



Prediction of electric heating load component of distribution feeder loads using statistical modeling
by Vijay Singh

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Electrical Engineering

Montana State University

© Copyright by Vijay Singh (1992)

Abstract:

A statistical model has been developed to represent the thermodynamic behavior of buildings. The model uses an analogous electric circuit to represent the thermodynamic characteristics of each building. The building model parameters are gaussian distributed, each with a specific mean and standard deviation. A computer program has been written which uses this model to predict the heating load component of the distribution load via simulation. The program can predict the feeder electric heating load demand under continuous power supply as well as after a power outage. It is specifically useful for predicting the feeder cold load pickup currents on cold winter days. The program has several features including allowing for changes in the ambient temperature during the outage and changes in the service voltage. Simulation results are presented and some experiments towards validating the computer program are included.

PREDICTION OF ELECTRIC HEATING LOAD
COMPONENT OF DISTRIBUTION FEEDER LOADS
USING STATISTICAL MODELING

by

Vijay Singh

A thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Electrical Engineering

MONTANA STATE UNIVERSITY
Bozeman, Montana

August, 1992

N378
Si 64

ii

APPROVAL
of a thesis submitted by
Vijay Singh

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

8/24/92
Date

M. Hashem Behar
Chairperson, Graduate Committee

Approved for the Major Department

8-24-92
Date

[Signature]
Head, Major Department

Approved for the College of Graduate Studies

8/31/92
Date

R. Brown
Graduate Dean

STATEMENT OF PERMISSION TO USE

In presenting this thesis in fulfillment of the requirements for a master's degree at Montana State University, I agree that the Library shall make it available to borrowers under rules of the Library.

If I have indicated my intention to copyright this thesis by including a copyright notice page, copying is allowable only for scholarly purposes, consistent with "fair use" as prescribed in the U.S. Copyright Law. Requests for permission for extended quotation from or reproduction of this thesis in whole or in parts may be granted only by the copyright holder.

Signature Vijay Singh

Date 8/20/92

ACKNOWLEDGMENT

The author would like to express his gratitude to his advisor Dr. Hashem M. Nehrir and members of the committee, Dr. Baldev Thapar, Dr. Murari Kejariwal and Dr. Victor Gerez for their guidance and encouragement.

For his constant support and inspiration this work is dedicated to the author's uncle Dr. D.S. Mohindra.

TABLE OF CONTENTS

LIST OF FIGURES.....	vi
ABSTRACT.....	ix
1. INTRODUCTION.....	1
2. COLD LOAD PICKUP LITERATURE REVIEW.....	6
Survey.....	6
Overview.....	9
3. MODELING.....	10
4. AGGREGATION.....	24
Algorithm for Sectionalizing.....	26
Algorithm for change in Voltage.....	28
Algorithm for change in Temperature.....	30
5. RESULTS.....	33
Effect of Change in Thermodynamic Parameters.....	36
Change in Voltage.....	41
Sectionalizing.....	44
Effect of Change in Ambient Temperature.....	46
6. EXPERIMENTAL RESULTS AND VALIDATION.....	49
A Continuous Power Supply Experiment on a House.....	49
A Cold Load Pickup Experiment on a House.....	52
A Continuous Power Supply Experiment on a Montana Power Company Feeder.....	54
7. SUMMARY AND CONCLUSIONS.....	59
Summary.....	59
Conclusions.....	60
Future Research.....	60
REFERENCES CITED.....	62
APPENDICES.....	64
A. Development of Equations.....	65
B. Effect of Change in Voltage.....	69
C. Data Used.....	71

LIST OF FIGURES

Figure		Page
1.	Heated building electrical equivalent.....	10
2.	Analogous electric circuit used for thermodynamic behavior of a house.....	11
3.	Hysteresis loop of a thermostat.....	13
4.	House ON/OFF state before and after a power outage.....	14
5.	Variation of house temperature under normal conditions at ambient temperature of -10°F	16
6.	Effect of thermal conductance on the temperature variation of the house.....	17
7.	Effect of change in thermal mass on the temperature variation of the house.....	18
8.	Effect of change in furnace size on the temperature variation of the house.....	19
9.	Variation of house temperature under very cold ambient temperature.....	20
10.	Drop and recovery of house temperature at ambient temperature of -10°F	22
11.	Effect of ambient temperature on the duty cycle.....	25
12.	Sectionalized feeder.....	26
13.	Number of houses demanding power at ambient temperature of 10°F and after a power outage of 30 minutes.....	28
14.	Effect of voltage on the on-time of the furnace at ambient temperature of -10°F	29
15.	Effect of ambient temperature on the on- time and off-time of the furnace.....	31
16.	Change in ambient temperature during the simulation period.....	32

LIST OF FIGURES....continue

Figure	Page
17. Steady state response at ambient temperature of -10°F	34
18. Effect of change in conductance at ambient temperature of 10°F and after a power outage of 30 minutes.....	36
19. Effect of change in capacitance at ambient temperature of 10°F and after a power outage of 30 minutes.....	37
20. Effect of change in heat injected into the house at ambient temperature of 10°F and after a power outage of 30 minutes.....	38
21. Effect of change in set point at ambient temperature of 10°F and after a power outage of 30 minutes.....	40
22. Effect of change in dead band at ambient temperature of 10°F and after a power outage of 30 minutes.....	41
23. Effect of change in voltage on the power supplied at ambient temperature of 10°F and after a power outage of 30 minutes.....	42
24. Effect of change in voltage on the current supplied at ambient temperature of 10°F and after a power outage of 30 minutes.....	43
25. Automatic sectionalizing. Time of closure for the sectionalizer was 64 minutes.....	44
26. Manual sectionalizing. Time of closure for the sectionalizer was 30 minutes.....	45
27. Steady state response at three different ambient temperatures.....	47
28. Effect of change in ambient temperature before and after a power outage.....	48
29. Simulated and measured values of current requirement of 14 rooms.....	51
30. Simulated current demand after an outage.....	52

LIST OF FIGURES....continue

Figure	Page
31. Current demand recorded by a strip chart recorder.....	53
32. Simulated value of the number of houses present on the feeder and the simulated value of the houses demanding power.....	56
33. Simulated value of the number of houses present on the feeder and the simulated value of the houses demanding power.....	57

ABSTRACT

A statistical model has been developed to represent the thermodynamic behavior of buildings. The model uses an analogous electric circuit to represent the thermodynamic characteristics of each building. The building model parameters are gaussian distributed, each with a specific mean and standard deviation. A computer program has been written which uses this model to predict the heating load component of the distribution load via simulation. The program can predict the feeder electric heating load demand under continuous power supply as well as after a power outage. It is specifically useful for predicting the feeder cold load pickup currents on cold winter days. The program has several features including allowing for changes in the ambient temperature during the outage and changes in the service voltage. Simulation results are presented and some experiments towards validating the computer program are included.

CHAPTER 1

INTRODUCTION

Restoring power to a circuit after a power outage during cold winter days which is commonly called Cold Load Pickup can result in feeder over currents, which may cause circuit breaker tripping. This is because in cold days a large number of buildings will demand heating power at the same time after the outage, thus making the load undiversified and causing excess currents to flow. It is therefore of interest to predict the magnitude and duration of the overload following an outage.

According to Audlin and Pratt [1] there are four separate time phases of the increased restoral pickup demand on any feeder after an outage.

- 1) The first phase provides inrush current to cold lamp filaments and to the distribution transformers.
- 2) The second phase lasts for about 1 second or less and provides the motor starting current.
- 3) The third phase lasts for about 15 seconds and provides the motor acceleration currents.
- 4) The fourth phase can last for several hours and is due to an abnormal number of appliances energized.

These appliances are mostly the heating furnaces. The solution to the first three phases is relays with extremely inverse settings.

This thesis makes an analysis of the fourth phase current, which is the heating current during cold ambient temperatures. The duration for which this fourth phase current or the Cold Load Pickup current lasts is dependent on the following factors:

- 1) Ambient temperature;
- 2) Length of outage;
- 3) Number of customers on the feeder;
- 4) Number and type of connected load like washing machines etc.;
- 5) Thermal parameters of the affected houses;
- 6) Type of housing (bungalow, apartment, etc.);
- 7) Water heater size, set point, consumption, etc.

Analysis of the power required for heating is not a simple problem. It is required to model the thermodynamics of a house and the behavior of the thermostat. House thermodynamics is modeled by using an R-C network connected by a switch to a current source Q . The resistor R represents the house insulation level. The capacitor C represents the thermal mass of the house. The current source represents the heat input from the furnace. The switch is controlled by a thermostat, the setting of which is manually adjustable. When the switch is open the charged capacitor provides the current to the resistor. Charge across the capacitor is analogous to the house temperature and so the discharge of the capacitor would mean cooling of the house.

The parameters which determine the current requirement of a house are the resistor R , the capacitor C , the current source Q and the thermostat setting. To model a large number of unique houses a random number generator is used to select the values of the parameters. A specific mean value is specified for each of these parameters and noise is added to it. The noise is a random number with gaussian distribution, mean of zero, and a specific standard deviation.

If the utility does not have an idea of the value of the Cold Load Pickup current the usual way to overcome the unsuccessful reclosure of the circuit breaker would be to disarm protective relays. There are two drawbacks to this.

- 1) The relay may allow the fault current to flow along with the Cold Load Pickup current.
- 2) If the disarmed relay is left in place even after the Cold Load Pickup is completed, it could be a severe danger to the protective system.

This thesis recommends two different ways to restore the electrical service successfully.

- 1) After an estimate of the Cold Load Pickup the relays can be disarmed to an extent to allow the Cold Load Pickup current and not the fault current.
- 2) Reducing the Cold Load Pickup current by means of feeder sectionalizing. A simulation program has been developed that provides the option of automatic sectionalizing and manual sectionalizing. In

automatic sectionalizing the computer program calculates the time when there is minimum current flowing through the feeder. It then simulates further as if the next load has been connected. In manual sectionalizing the operator has to enter the time for the next section of load to be connected.

The computer program has the provision of simulating for:

- 1) Change of ambient temperature during and after power outage;
- 2) Step change in voltage at which power is supplied.

Since the utilities are reluctant to intentionally disrupt electric service, not much validation of the cold load pickup current has been done. In this thesis data was collected from three different experiments towards the validation of the model.

Content of this thesis is contained within seven chapters described below.

Chapter 1 is a general introduction to the Cold Load Pickup problem.

Chapter 2 discusses the work done in the field of Cold Load Pickup so far.

Chapter 3 deals with the thermodynamic model of a house.

Chapter 4 deals with the aggregate load demand of the feeder.

Chapter 5 deals with the simulated results.

Chapter 6 deals with the results of the experiment

performed to validate Cold Load Pickup model.

Chapter 7 is the summery and conclusions of the study of Cold Load Pickup. Also suggestions for further research are presented.

CHAPTER 2

COLD LOAD PICKUP LITERATURE REVIEW

An analysis of the overload after a power outage is important for the utilities in order to prevent damage to their equipment. It is required to efficiently model the thermal characteristics of the house on the feeder in order to determine the magnitude and duration of this overload. In this chapter literature pertaining to Cold Load Pickup current and thermodynamics of a house is reviewed.

Survey

In 1949 Audlin et al. [1] performed a 15 minute outage on a feeder serving 3500 customers. Four time phases of the Pickup current was recognized. McDonald and Bruning [2] did experimentation to determine the value of the parameters of a house. House temperature was monitored for 24 hours. The conductance was determined by the steady state test which requires the house temperature to remain constant over a certain period of time. The heat input to the house, the house temperature and the ambient temperature need to be recorded to determine the conductance of the house. The time constant which is the product of the thermal resistance and the thermal mass is determined by a cool down test. In this test power is shut off for a certain period of time. The temperature of the house after the test period, the initial temperature of the house and the outside temperature are

required for the calculation of the time constant. The thermal mass of the house is then calculated by multiplying the conductance to the time constant. He then comes up with the average and standard deviation values of the parameters by conducting the individual tests in six electrically heated homes. His results include showing the average steady state power demand per house at different ambient temperatures and the effect of change in ambient temperature on the Cold Load Pickup demand and the duration of its peak.

Ihara and Schweppe [3] developed a thermodynamic model for a house and derived the equation for house on-time (T_{on}) and house off-time (T_{off}). They point out that the heater is either on with a probability $T_{on}/(T_{on} + T_{off})$ or off with a probability $T_{off}/(T_{on} + T_{off})$. An experiment was conducted to verify the results. House temperature was monitored for 80 and 140 minutes. Model and test results have been compared. The effect of change in ambient temperature, change in outage duration and change in the standard deviation of the thermal parameters of the house on the aggregate power demand is shown.

Lang and Anderson come up with the following values, for the model parameters of a house [4].

Thermal Conductance = .318 kw/°C (.177 kw/°F)

Thermal Mass = 3.21 kwh/°C (1.783 kwh/°F)

Reference [5] models an R-C circuit and derives the equation for house on-time and house off-time. Random numbers

are generated for the five parameters which affect the current demand of the house. The effect of change in furnace size, ambient temperature, outage duration and service voltage on the Cold Load Pickup current is shown.

Mortensen and Haggerty [6] modeled an R-C circuit using the same basic equation as Ihara and Schweppe [3]. Time is discretized by using a sampling period h . It is assumed that the thermostat always switches exactly at a sampling instant. An equation to find the temperature of a house after each sampling interval is developed. A discrete white noise term was added to the equation to include the effect of the random influences in the environment.

Mortensen [7] has discussed five models to estimate heating and cooling loads. The models are:

- 1) Deterministic differential equation;
- 2) Stochastic difference equation;
- 3) Markov Chain matrix equation;
- 4) Hybrid partial differential equation;
- 5) Alternating renewal process.

According to N.W.P.P.C. [8] the following are the typical values for the house parameters in the northwest United States.

Thermal Mass = 3223.0 wh/°F

Thermal conductance = 180 w/°F

Overview

Little work has been done towards validating the results of Cold Load Pickup current. Also the difference in the Cold Load Pickup current due to the type of construction of the houses has not been recognized.

This thesis uses the actual values of the house parameters best suited for the northwest United States. Three different experiments were performed towards validation of the Cold Load Pickup model.

CHAPTER 3

MODELING

A house model has been represented in the past by a multi node network. Lang and Anderson [4] have represented the thermodynamic model of a house by the following circuit.

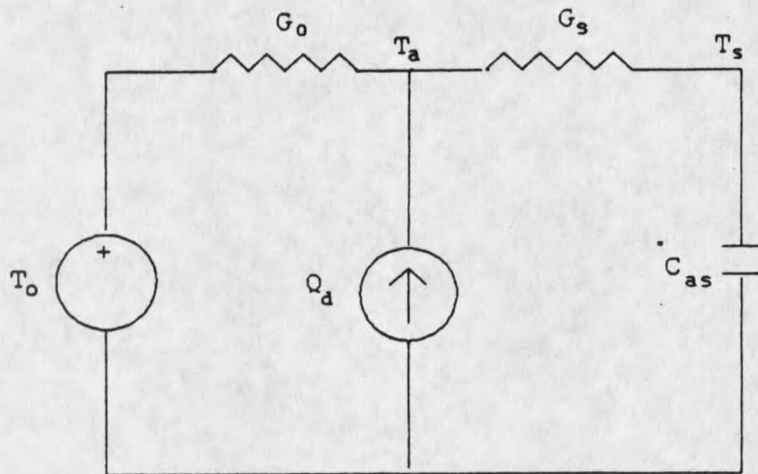


Fig. 1 Heated building electrical equivalent.

Where

T_a is the temperature of the living space air ($^{\circ}\text{C}$)

T_s is the temperature of the equivalent thermal mass ($^{\circ}\text{C}$)

T_o is the outdoor temperature ($^{\circ}\text{C}$)

C_{as} is the equivalent thermal mass representing the entire building ($\text{kwh}/^{\circ}\text{C}$)

G_s is the thermal conductance between the living space and the equivalent thermal mass ($\text{kw}/^{\circ}\text{C}$)

G_o is the thermal conductance between the living space

and the outdoors ($\text{kw}/^{\circ}\text{C}$)

Q_d is the total heat injected into the living space (kw)

According to Mortensen and Haggerty [7] only one or two sections are required due to the forgiving nature of highly aggregated configuration. The load on a power system is the aggregation over hundreds or thousands of houses, such that the individual details largely washout, and only consistent trends remain. Most of our references have used a simple RC circuit. Our model of the RC circuit is as follows.

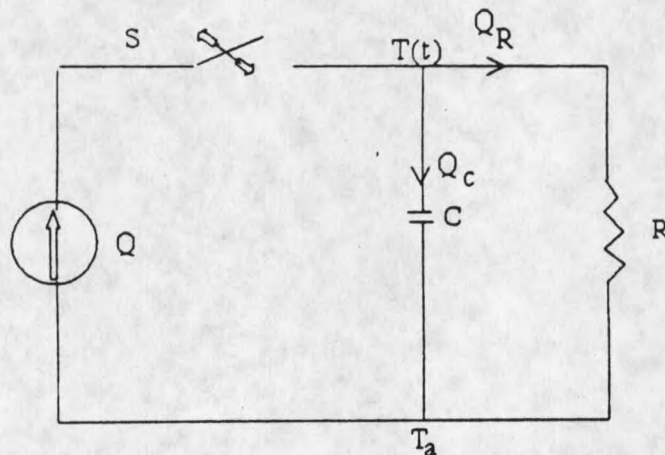


Fig. 2 Analogous electric circuit used for thermodynamic behavior of a house.

In the model

T is the house temperature ($^{\circ}\text{F}$)

T_a is the ambient temperature ($^{\circ}\text{F}$)

C is the equivalent thermal mass of the house ($\text{kwh}/^{\circ}\text{F}$)

R is the equivalent thermal resistance of

the house ($^{\circ}\text{F}/\text{kw}$)

G is the equivalent thermal conductance of the house ($\text{kw}/^{\circ}\text{F}$)

Q is the total heat injected into the house (kw)

Q_c is the heat flowing into the thermal mass C of the house (kw)

Q_R is the heat loss to the outside environment (kw)

$$Q = Q_c + Q_R$$

The thermal quantities used in this model are analogous to electrical quantities as follows.

<u>Thermal Quantity</u>	<u>Electrical Quantity</u>
Temperature	Voltage
Heat Flow	Current
Thermal Conductance	Electrical Conductance
Thermal Mass	Electrical Capacitance
Time	Time

According to N.W.P.P.C. [8] the following are typical values of the house parameters in the northwest United States.

$$\text{Thermal Mass } C = 3223 \text{ wh}/^{\circ}\text{F}$$

$$\text{Thermal Conductance } G = 180 \text{ }^{\circ}\text{F}/\text{w}$$

A value of heat injected $Q = 20.5 \text{ kw}$ was found likely in this region.

This thesis uses the above defined value for all plots unless otherwise stated.

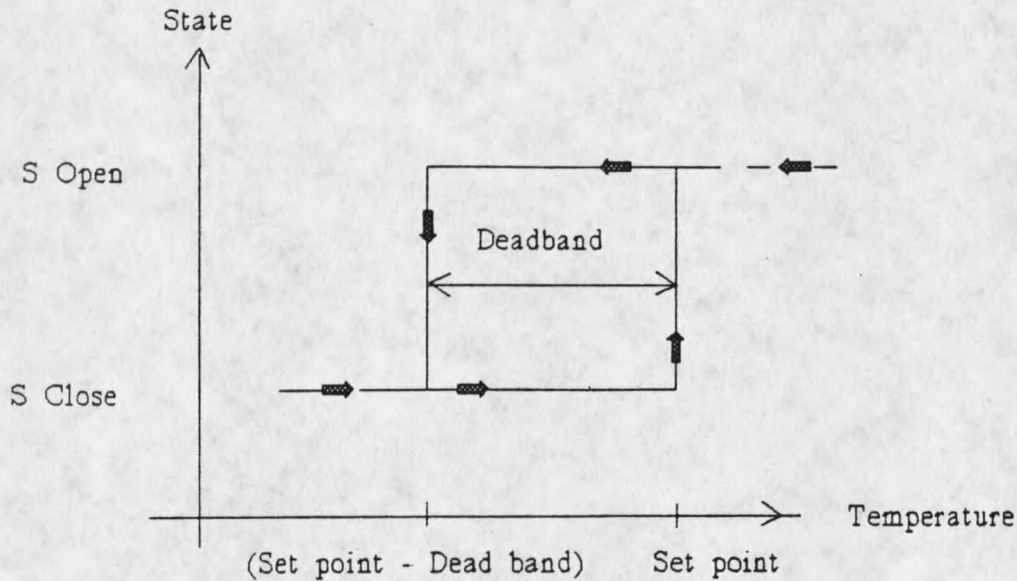


Fig. 3 Hysteresis loop of a thermostat.

Fig. 3 shows the hysteresis characteristics of the thermostat to which the switch S is connected. The upper setting of the thermostat is the setpoint and the lower setting is the setpoint minus the dead band. The switch S closes when the house temperature drops below the lower thermostat setting and opens when the house temperature reaches the upper setting.

It is assumed in this thesis that during power outage people do not add heat to their houses by any other source

like wood burning or any gas heating appliance. Also the effect of solar heat is not considered. The following range of values of the thermostat were found most likely.

Set point = 67-70 °F

Dead band = 2 °F

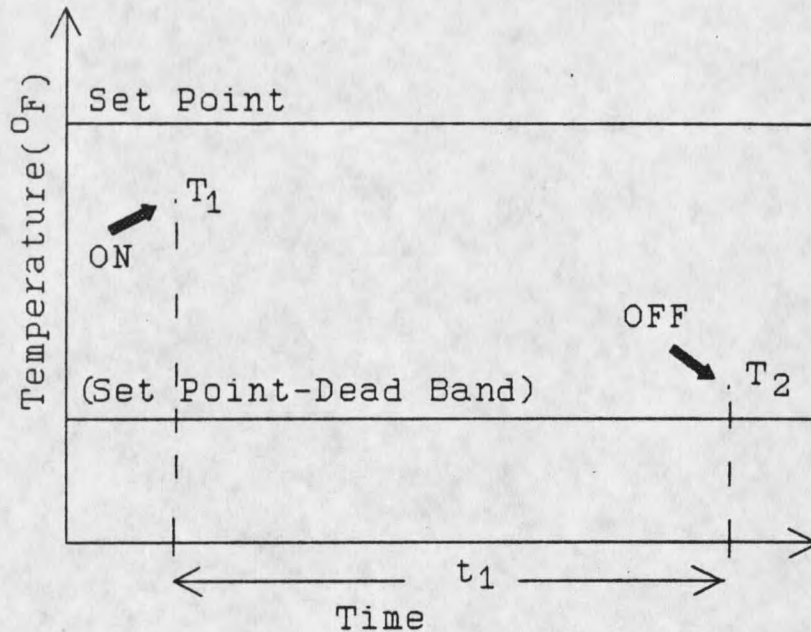


Fig. 4 House ON/OFF state before and after a power outage.

In Fig. 4 house A has a room temperature of T_1 and it is in the on state. There is a power outage for time t_1 after which power is restored. During this time t_1 the house has cooled to a temperature T_2 . Since the house temperature is

above the lower setting of the thermostat it will be in the cooling state. The house will be switched on once its temperature drops below the lower setting of the thermostat. It is for this reason that for short outages power demand at the instant of power restoration is less than the power demand a short time after power restoration.

From the equivalent circuit of a house given in Fig. 2 the temperature T of a house at any time t is given by the following equation. See Appendix A for details.

$$T(t) = T_a + (T_{ho} - T_a) e^{-t/RC} + wQR (1 - e^{-t/RC}) \quad (M-1)$$

where

T_{ho} is the initial house temperature

T_a is the ambient temperature

$w = 1$ when S is closed or

$w = 0$ when S is open

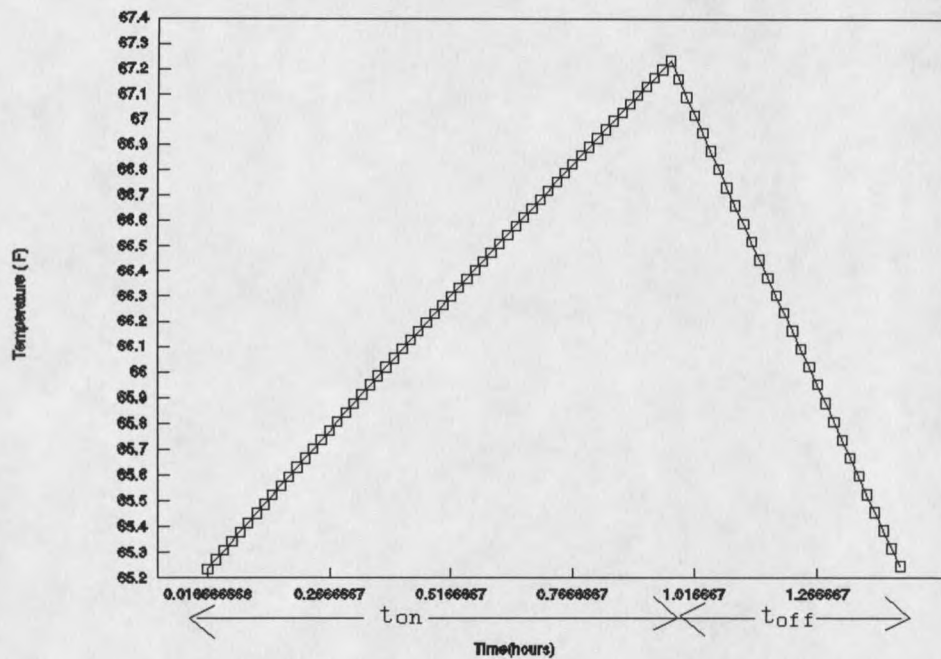


Fig. 5 Variation of house temperature under normal conditions at ambient temperature of -10°F .

Fig. 5 shows the variation of house temperature during one cycle of thermostat operation under normal conditions. It has been obtained using equation M-1. The time interval during which the house temperature rises from the lower thermostat setting to the upper thermostat setting is the on-time of the house. It is denoted by T_{on} . The time interval during which the house temperature falls from the upper setting to the lower setting is called the off-time of the house. It is denoted by T_{off} . It is shown in Appendix A that:

$$T_{on} = -RC \ln \frac{\text{setpoint} - T_a - QR}{\text{setpoint} - \text{deadband} - T_a - QR} \quad (\text{M-2})$$

$$T_{off} = -RC \ln \frac{\text{setpoint} - \text{deadband} - T_a}{\text{setpoint} - T_a} \quad (\text{M-3})$$

The on-time and off-time at ambient temperature of -10°F calculated from the above equations are .95 hours and .47 hours. These values match with those found from Fig. 5.

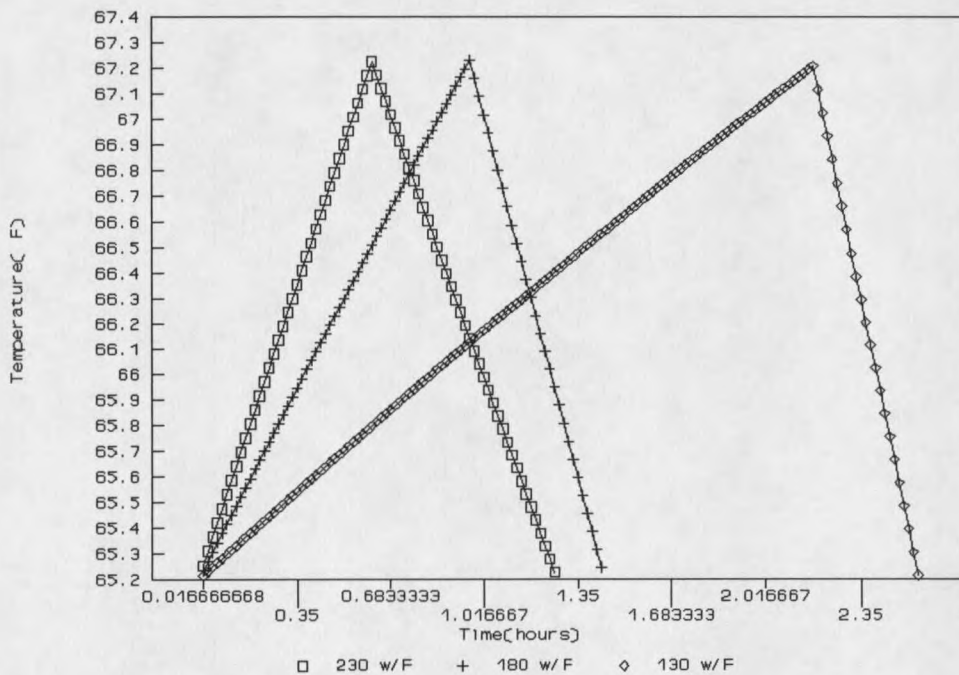


Fig. 6 Effect of thermal conductance on the temperature variation of the house.

Fig. 6 shows the effect of thermal conductance at ambient temperature -10°F . As Thermal Conductance increases, Q_R

increases. Since Q is constant and is the sum of Q_R and Q_C , an increase in Q_R would mean a decrease in Q_C . Q_C is the current responsible for heating the thermal mass of the house. Due to decrease in Q_C the time taken to heat the house increases meaning that T_{on} increases. An increase in Q_R would mean more heat is lost to the environment, meaning that the house will cool down faster.

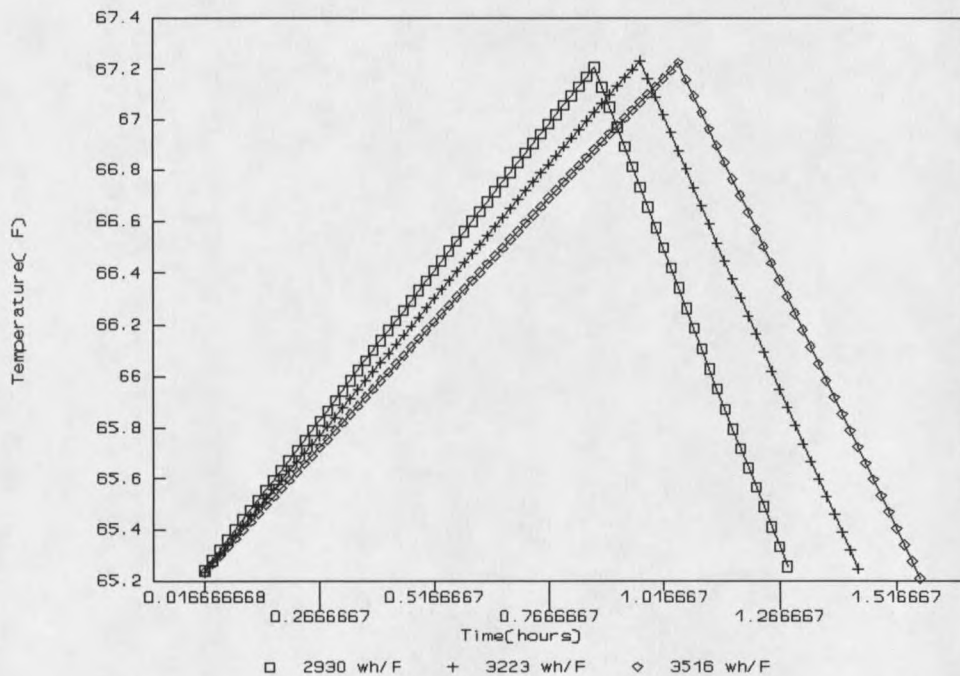


Fig. 7 Effect of change in thermal mass on the temperature variation of the house.

Fig. 7 shows the effect of Thermal Mass at ambient temperature of -10°F . Equations (M-2) and (M-3) show that on-

time and off-time are directly proportional to the thermal mass of the house. An increase in thermal mass would mean an increase in both on-time and off-time.

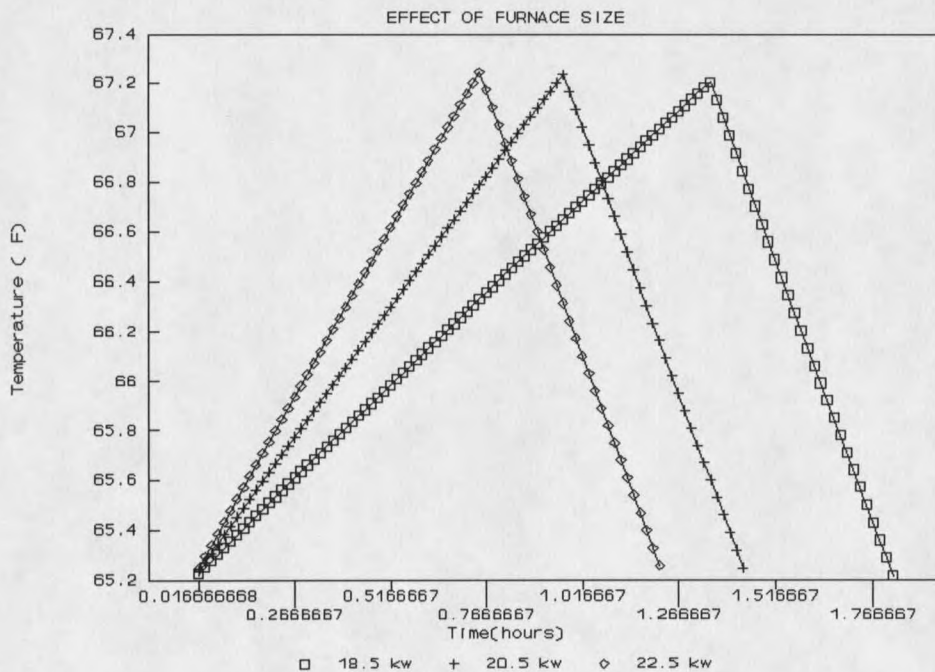


Fig. 8 Effect of change in furnace size on the temperature variation of the house.

Fig. 8 shows the effect of furnace size at ambient temperature of -10°F . An increase in furnace size would mean more heat is being supplied to the house and so the thermostat on-time will decrease. The off-time remains the same

irrespective of the furnace size as during this time the furnace is off. Note that equation (M-3) which shows the house thermostat off-time does not contain the term Q .

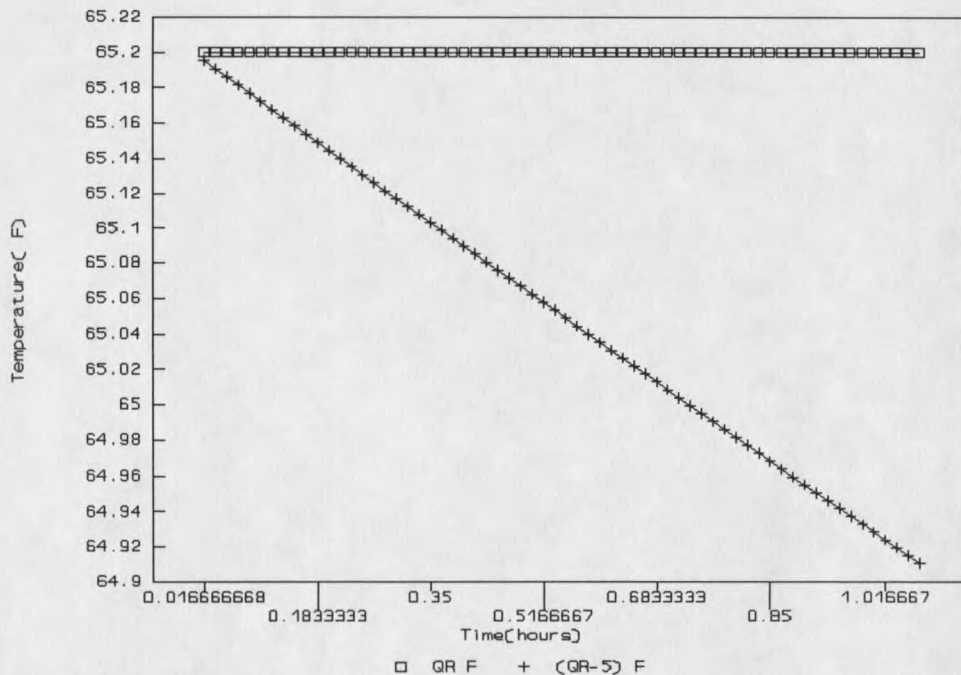


Fig. 9 Variation of house temperature under very cold ambient temperatures.

According to Mortensen and Haggerty [6] when a house furnace is unable to heat the house, the difference in house temperature and ambient temperature is called the temperature gain of the heater. It is denoted by T_g . This is because all the heat input is lost to the environment and does not add to the house temperature. In other words Q_c is zero. Therefore,

$$Q = Q_R + Q_C = Q_R$$

From our model temperature T of the house is given by

$$T = Q_R R + T_a$$

$$= QR + T_a$$

$$T_g = QR = T - T_a$$

$$T_a = T - QR \quad (M-4)$$

Substituting the above value of ambient temperature T_a in (M-1)

$$\begin{aligned} T(t) &= T - QR + (T_{ho} - (T - QR)) e^{-t/RC} + wQR (1 - e^{-t/RC}) \\ &= T_{ho} \end{aligned} \quad (M-5)$$

Equation (M-5) shows that the house temperature T at any time t remains equal to the initial temperature when ambient temperature is given by (M-4). Therefore line with square symbols in Fig. 9 shows the house temperature does not rise even though the furnace is on. The house temperature falls even though the furnace is on when the difference in the house temperature and ambient temperature is more than T_g . This is shown by the line with plus symbols in Fig. 9.

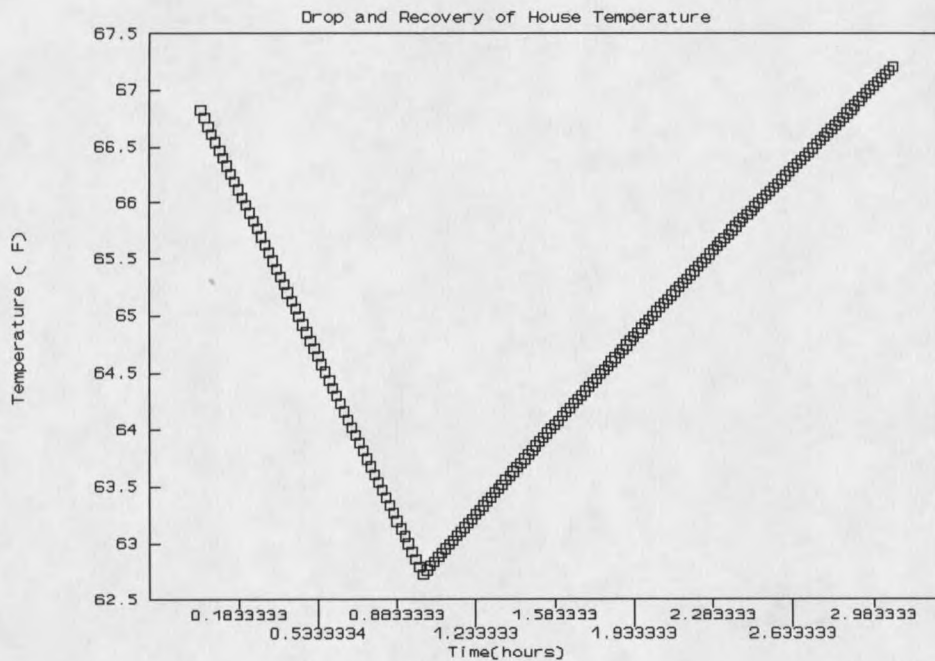


Fig. 10 Drop and recovery of house temperature at ambient temperature of -10°F .

In Fig. 10 the house temperature falls due to a power outage of one hour at ambient temperature of -10°F . The initial temperature of the house was under steady state conditions and so it has to be within the thermostat settings. It was generated using a random number within the thermostat setting. After the power is restored the house temperature

recovers. Note that the time for which the house temperature recovers is the T_{on} of the house under conditions of outage. Also the time of the temperature drop is the T_{off} of the house under conditions of outage.

CHAPTER 4**AGGREGATION**

This chapter deals with the determination of the total heating current supplied by a feeder. The current requirement of a house is dependent on the following.

- 1) Ambient Temperature (T_a)
- 2) Thermal Conductance (G)
- 3) Thermal Mass (C)
- 4) Capacity of the furnace (Q)
- 5) Thermostat set point (setpoint)
- 6) Thermostat dead band (deadband)

No two houses are the same. Thermal mass, thermal conductance and capacity of the furnace of each house depends on its construction and size. Since the thermostat is manually operated different people adjust it to a different setting depending on their lifestyle.

Due to the above reasons a different value has to be provided for the above parameters for each house. The simulation program adds noise to the specified mean of each parameter. This noise is a random number with a gaussian distribution, mean of zero, and a specified standard deviation. The on-time and off-time of each house depend on these parameters and so a different on-time and off-time is assigned to each house. In the simulation program these on-times and off-times are used to make the estimation of the current requirement.

Current demand of a feeder I_T is given by:

$$I_T = \sum I_i$$

where I_i is the current demand of each house on the feeder. The current I_i of each house could be either zero or have a certain value. According to Ihara and Schweppe [3] the probability that the current is not zero is given by

$$D = \frac{T_{on}}{T_{on} + T_{off}}$$

where D is called the duty cycle

Depending on the values of the parameters, D can take a value anywhere between 1 and 0.

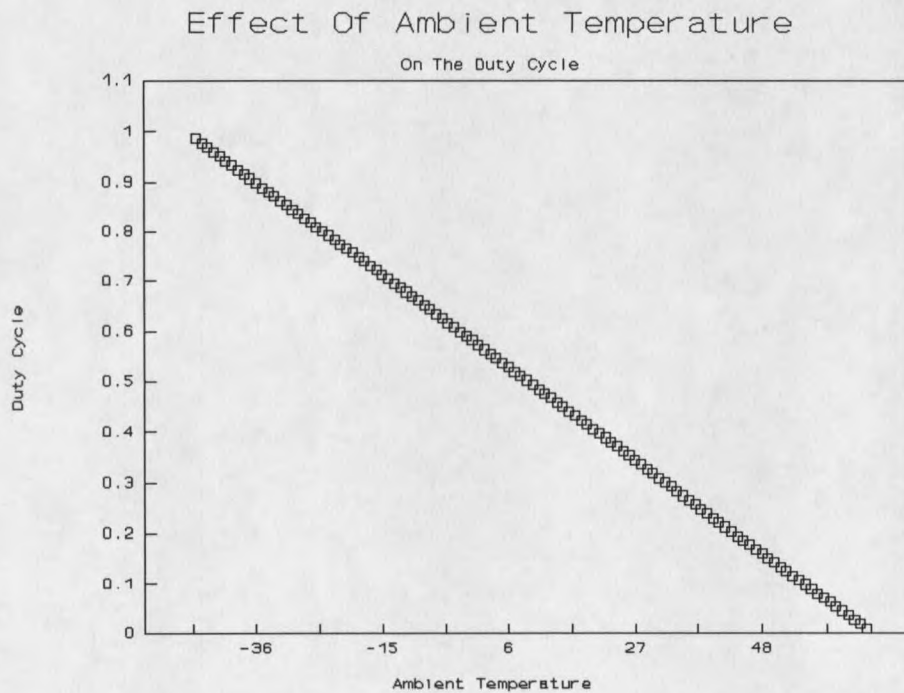


Fig. 11 Effect of ambient temperature on the duty cycle.

Duty cycle takes a value of 0 at ambient temperature of 65.2 °F. Duty cycle becomes 0 when the lower setting of the thermostat is less than the ambient temperature. Duty cycle becomes 1 when the house temperature is equal to or greater than $T_g + T_a$. Where $T_g = QR$ and is called the temperature gain of the heater. See chapter 3 for a complete description.

Algorithm for Sectionalizing

Sectionalizing means to restore power to different sections of a feeder at different times in order to minimize the increased restoral current. Fig. 12 shows power source S supplying power to a feeder. The feeder has two breakers, B1 and B2. Closure of B1 would mean that power is restored to upstream feeder which is the part of the feeder between B1 and B2. Closure of B2 after B1 has been closed would mean power is restored to the entire feeder.

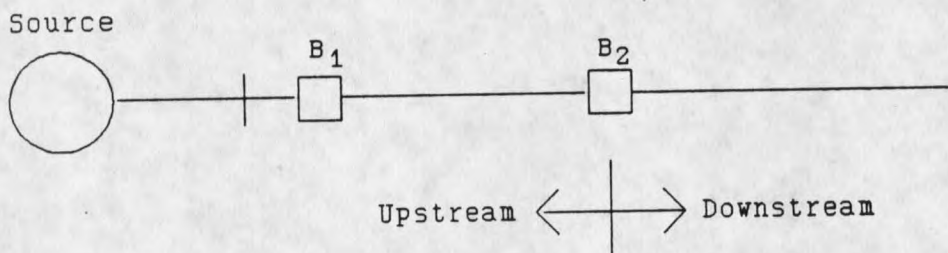


Fig. 12 Sectionalized feeder.

In Fig. 12 for sectionalizing, after a power outage,

power is restored on upstream portion of the feeder. The time of peak current requirement is allowed to pass after which power is restored to the downstream feeder. In order to decide the time of restoration of the downstream feeder it is required to know the percentage of electric space heating load on it. Note that the time of outage for the downstream feeder would be the time of normal outage plus extra time for which power was restored on the upstream feeder but not on the downstream feeder.

The simulation program has the provision of automatic sectionalizing as well as manual sectionalizing. In automatic sectionalizing the computer program makes simulations as if the downstream feeder was connected when the current on the upstream feeder is at its minimum. Fig. 13 shows that 64 minutes after power is restored the current is at its minimum so this is the time when the downstream feeder is connected. In manual sectionalizing the time of the closure of breaker B2 is decided by the operator.

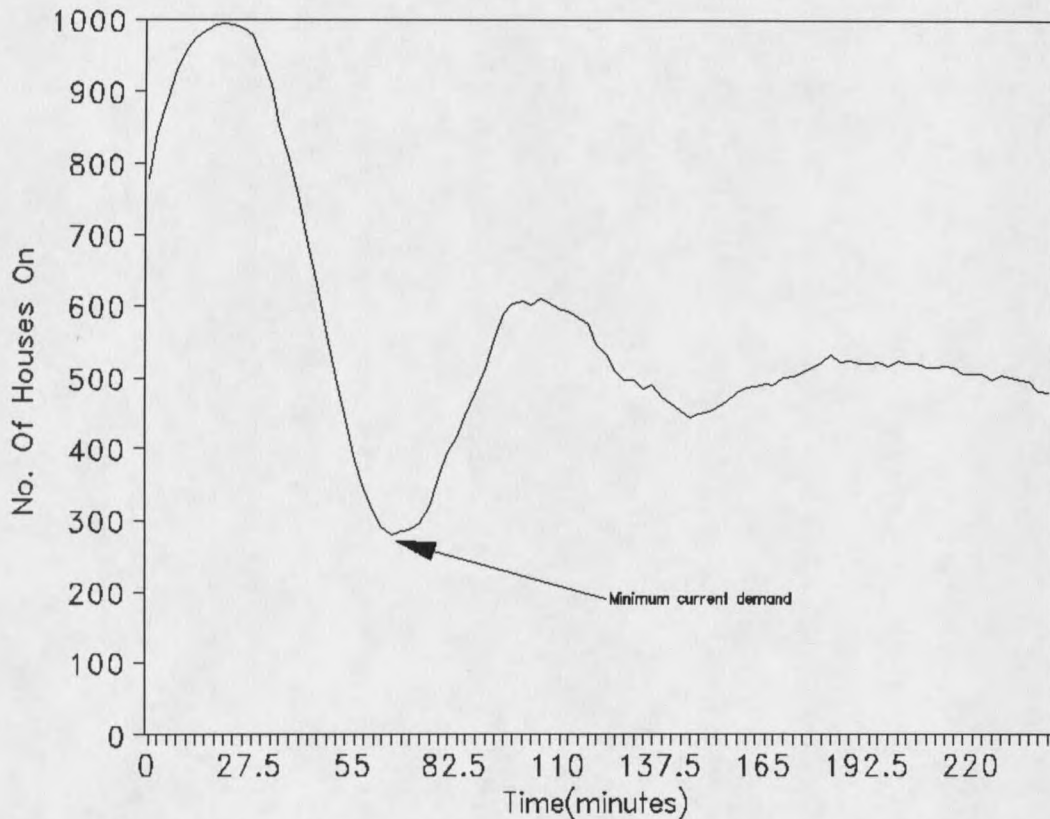


Fig. 13 Shows the number of houses demanding power at ambient temperature of 10°F and after a power outage of 30 minutes.

Algorithm for Change in Voltage

This thesis discusses the effect of change in voltage on the electrical power being supplied to a feeder under steady state conditions and when power is restored after an outage.

Q the current source of the R-C circuit is measured in kw and is the term for the electrical power supplied to the furnace. The power P is given by

$$P = \frac{V^2}{R}$$

If there is a change in voltage the new power will be given

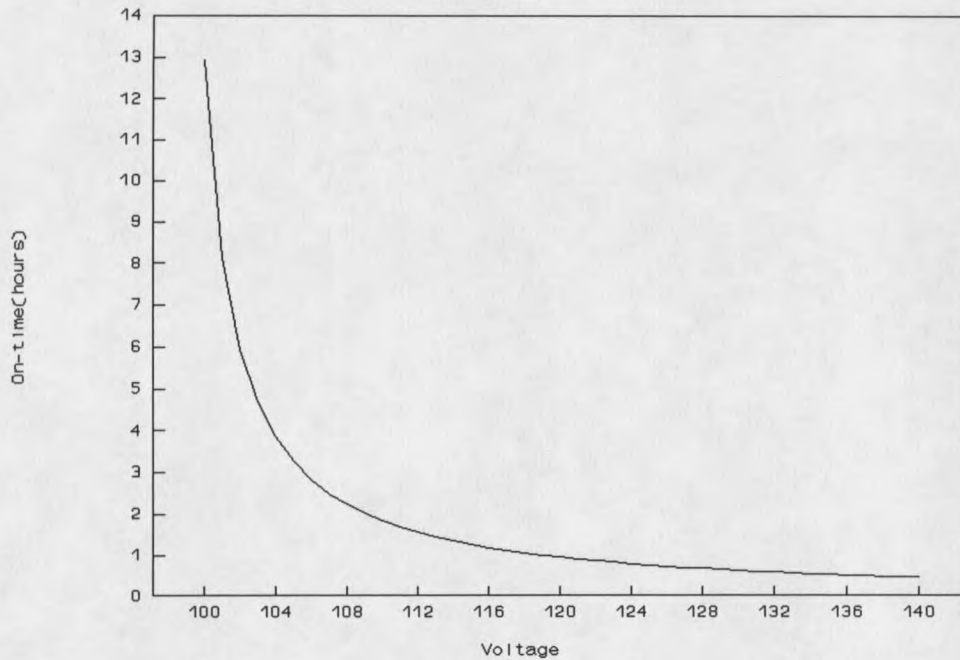


Fig. 14 Effect of voltage on the on-time of the furnace at ambient temperature of -10°F .

by

$$P_{new} = \frac{V_{new}^2 * P_{old}}{V_{old}^2}$$

From Appendix A, the on-time and off-time are given by

$$T_{on} = -RC \ln \frac{(\text{setpoint} - T_a - QR)}{(\text{setpoint} - \text{deadband} - T_a - QR)}$$

$$T_{off} = -RC \ln \frac{(\text{setpoint} - \text{deadband} - T_a)}{(\text{setpoint} - T_a)}$$

Inspection of the above equations show that on-time is

dependent on the power where as off-time is not. Fig. 14 shows that an increase in voltage decreases the house on-time and vice versa. This means that a change of voltage affects the rate at which the houses get heated.

If power is restored after an outage, an increase in voltage will have the effect of increase in the demand for power on the feeder. Under steady state conditions houses require a fixed amount of electrical energy to compensate for the heat lost to the environment. This means that the power demand does not change with a change in voltage under steady state conditions. However, the current demand decreases with increase in voltage since the power remains constant.

Algorithm for Change in Temperature

There is a possibility that the ambient temperature might change during the outage or after the outage. Inspection of equations for on-time and off-time show that both are dependent on the ambient temperature. Fig. 15 shows that with a decrease in ambient temperature on-time increases whereas off-time decreases.

The simulation program has the provision of changing the temperature twice. This is explained with the help of Fig. 16 as follows:

- 1) At time=0 ambient temperature is T_1 and there is a power outage.
- 2) At time t_1 ambient temperature changes to T_2 .
- 3) At time t_2 power is restored.

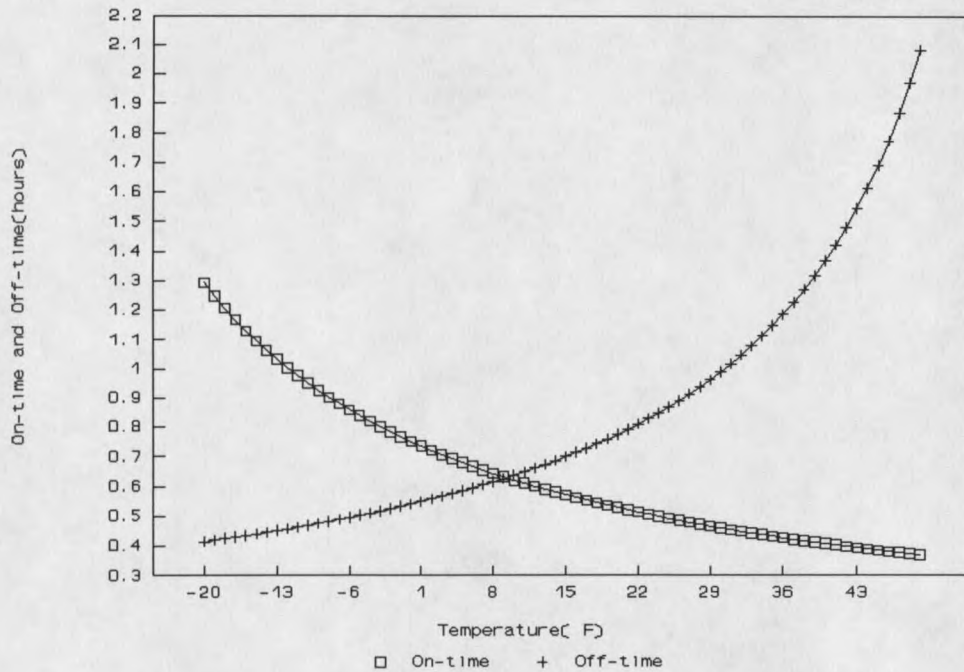


Fig. 15 Effect of ambient temperature on the on-time and the off-time of the furnace.

4) At time t_3 ambient temperature changes to T_3 .

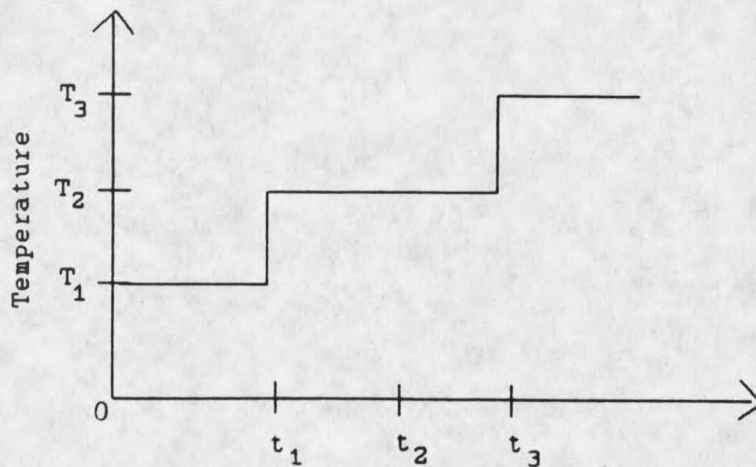


Fig. 16 Change in ambient temperature during the simulation period.

The value of the three ambient temperatures T_1, T_2, T_3 and the time t_1, t_2, t_3 is entered by the operator. Each time the ambient temperature is changed, the old value of the ambient temperature is replaced by the new value in the equation for on-time and off-time. All further simulations are made with the new values of on-time and off-time.

CHAPTER 5

RESULTS

In this thesis all simulations were done for 1000 houses. Random numbers with a specific mean and standard deviation were generated for the following five parameters of the house equivalent electrical model through which the current demand of each house can be determined.

- 1) Thermal Conductance (G)
- 2) Thermal Mass (C)
- 3) Heat Injected to the house (Q)
- 4) Thermostat Set Point
- 5) Thermostat Dead Band

Each house on the feeder is randomly assigned a value for the above parameters. The values assigned have a gaussian distribution with specific mean and standard deviation. Listed below are the mean values as well as the standard deviation of the parameters.

Mean G=180.0 w/°F	G Standard Deviation=10%
Mean C=3223.0 wh/°F	C Standard Deviation=16%
Mean Q=20.5 kw	Q Standard Deviation=12%
Mean Set point=67.2°F	Set point Standard Deviation=2%
Mean Dead band=2°F	Dead Band Standard Deviation=3%

The mean values of G and C were estimated based on N.W.P.P.C. report [8]. The mean value of Q, set point and deadband was found most likely. It is assumed that the thermodynamic parameters are statistically independent meaning that the

generated values of one parameter are not dependent on values of other parameters.

It is assumed that the random numbers generated are statistically independent meaning that they are not biased. If this is true then the number of houses demanding power are:

$$\frac{T_{on}}{T_{on} + T_{off}} * \text{No. of houses on the feeder}$$

Fig. 17 shows the number of houses demanding power under steady state. The line for Lang uses the value of G and C obtained from Lang and Anderson [4], and the line for N.W.P.P uses the value of G and C obtained from N.W.P.P.C. [8]. The values obtained from Lang and Anderson [4] are $C=1783.8 \text{ wh/}^\circ\text{F}$ and $G=176.6 \text{ w/}^\circ\text{F}$.

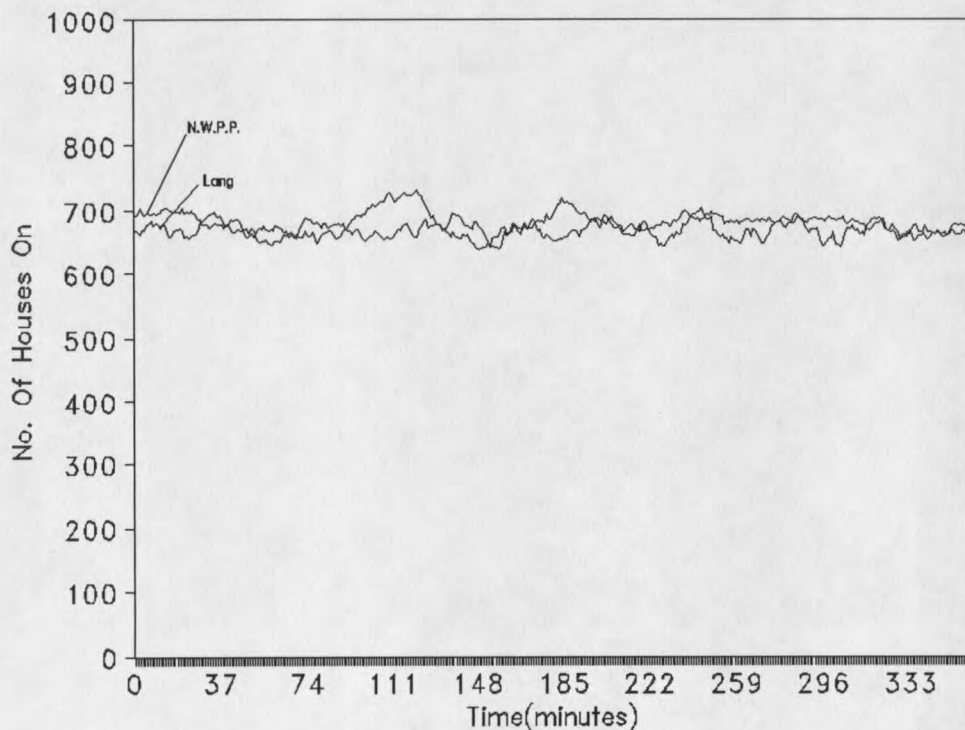


Fig. 17 Steady state response at ambient temperature of -10°F .

The average number of houses demanding power in Fig. 17 are;

669.8 for Lang and Anderson [4]

681.7 for N.W.P.P.C.[8]

The average number of houses demanding power calculated by substituting the values of the parameters in the equation for on-time and off-time are:

For Lang and Anderson [4]:

The on-time and off-time is calculated using equation (M-2) and (M-3)

$$T_{on} = .51 \text{ Hr}$$

$$T_{off} = .26 \text{ Hr}$$

Average number of houses demanding power

$$= \frac{T_{on}}{T_{on} + T_{off}} * 1000 = 662.33$$

$$\text{Error} = 1.1\%$$

For N.W.P.P.C. [8]:

The on-time and off-time is calculated using equation (M-2) and (M-3)

$$T_{on} = .95 \text{ Hours}$$

$$T_{off} = .47 \text{ Hours}$$

Average no. of houses demanding power

$$= \frac{T_{on}}{T_{on} + T_{off}} * 1000 = 669.01$$

$$\text{Error} = 1.9\%$$

The above results prove that the random numbers generated are statistically balanced.

In Fig. 17 the steady state values are close in the two

cases where as house on-time and off-time is not. This is because the value of thermal conductance in the two cases is close where as thermal mass is not.

Effect of Change in Thermodynamic Parameters

In order to see the effect of change in parameters on the power demand, this thesis changes the mean value of each of the parameters and discusses the results. The standard deviation in percentage of mean value is kept constant.

Decreasing the value of thermal conductance G means that there is extra insulation. This conserves heat energy. Since

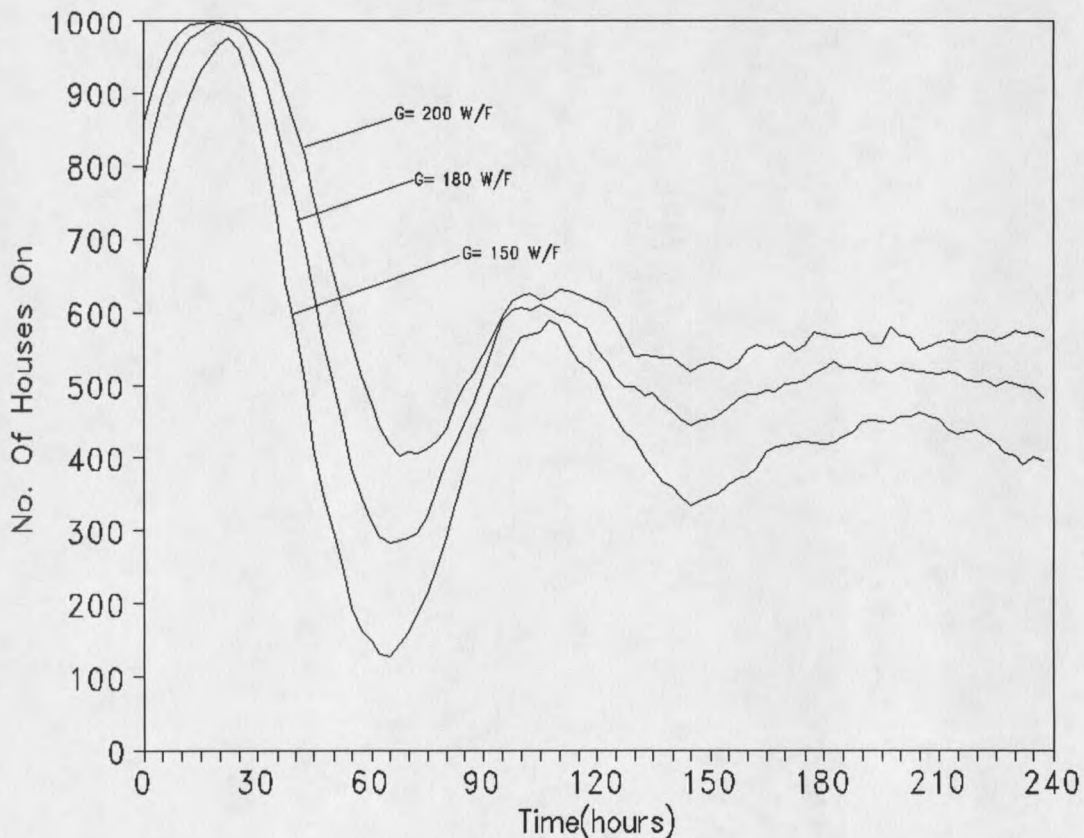


Fig. 18 Effect of change in conductance at ambient temperature of 10°F and after a power outage of 30 minutes.

the loss of heat energy to the environment is less, each house will stay on demanding power for a shorter period of time, therefore the number of houses demanding power would be less both when power is restored after an outage and under steady state conditions. Fig. 18 shows the simulated results for a 30 minutes outage at 10°F.

Electrical capacitance C of the R-C circuit is analogous to the thermal mass of the house. The size of the capacitance

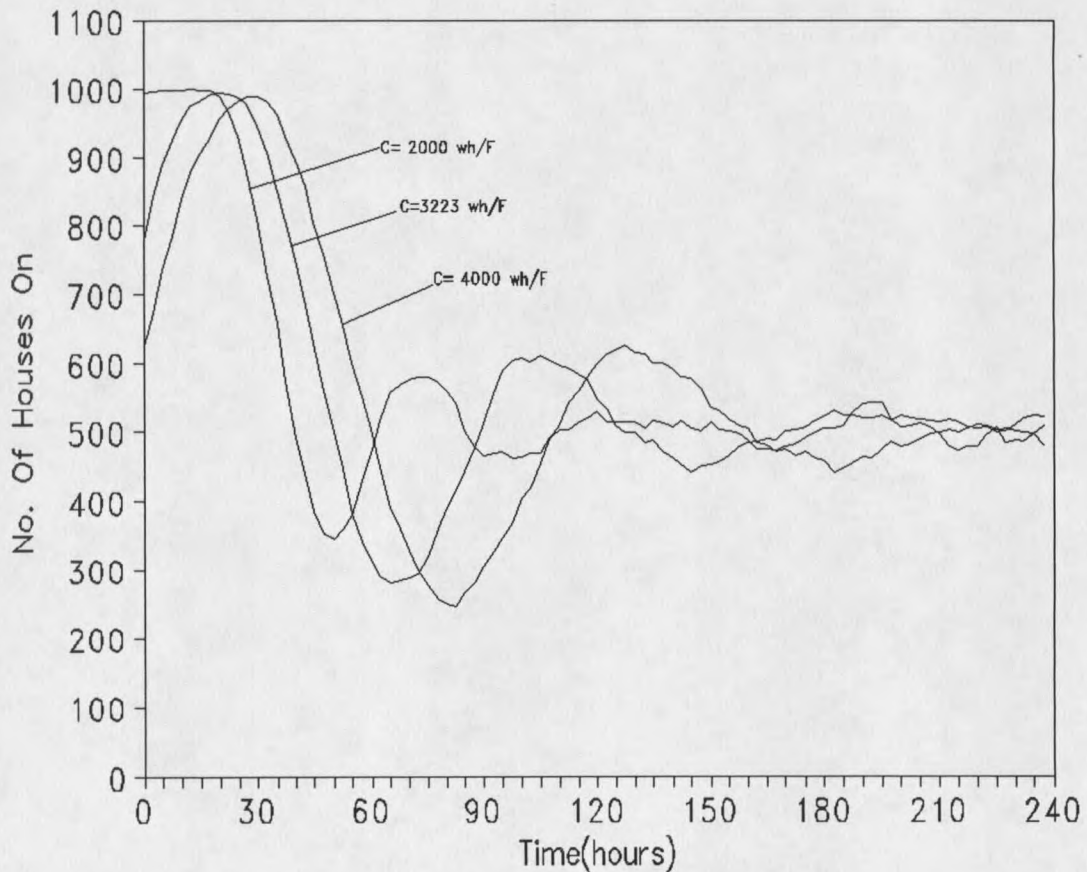


Fig. 19 Effect of change in capacitance at ambient temperature of 10°F and after a power outage of 30 minutes.

is a measure of the amount of electrical energy that can be stored. With a higher value of capacitance the houses will become slow in heating up and cooling down meaning that the on-time and off-time increase. Due to an increase in on-time of the house the peak current demand is delayed. This can be seen in Fig. 19.

With an increase in the value of the heat injected Q , houses get heated up faster. When power is restored after an outage, the power demand of the feeder increases because the

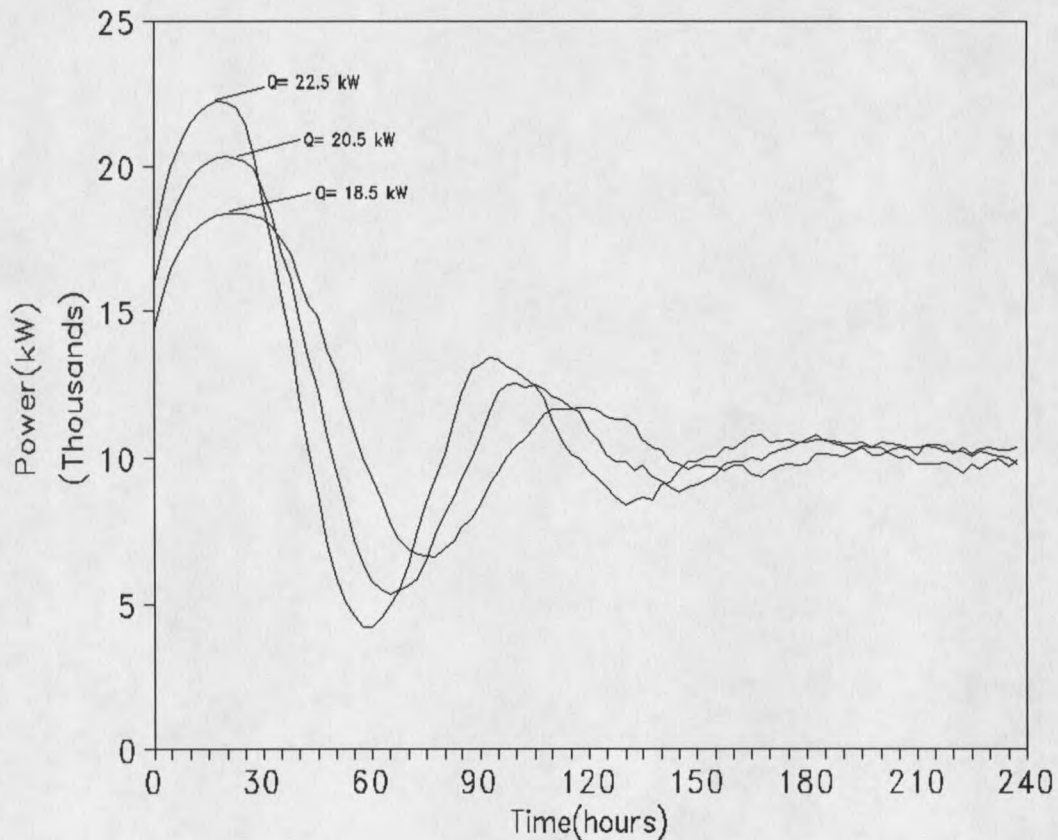


Fig. 20 Effect of change in heat injected into the house at ambient temperature of 10°F and after a power outage of 30 minutes.

furnace capacity of each house has increased. As seen in Fig 20, this produces a higher peak. Under steady state conditions the amount of electrical energy required to keep all the houses at a temperature within their thermostat setting is fixed and so a change in furnace has no effect on the aggregate steady state power demand.

Increase in thermostat set point has the effect of increase in the on-time and a decrease in off-time. This is better explained by an example at ambient temperature of -10°F .

For Set point = 67.2°F and using the mean values for other parameters

$$T_{\text{on}} = .95 \text{ Hr}$$

$$T_{\text{off}} = .47 \text{ Hr}$$

For Set point = 70.2°F

$$T_{\text{on}} = 1.03 \text{ Hr}$$

$$T_{\text{off}} = .45 \text{ Hr}$$

This means that with an increase in set point, energy is supplied for longer time and hence more houses will demand power as shown in Fig 21.

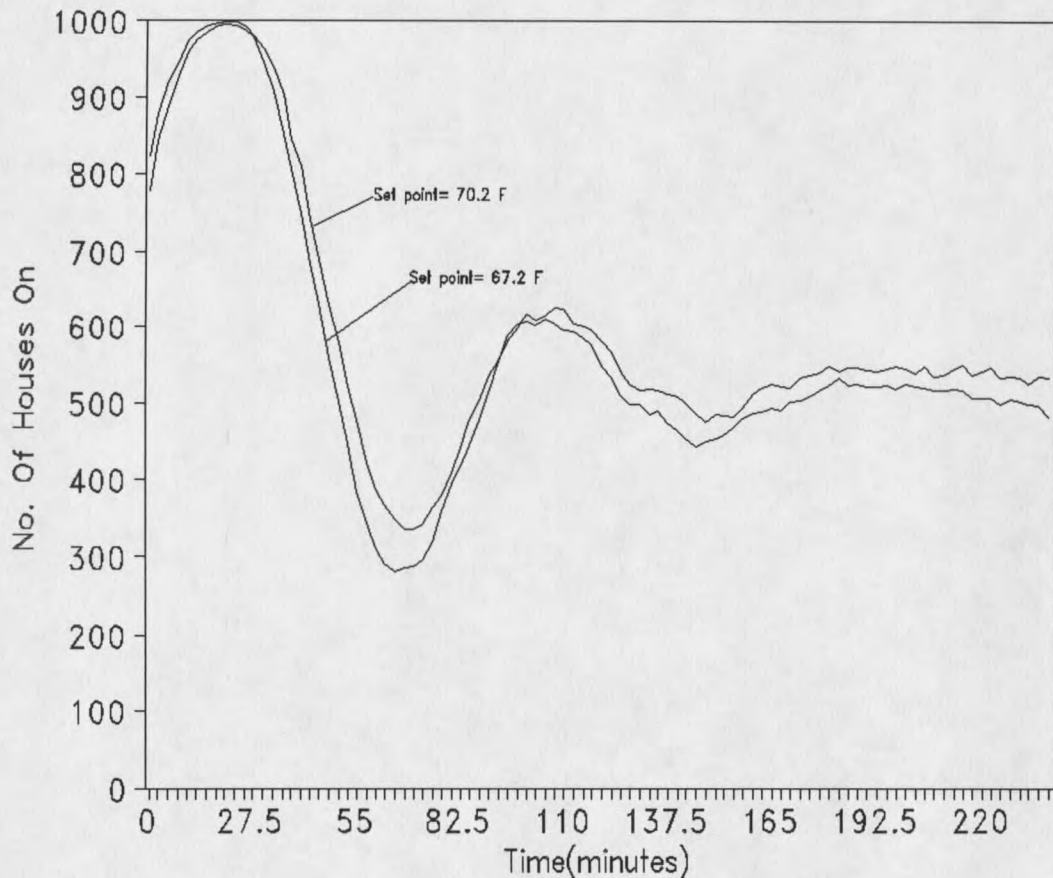


Fig. 21 Effect of change in set point at ambient temperature of 10°F and after a power outage of 30 minutes.

A smaller thermostat deadband would mean lower value of on-time and off-time. When power is restored more houses will be turned on because the lower setting of the thermostat has been raised. It is explained with an example.

Set point = 67.2°F

Deadband = 2.0°F

So, Upper setting = 67.2°F

and Lower setting = 65.2°F

Deadband is changed to 1.0°F

Then Upper setting = 67.2°F

Lower setting = 66.2°F

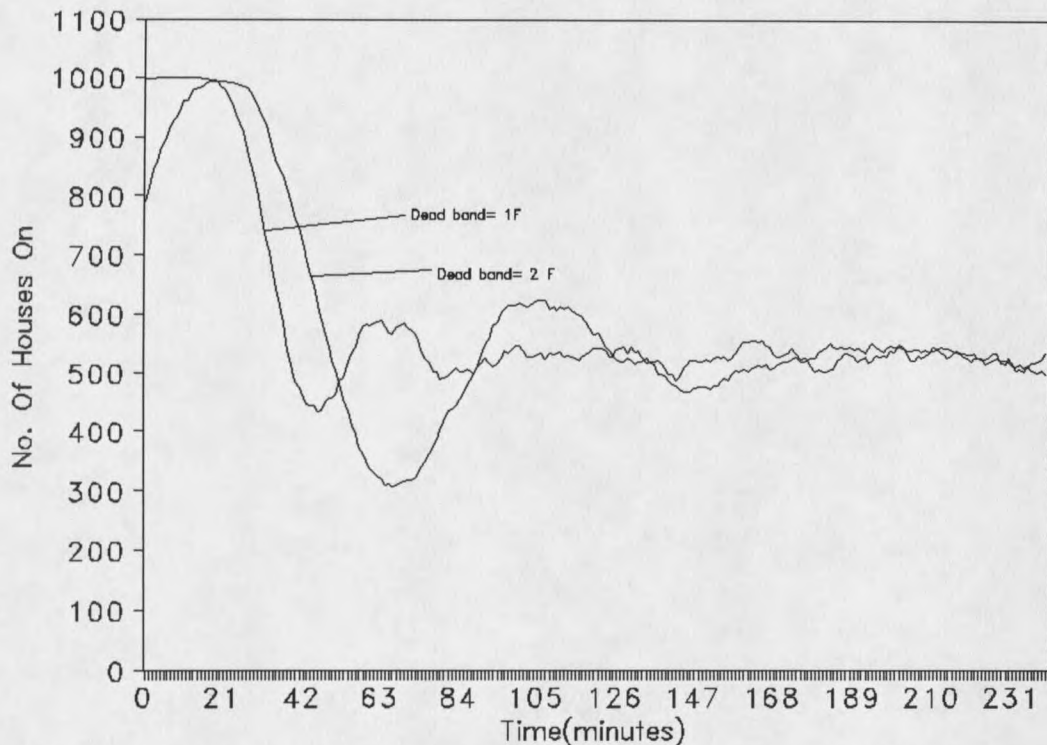


Fig. 22 Effect of change in dead band at ambient temperature of 10°F and after a power outage of 30 minutes.

When power is restored after an outage, there will be houses with temperature between 65.2°F and 66.2°F which will be the extra houses demanding power due to the change in thermostat deadband. Fig 22 shows more number of houses demanding power after an outage with a decrease in dead band.

Change in Voltage

Fig. 23 and Fig. 24 show the effect of voltage change on the power and the current supplied by the feeder. The electrical power is directly proportional to the square of the

voltage assuming that the resistance of the electric space heater is independent of the service voltage. When power is restored after an outage an increase in voltage would mean an

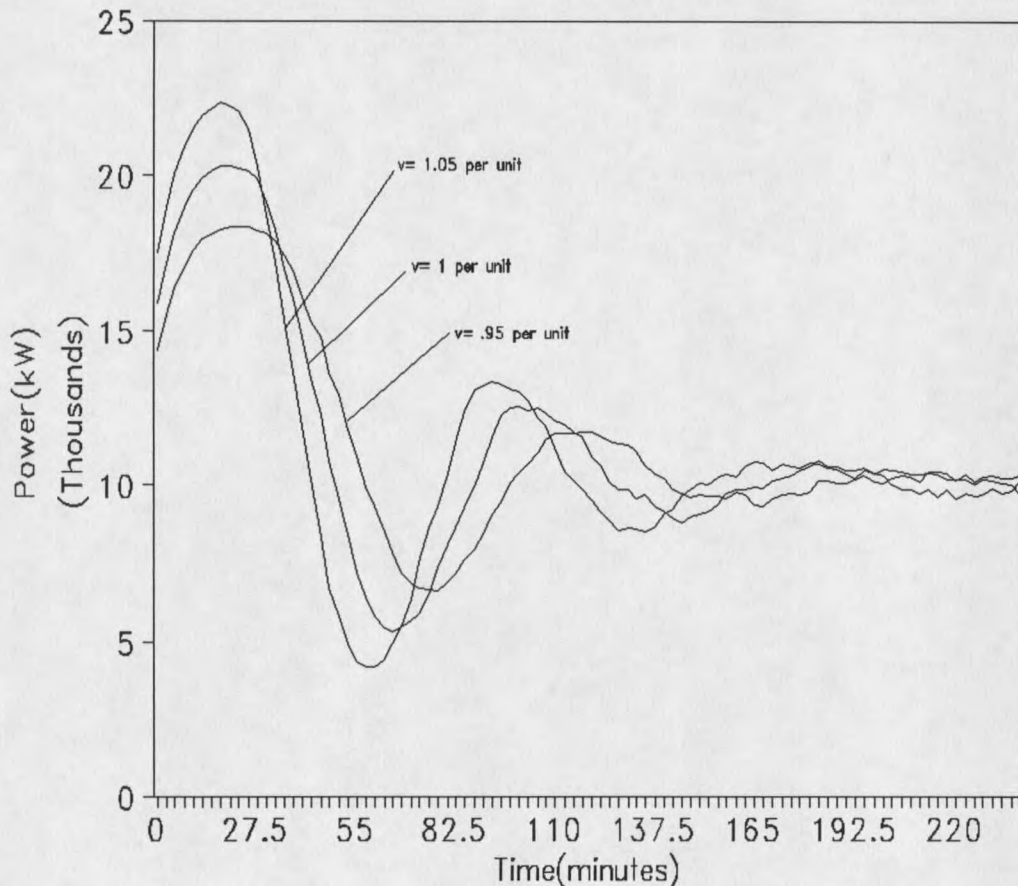


Fig. 23 Effect of change in voltage on the power supplied at ambient temperature of 10°F and after a power outage of 30 minutes.

increase in the power supplied. When steady state is reached houses will require the same quantity of electrical energy to replace the heat lost to the environment and hence the electrical power required will be a constant. In other words power required by a house over a period of time is fixed. If the instantaneous value of the power requirement of a house changes then the time during which the power is required also

changes so that the amount of power required over a period of time remains constant. It can be noticed in Fig. 23 that the power demand increases when power is restored, however, the total power demand under steady state conditions is independent of the service voltage.

In Fig. 24 after power is restored the current demand increases because the power demand has increased with an increase in voltage. Under steady state conditions the power demand becomes constant due to the reasons explained above.

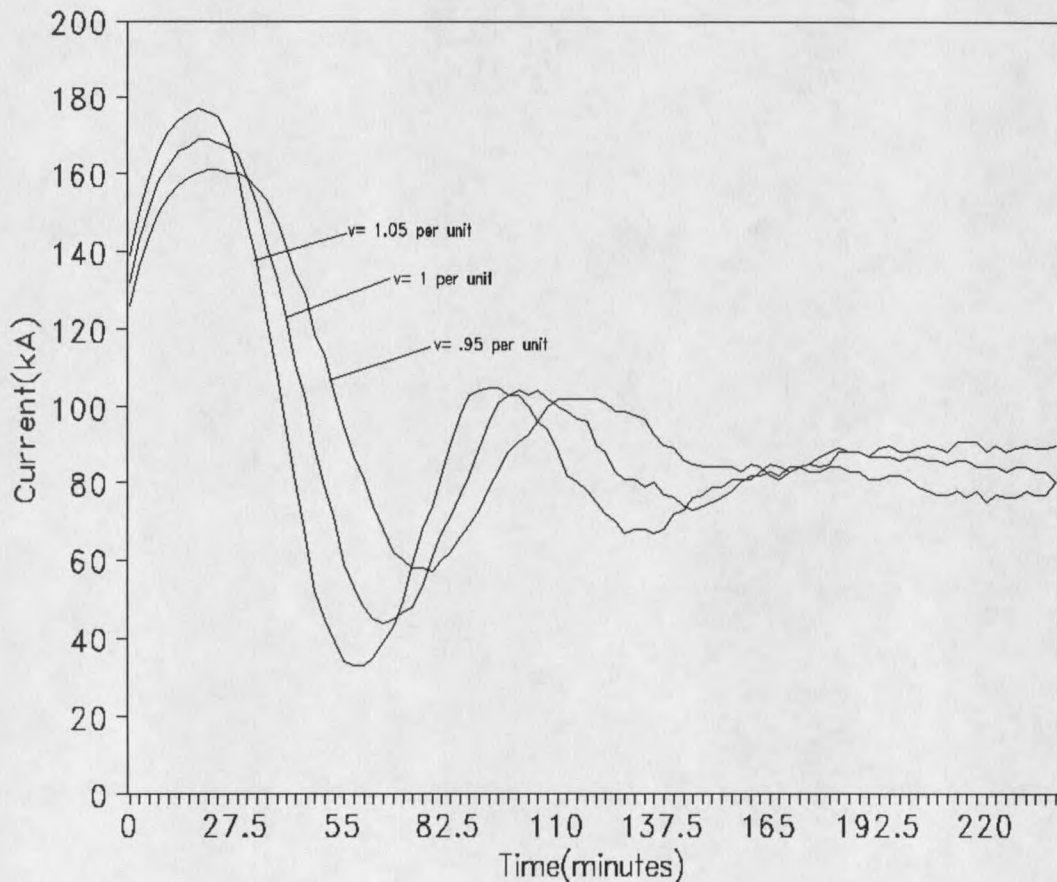


Fig. 24 Effect of change in voltage on the current supplied at ambient temperature of 10°F and after a power outage of 30 minutes.

$$\text{Power} = \text{Voltage} * \text{Current} \quad (\text{R-1})$$

In equation (R-1) an increase in voltage would mean a decrease in current since power is constant. This is the reason for the reduced current demand under steady state in Fig. 24.

Sectionalizing

When overcurrents at the time of power restoration are unavoidable sectionalizing may become necessary to restore service to the feeder. For Figures 25 and 26 the sectionalizer had 50% of the total load of the feeder. The ambient temperature was 10°F and power was interrupted for 30 minutes.

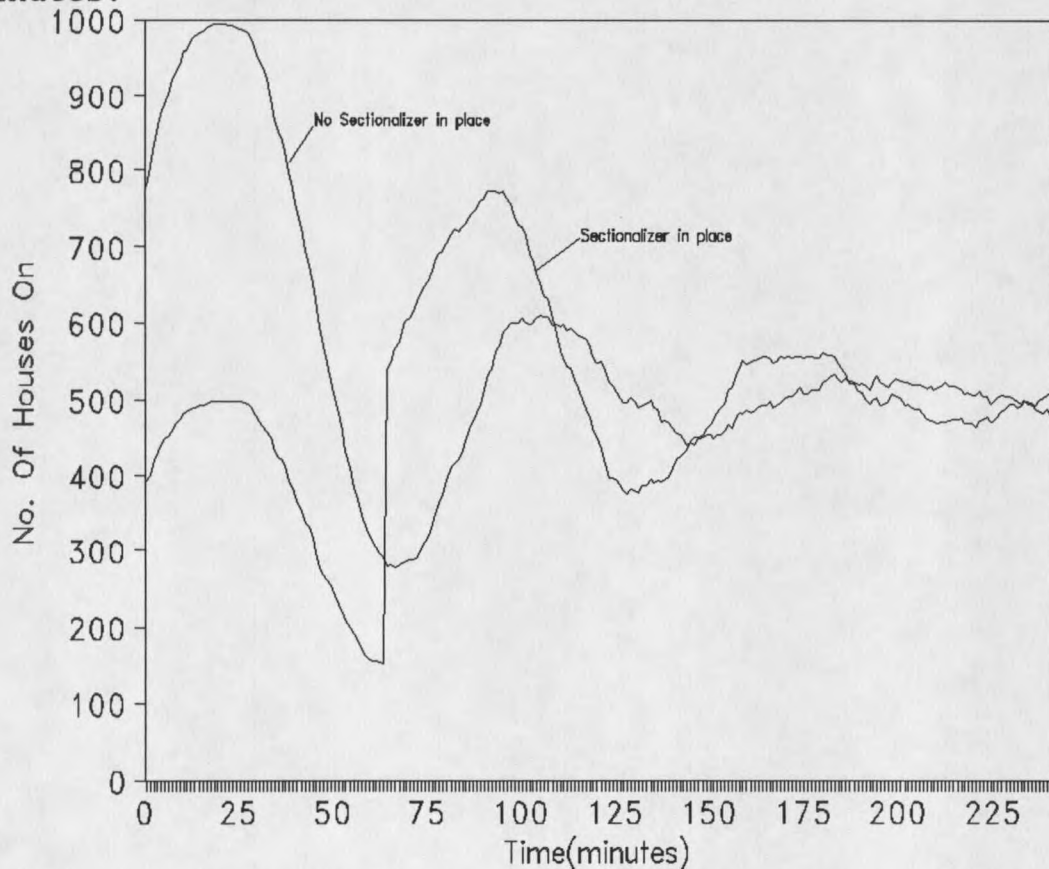


Fig. 25 Automatic sectionalizing. Time of closure for the sectionalizer was 64 minutes.

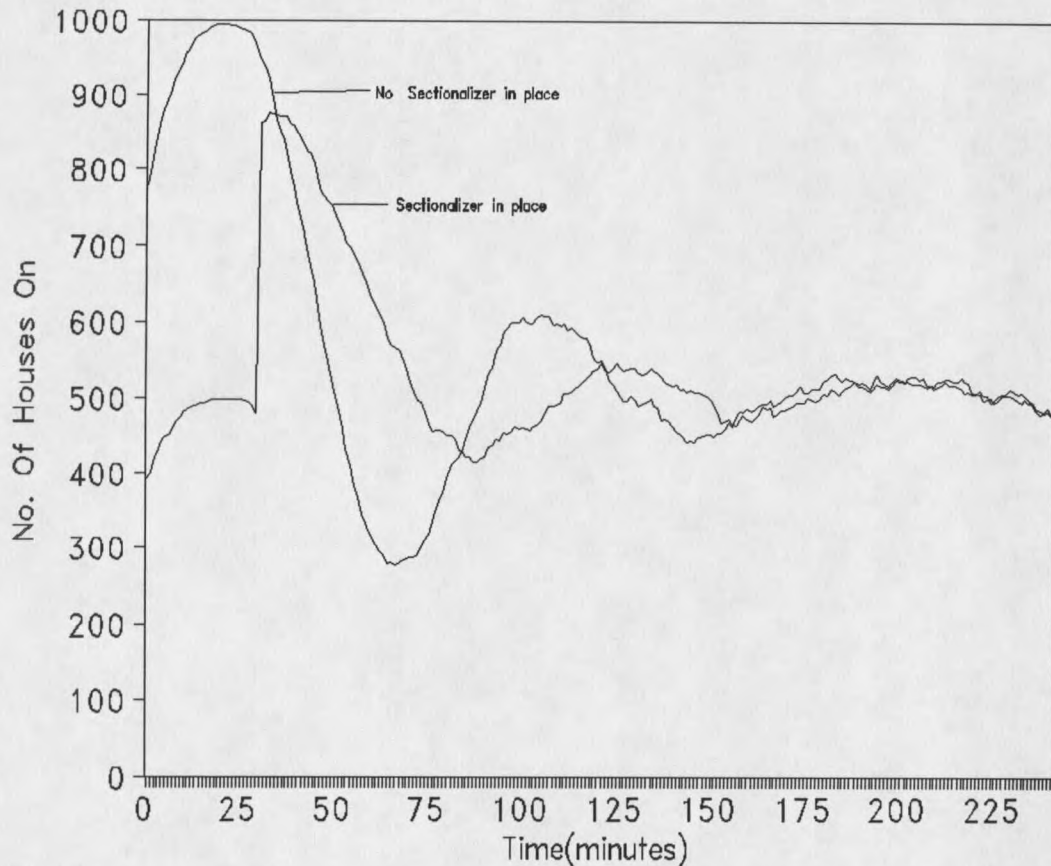


Fig. 26 Manual sectionalizing. Time of closure for the sectionalizer is 30 minutes.

Fig. 25 shows automatic sectionalizing in which the time of closure for the sectionalizer was calculated by the computer program. When power is restored after an outage, there comes a time when most of the houses have recovered and the current demand is at its minimum. Closing of the sectionalizer at this time allows houses on the downstream portion of the feeder to recover while houses upstream of the sectionalizer don't demand excess power. This time is calculated by the computer program and further simulations are made as if the downstream feeder was connected at this time.

Sometimes it might not be convenient for the utilities to wait for this time. Fig. 26 shows manual sectionalizing in which the time of closure has been entered by the operator.

Effect of Change in Ambient Temperature

The lower the ambient temperature the higher is the demand for power. With a decrease in ambient temperature the on-time increases where as the off-time decreases. This means that a house furnace will remain on for longer time. Fig. 27 shows the number of houses demanding power, under continuous power supply at different ambient temperatures.

It is likely that the ambient temperature might change during the period of simulation. In Fig. 28 no change in temperature line shows that the ambient temperature has remained at 10°F throughout the simulation period. The following scenario has taken place for the change of temperature line.

- 1) Ambient temperature is 10°F and electric service is interrupted.
- 2) Ambient temperature has changed to -10°F after 20 minutes.
- 3) Electric service is restored 60 minutes after it was interrupted.
- 4) 15 minutes after service was restored ambient temperature has changed to -15°F.

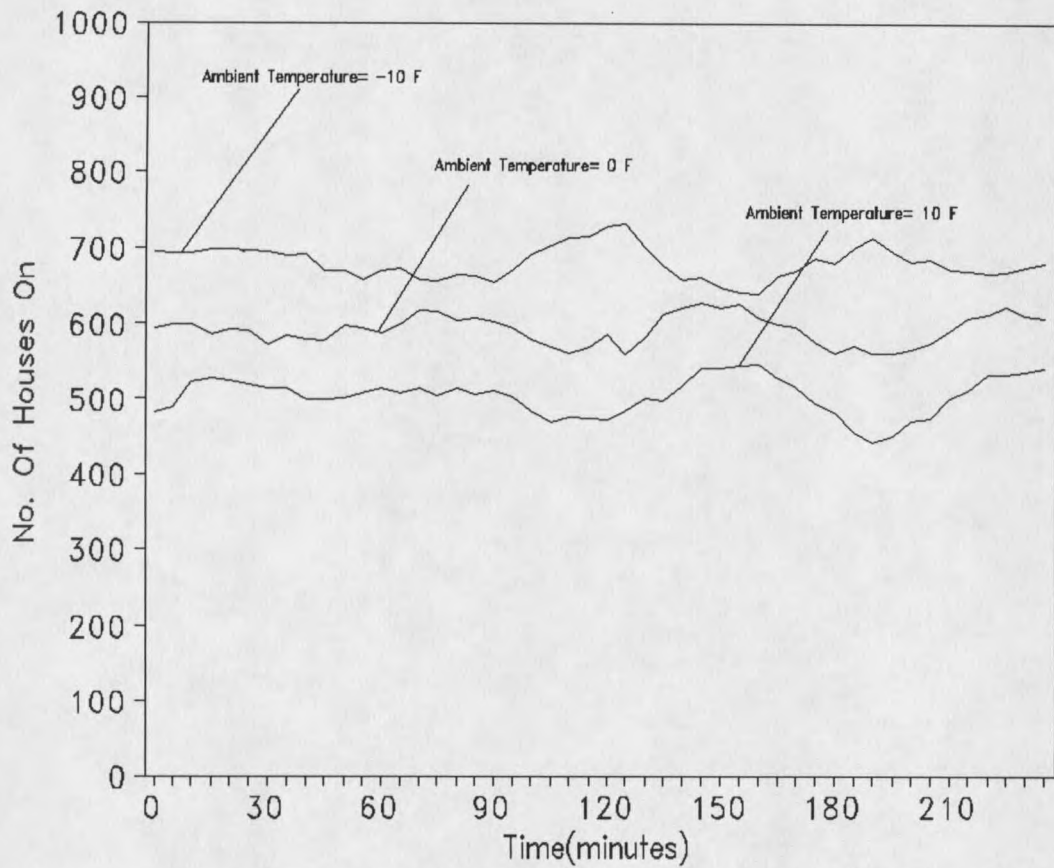


Fig. 27 Steady state response at three different ambient temperatures.

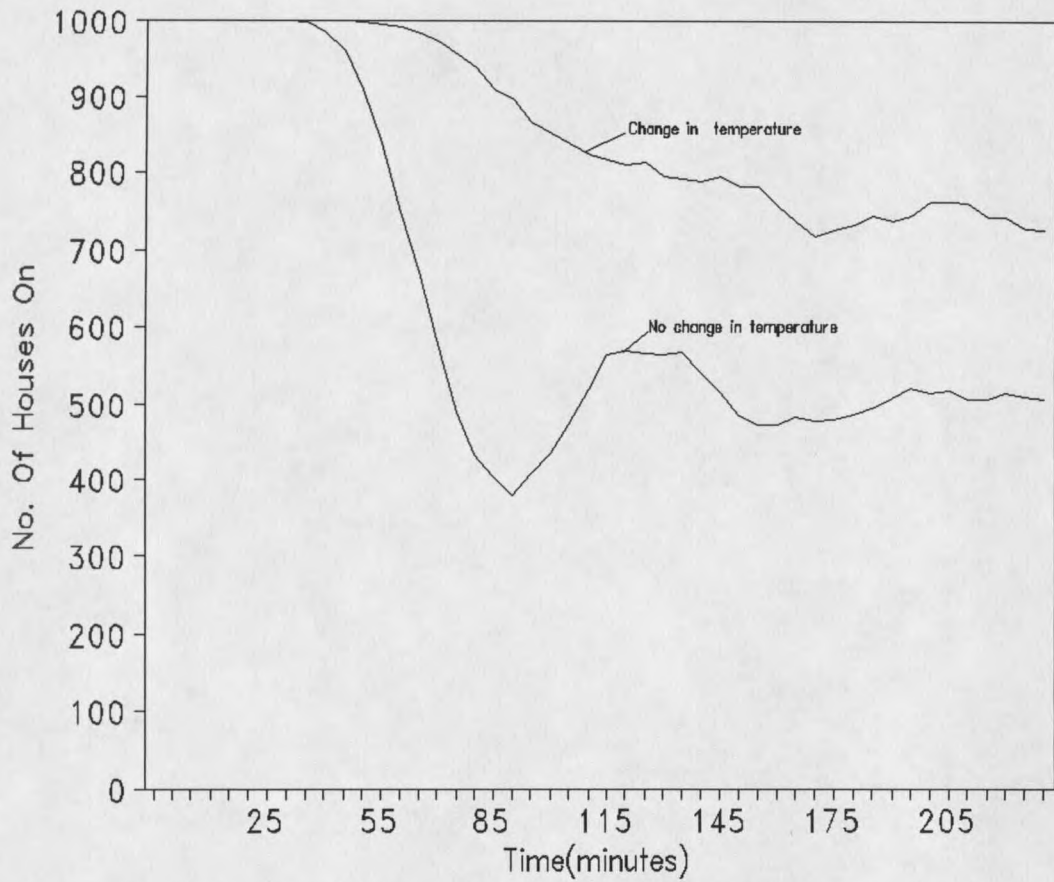


Fig. 28 Effect of change in ambient temperature before and after a power outage.

CHAPTER 6

EXPERIMENTAL RESULTS AND VALIDATION

Not much work has been done towards validating the results of Cold Load Pickup models because the utilities are reluctant to intentionally disrupt electric service. The following three experiments were performed towards validation of the Cold Load Pickup model developed.

A Continuous Power Supply Experiment On a House

In order to experimentally determine the mean values of thermal conductance and thermal mass of a room the steady state current required to heat a fairly big house in Bozeman, Montana which has electric baseboard heat was monitored for 40 minutes. The heated area of the house was divided into 14 different sub-areas and the house power demand was recorded at ambient temperature of -14°F . The average thermostat set-point was 59.5°F . The value of the dead band used was 2°F . The kw capacity of all room heaters was known and these actual values were used in the program. These values are given in Appendix C. The maximum heating demand of the house was 23.25 kw at a voltage of 240 Volts. Each room was simulated as a separate house. To convert the percentage of houses demanding power, obtained from the simulation program, to the current requirement under steady state, the following calculation was made.

$$I_{\text{required}} = (\% \text{ Houses on} * 14 * \text{ave. kw/room}) / 240$$

The amount of heat lost to the environment is

proportional to the thermal conductance G of the house. This in turn is proportional to the surface area exposed to the environment. In a room all walls may not be exposed. Since the exposed area of a room would be less than that of a house, one would expect a lower value of thermal conductance for a room. The lesser the air enclosed in a house the lower the thermal mass of the house. Therefore, a lower value of thermal mass would also be expected for an average room relative to that of a house.

The Cold Load Pickup program was run to calculate the values of the thermal conductance G and the thermal mass C for a room. The program was initially set for values of $G = 1$ w/°F and $C = 1$ wh/°F. Keeping G constant, C was incremented by one each time until a value of two thousand. The average simulated current was calculated for each of these values and compared with the average measured current. G was incremented by one and again all values of C from 1 to 2000 were tried. This process was continued till the average simulated current was within one ampere of the experimental current. The 14 random numbers for G and C generated are given in Appendix C. The selected values of G and C are 5 w/°F and 208 wh/°F. Typical values for G and C for a well insulated and approximately 2000 ft² in area house are 180 w/°F and 3223 wh/°F. As expected the value of G and C for a room is less compared to that of a house.

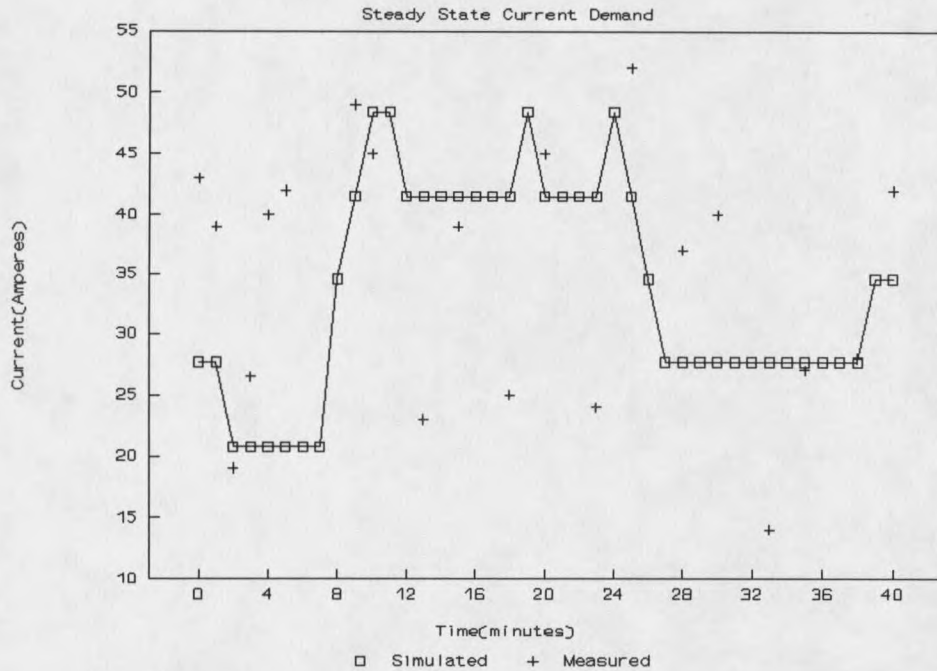


Fig. 29 Simulated and measured values of current requirement of 14 rooms.

In Fig. 29 the plus symbols show the actual current requirement and the line with square symbols shows the simulated current requirement of the house at steady state. The average simulated current demand is 33.75 Amperes and the measured current demand is 34.60 Amperes. The simulated and the measured readings are given in Appendix C.

With the help of this experiment it was possible to calculate the value of the parameters G and C for a room.

These values are used in the Cold Load Pickup experiment on a house.

A Cold Load Pickup Experiment on a House

In order to verify the simulated Cold Load Pickup current a strip chart recorder was used to monitor the current demand of the same house used previously. Random numbers for G and C generated for steady state were used. The house had been subjected to a power outage at 7:45 a.m. Power was restored at 6:06 p.m. The ambient temperature was 32°F at the time of outage and was 36°F at the time of restoration of power. The average setpoint of the house was 59.5°F.

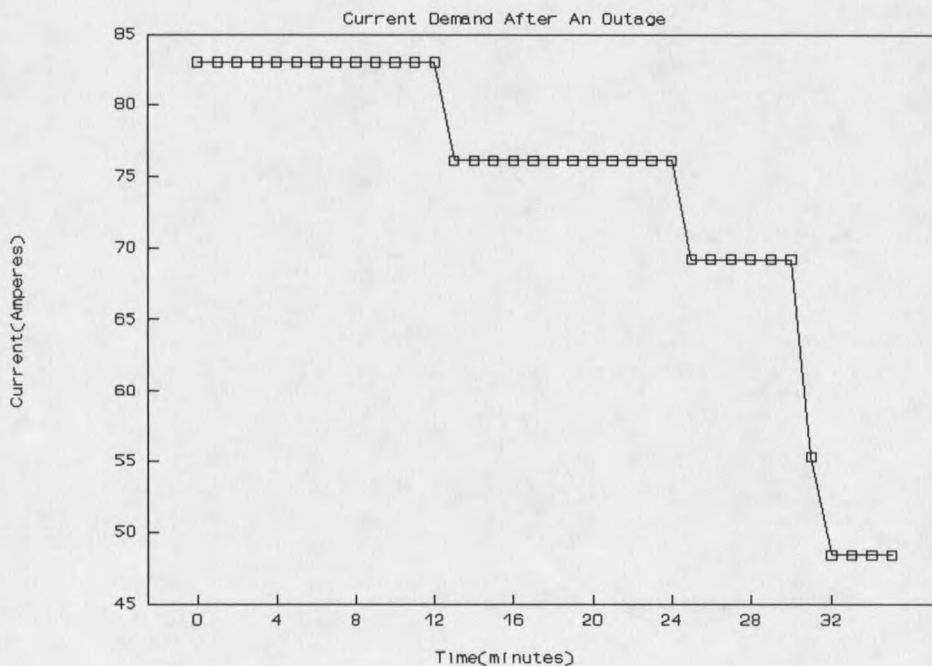


Fig. 30 Simulated current demand after an outage.

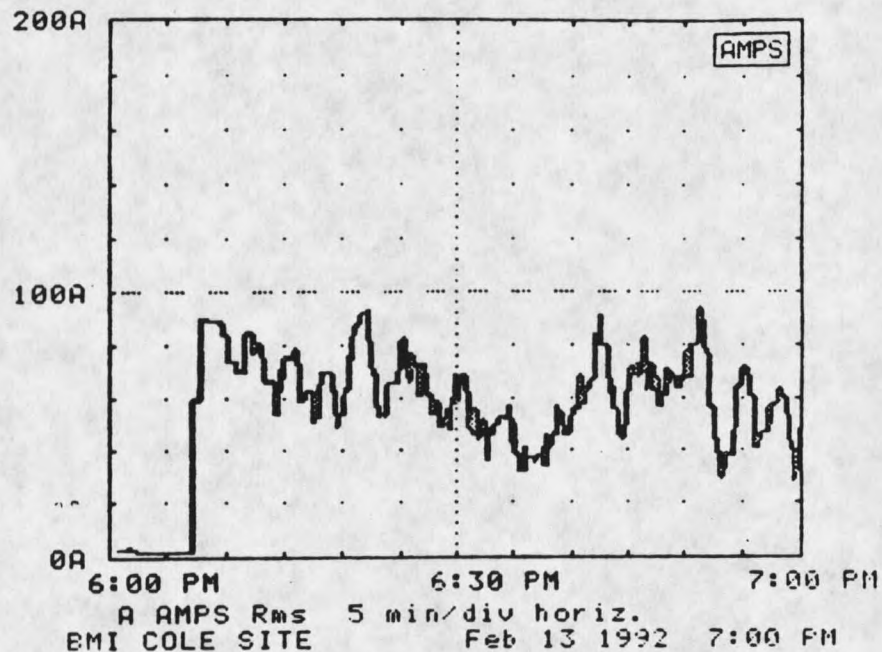


Fig. 31 Current demand recorded by strip chart recorder.

Fig. 31 shows the current demand as recorded by the strip chart recorder. Fig. 30 shows the simulated current for 34 minutes. Although the simulated current and actual current values do not exactly match, they have the same trend. The reasons for the discrepancy are listed below.

- 1) In our program we simulate each room as a house. It is expected to have all the walls exposed to the environment which is not true for the case of a room.
- 2) Since Cold Load Pickup program simulates houses, it is expected that each unit is independent and there is no transfer of heat from the heater of one unit to another. In the experiment there was transfer of heat between

rooms through the open doors and also the walls.

3) Only 14 random numbers for G and C were to be generated. They could be biased and may not be statistically balanced. The program normally runs for 1000 houses.

4) The average ambient temperature during the experiment was 36.42°F. There could be a considerable effect of solar heat.

A continuous Power Supply Experiment on a Montana Power Company Feeder

In this experiment the actual data taken on a feeder is used at three different times and the program is used to predict the total number of houses using electric heat on the feeder.

Data was obtained from the Montana Power Company for the steady state current demand of each phase of a feeder in Great Falls on a cold day in winter and a day in fall. This current was being supplied at a phase voltage of 7.2 KV. On a day in winter the power being supplied is expected to be used for heating and other activities like lighting, refrigeration, etc. On a day in the fall, the power being supplied is assumed to be used for similar activities other than heating. If the fall current demand is subtracted from the winter current demand an estimate of the heating current on the cold day is obtained. This was done twice. The two cold days

chosen were January 5 and January 8, 1991. The two days of fall chosen were September 30 and September 29, 1990. The temperature on the two days of fall was high enough to assume that heating power was not being supplied. The average temperature on September 30 was 43.33°F and on September 29 was 49.33°F. In order to estimate the number of houses demanding power (NHO) the following calculation was made.

$$NHO = I_H * V/Q$$

where

I_H is the heating current (Amperes)

V is the voltage at which the current is being supplied
(KV)

Q is the average value of the heat injected into the
house (kw)

The values of V and Q used are 7.2 KV and 20.5 kw. Odd timings such as 11 p.m., 2 a.m. and 5 a.m. were used. This is because there is less consumption of power for appliances like washing machine, electric oven, water heater, etc. during this time.

Fig. 32 and Fig. 33 show the estimated total number of houses on the feeder that use electric heating and the number of houses demanding power at 11 p.m, 2 a.m. and 5 a.m. The number of houses demanding power was calculated by dividing the total power demanded from the feeder by the average furnace capacity of a house. The temperature at these timings was known and is given in Appendix C. The parameter values given by N.W.P.P.C. [8] were used. Using the above

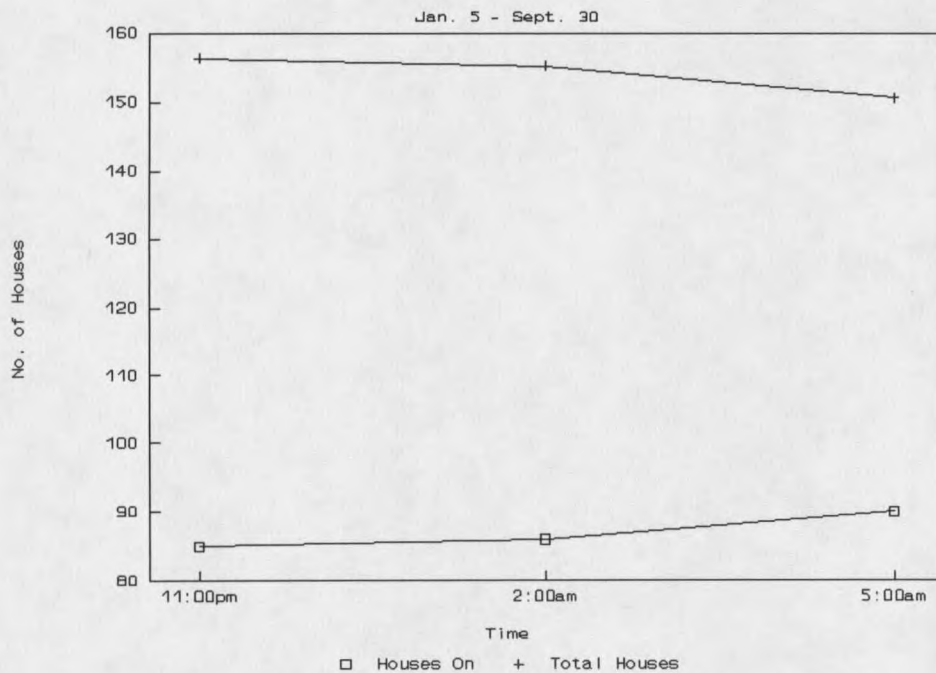


Fig. 32 The Fig. shows the simulated value of the number of houses present on the feeder and the simulated value of the houses demanding power.

information, the expected value of the total number of houses on the feeder was calculated. The plus signs show the simulated total number of houses on the feeder. Error was calculated as follows.

$$\text{Error} = \frac{(\max(\text{NOH}) - \min(\text{NOH})) * 100}{\max(\text{NOH})}$$

where

NOH = Total number of houses

Error for Fig. 32 was 3.8% and for Fig. 33 was 6.2%

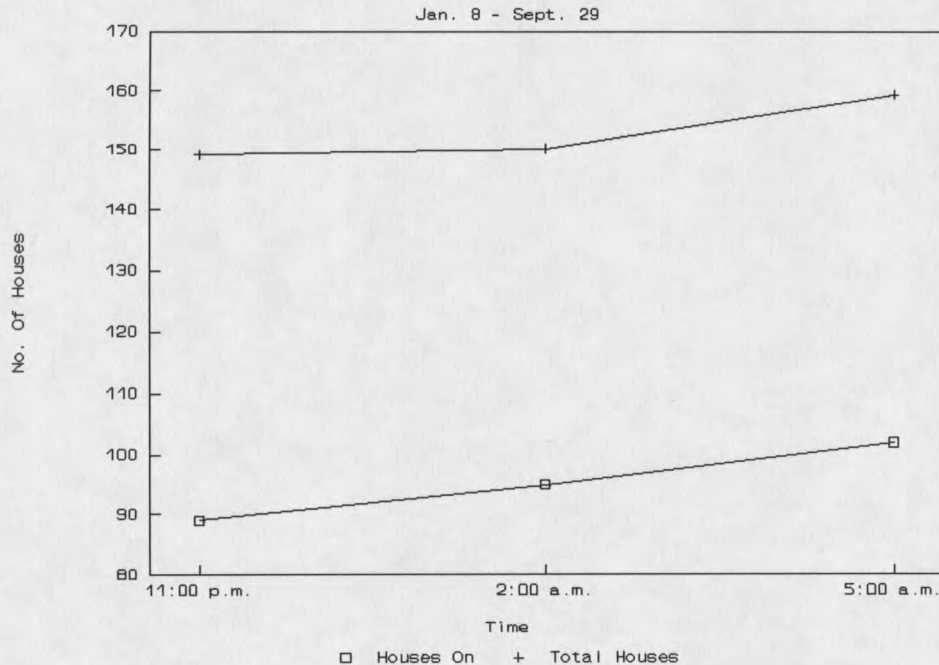


Fig. 33 The Fig. shows the simulated value of the number of houses present on the feeder and the simulated value of the houses demanding power.

In the experiment the number of houses using electric heat was simulated under three different conditions. It was noticed that the total number of houses using electric heat remains fairly stable for the late evening/very early morning times, but not stable as daily activities start early in the morning. In an ideal case where we have only power demand from the feeder for electric heat, all three cases should yield the same total number of houses. This is the case for the 11 p.m. and 2 a.m. runs and not for the 5 a.m. run in

Figures 33 and 34. The reason for this is that there is little power usage besides that used for electric heating for the first two cases. However, as power demand for other purposes besides that for heating increases, the program is unable to simulate the correct value of the houses using electric heat on the feeder.

CHAPTER 7

SUMMARY AND CONCLUSIONS

The summary and conclusions of the study and suggestions for further research are presented in this chapter.

Summary

The total connected electric heating load on a feeder is never demanded at any one time under normal conditions. During a power outage the temperature of the controlled space will decay towards the local ambient temperature. After power is restored the restoral current has two components; a transient component and an enduring component. The transient components can be taken care of by simple methods like relays with extremely inverse settings. The magnitude of the enduring component is dependent on the load, ambient temperature, outage duration, size and type of houses. This thesis makes an analysis of the enduring component by taking into consideration all the factors affecting it.

House thermodynamics was modeled using an analogous R-C electric circuit. In order to observe the effect of the house parameters on the aggregate power, the simulation program is run with a change in the mean value of the parameters.

Depending on suitability, utilities can adapt one of the many ways to restore power after an outage.

Current demand peak is dependent on the service voltage. A decrease in the service voltage could reduce the peak

current demand, at the time of restoration to some extent. The value of the peak current demand, after an outage, at .95 per unit voltage was simulated and found approximately 5 % less than the simulated value of the peak current at 1 per unit voltage. This can be seen in Figures 23 and 24.

Sectionalizing is an effective way to reduce overcurrents. Figures 25 and 26 show a considerable reduction in the peak power demand by sectionalization.

In very cold areas it is recommended that the utilities should advise on the construction of the houses to provide extra insulation. This will not only reduce the peak current demand after an outage but also reduce the steady state power demand.

Conclusions

A model of electric space heating load on a distribution feeder is presented. A simulation program is developed which estimates the heating power demand after a power outage. This could be used by a utility substation to calculate the magnitude and duration of the heating power demand during the recovery period following a feeder reclosure.

Future Research

A topic of further research could be validation of the Cold Load Pickup model. It is suggested that data be collected with the readings of the pickup current after a power interruption.

Modeling other heating loads such as water heater could

also be a topic for further research.

REFERENCES CITED

- 1) L. J. Audlin, M. H. Pratt, and A.J. McConnell, "New Relay Assures Feeder Resumption after outage", Part 1, Electrical World, September 10, 1949, pp. 99-103
- 2) J. E. McDonald, A. M. Bruning, and William R. Mahieu, "Cold Load Pickup", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, No. 4, July 1979, pp. 1384-1386
- 3) S. Ihara, and F. C. Schweppe, "Physically Based Load Modeling of Cold Load Pickup", IEEE Transaction on Power Apparatus and systems, PAS-100, No. 9, Sept. 1981, pp 4142-4150.
- 4) W. W. Lang and M. D. Anderson, "An analytical method for quantifying the Electrical space heating component of Cold Load Pick Up", IEEE Transactions on Power Apparatus and Systems, PAS-101, No. 4, April 1982, pp 924-932.
- 5) D. Athow, "Development and Applications of a Model for Cold Load Pickup", M.S. Thesis, Department of Electrical Engineering, University of Idaho, Jan. 1987
- 6) R.E. Mortensen and K.P. Haggerty, "A Stochastic Computer Model for heating and cooling loads", IEEE Transactions on Power Systems, Vol. 3, No. 3, August 1988 pp 1213-1219
- 7) R.E. Mortensen and K.P. Haggerty, "Dynamics of heating and cooling loads, simulation, and actual utility data", IEEE Transactions on Power Systems, Vol. 5, No. 2, Feb. 1990.
- 8) 1991 North West Conservation and Electric Power Plan, vol. 2 part 1, N.W. Power Planning Council, Portland, OR.

APPENDICES

APPENDIX A

Development of Equations

This appendix covers the mathematical equations of the analogous electric circuit for modeling the thermodynamic behavior of a house.

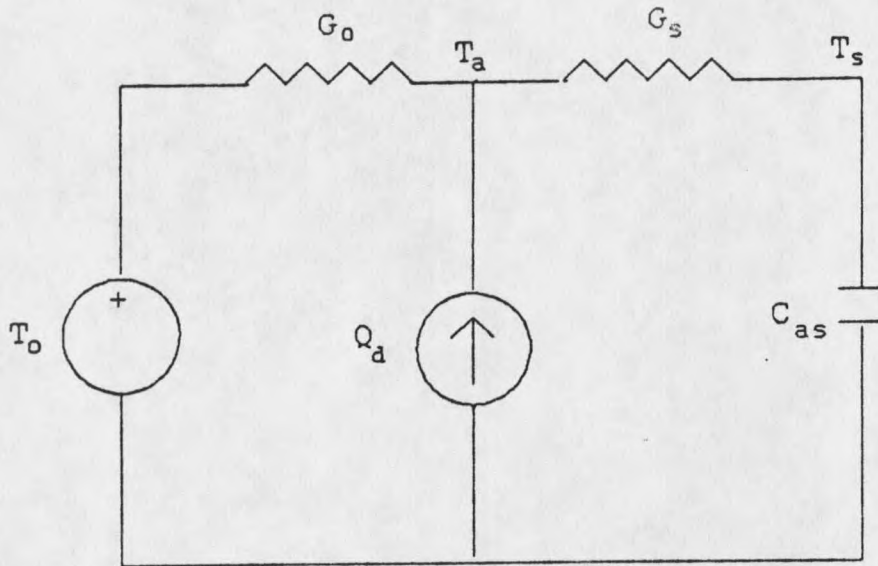


Fig. 2 Analogous electric circuit used for thermodynamic behavior of a house.

$$Q = Q_c + Q_R \quad (A-1)$$

By Kirchoff's current law

$$Q(t) = \frac{T(t) - T_a(t)}{R} + C \frac{dT(t)}{dt} \quad (A-2)$$

$T_a(t)$ is a constant in the model

$$Q(t) = \frac{T(t) - T_a}{R} + C \frac{d}{dt} T(t) \quad (\text{A-3})$$

$$\frac{d}{dt} T(t) = \frac{Q(t)}{C} - \frac{T(t) - T_a}{RC} \quad (\text{A-4})$$

Taking Laplace on both sides

$$sT(s) - T_{ho} = \frac{Q(s)}{C} - \frac{T(s)}{RC} + \frac{T_a(s)}{RC} \quad (\text{A-5})$$

Where T_{ho} is the initial house temperature

$$sT(s) - T_{ho} = \frac{Q}{sC} - \frac{T(s)}{RC} + \frac{T_a}{sRC} \quad (\text{A-6})$$

$$T(s) \left(s + \frac{1}{RC} \right) = T_{ho} + \frac{Q}{sC} + \frac{T_a}{sRC} \quad (\text{A-7})$$

$$T(s) = \frac{T_{ho}}{s + \frac{1}{RC}} + \frac{Q}{sC \left(s + \frac{1}{RC} \right)} + \frac{T_a}{sRC \left(s + \frac{1}{RC} \right)} \quad (\text{A-8})$$

$$T(s) = \frac{T_{ho}}{s + \frac{1}{RC}} + QR \left(\frac{1}{s} - \frac{1}{s + \frac{1}{RC}} \right) + T_a \left(\frac{1}{s} - \frac{1}{s + \frac{1}{RC}} \right) \quad (\text{A-9})$$

Taking inverse laplace

$$T(t) = T_{ho} e^{-\frac{t}{RC}} + QR - QR e^{-\frac{t}{RC}} + T_a - T_a e^{-\frac{t}{RC}} \quad (A-10)$$

$$T(t) = T_a + (T_{ho} - T_a) e^{-\frac{t}{RC}} + QR(1 - e^{-\frac{t}{RC}}) \quad (A-11)$$

On-time is the time the house takes to reach the upper setting of the thermostat starting from the lower setting. So

$$T_{ho} = T_l$$

$$T(t) = T_u$$

where

T_l is the lower setting of the thermostat

T_u is the upper setting of the thermostat

Substituting the above values of T_l and T_u in equation (A-11)

$$T_u = T_a + (T_l - T_a) e^{-\frac{t}{RC}} + QR(1 - e^{-\frac{t}{RC}}) \quad (A-12)$$

$$T_u - T_a - QR = (T_l - T_a - QR) e^{-\frac{t}{RC}} \quad (A-13)$$

Solving for t which is the on-time

$$t_{on} = -RC \ln \frac{T_u - T_a - QR}{T_l - T_a - QR} \quad (A-14)$$

Off-time is the time the house takes to reach the lower

setting of the thermostat starting from the upper setting when the furnace is off. So

$$T_{ho} = T_u$$

$$T(t) = T_l$$

$$Q = 0$$

Substituting the above values of T_{ho} , $T(t)$ and Q in equation (A-11)

$$T_l = T_a + (T_u - T_a) e^{-\frac{t}{RC}} \quad (A-15)$$

$$T_l - T_a = (T_u - T_a) e^{-t/RC} \quad (A-16)$$

Solving for t which is the off-time

$$t_{off} = -RC \ln \frac{T_l - T_a}{T_u - T_a} \quad (A-17)$$

APPENDIX B**Effect of Change in Voltage**

This appendix covers the mathematical equations which show the effect of service voltage on the power and current supplied by the feeder.

Q the current source in the analogous R-C circuit is the power in kw demanded by the furnace. Therefore

$$Q = \frac{V^2}{R} \quad (B-18)$$

Service voltage is changed and the new voltage is given by V_{new} . The new power Q_{new} is

$$\frac{V_{new}^2}{R} \quad (B-19)$$

So

$$\frac{Q_{new}}{Q} = \frac{V_{new}^2}{V^2} \quad (B-20)$$

The current I is given by

$$\frac{Q}{V} \quad (B-21)$$

So

$$\frac{I_{new}}{I} = \frac{V_{new}}{V_{old}} \quad (B-22)$$

APPENDIX C

Data Used

The following are the values of the thermal conductance G , thermal mass C and heat injected to the house Q for the 14 rooms of the house in which the experiment was performed. Random numbers were generated for G and C . The actual values of Q were used.

$G(w/^{\circ}F)$	$C(wh/^{\circ}F)$	$Q(watts)$
3.175648	136.0470	1500.000
1.441082	612.9801	1200.000
4.101494	279.6719	500.0000
4.196769	328.2234	1250.000
4.069674	183.5163	2500.000
4.946783	343.3462	2500.000
6.733378	285.4150	500.0000
3.273767	392.9054	750.0000
4.866119	101.9610	750.0000
7.8993320E-02	45.54129	2500.000
6.211481	112.8586	750.0000
2.263757	428.1706	500.0000
2.715868	43.56509	1000.000
5.248510	267.3224	7050.000

The following is the current demand in amperes for phase A, phase B and phase C of a feeder in Great Falls, Montana on Jan. 5, 1991, Jan. 8, 1991, Sept. 30, 1990 and Sept 29, 1990.

The current was supplied at a phase voltage of 7.2 kV. These readings were used in the continuous power supply experiment on a feeder.

	<u>Phase A</u>	<u>Phase B</u>	<u>Phase C</u>
<u>Jan. 5, 1991</u>			
11 p.m(Jan. 4, 1991)	181	190	242
2 a.m.	165	169	218
5 a.m	171	179	219
<u>Jan. 8, 1991</u>			
11 p.m(Jan. 7, 1991)	183	202	249
2 a.m.	165	182	232
5 a.m.	171	195	244
<u>Sept. 30, 1990</u>			
11 p.m(Sept. 29, 1990)	104	146	119
2 a.m.	91	116	100
5 a.m.	94	120	98

Sept. 29, 1990

11 p.m. (Sept. 28, 1990)	111	144	124
2 a.m.	90	119	99
5 a.m.	92	123	103

The following temperatures were recorded in Great Falls.

Jan. 5

11 p.m. (Jan. 4, 1991)	4°F
2 a.m.	3°F
5 a.m.	-2°F

Jan. 8

11 p.m. (Jan. 7, 1991)	4°F
2 a.m.	-6°F
5 a.m.	-2°F

The following are the values measured on Dec. 22, 1990 and the simulated values of the current demand of 14 rooms. The following data was used in the continuous power supply experiment on a house.

Time (minutes)	Simulated (Amperes)	Measured (Amperes)
0.000000	27.67857	43.0
1.000000	27.67857	39.0
2.000000	20.75893	19.0
3.000000	20.75893	26.5
4.000000	20.75893	40.0
5.000000	20.75893	42.0
9.000000	41.51786	49.0
10.00000	48.43750	45.0
13.00000	41.51786	23.0
15.00000	41.51786	39.0
18.00000	41.51786	25.0
20.00000	41.51786	45.0
23.00000	41.51786	24.0
25.00000	41.51786	52.0
28.00000	27.67857	37.0
30.00000	27.67857	40.0
33.00000	27.67857	14.0
35.00000	27.67857	27.0
38.00000	27.67857	28.0
40.00000	34.59821	42.0

MONTANA STATE UNIVERSITY LIBRARIES



3 1762 10171280 8