

THE EVOLUTION OF PROSPECTIVE ELEMENTARY TEACHERS'
COMPETENCIES: PROCEDURAL KNOWLEDGE, MATHEMATICAL
KNOWLEDGE FOR TEACHING, ATTITUDES, AND
ENACTMENT OF MATHEMATICAL PRACTICES

by

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ABSTRACT

The purpose of this research was to explore the evolution of prospective elementary teachers' competencies (in practices, knowledge, and attitudes); examine the relationships that occur between knowledge, attitudes and practices; and develop an idea of how certain prospective elementary teachers grow and progress in their enactment of two of the Common Core Standards for Mathematical Practice, persevering in problem solving and constructing viable arguments. This was conducted as a case study of the first two of three inquiry-based mathematical content courses for elementary teachers. Both qualitative and quantitative data was collected from a cohort of students moving through the curriculum over the course of a year. Results showed there was an increase in prospective elementary teachers' mathematical knowledge for teaching scores over time, but no change in their procedural knowledge or attitude scores. Positive, linear relationships existed between all of the pair-wise comparisons between mathematical knowledge for teaching, procedural knowledge, and attitudes toward mathematics. Overall, students grew in their ability to problem solve and construct viable arguments in mathematics while moving through the curriculum, with a few exceptions. Three factors contributed to students' learning in the curriculum: the amount of effort made by the student, the atmosphere and attitudes of students in the class, and the nature of the content and questions asked in the curriculum. Another important consideration which arose from the data analysis was the opportunities the curriculum allowed for the practice of written versus verbal explanations, and what was formally assessed. Designers of teacher education programs using a similar curriculum should evaluate the importance of written versus verbal explanations in the goals of the course, and appropriately assess the students.

1. STATEMENT OF THE PROBLEM

Introduction

The purpose of this research was to explore the evolution of prospective elementary teachers' competencies (in practices, knowledge, and attitudes); examine the relationships that occur between knowledge, attitudes and practices; and develop an idea of how certain prospective elementary teachers grow and progress in their enactment of two of the Common Core Standards for Mathematical Practice (CCSMP). This was conducted as a case study of the first two of three inquiry-based mathematical content courses for elementary teachers. This study informs those who seek to improve teaching quality through attention to teacher education programs and the students they serve. With the widespread adoption of the Common Core State Standards (CCSS), it is an important step in developing a better understanding of the CCSMP and how those are brought to life for prospective teachers through "doing" mathematics.

In 2008, the National Council of Teachers of Mathematics (NCTM) held a research agenda conference that was focused on linking research to practice. The organization identified several questions to guide mathematics education research. The first question of interest was "What should be the goals of professional learning, and how we will measure attainment of the goals in terms of teacher growth?" (Arbaugh, Herbel-Eisenmann, Ramirez, Knuth, Kranendonk, & Quander, 2008, p. 19). Arbaugh et al. (2008) indicated that for researchers to answer this question, the field should "develop a more elaborated trajectory of teachers' evolution of their competencies (e.g. knowledge, beliefs, dispositions, and practices), beginning from one end of the continuum when

teachers enter teacher preparation programs to the other end of the continuum when teachers establish themselves as effective teacher leaders” (p. 19). This research project examined the beginning of the continuum, when prospective elementary teachers were taking their first two mathematics content courses. The second question of interest was “What relationships exist among procedural knowledge, conceptual knowledge, and mathematical thinking?” (p. 53). This question was also examined in this study with regard to prospective elementary teachers.

Improving teaching quality is seen as one of the best paths to improving student achievement (Conference Board of the Mathematical Sciences, 2012; National Commission on Mathematics and Science, 2000). The U.S. Department of Education and the National Science Foundation have funded projects to improve teachers’ content knowledge as there has been increased attention to its ability to raise student achievement (Ball, Hill, & Bass, 2005; Matthews, Rech, & Grandgenett, 2010). Two modes of improving teaching quality are professional development and teacher education programs (NCMS, 2000). “According to the National Science Board (NSB), updating current teacher knowledge is essential, and improving teacher preparation programs is crucial to developing world-class mathematics instruction in the United States” (NSB as cited in Mizell & Cates, 2004, p. 1). This study supports these goals by focusing attention on an inquiry-based approach to mathematics content in teacher education programs.

To better understand the progression of prospective teachers’ competencies, teacher educators should examine in more detail prospective teachers’ competencies (in terms of knowledge, attitudes, and practices) in the beginning of their college education, the relationships between different components of their competencies, and how they

develop throughout different courses. Past research has shown preservice teachers’ “understanding of mathematics [to be] rule bound and compartmentalized” (Ball, 1990, p. 453). The mathematics courses offered to prospective elementary teachers should aim toward developing in them a deeper understanding of the mathematics content they will be teaching (Beckmann et al., 2004; Ball & Forzani, 2010) from the perspective of a teacher (CBMS, 2012). According to Ball (1990), teachers need to have absolute knowledge *of* mathematics (concepts, procedures, underlying principles, meanings, and connections) as well as knowledge *about* mathematics (the nature of mathematics and how it is done).

In order to teach mathematics, teachers need both common and specialized mathematics subject matter knowledge which “requires a conceptual understanding of the relevant mathematical concepts and procedures as well as awareness and understanding of interconnections between them” (Hourigan & O’Donoghue, 2013, p. 37). Teachers need to possess a deep understanding of the content they will teach and understand their students’ thought process in order to convey the material in a way that will promote meaningful learning (Ball & Forzani, 2010). In order for the mathematics courses offered to prospective elementary teachers to try and meet all of these demands and fix the problems of poor achievement and attitudes, researchers need to better understand the interplay between knowledge, attitudes, and practices.

Content and Standards

When examining these relationships, researchers need to consider what content and demands new teachers will face. An obstacle once faced in seeking the improvement

of teacher education programs was the lack of a common K-12 curriculum (Ball & Forzani, 2010). With the implementation of the Common Core State Standards of Mathematics (CCSSM) in the forty-four states that have adopted them, mathematics teacher educators should be able to continue with more consistency.

Teacher educators need to prepare prospective teachers to not only be aware of the CCSSM, but to have a deep understanding of them. “Like their students, teachers need to have the varieties of expertise described in these standards” (CBMS, 2012, p. 1). Teachers not only need to understand the standards, but also need to foster the learning and practice of them with their future students. The CCSSM “define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it” (CCSS, 2011). This is not an easy task, especially when a teacher does not understand the standard or does not know what understanding of the standard looks like. Teacher educators should be cognizant of this and allow prospective teachers the opportunities to develop their understandings and sophistication in mathematics while exposing them to the CCSSM during their mathematics content courses.

An important dimension of the CCSSM is the inclusion of both Content Standards and Standards for Mathematical Practice. The Standards for Mathematical Practice describe what “doing” mathematics should look like whereas the content standards describe “what” students should know and understand (CCSS, 2011). For this reason it is important for teacher educators to make sure prospective teachers have not only adequate procedural and content knowledge that align with the CCSSM, but also adequate exposure and opportunity to work on and develop their mathematical practices.

Examining the content standards and deciding what concepts need to be taught to prospective teachers is straightforward while deciding how to enable and allow for growth in their enactment of the Mathematical Practices is more difficult. It is also more difficult to assess attainment of sophisticated mathematical practice because it is not necessarily a measurable attribute like attainment of the ability to add two digit numbers. It is nonetheless still important to learn more about how prospective teachers may develop and grow in their enactment of the Mathematical Practices.

Problem Statement

Teacher education programs should foster growth in prospective teachers' competencies (knowledge, practices, and attitudes) in order to prepare them to enter the field of teaching. Given the inconsistencies of teacher education program course load requirements across the country (NCMS, 2000), it is important for researchers in mathematics education to explore the effects different components of teacher education programs have on prospective teachers' learning and growth. Some programs only require prospective elementary teachers to take a general mathematics content course while others require specialized content courses. The number of mathematics content courses required also varies. Ball & Forzani (2010) and Burton, Daane, and Giesen (2008) call for more research to be done in order to establish characteristics of effective teacher education programs and determine associated outcomes. Teacher educators need to ensure that graduates of teacher education programs exit with expertise (National Council on Teacher Quality, 2008). "Allowing teachers to learn at children's expense is unethical. We must build a system for ensuring that new teachers have the requisite

professional skills and know how to use them” (Ball & Forzani, 2010, p. 8). This research will help teacher educators better understand the ability of inquiry-based mathematics content courses for elementary teachers to foster growth in prospective mathematics teachers’ competencies. This gives a basis for changes and improvements to be made to similar programs, and provides rationale for creating more consistencies between teacher education programs across the nation.

The mathematics curriculum for prospective teachers should be focused around the CCSSM as they are currently being implemented in forty-four states. The CCSSM include both Content Standards and Standards for Mathematical Practice (CCSMP), which teachers need to understand and be able to implement in their classrooms. Teacher education programs need to be aware of and understand how prospective elementary teachers grow and develop in their mathematical knowledge and practices. They also need to ensure prospective elementary teachers develop these practices through their coursework because, “Like their students, teachers need to have the varieties of expertise described in these standards” (CBMS, 2012, p. 1). Teachers not only need to understand the standards, but also need to foster the learning and practice of them with their future students, so teacher educators must train and develop prospective teachers accordingly.

Mathematics content courses for elementary teachers, as part of a teacher education program, should also foster growth in prospective teachers’ competencies and build opportunities for the students to learn about and practice the CCSSM. It is important to pay attention to the relationship between attitudes and cognition in the prospective teacher population, as teachers’ attitudes and beliefs have a big influence on their practices in the classroom, how they teach mathematics, and the curriculum they

implement (Mizell & Cates, 2004). “Teachers’ beliefs, behaviors, and attitudes are invaluable variables to student learning” (White-Clark, DiCarlo, & Gilchrist, 2008, p. 40). Many researchers claim that the body of mathematics education research could be strengthened if attention was paid to the relationship between affect and cognition (McLeod, 1992 as cited in Philip, 2007; Zan, Brown, Evans, & Hannula 2006). It is also important to consider the relationship between procedural and conceptual knowledge “because it seems to hold the key to many learning processes and problems. If we understood more about the acquisition of these kinds of knowledge and the interplay between them in mathematical performance, we surely could unlock some doors that have until now hidden significant learning problems in mathematics” (Hiebert & Lefevre, 1986, p. 22).

The concept of an inquiry-based curriculum, where prospective teachers are given the opportunity to practice doing mathematics through productive struggle, seems to lend itself to fostering growth in the learning and practice of mathematics. To have evidence that this approach is appropriate, however, mathematics education research needs to explore the competencies of prospective elementary teachers as they move through this type of curriculum.

Research Questions

This study and the following questions were designed to provide evidence about the evolution of prospective elementary teachers’ competencies. Competencies addressed in this study included procedural knowledge, mathematical knowledge for teaching, attitudes toward mathematics, and enactment of the mathematical practices of

persevering in problem solving and constructing viable arguments. The evolution of these competencies was examined in the first two of three mathematics content courses for teachers at a university in the Mountain West, designed to foster growth in these competencies. The questions that guided this research were:

1. How do certain prospective elementary teachers progress in their enactment of the mathematical practices of persevering in problem solving and constructing viable arguments as they move through inquiry-based mathematics content courses?
2. What relationship (if any) exists between prospective elementary teachers' procedural knowledge and mathematical knowledge for teaching, and how does this relationship change over time as they move through inquiry-based mathematics content courses?
3. How do prospective elementary teachers' attitudes toward mathematics associate with their procedural knowledge and their mathematical knowledge for teaching, and how does this relationship change over time as they move through inquiry-based mathematics content courses?

Definition of Terms

Curriculum is defined as the text and the teacher's role in implementing the course material. *Inquiry-based curriculum* is one in which the text provides mathematical tasks and activities for the students to work on in groups and the teacher's role is that of a facilitator rather than lecturer. The focus is placed on *why* mathematical concepts are true rather than just *what* the algorithms are or *how* to compute.

Progress is defined as a noun of movement over time rather than a verb describing individuals' competencies.

Competencies in this study included procedural knowledge (basic algebra skills), mathematical knowledge for teaching, attitudes toward mathematics, and the Mathematical Practices of Persevering in Problem Solving and Constructing Viable Arguments.

Significance of the Study

This study will inform teacher educators about the development of prospective elementary teachers' competencies and the relationships between them as they move through a series of mathematics content courses for elementary teachers using an inquiry-based curriculum. It will further illuminate whether this type of inquiry-based curriculum is effective at fostering growth in prospective elementary teachers' competencies, and offer suggestions for improving the curriculum. The descriptions and discussions in this study will allow teacher educators using a similar curriculum to make appropriate comparisons to their own situation and use the results to make informed decisions about and improvements to their own curriculum.

This research study will shed light onto how prospective elementary teachers develop competency in the Mathematical Practices of persevering in problem solving and constructing viable arguments (CCSS, 2011). Qualitatively examining prospective elementary teachers' use of Mathematical Practices begins to paint a picture of how they are developed (or not) through an inquiry-based curriculum of mathematics content courses for elementary teachers. The results of this part of the study will be relevant to

teacher educators, who can use this information to better prepare prospective teachers in developing and later implementing the CCSMP.

This study will also explore the underlying relationships that might exist between prospective elementary teachers' knowledge, attitudes, and practices. If such relationships exist, then teacher educators need to be aware of this. Perhaps a certain curriculum or program is very good at fostering growth in one specific competency but not another. If teacher educators better understand the relationship between the two competencies, they may be able to use this knowledge to their advantage by focusing on the relationship between the competencies and making necessary adjustments to the curriculum. These adjustments to the curriculum may result in all competencies flourishing rather than just one or two. For example, if the program focuses more on building sophisticated practices with students, the procedural and content knowledge may follow suit.

This study will show whether or not prospective elementary teachers are given opportunity to practice “doing” mathematics according to the Common Core Standards for Mathematical Practice in inquiry-based mathematics content courses. Ultimately it is very important for prospective elementary teachers to develop the Mathematical Practices that they will in turn have to foster with their students in the future. “Like their students, teachers need to have the varieties of expertise described in these standards” (CBMS, 2012, p. 1). Teachers not only need to understand the standards, but also need to foster the learning and practice of them with their future students. The CCSSM “define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has

understood it” (CCSS, 2011). If prospective elementary teachers are not given opportunities to learn about and develop their Mathematical Practices, they will not be prepared for entering the field and developing the Practices in their students.

Limitations

The population of this study was confined to prospective elementary teachers from one university in the Mountain West and so did not display a diverse background of participants. The instructors for the studied courses varied, and although I as the researcher found these variations to be minimal, the variations will be discussed in chapter 4 so the reader can decide the degree to which the instructor variation may have had an effect. Finally, the act of being in the study could have had an effect on the participants. This is especially true of those participants who were involved with interviews as this provided a place and setting for further reflection on the coursework.

Conclusion

This study will examine how prospective elementary teachers learn and grow throughout their experience in specialized mathematics content courses. This includes taking a closer look at their procedural knowledge, conceptual knowledge, use of mathematical practices, and attitudes toward mathematics along with the relationships that exist between these competencies. The following review of the literature will first provide definitions of procedural and conceptual knowledge and the path to uncovering what specialized content knowledge may be required for teachers beyond that of pure content knowledge. Then, results from studies of additional content courses and different

types of courses on prospective elementary teachers' content knowledge for teaching will be shared. A discussion of attitudes toward mathematics and their relationship with cognition will follow, focusing on the impact of specialized content courses on prospective elementary teachers' attitudes.

This research will also scrutinize in more detail how prospective teachers progress in their use of mathematical practices. In particular, to examine how they grow in their enactment of two of the Common Core Standards for Mathematical Practice: 1) Make sense of problems and persevere in solving them, and 2) Construct viable arguments. The end of the next section will be dedicated to a description of events that led to the writing of the CCSSM, and will include a discussion of what the education community is saying about them and how they should be implemented.

2. LITERATURE REVIEW

Introduction

The literature review will begin with a discussion of the differences between procedural and conceptual knowledge. Then, the discussion moves into the increased attention to the specialized knowledge needed for teaching and instruments developed for measuring this specialized knowledge - in particular, content knowledge for teaching mathematics (CKT-M). Studies that have incorporated the use of this measure will be discussed next. These studies examined whether teachers' CKT-M was related to student achievement, whether prospective teachers' CKT-M was related to attitudes toward mathematics, and what effect different types of mathematics content courses on prospective teachers' CKT-M. Next is a discussion of the relationship between affect and cognition which moves into the effect of specialized content courses on prospective elementary teachers' attitudes toward mathematics. Then, the literature review will discuss the arrival of the Common Core State Standards for Mathematics and what practitioners are suggesting for implementing the Standards for Mathematical Practice, including the use of mathematical tasks. The final section will be dedicated to a discussion of the context of where we are in the history of mathematics education and the theoretical framework used in this study.

Mathematical Knowledge

Pure Content Knowledge – Procedural and Conceptual Knowledge

The most widely recognized label for conceptual vs. procedural knowledge is skill vs. understanding (Hiebert & Lefevre, 1986). Both of these types of knowledge are important to consider given the reform movement in education towards understanding and away from skill alone. As a disclaimer before defining procedural and conceptual knowledge, Hiebert and Lefevre (1986) stated: “We do not believe, however, that the distinction provides a classification scheme into which all knowledge can or should be stored. Not all knowledge can be usefully described as either conceptual or procedural. Some knowledge seems to be a little of both, and some knowledge seems to be neither” (p. 3).

According to Hiebert and Lefevre (1986), conceptual knowledge is based on the connections and relationships within the discipline. This knowledge is achieved through both constructing relationships between pieces of information and creating new relationships between existing knowledge and new information. There are two levels at which relationships can be established: primary, where the relationship and content are at the same level of abstractness; and reflective, where connections are at a more abstract level than the content – one has to step back and see the big picture (Hiebert & Lefevre, 1986). Skemp (1987) believed that forming conceptual understanding involves abstracting (finding similarities via experiences), classifying (putting together experiences by similarities), and forming an abstraction (recognizing a new experience as similar to prior experiences – connecting).

Hiebert and Lefevre (1986) stated procedural knowledge consists of language and symbols along with the algorithms and rules for completing tasks. Symbol manipulation and problem solving strategies are included here. Hiebert and Lefevre (1986) describe the main relationship of procedural knowledge as “after” or thinking about what is the next step to solve the problem. With conceptual knowledge, however, the relationships are not linear but more like a web. Without a connection being made between conceptual and procedural knowledge, one cannot be fully competent in mathematics. The relationship between procedural and conceptual knowledge is important in uncovering progressions and problems associated with how people learn and practice mathematics (Hiebert & Lefevre, 1986). Deep understanding arises when connections are made between procedural and conceptual knowledge (Beckmann et al., 2004).

This study aimed to examine this relationship between procedural knowledge and conceptual knowledge in prospective elementary teachers. Because of the unique population of prospective elementary teachers, a unique conceptual knowledge was used – a construct that arose from the need to define the knowledge necessary for teachers, mathematical knowledge for teaching. This construct will be discussed next.

Knowledge for Teaching

Part of the shift in research and increased attention to teacher knowledge was due to Shulman’s (1986) presidential address to the American Educational Research Association in which he shared a new conceptualization of content knowledge. Shulman (1986) categorized three different types of content knowledge needed for teaching: subject matter content knowledge, pedagogical content knowledge, and curricular

knowledge. Subject matter content knowledge included knowledge of facts and content along with an understanding of the underlying structure of the subject. This requires holding an understanding of both what is true and why it is true. Pedagogical content knowledge is the content knowledge needed for teaching which includes knowing how to make the content understandable for others and being aware of common preconceptions and misconceptions. Curricular knowledge involves being aware of alternative curricular materials along with both the lateral and vertical curriculum (within and across grade levels). Shulman (1986) made the argument there needed to be a combination of types of knowledge assessed in the competence of teachers.

The 1990s marked the beginning of a focus on teachers' mathematical knowledge, rather than general knowledge, in relation to student achievement. Researchers began to study this specialized knowledge needed for teaching in more detail, mainly through a close examination of teachers and teaching – in other words, by examining the work of a teacher (Hill, Dean, & Goffney, 2007). An early study using tasks and interviews was conducted by Ball (1990) who studied the mathematical understandings of a sample of prospective teachers, both elementary ($n=217$) and secondary ($n=35$). The mathematical context was division, which the participants examined from the perspective of students – knowing they will one day approach the context as teachers. Prospective teachers were asked to answer a multiple-choice question on the topic of division. During the interview they were asked how they learned to do a particular division problem and then asked to represent that division problem. Questionnaires and interviews were used for data collection. Frequencies of questionnaire responses were calculated and substantive analyses were conducted to find categories within the responses to interview questions.

Questions were cross-analyzed with respect to subject matter understanding; ideas about teaching, learning, and the teacher's role; and feelings or attitudes about mathematics, pupils, or self. Results indicated all prospective teachers had considerable difficulty in understanding the meaning of division with fractions while almost all were able to compute a fraction division problem. Other questionnaire items asked participants whether certain mathematical ideas could be explained, must be memorized, or whether they were not sure. Many participants thought ideas could not be explained and during interviews it became apparent that those who thought explanations could be offered turned out to cite rules rather than offer a true explanation (Ball, 1990).

With the awareness of mathematical knowledge for teaching came some attention to creating instruments to measure this knowledge. Starting in 1999, the Study of Instructional Improvement/Learning Mathematics for Teaching Project (SII/LMT) began to create quantitative measures to assess the knowledge necessary for teaching elementary school mathematics. Although the importance of mathematical knowledge for teaching was recognized and studies had shown student achievement gains related to teachers' mathematical knowledge, little was known or agreed upon as to the details and scope of this knowledge (Ball et al., 2005; Hill, Schilling, & Ball, 2004). In order to create questions pertaining to this specialized knowledge for teaching, researchers aimed to examine the actual work involved with teaching elementary school mathematics (Ball, Thames, & Phelps, 2008). This produced a picture of what mathematical knowledge for teaching was necessary beyond that of basic skills and understanding. For example, teachers must: deal with students' alternative methods and errors and determine how to respond appropriately; be able to explain why; use appropriate representations; make

connections; use strategic examples; and use appropriate mathematical language and symbols (Ball et al., 2005). These tasks involve mathematical skill and are in the context of student interaction yet do not require knowledge of students and pedagogy and are not typically taught in regular university mathematics courses (Ball et al., 2008).

The measure created, Content Knowledge for Teaching Mathematics (CKT-M), was thought to incorporate both the early visions of Shulman and colleagues as well as the qualitative research that followed in the 1990s (Hill, Rowan, & Ball, 2005). The focus was on numbers and operations (as this is most of the elementary school curriculum focus) and patterns, functions, and algebra, with the later addition of geometry. The two categories of content knowledge which arose out of this study of mathematical knowledge for teaching were denoted common content knowledge (CCK) and specialized content knowledge (SCK) (Ball et al., 2005). Ball et al. (2008) “hypothesized that Shulman’s content knowledge could be subdivided into common content knowledge and specialized content knowledge” (p. 399).

Since the creation of the CKT-M, many studies have incorporated its use. Two studies in particular highlight the important relationships between a teacher’s and prospective teacher’s mathematical knowledge for teaching and student achievement gains and attitudes toward mathematics, specifically anxiety, respectively. In 2005, Hill et al. aimed to examine teachers’ scores on the CKT-M which included items related to “explaining terms and concepts to students, interpreting students’ statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs” (p. 373). The scores on this assessment

were used as a predictor for student achievement gains in mathematics. First and third grade student achievement gains were compared to the previous year. Teachers' CKT-M was a predictor in student achievement gains, more so than experience and prior coursework. This held true even after controlling for certain variables such as student SES and absence rates, teacher credentials, and experience (Hill et al., 2005).

Examining prospective elementary teachers' attitudes, Gleason's (2007) study focused on two components of attitudes toward mathematics – mathematics test anxiety and numerical anxiety – and their relationship with prospective elementary teachers' mathematical content knowledge for teaching. To test this relationship, participants in this study took a shortened version of the Mathematics Anxiety Rating Scale (MARS 30-item) and the CKT-M. Correlations between scores on the Mathematics Test Anxiety, Numerical Anxiety, Number and Operations Content Knowledge, Geometry Content Knowledge, and Patterns, Functions, and Algebra Content Knowledge scales were computed using two-tailed Pearson correlations. There were strong correlations between anxiety measures and knowledge measures, but correlations between knowledge measures were weak enough to support the distinctness of the constructs as Hill et al. (2005) reported. Mathematics test anxiety was negatively correlated with mathematical knowledge for teaching – indicating a higher anxiety level would correspond to a lower mathematical knowledge for teaching level – while a negative correlation between numerical anxiety and mathematical knowledge for teaching existed but was much weaker (Gleason, 2007).

Hill (2010) took a different approach and wanted to learn more about the nature and predictors of elementary teachers' mathematical knowledge for teaching. This study

used data from a national sample of elementary mathematics teachers. Their performance on items measuring mathematical knowledge for teaching (CKT-M) was examined and data on both teacher and student characteristics were also gathered. The CKT-M measure incorporated both CCK and SCK items and was focused on number and operations. Item response theory (IRT) was used to standardize the scores and descriptive analyses were used to examine the relationship between teacher characteristics and CKT-M scores. No differences in item difficulty by content (whole numbers, integers, and rational numbers) were found which was in contrast to prior research which showed difficulties with rational numbers. CCK items were found to be easier than SCK items, especially those that focused on explanation. Taking more mathematics content and methods courses was indicative of a slightly higher MKT score and teachers' self-concept was correlated with MKT score. In the regression analysis, teacher participation in additional mathematics courses was positively associated with MKT score (Hill, 2010).

The special mathematical knowledge needed for teaching was discovered and an assessment was made to measure this. Research showed a teachers' MKT score was a predictor of student achievement (Hill et al., 2005), and negatively associated with attitudes toward mathematics – in particular text anxiety (Gleason, 2007). The importance for teachers to have this knowledge is evident and so prospective teachers must develop this in their teacher preparation programs.

This study aimed to examine the MKT scores of prospective elementary teachers as they were at the very beginning of the program in the first two of three mathematics content courses using an inquiry-based curriculum. Prospective teachers' MKT has been

tested in other research studies with their own unique curriculum changes, as are described in the next section.

Content Courses for Prospective Teachers

In the past, many prospective elementary teachers were only required to take general mathematics courses. Now, there are recommendations made by organizations such as the Conference Board of Mathematical Sciences (CBMS), the National Research Council, and the Mathematics Learning Study Committee that prospective elementary teachers enroll in courses specializing in the mathematics taught at the elementary school (Matthews & Seaman, 2007). The CBMS (2012) believe teachers should learn the content they will teach at a deeper level and from the perspective of a teacher. Even with these recommendations, there are many inconsistencies between teacher education programs across the nation (Ball & Forzani, 2010; Goodwin & Oyler, 2008; Hill, Sleep, Lewis, & Ball, 2007; Matthews et al., 2010; NCMS, 2000; NCTQ, 2008). Due to this discrepancy, there have been several studies which have considered the impact of specialized content courses vs. general content courses, additional content courses, and blending content with a methods course on the attitudes and content knowledge of prospective elementary teachers.

Matthews and Seaman (2007) aimed to examine the effects of different undergraduate mathematics courses on the content knowledge and attitudes towards mathematics of prospective elementary teachers. This study consisted of two groups of prospective elementary teachers, an experimental group who took a course called Logic of Arithmetic (LOA) and a control group who took a general mathematics course. Each

group took a Mathematical Content Knowledge for Elementary Teachers test (MCK) – designed by a group of experts – and the Aiken’s Revised Mathematics Attitude Scale. Independent two-sample t-tests were used to compare groups. Linear regression was also used in order to control for ability levels between groups (ACT comprehensive score and cumulative university GPA). There was a significant difference between the groups’ average mathematical content knowledge scores and average attitudes toward mathematics scores, with the experimental group performing better than the control group on both scales. This indicated their LOA course, which focused on conceptual understandings and used a combination of lecture and in-depth group discussions, was effective at improving prospective elementary teachers’ knowledge and attitudes.

Matthews, Rech, and Grandgenett (2010) completed a similar study at a school changing the requirements of prospective elementary teachers from completing a general mathematics content course to completing a specialized mathematics content course. In this study, data were collected over a two year period. The control group consisted of those students who never enrolled in a mathematics content course designed for elementary teachers while the experimental group consisted of those who enrolled in at least one of the two mathematics content courses offered for elementary teachers, either Number and Operations or Geometry. Groups were shown to be the same in terms of demographic variables. The Content Knowledge for Teaching Mathematics measure (CKT-M) was used to measure the content knowledge of both elementary number concepts and operations and elementary geometry. To measure attitudes toward mathematics, the Aiken’s Revised Mathematics Attitude Scale was used. After conducting independent two-sample t-tests, the researchers found the experimental group

scored significantly better on the CKT-M; however, no significant differences were found between their attitudes. On average, both groups showed an overall neutral attitude toward mathematics.

Burton, Daane, and Giesen (2008) showed concern with the lack of mathematical knowledge of elementary teachers and wanted to find out how to better prepare elementary mathematics teachers. This study aimed to examine how an intervention of an additional 20 minutes of mathematics content instruction interwoven into a methods course affected the mathematics content knowledge for teaching of prospective elementary teachers. The experimental group received the intervention while the control group did not. The CKT-M was used for a pre-test and post-test (2 different forms). The changes in the experimental group's pre- and post-test scores were significant for the experimental group, but they were not for the control group. Differences between CKT-M scores of the experimental group and the control group were not significant on the pre-test, but they were significant on the post-test. Overall, the experimental group outperformed the control group.

The studies of Matthews and Seaman (2007) and Matthews, Rech, and Grandgenett (2010) showed prospective elementary teachers taking a specialized mathematics content course score higher on the CKT-M than prospective teachers not taking a specialized content course. Burton, Daane, and Giesen (2008) found a difference in mathematical knowledge for teaching scores between prospective elementary teachers who received 20 minutes of content knowledge coverage in a methods course compared to those who did not. These studies highlight the importance of specialized content courses to help increase prospective elementary teachers' mathematical knowledge for

teaching. This study did not compare results from two groups in different curricula, but aimed to examine the evolution of prospective elementary teachers' mathematical knowledge for teaching as they move through a particular type of curriculum (inquiry-based). It also took this approach for examining the relationship between mathematical knowledge for teaching and attitude, a competency that will be explored next.

Attitudes Toward Mathematics

Many agree upon the importance of considering affect when examining cognition. In their Curriculum and Evaluation Standards (1989), NCTM emphasizes “becoming confident in one’s own ability” (p. 6). Skemp (1987) believed “the separation of cognitive from affective processes is an artificial one which does not accurately reflect human experiences” (p. 189). This statement was based upon his interaction with students and what the students reported—their learning was affected by their emotions. Ball (1990) wrote, “People’s *understandings* of mathematics are interrelated with how they *feel* about themselves and about mathematics” (p. 461), and Chamberlain (2010) claimed, “Affect...is arguably the single greatest factor that impacts the learning process” (p. 169). An analysis of research on affect in mathematics education caused McLeod (1992) to conclude that mathematics education research would be better if attention was paid to the relationship between affect and cognition (as cited in Philip, 2007). Many researchers have found, either through their own studies or reviews of literature, a link between attitudes and achievement in mathematics (Lim & Chapman, 2010; Ma & Kishor, 1997; Matthews et al., 2010; Randhawa, Beamer, & Lundberg, 1993; Tapia & Marsh, 2004). In their meta analysis, Ma & Kishnor (1997) investigated the relationship

between attitude towards mathematics and achievement in mathematics in grades K-12. From 113 studies chosen, the overall relationship found between attitude and achievement was positive and reliable, but not strong.

It is important to study this relationship between affect and cognition in teachers and prospective teachers moving through teacher education programs. “Teacher beliefs, behaviors, and attitudes are invaluable to student learning” (White-Clark et al., 2008, p. 40), and the mathematical content knowledge of elementary teachers seems to be intertwined with their mathematical attitudes (Matthews et al., 2010). In her research regarding prospective teachers, Ball (1990) found that “prospective teachers’ feelings are *part of* the way they participate in and understand mathematics, *not* a separate affective dimension called “attitude”, and are a critical area of focus for teacher education” (p. 462). Ball’s quote not only highlights the importance of the relationship between affect and cognition, but points to the difficulties that arise when trying to define affective domains.

Existing literature contains much ambiguity in constructs and discrepancies in definitions when it comes to affect (Gleason, 2007; Philip, 2007). Chamberlain (2010) argued that affect is a complex construct with many sub-components with non-measurable attributes, making it difficult to assess affect. He went on to say this difficulty can be overcome by creating definitions and using statistical methods to justify appropriate assessments (Chamberlain, 2010).

In his review of the literature, Philip (2007) used dictionary definitions and distinctions in the literature to describe and capture the essence of certain terms related to affect.

Affect – a disposition or tendency or an emotion or feeling attached to a certain idea or object. Affect is comprised of emotions, attitudes, and beliefs.

Attitudes – manners of acting, feeling, or thinking that show one's disposition or opinion. Attitudes change more slowly than emotions, but they change more quickly than beliefs. Attitudes, like emotions, may involve positive or negative feelings, and they are felt with less intensity than emotions. Attitudes are more cognitive than emotion but less cognitive than beliefs (p. 259).

Many studies involving attitudes also involve beliefs, anxiety, and self-efficacy as these terms are intertwined and dependent upon one another (Matthews & Seaman, 2007).

“Attitude toward mathematics is no doubt a complex idea that interacts with other important belief structures of a teacher” (Matthews et al., 2010, p. 3), but regardless of the ambiguity or lack of agreement when it comes to defining affect or attitude, many studies examine and assess them in their participants.

Mathematics Content Courses and Attitudes of Prospective Elementary Teachers

Whether by examining current mathematics content courses or comparing the impacts of new and/or additional courses, many researchers examine the beliefs and attitudes toward mathematics of prospective elementary teachers. The following two studies are examples of how changes in the mathematics curriculum for prospective elementary teachers improved the prospective teachers' attitudes toward mathematics. The first study incorporated standards-based mathematics pedagogy into a content course for elementary teachers, and the second study created a new course specifically designed for elementary teachers in lieu of a general mathematics course. The final study offers an

example of when a specialized content course for elementary teachers had no effect on their attitudes toward mathematics.

Lubinski and Otto (2004) aimed to examine the effects of preparing prospective elementary teachers in a content course focusing on standards-based mathematics pedagogy. They believed that to realize the vision of NCTM and their standards-based curriculum, the teacher “must not only have a conceptual understanding of the mathematics, but [also]...employ a pedagogy utilizing problem solving, reasoning, verifying, using different strategies, making connections, and communicating ideas” (Lubinski & Otto, 2004, p. 336). Based on the belief that teachers will teach how they were taught, the authors designed a course for prospective elementary teachers that would incorporate standards-based curriculum. This included opportunities for active learning, investigating, conjecturing, reflecting, reasoning, and developing a deep understanding of a fewer number of topics. Their underlying pedagogical theory was that of constructivism. The course did not have a textbook but was problem driven, and the instructor was facilitating and asking questions while emphasizing quantitative reasoning. Surveys and interviews were used to collect data from students in the newly designed course. Overall, students’ beliefs, attitudes, and perceptions of mathematics were positively influenced by the course.

Matthews and Seaman (2007) aimed to examine the effects of taking a specialized content course on prospective elementary teachers’ attitudes towards mathematics by comparing them to others taking a general mathematics course. Each group of prospective elementary teachers took the Aiken’s Revised Mathematics Attitude Scale, and through independent sample t-tests and linear regression to control for ability levels,

a difference between groups in attitude toward mathematics was found. The experimental group, who took the specialized course, scored higher than the control group, who took the general mathematics course.

This positive impact of specialized mathematics courses on prospective elementary teachers is not always seen, however. Matthews et al. (2010) also studied the impact of specialized content courses on prospective elementary teachers' attitude towards mathematics. Independent two-sample t-tests on scores from the Aiken's Revised Mathematics Attitude Scale did not indicate a difference in attitudes between students who took the specialized course and those who did not. Both groups showed an overall neutral attitude toward mathematics. These mixed results indicate more research needs to be done in this area to understand what about the specialized content courses has an effect on attitude.

These examples show mixed results in terms of the effect a specialized content course has on prospective elementary teachers' attitudes. This study aimed to examine attitudes of prospective elementary teachers moving through a curriculum similar to that in Lubinski and Otto's (2004) study, which found the attitudes of prospective elementary teachers' were positively influenced by the course. In addition, this study went beyond investigating the evolution of prospective elementary teachers' attitudes moving through an inquiry-based curriculum by also exploring the relationships between attitude and other competencies, such as enactment of mathematical practices, which will be discussed next.

Mathematical Practices

The Common Core State Standards for Mathematics (CCSSM) include both content standards and Standards for Mathematical Practice (CCSMP). The CCSMP, which are intended for use with all grade levels, represent ways in which students should engage in and “do” mathematics and are especially useful in connection with content standards involving student understanding (CCSS, 2011). The CCSMP were derived from both NCTM’s process standards and the mathematical proficiencies of the National Research Council’s (NCR) report, *Adding It Up* (Kilpatrick, Swafford, Findell & NRC, 2001). NCTM’s process standards include: problem solving, reasoning and proof, communication, connections, and representations. In *Adding it Up*, there are five strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These processes and proficiencies came together to form the eight CCSMP: 1) Make sense of problems and persevere in solving them; 2) Reason abstractly and quantitatively; 3) Construct viable arguments and critique the reasoning of others; 4) Model with mathematics; 5) Use appropriate tools strategically; 6) Attend to precision; 7) Look for and make use of structure; and 8) Look for and express regularity in repeated reasoning (CCSS, 2011).

There is some concern that the CCSMP will be brushed aside or ignored. “If the Standards for Mathematical Practice are taken seriously, we must focus on them in the same way we focus on any other standards—with targeted, intentional, planned instruction” (Russell, 2012, p. 52). “They are, as a group, the foundational skills for working in any of the domains” (p. 39), and as such, they should penetrate every facet of

classroom instruction (Strom, 2013). The practices need to be embedded in content, so once a particular area or topic is chosen, the teacher should determine which practice standards work best with the topic (Russell, 2012) as not all of the standards need to be addressed in every lesson (Burns, 2013). In an interview with Strom (2013), Burns advised teachers they should not be teaching the practices but rather using them to do mathematics. Burns' mantra for a classroom lesson is this: "If kids could be successful without having to think or reason, then the lesson is not good enough" (Strom, 2013, p. 41).

Reasoning and sense making was a focus in NCTM's process standards and aligns with the CCSMP of constructing viable arguments – a competency examined in this study. Because of NCTM's focus on sense making and reasoning, Beckmann (2002) decided it was important to teach prospective elementary teachers to make sense through "explaining why." Getting prospective teachers to become comfortable explaining mathematics is not only important for their own benefit, but for the benefit of their future students. Beckmann (2002) discussed how teacher educators need to be sensitive to explanations by prospective teachers. She noted that although rigorous proofs do offer an explanation of *why* through reasoning and sense making, they probably do not benefit the understanding of the prospective teachers' future students. She suggests a better approach may be to develop reasoning through a common sense approach, a combination of logical reasoning and sense making. Beckmann (2002) listed the following desirable features of explanations: 1) The explanation is logical; 2) The explanation explains in a common-sense way (convincing to both the person explaining and the intended audience); and 3) If possible, there are several coordinated explanations (e.g., an equation

and a picture). It is important for educators of prospective elementary teachers to not assume their students know what a proper explanation is. It is common for an explanation showing how (describing procedures) to be confused with explaining why. Time and experience need to be offered for prospective teachers to develop their reasoning and sense making skills. This is especially important with the new focus of the CCSSM on understanding rather than procedure alone. It is also relevant to this study as this idea of “explaining why” aligns with the goals of the curriculum in this study; the text used was written by Beckmann (2012).

Mathematical Tasks

Another important part of the curriculum in this study is mathematical tasks and activities. Because the CCSMP describe how one should “do” mathematics, students need opportunities to “do” mathematics. Rich, cognitively demanding mathematical tasks are at the core of getting students to actively engage in doing mathematics (Barlow & Harmon, 2012; Graves, 2011; Polly & Orrill, 2012; Russell, 2012). To support students in meeting the CCSMP, the tasks should be designed to engage students in exploring mathematics through problem solving with context; tying the content and practice standards together (Barlow & Harmon, 2012; Polly & Orrill, 2012; Russell, 2012). Before the students start on the problems, the teacher should go over expectations and make sure everyone understands the task (Barlow & Harmon, 2012). This introduction could include a group reading of the task where initial examinations of assumptions are made and students are asked to think about key ideas and important information (Billings, Coffey, Golden, & Wells, 2013; Graves, 2011). Once everyone is

on the same page, students should be allowed to start on and engage in the activity or task at hand.

While the students are working on the task, teachers need to listen to their students' reasoning and pay attention to the strategies they are employing toward the task (Burns, 2013; Strom, 2013; Wenrick, Behrend, & Mohs, 2013). In this way, teachers can start to recognize strengths and weaknesses or understandings and misunderstandings their students are having (Burns, 2013; Strom, 2013). They can then use this information to further guide their instruction (Burns, 2013; Graves, 2011). This could include determining how to approach the upcoming discussion, which students—based on strategies used—will share, and what questions to ask in order to strengthen students' arguments and understanding (Barlow & Harmon, 2012; Burns, 2013; Graves, 2011; Polly & Orrill, 2012; Wenrick et al., 2013). All of these decisions will lead into the next important feature of implementing mathematical tasks which is examining student generated solutions.

An overall debriefing should occur where different strategies are shown by students purposefully selected by the teacher (Polly & Orrill, 2012). Students should be allowed to share their work while offering an explanation of what they did, why they did it, and why it makes sense (Barlow & Harmon, 2012; Polly & Orrill, 2012). Students should be encouraged to discuss their solutions and describe any difficulties they may have had (Graves, 2011). There should also be opportunity for students to analyze the work of other students through discussion (Barlow & Harmon, 2012). Communication is the key for students to develop their own reasoning and learn from others. Teachers should make sure the focus of the discussion is on mathematics and, whenever

appropriate, should ask supporting questions to help students strengthen their arguments and make connections to key mathematics (Barlow & Harmon, 2012; Polly & Orrill, 2012). Students should be asked to make connections between different strategies, approaches, and representations, perhaps leading towards generalizations (Barlow & Harmon, 2012; Wenrick et al., 2013).

The following are the two Common Core Standards for Mathematical Practice that were a focus in this study.

**Make Sense of Problems
and Persevere in Solving Them**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves,

“Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches (CCSS, 2010).

Construct Viable Arguments
and Critique the Reasoning of Others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments (CCSS, 2010). This study focuses on constructing viable arguments and not on critiquing the reasoning of others.

Historical Context and Theoretical Framework

This research study was designed around studying students' Mathematical Practices as laid out by the Common Core State Standards for Mathematics and used the theoretical framework of constructivism. This section will discuss the history of mathematics curriculum in the U.S. and how this influenced the development of the standards-based reform movement in mathematics education and the constructivist revolution in research and practice.

The New Math movement of the 1960s was brought about by the Soviet's launch of *Sputnik* and the resulting public doubt that the U.S. mathematics and science curricula were rigorous enough. The New Math movement brought about a focus on understanding rather than basic skills and pushed for discovery learning. The New Math movement never gained enough public and teacher support so with the 1970s began public outcry and the beginning of a Back-to-Basics movement. This Back-to-Basics movement pushed for a focus on procedural skills and direct instruction (Fey & Graeber, 2003). According to Steffe and Kieren (1994), though the reform movement from New Math to Back-to-Basics was felt at the teaching level, it was not felt at the researcher level as this movement did not conflict with the empiricist assumption that was used in research. "More than any other single factor, the separation between the practice of teaching and the practice of research paved the way for the emergence of constructivism in mathematics education" (p. 72).

With the 1980s came dissatisfaction with the Back-to-Basics movement and there was a push towards standards-based education. NCTM's *Agenda for Action* (1980)

called for more problem solving and a broader definition of basic skills. The National Commission on Excellence in Education called for both an increase in and adoption of more rigorous and measurable standards in their 1983 report, *A Nation at Risk* which was written in response to the Cold War and low student achievement. It was this same year in which the first article with “constructivist” in the title was published in JRME (Steffe & Kieren, 1994). The standards-based movement, along with constructivism, once again called for a focus on conceptual understanding (*why*) rather than procedural skill alone (*what*). NCTM published Curriculum and Evaluation Standards in 1989, followed by the revisions in their 1990 Principles and Standards for School Mathematics. Issues with the standards offering a “mile wide, inch deep” curriculum and a need for consistency across the nation led to the creation and implementation of the Common Core State Standards (Cobb & Jackson, 2011).

According to Steffe and Kieren (1994), the constructivist revolution occurred first in research and then in practice. When researching from a constructivist assumption, “one is studying the construction of mathematical reality by individuals within the space of their experience” (p. 75). The assumption of constructivism is that students create their own knowledge based on their histories of interactions and reflections on these interactions. This idea of studying students as they construct knowledge in the classroom environment through interactions led to a problem centered instructional approach where tasks, group work, and discussion were a part of the curriculum. “Observing and listening to the mathematical activities of students is a powerful source and guide for teaching, for curriculum, and for ways in which growth in student understanding could be evaluated” (p. 75).

As this research study involved examining prospective elementary teachers' competencies in an inquiry-based curriculum where students were working on mathematical tasks in small groups, it lent itself to the use of constructivism as a theoretical framework. Studying a curriculum with the goal for students to build an understanding *why* rather than just *what*, this study offers the potential to be a "powerful source and guide" for teacher education programs, teachers, and researchers studying the Common Core Standards for Mathematical Practice.

Conclusion

This literature review examined a variety of mathematics teacher competencies. There is an important distinction between procedural knowledge and conceptual knowledge. With the adoption of the CCSSM, more emphasis has been placed on conceptual knowledge. Also, for the past two decades, particular attention has been paid to the mathematical knowledge needed for teaching. Instrument development has helped to measure separate domains between common content knowledge, specialized content knowledge, and knowledge of content related to students. Another important competency for teachers is attitude. A definite link has been shown between cognition and attitude. Where there is currently a gap in the research is in relation to the Common Core Standards for Mathematical Practice and how they are linked to other competencies such as procedural knowledge, mathematical knowledge for teaching, and attitudes. This study will explore these associations among prospective elementary teachers' competencies, and the methodology to do so will be discussed in the next chapter.

3. METHODOLOGY

Introduction

This study was designed to provide evidence about the evolution of prospective elementary teachers' competencies. The competencies addressed in this study include procedural knowledge, mathematical knowledge for teaching, attitudes toward mathematics, and enactment of the mathematical practices of persevering in problem solving and constructing viable arguments. The evolution of these competencies was examined in the first two of three mathematics content courses for teachers at a university in the Mountain West, in a program designed to foster growth in these specific competencies. The questions that guided this research were:

1. How do prospective elementary teachers progress in their enactment of the mathematical practices of persevering in problem solving and constructing viable arguments as they move through inquiry-based mathematics content courses?
2. What relationship (if any) exists between prospective elementary teachers' procedural knowledge and mathematical knowledge for teaching, and how does this relationship develop over time as they move through inquiry-based mathematics content courses?
3. How do prospective elementary teachers' attitudes toward mathematics interact with their procedural knowledge and their mathematical knowledge for teaching, and how does this relationship develop over time as they move through inquiry-based mathematics content courses?

The Researcher

This section is to share with the reader my background, highlight what biases I bring to the study, and describe what action I took to try and limit this bias. I received a Bachelor of Science in Education degree with a Field Endorsement in Mathematics (Grade 7-12) in May 2007. Immediately after this, I enrolled in a Master of Arts in Education (MAE) program with an Emphasis in Mathematics. As a part of this program, I was able to act as a Supplemental Instruction leader for College Algebra students and taught two different courses, Mathematics for the Elementary Teacher I and Math Topics for the Elementary Teacher. After I completed the MAE degree in May 2009, I enrolled in a Mathematics Ph.D. program with a mathematics education option. Through my graduate teaching assistantship for this degree, I have taught the following courses: College Algebra, Mathematics for the Liberal Arts, Calculus I, Calculus II, Introduction to Statistics, Number and Operations for K-8 Teachers, and Geometry and Measurement for K-8 Teachers. As part of this degree program, I also completed an internship at a local elementary school where I spent a semester in one first grade and one fourth grade classroom. This was my only experience in a K-8 setting.

My history and experiences learning about and working in the field of mathematics education present a potential for bias to this research study. Having taught several different mathematics courses over six years, I have preconceived notions of the types of students who take these courses and how they behave and perform in the class. I did my best to set aside my teacher hat in place of my researcher hat. This meant not analyzing the participants actions as what I would like to see as a teacher, but analyzing

them through the eyes of a researcher where I strictly take things for what they are, no added value. I treated and analyzed all of the participants the same, regardless of their behavior and ability levels.

As a mathematics educator, I have a vested interest in K-12 mathematics education and have taken note of the widespread adoption of the Common Core State Standards for Mathematics (CCSSM). I feel there needs to be a great focus on the implementation of these standards for both in-service and pre-service teachers. I believe the curriculum of the mathematics content courses for elementary teachers at the university in this study lends itself to helping reverse the problems of poor attitudes and performance of elementary education majors in mathematics and incorporates some important components of the CCSSM. Even though I believed the curriculum was structured to help foster growth in prospective elementary teachers' competencies, I put this belief aside and took care to not insert any false growth or achievement in the participants that I felt should have occurred as a result of the curriculum.

Background and Setting – The Case

Course Offerings and Objectives

Participants in this study were students majoring in K-8 Elementary Education and they were required to take three, 3-credit hour mathematics content courses for teachers. This aligns with one of the recommendations of the Conference Board of the Mathematical Sciences (CBMS) in the 2001 report, *The Mathematics Education of Teachers* (CBMS, 2001). Recently, however, this recommendation has been increased to at least 12 semester hours (CBMS, 2012). The three mathematics content courses for K-8

teachers at this university are Numbers and Operations, Geometry and Measurement, and Advanced Topics.

These are mathematics content courses for students preparing to become K-8 elementary teachers. They are designed to provide experiences to help students broaden and deepen their own understanding of mathematics. This course sequence aims to provide mathematical knowledge that will prepare these students, as professionals, to help children become confident problem solvers and powerful mathematical thinkers. The courses focus on making sense of the mathematics that children do and being able to explain the *why* questions. Students in these courses spend considerable time communicating their mathematical ideas, both orally and in writing.

To gain the most from the class activities, it is critically important that the students keep up with all of the assigned work and attend every class. Course objectives include the following four items: 1) **Explain**, through writing and speaking, fundamental concepts and processes important in (K-8) mathematics, particularly those related to number operations; 2) **Represent** quantity and relationships between quantities in problem situations using symbols, words, and diagrams; 3) **Solve** problems through quantitative reasoning; 4) **Construct** viable mathematical arguments and evaluate the reasoning of others. Instruction in this course is intended to be consistent with the vision of the Common Core State Standards (CCSS) for Mathematics including the eight Standards for Mathematical Practice. Although the three K-8 mathematics content courses (M1001, M1002, M1100) are not methods courses on how to teach elementary school mathematics, the variety of teaching methods used in the course helps students build a solid pedagogical framework for their future teaching.

Number and Operations for K-8 Teachers (M1001) is the study of number and operations at the elementary and middle school levels, including whole numbers, decimals, fractions, percents, integers, operations, numeration systems, and problem solving. Geometry and Measurement for K-8 Teachers (M1002) is the study of geometry and geometric measurement at the elementary and middle school levels, including synthetic, transformational, and coordinate geometry; constructions; congruence and similarity; 2-dimensional and 3-dimensional measurement; and problem solving. Advanced Topics for K-8 teachers (M1100) is the study of algebra, number theory, probability, and statistics, largely at the middle school level. Topics include proportional reasoning, functions, elementary number theory, statistical modeling and inference, and elementary probability theory. The reason this study focused only on the first two mathematics content courses was because the third course was in its beginning stages of being implemented and many other universities only offer a two course sequence.

Curriculum Text

The curriculum for these courses is based on *Mathematics for Elementary Teachers with Activity Manual (3rd Edition)* authored by Sybilla Beckmann (2012). This particular text with class activities uses an inquiry-based approach to learning, allowing students to engage with and explore the material while gaining a deeper understanding of the mathematics involved. Beckmann believes this deeper understanding comes from knowing more than just *how*; students must also be able to explain *why*. It is more about understanding concepts and less about rote memorization and procedures alone. The material is related to the content teachers will be certified to teach and includes some

focus on common student misconceptions. According to Beckmann, “Prospective elementary school teachers will learn to explain why the standard procedures and formulas of elementary mathematics are valid, why nonstandard methods can also be valid, and why other seemingly plausible ways of reasoning are not correct” (p. xii). Beckmann was a member of the writing committee for the Common Core State Standards of Mathematics and realizes the importance of incorporating these standards into teacher education. She believes in order to develop a deep understanding of mathematics, students and teachers alike must actively engage in mathematical practices.

Unlike many other mathematics texts for elementary teachers, Beckmann’s book is organized around operations rather than number type. This gives students opportunities to revisit typical weaknesses (fractions) and to unify their understanding of operations. Each section in a chapter includes a summary section to help focus students on the key ideas of the chapter. There are also practice problems that include solutions for students to see what appropriate explanations look like, and problems without solutions for students to check their comprehension. A subset of these problems is highlighted as core problems that cover the crucial concepts of the section. These core problems represented the majority of homework assigned to students at the university in this study. Finally, each chapter ends with a summary tying together all of the important concepts from each section (Beckmann, 2012).

This text not only includes the regular chapters typical in most books, but also includes a section of in-class activities. Throughout the regular chapters, students are guided to related class activities in the activities section. Here students are expected to first work alone or in small groups, and then discuss as a class. These activities provide

an opportunity for students to develop a deeper understanding in that they have to explain their solutions to someone else. The activities include practice with common misconceptions and examination of calculation methods that are not standard but correct.

The following is part of class activity 2C, relating fractions to wholes:

1. At a neighborhood park, $\frac{1}{3}$ of the area of the park is to be used for a new playground. Swings will be placed on $\frac{1}{4}$ of the area of the playground. What fraction of the neighborhood park will the swing area be?
 - a. Make a mathematics drawing to help you solve the problem and explain your solution. Use our definition of fraction in your explanation and attend to the whole (unit amount) that each fraction is *of*.
 - b. Describe the different wholes that occur in part (a). Discuss how one amount can be described with two different fractions depending on what the whole is taken to be (p. CA-26, Beckmann, 2014).

Setting

In this study, all sections of M1001 and M1002 were offered in the same classroom. The classroom was set up with pairs of desks together at which four students could sit – two facing two. Altogether there were seven pairs of desks making accommodations for 28 total students in the room. On the first day of class, students could sit wherever they wanted. Then, after each chapter, students were asked to change things up by switching groups and moving seats. In this way they got the benefit of working on mathematics with many different people and had the opportunity to learn new perspectives. The classroom was set up with several white boards, a SMART Board, a document camera, a set of laptops, and several bookcases full of materials and manipulatives. All of these provided easy opportunities for exploration and for student work to be shared.

Students in these classes usually worked in small groups (up to four), exploring and discussing the material in the class activity section of the book. Because many of the activities include thoughtful questions where the students are asked to explain *why* and justify their responses, explaining why became a classroom norm. It was also an expectation for all assignments and assessments. The intention of these classes was for students to learn and discover mathematical ideas and concepts on their own and with peers through guided activities. The role of the instructor was that of a facilitator, rather than lecturer. The instructor was there not to give answers, but to ask more questions of the students, guiding them in the right direction for discovery. Discussions as a whole class were orchestrated so students could explain and discuss with each other, rather than looking to the teacher as the definitive voice of authority. A focus of the class, built into the curricular materials, was bringing attention to and working on developing the Common Core Content Standards and Standards for Mathematical Practice.

On a typical day of class, the instructor would wrap up the previous day's materials, if necessary, and introduce the new topic of the day. This could include summaries and thoughts from the students. Students would then begin to work in groups on the class activities. During this time, the instructor walked around the room, paying attention to group discussions and facilitating if necessary. If several groups were struggling or if every group appeared to be done with an activity, the instructor usually had students share their work with the class as a whole and facilitated the discussion. Students were expected to be in charge of their own learning and thus the instructor did not give out answers but looked for students to lead most of the discussion. Several activities were explored during class with this process repeated throughout the class

period. If time allowed, a wrap up discussion of the day's materials would take place. If there was not enough time, the wrap up would begin the next class period.

These courses are typically taught by graduate teaching assistants majoring in mathematics education, although on occasion they are taught by mathematics education professors or the teacher-in-residence, who has extensive experience as a middle school teacher. Each semester, there are usually three or four sections of M1001 offered and two or three sections of M1002. During the spring 2014 semester there were three sections of M1001, and during the fall 2014 semester there were three sections of M1002.

Assignments and Grading

Students involved in this study were expected to attend class regularly, participate in classroom activities and discussions, complete homework (usually all of the core problems from each section in the chapter), have regular pen pal correspondence with local elementary students, take chapter exams, and take a comprehensive final at the end of the semester. Because class participation in activities was so important, students were penalized in participation points if they missed class. They were still expected to make up the activities from the day they missed and, depending on the instructor, were able to earn some of the lost points back. If a student missed more than four classes in a semester, they would be required to retake the course. A few times throughout the semester the students were required to fill out a participation rubric indicating the level of participation they felt they gave and also how many classes they had missed. The instructor then either agreed or disagreed with their self-assessment and gave feedback, if necessary, on how to improve.

Students were also expected to complete homework for each section in a chapter. During the study, M1001 students had access to a Livescribe Pencast (an online post made via a Smartpen) with the solutions to homework problems. Students were expected to independently complete the homework problems and then enhanced their solutions with corrections or additions based on the Pencast. As long as the students completed the homework and made the enhancements, they would receive full credit. In M1002, student-led discussion boards were used for homework problems. Students were assigned specific homework problems and were expected to post solutions on the discussion board. Another student would then be assigned to critique the work of the student who initially posted, who would then respond to the critique. Other students were welcome to join the discussion and all were expected to at least read through the discussion.

At the end of each chapter studied, students were expected to take an exam over the chapter material. The exam questions were set up in a similar fashion to the questions in the text, taken either from the problems at the end of a chapter section or from the classroom activities. They almost always included directions to justify or explain *why*.

For example, the following is an exam question from a previous semester of M1001:

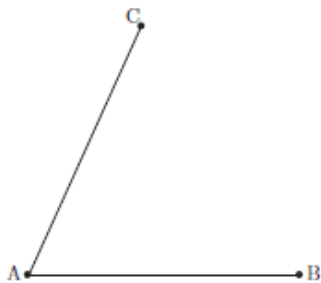
3. Consider the story problem:

One fourth of the beads in Alex's collection are red. One fifth of the beads in Alex's collection are oblong. What fraction of the beads in Alex's collection are either red or oblong?

Can the problem be solved by adding $\frac{1}{4} + \frac{1}{5}$? If yes, explain why. If no, explain why not.

A geometry question from a M1002 exam follows:

7. Use a compass and straightedge to divide the angle BAC (below) in half. Then, show a rhombus that arises naturally from your construction. Use the definition of rhombus, and the way you carried out your construction, to explain why your shape really is a rhombus.



Students were expected to problem solve and reason to arrive at an appropriate explanation. On each problem, students would receive a score of 4 - Expert, 3 - Practitioner, R – Apprentice (Redo), 1 - Novice, or 0 – No credit. Usually, if the student attempted the problem but was off the mark, they would receive an R and be required to redo the problem. They were given a certain amount of time, usually before the next exam, to get all of their redos done. If the students completed a redo, the R was replaced by a score of 3. Students were expected to correct the solution to the problem and also write a reflection about why their original solution was wrong and how they knew the revised solution was correct. Students were expected to complete all redos. If they failed to complete more than two redos, they would not pass the course.

CBMS Recommendations

With all of the variation across teacher education programs, it is important to note how the mathematics content courses for K-8 teachers at this particular university follow research- based and professional recommendations for high quality teacher education programs. The CBMS (2012) recommends teachers should learn the content they will

teach at a deeper level and from the perspective of a teacher. For teacher preparation, the CBMS (2012) recommends that prospective teachers need mathematics courses that develop a solid understanding of the mathematics they will teach; coursework should allow time to engage in reasoning, explaining, and making sense of the mathematics that prospective teachers will teach; and all courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. These recommendations are all goals of the curriculum of the courses in this study. As described above, the curriculum goes into understanding rather than procedure alone and includes components focused on student misconception. This is also in line with the goals of the CCSSM – that of aiming towards understanding.

Research Design – Embedded Case Study

According to Yin (1994), case studies are appropriate for *how* and *why* questions.

The first research question,

1. How do prospective elementary teachers progress in their enactment of the mathematical practices of persevering in problem solving and constructing viable arguments as they move through inquiry-based mathematics content courses?

lends itself to a case study approach. In particular, an embedded case study approach was used for the research design of this study. Yin (1994) defines an embedded case study as one in which more than one unit of analysis is involved, or when attention is given to a subunit or subunits. The mathematics content course sequence at the university in this study was designed to foster growth in prospective elementary teachers'

competencies. The two mathematics content courses were the unit of analysis – the case – for this study as they offered a boundary for which the research question could be answered. In other words, the students taking these courses could be distinguished from those that were not. This study also paid attention to individual students within these courses – the subunits of analysis, or participants of the study.

Both qualitative and quantitative data were collected to help illuminate the case. Quantitative data were gathered where valid and reliable instruments have been developed – namely for procedural knowledge, mathematical knowledge for teaching, and attitudes toward mathematics – and associations between variables were examined. There was no existing instrument to validly and reliably measure the enactment of the Common Core Standards for Mathematical Practice (CCSMP), and according to Mertens (1998), “Qualitative methods may...be chosen when no quantitative measure is available for the desired outcomes of a program” (p. 163). Also, qualitative research affords one to take a deeper look at people’s thoughts and behaviors and allows the researcher to ask them why they responded the way they did (Creswell, 2007). With qualitative analysis, there is opportunity for rich descriptions and deeper understanding that go beyond what a quantitative measure can “tell us.” For these reasons, a qualitative approach was used to gain a better understanding of how prospective elementary teachers’ mathematical practices develop as they progress through the two mathematics content courses described in the previous section. The quantitative data served to aid in the selection of the participants for the qualitative analysis.

I collected quantitative data from the entire cohort of students moving through the two courses. I selected a smaller subunit of students from the cohort from which to

gather qualitative data, taking a deeper look at their mathematical practices. Depending on prospective elementary teachers' beginning procedural knowledge and mathematical knowledge for teaching scores, there could be differences in where they begin and develop in their mathematical practices. Understanding the developments of different prospective elementary teachers' mathematical practices as they move through two mathematics content courses could offer greater insight into whether development of mathematical practices looks similar or different for a variety of students moving through the same courses. A description of how the participants for the qualitative portion were selected will be discussed in the next section.

Participants

I collected data from students enrolled in M1001 during the spring 2014 semester and those enrolled in M1002 during the fall 2014 semester at a mid-size university in the Mountain West. There were three sections of M1001 in the spring 2014 semester and three sections of M1002 in the fall 2014 semester with enrollment for each section capped at 28. All students enrolled in M1001 in spring 2014 and M1002 in fall 2014 were asked to complete two multiple choice exams and a likert-scale survey at the beginning and end of the semester as part of the curriculum for these courses. The second multiple-choice exam assessing procedural knowledge was required to be taken by the students in the mathematics testing center on campus. Participants for the quantitative portion of this research were all enrolled students who gave permission for their data to be used. During the first week of class, each student received a short consent form that provided, in writing, a short description of the study, of how there were no

foreseen risks or benefits to them, and of how their data would remain secure and confidential. I was also there to address concerns and gave them a quick description orally. Students were asked to sign the form if they were willing to participate. The data from those students who signed the consent form was used for quantitative analysis.

Participants for the qualitative portion of this research project were selected from the students enrolled in M1001 in spring 2014 who planned to enroll in M1002 in fall 2014. I employed stratified purposeful sampling based on the results of the quantitative outcomes at the beginning of the spring 2014 semester for consenting students in M1001. Mertens (1998) described stratified purposeful sampling as “a combination of sampling strategies such that subgroups are chosen based on specified criteria, and a sample of cases is then selected within those strata” (p. 263). I used the quantitative data to identify potential information-rich subunits to highlight the case. I wanted to select subunits that represented the full spectrum of procedural knowledge and mathematical knowledge for teaching among students in these classes. This helped illuminate any differences in how prospective elementary teachers progress in their enactment of certain mathematical practices defined by the Common Core. These subunits provided an opportunity to learn more about and inform a better understanding of the research problem, important aspects of case selection (Creswell, 2007; Stake, 2005). The intention was to provide a detailed description of the different subunits, including their beginning procedural knowledge and mathematical knowledge for teaching, both individually and related to the group.

Once the quantitative data were gathered, the pair-wise association between procedural knowledge scores and mathematical knowledge for teaching scores was examined. That is, a scatter plot of procedural knowledge vs. mathematical knowledge

for teaching was created. I believed there would not be as much variability within the mathematical knowledge for teaching scores as compared to the procedural knowledge scores as these prospective elementary teachers were just beginning their coursework and probably had not taken any teaching courses. For this reason, I stratified the mathematical knowledge for teaching scores at the median and stratified the procedural knowledge scores into thirds. This split the scatter plot of procedural knowledge vs. mathematical knowledge for teaching into six regions. The choice of dividing the scatter plot into six regions rather than four was to achieve more variation and account for possible attrition of participants.

Once the scatter plot was divided into the six regions, an examination was made to see if there were students represented in each region. It was reasonable to assume most of the regions would contain students as there were more than fifty students represented. If there were not students represented in each region, I would have focused on splitting the procedural knowledge scores into thirds. Because I wanted variation, the mathematical knowledge for teaching scores would have then been used to select participants toward the high and low end of what was represented within each of those three groups. If there was not much variation within mathematical knowledge for teaching (no relatively low or high scores) within procedural knowledge bins, then I would have examined attitude scores. This process was to ensure a wide variety of participants in relation to procedural knowledge, mathematical knowledge for teaching, and possibly attitudes toward mathematics.

A total of twelve students were to be chosen and contacted. If there were students represented in each of the six regions, then two from each region would be contacted. If

division into just three regions was necessary (procedural knowledge thirds), then four students from each region would be contacted. There were a few additional considerations that were made. First, I did not want to select any students who were repeating the course as they were probably more likely to drop out of the study or the class. Second, for ease of later being able to observe the students in the classroom, an equal number of students from each course section were to be selected. Finally, if there were any unusual or interesting cases who did not fall in line with the trend of the other students, they would be selected as well. An example of this interesting case would be a student scoring low in procedural knowledge but high in mathematical knowledge for teaching (relative to the other students in the study) as this was not expected or considered typical. I planned to contact all twelve of these students, ask for their participation in the research, and ask whether they planned to take M1002 in the fall 2014 semester. Based on their responses, I would select six participants for the study. If six participants did not agree to participate, I planned to go back through the data and choose more cases based on the same selection process.

Quantitative Data Collection

This study used quantitative measures to examine prospective elementary teachers' procedural knowledge, conceptual knowledge, beliefs, and practices. Quantitative data gathered in this study measured prospective elementary teachers' procedural knowledge, mathematical knowledge for teaching, and attitudes toward mathematics. The reason for examining mathematical knowledge for teaching rather than conceptual knowledge was the lack of a specific conceptual knowledge instrument in the

literature and the increased importance of the new domain of mathematical knowledge for teaching, for which valid and reliable measurements exist. A discussion of the instruments chosen for each measure will follow.

Procedural Knowledge Measure

The Mathematical Association of America (MAA) offers colleges and universities a web-based suite of mathematics placement tests. These tests were developed by a panel of college mathematics teachers involved with courses requiring placement. Final approval of the tests was made by the MAA. These tests are intended to quickly and efficiently help schools place students into appropriate mathematics courses in which they will be successful. According to Maplesoft (2011), before an MAA placement test is approved, it must undergo piloting where the results are carefully analyzed and necessary adjustments are made. Although every version of the exam cannot undergo detailed analysis, the algorithmic tests are based on the original algorithms to create parallel forms. The placement tests have high content validity, according to college instructors, and have been found valid and reliable for mathematics departments placing students for three decades. The placement tests have been used by hundreds of schools in the U.S. since 1977 (Maplesoft, 2011).

At the university in this study, these placement tests have been used since 2010. There are four levels of calculator-based tests offered; Arithmetic and Skills (Level II), Basic Algebra (Level III), Algebra (Level IV), and Calculus Readiness (Level V) (Maplesoft, 2011). In order to meet the prerequisite requirements to enroll in M1001 (the first mathematics content course for prospective elementary teachers), students must meet

one of the following requirements: Pass the Level III mathematics placement exam (65% or better), pass College Algebra (C- or better), achieve an ACT mathematics score of 23 or better, or achieve an SAT mathematics score of 540 or better.

The Level III mathematics placement exam was used to measure prospective elementary teachers' procedural knowledge and provide a baseline level of prerequisite knowledge held at the beginning of the semester. The Level III mathematics placement exam has 25 multiple choice questions measuring simple computational skills and manipulations of basic algebra (Maplesoft, 2011). A raw score (percentage) was used to report the scores of the prospective elementary teachers. It was reasonable to think that the percentage scores from the M1001 students in spring 2014 would range from 50% to 100%, considering that a score of 65% is one of the prerequisite requirements for the M1001 course.

I compared the items at Level III to the CCSSM. All items were found to align with the CCSSM. Fourteen of the questions aligned with grade 6-8 standards, and the remaining eleven questions aligned with high school algebra standards. Example items include:

1. Which of the following points lies on the line $2x + 3y + 4 = 0$?
 - a. $(0, 4/3)$
 - b. $(-3, 5/2)$
 - c. $(-3, 2/3)$
 - d. $(3, 10/3)$
 - e. $(3, -13/2)$

2. $4[5 - 2(6 - 7)] =$

- a. 12
- b. 8
- c. -28
- d. 28
- e. -12

As is illustrated, these items measure basic procedural knowledge through a multiple choice format. The first sample item is aligned with an eighth grade Common Core Function standard while the second sample item aligns with a sixth grade Number System standard. All items are aligned with the following Domains of standards: high school Algebra and Number and Quantity, eighth grade Functions and Expressions and Equations, seventh grade Expressions and Equations and Geometry, and sixth grade Number System, Ratios and Proportional Relationships, and Expressions and Equations.

Mathematical Knowledge for Teaching Measure

The Learning Mathematics for Teaching/Study for Instructional Improvement project at the University of Michigan developed test items that measure not only mathematical knowledge, but also mathematical knowledge needed for teaching. “These items probe whether teachers can solve mathematical problems, evaluate unusual solution methods, use mathematical definitions, and identify adequate mathematical explanations” (Hill & Ball, 2006). These measures can be used to examine how a groups’ mathematical knowledge for teaching develops over time and how this knowledge relates to other competencies (Hill & Ball, 2006). This purpose aligned well with the goals of this study

which were to examine the relationships between prospective elementary teachers' mathematical knowledge for teaching and other competencies as well as how this knowledge and the relationships change over the course of two semesters of mathematics content courses.

I found no specific instruments measuring conceptual knowledge. The MKT provided a reasonable substitute as it measured mathematical knowledge for teaching in prospective elementary teachers using items that addressed conceptual mathematics. Hill, Dean, and Goffney (2007) wrote that analysis of this instrument “has allowed us to rule out common problems and critiques of multiple-choice items. It does not appear, for instance, that CK [content knowledge] items draw mainly on respondents' ability to recall rules or algorithms. Instead, mathematical reasoning—and in some cases, justification—are required to come to an appropriate answer” (p. 92). This indicates the measure examines more than just procedural knowledge; it also assesses mathematics content knowledge at a deeper and more conceptual level.

The following are examples of released MKT items.

4. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

- a) Four is an even number, and odd numbers are not divisible by even numbers.
- b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).
- c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.
- d) It only works when the sum of the last two digits is an even number.

7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have \$1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

Hill et al. (2007) validated this measure by providing “positive evidence on a key assumption in our measures development: that teachers’ scores predict mathematical characteristics of their classroom instruction and student learning from that instruction” (p. 117). If this measure is truly indicative of teaching performance and student gains, then it is important to apply it to the prospective teacher population as well.

Several forms of this assessment exist, measuring content knowledge in number and operations; geometry; and patterns, functions, and algebra. Number concepts and operations are the most dominant topics in K-6 education. These are fundamental topics in M1001 and related to the content prospective elementary teachers will see in M1100. Prospective elementary teachers in this study were assessed using Form 2001 A, which has 26 multiple choice items and has been shown to be both valid and reliable. This form was piloted with California teachers who had participated in state-wide professional development. With a sample of 411 teachers, the reliability for this form was found to be

0.72 (Hill & Ball, 2006). Scores were reported as Item Response Theory (IRT) standardized scores (z-scores) with possible outcomes ranging from -2.976 to 2.450. IRT scale scores represent standard deviation units for a standardized distribution with a mean of 0 and standard deviation of 1. This distribution follows the 68-95-99.7 rule where approximately 68% of participants will fall within one standard deviation of the mean, 95% within 2 standard deviations, etc. The reason for reporting IRT scores was because the items on this assessment are not criterion-referenced or norm-referenced but instead are intended to measure average teacher ability – “which means deliberately making about half of the items more difficult than most teachers’ ability” (Hill & Ball, 2006). For this study, it is reasonable to think student MKT scores will be below zero as this measure was created for practicing teachers.

Attitudes Toward Mathematics Inventory

Many instruments are available that measure student attitudes toward mathematics. Chamberlin (2010) conducted a review of existing instruments. The three criteria used to select the instruments for review were: 1) Statistical data showing validity and reliability of the instrument, 2) Innovation in regard to new facets of affect, and 3) Amount of use as seen in follow-up studies and literature reviews. Chamberlin (2010) found the most widely used attitude scale across disciplines to be the Fennema-Sherman Mathematics Attitude Scale, but because it is so old, the reliability and validity are less stable. The Attitudes Toward Mathematics Inventory (ATMI) is more recent, created in 1996, and also considers multiple components of affect. Although not as

widely used as the Fennema-Sherman instrument, Chamberlain (2010) supported its potential to become the most widely used.

The ATMI is shorter than the Fennema-Sherman Mathematics Attitude Scale with a simple four factor structure rather than nine factors (Tapia & Marsh, 2004). The ATMI consists of 40 items based on a five-point Likert scale (see Appendix A). There are fifteen items associated with self-confidence (anxiety), ten for value, ten for enjoyment, and five for motivation. The five responses to each item are: A – Strongly Disagree, B – Disagree, C – Neutral, D – Agree, and E – Strongly Agree. There are 29 regular items and 11 reversed items. The regular item responses are given a numerical value from one to five, with one being assigned to a response of A and five being assigned to a response of E. The reversed item values are found by subtracting the regular value from six. Therefore an A would be scored as one less than six, or five (Tapia, 1996). Higher scores are associated with positive attitudes. I used the composite ATMI score (sum total) – which can range from 40 to 200 – along with the overall mean and standard deviation of scores.

Reliability and validity of the ATMI was initially examined on a sample of 544 high school students in Mexico City. A maximum Cronbach alpha of .9667 was found after deleting nine of the original 49 items. This indicated a good reliability and internal consistency of the instrument. Content validity was determined by having blueprints of the factors to be measured and having experienced mathematics teachers check items and give feedback. Construct validity was achieved because item-to-total correlation was higher than .49 for all items. This indicated only one construct was being measured (Tapia, 1996). Tapia and Marsh (2002) confirmed the use of the instrument and its four-

factor structure with U.S. college students. The sample of college students was 80% Caucasian and 20% African-American who ranged in age from 17-34 years old. Confirmatory factor analysis was used and showed justification for using the four-factor assessment on older students, and reliability estimates remained good.

The reasoning for reporting the overall sum rather than focusing on the four factor structure was influenced by the results of a pilot study conducted the semester prior to the study. The ATMI was given to prospective elementary teachers in M1001 and M1002. A factor analysis conducted with the data from this pilot study did not show the same four factor structure. This is probably due to the fact that prospective elementary teachers are a unique population of mathematics students.

Qualitative Data Collection

In qualitative data collection, “instead of using a test or questionnaire to collect data, the researcher is the instrument that collects data by observing, interviewing, examining records and documents in the research setting, or using some combination of these methods” (Mertens, 1998, p. 317). In the case study portion of this research, observations, interviews, and documents contributed to the body of data. Having multiple data sources allowed for triangulation, strengthening the validity of results. This was partially due to being able to fill in the gaps that could be seen from one source of data but not another. This idea will be illustrated in the data matrices at the end of this section. The documents examined were chapter exams from M1001 and M1002. These documents and the observations were used to guide the interviews. Detail about each of these qualitative data sources follows.

Exam Questions

The qualitative portion of this research focused on gaining a better understanding of how prospective elementary teachers develop in their mathematical practices of persevering in problem solving and constructing viable arguments. Chapter exam questions were used as a form of documentation data because they offered much insight into the problem solving and argumentation processes of the students. The exam questions in M1001 and M1002 were designed to test students' understanding of the material and almost always required an explanation or justification. All of the students across course sections took a common exam. Chapter 2 and chapter 5 exams were collected in M1001 during spring 2014 while chapter 11 and chapter 13 exams were collected in M1002 during fall 2014. These chapters were selected because they were in the middle of the semester rather than at the very beginning or the very end. Also, the practice standard of constructing viable arguments or making sense of problems and persevering in solving them was a highlighted Common Core standard in the selected chapters.

As soon as the participants completed their chapter exams, photocopies were made to analyze and use in the interviews. Their written work on each question was analyzed using the same protocol as for the observation and guided the questioning in the interview. The protocol was a matrix which used the language of the Common Core Standards for Mathematical Practice of persevering in problem solving and constructing viable arguments. For example, the Common Core practice standard "Making sense of problems and persevering in problem solving" states, "Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs

or draw diagrams of important features and relationships, graph data, and search for regularity or trends” (CCSS, 2011). Therefore, in the protocol there was a section for evidence of relating multiple representations. Another example is from the Common Core practice standard of constructing viable arguments. The standard states, “Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments” (CCSS, 2011). Therefore, the protocol contained a section for evidence of understanding and use of prior knowledge and/or stated assumptions (see Figures 2 and 3).

Observations

Qualitative observation should take place in a naturalistic setting (Mertens, 1998). Case study participants were observed in their mathematics classroom where they were, on a regular basis, working together in groups on solving problems and answering questions. These observations allowed me to see how they problem solved and made arguments in a group setting. Because this was a group setting, it was important for several observations to be made. In these classes, the students are often asked to switch groups, usually at the beginning of a new chapter. Students may behave differently depending upon the group in which they are working, and their comfort level may change throughout the chapter as they get used to their group members. Notes about group dynamics were included in the observation protocol.

Case study participants were observed throughout the chapter 2 material (six days) and chapter 5 material (four days) during the spring 2014 semester in M1001. I positioned myself near the student I was observing, but did not linger over their shoulder.

I constructed an observation protocol (see Appendix B) to guide me in looking for evidence of students meeting the guidelines of the Common Core Mathematical Practices of making sense of problems and persevering in solving them and constructing viable arguments. Because the problem solving standards say students should “monitor and evaluate their progress and change course if necessary” (CCSS, 2011), the observation protocol included a section looking for evidence of this in the student. Notes were taken if the student demonstrated this or discussed their progress within their group. For example, if multiple attempts were made because no progress was being made, this was evidence and notes were taken about the situation and how it came to fruition.

Researcher observation notes were supplemented by an audio recording of the group interactions through the use of a Smartpen. Use of the Smartpen was discussed with the case study participants when they were contacted about participation in the study. As soon as the observation was done, I made reflections and took additional notes. I reflected and analyzed the overall class period and what happened, e.g., what activities took place, what material was covered, how the group interacted together, the extent to which the participant was engaged (or not), and the degree of problem solving and argumentation. I wrote personal reflections about why I think things transpired the way they did as well as ideas for better future observations. I transcribed all audio from the Smartpens. Once this was done, I made a comparison between the transcriptions, the observation protocol notes and the marginal notes, and made further additions and analysis.

Interviews

Interviewing allows one to find out about things they cannot observe and to learn about the perspective of others (Patton, 2002). Without the interaction of the interview process, there would be a lot left unsaid and unlearned from the documents and observations. Inspecting the chapter exams and conducting the observations before the interviews provided time and material to develop relevant questions to uncover what remained hidden from view. I used a combination of what Patton (2002) calls an interview guide approach and a standardized format. An interview guide approach is where the interviewer has a checklist of topics to cover rather than a list of several questions. A standardized format is one in which all of the key questions are written out ahead of time. Using a combined approach allowed more flexibility for the incorporation of chapter exam and observation material into the interview. The checklist served to make sure everything that needed to be covered was covered.

The interviewing process in this study was meant to tie together my observations of participants with their written work on exams and to gain perspective directly from the participant. Participants were asked about occurrences in the classroom where they were problem solving or arguing. They were also asked about their solutions to exam questions. The goal was to better understand their thought process as they worked through and solved problems and created arguments and justifications. The interviews were intended to uncover the thought process and problem solving processes that could not be seen on the written exam solutions and better understand the participants' viewpoints of classroom group work, problem solving, and discussion. The interviews were closely based around the observations and exams.

The interview protocol (see Appendix C) included two “get to know you” questions in order to build rapport with the student and guide them in the direction of offering descriptive answers, two important components of interviewing laid out by Patton (2002). Participants were also asked how they felt they were progressing with regard to problem solving and justifying and to what they attributed this. I included two overarching questions, aimed at gaining perspective into the problem solving and argumentation processes as the students experienced them on the exam. The interview checklists consisted of components from the protocol to ensure a holistic picture of evidence of the Standards for Mathematical Practice of persevering in problem solving and constructing viable arguments was gained. In anticipation that some students would not offer much verbal description, supplemental questions were created to provide evidence for the items in the checklist. The interview questions in relation to the observation were less structured but were refined based on each observation. Participants were also asked to describe their perception of the dynamics of their group which was compared with the observation protocol notes.

Following Patton’s (2002) suggestions for writing good interview questions, the interview used open-ended questions in an effort to avoid a dichotomous response. A closing question asked the participant if they had anything to add. I remained neutral, not ever taking the role of instructor (by showing satisfaction or dissatisfaction or commenting on the correctness of their solutions) and not showing signs of favor or disfavor. Probes, reinforcement, and summarizing transitions were used as they were all recommendations made by Patton (2002). I used probing to gain more insight or detail where needed and used of statements such as “will you please tell me more about...” I

provided head nods as a reinforcement technique during responses and thanked the participants for answering the questions throughout the interview. I provided summarizing transitions to inform the participant one section of the interview was ending and another was beginning. For example, after the participants were asked about their solutions to exam questions, they were asked about classroom observations. Summarizing transitions such as, “We’ve been talking about your solutions to some exam questions and how you solved the problems and justified your answers. Before I ask you some questions about the classroom observations, are there any additional strategies you used or anything else you would like to add?” were used.

In total, I conducted four interviews per case. Each of the interviews were conducted as soon after the chapter exam as possible so there was little time for them to forget what they did on the exams or during the chapter material. The interviews took place at a neutral location, in a conference room. All interviews were audio-taped and videotaped using a hover cam. The hover cam was not focused on the student but rather on the exam questions being discussed during the interview. At the completion of each interview, I analyzed the interview and the process and made any additional notes. The interviews were transcribed and afterwards compared with my initial notes. Then, I made further notes and analysis.

Data Collection Matrices

Figure 1 represents a data matrix linking research questions to the data sources that helped answer them. Figures 2 and 3 focus on question 1 and show what qualitative data source(s) would most likely show evidence of how specific aspects of the Common

Core mathematical practice standards #1 and #3 are enacted. I took the two mathematical practices and broke them down into specific components, which students were expected to demonstrate in order to meet the standard. Figure 2 relates to the mathematical practice of persevering in problem solving and Figure 3 relates to the practice of constructing viable arguments.

Procedures

A meeting with the instructors of M1001 was scheduled the day before the spring 2014 semester started to discuss scheduling, exam administration, the use of Smartpens in class, observations, and any conflicts that might arise. Throughout the semester, I remained in close contact with the instructors, making sure everything was in order. During the first week of the spring 2014 semester, the procedural knowledge exam, MKT, and ATMI were administered and consent forms were given to all students in M1001. The MKT and ATMI were administered in class and turned in along with the consent form. The procedural knowledge exam was taken in a testing center outside of class.

		Data Sources					
		PK	MKT	ATMI	Exam Questions	Observations	Interviews
Research Questions	1. How do certain prospective elementary teachers demonstrate progress in their enactment of the mathematical practices of persevering in problem solving and constructing viable arguments?				*	*	*
	2. What relationship (if any) exists between prospective elementary teachers' procedural knowledge and mathematical knowledge for teaching, and how does this relationship change?	*	*				
	3. How do prospective elementary teachers' attitudes toward mathematics associate with their procedural knowledge and their mathematical knowledge for teaching, and how does this relationship change?	*	*	*			

Figure 1. Data Matrix.

		Data Source		
		Exam Questions	Observations	Interviews
Evidence of Persevering in problem solving	Attempting to understand the problem	*	*	*
	Devising a plan prior to action		*	*
	Evaluating progress and changing plans (if necessary)		*	*
	Relating multiple representations	*	*	*
	Sense making		*	*

Figure 2. Data sub-matrix for persevering in problem solving.

		Data Source		
		Exam Questions	Observations	Interviews
Evidence of constructing viable arguments	Understanding and use of prior knowledge and/or stated assumption	*	*	*
	A logical progression of arguments	*		*
	Justification/Explaining why	*	*	*

Figure 3. Data sub-matrix for constructing viable arguments.

I spent the next week examining data from scatter plots and selecting participants. Originally twelve participants (four per course section) were to be identified, but thirteen participants were actually identified. They were each contacted, either via email or a visit to their class, about potential participation in the study. They were asked whether or not they planned to take M1002 in the fall 2014 semester, and if they were willing to participate in four interviews (two per semester) which were to be approximately 30 minutes each. I planned to choose six willing participants, two participants per section, and was able to do so as twelve of the thirteen asked agreed to participate.

Once the participants were identified, I conducted classroom observations throughout the Chapter 2 material in M1001. I set up interviews with each case as soon after their Chapter 2 exam as possible. All but one of the interviews were conducted the same day the participants took their chapter exam. The interviews took place in a conference room on campus. Beginning with the first observations, I transcribed and analyzed the data, reflected on results, and decided on any necessary adjustments to improve the design and protocols for the observations and interviews. The cycle of observations and interviews was repeated for Chapter 5 material. During the final week of classes, I administered the procedural knowledge exam, MKT, and ATMI to all students again, in the same fashion as before. This process worked very similarly in the fall 2014 semester when the students were in M1002 (see schedule in Appendix D).

Data Analysis

Quantitative Analysis

Quantitative analysis was used to help answer the second and third research questions, which explored potential associations between prospective elementary teachers' procedural knowledge, mathematical knowledge for teaching, and attitudes toward mathematics. Scores on the measure for procedural knowledge were given as raw scores (overall percentage correct), scores on the mathematical knowledge for teaching instrument were given as scaled IRT scores, and scores on the attitude instrument were given as overall sums. Pair-wise associations were examined through the use of basic scatter plots. Because there appeared to be a correlation between the pair-wise

comparisons, further analysis was conducted. This included calculating correlation coefficients.

There were a total of three scatter plots for each time period (January, April, August, and December). One scatter plot represented the association between procedural knowledge scores and attitudes toward mathematics scores. I assumed there would be a positive correlation between these variables as there is a tendency to like mathematics if you are good at math. Another scatter plot represented the association between procedural knowledge scores and mathematical knowledge for teaching scores. I was unsure whether or not there would be a relationship between these variables. I assumed there would be a positive relationship, even though these students did not have any experience with mathematical knowledge for teaching prior to M1001. Therefore, I was thought there would probably be more variation with procedural knowledge scores than there would be with mathematical knowledge for teaching scores. The final scatter plot represented the association between mathematical knowledge for teaching scores and attitudes toward mathematics scores. I assumed there would be a positive correlation between these variables as it has been shown in previous studies (Gleason, 2007). In addition to the scatter plots, boxplots also helped paint a picture of overall group performance and how this changed throughout the two course sequence.

I created a total of four snapshots (scatter plots comparing competencies at a specific time) for each pair-wise comparison. One was at the beginning of the spring 2014 semester for M1001 students, one at the end of the spring 2014 semester, one at the beginning of the fall 2014 semester for M1002 students, and one at the end of the fall 2014 semester. I also created four boxplots for overall group performance on each

measure and four profile plots showing individual student change over time. Change in the associations over time was inspected by looking for patterns in the sequence of scatter plots and comparing the correlation coefficients.

Qualitative Analysis

Qualitative analysis was used to answer the first research question, which was related to the evolution of prospective elementary teachers' enactment of the mathematical practices of persevering in problem solving and constructing viable arguments. "Data analysis in qualitative studies is an ongoing process. It does not occur only at the end of the study as is typical in most quantitative studies" (Mertens, 1998, p. 348). By following a set of prospective elementary teachers throughout the course of two semesters, I was able to conduct analysis as an ongoing, nonlinear process. This is what Creswell (2007) calls the data analysis spiral, where the researcher moves in analytical loops rather than linearly. The spiral starts with data collection and ends with a narrative. In between, the researcher organizes data, reads everything several times while writing reflective notes, starts to define initial categories and classifications while interpreting and describing what they see.

After each day of observations, I reflected on the entire day and took additional notes of things not written initially on the protocols. I analyzed why things happened the way they did that day, took notes of the participants' behavior and the group dynamics, and made comparisons between course sections. I uploaded the audio from the Smartpen to my computer and transcribed each of the six participants' classroom interactions from the day of observation. I then read through each transcript, making margin notes along

the way. I then read through it again and made comparisons between the transcriptions and notes from the observation protocol to make sure I had filled the protocol out completely. Finally, I read through everything again and looked for initial codes, which came from the margin notes and items from the protocol. For example, an initial code was: Asked questions for clarification, which was an item in the row of Attempting to Understand the Problem in the protocol matrix. Once I completed the observations for the chapter I filled in details in the interview protocol. That is, I created specific questions for the interview based upon what happened in the observations.

The next step of data collection involved gathering the participants' chapter exams. After participants took the chapter exam, I obtained a copy from their instructor. I made two photocopies, one for my notes and one for use during the interview. The same protocol used in the observations was also used to analyze the exam questions, looking for evidence of enactment of the Mathematical Practices. This allowed me to look for evidence of the participants meeting specific components of Common Core Standards for Mathematical Practice 1 and 3 in their written exam solutions. Once I took margin notes on the exam and filled out the protocol, I reflected on the participants' overall performance (in relation to the Mathematical Practices) and took any additional notes. I also analyzed which components of the Mathematical Practices I could and could not see on the written exam. I looked for initial codes, which again came from margin notes and items in the protocol (evidence of meeting the standard) and made comparisons between the codes of observations and chapter exams. Responses on exam questions guided the interview process and were used as a visual aid in the interview. I

pointed to the actual exam questions and the student's response and asked about them in the interview.

For the final step of the first cycle of data collection, I conducted the interview. As mentioned above, the observations and exam responses were used to guide the interview and add more detail to the interview protocol. If I noticed participants have an "Aha!" moment in class, where they came to the solution to a problem without me seeing what happened, I would ask them about it in the interview. I also asked them about specific behaviors and attitudes they exhibited in class. Right after the interview, I took notes of things I forgot to ask or what I forgot to write down during the interview. I recorded my impressions of how the interview went and took note of anything unusual or interesting that happened. I then uploaded audio and visual recordings to my computer and started transcribing. I read each transcript several times, and took notes in the margins. I then read through the transcript and margin notes again and filled out the protocol. I examined my initial codes from the margin notes and evidence in the protocols, and made comparisons to the codes from the observations and chapter exams.

After the first cycle of data collection was complete, I decided the data I was collecting was adequate enough to be able to answer my research questions. I decided to adjust the protocol by adding more detail and language from the CCSMPs. I also rephrased some of the interview questions in order to gain more information from the participants with regard to their problem solving process. When the first round of data collection was complete, I began to look at refining initial codes and categories with the ultimate goal of breaking down the many codes and categories into a few themes (Creswell, 2007).

This process continued for the next round of data collection – the second round of classroom observations, exam collection, and interviews. I looked for new codes and new similarities and differences within and between participants. Over the summer I worked toward moving from codes to categories. It seemed natural that some of the categories come from the rows of the protocol I created. I also discovered the categories of student behavior, characteristics, and attitudes. I analyzed each category and decided if I was gaining enough information from the participants and how I could improve in data collection. For example, I decided I needed more information about what types of representations participants were using and if they were making appropriate connections between representations. I also realized the need to be more specific in my codes for participants' argumentation. For example, I decided I needed to go into more detail about what makes an argument good or vague.

I went into the next semester with these adjustments made and in mind and continued the cycle of data collection two more times. Once this data was transcribed and analyzed, I decided the need to go through all of the transcriptions and protocols again. I went through the second time with the refined coding scheme, making sure I did not miss anything from the first round of analysis. I also decided to make note cards to use for building categories from the codes. I made one note card per margin note or item in the protocol.

Once I had note cards created from the second round of analysis, I began to organize all of the note cards into categories. I ended up with similar categories to the ones I had come up with over the summer: there were one or two categories for each row of the protocol, one for student characteristics, and one for attitudes. For example, within

the protocol section of Building Logical Arguments I found the categories of using faulty logic and using inappropriate vocabulary. Upon this further analysis, I also decided the protocol sections of understanding the problem and devising a plan were too intertwined to be considered separate categories. After uncovering these categories, I stepped back to look at the big picture and pick out overarching themes. These themes came from analyzing patterns within and across participants across categories. One example of this was noticing a difference between the arguments offered in class to those offered on the exam, and how this related to the overall curriculum.

In addition to analyzing data, I created a detailed description of the participants and any discrepancies between the intended curriculum, as described in the beginning of this chapter, and the enacted curriculum. This included the participants' background knowledge and attitudes coming into this course, how they progressed through the initial mathematics content course for elementary teachers in terms of their practices, where they left the first course and began the second course in terms of knowledge and attitudes, how they progressed through the second mathematics content course in terms of their practices, and where they left the second course in terms of knowledge and attitudes.

Issues of Transferability and Credibility

It is the job of the qualitative researcher to supply a “thick description” about each case and the contexts involved in order for the reader to generalize subjectively. There is no statistical inference but “instead, generalizing from case studies reflects substantive topics or issues of interest, and the making of logical inferences (*analytic generalization*)” (Yin, 2006, p. 114). This type of generalization is often referred to as

transferability (Guba & Lincoln, 1989 as cited in Mertens, 1998). Because the case involves the use of a popular text for mathematics content courses for elementary teachers, there is opportunity for other universities to make an analytic generalization to their situation. I ensured transferability by providing an in-depth description of the case, both the intended and enacted curriculum, and all of the participants. I chose a purposeful sample which allowed me to describe what makes the participants different from one another. I also built enough rapport with these students to build an even better picture of both the uniqueness and similarities between participants. Having been a part of teaching these types of mathematics education courses for elementary teachers for six years, I had a good sense of the curriculum and environment of these courses. This allowed me to provide the thick description necessary for the setting involved with the curriculum.

According to Stake (2005), credibility of a qualitative study comes from triangulation. Triangulation “serves...to clarify meaning by identifying different ways the case is being seen” (p. 454). In other words, we are gaining “diversity of perception.” By using the data sources of exam questions, observations, and interviews, I was able to “see” the participants and case in several different ways. The exam questions let me see how the participant handled problem solving and argumentation in a high stakes situation, but the questions did not allow me to see all of the thought processes involved. The observations allowed me to see how the participant problem solved and argued in a group setting. Finally, the interviews allowed me to get at the hidden thought processes involved in answering the exam questions. The interactions in the interview gave me insight into each participant’s perspective about problem solving and arguing in a

mathematics class. These three sources of data allowed me to see the participants from multiple angles.

Also adding to the credibility of the study are my prolonged and substantial engagement, peer debriefing, and member checks (Mertens, 1998). I spent time with my participants over two semesters. I used my peers, fellow graduate students, as listeners and questioners in order to get my thoughts and reflections on the data in the open and to gain more insight and guidance on the process of analyzing data. Finally, I used member checks, mostly by incorporating summarizing transitions in the interviews where I summarized what had been said in order to check for credibility. In addition, I created a vignette for each participant after a round of observations, collecting exam questions, and interviewing to send to them via email for review. They were asked to read through the vignette and then either respond through email or a meeting time to discuss whether or not they agreed and what they would like added or adjusted.

Conclusions

This chapter described the embedded case study design used in this research. The case consisted of a two course sequence of inquiry-based mathematics content courses for elementary teachers. Quantitative and qualitative data was collected from the students, or participants embedded within the case. This chapter discussed the selection process for participants in the study. The participants who contributed to the quantitative data consisted of the cohort of students who moved through the two course sequence over the course of a year. The participants who contributed to the qualitative data were selected by choosing a wide variety of students based on their initial competency levels relative to

the rest of the class. This chapter offered descriptions of and justifications for the use of the quantitative measures, namely the MKT, ATMI, and procedural knowledge exam. Next, the qualitative data sources of observations, chapter exams, and interviews were discussed. The chapter continued with an explanation of my quantitative and qualitative data analysis processes and concluded with a discussion of issues of transferability and credibility.

Chapter 4 will include a discussion of how the enacted curriculum compared to the intended curriculum and the variations found between instructors. Descriptions of each of the participants who contributed to the qualitative data will be offered. Finally, results from both the qualitative and quantitative data that helped to answer the research questions will be shared.

4. RESULTS

Introduction

This study examined the evolution of prospective elementary teachers' competencies as they moved through the first two of three mathematics content courses for elementary teachers using an inquiry-based curriculum. These competencies included Mathematical Knowledge for Teaching (MKT), procedural knowledge, attitudes towards mathematics, and enactment of two of the Common Core Standards for Mathematical Practice (CCSMP), the practices of persevering in problem solving and constructing viable arguments. The data collection for this study was completed in four phases for both the quantitative and qualitative portions. Quantitative data relating to the students' MKT, procedural knowledge, and attitudes toward mathematics were collected at the beginning and end of each semester (in January, April, August, and December). The qualitative data from classroom observations, exams, and interviews were collected during the second and second-to-last chapters covered during each of the two semesters. These data related to the students' practices of problem solving and constructing viable arguments.

The questions that guided this research were:

1. How do certain prospective elementary teachers' progress in their enactment of the mathematical practices of persevering in problem solving and constructing viable arguments as they move through inquiry-based mathematics content courses?

2. What relationship (if any) exists between prospective elementary teachers' procedural knowledge and mathematical knowledge for teaching, and how does this relationship change over time as they move through inquiry-based mathematics content courses?
3. How do prospective elementary teachers' attitudes toward mathematics associate with their procedural knowledge and their mathematical knowledge for teaching, and how does this relationship change over time as they move through inquiry-based mathematics content courses?

This chapter will first discuss the qualitative aspects of the study and answers the first research question and then move on to the quantitative results used to answer the second two research questions. This first part of the chapter will begin with a review of the intended curriculum and a discussion of the changes that occurred in the enacted curriculum. Next, the selection process for the participants (students moving through the curriculum) will be discussed, followed by a description of each of the participants and who they were as a member of the class. Finally, the chapter will address qualitative results regarding how the students were problem solving and constructing viable arguments throughout the curriculum. The second part of the chapter will begin with a discussion of the results from each individual quantitative assessment, then move on to an examination of the pair-wise relationships between the variables of interest.

The Enacted Curriculum – The Case

Review of the Intended Curriculum Content, Text, and Objectives

The case for this study was the first two of three mathematics content courses for elementary teachers at a university in the Mountain West. The first course, Number and Operations for K-8 teachers (M1001), is the study of number and operations for prospective elementary and middle school teachers, including whole numbers, decimals, fractions, percents, integers, operations, numeration systems, and problem solving. The second course, Geometry and Measurement for K-8 teachers (M1002), is the study of geometry and geometric measurement for prospective elementary and middle school teachers, including synthetic, transformational, and coordinate geometry, constructions, congruence and similarity, 2-dimensional and 3-dimensional measurement, and problem solving.

The text used in M1001 and M1002 is *Mathematics for Elementary Teachers with Activity Manual (3rd Edition)* (Beckmann, 2012). This particular text with class activities uses an inquiry-based approach to learning, allowing students to engage with and explore the material while gaining a deeper understanding of the mathematics involved. M1001 and M1002 are intended to provide experiences to help students broaden and deepen their own understanding of mathematics and to allow students to acquire mathematical knowledge that will prepare them, as professionals, to help children become confident problem solvers and powerful mathematical thinkers. These courses allow students to make sense of the mathematics that children do and be able to explain the “why” questions. Students are intended to spend considerable time communicating their

mathematical ideas, both orally and in writing. Course objectives for M1001 and M1002 include the following four items: 1) **Explain**, through writing and speaking, fundamental concepts and processes important in (K-8) mathematics, particularly those related to number operations. 2) **Represent** quantity and relationships between quantities in problem situations using symbols, words, and diagrams. 3) **Solve** problems through quantitative reasoning. 4) **Construct** viable mathematical arguments and evaluate the reasoning of others. Instruction in this course is intended to be consistent with the vision of the Common Core State Standards (CCSS) for Mathematics including the eight Standards for Mathematical Practice.

The Enacted Curriculum

There were three sections of M1001 taught by two graduate teaching assistants and one adjunct instructor in the spring 2014 semester. All of these were included in the study. Only two of the three sections of M1002 in the fall 2014 semester were a part of the qualitative portion of the study because the third section did not contain any of the participants being followed (all sections contributed to the quantitative portion of the study). The two sections of M1002 that were part of this study were taught by two graduate teaching assistants, one of whom was also an instructor of M1001 the previous semester. With the exception of some minor differences due to instructor preference, the enacted curriculum aligned with the intended curriculum discussed in chapter 3.

Students worked in small groups of two to four on activities from the text, exploring the *why* of many mathematical concepts. Students were expected to discuss in their groups, participate in class discussions, reason with the material, and explain *why*.

Each chapter (or so), each of the instructors would mix up the groups, giving the students the opportunity to work with new people. The instructors did act as facilitators, not giving away answers, but asking more questions of students and guiding them in the right direction. Some instructors offered more guidance and direction than others. Although there were often whole class discussions and wrap-ups of the activities, this was sometimes a missing component of the class. Sometimes there was no wrap-up, either because the class period ended too soon or because the instructor would deem it unnecessary after having visited with all of the groups. When sharing with the class, students were given the opportunity to use the white board, SmartBoard, and document camera. There were also occasions when the students were given the opportunity to work with manipulatives to explore activities.

There was a tendency of the instructors to allow group presentations at the beginning of M1001 in order to ease the students into being comfortable explaining on their own. One minor difference in instructors was whether they preferred to ask for volunteers during classroom discussion or select students to present. One of the instructors used random selection of students while a couple of the other instructors would use purposeful selection in order to sequence strategies or ensure that several strategies were presented. For the most part, instructors used the activities in the text, but there were some minor deviations and supplemental material used. There was one instance when a M1001 instructor deviated from the activity manual for two days, and it did cause frustration and confusion with the students.

The Participants

Selecting Participants

To select the participants, I made a scatter plot of the initial scores on the procedural knowledge exam and the mathematical knowledge for teaching (MKT) assessment with the data from all students enrolled in M1001 during the fall 2014 semester. I identified the median MKT score and the thirds of the procedural knowledge exam scores. This created six regions, all of which contained multiple data points (students). As there were existing students in each of the six regions, one was to be selected from each region using the design laid out in chapter 3. When thinking about how to select a participant from each region, I took into consideration patterns found within the scatter plots, potential attrition of participants, initial attitude scores of students, and course section.

I noticed a pattern, that there were rows of students in roughly the middle of the upper half of the MKT scores and roughly the middle of the lower half of MKT scores (see Figure 4). I wanted to make use of this for two reasons. One, I wanted to select participants toward the middle of the region rather than the perimeter, and two, taking students in a line would provide groups for comparison based on initial MKT score. Once the two rows of MKT scores were selected, three students were selected from each row (see Figure 4). There were only three students in the top row of MKT scores, so I identified all of them for selection. There were seven students in the bottom row of MKT scores, so only a subset would be identified for possible selection. When looking at the region of scores that were below the median MKT score and in the lower third of

procedural knowledge scores, I considered the potential for attrition. I thought these students would be the most likely to drop or not make it through the first course, so I identified the student with the highest procedural knowledge in that region for selection. I identified the only student in the middle of the lower middle and then finally identified the student with a perfect procedural knowledge score as I thought this would provide an interesting case. Throughout and at the end of this initial process, I paid attention to which course section each student was in so I could select two students from each course section. Also, I looked at the six identified participants and what their initial attitude scores were relative to the class. There was at least one student from each third represented so these were determined to be the top six choices for participants in the study.

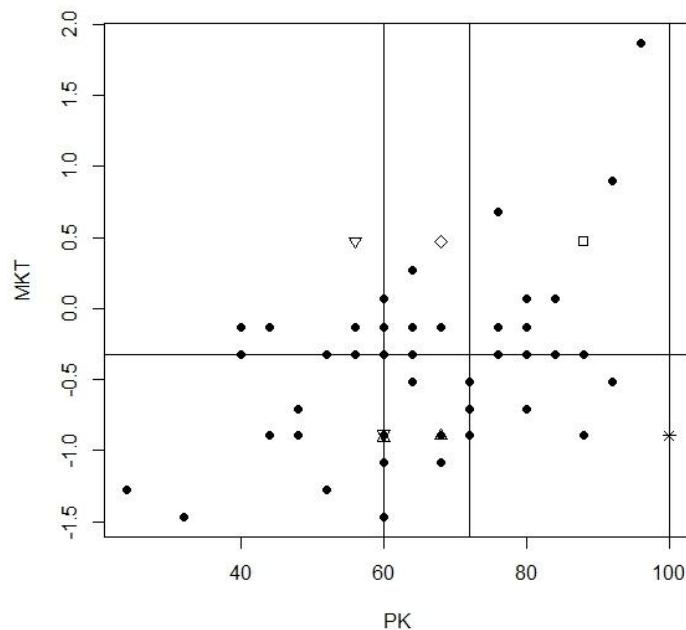


Figure 4. Initial MKT and PK levels divided into 6 regions.

Uncertain of the students' willingness to participate in the study, I decided to send an initial email to thirteen students in hopes that at least six would be willing to participate. I identified seven more students, each of whom was close to one of the original six identified. One extra person was identified in the low MKT and procedural knowledge region relative to the class, as I thought there might be more hesitation to participate from these students. Once these thirteen students were identified, I sent an email to each of them describing my study and what it would require for them to participate. If I did not hear back from them, I stopped by at the end of class and visited with them face-to-face to see if they would be willing to participate. Of the thirteen contacted, twelve agreed to participate. The only student not willing to participate was not one of the top six choices. I sent an email informing the top six choices they had been selected for the study and the other six that they had not, and thanked them for their willingness to participate.

Attrition

Of the six participants selected to contribute to the qualitative data, only four participated throughout the entire study, from the beginning of the spring 2014 semester to the end of the fall 2014 semester. There were initially six participants chosen in anticipation of some attrition, and it was the goal for at least four participants to remain through the entirety of the study. I thought that four participants would provide ample evidence and rich enough data to illuminate the case and answer the research questions. At the end of the spring semester, during our last interview together, I found out Joey would be transferring to a new school next year, leaving only five participants. During

administration of the beginning of the semester assessments in the spring, I noticed Ande had not yet taken the procedural knowledge assessment in the testing center. I sent Ande a reminder email and found out Ande had changed majors. Two weeks into the fall semester I was down to four participants, who continued to the end of the spring semester. The following is a description of each of the six participants selected to be in this study and how they acted as a member of the class. All six participants are described because they all contributed to the analysis of the qualitative data. First, though, are some tools and information the reader may find helpful to use for reference throughout the analysis.

Initial Competency Levels

Figures 5 and 6 below are provided for reference to the reader. Figure 5 illustrates where each of the participants entered M1001 in terms of their MKT, procedural knowledge, and attitude. This was the basis for the selection of the participants and gives the reader an idea about how the students compare relative to the rest of the class. The profile plots in the next figure highlight the paths of the participants' MKT, procedural knowledge, and attitude scores over the course of the year. These profile plots offer a glimpse at how the participants progressed in their competencies and how they compare to the rest of their classmates (the gray lines in Figure 6).

MKT	Taylor - Top 50% MKT, Bottom third PK (Top 50% attitude)	Ande - Top 50% MKT, Middle third PK (Bottom 50% attitude)	Alex - Top 50% MKT, Top third PK (Top 50% attitude)
	Jamie - Bottom 50% MKT, Bottom third PK (Bottom 50% attitude)	Joey - Bottom 50% MKT, Middle third PK (Top 50% attitude)	Jordan - Bottom 50% MKT, Top third PK (Top 50% attitude)
PK			

Figure 5. Initial competency levels of the six participants.

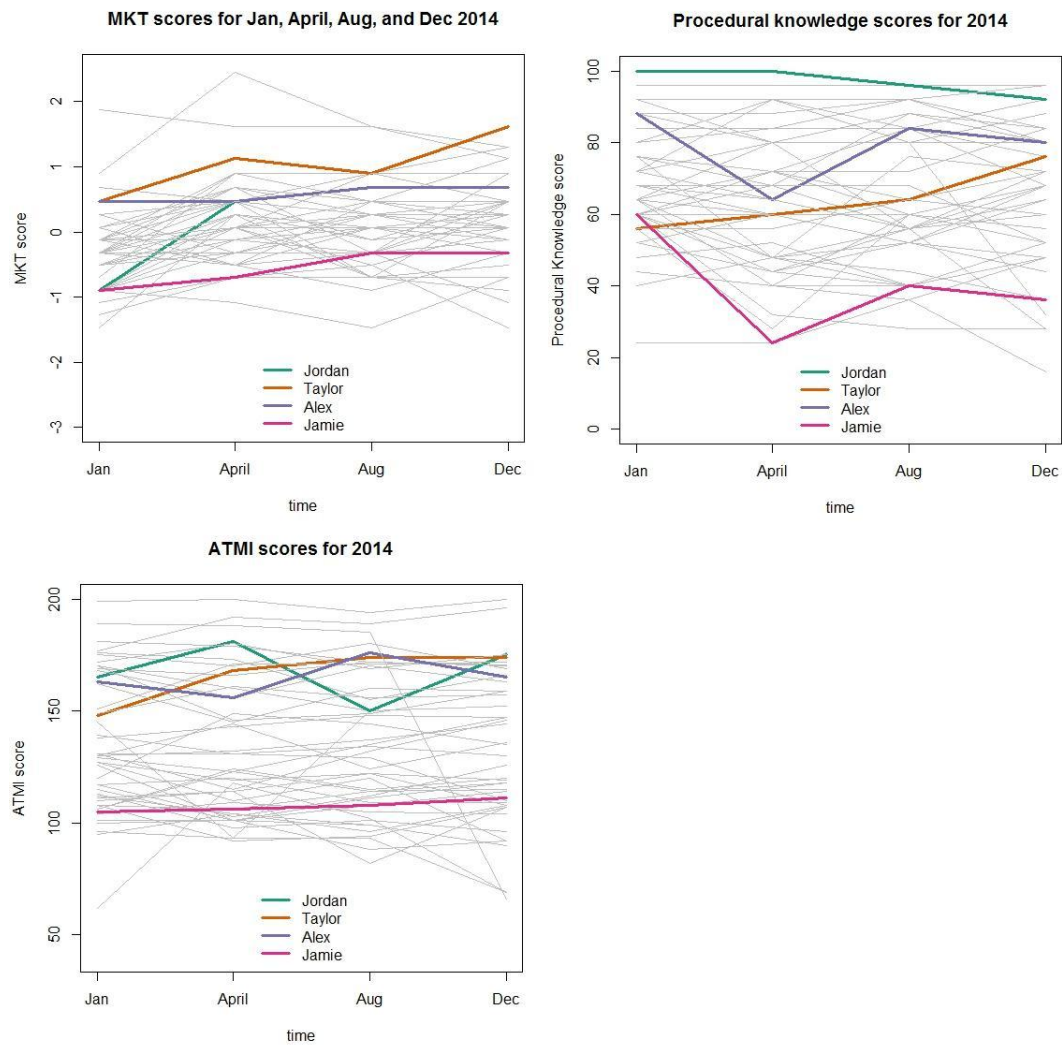


Figure 6. Profile plots of participants' MKT, PK, and ATMI scores over time.

Gender

Both female and male students were participants in the qualitative portion of the study; however, gender and its influence on prospective teacher preparation was not a focus of this study. In order to better maintain the confidentiality of the participants, their gender will not be shared. Gender neutral pseudonyms have been chosen for all of the participants.

Curriculum Observed

I was in the classroom conducting observations during a total of four chapters. The class met three days a week for 50 minutes. I observed for a total of ten days during chapters 2 and 5 in M1001. I spent a total of six days observing during chapter 2 which focused on the meaning of fractions, fraction equivalence, and common misconceptions held by students. I spent a total of four days observing during chapter 5 which focused on the multiplication of fractions, decimals, and integers, and why the algorithms and rules work the way they do. I observed for a total of ten days during chapters 11 and 13 in M1002. I spent a total of four days observing during chapter 11 which focused on measurement and common misconceptions held by students. I spent a total of six days observing during chapter 13 which focused on the surface area and volume of different shapes.

Alex

At the beginning of M1001, Alex had an MKT score in the upper half of the class and a procedural knowledge score in the upper third of the class. Alex started with an attitude score in the upper half of the class. This score remained in the upper half of the

class in April, moved into the upper 25% in August, then back to the upper half of the class in December. Alex started with a procedural knowledge score in the top 25% of the class, took a 24 percentage point drop from January to April, but remained in the top half of the class. From April to August, Alex gained 20 percentage points, jumped back into the top 25% of the class, and remained there in December. This indicates that the April score was not an accurate measure of Alex's knowledge. Alex's initial MKT score began in the top 25% of the class and the MKT scores in April, August, and December remained in the top quarter of the class. Alex made no gain in MKT score from January to April, a slight gain from April to August, and no gain again from August to December.

Alex was a sophomore enrolled in the first mathematics content course for elementary teachers. Alex thought of becoming a performer, but was advised by friends and family to pursue teaching. Initially unsure about the idea of teaching, Alex soon identified enjoying spending time with kids and a desire to inspire them and help them learn as reasons for pursuing teaching. Alex was considering going into special education, wanting to help these types of students and act as a role model. Throughout the years, Alex became more invested in majoring in education, especially after having not done well in the few engineering classes in college and then participating in a job shadowing for education.

During the beginning of M1001, Alex's chapter 2 group generally attacked class work by quietly working and then discussing, instead of all discussing a problem and then moving to individual drawings and write-ups. The group worked well together, and Alex was definitely a dominant voice in the group. Alex volunteered a couple times during class discussion, tried to relate class work to future teaching, and did not follow

the group in participating in negative talk. Alex seemed comfortable with the material, even an abstract question that troubled the rest of the participants. During the chapter 2 interview, Alex reflected on a couple of the exam questions, realizing one of the explanations could have been written better and admitting that sometimes it is hard to come up with the appropriate wording and vocabulary for an explanation. Alex really liked the set-up from the class and found “finding your own reasoning” very “valuable.”

Toward the end of M1001, Alex was still very vocal in the new group. Alex came in late the very first day of chapter 5, but caught right up and even shared with the class when their group was called on. Towards the end of the chapter, there was a lot of confusion and frustration shown in the group and the class as a whole, but Alex remained positive and continued to work toward understanding. During the interview, Alex talked about how the negative attitude of this group and the last made it difficult to stay positive and learn, especially when the others would rather complain about the class than do the work. Alex shared, “I realized how little input we did get from [the instructor] and how little I was learning from it.” Alex was nervous about the chapter 5 exam, feeling the class was growing in difficulty, but ultimately performed well.

During the beginning of the semester in M1002, Alex was in a group with only one other person. Alex was definitely the more vocal of the two, but both contributed in discussing the material. Alex started adding to notes based on class discussion and volunteered in class discussion on several days. At the beginning of chapter 11, Alex showed a good attitude about and confidence in the class, saying, “I feel cool that I’m getting to see this side of it and being able to understand it. It’s interesting.” Toward the end of the chapter, however, Alex was not confident about solutions and explanations,

even though they were correct. I also saw this up and down confidence during the interview, when Alex would go from feeling confident in an answer to feeling like there might be something missing. Most of the time, it was a matter of not feeling confident in an explanation as opposed to not understanding the problem itself. Alex self-identified some explanations as “wonky.” Overall, Alex had a good attitude about the class and doing math, saying M1002 was “one of my favorite classes to go to.”

Towards the end of M1002, Alex was still participating in group and class discussions and would supplement solutions and explanations based on class discussion. Alex did not show much confidence in some of the solutions and ideas from class work, saying things like “I don’t know if that’s right” and “sometimes I feel like I talk and I make no sense,” even when the ideas and explanations were good. Alex was not confident going into the chapter 13 exam, especially about being able to explain *why* on some concepts. On the exam Alex did well and felt better about it overall. Alex felt the nature of the problems in M1001 called for a more in-depth explanation of *why* as compared to the problems this semester, which were more calculation based. Overall, Alex showed a good understanding of the formulas used in M1002 and why they make sense. Alex said, “I would have thought I had to look up a formula or something...but I think now from the class, now I could look at any shape and be able to figure out the volume.”

Ande

At the beginning of M1001, Ande’s MKT score was in the upper half of the class while Ande’s procedural knowledge score was in the middle third of the class. Ande’s

initial attitude score was in the lower half of the class and remained so to the end of the semester and over the summer, with little change. Ande's procedural knowledge score increased 4 percentage points from the beginning to the end of the semester and remained in the upper 50% of the class. Ande's MKT score from the beginning to the end of the semester increased by just over 0.6 standard deviations and remained in the top 25% of the class.

Ande was a sophomore who enrolled in the first mathematics content course for elementary teachers. Ande was originally a social studies broad field education major, but had now switched to an elementary education major while working toward both Spanish and History teaching minors. After the first semester in M1001, Ande decided to switch back to majoring in secondary social science education and so dropped out of the study. When I asked, "Why major in elementary education?" Ande responded by recalling the senior year of high school, which was spent studying abroad in Finland. Ande spent a lot of time teaching English while there and really enjoyed it. Also influencing the decision was the fact that Ande's host parents were both practicing teachers.

During the beginning of the semester of M1001, most of the work Ande did in class was done independently. Ande would check answers with the other members of the group and make sure they understood, but most of the work in the group was done quietly. When reminded by the instructor, Ande had no problem discussing with the group and working on explanations, but sometimes more discussion led to less written work. Ande would occasionally volunteer in class discussions, but ultimately was not big on sharing ideas. Ande confessed, "I usually come to conclusions in very odd ways, so I

would have an argument on my hands to try and convince people that that's how we got there."

I found this interesting because on the first day of my observations at the end of the semester, Ande's group mates were asking a lot of questions and encouraging much discussion. Ande showed some frustration, but ultimately persevered in coming up with answers and an appropriate explanation to the group's questions. I asked Ande about this day during the end of semester interview, and Ande again spoke about reaching conclusions in different ways and so not wanting to explain to others. Ande said, "Some of the connections I make don't make sense to some people and then I just end up confusing them...It gets really complicated sometimes trying to figure out how to explain it otherwise." The groups switched the next class period, and I saw more independent work from Ande at the end of M1001. Ande would still reach out to the group if stuck and on occasion would volunteer in class discussion. During the interview at the end of the semester, I found out Ande was debating about switching majors back to social studies. I asked why, and Ande replied, "I like the History and the Political Science more than the everything, so I decided I'd be better off teaching...one thing that I really like than a bunch of things I don't really like so much." After that, Ande confessed to not being a fan of math, especially "higher level" mathematics like trigonometry. Ande told me the explanations required in these courses are good "because you're going to have to be able to explain it to little kids and it makes you look at it in more than one way."

Taylor

At the beginning of M1001, Taylor's MKT score was in the upper half of the class and procedural knowledge score was in the bottom third. Taylor started out with an initial attitude score in the top half of the class which jumped to the top 25% of the class in April, August, and December. The gain in attitude score was 20 points from January to April. Taylor's procedural knowledge score went from being in the lowest 25% of the class in January to the upper 50% of the class in August and December. This increase may be attributed to the fact that Taylor was taking other mathematics content courses for a concentration in mathematics in conjunction with the elementary education courses. Finally, Taylor's MKT score began at and remained in the top 25% of the class throughout the entire year. By December, Taylor was one of the two top scoring students in the class and could be considered an outlier in relation to the class as a whole.

Taylor was a junior who enrolled in the first mathematics content course for elementary teachers. Taylor was majoring in both elementary and secondary English education. This decision to major in elementary education was in part because of job availability and moving a lot, but a bonus was in not having to narrow down to one age group only. When I asked Taylor about the decision to go into elementary education, Taylor shared, "I like teaching people things. When they ask me a question I will like sit there and explain to them and I enjoy the process." Taylor also loved English, reading and writing and discussing books. Taylor felt this route was beneficial too, with reading and writing being the basis of all the subjects and the move of teaching mathematics more toward theory through the Common Core Standards.

During the first part of the semester in M1001, Taylor volunteered in class discussion almost every day and seemed to really enjoy helping and explaining things to the group, as well as to the instructor, when individual group checks were being made. The group tended to work independently first and then would come together and discuss the problem. There were no complaints made by the group, and Taylor did not appear to struggle with the material. Initially, Taylor was not initially confident in the answers given on the chapter 2 exam, but after discussing the problems in the interview, felt more confident about them. As a result of reflecting on this chapter, Taylor planned to use more pictures when teaching in the future to help students.

Taylor's chapter 5 group at the end of the semester in M1001 would all work independently and then come together to discuss. Taylor once again showed a desire to share explanations to both the group and to the class. I witnessed Taylor volunteering to share with the class several times each day. Taylor reflected on making a conscious effort to sit back more and try to let the others explain because they might not get it as fast and just end up copying what Taylor said. Taylor described both a love of mathematics and the fact of being an English major and thus being good at explaining things as possible explanations for success at understanding and explaining quickly. I found out during the chapter 5 interview that Taylor decided to add a mathematics concentraion and so was planning to take Pre-Calculus in the summer. This decision was made in part due to job security and a renewed love of mathematics found through taking M1001.

At the beginning of the next semester in M1002, Taylor was still a very vocal participant in the group, to the instructor, and to the class. Taylor would get done with

the activities before the rest of the group and then wait for them to get caught up to discuss. Taylor almost always stayed on task, even when the others in the group did not. During the chapter 11 interview, Taylor talked about liking geometry and it making sense. Compared to last semester in M1001, Taylor felt the material was much easier and more straightforward. Taylor also added at the end of the interview how calculus and mathematics for the elementary teacher “help each other.” Taylor made connections between the two and felt it helped in learning the “fundamental things” for a “deeper understanding of more complex topics.” Taylor related shapes and angles to Trigonometry and Pre-Calculus and also spoke about algebra skills being useful.

Towards the end of the semester in M1002, Taylor was still showing a desire to share with the group, instructor, and class as a whole. Often, Taylor was the first to share when the instructor would stop by to check on the group. The group mostly did individual work and then discussed with one another. Taylor chose to be quiet in the group and explained the reason as trying not to “dominate the conversation.” Taylor was gone the last two days before the chapter 13 exam and as a result felt a lack of confidence on the exam. Taylor shared, “Normally I just explain and you have a question and I answer it really promptly and then we move on to whereas this I don’t have any answers. I just am not sure.” Overall, Taylor performed well on the exam, had a good attitude about the class, and thought it was “interesting to see why” things are the way they are.

Jordan

Relative to the rest of the class at the beginning of M1001, Jordan scored in the lower half of the class on the MKT assessment and the upper third of the class on the

procedural knowledge exam. Jordan actually received a perfect score, 100%, on the procedural knowledge exam both at the beginning and end of the first semester and received above a 90% at the beginning and end of the second semester, staying and remaining in the top 25% of the class. Jordan started with an attitude score in the upper 25% of the class and remained there at the end of the first semester. At the beginning of the second semester, Jordan's attitude score dropped into the upper 50% of the class, but then by the end of the second semester it was back in the top 25% of the class. From the beginning to the end of the first semester in M1001, Jordan went from being in the bottom 50% of the class to the cusp of the upper 25% of the class in terms of MKT score. Over the summer, from April to August, Jordan made a slight improvement in MKT score and maintained the same score at the end of the semester in December, remaining in the top 25% of the class in M1002.

Jordan was a sophomore who enrolled in the first mathematics content course for elementary teachers. During our first interview together, Jordan shared about being valedictorian when graduating high school and about having strengths in music, math, and science. Jordan started out as an animal science major, then music education, and finally decided to switch to elementary education for the variety and planned on completing a mathematics focus. Jordan decided to go into elementary education, growing up wanting to be a teacher and loving kids. Jordan shared, "I just love how children are so, they want to learn and they're so moldable...it's just a really important aspect, I think, to be one of those people that helps influence children...I feel like it's important for schools to have educators who are passionate about what they're doing and...so for me, going into that, I want to be able to have an impact on our youth

because...you're not just a teacher, but you're also a role model." Jordan was excited at the potential to have an impact and was enjoying the variety of the elementary education curriculum.

During the beginning of the semester in M1001, Jordan definitely acted as the group motivator and worked hard to get group discussions going. Jordan explained to me during the chapter 2 interview, "I'm just very task oriented." This definitely showed when Jordan would be the only one in the group on task. Some work was done independently, but Jordan would ask for clarification if needed and sometimes try to get group discussion going. Jordan would always add to and enhance solutions and explanations based on relevant class discussions. Jordan worked hard to understand the problems and tried to relate them to future students and teaching. Jordan often volunteered in class discussion, but would get frustrated when the explanation did not make as much sense as it did in the small group setting and others in the class would start asking questions. Towards the end of chapter 2, Jordan was showing more of a negative attitude toward the tasks, saying things like "this is so stupid" and showing more frustration with explaining. Jordan admitted in the chapter 2 interview that a lot of this frustration stemmed from feeling very capable performing in more complicated mathematics classes and thinking these more basic mathematics ideas should be easier.

During the end of the semester in M1001, I never witnessed Jordan off task. Jordan was a vocal member of the group, always enhanced notes based on class discussion, and once made reference to a homework problem that was completed before the class. Jordan volunteered in class discussion once at the beginning of chapter 5 and made a few negative comments about the task at hand throughout chapter 5. Jordan was

frustrated at the fact they had to explain how and why multiplication of integers works. In class Jordan commented, “So here’s my thing. We were all students at one time and the fact that we have to relearn this is ridiculous. It’s just like, stupid because we don’t remember as students, like why. We don’t even know why.” This contradicted some of Jordan’s words during the interview, of the importance of this class and the frustration felt when others do not take it seriously. Jordan admitted to showing some negative attitude in class, and attributed the behavior to others’ negative attitudes. Jordan said, “I try not to complain, but I do definitely understand where they’re coming from for some of these things because it can be a little more challenging to explain the rationale than um, just to tell the kids this is the way it is.” Jordan also admitted that just because mathematics “clicks” does not necessarily mean it is personally enjoyable. Overall, Jordan felt confident in the performance on the chapter 5 exam and during our chapter 5 interview, was able to extend on and generalize some of the strategies used and reflect on other representations that could be incorporated.

During the beginning of the second semester in M1002, Jordan was once again acting as the group leader and initiating discussions. I never witnessed Jordan off task, and Jordan again made additions to notes based on class discussions. Jordan volunteered in class discussions a few times towards the end of chapter 11 but also expressed to the group a lack of confidence felt in this class, in particular with explaining. During our interview together, Jordan said the lack of confidence and willingness to volunteer came from being shy which made being in front of the class “uncomfortable.” I also found out during the interview Jordan is a proclaimed “numbers” person and does not like geometry. Although Jordan did well on the chapter 10 exam, this did not create

confidence going into the chapter 11 exam. Overall, Jordan was a very grade- and test-driven student and so saw the instructor as the expert and the one who hands out the grades. This became very apparent when we were discussing one of the exam problems during the interview and Jordan admitted to doing the problem in the way the instructor showed rather than a way that made more personal sense.

Jordan's pattern of showing a lack of self confidence in geometry and explaining as well as having a lack of confidence in group members continued into the end of M1002. In class, Jordan described explaining as "frustrating" and said, "I don't know if any of mine make sense." One day in class, Jordan had volunteered during a class discussion and had gone to the board. Some of the students did not understand Jordan's explanation completely and so were asking some clarifying questions. Jordan tried to answer the questions and then became very apologetic about it, saying, "I'm sorry if I just completely confused everyone...I think algebraically." Along these same lines, Jordan told me in the interview, "I'm really good at numbers and I think algebraically, but explanation-wise is always a little more difficult to word it in such a way that would make sense." Jordan's group used a combination of independent work and discussion. Jordan frequently wanted to reference the book and continued to make additions to solutions and explanations based on class discussion.

Joey

Relative to the rest of the class at the beginning of M1001, Joey scored in the lower half of the class on the MKT assessment and in the middle third of the class on the procedural knowledge exam. Joey started with an attitude score in the upper half of the

class, but it dropped to the lower 25% of the class by the end of the semester. Overall, there was a 32 point drop in Joey's attitude score. Joey scored in the bottom 25% of the class on the MKT both at the beginning and the end of M1001, though the raw score change from the beginning to the end of the semester was roughly +0.2 standard deviation. There was a 20 percentage point drop in Joey's procedural knowledge score from the beginning to the end of the semester and relative to the class; Joey went from being in the top 50% of the class to the bottom quarter.

Joey was a typical freshman who enrolled in the first mathematics content course for elementary teachers. The inspiration to become an elementary education major came from Joey's mother, who worked as an English teacher, assistant principal, principal, IEP director for a school district, and an executive director of student support services. Joey's mother was told by several employees how great Joey was at teaching in the three years Joey spent working at the same school. Joey figured this was a sign to go into elementary education in addition to it being seen as a rewarding and fun job. Joey was also planning to go into special education and, in particular, wanted to teach middle school special education.

During the beginning of the semester in M1001, Joey was often late to the early morning class. Joey seemed tired and sick throughout the class periods, coughing, getting up to leave the room, and one day, even resting on the table. Some days at the beginning of class, Joey was too tired to work through the problems with the group and so would quietly sit and copy answers in order to stay caught up. Towards the later part of the hour of class, Joey would participate better in group discussions, giving input, asking questions, and trying hard to understand. Joey never seemed very confident in

personal explanations and would never volunteer in class discussions. Joey even had a moment of panic when the group was called on to present one of the problems and ended up leaving the room and not presenting with the group. Joey was very reliant on the group members, often copying their explanations verbatim during the beginning of the semester, and asking questions to get caught up later in the semester.

During the later part of the semester in M1001, Joey was chronically behind the other two members in the group and as such was often off task with them when not yet finished with the problem. Towards the end, Joey was finally getting more involved and asking questions to understand and get caught up. During the second interview with Joey, I asked how the class went during the two chapters I was not observing. Joey pulled out the old exams and discussed the difficult problems that had to be redone. Joey could not make sense of one of the problems and so after several attempts at a redo, finally just memorized the work of another in the class. Joey admitted to still being confused about the problem, yet never consulted the instructor. I asked if Joey ever checked with the instructor to understand a problem better and get help on a redo. Joey did for the first two tests. When looking over the chapter 4 exam with me, Joey decided visiting with the instructor would be a good idea as Joey did not understand why some of the questions were wrong. Overall, Joey found receiving so many redos very frustrating and discouraging. Finally, when discussing group work during the interview, Joey shared, “You have to want to be there...and want to learn. I mean, you don’t want to learn this, but you have to for education majors so I guess a part of me does want to.” This admission of an overall weak investment in the class and into learning the material aligned with Joey’s behavior in the classroom. When I was wrapping up the interview at

the end of M1001, Joey informed me about transferring to another school. Joey was not a part of the study in M1002.

Jamie

At the beginning of M1001, Jamie started out in the bottom half of the class for MKT score and the bottom third for both procedural knowledge and attitude scores. Jamie improved in MKT score from the beginning of the first semester to the beginning of the second semester (with a change of $+.572$ SD) and remained the same from the beginning to the end of the fall semester. Jamie's attitude scores followed a similar pattern of slow and steady growth throughout the two semesters. Jamie's procedural knowledge score took a big dip from the beginning to the end of the first semester, falling by 36 percentage points. The score came back up a little and then leveled off at 36% at the end of M1002. During our last interview together, Jamie expressed dissatisfaction with having performance on an exam like this as a prerequisite to get into this course as it did not seem relevant to the material covered in M1001 and M1002. This is interesting to me as the researcher, knowing algebra will be covered in M1100, the next course in the sequence, but this is out of the scope of this study. Jamie admitted cramming to pass the entrance test into M1001 and felt the initial score did not reflect Jamie's retained knowledge.

Jamie was a typical freshman who enrolled in the first mathematics content course for elementary teachers. Jamie told me right away about being part of a long line of teachers in the family. This included Jamie's father, grandma, grandpa, aunt, and uncle. Jamie always liked teaching and working with little kids and discovered this by tutoring

elementary school students one on one and teaching swimming and tennis lessons. Jamie went on to admit that mathematics was not a strong suit and so this class was very challenging, especially the explanations. Jamie did feel somewhat prepared for this challenge, however, because some of the tutoring done at the elementary school was with a troubled student who needed help with math. Jamie learned how difficult it can be to explain to a young student.

Jamie was very outspoken and reminded me of a class clown, trying to make class fun and humorous. Every time I went into the classroom for observations and handed out the Smartpens, Jamie would get excited and leave messages for me on the pen, signing on and off each day. At the beginning of the semester in M1001, Jamie was always a part of the group discussion, even when it went off on tangents. Sometimes Jamie would act as a leader, trying to keep the group moving through the problems. Jamie hated it when the class period ended and there were problems left unfinished. Sometimes this desire to move on quickly would backfire as some of the problems that were worked through required more thinking and sense making. Frustration and a lack of confidence were sometimes shown when Jamie “understood” a problem but could not explain it. I asked Jamie about this lack of confidence in our interview together. Jamie explained, “I guess I don’t feel comfortable with my knowledge of it. I may know it, and I may know how to explain it, but I feel like...they have a better understanding of it...where me who still has a few questions and I’m not sure why some mechanics work, I’d rather have [them] go up there and explain it. It gets hard with me when people start asking questions because then I’m like,...I’m not sure why this works...If I go up to the board I usually have like a 100% confidence rate.” Jamie did volunteer once during class discussion at the end of

the chapter 2 material. Jamie worked very hard to understand the problems, asking group members lots of questions, but would hardly ever write anything down besides a drawing or a picture. Jamie was very creative and loved to draw, as shown by the group logos made in class and doodles made on the exam. Jamie said drawing helped to “refresh” the brain in order to maintain focus and clarity when working through problems.

During the later part of the semester in M1001, Jamie was showing much more frustration with the class and the instructor. This was reiterated during the interview when Jamie shared frustration with the amount of time necessary for this class, due to the amount of redos that needed to be completed. Jamie did not like having to be thinking about three chapters at the same time, which was what happened with the redo system. Jamie admitted to just finding somebody who got the answer right on the exam and copying it to complete the redo. Jamie felt “there’s no standard to what’s right and wrong in this class” and that they “never got the answer” or any “guiding” to the answer in class and never got their questions answered. Jamie was still very vocal in the group, but still only did the bare minimum on most of the problems in class rather than putting in more thought.

During the beginning of the next semester in M1002, Jamie seemed to have lost the frustration and gained back a sense of humor. Jamie said during the interview that it was easier now knowing the way things work and expectations of the class. Jamie was again a dominant voice in group discussions, tended to get off topic several times, and continued to hardly write anything down, except when reminded by the instructor. A difference I noticed from last semester to this was an increase in the amount of times Jamie volunteered during class discussion. Jamie still had some admitted struggles

during class with dimension and conversion. It was also telling how Jamie felt about personal abilities in mathematics when after another student presented a very mathematical, algebra-based approach, Jamie followed with, “That is far beyond my comprehension ability.” Jamie felt confident about some of the chapter 11 exam problems, not about others, and admitted to struggling with exam questions that were unfamiliar. “Seeing something I wasn’t totally familiar with kind of hurts me on tests because I kind of almost have a moment where I freak out and I kind of – everything I know leaves and then that kind of makes it more difficult to come up with the answer.”

Towards the end of the semester in M1002, Jamie was still very vocal in group discussions, which oftentimes went off topic. During our interview together, Jamie blamed much of the off topic behavior in class to one group mate in particular, who Jamie said just wanted to argue about everything. This tension with a group member escalated on the last day of the chapter 13 material and so Jamie ended up moving to another group. Aside from the difficult group dynamics, Jamie was volunteering and contributing to class discussions more than at the beginning of the semester, but still had some struggles with the material. In the interview, Jamie described conversions as a “mental barrier” and admitted to struggling with mixing 2-D and 3-D within the same problem. Overall, however, in our final interview together, Jamie expressed confidence in M1002, having a better understanding of how the class works and a knowledge base from M1001 to use.

Results from the Qualitative Data – Problem Solving

Understanding and Devising a Plan

The first part of the CCSMP1 states that “mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution” (CCSS, 2011). As the participants in this study moved through the curriculum, I witnessed all of them actively attempting to understand the problems, through either individual work, discussing as a group, or a combination of the two. I could not witness grappling with understanding the problem and devising a plan of action if they were working individually in class, however, I could if they were discussing as a group. If they were struggling to understand, they would always ask their group for help. I was also able to get a better understanding of this process with some of the more quiet participants during my interviews with them.

During the first semester in M1001, all of the participants were reading through the problems, rereading if necessary, and picking out key information. They were asking clarifying questions to get at the goal of the problem or what the question was asking, relating the current problem to similar problems and sometimes personal experience, and considering constraints of the problem to determine an appropriate strategy. Several of the participants would circle or underline key words in the problem – especially on

exams. In a fractions problem about gardening, Joey asked whether the garden sections contained equal parts in order to get at how to find the correct fraction of corn there was in total. Jordan and Taylor were able to relate to their personal tipping and shopping experiences when working on percentage problems involving mental math. Jordan's prior experience in mathematics classes and strong algebra background was helpful for understanding the problems about exponents. For one particular problem, all participants recognized in the directions that they were not allowed to use multiplication by 1 to explain and thus chose a different approach for the problem. All participants but Joey followed the directions of not using decimals, cross multiplication, or common denominators to compare fractions in a class activity, which ended up hurting Joey on the chapter exam as Joey did not have an appropriate strategy to use on the exam. On a few occasions during chapter five, when Jamie and Alex were really confused about the problems and were not working out of the activity manual, they referenced the book to try and understand the problem better.

To examine understanding when analyzing exam questions, I was only able to see if participants circled or underlined key words, followed the directions and constraints of the problem, used the given information, and got the correct answer for a problem in an appropriate way. All the participants drew pictures if asked and used the given examples in their work. In one instance, Jamie initially followed the directions in carefully stating the whole of each fraction, and then at the end of the problem, the final answer was incorrect because it was not related back to the initial whole.

During the interviews, I was able to hear how the participants went about understanding (or not understanding) the problem at hand. In many instances, the

participants referred to doing a similar problem in class or in the homework, or in the case of Alex, additional practice problems in the book done for studying purposes. They also referred to similar problems done in the chapter summary videos and Pencasts. There were a few instances in which Joey and Jamie did not recall doing a similar problem in class even though they did. One of these instances was when Joey was using a calculator to compare fractions instead following the directions of not using decimals, common denominators, or cross multiplication. Participants also referenced coming to an understanding by comparing the problem at hand to a similar problem and paying close attention to the wording. They were careful about following directions and answering what the problem was asking.

During the second semester, in M1002, the participants were still engaging in discussion with their groups to try and understand the problems. The participants would read through the problem and reread if necessary. They would ask their group clarifying questions, trying to get at the goal of the problem and what it was asking. These questions were mostly aimed at their group mates, but on occasion they would ask the instructor or reference the book, which was done by Jordan on several occasions. I witnessed the participants rephrasing the question and pointing out specific key information and givens more during this semester than last semester. Again, there were a few occasions where the participants would underline or circle key information, mostly on the exams. The participants compared similar problems in their small groups, and Jamie and Alex were able to relate the conversion and water displacement problems in chapter 13 to their experience in a physics class. During the interviews, the participants again related to similar problems they had done in class and in the homework.

The process of understanding the problem and planning what to do to solve the problem were often intertwined. The plans were often made via group discussion. The participants would almost always use a picture as a strategy to understand the problem better and build a base for continuing on towards an explanation. They would either draw the picture themselves or analyze the picture that was given. Jamie said, “I tend to think better after I’ve drawn something,” and Alex shared, “I always do well looking at or visualizing it.” They also used the strategy of comparing and contrasting. They would compare similar problems and pay attention to differences in their wording. They decided whether a similar strategy could be used and recognized how the problems built upon one another. For example, changing dimensions from 1-D to 2-D or 2-D to 3-D. On the chapter 11 exam, Jamie knew there needed to be an extension of knowledge from 2-D to 3-D on the last problem, but was unable to actually make the jump. On this same problem, Jordan realized they needed to “combine concepts” in order to solve the problem. Sometimes participants would just “know” what the answer was but not why or how to explain it, so oftentimes they would do the required calculation, create or examine a picture, and move on to the explanation from there. One strategy that seemed to be unique to Jamie on the exam was just writing down everything known and then revisiting the question and making sure it was answered. Finally, for the chapter 13 material which mostly involved equations of volume and surface area, the participants would figure out what they knew, what they needed to know, and figure out a formula or formulas to use to find the unknown, sometimes by working backwards. Again, the planning process was not visible to me when analyzing exam questions, but I was able to see it during group discussions and the interviews.

Evaluating Progress
and Changing Plans if Necessary

The next part of CCSMP1 states that “mathematically proficient students...monitor and evaluate their progress and change course if necessary” (CCSS, 2011). There were occasions when the participants should have modified their strategy or process but did not. Sometimes they followed along with their group mates without question while other times they went down the wrong path on their own. There were also times when the participants knew they were going down the wrong path, but could not think of anything else to do. This was especially true on exams, when the stakes were high and there was nobody else to consult. At other times the participants were able to appropriately change strategies or paths when necessary. Sometimes they would do this with the guidance of their group mates or instructors, and other times they came to the understanding by themselves. There were also occasions when the participants would realize their errors on the exam during the interview with me and reflect on how they could have done better.

With the exception of Joey, of the times I witnessed where the participants should have changed strategies or paths, there were only two times a participant went down the wrong path by just following their group mates without question. These occurrences were at the beginning of the first semester during the chapter 2 material. Several times participants would either not follow the directions or would not answer the question in full. For example, in a problem about discovering that a positive number times a negative number is negative, Ande arrived at the correct solution, but did not use the distributive property as instructed in the directions. Several of the participants did not

pay close enough attention to the wording of some of the problems or used an inappropriate strategy on exam questions, and as such, did or did not arrive at the correct answer by using faulty logic. Several of the participants missed questions asking whether a story problem was able to be solved with fraction multiplication by not paying close enough attention to the wording, and Joey and Jamie both used an inappropriate strategy for the specific pair of fractions they were comparing. Most of these incidents, when the participants did not choose an appropriate strategy for the problem, occurred within the first semester. The exception was Jamie, who continued to proceed with some incorrect problem solving strategies throughout both semesters. However, by the second semester, Jamie was realizing a need for a change in strategy on some problems and following through with a correct strategy for several problems.

This phenomenon of knowing an error is being made yet not knowing how to proceed correctly was discussed by both Joey and Jamie during my interviews with them. Both discussed knowing they were going down the wrong path on an exam problem, but just had to choose a direction and go with it. Joey knew that a number line was not an accurate and effective way to compare fractions, as was discovered in class, but could not think of another way to compare without using common denominators, decimals, or cross multiplication (these were excluded strategies stated in the directions). Jamie knew it was necessary to extend the idea of conversion from 2-D to 3-D on a chapter 11 exam problem, but could not make the necessary mathematical connection to appropriately calculate the conversion and solve the problem. Jamie was also unsure about a chapter 13 problem, unable to make the correct interpretation of the given picture. Jamie had the right overall strategy, to find the surface area of the pyramid, but was not finding the area

of the appropriate triangle. The exam setting did not allow students to discuss the problem with anyone, which is sometimes how they would realize a need to adjust their strategy during class.

I witnessed several occasions where a participant would be either stuck or going down the wrong path on a problem yet was able to make the necessary adjustments or changes to arrive at the correct solution with help from their group mate or guidance from the instructor. Sometimes it was a matter of getting help to make connections between representations, such as a story problem and an equation, or a picture and an explanation why. Other times it was the participant not understanding the necessity of a certain step or not paying attention to the directions of the problem without prompting from their group. With their group mates there to discuss with and the instructor there to answer questions or provide guidance, the participants were able to make necessary corrections and adjustments to their strategies.

When the participants made the necessary adjustments for problem solving on their own, I could not see the internal thought process behind it, but still saw some cues and actions that were a part of the process. There were a few times when the participant would say something about the directions or explanation they were writing being confusing and would then rephrase it so it would make more sense. Jamie even went as far as to refine the directions so “a student not in this mathematics class” could understand. There were also times when after rereading the question, the participant would erase or modify what they were doing. Before making appropriate revisions, Ande said, “My picture doesn’t make sense.” Taylor was trying to use a number line to compare two fractions, was not getting anywhere, and then crossed out the number line

and moved on to using the definition of a fraction and what the numerator and denominator mean.

During the interviews, the participants were able to shed more light on how they thought they knew they were going down the wrong path and what they did to fix it. In a couple of instances, the participants thought they were given certain information and later realized the mistake, such as when Jordan realized the perimeter of the pyramid was given rather than the area of the base. Taylor knew it was easy to get the inequality wrong when comparing fractions considering the fraction with the smaller pieces could relate to the larger overall fraction, so took extra time to reason and think through those types of problems. An example would be when comparing the fractions of $\frac{45}{47}$ and $\frac{82}{84}$ where both are 2 pieces away from the whole, but $\frac{1}{84}$ is smaller than $\frac{1}{47}$ so $\frac{82}{84}$ is closer to the whole. Ande tried to compare fractions using the size of the pieces, realized it wouldn't work, and switched to comparing both of the given fractions to $\frac{1}{4}$. Jordan and Ande both knew one of their answers on the exam was wrong because it was the same fraction as was given originally in the problem. They both erased what they had drawn and written, started over, and finally realized the whole changed for the final answer of the problem. These glimpses into the process of evaluating progress and changing course if necessary were accompanied by some reflections of mistakes made on the exam and not caught until the interview.

All participants made at least one reflection during my interviews with them. Taylor, Jamie, and Jordan shared additions they could have made to strengthen their arguments, such as adding a number line, multiplication tree, equation, or an array model. If there was a minor mistake made, such as a calculation error, it was more often than not

noted during the interview. Taylor, Jamie, and Alex thought their explanations could be better, Jamie thinking the explanation was “too vague” and Alex thinking if a picture had been drawn first rather than last, the explanation would have been better connected to it.

Using and Making Connections Between Multiple Representations

The next part of CCSMP1 states “mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem” (CCSS, 2011). Throughout my observations, I witnessed participants using pictures, equations, diagrams, patterns, manipulatives, descriptions, and written and verbal explanations. Sometimes several of these were used together in one problem, sometimes only one or two were used. There were times when the connections made between the representations were strong and other times when they were not (examples to follow). The use of multiple representations and connections made between them was one part of the practice of problem solving that I was able to see through all forms of my data sources – classroom observations, exams, and interviews.

As discussed earlier, drawing a picture was a method often used by participants in order to gain an understanding of the problem and work their way towards an explanation. This was a method encouraged by the instructors and the textbook activities and was also a personal strategy of choice by some of the participants. Jamie and Alex described themselves as visual learners, and Jamie used sketching as a tool to make sense of the problem and try to move on if stuck. Sometimes the students were asked to

analyze or modify a given picture, and other times they were asked to create their own picture. The participants paid attention to detail by carefully labeling their pictures, including units if necessary. There were times, however, when the participants made errors labeling or drawing pictures involving dimensions in M1002. Jordan labeled a square foot as 1 ft in class, Jamie represented 4 centimeters squared by a line 16 centimeters long on the exam, and several participants were representing a linear length with a 2-dimensional drawing both in class and on the exam, although Alex was purposely trying to make it look like a yard stick.

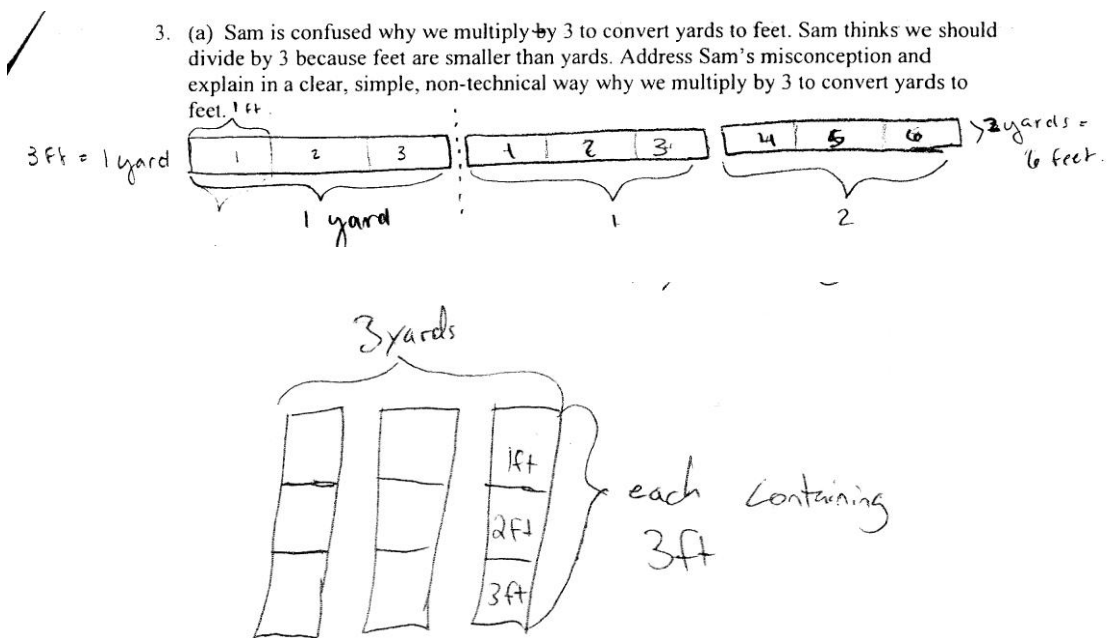


Figure 7. Alex (above) and Taylor (below) representing linear measurements with 2-dimension on the Chapter 11 exam

Another aspect of the pictures was adding action, or building in a progression of pictures to represent what was happening in the problem. As can be seen in the figure below, Jordan showed what happens when you multiply a fraction by $\frac{4}{4}$ and why it is

equivalent to the original and Jamie showed the action of multiplying fractions, both through a series of pictures. This aspect of action with representations was also carried out with equations as will be described next.

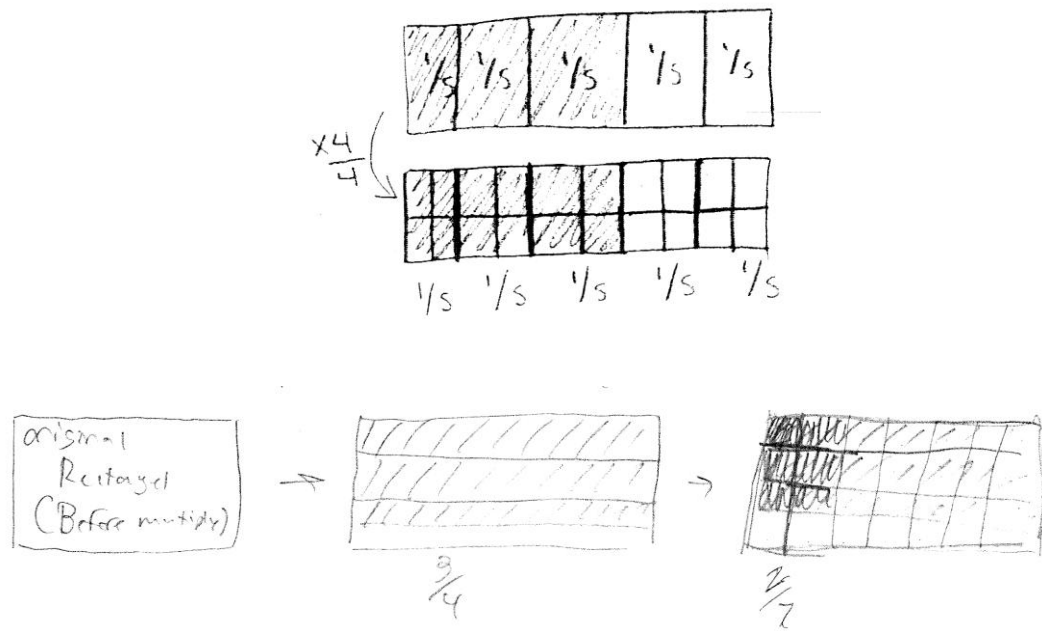


Figure 8. Jordan (above) and Jamie (below) showed the action of multiplying fractions with their pictures on the chapter 2 exam and the chapter 5 exam, respectively

Equations were used as a way to find the answer to a problem and to build an explanation. They were used as a check for reasonableness and to make sure the answer found made sense. Equations were represented with a story problem, used to find unknowns, and explained through pictures and words as to why the formula for multiplication of fractions works the way it does. Sometimes the equations involved action, showing the steps of working out the problem. Alex showed the hidden process of removing decimals and then putting them back in when multiplying decimals through an equation with action, as seen below.

$2.5 \times 10 \rightarrow 25$
 $\times 3.4 \times 10 \rightarrow 34$
 8.509
 $\uparrow \leftarrow \div 10 \div 10 \leftarrow 850$
 we don't need to show
 this zero when in decimal
 form.

Figure 9. Alex showed the process of decimal multiplication through equation actions on the chapter 5 exam

Participants also used manipulatives and patterns. They were given the opportunity to use manipulatives in chapters 2 and 13. In chapter 2, the participants used pattern blocks to explore fractions. They used the pattern blocks to solve fraction problems involving different wholes and parts of the whole. They also used the blocks to build shapes given specific proportions of certain colors to use. In chapter 13, participants used toothpicks and marshmallows to build polyhedra and used them to figure out the number of vertices, edges, and faces. They also used a building block to find a pattern to these numbers based on whether the figure was a prism or a pyramid. Another time participants examined patterns was in interpreting and explaining a given table of equations involving integer multiplication. Participants found the pattern in the table of equations in order to motivate why a positive times a negative is a negative and a negative times a negative is a positive.

Much of what was done in class involved verbal descriptions and explanations of different representations with minimal written words. On the exam, there was more

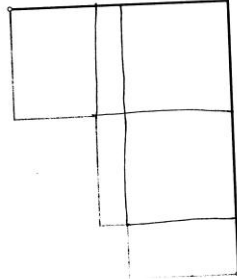
written work and explanation than typically seen in class – as could be expected on a written exam. The exceptions to this rule were when the students were reminded by their instructor to focus on their writing or told to write up their explanations as if it were an exam and with the participants who did mostly independent work, rather than focusing on discussing as a group. This lack of much written work could be part of the contribution to the apparent lack of connection between representations, but there were also times when the connections made were great.

A majority of the time, participants made good connections between representations. So good sometimes, that I was unable to understand their explanation without seeing the actions of the participants who were either pointing at a picture or equation or referring to the manipulative they were using, like when I was listening to the Pencasts from the day. During an observation, I noticed Taylor doing a great job connecting a verbal explanation to a picture that was drawn in order to find the surface area required for the label on a soup can. Taylor was explaining the process using the picture as a reference, so without seeing the reference, hearing the explanation, “You use the circumference... $2\pi r$ and the r is going to be 1.5 because it’s not the whole thing up there” sounded vague, but put into context it was appropriate with the connection being made to the picture.

On occasion there were multiple representations, some connected to one another and others not. Comparing Taylor, Jamie, and Jordan’s work on a homework problem in the figures below, there are some differences in the connections made in each one. In Taylor’s work, there is almost no connection being made between the picture and the explanation aside from having looked at the picture to come to the conclusion. On the

same problem, Jamie referenced the picture using the labels in the description, but does not tie the equation into the description as well as Jordan did. Jordan was able to connect the picture, equation, and description.

6. Which wire is longer, the black one, the gray one, or are they the same length?

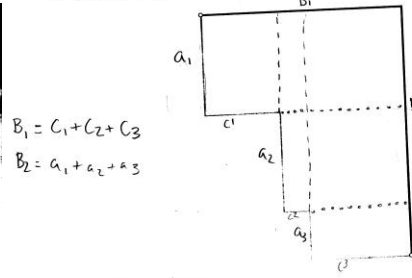


Answer: They are the same

Describe how you found your answer:

I looked at the vertical and horizontal lines separately, and realized they were equal for the black and gray lines.

6. Which wire is longer, the black one, the gray one, or are they the same length?



$$B_1 = C_1 + C_2 + C_3$$

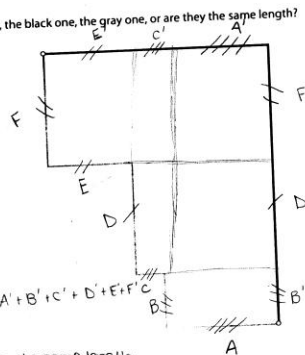
$$B_2 = a_1 + a_2 + a_3$$

Answer: Same length

Describe how you found your answer:

I found my answer by comparing the two sides (labeled B_1 and B_2) against a_1 - a_3 and C_1 - C_3 to see size relation. I discovered that they must be equal because each section combined equals one of two sides of the black wire. This was the simplest way to figure out the solution without tools such as a ruler or measuring device.

6. Which wire is longer, the black one, the gray one, or are they the same length?



$$A + B + C + D + E + F = A' + B' + C' + D' + E' + F'$$

Answer: they are the same length

Describe how you found your answer:

I sectioned the shape into rectangles and then marked each of the parallel sides that are equal. After that, it turns out that the sum of the black (prime) sides equal the gray sides.

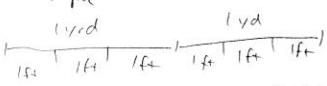
$$\begin{array}{c} A + B + C + D + E + F \\ \text{gray} \end{array} = \begin{array}{c} A' + B' + C' + D' + E' + F' \\ \text{black} \end{array}$$

Figure 10. Taylor's (top left), Jamie's (top right), and Jordan's (bottom left) work illustrating participants' varying efforts to make connections between representations.

Jamie labeled pictures well and paid attention to using those labels in an explanation, both in class and on exams. This connection between the explanation and picture can be seen in Jamie's work on problem #3b on the chapter 11 exam, but in comparison, Jamie made no reference to the picture in problem #3a (Figure 11). On the same problem, Alex made several references to the picture and also tied in an example (Figure 12).

3. (a) Sam is confused why we multiply by 3 to convert yards to feet. Sam thinks we should divide by 3 because feet are smaller than yards. Address Sam's misconception and explain in a clear, simple, non-technical way why we multiply by 3 to convert yards to feet.

If we have a distance say 2 yards long and we want to know how far it is in feet, you must know that it takes more feet to make up the same distance therefore you will have more feet, so you must multiply. Since the units size is smaller (feet smaller than yards) you must multiply by 3 because that is the amount of feet that fit into a yard.

Example

 If we were to divide by 3, you would get $\frac{2}{3}$ which doesn't simplify the distance in feet.

(b) Sam is now trying to convert square yards to square feet. His first guess is to multiply the square yards by 3 to get square feet. Is he correct? Explain in clear terms how you would convert square yards to square feet and why.

SAM'S Idea ↓

 1 side equals 1 yard

Sam is not correct because in a 1 by 1 yards box you must keep in mind that for each side (A, B, C, D) you have 3 ft boxes. EXAMPLE:

 I would convert by multiplying by 9 because you have 9 1 by 1 foot boxes in a 1 by 1 yd box.

B ↑ this whole box is 1 by 1 yd² therefore for each square yard you have 9 ft².

Figure 11. Jamie's chapter 11 exam solutions, one with good connections between the explanation and picture (3b) and the other without (3a).

3. (a) Sam is confused why we multiply by 3 to convert yards to feet. Sam thinks we should divide by 3 because feet are smaller than yards. Address Sam's misconception and explain in a clear, simple, non-technical way why we multiply by 3 to convert yards to feet.

Since feet are smaller than yards we can see from the picture it takes more feet to make up one yard or multiple yards. We see there are 3 ft in 1 yard from the picture above. If we wanted to know how many feet were in 2 yards we would need to have 2 times the amount of feet per yard. Thus it makes sense to multiply $2 \text{ yd} \times 3 \text{ ft}$ to get 6 ft for 2 yards. Yards are the number of groups while feet are the number in each group which can also help us since this is the definition of multiplication.

Figure 12. Alex's chapter 11 exam solution with explicit connection made between the picture and explanation.

Sense Making

The last part of CCSMP1 states “mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, ‘Does this make sense?’ They can understand the approaches of others to solving complex problems and identify correspondences between different approaches” (CCSS, 2011). During class, the participants would often ask their group mates, “Does that make sense?” after giving a verbal explanation of what they did and why. Other sense making strategies I witnessed were reading back through the question and making sure the problem was answered appropriately, seeing if the answer was reasonable in terms of its value, double checking calculations, using estimation, and relating to relevant prior knowledge.

The participants would also try and make sense of others' approaches during class activities. If the participants did not understand another's approach, they would ask clarifying questions. If there was fault in the approach of another, they would sometimes correct them. Sometimes they would restate what had been said in their own words, add to them, and, on occasion, use the other's strategy in a later problem. The participants would compare the other's strategy to their own, noting similarities and differences in the approaches.

I could not see how the participants made sense of or checked the reasonableness of their solution on the exam. I could, however, witness their approach to understanding others when the exam problem posed was interpreting the incorrect work of a student. I was able to see how they made sense of or checked the reasonableness of their solutions during the interviews. When asked during the interview if they were satisfied with their solutions an exam question, participants had several reasons why they were or were not. Several participants double checked their calculations, just "knew" the answer was right, related to what they had done in class, or made sure the question was being answered and the directions followed in order to feel satisfied with their solutions. Taylor felt satisfied if the picture, explanation, and solution all aligned with one another. Jordan and Ande both talked about being satisfied with their explanations because they could be generalized, or explained with any set of numbers. Finally, participants made a few reflections about multiple strategies or approaches to a problem. Ande thought a different approach to solving a problem on the chapter 5 exam should have been used, and Taylor also thought there was probably a better approach to take on one of the chapter 13 exam questions. Alex even went as far as to explain a few different

approaches that could have been taken on one of the chapter 11 exam questions, but was most sure about the one that was taken.

Summary

All participants actively attempted to understand and devise a plan to solve the problems given in class and on the exams. More often than not, participants used pictures to try and understand the problem better and work towards an explanation, and they also made comparisons to similar problems. As the year progressed, the participants improved in evaluating their progress and determining if another strategy needed to be used to solve the problem. All participants made reflections during the interviews either about what they could have done better or realized they should have done something else but did not know what that was. Throughout both semesters participants were using pictures, equations, patterns, manipulatives, and explanations to represent and solve the problem at hand. Participants were actively asking group mates and themselves if their solution made sense and if it answered the question at hand. They also worked to try and understand the approach of others by asking questions and making comparisons and connections between approaches.

Results from the Qualitative Data – Constructing Viable Arguments

Using Key Information in Justifying

The first part of CCSMP3 states, “Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments” (CCSS, 2011). Throughout both semesters, I witnessed the participants using

definitions, given information, prior knowledge of formulas and personal experience, and conversion equations to help build arguments. I was able to see what they were using in their verbal and written arguments in class, in the written arguments on the exam, and in their verbal arguments during the interview. Participants also used the information and results from class on the exams. This included using information learned in previous chapters.

At the beginning of the semester in M1001, students were working with fractions in chapter 2 of the text. During class, students were to work in groups to come up with a definition of a fraction A/B . They then came together as a class to work on refining the definition. I witnessed students using parts of the definition of a fraction to build arguments both in class and on the exam. Participants used the meaning of the numerator and denominator to both describe fractions and make comparisons between fractions. They also understood and used the importance of the whole being considered for each fraction and knew that to compare fractions, the same whole must be used. None of the participants were consistent in using the equal parts portion of the definition and Jamie latched on to using the idea of a unit fraction in explanations. Participants also used their knowledge of fraction operations, finding common denominators, and building equivalent fractions when solving problems and building arguments.

Toward the end of the semester in M1001, students were working on multiplication of different sets of numbers in chapter 5 of the text. When examining how multiplication of fractions works, students were told to use the definitions of multiplication and fractions to explain. This question also appeared on the chapter 5 exam without explicit directions to use the appropriate definitions. Alex and Ande

verbally recognized the use of the definitions of multiplication and fractions in their explanations during their interview with me. When examining story problems, the participants used cues and wording such as “take away” and “of” to determine what fraction operation was implied. Similarities and differences in the wording of questions was identified, such as if it was asking for “what fraction” or “how many.” Some participants did a better job with interpreting and paying careful attention to details and wording of the problem than others. Participants used and built upon their knowledge of the standard algorithm for decimal multiplication and our base 10 system and place value to explain why the algorithm for decimal multiplication works. Finally, I witnessed the participants incorporating the commutative and distributive property to work through and explain problems involving integer multiplication.

During my observations near the beginning of the semester in M1002, students were studying measurement in chapter 11 of the text. During class, participants used definitions of shapes, dimensional knowledge, formulas for finding the area and volume of different shapes, conversion formulas, the meaning of multiplication and division, the difference between measuring and counting, and the size and number of a unit in their work and explanations of problems. Alex and Jordan also used some proportional reasoning, and Alex and Taylor used the relationship between fractions and decimals. The participants used given information and followed the constraints of the problem, sometimes at the guidance of the group or instructor. I also saw all of the abovementioned knowledge being used on the exams and discussed during the interviews, and Alex verbally recognized using the definition of multiplication in one of the arguments on the exam.

Toward the end of the semester in M1002, students were studying volume and surface area of solid shapes in chapter 13 of the text. The activities and questions from this chapter tended to be more description and less explanation. Regardless, students were using the definitions of shapes, patterns (Alex only), formulas for the surface area and volume of different shapes, conversion formulas, Archimedes principle, angle measures, and given information in the problem. They were also building with and using manipulatives. Participants were also comparing similar problems and deciding whether a similar strategy was appropriate. Alex and Jamie were able to use their prior knowledge from a physics class on class activities and exam questions. Alex once again used proportional reasoning. All of these pieces of knowledge were also used in constructing solutions and explanations on the exam and explained during the interviews.

Building Logical Progressions – The Argument

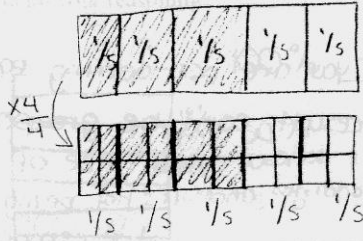
The next part of CCSMP3 states, “Mathematically proficient students...make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples...Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions” (CCSS, 2011). All participants were given the opportunity to make conjectures and explore during class time. At one time or another, all participants made good arguments through a logical progression of steps, and I noticed Jordan, Taylor, and Jamie using counterexamples or a simpler case to build an argument. The participants would use combinations of representations to build their arguments, including pictures, equations, tables, and both

verbal and written explanations. Oftentimes in class, verbal explanations were used more frequently than written. What was written, if anything, tended to be less detailed. Even though the participants each provided good explanations through a logical progression of statements, there were times when the explanations offered could have been better through the addition of more detail or precision. Sometimes the arguments were just wrong, following faulty logic, and sometimes they were confusing due to gaps in reasoning or an imprecise use of appropriate vocabulary. I noticed that sometimes an explanation of *what* or *how* was given instead of *why*.

With the exception of Joey, all participants had good, well thought out and logical explanations throughout their experiences in M1001 and M1002. This indicated the effort participants put into the coursework made a difference. To build these logical explanations, participants would oftentimes incorporate pictures and equations. There were no gaps in reasoning and a good understanding of the problems was shown. One example of good explanations is included in the figure below. Jordan was very specific in the chapter 2 exam explanation, addressing all aspects of the problem, representing the action of multiplication with a set of well labeled pictures using the given example and describing the process and why it creates an equivalent fraction. On the chapter 13 exam, Alex began with the conjecture of which soup container would be the better choice and then explained why it would be the better choice. Alex included well labeled pictures and equations along with the explanation, which connected to the picture and equations and followed a logical progression of steps using known and given information. On the chapter 2 exam (Figure 14), Ande used a series of well labeled pictures to show the process of solving the problem which represented operations with fractions. Ande

followed by explaining the step by step process in arriving at the answer seen in the series of pictures.

2. Without using multiplication by 1, explain why multiplying the numerator and denominator of a fraction by the same number produces an equivalent fraction. You may use the example $\frac{3}{5} = \frac{3 \times 4}{5 \times 4}$. Draw a picture.

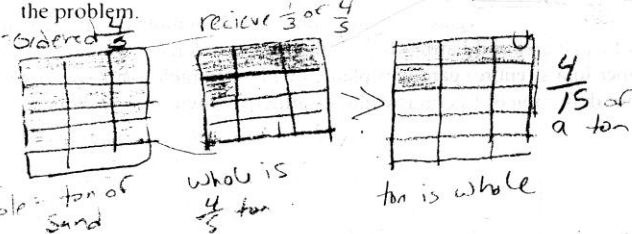


It is equivalent because you are dividing each $\frac{1}{5}$ piece into 4x as many pieces, but you are also shading in 4x as many pieces to compensate for the increased number of pieces.

- numerator - 4x as many shaded
- denominator - divided into 4x as many pieces

Figure 13. Jordan's chapter 2 exam – good argument.

3. Eta ordered $\frac{4}{5}$ of a ton of sand. Eta wants to receive $\frac{1}{3}$ of the order now (and $\frac{2}{3}$ of the order later). What fraction of a ton of sand should Eta receive now? Draw a picture to help you solve the problem. Explain how your picture helps you to solve the problem. Be clear about what the whole is for each fraction in the problem.



Ordered $\frac{4}{5}$ receive $\frac{1}{3}$ or $\frac{4}{5}$

whole = ton of sand whole is $\frac{4}{5}$ ton ton is whole

Eta ordered $\frac{4}{5}$ a ton of sand. $\frac{4}{5}$ is $\frac{12}{15}$. She only wanted $\frac{1}{3}$ of her total 12 pieces now and $\frac{1}{3}$ of 12 is 4. She wants now $\frac{4}{15}$ of a ton of sand. She started by ordering $\frac{12}{15}$ a ton, then she wanted $\frac{4}{12}$ of her total order then you put it back into the ton and she wanted $\frac{4}{15}$ of a ton.

Figure 14. Ande's chapter 2 exam – good argument.

There were also instances in which the participants used counterexamples or considered simpler cases when building their arguments. On a chapter 11 exam question,

Jamie and Taylor both extended a given picture as a counterexample to show why a student's reasoning was incorrect. In class, Taylor and Jordan used counterexamples to explain why it is inappropriate to say that area is length times width. When Taylor's group was struggling to figure out how to use the distributive property with integers, Taylor made a simpler example without negative numbers to highlight the process. Finally, Jordan built an argument of what to do to find the number of feet in 6 yards by working up to it and starting with the simpler cases of 1 yard and 2 yards to highlight the process and operation that should be used.

Verbal or Written Explanations. Oftentimes in class, there was much more verbal explanation given than written. When considering the work of a teacher, this may seem appropriate. Given the students have to take written exams, however, this meant that some students were not getting appropriate practice for exams during class time. Sometimes the students would be instructed to write more in class, but oftentimes writing put an end to discussion. When Ande, who usually did independent work, was reminded by the instructor to work with the group, the writing stopped. Much of what I saw written in class was a picture or equation with a few words and then a verbal explanation given to the group or class. This is with the exception of Ande who did most of the work independently and thus was not giving as many verbal explanations to the group as the other participants. Most of this lack of written work was illustrated in class, but on occasion showed up on exam questions. Jordan and Taylor both had very little written explanation on some of the chapter 5 exam questions as can be seen in the figures below.

1. Use pictures to help you explain why we multiply the numerators and why we multiply the denominators when we multiply fractions. Use the following example in your explanation:

$$\frac{2}{7} \times \frac{3}{4} = \frac{2 \times 3}{7 \times 4} = \frac{6}{28}$$

2 groups of 3 items

7 groups of 4 items

you have 6 copies of $1/28$ items

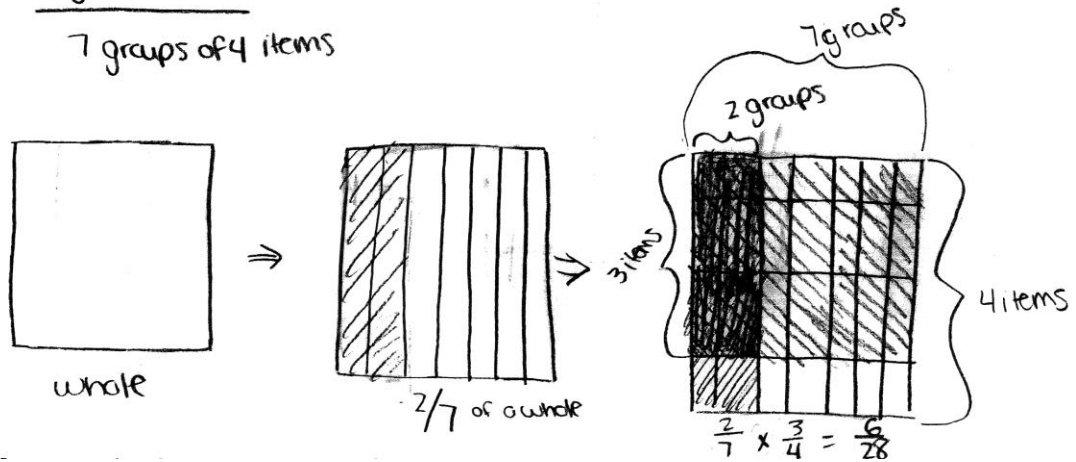
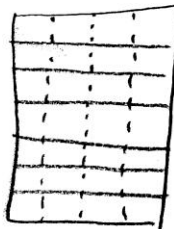


Figure 15. Jordan's chapter 5 exam #1.

1. Use pictures to help you explain why we multiply the numerators and why we multiply the denominators when we multiply fractions. Use the following example in your explanation:

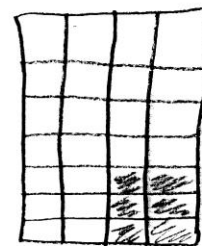
$$\frac{2}{7} \times \frac{3}{4} = \frac{2 \times 3}{7 \times 4}$$

denominator = total pieces that make up the whole.



$7 \times 4 = 28$ is the new whole

numerator = # of parts of the whole $2 \times 3 = 6$



$$\frac{6}{28}$$

Figure 16. Taylor's chapter 5 exam #1.

Lack of Precision. Some of the arguments followed appropriate logic, but were either missing little details that could strengthen the argument or used imprecise words or phrasing. On the chapter 2 exam, question 5 asked students to compare different pairs of fractions. Comparing the work of Alex, Jordan, and Taylor, some of these missing details become evident. Alex's explanation and solution to the question were good and included some of the details that are lacking in Jordan and Taylor's similar explanations. Taylor and Jordan said $21/80$ is one (piece) larger than $1/4$, but do not show the comparison of $20/80=1/4$. Taylor does explain that $1/4$ of 30 is 7.5, so $7/30$ is less than $1/4$, but Jordan again does not go into that explanation. Along the same line, there was a time in class when Jordan's explanation for comparing fractions was imprecise. Jordan wrote, "When the numerators are the same look at the denominator. The smaller denominator is bigger because it is divided into fewer pieces making them bigger pieces than if it was divided into more pieces." The overall right idea and understanding of fractions is there, but instead of saying "the smaller denominator is bigger", Jordan should have said the smaller denominator indicates the overall fraction is bigger.

The arguments that were the most confusing came from the beginning of M1001, during chapter 2, with the exception of some issues with vocabulary and dimension in M1002. These arguments either had some gaps in reasoning or used language and a progression (set of statements) that was vague and hard to follow. On the chapter 2 exam when Ande was justifying which fraction was larger, Ande determined one of the fractions was larger than $1/4$ and the other was larger than $1/5$, but was missing additional justification as to why this made the fraction larger than $1/4$ the biggest. During chapter 2, Jamie responded to Anna, a student in the class problem, who was confused about why

you cannot add the same number to the numerator and denominator to get an equivalent fraction by writing, “You have to multiply not add and you have to multiply by different numbers.” When answering another problem in class about a student who thought $6/6$ was bigger than $5/5$, Jamie responded, “No because both $5/5$ and $6/6$ are a whole just with more pieces and numbers. Both of these arguments are confusing because Jamie does not expand upon some of the reasoning such as what “you have to multiply by different numbers” means or what “a whole with just more pieces and numbers” means. They are also missing much of the *why* component which will be discussed further at the end of this section.

Faulty Logic. Many of the errors made by students in class were corrected by classmates, class discussions, or further interpretation of the problem. I did, however, notice participants using faulty logic on some exam problems. One example was when Joey and Alex did not pay attention to the importance of equal parts on a chapter 2 exam question. There were two garden plots of different sizes, each with 8 pieces and Joey and Alex treated the overall fraction out of 8. Alex realized this as an error during the interview, but was struggling with the idea of combining different shaped pieces together to make an equal piece of the whole (the garden plots consisted of a rectangle and a circle). Several participants struggled with the same chapter 5 exam question relating to fraction multiplication. Students were asked:

Can the following problem be solved by calculating $1/3 \times 2/3$? Explain why or why not. Ed put $2/3$ of a bag of candies in a batch of cookies that he made. Ed ate $1/3$ of the batch of cookies. How many candies did Ed eat (assuming the candies were distributed evenly throughout the cookies)?

Taylor, Ande, and Joey all answered the same problem with incorrect logic. Ande said it does not work because the whole for each is different, not realizing it can be solved by calculating $1/3 \times 2/3$ and ignoring the fact it is asking for how many candies rather than what fraction. Joey thought the story problem could be solved by calculating $2/3 - 1/3$. Taylor thought the problem could be solved by calculating $1/3 \times 2/3$, ignoring the fact that it was asking for how many candies.

Jamie, Jordan, and Joey (low initial MKT scores) sometimes explained *what* or *how* instead of *why* on a few problems. Jamie did this on both the chapter 2 and the chapter 11 exams where Jamie stated *what* was done, but not *why* it was done. Jordan had some of the same issues as Jamie, leaving most of the *why* out of the argument for the same chapter 11 exam problem as Jamie did. Joey was also missing some detail and the *why* of the explanation in a chapter 2 exam problem. Joey started in the right direction on a problem asking for an explanation why you multiply and not add the same value to the top and bottom of a fraction to get an equivalent fraction. Joey started with comparing multiplication and addition by 1, but did not include why one changes the value and the other does not. Joey added a picture to strengthen the argument, but did not explain the process creating the pictures or justify the ending statements made. During the interview over chapter 11, Jordan admitted that explaining why was still a weak point and said, “I’m really good explaining how I did it, but my reason why is still vague in some areas.” When I asked Jordan the difference between a good explanation and a vague explanation, Jordan said there had to be “enough detail.”

Summary

Throughout both semesters, participants used definitions, given information, and known formulas and equations to help build their explanations or arguments. Participants constructed their explanations on exam questions based on information and experience from curricular materials and class activities, including information and results from previous chapters. As time went on, more of the participants verbally acknowledged using definitions to construct their explanations. Participants also used multiple representations to build their arguments, such as pictures and equations, sometimes at the expense of much written work. The participants relied on using verbal arguments in class, and the written words, if any, tended to be less detailed. For the most part, participants constructed arguments through a logical progression of statements (with the exception of Joey). Arguments that were confusing and missing detail were most prevalent in chapter 2. Arguments following faulty logic were exclusively seen on the M1001 exams with the exception of Jamie.

Summary of the Qualitative Results

The qualitative data in this study were gathered to answer the first research question:

1. How do certain prospective elementary teachers progress in their enactment of the mathematical practices of persevering in problem solving and constructing viable arguments as they move through inquiry-based mathematics content courses?

In moving through the curriculum, participants examined problems and constructed explanations of mathematical concepts using multiple representations, definitions, and previously established results. The curriculum provided opportunity for the students to work together in groups and as a class to discuss the activities and work towards an understanding of the material good enough to be able to explain important concepts – that is, to the degree that the students were willing to put in the work. Joey put the least amount of effort into the class and had the most problems with understanding, solving problems, and constructing explanations. Compared to Jamie, who also started with an MKT in the bottom half of the class and an even lower PK score, Joey put in less effort, made less gains in competencies, and was less successful in the course.

There were two items of importance that came up related to the participants and their learning and practice in this curriculum. One element that came into play was the overall atmosphere of the classroom and the attitudes of the students. I was witness to this and some of the participants discussed this facet with me during our interviews together. Jamie, Alex, and Jordan all commented on how difficult it was to stay positive and motivated when others in their group had a negative attitude and would rather complain than do the work. The other element that came into play with regard to participants' learning and practice was the nature of the content in M1001 compared to M1002. Many of the questions, especially in chapter 13, were asking for descriptions rather than explanations and as such were missing the *why* component that is a part of constructing an argument. Alex and Taylor also made note of this difference during our last interview together.

Quantitative Results

Introduction

This section reviews the individual assessments used for the quantitative portion of this study and describes the scores over time of the individual assessments. Next, each of the pair-wise relationships will be described, with connection to the participants in the qualitative portion of the study. Finally, a summary of the quantitative results will be presented with attention given to the second and third research questions.

There were a total of 59 students who originally enrolled in M1001 in the spring 2014 semester. Four of those 59 students did not complete M1001, either due to poor performance or a high number of absences. Of the remaining 55 students at the end of M1001, 7 did not enroll in M1002 in the fall. These students either did not pass M1001, switched majors, transferred, or did not register for unknown reasons. Ande and Joey were part of this group. Of the 48 students remaining at the beginning of the fall 2014 semester in M1002, only one more dropped out due to poor performance. The scores of the 47 students who took M1001 in the spring 2014 semester and M1002 in the fall 2014 semester were used to create the boxplots of each individual assessment and also used in the calculation of correlations. The students who initially enrolled in M1001 in the spring but dropped out at some point of the study are, however, included in the scatter plots of each of the pair-wise comparisons for discussion purposes. These students were represented by different colored points than the rest of the students, as will be described in the relevant section. All graphs and statistical analyses were made using R (R Core

Team, 2014), and repeated measures ANOVA was performed using the `lme()` function in the package `{nlme}` (Pinheiro J, Bates D, DebRoy S, Sarkar D and R Core Team, 2015).

Mathematical Knowledge for Teaching

The Mathematical Knowledge for Teaching (MKT) assessment used in this study focused on K-6 Number and Operations content. “These items probe whether teachers can solve mathematical problems, evaluate unusual solution methods, use mathematical definitions, and identify adequate mathematical explanations” (Hill & Ball, 2006). The assessment consisted of 26 multiple choice problems and the scores reported are Item Response Theory (IRT) standardized scores with possible outcomes ranging from -2.976 to 2.450. The profile plot and boxplots in Figure 17 below represent the MKT scores of the 47 students moving through both M1001 and M1002 over time. The profile plots show the path taken by each student whereas the boxplots show the overall results from the class as a whole. Following the figure is a table of summary statistics for the MKT scores of the same 47 students. The participants in the qualitative portion of the study are represented by the colored lines and points. Students were given the assessment during the first and last week of classes during both semesters. As the boxplots show, there was a big jump in median MKT score from January to April. This makes sense as the content focus in M1001, Numbers and Operations, is also the focus of the exam. There was a slight drop in the median score from April to August (over the summer) but the median score went back up in December. This indicates the knowledge lost over the summer was gained back through the course of M1002 which was focused on Geometry and Measurement.

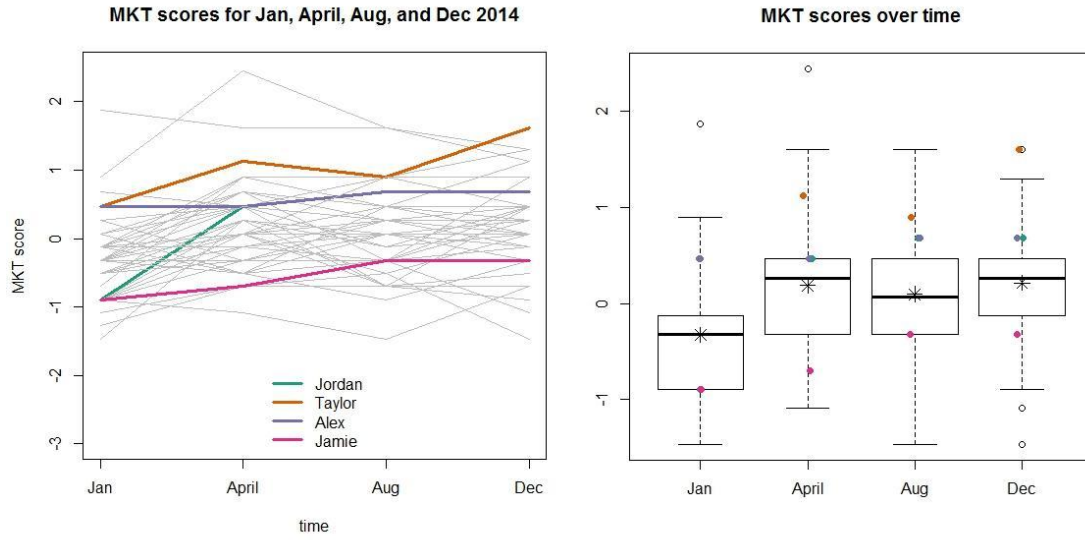


Figure 17. Profile plot and boxplots of students' MKT scores over time.

MKT	Jan	April	Aug	Dec
Mean	-0.326	0.188	0.098	0.217
Median	-0.322	0.268	0.068	0.268
Minimum	-1.47	-1.085	-1.47	-1.47
Maximum	1.872	2.45	1.61	1.61
S.D.	0.628	0.652	0.62	0.661

Table 1. Summary statistics of MKT scores over time.

Because there were multiple observations over time for the same students, I conducted a repeated measures ANOVA to test the null hypothesis that there are no differences among the mean MKT scores across all testing times, while accounting for the dependencies in the observed data. The repeated measures ANOVA provided sufficient evidence against the null hypothesis ($p=0.049$), suggesting the mean MKT scores of at least one of the testing times was different from the rest. To follow up, I investigated all pairwise comparisons of mean MKT scores across the testing times using Tukey's Honest Significant Difference procedure (still accounting for repeated measures

within participants). Using familywise 95% confidence intervals for each pairwise difference in means, I concluded that there is a difference in scores between January and all other months, as is illustrated in Figure 18 below.

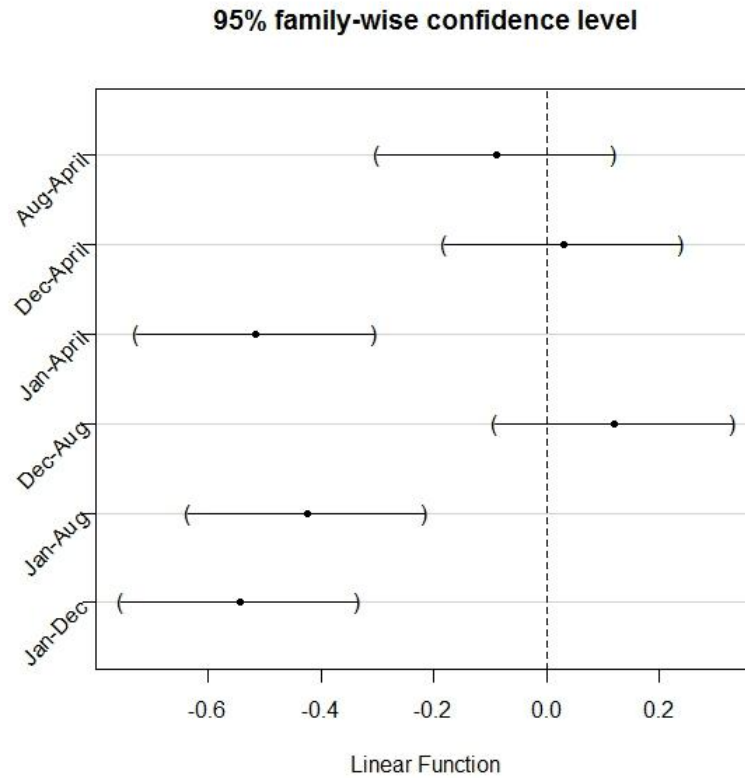


Figure 18. 95% family-wise confidence intervals for pair-wise differences in MKT score means.

Procedural Knowledge Exam

The procedural knowledge (PK) exam used in this study consisted of 25 multiple choice questions measuring simple computational skills and manipulations of basic algebra (Maplesoft, 2011). A raw score (percentage) was used to report the scores of the prospective elementary teachers. The range of possible scores is 0% to 100%. The

profile plot and boxplots in Figure 19 below represent the PK scores of the 47 students moving through both M1001 and M1002, over time. Following the figure is a table of summary statistics for the PK scores of the same 47 students. Students were given the assessment during the first and last week of classes during both semesters. The median PK score was a little higher in January and December, but overall there was little change in the median score. The spread of scores was smallest in January. This could be in part because some students had studied the material on the procedural knowledge exam in order to get a 65% or better to meet the prerequisite for entry into M1001. Jamie (pink dot) was one of these people, admitting to forgetting the material soon after the exam was taken. Given that students will learn algebra in M1100, the next course in the three-course sequence, it would be interesting to see if there are gains made in students' PK scores after taking that course.

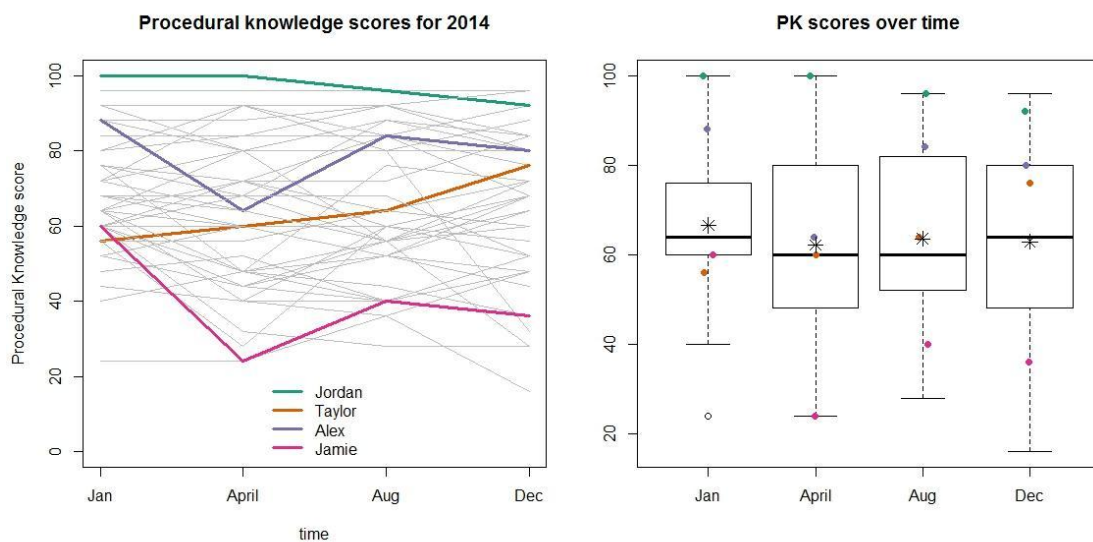


Figure 19. Profile plot and boxplots of students' PK scores over time.

PK	Jan	April	Aug	Dec
Mean	66.55	62.13	63.57	62.89
Median	64	60	60	64
Minimum	24	24	28	16
Maximum	100	100	96	96
S.D.	15.97	19.86	18.93	19.54

Table 2. Summary statistics of PK scores over time.

Attitudes Toward Mathematics Inventory

The Attitudes Toward Mathematics Inventory (ATMI) instrument used in this study was a 40 item, five-point Likert scale survey with responses ranging from strongly agree to strongly disagree. There were fifteen items associated with self-confidence (anxiety), ten for value, ten for enjoyment, and five for motivation. Higher scores are indicative of more positive attitudes. The ATMI scores are reported as a composite sum, which can range from 40 to 200. The boxplots in Figure 20 below represent the ATMI scores of the 47 students moving through both M1001 and M1002, over time. Following the figure is a table of summary statistics for the ATMI scores of the same 47 students. Students were given the assessment during the first and last week of classes during both semesters. Overall, there was very little change in the median ATMI score. The spread of scores is smaller in April with the minimum ATMI score increasing by approximately 30 points. The phenomenon faded over the second semester, with the spread getting larger and the minimum score getting lower in August and then December. This could be related to the nature of the content in M1001 compared to M1002. I noticed a difference in the nature of the types of questions asked, as did two of the participants. Also, as was indicated by Jordan, some students like numbers better than geometry.

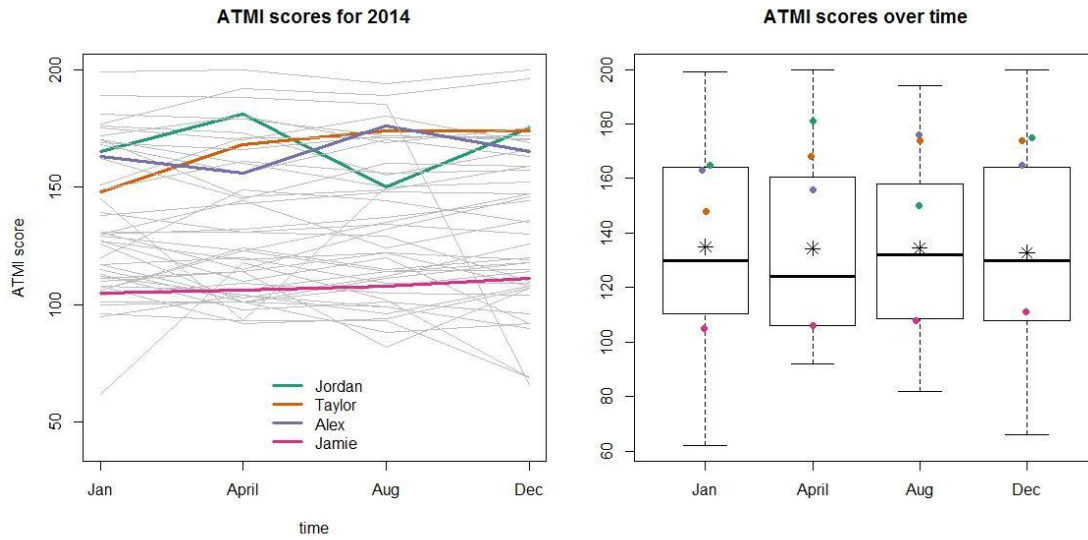


Figure 20. Profile plot and boxplots of students' ATMI scores over time.

ATMI	Jan	April	Aug	Dec
Mean	134.96	134.34	134.45	132.80
Median	130.00	124.00	132.00	130.00
Minimum	62.00	92.00	82.00	66.00
Maximum	199.00	200.00	194.00	200.00
S.D.	30.59	31.24	30.96	34.52

Table 3. Summary statistics of ATMI scores over time.

Pair-wise Comparisons

Mathematical Knowledge for Teaching v Procedural Knowledge. There was a consistent positive and fairly linear relationship between MKT scores and procedural knowledge scores throughout the entire year, from when students entered M1001 to when they left M1002 (see Figure 21). The upper left corners of the plots consistently stay empty, implying no students scored very high on the MKT assessment yet very low on the procedural knowledge assessment. From January to April, the space in the bottom right corner cleared, indicating students who started with low MKT scores and high

procedural knowledge scores no longer remained in that category by the end of the first semester. The students who either dropped in the middle of the semester (red points) or did not enroll in M1002 (green points) represented a wide range of MKT and procedural knowledge scores in January. In April, however, the students who made it to the end of the first semester but did not take M1002 (green points) represented the lower range of both MKT and procedural knowledge scores. This is with the exception of Ande, who was mid range in both, but ended up switching majors and dropping out of the study at the end of the first semester.

Taking a closer look at the path of the participants in the qualitative portion of the study (shapes), the students who made bigger improvements in their MKT scores during the first semester (Taylor and Ande) also made gains in their procedural knowledge scores. This is with the exception of Jordan who went from a perfect score to a near perfect score on the procedural knowledge exam. It is also the opposite movement made by those participants who only made minor improvements in their MKT score (Alex, Jamie and Joey). Their procedural knowledge scores were lower at the end of the first semester. From April to August all four of the participants remaining in the study made improvements in both their MKT and procedural knowledge scores. From the beginning to the end of the second semester, the only gain in PK score was made by Taylor. This could be, in part, due to the fact that Taylor was enrolled in other mathematics courses during this study.

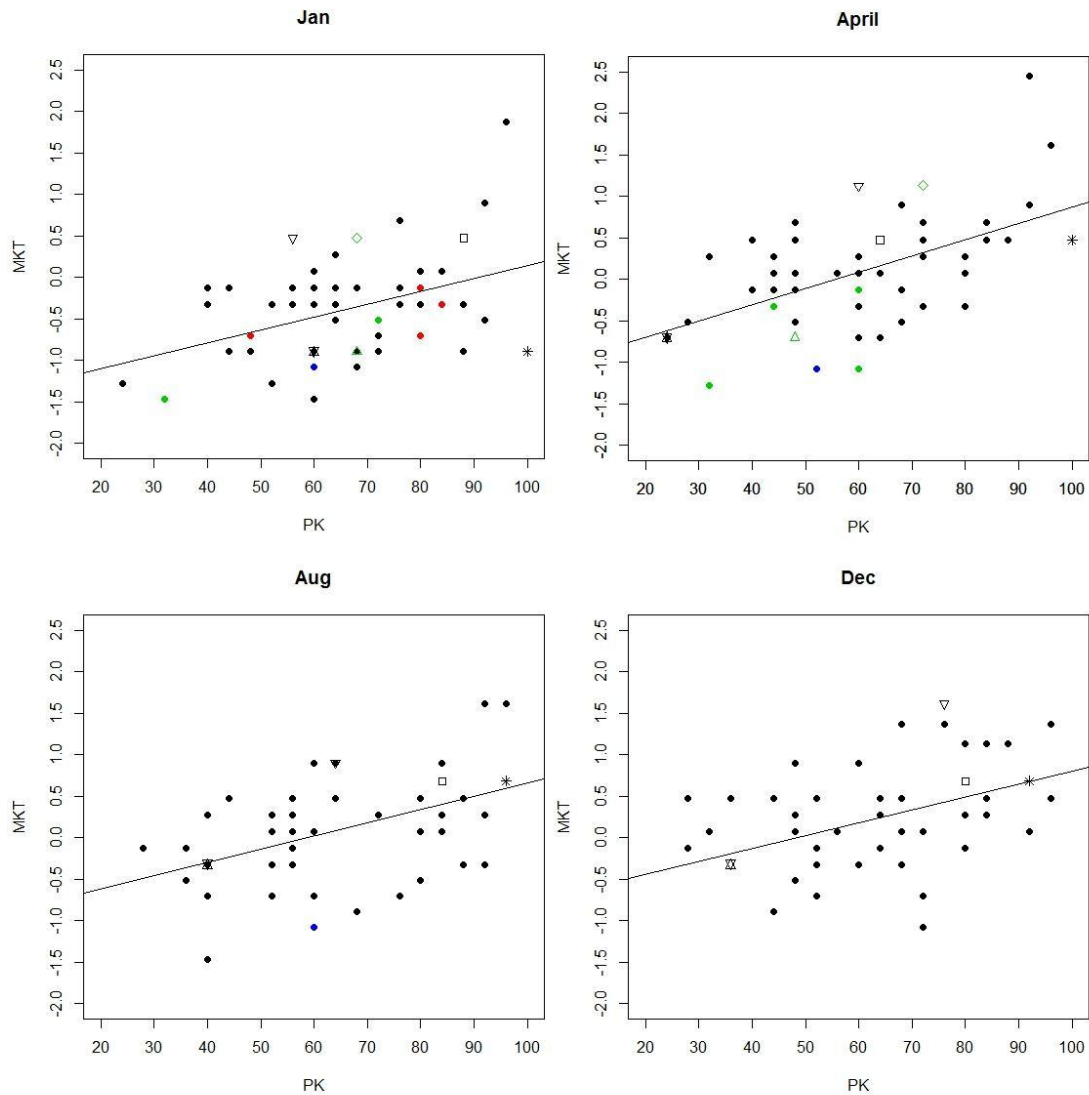


Figure 21. Scatter plots of the pair-wise relationship between MKT and PK over time.

Mathematical Knowledge for Teaching v Attitude. There was a consistent positive and fairly linear relationship between Mathematical Knowledge for Teaching (MKT) scores and Attitudes Toward Mathematics scores throughout the year. From January to April there was an upward shift in the minimum attitude score and a small shift up in the minimum MKT score. Consistent in all four time periods was an empty

space in the upper left corner, indicating there were no students with very low attitude toward mathematics scores and high MKT scores. From January to April there was a shift of the students with high attitude scores and low MKT scores, creating an empty space in the lower right corner. This empty space continued to be prevalent into August and December, indicating a lack of students with both high attitudes toward mathematics and low MKT. The students who dropped during the first semester (red points) represented a wide range of initial attitude scores, but none were above a score of zero for MKT (standardized score). In April, those students who did not move on to M1002 (green points) represented the middle of the class in terms of attitude and the lower half of the class in terms of MKT. Again, this is with the exception of Ande (green rhombus).

When looking more closely at the paths of the participants in the study (the shapes), the students who made big improvements in MKT score (Taylor, Jordan, and Ande) also made improvements in attitude score from January to April. Those students making minor improvements in MKT score (Alex, Jamie, and Joey) showed drops in their attitude scores from January to April. From April to August to December, Taylor and Jamie made continual improvements in MKT and attitude scores, Taylor at a faster pace than Jamie. Alex and Jordan both continued to improve slightly in their MKT scores, but were inconsistent with their attitude scores.

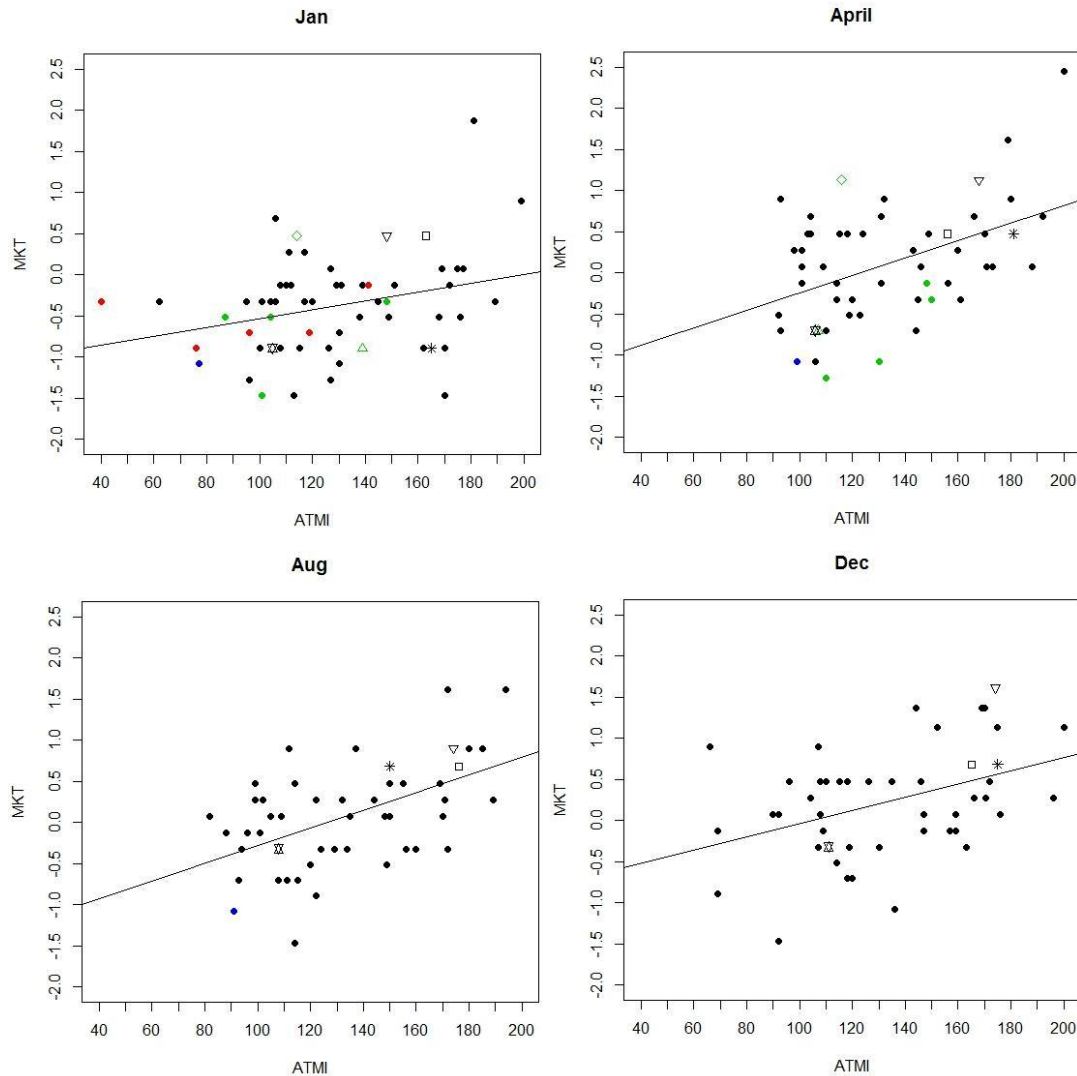


Figure 22. Scatter plots of the pair-wise relationship between MKT and ATMI over time

Procedural Knowledge v Attitude. There was a fairly consistent positive linear relationship between procedural knowledge and attitude toward mathematics scores throughout the year and the strength of the linear relationship appeared to get stronger over time. From January to April, there was a shift up in attitude scores. This shift was continued into August and then a few students dropped back down to lower attitude

scores by December. In addition to the empty space created by a lack of very low attitude scores, there are also blank spaces in the upper left and lower right corners of the scatter plots. The empty space in the upper left corner, which became more pronounced over time, indicates there were no students with very high procedural knowledge and very low attitude toward math. The empty space in the lower right corner, which also becomes more pronounced over time aside from one student in December, indicates there were no students with very high attitude toward mathematics and very low procedural knowledge. Students who dropped out during the first semester (red points) represented a wide range of both attitude and procedural knowledge scores, although did not represent the highest range of attitude scores. The students who made it to the end of the first semester but did not take M1002 (green points), had mid-range attitudes and low procedural knowledge scores when compared to the rest of the class.

When examining the path of the participants in the qualitative portion of the study (shapes), Alex and Joey decreased in both of their scores from January to April while Jordan, Ande, and Taylor increased in both of their scores. Jamie increased slightly in attitude score while decreasing majorly in procedural knowledge score. Overall from the beginning of M1001 in January to the end of M1002 in December, Jordan only made small fluctuations in both attitude and procedural knowledge scores. Alex went from decreasing in both scores at the end of the first semester to making nearly equivalent gains over the summer and then maintaining scores to the end of the second semester. This decrease seen in scores at the end of the first semester could be attributed to end of the semester stress and little investment in the assessments. The same dip and gain in

procedural knowledge score from April to August was also seen by Jamie. Taylor continued to show growth in both August and December.

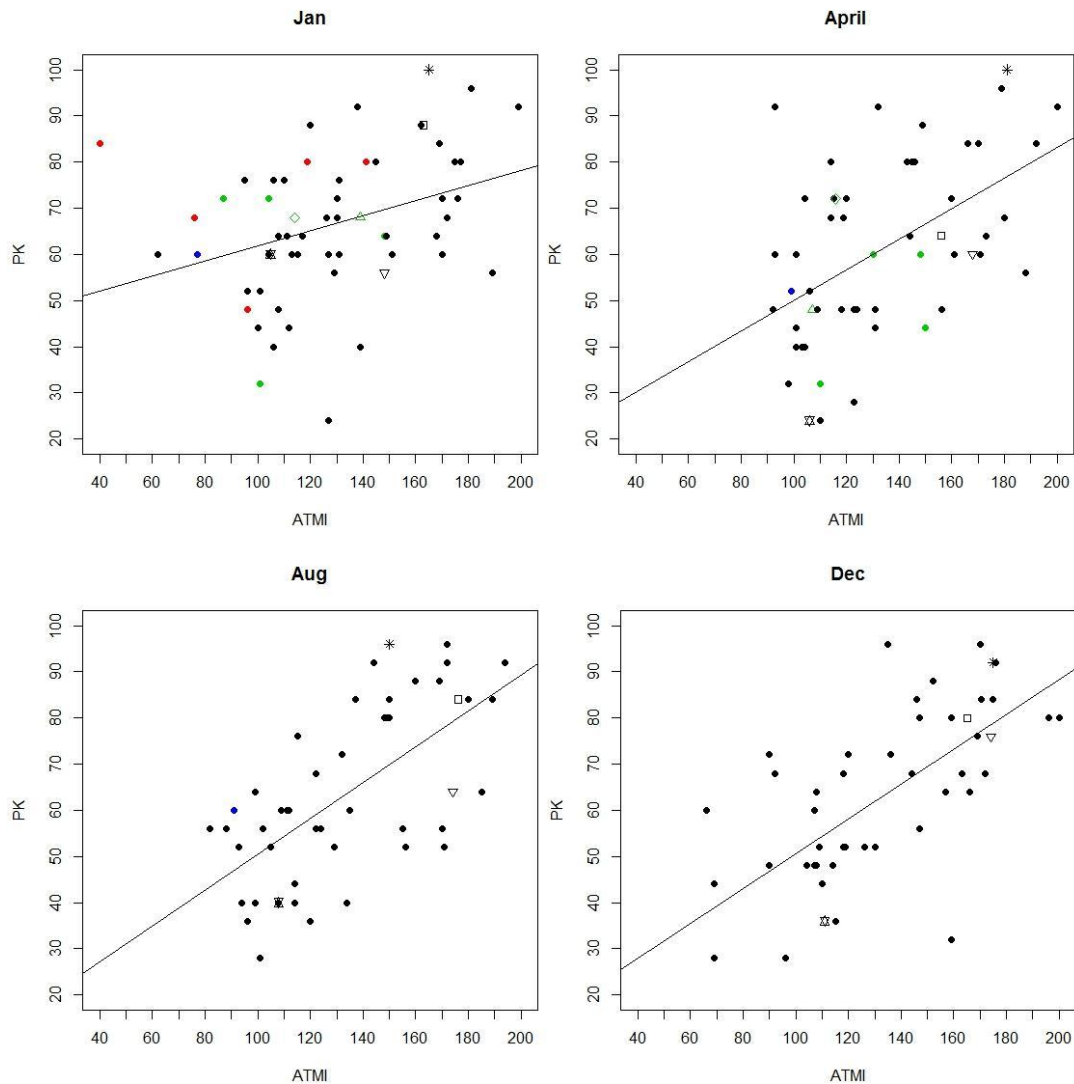


Figure 23. Scatter plots of the pair-wise relationship between PK and ATMI over time

Correlations. Because all of the pair-wise comparisons for all time periods represented a somewhat positive linear relationship, correlations were calculated for each pair during each time period. The following figure shows the correlation coefficient, r ,

for each pair-wise comparison in January, April, August, and December. They all begin with a weak to moderate positive correlation, with the relationship between MKT and PK being the strongest. All of the relationships strengthened from the beginning to the end of the first semester. Over time, however, the correlation coefficients between MKT and PK as well as between MKT and ATMI decline and level off. The relationship between PK and ATMI continues to grow, ending at approximately 0.67 which is nearing a strong positive relationship.

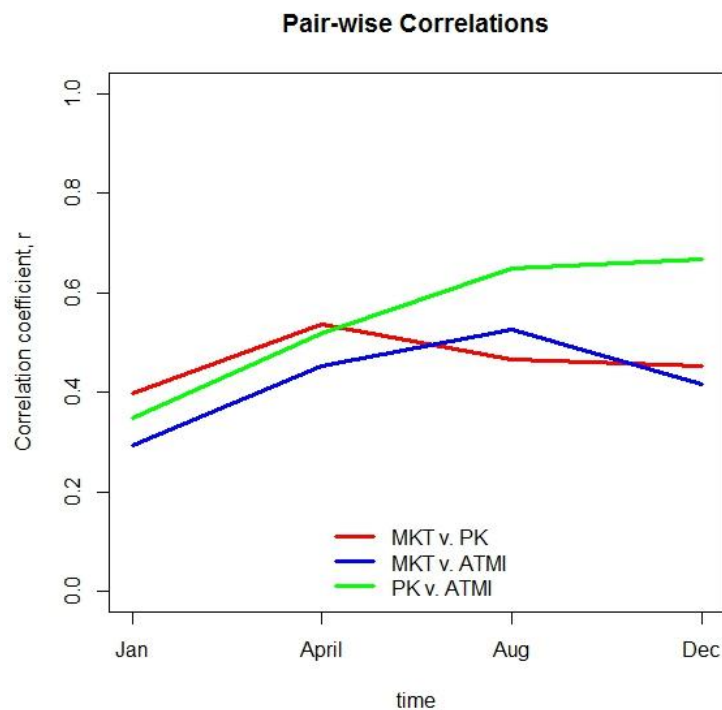


Figure 24. Correlation coefficients for pair-wise comparisons over time

Summary

The quantitative data collected in this study were to help answer the last two research questions in this study:

2. What relationship (if any) exists between prospective elementary teachers' procedural knowledge and mathematical knowledge for teaching, and how does this relationship develop over time as they move through inquiry-based mathematics content courses?
3. How do prospective elementary teachers' attitudes toward mathematics interact with their procedural knowledge and their mathematical knowledge for teaching, and how does this relationship develop over time as they move through inquiry based mathematics content courses?

The results of the study showed growth in students' MKT scores over the first semester which was maintained into the end of the second semester, even though the course content had changed from a focus on numbers and operations (aligned with the assessment) to geometry and measurement.

In contrast, the students' procedural knowledge and attitude scores remained fairly consistent over time, with the exception of some differences in spread. The initial spread of procedural knowledge scores in January was smaller than that in April, August, and December, which is most likely due to the fact that a similar version of the same assessment is used as a prerequisite measure to determine entrance into M1001. The initial spread in attitude score decreased in April, but then gradually increased, indicating students' attitudes may change due to differences between M1001 and M1002.

There was a positive linear relationship between each pair-wise comparison of the assessment scores. The relationship between procedural knowledge and attitude continued to gain strength over time whereas the strength of the relationship between MKT and both procedural knowledge and attitude showed some growth and decline in strength. This is reasonable considering the procedural knowledge and attitude scores stayed fairly constant over time while the MKT scores increased over time. Starting with higher MKT appeared to allow for greater growth in MKT and PK, at least initially. Jordan was one exception to this case, showing a greater increase in MKT score than others with a relatively low beginning MKT score, but did initially start with a perfect procedural knowledge score. There were more fluctuations with the participants' procedural knowledge and attitude scores, but overall from the beginning to the end of the year, all four of the participants either gained in or maintained their attitude scores and procedural knowledge scores. The only exception is Jamie, who did slowly gain in attitude score but overall declined in procedural knowledge score. It would have been interesting to see how Jamie performed in M1100 and if there would have been improvement in the procedural knowledge score as algebra is an included topic in the third course of the sequence. These results indicate that gains can be made by students in all of these competencies, regardless of where they enter the program.

Overall Summary

The objectives of the curriculum were to have students explain fundamental concepts in mathematics through writing and speaking; represent quantities in problem situations with symbols, words, and diagrams; solve problems through quantitative

reasoning; and construct viable arguments and evaluate the reasoning of others – objectives strongly aligned with the Common Core Standards of Mathematical Practices of persevering in problem solving and constructing viable arguments. Students moving through this curriculum were afforded the opportunity to grapple with and discuss problems as a group and class, with manipulatives, pictures, equations, and other representations, in order to reach an understanding and be able to explain important mathematical concepts. There were some minor issues with the clarity of arguments and the connections made between representations, but overall students grew in their practice of problem solving and arguing in mathematics while moving through the curriculum. The amount of growth depended, in part, on effort. Two other factors I observed to have an effect on learning in the curriculum were the atmosphere and attitude of students in the class and the nature of the content and questions in the curriculum. I will explore this further in chapter 5.

The median MKT score of the prospective elementary teachers increased throughout the year, while the median procedural knowledge and attitude scores remained fairly constant. The results of a repeated measures ANOVA and follow-up Tukey's Honest Significant Difference procedure showed there was a difference in scores between January and all other months. There were positive linear relationships between each pair of assessment scores – MKT, procedural knowledge, and attitude. The strongest relationship was between procedural knowledge and attitude, which makes sense because neither of their median scores changed much over time and the median MKT score increased over time. Because there was a positive relationship between the pair-wise comparisons and that relationship was maintained over time, there is reason for

the curriculum to foster growth in the competencies of MKT, procedural knowledge, and attitude toward mathematics. The curriculum did foster growth in prospective teachers' MKT, but had a lesser effect on their procedural knowledge and attitudes. Considering the third course in the sequence, M1100, is where prospective elementary teachers will learn about algebra, there may still be potential for growth in their procedural knowledge scores. In chapter 5 I will explore the possibility of the course sequence fostering growth in attitudes. I witnessed that attitudes have an effect on classroom learning, and this was felt by the participants in this study.

5. CONCLUSIONS

Introduction

This research study was motivated by a desire to learn more about mathematics programs for prospective elementary teachers, especially those employing an inquiry-based approach, in order to understand better how prospective elementary teachers learn and grow in this type of curriculum and to be able to make suggestions and recommendations for teacher educators and teachers. NCTM's 2008 research agenda focused on linking research to practice stated that in order to be able to determine goals of professional growth researchers need to "develop a more elaborated trajectory of teachers' evolution of their competencies (e.g. knowledge, beliefs, dispositions, and practices), beginning from one end of the continuum when teachers enter teacher preparation programs to the other end of the continuum when teachers establish themselves as effective teacher leaders" (Arbaugh et al., 2008, p. 19). This study examined the beginning of this continuum as students were in the first two of three mathematics content courses for elementary teachers and aimed to answer the following research questions:

1. How do certain prospective elementary teachers progress in their enactment of the mathematical practices of persevering in problem solving and constructing viable arguments as they move through inquiry-based mathematics content courses?
2. What relationship (if any) exists between prospective elementary teachers' procedural knowledge and mathematical knowledge for teaching, and how

does this relationship change over time as they move through inquiry-based mathematics content courses?

3. How do prospective elementary teachers' attitudes toward mathematics associate with their procedural knowledge and their mathematical knowledge for teaching, and how does this relationship change over time as they move through inquiry-based mathematics content courses?

This chapter will review the results of this study, and discuss implications for teachers, teacher education programs, and researchers studying the Common Core Standards for Mathematical Practice.

Problem Solving and Constructing Viable Arguments

The results related to the first research question showed the curriculum used in this study afforded students ample opportunity to tackle mathematical tasks and problems in class; discuss problems as a small group and whole class; and use manipulatives, pictures, equations, and other representations, in order to come to an understanding that allowed them to be able explain important mathematical concepts. Overall students grew in their ability to problem solve and construct viable arguments in mathematics while moving through the curriculum, with the exception of some difficulty related to specific content. Three factors contributed to students' learning in the curriculum. First, the amount of growth did, in part, depend on the effort made by the student. Second, the atmosphere and attitude of students in the class had an effect on learning. Finally, the nature of the content and questions in the curriculum had an effect on learning. Each factor will be discussed below, followed by a discussion of the importance of deciding

whether verbal or written arguments hold more significance in teacher preparation and how that plays into classroom assessment.

Effort

The amount of growth seen in the participants did depend, in part, on effort. When talking about effort, I mean the amount of work and participation students put into class, actively attempting to work toward a personal understanding of the concepts. When comparing the outcomes for Joey and Jordan, the two participants who started with low MKT and procedural knowledge, Jamie put in more effort and attained better results. It is hard to be sure exactly what individual characteristics and factors play a role in the amount of effort put forth by the students. It seemed to me that part of the problem with Joey's effort was due to the early morning class time. Because Joey was not a part of the study for the entire year, it would have been interesting to see if Joey's effort improved at all during the second semester.

Something I noticed during my classroom observations was a pattern of the same students speaking on behalf of the group when the instructor came around to check in. This allowed for other students to avoid having to share and to become reliant on the members of the group who understood the material well and liked explaining. This meant they could put very little effort into understanding the class activities and concepts. Taylor was aware of this too, and made active attempts to not to dominate the group discussions. In order to avoid letting the students who struggle more with the material or do not like to explain slip through the cracks, instructors should ensure everybody in the group can explain and has opportunity to do so. This could mean making a written or

mental check list of who explains each time the group is visited, or having each person in the group rephrase what the other has said in their own words.

Atmosphere and Attitudes

Students in this type of curriculum spend a lot of time working in groups on activities. They struggle to come to an understanding of and to be able to explain mathematical concepts. It is very important for the struggle to be productive and not lead to a place of unproductive frustration. During my observations, I was witness to some frustration and negative attitudes and comments made toward the class, both by the participants in the study and by other students in the class. The frustrations and negative attitudes of students in the class were also brought up by the participants during the interviews, who said it did have an impact on their learning.

The times I witnessed the most frustration from the students was when there was deviation from the activity manual and confusion about directions or the goal of the activity, and when the students felt their questions were not being addressed by the instructor or they were not receiving any guidance. This indicates there is a delicate balance between productive and unproductive struggle that is vital for instructors to be aware of and handle appropriately. As the text is set up, there are scaffolds in place in to help guide students in an appropriate direction. Deviation from the text activities, if done, should be done with much thought and care. The instructor should make the goals clear and set scaffolds in place or at least have thought of how to guide students if the struggle is unproductive.

Attitudes of students in the class also play into the classroom environment and student learning. Most of the participants commented on how hard it is to maintain a positive attitude in class when others in the group are making negative comments. Much of the negative attitudes seen had to do with the nature of the course. Many students did not like having to explain *why* and would rather just state the formulas and rules than explain *why* the rules work the way they do. In order to try and combat this complex issue of attitudes, which the results of the study showed was not changed as a result of the curriculum, instructors may want to spend more time discussing the importance of knowing *why* and how it relates to teaching. Many prospective elementary teachers' mathematics experience in school involved the *what* and not the *why*, so it is reasonable for them to want to continue this pattern. It is important that teacher educators work to break this cycle and have serious discussions with prospective elementary teachers on the importance of having a deep understanding of *why*.

Nature of the Content

There appeared to be a difference in the nature of the content and the questions being asked by the text and the instructors from M1001 to M1002, which were noted by both me and the participants in the study. Although some of the questions in M1002 still involved conceptual understanding and explaining *why*, many of the questions focused more on calculations and descriptions. This was particularly true of my observations in chapter 13, where students spent most of the time building with manipulatives or cutting out nets and folding them into their 3-dimensional shapes. I witnessed students spending much less time discussing the material and taking notes, as they seemed to be distracted

with the hands-on nature of the activities. Instructors should encourage students to maintain the same amount of discussion and focus normally put into class time, even when they are building or cutting. Along the same line, the nature of the questions on the chapter 13 exam also seemed to lack a focus on *why*. The directions said mostly to “describe” and “explain how” rather than “explain why.”

Because I was not observing during the entire semester, it could be that chapter 13 in particular loses the focus on *why* while the others do not. This could also have to do with the fact that number and operations is more prevalent in K-12 mathematics curriculum, meaning students would more likely have more knowledge and experience with it. This would make it easier to go deeper into the conceptual understanding and *why* of the content, rather than having to focus on the basics. This was true for Jamie, who admitted to struggling with dimensions and conversions. Regardless, this is something instructors of a curriculum like this need to take into consideration while making decisions about whether the focus of the content needs to be on *how* rather than *why*, or both and how to ensure the activities and questions reflect this emphasis. The quantitative results of the study showed an increase in the spread and overall minimum attitude scores from the end of M1001 to the end of M1002 which could be related to these differences in the nature of the content and questions asked. Some students just do not like geometry, like Jordan, and while the instructor does not have control over the content itself, they do have control over the types of activities used and questions asked in relation to the content.

Written or Verbal Explanations

During classes I witnessed most of the participants verbally discussing the problem at hand and working toward explanations with their small groups. Although the participants were reaching an understanding of the problem and verbally stating good explanations, there was often very little written and what was written was more often less detailed. This was with the exception of Ande, who did mostly quiet work during class and had more thorough written explanations. Given the nature of the work of a teacher, verbal explanations are a very important practice for prospective elementary teachers to develop. This was an objective of the curriculum, along with having students write explanations of mathematical concepts. There were times when the instructor would remind students to write explanations, even explicitly telling students to write up their explanations like they would for an exam. When students were reminded to do more writing, almost all discussion would stop. I saw the reverse with Ande. When Ande was reminded to discuss with the group, almost all written work stopped. Instructors should be aware of this student behavior and may want to set a specific schedule where students are given the opportunities to both discuss and write during class time to ensure they get practice with both methods of explaining.

This also brings into question how we should assess students in this type of curriculum. If the goal is to have students be able to explain both verbally and in writing, then perhaps both need to be assessed, and even given the same weight in relation to their course grade. The curriculum in this study placed an emphasis, at least grade-wise, on the written explanations on the exams. This seems interesting given that during class, many of the participants were not writing down explanations to the extent they would

need to be done on an exam. Also, what I noticed during the interviews is the written exam responses did not offer a full picture of what the students really knew and understood about the problems and mathematical concepts. As the participants were reflecting on the exam, they would sometimes pick up on errors they made, make extensions regarding how things could be generalized or what representations could be added to strengthen the argument (or help in explaining to future students), or admit they really did not understand what was going on. This indicates that there needs to be some other measure of student understanding, perhaps by a similar process to this study - an individual interview after an exam is taken, where students can reflect on the explanations they gave. The participants in the study often commented on how limited time and the lack of space to write on the exam prevented them from doing their best. It is very important for instructors of a curriculum like this to decide what is valued more (if either), writing or speaking, in explaining and appropriately assess their students.

Mathematical Knowledge for Teaching

The results of this study indicated prospective elementary teachers grow in their mathematical knowledge for teaching related to Numbers and Operations. This is no surprise given this is the main content focus in M1001. What was interesting was the fact that the growth was maintained, even by the end of M1002. It is one of an educator's greatest fears that their students are going to forget what they learned at the end of the semester. Even though the material in M1002, Geometry and Measurement, did not have a focus on Number and Operations, these are not isolated topics. I was witness to students using important ideas and concepts from M1001 in M1002. In interviews, some

of the participants also commented on doing so. This then makes me wonder about the continued trajectory of prospective elementary teachers' mathematical knowledge for teaching. Will they lose those gains in the next few years of the teacher education program as they move from content courses into methods courses?

The studies of Matthews and Seaman (2007) and Matthews, Rech, and Grandgenett (2010) showed prospective elementary teachers taking a specialized mathematics content course score higher on the CKT-M than prospective teachers not taking a specialized content course. This study did not compare two cohorts of students, but did show prospective elementary teachers mathematical knowledge for teaching scores increased after taking a specialized mathematics content course.

Procedural Knowledge

There was no change in the prospective elementary teachers' procedural knowledge scores over the course of the year. The only notable difference was in the spread of the scores, which was smallest at the beginning of the study. This was probably due to the fact that some students studied this material to pass a similar prerequisite exam to gain entrance into M1001. Jamie was one of these students, and complained to me about how it does not seem fair to have an exam like that as a prerequisite into M1001 as the material was not relevant to what was learned in M1001 or M1002. It is unfortunate this study did not follow students into the third course of the sequence, M1100, as this is where students learn about algebra (along with some other topics).

Attitudes Toward Mathematics

There was no change in the prospective elementary teachers' attitude scores over the course of the year. The only notable difference was in the spread and the minimum scores over time. From the beginning to the end of the first semester, the spread of attitude scores decreased while the minimum score increased by 30 points. Over the summer and the second semester, the reverse of this happened. The spread of attitude scores increased while the minimum score decreased. This change in scores over the course of the second semester could be due to students' dislike of geometry or to the nature of the content and the questions asked, as discussed earlier in this chapter.

This study contributed to the mixed results of several other studies which aimed to examine the effect a specialized content course has on prospective elementary teachers' attitudes (Lubinski and Otto, 2004; Matthews & Seaman, 2007 ; Matthews et. al., 2010). In contrast to Lubinski and Otto's (2004) findings that attitudes of prospective elementary teachers' were positively influenced by taking a specialized mathematics content course similar to those in this study, the results of this study showed no change in prospective elementary teachers' attitudes.

Relationships Between Competencies

It is important for mathematics content courses for prospective elementary teachers to pay attention to and foster growth in students' procedural knowledge, mathematical knowledge for teaching, and attitudes toward mathematics as this study showed they are all pair-wise positively (and linearly) related. Although prospective

elementary teachers' mathematical knowledge for teaching scores increased over the course of this study, their procedural knowledge and attitude scores did not. This could explain why the strength in the linear relationship between procedural knowledge scores and attitude scores continued to increase over time while the linear relationship between mathematical knowledge for teaching scores and both attitude and procedural knowledge scores was less consistent. The pair-wise linear relationships between mathematical knowledge for teaching and attitude scores, and mathematical knowledge for teaching and procedural knowledge scores both gained in strength initially, but then declined by the end of the year. Both of these relationships did, however, make slight overall gains over the course of the year. Future research should look into how these relationships continue to develop as students continue to move through the teacher education program.

Because I selected participants contributing to the qualitative data related to the Mathematical Practices of persevering in problem solving and constructing viable arguments using their initial mathematical knowledge for teaching and procedural knowledge scores, I was able to draw certain connections between these competencies. In analyzing the qualitative data, I found that the participants who began with lower mathematical knowledge for teaching scores had a greater tendency to explain *what* or *how* rather than *why* than those who began with higher mathematical knowledge for teaching scores. Aside from this difference, there were no other notable differences in the participants' progressions of their practices dependent on their initial mathematical knowledge for teaching and procedural knowledge scores, just those due to effort and difficulties with specific content as were discussed earlier in the chapter.

Implications

Implications for Teachers

It is important for teachers of this type of inquiry-based curriculum to build a positive classroom atmosphere where students can productively work towards building understandings of important mathematical concepts through the classroom activities and interactions. Many of the students who were a part of this study both exhibited negative attitudes and behaviors in the classroom and spoke about its effect on their learning. Some of this negative attitude and frustration stemmed from the difficulty of being able to explain important mathematical concepts and so instructors may want to talk about this with their students. Instructors should speak with their students about the difficulty of the content and encourage an atmosphere of support, where students work together to help each other, the teacher is there to offer guidance, and the class as a whole works to build a community of knowledge.

Instructors should be aware of the delicate balance between productive and unproductive struggle in an inquiry-based curriculum. The text used in this study offers scaffolds for student, guiding them towards understanding the big picture. Deviation from the text activities, if done, should be done with much thought and care. The instructor should make the goals clear and set scaffolds in place or at least have thought of how to guide students if the struggle is unproductive. Many of the frustrations and negative attitudes from students in this study arose either from the feeling of not being supported, which can be alleviated by instructors listening to their questions and guiding them in the right direction, or not understanding the importance of knowing *why*.

Instructors may want to spend more time discussing the importance of knowing *why* and how it relates to teaching. Many prospective elementary teachers' mathematics experience in school involved the *what* and not the *why*, so it is reasonable for them to want to continue this pattern. It is important that teacher educators work to break this cycle and have serious discussions with prospective elementary teachers on the importance of having a deep understanding of *why*. This falls in line with the reform movement in education and the emergence of constructivism and the Common Core State Standards. It is important for the prospective elementary teachers to understand that this is the type of understanding they will be expected foster in their future students.

Finally, to combat the issue of hindered student growth due, in part, to a lack of effort (as was seen in this study), instructors need to ensure participation and effort from all of their students. In order to avoid letting the students who struggle more with the material or do not like to explain slip through the cracks, instructors should ensure everybody in the group can explain and has opportunity to do so. This could mean making a written or mental check list of who explains each time the group is visited, or having each person in the group rephrase what the other has said in their own words. All students should be held accountable to explain both within their small groups and to the class as a whole.

Implications for Teacher Preparation

The inquiry-based curriculum for the mathematics content courses in this study did increase prospective elementary teachers' mathematical knowledge for teaching scores, but their procedural knowledge and attitude scores did not change. This indicates

there may need to be additions made to the curricular material to try and foster growth in these particular competencies, especially seeing as they are all pair-wise linearly related. Perhaps it is the case that prospective elementary teachers' procedural knowledge scores would increase after taking the third course in the sequence. Not knowing this, however, it might be a good idea for algebra to be woven into the first two courses.

Teacher education programs using this same curriculum text should consider the differences in the nature of the content and the questions asked as it was shown to have an effect on students' learning in this study. Decisions about whether the focus of the content needs to be on *how* rather than *why*, or both, and how to ensure the activities and questions reflect this emphasis need to be considered in the design and implementation of this and similar curricula.

Another important consideration of the curriculum highlighted by this study was the opportunities allowed for the practice of written versus verbal explanations, and what was formally assessed. As was shown in this study, if the focus in class was placed on writing explanations in class, group discussion tended to stop. If the focus in class was placed on discussing and explaining in groups, then written explanation tended to stop. Specific schedules where students are given the opportunities to both discuss and write during class time would ensure they get practice with both methods of explaining. If the goal of the curriculum is to have students be able to explain both verbally and in writing, then perhaps both need to be assessed, and even given the same weight in relation to their course grade. Although verbal explanations are a big part of the work of a teacher, the tendency is to assess via a written exam.

Along these same lines, this study highlighted that the written assessments did not give offer a full and accurate picture of the knowledge and understanding held by the students. The reflections students offered during the interviews with regard to exam questions showed a much better picture of their full understanding of the concepts being assessed. This indicates that there needs to be some other measure of student understanding, perhaps by a similar process to this study - an individual interview after an exam is taken, where students can reflect on the explanations they gave. It is very important for teacher education programs designing a curriculum like this to decide what is valued more (if either), writing or speaking, in explaining and appropriately assess their students.

Implications for Researchers

When researching the Common Core Standards for Mathematical Practice, it is important for researchers to realize that students' enactment of the practices is an ongoing and continual process that is always changing, especially in relation to the specific content being studied. The most valuable source of data for me in studying prospective elementary teachers' enactment of Mathematical Practices was the interview. It was here I could ask about the students' problem solving process which is most often internalized. Observations in an inquiry-based curriculum where students are working in groups on mathematical tasks are also helpful, but there are still some processes of problem solving and constructing viable arguments that remain internal to the students. The interviews, in which the students were asked about their processes in problem solving process and writing explanations on exam questions, were a vital component of the research data. It

was during these interviews that the normally hidden processes of enacting the Mathematical Practices became visible. This was also where I witnessed further enactment of the practices when the students reflected and expanded upon their solutions and explanations on a written exam.

For this research, I used the language of the Common Core Standards for Mathematical Practice to help focus my data collection on specific aspects of the practices. Future researchers may not want to include this structure in their data collection techniques and instead just focus on problem solving and constructing viable arguments in general. In this way, one could generate categories in the analysis that were not dependent upon the specific Common Core Standard for Mathematical Practice and see how the categories align with the practice itself.

Future Research

Future research should expand the scope of this study to see if any changes are made with regard to students' attitudes and procedural knowledge after having taken the third mathematics content course. It would also be beneficial to know how far into the future prospective elementary teachers maintain the gains seen in their mathematical knowledge for teaching. Future research expanding the longitude of this study by following prospective elementary teachers further into their educational programs would be beneficial in showing how these competencies and relationships between them continue to change (or not) over time. Future research should also consider the trajectory of prospective elementary teachers' competencies moving through a similar curriculum

with only a two course sequence (algebra and probability mixed into the two courses) to see if there are differences in those trajectories as compared to this study.

Conclusion

The results of this study showed the trajectory of competencies for a small group of prospective elementary teachers moving through the first two of three mathematics content courses using an inquiry-based curriculum. Institutions employing a similar curriculum and course sequence should be able to expect similar results as those found in this study. The students in this study did not represent much diversity in terms of ethnic and racial diversity. They did represent diversity in their beginning levels of mathematical knowledge for teaching, procedural knowledge, and attitudes – and as seen in the participants contributing to the qualitative data, different types of students. This curriculum fostered growth in the participants' practices of problem solving and constructing viable arguments as well as growth in their mathematical knowledge for teaching in numbers and operations. The considerations and recommendations made in the sections above would apply to any institution implementing an inquiry-based approach to mathematics content courses for elementary teachers.

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APPENDICES

APPENDIX A

ATTITUDES TOWARD MATHEMATICS INVENTORY

ATTITUDES TOWARD MATHEMATICS INVENTORY

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Choose the response that most closely corresponds to how the statements best describes your feelings. Use the following response scale to respond to each item.

PLEASE USE THESE RESPONSE CODES:

- A – Strongly Disagree
- B – Disagree
- C – Neutral
- D – Agree
- E – Strongly Agree

Complete your responses for all 40 statements.

1. Mathematics is a very worthwhile and necessary subject.
2. I want to develop my mathematical skills.
3. I get a great deal of satisfaction out of solving a mathematics problem.
4. Mathematics helps develop the mind and teaches a person to think.
5. Mathematics is important in everyday life.
6. Mathematics is one of the most important subjects for people to study.
7. College mathematics lessons would be very helpful no matter what I decide to study in the future.
8. I can think of many ways that I use mathematics outside of school.
9. Mathematics is one of my most dreaded subjects.
10. My mind goes blank and I am unable to think clearly when working with mathematics.
11. Studying mathematics makes me feel nervous.
12. Mathematics makes me feel uncomfortable.
13. I am always under a terrible strain in a mathematics class.
14. When I hear the word mathematics, I have a feeling of dislike.
15. It makes me nervous to even think about having to do a mathematics problem.
16. Mathematics does not scare me at all.
17. I have a lot of self-confidence when it comes to mathematics.
18. I am able to solve mathematics problems without too much difficulty.
19. I expect to do fairly well in any mathematics class I take.
20. I am always confused in my mathematics class.
21. I feel a sense of insecurity when attempting mathematics.
22. I learn mathematics easily.
23. I am confident that I could learn advanced mathematics.
24. I have usually enjoyed studying mathematics in school.
25. Mathematics is dull and boring.
26. I like to solve new problems in mathematics.
27. I would prefer to do an assignment in mathematics than to write an essay.
28. I would like to avoid using mathematics in college.

29. I really like mathematics.
30. I am happier in a mathematics class than in any other class.
31. Mathematics is a very interesting subject.
32. I am willing to take more than the required amount of mathematics.
33. I plan to take as much mathematics as I can during my education.
34. The challenge of mathematics appeals to me.
35. I think studying advanced mathematics is useful.
36. I believe studying mathematics helps me with problem solving in other areas.
37. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.
38. I am comfortable answering questions in mathematics class.
39. A strong mathematics background could help me in my professional life.
40. I believe I am good at solving mathematics problems.

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APPENDIX B

OBSERVATION PROTOCOL

Date:_____ Participant:_____
Class:_____

Chapter:_____ Topic:_____

Description of events:

Group Dynamics:

Evidence of:	Examples
Attempting to understand the problem	Underlined key ideas
Devising a plan prior to action	Made a conjecture Interpreted the meaning of the solution
Evaluating progress and changing plans (if necessary)	Made multiple attempts
Relating multiple representations	Described diagrams and/or equations with words
Sense making	Checked the reasonableness and/or correctness of solution
Understanding and use of prior knowledge and/or stated assumption	Used formula for... Used definition of...
A logical progression of arguments	
Justification/ Explaining why	Used concrete examples
Noteworthy/ Unexpected:	

APPENDIX C

INTERVIEW PROTOCOL

Project: How prospective elementary teachers problem solve and explain why

Time:

Place:

Interviewee:

(Briefly describe the project)

Tell me a little bit about yourself.

What influenced your decision to become an elementary education major?

How do you feel you're progressing in problem solving and justifying in mathematics class?

What specifically about the class do you attribute this to?

Exam Questions

Problem Solving

1. Will you walk me through your thought process while you were working on this exam question?

- ✓ Understanding
- ✓ Plan
- ✓ Multiple Attempts
- ✓ Representation
- ✓ Sense Making

(If necessary)

- a. **What was your understanding of the problem?**
- b. **Where, if anywhere, did you get stuck and how were you able to move on?**
- c. **What did you use to help you find a solution to this problem?**
- d. **Why were or weren't you satisfied with the solution you gave?**

Constructing Viable Arguments

2. You were asked to explain why/justify your response to this question. How did you decide what to write?

- ✓ Prior knowledge/assumptions
- ✓ Logical progression

(If necessary)

- a. **What did you use to help write an explanation for this problem?**
- b. **Why were or weren't you satisfied with the explanation you gave?**
- c. **If I was a stranger, unfamiliar with this material, how would your explanation help me to understand better?**

Observation

3. Please, describe the dynamics of your group for me.

4. I noticed _____ in the observation. Tell me more about _____.

Thank you! I believe you answered all of my questions. Is there anything else you care to add or think I've missed?

Thanks again! (Briefly speak about next observation and interview)

APPENDIX D

RESEARCH SCHEDULE

Spring 2014 - M1001		
Week 1	8-Jan	Implement PK, MKT, and ATMI in M1001
	10-Jan	
Week 2	13-Jan	
	15-Jan	Run data analysis/Select and contact participants
	17-Jan	
Week 3	22-Jan	
	24-Jan	Chapter 2 - Observations/Schedule Interviews
Week 4	27-Jan	
	29-Jan	
	31-Jan	
Week 5	3-Feb	
	5-Feb	
	7-Feb	Chapter 2 Exam
Week 6	10-Feb	Interviews M1001
	12-Feb	
	14-Feb	
Week 7	19-Feb	
	21-Feb	
Week 8	24-Feb	
	26-Feb	
	28-Feb	
Week 9	3-Mar	
	5-Mar	
	7-Mar	
Week 10	17-Mar	
	19-Mar	
	21-Mar	Chapter 5 - Observations/Schedule Interviews
Week 11	24-Mar	
	26-Mar	
	28-Mar	
Week 12	31-Mar	Chapter 5 Exam
	2-Apr	Interviews M1001
	4-Apr	
Week 13	7-Apr	
	9-Apr	
	11-Apr	
Week 14	14-Apr	
	16-Apr	
Week 15	21-Apr	Implement PK, MKT, and ATMI in M1001
	23-Apr	
	25-Apr	
Finals Week		

	Fall 2014 - M1002	
Week 1	25-Aug	Implement PK, MKT, and ATMI in M1002
	27-Aug	
	29-Aug	
Week 2	3-Sep	
	5-Sep	
Week 3	8-Sep	
	10-Sep	
	12-Sep	
Week 4	15-Sep	
	17-Sep	
	19-Sep	Observe for Ch 11
Week 5	22-Sep	
	24-Sep	
	26-Sep	
Week 6	29-Sep	
	1-Oct	Ch 11 exam
	3-Oct	Interviews
Week 7	6-Oct	
	8-Oct	
	10-Oct	
Week 8	13-Oct	
	15-Oct	
	17-Oct	
Week 9	20-Oct	
	22-Oct	
	24-Oct	Observe for Ch 13
Week 10	27-Oct	
	28-Oct	
	31-Oct	
Week 11	3-Nov	
	5-Nov	
	7-Nov	
Week 12	10-Nov	Ch 13 exam
	12-Nov	Interviews
	14-Nov	
Week 13	17-Nov	
	19-Nov	
	21-Nov	
Week 14	14-Nov	
Week 15	1-Dec	
	3-Dec	
	5-Dec	Implement PK, MKT, and ATMI in M1002
Finals Week		