DEVELOPMENT OF THE MATHEMATICS CURRICULUM IN THE UNITED STATES SINCE 1950

A Professional Paper

Presented to

The Faculty of the Department of Teacher Education

Montana State College

In Partial Fulfillment

of the Requirements for the Degree

Master of Education

by

Erwin L. Miller

July, 1962

P378
M613

TABLE OF CONTENTS

Chapter		Page
I.	STATEMENT OF THE PROBLEM	1
	The Problem	1
	Statement of the Problem	1
	Significance of the Study	1
	Limitations of the Study	2
	The Procedure Used	2
II.	DEVELOPMENTS IN MATHEMATICS CURRICULUM SINCE 1950	4
III.	THE UNIVERSITY OF ILLINOIS COMMITTEE ON SCHOOL MATHEMATICS PROGRAM	15
	History of the Program	15
	Philosophy of the Project	17
	Basic Outline of the Course	18
	Methodology of UICSM	20
	Conclusions	22
IV.	THE SCHOOL MATHEMATICS STUDY GROUP	24
	History of the Program	24
	Philosophy of the SMSG	26
	Basic Outline of the Course	27
	Methodology of the SMSG	30
	Conclusion	32
V.	IMPLICATIONS OF THE CONTEMPORARY MATHEMATICS CURRICULUMS FOR TEACHER EDUCATION	34
	The Need for Changes in Teacher Education	34
	Pre-service Education	36
	In-service Training	39

Chapter Page

IV. SUMMARY, CONCLUSIONS, AND IMPLICATIONS 48

Summary 48

Conclusions 50

Implications 51

BIBLIOGRAPHY 53

CHAPTER I

STATEMENT OF THE PROBLEM

Although curriculum development is a continuous process in our schools, from time to time increased emphasis appears to be placed upon some particular school subject. For the past few years this emphasis has fallen on the field of mathematics, and one result of this emphasis has been the emergence of a new mathematics curriculum which has received widespread recognition.

I. The Problem

Statement of the problem. It was the purpose of this study to: (1) survey the literature which sheds light on the reasons behind the development of this new mathematics curriculum and demonstrates the soundness of the theory behind it; (2) compare the basic course outlines and the teaching procedures developed by two different groups working with the mathematics curriculum; and (3) investigate the implications for teacher education which are created by the changes in high school mathematics curriculum.

Significance of the study. Any proposed change in our school curriculum must necessarily be studied carefully before its adoption to determine whether its value is sufficient for it to warrant a place in the school curriculum. Inevitably these changes are discussed and evaluated by the lay public because that is the nature of our public school system, but it should be the responsibility of the school

personnel, both faculty and administration, to lead in the study of curriculum development. Teachers of mathematics should become aware of the currently proposed changes in the curriculum, not only so they may be adequately prepared to teach the subject matter involved, but also so that they may be equipped to help make the decision of adoption or rejection of the proposed new curriculum by their school.

Limitations of the study. The currently suggested change in the mathematics curriculum effects the entire school mathematics curriculum beginning with the first grade. However, the scope of this study includes only the problem as it effects the secondary school.

Undoubtedly many individual school districts have undertaken experiments to modify their mathematics curriculum, but a second limitation of this study was to select for analysis only two proposed plans. The University of Illinois Committee on School Mathematics project and the School Mathematics Study Group were selected because they are the two most widely accepted plans and because of the authority of the groups which devised the plans.

II. The Procedure Used

The method used in this study was to first review the literature written since 1950 which is related to the development of the new mathematics curriculum. In the light of this historical background two currently used programs, the SMSG plan and the UICSM plan, were studied in detail to determine the general trends which are being followed.

Chapter Five is devoted to the implications of these programs for teacher education. The final chapter is a summary of the ideas presented in this study. In the last chapter the questions which are raised, but not answered by this study, are presented as a starting point for further investigation.

CHAPTER II

DEVELOPMENTS IN MATHEMATICS CURRICULUM SINCE 1950

The mathematics curriculum in the secondary schools has reached a turning point. There has been rigorous thinking and experimentation in the past ten years concerning a change which will place mathematics in its proper perspective in the curriculum. There is a definite need for a change in the high school mathematics curriculum, and this change does not necessarily mean the scrapping of all traditional mathematics, but rather an enlivenment of the traditional mathematics with the introduction of new concepts, ideas, and new approaches in teaching. A review of the literature since 1950 will show various ideas and approaches to the revision of the mathematics curriculum.

Zant[1] observed that the mathematics curriculum of most high schools is largely traditional, and that people who attend school 25 or 30 years ago would have slight difficulty in recognizing the material which is being taught in mathematics classes today. One would have little difficulty, for example, in recognizing the geometry of today in a copy of Playfair's Euclid, which was written in 1813, one hundred and fifty years ago. In relation to this idea of the outdated mathematics curriculum Zant said that:

> This is not necessarily bad. If students need the mathematics that was taught 75 years ago or 2000 years ago, as in the

---

[1]James H. Zant, "A Program for Determining the Mathematical Needs of Engineering Students," The Mathematics Teacher, 43:91, March, 1950.

> case of Euclid's geometry, there is every reason that such subject matter should continue to be included in our school curriculum. The important thing is the pupils' needs now and in the future. We should find what these needs are for all citizens and for special groups who are segregated for special training.

Betz[2] in examining what happened to mathematics from 1900 to 1950 shows that it declined in importance with educational policy makers. From 1900 to 1923 there was a great flowering of interest in mathematics with forward looking readjustments and hopeful reogranization. During the early 1920's the educational policy was dominated by the ever more vexing problem of mass education, and this led to the creation of thousands of curriculums in mathematics. In the 1930's mathematics was removed from its position of prominence and relegated to a position of merely being a tolerated elective. The 1940's began a fourth period which will witness a further elimination of mathematics or will see mathematics restored to the position it deserves. In many states not a single hour of credit in mathematics is required for graduation from high school. Betz concludes that we have entered an era of "mathematics without mathematics" and that:

> A two fold correction is needed. On the one hand we must insist on a sound mathematical foundation in terms of the recognized fundamentals of basic mathematics. And we must enable all capable students to attain the maximum of mathematical training that can be achieved in high school. On the other hand, for those whose chief concern is immediate life preparation, we must

---

[2]William Betz, "Five Decades of Mathematical Reform--Evaluation and Challenge," The Mathematics Teacher, 43:377-378, December, 1950.

> provide, not a pauperized type of mathematics, but a mathematical program that shall be both vocationally significant and culturally worthwhile.[3]

The questions raised by teachers of mathematics during the early 1950's showed a need for a comprehensive study of the entire mathematics program. Wren[4] raised the question of a need for such a comprehensive study by the following statement:

> The study must be made in the perspective of the nature of our school population and the mathematical needs of the pupils which constitute this population. It should be a study projected on the basis of professional cooperation for the purpose of designing an educationally significant program in mathematics for our schools.

According to Rosskopf and Exner[5] by the middle 1950's the movement toward a change in the traditional mathematics taught in the secondary school had gained impetus. The emphasis away from purely manipulative mathematics was the emerging trend in secondary school mathematics with more stress than formerly being given to mathematical logic and the foundations of mathematics. This shift was brought about because the application of mathematics stressed the use of fundamental concepts more often than it did straight-forward manipulation.

---

[3] Ibid., p. 386-387.

[4] F. Lynwood Wren, "What About the Structure of the Mathematics Curriculum?" The Mathematics Teacher, 44:169, March, 1951.

[5] Myron F. Rosskopf and Robert M. Exner, "Some Concepts of Logic and Their Application in Elementary Mathematics," The Mathematics Teacher, 48:290, May, 1955.

The dissatisfactions with the high school mathematics curriculum that mathematicians had were supported by Northrop.[6]

> Secondary school mathematics has become, and is continuing to become, less and less relevent to modern mathematics and its application to modern science--physical, biological, and social.

Since 1900 there has been a dynamic development of mathematical knowledge, and in the course of this development concepts and methods which formerly had been neatly compartmentalized into algebra, geometry, and analysis were mixed beyond recognition. Even as late as 1952 the United States Office of Education's report on mathematics, Mathematics in the Public High Schools and Curriculum Materials in High School Mathematics, made references to mathematics only in terms of conventional course labels: "algebra," "plane geometry," and the like. The problem that existed was one of lack of communication between mathematicians who had been pursuing new ideas with great intensity and teachers of high school mathematics who had been concentrating on the reformulation and improved presentation of old ideas. This mere changing around of the conventional mathematics curriculum has been a misleading device and has left high school mathematics in a state far more antiquated than any other subject in the curriculum.[7]

---

[6] E. P. Northrop, "Modern Mathematics and the Secondary School Curriculum," The Mathematics Teacher, 48:386, October, 1955.

[7] Ibid., p. 390.

The reason given by MacLane[8] for this antiquation of the mathematics curriculum was that there has been very little exchange of ideas between mathematicians and teachers. Before 1940 practically all professional mathematicians were teaching in addition to doing research. However, in the period following the 1940's a great percentage of the professional mathematicians no longer had any connection with teaching, but were now working in industry. The new mathematical ideas that were developed after this time failed to reach the high school curriculum. Most of the mathematics being taught consisted of ideas already well known 200 years ago, and many of them 2000 years ago. The basic concept of modern mathematics such as modulo arithmetic, changing of base numbers, and the algebra of sets, however, are concepts which could be understood by secondary school pupils. MacLane further stated that:

> It no longer suffices to continue to teach the old ideas; rather, one must start afresh to determine which ideas should be taught and in what perspective. In this way school mathematics could become fully relevant to the modern world.9

Meder[10] made the following observations about modern mathematics. "Modern mathematics is both a point of view and a new subject matter."

---

[8]Saunders MacLane, "The Impact of Modern Mathematics on Secondary Schools," The Mathematics Teacher, 49:67, February, 1956.

[9]Ibid., p. 69.

[10]Albert E. Meder Jr., "Modern Mathematics and Its Place in the Secondary School," The Mathematics Teacher, 50:418, October, 1957.

It is a point of view in that it consists of deductive development of certain propositions from a list of assumptions accepted without proof about terms which are undefined except as limited by the conditions of the assumptions. Modern mathematics is also new in subject matter. Topology, a modern point of view in geometry; the mathematical study of logic which has led to symbolic logic; and a variety of concepts such as groups and fields, which deal with structural rather than manipulative aspects of algebra are examples of this new subject matter. Another mathematical development that can be considered as new is the theory of probability and statistical inferences. But, perhaps the most important new concept in all modern mathematics is the notion of set, which is used in all branches of mathematics. It is appropriate that the set be studied in its own right as a concept which may be used to study otherwise diverse branches of mathematics.

Research and new applications of mathematics have called for new goals, new curriculums, new nexts, and new methods of teacher training. These goals cannot be achieved effectively by modified textbooks reprinted in a more attractive form. Rourke[11] made the following statement concerning this phase of the problem:

> Of the roadblocks that stand in the way of a really new curriculum, I shall mention two: the difficulties of semantics, and the resistance offered by the defenders of the old-time religion. The first can be overcome by care and patience; the second by exhibiting facts.

---

[11]Robert E. K. Rourke, "Some Implications of Twentieth Century Mathematics for High Schools," The Mathematics Teacher, 51:74, February, 1958.

The main points emphasized by Rourke[12] were that the traditional mathematics should not be scrapped, but should be enlivened with a new point of view. If the criticism that manipulative operations are being abandoned is to be avoided, those putting across the new program should have a sincere belief in the traditional, and they must show that nothing is wrong with the traditional system except that it is outdated. Those teachers working with the new program must show that with the use of modern mathematics in the mathematics curriculum many ideas and points can be clarified, many useless and vague terms will be deleted, and integrating concepts for unifying all branches of mathematics are to be instituted. It should be pointed out, however, that the high school is not to teach college mathematics, but to prepare students for college mathematics. It is not intended that these concepts of modern mathematics should be taught in full array to high school students.

Denbow[13] agreed in essence with this point of view, and confirmed it when he wrote:

> We do not need to choose between traditional and modern mathematics; rather, they blend together naturally and organically.

Denbow went on to point out that students have had the conviction that very little thinking was required in the learning of mathematics except

---

[12] Ibid., p. 74-86.

[13] Carl Denbow, "To Teach Modern Algebra," The Mathematics Teacher, 52:162, March, 1959.

as it was applied. Thus mathematics was on a one way track operated by a set of rules and algorisms committed to memory by the student. Somewhere between the two extremes of the modern mathematics and the traditional mathematics there should be found a workable and worthwhile compromise.[14]

Meder[15] made the following three suggestions concerning modern mathematics in the high school curriculum. First, the goal must be mathematics in which insight is more important than reasoning. The concept of mathematics is not novel or new, and the specific subject matter through which it can be developed is relatively unimportant. Second, teachers must be willing to learn more mathematics because this kind of instruction requires deep background knowledge. Third, curriculum revision must be recognized as a long range project. A new curriculum is no substitute for creative teaching, and the temptation to simply "teach sets" in response to popular clamor is only substituting one dull routine for another.

Adler[16] distinguished contemporary mathematics as having the following characteristics:

> It is classical mathematics grown mature.
> It is classical mathematics grown self-conscious and self critical.

---

[14]Ibid., p. 162-170.

[15]Albert E. Meder Jr., "Sets, Sinners, and Salvation," The Mathematics Teacher, 52:434, October, 1959.

[16]Irving Adler, "The Changes Taking Place in Mathematics," The Leadership Role of State Supervisors of Mathematics (Washington: Government Printing Office, 1962) p. 3.

> It is modern mathematics developed as a more efficient way of dealing with the content of classical mathematics.
>
> It is mathematics related more and more intimately to man's activities in industry, social life, science, and philosophy.

Gundlach[17] stated that mathematics has always had a basic scheme. Mathematicians call it the "structure of mathematics," and it is to be found throughout all phases of mathematics from the most elementary to the most advanced level. This scheme will be basic as long as men make mathematics. Rather than give the student a specific set of tools for solving specific problems, modern mathematics provides the learner with the ability to make his own mathematical tools to fit a situation. In brief, while traditional mathematics focused mainly on use, the modern approach gives attention to how mathematics is made.

The challenges presented by the new mathematics have given rise to a controversy between the traditionalists and the moderns which Rosenberg[18] describes effectively. The modern school of thought recommends inclusion of as much mathematics uncovered in recent centuries as possible in hopes of providing more illuminating, interesting, and less obsolete material while the traditionalists recommend retaining a maximum amount of mathematics uncovered in the earlier history of mathematics

---

[17]Bernard H. Gundlach, "How the State Supervisor Can Promote Preservice and Inservice Education for Mathematics Teachers," The Leadership Role of State Supervisors of Mathematics (Washington: Government Printing Office, 1962) p. 65.

[18]Herman Rosenberg, "Great Challenges of Mathematics Education," The Mathematics Teacher, 55:361, May, 1962.

to give students material less abstract and more fundamental and useful. The dividing line between the two schools of thought is not a hard and fast one, however. Some traditionalists favor modern concepts of sets as a possibility for unifying and illuminating much of basic traditional mathematics. Others favor certain other modern aspects as non-decimal numerals and/or vector concepts, but oppose sets because in the hands of untrained and unmotivated teachers and immature students the concept may be of little value. The basic challenge is to select the most useful mathematics from both schools which can be taught in a very limited period of education.

The changes that have taken place in mathematics are so extensive and far reaching that they can only be described as revolutionary. Price[19] suggests that there are three fundamental causes behind these changes. First a more profound mathematics has been created in the twentieth century than during all the rest of history. Second, the automaton revolution which has introduced machines that control machines has created a greater demand on mathematics. And third, the automatic digital computing machine, which is a product of the automaton revolution, has made it possible for mathematical theory to be teamed with the computing machine to produce the answers required by physicists and engineers.

These revolutionary changes in mathematics have important implications for the mathematics being taught in the high schools.

---

[19]G. Baley Price, "Progress in Mathematics," Education Digest, 27:20, February, 1962.

Mathematics courses must include the proper mathematical content. The schools must find well qualified teachers who know a great deal of mathematics. It must become a job of the school counselors to make sure that the students who have the interest and the ability to do so take four years of good high school mathematics.[20]

Mayor's[21] review of the studies for the necessary changes in the mathematics curriculum indicated that curriculum changes for grades nine through twelve were started soon after 1950 at the University of Illinois and several years later changes for grades seven and eight were begun at the University of Maryland. Both of these projects were supported financially by the Carnegie Corporation of New York. Other investigations have been conducted by Stanford University, Southern Illinois University, Ball State Teachers College, and Syracuse University. The major national effort in mathematics education has been the School Mathematics Study Group, supported by the National Science Foundation.

This chapter has reviewed chronologically some of the significant literature in the field of curriculum development in high school mathematics. In the succeeding two chapters the UICSM project and the SMSG plan for mathematics curriculum will be critically examined.

---

[20] Ibid., p. 23.

[21] John R. Mayor, "Mathematics Education," Educational Leadership, 19:395, March, 1962.

CHAPTER III

THE UNIVERSITY OF ILLINOIS COMMITTEE ON SCHOOL MATHEMATICS PROGRAM

One of the earliest attempts to create a new mathematics curriculum which would include modern mathematics was made by the University of Illinois. It is useful to consider how this program developed and what its basic principles are as well as some of its methods for presenting the new material.

I. The History of the Program

The University of Illinois Committee on School Mathematics under the leadership of Max Beberman started work on a mathematics curriculum in 1951 supported by a grant from the Carnegie Corporation of New York. The undertaking was sponsored jointly by the College of Education, the College of Engineering, and the Department of Mathematics. The committee was seeking ways and means of correcting what they felt sure to be certain deficiencies in conventional college preparatory mathematics.[1]

The materials for the UICSM program were produced by high school mathematics teachers and by mathematicians on the staff at the University of Illinois. The first experiment in the use of the materials was a ninth grade course which was taught in one class at the University of Illinois High School in 1952-1953. It was considered so successful

---

[1]Daniel W. Snader, "Secondary School Mathematics in Transition," School Life, 42:9, March, 1960.

that two Illinois high schools entered the program in the fall of 1953.[2]

The UICSM went on then to develop students' textbooks and teachers' guides for all four high school grades; these were used in 1957-1958 by twelve pilot schools involving forty teachers and approximately 1,700 pupils in the states of Illinois, Missouri, and Massachusetts. During the 1958-1959 school year some additional thirty-three schools participated in the program by offering the First Course. Most of the students taking the UICSM First Course were those who would ordinarily be taking conventional college preparatory mathematics. In a few cases gifted eighth grade pupils and highly gifted seventh grade pupils were allowed to take the First Course.[3]

Since 1952 more than 400 high school and junior high school teachers in cooperating schools have been directly involved in the Project.[4] A cooperating school is one in which teachers have had specialized training and submit reports to the Project Center in Urbana, Illinois. These teachers contribute suggestions for revisions of the text and the Teacher's Commentary. In 1960-1961 over 28,000 students were using UICSM materials. About 12,000 of these students were in 500 cooperating schools, and the rest were in schools which operate independently of the Project.

---

[2]UICSM Project Staff, "The University of Illinois School Mathematics Program", The School Review, 65:457, Winter, 1957.

[3]M. Eleanor McCoy, "A Secondary School Mathematics Program," The Bulletin of the National Association of Secondary School Principals, 43:12, May, 1959.

[4]William T. Hale, "UICSM's Decade of Experimentation," The Mathematics Teacher, 54:617, December, 1961.

II. The Philosophy of the Project

The philosophy behind the Project was based upon two questions the group has been debating since 1951: (1) "What mathematical ideas should be taught in high school? and (2) What are the most effective ways to teach these ideas?"[5]

The following are some pertinent quotations from the principles under which the group operates as they are stated in the preface to the first volume of the textbook:

> We believe that students should be given the opportunity to discover a great deal of mathematics which they are expected to learn. Mathematical ideas which a student discovers make sense to him. Discovery will lead him to feel that mathematics is a human and growing subject. . . . We also believe that the amount of pleasure a student derives from his learning of mathematics depends on the extent to which he finds opportunity for creative activity.
>
> . . . . . .
>
> Since we believe that interest is a necessary condition for learning we have set the development of mathematical ideas in situations which are inherently interesting to young people. . . . .. We have tried to provide in our material an intellectual challenge for a wide range of ability. . . . There is a time in the students' mathematical career when he needs to bring rigor into the pursuit of mathematics, and the mathematics he learns prior to this time must be so organized that when rigor is finally introduced it will not be necessary for him to throw out anything he has learned at an earlier level. One of the ways in which the mathematician contributes heavily to these courses is the establishment of this sound mathematical background.[6]

---

[5]UICSM, "Unit One: The Arithmetic of the Real Numbers," Board of Trustees, University of Illinois, 1958-1959, p. i.

[6]Ibid, p. i-ii.

The basic operating principles of the group have been summarized in the following three theses by the UICSM Project Staff. First, a consistent exposition of high school mathematics must be found. The traditional curriculum is riddled with inconsistencies, and the student who is interested in mathematics must learn it almost in spite of teacher and text. Second, high school students are profoundly interested in ideas; they are speculative and like working with abstractions. The high school student is not motivated to learn mathematics because of potential needs for it in his career some time in the future, but because it stimulates his imagination and gives him access to a kind of intellectual adventure which is enticing and satisfying. Third, manipulative tasks should be used primarily to cast light on basic concepts. Although there is a need for dexterity in carrying out routine tasks, a curriculum based on such tasks is not attractive to high school youth.[7]

III. The Basic Outline of the Course

The UICSM courses are in a state of continual revision through the expression of teachers' ideas, students' reactions, samples of their work, and the complaints and praise of parents. As reported in School Life,[8] the official journal of the Office of Education, the following is a brief outline of the major topics presented in the courses.

---

[7]UICSM Project Staff, op. cit., 458.

[8]Snader, op. cit., p. 11.

The First Course

- Distinction between numbers and numerals
- Real numbers
- Principles of real numbers
  - Associativity
  - Commutativity
- Inverse operations
- Relation of inequality
- Numerical variables (pronumerals)
- Generalizations about real numbers
- Notations and some concepts of the algebra of sets
- Solution of equations
  - Linear
  - Quadratic
- Solution of "worded" problems
- Ordered pairs of numbers
- Graphing equations and inequations

The Second Course

- Sets and relations
- Measures of intervals
- Arcs, angles, polygons, and circles

The Third Course

- Mathematical induction
- Exponents and logarithms
- Complex numbers
- Polynomial functions
  - Factor theorem
  - Synthetic division
  - Curve tracing

The Fourth Course

- Circular functions
  - Winding functions
  - Periodicity
  - Evenness and oddness
  - Monotoneity
  - Analytical trigonometry rather than triangle solving
  - Inverse circular functions
- Deductive theories
  - Abstraction of postulates from a model
  - Deduction of theorems from these postulates without reference to models
  - Reinterpretation of the theory to yield information about other models
- Analytical geometry

IV. The Methodology of UICSM

The members of the UICSM have been strong in their beliefs concerning how students learn mathematics. They felt that the best of curriculum plans are ineffective if teachers use inappropriate techniques. The teacher must foster within his students the belief that mathematics is not a complete activity, but an activity to which each student can contribute. This belief can be cultivated by actual creative experiences in mathematics.[9] As Max Beberman, the director of the Project, pointed out,

> Objections to the new programs should not center on questions concerning new subject matter. The real question is whether or not students should understand the skills which are taught in both the conventional and the new programs.[10]

The principle involved in the UICSM program of learning is that of intuitive understanding.[11] The UICSM takes the position that verbalization alone is not enough, and at some stages a student learns better if he is not required to read or produce verbalizations. It may be that the principle has been overemphasized, but the emphasis serves to illustrate in UICSM the avoidance of the use of abstract preachments.

If a teacher is committed to such a discovery approach, the question

---

[9] UICSM Project Staff, op. cit., p. 462.

[10] Max Beberman, "The Old Mathematics in the New Curriculum," Educational Leadership, 19: 375, March, 1962.

[11] Edwin Moise, "The New Mathematics Programs," The School Review, 70: 44, Spring, 1962.

of time becomes important. In the conventional approach it will take only a few minutes to explain to students how to solve a simple equation. It would take the better part of a class period in the UICSM approach. The discovery approach requires much more in the way of preparation on the part of the teacher.[12]

The following are examples which illustrate the kind of teaching espoused by the UICSM.[13] To illustrate the discovery of information by the students, for example, in the treatment of positive and negative numbers. Reasonable interpretations of the symbols used are given to the students, and the students then develop rules for operating with such numbers. The students are not asked to state any rules which are conceived at this stage since any such statements may contain technical errors. In later units there is formulation of these rules, and the students are asked to check these against his previous experience. Teachers are required to avoid verbalizations, and if asked for verbalizations by the students, the teacher is to tell the students that it is their job to make up the rules. In this sort of process the student is led to believe that he is not dependent upon textbooks or teachers to operate successfully. An example of a technique used to allow the students more freedom is related to the amount of formalism found. Much of traditional mathematics taught in high school is extremely formal, but formalism has no relation to the application of mathematics outside

---

[12]Beberman, op. cit., p. 375.

[13]UICSM Project Staff, op. cit., p. 462-464.

the classroom or to mathematics on an advanced level. Students, often very able ones, develop a dislike for mathematics because of this petty formalism. Teachers in the UICSM Project are encouraged to give their students freedom in solving problems. As an example of this in the treatment of equations and inequalities in the first course students are shown several techniques for solving problems of the "story" type. The students are not compelled to use equations to solve the problems. Teachers are to encourage ingenious solutions in conjunction with techniques of using equations. This atmosphere of freedom gives rise to creativity which is indicative of genuine mathematical talent. This atmosphere of freedom does not mean permissiveness on the part of the teacher. The teacher must be keenly receptive to students' ideas and have an attitude of delight in the intellectual adventures of the student.

V. Conclusion

The materials in the UICSM are similar to those taught in traditional mathematics courses. There is an introduction of the more modern concepts such as set theory and the terminology and ideas used in conjunction with the use of sets. The real emphasis is placed upon the approaches and techniques of teaching. The progression through the mathematics covered in a single year proceeds at a slower pace, but materials are studied more thoroughly. There appears to be less repetition of materials in succeeding courses than is found in traditional high school mathematics courses. The Teachers' Commentary which is used as a teaching guide to each unit of material is as long as the text itself, and techniques

to be used by the teachers are carefully prescribed. Some of these approaches and techniques are also to be found practiced by teachers in the traditional courses. The UICSM seems to consolidate these valuable techniques along with valid modern ideas in an effort to produce a more sound approach to the teaching of mathematics on the secondary level.

The next chapter will examine the work being done by the School Mathematics Study Group to develop a new mathematics curriculum for the secondary school in the same manner as this chapter has examined the work of the University of Illinois Committee on School Mathematics.

CHAPTER IV

THE SCHOOL MATHEMATICS STUDY GROUP

One of the most recent attempts to create a new mathematics curriculum was made by the School Mathematics Study Group. It has been a crash program in which the group has operated on the premise that if a single year were saved in improving the mathematics curriculum, the large expense incurred by their program would be justified.

I. The History of the Program

The School Mathematics Study Group began its work at Yale University in the summer of 1958 under a grant from the National Science Foundation. The director is E. G. Begle, formerly of Yale University and now at Stanford University. At the start the writing team consisted of forty people about half of whom were teachers in secondary schools and the other half mathematicians from Universities. This collaboration between mathematicians and classroom teachers has reflected in the fundamental policy of SMSG ever since.[1] The SMSG began as an experimental program with a variety of short units chosen for their mathematical value, their importance to general education, and their interest to students.[2] At the first writing session at Yale

---

[1] Edwin Moise, "The New Mathematics Program," The School Review, 70: 87, Spring, 1962.

[2] E. G. Begle, "The School Mathematics Study Group," The Bulletin of the National Association of Secondary School Principals, 43: 29, May, 1959.

University there was a division of labor scheme with one writing group for each of the four high school grades. Consultation was held between the writing teams to avoid serious duplication. In the summer of 1959 all four groups met again at the University of Colorado and finished four textbooks. Each of the textbooks was tried out at about 70 experimental centers throughout the country. Consultants were available to help the teacher who had no special training except for those participating in the writing project. The teacher using the materials in the fall of 1959 had not even seen the books until a week before the opening of school.[3]

During the 1959-1960 school year SMSG assigned seven of its experimental centers to Oklahoma.[4] These involved the use of the texts for grades nine, ten, and eleven. Twenty-three school systems were included, with 42 teachers, 84 classes, and approximately 2,500 students. Eight college staff members served as consultants. The participants in the SMSG program in Oklahoma considered it very successful. Teachers reported more interest and understanding on the part of the students than was usual in the traditional program. School administrators and parents were enthusiastic about the program and the results obtained.

The results of reports from the experimental centers formed a basis for the revisions made at Stanford University in the summer of

---

[3]Moise, op. cit., p. 88.

[4]James H. Zant, "Improving the Program in Mathematics in Oklahoma Schools," The Mathematics Teacher, 54: 597, December, 1961.

1960. This revision was placed on the market for use in the 1960-1961 school year. Again favorable responses were received from participants in the 1960-1961 program. The 1961-1962 circulation of these texts approximately tripled the 150,000 used during the first year.[5]

The existence of teaching materials written for the classroom by the SMSG should make it possible for private and independent publishers to move rapidly into a complete revision or writing of new commercial textbooks.[6] The SMSG program has as one of its objectives the filling of this transition period and to indicate a point of view in the writing of commercial textbooks.

II. The Philosophy of the SMSG

Two chief aims of the basic operating philosophy of the SMSG as stated by Begle, director of the group, are to improve the teaching of mathematics in the secondary school and to interest a greater number of students to study mathematics and to insure the mathematics studied is more adequate for life in the world today. In considering the problem of the improvement of the mathematics curriculum the SMSG was guided by the following points:

1. No one can predict exactly what mathematical skills will be important and useful in the future.

2. No one can predict exactly what career any particular student will choose when he leaves school.

---

[5]Moise, op. cit., p. 88.

[6]Zant, op. cit., p. 578.

3. Teaching which emphasizes understanding, insight, and imagination, without neglecting basic skills, is the best for all students of whatever ability and makes the best preparation for any vocation which uses mathematics.

4. An understanding of the role of mathematics in our society is essential for intelligent citizenship.

5. Any normal individual can appreciate some, at least of the beauty and power of mathematics, and this appreciation is an important part of a civilized person's cultural background.[7]

III. The Basic Outline of the Course

The content of new courses and the ways in which each new course differs from the traditional as summarized by the chairmen of the writing teams for the SMSG mathematics courses in grades nine through twelve are given in the following sections.[8]

The SMSG ninth grade course, First Course in Algebra, is different from the conventional texts in a number of ways. It is based upon structure properties of the real number system and is designed to bring out the nature of mathematics and strengthen the student's conceptions by relating them to basic ideas. Definitions and properties of the number system are carefully formulated, and there is work with simple proofs. The reading material used is designed so the student might discover the ideas involved. The number line and simpler concepts concerning sets are used to help the student in the understanding of ideas.

---

[7]Begle, op. cit., p. 27-28.

[8]John A. Brown and John R. Mayor, 'Mathematical Education Notes,' American Mathematical Monthly, 68: 283-285, March, 1961.

The material covered is practically the same as that covered by conventional first year algebra textbooks. It teaches the student how to perform fundamental operations with real numbers and with variables and how to use certain algebraic manipulations, such as factoring, work with powers and roots, and polynomial fractional expressions. The solution of linear equations in two variables and the solution of quadratic equations in one variable are studied. There is also considerable experience given in the solution of word problems.

The SMSG geometry text in the tenth grade differs from the conventional geometry courses in content, postulational scheme, and manner of treatment. There is no artificial difference made between plane and solid geometry, and considerable attention is given to the second. There is also an introduction to analytic plane geometry. The postulational system is a modification of Birkhoff's, which is the point of view that geometry be arithmetized as much as possible and built upon the student's knowledge of arithmetic, elementary algebra, and his ability to use a ruler and protractor. The use of real numbers in the theory and problems is widespread throughout the textbook. In the use of postulates, definitions, and theorems accuracy in use and statement is emphasized. In other respects the text is a treatment of Euclidean geometry, and a treatment of the usual topics: congruence, similarity, parallelism, and perpendicularity, area, circles, and construction using straight edge and compass. There is a main sequence of proved theorems, some of the minor ones are left as exercises for the students, and a long list of original proofs as exercises. Intuition, practical, and compu-

tational problems are used as motivational factors in the study of the basic postulates, definitions, and theorems.

The SMSG eleventh grade textbook varies from traditional textbooks in the following aspects. The carefully worded expositions are intended to place greater demands upon the students' ability to learn by reading. A continuation of the study of the number system in a rigorous manner is studied to develop a foundation for better understanding of elementary and advanced mathematics. An emphasis is placed upon the logical structure of algebra which is built on a relatively few fundamental principles. The student is led to make certain discoveries for himself. The nature of a valid mathematical argument is emphasized in all phases of study. The concept of a function is developed throughout the text. Coordinate geometry is introduced early and is used subsequently in the study of trigonometry and to aid in the understanding of logarithms. The treatment of trigonometry emphasizes identities equations, and graphs of trigonometric functions rather than the solution of triangles. Vectors are developed as a mathematical system and are used in the solution of a wide variety of problems. These new approaches are meant to give the students tools to handle unforseen problems which might arise. In some respects the eleventh grade textbook is similar to conventional textbooks. The book gives a review of the basic skills of first year algebra. Its content is basically that found in traditional advanced algebra and trigonometry courses. Practical applications of the material are given the same emphasis as in traditional courses. There are sufficient exercises which have been carefully graded as to difficulty.

The twelfth grade subject matter presented in a textbook entitled Elementary Functions is conventional in its approach to topics in the theory of equations as remainder and factor theorems and methods for finding rational roots. There are included laws of exponents and logarithms and rules for changing base. The chapter on circular functions contains familiar formulas and the consequences of their addition and subtraction, the identities and equations, and inverse trigonetric functions. Elementary Functions uses approaches which are new to twelfth grade mathematics. The concept of a mapping is introduced to aid in the understanding of polynomial, exponential, logarithmic, and circular functions. The treatment of tangents is intuitive, elementary, and rigorous thus permitting an introduction of Newton's method, and applications to maximum-minimum problems and to graphing. Thus the treatment lays the groundwork for the calculus without trespassing. A noval, clear, and thorough explanation is given of exponentials and logarithms. Trigonometry is developed using the mapping idea which is used informally, with full concrete explanations that convey the intent of mathematical thinking.

A topic closely related to the course outline is the technique used in teaching the materials.

IV. The Methodology of the SMSG

The SMSG is a plan for a mathematics curriculum that attempts to match sound mathematics with sound teaching, and the contention that much

of the material is new is not correct.[9] The SMSG organizes the traditional material along logical lines rather than introducing new subject material. Time which heretofore was spent learning an assortment of rules and techniques which were forgotten or confused is spent discovering a few basic principles and considering the logical consequences of them. Thus the student has learned the same rules and techniques as before, but his reaction to them is quite different.

The materials are aimed at the development of concepts of mathematics through the structure of mathematics, not through a series of disconnected manipulative tricks. The algebra student will understand, for example, that $6a+2a=8a$ because of a basic law, the distributive law, and not because 6 apples + 2 apples = 8 apples. This is an important concept from the standpoint of the structure of mathematics.[10]

In geometry other issues are involved. The SMSG textbook simplifies the logic of geometry and there is a transition from algebra to geometry. Thus the course moves at a proper speed and is more easily understood by the student.[11]

The mathematical theory used throughout the courses is simple enough so that definition of the theory is intelligible to the immature student.[12] The SMSG textbooks seldom explain the subtle and often

---

[9]Max Beberman, "The Old Mathematics in the New Curriculum," Educational Leadership, 19: 273, March, 1962.

[10]Isabelle P. Rucker, "Mathematics in High School," Educational Leadership, 19: 370, March, 1962.

[11]Moise, op. cit., p. 96.

[12]Moise, op. cit., p. 99-100.

obscure theorizing behind the methods they use; however, such explanations are found in the teacher's manuals. In trying to develop the concepts, the courses designed by SMSG take care of the range of abilities of students studying them.

In studying the materials available by the SMSG and the vast amount of literature written about it certain conclusions can be made concerning the SMSG program.

V. Conclusion

The inspection of the SMSG textbooks has a tendency to convey the idea that they have neglected the traditional materials. Many of the changes are curricular, but a great deal of emphasis is placed upon treatment of the material. The teacher apparently has greater freedom in the use of the SMSG materials than in the materials of the UICSM Project. The literature tends to indicate that the SMSG plan is receiving the most widespread usage.

The SMSG materials seem to cover more material than would normally be covered in a traditional four year high school mathematics program. It must be emphasized that many of the techniques used in SMSG are also used by teachers of traditional mathematics courses. The SMSG materials are an attempt to bring together new mathematical ideas and teaching techniques in a systemized manner for the maximum benefit of the students. Far reaching consequences in the mathematics curriculum of the secondary schools should result from this highly progressive and hard working group's efforts.

The efforts of the new programs to improve the mathematics curriculum and the teaching of mathematics have certain implications for teacher training. These implications will be studied in the next chapter.

CHAPTER V

IMPLICATIONS OF CONTEMPORARY MATHEMATICS CURRICULUMS FOR TEACHER EDUCATION

The proposed changes in the high school mathematics curriculum imply the need for changes in the teacher training programs also. The significance of this implication has been seen by both the UICSM program staff and by the SMSG staff which are leading the way in research in the area of teacher education.

I. The Need for Changes in Teacher Education

Fehr[1] points out that no other single factor will prove to be so important in carrying out a new mathematics curriculum at the secondary school level as the high school mathematics teachers. Upon them will ultimately rest the success or failure of any proposed program. The availability of teachers who are able and willing, not only to offer instruction in the subject matter included in the curriculum, but to give it the spirit of contemporary thinking is a consideration of utmost importance. Many teachers are convinced that the arguments set forth by groups and individuals interested in curriculum reform are valid, and that these teachers are genuinely interested in bringing their programs of teaching up to highest possible standards. However, the problem of the various types of preparation of mathematics teachers is immediately

---

[1]Howard Fehr, "The Education of Teachers of Mathematics," The Bulletin of the National Association of Secondary School Principals, 43: 170, May, 1959.

confronted:

> Only a small percentage of in-service teachers can possibly have had the up-to-date training required for the task. Only those who have begun their teaching careers recently have had an opportunity to take courses modern in content and point of view, and even most of these teachers have had collegiate mathematics of a most traditional character. It is very difficult to make changes in educational procedure. It is easy to teach what we were taught and to teach it in the way we were taught. Most teachers teach what is in the books in the way it is presented in the books. To effect a change, we need the cooperation of mathematicians, educators, and teachers in producing new courses and new teaching material.[2]

In a panel talk to the New Jersey Section of the Mathematical Association of America, Meder also expressed the need for a change in teacher education:

> The problem of teacher education is twofold. There is the problem of pre-service education: what mathematics should be taught to the undergraduate who is preparing to be a secondary school teacher of mathematics? There is also the problem of in-service education: what mathematics should be taught to the teacher who completed his education some time past, but who needs to bring his knowledge of mathematics up to date in order to modernize the instruction given in his classroom?

An examination of the present training of high school mathematics teachers makes it evident that drastic changes are needed in the area of pre-service education.[3] It is in this area, pre-service education, that will be examined next with respect to what has already been done or proposed.

---

[2]A. E. Meder Jr., "The Education of Mathematics Teachers," American Mathematical Monthly, 66: 805, November, 1959.

[3]Fehr, op. cit., p. 170-171.

II. Pre-Service Education

To produce teachers who will be adequately equipped to deal with the new mathematics curriculum the Commission on Mathematics has stated that it is imperative that the undergraduate programs of teacher training institutions begin at once to provide their potential teachers with a sound background of study of contemporary mathematics.[4]

Many different suggestions have been made in the past few years as to the nature of the curriculum which should be followed by a college student who is preparing to teach high school mathematics. Estes[5] in reviewing the issue of teacher training pointed out that the undergraduate program in mathematics of the Mathematical Association of America and the Board of Governors of the Mathematical Association of America endorsed the following requirements as minimal for high school mathematics teachers: (1) nine semester hours of analysis: three in analytic geometry and six in calculus; (2) six semester hours in abstract algebra: three in introduction (groups, fields, rings, etc.) and the remainder in linear algebra; (3) six semester hours in geometry beyond analytic geometry; (4) six semester hours in the area of probability and statistics; and (5) six semester hours in advanced electives, e.g. theory of numbers, real variables, history of mathematics, topology, and numerical analysis (computers).

---

[4]Commission on Mathematics, Program for College Preparatory Mathematics, (New York: College Entrance Examination Board, 1959), p. 54.

[5]Ronald V. Estes, "A Review of Research Dealing with Current Issues in Mathematics," School Science and Mathematics, 61: 625, November, 1961.

Four other studies by the Cooperative Committee on the Teaching of Science and Mathematics of the American Association for the Advancement of Science, the Mathematics Section of the Midwest Regional State College Conference on Science and Mathematics Teacher Education, the AAAS Mideast Regional State College Conference on Mathematics Teacher Education, and a survey by Arthur Siessal and Robert C. Fisher of 280 mathematics teachers are in almost complete agreement with this list of minimal requirements as determined by Mathematics Association of America insofar as the academic subjects to be included in the teacher education program is concerned as well as the total number of hours of mathematics to be included.[6]

Fehr[7] has proposed a program which is based on the idea that the more advanced mathematics a teacher is going to teach the more mathematics he must know himself. His proposal is for a hierarchy of mathematics courses for teachers at which all start at the bottom, and as the ladder is scaled a minimum height must be reached for a junior high school teacher and a higher level for a senior high school teacher. His proposed hierarchy is first a prerequisite of a four year high school college preparatory mathematics background equivalent to that recommended the Commission on Mathematics of the College Entrance Examination Board. To attain the first level which would entitle a teacher to teach junior high school mathematics he would require eight to twelve semester hours

---

[6]Ibid., p. 627.

[7]Howard F. Fehr, "How Much Mathematics Should Teachers Know?" The Mathematics Teacher, 52: 299-300, April, 1959.

in coordinate geometry, integral calculus, and elementary differential equations; and six to eight semester hours of polynomial, linear, and modern algebra. To attain the level which would enable a teacher to teach senior high school mathematics an additional six to eight semester hours of geometry and six semester hours of permutation, combination, probability, and statistical reference would be required.

In spite of the great interest which is evidenced by these studies in the area of pre-service education, a study by Schumaker[8] appears to insert a pessimistic note. The purpose of his study was to determine the relationship between trends in teacher education and the reports and reccommendations of committees and individual educators between 1920 and 1958. Schumaker drew three conclusions from his results. First, the implementation of any desired changes in teacher education is likely to be a long slow process. Second, the secondary school mathematics curriculum is a principal factor in determining teacher-education curriculum. Third, in the absence of definite changes in the secondary school mathematics there is little likelihood that any change will take place in teacher education.

This evidence would appear to indicate that teachers trained in the new mathematics may not be immediately forthcoming from the teacher training institutions. According to the Commission on Mathematics[9] other

---

[8]John A. Schumaker, "Trends in the Education of Secondary School Mathematics Teachers," The Mathematics Teacher, 54: 413-422, October, 1961.

[9]Commission on Mathematics, op. cit., p. 56.

possible difficulties which may be encountered in pre-service training of teachers. The graduate courses in mathematics have traditionally been designed to bring the student who has ability to the frontiers of mathematical knowledge as rapidly as possible. This is as it should be for those interested in mathematical research. However, these courses may not be suitable either in content or level of difficulty for the secondary school teacher who needs to place more emphasis on a broad understanding of the elementary aspects of a field of study and less emphasis upon the mastery of advanced details or complicated proofs. For the teacher the importance of making certain fine distinctions, the need for precise terminology and adequate symbolism and an appreciation for mathematical rigor are all more important than the precise content of a course, since the content itself will be of less professional significance to the teacher. Close cooperation between the departments of education and the departments of mathematics of the colleges and universities is mandatory if courses of the correct content and difficulty are to be provided for students who plan to teach mathematics.

In addition to the problems involved with pre-service education of teachers there remains the problem presented by the vast number of teachers who have already completed their education, and now find themselves inadequately prepared to teach the new mathematics curriculum.

III. In-Service Training

A great many kinds of in-service education for high school mathematics teachers have been proposed and already innovated to help prepare

the present teachers to teach the new curriculum better. Both UICSM and SMSG have quite intensive in-service programs related directly to their own materials. Other programs of in-service training are sponsored by colleges and universities and by private industry as well.

Brown[10] reported that a group of about 50 specialists in the field of mathematics met in Washington D. C. in March, 1960 at a meeting called by the Office of Education and the National Council of Mathematics Teachers specifically to consider the problem of in-service education of teachers. From this meeting came 16 suggestions which would be found in any good program of in-service training. These characteristics are based on the idea, however, that no single set of standards will fit the situations found in all local schools.

1. It meets the need of all persons responsible for the quality of instruction in mathematics.
2. It is schoolwide or systemwide.
3. It coordinates various school levels to insure continuity of instruction.
4. It is continuous, and is adjustable to the needs of the teacher and the demands of the curriculum.
5. It recognizes the differences among teachers--differences that reflect pre-service training, length and variety of experience, and grasp of subject matter.
6. It stimulates the teacher's growth and shows him various viewpoints.
7. It permits teachers to take part--through surveys, suggestions, and expression of preferences--in setting up the in-service program.
8. It is developed in a climate that shows administrative support and gives confidence to participants.
9. It takes into account the needs and abilities of all the children--the slow learner, the retarded, the under-achiever, the average, and the gifted.

---

[10]Kenneth E. Brown, "The Mathematics Teacher Needs In-service Education," School Life, 42: 25-26, April, 1960.

10. It is actively supported by the administrator.
11. It is closely integrated with the supervisory and guidance program.
12. It develops a variety of approaches to teacher growth (workshops, seminars, committee work, etc.).
13. Its effectiveness is judged regularly through various techniques.
14. It draws on the resources of the community, the colleges and universities, and the professional staff itself.
15. It takes into account the limitations on teacher time and load.
16. It recognizes and encourages active participants with promotional opportunities, financial renumeration, and professional status.

Fehr suggests that many of these teachers have been away from the classroom mathematics for years which has resulted in their having forgotten how to study as well as allowing their college mathematics to grow very rusty. For these teachers he proposes abbreviated courses in the following areas:

1. Modern analysis including concepts of variables, function, relation, mapping, sentences, propositions, and point-set theory. Especially the setting up of functions and their inverses.
2. Modern algebra including matrices, groups, rings, fields, number systems, Boolean algebra, and vectors, and also polynomial algebra.
3. Modern geometry and its foundations and the study of several types of geometries including a finite geometry.
4. Symbolic logic, axiomatic foundation, and nature of proof.
5. Probability, including combinational analysis of both finite and continuous variables.
6. Mathematical statistics, stressing statistical inference, tests of hypotheses, and correlation theory.[11]

The next question to be considered is the one of how these courses

---

[11]Fehr, op. cit., p. 172-173.

can best be presented to teachers. Some of the possible ways include full-time study, summer sessions, conferences, continuing study plans, and independent study. Each of these possibilities has its advantages as well as its disadvantages.

The National Science Foundation has instituted Academic Year Institutes which afford a relatively small number of teachers the opportunity to benefit from more intensive study over a period of nine months. The first two Academic Year Institutes were held in 1956-1957 at the University of Wisconsin and at Oklahoma A and M. In 1957-1958 there were 16 such institutes, and by 1959-1960 there were about 30 Academic Year Institutes throughout the United States. Each institute has about 50 participants each of whom receives a maximum stipend of three thousand dollars plus tuition costs and allowances for dependents, books, and travel. The benefits of these programs are not limited to the teachers attending who receive renewed interest, new insights, and a feeling of confidence, but they are extended to the university teachers who become acquainted with the problems and needs of the high school teachers and to the attending teacher's colleagues who are inspired by his experience and enthusiasm when he returns to his classroom.[12]

At Ohio State University an analysis was made of the effect of Academic Year Institutes on 66 experienced mathematics teachers in four institutes. Mayor[13] reported three conclusions from the study. First,

---

[12]Burton W. Jones, "Academic Year Institutes," The Bulletin of the National Association of Secondary School Principals, 43: 179-180, May, 1959.

[13]John R. Mayor, "Mathematics Education," Educational Leadership, 19: 403, March, 1962.

teachers acquired greatly increased interest in the study of mathematics and in taking further graduate work for advanced degrees. Second, the institute did not attract the teacher away from high school level teaching. Third, what the teachers learned at the institute brought about noticeable improvement in their own teaching. An additional conclusion derived from this analysis was that while the participants in the Academic Year Institutes returned home willing to contribute to the improvement of the mathematics programs in their schools, they had little effect on the other teachers.

Another type of plan features 'release-time' study for teachers. In this plan, which is known as the Syracuse Plan, the teachers teach approximately half their normal teaching load, study one-half time at a local university, receive their usual full-time salary, and do not pay the costs of tuition. One distinctive feature of this plan as it is described by Davis[14] is that it selects only the strongest and best teachers in their respective systems. This plan is financed partly by the Alfred P. Sloan Foundation and partly by the participating community.

Summer school courses, according to the Commission on Mathematics,[15] are more readily available to a larger number of teachers than either Academic Year Institutes or release-time plans, but they too have some serious drawbacks. Not all colleges and universities have yet begun an effective program of cooperation between the education and mathematics

---

[14]Robert Davis, "A University Assists in an In-Service Program," The Bulletin of the National Association of Secondary School Principals, 43: 187-189, May, 1959.

[15]Commission on Mathematics, op. cit., p. 51.

departments to provide courses specifically designed for teachers rather than for students who will do research, and this cooperation is essential in planning summer school courses for teachers.

One very interesting summer school program was undertaken in 1945 by General Electric. During recent summers 150 high school mathematics teachers have taken six week, all expense, graduate credit courses at three universities under this program undertaken by General Electric. Some 2200 teachers have graduated from these programs in their first 14 years of operation. Rietz[16] described the General Electric program this way. General Electric first started its summer fellowship programs when high school mathematics and science teachers were asked what would enable them to do a better job, and the response of the teachers was (1) to know more about what they were teaching, and (2) to know how the subject matter was going to be useful to their students in later life. In 1958 General Electric embarked on an additional and somewhat different program specifically designed to meet the need for teachers when more advanced courses and some of the more modern mathematics were introduced in the high school curriculum.

The UICSM Project Staff has conducted an intensive summer school program at Urbana for the teachers who will be using their materials the following year. In 1961 there were 225 teachers enrolled in this program

---

[16] G. A. Rietz, "Industry Lends a Hand," The Bulletin of the National Association of Secondary School Principals, 43: 182-183, May, 1959.

under National Science Foundation Sponsorship.[17] The purpose of these summer schools has been to strengthen the teachers' advanced mathematics background and to improve their teaching techniques by giving them the opportunity to observe skilled teachers in demonstration classes. These demonstration classes proved to be one of the most valuable parts of the training, but they also proved to be very difficult to arrange. A solution to this difficulty of having demonstration classes available for observation during the summer was described by Hendrix and Sims.[18] The UICSM developed 18 films tracing the development of one topic through the entire course. The teacher-training films are designed to replace 20 sessions of live demonstration classes which were a part of the four week intensive teacher training conference. The purpose of the films is two-fold: first, to prove the material is teachable to junior high school children, and second, to present vivid and repeated illustrations of a teaching method and teaching devices known to be successful with UICSM First Course materials. These films were made in actual classrooms with three cameras (one facing the teacher, one facing the front of the class, and one on the profile of the class). Each film is an edited sequence of the three which are shot simultaneously. Although the UICSM has experienced great success with using these films in their teacher training program, the many requisites make the production of the films a difficult

---

[17]William T. Hale, "UICSM's Decade of Experimentation," The Mathematics Teacher, 54: 617, December, 1961.

[18]Gertrude Hendrix and Byrl Sims, "The UICSM Teacher Training Films," The American Mathematics Monthly, 67: 686, August-September, 1960.

task. In addition to a teacher who is familiar with the course content and experienced in handling the situations likely to arise in class, and a programmer who knows which topics a new teacher will need help with, it is also necessary to have a class which is strong enough not to require remedial teaching of arithmetic interspersed with the demonstration material and another class taught by the same teacher which can be kept about two weeks ahead of the film class. In spite of the difficulties in producing films of this type, it may be possible for others to be made which are not specifically oriented to the UICSM materials.

For many teachers the attendence at even full length summer sessions is impossible. The Commission on Mathematics[19] suggests several possibilities which would meet the needs of these teachers in obtaining in-service training. Regional groups of teachers have held short summer conferences or institutes, or workshops lasting from three or four days to a week. In some places study groups have been organized to meet once a week or every two weeks or once a month for somewhat less formal programs of lectures, seminars, or other kinds of cooperative study. In localities near a university it is often possible to arrange a special lecture series at a time convenient for high school teachers. Professional organizations also provide opportunities at their national and sectional meetings for sessions which can be informative and stimulating though necessarily brief and sporadic. In some school systems the number of teachers desiring to take the same mathematics course is large enough to make it convenient and economical to bring a college instructor to the high school for off-

---

[19] Commission on Mathematics, op. cit., p. 52-53.

campus courses.

For teachers who cannot arrange a program provided by the shorter training sessions, it is still possible to formulate a plan for independent study. The Commission on Mathematics has prepared a number of appendices to accompany their report which contain the answers to many questions which independent study groups will have. Another source of information for this kind of a group was being prepared in 1959 by the Commission on Mathematics and the School Mathematics Study Group.[20] These guides list specific readings with commentary and indicate how to read the bibliographical listings and what to look for in them.

In the field of pre-service training steps are just beginning to be taken by the colleges and universities to improve the quality of the education of future high school mathematics teachers. In the area of in-service training more progress has been made, and it should prove to be relatively easy for interested mathematics teachers to find ways to further their own education in the areas they most desire help. The implications for teacher education which are to be found in relation to the new curriculum in high school mathematics are indeed far reaching.

In the final chapter the findings of this paper will be summarized and the conclusions drawn will be discussed.

---

[20]Commission on Mathematics, op. cit., p. 53-54.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

I. Summary

The developments which have taken place in the mathematics curriculum since 1950 are bringing a new era in the teaching of secondary school mathematics. It was the purpose of this study to trace the development of the new mathematics curriculum since 1950, to examine in detail the course outlines and teaching procedures conceived by the UICSM and the SMSG, and to survey the implications for teacher training involved with the changes in the high school mathematics curriculum.

The review of the literature showed that the development of the new mathematics curriculums was created by two conditions (1) expanded use of mathematics in more areas of our society and (2) the failure of traditional mathematics with its roots in the past to keep up with the demands of a rapidly changing technological world. The failure of secondary school mathematics to keep pace with the modern trends in mathematics was caused by a lack of communication between pure mathematicians who were dealing with developments in modern mathematics and secondary school teachers who were dealing with problems of an educational nature.

During the 1950's the need for a change in the mathematics curriculum was recognized by teachers, mathematicians from colleges and from industry, and scientists who joined forces to study objectively the current mathematics curriculum. Out of these joint efforts came different plans and materials for revitalized mathematics curriculums. Two plans,

the SMSG plan and the UICSM plan, for mathematics curriculum were examined in this study.

The earliest plan to create a new mathematics curriculum was a cooperative effort of the College of Education, the College of Engineering, and the Department of Mathematics at the University of Illinois. This group, the University of Illinois Committee on School Mathematics, saw the traditional mathematics curriculum riddled with inconsistencies. The plan developed by the UICSM placed emphasis upon basic concepts, and manipulative tasks were used only as an aid to learning the basic concepts. The techniques used in teaching the material were highly important in the Illinois method, and the main emphasis was placed on the discovery method of learning. The student is led away from reliance upon rules, textbooks, and teachers and encouraged to develop his own rules and methods. The materials taught in the Illinois plan are similar to those taught in traditional mathematics courses, however, there is an introduction of more modern concepts.

The most recent large scale effort to develop a more modern mathematics curriculum was begun in 1958 by the School Mathematics Study Group as a crash program which was provided funds by the National Science Foundation. This project was a joint effort of mathematicians and teachers of secondary school mathematics. Early try-outs of the SMSG materials appeared to be successful, and revisions of the material were made based on the results of these early classroom trials. At the present time the SMSG mathematics curriculum plan is the most widely accepted of the new mathematics curriculums which break with the traditional mathe-

matics taught in the high school. SMSG, although it uses more contemporary mathematics in its courses than the UICSM, also places strong emphasis on discovering basic principles of mathematics and on considering the logical consequences of these basic principles.

Since the ultimate success of any proposed change in the mathematics curriculum rests with the ability of the teachers to use the new materials and the new methods of teaching, a two-fold problem arises. The undergraduate training of prospective teachers must provide a broader mathematical background and an in-service training program will have to be effected to introduce the new ideas and methods to those already teaching.

II. Conclusions

The efforts of the SMSG and the UICSM have produced plans which have their own particular stamp of originality in their approach to teaching and in their content of subject matter. The goals of these plans are the same although the approaches are different. The UICSM and the SMSG plans for revision of the mathematics curriculum emphasize the unifying ideas of structure, measure, graphical representation, systems of numeration, properties of numbers, terminology, operations and their inverse logical deductions, and valid generalizations.

A second conclusion which can be made from this study is related to teacher education. The college and university courses which are being taught to undergraduate students preparing to teach high school mathematics are too often not suitable either in content or in level of

difficulty. The prospective teacher needs a broad background including precise terminology and adequate symbolism rather than a mastery of advanced details and complicated proofs.

A final conclusion can be drawn in regard to in-service training. There are a number of possibilities for in-service training of teachers who are inadequately prepared to teach the new mathematics curriculum, and any particular school system can fulfill the needs of its teachers by some combination of these possibilities. Academic year institutes, release-time programs, summer schools, conferences or workshops, or programs of continued study may be selected as the most promising type of in-service training by a school system depending upon such factors as the size of the school system, its proximity to a college, and the mathematical background of the teachers in the system.

III. Implications

There are implications to be considered involving students, teachers, and the administration of a school intending to introduce a plan for a revised mathematics curriculum. The present curriculum and teaching methods being used should be examined to see if they are meeting the needs of the students as well as they could be met by a new mathematics curriculum. If critical examination of the present curriculum reveals the desirability of adopting a revised curriculum, certain other factors must be considered. The administration must determine the ability and willingness of the present staff to teach a revised mathematics curriculum and if necessary provide the additional training they

need to do so. It is a further responsibility of the administration to determine the grade level at which the new program should be introduced and to coordinate the program so that a student will make a smooth transition from the elementary school to the junior high school to the high school without repetition or omission of important materials. It may also be necessary to educate the parents and the community to the need for modern mathematics in the public school curriculum.

There appeared to be a significant ommission in the literature which has implications for the development of a revised mathematics curriculum. Any decision as to the ultimate superiority of the modern approach or the traditional approach to mathematics teaching should be based upon controlled experimentation, and to the present time controlled experiments for this purpose have not been conducted to a significant degree.

The following statement succinctly expresses the viewpoint of mathematics teachers as they face the impending changes in the mathematics curriculum.

> There are, it seems to me, several things which it is essential to recognize. First, change is inevitable in mathematics itself and in the teaching of mathematics at any particular level. Second, although the urge to change is usually resisted somewhat, and perhaps a great deal, the resistance to change is not all of one complexion, and it is certainly wrong to label such resistance uniformly bad. Some of the resistance surely stems from pure adherence to custom and convention--from human inertia. Some of it stems from fear and ignorance of the new ways which are in prospect. But some of it stems from wisdom of experience and from a desire to moderate the force of revolt. Third, nothing is perfect--neither the outmoded present nor the shining promise of tomorrow.[1]

---

[1]Angus E. Taylor, "Convention and Revolt in Mathematics," The Mathematics Teacher, 55: 8, January, 1962.

BIBLIOGRAPHY

Adler, Irving, "The Changes Taking Place in Mathematics," The Leadership Role of State Supervisors of Mathematics, Washington: United States Government Printing Office, 1962, p. 11-27.

Beberman, Max, "The Old Mathematics in the New Curriculum," Educational Leadership, 19: 373-375, March, 1962.

Begle, E. G., "The School Mathematics Study Group," The Bulletin of the National Association of Secondary School Principals, 43: 26-31, May, 1959.

Betz, William, "Five Decades of Mathematical Reform--Evaluation and Challenge," The Mathematics Teacher, 43: 377-387, December, 1950.

Brown, John A. and Mayor, John R., "Mathematical Education Notes," American Mathematical Monthly, 68: 283-285, April, 1961.

Brown, Kenneth E., "The Mathematics Teacher Needs Inservice Education," School Life, 42: 25-26, April, 1960.

Commission on Mathematics, Program for College Preparatory Mathematics, New York: College Entrance Examinations Board, 1959.

Curtis, Charles W., Daus, Paul H., and Walker, Robert J., Euclidean Geometry Based on Ruler and Protractor Axioms, Volume II of the School Mathematics Study Group Studies in Mathematics, New Haven: Yale University Press, 1959.

Davis, Robert, "A University Assists in an In-Service Program," The Bulletin of the National Association of Secondary School Principals, 43: 187-189, May, 1959.

Denbow, Carl, "To Teach Modern Algebra," The Mathematics Teacher, 52: 162-170, March, 1959.

Estes, Ronald V., "A Review of Research Dealing with Current Issues in Mathematics Education," School Science and Mathematics, 61: 623-630, November, 1961.

Fehr, Howard F., "How Much Mathematics Should Teachers Know?" The Mathematics Teacher, 52: 299-300, April, 1959.

Fehr, Howard F., "The Education of Teachers of Mathematics," The Bulletin of the National Association of Secondary School Principals, 43: 169-173, May, 1959.

Gundlach, Bernard H., "How the State Supervisor Can Promote Pre-service and In-service Education for Mathematics Teachers," The Leadership Role of State Supervisors of Mathematics, Washington: United States Government Printing Office, 1962, p. 63-79.

Hale, William T., "UICSM's Decade of Experimentation," The Mathematics Teacher, 54: 613-616, December, 1961.

Hendrix, Gertrude and Sims, Byrl, "The UICSM Teacher Training Film," The American Mathematical Monthly, 67: 686-687, August-September, 1960.

Jones, Burton W., "Academic Year Institutes, "The Bulletin of the National Association of Secondary School Principals, 43: 179-181, May, 1959.

Luce, R. D., Some Basic Mathematical Concepts, Volume I of the School Mathematics Study Group Studies in Mathematics, the General Electric Company, 1959.

Mayor, John R., "Mathematics Education," Educational Leadership, 19: 395-405, March, 1962.

MacLane, Saunders, "The Impact of Modern Mathematics on Secondary Schools," The Mathematics Teacher, 49: 66-69, February, 1956.

McCoy, M. Eleanor, "A Secondary School Mathematics Program," The Bulletin of the National Association of Secondary School Principals, 43: 12-18, May, 1959.

Meder, Albert E., "Modern Mathematics and Its Place in the Secondary School," The Mathematics Teacher, 50: 418-423, October, 1957.

Meder, Albert E., "Sets, Sinners, and Salvation," The Mathematics Teacher, 52: 434-438, October, 1959.

Meder, Albert E., "The Education of Mathematics Teachers," American Mathematical Monthly, 66: 805-809, November, 1959.

Moise, Edwin, "The New Mathematics Programs," The School Review, 70: 82-101, Spring, 1962.

Northrop, E. P., "Modern Mathematics and the Secondary School Curriculum," The Mathematics Teacher, 48: 386-393, October, 1955.

Price, G. Baley, "Progress in Mathematics," Education Digest, 27: 20-23 February, 1962.

Rietz, G. A., "Industry Lenda a Hand," The Bulletin of the National Association of Secondary School Principals, 43: 181-186, May, 1959.

Rosenberg, Herman, "Great Challenges of Mathematics Education," The Mathematics Teacher, 55: 360-368, May, 1962.

Rosskopf, Myron F., and Exner, Robert, M. "Some Concepts of Logic and Their Application in Elementary Mathematics," The Mathematics Teacher, 48: 290-298, May, 1955.

Rourke, Robert E. K. "Some Implications of Twentieth Century Mathematics for High Schools," The Mathematics Teacher, 51: 74-86, February, 1958.

Rucker, Isabelle P., "Mathematics in High School," Educational Leadership, 19: 370, March, 1962.

Schumaker, John A., "Trends in the Education of Secondary School Mathematics Teacher," The Mathematics Teacher, 54: 413-422, October, 1961.

Snader, Daniel W., "Secondary School Mathematics in Transition," School Life, 42: 9-13, March, 1960.

Taylor, Angus, "Convention and Revolt in Mathematics," The Mathematics Teacher, 55: 2-9, January, 1962.

University of Illinois Committee on School Mathematics, The Arithmetic of the Real Numbers, Unit One of High School Mathematics, Urbana, Illinois: University of Illinois Press, 1959.

University of Illinois Committee on School Mathematics, The Pronumerals, Generalizations, and Algebraic Manipulation, Unit Two of High School Mathematics, Urbana, Illinois: University of Illinois Press, 1959.

University of Illinois Committee on School Mathematics, Equations and Inequations: Applications, Unit Three of High School Mathematics, Urbana, Illinois: University of Illinois Press, 1959.

University of Illinois Committee on School Mathematics, Ordered Pairs and Graphs, Unit Four of High School Mathematics, Urbana, Illinois: University of Illinois Press, 1959.

University of Illinois Committee on School Mathematics, Relations and Functions, Unit Five of High School Mathematics, Edited by Herbert E. Vaughan, Urbana, Illinois: University of Illinois Press, 1960.

University of Illinois Committee on School Mathematics Project Staff, "The University of Illinois School Mathematics Program", The School Review, 65: 457-465, Winter, 1957.

Wren, F. Lynwood, "What About the Structure of the Mathematics Curriculum?" The Mathematics Teacher, 44: 161-169, March, 1951.

Zant, James H., "A Program for Determining the Mathematical Needs of Engineering Students," The Mathematics Teacher, 43: 91-94, March, 1950.

Zant, James H., "Improving the Program in Mathematics in the Oklahoma Schools," The Mathematics Teacher, 54: 594-599, December, 1961.