

SPECTRAL CHARACTERISTICS OF KILOGAUSS PHOTOSPHERIC
MAGNETIC STRUCTURES ON SCALES NEAR
THE PHOTON MEAN FREE PATH

by

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TABLE OF CONTENTS

1. INTRODUCTION	1
2. RADIATIVE TRANSFER	5
Basic RE and Line Formation.....	5
The Non-LTE Problem	6
Radiative Transfer at or Below the Photon Mean Free Path Length	8
3. METHOD	10
Atmospheres and Magnetic Elements.....	10
Radiative Transfer Code	12
Atomic Model	13
4. RESULTS AND DISCUSSION	14
2D Results	14
3D Results	18
Comparison of 2D and 3D Results	20
Degradation with Instrumental Profile.....	21
Comparison with Hinode Data.....	22
5. CONCLUSION.....	26
REFERENCES CITED.....	28
APPENDIX A: Accuracy of Milne-Eddington Inversions.....	34

LIST OF TABLES

Table	Page
A.1 Inversion Parameters.....	34

LIST OF FIGURES

Figure	Page
2.1 Formation of spectral emission and absorption lines.....	6
3.1 Vertical magnetic field of selected magnetic structures	11
3.2 Locations of the flux tube, flux sheet, and umbral dot	12
4.1 Stokes profiles near the center of the flux tube.....	14
4.2 The effect of the velocity field on the intensity	16
4.3 Structure of the vertical velocity field in the flux tube	17
4.4 Intensity for Fe I 630.2 as a percentage of continuum intensity	18
4.4 Magnitude of Stokes V/I for the 630.2 line as a function of position.....	18
4.6 Comparison of 1D Stokes profiles to the 3D profile	19
4.7 Variations with x of intensity at the 630.2 line core	21
4.8 Differences between 3D and 2D spectra.....	21
4.9 Original (left) and degraded (right) continuum intensity.....	22
4.10 Longitudinal component of apparent flux.....	23
4.11 Correlation between the simulated profiles and SP data	24
A.1 2cmo and BayesME fits to the RH-generated Stokes spectra.....	35
A.2 2cmo inversion: inverted parameter values vs. actual values	36
A.3 Confidence intervals obtained with BayesME.....	38
A.4 Magnetic field strength calculated by the inversion	40

ABSTRACT

Modern telescopes are beginning to resolve structure in the Sun's photosphere at scales near and below 100 km, the length of the typical photon mean free path. In this thesis we discuss how LTE and non-LTE conditions affect the polarized spectral signatures of very small solar magnetic structures. Emergent spectra produced by atmospheres in LTE conditions are dominated by the integral along the entire line of sight, while under non-LTE conditions they originate from volumes on scales of the mean free photon path or longer. On scales approaching the mean free path, we expect non-LTE effects to result in a 'smearing' of the spectroscopic data.

We examined magnetic and atmospheric features from numerical sunspot models created by Rempel (2012), which simulate the photosphere with spatial resolutions down to 12 km horizontally and 8 km vertically (Rempel et al., 2009a,b). Using a multi-dimensional non-LTE radiative transfer code and a multi-level iron atom, we generated polarization profiles of small magnetic structures in both 2- and 3D.

The common diagnostic Fe I line pair at 630 nm allowed us to investigate the implications for our ability to accurately measure small-scale kilogauss magnetic elements. We searched a Hinode/SOT 2006 observation of a sunspot for matches to the simulated polarimetric data and examined the results of the MERLIN Milne-Eddington inversion code in light of our findings.

INTRODUCTION

The Sun's photosphere is a thin (~ 100 km) stratified boundary layer from which the entire solar luminosity is radiated into space. The pattern of granulation on the surface is the signature of underlying convection, in which upwellings of rapidly cooling fluid on scales of 1-2,000 km are accompanied by narrow, rapid downdrafts which carry much smaller-scale embedded magnetic fields. These fields are further collected and perhaps intensified by a "supergranular" flow into an associated magnetic network pattern, whose elements appear to be tube- or sheet- like according to their geometry. The advent of large solar telescopes with resolutions of $0.1''$, or 73 km when seen from Earth, has made possible the direct observation of such small magnetic structures in the solar photosphere. Nagata (2008) demonstrated that small-scale flux abounds in the photosphere, and proposed that flux concentrations of 100 km or larger form by flux expulsion followed by convective collapse (Parker (1978), Spruit (1979)). These kilogauss-strength flux tubes and sheets have significant structure at the diffraction limit of modern telescopes, and magnetohydrodynamic simulations predict finer structure still. The Advanced Technology Solar Telescope, with a diffraction limit of 14 km on the solar surface, is being designed in part to study the interaction of magnetic fields and plasmas on these fine scales.

The study of small scale magnetic fields is crucial to our understanding of solar magnetism. Turbulence and reconnection allow for the generation of magnetic fields and dissipation of energy on the Sun over a large range of scales (Parker, 2009). From global scales to the finest resolution yet observed, the magnetic field can be quite non-uniformly distributed. A close look at the quiet Sun reveals kilogauss strength fields clustered together in structures with dimensions comparable to a typical photon mean free path $\lambda \sim 100\text{km}$, surrounded by areas of much weaker fields.

Numerical simulations of the solar photosphere currently surpass instrumental resolutions. The highest resolution of the Rempel (2012) sunspot models has a spatial horizontal [vertical] grid spacing of 12 [8] km. By imposing suitable boundary conditions below and above the model, Rempel not only reproduced the well-known characteristics of a sunspot but also revealed a richness in detail at smaller scales than current instruments can resolve.

Because strong magnetic fields suppress convection (Parker (1963a,b), Spruit (1976), Weiss (1966)), radiation is the principal means of heating in areas where the field is strong. The radiative transfer equation describes the change in intensity of light as it propagates through an absorbing, emitting and scattering medium. Key quantities that determine the transfer of radiation are the source function and opacity. The transfer of radiation along a ray is described by

$$dI_\nu/ds = j_\nu - \alpha_\nu I_\nu \quad \text{or} \quad dI_\nu/d\tau_\nu = I_\nu - S_\nu$$

with $S = j_\nu/\alpha_\nu$, where j_ν is an emission coefficient, S the source function, α the opacity in cm^{-1} . Here $d\tau = -\alpha_\nu dz$ is an increment of optical depth. The photon mean free path is simply $\lambda = 1/\alpha_\nu$, from which it is seen that $S = j_\nu\lambda$ is the emission coefficient ‘integrated’ over one mean free path.

In this work we focused on scales less than or on the order of λ . On this scale the local source function cannot be expected to remain coupled to the local radiation field I . The well known formal solutions to the transfer equation above show that I is an integral of the source function over at least one mean free path, and it includes any boundary conditions from above the solar surface. In one dimension, this integral is usually along the line of sight. In the three dimensional case, it is an integral of many different rays, of conditions within a radius of at least λ seen by a point in the atmosphere, including an accompanying set of boundary conditions.

Therefore whether the source function is fully locally controlled (LTE) or whether it has contributions from scattered radiation (non-LTE), I is not generally equal to S below the photon mean free path. This is especially true when

thermodynamic structure exists on scales below the mean free path. Significant scattering contributions are also enough to render the LTE assumption invalid. This can be particularly important in certain spectral lines where incoherent scattering greatly enhances photon transport (‘complete redistribution’ where photon absorption and re-emission frequencies are uncorrelated is an extreme, but often realistic, case) over and above coherent scattering. In such lines the source functions can become ‘smeared’ over many mean free paths.

The importance of performing 3D non-LTE radiative transfer calculations has been demonstrated by Holzreuter and Solanki (2012). They found strong non-LTE radiative transfer effects in four commonly used visible Fe I lines. Their calculations included not just the intensity I but all of the parameters defining the state of polarization of the emitted light from the Sun’s atmosphere - Stokes Q and U describing linear polarization, and Stokes V describing circular polarization. It is expected that the same arguments concerning mean free paths presented above will apply to all of the Stokes parameters. Holzreuter and Solanki (2012) stressed that failing to account for multiple dimensions or non-LTE conditions can lead to errors of up to 20% in calculating the magnetic field strength. Their data also showed strong peaks in the radiation emerging from the boundaries of simulated flux tubes and flux sheets, and they argued that these ought to be visible in the photosphere even in the presence of horizontal radiative transfer.

In this work we employed a full 3D non-LTE treatment of radiative transfer in and around a realistic simulation of a small magnetic flux tube, as well as using the 2D non-LTE version to investigate other small magnetic elements. We abandoned the symmetry and length scale of Holzreuter & Solanki’s (2012) idealized flux tube and used the resulting polarization profiles to examine what it means to observe length scales below the photon mean free path. We investigated how much useful information the emitted radiation contains, and if future telescopes will be able to resolve it.

First we will discuss radiative transfer and the non-LTE problem in chapter 2.

We detail our magnetic structures, atomic model, and numerical simulations in chapter 3. In chapter 4 we present our results and discuss their implications, before concluding in chapter 5.

RADIATIVE TRANSFER

Basic Radiative Transfer and Line Formation

The equation for radiative transfer in an atmosphere is

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu \quad (2.1)$$

where I_ν is the specific intensity in $\text{ergs s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ ster}^{-1}$, j_ν is the emission coefficient and α_ν is the extinction coefficient (for more background on radiative transfer, see Mihalas (1978)). The equation represents conservation of energy along a line of sight: j_ν represents additions to the radiant energy while $\alpha_\nu I_\nu$ describes subtractions.

Defining the source function S_ν as the ratio of the emission and extinction coefficients and the optical depth as

$$\tau_\nu(s) = - \int_0^s \alpha_\nu(s) ds, \quad (2.2)$$

the transfer equation can be re-written as

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu. \quad (2.3)$$

The source function depends on the thermodynamic properties of the atmosphere, which are due to the populations of particles and photons. In the case of local thermodynamic equilibrium (LTE), it is given by the Planck function.

A formal solution to the transfer equation follows from integration:

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau'_\nu)e^{-(\tau_\nu - \tau'_\nu)} d\tau'_\nu \quad (2.4)$$

The formation of spectral lines in the photosphere is determined by the behavior of α_ν . Transitions within an atom or molecule may be due to spontaneous

or induced radiative deexcitation, collisional excitation or deexcitation, or absorption.

Together these processes determine the extinction and emission as a function of wavelength. Imagine an object emitting blackbody radiation surrounded by a shell of material whose absorption coefficient becomes large at some frequency ν_0 (Rybicki and Lightman, 1979). In an optically thin medium, if the intensity incident on the shell, I_ν , is greater than [less than] the source function in the material, an absorption [emission] line is formed (Figure 2.1). The point where the plasma becomes optically thick is said to be the formation depth of a spectral line in the photosphere. This occurs at various heights depending on the atomic transition. Wavelengths close to the line center, which compose the line core, form higher in the atmosphere and at cooler temperatures than the line wings, which form deepest in the atmosphere. Formation depth is also sensitive to thermodynamic conditions of the atmosphere, such as temperature and density.

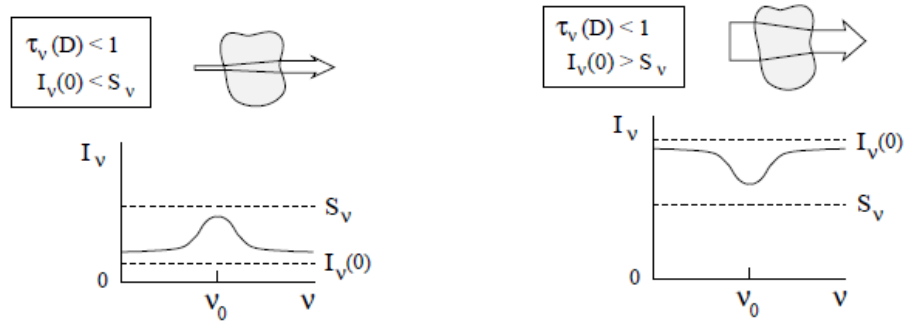


Figure 2.1: Formation of spectral emission and absorption lines in an optically thin atmosphere. Figure copied from Rutten (2003)

The Non-LTE Problem

In general, the photosphere can be considered thin, flat, and in LTE. However, in order to obtain an accurate description of radiative transfer in an atmosphere with both large and small scale thermal and magnetic structures, it is necessary to use the full non-LTE treatment.

Spectral line formation depends on the probability of a certain atomic tran-

sition, which in turn depends on the excitation state of the gas as well as the properties of the photons in the medium. When collisions are very rapid and the medium is well mixed, such as in dense atmospheric layers, thermodynamics shows us that the source function is given by the Planck function. However, in the photosphere, collisional coupling is not always complete. When this is the case, the source function can no longer be described by the Planck function, level populations can no longer be computed by the Saha-Boltzmann equation and the LTE assumption fails. The atomic level population at a certain frequency, itself dependent on the radiation field, may now be dependent on the populations of other levels which in turn are a product of the radiation field at a different frequency. The radiation field is no longer equal to its average value, and the radiation field becomes a function of position. The problem becomes non-linear and non-local, and we must use numerical methods to find a solution.

The goal of a non-LTE radiative transfer code is to use the provided atomic and atmospheric information to solve for a radiation field that is consistent with the thermal properties of the atmosphere. A commonly employed technique for solving the non-linear transfer equation is lambda iteration. Lambda iteration re-writes the formal solution to the transfer equation (2.4) with a linear operator, $\Lambda_{\mu\nu}$, where μ is the cosine of the angle with respect to the surface normal:

$$I_{\mu\nu} = \Lambda_{\mu\nu} S_{\nu} \quad (2.5)$$

Together with the statistical equilibrium equation, which for a two-level atom is

$$S_l = (1 - \epsilon)\bar{J} + \epsilon B \quad (2.6)$$

where S_l is the line source function, ϵ is the collisional destruction probability, $\bar{J}$ is the mean intensity and B is the Planck function, this can be solved using an iterative procedure where

$$S_l^{(n+1)} = (1 - \epsilon)\Lambda[S_l^{(n)}] + \epsilon B \quad (2.7)$$

Starting with the current intensity, one evaluates the source function and then solves the transfer equation, which results in an updated intensity, then iterates until convergence (Hubeny, 2003). This procedure converges slowly and stabilizes before the true solution is reached, so there are many techniques for accelerating the process. The end result is a self-consistent description of the radiation field and atomic population levels.

Radiative Transfer at or Below the Photon Mean Free Path

In the photosphere, the mean free path of a photon, given by $\lambda = 1/\alpha_\nu$ is on the order of 10^2 km. It is easy to see where this value comes from. The atmosphere is stratified by gravity ($dP/dz = -\rho g$) and the scale height is about 100 km. This means that radiation from the photosphere always originates from about one pressure scale height since $\alpha \sim \rho\sigma$ where σ , the scattering cross-section, is a property of the individual atom.

The source function, and hence the transfer equation, depends directly on λ . We can see that if non-LTE effects are present the source function is found by integrating the total contribution to the emergent intensity over one photon mean free path (Judge, 2014, Rybicki and Lightman, 1979). Because the mean free path is an average of the paths of a large number of photons, the intensity and source function, which describe the local radiation field, are also average quantities. The source function at a single point need not be identical to the average source function integrated over the length λ . Therefore the radiation field, which accounts for all the radiation within a radius λ seen by a point in the atmosphere, cannot be expected to remain coupled to the local source function (Judge, 2014).

At short length scales, photon scattering plays a dominant role on the emergent flux from the atmosphere. Consider equation (2.6). This can be re-written using

the scattering probability per extinction, $\eta = 1 - \epsilon$, as

$$S_l = (1 - \eta)B + \eta\bar{J} \quad (2.8)$$

Again we can find the mean intensity at any wavelength by operating on the source function with some integral operator Λ . Λ has a certain range of operation, which in one dimension is the thermalization length $\tau_{th} \approx \epsilon^{-\alpha}$, with opacity $\alpha = 1/2$ for coherent scattering and $\alpha = 1$ for spectral lines. τ_{th} is number of mean free paths it takes for the radiation field to thermalize and $\bar{J} \approx B$ (Rutten, 2003). If scattering dominates ($\epsilon \ll 1$), the source function becomes 'smeared' over many mean free paths. For spectral lines where τ_{th} is large, such as in many strong permitted photospheric or chromospheric transitions, the LTE assumption is no longer valid. In three dimensions, we must think of a thermalization volume rather than a length, and by the same argument as above conclude that the source functions in non-LTE become non-locally dependent of the volumes around them.

The photons we observe emerging from the photosphere mostly come from a layer which has an optical depth $\tau = 1$. Belluzzi (2013) points out that they must also come from a region with a thickness of approximately one pressure scale height, which is on the order of the mean free path in the photosphere. If we form an image of the solar surface, we ought to still be able to resolve structures below the mean free path length; however, the image will not be as sharp as it is for larger structures because it is possible for some photons to pass entirely through the small structure without being affected by it while for other photons might thermalize before they are able to emerge from the region.

METHOD

Here we introduce the various components of our simulation and radiative transfer calculations. First we mention the thermal and magnetic properties of the atmospheres describing our chosen magnetic structures. The details of the Rybicki-Hummer radiative transfer code and the atomic model used in conjunction with it follow.

Atmospheres and Magnetic Elements

We obtained models of magnetic and thermal structures from Mathias Rempel's (2012) sunspot simulation, with spatial resolution of $12 \text{ km} \times 12 \text{ km}$ per pixel in the horizontal dimension and 8 km per pixel in the vertical. His simulations solved conservation equations for the mass, momentum, and energy of the fluid along with an induction equation for the magnetic field. The energy equation contains the gain and loss terms from radiative transport, which in the photosphere are the dominant processes. To make calculations tractable yet realistic, Rempel adopted the frequency-independent prescription ("gray") radiative transfer approximation to simulate a low-resolution sunspot, before regridding and extending to several representative frequencies ("multi-gray") to produce the final product.

Figure 3.1 shows three small structures representative of what one might find in the photosphere: a flux tube, flux sheet, and an umbral dot. We took the tube and sheet from network areas surrounding the main sunspot simulation, as shown in Figure 3.2.

The flux tube and sheet both have vertical magnetic field strengths ranging from 3 to 5 kilogauss. The strong fields in the interiors of these elements are accompanied by a drop in density and temperature, while the vertical velocity component shows strong up- and down-flows of up to 2 km/s. These properties are

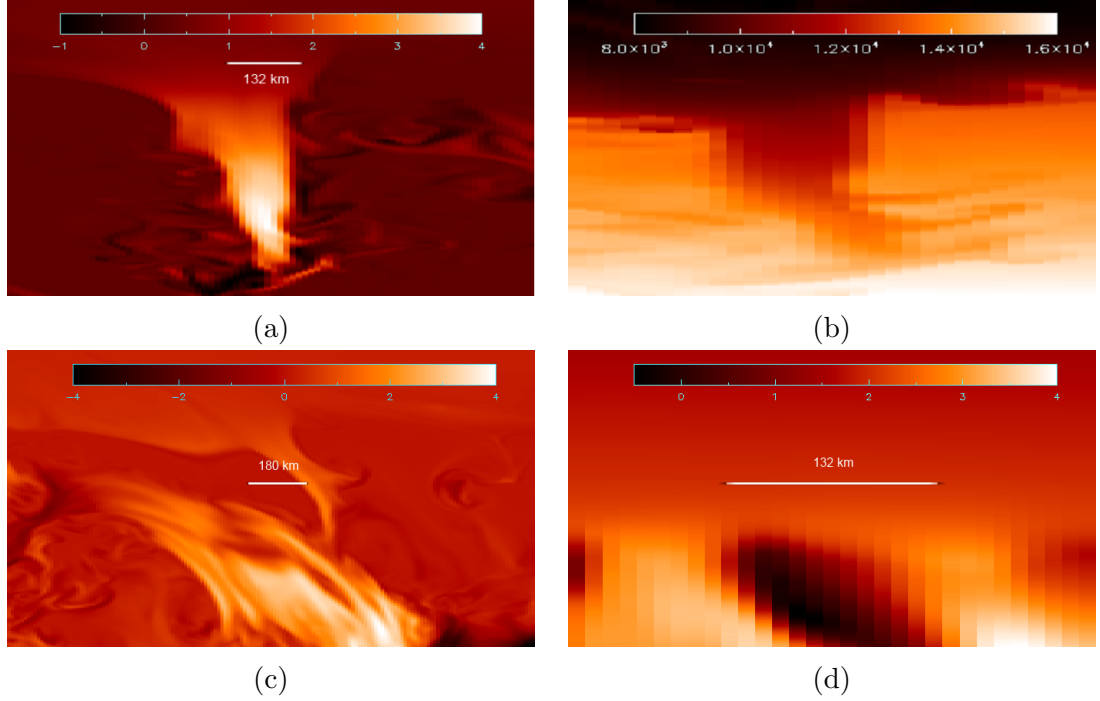


Figure 3.1: Vertical magnetic field in kilogauss of the flux tube, flux sheet, and umbral dot (3.1a, 3.1c, 3.1d). Also shown in 3.1b is the temperature distribution (K) in the flux tube. These 2D images show slices through the center of the features' 3D atmospheres

characteristic of magnetic fields concentrated through convective collapse. Flows of adiabatically cooling gas evacuate a flux tube, causing a drop in density and temperature. To balance the higher gas pressure outside the tube, the interior magnetic pressure must increase, leading to a concentration of strong magnetic fields (Parker, 1978).

The width of the flux sheet and diameters of the tube and umbral dot are on the same order as or smaller than the photon mean free path, at 84, 132, and 132 km respectively where $\tau = 1$. The atmospheric models became optically thin several scale heights below the top of Rempel's simulation. To reduce computation time, we simply removed the uppermost 390 km of the model, while still ensuring that the upper boundary condition we used for the radiative transfer - that of an optically thin medium with no incoming radiation - was appropriate.

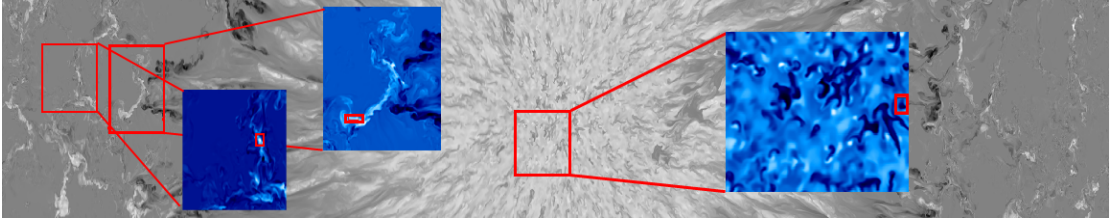


Figure 3.2: Locations of the flux tube, flux sheet, and umbral dot (left to right) on the high-resolution Rempel sunspot simulation. The vertical magnetic field is shown.

Radiative Transfer Code

To generate synthetic Stokes profiles, we used Han Uitenbroek’s Rybicky-Hummer (RH) radiative transfer and chemical equilibrium code (Uitenbroek, 2000). RH is based on the MALI (Multi-level Approximate Lambda Iteration) method described in Rybicki and Hummer (1991, 1992), which is sped up with Ng’s (1974) mathematical acceleration method. To solve for the Stokes vector, it uses the Socas-Navarro et al. (2000) version of the short characteristics method of Olson (1987) and Kunasz (1988) for Stokes I and Rees and Murphy (1989) for the full vector. It can solve for the full Stokes spectrum under general non-LTE conditions in four different geometries - here we only concern ourselves with the 2- and 3-D Cartesian geometries. We were able to use the full 3-D treatment for the flux tube, but due to computational expense we limited the rest of the calculations to the 2-D geometry.

When using a multi-dimensional version of RH, it is necessary to increase the angular resolution in order to reach the same accuracy of angular quadrature as in a 1-D problem (Holzreuter and Solanki, 2012). We used a Gaussian-Legendre angle set of 5 inclination and 2 azimuthal angles per octant, for a total of 80 rays. This proved to be sufficient for both the 2- and 3-D cases, with differences of at most a percent when the number of azimuthal angles was doubled. The standard angle set, A6, with 96 rays, lead to a maximum difference of 5% in the 2D case, but we decided that the corresponding increase in computational time, especially

for the 3D simulation, was not worth such a fractional increase in accuracy.

We enforced periodic boundary conditions in the horizontal x- (2D) and horizontal x- and y- (3D) dimensions by mirroring the atmosphere in these directions. This enabled us to avoid discontinuities or non-vertical rays leaving the boundaries. We neglected magneto-optical effects and solved for the line-of-sight spectral profile.

Atomic Model

For realistic non-LTE calculations, we used a 23 level iron atom, with 22 Fe I levels and the ground state of Fe II. The atomic model included 22 continuum and 33 line transitions. Data was provided by the following sources: energy levels from Corliss (1982) line data from Fuhr (1988), and oscillator strengths from Thévenin (1989, 1990), all compiled by J. H. M. J. Bruls. The solar iron abundance was 7.44, following Asplund (2000).

For our analysis we were interested in the spectra of the commonly used diagnostic lines at 630.15 and 630.25 nm. Their oscillator strengths, given by R.L. Kurucz, are 4.26×10^{-2} and 2.27×10^{-2} respectively. Calculations used 3668 wavelength points to solve the whole spectrum, and the 224 in the range of interest proved sufficient to produce a smooth spectrum.

RH solves for the radiative effects of Fe I transitions as well as the effects of interactions with the hydrogen and helium 'background'. These include scattering off of the neutral atoms and molecular hydrogen, bound-free and bound-bound processes involving singly ionized hydrogen and OH and CH molecules. A solution under non-LTE conditions must solve the statistical equilibrium equations and therefore account for collisions. In the ultraviolet regime, overionization leads to incorrect population numbers because of missing iron line opacity. We accounted for this with opacity fudging described in Bruls et al. (1992).

RESULTS AND DISCUSSION

2D Results

We took 2D slices of the 3D atmospheres at two different positions for each magnetic configuration. For the flux tube and umbral dot, these were where the magnetic field strength was at an extremum, in the middle of the structure ($y=4$ and $y=3$ respectively), and where we supposed the wall of the structures to be at the $\tau = 1$ level ($y=7$ and $y=6$). Two atmospheres of differing properties were created by slicing the flux sheet across the value of maximum field strength - one by slicing vertically at $x=9$ and the other by slicing horizontally at $y=5$.

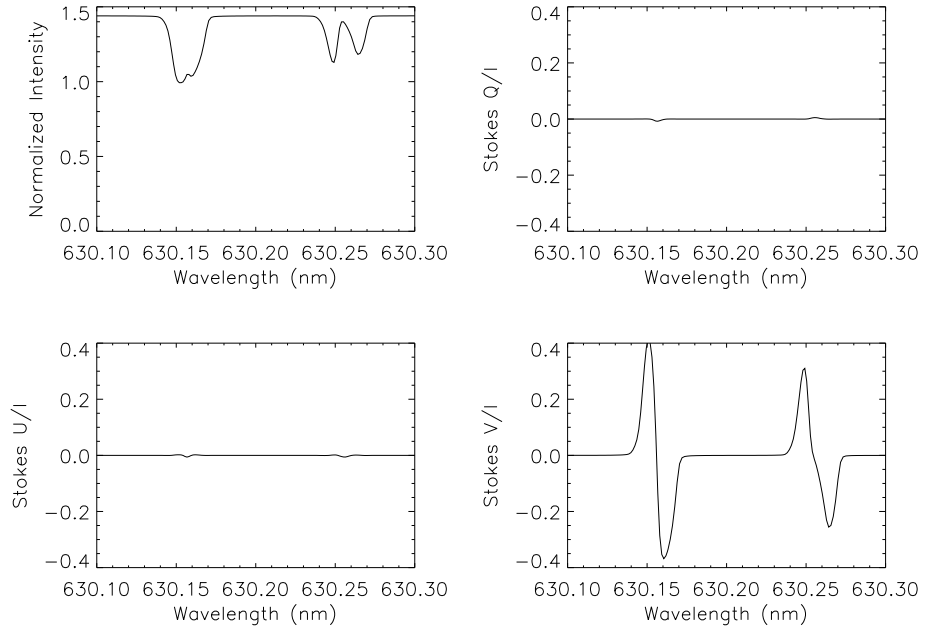


Figure 4.1: Stokes profiles near the center of the flux tube. The intensity is normalized to the quiet Sun continuum intensity.

The Stokes profiles generated by RH were largely what we expected to see from an atmosphere with strong vertical magnetic fields. Continuum intensity for the flux tube was higher than the quiet Sun continuum intensity, hence the common description of such features as bright points. The umbral dot, while brighter

than the surrounding umbra, still displayed a lower continuum intensity than the quiet Sun. Zeeman splitting was greater for the umbral dot than the tube and sheet (Figure 4.5). Since the umbral dot was in a cooler region of the sunspot with very strong (~ 4000 G) surrounding magnetic fields, these properties came as no surprise. Linear polarization of the emergent light, measured by Stokes Q and U, was almost non-existent due to the vertical nature of the magnetic field (Figure 4.1). The simulated atmospheres were not subject to stray light from the quiet Sun, seeing or instrumental effects, and therefore in areas of strong vertical magnetic field we saw no central (π) component in Stokes I resulting from the Zeeman effect.

The shape of the intensity profiles themselves is notable. Both the 630.1 and 630.2 spectra for the flux tube and showed a slightly lopsided shape (Figure 4.1). We know that red- and blue-shifts in spectral lines are due to the movement of the plasma towards or away from the observer. Strong km/s upflows and downflows are often present in flux tubes, for example the flows involved in convective collapse. In order to understand the asymmetry of the intensity profile, we examined the effect of the velocity field on the spectra. We repeated our 2D calculations with atmospheres where the velocity components were set to zero but the temperature, pressure, density and magnetic field remained the same.

A downflow of ~ 5 km/s, resulting in a redshift of ~ 0.01 nm appeared to be the dominant velocity component in the line-forming region of the atmosphere. This was particularly apparent in the 6302 line, where we saw much more broadening on the blue side than on the red side. In the 6301 line this same effect exhibited as a 'kink' in the blue wing (Figure 4.2c). Kinks in a spectral line core can also be interpreted as evidence of the reversal of the Stokes V line core due to the magneto-optical effect, so it is interesting to note that this line shape can be due to just the velocity field. Because Zeeman splitting is incomplete and there is more natural broadening in this line than in its partner, it is hard to tell if there is actually more broadening in the blue wing of the line than the red.

If we compare the effects of velocity at the center and edge of the flux tube (Figure 4.2a, Figure 4.2b), we see a greater change in the spectrum at the center of the feature than closer to the edges. From Figure 4.3 we see that the downflows contributing to the overall redshift are dominant at the edges of the structure, and this is reflected in the spectrum at those places (Figure 4.2b). The drop in continuum intensity near the center of the tube cannot be directly accounted for by any vertical velocity flows, although we know that the interior of the tube is a cooler region than its surroundings, originally formed by a downflow of plasma leading to the evacuation of the tube. The fact that we see only upflows in that area indicates some sort of convective motion, most likely combined with non-line-of-sight flows into and out of the tube.

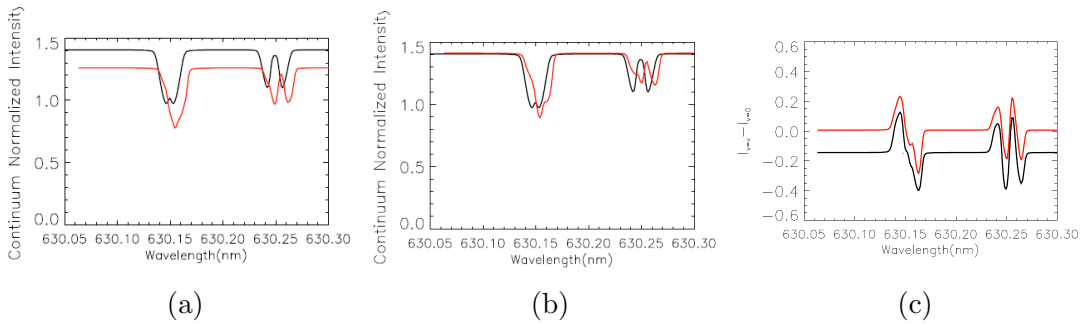


Figure 4.2: The effect of the velocity field on the intensity is shown at the center (a) and edge (b) of the flux tube. Here the black [red] lines indicate the spectra without [with] velocity. (c) shows the differences between the profiles with and without velocity at the center [black] and edge [red] of the tube.

Atmospheres with multiple components are known to produce asymmetric line profiles. For example, penumbral magnetic fields are often modeled with two different atmospheres - a horizontal flux tube embedded in an inclined magnetic field - as suggested by the uncombed penumbral model (Solanki and Montavon, 1993). Observations as well as forward modeling, such as in Bello González et al. (2005), produce blueshifted intensity profiles with a 'kink' in the red wing of the 630.1 line, similar to what we find. The magnitude of Stokes V is also comparable, although we expect Stokes Q and U to be much greater in the penumbral case than in our simulated flux tube. It is surprising that atmospheres with a very different

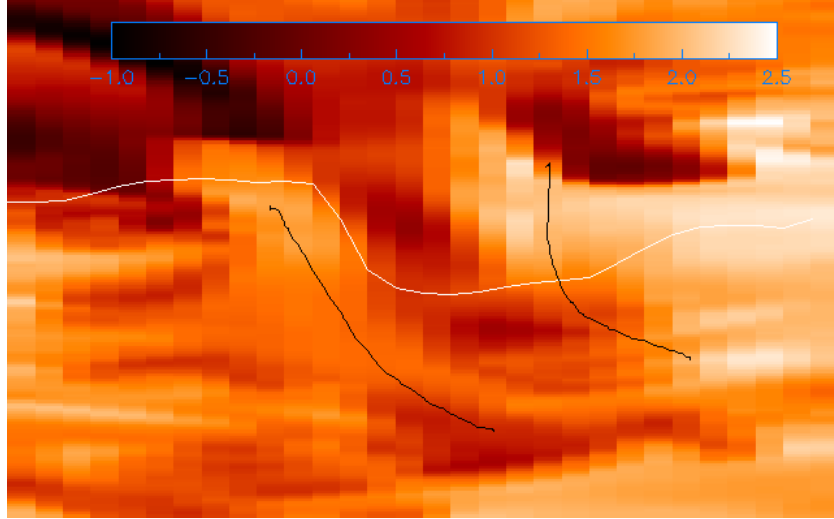


Figure 4.3: Structure of the vertical velocity field in the flux tube (km/s). The white line shows the $\tau = 1$ height in the atmosphere, where most of the emergent radiation originates from. The black lines show approximately the boundaries of the tube as seen in the magnetic field map.

magnetic field configuration can result in some of the same spectral properties, especially if we note the difference in scale between the observations (~ 700 km) and the simulation (~ 100 km).

The variation in the spectra as a function of position can tell us about the properties of the structures' boundaries. Figure 4.4 shows the variation in the 630.2 line core intensity with x , centered around the position of maximum field strength, and Figure 4.5 similarly displays the variation of the amplitude of V/I . Holzreuter and Solanki (2012) show that the hot walls of a simulated flux tube correspond to increases in the intensity of up to 400% compared to the intensity in the tube's interior. While we do see an increase in I between $x=-3$ and $x=-8$ for the flux tube, near the left-hand boundary of the structure, it is not as extreme as Holzreuter and Solanki suggested. There is also no corresponding increase on the right-hand boundary. The flux tube that Holzreuter and Solanki studied possessed a symmetrically distributed magnetic and thermal structure, ideal conditions not seen in the Rempel simulation. As can be seen from Figure 3.1b, our more realistic flux tube has a much cooler area on the right-hand side of the tube than on the left-hand which corresponds to the drop in intensity.

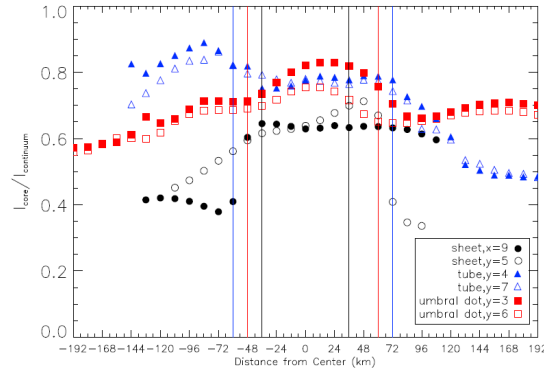


Figure 4.4: Intensity for Fe I 630.2 as a percentage of continuum intensity. The vertical blue, red, and black lines show where the approximate boundaries of the flux tube, umbral dot, and vertical dimension of the flux sheet are.

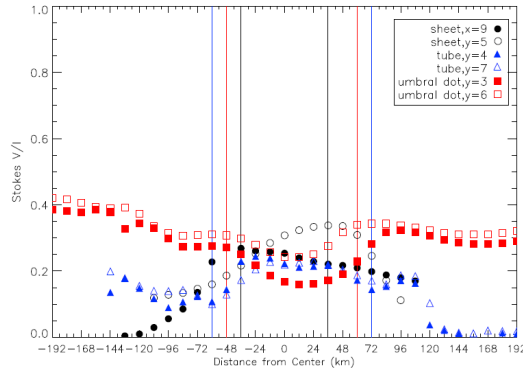


Figure 4.5: Magnitude of Stokes V/I for the 630.2 line as a function of position. This is directly proportional to the amount of Zeeman splitting, and therefore the strength of the underlying magnetic field.

3D Results

We were able to use the full 3D version of RH to obtain spectra for the flux tube, using a reduced area of 10×10 pixels which encompassed most of the important thermal and magnetic characteristics of the region. To better interpret the features of the spectra, we generated the 1D LTE Stokes profiles for each point in this area. Figure 4.6 shows the shape of the LTE profiles adjacent to the point of the overlaid 3D profile. The LTE profiles exhibit deeper line cores, less Zeeman splitting, and less line broadening. We were not able to remove the irregular points in the intensity without causing discontinuities in QUV, hence the odd points in the blue wing of the lines.

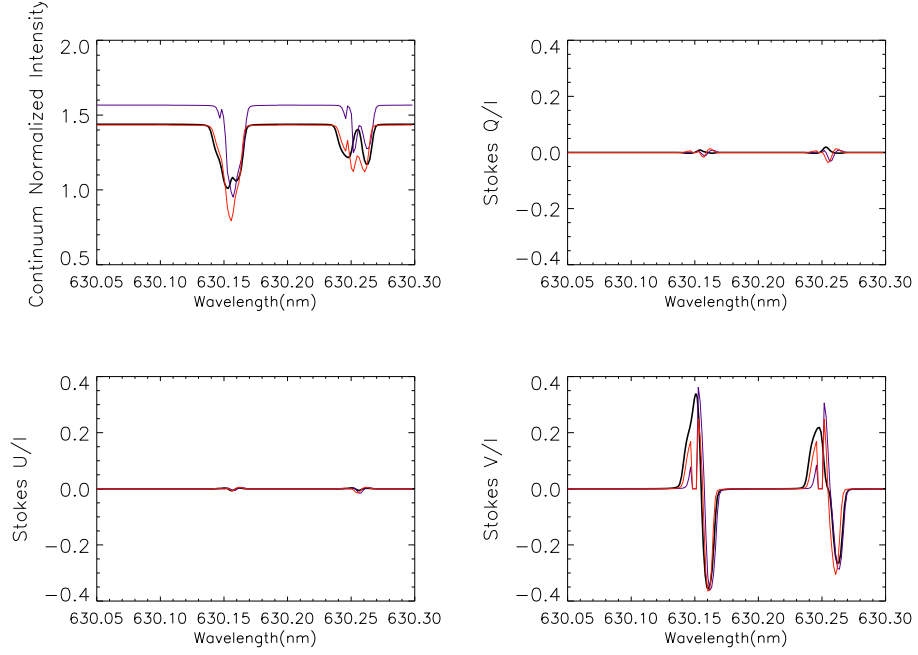


Figure 4.6: Comparison of 1D Stokes profiles to the 3D profiles at points to the right (red) and left (blue) of the location of the 3D profile (black)

In a large-scale, well-mixed atmosphere, we might expect that the 3D profile would resemble a combination of the nearby 1D spectra. A 1D treatment of radiative transfer essentially equates to replicating the properties of a single column of material over an entire uniform atmosphere. Therefore the spectra produced in this way ought to produce the Stokes characteristics of a certain point in an atmosphere, under the LTE assumption. It is clear from Figure 4.6 however that the 3D profile is not merely a combination of the spectra of surrounding points.

None of the LTE lines are as broad as the lines produced by the 3D simulation. We saw in our examination of 2D data that velocity plays quite a role in determining the width and asymmetry of the intensity profile. There is very little asymmetry in the 1D spectra, as expected. The narrower lines are also a consequence of assuming that the velocity vector at each point in the column represents the velocity at corresponding heights elsewhere in the atmosphere.

The most significant difference between the 1D LTE and 3D non-LTE simulation is in the Stokes Q profile, where the 3D treatment results in the opposite shape than the 1D - and as we will see in the next section - 2D cases. This

hints at the importance of non-LTE radiative transfer effects, and particularly that of horizontal radiative transfer, in determining the spectra of small magnetic features.

Comparison of 2D and 3D Results

A comparison of the overall intensity is shown in Figure 4.7. Although the intensity is greater for the 3D case, the relative intensity as a function of spatial position remains approximately constant. The 3D simulation must account for all incoming radiation within a certain volume, and so a corresponding increase in intensity is reasonable. Zeeman splitting in the 630.1 line is much more pronounced in the 3D case, although the shapes of the Stokes profiles remain about the same.

A glance at Figure 4.7 shows that the spacing between the black solid and clear triangles (2D and 3D, center of tube) is always greater than that between the blue solid and clear triangles (2D and 3D, edge of tube). The intensity calculated using a 3D geometry in the interior of the flux tube increases more relative to that in the 2D calculation than it does closer to the edge of the structure. This becomes especially evident when we take the difference of the spectra through the center of the flux tube ($x=0$) at the corresponding center in y ($y=4$) and on the edge at $y=7$ (Figure 4.8). Given that overall intensity drops when one encounters the cool interior of tube, we see that using the 3D treatment makes less of a distinction between the center and edges of the tube. Were we to extend our 3D simulation to a larger area, we would not be surprised to find a noticeable reduction in the hot wall effect.

There is also an increase in linear polarization when adding the extra dimension (Figure 4.8). This can only be due to horizontal radiative transfer that a two-dimensional simulation is unable to account for. Radiation flowing in or out of the walls of the structure is contributing to the emergent intensity and polarization. This helps explain the spectroscopic smearing mentioned above which makes it harder to distinguish features of the flux tube. Because the tube only has a

diameter of about 100 km, it is possible that radiation coming in from many points outside the tube is contributing to the spectrum that emerges from the interior; the source function might be determined by points in both the walls. In a larger structure we might expect that the only horizontal radiative transfer effects will come from the regions in the wall closest to the interior point of interest; that is, only the wall nearest to an interior point contributes to its spectrum.

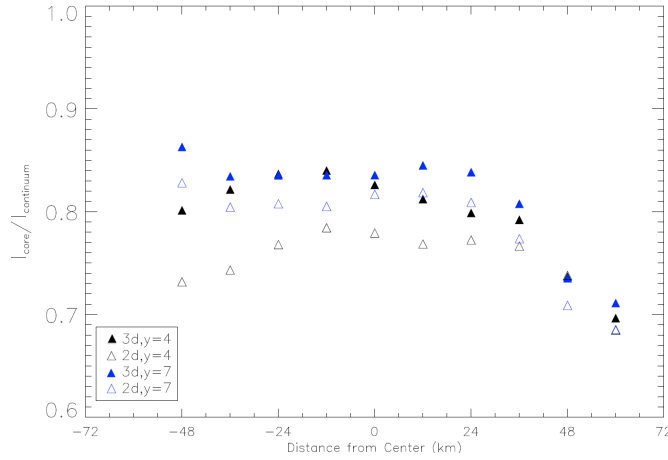


Figure 4.7: Variations with x of intensity at the 6302 line core.

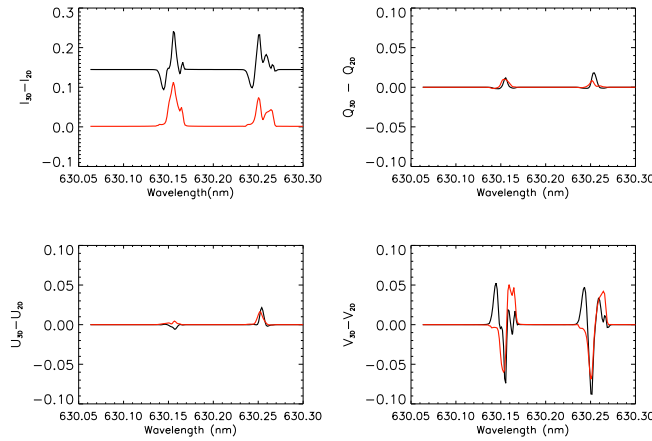


Figure 4.8: Differences between 3D and 2D spectra at the center (black) and edge (red) of the flux tube

Degradation with Instrumental Profile

In our analysis, we convolved the simulated Stokes profiles with the Hinode SOT spatial point-spread function for the red continuum as characterized by four Gaussians listed in Mathew et al. (2009), and the spectral point-spread function described by Lites and Ichimoto (2013). The spatial Gaussians describe both the diffraction-limited PSF and scattered light contributions from the instrument.

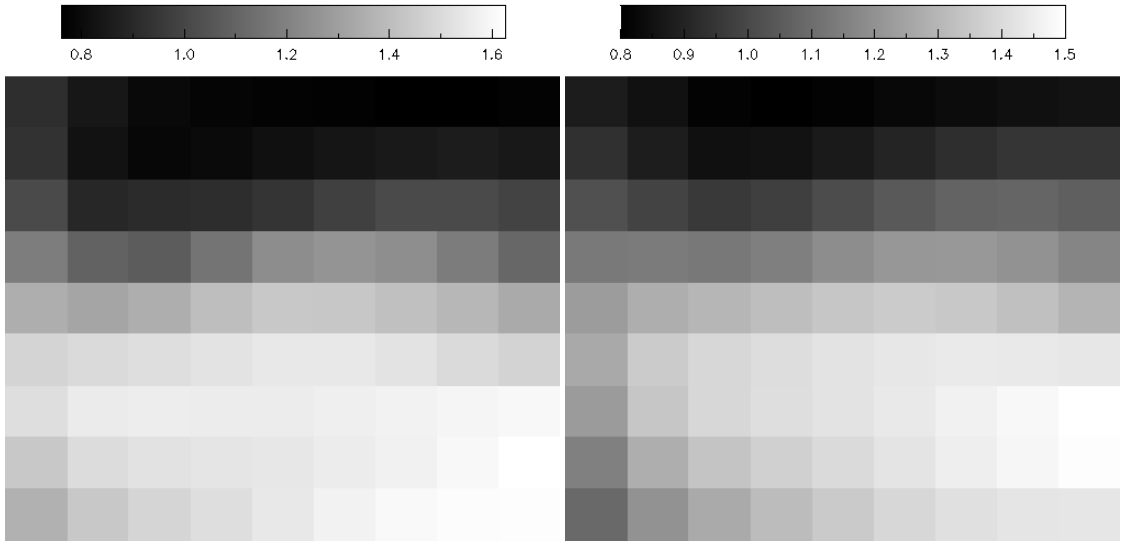


Figure 4.9: Original (left) and degraded (right) continuum intensity. Although the data was mirrored when we performed the convolution, we only show the unmirrored images here. Note the difference in contrast and intensity, evident by looking at the scale bars

The resulting spectra that appeared smoothed, with smaller asymmetries and an overall decrease in the amplitudes of all the Stokes components. The reduction in brightness and contrast, shown in Figure 4.9, is in accordance with Danilovich’s (2008) degradation of MHD simulation data. The further decrease of signal amplitude in an already small Stokes Q and U begs the question of whether these might, in the presence of noise associated with an actual observation, be entirely indistinguishable from the background.

Comparison with Hinode Data

We chose the December 10, 2006 observation of [NOAA] AR0930, a large sunspot near heliospheric disk center with a good view of the umbral area. This scan was taken with the SOT spectropolarimeter (SP), a modified Littrow spectrometer that creates high-precision Stokes profiles of the Fe I 630.1 and 630.2 nm line pair. The SP scans the solar disk through a 0.16'' x 164'' slit oriented North-South, with this particular scan covering the widest range of 2047 steps, or 328'' in the East-West direction. Its spectral resolution is 30 mÅ.

This two-hour scan started at 10:55 UTC. The penumbral region of this round sunspot has a diameter of $\sim 72''$, or about 52 Mm. The longitudinal magnetogram in Figure 4.10 shows a section of opposite polarity in the penumbra, as well as a few concentrations of opposite polarity just outside of the sunspot. This active region later resulted in an X-class flare (Myagkova et al., 2009, Ning, 2008).

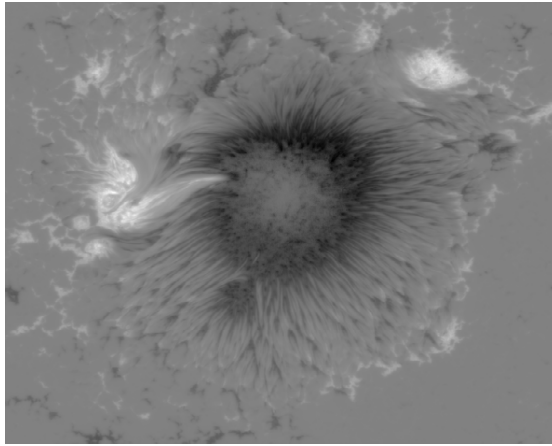


Figure 4.10: Longitudinal component of apparent flux of December 10, 2006 sunspot. Outward directed polarity is indicated by white while inward directed polarity is black.

We determined the regions of maximum similarity between the degraded Stokes IQUV and the SP observations of the Stokes parameters by using a series of correlations between the two spectral data sets. The correlation between two discrete functions is defined as $f \star g = \sum_{m=-\infty}^{\infty} f^*(m)g(n+m)$, and becomes identical to the convolution when the kernel (function f to be convolved with image g) is

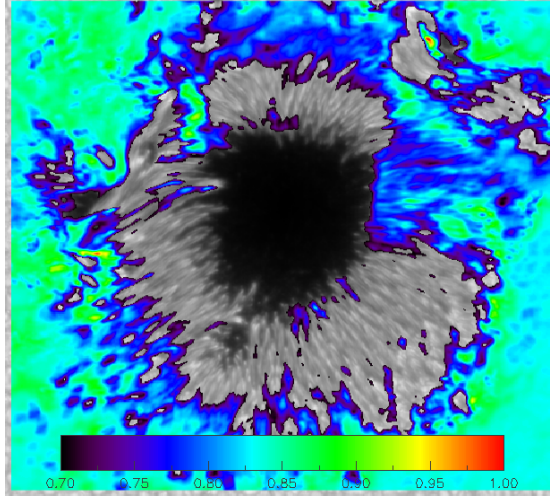


Figure 4.11: Correlation between the simulated Stokes profiles and the Hinode SP data, overlaid on a map of the continuum intensity. We show all positions where the value of the correlation is greater than the mean correlation of .708.

odd (Wiesstien). At each wavelength we created a 2D kernel from the spatial patterns of the simulated Stokes profiles - one kernel from i , one kernel from Q , etc. The offset, n in the above equation, is spatial and corresponds to the pixel size of the Hinode data. We ensured the compatibility of the kernel and the observed data by interpolating the simulated Stokes profiles onto the same wavelength grid as the spectropolarimeter.

Using the kernels and the spatial images of the SP Stokes parameters, we found at each wavelength the correlation between the simulated and observed profiles for Stokes I and V . Because the direction of the the azimuthal magnetic field is arbitrary, we rotated the linear polarization vector in increments of 22.5 degrees for a full 180 degrees and found the correlation in Q and U for each of these angles. For each of the Stokes parameters, we normalized the data such that the maximum possible correlation, calculated using the self-correlation of the kernel, was one over. Because a negative value of the correlation is a correlation to the mirror image of the kernel, we took the absolute value of the correlation for symmetric Stokes Q , U , and V .

Locations of maximum similarity were found by simply adding all the normalized values of the correlation together and then dividing by the number of Stokes

parameters, so that the maximum overall correlation possible was one. The result is a map where the brightest points indicate areas of greatest correlation. Figure 4.11 shows the correlation between the 3D RH simulation of the flux tube overlaid on the continuum intensity.

Most matches to the simulated flux tube spectra appear to occur in the plage area outside the sunspot and in the penumbra. We believe that the primary reason for high correlation in areas of weak magnetic field is that signals in Stokes Q and U are lost among the noise and therefore close enough to zero to result in a high value of correlation, despite any mismatches in Stokes I and V. The correlation process returns the best matches to the shape of the spectral profile, regardless of the magnitude of the signal itself.

Penumbral Stokes profiles are known to show asymmetries (Beckers and Schröter, 1969, Bello González et al., 2005), as discussed previously. Although this is a region of strong, small-scale magnetic fields, they can hardly be described as vertical. The correlation process returns the best matches to the shape of the spectral profile, regardless of the magnitude of the signal itself. Thus areas such as the penumbra with strong linear polarization with similar form but not magnitude as the synthetic profiles, appear to be highly correlated. This also explains the amount of correlations in the umbra, where the intensity is very low compared to that of the simulated tube.

To check the accuracy of our method, we investigated the areas of high correlation in light of the results of the MERLIN Milne-Eddington inversion that is used to interpret Hinode SP data. In areas where the inversion calculated strong values of the magnetic field, we also found high correlation, with the exception the umbra which possessed very strong umbral magnetic fields but very low correlation. Likewise, in areas with magnetic field inclinations close to zero we found strong correlation, which tells us that the correlation process, also dependent on clearly distinguishable shapes of the Stokes profiles, is indeed finding the types of profiles we are searching for. The MERLIN inversion often fails to produce a solution in

regions of low polarization or unusual intensity profile shape, calling into question its ability to describe the penumbra or quiet Sun. We look at the limitations of using a Milne-Eddington inversion code to extract accurate information about a complex atmosphere in the appendix.

A look at the areas of highest correlation shows intensity profiles with deeper line cores and less Zeeman splitting than in the simulated data. In the penumbra the amplitudes of Stokes I and V are much more similar to those in the simulated data. Stokes Q and U can be up to 10^2 times greater in magnitude than in the simulations, because the shape not the amplitude of the signal is what determines the amount of correlation. Generally we also find that the amount of Zeeman splitting exhibited in the Hinode data is not as great as in the simulations, and the line core depth in both lines is deeper. We must remember, however, that the pixel size of the spectropolarimeter is about five times the size of the simulated pixels, so that the entire small flux tube will be described by two pixels by two pixels were it to exist and be observed by the satellite. Therefore the portion of the tube that contributes to the radiation that Hinode observes is much larger than the volume the simulation uses to calculate its spectra. This might account for the deeper line cores in the Hinode data. It appears that, currently, confidently identifying a spectrum belonging to a very small magnetic feature is difficult because of the abundance of alternative possible interpretations of the data.

CONCLUSION

By examining the spectral profiles of small magnetic structures under 2- and 3-D treatments of radiative transfer, we demonstrated the importance of using the full 3D non-LTE simulation to account for important horizontal transfer effects. This general result was shown by Holzreuter and Solanki (2012) but we show additionally how crucial this is when looking at scales close to or less than the photon mean free path. The ability to distinguish the hot exterior boundary of a tube-like structure from its interior, predicted to persist even at small scales (Holzreuter and Solanki, 2012), proved to diminish considerably when the 3D version of RH was used. We believe that a major cause of this smoothing of the spectroscopic data originates in the degree to which the radiation from surrounding regions contributes to the source function at any one point. On such small scales, the properties of up to ten pixels - 120 km - worth of atmospheric information can determine the source function used in calculating the emergent radiation field. Because the local source function at each of these pixels is not necessarily equivalent to the 'average' source function, we can say that the loss of distinction between the spectra at different positions in a small-scale feature is due to the decoupling of the source function from the radiation field.

As modern telescopes begin to achieve diffraction limits in the photosphere on scales less than the photon mean free path, it is important to ask what we might expect to be able to reliably identify. The polarization profile of a 120 km flux tube displays the characteristics of an atmosphere containing strong vertical magnetic fields, as well as asymmetries in the intensity profile due to strong down-flows. Degradation of the spectra with the Hinode instrumental profile resulted in a smoothing of the spectra that nonetheless preserved the major characteristics of the original high-resolution simulated spectra. Comparison with a Hinode spectropolarimetric observation of a sunspot revealed that the regions with spectra

most similar to that of a very small flux tube were in the penumbra. Penumbral spectra are known to display asymmetries and have been modeled by a flux tube embedded in an inclined magnetic field. Similarities between the spectra of these two very different types of magnetic configuration indicate that with current instrumentation it would be difficult to ascertain if the underlying structure were indeed a small scale feature or not.

Convolution of the data with the spatial and spectral profiles of other instruments, especially that of the upcoming ATST, would be a useful continuation of this work. Analysis of the effect of photon scattering on the shape of the spectra would also help to quantify the role it plays in the formation of the spectral lines.

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APPENDIX A

ACCURACY OF MILNE-EDDINGTON INVERSIONS

One-Dimensional Atmospheres

Milne-Eddington inversions, used almost exclusively in the photosphere, assume a thin, flat atmosphere and are one of the most widely used inversion techniques. To investigate the potential weaknesses of parameterizing a stratified atmosphere using a single set of properties, we examine the consequences of using a Milne-Eddington inversion to invert spectra of complex atmospheres. A Milne-Eddington inversion treats the source function as linear with optical depth. Thus line formation occurs at a single height, absorption is constant, and LTE conditions exist. This makes it simple to calculate the properties of the atmosphere and magnetic field that result in fits to Stokes I, Q, U, and V spectral line profiles.

We used a series of one-dimensional model atmospheres with characteristic temperatures within 1000 K of the quiet Sun temperature and predetermined uniform magnetic field configurations with strengths up 3000 G and inclination and azimuthal angles from 0 to 180 degrees. Each uniformly stratified atmosphere consisted of temperature, electron density, velocity, and hydrogen ionization population data for a column approximately 0.1 Mm deep. Table 5.1 lists the properties of the atmosphere, magnetic field, and spectral line used in the inversion of the profiles.

Table 5.1: Inversion parameters

Parameter	Physical meaning	Typical range
B_0	Source function	$0 \leq B_0 \leq 1$ pixels
B_1	Gradient of source function	$0 \leq B_1 \leq 1$ pixels
B	Field strength	$100 \leq B_1 \leq 3000$ G
lc	Line center	15648.5 Å
dw	Doppler width	0.13 pixels
r_0	Inclination angle	$0 \leq r_0 \leq 90$ degrees
ρ_0	Azimuthal angle	$0 \leq \rho_0 \leq 180$ degrees
η_0	Line-to-continuum absorption ratio	10
a	Damping parameter	10 nm

We examined the Stokes profiles of the Fe I 15648.5 line, which with a Landé

g-factor of 3.0 is highly sensitive to the magnetic field. To invert the synthetic spectra, we used two simple Milne-Eddington inversion codes. The first, named the Two-Component Magneto-Optical code (2cmo), was developed in Jaeggli (2011). It uses chi-squared curve fitting to Faraday-Voigt profiles to quickly invert Stokes profiles. 2cmo can fit multiple lines simultaneously in order to better constrain the solution.

Most Milne-Eddington inversion codes are unable to provide statistical information about confidence intervals. Using a Bayesian approach to the inversion such as in Ramos et al. (2007) makes this possible. Bayes ME uses the error associated with each wavelength point to determine the confidence intervals. We used a photon noise distribution to estimate error. The Markov Chain Monte Carlo used in calculation requires much more CPU time than 2cmo, so although BayesME is also capable of fitting more than one line profile at a time, we opted to use a reduced wavelength range centered on Fe I 15648. Figure 5.1 demonstrates the fits to the profiles of the different codes. It is clear that small signals in Stokes Q, U and V may be lost among the noise.

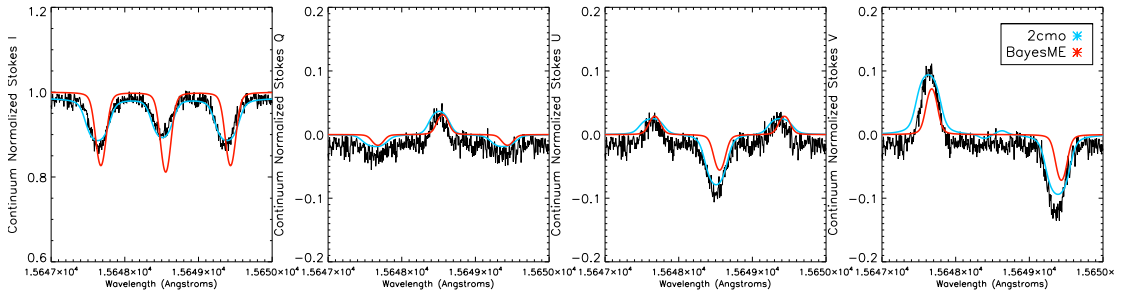


Figure 5.1: 2cmo and BayesME fits to the RH-generated Stokes spectra with the addition of Poissonian noise.

Figures 5.2 and 5.3 show how the values obtained from the inversion compare to the known magnetic field properties. The magnetic field strength is divided into two categories, high (≥ 1000 G) and low (≤ 500 G). The solutions for field intensity are sorted according to the characteristic temperature of the atmosphere, while the angular solutions are color coded with the known magnetic field strength. We show both the values obtained by the inversion versus the known quantities and

the residuals, $parameter_{inverted} - parameter_{actual}$ for a selected range.

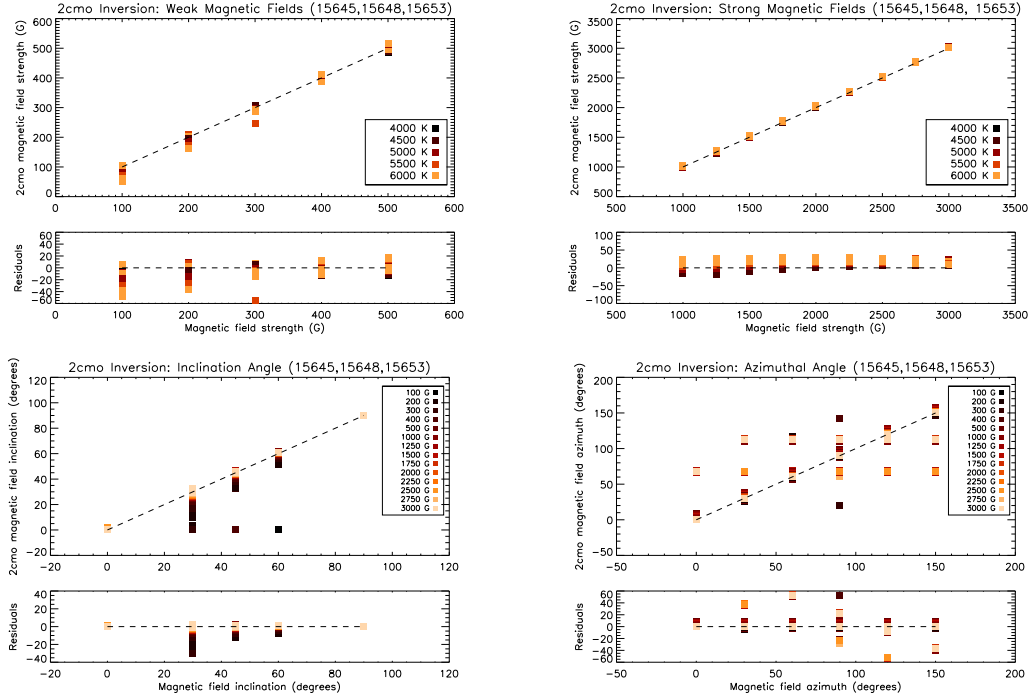


Figure 5.2: 2cmo inversion: inverted parameter values vs. actual values

From Figure 5.2 we can see several characteristics of the inverted parameters obtained by 2cmo. In general, the inversion is far less accurate for small values of B (≤ 500 G) than for large ones (≥ 1000 G). The magnetic field magnitude, inclination, and azimuth all demonstrated this property. This is to be expected because the amount of Zeeman splitting, which determines the amount of signal in the polarization profiles, is directly proportional to the strength of the magnetic field. Atmospheric temperature also impacted the calculation of magnetic field strength, with hotter atmospheres producing better results. This was not an observable trend in the angular data. Azimuthal angle was determined with less accuracy than the inclination angle. There is an inherent 180 degree ambiguity in this parameter that we accounted for in our analysis. The relative inaccuracy of the azimuthal inversion can be attributed to the strength of polarization signals in Stokes Q and U being on an order of magnitude smaller than those in Stokes V. Q and U ($0^\circ - 90^\circ$ and $45^\circ - -45^\circ$ linear polarization) together determine the azimuthal direction of the field, while V (LCP - RCP) is used to calculate the

inclination angle.

Results of the BayesME inversion help us quantify the impact of signal strength on inversion accuracy. To understand the significance of the confidence intervals calculated by BayesME, Figure 5.3 presents a histogram of the one-sigma errors associated with the inverted parameter. We follow the color coding of Figure 5.2, and similarly include a plot of the residuals. To reduce clutter in the graph, we only show the residuals for selected atmospheric temperatures and field strengths. Confidence intervals are represented with error bars; in most cases they are quite small and barely visible.

We find that small one-sigma confidence intervals do not always mean a more accurate inversion. For example, for residuals greater than 100 G, confidence intervals were between 10 and 15 G. This indicates that although in fact the inversion is greatly over- or under-estimating field strength, it is convinced that this is the solution that best fits the data. 90% of the one-sigma errors for the field strength calculation were within two orders of magnitude of the residuals, with the majority of outliers coming from $B \leq 500$ G. This is remarkable when we note that for $B \geq 1000$ G, 93% of the one-sigma confidence intervals were on the same order of magnitude as the residuals, while this was true for only 2% of the low B inversions. Thus for a magnetic field strength of 500 G with the inversion giving 510 G, confidence intervals could easily range from 400 G to 600 G, with an even greater range more likely. For such small field magnitudes to begin with, this is at the same time reassuring that the inversion produces a result so close to the actual value and alarming how great the range of possible solutions is. We note a similarly worrying trend for the azimuthal angle data, where only 9% of the confidence intervals were on the order of magnitude of the residuals. Inclination angle is much better determined, as expected, with 28% of the confidence intervals falling close to the value of the residuals.

Milne-Eddington inversions of one-dimensional atmospheres are very well suited to regions with magnetic field strengths greater than 1000 G with temperatures

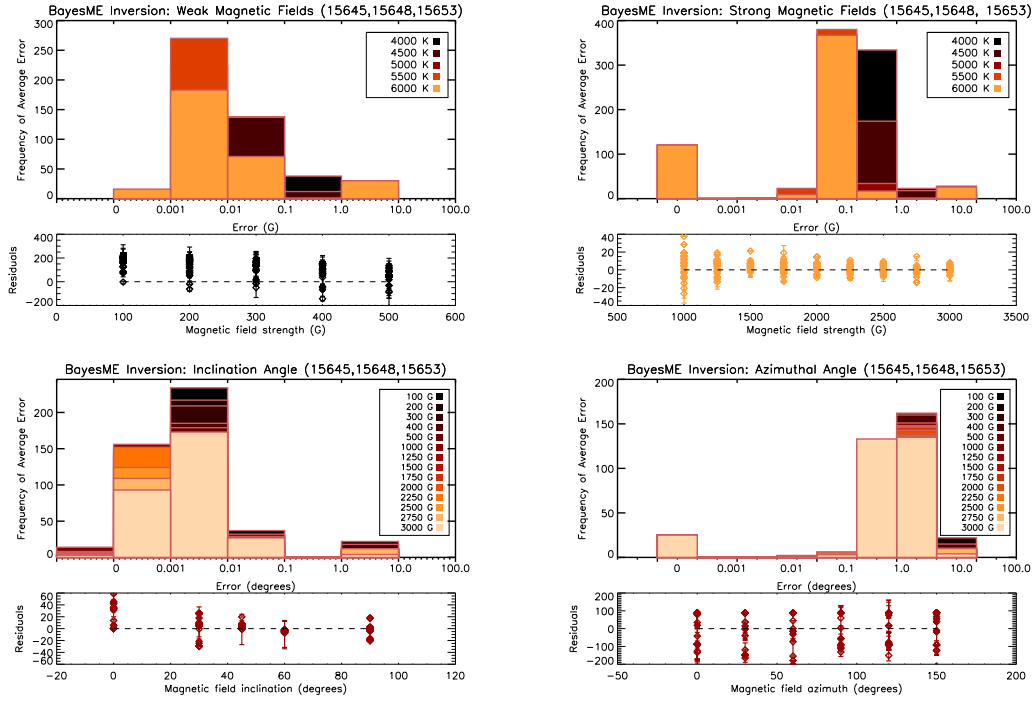


Figure 5.3: Confidence intervals obtained with BayesME and their frequency for inverted parameters

near or above the quiet Sun temperature. This is due to the formation height and amount of Zeeman splitting of our chosen $g=3$ Fe I line. Noise of any sort significantly reduces the accuracy of the inversions, especially in calculating the azimuthal angle which depends on Stokes Q and U.

The assumption of a thin, flat atmosphere in LTE made by the Milne Eddington approximation can be very accurate when examining large, fairly uniform regions of the photosphere. However, even a simple stratified atmosphere has more complexity than the Milne-Eddington assumption accounts for, which can lead to a substantial difference between the actual atmospheric parameters and those calculated by an inversion. If errors of this sort exist even under LTE conditions, one wonders if Milne-Eddington inversions would be any use at all in a much more complex atmosphere that cannot be treated in LTE.

Complex, Multi-Dimensional Atmospheres

Milne-Eddington inversions are commonly used to study magnetic field configurations in active regions. The Milne-Eddington Grid Linear Inversion Network

(MERLIN) code uses the Levenberg-Marquardt algorithm to perform a least-squares fitting to invert data from Hinode’s spectropolarimeter (SP). We were curious to see what a Milne-Eddington inversion would make of the RH-generated Stokes profiles for our small flux tube, flux sheet, and umbral dot.

Before and after degrading the synthesized Stokes profiles with the Hinode SOT profile, we used 2cmo to invert the data. The code allows for a maximum magnetic field value of 4000 G. Often the inversion failed when trying to fit the original spectra; perhaps the disproportionately strong Stokes V signal or the asymmetry of the line profiles was to blame. Certainly the LTE assumption does not allow for the strong directional velocity fields that contribute to the unique shape of Stokes I. Even smoothing the spectra with the Hinode PSFs did not guarantee a successful inversion, although failure was less common. Attempts to use BayesME on the "bad" spectra were also unsuccessful. From looking at any given map of the SP’s MERLIN inversion results, we can see that a failed inversion is nothing unusual, especially in areas of strong magnetic field such as the umbra, or in the quiet Sun, where the polarization signals are small.

A comparison of the inversion results with the known magnetic field configuration reveals how much information we lose in going from a multidimensional atmosphere to one that is presumed flat and in LTE (Figure 5.4). The field strength calculated by the inversion tends towards values greater than but close to that of the $\tau = 1$ magnetic field. The atmospheres do become more uniform above this height, from which most of the photons which escape to be detected by instruments originate, although the magnetic field strength decreases (or increases in the case of the umbral dot). It appears that even a magnetic element of four times the $\tau = 1$ field strength does not contribute distinctly enough to the emergent radiation to produce the corresponding spectral signature expected by a Milne-Eddington inversion. The closer-to-LTE plasma above the $\tau = 1$ level results in a spectrum more easily interpreted by the inversion, and thus plays the dominant role in the calculation of magnetic field properties. Degradation with the

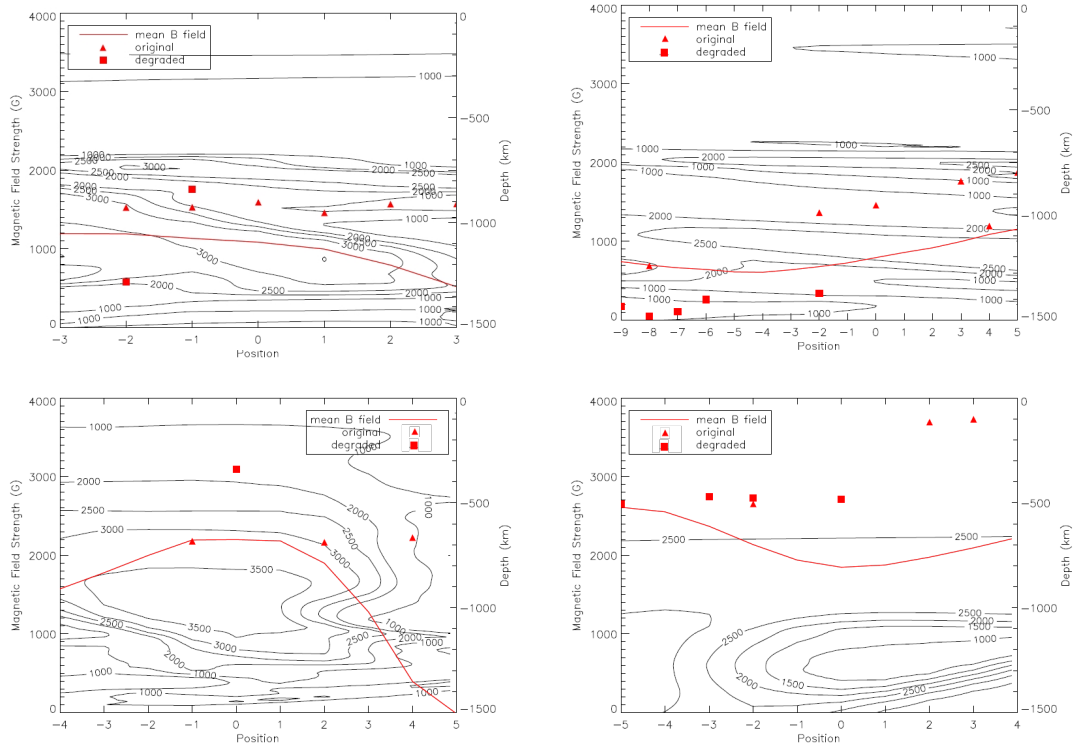


Figure 5.4: Magnetic field strength calculated by the inversion before and after convolution with Hinode PSF, compared to the magnetic field strength at $\tau = 1$ as a function of position

Hinode profile generally results in lower magnetic field strength values, doubtless due to the decrease in amplitude of the Stokes signals.

Two things about these results are striking. Firstly, the frequency of failure of the different curve-fitting algorithms emphasizes how the standard Voigt or Faraday-Voigt profile is not enough to describe the features evident in the Stokes profiles of small magnetic elements. Even the degradation and resultant smoothing of the spectroscopic data via the SOT instrumental profile resulted in failed inversions more often than not. Allowing for the multiple velocity components that we observe from these structures might perhaps improve the success rate of the fitting algorithms, but this would require a relaxation of the LTE assumption. A more complex inversion code is needed if one hopes to detect the presence of small magnetic structures in this way. Secondly, reducing a complex multi-dimensional atmosphere to one point loses the detail of a stratified atmosphere. The most important contribution to the solution appears to come from an area

close to the $\tau = 1$ level, and any information about features of the atmosphere outside that region, no matter how significant, is lost. We have seen that certain properties of a multi-dimensional, complex atmosphere - for example, the velocity field - play a large role in producing the unusual spectral signatures of small-scale magnetic elements. The Milne-Eddington approximation must ignore these contributions and as a result is only capable of providing approximate information about a very small portion of the atmosphere that fails to characterize the whole. It is clear that, at least in the case of very small photospheric features, solutions to the inverse problem can in no way provide as much or as accurate information as forward modeling can.