



Modeling and analysis of thin-film, piezoelectric actuators
by Ashley Erin Childs

Thesis submitted in partial fulfillment of the requirements for the degree of Master of Science . in
Mechanical Engineering
Montana State University
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Abstract:

This paper contains a description of the modeling and analysis of thin-film, piezoelectric actuators that were designed by a multi-discipline research team at Montana State University (MSU). The actuators consist of layers of poly(vinylidene fluoride) that are joined by epoxy and are formed into double S-curve shapes. The actuators were developed to actively control vibrations on experimental platforms in micro-gravity environments. To develop a robust active control algorithm, information about the actuator behavior was required. Analytical and finite element models were developed to provide this information, and analyses using these models were performed. The following results of the analyses are presented: the linear range of the actuator's behavior, the optimum radii of curvature, the effects of initial curvature, and the effect of the epoxy—either a uniform or non-uniform layer. This paper also shows that the analytical and finite element results correlate with each other and are reasonable when compared with experimental data. The models can therefore be used as design tools for future applications of the actuator.

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LIST OF SYMBOLS

CHAPTER 2

Symbol.....	Description
N	internal resultant normal force in a curved beam
$d\phi$	differential arc angle of a curved beam segment (cbs)
Q	internal resultant shear force in a curved beam
ds	differential arc length of a cbs
R	radius of curvature of the middle line (undeformed)
q	distributed load on a curved beam
M	internal resultant moment in a curved beam
$\bar{u}$	displacement field of a point in a cbs
$\bar{x}$	position of point P'
$\bar{X}$	position of point P
r	radius of curvature of a deformed cbs
FR	fixed reference frame
P	point in the undeformed cbs
P'	point P in the deformed cbs
$\bar{X}_o$	position of point A
A	point on the middle line of the undeformed cbs
$\bar{e}_r$	unit vector, radial direction, undeformed coordinates
y	radial distance from A to P
R'	radius to P
A'	point A in the deformed cbs

LIST OF SYMBOLS - Continued

$\vec{e}_\phi$	unit vector $\perp$ to $\vec{e}_r$, undeformed coordinate system (cs)
β	angle between $\vec{e}_r$ and line A'P'
$\vec{x}_o$	position of point A'
v	tangential component of the middle line displacement
w	radial component of the middle line displacement
u_r	displacement component in the radial direction
u_ϕ	displacement component in the tangential direction
ϵ_r	radial strain
ϵ_ϕ	tangential strain
$\gamma_{r\phi}$	shear strain
h	thickness of the curved beam
ϵ_o	strain at the middle line
$d\phi'$	differential arc angle of a cbs (deformed)
ds'	differential arc length of a cbs (deformed)
d_{ij}	piezoelectric strain coefficient: i dir. E field, j dir. strain
σ_ϕ	tangential stress
E_i	electric field intensity: i direction
Y	Young's modulus
b	width of a beam's cross-section
A	cross-sectional area of a beam
I	moment of inertia of the electroded film about the 2 axis
y_i	radial distance from the middle line of the i^{th} layer
Q_i	resultant internal shear force in the i^{th} layer
N_i	resultant internal normal force in the i^{th} layer

LIST OF SYMBOLS - Continued

M_i	resultant internal moment in the i^{th} layer
R_1	radius of curvature of the middle line, inner layer (1)
R_3	radius of curvature of the middle line, outer layer (3)
R	radius of curvature to the middle line of the bimorph
h_g	thickness of the epoxy layer
$\Delta\phi$	small segment of a bimorph
$\varepsilon_{\phi i}$	tangential strain in the i^{th} layer
E_{31}	electric field intensity in the 3 direction in layer 1
E_{33}	electric field intensity in the 3 direction in layer 3
Y_g	Young's modulus of the epoxy layer
A_g	cross-sectional area of the epoxy layer
I_g	moment of inertia of the epoxy layer about the 2 axis
$\Delta E_3 = E_{33} - E_{31}$	difference in the electrical field intensities, layers 3 and 1
w_i	radial displacement of the middle line of the i^{th} layer
v_i	tangential displacement of the middle line of the i^{th} layer
ϕ_i	arc angle of the small bimorph segment for the i^{th} layer
I_{eq}	equivalent moment of inertia of bimorph about the 2 axis
h_{eq}	equivalent thickness of the bimorph
$E_3 = E_{33} + E_{31}$	sum of the electrical field intensities in layers 3 and 1
ε_{oi}	strain at the middle line of the i^{th} layer
A_{eq}	equivalent cross-sectional area of the bimorph

CHAPTER 3

Symbol.....	Description
d_{ij}	piezoelectric strain coefficient: i dir. E field, j dir. strain

LIST OF SYMBOLS - Continued

S_{ij}	elastic compliance matrix components
V	wave speed
Δt	time between reflections
h	sample thickness
V_T	transverse wave speed
V_L	longitudinal wave speed
ν	Poisson's ratio
G	shear modulus
Y	Young's modulus
ρ_{avg}	mass density averaged over several samples
$U_x^{(R)}$	reflected wave field
AR	magnitude of $U_x^{(R)}$
ρ^i	mass density of medium i
V_L^i	longitudinal wave speed in medium i
I	magnitude of the incident wave
G_g	shear modulus of the epoxy
Y_g	Young's modulus of the epoxy

CHAPTER 4

Symbol.....	Description
∂V	differential of the voltage
ΔV	voltage gradient
r_e	distance between electrical charges

LIST OF SYMBOLS - Continued

P	poling direction
L	length of the piezoelectric film used
ϕ_0	arc angle of each beam segment
ϕ_i	arc angle along beam segment i
β_i	angle that the middle line of beam segment i rotates
v_i	tangential displacement of beam segment i
w_i	radial displacement of beam segment i
Q_i	resultant internal shear force in beam segment i
N_i	resultant internal normal force in beam segment i
M_i	resultant internal moment in beam segment i
Δ	total vertical displacement of an actuator
δ	vertical displacement of a bimorph
R_i	radius of curvature of beam segment i
N_{oi}	reaction in the tangential direction in beam segment i
Q_{oi}	reaction in the radial direction in beam segment i
M_{oi}	reaction moment in beam segment i
M^*	magnitude of the constant moment
Λ_i	electrical terms for beam segment i
Γ_i	electrical term for beam segment i
A_i	constant, differential equation solution, beam segment i
B_i	constant, differential equation solution, beam segment i
C_i	constant, differential equation solution, beam segment i
α_i	particular solution for beam segment i
M_i^*	magnitude of the constant moment in beam segment i

LIST OF SYMBOLS – Continued

s_i	arc length along beam segment i
L_i	arc length of beam segment i at ϕ_o
E_{3i}	E_3 , beam segment i
ΔE_{3i}	ΔE_3 , beam segment i
$l = L / 2$	length of the quarter model
$r = R_2 / R_1$	ratio of the radii of curvature of the beam segments
M_A	externally applied moment at A
F	externally applied force at A
A	point on the middle line, free end, cantilevered beam
B	point on the middle line, inflection point of the beam
ϕ_o	arc angle of each beam segment
ϕ	arc angle along beam segment
R_i	radius of curvature of beam segment i
Q_i	resultant internal shear force in beam segment i
N_i	resultant internal normal force in beam segment i
M_i	resultant internal moment in beam segment i
M_o	moment at the inflection point, left-hand segment, 1
M_{vi}	piezoelectric pseudo-moment in beam segment i
M_{totali}	total moment in the beam segment i
C	point on the middle line, slightly before the fixed end
U_i	total strain energy in beam segment i
U_{total}	total strain energy in both beam segments
θ_A	total slope at A
δ_A	total vertical displacement at A
$K = F / \delta_A$	stiffness of the bimorph

LIST OF SYMBOLS - Continued

K_g	stiffness of the bimorph with an epoxy layer
dV	applied voltage gradient
K_{vi}	slope of the voltage-displacement curves
K_{vp}	% difference, K_{vi} for bimorph w/ and w/out epoxy

CHAPTER 5

Symbol.....	Description.
u	displacement field approximating polynomial
$\{T\}$	mechanical stress matrix
$[c^E]$	stiffness matrix at constant E field
$\{S\}$	mechanical strain matrix
$[e^S]$	piezoelectric matrix at constant strain
$\{E\}$	electrical field matrix
$\{D\}$	electrical displacement matrix
$[\epsilon^S]$	dielectric matrix at constant strain
$[s^E]$	compliance matrix at constant E field
$[d^T]$	piezoelectric matrix at constant stress
$[\epsilon^T]$	dielectric matrix at constant stress
ϵ_{ij}	strain components
S_{ij}	compliance matrix components
T_{ij}	stress matrix components
d_{ij}	piezoelectric matrix components

LIST OF SYMBOLS - Continued

E_i	electric field intensities
σ_{ij}	stress components
Y_i	Young's moduli in the i^{th} material direction
G_i	shear moduli in the i^{th} material direction
ν_{ij}	Poisson's ratios

ABSTRACT

This paper contains a description of the modeling and analysis of thin-film, piezoelectric actuators that were designed by a multi-discipline research team at Montana State University (MSU). The actuators consist of layers of poly(vinylidene fluoride) that are joined by epoxy and are formed into double S-curve shapes. The actuators were developed to actively control vibrations on experimental platforms in micro-gravity environments. To develop a robust active control algorithm, information about the actuator behavior was required. Analytical and finite element models were developed to provide this information, and analyses using these models were performed. The following results of the analyses are presented: the linear range of the actuator's behavior, the optimum radii of curvature, the effects of initial curvature, and the effect of the epoxy—either a uniform or non-uniform layer. This paper also shows that the analytical and finite element results correlate with each other and are reasonable when compared with experimental data. The models can therefore be used as design tools for future applications of the actuator.

CHAPTER 1

INTRODUCTION

A multi-discipline research team at Montana State University (MSU) has developed a vibration isolation system designed for use in micro-gravity environments that exist on space vehicles. The system operates using a piezoelectric actuator designed at MSU. The actuator consists of thin, piezoelectric-polymer films formed into a double S-curve shape.

The research team also developed an active control algorithm for the system. To make the algorithm more robust and to optimize the system's performance, the researchers needed to answer several questions. What were the optimum radii of curvature for the actuator? Should the radii be the same or should they be some ratio of each other? Should there be any initial curvature? Answers to these questions lay in analytical models. The analytical model could also describe the epoxy layer that joins the thin films, and it could provide valuable information about the epoxy layer's effect on actuator behavior.

The analytical models, limited due to their complexity, could not provide all the answers. A finite element model, however, could describe other effects that the analytical model could not. It could illustrate the effects of non-uniformity in the epoxy layer thickness, and perhaps more importantly, the finite element model could also answer questions about the linearity of the actuator response. Determining the linear range of the actuator's force-deflection curve was important for developing a robust control algorithm.

Finite element and analytical models were developed to eliminate unknowns about the current actuator design, but the team can also use these models in the future as design tools. They can use these design guides to create actuators for different force-deflection responses and customize them for specific applications. This paper describes the development of these models. It answers questions regarding the current actuator design, and describes how to use the tools for

future applications. The paper begins with a brief history of piezoelectricity, provides some background information, and then it develops the theory behind the analytical models. Next, the paper describes the finite element model, and finally, it discusses the results.

In the 1880's, Pierre and Jacques Curie applied pressure to a quartz crystal and discovered an electric charge was produced. Conversely, when the Curies applied an electric field to the crystal, it changed shape. They named this effect "piezoelectricity," derived from the Greek verb "piezen," to press. They determined that the piezoelectric effect is a characteristic of crystals lacking a center of symmetry [1]. This characteristic causes charges to displace asymmetrically under mechanical stress. Asymmetrically distributed charges induce dipole moments in crystals. The dipole moment causes the material surfaces to develop an effective charge. On the other hand, under the presence of an electrical field, repulsion or attraction forces displace the charges, producing mechanical strain.

Scientists failed to realize the benefits of piezoelectricity until more than 40 years later, during World War I, when Langevin used quartz to generate and receive sound waves under water. This work led to the development of sonar. Other applications followed this, such as microphones and telephone receivers [1]. Further research revealed piezoelectric behavior in many biological substances, such as proteins, cellulose, and items containing collagen. Examples of these organic substances include wool, silk, DNA, RNA, bone, tendon, muscle, and skin [1, 2].

In 1969, Kawai discovered the effect in other organic materials [3]. He found that elongated, polarized films of synthetic polymers were piezoelectric. Poly(vinylidene fluoride) (PVDF) showed the largest effect. PVDF is inherently polar because the fluorine ions are negatively charged and the hydrogen ions positively charged, and lie on opposite sides of the carbon backbone. Despite its polar molecular structure, the average dipole moment in an amorphous region of PVDF is zero due to the random orientation of the molecules along the polymer chain. However, PVDF is semi-crystalline, and it has tiny crystals embedded in the amorphous matrix. Elongating films of PVDF transforms the crystals into the polar "beta" phase.

Poling¹ orients the dipole moments of all the crystals in the same direction to produce a macroscopic polarization. Poly(vinyl chloride) (PVC) and nylon also exhibit piezoelectric properties, but the effects are not as significant as that found in PVDF.

Today piezoelectric polymers and ceramics are used in a broad range of applications: biomedical devices, ink-jet printers, structures, traffic systems, robotics, aerodynamics, vibration control, and recreational equipment [4, 5, 6, 7, 8 and 9]. In any given system, the piezoelectric material can be used as a sensor and/or an actuator. The material acts as a sensor if pressure is applied to produce a signal sent to a control system. For example, piezoelectric cables can be placed on road surfaces [5]. When a car drives over them and applies pressure, the piezoelectric material experiences a change in electric field. This produces a signal that is then transmitted to a control system, telling it a car is present. Conversely, a piezoelectric material acting as an actuator deforms if a voltage is applied. An example of piezoelectric actuators is found in some ink-jet printers [8, 9]. A miniature piston made of piezoelectric ceramic expands when a voltage is applied, reducing the volume of the ink chamber and forcing ink out of a tiny hole.

Piezoelectric ceramics are useful as actuators, but since PVDF is a compliant polymer, it is not as useful for actuator applications, especially at low frequencies [2]. Several researchers have created certain configurations of PVDF, however, which do work well as actuators for some applications. These actuators involve layers of thin films that form either straight or curved elements. Ervin and Brei [10] indicate that stacks of straight, thin film actuators can be configured to customize force-deflection behavior or frequency response. These actuators are straight elements that are joined at their ends by a spacer, illustrated in Figure 1 (modified from [10]). Each half of the actuator, L , consists of two segments of equal length, $L/2$, to which opposing piezoelectric pseudo-moments are applied.

¹ Poling is a manufacturing process. The material is heated to near or above its Curie temperature. A large voltage is then applied, which aligns the dipoles, while the material is cooled back down to ambient temperature. The dipoles are then "locked" into the new position.

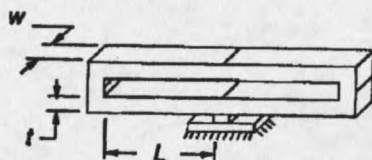


Figure 1. Straight actuator.

This design incorporates two layers of PVDF film joined by a layer of epoxy, called a bimorph. If more than two layers of PVDF film are used to create each layer of thickness " t ," then the group of layers is referred to as a multi-morph.

Typically a bimorph is used in bending mode. In Figure 2 (modified from [4]), one bimorph in bending mode and one in extension mode are illustrated. To create a bimorph that operates in bending mode, the polarity (denoted P on each layer) of the material must be in opposite directions with the same electric field applied across each layer, as is shown in the figure, part A. Bending modes also exist if the polarity of the layers is in the same direction but with the electric field in each layer equal and in opposite directions. In either case, one layer will try to contract and the other layer will extend, creating the bending motion.

In the extension mode, both layers either contract or extend together. To create a bimorph that operates in this mode, the polarity of the material must be in the same direction, as is shown in Figure 2, part B. Extension mode also exists if the material polarity of each layer is opposite the other, as long as the electric field applied is equal and opposite across each layer. Figure 2 shows the cases where the electric field is equal and in the same direction in both part A and B.

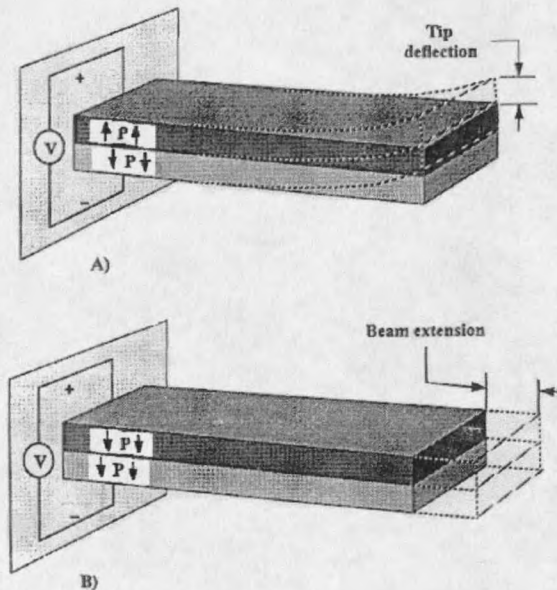


Figure 2. A) Bending mode. B) Extension mode.

Stampleman and von Flotow [12] have proposed an actuator design using the bending mode of a single circular arc, shown in Figure 3. This actuator consists of a bi- or multi-morph. The researchers specifically designed it for use in isolating vibrations in micro-gravity. Multiple actuators would be placed around an experimental platform and controlled appropriately to isolate the experiment from unwanted accelerations.



Figure 3. Single Circular Arc Actuator.

The MSU research team also developed their actuator design for vibration isolation in micro-gravity environments, like that on the space shuttle in orbit, or on the international space station. The future space station will introduce an opportunity for scientists to perform more

experiments in zero gravity, studies that might otherwise be difficult to perform on the earth due to gravity's effects. Examples of such research include the following: producing defect-free crystals, phase transitions, and combustion. Without the effects of gravity, scientists could learn more about the fundamental behavior of gases and find ways to produce more efficient combustion processes that are so critical to human life here on Earth.

Neither the space station nor the space shuttle provides a true zero-gravity environment. Such experiments would require a vibration isolation system to eliminate unwanted accelerations. These unwanted accelerations stem from both natural and man-made sources. Man-made sources include pumps operating, exercise equipment, thrusters, and closing hatches. Bodies in low-earth orbit also experience orbital disturbances. These disturbances arise when the center of gravity of the payload is not at the same altitude as that of the platform supporting it. The payload then orbits the earth at a different speed and angular rate than the platform, creating restraining forces in the supports [12].

MSU's current actuator design, shown in Figure 4 [13], consists of two bimorphs joined at their ends with no spacer between them. Note that the thicknesses of the layers are exaggerated for clarity. Each bimorph has initial curvature in a double S-shape with the inflection point at quarter length. The radius of curvature of each quarter of the bimorph is 31 millimeters. The bimorphs operate in bending mode to create an extension or contraction of the curved structure.

The bimorphs used in the current design are two inches square, made of two sheets of 28 μm PVDF film joined together by a layer of 30-minute epoxy. Silk-screened silver electrodes coat the 28- μm film, making the total thickness of the layer 45 μm . A complete bimorph has a total average thickness of 115 μm , so in the current design, the epoxy layer is 25 μm thick.

Stampleman and von Flotow suggest, through an undisclosed manufacturing process, that they are able to achieve an epoxy layer thickness of 2 μm [12]. They point out that the epoxy layers produce a stiffening effect, and they indicate that minimizing the thickness of this layer is beneficial for maximizing actuator deflection. In an effort to minimize the epoxy layer thickness in

the MSU actuator, the team mixed acetone with the epoxy during manufacturing. The effect of mixing acetone with the epoxy is also studied in this paper.

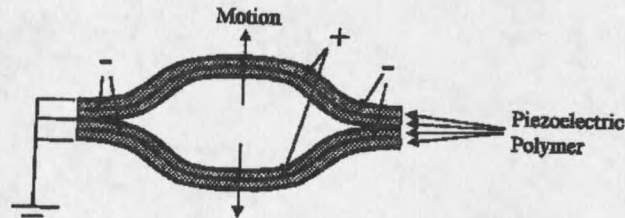


Figure 4. MSU Actuator (Double-Curvature Actuator)

A balsa wood stiffener in the shape of an X joins two actuators together. The actuator pairs are placed so that one line of the balsa wood X lies along the highest point on the curve. See Figure 5 for an illustration of the actuator stack. These actuator systems attach to a rigid frame on one side and an internal "experiment box" on the other side using the balsa wood blocks and a monofilament hinge. The experiment box houses the payload that requires isolation. This box is also equipped with accelerometers and Position Sensitive Devices (PSDs), which provide feedback on the box's displacement. In addition, accelerometers on the rigid frame enable control of the relative acceleration.

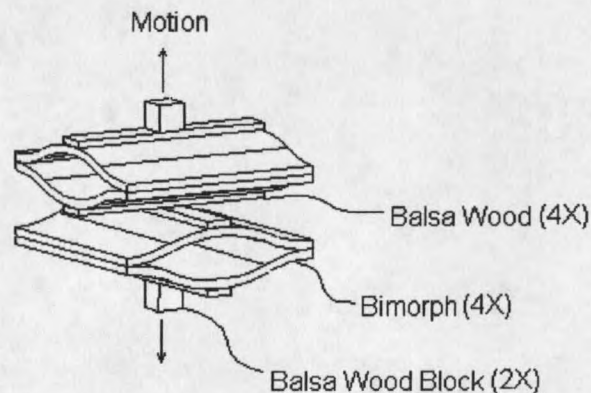


Figure 5. Actuator Stack.

The MSU team designed this actuator based on engineering judgment with a specific application in mind. Models were required to enable optimization of the design, justification for the pre-curvature shape, and extension of the design to other applications. Just as Ervin and Brei [10] developed models for their straight bimorph actuator to enable the custom configuration of the actuator force-deflection and frequency response, models for curved bimorph actuators that MSU designed enable that same type of design customization.

Both Ervin and Brei [10], with their straight design, and Stampleman and von Flotow [12], with their single-curved design, modeled their actuators as cantilevered beams. Ervin and Brei [10] include the capability to model multiple layers, but Stampleman and von Flotow [12] do not. They instead assumed that the epoxy and layers of PVDF had the same mechanical properties. In an equation for the electrically induced moment, they do mention, however, that the moment must be corrected for the regions of inactive piezoelectricity, the epoxy layers.

Although Stampleman and von Flotow [12] did develop their model including the effects of the curvature, they did not combine the electrical and mechanical behavior in a single model. A model that included both the piezoelectric and mechanical behavior was needed for a curved beam. A model that enabled the use of different boundary conditions—other than cantilevered conditions—was also desirable, since the sensitivity of the model to different boundary conditions was unknown. Larson and Vinson [14] developed curved beam equations that included piezoelectric behavior using general cylindrical shell theory and simplifying them to a curved beam with multiple layers. Various boundary conditions can also be applied to their model. Using their general equations, Larson and Vinson [14] solve several simple problems, such as a quarter ring with various end constraints and a full ring solution. Larson and Vinson's method, however adequate for our application, did not provide a good description of the physics behind curved, piezoelectric beams. In the next section, the development of another method more suited to MSU's actuator design concept will be described.

CHAPTER 2

THEORY DEVELOPMENT

Since there is no significant variation in the behavior of the actuator along its depth, the actuator can be modeled as a two-dimensional problem. Any cross-section of the actuator, as shown in Figure 4, has a high length to thickness ratio because the bimorphs are very thin. In fact, the length to thickness ratio, or aspect ratio, for the MSU actuator is on the order of 1000:1. Therefore, the slice can be appropriately modeled as a beam or group of beams. In this case, curved beams with constant radii are used, since the bimorphs have initial curvature. Also due to the extremely high aspect ratio, shear deflection in the beams could be neglected. Thus, elementary beam theory can be applied to develop the model. Therefore, plane sections normal to the middle plane before bending are assumed to remain plane and normal to the middle line after bending, and lines that are normal to the middle line are assumed not to change in length.

The goal of this section is to develop equations that describe the force-deflection behavior of the actuator, modeled as a set of curved beams. In general, for static analyses, these equations must satisfy three sets of fundamental equations: equilibrium, constitutive, and strain-displacement. This section begins with a derivation of equilibrium equations for a curved beam. Next, the strain in a curved beam is expressed in terms of the middle surface displacements. These strain-displacement relationships are substituted into the constitutive law for piezoelectric materials, which are then integrated over the cross-section to obtain stress resultants. These result in the governing equations for a single piezoelectric layer.

Then layers are combined to form a bimorph. This means that the tangential strain at the interfaces between the layers in the bimorph must be equal. The final result is a system of two equations that define the middle surface displacements in terms of the stress resultants and applied electric field.

If only an electrical field is applied to the structure, the moment is constant throughout the curved beams, since the shear and normal forces are zero. If mechanical loads are applied in addition to electrical loads, the solution becomes extremely tedious, even for simple boundary condition such as fixed-free ends. The shear and normal forces are no longer zero. In fact, they become functions of position along the beams. The moment then also becomes a function of position along the beam. A second model was therefore developed, using Castigliano's theorem, which yielded equations that were much easier to solve when mechanical loads are applied.

Although Castigliano's theorem provided a simple way to include mechanical loads, the differential equation solution provided valuable insight into the actuator's behavior. The differential equation method proved useful for optimization calculations, but Castigliano's theorem was useful when developing a force-deflection relationship for the actuator. This section will first develop the differential equation method, which is followed by the Castigliano's theorem approach.

Equilibrium Equations for a Curved Beam

To derive the equilibrium equations for a curved beam, consider a differential element within a curved beam as shown below in Figure 6.

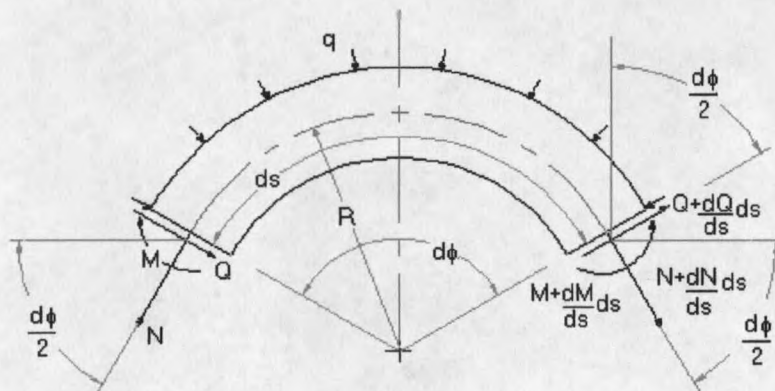


Figure 6. Equilibrium Segment.

By summing forces in the horizontal direction, the following equation can be written as follows:

$$-N \cos\left(\frac{d\phi}{2}\right) + Q \sin\left(\frac{d\phi}{2}\right) + \left(Q + \frac{dQ}{ds} ds\right) \sin\left(\frac{d\phi}{2}\right) + \left(N + \frac{dN}{ds} ds\right) \cos\left(\frac{d\phi}{2}\right) = 0.$$

If small angle approximations are used and products of small quantities are neglected, this equation simplifies to

$$Qd\phi + \frac{dN}{ds} ds = 0.$$

For a curved beam with constant radius of curvature, a circular arc, the differential arc length is

$$ds = R d\phi \quad (2.1-1).$$

With this substitution, the above equation becomes

$$\frac{dN}{d\phi} + Q = 0 \quad (2.1-2).$$

By summing forces in the vertical direction and performing similar simplifications as were done in the horizontal direction, the following equation results:

$$\frac{dQ}{d\phi} - N = qR \quad (2.1-3).$$

Now summing moments about point O and simplifying, one can write the expression below:

$$\frac{dM}{d\phi} + R \frac{dN}{d\phi} = 0 \quad (2.1-4).$$

Note that if Equation 2.1-3 is differentiated with respect to ϕ and Equation 2.1-2 is used, Equations 2.1-2 and 2.1-3 can be uncoupled. If Equation 2.1-2 is differentiated with respect to ϕ and Equation 2.1-3 is used, then the following two expressions result:

$$\frac{d^2 N}{d\phi^2} + N = -qR \quad (2.1-5)$$

$$\frac{d^2 Q}{d\phi^2} + Q = qR \quad (2.1-6).$$

Note that, from Equation 2.1-4, the quantity $M+RN$ must be a constant. Thus, if M increases, N must decrease correspondingly.

Strain-Displacement Relations for Curved Beams

This section develops the relationships between strain and displacement for curved beams, and it derives an expression for the change in curvature of such a beam due to deformation. It begins by defining the position of a particle before deformation and after deformation occurs. Refer to Figure 7 for an illustration of the particle's position.

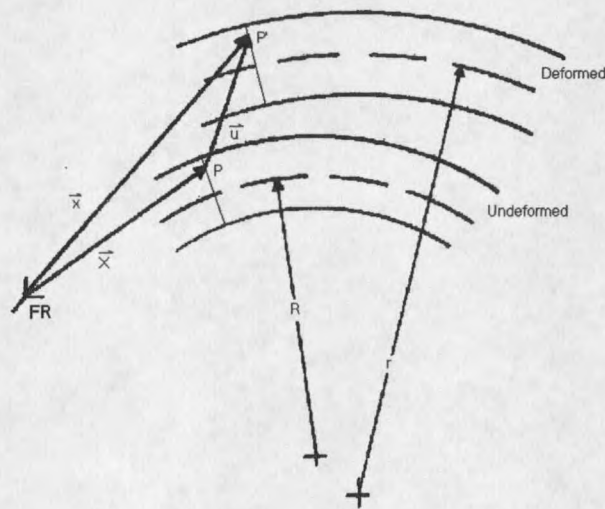


Figure 7. Positions P and P'.

Relative to a fixed reference frame at FR , the vector describing the position of a particle at P in the undeformed configuration is $\bar{X}$. The vector that describes this same particle's position, now

called P' , in the deformed configuration is $\vec{x}$. The vector $\vec{u}$ denotes the displacement of the particle. R is the undeformed radius of curvature of the middle line, and r is the radius of curvature of the middle line in the deformed configuration. From the figure, the following equation can be written:

$$\vec{u} = \vec{x} - \vec{X} \quad (2.2-1).$$

Examine first the undeformed configuration. $\vec{X}_o$ is a position vector to point A, the intersection of the middle line and a radial line through P. $\vec{e}_r$ is a unit vector in the radial direction, referred to the *undeformed* configuration. The quantity y is the radial distance from A to P. These are illustrated in Figure 8 below.

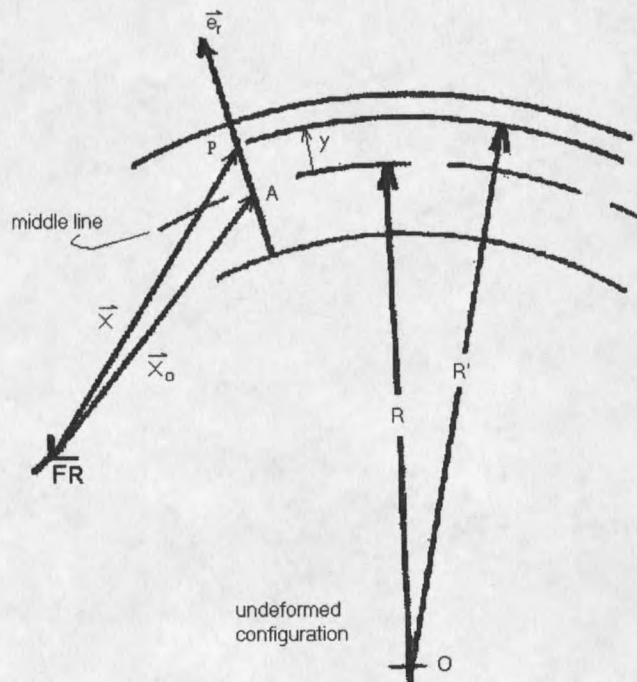


Figure 8. Undeformed Configuration.

With these quantities, $\vec{X}$ can be written as

$$\vec{X} = \vec{X}_o + y\vec{e}_r \quad (2.2-2).$$

In the deformed configuration, one can write an expression for the position of the particle at P' . The vector $\vec{x}_o$ defines the position point A' , which is the location of point A after deformation, relative to the fixed reference at FR . The unit vector $\vec{e}_\phi$ is normal to $\vec{e}_r$, and the angle β , which is assumed small, is the angle between $\vec{e}_r$ and $A'P'$. With these as shown in Figure 9, the following equation for $\vec{x}$ results:

$$\vec{x} = \vec{x}_o + y\vec{e}_r + y\beta \vec{e}_\phi \quad (2.2-3).$$

Note β is referred to the *undeformed* configuration and gives the rotation of line AP through deformation.

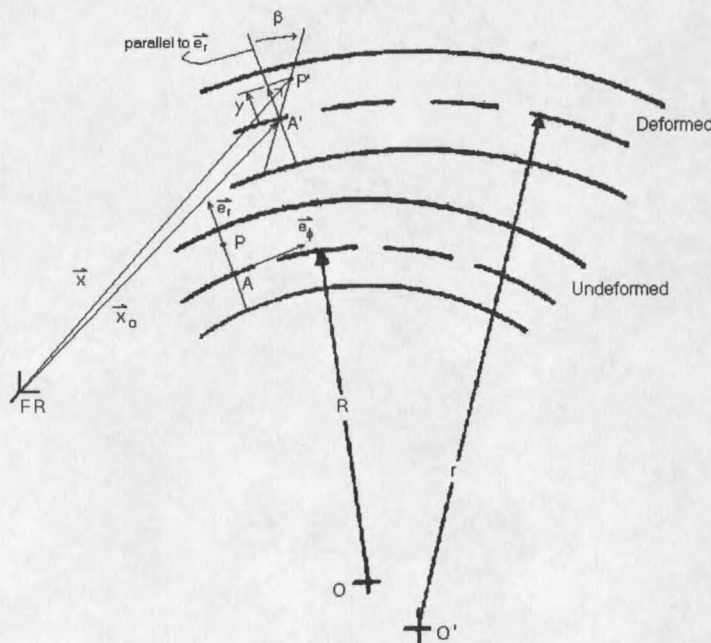


Figure 9. Angle β .

Next, using Figure 10, an expression can be written for $\vec{x}_o$ in terms of $\vec{X}_o$ and the tangential and radial components of the middle line displacements, v and w , respectively:

$$\vec{x}_o = \vec{X}_o + v\vec{e}_\phi + w\vec{e}_r \quad (2.2-4).$$

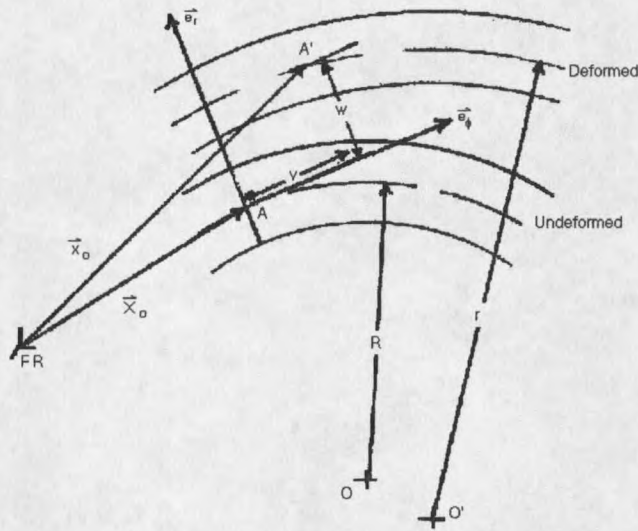


Figure 10. Displacements v and w .

Equations 2.2-2, 2.2-3 and 2.2-4 can be substituted into Equation 2.2-1 to obtain the following expression for the displacement vector:

$$\vec{u} = w\vec{e}_r + (v + y\beta)\vec{e}_\phi \quad (2.2-5).$$

This can be written as

$$\vec{u} = u_r\vec{e}_r + u_\phi\vec{e}_\phi$$

where

$$u_r = w \quad (2.2-6)$$

and

$$u_\phi = v + y\beta \quad (2.2-7).$$

The strain-displacement equations in polar coordinates for a differential element at P are [15, pg. 389]:

$$\varepsilon_r = \frac{\partial u_r}{\partial R'} \quad (2.2-8a)$$

$$\varepsilon_\phi = \frac{1}{R'} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{R'} \quad (2.2-8b)$$

$$\gamma_{r\phi} = \frac{1}{2} \left[\frac{\partial u_\phi}{\partial R'} + \frac{1}{R'} \frac{\partial u_r}{\partial \phi} - \frac{u_\phi}{R'} \right] \quad (2.2-8c)$$

where R' , from Figure 8, is the radius to P:

$$R' = R + y = R \left(1 + \frac{y}{R} \right) \quad (2.2-9).$$

The differential with respect to R' can be written as follows:

$$\frac{\partial}{\partial R'} = \frac{\partial}{\partial y} \frac{\partial y}{\partial R'} = \frac{\partial}{\partial y} \quad (2.2-10).$$

Note that

$$w = w(\phi) \quad v = v(\phi) \quad \beta = \beta(\phi). \quad (2.2-11).$$

Recall that lines normal to the middle plane are assumed not to change in length; therefore, the radial strain must be zero. Keeping this in mind and using Equations 2.2-6, 2.2-7 and 2.2-9 through 2.2-11 in Equations 2.2-8, the following expressions for the strains can be written as shown below:

$$\varepsilon_r = 0 \quad (2.2-12a)$$

$$\varepsilon_\phi = \frac{1}{R\left(1 + \frac{y}{R}\right)} \left(w + \frac{d}{d\phi} (v + y\beta) \right) \quad (2.2-12b)$$

$$\gamma_{r\phi} = \frac{1}{R\left(1 + \frac{y}{R}\right)} \frac{dw}{d\phi} + \beta - \frac{v + y\beta}{R\left(1 + \frac{y}{R}\right)} \quad (2.2-12c).$$

The lines normal to the middle line are also assumed to remain normal to the middle line before and after deformation. The shear strain given by Equation 2.2-12c must therefore also be zero.

This leads to the following expression for β :

$$\beta = \frac{v}{R} - \frac{1}{R} \frac{dw}{d\phi} \quad (2.2-13).$$

Note that β is the angle through which the middle line rotates during deformation. Since the thickness of the curved beam is significantly smaller than the radius of curvature, the ratio of y/R is much less than one. A binomial series expansion can then be used to simplify Equation 2.2-12. Since

$$\frac{1}{(1+x)} \approx 1 - x \quad x^2 < 1$$

ε_ϕ can be written as

$$\varepsilon_\phi = \left(\varepsilon_o + \frac{y}{R} \frac{d\beta}{d\phi} \right) \left(1 - \frac{y}{R} \right). \quad (2.2-14).$$

In Equation 2.2-14, ε_o is the strain at the middle line and is defined as follows:

$$\varepsilon_o = \left(\frac{w}{R} + \frac{1}{R} \frac{dv}{d\phi} \right) \quad (2.2-15).$$

The equation for the tangential strain can be further simplified by expanding the expressions in brackets to get the following result:

$$\varepsilon_{\phi} = \varepsilon_o + \frac{y}{R} \left(\frac{d\beta}{d\phi} - \varepsilon_o - \frac{y}{R} \frac{d\beta}{d\phi} \right) \quad (2.2-16).$$

Note that the last term in the expression is of higher order, since it is a derivative multiplied by

$\left(\frac{y}{R}\right)^2$. If this term is neglected, the tangential strain can be written as shown below:

$$\varepsilon_{\phi} = \varepsilon_o + \frac{y}{R} \left(\frac{d\beta}{d\phi} - \varepsilon_o \right).$$

Now consider the strain at the outer fiber in the beam, where $y=h/2$, and h is the thickness:

$$\varepsilon_{\phi h/2} = \varepsilon_o + \left[\frac{h}{2R} \left(\frac{d\beta}{d\phi} - \varepsilon_o \right) \right].$$

Note that the quantity in square brackets is of the same order of magnitude as ε_o . From this reasoning, an expression follows:

$$\frac{d\beta}{d\phi} \approx \varepsilon_o \left(1 + \frac{2R}{h} \right).$$

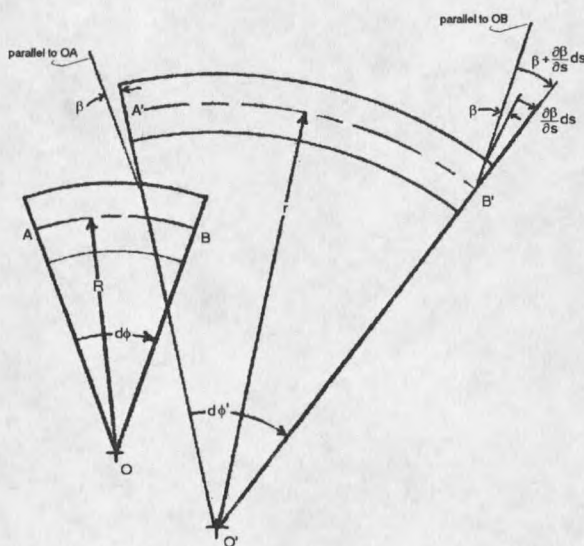
Again, since the thickness of the beam is much smaller than the radius of curvature, the following relationship can therefore be written:

$$\frac{d\beta}{d\phi} \gg \varepsilon_o \quad (2.2-17).$$

Thus, since $2R/h \gg 1$, Equation 2.2-16 can be written as follows:

$$\varepsilon_{\phi} = \varepsilon_o + \frac{y}{R} \left(\frac{d\beta}{d\phi} \right) \quad (2.2-18).$$

$$(\beta + d\phi') - \left(\beta + \frac{d\beta}{ds} ds \right) = d\phi.$$

$$d\phi' = \frac{d\beta}{ds} ds + d\phi \quad (2.2-19).$$


From the definition of strain within small deformation theory in linear elasticity, the following relationship is true:

$$ds' = (1 + \varepsilon_o) ds = r d\phi'.$$

$$\frac{(1 + \varepsilon_o) ds}{r} = \frac{d\beta}{ds} ds + \frac{ds}{R}.$$

By simplifying this expression, the following equation can be written:

$$\frac{1}{r} - \frac{1}{R} = \frac{d\beta}{ds} - \frac{1}{r} \varepsilon_o = \frac{1}{R} \left(\frac{d\beta}{d\phi} - \frac{R}{r} \varepsilon_o \right).$$

Using Equation 2.2-17 in the above expression, one can describe the change in curvature between the deformed and undeformed configurations with the following equation:

$$\frac{1}{r} - \frac{1}{R} = \frac{d\beta}{ds} \quad (2.2-20).$$

This expression shows that the change in curvature of the beam due to deformation is given by

$$\frac{d\beta}{ds}.$$

Constitutive Laws

In this section, the constitutive law for a piezoelectric material is presented. First, the piezoelectric axes and coefficients must be defined. The standard numbering of the axes is illustrated in Figure 12 [2]. For example, d_{31} is the piezoelectric strain coefficient. The first subscript is the direction of the electric field, and the second is the strain direction. For thin films, electrodes can only be applied to the top and bottom surfaces. Therefore there can only be an electric field in the 3 direction. In the case of PVDF, which is an orthotropic material, d_{32} is about an order of magnitude less than d_{31} and, although d_{33} is on the same order of magnitude as d_{31} , the thickness in the 3 direction is small. Consequently, significant deflection occurs only in the 1 direction. This is the direction along which the thin films were stretched during manufacturing. The mechanical properties of PVDF will be discussed in more detail in a later section.

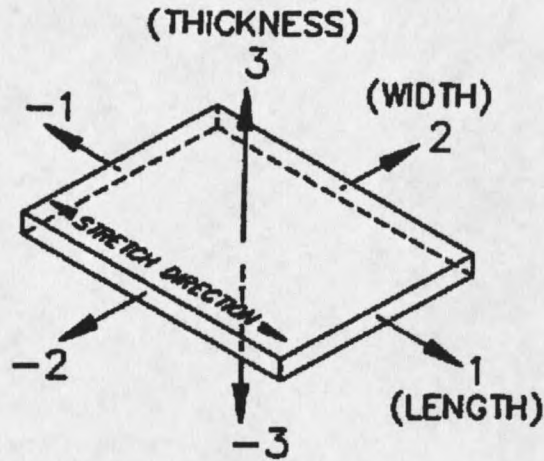


Figure 12. Axes labels for piezoelectric materials.

The constitutive law in one dimension for a piezoelectric material is [from 16, Equation 1]

$$\varepsilon_\phi = \frac{\sigma_\phi}{Y} + d_{31}E_3 \quad (2.3-1)$$

where Y is Young's modulus, σ_ϕ is the tangential stress, and E_3 is the electric field intensity.

Equation 2.3-1 can be rearranged to give

$$\sigma_\phi = \varepsilon_\phi Y - d_{31}E_3 Y \quad (2.3-2).$$

The resultant normal force, N , in a curved beam is found by integrating the tangential stress over the cross-sectional area:

$$N = \int_{-h/2}^{h/2} \sigma_\phi b dy \quad (2.3-3).$$

By substituting in Equation 2.3-2, the following expression results:

$$N = Y \int_{-h/2}^{h/2} (\varepsilon_\phi - d_{31}E_3) b dy.$$

If Equation 2.2-18 is used, the equation for the normal force simplifies to

$$N = AY(\varepsilon_o - d_{31}E_3) \quad (2.3-4)$$

where A , the cross-sectional area of the electroded piezoelectric film, is defined as

$$A = bh \quad (2.3-5).$$

Similarly, the resultant moment can be found by integrating the moment of the tangential stress over the cross-sectional area:

$$M = \int_{-h/2}^{h/2} \sigma_{\phi} y b dy.$$

By making the substitution for the tangential stress given in Equation 2.3-2 and by substituting in Equation 2.2-18 for the tangential strain, the following integral results:

$$M = bY \int_{-h/2}^{h/2} \left(\varepsilon_o + \frac{y}{R} \left(\frac{d\beta}{d\phi} \right) - d_{31}E_3 \right) y dy.$$

After integrating and using Equations 2.2-11, the expression for the moment reduces to

$$M = \frac{bh^3}{12} \frac{Y}{R} \frac{d\beta}{d\phi} = YI \frac{1}{R} \frac{d\beta}{d\phi} = YI \frac{d\beta}{ds} \quad (2.3-6)$$

where I is the moment of inertia of the electroded piezoelectric film about the 2 axis, and β is defined by Equation 2.2-13.

Applying Equations to Bimorphs

In this section, the strain-displacement relationships are substituted into the constitutive law. A bimorph with an epoxy layer will be considered, as shown in Figure 13. First, the

equations that describe the behavior of a single layer of piezoelectric film are developed. These equations will then be used when deriving expressions for bimorphs.

First, Equation 2.2-15 can be used in Equation 2.3-4, and Equation 2.2-13 is used in Equation 2.3-6 to give the following:

$$\frac{w}{R} + \frac{1}{R} \frac{dv}{d\phi} = \frac{N}{AY} + d_{31}E_3 \quad (2.4-1)$$

$$\frac{1}{R} \frac{dv}{d\phi} - \frac{1}{R} \frac{d^2w}{d\phi^2} = \frac{MR}{YI} \quad (2.4-2).$$

The moment, M , is then obtained by eliminating $\frac{dv}{d\phi}$ between Equations 2.4-1 and 2.4-2:

$$M = \frac{YI}{R} \left(-\frac{w}{R} + \frac{N}{AY} + d_{31}E_3 - \frac{1}{R} \frac{d^2w}{d\phi^2} \right) \quad (2.4-3).$$

Next, Equations 2.3-4 and 2.3-6 can be substituted into Equation 2.2-18 to give the expression for the tangential strain,

$$\varepsilon_\phi = \frac{N}{YA} + d_{31}E_3 + \frac{My}{YI} \quad (2.4-4).$$

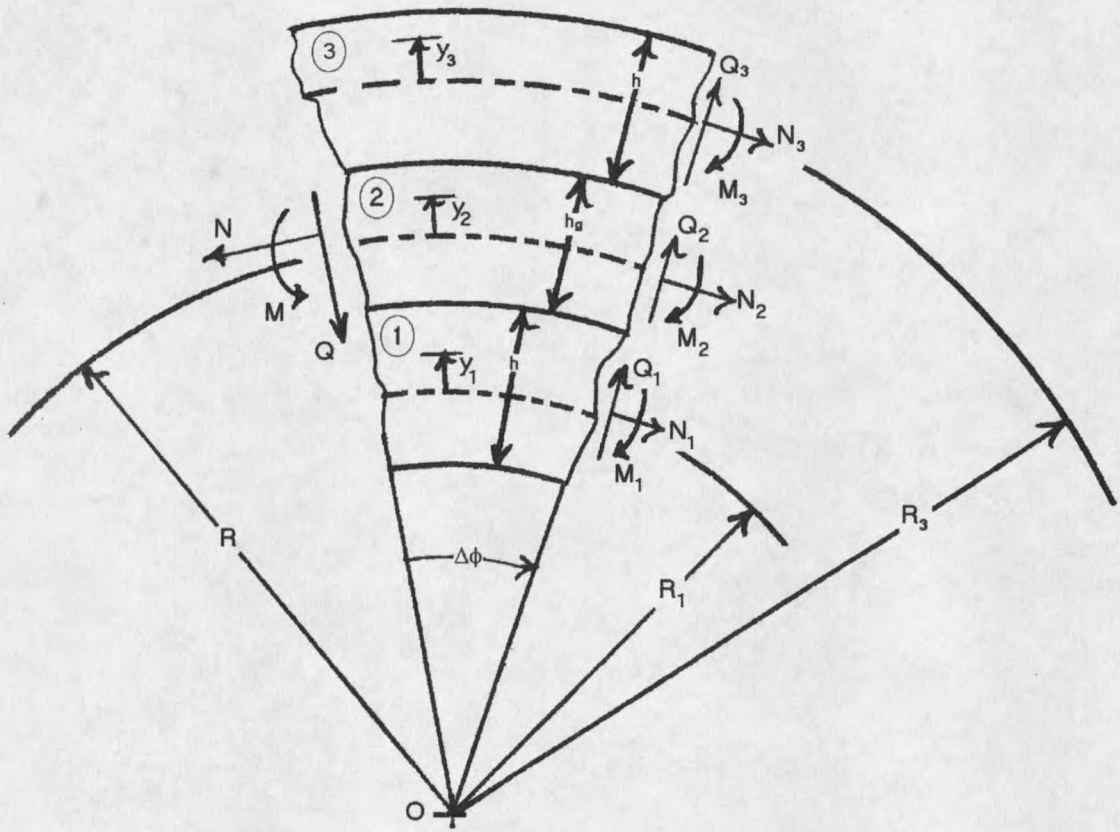


Figure 13. Bimorph with an Epoxy Layer.

The variables M_i , N_i , and Q_i , ($i=1,2,3$) represent the moment, tangential force, and shear force, respectively, in each layer. M , N , and Q represent the resultant quantities for the entire bimorph.

As $\Delta\phi$ goes to zero, the following relationships must be satisfied:

$$N = N_1 + N_2 + N_3 \quad (2.4-5)$$

$$Q = Q_1 + Q_2 + Q_3 \quad (2.4-6)$$

$$M = M_1 + M_2 + M_3 + \frac{h + h_g}{2} (N_3 - N_1) + (N_1 + N_2 + N_3 - N)R \quad (2.4-7).$$

By using Equation 2.4-5, Equation 2.4-7 can be simplified to

$$M = M_1 + M_2 + M_3 + \frac{h + h_g}{2} (N_3 - N_1) \quad (2.4-8).$$

At the interfaces between the layers, the tangential strains must be equal so that

$$\left[\varepsilon_{\phi_1} \right]_{y_1 = \frac{h}{2}} = \left[\varepsilon_{\phi_2} \right]_{y_2 = -\frac{h_g}{2}} \quad (2.4-9)$$

$$\left[\varepsilon_{\phi_2} \right]_{y_2 = \frac{h_g}{2}} = \left[\varepsilon_{\phi_3} \right]_{y_3 = -\frac{h}{2}} \quad (2.4-10).$$

Equation 2.4-4 can be used to write the following two equations for the tangential strain in each piezoelectric layer, assuming that the Young's modulus and cross-sectional dimensions are the same for both:

$$\varepsilon_{\phi_1} = \frac{N_1}{YA} + d_{31}E_{31} + \frac{M_1 y_1}{YI} \quad (2.4-11a)$$

$$\varepsilon_{\phi_3} = \frac{N_3}{YA} + d_{31}E_{33} + \frac{M_3 y_3}{YI} \quad (2.4-11b)$$

where E_{31} is the applied electrical field in the 3 direction of the piezoelectric material in layer 1, and E_{33} is the applied electrical field in the 3 direction in layer 3. Equation 2.4-4 can also be written for the epoxy layer to give

$$\varepsilon_{\phi_2} = \frac{N_2}{Y_g A_g} + \frac{M_2 y_2}{Y_g I_g} \quad (2.4-12).$$

By substituting Equations 2.4-11 and Equation 2.4-12 into Equations 2.4-9 and 2.4-10 and evaluating them at the appropriate values of y , the following expressions result:

$$\frac{N_1}{YA} + d_{31}E_{31} + \frac{M_1 h}{2YI} = \frac{N_2}{Y_g A_g} - \frac{M_2 h_g}{2Y_g I_g} \quad (2.4-13)$$

$$\frac{N_2}{Y_g A_g} + \frac{M_2 h_g}{2Y_g I_g} = \frac{N_3}{YA} + d_{31}E_{33} - \frac{M_3 h}{2YI} \quad (2.4-14).$$

Equations 2.4-13 and 2.4-14 can be rearranged and subtracted to yield the following equation:

$$\frac{(N_3 - N_1)}{YA} = \frac{M_2 h_g}{Y_g I_g} - d_{31}(E_{33} - E_{31}) + \frac{h}{2YI}(M_1 + M_3) \quad (2.4-15).$$

By substituting Equation 2.4-15 into Equation 2.4-8, the equation for the total moment can be written as

$$M = (M_1 + M_3) \left(1 + \frac{A h}{I 2} \frac{(h + h_g)}{2} \right) + M_2 \left(\frac{YA}{Y_g I_g} h_g \frac{(h + h_g)}{2} + 1 \right) - Y d_{31} \frac{A(h + h_g)}{2} \Delta E_3 \quad (2.4-16)$$

where

$$\Delta E_3 = (E_{33} - E_{31}) \quad (2.4-17).$$

If Equation 2.4-3 is written for each layer, the following set of expressions result:

$$\frac{M_1}{YI} = - \left(\frac{w_1}{R_1^2} + \frac{1}{R_1^2} \frac{d^2 w_1}{d\phi_1^2} \right) + \frac{N_1}{AYR_1} + \frac{d_{31}E_{31}}{R_1} \quad (2.4-19)$$

$$\frac{M_2}{Y_g I_g} = - \left(\frac{w_2}{R^2} + \frac{1}{R^2} \frac{d^2 w_2}{d\phi_2^2} \right) + \frac{N_2}{A_g Y_g R} \quad (2.4-20)$$

$$\frac{M_3}{YI} = -\left(\frac{w_3}{R_3^2} + \frac{1}{R_3^2} \frac{d^2 w_3}{d\phi^2}\right) + \frac{N_3}{AYR_3} + \frac{d_{31}E_{33}}{R_3} \quad (2.4-21).$$

Since lines normal to the middle line remain unstrained, it follows that

$$w_1 = w_2 = w_3 = w \quad (2.4-22).$$

where w is the radial displacement of the middle line. Note that $h \ll R$ and therefore $R_1 \approx R_2 \approx R_3$, so that when Equations 2.4-19, 20, 21 and 2.4-22 are substituted into Equation 2.4-16, the following expression for M results

$$\begin{aligned} M = & -\frac{1}{R^2} \left(\frac{d^2 w}{d\phi^2} + w \right) YI_{eq} + \frac{(N_1 + N_3)}{AR} \left(I + A \frac{h}{2} h_{eq} \right) + N_2 \frac{YA}{Y_g A_g} \frac{h_g}{R} h_{eq} + \dots \\ & + Y \frac{d_{31}}{R} E_3 \left(I + A \frac{h}{2} h_{eq} \right) - Y d_{31} A h_{eq} \Delta E_3 \end{aligned} \quad (2.4-23)$$

where R is the radius to the middle line of the bimorph. E_3 is defined as

$$E_3 = E_{31} + E_{33} \quad (2.4-24).$$

The equivalent moment of inertia is

$$I_{eq} = 2 \left(I + h_{eq}^2 A \right) + \frac{Y_g}{Y} I_g \quad (2.4-25)$$

where

$$h_{eq} = \frac{h + h_g}{2}$$

$$I_g = \frac{b h_g^3}{12}$$

and h_g is the thickness of the epoxy layer. By writing Equation 2.3-4 for each layer, the following expressions for the normal forces result:

$$N_1 = YA(\varepsilon_{o1} - d_{31}E_{31}) \quad (2.4-26)$$

$$N_2 = Y_g A_g \varepsilon_{o2} \quad (2.4-27)$$

$$N_3 = YA(\varepsilon_{o3} - d_{31}E_{33}) \quad (2.4-28)$$

If the above equations are substituted into Equation 2.4-5, the following equation can be written:

$$N = YA(\varepsilon_{o3} - \varepsilon_{o1}) + Y_g A_g \varepsilon_{o2} - YAd_{31}E_3$$

If Equation 2.2-15 and 2.4-22 are used in the above, and since $R_1 \approx R_2 \approx R_3 \approx R$, the following expression for the total normal force results:

$$N = YA \left(2 \frac{w}{R} + \frac{1}{R} \frac{\partial v_1}{\partial \phi} + \frac{1}{R} \frac{\partial v_3}{\partial \phi} \right) + Y_g A_g \left(\frac{w}{R} + \frac{1}{R} \frac{\partial v_2}{\partial \phi} \right) - YAd_{31}E_3 \quad (2.4-29).$$

Note that the average of the piezoelectric layers' tangential displacements is approximately equal to the tangential displacement in the epoxy layer, thus

$$v_1 + v_3 \approx 2v_2 \approx 2v$$

where v is the tangential displacement of the middle line. Equation 2.4-29 can therefore be simplified to

$$N = YA_{eq} \left(\frac{w}{R} + \frac{1}{R} \frac{dv}{d\phi} \right) - YAd_{31}E_3 \quad (2.4-30)$$

where

$$A_{eq} = 2A + \frac{Y_g}{Y} A_g \quad (2.4-31)$$

and A_g is the cross-sectional area of the epoxy layer. Now by adding Equations 2.4-26 and 2.4-28, and noting that

$$\varepsilon_{o1} + \varepsilon_{o3} = 2\varepsilon_{o2} = 2\varepsilon_o$$

then the following expression can be written

$$N_1 + N_3 = YA (2\varepsilon_o - d_{31}E_3)$$

and Equation 2.4-27 becomes

$$N_2 = Y_g A_g (\varepsilon_o)$$

where ε_o is the strain of the middle line. By using the above expressions, Equation 2.4-23 can be simplified to the following:

$$M = -\frac{1}{R^2} \left(\frac{d^2 w}{d\phi^2} + w \right) YI_{eq} + \frac{N}{R} \frac{2(I + Ah_{eq}^2)}{A_{eq}} + \frac{A}{A_{eq}} \frac{1}{R} Y \left[2(I + Ah_{eq}^2) \right] d_{31} E_3 - AYh_{eq} d_{31} \Delta E_3 \quad (2.4-32)$$

where Equation 2.4-25 defines I_{eq} . By rearranging Equation 2.4-30 and Equation 2.4-32, the following expressions can be written:

$$\frac{w}{R} + \frac{1}{R} \frac{dv}{d\phi} = \frac{N}{YA_{eq}} + \frac{Ad_{31}E_3}{A_{eq}} \quad (2.4-33)$$

$$\frac{1}{R^2} \left(\frac{d^2 w}{d\phi^2} + w \right) = -\frac{M}{YI_{eq}} + \frac{N}{R} \frac{2(I + Ah_{eq}^2)}{A_{eq}YI_{eq}} + \frac{A}{A_{eq}I_{eq}} \frac{1}{R} \left[2(I + Ah_{eq}^2) \right] d_{31}E_3 - A \frac{h_{eq}d_{31}\Delta E_3}{I_{eq}}$$

(2.4-34)

If there is no epoxy layer, then $h_g = 0$, and these equations simplify as follows:

$$\frac{w}{R} + \frac{1}{R} \frac{dv}{d\phi} = \frac{N}{AY} + d_{31}E_3$$

$$\frac{1}{R^2} \left(\frac{d^2 w}{d\phi^2} + w \right) = -\frac{M}{YI_{eqne}} + \frac{N}{RAY} + \frac{d_{31}}{R} E_3 - \frac{Ahd_{31}}{2I_{eqne}} \Delta E_3$$

where

$$I_{eqne} \equiv 2 \left(I + A \left(\frac{h}{2} \right)^2 \right).$$

Together, Equations 2.4-33 and 2.4-34 describe the behavior of a curved, piezoelectric bimorph. These equations are valid if the layers are thin, and it is assumed that the Young's modulus and cross-sectional area of the two piezoelectric layers are the same. The solution of these two equations, using boundary conditions appropriate for the double-S curve actuator designed at MSU, will be developed in a later chapter of this thesis.

Castigliano's Theorem Method

An alternative method for analyzing the curved actuator uses Castigliano's second theorem. It is often used to determine the deflections of structures, and it provides a relatively simple means of solving for the displacements of statically indeterminate structures. The theorem states that if rigid-body motion of a structure is prevented, the deflection at the point of application

of the concentrated load can be determined by differentiating the strain energy. Many authors, for example Boresi [17], cover Castigliano's second theorem on deflections.

Some simplifying assumptions using Castigliano's second theorem were applied specifically for the MSU actuator and are important to note. First, the radial stress term could be neglected, since its effect is small for large values of R/h , which is well over 100 for the MSU actuator. If the ratio $R/h > 2.0$, then the strain energy due to bending can be approximated by that for a straight beam. Therefore, the straight beam form of Castigliano's theorem was used.

Next, the shear deflection was also neglected, to be consistent with the differential equation method discussed previously. This is valid for ratio $R/h > 10$. This left only one term in the equation for the strain energy density, the one due to the tangential stress. Both internal moments and normal forces affect the tangential stress, which are both types of mechanical loads considered in the differential equation method.

The other type of load considered here includes the electrical, or piezoelectric, loads. The electrical loads are incorporated into Castigliano's theorem as piezoelectric pseudo-moments. A piezoelectric pseudo-moment is an electrically induced internal moment due to the piezoelectric behavior of the bimorph. This approach has been used previously by Ervin and Brei [10], but since the development of this method was not shown, it will be developed here for an analogous case.

Consider an infinitesimal element subjected to a stress, σ_x , and applied temperature gradient, ΔT . The strain energy per unit volume stored in the element is

$$W = \frac{1}{2}(\varepsilon_x - \alpha\Delta T)\sigma_x$$

where ε_x is the total strain, that is, mechanical plus thermal strain. The complimentary strain energy per unit volume is, by definition,

$$W^* = \sigma_x \varepsilon_x - W$$

or

$$W^* = \sigma_x(\varepsilon_x - \alpha\Delta T) + \alpha\Delta T\sigma_x - W = 2W + \alpha\Delta T\sigma_x - W,$$

which simplifies to

$$W^* = W + \alpha\Delta T\sigma_x.$$

Since in one dimension,

$$\sigma_x = Y(\varepsilon_x - \alpha\Delta T),$$

and therefore

$$\varepsilon_x = \frac{\sigma_x}{Y} + \alpha\Delta T,$$

the complimentary strain energy per unit volume can therefore be written as

$$W^* = \frac{\sigma_x^2}{2Y} + \alpha\Delta T\sigma_x \quad (2.5-1).$$

This equation for W^* can be found in numerous texts on thermal stress analysis [18]. Note that the temperature is analogous to electric field in the problem considered in this thesis, and the constant d_{31} is analogous to the thermal expansion coefficient α . The total complimentary strain energy is the integral of W^* over the volume:

$$U^* = \int_V W^* dV$$

which can be written for a beam of length L by integrating over the cross-sectional area and then over the length:

$$U^* = \int_0^L \int_A W^* dA dx. \quad (2.5-2).$$

Note that for a beam with an internal bending moment, M , and an internal normal force N , the following is true

$$\sigma_x = \frac{My}{I} + \frac{N}{A} \quad (2.5-3).$$

If a linear temperature distribution is assumed, then

$$\Delta T = 2T_o \frac{y}{h} \quad (2.5-4)$$

where h is the beam thickness, y is the distance from the middle line, and $\pm T_o$ are the temperatures at the outer fiber on the top and bottom, respectively. By using Equations 2.5-1, 2.5-3 and 2.5-4 in Equation 2.5-2, the following results

$$U^* = \int_0^L \int_A W^* dA dx = \int_0^L \frac{1}{2Y} \left(\frac{M^2}{I} + \frac{N^2}{A} + 4Y\alpha \frac{T_o}{h} M \right) dx \quad (2.5-5)$$

where I is the moment of inertia of the beam's cross-section given by

$$I = \int_A y^2 dA.$$

If we define M_T as

$$M_T = 2YI\alpha \frac{T_o}{h}$$

then Equation 2.5-5 can be rewritten as

$$U^* = \int_0^L \int_A W^* dA dx = \int_0^L \left[\frac{1}{2Y} \left(\frac{M^2}{I} + 2M_T \frac{M}{I} \right) + \frac{N^2}{2YA} \right] dx \quad (2.5-6).$$

Note that the term inside the curved brackets is almost factorable. To make the term factorable, a term M_T^2 can be added and another term M_T^2 subtracted from the expression. By factoring the term inside the curved brackets, Equation 2.5-6 can be written as follows:

$$U^* = \int_0^L \left[\frac{(M + M_T)^2}{2YI} + \frac{N^2}{2YA} - \frac{M_T^2}{2YI} \right] dx \quad (2.5-7).$$

Since Castigliano's second theorem on deflections states that

$$\delta_i = \frac{\partial U^*}{\partial F_i}$$

where δ_i is the deflection or slope at the point of application of a concentrated force, F_i , or moment, respectively. Since M_T is not a concentrated moment, then

$$\frac{\partial}{\partial F_i} M_T = 0$$

always. This term therefore contributes nothing to the deflection or slope and can therefore be neglected and Equation 2.5-7 can be simplified to the following:

$$U^* = \int_0^L \left[\frac{(M + M_T)^2}{2YI} + \frac{N^2}{2YA} \right] dx \quad (2.5-8).$$

Note that the complimentary strain energy must be computed to use Castigliano's theorem, and that M_T is a thermal moment analogous to the piezoelectric pseudo-moment.

Thus, using the piezoelectric pseudo-moments and these assumptions, Castigliano's second theorem on deflections can be used to determine the deflection of the curved actuator by differentiating the expression with respect to the applied loads. Finding the deflection in terms of the applied loads provides the load-deflection relationship. Following similar procedures outlined previously for the other analytical model, one can include an epoxy layer. The analysis of the bellows actuator using Castigliano's theorem is performed in a later section of this thesis, along with the solution to the differential equations. Both methods are then compared to the finite element solution for the linear elastic case.

CHAPTER 3

MODEL INPUTS

Several inputs are required to develop the models. Material properties and the physical dimensions of the actuator are both required, which includes the properties of the PVDF layers and of the epoxy layer that joins them.

Geometry

Each bimorph is two inches square, and the double-curvature shape consists of 31-millimeter curves with the inflection point at quarter length. The radius of curvature, 31 millimeters, is the radius of the molds used to manufacture bimorphs. The other physical dimensions of the actuator were determined by taking measurements.

The PVDF film was measured as a clear film and also with its silver electrode layer. The manufacturer rates the clear film as 28 μm thick, but the value measured by the team at MSU indicated that the film was consistently 30 μm thick on average. The PVDF film with silver, silk-screened electrodes was measured to be 45 μm thick on average. From this, both electrode layers are assumed to be equally 7.5 μm thick.

Next, the thicknesses of bimorphs were measured by Erica McKenzie to determine the epoxy layer thickness. Four bimorphs were constructed from layers of silver-electroded PVDF film and their thicknesses were measured. During construction, a single batch of epoxy was mixed and then used as consecutive bimorphs were made. The time elapsed between bimorph construction was a maximum of 15 minutes. The epoxy was curing during this time, which could effect the epoxy layer thickness in the bimorphs. Thickness measurements were taken on the first-made bimorph, the second, and so on. The average total thickness of the first-made bimorph was 110 μm . The average total thickness of the second-made bimorph was 119.8 μm . The

average of these two thicknesses, 115 μm , was used to determine the epoxy layer thickness for modeling purposes—25 μm thick. Data on the bimorph thicknesses are given in Appendix A.

The difference between the average epoxy thickness for the first-made bimorph and the second-made bimorph were found to be statistically significant. This means that once the epoxy is mixed a time delay of 15 minutes results in an epoxy layer whose thickness increases by a significant amount. The calculations to determine statistical significance of the mean thickness of the first-made bimorph and the second-made bimorph are given in Appendix B.

The uniformity of the bimorphs was also examined. The layers of electroded PVDF were found to be uniform within one μm . This small deviation was attributed to measurement error, so the layers were therefore assumed to be completely uniform. Any non-uniformities measured for the bimorphs then were attributed completely to the epoxy layer. The epoxy layer's overall standard deviation was found to be 9 μm for the first-made bimorph. The uniformity of the first-made bimorph was not statistically different than the second-made. Therefore, a random variation of 9 μm from the mean thickness of 25 μm was used to develop the finite element model used to test the effects of a non-uniform epoxy layer.

Since the MSU team mixed acetone with the epoxy during manufacturing in an effort to minimize epoxy layer thickness, measurements on bimorphs made with acetone-thinned epoxy were also made. Four bimorphs were constructed for each thinning ratio. The ratios tested included the following: one part epoxy and one part hardener to one part acetone (1:1:1), one part epoxy and one part hardener to two parts acetone (1:1:2), and one part epoxy and one part hardener to three parts acetone (1:1:3). Through statistical analysis of the data, the following conclusions could be drawn. First, the acetone does not reduce the overall mean thickness of the epoxy layer unless a thinning ratio of 1:1:3 or above is used. The acetone does, however, significantly improve the uniformity of the epoxy layer, even for a thinning ratio of 1:1:1. Third, the first-made bimorph has a smaller mean epoxy layer thickness than the second-made bimorph, just as without thinning. Lastly, acetone thinning degrades the uniformity of the epoxy layer

thickness between the first-made to the second-made bimorphs. Statistical calculations to determine these conclusions are given in Appendix B.

PVDF Material Properties

Material properties for PVDF are taken from the literature. Numerous studies have been performed to characterize the material properties of PVDF, including determining the properties at two different frequencies, 10 Hertz and 25 kilohertz [16]. These studies have been compiled in a publication by H. Wang [19], but they are for various manufacturing processes, different electrode types, and various sample shapes. Some studies were performed on bulk PVDF. Others were done on thin film PVDF with sputtered copper electrodes, sputtered aluminum electrodes, or silk-screened silver electrodes.

The piezoelectric and mechanical properties chosen for this paper were taken from the experiments performed by Schewe [16], since this was the most complete set of properties from a single source. These experiments were performed on 30- μm thick aluminum-electroded PVDF films. Since PVDF is generally orthotropic, the elastic properties are not the same in every material direction, as they are for an isotropic material. The elastic constant in the stretch direction, direction 1, for the aluminum-electroded films from Schewe [16] was within ten percent of the elastic constant measured in the stretch direction for a silver-electroded film tested by Tamura [20]. The properties for the aluminum-electroded film from Schewe then provide a good approximation of the behavior of the silver-electroded film used to make the MSU actuator. The piezoelectric strain coefficients, d_{ij} , are taken from Wang [19]. The piezoelectric strain coefficients, along with the elastic compliance matrix components [16], S_{ij} , are summarized in Table 1.

Table 1. Material Properties for PVDF

Property	Value
d_{31}	$2.1\text{E-}11 \text{ mm/(mmV/m)}$
d_{32}	$2.3\text{E-}12 \text{ mm/(mmV/m)}$
d_{33}	$-2.6\text{E-}11 \text{ mm/(mmV/m)}$
d_{24}	$-2.3\text{E-}11 \text{ mm/(mmV/m)}$
d_{15}	$-2.7\text{E-}11 \text{ mm/(mmV/m)}$
S_{11}	$3.65\text{E-}10 \text{ m}^2/\text{N}$
S_{22}	$4.24\text{E-}10 \text{ m}^2/\text{N}$
S_{33}	$4.72\text{E-}10 \text{ m}^2/\text{N}$
S_{12}	$-1.1\text{E-}10 \text{ m}^2/\text{N}$
S_{23}	$-1.92\text{E-}10 \text{ m}^2/\text{N}$
S_{13}	$-2.09\text{E-}10 \text{ m}^2/\text{N}$

Epoxy Material Properties

The material properties of the 30-minute epoxy were also needed. These properties were determined through ultrasonic testing at MSU. First, molds were constructed, to make plug-shaped epoxy samples one-half inch in diameter.

The following process was developed by Erica McKenzie to minimize entrained bubbles. First, 10 mL each of epoxy resin and hardener were mixed together in plastic bowl for approximately 2 minutes. The bowl was then placed in a vacuum chamber that was subsequently depressurized to 100 hPa. By tilting the bowl part way on its side, the epoxy was spread out across a larger surface. Through this tilted angle and by leaving the epoxy mixture in the vacuum for approximately ten minutes, virtually all of the bubbles were eliminated. The epoxy was then poured into the mold. The mold was placed back in the vacuum for an additional 5 minutes, during which few bubbles surfaced. Then the samples were allowed to cure overnight.

Using this process, epoxy samples without air bubbles could be successfully produced and tested. The plug samples, once cured, were machined down to a thickness of 2.89 millimeters, or approximately 7/64 inch. These samples were then tested using both shear and longitudinal 0.5-inch diameter, 2.25-megahertz ultrasonic transducers. The longitudinal wave travel time was measured using the pulse-echo mode and the shear wave travel time was

measured using the through-transmission mode of the pulser-receiver control box. A sketch of the pulser-receiver control box is shown below:

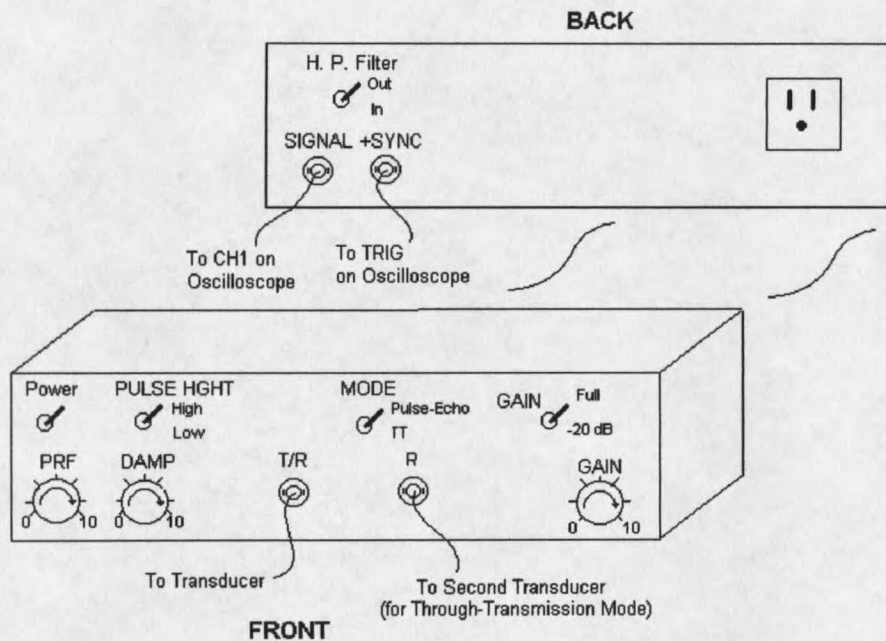


Figure 14. Pulser-Receiver Control Box.

A list of settings that were used for the pulser-receiver and oscilloscope are given in Appendix C. The wave speeds were determined by measuring the time between reflections on an oscilloscope, and by measuring the sample thickness. See Figure 15 at the end of the chapter for a sample oscilloscope trace for the wave form. For both the pulse-echo and through-transmission modes, the speed is calculated using

$$V = \frac{2h}{\Delta t} \quad (3.3-1)$$

where h is the sample thickness and Δt is the time between reflections. The elastic properties can then be determined from the shear and longitudinal wave speeds, V_T and V_L , respectively, and the density of the sample using the following equations [21]:

$$\nu = \frac{1 - 2 \left(\frac{V_T^2}{V_L^2} \right)^2}{2 - 2 \left(\frac{V_T^2}{V_L^2} \right)^2} \quad (3.3-2)$$

$$G = V_T^2 \rho_{avg}$$

$$Y = V_L^2 \rho_{avg} \frac{(1 + \nu)(1 - 2\nu)}{1 - \nu}$$

where ν is Poisson's ratio, Y is Young's modulus, G is the shear modulus, and ρ_{avg} is the average density of all the samples.

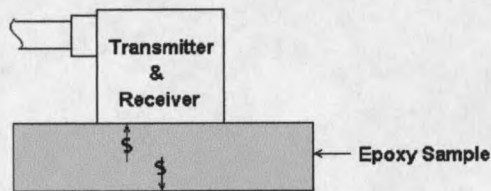


Figure 16. Part A) Pulse-Echo Mode.

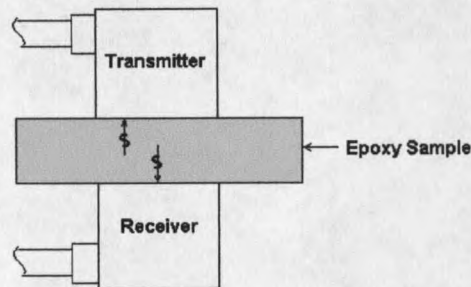


Figure 16. Part B) Through-Transmission Mode.

Note in Figure 15 in pulse-echo mode the reflected waves are inverted, or 180 degrees out of phase with the previous reflection. Figure 16, Part A, shows a single transducer used in pulse-echo mode. In the through-transmission mode, two transducers are used as shown in

Figure 16, part B. With through-transmission mode, the reflected waves are in phase with the previous reflection. This can be explained by examining the reflected waves, assuming normal incidence. Figure 17 illustrates a normally incident wave traveling from one medium to another.

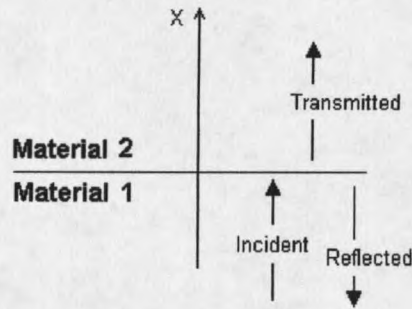


Figure 17. Reflection and Transmittal of a Wave Entering Another Medium.

Part of the incident wave is transmitted and remainder is reflected. The reflected wave field has the following form [22]

$$u_x^{(R)} = A_R e^{-i(k_1 x + \omega t)} \quad (3.3-3)$$

where

$$A_R = \frac{\rho^1 V_L^1 - \rho^2 V_L^2}{\rho^1 V_L^1 + \rho^2 V_L^2} I \quad (3.3-4)$$

The product ρV is the acoustic impedance, and I is the magnitude of the incident wave. In pulse-echo mode, when the pulse leaves the transducer and travels through the epoxy sample, it is reflected at an interface with air. If medium 1 is the epoxy and medium 2 is the air, then

$$\rho^2 V_L^2 < \rho^1 V_L^1$$

and the ratio of the amplitudes given by Equation 3.3-4 is positive, therefore the first reflected wave is in phase with the original pulse. This wave is then received in phase. Its reflection from

the transmitter/receiver is 180 degrees out of phase with the original, since Equation 3.3-4 gives a negative quantity, because

$$\rho^2 V_L^2 > \rho^1 V_L^1$$

where now material 2 is the steel transducer and the material 1 is the epoxy sample. The wave travels again through the epoxy sample and is reflected at the epoxy/air interface. Here again, it is not inverted, and therefore remains 180 out of phase. When this second reflection arrives at the receiver, it is still inverted. Therefore, the second reflection will be out of phase with the first for the pulse-echo mode. This process continues until the wave is completely attenuated.

In the case of through-transmission, however, a pulse is generated by the transducer and is reflected back at the surface of the sample. It is inverted when it passes from the epoxy sample to the transducer since

$$\rho^2 V_L^2 > \rho^1 V_L^1$$

where medium 2 is the steel transducer and medium 1 is the epoxy sample. The ratio of the amplitudes given by Equation 3.3-4 is therefore a negative quantity. The reflected wave is now 180 degrees out of phase with the original. The wave absorbed by the receiver, or the transmitted wave, however, is in phase with the incident wave. The reflected, inverted wave travels back through the epoxy sample, however, and is reflected back by the transmitter. Again, the reflection is inverted 180 degrees since the quantity given by Equation 3.3-4 is a negative quantity. Now the wave is in phase with the original, so when the second reflection reaches the receiver, it is in phase with the original wave and in phase with the first pulse received. This process repeats until the wave is completely attenuated.

Since shear waves attenuate more quickly than longitudinal waves, the shear wave measurements could not be made using pulse-echo mode. The wave was lost in the noise before two reflections could be received. Through-transmission was therefore used for measuring the shear wave speed, because the wave only has to travel once through the

thickness of the sample before the first pulse reaches the receiver (instead of twice, as in pulse-echo mode). This is illustrated in Figure 18 below.

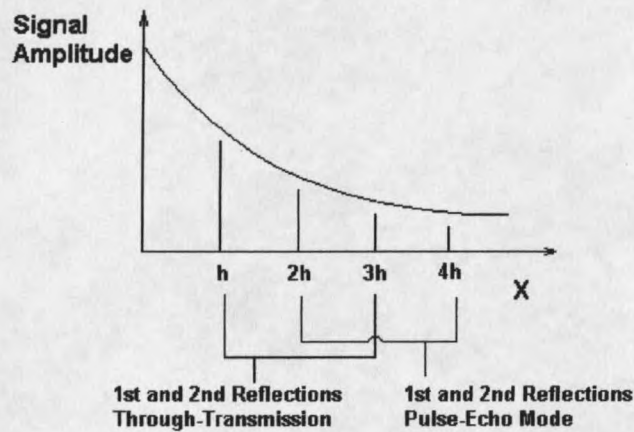


Figure 18. Signal Attenuation.

The wave still must travel through the sample thickness twice before the next reflection is received. By shortening the initial distance traveled, however, two good reflections could be observed, and the time between them could be measured.

The diameter and thickness of six epoxy samples were measured using a micrometer, and the masses were measured on a scale. By calculating the volume of a sample and by using the mass, the density could be determined. The average density for the six epoxy samples was 1186 kg/m^3 . The time between two reflections was measured from the oscilloscope traces for each sample, and average times were calculated. For samples that were 4.25 mm thick, the travel time for the longitudinal wave was $3.25\text{E-}6$ seconds, and the shear wave's travel time was $7.5\text{E-}6$ seconds. Only one sample 2.89 mm thick was made, and the travel time for this sample was $2.16\text{E-}6$ seconds for the longitudinal wave. The travel time for the shear wave of this sample was $4.8\text{E-}6$ seconds. These times were used in Equations 3.3-1 to calculate the shear and longitudinal wave speeds. The wave velocities that were calculated for the 2.89-mm samples and the 4.25-mm samples were approximately 10% different. This difference can be attributed to experimental error. The thinner samples, however, provided clearer reflections and should be

used for any future measurements. The Young's and shear moduli, calculated using Equations 3.3-2, are summarized in Table 2.

Table 2. Epoxy Material Properties

Epoxy Material Properties	
Y_g	4.2E9 Pa
G_g	1.5E9 Pa
ν	0.38

Epoxy samples that were thinned with acetone were also prepared. Testing these samples was not possible. Two reflections could not be distinguished on the oscilloscope due to the high attenuation. The epoxy samples thinned with acetone were noticeably rubbery in texture and yellowish in color compared to the un-thinned epoxy samples, which were quite hard and translucent. Attempts to optimize the test conditions using thinner samples, through-transmission mode, and a delay line between the sample and the transducer failed to yield two good reflections. The effect of acetone on the epoxy is not quantified, but it is known that it does increase the attenuating capabilities of the epoxy.

hp running

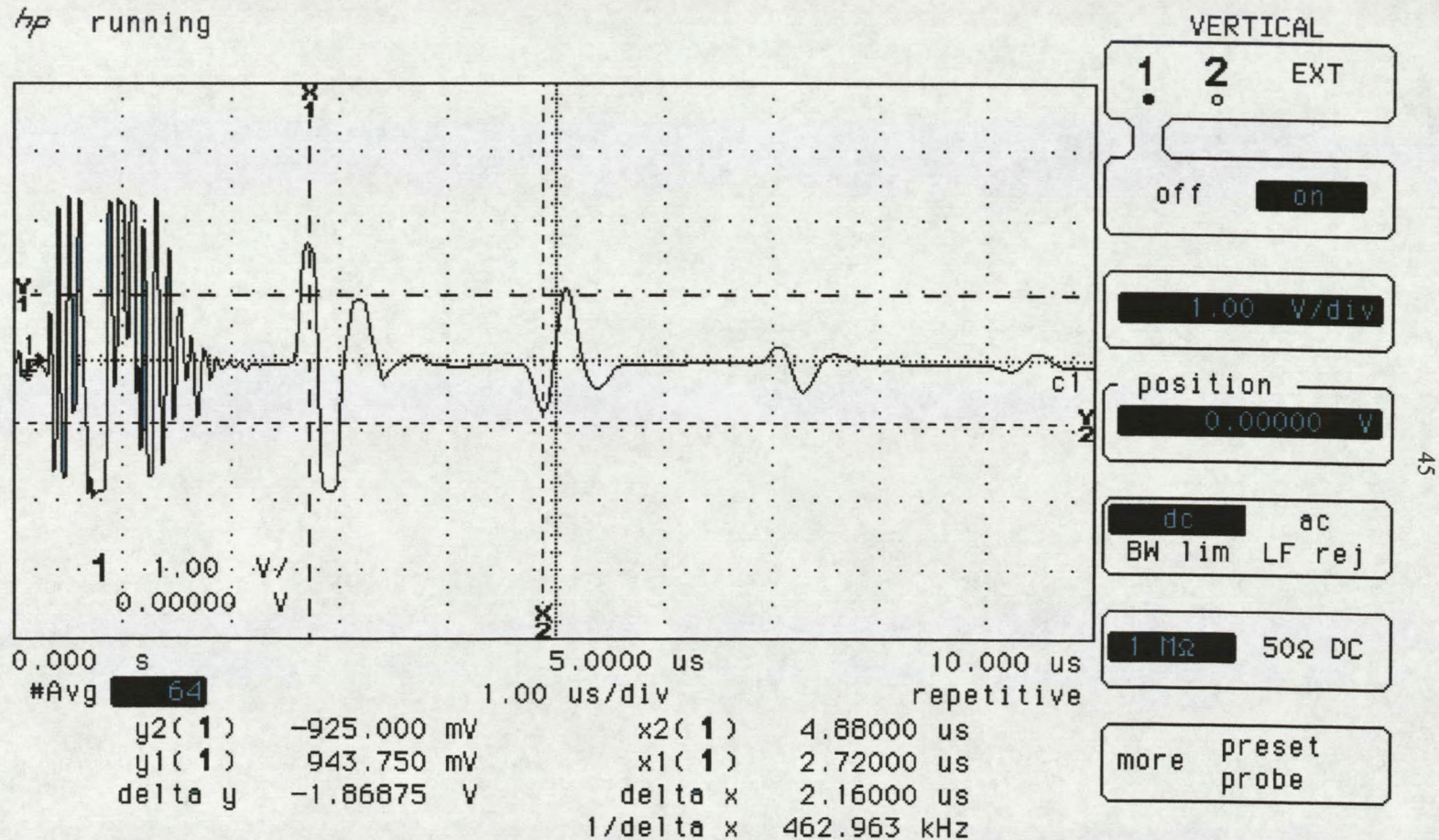


Figure 15. Longitudinal Wave Response for 2.89 mm Thick Epoxy Samples with No Acetone Thinning.

CHAPTER 4

ANALYTICAL SOLUTION

This section applies the theory that was developed in Chapter 2 to the MSU actuator. It shows the solution to Equations 2.4-33 and 2.4-34 and the optimization work using the differential equation method. It also describes the Castigliano's theorem model in more detail, and it presents the solution to the resulting equations.

Due to symmetry, only one quarter of an actuator is considered. This portion is shown in Figure 19. Obtaining a solution to Equations 2.4-33 and 2.4-34 requires using the differential equations separately on each quarter-length segment of the actuator. The inflection points of the double-S curve shape define these quarter-length segments.

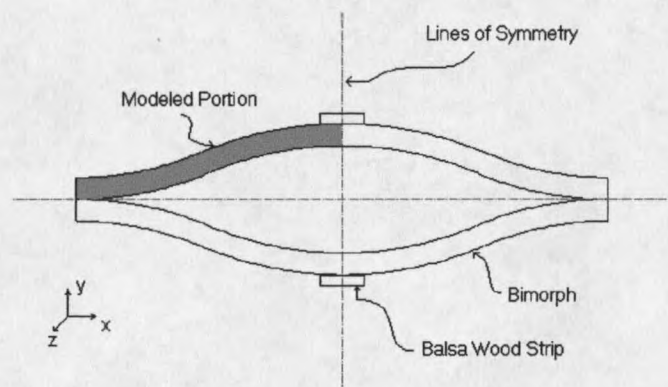


Figure 19. Actuator Symmetry.

Appropriate interface conditions must be assigned at the inflection point. In addition, other boundary conditions must be imposed that reflect the actuator's constraints. Once the boundary conditions are established, the solution of the equations is relatively simple.

Voltages can be applied to the bimorphs in many ways, depending on the electrode pattern on the piezoelectric polymer layers. The electrode pattern used on MSU's actuators is custom-made to enable the application of different voltages on each quarter length of a bimorph. The configuration of the applied voltages on each bimorph is illustrated in Figure 20, shown for 100 Volts. Note that the figure shows just one quarter of an actuator (half a bimorph) and that the actuator is symmetric about the horizontal and vertical axes shown. This figure also depicts the direction of the resulting electrical fields, E_3 , across the PVDF layers, and the poling direction, P , in each layer. Note that the electric field induced in each layer is

$$\bar{E} = -\frac{\partial V}{\partial r_e} \approx -\frac{\Delta V}{h} \quad (4.0-1).$$

where h is the thickness of the piezoelectric material in a layer, r_e is the distance between electrical charges, and ΔV is the voltage gradient across it.

The voltages applied to the inside surface of each layer in the bimorph result in electrical fields that are in opposite directions in the top and bottom layers. Since the poling direction is the same for both layers, each layer has a section where the electric field and polarity lie in the same directions. This will create an extension in that segment. On the other segments, the electric field and the polarity are in opposite directions, which will contract that portion. The combination of the extension and contraction creates bending in that segment of the bimorph. With the voltages applied as shown, each quarter of the bimorph increases in curvature, thus creating an upward displacement at the center.

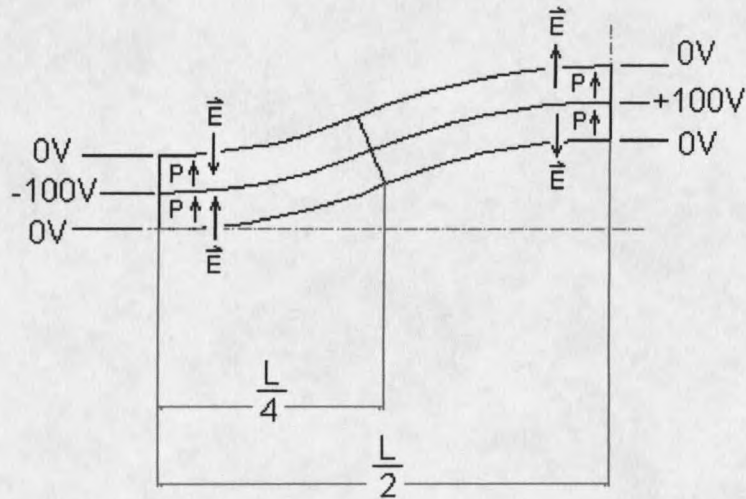


Figure 20. Configuration of Applied Voltages in the Actual System.

Both the individual layers within the bimorphs and the top and bottom bimorphs in an actuator are electrically connected using copper tape tabs attached to the edges of the film. These copper tabs are then coated with an insulator for safety. Note also that the outer layers of the bimorphs are always grounded for safety reasons. If the outer layers were *not* always grounded, the voltages could be applied as shown in Figure 21, which is equivalent to that shown in Figure 20. The configuration shown in Figure 21 was selected for use in the models, since all the applied voltages were of the same sign.

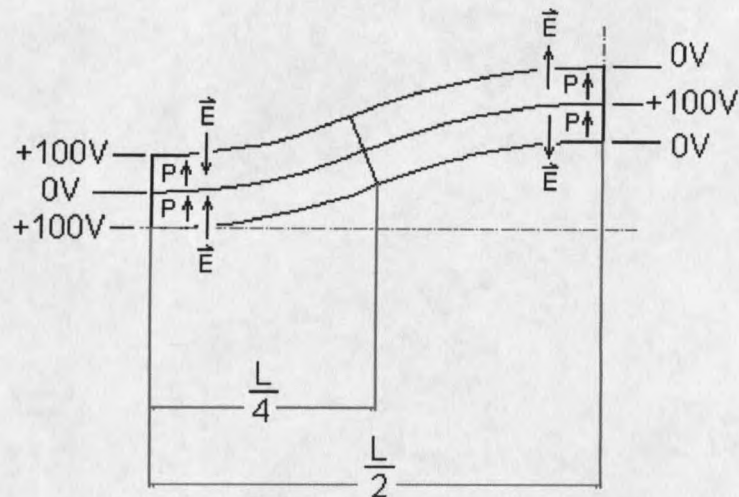


Figure 21. Applied Voltages Used in Modeling.

Differential Equation Model Solution

The differential equation model has both interface and boundary conditions, since compatibility conditions must be applied at the inflection point, where the two quarter-length segments join. Each quarter-length segment of a bimorph is modeled as a curved beam. Figure 22 illustrates the two segments, the end and interface constraints, and the internal forces and moments in the system.

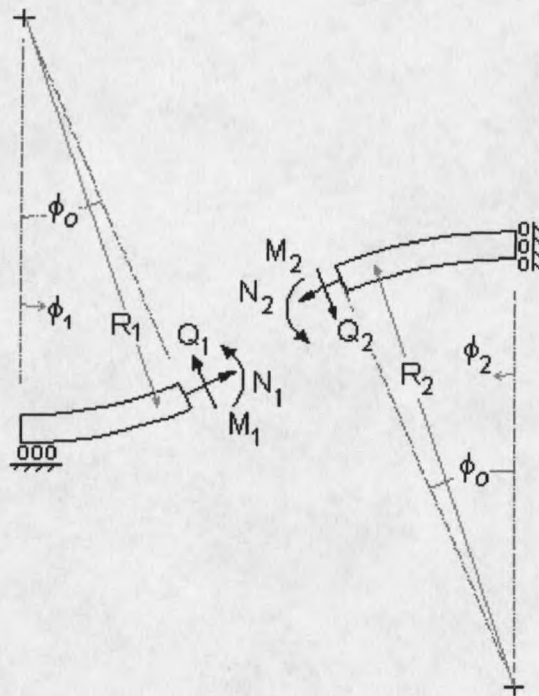


Figure 22. Differential Equation Model.

At $\phi_2 = 0$, symmetry conditions exist. The slope at this point, $\beta_2 = 0$, and the tangential displacement, v_2 , also is zero. Recall that the slope is given by Equation 2.2-13. The shear at this end, Q_2 , is zero since there is no restraint to vertical motion at this end. At $\phi_1 = 0$, there is no restraint to horizontal movement. Therefore, at this end, $N_1 = 0$. Since the model will predict displacements relative to the horizontal line of symmetry of the actuator, vertical motion at this end is prevented. Therefore, at $\phi_1 = 0$, the radial displacement, $w_1 = 0$. The slope at $\phi_1 = 0$ is also zero; therefore, $\beta_1 = 0$. The total displacement for an actuator is given by

$$\Delta = 2\delta$$

where δ is the vertical displacement of one quarter at $\phi_2 = 0$. The other quarter of the actuator at this point (right quarter of the top bimorph in Figure 19) has the same vertical displacement. The total vertical displacement for the actuator therefore is the sum of the top and bottom halves.

At ϕ_0 compatibility conditions exist. The radial displacements, tangential displacements, and slopes of the two segments must be compatible. Additionally, equilibrium holds across the interface. The boundary and interface conditions are summarized in Table 3.

Table 3. Boundary and Interface Conditions for Differential Equation Model.

	A	B	C
	at $\phi_1=0$	at $\phi_1=\phi_2=\phi_0$	at $\phi_2=0$
I	$w_1(0)=0$	$w_1(\phi_0)=-w_2(\phi_0)$	$v_2(0)=0$
II	$\beta_1(0)=0$	$\beta_1(\phi_0)=\beta_2(\phi_0)$	$\beta_2(0)=0$
III	$N_{o1}(0)=0$	$N_1(\phi_0)=-N_2(\phi_0)$	---
IV	---	$Q_1(\phi_0)=-Q_2(\phi_0)$	$Q_{o2}(0)=0$
V	---	$M_1(\phi_0)=-M_2(\phi_0)$	---
VI	---	$v_1(\phi_0)=-v_2(\phi_0)$	

With the boundary and interface conditions defined, Equations 2.4-33 and 2.4-34 can be solved. First, expressions for the normal and shear forces were developed as functions of the angles ϕ_1 and ϕ_2 . Summing forces on each beam segment accomplishes this. Figure 23 illustrates the two beam segments with all the internal forces and the constraints replaced with reactions.

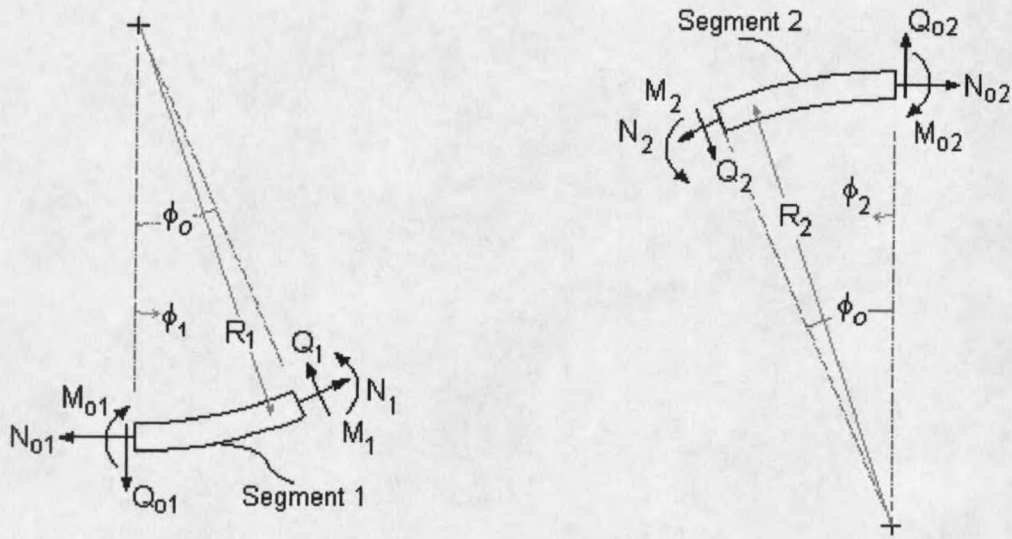


Figure 23. Beam Segments with Reactions.

By summing forces for each segment in the horizontal and vertical directions, four equations can be written, which define N_1 , Q_1 , N_2 , and Q_2 as functions of ϕ_1 and ϕ_2 , respectively. By using the boundary conditions IIIA and IVC, the four equations can then be simplified to the following:

$$\begin{aligned} N_1(\phi_1) &= Q_{o1} \cdot \sin(\phi_1) & N_2(\phi_2) &= N_{o2} \cdot \cos(\phi_2) \\ Q_1(\phi_1) &= Q_{o1} \cdot \cos(\phi_1) & Q_2(\phi_2) &= N_{o2} \cdot \sin(\phi_2) \end{aligned}$$

Due to the equilibrium conditions at the joint of the two beam segments, $Q_1(\phi_o) = -Q_2(\phi_o)$ and $N_1(\phi_o) = -N_2(\phi_o)$. Using these expressions, the equations for equilibrium reduce to the following:

$$\begin{bmatrix} \sin(\phi_o) & -\cos(\phi_o) \\ \cos(\phi_o) & \sin(\phi_o) \end{bmatrix} \begin{bmatrix} Q_{o1} \\ N_{o2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

which require that

$$Q_{o1} = 0$$

$$N_{o2} = 0$$

since the determinant of the coefficient matrix is non-zero. Thus, using Equations 2.1-5 and 2.1-6

$$N_1(\phi_1) = Q_1(\phi_1) = N_2(\phi_2) = Q_2(\phi_2)$$

It therefore follows from Equation 2.1-4 that

$$M = \text{constant} = M^*$$

throughout each segment. Note that if a mechanical load were applied in the vertical direction at the center of the actuator, that is at $\phi_2 = 0$, then Q_{02} would no longer be zero. Thus Q_2 , Q_1 , N_2 , and N_1 would become functions of their arc angles, and M would no longer be constant. This would lead to a particular solution that is not a constant, which adds a substantial number of terms to the model solution. Therefore, this model is limited to solutions involving only electrical loads.

Since the internal normal force is zero throughout the beam segments in the absence of applied mechanical loads, Equations 2.4-33 and 2.4-34 reduce to

$$\frac{d^2 w}{ds^2} + \frac{w}{R^2} = -\frac{M^*}{YI_{eq}} + \Lambda \quad (4.1-1a)$$

$$\frac{w}{R} + \frac{dv}{ds} = \Gamma \quad (4.1-1b).$$

where

$$\Lambda = \frac{\Gamma}{R} - d_{31} \frac{Ah}{I_{eq}} \Delta E_3 \quad (4.1-2).$$

$$\Gamma = \frac{d_{31} E_3}{2} \quad (4.1-3).$$

where E_3 , ΔE_3 , and I_{eq} are given by Equations 2.4-24, 2.4-17, and 2.4-25, respectively, and Y is Young's modulus of the electroded PVDF film. Since the electrical fields in the layers of the bimorphs are always in opposite directions with the same magnitude, E_3 will be zero. For now,

this term will be included for completeness. Equations 4.1-1 can be written for each segment shown in Figure 23:

$$\frac{d^2 w_1}{ds_1^2} + \frac{w_1}{R_1^2} = -\frac{M_1^*}{YI_{eq}} + \Lambda_1 \quad (4.1-4)$$

$$\frac{w_1}{R_1} + \frac{dv_1}{ds_1} = \Gamma_1 \quad (4.1-5)$$

$$\frac{d^2 w_2}{ds_2^2} + \frac{w_2}{R_2^2} = -\frac{M_2^*}{YI_{eq}} + \Lambda_2 \quad (4.1-6)$$

$$\frac{w_2}{R_2} + \frac{dv_2}{ds_2} = \Gamma_2 \quad (4.1-7)$$

where the subscripts 1 and 2 refer to the segments 1 and 2, respectively. The complete solution of Equations 4.1-4 through 4.1-7 for each beam segment is given below:

$$w_1(s_1) = A_1 \cos\left(\frac{s_1}{R_1}\right) + B_1 \sin\left(\frac{s_1}{R_1}\right) - \frac{R_1^2 M_1^*}{YI_{eq}} + \Lambda_1 R_1^2 \quad (4.1-8)$$

$$w_2(s_2) = A_2 \cos\left(\frac{s_2}{R_2}\right) + B_2 \sin\left(\frac{s_2}{R_2}\right) - \frac{R_2^2 M_2^*}{YI_{eq}} + \Lambda_2 R_2^2 \quad (4.1-9)$$

By applying the boundary condition IA from Table 3, Equation 4.1-8 can be rewritten as

$$w_1(s_1) = \alpha_1 \left(1 - \cos\left(\frac{s_1}{R_1}\right)\right) + B_1 \sin\left(\frac{s_1}{R_1}\right) \quad (4.1-10)$$

where

$$\alpha_1 = -\frac{R_1^2 M_1^*}{YI_{eq}} + \Lambda_1 R_1^2.$$

Integrating Equation 4.1-5 and using Equation 4.1-10 in the result gives the following expression for the tangential displacement.

$$v_1 = \Gamma_1 s_1 - \frac{\alpha_1}{R_1} (s_1) + \alpha_1 \sin\left(\frac{s_1}{R_1}\right) + B_1 \cos\left(\frac{s_1}{R_1}\right) + C_1 \quad (4.1-11)$$

Note that using Equations 2.2-13, 4.1-10, and 4.1-11, and applying the boundary condition IIA from Table 3 shows that $C_1 = 0$. Similarly, with boundary conditions IC and IIC from Table 3 and Equations 2.2-13, 4.1-7, Equation 4.1-9 reduces to

$$w_2(s_2) = A_2 \cos\left(\frac{s_2}{R_2}\right) + \alpha_2 \quad (4.1-12)$$

Equation 4.1-7 then gives

$$v_2 = \Gamma_2 s_2 - \frac{\alpha_2}{R_2} (s_2) - A_2 \sin\left(\frac{s_2}{R_2}\right) \quad (4.1-13)$$

where

$$\alpha_2 = -\frac{R_2^2 M_2}{YI_{eq}} + \Lambda_2 R_2^2$$

By applying the interface condition IIB from Table 3, and using Equations 4.1-11 through 4.1-13, the following relationship between the internal moments in the two beam segments can be derived:

$$M_1^* = \frac{Y}{R_1} d_{31} h_{eq} A (R_2 \Delta E_{3_2} - R_1 \Delta E_{3_1}) + \frac{R_2}{R_1} M_2^*$$

By using the interface condition VB from Table 3 in the above expression, the following two equations for the internal moments can be written:

$$M_1^* = -Yd_{31}h_{eq}A\left(\frac{-R_1\Delta E_{3_1} + R_2\Delta E_{3_2}}{R_1 + R_2}\right) \quad (4.1-14)$$

$$M_2^* = Yd_{31}h_{eq}A\left(\frac{-R_1\Delta E_{3_1} + R_2\Delta E_{3_2}}{R_1 + R_2}\right) \quad (4.1-15)$$

Now apply interface conditions IB and VIB from Table 3. By solving the two resulting equations simultaneously, the unknown coefficients from Equations 4.1-10 through 4.1-13 can be written in terms of known quantities as shown below:

$$A_2 = -\alpha_2(\phi_o \sin \phi_o + \cos \phi_o) + \alpha_1(1 - \phi_o \sin \phi_o - \cos \phi_o) + \Gamma_1 L_1 \sin \phi_o + \Gamma_2 L_2 \sin \phi_o$$

$$B_1 = \alpha_2(\phi_o \cos \phi_o - \sin \phi_o) + \alpha_1(\phi_o \cos \phi_o - \sin \phi_o) - \Gamma_1 L_1 \cos \phi_o - \Gamma_2 L_2 \cos \phi_o$$

where L_i are the arc lengths at ϕ_o . This completes the solution for the general case. For the MSU actuator model, however, some further simplifications can be made. Recall that $E_3 = 0$ since the electric fields in the bimorph's layers are in opposite directions and have the same magnitude. Therefore, the quantities E_{3_1} and E_{3_2} are both equal to zero. Thus, $\Gamma_1 = \Gamma_2 = 0$. Also note that $\Delta E_{3_2} = -\Delta E_{3_1}$ if the electric fields in the bimorph layers have the same magnitude and opposite sign. Making these simplifications and substituting in the expressions for α_i the following final expressions are

$$w_1(s_1) = \alpha_1\left(1 - \cos\left(\frac{s_1}{R_1}\right)\right) + B_1 \sin\left(\frac{s_1}{R_1}\right) \quad (4.1-16)$$

$$v_1(s_2) = -\frac{\alpha_1}{R_1}(s_1) + \alpha_1 \sin\left(\frac{s_1}{R_1}\right) + B_1 \cos\left(\frac{s_1}{R_1}\right) \quad (4.1-17)$$

$$w_2(s_2) = A_2 \cos\left(\frac{s_2}{R_2}\right) + \alpha_2 \quad (4.1-18)$$

$$v_2(s_2) = -\frac{\alpha_2}{R_2}(s_2) - A_2 \sin\left(\frac{s_1}{R_1}\right) \quad (4.1-19)$$

where

$$A_2 = -\alpha_2(\phi_o \sin \phi_o + \cos \phi_o) + \alpha_1(1 - \phi_o \sin \phi_o - \cos \phi_o)$$

$$B_1 = \alpha_2(\phi_o \cos \phi_o - \sin \phi_o) + \alpha_1(\phi_o \cos \phi_o - \sin \phi_o)$$

The other variables α_1 and α_2 simplify to the below:

$$\alpha_2 = -\frac{R_2^2 M_2^*}{YI_{eq}} - d_{31} \frac{Ah_{eq}}{I_{eq}} \Delta E_{32} R_2^2$$

$$\alpha_1 = -\frac{R_1^2 M_1^*}{YI_{eq}} - d_{31} \frac{Ah_{eq}}{I_{eq}} \Delta E_{31} R_1^2$$

The internal moments, M_1^* and M_2^* , are given by the following:

$$M_1^* = Yd_{31}h_{eq}A\Delta E_{31} \quad (4.1-20)$$

$$M_2^* = -Yd_{31}h_{eq}A\Delta E_{32} \quad (4.1-21)$$

The vertical deflection, δ , is given by

$$\delta = w_2(0) \quad (4.1-22)$$

since the radial direction is referred to the undeformed configuration, where $w_2(0)$ is the radial deflection. Substituting in all the expressions for the coefficients in terms of the applied electric field and material constants, the following expression results:

$$\delta = w_2(0) = \frac{-R_1 R_2 A h_{eq} d_{31} \Delta E_{31}}{I_{eq}} [\cos \phi_o + \phi_o \sin \phi_o - 1] \quad (4.1-23)$$

Equation 4.1-23 provides the vertical displacement of the top bimorph of the actuator (at its center) for a given radius of curvature, moment of inertia, applied electric field, and cross-sectional area.

The angle ϕ_o is a function of R_1 , R_2 , and of the arc length of the quarter model, since actuators are made with a film of a *fixed* length L . The length of film available for the quarter model is

$$l = \phi_o R_1 + \phi_o R_2$$

therefore

$$\phi_o = \frac{l}{(R_1 + R_2)} \quad (4.1-24)$$

where

$$l = \frac{L}{2}.$$

What are the optimal radii of curvature that would maximize the vertical displacement at the center of the bimorph? To answer this, first note that δ is a function of R_1 and R_2 . At the extrema of Equation 4.1-23,

$$d\delta = \frac{\partial \delta}{\partial R_1} dR_1 + \frac{\partial \delta}{\partial R_2} dR_2 = 0$$

and since R_1 and R_2 are arbitrary, the following must therefore be true:

$$\frac{\partial \delta}{\partial R_1} = 0 \quad (4.1-25)$$

$$\frac{\partial \delta}{\partial R_2} = 0 \quad (4.1-26)$$

By taking the two derivatives and subtracting the resulting equations, the following equation can be written after some simplifications:

$$(R_1 - R_2) \left[\frac{l}{(R_2 + R_1)} \sin \left(\frac{l}{R_2 + R_1} \right) + (R_1 - R_2) \cos \left(\frac{l}{R_2 + R_1} \right) + (R_2 - R_1) \right] = 0$$

therefore, $R_1 = R_2$. Therefore, to maximize vertical displacement, the two radii of curvature must be equal. Given l , however, what value should this radius be? This question can be answered by using Equation 4.1-23 and 4.1-25 and setting $R_1 = R_2 = R$. Making this substitution and simplifying the result, the following expression is obtained:

$$R \left[1 - \cos \left(\frac{l}{2R} \right) \right] - \frac{l}{2} \sin \left(\frac{l}{2R} \right) + \frac{l^2}{8R} \cos \left(\frac{l}{2R} \right) = 0 \quad (4.1-27)$$

The length of the quarter model is 1 inch. Setting l equal to 1 inch, Equation 4.1-27 can be plotted for a range of values for R as shown in Figure 24.

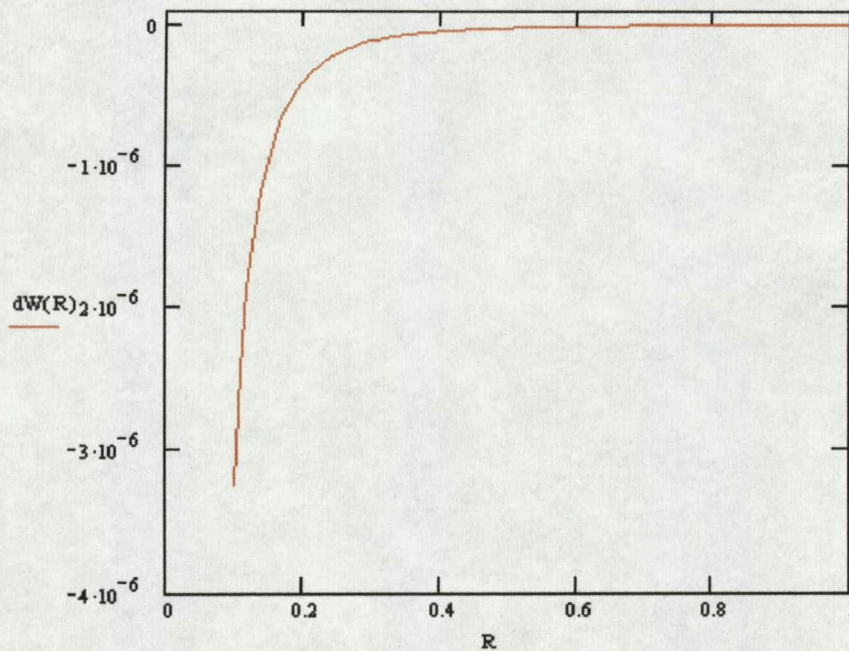


Figure 24. Optimum Radius of Curvature Plot.

Note that from the graph, it appears that for any radius greater than 0.5 m, the optimal value is reached. The curve appears to be equal to zero after this point for larger radii, but continues to approach it asymptotically as the radius goes to infinity. Therefore, a straight beam is optimal for maximizing vertical displacement.

By looking at the scale on the graph, however, and choosing an appropriate tolerance that will be considered equal to zero, a close approximation to the optimal value for the bimorph's initial curvature can be obtained. For a tolerance value of $1.0\text{E-}6$, any value of $R > 0.15$ m is sufficient to reach the optimum value. That is, $R > 0.15$ m will ensure that the derivative of the vertical deflection at the center of the bimorph equals zero within $1.0\text{E-}6$. Thus, the vertical deflection will essentially be at a maximum.

If both sides of Equation 4.1-27 are divided by l , the dimensionless parameter R/l can be used to generalize Figure 24. Figure 25 shows a plot of Equation 4.1-17 versus R/l . Note that R/l for the MSU actuator is 1.22. Note also that in order to form a double s-curve shape,

$$\phi_o \leq \frac{\pi}{2}.$$

Therefore, from Equation 4.1-24, and equal radii, the quantity R/l must be greater than or equal to

$$\frac{R}{l} \geq \frac{1}{\pi} \approx 0.318$$

If the right-hand side of Equation 4.1-17 is evaluated at $R = 31\text{mm}$ and $l = 1$ inch, the result is $-4.22\text{E-}3$. If R is increased to 34 mm or decreased to 31 mm, then the right-hand side of Equation 4.1-17 changes by approximately 35% (as the radius increases, the value decreases).

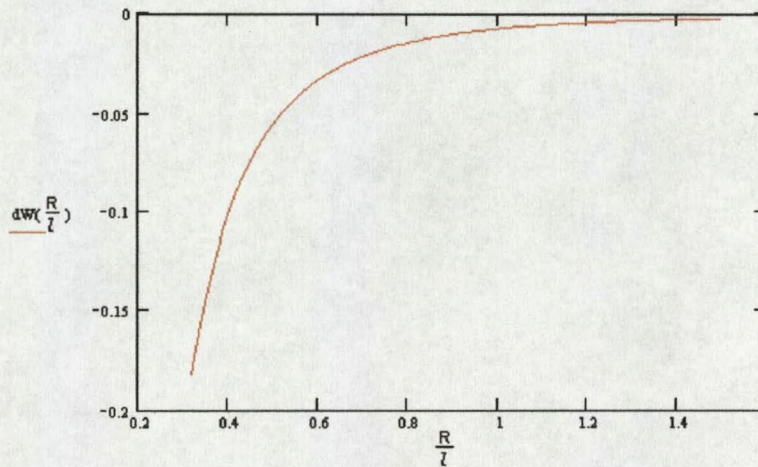
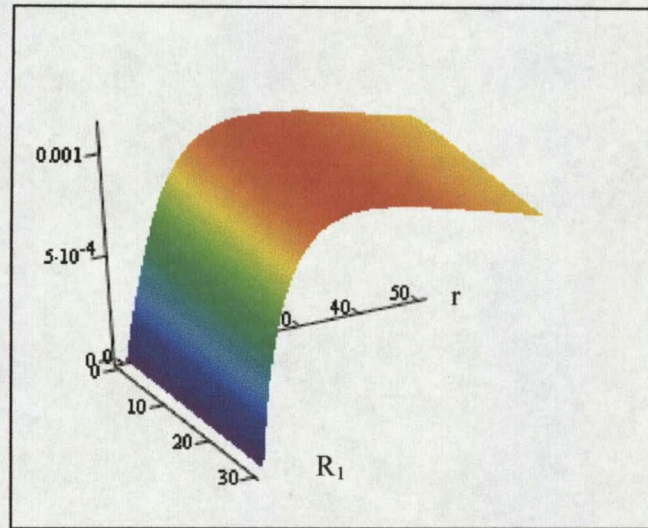


Figure 25. Generalized Optimal Radius/Model Length.

Note also that Equation 4.1-23 can be written in terms of R_1 and the ratio

$$r = \frac{R_2}{R_1}.$$

Figure 26 shows Equation 4.1-23 plotted versus R_1 and r . Note that the peak of the displacement occurs over a wide range of r . Therefore, the displacement does not appear to be very sensitive to small differences between R_1 and R_2 that might be incurred during manufacturing.



W2

Figure 26. Vertical Displacement versus Radius and Radius Ratio.

Although the optimal radius of curvature is now known, a force-deflection relationship for the bimorph is still required. This relationship will be derived using a different modeling technique. The solution using the differential equation model becomes too complex due to the increased number of terms that must be carried, since the moment is no longer constant throughout the beam segments.

Castigliano's Theorem Model Solution

The solution of the Castigliano's theorem model is presented in this section, resulting in a force-deflection relationship for the bimorph. Figure 27 illustrates the curved, cantilevered-beam model to which Castigliano's theorem will be applied. The end at A is also constrained to have zero slope. However, this leads to a problem that is statically indeterminant. Therefore, the slope constraint is replaced by a moment, M_A , which is applied at this end. Subsequently, the value of the moment will be chosen in order to force the slope at A to be zero.

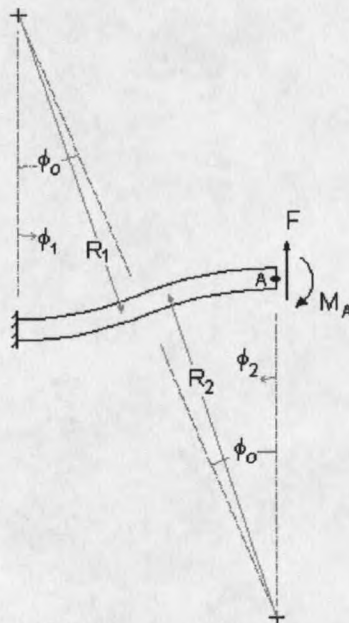


Figure 27. Castigliano's Theorem Model.

The vertical force F and the moment M_A shown in Figure 27 are externally applied mechanical loads. The single, cantilevered beam above is cut at the inflection point of the double-S curve, and the resulting free body diagrams are drawn in Figures 28 and 29. Figure 28 shows the right-hand segment of the beam, from the free end to the inflection point.

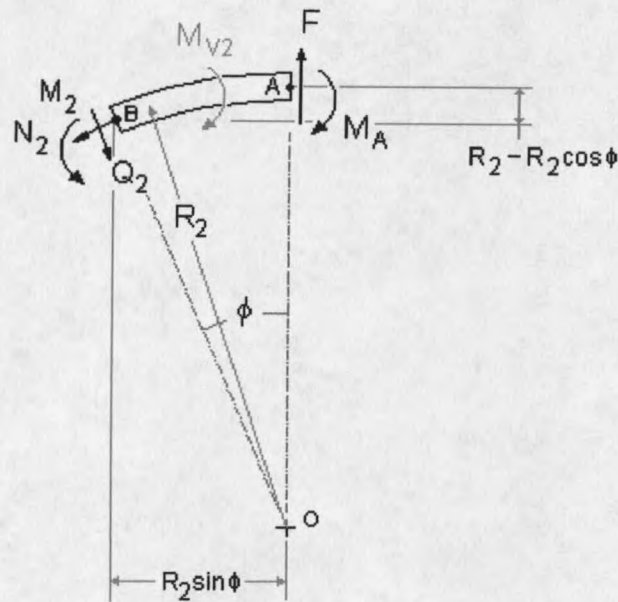


Figure 28. Free Body Diagram of the Right-Hand Segment.

By summing forces in the directions parallel with N and Q , the following two equations result:

$$N_2 = F \sin \phi \quad (4.2-1).$$

$$Q_2 = F \cos \phi \quad (4.2-2).$$

Summing moments about point B gives

$$M_{total2} = M_A - M_{V2} - FR_2 \sin \phi \quad (4.2-3)$$

where

$$M_{total2} = M_2 - M_{V2} \quad (4.2-4)$$

The moment M_{V2} is a pseudo-moment due to the electric field, and is equal to the internal moment, M_2^* , given by Equation 4.1-21. Figure 29 shows the left-hand side of the beam, from

the inflection point to just before the fixed end, with its pseudo-moment, M_{v1} , which is given by Equation 4.1-20.

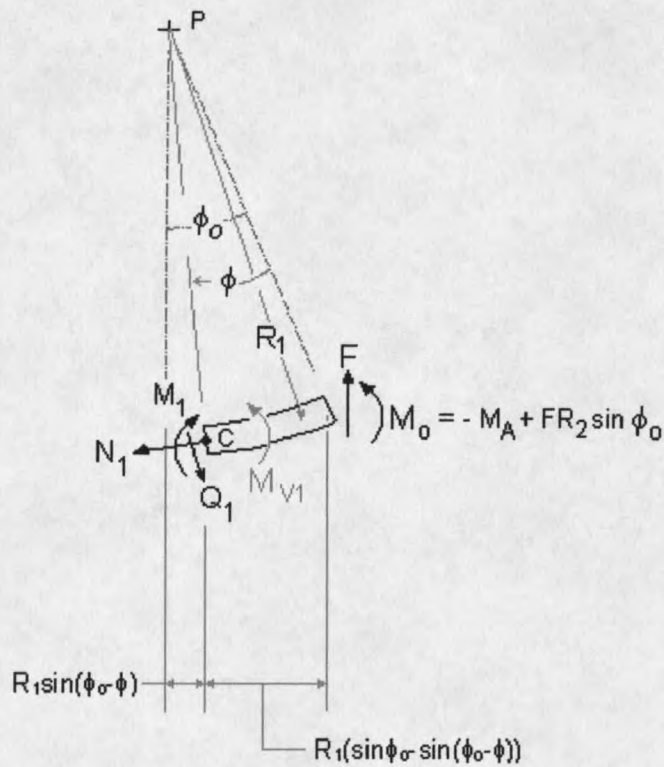


Figure 29. Free Body Diagram of the Left-Hand Segment.

By summing forces in the directions parallel with N and Q , the following two equations result:

$$N_1 = F \sin(\phi_o - \phi) \quad (4.2-8)$$

$$Q_1 = F \cos(\phi_o - \phi) \quad (4.2-9)$$

Summing moments about point C gives

$$M_{total1} = M_O - M_{v1} - FR_1[\sin(\phi_o) - \sin(\phi_o - \phi)] \quad (4.2-10)$$

where

$$M_{total1} = M_1 - M_{V1} \quad (4.2-11)$$

The moment M_o is given by the following by equilibrium applied at the interface between the two segments:

$$M_o = -M_2(\phi_o) = -M_A + FR_2 \sin \phi_o \quad (4.2-12)$$

The total strain energy in the right-hand beam is given by the following integral equation.

$$U_2^* = \int_0^{\phi_o} \left(\frac{M_{total2}^2}{2YI_{eq}} + \frac{N_2^2}{2YA_{eq}} \right) R_2 d\phi \quad (4.2-13)$$

Note that since the radius of curvature is greater than two times the beam thickness, the straight-beam form of the strain energy integral can be used. Also, since the length to thickness ratio for the bimorph is greater than 10, the shear effects can be neglected [17]. The total strain energy in the left-hand beam is given by the integral equation below:

$$U_1^* = \int_0^{\phi_o} \left(\frac{M_{total1}^2}{2YI_{eq}} + \frac{N_1^2}{2YA_{eq}} \right) R_1 d\phi \quad (4.2-14)$$

The total strain energy in the cantilevered beam then is

$$U_{total}^* = U_1^* + U_2^*.$$

The total slope at the free end of the model, point A as shown in Figure 12, is given by

$$\theta_A = \frac{\partial U_{total}^*}{\partial M_A} = \frac{\partial U_1^*}{\partial M_A} + \frac{\partial U_2^*}{\partial M_A} \quad (4.2-15)$$

Differentiating Equations 4.2-13 and 4.2-14 with respect to M_A (while using Equations 4.2-1, 3, 8, 10 and 4.2-12), performing the integration and adding the two results gives the following expression for the total slope at point A:

$$\theta_A = R_1 \left[\frac{(-R_1 \sin(\phi_o) \cdot \phi_o + R_1 - R_2 \sin(\phi_o) \cdot \phi_o)}{(Y \cdot I_{eq})} - R_1 \frac{\cos(\phi_o)}{(Y \cdot I_{eq})} \right] \cdot F + R_1 \frac{(M_{V1} \phi_o + M_A \phi_o)}{(Y \cdot I_{eq})} \dots$$

$$+ R_2 \left[\cos(\phi_o) \cdot \frac{R_2}{(Y \cdot I_{eq})} - \frac{R_2}{(Y \cdot I_{eq})} \right] \cdot F + R_2 \frac{(M_A \phi_o - M_{V2} \phi_o)}{(Y \cdot I_{eq})}$$

Since the slope at point A is constrained to be zero, the above expression can be set equal to zero and solved for M_A to give:

$$M_A = \frac{[\sin(\phi_o) \cdot \phi_o \cdot R_2 \cdot R_1 + (-\cos(\phi_o) + 1) \cdot R_2^2 + (\sin(\phi_o) \cdot \phi_o - 1 + \cos(\phi_o)) \cdot R_1^2]}{[(R_1 + R_2) \cdot \phi_o]} \cdot F + \frac{(R_2 \cdot M_{V2} - M_{V1} \cdot R_1)}{((R_1 + R_2))}$$

(4.2-16)

This is the moment reaction necessary to maintain zero slope at point A. The vertical deflection at point A is given by

$$\delta_A = \frac{\partial U_{total}^*}{\partial F} = \frac{\partial U_1^*}{\partial F} + \frac{\partial U_2^*}{\partial F} \quad (4.2-17)$$

Differentiating Equations 4.2-13 and 4.2-14 with respect to the externally applied force, F , using Equations 4.2-1, 3, 8, 10, 12 and 4.2-16, and performing the integration and adding the two results gives, when simplified for $R_1=R_2=R$:

$$\delta_A = (\sin(\phi_o) \cdot \phi_o - 1 + \cos(\phi_o)) \cdot \frac{R^2}{(Y \cdot I_{eq})} \cdot (M_{V1} + M_{V2}) \dots$$

$$+ \left[\frac{(-3 \cdot \phi_o - 3 \cdot \cos(\phi_o) \cdot \sin(\phi_o) + 4 \cdot \sin(\phi_o) + 2 \cdot \phi_o \cdot \cos(\phi_o)^2)}{(Y \cdot I_{eq})} \cdot R^3 \right] \cdot F \dots$$

$$+ \left[R \cdot \frac{(\cos(\phi_o) - 1 - \phi_o + \cos(\phi_o) \cdot \sin(\phi_o))}{(Y \cdot A_{eq})} \right] \cdot F$$

(4.2-18)

The piezoelectric pseudo-moments, M_{V1} and M_{V2} , are equal to the internal moments given by Equations 4.1-20 and 4.1-21. The quantities A_{eq} and I_{eq} are given by Equations 2.4-31 and 2.4-25, respectively. Note that if the bimorph length is fixed, then with Equation 4.1-26, the second term in Equation 4.2-18 behaves as R^2 for large R , whereas the remaining terms are bounded. This unusual behavior is also seen in beams consisting of a single circular arc, as can be seen in [23]. If the applied mechanical force, F , is set equal to zero then Equation 4.2-18 becomes

$$\delta_A = -\left(\sin(\phi_o) \cdot \phi_o - 1 + \cos(\phi_o)\right) \cdot \frac{R^2}{Y \cdot I_{eq}} \cdot (M_{V1} + M_{V2}) \quad (4.2-19)$$

which is equivalent to Equation 4.1-23, the applied electrical field solution using the differential equation method.

The stiffness of the bimorph is given by the following expression:

$$K = \frac{F}{\delta_A} = \left[\frac{-\left(-3 \cdot \phi_o - 3 \cdot \cos(\phi_o) \cdot \sin(\phi_o) + 4 \cdot \sin(\phi_o) + 2 \cdot \phi_o \cdot \cos(\phi_o)^2\right) \cdot (Y \cdot I_{eq})}{\left(\cos(\phi_o) - 1 - \phi_o + \cos(\phi_o) \cdot \sin(\phi_o)\right)} \cdot R^3 + \left[R \cdot \frac{\left(\cos(\phi_o) - 1 - \phi_o + \cos(\phi_o) \cdot \sin(\phi_o)\right)}{(Y \cdot A_{eq})} \right] \right]^{-1}$$

(4.2-20).

The stiffness can be plotted as a function of radius for a model with no epoxy layer, as shown in Figure 30:

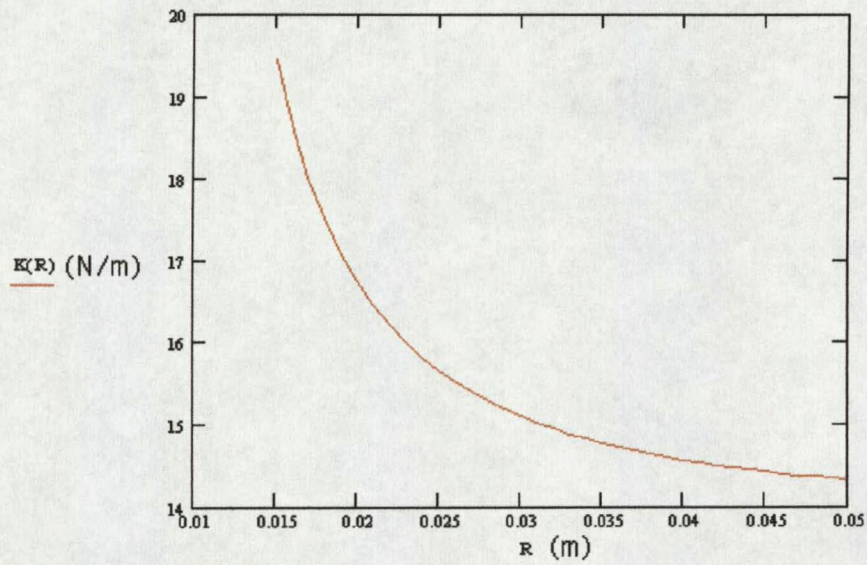


Figure 30. Stiffness versus R for a Bimorph with No Epoxy Layer.

By including an epoxy layer that is 25 μm thick, a plot of the stiffness as a function of radius can be generated to produce the following, where K_g denotes the stiffness with the epoxy layer:

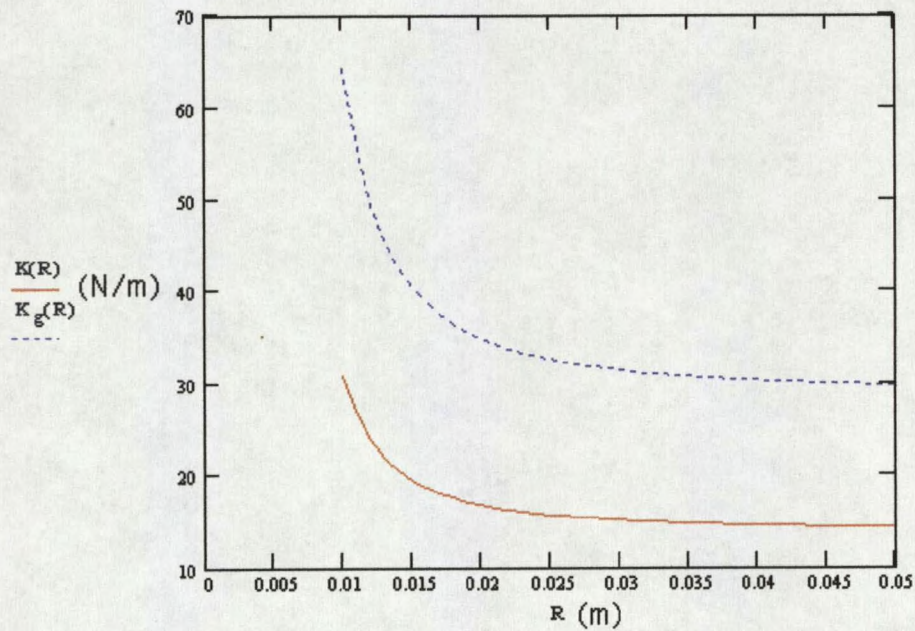


Figure 31. Stiffness versus Radius for a Bimorph With an Epoxy Layer 25 μm Thick.

If the vertical force is set equal to zero in Equation 2.4-15, the resulting equation can be solved for the voltage-displacement ratio. This ratio gives the displacement per given voltage gradient applied across the bimorph layers:

$$\frac{\delta_A}{dV} = \frac{2 \cdot (\sin(\phi_0) \cdot \phi_0 - 1 + \cos(\phi_0)) \cdot R^2 \cdot d_{31} \cdot A \cdot h_{eq}}{(I_{eq} \cdot h_p)} \quad (4.2-21)$$

The thickness h_p is the thickness of the clear PVDF film, without electrodes, and the thickness h is electroded film thickness. This slope can be plotted versus epoxy layer thickness for various radii, as shown in Figure 30. In the figure, K_{V1} is for a radius of 0.05 m, K_{V2} is for a radius of 0.0315 m, and K_{V3} is for a radius of 0.01 m.

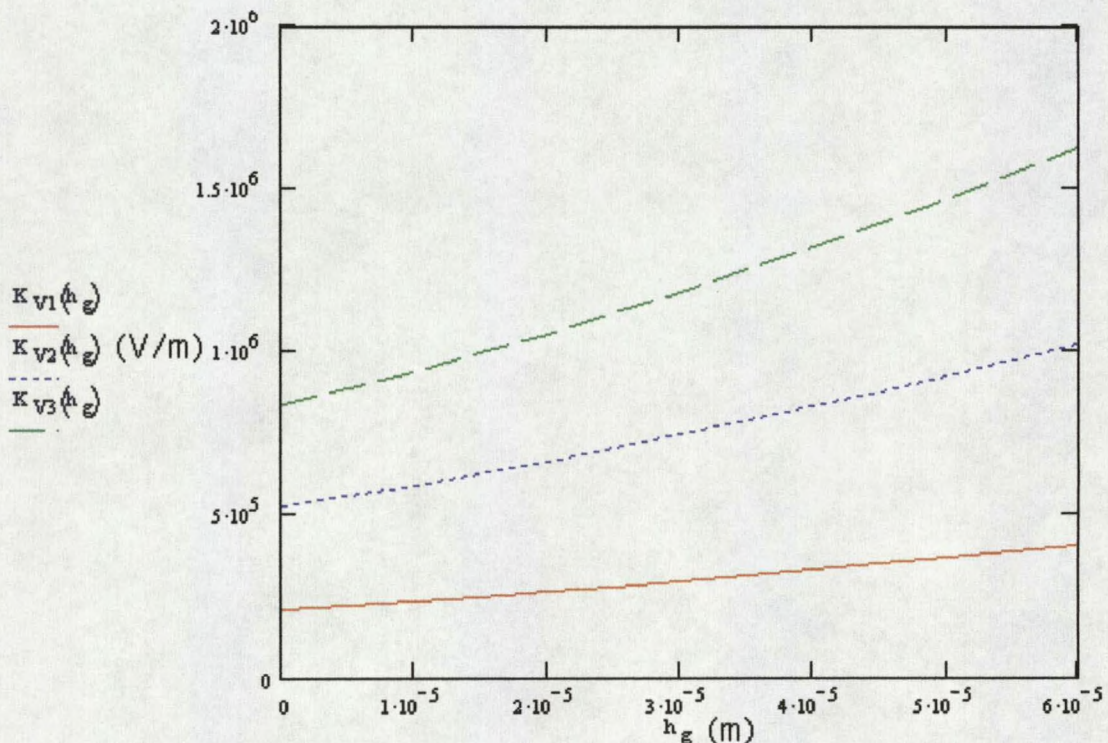


Figure 32. Voltage-Displacement Curve versus Epoxy Layer Thickness for Various R.

The percent difference between the voltage-displacement slope with an epoxy layer relative to the slope without one can be calculated. This percent difference, which is independent of radius of curvature, can be plotted as a function of epoxy layer thickness. In Figure 33, the percent difference is plotted. Note that in order to achieve less than a 10% difference between the behavior with an epoxy layer and without one, the epoxy layer thickness must be at or below $9\text{ }\mu\text{m}$ thick.

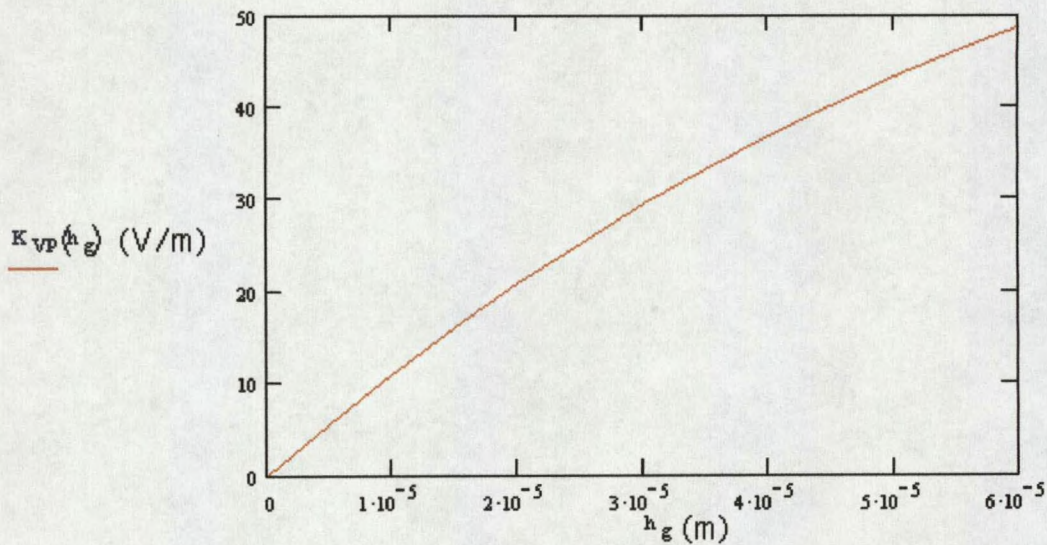


Figure 33. Percent Difference in Voltage-Displacement Slope for Epoxy Layer Relative to No Epoxy Layer as a Function of Epoxy Layer Thickness.

Details of the above optimization and stiffness calculations can be found in Appendix D.

CHAPTER 5

FINITE ELEMENT MODEL

The finite element model was developed using ANSYS finite element software. Due to symmetry, only one quarter of an actuator was modeled. Figure 19 illustrates that portion and the planes of symmetry. The reinforcing strip of balsa wood along the actuator's top surface distributes any applied loads along that surface. The actuator is always attached—to the instrument box that it isolates on one side and a rigid frame on the other—at a point at the mid-span of this strip; therefore, the deflections at point O in Figure 34, will be the same as at point I. In other words, any cross-section through the actuator in a plane where z is constant will experience the same deflections as any other. Therefore, a two-dimensional model will satisfactorily describe the actuator's behavior.

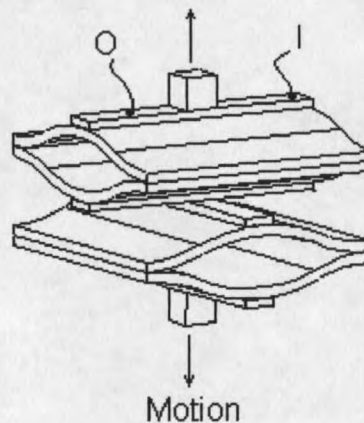


Figure 34. Actuator Behavior Through the Depth.

Three models were developed: one with no glue layer, one with a glue layer with uniform 25- μm thickness, and one with a non-uniform glue layer with 9 μm random variation from a mean thickness of 25 μm . The geometry of each model was defined using several keypoints in Ansys. These keypoints are simply points in space from which areas can be defined. Determining the location of the keypoints required some calculations to define points that matched the actuator shape. Mathcad programs were developed to perform these calculations, and are given in Appendix E for each model. These programs calculate the location of keypoints from inputs such as the PVDF layer thickness, epoxy layer thickness, radius of curvature, and length of the actuator. Defined areas can then be meshed with finite elements, and thus, a model is generated.

Elements

To perform a piezoelectric analysis in Ansys, coupled-field elements must be used. Some examples of coupled-field analyses include thermo-electric, thermal-structural, electro-magnetic, and piezoelectric. There are three elements available in Ansys for piezoelectric analyses: SOLID5, PLANE13, and SOLID98. The solid elements are three-dimensional bricks, SOLID5 being a linear element and SOLID98 being quadratic. For a two-dimensional analysis, Ansys provides only the PLANE13 element, which is a four-node quadrilateral.

Ansys does not provide coupled-field beam elements, which would be the most desirable element with which to model the actuator since the analytical solution treats the actuator as a beam. Beam elements predict a linear distribution of stress through the thickness, as is the case in elementary beam theory. The PLANE13 element can approximate the behavior of a beam since it has only four nodes. As such, it uses a bilinear displacement field of the form:

$$u = A_0 + A_1x + A_2y + A_3xy$$

Therefore, only linear strain variations can be predicted. Thus, the four-node linear quadrilateral element can produce a linear stress distribution through its thickness. However, if there is more than one element in the mesh through the beam's thickness, then non-linear stress distributions can be approximated as piecewise linear. See Figure 35 for an illustration of a beam cut at some point near the fixed end, and note the stress distribution plotted along the cut end. In part A, the single element accurately describes the linear distribution through the thickness. In part B, the multiple elements allow a piecewise linear distribution through the thickness of the beam.

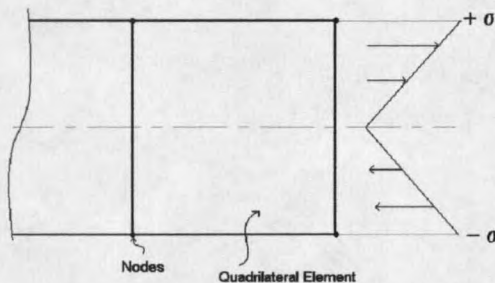


Figure 35. Part A) One Element.

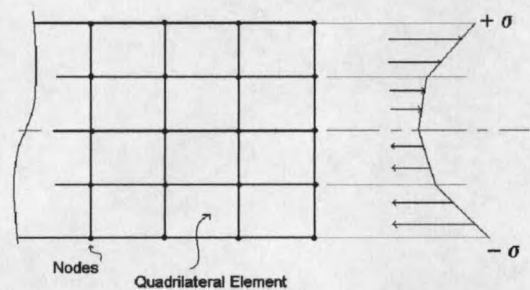


Figure 35. Part B) Multiple Elements.

The finite element model should be compared to the analytical model to verify that it was set up properly. The finite element model is also capable of predicting a non-linear stress distribution through the thickness of the beam, since plane elements are used. The analytical model predicts a linear stress distribution through the beam thickness. It was determined how closely the finite element model agrees. Once the finite element model has been verified, however, it can more accurately represent the actuator's behavior since it can provide non-linear analysis, include shear deflections, and use orthotropic material properties.

What minimum number of elements should be used, then, through the thickness of the actuator? If too few elements were used, a misleading linear distribution would be produced automatically, because the plane elements could not predict anything else. To determine how linear the distribution would be for a beam with an aspect ratio comparable to that of the actuator model, a concentrated load was applied to the free end of a straight, cantilevered beam with a length 100 times its thickness. A very linear stress distribution resulted, even with ten elements

through the thickness. The actuator's length is on the order of 1000 times its thickness, so this study provided a conservative estimate of the stress linearity through the thickness. The finite element model therefore also produced a linear distribution through the thickness, even when using a large number of elements.

The thinness of the actuator made it easy to model the layers each with just one element through the thickness. Thus, only five elements would exist through the entire thickness of the actuator (if both the electrode layers and the epoxy layer were modeled separately from the layers of PVDF). One element through the thickness of each layer produces, using the automatic area meshing function in Ansys, square elements on the order of ten to 30 microns long. In order to mesh one quarter of an actuator, thousands of elements would be required to span the one-inch length.

In order to reduce the model size, rectangular-shaped elements are more desirable than square ones. Some sources indicate that using higher aspect ratio—length over height—elements compromises solution accuracy [24]. Other sources indicate that aspect ratio has no correlation with solution accuracy, as long as a sufficiently fine mesh of elements is used in regions with high stress gradients [25]. Another study was done to determine the maximum aspect ratio for elements used to model the actuator. Keeping the total number of elements constant, straight, cantilevered beams meshed with PLANE13 elements of varying aspect ratios were analyzed when a concentrated load was applied at the free end. For an aspect ratio of six, the results were within two percent of the results for an aspect ratio of one. An aspect ratio of three or less was used in the models discussed in this paper.

Another point that must be considered when modeling with the PLANE13 element is that Ansys allows it to be either a four-node quadrilateral or a three-node triangular element. However, the triangular element results in constant strain in the element, which will under-predict displacements in bending. In other words, the element is too stiff. Ansys provides the option to set either of the shapes as the only form to be used in meshing, or the user can choose to let Ansys determine the appropriate form. To perform an analysis of the actuator, the quadrilateral form must be chosen as the default.

By paying attention to the element shape, its aspect ratio, and recognizing that the stress distribution may not be entirely linear for shorter aspect ratio beams, the PLANE13 element can be successfully used and compared to the analytical beam solutions. There is one potential cause for differences between the finite element and analytical methods that must be kept in mind, however. The analytical solution should be slightly stiffer than the finite element solution, since shear deflection is not accounted for in the analytical solution. It is implicitly included in the finite element solution, however, if PLANE13 elements are used. The analytical solution should therefore predict smaller deflections than the finite element model. Due to the high aspect ratio of the actuators, however, the effects of shear should also be negligible in the finite element solution.

Linear and Non-linear Analyses

One of the major benefits of the finite element method over the analytical method is that the limits of the actuator's linear behavior can be determined using non-linear analyses. Ansys has the capability to perform non-linear analyses for mechanical degrees of freedom only, that is, only displacement degrees of freedom. For the PLANE13 element, that means only UX and UY may be active, and the VOLT degree of freedom must be excluded. This is in contrast to a linear analysis, in which the electrical degree of freedom may be active.

Non-linearities may be introduced by material behavior or by geometry. An example of non-linear material behavior is plastic deformation. Geometric non-linearities arise when the stiffness of the structure is dependent upon the deflections. A classic example of this is the fishing rod. The fishing rod is a straight, cantilevered beam fixed in the fisherman's hand. When a load is applied at the free end, such as when a fish is caught, the beam deflects downward and a resisting moment is produced at the fixed end. As the rod deflects downward, it also deflects laterally, thus decreasing the moment arm of the load. Therefore, the moment at the fixed end does not change in direct proportion to the load.

Ansys can perform analyses including the effects of geometric non-linearities by applying the loading in small substeps. A "load step" in Ansys is a particular configuration of loads that will

be applied to a model. In linear static analyses, multiple load steps can be used to apply different load types, such as wind or gravity loads. In transient analyses, different load steps can be used to apply loads sequentially in time. "Substeps" are points within a load step at which Ansys will calculate a solution. In a substep, a fraction of the total load is applied to the geometry and displacements are calculated. These displacements are stored, so that in the next substep, another fraction of the load is applied to the geometry that has already been displaced by the previous substep. The displacements are calculated again. Now the total load that the structure has been exposed to is the sum of the first and second substeps. The geometry of the model is essentially updated between each substep, but this process is transparent to the user. This is the same as applying a load to a structure and calculating the displacement due to that load, then updating the geometry of the structure to the deformed configuration and applying a higher load, and so on. In either case, if the load were plotted versus the deflection at each substep, any non-linearity would be evident.

Ansys Material Properties

Anisotropic material properties must be input into Ansys differently than isotropic properties. A material property table must be used for the mechanical properties and another table for the piezoelectric properties. Each property table corresponds to a material property matrix in the set of piezoelectric constitutive laws used in Ansys, which are

$$\{T\} = [c^E]\{S\} - [e^S]\{E\} \quad (5.3-1a)$$

$$\{D\} = [e^S]^t\{S\} + [\epsilon^S]\{E\} \quad (5.3-1b)$$

where, $\{T\}$ is the 6 x 1 stress matrix, $[c^E]$ is the 6 x 6 stiffness matrix at constant electric field, and $\{S\}$ is the 6 x 1 mechanical strain matrix. $[e^S]$ is the 6 x 3 piezoelectric matrix at constant mechanical strain, $\{E\}$ is the 3 x 1 electric field matrix, and $\{D\}$ is the 3 x 1 electric displacement

matrix. The 3 x 3 dielectric matrix at constant mechanical strain is given by $[\epsilon^S]$. The superscript t denotes matrix transposition. Equations 5.3-1 can be rearranged into a form that is more commonly used in published work:

$$\{S\} = [s^E]\{T\} + [d^T]^t \{E\} \quad (5.3-2a)$$

$$\{D\} = [d^T]\{T\} + [\epsilon^T]\{E\} \quad (5.3-2b).$$

In the above, $[s^E]$ is the 6 x 6 compliance matrix at constant electric field, $[d^T]$ is the 3 x 6 piezoelectric matrix at constant stress, and $[\epsilon^T]$ is the dielectric matrix at constant stress.

Material properties are usually reported in the literature in terms of the $[s^E]$, $[d^T]$, and $[\epsilon^T]$ matrices (see, for example, Schewe [16]). These matrices must be converted to $[c^E]$, $[e^S]$, and $[\epsilon^S]$. If Equation 5.3-2a is solved for $\{T\}$ and the result substituted into Equation 5.3-2b, this new set of equations can be compared with Equations 5.3-1 to get the following matrix relationships:

$$[c^E] = [s^E]^{-1} \quad (5.3-3a)$$

$$[e^S] = [s^E]^{-1}[d^T]^t = [c^E] \cdot [d^T]^t \quad (5.3-3b)$$

$$[\epsilon^S] = [\epsilon^T] - [d^T] \cdot [c^E] \cdot [d^T]^t \quad (5.3-3c).$$

Equation 5.3-2a, in expanded form, is

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ s_{12} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ s_{13} & s_{23} & s_{33} & s_{34} & s_{35} & s_{36} \\ s_{14} & s_{24} & s_{34} & s_{44} & s_{45} & s_{46} \\ s_{15} & s_{25} & s_{35} & s_{45} & s_{55} & s_{56} \\ s_{16} & s_{26} & s_{36} & s_{46} & s_{56} & s_{66} \end{bmatrix} \begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{31} \\ T_{12} \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \\ d_{41} & d_{42} & d_{43} \\ d_{51} & d_{52} & d_{53} \\ d_{61} & d_{62} & d_{63} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

In the above expression, ε_{ij} denote the strains, s_{ij} are the compliance matrix components, T_{ij} are the stresses, d_{ij} are the piezoelectric matrix components, and E_i indicate the electric fields. Since PVDF is composed of orthorhombic crystals with $mm2$ structure [19], the matrices in the above equation reduce to the following [26]:

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{13} & s_{23} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{31} \\ T_{12} \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{13} \\ 0 & 0 & d_{23} \\ 0 & 0 & d_{33} \\ 0 & d_{42} & 0 \\ d_{51} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

Since the finite element model considers a plane problem, the matrices further reduce for two dimensions. Usually, to reduce three-dimensional constitutive equations for a two-dimensional problem, plane stress or plane strain is assumed. Then out-of-plane components are eliminated from the constitutive equations. However in this case, since Ansys retains the out-of-plane normal stress and strain components, the same formulation can be used for either plane stress or plane strain. When element types are defined, plane stress or plane strain can be chosen, and Ansys makes the appropriate reductions to eliminate one of the normal components and two shear components. The stiffness or compliance matrices for plane problems in Ansys

are therefore 4 x 4 instead of 3 x 3 matrices, as they would be if that normal component were eliminated.

Recall Figure 12, and that the electrodes are applied such that the 3 direction is the only active piezoelectric axis in the piezoelectric film. Therefore, the strains ϵ_{23} and ϵ_{12} and the electric field E_2 are eliminated for the two-dimensional problem, but the out-of-plane normal strain, ϵ_{22} , and normal stress, σ_{22} , are retained.

$$\begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{31} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 \\ s_{12} & s_{22} & s_{23} & 0 \\ s_{13} & s_{23} & s_{33} & 0 \\ 0 & 0 & 0 & s_{55} \end{bmatrix} \begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{31} \end{bmatrix} + \begin{bmatrix} 0 & d_{31} \\ 0 & d_{23} \\ 0 & d_{33} \\ d_{51} & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_3 \end{bmatrix} \quad (5.3-4)$$

Note then that the two-dimensional slice of the actuator has the 3 direction aligned with the vertical axis, which is defined as the y-axis in Ansys. The 2-axis of the piezoelectric film is the out-of-plane axis, which would be the z-axis in Ansys. See Figure 36 below.

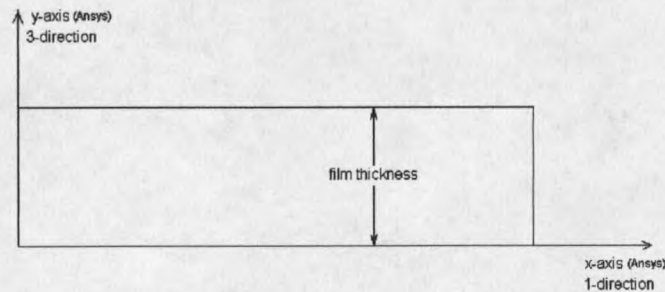


Figure 36. Definition of Axes in the Ansys Model.

The components of the compliance matrix are input in terms of the Young's moduli, Y_i , the Poisson's ratios, ν_{ij} , and the shear moduli, G_i , in which case Equation 5.3-4 becomes [27]

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{31} \end{bmatrix} = \begin{bmatrix} \frac{1}{Y_1} & \frac{-\nu_{21}}{Y_2} & \frac{-\nu_{31}}{Y_3} & 0 \\ \frac{-\nu_{21}}{Y_2} & \frac{1}{Y_2} & \frac{-\nu_{32}}{Y_3} & 0 \\ \frac{-\nu_{31}}{Y_3} & \frac{-\nu_{32}}{Y_3} & \frac{1}{Y_3} & 0 \\ 0 & 0 & 0 & \frac{1}{G_{31}} \end{bmatrix} \cdot \begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{31} \end{bmatrix} + \begin{bmatrix} 0 & d_{31} \\ 0 & d_{23} \\ 0 & d_{33} \\ d_{51} & 0 \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_3 \end{bmatrix}$$

The matrix equation can therefore be written in terms of direction 1=x, 2=z, and 3=y:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \varepsilon_{yy} \\ \varepsilon_{yx} \end{bmatrix} = \begin{bmatrix} \frac{1}{Y_x} & \frac{-\nu_{zx}}{Y_z} & \frac{-\nu_{yx}}{Y_y} & 0 \\ \frac{-\nu_{zx}}{Y_z} & \frac{1}{Y_z} & \frac{-\nu_{yz}}{Y_y} & 0 \\ \frac{-\nu_{yx}}{Y_y} & \frac{-\nu_{yz}}{Y_y} & \frac{1}{Y_y} & 0 \\ 0 & 0 & 0 & \frac{1}{G_{yx}} \end{bmatrix} \cdot \begin{bmatrix} T_{xx} \\ T_{zz} \\ T_{yy} \\ T_{yx} \end{bmatrix} + \begin{bmatrix} 0 & d_{31} \\ 0 & d_{23} \\ 0 & d_{33} \\ d_{51} & 0 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

Ansys expects the order of the terms in the stress and strain matrices to be

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yx} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} T_{xx} \\ T_{yy} \\ T_{zz} \\ T_{yx} \end{bmatrix}.$$

Therefore, the material property matrices must be re-ordered using a simple matrix multiplication shown in Appendix F. This calculation also illustrates how to convert the matrices into the form of the constitutive laws that Ansys uses.

Material properties in Ansys can be assigned in the global coordinate system, or they can be assigned in the local coordinate system for each element. For an isotropic material, the two are equivalent. For PVDF, however, the difference is important.

Since the actuator model has curvature, and the direction of polarity of the material is always in the direction of the surface normal, the material properties must be defined in the element coordinate system. See Figure 37 for an illustration of a quarter actuator and the coordinate systems. If the material properties are aligned parallel to the element coordinate systems, then the correct direction for the polarity of the material is maintained. The polarity is always perpendicular to the electrode surfaces. This changes direction, relative to the global coordinate system, along the curved shape of the actuator. The element coordinate systems remain parallel with two sides of the element.

The element coordinate systems are defined by the order the nodes are selected when creating elements. If automatic area meshing is used, then all the element coordinate systems automatically are all in the same direction if the areas are defined the same. Areas are defined using the "keypoints", and these keypoints must be selected in the same order. Similarly, if area meshing is not used and the elements are created by hand, the user must be careful to always select nodes in the same clockwise or counter-clockwise direction around the quadrilateral region.

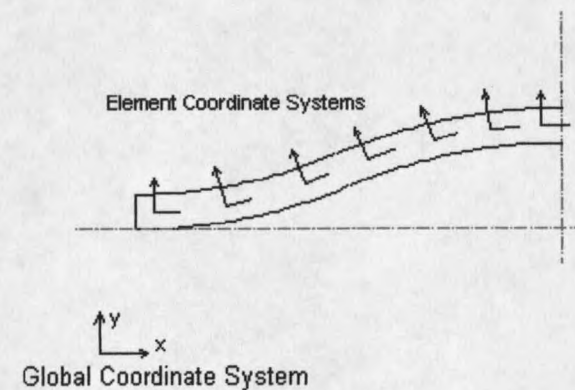


Figure 37. Element versus Global Coordinate System.

Once all the material property tables for the finite element model were established, macro programs were written that defined the geometry, elements, material properties, and solution parameters. These macro programs are then executed within the Ansys software to produce a solution. These macro programs are easy to modify and take up much less disk storage space than an entire model, which requires thousands of nodes and elements.

CHAPTER 6

RESULTS

Three finite element models were solved: one with no glue layer, one with a 25- μm uniform epoxy layer, one with a non-uniform epoxy layer with 9- μm random variation from a mean thickness of 25 μm . Non-linear analyses were performed for models without electrical degrees of freedom. Linear analyses were also performed that included the voltage degrees of freedom. The complete finite element models, described by macro programs, are given in Appendix G. These must be executed within Ansys finite element software. They contain comments that describe each command in detail.

The force-deflection curves generated using these models are shown in the following figures. The mechanical stiffness of the actuator predicted by the Ansys model without an epoxy layer was 13.9 N/m. Castigliano's theorem predicts a stiffness of 13.85 N/m for the actuator without an epoxy layer. This gives an error of around 0.4% between the two linear models. The Ansys model with a 25- μm , uniform epoxy layer had a stiffness of 28.7 N/m, and the stiffness predicted by Castigliano's theorem is 28.9 N/m. This gives a difference of approximately 0.7%.

The analytical and Ansys results for models with an applied mechanical load, but without an epoxy layer, are shown in Figure 38. The error between the linear and non-linear solution at a load of 0.05 N is 19.5%. The load, 0.05 N, corresponds to a deflection of the bimorph of 3.04 mm, as predicted by the non-linear Ansys model. Note that for an actuator, a load of 0.1N would correspond to 6.0-mm displacement for the linear solution. The comparison is performed at this point since an actuator deflection of 6.0 mm is the maximum that the actuator is required to provide in the current application.

The analytical and Ansys results for models with an epoxy layer and an applied mechanical load, both linear and non-linear solutions, are shown in Figure 38. The difference

between the linear and non-linear results for a model with the epoxy layer is 7.8% at a load of 0.0935 N, for a deflection of 3.01 mm.

Figure 39 shows the results of the Ansys and analytical model for an applied electric field. Both models with and without an epoxy layer are shown. Experimental data gathered by Andy Ross and Michelle Galvin in November 1999 are also shown on this chart. The actuators tested experimentally had an average epoxy layer thickness of 9 μm , instead of the epoxy layer measured on previous actuators of 25 μm . This is because a new manufacturing technique was used. Note, however, that two of the curves fall approximately between the model results for an epoxy layer of 25 μm and no epoxy layer, as they should. Two other curves fall entirely out of range of the models, which could be attributed to lack of consistency in the new manufacturing process.

The electrical stiffness that is predicted by the analytical model for an actuator without an epoxy layer is 150 V/mm. The electrical stiffness that is predicted by the finite element model without an epoxy layer is 149 V/mm. The analytical and finite element electrical stiffnesses for a model with a 25- μm epoxy layer and 45- μm PVDF layers are 530 V/mm and 527 V/mm, respectively. These results validate the finite element models. In all the models, the voltage was applied along the outside surface of the PVDF layer on the finite element models for comparison, since the electrode layers were not modeled separately in Ansys. The keypoints calculations have the capability to generate points along these layers for the Ansys model; however, since no material properties were available for the electrode layer, these were not included separately in the model. Therefore, the electrical stiffness that is predicted by the Ansys model is higher than the actual electrical stiffness for a 45- μm PVDF layer with a 30- μm unelectroded thickness. Figure 39 shows the Castigliano's theorem model results for a bimorph constructed from two 45- μm PVDF layers and a 25- μm epoxy layer. The electric fields are calculated using the 30- μm unelectroded PVDF thickness, which more accurately represents the bimorph. The electrical stiffness predicted using this model is 353 V/mm.

Figure 40 illustrates the results of the Ansys model with a non-uniform epoxy layer. Note that the mechanical stiffness is 29.86 N/m, which is slightly higher than the uniform epoxy layer. This is simply due to the random variation, which caused the epoxy layer to be slightly thicker along the bimorph than the uniform layer. The error between the linear and non-linear results for the model with a non-uniform epoxy layer is 7.6% in extension at 3.0 mm of displacement. The uniformity of the epoxy layer, therefore, does not significantly affect the linear range of the actuator's behavior. It also does not have a significant effect on the actuator's stiffness; it only differs from the stiffness of the uniform epoxy layer results by 4%. Figure 41 shows an example of the shape of the deformed Ansys model compared to the undeformed model. In Figure 41, an extension load is applied, so the deformed model is the contoured, top S-curve. Figure 42 shows the results with a compression load applied.

Note in the graphs of the finite element results, that the force-deflection curve in the compression region is essentially linear. In the extension region, the curves become increasingly non-linear. The stiffness increases with increasing displacement, because as the displacement gets larger, the bimorph's radius of curvature decreases. The increasing stiffness with decreasing radius of curvature is also illustrated in Figure 31 from Chapter 4. Consider a semi-circular arc of fixed length that is loaded in the center. If the radius of curvature is small, the arc will be stiffer than for a larger radius of curvature. Likewise, the bimorph's stiffness changes with changing radius of curvature.

Load vs. UY End Displacement Ansys Model Results
 No Epoxy Layer and 45 micron PVDF layers, 25 Micron Epoxy Layer and 45 micron PVDF layers

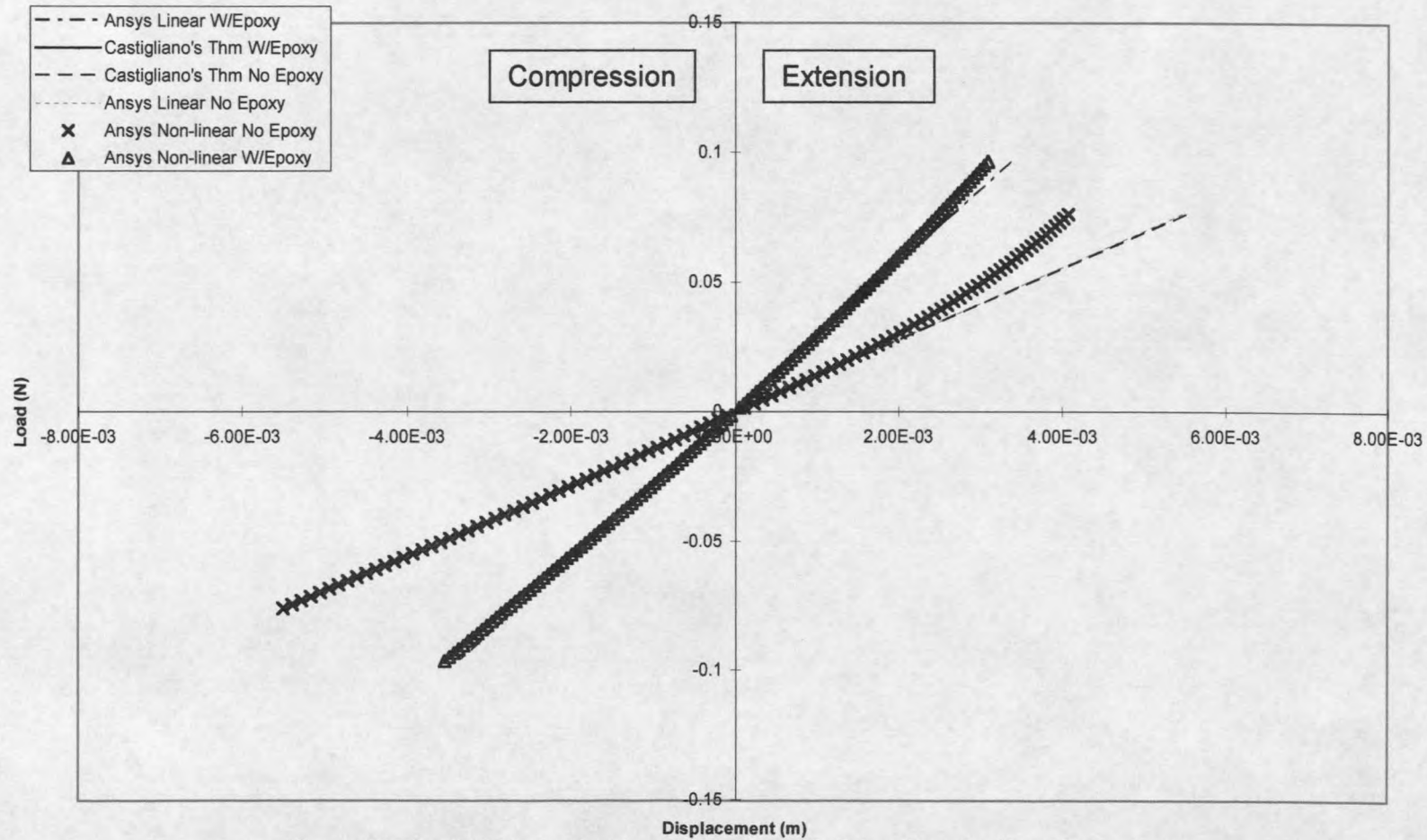


Figure 38. Applied Mechanical Load Results for Models with and without Epoxy layers.

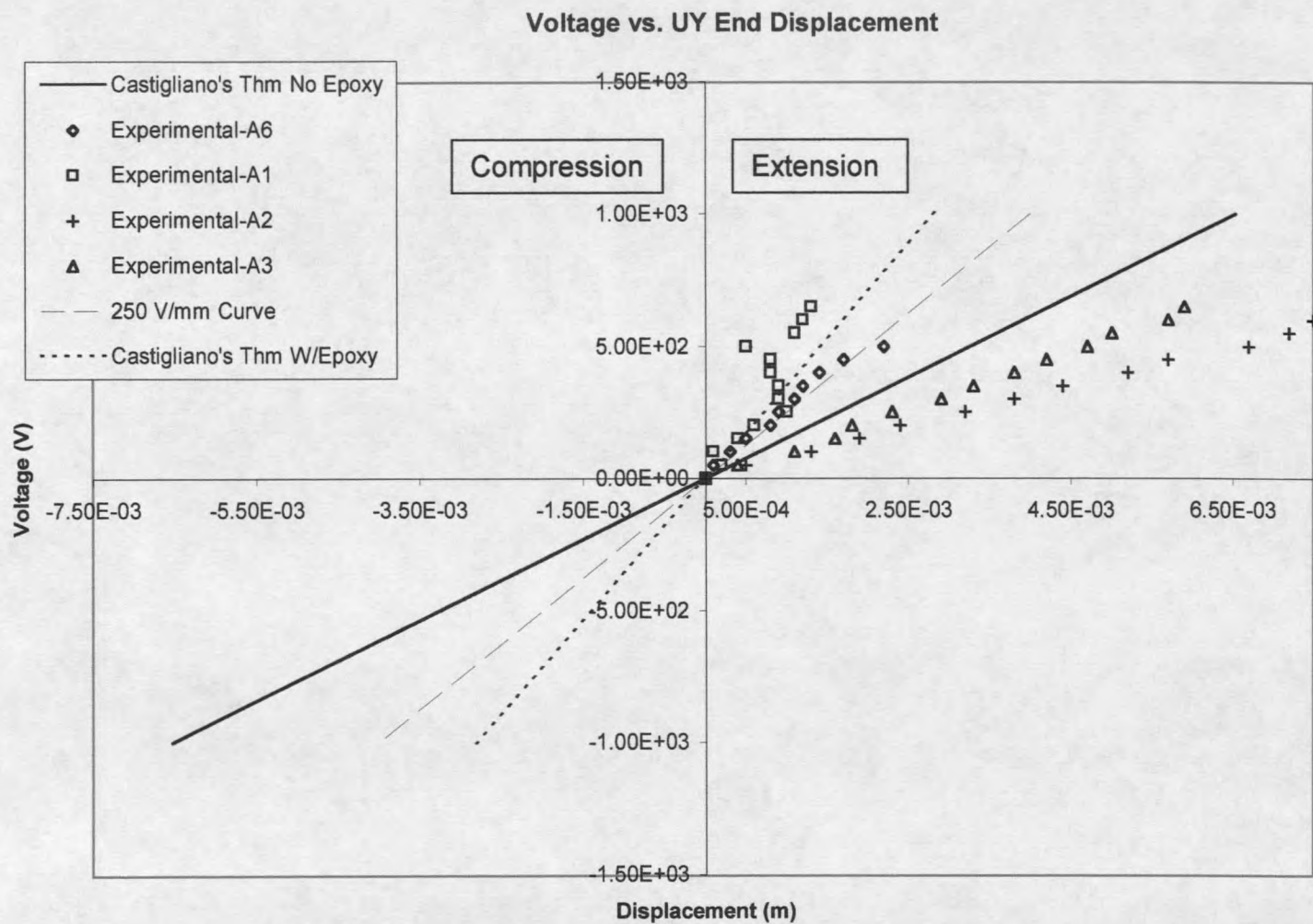


Figure 39. Applied Electric Field Results for Models with and without Epoxy Layers.

Non-uniform Epoxy Layer Stiffness

Nonuniform 25 micron Epoxy layer with 9 micron random deviation and 45 micron PVDF layers

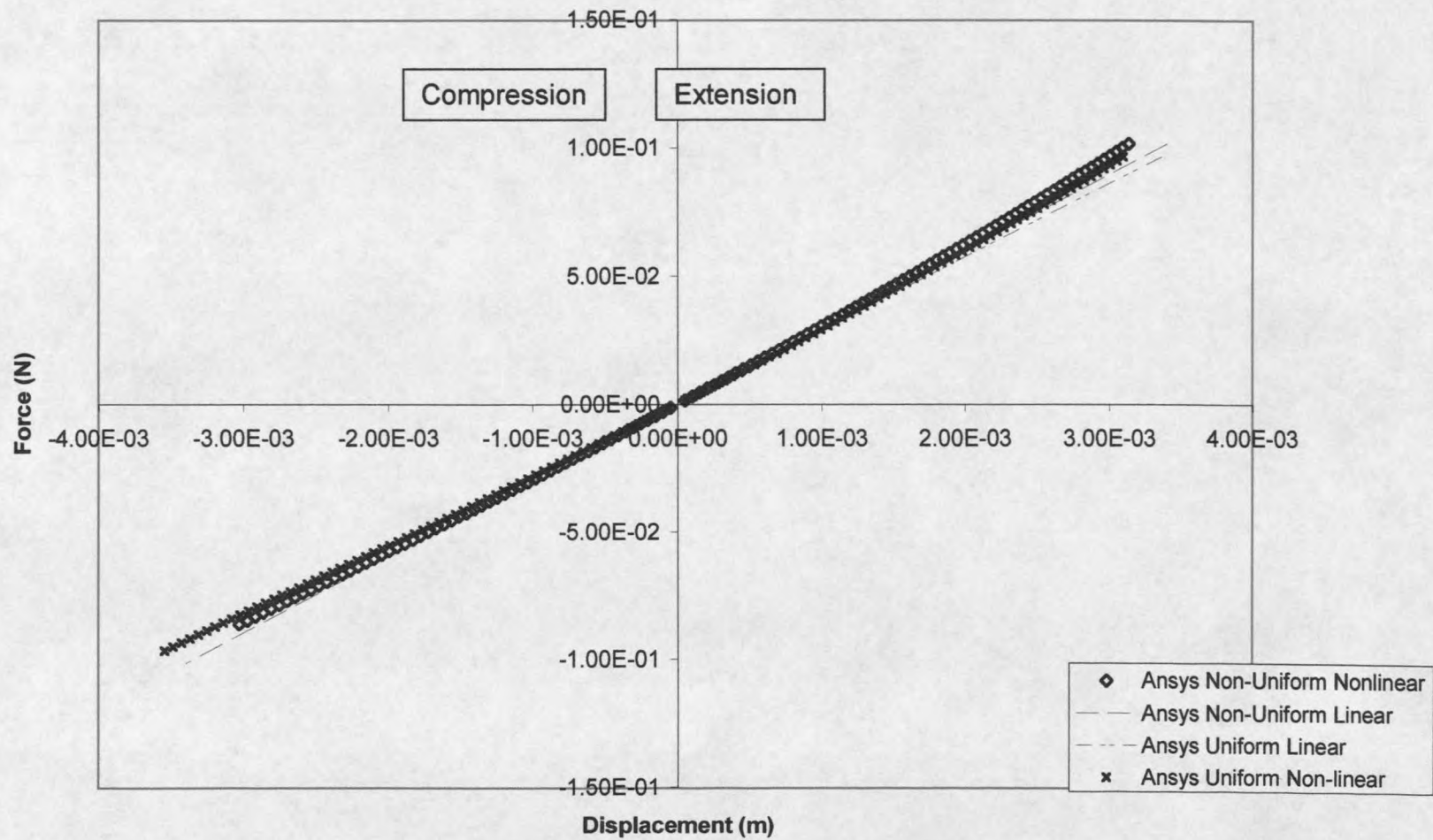


Figure 40. Applied Mechanical Load Results for Model with Non-Uniform Epoxy Layer.

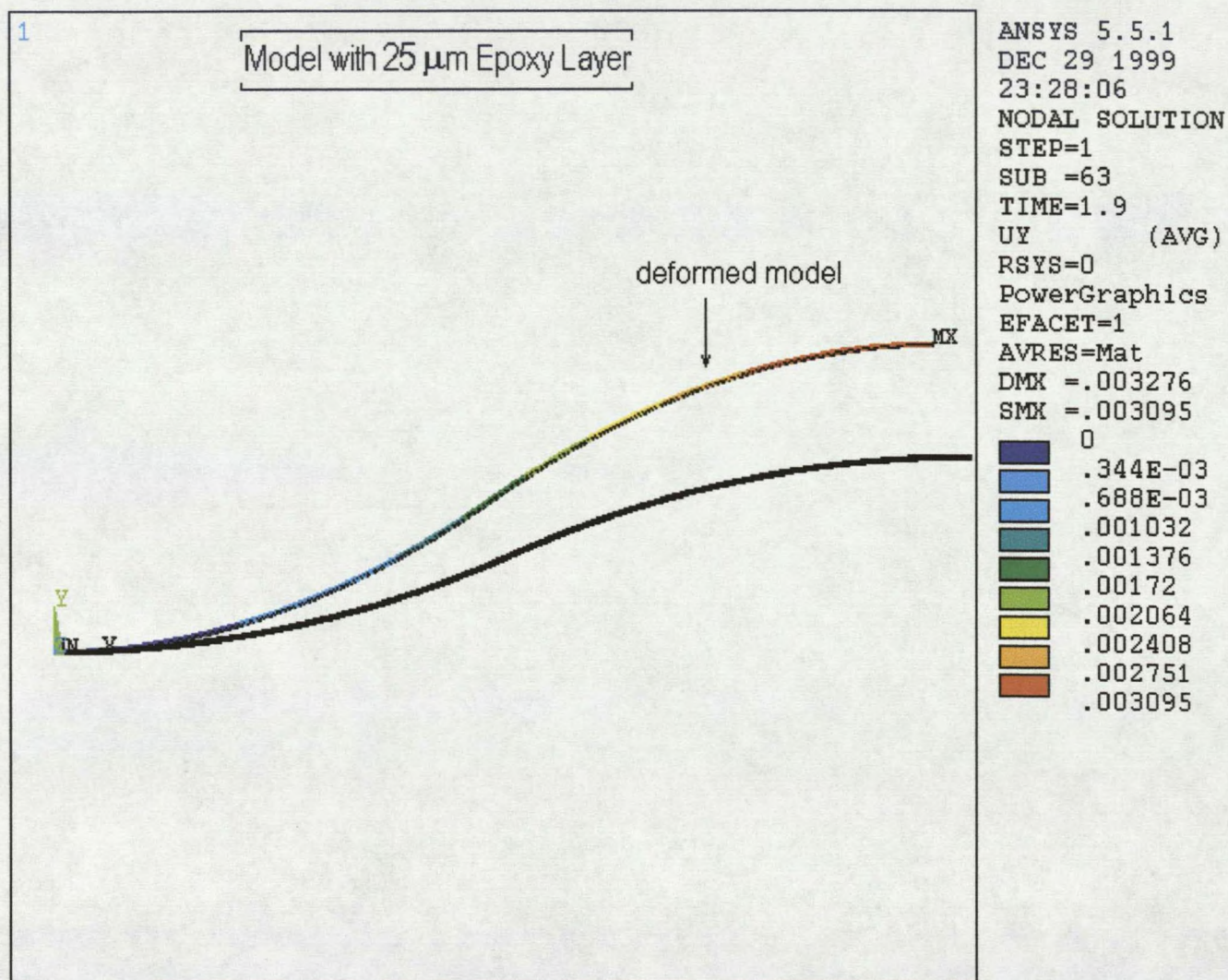


Figure 41. Ansys Model Results with Applied Extension Load.

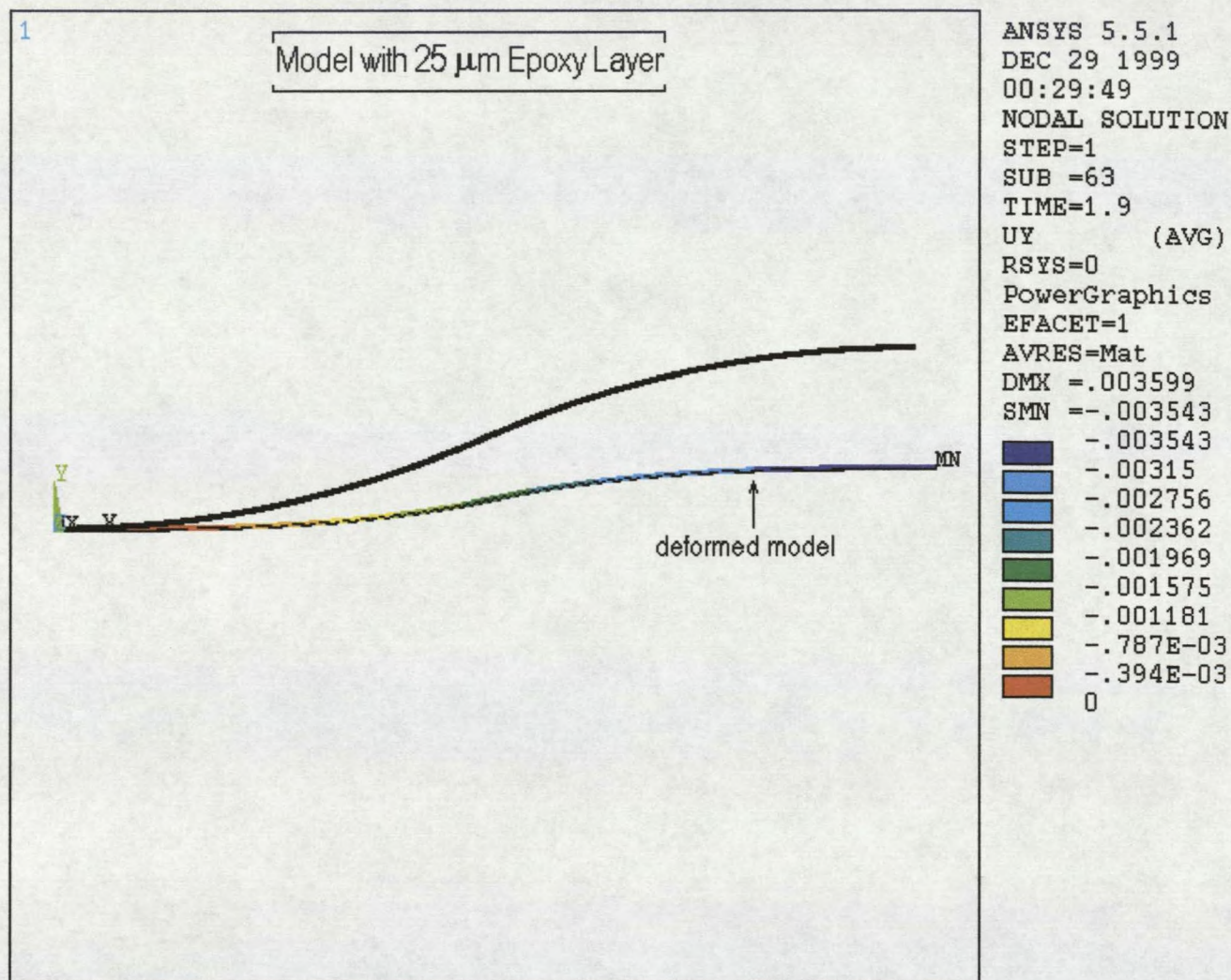


Figure 42. Ansys Model Results with Applied Compression Load.

CHAPTER 7

CONCLUSIONS

The graphs in the previous section show that the finite element model agrees with the analytical solution within 2% or less. Error could be attributed to the inclusion of shear deflections by the finite element model or the small inherent error in the finite element method. From these graphs, it is also evident that the model can reasonably predict the behavior of a bimorph, but that due to inconsistent manufacturing there is currently large variation in experimental results. Adequate measurement techniques also do not exist to determine if bimorphs can be manufactured to a particular specification. Therefore, the model inputs do not necessarily reflect the actual geometry of the bimorphs tested.

Also, from these graphs, it is evident that linear behavior exists in the region of the force-deflection curve applicable to the current applications for the MSU actuator, or 6.0 mm (3.0 mm per bimorph), and that the linear region can be extended by maximizing the epoxy layer thickness. Minimizing the epoxy layer thickness is beneficial, however, to maximizing the displacements. A tradeoff exists therefore between reducing or increasing the epoxy layer thickness, that is, maximizing displacement or extending the linear range of force-deflection response.

With the current manufacturing process, if the epoxy is mixed and bimorphs are sequentially built using the same batch of epoxy, the mean thickness of the epoxy layer in the bimorphs significantly increases from the first-made bimorph to the second-made, and so on. Thinning the epoxy with acetone does not significantly reduce the mean thickness of the epoxy layer, but it does improve the uniformity by a statistically significant amount. However, the uniformity of the epoxy layer does not affect its behavior, as is shown in the previous section's finite element results. Therefore, thinning the epoxy with acetone does not provide any benefit.

The linear region on the force-deflection curve is not affected either, so the bimorph with a non-uniform epoxy layer simply behaves as a bimorph with a uniform epoxy layer with a slightly different mean thickness. Note however that thinning with acetone increases the attenuating properties of the epoxy. Perhaps greater attenuation is perceived as a benefit to system performance. Recall that the elastic properties of the thinned epoxy could not be measured using ultrasonic techniques due to the high attenuation.

The analytical calculations demonstrate that the optimum ratio of radii of curvature for the double-S curve actuator is one to one, when maximizing vertical displacement. Note, however, that a *straight* beam is optimal when maximizing vertical displacement. The initial curvature is detrimental for maximizing displacements, since it effectively increases the stiffness of the actuator over that of a straight actuator design.

In summary, the following accomplishments were illustrated in this thesis. A mathematical model for curved (circular arc) piezoelectric actuators was developed. This model can serve as a design tool for future actuator design optimization with respect to other variables, such as force. The stiffness of the actuator was determined as a function of radius of curvature, taking into account the epoxy layer. The vertical displacements can be predicted for both mechanical and electrical loads, or a combination of the two. The linear region of behavior for the actuator was determined. Likewise, the Ansys finite element models that were developed can be easily modified to determine the stiffness and linear regions of behavior for other actuator shapes, varying number of layers, different types of electrode layers, or PVDF films with different thickness.

CHAPTER 8

RECOMMENDATIONS FOR FUTURE WORK

A manufacturing process should be developed for building actuators. This process should operate within process control limits and within certain specifications. That is, it should be able to produce actuators that have dimensions that are both accurate and precise. Techniques for measuring the geometry of the actuator will therefore be required.

The material properties of the acetone-thinned epoxy are unknown. If it is desirable for this layer to be dissipative, then thinning with acetone is beneficial to the system and its mechanical properties should be determined. How to control the attenuating properties with different thinning ratios could also be determined and a correlation between thinning ratio and mechanical properties could be developed. Although the keypoints calculations are capable of producing the geometry to model the electrode layers separately, the properties of the different types of electrode layer materials are not known. If these properties were determined, the electrode type and thickness could also be optimized for a particular application.

The boundary conditions should be verified to determine that what was assumed is what the actuator sees. For example, the epoxy joint where two bimorphs are connected at their ends may not be adequately represented. Also, the interaction between the bottom bimorph and the top bimorph in an actuator has not been considered. When the actuator is compressed, the inside surfaces of the bimorphs begin to touch each other and interact. The greater the compression, the more contact the two bimorphs have—which rolls from the outside edge where they are connected inward. Once the actuator is fully collapsed, the bimorphs contact each other completely along their entire areas. This interaction could perhaps be modeled as a beam on an elastic foundation, or by using contact elements in a finite element model.

REFERENCES CITED

1. Dunn, P. E., and Carr, S. H., "A Historical Perspective on the Occurrence of Piezoelectricity in Materials", *MRS Bulletin*, February 1989, pg. 22-31.
2. Measurement Specialties, Inc., *Piezo Film Sensors Technical Manual*, Part 1, August 1998.
3. Kawai, H., *Japanese Journal of Applied Physics*, Vol. 8, No. 975, 1969.
4. Wang, T.T., Herbert, J. M., and Glass, A. M., *The Applications of Ferroelectric Polymers*, Chapman and Hall, New York.
5. Berardinis, Larry, "Putting Piezo Polymer Film to Work", *Machine Design*, July 25, 1994.
6. Ashley, Steven, "Smart Skis and Other Adaptive Structures", *Mechanical Engineering*, November 1995.
7. Harvey, Dr. Jim, "Smart Materials and Structures", *43rd International SAMPE Symposium and Exhibition*, Society for the Advancement of Materials and Process Engineering, June 1, 1998.
8. DeYoung, T. W., and Maltsev, V. B., *United States Patent No. 4,439,780*, 1984.
9. Howkins, S. D., *United States Patent No. 4,459,601*, 1984.
10. Ervin, James D., and Brei, Diann, "Recurve Piezoelectric-Strain-Amplifying Actuator Architecture", *IEEE/ASME Transactions on Mechatronics*, Vol. 3, No. 4, December, 1998, pg 293-301.
11. Gandhi, Mukesh V. and Thompson Brian S., *Smart Materials and Structures*, Chapman and Hall, New York 1992.
12. Stampleman, David S. and von Flotow, Andreas H., "Micro-gravity Isolation Mounts Based Upon Piezoelectric Film", *Active Noise and Vibration Control*, Annual Meeting of ASME, Dallas, Texas, November 25-30, 1990, pg. 57-65.
13. Bohannon, G., Schmidt, H., Brandt, D., and Mooibroek, M., "Piezoelectric Polymer Actuators for Active Vibration Isolation in Space Applications", *Ferroelectrics*, 1999, Vol. 224, pp. 211-217.
14. Larson, Philip H. and Vinson, Jack R., "The Use of Piezoelectric Materials in Curved Beams and Rings", *Adaptive Structures and Material Systems*, AD-Vol. 35, ASME, 1993, pg. 277-285.
15. Shames, Irving H. and Cozzarelli, Francis A., *Elastic and Inelastic Stress Analysis*, Revised Printing, Taylor and Francis, Washington, DC, 1997.
16. Schewe, H., "Piezoelectricity of Uniaxially Oriented Polyvinylidene Fluoride", *IEEE Ultrasonics Symposium Proceedings*, 1982, pg. 519-524.
17. Boresi, Arthur P., *Advanced Mechanics of Materials*, Fifth Edition, John Wiley and Sons, New York, 1993.
18. Boley, B. A., and Weiner, J. H., *Theory of Thermal Stresses*, John Wiley and Sons, New York, 1960, pg. 264, equation 8.11.7 and pg. 364, 11.4.5.
19. Wang, H., "Piezoelectric, Dielectric, and Elastic Properties of P(VDF/Trifluoroethylene)", *Journal of Applied Physics*, Vol. 74, No. 5, 1 September, 1993, pg. 3394-3396.

20. Tamura, M., Ogasawara, K., Ono, N., Hagiwara, S., "Piezoelectricity in Uniaxially Stretched Poly(vinylidene Fluoride)", *Journal of Applied Physics*, Vol. 45, No. 9, September 1974, pg. 3768-3771.
21. Krautkramer, J. and Krautkramer, H., *Ultrasonic Testing of Materials*, 4th Edition, Springer-Verlag, Berlin, 1990, pg. 13-14.
22. Rose, Joseph L., *Ultrasonic Waves in Solid Media*, Cambridge University Press, New York, 1999, pg. 40-44.
23. R. J. Roark, *Formulas for Stress and Strain*, 4th ed., McGraw-Hill Book Company, New York (1965), pg. 180, case 35.
24. Logan, Daryl L., *A First Course in the Finite Element Method*, PWS Engineering, Boston, 1986.
25. *Ansys Theory Reference*, Ninth Edition, Chapter 13, Section 7, SAS IP, Inc.
26. "IEEE Standard on Piezoelectricity: ANSI/IEEE Std 176-1978", *IEEE Transactions on Sonics and Ultrasonics*, Vol. SU-31, No. 2, March 1984, IEEE Inc., New York, 1978 and 1984.
27. Jones, Robert M., *Mechanics of Composite Materials*, McGraw-Hill, New York, 1975, pg. 37-38.

APPENDICES

APPENDIX A

EPOXY LAYER THICKNESS DATA

Bimorph Thickness Data

Epoxy:Hardener:Acetone Ratio 1:1:0

Four samples of silver, silk-screened-electroded PVDF. A grid was laid out on each of four piezoelectric layers and the thickness measured in the center points of each grid square. Total of 25 points measured per layer. The matrices are the thicknesses measurements of one piezo-electric layer, laid out in the same manner they were measured on the grids.

$$pe_{4.1} := \begin{bmatrix} 44 & 46 & 43 & 46 & 45 \\ 47 & 46 & 45 & 47 & 47 \\ 43 & 45 & 47 & 46 & 45 \\ 43 & 46 & 44 & 44 & 44 \\ 43 & 44 & 43 & 46 & 43 \end{bmatrix}$$

$$pe_{4.2} := \begin{bmatrix} 45 & 44 & 43 & 45 & 46 \\ 43 & 46 & 42 & 43 & 42 \\ 47 & 46 & 45 & 44 & 43 \\ 45 & 42 & 44 & 43 & 45 \\ 46 & 45 & 46 & 47 & 45 \end{bmatrix}$$

$$pe_{4.3} := \begin{bmatrix} 47 & 46 & 45 & 47 & 45 \\ 45 & 43 & 46 & 44 & 45 \\ 43 & 44 & 46 & 45 & 42 \\ 46 & 44 & 43 & 42 & 46 \\ 47 & 45 & 47 & 48 & 45 \end{bmatrix}$$

$$pe_{4.4} := \begin{bmatrix} 46 & 45 & 46 & 45 & 44 \\ 46 & 42 & 45 & 46 & 42 \\ 43 & 44 & 45 & 46 & 45 \\ 45 & 44 & 45 & 47 & 46 \\ 46 & 46 & 47 & 45 & 46 \end{bmatrix}$$

$$aver_{4.1} := \text{mean}(pe_{4.1}) \quad sd := \text{stdev}(pe_{4.1})$$

$$aver_{4.1} = 44.88 \quad sd = 1.5$$

$$aver_{4.2} := \text{mean}(pe_{4.2}) \quad sd := \text{stdev}(pe_{4.2})$$

$$aver_{4.2} = 44.48 \quad sd = 1.5$$

$$aver_{4.3} := \text{mean}(pe_{4.3}) \quad sd := \text{stdev}(pe_{4.3})$$

$$aver_{4.3} = 45.04 \quad sd = 1.294$$

$$aver_{4.4} := \text{mean}(pe_{4.4}) \quad sd := \text{stdev}(pe_{4.4})$$

$$aver_{4.4} = 45.08 \quad sd = 1.294$$

$$av := \frac{(aver_{4.1} + aver_{4.2} + aver_{4.3} + aver_{4.4})}{4}$$

"av" indicates the average thickness of one, silver silk-screened-electroded piezo-electric layer.

$$av = 44.87$$

$$average := \begin{bmatrix} av & av & av & av & av \\ av & av & av & av & av \\ av & av & av & av & av \\ av & av & av & av & av \\ av & av & av & av & av \end{bmatrix}$$

Matrix set up for further calculations with bimorphs.

Four bimorphs constructed from two layers of silver, silk-screened-electroded PVDF and epoxy. Bimorphs were constructed consecutively and are numbered according to manufacturing order. Total of 25 thicknesses measured per bimorph.

$$\text{bim}_{0.1} := \begin{bmatrix} 102 & 103 & 108 & 114 & 102 \\ 101 & 117 & 110 & 117 & 135 \\ 112 & 119 & 109 & 110 & 103 \\ 102 & 105 & 108 & 111 & 122 \\ 103 & 111 & 103 & 131 & 100 \end{bmatrix}$$

$$\text{mean}(\text{bim}_{0.1}) = 110.32$$

$$\text{stdev}(\text{bim}_{0.1}) = 8.97$$

$$\text{bim}_{0.2} := \begin{bmatrix} 111 & 131 & 129 & 111 & 108 \\ 109 & 118 & 119 & 114 & 109 \\ 132 & 138 & 128 & 122 & 108 \\ 120 & 127 & 133 & 119 & 109 \\ 127 & 119 & 139 & 111 & 105 \end{bmatrix}$$

$$\text{mean}(\text{bim}_{0.2}) = 119.84$$

$$\text{stdev}(\text{bim}_{0.2}) = 10.055$$

$$\text{bim}_{0.3} := \begin{bmatrix} 150 & 128 & 122 & 121 & 125 \\ 139 & 120 & 116 & 132 & 121 \\ 132 & 127 & 116 & 135 & 122 \\ 126 & 110 & 148 & 147 & 128 \\ 130 & 122 & 149 & 164 & 163 \end{bmatrix}$$

$$\text{mean}(\text{bim}_{0.3}) = 131.72$$

$$\text{stdev}(\text{bim}_{0.3}) = 14.09$$

$$\text{bim}_{0.4} := \begin{bmatrix} 159 & 156 & 148 & 137 & 151 \\ 172 & 166 & 152 & 151 & 151 \\ 148 & 159 & 151 & 163 & 170 \\ 159 & 137 & 142 & 172 & 159 \\ 138 & 132 & 143 & 158 & 135 \end{bmatrix}$$

$$\text{mean}(\text{bim}_{0.4}) = 152.36$$

$$\text{stdev}(\text{bim}_{0.4}) = 11.38$$

$$\frac{8.97 + 10.055 + 14.09 + 11.38}{4} = 11.124$$

$$t_{0.1} := \text{bim}_{0.1} - 2 \cdot \text{average}$$

Epoxy layer thickness across a bimorph.

$$t_{0.1} = \begin{bmatrix} 12.26 & 13.26 & 18.26 & 24.26 & 12.26 \\ 11.26 & 27.26 & 20.26 & 27.26 & 45.26 \\ 22.26 & 29.26 & 19.26 & 20.26 & 13.26 \\ 12.26 & 15.26 & 18.26 & 21.26 & 32.26 \\ 13.26 & 21.26 & 13.26 & 41.26 & 10.26 \end{bmatrix}$$

$$\text{stdev}(t_{0.1}) = 8.97$$

Standard deviation of the epoxy layer thickness across a bimorph.

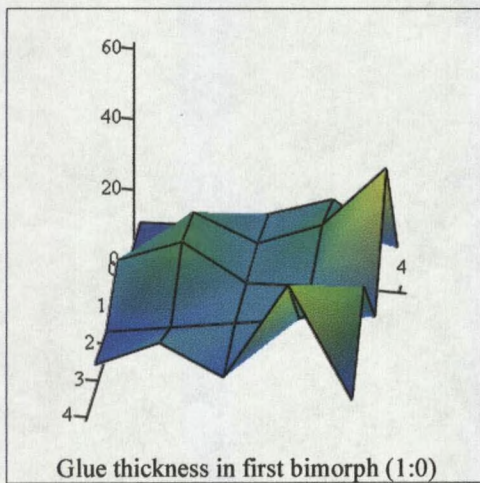
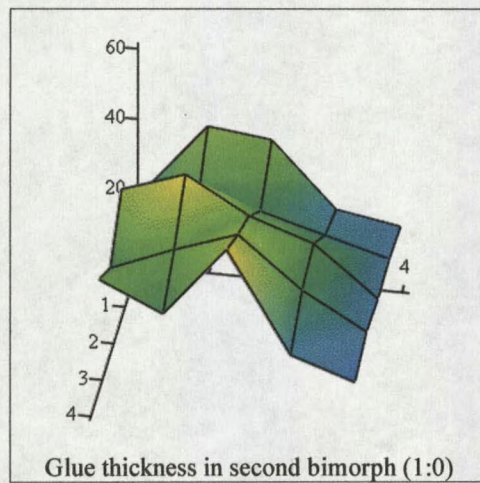
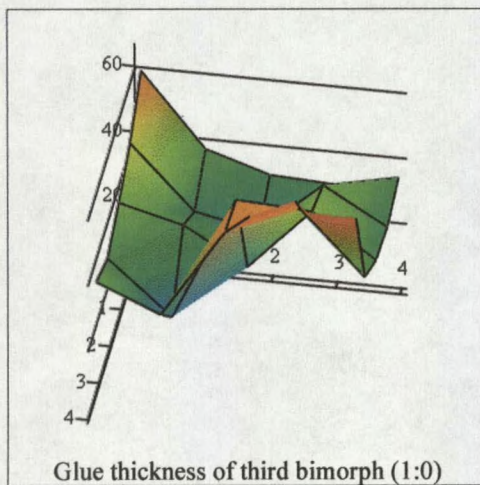
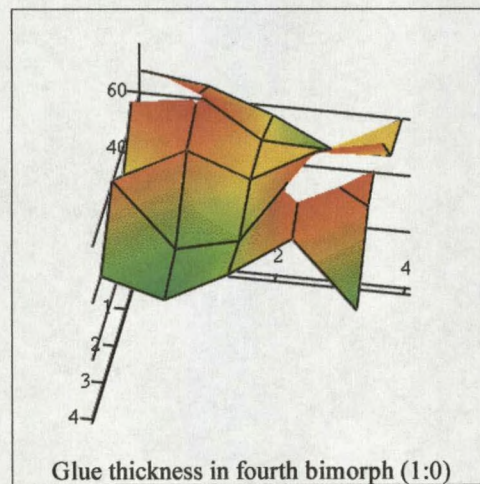
$$\text{mean}(t_{0.1}) = 20.58$$

Mean epoxy layer thickness across a bimorph.

$$t_{0.2} := \text{bim}_{0.2} - 2 \cdot \text{average} \quad \text{mean}(t_{0.2}) = 30.1 \quad \text{stdev}(t_{0.2}) = 10.055$$

$$t_{0.3} := \text{bim}_{0.3} - 2 \cdot \text{average} \quad \text{mean}(t_{0.3}) = 41.98 \quad \text{stdev}(t_{0.3}) = 14.09$$

$$t_{0.4} := \text{bim}_{0.4} - 2 \cdot \text{average} \quad \text{mean}(t_{0.4}) = 62.62 \quad \text{stdev}(t_{0.4}) = 11.38$$


 $t_{0.1}$

 $t_{0.2}$

 $t_{0.3}$

 $t_{0.4}$

Bimorph Thickness Data

Epoxy:Hardener:Acetone Ratio 1:1:1

Four samples of silver, silk-screened-electroded PVDF. A grid was laid out on each of four piezoelectric layers and the thickness measured in the center points of each grid square. Total of 25 points measured per layer. The matrices are the thicknesses measurements of one piezo-electric layer, laid out in the same manner they were measured on the grids.

$$pe_{4.1} := \begin{bmatrix} 44 & 46 & 43 & 46 & 45 \\ 47 & 46 & 45 & 47 & 47 \\ 43 & 45 & 47 & 46 & 45 \\ 43 & 46 & 44 & 44 & 44 \\ 43 & 44 & 43 & 46 & 43 \end{bmatrix}$$

$$pe_{4.2} := \begin{bmatrix} 45 & 44 & 43 & 45 & 46 \\ 43 & 46 & 42 & 43 & 42 \\ 47 & 46 & 45 & 44 & 43 \\ 45 & 42 & 44 & 43 & 45 \\ 46 & 45 & 46 & 47 & 45 \end{bmatrix}$$

$$pe_{4.3} := \begin{bmatrix} 47 & 46 & 45 & 47 & 45 \\ 45 & 43 & 46 & 44 & 45 \\ 43 & 44 & 46 & 45 & 42 \\ 46 & 44 & 43 & 42 & 46 \\ 47 & 45 & 47 & 48 & 45 \end{bmatrix}$$

$$pe_{4.4} := \begin{bmatrix} 46 & 45 & 46 & 45 & 44 \\ 46 & 42 & 45 & 46 & 42 \\ 43 & 44 & 45 & 46 & 45 \\ 45 & 44 & 45 & 47 & 46 \\ 46 & 46 & 47 & 45 & 46 \end{bmatrix}$$

$$aver_{4.1} := \text{mean}(pe_{4.1})$$

$$aver_{4.1} = 44.88$$

$$aver_{4.2} := \text{mean}(pe_{4.2})$$

$$aver_{4.2} = 44.48$$

$$aver_{4.3} := \text{mean}(pe_{4.3})$$

$$aver_{4.3} = 45.04$$

$$aver_{4.4} := \text{mean}(pe_{4.4})$$

$$aver_{4.4} = 45.08$$

$$av := \frac{(aver_{4.1} + aver_{4.2} + aver_{4.3} + aver_{4.4})}{4}$$

$$av = 44.87$$

"av" indicates the average thickness of one, silver silk-screened-electroded piezo-electric layer.

$$average := \begin{bmatrix} av & av & av & av & av \\ av & av & av & av & av \\ av & av & av & av & av \\ av & av & av & av & av \\ av & av & av & av & av \end{bmatrix}$$

Matrix set up for further calculations with bimorphs.

Four bimorphs constructed from two layers of silver, silk-screened-electroded PVDF and epoxy w/acetone. Bimorphs were constructed consecutively and are numbered according to manufacturing order. Total of 25 thicknesses measured per bimorph.

$$\text{bim}_{5.1} := \begin{bmatrix} 119 & 118 & 109 & 108 & 101 \\ 112 & 108 & 109 & 102 & 106 \\ 108 & 109 & 109 & 107 & 105 \\ 110 & 112 & 109 & 98 & 105 \\ 120 & 120 & 111 & 109 & 111 \end{bmatrix}$$

$$\text{bim}_{5.2} := \begin{bmatrix} 109 & 115 & 110 & 117 & 110 \\ 115 & 113 & 110 & 119 & 119 \\ 111 & 106 & 112 & 111 & 108 \\ 112 & 104 & 119 & 114 & 111 \\ 104 & 118 & 120 & 136 & 138 \end{bmatrix}$$

$$\text{bim}_{5.3} := \begin{bmatrix} 118 & 150 & 133 & 118 & 132 \\ 137 & 123 & 118 & 132 & 130 \\ 130 & 131 & 151 & 152 & 119 \\ 139 & 118 & 129 & 128 & 137 \\ 140 & 134 & 129 & 121 & 120 \end{bmatrix}$$

$$\text{bim}_{5.4} := \begin{bmatrix} 113 & 123 & 116 & 108 & 107 \\ 111 & 145 & 118 & 108 & 113 \\ 117 & 122 & 119 & 121 & 121 \\ 122 & 110 & 112 & 112 & 108 \\ 126 & 110 & 112 & 111 & 111 \end{bmatrix}$$

$$\text{mean}(\text{bim}_{5.1}) = 109.4$$

$$\text{mean}(\text{bim}_{5.3}) = 130.76$$

$$\text{stdev}(\text{bim}_{5.1}) = 5.396$$

$$\text{stdev}(\text{bim}_{5.3}) = 10.112$$

$$\text{mean}(\text{bim}_{5.2}) = 114.44$$

$$\text{mean}(\text{bim}_{5.4}) = 115.84$$

$$\text{stdev}(\text{bim}_{5.2}) = 8.005$$

$$\text{stdev}(\text{bim}_{5.4}) = 8.028$$

$$\frac{5.396 + 10.112 + 8.005 + 8.028}{4} = 7.885$$

$$t_{5.1} := \text{bim}_{5.1} - 2 \cdot \text{average}$$

Epoxy layer thickness across a bimorph.

$$t_{5.1} = \begin{bmatrix} 29.26 & 28.26 & 19.26 & 18.26 & 11.26 \\ 22.26 & 18.26 & 19.26 & 12.26 & 16.26 \\ 18.26 & 19.26 & 19.26 & 17.26 & 15.26 \\ 20.26 & 22.26 & 19.26 & 8.26 & 15.26 \\ 30.26 & 30.26 & 21.26 & 19.26 & 21.26 \end{bmatrix}$$

$$\text{stdev}(t_{5.1}) = 5.396$$

Standard deviation of the epoxy layer thickness across a bimorph.

$$\text{mean}(t_{5.1}) = 19.66$$

Mean epoxy layer thickness across a bimorph.

$$t_{5.1} := \text{bim}_{5.1} - 2 \cdot \text{average}$$

$$\text{mean}(t_{5.1}) = 19.66$$

$$t_{5.2} := \text{bim}_{5.2} - 2 \cdot \text{average}$$

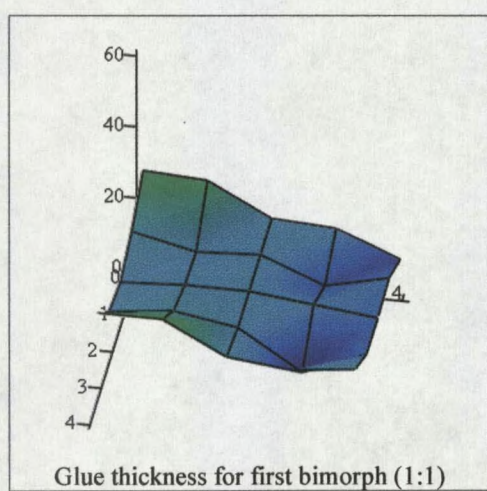
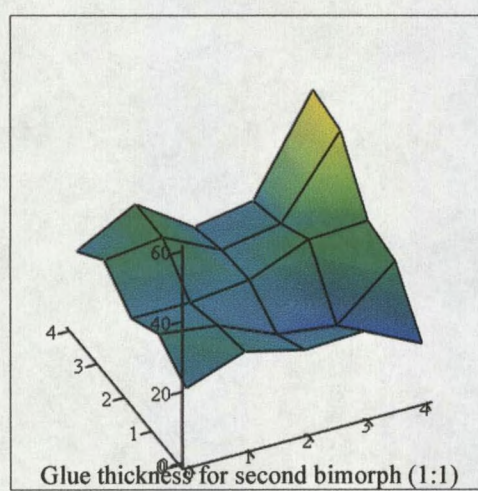
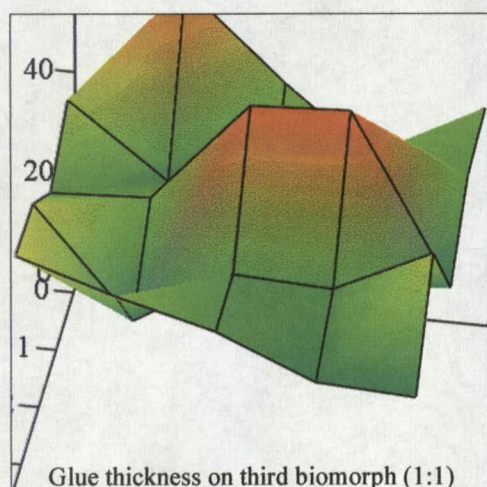
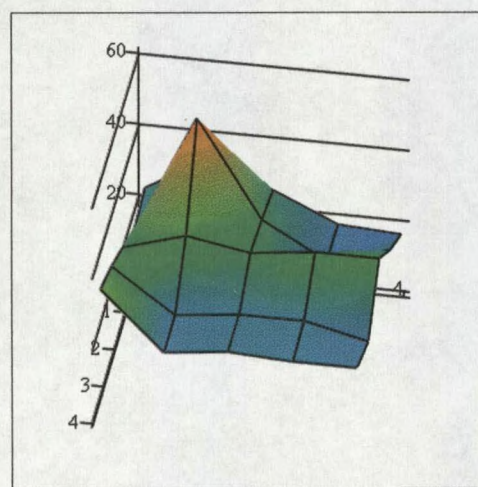
$$\text{mean}(t_{5.2}) = 24.7$$

$$t_{5.3} := \text{bim}_{5.3} - 2 \cdot \text{average}$$

$$\text{mean}(t_{5.3}) = 41.02$$

$$t_{5.4} := \text{bim}_{5.4} - 2 \cdot \text{average}$$

$$\text{mean}(t_{5.4}) = 26.1$$

t_{5.1}t_{5.2}t_{5.3}t_{5.4}

Bimorph Thickness Data

Epoxy:Hardener:Acetone Ratio 1:1:2

Four samples of silver, silk-screened-electroded PVDF. A grid was laid out on each of four piezoelectric layers and the thickness measured in the center points of each grid square. Total of 25 points measured per layer. The matrices are the thicknesses measurements of one piezo-electric layer, laid out in the same manner they were measured on the grids.

$$pe_{4.1} := \begin{bmatrix} 44 & 46 & 43 & 46 & 45 \\ 47 & 46 & 45 & 47 & 47 \\ 43 & 45 & 47 & 46 & 45 \\ 43 & 46 & 44 & 44 & 44 \\ 43 & 44 & 43 & 46 & 43 \end{bmatrix}$$

$$pe_{4.2} := \begin{bmatrix} 45 & 44 & 43 & 45 & 46 \\ 43 & 46 & 42 & 43 & 42 \\ 47 & 46 & 45 & 44 & 43 \\ 45 & 42 & 44 & 43 & 45 \\ 46 & 45 & 46 & 47 & 45 \end{bmatrix}$$

$$pe_{4.3} := \begin{bmatrix} 47 & 46 & 45 & 47 & 45 \\ 45 & 43 & 46 & 44 & 45 \\ 43 & 44 & 46 & 45 & 42 \\ 46 & 44 & 43 & 42 & 46 \\ 47 & 45 & 47 & 48 & 45 \end{bmatrix}$$

$$pe_{4.4} := \begin{bmatrix} 46 & 45 & 46 & 45 & 44 \\ 46 & 42 & 45 & 46 & 42 \\ 43 & 44 & 45 & 46 & 45 \\ 45 & 44 & 45 & 47 & 46 \\ 46 & 46 & 47 & 45 & 46 \end{bmatrix}$$

$$aver_{4.1} := \text{mean}(pe_{4.1})$$

$$aver_{4.2} := \text{mean}(pe_{4.2})$$

$$aver_{4.1} = 44.88$$

$$aver_{4.2} = 44.48$$

$$aver_{4.3} := \text{mean}(pe_{4.3})$$

$$aver_{4.4} := \text{mean}(pe_{4.4})$$

$$aver_{4.3} = 45.04$$

$$aver_{4.4} = 45.08$$

$$\text{average} := \frac{(\text{aver}_{4.1} + \text{aver}_{4.2} + \text{aver}_{4.3} + \text{aver}_{4.4})}{4}$$

"average" indicates the average thickness of one, silver silk-screened-electroded piezo-electric layer.

$$\text{average} = 44.87$$

$$bim_{7.1} := \begin{bmatrix} 105 & 114 & 111 & 106 & 110 \\ 108 & 108 & 110 & 111 & 109 \\ 110 & 109 & 112 & 116 & 108 \\ 109 & 117 & 121 & 111 & 112 \\ 114 & 109 & 112 & 109 & 107 \end{bmatrix}$$

$$bim_{7.2} := \begin{bmatrix} 109 & 109 & 110 & 111 & 110 \\ 110 & 112 & 114 & 108 & 108 \\ 109 & 111 & 110 & 108 & 106 \\ 110 & 110 & 110 & 111 & 113 \\ 103 & 110 & 109 & 118 & 121 \end{bmatrix}$$

$$\text{bim}_{7.3} := \begin{bmatrix} 106 & 112 & 122 & 110 & 110 \\ 109 & 121 & 127 & 122 & 115 \\ 113 & 118 & 115 & 111 & 103 \\ 102 & 103 & 111 & 109 & 109 \\ 103 & 110 & 109 & 118 & 121 \end{bmatrix}$$

$$i := 0..4$$

$$j := 0..4$$

$$\text{gt}_{7.1_{i,j}} := \text{bim}_{7.1_{i,j}} - 2 \cdot \text{average}$$

$$\text{mean}(\text{gt}_{7.1}) = 20.98$$

$$\text{gt}_{7.3_{i,j}} := \text{bim}_{7.3_{i,j}} - 2 \cdot \text{average}$$

$$\text{mean}(\text{gt}_{7.3}) = 22.62$$

$$\text{stdev}(\text{gt}_{7.1}) = 3.505$$

$$\text{stdev}(\text{gt}_{7.2}) = 3.429$$

$$\text{bim}_{7.4} := \begin{bmatrix} 110 & 122 & 109 & 110 & 119 \\ 111 & 110 & 119 & 116 & 113 \\ 107 & 110 & 141 & 110 & 121 \\ 112 & 112 & 113 & 118 & 108 \\ 112 & 108 & 109 & 109 & 102 \end{bmatrix}$$

$$\text{gt}_{7.2_{i,j}} := \text{bim}_{7.2_{i,j}} - 2 \cdot \text{average}$$

$$\text{mean}(\text{gt}_{7.2}) = 20.66$$

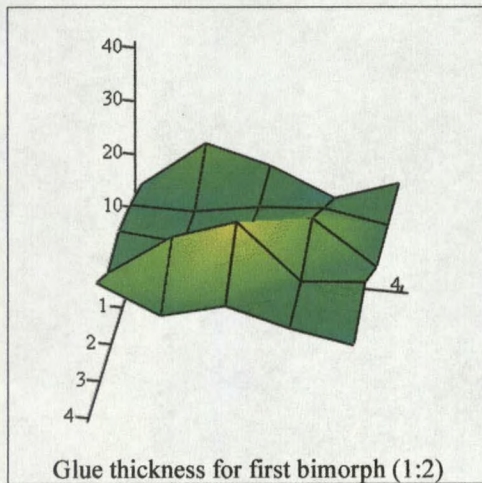
$$\text{gt}_{7.4_{i,j}} := \text{bim}_{7.4_{i,j}} - 2 \cdot \text{average}$$

$$\text{mean}(\text{gt}_{7.4}) = 23.5$$

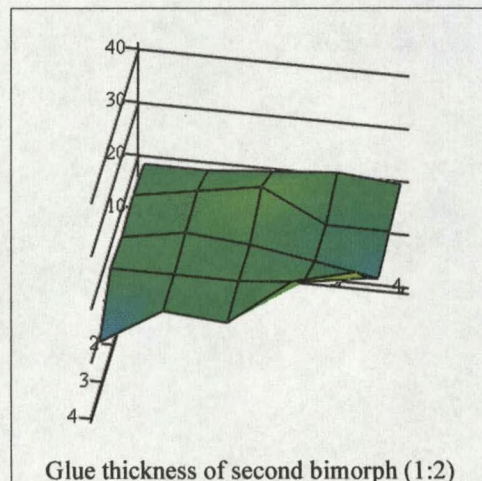
$$\text{stdev}(\text{gt}_{7.3}) = 6.645$$

$$\text{stdev}(\text{gt}_{7.4}) = 7.323$$

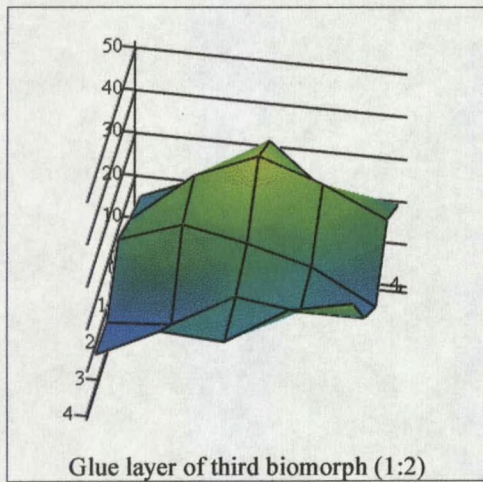
$$\frac{20.98 + 20.66 + 22.62 + 23.5}{4} = 21.94$$



gt 7.1



gt 7.2



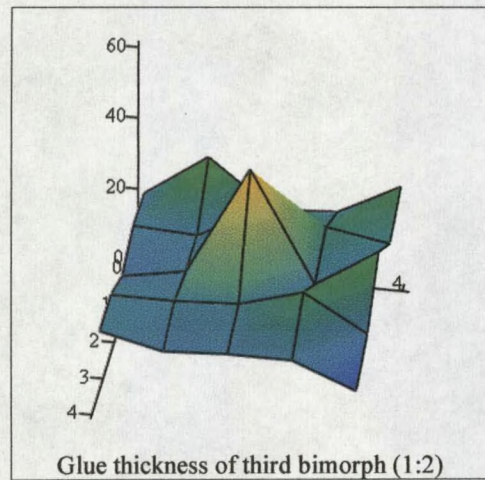
gt 7.3

$$gt_{7.1} = \begin{bmatrix} 15.26 & 24.26 & 21.26 & 16.26 & 20.26 \\ 18.26 & 18.26 & 20.26 & 21.26 & 19.26 \\ 20.26 & 19.26 & 22.26 & 26.26 & 18.26 \\ 19.26 & 27.26 & 31.26 & 21.26 & 22.26 \\ 24.26 & 19.26 & 22.26 & 19.26 & 17.26 \end{bmatrix}$$

$$\text{mean}(gt_{7.1}) = 20.98$$

$$gt_{7.3} = \begin{bmatrix} 16.26 & 22.26 & 32.26 & 20.26 & 20.26 \\ 19.26 & 31.26 & 37.26 & 32.26 & 25.26 \\ 23.26 & 28.26 & 25.26 & 21.26 & 13.26 \\ 12.26 & 13.26 & 21.26 & 19.26 & 19.26 \\ 13.26 & 20.26 & 19.26 & 28.26 & 31.26 \end{bmatrix}$$

$$\text{mean}(gt_{7.3}) = 22.62$$



gt 7.4

$$gt_{7.2} = \begin{bmatrix} 19.26 & 19.26 & 20.26 & 21.26 & 20.26 \\ 20.26 & 22.26 & 24.26 & 18.26 & 18.26 \\ 19.26 & 21.26 & 20.26 & 18.26 & 16.26 \\ 20.26 & 20.26 & 20.26 & 21.26 & 23.26 \\ 13.26 & 20.26 & 19.26 & 28.26 & 31.26 \end{bmatrix}$$

$$\text{mean}(gt_{7.2}) = 20.66$$

$$gt_{7.4} = \begin{bmatrix} 20.26 & 32.26 & 19.26 & 20.26 & 29.26 \\ 21.26 & 20.26 & 29.26 & 26.26 & 23.26 \\ 17.26 & 20.26 & 51.26 & 20.26 & 31.26 \\ 22.26 & 22.26 & 23.26 & 28.26 & 18.26 \\ 22.26 & 18.26 & 19.26 & 19.26 & 12.26 \end{bmatrix}$$

$$\text{mean}(gt_{7.4}) = 23.5$$

Bimorph Thickness Data

Epoxy:Hardener:Acetone Ratio 1:1:3

Four samples of silver, silk-screened-electroded PVDF. A grid was laid out on each of four piezoelectric layers and the thickness measured in the center points of each grid square. Total of 25 points measured per layer. The matrices are the thicknesses measurements of one piezo-electric layer, laid out in the same manner they were measured on the grids.

$$pe_{4.1} := \begin{bmatrix} 44 & 46 & 43 & 46 & 45 \\ 47 & 46 & 45 & 47 & 47 \\ 43 & 45 & 47 & 46 & 45 \\ 43 & 46 & 44 & 44 & 44 \\ 43 & 44 & 43 & 46 & 43 \end{bmatrix}$$

$$pe_{4.2} := \begin{bmatrix} 45 & 44 & 43 & 45 & 46 \\ 43 & 46 & 42 & 43 & 42 \\ 47 & 46 & 45 & 44 & 43 \\ 45 & 42 & 44 & 43 & 45 \\ 46 & 45 & 46 & 47 & 45 \end{bmatrix}$$

$$pe_{4.3} := \begin{bmatrix} 47 & 46 & 45 & 47 & 45 \\ 45 & 43 & 46 & 44 & 45 \\ 43 & 44 & 46 & 45 & 42 \\ 46 & 44 & 43 & 42 & 46 \\ 47 & 45 & 47 & 48 & 45 \end{bmatrix}$$

$$pe_{4.4} := \begin{bmatrix} 46 & 45 & 46 & 45 & 44 \\ 46 & 42 & 45 & 46 & 42 \\ 43 & 44 & 45 & 46 & 45 \\ 45 & 44 & 45 & 47 & 46 \\ 46 & 46 & 47 & 45 & 46 \end{bmatrix}$$

$$aver_{4.1} := \text{mean}(pe_{4.1})$$

$$aver_{4.1} = 44.88$$

$$aver_{4.2} := \text{mean}(pe_{4.2})$$

$$aver_{4.2} = 44.48$$

$$aver_{4.3} := \text{mean}(pe_{4.3})$$

$$aver_{4.3} = 45.04$$

$$aver_{4.4} := \text{mean}(pe_{4.4})$$

$$aver_{4.4} = 45.08$$

$$av := \frac{(aver_{4.1} + aver_{4.2} + aver_{4.3} + aver_{4.4})}{4}$$

$$av = 44.87$$

$$bim_{9.1} := \begin{bmatrix} 100 & 108 & 109 & 101 & 98 \\ 100 & 110 & 105 & 102 & 99 \\ 101 & 102 & 108 & 101 & 100 \\ 103 & 104 & 101 & 100 & 92 \\ 104 & 101 & 109 & 100 & 98 \end{bmatrix}$$

$$bim_{9.2} := \begin{bmatrix} 100 & 102 & 101 & 97 & 101 \\ 109 & 111 & 107 & 110 & 100 \\ 107 & 111 & 103 & 117 & 109 \\ 107 & 101 & 109 & 106 & 97 \\ 110 & 101 & 102 & 100 & 99 \end{bmatrix}$$

$$\text{bim}_{9,3} := \begin{bmatrix} 96 & 102 & 104 & 100 & 99 \\ 101 & 120 & 111 & 109 & 104 \\ 103 & 110 & 112 & 109 & 103 \\ 118 & 111 & 109 & 101 & 104 \\ 102 & 111 & 110 & 106 & 105 \end{bmatrix}$$

$$i := 0..4 \quad j := 0..4$$

$$\text{gt}_{9,1}_{i,j} := \text{bim}_{9,1}_{i,j} - 2 \cdot \text{av}$$

$$\text{mean}(\text{gt}_{9,1}) = 12.5$$

$$\text{stdev}(\text{gt}_{9,1}) = 4.082$$

$$\text{gt}_{9,3}_{i,j} := \text{bim}_{9,3}_{i,j} - 2 \cdot \text{av}$$

$$\text{mean}(\text{gt}_{9,3}) = 16.66$$

$$\text{stdev}(\text{gt}_{9,3}) = 5.671$$

$$\text{bim}_{9,4} := \begin{bmatrix} 101 & 103 & 103 & 109 & 117 \\ 109 & 110 & 109 & 107 & 101 \\ 102 & 102 & 108 & 112 & 101 \\ 110 & 111 & 110 & 101 & 103 \\ 104 & 105 & 118 & 109 & 101 \end{bmatrix}$$

$$\text{gt}_{9,2}_{i,j} := \text{bim}_{9,2}_{i,j} - 2 \cdot \text{av}$$

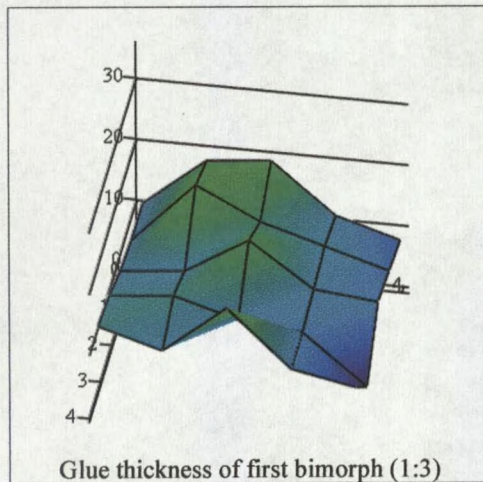
$$\text{mean}(\text{gt}_{9,2}) = 14.94$$

$$\text{stdev}(\text{gt}_{9,2}) = 5.097$$

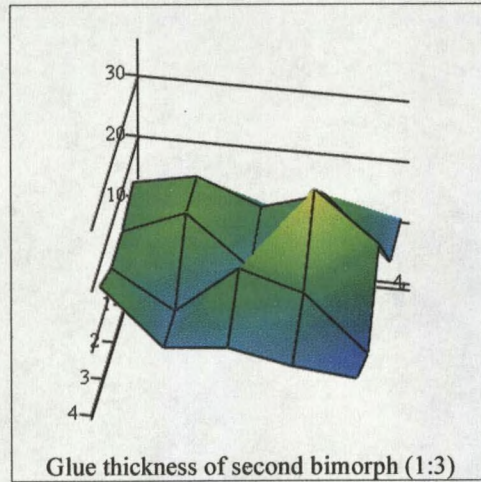
$$\text{gt}_{9,4}_{i,j} := \text{bim}_{9,4}_{i,j} - 2 \cdot \text{av}$$

$$\text{mean}(\text{gt}_{9,4}) = 16.9$$

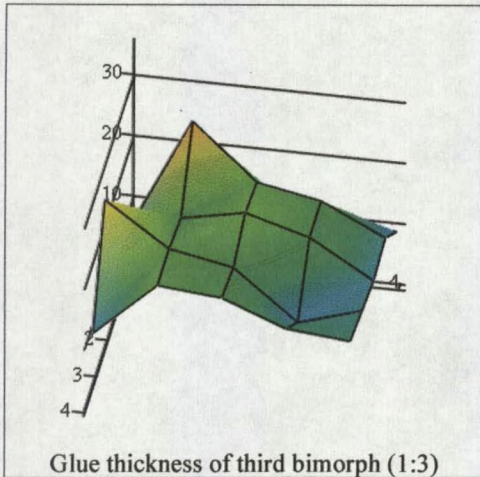
$$\text{stdev}(\text{gt}_{9,4}) = 4.873$$



gt 9,1



gt 9,2



gt 9.3

$$gt_{9.1} = \begin{bmatrix} 10.26 & 18.26 & 19.26 & 11.26 & 8.26 \\ 10.26 & 20.26 & 15.26 & 12.26 & 9.26 \\ 11.26 & 12.26 & 18.26 & 11.26 & 10.26 \\ 13.26 & 14.26 & 11.26 & 10.26 & 2.26 \\ 14.26 & 11.26 & 19.26 & 10.26 & 8.26 \end{bmatrix}$$

$$\text{mean}(gt_{9.1}) = 12.5 \quad \text{stdev}(gt_{9.1}) = 4.082$$

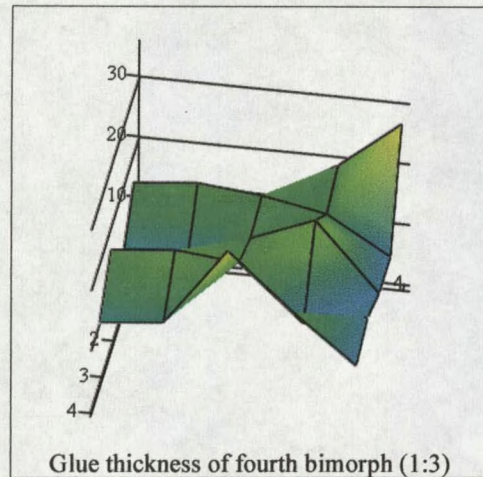
$$\text{max}(gt_{9.1}) = 20.26 \quad \text{min}(gt_{9.1}) = 2.26$$

$$gt_{9.3} = \begin{bmatrix} 6.26 & 12.26 & 14.26 & 10.26 & 9.26 \\ 11.26 & 30.26 & 21.26 & 19.26 & 14.26 \\ 13.26 & 20.26 & 22.26 & 19.26 & 13.26 \\ 28.26 & 21.26 & 19.26 & 11.26 & 14.26 \\ 12.26 & 21.26 & 20.26 & 16.26 & 15.26 \end{bmatrix}$$

$$\text{mean}(gt_{9.3}) = 16.66 \quad \text{stdev}(gt_{9.3}) = 5.671$$

$$\text{max}(gt_{9.3}) = 30.26$$

$$\text{min}(gt_{9.3}) = 6.26$$



gt 9.4

$$gt_{9.2} = \begin{bmatrix} 10.26 & 12.26 & 11.26 & 7.26 & 11.26 \\ 19.26 & 21.26 & 17.26 & 20.26 & 10.26 \\ 17.26 & 21.26 & 13.26 & 27.26 & 19.26 \\ 17.26 & 11.26 & 19.26 & 16.26 & 7.26 \\ 20.26 & 11.26 & 12.26 & 10.26 & 9.26 \end{bmatrix}$$

$$\text{mean}(gt_{9.2}) = 14.94 \quad \text{stdev}(gt_{9.2}) = 5.097$$

$$\text{max}(gt_{9.2}) = 27.26 \quad \text{min}(gt_{9.2}) = 7.26$$

$$gt_{9.4} = \begin{bmatrix} 11.26 & 13.26 & 13.26 & 19.26 & 27.26 \\ 19.26 & 20.26 & 19.26 & 17.26 & 11.26 \\ 12.26 & 12.26 & 18.26 & 22.26 & 11.26 \\ 20.26 & 21.26 & 20.26 & 11.26 & 13.26 \\ 14.26 & 15.26 & 28.26 & 19.26 & 11.26 \end{bmatrix}$$

$$\text{mean}(gt_{9.4}) = 16.9 \quad \text{stdev}(gt_{9.4}) = 4.873$$

$$\text{max}(gt_{9.4}) = 28.26$$

$$\text{min}(gt_{9.4}) = 11.26$$

APPENDIX B

STATISTICAL CALCULATIONS

Decisions are subject to the risk of making the "wrong" one due to variance.

α -error ---- falsely accepting H_a when H_0 is in fact true (chasing a wild goose)

β -error ---- falsely accepting H_0 when H_a is in fact true (failing to chase a fat goose)

Confidence level: Probability of failing to reject H_0 when H_0 is true ($1-\alpha$)-- want this to be high!!

Consequence of α -error is wasted resources.

Consequence of β -error is a missed opportunity.

Rule of thumb for α -error choice is between 1% and 10% (0.01 and 0.1).

Rule of thumb for β -error choice is between 5% and 0.01% (0.05 and 0.001).

α - error is not as critical as β -error since the truth will generally emerge anyway at the expense of cost without any benefit. With β -error, you conclude that something doesn't have an effect when it really does. You've made a decision about something you won't go back to.

β can be considered a probability of missing a change.

First test the average thicknesses between a ratio of epoxy:acetone of 1:0 and 1:1 (four samples for each type, the value given for each sample is the average thickness across the surface of the sample).

Choose α value: $\alpha := 0.1$

Null Hypothesis: $x_1 = x_2$

Alternative Hypothesis: $x_1 \neq x_2$

The "not equal" is considered double-sided.

$i := 1..4$

$x_{1,i} :=$ $x_{2,i} :=$

20.58	19.66
30.1	24.7
41.98	41.02
62.62	26.1

Average thicknesses over each of 4 different bimorphs, in order of their manufacture.

$N_1 := 4$

Number of data points in each set.

$N_2 := 4$

Since these were two different samples, then the following procedure would be used.

$$X_1 := \frac{1}{N1} \sum_{i=1}^{N1} x1_i \quad X_1 = 38.82 \quad \text{Mean of data set 1.}$$

$$X_2 := \frac{1}{N2} \sum_{i=1}^{N2} x2_i \quad X_2 = 27.87 \quad \text{Mean of data set 2.}$$

$$\text{Var1} := \frac{1}{N1-1} \sum_{i=1}^{N1} (x1_i - X_1)^2 \quad \text{Var1} = 328.387 \quad \text{Variance of data set 1.}$$

$$\text{Var2} := \frac{1}{N2-1} \sum_{i=1}^{N2} (x2_i - X_2)^2 \quad \text{Var2} = 84.503 \quad \text{Variance of data set 2.}$$

$$\sigma_1 := \sqrt{\text{Var1}} \quad \sigma_1 = 18.121 \quad \text{Standard deviation of data set 1.}$$

$$\sigma_2 := \sqrt{\text{Var2}} \quad \sigma_2 = 9.193 \quad \text{Standard deviation of data set 2.}$$

$$s_D := \sqrt{\frac{\sum_{i=1}^{N1} (x1_i - X_1)^2 + \sum_{i=1}^{N2} (x2_i - X_2)^2}{N1 + N2 - 2} \cdot \left(\frac{1}{N1} + \frac{1}{N2} \right)} \quad \text{Pooled variance.} \quad s_D = 10.16$$

$$t := \frac{X_1 - X_2}{s_D} \quad t = 1.078$$

Now evaluate the significance of this t value for the Student's t-distribution with $N1+N2-2$ df.

$$\text{df} := N1 + N2 - 2$$

$$\text{df} = 6$$

We reject H_0 if: $|t| > t_{\frac{\alpha}{2}}$ where the α values are double-sided because the hypothesis is.

Using the Student's t-distribution, and for a double-sided hypothesis, values of $t_{\frac{\alpha}{2}}$ are used.

$$t_{\frac{\alpha}{2}} = t_{0.05} = 1.943$$

$$t = 1.078$$

Since t is not greater than $\frac{t_{\alpha}}{2}$ then we accept H_0 , that the two means are the same; they are equal.

Did the standard deviation change?

$$\sigma_1 = 18.121$$

$$\text{Var1} = 328.387$$

$$\sigma_2 = 9.193$$

$$\text{Var2} = 84.503$$

Once again, it appears the variance has been substantially decreased for the second data set. Is this difference statistically significant?

Choose $\alpha = 0.1$

First state the hypotheses:

$H_0: \sigma_1^2 = \sigma_2^2$ Null hypothesis--the variances are the same (equal).

$H_a: \sigma_1^2 > \sigma_2^2$ Alternative hypothesis--the variances are not equal.

$F := \left(\frac{\text{Var1}}{\text{Var2}} \right)$ $F = 3.886$ F value is the ratio of the variances--the larger of the two on top.

$$\text{df1} := N1 - 1 \quad \text{df1} = 3$$

$$\text{df2} := N2 - 1 \quad \text{df2} = 3$$

What we want to check now is the following:

$F > F_{\alpha}(\text{df2}, \text{df1})$ If this inequality is true, we accept H_a , the alternative hypothesis.

Using a table for the f-variate:

$$F_{0.10}(\text{df1}, \text{df2}) = 5.39$$

$F = 3.886$ This value of F is not greater than F_{α} , therefore we accept the null hypothesis.

We conclude that the variances are equal.

If we say that the variances are equal, then the uniformity hasn't changed from an epoxy:acetone ratio of 1:0 to 1:1. We can conclude then, that the acetone does not make the epoxy layer more consistent during manufacturing (that the deviation between the mean thicknesses of the 4 samples within one kind of epoxy/acetone mixture is not smaller with acetone). That means that greater consistency in manufacturing cannot be achieved with acetone.

Next, test the average thicknesses between samples 1 and 2 for an epoxy ratio of 1:0 acetone.

Choose α value: $\alpha := 0.1$

Null Hypothesis: $x_1 = x_2$

Alternative Hypothesis: $x_1 < x_2$

The less than is considered single-sided.

$N_1 := 25$ Number of data points in each set.

$N_2 := 25$

Since these were two different samples, then the following procedure would be used.

$$X_1 := \frac{1}{N_1} \cdot \sum_{i=1}^{N_1} x_{1,i} \quad X_1 = 20.579 \quad \text{Mean of data set 1.}$$

$$X_2 := \frac{1}{N_2} \cdot \sum_{i=1}^{N_2} x_{2,i} \quad X_2 = 30.1 \quad \text{Mean of data set 2.}$$

$i := 1..25$

$x_{1,i} :=$ $x_{2,i} :=$

12.26	21.26
13.26	41.26
18.26	39.26
24.26	21.26
12.26	18.26
11.26	19.26
27.26	28.26
20.26	29.26
27.26	24.26
45.26	19.26
22.26	42.26
29.26	48.26
19.26	38.26
20.26	32.26
13.26	18.26
12.26	30.26
15.26	37.26
18.26	43.26
21.26	29.26
32.23	19.26
13.26	37.26
21.26	29.26
13.26	49.26
41.26	21.26
10.26	15.26

$$\text{Var1} := \frac{1}{N_1 - 1} \cdot \sum_{i=1}^{N_1} (x_{1,i} - X_1)^2 \quad \text{Var1} = 83.781$$

Variance of data set 1.

$$\text{Var2} := \frac{1}{N_2 - 1} \cdot \sum_{i=1}^{N_2} (x_{2,i} - X_2)^2 \quad \text{Var2} = 105.307$$

Variance of data set 2.

$$\sigma_1 := \sqrt{\text{Var1}} \quad \sigma_1 = 9.153$$

Standard deviation of data set 1.

$$\sigma_2 := \sqrt{\text{Var2}} \quad \sigma_2 = 10.262$$

Standard deviation of data set 2.

$$s_D := \sqrt{\frac{\sum_{i=1}^{N1} (x1_i - X_1)^2 + \sum_{i=1}^{N2} (x2_i - X_2)^2}{N1 + N2 - 2} \cdot \left(\frac{1}{N1} + \frac{1}{N2} \right)}$$

Pooled variance.
 $s_D = 2.75$

$$t := \frac{X_1 - X_2}{s_D} \quad t = -3.462$$

Now evaluate the significance of this t value for the Student's t-distribution with $N1+N2-2$ df.

$$df := N1 + N2 - 2$$

$$df = 48$$

We reject H_0 if: $|t| > t_\alpha$ where the α values are single-sided because the hypothesis is.

Using the Student's t-distribution, and for a single-sided hypothesis, values of t_α are used.

$$t_\alpha = t_{0.1} = 1.299$$

$$|t| = 3.462$$

Since t is greater than t_α then we reject H_0 and accept the alternative, that the mean of population one is less than the mean of population two.

We can conclude therefore that the second manufactured actuator will have a greater mean thickness than the first made actuator.

Did the standard deviation change?

$$\sigma_1 = 9.153$$

$$\text{Var1} = 83.781$$

$$\sigma_2 = 10.262$$

$$\text{Var2} = 105.307$$

Once again, it appears the variance has increased for the second data set, indicating a decrease in epoxy layer uniformity.

Is this difference statistically significant?

Choose $\alpha = 0.1$

First state the hypotheses:

Ho: $\sigma_1^2 = \sigma_2^2$ Null hypothesis--the variances are the same (equal).

Ha: $\sigma_1^2 < \sigma_2^2$ Alternative hypothesis--the variance in population 2 is greater than in 1.

$F := \left(\frac{\text{Var2}}{\text{Var1}} \right)$ $F = 1.257$ F value is the ratio of the variances--the larger of the two on top.

$df1 := N1 - 1$ $df1 = 24$

$df2 := N2 - 1$ $df2 = 24$

What we want to check now is the following:

$F > F_{\alpha}(df2, df1)$ If this inequality is true, we accept Ha, the alternative hypothesis.

Using a table for the f-variate:

$F_{0.10}(df1, df2) = 1.98$

$F = 1.257$ This value of F is less than 1.98, therefore we accept the null hypothesis.

We conclude that the variances are equal.

We conclude then that the uniformity has not changed from the first made actuator to the second.

Next, test the average thicknesses between sample 1 for an epoxy ratio of 1:0 acetone and sample 1 for a ratio of 1:1.

Choose α value: $\alpha := 0.1$

Null Hypothesis: $x_1 = x_2$

Alternative Hypothesis: $x_1 < x_2$

The less than is considered single-sided.

$N_1 := 25$ Number of data points in each set.

$N_2 := 25$

Since these were two different samples, then the following procedure would be used.

$$X_1 := \frac{1}{N_1} \cdot \sum_{i=1}^{N_1} x_{1,i} \quad X_1 = 20.579 \quad \text{Mean of data set 1.}$$

$$X_2 := \frac{1}{N_2} \cdot \sum_{i=1}^{N_2} x_{2,i} \quad X_2 = 19.66 \quad \text{Mean of data set 2.}$$

$$\text{Var1} := \frac{1}{N_1 - 1} \cdot \sum_{i=1}^{N_1} (x_{1,i} - X_1)^2 \quad \text{Var1} = 83.781$$

Variance of data set 1.

$$\text{Var2} := \frac{1}{N_2 - 1} \cdot \sum_{i=1}^{N_2} (x_{2,i} - X_2)^2 \quad \text{Var2} = 30.333$$

Variance of data set 2.

$$\sigma_1 := \sqrt{\text{Var1}} \quad \sigma_1 = 9.153$$

Standard deviation of data set 1.

$$\sigma_2 := \sqrt{\text{Var2}} \quad \sigma_2 = 5.508$$

Standard deviation of data set 2.

$$s_D := \sqrt{\frac{\sum_{i=1}^{N_1} (x_{1,i} - X_1)^2 + \sum_{i=1}^{N_2} (x_{2,i} - X_2)^2}{N_1 + N_2 - 2} \cdot \left(\frac{1}{N_1} + \frac{1}{N_2} \right)}$$

Pooled variance.

$$s_D = 2.136$$

$i := 1..25 \quad x_{1,i} := \quad x_{2,i} :=$

12.26	29.26
13.26	28.26
18.26	19.26
24.26	18.26
12.26	11.26
11.26	22.26
27.26	18.26
20.26	19.26
27.26	12.26
45.26	16.26
22.26	18.26
29.26	19.26
19.26	19.26
20.26	17.26
13.26	15.26
12.26	20.26
15.26	22.26
18.26	19.26
21.26	8.26
32.23	15.26
13.26	30.26
21.26	30.26
13.26	21.26
41.26	19.26
10.26	21.26

$$t := \frac{X_1 - X_2}{s_D} \quad t = 0.43$$

Now evaluate the significance of this t value for the Student's t -distribution with $N_1 + N_2 - 2$ df.

$$df := N_1 + N_2 - 2$$

$$df = 48$$

We reject H_0 if: $|t| > t_\alpha$ where the α values are single-sided because the hypothesis is.

Using the Student's t -distribution, and for a single-sided hypothesis, values of t_α are used.

$$t_{\alpha=0.1} = 1.299$$

$$|t| = 0.43$$

Since t is not greater than t_α then we accept H_0 , that the mean of population one is the same as the mean of population two.

We can conclude therefore that the thinning using epoxy does not change the overall mean thickness of the first made actuator.

Did the standard deviation change?

$$\sigma_1 = 9.153 \quad \text{Var1} = 83.781$$

$$\sigma_2 = 5.508 \quad \text{Var2} = 30.333$$

Once again, it appears the variance has increased for the second data set, indicating a decrease in epoxy layer uniformity.

Is this difference statistically significant?

Choose $\alpha = 0.1$

First state the hypotheses:

$H_0: \sigma_1^2 = \sigma_2^2$ Null hypothesis--the variances are the same (equal).

$H_a: \sigma_1^2 > \sigma_2^2$ Alternative hypothesis--the variance in population 1 is greater than in 2.

$$F := \left(\frac{\text{Var1}}{\text{Var2}} \right) \quad F = 2.762 \quad \text{F value is the ratio of the variances--the larger of the two on top.}$$

$$df1 := N1 - 1 \quad df1 = 24$$

$$df2 := N2 - 1 \quad df2 = 24$$

What we want to check now is the following:

$F > F_{\alpha}(df2, df1)$ If this inequality is true, we accept H_a , the alternative hypothesis.

Using a table for the f-variate:

$$F_{0.10}(df1, df2) = 1.70$$

$F = 2.762$ This value of F is greater than F_{α} , therefore we reject the null hypothesis and accept the alternative.

We conclude that the variances of the first population is greater than the second.

We conclude then that the uniformity is better for the thinned epoxy 1:1 than for 1:0 (no acetone), for the first sample made.

Next, test the average thicknesses between samples 1 and 2 for an epoxy ratio of 1:1 acetone.

Choose α value: $\alpha := 0.1$

Null Hypothesis: $x_1 = x_2$

Alternative Hypothesis: $x_1 < x_2$

The less than is considered single-sided.

$N_1 := 25$ Number of data points in each set.

$N_2 := 25$

Since these were two different samples, then the following procedure would be used.

$$X_1 := \frac{1}{N_1} \sum_{i=1}^{N_1} x_{1i} \quad X_1 = 18.62 \text{ Mean of data set 1.}$$

$$X_2 := \frac{1}{N_2} \sum_{i=1}^{N_2} x_{2i} \quad X_2 = 24.7 \text{ Mean of data set 2.}$$

$$\text{Var1} := \frac{1}{N_1 - 1} \sum_{i=1}^{N_1} (x_{1i} - X_1)^2 \quad \text{Var1} = 27.157 \text{ Variance of data set 1.}$$

$$\text{Var2} := \frac{1}{N_2 - 1} \sum_{i=1}^{N_2} (x_{2i} - X_2)^2 \quad \text{Var2} = 66.757 \text{ Variance of data set 2.}$$

$$\sigma_1 := \sqrt{\text{Var1}} \quad \sigma_1 = 5.211 \quad \text{Standard deviation of data set 1.}$$

$$\sigma_2 := \sqrt{\text{Var2}} \quad \sigma_2 = 8.17 \quad \text{Standard deviation of data set 2.}$$

$$s_D := \sqrt{\frac{\sum_{i=1}^{N_1} (x_{1i} - X_1)^2 + \sum_{i=1}^{N_2} (x_{2i} - X_2)^2}{N_1 + N_2 - 2}} \cdot \left(\frac{1}{N_1} + \frac{1}{N_2} \right) \quad \text{Pooled variance.}$$

$$s_D = 1.938$$

$i := 1. \quad x_{1i} := \quad x_{2i} :=$

29.26	19.26
28.26	25.26
19.26	20.26
18.26	27.26
11.26	20.26
22.26	25.26
18.26	23.26
19.26	20.26
12.26	29.26
16.26	29.26
18.26	21.26
19.26	16.26
12.26	22.26
16.26	21.26
18.26	18.26
19.26	22.26
19.26	14.26
17.26	29.26
15.26	24.26
20.26	21.26
22.26	14.26
19.26	28.26
8.26	30.26
15.26	46.26
30.26	48.26
30.26	
21.26	
19.26	
21.26	

$$t := \frac{X_1 - X_2}{s_D} \quad t = -3.137$$

Now evaluate the significance of this t value for the Student's t-distribution with $N1+N2-2$ df.

$$df := N1 + N2 - 2$$

$$df = 48$$

We reject H_0 if: $|t| > t_\alpha$ where the α values are single-sided because the hypothesis is.

Using the Student's t-distribution, and for a single-sided hypothesis, values of t_α are used.

$$t_{\alpha=0.1} = 1.299$$

$$|t| = 3.137$$

Since t is greater than t_α then we reject H_0 and accept the alternative, that the mean of population one is less than the mean of population two.

We can conclude therefore that the second manufactured actuator will have a greater mean thickness than the first made actuator.

Did the standard deviation change?

$$\sigma_1 = 5.211 \quad \text{Var1} = 27.157$$

$$\sigma_2 = 8.17 \quad \text{Var2} = 66.757$$

Once again, it appears the variance has increased for the second data set, indicating a decrease in epoxy layer uniformity.

Is this difference statistically significant?

Choose $\alpha = 0.1$

First state the hypotheses:

$H_0: \sigma_1^2 = \sigma_2^2$ Null hypothesis--the variances are the same (equal).

$H_a: \sigma_1^2 < \sigma_2^2$ Alternative hypothesis--the variance in population 2 is greater than in 1.

$$F := \left(\frac{\text{Var2}}{\text{Var1}} \right)$$

$$F = 2.458$$

F value is the ratio of the variances--the larger of the two on top.

$$\text{df1} := N1 - 1$$

$$\text{df1} = 24$$

$$\text{df2} := N2 - 1$$

$$\text{df2} = 24$$

What we want to check now is the following:

$$F > F_{\alpha}(\text{df2}, \text{df1})$$

If this inequality is true, we accept H_a , the alternative hypothesis.

Using a table for the f-variate:

$$F_{0.10}(\text{df1}, \text{df2}) = 1.98$$

$F = 2.458$ This value of F is greater than 1.98, therefore we accept the alternative.

We conclude that the variances are NOT equal.

We conclude then that the uniformity has actually INCREASED from the first made actuator to the second with an acetone ratio of 1:1.

Test the average thicknesses between a ratio of epoxy:acetone of 1:0 and 1:3 (four samples for each type, the value given for each sample is the average thickness across the surface of the sample).

Choose α value: $\alpha := 0.1$

Null Hypothesis: $x_1 = x_2$

Alternative Hypothesis: $x_1 \neq x_2$

The "not equal" is considered double-sided.

$i := 1..4$

$x_{1,i} :=$ $x_{2,i} :=$

20.58
30.1
41.98
62.62

12.5
14.94
16.66
16.9

Average thicknesses over each of 4 different bimorphs, in order of their manufacture.

$N_1 := 4$

Number of data points in each set.

$N_2 := 4$

Since these were two different samples, then the following procedure would be used.

$$X_1 := \frac{1}{N_1} \sum_{i=1}^{N_1} x_{1,i} \quad X_1 = 38.82 \quad \text{Mean of data set 1.}$$

$$X_2 := \frac{1}{N_2} \sum_{i=1}^{N_2} x_{2,i} \quad X_2 = 15.25 \quad \text{Mean of data set 2.}$$

$$\text{Var1} := \frac{1}{N_1 - 1} \sum_{i=1}^{N_1} (x_{1,i} - X_1)^2 \quad \text{Var1} = 328.387 \quad \text{Variance of data set 1.}$$

$$\text{Var2} := \frac{1}{N_2 - 1} \sum_{i=1}^{N_2} (x_{2,i} - X_2)^2 \quad \text{Var2} = 4.123 \quad \text{Variance of data set 2.}$$

$$\sigma_1 := \sqrt{\text{Var1}} \quad \sigma_1 = 18.121 \quad \text{Standard deviation of data set 1.}$$

$$\sigma_2 := \sqrt{\text{Var2}} \quad \sigma_2 = 2.031 \quad \text{Standard deviation of data set 2.}$$

$$s_D := \sqrt{\frac{\sum_{i=1}^{N1} (x1_i - X_1)^2 + \sum_{i=1}^{N2} (x2_i - X_2)^2}{N1 + N2 - 2} \cdot \left(\frac{1}{N1} + \frac{1}{N2}\right)}$$

Pooled variance.
 $s_D = 9.117$

$$t := \frac{X_1 - X_2}{s_D} \quad t = 2.585$$

Now evaluate the significance of this t value for the Student's t-distribution with $N1+N2-2$ df.

$$df := N1 + N2 - 2$$

$$df = 6$$

We reject H_0 if: $|t| > t_{\frac{\alpha}{2}}$ where the α values are double-sided because the hypothesis is.

Using the Student's t-distribution, and for a double-sided hypothesis, values of $t_{\frac{\alpha}{2}}$ are used.

$$t_{\frac{\alpha}{2}} = t_{0.05} = 1.943$$

$$t = 2.585$$

Since t is greater than $t_{\frac{\alpha}{2}}$ then we accept H_a , that the two means are NOT the same; they are NOT equal.

Did the standard deviation change?

$$\sigma_1 = 18.121 \quad \text{Var1} = 328.387$$

$$\sigma_2 = 2.031 \quad \text{Var2} = 4.123$$

Once again, it appears the variance has been substantially decreased for the second data set. Is this difference statistically significant?

Choose $\alpha = 0.1$

First state the hypotheses:

$H_0: \sigma_1^2 = \sigma_2^2$ Null hypothesis--the variances are the same (equal).

Ha: $\sigma_1^2 > \sigma_2^2$ Alternative hypothesis--the variances are not equal.

$F := \left(\frac{\text{Var1}}{\text{Var2}} \right)$ $F = 79.646$ F value is the ratio of the variances--the larger of the two on top.

$df1 := N1 - 1$ $df1 = 3$

$df2 := N2 - 1$ $df2 = 3$

What we want to check now is the following:

$F > F_{\alpha}(df2, df1)$ If this inequality is true, we accept Ha, the alternative hypothesis.

Using a table for the f-variate:

$F_{0.10}(df1, df2) = 5.39$

$F = 79.646$ This value of F is greater than F_{α} , therefore we accept the alternative.

We conclude that the variances are NOT equal, that the variance of the 1:0 ratio is higher than the variance of the 1:3 ratio..

We can concluded then before, that for an acetone ratio of 1:1, the average of the mean thicknesses of all the samples was not any different than for 1:0 ratio, but it IS different for a ratio of 1:3 (compared to 1:0), AND the uniformity is improved (smaller deviation).

Test the average thicknesses between a ratio of epoxy:acetone of 1:0 and 1:2 (four samples for each type, the value given for each sample is the average thickness across the surface of the sample).

Choose α value: $\alpha := 0.1$

Null Hypothesis: $x_1 = x_2$

Alternative Hypothesis: $x_1 \neq x_2$

The "not equal" is considered double-sided.

$i := 1..4$

$x_{1_i} :=$ $x_{2_i} :=$

20.58
30.1
41.98
62.62

20.98
20.66
22.66
23.5

Average thicknesses over each of 4 different bimorphs, in order of their manufacture.

$N_1 := 4$

Number of data points in each set.

$N_2 := 4$

Since these were two different samples, then the following procedure would be used.

$$X_1 := \frac{1}{N_1} \sum_{i=1}^{N_1} x_{1_i} \quad X_1 = 38.82 \quad \text{Mean of data set 1.}$$

$$X_2 := \frac{1}{N_2} \sum_{i=1}^{N_2} x_{2_i} \quad X_2 = 21.95 \quad \text{Mean of data set 2.}$$

$$\text{Var1} := \frac{1}{N_1 - 1} \sum_{i=1}^{N_1} (x_{1_i} - X_1)^2 \quad \text{Var1} = 328.387 \quad \text{Variance of data set 1.}$$

$$\text{Var2} := \frac{1}{N_2 - 1} \sum_{i=1}^{N_2} (x_{2_i} - X_2)^2 \quad \text{Var2} = 1.837 \quad \text{Variance of data set 2.}$$

$$\sigma_1 := \sqrt{\text{Var1}} \quad \sigma_1 = 18.121 \quad \text{Standard deviation of data set 1.}$$

$$\sigma_2 := \sqrt{\text{Var2}} \quad \sigma_2 = 1.355 \quad \text{Standard deviation of data set 2.}$$

$$s_D := \sqrt{\frac{\sum_{i=1}^{N1} (x1_i - X_1)^2 + \sum_{i=1}^{N2} (x2_i - X_2)^2}{N1 + N2 - 2}} \cdot \left(\frac{1}{N1} + \frac{1}{N2} \right)$$

Pooled variance.
 $s_D = 9.086$

$$t := \frac{X_1 - X_2}{s_D} \quad t = 1.857$$

Now evaluate the significance of this t value for the Student's t-distribution with $N1+N2-2$ df.

$$df := N1 + N2 - 2$$

$$df = 6$$

We reject H_0 if: $|t| > t_{\frac{\alpha}{2}}$ where the α values are double-sided because the hypothesis is.

Using the Student's t-distribution, and for a double-sided hypothesis, values of $t_{\frac{\alpha}{2}}$ are used.

$$t_{\frac{\alpha}{2}} = t_{0.05} = 1.943$$

$$t = 1.857$$

Since t is NOT greater than $t_{\frac{\alpha}{2}}$, then we accept H_0 , that the two means are the same; they are equal.

Did the standard deviation change?

$$\sigma_1 = 18.121 \quad \text{Var1} = 328.387$$

$$\sigma_2 = 1.355 \quad \text{Var2} = 1.837$$

Once again, it appears the variance has been substantially decreased for the second data set. Is this difference statistically significant?

Choose $\alpha = 0.1$

First state the hypotheses:

$H_0: \sigma_1^2 = \sigma_2^2$ Null hypothesis--the variances are the same (equal).

$H_a: \sigma_1^2 > \sigma_2^2$ Alternative hypothesis--the variances are not equal.

$$F := \left(\frac{\text{Var1}}{\text{Var2}} \right) \quad F = 178.743 \quad \text{F value is the ratio of the variances--the larger of the two on top.}$$

$$\text{df1} := N1 - 1 \quad \text{df1} = 3$$

$$\text{df2} := N2 - 1 \quad \text{df2} = 3$$

What we want to check now is the following:

$$F > F_{\alpha}(\text{df2}, \text{df1}) \quad \text{If this inequality is true, we accept } H_a, \text{ the alternative hypothesis.}$$

Using a table for the f-variate:

$$F_{0.10}(\text{df1}, \text{df2}) = 5.39$$

$F = 178.743$ This value of F is greater than F_{α} , therefore we accept the alternative.

We conclude that the variances are NOT equal, that the variance of the 1:0 ratio is higher than the variance of the 1:2 ratio.

We can conclude then before, that for an acetone ratio of 1:1, the average of the mean thicknesses of all the samples was not any different than for 1:0 ratio, and it is NOT different for a ratio of 1:2 (compared to 1:0), BUT the uniformity is improved (smaller deviation).

Next, test the average thicknesses between sample 1 for an epoxy ratio of 1:0 acetone and sample 1 for a ratio of 1:2.

Choose α value: $\alpha := 0.1$

Null Hypothesis: $x_1 = x_2$

Alternative Hypothesis: $x_1 < x_2$

The less than is considered single-sided.

$N_1 := 25$ Number of data points in each set.

$N_2 := 25$

Since these were two different samples, then the following procedure would be used.

$$X_1 := \frac{1}{N_1} \sum_{i=1}^{N_1} x_{1i} \quad X_1 = 20.579 \quad \text{Mean of data set 1.}$$

$$X_2 := \frac{1}{N_2} \sum_{i=1}^{N_2} x_{2i} \quad X_2 = 20.98 \quad \text{Mean of data set 2.}$$

$$\text{Var1} := \frac{1}{N_1 - 1} \sum_{i=1}^{N_1} (x_{1i} - X_1)^2 \quad \text{Var1} = 83.781 \quad \text{Variance of data set 1.}$$

$$\text{Var2} := \frac{1}{N_2 - 1} \sum_{i=1}^{N_2} (x_{2i} - X_2)^2 \quad \text{Var2} = 12.793 \quad \text{Variance of data set 2.}$$

$$\sigma_1 := \sqrt{\text{Var1}} \quad \sigma_1 = 9.153 \quad \text{Standard deviation of data set 1.}$$

$$\sigma_2 := \sqrt{\text{Var2}} \quad \sigma_2 = 3.577 \quad \text{Standard deviation of data set 2.}$$

$$s_D := \sqrt{\frac{\sum_{i=1}^{N_1} (x_{1i} - X_1)^2 + \sum_{i=1}^{N_2} (x_{2i} - X_2)^2}{N_1 + N_2 - 2} \cdot \left(\frac{1}{N_1} + \frac{1}{N_2} \right)} \quad \text{Pooled variance.}$$

$$s_D = 1.965$$

$$t := \frac{X_1 - X_2}{s_D} \quad t = -0.204$$

$i := 1..25$ $x_{1i} :=$ $x_{2i} :=$

12.26	15.26
13.26	24.26
18.26	21.26
24.26	16.26
12.26	20.26
11.26	18.26
27.26	18.26
20.26	20.26
27.26	21.26
45.26	19.26
22.26	20.26
29.26	19.26
19.26	22.26
20.26	26.26
13.26	18.26
12.26	19.26
15.26	27.26
18.26	31.26
21.26	21.26
32.23	22.26
13.26	24.26
21.26	19.26
13.26	22.26
41.26	19.26
10.26	17.26

Now evaluate the significance of this t value for the Student's t-distribution with $N1+N2-2$ df.

$$df := N1 + N2 - 2$$

$$df = 48$$

We reject H_0 if: $|t| > t_{\alpha}$ where the α values are single-sided because the hypothesis is.

Using the Student's t-distribution, and for a single-sided hypothesis, values of t_{α} are used.

$$t_{\alpha} = t_{0.1} = 1.299$$

$$|t| = 0.204$$

Since t is not greater than t_{α} then we accept H_0 , that the mean of population one is the same as the mean of population two.

We can conclude therefore that the thinning using epoxy does not change the overall mean thickness of the first made actuator.

Did the standard deviation change?

$$\sigma_1 = 9.153 \quad \text{Var1} = 83.781$$

$$\sigma_2 = 3.577 \quad \text{Var2} = 12.793$$

Once again, it appears the variance has increased for the second data set, indicating a decrease in epoxy layer uniformity.

Is this difference statistically significant?

Choose $\alpha = 0.1$

First state the hypotheses:

$H_0: \sigma_1^2 = \sigma_2^2$ Null hypothesis--the variances are the same (equal).

$H_a: \sigma_1^2 > \sigma_2^2$ Alternative hypothesis--the variance in population 1 is greater than in 2.

$F := \left(\frac{\text{Var1}}{\text{Var2}} \right)$ $F = 6.549$ F value is the ratio of the variances--the larger of the two on top.

$$df1 := N1 - 1 \quad df1 = 24$$

$$df_2 := N_2 - 1 \quad df_2 = 24$$

What we want to check now is the following:

$$F > F_{\alpha}(df_2, df_1)$$

If this inequality is true, we accept H_a , the alternative hypothesis.

Using a table for the f-variate:

$$F_{0.10}(df_1, df_2) = 1.70$$

$F = 6.549$ This value of F is greater than F_{α} , therefore we reject the null hypothesis and accept the alternative..

We conclude that the variances of the first population is greater than the second.

We conclude then that the uniformity is better for the thinned epoxy 1:2 than for 1:0 (no acetone), for the first sample made.

Next, test the average thicknesses between sample 1 for an epoxy ratio of 1:0 acetone and sample 1 for a ratio of 1:3.

Choose α value: $\alpha := 0.1$

Null Hypothesis: $x_1 = x_2$

Alternative Hypothesis: $x_1 > x_2$

The less than is considered single-sided.

$N_1 := 25$ Number of data points in each set.

$N_2 := 25$

Since these were two different samples, then the following procedure would be used.

$$X_1 := \frac{1}{N_1} \sum_{i=1}^{N_1} x_{1i} X_1 = 20.579 \quad \text{Mean of data set 1.}$$

$$X_2 := \frac{1}{N_2} \sum_{i=1}^{N_2} x_{2i} X_2 = 12.5 \quad \text{Mean of data set 2.}$$

$$\text{Var1} := \frac{1}{N_1 - 1} \sum_{i=1}^{N_1} (x_{1i} - X_1)^2 \quad \text{Var1} = 83.781 \quad \text{Variance of data set 1.}$$

$$\text{Var2} := \frac{1}{N_2 - 1} \sum_{i=1}^{N_2} (x_{2i} - X_2)^2 \quad \text{Var2} = 17.357 \quad \text{Variance of data set 2.}$$

$$\sigma_1 := \sqrt{\text{Var1}} \quad \sigma_1 = 9.153$$

Standard deviation of data set 1.

$$\sigma_2 := \sqrt{\text{Var2}} \quad \sigma_2 = 4.166$$

Standard deviation of data set 2.

$$s_D := \sqrt{\frac{\sum_{i=1}^{N_1} (x_{1i} - X_1)^2 + \sum_{i=1}^{N_2} (x_{2i} - X_2)^2}{N_1 + N_2 - 2} \cdot \left(\frac{1}{N_1} + \frac{1}{N_2} \right)}$$

Pooled variance.

$$s_D = 2.011$$

$$t := \frac{X_1 - X_2}{s_D} \quad t = 4.017$$

$i := 1..25 \quad x_{1i} := \quad x_{2i} :=$

12.26	10.26
13.26	18.26
18.26	19.26
24.26	11.26
12.26	8.26
11.26	10.26
27.26	20.26
20.26	15.26
27.26	12.26
45.26	9.26
22.26	11.26
29.26	12.26
19.26	18.26
20.26	11.26
13.26	10.26
12.26	13.26
15.26	14.26
18.26	11.26
21.26	10.26
32.23	2.26
13.26	14.26
21.26	11.26
13.26	19.26
41.26	10.26
10.26	8.26

Now evaluate the significance of this t value for the Student's t-distribution with $N1+N2-2$ df.

$$df := N1 + N2 - 2$$

$$df = 48$$

We reject H_0 if: $|t| > t_{\alpha}$ where the α values are single-sided because the hypothesis is.

Using the Student's t-distribution, and for a single-sided hypothesis, values of t_{α} are used.

$$t_{\alpha} = t_{0.1} = 1.299$$

$$|t| = 4.017$$

Since t is greater than t_{α} then we accept H_a , that the mean of population one is greater than the mean of population two. The thickness decreased with an epoxy ratio of 1:3 compared to 1:0 (no acetone).

We can conclude therefore that the thinning using epoxy does not change the overall mean thickness of the first made actuator unless a ratio of 1:3 is used, since we concluded before that there was no change in the thickness for ratios of 1:1 and 1:2.

Did the standard deviation change?

$$\sigma_1 = 9.153 \quad \text{Var1} = 83.781$$

$$\sigma_2 = 4.166 \quad \text{Var2} = 17.357$$

Once again, it appears the variance has increased for the second data set, indicating a decrease in epoxy layer uniformity.

Is this difference statistically significant?

Choose $\alpha = 0.1$

First state the hypotheses:

$H_0: \sigma_1^2 = \sigma_2^2$ Null hypothesis--the variances are the same (equal).

$H_a: \sigma_1^2 > \sigma_2^2$ Alternative hypothesis--the variance in population 1 is greater than in 2.

$$F := \left(\frac{\text{Var1}}{\text{Var2}} \right) \quad F = 4.827 \quad \text{F value is the ratio of the variances--the larger of the two on top.}$$

$$df1 := N1 - 1 \quad df1 = 24$$

$$df2 := N2 - 1 \quad df2 = 24$$

What we want to check now is the following:

$$F > F_{\alpha}(df2, df1)$$

If this inequality is true, we accept H_a , the alternative hypothesis.

Using a table for the f-variate:

$$F_{0.10}(df1, df2) = 1.70$$

$F = 4.827$ This value of F is greater than F_{α} , therefore we reject the null hypothesis and accept the alternative.

We conclude that the variances of the first population is greater than the second.

We conclude then that the uniformity is better for the thinned epoxy 1:3 than for 1:0 (no acetone), for the first sample made, as it is for any use of acetone.

APPENDIX C

PULSER-RECEIVER AND OSCILLOSCOPE SETTINGS

OSCILLOSCOPE SETTINGS

6/28/99

LONGITUDINAL WAVE MEASUREMENTS

Time Base (time button)2 μ sec/divison

Delay 0

Reference Left

Voltage (chan button)

100 mV/division

Offset 0

Trigger (trig button)

Auto

edge

Ext Source Selected

Level 0

Averaging on 32 samples (Display button)

PULSER-RECEIVER SETTINGS

6/28/99 and 7/19/99

LONGITUDINAL & SHEAR WAVE

PRF = 6

Pulse Height = High

Damping = 5

Gain Switch = -20dB

Gain = 9

HP Filter = Out (setting on the back of the box)

OSCILLOSCOPE SETTINGS

7/19/99

SHEAR WAVE

Time Base (time button)5 μ sec/divisonDelay -10 μ sec

Reference Left

APPENDIX D

STIFFNESS CALCULATIONS AND OPTIMIZATION WORK

The following expressions are derived using Castigliano's Second Theorem for two curved beams of equal radius, with both an applied concentrated load at the right end of the model (quarter model of a single actuator) and the piezoelectric effects. The model considers two layers of piezoelectric material and also the epoxy layer between them. NOTE: PVDF is considered to be isotropic in this calculation.

BIMORPH: NO EPOXY LAYER

$$R := 0.0315 \cdot \text{m}$$

$$b := 2 \cdot \text{in}$$

b is the width of the actuator into the plane of the paper, R is the radius of curvature of the bimorph

$$\phi_0 := 23.72 \cdot \text{deg}$$

$$h := 28 \cdot 10^{-6} \cdot \text{m}$$

Thickness of the PVDF film with electrodes (for a bimorph with no epoxy layer, there is no capability to model the electrode layers developed, so the voltages have to as yet be applied on the surface of the film so the total thickness can't be modeled)

$$h_p := 28 \cdot 10^{-6} \cdot \text{m}$$

Thickness of the PVDF (clear part, the dielectric thickness)

$$A := b \cdot h \quad \text{Cross-sectional area of one layer of PVDF in the bimorph.}$$

$$d := \frac{h}{2} \quad \text{Distance to the neutral axis when no epoxy layer is present.}$$

$$I := \frac{b \cdot h^3}{12} \quad \text{Moment of inertia of one layer of PVDF in the bimorph.}$$

$$I_{eq} := 2 \cdot (I + A \cdot d^2) \quad \text{Equivalent moment of inertia of the two-layer bimorph.}$$

$$Y := 6.154 \cdot 10^9 \cdot \frac{\text{newton}}{\text{m}^2} \quad \text{Young's modulus of the PVDF layers with electrodes.}$$

$$d_{31} := 21 \cdot 10^{-12} \cdot \frac{\text{m}}{\text{V}} \quad \text{Piezoelectric strain coefficient.}$$

Below are the voltage gradients across the inner and outer layers of each beam segment of the bimorph (outer surface voltage minus inner surface voltage relative to the arc center):

$$dV1i := -1000 \cdot \text{V}$$

$$dV1o := 1000 \cdot \text{V}$$

$$dV2i := 1000 \cdot \text{V}$$

$$dV2o := -1000 \cdot \text{V}$$

Below are the electric fields resulting from the applied voltage gradients:

$$E_{inner1} := \frac{-dV1i}{h_p}$$

$$E_{outer1} := \frac{-dV1o}{h_p}$$

$$E_{inner2} := \frac{-dV2i}{h_p}$$

$$E_{outer2} := \frac{-dV2o}{h_p}$$

$$E_{inner1} = 3.571 \cdot 10^7 \cdot \frac{\text{V}}{\text{m}} \quad E_{outer1} = -3.571 \cdot 10^7 \cdot \frac{\text{V}}{\text{m}} \quad E_{inner2} = -3.571 \cdot 10^7 \cdot \frac{\text{V}}{\text{m}} \quad E_{outer2} = 3.571 \cdot 10^7 \cdot \frac{\text{V}}{\text{m}}$$

Calculating the electrical internal moments in each beam segment:

$$dE1 := E_{\text{outer1}} - E_{\text{inner1}}$$

$$dE2 := E_{\text{outer2}} - E_{\text{inner2}}$$

$$dE1 = -7.143 \cdot 10^7 \frac{\text{V}}{\text{m}}$$

$$dE2 = 7.143 \cdot 10^7 \frac{\text{V}}{\text{m}}$$

$$M_{V1} := d_{31} \cdot A \cdot Y \cdot \frac{h}{2} \cdot dE1$$

$$M_{V2} := -d_{31} \cdot A \cdot Y \cdot \frac{h}{2} \cdot dE2$$

$$M_{V1} = -1.838 \cdot 10^{-4} \text{ newton} \cdot \text{m}$$

$$M_{V2} = -1.838 \cdot 10^{-4} \text{ newton} \cdot \text{m}$$

$$F := 0 \cdot \text{newton}, 0.01 \cdot \text{newton} \dots 0.03 \cdot \text{newton}$$

Range of forces considered.

Displacement from Castigliano's Theorem method, including electrical and mechanical loads:

$$\begin{aligned} \delta_A(F) := & -(\sin(\phi_0) \cdot \phi_0 - 1 + \cos(\phi_0)) \cdot \frac{R^2}{(Y \cdot I_{eq})} \cdot (M_{V1} + M_{V2}) \dots \\ & + \left[\frac{-(-3 \cdot \phi_0 - 3 \cdot \cos(\phi_0) \cdot \sin(\phi_0) + 4 \cdot \sin(\phi_0) + 2 \cdot \phi_0 \cdot \cos(\phi_0)^2)}{(Y \cdot I_{eq})} \cdot R^3 \right] \cdot F \dots \\ & + \left[\frac{R \cdot (\cos(\phi_0) - 1 - \phi_0 + \cos(\phi_0) \cdot \sin(\phi_0))}{(Y \cdot A)} \right] \cdot F \end{aligned}$$

$$\delta_A(0 \cdot \text{newton}) = 6.543 \cdot 10^{-3} \text{ m}$$

Displacement due to electrical part only at the voltages applied above.

Note that this does not include the effects of the glue layer, which will make it more stiff.

Now look at a mechanical load, concentrated at the center of the bimorph only:

$$R := 0.0315 \cdot \text{m}$$

$$b := 2 \cdot \text{in}$$

b is the width of the actuator into the plane of the paper

$$\phi_0 := 23.72 \cdot \text{deg}$$

$$h := 45 \cdot 10^{-6} \cdot \text{m}$$

Thickness of the PVDF film with electrodes.

$$h_p := 30 \cdot 10^{-6} \cdot \text{m}$$

Thickness of the PVDF (clear part, the dielectric thickness)

$$A := b \cdot h \quad \text{Cross-sectional area of one layer of PVDF in the bimorph.}$$

$$d := \frac{h}{2} \quad \text{Distance to the neutral axis when no epoxy layer is present.}$$

$$I := \frac{b \cdot h^3}{12} \quad \text{Moment of inertia of one layer of PVDF in the bimorph.}$$

$$I_{eq} := 2 \cdot (I + A \cdot d^2) \quad \text{Equivalent moment of inertia of the two-layer bimorph.}$$

$$Y := 6.154 \cdot 10^9 \cdot \frac{\text{newton}}{\text{m}^2} \quad \text{Young's modulus of the PVDF layers with electrodes.}$$

$$d_{31} := 21 \cdot 10^{-12} \cdot \frac{\text{m}}{\text{V}} \quad \text{Piezoelectric coefficient.}$$

$$M_{V1} := 0 \cdot \text{newton} \cdot \text{m}$$

Set the piezoelectric moments equal to zero, since applying only a mechanical force.

$$M_{V2} := 0 \cdot \text{newton} \cdot \text{m}$$

$$3 \cdot \text{gm} \cdot 9.81 \cdot \frac{\text{m}}{\text{sec}^2} = 0.029 \cdot \text{newton}$$

$$F := 0 \cdot \text{newton}, 0.1 \cdot 0.0508 \cdot \text{newton}.. 0.3 \cdot 0.0508 \cdot \text{newton} \quad \text{Range of forces considered.}$$

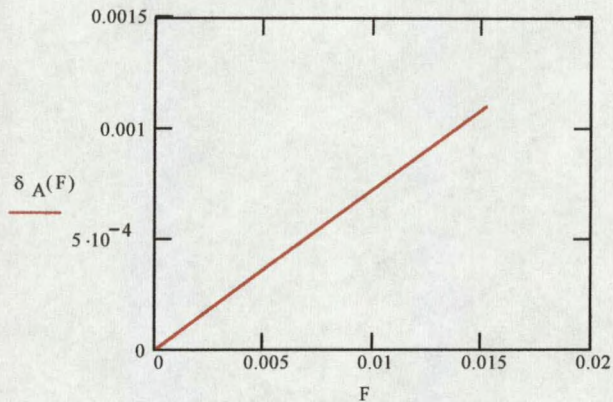
Displacement from Castigliano's Theorem method, mechanical load:

$$\begin{aligned} \delta_A(F) := & -(\sin(\phi_0) \cdot \phi_0 - 1 + \cos(\phi_0)) \cdot \frac{R^2}{(Y \cdot I_{eq})} \cdot (M_{V1} + M_{V2}) \dots \\ & + \left[\frac{-(-3 \cdot \phi_0 - 3 \cdot \cos(\phi_0) \cdot \sin(\phi_0) + 4 \cdot \sin(\phi_0) + 2 \cdot \phi_0 \cdot \cos(\phi_0)^2)}{(Y \cdot I_{eq})} \cdot R^3 \right] \cdot F \dots \\ & + \left[R \cdot \frac{(\cos(\phi_0) - 1 - \phi_0 + \cos(\phi_0) \cdot \sin(\phi_0))}{(Y \cdot A)} \right] \cdot F \end{aligned}$$

$$\delta_A(0.3 \cdot 0.0508 \cdot \text{newton}) = 1.098 \cdot \text{mm}$$

$\delta_A(F) =$	m	F =	kg·m·s ⁻²
0		0	
$3.659 \cdot 10^{-4}$		$5.08 \cdot 10^{-3}$	
$7.317 \cdot 10^{-4}$		0.01	
$1.098 \cdot 10^{-3}$		0.015	

Total displacement vs mechanical load for no applied voltage.



$i := 0..1$

$d_i :=$

0·m
0.001098·m

$F_i :=$

0·newton
0.3·0.0508·newton

$$K_{wo} := \text{slope}(d, F) \quad K_{wo} = 13.88 \frac{\text{newton}}{\text{m}}$$

Stiffness of the bimorph in N/m with no epoxy layer, analytical result.

Ansys results, no epoxy, 45 micron layers.

$dan_i :=$

0
0.001093

$Fan_i :=$

0
0.3·0.0508

$$K_{woa} := \text{slope}(dan, Fan) \quad K_{woa} = 13.943$$

Stiffness of the bimorph in N/m with no epoxy layer, Ansys result.

$$\frac{K_{woa} \frac{\text{newton}}{\text{m}} - K_{wo}}{K_{wo}} \cdot 100 = 0.457$$

Percent difference between the stiffness predicted by the Ansys model and the analytical model.

Now consider the mechanical part as a function of radius:

The purely mechanical behavior is the following (when the electrical part is set to zero):

$$\delta_A = \left[\frac{-(-3 \cdot \phi_0 - 3 \cdot \cos(\phi_0) \cdot \sin(\phi_0) + 4 \cdot \sin(\phi_0) + 2 \cdot \phi_0 \cdot \cos(\phi_0)^2) \cdot R^3}{(Y \cdot I_{eq})} \cdot F \dots \right. \\ \left. + \left[R \cdot \frac{(\cos(\phi_0) - 1 - \phi_0 + \cos(\phi_0) \cdot \sin(\phi_0))}{(Y \cdot A)} \right] \cdot F \right]$$

The stiffness, then, of the actuator is given by the following expression:

$$\frac{F}{\delta_A} = \left[\left[\frac{-(-3 \cdot \phi_0 - 3 \cdot \cos(\phi_0) \cdot \sin(\phi_0) + 4 \cdot \sin(\phi_0) + 2 \cdot \phi_0 \cdot \cos(\phi_0)^2) \cdot R^3}{(Y \cdot I_{eq})} \dots \right] \right]^{-1} \\ \left[+ \left[R \cdot \frac{(\cos(\phi_0) - 1 - \phi_0 + \cos(\phi_0) \cdot \sin(\phi_0))}{(Y \cdot A)} \right] \right]$$

Or the compliance is given by the following:

$$\frac{\delta_A}{F} = \frac{-(-3 \cdot \phi_0 - 3 \cdot \cos(\phi_0) \cdot \sin(\phi_0) + 4 \cdot \sin(\phi_0) + 2 \cdot \phi_0 \cdot \cos(\phi_0)^2) \cdot R^3}{(Y \cdot I_{eq})} \dots \\ + \left[R \cdot \frac{(\cos(\phi_0) - 1 - \phi_0 + \cos(\phi_0) \cdot \sin(\phi_0))}{(Y \cdot A)} \right]$$

Plot now the stiffness as a function of radius, R:

R := 0.015·m, 0.016·m.. 0.05·m

$$K(R) := \left[\left[\frac{-(-3 \cdot \phi_0 - 3 \cdot \cos(\phi_0) \cdot \sin(\phi_0) + 4 \cdot \sin(\phi_0) + 2 \cdot \phi_0 \cdot \cos(\phi_0)^2) \cdot R^3}{(Y \cdot I_{eq})} \dots \right] \right]^{-1} \\ \left[+ \left[R \cdot \frac{(\cos(\phi_0) - 1 - \phi_0 + \cos(\phi_0) \cdot \sin(\phi_0))}{(Y \cdot A)} \right] \right]$$

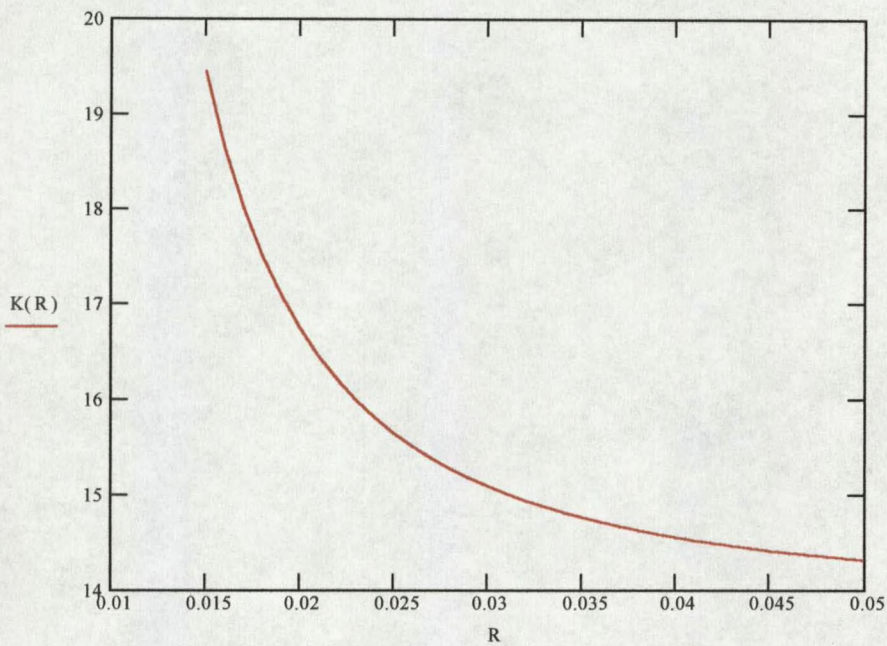
$$K(0.0315 \cdot m) = 13.885 \frac{\text{newton}}{m}$$

Note, however, that the angle ϕ_0 is also a function of R, if the length of PVDF film is taken to be fixed (that is, you have a certain size actuator and are allowing R to vary, therefore ϕ_0 must vary).

L := 1·in

$$\phi\alpha(R) := \frac{L}{2 \cdot R}$$

$$K(R) := \left[\left[\frac{-(-3 \cdot \phi\alpha(R) - 3 \cdot \cos(\phi\alpha(R)) \cdot \sin(\phi\alpha(R)) + 4 \cdot \sin(\phi\alpha(R)) + 2 \cdot \phi\alpha(R) \cdot \cos(\phi\alpha(R))^2)}{(Y \cdot I_{eq})} \cdot R^3 \dots \right] + - \left[R \cdot \frac{(\cos(\phi\alpha(R)) - 1 - \phi\alpha(R) + \cos(\phi\alpha(R)) \cdot \sin(\phi\alpha(R)))}{(Y \cdot A)} \right] \right]^{-1}$$



Note the larger the curvature (smaller the radius), the more stiff the actuator will behave, as one would expect for a circular arc.

BIMORPH: WITH EPOXY LAYER

Plot now the stiffness as a function of radius, R, if the glue layer is included:

$h := 45 \cdot 10^{-6} \cdot \text{m}$ Total thickness of one layer of PVDF film (including electrodes).

$h_p := 30 \cdot 10^{-6} \cdot \text{m}$ Total thickness of the unelectroded PVDF film (no capability to model the electrode layers separately in Ansys unless have material properties, so are not taken into account in results compared to Ansys). Keypoints file can calculate the points to create the areas, however, and here they will be considered.

$h_g := 25 \cdot 10^{-6} \cdot \text{m}$ Thickness of the glue layer.

$h_e := h - h_p$ $h_e = 1.5 \cdot 10^{-5} \text{ m}$ Thickness of the electrodes (for both sides of one layer of film).

$I := \frac{b \cdot h^3}{12}$ Moment of inertia of one layer of PVDF in the bimorph.

$I_g := \frac{b \cdot h_g^3}{12}$ Moment of inertia of the epoxy layer in the bimorph.

$A := b \cdot h$ Cross-sectional area of one layer of PVDF in the bimorph.

$A_g := b \cdot h_g$ Cross-sectional area of the epoxy layer in the bimorph.

$Y_g := 4.235 \cdot 10^9 \frac{\text{newton}}{\text{m}^2}$ Young's modulus of the epoxy.

This value for the Young's modulus of the glue was tested by Ashley, July 19, 1999. The value for Young's modulus of the PVDF is taken from paper by H.Wang. Here only isotropic material behavior is accounted for.

$A_{eq} := 2 \cdot A + \frac{Y_g}{Y} \cdot A_g$ Equivalent area.

$d := \frac{h}{2} + \frac{h_g}{2}$ Distance from the centroid of the PVDF film layer to the neutral axis if an epoxy layer exists.

$I_{eq} := 2 \cdot (I + A \cdot d^2) + I_g \cdot \frac{Y_g}{Y}$ Equivalent moment of inertia for the bimorph with the glue layer.

$Y = 6.154 \cdot 10^9 \text{ Pa}$ Young's modulus of the PVDF layers with electrodes.

$\phi_0 := 0.414$ Arc angle used in the Ansys models, or 23.72 degrees.

$$h_{eq} := d$$

Plot now the stiffness as a function of radius:

$$R := 0.01 \cdot m, 0.011 \cdot m, \dots, 0.05 \cdot m$$

$$K_{g1}(R) := \left[\frac{-\left(-3 \cdot \phi_0 - 3 \cdot \cos(\phi_0) \cdot \sin(\phi_0) + 4 \cdot \sin(\phi_0) + 2 \cdot \phi_0 \cdot \cos(\phi_0)^2\right) \cdot R^3}{(Y \cdot I_{eq})} + \left[R \cdot \frac{(\cos(\phi_0) - 1 - \phi_0 + \cos(\phi_0) \cdot \sin(\phi_0))}{(Y \cdot A_{eq})} \right] \right]^{-1}$$

$$K_{g1}(0.0315 \cdot m) = 28.874 \frac{\text{newton}}{m}$$

This value of K is in the range of previously measured K values for the actuators (by Mike M.).

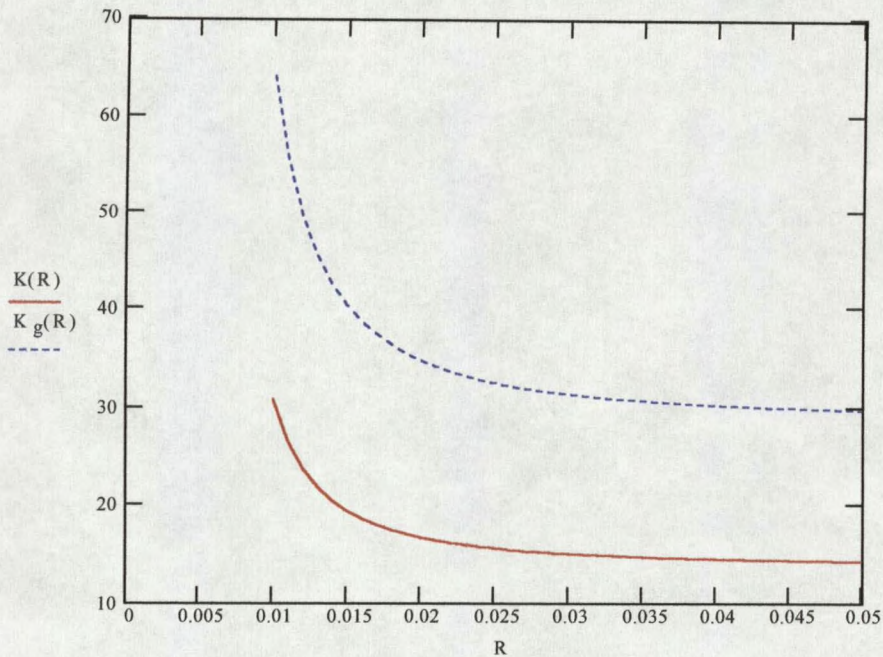
Again, $\phi_0(R)$, so plotting the $K_g(R)$ as such:

$$\phi_0(R) := \frac{L}{2 \cdot R}$$

$$K_g(R) := \left[\frac{-\left(-3 \cdot \phi_0(R) - 3 \cdot \cos(\phi_0(R)) \cdot \sin(\phi_0(R)) + 4 \cdot \sin(\phi_0(R)) + 2 \cdot \phi_0(R) \cdot \cos(\phi_0(R))^2\right) \cdot R^3}{(Y \cdot I_{eq})} + \left[R \cdot \frac{(\cos(\phi_0(R)) - 1 - \phi_0(R) + \cos(\phi_0(R)) \cdot \sin(\phi_0(R)))}{(Y \cdot A_{eq})} \right] \right]^{-1}$$

Note that $K_g(R)$ when $\phi_0(R)$ is calculated, the ϕ_0 value for $R = 0.0135 \text{ m}$ is 0.403 radians, or 23.1 degrees. The angle used to develop the Ansys models was 23.72 degrees, so the difference in the stiffness between the $K_{g1}(R)$ and $K_g(R)$ is due to this difference in angles.

$$\frac{L}{2 \cdot 0.0135 \cdot m} = 0.403$$



Ansys results, with epoxy, 45 micron PVDF layers, 25 micron epoxy.

$$K_{g1}(0.0315 \cdot \text{m}) = 28.874 \frac{\text{newton}}{\text{m}}$$

$$K_{\text{ansys}} := \frac{.09652 \cdot \text{newton}}{3.3363 \cdot 10^{-3} \cdot \text{m}} \quad K_{\text{ansys}} = 28.93 \frac{\text{newton}}{\text{m}}$$

$$\frac{K_{\text{ansys}} - K_{g1}(0.0315 \cdot \text{m})}{K_{g1}(0.0315 \cdot \text{m})} \cdot 100 = 0.196 \quad \text{percent difference between the Ansys and analytical results}$$

Now consider the electrical field applied to a bimorph with an epoxy layer:

Below are the voltage gradients across the inner and outer layers of each beam segment of the bimorph (outer surface voltage minus inner surface voltage relative to the arc center):

$$dV1i := -1000 \cdot \text{V}$$

$$dV1o := 1000 \cdot \text{V}$$

$$dV2i := 1000 \cdot \text{V}$$

$$dV2o := -1000 \cdot \text{V}$$

Below are the electric fields resulting from the applied voltage gradients:

$$E_{\text{inner1}} := \frac{-dV1i}{h_p}$$

$$E_{\text{outer1}} := \frac{-dV1o}{h_p}$$

$$E_{\text{inner2}} := \frac{-dV2i}{h_p}$$

$$E_{\text{outer2}} := \frac{-dV2o}{h_p}$$

$$E_{\text{inner1}} = 3.333 \cdot 10^7 \frac{\text{V}}{\text{m}} \quad E_{\text{outer1}} = -3.333 \cdot 10^7 \frac{\text{V}}{\text{m}} \quad E_{\text{inner2}} = -3.333 \cdot 10^7 \frac{\text{V}}{\text{m}} \quad E_{\text{outer2}} = 3.333 \cdot 10^7 \frac{\text{V}}{\text{m}}$$

Calculating the electrical internal moments in each beam segment:

$$dE1 := E_{\text{outer1}} - E_{\text{inner1}}$$

$$dE2 := E_{\text{outer2}} - E_{\text{inner2}}$$

$$dE1 = -6.667 \cdot 10^7 \frac{\text{V}}{\text{m}}$$

$$dE2 = 6.667 \cdot 10^7 \frac{\text{V}}{\text{m}}$$

$$M_{V1} := d_{31} \cdot A \cdot Y_{\text{eq}} \cdot dE1$$

$$M_{V2} := -d_{31} \cdot A \cdot Y_{\text{eq}} \cdot dE2$$

$$M_{V1} = -6.893 \cdot 10^{-4} \text{ newton} \cdot \text{m}$$

$$M_{V2} = -6.893 \cdot 10^{-4} \text{ newton} \cdot \text{m}$$

$$F := 0 \cdot \text{newton}, 0.01 \cdot \text{newton}.. 0.03 \cdot \text{newton}$$

Range of forces considered.

$$\phi_0 := 0.413$$

Perform calculation for a single bimorph design.

$$R := 0.0315 \cdot \text{m}$$

Displacement from Castigliano's Theorem method, including electrical and mechanical loads:

$$\begin{aligned} \delta_A(F) := & -(\sin(\phi_0) \cdot \phi_0 - 1 + \cos(\phi_0)) \cdot \frac{R^2}{(Y \cdot I_{\text{eq}})} \cdot (M_{V1} + M_{V2}) \dots \\ & + \left[\frac{-(-3 \cdot \phi_0 - 3 \cdot \cos(\phi_0) \cdot \sin(\phi_0) + 4 \cdot \sin(\phi_0) + 2 \cdot \phi_0 \cdot \cos(\phi_0)^2) \cdot R^3}{(Y \cdot I_{\text{eq}})} \cdot F \dots \right. \\ & \left. + \left[R \cdot \frac{(\cos(\phi_0) - 1 - \phi_0 + \cos(\phi_0) \cdot \sin(\phi_0))}{(Y \cdot A_{\text{eq}})} \right] \cdot F \right] \end{aligned}$$

$$\delta_A(0 \cdot \text{newton}) = 2.829 \cdot 10^{-3} \text{ m}$$

Displacement due to electrical part only at the voltages applied above.

$$\frac{1000 \cdot \text{V}}{\delta_A(0 \cdot \text{newton})} = 353.453 \frac{\text{V}}{\text{mm}}$$

Taking the two derivative equations (derivative with respect to R1 and then with respect to R2) and making the cosine and sine terms dimensionless by dividing top and bottom by 1/L:

$$-R_2 \cdot d_{31} \cdot A \cdot h \cdot \frac{dE}{I_{eq}} \cdot \left[\cos \left[\frac{\frac{L}{L}}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_1 + R_2)} \cdot \sin \left[\frac{\frac{L}{L}}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] \dots = 0$$

$$+ R_2 \cdot d_{31} \cdot A \cdot h \cdot R_1 \cdot \frac{dE}{I_{eq}} \cdot \frac{L^2}{(R_1 + R_2)^3} \cdot \cos \left[\frac{\frac{L}{L}}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right]$$

Collecting terms to simplify:

$$\left[-R_2 \cdot \left[\cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] \right] \cdot d_{31} \cdot A \cdot h \cdot \frac{dE}{I_{eq}} = 0$$

Dividing out the constants:

$$\left[-R_2 \cdot \left[\cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] \right] = 0$$

For the other derivative equation:

$$-d_{31} \cdot A \cdot h \cdot R_1 \cdot \frac{dE}{I_{eq}} \cdot \left[\cos \left[\frac{\frac{L}{L}}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_1 + R_2)} \cdot \sin \left[\frac{\frac{L}{L}}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] \dots = 0$$

$$+ R_2 \cdot d_{31} \cdot A \cdot h \cdot R_1 \cdot \frac{dE}{I_{eq}} \cdot \frac{L^2}{(R_1 + R_2)^3} \cdot \cos \left[\frac{\frac{L}{L}}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right]$$

Collecting terms to simplify:

$$\left[-R_1 \cdot \left[\cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] \right] \cdot d_{31} \cdot h \cdot A \cdot \frac{dE}{I_{eq}} = 0$$

Dividing out the constants:

$$\left[-R_1 \cdot \left[\cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] \right] = 0$$

Now we have two equations and two unknowns, R1 and R2:

$$\left[-R_2 \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] = 0$$

$$\left[-R_1 \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] = 0$$

Subtracting these two equations from each other, get the following:

$$\left[-R_2 \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] \dots = 0$$

$$+ - \left[\left[-R_1 \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] \right]$$

Simplifying and collecting terms:

$$\frac{(-R_2 + R_1)}{(R_2 + R_1)} \cdot \sin \left[\frac{L}{(R_2 + R_1)} \right] \cdot L + \frac{(-R_2^2 + R_1^2)}{(R_2 + R_1)} \cdot \cos \left[\frac{L}{(R_2 + R_1)} \right] + \frac{(R_2^2 - R_1^2)}{(R_2 + R_1)} = 0$$

Note that for the left hand side to equal zero, that R2 must equal R1, for maximum displacement.

Take now the first equation out of the set of two that resulted from taking derivatives of W2:

$$\left[-R_2 \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] + \frac{L}{(R_2 + R_1)} \cdot \sin \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] - 1 \right] + R_2 \cdot R_1 \cdot \frac{L^2}{(R_2 + R_1)^3} \cdot \cos \left[\frac{1}{\left(\frac{R_1}{L} + \frac{R_2}{L} \right)} \right] = 0$$

Set $R_1 = R_2 = R$:

$$\left[-R \cdot \cos \left[\frac{1}{\left(\frac{R}{L} + \frac{R}{L} \right)} \right] + \frac{L}{(R + R)} \cdot \sin \left[\frac{1}{\left(\frac{R}{L} + \frac{R}{L} \right)} \right] - 1 \right] + R \cdot R \cdot \frac{L^2}{(R + R)^3} \cdot \cos \left[\frac{1}{\left(\frac{R}{L} + \frac{R}{L} \right)} \right] = 0$$

Simplifying the above now we get:

$$\left[-\cos \left[\frac{1}{(2 \cdot R)} \cdot L \right] + 1 \right] \cdot R - \frac{1}{2} \cdot L \cdot \sin \left[\frac{1}{(2 \cdot R)} \cdot L \right] + \frac{1}{(8 \cdot R)} \cdot L^2 \cdot \cos \left[\frac{1}{(2 \cdot R)} \cdot L \right] = 0$$

Since we can't solve this symbolically for R, then find the numerical result for R given L:

$$L := 0.0254 \cdot \text{m}$$

$$\text{Initial guess R: } R := .5 \cdot \text{m}$$

Given

$$\left[-\cos\left[\frac{1}{(2 \cdot R)} \cdot L\right] + 1 \right] \cdot R - \frac{1}{2} \cdot L \cdot \sin\left[\frac{1}{(2 \cdot R)} \cdot L\right] + \frac{1}{(8 \cdot R)} \cdot L^2 \cdot \cos\left[\frac{1}{(2 \cdot R)} \cdot L\right] = 0$$

Find(R) = 3.03 m

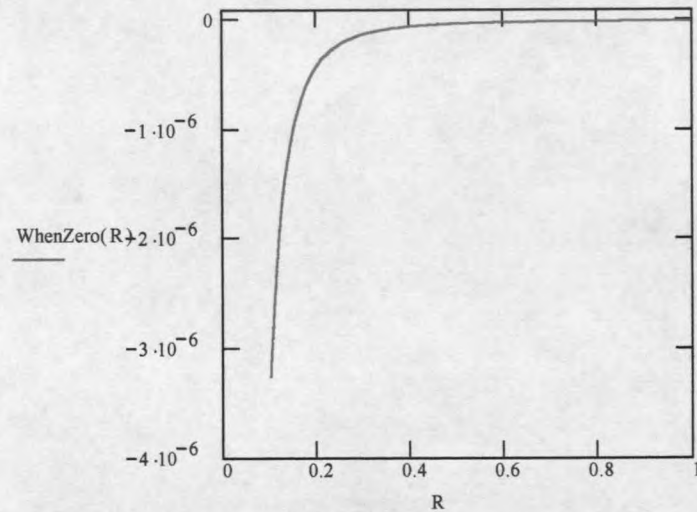
Note that by choosing a different guess value, numerous optimal R's result.

Let's try to get an enveloping value for R, such as a minimum required R or a maximum required R such that an optimal displacement is achieved.

$R := 0.1 \cdot m, 0.11 \cdot m, 1 \cdot m$

Plot the above to see for which values of R it equals zero:

$$\text{WhenZero}(R) := \left[\left[-\cos\left[\frac{1}{(2 \cdot R)} \cdot L\right] + 1 \right] \cdot R - \frac{1}{2} \cdot L \cdot \sin\left[\frac{1}{(2 \cdot R)} \cdot L\right] \right] + \frac{1}{(8 \cdot R)} \cdot L^2 \cdot \cos\left[\frac{1}{(2 \cdot R)} \cdot L\right]$$



It appears from this graph that for $R > 0.5$ m that the optimal value is reached (since the derivative is equal to zero after that point for any R value greater than this). By looking at the scale of the graph, however, and choosing a tolerance that will be considered equal to zero (choose $1E-6$), then it can be seen from the below that $R > 0.15$ m is sufficient to reach the optimal value, that is, is sufficient to make the derivative of $W2$ with respect to R equal to zero---a max/min point.

$R := 0.0000001 \cdot m, 0.01 \cdot m, 0.3 \cdot m$

WhenZero(R) = m	R = m
-353.536	$1 \cdot 10^{-7}$
$-2.703 \cdot 10^{-3}$	0.01
$-3.885 \cdot 10^{-4}$	0.02
$-1.181 \cdot 10^{-4}$	0.03
$-5.024 \cdot 10^{-5}$	0.04
$-2.583 \cdot 10^{-5}$	0.05
$-1.498 \cdot 10^{-5}$	0.06
$-9.446 \cdot 10^{-6}$	0.07
$-6.334 \cdot 10^{-6}$	0.08
$-4.451 \cdot 10^{-6}$	0.09
$-3.246 \cdot 10^{-6}$	0.1
$-2.44 \cdot 10^{-6}$	0.11
$-1.88 \cdot 10^{-6}$	0.12
$-1.479 \cdot 10^{-6}$	0.13
$-1.184 \cdot 10^{-6}$	0.14
$-9.628 \cdot 10^{-7}$	0.15

APPENDIX E

KEYPOINTS CALCULATIONS

CALCULATE KEYPOINTS: MODEL WITH NO GLUE LAYER

Define input data.

$$L := 1 \cdot \text{in}$$

$$h := 0.21 \cdot \text{in}$$

$$L = 0.025 \text{ m}$$

$$h = 5.334 \cdot 10^{-3} \text{ m}$$

Length and height of actuator
(quarter model).

$$t := 28.0 \cdot 10^{-6} \cdot \text{m}$$

$$t = 2.8 \cdot 10^{-5} \text{ m}$$

Thickness of a single PVDF film layer.

$$L_{\text{pad}} := 0 \cdot \text{m}$$

$$t_{\text{pad}} := 0 \cdot \text{m}$$

Length and height of the epoxy pad
at the left end.

Compute R and ϕ using solve block:

Initial guess values for R and ϕ :

$$R := 5 \cdot \text{in}$$

$$\phi := 10 \cdot \text{deg}$$

Given

$$R - \frac{h}{2} = R \cdot \cos(\phi)$$

$$\sin(\phi) = \frac{L}{2 \cdot R}$$

$$\begin{bmatrix} R \\ \phi \end{bmatrix} := \text{Find}(R, \phi)$$

$$R = 0.0315716 \text{ m}$$

$$\phi = 23.7195582 \cdot \text{deg}$$

Get x and y values for keypoints.

$$\begin{aligned}
 x_1 &:= 0 \cdot m & y_1 &:= 0 \cdot m \\
 x_2 &:= \frac{L}{2} & y_2 &:= \frac{h}{2} \\
 x_3 &:= L & y_3 &:= h \\
 x_4 &:= x_1 & y_4 &:= y_1 + t \\
 x_5 &:= x_2 - t \cdot \sin(\phi) & y_5 &:= y_2 + t \cdot \cos(\phi) \\
 x_6 &:= x_3 & y_6 &:= y_3 + t \\
 x_7 &:= x_1 & y_7 &:= y_1 + 2 \cdot t \\
 x_8 &:= x_2 - (2 \cdot t) \cdot \sin(\phi) & y_8 &:= y_2 + (2 \cdot t) \cdot \cos(\phi) \\
 x_9 &:= x_3 & y_9 &:= y_3 + 2 \cdot t
 \end{aligned}$$

Offset points to account for the flat section:

$$i := 1..9$$

$$x_i := x_i + L_{\text{pad}} \quad y_i := y_i + t_{\text{pad}}$$

Append the data set for the flat section and the glue pad.

$$\begin{aligned}
 x_{10} &:= 0 \cdot m & y_{10} &:= t_{\text{pad}} \\
 x_{11} &:= 0 \cdot m & y_{11} &:= t + t_{\text{pad}} \\
 x_{12} &:= 0 \cdot m & y_{12} &:= 2 \cdot t + t_{\text{pad}} \\
 x_{13} &:= 0 \cdot m & y_{13} &:= 0 \cdot m \\
 x_{14} &:= L_{\text{pad}} & y_{14} &:= 0 \cdot m
 \end{aligned}$$

Get radii.

$r_1 := R$ Outside radius of the left hand curve.

$r_2 := R - t$

$r_3 := R - 2 \cdot t$ Inside radius of the left hand curve.

$r_4 := R$ Inside radius of the right hand curve.

$r_5 := R + t$

$r_6 := R + 2 \cdot t$ Outside radius of the right hand curve.

Display results:

Note that in calculating the radii this way, the inside radius of the left curve doesn't have the same radius as the inside of the right hand curve. The coordinates of the keypoints are not determined by this radius calculation, so it doesn't affect the finite element model except for the calculation of the center points of the two curves (used for developing cylindrical coordinate systems). This is done because if the actuators are made in a mold, both curves of the mold with the same radii, then these radii are correct for that case.

	1
1	0
2	0.0127
3	0.0254
4	0
5	0.01269
6	0.0254
7	0
8	0.01268
9	0.0254
10	0
11	0
12	0
13	0
14	0

x = m

	1
1	0
2	0.00267
3	0.00533
4	0.00003
5	0.00269
6	0.00536
7	0.00006
8	0.00272
9	0.00539
10	0
11	0.00003
12	0.00006
13	0
14	0

y = °m

$$r = \begin{bmatrix} 0.03157 \\ 0.03154 \\ 0.03152 \\ 0.03157 \\ 0.0316 \\ 0.03163 \end{bmatrix} \text{ m}$$

Now find the center of the arc for both curves. The origin for the global coordinate system is at 0,0
The centers of the arcs will be used to generate a cylindrical coordinate system for the curves in ANSYS to mesh the areas defined by the keypoints calculated above.

Below is the calculation of the center for the first curve (left hand) (valley).

$h1 := 0.1 \cdot m$ $k1 := 0.1 \cdot m$ $R1 := r_3$ Inside radius of the left hand curve.

Given

$$(x_6 - h1)^2 + (y_6 - k1)^2 = R1^2$$

$$(x_7 - h1)^2 + (y_7 - k1)^2 = R1^2$$

$$\begin{bmatrix} h1 \\ k1 \end{bmatrix} := \text{Find}(h1, k1)$$

$$h1 = 0.00683 \cdot m$$

$$k1 = 0.0308233 \cdot m$$

Now repeat the above for the next curve (right hand side) (hill).

$h2 := 0.1 \cdot m$ $k2 := -0.1 \cdot m$ $R2 := r_4$ Inside radius of the right hand curve.

Given

$$(x_1 - h2)^2 + (y_1 - k2)^2 = R2^2$$

$$(x_2 - h2)^2 + (y_2 - k2)^2 = R2^2$$

$$\begin{bmatrix} h2 \\ k2 \end{bmatrix} := \text{Find}(h2, k2)$$

$$h2 = 0.0127 \cdot m$$

$$k2 = -0.029 \cdot m$$

Now add two more keypoints to the list, with the coordinates (h1,k1) and (h2,k2).

$$x_{15} := h1 \quad y_{15} := k1$$

$$x_{16} := h2 \quad y_{16} := k2$$

`M := augment(x, y)` Combine the coordinates into one matrix in order to write to a file.

	1	2
1	0	0
2	0.0127	0.002667
3	0.0254	0.005334
4	0	0.000028
5	0.012689	0.002693
6	0.0254	0.005362
7	0	0.000056
M = 8	0.012677	0.002718
9	0.0254	0.00539
10	0	0
11	0	0.000028
12	0	0.000056
13	0	0
14	0	0
15	0.006827	0.030823
16	0.0127	-0.028905

Note that if there is no glue pad at the left end (with which to attach another bimorph) then all the keypoints from 10 through 14 can be disregarded.

Write the x-y coordinates to a file that can be easily cut and pasted into the macro:

The units of the numbers in this file are meters.

`WRITEPRN("c:\temp\x_y_coordkeypt_noglue.prn") :=  $\frac{M}{m}$`

The file listed above contains all the keypoints required to generate a model in ANSYS of an actuator with no epoxy layer, two layers of piezoelectric film, and a left hand glue pad to attach another bimorph to it.

CALCULATE KEYPOINTS: WITH GLUE LAYER

Define input data.

$L := 1 \cdot \text{in}$	$h := 0.21 \cdot \text{in}$	
$L = 0.0254 \text{ m}$	$h = 0.005334 \text{ m}$	Length and height of actuator (quarter model).
$t_{\text{un}} := 45 \cdot 10^{-6} \cdot \text{m}$	$t_{\text{un}} = 0.000045 \text{ m}$	Thickness of a single layer of unelectroded PVDF film.
$t := 45.0 \cdot 10^{-6} \cdot \text{m}$	$t = 0.000045 \text{ m}$	Thickness of a single layer of electroded PVDF film (silver ink electrodes).
$t_g := 25 \cdot 10^{-6} \cdot \text{m}$	$t_g = 0.000025 \text{ m}$	Thickness of an epoxy layer.
$t_e := \frac{t - t_{\text{un}}}{2}$	$t_e = 0 \text{ m}$	Thickness of each electrode layer.
$L_{\text{pad}} := 0 \cdot \text{m}$	$t_{\text{pad}} := 0 \cdot \text{m}$	Length and thickness of the epoxy pad at the left end.

Compute R and ϕ using solve block:

Initial guess values for R and ϕ :

$$R := 5 \cdot \text{in}$$

$$\phi := 10 \cdot \text{deg}$$

Given

$$R - \frac{h}{2} = R \cdot \cos(\phi)$$

$$\sin(\phi) = \frac{L}{2 \cdot R}$$

$$\begin{bmatrix} R \\ \phi \end{bmatrix} := \text{Find}(R, \phi)$$

$$R = 0.0316 \text{ m}$$

$$\phi = 23.7195582 \cdot \text{deg}$$

Get x and y values for keypoints.

$$\begin{aligned}
 x_1 &:= 0 \cdot m & y_1 &:= 0 \cdot m \\
 x_2 &:= \frac{L}{2} & y_2 &:= \frac{h}{2} \\
 x_3 &:= L & y_3 &:= h \\
 x_4 &:= x_1 & y_4 &:= y_1 + t \\
 x_5 &:= x_2 - t \cdot \sin(\phi) & y_5 &:= y_2 + t \cdot \cos(\phi) \\
 x_6 &:= x_3 & y_6 &:= y_3 + t \\
 x_7 &:= x_1 & y_7 &:= y_1 + t + t_g \\
 x_8 &:= x_2 - (t + t_g) \cdot \sin(\phi) & y_8 &:= y_2 + (t + t_g) \cdot \cos(\phi) \\
 x_9 &:= x_3 & y_9 &:= y_3 + t + t_g \\
 x_{10} &:= x_1 & y_{10} &:= y_1 + 2 \cdot t + t_g \\
 x_{11} &:= x_2 - (2 \cdot t + t_g) \cdot \sin(\phi) & y_{11} &:= y_2 + (2 \cdot t + t_g) \cdot \cos(\phi) \\
 x_{12} &:= x_3 & y_{12} &:= y_3 + 2 \cdot t + t_g
 \end{aligned}$$

Keypoints that can be used to create the electrode layers inside the thickness of the electroded film that was defined above:

$$\begin{aligned}
 x_{13} &:= x_1 & y_{13} &:= y_1 + t_e \\
 x_{14} &:= x_2 - t_e \cdot \sin(\phi) & y_{14} &:= y_2 + t_e \cdot \cos(\phi) \\
 x_{15} &:= x_3 & y_{15} &:= y_3 + t_e \\
 x_{16} &:= x_1 & y_{16} &:= y_1 + 2 \cdot t + t_g - t_e \\
 x_{17} &:= x_2 - (2 \cdot t + t_g - t_e) \cdot \sin(\phi) & y_{17} &:= y_2 + (2 \cdot t + t_g - t_e) \cdot \cos(\phi) \\
 x_{18} &:= x_3 & y_{18} &:= y_3 + 2 \cdot t + t_g - t_e
 \end{aligned}$$

Offset points to account for the flat Lpad section:

$$i := 1 \dots 18$$

$$x_i := x_i + L_{\text{pad}}$$

$$y_i := y_i + t_{\text{pad}}$$

NOTE: Now the total length of the actuator is longer than L if Lpad is greater than zero:

$$L_{\text{tot}} := L + L_{\text{pad}}$$

$$t_{\text{pad}} = 0 \text{ m}$$

$$L_{\text{tot}} = 1 \text{ m}$$

Append the data set for the flat section and the glue pad.

$$x_{19} := 0 \cdot m$$

$$y_{19} := t_{\text{pad}}$$

$$x_{20} := 0 \cdot m$$

$$y_{20} := t_{\text{pad}} + t$$

$$x_{21} := 0 \cdot m$$

$$y_{21} := t_{\text{pad}} + t_g + t$$

$$x_{22} := 0 \cdot m$$

$$y_{22} := t_{\text{pad}} + t_g + 2 \cdot t$$

$$x_{23} := 0 \cdot m$$

$$y_{23} := 0 \cdot m$$

$$x_{24} := L_{\text{pad}}$$

$$y_{24} := 0 \cdot m$$

Get radii.

$$r_1 := R$$

Outside radius of the left curve.

$$r_2 := R - t$$

$$r_3 := R - t - t_g$$

$$r_4 := R - 2 \cdot t - t_g$$

Inside radius of the left curve.

$$r_5 := R$$

Inside radius of the right curve.

$$r_6 := R + t$$

$$r_7 := R + t + t_g$$

$$r_8 := R + 2 \cdot t + t_g \quad \text{Outside radius of the right curve.}$$

Note that in calculating the radii this way, the inside radius of the left curve doesn't have the same radius as the inside of the right hand curve. The coordinates of the keypoints are not determined by this radius calculation, so it doesn't affect the finite element model except for the calculation of the center points of the two curves (used for developing cylindrical coordinate systems). The reason the radii are calculated this way is because it is assumed that the mold would be made with the two radii the same, and that they wouldn't be corrected for the small difference that the thickness of the layers makes.

Note that these radii do not include the radii of the electrode layers (which are internal to the layers of PVDF total film thickness).

Display results:

	1		1			
1	0		1	0	$r = \begin{bmatrix} 0.03157 \\ 0.03153 \\ 0.0315 \\ 0.03146 \\ 0.03157 \\ 0.03162 \\ 0.03164 \\ 0.03169 \end{bmatrix} m$	
2	0.0127		2	0.002667		
3	0.0254		3	0.005334		
4	0		4	0.000045		
5	0.0126819		5	0.0027082		
6	0.0254		6	0.005379		
7	0		7	0.00007		
x = 8	0.0126718	m	y = 8	0.0027311		m
9	0.0254		9	0.005404		
10	0		10	0.000115		
11	0.0126537		11	0.0027723		
12	0.0254		12	0.005449		
13	0		13	0		
14	0.0127		14	0.002667		
15	0.0254		15	0.005334		
16	0		16	0.000115		

Note that if Lpad and tpad are both zero (no glue pad at the end), then the only keypoints required are the first ones up until another keypoint at the origin is encountered going down the list. For example, everything after and including the second keypoint at (0,0) should be ignored. The last two keypoints in the list, however, after the end of this sheet, will be the centers of the curves, which are needed. See the end of this sheet for the name of the file that includes all the coordinates of the keypoints.

Now find the center of the arc for both curves. The origin for the coordinate system is at 0,0.

The centers of the arcs will be used to generate a cylindrical coordinate system for the curves in ANSYS to mesh the areas defined by the keypoints calculated above.

Below is the calculation of the center for the first curve (left hand) (valley).

$$h1 := 0.1 \cdot m \quad k1 := 0.1 \cdot m \quad R1 := r_4 \quad \text{Inside radius of the left curve.}$$

Given

$$(x_{10} - h1)^2 + (y_{10} - k1)^2 = R1^2$$

$$(x_{11} - h1)^2 + (y_{11} - k1)^2 = R1^2$$

$$\begin{bmatrix} h1 \\ k1 \end{bmatrix} := \text{Find}(h1, k1)$$

$$h1 = -1.19133 \cdot 10^{-10} \cdot m$$

$$k1 = 0.0315716 \cdot m$$

Now repeat the above for the next curve (right hand side) (hill).

$$h2 := 0.1 \cdot m \quad k2 := -0.1 \cdot m \quad R2 := r_5 \quad \text{Inside radius of the right curve.}$$

Given

$$(x_2 - h2)^2 + (y_2 - k2)^2 = R2^2$$

$$(x_3 - h2)^2 + (y_3 - k2)^2 = R2^2$$

$$\begin{bmatrix} h2 \\ k2 \end{bmatrix} := \text{Find}(h2, k2)$$

$$h2 = 0.0254 \cdot m$$

$$k2 = -0.0262376 \cdot m$$

Now add two more keypoints to the list, with the coordinates (h1,k1) and (h2,k2).

$$x_{25} := h1 \quad y_{25} := k1$$

$$x_{26} := h2 \quad y_{26} := k2$$

$$M := \frac{\text{augment}(x, y)}{m}$$

Combine the coordinates into one matrix in order to write to a file.

	1	2
1	0	0
2	0.0127	0.002667
3	0.0254	0.005334
4	0	0.000045
5	0.0126819	0.0027082
6	0.0254	0.005379
7	0	0.00007
8	0.0126718	0.0027311
9	0.0254	0.005404
10	0	0.000115
11	0.0126537	0.0027723
12	0.0254	0.005449
13	0	0
14	0.0127	0.002667
15	0.0254	0.005334
16	0	0.000115
17	0.0126537	0.0027723
18	0.0254	0.005449
19	0	0
20	0	0.000045
21	0	0.00007
22	0	0.000115
23	0	0
24	0	0
25	-1.1913253·10 ⁻¹⁰	0.0315716
26	0.0254	-0.0262376

M = $m^{-1} \cdot m$

Write the x-y coordinates to a file that can be easily cut and pasted into the macro:

The numbers in this file have the units of meters.

WRITEPRN("c:\temp\x_y_coordkeypt_w_glue.prn") := M

The file listed above contains all the keypoints required to generate a model in ANSYS of an actuator with an epoxy layer, two layers of piezoelectric film, electrode layers, and a left hand glue pad to attach another bimorph to it.

Calculate Keypoints: Non-Uniform Glue Layer

Define input data.

$$L := 1 \cdot \text{in} \quad L = 0.0254 \text{ m}$$

Length and height of actuator
(quarter model).

$$h := 0.21 \cdot \text{in} \quad h = 0.005334 \text{ m}$$

$$t_{\text{un}} := 30 \cdot 10^{-6} \cdot \text{m}$$

$$t_{\text{g1}} := 20.14 \cdot 10^{-6} \cdot \text{m}$$

Random variation of the epoxy
layer at three points along the
length.

$$t := 45 \cdot 10^{-6} \cdot \text{m}$$

$$t_{\text{g2}} := 33.83 \cdot 10^{-6} \cdot \text{m}$$

$$t_{\text{e}} := \frac{t - t_{\text{un}}}{2}$$

$$t_{\text{e}} = 0.0000075 \text{ m}$$

$$t_{\text{g3}} := 28.48 \cdot 10^{-6} \cdot \text{m}$$

$$L_{\text{pad}} := 0 \cdot \text{m}$$

$$t_{\text{pad}} := 0 \cdot \text{m}$$

Compute R and ϕ using solve block:

Initial guess values for R and ϕ :

$$R := 5 \cdot \text{in}$$

$$\phi := 10 \cdot \text{deg}$$

Given

$$R - \frac{h}{2} = R \cdot \cos(\phi)$$

$$\sin(\phi) = \frac{L}{2 \cdot R}$$

$$\begin{bmatrix} R \\ \phi \end{bmatrix} := \text{Find}(R, \phi)$$

$$R = 0.03157 \text{ m}$$

$$\phi = 23.7195582 \cdot \text{deg}$$

Get x and y values for the keypoints.

$$x_1 := 0 \cdot m$$

$$y_1 := 0 \cdot m$$

$$x_2 := \frac{L}{2}$$

$$y_2 := \frac{h}{2}$$

$$x_3 := L$$

$$y_3 := h$$

$$x_4 := x_1$$

$$y_4 := y_1 + t$$

$$x_5 := x_2 - t \cdot \sin(\phi)$$

$$y_5 := y_2 + t \cdot \cos(\phi)$$

$$x_6 := x_3$$

$$y_6 := y_3 + t$$

$$x_7 := x_1$$

$$y_7 := y_1 + t + t_{g1}$$

$$x_8 := x_2 - (t + t_{g2}) \cdot \sin(\phi)$$

$$y_8 := y_2 + (t + t_{g2}) \cdot \cos(\phi)$$

$$x_9 := x_3$$

$$y_9 := y_3 + t + t_{g3}$$

$$x_{10} := x_1$$

$$y_{10} := y_1 + 2 \cdot t + t_{g1}$$

$$x_{11} := x_2 - (2 \cdot t + t_{g2}) \cdot \sin(\phi)$$

$$y_{11} := y_2 + (2 \cdot t + t_{g2}) \cdot \cos(\phi)$$

$$x_{12} := x_3$$

$$y_{12} := y_3 + 2 \cdot t + t_{g3}$$

$$x_{13} := x_1$$

$$y_{13} := y_1 + t_e$$

$$x_{14} := x_2 - t_e \cdot \sin(\phi)$$

$$y_{14} := y_2 + t_e \cdot \cos(\phi)$$

$$x_{15} := x_3$$

$$y_{15} := y_3 - t_e$$

$$x_{16} := x_1$$

$$y_{16} := y_1 + 2 \cdot t + t_{g1} - t_e$$

$$x_{17} := x_2 - (2 \cdot t + t_{g2} - t_e) \cdot \sin(\phi)$$

$$y_{17} := y_2 + (2 \cdot t + t_{g2} - t_e) \cdot \cos(\phi)$$

$$x_{18} := x_3$$

$$y_{18} := y_3 + 2 \cdot t + t_{g3} - t_e$$

Offset points to account for the flat Lpad section:

$$i := 1..18$$

$$x_i := x_i + L_{\text{pad}}$$

$$y_i := y_i + t_{\text{pad}}$$

NOTE: Now the total length of the actuator is longer than L if Lpad is greater than zero:

$$L_{\text{tot}} := L + L_{\text{pad}}$$

$$t_{\text{pad}} = 0 \text{ m}$$

$$L_{\text{tot}} = 1 \text{ in}$$

Append the data set for the flat section and the glue pad.

$$x_{19} := 0 \cdot \text{m}$$

$$y_{19} := t_{\text{pad}}$$

$$x_{20} := 0 \cdot \text{m}$$

$$y_{20} := t_{\text{pad}} + t$$

$$x_{21} := 0 \cdot \text{m}$$

$$y_{21} := t_{\text{pad}} + t_{g1} + t$$

$$x_{22} := 0 \cdot \text{m}$$

$$y_{22} := t_{\text{pad}} + t_{g1} + 2 \cdot t$$

$$x_{23} := 0 \cdot \text{m}$$

$$y_{23} := 0 \cdot \text{m}$$

$$x_{24} := L_{\text{pad}}$$

$$y_{24} := 0 \cdot \text{m}$$

Get radii.

$$r_1 := R$$

Outside radius of the left curve.

$$r_2 := R - t$$

$$r_3 := R - t - \left(\frac{t_{g1} + t_{g2}}{2} \right)$$

$$r_4 := R - 2 \cdot t - \frac{t_{g1} + t_{g2}}{2}$$

Inside radius of the left curve.

$$r_5 := R$$

Inside radius of the right curve.

$$r_6 := R + t$$

$$r_7 := R + t + \frac{t_{g2} + t_{g3}}{2}$$

$$r_8 := R + 2 \cdot t + \frac{t_{g2} + t_{g3}}{2} \quad \text{Outside radius of the right curve.}$$

Note that in calculating the radii this way, the inside radius of the left curve doesn't have the same

radius as the inside of the right hand curve. The coordinates of the keypoints are not determined by this radius calculation, so it doesn't affect the finite element model except for the calculation of the center points of the two curves (used for developing cylindrical coordinate systems). The reason the radii are calculated this way is because it is assumed that the mold would be made with the two radii the same, and that they wouldn't be corrected for the small difference that the thickness of the layers makes.

Note that these radii do not include the radii of the electrode layers (which are internal to the layers of PVDF total film thickness).

Note that the radii are using the average glue layer thickness along each of the radii or curvature

Display results:

	1
1	0
2	0.0127
3	0.0254
4	0
5	0.0126819
6	0.0254
7	0
8	0.0126683
9	0.0254
10	0
11	0.0126502
12	0.0254
13	0
14	0.012697
15	0.0254
16	0
17	0.0126532
18	0.0254
19	0
20	0
21	0
22	0
23	0
24	0

x = m

	1
1	0
2	0.002667
3	0.005334
4	0.000045
5	0.0027082
6	0.005379
7	0.0000651
8	0.0027392
9	0.0054075
10	0.0001101
11	0.0027804
12	0.0054525
13	0.0000075
14	0.0026739
15	0.0053265
16	0.0001026
17	0.0027735
18	0.005445
19	0
20	0.000045
21	0.0000651
22	0.0001101
23	0
24	0

y = m

$$r = \begin{bmatrix} 0.03157 \\ 0.03153 \\ 0.0315 \\ 0.03145 \\ 0.03157 \\ 0.03162 \\ 0.03165 \\ 0.03169 \end{bmatrix} \text{ m}$$

Note that if Lpad and tpad are both zero (no glue pad at the end), then the only keypoints required are the first ones up until another keypoint at the origin is encountered going down the list. For example, everything after and including the second keypoint at (0,0) should be ignored. The last two keypoints in the list, however, after the end of this sheet, will be the centers of the curves, which are needed. See the end of this sheet for the name of the file that includes all the coordinates of the keypoints.

Now find the center of the arc for both curves. The origin for the coordinate system is at 0,0.

The centers of the arcs will be used to generate a cylindrical coordinate system for the curves in ANSYS to mesh the areas defined by the keypoints calculated above.

Below is the calculation of the center for the first curve (left hand) (valley).

$$h1 := 0.1 \cdot m \quad k1 := 0.1 \cdot m \quad R1 := r_4 \quad \text{Inside radius of the left curve.}$$

Given

$$(x_{10} - h1)^2 + (y_{10} - k1)^2 = R1^2$$

$$(x_{11} - h1)^2 + (y_{11} - k1)^2 = R1^2$$

$$\begin{bmatrix} h1 \\ k1 \end{bmatrix} := \text{Find}(h1, k1)$$

$$h1 = -0.00003 \cdot m$$

$$k1 = 0.0315647 \cdot m$$

Now repeat the above for the next curve (right hand side) (hill).

$$h2 := 0.1 \cdot m \quad k2 := -0.1 \cdot m \quad R2 := r_5 \quad \text{Inside radius of the right curve.}$$

Given

$$(x_2 - h2)^2 + (y_2 - k2)^2 = R2^2$$

$$(x_3 - h2)^2 + (y_3 - k2)^2 = R2^2$$

$$\begin{bmatrix} h2 \\ k2 \end{bmatrix} := \text{Find}(h2, k2)$$

$$h2 = 0.0254 \cdot m$$

$$k2 = -0.0262376 \cdot m$$

Now add two more keypoints to the list, with the coordinates (h1,k1) and (h2,k2).

$$x_{25} := h1 \qquad y_{25} := k1$$

$$x_{26} := h2 \qquad y_{26} := k2$$

M := augment(x,y)

WRITEPRN("c:\temp\x_y_coordkeypt_nonunif_glue.prn") := $\frac{M}{m}$

APPENDIX F

MATERIAL PROPERTY MATRIX MANIPULATION

Matrix Multiplication for Ansys Format

$$S_E := \begin{bmatrix} 3.65 \cdot 10^{-10} & -1.1 \cdot 10^{-10} & -2.09 \cdot 10^{-10} & 0 \\ -1.1 \cdot 10^{-10} & 4.24 \cdot 10^{-10} & -1.92 \cdot 10^{-10} & 0 \\ -2.09 \cdot 10^{-10} & -1.92 \cdot 10^{-10} & 4.72 \cdot 10^{-10} & 0 \\ 0 & 0 & 0 & 96.3 \cdot 10^{-10} \end{bmatrix}$$

Compliance matrix components from paper by H. Wang.

$$d_T := \begin{bmatrix} 0 & 0 & 0 & -2.7 \cdot 10^{-11} \\ 2.1 \cdot 10^{-11} & 2.3 \cdot 10^{-12} & -2.6 \cdot 10^{-11} & 0 \end{bmatrix}$$

Piezoelectric matrix components from paper by H. Wang.

$$\epsilon_0 := 8.854187817 \cdot 10^{-12} \frac{\text{farad}}{\text{m}}$$

Permittivity of free space.

$$\epsilon_0 := 8.854187817 \cdot 10^{-12}$$

$$\epsilon_T := \begin{bmatrix} 6.9 \cdot \epsilon_0 & 0 \\ 0 & 7.6 \cdot \epsilon_0 \end{bmatrix} \quad \epsilon_T = \begin{bmatrix} 6.109 \cdot 10^{-11} & 0 \\ 0 & 6.729 \cdot 10^{-11} \end{bmatrix}$$

Permittivity of PVDF (relative permittivities taken from paper by H.Wang).

$$I_1 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix to switch the order of the matrix components for the Ansys input.

Stress equation: $T = C_E \cdot S - e_S \cdot E$

The stiffness matrix

$$C_E := S_E^{-1}$$

$$C_E = \begin{bmatrix} 6.154 \cdot 10^9 & 3.469 \cdot 10^9 & 4.136 \cdot 10^9 & 0 \\ 3.469 \cdot 10^9 & 4.847 \cdot 10^9 & 3.508 \cdot 10^9 & 0 \\ 4.136 \cdot 10^9 & 3.508 \cdot 10^9 & 5.377 \cdot 10^9 & 0 \\ 0 & 0 & 0 & 1.038 \cdot 10^8 \end{bmatrix}$$

The piezoelectric matrix

$$e_S := S_E^{-1} \cdot d_T^T$$

$$e_S = \begin{bmatrix} 0 & 0.03 \\ 0 & -7.2 \cdot 10^{-3} \\ 0 & -0.045 \\ -2.804 \cdot 10^{-3} & 0 \end{bmatrix}$$

Electric displacement equation: $D = e_S \cdot S + \epsilon_S \cdot E$

Now converting into the order required to input into ANSYS (so that the 3 material direction will be considered the y-direction as the model is set up in ANSYS).

Transformed stress equation: $T_a = I_1 \cdot C_E \cdot I_1 \cdot S_a - I_1 \cdot e_S \cdot E$

The stiffness matrix

$$C_A := I_1 \cdot C_E \cdot I_1$$

$$C_A = \begin{bmatrix} 6.154 \cdot 10^9 & 4.136 \cdot 10^9 & 3.469 \cdot 10^9 & 0 \\ 4.136 \cdot 10^9 & 5.377 \cdot 10^9 & 3.508 \cdot 10^9 & 0 \\ 3.469 \cdot 10^9 & 3.508 \cdot 10^9 & 4.847 \cdot 10^9 & 0 \\ 0 & 0 & 0 & 1.038 \cdot 10^8 \end{bmatrix}$$

The piezoelectric matrix

$$e_A := I_1 \cdot e_S$$

$$e_A = \begin{bmatrix} 0 & 0.03 \\ 0 & -0.045 \\ 0 & -7.2 \cdot 10^{-3} \\ -2.804 \cdot 10^{-3} & 0 \end{bmatrix}$$

Transformed electric displacement equation:

$$\mathbf{D} = \mathbf{e}_S \cdot \mathbf{I}_1 \cdot \mathbf{S}_a + \boldsymbol{\varepsilon}_S \cdot \mathbf{E} = \mathbf{e}_A \cdot \mathbf{S}_a + \boldsymbol{\varepsilon}_S \cdot \mathbf{E}_a$$

The dielectric matrix

$$\boldsymbol{\varepsilon}_S := \boldsymbol{\varepsilon}_T - \mathbf{d}_T \cdot \mathbf{C}_E \cdot \mathbf{d}_T^T$$

$$\boldsymbol{\varepsilon}_S = \begin{bmatrix} 6.102 \cdot 10^{-11} & 0 \\ 0 & 6.552 \cdot 10^{-11} \end{bmatrix}$$

Relative permittivities are the permittivities divided by the permittivity of free space. Ansys uses the relative permittivity.

$$\boldsymbol{\varepsilon}_S := \frac{\boldsymbol{\varepsilon}_S}{\boldsymbol{\varepsilon}_0}$$

$$\boldsymbol{\varepsilon}_S = \begin{bmatrix} 6.891 & 0 \\ 0 & 7.4 \end{bmatrix}$$

In the Ansys property table PIEZ, the transposed matrix must be input. Use this as a guide:

$$\mathbf{d} := \begin{bmatrix} 0 & 0 & 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 24 & 0 & 0 \\ 31 & 32 & 33 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{d}^T = \begin{bmatrix} 0 & 0 & 31 \\ 0 & 0 & 32 \\ 0 & 0 & 33 \\ 0 & 24 & 0 \\ 15 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix below shows where the constants go in which location in the table. Remember that the second column is missing, since it is a 2-D problem, so the constants that are input are actually only the first and third columns. The third column in the matrix below must be input into the second column in the Ansys property table.

Use the transformed matrix below as input into the PIEZ data table:

$$\mathbf{e}_A = \begin{bmatrix} 0 & 0.03 \\ 0 & -0.045 \\ 0 & -7.2 \cdot 10^{-3} \\ -2.804 \cdot 10^{-3} & 0 \end{bmatrix}$$

Use the transformed matrix below as input into the ANEL data table:

$$C_A = \begin{bmatrix} 6.154 \cdot 10^9 & 4.136 \cdot 10^9 & 3.469 \cdot 10^9 & 0 \\ 4.136 \cdot 10^9 & 5.377 \cdot 10^9 & 3.508 \cdot 10^9 & 0 \\ 3.469 \cdot 10^9 & 3.508 \cdot 10^9 & 4.847 \cdot 10^9 & 0 \\ 0 & 0 & 0 & 1.038 \cdot 10^8 \end{bmatrix}$$

Use the MP command to input the following relative permittivities:

MP,PERX, ϵ S11

$$\epsilon_S = \begin{bmatrix} 6.891 & 0 \\ 0 & 7.4 \end{bmatrix}$$

MP,PERY, ϵ S22

APPENDIX G

ANSYS MACRO PROGRAMS

```
! Model: No Epoxy Layer, Electrical Field Applied Only
! give the file a name for use in other commands, such as upgeom, which
! needs a restart file
```

```
/FILENAME,noglu-fullelec
```

```
! open the preprocessor in ANSYS to begin modeling
```

```
/PREP7
```

```
! MODEL DEFINITION
```

```
! define keypoints for a curved actuator (full width 2 inches)
! half width model, radius of curvature 31 mm
! without epoxy pad at the left end for attaching to other
! bimorph
! units are in meters (coordinates)
!
! this model has no glue layer
```

```
k,1,0,0
k,2,0.0127, 0.002667
k,3,0.0254, 0.005334
k,4,0, 2.8e-005
k,5,0.01269, 0.002693
k,6,0.0254, 0.005362
k,7,0, 5.6e-005
k,8,0.01268, 0.002718
k,9,0.0254, 0.00539
```

```
! define keypoints that are the arc centers
! valley curve (left hand curve of actuator)
```

```
k,10,0.006827,0.03082
```

```
! hill curve (right hand curve of actuator)
```

```
k,11,0.0127, -0.0289
```

```
! define cylindrical coordinate system with origin at second
! to the last keypt
! use command cskp,a,b,c,d,e
! a = number designator of coordinate system (must start with 11)
! b = 0 if cartesian, 1 if cylindrical, 2 if spherical coord. system
! c = keypoint defining the origin of the coord. system
! d = keypoint defining the positive x direction of the coord. sys.
! e = keypoint defining the x-y plane
```

```
cskp,11,1,10,7,8
```

```
! make the new coordinate system active
```

```
csys,11
```

```

! define lines so can track their number for applying voltages

1,1,2
1,2,5
1,5,4
1,1,4
1,5,8
1,8,7
1,4,7

! define the areas in the new coordinate system
! these are the curves (valley) on the left

a,1,2,5,4
a,4,5,8,7

! make the cartesian coordinate system active

csys,0

! define cylidrical coordinate system with origin at last keypt

cskp,12,1,11,2,3

! make the new system active

csys,12

! define lines so can track their number for applying voltages

1,2,3
1,3,6
1,6,5
1,6,9
1,9,8

! define areas in the new coordinate system
! these are the curves (hill) on the right

a,5,6,9,8
a,2,3,6,5

! ELEMENT DEFINITION

! Define the element type to be used (must be done to make element
! available for meshing)
! Use the element type command:
! ET, ITYPE, ENAME, KOP1, KOP2, KOP3, KOP4, KOP5, KOP6

! KOP# = key options which can be set to various values to define
! specific
! options for elements. These are defined in the ANSYS help manual

```

```

! for each element type
! ITYPE = number designating element type (defaults to current max +1)
! ENAME = name of the element type to be added
! Here, the PLANE13 element will be used: a plane, coupled-field
! element
! Keyop 1 = 3 makes UX and UY degrees of freedom the active ones
! Keyop 1 = 7 makes UX, UY, VOLT the degrees of freedom
! Keyop 2 = 0 includes extra quadratic shape functions (1 excludes
! them)
! Keyop 3 = 2 makes the element plane stress (0 makes plane strain)
! Keyop 4 = 1 element coordinate system based on element I-J side
! Keyop 4 = 0 element coordinate system parallel to global
! Keyop 5 = 0 basic element printout
! Keyop 5 = 1 repeats basic solution for all integration points
! Keyop 5 = 2 nodal stress printout
! Keyop 6 = 0 means that the material data matrix input is the
! stiffness matrix
! Keyop 6 = 1 means that the material data matrix input is the
! compliance matrix

```

```
ET,,PLANE13,7,0,2,1,,0
```

```

! Define the global element size to be used in the mesh (length of
! element side in meters)

```

```
ESIZE,0.000056
```

```
! MATERIAL PROPERTY INPUT
```

```

! Define material properties (in SI units)
! Use the linear material property command: MP,LABEL,MAT#,C
! LABEL = name of the property to be input, a list given in Commands
! Manual in ANSYS help
! MAT# = number of the material table (since can have multiple \
! materials defined)
! C = value of the constant
! One can also input temperature dependent properties using further
! constants
! see the Commands Manual for this command in the ANSYS help for more
! information

```

```
MP,PERX,1,6.9
```

```
MP,PERY,1,7.6
```

```
! For input of the stiffness matrix, use a data table command, TB
```

```
TB,ANEL,1
```

```
TBDATA,1,6.154E9,4.136E9,3.469E9,0,0,0
```

```
TBDATA,7,5.377E9,3.508E9,0,0,0,4.87E9
```

```
TBDATA,13,0,0,0,0.1038E9,0,0
```

```
TBDATA,19,0,0,0
```

```
TB,PIEZ,1
```

```
TBDATA,1,0,0.03,0,0,-0.045,0
```

```
TBDATA,7,0,-7.2E-3,0,-2.804E-3,0
```

TBDATA,13,0,0,0,0,0,0

! MESHING THE MODEL

! Define whether mapped or free mesh will be used
! (0=free, 1=mapped)

MSHKEY,1

! Define the element shape to be used if the element has the option
! (quadrilaterals = 0, triangular = 1) and whether 2D or 3D

MSHAPE,0,2D

! Mesh the areas from 1 through 4

! first activate the material number you want the elements to have

MAT,1

AMESH,1,2

AMESH,3,3

AMESH,4,4

! open the solution processor

/SOLU

! set the analysis type to a new static analysis
! (this is the default)

ANTYPE,STATIC,NEW

NSEL,ALL

csys,0

! select nodes to apply constraints
! first select nodes with x coordinate equal to zero

dx=0.0000001

dy=0.0000001

NSEL,S,LOC,X,0-dx,0+dx

! constrain the x displacement to be zero for these nodes

D,ALL,UY,0

D,ALL,UX,0

! now create a coupled set of nodes, so that they can be
! constrained for zero slope at the far right end of the beam

XMAX=0.0254

YMAX=0.0539

```
NSEL,S,LOC,X,XMAX-dx,dx+XMAX
```

```
CP,NEXT,UX,ALL
```

```
! reselect all nodes so they all become active again
```

```
NSEL,ALL
```

```
! next apply voltage constraints to the lines
```

```
! the DL command applies a degree of freedom constrain to a line
```

```
! DL,a,b,c,d where
```

```
! a = line # at which constraint is to be applied
```

```
! b = area to which the line belongs
```

```
! c = label of the degree of freedom constrained = VOLT
```

```
! d = value of the constraint (volts applied)
```

```
DL,6,2,VOLT,-1000
```

```
DL,1,1,VOLT,-1000
```

```
DL,3,1,VOLT,0
```

```
DL,8,4,VOLT,0
```

```
DL,12,3,VOLT,0
```

```
DL,10,3,VOLT,-1000
```

```
! the above applies -1000 V to the outer surfaces on the left beam
```

```
! and 0 V to the center, and it applies -1000 V to the center of the
```

```
! right beam, with 0 V on the outer surfaces of the right beam
```

```
EPLOT
```

```
SOLVE
```

! Model: No Epoxy Layer, Applied Mechanical Load, Linear Analysis
 ! give the file a name for use in other commands

/FILENAME,noglue-P-lin

! open the preprocessor in ANSYS to begin modeling

/PREP7

! MODEL DEFINITION

! define keypoints for a curved actuator (full width 2 inches)
 ! half width model, radius of curvature 31 mm
 ! without epoxy pad at the left end for attaching to other
 ! bimorph
 ! units are in meters (coordinates)
 !
 ! this model has no glue layer

k,1,0,0
 k,2,0.0127,0.002667
 k,3,0.0254, 0.005334
 k,4,0, 4.5e-005
 k,5,0.01268, 0.002708
 k,6,0.0254, 0.005379
 k,7,0, 9e-005
 k,8,0.01266, 0.002749
 k,9,0.0254, 0.005424

! define keypoints that are the arc centers
 ! valley curve (left hand curve of actuator)

k,10,0.006852,0.03082

! hill curve (right hand curve of actuator)

k,11,0.0127, -0.0289

! define cylindrical coordinate system with origin at second
 ! to the last keypt
 ! use command cskp,a,b,c,d,e
 ! a = number designator of coordinate system (must start with 11)
 ! b = 0 if cartesian, 1 if cylindrical, 2 if spherical coord. system
 ! c = keypoint defining the origin of the coord. system
 ! d = keypoint defining the positive x direction of the coord. sys.
 ! e = keypoint defining the x-y plane

cskp,11,1,10,7,8

! make the new coordinate system active

csys,11

```

! define lines so can track their number for applying voltages

! define the areas in the new coordinate system
! these are the curves (valley) on the left

a,1,2,5,4
a,4,5,8,7

! make the cartesian coordinate system active

csys,0

! define cylindrical coordinate system with origin at last keypt

cskp,12,1,11,2,3

! make the new system active

csys,12

! define lines so can track their number for applying voltages

! define areas in the new coordinate system
! these are the curves (hill) on the right

a,5,6,9,8
a,2,3,6,5

! ELEMENT DEFINITION

! Define the element type to be used (must be done to make element
! available for meshing)
! Use the element type command:
! ET, ITYPE, ENAME, KOP1, KOP2, KOP3, KOP4, KOP5, KOP6

! KOP# = key options which can be set to various values to define
! specific
! options for elements. These are defined in the ANSYS help manual
! for each element type
! ITYPE = number designating element type (defaults to current max +1)
! ENAME = name of the element type to be added
! Here, the PLANE13 element will be used. It is a plane, coupled-
! field element
! Keyop 1 = 3 makes UX and UY degrees of freedom the active ones
! Keyop 1 = 7 makes UX, UY, VOLT the degrees of freedom
! Keyop 1 = 4 makes UX, UY, TEMP, and AX the degrees of freedom
! Keyop 2 = 0 includes extra quadratic shape functions (1 excludes
! them)
! Keyop 3 = 2 makes the element plane stress (0 makes plane strain)
! Keyop 4 = 1 element coordinate system based on element I-J side
! Keyop 4 = 0 element coordinate system parallel to global
! Keyop 5 = 0 basic element printout
! Keyop 5 = 1 repeats basic solution for all integration points

```

```
! Keyop 5 = 2 nodal stress printout
! Keyop 6 = 0 means that the material data matrix input is the
!   stiffness matrix
! Keyop 6 = 1 means that the material data matrix input is the
!   compliance matrix
```

```
ET,,PLANE13,3,0,2,1,,0
```

```
! Define the global element size to be used in the mesh (length of
!   element side in meters)
```

```
ESIZE,0.000045
```

```
! MATERIAL PROPERTY INPUT
```

```
! Define material properties (in SI units)
! Use the linear material property command: MP,LABEL,MAT#,C
! LABEL = name of the property to be input, a list given in Commands
!   Manual in ANSYS help
! MAT# = number of the material table (since can have multiple
!   materials defined)
! C = value of the constant
! One can also input temperature dependent properties using further
!   constants
! see the Commands Manual for this command in the ANSYS help for more
!   information
```

```
! Considering only isotropic material properties, input Young's
!   Modulus as follows
```

```
MP,EX,1,6.154E9
```

```
! OR could consider the orthotropic properties of PVDF
! For input of the entire stiffness matrix, use a data table command,
!   TB
```

```
!TB,ANEL,1
!TBDATA,1,6.154E9,4.136E9,3.469E9,0,0,0
!TBDATA,7,5.377E9,3.508E9,0,0,0,4.87E9
!TBDATA,13,0,0,0,0.1038E9,0,0
!TBDATA,19,0,0,0
```

```
! MESHING THE MODEL
```

```
! Define whether mapped or free mesh will be used
! (0=free, 1=mapped)
```

```
MSHKEY,1
```

```
! Define the element shape to be used if the element has the option
! (quadrilaterals = 0, triangular = 1) and whether 2D or 3D
```

```
MSHAPE,0,2D
```

```
! Mesh the areas from 1 through 4
```

```

! first activate the material number you want the elements to have
! and the element type
MAT,1
TYPE,1
AMESH,1,2
AMESH,3,3
AMESH,4,4

! open the solution processor

/SOLU

! set the analysis type to a new static analysis
! (this is the default)
ANTYPE,STATIC,NEW

! define the maximum load that will be applied
L_MAX=0.3

! Select all nodes and reset the active coordinate system to cartesian
NSEL,ALL

csys,0

! select nodes to apply constraints
! first select nodes with x coordinate equal to zero

dx=0.0000001
dy=0.0000001

NSEL,S,LOC,X,0-dx,0+dx

! constrain the x displacement to be zero for these nodes

D,ALL,UY,0
D,ALL,UX,0

! now create a coupled set of nodes, so that they can be
! constrained for zero slope at the far right end of the beam

XMAX=0.0254
YMAX=0.005424

NSEL,S,LOC,X,XMAX-dx,dx+XMAX

CP,NEXT,UX,ALL

NSEL,ALL

! apply a point load to the upper most end node

```

F,1753,FY,-L_MAX

! Plot the elements

EPLOT

! Solve the model
SOLVE

! Model: No Epoxy Layer, Applied Mechanical Load, Non-linear Analysis
! give the file a name for use in other commands

/FILENAME,noglue-P-nonlin

! open the preprocessor in ANSYS to begin modeling

/PREP7

! MODEL DEFINITION

! define keypoints for a curved actuator (full width 2 inches)
! half width model, radius of curvature 31 mm
! without epoxy pad at the left end for attaching to other
! bimorph
! units are in meters (coordinates)
!
! this model has no glue layer

k,1,0,0
k,2,0.0127, 0.002667
k,3,0.0254, 0.005334
k,4,0, 4.5e-005
k,5,0.01268, 0.002708
k,6,0.0254, 0.005379
k,7,0, 9.0e-005
k,8,0.01266, 0.002749
k,9,0.0254, 0.005424

! define keypoints that are the arc centers
! valley curve (left hand curve of actuator)

k,10,0.006827,0.03082

! hill curve (right hand curve of actuator)

k,11,0.0127, -0.0289

! define cylindrical coordinate system with origin at second
! to the last keypt
! use command cskp,a,b,c,d,e
! a = number designator of coordinate system (must start with 11)
! b = 0 if cartesian, 1 if cylindrical, 2 if spherical coord. system
! c = keypoint defining the origin of the coord. system
! d = keypoint defining the positive x direction of the coord. sys.
! e = keypoint defining the x-y plane

cskp,11,1,10,7,8

! make the new coordinate system active

csys,11

! define lines so can track their number for applying voltages

```

1,1,2
1,2,5
1,5,4
1,1,4
1,5,8
1,8,7
1,4,7

```

```

! define the areas in the new coordinate system
! these are the curves (valley) on the left

```

```

a,1,2,5,4
a,4,5,8,7

```

```

! make the cartesian coordinate system active

```

```

csys,0

```

```

! define cylidrical coordinate system with origin at last keypt

```

```

cskp,12,1,11,2,3

```

```

! make the new system active

```

```

csys,12

```

```

! define lines so can track their number for applying voltages

```

```

1,2,3
1,3,6
1,6,5
1,6,9
1,9,8

```

```

! define areas in the new coordinate system
! these are the curves (hill) on the right

```

```

a,5,6,9,8
a,2,3,6,5

```

```

! ELEMENT DEFINITION

```

```

! Define the element type to be used (must be done to make element
! available for meshing)

```

```

! Use the element type command:

```

```

! ET, ITYPE, ENAME, KOP1, KOP2, KOP3, KOP4, KOP5, KOP6

```

```

! KOP# = key options which can be set to various values to define
! specific

```

```

! options for elements. These are defined in the ANSYS help manual
! for each element type

```

```

! ITYPE = number designating element type (defaults to current max +1)
! ENAME = name of the element type to be added
! Here, the PLANE13 element will be used. It is a plane, coupled-
!   field element
! Keyop 1 = 3 makes UX and UY degrees of freedom the active ones
! Keyop 1 = 7 makes UX, UY, VOLT the degrees of freedom
! Keyop 1 = 4 makes UX, UY, TEMP, and AX the degrees of freedom
! Keyop 2 = 0 includes extra quadratic shape functions (1 excludes
!   them)
! Keyop 3 = 2 makes the element plane stress (0 makes plane strain)
! Keyop 4 = 1 element coordinate system based on element I-J side
! Keyop 4 = 0 element coordinate system parallel to global
! Keyop 5 = 0 basic element printout
! Keyop 5 = 1 repeats basic solution for all integration points
! Keyop 5 = 2 nodal stress printout
! Keyop 6 = 0 means that the material data matrix input is the
!   stiffness matrix
! Keyop 6 = 1 means that the material data matrix input is the
!   compliance matrix

```

```
ET,,PLANE13,3,0,2,1,,0
```

```

! Define the global element size to be used in the mesh (length of
!   element side in meters)

```

```
ESIZE,0.000045
```

```
! MATERIAL PROPERTY INPUT
```

```

! Define material properties (in SI units)
! Use the linear material property command: MP,LABEL,MAT#,C
! LABEL = name of the property to be input, a list given in Commands
!   Manual in ANSYS help
! MAT# = number of the material table (since can have multiple
!   materials defined)
! C = value of the constant
! One can also input temperature dependent properties using further
!   constants
! see the Commands Manual for this command in the ANSYS help for more
!   information

```

```
MP,EX,1,6.514E9
```

```

! Alternative method, if using orthotropic properties
!   input of the stiffness matrix, use a data table command, TB

```

```

!TB,ANEL,1
!TBDATA,1,6.154E9,4.136E9,3.469E9,0,0,0
!TBDATA,7,5.377E9,3.508E9,0,0,0,4.87E9
!TBDATA,13,0,0,0,0.1038E9,0,0
!TBDATA,19,0,0,0

```

```
! MESHING THE MODEL
```

```
! Define whether mapped or free mesh will be used
! (0=free, 1=mapped)
```

```
MSHKEY,1
```

```
! Define the element shape to be used if the element has the option
! (quadrilaterals = 0, triangular = 1) and whether 2D or 3D
```

```
MSHAPE,0,2D
```

```
! Mesh the areas from 1 through 4
```

```
! first activate the material number you want the elements to have
! and the element type
```

```
MAT,1
```

```
TYPE,1
```

```
AMESH,1,2
```

```
AMESH,3,3
```

```
AMESH,4,4
```

```
! open the solution processor
```

```
/SOLU
```

```
! set the analysis type to a new static analysis
```

```
! (this is the default)
```

```
! define the maximum load that will be applied and what the step size
! is
```

```
L_MAX=0.3
```

```
L_INC=0.03
```

```
ANTYPE,STATIC,NEW
```

```
SOLCONTROL,ON
```

```
AUTOTS,OFF
```

```
NLGEOM,ON
```

```
! set the total time (load) to be the same as the total load applied
```

```
TIME,L_MAX
```

```
DELTIM,L_INC
```

```
NSUBST,L_MAX/L_INC,30,2
```

```
! set load application to ramped mode
```

```
KBC,0
```

```
OUTRES,BASIC,ALL
```

```
NSEL,ALL
```

```
csys,0
```

```
! select nodes to apply constraints
```

```
! first select nodes with x coordinate equal to zero
```

```
dx=0.0000001
```

```
dy=0.0000001
```

```
NSEL,S,LOC,X,0-dx,0+dx
```

! constrain the x displacement to be zero for these nodes

D,ALL,UY,0

D,ALL,UX,0

! now create a coupled set of nodes, so that they can be

! constrained for zero slope at the far right end of the beam

XMAX=0.0254

YMAX=0.05424

NSEL,S,LOC,X,XMAX-dx,dx+XMAX

CP,NEXT,UX,ALL

NSEL,ALL

! apply a point load to the upper most end node

F,1753,FY,-L_MAX

! reselect all nodes so they all become active again

NSEL,ALL

! Plot the elements

EPLOT

SOLVE

/POST26

NSOL,2,1753,U,Y,UY_end

NSOL,3,1753,U,X,UX_end

! plot time (since static case, equivalent to load) as the x-variable
XVAR,1

! plot UY at the right end vs load and UX at the right end vs load

PLVAR,2,3

PRVAR,2,3

```
! give the file a name for use in other commands, such as upgeom, which
! needs a restart file
```

```
/FILENAME,glue-fullelec
```

```
! open the preprocessor in ANSYS to begin modeling
```

```
/PREP7
```

```
! MODEL DEFINITION
```

```
! define keypoints for a curved actuator (full width 2 inches)
! half width model, radius of curvature 31 mm
! without epoxy pad at the left end for attaching to other
! bimorph
! units are in meters (coordinates)
!
! this model has a glue layer and electrode layers
```

```
k,1,0,0
k,2,0.0127,0.002667
k,3,0.0254,0.005334
k,4,0, 4.5e-005
k,5,0.0126819,0.0027082
k,6,0.0254,0.005379
k,7,0,0.00007
k,8,0.0126718,0.0027311
k,9,0.0254,0.005404
k,10,0,0.000115
k,11,0.0126537,0.0027723
k,12,0.0254,0.005449
k,13,0,7.5e-006
k,14,0.0127,0.002667
k,15,0.0254,0.005334
k,16,0,0.000115
k,17,0.0126537,0.0027723
k,18,0.0254,0.005449
```

```
! define keypoints that are the arc centers
! valley curve (left hand curve of actuator)
```

```
k,19,-2.33e-011,0.0315716
```

```
! hill curve (right hand curve of actuator)
```

```
k,20,0.0254,-0.0262376
```

```
! define cylindrical coordinate system with origin at second
! to the last keypt
! use command cskp,a,b,c,d,e
! a = number designator of coordinate system (must start with 11)
! b = 0 if cartesian, 1 if cylindrical, 2 if spherical coord. system
! c = keypoint defining the origin of the coord. system
! d = keypoint defining the positive x direction of the coord. sys.
```

```

! e = keypoint defining the x-y plane

cskp,11,1,19,10,11

! make the new coordinate system active

csys,11

! define the areas in the new coordinate system
! these are the curves (valley) on the left

a,1,2,5,4
a,4,5,8,7
a,7,8,11,10

! make the cartesian coordinate system active

csys,0

! define cylindrical coordinate system with origin at last keypt

cskp,12,1,20,2,3

! make the new system active

csys,12

! define areas in the new coordinate system
! these are the curves (hill) on the right

a,2,3,6,5
a,5,6,9,8
a,8,9,12,11

! ELEMENT DEFINITION

! Define the element type to be used (must be done to make element
! available for meshing)
! Use the element type command:
! ET, ITYPE, ENAME, KOP1, KOP2, KOP3, KOP4, KOP5, KOP6

! KOP# = key options which can be set to various values to define
! specific
! options for elements. These are defined in the ANSYS help manual
! for each element type
! ITYPE = number designating element type (defaults to current max +1)
! ENAME = name of the element type to be added
! Here, the PLANE13 element will be used. It is a plane, coupled-
! field element
! Keyop 1 = 3 makes UX and UY degrees of freedom the active ones
! Keyop 1 = 7 makes UX, UY, VOLT the degrees of freedom
! Keyop 2 = 0 includes extra quadratic shape functions (1 excludes
! them)
! Keyop 3 = 2 makes the element plane stress (0 makes plane strain)

```

```

! Keyop 4 = 1 element coordinate system based on element I-J side
! Keyop 4 = 0 element coordinate system parallel to global
! Keyop 5 = 0 basic element printout
! Keyop 5 = 1 repeats basic solution for all integration points
! Keyop 5 = 2 nodal stress printout
! Keyop 6 = 0 means that the material data matrix input is the
!   stiffness matrix
! Keyop 6 = 1 means that the material data matrix input is the
!   compliance matrix

```

```
ET,1,PLANE13,7,0,2,1,,0
```

```

! define the element type with only UX and UY degrees of freedom for
! the glue layer
! and electrode layers

```

```
ET,2,PLANE13,3,0,2,1,,0
```

```

! Define the global element size to be used in the mesh (length of
! element side in meters)

```

```
ESIZE,0.000056
```

```
! MATERIAL PROPERTY INPUT
```

```

! Define material properties (in SI units)
! Use the linear material property command: MP,LABEL,MAT#,C
! LABEL = name of the property to be input, a list given in Commands
! Manual in ANSYS help
! MAT# = number of the material table (since can have multiple
! materials defined)
! C = value of the constant
! One can also input temperature dependent properties using further
! constants
! see the Commands Manual for this command in the ANSYS help for more
! information

```

```
MP,PERX,1,6.9
```

```
MP,PERY,1,7.6
```

```
! For input of the stiffness matrix, use a data table command, TB
```

```
TB,ANEL,1
```

```
TBDATA,1,6.154E9,4.136E9,3.469E9,0,0,0
```

```
TBDATA,7,5.377E9,3.508E9,0,0,0,4.87E9
```

```
TBDATA,13,0,0,0,0.1038E9,0,0
```

```
TBDATA,19,0,0,0
```

```
TB,PIEZ,1
```

```
TBDATA,1,0,0.03,0,0,-0.045,0
```

```
TBDATA,7,0,-7.2E-3,0,-2.804E-3,0
```

```
TBDATA,13,0,0,0,0,0,0
```

```
! input the material properties of the glue layer
```

MP,EX,2,4.235E9
 MP,GXY,2,1.531E9

! MESHING THE MODEL

! Define whether mapped or free mesh will be used
 ! (0=free, 1=mapped)

MSHKEY,1

! Define the element shape to be used if the element has the option
 ! (quadrilaterals = 0, triangular = 1) and whether 2D or 3D

MSHAPE,0,2D

! Mesh the areas from 1 through 4

! first activate the material number you want the elements to have

! first make the entire thickness of the PVDF layer have the
 ! properties of PVDF, don't separate the electrode layer properties

! set material pointer to material 1 and element type to type 1

MAT,1
 TYPE,1
 AMESH,1,1

MAT,2
 TYPE,2
 AMESH,2,2

MAT,1
 TYPE,1
 AMESH,3,3

MAT,1
 TYPE,1
 AMESH,4,4

MAT,2
 TYPE,2
 AMESH,5,5

MAT,1
 TYPE,1
 AMESH,6,6

! open the solution processor

/SOLU

! set the analysis type to a new static analysis
 ! (this is the default)

```
ANTYPE,STATIC,NEW
```

```
NSEL,ALL
```

```
csys,0
```

```
! select nodes to apply constraints
! first select nodes with x coordinate equal to zero
```

```
dx=0.0000001
```

```
dy=0.0000001
```

```
NSEL,S,LOC,X,0-dx,0+dx
```

```
! constrain the x displacement to be zero for these nodes
```

```
D,ALL,UY,0
```

```
D,ALL,UX,0
```

```
! now create a coupled set of nodes, so that they can be
! constrained for zero slope at the far right end of the beam
```

```
XMAX=0.0254
```

```
YMAX=0.05419
```

```
NSEL,S,LOC,X,XMAX-dx,dx+XMAX
```

```
CP,NEXT,UX,ALL
```

```
NSEL,ALL
```

```
! apply a point load to the upper most end node
```

```
!NSEL,R,LOC,Y,YMAX-dy,dy+YMAX
```

```
! reselect all nodes so they all become active again
```

```
NSEL,ALL
```

```
! next apply voltage constraints to the lines
```

```
! the DL command applies a degree of freedom constrain to a line
```

```
! DL,a,b,c,d where
```

```
! a = line # at which constraint is to be applied
```

```
! b = area to which the line belongs
```

```
! c = label of the degree of freedom constrained = VOLT
```

```
! d = value of the constraint (volts applied)
```

```
DL,1,1,VOLT,1000
```

```
DL,3,1,VOLT,0
```

```
DL,6,3,VOLT,0
```

```
DL,9,3,VOLT,1000
```

```
DL,11,4,VOLT,0
```

DL,13,4,VOLT,1000

DL,15,6,VOLT,1000

DL,17,6,VOLT,0

SOLVE

```
! Model: WITH Epoxy Layer, Applied Mechanical Load, Linear Analysis
! give the file a name for use in other commands, such as upgeom, which
! needs a restart file
```

```
/FILENAME,glue-P-lin
```

```
! open the preprocessor in ANSYS to begin modeling
```

```
/PREP7
```

```
! MODEL DEFINITION
```

```
! define keypoints for a curved actuator (full width 2 inches)
! half width model, radius of curvature 31 mm
! without epoxy pad at the left end for attaching to other
! bimorph
! units are in meters (coordinates)
!
! this model has a glue layer and electrode layers
```

```
k,1,0,0
k,2,0.0127,0.002667
k,3,0.0254,0.005334
k,4,0, 4.5e-005
k,5,0.0126819,0.0027082
k,6,0.0254,0.005379
k,7,0,0.00007
k,8,0.0126718,0.0027311
k,9,0.0254,0.005404
k,10,0,0.000115
k,11,0.0126537,0.0027723
k,12,0.0254,0.005449
k,13,0,7.5e-006
k,14,0.012697,0.0026739
k,15,0.0254,0.0053415
k,16,0,0.0001075
k,17,0.0126568,0.0027654
k,18,0.0254,0.0054415
```

```
! define keypoints that are the arc centers
! valley curve (left hand curve of actuator)
```

```
k,19,-2.33e-011,0.03157
```

```
! hill curve (right hand curve of actuator)
```

```
k,20,0.0254,-0.02624
```

```
! define cylindrical coordinate system with origin at second
! to the last keypt
! use command cskp,a,b,c,d,e
! a = number designator of coordinate system (must start with 11)
! b = 0 if cartesian, 1 if cylindrical, 2 if spherical coord. system
```

```

! c = keypoint defining the origin of the coord. system
! d = keypoint defining the positive x direction of the coord. sys.
! e = keypoint defining the x-y plane

```

```

cskp,11,1,19,10,11

```

```

! make the new coordinate system active

```

```

csys,11

```

```

! define the areas in the new coordinate system
! these are the curves (valley) on the left

```

```

a,1,2,5,4
a,4,5,8,7
a,7,8,11,10

```

```

! make the cartesian coordinate system active

```

```

csys,0

```

```

! define cylindrical coordinate system with origin at last keypt

```

```

cskp,12,1,20,2,3

```

```

! make the new system active

```

```

csys,12

```

```

! define areas in the new coordinate system
! these are the curves (hill) on the right

```

```

a,2,3,6,5
a,5,6,9,8
a,8,9,12,11

```

```

! ELEMENT DEFINITION

```

```

! Define the element type to be used (must be done to make element
! available for meshing)

```

```

! Use the element type command:

```

```

! ET, ITYPE, ENAME, KOP1, KOP2, KOP3, KOP4, KOP5, KOP6

```

```

! KOP# = key options which can be set to various values to define
! specific

```

```

! options for elements. These are defined in the ANSYS help manual
! for each element type

```

```

! ITYPE = number designating element type (defaults to current max +1)

```

```

! ENAME = name of the element type to be added

```

```

! Here, the PLANE13 element will be used. It is a plane, coupled-
! field element

```

```

! Keyop 1 = 3 makes UX and UY degrees of freedom the active ones

```

```

! Keyop 1 = 7 makes UX, UY, VOLT the degrees of freedom

```

```

! Keyop 2 = 0 includes extra quadratic shape functions (1 excludes

```

```

!      them)
!      Keyop 3 = 2 makes the element plane stress (0 makes plane strain)
!      Keyop 4 = 1 element coordinate system based on element I-J side
!      Keyop 4 = 0 element coordinate system parallel to global
!      Keyop 5 = 0 basic element printout
!      Keyop 5 = 1 repeats basic solution for all integration points
!      Keyop 5 = 2 nodal stress printout
!      Keyop 6 = 0 means that the material data matrix input is the
!      stiffness matrix
!      Keyop 6 = 1 means that the material data matrix input is the
!      compliance matrix

ET,1,PLANE13,3,0,2,1,,0

!      define the element type with only UX and UY degrees of freedom for
!      the glue layer

ET,2,PLANE13,3,0,2,1,,0

!      Define the global element size to be used in the mesh (length of
!      element side in meters)

ESIZE,0.000025

!      MATERIAL PROPERTY INPUT

!      Define material properties (in SI units)
!      Use the linear material property command:  MP,LABEL,MAT#,C
!      LABEL = name of the property to be input, a list given in Commands
!      Manual in ANSYS help
!      MAT# = number of the material table (since can have multiple
!      materials defined)
!      C = value of the constant
!      One can also input temperature dependent properties using further
!      constants
!      see the Commands Manual for this command in the ANSYS help for more
!      information

!      Input the Young's modulus.

MP,EX,1,6.154E9

!      OR if considering the orthotropic properties of PVDF
!      For input of the stiffness matrix, use a data table command, TB

!TB,ANEL,1
!TBDATA,1,6.154E9,4.136E9,3.469E9,0,0,0
!TBDATA,7,5.377E9,3.508E9,0,0,0,4.87E9
!TBDATA,13,0,0,0,0.1038E9,0,0
!TBDATA,19,0,0,0

!      input the material properties of the glue layer

MP,EX,2,4.235E9
MP,GXY,2,1.531E9

```

```
! MESHING THE MODEL

! Define whether mapped or free mesh will be used
! (0=free, 1=mapped)

MSHKEY,1

! Define the element shape to be used if the element has the option
! (quadrilaterals = 0, triangular = 1) and whether 2D or 3D

MSHAPE,0,2D

! Mesh the areas from 1 through 4

! first activate the material number you want the elements to have

! first make the entire thickness of the PVDF layer have the
! properties of PVDF, don't separate the electrode layer properties

! set material pointer to material 1 and element type to type 1

MAT,1
TYPE,1
AMESH,1,1

MAT,2
TYPE,2
AMESH,2,2

MAT,1
TYPE,1
AMESH,3,3

MAT,1
TYPE,1
AMESH,4,4

MAT,2
TYPE,2
AMESH,5,5

MAT,1
TYPE,1
AMESH,6,6

! open the solution processor

/SOLU
```

```
! set the analysis type to a new static analysis
! (this is the default)
```

```
ANTYPE,STATIC,NEW
```

```
L_MAX=0.3
```

```
NSEL,ALL
```

```
csys,0
```

```
! select nodes to apply constraints
! first select nodes with x coordinate equal to zero
```

```
dx=0.0000001
```

```
dy=0.0000001
```

```
NSEL,S,LOC,X,0-dx,0+dx
```

```
! constrain the x displacement to be zero for these nodes
```

```
D,ALL,UY,0
```

```
D,ALL,UX,0
```

```
! now create a coupled set of nodes, so that they can be
! constrained for zero slope at the far right end of the beam
```

```
XMAX=0.0254
```

```
YMAX=0.05449
```

```
NSEL,S,LOC,X,XMAX-dx,dx+XMAX
```

```
CP,NEXT,UX,ALL
```

```
! reselect all nodes so they all become active again
```

```
NSEL,ALL
```

```
! apply a point load to the upper most end node
```

```
F,6289,FY,-L_MAX
```

```
! Plot the elements
```

```
EPLOT
```

```
SOLVE
```

```
! Model: WITH Epoxy Layer, Applied Mechanical Load, Non-linear
! Analysis
! give the file a name for use in other commands, such as upgeom, which
! needs a restart file
```

```
/FILENAME,glue-P-nonlin
```

```
! open the preprocessor in ANSYS to begin modeling
```

```
/PREP7
```

```
! MODEL DEFINITION
```

```
! define keypoints for a curved actuator (full width 2 inches)
! half width model, radius of curvature 31 mm
! without epoxy pad at the left end for attaching to other
! bimorph
! units are in meters (coordinates)
!
! this model has a glue layer and electrode layers
```

```
k,1,0,0
k,2,0.0127,0.002667
k,3,0.0254,0.005334
k,4,0, 4.5e-005
k,5,0.0126819,0.002708
k,6,0.0254,0.005379
k,7,0,0.00007
k,8,0.0126718,0.0027311
k,9,0.0254,0.005404
k,10,0,0.000115
k,11,0.0126537,0.0027723
k,12,0.0254,0.005449
k,13,0,7.5e-006
k,14,0.012697,0.0026739
k,15,0.0254,0.0053415
k,16,0,0.0001075
k,17,0.0126568,0.0027654
k,18,0.0254,0.0054415
```

```
! define keypoints that are the arc centers
! valley curve (left hand curve of actuator)
```

```
k,19,-2.33e-011,0.03157
```

```
! hill curve (right hand curve of actuator)
```

```
k,20,0.0254,-0.02624
```

```
! define cylindrical coordinate system with origin at second
! to the last keypt
! use command cskp,a,b,c,d,e
! a = number designator of coordinate system (must start with 11)
! b = 0 if cartesian, 1 if cylindrical, 2 if spherical coord. system
```

```
! c = keypoint defining the origin of the coord. system
! d = keypoint defining the positive x direction of the coord. sys.
! e = keypoint defining the x-y plane
```

```
cskp,11,1,19,10,11
```

```
! make the new coordinate system active
```

```
csys,11
```

```
! define the areas in the new coordinate system
! these are the curves (valley) on the left
```

```
a,1,2,5,4
a,4,5,8,7
a,7,8,11,10
```

```
! make the cartesian coordinate system active
```

```
csys,0
```

```
! define cylindrical coordinate system with origin at last keypt
```

```
cskp,12,1,20,2,3
```

```
! make the new system active
```

```
csys,12
```

```
! define areas in the new coordinate system
! these are the curves (hill) on the right
```

```
a,2,3,6,5
a,5,6,9,8
a,8,9,12,11
```

! ELEMENT DEFINITION

```
! Define the element type to be used (must be done to make element
! available for meshing)
! Use the element type command:
! ET, ITYPE, ENAME, KOP1, KOP2, KOP3, KOP4, KOP5, KOP6
```

```
! KOP# = key options which can be set to various values to define
! specific
! options for elements. These are defined in the ANSYS help manual
! for each element type
! ITYPE = number designating element type (defaults to current max +1)
! ENAME = name of the element type to be added
! Here, the PLANE13 element will be used. It is a plane, coupled-
! field element
! Keyop 1 = 3 makes UX and UY degrees of freedom the active ones
! Keyop 1 = 7 makes UX, UY, VOLT the degrees of freedom
! Keyop 2 = 0 includes extra quadratic shape functions (1 excludes
```

```

!   them)
!   Keyop 3 = 2 makes the element plane stress (0 makes plane strain)
!   Keyop 4 = 1 element coordinate system based on element I-J side
!   Keyop 4 = 0 element coordinate system parallel to global
!   Keyop 5 = 0 basic element printout
!   Keyop 5 = 1 repeats basic solution for all integration points
!   Keyop 5 = 2 nodal stress printout
!   Keyop 6 = 0 means that the material data matrix input is the
!   stiffness matrix
!   Keyop 6 = 1 means that the material data matrix input is the
!   compliance matrix

ET,1,PLANE13,3,0,2,1,,0

!   define the element type with only UX and UY degrees of freedom for
!   the glue layer

ET,2,PLANE13,3,0,2,1,,0

!   Define the global element size to be used in the mesh (length of
!   element side in meters)

ESIZE,0.000025

!   MATERIAL PROPERTY INPUT

!   Define material properties (in SI units)
!   Use the linear material property command:  MP,LABEL,MAT#,C
!   LABEL = name of the property to be input, a list given in Commands
!   Manual in ANSYS help
!   MAT# = number of the material table (since can have multiple
!   materials defined)
!   C = value of the constant
!   One can also input temperature dependent properties using further
!   constants
!   see the Commands Manual for this command in the ANSYS help for more
!   information

!   Input the Young's modulus

MP,EX,1,6.154E9

!   OR if considering the orthotropic properties of PVDF
!   For input of the stiffness matrix, use a data table command, TB

!TB,ANEL,1
!TBDATA,1,6.154E9,4.136E9,3.469E9,0,0,0
!TBDATA,7,5.377E9,3.508E9,0,0,0,4.87E9
!TBDATA,13,0,0,0,0.1038E9,0,0
!TBDATA,19,0,0,0

!   input the material properties of the glue layer

MP,EX,2,4.235E9
MP,GXY,2,1.531E9

```

```
! MESHING THE MODEL

! Define whether mapped or free mesh will be used
! (0=free, 1=mapped)

MSHKEY,1

! Define the element shape to be used if the element has the option
! (quadrilaterals = 0, triangular = 1) and whether 2D or 3D

MSHAPE,0,2D

! Mesh the areas from 1 through 4

! first activate the material number you want the elements to have

! first make the entire thickness of the PVDF layer have the
! properties of PVDF, don't separate the electrode layer properties

! set material pointer to material 1 and element type to type 1

MAT,1
TYPE,1
AMESH,1,1

MAT,2
TYPE,2
AMESH,2,2

MAT,1
TYPE,1
AMESH,3,3

MAT,1
TYPE,1
AMESH,4,4

MAT,2
TYPE,2
AMESH,5,5

MAT,1
TYPE,1
AMESH,6,6

! open the solution processor

/SOLU
```

```
! set the analysis type to a new static analysis
! (this is the default)
```

```
ANTYPE,STATIC,NEW
L_MAX=1.7
L_INC=0.03
SOLCONTROL,ON
AUTOTS,OFF
NLGEOM,ON
! set the total time (load) to be the same as the total load applied
TIME,L_MAX
DELTIM,L_INC
NSUBST,L_MAX/L_INC,60,2
! set load application to ramped mode
KBC,0
OUTRES,BASIC,ALL
NSEL,ALL
```

```
csys,0
```

```
! select nodes to apply constraints
! first select nodes with x coordinate equal to zero
```

```
dx=0.0000001
dy=0.0000001
```

```
NSEL,S,LOC,X,0-dx,0+dx
```

```
! constrain the x displacement to be zero for these nodes
```

```
D,ALL,UY,0
D,ALL,UX,0
```

```
! now create a coupled set of nodes, so that they can be
! constrained for zero slope at the far right end of the beam
```

```
XMAX=0.0254
YMAX=0.05449
```

```
NSEL,S,LOC,X,XMAX-dx,dx+XMAX
```

```
CP,NEXT,UX,ALL
```

```
NSEL,ALL
```

```
! apply a point load to the upper most end node
```

```
F,6289,FY,L_MAX
```

```
! reselect all nodes so they all become active again
```

```
EPLOT
```

SOLVE

! Plot the elements

/POST26

NSOL,2,6289,U,Y,UY_end

NSOL,3,6289,U,X,UX_end

! plot time (since static case, equivalent to load) as the x-variable
XVAR,1

! plot UY at the right end vs load and UX at the right end vs load
PLVAR,2,3
PRVAR,2,3

```
! Model: Non-uniform Epoxy Layer, Applied Mechanical Loads, Linear
! Analysis
! give the file a name for use in other commands, such as upgeom, which
! needs a restart file
```

```
/FILENAME,nunfglue-P-lin
```

```
! open the preprocessor in ANSYS to begin modeling
```

```
./PREP7
```

```
! MODEL DEFINITION
```

```
! define keypoints for a curved actuator (full width 2 inches)
! half width model, radius of curvature 31 mm
! without epoxy pad at the left end for attaching to other
! bimorph
! units are in meters (coordinates)
!
! this model has a glue layer and electrode layers
```

```
k,1,0,0
k,2,0.0127,0.002667
k,3,0.0254,0.005334
k,4,0, 4.5e-005
k,5,0.0126819, 0.002708199
k,6,0.0254, 0.005379
k,7,0, 6.514e-005
k,8,0.01266829, 0.002739171
k,9,0.0254, 0.00540748
k,10,0, 0.00011014
k,11,0.01265019, 0.00278037
k,12,0.0254, 0.00545248
k,13,0, 7.5e-006
k,14,0.01269698, 0.002673866
k,15,0.0254, 0.0053265
k,16,0, 0.00010264
k,17,0.01265321, 0.002773503
k,18,0.0254, 0.00544498
```

```
! define keypoints that are the arc centers
! valley curve (left hand curve of actuator)
```

```
k,19,-3.259167e-005, 0.03156473
```

```
! hill curve (right hand curve of actuator)
```

```
k,20,0.0254,-0.0262376
```

```
! define cylindrical coordinate system with origin at second
! to the last keypt
! use command cskp,a,b,c,d,e
! a = number designator of coordinate system (must start with 11)
```

```

! b = 0 if cartesian, 1 if cylindrical, 2 if spherical coord. system
! c = keypoint defining the origin of the coord. system
! d = keypoint defining the positive x direction of the coord. sys.
! e = keypoint defining the x-y plane

```

```

cskp,11,1,19,10,11

```

```

! make the new coordinate system active

```

```

csys,11

```

```

! define lines so can track their number for applying voltages

```

```

1,1,2
1,2,14
1,14,5
1,14,13
1,5,4
1,4,13
1,13,1

```

```

1,5,8
1,8,7
1,7,4

```

```

1,8,17
1,17,11
1,17,16
1,11,10
1,10,16
1,16,7

```

```

! define the areas in the new coordinate system
! these are the curves (valley) on the left

```

```

a,1,2,5,4
a,4,5,8,7
a,7,8,11,10

```

```

! make the cartesian coordinate system active

```

```

csys,0

```

```

! define cylindrical coordinate system with origin at last keypt

```

```

cskp,12,1,20,2,3

```

```

! make the new system active

```

```

csys,12

```

```

! define lines so can track their number for applying voltages

```

```

1,2,3

```

```

1,3,15
1,15,6
1,15,14
1,6,5
1,6,9
1,9,8
1,9,18
1,18,12
1,18,17
1,12,11

```

```

! define areas in the new coordinate system
! these are the curves (hill) on the right

```

```

a,2,3,6,5
a,5,6,9,8
a,8,9,12,11

```

``` ! ELEMENT DEFINITION ```

```

! Define the element type to be used (must be done to make element
! available for meshing)
! Use the element type command:
! ET, ITYPE, ENAME, KOP1, KOP2, KOP3, KOP4, KOP5, KOP6

! KOP# = key options which can be set to various values to define
! specific
! options for elements. These are defined in the ANSYS help manual
! for each element type
! ITYPE = number designating element type (defaults to current max +1)
! ENAME = name of the element type to be added
! Here, the PLANE13 element will be used. It is a plane, coupled-
! field element
! Keyop 1 = 3 makes UX and UY degrees of freedom the active ones
! Keyop 1 = 7 makes UX, UY, VOLT the degrees of freedom
! Keyop 2 = 0 includes extra quadratic shape functions (1 excludes
! them)
! Keyop 3 = 2 makes the element plane stress (0 makes plane strain)
! Keyop 4 = 1 element coordinate system based on element I-J side
! Keyop 4 = 0 element coordinate system parallel to global
! Keyop 5 = 0 basic element printout
! Keyop 5 = 1 repeats basic solution for all integration points
! Keyop 5 = 2 nodal stress printout
! Keyop 6 = 0 means that the material data matrix input is the
! stiffness matrix
! Keyop 6 = 1 means that the material data matrix input is the
! compliance matrix

```

```
ET,1,PLANE13,3,0,2,1,,0
```

```

! define the element type with only UX and UY degrees of freedom for
! the glue layer

```

```
ET,2,PLANE13,3,0,2,1,,0
```

```
! Define the global element size to be used in the mesh (length of
! element side in meters)
```

```
ESIZE,0.000025
```

```
! MATERIAL PROPERTY INPUT
```

```
! Define material properties (in SI units)
! Use the linear material property command: MP,LABEL,MAT#,C
! LABEL = name of the property to be input, a list given in Commands
! Manual in ANSYS help
! MAT# = number of the material table (since can have multiple
! materials defined)
! C = value of the constant
! One can also input temperature dependent properties using further
! constants
! see the Commands Manual for this command in the ANSYS help for more
! information
```

```
MP,EX,1,6.154E9
```

```
! OR consider the orthotropic properties of PVDF
! For input of the stiffness matrix, use a data table command, TB
```

```
!TB,ANEL,1
!TBDATA,1,6.154E9,4.136E9,3.469E9,0,0,0
!TBDATA,7,5.377E9,3.508E9,0,0,0,4.87E9
!TBDATA,13,0,0,0,0.1038E9,0,0
!TBDATA,19,0,0,0
```

```
! input the material properties of the glue layer
```

```
MP,EX,2,4.235E9
MP,GXY,2,1.531E9
```

```
! MESHING THE MODEL
```

```
! Define whether mapped or free mesh will be used
! (0=free, 1=mapped)
```

```
MSHKEY,1
```

```
! Define the element shape to be used if the element has the option
! (quadrilaterals = 0, triangular = 1) and whether 2D or 3D
```

```
MSHAPE,0,2D
```

```
! Mesh the areas from 1 through 4
```

```
! first activate the material number you want the elements to have
```

```
! first make the entire thickness of the PVDF layer have the
! properties of PVDF, don't separate the electrode layer properties
```

! set material pointer to material 1 and element type to type 1

MAT,1
TYPE,1
AMESH,1,1

MAT,2
TYPE,2
AMESH,2,2

MAT,1
TYPE,1
AMESH,3,3

MAT,1
TYPE,1
AMESH,4,4

MAT,2
TYPE,2
AMESH,5,5

MAT,1
TYPE,1
AMESH,6,6

! open the solution processor

/SOLU

! set the analysis type to a new static analysis
! (this is the default)

ANTYPE,STATIC,NEW

L_MAX=0.3

! set the total time (load) to be the same as the total load applied
NSEL,ALL

csys,0

! select nodes to apply constraints
! first select nodes with x coordinate equal to zero

dx=0.0000001
dy=0.0000001

NSEL,S,LOC,X,0-dx,0+dx

! constrain the x displacement to be zero for these nodes

```
D,ALL,UY,0
D,ALL,UX,0
```

```
! now create a coupled set of nodes, so that they can be
! constrained for zero slope at the far right end of the beam
```

```
XMAX=0.0254
YMAX=0.05449
```

```
NSEL,S,LOC,X,XMAX-dx,dx+XMAX
```

```
CP,NEXT,UX,ALL
```

```
NSEL,ALL
```

```
! apply a point load to the upper most end node
F,6289,FY,L_MAX
```

```
! reselect all nodes so they all become active again
```

```
NSEL,ALL
```

```
! Plot the elements
EPLLOT
```

```
SOLVE
```

```
! Model: Non-uniform Epöxy Layer, Applied Mechanical Load, Non-linear
! Analysis
! give the file a name for use in other commands, such as upgeom, which
! needs a restart file
```

```
/FILENAME,nunf glue-P-nonlin
```

```
! open the preprocessor in ANSYS to begin modeling
```

```
/PREP7
```

```
! MODEL DEFINITION
```

```
! define keypoints for a curved actuator (full width 2 inches)
! half width model, radius of curvature 31 mm
! without epoxy pad at the left end for attaching to other
! bimorph
! units are in meters (coordinates)
!
! this model has a glue layer and electrode layers
```

```
k,1,0,0
k,2,0.0127,0.002667
k,3,0.0254,0.005334
k,4,0, 4.5e-005
k,5,0.0126819, 0.002708199
k,6,0.0254, 0.005379
k,7,0, 6.514e-005
k,8,0.01266829, 0.002739171
k,9,0.0254, 0.00540748
k,10,0, 0.00011014
k,11,0.01265019, 0.00278037
k,12,0.0254, 0.00545248
k,13,0, 7.5e-006
k,14,0.01269698, 0.002673866
k,15,0.0254, 0.0053265
k,16,0, 0.00010264
k,17,0.01265321, 0.002773503
k,18,0.0254, 0.00544498
```

```
! define keypoints that are the arc centers
! valley curve (left hand curve of actuator)
```

```
k,19,-3.259167e-005, 0.03156473
```

```
! hill curve (right hand curve of actuator)
```

```
k,20,0.0254,-0.0262376
```

```
! define cylindrical coordinate system with origin at second
! to the last keypt
! use command cskp,a,b,c,d,e
! a = number designator of coordinate system (must start with 11)
```

```

! b = 0 if cartesian, 1 if cylindrical, 2 if spherical coord. system
! c = keypoint defining the origin of the coord. system
! d = keypoint defining the positive x direction of the coord. sys.
! e = keypoint defining the x-y plane

```

```

cskp,11,1,19,10,11

```

```

! make the new coordinate system active

```

```

csys,11

```

```

! define lines so can track their number for applying voltages

```

```

1,1,2
1,2,14
1,14,5
1,14,13
1,5,4
1,4,13
1,13,1

```

```

1,5,8
1,8,7
1,7,4

```

```

1,8,17
1,17,11
1,17,16
1,11,10
1,10,16
1,16,7

```

```

! define the areas in the new coordinate system
! these are the curves (valley) on the left

```

```

a,1,2,5,4
a,4,5,8,7
a,7,8,11,10

```

```

! make the cartesian coordinate system active

```

```

csys,0

```

```

! define cylindrical coordinate system with origin at last keypt

```

```

cskp,12,1,20,2,3

```

```

! make the new system active

```

```

csys,12

```

```

! define lines so can track their number for applying voltages

```

```

1,2,3

```

```

1,3,15
1,15,6
1,15,14
1,6,5
1,6,9
1,9,8
1,9,18
1,18,12
1,18,17
1,12,11

```

```

! define areas in the new coordinate system
! these are the curves (hill) on the right

```

```

a,2,3,6,5
a,5,6,9,8
a,8,9,12,11

```

``` ! ELEMENT DEFINITION ```

```

! Define the element type to be used (must be done to make element
! available for meshing)
! Use the element type command:
! ET, ITYPE, ENAME, KOP1, KOP2, KOP3, KOP4, KOP5, KOP6

! KOP# = key options which can be set to various values to define
! specific
! options for elements. These are defined in the ANSYS help manual
! for each element type
! ITYPE = number designating element type (defaults to current max +1)
! ENAME = name of the element type to be added
! Here, the PLANE13 element will be used. It is a plane, coupled-
! field element
! Keyop 1 = 3 makes UX and UY degrees of freedom the active ones
! Keyop 1 = 7 makes UX, UY, VOLT the degrees of freedom
! Keyop 2 = 0 includes extra quadratic shape functions (1 excludes
! them)
! Keyop 3 = 2 makes the element plane stress (0 makes plane strain)
! Keyop 4 = 1 element coordinate system based on element I-J side
! Keyop 4 = 0 element coordinate system parallel to global
! Keyop 5 = 0 basic element printout
! Keyop 5 = 1 repeats basic solution for all integration points
! Keyop 5 = 2 nodal stress printout
! Keyop 6 = 0 means that the material data matrix input is the
! stiffness matrix
! Keyop 6 = 1 means that the material data matrix input is the
! compliance matrix

```

```
ET,1,PLANE13,3,0,2,1,,0
```

```

! define the element type with only UX and UY degrees of freedom for
! the glue layer

```

```
ET,2,PLANE13,3,0,2,1,,0
```

```
! Define the global element size to be used in the mesh (length of
! element side in meters)
```

```
ESIZE,0.000025
```

```
! MATERIAL PROPERTY INPUT
```

```
! Define material properties (in SI units)
! Use the linear material property command: MP,LABEL,MAT#,C
! LABEL = name of the property to be input, a list given in Commands
! Manual in ANSYS help
! MAT# = number of the material table (since can have multiple
! materials defined)
! C = value of the constant
! One can also input temperature dependent properties using further
! constants
! see the Commands Manual for this command in the ANSYS help for more
! information
```

```
MP,EX,1,6.154E9
```

```
! For input of the stiffness matrix, use a data table command, TB
```

```
!TB,ANEL,1
!TBDATA,1,6.154E9,4.136E9,3.469E9,0,0,0
!TBDATA,7,5.377E9,3.508E9,0,0,0,4.87E9
!TBDATA,13,0,0,0,0.1038E9,0,0
!TBDATA,19,0,0,0,0
```

```
!TB,PIEZ,1
!TBDATA,1,0,0.03,0,0,-0.045,0
!TBDATA,7,0,-7.2E-3,0,-2.804E-3,0
!TBDATA,13,0,0,0,0,0,0
```

```
! input the material properties of the glue layer
```

```
MP,EX,2,4.235E9
MP,GXY,2,1.531E9
```

```
! MESHING THE MODEL
```

```
! Define whether mapped or free mesh will be used
! (0=free, 1=mapped)
```

```
MSHKEY,1
```

```
! Define the element shape to be used if the element has the option
! (quadrilaterals = 0, triangular = 1) and whether 2D or 3D
```

```
MSHAPE,0,2D
```

```
! Mesh the areas from 1 through 4
```

```

! first activate the material number you want the elements to have

! first make the entire thickness of the PVDF layer have the
! properties of PVDF, don't separate the electrode layer properties

! set material pointer to material 1 and element type to type 1

```

```

MAT,1
TYPE,1
AMESH,1,1

```

```

MAT,2
TYPE,2
AMESH,2,2

```

```

MAT,1
TYPE,1
AMESH,3,3

```

```

MAT,1
TYPE,1
AMESH,4,4

```

```

MAT,2
TYPE,2
AMESH,5,5

```

```

MAT,1
TYPE,1
AMESH,6,6

```

```

! open the solution processor

```

```

/SOLU

```

```

! set the analysis type to a new static analysis
! (this is the default)

```

```

ANTYPE,STATIC,NEW
L_MAX=2.0
L_INC=0.03
SOLCONTROL,ON
AUTOTS,OFF
NLGEOM,ON

```

```

! set the total time (load) to be the same as the total load applied
TIME,L_MAX
DELTIM,L_INC
NSUBST,L_MAX/L_INC,100,2
! set load application to ramped mode

```

```
KBC,0
OUTRES,BASIC,ALL
NSEL,ALL
```

```
csys,0
```

```
! select nodes to apply constraints
! first select nodes with x coordinate equal to zero
```

```
dx=0.0000001
dy=0.0000001
```

```
NSEL,S,LOC,X,0-dx,0+dx
```

```
! constrain the x displacement to be zero for these nodes
```

```
D,ALL,UY,0
D,ALL,UX,0
```

```
! now create a coupled set of nodes, so that they can be
! constrained for zero slope at the far right end of the beam
```

```
XMAX=0.0254
YMAX=0.05449
```

```
NSEL,S,LOC,X,XMAX-dx,dx+XMAX
```

```
CP,NEXT,UX,ALL
```

```
NSEL,ALL
```

```
! apply a point load to the upper most end node
```

```
F,6289,FY,L_MAX
```

```
! reselect all nodes so they all become active again
```

```
NSEL,ALL
```

```
EPLOT
```

```
SOLVE
```

```
! Plot the elements
```

```
EPLOT
```

```
/POST26
```

```
NSOL,2,6289,U,Y,UY_end
NSOL,3,6289,U,X,UX_end
```

```
! plot time (since static case, equivalent to load) as the x-variable
XVAR,1
```

! plot UY at the right end vs load and UX at the right end vs load
PLVAR,2,3
PRVAR,2,3

