Bubble mechanics by dimensional analysis

by Frederick William Steele

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in MECHANICAL ENGINEERING
Montana State University

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2.) The use of liquids for high heat transfer processes in nuclear reactors.

Ihis thesis concentrates on two specific areas in the general field of bubble mechanics. Ihese two areas are bubble motion and bubble inception or nucleation. Because of the complexity of the processes, dimensional analysis was found to be the most effective method for establishing relationships. General equations were derived for both bubble motion and bubble inception. Experimental results obtained by other investigators were then used to verify the general equations for certain special cases. Some predictions were made for the phenomena for which no former experimental evidence was available.

It was concluded that the general equations which were derived would provide a systematic approach to the study of bubble motion and inception.

Ihe limitations of the general equations were stated and some possible experimental methods were suggested.

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Abstract

The study of bubble mechanics has been of interest to scientific investigators for over a century. However, not until very recently has there been any intensive study of the subject. The increased interest has been caused mainly by two developments:

1.) The problems caused by cavitation in the liquid fuel systems of space vehicles.

2.) The use of liquids for high heat transfer processes in nuclear reactors.

This thesis concentrates on two specific areas in the general field of bubble mechanics. These two areas are bubble motion and bubble inception or nucleation. Because of the complexity of the processes, dimensional analysis was found to be the most effective method for establishing relationships. General equations were derived for both bubble motion and bubble inception. Experimental results obtained by other investigators were then used to verify the general equations for certain special cases. Some predictions were made for the phenomena for which no former experimental evidence was available.

It was concluded that the general equations which were derived would provide a systematic approach to the study of bubble motion and inception. The limitations of the general equations were stated and some possible experimental methods were suggested.

CHAPTER 1

INTRODUCTION

A bubble is defined as a small body of gas within a liquid. More generally, it is the existence of a closed surface which divides the region of concern into two parts, each occupied by a homogeneous fluid (12). Bubble mechanics is thus concerned with the behaviour of bubbles and their interaction with the surrounding fluid.

The study of bubble mechanics has received increasing attention in the past few years. This is due mainly to the importance of bubble mechanics in the design of systems for space applications.

Cryogenic fluids are used as oxidizers in the first stage of the Saturn I, Saturn IB and Saturn V space vehicles. They are used as both oxidizers and fuel in the upper stages of these vehicles. All of these fluids have very small latent heats of vaporization and low boiling points in comparison to water. Because of these properties, they are very susceptible to phase change and boiling. This creates many engineering problems, such as cavitation, vortexing, stratification, two-phase flow in suction lines, pressure losses in lines, the coalescence of bubbles under conditions of extreme vibration, and intermittent firing (3).

In the study of bubble mechanics more and more investigators are turning to the use of dimensional analysis. This is probably due to the many factors present in bubble mechanics which make it too complex for strictly analytical methods. Most of the really significant studies in the past few years have used some method of dimensional analysis.

One of the principal advantages of dimensional analysis is the prediction of similarities which allow small-scale modeling of full-scale operations. In a dimensional analysis the problem being considered is shown to depend on a particular set of dimensionless numbers. These dimensionless numbers can be considered as ratios (usually force ratios). A familiar example of a dimensionless number as a force ratio is the Reynolds number, which is a ratio of two forces in flow: the inertia force, resulting from the motion of the liquid, and the viscous force. Table No. 2 contains a list of dimensionless numbers expressed as force ratios.

Whenever a model has force ratios similar to a prototype, it can be used to simulate the prototype. An example which is illustrative of this principal was presented by Clark (30). When dealing with low gravity systems, the Bond number often becomes important in predicting the phenomena involved. The Bond number is a ratio of body or net bouyant force to surface tension force. At low Bond numbers the surface tension forces predominate over the body forces, which corresponds to the case where low gravity exists. By dipping a slender tube one-millimeter in diameter into water, a situation can be achieved with a Bond number of about 0.036. The surface tension force is the predominate force in the system. In comparison, a container holding liquid hydrogen with a diameter of ten feet will have a Bond number of 0.036 in a gravitational field of about 10^{-7} feet per second squared.

Therefore, the slender tubing can be used as a model to represent the hydrogen container in a simulated low gravity experiment. With this experiment it is possible at terrestial gravity to predict the shape of the liquid interface at low gravities.

Bubble mechanics was first attacked by Euler in the 18th century. He formed the equation of motion for a perfect liquid. In connection with this he recognized that a small reduced pressure localized at some point might create small voids (1).

The collapse or growth of a spherical bubble under adiabatic conditions was first treated for an ideal fluid by Besant in 1859. Lord Rayleigh continued this work on "cavitation bubbles" in 1917 (10). Rayleigh worked with the motion of a single cavity in a liquid. Resurrecting the solution derived by Besant, he attempted to derive relationships which would describe the phenomena occuring when a spherical cavity is suddenly created in a fluid. Equating expressions for the kinetic energy and work involved in the process gave him expressions for the velocity of the fluid and the time for complete collapse. He assumed a constant pressure process at first and later an isothermal process. B. E. Noltinkg and E. A. Neppiras extended Rayleigh's study to include adiabatic processes.

Later experimentation revealed that Rayleigh's work was not in accord with data obtained from real fluids. However, his work established the dynamics of cavities in fluids as a respectable subject for scientific inquiry. His work was also the starting point from which many later investigators began (1).

A few years before Rayleigh's work, Osborne Reynolds investigated the flow of water through constricted tubes, and found that in regions of vanishing pressure small bubbles form and collapse noisily. His study was instigated by his desire to learn why kettles sing. At approximately the same time Sir John Thornycroft and Sidney W. Barnaby proposed the formation of cavities around marine propellers as an explanation for the failure of their destroyer design to meet its predicted speed. Sir Charles Parsons verified their hypothesis by experimentation. Robert C. Froude, then Director of Experimental Research in the British Admiralty, coined the word "cavitation." (1)

Stokes presented one of the earliest bubble mechanics studies in 1880, which proved to be of particular significance. He solved for the velocity of fluid past rigid spheres at low Reynolds number. The expression he obtained is now referred to as "Stokes' Law." Allen and Robinson showed Stokes Law to be restricted to Reynolds number of less than two (8).

Miyagi studies the motion of air bubbles in water. He showed the terminal velocity of bubbles to be independent of size for bubbles with radii of less than 0.01 foot. Byrn classified the motion of bubbles in aqueous solutions and established three distinct types:

1.) Small, spherical bubbles rising in straight lines.

2.) Medium sized, horizontally flattened bubbles rising with an oscillatory motion.

3.) Large, mushroom-shaped bubbles rising relatively straight (1).

O'Brien and Gosline presented one of the first studies which relied entirely on dimensional analysis in 1935. They found the drag coefficient of bubbles to be a function of Reynolds number, Weber number and bubble radius to tube radius ratio. However, they did not present any experimental work to establish the functional relationship between the dimensionless groups (1).

Lapple and Shepherd used dimensional analysis to make calculations of particle trajectories. The equations they developed took into account the effect of fluid friction. Curves were presented for various shaped particles ranging from spherical to disk (10). This study was published in 1940. Many investigators since then have applied the results to bubble mechanics.

Several years later Kaissling used dimensional analysis to study the motion of bubbles in vertical boiler tubes. Functional relationships between the dimensionless groups which he derived were not determined. In 1948 Wigner used dimensional analysis to make some speculations with regard to the velocity of gas bubbles in liquids. The following year Levich applied boundary layer theory, with reference to liquid-gas interfaces, to the computation of total resisting forces acting on a bubble rising in a tube. He obtained an equation for the velocity of the bubble, but was unable to verify his results. The same year Gorodetskaya attempted to verify Levich's equation by experimental methods. His results were 30% off the predicted values (8).

In 1950 several studies were published on bubble mechanics. Verschoor unsuccessfully attempted to establish relationships using the groups obtained by Kaissling. Van Drevelen and Hoftijzer verified the work done by O'Brien and Gosline. Davies and Taylor were the first to use photographic methods to study bubble behaviour. They used it to measure the terminal velocity of bubbles. Rosenberg grouped bubbles into three types similar to those presented by Byrn. He added to Byrn's groups by assigning ranges of Reynolds number to each (1).

In 1951 Rohsenow presented the first of several papers by him on bubble mechanics. In this paper he used data obtained from 0.024 inch platinum wire in distilled water to establish the relationship between several dimensionless numbers. The equation he derived is known as the "Rohsenow correlation."

His relationship was found to agree remarkably well with data for other surface-fluid combinations (7).

Forster and Zuber assumed the movement of the bubble boundary to be of prime importance. Proceeding from this assumption, they obtained an equation known as the "Forster-Zuber correlation." Their results agreed principally with that obtained by Rohsenow. Data obtained by Cichelli, Bonilla and Kazakova agreed well with their equation also (7).

In 1953 Peelbes and Garber presented an extensive study which attempted to verify the work done by earlier investigators. The experimentation consisted of the determination of the steady state velocity of air bubbles in twenty-two liquids. The liquids were chosen so that the variation of liquid density, viscosity, and surface tension were evident. Four distinct regions of bubble motion were noted, each of which could be referred to previous experimental work (8).

Corty and Foust did experimental work in 1955. They found that, if a surface was kept clear of bubbles for a period of time, it was possible to retain free convection at much higher heat transfers than were normally necessary to produce nucleate boiling. This phenomena is known as "aging" or "hysteresis" (7).

With the advent of space travel in the sixties, the volume of work done on bubble mechanics took a sharp increase. The effects of surface roughness on nulceate boiling were studied by Avery. He used boiling mercury in his experiments and found that the nucleate boiling heat transfer was greatly increased by increasing the surface roughness (7).

C. M. Usiskin and R. Siegel made one of the earliest studies of bubble mechanics in reduced gravity fields. They used a pool boiling apparatus which could be dropped a distance of nine feet to achieve reduced gravity. The bubble departure diameters were found to increase as gravity was decreased (31).

J. A. Merte and H. Clark were interested in the effects of increased gravity on pool boiling. By use of a centrifugal motion apparatus they produced system accelerations of from one to 21 times normal gravity. Their results showed that much larger heat fluxes could be obtained at a given surface temperature with increased acceleration. No attempt was made to measure bubble velocity or size (27).

In 1964 R. Siegel and E. G. Keshick completed another study on nucleate boiling bubble mechanics at reduced gravity. In this experiment a 12.5 foot drop tower was used to vary the gravitational field between 1.4 and 100% of earth gravity. In this experiment the departure diameters were found to increase as gravity to the minus 2/7 power. The rise velocity of the bubbles was found to decrease greatly with decreasing gravitational field, due to decreased bouyancy. At very small gravitational fields the bubbles tended to remain close to the surface and act as reservoirs into which newly formed bubbles would pump. This greatly increased the bubble frequency (6).

Recently, F. Numachi did an experimental study on accelerated cavitation induced by ultrasonics. He created clouds of cavitation bubbles on a vibrating cylindrical vessel. These he studied by means of a high speed camera. He found that when a certain frequency was attained, bubbles were generated in a vertical direction. The bubbles were spheroidal in form. The cloud formation could be prevented if the wall thickness was sufficient (16).

Another recent paper was done by T. H. K. Frederking and D. J. Daniels. They studied the kinematics of vapor removal from a sphere during film boiling. A heated glass sphere was submerged in liquid nitrogen. The dynamics of the bubbles formed was studied by means of a high-speed camera. They found that the vapor was removed by gravity regardless of the magnitude of the heat flux. They also established a relationship between bubble departure diameter and frequency of removal (22).

Some of the most recent work with bubble mechanics has been done by C. G. Fritz. His first paper presented a dimensional analysis approach to the study of bubble size and velocity (1). His second papr used an experimental investigation to determine relationships between different dimensionless groups (2). His most recent paper will be published in the coming year in <u>Advances in Cryogenic Engineering</u>. It presents a dimensional analysis of bubble motion in liquid nitrogen. The analysis is followed by experimental data which gives trends of various force ratios (3).

CHAPTER 2

ANALYTICAL PROCEDURE

The field of bubble mechanics is very large and contains almost unlimited possibilities for investigation and experimentation. A paper of this scope, therefore, must be limited to specific areas in the general field.

This paper deals with two aspects of bubble mechanics which are of importance to the space program. The first aspect is the prediction of bubble size and velocity in a fluid under different conditions or environments. The second aspect is the prediction of the rate of inception or nucleation of bubbles under different environments. This aspect is especially vital, since an understanding of the causation of bubbles would allow the more effective use of preventive measures to minimize the engineering problems which they create.

Special consideration is given in this paper to conditions which would be encountered in space vehicles. For this reason the containers are considered to be cylindrical in order to simulate propellant tanks or pipe lines. Also, the fluid is analyzed under both static and dynamic conditions.

One of the most important conditions to be considered with respect to space vehicles is that of weightlessness or reduced gravity. In an orbiting satellite or in a space vehicle which is distant from planetary bodies, the gravity will approzch zero. In a slightly accelerating system in space or on a body such as the moon, the gravity field will be greatly reduced. Since heat transfer processes such as boiling and convection are gravity dependent, they would be expected tovary with reduced gravity. Prime consideration is, therefore, given to the effects of reduced gravity.

The mechanisms of bubble mechanics are so complicated that purely analytical approaches have generally failed to produce significant results. In attempting to simplify the problems into more manageable forms, many investigators have assumed the existence of either ideal fluids or ideal environments. Since their results are contrary to actual experience with all real fluids, they appear to be of academic use only. When the analytical approach fails to yield the solution to a problem, the method of dimensional analysis is often an effective alternative.

Dimensional analysis has proved to be a very useful tool, especially in the field of fluid mechanics. In an equation which expresses a physical relationship, absolute numerical and dimensional equality must exist. In general, all physical relationships can be reduced to the fundamental quantities of force (F), mass (M), length (L), time (t), and heat (Q).

There are three steps in solving a problem by the use of dimensional analysis:

1.) The parameters which will affect the problem must be determined. This is often the most crucial step in the analysis. These parameters must be determined from experiments, by analogy, or from a basic understanding of the processes involved. If all of the parameters are not included, the solution will be faulty. If too many are included, the solution will at least be complicated and unwieldy.

2.) After the parameters are chosen, the relations between the dimensions are established in expressing a physical law. The most common method used for this step is the Buckingham Pi Method. In this paper the ARDA Method discussed in reference 9 is used. This method introduces new procedures which make possible simpler, yet more comprehensive analysis.

3.) Finally, the coefficients and exponents in the relationship must be determined. This is done by empirical methods. Since no appropriate experimental apparatus was available for this problem, the third step had to be foregone. Instead, the paper attempts to correlate previous experiments into regions or domains of applicability. This is done for both the size-velocity analysis and the nucleation rate analysis.

CHAPTER 3

DIMENSIONAL ANALYSIS OF BUBBLE MOTION

The first step in making the dimensional analysis of bubble motion was to choose the parameters which would have a significant effect on the problem. It has been assumed that the liquid is contained in a cylindrical vessel of variable size. The length (L) and the diameter (D_c) of the container should, therefore, be significant parameters. It has also been assumed that the vessel will be subject to variable gravitational field. The gravity (g) acting on the system is consequently included.

Two parameters which have been found important in fluid and bubble mechanics are surface tension (T) and viscosity (u_f). The viscosity of a fluid is a property which measures its resistance to shear stress, and has direct effect on most problems which deal with the flow of real fluids. Surface tension is the force which keeps a bubble from collapsing.

The pressure (P) at the fluid-cylinder interface would probably affect the problem, as would the density (p) of the fluid involved. The amount of heat flux (Q) passing through the cylinder wall would affect the bubble size and velocity also. This is especially evident at high heat fluxes, as the bubbles depart from the surface with large areas of contact.

Two factors which are important in problems dealing with heat transfer, such as boiling, are thermal conductivity (k) and specific heat (c_p). Thermal conductivity is the measure of a material's ability, in this case that of the liquid, to resist the transfer of heat. Specific heat is the ability of a material to store heat.

Since it has been assumed that the fluid could be in motion within the container, as in the case of a pipeline, the relative velocity of the fluid with respect to the vessl wall (V_L) must be considered to be a factor. The effect of the liquid velocity at reduced gravity would be particularly evident. Under reduced gravitational fields the bubbles are removed very rapidly from the surface at reduced size by previously detached bubbles that momentarily remain close to the surface, due to their decreased bouyancy. This effect is illustrated in Fig. 1. If the fluid were in motion, this

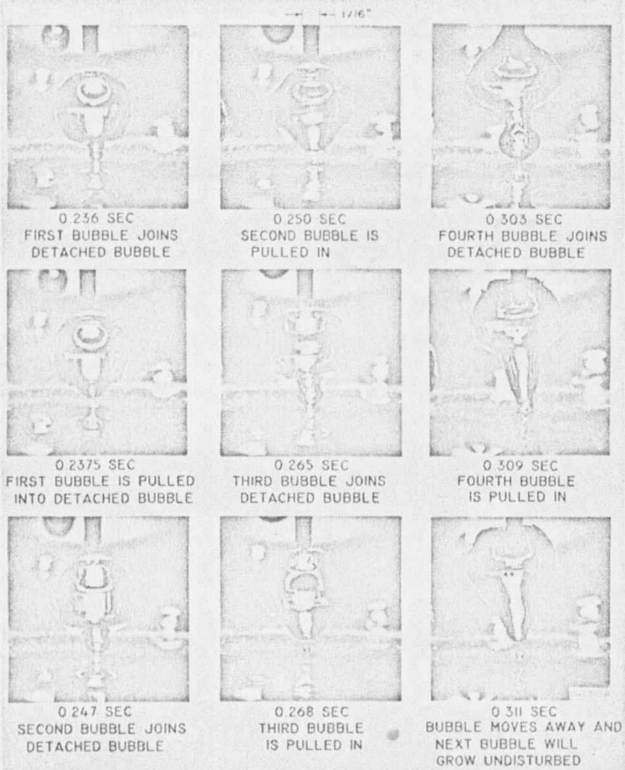

Fig. 1. The growth of a bubble at reduced gravity ($0.061g_n$). It shows the merging of successive bubbles with undisturbed bubble (6).

reservoir for the bubbles would be removed. This velocity is taken as an average velocity of the fluid, realizing that the velocity varies across the cross-sectional area.

Until a certain temperature difference is reached between the cylinder wall and the liquid, the heat is transferred completely by convection. When this difference becomes sufficient the liquid will begin to vaporize at the surface, creating bubbles. This difference ($\Delta\theta$) must, therefore, be included in the analysis.

Whenever and analysis contains both force and mass units, as does this particular problem, a conversion factor (g_c) must be inserted. Also, since both seconds and hours are used as units for time in the various parameters, a conversion factor of 3600 sec/hr is inserted.

The above mentioned parameters were chosen as the most influential factors present in this particular analysis. Their relation to the bubble diameter (D_b) and the velocity (V_b) was then determined by the ARDA method.

Any of the above parameters can be considered to be a function of the others in determining the relationship involved. For convenience, surface tension was chosen. The following general equation was then set up:

$$T = \text{fcn}\ (p, D_b, V_b, g, g_c, P, L, \mu_f, Q, \Delta\theta, V_L, D_c, c_p, k, 3600)$$

or

$$T = C\ (p)^a (D_b)^b (V_b)^c (g)^d (g_c)^e (P)^g (L)^h (\mu_f)^f$$
$$(Q)^i (\Delta\theta)^j (V_L)^k (D_c)^m (c_p)^n (k)^r (3600)^s$$

where C is an arbitrary constant and the exponents a, b, c, ... are arbitrary.

Inserting proper dimensions gave the following:

$$T \frac{lbf}{ft} = C \left(\rho \frac{lbm}{ft^3}\right)^a (D_b ft)^b \left(V_b \frac{ft}{sec}\right)^c \left(g \frac{ft}{sec^2}\right)^d \left(g_c \frac{lbm}{lbf} \frac{ft}{sec^2}\right)^e$$

$$\left(\mu_f \frac{lbf-sec}{ft^2}\right)^f \left(P \frac{lbf}{ft^2}\right)^g (L\ ft)^h \left(Q \frac{Btu}{hr}\right)^i (\Delta\theta\,^oR)^j$$

$$\left(V_L \frac{ft}{sec}\right)^k (D_c ft)^m \left(c_p \frac{Btu}{lbm-{}^oR}\right)^n \left(k \frac{Btu}{hr-ft-{}^oR}\right)^r \left(3600 \frac{sec}{hr}\right)^s$$

Two laws were then used to solve for the relationships involved:

1.) When multiplying values taken to a power, the exponents are added.

2.) Dimensional equality must exist in the equation.

Using these laws, six independent equations were obtained, one for each individual dimension present:

<u>for feet</u>

$-1 = -3a+b+c+d+e-2f-2g+h+k+m-r$

<u>for seconds</u>

$0 = -c-2d-2e+f-k+s$

<u>for lbm</u>

$0 = a+e-n$

<u>for lbf</u>

$1 = -e+f+g$

<u>for Btu</u>

$0 = n+r+i$

<u>for hours</u>

$0 = -r-s-i$

<u>for</u> $\underline{^{o}R}$

$$0 = -n-r+j$$

Simultaneous solutions of these equations gave the following:

$$r = j-n$$

$$s = n$$

$$i = -i$$

$$e = f+g-1$$

$$a = -f-g+1+n$$

$$c = -2d-2g-f-k+2+n$$

$$b = -f+n+d+1-h-m+j$$

Substitution of these sums for their respective exponents in the original equation gave:

$$T = C\,(p)^{-f-g+1+n}\,(D_b)^{-f+n+d+1-h-m+j}\,(V_b)^{-2d-2g-f-k+2+n}\,(g)^{d}$$
$$(g_c)^{f+g-1}\,(\;f)^{f}\,(P)^{g}\,(L)^{h}\,(Q)^{-j}\,(\;0)^{j}\,(V_L)^{k}\,(D_c)^{m}\,(c_p)^{n}$$
$$(k)^{j-n}\,(3600)^{n}$$

Terms of like exponents were then grouped:

$$\frac{g_c T}{\qquad} = C\left(\frac{gD_b}{V_b^2}\right)^{d}\left(\frac{\mu f g_c}{pV_bD_b}\right)^{f}\left(\frac{Pg_c}{pV_b^2}\right)^{g}\left(\frac{L}{D_b}\right)^{h}\left(\frac{k\Delta\theta}{QD_b}\right)^{j}\left(\frac{V_L}{V_b}\right)^{k}$$
$$\left(\frac{D_c}{D_b}\right)^{m}\left(\frac{3600pV_bD_bc_p}{k}\right)^{n}$$

The groups in parenthesis can be recognized as forms of familiar dimensionless numbers in most cases:

$$\frac{g_c T}{pV_b^2 D_b} = \frac{1}{\text{Weber Number (We)}}$$

$$\frac{gD_b}{V_b^2} = \frac{1}{\text{Froude Number (Fr)}}$$

$$\frac{\mu f g_c}{pV_b D_b} = \frac{1}{\text{Reynolds Number (Re)}}$$

$$\frac{Pg_c}{pV^2_b} = \text{Drag Coefficient } (C_D)$$

$$\frac{K\Delta\Theta}{QD_b} = \frac{1}{\text{Nusselt Number for Bubbles (Nub)}}$$

$$\frac{3600pV_b D_b cp}{k} = \text{Peclet Number (Pe)}$$

The Peclet number is redundant in that it can be expressed as two more commonly used dimensionless numbers:

$$Pe = (Re)(Pr)$$

For this reason the Peclet number was replaced by the Prandtl number and the Reynolds number. The Reynolds number was then incorporated into the Reynolds number already obtained in the analysis.

Because the exponents are arbitrary, the numbers which appeared inverted from their normal form could be restored to usual by letting the sign of the exponent be reversed. Example : let $f = -f$, so that $(\frac{1}{We})^f = (\frac{1}{We})^{-f} = (We)^f$. It was also assumed that $m = -h$ so that: $(\frac{1}{D_b})^h(\frac{D_c}{D_b})^{-h} = (\frac{L}{D_c})^h$ which is the familiar shape factor.

Finally, both sides of the equation were taken to the 1/d power so as to solve for the Froude number, which contains the parameters most basic to bubble motion. The final general equation obtained was then:

$$Fr = C(We)^d\ (Re)^f\ (C_D)^g\ (Nu_b)^j\ (Pr)^n\ (L/D_c)^h\ (V_L/V_d)^k$$

CHAPTER 4

BUBBLE MECHANICS FLOW REGIONS

In Chapter 3 a general equation was developed which established six dimensionless numbers as describing the motion of bubbles. The relation was:

$$Fr = C(We)^d(Re)^f(C_D)^g(Nu_b)^j(Pr)^n(L/D_c)^h(V_L/V_d)^k$$

For this chapter the general equation was modified to the following form:

$$C = (Fr)^a(We)^d(Re)^f(C_D)^g(Nu_b)^j(Pr)^n(L/D_c)^h(V_L/V_b)^k$$

This was done by assigning the Froude number an arbitrary exponent and reversing the positions of the constant (C) and the Froude number. Both of these operations are possible because of the arbitrary nature of the exponents and the constant. The new equation facilitates easier manipulation in later developments.

The next step in a dimensional analysis is to determine the exponents and constants involved in the relationship by empirical methods. Since no appropriate experimental apparatus was available, the data necessary for this step was taken from the material of preceding investigators.

Depending on the environment or conditions present, different sub-groups of the general equation established will dominate the relationship. That is, certain dimensionless numbers vary so slightly, their contribution to the equation becomes insignificant. Mathematically it can be said that either their exponent approaches zero, or that they become part of the constant in the equation. This is an important consideration when reducing the general equation to relationships established by other investigators.

One of the earliest and most important works on bubble mechanics was done by Stokes. He presented a formula for flow past a rigid sphere. His formula, known as Stoke's Law, is generally accepted to be valid for bubbles when the flow has a Reynolds number of less than two. His formula is as follows: $C_D = 24/Re.$ (1)

This equation fits the general equation obtained in Chapter 3 when all of the exponents approach zero except f and g, both of which equal one. This means that all of the dimensionless numbers except the coefficient of drag and the Reynolds number are relatively insignificant in this region. The constant (C) in the equation is equal to 24. The form of the general equation is then:

$$24 = (Fr)^0 (We)^0 (Re)^1 (C_D)^1 (Nu_b)^0 (Pr)^0 (L/D_c)^0 (V_L/V_b)^0$$

When the Reynolds number becomes much greater than two, the flow around the bubbles becomes more turbulent and there is significant departure from Stoke's Law. Allen's experimental results gave the following equation for Reynolds number bounded between two and 200:

$$C_D = 18.5\ Re^{-.6} \qquad (8)$$

Lapple and Shepherd experimented with particle trajectories and obtained relationships between the coefficient of drag and the Reynolds number for different shapes of particles. These relationships have been shown to satisfactorily predict bubble motion for certain ranges of Reynolds number. Using their results Fritz obtained an equation for Reynolds number between 2 and 720. His equation was as follows:

$$C_D = 18.7/Re^{.68} \qquad (1)$$

Both of the preceding equations can again be seen to fit into the general equation, if proper values for the exponents and constant are inserted as follows:

$$18.5 = (Fr)^0 (We)^0 (Re)^{.6} (C_D)^1 (Nu_b)^0 (Pr)^0 (L/D_c)^0 (V_L/V_b)^0$$

for Allen's equation, and

$$18.7 = (Fr)^0(We)^0(Re)^{.68}(C_D)^1(Nu_b)^0(Pr)^0(L/D_c)^0(V_L/V_b)^0$$

for the Fritz equation.

Kaissling and Rosenberg claimed the following relationship to exist for the region of Reynolds number greater than 400:

$$\frac{D_b V p}{\mu f} = 1.91 \left(\frac{D_b p T g_c}{\mu f}\right)^{\frac{1}{2}} \qquad (1)$$

The group on the left of the equation is obviously the Reynolds number. The group in parenthesis is:

$$\frac{D_b p T g_c}{\mu^2 f} = \frac{D^2{}_b V^2 p^2}{\mu^2 f} \; \frac{T g_c}{V^2 D_b p}$$

$$= (Re)^2 \left(\frac{1}{We}\right)$$

The equation can therefore be reduced to the form:

$$Re = 1.91 \; (Re^2/We)^{\frac{1}{2}}$$

which simplifies to:

$$We^{\frac{1}{2}} = 1.91$$

This again fits the general equation providing all exponents are zero except for d, which equals $\frac{1}{2}$, and C equals 1.91.

Fritz derived another equation which he claimed to be valid for bubble motion in the region bounded by Reynolds numbers of 700 and 1500. This equation was:

$$C_D = 0.0275 \; \frac{g\mu_f{}^4 g_c{}^4}{pT^3 g_c{}^3} \; Re^4 \qquad (1)$$

This can be simplified as follows:

$$C_D = 0.0275 \; \frac{g\mu_f{}^4 g_c{}^4}{pT^3 g_c{}^3} \; \frac{D^4 V^4 p^4}{\mu f^4 g_c{}^4}$$

$$= 0.0275 \; \frac{p^3V^6D^3}{g_c{}^3T^3} \; \frac{gD}{V^2}$$

$$= 0.0275 \; We^3/Fr$$

The form of the general equation in this case is:

$$0.0275 = (Fr)^1(We)^{-3}(Re)^0(C_D)^1(Nu_b)^0(Pr)^0(L/D_c)^0(V_L/V_b)^0$$

Working with a dimensional grouping suggested by Kaissling, Cooper derived the following equation for very large Reynolds number:

$$V = 2\left[\frac{Tgg_c}{P}\right]^{\frac{1}{4}} \qquad (8)$$

This can be written:

$$\frac{1}{2} = \left[\frac{Tgg_c}{PV^4}\right]^{\frac{1}{4}}$$

$$= \left[\frac{Tg_c}{pV^2D}\right]^{\frac{1}{4}}\left[\frac{gD}{V^2}\right]^{\frac{1}{4}}$$

$$= We^{-\frac{1}{4}}Fr^{-\frac{1}{4}}$$

The equation is then equivalent to:

$$WeFr = 16$$

which again is the general equation with proper values assumed for the exponents and constant.

Several of the preceding developments were separated into arbitrary regions and graphed by Fritz. This graph has been reproduced in Figure 2.

It is evident that the two dimensionless numbers in the general equation dealing with heat transfer, the Prandtl number and the Nusselt number, were not present in any of the equations discussed to this point. This is because the experimenters were concerned with the motion of individual bubbles. Great care was often taken to produce single columns of bubbles in order to

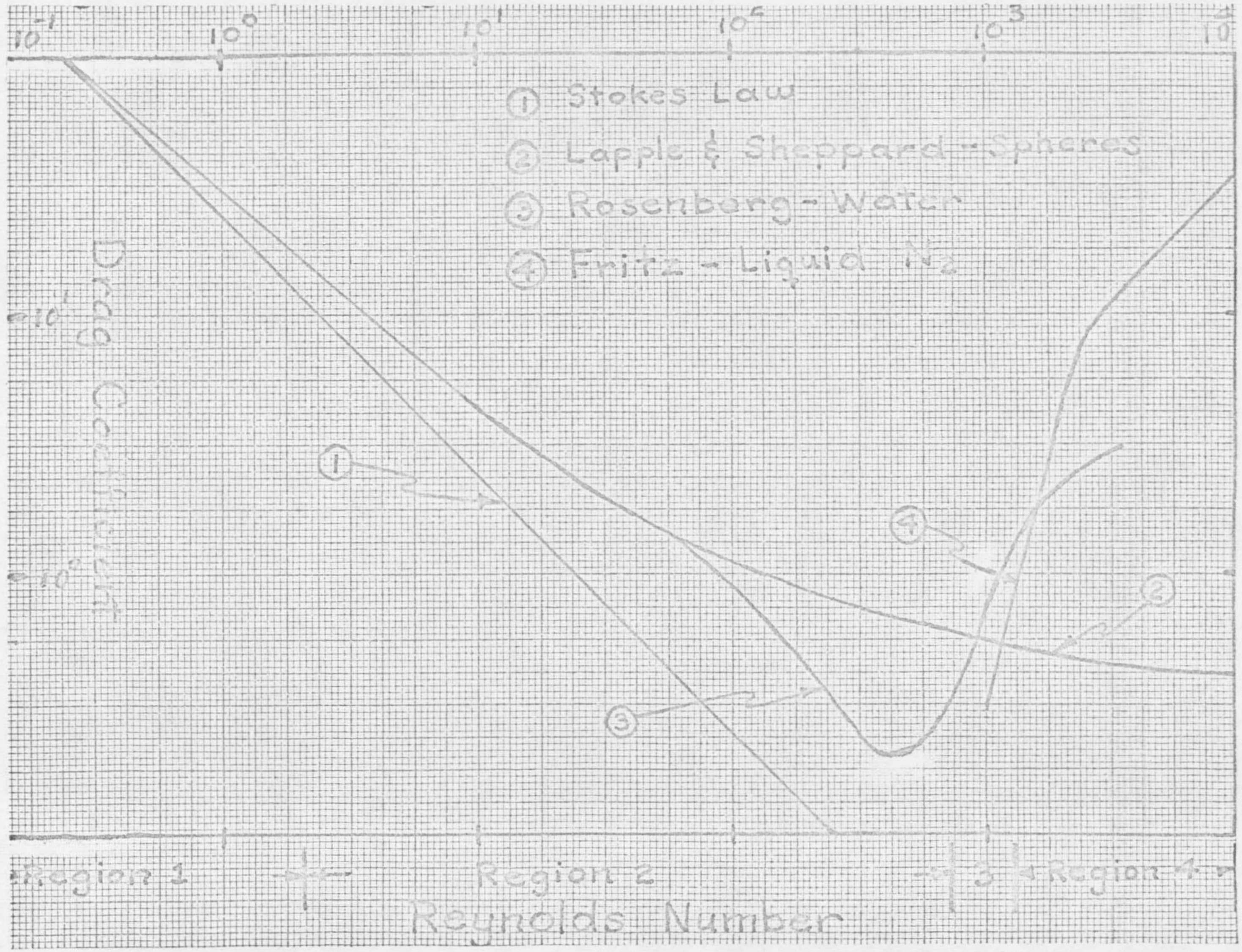

Fig. 2 Flow Regions for Bubbles(1).

facilitate their observation. The actual boiling process was not of concern to them. Heat transfer processes were, therefore, of little consequence also. Two investigators who were concerned with the boiling phenomena were Rohsenow and Forster.

One of the most frequently referred to equations dealing with liquid boiling mechanics is the Rohsenow correlation. Rohsenow assumed that three dimensionless numbers, the Reynolds, Nusselt and Prandtl numbers, would adequately describe the movement of bubbles at the instant of breaking away from a heating surface. His general equation was:

$$Nu_b = \text{fcn}\ (Re, Pr)$$

Empirically he found that

$$\frac{RePr}{Nu_b} = C(Re)^n(Pr)^m \qquad (7)$$

Manipulation of this equation gives the following equivalent:

$$C = (Re)^{1-n}(Pr)^{1-m}(Nu_b)^{-1}$$

This agrees with the general equation of Chapter 3 assuming all of the dimensionless groups except the Reynolds, Prandtl and Nusselt numbers are insignificant for the circumstances of the problem. Rohsenow used 14 different surface-fluid combinations to determine the exponents and constants in his equation. Once the exponents were set, he could achieve remarkably good results by using different values of C for different combinations.

Another well known equation dealing with liquid boiling mechanics is the Forster-Zuber correlation. It is similar to the Rohsenow correlation in that the same dimensionless numbers are used. The Forster-Zuber correlation is as follows:

$$Nu_b = 0.0015\,(Re)^{.62}(Pr) \qquad (7)$$

This again agrees with the general equation if the exponents are assigned as follows:

$$0.0015 = (Fr)^0(We)^0(Re)^{-.62}(C_D)^0(Nu_b)^1(Pr)^{-}\ (L/D_c)^0(V_L/V_b)^0$$

Some of the data verifying the Forster-Zuber correlation is shown in Figure 3.

As was stated earlier in the paper, special emphasis was to be given the condition of reduced gravity fields. It has been shown that bubble departure diameter increases with decreasing gravitational field. This was illustrated by the photographic work done by Usiskin and Seigel (Figure 4). Several authors have proposed equations for predicting the size of bubbles at departure from a horizontal surface in relation to the gravitational field present. The best known equation is probably the Fritz equation:

$$D_b = 0.02080\left[\frac{Tg_c}{gp}\right]^{\frac{1}{2}} \qquad (6)$$

This can be expressed as

$$\frac{1}{(0.02080)^2} = \frac{Tg_c}{gpD_b^{\,2}}$$

$$= \left[\frac{Tg_c}{pV_b^{\,2}D_b}\right]\left[\frac{V_b^{\,2}}{gD_b}\right]$$

or $$(0.02080)^2 = (We)\left(\frac{1}{Fr}\right) = Bo$$

where Bo is the Bond number (Table 2). In this case the general equation of Chapter 3 takes the form:

$$(0.02080)^2 = (Fr)^{-1}(We)^1(Re)^0(C_D)^0(Nu_b)^0(Pr)^0(L/D_c)^0(V_L/V_b)^0$$

Fig. 3 Forster-Zuber Correlation

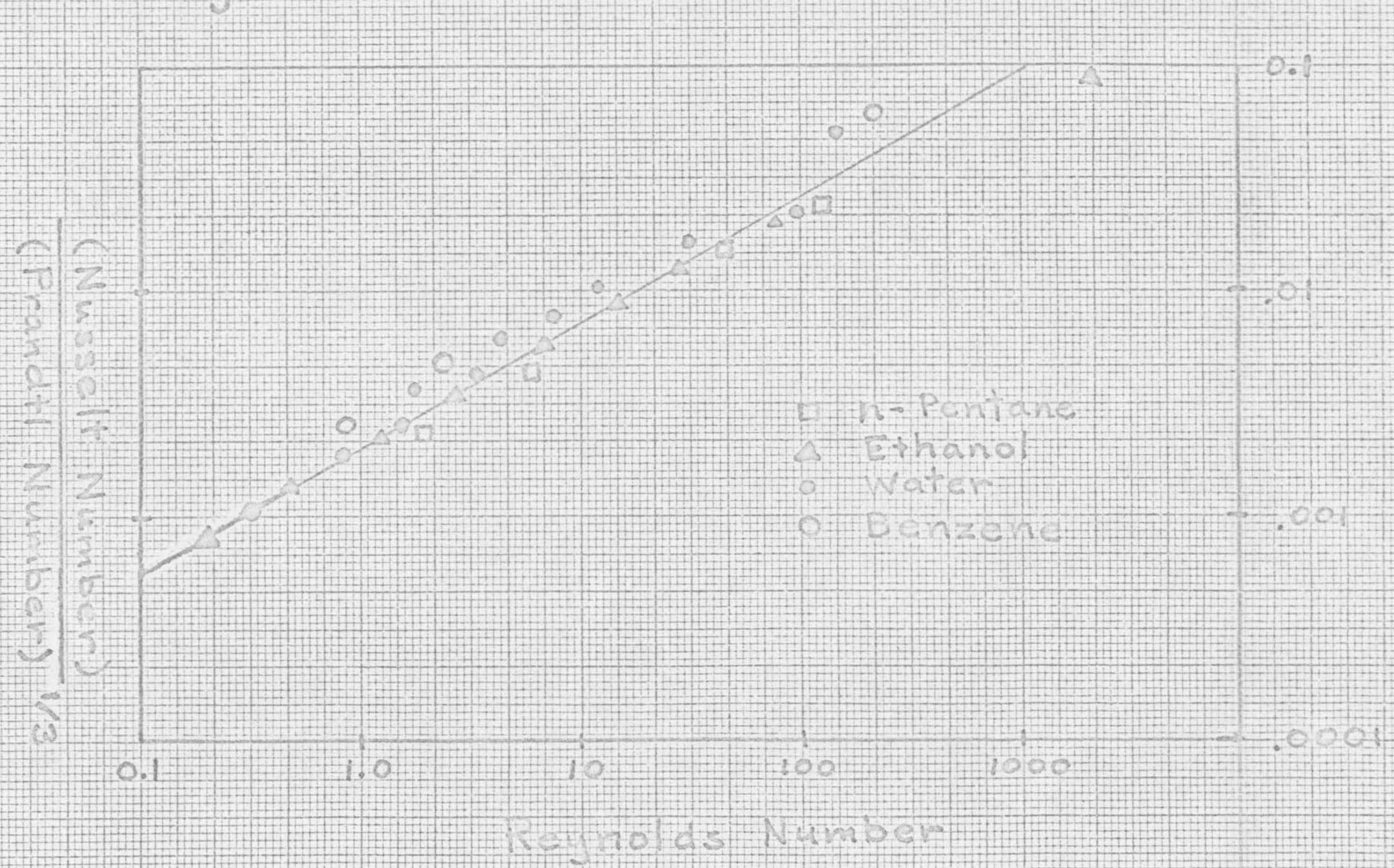

Fig. 4 Photographs by Usiskin and Seigel showing the effects of reduced gravity on bubble departure diameter (31).

Another well known correlation was developed by Zuber:

$$D_b = \frac{6 T g_c k \Delta\theta}{g p Q} \qquad (6)$$

This can be expressed as

$$1/6 = \frac{T g_c k \Delta\theta}{g p Q D_b^3}$$

$$= \left[\frac{k \Delta\theta}{Q D_b}\right]\left[\frac{T g_c}{p V_b^2 D_b}\right]\left[\frac{V_b^2}{g D_b}\right]$$

$$= \left[\frac{1}{Nu_b}\right]\left[\frac{1}{We}\right] Fr$$

As before, this is a form of the general equation.

The final equation examined is the correlation developed by Cole:

$$D_b = 0.040 \; \frac{T g_c}{g p} \; \frac{g T g_c}{V_b^4 p}^{-.22} \qquad (6)$$

This can be rewritten:

$$\frac{1}{(0.040)^2} = \left[\frac{T g_c}{g p D_b^2}\right]\left[\frac{g T g_c}{V_b^4 p}\right]^{-.44}$$

$$= \left[\frac{T g_c}{p V_b^2 D_b}\right]\left[\frac{V^2}{g D_b}\right]\left[\frac{T g_c}{p V_b^2 D_b}\right]^{-.44}\left[\frac{g D_b}{V_b^2}\right]^{-.44}$$

$$= \left[\frac{T g_c}{p V_b^2 D_b}\right]^{.56}\left[\frac{V_b^2}{g D_b}\right]^{1.44}$$

$$= \left(\frac{1}{We}\right)^{.56} Fr^{.44}$$

Once again this is a form of the general equation.

The Fritz equation predicted the departure diameter of the bubbles to increase as gravity to the minus one-half power. The Zuber and Cole equations predicted the departure diameter to increase as gravity to the minus one-third and minus .72 power respectively. Using experiments conducted with a drop tower for reducing the gravitational field, Seigel and Keshock found the departure diameter to increase as gravity to the minus 0.286 power (Figure 5). This agrees closely to the prediction of Zuber.

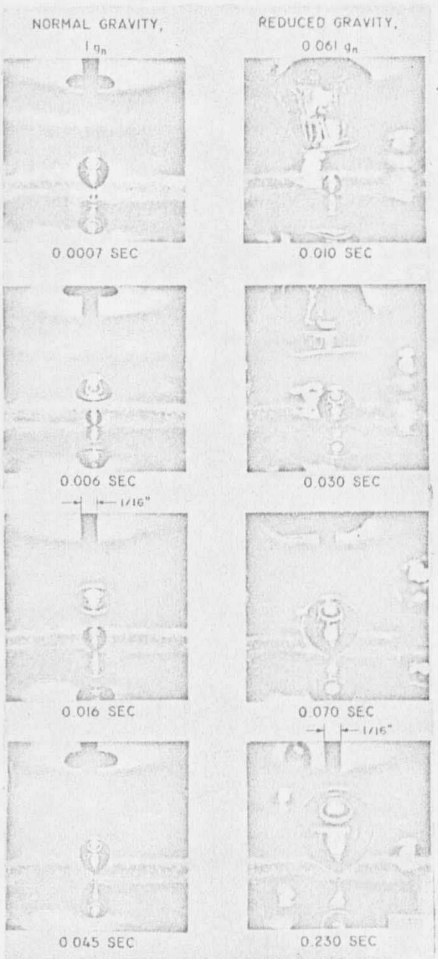

Fig. 5 Photographs used by Seigel and Keshock to determine the relationship between bubble departure diameter and gravity (6).

The most commonly used equations dealing with bubble motion have all been shown to be some form of the general equation of Chapter 3. The work of other investigators thus seems to verify the equation. However, two of the dimensionless groupings in the equation were not present in any of the developments. These were the shape factor (D_c/L) and the velocity ratio (V_L/V_b). Although it is possible that these ratios never become significant enough to affect the bubble motion, there are conditions under which they conceivably could.

Asch (5) stated that "heat-transfer characteristics are independent of liquid velocity once boiling begins..." since the "turbulence produced by bubble formation overshadows any flow turbulence." However, this does not eliminate the possibility of the liquid motion affecting the motion of the bubbles. Very little experimentation has been done for bubble mechanics with the liquid in motion. A special condition mentioned earlier in which the liquid velocity would seem to affect the bubble motion is at reduced gravity. Under this condition the bubbles tend to remain close to the surface due to decreased bouyancy. Subsequent bubbles are then pumped into these bubbles at increased velocity and decreased size. If the liquid was in motion, this effect would presumably be decreased or eliminated as the bubbles would be removed from this position.

The shape factor could influence the bubble motion also. The heat transfer characteristics of a cube or sphere are certainly different than those of a slender tube. Since the heat transfer characteristics have been shown to affect bubble mechanics, as evidenced by the use of Nusselt and Prandtl numbers, the shape factor should be of influence.

CHAPTER 5

BUBBLE INCEPTION

One area of bubble mechanics in which there has been a limited amount of investigation is the inception or nucleation of bubbles. This fact was confirmed by C. G. Fritz (3) in one of his most recent papers on bubble mechanics. Fritz, who has been working on bubble mechanics at the George C. Marshall Space Flight Center, reported, "Additional analytical and experimental studies are required to provide a basic understanding of bubbles in cryogenic fluids.....in the area of inception by a nucleation theory...." The importance of nucleation to the space program was emphasized earlier in this paper. The boiling phenomena has also received additional attention in the past few years because of its importance in nuclear reactors. It is desirable to achieve high rates of heat transfer in these reactors. Because of this boiling is usually present.

Boiling occurs in three regions. Nucleate boiling involves the generation of bubbles on a heated surface (Figure 6.). It is characterized by individually distinguishable bubbles. Film boiling involves high rates of heat transfer from a surface (Figure 7). The liquid vaporizes so quickly and at so many places, that a film or blanket is formed on the surface. The third region is transition boiling, which exists between nucleate and film boiling (Figure 8). This analysis was necessarily limited to nucleate boiling since a rate of inception would have significance only when individually distinguishable bubbles are being formed.

The object of this analysis was to determine a nucleation rate (NR), or the number of bubbles forming per unit area per second, in terms of dimensionless numbers. The liquid was again assumed to be contained in a cylin-

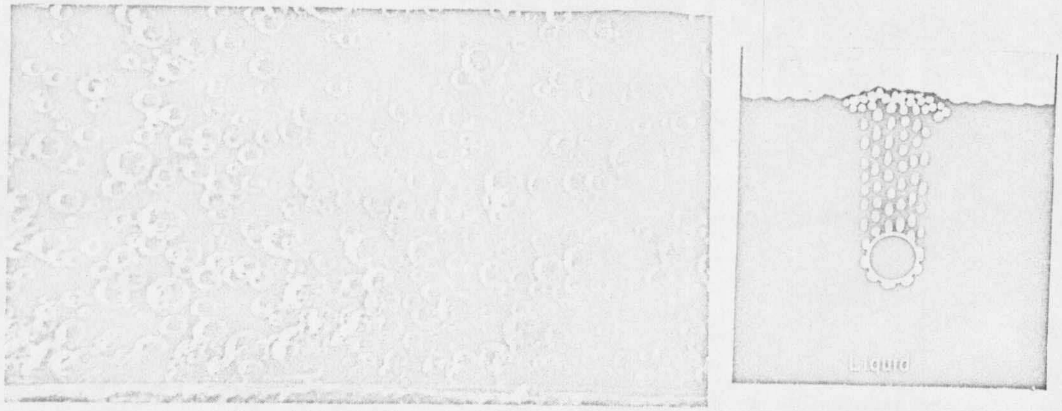

Fig. 6 Nucleate Boiling, bubbles are spherical in form(5).

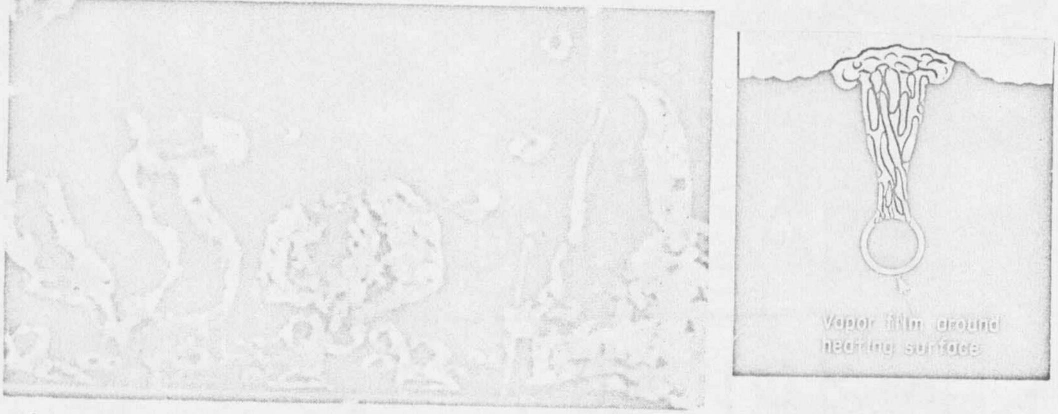

Fig. 7 Film Boiling, a vapor blanket forms on the surface(5).

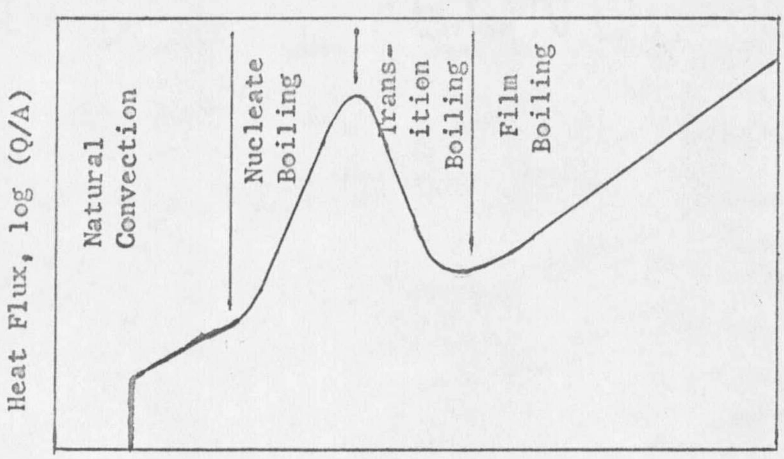

Fig. 8 Boiling Characteristics of a Liquid, log scale.

drical vessel subject to a variable gravitational field.

All of the parameters which were used in the analysis of Chapter 1 were important in this analysis also, for similar reasons. However, several new parameters became significant due to the increased importance of heat transfer and surface conditions.

Thermal conductivity and specific heat were again important since heat transfer and storage were involved. In addition, the latent heat of vaporization (H_{fg}) were included. It measures the amount of heat necessary to vaporize a unit mass of the particular fluid under consideration. Also, the convection coefficient (h_c) was included. It measures the heat transfer at the liquid-cylinder interface due to convection, which is important for boiling processes.

Extreme vibration is caused by the firing of a rocket. Since vibration can cause bubble formation under certain conditions, parameters describing vibration had to be included. A good example of vibration causing bubble formation is Numachi's experiment with ultrasonic impulses (16). The cavitation cloud he produced is shown in Figure 9. Vibration can be adequately

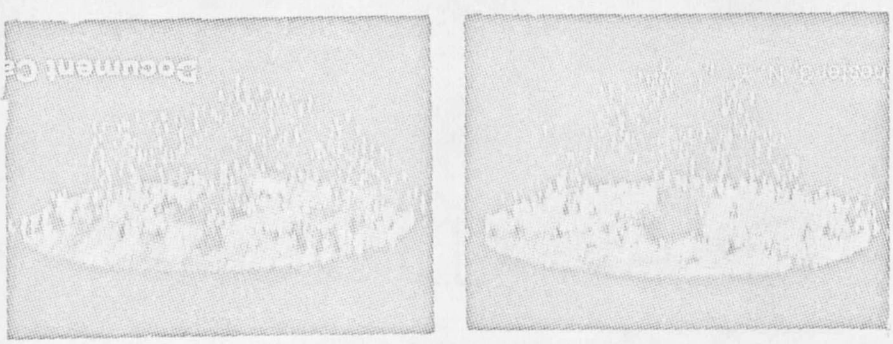

Fig. 9 High-speed photos of cavitation
cloud formed over vibrating vessel.

represented by an amplitude of vibration (Ym) and a frequency of vibration (f).

Any surface contains irregularities where small quantities of trapped fluid act as nuclei for the buildup of bubbles. Therefore, the root-mean-square height of roughness (e) was included to account for this effect.

Because factors were included with dimensions of both seconds and hours, a conversion factor of 3600 seconds per hour was again needed.

The following general equation was set up:

$$NR \frac{1}{\text{sec-ft}^2} = C\left(\rho\frac{\text{lbm}}{\text{ft}^3}\right)^a \left(V\frac{\text{ft}}{\text{sec}}\right)^b (D\ \text{ft})^c (\Delta\theta\,{}^{o}R)^d \left(H_{fg}\frac{\text{Btu}}{\text{lbm}}\right)^e$$

$$\left(g_c\frac{\text{lbm}}{\text{lbF}}\frac{\text{ft}}{\text{sec}^2}\right)^f \left(\mu_f\frac{\text{lbf-sec}}{\text{ft}^2}\right)^g \left(k\frac{\text{Btu}}{\text{hrft}\,{}^{o}R}\right)^h \left(g\frac{\text{ft}}{\text{sec}^2}\right)^k$$

$$\left(P\frac{\text{lbf}}{\text{ft}^2}\right)^m \left(c_p\frac{\text{Btu}}{\text{lbm-}{}^{o}R}\right)^n \left(T\frac{\text{lbf}}{\text{ft}}\right)^p \left(Q\frac{\text{Btu}}{\text{ft}^2\text{-hr}}\right)^r (L\text{ft})^s$$

$$\left(h_c\frac{\text{Btu}}{\text{hr-ft-}{}^{o}R}\right)^v \left(3600\frac{\text{sec}}{\text{hr}}\right)^w (e\ \text{ft})^x (Y_m\ \text{ft})^y \left(f\frac{1}{\text{sec}}\right)^z$$

Equating exponents in the equation gave the following seven independent equations:

<u>for</u> <u>feet</u>
$-2 = -3a+b+c+f-2g-h+k-2m-p-2r+s-2v+x+y$

<u>for</u> <u>seconds</u>
$-1 = -b-2f+g-2k+w-z$

<u>for</u> <u>lbm</u>
$0 = a-e+f-n$

<u>for</u> <u>lbf</u>
$0 = -f+g+m+p$

<u>for</u> <u>Btu</u>
$0 = e+h+n+r+v$

<u>for</u> <u>hours</u>
$0 = -h-r-v-w$

$$\frac{\text{for}}{0 =}\ \frac{{}^{o}R}{d-h-n-v}$$

Solving these equations:

$h = -r-v-w$

$n = w-e$

$d = -r-e$

$f = g+m+p$

$a = w-g-m-p$

$b = 1-g-2m-2p-2k+w+z$

$c = -3+w+x-g-p+k+r+v-s-x-y+z$

Substituting and grouping terms of like exponent:

$$\frac{NR\ (D^3)}{V} = C\left(\frac{Hfg}{C_p\Delta\Theta}\right)^e \left(\frac{\mu f g_c}{pVD}\right)^g \left(\frac{gD}{V^2}\right)^k \left(\frac{Pg_c}{pV^2}\right)^m \left(\frac{Tg_c}{V^2pD}\right)^p \left(\frac{QD}{k\Delta\Theta}\right)^r \left(\frac{L}{D}\right)^s \left(\frac{h_cD}{K}\right)^v \left(\frac{3600c_ppVD}{K}\right)^w \left(\frac{e}{D}\right)^x \left(\frac{Y_m}{D}\right)^y \left(\frac{fD}{V}\right)^z$$

The subscripts in this relationship have been neglected to this point. This was done purposefully so that meaningful values could be grouped in each particular dimensionless number. In an analysis of this size it is impossible to predesignate all of the parameters and still obtain a sensible result. Waiting until the analysis has been completed to assign the parameters is a valid operation, since the dimensional analysis only requires dimensional equality. It does not indicate what each particular dimension represents.

Assigning appropriate subscripts gave the following dimensionless numbers where v = vapor, L = liquid, b = bubble, w = wall and c = cylinder.

$$\frac{V_bD_bp_v}{\mu_f g_c} = \text{Reynolds number for a bubble } (Re)_b$$

$\frac{V_b^{\,2}}{gD_b}$ = Froude number for a bubble $(Fr)_b$

$\frac{g_c P}{p_v V_b^{\,2}}$ = Drag Coefficient (C_D)

$\frac{Tg_c}{p_v V_b^{\,2} D_b}$ = Weber number (We)

$\frac{D_b Q}{k_L(\theta_w - \theta_b)}$ = Nussult number for a bubble $(Nu)_b$

$\frac{h_c D_c}{k_L}$ = Nussult number for the liquid $(Nu)_L$

$\frac{3600 c_p P V_L D_c}{k_L}$ = Peclet number for liquid (Pe)

$\frac{L}{D_c}$ = shape factor for the cylinder

$\frac{fD}{V}$ = Strouhal number (St)

Assuming a density ratio is present in the $(\frac{H_{fg}}{c_p\theta})$ term gives:

$\frac{c_p P_l(\theta_w - \theta_b)}{H_{fg} P_v}$ = Jakob number (Ja)

The Peclet number is redundant in that it can be expressed as the product of 2 more common dimensionless numbers:

$$Pe = Re_L Pr$$

where: $Pr = \frac{3600 \mu_f c_p g_c}{K}$

and $Re_L = \frac{V\ D\ P}{\mu_f g_c}$

The final solution obtained was:

$$\frac{NR\ (D_b)^3}{V_b} = C(Ja)^e (Re)_b^{\,g} (Fr)^k (C_D)^m (We)^p (Nu)_b^{\,r} (Nu)_L^{\,v} \left(\frac{L}{D_c}\right)^s (Pr)^w$$

$$(Re)_L^{\,v} (St)^x \left(\frac{Y_m}{D_c}\right)^y \left(\frac{e}{D_c}\right)^z$$

CHAPTER 6

GENERAL BUBBLE NUCLEATION DOMAIN

In Chapter 5 a general formula was derived by dimensional analysis for the nucleation rate of bubbles under different environments. The equation was as follows:

$$\frac{NR\ (D_b\ )}{V_b} = C(Ja)^e(Re)_b{}^g(Fr)^k(C_D)^m(We)^p(Nu)_b{}^r(Nu)_L{}^v(\frac{L}{D_c})^s(Pr)^w (Re)_L{}^v(St)^x(\frac{Y_m}{D_c})^y(\frac{e}{D_c})^z$$

It would have been desirable to perform experiments using liquids with a wide variation of physical properties under a wide range of physical environments to determine specific relationships. Since no adequate facilities were available for this, an alternate method would be to use the results of other investigators for developing specific relationships, as was done for the bubble mechanics flow regions. However, there has been very little work done on bubble nucleation.

McFadden and Grassmann developed the following equation for bubble nucleation by semi-empirical methods:

$$f_b\ (D_b)^{\frac{1}{2}} = 0.56\left[\frac{g(p_L-p_V)}{p_L}\right]^{\frac{1}{2}} \qquad (22)$$

where, f = bubble frequency (bubbles/sec.)

$$= NR\ (D_b)^2$$

Assuming that the density of the liquid (p_L) is much greater than the density of the vapor (p_v), so that $(p_v-p_L)/p_L$ is approximately equal to one, the relation reduces to the following:

$$NR.D_b{}^2\ D_b{}^{\frac{1}{2}} = 0.56\ (g)^{\frac{1}{2}}$$

Therefore,

$$(NR)^2\ D_b{}^5 = (0.56)g$$

$$\frac{(NR)^2 D_b^{\,6}}{V_b^{\,2}} = (0.56)^2 \left(\frac{gD_b}{V_b^{\,2}}\right)$$

$$\frac{NR\ (D^3{}_b)}{V_b} = 0.56\ (Fr)^{\frac{1}{2}}$$

This is equivalent to the general equation when all of the exponents on the right side of the equation are zero except k, which equals one-half, and the constant C equals 0.56.

Jakob developed the only other available equation for bubble nucleation, which is:

$$D_b f_b = 0.59 \quad Tg_c g\left[\frac{(p_L - p_V)}{p_L^{\,2}}\right]^{\frac{1}{4}} \qquad (7)$$

Again using $f = NR\ (D_b)^2$ and assuming $(p_L - p_V)/p_L = 1$, gives

$$D_b NR D_b^{\,2} = 0.59\left[\frac{Tg_c g}{p}\right]^{\frac{1}{4}} \quad \text{or,}$$

$$\left(\frac{1}{.59}\right)^4 = \frac{Tg_c g}{pD_b^{\,2}(NR)^4}$$

$$= \left[\frac{V_b^{\,4}}{(NR)^4 D_b^{\,2}}\right]\left[\frac{Tg_c g}{pV_b^{\,4}}\right]$$

$$= \left[\frac{V_b}{(NR) D_b^{\,3}}\right]\left[\frac{Tg_c}{V_b^{\,2} p D_b}\right]\left[\frac{gD_b}{V_b^{\,2}}\right]$$

Therefore,

$$\frac{NR(D_b)^3}{V_b} = 0.59\ (We)^{-\frac{1}{4}}(Fr)^{-\frac{1}{4}}$$

This again is a form of the general equation derived in Chapter 5 when all of the exponents approach zero except k and p, both of which equal minus one-fourth. The constant in this case is equal to 0.56.

It is obvious that these two equations alone would be very inadequate for describing all of the phenomena which could be involved. Additional empirical evidence is needed. However, without further experimentation some qualitative predictions can be made.

Several of the groups in the general equation would be of significance only under special circumstances. The dimensionless groups describing vibration, (Y_m/D_c) and the Strouhal number, would affect the problem only when sufficient vibration was present. Fritz, Ponder and Blount (32) were able to create bubble formation under random vibration of from 20 to 2000 cycles per second. Another investigator found the heat transfer across the interface to increase significantly in a similar range of vibration (6). Since this range closely simulates the vibration that occurs during the firing of large rocket engines, the groups describing vibration should certainly be included when these circumstances are of interest. However, under normal conditions they would be omitted.

The Reynolds number for the liquid (Re_L) would have an effect on the equation only when liquid flow was present. If the liquid was stationary, as in pool boiling, or if the liquid velocity was negligible, the liquid Reynolds number would be omitted. Experimental data would be required to determine the point at which the liquid velocity became of sufficient magnitude to be an influence.

The shape factor (L/D_c) would probably affect the problem only when extreme shapes, such as slender tubing were involved. Omitting the preceding dimensionless groups would reduce the general equation to the following somewhat simpler form:

$$\frac{NR(D_b)^3}{V_b} = C(Ja)^e(Re)_b{}^g(Fr)^k(C_D)^m(We)^p(Nu)_b{}^r(Nu)_L{}^v(Pr)^w(e/D_c)^z$$

This would then be the form of the equation for normal pool boiling with the other dimensionless groups being absorbed into the constant.

Some of the other dimensionless groups would undoubtedly become insignificant under certain circumstances. However, it is impossible to predict which groups dominate the equation in what regions without experimental evidence. All of the remaining groups contain parameters which have been shown to affect the nucleation of bubbles.

A decrease of the pressure at the interface has been shown to decrease the temperature of the surface needed to begin boiling (15). Since the coefficient of drag (C_D) is the only dimensionless number which contains the pressure parameter, it must be included.

The Nusselt number for bubbles (Nu_b) alone contains the heat flux parameter (Q). Borishanski (27) indicated that the nucleation rate increased as the heat flux squared, other parameters being held constant. Others have indicated a direct proportion. Since an increase of heat flux increases the nucleation rate, this dimensionless number must be included.

The Jakob number (Ja) is the only dimensionless number containing the latent heat of vaporization (H_{fg}). The greater the latent heat of vaporization of a fluid, the greater the heat flux required to produce nucleation. The Jakob number must be included.

The parameter of gravitational field (g) is contained solely by the Froude number (Fr). Since it has been assumed that the problem is subject to a variable gravitational field, this dimensionless number is also included.

Pockmarks, crevices and irregularities in the surface act as gas traps where nucleation can be initiated. Several authors have observed the effect of the degree of roughness of the container's surface on the nucleation rate (7). The number of nucleation sites increases with increasing surface roughness. This degree of roughness is usually measured by the root-mean-square height of roughness. The ratio containing this parameter (e/D_c) will, therefore, be of significance.

The Reynolds number for bubbles (Re_b) contains the viscosity parameter. The velocity of bubbles has been found to decrease with increasing viscosity of a fluid (23). Since the frequency of bubble removal and, consequently, the nucleation rate vary with the bubble velocity, the Reynolds number for bubbles should influence the phenomena.

Surface tension has been found to influence the nucleation rate of bubbles (7). The departure diameter of bubbles varies directly with the surface tension of the liquid. Since the product of departure diameter and bubble frequency is relatively constant, bubble frequency and nucleation rate will decrease with increasing surface tension. The Weber number (We), as the only dimensionless group containing viscosity, is therefore included.

Even at high rates of heat flux a large percentage of the heat transfer at the interface is by convection. The greater the heat transport by convection the less heat is available for bubble creation. The Nusselt number for liquids alone contains this parameter. Because of this it must also be included.

The only remaining dimensionless number in the general equation is the Prandtl number. It is included since it alone contains the specific heat of the fluid. Although no experimental evidence was available to establish a dependence of bubble nucleation on the liquid specific heat, it is assumed that it exists. The phenomena of bubble nucleation is highly dependent on heat transfer so that an important heat transfer parameter, such as specific heat, should affect it.

CHAPTER 7

LIMITATIONS AND CONCLUSIONS

In this paper dimensional analysis was used to derive equations for the motion and inception of bubbles. These equations contained a large number of parameters so that they would cover a wide variety of possible conditions. However, it must be realized that under certain situations there are other factors which will affect bubble motion and inception. The general equations are thus limited by the assumption that these factors will be taken into consideration if they are of significant influence.

One important factor which was not accounted for in either of the general equations but which has been found to exert significant influence, is the orientation of the heating surface. In this paper the heating surface was assumed to be cylindrical and in the normal horizontal position. Departure from the horizontal position will produce considerable changes in the boiling heat transfer. Githinji and Sabersky (24) found that for a given surface temperature the heat transfer characteristics degenerated as the surface was rotated from a perpendicular orientation to the gravitational field. Bernath (7) achieved similar results. This was caused by the bubbles from the lower sections of the surface rising and passing over the upper sections.

Another factor which has been shown to affect boiling, but could not be accounted for in the equations, is the aging or hysteresis effect. Corty and Foust (7) reported that after a surface had been kept free of bubbles for 10 to 15 minutes, it was possible to retain free-convection heat transfer at higher heat fluxes than would normally be required to produce boiling. Wall temperatures of from 40 to 50 degrees fahrenheit were obtained with no bubble formation, where nucleate boiling normally occurred at 25 degrees fahrenheit.

Further increases of heat flux produced spontaneous vigorous nucleate boiling. This effect is somewhat analogous to the supersaturation of solutions.

Mead (7) found the type of surface material to affect boiling. This effect could be due entirely to the difference in surface roughness of various materials. If so, the root-mean-square height of roughness parameter in the bubble inception equation would account for the phenomena. However, it would not account for the variation if it was due to a basic difference in molecular structure.

Costello and Frea (7) found the boiling heat transfer to increase with time for extended periods of boiling. They theorized that this was due to a build up of a surface coating from deposits. This factor could then again be accounted for by the surface roughness parameter.

Another phenomena which is probably related to the surface roughness is the effect of "cleaning." Owens (25) found that surfaces cleaned with carbon tetrachloride and acetone gave higher boiling heat transfer in stainless steel tubes.

The addition of wetting agents to the liquid have been found to increase the boiling heat transfer rate and the bubble frequency. The addition of oleic acid increased the bubble frequency by a factor of three in one experiment (5). The wetting ability of a liquid is a measure of how well it clings to a surface. It is closely associated with surface tension, in that high wetting ability is synonymous with low surface tension. This phenomena could, therefore, be accounted for in the surface tension parameter which is included in both general equations.

It can be seen from the large number of parameters present in the two derivations, and from the above limitations, that bubble motion and inception are very involved processes. Attempts to create simplified expressions are, therefore, subject to error unless the limits of their applicability are specified. By examining the equations of other investigators presented in this paper, it can also be seen that there is often very little agreement in their results. Also, there are many situations for which no experimental evidence is available. An example of this is the effect of vibration on bubble nucleation at reduced gravity. From these observations it can be concluded that much more empirical work is necessary before accurate predictions can be made for bubble motion and inception.

It is suggested that a systematic method of obtaining relationships for bubble motion and inception would be to use the general equations derived in this paper. By experimenting with a wide range of liquids and environments, the exponents and constants could be evaluated in these expressions for particular ranges of applicability. These results could then be tabulated, noting the limitations stated earlier in this chapter. Any particular problem could then be solved by referring to the table and inserting proper values into the general equation.

In order to construct such a table, the effect of varying each individual parameter would have to be determined. Some of the parameters could be varied and measured easily by conventional means. Others would be more difficult. The parameters of the liquid, such as thermal conductivity, viscosity and density, could be varied by using different liquids. However, it would have to be realized that these parameters are dependent on other variables, such as temperature. High-speed photography would be a very necessary piece of measuring equipment for parameters such as bubble velocity and diameter.

Probably the most difficult parameter to vary would be the gravitational field. However, this would also be of prime interest. The first reduced gravity experiments were performed in aircraft using the Keplerian trajectory. Experiments were also conducted in the nose cone of the Atlas missiles and the Aerobee. All of these methods were very expensive and the latter two required the telemetering of data. A recent method developed by Lyons (32) used magnetic levitation. Magnetically susceptible fluids were placed in fields to produce conditions equivalent to reduced gravity. This method has a serious drawback for the study of bubble mechanics in that the vapor often has different magnetic properties than the liquid. Therefore, as the liquid went to zero gravity, the vapor in the bubble might not.

The most practical method from an economic standpoint for reducing the gravity field appears to be by the use of drop towers. At the George Marshall Space Flight Center a 32 foot high tower was used. It gave a 1.4 second period of low gravity. An 88 foot tower was constructed in Cleveland, which produced a 2.2 second period of low gravity. Although these periods were relatively short, transient problems did not seem to be evident. The main limitation was the inability to make experiments for extended periods of low gravity. The ultimate in low gravity experimentation would be the manned orbital laboratory.

APPENDIX

Table I

Definition of Symbols

Symbol	Name	Units
$A = L^2$	area	ft^2
B	coefficient of volume expansion	$\frac{ft^3}{ft^3\ F} = \frac{1}{F}$
$D = 2R$	diameter	ft
F	force	lbf
h	enthalpy evaporation	$\frac{Btu}{lb}$
Q	heat flux	$\frac{Btu}{hr\text{-}ft^2}$
C	constant	
L	length	ft
m	mass	lbm
M	moles	moles
$R = L$	radius	ft
O	temperature	$^oR = F\ abs = 460\ ^oF$
O	temperature difference	^oF or ^oR
t	time	sec
$V = L^3$	volume	ft^3
$V = V_2 - V_1$	volume difference	ft^3
PE	potential energy	ft lbf
R	gas constant	$\frac{ft\ lbf}{lbm\ Fabs}$
R_c	universal gas constant	$1545\ \frac{ft\ lbf}{mole\ Fabs}$

Table I (cont.)

T	surface tension	$\frac{ft\ lbf}{ft^2}$ or $\frac{lbf}{ft}$
μ_f	viscosity	$\frac{lbf\ ft\ sec}{ft^2 ft} = \frac{lbf\ sec}{ft^2}$
v	velocity	$\frac{ft}{sec}$
$wB\Delta T$	buoyancy force	lbf
H_{fg}	latent heat of vaporization	$\frac{Btu}{lbm}$
g_c	acceleration of mass conversion factor	$32.2\ \frac{lbm\text{-}ft}{lb_f\text{-}sec^2}$
e	root-mean-square height of roughness	ft
Y_m	amplitude	ft
f	frequency	$\frac{1}{sec}$
a	acceleration	$\frac{ft}{sec^2}$
c_p	specific heat (constant p)	$\frac{Btu}{lbmF}$ or $\frac{Btu}{lbwF}$
fcn	function of	
g	gravity acceleration	$32.2\ \frac{ft}{sec^2}$ (substitute local g if not on surface of earth, or use a for acceleration field)
h_c	convection coefficient	$\frac{Btu}{hrft^2F}$
k	thermal conductivity	$\frac{Btuft}{hrft^2F} = \frac{Btu}{hrftF}$
KE	kinetic energy	ft lbf

Table I (cont.)

$\frac{m}{M}$	molecular weight	$\frac{lbm}{mole}$
$p = \frac{m}{V} = d$	mass density	$\frac{lbm}{ft^3}$
P	pressure	$\frac{lbf}{ft^2}$
NR	nucleation rate	$\frac{1}{ft^2\text{-}sec}$

Table II

Dimensionless Numbers

Symbol	Name	Formula = 1	Ratio	Equivalent
Ne	Newton	$\left(\frac{ma}{g_c F}\right)$	Inertial Force / Imposed Force	
Nc	natural convection	$\left[\frac{D^2 \rho g B \Delta\theta}{\mu_f g_c v}\right]$	Buoyancy Force / Viscous Force	
Nu	Nusselt	$\left(\frac{hD}{k}\right)$	Total Heat Transfer / Conductive Heat Transfer	
Pe	Peclet	$\left[\frac{D\rho v c_p}{k}\right]$		(Re)(Pr)
Pr	Prandtl	$\left[\frac{3600 c_p \mu_f g_c}{k}\right]$	Kinematic Viscosity / Thermal Diffusivity	
Ra	Raleigh	$\left[\frac{D^3 \rho^2 (3600) g B \Delta\theta c_p}{k \mu_f g_c}\right]$		(Re)(Pr)(Nc)
	Reynolds	$\left[\frac{\rho v D}{\mu_f g_c}\right]$	Inertial Force / Viscous Force	
$\left(\frac{L}{D}\right)$ or $\left(\frac{L}{H}\right)$	shape factor	$\left(\frac{L}{D}\right)$		
St	Staunton	$\left[\frac{h}{3600 v \rho c_p}\right]$		$\left[\frac{(Nu)}{(Pr)(Re)}\right]$
St	Stokes	$\left[\frac{\mu v g_c}{\rho R^2 g}\right]$		$\frac{(Fr)}{(Re)}$
We	Weber	$\left[\frac{\rho v^2 L}{T g_c}\right]$	Inertial Force / Surface Tension	
B_n	Boiling	$\left[\frac{Q}{H_{fg} V \rho}\right]$	Vapor Velocity Force / Flow Velocity	
Ja	Jakob	$\left[\frac{c_p \rho \Delta\theta}{H_{fg} \rho v}\right]$	Liquid Sensible Heat / Bubble Latent Heat	

Table II (cont.)

Symbol	Name	Formula = 1	Ratio	Equivalent
Nu	Nusselt for Bubbles	$\left[\frac{D\ Q}{K\Delta\theta}\right]$	Boiling Heat Transfer / Conduction Heat Transfer	
Bo	Bond	$\left[\frac{mg}{T^{L}g_c}\right]$	Gravitational Force / Surface-tension Force	$\left(\frac{We}{Fr}\right)$
C_D	drag coefficient	$\left[\frac{Pg_c}{pv^2}\right]$	Gravitational Force / Inertial Force	
Co	condensation	$\left[\frac{D^3 p\Delta hg}{\mu_f g_c k\Delta\theta}\right]$	Gravitational Force / Viscous Force	Latent heat / Sensible Heat
				Eu
De	density	$\left[\frac{1545\theta p}{\left(\frac{m}{M}\right)\ P}\right]$	Solid Density / gas density	
Eu	Euler	$\left[\frac{Pg_c}{pv^2}\right]$	Friction Head / Velocity Head	C_D
Fo	Fourier	$\left[\frac{k}{c_p p L^2}\right]$	Elapsed time / Time to bring solid to final temperature	
Fr	Froude	$\left[\frac{v^2}{2gL}\right]$	Inertial force / Gravitational Force	
Gc	gas constant	$\left[\frac{\left(\frac{m}{M}\right)R}{1545}\right]$		
Gr	Grashof	$\left[\frac{D^3 p^2 gB\Delta\theta}{\mu_f^2 g_c^2}\right]$		(Re)(Nc)
Gz	Graetz	$\frac{3600\pi}{4}\left[\frac{D^2 vpc_p}{kL}\right]$		$\left[\frac{\pi}{4}(Re)(Pr)\left(\frac{D}{L}\right)\right]$
Ka	Kaissling	$\left[\frac{g\mu_f^{4}g_c}{p_T^{3}}\right]$		$\frac{(We)^3}{(Fr)(Re)^4}$
St	Strouhal	$\left[\frac{fD}{V}\right]$		
	Roughness Factor	$\left[\frac{e}{D}\right]$	Height of roughness / Diameter of Channel	

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