Entrance level competencies needed for beginning mathematics courses at Montana State University

by John Dail Whitesitt

A thesis submitted in partial fulfillment of the requirements for the degree of DOCTOR OF EDUCATION
Montana State University

Abstract:
The problem of this study was: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

Students enrolled in Math 100, an intermediate algebra course; Math 115, a trigonometry course; Math 117, a one quarter survey calculus course; and Math 121, a regular beginning calculus course were tested at the beginning of fall quarter to identify entrance level competencies. At the end of the quarter these students were classified as successful or not successful based on course grades. The independence of success and entrance level competencies in various areas was tested using a Chi Square Test of Independence. The effectiveness of the remedial courses was considered by estimating the proportion of successful students in Math 001, a beginning algebra course; Math 100; and Math 115 who demonstrated competence in the areas identified as important for success in subsequent courses. All hypotheses were tested at the .01 level of significance and the proportions were estimated using 95 percent confidence intervals.

Three of the nine competency areas tested in Math 100 were found to be significantly related to success. One of the ten areas tested in Math 115 and Math 117 was found to be significantly related to success, and seven of the thirteen areas tested in Math 121 were found to be significantly related to success. All of the remedial mathematics courses at Montana State University were ineffective in developing competency in most areas identified as important for success in subsequent courses.

More emphasis needs to be placed on developing competence in algebra for college bound students and less needs to be placed on trigonometry. Many competency areas in algebra were found to be significantly related to success in beginning mathematics courses, but only one trigonometry area was significantly related to success. All high school students should be encouraged to take at least two years of algebra before they graduate because of the ineffectiveness of college remedial courses.

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ABSTRACT

The problem of this study was: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

Students enrolled in Math 100, an intermediate algebra course; Math 115, a trigonometry course; Math 117, a one quarter survey calculus course; and Math 121, a regular beginning calculus course were tested at the beginning of fall quarter to identify entrance level competencies. At the end of the quarter these students were classified as successful or not successful based on course grades. The independence of success and entrance level competencies in various areas was tested using a Chi Square Test of Independence. The effectiveness of the remedial courses was considered by estimating the proportion of successful students in Math 001, a beginning algebra course; Math 100; and Math 115 who demonstrated competence in the areas identified as important for success in subsequent courses. All hypotheses were tested at the .01 level of significance and the proportions were estimated using 95 percent confidence intervals.

Three of the nine competency areas tested in Math 100 were found to be significantly related to success. One of the ten areas tested in Math 115 and Math 117 was found to be significantly related to success, and seven of the thirteen areas tested in Math 121 were found to be significantly related to success. All of the remedial mathematics courses at Montana State University were ineffective in developing competency in most areas identified as important for success in subsequent courses.

More emphasis needs to be placed on developing competence in algebra for college bound students and less needs to be placed on trigonometry. Many competency areas in algebra were found to be significantly related to success in beginning mathematics courses, but only one trigonometry area was significantly related to success. All high school students should be encouraged to take at least two years of algebra before they graduate because of the ineffectiveness of college remedial courses.

Chapter 1

INTRODUCTION

Mathematics education for the college bound student has undergone a rapid and often diverse reorganization during the past 20 years. The success of this reorganization has been questioned widely by mathematics educators such as Morris Kline (1973) and by parents and students as well. Declining achievement test scores during the past 15 years have focused public attention on the question and have led many people, including Kline, to conclude that the reorganization, commonly called "New Math," is the cause of the problem. "New Math" is a term that refers to all of the mathematics programs during a period of time spanning two decades. As such, it certainly is not a single entity and, therefore, is most likely not either entirely good or entirely bad.

As Ferguson (1976) and many others have pointed out, the nature of the college bound student has changed drastically during this time. The growing belief in free access to higher education and accompanying open admissions policies have had a definite effect on the variety and number of students attending college. The preparation of students for college level mathematics has, correspondingly, become much more complex than ever before. According to Lindberg (1974), Lindquist (1965), and Maxwell (1975), universities, colleges, and junior colleges have become increasingly involved in the process. Increased coordination of efforts has recently been called for in a joint publication of the Mathematical

Association of America and the National Council of Teachers of Mathematics (1978). It was within this rather complex setting that this study took place.

At Montana State University there are basically five courses that students can take as beginning mathematics courses. These courses are: (1) Math 001, a course in elementary algebra; (2) Math 100, a course in intermediate algebra; (3) Math 115, a course in trigonometry; (4) Math 117, a terminal one quarter calculus course for students who need only a broad overview of calculus; and (5) Math 121, the first quarter of the standard six quarter calculus sequence. A student and his advisor usually make the decision as to which course is an appropriate starting point for that student. It was on these five courses that this study centered.

Statement of the Problem

The problem of this study was two-fold: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

Need or Purpose of the Study

The first indication of a need for this study was provided by the low success rate of students in beginning mathematics courses at Montana State University. For the purpose of this study, a successful student was defined to be any student who received a course grade of A, B, or C. Data was available only for students who stayed in a course long enough to be listed on the registrar's final grade sheet. This gave success rates that appeared higher than they would have if all students who started the courses could have been considered. Even considering only those students, during fall quarter of 1977, 32 percent of the basic algebra students were unsuccessful; 26.5 percent of the intermediate algebra students were unsuccessful; 29 percent of the trigonometry students were unsuccessful; and 15 percent of the beginning calculus students were unsuccessful. There is no question that including students who dropped courses before final grade sheets were printed would have significantly increased these figures. This situation is fairly common at other institutions as well. Lindberg (1974), in a study of 450 universities, colleges, and junior colleges, found that 57 percent of the mathematics departments surveyed rated at least 25 percent of their beginning mathematics students as unsuccessful. Identification of the competencies needed for success in beginning mathematics courses was needed to help develop an understanding of the situation.

Another reason for doing this study arose from the fact that the tremendous variety of mathematics courses now being offered to college bound students at the secondary level may lead to poor preparation of these students. The variety is pointed out by the following examples. Bozeman Senior High School offered only four mathematics courses during the 1957-1958 school year: Algebra I, Plane Geometry, Algebra II, and Math IV. During the 1977-1978 school year they offered 13 mathematics courses: Basic Computer Applications, Data Processing, Algebra I-A, Algebra I-B, Algebra I, Geometry, Trigonometry, Calculus, Industrial Mathematics, Math Topics, Statistics, Algebra II, and Business Math. Bozeman Senior High School is not alone in this move. As another example, Helena Senior High School moved from four courses similar to those at Bozeman Senior High School in the 1957-1958 school year to a total of 13 courses in the 1978-1979 school year. This presents the college bound secondary student with a confusingly large number of alternatives. Which of these will provide the student with the best chance for success when he gets to college? Can the student wait until he gets to college to take algebra and still have a reasonable chance for success? Will computer programming or statistics be more valuable than a second year of algebra? The answers to these and many similar questions depend not only on the future plans of the student, but also on the identification of needed competencies and the evaluation of the remedial courses offered at the college level. The Carnegie Foundation for the Advancement of Teaching believes that "colleges have an obligation to make clear to the high schools which skills they expect their students to acquire before they are admitted" (1978).

Effective placement of students once they get to college also depends on the identification of competencies needed for success in various possible beginning courses. No advanced placement testing program can be useful if the competencies being tested don't coincide with those needed by the student once he is placed. A recent study by Bers and Jaffe (1977) showed that there was no significant relationship between prerequisites for beginning chemistry courses at Oakton Community College and course grades. Prerequisites must provide the student with needed competencies in order to be meaningful.

Declining achievement test scores also point to a need for establishing the importance of prerequisite skills for success in college mathematics. There has been a significant decline in mean scores in both the American College Testing Program (ACT) test scores and the Scholastic Aptitude Test (SAT) scores during the past 15 years. Ferguson (1976) and Ferguson and Schmeiser (1978), discussing the ACT test score decline, and Braswell (1978) and Edson (1976), discussing the SAT score decline, list a number of probable causes, but point out that real evidence of a cause-effect relationship has not been found. Among the probable causes that they present are: (1) an increase in the number of elective courses being offered in secondary schools, (2) a lowering of academic standards, (3) changes in our society, and (4) declining student motivation. In a survey of incoming freshmen at Berkeley, Maxwell (1975) found that although achievement test scores were declining, average grade point averages were increasing for college freshmen. She attributed this trend to grade inflation because of her belief that performance in college courses had also declined. Maxey and Others (1976) found a similar trend in high school grade point averages. Their findings were based on self-reported grades in four subject areas obtained from the ACT test results for 1970-1971 through 1974-1975.

Finally, there exists a controversy over whether the responsibility for remedial education belongs at the universities, the colleges, the junior colleges, or the secondary schools. Maxwell (1975), a professor at Berkeley, states:

> I do not see the University's function to be one of offering basic elementary and junior high skills and subjects. It is too costly and too time consuming for both staff and students.

In a recent article to Montana mathematics teachers, Bill Stannard (1978), a member of the Board of Directors of the National Council of Teachers of Mathematics, said:

> Clearly, the proper place to learn high school mathematics is in high school, not on a college campus. The pace in college for this kind of course is about four times that for the same material in high school.

If students are to be convinced to come to college better prepared than they do at present, it seemed critical to identify necessary entrance level competencies as well as to provide some evidence that remedial mathematics courses at the college level are not as effective as they might be.

General Questions Answered

This study attempted to answer eight major questions. They were:

1. Are there certain mathematical competencies that a student should have before taking a beginning mathematics course at Montana State University if he or she expects to be successful?

2. Do the remedial mathematics courses at Montana State University effectively develop the competencies needed for success in subsequent courses?

3. What proportion of the students who register for and attend beginning mathematics courses at Montana State University are actually successful in that course?

4. What proportion of the students who register for and attend remedial mathematics courses at Montana State University have not already passed an equivalent course?

5. Do students who wait until they get to Montana State University to take their first algebra course have a reasonable chance for success in that course?

6. Does the age of the student have any effect on his or her chances for success in beginning mathematics courses at Montana State

University?

7. Does the sex of the student have any effect on his or her chances for success in beginning mathematics courses at Montana State University?

8. Does the method of instruction used for Math 100 have any effect on the success rate for students in that course?

Sex, age, and method of instruction were included in the study because they had been reported in other research as important factors and because they were of particular interest to either the researcher or the Mathematics Department at Montana State University.

General Procedures

Arrangements were made through the chairman of the Mathematics Department at Montana State University for the sampling and testing of mathematics students taking beginning mathematics courses during fall quarter of the 1978-1979 academic year.

Students were given a test of mathematical competencies at the beginning of fall quarter. The competencies of interest in this study were those competencies that a student could acquire from courses that precede the one for which he was currently registered. The competencies covered in the course for which the student was registered were not the subject of this investigation. Students who demonstrated a prior knowledge of the competencies covered by this course were eliminated from consideration for questions involving entrance level competencies.

This procedure was necessary because many students were required only to take a certain number of mathematics courses and often took the easiest course offered even though they already knew the material. Their success would not have been the result of needed competencies but of prior knowledge of the course itself.

In addition to the test, the students were asked to fill out a short personal data form which allowed the researcher to consider questions pertaining to the effectiveness of previous courses, the effect of age, the effect of sex, and the effect of instructional method.

At the end of the quarter, students were classified as either successful or unsuccessful according to the course grade that they received. Using the Chi Square Test of Independence, the researcher checked to see if the success of the students was related to competence as measured by the tests discussed earlier.

The researcher also checked to see what proportion of the successful students in each remedial course demonstrated competence in the areas identified as important for success in the next sequential course.

Limitations and/or Delimitations

The delimitations of this study were:

1. The study considered only the mathematics area and not any other areas subject to remediation at the college level.

2. The study considered only students who were taking mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year.

3. The study was supported by the resources available through the Montana State University library.

Definition of Terms

The following definitions were provided by the researcher or correspond to those used in other research as indicated by citation and are to be considered as operational definitions.

College level mathematics. Courses at or above the level of calculus or calculus and analytic geometry (Mathematical Association of America and the National Council of Teachers of Mathematics, 1978).

Remedial mathematics. Courses taught by colleges which cover one or more of arithmetic, elementary algebra, intermediate algebra, trigonometry, and high school geometry.

Successful student. Any student who received a course grade of A, B, or C (Lindberg, 1974). In pass-fail courses, students who passed were considered successful.

Unsuccessful student. Any student who registered for and attended class but failed to receive a course grade of A, B, or C with the following exception. Students who received an I (Incomplete) grade were eliminated from further consideration but students who withdrew, for any reason, were considered unsuccessful. In pass-fail courses, students who failed or withdrew were considered unsuccessful and those who received an I were again eliminated.

Entrance level competency. A skill or an understanding based on material covered in previous courses that a student demonstrated at the beginning of a given course.

Exit level competency. A skill or an understanding that a student demonstrated at the end of a given course.

Younger student. Any student who was less than 25 years old.

Older student. Any student who was 25 years old or older.

Math 001. A course in elementary algebra offered at Montana State University for no credit.

Math 100. A course in intermediate algebra offered at Montana State University for five quarter credits.

Math 115. A course in trigonometry offered at Montana State University for five quarter credits.

Math 117. A one quarter, terminal calculus course offered at Montana State University for five quarter credits.

Math 121. A beginning calculus course offered at Montana State University for four quarter credits.

Competence. The ability to answer correctly the items on the tests in Appendix B as follows: answer both items correctly for areas covered by only two items, answer two out of three items correctly for areas covered by three items, and answer three out of four items correctly for areas covered by four items.

Summary

Changing programs in mathematics at all levels during the past 20 years have created a confusing picture for college bound secondary students. This study attempted to clarify the situation by identifying those mathematical competencies that a student needs in order to be successful in beginning mathematics courses at Montana State University. The study also evaluated the remedial mathematics courses at Montana State University in an attempt to convince college bound students and their advisors of the need for such students to take certain mathematics courses in high school.

Chapter 2

REVIEW OF LITERATURE

For the purpose of this study, the related literature was considered under three main topics. These topics were: college preparatory high school programs, college remedial programs, and prediction of college success. National trends as well as the situation in Montana were considered within each of the first two topics. Prediction of college success was broken down in order to examine literature related to general college success as well as that related specifically to success in mathematics courses.

College Preparatory High School Programs

High school mathematics programs in the late 1950's consisted primarily of Algebra I for freshmen, Plane Geometry for sophomores, Algebra II for juniors, and some course for seniors that commonly included trigonometry and solid geometry (Fehr, 1957). Fehr predicted a drastic change in secondary mathematics in his 1957 article "Mathematics in Ragged Clothes." The Commission on Mathematics of the College Entrance Examination Board (CEEB), in its report Program for College Preparatory Mathematics (1959), made eight specific recommendations pertaining to the content of a college preparatory mathematics program. Their first recommendation indicated that college bound seniors should be prepared to take college mathematics courses beginning at the level of calculus and analytic geometry. Another indicated a definite need for trigonometry at the junior level. The combination of plane and solid geometry was recommended along with the inclusion of inequalities in the consideration of equations. They recommended that seniors should study elementary functions and either probability or modern algebra from an abstract point of view. The other recommendations included stress on deductive reasoning in algebra as well as geometry and stress on the structure of mathematics and the use of sets, variables, functions, and relations as unifying concepts throughout the curriculum. This report represented the beginning of a major movement in mathematics curriculum reform that has been commonly called "New Math."

In 1974, following a period of rapid and varied change in the secondary mathematics curriculum, the Conference Board of the Mathematical Sciences appointed the National Advisory Committee on Mathematical Education (NACOME) to study the changes that had occurred in mathematics education during the preceding fifteen years. In the NACOME report, Overview and Analysis of School Mathematics Grades K-12 (1975), the changes that followed the CEEB recommendations are presented and discussed. According to the NACOME report, some of these recommendations were widely implemented. Plane and solid geometry have been combined, inequalities are considered with equations, and trigonometry is generally taught in college preparatory programs. Many other recommendations were less widely implemented and the result, as indicated by this quotation from the NACOME report (1975), has been a wide diversity in college preparatory secondary mathematics programs.

> As part of the NACOME effort to survey the status of mathematics education in the United States, we have collected whatever published mathematics objectives were available in each of the 50 states. The diversity of this collection defies detailed quantitative analysis.

Diversity of program offerings seems to be the present trend in secondary mathematics programs. The NACOME report also makes eight recommendations for mathematics curriculum change. It is important here to notice the similarities and differences between the NACOME recommendations and the CEEB recommendations presented fifteen years earlier. The NACOME report does not recommend that college bound seniors be prepared to take calculus and also fails to mention those items that it found to have been widely implemented. There is continued stress on deductive reasoning, structure, unifying concepts, and probability. Several recommendations in the NACOME report deal with mathematics for non college bound students, an area that was not covered in the CEEB report. Specific suggestions for college preparatory programs corresponding to those in the CEEB report are missing from the NACOME report, along with any indication of the desired level for college freshmen to start their mathematics work.

Recently, the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) jointly made some specific recommendations for college bound high school students in their publication, <u>Recommendations for the Preparation of High School Students</u>

for College Mathematics Courses (1978). This document again indicates that college mathematics begins at the calculus or calculus and analytic geometry level. The authors go on to say:

> Any student who is unable to perform arithmetic calculations and algebraic operations with accuracy and reasonable speed, to understand which operations to use in a given problem, and to determine whether the results have meaning is severely handicapped in the study and applications of mathematics.

They further recommend a de-emphasis on trigonometry and a greater emphasis on algebra. A final recommendation of interest suggests more cooperation between secondary schools and colleges in the areas of remedial programs in schools and colleges and the preparation of students for college mathematics. This recommendation was foreshadowed by Rippey (1976) who claimed that confusing aims in secondary schools have contributed to poor preparation of high school students for college mathematics. In addition, the Carnegie Foundation for the Advancement of Teaching claims "colleges have an obligation to make clear to the high schools which skills they expect their students to acquire before they are admitted" (1978).

The college preparatory programs that have evolved during the past twenty years have not been totally successful. A study (Blai, 1976) of 272 freshmen (73 percent of the freshman class) at Harcum Junior College indicated that 61 percent of the entering freshmen felt that they were not adequately prepared for their college mathematics courses. In 1974, 41 percent of the incoming freshmen at Berkeley felt that they needed remedial mathematics (Maxwell, 1975). Rosovsky (1976) reports that 35 percent of the freshmen entering Harvard in the fall of 1977 failed to achieve a score of 50 percent on the placement tests covering basic pre-calculus mathematics.

College Remedial Programs

The enrollment in remedial mathematics courses at the college level has increased dramatically in recent years. In a study of 450 colleges, junior colleges, and universities (Lindberg, 1974), 75 percent of the mathematics departments reported offering "preparatory mathematics programs." Lindberg defines preparatory mathematics as mathematics that is generally available to high school students including arithmetic, algebra, and geometry. The percentages varied from 65 percent in institutions with graduate programs to 96 percent in two year colleges. More than one third of the departments in this study reported that the enrollment in preparatory mathematics courses accounted for at least 30 percent of their total enrollment. Lindquist (1965), in a study of 855 post-secondary institutions, reported that 77 percent of them offered remedial mathematics courses. In a study of 194 junior colleges conducted by the American Mathematical Association of Two Year Colleges, Baldwin and Others (1975) found that 91 percent of the junior colleges actually offered "developmental mathematics" courses but 96 percent felt that there was a need for such courses. Developmental mathematics was defined to be arithmetic and basic algebra.

According to final enrollment figures kept by the Office of the Registrar at Montana State University, the enrollment in remedial mathematics courses at Montana State University has been rising faster than the total enrollment in mathematics courses in general. While the ratio held fairly constant during the early sixties, it has risen dramatically during the seventies. Table 2.1 shows these figures based on fall quarter ending enrollments for every third year beginning with 1961 and ending with 1979.

Table 2.1

Total Enrollment in Mathematics Compared to Enrollment in Remedial Mathematics at Montana State University

Year	Total Mathematics Enrollment	Remedial Mathematics Enrollment	Ratio Remedial/ Total
1961	1600	429	.268
1964	1956	526	.269
1967	2287	557	.244
1970	2415	922	.382
1973	2434	773	.318
1976	2321	1068	.460
1979	2769	1461	.528

There appears to be widespread concern about the effectiveness of college level remedial mathematics programs. Lindberg (1974), in the study previously discussed, reported that 67 percent of the departments responding to his questionnaire rated their preparatory mathematics programs as only "somewhat successful." He also reported that almost 57 percent of the departments rated fewer than 75 percent of their preparatory mathematics students as successful. Here, again, success was considered to be a course grade of A, B, or C. Baldwin and Others (1975), also previously discussed, found that only six percent of the junior colleges in their study reported more than an 80 percent success rate for developmental mathematics students in subsequent mathematics courses. Almost 50 percent of the institutions had never evaluated their remedial progrmas. Edmond (1976) reported that a majority of the remedial programs in junior colleges in the United States are unsuccessful in preparing students for subsequent non-remedial courses.

Maxwell (1975) of Berkeley shares the feeling of many mathematics educators that students who are admitted to Universities "should be able to write at an eighth grade level and have mastered fundamental arithmetic skills and basic algebra concepts."

The Carnegie Foundation for the Advancement of Teaching (1978) sums up the situation that college remedial programs are in as follows:

> For the student who reaches college without such skills [elementary skills], even the best of remedial education may be too

little and too late. We therefore urge all states to take such measures as may be needed to ensure that elementary and high school students acquire proficiencies consistent with their abilities and grade levels in reading, computation, and written English.

Prediction of Success in College

Many authors have written about the prediction of success for college students. Armstrong (1976) reported that high school grade point averages (GPA), ACT test scores, and SAT test scores were all useful in predicting college success measured by college GPA. She used the entire freshman class of the University of Minnesota during the 1973-1974 academic year as her subjects. Mauger and Kolmodin (1975) showed that SAT test scores give valid prediction of success in college course work. Croft (1976) also reported high school GPA to be useful in predicting college success but did not find ACT test scores to be useful. Croft was, however, considering only multicultural students with low prior success in educational endeavors.

More than 20 years ago, Paul Horst (1956) recognized the need for predicting success in particular course areas in addition to predicting general success in college.

> The prediction of general or over-all college success is certainly worthwhile, but the differential prediction of success for a wide variety of college course areas should be much more valuable to the student in helping him avoid the areas where he is weak and choosing those for which his chances for success are greater.

Achievement test scores, ranking in the high school graduating class, high school mathematics grades, and I.Q. scores have all been shown to be useful predictors of success in college mathematics courses (Troutman, 1977). In a study of 142 women selected at random from a class of 706 freshmen at Longwood College, Gussett (1974) found all three of the Scholastic Aptitude Test scores to be useful predictors of mathematics course grades. The correlation coefficients between each test and student mathematics grades were .63 for SAT-T, .62 for SAT-M, and .48 for SAT-V. Troutman (1977) found only a .50 correlation between SAT-M scores and mathematics grades. His study used 123 students at York College. SAT-M was, however, the best predictor of mathematics grades that he found with high school rank, high school mathematics grades, and I.Q. scores also significant at the .01 level.

Having the necessary prerequisites would, by its very nature, seem to be important for the success of college mathematics students since all college mathematics courses require some background. Dahlke (1974) found that previous achievement in arithmetic was the best predictor of success in a college arithmetic review. However, in a study of 120 chemistry students at Oakton Community College, Bers and Jaffe (1977) examined the relationship between prerequisites and success in beginning chemistry courses and did not find it to be significant. In fact, the successful and unsuccessful groups each had 69 percent who had the necessary prerequisites and 31 percent who did not.

Summary

The college preparatory mathematics curriculum has undergone a rapid and often diversified reorganization in recent years. The result has been a very non-uniform preparation of students for college mathematics. College mathematics programs have expanded their remedial course offerings in an attempt to meet the changing needs of their students. The efforts, however, have not been as successful as they could have been. In trying to properly advise college bound students, many methods have been developed for the prediction of success in college mathematics and achievement test scores seem to be the best single predictor.

Chapter 3

PROCEDURES

The problem of this study was two-fold: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

This chapter begins with a description of the population and of the sampling procedure that was used. This is followed by an explanation of the categories that were compared and the methods that were used to account for contaminating variables. The next section of the chapter discusses the data collection procedure and the test instrument. Next, the statistical hypotheses that were tested are listed. Finally, the actual analysis of the data is explained along with the precautions taken to insure accuracy in the collection, analysis and presentation of the data.

Population Description and Sampling Procedure

This study took place on the campus of Montana State University, a land grant institution established in 1893 under the authorization of the Morrill Act of 1862. The University is located in Bozeman, Montana, a community of approximately 20,000 people, and during fall quarter of 1978 had an undergraduate student population of approximately 9,400 with a graduate student population of approximately 520. The academic faculty numbers over 500 with an additional 200 professional staff members in the research and extension areas.

The Department of Mathematics at Montana State University consists of approximately 25 full time instructional faculty members and 30 part time graduate assistants. During fall quarter of 1978, there were 3,388 students enrolled in mathematics courses with 1,366, approximately 40 percent, enrolled in 39 sections of remedial algebra and trigonometry.

The population under consideration in this study consisted of the students enrolled in Math 001, Math 100, Math 115, Math 117, and Math 121 at Montana State University during fall quarter of the 1978-1979 academic year. There were 2,169 students enrolled in those courses at that time.

The statistics that were estimated in this study were to be used as estimates of parameters within each course and not as estimates of overall population parameters. For this reason, sample size calculations were made for each course separately. Since the sample size requirements for using the Chi Square Test of Independence are minimal, the sample sizes were determined by considering the accuracy desired for estimating the proportions of interest. The researcher wanted to be able to estimate the proportion in question accurately to within five percent with 95 percent confidence, but was willing to settle for less accurate estimation due to reduction in sample size caused by elimination of students, etc. Because the researcher had reason to doubt the prior estimates of p, the proportion being estimated, and q, 1-p, values of p = q = .50 were used for sample size calculations to insure that the sizes were large enough for the stated purpose. Cochran (1963) recommends the use of the following formulas for sample size calculations of this type:

$$(1)\quad n_0 = (t^2)(p)(q)/(d^2)$$

$$(2)\quad n \doteq (n_0)/(1+[n_0/N])$$

The first formula gives an initial approximation of the sample size where t is the appropriate value of Student's t Statistic, p and q are both .50, and d is the allowable variation in the proportion to be estimated. The second formula corrects the initial estimate for finite populations of size N. The calculations of sample sizes for each course are summarized in Table 3.1.

The sample size selected in each case was the first multiple of 40 larger than n. The extra sample size was an allowance for students who would later have to be eliminated from the study and multiples of 40 were used to allow sampling by sections of 40 students each. These calculations indicated that samples of 200 for Math 001, 280 for Math 100, 200 for Math 115, 160 for Math 117, and 280 for Math 121 would be sufficient.

Table 3.1

Sample Size Calculations

Course	Estimated Enrollment N	n_0	n	Projected Sample Size
Math 001	350	384	183	200
Math 100	850	384	265	280
Math 115	375	384	190	200
Math 117	225	384	142	160
Math 121	700	384	248	280

The Math 001 and Math 100 students presented a different sampling problem than the other students. Math 001 did not start until two weeks after the beginning of fall quarter to allow those Math 100 students who had difficulty to drop back to Math 001 without missing part of the course. All of the Math 001 students and part of the Math 100 students received individualized instruction and were not assigned to sections. A simple random sample of 200 Math 001 students was to be selected during the first week of the course. The computer fills sectioned courses to 40 students per section so four sections of Math 100 were chosen at random to get the first 160 students in that group and the remaining 120 were selected at random from the students who received individualized instruction. Five, four, and seven sections of Math 115, Math 117, and Math 121 respectively, were chosen at random to produce the desired samples for those courses.

Categories Compared and Methods Used to Account for Contaminating Variables

Students in Math 100, Math 115, Math 117, and Math 121 were tested at the start of fall quarter and, based on the results of this pre-test, were divided into two groups relative to each competency being tested. Math 100 students took Test I, Math 115 and Math 117 students took Test II, and Math 121 students took Test III. The competency areas tested by each of these tests are listed in Table 3.3. One group, identified by this process, consisted of those students who demonstrated competence in the area and the other group consisted of those students who did not. For the purpose of this study, competence was defined in terms of the number of test items that the student answered correctly. Each area of competency was covered by from two to four test items as listed in Table 3.3. To be considered competent in a given area a student had to answer correctly both items for an area covered by only two items, two out of three items or better for an area covered by three items, and three out of four items or better for an area covered by four items. At the end of fall quarter the students involved in the study were classified as successful or unsuccessful based on their performance in the course. Students who received a course grade of A, B, or C were considered successful while students who dropped the course or received a course grade of D or F were considered unsuccessful. Students who received an I (Incomplete) were eliminated from the study. In Math 001, which was offered only on a pass-fail basis, students who passed were considered successful and those who did not were considered unsuccessful. The same method applied to students who took one of the other courses on a pass-fail basis. The independence of student success and entrance level competence was then checked.

The students in Math 001, Math 100, and Math 115 were tested at the end of fall quarter to determine whether they had developed competence in the areas previously identified as important to success in subsequent courses. For this purpose Math 001 students took Test I, Math 100 students took Test II, and Math 115 students took Test III.

In addition to the comparisons mentioned in preceding paragraphs, sex, age, and method of instruction were compared to success. Finally, proportions were estimated to answer the remaining general questions in this study.

Student differences other than entrance level mathematics competencies were controlled by randomization. Students who took courses by an individualized method were actually selected at random. Students who took courses by the traditional lecture method were selected in groups by section. The sections were chosen at random and since the students were assigned to sections by the computer without consideration being given to any variables that were considered contaminating, this process was also viewed as a random process in relation to contaminating variables.

The teacher effect was also controlled by randomization since the sections were selected at random. In some situations the teacher variable might well have deserved more careful control as a contaminating variable. The courses under consideration in this study, however, were all subjected to careful standardization because of their purpose. Since these are service courses, students go on from the numerous sections of these courses to subsequent courses that depend on the abilities developed in these prerequisite courses. To make possible the planning of the later courses, a great deal of effort is taken to minimize the effect of the teacher variable for these courses. Sequencing and scheduling of topics for these courses is done by the course supervisor and common final exams as well as common hour exams are used extensively.

One of the most important contaminating variables was the prior knowledge that a student had of the course for which he or she was registered. Many students are required to take a certain number of unspecified mathematics credits. For obvious reasons, some students take the easiest courses offered for credit regardless of their mathematics background. A student, for example, who did well in high school Algebra II might take Math 100 for an "easy A" and would most likely wind up in the successful group. The success of this student would be strongly influenced by the fact that he had taken the course previously and would, consequently, shed very little light on the importance of the entrance level competencies to success in the course. Because of this problem, students were asked to indicate the courses that they had taken previously and those students who had already been exposed to the material covered in the course for which they were registered were eliminated from further consideration in relation to identification of needed entrance level competencies.

Data Collection Procedures and Test Instruments

All students were read the same cover letter and asked to fill out a short questionnaire at the beginning of fall quarter. This questionnaire provided the necessary background information for completing the analysis discussed later in this chapter. A copy of this questionnaire and the cover letter appear in Appendix A. Figure 3.1 shows the course sequence for the courses under consideration in this study. Any of these courses could be taken as a first course at Montana State University depending on a student's background. Each course in the sequence has the preceding course or its equivalent as a prerequisite. Three tests were used to measure mathematical competencies at various times and in various courses. Table 3.2 shows the data collection schedule used during fall quarter of 1978.

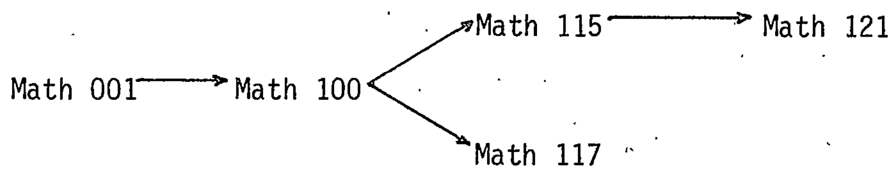

Figure 3.1

Course Sequence

Table 3.2

Data Collection Schedule

Course	Time: Beginning of Quarter	Time: End of Quarter
Math 001	Questionnaire Only	Test I
Math 100	Test I & Questionnaire	Test II
Math 115	Test II & Questionnaire	Test III
Math 117	Test II & Questionnaire	
Math 121	Test III & Questionnaire	

The three tests were written by the researcher and appear in Appendix B. Content validity for these test instruments was established by the construction procedure. The competency areas to be tested and the number of items to be used for each appear in Table 3.3 and were selected following examination of high school texts, examination of the texts used in Math 001, Math 100, and Math 115, and after consultation with the faculty supervisors for the college courses. From Table 3.3 the test items were written using tests from the texts and old hour exams and final exams from the college courses as models. The tests were then examined by the course supervisors and altered according to their recommendations. The predictive validity of the instruments was demonstrated by the results of the study and is discussed in a later chapter. The reliability of the test instruments was established through pre-testing of the instruments during summer quarter of 1978.

A test-retest technique was used and followed by the calculation of the standard correlation coefficient for the two sets of scores. Ferguson (1976) recommends the following formula:

$$r = \frac{N\Sigma XY - \Sigma X\Sigma Y}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}}$$

In the formula, X is a first test score and Y is a second test score for the same person; N is the number of students who were tested. Students in education classes during summer quarter of 1978 were used as subjects to establish this reliability. The results of these calculations are summarized in Table 3.4. For Test I the correlation coefficient was .874, for Test II it was .802, and for Test III it was .805.

Table 3.3

Number of Test Items Per Competency

Competencies	Test I	Test II	Test III
Arithmetic	4		
Solution of Linear Equations	4	2	2
Graphing (Points and Lines)	4	2	2
Solution of Linear Inequalities	3	2	2
Rational Expressions	3	3	
Polynomials and Factoring	3	3	2
Exponents and Radicals	4	3	
Solution of Systems of Equations	3	3	
Solution of Quadratic Equations	2	4	2
Exponential and Logarithmic Functions		4	2
Conic Sections and Their Graphs		4	2
Trigonometric Functions (Definitions)			3
Trigonometric Identities (Basic)			3
Graphing (Trigonometric Functions)			3
Inverse Trigonometric Functions			2
Complex Numbers			3
Vectors			2
Total Number of Items	30	30	30

Table 3.4

Summary of Reliability Calculations for Test Instruments

Test	Number of Students	Pearson Product - Moment Correlation Coefficient
I	21	.874
II	14	.802
III	12	.805

Statistical Hypotheses Tested

The following statistical hypotheses, in null hypotheses form, were tested in this study:

1. The success of a student in the courses considered is independent of entrance level competence in each competency area tested.

2. The success of a student in the courses considered is independent of the sex of the student.

3. The success of a student in the courses considered is independent of the age group to which the student belongs.

4. The success of a student in the courses considered is independent of the method of instruction that the student received.

All of the hypotheses were tested using a Chi Square Test of Independence. Snedecor and Cochran (1967) recommend the following formula:

$$\chi^2 = \frac{N(|AD-BC|-N/2)^2}{(A+B)(C+D)(A+C)(B+D)}$$

In this formula, N is the total number of students in a given sample and A, B, C, and D are the numbers of students who wound up in various categories as determined by the two variables of interest. For example, for hypothesis number one there are two levels of success, successful and unsuccessful, and two levels of competence, competent and not competent. Table 3.5 shows the relationship of A, B, C, and D to these categories. In this case A, for example, is the number of students who were both competent and successful. The term N/2 is a continuity correction. The remaining general questions were answered by estimating the proportions of interest using interval estimation techniques and not by hypothesis testing.

Table 3.5

2x2 Contingency Table

	Successful	Not Successful	Total
Competent	A	B	A + B
Not Competent	C	D	C + D
Total	A + C	B + D	N

Analysis

The statistical hypotheses were tested at the $\alpha = .01$ level. The decision to use this level of significance was based on a consideration of the impact of the results of this study. With relatively large sample sizes used in order to get accurate estimates of the proportions only highly significant results can be considered meaningful. Those relationships that were found to be significant at the $\alpha = .01$ level formed the basis for the recommendations that accompany this study.

As mentioned in the discussion of sample size, the researcher wanted to be able to estimate proportions to within five percent with 95 percent confidence. This lead to 95 percent confidence intervals for p whose width was approximately .10. A normal approximation to the binomial was used to set these intervals. Cochran (1963) suggests the following formula:

$$p \pm \{t\sqrt{1 - f}\,\sqrt{pq/(n - 1)} + 1/2n\}$$

In this formula p is the observed proportion, t is the appropriate value of Student's t Statistic, $f = n/N$ where n is the sample size and N is the population size, and $q = 1 - p$. The factor $\sqrt{1 - f}$, is a finite population correction and the term $1/2n$ is a continuity correction. In addition, as Cochran points out, the $n - 1$ is usually replaced by n to simplify computations. The researcher used the simplified version with n replacing $n - 1$.

Summary

A sample of the students in beginning mathematics courses at Montana State University during fall quarter of 1978 was taken. These students were given pre-tests and post-tests of mathematical competency. By comparing the success rates for students who demonstrated competence and those who did not, those competencies that were important for success in beginning courses were identified. The exit level competencies of successful students in each course were measured to see if the remedial courses actually were effective in developing those competencies identified as important. Comparisons were also made of males and females, of younger and older students, and of students receiving individualized instruction and those receiving traditional lectures. Answers to other questions were gained by the use of descriptive statistics available in the data that was collected.

Chapter 4

RESULTS AND FINDINGS

This study produced data concerning mathematical competencies related to beginning mathematics classes at Montana State University. The results are presented under three major headings. First, the population and the samples that were drawn are discussed. Second, the results from the testing of the four statistical hypotheses are presented. Finally, the general questions not answered by hypothesis testing are each discussed.

Population and Samples

The population under consideration in this study was the students enrolled in Math 001, Math 100, Math 115, Math 117, and Math 121 at Montana State University during fall quarter of the 1978-1979 academic year. As mentioned in Chapter 3, the statistics that were calculated by this study were used as estimates of parameters within courses and not as estimates of overall population parameters.

Sample size calculations were based on estimated starting enrollments in each of the courses under consideration. Actual enrollments were not used because they were not available until after the collection of data had been started. Table 4.1 shows estimated starting enrollments, actual starting enrollments, calculated sample sizes, and actual sample sizes for each course that was considered. The actual sample sizes differ from the calculated sample sizes in Math 100, Math

115, Math 117, and Math 121 because students were sampled by sections and the sections were not all filled to 40 students each as had been expected. The desired level of accuracy was maintained because the actual total enrollments were also smaller than expected. Because the total enrollment for Math 001 was so close to the calculated sample size for that course and because all of these students were in one individualized instruction section, all of the Math 001 students were tested.

Table 4.1

Enrollments and Sample Sizes

Course	Estimated Starting Enrollment	Actual Starting Enrollment	Calculated Sample Size	Actual Sample Size
Math 001	350	240	200	240
Math 100	850	809	280	264
Math 115	375	344	200	169
Math 117	225	208	160	136
Math 121	700	568	280	234

Students in Math 117 and Math 121 were sampled only at the beginning of the quarter while students in Math 001, Math 100, and Math 115 were sampled both at the beginning of the quarter and at the end of the quarter. Not all of the students selected in any one sample could be used in the analysis. As discussed in Chapter 3, students who had previously taken a given course were excluded from consideration when determining entrance level competencies needed for success in that course. Only successful students from samples taken at the end of the quarter were used for measuring the effectiveness of a course in providing needed competencies. Because forced participation was not possible, a few students also declined to respond after they had been selected as part of a sample. The number of students actually used for answering each question are given as part of the discussion for that question.

Table 4.2 gives the section numbers for the sections that were selected as part of the sample for each of the courses considered. These sections were selected at random from all sections of each course.

Table 4.2

Sections Sampled for Various Courses

Course	Sections Sampled
Math 100 (Lecture)	2, 7, 10, 12, 14
Math 115	3, 4, 5, 8, 9
Math 117	1, 2, 5, 6
Math 121	1, 3, 6, 8, 13, 14, 16

Statistical Hypotheses

All four of the statistical hypotheses that were tested in this study were tested using a Chi Square Test of Independence. The level of significance used for all tests was $\alpha = .01$.

Null Hypothesis One: The success of a student in the courses considered is independent of entrance level competence in each competency area tested.

Table 4.3 lists the competency areas tested for significance relative to success in Math 100 and also summarizes the results of the Chi Square analysis. Competencies 1, 3, and 8 were found to be significantly related to success in Math 100 when tested at the .01 level of significance. These competencies were in the areas of arithmetic, graphing points and lines, and solutions of systems of equations respectively.

Tables 4.4 through 4.12 are the 2x2 contingency tables used to produce the figures shown in Table 4.3. In each case the critical value of Chi Square with one degree of freedom at the .01 level of significance is 6.63. After students who had previously taken Math 100 or an equivalent course were removed from the sample there were 115 students left (N = 115). Each Chi Square value that was significant at the .01 level is identified as significant in the table.

Table 4.3

Summary of Chi Square Analysis for Competencies Related to Success in Math 100

Competency	Critical Value of Chi Square	Calculated Value of Chi Square
1. Arithmetic	6.63	26.54*
2. Solution of Linear Equations	6.63	2.734
3. Graphing (Points and Lines)	6.63	8.458*
4. Solution of Linear Inequalities	6.63	.5610
5. Rational Expressions	6.63	5.615
6. Polynomials and Factoring	6.63	.8571
7. Exponents and Radicals	6.63	2.606
8. Solution of Systems of Equations	6.63	6.830*
9. Solution of Quadratic Equations	6.63	4.748

*Significant at the $\alpha = .01$ level.

Table 4.4

2x2 Contingency Table: Competence in Arithmetic and Success in Math 100

	Successful	Not Successful
Competent	41	12
Not Competent	17	45

Calculated $\chi^2 = 26.54^*$ N = 115

*Significant at the $\alpha = .01$ level.

Table 4.5

2x2 Contingency Table: Competence in Solution of Linear Equations and Success in Math 100

	Successful	Not Successful
Competent	27	17
Not Competent	31	40

Calculated $\chi^2 = 2.734$ N = 115

Table 4.6

2x2 Contingency Table: Competence in Graphing (Points and Lines) and Success in Math 100

	Successful	Not Successful
Competent	27	11
Not Competent	31	46

Calculated $\chi^2 = 8.458$* N = 115

*Significant at the $\alpha = .01$ level.

Table 4.7

2x2 Contingency Table: Competence in Solution of Linear Inequalities and Success in Math 100

	Successful	Not Successful
Competent	20	15
Not Competent	38	42

Calculated $\chi^2 = .5610$ N = 115

Table 4.8

2x2 Contingency Table: Competence with Rational Expressions and Success in Math 100

	Successful	Not Successful
Competent	16	5
Not Competent	42	52

Calculated χ^2 = 5.615 N = 115

Table 4.9

2x2 Contingency Table: Competence in Polynomials and Factoring and Success in Math 100

	Successful	Not Successful
Competent	22	16
Not Competent	36	41

Calculated χ^2 = .8571 N = 115

Table 4.10

2x2 Contingency Table: Competence in Exponents and Radicals and Success in Math 100

	Successful	Not Successful
Competent	15	7
Not Competent	43	50

Calculated $\chi^2 = 2.606$ N = 115

Table 4.11

2x2 Contingency Table: Competence in Solution of Systems of Equations and Success in Math 100

	Successful	Not Successful
Comptetent	45	30
Not Competent	13	27

Calculated $\chi^2 = 6.830$* N = 115

*Significant at the $\alpha = .01$ level.

Table 4.12

2x2 Contingency Table: Competence in Solution of Quadratic Equations and Success in Math 100

	Successful	Not Successful
Competent	12	3
Not Competent	46	54

Calculated $\chi^2 = 4.748$ N = 115

Table 4.13 lists the competency areas tested for significance relative to success in Math 115 and also summarizes the results of the Chi Square analysis. Only competency 5, which covers polynomials and factoring, was found to be significantly related to success in Math 115 when tested at the .01 level of significance.

Table 4.13

Summary of Chi Square Analysis for Competencies Related to Success in Math 115

	Competency	Critical Value of Chi Square	Calculated Value of Chi Square
1.	Solution of Linear Equations	6.63	.1578
2.	Graphing (Points and Lines)	6.63	1.340
3.	Solution of Linear Inequalities	6.63	.9999
4.	Rational Expressions	6.63	.9872
5.	Polynomials and Factoring	6.63	11.27*
6.	Exponents and Radicals	6.63	.2297
7.	Solution of Systems and Equations	6.63	.0047
8.	Solution of Quadratic Equations	6.63	.0298
9.	Exponential and Logarithmic Functions	6.63	.0632
10.	Conic Sections and Their Graphs	6.63	.2945

*Significant at the α = .01 level.

Tables 4.14 through 4.23 are the 2x2 contingency tables used to produce the figures shown in Table 4.13. In each case the critical value of Chi Square with one degree of freedom at the .01 level of significance is 6.63. After students who had previously taken Math 115 or an equivalent course were removed from the sample there were 85 students left (N = 85). The Chi Square value that was significant at the .01 level is identified as significant in the table.

Table 4.14

2x2 Contingency Table: Competence in Solution of Linear Equations and Success in Math 115

	Successful	Not Successful
Competent	20	13
Not Competent	35	17

Calculated $\chi^2 = .1578$ N = 85

Table 4.15

2x2 Contingency Table: Competence in Graphing Points and Lines and Success in Math 115

	Successful	Not Successful
Competent	19	6
Not Competent	36	24

Calculated χ^2 = 1.340   N = 85

Table 4.16

2x2 Contingency Table: Competence in Solution of Linear Inequalities and Success in Math 115

	Successful	Not Successful
Competent	24	9
Not Competent	31	21

Calculated χ^2 = .9999   N = 85

Table 4.17

2x2 Contingency Table: Competence with Rational Expressions and Success in Math 115

	Successful	Not Successful
Competent	18	6
Not Competent	37	24

Calculated $\chi^2 = .9872$ N = 85

Table 4.18

2x2 Contingency Table: Competence in Polynomials and Factoring and Success in Math 115

	Successful	Not Successful
Competent	37	8
Not Competent	18	22

Calculated $\chi^2 = 11.27$* N = 85

*Significant at the $\alpha = .01$ level.

Table 4.19

2x2 Contingency Table: Competence in Exponents and Radicals and Success in Math 115

	Successful	Not Successful
Competent	9	3
Not Competent	46	27

Calculated χ^2 = .2297 N = 85

Table 4.20

2x2 Contingency Table: Competence in Solution of Systems of Equations and Success in Math 115

	Successful	Not Successful
Competent	21	12
Not Competent	34	18

Calculated χ^2 = .0047 N = 85

Table 4.21

2x2 Contingency Table: Competence in Solution of Quadratic Equations and Success in Math 115

	Successful	Not Successful
Competent	8	4
Not Competent	47	26

Calculated χ^2 = .0298 N = 85

Table 4.22

2x2 Contingency Table: Competence in Exponential and Logarithmic Functions and Success in Math 115

	Successful	Not Successful
Competent	6	2
Not Competent	49	28

Calculated χ^2 = .0632 N = 85

Table 4.23

2x2 Contingency Table: Competence in Conic Sections and Their Graphs and Success in Math 115

	Successful	Not Successful
Competent	1	2
Not Competent	54	28

Calculated $\chi^2 = .2945$ N = 85

Table 4.24 lists the competency areas tested for significance relative to success in Math 117 and also summarizes the results of the Chi Square analysis. Only competency 5, which covers polynomials and factoring, was found to be significantly related to success in Math 117 when tested at the .01 level of significance.

Table 4.24

Summary of Chi Square Analysis for Competencies Related to Success in Math 117

Competency	Critical Value of Chi Square	Calculated Value of Chi Square
1. Solution of Linear Equations	6.63	3.602
2. Graphing (Points and Lines)	6.63	1.419
3. Solution of Linear Inequalities	6.63	.0131
4. Rational Expressions	6.63	1.164
5. Polynomials and Factoring	6.63	6.981*
6. Exponents and Radicals	6.63	.3928
7. Solution of Systems of Equations	6.63	.8146
8. Solution of Quadratic Equations	6.63	3.671
9. Exponential and Logarithmic Functions	6.63	5.368
10. Conic Sections and Their Graphs	6.63	.9227

*Significant at the $\alpha = .01$ level.

Tables 4.25 through 4.34 are the 2x2 contingency tables used to produce the figures shown in Table 4.24. In each case the critical value of Chi Square with one degree of freedom at the .01 level of significance is 6.63. After students who had previously taken Math 117 or an equivalent course were removed from the sample there were 103 students left (N = 103). The Chi Square value that was significant at the .01 level is identified as significant in the table.

Table 4.25

2x2 Contingency Table: Competence in Solution of Linear Equations and Success in Math 117

	Successful	Not Successful
Competent	33	10
Not Competent	34	26

Calculated $\chi^2 = 3.602$ N = 103

Table 4.26

2x2 Contingency Table: Competence in Graphing Points and Lines and Success in Math 117

	Successful	Not Successful
Competent	28	10
Not Competent	39	26

Calculated χ^2 = 1.419 N = 103

Table 4.27

2x2 Contingency Table: Competence in Solution of Linear Inequalities and Success in Math 117

	Successful	Not Successful
Competent	17	8
Not Competent	50	28

Calculated χ^2 = .0131 N = 103

Table 4.28

2x2 Contingency Table: Competence in Rational Expressions and Success in Math 117

	Successful	Not Successful
Competent	19	6
Not Competent	48	30

Calculated χ^2 = 1.164 N = 103

Table 4.29

2x2 Contingency Table: Competence in Polynomials and Factoring and Success in Math 117

	Successful	Not Successful
Competent	52	18
Not Competent	15	18

Calculated χ^2 = 6.981* N = 103

*Significant at the α = .01 level.

Table 4.30

2x2 Contingency Table: Competence in Exponents and Radicals and Success in Math 117

	Successful	Not Successful
Competent	17	12
Not Competent	50	24

Calculated χ^2 = .3928 N = 103

Table 4.31

2x2 Contingency Table: Competence in Solution of Systems of Equations and Success in Math 117

	Successful	Not Successful
Competent	26	10
Not Competent	41	26

Calculated χ^2 = .8146 N = 103

Table 4.32

2x2 Contingency Table: Competence in Solution of Quadratic Equations and Success in Math 117

	Successful	Not Successful
Competent	15	2
Not Competent	52	34

Calculated $\chi^2 = 3.671$ N = 103

Table 4.33

2x2 Contingency Table: Competence in Exponential and Logarithmic Functions and Success in Math 117

	Successful	Not Successful
Competent	34	9
Not Competent	33	27

Calculated $\chi^2 = 5.368$ N = 103

Table 4.34

2x2 Contingency Table: Competence in Conic Sections and Their Graphs and Success in Math 117

	Successful	Not Successful
Competent	4	0
Not Competent	63	36

Calculated χ^2 = .9227 N = 103

Table 4.35 lists the competency areas tested for significance relative to success in Math 121 and also summarizes the Chi Square analysis. Competencies 2, 3, 5, 6, 7, 9, and 12 were all found to be significantly related to success in Math 121 when tested at the .01 level of significance. These competencies were in the areas of graphing points and lines, solution of linear inequalities, solution of quadratic equations, exponential and logarithmic functions, conic sections and their graphs, basic trigonometric identities, and complex numbers respectively.

Tables 4.36 through 4.48 are the 2x2 contingency tables used to produce the figures shown in Table 4.35. In each case the critical value of Chi Square is 6.63. After students who had previously taken Math 121 or an equivalent course were removed from the sample there were 160 students left (N = 160). The Chi Square values that were significant at the .01 level are identified in the tables.

Table 4.35

Summary of Chi Square Analysis for Competencies Related to Success in Math 121

	Competency	Critical Value of Chi Square	Calculated Value of Chi Square
1.	Solution of Linear Equations	6.63	.2808
2.	Graphing (Points and Lines)	6.63	15.81*
3.	Solution of Linear Inequalities	6.63	9.913*
4.	Polynomials and Factoring	6.63	.3684
5.	Solution of Quadratic Equations	6.63	19.02*
6.	Exponential and Logarithmic	6.63	17.07*
7.	Conic Sections and Their Graphs	6.63	8.727*
8.	Trigonometric Functions (Definitions)	6.63	4.818
9.	Trigonometric Identities (Basic)	6.63	12.48*
10.	Graphing (Trigonometric Functions)	6.63	1.811
11.	Inverse Trigonometric Functions	6.63	.8644
12.	Complex Numbers	6.63	25.23*
13.	Vectors	6.63	6.545

*Significant at the $\alpha = .01$ level.

Table 4.36

2x2 Contingency Table: Competence in Solution of Linear Equations and Success in Math 121

	Successful	Not Successful
Competent	32	19
Not Competent	62	47

Calculated χ^2 = .2808 N = 160

Table 4.37

2x2 Contingency Table: Competence in Graphing (Points and Lines) and Success in Math 121

	Successful	Not Successful
Competent	42	9
Not Competent	52	57

Calculated χ^2 = 15.81* N = 160

*Significant at the $\alpha = .01$ level.

Table 4.38

2x2 Contingency Table: Competence in Solution of Linear Inequalities and Success in Math 121

	Successful	Not Successful
Competent	44	14
Not Competent	50	52

Calculated $\chi^2 = 9.913$* N = 160

*Significant at the $\alpha = .01$ level.

Table 4.39

2x2 Contingency Table: Competence in Polynomials and Factoring and Success in Math 121

	Successful	Not Successful
Competent	84	56
Not Competent	10	10

Calculated $\chi^2 = .3684$ N = 160

Table 4.40

2x2 Contingency Table: Competence in Solution of Quadratic Equations and Success in Math 121

	Successful	Not Successful
Competent	28	1
Not Competent	66	65

Calculated χ^2 = 19.02*

*Significant at the α = .01 level.

Table 4.41

2x2 Contingency Table: Competence in Exponential and Logarithmic Functions and Success in Math 121

	Successful	Not Successful
Competent	31	3
Not Competent	63	63

Calculated χ^2 = 17.07*   N = 160

*Significant at the α = .01 level.

Table 4.42

2x2 Contingency Table: Competence in Conic Sections and Their Graphs and Success in Math 121

	Successful	Not Successful
Competent	28	6
Not Competent	66	60

Calculated $\chi^2 = 8.727$* N = 160

*Significant at the $\alpha = .01$ level.

Table 4.43

2x2 Contingency Table: Competence in Definition of Trigonometric Functions and Success in Math 121

	Successful	Not Successful
Competent	59	29
Not Competent	35	37

Calculated $\chi^2 = 4.818$ N = 160

Table 4.44

2x2 Contingency Table: Competence in Basic Trigonometric Identities and Success in Math 121

	Successful	Not Successful
Competent	71	31
Not Competent	23	35

Calculated χ^2 = 12.48* N = 160

*Significant at the α = .01 level.

Table 4.45

2x2 Contingency Table: Competence in Graphing Trigonometric Functions and Success in Math 121

	Successful	Not Successful
Competent	38	19
Not Competent	56	47

Calculated χ^2 = 1.811 N = 160

Table 4.46

2x2 Contingency Table: Competence in Inverse Trigonometric Functions and Success in Math 121

	Successful	Not Successful
Competent	11	4
Not Competent	83	62

Calculated χ^2 = .8644 N = 160

Table 4.47

2x2 Contingency Table: Competence in Complex Numbers and Success in Math 121

	Successful	Not Successful
Competent	41	4
Not Competent	53	62

Calculated χ^2 = 25.23* N = 160

*Significant at the α = .01 level.

Table 4.48

2x2 Contingency Table: Competence in Vectors and Success in Math 121

	Successful	Not Successful
Competent	36	12
Not Competent	58	54

Calculated $\chi^2 = 6.545$ N = 160

In summary, three of the nine entrance level competencies tested in Math 100 showed a significant relationship to success in that course. Those competencies were in the areas of arithmetic, graphing points and lines, and solution of systems of equations. Only one of the 10 entrance level competencies tested in each of Math 115 and Math 117 showed a significant relationship to success in those courses. For both Math 115 and Math 117 the competency that was significantly related to success was in the area of polynomials and factoring. Finally, seven of the 13 entrance level competencies tested in Math 121 showed a significant relationship to success in that course. Those competencies that were significantly related to success in Math 121 were in the areas of graphing points and lines, solution of linear inequalities, solution of quadratic equations, exponential and logarithmic functions, conic sections and their graphs, basic trigonometric identities, and complex numbers. Table 4.49 summarizes the findings for hypothesis number one by showing the competencies tested for each course and those competencies that were significantly related to success in each course.

The data analysis for null hypothesis one was done on Montana State University's Scientific Data Systems (SDS) Sigma Seven computer using a computer program written by the researcher. This program appears in Appendix C.

Table 4.49

Competencies Tested and Competencies Found to be Important to Success in Each of the Courses Considered

Competency	Important for Math 100	Important for Math 115	Important for Math 117	Important for Math 121
Arithmetic	X	-	-	-
Solution of Linear Equations				
Graphing (Points and Lines)	X			X
Solution of Linear Inequalities				X
Rational Expressions				-
Polynomials and Factoring		X	X	
Exponents and Radicals				-
Solution of Systems of Equations	X			-

Table 4.49 (continued)

Competency	Important for Math 100	Important for Math 115	Important for Math 117	Important for Math 121
Solution of Quadratic Equations				X
Exponential and Logarithmic Functions	-			X
Conic Sections and Their Graphs	-			X
Trigonometric Functions (Definitions)	-	-	-	
Trigonometric Identities (Basic)	-	-	-	X
Graphing (Trigonometric Functions)	-	-	-	
Inverse Trigonometric Functions	-	-	-	
Complex Numbers	-	-	-	X
Vectors	-	-	-	

X = Important to Success

- = Not Tested

Null Hypothesis Two: The success of a student in the courses considered is independent of the sex of the student.

This hypothesis was tested for Math 100, Math 115, Math 117, and Math 121 using a Chi Square Test of Independence. The hypothesis for each course was tested at the .01 level of significance and the critical value of Chi Square with one degree of freedom was 6.63. This study did not find sufficient evidence to reject the null hypothesis. Therefore, based on the evidence gathered by this study, the researcher cannot conclude that the success of a student in any beginning mathematics course at Montana State University depends on the sex of the student. Tables 4.50 through 4.53 give the 2x2 contingency tables used to carry out the Chi Square Tests of Independence. After removing students who had taken the course previously from the samples, the sample sizes were 115, 85, 103, and 160 respectively for Math 100, Math 115, Math 117, and Math 121.

The Chi Square analysis for null hypothesis two was done on Montana State University's SDS Sigma Seven computer using a computer program written by the researcher. This program appears in Appendix C.

Table 4.50

2x2 Contingency Table: Sex of a Student and Success in Math 100

	Successful	Not Successful
Male	36	32
Female	22	25

Calculated χ^2 = .2088 N = 115

Critical χ^2 = 6.63

Table 4.51

2x2 Contingency Table: Sex of a Student and Success in Math 115

	Successful	Not Successful
Male	39	25
Female	16	5

Calculated χ^2 = 1.012 N = 85

Critical χ^2 = 6.63

Table 4.52

2x2 Contingency Table: Sex of a Student and Success in Math 117

	Successful	Not Successful
Male	36	28
Female	31	8

Calculated χ^2 = 4.779 N = 103

Critical χ^2 = 6.63

Table 4.53

2x2 Contingency Table: Sex of a Student and Success in Math 121

	Successful	Not Successful
Male	72	55
Female	22	11

Calculated χ^2 = .7030 N = 160

Critical χ^2 = 6.63

Therefore, in summary, the researcher did not find sufficient evidence to reject the hypothesis that the success of a student is independent of the student's sex.

<u>Null Hypothesis Three</u>: The success of a student in the courses considered is independent of the age group to which the student belongs.

The students were classified as "older students" if they were 25 years old or older and as "younger students" if they were less than 25 years old. This hypothesis was tested for each of Math 100, Math 115, Math 117, and Math 121 using a Chi Square Test of Independence. The hypothesis for each course was tested at the .01 level of significance and the critical value of Chi Square with one degree of freedom was 6.63. This study did not find sufficient evidence to reject the null hypothesis. Therefore, based on the evidence gathered by this study, the researcher cannot conclude that the success of a student in any beginning mathematics course at Montana State University depends on the age of the student. Tables 4.54 through 4.57 give the 2x2 contingency tables used to carry out the Chi Square Tests of Independence. After removing students who had taken the course previously from the samples, the sample sizes were 115, 85, 103, and 160 respectively for Math 100, Math 115, Math 117, and Math 121.

The Chi Square analysis for null hypothesis three was done on Montana State University's SDS Sigma Seven computer using a computer program written by the researcher. This program appears in Appendix C.

Table 4.54

2x2 Contingency Table: Age of a Student and Success in Math 100

	Successful	Not Successful
Older	4	1
Younger	54	56

Calculated χ^2 = .8005 N = 115

Critical χ^2 = 6.63

Table 4.55

2x2 Contingency Table: Age of a Student and Success in Math 115

	Successful	Not Successful
Older	8	2
Younger	47	28

Calculated χ^2 = .5259 N = 85

Critical χ^2 = 6.63

Table 4.56

2x2 Contingency Table: Age of a Student and Success in Math 117

	Successful	Not Successful
Older	5	2
Female	62	34

Calculated χ^2 = .0019   N = 103

Critical χ^2 = 6.63

Table 4.57

2x2 Contingency Table: Age of a Student and Success in Math 121

	Successful	Not Successful
Older	4	6
Younger	90	60

Calculated χ^2 = .8322   N = 160

Critical χ^2 = 6.63

Therefore, in summary, the researcher did not find sufficient evidence to reject the hypothesis that the success of a student is independent of the age group to which the student belongs.

<u>Null Hypothesis Four</u>: The success of a student in Math 100 is independent of the method of instruction that the student received.

Math 100 was taught using both the traditional lecture method and the Personalized System of Instruction (PSI) method during fall quarter of 1978. The hypothesis was tested using a Chi Square Test of Independence. The level of significance used was .01 and the critical value of Chi Square with one degree of freedom was 6.63. This study did not find sufficient evidence to reject the null hypothesis. Therefore, based on the evidence gathered by this study, the researcher cannot conclude that the success of a student in Math 100 at Montana State University depends on the method of instruction that the student received. Table 4.58 gives the 2x2 contingency table used to carry out the Chi Square Test of Independence. After removing students who had taken the course previously from the sample there were 115 students left for this test.

The Chi Square analysis for null hypothesis four was done on Montana State University's SDS Sigma Seven Computer using a computer program written by the researcher. This program appears in Appendix C.

Table 4.58

2x2 Contingency Table: Method of Instruction and Success in Math 100

	Successful	Not Successful
PSI	26	34
Lecture	32	23

Calculated χ^2 = 1.972 N = 115

Critical χ^2 = 6.63

Therefore, in summary, the researcher did not find sufficient evidence to reject the hypothesis that the success of a student in Math 100 is independent of the method of instruction that the student received.

General Questions

Four general questions were answered by methods other than hypothesis testing. They were:

1. Do remedial mathematics courses at Montana State University effectively develop the competencies needed for success in subsequent courses?

2. What proportion of the students who register for and attend beginning mathematics courses at Montana State University are actually successful in that course?

3. What proportion of the students who register for and attend beginning mathematics courses at Montana State University have not already passed an equivalent course?

4. Do students who wait until they get to Montana State University to take their first algebra course have a reasonable chance for success in that course?

Each of these questions will be discussed separately.

General Question One: Do remedial mathematics courses at Montana State University effectively develop the competencies needed for success in subsequent courses?

The reader is reminded that for the purpose of this study a successful student was defined to be any student who received a course grade of A, B, or C. In pass/fail courses students who passed were considered successful. In all cases students who received I (Incomplete) grades were eliminated from consideration.

Math 001 or its equivalent is required as a prerequisite for Math 100. The first general question has, then, as its first part the question of how well Math 001 develops the competencies needed for Math 100. In the previous section of this chapter, three competencies were identified as important to success in Math 100. They were in the areas of arithmetic, graphing of points and lines, and solution of systems of equations. Students who were successful in Math 001 were tested to determine their competence in these areas. As in the previous section, all of the Math 001 students were tested because of the small total number of students in this course. A slight reduction in the total number of successful students was observed because of the fact that some students declined to be tested at the end of the course. Of the 53 Math 001 students who were successful by the end of fall quarter, 41 were tested. Because all of the students in the population of successful Math 001 students were used the exact proportion that demonstrated competence in each area can be reported. In arithmetic, the proportion of successful Math 001 students who were competent was .5854. In graphing points and lines the proportion was .6341, and in solution of systems of equations the proportion was .7073. Table 4.59 summarizes these results.

Table 4.59

Performance of Successful Math 001 Students on Competencies Identified as Important for Math 100

Competency Area	Proportion Demonstrating Competence
Arithmetic	.5854
Graphing (Points and Lines)	.6341
Solution of Systems of Equations	.7073

N = 41

Math 100 is a prerequisite for Math 115, Math 117, and for Math 121. Therefore, a second part of the first general question deals with how well Math 100 develops the competencies needed for Math 115, Math 117, and Math 121. In the previous section of this chapter, only one competency was identified as important to success in Math 115 and Math 117. This competency was in the area of polynomials and factoring. Seven competencies, however, were identified as important to success in Math 121. These were in the areas of graphing of points and lines, solution of linear inequalities, solution of quadratic equations, exponential and logarithmic functions, conic sections and their graphs, basic trigonometric identities, and complex numbers. Math 100 students were not tested for competence on the last two of these areas because those areas are not covered in Math 100. These topics are covered in Math 115. For each of the other areas, however, 95 percent confidence intervals were set for the true proportion of successful Math 100 students who demonstrated competence in these areas. As with Math 001, the sample size was reduced because some students declined to participate. A total of 188 successful Math 100 students were tested. Table 4.60 summarizes these results.

Table 4.60

Performance of Successful Math 100 Students on Competencies Identified as Important for Math 115, Math 117, and Math 121

Competency	Course Requiring Competency	Proportion Demonstrating Competency	95 Percent C.I. for True Proportion	
			Lower Limit	Upper Limit
Graphing (Points and Lines	121	.3830	.3257	.4403
Solution of Linear Inequalities	121	.4681	.4093	.5269
Polynomials and Factoring	115, 117	.7234	.6704	.7764
Solution of Quadratic Equations	121	.3138	.2590	.3687
Exponential and Logarithmic Functions	121	.6436	.5871	.7001
Conic Sections and Their Graphs	121	.0213	.0024	.0402
Trigonometric Identities (Basic)	121	-	-	-
Complex Numbers	121	-	-	-

- = Not Tested

Math 115 or its equivalent is also a prerequisite for Math 121. This leads to the consideration of how well Math 115 develops competencies needed for success in Math 121 as a final part of general question one. While the competencies needed for Math 121 are not all covered as part of Math 115, some that are not covered could still be developed through application as the student covers course material. All of the competencies important to Math 121, therefore, were tested in the sample of successful Math 115 students. For each of these seven areas, 95 percent confidence intervals were set for the true proportion of successful Math 115 students who demonstrated competence in these areas. After a slight reduction in size due to nonparticipation, there were 118 students used for these calculations. Table 4.61 summarizes the results of these calculations.

The proportions and confidence intervals for general question one were produced by Montana State University's SDS Sigma Seven computer using a computer program written by the researcher. This program appears in Appendix C.

In summary, approximately 59 percent, 63 percent, and 71 percent of the successful Math 001 students respectively, demonstrated competence in arithmetic, graphing of points and lines, and solution of systems of equations. While approximately 72 percent of the successful Math 100 students demonstrated competence in polynomials and factoring, the only competency identified as important to success in Math 115 and

Math 117, the proportions demonstrating competence in the seven areas identified as important for Math 121 varied greatly. These results ranged from approximately two percent who demonstrated competence in conic sections and their graphs to approximately 64 percent who demonstrated competence in exponential and logarithmic functions. Two basic types of competencies were identified as important for success in Math 121. They were algebraic competencies and trigonometric competencies. More than 90 percent of the successful Math 115 students demonstrated competence in the trigonometry areas but less than 30 percent of them demonstrated competence in even the best of the algebra competency areas. Possible interpretations of this data are discussed in Chapter 5 of this study.

Table 4.61

Performance of Successful Math 115 Students on Competencies Identified as Important for Math 121

Competency	Proportion Demonstrating Competence	95 Percent C.I. for True Proportion	
		Lower Limit	Upper Limit
Graphing (Points and Lines)	.2203	.1612	.2795
Solution of Linear Inequalities	.2712	.2081	.3343
Solution of Quadratic Equations	.0763	.0369	.1157
Exponential and Logarithmic Functions	.1695	.1156	.2234
Conic Sections and Their Graphs	.2542	.1923	.3162
Trigonometric Identities (Basic)	.9661	.9379	.9943
Complex Numbers	.9237	.8843	.9631

General Question Two: What proportion of the students who register for and attend beginning mathematics courses at Montana State University are actually successful in that course?

While it was planned to answer this question by setting a 95 percent confidence interval for the true proportion of students that were successful, the data collected allowed for the calculation of the exact proportion in three of the five courses of interest. The exact proportions in the other two courses were available for relatively little additional effort and, therefore, exact proportions were reported for all courses considered. Table 4.62 summarizes these proportions.

Table 4.62

Exact Proportions of Students Successful in
Math 001, Math 100, Math 115, Math 117, and Math 121

Course	Starting Enrollment	Total Number Successful	Proportion Successful
Math 001	240	53	.2208
Math 100	809	493	.6094
Math 115	344	256	.7442
Math 117	208	149	.7163
Math 121	568	394	.6937

The proportions for general question two were calculated by hand with arithmetic done using a Texas Instruments TI-30 hand held calculator.

In summary, the exact proportion of students registered for and attending beginning mathematics classes at Montana State University during fall quarter of 1978 was calculated for each course of interest. Approximately 22 percent of the Math 001 students were successful; approximately 61 percent of the Math 100 students were successful; approximately 74 percent of the Math 115 students were successful; approximately 72 percent of the Math 117 students were successful; and approximately 69 percent of the Math 121 students were successful. A discussion of these results is given in Chapter 5 of this study.

General Question Three: What proportion of the students who register for and attend beginning mathematics courses at Montana State University have not already passed an equivalent course?

This question was answered by setting 95 percent confidence intervals for the true proportions in Math 100, Math 115, Math 117, and Math 121. For Math 001, all of the students were questioned and therefore no estimation is needed. All of the students in the original samples were used for this question giving sample sizes of 240 (all students), 264, 169, 136, and 234 respectively for Math 001, Math 100, Math 115, Math 117, and Math 121. Table 4.63 summarizes the data.

Table 4.63

Proportions of Students Who Have Not Passed Equivalent Courses Prior to Math 001, Math 100, Math 115, Math 117, and Math 121

Course	Starting Enrollment	Number Not Repeating	Proportion Not Repeating	95 Percent C.I. for True Proportion	
				Lower Limit	Upper Limit
Math 001	240	0	0	---*	---*
Math 100	264	115	.4356	.3846	.4866
Math 115	169	85	.5030	.4462	.5598
Math 117	136	103	.7574	.7113	.8035
Math 121	234	160	.6838	.6350	.7326

*Confidence interval not needed because sample = population.

In Math 001, none of the 240 students indicated never having taken an algebra course. The researcher feels that this figure may be misleading and comments further on the subject in Chapter 5 of this study. Approximately 44 percent of the Math 100 students have never had an equivalent course, approximately 50 percent of the Math 115 students and approximately 76 percent of the Math 117 students have never had an equivalent course, and approximately 68 percent of the Math 121 students have never had an equivalent course.

<u>General Question Four</u>: Do students who wait until they get to

Montana State University to take their first algebra course have a reasonable chance for success in that course?

None of the 240 Math 001 students admitted to never having taken an algebra course. This study, therefore, does not directly provide any data with which to answer this question. If, in fact, very few Math 001 students have never taken an algebra course then the question is not an important one. If, on the other hand, there are regularly a number of Math 001 students with no algebra background, then this question bears further consideration. The researcher discusses possible implications of other data from this study on this question in Chapter 5 of this study.

Summary

The researcher first tested the hypothesis that the success of a student in the courses considered is independent of the entrance level competencies that the student has. For Math 100 the evidence suggests that success depends on competence in arithmetic, graphing points and lines, and solution of systems of equations. For Math 115 and Math 117 the evidence suggests that only competence in polynomials and factoring is significantly related to success. For Math 121 competence in graphing points and lines, solution of linear inequalities, solution of quadratic equations, exponential and logarithmic functions, conic sections and their graphs, basic trigonometric identities, and complex numbers all appear to be important to success. The researcher next tested the independence of success in each of the courses considered and 1) the sex of the student, 2) the age of the student, and 3) the method of instruction used. None of these was found to be significantly related to success in any of the courses considered. The researcher next estimated the proportion of successful students in each course who were able to demonstrate competence in the areas identified as important to success in subsequent courses. These proportions varied from as low as two percent to as high as 97 percent. The researcher then calculated the exact proportion of students who were successful in each of the courses considered. These proportions were approximately .22, .61, .74, .72, and .69 respectively for Math 001, Math 100, Math 115, Math 117, and Math 121. The researcher finally estimated the proportion of students in beginning mathematics courses who had not previously passed an equivalent course. These proportions were approximately 0.0, .44, .50, .76, and .68 respectively for Math 001, Math 100, Math 115, Math 117, and Math 121. The researcher was not able to determine what chance a student had for success if he waited until college to take his first algebra course because none of the students admitted to being in this category.

Chapter 5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This chapter summarizes the study "Entrance Level Competencies Needed for Beginning Mathematics Courses at Montana State University." It also presents conclusions drawn from the analysis of data collected in the study and makes recommendations for further study and for action suggested by the data.

Summary

The problem of this study was two-fold: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

Related literature was reviewed in Chapter 2 under three main topics. These topics were: college preparatory high school programs, college remedial programs, and prediction of college success. National trends as well as the situation in Montana were considered within each of the first two topics.

College preparatory high school programs have undergone a rapid diversification during the past 20 years resulting in a wide variety of backgrounds among the students who presently go to college.

College remedial programs have expanded greatly in response to the increasing demand for such courses from the incoming students. Concern with respect to the effectiveness of college remedial programs as well as concern with respect to the appropriateness of such programs at the college level has increased dramatically. Prediction of overall college success has been widely attempted during this period but far less has been said about success in specific subject matter areas. Declining achievement test scores and rising grade point averages among high school students called public attention to the problem and helped to motivate this study.

A complete description of the methods used to collect and analyze the data was presented in Chapter 3. Sample sizes were calculated based on projected enrollments in courses of interest. Tests were developed cooperatively with members of the Department of Mathematics at Montana State University and students were tested at the beginning and at the end of the quarter. Statistical hypotheses were tested using a Chi Square Test of Independence and the remaining general questions were answered using standard interval estimation techniques.

The data collected during this study was presented in Chapter 4. Three of the nine competencies tested in Math 100 were found to be significantly related to success. One of the ten competencies tested for Math 115 and Math 117 was found to be significantly related to success in these courses. In Math 121, seven of the thirteen competencies tested were found to be significantly related to success. Sex, age, and method of instruction were not found to be significantly related to success in any of the courses considered. Confidence intervals were set for the true proportion of successful students demonstrating competence in each area that was significantly related to success. The exact proportion of students who succeeded in each course was calculated and confidence intervals were set for the true proportion of students taking these courses who had not already passed an equivalent course.

Conclusions

The researcher has reached the following conclusions based on an analysis of the data collected in this study.

1. Success in each of the beginning mathematics courses considered depends on the student's competence in certain areas prior to taking the course. For Math 100, an intermediate algebra course, arithmetic, graphing of points and lines, and solving systems of equations were all significantly related to success. The researcher feels that the last two of these areas were significant because of the specific content and emphasis applied to various topics in Math 100 at Montana State University. Several other algebra areas came close to being significant at the $\alpha = .01$ level and under slightly different circumstances might also be significantly related to success. The area of arithmetic, however, is different for two reasons. First, arithmetic skills are not covered to any great extent in intermediate algebra

courses in general. Therefore the significance of this area in relation to student success is likely to be found in most intermediate algebra courses. Secondly, the Chi Square value for this area was 26.54, the largest value found in the entire study. This also suggests that the role played by arithmetic is more than a result of the particular course considered at Montana State University. Polynomials and factoring was the only area significantly related to success in either Math 115, a trigonometry course, or Math 117, a survey calculus course. This similarity is most likely not a coincidence. Both of these courses are introductory courses in areas of mathematics that are very different from the algebra covered in previous courses. The researcher feels that the low number of competencies that were found to be important to success results from the fact that in both courses elementary consideration of new ideas prevents in depth consideration that might call for more algebra background. Seven of the thirteen competencies tested in Math 121, a standard first quarter calculus course, were found to be significantly related to success. Six of these seven competencies were standard algebra topics and only one was a trigonometry topic. The researcher concludes that mastery of algebra is critical to success in first quarter calculus and further, that very little trigonometry is actually important for success in first quarter calculus. Closer examination of the data reveals that the one trigonometry topic that was important was knowledge of basic trigonometry identities, a topic covered in an appendix of the calculus text used at Montana State University (Riddle, 1974).

2. The success of a student is independent of the student's sex in beginning mathematics courses at Montana State University. No evidence was found to support the idea that one sex is more successful than the other in these courses.

3. The success of a student is independent of the age group to which the student belongs. No evidence was found to suggest that older students are more successful or less successful than younger students.

4. The success of a student is independent of the method of instruction used when the choices are a standard lecture approach or a PSI (Personalized System of Instruction) approach. This is not a claim that the two methods are equivalent but only a claim that they are unrelated to a student's chances for success.

5. The remedial mathematics courses at Montana State University do not effectively develop the competencies needed for success in subsequent courses. If all successful students in a given course do not demonstrate competence in all areas covered by that course that are important for success in subsequent courses, the course cannot be labeled as effective. A number of possible reasons exist for the ineffectiveness of these courses. The first possibility is the lack of time. The remedial courses at Montana State University attempt to cover at least a full year's worth of high school or grade school material in

one quarter. A university cannot spend unlimited time and resources on teaching material that students should have learned before they came to college. Another possibility is that the course content is not appropriate and needed areas are simply not covered in remedial courses. The area of conic sections and their graphs is an example of an area found to be important for success in Math 121 but not covered in any prerequisite course at Montana State University. Again, limited time may be a factor in problems of this type. A third possible reason for the ineffectiveness of these remedial courses is that students cannot be forced to take all of the remedial courses that they need. Because prerequisite skills have been shown to be important for success in these remedial courses, it would not be surprising to find that they were also important for developing competence in various specific areas within courses. Finally these courses may be poorly taught. Remedial courses at Montana State University have been taught primarily by graduate teaching assistants and not by senior staff members. The researcher feels that, based on the fact that many graduate teaching assistants are trained, experienced teachers, this is most likely not the problem. The true reason for the ineffectiveness of these courses is likely a combination of several of the possibilities presented and no final conclusion can be made based on the results of this study. The fact remains that these courses are not effective in developing needed competencies.

6. The proportion of students who are successful in introductory and intermediate algebra courses is extremely low. While approximately 70 percent or more of the students in Math 115, Math 117, and Math 121 were successful, only approximately 61 percent of the Math 100 students were successful and only approximately 22 percent of the Math 001 students were successful. Many of the unsuccessful students in these courses may eventually master the material and be successful, but the cost to the student in terms of time, disruption of program, and actual dollars is tremendous.

7. A large proportion of the students in beginning mathematics courses have already passed equivalent courses. Almost 60 percent of the Math 100 students were repeating an intermediate algebra course, while approximately 50 percent of the Math 115 students were repeating a trigonometry course. Only approximately 30 percent or less of the calculus students were repeating similar courses. While none of the introductory algebra students admitted to never having taken the course previously, the researcher feels that this is not the case. The questionnaire (Appendix A) did not allow students the opportunity to check courses taken prior to introductory algebra and therefore may have forced some to check introductory algebra who had not taken it. The proportion that were repeating this course would most likely still have been very high.

Recommendations for Further Study

The researcher recommends the following additional research:

1. A study should be conducted to investigate the possibility that a sizable proportion of beginning mathematics students at the college level have never taken an algebra course. This study should include an investigation of the success rates for these students in their first college mathematics course.

2. Each course considered in this study should be re-examined in greater detail individually to determine if there are other necessary entrance level competencies not identified by this study.

3. A study should be conducted to examine the effectiveness of remedial courses for students who have previously taken an equivalent course. It seems possible that college remedial courses might be very effective as a review for students who have been out of school for several years even though they are not very effective for the average student.

4. This study should be replicated to establish the universiality of the findings.

Recommendations for Action

Based on the findings of this study, the researcher recommends that the following actions be taken:

1. Change the present set of prerequisites for Math 121 at

Montana State University to include a stronger emphasis on algebra and less of an emphasis on trigonometry. Specifically, conic sections and their graphs need to be included in the pre-calculus algebra requirements. This recommendation might be implemented by changing the required trigonometry courses (Math 115) to a required college algebra and trigonometry course. This would allow the topics of complex numbers and basic trigonometric identities now included in Math 115 to be kept as part of the prerequisite material. This recommendation agrees with the joint recommendation of the Mathematical Association of America and the National Council of Teachers of Mathematics discussed in Chapter 2 (1978).

2. Encourage all high school students to take two years of algebra. Most high school students don't know for sure that their future plans do not include college. Because the college remedial algebra courses are ineffective for the general student, it is important for students who may go on to take as much algebra as possible in high school.

3. Encourage all prospective elementary mathematics teachers to take algebra because of the critical role that arithmetic plays in algebra. Knowledge of how their students will use what they are teaching may help them to prepare their students now.

4. Insist that students at all levels who take mathematics do, in fact, master the material before they pass the course. One possible

way to implement this recommendation is through the use of Mastery Learning (Block, 1977). The Department of Mathematics at Montana State University has gone to a Mastery Learning approach for all remedial algebra courses. Preliminary evidence suggests that this approach does produce students with a better background than previous approaches did.

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APPENDIX A

COVER LETTER

Dear Students:

The test that you are going to take is part of a study that is being conducted in order to improve the mathematics education being offered to beginning mathematics students at Montana State University. Your performance on this test will be confidential and will not affect your course grade. Please do your best on the test so that appropriate changes in the mathematics program can be recommended. Your cooperation is essential if the quality of educational opportunities at Montana State University is to continue to improve.

Thank you for your participation in this study.

John Whitesitt

PERSONAL DATA

Name: ______________________________

Social Security Number: ____________________

Sex (check one):

_______ Male

_______ Female

Age Group (check one):

_______ Under 25

_______ 25 or Older

Previous Courses (check those courses that you have taken previously):

_______ Math 001 (or equivalent)

_______ Math 100 (or equivalent)

_______ Math 115 (or equivalent)

_______ Math 117 (or equivalent)

_______ Math 121 (or equivalent)

_______ Algebra I (high school)

_______ Algebra II (high school)

_______ Trigonometry (high school)

_______ Calculus (high school)

APPENDIX B

Test I

Please select the best response to each item and mark the corresponding position on the answer sheet provided.

1. $7 - (-3(2 - 5) + 3) =$

   (a) 1 (b) -5 (c) 21 (d) 13 (e) -1

2. If $x = 2$ and $y = -3$ then $x - y =$

   (a) 5 (b) -6 (c) -5 (d) -1 (e) 1

3. If $a = 5$ and $b = -7$ then $|a + b| =$

   (a) -12 (b) -2 (c) 2 (d) 12 (e) 35

4. If $a = -12$, $b = 9$, and $c = -3$ then $\frac{c}{a + b} =$

   (a) -1 (b) 1/12 (c) -1/12 (d) 1 (e) None of these

5. If $2x + 4 = 14$ then $x =$

   (a) 4 (b) 8 (c) 9 (d) 5 (e) 16

6. If $\frac{4}{x} = \frac{5}{3}$ then $x =$

   (a) 5/12 (b) 20/3 (c) 12/5 (d) 3/20 (e) 7

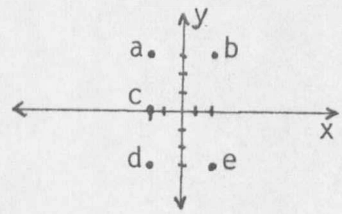
7. If $x - 3(x - 2) = 4(x + 1) - 8$ then $x =$

   (a) 5/3 (b) 2 (c) -5/3 (d) 2/3 (e) -1/3

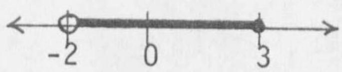
8. The solution set of $\frac{5}{x-5} - 3 = \frac{x}{x-5}$ is

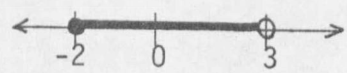
(a) { -5 } (b) { 0 } (c) { 5 } (d) { 8 } (e) The empty set

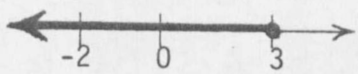
9. Which point in the picture is the graph of the point (2, -3)?

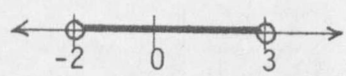


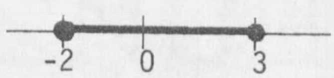
10. Which of the following represents the graph of the two conditions $x \leq 3$ and $x > -2$?

(a)


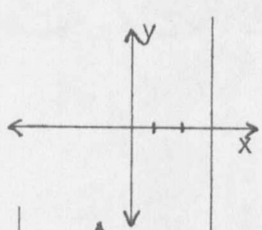
(b)


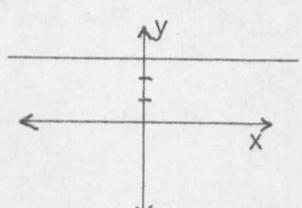
(c)


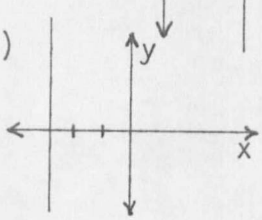
(d)


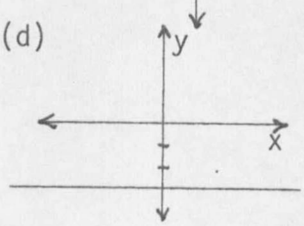
(e)


11. Which of the following represents the graph of $y = -3$?

(a)


(b)


(c)


(d)


(e) None of these

12. The slope of the line whose equation is $y = 2x - 3$ is

(a) -2 (b) -3 (c) 2 (d) 3 (e) -2/3

13. The solution set for $-2x < 6$ consists of all x such that

(a) $x < 3$ (b) $x < -3$ (c) $x \leq -3$ (d) $x > 3$ (e) $x > -3$

14. The graph of $2x + 3 \leq 5$ contains one point that the graph of $2x + 3 < 5$ does not contain. That point is x =

(a) 5 (b) -4 (c) -1 (d) 1 (e) 4

15. The graph of the solution set for $3(x - 2) \leq 2x + 1$ is

(a) a point (b) a ray (c) two points (d) a ray without its endpoint (e) a segment

16. When simplified, $\frac{3a + 6a^2}{3a} =$

(a) $2a$ (b) $1 + 2a$ (c) $6a^2$ (d) $1 + 2a^2$ (e) $1 + 6a^2$

17. $\frac{2}{x} - \frac{1}{y} =$

(a) $\frac{1}{xy}$ (b) $\frac{1}{x - y}$ (c) $\frac{1}{x + y}$ (d) $\frac{2x - y}{xy}$ (e) $\frac{2y - x}{xy}$

18. $\frac{x - 2}{2} \cdot \frac{6}{x - 2} =$

(a) $(x - 1)(\frac{6}{x - 2})$ (b) $3(x - 1)^2$ (c) 3 (d) -3 (e) 12

19. Which of the following is a factor of $rx - rsy$?

(a) s (b) r (c) x (d) y (e) rx

20. Which of the following is <u>not</u> a factor of $2x^2 + 6x + 4$?

(a) $6x$ (b) 2 (c) $(x + 1)$ (d) $(x + 2)$ (e) $x^2 + 3x + 2$

21. Which of the following is (are) factorable ?

(a) $4x^2 - 9y^2z^4$ (b) $4x^2 + y^4$ (c) $4x^2 + 2y^3$

(d) both a and c are factorable (e) both a and b are factorable

22. $2^0 2^3 =$

(a) 0 (b) 1 (c) 6 (d) 8 (e) 16

23. $3^4/3^5 =$

(a) 1/3 (b) 3 (c) -3 (d) -1/3 (e) -1

24. $\sqrt{(5^2)^3} =$

(a) 125 (b) 25 (c) $25\sqrt{5}$ (d) $5\sqrt{5}$ (e) None of these

25. $2/\sqrt{6} =$

(a) $\sqrt{6}/6$ (b) $\sqrt{6}/3$ (c) $1/\sqrt{3}$ (d) $\sqrt{6}/2$ (e) $\sqrt{3}$

26. The solution set for the system $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$ is

(a) { (2, 1) } (b) { (2, 2) } (c) { (1, 2) } (d) { 1 }

(e) The empty set

27. The solution set for the system $\begin{cases} y = 2x + 3 \\ y = 2x + 4 \end{cases}$ is

(a) { (1, 5) } (b) { (2, 1) } (c) { (1, 6) }

(d) Infinite (e) The empty set

28. The solution set for the system $\begin{cases} 2x - y = 1 \\ 4x - 2y = 2 \end{cases}$ is

(a) { (1, 1) } (b) { (2, 3) } (c) { (-1, -3) }

(d) Infinite (e) The empty set

29. The solution set for $x^2 - 4 = 0$ is

(a) { 2 } (b) { -2 } (c) { 4, -4 } (d) { 2, -2 }

(e) The empty set

30. The solution set for $x^2 = 2x$ is

(a) { 0 } (b) { 0, 2 } (c) { 2 } (d) { $\sqrt{2x}$ }

(e) None of these

Test II

Please select the best response to each item and mark the corresponding position on the answer sheet provided.

1. If $x - 2(x + 3) = 2x + 3$ then $x =$

   (a) -3 (b) 3 (c) 1 (d) -1 (e) 9

2. If $\frac{x - 2}{x} = \frac{3}{2}$ then $x =$

   (a) 0 (b) 1 (c) 4 (d) -4 (e) -1

3. The graph of $2x + 3y = 5$ crosses the x-axis at $x =$

   (a) 2 (b) 3 (c) 5/2 (d) 5/3 (e) 5

4. If line l is parallel to the y-axis and the point (2, 3) is on l, which of the following points is also on l ?

   (a) (-2, 5) (b) (3, 3) (c) (3, -3) (d) (2, 5) (e) (0, 0)

5. The solution set for $2x - 3 \leq 3x + 2$ contains all x such that

   (a) $x > -5$ (b) $x \geq -5$ (c) $x < 5$ (d) $x \leq -5$ (e) $x \geq 5$

6. The solution set for $|x - 2| \leq 3$ contains all x such that

   (a) $x \leq -1$ and $x \geq 5$ (b) $-1 \leq x \leq 5$ (c) $x \leq -5$ and $x \geq 5$
   (d) $-3 \leq x \leq 3$ (e) $-3 \leq x \leq 5$

7. $\frac{2x}{x^2 - 1} - \frac{1}{x - 1} =$

(a) $\frac{1}{x - 1}$ (b) $\frac{1}{x + 1}$ (c) $\frac{2x - 1}{x^2 - 1}$ (d) $x - 1$ (e) $x^2 - 2x + 1$

8. $\frac{x - 3}{x^2 - x - 2}$ is undefined for $x =$

(a) 3 (b) 2 (c) -1 (d) b and c (e) a, b, and c

9. When simplified $\frac{2a + 6a^2}{2a} =$

(a) $1 + 6a^2$ (b) $5a$ (c) $1 + 3a^2$ (d) $1 + 3a$ (e) $6a^2$

10. Which of the following is a factor of $xy^2 + xyz$?

(a) xyz (b) xy^2 (c) xy (d) Both a and b are factors

(e) None of these are factors

11. Which of the following is not a factor of $3x^2 - 15x + 18$?

(a) 3 (b) 18 (c) $(x - 2)$ (d) $(x - 3)$ (e) 1

12. $x^5 - 2x^2 + 1$ is a polynomial of degree

(a) 8 (b) 7 (c) 5 (d) 3 (e) 2

13. The correct value for $\sqrt{(x - 5)^2}$ is

(a) $x - 5$ (b) $\pm(x - 5)$ (c) $|x - 5|$ (d) $5 - x$ (e) None of these

14. $\left(\frac{x^3}{2y^{-1}}\right)^{-2} =$

(a) $4xy^3$ (b) $\frac{xy^3}{4}$ (c) $\frac{1}{4x^6y^2}$ (d) $\frac{x^6}{4y^2}$ (e) $\frac{4}{x^6y^2}$

15. $2^0 2^3 =$

(a) 0 (b) 1 (c) 6 (d) 8 (e) 16

16. The system of equations $\begin{cases} x + 2y = 5 \\ 2x + 4y = 10 \end{cases}$

(a) has many solutions (b) has no solutions (c) has exactly one solution (d) has exactly two solutions (e) None of these

<u>Note</u>: A solution to a system of equations is a pair (x, y) of numbers.

17. The system of equations $\begin{cases} y = x \\ x^2 + y^2 = 4 \end{cases}$

(a) has many solutions (b) has no solutions (c) has exactly one solution (d) has exactly two solutions (e) None of these

18. The solution set for the system $\begin{cases} 2x - y = -1 \\ x + 3y = 10 \end{cases}$ is

(a) { (2, 5) } (b) { (4, 2) } (c) { (-1, 10) }

(d) { (1, 3) } (e) None of these

19. The solution set for $x^2 = 5x$ is

(a) { 5 } (b) { 0 } (c) { 5, 0 } (d) { $\sqrt{5x}$ }

(e) None of these

20. The solution set for $2x^2 + 6x + 4 = 0$ is

(a) { 2, 1 } (b) { 4, 2 } (c) { -4, -2 }

(d) { 0, -2, -1 } (e) None of these

21. For $0 = ax^2 + bx + c$, if $b^2 - 4ac = 0$ then the equation has

(a) many solutions (b) no solutions (c) exactly one solution

(d) exactly two solutions (e) no real solutions

22. The solution set for $x^2 + 3x + 1 = 0$ is

(a) { $(1/2)(3 + \sqrt{5})$, $(1/2)(3 - \sqrt{5})$ }

(b) { $(1/2)(-3 + \sqrt{13})$, $(1/2)(-3 - \sqrt{13})$ }

(c) { $(1/2)(-3 + \sqrt{5})$, $(1/2)(-3 - \sqrt{5})$ }

(d) { $(1/2)(3 + \sqrt{13})$, $(1/2)(3 - \sqrt{13})$ }

(e) None of these

23. $\log_2 32 =$

(a) 5 (b) 16 (c) 64 (d) $4\sqrt{2}$ (e) Must have a table to answer

24. $y = \log_3 x$ is equivalent to

(a) $y = x^3$ (b) $x = y^3$ (c) $x/y = 3$ (d) $x = 3^y$ (e) $x = 3y$

25. $f(x) = a^x$ is an increasing function of x for

(a) all values of a (b) no values of a (c) all $a > 0$

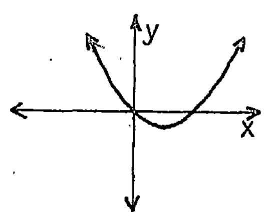
(d) all $a > 1$ (e) all a such that $0 < a < 1$

26. $\log_2(-3)$ is

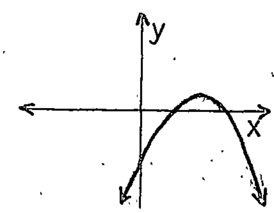
(a) $-\log_2(3)$ (b) $\log_2(3)$ (c) $1/\log_2(3)$ (d) 0 (e) Undefined

27. For $y = ax^2 + bx + c$, you are told that $b^2 - 4ac > 0$, $a < 0$, and $c = 0$. Which of the following is a possible graph of the equation?

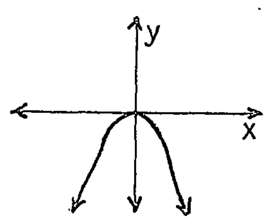
(a)



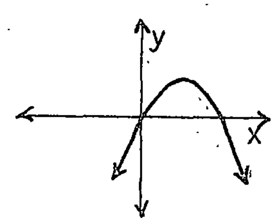
(b)



(c)



(d)



(e) None of these

28. The equation for a circle with center at (1, 2) and radius 2 is

(a) $(x + 1)^2 + (y + 2)^2 = 4$ (b) $(x - 1)^2 - (y - 2)^2 = 4$

(c) $(x - 1)^2 + (y - 2)^2 = 4$ (d) $(x + 1)^2 - (y + 2)^2 = 4$

(e) $x^2 + y^2 = 4$

29. $x^2 + 2x - y^2 = 0$ has a graph that is

(a) a circle (b) a parabola (c) an ellipse (d) a hyperbola

(e) a line

30. How many times does the graph of $y = 4x^2 + 12x + 9$ intersect the x-axis?

(a) 3 (b) 2 (c) 1 (d) 0 (e) More than 3

Test III

Please select the best response to each item and mark the corresponding position on the answer sheet provided.

1. If $x - 3(x - 2) = 4(x + 1) - 8$ then $x =$

   (a) 5/3 (b) 2 (c) -5/3 (d) 2/3 (e) 5/6

2. The solution set for $\frac{5}{x - 5} - 3 = \frac{x}{x - 5}$ is

   (a) { -5 } (b) { 0 } (c) { 5 } (d) { 8 } (e) The empty set

3. The slope of the line whose equation is $2x + 3y = 5$ is

   (a) 2/5 (b) -2/3 (c) -3/2 (d) 5/3 (e) -3/5

4. The graph of $2x + 3y = 5$ crosses the x-axis at $x =$

   (a) 2 (b) 3 (c) 3/2 (d) 5/3 (e) 5/2

5. The solution set for $-3x \leq 9$ contains all x such that

   (a) $x \geq -3$ (b) $x \leq -3$ (c) $x \leq 3$ (d) $x > -3$ (e) $x < -3$

6. The solution set for $|x + 1| \leq 5$ contains all x such that

   (a) $x \leq -6$ and $x \geq 4$ (b) $-6 \leq x \leq 6$ (c) $-4 \leq x \leq 4$

   (d) $-6 \leq x \leq 4$ (e) $x \leq 6$

7. When factored, $4x^2 + 16x - 9 =$

   (a) $(2x - 3)(2x + 3)$ (b) $(2x + 3)(2x + 3)$ (c) $(2x - 3)(2x - 3)$

   (d) $(2x - 9)(2x + 1)$ (e) $(2x + 9)(2x - 1)$

8. Which of the following is a factor of rs - rxy ?

(a) r (b) s (c) x (d) y (e) rs

9. For $0 = ax^2 + bx + c$, you are told that $b^2 - 4ac < 0$. You know, therefore, that the equation has

(a) many solutions (b) no solutions (c) no real solutions

(d) exactly one real solution and one imaginary solution

(e) exactly two real solutions

10. If asked to complete the square for $x^2 - 3x = 7$, what would you add to both sides of the equation?

(a) -3/2 (b) 3/2 (c) -9/4 (d) 9/4 (e) None of these

11. $\log_3 81$ is

(a) 27 (b) 9 (c) 4 (d) 3 (e) Undefined

12. $y = \log_2 x$ is equivalent to

(a) $y = x^2$ (b) $x = y^2$ (c) $x/y = 2$ (d) $x = 2^y$ (e) $x = 2y$

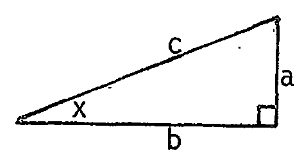
13. The graph of $y = 2x^2 + 3x + 1$ intersects the x-axis how many times?

(a) 0 (b) 1 (c) 2 (d) 3 (e) More than 3

14. For $y = ax^2 + bx + c$, $c = 0$ implies that

(a) the graph does not intersect the x-axis (b) the graph intersects the x-axis in only one point (c) the graph passes through the point (0, 0) (d) the graph is concave downward (e) None of these

15. In the triangle at the right,



sin x =

(a) a/b (b) a/c (c) b/c (d) c/a (e) c/b

16. For the triangle in number 15, sec x =

(a) a/b (b) a/c (c) b/c (d) c/a (e) c/b

17. If csc x = 5/3 and sec x = 5/4, then tan x =

(a) 5/4 (b) 5/3 (c) 3/4 (d) 4/3 (e) None of these

18. $\sin^2 x + \cos^2 x = 1$ for

(a) all values of x (b) all $x > 0$ (c) all x such that $0 < x < \pi/2$ (d) no values of x (e) one unknown value of x

19. tan x =

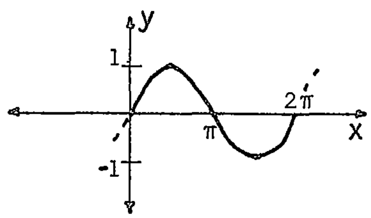
(a) cos x/sin x (b) sin x/cos x (c) 1/sin x (d) 1/cos x (e) $1/\sin x \cdot \cos x$

20. sin 2x =

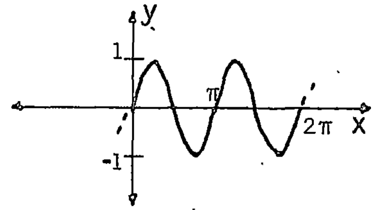
(a) $2\sin x \cdot \cos x$ (b) $2\sin x$ (c) $\sin^2 x$ (d) $\sin x + \cos x$
(e) $\sin^2 x + \cos^2 x$

21. Which drawing represents the graph of $y = \sin 2x$?

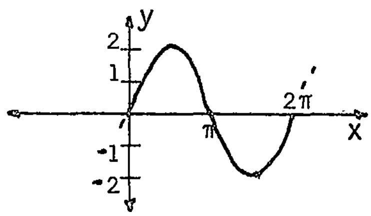
(a)



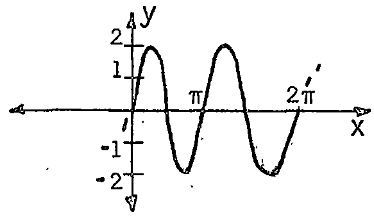
(b)



(c)



(d)



(e) None of these

22. If $y = \sin x$, then which of the following can be said about y?
(a) $y \leq 1$ for all values of x (b) $y \geq -1$ for all values of x
(c) $y = 0$ for infinitely many values of x (d) Only a and b are true (e) All of a, b, and c are true

23. For which of the following is y a function of x?
(a) $y = \sin x$ (b) $y = \tan x$ (c) $y = \sec x$ (d) All are functions (e) Only a and b are functions, c is not

24. $y = \sin^{-1}x$ is equivalent to

(a) $y = 1/\sin x$ (b) $1/y = \sin x$ (c) $y = -\sin x$

(d) $x = \sin y$ (e) $x = 1/\sin y$

25. $\sin^{-1}(1) =$

(a) π (b) 2π (c) $\pi/2$ (d) 0 (e) 1

26. $i^{206} =$

(a) 1 (b) -1 (c) i (d) -i (e) None of these

27. $(\sqrt{-3})(\sqrt{-12}) =$

(a) 6i (b) -6i (c) -6 (d) 6 (e) ±6

28. $(1 + 3i)(2 - 4i) =$

(a) 14 + 2i (b) -10 + 2i (c) 2 - 12i (d) 14 + 10i

(e) None of these

29. If $\vec{v}$ is a geometric vector with initial point (0, 0) and terminal point (-1, -1), then $||\vec{v}||$ (the magnitude of $\vec{v}$) is

(a) 1 (b) -1 (c) $\sqrt{2}$ (d) $-\sqrt{2}$ (e) None of these

30. For the vector $\vec{v}$ in number 29, $||\vec{v}_x||$ (the magnitude of the horizontal component of $\vec{v}$) is

(a) 1 (b) -1 (c) $\sqrt{2}$ (d) $-\sqrt{2}$ (e) None of these

APPENDIX C

Computer Program Used for Null Hypothesis One

```
      DIMENSION IA(13), IB(13), IC(13), ID(13), NC(13), CS(13),
     1N(13), TCS(13), DCS(13)
      DO 3 I=1,13,1
      IA(I)=0
      IB(I)=0
      IC(I)=0
      ID(I)=0
 3    CONTINUE
 4    READ(1,200,END=40)(NC(I),I=1,13),NS
      IF(NS.EQ.0)GO TO 20
      DO 10 I=1,13,1
      IF(NC.EQ.0)GO TO 5
      IA(I)=IA(I)+1
      GO TO 10
 5    IC(I)=IC(I)+1
10    CONTINUE
      GO TO 4
20    DO 30 I=1,13,1
      IF(NC(I).EQ.0)GO TO 25
      IB(I)=IB(I)+1
      GO TO 30
25    ID(I)=ID(I)+1
30    CONTINUE
      GO TO 4
40    DO 50 I=1,13,1
      N(I)=IA(I)+IB(I)+IC(I)+ID(I)
      TCS(I)=N(I)*((ABS(IA(I)*ID(I)-IB(I)*IC(I))-(N(I)/2))**2)
      DCS(I)=(IA(I)+IB(I))*(IC(I)+ID(I))*(IA(I)+IC(I))*(IB(I)+ID(I))
      IF(DCS(I).EQ.0)CS(I)=999.;GO TO 50
      CS(I)=TCS(I)/DCS(I)
50    CONTINUE
      WRITE(2,201)
      DO 60 I=1,13,1
      WRITE(2,202)
      WRITE(2,203)
      WRITE(2,204)IA(I),IB(I)
      WRITE(2,205)
      WRITE(2,206)I
      WRITE(2,207)IC(I),ID(I)
60    WRITE(2,210)CS(I)
```

Computer Program Used for Null Hypothesis One (continued)

```
200     FORMAT(10X,14I1)
201     FORMAT('MATH 121')
202     FORMAT(22X,'SUCCESS')
203     FORMAT(21X,'YES",4X,'NO')
204     FORMAT(11X,'COMPETENT',1X,I3,3X,I3)
205     FORMAT('COMPETENCY')
206     FORMAT(1X,'NUMBER',I3)
207     FORMAT(11X,'NOT COMP',2X,I3,3X,I3,//)
210     FORMAT('CHI SQUARE = ',F8.4,////)
        STOP
        END
```

Note: This program was used to do the analysis of null hypothesis one for each of the courses considered. The course number in statement 201 was changed to reflect the course that was being analyzed.

Computer Program Used for
Null Hypotheses Two
Three and Four

```
      IA=0;IB=0;IC=0;ID=0
  4   READ(1,200,END=40)NC,NS
      IF(NS.EQ.0)GO TO 20
      IF(NC.EQ.0)GO TO 5
      IA=IA+1
      GO TO 4
  5   IC=IC+1
      GO TO 4
 20   IF(NC.EQ.0)GO TO 25
      IB=IB+1
      GO TO 4
 40   N=IA+IB+IC+ID
      TCS=N*((ABS(IA*ID-IB*IC)-N/2)**2)
      DCS=(IA+IB)*(IC+ID)*(IA+IC)*(IB+ID)
      IF(DCS.EQ.0)CS=999.;GO TO 50
      CS=TCS/DCS
 50   CONTINUE
      WRITE(2,201)
      WRITE(2,202)
      WRITE(2,203)
      WRITE(2,204)IA,IB
      WRITE(2,205)
      WRITE(2,206)
      WRITE(2,207)IC,ID
      WRITE(2,210)CS
200   FORMAT(7X,I1,T24,I1)
201   FORMAT('MATH 121')
202   FORMAT(22X,'SUCCESS')
203   FORMAT(21X,'YES',4X,'NO')
204   FORMAT(16X,'MALE',I4,2X,I4)
205   FORMAT('SEX OF')
206   FORMAT('INDIVIDUAL')
207   FORMAT(14X,'FEMALE',I4,2X,I4,//)
210   FORMAT('CHI SQUARE = ',F8.4////)
      STOP
      END
```

Computer Program Used for Null Hypotheses Two Three and Four (continued)

Notes: (1) This program was used to do the analysis in each course and, therefore, the course number that appears in statement 201 was changed to reflect the course that was being analyzed. (2) This program was used to do the analysis for null hypotheses two, three, and four. The FORMAT statements vary for each of these hypotheses. The FORMAT statements that appear on the previous page are for hypothesis number two. Those used for the other null hypotheses appear below.

FORMAT Statements Used for Null Hypothesis Three:

```
 200   FORMAT(8X,I1,T24,I1)
 201   FORMAT('MATH 121')
 202   FORMAT(22X,'SUCCESS')
 203   FORMAT(21X,'YES',4X,'NO')
 204   FORMAT(17X,'OLD',I4,2X,I4)
 205   FORMAT('AGE OF')
 206   FORMAT('INDIVIDUAL')
 207   FORMAT(15X,'YOUNG',I4,2X,I4,//)
 210   FORMAT('CHI SQUARE = ',F8.4,////)
```

FORMAT Statements Used for Null Hypothesis Four:

```
 200   FORMAT(9X,I1,T24,I1)
 201   FORMAT('MATH 100')
 202   FORMAT(22X,'SUCCESS')
 203   FORMAT(21X,'YES',4X,'NO')
 204   FORMAT(17X,'PSI',I4,2X,I4)
 205   FORMAT('METHOD OF')
 206   FORMAT('INSTRUCTION')
 207   FORMAT(13X,'LECTURE',I4,2X,I4,//)
 210   FORMAT('CHI SQUARE = ',F8.4,////)
```

Computer Program Used for
General Question One

```
      DIMENSION INC(13), IC(13), P(13), R(13), F2(13), Q(13), UL(13),
     1BL(13), C(13)
      DO 5 I=1,13,1
    5 INC(I)=0
      ICT=0
   10 READ(1,200,END=100)(IC(I),I=1,13)
      DO 20 I=1,13,1
      IF(IC(I).EQ.1) GO TO 20
      INC(I)=INC(I)+1
   20 CONTINUE
      ICT=ICT+1
      GO TO 10
  100 F1=.4758
      T1=1./(2.*ICT)
      DO 110 I=1,13,1
      CT=ICT
      C(I)=INC(I)
      P(I)=(CT-C(I))/CT
      R(I)=(P(I)*(1-P(I)))/ICT
      IF(R(I).EQ.0)F2(I)=0.;GO TO 105
      F2(I)=SQRT(R(I))
  105 Q(I)=(1.96)*(F1)*(F2(I))+T1
      UL(I)=P(I)+Q(I)
      BL(I)=P(I)-Q(I)
  110 CONTINUE
      DO 120 I=1,13,1
      WRITE(2,201)I,P(I)
      WRITE(2,202)BL(I),UL(I)
  120 CONTINUE
  200 FORMAT(10X,13I1)
  201 FORMAT('COMPETENCY NO ',I2,5X,'PROP COMP = ',F5.4,/)
  202 FORMAT(20X,'95 PERCENT C I (',F5.4,' - ',F5.4,')',//)
      STOP
      END
```

Note: The constant, F1, was different for each course that was analyzed. The one shown in the program (statement 100) is for Math 001. For Math 100, F1 = .7866 and for Math 115 F1 = .7342.