



A finite difference groundwater model for the East Decker, Montana mine
by Richard H Engelmann

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
in Civil Engineering
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Abstract:

As part of a U. S. Environmental Protection Agency funded project, a groundwater model was developed for the proposed East Decker coal mine. The model is capable of simulating flow before and during mining, accounting for drainage into the mine cut and resultant aquifer drawdowns.

The literature was reviewed for previous field research on groundwater flow at existing mines, modeling attempts made at existing mines, and for available groundwater models. On the basis of the review, a two-dimensional, finite difference model based on the Prickett and Lonquist model was chosen for application at the East Decker site. Modifications of the Prickett and Lonquist model were developed to handle multiple aquifers, merging aquifers, regions of confined and unconfined flow, and aquifer drainage into the mine cut, all conditions which would exist in the aquifer-mining environment.

Using the site data available, a simulation of the premining conditions was run in order to roughly calibrate the model for aquifer recharge. This simulation was followed by a simulation of the aquifer system with the first mine cut in place. The mining simulation produced realistic drawdowns in the affected aquifers, although no data existed that could be used to verify the results.

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December 19, 1980

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FOR THE EAST DECKER, MONTANA MINE

by

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A thesis submitted in partial fulfillment
of the requirements for the degree

of

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ABSTRACT

As part of a U. S. Environmental Protection Agency funded project, a groundwater model was developed for the proposed East Decker coal mine. The model is capable of simulating flow before and during mining, accounting for drainage into the mine cut and resultant aquifer drawdowns.

The literature was reviewed for previous field research on groundwater flow at existing mines, modeling attempts made at existing mines, and for available groundwater models. On the basis of the review, a two-dimensional, finite difference model based on the Prickett and Lonquist model was chosen for application at the East Decker site. Modifications of the Prickett and Lonquist model were developed to handle multiple aquifers, merging aquifers, regions of confined and unconfined flow, and aquifer drainage into the mine cut, all conditions which would exist in the aquifer-mining environment.

Using the site data available, a simulation of the premining conditions was run in order to roughly calibrate the model for aquifer recharge. This simulation was followed by a simulation of the aquifer system with the first mine cut in place. The mining simulation produced realistic drawdowns in the affected aquifers, although no data existed that could be used to verify the results.

Chapter I

INTRODUCTION

In the process of surface mining coal in the Northern Great Plains of the United States, large amounts of overburden and interburden material are stripped and moved to reach coal seams. As a result, groundwater aquifers that may exist in coal, overburden, and interburden are truncated, so that a mine cut can act as a very large groundwater sink for these aquifers. Should confined aquifers lie below a mine cut, they may also discharge into the cut by leaking through the material above them. The aquifer discharges will produce drawdowns that can dry up springs and wells some distance from a mine.

After the removal of coal, reclamation of mined-out areas is begun by backfilling old cuts with overburden and interburden materials (spoils) stripped from new, adjacent mine cuts. Thus, aquifers destroyed by mining are rebuilt with new materials that could have hydraulic properties differing from the original aquifer material. If the properties are significantly different, the lateral groundwater flow through a completely reclaimed mine site may be quite different from the flow in the original site. This could also hold true for the vertical flow of groundwater in a reclaimed mine.

A capability to predict the effects of mining on groundwater flow is desirable for several reasons. Estimation of aquifer discharge rates into a mine cut would be useful in planning for the pumping and treatment of mine water. A prediction of resulting drawdowns would indicate

what wells and springs might be adversely affected by a mine. In addition, a prediction of the quantity and depth of flow through spoils material would aid in predicting the degree of groundwater contamination by the spoils material, and in how deeply toxic spoils (i.e. spoils containing soluble metals, etc.) could be placed.

THESIS OBJECTIVE

The intent of this thesis is to produce a model capable of simulating flow in the saturated, shallow groundwater aquifer system surrounding the East Decker coal mine in southeastern Montana. The location of the mine is shown in Figures 1 and 2. The model should be capable of simulating flow before, during, and after mining. This includes the capacity to approximate the aquifer discharges into the mine cut and to estimate the aquifer drawdowns.

SCOPE OF WORK

The System to be Modeled

The aquifer system at East Decker contains several aquifers that will be significantly affected by mining and that must be included in the model. These include the three coal bed aquifers that are the object of mining, the clinker aquifers, and the alluvial aquifers that lie along the shore of the Tongue River Reservoir and through the valleys (Van Voast and Hedges, 1975; USGS, 1977). The clinker, alluvial, and coal aquifers, with their very different and variable hydraulic

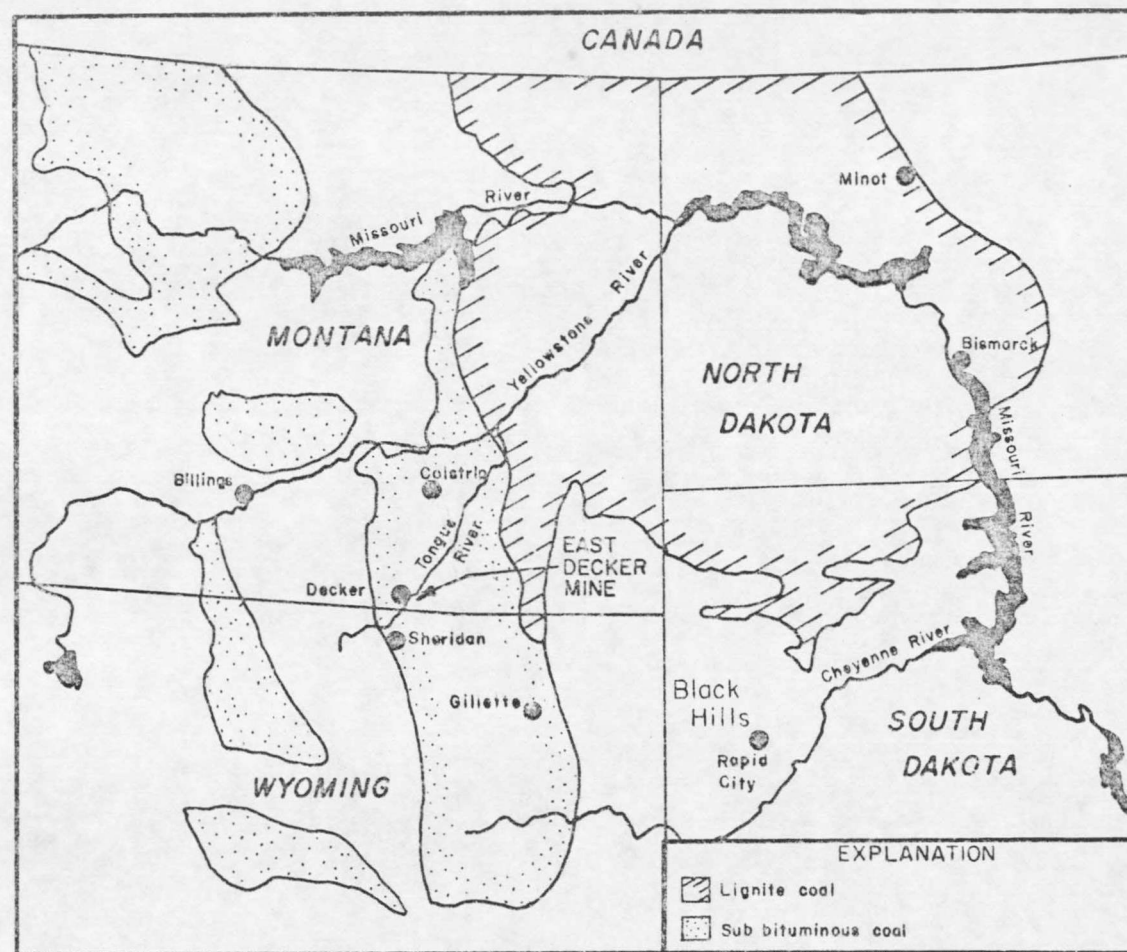


Fig. 1. Map of the Northern Great Plains coal fields showing the location of the East Decker mine in southeastern Montana (from Permack, 1973).

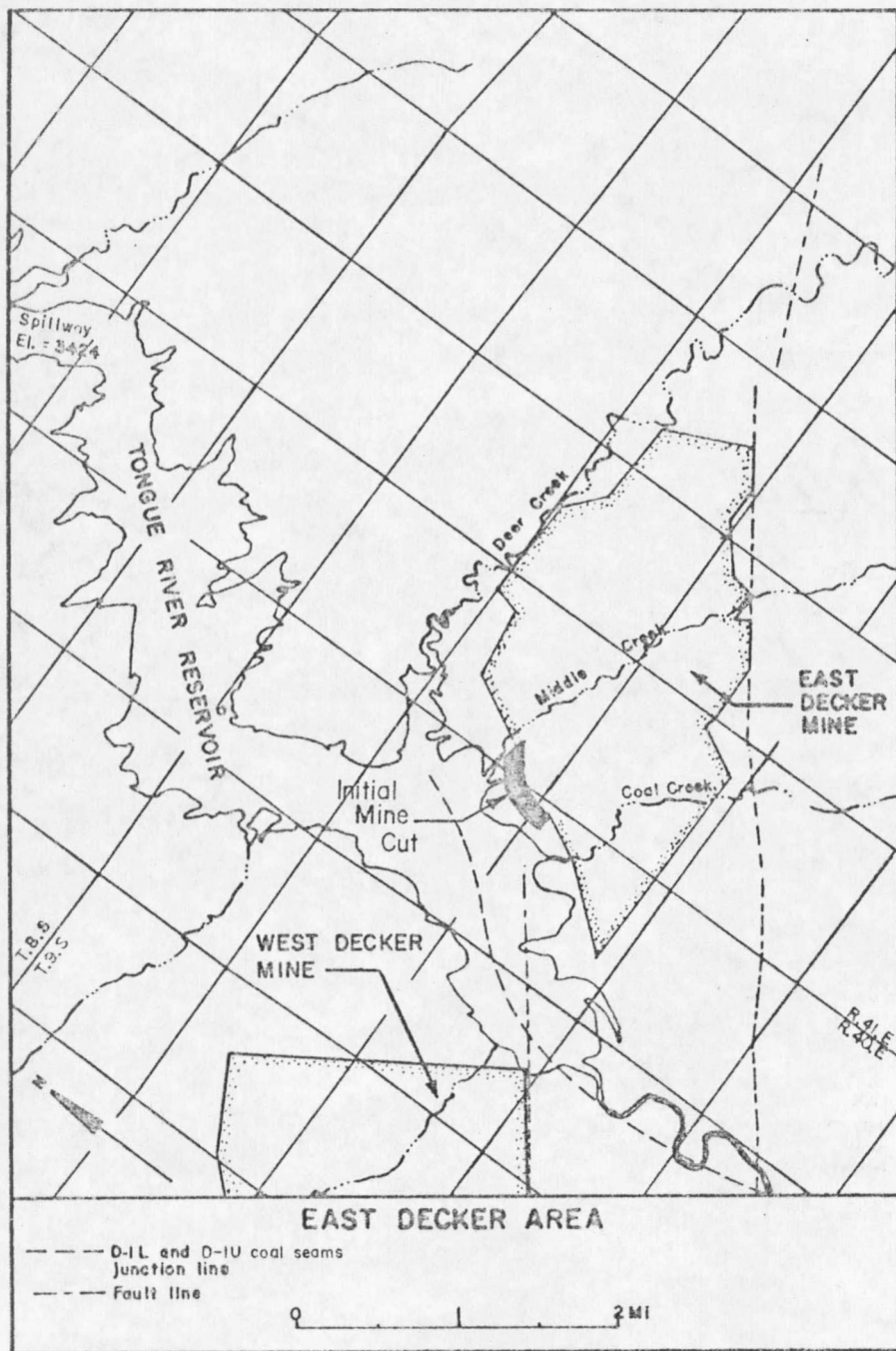


Fig. 2. Map of the East Decker area showing locations of the East and West Decker mines. (Base map from Van Voast and Hedges (1975)).

properties (Van Voast and Hedges, 1975, 1976), are linked together to form an extremely complex groundwater system.

In addition to the nonhomogenous characteristics of the aquifers, the coal hydraulic conductivities are probably anisotropic, research at the AMAX Belle Ayr and Black Thunder mines south of Gillette, Wyoming indicates that flow in coal is limited to the fractures and is thus strongly controlled by the coal fracture pattern (Hasfurther and Rechar, 1976).

A fracture study in the East Decker area found the major rock fracture direction to be in the northwest-southeast direction, with a significant secondary fracture direction that is northeast-southwest (EPA Quarterly Report, 1976). Flow in the coal may be expected to follow these fractures.

The East Decker aquifer boundaries are also quite complicated. On the southeast side of the site there is a significant fault that forms a break in the coal aquifers (a low permeability boundary), while on the northwest side the aquifers are linked, directly or indirectly, with the Tongue River Reservoir (USGS, 1977). During mining, the mine cut will form a large, complete drawdown boundary in the middle of the system. This boundary will be moving as mining progresses. Additionally, inter-aquifer leakage may be occurring between the overlying coal beds. Leakage is believed to be occurring between two coal seams on the west side

of the Tongue River Reservoir (Van Voast, 1974), an area which has a lithostratigraphy similar to the east side.

Type of Modeling

The complexity of the East Decker aquifer system dictates the use of a computer modeling technique. Electrical analog, digital, and hybrid (a digital-analog combination) computers have all been successfully used for modeling complex aquifers (Glover, 1974) and could be applied to the problem. Due to readily available equipment, it was decided to use a digital computer (numerical) technique for the East Decker site.

There are two types of digital models in use for groundwater modeling, finite difference models and finite element models (Prickett, 1975). The finite difference type uses discrete forms of the differential equations of flow and solves them with standard matrix methods. Finite element models use variation formulations of the flow equations. Because finite difference models have a more straight forward and intuitively satisfactory mathematical basis and because of their easier data input, it was decided to use this type of modeling for the East Decker aquifers.

Chapter II

LITERATURE REVIEW

Groundwater Flow at Existing Mines

In order to indicate what problems may be encountered at East Decker, the literature was surveyed for research on groundwater flow and mining impacts at existing surface mines.

Aquifer Drainage

An obvious result of mining is the drainage of aquifers into the mine cut. Van Voast (1974) and Van Voast and Hedges (1975) have studied this at the West Decker coal mine, located across the Tongue River Reservoir from the East Decker site. Observation wells were drilled to observe the drawdowns in coal seam aquifers, the D-1, which was being mined, and the D-2 (Decker Coal Company designations), a thinner coal seam that lies below. Both aquifers are confined and flow generally from west to east, discharging into the Tongue River Reservoir. It is believed that the aquifers are recharged in the hills west of the mine. In the study area the D-2 aquifer is believed to recharge the D-1 by vertical leakage (Van Voast and Hedges, 1975).

Mining began at West Decker early in the summer of 1974. During the first year of mining, discharge into the cut was estimated to be 350,000 to 400,000 gpd (Van Voast, 1974). The discharge declined only slightly in the following two years. After three years of mining, head

drops of up to 20 feet had occurred in the D-1 coal about a mile west of the mine cut. One well over two miles from the cut had a three foot drop from mid 1974 to mid 1975. The rate of decline in the wells began to decrease after two years of mining. Lesser head drops were also recorded in the D-2 coal. This was attributed to increased leakage from the D-2 to the D-1 coal caused by greater head differences between the two aquifers (Van Voast, 1974).

The work of Van Voast and Hedges at West Decker is the only field research of consequence reported in the literature on aquifer drainage. However, due to the great similarity of the East and West Decker sites, it seems safe to say that aquifer drainage at East Decker can be expected to be similar to the drainage at West Decker.

Flow Through Spoils

The hydraulic characteristics of the spoils materials have an important effect on post mining groundwater flow. The type of material spoiled is important, as well as how the spoils are placed. Area strip mining, the most common type of mining, is usually done by one of two methods. Many operations use large draglines for stripping and placing spoils. Others, due to a shortage of dragline equipment, use power shovels for excavation and haul trucks for transporting and placing spoils. In either case, spoils are leveled by bulldozers before being topsoiled (Mathematica, Inc., 1975).

In spoils, lateral hydraulic conductivities are often assumed to be similar to hydraulic conductivities in coals (Van Voast and Hedges, 1976; Davis, 1976). Van Voast and Hedges (1976) have reported pump test results from dragline spoils in the Colstrip, Montana area that support this assumption. Lateral hydraulic conductivities in spoils ranged from 0.04 to 5.7 ft/day. Coal hydraulic conductivities in the same area had values ranging from 0.07 to 3.1 ft/day.

Rahn (1975, 1976) has conducted research on the hydraulic properties of spoils materials in Powder River Basin mines. Pump tests were performed in 20-year old spoils at two mines which had lithologically similar overburdens of yellowish-brown sandstone interbedded with carbonaceous shale. The mines were the abandoned Hidden Water Creek mine, a dragline operation near Monarch, Wyoming, and the active Big Horn mine, a shovel-truck operation northwest of Sheridan, Wyoming.

Pump tests produced an oval shaped drawdown pattern in the dragline spoils. The oblong pattern was apparently caused by the distribution of spoils materials. Winczewski (1977) observed significant sorting and segregation of material in dragline spoil banks in North Dakota mines. A fairly continuous, permeable basal rubble zone was formed between the parallel spoil banks, while finer material was found inside the banks. If the material in the Hidden Water Creek mine spoils was segregated in a similar manner, an oval drawdown with the

long axis parallel to the spoil banks would be expected (i.e., drawdown would be greater along the interbank rubble, and less in the banks).

The drawdown pattern observed in the Big Horn mine spoils, where a shovel-truck operation was employed, was very irregular, suggesting an irregular lateral distribution of material types. Overall, though, the same oblong pattern as that observed in the Hidden Creek spoils was seen.

On the basis of these tests along with pump tests in coal, Rahn (1976) reported an approximate ratio of hydraulic conductivities of dragline spoils to coal to shovel-truck spoils of 2/1/0.3. The great difference in dragline and shovel-truck hydraulic conductivities was attributed to the greater material consolidation produced by the latter type of operation.

Conclusions

The following conclusions may be drawn from the mine site research reviewed:

1. The aquifer drawdowns produced by mining can extend several miles from the mine cut. At West Decker, Montana, significant drawdowns have been observed over two miles from the mine.
2. Lower aquifers not cut by mining may also experience significant drawdowns due to vertical leakage.

3. The spoils material and the type of mining operation have a strong impact on the magnitude and variability of spoils lateral hydraulic conductivity.

Finite Difference Groundwater Aquifer Models

Basic Methodology

The equation most commonly used in finite difference models for the description of saturated groundwater flow is some form of the two-dimensional equation (Prickett, 1975)

$$\frac{\partial}{\partial x} (a K_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (a K_y \frac{\partial h}{\partial y}) = S \frac{\partial h}{\partial t} + W \quad (1a)$$

or

$$\frac{\partial}{\partial x} (T_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (T_y \frac{\partial h}{\partial y}) = S \frac{\partial h}{\partial t} + W \quad (1b)$$

where

x, y = distances in the x and y directions in the horizontal plane (L)

a = saturated thickness of the aquifer (L)

K_x, K_y = aquifer hydraulic conductivities in the x and y directions (L/T)

$K_x \frac{\partial h}{\partial x}, K_y \frac{\partial h}{\partial y}$ = flow velocities in the x and y directions, respectively (L/T)

S = aquifer storage coefficient (dimensionless)

h = head (L)

t = time (T)

W = source/sink function (L/T) (pumpage, recharge, leakage, etc., a flow rate per unit surface area)

T_x, T_y = aquifer transmissivities in the x and y directions (L²/T)

Equation (1) is the equation of continuity for saturated, nonsteady flow through an aquifer in which Darcy's law for flow through porous media holds and the approximation of horizontal flow is valid. Models of three-dimensional flow often use the equation

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t} \quad (2)$$

where K_z is the hydraulic conductivity in the z direction, S_s is specific storage (L⁻¹), and other terms are as previously defined. For steady flow, $\frac{\partial h}{\partial t} = 0$ in both Equations (1) and (2). Finite difference solutions of the continuity equations are obtained through numerical methods that are briefly discussed below. The equations of flow are more extensively reviewed in the following section.

In finite difference models the groundwater system is represented by (discretized into) finite volumes, or nodes, usually laid out in a rectangular array (grid). For a horizontal flow approximation there is only one layer of nodes, each node representing an entire aquifer thickness. Three-dimensional models use a three-dimensional array. An example of a square node grid for an aquifer horizontal flow approximation is shown in plan view in Figure 3.

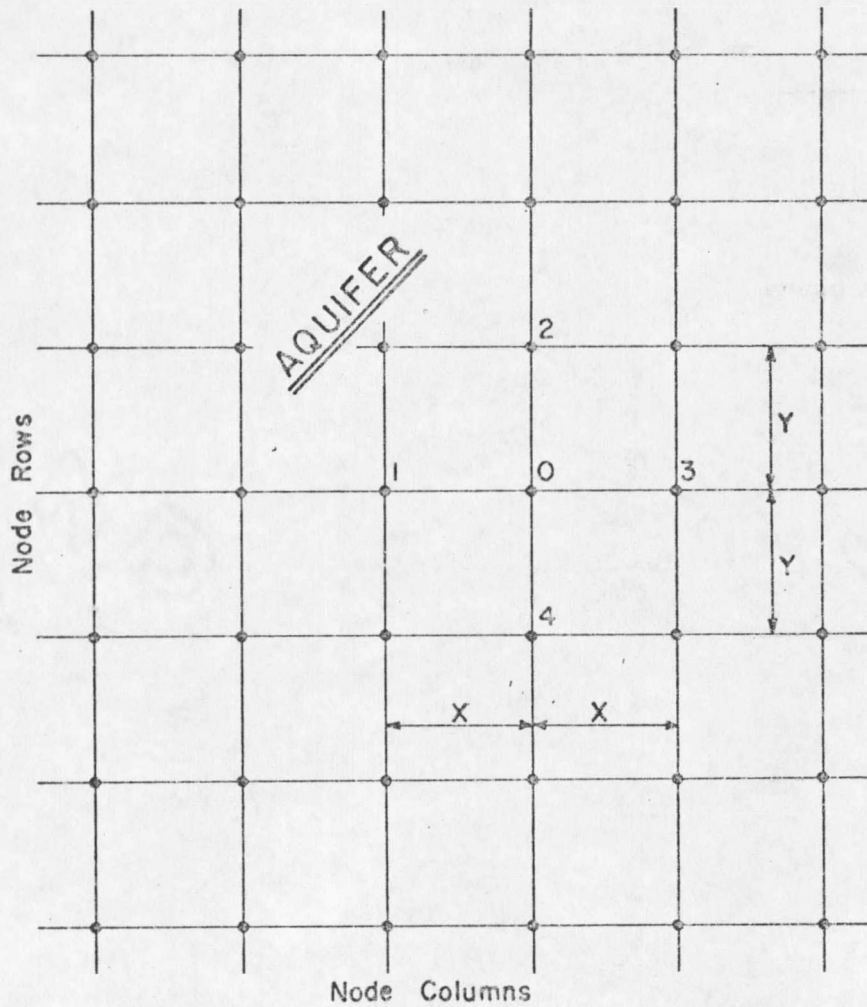


Fig. 3. Plan view of a typical node layout for a two-dimensional finite difference representation of an aquifer.

Using the discretized aquifer of Figure 3 and discretized time, Equation (1b) can be written for node 0, employing central difference approximations for the second order partial differential terms, as (Stallman, 1962)

$$\begin{aligned} & \frac{\frac{T_{x_{0-3}}(h_3 - h_0)}{X} - \frac{T_{x_{1-0}}(h_0 - h_1)}{X}}{X} + \\ & \frac{\frac{T_{y_{0-2}}(h_2 - h_0)}{Y} - \frac{T_{y_{4-0}}(h_0 - h_4)}{Y}}{Y} \\ & = S_0 \frac{h_{0_{t+\Delta t}} - h_{0_t}}{\Delta t} + W_0 \end{aligned} \quad (3)$$

a linear equation in which

$T_{x_{0-3}}, T_{x_{1-0}}$ = aquifer transmissivities between nodes 0 and 3 and nodes 1 and 0 (L^2/T)

$T_{y_{0-2}}, T_{y_{4-0}}$ = aquifer transmissivities between nodes 0 and 2 and nodes 4 and 0 (L^2/T)

h_0, h_1, h_2, h_3, h_4 = aquifer heads at nodes 0, 1, 2, 3, and 4 (L)

X, Y = grid dimensions (L)

S_0 = aquifer storage coefficient at node 0 (dimensionless)

$h_{0_{t+\Delta t}}$ = head at node 0 at time $t+\Delta t$, unknown (L)

h_{0_t} = head at node 0 at time t , known (L)

Δt = time increment (T)

W_0 = aquifer source/sink function at node 0 (L/T)

This is essentially a water balance equation for node 0, accounting for flow into and out of the node and the resultant change in storage. In an aquifer simulation using a model of N nodes, Equation (3) is applied to every node for each successive time increment. Because all node heads at the end of a time increment are unknown, this yields N equations in N unknown node heads.

For nonsteady flow problems Equation (3) is usually written, with respect to time, in either a forward difference explicit form, a backward difference implicit form, or by some combination of the two (Huntoon, 1974; Remson et al., 1971). In the forward difference explicit form, the heads at nodes adjacent to node 0 (h_1 , h_2 , h_3 , and h_4) are values at time t, known values. Thus, the approximations

$$\left(\frac{T_{x_{0-3}} \frac{h_3 - h_0}{X} - T_{x_{1-0}} \frac{h_0 - h_1}{X} \right) \bigg/ X \bigg|_t \approx T_x \frac{\partial^2 h}{\partial x^2} \quad (4)$$

and

$$\left(\frac{T_{y_{0-2}} \frac{h_2 - h_0}{Y} - T_{y_{4-0}} \frac{h_0 - h_4}{Y} \right) \bigg/ Y \bigg|_t \approx T_y \frac{\partial^2 h}{\partial y^2} \quad (5)$$

are used in Equation (3). This leaves only one unknown head in Equation (3) to be solved for, $h_{0_{t+\Delta t}}$. However, to insure solution

stability and convergence, very small time increments must be used (Prickett, 1975). These must meet the criterion (Smith, 1965)

$$\Delta t \leq \frac{(X_a Y_a) S_a}{2T_a}$$

where

X_a, Y_a = average node dimensions (L)

S_a = average node storage coefficient (dimensionless)

T_a = average node transmissivity (L^2/T)

The backward difference implicit form uses the approximations

$$\left(\frac{T_{x_{0-3}} \frac{h_3 - h_0}{X} - T_{x_{1-0}} \frac{h_0 - h_1}{X}}{X} \right)_{t+\Delta t} \approx T_x \frac{\partial^2 h}{\partial x^2} \quad (6)$$

$$\left(\frac{T_{y_{0-2}} \frac{h_2 - h_0}{Y} - T_{y_{4-0}} \frac{h_0 - h_4}{Y}}{Y} \right)_{t+\Delta t} \approx T_y \frac{\partial^2 h}{\partial y^2} \quad (7)$$

In Equation (3). In writing Equation (3) exclusively in terms of heads at time $t+\Delta t$, an equation in five unknown node heads results (only h_{o_t} is known), necessitating a simultaneous solution of all the node equations for each time increment. Although the backward difference method requires a more difficult solution, it is unconditionally stable and convergent for any Δt used (Huntoon, 1974).

Combinations of the forward difference and backward difference schemes are essentially averages of the two. Examples of these are the

Crank-Nicholson implicit method and the alternating direction implicit method of Peaceman and Rachford (1955). Combinations and the implicit form are used almost exclusively in two-dimensional aquifer models.

If implicit or combination methods are used, a large number of simultaneous linear node equations must be solved for the heads at time $t+\Delta t$. Methods have long existed for their solution. Direct methods such as Gauss-Jordan elimination and matrix inversion are effective for small sets of equations. Larger sets are more efficiently solved with iterative techniques. Some of these are the Gauss-Seidel and Jacobi iteration methods, the Successive Overrelaxation (SOR) technique, and the Strongly Implicit Procedure (SIP) (see Carnahan et al., 1969). The iterative alternating direction implicit procedure (IADI) (Washpress and Habetler, 1960) is a particularly powerful method of solution.

For even a small set of equations, application of any of the solution methods can lead to a prohibitive number of hand calculations. With the advent of the large capacity digital computer, however, solutions of large matrix problems became practical. Digital computers became generally available by 1960 (Fleming, 1975), with the first computer models for regional groundwater flow being developed and published soon after. There has been concurrent development of petroleum reservoir models, which use similar equations of flow (Briggs and Dixon, 1968; Bjordammen and Coats, 1969).

The primary difference in finite difference computer models published since 1962 has been in the numerical method used for solution of the finite difference equations. The models also differ in what is included in W , the source/sink function. Interaquifer leakage, aquifer recharge, pumpage, and evapotranspiration losses may all be represented by W . Existing models simulate some or all of these with various degrees of sophistication. In addition, some models are designed to handle unconfined aquifer flow or a combination of confined-unconfined flow. A few three-dimensional models couple saturated flow with unsaturated flow for a more complete description of a groundwater system.

Published Models

One of the first digital, regional groundwater models published was that of Fayers and Sheldon (1962). Their model employs a three-dimensional form of the continuity equation for the general problem of simulating steady, confined flow in an isotropic, nonhomogeneous geologic basin. A curvilinear, three-dimensional grid is used because it was felt this would best represent curving basin boundaries. The simultaneous equations of flow are solved by successive point over-relaxation. No evidence of model verification was given in their paper.

Tyson and Weber (1964) used a polygonal grid and an implicit numerical integration technique to model the Los Angeles coastal plain

groundwater basin. The model is capable of simulating confined or unconfined transient flow in the nonhomogeneous, isotropic basin aquifer. For each time step of simulation the equations of flow are simultaneously solved with the Gauss-Seidel iteration procedure. Results of a basin simulation with the model compared favorably with a resistance-capacitance (R-C) analog simulation.

The earliest use of a rectangular node system was for the analysis of the effects of a proposed reservoir on water levels in a New Jersey aquifer (Remson et al., 1965). Steady aquifer flow was simulated using Equation (3) with a grid of variable dimensions. Solution of the simultaneous equations was by Gauss-Seidel iteration. Confined or unconfined flow in a nonhomogeneous, isotropic aquifer can be handled by this model.

Freeze has done considerable work with three-dimensional modeling of groundwater basins. Freeze and Witherspoon (1966) used a three-dimensional rectangular grid with nonuniform dimensions for the modeling of unconfined, steady, saturated flow in a nonhomogeneous, anisotropic basin. The equation of flow is a finite difference form of Equation (2). The extrapolated Liebmann method of overrelaxation is used to solve the set of finite difference equations. In order to verify the model, a groundwater basin in Saskatchewan, Canada was simulated. Results of the simulation were close to field observations. A

more thorough discussion of the model and demonstrations of its capabilities are given in Freeze (1969).

Freeze (1971, 1972) expanded on the groundwater basin model by including the capacity to simulate unsaturated regions of flow and coupling the model with surface infiltration, recharge, and stream flow. Integrated forms of Equation (2), for saturated flow, and Richards' equation (Richards, 1931)

$$\frac{\partial}{\partial x} [K_x(\theta) \frac{\partial h}{\partial x}] + \frac{\partial}{\partial y} [K_y(\theta) \frac{\partial h}{\partial y}] + \frac{\partial}{\partial z} [K_z(\theta) \frac{\partial h}{\partial z}] = \frac{\partial \theta}{\partial t} \quad (8)$$

(where hydraulic conductivity is a function of the moisture content, θ), for unsaturated flow, are used with a nonuniform rectangular grid. The simultaneous equations are solved by line successive overrelaxation.

Pinder and Bredehoeft (1968) developed a two-dimensional model for a nonhomogeneous, isotropic, confined leaky aquifer. A uniform rectangular grid is used for the application of a form of Equation (3). Node equations are solved using the alternating direction implicit method. The results of simulations of several hypothetical test cases compared favorably with analytical solutions.

Bredehoeft and Pinder (1970) presented a quasi-three-dimensional model for simulating nonsteady flow in multiple, overlying, nonhomogeneous, isotropic aquifers that leak to each other at nonsteady rates through their separating layers. Shying away from use of Equation (2) because of the excessive number of nodes, calculations, and computer

core that would be required, they modeled each aquifer with Equation (3) and a uniform, rectangular grid. An analytical relation that accounted for separating layer storage was used for calculating the nonsteady flow between aquifers. Many of the assumptions of Hantush (1960) were used for interaquifer leakage. Linking overlying nodes with the analytical interaquifer leakage equations and simultaneously solving the node equations in all aquifers permits simulation of an entire system. The model uses an iterative alternating direction implicit technique for solution of the equations. The results of simulations of several simple problems compared favorably with analytical solutions.

Prickett and Lonquist's two-dimensional aquifer model (1971) is a particularly versatile one. It uses a rectangular grid with nonuniform dimensions for application of Equation (3). Node equations are solved by the modified iterative alternating direction implicit method (MIADI) developed by the authors. With the model variations presented, it is possible to simulate transient flow in a nonhomogeneous, anisotropic aquifer that is confined, unconfined, or that is partly confined and partly unconfined. In addition, it is designed to handle multiple aquifers that are leaking to each other at a steady rate (through a quasi-three-dimensional approach, similar to the Bredehoeft and Pinder model), variable well pumping rates, evapotranspiration, and stream-aquifer recharge/discharge. Evapotranspiration and stream-aquifer

recharge/discharge are handled analytically with linear relations. The model was verified with a variety of theoretical and field test cases.

The model of Trescott et al. (1976) was built on the earlier work of Pinder (1969, 1970), Bredehoeft and Pinder (1970), and Trescott (1973). Very similar to the Prickett and Lonquist model in its capabilities, it can also simulate nonsteady interaquifer leakage. A rectangular grid with nonuniform dimensions and Equation (3) are used for representing aquifer flow. Three methods of solution for the equations are provided: Line successive overrelaxation (LSOR), a strongly implicit procedure (SIP), and an iterative alternating direction implicit method (IADI). Comparison of solutions indicated that SIP, the most complex and computer core demanding of the three, is generally the most rapid solution procedure, followed by IADI and LSOR. The comparisons are more thoroughly discussed in Trescott et al. (1977).

Rushton has compared solution techniques for two-dimensional models and analyzed their limitations in several studies. Rushton and Tomlinson (1971) presented time discretization criteria for simulating various types of well discharge problems with a nonuniform, rectangular grid model employing ADI. Rushton (1973), using the nonuniform, two-dimensional grid model, developed further time discretization criteria for the Crank-Nicholson and ADI solutions of well problems. In comparing the systematic overrelaxation method and the modified iterative

alternating direction implicit method of Prickett and Lonquist (1971), Rushton (1974) found little difference in solution efficiency.

Other models and studies of note are Thrialkili's (1972), a model that can simultaneously simulate laminar and turbulent (non Darcian) flow in limestone aquifers, and Lin's (1972) model that uses Boussinesq's equation for unconfined flow problems. Davis (1976) has used the MIADI method of Prickett and Lonquist (1971) with a two-dimensional model and applied it to a coal strip mine site in Wyoming. Some of the details of this model will be discussed in later sections.

Equations of Flow

As indicated in the previous section, Equation (1b)

$$\frac{\partial}{\partial x} \left(T_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(T_y \frac{\partial h}{\partial y} \right) = S \frac{\partial h}{\partial t} + W \quad (1b)$$

the equation of continuity for horizontal groundwater flow, is often used in finite difference groundwater models. Different forms of this equation are used for confined and unconfined groundwater flow. The two equation forms and their theoretical basis are reviewed in this section.

Confined Aquifer Flow

The continuity equation for three dimensional, nonsteady water flow through a homogeneous, isotropic, fully saturated porous medium in which Darcy's law applies is (DeWeist, 1969)

$$\frac{\partial}{\partial x} \left(K \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t} \quad (9a)$$

or

$$K \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right) = S_s \frac{\partial h}{\partial t}, \quad (9b)$$

where

x, y, z = cartesian distances (L)

K = porous medium hydraulic conductivity (L/T)

h = piezometric head, measured with respect to an arbitrary datum (L)

$S_s = \rho g [n\beta + (1-n)\alpha]$ = specific storage, the volume of water released from a unit volume of the porous medium per unit head drop (L⁻¹)

ρ = mass density of water (M/L³)

g = gravity acceleration (L/T²)

n = porous medium porosity (dimensionless)

β = compressibility of water (LT²/M)

α = vertical compressibility of aquifer (LT²/M)

t = time (T)

Equations (9a) and (9b) assume that the medium is perfectly elastic with negligible deformation in the horizontal (x,y) plane.

For confined flow in a nearly horizontal aquifer, flow is often assumed to be horizontal. Dropping the vertical flow component from Equation (9b), the equation becomes

$$K \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = S_s \frac{\partial h}{\partial t} \quad (10)$$

Rewriting in terms of the entire aquifer thickness b (a constant),

$$Kb \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = S_s b \frac{\partial h}{\partial t} = S_c \frac{\partial h}{\partial t} \quad (11)$$

where S_c is the aquifer confined storage coefficient, the volume of water released from a vertical aquifer column of unit cross sectional area when a unit head drop occurs. $S_c = S_s b$, and thus is a dimensionless number.

If a horizontal aquifer is isotropic but not homogeneous, then K and S_c become functions of x and y . In this case, Equation (11) must be rewritten as (Bear, 1972)

$$\frac{\partial}{\partial x} \left(Kb \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(Kb \frac{\partial h}{\partial y} \right) = S_c \frac{\partial h}{\partial t} \quad (12)$$

In addition, if the aquifer medium is also anisotropic (i.e., K is a function of direction as well as x and y), and the directions of maximum and minimum hydraulic conductivities lie along the coordinate axes, then Equation (12) is rewritten as

$$\frac{\partial}{\partial x} \left(K_x b \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y b \frac{\partial h}{\partial y} \right) = S_c \frac{\partial h}{\partial t} \quad (13a)$$

or

$$\frac{\partial}{\partial x} \left(T_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(T_y \frac{\partial h}{\partial y} \right) = S_c \frac{\partial h}{\partial t} \quad (13b)$$

where

K_x = hydraulic conductivity in the x direction (L/T)

K_y = hydraulic conductivity in the y direction (L/T)

$T_x = K_x b$ = aquifer transmissivity in the x direction (L^2/T)

$T_y = K_y b$ = aquifer transmissivity in the y direction (L^2/T)

To complete the horizontal aquifer flow equation, vertical aquifer discharge or recharge must be accounted for. Recharge or discharge may be caused by well pumping, leakage out of or into the aquifer, evapotranspiration, infiltration, etc. The sum of these is included in the term W, volumetric flux per unit area of aquifer surface (L/T). Adding W to Equation (13b) (Trescott et al., 1976),

$$\frac{\partial}{\partial x} (T_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (T_y \frac{\partial h}{\partial y}) = S_c \frac{\partial h}{\partial t} + W. \quad (14)$$

This is the confined aquifer form of Equation (1b). W is positive for discharge flux.

Unconfined Aquifer Flow

Unconfined, nonsteady, saturated groundwater flow below the phreatic surface in a homogeneous, isotropic porous medium is also described by Equations (9a) and (9b), subject to the assumptions given with them. However, at the phreatic surface, the boundary condition (Bear, 1972)

$$K \left(\frac{\partial h}{\partial x} \right)^2 + K \left(\frac{\partial h}{\partial y} \right)^2 + K \left(\frac{\partial h}{\partial z} \right)^2 - K \frac{\partial h}{\partial z} = S_y \frac{\partial h}{\partial t} \quad (15)$$

must be satisfied. The specific yield (also known as effective porosity), S_y , is defined as

$$S_y = \frac{V_w}{V} \text{ (dimensionless)}$$

where

V_w = volume of water that can be drained by gravity from V
(L^3)

V = volume of porous medium (L^3)

In order to avoid the use of Equation (15), which is very difficult to evaluate, the Dupuit assumptions (as discussed in Viessman et al., 1972) can be employed. If the aquifer is nearly horizontal and the slope of the phreatic surface mild, Dupuit assumed that the flow lines are essentially horizontal [i.e., $\frac{\partial h}{\partial z} = 0$ (Streeter, 1971)]. Hydraulic gradient is then a function of x, y , and t only. Dupuit also assumed that the hydraulic gradient of flow equals the slope of the phreatic surface in the direction of flow.

Figure 4 will be used to illustrate the use of the assumptions for unconfined flow through a vertical section of unit thickness. Flow lines in the section are in the x, z plane.

By assuming that stream lines are horizontal, that hydraulic gradients are equal to the slope of the phreatic surface, and that Darcy's law holds for the medium, the transient flow through the elemental volume of unit thickness, height h_s , and width dx may be described for one-dimensional flow as

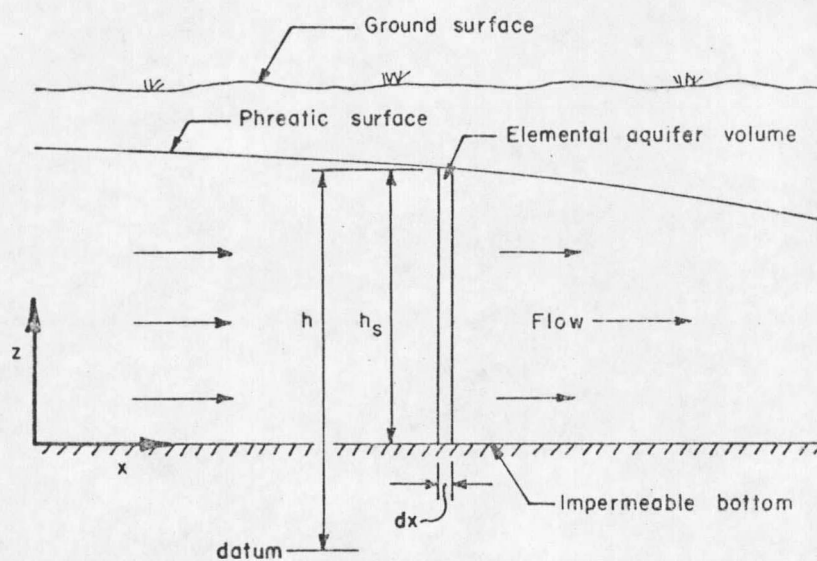


Fig. 4. A vertical cross section of unit thickness through an unconfined aquifer.

$$\frac{\partial}{\partial x} h_s \left(K_x \frac{\partial h}{\partial x} \right) = S_y \frac{\partial h}{\partial t} \quad (16)$$

where

K_x = hydraulic conductivity of the medium in the x direction (L/T)

h = height of the phreatic surface above the datum (L)

h_s = saturated thickness of the aquifer (l)

In using this equation it must be assumed that aquifer dewatering (or rewatering) caused by the lowering (or raising) of the phreatic surface occurs instantaneously.

For two-dimensional flow in an anisotropic, nonhomogeneous aquifer medium, the flow equation is written as

$$\frac{\partial}{\partial x} h_s \left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} h_s \left(K_y \frac{\partial h}{\partial y} \right) = S_y \frac{\partial h}{\partial t} \quad (17a)$$

or

$$\frac{\partial}{\partial x} \left(T_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(T_y \frac{\partial h}{\partial y} \right) = S_y \frac{\partial h}{\partial t} \quad (17b)$$

where

$K_x = f(x,y)$ = hydraulic conductivity in the x direction (L/T)

$K_y = f(x,y)$ = hydraulic conductivity in the y direction (L/T)

$T_x = h_s K_x = f(x,y,t)$ = transmissivity in the x direction (L²/T)

$T_y = h_s K_y = f(x,y,t)$ = transmissivity in the y direction (L²/T)

The horizontal, unconfined flow approximation is completed by adding the W term to Equation (17b):

$$\frac{\partial}{\partial x} \left(T_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(T_y \frac{\partial h}{\partial y} \right) = S_y \frac{\partial h}{\partial t} + W \quad (18)$$

This is the unconfined form of Equation (1b).

Forcheimer used the Dupuit assumptions to formulate the Dupuit-Forcheimer well discharge (radial flow) steady flow equation. Although the equation does not accurately describe the phreatic surface near a well (it overestimates the surface drawdown), for given heads it does closely approximate the flow rate (Hantush, 1962). The flow rate computed from the equation is believed to be accurate within one percent (Boulton, 1951).

Equation (18) approximates saturated flow under the phreatic surface only. It ignores flow occurring above the surface, in the capillary area, which can be significant if the capillary zone is thick (Mavis and Tsui, 1939). Furthermore, it should be noted that the capillary zone can be saturated for some height above the phreatic surface (Todd, 1959).

Aquifer Boundaries

All aquifers possess flow boundaries, lateral boundaries as well as vertical (top and bottom) boundaries. For a completely confined aquifer, the top and bottom impermeable boundaries are already accounted for in Equation (14). Similarly, in an unconfined aquifer

which has an impermeable bottom boundary and a moving top boundary (the phreatic surface), Equation (18), subject to the assumptions previously stated, automatically accounts for the boundaries. Lateral impermeable and constant head aquifer boundaries are easily handled in finite difference solutions of Equations (14) and (18), as will be described in the next chapter. However, complete drawdown lateral boundaries (the mine cut boundary) and semipermeable vertical boundaries, which will have to be modeled in the East Decker aquifers, are much more difficult to treat. A review of methods that have been used for these boundaries is given in this section.

Complete Drawdown Boundary

At an aquifer mine cut boundary, a situation such as that shown in cross section in Figure 5 is encountered. The unconfined aquifer has been completely cut through, resulting in its drainage into the cut. A flow pattern similar to that shown with the flow net would occur some time after the aquifer was truncated. This type of discharge and drawdown is expected to happen in the unconfined alluvial and clinker aquifers at the East Decker mine.

Figure 6 illustrates a truncated confined aquifer. Drainage causes a conversion from confined to unconfined flow to occur initially at the cut face and then to move away from the cut with time. This is expected to happen in the confined coal aquifers at East Decker.

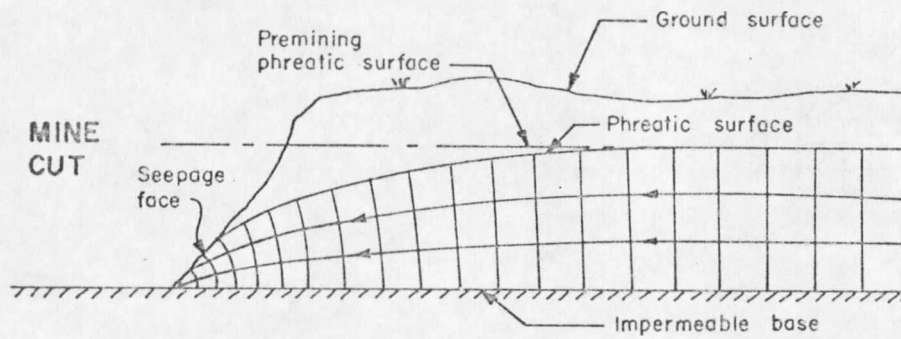


Fig. 5. Drainage of an unconfined aquifer into a mine cut.

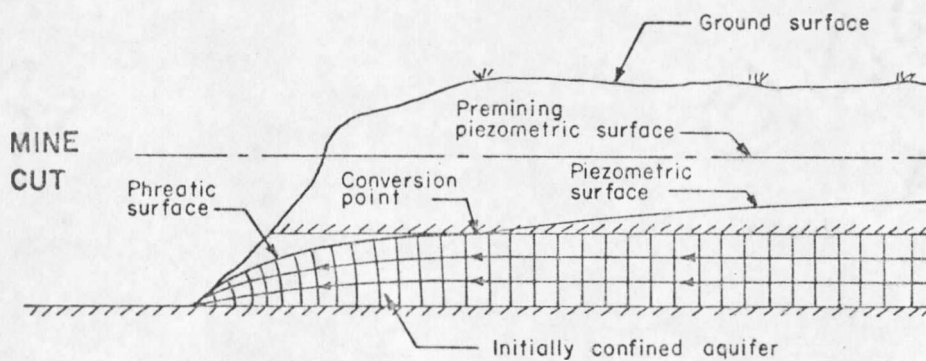


Fig. 6. Drainage of an initially (premining) confined aquifer into a mine cut.

In the flow near an aquifer seepage face, equipotential lines are normal to the face. This results in the top flow line tangentially approaching the face slope (Lambe and Whitman, 1969). Thus, near the face, curvilinear flow with a significant vertical component will occur. In evaluating the effect of a seepage face on an aquifer, a three dimensional analysis is obviously called for. However, because of the size of the site to modeled and the amount and detail of data required for a three-dimensional model, it was decided that a two-dimensional, horizontal approximation model would be used. Within this constraint, the literature was reviewed for methods for approximating drawdown and discharge at complete drawdown boundaries. With a good discharge approximation it would be possible to pump a mine cut at the computed rate and produce realistic drawdowns in the aquifers.

Van Voast and Hedges (1975) and the USGS (1977) believe that the groundwater discharged into the East Decker mine will come from three sources. These are intercepted natural groundwater flow, induced flow from the Tongue River Reservoir to the cut, and flow derived from storage in the aquifers. The natural flow can be estimated from an application of Darcy's law using the natural groundwater gradients and aquifer transmissivities (Van Voast and Hedges, 1975; USGS, 1977). The reservoir to cut flow and the flow from aquifer storage, however, require more complex approximations since they are affected by the complete drawdown boundary.

Flow derived from reservoir. The East Decker mine will be opened near the Tongue River Reservoir in clinker and alluvial aquifers that are linked with the reservoir (USGS, 1977). Because the initial cut will be so close to the reservoir (Figure 2 shows the initial cut location), the USGS believes that an assumption of steady flow from the reservoir to the mine is valid.

Using the Dupuit assumptions, the steady flow rate through a unit width of unconfined aquifer with hydraulic conductivity K is

$$q = -Kh_s \frac{dh_s}{dx}, \quad (19)$$

where h_s is the height of the water table above the aquifer bottom.

Assuming the mine cut completely penetrates the aquifer, integration of Equation (19) from the reservoir to the mine yields the Dupuit equation (Todd, 1959).

$$q_r = \frac{K}{f} \left[\frac{h_s^2}{2} \right]_0^{H_r} \quad (20)$$

where

q_r = discharge per unit length of mine cut from the reservoir
(L^2/T)

f = distance between the mine cut and reservoir (L)

H_r = height of the reservoir surface above the aquifer bottom
(L)

The Dupuit equation was used by the USGS (1977) for estimating reservoir to mine cut flow.

When the mine cut reaches the level of the D-2 coal (the D-1 upper and D-1 lower seams are not present between the reservoir and the cut), flow will begin through the coal from the reservoir to the cut. Since the D-2 coal is below the level of the reservoir, once the cut truncates the seam, flow to the cut will be as in Figure 6; unconfined flow conditions will exist near the cut face, while flow will be confined back towards the reservoir. Nothing was found in the literature dealing with such a steady flow problem.

Flow derived from storage. Davis (1976) suggested that the rate of discharge from aquifer storage can be computed with a method used by Stallman (in Ferris et al., 1962) for calculating nonsteady flow from a confined aquifer into a stream whose stage has suddenly dropped. The USGS (1977) used this method for estimating coal aquifer discharges at East Decker. The method assumes the stream is straight, of infinite length, and fully penetrates a semi-infinite, homogeneous, horizontal confined aquifer that has an initial head of 0.0 throughout (see Figure 7). At time $t = 0$ the stream stage instantly drops H_0 from its original stage (0.0). The resulting aquifer discharge is normal to the stream direction (one-dimensional) and is derived entirely from aquifer storage.

Rewriting the solution of an analogous one-dimensional heat flow problem (Ingersoll et al., 1948) in groundwater terms, the aquifer drawdown at any time t at distance x from the stream is

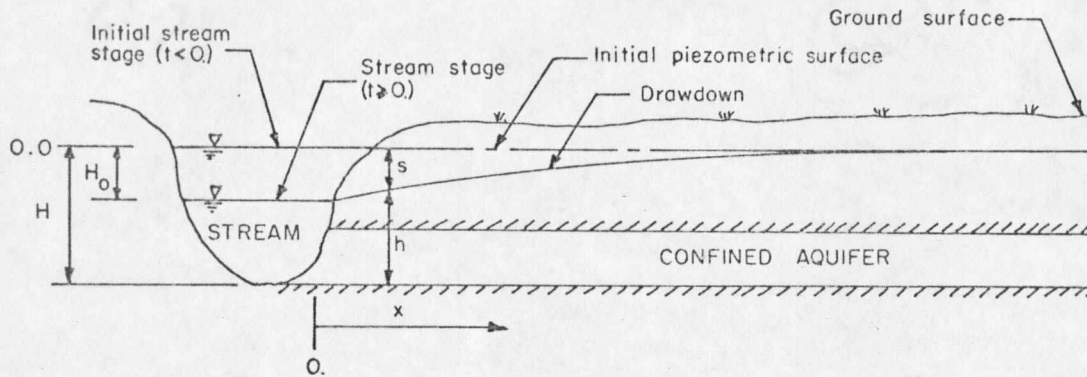


Fig. 7. Confined aquifer outflow and drawdown caused by a sudden fall in stream stage.

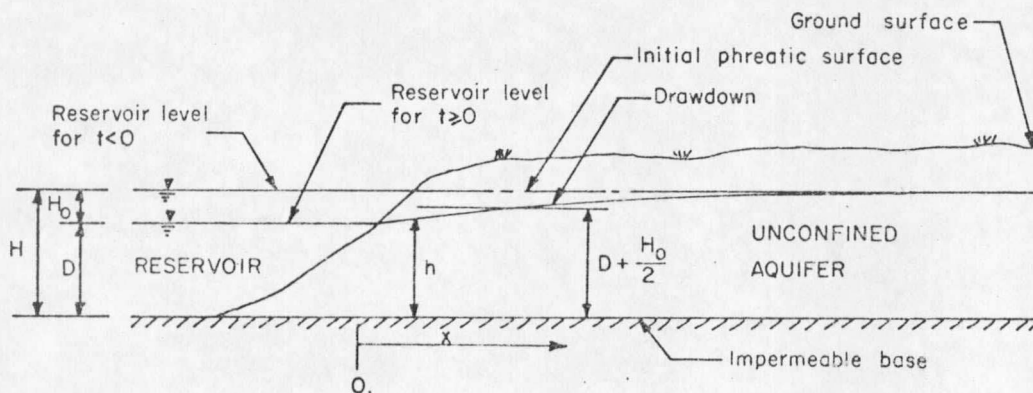


Fig. 8. Unconfined aquifer bank storage outflow due to a sudden reservoir level drop (after Glover, 1974).

$$s = H_0 \left[1 - \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du \right] \quad (21)$$

where

T = aquifer transmissivity (L^2/T)

$$u = \frac{x}{(4Tt/S_c)^{1/2}}$$

and other terms are as previously defined. This is the Stallman solution.

Evaluating the derivative of s with respect to x at $x = 0$ and multiplying by the transmissivity yields, by Darcy's law, the aquifer discharge rate from storage per unit length of stream:

$$q_{st} = T \left(\frac{\partial s}{\partial x} \right)_{x=0} = H_0 \left(\frac{S_c T}{\pi t} \right)^{1/2} \quad (22)$$

Davis (1976) observed that for complete drawdown $H_0 = H$. This results in

$$q_{st} = H \left(\frac{S_c T}{\pi t} \right)^{1/2} \quad (23)$$

Note that in the problem in Figure 7 the aquifer is discharging into a body of water. Unlike a seepage face, the head is constant over the entire aquifer-stream interface; aquifer flow is confined and horizontal right up to the stream, and the transmissivity remains constant in the direction of flow. In a mine cut these conditions may be approximated when a confined coal aquifer is first exposed. However,

once the coal is cut through and removed, the aquifer will discharge through a seepage face as shown in Figure 6.

Equations (21) and (22) are similar to drawdown and discharge equations presented by Glover (1974) for an idealized reservoir bank storage problem. This problem is illustrated in Figure 8.

It is assumed that a semi-infinite unconfined aquifer overlying an impermeable layer is bounded and fully penetrated on one side by a reservoir whose shore is straight and of infinite length. Initially the reservoir and water table are at elevation H . At $t = 0$ the reservoir level is instantly dropped H_0 , producing discharge from the aquifer that is derived solely from aquifer storage.

Using Dupuit-Forcheimer assumptions, flow may be described by the differential equation

$$K \frac{\partial}{\partial x} h \left(\frac{\partial h}{\partial x} \right) = S_y \frac{\partial h}{\partial t} \quad (24)$$

with the conditions

$$h_{x \geq 0} = H \text{ at } t = 0$$

$$h_{x=0} = D \text{ for } t > 0.$$

By assuming that $D \gg H_0$ and

$$K \frac{\partial}{\partial x} h \left(\frac{\partial h}{\partial x} \right) \approx KD \frac{\partial^2 h}{\partial x^2},$$

Glover (1974) gave the following approximate solution for the head at any t at distance x from the reservoir:

$$h \approx H_0 \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du \quad (25)$$

where

$$u = \frac{x}{(4KDt/S_y)^{1/2}} \text{ (dimensionless)}$$

The resulting aquifer discharge per unit length of reservoir shore is obtained by differentiating Equation (25) with respect to x . Then

$$q_{st} = KD \left(\frac{\partial h}{\partial x} \right)_{x=0} = H_0 \left(\frac{S_y KD}{\pi t} \right)^{1/2} \quad (26)$$

Because Equation (25) assumes that transmissivity through the length of flow is a constant KD , Equations (21) and (22) and Equations (25) and (26) have similar forms. The only difference between Equations (21) and (25) is that Equation (25) is the solution for h rather than for drawdown s .

Since $D = 0$ for complete drawdown, Equation (25) cannot be used for this problem. However, Haushild and Kruse (1962) presented another approximate solution of Equation (24) which is applicable to a complete drawdown. Assuming that

$$K \frac{\partial}{\partial x} h \left(\frac{\partial h}{\partial x} \right) \approx K \left(D + \frac{H_0}{2} \right) \frac{\partial^2 h}{\partial x^2},$$

the solution for h for a complete drawdown ($D=0$) is

$$h = H \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du \quad (27)$$

where

$$u = \frac{x}{(4KHt/2S_y)^{1/2}} \quad (\text{dimensionless})$$

The corresponding discharge per unit length of reservoir is

$$q_{st} = K \frac{H}{2} \left(\frac{\partial h}{\partial x} \right)_{x=0} = H \left(\frac{S_y KH}{2\pi t} \right)^{1/2} \quad (28)$$

Haushild and Kruse compared solutions from Equation (27) to a complete drawdown experiment ($D=0$, $H_0=H$) performed by Keller and Robinson (1959). The Keller and Robinson experiment involved drainage of a sand filled, horizontal flume from the bottom of one end of the flume floor. The drawdown after 6.5 minutes of drainage is shown in Figure 9. Also shown is the drawdown curve computed from Equation (27) for $t=6.5$ minutes. Equation (27) overestimated the actual drawdown, but not drastically so.

Haushild and Kruse (1962) presented another approximate solution to Equation (24). By using the approximation employed in deriving Equation (27) to estimate the flow rate (called here approximation "a")

$$q_a = K \left(D + \frac{H_0}{2} \right) \frac{\partial h_a}{\partial x} \quad (29)$$

and equating it to the more accurate approximation "b"

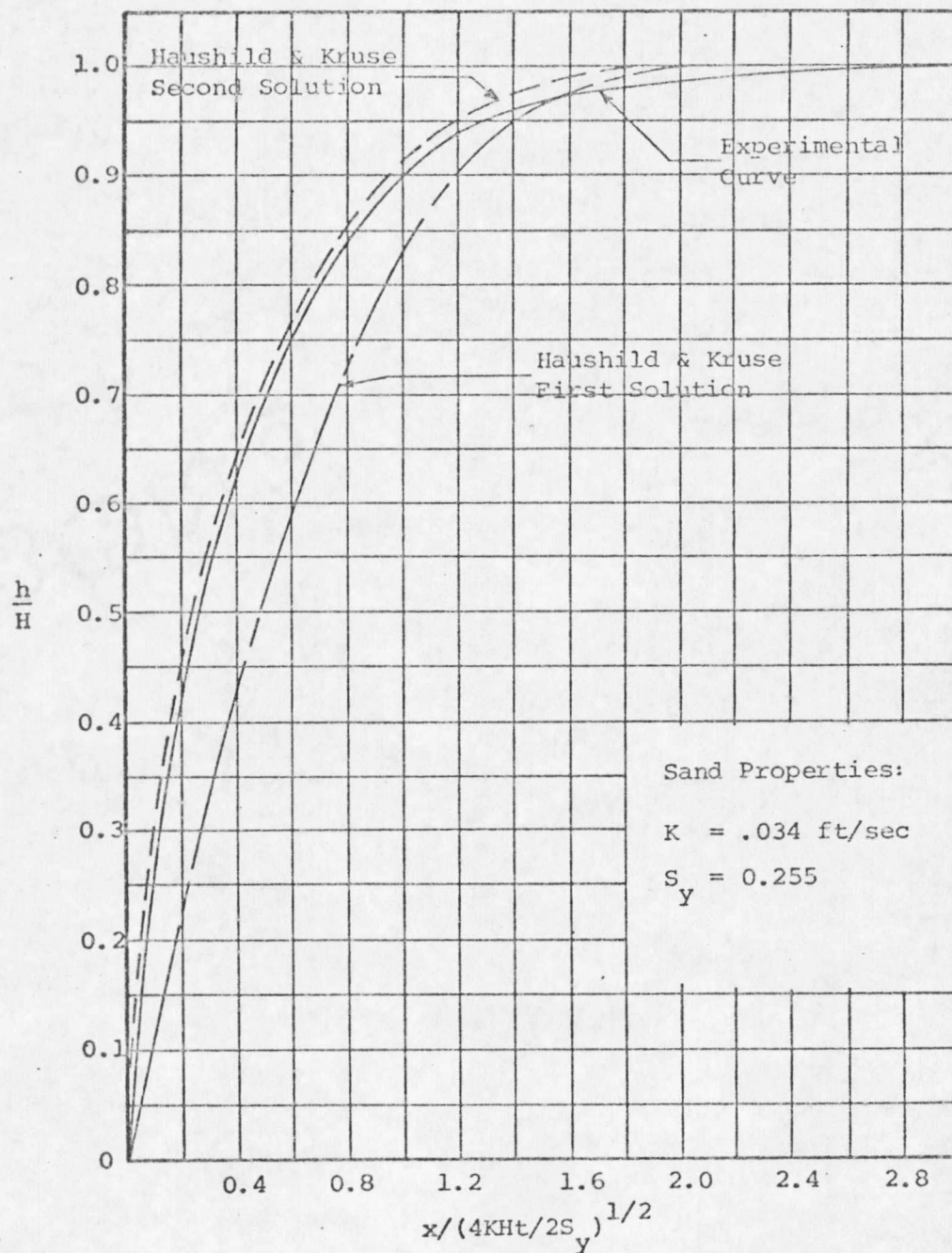


Fig. 9. Comparisons of Haushild and Kruse (1962) solutions with Keller and Robinson (1959) sand flume--complete drawdown experiment. From Haushild and Kruse (1962).

$$q_b = Kh_b \frac{\partial h_b}{\partial x} \quad (30)$$

and solving for h_b , h can be approximated for a complete drawdown ($D=0$, $H_o=H$). This is done as follows:

$$Kh_b \frac{\partial h_b}{\partial x} = K\left(\frac{H}{2}\right) \frac{\partial h_a}{\partial x}$$

Simplifying

$$h_b \frac{\partial h_b}{\partial x} = \frac{H}{2} \frac{\partial h_a}{\partial x}$$

and then integrating results in

$$\frac{h_b^2}{2} = \frac{Hh_a}{2} + C$$

Because h must equal 0 at $x=0$, C must also equal 0. Therefore,

$$h_b = (Hh_a)^{1/2}$$

Since Equation (27) is the approximate head solution for flow approximation (a),

$$h = h_b \approx H \left[\frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du \right]^{1/2} \quad (31)$$

where

$$u = \frac{x}{(4Kht/2S_y)^{1/2}} \quad (\text{dimensionless})$$

Applying this equation to the Keller and Robinson experiment produced the drawdown shown in Figure 9. This solution is much closer to

the actual drawdown than solution (27). Taking the derivative of Equation (31) with respect to x and multiplying by the transmissivity would give q_{st} , the discharge rate.

As noted earlier, both confined and unconfined aquifers will be draining into the East Decker mine. Equation (21), the Stallman equation, is applicable to a confined aquifer when it is first exposed. The Haushild and Kruse equations, Equations (27) and (31), are suitable for complete drawdown boundaries in unconfined aquifers. None of the equations reviewed, however, are meant for application to a truncated confined aquifer. In such an aquifer the region near the mine cut will become unconfined. In the unconfined region, transmissivity varies with x and t , and storage effects are dependent upon specific yield. A constant transmissivity and the confined storage coefficient are applicable in the confined region of flow. The previously described equations do not account for flow in the two different regions.

It is possible that an assumption of steady discharge into the mine cut is valid. Davis (1977), on the basis of observations at two mines, believes that mines are opened so slowly that the discharge into a cut is closer to steady state than it is to rates computed with complete drawdown equations. The steady discharge is best estimated by monitoring the pumpage from a cut.

Interaquifer Leakage

For a rigorous solution of nonsteady aquifer flow and interaquifer leakage, a three-dimensional analysis of the aquifers and their confining layers should be used. However, for certain cases an assumption of steady, strictly vertical flow through confining layers is reasonable, even during transient conditions (Hantush, 1960). This assumption was used by Pinder and Bredehoeft (1968), Prickett and Lonquist (1971), and Rushton (1974) for interaquifer leakage in their two dimensional models.

In discussing interaquifer leakage, reference is made to Figure 10, a cross section of an idealized, three aquifer system. The aquifers are confined and separated by semipermeable layers. Pumping is begun out of aquifer 2 at time $t = 0$, disturbing the initial equilibrium of the system.

If storage in the semipermeable layers is negligible and the ratio of aquifer hydraulic conductivities to confining layer conductivities is greater than 100, interaquifer leakage may be considered vertical and steady (Hantush, 1967). Assuming elastic, homogeneous, isotropic aquifers whose hydraulic properties remain constant with time, and steady vertical interaquifer leakage, by use of Darcy's law, flow in aquifer 1 can be approximated as (Hantush, 1967)

$$T_1 \left(\frac{\partial^2 h_1}{\partial x^2} + \frac{\partial^2 h_1}{\partial y^2} \right) + (h_2 - h_1) (K_1' / b_1') = S_{c1} \frac{\partial h_1}{\partial t} \quad (32)$$

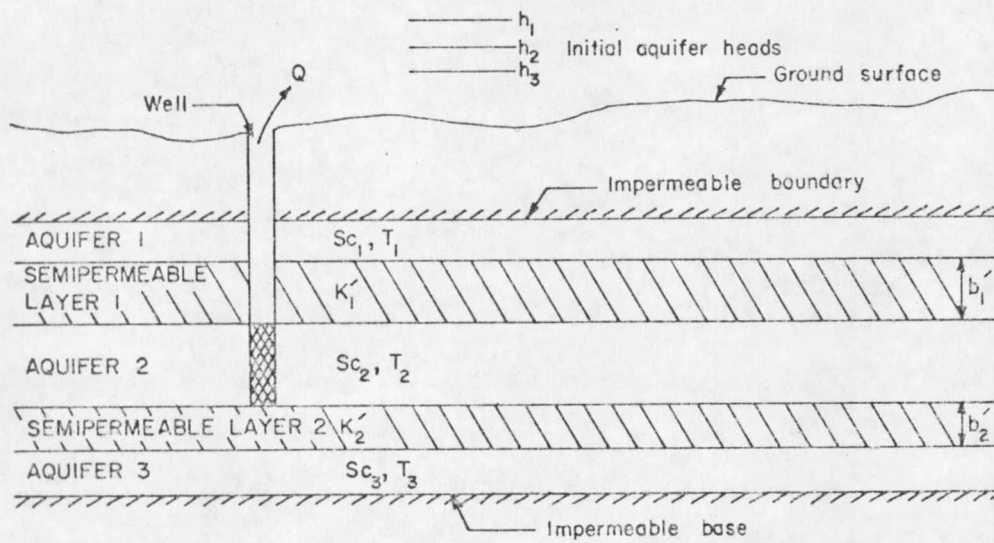


Fig. 10. An idealized system of three confined aquifers linked to each other by leakage through the separating beds.

where

h_1, h_2 = heads of aquifers 1 and 2 (L)

K_1' = vertical hydraulic conductivity of confining layer 1 (L/T)

b_1' = thickness of confining layer 1 (L)

A similar equation can be written for aquifer 3. Aquifer 2, however, is linked with both aquifers 1 and 3. The flow approximation for it is

$$T_2 \left(\frac{\partial^2 h_2}{\partial x^2} + \frac{\partial^2 h_2}{\partial y^2} \right) + (h_1 - h_2) (K_1' / b_1') + (h_3 - h_2) (K_2' / b_2') = S_{c2} \frac{\partial h_2}{\partial t} + W \quad (33)$$

If the leakage terms in Equations (32) and (33) are placed to the right of the equalities, the isotropic, homogeneous form of Equation (14) results. The leakage terms would form part of the W term.

Semipervious layers of elastic clay or silt may have large storage capacities. After pumping has begun in one aquifer, an assumption of steady flow through the confining layers is not then valid (Hantush, 1960). Brocdehoeft and Pinder (1970) use the parameter "dimensionless time"

$$t' = \frac{K't}{S'b'^2} \quad (34)$$

where t is the time since pumping began and S'_s is the specific storage of the layer, to determine the type vertical flow (steady or transient). For t' less than 0.5, nonsteady flow is assumed to occur.

Trescott et al. (1976) compute transient leakage from aquifer 1 to aquifer 2 per unit area at time t as

$$q'_{1,t} \approx (h_{2,o} - h_{2,t}) \frac{K'_1}{\left(\frac{K'_1 t}{3b_1^2 S'_1}\right)^{1/2} b'_1} \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp \left[-\frac{n^2}{\left(\frac{K'_1 t}{3b_1^2 S'_1}\right)} \right] \right\} \quad (35)$$

$$+ \frac{K'_1}{b'_1} (h_{1,o} - h_{2,o})$$

where

$h_{1,0}$ = head in aquifer 1 at $t = 0$ (L)

$h_{2,0}$ = head in aquifer 2 at $t = 0$ (L)

$h_{2,t}$ = head in aquifer 2 at $t > 0$ (L)

S'_s = specific storativity of confining layer 1 (L^{-1})

and other terms are as previously defined. The first term represents transient leakage. Beyond $t' = 0.5$ this term becomes nearly constant (steady flow). The second term is initial steady leakage between the aquifers.

Chapter III

AQUIFER MODEL METHODOLOGY

Equations (14) and (18), the two-dimensional approximations for confined and unconfined aquifer flow, were chosen to model aquifer flow at East Decker. Two-dimensional modeling was chosen rather than three-dimensional because the latter type of modeling requires considerably more data input and computer time (Freeze, 1969). Since relatively little is known of the subsurface hydraulic characteristics of the East Decker site, it was felt that detailed, three-dimensional modeling was not warranted.

This chapter presents the development of the finite difference flow equations, outlines their method of solution, and shows how the boundary conditions are incorporated into the solution. The equations are written to conform with the modified iterative alternating direction implicit (MIADI) method developed by Prickett and Lonquist (1971). This method was picked because it has been used successfully for many groundwater problems that are similar to those encountered at East Decker.

Finite Difference Forms of Flow Equations

Confined Flow

Confined, saturated aquifer flow in the model is described by the horizontal approximation

$$\frac{\partial}{\partial x} \left(T_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(T_y \frac{\partial h}{\partial y} \right) = S_c \frac{\partial h}{\partial t} + W \quad (14)$$

In writing Equation (14) in a discrete, finite difference form, reference is made to Figures 11 and 12. Figure 11 illustrates the rectangular, horizontal node layout that will be used for the two dimensional description of an aquifer. Figure 12 gives the dimensions associated with each node i,j . Except for the dimensioning, node notation follows the form used by Huntton (1974).

The best practical approximation for the space derivatives in Equation (14) are central difference approximations (Gerald, 1974). With these and an approximation of the time derivative, Equation (14) is approximated at node i,j as

$$\begin{aligned} & \frac{TX'_{i+1/2,j} \frac{h_{i+1,j} - h_{i,j}}{X1_{i,j}} - TX'_{i-1/2,j} \frac{h_{i,j} - h_{i-1,j}}{X1_{i-1,j}}}{X2_{i,j}} \\ & + \frac{TY'_{i,j+1/2} \frac{h_{i,j+1} - h_{i,j}}{Y1_{i,j}} - TY'_{i,j-1/2} \frac{h_{i,j} - h_{i,j-1}}{Y1_{i,j-1}}}{Y2_{i,j}} \\ & = S_{c,i,j} \frac{h_{i,j} - h_{o,i,j}}{\Delta t} + W_{i,j} \end{aligned} \quad (36)$$

where

$$TX'_{i+1/2,j} = \text{aquifer transmissivity between nodes } i,j \text{ and } i+1,j \quad (L^2/T)$$

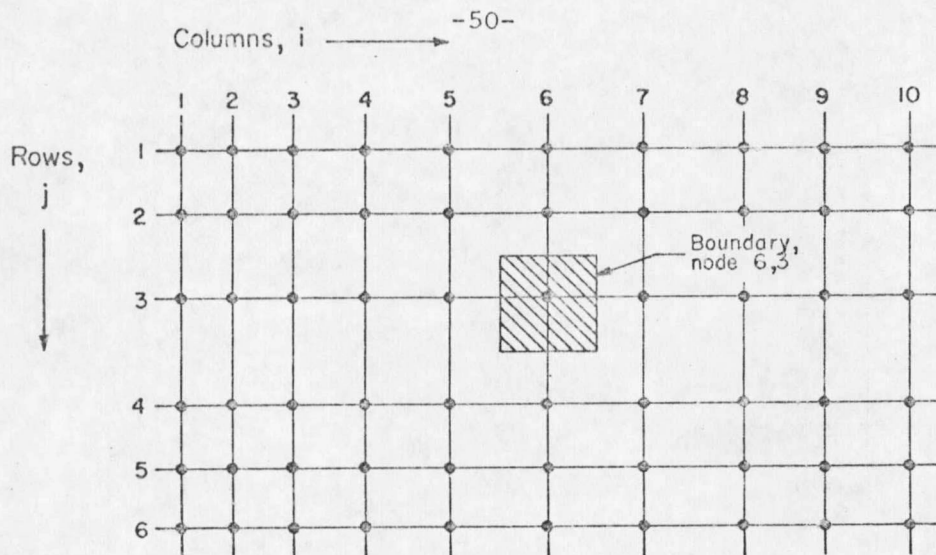


Fig. 11. Example of the finite difference grid used in the model.

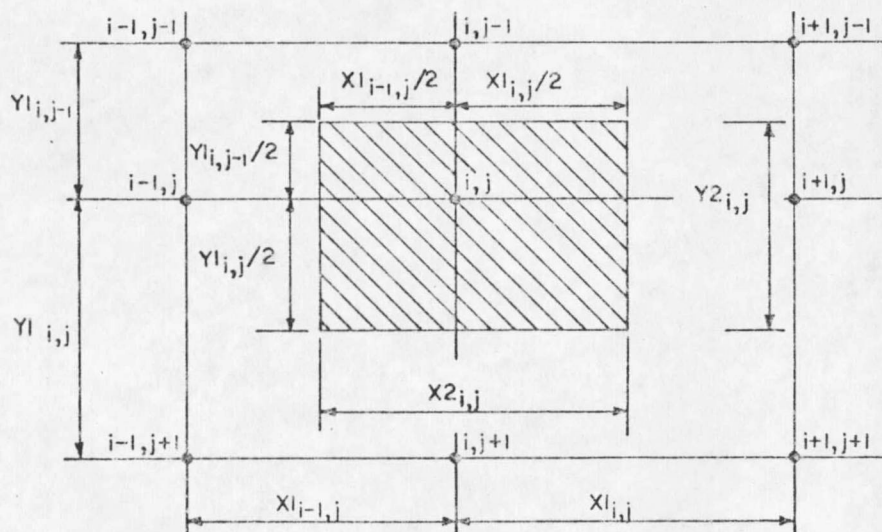


Fig. 12. Notation and dimensions associated with each node i, j .

$TX'_{i-1/2,j}$ = aquifer transmissivity between nodes $i-1,j$ and i,j (L^2/T)

$TY'_{i,j+1/2}$ = aquifer transmissivity between nodes i,j and $i,j+1$ (L^2/T)

$TY'_{i,j-1/2}$ = aquifer transmissivity between nodes $i,j-1$ and i,j (L^2/T)

$h_{i,j}$ = aquifer head at node i,j at the current time, time $t+\Delta t$ (L)

$h_{o,i,j}$ = aquifer head at node i,j at the previous time, time t (L)

$S_{c,i,j}$ = confined storage coefficient of node i,j (dimensionless)

t = time (T)

Δt = time increment (T)

$X1, X2, Y1, Y2$ = as defined in Figure 12

$W_{i,j}$ = discharge flux from node i,j (LT^{-1})

Equation (36) is written implicitly with respect to the node heads, i.e., all of the head values (except $h_{o,i,j}$) represent heads at the present time, at time $t+\Delta t$, and therefore are unknown values. This is done to conform to the MIADI solution method.

Following the MIADI method, both sides of the equation are multiplied by the area of node i,j , ($X2_{i,j} Y2_{i,j}$), giving

$$TX'_{i+1/2,j} (h_{i+1,j} - h_{i,j}) \frac{Y2_{i,j}}{X1_{i,j}} -$$

$$\begin{aligned}
 & TX'_{i-1/2,j} (h_{i,j} - h_{i-1,j}) \frac{Y2_{i,j}}{X1_{i-1,j}} + \\
 & TY'_{i,j+1/2} (h_{i,j+1} - h_{i,j}) \frac{X2_{i,j}}{Y1_{i,j}} - \\
 & TY'_{i,j-1/2} (h_{i,j} - h_{i,j-1}) \frac{X2_{i,j}}{Y1_{i,j-1}} \\
 & = S_{c,i,j} \frac{h_{i,j} - h_{o,i,j}}{\Delta t} (X2_{i,j} Y2_{i,j}) + W_{i,j} (X2_{i,j} Y2_{i,j})
 \end{aligned} \tag{37}$$

For simplifying the solution of the node equations, several of the terms are combined in the following way to form "factors":

$$TX_{i+1/2,j} = TX'_{i+1/2,j} \frac{Y2_{i,j}}{X1_{i,j}}, (L^2/T) \tag{38a}$$

$$TX_{i-1/2,j} = TX'_{i-1/2,j} \frac{Y2_{i,j}}{X1_{i-1,j}}, (L^2/T) \tag{38b}$$

$$TY_{i,j+1/2} = TY'_{i,j+1/2} \frac{X2_{i,j}}{Y1_{i,j}}, (L^2/T) \tag{38c}$$

$$TY_{i,j-1/2} = TY'_{i,j-1/2} \frac{X2_{i,j}}{Y1_{i,j-1}}, (L^2/T) \tag{38d}$$

$$SF1_{i,j} = S_{c,i,j} (X2_{i,j} Y2_{i,j}), (L^2) \tag{38e}$$

$$Q_{i,j} = W_{i,j} (X2_{i,j} Y2_{i,j}), (L^3/T) \tag{38f}$$

The Q factor can represent several things, including well discharge or recharge, evapotranspiration, infiltration, etc. Computation of these components of Q will be defined as they are encountered.

Inserting these factors into Equation (37) and rearranging terms,

$$\begin{aligned} & TX_{i+1/2,j} (h_{i+1,j} - h_{i,j}) + TX_{i-1/2,j} (h_{i-1,j} - h_{i,j}) + \\ & TY_{i,j+1/2} (h_{i,j+1} - h_{i,j}) + TY_{i,j-1/2} (h_{i,j-1} - h_{i,j}) \\ & = SFl_{i,j} \frac{h_{i,j} - h_{o,i,j}}{\Delta t} + Q_{i,j} \end{aligned} \quad (39)$$

For a confined aquifer, Equation (39) is written for all N nodes, resulting in N node equations in N unknown heads.

As long as flow is confined, the transmissivity and storage coefficient remain constant with time. Combining these values with the dimensions at the beginning of a simulation eliminates the need to carry the dimensions through all subsequent time increment calculations.

Unconfined Flow

Unconfined, saturated aquifer flow is simulated in the model with the horizontal approximation

$$\frac{\partial}{\partial x} (T_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (T_y \frac{\partial h}{\partial y}) = S_y \frac{\partial h}{\partial t} + W \quad (18)$$

As with the confined aquifer equation, Equation (18) is written in an implicit, central difference form for node i,j. Combining and rearranging terms yields

$$\begin{aligned}
 & TX_{i+1/2,j} (h_{i+1,j} - h_{i,j}) + TX_{i-1/2,j} (h_{i-1,j} - h_{i,j}) + \\
 & TY_{i,j+1/2} (h_{i,j+1} - h_{i,j}) + TY_{i,j-1/2} (h_{i,j-1} - h_{i,j}) \\
 & = SF2_{i,j} \frac{h_{i,j} - h_{o,i,j}}{\Delta t} + Q_{i,j}
 \end{aligned} \tag{40}$$

where

$$SF2_{i,j} = S_{y_{i,j}} (X2_{i,j} \quad Y2_{i,j})$$

Equation (40) is the same as Equation (39), except that the transmissivity factors are functions of water table height which varies with time, and $SF2_{i,j}$, the unconfined storage factor, is used. Calculation of the transmissivity factors will be discussed in a later section.

Confined-Unconfined Flow

A more general flow problem occurs in an aquifer which is confined in some areas and unconfined in others. For this problem, the equation

$$\begin{aligned}
 & TX_{i+1/2,j} (h_{i+1,j} - h_{i,j}) + TX_{i-1/2,j} (h_{i-1,j} - h_{i,j}) + \\
 & TY_{i,j+1/2} (h_{i,j+1} - h_{i,j}) + TY_{i,j-1/2} (h_{i,j-1} - h_{i,j}) \\
 & = SF_{i,j} \frac{h_{i,j} - h_{o,i,j}}{\Delta t} + Q_{i,j}
 \end{aligned} \tag{41}$$

is used for all nodes. Equation (41) is essentially the same as Equations (39) and (40), except that the transmissivity factors are

determined from average saturated thicknesses in the unconfined regions, and from aquifer thicknesses in the confined regions. This is discussed in the next section. Also, the value used for $SF_{i,j}$ is $SF1_{i,j}$ or $SF2_{i,j}$, depending on whether or not a node is confined.

During transient conditions, some of the nodes will be undergoing conversion from confined to unconfined conditions (or vice versa) from one time increment to the next. If this occurs, switching directly from $SF1$ to $SF2$ (or vice versa) can lead to instability in the numerical solution due to the great difference in the magnitudes of the two values. Head values at a node undergoing conversion can behave erratically. To minimize this problem, storage factors are apportioned during the transition. For example, for node i,j in Figure 13, which is changing from confined to unconfined conditions during time Δt , the storage coefficient conversion is smoothed by apportioning the $SF1$ and $SF2$ values as follows (after Prickett and Lonquist, 1971):

$$SF_{i,j} = \frac{(h_{o,i,j} - CH_{i,j}) SF1_{i,j} + (CH_{i,j} - h_{i,j}) SF2_{i,j}}{(h_{o,i,j} - h_{i,j})} \quad (42)$$

where $SF_{i,j}$ is the smoothed storage coefficient factor and $CH_{i,j}$ is the elevation of the aquifer top. Similarly, if conversion from unconfined to confined conditions is occurring,

$$SF_{i,j} = \frac{(h_{i,j} - CH_{i,j}) SF1_{i,j} + (CH_{i,j} - h_{o,i,j}) SF2_{i,j}}{(h_{i,j} - h_{o,i,j})} \quad (43)$$

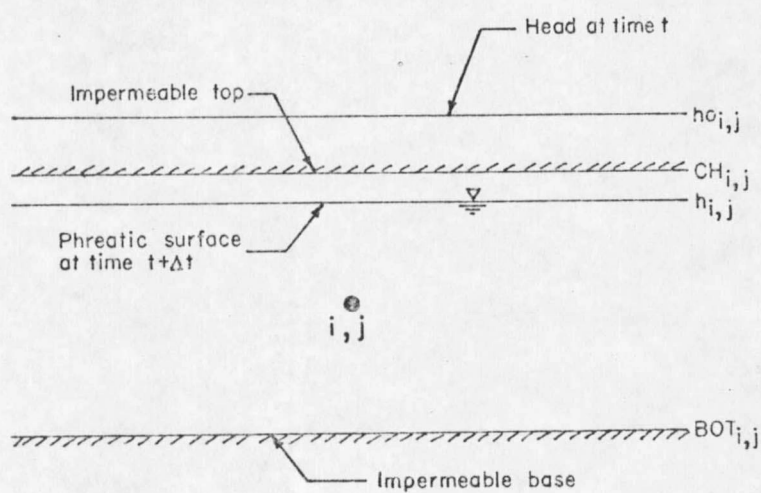


Fig. 13. Cross sectioned view of node i,j , which is undergoing conversion from confined to unconfined conditions.

Calculation of Transmissivity Factors

Unconfined, Homogeneous Aquifer Flow

For regions of unconfined flow in a homogeneous aquifer, the transmissivity at any point between two nodes A and B varies directly with the saturated aquifer thickness. The problem is to determine an average transmissivity that is representative of the length of flow between the nodes. The problem is summarized in Figure 14, a cross section of an unconfined aquifer through nodes A and B.

Butler (1957) used a geometric mean for calculating a representative transmissivity for flow from A to B. In this case.

$$T_{A-B} = [T_A \cdot T_B]^{1/2} \quad (44)$$

Using this in Equation (41) for calculating $TX_{i+1/2,j}$, for example,

$$TX_{i+1/2,j} = [(h_{i,j} - BOT_{i,j}) (HCX_{i,j}) \\ (h_{i+1,j} - BOT_{i+1,j}) (HCX_{i+1,j})]^{1/2} \frac{Y2_{1,j}}{X1_{i,j}}$$

where

BOT = elevation of aquifer bottom (L)

HCX = hydraulic conductivities in the x direction (L/T).

If the aquifer is homogeneous, $HCX_{i,j} = HCX_{i+1,j}$

Note that $h_{i,j}$ and $h_{i+1,j}$ are unknown values because they are heads at time $t+\Delta t$.

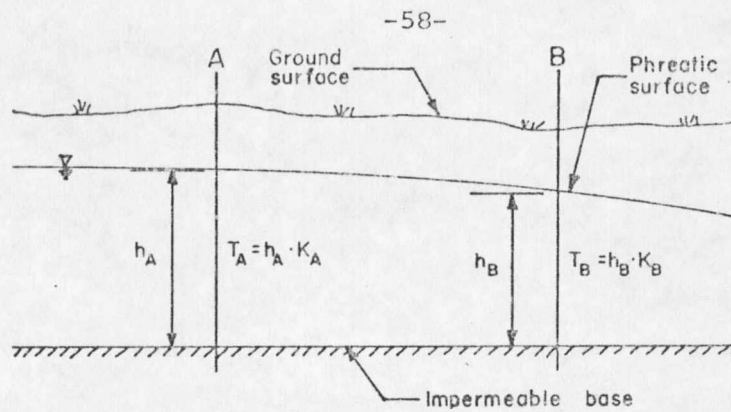


Fig. 14. Vertical cross section through nodes A and B in an unconfined aquifer.

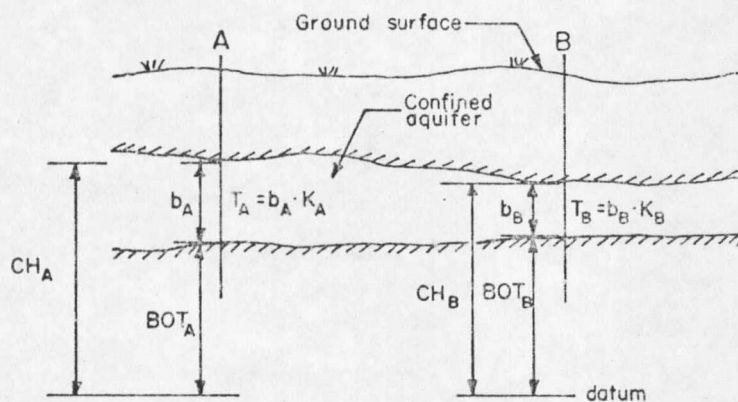


Fig. 15. Vertical cross section through nodes A and B in a confined aquifer.

Prickett and Lonquist (1971,1973) used geometric means for computing transmissivities in simulations of unconfined aquifer well flow problems. Their simulated drawdowns agreed closely with the theoretical drawdowns computed from Jacob's (1944) method for adjusting the Theis (1935) equation for variable transmissivity.

The geometric mean is used in the model for unconfined flow transmissivities in homogeneous regions.

Unconfined, Nonhomogeneous Aquifer Flow

In a nonhomogeneous, unconfined region where $K_A \neq K_B$, Huntton (1974) suggested that a harmonic mean of T_A and T_B is more representative of the transmissivity between A and B than the geometric mean because the former heavily weights the smaller hydraulic conductivity and saturated thickness. For the problem in Figure 14,

$$T_{A-B} = \frac{2T_A T_B}{T_A + T_B} \quad (45)$$

The harmonic mean is used for calculating transmissivities in nonhomogeneous, unconfined regions in the East Decker model, such as at the interface between clinker and alluvium.

Confined, Homogeneous or Nonhomogeneous Aquifer Flow

The harmonic mean is also used for computing internodal transmissivities in all regions of confined flow. For the

example situation shown in Figure 15, the transmissivity between A and B is

$$T_{A-B} = \frac{2T_A T_B}{T_A + T_B} \quad (45)$$

If the aquifer is homogeneous ($K_A = K_B$) and if $b_A = b_B$, then Equation (45) reduces to

$$T_{A-B} = T_A = T_B$$

Modeling confined aquifer flow with Equation (41), $TX_{i+1/2,j}$ is

$$TX_{i+1/2,j} = \frac{2[(CH_{i,j} - BOT_{i,j})(HCX_{i,j})][(CH_{i+1,j} - BOT_{i+1,j})(HCX_{i+1,j})]}{[(CH_{i,j} - BOT_{i,j})(HCX_{i,j})] + [(CH_{i+1,j} - BOT_{i+1,j})(HCX_{i+1,j})]} \cdot \frac{Y2_{i,j}}{X1_{i,j}}$$

As in the nonhomogeneous, unconfined flow case, the harmonic mean gives heavy weight to the smaller transmissivity when the node transmissivities are not equal.

Confined-Unconfined Flow Boundary

In an aquifer that is partly confined and partly unconfined, a problem in determination of transmissivity arises along the boundary between the confined and unconfined zones. Such an aquifer is shown in plan view in Figure 16. Figure 17 is a cross-sectional view through confined node i,j and unconfined node $i+1,j$.

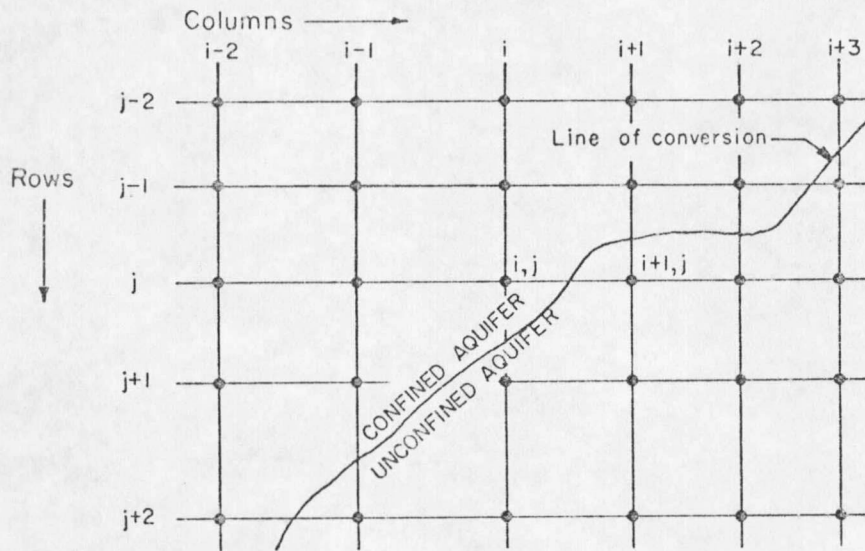


Fig. 16. Plan view of an aquifer that is confined in one portion and unconfined in the other.

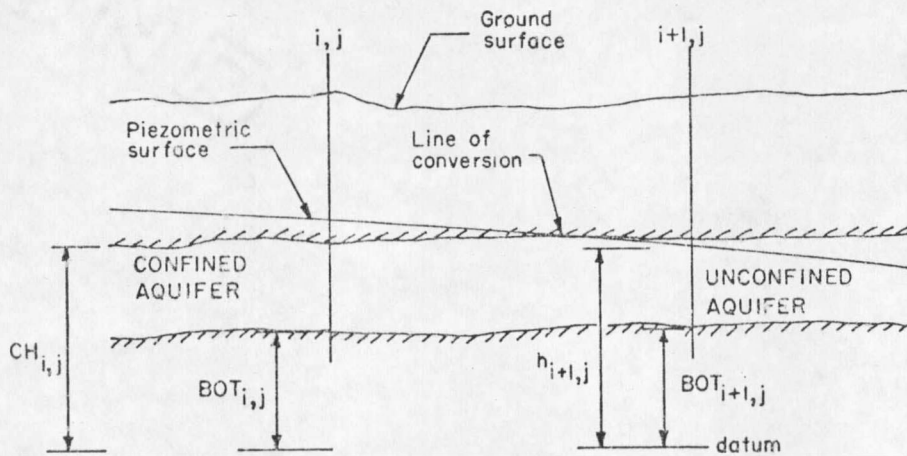


Fig. 17. Vertical cross section through nodes i, j and $i+1, j$ of the aquifer shown in Figure 16.

Here again, a harmonic mean is used for computing the internodal transmissivity. For example, $TX_{i+1/2,j}$ is calculated as

$$TX_{i+1/2,j} = \frac{2[(CH_{i,j} - BOT_{i,j})(HCX_{i,j})][(h_{i+1,j} - BOT_{i+1,j})(HCX_{i+1,j})]}{[(CH_{i,j} - BOT_{i,j})(HCX_{i,j})] + [(h_{i+1,j} - BOT_{i+1,j})(HCX_{i+1,j})]}$$

$$\frac{Y2_{i,j}}{X1_{i,j}}$$

Solution of Equation (41)

At every new time increment, Equation (41), with the appropriate factors, is applied to each node in the aquifer system. For even a small nodal array this will result in a large number of simultaneous equations. As pointed out by Remson et al. (1971), solution of such a set of equations calls for the use of an iterative solution that solves the entire equation matrix by parts. The alternating direction implicit method of Peaceman and Rachford (1955) coupled with an iterative procedure is such a solution. It can use fairly large time increments and still converge to an accurate solution rapidly. It is also accurate in dealing with problems involving time dependent sources and sinks (Huntoon, 1974).

For solution of the node equations in the model presented here, the modified iterative alternating direction implicit method (MIADI) of Prickett and Lonquist (as discussed in Prickett, 1975), which uses

a combination of the Peaceman and Rachford alternating direction implicit method (Peaceman and Rachford, 1955) and the Gauss-Seidel iterative method (see Carnahan et al., 1969), was chosen. The method is an alternating direction technique because it first solves for all node heads by calculations involving columns of nodes (i), then for all node heads by row calculations (j). One column-row solution cycle constitutes one full iteration. The method keeps cycling through iterations until the head solutions converge within a specified error criterion.

Solution for the node heads at time $t+\Delta t$ is begun by roughly predicting the heads with a linear extrapolation from the heads at time t and $t-\Delta t$. Then the first column-row iteration is performed using the predicted head values. The linear prediction (Prickett and Lonquist, 1971) was found to greatly decrease the number of iterations required for convergence in well flow problems.

In calculations by node columns, for every other iteration node columns are solved column by column in a sequence from left to right. Columns are solved in a sequence from right to left in alternate iterations. Alternating the direction of column processing increases the overall convergence efficiency.

Before solving for heads in any column i , the equations for all NR (Number of Rows in grid array) nodes in the column are written using Equation (41). The node heads from columns $i-1$ and $i+1$ that are

included in the column i node equations are the best current estimates of those heads at time $t+\Delta t$; they are head solutions from the most recent iteration or current half iteration (or the predicted heads if this is iteration one) and therefore are known values. As a result, each node equation in column i , except for the equations for nodes $i,1$ and i,NR , contains three unknown head values, $h_{i,j-1}$, $h_{i,j}$, and $h_{i,j+1}$. Because the outer boundary of the grid is automatically treated as an impermeable boundary, the node equations for $i,1$ and i,NR contain only two unknowns each, respectively $h_{i,1}$ and $h_{i,2}$, and $h_{i,NR-1}$ and $h_{i,NR}$. (This point will be covered more fully later in this chapter.) Therefore, column i has NR node equations in NR unknowns.

As an example, for column calculations where columns are being processed in a sequence from left to right, Equation (41) is written for node i,j (assuming i,j is not a boundary node) in the solution of column i heads (see Figure 18) as

$$\begin{aligned}
 & TX_{i+1/2,j} (h_{i+1/2,j}^n - h_{i,j}^{n+1/2}) \\
 & + TX_{i-1/2,j} (h_{i-1/2,j}^{n+1/2} - h_{i,j}^{n+1/2}) \\
 & + TY_{i,j+1/2} (h_{i,j+1/2}^{n+1/2} - h_{i,j}^{n+1/2}) \\
 & + TY_{i,j-1/2} (h_{i,j-1/2}^{n+1/2} - h_{i,j}^{n+1/2}) \\
 & = SF_{i,j} \frac{h_{i,j}^{n+1/2} - h_{o,i,j}}{\Delta t} + Q_{i,j}
 \end{aligned} \tag{46}$$

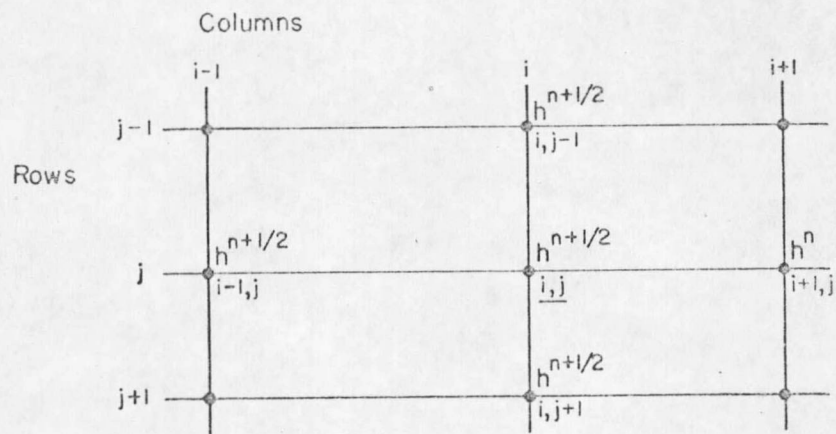


Fig. 18. The node head values included in Equation (46).

where superscript n indicates a value from the previous iteration, iteration n , superscript $n+1/2$ indicates a value from the current half iteration (column or row calculations constitute a half iteration), and therefore

$h_{i+1,j}^n$ = known node head at time $t+\Delta t$, from preceding iteration (L)

$h_{i-1,j}^{n+1/2}$ = known node head at time $t+\Delta t$, from preceding solution of column $i-1$ (L)

$h_{i,j}^{n+1/2}$, $h_{i,j+1}^{n+1/2}$, $h_{i,j-1}^{n+1/2}$ = unknown node head values associated with node i,j in column i at time $t+\Delta t$ (L)

$h_{o,i,j}$ = known head at node i,j at time t (L)

Before column processing is ever begun, all of the internodal transmissivity factors for the unconfined zones in the array are calculated by using the best available estimates of the node heads at time $t+\Delta t$. For example, $TY_{i,j-1/2}$ is calculated from $h_{i,j-1}^n$ and $h_{i,j}^n$, and $TX_{i-1/2,j}$ is calculated from $h_{i-1,j}^n$ and $h_{i,j}^n$. Doing this rather than using unknown node heads avoids turning Equation (46) into a second order equation. It must be stressed that these factors remain constant during the processing of all columns.

By combining known and unknown terms in Equation (46) and simplifying, the following equation results:

$$AA_{i,j} h_{i,j-1}^{n+1/2} + BB_{i,j} h_{i,j}^{n+1/2} + CC_{i,j} h_{i,j+1}^{n+1/2} = DD_{i,j} \quad (47)$$

where

$$AA_{i,j} = TY_{i,j-1/2}$$

$$BB_{i,j} = -TX_{i+1/2,j} - TX_{i-1/2,j} - TY_{i,j+1/2} - TY_{i,j-1/2} \\ - (SF_{i,j}/\Delta t)$$

$$CC_{i,j} = TY_{i,j+1/2}$$

$$DD_{i,j} = -(TX_{i+1/2,j} h_{i+1,j}^n) - (TX_{i-1/2,j} h_{i-1,j}^{n+1/2}) \\ - (SF_{i,j} \frac{h_{i,j}^{ho}}{\Delta t}) + Q_{i,j}$$

Writing all of the column i node equations in this form produces the following tridiagonal matrix system for column i:

$$\begin{bmatrix} BB_{i,1} & CC_{i,1} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ AA_{i,2} & BB_{i,2} & CC_{i,2} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & AA_{i,3} & BB_{i,3} & CC_{i,3} & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & & & & & & & & \\ \cdot & & & & & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & AA_{i,NR-1} & BB_{i,NR-1} & CC_{i,NR-1} \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & & AA_{i,NR} & BB_{i,NR} \end{bmatrix}$$

$$\begin{bmatrix} h_{i,1}^{n+1/2} \\ h_{i,2}^{n+1/2} \\ h_{i,3}^{n+1/2} \\ \cdot \\ \cdot \\ h_{i,NR-1}^{n+1/2} \\ h_{i,NR}^{n+1/2} \end{bmatrix} = \begin{bmatrix} DD_{i,1} \\ DD_{i,2} \\ DD_{i,3} \\ \cdot \\ \cdot \\ DD_{i,NR-1} \\ DD_{i,NR} \end{bmatrix}$$

In the MIADI method the matrix is solved by Gaussian elimination using a Thomas algorithm (described in Van Rosenberg, 1969). After solution of the column i heads, columns $i+1$ through NC (Number of Columns in grid array) are solved, column by column, in an identical manner.

On completion of all column calculations (the $n+1/2$ iteration), the row calculations (the $n+1$ iteration) are begun by first calculating all internodal transmissivity factors using the column calculation head solutions. Then the node head equations are solved, row by row, in a manner exactly identical to that of the column solutions. Completion of row calculations completes one full column-row iteration. If the solution convergence criterion has not been met, another iteration is begun.

The convergence check used with the MIADI method involves checking the sum of the changes in head solutions during a full iteration. If the sum of the absolute values of the head solution changes for all nodes (i.e. $|h^n - h^{n+1/2}| + |h^{n+1/2} - h^{n+1}|$ summed for all nodes) during an iteration is less than a specified convergence criterion, convergence is assumed to have occurred. Calculations for the next time increment are then begun.

Prickett and Lonnquist (1971) use the following empirical relation for calculating a simulation convergence criterion:

$$\text{error} = \frac{|Q| \Delta t_i}{10SF_a} \quad (49)$$

where

error = convergence criterion

$|Q|$ = total average net withdrawal rate during simulation
(L^3/T)

Δt_i = initial time increment used in simulation (T)

SF_a = average storage factor for all nodes (L^2)

This equation is based on the use of time increments that are successively increased by a factor of 1.2. In other words,

$$\Delta t_1 = 1.2 \Delta t_0$$

$$\Delta t_2 = 1.2 \Delta t_1$$

Because Q (discharge into the mine cut) varies during a simulation of the East Decker site, Equation (49) could be used only as a rough approximation. Q was estimated by making an initial run with the model, getting discharge rates printed, and then selecting an average discharge value for the period of simulation. This value was then inserted into Equation (49), and the resulting error criterion was used in subsequent computer runs.

Aquifer Boundaries

Zero Flux (Impermeable) Boundaries

Zero flux lateral boundaries occur where an aquifer is bounded by impermeable material. At the East Decker site, such a boundary is believed to exist along the faults on the southeast side of the area (Van Voast and Hedges, 1976). The fault is shown in Figure 2.

Across a zero flux boundary, $K = 0$. Referring to Figure 19, in applying Equation (41) to node i, j ,

$$TX_{i+1/2, j} = 0$$

Similarly, for node $i, j+1$,

$$TX_{i+1/2, j+1} = 0$$

$$TY_{i, j+3/2} = 0$$

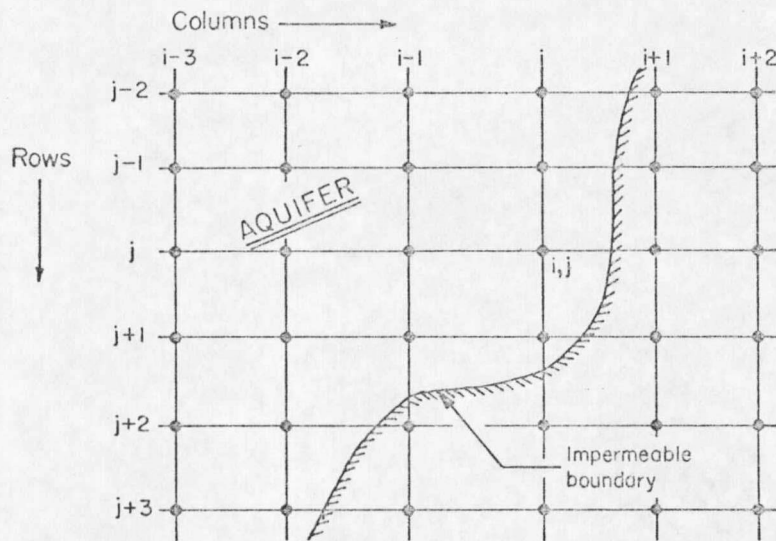


Fig. 19. Aquifer with an impermeable lateral boundary.

It should be noted that the model automatically treats the outer edges of the grid as impervious boundaries. This must be done in order to permit the outer node equations to be written.

Surface Water Boundaries

Surface water boundaries occur where an aquifer is directly linked with a surface water source. At East Decker, the alluvial aquifers along the shore of the Tongue River Reservoir are considered to be directly linked with the reservoir (USGS, 1977). It is assumed that the reservoir level is relatively independent of aquifer inflow/outflow.

If the reservoir level is independent of groundwater flow, then at the junction of the aquifer and reservoir

$$\frac{\partial h}{\partial t} = 0$$

in either Equation (14) or (18).

A surface water boundary is handled by assigning very large storage coefficients to boundary nodes (Prickett and Lonquist, 1971). Using a very large storage coefficient in Equation (41) maintains an essentially constant head $[(h_{i,j} - h_{o,i,j}) \approx 0]$.

Should the boundary water level change with time (such as during reservoir filling in the spring), the head values at the boundary nodes must be reassigned during a simulation.

Interaquifer Boundaries

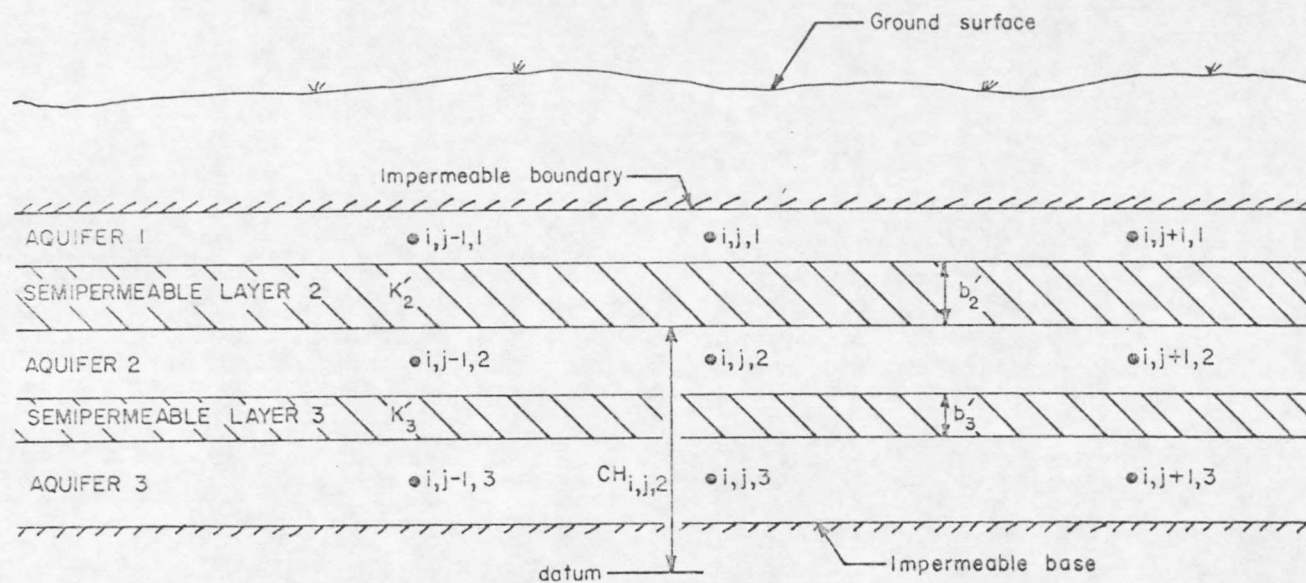
Because the beds separating the East Decker coal seams are siltstones and sandstones (USGS, 1977), and because so little is known of the hydraulic properties of these layers, a steady state approximation for interaquifer leakage was utilized. If the interaquifer material is not a clay or silt material with a high storage coefficient, this assumption is often valid (Hantush, 1960).

Figure 20 is a cross section through column i of a three layer node array that would be used for modeling three confined aquifers that leak to each other through semipermeable layers. It should be noted that the nodes for each aquifer lie directly above and below the nodes of the other aquifers. Therefore, all nodes in a "stack" of aquifer nodes have the same horizontal dimensions associated with them.

Interaquifer leakage is handled in the model by modeling each aquifer individually and linking overlying nodes with the steady state leakage terms of Equations (32) and (33). As an example, in writing Equation (41) for node i,j,2 (2 indicates aquifer 2), which is linked by leakage to aquifers 1 and 3, leakage is calculated as

$$L_{i,j,2} = [K_2'(h_{i,j,2} - h_{i,j,1})/b_2' + K_3'(h_{i,j,2} - h_{i,j,3})/b_3'] (X_{i,j}^2 - Y_{i,j}^2)$$

This is included in term $Q_{i,j,2}$.



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Fig. 20. Vertical cross section through column i of a three-layer node array used for modeling a three aquifer system with semipermeable interaquifer layers.

In the model the vertical conductivity terms K'_2 and K'_3 , the layer thicknesses b'_2 and b'_3 , and the node dimensions are combined into the factors (after Prickett and Lonquist, 1971)

$$R_{i,j,2} = (K'_2/b'_2) (X_{i,j}^2 Y_{i,j}^2) \quad (51a)$$

$$R_{i,j,3} = (K'_3/b'_3) (X_{i,j}^2 Y_{i,j}^2), \quad (51b)$$

values which remain constant with time. Doing this minimizes the number of calculations and computer memory required.

It is possible that some or all of the aquifers are unconfined. If, for example, aquifer 2 is unconfined at node $i,j,2$ ($h_{i,j,2} < CH_{i,j,2}$, the elevation of the aquifer top) then (Trescott et al., 1976)

$$L_{i,j,2} = (CH_{i,j,2} - h_{i,j,1}) (R_{i,j,2}) + (h_{i,j,2} - h_{i,j,3}) (R_{i,j,3}) \quad (52)$$

This relation assumes that the vertical hydraulic conductivity of the aquifer 2 material is much greater than K'_2 , in which case the head drop due to vertical leakage through the aquifer 2 material above the phreatic surface is negligible in comparison to the head drop across semipermeable layer 2.

Solution of a multi-aquifer system for heads at time $t+\Delta t$ is obtained with a simple variation of the MIADI method (Prickett and Lonquist, 1971). An iteration is begun by solving by columns all of the node equations in the top aquifer ($k=1$), followed by column

solutions of all aquifer 2 ($k=2$) node equations, and so on through the bottom aquifer ($k=NL$, the total number of aquifer layers). Row solutions are done in an identical manner, starting with aquifer 1.

The problem in Figure 20 will be used as an example of how node equations are written for multi-aquifer systems. Applying Equation (46) to node $i,j,2$ for column calculations (iteration $n+1/2$) that are processed in a sequence from column $i=1$ to $i=NC$,

$$\begin{aligned}
 & TX_{i+1/2,j,2} (h_{i+1,j,2}^n - h_{i,j,2}^{n+1/2}) + \\
 & TX_{i-1/2,j,2} (h_{i-1,j,2}^{n+1/2} - h_{i,j,2}^{n+1/2}) + \\
 & TY_{i,j+1/2,2} (h_{i,j+1,2}^{n+1/2} - h_{i,j,2}^{n+1/2}) + \\
 & TY_{i,j-1/2,2} (h_{i,j-1,2}^{n+1/2} - h_{i,j,2}^{n+1/2}) \\
 & = SF_{i,j,2} \frac{h_{i,j,2}^{n+1/2} - h_{i,j,2}^n}{\Delta t} + \quad (53) \\
 & (h_{i,j,2}^{n+1/2} - h_{i,j,1}^{n+1/2})R_{i,j,2} + (h_{i,j,2}^{n+1/2} - h_{i,j,3}^n)R_{i,j,3} \\
 & + Q_{i,j,2}
 \end{aligned}$$

where

$h_{i+1,j,2}^n, h_{i,j,3}^n$ = known node heads from previous iteration, iteration n (L)

$h_{i-1,j,2}^{n+1/2}$ = known head from preceding solution of column $i-1$ (L)

$h_{i,j,1}^{n+1/2}$ = known head from iteration $n+1/2$ solution of the overlying nodes (L)

$h_{o,i,j,2}$ = known head at time t (L)

$h_{i,j-1,2}^{n+1/2}$, $h_{i,j,2}^{n+1/2}$, $h_{i,j+1,2}^{n+1/2}$ = unknown node heads in column i , aquifer 2 (L)

As with Equation (46), Equation (53) can be rearranged into the form of Equation (47),

$$AA_{i,j,2} h_{i,j-1,2}^{n+1/2} + BB_{i,j,2} h_{i,j,2}^{n+1/2} + CC_{i,j,2} h_{i,j+1,2}^{n+1/2} = DD_{i,j,2}$$

in which

$$AA_{i,j,2} = TY_{i,j-1/2,2}$$

$$BB_{i,j,2} = -TX_{i+1/2,j,2} - TX_{i-1/2,j,2} - TY_{i,j+1/2,2} - TY_{i,j-1/2,2} - (SF_{i,j,2}/\Delta t) - R_{i,j,2} - R_{i,j,3}$$

$$CC_{i,j,2} = TY_{i,j+1/2,2}$$

$$DD_{i,j,2} = -(TX_{i+1/2,j,2} h_{i+1,j,2}^n) - (TX_{i-1/2,j,2} h_{i-1,j,2}^{n+1/2}) - (SF_{i,j,2} \frac{h_{o,i,j,2}}{\Delta t}) - (R_{i,j,2} h_{i,j,1}^{n+1/2}) - (R_{i,j,3} h_{i,j,3}^n) + Q_{i,j,2}$$

This conforms with the MIADI solution method.

Equations for layers 1 and 3 will be the same except one or the other of the leakage terms will not be present.

Aquifer Contact Boundaries

At East Decker the two upper coal seam aquifers, the D-1 Upper and D-1 Lower seams (Decker Coal Company designations), are in contact in the alluvial material along Deer Creek and are also joined together where the seams combine at the south end of the Tongue River Reservoir (Van Voast and Hedges, 1975; see Figure 2). During and after mining all three coal seam aquifers will contact in the spoils material, where a single aquifer is expected to form. In order for the aquifer junctions to be realistically modeled, the aquifer flows must be simulated simultaneously.

A plan view of a three dimensional node array that would be used for modeling two aquifers that join together is shown in Figure 21. Figure 22 is a cross-sectional view through the column i nodes of both aquifers. Note that no nodes are shown for $(i,j,1)$, $(i,j-1,1)$, etc. in Figure 22. Though these nodes would actually be in the computer memory, they would be "dummy" nodes - H_{CX} and H_{CY} (hydraulic conductivities in the x and y directions) would be set equal to 0.

This multi-aquifer system would be modeled in a fashion identical to that of multiple leaky aquifers, solution of the node equations being obtained with the variation of the MIADI method used for leaky aquifers.

A combined aquifer node adjacent to the aquifer junction must be linked with adjacent nodes in both aquifers. Node $i,j,2$, for instance,

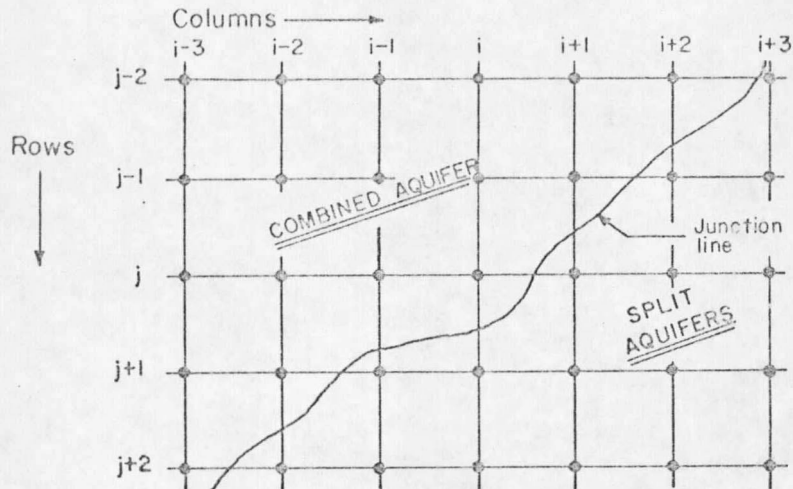


Fig. 21. Plan view of node array used to represent two aquifers that join together.

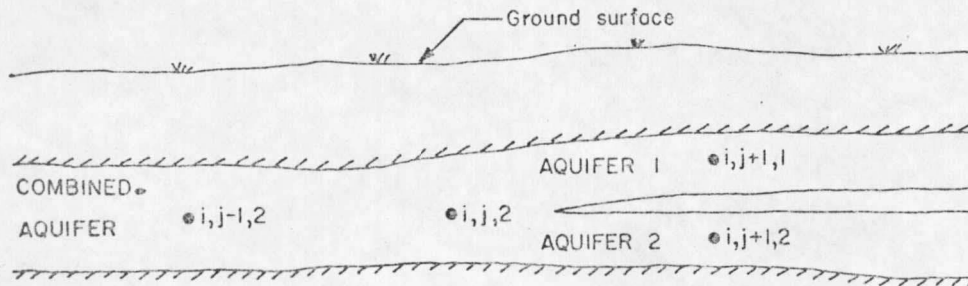


Fig. 22. Vertical cross section through column i of the node array shown in Figure 21.

must be linked with nodes $i, j+1, 1$ and $i+1, j, 1$, as well as with the adjacent nodes in aquifer 2. In applying Equation (46) to node $i, j, 2$, additional terms for $h_{i, j+1, 1}$ and $h_{i+1, j, 1}$ must be included. For column calculations (iteration $n+1/2$) that are processed in a left to right column sequence, the node equation for $i, j, 2$ is

$$\begin{aligned}
 & TX_{i+1/2, j, 2} (h_{i+1, j, 2}^n - h_{i, j, 2}^{n+1/2}) \\
 & + TX_{i-1/2, j, 2} (h_{i-1, j, 2}^{n+1/2} - h_{i, j, 2}^{n+1/2}) \\
 & + TY_{i, j+1/2, 2} (h_{i, j+1, 2}^{n+1/2} - h_{i, j, 2}^{n+1/2}) \\
 & + TY_{i, j-1/2, 2} (h_{i, j-1, 2}^{n+1/2} - h_{i, j, 2}^{n+1/2}) \\
 & = TR_{i+1/2, j, 3/2} (h_{i, j, 2}^{n+1/2} - h_{i+1, j, 1}^{n+1/2}) \\
 & + TB_{i, j+1/2, 3/2} (h_{i, j, 2}^{n+1/2} - h_{i, j+1, 1}^{n+1/2}) \\
 & + SF_{i, j, 2} \frac{h_{i, j, 2}^{n+1/2} - h_{o, i, j, 2}}{\Delta t} + Q_{i, j, 2} \quad (55)
 \end{aligned}$$

where

$TR_{i+1/2, j, 3/2}$ = the internodal transmissivity factor between nodes $i, j, 2$ and $i+1, j, 1$, computed as a harmonic mean (L^2/T)

$TB_{i, j+1/2, 3/2}$ = the internodal transmissivity factor between nodes $i, j, 2$ and $i, j+1, 1$, computed as a harmonic mean (L^2/T)

$h_{i+1, j, 2}^n$ = known head value from previous iteration, iteration n (L)

$h_{i-1,j,2}^{n+1/2}$ = known head value from preceding solution of column $i-1$ in aquifer 2 (L)

$h_{i,j+1,1}^{n+1/2}, h_{i+1,j,1}^{n+1/2}$ = known heads from current iteration $(n+1/2)$ solution of aquifer 1 nodes (L)

$h_{o,i,j,2}$ = known head at time t (L)

$h_{i,j-1,2}^{n+1/2}, h_{i,j,2}^{n+1/2}$ = unknown node heads in column i , aquifer 2 (L)

$h_{i,j+1,2}^{n+1/2}$

Combining known and unknown terms and rearranging Equation (55)

results in

$$\begin{aligned} & AA_{i,j,2} h_{i,j-1,2}^{n+1/2} + BB_{i,j,2} h_{i,j,2}^{n+1/2} + CC_{i,j,2} h_{i,j+1,2}^{n+1/2} \\ & = DD_{i,j,2} \end{aligned} \quad (54)$$

where

$$AA_{i,j,2} = TY_{i,j-1/2,2}$$

$$\begin{aligned} BB_{i,j,2} = & -TX_{i+1/2,j,2} - TX_{i-1/2,j,2} - TY_{i,j+1/2,2} \\ & - TY_{i,j-1/2,2} - (SF_{i,j,2}/\Delta t) - TR_{i+1/2,j,3/2} \\ & - TB_{i,j+1/2,3/2} \end{aligned}$$

$$CC_{i,j,2} = TY_{i,j+1/2,2}$$

$$\begin{aligned} DD_{i,j,2} = & -(TX_{i+1/2,j,2} h_{i+1,j,2}^n) - (TX_{i-1/2,j,2} h_{i-1,j,2}^{n+1/2}) \\ & - (SF_{i,j,2} \frac{h_{i,j,2}^{ho}}{\Delta t}) - (TR_{i+1/2,j,3/2} h_{i+1,j,1}^{n+1/2}) \\ & - (TB_{i,j+1/2,3/2} h_{i,j+1,1}^{n+1/2}) + Q_{i,j,2} \end{aligned}$$

Equations for all nodes in aquifers 1 and 2 adjacent to the aquifer junction would be written in a similar manner.

Mine Cut Boundary

The mine presents the most difficult boundary in the system to treat. In discussing how it is handled in the model, reference will be made to Figure 23, a plan view of an aquifer grid with an initial mine cut (an idealization of the East Decker mine), and Figure 24, a cross section through the row j nodes. The flow arrows in Figure 23 indicate the direction of premining groundwater flow. Figure 24 is an idealized cross section of the East Decker stratigraphy near the mouth of Middle Creek.

As pointed out in Chapter II, it is believed that flow into the mine cut can be split into three components (Van Voast and Hedges, 1975; USGS, 1977): intercepted natural flow, flow from storage, and induced flow from the Tongue River Reservoir. Flow to the cut from the reservoir will begin through the alluvium when the cut floor reaches below the reservoir surface level (see Figure 24). When the initial cut reaches the D-2 coal, water will also begin to

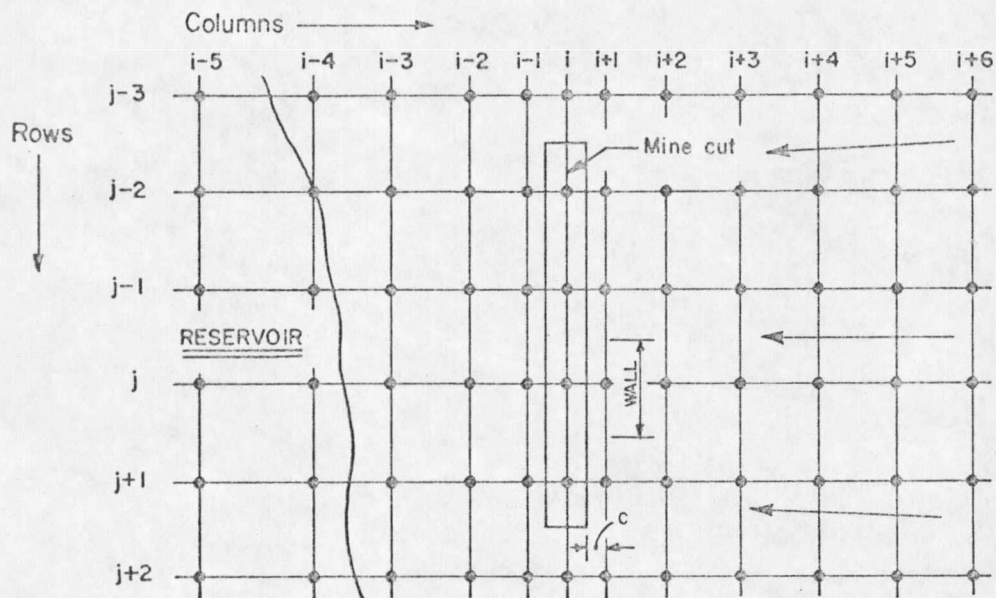


Fig. 23. Plan view of a mine cut with an aquifer node grid.

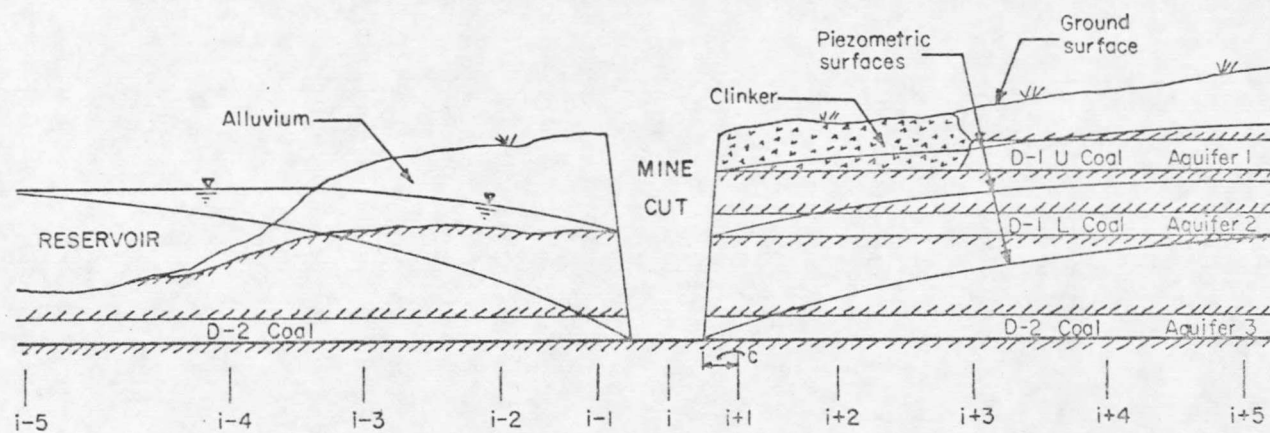


Fig. 24. Cross section through row j of the node grid in Figure 23.

flow through it from the reservoir. Water discharged on the other side of the cut will consist of intercepted natural flow plus flow derived from storage in the aquifers.

Induced reservoir to mine cut flow. Once the initial mine cut is below the reservoir level, flow will begin through the alluvium from the reservoir to the mine cut. This flow is calculated in the manner of USGS (1977) with the steady flow Dupuit equation (Todd, 1959).

$$q_r = \frac{K}{f} \left[\frac{h_s^2}{2} - \frac{H_r^2}{2} \right] \quad (20)$$

where

q_r = discharge rate from the reservoir per unit length of mine cut (L^2/T)

K = hydraulic conductivity of the alluvium (L/T)

f = distance from the mine cut to the reservoir (L)

h_s = height of the water table above the aquifer floor, which is assumed horizontal (L)

H_r = height of the reservoir surface above the aquifer bottom (L)

Note that this equation assumes an instantaneous opening of the cut down to and below the level of the D-1L floor.

The D-2 coal is also linked with the reservoir (Van Voast and Hedges, 1975). Steady flow from the reservoir to the mine through the coal per unit length of mine cut is estimated with Darcy's law as

$$q_r = T \frac{H_r - C}{f} \quad (55)$$

where

T = transmissivity of coal $\cdot (L^2/r) = Kb$

b = thickness of coal seam (L)

H_{r-c} = height of reservoir surface above the top of the coal seam (L)

and other terms are as defined with the previous equation. Equation (55) can only be considered a rough estimate of flow through the coal: it does not account for the seepage face that will develop at the cut face (see Figure 24. Unfortunately, no analytical method exists for determining steady flow rates through a length of confined-unconfined aquifer.

To simulate reservoir to mine cut flow in the problem in Figure 23, the $k=2$ level (D-1L) and $k=3$ level (D-2), 1-1 nodes adjacent to the cut would be pumped at the rate of

$$Q_r = q_r \text{ WALL}, \quad (56)$$

where q_r is calculated from either Equation (20) (for the D-1L nodes) or Equation (55) (for the D-2 nodes), and WALL is the length of mine cut adjacent to a node. This Q would be included in Equation (41) for each of these nodes.

Steady flow rates are used for reservoir to initial mine cut flow because the initial cut will be very close to the reservoir and there is little water in storage between the cut and the reservoir.

Intercepted natural flow. Flow into the other side of the mine cut will consist of intercepted flow plus flow from storage from both the coal and the overlying alluvium and clinker. However, because of the limited extent of the alluvial and clinker aquifers on the east side of the cut, their contribution to mine cut flow is ignored.

The natural flow rate in the coal is calculated with Darcy's law as

$$q_n = TI \quad (57)$$

where

- q_n = the natural flow rate per unit length of cut (L^2/T)
- I = natural (premining) hydraulic gradient (dimensionless)
- T = transmissivity of coal (L^2/T)

To simulate intercepted flow in the Figure 23 problem, column $i+1$ nodes adjacent to the mine cut (at the $k=2$ and $k=3$ levels) would be pumped at the rate of

$$Q_{n,i+1,j} = q_n \text{ WALL} \quad (58)$$

where WALL is the length of the mine cut adjacent to the node. Intercepted natural flow is assumed steady.

Flow from storage. As stated earlier (Chapter II), flow from storage in the coal aquifers can be simulated by pumping adjacent nodes at steady rates (Davis, 1977). However, this requires knowledge of the actual drainage rates into the mine cut. Since there is no such data for the East Decker Mine, a nonsteady simulation for flow from storage was used.

With the nonsteady simulation, flow from storage is estimated with a bank storage discharge equation, as Davis (1976) and USGS (1977) have done. If an aquifer is unconfined, or if it contains relatively little water in confined storage (i.e., when the piezometric surface is not much higher than the aquifer top, and $S_y \gg S_c$), then the Haushild and Kruse (1962) second drawdown solution for unconfined conditions (see Chapter 2) may be used:

$$h = H \left[\frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du \right]^{1/2} \quad (31)$$

where

h = height of the water table above the aquifer floor (L)

H = original height of the water table above the aquifer floor, assumed uniform (L)

$$u = \frac{x}{(4Kht/2S_y)^{1/2}}$$

K = aquifer hydraulic conductivity (L/T)

x = distance from the mine cut

t = time since opening of the cut (T)

S_y = specific yield (dimensionless)

Nonsteady discharge from storage is calculated with Darcy's law using the derivative of h with respect to x at some $x > 0$. For example, the nonsteady discharge rate from storage for node $i+1, j, 2$ (see Figure 24) is calculated as

$$Q_{st\ i+1,j,2} = T_{i+1,j,2} \text{ WALL } \left(\frac{\partial h}{\partial x} \right)_{x=c} \quad (59)$$

where $T_{i+1,j,2}$ is the transmissivity at the center of node $i+1,j,2$ (calculated with the saturated thickness of the node), c is the distance from the mine cut to the center of node $i+1,j,2$, and $\frac{\partial h}{\partial x}$ is obtained by differentiating Equation (31). The derivative of Equation (31) with respect to x is (Buck, 1965)

$$\begin{aligned} \frac{\partial h}{\partial x} &= \frac{\partial h}{\partial u} \frac{\partial u}{\partial x} \\ &= \frac{1}{2} \left(\frac{2}{\sqrt{\pi}} \right)^{1/2} H \left(\int_0^u e^{-u^2} du \right)^{-1/2} e^{-u^2} \frac{1}{(4Kht/2S_y)^{1/2}} \end{aligned} \quad (60)$$

Since Equation (60) cannot be evaluated at $x=0$, it must be evaluated at some distance from the mine cut, distance c in this case. Furthermore, since there is no analytical solution for Equation (60), it must be solved by a numerical technique. Simpson's One-Third Rule was used for solution, and it is contained in a computer program subroutine (subroutine EST).

Nonsteady flow from an aquifer containing a significant amount of water in confined storage (i.e., the piezometric surface is well above the aquifer top), such as the D-2 coal aquifer at East Decker, may be

initially estimated with the use of Stallman's solution for aquifer drawdown. Stallman's solution for complete drawdown ($H=H_0$) is

$$s = H \left[1 - \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du \right] \quad (21)$$

where

s = drawdown (L)

H = original height of the aquifer potential head, measured from the aquifer floor (L)

$$u = \frac{x}{(4T_0 t/S_c)^{1/2}}$$

T_0 = original aquifer transmissivity

S_c = confined storage coefficient (dimensionless)

It is believed (see Chapter 2) that this equation is valid when a coal seam is first exposed. The flow rate from storage is calculated using Darcy's law and the slope (gradient) of s . Calculating the discharge from storage for node $i+1, j, 3$ (see Figure 24), for example,

$$Q_{st, i+1, j, 3} = T_{i+1, j, 2} \text{ WALL} \cdot \left(\frac{\partial s}{\partial x} \right)_{x=c} \quad (61)$$

where (Buck, 1965)

$$\frac{\partial s}{\partial x} = -H \left[\frac{2}{\sqrt{\pi}} e^{-u^2} \frac{1}{(4T_0 t/S_c)^{1/2}} \right] \quad (62)$$

Again, $Q_{st\ i+1,j,3}$ is added to $Q_{n\ i+1,j,3}$ to give the total discharge rate from node $i+1,j,3$.

As time goes on, Equation (21) will become less valid as more of the aquifer converts to unconfined conditions and more flow comes out of unconfined storage. It is possible that this can be accounted for by either switching to the use of Equation (59), or by combining Equations (59) and (61). This is discussed further in the next chapter.

In the solution of the node equations, nonsteady flow rates are recalculated before each half iteration (column or row solution) using the best available estimates for transmissivities at time $t+\Delta t$. They are calculated at the same time internodal transmissivities are calculated.

Computer Program Listing

In Appendix B the computer program for the East Decker Groundwater model is listed. It is written in Fortran IV compatible with the University's Xerox Sigma 7 Fortran IV compiler.

The program consists of the main program CENTRAL and five subprograms. CENTRAL contains the basic MIADI solution procedure and prints out solution results. It calls on five subprograms for input into the solution procedure.

Subroutine VALUE reads the initial node data into computer memory, and changes data when appropriate. Subroutine TRANS calculates inter-nodal transmissivities before each half iteration (column and row solutions). FLOW calculates nonsteady discharge into the mine cut, calling on subroutine EST to solve the integral in Equation (60). FLOW is called at the same point in the program as TRANS, just before each half iteration. Subprogram LINK combines (joins) aquifer flows where needed, and is called during each half iteration.

Given with the listing are the definitions of the program variables. The variables match text variables as closely as possible. A fairly complete explanation of the program routines is also given in the listing.

CHAPTER IV

APPLICATION OF THE MODEL TO THE EAST DECKER SITE

The Premining Aquifer System

The East Decker aquifers that will be significantly affected by mining are the alluvial aquifers along Deer Creek and the shore of the Tongue River Reservoir, the clinker aquifers, and the coal and sandstone aquifers in the upper portion of the Tongue River Member of the Fort Union Formation (Van Voast and Hedges, 1975). The most extensive and continuous of these are the three coal aquifers, the object of the mining. Some of the coal aquifers are linked directly with the alluvial and clinker aquifers. The sandstone aquifers occur in discontinuous lenses that appear to be isolated from the other aquifers (USGS, 1977).

Geologic Features

The D-2 (Decker Coal Co. designation) coal is the deepest of the three coal seams. In some parts of the mine area the seam is well over 350 feet below land surface. It is the most laterally continuous of the three seams. Minor outcropping occurs along the reservoir shore north of the mine (Van Voast and Hedges, 1975).

The D-1L and D-1U coal seams are much closer to the surface and are more discontinuous. Both seams subcrop into the alluvium along Deer Creek and along the Tongue River Reservoir shore, resulting in the alluvium acting as a junction between the two coal aquifers (based

on interpretation of Peter Kiewit Sons, Inc. drill hole data; see Figures 25 and 26). The two seams merge together as a single bed within the modeled area, southwest of the mine proper, along a line roughly following the course of the Tongue River.

Extensive clinker deposits occur where the D-IU coal has burned, mostly in the outcrop area along Deer Creek (see Figure 25). Based on structural maps (Peter Kiewit Sons, Inc.) and groundwater maps (Van Voast and Hedges, 1975), it is believed that upward folding of the strata has left much of the clinker south of Deer Creek dry. This folded area forms a high, dry pocket in the D-IU aquifer.

The interburden between coal seams changes considerably in thickness, varying as much as 100 feet over a distance of a mile (USGS, 1977). Interburden thicknesses are greatest in the eastern mine area and tend to decrease in the westward direction. Since the Tongue River Member was formed from sediments deposited by an alluvial flood plain system (Widmayer, 1977), the deposits of sandstone, siltstone, and shale that make up the interburden and overburden tend to be limited in areal extent (USGS, 1977).

Groundwater Flow

Except for the sandstone aquifers, all of the previously mentioned aquifers are included in the East Decker groundwater model. The sandstone lenses are ignored because of their very limited extent.

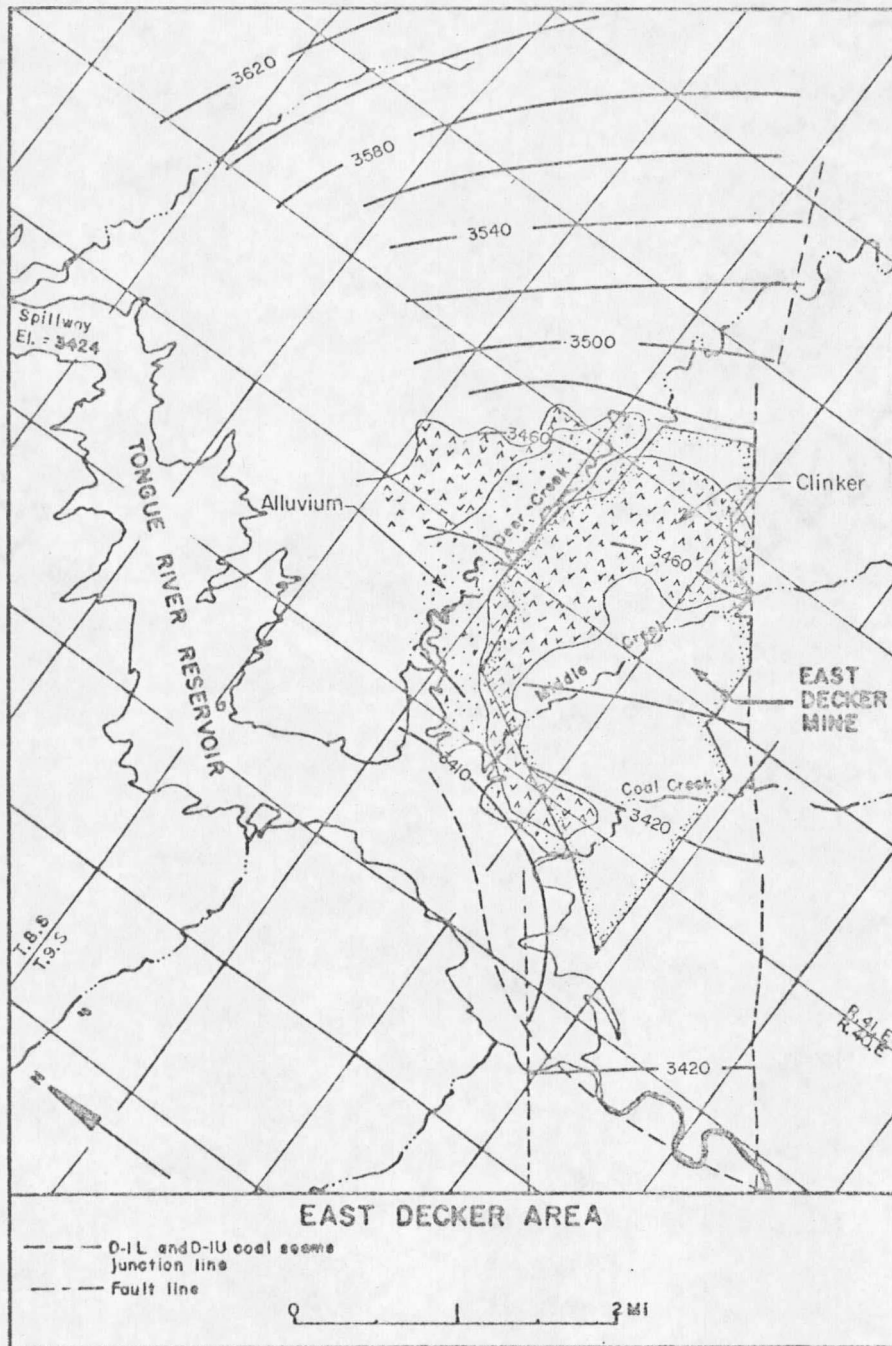


Fig. 25. Piezometric surface contours of the D-1U aquifer produced by the pre-mining, steady state simulation. Also shown are the alluvium and clinker beds adjacent to the D-1U aquifer.

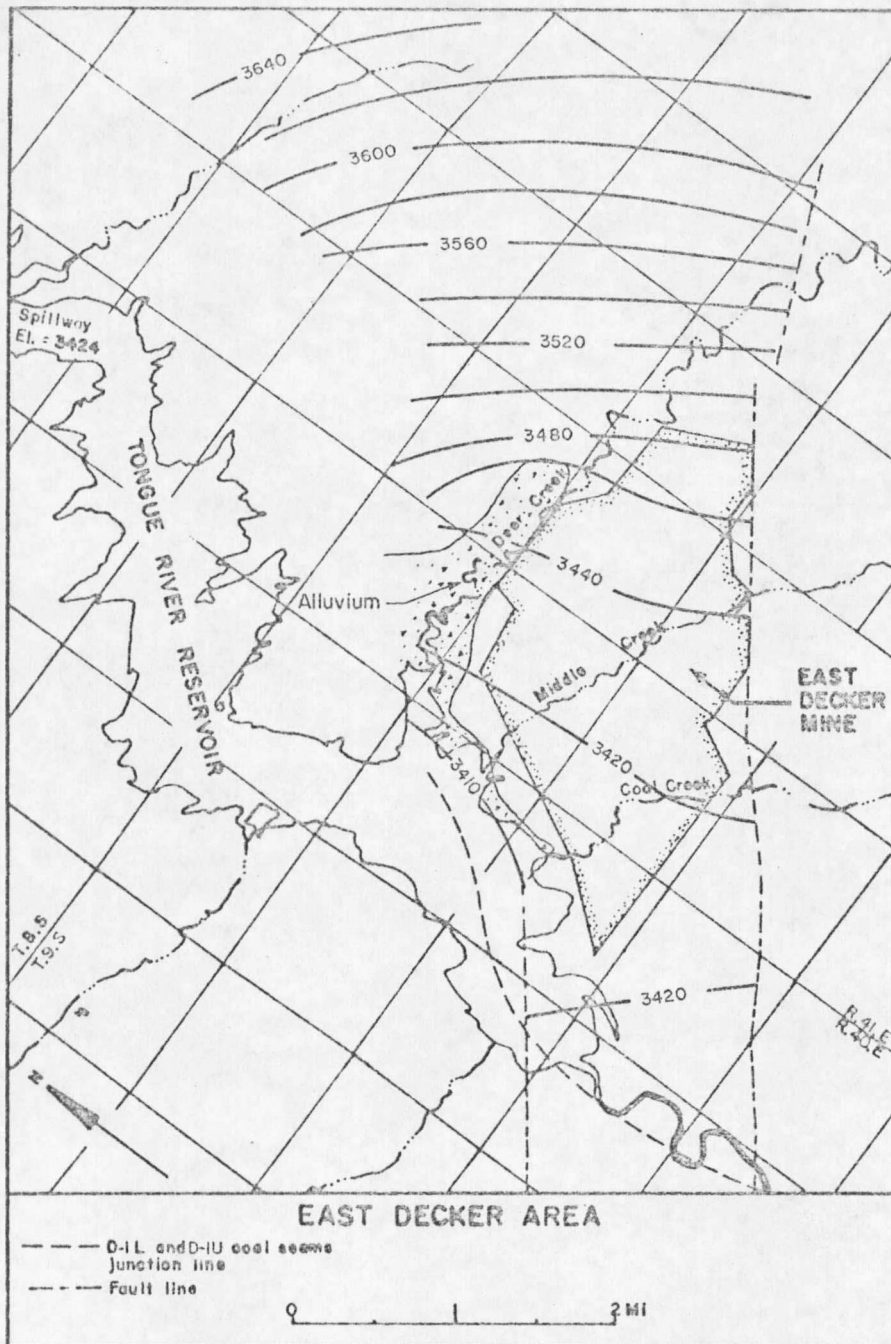


Fig. 26. Piezometric surface contours of the D-1L aquifer produced by the pre-mining, steady state simulation. Also shown is the alluvium that forms a junction between the D-1U and D-1L aquifers.



Fig. 27. Piezometric surface contours of the D-2 aquifer produced by the pre-mining, steady state simulation.

In describing the modeled aquifers, Figures 25 to 27 will be used. These are maps showing simulated, premining piezometric surfaces for the D-2, D-1 Lower, and D-1 Upper coal aquifers. (The simulations are described later in this chapter.)

In general, all three coal aquifers recharge in areas northeast and southwest of the mine, discharging into the Tongue River Reservoir in the area around the mouths of Deer, Middle, and Coal Creeks (Van Voast and Hedges, 1975). The region of interest is bounded in part by two major northeast trending faults, one running through the upstream end of the reservoir, the other about one and one-half miles southeast of the upper reservoir (see Figures 25 to 27 for locations). Strata have dropped as much as 200 feet on the southeast, downthrown sides of these faults (USGS, 1977), producing breaks in the coal seams. Van Voast and Hedges (1975), on the basis of well observations, believe that these form impermeable boundaries for the coal aquifers. Because of a regional southwestward dip of the strata (USGS, 1977), the coal seams north of the mine area are high and probably contain little or no water (based on analysis of structural and observation well data in Van Voast and Hedges, 1975).

Hydraulic Characteristics

Except for the coal, the hydraulic properties of the aquifers are only sketchily known. Van Voast and Hedges (1975) conducted eight short pump tests in the East Decker coal, reporting hydraulic

conductivities ranging from 0.5 to 19 ft/day. Most values were around 1 foot/day. They also estimated the confined storage coefficient of the coal as 0.00003 using Lohman's method (Lohman, 1972), and 0.00001 to 0.00002 using Jacob's method (described in Ferris et al., 1962). USGS (1977) estimated coal hydraulic conductivities to be three to five feet per day, with a confined storage coefficient of 0.00001.

It is believed that flow through the coal is controlled by fractures (Hasfurther and Rechard, 1976). A fracture study at East Decker (U.S. E.P.A. Quarterly Report, 1976) indicates two general fracture directions. The major fracture trend is northwest-southeast, with a significant northeast-southwest trend present also. These fractures probably give the coal strongly directional hydraulic conductivities.

USGS (1977) estimates that the alluvium has a hydraulic conductivity of 50 to 100 ft/day, and a specific yield of 0.20. Clinker is estimated to have a hydraulic conductivity of 150-200 feet/day, and a specific yield of 0.10.

The Model

The East Decker aquifer system is represented by a three layer node grid, with each layer representing a single coal seam. The grid is shown in plan view in Figure 28 superimposed over the East Decker area map. Node columns and rows are oriented to roughly match the

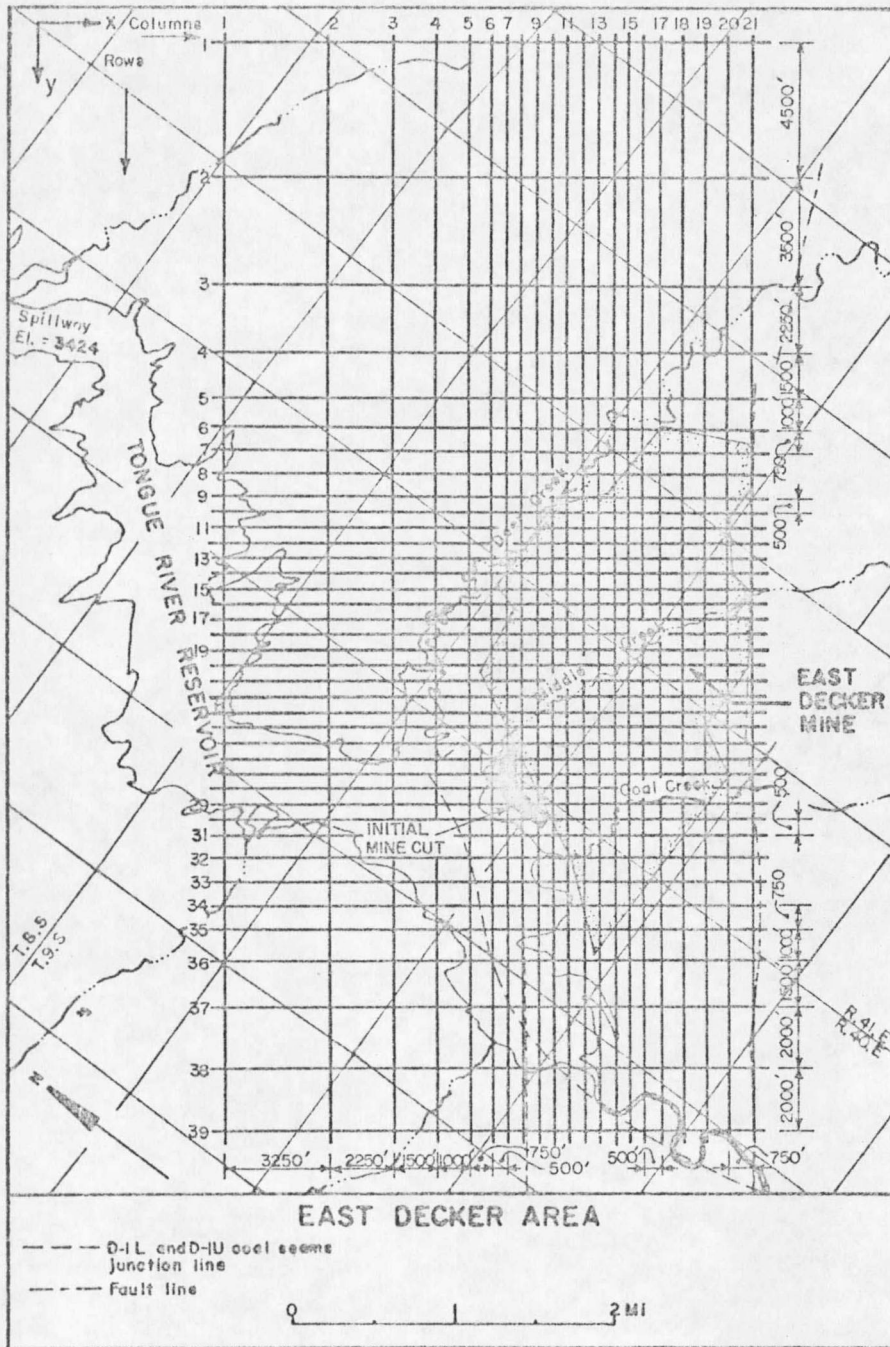


Fig. 28. The model grid.

northeast-southwest and northwest-southeast fracture directions. Rows run in the X direction, columns in the Y direction.

The Node Grid

The number of grid nodes used is limited by the memory (core) of the university's digital computer, which can handle no more than 2500 nodes and their associated data. Within this constraint, a three-layer grid system of 21 x 39 nodes was designed. Each layer contains 819 nodes, with a total of 2457 nodes for all layers. The grid size is not uniform. The smallest nodal areas (500 x 500 feet) are on the mine site; and the largest are beyond the mine site proper, towards the grid boundaries. Grid spacings are based on suggestions by Trescott et al. (1976); smaller grid spacings are used in areas of expected steep drawdown.

Column 21 forms the impermeable boundary along the fault southeast of the mine. As noted in Chapter III, the model automatically treats outer columns and rows as impermeable boundaries. The impermeable fault boundary line in the upper reservoir, running along column 8, is handled by setting TX factors for the fault nodes equal to 0.

The grid extends out several miles to the northeast and southwest of the mine into the recharge areas. Column 1 extends out several miles northeast of the mine into the high, dry boundaries of the aquifers.

Node layer three ($k = 3$), the lowest layer, represents the D-2 coal aquifer. Layer 2 ($k = 2$) is used to represent the D-1L coal aquifer, the D-1L and D-1U combined coal aquifer southwest of the mine, and the alluvial aquifer along Deer Creek, where the alluvium contacts the D-1L. Layer 1 ($k = 1$) represents the D-1U coal and clinker aquifers. Where the D-1U coal and clinker aquifers combine with the D-1L (in the Deer Creek alluvium and at the D-1U and D-1L seam junction), the aquifers are linked as described in Chapter III. All three aquifers are linked to the reservoir in the areas of the Deer, Middle, and Coal Creeks mouths. This linkage is accomplished as described in Chapter III. The reservoir elevation is assumed to be a constant 3410 feet above MSL (USGS, 1977).

Basic Node Data

Data input for each node includes the elevations of the aquifer top and bottom, the premining aquifer piezometric head, the hydraulic conductivities in the X and Y directions, the confined storage coefficient, the specific yield, the discharge or recharge flux (if any), and the node integer identification number. The identification number (NOD) is used in several ways. It identifies the type of aquifer material (coal, clinker, etc.). It is also used to identify nodes in and adjacent to the mine cuts, and nodes at which aquifers are linked together. Identification of these nodal characteristics is necessary to the model operation.

Aquifer base elevations were obtained from structural data from Van Voast and Hedges (1975), and from structural maps and drill hole data provided by Peter Kiewit Sons, Inc. The most complete data exists for the mining area proper, with lesser amounts for the surrounding areas.

Since the coal seams display little areal variation in thickness, they were assigned constant thicknesses throughout. The aquifer tops of the D-2 and D-1L coals were set 15 feet above their respective node bases, while in the D-1U coal the tops were set 23 feet above the node bottoms. In the unconfined alluvial and clinker aquifers, the aquifer tops were set at the node surface elevations as determined from topographic maps.

Premining Piezometric heads for the D-2 and D-1L aquifer nodes were obtained from piezometric contour maps presented by Van Voast and Hedges (1975). Heads for the D-1U aquifer were determined from a piezometric contour map prepared by the author from observation well data provided by Van Voast and Hedges. The maps were based on water level observations from six wells finished in the D-2 coal, ten wells finished in the D-1L aquifer, and nine finished in the D-1U aquifer. The wells were concentrated in the proposed mine area.

Because so little is known of the hydraulic properties of the aquifers, uniform properties were assigned within the aquifer material types. Clinker nodes were assigned hydraulic conductivities of 100

feet per day and specific yields of 0.1, as estimated by USGS (1977). (Clinker hydraulic conductivity was assumed isotropic because clinker is a material that has been fractured primarily by burning coal beds rather than directional earth stresses.)

The USGS estimates of the hydraulic conductivity (50-100 feet per day) and specific yield (0.20) for alluvium were not used, however, because drill hole logs indicated that alluvium along Deer Creek was best described as a sandy silt. Tables in Todd (1959) and Lohman (1972) give an approximate specific yield of 0.05 and a hydraulic conductivity of 20 feet per day for this type of material. Alluvium nodes were assigned these values.

Coal nodes were assigned directional hydraulic conductivities based on the previously mentioned fracture study (U. S. EPA Quarter Report, 1976) and also on hydraulic conductivity values given in USGS (1977) and Van Voast and Hedges (1975). These values averaged 2 to 3 feet per day. Because fracture counts indicated about twice as many fractures in the northwest-southeast direction as in the northeast-southwest direction, a ratio of HCX to HCY of 2:1 was used. HCX was assigned a value of 3 feet per day.

The coal node confined storage coefficients were assigned values of 0.00002, a value estimated by Van Voast and Hedges (1975). The specific yield of coal, for which no field measurements were available, was estimated to be 0.005. This was based on the fact that coal has a

hydraulic conductivity equivalent to that of silty clay (Lohman, 1972). A graph in Todd (1959) indicates a specific yield of less than .01 is applicable for this type of material.

Recharge flux values, which were required at some boundary nodes, were based on a premining simulation and are discussed in the following section.

Steady State (Premining) Simulation

Before a simulation with the mine cut was attempted, a simulation of nearly two years was run in order to approximate steady state, premining conditions. This was necessary because it was recognized that there were very probably inconsistencies in the basic data, inconsistencies which might result in an initially unstable and unrealistic model performance. Once the piezometric surface positions had stabilized, the mine cut could be simulated; head changes occurring from this point on would be attributable to the mine cut.

To attain an approximate steady state condition within the model area, it was necessary to provide lateral recharge at some model boundaries. As noted earlier, recharge in all three coal aquifers appears to take place in areas northeast and southwest of the mine. To approximate the recharge rate for the steady state condition, flow out of the recharge areas was estimated with Darcy's law using the coal transmissivities and hydraulic gradients from the piezometric contour

maps of Van Voast and Hedges (1975). The recharge rates, given in Table 1, were applied uniformly to the rows 1 and 39 nodes, the nodes that border the recharge areas.

Beginning with the initially assigned node piezometric heads and using the Table 1 recharge rates, 600 days of system simulation produced flow that was essentially steady (minimal fluctuation in node heads). The simulated steady state piezometric surfaces for the D-1U, D-1L, and D-2 aquifers are shown in Figures 25 to 27, pp. 97-99. These simulated surfaces exhibit the same general flow patterns shown by the piezometric surface maps of Van Voast and Hedges (1975). Surface differences, which were not extreme (maximum of 15 feet), were ignored since the primary objective of this research effort was the development of a modeling methodology rather than the fine tuning of the model.

TABLE 1: Node Recharge Rates.

Aquifer	Row 1	Row 39
	recharge rate per unit node area	recharge rate per unit node area
D-1U	.00006 ft ³ /ft ² /day	.00003 ft ³ /ft ² /day
D-1L	.00005	.000016
D-2	.000032	.00003

Mine Cut Simulation

First Simulation Attempt

Procedure. In the first simulation, nodes immediately adjacent to the cut on the side away from the reservoir (all coal nodes) were coded so that they would be pumped at rates computed from the sum of Equation (58) (intercepted natural flow rate),

$$Q_n = (q_n) (WALL) \quad (58)$$

where

q_n = the natural (premining) flow rate per unit length of cut (L^2/T)

WALL = length of the mine cut adjacent to the node (L)

and Equation (59) (nonsteady flow rate from storage),

$$Q_{st} = (T) (WALL) \left(\frac{\partial h}{\partial x} \right)_{x=c} \quad (59)$$

where

$T = K(h-BOT)$ = transmissivity in the direction of flow at the node (L^2/T)

K = hydraulic conductivity in the direction of flow (L/T)

h = head elevation at the node (L)

BOT = elevation of the aquifer bottom at the node (L)

and $(\partial h / \partial x)_{x=c}$ is evaluated with Equation (60)

$$\frac{\partial h}{\partial x} = \frac{1}{2} \left(\frac{2}{\sqrt{\pi}} \right)^{1/2} H \left(\int_0^u e^{-u^2} du \right)^{-1/2} e^{-u^2} \frac{1}{(4Kht/2S_y)^{1/2}} \quad (60)$$

Equation (60) is the derivative with respect to x (flow direction) of Equation (31), the second Haushild and Kruse unconfined aquifer head approximation (Equations (31), (58), (59), and (60) are discussed in Chapter III.)

Because intercepted natural flow rates were assumed constant, flow rates were calculated outside of the program with Equation (58) and placed in computer memory with the other node data.

Equation (31), which is for unconfined nonsteady flow, was assumed applicable to the D-1U and D-1L node calculations because these aquifers did not have significant amounts of water in confined storage before mining: around the initial mine cut the D-1U was naturally unconfined, while the D-1L had a head of only about 20 to 30 feet above the aquifer top. Although the D-2 coal had a high initial head, approximately 80 to 100 feet above the seam with correspondingly more confined storage, it was felt that Equation (31) might be applicable since initial drainage would rapidly lead to unconfined conditions around the mine.

Because $(\partial h / \partial x)$ is a function of time, Equation (60) had to be reevaluated for successive time increments in the simulation. For nodes on the east side of the mine cut, where flow would be in a westerly direction, K in Equation (60) was set equal to HCX , the

hydraulic conductivity in the X direction. Figure 29 shows a plot of $(\partial h / \partial x)_{x=c}$ versus time for $K = HCX = 3.0$ feet/day. The nodes on the north and south ends of the cut would experience flow in the y direction. Figure 30 plots $(\partial h / \partial x)_{x=c}$ versus time for $K = HCY = 1.5$ feet/day. Both curves were calculated from Equation (60) for $x = c = 250$ feet, half a node width (the distance from the mine cut to an adjacent node center) and $H = 100$ feet.

Calculations for the nonsteady pumping rates for all nodes immediately adjacent to the cut on the side away from the reservoir were performed in Subroutine FLOW of the computer program. FLOW is listed in Appendix 2.

Nodes adjacent to the cut on the reservoir side were coded so that they could be pumped at steady rates. For the alluvium which contacts and recharges the D-1U and D-1L coals, Equation (20) was used:

$$q_r = \frac{K}{f} \left[\frac{h_s^2}{2} \right] H_r \quad (20)$$

where

q_r = discharge rate from the reservoir per unit length of mine cut (L^2/T)

K = hydraulic conductivity of the alluvium in the direction of flow (L/T)

f = distance from the mine cut to the reservoir (L)

h_s = height of the water table above the aquifer floor. Watertable is assumed initially horizontal (L)

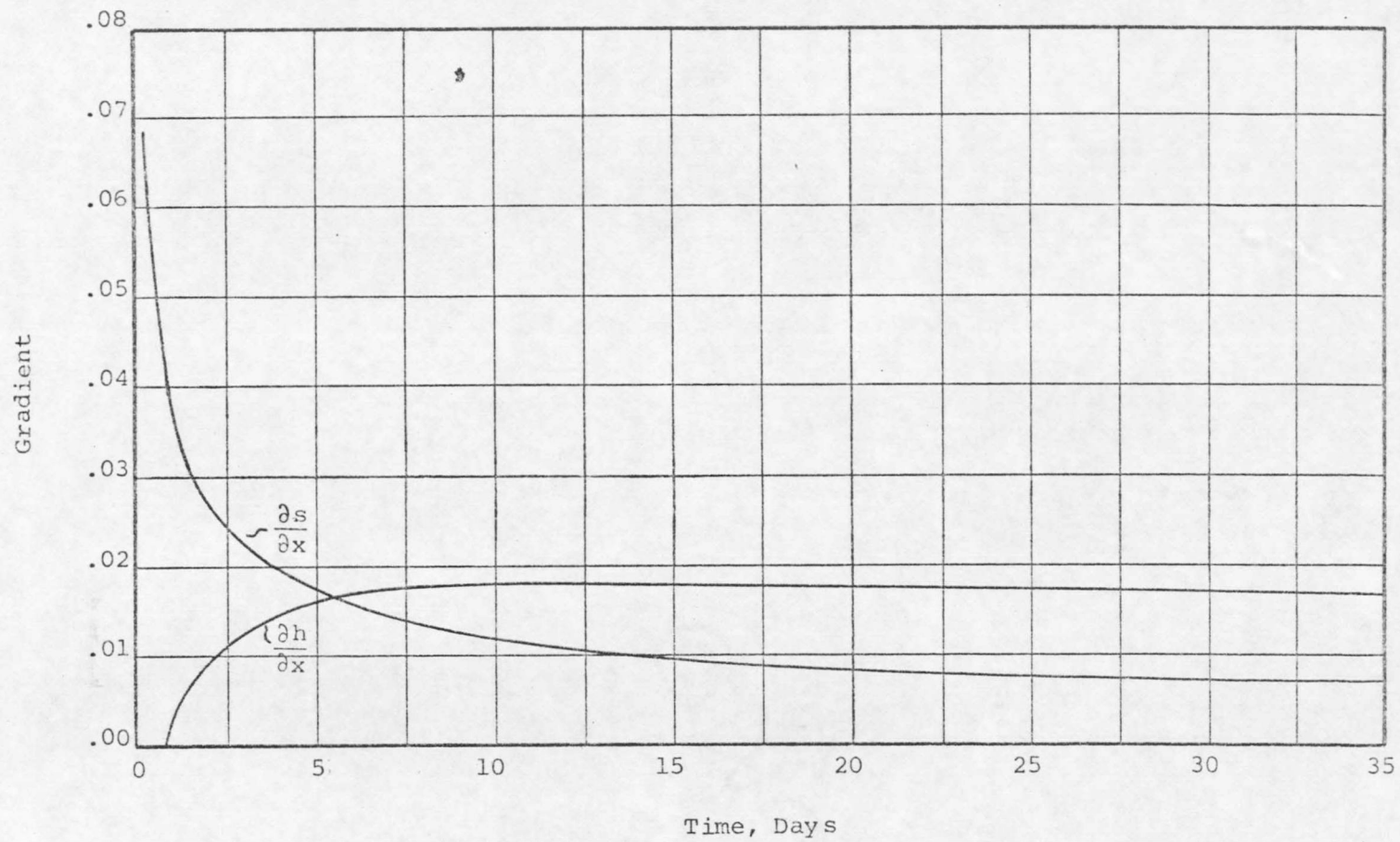


Fig. 29. Plots of $\frac{\partial h}{\partial x} |_{x=250}$ and $\frac{\partial s}{\partial x} |_{x=250}$ vs time for $H_{CX}=3.0$ ft/day.

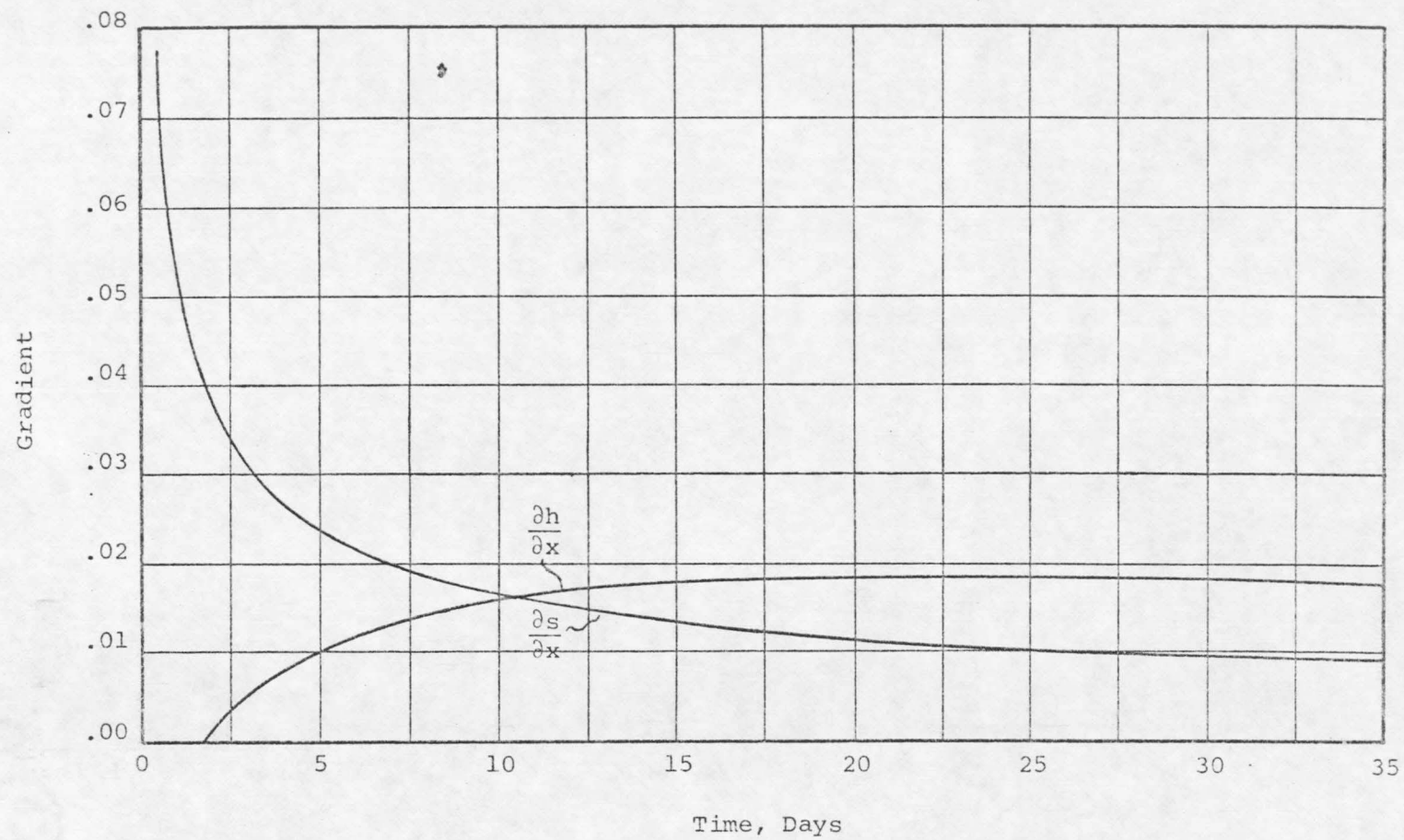


Fig. 30. Plots of $\frac{\partial h}{\partial x} |_{x=250}$ and $\frac{\partial s}{\partial x} |_{x=250}$ vs. time for HCY = 1.5 ft/day.

H_r = height of the reservoir surface above the aquifer floor (L)

For the D-2 coal, Equation (55) for confined flow was employed:

$$q_r = T \frac{H_{r-c}}{f} \quad (55)$$

where

T = transmissivity of the coal (L^2/T)
 = (coal hydraulic conductivity in the direction of flow)
 (coal seam thickness)

H_{r-c} = height of the reservoir surface above the top of the coal seam (L)

and other terms are as defined above.

Equations (20) and (55), which are discussed in Chapter III, were used to calculate the steady node discharge flux rates (discharge rates per unit surface area) given in Table 2. To avoid repeated

TABLE 2: Steady Drainage Flux Rates

Node	Steady State Drainage Rate, ft^3 per square foot of node per day
6,27,1	.002 $ft^3/ft^2/day$
7,29,1	.005
8,30,1	.006
8,31,1	.024
9,31,1	.024
10,31,1	.005
6,25,2	.001
6,26,2	.0007
6,27,2	.0005
6,28,2	.0006
7,29,2	.001
8,30,2	.001
8,31,2	.001

(cont.)

Table 2. (Cont.)

-113-

Node	Steady State
	Drainage Rate, ft ³ per square foot of node per day
6,25,3	.01
6,26,3	.01
6,27,3	.01
6,28,3	.01

calculation of these steady values, the values were calculated outside of the model and placed in computer memory with the other node data.

The program automatically calculated the Q_r values, the net node reservoir to cut discharge rates, from each node's dimensions.

Results. This first simulation produced drawdowns in all three aquifers. However, the drawdowns produced were unrealistic, especially in the D-2 aquifer: extrapolation of drawdown curves to the cut wall yielded drawdowns much less than the expected full drawdown. Furthermore, after about 15 days of simulation the D-1U and D-1L nodes immediately southeast of the mine began to fill. Apparently discharge rates were being underestimated, probably in large part because Equation (59) did not account for water released from confined storage.

Second Simulation Attempt

Procedure. This simulation was run in the same way as the first simulation, except minor changes were made to account for water released from confined storage.

To account for the significant amount of water in confined storage in the D-2 coal, Equation (61) (see Chapter III)

$$Q_{st} = (T) (WALL) \left[\frac{\partial s}{\partial x} \right]_{x=c} \quad (61)$$

was employed to calculate the discharge from confined storage for D-2 nodes pumped at nonsteady rates (nodes adjacent to the mine cut on the side away from the reservoir). This flow rate was added to the flow rate calculated from the sum of Equations (58) and (59) to yield the total pumping rate for the nonsteadily discharging D-2 nodes.

As was the case with Equation (59), Equation (61) had to be recomputed at each new time increment since $(\partial s / \partial x)$ is a time dependent function. The $(\partial s / \partial x)$ gradients were evaluated with Equation (62)

$$\frac{\partial s}{\partial x} = -H \left[\frac{2}{\sqrt{\pi}} e^{-u^2} \frac{1}{(4T_o t / S_c)^{1/2}} \right] \quad (62)$$

where

$$u = \frac{x}{(4T_o t / S_c)^{1/2}} \quad \text{alternate} \quad \frac{x}{\sqrt{4T_o t / S_c}}$$

T_o^{∞} = confined aquifer transmissivity in the direction of flow
 (L^2/T)
 $= (K)$ (coal thickness)

and other terms are as defined in Chapter III. Plots of $(\partial s / \partial x)_{x=c}$ versus time are given in Figures 29 and 30 for $H_{CX} = 3.0$ feet/day and $H_{CY} = 1.5$ feet/day. These curves were calculated for $x = 250$ feet (half a node width) and $H = 100$ feet (the initial head of the D-2 aquifer in the immediate mine area was about 100 feet above the base of the coal seam).

Because the D-1U and D-1L nodes southeast of the cut began to fill after some time during the first simulation, it was apparent that the discharge rates computed from the sum of Equations (58) and (59) for these D-1U and D-1L nodes were underestimates, at least after some time t . Though this was probably due to the fact that confined storage was being ignored, it was felt that use of Equation (61) to compute flow out of confined storage in these aquifers would be invalid; most of the water in confined storage was in areas away from the mine, and the situation did not come near to the ideal conditions for which Equation (61) was derived. Therefore, to deal with the underestimation, it was decided to switch to an essentially steady discharge into the cut when $t = 10$ days, shortly before the nonsteady simulation would cause nodes to refill. For $t \geq 10$ days, those nodes in the D-1U, D-1L, and D-2 that had been pumped at nonsteady rates were pumped at rates calculated using constant gradients $\partial h / \partial x$ and $\partial s / \partial x$ (the gradient values at $t = 10$ days) in the appropriate equations. (D-1L and D-1U node discharge rates were calculated from the sum of the steady rate of Equation (58) and the nonsteady rate of Equation (59), and the D-2 node discharge rates were calculated from the sum of the steady rate of Equation (58) and the nonsteady rate of Equations (59) and (61).) Although the D-2 nodes had experienced no refilling during the first simulation attempt, the D-2 nodes were also pumped at steady rates for the sake of consistency.

Holding the gradients constant permitted node discharges to fluctuate in direct proportion to the saturated thickness, thus preventing any numerical instabilities if a pumped node dried up (It should be mentioned that although $\partial h/\partial x$ and $\partial s/\partial x$ rather than Q_{st} were held constant, it turned out that total discharge to the cut held fairly steady for $t \geq 10$ days).

Though the decision to use an essentially steady discharge after $t = 10$ days may appear rather arbitrary, there is a rational basis for it. As noted previously in Chapters II and III, Davis (1977) has observed that drainage to mine cuts can be assumed as essentially steady after a short period of time.

In the first attempts to run this simulation procedure, a significant problem appeared, a problem which had not shown up in the first simulation. Due to the higher discharge rates in this simulation, large drawdowns occurred in the D-2 aquifer at the southern grid boundary, drawdowns on the order of 30 feet. Drawdowns also occurred in the D-1L aquifer in this area. Thus, the initial assumption that row 39 would lie beyond the area of significant drawdown was false; row 39 could not be treated as a constant recharge boundary as originally conceived. To compensate for this without completely reworking the grid and initial data, recharge along the row 39 boundary, columns 9-21 (see Figure 28), was recalculated for the D-2 and D-1L nodes for

each new time increment by a quasi-steady state method. Head values from the previous time increment were used in the Darcy equation to estimate recharge for the subsequent time increment. This tactic allowed the recharge of this boundary to vary with time when the pit drawdowns reached the boundary.

For example, recharge for D-2 node (15, 39, 3) was calculated for each time increment as

$$\text{Recharge} = \text{HCY}_{15,38,3} \left[\frac{h_{15,39,3} - h_{15,38,3}}{y_{15,38}} \right] \\ \times X_{15,39}^{2} [CH_{15,39,3} - BOT_{15,39,3}]$$

where terms are as previously defined.

During the first portion of a simulation, before drawdowns reached the boundary, this procedure yielded essentially constant recharge rates; rates were being calculated from the premining gradient, just as row 39 recharge was calculated for the previous computer runs. However, once drawdowns moved out to rows 38 and 39, the gradients began to steepen and the calculated recharge increased. Since the increased recharge was calculated with a steady state equation for each time increment (hence the term "quasi-steady state"), the procedure can only be considered approximate.

Results. With the changes described above, the second simulation produced the drawdowns shown in Figures 31 - 33 after 356 days of simulation with



Fig. 31. Drawdown in the D-1U aquifer 356 days after introduction of the mine cut.

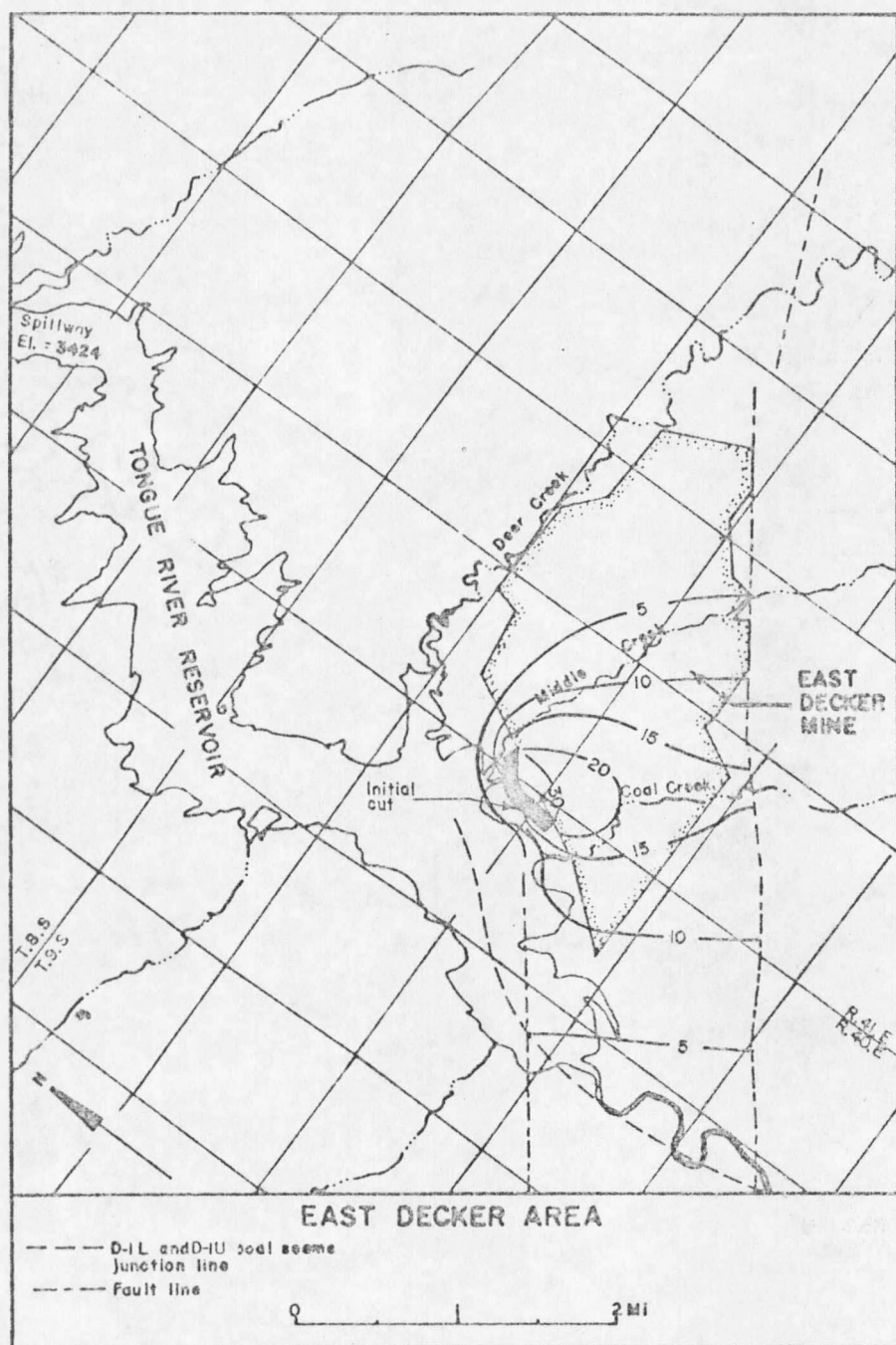


Fig. 32. Drawdown in the D-1L aquifer 356 days after the introduction of the mine cut.

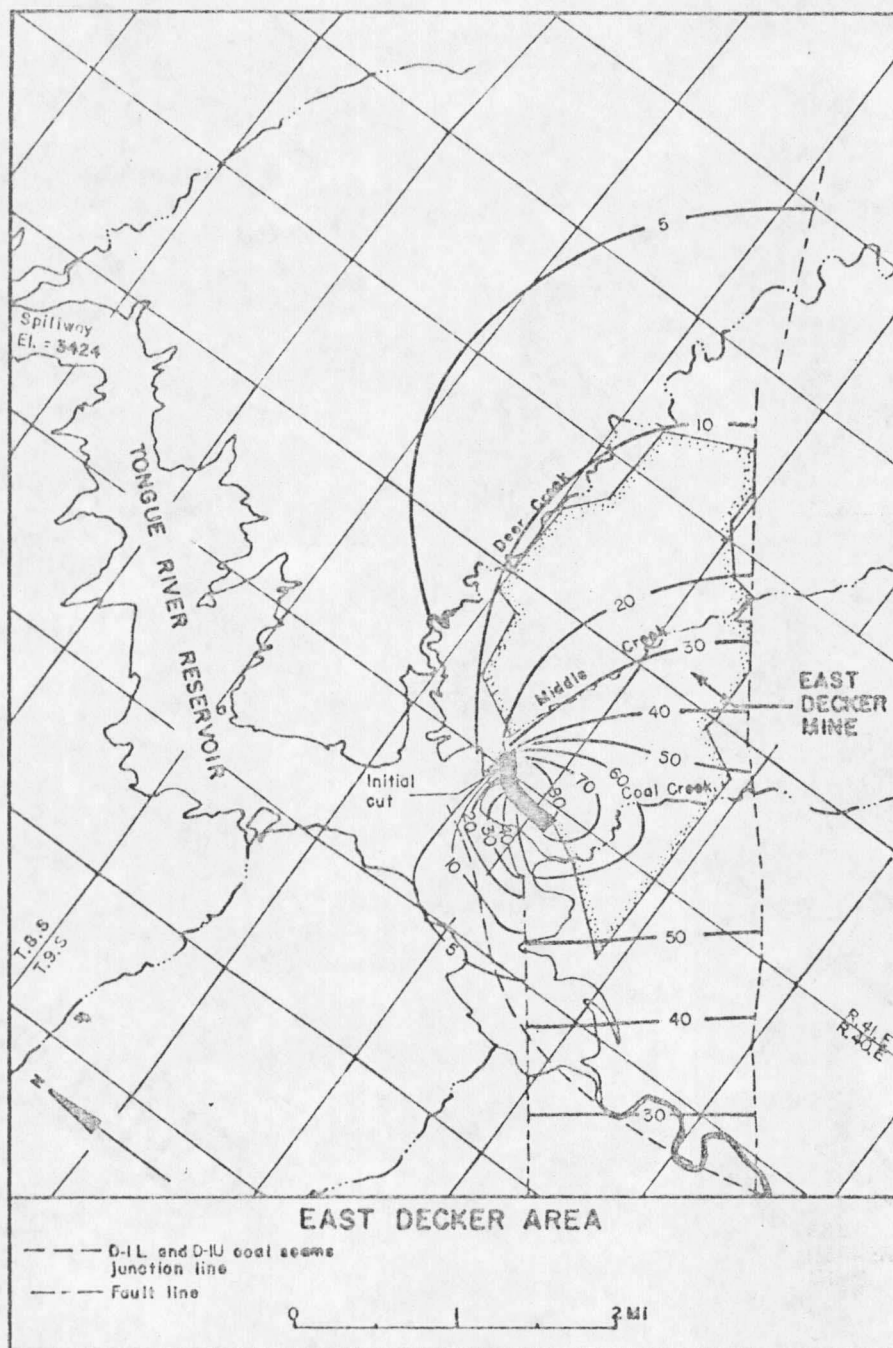


Fig. 33. Drawdown in the D-2 aquifer 356 days after introduction of the mine cut.

the mine cut. As was anticipated, the greatest drawdowns were produced in the D-2 aquifer (see Figure 33), the aquifer with the greatest initial piezometric head. Drawdowns in the D-2 aquifer were as much as 80 feet. The D-1L aquifer, with its somewhat lower initial head, exhibited much less drawdown (see Figure 32). Maximum drawdown in the D-1L aquifer was 30 feet. Very little drawdown (Figure 31) was seen in the D-1U aquifer. Only at its southern end did the mine pit cut into the water bearing portions of D-1U coal and clinker; at the northern end the pit cut into dry clinker.

The second simulation proved to be far more realistic than the first simulation: extrapolation of drawdowns to the cut walls yielded the expected full drawdown at the wall face. In addition, nodes near the cut experienced a decreasing rate of dewatering, something to be expected in a system that is attempting to regain equilibrium.

Chapter V

SUMMARY

The object of this thesis work was to produce a digital computer model capable of simulating groundwater flow at the East Decker coal mine in southeastern Montana. The computer model was intended to predict the drawdowns and mine drainage rates that would be expected to result from proposed mining operations.

A two dimensional, finite difference numerical modeling approach was selected for the application. The Modified Iterative Alternating Direction Implicit numerical method (MIADI) developed by Prickett and Lonquist (1971) for the solution of the nonsteady groundwater flow equation was adopted as the model basis. To this basic program, numerous routines were added to deal with the site specific problems at the East Decker mine (special boundary conditions, etc.) A three layer array of 2457 nodes was used to represent the three aquifers of interest.

Using somewhat different approaches to modeling the mine cut boundary, two attempts were made to simulate the groundwater flow changes that would result from the initial East Decker mine cut. The first attempt used reservoir bank storage equations for unconfined aquifers for estimating the nonsteady discharge from aquifers exposed by the initial mine cut. Nodes immediately north, east, and south of the mine cut (the sides away from the reservoir) were pumped at rates

calculated from the bank storage equations in order to produce aquifer drawdowns. Flows into the west side of the cut were assumed steady due to the close proximity of the reservoir.

The first simulation attempt produced significant drawdowns in all three aquifers. However, there were problems with the simulation results: extrapolation of drawdowns to the cut walls indicated that the simulated drawdowns were much less than would be expected, and, furthermore, some drawn down nodes began to refill after some time. It was felt that these problems indicated that aquifer discharge rates were being underestimated, probably because water released from confined storage was ignored in the simulation.

The second simulation was different from the first in two respects. First, nonsteady discharge to the mine cut was estimated by using the sum of the discharges yielded by the confined and unconfined bank storage equations. Secondly, when the aquifer discharge began to approach a constant rate after a period of simulated time, the discharge rates were held essentially constant for all succeeding time increment calculations. This second change followed a suggestion by Davis (1976) who had observed that discharge to a mine cut could be assumed as essentially steady within a short time after the cut opening.

Drawdowns produced by the second simulation procedure were realistic. Extrapolation of simulated drawdowns to the edge of the cut yielded the expected full drawdown. Aquifer drawdowns

diminished with time, something to be expected in a system attempting to reestablish an equilibrium condition. The drawdowns after 356 days of simulation are shown in Figures 31 to 33.

RECOMMENDATIONS FOR FURTHER STUDY AND DEVELOPMENT

It is felt that the following represent the most important points warranting further work:

1. A better way of handling the mine cut boundary would be desirable. The author feels that the two-dimensional analytical equations used in the model to represent this boundary were used at their limit of applicability. It is suggested that a three-dimensional approach (possibly a numerical method) holds the most promise.
2. Before any further work is done, the node grid should be expanded to the southwest to reach beyond the area of significant aquifer drawdown.
3. To give the model the capability for simulating the long term effects of mining, a methodology must be developed for moving the mine cut during a simulation. This must include development of methods for handling flow through spoils material.
4. Graphical printout, showing potentiometric contours, drawdown contours and flow direction, would greatly aid in interpretation of simulation results.
5. If possible, an attempt should be made to collect more and better site data. Out of necessity, the data used for the previously

discussed simulations involved some approximations. The practical applicability of the model is of course limited by these approximations.

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APPENDICES

APPENDIX A

Symbols

AA = coefficient defined on p. 69 (L^2/T)

a = aquifer saturated thickness (L)

BB = coefficient defined on p. 69 (L^2/T)

BOT = elevation of aquifer bottom (see Figure 13, p. 58) (L)

b = thickness of a confined aquifer (L)

bⁱ = thickness of an aquifer confining layer (aquiterd) (L)

CC = coefficient defined on p. 69 (L^2/T)

CH = elevation of aquifer top (see Figure 13, p. 58) (L)

D = as defined in Figure 8, p. 38 (L)

DD = coefficient defined on p. 69 (L^2/T)

f = distance from mine cut to reservoir (L)

g = gravity acceleration (L/T)

H = original aquifer head level (L)

H_O = stream or reservoir level drop (L)

H_r = height of reservoir surface above aquifer bottom (L)

H_{r-c} = height of reservoir surface above a coal seam (L)

HC_x = node hydraulic conductivity in the x direction (L/T)

HC_y = node hydraulic conductivity in the y direction (L/T)

h = head at time t+ t(L)

h_O = head at time t (L)

h_s = saturated thickness of an aquifer (L)

i = node column number (see Figure 11, p. 52)

j = node row number (see Figure 11, p. 52)

- K = hydraulic conductivity (L/T)
- K_x = hydraulic conductivity in the x direction (L/T)
- K_y = hydraulic conductivity in the y direction (L/T)
- K_z = hydraulic conductivity in the z direction (L/T)
- K' = vertical hydraulic conductivity of an aquifer confining layer (L/T)
- k = node grid layer number (see Figure 20, p. 76)
- L = node interaquifer leakage rate (L^3/T)
- NC = number of columns in node grid array
- NL = number of layers in node grid array
- NR = number of rows in node grid array
- n = porosity (dimensionless)
- $n(\text{superscript})$ = iteration number
- Q = node source/sink factor, as defined on p. 54 (L^3/T)
- $Q_n = q_n \text{ Wall}$ (L^3/T)
- $Q_r = q_r \text{ Wall}$ (L^3/T)
- $Q_{st} = q_{st} \text{ Wall}$ (L^3/T)
- q = discharge rate per unit length (L/T)
- q_n = natural flow rate per unit length of mine cut (L/T)
- q_r = aquifer discharge rate per unit length of mine cut from reservoir (L/T)
- q_{st} = aquifer discharge rate per unit length of mine cut from aquifer storage (L/T)
- R = node vertical hydraulic conductivity factor, as defined on p. 77 (L^2/T)

- S = aquifer storage coefficient (dimensionless)
- S_a = average node storage coefficient (dimensionless)
- S_c = confined aquifer storage coefficient = $S_s b$ (dimensionless)
- S_s = specific storage = $\rho g (n\beta + (1-n)\alpha)$ = the volume of water released from a unit volume of porous medium per unit head drop (1/L)
- S'_s = specific storage of an aquifer confining layer (1/L)
- S_y = specific yield (effective porosity) = V_w/V (dimensionless)
- SF = node storage coefficient factor as defined on p. 54 (L^2)
- SF1 = node storage coefficient factor for confined conditions (L^2)
- SF2 = node storage coefficient factor for unconfined conditions (L^2)
- s = aquifer drawdown (L)
- T = transmissivity (L^2/T)
- T_a = average node transmissivity (L^2/T)
- T_x = transmissivity in the x direction (L^2/T)
- T_y = transmissivity in the y direction (L^2/T)
- TB = internodal transmissivity factor between adjacent nodes in the same column but not in the same node layer (see p. 82) (L^2/T)
- TR = internodal transmissivity factor between adjacent nodes in the same row but not in the same node layer (see p. 82) (L^2/T)
- TX = aquifer transmissivity factor between adjacent nodes in the same row (see p. 54) (L^2/T)
- TX' = aquifer transmissivity between adjacent nodes in the same row (L^2/T)
- TY = aquifer transmissivity factor between adjacent nodes in the same column (see p. 54) (L^2/T)

TY' = aquifer transmissivity between adjacent nodes in the same column (L /T)

t = time (T)

t' = "dimensionless time" (see p. 48)

u = integral argument, variously defined

V = volume of porous medium (L^3)

V_w = volume of water that can be drained by gravity from V (L^3)

W = source/sink function (pumpage, recharge, etc.) (L/T)

$WALL$ = length of mine cut immediately adjacent to a node (L)

X = node grid dimension along x axis (L)

X_a = average node dimension in x direction (L)

$X1$ = node dimension as defined in Figure 12, p. 52 (L)

$X2$ = node dimension as defined in Figure 12, p. 52 (L)

x = distance along x axis (L)

Y = node grid dimension along y axis (L)

Y_a = average node dimension in y direction (L)

$Y1$ = node dimension as defined in Figure 12, p. 52 (L)

$Y2$ = node dimension as defined in Figure 12, p. 52 (L)

y = distance along y axis (L)

z = distance along z axis (L)

α = vertical compressibility of an aquifer (LT^2/M)

β = compressibility of water (LT^2/M)

Δt = time increment (T)

α = moisture content (dimensionless)

ρ = mass density of water (M/L^3)

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C      (FT**2/DAY)
C      BY - AQUIFER TRANSMISSIVITY AT NODE I,J+1,K IN THE Y DIRECTION
C      (FT**2/DAY)
C      BOT(I,J,K) - ELEVATION OF AQUIFER BOTTOM (FT)
C      BOTT(K) - DEFAULT VALUE FOR BOT(I,J,K)
C      CH(I,J,K) - ELEVATION OF AQUIFER TOP (FT)
C      CCH(K) - DEFAULT VALUE FOR CH(I,J,K) (FT)
C      DELTA - TIME INCREMENT (DAYS)
C      DH(I,J,K) = H*WALL*(1/X)*K (SEE EQUATION (60))
C      DL(I,J,K) - INTERMEDIATE CALCULATIONAL VALUE IN HEAD PREDICTION
C      ROUTINE
C      ERF - INTEGRAL IN EQUATION (60)
C      ERROR - HIGHEST SUMMED ERROR PERMITTED TO MEET CONVERGENCE
C      CRITERION
C      FK(I,J,K) = (X/S)*PI(4*K*H/25)) (SEE EQUATION (60))
C      G(I,K), OR GC(J,K) - INTERMEDIATE CALCULATIONAL VALUE IN THE THOMAS
C      ALGORITHM
C      H(I,J,K) - HEAD AT END OF CURRENT TIME INCREMENT (FT)
C      HA - INTERMEDIATE HEAD VALUE IN THE THOMAS ALGORITHM
C      HCX(I,J,K) - HYDRAULIC CONDUCTIVITY IN THE X DIRECTION (FT/DAY)
C      HCY(I,J,K) - HYDRAULIC CONDUCTIVITY IN THE Y DIRECTION (FT/DAY)
C      HH(K) - DEFAULT VALUE FOR H(I,J,K) AND H0(I,J,K) (FT)
C      H0(I,J,K) - HEAD AT END OF PREVIOUS TIME INCREMENT (FT)
C      I - GRID COLUMN NUMBER
C      ISTEP - TIME INCREMENT NUMBER
C      ITER - MIADI ITERATION NUMBER
C      J - GRID ROW NUMBER
C      K - AQUIFER NUMBER: 1 = D-1 U COAL
C      2 = D-1 L COAL
C      3 = D-2 COAL
C      KSTEP - ISTEP FOR WHICH PRINT OUT OF RESULTS IS DESIRED
C      MSTEP - ISTEP FOR WHICH DATA CHANGE (MINING CHANGED) OCCURS
C      NC - NUMBER OF GRID COLUMNS = 21
C      NL - NUMBER OF GRID LAYERS = 3
C      NR - NUMBER OF GRID ROWS = 39
C      NOD(I,J,K) - NODE IDENTIFICATION NUMBER:
C      1 = NODE PUMPED AT NONSTEADY RATE (ADJACENT TO MINE)
C      2 = RESERVOIR BOUNDARY NODE
C      3 = ALLUVIUM NODE
C      4 = CLINKER NODE
C      6 = COAL NODE

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C      9 = DUMMY NODE (HYDRAULIC CONDUCTIVITIES = 0)
C      10 = TRIGGER NUMBER TO EXIT READ ROUTINE IN "VALUE"
C      11 = MINE CUT NODE (HYDRAULIC CONDUCTIVITIES = 0)
C      20 = TRIGGER NUMBER TO EXIT READ ROUTINE IN "VALUE"
C      21 = INDICATES AQUIFER 2 NODE SHOULD BE LINKED WITH
C            SURROUNDING AQUIFER 1 NODES
C      31 = INDICATES AQUIFER 3 NODE SHOULD BE LINKED WITH
C            SURROUNDING AQUIFER 2 NODES
C      32 = INDICATES AQUIFER 3 NODE SHOULD BE LINKED WITH
C            SURROUNDING AQUIFER 2 AND AQUIFER 1 NODES
C
C      NSTEPS - TOTAL NUMBER OF TIME INCREMENTS FOR WHICH SOLUTION IS
C              DESIRED
C      PP(K) - DEFAULT VALUE FOR HXX(I,J,K) AND HCY(I,J,K) (FT/DAY)
C      Q(I,J,K) - DISCHARGE FACTOR (FT**2/DAY)
C      QMC(K) - DISCHARGE FROM STORAGE INTO MINE CUT FROM EACH AQUIFER
C              (FT**3/DAY)
C      QQ(K) - DEFAULT VALUE FOR Q(I,J,K) (FT**3/DAY)
C      R(I,J,K) - INTERAQUIFER LEAKAGE FACTOR (FT**2/DAY). ALSO USED FOR
C              NONSTEADY PUMPED NODES FOR THE INTERCEPTED NATURAL FLOW
C              RATE (FT**3/DAY). SEE CHAPTER 4.
C      RISE(I,J,K) - CHANGE IN NODE HEAD SINCE BEGINNING OF SIMULATION
C                  (FT)
C      RR(K) - DEFAULT VALUE FOR R(I,J,K)
C      SF1(I,J,K) - STORAGE FACTOR FOR CONFINED CONDITIONS (FT**2)
C      SF2(I,J,K) - STORAGE FACTOR FOR UNCONFINED CONDITIONS (FT**2)
C      ST - SATURATED THICKNESS OF AQUIFER AT NODE I,J,K (FT)
C      S1(K) - DEFAULT VALUE FOR SF1(I,J,K) (FT**2)
C      S2(K) - DEFAULT VALUE FOR SF2(I,J,K) (FT**2)
C      TIME - TIME SINCE BEGINNING OF SIMULATION (DAYS)
C      TX(I,J,K) - AQUIFER TRANSMISSIVITY FACTOR BETWEEN NODES I,J,K AND
C                  I+1,J,K (FT**2/DAY)
C      TY(I,J,K) - AQUIFER TRANSMISSIVITY FACTOR BETWEEN NODES I,J,K AND
C                  I,J+1,K (FT**2/DAY)
C      X1(I), X2(I) - NODE DIMENSIONS DEFINED IN FIGURE 12 (FT)
C      Y1(J), Y2(J) - NODE DIMENSIONS DEFINED IN FIGURE 12 (FT)
C      W - INTERMEDIATE CALCULATIONAL VALUE IN THE THOMAS ALGORITHM
C
C*****
C*****
C

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C
C
C
C
C
C

CENTRAL

COMMON H(21,39,3),H0(21,39,3),SF1(21,39,3),QC(21,39,3),TY(21,39,3),
1TX(21,39,3),R(21,39,3),G(21,39,3),DL(21,39,3),SF2(21,39,3),CH(21,39,3),H
2CY(21,39,3),HCX(21,39,3),ROT(21,39,3),MOD(21,39,3),RISE(21,39,3),F
3K(21,39,3),K(21,39,3),I,J,K,NC,NR,NL,IYER,TIME,DELTA,MSTEP,S1(3),S
42(3),HH(3),CCH(3),PP(3),BOTT(3),RR(3),QQ(3),DH(21,39,3),X1(21),X2(
521),Y1(39),Y2(39),QMC(3)

C
C
C
C
C
C
C

C 1. THE FOLLOWING SECTION READS IN SIMULATION PARAMETERS, DEFAULT
VALUES, NODE DIMENSIONS, AND INITIAL NODE DATA. ALL DATA IS
READ OUT OF FILES 105 AND 106.

C**** READ SIMULATION PARAMETERS: NUMBER OF TIME INCREMENTS, INITIAL TIME
INCREMENT, ERROR CRITERION, AND NUMBER OF NODE COLUMNS, ROWS, AND
LAYERS.

READ(105,10)NSTEPS,DELTA,ERROR,NC,NR,NL
10 FORMAT(16,2F6.0,3I4)

C**** READ NODE DEFAULT VALUES.

DO 11 K=1,NL
11 READ(105,12)S1(K),HH(K),QQ(K),S2(K),CCH(K),PP(K),BOTT(K),RR(K)
12 FORMAT(9F6.0)

C**** FILL ARRAYS WITH DEFAULT VALUES.

DO 20 K=1,NL
QMC(K)=0.0
DO 20 I=1,NC
DO 20 J=1,NR
SF1(I,J,K)=S1(K)
H(I,J,K)=HH(K)
H0(I,J,K)=HH(K)
DL(I,J,K)=0.0
SF2(I,J,K)=S2(K)
CH(I,J,K)=CCH(K)
HCY(I,J,K)=PP(K)
HCX(I,J,K)=PP(K)

```

      BOT(I,J,K)=BOTI(K)
      NOC(I,J,K)=7
      PISE(I,J,K)=0.0
      FK(I,J,K)=0.0
      R(I,J,K)=RP(K)
      DM(I,J,K)=0.0
      Q(I,J,K)=QQ(K)
20  Q(I,J,K)=QQ(K)
C**** FILL NODE DIMENSION ARRAYS.
      READ(105,30)(X1(I),I=1,NC)
      READ(105,30)(X2(I),I=1,NC)
30  FORMAT(7F10.0)
      READ(105,31)(Y1(J),J=1,NR)
      READ(105,31)(Y2(J),J=1,NR)
31  FORMAT(15F6.0)
C**** COMPUTE SF1, SF2, R, AND Q FACTORS FROM THEIR BASIC INPUT VALUES.
50  DO 49 K=1,NL
      DO 49 I=1,NC
      DO 49 J=1,NR
          SF1(I,J,K)=SF1(I,J,K)*X2(I)*Y2(J)
          SF2(I,J,K)=SF2(I,J,K)*X2(I)*Y2(J)
          R(I,J,K)=R(I,J,K)*X2(I)*Y2(J)
49  CONTINUE
C**** REPLACE DEFAULT VALUES WITH ACTUAL NODE DATA, READ MSTEP VALUE.
      CALL VALUE
C**** START THE SIMULATION.
      KSTEP=1
      TIME=0.
C
C
C II. SIMULATION WITH THE MIADI METHOD. THIS IS CONTAINED IN THE DO
C 311 LOOP.
C
C
      DO 311 ISTEP=1,NSTEPS
C**** DOES A MINING CHANGE OCCUR? IF SO, READ NEW NODE DATA BY CALLING
C SUPERROUTINE "VALUE."
      IF(ISTEP.LT.MSTEP)GO TO 56
      CALL VALUE
C**** PREDICT HEADS FOR THE NEXT TIME INCREMENT WITH THE PRICKEY AND
C LONGQUIST PREDICTION ROUTINE.
      56 TIME=TIME+DELTA

```

```

DO 70 K=1,NL
DO 70 I=1,NC
DO 70 J=1,NR
D=H(I,J,K)-HO(I,J,K)
HO(I,J,K)=H(I,J,K)
F=1.
IF(DL(I,J,K).EQ.0.0)GO TO 60
IF(ISTEP.GT.2)F=D/DL(I,J,K)
IF(F.GT.5)F=5.0
IF (F.LT.0.0)F=0.0
60 DL(I,J,K)=D
H(I,J,K)=H(I,J,K)+D*F
70 IF(H(I,J,K).LE.BOT(I,J,K))H(I,J,K)=BOT(I,J,K)+0.01
C**** REFINE ESTIMATES WITH THE MIADI METHOD.
ITER=0
80 IF(ITER.EQ.7)GO TO 305
E=0.0
ITER=ITER+1
C**** ASSIGN INTERNUAL TRANSMISSIVITY FACTORS.
CALL TRANS
C**** RECALCULATE Q(I,J,K) VALUES.
CALL FLOW
C
C A. COLUMN CALCULATIONS (FIRST HALF ITERATION), THE DO 190 LOOP.
C
DO 190 K=1,NL
DO 190 I1=1,NC
I=I1
IF(MOD(ISTEP+ITER,2).EQ.1) I=NC-I+1
C**** WRITE THE NODE EQUATIONS FOR ALL COLUMN I NODES. THIS IS THE
C FUNCTION OF THE DO 170 LOOP.
DO 170 J=1,NR
ST=H(I,J,K)-BOT(I,J,K)
C**** ASSIGN NODE STORAGE FACTOR.
S=SF1(I,J,K)
IF(H(I,J,K).LT.CH(I,J,K))S=SF2(I,J,K)
BB=S/DLTA
DD=HO(I,J,K)*S/DLTA-Q(I,J,K)
IF((H(I,J,K)-CH(I,J,K))*(CH(I,J,K)-CH(I,J,K)).LT.0.0)DD=DD+(HO(I,J
1,K)-CH(I,J,K))*(SF1(I,J,K)-SF2(I,J,K))/DLTA
C**** THE FOLLOWING SECTION ACCOUNTS FOR INTERAQUIFER LEAKAGE. BECAUSE

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C   THE R ARRAY IS ALSO USED FOR INTERCEPTED NATURAL FLOW (FLOW OUT
C   OF NODES ADJACENT TO THE MINE CUT, NODES CODED WITH NOD = 1), THE
C   CONDITIONALS ABOUT NOD ARE INCLUDED.
      IF(NOD(I,J,K).LT.2)GO TO 88
      IF(X.EQ.1)GO TO 88
      IF(NOD(I,J,K-1).LT.2)GO TO 88
      IF(H(I,J,K-1)-DOT(I,J,K-1).LT..02.AND.H(I,J,K-1).GT.H(I,J,K))GO TO
188
      IF(H(I,J,K).GT.CH(I,J,K))GO TO 87
      DD=DD+(P(I,J,K)+(H(I,J,K-1)-CH(I,J,K)))
      GO TO 88
87  BB=BB+R(I,J,K)
      DD=DD+(R(I,J,K)+H(I,J,K-1))
88  AA=0.0
      CC=0.0
      IF(NOD(I,J,K).LT.2)GO TO 89
      IF(X-AL.EQ.0)GO TO 89
      IF(NOD(I,J,K+1).LT.2)GO TO 89
      IF(ST.LT..02.AND.H(I,J,K).GT.H(I,J,K+1))GO TO 89
      IF(H(I,J,K+1).GT.CH(I,J,K+1))GO TO 881
      BB=BB+P(I,J,K+1)
      DD=DD+(R(I,J,K+1)+CH(I,J,K+1))
      GO TO 89
881 BB=BB+R(I,J,K+1)
      DD=DD+(P(I,J,K+1)+H(I,J,K+1))
C**** COMPLETE THE NODE EQUATION. THE CONDITIONALS CONCERNING
C   SATURATED THICKNESSES OF NODE I,J,K AND SURROUNDING NODES ARE
C   DESIGNED TO PREVENT FLOW OUT OF DRY NODES.
89  IF(J-1)90,100,90
90  IF(H(I,J-1,K)-BOT(I,J-1,K).LT..02.AND.H(I,J-1,K).GT.H(I,J,K))GO TO
1100
      IF(ST.LT..02.AND.H(I,J,K).GT.H(I,J-1,K))GO TO 100
      AA=-TY(I,J-1,K)
      BB=BB+TY(I,J-1,K)
100 IF(J-NE)110,120,110
110 IF(H(I,J+1,K)-BOT(I,J+1,K).LT..02.AND.H(I,J+1,K).GT.H(I,J,K))GO TO
1120
      IF(ST.LT..02.AND.H(I,J,K).GT.H(I,J+1,K))GO TO 120
      CC=-TY(I,J,K)
      BP=BP+TY(I,J,K)
120 IF(I-1)130,140,130

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130 IF(H(I-1,J,K)-ROT(I-1,J,K).LT..02.AND.H(I-1,J,K).GT.H(I,J,K))GO TO
1140
IF(ST.LT..02.AND.H(I,J,K).GT.H(I-1,J,K))GO TO 140
PB=PB+TX(I-1,J,K)
UD=UD+H(I-1,J,K)*TX(I-1,J,K)
140 IF(T-NC)150,160,150
150 IF(H(I+1,J,K)-ROT(I+1,J,K).LT..02.AND.H(I+1,J,K).GT.H(I,J,K))GO TO
1160
IF(ST.LT..02.AND.H(I,J,K).GT.H(I+1,J,K))GO TO 160
BR=BR+TX(I,J,K)
UD=UD+H(I+1,J,K)*TX(I,J,K)
160 ASSIGN 162 TO K2
C**** GO TO "LINK" TO CONNECT AQUIFERS IF NECESSARY.
GO TO 500
162 W=PB-AA+H(J-1,K)
F(J,K)=CC/W
170 G(J,K)=(OD-AA+G(J-1,K))/W
C**** RE-ESTIMATE COLUMN 1 NODE HEADS WITH THE THOMAS ALGORITHM.
E=E+ABS(H(I,NP,K)-G(NR,K))
H(I,NP,K)=G(NR,K)
N=NP-1
180 HA=E(N,K)-R(N,K)+H(I,N+1,K)
E=E+ABS(HA-H(I,N,K))
H(I,N,K)=HA
N=N-1
IF(N.GT.0)GO TO 180
DO 190 N=1,NR
IF(H(I,N,K).GT.ROT(I,N,K))GO TO 190
E=E-ROT(I,N,K)+0.01+H(I,N,K)
H(I,N,K)=ROT(I,N,K)+0.01
190 CONTINUE
OUTPUT E,ITER
C**** ASSIGN INTERNODAL TRANSMISSIVITY FACTORS.
CALL TRANS
C**** RECALCULATE Q(I,J,K) VALUES.
CALL FLOW
C
C B. ROW CALCULATIONS (SECOND HALF ITERATION). THIS HAS THE SAME FORM
C AS THE COLUMN CALCULATIONS AND IS ALL INCLUDED IN THE DO 300 LOOP.
C
DO 300 K=1,NL

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DO 300 JJ=1,NR
J=JJ
IF (MOD(ISTEP+ITER,7).EQ.1) J=NR-J+1
C**** WRITE THE NODE EQUATIONS FOR ALL ROW J NODES. THIS IS THE FUNCTION
C OF THE DO 280 LOOP.
DO 280 I=J,NC
ST=F(I,J,K)-ROT(I,J,K)
C**** ASSIGN NODE STORAGE FACTOR.
S=SF1(I,J,K)
IF(F(I,J,K).LT.CH(I,J,K))S=SF2(I,J,K)
PP=S/DELTA
DD=HO(I,J,K)*C/DELTA-Q(I,J,K)
IF((HO(I,J,K)-CH(I,J,K))*CH(I,J,K)-CH(I,J,K)).LT.0.0)DD=DD+(HO(I,J
1,K)-CH(I,J,K))*(SF1(I,J,K)-SF2(I,J,K))/DELTA
C**** THE FOLLOWING SECTION ACCOUNTS FOR INTERAQUIFER LEAKAGE.
IF(NOD(I,J,K).LT.2)GO TO 198
IF(K.EQ.1)GO TO 198
IF(NOD(I,J,K-1).LT.2)GO TO 198
IF(CH(I,J,K-1)-HOT(I,J,K-1).LT..02.AND.H(I,J,K-1).GT.H(I,J,K))GO TO
1198
IF(H(I,J,K).GT.CH(I,J,K))GO TO 197
DD=DD+(H(I,J,K)*(H(I,J,K-1)-CH(I,J,K)))
GO TO 198
197 RR=RR+R(I,J,K)
DD=DD+(R(I,J,K)+H(I,J,K-1))
198 AA=0.0
CC=0.0
IF(NOD(I,J,K).LT.2)GO TO 199
IF(K-PL.EQ.0)GO TO 199
IF(NOD(I,J,K+1).LT.2)GO TO 199
IF(ST.LT..02.AND.H(I,J,K).GT.H(I,J,K+1))GO TO 199
IF(CH(I,J,K+1).GT.CH(I,J,K+1))GO TO 1981
RR=RR+R(I,J,K+1)
DD=DD+(R(I,J,K+1)+CH(I,J,K+1))
GO TO 199
1981 RR=RR+R(I,J,K+1)
DD=DD+(R(I,J,K+1)+H(I,J,K+1))
C**** COMPLETE THE NODE EQUATION.
199 IF(J-1)200,210,200
200 IF(H(I,J-1,K)-HOT(I,J-1,K).LT..02.AND.H(I,J-1,K).GT.H(I,J,K))GO TO
1210

```

```

IF(ST.LT..02.AND.H(I,J,K).GT.H(I,J-1,K))GO TO 210
HB=HB+TY(I,J-1,K)
DU=DU+H(I,J-1,K)+TY(I,J-1,K)
210 IF(J-NK)220,230,220
220 IF(H(I,J+1,K)-BOT(I,J+1,K).LT..02.AND.H(I,J+1,K).GT.H(I,J,K))GO TO
1230
IF(ST.LT..02.AND.H(I,J,K).GT.H(I,J+1,K))GO TO 230
DB=DB+H(I,J+1,K)+TY(I,J,K)
HB=HB+TY(I,J,K)
230 IF(I-1)240,250,240
240 IF(H(I-1,J,K)-BOT(I-1,J,K).LT..02.AND.H(I-1,J,K).GT.H(I,J,K))GO TO
1250
IF(ST.LT..02.AND.H(I,J,K).GT.H(I-1,J,K))GO TO 250
BP=BP+TX(I-1,J,K)
AA=-TX(I-1,J,K)
250 IF(I-NC)260,270,260
260 IF(H(I+1,J,K)-BOT(I+1,J,K).LT..02.AND.H(I+1,J,K).GT.H(I,J,K))GO TO
1270
IF(ST.LT..02.AND.H(I,J,K).GT.H(I+1,J,K))GO TO 270
BP=BP+TX(I,J,K)
CC=-TX(I,J,K)
270 ASSIGN 272 TO K2
C*** GO TO "LINK" TO CONNECT AQUIFERS IF NECESSARY.
GO TO 500
272 W=EP-AA+B(I-1,K)
B(I,K)=CC/W
280 G(I,K)=(DD-AA+G(I-1,K))/W
C*** RE-ESTIMATE KOW J HEADS WITH THE THOMAS ALGORITHM.
F=E+ABS(H(NC,J,K)-G(NC,K))
H(NC,J,K)=G(NC,K)
N=NC-1
290 HA=G(N,K)+H(N,K)+H(N+1,J,K)
E=E+ABS(H(N,J,K)-HA)
H(N,J,K)=HA
N=N-1
IF(N.GT.0)GO TO 290
DO #00 N=1,NC
IF(H(N,J,K).GT.BOT(N,J,K))GO TO 300
F=F-POT(N,J,K)-0.01+H(N,J,K)
H(N,J,K)=BOT(N,J,K)+0.01
300 CONTINUE

```

```

      OUTPUT E,ITER
C
C C. DOES ERROR MEET CRITERION? IF NOT, RETURN TO 80.
      IF(E.GT.ERROR) GO TO 80
      IF(ITER.LF.100) GO TO 80
      IF(ITER.LI.7.AND.ISTEP.EQ.1) GO TO 80
305 WRITE(108,310)TIME,E,ITER
310 FORMAT('TIME =',F8.4/,'ERROR =',E20.7/,'ITER =',I2//)
C
C D. CALCULATE HEAD RISES.
C
      DO 400 K=1,NL
      DO 400 I=1,NC
      DO 400 J=1,NR
      RTSE(I,J,K)=RISE(I,J,K)+(H(I,J,K)-HO(I,J,K))
400 CONTINUE
      DELTA=DELTA+1.2
C*** DELTA WAS LIMITED TO A MAXIMUM OF 10 DAYS BECAUSE OF CONVERGENCE
C PROBLEMS.
      IF(DELTA.GT.10.0)DELTA=10.0
C
C E. PRINT SOLUTIONS IF DESIRED.
C
      IF(ISTEP.NE.KSTEP)GO TO 311
C*** PRINT OUT IS DESIRED AT EVERY 17TH ISTEP.
      KSTEP=KSTEP+17
      DO 314 K=1,NL
      WRITE(108,401)K
401 FORMAT('1',45X,'RESULTS FOR AQUIFER ',I1)
      WRITE(108,402)TIME
402 FORMAT(47X,'AT TIME=',1F5.1,' DAYS'//)
      WRITE(108,403)QMC(K)
403 FORMAT(1X,'DRAINAGE RATE=',1F7.0,' CFD'/)
      WRITE(108,404)
404 FORMAT(1X,'CHANGES IN NODE HEADS')
      WRITE(108,405)
405 FORMAT(1X,'SINCE MINE OPENING (TIME=0. DAYS):')
      WRITE(108,406)
406 FORMAT(49X,'COLUMNS 1 - 21')
      WRITE(108,407)

```


[illegible]

```

C      MEAN. THE CONDITIONALS CONCERNING SATURATED THICKNESSES ARE TO
C      PREVENT FLOW OUT OF DRY NODES.
C
501 IF(I.EQ.NC)GO TO 502
   IF(H(I+1,J,K-1)-BOT(I+1,J,K-1).LT..02)GO TO 502
   R1=(H(I+1,J,K-1)-BOT(I+1,J,K-1))*HCX(I+1,J,K-1)
   IF(H(I+1,J,K-1).GT.CH(I+1,J,K-1))R1=(CH(I+1,J,K-1)-BOT(I+1,J,K-1))
   I*HCX(I+1,J,K-1)
   TP1=(2.*AX+R1*Y2(J)/X1(I))/(AX+R1)
   DD=DD+H(I+1,J,K-1)*TR1
   BR=BR+TR1
502 IF(I.EQ.1)GO TO 503
   IF(H(I-1,J,K-1)-BOT(I-1,J,K-1).LT..02)GO TO 503
   L1=(H(I-1,J,K-1)-BOT(I-1,J,K-1))*HCX(I-1,J,K-1)
   IF(H(I-1,J,K-1).GT.CH(I-1,J,K-1))L1=(CH(I-1,J,K-1)-BOT(I-1,J,K-1))
   I*HCX(I-1,J,K-1)
   TL1=(2.*AX+L1*Y2(J)/X1(I-1))/(AX+L1)
   DD=DD+H(I-1,J,K-1)*TL1
   BR=BR+TL1
503 IF(J.EQ.1)GO TO 504
   IF(H(I,J-1,K-1)-BOT(I,J-1,K-1).LT..02)GO TO 504
   T1=(H(I,J-1,K-1)-BOT(I,J-1,K-1))*HCY(I,J-1,K-1)
   IF(H(I,J-1,K-1).GT.CH(I,J-1,K-1))T1=(CH(I,J-1,K-1)-BOT(I,J-1,K-1))
   I*HCY(I,J-1,K-1)
   TT1=(2.*AY+T1*X2(I)/Y1(J-1))/(AY+T1)
   DD=DD+H(I,J-1,K-1)*TT1
   BR=BR+TT1
504 IF(J.EQ.NP)GO TO 505
   IF(H(I,J+1,K-1)-BOT(I,J+1,K-1).LT..02)GO TO 505
   B1=(H(I,J+1,K-1)-BOT(I,J+1,K-1))*HCY(I,J+1,K-1)
   IF(H(I,J+1,K-1).GT.CH(I,J+1,K-1))B1=(CH(I,J+1,K-1)-BOT(I,J+1,K-1))
   I*HCY(I,J+1,K-1)
   TB1=(2.*AY+B1*X2(I)/Y1(J))/(AY+B1)
   DD=DD+H(I,J+1,K-1)*TB1
   BR=BR+TB1
C*** IS NODE I,J,K LINKED WITH THE NEXT AQUIFER UP? IF NOT, GO TO 509.
505 IF(NDD(I,J,K).NE.32)GO TO 509
C
C
C II. THIS SECTION LINKS NODE I,J,K TO THE NEXT AQUIFER UP. IT IS

```

C SIMILAR TO SECTION I.
C
C

```

IF(I.EQ.NC)GO TO 506
IF(H(I+1,J,K-2)-BOT(I+1,J,K-2).LT..02)GO TO 506
R2=(H(I+1,J,K-2)-BOT(I+1,J,K-2))*HCX(I+1,J,K-2)
IF(H(I+1,J,K-2).GT.CH(I+1,J,K-2))R2=(CH(I+1,J,K-2)-BOT(I+1,J,K-2))
1*HCX(I+1,J,K-2)
TP2=(2.*AX*Y2*Y2(J)/X1(I))/(AX+R2)
DD=DD+H(I+1,J,K-2)*TR2
BB=BB+TR2
506 IF(I.EQ.1)GO TO 507
IF(H(I-1,J,K-2)-BOT(I-1,J,K-2).LT..02)GO TO 507
L2=(H(I-1,J,K-2)-BOT(I-1,J,K-2))*HCX(I-1,J,K-2)
IF(H(I-1,J,K-2).GT.CH(I-1,J,K-2))L2=(CH(I-1,J,K-2)-BOT(I-1,J,K-2))
1*HCX(I-1,J,K-2)
TL2=(2.*AX*L2*Y2(J)/X1(I-1))/(AX+L2)
DD=DD+H(I-1,J,K-2)*TL2
BB=BB+TL2
507 IF(J.EQ.1)GO TO 508
IF(H(I,J-1,K-2)-BOT(I,J-1,K-2).LT..02)GO TO 508
T2=(H(I,J-1,K-2)-BOT(I,J-1,K-2))*HCY(I,J-1,K-2)
IF(H(I,J-1,K-2).GT.CH(I,J-1,K-2))T2=(CH(I,J-1,K-2)-BOT(I,J-1,K-2))
1*HCY(I,J-1,K-2)
TT2=(2.*AY*T2*X2(I)/Y1(J-1))/(AY+T2)
DD=DD+H(I,J-1,K-2)*TT2
BB=BB+TT2
508 IF(J.EQ.NR)GO TO 521
IF(H(I,J+1,K-2)-BOT(I,J+1,K-2).LT..02)GO TO 521
R2=(H(I,J+1,K-2)-BOT(I,J+1,K-2))*HCY(I,J+1,K-2)
IF(H(I,J+1,K-2).GT.CH(I,J+1,K-2))R2=(CH(I,J+1,K-2)-BOT(I,J+1,K-2))
1*HCY(I,J+1,K-2)
TB2=(2.*AY*R2*X2(I)/Y1(J))/(AY+R2)
DD=DD+H(I,J+1,K-2)*TB2
BB=BB+TB2
GO TO 521
509 IF(K.EQ.3)GO TO 521

```

C
C III. THIS SECTION LINKS NODE I,J,K WITH THE SURROUNDING NODE(S) IN THE
C AQUIFER BELOW THAT ARE CODED WITH NOD = 21, 31, OR 32.

C
C

```

IF(CH(I,J,K)-ROT(I,J,K).LT..02)GO TO 521
IF(I.EQ.1)GO TO 511
IF(NOD(I-1,J,K+1).EQ.31.OR.NOD(I-1,J,K+1).EQ.32)GO TO 510
IF(NOD(I-1,J,K+1).EQ.21)GO TO 510
GO TO 511
510 L1=(CH(I-1,J,K+1)-ROT(I-1,J,K+1))*HCX(I-1,J,K+1)
IF(CH(I-1,J,K+1).GT.CH(I-1,J,K+1))L1=(CH(I-1,J,K+1)-ROT(I-1,J,K+1))
1*HCX(I-1,J,K+1)
TL1=(2.*AX*L1*Y2(J)/X1(I-1))/(AX*L1)
DD=DD+H(I-1,J,K+1)*TL1
BB=BB+TL1
511 IF(I.EQ.NC)GO TO 513
IF(NOD(I+1,J,K+1).EQ.31.OR.NOD(I+1,J,K+1).EQ.32)GO TO 512
IF(NOD(I+1,J,K+1).EQ.21)GO TO 512
GO TO 513
512 R1=(CH(I+1,J,K+1)-ROT(I+1,J,K+1))*HCX(I+1,J,K+1)
IF(CH(I+1,J,K+1).GT.CH(I+1,J,K+1))R1=(CH(I+1,J,K+1)-ROT(I+1,J,K+1))
1*HCX(I+1,J,K+1)
TR1=(2.*AX*R1*Y2(J)/X1(I+1))/(AX*R1)
DD=DD+H(I+1,J,K+1)*TR1
LR=LR+TR1
513 IF(J.EQ.1)GO TO 515
IF(NOD(I,J-1,K+1).EQ.31.OR.NOD(I,J-1,K+1).EQ.32)GO TO 514
IF(NOD(I,J-1,K+1).EQ.21)GO TO 514
GO TO 515
514 T1=(CH(I,J-1,K+1)-ROT(I,J-1,K+1))*HCY(I,J-1,K+1)
IF(CH(I,J-1,K+1).GT.CH(I,J-1,K+1))T1=(CH(I,J-1,K+1)-ROT(I,J-1,K+1))
1*HCY(I,J-1,K+1)
TT1=(2.*AY*T1*X2(I)/Y1(J-1))/(AY*T1)
DD=DD+H(I,J-1,K+1)*TT1
BB=BB+TT1
515 IF(J.EQ.NR)GO TO 517
IF(NOD(I,J+1,K+1).EQ.31.OR.NOD(I,J+1,K+1).EQ.32)GO TO 516
IF(NOD(I,J+1,K+1).EQ.21)GO TO 516
GO TO 517
516 B1=(CH(I,J+1,K+1)-ROT(I,J+1,K+1))*HCY(I,J+1,K+1)
IF(CH(I,J+1,K+1).GT.CH(I,J+1,K+1))B1=(CH(I,J+1,K+1)-ROT(I,J+1,K+1))
1*HCY(I,J+1,K+1)
TB1=(2.*AY*B1*X2(I)/Y1(J+1))/(AY*B1)

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```

DD=DD+H(I,J+1,K+1)*TB1
BE=BE+TB1
C*** IF NODE I,J,K IS NOT IN AQUIFER 1 (K=1), GO TO 521.
517 IF(K.GT.1)GO TO 521
C
C
C IV. THIS SECTION LINKS NODE I,J,K WITH THE SURROUNDING NODE(S) IN
C AQUIFER 3 IF THEY ARE CODED WITH MOD = 32.
C
C
IF(I.EQ.1)GO TO 518
IF(NOD(I-1,J,K+2).NE.32)GO TO 518
L2=(H(I-1,J,K+2)-BOT(I-1,J,K+2))*HCX(I-1,J,K+2)
IF(H(I-1,J,K+2).GT.CH(I-1,J,K+2))L2=(CH(I-1,J,K+2)-BOT(I-1,J,K+2))
1*HCX(I-1,J,K+2)
TL2=(2.*AX*L2+Y2(J))/X1(I-1)/(AX+L2)
DD=DD+H(I-1,J,K+2)*TL2
BE=BE+TL2
518 IF(I.EQ.NC)GO TO 519
IF(NOD(I+1,J,K+2).NE.32)GO TO 519
P2=(H(I+1,J,K+2)-BOT(I+1,J,K+2))*HCX(I+1,J,K+2)
IF(H(I+1,J,K+2).GT.CH(I+1,J,K+2))P2=(CH(I+1,J,K+2)-BOT(I+1,J,K+2))
1*HCX(I+1,J,K+2)
TR2=(2.*AX*P2+Y2(J))/X1(I+1)/(AX+P2)
DD=DD+H(I+1,J,K+2)*TR2
BE=BE+TR2
519 IF(J.EQ.1)GO TO 520
IF(NOD(I,J-1,K+2).NE.32)GO TO 520
T2=(H(I,J-1,K+2)-BOT(I,J-1,K+2))*HCY(I,J-1,K+2)
IF(H(I,J-1,K+2).GT.CH(I,J-1,K+2))T2=(CH(I,J-1,K+2)-BOT(I,J-1,K+2))
1*HCY(I,J-1,K+2)
TT2=(2.*AY*T2+X2(I))/Y1(J-1)/(AY+T2)
DD=DD+H(I,J-1,K+2)*TT2
BE=BE+TT2
520 IF(J.EQ.NR)GO TO 521
IF(NOD(I,J+1,K+2).NE.32)GO TO 521
B2=(H(I,J+1,K+2)-BOT(I,J+1,K+2))*HCY(I,J+1,K+2)
IF(H(I,J+1,K+2).GT.CH(I,J+1,K+2))B2=(CH(I,J+1,K+2)-BOT(I,J+1,K+2))
1*HCY(I,J+1,K+2)
TR2=(2.*AY*B2+X2(I))/Y1(J)/(AY+B2)
DD=DD+H(I,J+1,K+2)*TB2

```

DA=BR+TBZ
C**** RETURN CONTROL TO "CENTRAL."
521 GO TO K2,(162,272)

C
600 OUTPUT TIME
END

C
C
C
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C

SUBROUTINE TRANS

THIS SUBROUTINE CALCULATES INTERNODAL TRANSMISSIVITY FACTORS.

SUBROUTINE TRANS-
COMMON HC(21,39,3),HC(21,39,3),SF1(21,39,3),CC(21,39,3),TY(21,39,3),
ITX(21,39,3),RC(39,3),G(39,3),OL(21,39,3),SF2(21,39,3),CH(21,39,3),H
ZCY(21,39,3),HCX(21,39,3),ROT(21,39,3),MOD(21,39,3),FISE(21,39,3),F
3K(21,39,3),PC(21,39,3),I,J,K,NC,NR,NL,ITER,TIME,DELTA,*STEP,S1(3),S
42(3),HH(3),CCH(3),PP(3),ROTT(3),RR(3),QQ(3),OH(21,39,3),X1(21),X2(21),
Y1(39),Y2(39),Q*CC(3)
DO 90 K=1,NL
DO 90 I=1,NC
DO 90 J=1,NR
TXX=0.0
TTY=0.0
C**** INITIALLY CALCULATE TXX AND TTY FOR NODE I,J,K WITH HARMONIC MEANS
C ASSUMING CONFINED CONDITIONS.
IF(I.EQ.NC)GO TO 79
AX=(CH(I,J,K)-ROT(I,J,K))*HCX(I,J,K)
BX=(CH(I+1,J,K)-ROT(I+1,J,K))*HCX(I+1,J,K)
IF(AX+BX.LE..01)GO TO 79
TXX=(2.*AX+BX)/(AX+BX)
79 IF(S.EQ.NR)GO TO 90
AY=(CH(I,J,K)-ROT(I,J,K))*HCY(I,J,K)
BY=(CH(I,J+1,K)-ROT(I,J+1,K))*HCY(I,J+1,K)
IF(AY+BY.LE..01)GO TO 80
TTY=(2.*AY+BY)/(AY+BY)

```

C**** IS NODE I,J,K CONFINED OR UNCONFINED?
80 IF(CH(I,J,K)-CH(I,J,K))C1,81,82
81 IF(I.EQ.NC)GO TO 85
C**** IS NODE I+1,J,K CONFINED OR UNCONFINED?
IF(CH(I+1,J,K)-CH(I+1,J,K))83,83,84
C
C
C 1. NODE I,J,K IS UNCONFINED.
C
C
C**** SINCE BOTH NODES ARE UNCONFINED, CALCULATE TXX WITH THE GEOMETRIC
MEAN. HOWEVER, IF THE NODES ARE NONHOMOGENOUS, RECALCULATE WITH
C THE HARMONIC MEAN.
83 AX=(CH(I,J,K)-ROT(I,J,K))*HCX(I,J,K)
BX=(CH(I+1,J,K)-ROT(I+1,J,K))*HCX(I+1,J,K)
TXX=SQRT(AX*BX)
IF(NOD(I,J,K).LT.3)GO TO 831
IF(NOD(I+1,J,K).LT.3)GO TO 831
IF(NOD(I,J,K).EQ.NOD(I+1,J,K))GO TO 831
IF(AX+BX.LE..01)GO TO 831
TXX=(2.*AX*BX)/(AX+BX)
831 AM=(CH(I,J,K)-ROT(I,J,K))/(CH(I+1,J,K)-ROT(I+1,J,K))
BM=(CH(I,J,K)-ROT(I,J,K)+CH(I+1,J,K)-ROT(I+1,J,K))/2.
IF(AM.LT..20.OR.AM.GT.5.)TXX=BM*(HCX(I,J,K)+HCX(I+1,J,K))/2.
GO TO 85
C**** BECAUSE NODE I+1,J,K IS CONFINED, USE THE HARMONIC MEAN FOR TXX.
84 AX=(CH(I,J,K)-ROT(I,J,K))*HCX(I,J,K)
BX=(CH(I+1,J,K)-ROT(I+1,J,K))*HCX(I+1,J,K)
IF(AX+BX.LE..01)GO TO 85
TXX=(2.*AX*BX)/(AX+BX)
85 IF(J.EQ.NR)GO TO 89
C**** IS NODE I,J+1,K CONFINED OR UNCONFINED?
IF(CH(I,J+1,K)-CH(I,J+1,K))86,86,87
C**** BECAUSE BOTH NODES I,J,K AND I,J+1,K ARE UNCONFINED, COMPUTE TYY
WITH THE GEOMETRIC MEAN. HOWEVER, IF THEY ARE NONHOMOGENOUS NODES,
C RECALCULATE WITH THE HARMONIC MEAN.
86 AY=(CH(I,J,K)-ROT(I,J,K))*HCY(I,J,K)
BY=(CH(I,J+1,K)-ROT(I,J+1,K))*HCY(I,J+1,K)
TYY=SQRT(AY*BY)
IF(NOD(I,J,K).LT.3)GO TO 861
IF(NOD(I,J+1,K).LT.3)GO TO 861

```

```

      IF(NOD(I,J,K).EQ.NOD(I,J+1,K))GO TO 861
      IF(AY+BY.LE..01)GO TO 861
      TYY=(2.*AY+BY)/(AY+BY)
861  AW=(HC(I,J,K)-BOT(I,J,K))/(HC(I,J+1,K)-BOT(I,J+1,K))
      BW=(HC(I,J,K)-BOT(I,J,K)+HC(I,J+1,K)-BOT(I,J+1,K))/2.
      IF(AW.LE..20.OR.AW.GT.5.0)TYY=(HCY(I,J,K)+HCY(I,J+1,K))*BW/2.
      GO TO 89
C**** SINCE NODE I,J+1,K IS CONFINED, COMPUTE TYY WITH THE HARMONIC MEAN.
87  AY=(HC(I,J,K)-BOT(I,J,K))*HCY(I,J,K)
      PY=(HC(I,J+1,K)-BOT(I,J+1,K))*HCY(I,J+1,K)
      IF(AY+PY.LE..01)GO TO 89
      TYY=(2.*AY+PY)/(AY+PY)
      GO TO 89
C
C
C II. NODE I,J,K IS CONFINED. IN THIS CASE, BOTH TXX AND TYY ARE CALCU-
C   LATED WITH THE HARMONIC MEAN.
C
C
82  IF(I.EQ.AC)GO TO 88
      IF(H(I+1,J,K)-CH(I+1,J,K).GE.0.0)GO TO 88
      AX=(CH(I,J,K)-BOT(I,J,K))*HCX(I,J,K)
      PX=(H(I+1,J,K)-BOT(I+1,J,K))*HCX(I+1,J,K)
      IF(AX+PX.LE..01)GO TO 88
      TXX=(2.*AX+PX)/(AX+PX)
88  IF(J.EQ.AR)GO TO 89
      IF(H(I,J+1,K)-CH(I,J+1,K).GE.0.0)GO TO 89
      AY=(CH(I,J,K)-BOT(I,J,K))*HCY(I,J,K)
      BY=(H(I,J+1,K)-BOT(I,J+1,K))*HCY(I,J+1,K)
      IF(AY+BY.LE..01)GO TO 89
      TYY=(2.*AY+BY)/(AY+BY)
C
C
C III. CONVERT TXX AND TYY TO TRANSMISSIVITY FACTORS TX(I,J,K) AND
C   TY(I,J,K).
C
C
89  TX(I,J,K)=TXX*Y2(J)/X1(I)
      TY(I,J,K)=TYY*X2(I)/Y1(J)
90  CONTINUE
      RETURN

```


END

SUBROUTINE FLOW

THIS SUBROUTINE CALCULATES RECHARGE ALONG ROW 39 AND CALCULATES DISCHARGE INTO THE MINE CUT.

```

SUBROUTINE FLOW
COMMON HC(21,39,3),HNC(21,39,3),SF1(21,39,3),QC(21,39,3),TY(21,39,3),
1TY(21,39,3),RC(19,3),GC(39,3),DL(21,39,3),SF2C(21,39,3),CHC(21,39,3),H
2CY(21,39,3),HCX(21,39,3),RUC(21,39,3),HND(21,39,3),PISEC(21,39,3),F
3K(21,39,3),K(21,39,3),I,J,K,XC,NR,NL,ITER,TIME,DELTA,STEP,SI(3),S
4Z(3),HC(3),CC(3),PP(3),DIT(3),RR(3),QQ(3),DH(21,39,3),XI(21),XZ
5(21),YI(39),YZ(39),ZC(3)

```

C
C
C
C
C

C I. CALCULATE RECHARGE ALONG ROW 39 USING DARCY'S LAW.

```

DO 5 K=1,NL
DO 5 I=1,NC
DO 5 J=1,NR
IF(J.LI.39)GO TO 5
A=Y2(J)**2
IF(K.LI.3)GO TO 4
Q(I,J,K)=- (H(I,J,K)-H(I,J-1,K))*22.5/A
Q(I,J,K)=Q(I,J,K)*X2(I)*Y2(J)
GO TO 5
4 IF(K.LI.2)GO TO 5
Q(I,J,K)=- (P(I,J,K)-H(I,J-1,K))*22.5/A
C=Q(I,J,K)*.000016
IF(C.GT.0.0)Q(I,J,K)=-.000016
IF(I.LI.15.AND.I.GT.7)Q(I,J,K)=Q(I,J,K)-.00003
IF(I.EQ.6)Q(I,J,K)=-.000036

```

```

      Q(I,J,K)=Q(I,J,K)*X2(I)*Y2(J)
5  CONTINUE
      QT=0.0

```

C
C
C
C
C

II. CALCULATE NONSTEADY AQUIFER OUTFLOW INTO THE MINE CUT. FOR FURTHER EXPLANATION, SEE CHAPTER 4.

```

      DO 10 K=1,NL
      DO 10 I=1,NC
      DO 10 J=1,NR
      IF(NOD(I,J,K).GT.1.AND.NOD(I,J,K).LT.33)GO TO 10
      T=TIME
      IF(TIME.GT.10.0)T=10.0
      Q(I,J,K)=0.0

```

C
C
C

A. COMPUTE OUTFLOW FROM UNCONFINED STORAGE.

```

      U=FK(I,J,K)*(1./SQRT(T))
C**** SOLVE THE INTEGRAL IN EQUATION (60) BY CALLING EST.
      CALL EST(U,ERF)
      EX=(FK(I,J,K)**2)/T
      IF(EX.GT.100.)GO TO 10
      GRAD=.5311*DH(I,J,K)*(1./SQRT(ERF*T))+(1./EXP(EX))*FK(I,J,K)
      Q(I,J,K)=(H(I,J,K)-BOT(I,J,K))*GRAD
      IF(H(I,J,K).GT.CH(I,J,K))Q(I,J,K)=(CH(I,J,K)-BOT(I,J,K))*GRAD
      IF(K.LT.3)GO TO 8

```

C
C
C
C

B. THE FOLLOWING IS USED TO COMPUTE OUTFLOW FROM CONFINED STORAGE IN AQUIFER 3.

```

      FKK=FK(I,J,K)*.0447
      EXX=(FKK**2)/T
      GRAD=.452*(1./EXP(EXX))*(1./SQRT(T))*FKK
      QC=(CH(I,J,K)-BOT(I,J,K))*16.7*DH(I,J,K)*GRAD
      IF(H(I,J,K).LT.CH(I,J,K))QC=(H(I,J,K)-BOT(I,J,K))*16.7*DH(I,J,K)*G
      IRAD
      Q(I,J,K)=Q(I,J,K)+QC

```

C
C

C. ADD INTERCEPTED NATURAL FLOW TO Q.

```
C
C      8 Q(I,J,K)=Q(I,J,K)+K(I,J,K)
C      9 QT=QT+Q(I,J,K)
C     10 CONTINUE
C
C C
C C III. CALCULATE TOTAL NONSTEADY OUTFLOW AND INTERCEPTED NATURAL FLOW
C C FOR EACH AQUIFER.
C C
C C
C C 11 DO 15 K=1,NL
C C    QMC(K)=0.0
C C    DO 15 I=1,NC
C C      DO 15 J=1,NR
C C        IF(Q(I,J,K).LT.0.0)GO TO 15
C C        IF(Q(I,J,K).GT.1)GO TO 15
C C        QMC(K)=QMC(K)+Q(I,J,K)
C C 15 CONTINUE
C C    OUTPUT QT
C C    RETURN
C C    END
C
C
C C
C C SUBROUTINE FST
C C *****
C
C C
C C THIS SUBROUTINE COMPUTES ERF (THE INTEGRAL IN EQUATION (60))
C C BY SIMPSON'S ONE - THIRD RULE.
C C
C C
C C SUBROUTINE EST(U,ERF)
C C   ERF=0.0
C C   IF(U.LE.0.50)E=0.0
C C   IF(U.GT.0.50.AND.U.LE.1.50)E=0.50
C C   IF(U.GT.1.50.AND.U.LE.3.00)E=1.50
C C   IF(U.GT.3.00)GO TO 5
C C   ERF=1./(EXP(U**2))
C C   F=0.01
```

```

1 IF(F.GT.0.99)GO TO 2
  EX=(1+(F*(U-1)))**2
  LRF=ERF+(4.*(1./(EXP(EX))))
  F=F+0.02
  GO TO 1
2 F=0.0
3 F=F+0.02
  IF(F.GT.0.995)GO TO 4
  EX=(1+(F*(U-1)))**2
  LRF=ERF+(2.*(1./(EXP(EX))))
  GO TO 3
4 IF(U.LE.0.50)ERF=ERF+1.0
  IF(U.GT.0.50.AND.U.LE.1.50)ERF=ERF+0.77880
  IF(U.GT.1.50.AND.U.LE.3.00)ERF=ERF+0.10540
  ERF=((U.01)*(U-F)*ERF/3.)
  IF(U.GT.0.50.AND.U.LE.1.50)ERF=ERF+0.46128
  IF(U.GT.1.50.AND.U.LE.3.00)ERF=ERF+0.85619
  GO TO 17
5 ERF=0.88623
17 RETURN
  END

```

SUBROUTINE VALUE

THIS SUBROUTINE READS IN NODE DATA AND THE MSTEP VALUE. IT ALSO APPENDS DATA WHEN NEEDED.

NOTE THAT BASIC VALUES FOR SF1, SF2, R, AND Q ARE READ INTO THE ARRAYS. THE SUBROUTINE THEN CALCULATES THE "FACTORS" USING NODE DIMENSIONS. FOR EXAMPLE, THE SPECIFIC YIELD IS READ IN FOR SF2 AND IS SUBSEQUENTLY MULTIPLIED BY (X2(I)*Y2(J)) TO PRODUCE THE STORAGE "FACTOR."

SUBROUTINE VALUE
COMMON P(21,39,3),HOC(21,39,3),SF1(21,39,3),QC(21,39,3),TY(21,39,3),
1TX(21,39,3),H(39,3),G(39,3),DL(21,39,3),SF2(21,39,3),CH(21,39,3),H


```
2CY(21,39,3),HCX(21,39,3),ROT(21,39,3),NOD(21,39,3),RISE(21,39,3),F
3K(21,39,3),P(21,39,3),I,J,K,NC,NR,NL,ITER,TIME,DELTA,STEP,SIC(3),S
42(2),HH(3),CCH(3),PP(3),BOTT(3),RR(3),QQ(3),DH(21,39,3),X1(21),X2(
521),Y1(39),Y2(39),JMC(3)
```

C
C
C
C
C

I. READ IN A FULL SET OF DATA FOR THE NODES.

```
54 READ(106,40)I,J,K,SF1(I,J,K),SF2(I,J,K),QC(I,J,K),HC(I,J,K),HOC(I,J,K
1),CH(I,J,K),HNT(I,J,K),HCX(I,J,K),HCY(I,J,K),NOD(I,J,K),DH(I,J,K),
7FK(I,J,K),R(I,J,K)
40 FORMAT(I2,3E9.3,4F8.2,2F7.1,1I3,3E9.3)
C**** CALCULATE SF1, SF2, P, AND Q FACTORS FROM THE BASIC DATA.
39 Q(I,J,K)=QC(I,J,K)*X2(I)*Y2(J)
SF1(I,J,K)=SF1(I,J,K)*X2(I)*Y2(J)
SF2(I,J,K)=SF2(I,J,K)*X2(I)*Y2(J)
R(I,J,K)=R(I,J,K)*X2(I)*Y2(J)
C**** IF NOD = 10 OR 20, LEAVE THIS READ ROUTINE.
IF(NOD(I,J,K).EQ.10)GO TO 55
IF(NOD(I,J,K).EQ.20.AND.TIME.NE.C.O)GO TO 56
GO TO 54
```

C
C
C
C
C
C
C

II. READ IN ALL NODE DATA EXCEPT FOR H AND HO. THIS READ ROUTINE IS USED WHEN IT IS DESIRED TO CHANGE NODE PROPERTIES BUT NOT NODE HEAD VALUES. THIS CAN HAPPEN WHEN A MINING CHANGE OCCURS.

```
56 READ(106,42)I,J,K,SF1(I,J,K),SF2(I,J,K),QC(I,J,K),CH(I,J,K),ROT(I,J
1,K),HCX(I,J,K),HCY(I,J,K),NOD(I,J,K),DH(I,J,K),FK(I,J,K),R(I,J,K)
42 FORMAT(I2,3E9.3,2F8.2,2F7.1,1I3,3E9.3)
C**** CALCULATE SF1, SF2, K, AND Q FACTORS FROM THE BASIC DATA.
Q(I,J,K)=QC(I,J,K)*X2(I)*Y2(J)
SF1(I,J,K)=SF1(I,J,K)*X2(I)*Y2(J)
SF2(I,J,K)=SF2(I,J,K)*X2(I)*Y2(J)
R(I,J,K)=R(I,J,K)*X2(I)*Y2(J)
C**** IF NOD = 10, LEAVE THE READ ROUTINE.
IF(NOD(I,J,K).EQ.10)GO TO 90
GO TO 56
90 DELTA=.25
```

-160-

```
      TIME=0.0
C
C
C III. READ THE MSTEP VALUE.
C
C      55 READ(106,41)MSTEP
      41 FORMAT(113)
C
C
C IV. PRINT OUT THE MOD VALUES FOR ALL NODES.
C
C
C      DO 58 K=1,NL
      DO 58 J=1,NR
58 WRITE(108,57)J,(MOD(I,J,K),I=1,NC)
57 FORMAT(15,5X,10I10/(12X10I10))
      RETURN
      END
```

Typical Node Data Input

133	3	.200E-04	.500E-02	.000E 00	429.42	429.42	340.00	345.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
233	3	.200E-04	.500E-02	.000E 00	429.47	429.47	340.00	325.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
333	3	.200E-04	.500E-02	.000E 00	429.54	429.54	335.00	320.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
433	3	.200E-04	.500E-02	.000E 00	429.59	429.59	335.00	320.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
533	3	.200E-04	.500E-02	.000E 00	429.61	429.61	325.00	210.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
633	3	.200E-04	.500E-02	.000E 00	429.62	429.62	320.00	305.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
733	3	.200E-04	.500E-02	.000E 00	429.63	429.63	315.00	301.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
833	3	.200E-04	.500E-02	.000E 00	437.27	437.27	65.00	50.00	.0	1.5	6	.000E 00	.000E 00	.000E 00
933	3	.200E-04	.500E-02	.000E 00	443.80	443.88	82.00	67.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1033	3	.200E-04	.500E-02	.000E 00	443.89	443.89	90.00	83.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1133	3	.200E-04	.500E-02	.000E 00	443.89	443.89	115.00	109.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1233	3	.200E-04	.500E-02	.000E 00	443.90	443.90	130.00	115.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1333	3	.200E-04	.500E-02	.000E 00	443.91	443.91	145.00	134.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1433	3	.200E-04	.500E-02	.000E 00	443.92	443.92	160.00	145.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1533	3	.200E-04	.500E-02	.000E 00	443.94	443.94	170.00	155.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1633	3	.200E-04	.500E-02	.000E 00	443.95	443.96	175.00	160.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1733	3	.200E-04	.500E-02	.000E 00	443.97	443.97	182.00	167.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1833	3	.200E-04	.500E-02	.000E 00	444.00	444.00	190.00	175.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1933	3	.200E-04	.500E-02	.000E 00	444.04	444.04	194.00	179.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
2033	3	.200E-04	.500E-02	.000E 00	444.08	444.09	197.00	182.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
2133	3	.200E-04	.500E-02	.000E 00	444.15	444.15	198.00	183.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
137	3	.200E-04	.500E-02	.100E-04	434.78	434.78	355.00	340.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
219	3	.200E-04	.500E-02	.300E-04	434.82	434.82	325.00	310.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
339	3	.200E-04	.500E-02	.100E-04	434.86	434.86	325.00	310.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
439	3	.200E-04	.500E-02	.300E-04	434.89	434.89	335.00	320.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
539	3	.200E-04	.500E-02	.100E-04	434.90	434.90	325.00	310.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
639	3	.200E-04	.500E-02	.300E-04	434.91	434.91	315.00	300.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
739	3	.200E-04	.500E-02	.300E-04	434.91	434.91	50.00	35.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
839	3	.200E-04	.500E-02	.100E-04	442.61	442.61	60.00	45.00	.0	1.5	6	.000E 00	.000E 00	.000E 00
939	3	.200E-04	.500E-02	.300E-04	449.22	449.22	65.00	50.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1039	3	.200E-04	.500E-02	.300E-04	449.22	449.22	80.00	65.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1139	3	.200E-04	.500E-02	.300E-04	449.22	449.22	95.00	80.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1239	3	.200E-04	.500E-02	.300E-04	449.22	449.22	110.00	95.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1339	3	.200E-04	.500E-02	.300E-04	449.22	449.22	120.00	105.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1439	3	.200E-04	.500E-02	.300E-04	449.22	449.22	130.00	115.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1539	3	.200E-04	.500E-02	.300E-04	449.22	449.22	141.00	126.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1639	3	.200E-04	.500E-02	.300E-04	449.21	449.21	155.00	140.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1739	3	.200E-04	.500E-02	.300E-04	449.20	449.20	165.00	150.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1839	3	.200E-04	.500E-02	.300E-04	449.19	449.19	171.00	156.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
1939	3	.200E-04	.500E-02	.300E-04	449.16	449.16	175.00	160.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00
2039	3	.200E-04	.500E-02	.300E-04	449.12	449.12	177.00	162.00	3.0	1.5	6	.000E 00	.000E 00	.000E 00

2133	3	.200E-04	.500E-02	.300E-04	449.06	449.06	178.00	163.00	4.0	4.0	10	.003E	00	.000E	00	.000E	00	
726	1	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
727	1	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
728	1	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
829	1	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
930	1	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
726	2	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
727	2	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
728	2	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
829	2	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
930	2	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
726	3	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
727	3	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
728	3	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
829	3	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
930	3	.100E	01	.100E	01	.000E	00	1.00	1.00	0.50	0.00	0.0	0.0	11	.000E	00	.000E	00
725	2	.100E-04	.250E-02	.000E	00	397.00	382.00	3.0	1.5	1	.450E	02	.186E	01	.150E-03			
825	2	.100E-04	.250E-02	.000E	00	304.00	369.00	3.0	1.5	1	.900E	02	.186E	01	.150E-03			
826	2	.100E-04	.250E-02	.000E	00	393.00	375.00	3.0	1.5	1	.900E	02	.186E	01	.150E-03			
827	2	.100E-04	.250E-02	.000E	00	389.00	374.00	3.0	1.5	1	.900E	02	.186E	01	.150E-03			
828	2	.100E-04	.250E-02	.000E	00	303.00	368.00	3.0	1.5	1	.900E	02	.186E	01	.150E-03			
929	2	.100E-04	.250E-02	.000E	00	305.00	370.00	3.0	1.5	1	.900E	02	.186E	01	.150E-03			
1030	2	.100E-04	.250E-02	.000E	00	370.00	355.00	3.0	1.5	1	.900E	02	.186E	01	.150E-03			
1031	2	.100E-04	.250E-02	.000E	00	371.00	354.00	3.0	1.5	1	.900E	02	.186E	01	.150E-03			
931	2	.100E-04	.250E-02	.000E	00	382.00	367.00	3.0	1.5	1	.450E	02	.263E	01	.186E-03			
831	2	.100E-04	.250E-02	.100E-02		390.00	375.00	3.0	1.5	6	.000E	00	.000E	00	.000E	00		
830	2	.100E-04	.250E-02	.100E-03		390.00	383.00	3.0	1.5	6	.000E	00	.000E	00	.000E	00		
729	2	.100E-04	.250E-02	.450E-03		405.00	390.00	3.0	1.5	6	.000E	00	.000E	00	.000E	00		
628	2	.100E-04	.250E-02	.600E-03		400.00	391.00	3.0	1.5	6	.000E	00	.000E	00	.000E	00		
627	2	.100E-04	.250E-02	.100E-02		398.00	383.00	3.0	1.5	6	.000E	00	.000E	00	.000E	00		
626	2	.100E-04	.250E-02	.100E-02		401.00	386.00	3.1	1.5	6	.000E	00	.000E	00	.000E	00		
625	2	.100E-04	.250E-02	.100E-02		402.00	387.00	3.0	1.5	6	.000E	00	.000E	00	.000E	00		
828	1	.100E	00	.100E	00	.000E	00	445.00	400.00	100.0	100.0	1	.900E	02	.186E	01	.150E-03	
929	1	.100E	00	.100E	00	.000E	00	460.00	400.00	100.0	100.0	1	.900E	02	.186E	01	.150E-03	
1030	1	.100E	00	.100E	00	.000E	00	460.00	405.00	100.0	100.0	1	.900E	02	.186E	01	.150E-03	
1031	1	.100E	00	.100E	00	.500E-02		440.00	392.00	100.0	100.0	4	.000E	00	.000E	00	.000E	00
931	1	.500E-01	.500E-01	.240E-01		440.00	383.00	20.0	20.0	3	.000E	00	.000E	00	.000E	00		
831	1	.500E-01	.500E-01	.240E-01		430.00	375.00	20.0	20.0	3	.000E	00	.000E	00	.000E	00		
830	1	.100E	00	.100E	00	.600E-02		445.00	395.00	100.0	100.0	4	.000E	00	.000E	00	.000E	00
729	1	.100E	00	.100E	00	.500E-02		440.00	390.00	100.0	100.0	4	.000E	00	.000E	00	.000E	00

627	1	.500E-01	.500E-01	.250E-02	440.00	400.00	20.0	20.0	3	.000E 00	.000E 00	.000E 00
729	3	.100E-04	.250E-02	.000E 00	319.00	304.00	3.0	1.5	1	.900E 02	.186E 01	.233E-03
929	3	.100E-04	.250E-02	.000E 00	315.00	300.00	3.0	1.5	1	.900E 02	.186E 01	.233E-03
830	3	.100E-04	.250E-02	.000E 00	315.00	300.00	3.0	1.5	1	.900E 02	.186E 01	.233E-03
1030	3	.100E-04	.250E-02	.000E 00	295.00	280.00	3.0	1.5	1	.900E 02	.186E 01	.233E-03
831	3	.100E-04	.250E-02	.000E 00	303.00	288.00	3.0	1.5	1	.900E 02	.186E 01	.233E-03
931	3	.100E-04	.250E-02	.000E 00	295.00	280.00	3.0	1.5	1	.450E 02	.263E 01	.233E-03
1031	3	.100E-04	.250E-02	.000E 00	289.00	274.00	3.0	1.5	1	.900E 02	.186E 01	.233E-03
828	3	.100E-04	.250E-02	.000E 00	315.00	300.00	3.0	1.5	1	.900E 02	.186E 01	.233E-03
827	3	.100E-04	.250E-02	.000E 00	312.00	297.00	3.0	1.5	1	.900E 02	.186E 01	.233E-03
826	3	.100E-04	.250E-02	.000E 00	311.00	296.00	3.1	1.5	1	.900E 02	.186E 01	.233E-03
825	3	.100E-04	.250E-02	.000E 00	309.00	294.00	3.1	1.5	1	.900E 02	.186E 01	.233E-03
725	3	.100E-04	.250E-02	.000E 00	312.00	297.00	3.0	1.5	1	.450E 02	.263E 01	.233E-03
625	3	.100E 21	.100E 21	.100E-01	314.00	299.00	20.0	20.0	2	.000E 00	.000E 00	.000E 00
626	3	.100E 21	.100E 21	.100E-01	314.00	299.00	20.0	20.0	2	.000E 00	.000E 00	.000E 00
627	3	.100E 21	.100E 21	.100E-01	315.00	300.00	20.0	20.0	2	.000E 00	.000E 00	.000E 00
628	3	.100E 21	.100E 21	.100E-01	317.00	302.00	20.0	20.0	10	.000E 00	.000E 00	.000E 00

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RESULTS FOR AQUIFER 1
AT TIME=356.3 DAYS

DRAINAGE RATE= 649. CFD

CHANGES IN NODE HEADS
SINCE MINE OPENING (TIME=0. DAYS):

		COLUMNS 1 - 21									
NODE ROWS											
1	1.9846 .6152 .4771	1.9294 .5933	1.2239 .5774	.9297 .5566	.8589 .5393	.7913 .5254	.7485 .5137	.7092 .4988	.6719 .4851	.6443 .4814	
2	.4377 .2061	-.0002	-.2646	-.1143	-.0405	.0168	.0474	.0811	.0959	.1228	
3	.0537 .0203 .0920	.0034 .0244	.4170 .0378	.2175 .0403	.0627 .0476	.0225 .0564	.0020 .0642	-.0039 .0796	.0015 .0781	.0120 .0852	
4	-.0022 -.1086 .0657	-.0054 -.0652	.0964 -.0398	-1.9036 -.0200	-1.0940 .0005	-.7068 .0173	-.5115 .0288	-.3667 .0532	-.2590 .0625	-.1741 .0637	
5	.0015 .2046 .1406	.0059 .2019	-.0056 .1970	-.1311 .1768	.0544 .1738	.0942 .1809	.1582 .1731	.1821 .1631	.1899 .1606	.1973 .1504	
6	-.0034 .3357 -.1334	.0063 .4150	-.0056 .2756	-.0027 .1316	.0054 .1169	1.2014 -.0210	1.1541 -.1375	.7920 -.1038	.4656 -.1038	.3250 -.1167	
7	.0068 -.5798 -.5698	-.0083 1.0117	.0042 .1802	-.0005 -.1248	-.1104 -.3787	2.7290 -1.9561	2.0903 -3.0857	1.2104 -2.5928	.4019 -.6975	.2854 -.5552	
8	-.0054 .1450 -1.1165	-.0024 -.0002	-.0078 -.0085	-.0024 .0244	.0046 -.2429	1.7632 -.6650	1.5444 -1.4224	.0000 -1.2925	.0000 -1.1511	.0000 -1.0994	
9	-.0022 .0000 -1.5171	-.0022 -.0027	-.0024 -.0027	-.0029 -.8142	-.0044 -1.3552	.0000 -1.4773	.0000 -1.4900	.0000 -1.5647	.0000 -1.5852	.0000 -1.5344	
10	.0049 -.0049	-.0039 -.0046	-.0024 -.0054	-.0049 -1.0386	-.0076 -1.1326	.0000 -1.2317	.0000 -1.3228	.0000 -1.4573	.0000 -1.5173	.0000 -1.5215	

-104-

	.8030	.6858	.5618	.4290	.3069	.1821	.0774	-.0461	-.1833	-.3059
	-.4155									
25	.0000	.0000	.0000	.0000	1.7322	-.0803	.9460	.9958	.6057	.4817
	.4219	.3560	.2827	.1990	.0967	.0032	-.0815	-.2424	-.4241	-.6331
	-.8271									
26	-.0107	-.0078	-.0042	-.0112	-.0049	-.0117	-.0083	-.0002	-.0645	.0110
	.0737	.0703	.0317	-.0183	-.0974	-.1902	-.2874	-.5256	-.8330	-1.1426
	-1.1553									
27	-.0107	-.0078	-.0042	-.0112	-.0049	-1.2700	.0002	-.2454	-3.8184	-1.1868
	-.3645	-.2258	-.1917	-.2270	-.2629	-.3569	-.4973	-.7849	-1.0901	-1.2397
	-1.2316									
28	-.0107	-.0078	-.0042	-.0112	-.0049	-.0117	.0002	-.4299	-.6755	-2.8391
	-4.8694	-1.0894	-.4807	-.4360	-.5305	-.6292	-.7566	-1.0127	-1.1851	-1.2373
	-1.2437									
29	-.0107	-.0078	-.0042	-.0112	-.0049	-1.1780	-2.5986	-.0005	-1.6145	-2.3829
	-1.4121	-.8901	-.9104	-1.2412	-1.1179	-1.1174	-1.0717	-1.1218	-1.2009	-1.2429
	-1.2451									
30	-.0107	-.0078	-.0042	-.0112	-.0049	-1.0994	-2.2676	-5.3752	-.0005	-5.9001
	-1.7703	-1.0750	-1.2268	-1.3496	-1.5142	-1.5908	-1.4966	-1.2263	-1.2131	-1.2649
	-1.2710									
31	-.0068	-.0112	-.0063	-.0037	-.0051	-.0112	-.0117	-6.6057	-11.4324	-9.0957
	-4.8875	-2.4570	-1.8013	-1.7214	-1.6973	-1.7078	-1.7075	-1.5156	-1.3425	-1.3152
	-1.3098									
32	-.0090	-.0112	-.0129	-.0056	-.0078	-.0046	-.0068	-.0068	-3.6692	-5.1750
	-4.1326	-2.9380	-2.3301	-1.9944	-1.7961	-1.6982	-1.6252	-1.5400	-1.4536	-1.3811
	-1.3594									
33	-.0090	-.0112	-.0129	-.0056	-.0078	-.0046	-.0068	-.0068	-.8813	-1.7036
	-1.7942	-1.7600	-1.6719	-1.5940	-1.5374	-1.4951	-1.4570	-1.4131	-1.3821	-1.3545
	-1.3438									
34	-.0090	-.0112	-.0129	-.0056	-.0078	-.0046	-.0068	-.0068	-.0068	-.0068
	-.3884	-.7832	-1.0029	-1.1255	-1.1951	-1.2380	-1.2649	-1.2791	-1.2827	-1.2820
	-1.2861									
35	-.0034	-.0054	-.0107	-.0044	-.0042	-.0115	-.0051	-.0051	-.0620	-.2029
	-.3950	-.5908	-.7544	-.8916	-.9927	-1.0618	-1.1211	-1.1748	-1.2014	-1.2180
	-1.2253									
36	-.0056	-.0137	-.0073	-.0112	-.0042	-.0054	-.0056	-.0056	-.0840	-.2485
	-.4023	-.5542	-.6821	-.7932	-.8843	-.9702	-1.0259	-1.0957	-1.1372	-1.1687
	-1.1753									
37	-.0051	-.0046	-.0127	-.0066	-.0046	-.0110	-.0042	-.0042	-.1589	-.3904
	-.5703	-.7078	-.8250	-.9119	-.9841	-1.0437	-1.0938	-1.1404	-1.1760	-1.1963
	-1.2000									

38	-.0117	-.0078	-.0063	-.0042	-.0117	-.0056	-.0115	.0000	.0000	-2.0410
	-1.8530	-1.7197	-1.6323	-1.5608	-1.5095	-1.4783	-1.4475	-1.4226	-1.3987	-1.3977
	-1.3904									
39	-.0117	-.0078	-.0063	-.0042	-.0117	-.0056	-.0115	.0000	.0000	.0000
	.0000	.0000	.0000	.0000	-1.6113	-1.5879	-1.5598	-1.5396	-1.5208	-1.5022
	-1.4988									

RESULTS FOR AQUIFER 2
AT TIME=356.3 DAYS

DRAINAGE RATE= 2873. CFD

CHANGES IN NODE HEADS
SINCE MINE OPENING (TIME=0. DAYS):

NODE ROWS		COLUMNS 1 - 21									
1		-3.3945	-3.3381	-3.1311	-2.9883	-2.8948	-2.8333	-2.7905	-2.7554	-2.7207	-2.6797
		-2.6550	-2.6204	-2.5994	-2.5754	-2.5527	-2.5366	-2.5217	-2.4956	-2.4841	-2.4744
		-2.4724									
2		-0.0063	-4.2373	-3.0564	-2.6062	-2.4019	-2.2830	-2.2212	-2.1577	-2.1084	-2.0693
		-2.0349	-2.0103	-1.9785	-1.9636	-1.9470	-1.9299	-1.9216	-1.9048	-1.8916	-1.8921
		-1.8805									
3		-0.0042	3.8196	.5605	-.0549	-.3379	-.4973	-.5957	-.6729	-.7446	-.8093
		-.8660	-.9148	-.9568	-.9941	-1.0293	-1.0669	-1.0903	-1.1204	-1.1487	-1.1616
		-1.1692									
4		-.0095	1.7744	.6826	.3086	.1074	-.0273	-.1025	-.1880	-.2576	-.3306
		-.4004	-.4658	-.5286	-.5918	-.6489	-.6929	-.7361	-.8013	-.8376	-.8684
		-.8762									
5		-.0078	.0066	.3555	.3042	.2507	.1926	.1396	.0867	.0195	-.0562
		-.1353	-.2263	-.3103	-.4023	-.4788	-.5571	-.6282	-.7195	-.7903	-.8345
		-.8440									
6		-.0044	.3657	.2644	.3604	.3875	.4202	.4075	.3694	.3013	.2026
		.0823	-.0461	-.1807	-.3035	-.4331	-.5425	-.6411	-.7749	-.8689	-.9150
		-.9377									
7		-.0095	-.0017	.2029	.3613	.5034	.6938	.7456	.7764	.6653	.4895
		.3013	.1055	-.0859	-.2700	-.4414	-.5886	-.7280	-.8926	-1.0054	-1.0815
		-1.1023									
8		-.0098	-.0032	2.3538	.5884	.9324	1.1794	1.2534	1.3020	.9919	.6943
		.5354	.2317	-.0452	-.2952	-.5156	-.7117	-.8872	-1.0906	-1.2432	-1.3337
		-1.3564									
9		-.0010	-.0046	-.0054	1.3010	1.4648	1.4856	1.4819	1.3767	1.1848	1.0063
		.7126	.2720	-.1118	-.4353	-.7156	-.9639	-1.1748	-1.4163	-1.5913	-1.6982
		-1.7288									
10		-.0059	-.0032	-.0049	.9629	1.6257	1.5610	1.4958	1.4482	1.3320	1.0439

	.5767	.1196	-.2795	-.6331	-.9446	-1.2195	-1.4497	-1.7139	-1.9102	-2.0239
	-2.0559									
11	.0051	-.0032	.0051	3.3018	1.2735	1.6624	1.5896	1.5203	1.4048	.9902
	.3933	-.1062	-.5457	-.9321	-1.2644	-1.5513	-1.7974	-2.0894	-2.2896	-2.4082
	-2.4546									
12	.0054	-.0032	-.0049	.0408	2.0496	1.7410	1.6897	1.6174	1.2302	.6155
	.0474	-.4705	-.9250	-1.3171	-1.6741	-1.9719	-2.2285	-2.5308	-2.7437	-2.8687
	-2.9194									
13	-.0015	-.0049	.0056	-.0020	1.9692	1.7549	1.6980	1.3696	.7327	.1296
	-.4270	-.9382	-1.4063	-1.8196	-2.1782	-2.4822	-2.7458	-3.0591	-3.2756	-3.4124
	-3.4468									
14	.0049	-.0032	-.0049	1.0759	1.7261	1.7361	1.4590	.8237	.1890	-.4275
	-.9938	-1.5193	-1.9949	-2.4075	-2.7708	-3.0903	-3.3577	-3.5777	-3.8943	-4.0251
	-4.0610									
15	.0049	-.0039	-.0063	-.0955	1.3774	1.7002	1.1384	.3103	-.4204	-1.0715
	-1.6692	-2.1985	-2.6875	-3.1106	-3.4802	-3.7891	-4.0588	-4.3713	-4.5813	-4.7078
	-4.7517									
16	.0049	-.0032	.0010	.4585	1.0586	.9666	.2988	-.4353	-1.1567	-1.8796
	-2.4453	-2.9963	-3.4905	-3.9175	-4.2783	-4.5945	-4.8462	-5.1467	-5.3530	-5.4751
	-5.5142									
17	.0049	-.0032	.0005	.3957	.9407	.3977	-.4053	-1.2146	-1.9873	-2.7112
	-3.3574	-3.9248	-4.4143	-4.8438	-5.1938	-5.4951	-5.7393	-6.0139	-6.1990	-6.3062
	-6.3472									
18	.0037	-.0046	.0034	.2354	.8359	-.0254	-1.0745	-2.0547	-2.9431	-3.7344
	-4.4153	-4.9963	-5.4858	-5.8967	-6.2329	-6.5078	-6.7244	-6.9673	-7.1233	-7.2144
	-7.2332									
19	.0037	-.0032	.0005	.9058	.7346	-.4949	-1.8796	-3.0854	-4.1013	-4.9673
	-5.6711	-6.2581	-6.7229	-7.0974	-7.3921	-7.6250	-7.8013	-7.9854	-8.1028	-8.1663
	-8.1792									
20	.0049	-.0039	.0020	.3074	.6519	-1.2847	-3.0518	-4.4634	-5.5977	-6.4927
	-7.1978	-7.7429	-8.1477	-8.4553	-8.6833	-8.8433	-8.9534	-9.0686	-9.1270	-9.1514
	-9.1646									
21	.0049	-.0032	.0029	.1531	-.6589	-3.2947	-4.9624	-6.3945	-7.5405	-8.4163
	-9.0427	-9.4856	-9.7847	-9.9814	-10.0979	-10.1594	-10.1951	-10.1902	-10.1790	-10.1626
	-10.1526									
22	.0049	-.0046	.0032	-.0056	-2.2974	-5.4365	-7.3328	-8.9063	-10.0530	-10.8062
	-11.2732	-11.5244	-11.6357	-11.6465	-11.6113	-11.5457	-11.4575	-11.3323	-11.2329	-11.1658
	-11.1394									
23	.0049	-.0039	.0027	-.0056	-3.4895	-7.8557	-10.3691	-12.2791	-13.3315	-13.7981
	-13.9156	-13.8418	-13.6677	-13.4414	-13.1946	-12.9558	-12.7383	-12.4578	-12.2458	-12.1206
	-12.0852									

24	-.0107	-.0039	.0010	-.0056	-4.3406	-10.8462	-14.5898	-17.3633	-17.7898	-17.4807
	-16.9478	-16.3630	-15.7922	-15.2578	-14.7898	-14.3330	-13.9697	-13.5205	-13.1990	-13.0122
25	-.0107	-.0078	-.0042	-.0056	-4.3926	-15.9983	-22.4651	-27.1719	-23.7185	-21.5986
	-20.0947	-18.8652	-17.8538	-16.9810	-16.2324	-15.6033	-15.0730	-14.4568	-14.0298	-13.7942
26	-.0107	-.0078	-.0042	-.0112	-2.7742	-10.8679	.0017	-33.0393	-26.9764	-24.5579
	-22.6074	-20.9829	-19.6089	-18.4558	-17.4756	-16.6655	-15.9958	-15.2339	-14.7156	-14.4106
27	-.0107	-.0078	-.0042	-.0112	-2.3347	-8.8201	.0007	-38.5366	-29.3667	-26.5178
	-24.4056	-22.6077	-20.9521	-19.5547	-18.3882	-17.4360	-16.6606	-15.7822	-15.2004	-14.8618
28	-.0107	-.0078	-.0042	-.0112	-1.7966	-6.1575	.0000	-39.9727	-31.9595	-28.8174
	-26.0925	-23.7454	-21.7988	-20.1907	-18.8921	-17.8430	-17.0110	-16.0840	-15.4675	-15.1172
29	-.0107	-.0078	-.0042	-.0112	-1.0251	-2.9450	-5.6260	-.0029	-36.1958	-30.3092
	-26.9023	-24.1294	-21.9382	-20.2271	-18.8950	-17.8462	-17.0076	-16.0981	-15.4944	-15.1587
30	-.0107	-.0078	-.0042	-.0112	-.0049	-.8745	-3.3955	-8.4788	-.0022	-31.5950
	-26.4609	-23.2847	-21.1145	-19.5344	-18.3315	-17.3979	-16.6528	-15.8384	-15.2996	-14.9954
31	-.0068	-.0112	-.0063	-.0037	-.0051	-.0112	-1.4932	-7.9470	-17.2170	-24.1025
	-22.1023	-20.4263	-19.1184	-18.0862	-17.2383	-16.5513	-15.9802	-15.3406	-14.9009	-14.6418
32	-.0090	-.0112	-.0129	-.0056	-.0078	-.0046	-.0068	-.6626	-5.4761	-11.0605
	-13.6394	-14.7585	-15.1487	-15.1743	-15.0271	-14.8008	-14.5698	-14.2366	-13.9858	-13.8306
33	.6375	.6340	.6292	.5955	.5708	.5642	.5596	-.6245	-9.8960	-10.7944
	-11.6970	-12.3704	-12.8018	-13.0381	-13.1335	-13.1389	-13.1018	-12.9958	-12.8879	-12.8076
34	1.7051	1.6995	1.6897	1.6611	1.6587	1.6533	1.6516	-.5591	-10.2747	-10.4888
	-10.7847	-11.0784	-11.3313	-11.5090	-11.6375	-11.7063	-11.7344	-11.7368	-11.7227	-11.6987
35	2.7649	2.7634	2.7485	2.7375	2.7307	2.7275	2.7266	-.4934	-9.7134	-9.7813
	-9.8953	-10.0217	-10.1536	-10.2688	-10.3618	-10.4397	-10.4905	-10.5291	-10.5557	-10.5586
36	4.1799	4.1819	4.1650	4.1597	4.1563	4.1545	4.1541	-.4028	-8.4883	-8.5120
	-8.5581	-8.6252	-8.6926	-8.7607	-8.8308	-8.8860	-8.9390	-8.9958	-9.0374	-9.0620
37	6.3162	6.3108	6.3064	6.3030	6.3013	6.2905	6.2905	-.2712	-6.2578	-6.2859
	-6.3462	-6.4111	-6.4829	-6.5522	-6.6309	-6.6892	-6.7485	-6.8271	-6.8743	-6.9136

	-6.9263											
38	9.1553	9.1519	9.1489	9.1470	9.1460	9.1458	9.1458	-0.0991	-2.3484	-2.7236		
	-3.0337	-3.2878	-3.4998	-3.6736	-3.8254	-3.9534	-4.0569	-4.1765	-4.2651	-4.3125		
	-4.3281											
39	11.9976	11.9951	11.9927	11.9915	11.9910	11.9907	11.9910	-0.1016	-1.3262	-1.3823		
	-1.4421	-1.5137	-1.5864	-1.6501	-1.7603	-1.8604	-1.9504	-2.0703	-2.1504	-2.2004		
	-2.2102											

RESULTS FOR AQUIFER 3
AT TIME=356.3 DAYS

DRAINAGE RATE= 8137. CFD

CHANGES IN NODE HEADS
SINCE MINE OPENING (TIME=0. DAYS):

NODE ROWS		COLUMNS 1 - 21									
1		-1.8047	-2.2361	-2.5950	-2.8237	-2.9656	-3.0754	-3.1443	-3.2007	-3.2614	-3.3228
		-3.3774	-3.4272	-3.4714	-3.5195	-3.5520	-3.5876	-3.6167	-3.6597	-3.6848	-3.7021
		-3.7012									
2		-4.3465	-1.9236	-2.6680	-3.1016	-3.3489	-3.5173	-3.6316	-3.7373	-3.8345	-3.9250
		-4.0095	-4.0884	-4.1626	-4.2227	-4.2793	-4.3328	-4.3838	-4.4431	-4.4814	-4.5051
		-4.5190									
3		-1.0330	-2.6968	-3.4976	-3.9983	-4.3247	-4.5515	-4.6978	-4.8352	-4.9756	-5.0994
		-5.2245	-5.3433	-5.4451	-5.5447	-5.6328	-5.7095	-5.7859	-5.8677	-5.9253	-5.9695
		-5.9810									
4		-1.0012	-3.3203	-4.1555	-4.7612	-5.1743	-5.4792	-5.6777	-5.8645	-6.0613	-6.2390
		-6.4287	-6.5710	-6.7268	-6.8662	-7.0000	-7.1184	-7.2217	-7.3440	-7.4338	-7.4915
		-7.5073									
5		-0.0337	-3.6453	-4.5166	-5.2568	-5.7803	-6.1749	-6.4338	-6.6921	-6.9397	-7.1965
		-7.4113	-7.6396	-7.8457	-8.0420	-8.2178	-8.3738	-8.5095	-8.6890	-8.8059	-8.8899
		-8.9114									
6		-0.0085	-3.5349	-4.6077	-5.5220	-6.1677	-6.6582	-6.9802	-7.2976	-7.6106	-7.9192
		-8.2024	-8.4810	-8.7437	-8.9805	-9.2014	-9.4055	-9.5732	-9.7915	-9.9436	-10.0391
		-10.0774									
7		0.0039	-3.2234	-4.5896	-5.6829	-6.4509	-7.0383	-7.4209	-7.8042	-8.1675	-8.5305
		-8.8730	-9.2043	-9.5137	-9.8003	-10.0637	-10.2935	-10.4990	-10.7637	-10.9463	-11.0559
		-11.0925									
8		-0.0054	-2.6677	-4.5046	-5.8152	-6.7288	-7.4241	-7.8897	-8.3389	-8.7844	-9.2148
		-9.6287	-10.0156	-10.3840	-10.7234	-11.0425	-11.3201	-11.5657	-11.8691	-12.0940	-12.2195
		-12.2646									
9		-0.0115	-2.3979	-4.3850	-5.9041	-6.9990	-7.8220	-8.3796	-8.9189	-9.4500	-9.9712
		-10.4714	-10.9387	-11.3813	-11.7878	-12.1563	-12.4956	-12.7847	-13.1506	-13.4109	-13.5635
		-13.6177									
10		-0.0046	-2.2478	-4.2488	-5.9341	-7.1541	-8.0896	-8.7158	-9.3398	-9.9407	-10.5261

	-11.0947	-11.6235	-12.1196	-12.5815	-13.0066	-13.3835	-13.7107	-14.1145	-14.4036	-14.5852
	-14.6392									
11	-1.0047	-2.0244	-4.0610	-5.9250	-7.2959	-8.3701	-9.0757	-9.7864	-10.4705	-11.1349
	-11.7764	-12.3811	-12.9458	-13.4673	-13.9326	-14.3604	-14.7280	-15.1826	-15.5022	-15.7041
	-15.7681									
12	-1.0046	-1.5398	-3.7817	-5.8604	-7.4197	-8.6375	-9.4536	-10.2554	-11.0388	-11.7991
	-12.5305	-13.2183	-13.8572	-14.4436	-14.9741	-15.4460	-15.8572	-16.3555	-16.7168	-16.9402
	-17.0042									
13	-1.0030	-1.0073	-3.4106	-5.7239	-7.5093	-8.9080	-9.8394	-10.7703	-11.6729	-12.5393
	-13.3708	-14.1409	-14.8625	-15.5310	-16.1218	-16.6519	-17.1094	-17.6641	-18.0588	-18.3035
	-18.3782									
14	-1.0078	-1.6689	-2.9102	-5.4817	-7.5439	-9.1672	-10.2551	-11.3201	-12.3584	-13.3572
	-14.3042	-15.1895	-16.0044	-16.7424	-17.4097	-18.0022	-18.5088	-19.1113	-19.5500	-19.8062
	-19.8909									
15	-1.0042	-1.4375	-2.1909	-5.1021	-7.5278	-9.4277	-10.6963	-11.9316	-13.1321	-14.2778
	-15.3698	-16.3679	-17.2868	-18.1157	-18.8547	-19.5032	-20.0603	-20.7209	-21.1921	-21.6683
	-21.5520									
16	-1.0081	-1.2683	-1.1362	-4.5427	-7.4490	-9.6960	-11.1711	-12.6226	-14.0159	-15.3420
	-16.5786	-17.7183	-18.7490	-19.6743	-20.4902	-21.1960	-21.7930	-22.5081	-22.9976	-23.2905
	-23.3840									
17	-1.0066	-1.1877	-1.3335	-3.7192	-7.3174	-9.9795	-11.7219	-13.4187	-15.0422	-16.5750
	-17.9873	-19.2749	-20.4307	-21.4507	-22.3350	-23.0957	-23.7378	-24.4780	-24.9873	-25.2915
	-25.3918									
18	-1.0056	-1.1777	-1.2920	-3.2227	-7.2183	-10.3232	-12.3730	-14.3701	-16.2688	-18.0347
	-19.6553	-21.1006	-22.3821	-23.4888	-24.4458	-25.2422	-25.8979	-26.6560	-27.1704	-27.6666
	-27.5703									
19	-1.0066	-1.1672	-1.2466	-2.6316	-7.0427	-10.6370	-13.1299	-15.4971	-17.7407	-19.8040
	-21.6509	-23.2742	-24.6726	-25.8552	-26.8481	-27.6594	-28.3231	-29.0642	-29.5515	-29.8257
	-29.9180									
20	-1.0066	-1.1455	-1.1931	-1.5215	-6.6167	-11.0393	-14.0122	-16.9023	-19.5828	-22.0059
	-24.1030	-25.8970	-27.3840	-28.6118	-29.6035	-30.3921	-31.0181	-31.6887	-32.1169	-32.3499
	-32.4343									
21	-1.0066	-1.1311	-1.0042	-1.0112	-5.7161	-11.3508	-15.0906	-18.6968	-21.9783	-24.8203
	-27.1909	-29.1145	-30.6436	-31.8403	-32.7551	-33.4604	-33.9917	-34.5247	-34.8521	-35.0298
	-35.0786									
22	-1.0066	-1.1091	-1.0042	-1.0112	-5.6229	-11.6665	-16.4553	-21.1401	-25.2322	-28.5769
	-31.1729	-33.1304	-34.5679	-35.6133	-36.3547	-36.8779	-37.2327	-37.5598	-37.7261	-37.8010
	-37.8259									
23	-1.0066	-1.0916	-1.0042	-1.0112	-4.2632	-11.4124	-18.0476	-24.7385	-30.0195	-33.8208
	-36.4312	-38.1655	-39.2866	-39.9856	-40.3896	-40.6135	-40.7134	-40.7266	-40.6792	-40.6333
	-40.6152									

24	-0.0107	-0.0698	-0.0042	-0.0112	-2.2703	-8.1685	-19.5574	-31.1367	-37.7554	-41.4470
	-43.4502	-44.4487	-44.8584	-44.9290	-44.8154	-44.5974	-44.3413	-43.9561	-43.6382	-43.4482
	-43.3804									
25	-0.0107	-0.0078	-0.0042	-0.0112	-0.0049	-0.0117	-20.6772	-47.4446	-51.3347	-52.4895
	-52.4595	-51.8999	-51.1189	-50.2705	-49.4304	-48.6489	-47.9666	-47.1155	-46.5000	-46.1443
	-46.0232									
26	-0.0107	-0.0078	-0.0042	-0.0112	-0.0049	-0.0117	-0.0020	-77.5164	-70.4253	-65.8918
	-62.5369	-59.8147	-57.5247	-55.5754	-53.9292	-52.5442	-51.3870	-50.0559	-49.1475	-48.6211
	-48.4490									
27	-0.0107	-0.0078	-0.0042	-0.0112	-0.0049	-0.0117	-0.0020	-93.7905	-84.1125	-76.9380
	-71.3386	-66.8562	-63.2390	-60.2959	-57.9063	-55.9641	-54.3977	-52.6411	-51.4634	-50.7922
	-50.5671									
28	-0.0107	-0.0078	-0.0042	-0.0112	-0.0049	-0.0117	.0005	-101.9529	-92.7922	-84.8247
	-77.8562	-72.1982	-67.6067	-63.9199	-60.9797	-58.6335	-56.7629	-54.7012	-53.3367	-52.5554
	-52.5022									
29	-0.0107	-0.0078	-0.0042	-0.0112	-6.4863	-18.3337	-39.3997	.0005	-101.5325	-90.6335
	-82.0132	-75.3193	-70.1360	-66.0813	-62.8774	-60.3538	-58.3467	-56.1411	-54.6892	-53.8672
	-53.5940									
30	-0.0107	-0.0078	-0.0042	-3.6736	-15.9492	-31.3848	-47.0327	-65.1807	-0.0017	-91.8970
	-82.2480	-75.4607	-70.4106	-66.5134	-63.4565	-61.0129	-59.0759	-56.9175	-55.4980	-54.6782
	-54.4050									
31	-0.0088	-0.0112	-0.0063	-7.2769	-20.6763	-35.8359	-49.6758	-67.0681	-76.1702	-82.4597
	-76.7412	-72.1331	-68.3806	-65.3076	-62.7715	-60.6912	-58.9927	-57.0613	-55.7603	-54.9950
	-54.7534									
32	-0.0071	-0.0095	-0.0129	-8.9639	-20.4536	-31.3638	-40.2634	-50.8926	-58.8975	-63.2480
	-64.3516	-63.9077	-62.8052	-61.4446	-60.0735	-58.7856	-57.6416	-56.2444	-55.2500	-54.6555
	-54.4548									
33	-0.4712	-0.9932	-2.3347	-9.6741	-16.2742	-21.0310	-23.1182	-46.5139	-58.1187	-58.6165
	-58.8730	-58.7554	-58.3245	-57.7024	-56.9795	-56.2480	-55.5457	-54.6328	-53.9347	-53.5149
	-53.3750									
34	-0.8433	-1.7866	-4.0938	-8.9274	-12.4448	-14.5957	-15.3547	-42.1396	-55.0953	-55.1235
	-58.1096	-54.9778	-54.7258	-54.3845	-53.9905	-53.5671	-53.1418	-52.5730	-52.1267	-51.8374
	-51.7485									
35	-1.2600	-2.2751	-4.5713	-7.5828	-9.5720	-10.6289	-10.9666	-37.7698	-51.9290	-51.9014
	-51.8420	-51.7310	-51.5679	-51.3591	-51.1267	-50.8733	-50.6201	-50.2720	-49.9844	-49.8135
	-49.7510									
36	-1.4363	-2.4919	-4.2807	-5.9458	-6.9055	-7.3782	-7.5173	-31.9521	-47.9124	-47.8965
	-47.8462	-47.7749	-47.6877	-47.5737	-47.4517	-47.3223	-47.1865	-47.0089	-46.8623	-46.7605
	-46.7283									
37	-1.4138	-2.1863	-3.2107	-3.9438	-4.3157	-4.5002	-4.5547	-23.2314	-42.3082	-42.2896
	-42.2732	-42.2507	-42.2144	-42.1672	-42.1223	-42.0635	-42.0149	-41.9399	-41.8865	-41.8359

38	-41.8286 -1.8757 -35.2417 -35.0430	-1.2383 -35.2407	-1.6467 -35.2363	-1.9048 -35.2290	-2.0247 -35.2285	-2.0793 -35.2148	-2.1023 -35.2075	-11.6160 -35.1882	-35.2432 -35.1631	-35.2493 -35.1106
39	-0.0520 -28.4558 -29.0869	-0.0930 -28.4797	-0.1340 -28.5115	-0.1646 -28.5513	-0.1748 -28.5991	-0.1848 -28.6448	-0.1848 -28.6982	-0.0190 -28.7886	-28.4321 -28.8967	-28.4399 -29.0029

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En33 A finite difference
cop.2 groundwater model for the
East Decker, Montana mine

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