

OBJECTIVE OR STATISTICAL EVIDENCE
SUPPORTING MODERN MATH
IN THE FIRST GRADE

BY

REGINA ANHELUK MCNAIR

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CHAPTER I

INTRODUCTION

The investigator has been concerned with the question as to whether or not the "modern" math program as used in the first grade, was meeting the needs and interests of the students. This investigation was designed to determine whether or not there was objective or statistical evidence ascertaining the value of incorporating the new math into the first grade curriculum.

Upton's, First Days With Numbers, a traditional math text, suggests the following topics for the first grade math program:

1. Reading and writing numbers
2. Counting by ones to five hundred
3. Counting by twos, fives, and tens
4. Ordinal numbers first to fifth
5. Simple vocabulary and number comparisons
6. One hundred addition facts
7. Single column addition with three and four addends
(no bridging)
8. Two-column addition with two and three addends
(no carrying)
9. One hundred subtraction facts
10. Two-place subtraction (no borrowing)
11. Place value through hundreds
12. The fraction one half and one fourth as used with
objects and groups of objects
13. The coins cent, nickel, dime, quarter, half dollar
14. Change-making
15. Reading of dollars and cents
16. Telling time
17. The calendar
18. Simple measurements¹

Three years later the following topics were suggested for

¹Upton, Clifford B., First Days With Numbers, p.G-5.

the first grade curriculum by Deans in a "modern" math textbook:

Chapter 1

Understanding numbers one through five; exploring sets and subsets; one to one correspondence; ordinals; comparing sets (more and fewer); completing sets (how many more are needed); reading and writing numerals through five; sequence; number words; combining and separating sets; pictured problems.

Chapter 2

Understanding numbers two through six; numerals through six; addition and subtraction facts for two through six; meaning of addition and subtraction; commutative property; comparisons; number stories.

Chapter 3

Understanding numbers three through seven; recognizing and using subsets of five; ordinals; reading number sentences; more and less; numerals through seven; sequence; before and after; addition and subtraction facts for three through seven; inverse operations; commutative property; problem solving (using number sentences).

Chapter 4

Understanding numbers four through eight; comparing numbers; numerals through eight; how many more are needed; addition and subtraction facts for four through eight; readiness for addition of three numbers; counting by twos, threes, fours; multiplication and division facts for twos and fours through eight; problem solving (originating number stories, writing number sentences); meaning and use of plus symbol, equal symbol, and equals symbol; doubles; adding and subtracting one.

Chapter 5

Understanding numbers five through nine; numerals through nine; number words six through nine; ordinals; missing numbers; inverse operations; commutative property; addition and subtraction facts for five through nine; multiplication and division facts for twos and fours through nine; adding three numbers; measures; problem solving.

Chapter 6

Completing sets; reviewing addition and subtraction facts through nine; vertical form for addition and subtraction; doubles; money; comparing sets; understanding, reading, writing ten; hand symbols for five and ten; column addition; using five; counting by ones, twos, threes, fours, fives.

Chapter 7

Understanding tens to one hundred; reading and writing tens to one hundred; "teen" numbers; tens and ones to fifty; before and after; number facts in problems; one half; one fourth; counting by ones, twos, fives, tens; time (hours); missing numbers.

Chapter 8

Understanding numbers to one hundred; tens and ones to one hundred; counting by twos, fives, tens; addition and subtraction facts for six through ten; multiplication and division facts for ones and fives through ten; comparing money; column addition involving money; doubles; numerals one through one hundred; sequence; before and after.

Chapter 9

Addition and subtraction facts for eleven through eighteen; doubles and near doubles; time (half hours); multiplication and division facts for twos and nines through 18; problem solving.²

Emphases in the early grades had changed from merely covering an assigned area of math to one of developing an understanding of the material covered. The study of the structure of our number system was essential if arithmetic was to have real meaning. There was emphasis on relationships. Relationships served to reduce the number of separate facts a child must learn. The concept of mathematical "ideas" was emphasized. Ideas defined as, numbers are abstract, numbers are represented by symbols and words which make it possible for us to communicate with others regarding the idea. The "discovery" system was stressed in "modern" math. If a child did not know an answer, he needed a means by which he can find the answer for himself. Many different ways of thinking were encouraged

²Deans, Edwina, Kane, Robert B., Oesterle, Robert A., Meeting Mathematics, table of contents, pages ii,iii,iv.

for this.³

Statement of the Problem

The writer had used a "modern" textbook series for four years. She had taught in the third grade one year and taught in the first grade three years. Mathematical background of the writer was limited to minimum college requirements and an in-service training program provided upon instituting the particular series used. It was during the period of teaching first grade that the writer had some uncertainty about the content in the "modern" math series. The uncertainties were due to studies concerning Piaget which dealt with stages of children's learning development. The writer felt there was a conflict between their philosophies and material presented in the math curriculum at the first grade level.

A preliminary review of literature on "modern" math confirmed the writer's intuitive feeling that these uncertainties were shared by other educators.

Riedesel and others who reviewed research on mathematics education on the elementary level, found growing numbers of studies concerned with individual differences and with ideas presented by

³Ibid., page vi, vii, viii.

Piaget.⁴

A master's thesis written at Montana State University by Messick, indicated that modern mathematics in the first grade was not accelerating children's number concepts. One of the objectives of this study was to note if "modern" math approaches accelerated children's number concepts.⁵

Due to the subjective feelings of the writer and the limited readings on the subject, the problem existed as to whether there was, or was not, objective or statistical evidence to support the use of "modern" math materials in the first grade curriculum.

Procedure

The procedure, established to determine through objective or statistical evidence whether or not there is value in a "modern" math program in the first grade, consisted of two steps: (1) Review of available literature and findings pertaining to the use and effectiveness of "modern" math materials in grade one, (2) Transmission of letters of inquiry to publishers of "modern" math series to establish whether they had established statistical or objective evidence to substantiate the inclusion

⁴Riedesel, C.A., Suydam, M.M., Pikaart, L., "Research on Mathematics Education, Grades k-8 for 1967", The Arithmetic Teacher Vol. 15:532.

⁵Messick, Theresa J., (citing Piagets Stages of Development Used to Evaluate Modern Mathematical Experiences, pages 19-20, Montana State University, Bozeman, Montana, thesis).

of "modern" math concepts in the first grade curriculum. (See Appendix).

The examination revealed a distinct difference in the content of the "traditional" and "modern" math textbooks for the first grade. A random selection of two textbooks published by the same company was made by the writer. The purpose of the selection was to present content from a "traditional" and "modern" math textbook.

Limitations

This study was limited to reviewing available published material at the Montana State University Library, Bozeman, Montana, about "modern" math in the first grade. Letters of inquiry were sent to ten publishers of "modern" math series.

Summary

An examination of "modern" math textbooks in the Montana State University Library revealed that most math texts were based on the "modern" approach. Further examination revealed the content of the "traditional" and "modern" texts differed in content and in the presentation of material. The emphases in math had changed from "drill" and "rote" methods, of learning a given amount of material, to an approach that provided greater meaning and understanding of mathematics. The scope of material

covered appeared to be greater. It was the intuitive feeling of the writer that perhaps there was a conflict between the material presented in "modern" math texts and the developmental stages of children's learning as suggested in studies by Piaget. Consequently, the writer attempted to determine whether or not there was objective or statistical evidence to support "modern" math in the first grade curriculum.

Chapter two will define modern math and review material that led to its development.

CHAPTER II

DEFINITION AND HISTORY OF "MODERN" MATH

There appeared to be a marked difference between "traditional" math and "modern" math. The difference may be resolved to some extent by establishing a definition for "traditional" math and "modern" math. The next question that had been raised by the writer was: Why the change from "traditional" to "modern" math? The purpose of this chapter was to establish a definition and review the history of "modern" math.

Definition

According to Ohmar, Aucoin, and Cortez, a definition should obey the following properties:

1. Any definition of a new word must be expressed in terms of the undefined words and/or the previously defined words and common nontechnical English words.
2. Any definition must be consistent with itself and with any other definitions.
3. Any definition must be meaningful.
4. Any definition must be expressed in such a manner that it includes all desired cases and excludes all undesired cases.¹

To meet the requirements of a correct definition, the writer found several sources that defined "modern" math and "traditional" math. The writer also defined the term "modern" and "traditional"

¹Ohmar, M.M., Aucoin, C.V., and Cortez, M.J., Elementary Contemporary Mathematics, page 2.

math as related to mathematics and arithmetic.

McDowell, gave the following definitions of "mathematics" and "arithmetic":

"Mathematics" is the science which includes arithmetic, algebra, trigonometry, and geometry. It treats of the exact relations existing between numbers, quantities, magnitudes or operations, and of the methods by which, in accordance with these relations, quantities sought are found from others known or supposed.

"Arithmetic" is the science which treats numbers and of the method or way of calculating by them.²

Webster gave the following as definitions of "mathematics" and "arithmetic":

"Mathematics" is the group of sciences, including arithmetic, geometry, algebra, calculus, etc. dealing with quantities, magnitudes, forms, and their relationships, attributes, etc., by the use of numbers and symbols. Abbreviated "math".

"Arithmetic" is the science or art of computing by positive, real numbers.³

The terms "new" or "modern" math did not appear in the dictionaries noted; however, the term was defined in several other sources.

Authors from Scott, Foresman and Company stated:

"Modern" mathematics generalizes classical math. Modern math deals with basic concepts that explain the rules and serve to unify the branches of math. The goal of new math is gradual growth in understanding the basic mathematical ideas and seeing the connections

²McDowell, C.H., A Short Dictionary of Mathematics, page 10.

³Webster's New World Dictionary of the American Language, pages 77, 906.

that exist among them so that there will be greater flexibility and creativity in application.⁴

Copeland explained "modern" math:

New mathematics is new in the sense that some basic mathematical ideas useful to students in advanced arithmetic are being introduced in the public elementary school. The mathematics taught today in the public school is also new in that greater emphasis is being placed on a continuous program of studies in mathematics from kindergarten through grade twelve. For example, percentage and proportion in the sixth grade, algebra in the primary grades, base two in the fifth grade.

Probably the most important part of new mathematics, from the stand point of elementary school, is an emphasis on what might be called the structure of our number system.

Number systems have certain properties or characteristics, and to understand a system we need to understand its properties. Such properties of a number system as the commutative, associative, and distributive were studied.

Mathematics at the elementary school level is a study of the structure of our number system (arithmetic), sentences and sets (algebra), and space and location (nonmetric geometry), and measurement (metric geometry).⁵

Another explanation of the "modern" math approach is given by Starr:

The major emphasis in the modern approach is on "understanding" of basic mathematical ideas which are sometimes hidden by computational techniques. Computation is an important part of mathematics and must be taught, but it should be taught only after a child understands the underlying mathematical ideas.

⁴An Administrator's Handbook on Mathematics, Scott, Foresman and Company, pages 3, 4, 5.

⁵Copeland, R. W., Mathematics and the Elementary Teacher, pages 5-10.

The child should make use of inductive and deductive reasoning as he progresses from intuitive learning to basic information and then to an understanding of more abstract concepts. The discovery approach to learning is employed: the child is given the opportunity to discover mathematical concepts for himself.⁶

"Traditional" math is a term used loosely to describe that which is not "new" math. Hartung and others presented this statement:

The "old" view was that arithmetic was largely a set of number facts and computational procedures governed by rules.⁷

Fehr and Hill define "traditional" math as:

The "traditional mathematics" program in the elementary school was for many years a course in computational arithmetic, based largely on the theory of learning which emphasizes associative skills.⁸

A review of the definitions presented would indicate that "modern" math was that which included arithmetic, geometry, algebra, and calculus. It dealt with quantities, magnitudes, forms, and their relationships and attributes by the use of numbers and symbols. Emphasis was on understanding the theories and properties. "Traditional" math is simply that which is not

⁶Starr III, J. W., The Teaching of Mathematics in the Elementary School, Chapter 1.

⁷Hartung, M. L.; Engen, H. V.; Gibb, E. G.; Stochl, J. E.; Knowles, L.; Welch, R.; Curriculum Foundation Series, "Seeing Through Arithmetic", page 7.

⁸Fehr, H. F. and Hill T. J., Contemporary Mathematics for Elementary Teachers, Preface.

"modern" math. It is the science of computing with real numbers.

Evolution of "Modern" Math

A number of study groups were fundamental in the instruction of modern math.

Mueller, author of an introductory math text, stated:

The seeds of "new" math were sown in the nineteen forties. World War II postponed any significant growth till the fifties. The movement was known as "meaningful arithmetic" movement. Leaders of the movement were mainly mathematic educators whose interests lay more in psychology and learning theory than advanced math---the reverse of the situation today. They were concerned with the rote and mechanical method the children used to learn arithmetic. Psychologists showed conclusively that the more the learner understood about what he was doing, the faster he learned the material and the longer he retained it.⁹

The School Mathematics Study Group (SMSG) was organized in 1958 at Yale University under the direction of E.G. Begle and was supported by the National Science Foundation. SMSG represented the greatest united effort for improvement of mathematics education in history. SMSG materials were unique in representing the combined thinking of psychologists, learning specialists, testmakers, mathematicians from both colleges and industry, biologists, and public school teachers. As early as 1959-1960, sample textbooks and teachers manuals were used in forty-five states by more than four

⁹Mueller, F.J., Understanding the New Elementary School Mathematics, page 4.

hundred teachers and forty-five thousand pupils. A continuing longitudinal study of SMSG texts, student achievement, and teacher attitude was conducted.

The Committee on the Undergraduate Program in Mathematics was a committee of the Mathematical Association of America. In 1953 the President of MAA appointed a committee to modernize undergraduate programs in mathematics. In 1961 the Panel on Teacher Training published its report, Recommendation for the Training of Teachers of Mathematics. These recommendations suggested the type and amount of training that should be required of teachers of mathematics at each of five levels. Level I was mathematics in the elementary school.

The Greater Cleveland Mathematics Program, GCMP, was founded in 1959 by the Educational Research Council. The program was concept-oriented and utilized discovery techniques in a kindergarten through twelve integrated approach. Films and other training materials for teachers and administrators, visual and manipulative aids for students and teachers were a part of the program. Science Research Associates, SRA, began publication of the program commercially in 1962.

The Nuffield Mathematics Project, directed by Loeffrey Mathews, St. Dunstons College, London, was one of the finest "modern" mathematics projects and is most closely based on careful child study. The stated objective of the Project was to produce a

contemporary course for children from age five to thirteen. This was being designed to help them connect together many aspects of the world around them, to introduce them gradually to the processes of abstract thinking, and to foster in them a critical, logical, but also creative turn of mind.

In the summer of 1963, a group of mathematicians met at Cambridge, Massachusetts, at the invitation of the Educational Services Incorporated, with financial support from the National Science Foundation, to discuss the current state and the possible future of mathematics in the elementary and secondary schools. It was not their purpose to evaluate "new" mathematics curricula and programs for these levels but to look ahead at what might be accomplished in a "second wave" of reform after new courses and fresh attitudes towards mathematics had become established and had been accepted by the public. The CCSM recognized that there might be intrinsic limitation on the ability of young children to handle mathematical ideas, but it was also recognized that there was little evidence at that time relative to the degree to which such limitation can be changed by the teaching process.

Marks, Purdy, and Kinney discussed other experimental programs as well as those previously mentioned. Reason for these programs was expressed as: "But with more mathematics to be learned, and to be learned more thoroughly, it is no longer

possible to delay the formal study of mathematics until secondary school."¹⁰ Marks, Purdy, and Kinney went on to state:

Groups which exerted powerful influence on the curriculum in mathematics at all grade levels were: School Mathematics Group (SMG), The University of Illinois Arithmetic Project, Geometry Project at Stanford University, Suppes Arithmetic Project, Minnesota School Mathematics Center.

Certain characteristics common to these experimental programs that provided for sound study were:

1. In most cases the leaders were college or university mathematicians or persons with equivalent training and experience. The result was that the programs were generally sound mathematically.
2. Their primary purpose was to improve mathematics education. This included attention not only to content but also to development of sound teaching procedures.
3. Most of the projects were supported by ample funds from private foundations and the federal government. The aid varied, in some cases supporting only classroom tryouts of the material; in others it provided for writers to develop texts or units.
4. An experimental approach was used in trying to improve effectiveness of materials. A typical pattern was to write texts, try them out in the classroom, rewrite on the basis of criticism, and try them again with pupils.
5. A program of teacher education accompanied some programs. This assumed various forms, such as teachers in the experimental program meeting with a consultant to discuss the mathematical background of the material to be taught; teachers viewing television or films regularly; teachers studying the extensive mathematical background that was the part of the teachers manual.
6. Reports concerning most materials indicated that they were designed to help pupils

¹⁰Marks, J.L., Purdy, R.C., Kinney, L.B., Teaching Elementary School Mathematics for Understanding, page 6.

discover ideas, challenged pupils to create mathematics, and aided them to form, verify, and in some cases prove generalizations.

7. Most material was of such nature that the pupil's ability to read was an important factor in his success. The new textbooks at all grade levels contained more exposition and explanation than traditional textbooks.

Conclusions reached by mathematics study groups were that:

1. Pupils can learn more mathematics
2. Pupils can learn mathematics earlier
3. More effective procedures are possible
4. Pupils learn related ideas
5. Pupils are enthusiastic

Questions about "new" math that still remain unanswered were:

1. At what age is this learning meaningful and permanent?
2. Does the child actually understand the generality of his statement or is it merely a verbal exercise which he can repeat but not understand?
3. Will he be equally successful and understand equally well if he has had an intuitive understanding derived through manipulating blocks, trying examples, examining tables and the like?
4. Is verbalization of this commutative property or symbolization of the property important?
5. Which results should be expected from all pupils and which from a select minority?¹¹

Summary

An examination of definitions of "modern" math led the writer to conclude that "modern" math included "traditional" arithmetic with the addition of all the elements of math into the elementary curriculum. Methodology was a deemphasis of "drill" and "rote", to an emphasis on "understanding" theories and processes.

¹¹Ibid., pages 6-10.

"Modern" math evolved from a need for change. The needs can be summarized as:

1. Rapid advance of knowledge in mathematics which makes increasingly greater demands on citizens.
2. The need for more effective articulation from one grade to the next and from elementary to high school.
3. The recognition that the regular elementary math program was somewhat barren of the fascinating, interesting phases of mathematics.
4. The need for better understanding of the structure of mathematics as essential to satisfactory and permanent grasp of subject matter.
5. The need for improved mathematics program for children of different abilities.¹²

Chapter three will discuss factors influencing the mathematics curriculum.

¹²Deans, Edwina, "Elementary School Mathematics, New Direction", U. S. Department of Health, Education, and Welfare, Office of Education, page 1.

CHAPTER III

FACTORS INFLUENCING THE MATHEMATICAL CURRICULUM

There were a number of indexes in education that pointed to the controversy concerning the use of "modern" math in the first grade curriculum. On examination, the controversy did not yield objective or statistical material supporting the value of modern math in the first grade, but gave evidence as to the existence of a relevant problem.

Child Development and Child Behavior

Jean Piaget, a Swiss psychologist, was mentioned frequently in journals of education and mathematics. His main concern had been with how the child acquired knowledge and what happened to mental processes during this acquisition.

Piaget maintained that there were five stages in the continuum in child development. Mair presented these stages as follows:

1. Sensor-motor phase, (roughly, ages: 0-2). The child depends upon sensory and body motivated experiences.
2. Preconceptual phase, (roughly, ages: 2-4). This is a period of investigation of the environment for the child between the time when he exhibits purely self-satisfying behavior and rudimentary socialized behavior.
3. Phase of intuitive thought, (roughly, ages: 4-7). The child has a widening social interest in the world around him. (Phase two and three can be combined and thought of as preparational thought.)
4. Phase of concrete operations, (roughly, ages: 7-11/12). The child has the mental capacity to order and relate experiences to an organized world.

Reversibility appears. This is the permanent possibility of returning to the starting point of operation in question and a child's ability to perform concrete operations dependent upon it.

5. Formal operations, (roughly, ages: 11/12 and up). Thinking beyond the present and the formation of theories becomes possible. This is cognitive thought.¹

Correlating closely with stages in child development were stages in child behavior. The Gesell Institute of Child Development had conducted extensive studies in child behavior in school. They found that a child moved through stages and cycles of growth. In summary, these six successive stages which occur repeatedly in the years from two to sixteen may be described as follows:

1. A stage of calm, of integration, of meshing with the environment.
2. A stage of boiling and bubbling begins to occur.
3. A stage when an organization of opposites takes place.
4. A stage of withdrawal. Nothing seems to be happening.
5. A stage of out of bounds, a time of expansion and release.
6. A stage of return to the beginning of the cycle.²

The average age of a child entering first grade in the United States was somewhere between five and one-half to six years. Based on the studies of Ilg and Ames, the writer followed a part of these six successive stages of child behavior for this age level for better understanding by the reader:

Half-year intervals still show striking changes,

¹Mair, H. W., Three Theories of Child Development, pages 91-141.

²Ilg, F. and Ames, L. B., School Readings, pages 9, 10.

and the five year old has changed a great deal by five and one-half. Parents are the first ones to recognize the changes, and the significant question they ask is, "What has gotten into him?"

What has happened is that their darling little cooperative five year old is beginning to explode, to oppose, to demand. This stage, just beginning at five and one-half and rising to a crescendo by six, often takes parents unawares.

Whatever the six year old does, he does with impulsive enthusiasm. His exuberance is contagious. When he is ready to use his forces positively he draws delightful scenes with paint or crayon, or he listens to stories with avidity, and spontaneously wishes to dramatize them, or he gives himself to any intellectual pursuit offered him. He is characteristically eager to learn.³

From the evidence gathered from Piaget's stages of development and Gesell's stages of child behavior the writer has developed an insight as to what stage the average six year old falls in developmentally and behaviorally. The question remained: Is "modern" math suitable for this age level?

Cambridge Report

Before proceeding with evidence supporting "modern" math the writer felt it would be of value to discuss the "Cambridge Conference Report". This report dealt with goals for "modern" math. The main conclusion of the report was:

A student who has worked through the full thirteen years of mathematics in Grades K to 12 should have a level of training comparable to three years of top-level college training today; that is, we shall expect him to have the

³Ibid., pages 266-267, 274-275.

equivalent of two years of calculus, and one semester each of modern algebra and probability theory. We propose to gain three years through a new organization of the subject matter and virtually total abandonment of drill for drill's sake, replacing the unmotivated drill of classical arithmetic by problems which illustrate new mathematical concepts.⁴

The Cambridge Report also proposed that the objectives for mathematics instruction in the elementary Grades K to 6 should be the development of familiarity with the real number system and the main ideas of geometry. To implement this proposal in Grades K to 2, it suggested that the number line, including negatives, be used from the very beginning. It suggested an early introduction to the idea of inequalities and to the idea of "the neighborhood of a point" on the number line. It emphasized the use of physical interpretations of addition and multiplication, and the discovery by the children of the commutative nature of addition and multiplication. It proposed that the children deal with problems such as multiplication of a number "a little bit more than three" by a number "a little bit less than five". It also proposed the use of a box as a placeholder in simple algebraic problems. It recommended an introduction to Cartesian coordinates through appropriate games; experience with the additive property of area, closely integrated with the operation of multiplication, and experience with the symmetries of certain plane and solid figures. It also suggested the early use of sets and the

⁴Adler, I., "The Cambridge Report: Blueprint or Fantasy?", The Arithmetic Teacher Vol 13: 179, 180, March, 1966.

concept of function. This, of course, is only a partial list of topics proposed for Grades K to 2. For these early grades the report emphasized that children can study mathematics more satisfactorily when each child has abundant opportunity to manipulate suitable physical objects. Content from "traditional" math was included in the program.

The question remained: Can the children really learn so much? Can they learn at an early age the sophisticated mathematics that has been traditionally taught at a later age? The consensus of the Cambridge Conference group was positive in the case of the "bright" child. The group felt that the average could respond if courses were attractive and meaningful, and the methods used were psychologically sound.

Another question was raised: Is a downward movement of the curriculum really psychologically sound? According to Piaget's stages of development the sequence of those stages are fixed. During each stage, the child acquired the abilities that prepared him for the next stage. Hence he cannot skip over a stage. However, the preparation for each stage depends on three principal factors: (1) Maturation of the nervous system, (2) Experience acquired in interaction with the physical environment, (3) The influence of the social milieu. At least two of these factors are subject to our control, since they are cultural rather than genetic. Consequently, while the sequence of the stages is constant, the ages at which they occur are not. Thus acceleration of learning is completely

possible.⁵

The summary presented of the Cambridge Conference is quite limited. The purpose of its inclusion was to establish evidence that "modern" math proposed to teach more math to the first grade child in the future.

Summary

Consideration of studies made by Jean Piaget in the area of child development and Arnold Gesell in child behavior were important factors to any curriculum change. The Cambridge Conference Report proposed changes in the mathematics curriculum. Changes proposed in mathematics were based on needs in a changing society. Psychological studies were considered in proposing these changes.

A review of materials received from publishers and of literature pertaining to the "modern" math in the first grade, was discussed in the following chapter.

⁵Ibid., pages 180-184.

CHAPTER IV

REVIEW OF LETTERS AND LITERATURE

The main purpose of this paper was to gather evidence supporting the "modern" math program in the first grade. The writer attempted to gather the evidence by reviewing available literature in the Montana State University Library and sending letters of inquiry to ten publishers. (See Appendix) Letters were sent to: D.C. Heath and Company; Harcourt, Brace and World, Inc.; Ginn and Company; Science Research Associates, Inc.; McCormick-Mathers Publishing Co.; L.W. Singer Company; American Book Company; Holt, Rinehart and Winston, Inc.; Scott, Foresman and Company; and Laidlaw Brothers Publishing Company. The writer received seven responses. Only the responses that pertained to first grade math were reviewed.

Review of Letters

An article presented by Scott, Foresman and Company discussed modern trends in mathematics today. Sister Mary Petronia, Ph.D., summarized her discussion with the following remarks:

I have discussed many of the modern trends in mathematics today. All are good, if used properly. "Modern mathematics", as I said before, aims to have children understand why they do what they do. What could not be good about that? Teachers, let us be diligent and use every effort to have those we teach understand the why of mathematics so that they may learn to appreciate and enjoy it.

In the teaching of mathematics, especially at the lower levels, I think there is the danger of

becoming too abstract too soon and not giving children enough use of physical materials in an arithmetic class, making the transition from the concrete to the abstract before children are really ready for it. Children need a great diversity of types of experiences. Of course teachers must be careful not to let the pendulum swing too far in either direction, the concrete or the abstract. In the first case children will become too dependent on physical things or pictures or tools and will never acquire the habit of thinking abstractly. In the second case they may be able to manipulate symbols and signs efficiently but never really understand what it is all about.

It has often been said, and it is true, that teachers are apt to teach the way they have been taught rather than the way they have been taught to teach. Teachers, let us not be in this category! Rather, let us constantly have a research attitude, continuously examining our methods of teaching with a view of improving them. A good mathematics program depends not so much on the text as on the teacher and his methods of teaching. The teacher is really the key.¹

The L.W. Singer Company, Inc., did a non-verbal approach to teaching of arithmetic study in the primary program. The study was done in Ghana, 1962, as a project of the African Education Study of Education Services Incorporated. This study emphasized the fact that math could be taught successfully to first grade children regardless of language and cultural factors. A summary of the study follows:

Eleven classes of Primary One, first grade children in Accra, Ghana began work in Sets and Numbers, Book 1A, at the mid-point of the academic year, 1961-62. Members of the Sets and Numbers project staff were in Accra in February and in May to provide the initial orientation for the classroom teachers and to aid in organizing the

¹Sister Mary Petronia, Ph.D., A Second Look At "Modern Mathematics", pages 20-21.

testing program at the end of the term. The regular classroom teachers taught the program and the same workbooks used in classes in the United States were used in the Ghanaian classes. The eleven classes were from five schools in and around Accra. Schools were elected by the Ministry of Education, Ghana, to provide a broad representation in type of background of the students. The program was under the direct supervision of the Ministry of Education. At the end of the term, approximately two-thirds of the students in each class had completed work in Book 1A. The same achievement test given first-grade experiment classes in the United States at the completion of Book 1A was administered to these children. Since the upper two-thirds only of each Ghanaian class were represented in the test results, these results were compared with the upper two-thirds of the children in each American class.

The sample size of the Ghanaian classes were 215; the American classes were 1195. Mean of the Ghanaian classes was 72.4; the American classes 73.8. The standard deviation of the Ghanaian classes was 9.16; the American classes 8.2. Difference of means was non-significant.²

Suppes published a report of six experiments with primary children dealing with mathematical concept formation. Tentative conclusions of the study that are important for pedagogical procedures were:

1. Formation of simple mathematical concepts in young children is approximately an all or none process.
2. Learning is more efficient if the child who makes an error is required to make an overt correction response in the presence of the stimulus to be learned.
3. Incidental learning does not appear to be an effective method of acquisition for young children.
4. Contiguity of response stimulus, and reinforcement enhances learning.
5. In the learning of related mathematical concepts the amount of overall transfer from the learning of one concept to another is surprisingly small.
6. Transfer of a concept is more effective if the

²A bulletin received from the L.W. Singer Company, Inc., 1969.

learning situation has required the subject to recognize the presence or absence of a concept in a number of stimulus displays than if learning has involved matching from a number of possible responses.³

The writer felt the importance of the study was consideration of how children learn. This was the area that raised the question of whether or not there was a conflict between "what" was in the first grade curriculum and "when" a child could readily comprehend these concepts.

Other responses from publishers indicated that research and experimental programs were constantly being pursued. Kingston replied: "There is much research to support any of a number of approaches at any given level. However, there is, at the same time, much that is a matter of taste and/or judgement, and which still requires research."⁴

The Greater Cleveland Mathematics Program published by SRA, Science Research Associates, Inc., was one of the first publishers of new math series. The program claimed these following advantages:

1. Program developed by outstanding mathematicians and educators.
2. A program pretested and revised in the classroom.
3. A program evaluated and proved effective with more

³Suppes, Patrick, "The Development of Mathematical Concepts in Children", Technical Report No. 63, Psychology Series, Institute for Mathematical Studies in the Social Sciences, pages 32-33.

⁴Letter received from Sibert W. Kingston, American Book Company, May, 1969.

than one hundred thousand pupils.

The worth of the new math program was well summarized in the following paragraphs:

The Greater Cleveland Mathematics Program content is mathematically correct, and its sequence in presentation is based on evidence that traditional programs have underestimated the mathematical capabilities of children. Today, it is clear that advanced concepts and ideas can be successfully introduced much earlier than had previously been thought possible or practical. In this regard, the developers of the program have taken into consideration the recommendations and experimental findings of the major curriculum study groups, and the best thinking of such noted mathematicians as Dr. John G. Kemeny, chairman of the Department of Mathematics, Dartmouth College.

GCMP content recognizes that there is an acute need for a higher level of achievement in mathematics during this nuclear-missile age. There is more demand for mathematical reasoning not only in the physical sciences, but in the social sciences as well. With the increasing need for mathematics skills, it now becomes necessary to teach pupils not only how to compute, but also how to think mathematically. This cannot be done by drill and rote learning.⁵

Review of Literature

The writer reviewed literature with the following objectives as guides:

1. What rationale was considered for the new math programs?
2. Were psychological studies on child development and child behavior considered in the development of the new math program?

⁵SRA, "The Wonderful World of Modern Math", a bulletin, page 1.

3. Were there any studies supporting the new math program?

Hartund and others, gave this justification for new math:

The old view that arithmetic was largely a set of number facts and computational procedures governed by rules was no longer dominant. The demands of a changing society have created the need for a new look at what are considered the fundamentals of arithmetic. Other reasons for diversion from computational procedures:

1. Increasing realization that emphasis on a set of basic ideas that are structured in a meaningful way increases the efficiency of learning.
2. The need to develop readiness for further study of mathematical ideas in the upper grades requires that the children be introduced to some of the fundamental mathematical concepts in the arithmetic program for primary grades.
3. In primary grades, it is important, it is wise, and it is possible to lay a foundation on mathematical concepts on which the program for the middle and upper grades can be built.⁶

Authorities in the psychology of learning differed in their opinions of new math. Some felt the approach should be more in harmony with what they have found about the learning process.

Bruner stated his ideas on this subject as follows:

Many people believe that the pre-school child should be exposed to manipulation, classification, and ordering as a foundation for mathematical experiences.

One wonders whether it might not be interesting to devote the first two years of school to a series of exercises in manipulating, classifying, and ordering objects in ways that highlight basic operations of logical addition, multiplication, inclusion, serial ordering, and the like.

Effects of this type of approach would be, we think, to put more continuity into science and mathematics and also

⁶Hartund, M. L., Engen, H. V., Gibb, E. G., Stochl, J. E., Knowles, L., Welch, R., Curriculum Foundation Series "Seeing Through Arithmetic", Introduction.

give the child a much better and firmer comprehension of the concepts which unless he has this early foundation, he will mough later without being able to use them in an effective way.

The abstract ideas of mathematics can be learned through concrete manipulative experiences. If a child is to function in the real world of mathematics, then his learning experiences must begin at the beginning. Counting is not the beginning.⁷

A review of the principles of new psychology by Wheeler summarized the trends of thought on learning. The writer noted a discrepancy in the list with the findings of Suppes with regard to incidental learning. The principles are:

1. Learning is a function of maturation and insight. It is a growth process that follows laws of dynamics.
2. Learning does not proceed by trial and error.
3. Personality and the relationship between the learner and teacher are by far more important. Atmosphere of class room is more important than textbooks.
4. Learning depends on the will to learn.
5. A large part of the most efficient learning of mathematics is incidental.⁸

The writer found Piaget's ideas, in regard to new math, reviewed in several sources. Most of the comments had to do with his five stages in the continuum in child development that were discussed in Chapter III. Several of the studies dealt with the specific topics that Piaget studied in his book entitled: The Child's Conception of Number.

⁷Lucas, J. S. Neufeld, E., Developing Pre-Number Ideas, Introduction.

⁸Wheeler, Raymond, "New Psychology of Learning", The National Council of Teachers of Mathematics, Tenth Yearbook, pages 237-239.

Stommel found that results generally agree with Piaget's developmental stages for number concept. There was great variability from child to child in the consistency of responses made.⁹

Support was found for Piaget's contention that the development of the logical operations of class and seriation (for cardinal and ordinal numbers) is required for number conceptualization by Benson.¹⁰

A study of the age for acquisition of conservation of length was between seven and eight years old as proposed by Piaget. The study was done by Sawad and Nelson.¹¹

Diggory conducted a study that compared Piaget's findings with American children. Diggory found:

The preoperational child is said to lack the concept of "reversibility". Preoperational stage lasts until a child is somewhere between six and eight years old in American middle class culture and older in a lower class or primitive culture.¹²

⁹Stommel, Lewis John, "The Prediction of First Grade Achievement in Arithmetic by an Instrument Based on Piaget's Theory of the Formation of Number Concept", Dissertation Abstracts XXVII, January-February, 1967, 2457A.

¹⁰Benson, Francis and Mitchell, Arthur, "An Examination over an Eight Month Period of Piaget's Concept of Number Development and the Presence or Absence of Certain Interrelated Tasks in a Group of First Grade Children", Dissertation Abstracts XXVII, March-April, 1967, 3300A.

¹¹Sawad, Daiyo and Nelson, L. Doyal, "Conservation of Length and the Teaching of Linear Measurement", The Arithmetic Teacher Vol 14, 345-348, May, 1967.

¹²Diggory, Sylvia Farnham, "On Readiness and Remedy in Mathematics Instruction", The Arithmetic Teacher Vol 15: 614-621, November, 1968.

Piaget found "reversibility" to occur about seven years old.

Jean Piaget made the following comment about the five stages in continuum in child development:

The stages are real and the sequence is constant, but it is a serious misinterpretation to place time boundries on the stages. Americans do put time boundries.¹³

To emphasize the importance of Piaget's study in relation to new math the writer found this statement by Piaget appropriate:

When adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth.¹⁴

Bruner, made this statement which reflected a theory of learning about new math in the first grade:

Any subject can be taught effectively in some intellectually honest form to any child at any stage of development.¹⁵

The writer found two studies that were related to new math in the first grade. One problem that existed was a testing device. Two tests were found that tested new math. One was a Contemporary Mathematics Test, Upper Elementary Form W, 1966, published by the California Test Bureau. This test consisted of a general cross section of modern mathematics understanding and concepts. Another

¹³Piaget, Jean, "How Children Form Mathematical Concepts", Scientific American, November, 1953, page 74.

¹⁴Schlinsog, George W., "Mathematics in Kindergarten?", The Arithmetic Teacher Vol 14: 292-295, April, 1967.

¹⁵Ibid., page 292.

test, Primary Arithmetic Understanding Test, by Jewell L. Garner was designed to assess basic arithmetical understanding. There was no written computation on the test. Responses were to one hundred fourteen multiple choice items based on fifty-nine arithmetic principles taught in the primary grades. The test consisted of four parts:

1. numbers and numeration
2. addition and subtraction
3. multiplication and division
4. meaning of fractional numbers.

The Metropolitan Achievement Test, was also used to evaluate one approach to teaching of arithmetic.

Flourney noted in a study of understanding of arithmetic in primary grades, using the Primary Arithmetic Understanding Test, that:

1. Third graders performed below expectations in comparison with pupils in grades one and two.
2. Grades two and three did about the same on number and numeration section. Therefore, he recommended the study should be extended for third graders.
3. Because some first and second graders had some ideas about multiplication and division perhaps it could be introduced in the second grade.
4. The study recommended more study of fractions for third graders.¹⁶

Another study pertaining to teaching of new math was done by Hunt. Nursery school children, three, four, five years old, were encouraged to play games involving numbers. The game or play approach was continued through the lower grades with formal instruction in

¹⁶Flournoy, Frances, "A Study of Pupils Understanding of Arithmetic in Primary Grades", The Arithmetic Teacher Vol 14; 481-485, October, 1967.

arithmetic processes given only when pupils evidenced curiosity about processes when they were used by the teacher. Hunt used the Metropolitan Achievement Test to evaluate his study. He found:

1. Grade one ranked in the bottom tenth
2. Grade two ranked in the bottom quarter
3. Grade three ranked in the bottom half
4. Grade four ranked in the upper two-thirds
5. Grade five ranked in the upper third.¹⁷

Other studies indirectly affecting new math dealt with concepts, socio-economic status, number experience prior to school entrance. Buchanan found that pupils were able to learn to write numbers legibly, but this did facilitate arithmetic conceptualization.¹⁸ Skypek noted the relationship of socio-economic status to concept-test scores was highly significant in favor of middle class children.¹⁹ It was established by Rickard, that children have acquired considerable number knowledge before entering first grade. Girls were consistently a little more successful than boys.²⁰

¹⁷Newport, John F., "Are Almost All Children Disadvantaged in Mathematics?", The Arithmetic Teacher Vol 14: 21-23, January, 1967.

¹⁸Buchanan, Van Dyk, "Beginning Instruction in Writing Numerals in Kindergarten", Dissertation Abstracts XXVII, March-April, 1967, 3217A.

¹⁹Skypek, Dora Helen, "The Relationship of Socio-Economic Status to the Development of Conservation of Number", Dissertation Abstract XXVII, September, 1967, 1012-13A.

²⁰Rickard, Esther E. Sidwell, "An Inventory of Number Knowledge of Beginning First Grade School Children, Based on the Performance of Selected Number Tasks", Dissertation Abstracts XXVIII, August, 1967, 406A.

Summary

A review of the letters of inquiry written by the writer discussed the philosophy of modern math, presented studies of experiments dealing with mathematical concept formation, and an actual study done with first grade children in Ghana, Africa.

The review of literature fulfilled the objective proposed by the writer.

1. Rationale was established for the modern math program.
2. Psychological studies pertaining to modern math were considered.
3. Studies pertaining to new math were conducted, but the writer found the number of studies limited.

The results of this study were summarized and conclusions and recommendations for further study were included in Chapter V.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

The purpose of this study had been to establish whether there was or was not statistical or objective evidence supporting modern math in the first grade curriculum. Evidence was gathered by reviewing literature in the Montana State University Library and letters of inquiry to publishers of modern math series.

Modern math was defined in Chapter II as a subject that included arithmetic as well as algebra, trigonometry, and geometry. Modern math dealt with basic concepts that explained rules and serve to unify the branches of math. The goal of new math was gradual growth in understanding the basic mathematical ideas and seeing the connections that existed among them so that there will be greater flexibility and creativity in application.

The movement toward a modern math program started in the nineteen forties. Leaders of the movement were mainly mathematic educators whose interests lay more in the psychology and learning theory than advanced math.

Much of the early modern math curriculum resulted from the work of study groups sponsored by private and governmental funds. Some of the leading groups were, the School Mathematics Study Group,

the Committee on the Undergraduate Program in Mathematics, the Greater Cleveland Mathematics Program, the Nuffield Mathematics Project, the Cambridge Conference of Study of Mathematics. These groups conducted experimental programs, published texts, developed teacher training curriculum. The mathematical correctness of the modern math program was attributed to these groups.

The development in psychology of learning was one of the early reasons for a complete revision in the math program. Jean Piaget, a Swiss psychologist contributed much of the early theory that was studied by American educators. Chapter III reviewed the philosophy of Piaget and Gesell. The review was followed by a summary of the Cambridge Report to reveal the correlation between future modern math and developments in the psychology of learning.

Review of letters and literature reinforced earlier statements about psychological findings and the modern math curriculum. Studies supporting modern math were very limited.

Conclusions

The study led to the following conclusions about objective or statistical evidence supporting modern math in the first grade:

1. No valid comparison can be made between modern math and traditional math as the writer found little if any, objective evidence to support the use of one in preference to the other.

2. The writer did not find any studies specifically comparing the modern math to the traditional math.
3. Studies concerning learning development of a child were considered in most programs reviewed.

Recommendations

Modern math has been introduced to the school curriculum. Rather than study whether the traditional math program was better than the modern math program is no longer the issue. What is important is constant evaluation of the modern math program and adapting it to the needs of the individual child. As new developments occur in learning, the writer feels it important to incorporate these developments to build a math program suitable to the level of development of the child.

APPENDIX

APPENDIX

LETTER OF INQUIRY

Dear Sir:

This letter is a letter of inquiry requesting statistical or objective evidence regarding your series, _____, supporting the fact that Modern Math, as you have presented it should be taught in the first grade.

At this time I am working on a professional paper for partial fulfillment of the requirements for a Masters Degree in Education at Montana State University, Bozeman, Montana.

Your assistance will be most helpful.

Sincerely yours,

Regina McNair

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