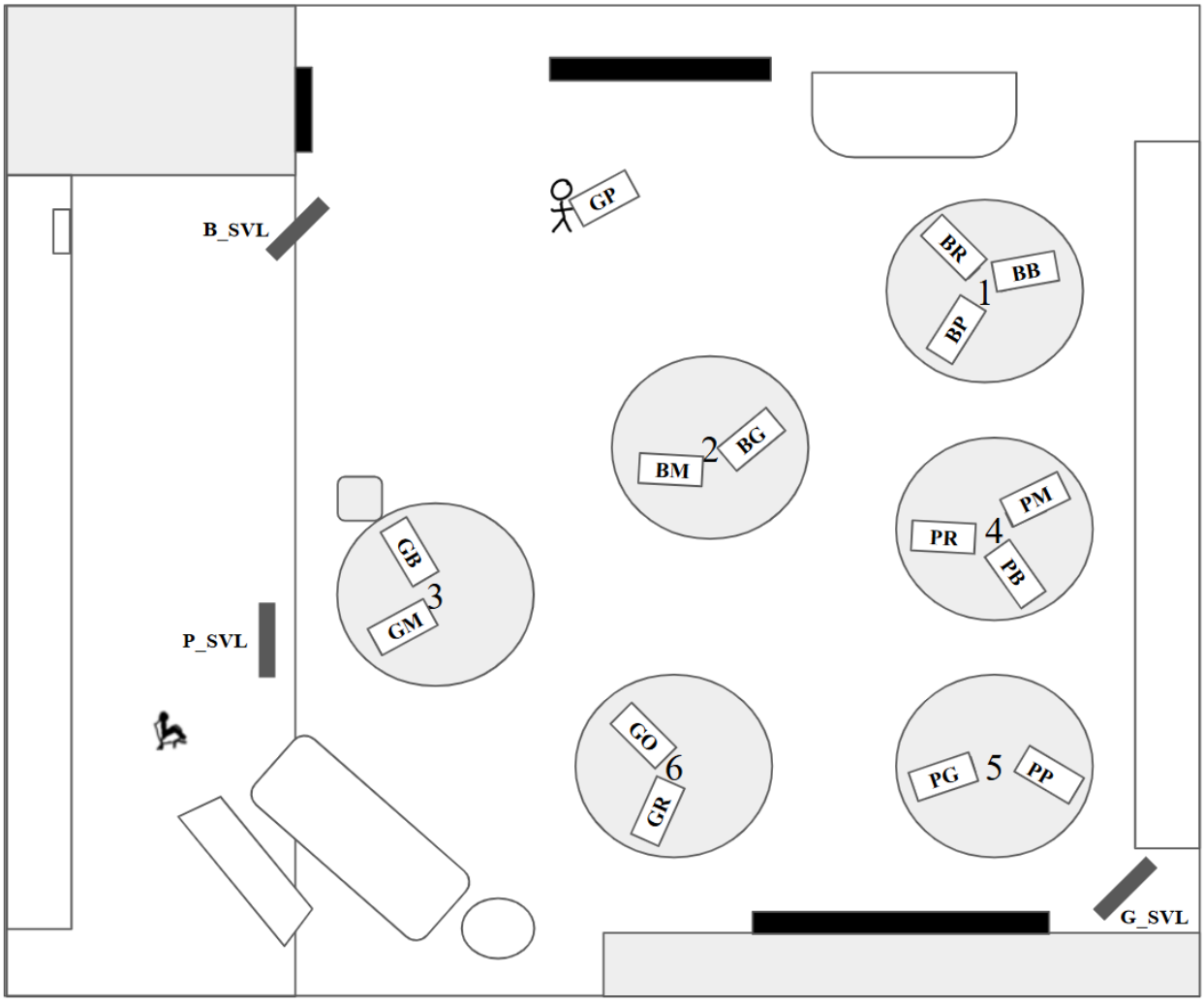
Figure 6. Sketch of the Fourth Grade Classroom



To collect classroom data, I brought three Swivl recording devices and color coded them as green, purple, and blue. I triangularly positioned the three Swivls throughout the classroom and in this way was able to capture audio and video for each student group as each Swivl focused on at least two student groups. The green (G), purple (P), and blue (B) Swivl (SVL) recording devices are indicated by G_SVL, P_SVL, and B_SVL, respectively. Additionally, each Swivl had five associated audio markers. I placed these audio markers by twos and threes on student tables and Ms. Kedrik wore one during the lesson. The sketch of the classroom in Figure 7

depicts the placement of the Swivl video recording devices and their associated audio markers. To indicate which Swivl an audio marker is linked with, I denote the color code of the Swivl device followed by the color of the audio marker. For example, the purple audio marker for the blue Swivl is marked as BP – as depicted on Group 1 in Figure 7.

Figure 7. Placement of the Swivl Audio and Video Recording Devices



Researcher Participation. For three consecutive weeks prior to the jump rope modeling lesson, I acted as a nonparticipant observer during the students' morning lesson. Once, the lesson

took place in another room in the building and I sat and observed from the back of the room. Twice the lesson took place in their regular classroom. For these two lessons I sat and observed from the same place as I did during the jump rope modeling lesson. By the time of the jump rope modeling task, the students knew I would be observing, where I would be sitting, and that I would not be participating in their lesson. This was done as an effort to minimize any effect my presence may have had on student participation during the time of the jump rope task.

For this study, I acted as a *nonparticipant observer* and did "not interact or participate in the life of the setting under study" (Mills & Gay, 2019, p. 550). The day of the mathematical modeling task I was on site to set up data collection devices, observe the case, and record field notes. I sat next to Ms. Kedrik's desk towards the back of the classroom occasionally checking that the three Swivls (Blue, Purple, and Green) were recording throughout the lesson. Once, during silent writing time for the students, I adjusted one of Group 3's audio markers to face it more towards the student seated at the desk (as depicted in Figure 7).

Data Types

This section describes the research data and their alignment with the sub-research questions. I used two types of data in this study – first-order data and second-order data. The former consisted of original data from the field and includes audiovisual (AV) data, student work and field notes. I generated second-order data from first-order data and included jottings, analytic memos, content logs, and transcripts.

First-Order Data: Raw Materials from the Field. First-order data consisted of the raw materials I collected from the field. For this case, it was AV data, student work, and field notes.

First-Order Data: Audiovisual Recordings. I collected AV data for the duration of the Jump Rope modeling lesson to contextualize the case study and capture external factors that influenced group work conversations. The use of positioning theory as the theoretical framework motivated my decision to record the entire modeling lesson as contextual information for group work conversations. Positioning theory addresses what happens in the moment, rather than after the fact (as in the case with interviews) and because of this contextual information surrounding the focus of the study can be collected and analyzed (Wood, 2013; Kayi-Aydar and Miller, 2018).

Positioning theory in this research supports "the analysis of the dynamics of person-to-person encounters" (Harré et al., 2009, p. 6). Erickson (2006) explains, "fine-grained information about the actual conduct of social interaction comes best from making audiovisual recordings of it from which either detailed transcriptions of the interaction can be prepared and analyzed or careful moment-by-moment coding can be done" (p. 77). Based on these principles, I collected AV data focusing on student groups to capture a comprehensive picture of student verbal and nonverbal interactions.

First-Order Data: Student Work. I collected student group work from each table to triangulate with group work conversations. Student work is an "aspect of the mathematical [modeling] task" that may be associated with group members' speech acts, positions, and story lines during modeling (Wood, 2013, p. 780). The student work considered alongside the AV data supports understanding the content of student conversations but also sheds light on some of their gestures – for instance, students pointing at each other's papers during conversation.

First-Order Data: Field Notes. Finally, I recorded field notes during and immediately after the lesson to collect my observations, thoughts, and comments related to the modeling

episode. In particular, I recorded the events of the day, including the sequence of the modeling lesson, a sketch of the classroom layout (number of student desks, students per desk, and the number of boys and girls in each group), placement of the AV recording devices, and conversations before and after the lesson – such as when I arrived and when I received consent forms. These notes provided contextual information and helped triangulate findings across data sources.

Second-Order Data: Products of Interpretation and Analysis. After collecting first-order data in the field, I developed second-order data through analysis of first-order data. Second-order data includes jottings, analytic memos, content logs, and transcripts.

Second-Order Data: Jottings and Analytic Memos. I recorded jottings and analytic memos throughout the duration of the study. These records provide an electronic paper trail of my thinking throughout the duration of the study. In particular, analytic memos contain my observations resulting from analysis and also provide a detailed record for the analysis process. For instance, after writing the content log of the modeling lesson, I recorded the memo displayed in Table 3 regarding whole class engagement in the modeling task.

Table 3. Sample Analytic Memo for Whole Class Engagement

Whole Class Level - Modeling
Ms. Kedrik created a situation for students to experience creating part of a jump rope by teaching them how to braid and by giving them time in class to braid three plastic bags together. She helped students use this experience as mathematical information. That is, Ms. Kedrik supported her students to represent their rope mathematically by asking them to measure its length. All tables reported the length of their rope to the class during whole class discussion. The most common rope length was one foot, so the students made the assumption that three plastic bags would braid into a one-foot jump rope. Collectively, the students used their personal knowledge about their own heights to determine the lengths of jump rope they would need. They stood back-to-back with each other comparing heights and used this information to estimate how many people would need a seven-foot rope, and how many would need an 8' jump rope. They eliminated ropes shorter than seven feet because they knew no one in fourth grade was shorter than 4'6". And they knew this because the shortest person in fourth grade is Johnny and he is 4'7". They eliminated any ropes longer than eight feet because they knew that no one in the fourth grade was taller than Ms. Kedrik – who is not over 5'5". Jump rope lengths and heights: Up to 4'6" tall then jump rope is 7' long. Between 4'6" and 5'5" then the rope is 8' long. Over 5'5" then the jump rope is 9' long.

Second-Order Data: Content Logs. Content logs are time-stamped documents describing the events of the modeling lesson. I developed two kinds of content logs, lesson-level content logs and group-level content logs.

The *lesson-level content log* (Table B2 in Appendix B) describes what occurred during the lesson generally, within student work groups specifically, and any relevant information from field notes (Erickson, 2006). It ties together all first-order data sources (general classroom AV data, group work AV data, field notes, and student work) and so provides a comprehensive overview of first-order data. Table 5 includes an excerpt from the lesson-level content log which includes time stamps for each Swivl color. The green, purple, and blue Swivl recording devices are indicated by G_SVL, P_SVL, and B_SVL, respectively. Each student table group (G) is denoted by G1, G3, G4, or G5 depending on the table number the student group is situated at. For example, consider G4_P_SVL from the sample content log in Table 4. G4 indicates Group 4. P_SVL indicates the audio markers were associated with the purple Swivl.

Table 4. Sample Lesson-Level Content Log

Time			**Description of Events**	
G_SVL	*P_SVL*	*B_SVL*		
0:00	0:00	0:00	Ms. Kedrik reminds them that they are going to make jump ropes. Recaps by showing them how to braid the ropes. I am finishing up with setting up the technology. Ms. Kedrik wears an audio marker too. A student asks about the audio marker on the table. Ms. Kedrik checks that there is at least one person at each table who knows how to braid. Shows them how to braid.	
…	…	…	…	
6:45 **Group Work**	6:30	6:07	"What do you know about jump ropes?" Talk in tables, what are some things they know. Group conversation	
G1_B_SVL		6:08-6:38	They're fun, you can do a lot of different challenges with them, can do it with a big, long piece of cording, two jump ropes for twice as fast, not twice as fast just twice as much.	
G3_G_SVL		6:46-7:17	Ropes are normally made of rope, plastic, or fiber glass or something like that, like climbing rope. Like the rope of horses.	
G4_P_SVL		6:33-7:03	They're fun, can do tricks on them, can have two people, can play with friends or by yourself.	
G5_P_SVL		6:33-7:03	They're fun, you jump like this, good way to exercise your body, lots of things to use a jump rope for, tug of war.	

Two of the 21 students present during data collection did not have guardian consent to participate in the study. These students were seated in Groups 2 and 6 so I did not include data from these two groups in the lesson-level content log. I further structured the lesson-level content log by organizing it into named sections, see Table 5.

Table 5. Named Sections of the Modeling Lesson

Time Length	Name of Section
4 minutes 25seconds	Introducing the lesson
2 minutes 20 seconds	What do you know about plastic bags?
3 minutes 30 seconds	What do you know about jump ropes?
6 minutes 55 seconds	What do you notice, what do you wonder?
9 minutes 50 seconds	What math questions can you ask?
3 minutes	Posing the modeling problem
10 minutes 25 seconds	Braiding plastic bags
7 minutes 50 seconds	Measuring the jump rope
14 minutes 10 seconds	What do you know about making jump ropes?
6 minutes 5 seconds	Estimating the number of plastic bags they will need
17 minutes 45 seconds	Building models to answer the question
14 minutes 25 seconds	Reporting out on models and solutions

The section names reflect what occurred during the lesson and so provide an outline of the modeling lesson. This outline also demonstrates that the lesson enacted in this study was indeed a modeling lesson. As discussed in Chapter Two, modeling involves translating between the real and mathematical worlds by mathematizing from the real to the mathematical and interpreting mathematics back within the real-world context. Prior research has shown that, in elementary classrooms, mathematizing and interpreting frequently occur as students move between these two worlds while addressing a modeling problem (Fulton, 2017). The same process was evident in this study's jump rope lesson, as reflected in Table 6.

Table 6. Classifying the Sections of the Lesson

Classification	Name of Section
real world	Introducing the lesson
	What do you know about plastic bags?
	What do you know about jump ropes?
	What do you notice, what do you wonder?
mathematizing	What math questions can you ask?
	Posing the modeling problem
real world	Braiding plastic bags
mathematizing	Measuring the jump rope
real world	What do you know about making jump ropes?
mathematical world	Estimating the number of plastic bags they will need
	Building models to answer the question
interpreting	Reporting out on models and solutions

After constructing the lesson-level content log of the modeling lesson, I created a more detailed *group-level content log* (Table B3 in Appendix B) for each student table group (excluding groups 2 and 6). The group-level content log is distinct from the first content log as it includes only those parts of the lesson where Ms. Kedrik gave her students time to work together in their groups. The purpose of the group-level content log was to provide a more focused look into each group's conversations during collaborative time. I used the group-level content log to begin the process of selecting lesson segments for transcription. Table 7 includes an excerpt from the group-level content log.

Table 7. Excerpt from the Group-Level Content Log

Building models to answer the question			
1:08:07 **Group Work**	1:07:53	1:07:29	"Ok. Alright, so..." They are going to do their work. Teacher recaps info and what they need to do. They understand what they are doing and begin working in groups. Kedrik visits tables as they work.
T1_B_SVL		1:08:03-1:16:25	First we need to figure out, yeah. Owen, put that in the middle. Figure out how many in a 7' rope. Don't do that, not good for you, you're eating lead. Not good for you. Ok then get sick, I don't care it's your fault. Yay, yay. Key is bags for foot. Idea, how many kids in our class. We know. Then count by the kids in our class to the other classes. Is that a good idea or not? I don't know. you can figure it out. it's a good idea but hard to figure out. How many in this class right now. They discuss numbers in the classes. They add them up. They got 65. Then 65*30. They work it out. 65+10, or add 65 ten times which would be harder. or add 65, 30 times. It's 75. That is not 75. We're doing 65*10. not plus 10. we're breaking it up. I have an idea. It's 950. Wait, no b/c 65*10=650. Adding, 1950 would be the answer. They have the answer. Discussion about how they got there. They all record it on their papers. They compare with the estimate they made at the beginning of class. They were close. Make sure to write bags. Height comparison with other students. Basketball comes up.

Second-Order Data: Transcripts. The final form of second-order data is transcripts (Appendix C). I generated these written records of group discussions from AV recordings and organized them into tables by assigning a separate column to each student. The table structure allowed me to look across a row and understand the simultaneous interactions of group members. The development of student transcripts is detailed in the data analysis section.

Data Management and Confidentiality

To protect the identities of the participants, I followed IRB-compliant processes for data storage and retrieval. All first-order data were downloaded from the Swivl collection technology and saved to my computer's hard drive and also uploaded to OneDrive. I uploaded AV data and copies of student work to the OneDrive drive for storage and future reference. I shredded the

hard copies of student work. I also saved second-order data, as well as jottings and analytic memos to my OneDrive account. I used Shotcut, a video editing program, to create AV data for transcription. This AV data was uploaded to a transcription company, Rev, to generate preliminary transcripts. Once I finalized the transcript data, I deidentified the data by assigning pseudonyms to the participants. After this point, all subsequent memos and jottings were also deidentified. I uploaded second-order data (such as deidentified transcripts and analytic memos) to my desktop version of NVivo which I used during with data analysis. Swivl, OneDrive, Rev, NVivo are all password protected. Shotcut is local to my computer's hard drive and so also password protected. As stated earlier, to protect the identity of the research participants, I deidentified all data appearing in this research by assigning pseudonyms to all research participants.

Analytical Approach to Data Analysis

Overview of the Analytical Approach.

The data analysis in this research followed a First and Second Cycle Coding approach. After collecting first-order data, I generated second-order data using preliminary analysis methods in order to create transcripts (second-order data) from student conversations during group work.

I began with holistic coding, a method often used to prepare data for more detailed first- and second-cycle coding and particularly useful when investigating specific concepts (Miles et al., 2020). Following this, I applied first cycle coding to transcripts, student work, and AV data. I then conducted second cycle coding to address the sub-research questions through analysis of first cycle coding results. Throughout the coding process, I recorded jottings to capture my

thoughts and impressions and used them to inform my analytic memos which supported the development of findings.

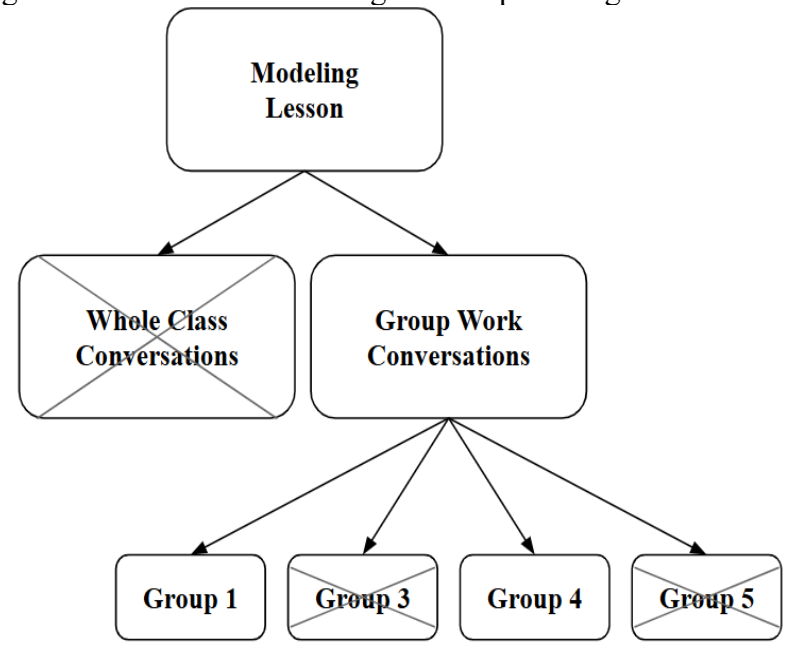
Holistic Coding: Preparing the Data. Guided by the research questions and the theoretical framework, I conducted preliminary analysis of all group conversations through holistic coding by initially considering larger units of data across the modeling lesson for the purpose of identifying parts of student group conversations to transcribe. I did not transcribe the entirety of each student group's conversation as I transcribed "only those strips that contain phenomena of research interest" (Erickson, 2006, p. 184). In this study, the *general* requirement for transcribing conversation segments was that students interacted with each other in groups (motivated by positioning theory) while working on the mathematical modeling task (motivated by mathematical modeling literature). More specifically, I identified group work conversation segments for transcription when the segment met one or both of the following criteria:

C1: student interactions relate to the context of the task
C2: student interactions reference the mathematics of the task

"Interactions" are defined as verbal and nonverbal interactions occurring between two or more students during group work. The "context of the task" concerns making jump ropes from plastic bags. What does not count as "context of the task" is a general discussion about plastic bags or jump ropes with no connection to the problem at hand. An example of what does not meet the first criterion is a comment one student made about the cost of plastic bags in Washington D.C. Another example from a student is the comment that his dog is afraid of plastic bags. While both of these students' comments are connected to plastic bags generally, they do not relate to the task of determining how many plastic bags are needed to make jump ropes for the entire fourth grade class.

Transcript Selection Process: Overview. In selecting group conversation segments to transcribe I applied the transcription criteria across the modeling lesson as a whole and then within student groups specifically. Figure 8 depicts the general process of selecting transcription segments.

Figure 8. Overview of Selecting Transcription Segments



The process of selecting segments of the AV recordings to transcribe involved looking across the whole modeling lesson and narrowing my focus to group work conversations. I then considered the conversations occurring within groups 1, 3, 4, and 5. I removed Groups 3 and 5 for transcription primarily due to impaired AV data. Within the two remaining student groups, Group 1 and Group 4, I applied the transcription criteria and identified those segments where student interactions met at least one of the criteria. While identifying lesson segments to transcribe, I also marked segments which would not be included in the transcript. Table 8 provides an overview of the segments of the lesson I did not consider for transcription along with a brief justification.

Table 8. Lesson Segments Eliminated from Transcription

Across the Modeling Lesson	
Eliminated	***Justification***
Whole Class Time	Student interactions are primarily between students and teacher
Silent Independent Thinking Time	Students are not interacting with each other
Across Student Groups	
Eliminated	***Justification***
Group 3	Impaired AV data, especially for the student at the adjoining desk
Group 5	Impaired AV data with squeaky audio markers, obscured visual, and significant time spent with Ms. Kedrik
Within Group 1 and Group 4	
Eliminated	***Justification***
Braiding Plastic Bags Measuring the Jump Rope Height Comparisons	Inconsistent AV data as students move away from their tables and interact with classmates from other groups
Teacher with Student Groups	Interactions are between Ms. Kedrik and the student(s)
What do you know about plastic bags? What do you know about jump ropes?	General prompt not connected to the context of the task and student responses reflected this
Students do not interact with each other Interactions do not relate to the task context or mathematics	Group interactions do not meet either of the transcription criteria.

In the sections that follow, I detail the process of selecting and eliminating lesson segments for transcription and include justification for these choices.

My first step was to select segments to transcribe from *across the modeling lesson*. To begin, I reviewed the content log of the entire modeling lesson by marking the times during the task when students were engaged in group conversations. By noting these instances, I also noted when students were engaged in whole class conversation – and thus, not engaged in group conversations. I removed whole class conversations from consideration for transcription as students were not interacting with each other in their groups during these times.

In my second step, I looked *across student groups* and eliminated Groups 3 and 5 from transcription. Group 3 was excluded due to a consistently inaudible student, and Group 5 due to impaired audio and visual data as well as limited student-to-student interactions resulting from Ms. Kedrik's frequent assistance.

After these eliminations, Group 1 and Group 4 remained. The third and last step I took was applying the transcription criteria *within Group 1 and Group 4*. Recall, the transcription criteria that student group work conversations meet one or both of the following criteria:

C1: student interactions relate to the context of the task
C2: student interactions reference the mathematics of the task

Using the AV data alongside the group work content log, I applied the transcription criteria to Group 1 and Group 4 and recorded a justification for whether each time segment met one or both criteria. Table 9 is an excerpt from Group 1 depicting either the reasoning for or against transcription of the indicated segment.

Table 9. Excerpt from Group 1 Conversation Content Log

G1_B_SVL	Description	Reasoning
1:09:00-1:11:06 126 sec	Everet - alright so Owen - yay Everet - alright so we're going to need, Wren - Key is bags for foot. Owen - Idea, how many kids in our class. Wren - We know. Eric - we already know Owen - Then count by the kids in our class to the other classes. Is that a good idea or not? Everet - I don't know. you can figure it out yourself. No, it's a good idea but hard to figure out. Owen - How many in this class right now. Everet, Owen, and Eric - discuss numbers in the classes. And which classes have how many students. Eric - we can add them up. easy. Everet and 4 - add them up. They got 65. Eric - does it, wait, let's see. Reworks it. 67. Everet - hey, two off not bad. (I wonder if they were comparing to their estimate of 65)	Transcribe – Bags per foot Kids in this class Number of classes Students in each class Estimate comparison
1:11:06-1:11:32 26 sec	That looks like lulu doesn't it. Looks like you too. I've seen that meme – looking at a cup.	Do not transcribe – unrelated to C1 or C2
1:11:32-1:13:35 123 sec	Everet - alright so, now we've got to do 65*30. Eric - I don't know what that is, just kidding. Everet, Owen, and 4 discuss math strategies. 65+10, or add 65 ten times which would be harder. or add 65, 30 times. It's 75. That is not 75. We're doing 65*10. not plus 10. we're breaking it up. Wren - is working silently on her paper. Everet, Owen, Eric - working I have an idea. It's 950. Wait, no because 65*10=650. Attach a zero at the end. 1,950 would be our answer. Wren - so like 65*10, so 65*10, 65*10. The boys keep talking over her the first two times she states her request. Owen - attach a zero at the end.	Transcribe – Math strategies Math computations
1:13:35-1:14:03 28 sec	Discussion about an explosion, someone sounds like a grandma.	Do not transcribe – unrelated to C1 or C2

The following paragraphs provide a few instances of student group work conversations that did not meet the transcription criteria and were therefore excluded from the final transcripts.

I did not transcribe the “Braiding plastic bags” and “Measuring the jump rope” segments, as students stood away from their tables to braid or moved around the room visiting other groups. For similar reasons, I also excluded the times when students walked away from their tables to compare heights with each other. Segments in which Ms. Kedrik stopped by a group were likewise omitted, since the interactions were primarily teacher–student rather than student–student. Although these portions were not transcribed, they were considered during analysis, as transcripts were not examined in isolation from the full modeling lesson.

I also excluded the “What do you know about plastic bags?” and “What do you know about jump ropes?” lesson sections. At that stage, students had not yet been introduced to the modeling problem, so their responses did not address the task’s context as required by the transcription criteria.

After systematically applying these transcription criteria and eliminating segments that did not meet them, I identified group work conversations that were to be transcribed. These segments came from the sections of the modeling lesson listed in Table 10.

Table 10. Lesson Segments where Conversations were Transcribed

Time Length	Name of Section
9 minutes 50 seconds	What math questions can you ask?
14 minutes 10 seconds	What do you know about making jump ropes?
6 minutes 5 seconds	Estimating the number of plastic bags they will need
17 minutes 45 seconds	Building models to answer the question

Preliminary Group Conversation Transcripts. After finalizing the conversation segments selected for transcription, I uploaded the AV data from Group 1 and Group 4 to a video editing program, Shotcut. Using Shotcut, I created two edited videos – one for each group – that

included only those conversation segments marked for transcription. I also adjusted each video to focus exclusively on the group whose conversation was to be transcribed.

I then submitted both videos to Rev for human transcription. Rev is a transcription company with a 99% accuracy rating for their human transcription services. I received the completed preliminary transcripts later that same day. I reviewed the transcripts by comparing them against the AV data and made corrections to ensure the speakers and their words were accurately recorded. There were instances where it was impossible to distinguish some of the words and these instances were noted in the transcripts – both by Rev and myself. Table 11 provides a sample from a preliminary transcript.

Table 11. Preliminary Transcript Sample Conversation

Owen: Yay, yay. Everet: ... 21 bags for seven feet rope. Seven-foot rope, all right, 28 bags for- Wren: 21 bags times 35 equals 1825. And the key is one bag equals one foot. Everet: I think with eight-foot rope... Owen: I have an idea, I have an idea. Everet: Ah. (screech sound) Eric: What was that? Owen: I have an idea. We need to figure out how many kids are in our class and then we could- Wren: We already know. Eric: We already know. Owen: Then, um, then you can count, um, by those kids in our class, all to the other classes. Everet: Ah. (screech) Eric: Whoa, okay (laughs). Owen: Is that a good idea or no? Wren: Guys, we need to stay on focus. Everet: Um, I don't know if that's a good idea. You figure that out yourself. So, well- Eric: Johnny. Everet: ... it would, it's a good idea, it's just kind of hard to figure out how many are in the other classes. Owen: How many are in this class right now? Eric: 24. Everet: 22. Eric: No I'm just kidding, just kidding.

Final Group Conversation Transcripts. While the preliminary transcripts included the conversants words, they did not include overlapping speech between any of the speakers and also lacked nonverbal forms of communication.

I first addressed the former issue by reorganizing the transcript records into a table, assigning one column to each student within a group. I then studied the preliminary transcripts in tandem with the AV recordings, analyzing the AV data moment by moment. When speakers overlapped, I documented their speech in the same row to accurately portray simultaneous dialogue. I also used numbered turns to indicate a shift between speakers, with overlapping words and actions recorded within the same turn. I then included nonverbal communication between students.

The inclusion of nonverbal forms of communication in the transcripts is demanded by the use of positioning theory as the theoretical lens. Even within studies using positioning theory, there is a need for research "that give[s] more fine-grained analytic attention to micro-level positioning features such as intonation, pausing, body movement and gestures – moving beyond consideration only of participants' words" (Kayi-Aydar & Miller, 2018, p. 91). I included nonverbal forms of communication in the transcript when they were part of a student's interaction with another student – such as, pointing at a paper, the use of hand gestures to emphasize words, writing, leaning across the table, changing seats to be nearer a group member, etc. Nonverbal gestures that I did *not include* were those that were not part of *and* did not become part of the student's interaction with another student. For instance, I did not transcribe Everet coughing into his elbow.

Lastly, I marked the transcript according to the transcript codebook in Table 12. The transcript codebook was adapted from the Jefferson transcription system and developed by my chair and myself by applying the system to EQ-STEMM project data.

Table 12. Transcript Codebook

Rule #	Notation	Rule	Example
1	.,?! … -	Standard use of English punctuation	But, I think we should round at least to like 300 maybe? (Lydia)
2	Underlining	Increased volume or emphasis	No, it's a guess, guys. Guys, it's a guess. (Owen)
3	(2 seconds)	A timed pause	25, so one dollar. (5 seconds) Three dollars equals eight. (Lydia)
4	sooo	Repeat letter(s) for stretched sound	Three dollllllarrrrrrssssss. (Lydia)
5	↑	Rising intonation (uparrow)	*[leaning towards Everet]* I ↑ went for 400 ↑. (Owen)
6	↓	Dropping intonation (darrow)	We don't do the math just yet ↓. (Owen)
7	[clarification]	Insert clarifying content	This [ruler] helps me think. (Owen)
8	*[description]*	Insert descriptions of verbal and nonverbal communications	No, not anymore. It, it, wait. Okay. So it's *[flips her paper around so Jason can read it from his view. Explains and demonstrates with the paper while speaking]* 17 times 24, which I would equal 408. (Lydia)
9	[undecipherable: "option(s)"]	Indicate when parts of the transcript are undecipherable, if able, include option(s) for transcript	Yeah [undecipherable: "yup"]. (Everet) It's diffi [undecipherable: "diffint" or "different"] (Eric) Not it, well, if you did your estimate, right? *[quietly]* If you did your work ↓ [undecipherable]. (Everet)

After marking the transcripts and including nonverbal communication, I finalized the transcripts for Group 1 and Group 4 by situating the transcribed content within the context of the modeling lesson. See Table 13 by way of example.

Table 13. Example of Final Transcript

Class regroup. Ms. Kedrik asks about a reasonable estimate. Asks about bags in a whole rope. Student volunteers 30, their estimates are 1,800 – friendly number. Ms. Kedrik shares thinking of a student 24 bags in an 8 foot rope, then 240 for 4th grade, but that's only 10 so using that number to come up with an estimate. Would 20k be reasonable? No. Be sure numbers are reasonable. 900, 3000. Different numbers. 450 bags. About 25 bags in a rope. Think quarters.				
"Ok. Alright, so..." They are going to do their work. Teacher recaps info and what they need to do. They understand what they are doing and begin working in groups.				
Turn	**Everet**	**Owen**	**Wren**	**Eric**
117	*[facing Owen]* Alright, so we need to figure out. Ok stop messing around *[takes away Owen's pencil, looks away from Owen.]*	*[pointing his pencil at Everet and running his finger up and down the pencil in Everet's direction]*	*[sitting facing the table, with her pencil in her left hand, elbow on the table, left hand up to her face. Facing Everet and Owen]*	
118	*[Gives the pencil back to Owen by putting it on the table in front of Owen]*			Yeah, we got it last year.
119		*[putting the ruler to his lips]*		*[chuckles]* It's diffi [undecipherable: "diffint" or "different"] -... *[looking towards Owen]* All right, Henry, put the ruler down.
120	All right, so-			
121		*[putting the ruler in the center of the table]* This [ruler] helps me think.		
122	No, it doesn't.			*[places the ruler on the supplies situated to his left and at the end of the table]*

Ethical Considerations

The primary ethical concern was parental consent. I obtained consent for the students in the groups I selected for analysis, and I secured a signed consent form from the teacher. Both the teacher and the student consent forms were IRB forms approved by the university (Appendix A).

Additionally, I secured participant privacy, confidentiality, and anonymity were protected by storing all data in a password-protected OneDrive folder, to which I was the only one with access. I protect participant identity by using pseudonyms in the research report, anonymized identifying information in the transcripts, and by ensuring that no identifying features of any participants are included in the final research report.

Analysis of the Data

Overview of Data Analysis

Data sources for analysis consist of first-order data (AV recordings, student work samples, field notes) and second order data (content logs, transcripts, analytic memos and jottings). I created two kinds of content logs. A lesson-level content log describing what occurred over the course of the entire modeling lesson and within each of the student groups. The lesson-level content log also contained student work samples. The second content log was a group-level content log focusing specifically on group work conversations. Using holistic coding I then developed transcripts of group work conversations from Group 1 and Group 4 when the conversation met one or both of the following criteria:

C1: student interactions relate to the context of the task
C2: student interactions reference the mathematics of the task

Recall that *interactions* are defined as verbal and nonverbal interactions occurring between two or more students during group work, and the *context of the task* concerns making jump ropes from plastic bags.

I engaged in data analysis through first and second cycle coding methods. During first cycle coding, I applied two qualitative coding methods: process coding and concept coding.

Process coding uses gerunds, process codes, to identify observable actions in the transcript data, particularly student interactions. These interactions were relevant to the construction of student positions. During concept coding, I applied concept codes to capture the underlying concept or idea for the interactions occurring in the moment. Specifically, I used concept codes to document student positions and mathematical modeling competencies. Taken together, my use of process and concept coding allowed me to investigate student conversations, focusing on positions and engagement in modeling competencies and their underlying interactions.

First and Second Cycle Coding. After generating transcript data through holistic coding, I applied first and second cycle coding methods to the transcripts. These coding methods align with the theoretical framework of positioning theory. Positioning theory emphasizes how the interactions between people engaged in a conversation contribute to the positions enacted by each of the conversation members. It is through these social interactions that participants engage in positioning themselves and others by reinforcing, negotiating, or accepting positions over the course of the conversational episode. Because the process of positioning (and resulting positions) is built by the moment-by-moment interactions, my coding focused on capturing student interactions as they unfolded over the course of the modeling task. Because the present study focuses on student interactions during modeling work, my coding also focused on the content of student conversations.

I used second cycle coding to address the central research question: *What is the nature of student engagement in mathematical modeling competencies during group work?* I developed second cycle codes by synthesizing first cycle codes into broader category codes which I used to identify findings from the data. In particular, I traced the evolution of student positions over the

course of the modeling lesson and studied the modeling competencies students displayed in relation to positioning. In other words, I addressed the overarching research question through the two sub-research questions: *How does student attention to the task context influence their engagement in mathematical modeling competencies? How do student positions influence student engagement in mathematical modeling competencies?* In addressing these questions, I examined the modeling competencies students evidenced within the social context of group work, and I considered how student engagement in these modeling competencies was shaped by student positions through their interactions.

Codebook Development. To manage the iterative development of the first and second cycle codebooks, I created and maintained a Microsoft Excel file where I recorded code definitions, examples, and notes. The codebooks were developed through iterative cycles of analysis. The development process was characterized by continuous comparison between the codes and the data to revise the codebook. I completed these iterative coding cycles until a stable codebook emerged. Throughout the coding process I documented my thinking through jottings and analytic memos. These analytic artifacts supported my understanding of the data, as well as the development and refinement of the codebooks. During the coding process, I discussed emerging codes with members of my dissertation committee, who provided feedback that informed the development and refinement of the codebooks. The following sections detail the process of first and second cycle coding.

First Cycle Coding

During first cycle coding I applied both process codes and concept codes to the transcripts from Group 1 and Group 4 student conversations. Process codes captured observable

student interactions which were relevant to building an understanding of student positioning. Concept codes captured the underlying ideas that emerged through these interactions – mathematical modeling competencies and enacted positions.

Because mathematical modeling competencies are explicitly stated in the literature, I used a priori concept codes and operationalized definitions when coding for modeling competencies. Recall the modeling competency framework from Chapter 2:

(1) simplify the context by formulating the modeling problem, identifying relevant quantities, and making needed assumptions
(2) build a model by constructing relationships between quantities and using the model to generate a solution to the problem context
(3) interpret the model in the context of the situation and compare model and solution with other models and solutions
(4) check that the model and solution appropriately address the problem context and carry out identified model revisions
(5) communicate the model and solution to others and consider other contexts the model may extend to

I organized this modeling competency framework into concept codes and developed operational definitions for each of them. Table 14 displays the initial competency codes along with their operational definitions and corresponding modeling competency.

Table 14. A Priori Concept Codes and Operationalized Definitions

Code	Operational Definition – Students are…	Modeling Competency
posing problems	formulating mathematical questions to help them address the modeling problem or state the modeling question	(1) simplify the context by formulating the modeling problem, identifying relevant quantities, and making needed assumptions
identifying quantities	determining which aspects of the situation must be included in their model	
making assumptions	addressing the openness of the situation by making assumptions that narrow the problem and enable progress	
building the model	developing mathematical relationships between identified quantities	(2) build a model by constructing relationships between quantities and using the model to generate a solution to the problem context
generating a solution	applying their model to develop a solution to the problem	
interpreting the model	connecting model components back to the context of the situation	(3) interpret the model in the context of the situation and compare model and solution with other models and solutions
analyzing the model	analyzing their own and others' models by considering how they are working in the context of the problem; students are comparing models and solutions with each other	
revising the model	revising their models based on a change in contextual understanding, altering a quantity, changing an assumption, etc.	(4) check that the model and solution appropriately address the problem context and carry out identified model revisions
reporting out	communicating their models and solutions to others (peers, teacher, whole class)	(5) communicate the model and solution to others and consider other contexts the model may extend to
generalizing the model	considering how their models could apply in other situations	

For the remainder of this research report, when I refer to analyzing the data by coding for mathematical modeling competencies, I specifically mean coding for the individual components of each modeling competency.

Developing First Cycle Codes. Through first cycle coding methods, I iteratively developed a first cycle codebook consisting of three primary coding categories: mathematical modeling competencies, interactions, and positions. The development of the codebook began with coding transcripts of student conversations by focusing on *micro-level interactions* – turn-by-turn exchanges occurring in the moment between individuals engaged in dialogue. Through this lens, I conducted iterative rounds of process and concept coding to identify student interactions and mathematical modeling competencies as they emerged during micro-level interactions. With each iteration of these codes – particularly interaction codes – I asked the following guiding questions:

(1) What is the meaning of this code?

(2) How is this code distinct from similar codes?

(3) How well does this code capture what is represented in the data?

(4) Would a different code better represent the data?

(5) Is this code distinct, or could it be collapsed into another?

After iterating on the competency and interaction codes until they stabilized (see Table 15 and Table 16), I shifted focus to analyzing the transcripts through the lens of *macro-level interactions* – broader participation interactions that unfolded across multiple turns within a dialogic episode.

Table 15. Competency Codes, Operational Definitions, and Examples

Competency Codes	**Operational Definition Students are….**	**Example(s) and Comment(s)**
posing problems	formulating relevant questions that can be answered using mathematics to address the problem context	Everet: How many jump ropes [bags] you need to make one rope. Jason: Uh how many jump ropes [bags] it would take to make one jump rope? *Comment: From the context of the conversation, it seems both Everet and Jason meant to say "bags" instead of "jump ropes."*
identifying quantities	determining which aspects of the situation need to be included in their model	Wren: We know that, um, three bags is at least a foot. Lydia: And, um, you need to figure out the length, because some people are taller than other people. *Comment: Wren states the variable "bags per foot" while Lydia identifies the variable "height for rope size."*
making assumptions	addressing the openness of the situation by making assumptions which narrow the context and allow them to move forward in building their model	Eric: Well, I guess 30 bags can make one jump rope. Lydia: Yeah. But, um, for our class, if Brendan was here, we - I feel like - almost everybody needs an eight-foot rope. *Comment: During the estimating part of the lesson, Eric makes the assumption that there are 30 bags in one jump rope. The group uses this assumption to estimate the total number of plastic bags needed. Lydia makes two assumptions: first, that everybody in the class will be present, second, that most of the class needs an eight-foot rope.*

Table 15. Continued.

Competency Codes	Operational Definition Students are….	Example(s) and Comment(s)
building the model	developing mathematical relationships between quantities to build their models	Everet: So – seve- so three bags is one foot and you need seven feet. So, seven times three. Lydia: So 24 times 18 *Comment: Everet develops a mathematical relationship between the number of bags per foot of rope (three) and the number of feet in a seven-foot rope (seven). Lydia develops a mathematical relationship between the number of bags in an eight-foot rope (24) and the number of people in her class who need an eight-foot rope (18).*
generating a solution	determining solutions using mathematical relationships to address the problem posed	Everet: 65 times 30. But we broke it up into 65 times 10 and add those three times. I got, 1950. *Comment: Everet works through the model of* 65×30 *to get 1950.*
interpreting the model	connecting model components back to the context of the situation	Everet: So – seve- so three bags is one foot and you need seven feet. So, seven times three. 21 bags. In a seven-foot rope. Lydia: I would think it would be around five- 1,600, maybe, if people were- if we had more people in different classes. *Comment: Everet interprets* $3 \times 7 = 21$ *in the context of the situation, Lydia interprets the answer of 1,600 bags in context of the problem.*
analyzing the model	analyzing components of their model by considering how they are working in the context of the problem	Lydia: Ummmm, maybe, probably. Around 2,000 if people are taller - Yeah. Yeah. There's - Better to have extras than less than, so. *Comment: Lydia responds to Jason's suggestion of 2000 bags total by analyzing how having 2000 bags would work in this context.*

Table 15. Continued.

Competency Codes	Operational Definition Students are….	Example(s) and Comment(s)
revising the model	revising their models based on a change in contextual understanding, quantities, or assumptions	Everet: 45… So *[short pause]* What's 40 plus 20? Or 40 times 20? *Comment: Group 1 revises their initial model of* 65×30 *to* $(45 \times 24) + (20 \times 21)$ *to account for the number of students needing seven- and eight-foot jump ropes and to align with their teacher's initial constraint that their model should not include too many extra jump ropes.*

Table 16. Interaction Codes, Operational Definitions, and Examples

Interaction Codes	Operational Definition Students are…	Example(s)
countering	pushing against an idea or the conversation by offering a differing idea, repeating a question or statement, questioning an idea, or by attempting to pause the conversation	Eric: No, no, what's. It's 235, it would be. Eric: Does it? Wait, let's see. Brittany: No. 330.
negating	categorically denying an idea without contributing an alternative idea	Owen: You don't actually know, know. Like, for sure. You're not for surrrre. Eric: No, no, not Ali- no Ali- Alia is not.
agreeing	concurring with another's thinking	Eric: Yeah. You could just be like, "Hey Alexa, how many bags?" Like different bags. Everet: Yes. Everet: Well yeah we could estimate at 30.
clarifying	seeking information from another	Owen: Can you say it again but slower? Jason: Wait, what did you say?
confirming	seeking confirmation of their thinking from another	Jason: So it's not 17 times 24? Owen: *[looking towards Wren*] Ain't I close?
validating	affirming themselves or others through recognition	Everet: Which I already know the answer to, but you would have to ask it. Owen: I think I'm close.
unknowing	expressing uncertainty or a gap in understanding or knowledge	Eric: I don't know how I got that wrong. Owen: Yeahhhh, I dont' think I'll gonna, be able to help youuu with that. Ummm, I'm gonna run away.

Table 16. Continued.

Interaction Codes	Operational Definition Students are…	Example(s)
redirecting	voicing thoughts or taking actions that shift or sustain interactions away from the modeling context	Brittany: Look how long my hair is. Except when it's in a ponytail and it's sticking out because I have so much of it. Jason: skibidi go, go, go. Yes, yes, yes. Skibidi, yes. Skibidi yes. Skibidi toilet.
explaining	sharing their thinking by responding to questions or by justifying their ideas or decisions	Lydia: And, um, you need to figure out the length, because some people are taller than other people and if we make short jump ropes they would keep fall -
mapping	planning the next step(s) in their modeling work	Everet: How - well - so we're gonna need to know how many jump ropes, how many bags go into one jump rope? And then, how many jump ropes can we make? Lydia: Okay. So first we need to figure out - like the height.
directing	giving instruction to another regarding a specific behavior or goal	Lydia: Just work with me now. Brittany: Use your mathematical brain. Eric: And make sure to write bags.
correcting	giving direction to correct another's behavior	Everet: Ok, stop messing around. Eric: All right, Owen, put the ruler down. Jason: Don't put that in your mouth, nasty rope.
sharing	communicating their ideas, observations, or information in a neutral non-directive manner	Brittany: We know that we can braid them [the jump ropes]? Owen: I brought a lot of bags. Wren: We know that, um, three bags is at least a foot-

In contrast to micro-level analysis, which focused on individual conversation turns, macro-level coding required attention to how interactions connected across multiple turns. At the macro level of coding, I focused on the progression of micro-level interactions in order to ascertain possible positions of the conversants. Consequently, while I considered each turn in the conversation, I focused on larger dialogic episodes to identify possible student positions during group work interactions. I used concept coding to analyze students' macro-level interactions to

identify student positions enacted during group work. Iterations of position concept codes (see Table 17) were guided by the following questions until I arrived at a stable set of codes:

(1) What possible positions are in play? Did someone position another?

(2) What is the content of the interaction?

(3) What actions are not occurring? (e.g., is someone not speaking)?

(4) If there is a change in participation or topic, how and by whom is it indicated?

Table 17. Position Codes, Operational Definitions, and Examples

Position Codes	**Operational Definition Students are…**	**Example(s)**
process contributor	mapping the next step(s) in the modeling work	Lydia: So, we need to realize how much -. Bags, um, we need in our classroom, first, then we can go on to the four grade- or the whole fourth grade. Everet: How - well - so we're gonna need to know how many jump ropes, how many bags go into one jump rope? And then, how many jump ropes can we make?
modeling contributor	contributing mathematizing and interpreting ideas related to the task	Jason: The average height of a fourth-grader is probably four-foot six. Lydia: Most of them [the jump ropes] would be eight-foot. So I'm guessing like- One, two, three *[pauses],* four? maybe four people need a eight- seven-foot ropes. Maybe?
mathematics contributor	contributing ideas related to the mathematics of the task	Eric: *[writing and looking towards Everet]* Okay, so two plus one would obviously be three, seven, and that equals s-seven? Everet: So what's 65- *[looks towards Owen for a second]*, *[speaks with his hand over his mouth]* just do 65 times 10, plus 65 times 10, plus 65 plus 10 *[mouth open smile towards Eric]*

Table 17. Continued.

Position Codes	Operational Definition Students are…	Example(s)
context contributor	contributing ideas related to the context of the task	Brittany: We know that we can braid them? Lydia: Me and Johnny might be the same height.
social contributor	contributing ideas unrelated to the modeling task	Brittany: Look how long my hair is. Brittany: Wait, no, I'm an avatar- Brittany: Oh my god, have you ever played with a measuring tape?
modeling supporter	acknowledging, revoicing or echoing mathematizing and interpreting ideas contributed by another	Jason: ... there'd only be like a couple uh that would neeeed aaa -a seven-foot rope, and most of them would need an eight-foot rope. Correct? Eric: Yeah. You could just be like, "Hey Alexa, how many bags?" Like different bags.
mathematics supporter	acknowledging, revoicing or echoing mathematical ideas contributed by another	Eric: Uhhhmmm, uhhhh- ... uhhhmmmm, um, um, umm mmmm *[very quickly]* Wa-, wait. *[writing on his paper, then looks up to Everet]* Wait, what? 65 times three?
context supporter	acknowledging, revoicing or echoing contextual ideas contributed by another	Eric revoices Wren's contribution: Or Belle. Jason agrees with Lydia's contribution: Oh yeah, definitely Belle. Brittany agrees with Lydia's contribution: Belle's definitelyyyyy-
instructor	explaining ideas to another, checking another's ideas, or assessing the merits of another's ideas with or without being asked by the recipient	Lydia: How many feet are on the jump rope and all. So, like, yeeah. *[takes one end of the jump rope while Brittany holds the other end]* (2 seconds) Like there could be seven-feet, eight-feet... (5 seconds) So we would need to figure out how many feet it is *[spoken more slowly]* ↑. (2 seconds) For different people.
learner	receiving another's explanations and assessments by invitation or by imposition	Jason: Wait, what did you say? Jason: So what are we estimating? Brittany: What are you writing down now?

Table 17. Continued.

Position Codes	Operational Definition Students are…	Example(s)
observer	present during the conversation without participating verbally or nonverbally in ways that engage with the conversation	Brittany: *[looks over to Lydia's paper, then writes on her own paper]* Wren: *[looking over towards Eric's paper then writing on her paper]* No interactions over a number of turns from Wren, Brittany and Owen.
social corrector	correcting another's perceived mistake regarding matters unrelated to the modeling context, with or without explanation	Brittany: *[addressing Jason regarding the jump rope]* Oh my God, stop you're unbraiding it! Jason: I wouldn't put that in your mouth because how many people do you think have touched that bag? Don't put that in your mouth, nasty rope.
task corrector	correcting another's perceived mistake regarding the modeling context, with or without explanation	Jason: That's definitely not going to be it Lydia. Owen: You don't actually know, know. Like, for sure. You're not for surrrre. Everet: You're off. *[quickly makes a small gesture where he waves his left hand at Owen and then shakes his head and looks back to Owen]* Owen: No, it's a guess, guys. Guys, it's a guess. We don't do the math just yet.
task wrongdoer	being wrong with respect to the task	Owen accepts Eric's correction: That is truuueee. Eric: I don't know how I got that wrong. Everet accepts Eric's correction: Plus…Oh, yeah. Well *[leans over the table to see Eric's paper and points at the paper with his pencil]* – yeah –235.
social wrongdoer	being wrong with respect to matters unrelated to the task	Lydia was corrected for her pronunciation of the word "figure" which she pronounced as "figger." Owen's pencil was taken away from him by Everet. Owen was told by Eric to put down the ruler he was holding.

During both micro- and macro-level analysis, I engaged in iterative cycles of coding and revision. I alternated between the two student groups so that neither was consistently coded first. For each draft of codes, I recorded code names, operational definitions, examples, and reflections in the form of jottings. These jottings, along with data excerpts, informed the refinement of operational definitions. When I was uncertain about how to code a conversation segment, I flagged it for reconsideration during subsequent cycles of coding. With each iteration, I systematically reviewed the codebook, merging, splitting, and revising entries as needed. I compared codes for similarities and differences, revisited flagged questions, and refined the coding structure to more accurately represent the modeling competencies, interactions, and positions emerging in the data.

After several iterations of first cycle codes, I arrived at a stable first cycle codebook containing the codes for student interactions, mathematical modeling competencies, and positions.

Applying First Cycle Codes. After completing the codebook iterations detailed in previous sections, I coded student transcripts using NVivo. I began by coding for modeling competencies (see Table 18), followed by student interactions (see Table 19), and finally student positions (see Table 20).

In Table 18, Lydia and Jason (Group 4) engage in the modeling competency of *making assumptions* when revisiting their group's earlier decision that four students would need a seven-foot jump rope. The dialogue in Turns 176-179 shows the students deciding that five students in their class need the shorter jump-rope size. Previously, the group had assumed that four students needed the shorter rope and 18 students required eight-foot ropes. As Lydia explains (Turn 182),

this assumption informed their model-building endeavors as the group continued building their model to include five students in the shorter-rope category and 17 in the longer one.

Table 18. Example Coding Competencies – Making Assumptions

Turn	Brittany	Lydia	Jason
176		*[addressing Jason]* Wait is Ow- which one is Owen, Johnny's height or no?	
177			Probably a little bit shorter.
178	I'm-	*[looking at Jason]* So should we	
179		count him in- The seven-foot- seven-foot, okay. *[writes on her paper]* So I'm going to change this up to seventy-wait, *[addressing Jason]* what if Brendan-Is Brendan um, taller than Owen?	
180			Yeah.
181		Yeah.	He's in.
182	*[looking around the room and also looking at Lydia's paper and then writing on her own paper]*	Okay, okay. *[working on her paper, writing and speaking aloud]* So we're gonna move that into five. So we're gonna change it up. So this would be 22, 25. So. Five minus two, 18, minus 10. Would equal, 408 ↑ . Then instead of that it would be one-oh ↓ -six. Okay.	*[sits in his seat but does not write on his paper]*

In Table 19, Turns 162 and 163 highlight Eric (Group 1) *countering* Owen's idea. Eric first resists Owen's suggestion with "Well, no" (Turn 162) and then introduces an alternative idea – that each of the three fourth-grade classes might have a different number of students (Turn 163). This interaction illustrates the interaction code countering, in which a student pushes against an idea or the direction of the conversation by proposing a differing idea.

Table 19. Example of Coding Interactions – Countering

Turn	Everet	Owen	Wren	Eric
157		How many are in this class right now?		
158				24.
159	22.			*[quickly]* No I'm just kidding, just kidding.
160		22		
161		*[looking up towards the ceiling] [slowly]* twenty-twooo, fourty-fourrr,		
162		sixty-sixx		Well, no.
163		*[accidentally flicks his pencil to the floor, moves chair back and leans back to retrieve it]*		You're adding up just our class. Don't count our class because y-, like, Ms. White's might have 21 in their whole class when everybody's there.

In Table 20, Lydia's dialogue in Turns 199-205 provides an example of enacting the *instructor position*. In this excerpt, Lydia has completed the mathematical work and determined that they would need 1,532 bags for the fourth grade. On Turn 198, Jason asks about estimating their total to 1,000 bags, adopting the learner position while positioning Lydia as an instructor. Lydia takes up this positioning on Turn 199, explaining her calculations to Jason by turning her paper toward him and pointing to her work as she speaks (Turns 199-205).

Table 20. Example of Coding Positions – Instructor

Turn	Brittany	Lydia	Jason
196		So,	Okay, so our est- the estimate would be around...
197		It would be around 450. 532.	
198			So, five- so probably like 1,000 ↑?
199	Yeeah.	No *[definitively]*.	
200		*[turns her paper upside down so Jason can read it, then points to her paper while explaining]* Because, I just did this, and so, *[gesturing with her arms]* people might be shorter, but that would be like minus 500. So ↓. Maybe we would need like 1,532 bags. *[lightly throws arms up in the air and drops them on the table]* I don't know.	*[leaning over to look at Lydia's paper]*
201	[*maybe says "Yeahh"]*		I think the estimate of 900 and 300 per class is a pretty good estimate.
202		Yeah, but, I figured out that just in this class it's 514. Just in this class. So when we times that by three we get this answer. So, it could be higher than that because we don't know if	*[plays with ruler by tapping it on the table]*
203	'Cause yeah-	people are	
204	*[speaks a split second after Lydia]* taller or shorter	Taller or shorter.	
205		So, it, but, *[exhales]*, our estimate would be about this. So.	*[tapping ruler]*

Before each round of coding, I revisited the relevant codes and operational definitions. After coding, I carefully reviewed how I applied the codes to the transcripts. Unlike earlier coding iterations where I focused primarily on code development, operational definitions, and associated examples, this stage of analysis emphasized the application of first cycle codes to student transcripts. I examined how competency, interaction, and position codes were assigned within the transcripts, made adjustments to operational definitions as needed, and checked for consistency and accuracy across the data.

In particular, when coding student positions, I considered both verbal and nonverbal forms of communication – that is, students' interactions. I analyzed how these interactions unfolded across multiple conversational turns and drew on additional contextual information beyond the transcripts – by looking at what occurred before or after the transcribed conversation.

At times, student positioning remained ambiguous. One such instance occurred with Lydia when she said "Umm" and "I don't know-." Her comments, the surrounding transcript, and the nontranscribed portion of the conversation did not provide enough information to determine a position. In this instance I determined that there was not enough information to reasonably name Lydia's position, and left these turns uncoded.

Conclusion to First Cycle Coding. Thus far, this chapter has detailed the application of first cycle coding methods to analyze student transcripts and develop a comprehensive first cycle codebook. I structured first cycle coding by first analyzing the micro-level interactions between students followed by the macro-level interactions. During analysis of micro-level interactions – turn-by-turn exchanges between students – I conducted several rounds of coding to iteratively developed and revise codes, operational definitions, and examples. I applied process and concept coding to student transcripts by capturing in-the-moment student interactions with process codes, and mathematical modeling competencies with concept codes.

I analyzed macro-level interactions by coding for student positions using concept codes over the course of several rounds of coding and codebook iterations. During these iterations, I developed and refined position codes, operational definitions, and relevant examples. Throughout the process, I documented my coding decisions, uncertainties, and the evolution of specific codes in memos containing explanations and examples excerpts from the data.

After finalizing the first cycle codebook, I uploaded student transcripts and codes into NVivo and conducted three additional coding iterations: 1) coding for modeling competencies, 2) coding for interactions, and 3) coding for positions. Before each round, I reviewed the relevant codes and operational definitions; after each round, I made necessary adjustments to refine the codes and their operational definitions. In particular, I reviewed *how* I applied the codes to student transcripts, checking for both accuracy and consistency. Throughout this process, I recorded memos containing examples of my coding decisions and jottings about the challenges encountered in each round. With first cycle coding complete, I turned to second cycle coding, where I synthesized the first cycle codes into broader categories and identified findings.

Second Cycle Coding

Second cycle coding primarily focused on identifying categories across first cycle codes. This phase of coding built on first cycle coding by moving from isolated instances of student engagement in modeling competencies, interactions, and positions to the development of broader conceptual categories. My approach to second cycle coding was informed by the central and sub-research questions, the theoretical framework of positioning theory, and the modeling competency framework (see Chapter Two).

To begin second cycle coding, I sorted the first-cycle codes by similarity and developed initial category codes. I then applied a layered analysis approach using the category codes to analyze student transcripts and memos over three layers – competencies, positions, and interactions. This approach supported an examination of the data (transcripts, student work, memos, jottings) and facilitated the generation of insights that address the central and sub-research questions.

As with first-cycle coding, I created analytic memos throughout the process of second cycle coding. With each layer of analysis, I continuously revisited the initial application of first-cycle codes and made refinements when necessary. After each revision, I reanalyzed student transcripts using updated category codes and adjusted my emerging codes to reflect these insights. The sections that follow provide an overview of the development of category codes, the application of layered analysis, and the subsequent cross-layers analysis that culminated in the development of findings.

Generating Category Codes from First-Cycle Codes. To begin second cycle coding, I examined how the modeling competency codes related to one another, identifying broader conceptual threads that connected them. I then repeated this process for both interaction and position codes, condensing first-cycle codes into larger conceptual categories.

Second Cycle Coding: Modeling Competency Codes. I reviewed the codes and operational definitions for modeling competencies by clustering competency codes around related ideas. After several rounds of reorganizing, regrouping, and reexamining clusters of competency codes, I developed the category codes: *contextual considerations, model development,* and *model recontextualization*.

I grouped the competency codes *posing problems, identifying quantities,* and *making assumptions* under *contextual considerations* because each of these modeling competencies involves reasoning directly with the context of the situation. These modeling competencies are actions students took to make sense of the real-world context to address the situation by building a mathematical model. The modeling competencies *building the model, revising the model,* and *generating a solution* were grouped under *model development* as these competencies concern

either constructing, calculating, or revising their mathematical models. The final grouping, *model recontextualization,* included the two remaining competencies, *interpreting,* and *analyzing*. These modeling competencies require students to re-engage with the context of the situation once their model or parts of the model have been constructed. Each of these competencies is focused on understanding and connecting the model or components of the model back to the context of the problem. Table 21 includes the *competency categories,* the list of competency category codes, definitions, and included first-cycle codes.

Table 21. Competency Categories, Operational Definitions, and Included First-Cycle Codes

Competency Categories – categories that capture the modeling content or conceptual ideas expressed during interactions		
Category Code	Operational Definition	First-Cycle Codes Included
Contextual considerations	students are grounding themselves in the context by formulating relevant questions, identifying important aspects, and narrowing the context with assumptions	Posing problems, Identifying quantities, Making assumptions
Model development	students are developing mathematical relationships, determining solutions, and revising their models	Building the model, Generating a solution, Revising the model
Model recontextualization	students are reframing or revising model components based on the problem context	Interpreting the model, Analyzing the model, Revising the model

Second Cycle Coding: Student Interaction Codes. As with modeling competency codes, I considered the interaction codes by looking for connections between them. After several iterations of various organizations and after further analysis, I eventually developed five category codes and definitions which I nested under the overarching code: *interactional categories.* Table 22 includes the list of interactional category codes, definitions, and included first-cycle codes.

Table 22. Interactional Categories, Operational Definitions, and Included First-Cycle Codes

Interactional Categories – categories that concern the structure of student interactions		
Category Code	Operational Definition	First-Cycle Codes Included
Oppositional moves	students are pushing against the conversation by offering alternative ideas, categorically denying ideas, or redirecting discussion away from the modeling context	Countering, Negating, Redirecting
Affiliative moves	students are supporting themselves or others through agreement or affirmation	Agreeing, Validating
Inquiry moves	students are expressing a need for information, understanding, or confirmation	Clarifying, Confirming, Unknowing
Guiding moves	students are influencing another's understanding, behavior, or the direction of the modeling work	Explaining, Mapping, Directing, Correcting
Knowledge moves	students are building the modeling conversation by sharing relevant ideas, observations, and information	Sharing

Second Cycle Coding: Student Position Codes. As with interaction and competency codes, I analyzed student position codes by considering their operational definitions and developing ideas that tied different positions together and trying various code groupings. After several attempts, I eventually reconsidered position codes by considering *how students participated during group conversations* and developed two more drafts of position groupings before deciding to sort position codes into the following categories: *contributor position, supporter position, recipient position, evaluator position,* and *viewer position.* I then developed a definition for each category code and grouped them under the overarching concept of *positional categories* – categories that describe how students participated in group conversations through enacted positions. I then reviewed and revised the competency, interactional, and positional

codes to build uniformity across all category codes. Table 23 includes the list of positional category codes, definitions, and included first-cycle codes.

Table 23. Positional Categories, Operational Definitions, and Included First-Cycle Codes

Positional Categories – categories that describe how students participated in group conversations through enacted positions		
Category Code	Operational Definition	First-Cycle Codes Included
Contributor	students are contributing ideas that build and shape the direction of the conversation	Process contributor, Modeling contributor, Mathematics contributor, Context contributor, Social contributor
Supporter	students are acknowledging, revoicing or echoing ideas contributed by others	Modeling supporter, Mathematics supporter, Context supporter
Recipient	students are receiving explanations, evaluations, or corrections from others or from themselves	Task wrongdoer, Social wrongdoer, Learner
Evaluator	students are administering explanations, evaluations, or corrections to others	Social corrector, Task corrector, Instructor
Viewer	students are present but not participating verbally or nonverbally in the conversation	Observer

While not related to modeling, the *social contributor* was a position students adopted to engage in non-modeling conversations – social conversations. This position identifies a way students contributed to their group's conversation, even if that contribution occurred outside the scope of mathematical modeling. In all second cycle analyses, I distinguish between contributor positions and clarify how each related – or did not relate – to the modeling process.

Using Category Codes to Examine First Cycle Coding. After developing second-cycle category codes, I included them in my NVivo research project. I then used the category codes to check for consistency with my first cycle coding. In NVivo, I reviewed my application of first cycle codes by using second cycle codes to run co-occurrence matrices within student

transcripts: competency categories by competency categories, positional categories by positional categories, and interactional categories by interactional categories. The resulting matrices showed where coding references for one category overlapped with another. I reviewed all instances of overlapping codes to determine whether each code had been appropriately applied or whether the coding application required revision. In cases where adjustments were necessary, I reconsidered the context and applied the most appropriate code. I then reran each matrix to confirm that the identified issues had been addressed and resolved. In this manner, I was able to check and confirm the accuracy of my coding.

Layered Analysis Using Category Codes. After refining my application of first-cycle codes using second-cycle category codes, I used the category codes to analyze student transcripts with the support of NVivo tools such as co-occurrence matrices and coding stripes (visual indicators of where codes were applied in the transcript). This analysis proceeded in three iterative and interconnected layers. Each layer built on insights from the previous one and involved multiple rounds of coding, memoing, and revision.

Layer 1: Competency Categories. In the first analytic layer, I examined how competency categories unfolded across the lesson, guided by the questions:

(1) What parts of the lesson do competency categories occur?

(2) How do competency categories change over the lesson?

This analysis addressed the CRQ – *What is the nature of student engagement in modeling competencies during group work?* – and SRQ1 – *How does student attention to the task context influence their engagement in mathematical modeling competencies?*

Layer 2: Positional and Competency Categories. During the second layer of analysis, I focused on positional categories as they related to competency categories, guided by the questions:

(1) What positional categories occur in relation to the competency categories?

(2) How do positional categories relate to mathematical modeling competencies?

This analysis primarily addressed SRQ2 – *How do student positions influence student engagement in mathematical modeling competencies?* and also contributed to the CRQ.

Layer 3: Interactional, Positional, and Competency Categories. In the third layer, I considered the positions students enacted and the interactions they used to engage in competency categories (contextual considerations, model development, and model recontextualization). The following questions guided my memoing and jotting during this analytic layer:

(1) How do interactional categories intersect with positional categories?

(2) How do interactional categories connect to engagement in modeling competencies?

This analysis addressed SRQ2 – *How do student positions influence their engagement in mathematical modeling competencies?* – as well as the CRQ.

Across all three layers, I documented analytic decisions and findings in memos and jottings, systematically revisited flagged questions, and refined interpretations through iterative review of transcripts. The integration of these layers supported the development of findings that respond to the central and sub-research questions (see Table 24).

Table 24. Alignment between Research Questions and Analysis

Analytic Layer	RQs Addressed	Guiding Focus / Questions
Layer 1	CRQ, SRQ1	What parts of the lesson do competency categories occur? How do competency categories change over the lesson?
Layer 2	CRQ, SRQ2	What positional categories occur in relation to the competency categories? How do positional categories relate to mathematical modeling competencies?
Layer 3	CRQ, SRQ2	How do interactional categories intersect with positional categories? How do interactional categories connect to engagement in modeling competencies?

Cross-Layer Analysis to Identify Findings. After completing the three analytic layers, I conducted iterative cross-layer analysis to identify findings. I uploaded memos into NVivo and systematically reviewed each memo and jotting to trace ideas for further investigation. This included examining original memos from transcript analysis, NVivo coding stripes, and co-occurrence matrices, as well as memos and jottings generated during subsequent reflection and analysis. Throughout, I used memoing to record my thinking, guided by the questions:

(1) What findings do the data reveal?

(2) What do these findings reveal about the nature of student engagement in modeling competencies?

Analytic Approach and Development of Findings. I first reviewed all analytic memos to identify potential findings, then examined them within each layer before comparing across layers. To support my development of the findings, I used NVivo to generate coding stripes and co-occurrence matrices, which highlighted overlaps across competency, positional, and interactional categories (i.e., modeling competencies, student positions, and interactions). In

determining findings, I considered whether the finding meaningfully addressed the central or sub-research questions; helped explain how students moved through the modeling process; and whether it recurred across the data rather than appearing in a single, isolated event. These criteria supported the development of robust findings related to the nature of student engagement in modeling competencies.

Conclusion to Second Cycle Coding. Second cycle coding built on first cycle coding by moving beyond isolated codes related to modeling competencies, student interactions, and student positions toward the development of broader analytic categories. To begin, I condensed first cycle codes into three overarching categories: competency categories (modeling competencies), interactional categories (student interactions), and positional categories (student positions).

With these categories in place, I conducted a layered analysis process, followed by cross-layer analysis, to examine how the categories addressed the central and sub-research questions. Across each analytic layer, I engaged in iterative cycles of coding, memoing, jotting, and revising, which supported the refinement of category codes and the development of findings. The cross-layer analysis likewise involved repeated cycles of data analysis but emphasized identifying and synthesizing findings across layers to deepen understanding of how students engaged in mathematical modeling competencies during group work conversations.

Conclusion to Analysis of the Data

This chapter presents a detailed account of the systematic analytic process I used to investigate how students engaged with mathematical modeling competencies. I began with holistic coding of audiovisual data collected while fourth-grade students enacted a modeling task

facilitated by their teacher, Ms. Kedrik. Through holistic coding, I generated transcripts of student group work conversations that met one or both of the following criteria:

C1: student interactions relate to the context of the task

C2: student interactions reference the mathematics of the task

In this study, *interactions* refer to verbal and nonverbal exchanges between two or more students during group work, and the *context of the task* refers to making jump ropes from plastic bags.

After generating transcripts, I applied first and second cycle coding methods. First cycle coding resulted in codes and operational definitions related to modeling competencies, student interactions, and student positions. I used second cycle coding, to analyze and condense first cycle codes into broader category codes, which I then organized into three overarching concepts: competency categories, positional categories, and interactional categories.

Competency categories include contextual considerations, model development, and model recontextualization. Positional categories described how students participated in conversations through enacted positions. These categories were developed from position codes and include contributor, supporter, recipient, evaluator, and viewer. Developed from interaction codes, interactional categories referred to the structure of student interactions and include affiliative moves (agreeing and validating), guiding moves (explaining, mapping, directing, and correcting), inquiry moves (clarifying, confirming, and unknowing), knowledge moves (sharing), and oppositional moves (countering, negating, and redirecting).

Using these second-cycle category codes, I conducted a layered analysis of student transcripts. To build an understanding of student engagement in modeling competencies during group work conversations, I analyzed the transcripts in three layers:

(1) competency categories

(2) competency categories with positional categories

(3) competency categories with both positional and interactional categories

Each layer of analysis expanded on the previous layer, adding analytic depth and contributing to addressing the central and sub-research questions. Throughout the analysis, I documented my thinking and observations through ongoing memoing and jotting, paying particular attention to emerging findings.

After completing the three analytic layers, I revisited the transcripts, memos, and jottings, to analyze emerging findings from the data. By working across these layers and consistently revisiting, comparing, and analyzing my memos, jottings, and transcripts, I identified emerging findings with how students engaged with mathematical modeling competencies during group work. In the following chapter, I present the findings developed during second cycle coding and illustrate how they contribute to understanding the nature of student engagement in modeling competencies during group work conversations.

CHAPTER FOUR

FINDINGS FROM ANALYSIS

Introduction

The purpose of this research study is to contribute to understanding how fourth-grade students engage with mathematical modeling competencies during group work by examining the positions they enact through social interactions, and how those positions influence their engagement in modeling competencies. This purpose is met through the following central and sub-research questions:

CRQ: What is the nature of student engagement in mathematical modeling competencies during group work?

SRQ1: How does student attention to the task context influence their engagement in mathematical modeling competencies?

SRQ2: How do student positions influence their engagement in mathematical modeling competencies?

The balance of this chapter provides a contextual overview of the Jump Rope task, followed by a presentation of the two main findings from analysis.

Contextual Overview

Enactment of the Jump Rope Task

This section provides a chronological description of the enactment of the Jump Rope modeling task in Ms. Kedrik's fourth-grade classroom. The Jump Rope Task is an upcycling activity in which students collect plastic bags and repurpose them into braided jump ropes.

The day before facilitating the lesson, the classroom teacher, Ms. Kedrik, taught her students how to braid plastic bags into jump ropes. On the day of the task, Ms. Kedrik began the lesson by reviewing the braiding technique and checking that at least one student at each of the six tables was able to braid. She then asked, “What do you know about plastic bags?” Students discussed this question in their table groups and shared their ideas during a whole class debrief. They mentioned that plastic bags are used for shopping, are recyclable, and are not eco-friendly.

Next, students discussed their responses to the question “What do you know about jump ropes?” They shared that jump ropes are a fun way to exercise, there are different techniques for jumping, they can be used to play games, and they come in different sizes. After a prompting from Ms. Kedrik, students elaborated that smaller jump ropes are for one jumper and longer jump ropes are for two jumpers.

Ms. Kedrik then asked students what they notice and wonder about a set of images depicting people jumping with different types of jump ropes. After silent thinking time where students independently record their thoughts on paper, students shared their noticings and then their wonderings with the class. They noticed that the jump ropes were different colors and sizes, there were different jumping speeds – some were jumping faster than others, and the jump ropes were made of different materials – like rubber bands. Students wondered why some jump ropes were smaller than others, how many kids were jumping rope in the pictures, and how many jumps the jumpers could complete.

Building on this discussion, Ms. Kedrik asked students to think about what math questions they could ask, and shared that the goal of the lesson was to create upcycled jump

ropes. After 30 seconds of quiet think time, students collaborated in their groups and then reported questions such as:

(1) How many plastic bags for everybody in the class?

(2) How thick are the ropes?

(3) How long does the jump rope need to be? Taller students need taller ones and shorter students might trip on a rope that is too long.

(4) How many plastic bags in all?

(5) How many people?

(6) Do we have enough bags?

(7) How long is each bag?

(8) How much time do we have?

Building on these questions, Ms. Kedrik stated the central modeling question:

How many plastic bags does it take to create jump ropes for all of fourth grade?

The students recorded the question on their papers and Ms. Kedrik played a video demonstrating how to braid plastic bags together. Ms. Kedrik asked students to braid three plastic bags and measure the length of the rope. The groups reported an average length of 12 inches and the class collectively agreed to assume that three bags produce one foot of jump rope.

Ms. Kedrik next provided the following information for determining rope length based on an individual's height:

Up to 4 feet 6 inches tall, then the jump rope is 7 feet long.

Between 4 feet 6 inches and 5 feet 5 inches, then the rope is 8 feet long.

Over 5 feet 5 inches, then the jump rope is 9 feet long.

Students commented that no one in the fourth grade would be over 5 foot 5 inches.

Within their groups, students discussed what they knew about making jump ropes. They shared that jump ropes come in different sizes for different people, that three bags make one foot of jump rope, and that based on the heights of students they knew, no one would need a nine-foot rope. Therefore, they assumed they would not need to make any nine-foot ropes. They also noted that they needed to know how many people were in the fourth grade.

Ms. Kedrik asked students if they could make an assumption or an estimate about the number of fourth graders at the school. The students proposed 66 as a possible number but agreed that 65 would be a friendlier number to use for calculations. When prompted by Ms. Kedrik to consider what else they knew or needed to know, students noted that estimating the heights of the shortest and tallest students and identifying the average height of students would be important. Ms. Kedrik asked if they could make an assumption about average height, and the class agreed that it was something they needed to do.

Following a whole-class discussion about how many students might need each size of jump rope, the students concluded that they would need more eight-foot ropes than seven-foot ones. Ms. Kedrik then recapped the task, reminding the class that they would work in groups to develop plans to address the scenario – given the constraint that their plan needed to show how they knew they had enough jump ropes without a lot of extras. Each group's plan could differ but needed to include the following:

(1) How they know they have enough jump ropes without a lot of extra,

(2) How many bags they need, and

(3) How others could use their plan to make jump ropes.

Students were instructed to begin by estimating the total number of plastic bags needed, based on the information that there were 65 students, each jump rope would be either seven or eight feet long, and three plastic bags produced one foot of jump rope.

In their groups, students estimated the number of plastic bags required, with proposed totals ranging from 450 to 1,800. To guide their reasoning, Ms. Kedrik led a class discussion regarding what might constitute a reasonable or unreasonable estimate, prompting students to reflect on whether figures like 20,000, 900, or 3,000 bags would make sense.

For the remainder of the lesson, students collaborated in their groups to generate solutions to the modeling problem. At the end of the lesson, some student groups presented their models and solutions to the class – including Group 1 and Group 4. Group 1 consisted of four students – three boys and one girl with the pseudonyms of Everet, Owen, Wren, and Eric. Group 4 included two girls and one boy: Brittany, Lydia, and Jason. How these students' modeling work developed is addressed in the first finding.

Finding 1: Contextually Grounded Mathematical Modeling

The findings in this section address the first sub-research question: *How does student attention to the task context influence their engagement in mathematical modeling competencies?* The analysis revealed that when students consistently connected their mathematical work to the situation, they developed models that were well-aligned with the problem context. In contrast, when students worked with mathematics disconnected from the context, their models became misaligned with the problem which prompted revision. Importantly, revising a model and making corrections within a model are distinct actions. Revising a model occurs when students make a change to some component of their mathematical

model based on their understanding of the context, relevant quantities, or assumptions. In contrast, making corrections involves correcting inaccuracies – such as computational errors. These findings illustrate the ways modeling competencies emerged during group work conversations and show how attention to the task context shaped the development of their mathematical models.

Modeling Competencies during Group Work Conversations

During group work conversations, students engaged in three categories of modeling competencies: contextual considerations (*problem posing, identifying quantities*, and *making assumptions*), model development (*building the model, generating a solution,* and *revising the model*), and model recontextualization (*interpreting the model*, and *analyzing the model*). They did not engage in *generalizing the model* or *reporting out* on the model.

Students engaged in *problem posing* by formulating questions that could be answered mathematically, such as "Uh, how many jump ropes [plastic bags] it would take to make one jump rope?" and "How many bags do we need for everyone?" They *identified quantities* by determining which aspects of the situation to include in their models and *made assumptions* that narrowed the openness of the task. Often, these two competencies were intertwined. For example, the assumption that three bags produced one foot of rope identified the quantity of bags per foot of rope, and the assumption that most students would need an eight-foot rope identified the quantity of people per rope length.

When engaged in the competency of *building their models*, students developed mathematical relationships between quantities that contributed to the development of their models (e.g., 7×3, where seven represents the seven-foot rope and three represents the number

of bags per foot of rope). They then engaged in the competency of *generating solutions* by finding quantities that resulted from these relationships (e.g., $7 \times 3 = 21$, where 21 represents the number of bags needed to produce a seven-foot jump rope). They *interpreted the model* when connecting model components back to the context of the situation (e.g., "So 21. 21, um, bags would equal a seven-foot"), and they *analyzed the model* by considering how components worked in the context of the problem. For instance, given their solution of 1,950 bags, Wren and Eric examined how many bags each person in school would need to collect. Finally, in Group 1, students engaged in *revising their model* to align with the problem context by accounting for the number of fourth-grade students needing seven- or eight-feet ropes while also meeting the constraint of not having too many extra jump ropes.

Contextual Grounding During Group Work Conversations

This finding highlights how students' modeling work was shaped by their attention to the task context – particularly how mathematical models and solutions developed when students were connected with or disconnected from the context of the situation.

At the start of the lesson, Ms. Kedrik asked, *"What math questions can you ask?"* Group 1 and Group 4 connected their modeling work to the context of the situation by engaging in contextual consideration competencies – *posing questions* such as "How many bags go into one jump rope?" "How many jump ropes can we make?" and "How many plastic bags do we need to make, uh, jump ropes?" They engaged in *identifying relevant quantities* by determining aspects of the situation they wanted to incorporate into their models – including number of bags per jump rope, jump rope length, and student height – and they engaged in *making assumptions* connected to the problem context, such as assuming that students would fall if they had jump

ropes that were too short. Similarly, in response to Ms. Kedrik's prompt, *"What do you know about making jump ropes?"* students continued to ground their modeling work in the situation by *identifying quantities* such as number of bags per foot of rope, total feet per rope, and rope length needed for different students, while also assuming that most people would need an eight-foot rope. The mathematics students used – primarily addition, multiplication, and ratios – was closely tied to their engagement in modeling competencies. For instance, these operations supported *determining quantities* (e.g., bags per rope length and number of students per class), *making assumptions* (e.g., three bags per foot of rope), and *building the model* by constructing mathematical relationships, such as 7×3, which contributed to the development of the overarching model but was not itself the model, and *generating solutions* by evaluating those relationships (e.g., $7 \times 3 = 21$, where 21 represents the number of bags for a seven-foot jump rope).

When Ms. Kedrik released the students to work in groups to *develop models* and *generate solutions*, they worked through the modeling process by engaging with modeling competencies to address the overarching question: *"How many plastic bags does it take to create jump ropes for all of fourth grade?"* while attending to the constraint that they could not produce too many extra jump ropes. In doing so, they engaged in contextual consideration, model development, and model recontextualization competencies. At the outset of their work, Ms. Kedrik had encouraged students to think about how other people might use their models – indicating an expectation that students would *generalize their models*. However, neither group generalized their models, and Ms. Kedrik did not revisit or press this request during group work or at the end of the lesson when Groups 1 and 4 reported on their models to their class.

The following sections highlight how each group's modeling-building conversations unfolded: Group 4 remained consistently connected to the context of the task while engaging in model development competencies, whereas Group 1 focused primarily on the mathematics, largely disconnecting from the context.

Group 4: Sustained Contextual Grounding of Mathematical Modeling. Group 4 demonstrated consistent attention to the task context while building and evaluating mathematical relationships, repeatedly interpreting their work in light of the situation and developing a mathematical model that reflected the problem context and constraint.

Group 4: Development of Model and Solution. Students in Group 4 engaged with the competency of *building the model* by initially outlining a modeling plan that was grounded in elements from the task situation. This plan was proposed by Lydia who said, "So. We need to realize how much. Bags, um, we need in our classroom, first, then we can go on to the fourth grade- or the whole fourth grade" (Turn 80, Appendix C, Table C2). Table 25 illustrates Lydia and Jason engaged in conversation *identifying relevant quantities* (e.g., 22 students in their class, Turn 91) and *making assumptions* about student heights to determine the number of students requiring each rope length – either a seven-foot or an eight-foot jump rope.

Table 25. Group 4 Sample Dialogue – Engaging in the Context

Turn	Brittany	Lydia	Jason
91		*[still looking around the room counting off and emphasizing her words by pointing with her arm and pencil – almost like making a tally mark in the air]* Um, 18. 22. So, there's 22 people in our class.	
92		Soo ↓ *[pause]* most of the- most of them would be eight-foot -	*[looks around the classroom]*
93	*[silently making faces with her mouth]*		There's 22 kids in our class.
94	*[playing with the braided jump rope]*	Yep s-	
95			and right now there's only *[haltingly]* 21.
96		Yeah.	because Brendan is gone.
97	*[playing with the braided jump rope, lifts the table a little bit but stops playing with the jump rope after lifting the table. Leaves the jump rope in front of her on the table.]*	Yeah. But, um, for our class, if Brendan was here, we - I feel like - almost everybody needs an eight-foot rope. Almost everybody except for like - you *[looks towards Brittany and pushes back down on the table slightly]*	
98	*[quietly]* Mee ↓		
99		*[thoughtfully]* M-Maybe, maybe ↓ , I don't know. Except for like, I know Brie. Because she's shorter than me.	*[very quietly, almost inaudible, addresses Brittany]* [undecipherable] can you stop lifting the table?
100	*[folds arms in front of her on the table and leans into the table]*	Maybe Owen. *[taps lips with pencil]* *[quietly]* He's shorter than ↓ me.	
101			Definitely Wren.
102	Definitely.	Yeah.	
103		*[looking out towards the classroom]* And also Belle.	*[looks out towards the same place as Lydia was looking]*

When determining the number of students in their class, Jason counted those present, noting across Turns 93-96 (Table 25) that while there are 22 students total, only 21 were currently in the room because Brendan was absent. On Turn 97, Lydia made an implicit assumption that their model should plan for everybody being present and include all 22 students. A second was made when Lydia said most students in the class will need an eight-foot jump rope (Turns 92, 97, Table

25). Finally, across Turns 97-103 (Table 25), Lydia led the group in *making assumptions* about student heights, using their knowledge of their peers to determine which would require seven-foot ropes. This dialogue excerpt (Table 25) illustrates how students in Group 4 connected their knowledge of the context of the task to inform their engagement in the modeling competencies of *identifying quantities* and *making assumptions*.

As they progressed in developing their mathematical work, Group 4 moved fluidly between the competencies of *building models* and *generating solutions*, consistently reconnecting to the context through the competency of *interpreting the model*. For example, after calculating $7 \times 3 = 21$ (Turns 136-138, Table 26), Lydia interpreted the result by connecting it back to the situation: "So 21. 21, um, bags would equal seven-foot" (Turn 144, Table 26). Similarly, the group determined that an eight-foot rope required 24 bags (Turns 146-148, Table 26).

Table 26. Group 4 Sample Dialogue – Interpreting Mathematics within the Context

Turn	Brittany	Lydia	Jason
136		Yeah, seven times-	*[addressing Brittany]* You put your mouth on that.
137		Three. That would equal *[pauses]*	
138		that would equal 21.	I wouldn't put that in your mouth
…			
143		So 21.	Don't put that in your mouth, nasty rope.
144	*[tosses the braided jump rope across the table]*	21, um, bags would equal seven-foot.	
145			*[touches the braided bag and addressing Brittany]* Pull it.
146	*[mouths something to Jason]*	And then, so that means, 24, then that would	
147		mean	*[responds to Brittany]* I don't know.
148	*[laughs inaudibly]*	18 time 3 equals 24.	

They then extended their work by setting up and evaluating the multiplicative relationships 24×18 and 21×4 . In the first relationship, they multiplied the number of bags in an eight-foot jump rope (24) by the number of students (18) in their class who they determined would need this size rope. In the second, they multiplied the number of bags in a seven-foot jump rope (21) by the number of students (four) they thought would need the shorter size.

Their mathematical work did not stop with these computations. Group 4 reengaged with the context by *interpreting* and *analyzing* their solutions in light of the situation, thereby recontextualizing their mathematical work through the modeling competencies of *interpreting and analyzing the model*. Specifically, they re-engaged in *making assumptions* by reconsidering their initial decision that only four students required the shorter rope, questioning whether one more student should be included (Turn 174, Table 27).

Table 27. Group 4 Sample Dialogue – Adjusting Assumptions

Turn	Brittany	Lydia	Jason
174	*[leans over her paper looking at what Lydia writes on her paper]*	Then, 24 times four, 21 times four *[writes on her paper and speaks under her breath]* 21 times 4 would equal... 80. 84. *[writes on her paper and speaks under her breath]* would equal 84. *[slowly and questioningly]* Uh, ↑ maybe Owen ↑ should be ↑ in herrrrre ↑ ?	
175		Mmmmmm	*[resumes his seat and rejoins his table group]*
176		*[addressing Jason]* Wait is Ow- which one is Owen, Johnny's height or no?	
177			Probably a little bit shorter.
178	I'm-	*[looking at Jason]* So should we	
179		count him in- The seven-foot- seven-foot, okay. *[writes on her paper]* So I'm going to change this up to seventy-wait, *[addressing Jason]* what if Brendan-Is Brendan um, taller than Owen?	
180			Yeah.
181		Yeah.	He's in.
182	*[looking around the room and also looking at Lydia's paper and then writing on her own paper]*	Okay, okay. *[working on her paper, writing and speaking aloud]* So we're gonna move that into five. So we're gonna change it up. So this would be 22, 25. So. Five minus two, 18, minus 10. Would equal, 408 ↑ . Then instead of that it would be one-oh ↓ -six. Okay.	*[sits in his seat but does not write on his paper]*

This analysis prompted them to adjust their assumption and re-engage in model development competencies by *building their model* to account for five students (instead of four) who would need seven-foot jump ropes (Turn 182). In *building their model and solution* (Table 28) Group 4 determined that $17 \times 24 = 408$ and that $21 \times 5 = 106$ (Turn 186). Adding the products of these two mathematical relationships ($408 + 105 = 514$), they initially determined

that their class would need 514 bags (Turn 186), which they then scaled by three to calculate the total number of bags needed for all three fourth-grade classes at their school – 1,532 bags.

Table 28. Group 4 Sample Dialogue – Building the Model

Turn	Brittany	Lydia	Jason
183		We've got	Okay, so what are we-
184		these numbers, so we've got-	
185			*[picks up his pencil]* So it's not 17 times 24 ↑?
186	*[looks away from the table towards another table but eventually turns back to see what Lydia is writing, then writes on her paper]*	No, not anymore. It, it, wait. Okay. So it's *[flips her paper around so Jason can read it from his view. Explains and demonstrates with the paper while speaking]* 17 times 24, which I would equal 408. Then, twel- 21 times five, would equal 106. *[waits for Jason to record on his paper]* Okay, now we're gonna-now we're gonna add *[flips her paper back around and writes on her paper while speaking]* 408 plus ↑ one-oh-six, which that would equal ↑, four, one, annnd five. 514. Now, we're gonna add 514 plu-times, *[pauses]* times ↑ three, would equal 1,532.	*[writes on his paper]*

Lydia recontextualized their solution by *interpreting* and *analyzing* her mathematical work in conversation with Jason, who proposed estimates of 1,000 (see Table 29), 2,000, and 1,600 bags.

Table 29. Group 4 Sample Dialogue – Recontextualizing the Mathematical Solution

Turn	Brittany	Lydia	Jason
198			So, five- so probably like 1,000 ↑?
199	Yeeah.	No [definitively].	
200		[turns her paper upside down so Jason can read it, then points to her paper while explaining] Because, I just did this, and so, [gesturing with her arms] people might be shorter, but that would be like minus 500. So ↓. Maybe we would need like 1,532 bags. [lightly throws arms up in the air and drops them on the table] I don't know.	[leaning over to look at Lydia's paper]

After each of Jason's suggested estimates, Lydia *interpreted* and *analyzed her model* to explain how the estimate did or did not align with the context. Lydia evaluated each estimate against the context, ultimately agreeing with 1,600 as a "friendly number" (see Table 30) and discussed how the estimate would fit the situation – if people were taller, if there were more people in the other classes, and that would probably have extras.

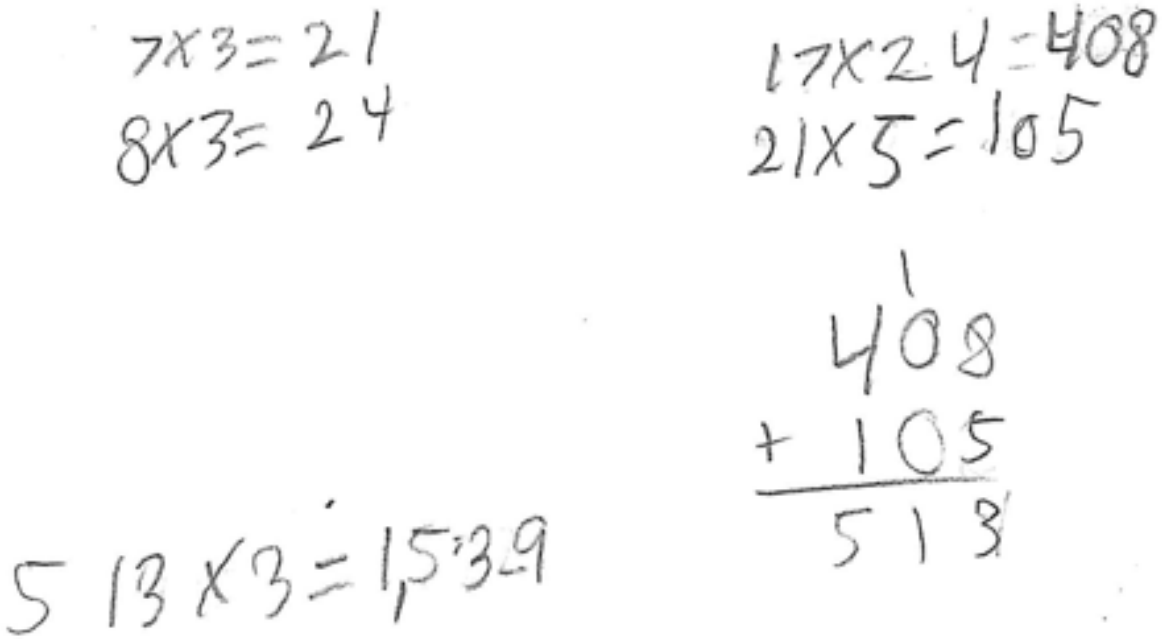
Table 30. Group 4 Sample Dialogue – Analyzing and Estimating the Solution

Turn	Brittany	Lydia	Jason
223			[questioningly] Maybe 1,600 ↑ ?
224		M- Yeah, probably, maybe. If	
225		people were tall.	For a friendly number.
226		Yep.	
227			1,600
228		Yep, friendly number. I th- I would think it would be around five- 1,600, maybe, if people were- if [gesturing with her arms] we had more people in	
229		different classes-	Yeah, we probably-
230			we probably have the people with the tallest. So. We probably have the most tall kids in our class.
231		So	So
232			Yeah, you should- probably.
233		It might -	
234			A friendly number would be 1,600. Yeah, I think that would work.
235		[gesturing with arms] Yeah, if 100 and 600 would have extras, probably. Unless other classes had more people, and, and, tall people. Like we have five people that are probably short ↑. [speaking slowly] Shorter than four-six. S- that, seven-foot ↓ [pause] that needs seven-foot	
236		long-	Okay so.
237		seven-feet long.	

Later, during whole-class *reporting on their model*, Group 4 corrected their arithmetic calculation with support from Ms. Kedrik and their peers. The group determined that $17 \times 24 = 408$ and $21 \times 5 = 105$ (instead of 106), resulting in a sum of $408 + 105 = 513$ (rather than 514). When multiplied by three, they calculated that they needed a total of 1,539 bags (instead of 1,532 bags). Their work, shown in Figure 9, demonstrates both this correction as well as their changes to their assumed number of students in each rope size through the recorded

computations of 17×24 and 21×5. Prior to adjusting their assumption, they had worked with 24×18 and 21×4, representing their assumptions that 18 students would require an eight-foot rope and four would need a seven-foot rope. Figure 9 reflects their assumptions that 17 students required an eight-foot rope and five needed a seven-foot rope.

Figure 9. Group 4's Mathematical Modeling Work



Group 4: Generalized Representation of the Model. Overall, Group 4's model relied on the parameter of three plastic bags per foot of rope. While the group did not state it explicitly, they used this parameter to construct the mathematical relationship of $3x = y$, where the independent variable x represented rope length in feet and the dependent variable y represented the total number of bags per rope. The discrete values of x were 7 and 8, yielding corresponding values of 21 and 24 bags. Building on this (although not directly formulated by the students) this group extended their work by developing the model:

$$n[21(x_{1,c}) + 24(x_{2,c})] = \mathrm{y_t}$$

where n was the number of fourth-grade classes, $x_{1,c}$ was the number of students needing a seven-foot rope in their class, $x_{2,c}$ was the number of students needing an eight-foot rope in their class, and y_t was the total number of plastic bags needed for all three classes. In this model formulation, the students used the initial parameter of three bags per one foot of rope to establish two new parameters, 21 and 24 bags, for the seven- and eight-foot ropes. Their independent variables were the number of fourth-grade classes and the number of students per rope length in their class, and their dependent variable, y_t, represented the solution to the modeling question: *How many plastic bags does it take to create jump ropes for all of fourth grade?*. While the students in Group 4 did not record this generalized form of their model, they developed and evaluated a specific application of this model incrementally, by first multiplying the number of bags per rope length by the number of students in their class needing that length (408 and 105), adding these totals to obtain 513 bags, and then scaling by the three fourth-grade classes, calculating that they needed 1,539 total bags (and then rounding to 1,600 bags).

Conclusion: Group 4's Sustained Contextual Grounding in Modeling. Overall, Group 4's sustained contextual grounding through model interpretation supported their engagement in *developing a model* and *generating a solution* and, consequently, the development of a mathematical model that was well aligned with the situation. They repeatedly *interpreted* their mathematics in relation to the context, which contributed to their revisiting initial assumptions and subsequent model development. For this group, continuous reference to the context of the problem guided their mathematical thinking, and, in turn, shaped their engagement in modeling competencies and the *development of their model* and final solution.

Group 1: Inconsistent Contextual Grounding of Mathematical Modeling. When engaged in model development competencies, Group 1's modeling work emphasized the mathematics of *building a model* and *generating solutions*, with limited attention to the context of the situation. They infrequently *interpreted* their mathematical work in relation to the problem context and *developed a model* that deviated from their assumptions – that most students would need an eight-foot jump rope, some would need seven-foot jump ropes, and none would need nine-foot ropes. Additionally, their initial model did not align with the constraint specified by Ms. Kedrik that their plan could not include too many extra jump ropes. After *revising their initial model* by reconnecting with the context of the task, Group 1 developed a model that aligned with the problem context.

Group 1: Development of Model and Solution. Group 1 initially grounded their mathematical thinking in the context by considering the number of bags needed to produce a seven-foot rope and the number of students in each fourth-grade class. Working from the assumption that three bags produced one foot of jump rope, they calculated that a seven-foot rope would require $7 \times 3 = 21$ bags (see Table 31).

Table 31. Group 1 Sample Dialogue – Connecting Mathematics to the Context

Turn	Everet	Owen	Wren	Eric
126	So – seve- so three bags is one foot and you need seven feet. So, seven times three *[looks towards Eric].*			
127				Is 20-
A	21 bags.			
129				*[quietly]* 21 bags, yeah.
130	In a seven-foot rope. *[Arms up but elbows on the table, with left palm out and open, Everet looks at each member of his group, first Eric, then Wren, then Owen]*	*[looking towards Everet]*		

With this calculation, students used their understanding of the context to build and evaluate the mathematical relationship $7 \times 3 = 21$. They recontextualized their work by *interpreting* each component of the relationship in terms of the situation: the seven represented the rope length in feet, the three represented the number of bags per foot, and the product, 21, represented the total number of bags needed for a seven-foot rope.

After this calculation, Everet stated, “28 bags equals eight-foot rope” without discussing with his peers the mathematical relationship that produced this result. The statement of “28 bags” suggests either an arithmetic error or a possible assumption. Either way, 28 bags represents a misalignment with the group’s assumption that three bags produced one foot of rope and illustrates how students *interpreted a result* in the context of the situation (“28 bags” as an eight-foot rope) without connecting it to a supporting mathematical relationship.

As the group progressed in their mathematical modeling work, they reconnected with the context of the task, grounding their mathematics in contextual information about class size (see Table 32). On Turns 146-149 (Table 32), Owen proposed extending their own class size to estimate the total across all fourth-grade classes: “you can count, um, byy those kids in our class, all to the other classes.”

Table 32. Group 1 Sample Dialogue – Context Supported Model Development

Turn	Everet	Owen	Wren	Eric
141		*[quickly]* I think		
142	↓ equals eight-foot rope	↓ *[quietly and quickly]* wait-w-w-w-w-wait	*[looks to Owen]*	
143		I have an idea, I have an idea.		
144	*[stops writing and looks up to Owen]* [screech sound]			
145				*[addressing Everet]* What was that?
146		I have an idea. We need to figure out how many kids are in our class		
147	*[writing on his paper]*	and then we could-	*[looking at Eric]* We already know!	
148				We already know.
149		*[looks at his table group members]* Thenn ↓, umm ↓ *[hesitates]* then um, you can count, um, byy ↑ those kids in our class, all to the other classes.	*[playing with the top of her water bottle]*	
150	[looking at Owen] [screech sound]			
151		*[looks at Eric]*		[with a laugh in his voice] Whoa, okay.
152		*[looking between Everet and Eric]* *Is that a good idea or no?*		Guys, we

Everet shared that their class had 22 students, and Owen began counting, “twenty-twooo, fourty-fourrr, sixty-sixx” (Turns 161-162, Table 33). At this point, Eric drew on contextual understanding, noting not all classes had the same number of students (Turns 162-163, Table 33).

Table 33. Group 1 Sample Dialogue – Countering Model Development with the Context

Turn	Everet	Owen	Wren	Eric
161		*[looking up towards the ceiling] [slowly]* twenty-twooo, fourty-fourrr,		
162		sixty-sixx		Well, no.
163		*[accidentally flicks his pencil to the floor, moves chair back and leans back to retrieve it]*		You're adding up just our class. Don't count our class because y-, like, Ms. White's might have 21 in their whole class when everybody's there.
164	*[looking at Eric]* Ms. Hen-Ms. Henderson has exactly 21.	*[emphatically* That is truuueee. *[moves his chair a little away from the table so that he is essentially sitting next to Wren]*		

Eric and Everet identified the number of students in each of the three classes (21, 22, and 24) and engaged in the competencies of *building a model* and *generating a solution* by constructing and evaluating a mathematical relationship using addition (Turns 173-178, Table 34). At first, they computed $20 + 22 = 42$, $42 + 1 = 43$, $43 + 24 = 65$, but Eric immediately checked their work (Turns 180-188, Table 34) and they calculated there were 67 students in the fourth grade.

Table 34. Group 1 Sample Dialogue – Developing Model Solutions

Turn	Everet	Owen	Wren	Eric
170	She has somewhere around 24.			
171	She has, like, 24-			So and then we could
172				add all of those up and w- it should be easy.
173		*[looking at the pencil in his left hand]* I thought, um, Ms. Henderson had the exact same number as us in her class.		20- 22, well, 22 plus 21. 20 plus 22 equals 42,
174				plus one would equal
175	43.			
176		*[looks towards Wren]* That would make it a lot easier though.		*[looking towards Eric]* 43. Plus 24 would beee?
177	65.			
178	*[loudly, excited]* Heyyyyy!			Yeah. *[almost surprised]* 65 ↑ *[lifts both hands, palms up towards Everet]*
179	*[arms open and out to the sides of his body, looks towards Owen]* Hey, it gets 65!			
180				Does it? Wait, let's see. *[looks back towards his paper]*
181	*[emphatically, half standing out of his seat, leans over the table towards Eric]* Yes, it does, bro. Just payy ↓ …		*[looking at Eric's paper, then writing on her paper]*	
182		*[pointing at something in the center of the table]* That's a weird side.		*[looking up towards Everet]* 20- and what?
183	24.			
184				24?
185	*[kneeling on his chair leaning over the table]* That's like bags that			
186	I know of-	[szszszszszs sound]		
187			*[still looking to Eric's paper and then writing on her own paper]*	*[writing and looking towards Everet]* Okay, so two plus one would obviously be three, seven, and that equals s-seven?

Table 34. Continued.

Turn	Everet	Owen	Wren	Eric
188	Yeah, seven, uh, 67. I, I w-			
189	↑ Hey ↑ yyy,		*[writing on her paper]*	*[arms out to his sides, elbows in towards his torso, palms up, slight shrug]* Hey, that's still close.
190	two off, not bad.			

As Group 1 continued their modeling work, they engaged in *building models* and *generating solutions* by developing and evaluating mathematical relationships to address the overarching modeling question, but did so without connecting their mathematics to the context of the situation. This disconnect became evident when Everet suggested they "do 65 times 30," (Turns 193-195, Table 35), and the group pursued this idea despite their earlier agreement on class sizes and rope lengths. The mathematical model of 65×30 was misaligned with the constraint given by Ms. Kedrik that they could not have too many extra jump ropes, and it also deviated from their contextual assumptions that no students needed a nine-foot jump rope, most needed an eight-foot rope, and some needed a seven-foot rope.

Table 35. Group 1 Sample Dialogue – Disconnecting Mathematics from the Context

Turn	Everet	Owen	Wren	Eric
191	All right,	Ok		
192	soo ↑		*[looks away from the table and out towards the class]*	*[looks away from the table and calls to Johnny]* Johnnnyy ↑
193	Now we've gotta do 65-			
194		[bowwowow sound]		
195	now we've gotta do 65 times 30.			
(4 seconds)				
196				Well, I don't know what that is. *[quietly, with a slight chuckle]* No I'm just kidding-
197	So what's 65- *[looks towards Owen for a second]*, *[speaks with his hand over his mouth]* just do 65 times 10, plus 65 times 10, plus 65 plus 10 *[mouth open smile towards Eric]*		*[turns back towards the table]*	

The resulting mathematical model and solution, $65 \times 30 = 1{,}950$, not only deviated from the established number of students (67) but, more seriously, departed from their earlier contextually grounded assumptions and mathematical work regarding jump rope size and bags per jump rope.

In setting up and evaluating $65 \times 30 = 1{,}950$, the group engaged exclusively in *model building* and *solution generation* without reconnecting their mathematics to the context of the problem – they did not engage in *interpreting* or *analyzing their model* and did not revisit their contextual understandings. Turns 221-224 in Table 36 illustrate their decision to compute 65×30 by breaking 30 into three groups of ten. In Turns 230-242 (Table 36) Everet and Eric have computed 65×10 three times, obtaining 650, 650, and 650, which they then added together to compute their total of 1,950 (Turns 244-248, Table 36).

Table 36. Group 1 Sample Dialogue – Evaluating Mathematical Relationships

Turn	Everet	Owen	Wren	Eric
221	*[looking towards Owen]* I knowwww. We're doing 65 times 10.			
222		*[silently opens mouth very wide as if to say "ohhhh"]*		
223	*[looking towards Eric]*			65 times 30.
224	I know, but we're breaking it up.		*[writing on her paper]*	*[bends over his paper]*
225	*[throws pencil in the air, doesn't catch it, pencil lands on the floor and he leans over to pick it up]* [undecipherable]	Yeahhhh, I don't think I'll gonna, be able	*[writing on her paper and glancing over at Eric's paper]* *[quietly]* So it's 65 times 10?	
226		to help youuu with that. Ummm, I'm gonna run away,		
227				I got an idea.
228			*[looking at her paper and writing]* Sooo	
(3 seconds)				
229			*[questioningly]* wouldn't it -	
230	*[looking towards Eric]* It's just 65 times-		*[writing on her paper]*	*[looking towards Everet]* It's nine hundred and f-
231				it's 950.
232		Wait.		
233	*[elbows on the table, moving arms straight up and down, hands straight out]* No, 'cause			
234	65	[begins to say something]		
235	times 10 is 650-			
236		*[emphatically and loudly]* Just add a		
237	plus - I diiiddd, okay? *[puts head in hands for a moment]*	zero at the enndd! *[extends both arms straight out towards the center of the table]*		
238	650.			No, no, no.
239	*[looking down towards the table]*	*[looking towards Everet]* attach.	*[writing on her paper]*	*[looking towards Everet]* 'Cause all you
240				have to do is add, if you
241	*[writing while speaking]* 650,		*[pauses writing, looks towards Eric]*	know 65-

Table 36. Continued.

Turn	Everet	Owen	Wren	Eric
242	650, 650.			
243				Ohhh, that's
244	You do zero,		*[writing on her paper]*	*[leans over his paper]* true. That's-
245	five, one and then 603 is 18 plus one, is			
246	1,950.			Yeah, okay.
247		*[playing with his pencil in his mouth]*		*[looks up from paper towards Everet]* 1,950 would be our
248			*[looking at her paper]* So like-	answer probably.
249			*[looks up a little from her paper]* 65	
250		*[holds up his paper in the direction of Eric and points with his pencil]* I was really off.	times 10	*[looks towards Owen's paper]*
251	*[writing and erasing on his paper]*		is?	
252		*[looking towards Wren]*	*[looks towards Owen]* So 65 times 10 is?	
253		65		
254		times 10. *[in a lively tone of voice]* Attach		*[looks away from the table and then back to the table]* So yeah, I think Everet's right ↓.
255		a zero at the end ↑!		

They *interpreted* their mathematical solution in terms of the context – 1,950 bags; but as with their earlier claim that "28 bags equals eight-foot rope," this final solution was ultimately misaligned with the problem situation.

At this point, Ms. Kedrik intervened. Through a series of questions and statements, she supported the students in *interpreting* and *analyzing* their mathematical work in light of the task context. Her prompts included:

How did you get that? How many bags in an eight-foot rope? Wren does not have 30, she has 24. How did you get 24? How many bags in one foot of rope? So in eight feet, how many would that be?

These prompts highlighted the misalignments between the group's mathematical calculations and their contextual reasoning, helping students recognize that their mathematical work did not follow from their initial assumptions and therefore did not address the problem scenario. Through this questioning, Ms. Kedrik guided students to reconnect their mathematical work to their assumption of three bags per foot of rope, leading them to determine that an eight-foot rope required 24 bags.

When the students re-engaged with the context of the situation by *interpreting* their model components in relation to the task, they recognized the need to *revise their model* to better align with the problem. They realized that the initial model, 65×30, meant that each jump rope consisted of 30 bags – too many for an eight-foot jump rope. In revising their model, the group engaged in *making assumptions* by deciding to use 65 total students (rather than 67) as a friendly number. They also revised their mathematical model by incorporating the information that a seven-foot jump rope required 21 bags and an eight-foot rope required 24 bags, adjusting their plan to ensure they did not have too many extra jump ropes.

Group 1 then set up and computed their revised model and solution incrementally (see Table 37). They multiplied $21 \times 20 = 420$, representing the number of bags needed for 20 students with the shorter ropes, and then $45 \times 24 = 820$ (Turn 309, Table 37), for 45 students with the longer ropes. Recall that at the time of this lesson, students had not yet learned multidigit multiplication. Their strategy for computing 45×24 was to multiply the digits in the

tens place, 4×2, multiply the digits in the ones place, 5×4, and then combined the two products, 8 and 20, to determine that $45 \times 24 = 820$. Adding 420 to 820, they determined that they would need a total of 1,240 bags – engaging in model recontextualization competencies by *interpreting* their revised solution in relation to the problem context (Turns 314–322, Table 37).

Table 37. Group 1 Sample Dialogue – Reconnecting Model Development with the Context

Turn	Everet	Owen	Wren	Eric
309			*[writing on her paper and looking over at Eric's paper]*	I know. If I know my basic, do the basic facts, if I know five times four is 20, well then, you've got a 20 here *[writing on his paper]*. And then if I know four times two is eight, so you've got 820 *[drops pencil on the table and leans back in his seat and looks towards Everet]*.
310	*[quietly]* Yep *[leans down and writes on his paper]*.			
311		*[addressing Eric]* Why does	*[very quietly]* Owen. *[pushes her paper towards Owen and points to something on the paper]*	
312		Lydia keep saying my name?		
313			*[very quietly, taps finger on the paper]* Owen.	
314		*[putting on his hat which falls off his head, looks towards Wren]*		*[writing on his paper while speaking]* So at 820...
315		What?		
316			*[indicating on her paper with her pencil]* What's the one that-	
317		*[gets his hat on his head, looks towards Wren's paper]*		... plus 420
318		Oh *[picks up his pencil and writes on his paper]*		
319		*[violently erases his paper so that the pencil squeaks against the page]*	*[looks over towards Eric's paper]*	would be-
(4 seconds)				
320		All this math.	*[writing on her paper]*	

Table 37. Continued.

Turn	Everet	Owen	Wren	Eric
321				1,240.
322	*[writing on his paper]*	*[getting up and leaving the table]* And my eraser broke, geesh.	*[looking over towards Eric's paper then writing on her paper]*	It would take 1,240 bags.

However, during whole-class *reporting on their model*, Group 1 presented a final solution of 1,530 bags instead of 1,240 (Figures 10 and 11). Figure 10 shows that the earlier result of $820 + 420 = 1{,}240$ had been erased, with the final answer recorded as 1,530 bags, while Figure 11 depicts their written work with 1,240 bags crossed out in favor of 1,530 bags. They may have reported a total of 1,530 bags as the previous group (not Group 4) reported a total of 1,530 bags.

Figure 10. Group 1's Mathematical Modeling Work – Sample 1

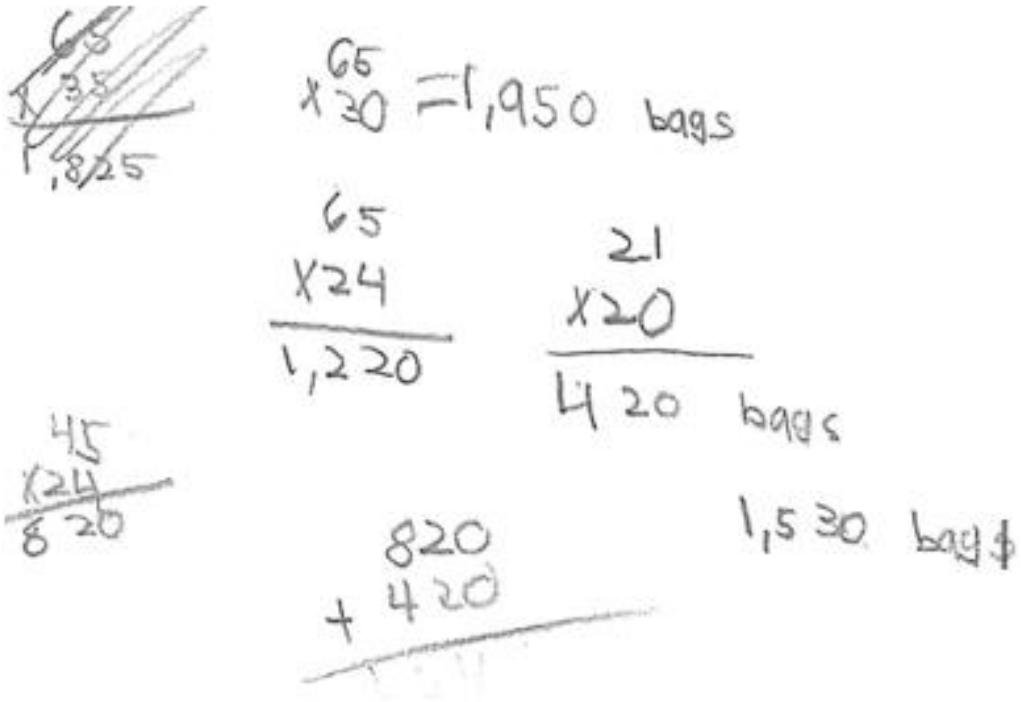
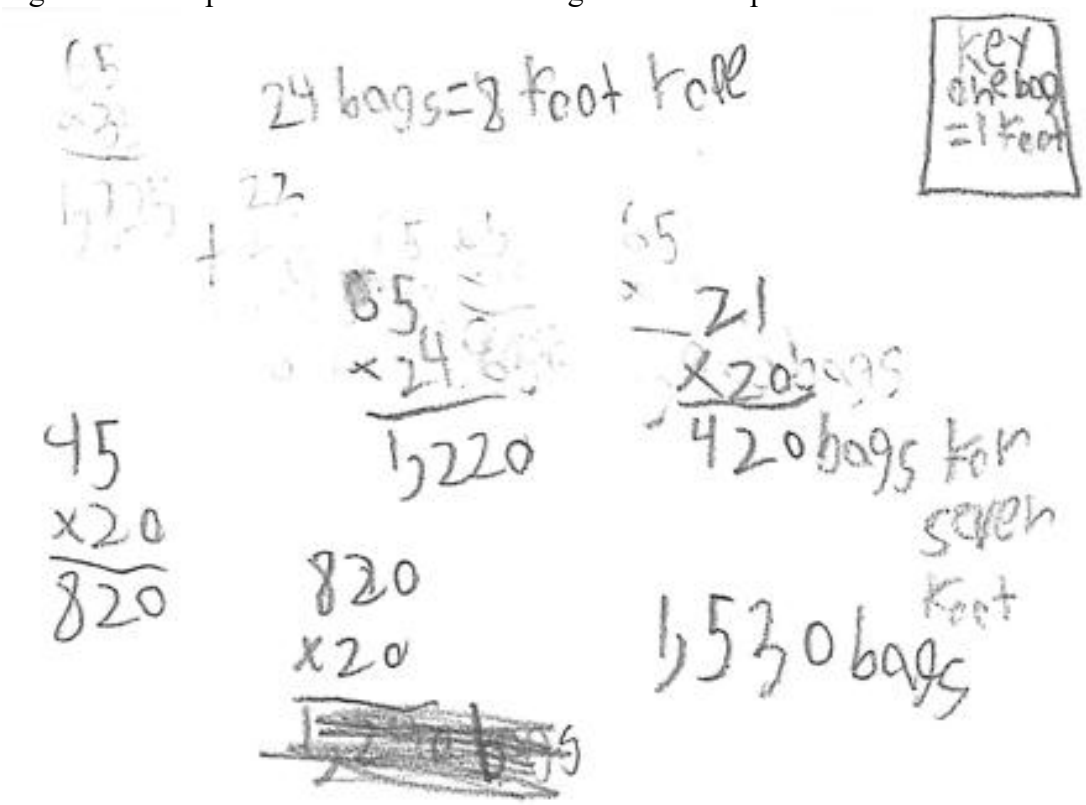


Figure 11. Group 1's Mathematical Modeling Work – Sample 2



In the discussion, the group explained that they had initially developed a model of 65×30, yielding 1,950 bags, but then recognized the need to revise their model to reflect the number of bags in seven- and eight-foot jump ropes. They explained their reasoning and discussed with their peers and Ms. Kedrik how to refine their calculations. Through recontextualizing their mathematical work, the group's revised model came to align with their assumptions and the problem's constraints.

Group 1: Generalized Representation of the Model. As with Group 4, Group 1 developed their final mathematical model incrementally, building it piece by piece using the assumption of three bags per foot of rope. Although not explicitly stated, they applied this assumption as an initial parameter in the relationship $3x = y$, where the independent variable, x, represents rope

length in feet and the dependent variable, y, represents the total number of bags per rope. For a seven-foot rope, $x = 7$, they calculated $y = 21$ bags, and for an eight-foot rope, $x = 8$, they calculated $y = 24$ bags. These computed values then served as parameters for their subsequent modeling work.

While Group 1 did not develop a generalized form of their model, their final, specific model could be expressed more generally as:

$$21(x_{1,t}) + 24(x_{2,t}) = \mathrm{y_t}$$

where the independent variables $x_{1,t}$ and $x_{2,t}$ represent, respectively, the total number of fourth-grade students needing seven-foot ropes and eight-foot ropes at the school. The dependent variable, $\mathrm{y_t}$, represents the total number of plastic bags needed for the entire fourth-grade. The values 21 and 24, determined from the initial parameter of three bags per foot of rope, functioned as fixed parameters in this generalized representation of their model.

Conclusion: Group 1's Inconsistent Contextual Grounding in Modeling. In conclusion, Group 1 infrequently grounded their mathematical thinking in the task context, initially constructing a model misaligned with the situation but later revised after reconnecting with the context through Ms. Kedrik's support. Although both student groups began their modeling work by consistently linking their mathematics to the context, they diverged in their contextual engagement with building models and generating solutions. Group 4 consistently reconnected with the task context through contextual consideration and model recontextualization competencies, whereas Group 1 focused primarily on the mathematics with limited attention to context. With Ms. Kedrik's guidance, Group 1 re-engaged with the context and revised their model. Overall, students' connection between the mathematics and the context of the situation

illustrates how attending to context during mathematical work can serve as a scaffold for developing context-aligned mathematical relationships and generating solutions that effectively address the problem of the situation.

Synthesis of Contextually Grounded Mathematical Modeling

Overall, the unfolding of each group's modeling conversations illustrates that attention to the task context supported students' mathematical work as they engaged in *developing models* and solutions. When students disconnected from the context and focused solely on their mathematical work they produced models and solutions that were misaligned with the context and ill-equipped to address the overarching problem scenario. In contrast, when students consistently connected their mathematics to relevant aspects of the problem context, their final models and solutions were grounded in the context, aligned with contextually formulated assumptions, and effectively addressed the problem scenario.

Finding 2: Positional Authority in Modeling Conversations

In this modeling context of upcycling plastic bags to make jump ropes, student interactions during group work produced two types of conversations: modeling and social (non-modeling). Conversations were constructed through interactions (verbal and nonverbal) between participants engaged in a dialogic episode. As students interacted with one another, they took up and discarded various positions – discursively produced locations in conversation. Over the course of the modeling lesson, students enacted five categories of positions: contributor (process, context, modeling, mathematics, and social), evaluator (instructor, task corrector, social corrector), recipient (learner, task wrongdoer, social wrongdoer), supporter (context, modeling,

mathematics), and viewer (observer). Underlying the enactment of these positions were five categories of interactional moves: affiliative, guiding, inquiry, knowledge, and oppositional. In conversation, students used these interactional moves to adopt, sustain, or discard positions, and in doing so, shaped the group's conversation through their enactment of the position in relation to the context of the conversation. Primarily addressing the second sub-research question – *How do student positions influence their engagement in mathematical modeling competencies?* – the findings indicate that some positions carried greater authority, which shaped students' access to modeling conversations and their engagement in mathematical modeling competencies.

Positional authority refers to the conversational influence students exert over the group's modeling process – indicated by and shaping who engages in modeling competencies, whose ideas are taken up, whose voices are heard, and who is positioned as a source of knowledge. Positional authority within group conversations emerged through student interactions – interactions related to the positions students enacted, and in particular their interruptions and questions. This authority was fluid, shifting as students enacted positions through various interactional moves during conversation. The findings indicate that positional authority was not only associated with the positions that influenced the modeling conversation through engagement in modeling competencies but could also be tied to the students who enacted these positions. The following sections illustrate how students exercised positional authority through their fluid enactment of positions and their use of authority-sharing and authority-reinforcing interactional mechanisms – specifically interruptions and questions – that both shaped and reflected their positional authority in modeling conversations and their engagement in modeling competencies.

Positions as Sources of Positional Authority

Of the five contributor positions, students took up the social contributor position by participating in social (non-modeling) conversations, whereas the other four contributor positions – process, context, modeling, and mathematics – were enacted in response to various aspects of the modeling task. These four positions carried the greatest positional authority within modeling conversations. While enacting these task-related positions, students shaped the modeling conversation through engagement in modeling competencies, communicating their ideas either by *mapping* the direction of the work as a process contributor, or by assuming one of the context, mathematics, and modeling contributor positions by *sharing* their thinking and advancing the modeling process. In contrast, the observer position reflected a shift into a silent presence during conversation; thus, while enacting this position, students held minimal influence over the modeling conversation and correspondingly lower positional authority. The findings presented in the following paragraphs illustrate that positional authority was not only associated with the positions students assumed but was also tied to the students enacting these positions.

Higher-Authority Positions in Modeling. Although the process contributor position was rarely adopted, students enacting this position exercised the greatest authority in their group's modeling work. Students taking up this position assumed the authority to shape the direction of their group's modeling work by *directing* and *mapping* the next steps of the modeling process. This authority was acknowledged by the other group members as they carried out the steps specified by the process contributor. The students who enacted this position – Everet and Lydia – held the most influence over their group's modeling conversations and thus exercised the greatest

levels of positional authority. Importantly, this authority was not only tied to the position itself but also attached to the students who enacted it.

Everet (Group 1) enacted the process contributor position twice. The first instance occurred at the beginning of the lesson when the group was estimating the total number of plastic bags they needed. Using the interactional move, *directing,* he told the group they would calculate "65 times 30" (Turn 59, Appendix C, Table C1) to estimate the total number of plastic bags, and the other group members followed his lead. Later in the lesson when Group 1 was building their model, Everet again assumed the process contributor position. In this instance, Group 1 worked with the context of the situation to make assumptions and perform preliminary calculations. They assumed, along with their class, that three bags produced one foot of rope, that most students would need an eight-foot jump rope, and that no students would require a nine-foot rope. They calculated 21 bags were required for a seven-foot rope and that there were 67 students in the fourth grade (Table 34). However, despite this contextual information, Everet enacted the process contributor position, *mapping* the group's direction by saying that they needed to do 65 times 30 (Turns 191-195, Table 35). No student in the group questioned Everet's decision and the group proceeded to carry out the calculation, even though it did not follow from their previous work. Only when Ms. Kedrik intervened did the group revise their model by aligning their mathematical work with their contextual understandings.

In Group 4, Lydia enacted the process contributor position by *mapping* the structure of their modeling approach: "So. We need to realize how much – Bags, um, we need in our classroom, first, then we can go on to the fourth grade – or the whole fourth grade" (Turn 80, Appendix C, Table C2). Her initial mapping of the group's modeling work resulted in

engagement in the competency of building a model and shaped all subsequent modeling endeavors, as the group proceeded to construct their model in exactly the way Lydia described. This is demonstrated in Finding 1 which presents Group 4 incrementally constructing their model by first determining the number of bags required for their class and then scaling up by three to predict the number needed for the fourth grade.

These instances of Everet and Lydia taking up the process contributor position demonstrate that, although enacted infrequently, this position carried significant influence over the direction of their groups' modeling work – and consequently, considerable positional authority. This authority was not only assumed by the student enacting the process contributor position but was also acknowledged by the other group members, who did not contest Everet or Lydia's mappings and instead followed the step(s) they outlined.

After the process contributor position, context, mathematics, and modeling contributor positions were higher-authority positions in modeling conversations as students enacting these positions held the greatest influence over the modeling conversation. It was through these three positions that students actively participated in advancing the modeling process by *sharing* their ideas and directly engaging in modeling competencies. In *sharing* their ideas, students enacting context, mathematics, and modeling contributor positions shaped the modeling discussion and consequently, the modeling work. For example, students in both groups enacted context and modeling contributor positions by sharing their thinking about which students would require a shorter jump rope. Through these sharings, they engaged in *making assumptions* and influenced the number of students the group decided to include in this category – thereby shaping the unfolding modeling work. Another example illustrates how Everet, enacting the modeling

contributor position, shared that three bags produce one foot of rope and so 21 bags would produce a seven-foot rope (Table 31). Through this contribution, he engaged in the competencies of building and interpreting the model, influencing the group's modeling work as they later incorporated his contribution into their model to predict the total number of plastic bags needed for the fourth-grade (see Figures 10 and 11, where Group 1's mathematical modeling work shows multiplying 21 bags by 20 students). As portrayed in these examples, it was often through their enactment of the context, modeling, and mathematics contributor positions that students engaged in modeling competencies and shaped their groups' modeling conversations.

Moderate to Lower-Authority Positions in Modeling. The instructor and learner positions were often enacted together – either when a student assumed the instructor position (often through the interactional move *explaining*) and positioned another as learner, or vice versa – when a student adopted the learner position (often through *clarifying* or *confirming* interactions) while positioning another as an instructor. In either positioning, the instructor position was associated with moderate positional authority while the learner position corresponded to lower positional authority. However, it often happened that the students enacting the position of instructor were given positional authority by those asking for *clarification* or *confirmation* of their thinking. In these instances, the instructor was positioned by the learner as a knowledgeable authority. Thus, the learner ceded positional authority to the other by positioning them as one who had knowledge to share. The example illustrated in Table 38 depicts Jason enacting the learner position through the question "So it's not 17 times 24?" on Turn 185 while placing Lydia in the position of instructor – which she assumes on Turn 186 by explaining the work to Jason. Through his question, Jason engaged in generating a solution by asking about their original

mathematical work of 17 times 24, while Lydia engaged in this competency through her detailed explanation of the mathematical relationships she used to determine the solution to the problem. Both of these examples illustrate how Jason and Lydia engaged in generating a solution through their respective positions of learner and instructor.

Table 38. Sample Dialogue – Learner and Instructor Positional Authority

Turn	Brittany	Lydia	Jason
183		We've got	Okay, so what are we-
184		these numbers, so we've got-	
185			*[picks up his pencil]* So it's not 17 times 24 ↑?
186	*[looks away from the table towards another table but eventually turns back to see what Lydia is writing, then writes on her paper]*	No, not anymore. It, it, wait. Okay. So it's *[flips her paper around so Jason can read it from his view. Explains and demonstrates with the paper while speaking]* 17 times 24, which I would equal 408. Then, twel- 21 times five, would equal 106. *[waits for Jason to record on his paper]* Okay, now we're gonna-now we're gonna add *[flips her paper back around and writes on her paper while speaking]* 408 plus ↑ one-oh-six, which that would equal ↑, four, one, annnd five. 514. Now, we're gonna add 514 plu-times, *[pauses]* times ↑ three, would equal 1,532.	*[writes on his paper]*

Students enacting either the social corrector or task corrector positions assumed the authority to correct another's ideas or behaviors – positioning themselves as right and the other as wrong. Sometimes this authority was acknowledged, and the positioning was accepted, such as when Owen accepted the position of task wrongdoer (Turn 164, Table 33) after Eric corrected his strategy for determining the number of people in their grade (Turns 161-162, Table 33). Other times, the positioning of task wrongdoer was resisted, as when Owen's corrected the group's

work saying that the group should not be doing the math yet (Turn 102, Table 39), but Eric rejected this positioning and continued his engagement in *generating a solution* from his position as instructor – by explaining how to determine 65 times 30 (Turns 104-108, Table 39).

Table 39. Sample Dialogue – Rejecting Positioning

Turn	Everet	Owen	Wren	Eric
95		*[first leaning towards Wren and then looking towards Everet]* I say 400 - because it's a guess.	*[stops drawing on her shoe and resituates herself in her seat to face the table and leans into the table]*	
96	*[Everet does not look at Owen but looks at Eric]*		*[picks up pencil and begins writing on her paper]*	*[immediately after Owen stops speaking]* No *[writing on his paper]*
97	*[immediately after Eric says "no"]* How do you do			
98	60 times 30?	*[somewhat emphatically, but not loudly. The words are well annunciated]* No, it's a guess,		
99		guys. Guys, it's a guess. We don't do the math		
100		just yet ↓.		*[looks towards Everet]* Well, let's do our basic
101				facts. If I know five times
102	Wait,			five is tw-
103	but we don't use			
104	the math just yet.			If I know,
105		*[looking towards Eric]* Yeah, we don't use the math just yet.	*[still writing on her paper]*	if I know five times fiiive- if I i-if I know *[looking towards Everet]*
106				five times five is 25, six times three
107	*[writing on his paper]* I'm just			is 18.
108	gonna say 1,189.			*[writing on his paper]*

Collectively, the social contributor, supporter, and observer positions were associated with lower positional authority in modeling conversations. Students enacting the social contributor position focused their interactions on topics unrelated to the modeling work (e.g., Brittany: "Look how long my hair is. Except when it's in a ponytail and it's sticking out because

I have so much of it," Turns 83-84, Appendix C, Table C2) and so did not engage with modeling competencies. Students enacting a supporter position supported others without assuming authority for themselves. Their contributions to the conversation often took shape through *affiliative* interactions – acknowledging, revoicing, or echoing another's idea rather than proffering their own thinking to the group (e.g., Everet saying, "How many jump ropes [plastic bags] you need to make one rope? Which I already know the answer to, but you would have to ask it," Turn 1, Appendix C, Table C1; Eric agreeing, "Yeah. You could just be like, "Hey Alexa, how many bags?" Turn 2, Appendix C, Table C1). While acting as observers, students maintained a silent presence in the conversation, contributing neither verbally nor nonverbally to the group's modeling work (e.g., Brittany in Table 30). Overall, when enacting social contributor, supporter, and observer positions, students minimally (if at all) engaged in modeling competencies and so exercised little to no influence over the modeling conversation, reflecting the lower positional authority associated with these positions.

Conclusion: Positions as Sources of Positional Authority. Although the process contributor position was enacted only briefly, it carried the greatest authority by actively shaping the direction of the group's modeling endeavors, which were then carried out through the modeling conversations. More commonly, students assumed the higher-authority positions of context, mathematics, and modeling contributor to advance their group's modeling work. Students who consistently enacted these positions demonstrated sustained engagement in modeling competencies and played active roles in developing and shaping the modeling conversation. By repeatedly taking up these positions, they maintained influence over the group's modeling process and reinforced their positional authority. Having examined the positions

students enacted and their corresponding levels of positional authority, I next illustrate how this authority was both reflected and shaped through students' use of interruptions and questions during modeling conversations. For students, interruptions and questions functioned as interactional mechanisms that could share or reinforce positional authority.

Interruptions and Positional Authority in Modeling

Interruptions emerged as a key mechanism through which students accessed the conversation, engaged in modeling competencies, and influenced the direction of the group's work. In this way, interruptions both reflected and exercised students' positional authority, allowing students to enter, sustain, or redirect the modeling discussion. Three primary types of interruptions occurred during group conversations: interjecting (inserting ideas without fully silencing another speaker), cutting off (taking the conversational floor), and transitional speech (speaking while another was finishing, resulting in overlap). Although occasional nonverbal interruptions occurred, most instances involved verbal interaction. Students enacting higher-authority positions primarily interrupted by *sharing* their own thinking, often reinforcing their own positional authority, whereas students enacting lower-authority positions tended to interrupt through *clarifying* or *confirming* questions. These question-based interruptions typically reinforced the authority of the peer being questioned by positioning them as the knowledge holder. Overall, interruptions served as both authority-sharing and authority-reinforcing mechanisms, though the latter was far more common. The findings suggest that interruptions not only reflected students' positional authority but also served as a resource for exercising it, operating as a mechanism through which students shared or reinforced authority and shaped their engagement in modeling competencies.

Interruptions as Authority-Sharing Mechanisms. Interruptions functioned as authority-sharing mechanisms when students used them to support, extend, or collaboratively build on one another's ideas. These instances reflected relatively balanced positional authority between the students engaged in the dialogic episode, where authority-sharing interruptions enabled students to collaboratively advance the modeling work and sustain mutual engagement in modeling competencies.

Authority-sharing interruptions occurred primarily between Everet (Group 1) and Eric (Group 1), who carried the bulk of their group's modeling conversation. Both students frequently interrupted each other from higher-authority contributor positions, most often using the interactional move of *sharing* and occasionally *guiding* and *oppositional* moves. Their mutual interruptions positioned them as their group's primary drivers of the modeling work, sustaining and sharing positional authority through active engagement in modeling competencies such as generating solutions, identifying quantities, and interpreting the model.

For instance, Table 40 shows Everet and Eric interrupting each other (Turns 168 and 171, Table 40) while engaged in the competency of generating a solution by collaboratively computing the number of students in the fourth grade.

Table 40. Sample Dialogue – Sharing Positional Authority by Interrupting

Turn	Everet	Owen	Wren	Eric
167		*[scoots his chair closer to his original location at the table]*		*[gestures with arms open out to his sides]* All right, so we can add that up with-
168	And then Ms. White has 24, I think.			
169				Okay.
170	She has somewhere around 24.			
171	She has, like, 24-			So and then we could
172				add all of those up and w- it should be easy.
173		*[looking at the pencil in his left hand]* I thought, um, Ms. Henderson had the exact same number as us in her class.		20- 22, well, 22 plus 21. 20 plus 22 equals 42,
174				plus one would equal
175	43.			
176		*[looks towards Wren]* That would make it a lot easier though.		*[looking towards Eric]* 43. Plus 24 would beee?
177	65.			

While Owen's attempt to contribute was unacknowledged (Turn 173, Table 40), Everet and Eric maintained a continuous exchange, demonstrating their shared authority through sustained engagement in modeling work through the competency of generating a solution. However, as illustrated in the following section, while interruptions sometimes served as authority-sharing interactions, they were more often a way for students to reinforce their own or another's authority within group modeling conversations.

Interruptions as Authority-Reinforcing Mechanism. In modeling conversations, interruptions frequently functioned as authority-reinforcing mechanisms when they advanced one's own ideas, positioned another as an instructor, or silenced another group member – actions that maintained or strengthened a student's positional authority. Additionally, how a student

managed being interrupted could also reinforce their own positional authority when they maintained connection with the modeling conversation through engagement in modeling competencies.

Lydia's (Group 4) management of interruptions illustrates how interruptions can serve as authority-reinforcing mechanisms. She was the only student to enact the process contributor position and frequently acted as a mathematics, modeling, or context contributor. Her sustained engagement in modeling competencies through these positions gave her the greatest positional authority and voice within the group's modeling work – even when interrupted. While being the most often interrupted student in Group 4, Lydia's treatment of these interruptions served to reinforce her own authority – she persisted with her engagement in modeling competencies, advancing the modeling conversation, while generally disregarding Jason and Brittany's social exchanges. On occasion, Lydia exercised her positional authority by interrupting Jason and Brittany. For example, while she and Brittany were both enacting the context contributor position making assumptions about Brittany's height, Brittany playfully shifted into the social contributor position thanking the world for making her taller than Lydia (Turn 130, Table 41).

Table 41. Sample Dialogue – Exercising Positional Authority with Interrupting

Turn	Brittany	Lydia	Jason
124			Definitely Wren.
125		Yeah, definitely Wren	
126	I'm definitely taller than Wren.	and Belle	*[sitting back in his chair at an angle to the table so he is half facing the back of the room while playing with a ruler]*
127		Me too.	
128	I might be taller than you.		
129		Yeah you probably are.	
130	*[leaning into the audio markers and annunciating each word]* Thank you, world for making me taller than Lydia. *[smiling and looking towards Lydia and moves her right arm in a circular motion mimicking hypnosis]*	*[looking towards Brittany]*	
131		Why?	
132	*[very quietly and hypnotically]* [undecipherable: *murmuring*] than Lydia, Lydia	Okay, but anyways- So, seven *[pauses]* seven-foot	
133	What?	would be	
134	*[begins playing with jump rope again]*	-so, seven, no, yes, no, no- *[writing on her paper while speaking out loud]*	
135	My spider like this.		

On Turn 132 (Table 41), Lydia interrupted Brittany by cutting her off, assuming the modeling contributor position and refocusing the conversation back to the modeling work. This example illustrates how Lydia's interruptions reinforced her positional authority and sustained her engagement in the competency of building the model.

Although Everet and Eric often used interruptions collaboratively, Everet exercised slightly greater authority by more frequently cutting off Eric to advance his own ideas. As such, interruptions reinforced Everet's authority in the group. Eric, by contrast, tended to use interjections or transitions to support, challenge, or extend Everet's thinking. For instance, Everet, enacting the modeling contributor position, cut Eric off by *sharing* that 21 bags were

needed for a seven-foot jump rope (Turn 128, Tabe 31), thereby engaging in the modeling competencies of generating a solution and interpreting the model. Another example includes Eric, acting as a context contributor, interjected by *sharing* the name of a student who would need a seven-foot rope while Everet was still speaking (Turn 37, Appendix C, Table C1). Another time, Eric, a modeling contributor, interrupted Everet (also a modeling contributor) with a transition – first *agreeing* with him and then *sharing* his own thinking: "Mo- yeah, most people. Unless there might be, like, maybee, maybe one" other person who would need an eight-foot jump rope (Turns 34-35, Appendix C, Table C1). These exchanges highlight how interruptions between Everet and Eric reflected and reinforced Everet's slightly greater authority within the modeling conversation.

Jason's (Group 4) interruptions reflected his lower authority in the modeling conversation, but also his higher authority in social interactions. He interrupted both Lydia and Brittany, sometimes by *negating* ideas or *sharing* his own as a modeling contributor, but most often by positioning himself as a learner and Lydia as an instructor through *clarifying* or *confirming* questions. While these interruptions occasionally allowed him to engage in modeling competencies by advancing the modeling work through his own ideas, they more often served as a way for him to engage with modeling ideas generated by another – particularly in competencies such as making assumptions, identifying quantities, and generating solutions. For instance, Jason assumed the learner position by interrupting Lydia with a *clarifying* question, "So it's not 17 times 24?" Through this question, he engaged in the competency of generating a solution by asking about Lydia's idea to include 17 students in the eight-foot jump rope category – while simultaneously reinforcing her authority by positioning her as an instructor and

knowledge holder. At other times, his interruptions were social, such as correcting Brittany for lifting the table or Lydia for her pronunciation of "figure." Collectively, these examples illustrate that Jason's interruptions reflected his more limited positional authority in modeling conversations as his interruptions often reinforced Lydia's positional authority in modeling conversations and provided him constrained engagement in modeling competencies.

Owen's (Group 1) interruptions were sporadic, granting him only brief influence in the modeling conversation. His interruptions came from a wide range of positions, including mathematics contributor, modeling contributor, task corrector, learner, social wrongdoer, and occasionally, task wrongdoer. He executed interruptions through interactions such as confirming, countering, agreeing, and directing, but these interruptions generally granted him limited and unsustained access to the group's conversation, as his contributions were often overruled by Everet or Eric. For instance, using the interactional move of *countering* (Turns 141-143, 146 Table 42), Owen assumed the modeling contributor position and interrupted Everet and Eric by insisting, "I have an idea, I have an idea," before *sharing* his thinking about how to calculate the number of students in their grade level – engaging in building the model and generating a solution (Turns 146-149, Table 42).

Table 42. Sample Dialogue – Interruptions Granting Limited Access to the Conversation

Turn	Everet	Owen	Wren	Eric
141		*[quickly]* I think		
142	↓ equals eight-foot rope	↓ *[quietly and quickly]* wait-w-w-w-w-wait	*[looks to Owen]*	
143		I have an idea, I have an idea.		
144	*[stops writing and looks up to Owen]* [screech sound]			
145				*[addressing Everet]* What was that?
146		I have an idea. We need to figure out how many kids are in our class		
147	*[writing on his paper]*	and then we could-	*[looking at Eric]* We already know!	
148				We already know.
149		*[looks at his table group members]* Thenn ↓, umm ↓ *[hesitates]* then um, you can count, um, byy ↑ those kids in our class, all to the other classes.	*[playing with the top of her water bottle]*	
150	*[looking at Owen]* [screech sound]			
151		*[looks at Eric]*		*[with a laugh in his voice]* Whoa, okay.
152		*[looking between Everet and Eric]* Is that a good idea or no?		Guys, we
153	[more quiet screech sound]			need to stay on focused.

A short time later, Eric interrupted to correct Owen's approach (Turns 162–163, Table 43), engaging in identifying quantities and building the model while using this interruption to reinforce his own authority within the conversation. Owen responded to the correction by shifting into the task wrongdoer position by *agreeing* with Eric (Turn 164, Table 43) and then *disengaged* from modeling competencies.

Table 43. Sample Dialogue – Interruptions Reinforcing the Distribution of Positional Authority

Turn	Everet	Owen	Wren	Eric
154	*[looking at Owen]* Umm, *[smiling, sounds as though he is speaking partially through his teeth]* I don't know if that's a good idea - you figure that out yourself. No, so, well-	*[looking at Everet]*		
155				*[turning away from the table and quietly calls to another student]* Johnnyy ↑ .
156	It would, it's a good idea, it's just kind of hard to figure out how many are in the other classes. *[points towards the classroom door]*			
157		How many are in this class right now?		
158				24.
159	22.			*[quickly]* No I'm just kidding, just kidding.
160		22		
161		*[looking up towards the ceiling] [slowly]* twenty-twooo, fourty-fourrr,		
162		sixty-sixx		Well, no.
163		*[accidentally flicks his pencil to the floor, moves chair back and leans back to retrieve it]*		You're adding up just our class. Don't count our class because y-, like, Ms. White's might have 21 in their whole class when everybody's there.
164	*[looking at Eric]* Ms. Hen- Ms. Henderson has exactly 21.	*[emphatically* That is truuueee. *[moves his chair a little away from the table so that he is essentially sitting next to Wren]*		
165			*[looking at Eric]*	*[looking at Everet]* Oh, she does?
166	Yeah [undecipherable: "yup"].			

Instances such as these illustrate that while Owen gained access to the modeling conversation through his interruptions, they did not grant him sustained voice or authority in the modeling conversation – even with some level of engagement in modeling competencies.

Brittany (Group 4) held little authority in modeling conversations, frequently enacting the observer position and minimally engaging in modeling competencies. While she was often interrupted by others – reinforcing their own positional authority – she did occasionally interrupt. These interruptions were typically brief interjections tied to context or social exchanges, such as "Yeah," "Thank you," "Knew you better," "Satisfying, satisfying, satisfying, satisfying" and "'Cause yeah – taller or shorter." These interjections positioned her as a social contributor or a supporter by aligning with Lydia's ideas, reinforcing Lydia's positional authority, but not supporting Brittany's participation in the modeling work or her own positional authority in modeling conversations. However, in one instance, Brittany, enacting the context contributor position, interrupted Lydia by interjecting and *sharing* that she was taller than Wren. Through this interruption, Brittany reinforced her own positional authority, directly engaged in the modeling competency of making assumptions, and influenced the modeling conversation as her contribution was taken up by Lydia. Although Brittany infrequently interrupted others, and infrequently participated in modeling conversations, she was often interrupted (and became silent) when attempting to contribute to modeling conversation (Turns 25 and 32, Table 46), indicating her lower positional authority within the group's hierarchy of authority.

Wren (Group 1) spoke infrequently and interrupted even less. When she did interrupt, she briefly enacted positions such as modeling contributor, task corrector, learner, or instructor, typically using the interactional moves of *sharing*, *countering*, and *clarifying*. These interruptions seldom involved engagement with modeling competencies through ownership of a modeling-related idea; she more often asking *clarifying* questions – reinforcing the positional authority of another student. For instance, when the group was evaluating the mathematical

relationship of 65 times 30, she interrupted, asking what the product of 65 times 10 is (Turns 225, 248-251, Table 36) and engaged in the competency of generating a solution by asking about Everet's original idea. At one point, when evaluating mathematical relationships, both Everet and Eric simultaneously took the conversational turn away from Wren (Turn 230, Table 36). After this interruption, Wren shifted into the observer position, demonstrating how interruptions of her functioned as authority-reinforcing mechanisms for the interrupter. Overall, Wren's infrequent interruptions and constrained engagement in the modeling conversation (and consequently, modeling competencies) reflected her limited positional authority in modeling conversations.

Conclusion: Interruptions and Positional Authority in Modeling. Overall, interruptions both reflected and shaped students' positional authority in modeling conversations, functioning as a mechanism through which students shared or reinforced authority. In Group 1, Everet and Eric's frequent, competency-driven interruptions positioned them as the group's central contributors, with Everet holding slightly more authority by cutting off Eric to advance his own ideas. Both frequently interrupted from contributor positions and engaged more consistently in modeling competencies than Owen or Wren. In contrast, even with persistence, Owen's interruptions functioned to grant him brief access to the modeling conversation with little influence over its direction, and Wren's rare interruptions marked her marginalized voice in the conversation.

Similarly, in Group 4, Lydia held the most authority as the primary contributor, consistently advancing the modeling conversation through sustained engagement in competencies. Jason exercised limited positional authority through his interruptions – sometimes contributing his own ideas and other times engaging with others' ideas, while also asserting

social authority. Brittany's interruptions afforded her minimal engagement in modeling competencies and reflected her minimal authority in modeling conversations. Her modeling contributions were often interrupted or silenced, leaving her with virtually no authority or voice in the modeling work. Brittany held greater authority in social conversations, frequently acting as a social contributor or social corrector, often alongside Jason who shared this social authority. The dynamics of interruptions illustrate that while Brittany and Jason carried more authority in social conversations, Lydia's sustained, competency-driven engagement afforded her authority in modeling conversations.

Across group conversations, interruptions occurred most often while students engaged in making assumptions and generating solutions, particularly by students enacting higher-authority context, mathematics, and modeling contributor positions. For these students, interruptions served as tools to participate in the modeling conversation and reinforce their positional authority by engaging in competencies to advance their own ideas within the group. By contrast, students enacting the learner position primarily interrupted through clarifying and confirming questions, engaging in modeling competencies via others' ideas and reinforcing their peers' authority. Occasionally, students enacting supporter positions interrupted to *agree* with students enacting contributor positions, briefly participating in the conversation without shaping the modeling work. In sum, interruptions enabled students to enter, sustain, or redirect modeling conversations. Although interruptions occasionally functioned as authority-sharing mechanisms, they more frequently reinforced authority through how students managed being interrupted and how they interrupted others. Interruptions both signaled and reinforced positional authority: students with higher authority often used them to sustain and advance the modeling work through engagement

in modeling competencies, while those with lower authority gained only brief or limited access to modeling conversations through their interruptions. Similarly, students' use of questions reflected their positional authority, as questions served both to share and to sustain authority within the modeling conversations.

Questions and Positional Authority in Modeling

Throughout the modeling lesson, students used question-asking as a flexible interactional resource that supported both engagement and authority within group modeling conversations. Depending on the position from which they were asked, questions functioned as authority-sharing or authority-reinforcing mechanisms – by engaging peers in advancing the modeling work (sharing authority) or by positioning another student as a knowledge holder (reinforcing authority).

Questions as Authority-Sharing Mechanisms. Students enacting context, modeling and mathematics contributor positions primarily used clarifying and confirming questions as authority-sharing mechanisms, positioning peers as collaborators in the modeling work and bouncing ideas of one another. Through these question-asking interactions, students enacting a contributor position often invited peers to step into a matching contributor position, thereby distributing (sharing) authority within the group.

For example, Brittany (Group 4) asked Lydia the clarifying question, "What about me?" – taking up the context contributor position while engaging in the modeling competency of making assumptions by asking about how her height might affect which size jump rope she would need (Turn 104, Table 44).

Table 44. Sample Dialogue 1 – Questions Sharing Positional Authority

Turn	Brittany	Lydia	Jason
104	*[addressing Lydia by turning her head and shifting her upper body to lean towards Lydia]* What about me?		
105	*[smiles]*	*[considers Brittany]* You might be taller than Belle *[looking out over the classroom]*	*[looking out over the classroom]*
106	*[facing out towards the classroom due to her location at the table]*		Oh yeah, definitely Belle.
107		Yeah, Belle definitely *[laughs]*.	
108	Belle's definitelyyyyy-		Belle definitely might need
109			yeah, a seven-foot rope.

Brittany's question supported Lydia's position as a higher-authority context contributor prompting her to immediately engage in making assumption by comparing Brittany's height with that of another student (Turn 105, Table 44). Later, Lydia enacted the same competency by asking the clarifying question, "Wait is Ow-which one is Owen, Johnny's height or no?" Jason responded, "Probably a little bit shorter" stepping into the position of context contributor himself (Turns 176-181, Appendix C, Table C2). In both cases, questions served to position another student as one with the authority to contribute meaningfully to the conversation (through engagement in modeling competencies), thereby supporting another's positional authority in the modeling work.

Similarly, students enacting the higher-authority mathematics contributor position frequently relied on confirming and clarifying questions to engage peers as co-collaborators in the competency of generating a solution. For example, Everet (Group 1) asked Eric, "What's 65 times three?" (Turns 61-64, Table 46) and in responding, Eric shifted from a supporter to a mathematics contributor (Turn 68, Table 46) as they worked out the computation together.

Table 46. Sample Dialogue 2 – Questions Sharing Positional Authority

Turn	Everet	Owen	Wren	Eric
61	[turns to address Eric]			... uhhhmmmm, um, um, umm
62	[looking at Eric] What's six times,	[looking towards Wren] I think I'm close.	[looks towards Owen]	mmmm
63	what's	Ain't I close? [shows paper to Wren and leans back in his seat toward her, but his chair still faces Everet]	[looks to Owen's paper]	[very quickly] Wa-, wait.
64	65 times three?		[looking at Owen] 400 for fourth grade!	
65		[nods head while facing Wren]		[writing on his paper, then looks up to Everet]
66	[leaning into the table towards Eric]			Wait, what?
67	65 times three.			65 times three?
68		[picking something up from the ground]	[has her left foot crossed over her right knee, adjusts her sock]	[looking at Everet] That would be, well, 65 plus 65 equals 180. [almost to himself] No. No.
69	[looking at Eric and gesturing with his pencil while speaking] 170.			
70				100- okay.
71	times 165			Plus [undecipherable]-
72	So plus six-			

Students acting from the mathematics contributor position also used questions to bounce their reasoning off one another: Eric asked Everet, "Okay, so two plus one would obviously be three, seven, and that equals s-seven?" (Turn 187, Table 34); Lydia proposed, "So I feel like it would be, so 20, so 18, times 24, okay?" (Turn 97, Appendix C, Table C2). These questions were not about positioning one student as instructor and another as learner, but about drawing peers into the mathematical work as a collaborator in a similar contributor position. In this way, students enacting contributor positions exercised positional authority not only by advancing their own ideas but also by creating opportunities for others to share in that authority, thereby supporting both their own and another's voice in shaping the modeling conversation through engagement in

modeling competencies.

Questions as Authority-Reinforcing Mechanisms. Most of the questions students asked during modeling conversations functioned as authority-reinforcing mechanisms. They were primarily clarifying or information-seeking questions typically asked by students enacting the lower-authority learner position. These questions reflected the limited authority associated with the learner position, as students enacting this position primarily engaged in modeling competencies by taking up ideas contributed by others rather than advancing their own. At the same time, these questions positioned another student as an instructor – a position carrying moderate authority in modeling conversations. In most cases, the students positioned as instructors (often Everet and Lydia) held higher positional authority through their consistent enactment of higher-authority contributor positions, and being asked questions reinforced their authority while creating space for their voices in the modeling conversation.

Lydia (Group 4), for example, never enacted the learner position, but Brittany and Jason frequently assumed this position by asking her clarifying questions. For instance, Jason asked Lydia, “Wait, what did you say?” (Turn 17, Appendix C, Table C2), “So what are we estimating?” (Turn 46, Appendix C, Table C2), “So it’s not 17 times 24?” (Turn 185, Table 28), “Okay, so our est- the estimate would be around... So, five- so probably like 1,000?” (Turn 198, Table 29), and, towards the end of the lesson, “So, now that we’re pretty much done, do we just read a book?” (Turn 238, Appendix C, Table C2). These exchanges illustrate how Lydia’s authority was reinforced through her positioning as an instructor while her peers simultaneously positioned themselves in the lower-authority learner position.

Similarly, Everet and Eric (Group 1) frequently exchanged confirming and clarifying

questions with each other and also received them from the other members in the group, reflecting the higher positional authority they held. They asked each other questions such as, “How do you do 60 times 30?” (Everet, Turns 97-98, Table 39), ”What’s 40 plus 20? Or 40 times 20? What’s 40 times 20?” (Everet, Turns 304-308, Appendix C, Table C1), and “43. Plus 24 would beee?” (Eric, Turn 176, Table 34). At the same time, Owen and Wren – who held less authority in the group – most often directed their questions towards Everet and Eric. For example, they asked, “So we do that [65 plus 10] three times?” (Owen, Turn 205, Appendix C, Table C1), “We’re doing 65 times 10, right? Or 30?” (Owen, Turns 266-267, Appendix C, Table C1), and “Then you get a hun-650. Because 65 times 10. *[looks towards Eric]* Right?” (Wren, Turn 258, Appendix C, Table C1). Rarely were such questions directed at Owen or Wren themselves, underscoring their lower positional authority.

Conclusion: Questions and Positional Authority in Modeling. The findings show that student questions functioned as flexible authority-sharing or authority-reinforcing interactional mechanisms that shaped participation and positional authority in their group conversations. Questions served two primary functions: sharing positional authority by inviting peers to collaborate in advancing the modeling work and reinforcing positional authority by positioning another as the recognized knowledge holder. When students enacted context, modeling, and mathematics contributor positions and asked confirming and clarifying questions, they exercised positional authority by inviting others to engage in the mathematical modeling competencies as collaborators in a similar higher-authority contributor positions. In these instances, the recipient of the question often assumed a matching contributor position, which supported their own positional authority within the group. However, most questions asked during groupwork

conversations were from students enacting the lower-authority learner position and took the form of clarifying or confirming questions. While reflecting the limited authority of the learner position, these questions reinforced the authority of others by positioning them as instructors with recognized knowledge to share. Taken together, these findings demonstrate that questions acted as mechanisms of positional authority, shaping how authority was exercised during group conversations. Questions either broadened participation by supporting other students to engage as fellow contributors or reinforced the authority of select knowledge holders. Across positions, questions impacted both student engagement in modeling competencies and the exercise of authority within group modeling conversations.

For both interruptions and questions, the findings indicate that students' use of these authority-sharing and authority-reinforcing interactional mechanisms both reflected their positional authority and influenced their participation in modeling conversations through engagement in mathematical modeling competencies. The following section examines how positional authority was distributed among students, as indicated by their enacted positions and their use of interruptions and questions.

Distribution of Positional Authority Among Students

Student differences in participation in during modeling conversations – through their enactment of positions, use of interruptions and questions, and engagement in modeling competencies – reflected that students held varying levels of positional authority. The findings in this section illustrate that students' consistent enactment of positions at a given authority level corresponded with their relative authority within group modeling conversations.

Students Holding Higher Positional Authority. Lydia (Group 4), Everet (Group 1), and Eric (Group 1) held higher positional authority than their peers, with Everet holding slightly more than Eric. Their authority was evident in their consistent influence on their groups' modeling work, in how their group members acknowledged and followed their ideas, and in the interactional dynamics of interruptions and questions. Notably, both Lydia and Everet exercised higher levels of authority even when enacting positions beyond the higher-authority process contributor position, and they were often positioned as instructors by other group members.

Both Everet and Lydia were the only students to enact the higher-authority process contributor position, which involved directing or mapping the next step(s) in the modeling work. As described in Finding 1, their peers followed their ideas without question, reflecting the authority they held. Everet, Lydia, and Eric frequently and consistently shifted among the context, modeling, and mathematics contributor positions. Each of these positions carried authority to shape the group's modeling process through engagement in modeling competencies. As they fluidly adopted and discarded these higher-authority positions, Everet, Lydia, and Eric advanced their groups' modeling through sustained engagement in modeling competencies, giving their voices influence in shaping the modeling conversation.

The higher positional authority of these students was sustained and reinforced through interruptions and questioning. For example, in Table 38 (Turns 183–185), Jason positioned Lydia as an instructor by asking, "Okay, so what are we- So it's not 17 times 24?" Lydia took up the instructor position by explaining the modeling work, illustrating how Jason's question reinforced her authority and voice within the group. Interruptions (either being interrupted or interrupting) also allowed Lydia to sustain her positional authority when she persisted in her engagement in

modeling competencies, such as when she interrupted Britttany, refocusing the conversation on the number of plastic bags in a seven-foot jump rope (Turn 132, Table 41). Likewise, Eric reflected and reinforced his higher positional authority by interrupting Owen to correct his method for determining the number of students in the fourth grade (Turn 162-163, Table 43). These examples demonstrate that interruptions and questions often functioned as authority-reinforcing mechanisms for students holding higher positional authority.

Interruptions and questions could also serve as authority-sharing mechanisms. For instance, Lydia invited Jason into collaboration when reconsidering the number of students requiring a seven-foot jump rope, asking, “Wait is Ow- which one is Owen, Johnny’s height or no?” In responding, Jason assumed the context contributor position (Turns 176-181, Appendix C, Table C2) showing how Lydia shared authority while positioning Jason in the same contributor position she occupied. Similarly, Everet and Eric’s overlapping and mutually interruptive exchanges while estimating the number of students in the fourth grade (Table 40) exemplify interruptions as authority-sharing interactions that advance the modeling work.

Together, these examples show that the positional authority associated with certain positions shaped the authority students held within their group modeling conversations. Lydia, Everet, and Eric, who consistently enacted higher-authority contributor positions, held greater positional authority than their peers, as reflected both in their engagement in modeling competencies and in their interactions with peers through interruptions and questions.

Students Holding Moderate or Lower Positional Authority. As with higher-authority positions, frequent enactment of lower-authority positions shaped the influence students carried even when taking up higher-authority positions. Wren (Group 1) and Brittany (Group 4)

frequently enacted the observer position, and when they shifted (or attempted to shift) into a position of higher authority related to the modeling task, their voices carried less weight than their peers – who often interrupted or ignored them. Owen (Group 1) experienced similar constraints and had to actively advocate for his contributions to be acknowledged. In contrast, Jason (Group 4) held moderate positional authority as his ideas were acknowledged by his peers, but he often enacted the lower authority position of learner.

For example, Wren repeatedly tried to participate in the modeling conversation, engaging in generating a solution by asking what 65 times 10 would be (Table 36). Before Owen responded (Turns 252–255, Table 36), she had asked the same question three times (Turns 225, 248–250, 252, Table 36) and was interrupted mid-question once (Turns 228–229, Table 36). For Brittany, interruptions minimized her voice as peers reinforced their own authority – for instance, during one of their earlier group conversations, Lydia and Jason were discussing peer heights to determine jump rope requirements (Table 46). Brittany, however, was interrupted twice when attempting to contribute and was not able to contribute her ideas to this conversation – and consequently, was not able to engage in modeling competencies.

Table 46. Sample Dialogue – Interruptions as Silencing Mechanisms

Turn	Brittany	Lydia	Jason
23		Umm *[begins writing on her paper].*	*[holding and playing with the jump rope]*
24	(4 seconds)		*[looking towards another table whose members are saying that everybody is about 5 foot. Looks back to his table]*
25	I would take allll-	I don't know-	
26			*[loudly with a shake of his head]* No, not every body's about five-foot.
27		No, definitely not-	No no
28			*[with a slight chuckle in his voice]* Noo
29		I'm not five-foot.	
30			*[with a slight chuckle in his voice]* Noo
31		*[looks behind her towards Table 5. A student says "four-foot-seven to five-foot"]*	
32	I'm gonna say-		The average height of a
33			fourth grader is probably four-foot-six.
34		Yeah.	
35			Somewhere in there.

Owen, who occasionally enacted the observer position, often took up learner or modeling contributor positions. Yet, like Wren and Brittany, his voice often carried less weight in the group's conversations, requiring him to actively advocate for his contributions to be acknowledged. For example, in Table 39, Owen enacts a position of higher authority as a mathematics contributor on Turn 95, before shifting into the task corrector position on Turn 98 through the oppositional move of countering. Despite acting from positions of higher authority, Owen's idea that their initial estimate for the total number of plastic bags should be a guess (Turn 95) was initially unacknowledged by his group members. Eventually, his idea was revoiced by Everet (Turns 102-104) after Owen advocated for it (Turns 98-100), but Eric disregarded the

contribution and continued with his original calculation (Turns 100-107). These examples demonstrate that more consistent enactment of lower-authority positions influenced the positional authority of the student enacting them and shaped their access to modeling conversations – and by extension, engagement in modeling competencies – as indicated by their experiences with interruptions and questions from peers.

Jason, by contrast, rarely enacted the observer position and more often adopted learner or contributor positions, enabling him to exercise moderate authority in modeling conversations. His use of inquiry moves while enacting the learner position and knowledge moves while enacting contributor positions afforded him access to the conversation. For instance, Jason enacted the modeling contributor position and engaged in making assumptions when sharing that there were 22 students total in their class, but only 21 present during the lesson (Turns 93-96, Table 23), and he collaborated with Lydia as a fellow context contributor when she engaged in making assumptions by asking Jason about Owen's height (Turns 176-181, Table 27). Jason also frequently used clarifying and confirming questions while enacting the lower-authority learner position, positioning Lydia in the higher-authority instructor position. For example, on Turn 185 (Table 28), he asked Lydia, "So it's not 17 times 24?" and on Turn 198 (Table 29), he asked about their overall solution to the modeling problem, "So, five- so probably like 1,000?" In response, Lydia enacted the instructor position, explaining that based on her work, they would need a total of 1,532 bags. Jason again assumed the learner position, asking, "Maybe, 1,600?" (Turn 223, Table 30), to which Lydia agreed that it could work as a friendly number. In these illustrations, Jason engaged in generating a solution through his questioning participation in determining the solution to the problem posed. Through her positioning as an instructor by

responding to Jason's questions – Lydia engaged in the competencies of generating a solution, analyzing the model, and interpreting the model. Overall, Jason's participation in modeling conversations – from contributor positions and often from the learner position – reinforced his moderate positional authority within the group.

Taken together, Wren, Brittany, Owen, and Jason illustrate that positional authority was not only associated with specific positions but also tied to the students consistently enacting these positions. Frequent enactment of lower authority positions constrained students' overall engagement in modeling conversations and competencies. The lower or moderate positional authority of these students was reflected not only in their engagement with modeling competencies through the positions they consistently enacted, but also in how interruptions and questions influenced their participation.

Conclusion to Distribution of Positional Authority Among Students. These findings demonstrate that consistent engagement in positions of higher authority (e.g., process contributor, context contributor, modeling contributor, or mathematics contributor) supported students' overall higher authority in the group. Likewise, frequent enactment of lower-authority positions (e.g., observer, learner, and supporter), supported students' lower authority in group modeling conversations – even when students sometimes adopted higher-authority positions.

Conclusion: Positional Authority and Student Voice in Modeling

A side effect of holding positional authority was student voice – students with higher positional authority were those whose voices were consistently heard and whose contributions taken up in group conversations. In this way, student voice functioned as a marker of positional

authority: students who were listened to held greater influence over the modeling work, while those whose voices carried less weight had fewer opportunities to shape the modeling process. Consequently, compared to peers whose voices were not heard, students whose voices carried in the conversation enjoyed greater access to and engagement in modeling competencies.

Everet and Lydia were the only students to enact the higher-authority process contributor position, and they enjoyed the corresponding authority in their groups. They also consistently enacted other higher-authority contributor positions, engaging in modeling competencies to advance the modeling work. Their positional authority was reinforced by other students who turned to them with questions, positioning them as knowledgeable sources within the group. Peers followed their ideas (e.g., mapping the development of their model), and their contributions were actively sought and taken up, reflecting the significance of their voices in their groups' modeling conversations.

While Eric did not enact the process contributor position, he still influenced the modeling process through his sustained engagement in modeling competencies while enacting higher-authority context, modeling, or mathematics contributor positions. His voice carried weight, and he maintained significant positional authority through these positions and his engagement in modeling competencies.

Jason enacted both higher- and lower-authority positions, including learner, social contributor, and observer. His voice carried some weight; he occasionally pushed back against Lydia's ideas and often positioned himself as a learner – and so engaged in modeling competencies primarily through questioning interactions. Jason occasionally exercised social authority, for example by correcting Lydia's pronunciation, but these instances were infrequent

and unrelated to modeling conversations.

Owen, Brittany, and Wren held lower positional authority within their groups' modeling conversations. Owen participated in both modeling and social conversations and enacted a variety of positions, but his modeling ideas were often acknowledged only after persistent effort. As when he repeatedly reminded the group that estimating the total number of plastic bags was "just a guess" before Everet acknowledged and revoiced his comment. Despite his persistence, his voice carried little weight in shaping the modeling process – indicated by his limited influence on the group's modeling work. Brittany had the least authority in her group, especially during modeling work. While she occasionally assumed authority through social positions (social corrector or social contributor), she infrequently enacted context, mathematics, or modeling contributor positions, and most often enacted the lower-authority observer position through her sustained silence. Accordingly, she engaged with modeling competencies only infrequently and exercised very little positional authority in the group's modeling conversations. Often, when she did attempt to speak, her voice was overruled by her peers. Similarly, Wren's voice carried little weight as she enacted the lower-authority observer position for nearly the entirety of the conversation.

Overall, the findings indicate that consistent engagement in higher-authority contributor positions – process, context, modeling, and mathematics – allowed students to advance their own ideas, maintain influence over the modeling process through engagement in modeling competencies, and support their positional authority. In contrast, students frequently enacting lower-authority positions – learner, observer, or social contributor – demonstrated constrained engagement in modeling competencies and limited voice in modeling conversations.

Interactional mechanisms such as interruptions and questions reflected the distribution of students' positional authority within their groups, serving as flexible interactional mechanisms that could either reinforce or share authority and, in turn, shape participation in modeling conversations and engagement in modeling competencies.

CHAPTER FIVE

DISCUSSION AND CONCLUSION

Mathematical modeling in educational settings can be understood as the process where students translate between the real and the mathematical worlds to address a real-world problem. The modeling process supports students in their ability to apply mathematics to the world around them, which in turn fosters their mathematical literacy by increasing their ability to reason mathematically about real-world problems by using mathematics to interpret and address the problem. Both mathematical modeling and mathematical literacy are concerned with student proficiency in recognizing, applying, and interpreting mathematics in real-world contexts, and also with student ability to make decisions and assumptions related to situations. Thus, facility with mathematical modeling supports mathematical literacy.

This research study focused on students during a mathematical modeling task by considering their engagement in modeling competencies – skills students use to work through the process of modeling. In particular, this research focused on elementary learners – specifically, fourth-grade students – working through a modeling task in small groups. These groups create space for collaboration, which hinges on student conversations carried out through verbal and nonverbal interactions. Student conversations can be understood through the theoretical framework of positioning theory, a neo-Vygotskyan learning theory. While neo-Vygotskyan learning theory "proposes that social interaction (including the use of language and other cultural tools) is the fundamental medium within which learning takes place" (Erickson, F., 2006, p. 181), positioning theory specifically attends to the positions enacted by students through social interactions during conversation.

This study then, and its findings, are broadly situated at the intersection of mathematical literacy and neo-Vygotskyan learning theory – through mathematical modeling competencies and positioning theory. The purpose of this study was to contribute to understanding how fourth-grade students engaged with mathematical modeling competencies during group work, the positions they enacted through social interactions, and how those positions influenced their engagement in modeling competencies. To address this purpose, I asked the following central research question (CRQ): What is the nature of student engagement in mathematical modeling competencies during group work? I addressed the CRQ through two sub-research questions (SRQs):

SRQ1: How does student attention to the task context influence their engagement in mathematical modeling competencies?

SRQ2: How do student positions influence their engagement in mathematical modeling competencies?

The balance of this chapter includes a summary and discussion of the findings, their implications, and the study's significance, limitations, quality of findings, and conclusion.

Summary and Discussion of Findings

Summary of the Findings in Relation to the Research Questions

The analysis of the data produced two primary findings: contextually grounded mathematical modeling, and positional authority in modeling conversations. Taken together, these findings address the central and sub-research questions.

Finding 1: Contextually Grounded Modeling. Finding 1 addresses the first sub-research question: *How does student attention to the task context influence their engagement in mathematical modeling competencies?* This finding revealed that student attention to the task context supported their mathematical work as they engaged in developing models and solutions.

When students consistently interpreted their mathematics in light of the task context, they developed models and solutions that aligned with the real-world situation. In contrast, when students focused only on the mathematical work of building models and generating solutions without connecting to the context, their models drifted from agreed-upon assumptions and required revision. By interpreting and analyzing their mathematical work in light of the task context students engaged in revising their model to align with the problem context. Thus, attention to the task context supported students' mathematical modeling work. Specifically, context-driven competencies – problem posing, identifying quantities, making assumptions, interpreting, and analyzing the model – supported mathematics-driven competencies – building the model, generating solutions, and revising the model. The work students completed when engaged in mathematics-driven competencies was supported by frequent engagement with context-driven competencies. Students who neglected the context produced models that were mis-aligned with the context because their mathematics did not follow from their contextual work.

Finding 2: Positional Authority in Modeling Conversations. Finding 2 addresses the second sub-research question: *How do student positions influence their engagement in mathematical modeling competencies?* This finding showed that some positions carried more

positional authority than others and that this authority was tied to students consistently enacting these positions.

Throughout student conversations, five categories of positions were taken up, carried out, and discarded through social interactions: contributor (process, context, modeling, mathematics, and social), evaluator (instructor, task corrector, social corrector), recipient (learner, task wrongdoer, social wrongdoer), supporter (context, modeling, mathematics), and viewer (observer).

Students consistently enacting higher-authority process, context, modeling, or mathematics contributor positions shaped the direction of the group's modeling work through sustained engagement in modeling competencies. The students consistently enacting these positions enjoyed greater authority in modeling conversations, allowing them to advance their own thinking and influence group modeling work through engagement in modeling competencies. The process contributor position, though infrequently enacted, carried significant positional authority: students mapping or directing their group's modeling process were often treated as higher-authority holders by their peers who turned to them with questions and listened to them in conversation by taking up their ideas.

Conversely, students frequently enacting lower-authority learner, supporter, or observer positions demonstrated limited access to modeling competencies. Student enacting the learner and supporter positions often reinforced another's positional authority – the former by positioning peers as instructors, and the latter by revoicing, acknowledging, or agreeing with another's ideas. The observer position carried with it especially low positional authority, students enacting this position maintained a silent presence in conversation – and so did not interact with

their peers to engage in modeling competencies. The lower-authority associated with learner, follower, and especially observer positions became connected to the students consistently enacting these positions – just as the higher-authority of the process, context, modeling, and mathematics positions was associated with the students consistently enacting these positions.

In answer to the second sub-research question then, positions influenced student engagement in modeling competencies because they were associated with some level of authority in modeling conversations. Higher authority positions afforded access to modeling competencies through social interactions which allowed students to shape the group's modeling work. Lower authority positions limited access to modeling conversations through social interactions, reducing opportunities to influence the group's modeling endeavors.

Connecting the Findings: Nature of Student Engagement in Modeling Competencies. Underlying both findings were the social interactions through which students grounded their modeling work in the context of the situation and fluidly enacted, sustained, and discarded positions during group modeling conversations. These interactions ultimately shaped how students engaged in modeling competencies. Taken together, these findings address the central research question: *What is the nature of student engagement in mathematical modeling competencies during group work?* The data indicate that the nature of student engagement in mathematical modeling competencies during group work was interactionally produced, shaped by positional authority, and dynamic.

Students engaged in group conversations using affiliative, guiding, inquiry, knowledge, and oppositional moves. In modeling conversations, students interacted with each other about the context of the situation and drew on their personal, contextually relevant knowledge to support

their modeling work. This contextual knowledge informed their modeling decisions as they engaged in modeling competencies. Student engagement in these competencies was interactionally produced: positions were taken up and enacted through ongoing interactions, shaping students' opportunities to participate in modeling conversations. The positions students enacted further influenced their access to and engagement in modeling competencies, as this engagement was shaped by the level of authority associated with each position. Students enacting higher-authority context, modeling, and mathematics contributor positions more frequently advanced their thinking within the conversation, actively shaping the development of the modeling work. When enacting lower authority positions, students engaged less frequently with modeling competencies, often by commenting on or asking about the ideas of another rather than communicating their own ideas. Engagement in the various modeling competencies was dynamic, shifting as students used the context to ground their modeling work and fluidly moved among positions with differing levels of authority. Overall, student engagement in modeling competencies during group work was interactionally produced, influenced by positional authority, and characterized by this dynamic nature.

Discussion of the Findings in Relation to the Relevant Literature

Discussion of Contextually Grounded Modeling. The finding of contextually grounded modeling showed that student attention to the context of the task supported their mathematical work as they engaged in the modeling competencies of developing models and generating solutions – and in one instance, revising the model. These findings align with and extend the modeling literature by illustrating, through the case of a fourth-grade class, how elementary students move through the process of modeling by engaging in modeling competencies during

group work conversations. In particular, this finding demonstrates that the context of the task served as a resource for supporting students' modeling endeavors, and that elementary students are capable of carrying out the entirety of the modeling process.

Supporting Functions of the Task Context. In this study, the task context referred to the upcycling project of building jump ropes from plastic bags. The students collected plastic bags and determined the total number needed for every fourth-grade student in their school to have a jump rope. The context of the task connected to their experiences with jumping rope as well as their community – the school they attended, the peers they knew, and the plastic bags they gathered from home.

These context-related factors supported students as they engaged in modeling competencies such as posing the problems, identifying quantities, and making assumptions. For instance, students made the assumption that three plastic bags produced one foot of rope based on a shared classroom experience of braiding and measuring plastic bags. In group conversations, students used personal experiences with jump-roping to vocalize that, depending on height, some people need taller or shorter jump ropes. They used their personal knowledge of their classmates to estimate who might need a shorter, seven-foot jump rope, even naming them individually. These examples highlight how the task context, rooted in students' community knowledge and experiences, influenced their engagement in modeling work. These examples illustrate Wickstrom and Yates's (2021) argument that modeling problems enable students to connect the real and mathematical worlds. Consistent with the broader modeling literature, drawing on relevant community contexts can make such problems meaningful and purposeful for students (English, 2021; Wickstrom & Yates, 2021). In this study, the findings reveal how

students “bring with them to the mathematics classroom a wealth of informal mathematical knowledge in their everyday cultural and social experiences, and that knowledge and those experiences are valuable resources for mathematics teaching and learning” (Berry et al., 2020), which served as a resource for grounding their modeling work in the context.

The context of the task not only functioned to connect students to the task, but also functioned as a support in model development, showing that through consistent interpretation of their mathematical work in light of the context, students developed models and solutions that followed from their contextual work around assumptions and aligned with the real-world situation. When this happened, students consistently recontextualized their mathematical work by interpreting and sometimes analyzing it in light of the context. By consistently interpreting their mathematical work in light of the context, students often moved between the real and the mathematical worlds and so experienced modeling as “an ongoing, iterative cycle of modeling and interpreting” (Flevares & Schiff, 2013, p. 39). This iterative movement aligns with the work of Turner et al. (2021) who found that “model-building considerations tightly connect to the context, so as students construct a model, they often continue to make sense of the situation or problem” (p. 240). For instance, when, as a result of analysis, students re-engaged in making assumptions by adjusting their assumption about the number of students who would need a seven-foot jump rope. In this instance, personal knowledge of their classmates informed their decision to adjust their initial assumption that only four students needed a seven-foot jump rope; they decided to include one more student in this category as he was probably short enough. This illustrates how students drew on their personal knowledge of their peers’ heights to guide modeling decisions, which reflects one of Turner et al.’s (2023) dimensions of culturally

responsive modeling, which highlights that student connections to their community contexts "inform modeling work – influencing decisions made or actions taken in any phase of [the] modeling process" (p. 979) – showing how both the context of the task and their personal connections to it supported students in their engagement with modeling competencies.

The context of the task not only supported student engagement with the modeling process and model development, but provided access to their group's conversations. Student modeling conversations sometimes focused on the context of the task, sometimes on the mathematics of the task, and sometimes on connections between the two. Students who did not contribute their own mathematical ideas may have benefited from their connection to the context, as their familiarity with the situation could support their understanding when peers explained mathematical ideas. This illustrates the opportunities present for students to learn from each other (Featherstone et al., 2011) and aligns with the idea that "the benefits of collaboration occur when a more-expert person enables a less-expert person […] to carry out a task that the student could not perform without assistance" (Webb et al., 2019, p. 177). This connects to and builds on the modeling work of Flevares and Schiff (2013) who found that in "[w]orking together on big questions, children can tackle bigger problems and more complex situations than they could individually, using the cognition distributed not only among team members" (p. 43). Interpreted through the lens of positioning theory, the context of the task created opportunities for students to enact positions (e.g., context contributor, learner, instructor, context supporter), providing access to and participation in group modeling conversations without having to consistently contribute their own mathematical ideas.

In particular, the context of the task supported students enacting the learner position. Students enacting this position used interactions such as clarifying and confirming questions to participate in mathematical exchanges by positioning another student as an instructor who could explain the group's mathematical work. This instructor positioning gave the student the opportunity to explain their mathematical thinking while giving the questioner the opportunity to engage in the mathematics through the learner position. The use of questions and explanations to enact learner and instructor positions demonstrates the idea in positioning theory that positions are both produced interactionally in conversation and dependent on the social context (Davies & Harré, 1990; McVee, 2011). By assuming the instructor position, a student took up the responsibility of explaining mathematical work, while the learner position carried the right to ask questions – both positions enabled access to mathematical conversation and participation in the modeling process. This extends positioning theory into mathematical modeling conversations by illustrating that positions are equipped with rights and responsibilities and influence access to the conversation by shaping how conversants interact with each other (Davies & Harré, 1990; Harré et al., 2009; Langer-Osuna et al., 2021; Van Langenhove, 2021).

Student Enactment of the Modeling Process. Over the course of the lesson, students' group- work conversations reflected their enactment of the full modeling process as they made sense of an open-ended, real-world scenario by making assumptions and using mathematics to generate a solution to the problem (Jung & Brand, 2021; Stohlmann & Albarracin, 2016). Students asked contextually relevant mathematical questions, identified important quantities to consider in building their model, made assumptions that enabled them to make progress on developing a model, constructed mathematical relationships between identified quantities,

generated solutions to their models, interpreted their mathematical work and results in the context of the task. Occasionally, they analyzed and revised their models. While students did not construct generalized versions of their models, they developed context-specific models that aligned with the context of the task and addressed the modeling problem, and so carried out the modeling process (Turner et al., 2021). This aligns with modeling literature that in the elementary grades, students engage in modeling by posing mathematical problems, using mathematics to investigate and represent situations (where they make assumptions), and interpreting and revising their solutions (Arnold et al., 2021; Jung, H. & Brand, S., 2021). This finding aligns with the growing body of work demonstrating that elementary students are capable of engaging in and successfully working through complex modeling problems (Carlson et al., 2016; English, 2021; English & Watters, 2005; Flevares and Schiff, 2013; Greer et al., 2007; Stankiewicz-Van Der Zanden et al., 2021; Suh et al., 2021; Turner et al., 2021).

Conclusion to Discussion of Contextually Grounded Modeling. Taken together, the findings related to contextually grounded modeling support and extend the modeling literature by demonstrating that elementary students are capable of engaging in and carrying out the full process of mathematical modeling. This modeling task, grounded in community contexts familiar and relevant to students, created space for students to take up positions and contribute contextually- relevant information, participating in the modeling work of their group. In this study, students engaged in posing problems, identifying relevant quantities, and making assumptions tied to the task context – context-driven competencies that, alongside interpreting and occasionally analyzing their models, supported their development of models, generation of solutions, and, in one case, revision of the model. As students built models and generated

solutions to address the problem, their personal connection and familiarity with the task context supported their mathematical work and enabled them to carry out the entirety of the modeling process. While students' connection to the context of the task supported their modeling work, and supported student access to the modeling conversation through their knowledge of the context, student interactions and positions also influenced access to and engagement in modeling competencies during group work conversations. Differences in who spoke and who stayed silent (Horn & National Council of Teachers of Mathematics, 2012), who directed and mapped the modeling process, and whose ideas were taken up indicated the positional authority associated with particular positions and with the students themselves. The following section discusses the influence of positional authority on student engagement in modeling competencies and examines how these findings align with and extend positioning theory.

Discussion of Positional Authority in Modeling. The findings from this study align with and extend positioning theory by illustrating how the positions students enacted during group work conversations influenced their engagement in mathematical modeling competencies. In positioning theory, conversation – "a form of social interaction" (Davies & Harré, 1990, p. 45) – is the medium through which social interaction occurs. Conversation consists of three core constructs: speech acts, positions, and story lines, all of which are grounded in interaction. The findings demonstrated that, through their ongoing interactions, students fluidly shifted between a range of positions, each carrying varying degrees of positional authority. This positional authority shaped student access to mathematical modeling competencies, at times supporting and at other times constraining their engagement. Moreover, the authority associated with consistently enacted positions was often attached to the students themselves, influencing their

positioning and participation in group modeling conversations. Overall, these findings reinforce the concept from positioning theory that positions function as a conversational resource to those who take them up (Davies & Harré, 1990; Harré et al., 2009). While the findings from this research represent only a part of what occurs during a modeling lesson, they provide insight into the nature of student engagement in modeling competencies during group work conversations. The following sections extend positioning theory within mathematics education by discussing how the findings relate to the constructs of authority and identity in mathematical modeling conversations, particularly as they emerge through students' engagement in modeling competencies during group work.

Authority-Shaped Modeling Engagement. In this study, positional authority refers to the conversational influence students exerted over the group's modeling process – indicated by and shaping who engaged in modeling competencies, whose ideas were taken up, and who was positioned as a source of knowledge. The findings revealed that students who regularly enacted higher-authority positions – such as process, context, modeling, and mathematics contributor – held higher positional authority, greater access to the modeling conversation and to engagement in modeling competencies. In contrast, students who frequently enacted lower-authority positions – such as learner, supporter, and observer – experienced lower positional authority and constrained or minimal engagement in modeling competencies. From a positioning theory perspective, these findings illustrate how micro-level interactions produce positions that afford or constrain participation in conversations. However, they also extend positioning theory by demonstrating how, in mathematical modeling conversations, the authority associated with

enacted positions shaped who had access to modeling conversations through engagement in mathematical modeling competencies.

These findings show that the authority of positions comes to life in a mathematics classroom and build on Edelen et al.'s (2022) assertion that it is through interactions that this occurs. Edelen et al. (2022) also note that while positioning theory "does not describe how individuals come to understand mathematical ideas, it does describe the social endeavors that influence the opportunities for students to learn mathematics" (p. 119). The present study extends this reasoning by showing that the authority of the enacted positions influenced students' opportunity to engage in modeling competencies. Students consistently enacting higher-authority positions in modeling conversations developed a 'footing' in the modeling conversation as these students' ideas were heard, taken up, and followed. This aligns with Harré et al.'s (2009) description in positioning theory that "[s]omeone with 'footing' [in a social interaction] is listened to. The force of their speech acts is taken notice of, and, even when unintended, 'taken up' by the members." (Harré et al., 2009, p. 12). Those with an established footing in modeling conversations held authority that was attached to them through the higher authority positions they consistently enacted. In contrast, students holding lower authority in modeling conversations, often struggled to establish footing in the conversation, as their voices were frequently interrupted or went unacknowledged. Students who were unable to establish footing engaged less frequently in modeling competencies, further impacting their opportunities to shape the group's modeling work. In this way, positions, and their corresponding authority, functioned as resources that opened or constrained participation in the modeling conversation through engagement in mathematical modeling competencies.

These findings extend the work of Langer-Osuna et al. (2021), who used positioning theory to examine how K-2 students enact authority relations during moments of struggle in a mathematics lesson. They found that young children "navigate lived worlds organized by storylines that afford particular relations of power" where some "characters take on more or less authority," and consequently, "deployment of different forms of authority shape possibilities for engagement" (pp. 98-99). Their findings emphasized that classroom context, and not grade level, was influential to student authority positioning which affected student engagement in the mathematical activity. The current study builds on this work by illustrating that, within mathematical modeling, positional authority associated with enacted positions similarly shaped who engaged in modeling competencies and how that engagement unfolded.

Student Development of Identity and Self. Positioning theory, as developed by Davies and Harré (1990), provides a lens for describing and understanding the formation of the individual, as an "individual emerges through the processes of social interaction" and develops their self from the positions adopted during conversation (Davies & Harré, 1990, p. 46). According to Davies and Harré (1990), a stationary or static notion of self cannot exist as the self is continuously revised and shaped through the enactment of positions. As such, the self constitutes a "diversity of selves" (p. 47). Positions can, therefore, be understood as micro-identities – short-lived interactionally produced, context-dependent enactments of self – that, over time, accumulate to form more stable macro or thickened identities (Edelen et al., 2022; Wood, 2013). As individuals repeatedly take up particular positions, they eventually come to understand themselves in light of those positions (Davies & Harré, 1990).

The findings from this study exemplify these ideas. Within group-work modeling conversations, students who repeatedly enacted positions of higher or lower authority were associated with that level of authority, as indicated by their influence on modeling conversations, engagement in modeling competencies, questions and interruptions, and the weight their voices carried within the group. For example, students who engaged in the mathematics-driven modeling competencies of building the model and generating solutions – as a mathematics contributor – experienced the micro-identity of being capable mathematical modelers. Over time, these repeated enactments could contribute to a more stable macro-identity in which students see themselves as mathematically capable modelers. Conversely, students who rarely (if ever) had opportunities to engage in these mathematics-driven competencies may not have experienced themselves as mathematically capable modelers within their group conversations. With limited enactment of the positions associated with sustained engagement in modeling competencies – primarily the context, modeling, and mathematics contributor positions – students experienced fewer opportunities to develop micro-identities as individuals who could meaningfully engage in modeling competencies by contributing ideas related to contextual, modeling, or mathematical aspects of the task. This, in turn, may have contributed – or may eventually contribute – to a narrower development of students' sense self in the context of mathematical modeling.

These findings extend the work of Wood (2013), who examined a fourth-grade student, Jakeel, during a mathematics lesson and observed the formation of three micro-identities. In two, he acted as a mathematically capable student, but in the third – "menial worker" – he engaged minimally with mathematics and showed "no evidence [of] communicating mathematical ideas and no evidence that he learned" (p. 802). Wood's work highlights that micro-identities shift over

the course of the lesson, are sensitive to context, and that repeated use of a learning-inhibiting micro-identity eventually thickens into a macro-identity. Similarly, the present study shows that repeated enactment of higher or lower-authority positions (i.e., micro-identities) shaped students' positional authority and engagement in modeling competencies. These enacted positions reflected students' in-the-moment micro-identities as modelers and may, over time, have implications for their developing macro-identities.

In positioning theory, Harre and Davies (1990) emphasize that selves are discursively produced through conversation and interaction with others. In this study, if a student did not or could not participate in the modeling conversation through engagement in modeling competencies, they did not enact micro-identities and develop the aspects of self associated with being a capable modeler. In other words, students' opportunities to "be the kind of person who models" were constrained. When students did not have a chance (or chose not) to interact in the conversation, they did not develop that part of their identity – the micro-identity of a participant in modeling competencies. These micro-identities represent students' in-the-moment sense of self; thus, their participation in modeling conversations shapes their immediate identity and informs how they understand themselves as modelers. Over time, these micro-identities may accumulate into macro-identities, developing into a more enduring sense of self in relation to modeling – and, more generally, to mathematical contexts.

Conclusion to Discussion of Positional Authority in Modeling. Social interactions during conversation contribute to the development of identity and of self (Davies & Harré, 1990; Harré et al. 2009; Harré & Van Langenhove, 1991). These interactions produced positions that students enacted, allowing them to experience the formation of micro-identities, which, through repeated

enactment, could thicken into a macro-identity and become a more enduring aspect of the student. In modeling conversations, positions carried varying degrees of positional authority, shaping both students' authority within the conversation and their engagement in mathematical modeling competencies. This authority impacted students' opportunities to experience themselves as capable modelers. Overall, the findings indicate that positions both reflected students' micro-identities and served as conversational resources, with consistent enactment shaping positional authority and influencing students' access to engagement in mathematical modeling competencies during group work conversations.

Conclusion to the Discussion of the Findings. Overall, students in this study carried out the Jump Rope task by collaborating in small groups, using the context of the task as a resource to work through the process of modeling. Through social interactions during modeling conversations, students enacted positions that either enabled or constrained their access to and engagement in mathematical modeling competencies. These inter-relational dynamics reveal that certain positions granted students greater influence over their group's modeling work, reflecting greater positional authority in modeling conversations. Taken together, these findings contribute to and extend both the mathematical modeling literature and positioning theory by illuminating how elementary students engage in modeling tasks through interactions that connect to the task context and support positioning, while also showing how the positional authority associated with specific positions shapes engagement in modeling competencies.

Implications and Significance of the Findings

The findings from this study indicate that elementary students' engagement in mathematical modeling competencies during group work conversations is influenced not only by the context of the task but also by the positions student enact through their interactions. The following sections discuss how these findings inform theory, practice, and research, and conclude with their significance for learners of mathematics.

Implications of the Findings

Implications for Theory. This research contributes to and extends existing theories of mathematical modeling and positioning.

Within mathematical modeling theory, the findings highlight that carrying out the modeling process through modeling competencies is not only situated in relevant, real-world contexts, but is also inextricably linked to the positions students enacted through interaction. As such, modeling competencies were interactionally produced through conversation. Students carried out the entirety of the modeling process through their continual re-engagement with the context – drawing upon their personal knowledge of their community and their lived experiences. This familiarity supported their modeling endeavors and their engagement in modeling competencies through the positions they enacted. Moreover, the context of the task directly supported the positions made available in modeling conversations as positions are connected to the substance of a conversation. However, the positions students assumed also influenced their engagement in these competencies. Taken together, these findings indicate that modeling theory in the elementary grades might more explicitly integrate community-connected contexts, recognizing that students' engagement in modeling competencies is informed and

supported by their personal knowledge of the context – which both shapes and is shaped by the positions they enact.

The findings also extend positioning theory into the space of mathematical modeling by illustrating how positions carry with them positional authority that influences student access to and engagement in modeling competencies. By enacting particular positions, students assumed levels of authority that shaped their participation – either affording or constraining access to the modeling work. Moreover, when students consistently enacted positions associated with authority, that authority became associated with the individual, further influencing their positioning and access to modeling competencies. These findings extend positioning theory by highlighting the influence of positional authority in shaping conversational engagement – particularly within mathematical modeling contexts.

Implications for Practice. Taken together, the findings have implications for the classroom, particularly for how teachers plan and facilitate modeling tasks with elementary learners.

When designing mathematical modeling tasks, teachers can support students' modeling endeavors by selecting or creating problems whose contexts connect to students' communities and experiences. Such tasks allow students to draw from their personal and community-based knowledge and experiences. This grounding in familiar contexts can deepen student engagement in modeling competencies and may support more students to enact positions related to the context of the task. By incorporating personally relevant, community-based modeling problems, teachers can position their students as knowledge holders in the mathematical modeling work, supporting both their authority and engagement in modeling competencies.

Awareness of positional authority also has practical implications for teachers facilitating modeling lessons. Positions function as resources that shape participation and access – students who enact positions with greater authority often have greater access to conversational resources. When planning mathematical modeling lessons with students conversing in small groups, teachers might intentionally plan for the types of positions the context could make space for, and plan authority-distributing moves. For instance, teachers might position students as higher-authority process contributors by inviting them to individually reflect on how they would approach the overarching problem before group discussion, and then ensure that all students share their ideas in turn. By holding an awareness of student positioning, teachers can more intentionally support equitable participation and promote student authority within mathematical modeling conversations.

Implications for Research. The findings presented from this study also suggest directions for research on modeling and positioning.

Currently, little research exists on how to assess students' modeling competencies, and the question of what should be assessed remains open. As Cevikbas et al. (2021) note, "[p]recise conceptualizations of mathematical modelling competence as a construct are needed to underpin reliable and valid measurement instruments and implementation studies to foster mathematical modelling competencies effectively" (p. 206). Future research could extend the present study by examining how elementary students engage with and develop modeling competencies across different contexts, thereby addressing this gap in the literature.

The present study did not directly examine how students' *learning* of mathematics was influenced through their engagement in modeling competencies. However, drawing on neo-

Vygotskian learning theory – which views interaction as the medium for learning – future research might explore not only how interactions and the resulting positions influence engagement in modeling competencies, but also what and how students learn through this interactionally produced engagement.

Additionally, research might further explore how individual characteristics shape positioning and corresponding positional authority. Harré and Langenhove (1991) emphasize that the traits and characteristics of individuals shape how they carry out that position and their power with positioning themselves and others. Understanding how individual traits interact with positioning processes in modeling contexts could offer valuable insight into how students become more skilled in positioning practices that support their access to and engagement in modeling competencies.

Significance of the Findings

This study contributes to the field of mathematics education by highlighting the importance of making visible the positions students enact – and the influence those positions hold over access to modeling conversations and engagement in modeling competencies. In doing so, this research bridges mathematical modeling and positioning theory by bringing them together in an elementary context. These positions shape the malleable and developing identities of students as they take up micro-identities that, through repeated enactment, evolve into more enduring macro-identities. The positions students adopt during modeling conversations influence how they see themselves as mathematical people – if only momentarily, but eventually, this awareness may become a part of their broader sense of self.

Recognizing and attending to these positions matters not only for students' mathematical development but also for their growth as mathematical human beings. When teachers and researchers learn to notice the positions students take up, they become better equipped to support students in developing as competent modelers – whether through classroom practice, or research that helps teachers in their work of shaping and supporting young learners. As Goleman (1997) reminds us:

> The range of what we think and do
> is limited by what we fail to notice.
> And because we fail to notice
> *that* we fail to notice,
> there is little we can do
> to change
> until we notice
> how failing to notice
> shapes our thoughts and deeds. (p. 24)

By examining how students engaged in modeling competencies during group work conversations – and how enacted positions influenced that engagement – this research responds to the call for a deeper understanding of elementary students' participation in mathematical modeling. It demonstrates that attention to positioning is essential in supporting student engagement in modeling competencies and developing their identities as capable mathematical modelers.

Limitations of the Research

The limitations of this research fall into two main categories: contextual and methodological boundaries, and interpretive and analytical constraints.

Contextual and Methodological Boundaries

One of the most apparent limitations of this study is the transferability of the findings. Transferability may be limited due to the specific participant demographics, students' modeling experiences, Ms. Kedrik's experience as a teacher of modeling, and the context of the modeling task. As a single case study centered on one modeling lesson, the positions enacted, the mathematical modeling competencies displayed, and the nature of student engagement in modeling competencies could differ significantly in other contexts.

Another limitation stems from the inherent challenges of collecting AV data in a classroom setting. Effectively capturing clear audio and video of student voices and gestures in a noisy setting – where students were talking and working with plastic bags – proved challenging. There were times when it was impossible to distinguish the speaker or their words. Student movement in their seats or around the room occasionally obscured the recording of other students. These limitations impacted the accuracy of the transcripts. Additionally, there were some technical issues where the audio files were squeaky and obscured student voices. Some students were seated facing a wall and it was impossible to record all of their gestures, or even their faces, on camera.

Interpretive and Analytical Constraints

A further limitation lies in my role as the primary instrument in data analysis. My understanding and interpretations were inevitably shaped by my personal knowledge and perspective, meaning the findings are bounded by these limits. In this study, I analyzed student interactions based on my comprehension of those interactions. Consequently, the analysis – and

the resulting findings – are limited to my interpretation of events and the ways in which I understood and made sense of the data.

Furthermore, I did not have baseline knowledge of the students and their interaction patterns. I could not determine whether students' interactions during this modeling lesson were typical or atypical. Additionally, if a student remained silent while others worked on the modeling problem, I could not know whether that student was disengaged or actively observing. In either case, the lack of visible interaction appeared the same.

Quality of the Findings

Coding "is not a precise science; it's primarily an interpretive act" (Miles et al., 2020, p. 83). Consequently, to ensure the soundness of the research presented in this study, I addressed reliability, confirmability, credibility, and transferability. I ensured *reliability* of the transcript codebook by grounding it in the Jefferson Transcription System, by adapting the codebook based on data from a first-grade modeling lesson, and by the codebook's collaborative development by two researchers. To ensure *confirmability*, I engaged in transparency by providing a complete, detailed description of the methods for data collection and analysis, I clearly articulated connections between the study design and the theoretical framework, and engaged in reflexivity by including a thorough account of my positionality in this research. I supported confirmability of the findings by tying the findings to the data, including several supporting excerpts, considering alternative interpretations, and by storing the data for reanalysis should the need arise. To support *credibility*, I used *thick description* in presenting the findings, engaged in discussions with my dissertation chair and committee members throughout the duration of the research process, and used *triangulation* of multiple data sources (audio and video recordings,

student work, field notes, content logs, transcripts, and analytic memos) in relation to both the research questions and positioning theory. I also used *negative* and *discrepant data* to inform code development during first and second cycle coding. This process helped refine and clarify emerging findings by capturing the nuance of student interactions, enacted positions, and their influence on engagement in mathematical modeling competencies. Finally, I supported *transferability* through rich, contextual descriptions of the case study setting and the participants, allowing readers to assess the applicability of the findings to similar situations.

Concluding Remarks

This research set out to deepen understanding of how fourth-grade students engaged with mathematical modeling competencies during group work, the positions they enacted through social interactions, and how those positions influenced their engagement in modeling competencies. The findings address this purpose by illustrating that personally relevant modeling contexts supported student engagement in modeling competencies, and that the positions students enacted through interaction carried levels of positional authority that either supported or constrained their access to modeling conversations – and, consequently, to modeling competencies themselves.

This research can be compared to a forest, where the study examined the nutrients present in the forest soil. These nutrients sustain the grasses, underbrush, and trees, yet they are invisible and often go unnoticed. Without them, the forest could not exist. As with nutrients, the positions students enact in conversation may seem unremarkable because they are not immediately visible, but they form the underlying fabric of social interaction. These positions influence students' access to conversation and, in turn, their engagement in mathematical modeling competencies.

Positions are thus the foundational nutrients that support students' engagement in mathematical modeling and their development as capable modelers who are able to recognize and use mathematics in their everyday lives.

Mathematical modeling competencies are nested within the broader process of modeling, which itself unfolds through mathematical conversation situated within the larger social fabric of conversation. While the positions identified in this study are specific to modeling conversations within the context of the Jump Rope task, the phenomenon of positioning is universal. Positions shape participation in all conversations—regardless of topic. Although these findings are situated within this single case, understanding how positioning influences conversational access has implications for broader contexts. This awareness enables individuals to notice how their own and others' positions affect access to the conversation. With this awareness comes agency: individuals can adjust their own positioning, influence the positioning of others, advocate for themselves and others, and shape conversations in which all participants have equitable opportunities to engage.

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APPENDICES

APPENDIX A

UNIVERSITY CONSENT FORMS

Table A1. Teacher Consent Form
IRB #1655
Page 1 of 3

Montana State University
Consent to Participate in Research

Study Title
Student Mathematical Modeling Competencies during Group Work

This is a consent form for participation in a research project.
Your participation in this research study is voluntary. This document contains important information about this study and what to expect. Please consider the information carefully. Feel free to ask questions before deciding whether or not to participate.

Summary of the research.
You are being invited to participate in a research study about mathematical modeling competencies. Mathematical modeling is a regular part of the elementary mathematics curriculum; it involves using mathematics to understand real world contexts and scenarios. Modeling competencies are the skills students use to work through a mathematical modeling task.

The study will take place during your regular mathematics instruction and will include recording a mathematical modeling lesson and collecting student work. The study will last for the duration of one modeling lesson.

Why is this study being done?
The study is conducted by Mary P. Greene under the direction of Mary Alice Carlson, PhD, Professor at Montana State University. Mary P. Greene is conducting this study to meet the requirements for her doctoral degree. The purpose of this research study is to understand how students engage with mathematical modeling competencies during group work. At your student's 4th grade level, mathematical modeling involves solving real-world problem-solving tasks. The results of this study may assist the teaching profession in better understanding how to support children's mathematical learning.

What will happen if I take part in this study?
Mary P. Greene will visit your classroom to observe during a mathematical modeling task. Student class work will be collected and the lesson will be videotaped to capture the students working in groups.

How long will I be in the study?
Your participation in the study will be for one mathematical modeling lesson on October 11, 2024.

Page 2 of 3

Can I stop being in the study?

You do not need to participate in this study. If you decide to take part in the study, you may leave the study at any time. No matter what decision you make, there will be no penalty to you and you will not lose any of your usual benefits. Your decision will not affect you or your future relationship with Montana State University. If you are a student or employee at Montana State University, your decision will not affect your grades or employment status.

What risks or benefits can I expect from being in the study?

There are no direct benefits to you. Your students may feel challenged by addressing unfamiliar questions during the modeling lesson.

Will I be paid for participating in the study or experience any costs?

You will be paid $200 for your participation in the study. Aside from the time required to plan and facilitate a mathematical modeling lesson, there are no costs for taking part in the study.

Will my study-related information be kept confidential?

Efforts will be made to keep you and your students' study-related information confidential. All computer files will be stored on a secure server and password protected. This consent form will be uploaded to the secure server and then shredded. Your name will not be used in any report. Identifiable research data will be encrypted and password protected.

The information collected in the study will be handled confidentially. However, there may be circumstances where this information must be released or shared as required by law. Montana State University Institutional Review Board; other federal, state, or international regulatory agencies; or the sponsor of the study, if any, may review the research records for monitoring purposes.

Will my study-related information be used for future research?

Information collected about you will not be used or shared for future research studies.

Who can answer my questions about the study?

For questions, concerns, or complaints about the study you may contact **Mary P. Greene**, doctoral student at Montana State University, via e-mail at marygreene1@montana.edu. For questions about your rights as a participant in this study or to discuss other study-related concerns or complaints with someone who is not part of the research study, you may contact Montana State University's Institutional Review Board at irb@montana.edu.

Page 3 of 3
Signing the consent form
***Please keep pages 1 and 2 for your records. Please return page three.**
By signing this consent form, you acknowledge that you have read (or someone has read to you) the details of this research study, and you freely give consent to participate. Also, as noted prior, participation is completely voluntary, and you may withdraw from the study at any time without penalty.

Date _____

Printed Name _____

Signature _____

Table A2. Student Consent Form
IRB #1655
Page 1 of 3

Montana State University
Parental Permission to Participate in Research

Study Title
Student Mathematical Modeling Competencies during Group Work

This is a consent form for participation in a research project.
Your child's participation in this research study is voluntary. This document contains important information about this study and what to expect. Please consider the information carefully. Feel free to ask questions before deciding whether or not to participate.

Summary of the research.
Your child is being invited to participate in a research study about mathematical modeling competencies. Mathematical modeling is a regular part of the elementary mathematics curriculum; it involves using mathematics to understand real world contexts and scenarios. Modeling competencies are the skills students use to work through a mathematical modeling task.

The study will take place during your child's regular mathematics instruction and will include recording a mathematical modeling lesson and collecting student work. The study will last for the duration of one modeling lesson.

Why is this study being done?
The study is conducted by Mary P. Greene under the direction of Mary Alice Carlson, PhD, Professor at Montana State University. Mary P. Greene is conducting this study to meet the requirements for her doctoral degree. The purpose of this research study is to understand how students engage with mathematical modeling competencies during group work. At your child's 4th grade level, mathematical modeling involves solving real-world problem-solving tasks. The results of this study may assist the teaching profession in better understanding how to support children's mathematical learning.

What will happen if your child take's part in this study?
Mary P. Greene will visit your child's classroom to observe your child during a mathematical modeling task. Student class work will be collected and the lesson will be videotaped to capture the students working in groups.

How long will your child be in the study?
Your child's participation in the study will be for one mathematical modeling lesson on October 11, 2024.

Can your child stop being in the study?
Your child does not need to participate in this study. If you decide to authorize your child's participation in the study, your child may withdraw from the study at any time. No matter what decision you make, there will be no penalty. Your decision will not affect you or your child's future relationship with Montana State University. Your decision will not affect your child's grades or class standing at Sunrise Elementary.

What risks or benefits can your child expect from being in the study?
There are no direct benefits to you. Your child may feel challenged by addressing unfamiliar questions during the modeling lesson. If your child is bothered as a result of this study you should contact your child's teacher or the principal investigator of this study.

Will your child be paid for participating in the study or experience any costs?
Your child will not be paid for participation in the study. Since all study activities occur as part of regular classroom mathematics instruction, there are no costs for taking part in the study.

Will your child's study-related information be kept confidential?
Efforts will be made to keep your child's study-related information confidential. All computer files will be stored on a secure server and password protected. This consent form will be uploaded to the secure server and then shredded. Your child's name will not be used in any report. Identifiable research data will be encrypted and password protected.

The information that you provide in the study will be handled confidentially. However, there may be circumstances where this information must be released or shared as required by law. Montana State University Institutional Review Board; other federal, state, or international regulatory agencies; or the sponsor of the study, if any, may review the research records for monitoring purposes.

Will your child's study-related information be used for future research?
Information collected about your child will not be used or shared for future research studies.

Who can answer my questions about the study?
For questions, concerns, or complaints about the study you may contact **Mary P. Greene**, doctoral student at Montana State University, via e-mail at marygreene1@montana.edu. For questions about your rights as a participant in this study or to discuss other study-related concerns or complaints with someone who is not part of the research study, you may contact Montana State University's Institutional Review Board at irb@montana.edu.

Page 3 of 3
Signing the consent form
***Please keep pages 1 and 2 for your records. Please return page three.**
By signing this consent form, you acknowledge that you have read (or someone has read to you) the details of this research study, and you freely give consent for your child to participate. Also, as noted prior, participation is completely voluntary, and your child may withdraw from the study at any time without penalty. Your child's choice to participate, or opt out, will not impact their grades or class standing in any way at Sunrise Elementary.

Date _____

Child's Name _____

Printed Name of Parent/Guardian _____

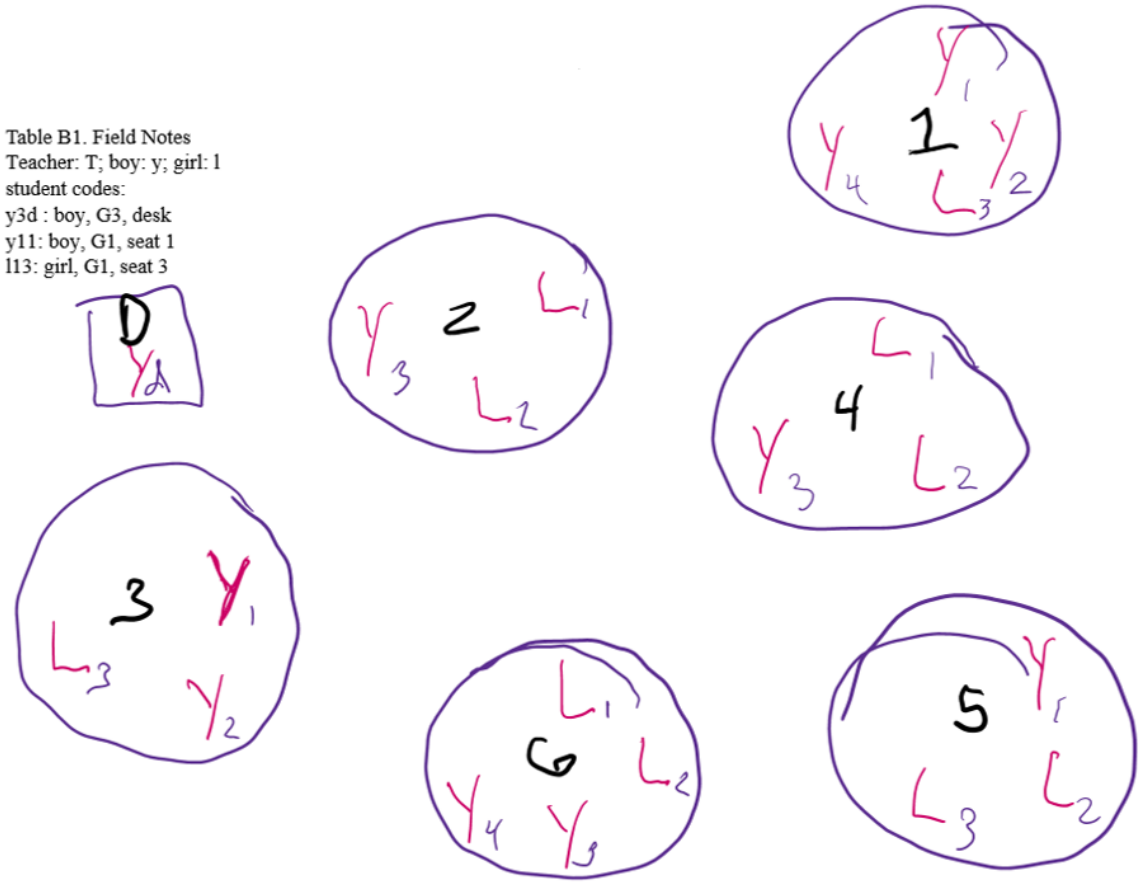
Signature of Parent/Guardian _____

Relationship to Child _____

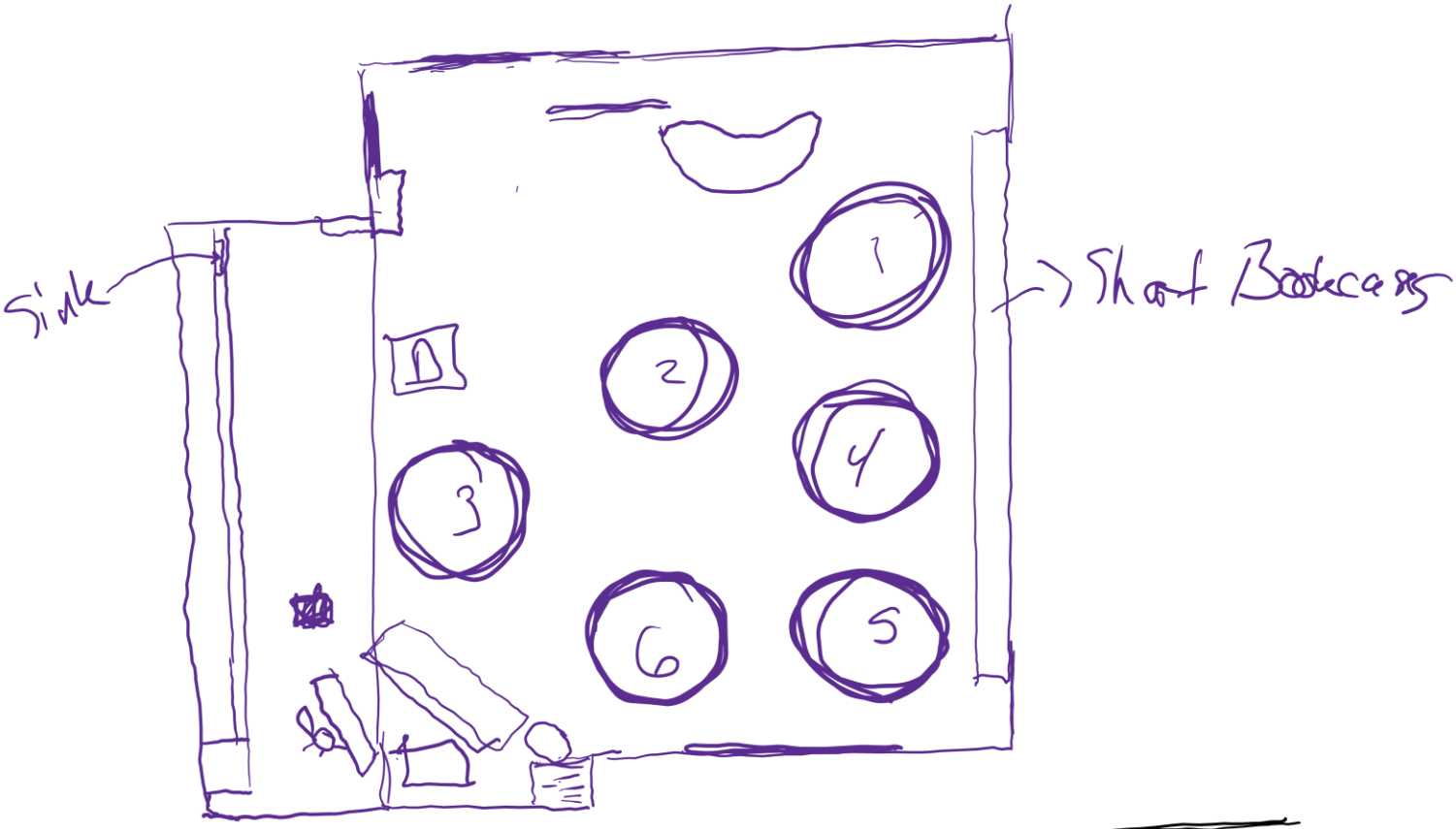
APPENDIX B

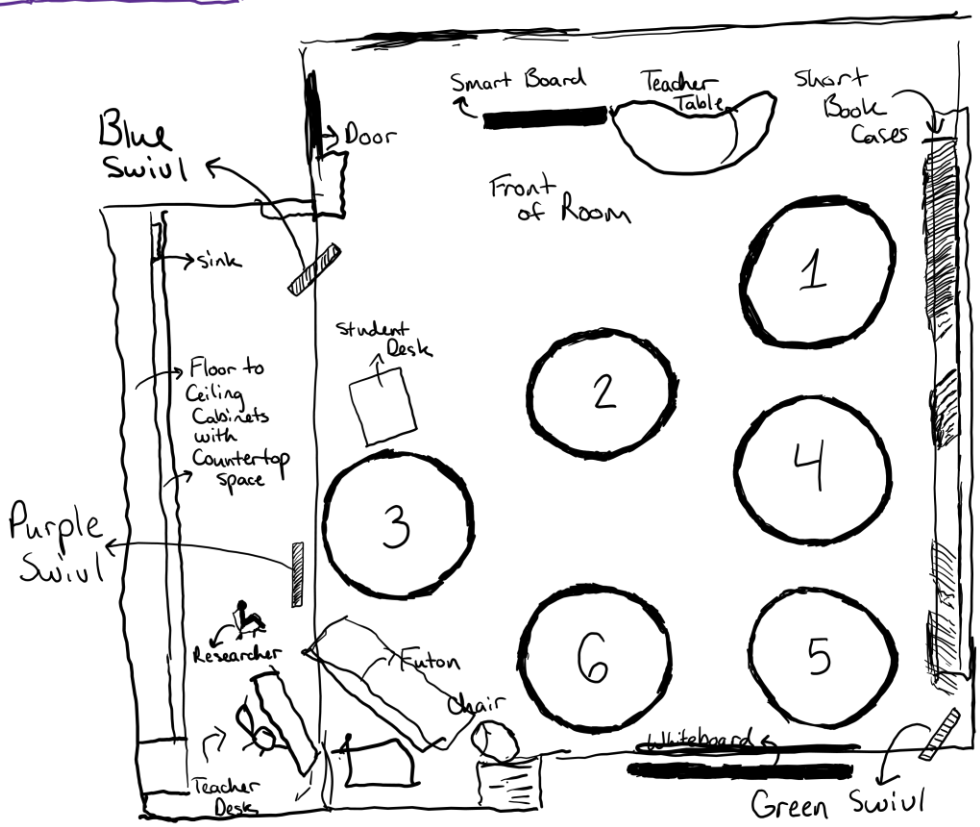
FIELD NOTES AND CONTENT LOGS

Table B1. Field Notes
Teacher: T; boy: y; girl: l
student codes:
y3d : boy, G3, desk
y11: boy, G1, seat 1
l13: girl, G1, seat 3



y11 Everet	y32 Emmet
y12 Owen	l33 Belle
l13 Wren	l41 Brittany
y14 Eric	l42 Lydia
l21 Brie	y43 Mason
l22 Ali	y51 Jacob
y23 Paxton	l52 Ellie
y3d Seamus	l53 Maria; l61 Melina; l62 Gemma
y31 Blaze	y63 Jay; y64 Johnny


Sink
Short Bookcases
1
2
3
4
5
6


Smart Board
Teacher Table
Short Book Cases
Door
Blue Swivel
Front of Room
sink
Student Desk
Floor to Ceiling Cabinets with Countertop Space
Purple Swivel
Researcher
Futon
Chair
Teacher Desk
Whiteboard
Green Swivel
1
2
3
4
5
6

room dimensions and notes
counting the ceiling tiles. from front of room with screen to back wall with dry erase boards: 14.5 tiles.

from front of room to window at back 15 tiles.
from front of room behind the door to window back: 12 tiles.
from door to wall, side to side: a little over 11 tiles.
from cabinet wall to opposite wall: a little over 15 tiles.
so the room is essentially square.
cabinets about 1 tile deep
G1 starts 5 and 6 ceiling tiles in from front to back.
book case along the opposite wall is a bit less than on tile in. But there is room to walk, so the desks start a little over one tile in.
G6 ends in line with the wall insert where the dry erase boards are at the back of the room.
Rug starts in line with the door insert away from cabinet wall.
Concrete where there is no rug.
G3 covers about 6, 7, 8, tiles from the window wall.

22 total students but one absent for the lesson.

I arrived at 8:05a, just in time for breakfast. I had planned to arrive closer to 8a but there was a train from 7:50-7:53 and then a red light immediately after that. T was sitting at her desk in the back of the classroom reviewing her slides. She wanted to go over a few things with me regarding the task, and she was a little worried about how it would go. The last time she facilitated the jump rope task with her 4th graders, they were farther along in their mathematics as they did the task in February. But it is mid-October and these kids have had very little exposure to multidigit multiplication. I asked if they could do repeated addition. Yes, but… well these kids are also more advanced mathematically as they are the first group she's taught that aren't affected by Covid. She was also concerned that they wouldn't have enough background knowledge to work through the task. The last group had done a jump rope unit in PE and knew that various lengths of rope were needed for different heights of children. We discussed that they may bring it up in their groups. Even if she only heard one of them say this, she could ask them to share with the class. Or, if no one mentions this, she could point out their different heights and ask if they could use the same rope. She felt better and was good to go after we talked.
T gave me the consent forms which I placed in my backpack.

Then I began setting up all the recording equipment. She said I had time. They don't start until 8:30 and they don't have to be at specials until 10:10a. I was setting up during the students' breakfast and several came over asking me questions about the technology. They seemed excited to be recorded.

I got them set up, and checked that the devices were recording. It was tricky lining up the audio markers with which camera b/c each camera covered about three tables. I did adjust the blue audio marker at G3 and the blue Swivl a little bit into the lesson during quiet writing time so as not to disrupt group work time. I moved the blue Swivl during a quiet writing time. I adjusted the audio marker to better capture the desk student when he leaned over to discuss with G3.

Before officially starting the lesson, T reminds them that they learned how to build these jump ropes. Does a quick demonstration.

T introduced lesson.

I tried to be as inconspicuous as possible. Sat in my usual place, took some notes. Tried not to make eye contact with any kids. But one girl, pink shirt with black heart at G6 looked at me, I gave a small smile. It was more difficult taking notes this time around than with previous observations. In this case I know the lesson is being recorded, so I don't need to write things that will show up on the camera. In my three prior observations, I didn't record anything so I wrote down everything I could about what was said and done. But I did think of questions to ask T after the lesson and noted those. It was challenging observing the students as I didn't want to observe a group too intensely and have them look at me. Even though I tried to avoid impacting their work, I think that my sitting near G3 and D may have influenced the way they communicated. A couple times it felt as though they were saying something a little louder for my attention. Or maybe I just felt that way because I was trying not to look at them too much.

T let them know that they can thank Miss Greene for cutting all the bags for them so they can do this. They thanked me. I said you're welcome.

It is 9:10a and the students are building the beginning of their jump ropes. The students are paying particular attention to the length of rope produced with three plastic bags. T walks around from group to group. She reminds them to only tape one side, not the other, and to braid to the end.

T: once braided all the way to the end, how long are three bags braided together. Measure once they get it braided. They need to know that.

The students are walking around, going from table to table building jump ropes. They don't seem bothered by the recording technology. Some students try to jump with their tiny jump rope. It feels like they are having fun. T asks a few not to jump with the ropes.

T regroups. They are going to collect info. The students go back to their tables. They've measured jump ropes and report on the length of their rope from the three plastic bags. Most common number is 12 inches, which is the same as one foot. T recaps the info they have, they

assume that they can braid 3 jump ropes into about 1 foot. During the discussion, the students bring up the info that taller people need a taller rope than a shorter person might. T gives them the table with person height and jump rope length. The students begin talking about heights.

They think T is over 5'5.
T: I'm not over 5'5 [said with a small smile]
T asks them to discuss in their groups, what they know about making jump ropes.
The groups begin discussing.

researcher thought: I should have put three audio markers at G3.

The students discuss and regroup.

T brings up assumptions and there is a class discussion around height. Some students share their heights with each other. y64 is 4'7. They compare several students back to back with heights and use that info to inform their assumption about jump rope lengths. T regroups the students who are comparing heights. She refocuses their attention on the assumption they are making about the height.

Now in groups, they are going to come up with a plan, the plans might be different. But at their tables, the students have to decide on what length of rope they need and how many bags they need.

T explicitly notes that their plans must show how they know what they need, what they need, any extra, how can others use their plan to make jump ropes.

There are 65 kids in 4th grade.

The students are supposed to start by writing an estimate, a guess, no work, about how many bags for all three classrooms. Their group estimates are different numbers as they report out. They did some work on their estimate.

They begin their group work. Groups compare across each other their different heights.
T wears microphone – the whole room can hear. But the groups don't seem to be listening in to each other.

They regroup and report out.

T collects their work, sends them off to PE and returns to give me the work. Then we chat for about 25 minutes or so. I record our conversation so that I'm not typing her answers to my questions. Multi-digit multiplication assess at the end of 2nd quart – end of January, G5 had trouble with the math.

I had mostly packed up after the lesson finished, but completely finished packing away the recording devices after talking with T. The students came back from PE and got their snacks sat down at their places. I was packed up ready to leave and the teacher recapped how many bags they thought they needed for the entire fourth grade it was around 1500.

She asked what they thought that they would need for just their class. They estimated around 500. She brought the two boxes of bags out and had me explain that for the first one I didn't think I took a minute per bag – closer to half of a minute per bag and I spent approximately 2 hours on the first box, and a little bit less for the other box.

Each student was going to make their jump rope. If they didn't have enough they would have a good enough start that they could finish them on their own at home. Then I left.

Part of me wished I had known they were going to make them immediately after PE. I would have left my recording devices up and recorded them making the jump ropes. However the other part of me was glad that I had already packed everything up, because making jump ropes is a little out outside the modeling competencies that I'm interested in looking at. But it is interesting, or it could have been interesting to see their interactions with each other. But from experience watching the EQ-STEMM making lessons, the craft part of the making task doesn't have that much mathematics – unless the students go back and revise their thinking. I said goodbye and thank you as I left – but the students thanked me!

Table B2. Lesson-Level Content Log

Lesson-Level Content Log

Notes

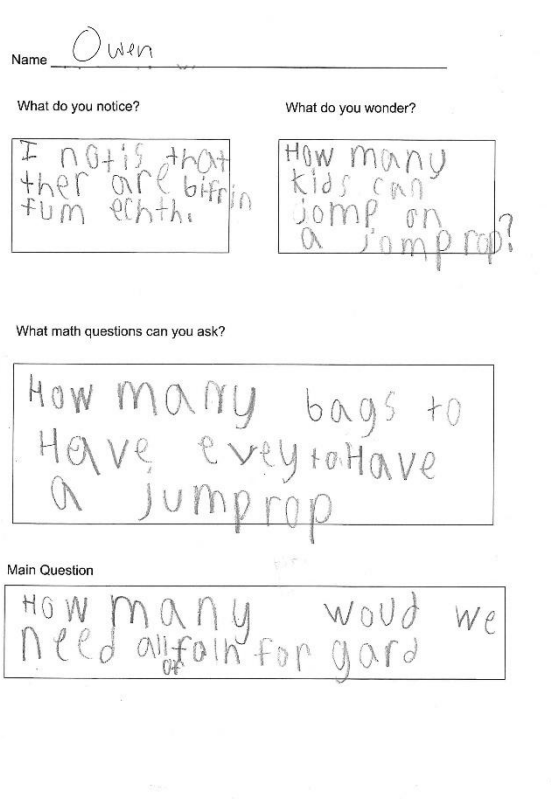
Acronyms
T = Teacher, Ms. Kedrik.
B_SVL = Blue Swivl Camera
P_SVL = Purple Swivl Camera
G_SVL = Green Swivl Camera
PE = Physical Education

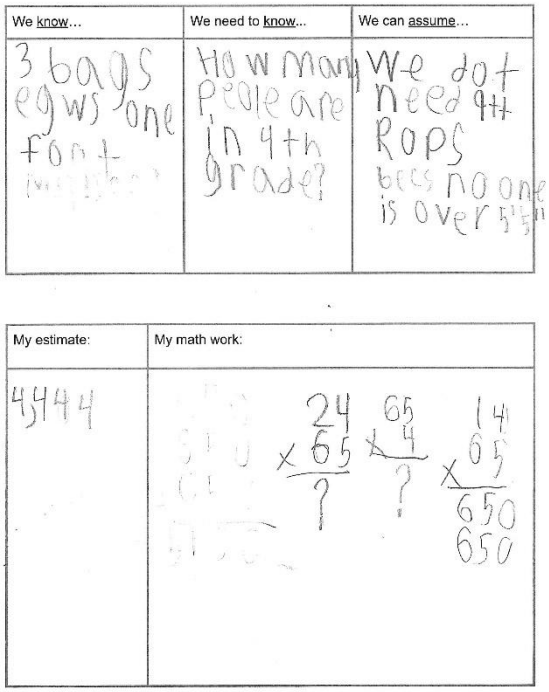
Lesson Time: about 1h 40m (per Swivl recording, three Swivl recordings)
Group Work Time: 42m 49s (per group)
Unconsented students at G2 and G6.

Group Work

Student groups (all names are pseudonyms)

Student Group Work – Group 1: Everet, Owen, Wren, Eric





Name Everet

What do you notice?

There are lots of different ways to jump rope.

What do you wonder?

I wonder who invented the jump rope.

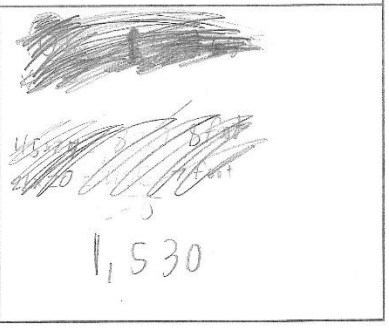
What math questions can you ask?

How many bags go in a rope?
How many ropes can we make?
Do we have enough jump ropes for the whole class?

Main Question

How many bags would it take to create jump ropes for the whole fourth grade?

We know...	We need to know...	We can assume...
• 3 bags = 1 foot • Everyone needs different sizes • up to 4'6 needs 7ft • 4'6 to 5'5 needs	• How many people in fourth grade. • How many bags do we have?	• we don't have 9ft ropes. • 65 students in 4th grade • Have 8ft rope

My estimate:	My math work:
1,189	

Name Wren

What do you notice?

There are a lot of diffrint kinds of jump ropes.

What do you wonder?

I wonder why different jump ropes are big or small and diferent colors?

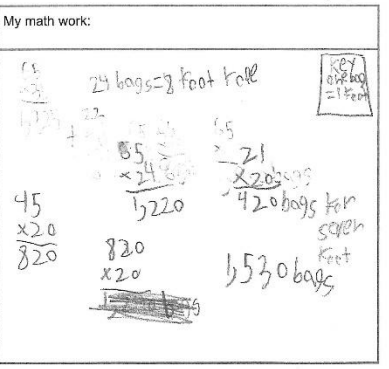
What math questions can you ask?

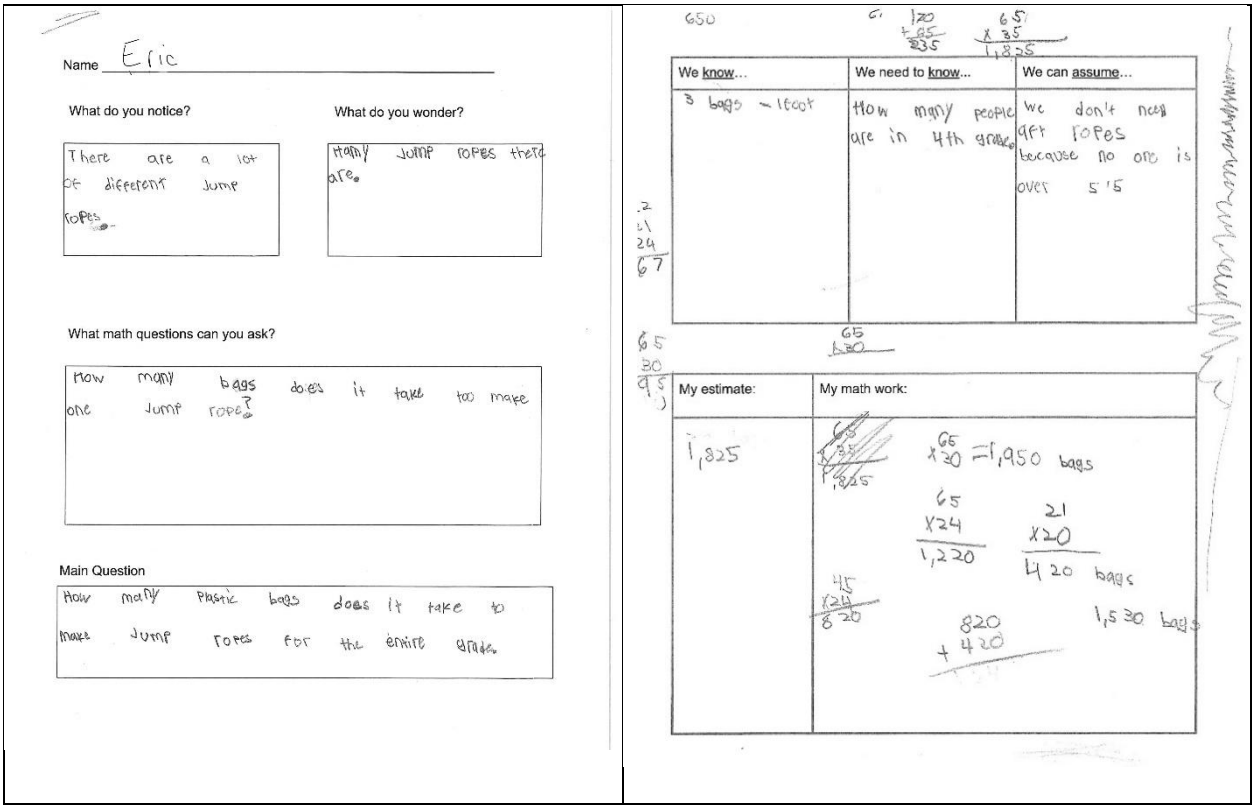
How many bags do you need to make one jump rope? How many bags do you need to make one big jump rope?

Main Question

How many bags does it take to make jump ropes for all of fourth grade?

We know...	We need to know...	We can assume...
3 bags = 1 foot. 7 feet, 8 feet.	How many pelle are in fourth grade? we need to know the avrege height of 4th grader. How many do we have?	we dont need 9 foot ropes. we can assume that there are sixty five students. we need 8 more ropes.

My estimate:	My math work:
1825	



Name Eric

What do you notice?

There are a lot of different jump ropes.

What do you wonder?

Many jump ropes there are.

What math questions can you ask?

How many bags does it take to make one jump rope?

Main Question

How many Plastic bags does it take to make jump ropes for the entire grade.

650

We know...	We need to know...	We can assume...
3 bags = 1 foot	How many people are in 4th grade.	We don't need 9ft ropes because no one is over 5'5

65 x 30 = 1,950 bags

65 x 24 = 1,220

21 x 20 = 420 bags

45 x 24 = 820

820 + 420 = 1,530 bags

My estimate:	My math work:
1,825	65 x 30 = 1,950 bags 65 x 24 = 1,220 21 x 20 = 420 bags 45 x 24 = 820 820 + 420 = 1,530 bags

Student Group Work – Group 3: Seamus, Blaze, Emmet, Belle

Name Belle

What do you notice?

difrent colors and difrent lahecths

What do you wonder?

what tipes of jomp ropes are there

What math questions can you ask?

How long do you need it and how many bags do you need?

Main Question

How to make your own plastic jump Rope

We know...	We need to know...	We can assume...
• difrent sises for difrent pepie	• we need to no how many pepl • avreg hite 50,49 feet • how mahy bags do we need	• we dont need 9 feet ropes • we assume that there is 60 pepie

My estimate:	My math work:
230 bags	15 × 51 = 765 50

Name Seamus

What do you notice?

maxeple size colers

What do you wonder?

is there real jump ropes

What math questions can you ask?

How large is reach BQ and how much time do we have to make the jump rope

Main Question

How to make a jump rope that reach the moon

We know...	We need to know...	We can assume...
	How many people don't know how to make a jump rope	

My estimate:	My math work:
24 BQs 1 ft rope 24 and 24 BQs = 1 ft rope	24 × 6 = 144 21 × 5 = 105

Name Emmet

What do you notice?

- That they are different lengths
- different colors
- some are thin some are thin
- makes dream mixed

What do you wonder?

- why are they different colors

What math questions can you ask?

- How long the jump ropes need to be?

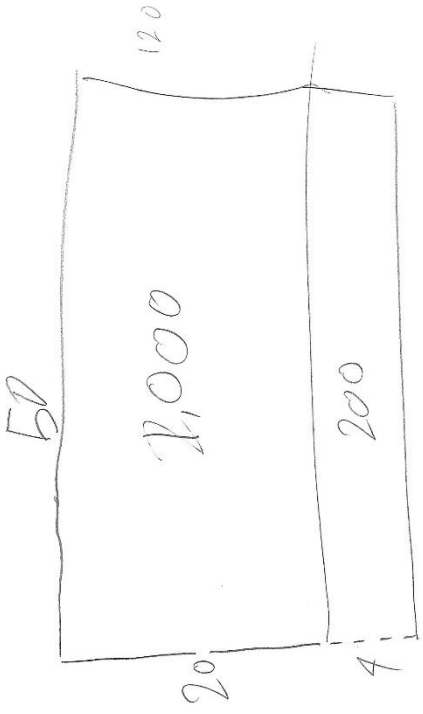
Main Question

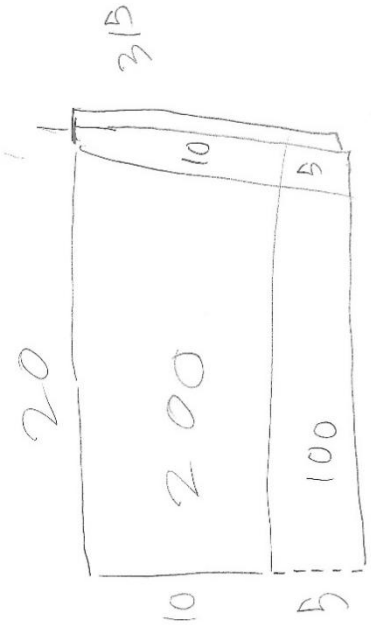
How many plastic bags do we need to make for the forth grade?

We know...	We need to know...	We can assume...
We need different sizes 7 total SH 3 bags = 1FT 4 pre to feet	How many people are in 4th grade	

21 x 15 = 315

My estimate:	My math work:
230 bags 1200 + 315 1,515	65 15 50 1200 + 315 1515





Name Blaze

What do you notice?

they are different colors and lagth

What do you wonder?

can you make a Rock Jump Rope

What math questions can you ask?

How many bags we nede for a Jamp rope? and How long

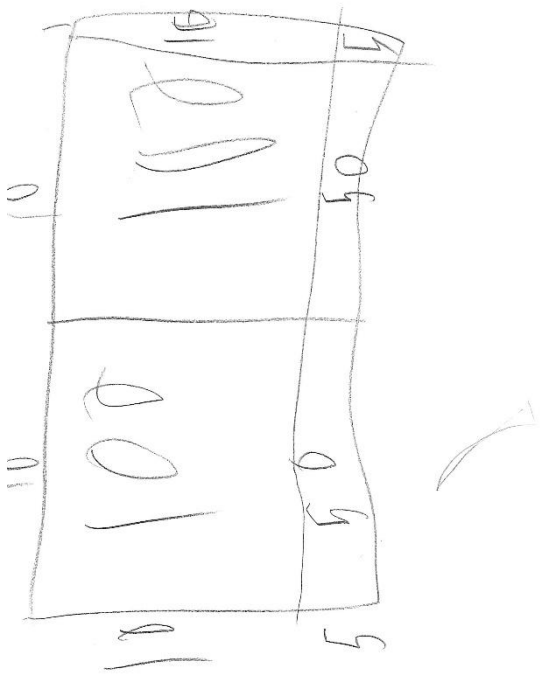
Main Question

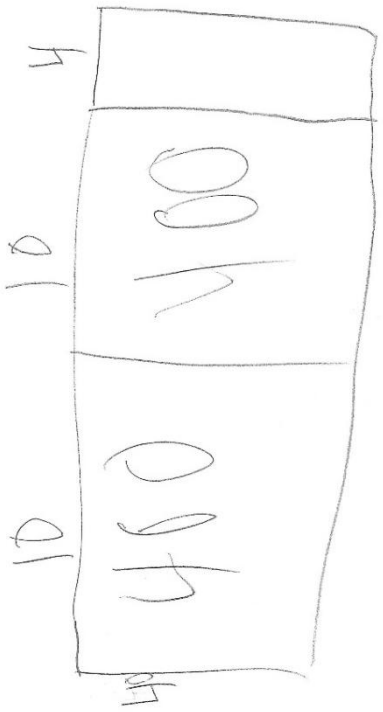
How many bags do we nede for the entir 4 grade

We know...	We need to know...	We can assume...
Diferent Sizes		

21
10

My estimate:	My math work:
1000 bags 315	24 bags in 6 15 6 30 1000 6 6





Student Group Work, Group 4: Brittany, Lydia, Jason

Name Brittany

What do you notice?

I notice that they are Diffrent sises.

What do you wonder?

What is Plactic made of?

What math questions can you ask?

How many Plactic Bags Dose it take to make evreone a Jumle rope?

Main Question

How many Plactic bags dose it take to create Jump ropes for the entire fourt grad?

We know...	We need to know...	We can assume...
Diffrent sises ropse	How many pep is apein foruth grade	We Dont need 9ft

My estimate:	My math work:
903	7X3=24 8X3=24 17X24=408 21X5=105 408 +105 513 513X3=1,539

Name Lydia

What do you notice?

there is a bigger size for some others

What do you wonder?

What math questions can you ask?

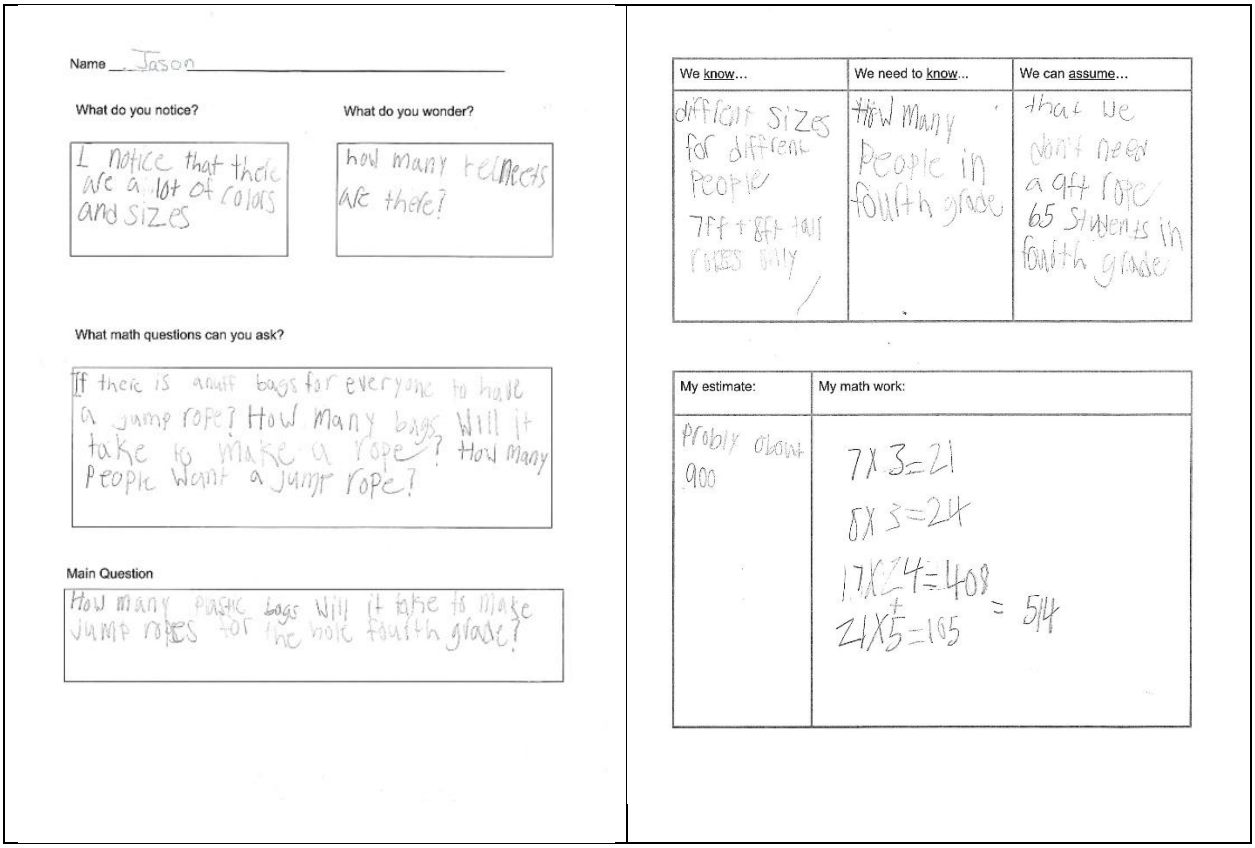
What length are the jump ropes? How many bags do we need? Is there the right amot?

Main Question

How many plastic bags does it take to create jump ropes for the entire fourth grade?

We know...	We need to know...	We can assume...
We need diffent size ropes.	how many people are in 4th grade?	We dont need 9ft ropes.

My estimate:	My math work:
900	7X3=21 8X3=24 17X24=408 21X5=105 408 +105 513 513X3=1,539



Name Jason

What do you notice?

I notice that there are a lot of colors and sizes

What do you wonder?

how many helmets are there?

What math questions can you ask?

If there is enuff bags for everyone to have a jump rope? How many bags will it take to make a rope? How many people want a jump rope?

Main Question

How many plastic bags will it take to make jump ropes for the hole fourth grade?

We know...	We need to know...	We can assume...
diffrint sizes for diffrent people 7ft + 8ft tall ropes only	How Many People in fourth grade	that we don't need a 9ft rope 65 Students in fourth grade

My estimate:	My math work:
Probly about 900	7 X 3 = 21 8 X 3 = 24 17 X 24 = 408 21 X 5 = 105 = 514

Student Group Work, Group 5: Jacob, Ellie, Maria

Name Ellie

What do you notice?

They are useing different teqnneeys.

What do you wonder?

How many can they do?

What math questions can you ask?

- How many plastic bags will we need?
- How many people will need to share?
- How many groups will we do?
- What sizes will we need?
- How much time?

Main Question

How many plastic bags will we need to make jumpropes for all of fourth grade?

We know...	We need to know...	We can assume...
3 bags are 12 inches up to 6,7 for teachs	How many 4th graders are here How many do we have	we don't need 9 feet jumpropes because no one is over 5'3. 65 students.

My estimate:	My math work:
480 bags	E-4-7 to 5-5

Name Maria

What do you notice?

They all diffent colors.

What do you wonder?

How many teknees are there?

What math questions can you ask?

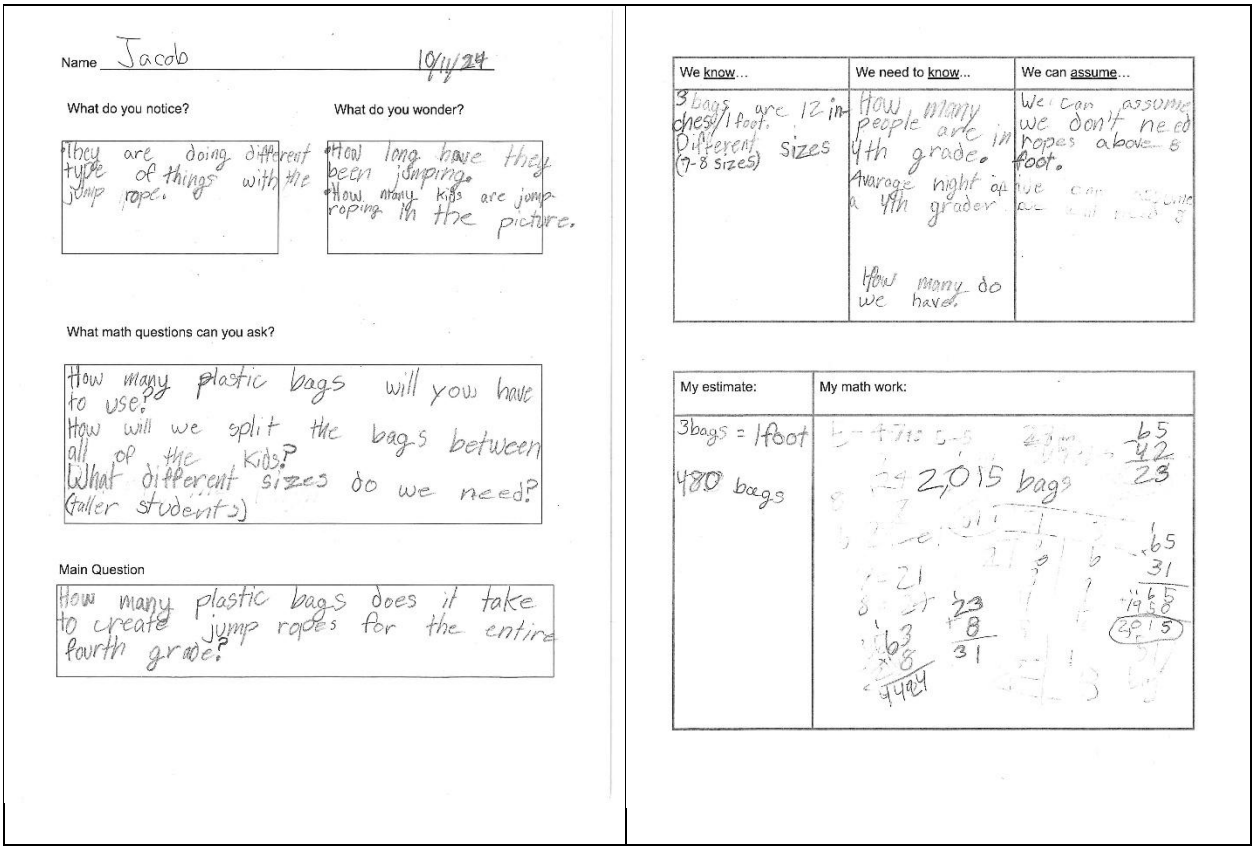
- How many bags do we need?
- How many jumpropes we need for every student?

Main Question

- How many jumpropes do we need to all of 4th gade?

We know...	We need to know...	We can assume...
3 bags are 12 inches.	How many peap are in 4th grade?	We doint have ony one that i taller than 5.5 There 65 kids in 4th gade.

My estimate:	My math work:
480 bags.	E-4-7



G2: Brenna, Alia, Paxton

G6: Melina, Gemma, Jay, Johnny

Introducing the Lesson

Time			**Description of Events**
G_SVL	***P_SVL***	***B_SVL***	
0:00	0:00	0:00	T reminds them that they are going to make jump ropes. Recaps by showing them how to braid the ropes. I am finishing up with setting up the technology. T wears an audio marker too. A student asks about the audio marker on the table. T checks that there is at least one person at each table who knows how to braid. Shows them how to braid. Explains that if you pull tight it will be a smaller section. They braid all the way to the end piece. They are going to do some measurements. T wears an audio marker associated with this Swivl (green)
3:14	3:00	2:37	T: “yes Seamus”

			Seamus asks about the audio recorders. T tells them about recording math conversations and that I'll hear any side conversations, they should only talk about math.
3:48	3:34	3:11	T: they are going to make a math task like the caramel apple one from before. First slide, what do you know about plastic bags. Think for a second and share with your table.

What do you know about plastic bags?

4:25 **GW**	4:06	3:43	Class erupts into group conversation.
G1_BR_BB_BP		3:43-4:17 Purple	Students discuss that they're good for shopping, other purposes like right now, garbage cans, bag is free – unless you're in California.
G2_BM_BG			Usually grocery stores have them there for shoppers, not the best for the environment, they burst their bottoms
G3_GB_GG		4:20-4:56 Blue	Made out of plastic for carrying groceries. T notes that that the desk is allowed to be in the video and be recorded, he thought he wasn't allowed initially. He jumps in sharing that they are recyclable and they scare his dog.
G4_PB_PM_PR		4:06-4:40	Plastic bags are in the ocean and sometimes kill animals like sea turtles, all sorts of animals, they're in stores, you can pay cents for a plastic bag.
G5_PG_PP		4:06-4:42 Green	Get them at the grocery store, lots of groceries in them, different logos, circle with target, different colors, different material, they're all made out of plastic.
G6_GR_GO			

4:56	4:41	4:18	T regroups. "Alright, what do we know about plastic bags?" Students share: use them for shopping, they are recyclable, not ecofriendly. T notes they are going to learn more about these in a book. They will take that idea, upcycling. Not recycling but where you use it for another purpose. They are going to upcycle their plastic bags they've been collecting into jump ropes.

What do you know about jump ropes?

6:45 **GW**	6:30	6:07	"What do you know about jump ropes?" Talk in tables, what are some things they know. Group conversation

G1_BR_BB_BP	6:08-6:38 Purple	They're fun, you can do a lot of different challenges with them, can do it with a big long piece of cording, two jump ropes for twice as fast, not twice as fast just twice as much.
G2_BM_BG		
G3_GB_GG	6:46-7:17 Blue	Ropes are normally made of rope, plastic, or fiber glass or something like that, like climbing rope. Like the rope of horses.
G4_PB_PM_PR	6:33-7:03 Blue	They're fun, can do tricks on them, can have two people, can play with friends or by yourself.
G5_PG_PP	6:33-7:03 Green	They're fun, you jump like this, good way to exercise your body, lots of things to use a jump rope for, tug of war.
G6_GR_GO		

7:20	7:05	6:42	T regroups: what do you know? Students: fun way to exercise, different techniques for jumping. Can do a lot of them. T: what do you mean? Different tricks, can play games with them, can do a long rope and two of them for someone to jump. They come in different sizes. T: tell me more about that. S: smaller for one person, longer for two. Can jump alone or with friends, make of different materials like plastic. T gives a moment to think

9:15 **GW**	9:00	8:37	Group conversation
G1_BR_BB_BP		8:37-9:10 Purple	There are different kinds of jump ropes, different kinds and different ways to do it, some are bad at jump roping, every girl is good at it.
G2_BM_BG			
G3_GB_GG		9:14-9:52 Blue - > main	The black one in front has one jump rope, some are shorter, some are thin, some are thick. You can make it, but it just takes forever.
G4_PB_PM_PR		9:00-9:29 (39?) blue	Notices there are different colors, they go at different speeds, yeah, with that one in the first picture there is a really big one for their jump roping
G5_PG_PP		9:00-9:39 Purple	They're different colors, different speeds of jumping, different techniques, some of the skipping ones, one foot, they cross sometimes, more than one person, yeah
G6_GR_GO			

What do you notice, what do you wonder?

9:53	9:39	9:15	"Alright" Regroup. What do they notice and wonder. They will fill it out on their paper Don't do the math questions yet, just do the notice and wonder at the very top.

10:25 **GW - silent**	10:11	9:47	Class is pretty quiet. T passes out their paper. They begin discussing with each other. T announces that they can do the notice and wonder, no math questions yet. I sit down to take notes. T walks around the room, but it's still pretty silent. The group conversations don't happen in this time segment although there are some whispers here and there. T asks for a thumbs up if they've had a chance to finish.
G1_BR_BB_BP			Papers shuffling, students whispering as they write. Notice that there are different kinds of jump ropes. Check in with T about doing notice and wonder, not math questions yet. Very Quiet.
G2_BM_BG			
G3_GB_GG		----silent	Some chatter here but it's pretty silent with students recording their thinking. Some whispers.
G4_PB_PM_PR		10:18-10:27 Blue	Student asks for Jason to hand the paper, he tosses it back, one group member says he doesn't have to through them.
G5_PG_PP		---silent during writing	Small talk about pens. Generally pretty quiet while student write.
G6_GR_GO			

13:25	13:11	12:47	T records some ideas. Notices first Students share: different colors of ropes, bigger sizes for different kind of jumping, all different sizes, different speeds – some fast than others, the jump ropes look different, they are lots of ..., one for the jump ropes was made of rubber bands so different materials. T asks about what they wonder Students: why some are smaller than others, how many kids jump roping there are in the pictures, how many jumps can they do

What math questions can you ask?

16:45	16:32	16:08	"We are going to think about..." T: what math questions can you ask. Goal today is to create these upcycled jump ropes, so what are some math questions you can ask that you might need to solve before we can do this. 30 seconds of quiet think time.

17:35 **GW**	17:22	16:55	Group discussion and T walks around
G1_BR_BB_BP		16:55-18:03 Purple	How many jump ropes for one rope which we know the answer but we'd have to ask it, you don't actually know know, not for sure. 30 bags for one jump rope, but maybe we can combine for a big one. how many bags go into one jump rope and how many we can make. if we know how many bags we have we would know exactly how to, we could divide by 30 right. how many bags do we need. my estimate could be 31. we could estimate at 30. how many bags do we need for everyone. my estimate would be 30.
G2_BM_BG			
G3_GB_GG		17:32-18:13 Main	How tall you are, how long you need it. Discussion around individuals' heights. After 18:13 they remind T that she forgot to pick a stick for sitting on the futon – which they will do after this lesson.
G4_PB_PM_PR		17:17-18:17 Blue	How many jump ropes to make one, figure out the lengths because some are taller than others, depends on height, ummmm how many plastic bags to make the jump ropes. T briefly stops by. If we have enough jump ropes to have everybody in the class have one. Some whispering sounds.
G5_PG_PP		17:17-18:27 green	How many kids are jumping, what kind of jump rope are they using. T stops by. reminds them about math questions. How many plastic bags will we need. how many jump ropes we need, how many kids will jump, how many kids will share, how many plastic bags we have to use.
G6_GR_GO			

18:40 **GW - silent**	18:27	18:03	T has them write down math questions. It's quiet in the room now. I move the blue Swivl and adjust an audio marker at G3. T sees their questions and will start typing them in.
G1_BR_BB_BP			Super silent in the group as they write. They start whispering. What do you think. I wrote how many bags do you have for everyone to have a jump rope. Probably 180 or something. So how many bags for everyone to have a jump rope. Doesn't really make sense.
G2_BM_BG			
G3_GB_GG		----silent	
G4_PB_PM_PR		----silent	
G5_PG_PP		----silent	

G6_GR_GO		

21:40	21:27	21:03	Students share: how many plastic bags for everybody in the class; what lengths are the jump ropes; how thick are the ropes; how long does it need to be different sizes, taller students need taller ones. Shorter need shorter might trip on a rope that is too long. Would change the number of bags, more bags; how many plastic bags in all, how many people, do we have enough bags, how many bags for a two person rope, how long is each bag and how much time do we have.

Posing the Modeling Problem

26:35 **GW - silent**	26:22	25:58	T: main question: How many plastic bags does it take to create jump ropes for all of 4th grade. Class starts talking. T regroups, has them record the question. They are going to do math to see what they would need if they wanted to do this. Students record on papers. T walks around.
G1_BR_BB_BP			Group gasps no! For the whole school. Ok, that makes sense. It'd be a waste of my time. Just go shopping every day. I would. I might have some [it's hard to hear students over the teacher's mic] pretty quiet as students record. You skipped, you said how many would we need. I know, but you said how many would we need, if someone doesn't know what we're talking about. T stops by, that's ok, I would say all of 4th grade.
G2_BM_BG			
G3_GB_GG		27:12-17:25 Main 28:38-	When everyone gasps about the modeling question, one student in this group says no, 400 is a good estimate. That's true. Otherwise, silent with recording. Whispering – I put 116, 115, 200, 300 or 400 max. You're supposed to write that. But I can't tell who's doing the whispering.
G4_PB_PM_PR		----silent	Silent except for a gasp with the main question and a comment "whoa, that's a lot"
G5_PG_PP		-----silent	General exclamations of excitement.
G6_GR_GO			

Braiding plastic bags

29:35	29:21	28:58	"So, we're going to do a little investigation here..." T thanks me that I prepped the bags. They will need tape for the handle. First is

			investigation. She shows them a video to show them how to braid, she showed them yesterday, but this will show as well. T plays video.
31:35	31:21	30:58	They are going to investigate. They are going to braid, T helps them think about getting the whole piece braided, want to see how long it is from the knot to the end, how long? They are going to braid and get the measurements.

32:44 **GW**	32:30	32:07	"So, I'm going to come to your table..." Group work time. T gives bags, students begin talking and braiding in groups. T goes from table to table checking in, asking how they're doing, supporting their braiding. T teaches how to braid as needed. T asks about measuring.

G1_BR_BB_BP	32:08-39:23 Red	Discussion about braiding. They know how to braid. Can we start? T: yes you can start. lots of rustling from the bags. Comments about types of bags – Albertson's, Walmart, target. Most times at target. You put the top bag under right, then you pull it. Yep, you did it. Wait how did you do that so quick? Ok, make a circle for Owen, how many would you use to make it tighter, but for one jump rope. they decide to make it a bit tighter. They're going really hard. don't paper cut yourself I've had it before when I was a little guy. Tuck that one though, yeah there you go. Put that through and pull here. then pull here too. Combine them all. That's so tiny. Each table is going to make one jump rope. how. The group walks around a bit. Out of range of mic but working on braiding bags together. T stops by to help them braid. Students wandering around the room. We're going to need to be, are we still braiding. Who wants to be the tester for this. Oh my gosh, that's so little. What the heck. Do you know what….. no do you know what…. I took a shower this morning, it looks like a wire for some reason, body wash, it's nice, it's not looking too bad. Discussion around making one whole big bag.
G2_BM_BG		
G3_GB_GG	32:48-39:45 Blue & main	Very hard to distinguish audio here. The markers pick up on so much background noise. They work on braiding their bags. Can't see them while the class is standing because G6 blocks the view. 35:00 audio is better. T stops by helps with assigning tasks for braiding. students wandering around the room as they braid the bags. They wander back. Suddenly the markers

		block out background noise more so it's easier to hear the students. The student at the desk stays at the desk when the other students walk around. They braid their rope.
G4_PB_PM_PR	32:37-39:30 Blue -> red	They each get two so they each get to make one. T hands them the bags. They arrange their bags. How do you do it? They help each other loop their bags. They get it and are excited for each other. Jason is the one who knows how to do it and helped the other two. Jason walks over to Group 1, but is called back to braid with his group mates. They braid the plastic bags. They're a bit far from the microphones on the table. General chatting amongst each other. T stops by asks how long the braid from three bags is.
G5_PG_PP	32:34-39:33 Purple <-> green	Discussion about roles while braiding, who gets to hold it. They do rock paper scissors, someone wants to braid it. Worries about cheating with rock paper scissors. They work with their bags. View on P_SVL is blocked by G6. Discussion around getting the bags together and braiding. T stops by. They say they are doing great. She watches for a minute and leaves. You do the first loop and I'll do the second. This is how you do it. I know how to do it guys. You're not letting me do something. My arm's getting sore. My turn, it doesn't need to be perfect, no judging. My turn. Some laughing. They get tape. This is not going to work. Yes it is. Shows how to tape it. Nice.
G6_GR_GO		

Measuring the Jump Rope

40:00 **(GW)**	39:46	39:23	T quickly recalls class and asks the class to measure how long three bags braided together are from the knot. Students continue working in groups and T moves from table to table asking them about measuring from the knot to end of braid.
G1_BR_BB_BP		39:23-41:53 Red	T gives directions. Hard to understand everything they're saying and they're stepping out of the frame of the video a little bit. Discussion around an eraser. they show their jump rope to other students. They note it's exactly 12 inches. Well not exactly. General discussion about things unrelated to math. The group is mulling around chatting with other groups.
G2_BM_BG			
G3_GB_GG		40:27-42:15 Primary & blue	Length is one foot. general chatter with each other and other group members as students walk around the room. T stops

		by, wants to know how long it is from the knot. They report 1 foot.
G4_PB_PM_PR	40:13-40:44 Red	They measure the braid. And walk around a little bit. They walk out of the picture at 40:44. The class generally walking around talking with each other.
G5_PG_PP	40:11-42:12 Purple <-> green	Mumbling, laughing. T stops by asking where their knots are at. How long is the braided section. T leaves. Some discussion, but hard to hear – there's a lot of background noise. They say hi to the microphone.
G6_GR_GO		

42:30	42:16	41:53	T regroups. Students back at tables to collect info. On their paper, they measured knots to end. Groups: 12 inches, 13.5 inches, 1 foot – not counting the handle, 9 inches, 12 inches, 13.5 inches. T what do we have the most of. Class realizes it's around 12 inches because 1 foot is the same as 12 inches. They agree it's a good number to use. T points out it's a friendly number to use, it might be different depending on the bags. T recaps info.
46:48	46:34	46:11	T gives jump rope measurements. Up to 4'6" tall then jump rope is 7' long. Between 4'6" and 5'5" then the rope is 8' long. Over 5'5" then the jump rope is 9' long. A student comments none are over 5'5" except the T. T comments that she's not over 5'5, good info for them.

What do you know about making jump ropes?

47:50 **GW**	47:36	47:13	"Ok, at your tables, go ahead and talk about..." T has them talk at their tables, what do they know about making jump ropes. They have some info. T walks around. Students work in groups.
G1_BR_BB_BP		47:23-48:14 Purple	We know that 3 bags is a foot, so, we also know that most people will need an 8 foot long jump rope, most unless there might be like 1 like Maria, Wren, they list other students who might need a shorter rope. Alia is like 4'8. I'm 4'11, almost 5'. I'm 4'11 and ½.
G2_BM_BG			
G3_GB_GG		48:00-48:53 Main & blue	Making jump ropes is twice as tall, if you go from the floor connected to arm span. Discussing heights. View is blocked for a bit by T.
G4_PB_PM_PR		47:45-48:40 Main	We know that we can braid them, that we need to do length and find out how many feet are on the jump rope in all.

		There could be 7 feet, 8 feet. We would need to figure out how many feet it is. Some inaudible mutterings.
G5_PG_PP	47:45-48:34 Purple <-> green	Three bags is 12 inches, or is that six bags? Six bags is. Measure here to hear. T stops by, how long is three bags, that's what we're asking. T leaves.
G6_GR_GO		

48:53	48:39	48:16	T regroups. What are some things they already know. Pulls popsicle sticks. Students share: different sizes for different people. 8, 7 and 5. T recalls nobody needs a 9 foot rope. Discussion around Paxton's height. T are we going to assume that no 4th grader will be taller. Class discussion about heights of people they know. They are talking about 4th grade class this year. No one is taller than Paxton. They assume they won't need a 9' rope.
51:11	50:57	50:34	T what else do we know Students: 3 bags is one foot; need to know how many people are in 4th grade. T: can we assume or make a guess about how many are in 4th grade? They decide on 66. But then discuss and land on 65 – it's an assumption and a friendly number.
53:00	52:46	52:23	T: anything else we know, need to know. Do you have all the info you need? Class: no. Because estimate of smallest and tallest, average height. T ok, can we make assumption about that? Student: yes, and something we do need to know. T: if we wanted to make them. But right now, what do we need if we want to make them. We won't make them for all of 4th grade, just our class, but we might not have enough for that. Class discussion around numbers and heights. They know up to 4'6 they need 7 feet, 4'7 to 5'5 is 8 feet. Add to what they know. Class discussion around numbers of students for each size.
57:28	57:14	56:51	Johnny is 4'7, they discuss how many people are shorter than that. Students move around and compare heights. T regroups. Assumption is that they need more 8 foot than 7 foot ropes.
59:03	58:49	58:26	T says they'll come up with plans in their groups, they might be different. They have what they know and need to know. Their plans must show: how they know they have enough for PE, without a lot of extra, how many bags they need, and how others could use your plan to make jump ropes. First part of paper, make an estimate about the number of bags, then show work. Decide at tables how many 7 and 8 ft ropes to make. 65 students total. Decide how many ropes for 7 and how

			many for 8. They know 3 bags for 1 foot of rope, use the info when deciding how many "jump ropes for a 7 foot rope and how many go into an 8 foot rope"
1:01:05	59:51	59:28	Student asks a question about not being able to make an 8 foot rope with 3 bags. Students, you have to use more. T wants to know how many bags in a rope. You have 6 bags, your rope is about 2ft long, think about how many bags in a full complete rope.

Estimating the number of plastic bags they will need

1:01:59 **GW**	1:01:45	1:01:22	[clapping sound from T] Before they work, put down estimate. Groups begin discussing. T walks around. T reminds them they are just writing an estimate.
G1_BR_BB_BP		1:01:35-1:03:48 Red	I would say 30. All three classes, how many bags. It takes 30 bags to make one rope. to make one rope yeah. so 65*30. that's my estimate. Ummmm what 65*3. that's 65+65=180, no 170, ok, that would be plus 40 is 200, then plus 25 and that's 225 that's 65*3. It's 235. I went for 400. I killed this pencil. 235. Yeah, 235, then add, then you do, so what is it 65*65. I say 400 because it's a guess. No, how do you do 65*3. It's a guess guys. How about we use our basic facts. If I know that 5*5 is 25, 6*3 is 18. I'm going to say 1189. I said 400. You're off. Yeah, you're way off. I think it's 1825. I think it's 1825.
G2_BM_BG			
G3_GB_GG		1:02:11-1:04:25 Main and blue	Discuss numbers. 65*4. It's over 100. 21 is a good estimate for a bag. Guess for the whole class is 230. For everybody, 230 bags. Why I'm doing this is because there's 61 is our estimate. 65+65 plus another 65. number discussion.
G4_PB_PM_PR		1:01:28-1:03:06 Blue	Still during class discussion but one group member notes that 6 bags is two feet. It's about 6 foot but the end, not 6. They work on an estimate, everybody is five foot, definitely not, maybe average is 4'6. Discussion of student heights. What are we estimating? I don't know, guess 135. How many bags for each class. That's not going to be it Lydia.
		1:03:06-1:04:11 Red	Has to be at least double the amount of classroom, let's say there was 30 kids in our class and average height is 4'10 we'd probably need 100. We should round, 330. No like 100 per classroom. Estimating 300.
G5_PG_PP		1:01:59-1:04:12 Purple ->green Squeaky audio markers, but pretty good audio here.	1 foot if everyone is around 5 foot, well not everyone is. Average height of 4th grade is 4'6. No, that's the height of Johnny. Discussion of heights. An estimate. 73. Let's do the work. Curly hair conversation. Everybody is 4'7".

G6_GR_GO		

1:04:25	1:04:11	1:03:48	T regroups, asks about a reasonable estimate. Asks about bags in a whole rope. Student volunteers 30, their estimates are 1,800 – friendly number. T shares thinking of a student 24 bags in an 8 foot rope, then 240 for 4th grade, but that's only 10 so using that number to come up with an estimate. Would 20k be reasonable? No. Be sure numbers are reasonable. 900, 3000. Different numbers. 450 bags. About 25 bags in a rope. Think quarters. 1000 – higher than 450. T helps with mathematics.

Building Models to Answer the Question

1:08:07 **GW**	1:07:53	1:07:29	"Ok. Alright, so..." They are going to do their work. T recaps info and what they need to do. They understand what they are doing and begin working in groups. T visits tables as they work.
G1_BR_BB_BP		1:08:03-1:16:25 Red -> Purple	First we need to figure out, yeah. Owen, put that in the middle. Figure out how many in a 7' rope. Don't do that, not good for you, you're eating lead. Not good for you. Ok then get sick, I don't care it's your fault. Yay, yay. Key is bags for foot. Idea, how many kids in our class. We know. Then count by the kids in our class to the other classes. Is that a good idea or not? I don't know. you can figure it out. it's a good idea but hard to figure out. How many in this class right now. They discuss numbers in the classes. They add them up. They got 65. Then 65*30. They work it out. 65+10, or add 65 ten times which would be harder. or add 65, 30 times. It's 75. That is not 75. We're doing 65*10. not plus 10. we're breaking it up. I have an idea. It's 950. Wait, no because 65*10=650. Adding, 1950 would be the answer. They have the answer. Discussion about how they got there. They all record it on their papers. They compare with the estimate they made at the beginning of class. They were close. Make sure to write bags. Height comparison with other students. Basketball comes up. After 1:16:25 They comment on the answer. But otherwise, they discuss dinosaurs. Wander around the room a bit. Chat, and make funny sounds for each other. Discussion of a spider. T stops by, they tell her their total. T asks about their work. Asks how they got their numbers. 3 bags in one foot, in 8 bags how many. 30 is too high. Can you go from here, how many more groups of 65 can you add. two more

		groups? Nope, but rather than do the whole problem over… T helps with math strategies. They discuss multiplication. They are back to working on math, using basic facts and they get 1,240 bags.
G2_BM_BG		
G3_GB_GG	1:08:37-1:25:54 Main and blue	Discussion about who needs what rope length. How many bags in all of it. So confused. A jump rope snapped and went into pieces. An actual rope, not bags. that means it was too big. So if you need a 7 foot rope, but most need 8. Discussion about who is taller or shorter. They compare heights with themselves and people from other groups. T reminds them to work in their groups. Mine was 8 feet so we tied the ends to make it smaller. Diamonds or crystals or rocks. Rock discussion. I don't know what you're talking about but focus on.. focus again on the task. They go back to discussing who needs what size rope. Number of students 6+15 is 21 people. T stops by. Where are we at. Reports on who needs what size rope and the number of people per rope size. T asks about all the fourth grade, not just this class. They go through individual students again. T counts. If there's 6 in our class, do you think it's similar in other classes. Does it have to be exact. They named 10 people. They think 15. Others might be an 8' rope. 50 and 15 is 65. T: how many bags for a 7' rope. They were doing 7*3 = 21 bags. So how many people will have a 21 bag rope. 15. They multiply. They get more paper for an area model. T leaves. They want to be exact and use an area model. They work through mathematics. Then plus the tall people. Don't forget to draw the rest. Working through multiplying. Tell the teacher, tell the teacher, tell the teacher. It's snack time.
G4_PB_PM_PR	1:08:24-1:15:01 Main	Need to figure out the height. Discussion about pronunciation of figure. Figure out we need to realize how much bags we need in our classroom first then we can go on to the whole 4th grade. Pencil tapping. One's an Avatar. One is counting their class. 21 people in the class right now. But 22 total. Almost everyone needs an 8 foot rope, except a few maybe. They discuss individual students who need shorter ropes. Some students from other tables block the view a little bit. But still able to see fairly well. 4 people need 7 foot ropes. Kids in the middle of the room

		comparing heights. T breaks them up after a little bit. The table view is back. 4 people maybe with a 7 foot rope. Discussion of individual heights. 7*3=21. 21 bags is a 7foot. So 24, then 8*3=24. So if everyone in our class, 24* most of them, only a couple that need a 7 foot rope, others need an 8 foot rope correct. Correct. 18*24. Work with me now. "Is that a booger?" Discussion of multiplying.
	1:15:01-1:16:06 Blue	T stops by reminds them all to work together and talk together and not just Lydia and not just copying. Lydia explains her thinking. T leaves.
	1:16:06-1:23:15 Blue	Working on multiplying. Another group of students moves to this table. Hard to distinguish voices. but Lydia is the only one working at her table for a bit. Brittany comes back to sit down. Jason moves to Group 6 and then wanders back. Lydia continues working through the problem asking Brittany and Jason questions. Lydia explains her math, recaps her math. Notes her next steps for adding. About 1532. So about 1k? No, because maybe we need this many bags. An estimate of 900 is pretty good. But just in this class it's about 500 something, so it could be higher than that. Estimate into a friendly number 1530. Around 1000. Mmmm if people are taller discussion.
	1:23:15-1:25:40 Main	Discussing estimates. 1600 for a friendly number. If more people in the classes. Would have extras probably. Unless other classes had more people. T stops by. Lydia starts explaining, Jason chimes in. T confirms with them and asks for them to share yes.
G5_PG_PP Heavily guided by T.	1:08:25-1:25:30 Purple & green	There are 23 students in each class. No 22, because Emery left. Discussion of class numbers per class and their class in particular. No listen, we are also talking about the other class. T stops by the table. T: 65 4th graders, of those how many would you need to assign the size ropes. T encourages them to pick numbers for 7 and 8 foot ropes and then work with that. T helps them see that three bags is one foot. And she needs 8 of them. T sends the students back to their tables. T goes to dry erase board to demonstrate. 1 foot for 3 bags. They have to figure out 7 and 8. T helps with mathematics. So would it be like 21. 7 is 21 so 8 is 24. T: decide how many 7 foot ropes you're going to make and how many 8 foot ropes you're going to make. Group discusses student heights. Asks a couple others about their heights. Regroup. Then distraction from two other students asking about what they requested. Scored 10 touch downs. Some general chatter. Then back to math. There are

		5. This is tedisome. If there are 24 bags for Owen and … some individuals. Plus 2, wait how many bags did we say there were? There should be 4, wait, 60, 65. The girls chatting. the boy is the one primarily doing the work. He points this out. "you guys are doing no work" Working through math, I got the answer. Walks away from the table to chat with G6. They write things down on their papers. Begin reading books.
G6_GR_GO		

Reporting Out on Models and Solutions

1:25:52	1:25:38	1:25:14	T regroups, they are going to wrap this up and share math thinking. G4: Jason. They did that there was 5 people with 7' rope, 17 people with 8 foot rope, just in their class. 17*24=408 then did 21*5=106, then 408+ T: how'd you get 106. Not sure. T: let's double check. 20*5=100, so 105. Then 24*17 with area model. They get 388. Just for their class. Then they realize it's 408 and 305. Add together, 513. Then they did 514 *3 but now it will change by 1 group of 3. T: why? Students: because there's three classes. T: ok, so most groups looking at 65, this group looked at one class and multiplied by 3 making the assumption that there would be similar class sizes. T helps with the total but they got 1539. They estimated to 1600, T you rounded to 1600 it will give us enough.
1:30:25	1:30:10	1:29:46	G6: Johnny. They figured out that 24 bags is 8 feet, so 24*65 for 65 kids in 4th grade. T: all the kids in 4th grade will have the same rope. Johnny: no. T: ok, how did you do that? M: area model. 1560 total. Then they thought 10 kids that are going to need a 7 foot rope, so subtracted 10 to get 1550. T: tell me more about this. Each rope is 24 feet. M: around 5-10 that need 7 foot rope which is 21. They need 21 bags. We thought, it's a 3 bag difference for 5 people. T: you said 10 people. What about the rest of the table. They want to use 10 people. So three bag difference, subtract 30, to get 1530. T comments that it's a close estimate to G4. T reexplains G6 subtraction strategy to the class. Good way, she didn't think about it that way.
1:34:15	1:34:00	1:33:36	T has one more group to share. Group1 65*30 doesn't work because not all kids, plus 3 bags is 1 foot. So that's 24. So we switched it.

			T: did you redo the problem or work with that? Worked with it. T: this is a good example of making a mistake with 30 bags in a rope, but worked with it. What next? They did 65*30=1950 bags. Then they 65*24=1220 T: ok, so you did do a new problem. That's different what method did you use? Basic facts. What are we missing here? We haven't done a lot of this multiplication yet. They need 5*20, and 4*60, then 1560 which is the same. T: how did you adjust for the smaller bags? Did you make them all the same size? Group: no about 3 people need 7 foot in our class, then another class is 4, and it's about 3, so about 10 people. So then half of 65. Well no I guess, T how'd you adjust for that. You have all 65 kids, so if they all had 8 feet. They did 45*24. T: tell me why you did this? Could you take the idea of Johnny's group and figure out the difference in bags and then subtract, that might make your math more efficient. Their answer 1530 final answer. Similar to another group.
1:38:43	1:38:28	1:38:04	They stop here. Name on paper. T will collect them as they line up for PE. T asks me if it's ok how they reported out rather than the whole group chart. Of course.
1:40:26	1:39:52	1:39:09	End of recording.

Concluding notes from the field

A note from the field notes.
I had mostly packed up after the lesson finished, but completely finished packing away the recording devices after the informal interview. The students came back from PE and got their snacks sat down at their places. I was packed up ready to leave and the teacher recapped how many bags they thought they needed for the entire fourth grade it was around 1500.
She asked what they thought that they would need for just their class. They estimated around 500. She brought the two boxes of bags out and had me explain that for the first one I didn't think I took a minute per bag – closer to half of a minute per bag and I spent approximately 2 hours on the first box, and a little bit less for the other box. Each student was going to make their jump rope. If they didn't have enough they would have a good enough start that they could finish them on their own at home.

Table B3. Group-Level Content Log

Group-Level Content Log

Notes

Acronyms
T = Teacher, Ms. Kedrik.
B_SVL = Blue Swivl Camera
P_SVL = Purple Swivl Camera
G_SVL = Green Swivl Camera
PE = Physical Education

Group Work Participants

G1: Everet, Owen, Wren, Eric
JumpRope_GW_G1.pdf
G3: Seamus, Blaze, Emmet, Belle
JumpRope_GW_G3.pdf
G4: Brittany, Lydia, Jason
JumpRope_GW_G4.pdf
G5: Jacob, Ellie, Maria
JumpRope_GW_G5.pdf

NOTE: "green" sticker color for G_SVL audio marker is actually the main/primary audio marker.

What do you know about plastic bags?

4:25 **GW**	4:06	3:43	Class erupts into group conversation.
G1_BR_BB_BP		3:43-4:17 Purple	Students discuss that they're good for shopping, other purposes like right now, garbage cans, bag is free – unless you're in California.
G2_BM_BG			Usually grocery stores have them there for shoppers, not the best for the environment, they burst their bottoms
G3_GB_GG		4:20-4:56 Blue	Made out of plastic for carrying groceries. T notes that that the desk is allowed to be in the video and be recorded, he thought he wasn't allowed initially. He jumps in sharing that they are recyclable, and they scare his dog.
G4_PB_PM_PR		4:06-4:40	Plastic bags are in the ocean and sometimes kill animals like sea turtles, all sorts of animals, they're in stores, you can pay cents for a plastic bag.

G5_PG_PP	4:06-4:42 Green	Get them at the grocery store, lots of groceries in them, different logos, circle with target, different colors, different material, they're all made out of plastic, that's why you recycle them. Yah.
G6_GR_GO		

What do you know about jump ropes?

6:45 **GW**	6:30	6:07	"What do you know about jump ropes?" Talk in tables, what are some things they know. Group conversation
G1_BR_BB_BP		6:08-6:38 Purple	They're fun, you can do a lot of different challenges with them, can do it with a big long piece of cording, two jump ropes for twice as fast, not twice as fast just twice as much.
G2_BM_BG			
G3_GB_GG		6:46-7:17 Blue	Ropes are normally made of rope, plastic, or fiber glass or something like that, like climbing rope. Like the rope of horses.
G4_PB_PM_PR		6:33-7:03 Blue	They're fun, can do tricks on them, can have two people, can play with friends or by yourself.
G5_PG_PP		6:33-7:03 Green	They're fun, you jump like this, good way to exercise your body, lots of things to use a jump rope for, tug of war.
G6_GR_GO			

9:15 **GW**	9:00	8:37	Group conversation
G1_BR_BB_BP		8:37-9:10 Purple	There are different kinds of jump ropes, different kinds and different ways to do it, some are bad at jump roping, every girl is good at it.
G2_BM_BG			
G3_GB_GG		9:14-9:52 Blue - > main	The black one in front has one jump rope, some are shorter, some are thin, some are thick. You can make it, but it just takes forever.
G4_PB_PM_PR		9:00-9:29 blue	Notices there are different colors, they go at different speeds, yeah, with that one in the first picture there is a really big one for their jump roping
G5_PG_PP		9:00-9:39 Purple	They're different colors, different speeds of jumping, different techniques, some of the skipping ones, one foot, they cross sometimes, more than one person, yeah
G6_GR_GO			

What math questions can you ask?

17:35 **GW**	17:22	16:55	Group discussion and T walks around
G1_BR_BB_BP		16:55-18:03 Purple	How many jump ropes for one rope which we know the answer but we'd have to ask it, you don't actually know know, not for sure. 30 bags for one jump rope, but maybe we can combine for a big one. how many bags go into one jump rope and how many we can make. if we know how many bags we have we would know exactly how to, we could divide by 30 right. how many bags do we need. my estimate could be 31. we could estimate at 30. how many bags do we need for everyone. my estimate would be 30.
G2_BM_BG			
G3_GB_GG		17:32-18:13 Main	How tall you are, how long you need it. Discussion around individuals' heights. After 18:13 they remind T that she forgot to pick a stick for sitting on the futon – which they will do after this lesson.
G4_PB_PM_PR		17:17-18:17 Blue	How many jump ropes to make one, figure out the lengths because some are taller than others, depends on height, ummmm how many plastic bags to make the jump ropes. T briefly stops by. If we have enough jump ropes to have everybody in the class have one. Some whispering sounds.
G5_PG_PP		17:17-18:27 green	How many kids are jumping, what kind of jump rope are they using. T stops by. reminds them about math questions. How many plastic bags will we need. how many jump ropes we need, how many kids will jump, how many kids will share, how many plastic bags we have to use.
G6_GR_GO			

Braiding plastic bags

32:44 **GW**	32:30	32:07	"So, I'm going to come to your table..." Group work time. T gives bags, students begin talking and braiding in groups. T goes from table to table checking in, asking how they're doing, supporting their braiding. T teaches how to braid as needed. T asks about measuring.
G1_BR_BB_BP		32:08-39:23 Red	Discussion about braiding. They know how to braid. Can we start? T: yes you can start. lots of rustling from the bags. Comments about types of bags – Albertson's, Walmart, target. Most times at target. You put the top bag under right, then you pull it. Yep, you did it. Wait how did you do that so quick? Ok, make a circle for Owen, how many would

		you use to make it tighter, but for one jump rope. they decide to make it a bit tighter. They're going really hard. don't paper cut yourself I've had it before when I was a little guy. Tuck that one though, yeah there you go. Put that through and pull here. then pull here too. Combine them all. That's so tiny. Each table is going to make one jump rope. how. The group walks around a bit. Out of range of mic but working on braiding bags together. T stops by to help them braid. Students wandering around the room. We're going to need to be, are we still braiding. Who wants to be the tester for this. Oh my gosh, that's so little. What the heck. Do you know what….. no do you know what…. I took a shower this morning, it looks like a wire for some reason, body wash, it's nice, it's not looking too bad. Discussion around making one whole big bag.
G2_BM_BG		
G3_GB_GG	32:48-39:45 Blue & main	Very hard to distinguish audio here. The markers pick up on so much background noise. They work on braiding their bags. Can't see them while the class is standing because G6 blocks the view. 35:00 audio is better. T stops by helps with assigning tasks for braiding. students wandering around the room as they braid the bags. They wander back. Suddenly the markers block out background noise more so it's easier to hear the students. The student at the desk stays at the desk when the other students walk around. They braid their rope.
G4_PB_PM_PR	32:37-39:30 Blue -> red	They each get two so they each get to make one. T hands them the bags. They arrange their bags. How do you do it? They help each other loop their bags. They get it and are excited for each other. Jason is the one who knows how to do it and helped the other two. Jason walks over to Table 1, but is called back to braid with his group mates. They braid the plastic bags. They're a bit far from the microphones on the table. General chatting amongst each other. T stops by asks how long the braid from three bags is.
G5_PG_PP	32:34-39:33 Purple <-> green	Discussion about roles while braiding, who gets to hold it. They do rock paper scissors, someone wants to braid it. Worries about cheating with rock paper scissors. They work with their bags. View on P_SVL is blocked by G6. Discussion around getting the bags together and braiding. T stops by. They say they are doing great. She watches for a minute and leaves. You do the first loop and I'll do the second. This is how you do it. I know how to do it guys.

		You're not letting me do something. My arm's getting sore. My turn, it doesn't need to be perfect, no judging. My turn. Some laughing. They get tape. This is not going to work. Yes it is. Shows how to tape it. Nice.
G6_GR_GO		

What do you know about making jump ropes?

47:50 **GW**	47:36	47:13	"Ok, at your tables, go ahead and talk about..." T has them talk at their tables, what do they know about making jump ropes. They have some info. T walks around. Students work in groups.
G1_BR_BB_BP		47:23-48:14 Purple	We know that 3 bags is a foot, so, we also know that most people will need an 8 foot long jump rope, most unless there might be like 1 like Maria, Wren, they list other students who might need a shorter rope. Alia is like 4'8. I'm 4'11, almost 5'. I'm 4'11 and ½.
G2_BM_BG			
G3_GB_GG		48:00-48:53 Main & blue	Making jump ropes is twice as tall, if you go from the floor connected to arm span. Discussing heights. View is blocked for a bit by T.
G4_PB_PM_PR		47:45-48:40 Main	We know that we can braid them, that we need to do length and find out how many feet are on the jump rope in all. There could be 7 feet, 8 feet. We would need to figure out how many feet it is. Some inaudible mutterings.
G5_PG_PP		47:45-48:34 Purple <-> green	Three bags is 12 inches, or is that six bags? Six bags is. Measure here to here. T stops by, how long is three bags, that's what we're asking. T leaves.
G6_GR_GO			

Estimating the number of plastic bags they will need

1:01:59 **GW**	1:01:45	1:01:22	[clapping sound from T] Before they work, put down estimate. Groups begin discussing. T walks around. T reminds them they are just writing an estimate.
G1_BR_BB_BP		1:01:35-1:03:48 Red	I would say 30. All three classes, how many bags. It takes 30 bags to make one rope. to make one rope yeah. so 65*30. that's my estimate. Ummmm what 65*3. that's 65+65=180, no 170, ok, that would be plus 40 is 200, then plus 25 and that's 225 that's 65*3. It's 235. I went for 400. I killed this pencil. 235. Yeah, 235, then add, then you do, so what is it 65*65. I say 400 because it's a guess. No, how do you do 65*3. It's a guess guys. How about we use our basic facts. If

		I know that 5*5 is 25, 6*3 is 18. I'm going to say 1189. I said 400. You're off. Yeah, you're way off. I think it's 1825. I think it's 1825.
G2_BM_BG		
G3_GB_GG	1:02:11-1:04:25 Main and blue	Discuss numbers. 65*4. It's over 100. 21 is a good estimate for a bag. Guess for the whole class is 230. For everybody, 230 bags. Why I'm doing this is because there's 61 is our estimate. 65+65 plus another 65. number discussion.
G4_PB_PM_PR	1:01:28-1:03:06 Blue	Still during class discussion but one group member notes that 6 bags is two feet, It's about 6 foot but the end, not 6. They work on an estimate, everybody is five foot, definitely not, maybe average is 4'6. Discussion of student heights. What are we estimating? I don't know, guess 135. How many bags for each class. That's not going to be it Lydia.
	1:03:06-1:04:11 Red	Has to be at least double the amount of classroom, let's say there was 30 kids in our class and average height is 4'10 we'd probably need 100. We should round, 330. No like 100 per classroom. Estimating 300.
G5_PG_PP	1:01:59-1:04:12 Purple ->green Squeaky audio markers, but pretty good audio here.	1 foot if everyone is around 5 foot, well not everyone is. Average height of 4th grade is 4'6. No, that's the height of Johnny. Discussion of heights. An estimate. 73. Let's do the work. Curly hair convo. Everybody is 4'7".
G6_GR_GO		

Building models to answer the question

1:08:07 **GW**	1:07:53	1:07:29	"Ok. Alright, so..." They are going to do their work. T recaps info and what they need to do. They understand what they are doing and begin working in groups. T visits tables as they work.
G1_BR_BB_BP		1:08:03-1:16:25 Red -> Purple	First we need to figure out, yeah. Owen, put that in the middle. Figure out how many in a 7' rope. Don't do that, not good for you, you're eating lead. Not good for you. Ok then get sick, I don't care it's your fault. Yay, yay. Key is bags for foot. Idea, how many kids in our class. We know. Then count by the kids in our class to the other classes. Is that a good idea or not? I don't know. you can figure it out. it's a good idea but hard to figure out. How many in this class right now. They discuss numbers in the classes. They add them up. They got 65. Then 65*30. They work it out. 65+10, or add 65 ten times which would be harder. or add 65, 30 times. It's 75. That is not 75. We're doing 65*10. not plus 10. we're breaking it

		up. I have an idea. It's 950. Wait, no because 65*10=650. Adding, 1950 would be the answer. They have the answer. Discussion about how they got there. They all record it on their papers. They compare with the estimate they made at the beginning of class. They were close. Make sure to write bags. Height comparison with other students. Basketball comes up. After 1:16:25 The comment on the answer. But otherwise, they discuss dinosaurs. Wander around the room a bit. Chat, and make funny sounds for each other. Discussion of a spider. T stops by, they tell her their total. T asks about their work. Asks how they got their numbers. 3 bags in one foot, in 8 bags how many. 30 is too high. Can you go from here, how many more groups of 65 can you add. two more groups? Nope, but rather than do the whole problem over… T helps with math strategies. They discuss multiplication. They are back to working on math, using basic facts and they get 1,240 bags.
G2_BM_BG		
G3_GB_GG	1:08:37-1:25:54 Main and blue	Discussion about who needs what rope length. How many bags in all of it. So confused. A jump rope snapped and went into pieces. An actual rope, not bags. that means it was too big. So if you need a 7 foot rope, but most need 8. Discussion about who is taller or shorter. They compare heights with themselves and people from other groups. T reminds them to work in their groups. Mine was 8 feet so we tied the ends to make it smaller. Diamonds or crystals or rocks. Rock discussion. I don't know what you're talking about but focus on.. focus again on the task. They go back to discussing who needs what size rope. Number of students 6+15 is 21 people. T stops by. Where are we at. Reports on who needs what size rope and the number of people per rope size. T asks about all the fourth grade, not just this class. They go through individual students again. T counts. If there's 6 in our class, do you think it's similar in other classes. Does it have to be exact. They named 10 people. They think 15. Others might be an 8' rope. 50 and 15 is 65. T: how many bags for a 7' rope. They were doing 7*3 = 21 bags. So how many people will have a 21 bag rope. 15. They multiply. They get more paper for an area model. T leaves.

		They want to be exact and use an area model. They work through mathematics. Then plus the tall people. Don't forget to draw the rest. Working through multiplying. Tell the teacher, tell the teacher, tell the teacher. It's snack time.
G4_PB_PM_PR	1:08:24-1:15:01 Main	Need to figure out the height. Discussion about pronunciation of figure. Figure out we need to realize how much bags we need in our classroom first then we can go on to the whole 4th grade. Pencil tapping. One's an Avatar. One is counting their class. 21 people in the class right now. But 22 total. Almost everyone needs an 8 foot rope, except a few maybe. They discuss individual students who need shorter ropes. Some students from other tables block the view a little bit. But still able to see fairly well. 4 people need 7 foot ropes. Kids in the middle of the room comparing heights. T breaks them up after a little bit. The table view is back. 4 people maybe with a 7 foot rope. Discussion of individual heights. 7*3=21. 21 bags is a 7foot. So 24, then 8*3=24. So if everyone in our class, 24* most of them, only a couple that need a 7 foot rope, others need an 8 foot rope correct. Correct. 18*24. Work with me now. "Is that a booger?" Discussion of multiplying.
	1:15:01-1:16:06 Blue	T stops by reminds them all to work together and talk together and not just Lydia and not just copying. Lydia explains her thinking. T leaves.
	1:16:06-1:23:15 Blue	Working on multiplying. Another group of students moves to this table. Hard to distinguish voices. but Lydia is the only one working at her table for a bit. Brittany comes back to sit down. Jason moves to Table 6 and then wanders back. Lydia continues working through the problem asking Brittany and Jason questions. Lydia explains her math, recaps her math. Notes her next steps for adding. About 1532. So about 1k? No, because maybe we need this many bags. An estimate of 900 is pretty good. But just in this class it's about 500 something, so it could be higher than that. Estimate into a friendly number 1530. Around 1000. Mmmm if people are taller discussion.
	1:23:15-1:25:40 Main	Discussing estimates. 1600 for a friendly number. If more people in the classes. Would have extras probably. Unless other classes had more people. T stops by. Lydia starts explaining, Jason chimes in. T confirms with them and asks for them to share yes.
G5_PG_PP Heavily guided by T.	1:08:25-1:25:30 Purple & green	There are 23 students in each class. No 22, because Emery left. Discussion of class numbers per class and their class in particular. No listen, we are also talking about the other

		class. T stops by the table. T: 65 4th graders, of those how many would you need to assign the size ropes. T encourages them to pick numbers for 7 and 8 foot ropes and then work with that. T helps them see that three bags is one foot. And she needs 8 of them. T sends the students back to their tables. T goes to dry erase board to demonstrate. 1 foot for 3 bags. They have to figure out 7 and 8. T helps with mathematics. So would it be like 21. 7 is 21 so 8 is 24. T: decide how many 7 foot ropes you're going to make and how many 8 foot ropes you're going to make. Group discusses student heights. Asks a couple others about their heights. Regroup. Then distraction from two other students asking about what they requested. Scored 10 touch downs. Some general chatter. Then back to math. There are 5. This is tedisome. If there are 24 bags for Owen and … some individuals. Plus 2, wait how many bags did we say there were? There should be 4, wait, 60, 65. The girls chatting. the boy is the one primarily doing the work. He points this out. "you guys are doing no work" Working through math, I got the answer. Walks away from the table to chat with G6. They write things down on their papers. Begin reading books.
G6_GR_GO		

APPENDIX C

TRANSCRIPTS

Table C1. Group 1 Transcript

Turn	Everet	Owen	Wren	Eric
Ms. Kedrik reminds the class that they are going to make jump ropes. Recaps by showing them how to braid the ropes. Ms. Kedrik checks that there is at least one person at each table who knows how to braid. Shows them how to braid. Explains that if you pull tight it will be a smaller section. They braid all the way to the end piece. They are going to do some measurements. A student from Group 3 asks about the audio recorders on their tables. Ms. Kedrik tells them about recording math conversations and they should only talk about math. Ms. Kedrik shares that they are going to do a math task like the caramel apple one they did. First slide in the presentation, what do you know about plastic bags?				
G1 conversation: Owen first mentions that they're good for shopping, Everet and Eric restate this. Eric notes you can carry things with them, Everet notes they have other purposes, Wren comments that it's like for shopping. Everet notes that you can recycle them, Eric says you can use them for what they're doing right. Owen: use them for garbage cans. Everet and Eric agree and comment on using them for mini garbage cans like for their room, plus the bag is free – unless you're in California.				
Ms. Kedrik regroups the class. Students share what they know about plastic bags. *Wren shares with the class after her name is drawn to be called on: you can use them for shopping and carry things in them and use them for trash cans.* *Other students share that they have different logos, they can be recycled, they're not ecofriendly – not good for the environment. Ms. Kedrik says that later on they'll read a book about plastic bags in her part of the world. Good to reuse, or recycle them.* *Ms. Kedrik shares that they are going to upcycle in this task, not recycle, but upcycle where they take the bags and use them for something else. She asks "What do you know about jump ropes?" Talk in tables, what are some things they know.*				
G1 conversation. Eric: They're fun and I suck at them. Owen: you can do a lot of different challenges with them. Everet: if you have a big long piece of cording. Wren: you can do both jump ropes with both people and like. Eric: I like big jump ropes. Wren: yeah. Owen: can do it with two jump ropes for twice as fast. Everet: not twice as fast just twice as much.				
Ms. Kedrik regroups: what do you know? *Students: fun way to exercise, different techniques for jumping.* *Wren: you can use a lot of them.* *Ms. Kedrik: what do you mean?* *Wren: Different tricks,* *Eric: can play games with them,* *Student: Can do a long rope and two of them for someone to jump.* *They come in different sizes.* *Ms. Kedrik: tell me more about that.* *Student: smaller for one person, longer for two.* *Lydia: Can jump alone or with friends.* *Student: Made of different materials like plastic.* *Ms. Kedrik gives a moment to think – what do you notice about the different jump ropes.*				
G1 conversation: *Owen: There are different kinds of jump ropes everywhere,*				

Everet: different kinds and different ways to do it, *Owen: yeah, basically what I said.* *Eric: I remember [inaudible] jump ropes* *Owen: I'm really bad at it.* *Eric: I know, I suck* *Everet: my sister is super good* *Eric: every girl is good at it, I swear to God* *Owen: yeah, my sister is really good at it* *Wren says nothing.*				
Ms. Kedrik has them think about notices and wonders silently in their groups. *They share out as a class. Ms. Kedrik records some ideas. Notices first. Students share: different colors of ropes, Lydia: bigger sizes for different kind of jumping, all different sizes, different speeds – some fast than others, Owen: the jump ropes look different from each other, Everet: there are lots of different ways to jump rope, one for the jump ropes was made of rubber bands so different materials.* *Ms. Kedrik asks about what they wonder.* *Students: Wren: why some are big or small and different colors, Jason: how many kids jump roping there are in the pictures, Student: wonder about jumping with three jump ropes, Student: how many jumps there are, Owen: how many kids can jump on a jump rope.*				
"We are going to think about..." Ms. Kedrik: what math questions can you ask. Goal today is to create these upcycled jump ropes, so what are some math questions you can ask that you might need to solve before we can do this.				
1	How many jump ropes [plastic bags] you need to make one rope? Which I already know the answer to, but you would have to ask it.	*[leaning forward looking at Everet]*	*[looking down at the table]*	*[leaning back in his chair looking at Everet]*
2				*[sitting up and leaning into the table]* Yeah. You could just be like, "Hey Alexa, how many bags?" Like different bags.
3	No, I already know.			
4		*[holding pencil and clasping hands flat together emphasizes words by moving*		

		hands up and down while keeping elbows on the table] You don't actually know, know. Like, for sure. You're not for surrrre.		
5		*[looks to Eric]*	*[looking at Eric]*	Well, I guess 30 bags can make one jump rope.
6	Yes.			
7				But maybe we can combine another
8	How many jump ropes...			30 to make a big one.
9	How - well ↓ - so we're gonna need to know how many			
10	jump ropes,			Okay
11	how many bags go into one jump rope? And then, how many jump ropes can we make?			
12	*[sitting upright, resting chin in the palm of his hand]*	*[leaning into the table]*	*[sitting upright, arms overlapping on the table, very still]*	*[elbows on table, gesturing with hands while speaking]* So, we can go 30 tiimmes ↓ we would just- if we knew how many bags we had,
13		30 times two? *[slight chuckle]*		we can know how many-
14				We can, we, like, if we know exactly the amount of bags *[points towards Ms. Kedrik's table at the front of the room]* we had, we would know h- exactly how to-

15		I brought		
16		a lot of bags.		We could just divide it
17				by 30, right? *[looks towards Everet]*
Unsure who says "Mm-hmm."				
18	*[looking towards Eric, removes head from palm, turns palm outward with fingers extended in a stopping gesture and pulses hand back and forth a little bit. Then puts his pointer finger to his lips to gesture silence. In the background, Ms. Kedrik is helping another table.]*			
19	*[Group 1 appears to listen to Ms. Kedrik]* *Teacher: how many we need to have everybody?* *Class: [undecipherable].*			
20	*[looks to Owen]*	*[slowly, holding pencil in his hands]* How many bags do we		
21		need ↓		*[quickly]* My estimate would be 30.
22		for everyone?		
23	*[quietly and looking towards Eric]* Well yeah, we could estimate at 30.	*[looking at Eric]*	*[looking down at the table]*	
24		*[stated matter-of-factly]* How many bags do we need for everyone? *[elbows on the table, palms flat together holding*		

		pencil and moving hands slightly up and down]		
25	*[softly]* What? *[looking at Owen]*		*[looks away from the table out towards the classroom]*	
26		*[scrunches up face by bringing eyebrows together]* How many bags do you need		
27		for everyone? *[rolls pencil in palm of hands]*		Oh, I, my estimate would be 30.
28	Well, yeah.			
Class regroup and share. Students share: Brittany: how many plastic bags for everybody in the class to make a jump rope; Lydia: what lengths are the jump ropes; how thick are the ropes; (Everet mutters that it depends on you braid it, Eric agrees), how long does it need to be different sizes; taller students need taller ones. Shorter need shorter might trip on a rope that is too long. Eric says he uses his mom's jump rope and it's pretty big. Ms. Kedrik: Would that change the number of bags? Students: more bags, how many plastic bags in all, Johnny: how many people want to jump rope, Student: do we have enough bags, Everet: three questions, how many bags go in a rope, how many ropes can we make, do we have enough jump ropes for the whole class. Owen mutters that he should have kept count of his bags. Wren: how many bags do you need for one big jump rope. G1 things this is interesting "ooo" they discuss quietly the whole class and the teachers. Student: how long is each bag and how much time do we have to make them. That's good, like in science when they have a time constraint.				
Ms. Kedrik: main question: How many plastic bags does it take to create jump ropes for all of 4th grade. Class starts talking. Ms. Kedrik regroups, has them record the question. Everet quietly helps Owen with writing the question because Owen forgot to include a word. They are going to do math to see what they would need if they wanted to do this. Students record on papers. Ms. Kedrik walks around.				
"So, we're going to do a little investigation here..." They are going to braid, Ms. Kedrik helps them think about getting the whole piece braided, want to see how long it is from the knot to the end, how long? They are going to braid and get the measurements. "So, I'm going to come to your table..." Group work time. Ms. Kedrik gives bags, students begin talking and braiding in groups.				
Group 1 conversation: The boys discuss braiding. They know how to braid. *Eric: Can we start?* *Ms. Kedrik: yes you can start. lots of rustling from the bags. Comments about types of bags. Comments about types of bags –* *Wren: I have Albertson's, Walmart, Target. Most times at Target.* *Eric: You put the top bag under right,*				

Everet: then you pull it. *Eric: Yep, you did it.* *Everet: Wait how did you do that so quick?* *Everet: Ok, make a circle for Owen, how many would you use to make it tighter, but for one jump rope. Up to this point, all four students at the table group were working with plastic bags, but when they (Everet leads this idea) decide to make it a bit tighter.* *Everet takes the bags from Wren that she had been working on.* *Everet to Wren: you've got the right idea, I'm just going to make them a little bit tighter.* *Owen asks about how many bags with the whole class and one jump rope. They're going really hard with flattening out the bag.* *Everet smooths out the bags and gives them bag to Wren.* *Eric tells her to pull really tight.* *Wren: yeah, into a little tiny knot.* *Eric helps Wren with the bags and then takes the knotted bags.* *Jason comes over and wonders to Eric about how they'll be able to jump rope with how short the two bags are.* *Eric explains they will combine them. The boys wander a little away from the table to braid the bags.* *Wren remains at her seat and does not participate in braiding, she doesn't talk with anyone initially.* *Wren eventually joins the boys as they are restarting their braid.* *Ms. Kedrik stops by giving additional instructions. It's hard to see and nearly impossible to hear, but it looks like Wren took hold of the bags and began to help Everet braid – Ms. Kedrik instructs as they braid.* *The class walks around the room. The video of G1 is obscured. It appears as though Wren traded places with Eric to takes over braiding for her while Everet holds the end of the rope. Wren and Owen observe the braiding and make some comments – but it's very difficult to hear.*				
Ms. Kedrik quickly recalls class and asks the class to measure how long three bags braided together are from the knot. Students continue working in groups.				
Group 1 conversation (impossible to hear clearly): Discussion around an eraser. Eric shows the jump rope to Jason noting that he thinks it got smaller. Students move around the room talking with each other as Ms. Kedrik checks on the groups. G1 They note it's exactly 12 inches. Well not exactly.				
Class regroup and share jump rope measurements. They agree that 12 inches and three bags is a friendly number to use. Ms. Kedrik gives jump rope measurements. *Up to 4'6" tall then jump rope is 7' long.* *Between 4'6" and 5'5" then the rope is 8' long.* *Over 5'5" then the jump rope is 9' long.* *A student comments none are over 5'5" except the T. Ms. Kedrik comments that she's not over 5'5, good info for them.* *Students discuss in groups what they know about making jump ropes.*				
29			*[leaning in towards the table, holds pencil in left hand*	

			and points it towards the center of the table in Everet's direction] We know that, um, three bags is at least	
30	*[begins writing]* Threeee		a foot-	
31	baaags *[quickly]* equals one foot. *[speaking while writing]* bags. equals. one. foot.		*[writing on her paper]*	*[writing on his paper]*
32		*[writing on his paper]*		*[speaks quietly under his breath while writing]* Three bags equals...
33	So *[undecipherable: "Dang" or "Damian"]* so - and then, and then we also know that, um, most people will need an eight-foot-long jump rope.	*[writing on his paper]*	*[looks away from the table out towards the classroom, turns back around and leans into the table, then sits back and writes on her paper]*	*[looks up towards Everet]*
34	Most people.			Mo-
35				yeah, most people. Unless there might be, like, maybee, *[looks out towards the classroom]* maybe one.
36	No,		*[finishes writing and looks up]*	
37	like three.			Like, Maria.
38	Three.			
39				Maria, aa- probably Wren.
40		*[looks up, mouth wide open and taps pencil against his cheek]*	Yeah, so ↓ .	

41	No offense, *[holds palm out towards Wren, fingers extended in a "whoa" or "stopping" gesture and pulses hand forward and back slightly and quickly]*			And probably Brie.
42		*[taps ruler to left side of his face]*	Maybe Belle ↑ *[looking towards Eric]*?	*[looks at Wren]*
43				*[turns to look at Everet]* Or Belle. Probably, and probably Brie.
44		Yeah, Brie is a little shorter		
45	Ali's tall.	than me.		
46	Ali's, like, four-eight.	*[holding ruler up towards face]*	*[leaning towards the table somewhat on her right arm. Left arm extended out to her paper]*	*[quickly]* No, <u>no</u>, not Ali-
47	Ali's, like, four-eight.			no Ali-
48				<u>Ali</u> is <u>not</u> ↓ .
49	*[raises left hand palm and fingers out in a stopping gesture]* 'Cause I'm, I'm four-eleven, five-foot.*[more quietly]* Almost five-foot.	*[taps ruler against face]*		
In the background, Ms. Kedrik begins counting down to regroup the class.				
50				*[quietly]* I'm four-eleven and half so I just say I'm five-foot.
51	*[very quietly but not whispering]* Yeah, same.			

Class regroup. Class discussion about heights of people they know. They are talking about 4th grade class this year. They share what they know. Ms. Kedrik: can we assume or make a guess about how many are in 4th grade? They decide on 66. But then discuss and land on 65 – it's an assumption and a friendly number. Ms. Kedrik: anything else we know, need to know. Do you have all the info you need? Class: no. Because estimate of smallest and tallest, average height. Ms. Kedrik: if we wanted to make them. But right now, what do we need if we want to make them. We won't make them for all of 4th grade, just our class, but we might not have enough for that. Class discussion around numbers and heights. They know up to 4'6 they need 7 feet, 4'7 to 5'5 is 8 feet. Add to what they know. Class discussion around numbers of students for each size. Assumption is that they need more 8 foot than 7 foot ropes. T says they'll come up with plans in their groups, they might be different. They have what they know and need to know.				
First part of paper, make an estimate about the number of bags, then show work. Decide at tables how many 7 and 8 ft ropes to make. 65 students total. Decide how many ropes for 7 and how many for 8. They know 3 bags for 1 foot of rope, use the info when deciding how many "jump ropes for a 7 foot rope and how many go into an 8 foot rope"				
52				*[quickly]* I would say 30. Are-
53	*[incredulously]* All three classrooms? How many bags it would take?			
54	In all three classrooms,	*[looking down towards his paper]*	*[looking around at the other team members – especially at Everet and Eric]*	Oh, it would only take 30. That's-
55	it takes <u>30</u> bags to make <u>one</u> rope.			
56		*[erasing and writing on his paper]*		It takes 30 bags, to
57	To make one *[emphasis with a slight chuckle]* rope.			make-
58				To *[slight chuckle]* make one rope, yes ↓ .
59	So then you do 65 times 30.			
60	*[looks at Owen but says nothing]*	*[turns to look at Everet while speaking]* That's my estimate. *[sharp inhale]*		Uhhhmmm, uhhhh-

61	*[turns to address Eric]*			... uhhhmmmm, um, um, umm
62	*[looking at Eric]* What's six times,	*[looking towards Wren]* I think I'm close.	*[looks towards Owen]*	mmmm
63	what's	Ain't I close? *[shows paper to Wren and leans back in his seat toward her, but his chair still faces Everet]*	*[looks to Owen's paper]*	*[very quickly]* Wa-, wait.
64	65 times three?		*[looking at Owen]* 400 for fourth grade!	
65		*[nods head while facing Wren]*		*[writing on his paper, then looks up to Everet]*
66	*[leaning into the table towards Eric]*			Wait, what?
67	65 times three.			65 times three?
68		*[picking something up from the ground]*	*[has her left foot crossed over her right knee, adjusts her sock]*	*[looking at Everet]* That would be, well, 65 plus <u>65</u> equals 180. *[almost to himself]* No. No.
69	*[looking at Eric and gesturing with his pencil while speaking]* 170.			
70				100- okay.
71	times 1<u>65</u>			Plus [undecipherable]-
72	So plus six-			
Ms. Kedrik reminds the class: So, you're just writing your estimate down right now.				
73	Um, plus, so that'd be plus 40 is, uh-			
74				*[while writing]* So it's-
75	*[emphatically]* <u>200</u>.			
76			*[playing with her shoe]*	*[matter-of-factly]* No.

77	Yeah, it would be 200.			
78				No, no, what's
79	*[insistently]* And then y- and then you			[undecipherable]
80	do plus 25 is 225, that's 60 times, 65 times 3 ↑ .			
81				*[with emphasis and slightly louder]* <u>It's 235</u>, *[slight pause]* it would be *[looks down at his paper]*.
82		*[leaning towards Everet]* I ↑ went for 400 ↑.	*[using her pencil to draw on the sole of her shoe]*	
83		And I killed		
84	*[looking at Eric]* Not it,	this pencil.		
85	well, if you did your estimate, right? *[quietly]* If you did your work ↓ [undecipherable].			
86			*[using her pencil to draw on the sole of her shoe]*	No, 170
87	*[Everet does not look at Owen, continues conversation with Eric]*	*[leans over to show Everet]* No, you could put your finger through it.		plus 65.
88	Plus…Oh, yeah. Well *[leans over the table to see Eric's paper and points at the paper with his pencil]* – yeah –235.			
89				*[quickly]* Yeah, it would be 235.
90	Yeah.			

91	An-			So that'd get you to 35, and then you add
92	And then you do,			another-
93	and then you do, um *[pauses and moves pencil over an area of his paper but not in a way so as to show it to any of the group members]*			
94				So [undecipherable: "say"] what is it? 65 times 35?
(4 seconds)				
95		*[first leaning towards Wren and then looking towards Everet]* I say 400 - because it's a guess.	*[stops drawing on her shoe and resituates herself in her seat to face the table and leans into the table]*	
96	*[Everet does not look at Owen but looks at Eric]*		*[picks up pencil and begins writing on her paper]*	*[immediately after Owen stops speaking]* No *[writing on his paper]*
97	*[immediately after Eric says "no"]* How do you do			
98	60 times 30?	*[somewhat emphatically, but not loudly. The words are well annunciated]* No, it's a guess,		
99		guys. Guys, it's a guess. We don't do the math		
100		just yet ↓.		*[looks towards Everet]* Well, let's do our basic
101				facts. If I know five times

102	Wait,			five is tw-
103	but we don't use			
104	the math just yet.			If I know,
105		*[looking towards Eric]* Yeah, we don't use the math just yet.	*[still writing on her paper]*	if I know five times fiiive- if I i-if I know *[looking towards Everet]*
106				five times five is 25, six times three
107	*[writing on his paper]* I'm just			is 18.
108	gonna say 1,189.			*[writing on his paper]*
109	*[stops writing and looks towards Owen]*			
110		I said 400.	*[done writing, still sitting facing the table]*	
111	You're off. *[Quickly makes a small gesture where he waves his left hand at Owen and then shakes his head and looks back to Owen.]*			
112				*[looking down and writing on his paper]* Yeah, you're way off.
113	*[looking at the thing Owen has]*	*[holding something hidden from view of the camera and talking about that thing]* It actually -		I think
114		*[drops the thing, the pencil, on the table]* Where...		it's 1,825.
115	*[quietly and to Owen]* Oh, it's this pencil.			
116				*[speaking while Ms. Kedrik in the back*

				begins regrouping the class. Teacher: Okay. Hey, hey, fourth grade.] ↓ Yeah. I think it's one - 1,825.
Class regroup. Ms. Kedrik asks about a reasonable estimate. Asks about bags in a whole rope. Student volunteers 30, their estimates are 1,800 – friendly number. Ms. Kedrik shares thinking of a student 24 bags in an 8 foot rope, then 240 for 4th grade, but that's only 10 so using that number to come up with an estimate. Would 20k be reasonable? No. Be sure numbers are reasonable. 900, 3000. Different numbers. 450 bags. About 25 bags in a rope. Think quarters.				
"Ok. Alright, so..." They are going to do their work. Ms. Kedrik recaps info and what they need to do. They understand what they are doing and begin working in groups.				
117	*[facing Owen]* Alright, so we need to figure out. Ok stop messing around. *[Takes away Owen's pencil, looks away from Owen.]*	*[pointing his pencil at Everet and running his finger up and down the pencil in Everet's direction]*	*[sitting facing the table, with her pencil in her left hand, elbow on the table, left hand up to her face. Facing Everet and Owen]*	
118	*[Gives the pencil back to Owen by putting it on the table in front of Owen]*			Yeah, we got it [perhaps referring to what Owen is holding] last year.
119		*[putting the ruler to his lips]*		*[chuckles]* It's diffi [undecipherable: "diffint" or "different"] -... *[looking towards Owen]* All right, Owen, put the ruler down.
120	All right, so-			
121		*[putting the ruler in the center of the table]* This [ruler] helps me think.		
122	No, it doesn't.			*[Takes the ruler and places on the supplies situated to his left and at the end of the table]*

123	All right, so *[brief pause]* we're gonna do – to figure out how many are in a seven-foot rope.			
124	*[very quietly]* Right?		*[excited, stands up a little from her chair and leans in more towards Everet]* Uh-huh.	
125	*[Looks towards Wren and then resituates himself a little by leaning towards Wren and into the table a little bit]*			Yes.
126	So – seve- so <u>three</u> bags is <u>one</u> foot and you need seven feet. So, seven times three *[looks towards Eric]*.			
127				Is 20-
128	21 bags.			
129				*[quietly]* 21 bags, yeah.
130	In a seven-foot rope. *[Arms up but elbows on the table, with left palm out and open, Everet looks at each member of his group, first Eric, then Wren, then Owen]*	*[looking towards Everet]*		
131				I don't know how I got that wrong.
Not transcribed G1 conversation: Eric talking to Owen, don't do that, not good for you, you're eating lead. Not good for you. Ok then get sick, I don't care it's your fault. Owen saying yay, yay.				
132	All right, so-			

133		*[looking towards Wren]* Yaaayyyy ↓ !	*[looking towards Owen]*	*[leaning into the table]*
134	*[leans over paper and begins writing]* All right, so, so we're			
135	gonna need-	Yaaayyyy ↓ !		
136	21	Yaayy ↓ !		
137	*[speaking as he writes]* bags for-	*[looks towards Everet]*		
138	equal seven feet rope. *[looking at his paper and speaking along with his writing, does not look at any of his other group mates]*	21 bagsss ↓	*[turns to her paper with pencil in left hand points to her paper while speaking to the table at large] [quickly]* We know 65 bags	
139	[a sound which sounds like "add"], 28 bags ↓		times 35 equals 1825.	
140			And the key is one bag equals one foot.	
141		*[quickly]* I think		
142	↓ equals eight-foot rope	↓ *[quietly and quickly]* wait-w-w-w-w-wait	*[looks to Owen]*	
143		I have an idea, I have an idea.		
144	*[stops writing and looks up to Owen]* [screech sound]			
145				*[addressing Everet]* What was that?
146		I have an idea. We need to figure out how many kids are in our class		
147	*[writing on his paper]*	and then we could-	*[looking at Eric]* We already know!	
148				We already know.

149		*[looks at his table group members]* Thenn ↓, umm ↓ *[hesitates]* then um, you can count, um, byy ↑ those kids in our class, all to the other classes.	*[playing with the top of her water bottle]*	
150	*[looking at Owen]* [screech sound]			
151		*[looks at Eric]*		*[with a laugh in his voice]* Whoa, okay.
152		*[looking between Everet and Eric]* Is that a good idea or no?		Guys, we
153	[more quiet screech sound]			need to stay on focused.
154	*[looking at Owen]* Umm, *[smiling, sounds as though he is speaking partially through his teeth]* I don't know if that's a good idea - you figure that out yourself. No, so, well-	*[looking at Everet]*		
155				*[turning away from the table and quietly calls to another student]* Johnnyy ↑ .
156	It would, it's a good idea, it's just kind of hard to figure out how many are in the other classes. *[points towards the classroom door]*			
157		How many are in this class right now?		

158				24.
159	22.			*[quickly]* No I'm just kidding, just kidding.
160		22		
161		*[looking up towards the ceiling] [slowly]* twenty-twooo, fourty-fourrr,		
162		sixty-sixx		Well, no.
163		*[accidentally flicks his pencil to the floor, moves chair back and leans back to retrieve it]*		You're adding up just our class. Don't count our class because y-, like, Ms. White's might have 21 in their whole class when everybody's there.
164	*[looking at Eric]* Ms. Hen- Ms. Henderson has exactly 21.	*[emphatically* That is truuueee. *[moves his chair a little away from the table so that he is essentially sitting next to Wren]*		
165			*[looking at Eric]*	*[looking at Everet]* Oh, she does?
166	Yeah [undecipherable: "yup"].			
167		*[scoots his chair closer to his original location at the table]*		*[gestures with arms open out to his sides]* All right, so we can add that up with-
168	And then Ms. White has 24, I think.			
169				Okay.
170	She has somewhere around 24.			
171	She has, like, 24-			So and then we could

172				add all of those up and w- it should be easy.
173		*[looking at the pencil in his left hand]* I thought, um, Ms. Henderson had the exact same number as us in her class.		20- 22, well, 22 plus 21. 20 plus 22 equals 42,
174				plus one would equal
175	43.			
176		*[looks towards Wren]* That would make it a lot easier though.		*[looking towards Eric]* 43. Plus 24 would beee?
177	65.			
178	*[loudly, excited]* Heyyyyy!			Yeah. *[almost surprised]* 65 ↑ *[lifts both hands, palms up towards Everet]*
179	*[arms open and out to the sides of his body, looks towards Owen]* Hey, it gets 65!			
180				Does it? Wait, let's see. *[looks back towards his paper]*
181	*[emphatically, half standing out of his seat, leans over the table towards Eric]* <u>Yes</u>, it <u>does,</u> bro. Just payy ↓ …		*[looking at Eric's paper, then writing on her paper]*	
182		*[pointing at something in the center of the table]* That's a weird side.		*[looking up towards Everet]* 20- and what?
183	24.			
184				24?

185	*[kneeling on his chair leaning over the table]* That's like bags that			
186	I know of-	[szszszszszs sound]		
187			*[still looking to Eric's paper and then writing on her own paper]*	*[writing and looking towards Everet]* Okay, so two plus one would obviously be three, seven, and that equals s-seven?
188	Yeah, seven, uh, 67. I, I w-			
189	↑ Hey ↑ yyy,		*[writing on her paper]*	*[arms out to his sides, elbows in towards his torso, palms up, slight shrug]* Hey, that's still close.
190	two off, not bad.			
Not transcribed G1 conversation. That looks like lulu doesn't it. Looks like you too. I've seen that meme [looking at a cup maybe].				
191	All right,	Ok		
192	soo ↑		*[looks away from the table and out towards the class]*	*[looks away from the table and calls to Johnny]* Johnnnyy ↑
193	Now we've gotta do 65-			
194		[bowwowow sound]		
195	now we've gotta do 65 times 30.			
(4 seconds)				
196				Well, I don't know what that is. *[quietly, with a slight chuckle]* No I'm just kidding-
197	So what's 65- *[looks towards Owen for a second]*, *[speaks*		*[turns back towards the table]*	

	with his hand over his mouth] just do 65 times 10, plus 65 times 10, plus 65 plus 10 *[mouth open smile towards Eric]*			
198			*[looking at Eric's paper]*	*[looks at Everet, speaks quickly and somewhat quietly]* Hold on, that's not- *[bends his head over his [paper]*
199	*[looking towards Owen]*	*[looking towards Everet]* I didn't hear anything you said ↓.	*[writing on her paper]*	*[head bent over his paper]*
200	*[leaning his head over towards Owen nearly touching Owen's head]* [small screech sound]	*[leans head over towards Everet nearly touching Everet's head]*		
201		*[looking towards Everet]* Can you say it again *[short pause]* <u>but slower</u>?		
202			*[almost to herself]* 65.	
203	*[looking towards Owen]* 65 plus 10, plus 65 plus 10, plus 65 plus 10 *[emphasis at the end of the sentence by tapping pencil eraser on the table]*.	*[looking towards Everet]*		
204			*[looks over towards Eric's paper]*	So, wait,
205		*[looking towards Everet]* So we do that three times?	*[writing on her paper]*	65 plus-

206	*[looking towards Owen]* Well, you do			*[looking towards Everet]* 65…
207	65 plus 10 and then you			
208	add, so-			Or you could
209				just add 65 ten times,
210		*[waving pencil in hand at Everet]* OOHH ↑ , oh oh oh oh oh oh		which would be harder.
211		oh oh ohhhoh ↑ . *[excitedly and moving in his seat looking towards Everet]* ohlaohlaohla ohlaohlaohla ↑ Uhh		
212		it's 76.		Or if you want,
213			*[erasing on her paper]*	you can just add 65, 30 times.
214		*[looking towards Everet, extends his right arm out towards Everet while holding a pencil]* S- it's 75!	*[looks towards Owen]*	
215	*[looks towards Eric]*	*[looks from Everet to Eric, excitedly]* So 75-		*[definitively]* That's not 75.
216		three times.	*[looks towards Eric]*	*[clearly enunciated and with a smile in his voice, looks towards Everet]* That is
217			*[looking towards Owen]*	not 75.
218		*[incredulously]* What ↑?		
219	That's 65 plus 10.			
(2 seconds)				
220		*[nodding head, looking towards*		*[speaking slowly and clearly]* 65.

		Everet, speaking quickly] Yeah, that equals 75.		
221	*[looking towards Owen]* I <u>knowwww</u>. We're doing 65 <u>times</u> 10.			
222		*[silently opens mouth very wide as if to say "ohhhh"]*		
223	*[looking towards Eric]*			65 times 30.
224	I <u>know</u>, but we're breaking it up.		*[writing on her paper]*	*[bends over his paper]*
225	*[throws pencil in the air, doesn't catch it, pencil lands on the floor and he leans over to pick it up]* [undecipherable]	Yeahhhh, I don't think I'll gonna, be able	*[writing on her paper and glancing over at Eric's paper]* *[quietly]* So it's 65 times 10?	
226		to help youuu with that. Ummm, I'm gonna run away,		
227				I got an idea.
228			*[looking at her paper and writing]* Sooo	
(3 seconds)				
229			*[questioningly]* wouldn't it -	
230	*[looking towards Eric]* It's just 65 times-		*[writing on her paper]*	*[looking towards Everet]* It's nine hundred and f-
231				it's 950.
232		<u>Wait.</u>		
233	*[elbows on the table, moving arms straight up and down, hands straight out]* No, 'cause			

234	65	[begins to say something]		
235	times 10 is 650-			
236		*[emphatically and loudly]* Just add a		
237	plus - I diiiddd, okay? *[puts head in hands for a moment]*	zero at the enndd! *[extends both arms straight out towards the center of the table]*		
238	650.			No, no, no.
239	*[looking down towards the table]*	*[looking towards Everet]* attach.	*[writing on her paper]*	*[looking towards Everet]* 'Cause all you
240				have to do is add, if you
241	*[writing while speaking]* 650,		*[pauses writing, looks towards Eric]*	know 65-
242	650, 650.			
243				Ohhh, that's
244	You do zero,		*[writing on her paper]*	*[leans over his paper]* true. That's-
245	five, one and then 603 is 18 plus one, is			
246	1,950.			Yeah, okay.
247		*[playing with his pencil in his mouth]*		*[looks up from paper towards Everet]* 1,950 would be our
248			*[looking at her paper]* So like-	answer probably.
249			*[looks up a little from her paper]* 65	
250		*[holds up his paper in the direction of Eric and points with his pencil]* I was really off.	times 10	*[looks towards Owen's paper]*
251	*[writing and erasing on his paper]*		is?	

252		*[looking towards Wren]*	*[looks towards Owen]* So 65 times 10 is?	
253		65		
254		times 10. *[in a lively tone of voice]* Attach		*[looks away from the table and then back to the table]* So yeah, I think Everet's right ↓.
255		a zero at the end ↑!		
Not transcribed Group 1 conversation: Discussion about an explosion, someone sounds like a grandma.				
256	*[smiles and looks questioningly at Eric.]* [undecipherable: "huh?" or "what?"]	*[opposite side of the table from his seat, picking up a pencil]*	*[looking towards Eric]* Then you get a hun-	*[looking towards Everet]*
257			<u>650.</u>	They're playing lie.
258		I'm gonna kill this *[places pencil on the ground]*.	*[more quietly, crosses left ankle over right knee, and looks down at her paper]* Because 65 times 10. *[looks towards Eric]* Right?	
259	*[leans over towards the pencil Owen is stomping on]*	*[stomping on pencil]* Die, die, die,		So, I think everyone w-
260	*[moves the pencil a little away from Owen's foot on the ground]*	die, die, die		*[looking towards Everet]* Do you know what my-Marcy says?
261	*[sits back up and looks towards Eric]*	*[walks away from the pencil leaving it on the grown and takes his seat at the table]*		All right, so yeah, I think we got the answer, though.
(4 seconds)				
262	*[smiling as though he's laughing but with no laughing sounds]*		*[looks at Eric and her face crinkles up with laughter]*	*[mumbling and chuckling a little bit]* So do we have

				nothing in there. *[pauses]* Sure.
263	*[small chuckle]* All right. So. so the an- so yeah, *[writes on his paper]*			
264		Here. *[leans over looking at Everet's paper]*	*[looks towards Everet and Owen]*	*[leaning over his paper]*
265	1,950 bags.			
266		*[addressing Everet]* We're doing 65 times 10, right?		
267		↓ or 30?		No.
268	*[looking towards Owen]* 65 times 30. But we broke it up into 65 times 10 and add those three times ↓.			
269	*[looks at Owen's paper]*	*[holds up his paper for Everet to see]*		
270	*[looks down towards his paper]*		*[writing on her paper]*	*[slowly while writing]* 30 equals,
271				*[looks towards Everet, speaks quickly]* wait, what was that?
272	*[speaking to Owen]* I got, 1950 ↓.			What was the answer again?
273	*[looking towards Eric]* 1,950.		*[writing on her paper]*	
274		*[writing on his paper]*		*[to Everet]* What did I say?
275	*[to Eric]* Well, the answer is one thousand. nine hundred. and fifty.			
276			*[looks over at Eric's paper]*	*[writing on his paper]* And so I was close.
277	Yeah, you were like...*[stands up*		*[writing on her paper]*	

	and leans over the table towards Eric] What was your estimate?			
278				Ii just-
279	825?			
280				W- uh, yeah.
281	Yeah, mine was 1,983.			
282				Wait, what was it? I'm so- it was one hundred, it was
283	1,950 *[nods his head towards Eric in a yes motion]*			1,950
284			*[looking over towards Eric's paper]*	Okay.
285		*[quietly to himself]* 18, 19	*[quietly, under her breath while writing]* 1,900…	
286				Bags.
287	*[leans over to look at Owen's paper]*	*[looks up from his paper]* Ok.	*[writing on her paper]*	*[leaning over and writing on his paper]*
288		*[addressing Everet]* Is that the		Bags, and make sure-
289		exact answer that you got?	*[looking towards Eric] [loudly]* That's a lot of bags.	
290	*[addressing Owen]* Yeah.			*[addressing Wren]* And make sure to write bags.
291				*[gesturing with his hand towards her paper]* You did? Okay, good job.
292	*[standing nearer to Owen's seat looking out towards the back of the room talking with someone telling them that*	*[standing a little away from his seat also looking out towards the back of the room]*	Uh, everybody in the whole school would have to bring, like, like-	

	Owen "is like 4'8".]			
293				100 bags.
294			Yeah.	
295		*[still addressing another table]* Take off my shoes.	Just for fourth grade.	
296				Yeah.
Not transcribed G1 conversation. Height comparison between 1 and 2 of the students. Noting that their height estimate was close. Basketball comes up from Johnny. Other students with height comparison. The students in the class moving around.				
297	*[standing to the side of his seat]*	Heeyyy, we have our answerrrr. *[shakes his paper in front of Wren]*	*[looking at Owen's paper]*	*[not at the table]*
298			*[addressing Owen]* What's our answer, then?	
299		*[addressing Wren]* 1,950.		
300			Yep.	
Not transcribed G1 conversation. They make sounds like baby dinosaurs. Interesting note about Everet – asking another group member to stop. the member apologizes, it just comes out of his mouth. Everet says it's fine, do what you have to do. Chat, and make funny sounds for each other. Discussion of a spider.				
Not transcribed Ms. Kedrik visits G1. *Ms. Kedrik stops by, how are you doing.* *Owen- we got 1950* *Everet restates* *Ms. Kedrik: how'd you get that.* *Eric starts to speak.* *Everet cuts in and begins explaining. 3 bags in one foot, in 8 bags [feet] how many.* *Ms. Kedrik: how many bags in an 8 foot.* *Everet- we got 30.* *Ms. Kedrik: but Wren doesn't have 30, she has 24.* *Wren – oh – begins erasing.* *Ms. Kedrik: hold on, stop. That's not incorrect. How did you get that?* *Wren – because ummmm (person) said 24 bags in an 8 foot rope.* *Ms. Kedrik: how many in one foot.* *Group: 3* *Ms. Kedrik: so in 8 feet? How many would that be?* *Eric- counts up to 24* *Ms. Kedrik: yeah, so 30 is too high. Can you go from here, how many more groups of 65 did you add?*				

Eric- 30 *Ms. Kedrik: but how many groups.* *Everet- two more groups?* *Ms. Kedrik: Nope, some discussion with 1 and 4.* *Ms. Kedrik: but rather than do the whole problem over… Ms. Kedrik helps with math strategies. They discuss multiplication with Ms. Kedrik help. Ms. Kedrik walks away.*				
301	*[back in his seat at the table]* 45…		*[writing on her paper]*	*[seated at the table, bending over his paper and writing]*
(3 seconds)				
302	So *[short pause]*			
303				Again, if I
304	What's			know
305	40 plus 20?			
306				No.
307	Or 40 times 20?			No. Here.
308	*[quickly]* What's 40 times 20?			No, I got it.
309			*[writing on her paper and looking over at Eric's paper]*	I know. If I know my basic, do the basic facts, if I know five times four is 20, well then, you've got a 20 here *[writing on his paper]*. And then if I know four times two is eight, so you've got 820 *[drops pencil on the table and leans back in his seat and looks towards Everet]*.
310	*[quietly]* Yep *[leans down and writes on his paper]*.			
311		*[addressing Eric]* Why does	*[very quietly]* Owen. *[pushes her paper towards Owen and points to something on the paper]*	

312		Lydia keep saying my name?		
313			*[very quietly, taps finger on the paper]* Owen.	
314		*[putting on his hat which falls off his head, looks towards Wren]*		*[writing on his paper while speaking]* So at 820...
315		What?		
316			*[indicating on her paper with her pencil]* What's the one that-	
317		*[gets his hat on his head, looks towards Wren's paper]*		... plus 420
318		Oh *[picks up his pencil and writes on his paper]*		
319		*[violently erases his paper so that the pencil squeaks against the page]*	*[looks over towards Eric's paper]*	would be-
(4 seconds)				
320		All this math.	*[writing on her paper]*	
321				1,240.
322	*[writing on his paper]*	*[getting up and leaving the table]* And my eraser broke, geesh.	*[looking over towards Eric's paper then writing on her paper]*	It would take 1,240 bags.
*Not transcribed G1 conversation. During another group's reporting out to the class, Owen asks Everet what 24*65 is.* *Everet repeated the question but didn't answer as he refocused his attention back to the class discussion.*				
Reporting out. Ms. Kedrik has this group share out as the last group to share. Eric raises his hand so that their group would be the one to share. Eric does the majority of the explaining. *65*30 doesn't work because not all kids, plus 3 bags is 1 foot. So that's 24. So we switched it.* *Ms. Kedrik: did you redo the problem or work with that?* *Worked with it.* *Ms. Kedrik: this is a good example of making a mistake with 30 bags in a rope, but worked with it. What next?* *They did 65*30=1950 bags. Then they 65*24=1220*				

Ms. Kedrik: ok, so you did do a new problem.
That's different what method did you use? Basic facts. What are we missing here? We haven't done a lot of this multiplication yet.
*They need 5*20, and 4*60, then 1560 which is the same.*
Ms. Kedrik: how did you adjust for the smaller bags? Did you make them all the same size?
Table: no about 3 people need 7 foot in our class, then another class is 4, and it's about 3, so about 10 people. So then half of 65. Well no I guess,
Ms. Kedrik how'd you adjust for that. You have all 65 kids, so if they all had 8 feet.
*They did 45*24.*
Ms. Kedrik: tell me why you did this? Could you take the idea of Johnny's group and figure out the difference in bags and then subtract, that might make your math more efficient.
Their answer 1530 final answer.

Table C2. Group 4 Transcript

Turn	**Brittany**	**Lydia**	**Jason**
Ms. Kedrik reminds the class that they are going to make jump ropes. Recaps by showing them how to braid the ropes. Ms. Kedrik checks that there is at least one person at each table who knows how to braid. Shows them how to braid. Explains that if you pull tight it will be a smaller section. They braid all the way to the end piece. They are going to do some measurements. A student from Group 3 asks about the audio recorders on their tables. Ms. Kedrik tells them about recording math conversations and they should only talk about math. Ms. Kedrik shares that they are going to do a math task like the caramel apple one they did. First slide in the presentation, what do you know about plastic bags?			
G4 conversation: Jason: Plastic bags are in the ocean and sometimes kill animals. Brittany: yeah, like sea turtles. Jason: All sorts of animals. Lydia: and plastic bags can be from stores. Brittan: Oh, did you know that they make you pay two cents for a plastic bag.			
Ms. Kedrik regroups the class. Students share what they know about plastic bags. *Wren shares with the class after her name is drawn to be called on: you can use them for shopping and carry things in them and use them for trash cans.* *Other students share that they have different logos, they can be recycled, they're not ecofriendly – not good for the environment. Ms. Kedrik says that later on they'll read a book about plastic bags in her part of the world. Good to reuse, or recycle them.* *Ms. Kedrik shares that they are going to upcycle in this task, not recycle, but upcycle where they take the bags and use them for something else. She asks "What do you know about jump ropes?" Talk in tables, what are some things they know.*			
G4 conversation. Brittany: they're really, really fun. Jason: They're fun and sometimes tricky to do. Lydia: they help you exercise. Brittany: yeah, and it's fun to do tricks on them. Lydia: and you can play with them with friends and by yourself. Brittany: yeah, you can have two people holding one end and then someone in the middle. Lydia: yeah, so you can either play by yourself or with other people. Brittany: yeah!			
Ms. Kedrik regroups: what do you know? *Students: fun way to exercise, different techniques for jumping.* *Wren: you can use a lot of them.* *Ms. Kedrik: what do you mean?* *Wren: Different tricks,* *Eric: can play games with them,* *Student: Can do a long rope and two of them for someone to jump.* *They come in different sizes.* *Ms. Kedrik: tell me more about that.* *Student: smaller for one person, longer for two.* *Lydia: Can jump alone or with friends.* *Student: Made of different materials like plastic.* *Ms. Kedrik gives a moment to think – what do you notice about the different jump ropes.*			
G4 conversation: Brittany: I notice that there are different colors, they all go at different speeds. Lydia: yeah, there's different colors. Like with that one in the first picture there is a really big one for their jump roping.			
Ms. Kedrik has them think about notices and wonders silently in their groups.			

They share out as a class. Ms. Kedrik records some ideas. Notices first. Students share: different colors of ropes, Lydia: bigger sizes for different kind of jumping, all different sizes, different speeds – some fast than others, Owen: the jump ropes look different from each other, Everet: there are lots of different ways to jump rope, one for the jump ropes was made of rubber bands so different materials. *Ms. Kedrik asks about what they wonder.* *Students: Wren: why some are big or small and different colors, Jason: how many kids jump roping there are in the pictures, Student: wonder about jumping with three jump ropes, Student: how many jumps there are, Owen: how many kids can jump on a jump rope.*			
"We are going to think about..." Ms. Kedrik: what math questions can you ask. Goal today is to create these upcycled jump ropes, so what are some math questions you can ask that you might need to solve before we can do this.			
1			Uh, how many jump ropes [plastic bags] it would take to make one jump rope?
2		*[gesturing with arms]* Yes. And, um, you need to figure out the length, because some people are taller than	
3	Yeahhh	other people	
4		and if we make short jump ropes	
5	And jump ropes	They would keep fall-	
6	are dependent on weight, uh height, I meant.		
7		Yeah, height.	
8		(3 seconds pause)	
9	*[looking at Lydia]*	Uhhhhhh *[moderate tone of voice as one thinking]*	
(8 seconds pause)			
10		Um. (2 seconds pause) How many plastic bags we need to make uh, jump ropes.	
11			That's what I said.
12		Yeah.	
Ms. Kedrik walks up to the table and stands by while students continue their discussion.			
13			If we have -
Teacher: What are we thinking?			
14			we also need to know - if we have enough jump ropes to make everybody in the class have one ↑.

Class regroup and share. Students share: Brittany: how many plastic bags for everybody in the class to make a jump rope; Lydia: what lengths are the jump ropes; how thick are the ropes; (Everet mutters that it depends on you braid it, Eric agrees), how long does it need to be different sizes; taller students need taller ones. Shorter need shorter might trip on a rope that is too long. Eric says he uses his mom's jump rope and it's pretty big. Ms. Kedrik: Would that change the number of bags? Students: more bags, how many plastic bags in all, Johnny: how many people want to jump rope, Student: do we have enough bags, Everet: three questions, how many bags go in a rope, how many ropes can we make, do we have enough jump ropes for the whole class. Owen mutters that he should have kept count of his bags. Wren: how many bags do you need for one big jump rope. G1 things this is interesting "ooo" they discuss quietly the whole class and the teachers. Student: how long is each bag and how much time do we have to make them. That's good, like in science when they have a time constraint.
Ms. Kedrik: main question: How many plastic bags does it take to create jump ropes for all of 4th grade. Class starts talking. Ms. Kedrik regroups, has them record the question. They are going to do math to see what they would need if they wanted to do this. Students record on papers. Ms. Kedrik walks around.
"So, we're going to do a little investigation here..." They are going to braid, Ms. Kedrik helps them think about getting the whole piece braided, want to see how long it is from the knot to the end, how long? They are going to braid and get the measurements. "So, I'm going to come to your table..." Group work time. Ms. Kedrik gives bags, students begin talking and braiding in groups.
Group 4 conversation: Jason: So we each get two. *Lydia: yeah we each get two so we each get to make one. But Brittany, you're not always going to [inaudible].* *Brittany: I know.* *Lydia: But you'll do it this time.* *Ms. Kedrik hands them the bags.* *Jason: I will hand them to you.* *Lydia: Thank you.* *The students arrange their bags.* *Lydia: How do you do it?* *Brittany: I did it!* *Jason: That's not how.* *Jason helps Brittany. Jason: see if you pulled a little more it would undo. They help each other loop their bags. Brittany very excited with figuring out how to do the knot. Lydia figured it out after Brittany. They get it and are excited for each other. Jason is the one who knows how to loop the bags together and helped the other two. Jason walks over to Table 1, but is called back by Lydia to braid with his group mates.* *Jason: Can I braid it.* *Lydia: No, Brittany is going to braid it first.* *Jason: I want to braid it. He takes the bags and tells Lydia to hold it for him while he braids it. But Jason quickly walks away and Brittany takes over braiding. Jason walks back and chats with Lydia and Brittany. It sounds as though they are talking about braiding hair and having sisters. But it's really hard to understand more than a word here and there. Jason reminds the*

girls they need room for a handle. Lydia holds about a hand space for it while Brittany braids. Jason walked away and walked back. Jason helps with setting up the handle. They finish braiding.			
Ms. Kedrik quickly recalls class and asks the class to measure how long three bags braided together are from the knot. Students continue working in groups.			
Group 4 conversation: Lydia asks Brittany if it's almost done. Brittany: Almost. Lydia asks Jason for the ruler to measure the rope. They measure the braided rope and then walk around a little bit.			
Class regroup and share jump rope measurements. They agree that 12 inches and three bags is a friendly number to use. Ms. Kedrik gives jump rope measurements. *Up to 4'6" tall then jump rope is 7' long.* *Between 4'6" and 5'5" then the rope is 8' long.* *Over 5'5" then the jump rope is 9' long.* *A student comments none are over 5'5" except the T. Ms. Kedrik comments that she's not over 5'5, good info for them.* *Students discuss in groups what they know about making jump ropes..*			
15	We know that we can braid them [the jump ropes]?		
16	*[picks up braided rope and plays with it]*	*[haltingly, using arms to gesture while speaking]* Uh, we know that, uh, we need ah do leeeennngth and uh, find out how many feet are on the jump rope in all ↑ *[palms open and facing upwards]*.	
17	Same as above.		Wait, what did you say?
18	*[braiding the end of the jump rope while Lydia holds the other end]*	How many feet are on the jump rope and all. So, like, yeeah. *[takes one end of the jump rope while Brittany holds the other end]*	
(2 seconds)			
19	*[playing with the jump rope]*	Like there could be seven-feet, eight-feet...	
(5 seconds)			
20	*[holding the jump rope]*	So we would need to figure *[pronounced "figger"]* out how many feet it is *[spoken more slowly]* ↑ .	
21	*[British accent]* That word-		

(2 seconds)			
22		For different people.	
In the background, Ms. Kedrik begins counting down to regroup the class.			
[undecipherable comments between the students as Ms. Kedrik regroups.]			
Class regroup. Class discussion about heights of people they know. They are talking about 4th grade class this year. They share what they know. Ms. Kedrik: can we assume or make a guess about how many are in 4th grade? They decide on 66. But then discuss and land on 65 – it's an assumption and a friendly number. Ms. Kedrik: anything else we know, need to know. Do you have all the info you need? Class: no. Because estimate of smallest and tallest, average height. Ms. Kedrik: if we wanted to make them. But right now, what do we need if we want to make them. We won't make them for all of 4th grade, just our class, but we might not have enough for that. Class discussion around numbers and heights. They know up to 4'6 they need 7 feet, 4'7 to 5'5 is 8 feet. Add to what they know. Class discussion around numbers of students for each size. Assumption is that they need more 8 foot than 7 foot ropes. T says they'll come up with plans in their groups, they might be different. They have what they know and need to know.			
First part of paper, make an estimate about the number of bags, then show work. Decide at tables how many 7 and 8 ft ropes to make. 65 students total. Decide how many ropes for 7 and how many for 8. They know 3 bags for 1 foot of rope, use the info when deciding how many jump ropes go into a 7 foot rope and how many go into an 8 foot rope.			
Not transcribed G4 conversation taking place during the whole class conversation: Lydia: I said two foot [saying that 6 bags is two feet]. Jason looks back to Lydia and asks to see the braided rope. Lydia gives it to him. Lydia: It's about 6 foot but the end, oh my god not 6 foot. Jason measures the rope.			
23		Umm *[begins writing on her paper].*	*[holding and playing with the jump rope]*
24	(4 seconds)		*[looking towards another table whose members are saying that everybody is about 5 foot. Looks back to his table]*
25	I would take allll-	I don't know-	
26			*[loudly with a shake of his head]* No, not every body's about five-foot.
27		No, definitely not-	No no
28			*[with a slight chuckle in his voice]* Noo
29		I'm not five-foot.	
30			*[with a slight chuckle in his voice]* Noo
31		*[looks behind her towards Table 5. A student says "four-foot-seven to five-foot"]*	
32	I'm gonna say-		The average height of a

33			fourth-grader is probably four-foot-six.
34		Yeah.	
35			Somewhere in there.
36		Yeah,	
(3 seconds)			
37	*[looking out towards the classroom]*	*[looking towards the classroom scanning the room]*	Yeah ↑ *[playing with the jump rope and looking away from his table to scan the classroom]* That's like the average height.
38		*[looking back towards her table group]* Me and Johnny might be the same height.	
39			*[playing with the jump rope]* There's definitely people-
A class member not in this group: Average is like four-ten.			
(8 seconds)			
40			He's four-nine? I'm like four-
41		Um.	
42			I'm like four eleven
43		My estimate,	without shoes on
44		sooo normally people are four-feet, eight-feet,	
Ms. Kedrik reminds the class: So, you're just writing your estimate down right now.			
45		*[haltingly]* Eight-feet, so- ↑ Maybe...135 ↑. I doo ↑ not ↓ know *[slight chuckle, writes on her paper]*.	
46			So what are we estimating?
47		Uhhh, I don't ↑ know. I just guessed 135. I don't know.	
48			*[playing with jump rope]* So, like, how many bags for each class? *[looking away from the table group]* That's definitely not going to be it Lydia.

49		*[gesturing with arms and hands, palm open, elbows remain on the table]* It has to be at least double the amount of classrooms. So	*[looks back to Lydia]*
50			*[playing with jump rope]* Yeah. So If it was like, let's say there is 30 kids in uhhhh in our classroom and the average height was four-ten. So, we'd probably need a 100 bags ↑...
51		At least -	
52	*[addressing Jason regarding the jump rope]* Oh my God, stop you're unbraiding it!		
53			*[in a whining tone of voice]* Nooo ↑ I'm noooott ↓
54	*[in a whining tone of voice]* Yes you arrrrrrre ↓.		ssssssuuuhh ↓
55	Okay, I'll just braid it up *[reaches for the jump rope but doesn't take it as Jason starts braiding]*		*[starts braiding the jump rope]*
56		*[addressing Jason who is braiding the jump rope]* Make it tight. *[lightly striking left fist on the table while holding her pencil]* But, I think we should round at least to like 300 maybe? I don't know.	*[continues braiding the jump rope]*
57	No. 330.		
58		Three-	*[lightly tosses one end of the jump rope at Brittany and then pulls it back]*
59		330?	
60			No, like a 100 per class at least.
61	Yeah.	Yeah, cu-	

62		*[lightly striking left fist on the table while holding her pencil]* ↑ So at least ↓ 300.	
63			*[lightly flipping one end of the jump rope off to the side of the table]* Yeah, at least 300 probably
64		So estimating 300, right?	[undecipherable]-
65		So, there should be- *[writes on her paper]*	Yeah, my estimate-
66	*[looks over to Lydia's paper, then writes on her own paper]*	because… *[writes on her paper]*-it has to be at least double the classroom *[gestures while speaking with her right arm, elbow on the table, palm facing upwards]*	*[writes on his paper]*
Ms. Kedrik regroups the class: Okay, hey, hey, fourth grade.			
Class regroup. Ms. Kedrik asks about a reasonable estimate. Asks about bags in a whole rope. Student volunteers 30, their estimates are 1,800 – friendly number. Ms. Kedrik shares thinking of a student 24 bags in an 8 foot rope, then 240 for 4th grade, but that's only 10 so using that number to come up with an estimate. Would 20k be reasonable? No. Be sure numbers are reasonable. 900, 3000. Different numbers. 450 bags. About 25 bags in a rope. Think quarters.			
"Ok. Alright, so..." They are going to do their work. Ms. Kedrik recaps info and what they need to do. They understand what they are doing and begin working in groups.			
67	*[plays with the braided jump rope]*	*[gestures with her right arm in circular motion indicating the table group]* Okay. So first we need to figure *[pronounced "figger"]* out-	*[turning from the front of the room back towards Lydia]*
68		like	Figure not figger.
69		the height	
70	Figger.		
71		Figure *[pronounced "figger"]* out-	
72			Figure.
73	Figurrre.		
74		*[looking towards Brittany]* That's what I'm saying.	
75	No, you're saying figger.		You're saying figger.
76	*[smiling and playing with the braided jump rope]*	*[in a light tone with a slight accent]* I don't care.	[undecipherable: *chuckles*]

		[arms open and gesturing while speaking] But it's- *[laughs]* it's fine *[laughs]* Okay. We need to- *[pauses]* We need tooo *[slight pause]*	
77	*[laughs]*	figure *[pronounced "figger"]* out.	
78	*[laughing]*	*[with a smile in her voice and arm extended towards Brittany]* Just- just shut up, okay?	*[head down appearing to look at his paper]*
(6 seconds)			
79	*[smiling with mouth wide open, playing with the braided jump rope]*	*[pausing, arms open with elbows on the table, shrugs her shoulder and shakes her head a little]*	*[leaning into the table facing his two group mates]*
80	*[playing with the braided jump rope]*	So ↓ *[pauses]* We need to *[pauses and uses hands to accompany her words]* <u>realize</u> how much- *[picks up her paper]*	
81	Thank you *[smiles and gestures towards Lydia with her hand, elbow on the table, palm open]*.		
82	*[playing with the braided jump rope]*	*[holding her pencil gesturing with her right arm with elbow on the table, left arm lays still on the table]* Bags, um, we need in our classroom, first, *[begins gesturing with both arms in an explanatory manner while speaking]* then we can go on to the fourth grade- or	*[leaning into the table facing his two group mates but hunched over towards his paper – not writing on his paper]*
83	*[quietly while using both arms to raise the braided rope behind her head]* Weeeeee.	the whole fourth grade.	
84	*[spoken slowly and somewhat quietly to the table while holding the*	*[erases on her paper with the eraser on her pencil]* So- there's like	*[looks up towards Brittany while still leaning into the table]*

	braided rope against her hair] Look how long my hair is ↓ .		
(4 seconds)			
85	*[spoken very quietly]* Except when it's in a ponytail and it's sticking out because I have so much of it.	*[looking straight ahead and occasionally tapping her pencil]*	[undecipherable: "that's what she'd say"]
86		*[looks away from the table and around the room counting and gesturing with her arms]* -eight, 11-	
87	*[still holding the braided rope to her hair]* Wait, no, I'm an avatar-		
88		14.	
89	*[laughs]*		
(7 seconds)			
90	*[either Brittany or Jason: high pitched humming]*		
91		*[still looking around the room counting off and emphasizing her words by pointing with her arm and pencil – almost like making a tally mark in the air]* Um, 18. 22. So, there's 22 people in our class.	
92		So<u>o</u> ↓ *[pause]* most of the- most of them would be eight-foot -	*[looks around the classroom]*
93	*[silently making faces with her mouth]*		There's 22 kids in our class.
94	*[playing with the braided jump rope]*	Yep s-	
95			and right now there's only *[haltingly]* 21.
96		Yeah.	because Brendan is gone.
97	*[playing with the braided jump rope, lifts the table a little bit but stops playing with the jump rope after lifting the table. Leaves the*	Yeah. <u>But</u>, um, for our class, if Brendan was here, we - I feel like - almost everybody needs an eight-foot rope. Almost	

	jump rope in front of her on the table.]	everybody except for like - you *[looks towards Brittany and pushes back down on the table slightly]*	
98	*[quietly]* Mee ↓		
99		*[thoughtfully]* M-Maybe, maybe ↓ , I don't know. Except for like, I know Brie. Because she's shorter than me.	*[very quietly, almost inaudible, addresses Brittany]* [undecipherable] can you stop lifting the table?
100	*[folds arms in front of her on the table and leans into the table]*	Maybe Owen. *[taps lips with pencil]* *[quietly]* He's shorter than ↓ me.	
101			Definitely Wren.
102	Definitely.	Yeah.	
103		*[looking out towards the classroom]* And also Belle.	*[looks out towards the same place as Lydia was looking]*
104	*[addressing Lydia by turning her head and shifting her upper body to lean towards Lydia]* What about me?		
105	*[smiles]*	*[considers Brittany]* You might be taller than Belle *[looking out over the classroom]*	*[looking out over the classroom]*
106	*[facing out towards the classroom due to her location at the table]*		Oh yeah, definitely Belle.
107		Yeah, Belle definitely *[laughs]*.	
108	Belle's definitelyyyyy-		Belle definitely might need
109			yeah, a seven-foot rope.
110		Yeah. Maybe ↑ Johnny -I don't know ↓ -	
111			Johnny said he's-Johnny needs an eight-foot rope 'cause he's uh, four-foot-seven.
112		Okay. So he's like right about.	

113	*[addressing Lydia]* Oh my god, have you ever played with a measuring tape?		
114		*[gestures with her arms while elbows remain on the table]* Well, okay, so-	
115	*[quickly]* Knew you better *[to the audio markers on the table]*		
116		*[gestures with arms while speaking]* Most of them would be eight-foot. So I'm guessing like-	
117		One,	Yeah, most-
118		two, three *[pauses]*, four↑? maybe four people need a eight- seven-foot ropes. Maybe?	
119		So, maybe, maybe four people *[haltingly]* would need seven-foot rope, maybe.	*[stands up and looks around the room]*
120			*[turning back to the table and resuming his seat]* Okay, so what?
121		*[pointing while speaking]* So, I think, maybe four people will need seven-foot ropes. Brie, Wren, Belle, Maria.	
123		Maybe Owen. Maybe. *[gestures with pencil in hand and puts pencil to her lips]*	Sooo
124			Definitely Wren.
125		Yeah, definitely Wren	
126	I'm definitely taller than Wren.	and Belle	*[sitting back in his chair at an angle to the table so he is half facing the back of the room while playing with a ruler]*
127		Me too.	
128	I might be taller than you.		
129		Yeah you probably are.	

130	*[leaning into the audio markers and annunciating each word]* Thank you, world for making me taller than Lydia. *[smiling and looking towards Lydia and moves her right arm in a circular motion mimicking hypnosis]*	*[looking towards Brittany]*	
131		Why?	
132	*[very quietly and hypnotically]* [undecipherable: *murmuring*] than Lydia, Lydia	Okay, but anyways- So, seven *[pauses]* seven-foot	
133	What?	would be	
134	*[begins playing with jump rope again]*	-so, seven, no, yes, no, no- *[writing on her paper while speaking out loud]*	
135	My spider like this.		
136		Yeah, seven times-	*[addressing Brittany]* You put your mouth on that.
137		Three. That would equal *[pauses]*	
138		that would equal 21.	I wouldn't put that in your mouth
139			because how many people do
140		So <u>21-</u>	you think have touched that bag?
141		yeah *[looks at Brittany]*	
142	[undecipherable: "I'm playing with it"]		
143		<u>So 21.</u>	Don't put that in your mouth, <u>nasty</u> rope.
144	*[tosses the braided jump rope across the table]*	<u>21</u>, um, bags would equal seven-foot.	
145			*[touches the braided bag and addressing Brittany]* Pull it.
146	*[mouths something to Jason]*	And then, so that means, 24, then that would	

147		mean	*[responds to Brittany]* I don't know.
148	*[laughs inaudibly]*	18 time 3 equals 24.	
149	Look at Emmet.	So, we just -	
150	He just put that chord around his neck.	*[looks up from her writing towards Emmet]*	*[looks towards Emmet]*
151	*[playing with the braided jump rope]*	*[turns back to her paper]* if everybody was in our classroom. So 24 times	
152		uhhhh let's do 18 ↑ ?	*[moves his chair back into the table and leans over his paper]* So most of the-
153	Okkkay ↑ ?		So, there'd-
154		*[writing on her paper]*	there'd only be like a couple uh that would neeeed aaa ↓ -a seven-foot rope,
155		Yeah.	and most
156	Yeahhh ↓	Yeah	of them would need an eight-foot rope. Correct?
157		Correct.	
158	Corrrrrrect.		
159		So I feel like it would be, so 20, so 18, times 24, okay? *[gesturing to her teammates using circular inclusive motions indicating a 'bringing in' or 'coming together' idea]* Just work with me now.	
160	Okay.		
161	*[plays with braided jump rope]*	*[gesturing with her arms]* So 24 times 18, we're gonna do the quarter trick which, that would be *[pauses and says something under her breath]*.	
Not transcribed. Brittany: [lays the braided rope on the table] look how long. Jason: [looking at Brittany's cup] "Is that a booger?" Lydia: a what? Brittany: Oh my god no that's a tea string. Jason: Oh that scared me I thought it was a booger. Lydia begins writing on her paper.			
		25, so that would be *[counting on her fingers]* one dollar,	

162		two dollars	*[addressing Brittany]* You actually like that tea?
163	Yeah. I like lavender tea.	Three dollllllarrrrrrssssss.	
164	Lavender tea's the best.	Four dollllllllarrrrrrssssss.	
165	With a pinch of honey.	Four-	
166		Four-fifty.	I like beer.
167	You like beer? Oh Lydia has dranken beer.	*[writing on her paper]* Then take away eight. 18	
*Ms. Kedrik visits G4. Ms. Kedrik stops by reminds them all to work together and talk together and not just Lydia and not just copying. Lydia explains her thinking 7*3=21 so 21 bags in 7 foot then 24 bags in 8 foot. 18*24 because I think 4 people in this class would need 7 foot ropes and the rest would need 8 and there's 22 people in this class. So 18*24 which I haven't done yet. Ms. Kedrik: figuring out our class and multiplying by three. Lydia: yes. Ms. Kedrik: alright. Ms. Kedrik leaves.*			
168		So.	
169	What ↑ are you writing down now ↓?		
170		*[quickly]* Hold on. That would be 450.	
171	Use your mathematical brain.		
172		*[speaking slowly along with her thinking and writing]* Four. Four. 20. Fouurr. Twenty. 422 maybe ↑ ?	
Not transcribed. Brittany and Jason leave the table. Lydia working on her paper and saying numbers. Class wandering around.			
173	*[sits back down at the table. Looks at Lydia's paper and then writes on her own paper. Sings a few notes]*	Okay. I'mma say. 400, uhhhh Yeah.	*[not seated with his table group but is standing behind the table along the bookshelves talking with another student]*
174	*[leans over her paper looking at what Lydia writes on her paper]*	Then, 24 times four, 21 times four *[writes on her paper and speaks under her breath]* 21 times 4 would equal... 80. 84. *[writes on her paper and speaks under her breath]* would equal 84.	

		[slowly and questioningly] Uh, ↑ maybe Owen ↑ should be ↑ in herrrrre ↑ ?	
175		Mmmmmm	*[resumes his seat and rejoins his table group]*
176		*[addressing Jason]* Wait is Ow-which one is Owen, Johnny's height or no?	
177			Probably a little bit shorter.
178	I'm-	*[looking at Jason]* So should we	
179		count him in- The seven-foot- seven-foot, okay. *[writes on her paper]* So I'm going to change this up to seventy-wait, *[addressing Jason]* what if Brendan-Is Brendan um, taller than Owen?	
180			Yeah.
181		Yeah.	He's in.
182	*[looking around the room and also looking at Lydia's paper and then writing on her own paper]*	Okay, okay. *[working on her paper, writing and speaking aloud]* So we're gonna move that into five. So we're gonna change it up. So this would be 22, 25. So. Five minus two, 18, minus 10. Would equal, 408 ↑ . Then instead of that it would be one-oh ↓ -six. Okay.	*[sits in his seat but does not write on his paper]*
183		We've got	Okay, so what are we-
184		these numbers, so we've got-	
185			*[picks up his pencil]* So it's not 17 times 24 ↑?
186	*[looks away from the table towards another table but eventually turns back to see what Lydia is writing, then writes on her paper]*	No, not anymore. It, it, wait. Okay. So it's *[flips her paper around so Jason can read it from his view. Explains and demonstrates with the paper while speaking]* 17 times 24,	*[writes on his paper]*

		which I would equal 408. Then, twel- 21 times five, would equal 106. *[waits for Jason to record on his paper]* Okay, now we're gonna- now we're gonna add *[flips her paper back around and writes on her paper while speaking]* 408 plus ↑ one-oh-six, which that would equal ↑, four, one, annnd five. 514. Now, we're gonna add 514 plu- times, *[pauses]* times ↑ three, would equal 1,532.	
187			Uhhhh sooo...
188		What?	
189			skibidi go, go, go. Yes, yes, yes. Skibidi, yes. Skibidi yes.
190		Eight.	
191		Yeah	Skibidi toilet.
192		*[looking at her paper]* This would equal it.	
193	Skibidi toilet.	Okay.	
194	*[quietly singing]* Skibidi skibidi toilet skibidi skibidi ↑	So, it would be about 153- no, oh my gosh- it would be about	
195	*[looking over at Lydia's paper]*	1,532. It would be about that.	
196		So,	Okay, so our est- the estimate would be around...
197		It would be around 450. 532.	
198			So, five- so probably like 1,000 ↑?
199	Yeeah.	No *[definitively]*.	
200		*[turns her paper upside down so Jason can read it, then points to her paper while explaining]* Because, I just did this, and so, *[gesturing with her arms]*	*[leaning over to look at Lydia's paper]*

		people might be shorter, but that would be like minus 500. So ↓. Maybe we would need like 1,532 bags. *[lightly throws arms up in the air and drops them on the table]* I don't know.	
201	[*maybe says "Yeahh"]*		I think the estimate of 900 and 300 per class is a pretty good estimate.
202		Yeah, but, I figured out that just in this class it's 514. Just in this class. So when we times that by three we get this answer. So, it could be higher than that because we don't know if	*[plays with ruler by tapping it on the table]*
203	'Cause yeah-	people are	
204	*[speaks a split second after Lydia]* taller or shorter	Taller or shorter.	
205		So, it, but, *[exhales]*, our estimate would be about this. So.	*[tapping ruler]*
(6 seconds)			
206		So, instead of 900 it would about 100- 532	
207		If we wanted	100 and-
208		to estimate into a friendly number 100- 530.	
209			*[looking at Lydia]* So, probably around 2,000?
210		↑ Ummmm ↑ , maybe, probably. Around 2,000. <u>If people</u> are taller *[gestures 'taller' with her arms]* -	
211			Maybe, it's better to have extras than not extras.
212		Yeah. Yeah. There's- Better to have extras than less than, so ↓. It would take about like maybe 200- 2,000, m<u>aybe</u>. If people are	

		taller. *[pauses]* But, it ↑ wouldn't- actually ↑ -	
213	*[sneezes and looks at Lydia in surprise]*	*[looks at Brittany]*	Okayyy.
214		I don't think it would, actually equal ↓ ...Okay, so if everybody had 22 kids, so 20- 22 times 24, would equal… one, so one dolllarrrr	
215	*[leaning over the braided jump rope towards the center of the table]* Satisfying satisfying ↓ satisfying ↓ satisfying ↓ *[fades to murmuring]*	one dollar.	
216		25, so one dollar.	
(5 seconds)			
217		Three dollars equals eight.	*[tapping ruler]*
218			Okay, sooo-
219		So it would. Actually	
220		if everybody- *[pause]*	It would probably- *[pause]*
221			it would probably be an estimate arounnnnd-d
222		1,500.	
223			*[questioningly]* Maybe 1,600 ↑ ?
224		M- Yeah, probably, maybe. If	
225		people were tall.	For a friendly number.
226		Yep.	
227			1,600
228		Yep, friendly number. I th- I would think it would be around five- 1,600, maybe, if people were- if *[gesturing with her arms]* we had more people in	
229		different classes-	Yeah, we probably-
230			we probably have the people with the tallest. So. We probably have the most tall kids in our class.
231		So	So

232			Yeah, you should-probably.
233		It might -	
234			A friendly number would be 1,600. Yeah, I think that would work.
235		*[gesturing with arms]* Yeah, if 100 and 600 would have extras, probably. U<u>nless</u> other classes had more people, and, and, tall people. Like we have five people that are probably short ↑. *[speaking slowly]* Shorter than four-six. S- that, seven-foot ↓ *[pause]* that needs seven-foot	
236		long-	Okay so.
237		seven-feet long.	
238			*[looking at Lydia]* So, now that we're pretty much done, do we just read a book?
239		Uhh maybe we should talk about it and see what they're doing *[looks up to see Ms. Kedrik approaching the table]* yeah.	
Ms. Kedrik visits G4. Ms. Kedrik stops by. Lydia starts explaining, Jason chimes in rounding to 1,600. Lydia explains the changes made from the last time she stopped by. Ms. Kedrik confirms with them and asks for them to share with the class.			

Reporting out. Ms. Kedrik requested this group share their work at the end. Jason reported out. Ms. Kedrik paused them, asking Jason a question and Lydia whispered it to Jason. Ms. Kedrik asked a follow up question about how he got the work, he said he wasn't sure.
Lydia starts working it out, tells Jason the answer who shares it out loud.
Ms. Kedrik works it out, but the class realizes a mistake so that Lydia's work was correct.
Jason continues explaining, but where adjustments are needed Lydia jumps in to explain. When Ms. Kedrik asks another follow up question "why" Lydia gives the reasoning.
Jason explains their estimate where they rounded. Brittany says nothing during this time. They did that there was 5 people with 7' rope, 17 people with 8 foot rope, just in their class.
*17*24=408 then did 21*5=106, then 408+*
Ms. Kedrik: how'd you get 106. Not sure.
*Ms. Kedrik: let's double check. 20*5=100, so 105. Then 24*17 with area model. They get 388. Just for their class. Then they realize it's 408 and 305. Add together, 513. Then they did 514 *3 but now it will change by 1 group of 3.*
Ms. Kedrik: why?
Lydia: because there's three classes.
Ms. Kedrik: ok, so most groups looking at 65, this group looked at one class and multiplied by 3 making the assumption that there would be similar class sizes. Ms. Kedrik helps with the total but they got 1539. They estimated to 1600, Ms. Kedrik you rounded to 1600 it will give us enough.