

LEVITATED OPTOMECHANICS IN THE FREE PARTICLE LIMIT

by

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## ABSTRACT

Both levitated and tethered systems are excellent platforms for measuring weak interactions. While tethered systems allow for the observation of free mass behavior, they are susceptible to intermodulation effects, which do not affect levitated optomechanical systems. However, levitated systems are typically modeled as oscillators rather than free masses. We have helped fill this gap by producing a levitated optomechanical system that enables observations within the free particle limit. Leveraging the low oscillation frequencies of a magneto-gravitational trap, we employed high-speed imaging techniques to observe behaviors beyond oscillator resonance. These techniques have enabled us to achieve observations just 25 times above the standard quantum limit for a free mass, achieving a displacement sensitivity of  $3 \text{ pm}/\sqrt{\text{Hz}}$ . The first use of a Bessel beam optical ruler was tested for distance calibration of a magneto-gravitational trap. With active feedback cooling, a continuous set of position data was collected with a duration of 48 h. This data was instrumental in evaluating force sensitivity for this system within the free particle limit, reaching a sensitivity of 22 yN. We devised a new mathematical method to characterize force sensitivity for a free mass subjected to stochastic noise. The advancement of these techniques and systems can serve as effective platforms for searching for dark matter candidates and testing quantum collapse models.

## INTRODUCTION

The field of physics has undergone significant changes over the past century. We have discovered that our classical descriptions fail to accurately describe many phenomena observed in nature. The advancements in quantum physics provide a more comprehensive understanding, but they often challenge our intuition. Quantum physics encompasses a wide range of concepts, a particularly interesting one being the Standard Quantum Limit (SQL). Formulated by Vladimir Braginsky in the 1960s, this limit defines the minimum level of uncertainty that is permissible in measurements as described by quantum mechanics, given by

$$\Delta x_{SQL} = \sqrt{\frac{\tau \hbar}{m 2}} \quad (1)$$

where  $\tau$  is the measurement time,  $\hbar$  is the reduced Planck constant, and  $m$  is the mass of the object being observed.

Later, Braginsky discovered a method to surpass this limit [1, 2]. The SQL, represented by Equation 1, offers little intuitive understanding of the subject. Even if we grasp the concept, how can we design an experiment that approaches these limits? In this dissertation, we will explain the free mass SQL, outline how to create an experiment that can get close to this limit, and describe the potential applications of such an experimental setup.

### Levitated Optomechanics

#### Types of Traps

In 1970, Arthur Ashkin became the first to publish results using what is now known as optical tweezers [3]. By employing a tightly focused beam, it is possible to levitate a particle

against the force of gravity. The scattering of photons from this focused beam creates a restoring force that pulls the particle back to the center of the beam. Dielectric particles, in particular, tend to move toward regions of high electric field strength found at the beam's center. The motion of a particle in an optical trap is akin to that of a harmonic oscillator, with frequencies typically on the order of kHz [4, 5]. Common dielectric materials, such as silicon dioxide ( $\text{SiO}_2$ ) [6, 7] and diamond [8], can be manipulated using optical tweezers.

Optical tweezers are versatile tools capable of trapping a wide variety of particles, ranging from the scale of 100 nm [9, 10] to several  $\mu\text{m}$  [11, 12]. Since the inception of optical tweezers, several advancements in trapping technology have been made. Earlier experiments were often conducted in mediums that hindered the reduction of thermal damping, while modern levitated optomechanical systems typically operate in vacuum environments. Achieving such conditions for optically levitated particles can be quite challenging. The optical trapping of dielectric materials has significantly contributed to the exploration of new physics, including the search for millicharged particles [13] and investigations into potential dark matter candidates [14–17].

Another type of particle trap is the quadrupole ion trap, commonly known as a Paul trap [18]. This device uses alternating electric fields to suspend a charged particle within a saddle-shaped potential that oscillates according to the driving frequency of the current source. These alternating fields are necessary due to Earnshaw's theorem, which states that no static fields proportional to  $r^{-2}$  can achieve a stable particle trap. Paul traps can accommodate a wide range of particle sizes [19–21] and have recently demonstrated exceptional ultra-high Q factors of approximately  $10^{10}$  [22]. However, if the objective is to probe weak forces or gravity, Paul traps may not be the best option, as their trapping strength and the requirement for charged particles make it challenging to observe weak interactions.

We may want to focus on trapping particles that are completely discharged. To

achieve this, we can use various magnetic trapping techniques. Similar to optical trapping, magnetic trapping is versatile and can accommodate a wide range of particle sizes, including nanometer-scale [23], micrometer-scale [23–25], and millimeter-scale particles [26]. The trapping mechanism may not rely solely on a magnetic field, such as in magneto-optical traps, where laser cooling combined with a time-varying magnetic field is required, but can effectively trap cold atoms. These traps can even isolate individual atoms [27].

Magneto-gravitational traps utilize a magnetic quadrupole configuration to create a trapping region enhanced by gravity [24]. These traps exploit diamagnetism, a quantum mechanical property of materials where high magnetic field gradients cause repulsion. With the proper setup, no additional beams or power sources are necessary to keep a particle trapped. Furthermore, these traps enable incredibly low oscillation frequencies [25, 28, 29], providing an ideal environment to probe weak forces.

### Landscape of High-Sensitivity Measurements

High-sensitivity measurements are not limited to levitated systems; tethered systems also provide an excellent environment for exploring quantum behavior. Various methods have been employed in attempts to observe behavior around the SQL. In some cases, these efforts have successfully surpassed the SQL through different measurement schemes [30–34]. One of the most notable sub-SQL measurements was achieved by LIGO, which recently exceeded the SQL by utilizing squeezed light [35]. Another instance of surpassing the SQL involved using an optical spring [33], with the ultimate goal of enhancing LIGO’s capability to detect gravitational waves. These systems utilize optical cavities that are necessarily tethered (physically connected) to their surroundings. However, intermodulation effects in these setups can cause problems due to their higher-order modes [36].

An alternative to these tethered systems is levitated optomechanics, discussed in the ‘types of traps’ section, these systems investigate particles that are electrically, optically,

or magnetically levitated, typically in the mesoscopic regime [37]. Trapping particles in a high vacuum environment provides a useful method for studying their dynamics and quantum mechanics [37–39]. Levitated systems benefit from extreme isolation from their surroundings. Furthermore, the absence of tethering allows for movement in six degrees of freedom (three translational and three rotational), reducing the environmental noise affecting measurements. To create an environment suitable for testing quantum mechanics, it is essential to reduce noise in the system so that it becomes quantum limited [40, 41]. In contrast to tethered systems, which allow for the analysis of both oscillator and free mass behavior, levitated systems primarily focus on oscillator behavior. Some measurement techniques that create conditions for free particles have been proposed and tested by temporarily disabling trapping mechanisms [42, 43].

Most particle trapping configurations utilize high oscillation frequencies for microscopic particles. These high frequencies lead to large spacings between the quantum energy levels of the oscillator and help avoid impacts from vibration [44]. Reducing thermal noise to observe quantum behavior in levitated systems presents significant challenges. Despite these challenges, there have been successful demonstrations of cooling to the quantum ground state through techniques such as cavity coupling [45] and feedback control [46].

Oscillators have certain limitations, including a restricted bandwidth associated with their resonance. Nevertheless, these systems serve as fairly ideal force sensors [12, 43, 47, 48]. With careful preparation, sensitivities on the order of attonewtons [49], zeptonewtons [10], and yoctonewtons [50] have been achieved. Furthermore, nano-g acceleration sensitivity has also been demonstrated [11, 28], along with the potential for sensitivity to high-frequency gravitational waves using levitated optomechanics [51].

One promising aspect of levitated optomechanics is its potential to probe weak forces, such as gravity, and search for new physics [52]. The ability to control single-electron charges in trapped particles [13, 25, 53] facilitates experiments that can investigate weak forces.

Furthermore, quantum gravity tests may become feasible by investigating gravitationally induced entanglement [54, 55].

However, studying weak forces presents challenges, particularly regarding the strength of particle traps and the small sizes of the particles. Utilizing larger particles with weaker trapping strengths has both advantages and disadvantages. One advantage is that larger particle sizes help reduce the effects of thermal noise, while the weaker trapping potential allows for easier detection of weak forces. On the downside, traps designed for larger masses and lower restoring forces result in lower oscillation frequencies, making the system more susceptible to low-frequency noise sources, such as vibrations.

Achieving a quantum-limited system is challenging. However, if accomplished, it would enable observations close to the SQL [56] and potentially create new thresholds in force sensitivity measurements. Some proposed experiments have force sensitivities reaching  $2 \times 10^{-21} \text{ N}/\sqrt{\text{Hz}}$  [48]. Table 1 displays measurement outcomes from various groups attempting to achieve low force sensitivity.

| Author           | Year | Vacuum (Torr)         | Force Sensitivity $\sqrt{S_F} \left( \frac{\text{N}}{\sqrt{\text{Hz}}} \right)$ |
|------------------|------|-----------------------|---------------------------------------------------------------------------------|
| Gieseler[57]     | 2013 | $7.5 \times 10^{-6}$  | $2 \times 10^{-20}$                                                             |
| Moore[13]        | 2014 | $1.5 \times 10^{-7}$  | $5 \times 10^{-17}$                                                             |
| Ranjit[49]       | 2015 | $5 \times 10^{-6}$    | $2.17 \times 10^{-16}$                                                          |
| Ranjit[10]       | 2016 | $5 \times 10^{-6}$    | $1.63 \times 10^{-18}$                                                          |
| Monteiro*[11]    | 2017 | $7.5 \times 10^{-7}$  | $4.8 \times 10^{-18}$                                                           |
| Hempston[58]     | 2017 | $1.2 \times 10^{-5}$  | $3.2 \times 10^{-20}$                                                           |
| Monteiro[59]     | 2020 | $7.5 \times 10^{-7}$  | $0.95 \times 10^{-18}$                                                          |
| Kawasaki[60]     | 2020 | $7.5 \times 10^{-7}$  | $1.7 \times 10^{-17}$                                                           |
| Lewandowski*[28] | 2020 | $10^{-7}$             | $9 \times 10^{-18}$                                                             |
| Liang[50]        | 2022 | $5.93 \times 10^{-9}$ | $6.33 \times 10^{-21}$                                                          |

Table 1: Force sensitivity reported as  $\sqrt{S_F}$  for various systems. \* indicates reported values were given as acceleration sensitivity. Force sensitivity was inferred by the reported masses used when determining acceleration sensitivity.

There has been interest in using levitated particles to place limits on collapse models in quantum mechanics [61–64], and in particular the continuous spontaneous localization

(CSL) model [65, 66].

### The Proposal

Most systems mentioned involve particles probed near their respective oscillators' resonance frequencies. These systems face narrow bandwidth limitations given oscillator behavior and often require powered traps for operation. In some instances, the constraints of the trapping environment limit the pressures achieved without feedback stabilization [67].

We propose an alternative method for achieving high sensitivity measurements using levitated optomechanics in the free particle limit. A highly isolated system within an ultra-high vacuum (UHV) environment is achieved by suspending micrometer-scale silica particles in a magneto-gravitational trap. The equation of motion for such a system simplifies to  $F = ma$  when observed at time scales shorter than the oscillation frequency. This concept is not entirely new; for instance, LIGO employs heavy suspended mirrors to form its interferometer, allowing them to be treated as free masses due to their low resonance frequencies [39]. Employing levitated optomechanics helps us avoid unwanted intermodulation effects from higher-order modes that can occur in tethered systems such as LIGO. By simplifying our mathematics to the free particle regime, concepts from gravitational wave detectors can be applied, even though our free mass is both untethered and  $\approx 10^{-16}$  times less massive than the LIGO mirrors.

We aim to explore the free particle limit by leveraging the low oscillation frequencies offered by magneto-gravitational traps and the absence of intermodulation effects in levitated systems. Furthermore, a new approach is taken to evaluate our system's force sensitivity within this free particle limit. Achieving success in these areas would demonstrate the potential of this new regime for studying levitated optomechanics.

## Dissertation Outline

This dissertation is organized into six chapters. Chapter 1 provides the background, context, and motivation for our work in the D’Urso lab. Chapter 2 offers a mathematical framework for the planned experiments that include a derivation of the SQL and examines two sources of quantum noise: radiation pressure shot noise (RPSN) and detector shot noise (DSN). Additionally, thermal noise, harmonic oscillator systems, and various analytical methods applicable to these systems are discussed.

I will describe the main experiment in Chapter 3. This chapter outlines our particle trapping methods, vacuum pumps, and optical systems. It also mentions various parameters that need to be determined, addressed in Chapter 4, showing how each unknown parameter discussed previously is measured experimentally, and how the systems outlined in Chapter 3 are controlled.

Finally, Chapters 5 and 6 present the results of the data collected according to the methods described earlier. This includes comparing several data sets, identifying various behaviors, and discussing expected and unexpected results. This document concludes with insights and an outline of future work and potential improvements for the existing systems.

## A MATHEMATICAL DESCRIPTION

### Chapter Overview

The previous chapter reviewed the current landscape of high-sensitivity measurements. I discussed key concepts such as the SQL, levitated and tethered systems, and force sensitivity. In this section, I will provide a mathematical description of the relevant topics in this dissertation and guide us toward achieving an experimental levitated optomechanical system that can produce free particle behavior.

To begin, we will explore the SQL and its significance in relation to quantum noise sources. Following this discussion, we will examine how thermal noise can be described in levitated optomechanical systems. Our exploration of the SQL and these various noise sources will lead to a damped harmonic oscillator model consistent with a magneto-gravitational trap.

Once we have established this foundation, we can focus on the two main goals of this dissertation. First, we aim to approach the SQL by observing a harmonic oscillator in the free-particle limit. Second, we will work towards achieving a new record in force sensitivity measurements within our levitated optomechanical system. Everything outlined in this chapter will be consolidated into a simulation based on the equation of motion for a damped harmonic oscillator. This simulation will illustrate our objectives and what we hope to achieve on an optical table.

### The Standard Quantum Limit

This section aims to provide a detailed and comprehensive assessment of the SQL and how to interpret it, particularly in the frequency domain.

Vladimir Braginsky first proposed the SQL in the 1960s. It addresses the question:

“How precisely can one measure the position of an object” In our everyday experiences, the uncertainty involved in position measurements is primarily classical. We must shift our perspective from classical to quantum physics to achieve the smallest possible uncertainty in these measurements.

As the uncertainty in position decreases, from using everyday measurement tools like rulers to employing more advanced measurement tools, the sizes approach the quantum regime. In this realm, the uncertainty in position can shrink to the size of individual atoms or even smaller! Photons serve as an ideal measurement tool for such situations; however, their usefulness comes with limitations imposed by the Heisenberg uncertainty principle. This principle restricts the uncertainty of compatible observables, thereby leading to the concept of the SQL.

To reach an approximate limit for the position uncertainty of an object, such as a free particle, we will follow series of straightforward steps.

Begin by imagining a particle in space that is not interacting with any other object. Consider a perfect microscope positioned to capture the most precise image of the particle that can be achieved, and it collects a photon that scatters off the object to determine its location. A photon approaches and scatters off the particle. This forms the basis of what is known as a “Heisenberg microscope” [80]. A diagram illustrating such a microscope can be seen in Figure 1.

The Heisenberg microscope thought experiment is flawed in that it produces a relation that exceeds the true uncertainty limit. The microscope in Figure 1 gives a position uncertainty as  $\Delta x \approx \frac{\lambda}{\sin \theta}$ , where  $\lambda$  is the photon wavelength and momentum uncertainty (along the x-axis) caused by the photon interaction is  $\Delta p_x \approx \frac{h}{\lambda} \sin \theta$  where  $h$  is Planck’s constant. The product of these uncertainties results in the relation  $\Delta x \Delta p_x \approx h$ . This thought experiment produces an approximation of the Heisenberg uncertainty principle, while the true uncertainty relation given by

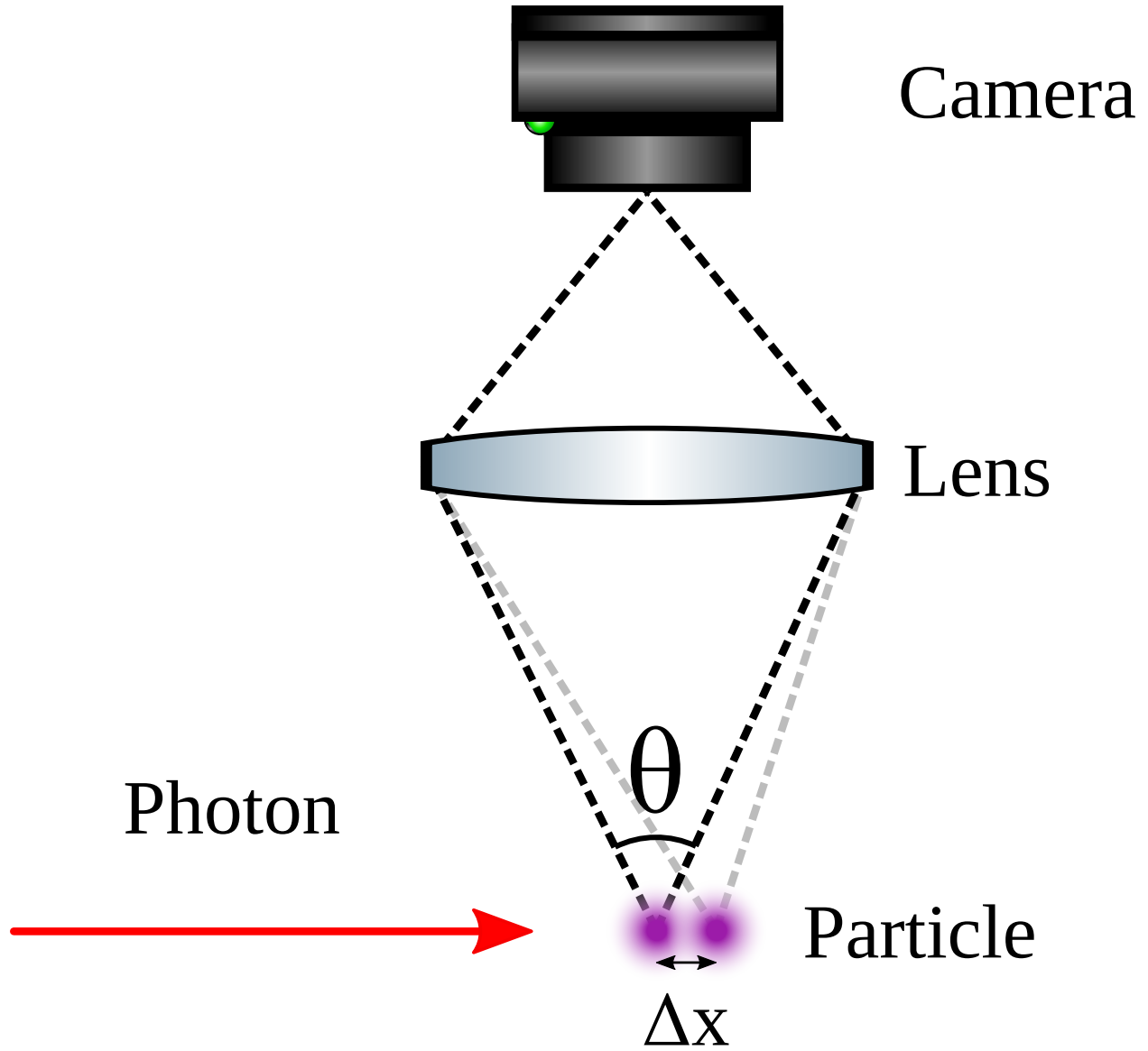


Figure 1: A diagram of Heisenberg's microscope. A single photon displaces a particle, and this scattered photon is collected by a detector (in this case, a CMOS sensor). This interaction results in uncertainty in both position and momentum.

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad (2)$$

where  $\hbar$  is reduced Plank's constant,  $\Delta x$  is position uncertainty, and  $\Delta p_x$  is momentum uncertainty. From this point onward, two things occur. First, the particle is displaced

due to the momentum transfer from photon scattering. Second, the microscope collects the scattered photon, which provides the position of the particle when the photon was scattered.

Let us assume that the microscope is perfect, meaning that the product of position uncertainty and momentum uncertainty meets the Heisenberg uncertainty limit, as shown in Equation 2. Consequently, the initial measurement has a position uncertainty described by

$$\Delta x_{init} = \frac{\hbar}{2\Delta p_{init}} \quad (3)$$

where  $\hbar$  is reduced Planck's constant, and  $\Delta p_{init}$  is the uncertainty of the particle's momentum at the time of scattering. This position uncertainty arises at the moment of scattering off the particle, and the CMOS sensor only observes the particle's location at that time.

If another single photon is sent at the particle after some later time  $\tau$ , the position uncertainty at that time would no longer be solely defined by Equation 3. This is a consequence of the relation  $[\hat{x}(t), \hat{x}(\tau)] \neq 0$ , since a precise measurement of  $\hat{x}(t)$  necessarily perturbed  $\hat{p}(t)$ . Thus, one must also consider the additional uncertainty resulting from the momentum transfer of the first scattered photon. Let's call our initial position uncertainty  $\Delta x_{init}$ , and our additional uncertainty from momentum transfer  $\Delta x_{add} = (\tau/m) \Delta p_{init}$ . To compute the total uncertainty of our position measurements use

$$\sigma_{tot} = \sqrt{\sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \sigma_{x_i} \right)^2}, \quad (4)$$

where  $f$  is the function of interest,  $x_i$  represents the independent variables, and  $\sigma_{x_i}$  is the uncertainty associated with the independent variable  $x_i$ . In this case, apply Equation 4 to our two considerations of position uncertainty, the sum of which is given as  $\Delta x(t + \tau)$ , and arrive at the following:

$$\Delta x_{tot} = \sqrt{\left(\frac{\partial x(t+\tau)}{\partial x} \Delta x_{init}\right)^2 + \left(\frac{\partial x(t+\tau)}{\partial p} \Delta p_{init}\right)^2} \quad (5)$$

$$\Delta x_{tot} = \sqrt{([1] \Delta x_{init})^2 + \left([\frac{\tau}{m}] \Delta p_{init}\right)^2} \quad (6)$$

$$\Delta x_{tot} = \sqrt{(\Delta x_{init})^2 + \left(\frac{\tau}{m} \Delta p_{init}\right)^2} \quad (7)$$

$$\Delta x_{tot} = \sqrt{(\Delta x_{init})^2 + \left(\frac{\tau}{m} \frac{\hbar}{2\Delta x_{init}}\right)^2} \quad (8)$$

Notice how there is now a total position uncertainty, which includes contributions from both the scattering of the photon and the resulting momentum transfer to the particle. This total uncertainty is expressed as a function of our initial position uncertainty,  $\Delta x_{init}$ , at the time of photon scattering. As our initial position uncertainty approaches zero, the total uncertainty—as seen in Equation 8 asymptotically approaches infinity. This outcome is unsurprising; according to the Heisenberg uncertainty principle, a decrease in position uncertainty naturally leads to a proportional increase in momentum uncertainty, contributing to the total uncertainty calculation.

Conversely, if the original position uncertainty increases while reducing the original momentum uncertainty, a similar problem is encountered. Between these two extremes lies an optimal point, a balance between initial position and the additional uncertainty from momentum transfer, that yields the lowest possible value for  $\Delta x_{tot}$ . This is shown in Figure 2. To find this minimum, start by taking the derivative of Equation 8 and setting it equal to zero.

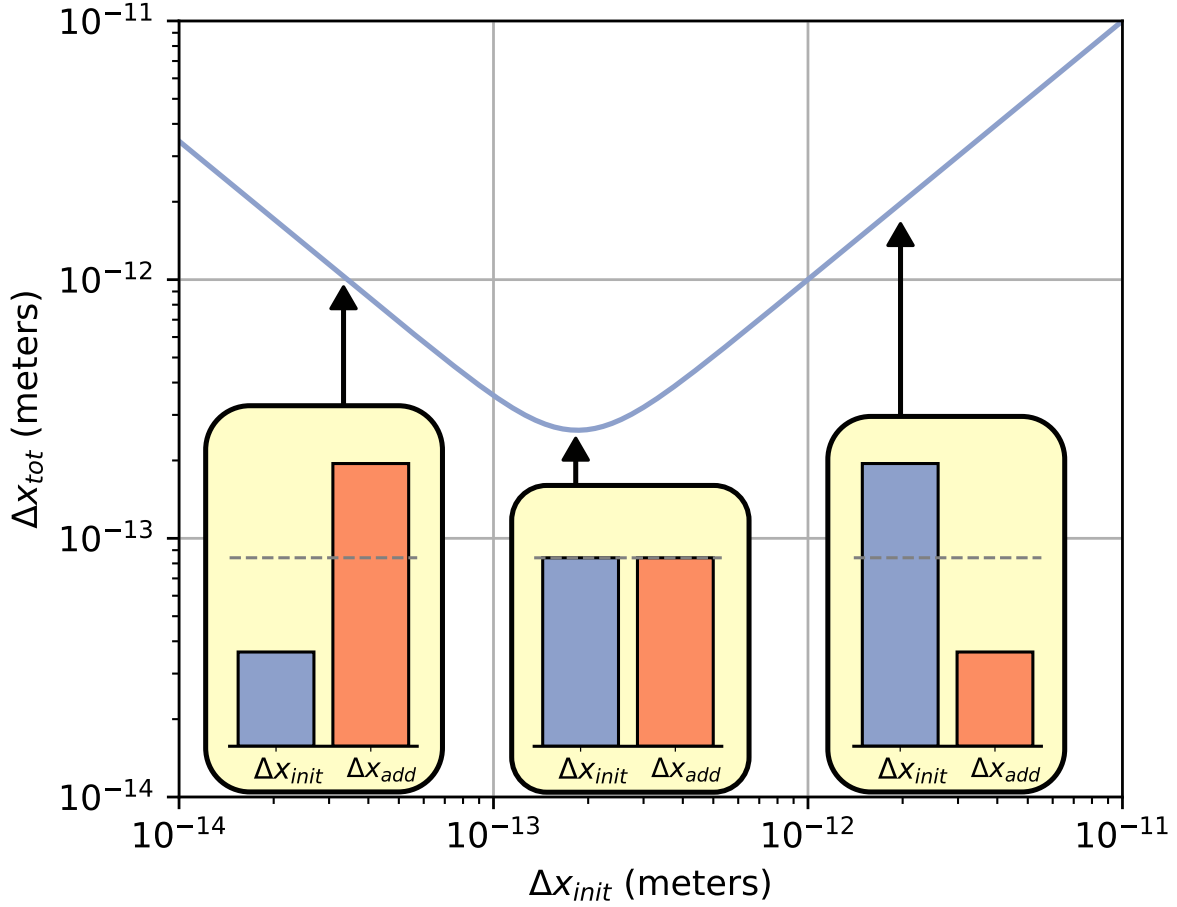


Figure 2: A plot of Equation 8, the total uncertainty as a function of the initial position uncertainty. Since the product of momentum and position must equal  $\hbar/2$  for an ideal Heisenberg microscope, any increase or decrease in the initial position uncertainty must be matched with an adjustment to the initial momentum uncertainty. This relationship results in a minimum in our total uncertainty. This plot was generated using  $\tau = 1 \mu\text{s}$  and  $m = 1.54 \times 10^{-15} \text{ kg}$ .

$$\frac{d}{d\Delta x}[\Delta x_{tot}] = \frac{d}{d\Delta x} \left[ \sqrt{(\Delta x)^2 + \left( \frac{\tau \hbar}{m 2\Delta x} \right)^2} \right] \quad (9)$$

$$\frac{d}{d\Delta x}[\Delta x_{tot}] = \frac{1}{2\sqrt{(\Delta x)^2 + \left( \frac{\tau \hbar}{m 2\Delta x} \right)^2}} \sqrt{2\Delta x - 2 \left( \frac{\tau \hbar}{m 2} \right)^2 \frac{1}{\Delta x^3}} \quad (10)$$

The denominator can be multiplied out, leaving us to solve for  $\Delta x_{SQL}$  in the numerator. The results of this process are given by

$$0 = \sqrt{2\Delta x_{min} - 2 \left( \frac{\tau \hbar}{m} \right)^2 \frac{1}{\Delta x_{min}^3}} \quad (11)$$

$$0 = 2\Delta x_{min} - 2 \left( \frac{\tau \hbar}{m} \right)^2 \frac{1}{\Delta x_{min}^3} \quad (12)$$

$$2\Delta x_{min}^4 = 2 \left( \frac{\tau \hbar}{m} \right)^2 \quad (13)$$

$$\Delta x_{min} = \sqrt{\frac{\tau \hbar}{m}} \quad (14)$$

where  $m$  is the mass of the free particle,  $\hbar$  is Planck's constant, and  $\tau$  is the time between measurements. With the minimum initial position uncertainty recovered, plug this back into Equation 14, resulting in

$$\Delta x_{SQL} = \sqrt{\frac{\hbar \tau}{m}} \quad (15)$$

This concludes the derivation of the SQL and leads to the next topic, quantum noise.

### Quantum Noise

In the previous section, the SQL for a free particle was derived. Let's continue using our 'perfect microscope' where our only interactions involve photons. As shown in Figure 2, the SQL necessitates a specific balance between our uncertainty in position and momentum. This balance depends on both the measurement time, denoted as  $\tau$ , and the mass of the particle,  $m$ . While this information is interesting, it might not be beneficial for our purposes.

A more productive approach is to express the SQL as a displacement power spectral density (PSD), which illustrates the lower limit of position uncertainty permitted by quantum mechanics as a function of frequency. A PSD gives us information about power distribution over various frequency components, and in this work double-sided PSDs are used. Spectral density plots are a common tool used for high-sensitivity measurement experiments in quantum mechanics, making it a reasonable path to explore.

The first step in this process is to convert our findings into a PSD. To evaluate the measure of the variance for the displacement at a given frequency, consider both the position variance  $|\Delta x_{SQL}|^2$  and our measurement time  $\tau$ . Given the time dependence of  $\Delta x_{SQL}$ , it should come as no surprise that our eventual displacement spectrum will scale inversely proportional to frequency, since  $\tau \propto \frac{1}{2\pi f}$ . Thankfully, this conversion is relatively straightforward, resulting in the relationship  $S_D = |\Delta x|^2(2\tau)$ , which denotes the double-sided displacement PSD as  $S_D$ . By substituting in the final result from Equation 14, the PSD for the SQL is recovered, given by

$$S_{SQL} = \frac{2\hbar\tau^2}{m} \quad (16)$$

For convenience and to match existing works in the field of levitated optomechanics, it is preferred to present this expression as a function of angular frequency. This leads to the SQL PSD, which is given by [39]

$$S_{SQL} = \frac{2\hbar}{m\Omega^2} \quad (17)$$

where  $\Omega$  is the angular frequency. A plot of this function can be seen in Figure 3.

This simple representation of the SQL opens the door to our further analysis. The line produced from our SQL PSD may represent a limit, but it does little to offer an

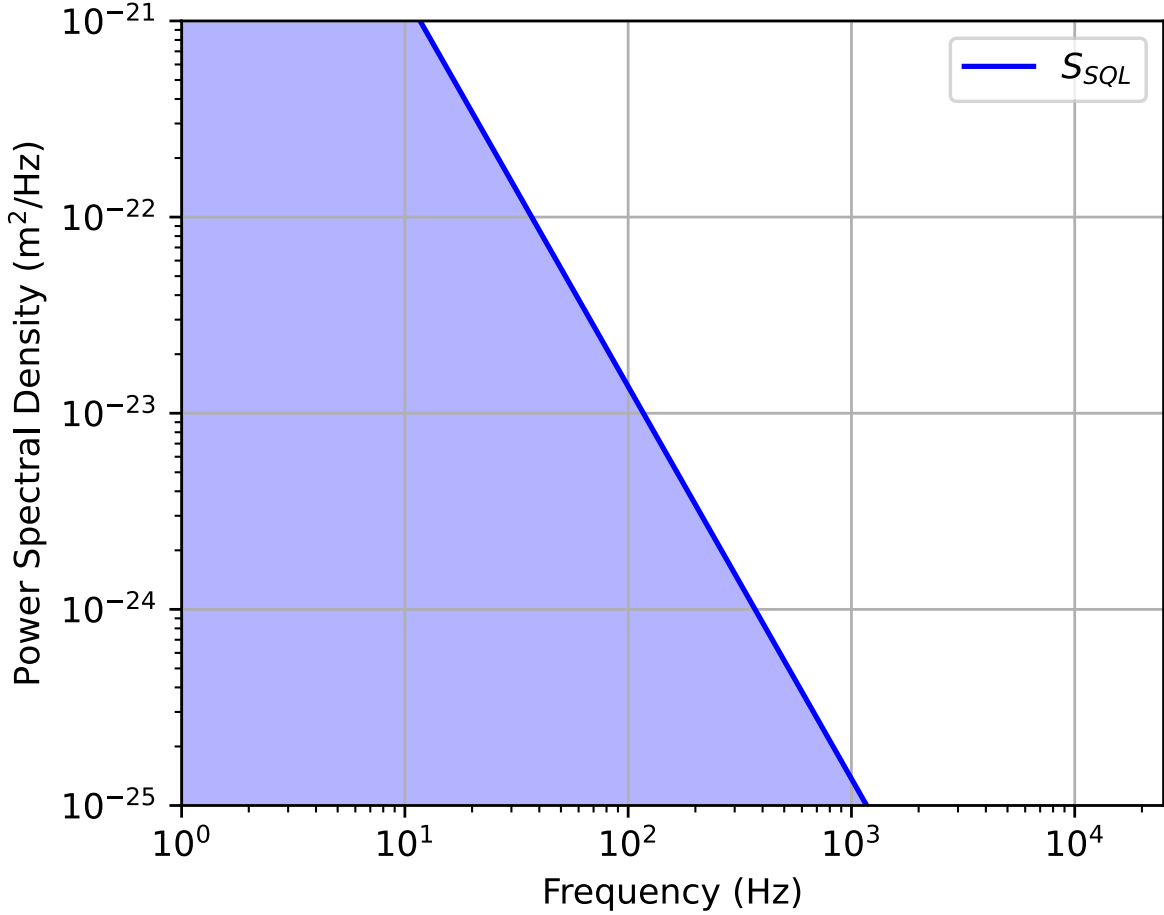


Figure 3: A plot of the displacement PSD for the SQL. Viewed in a log-log scale, a linear slope characteristic of the  $\frac{1}{\Omega^2}$  drop-off expected from the SQL derivation is observed. This highlights that as the frequency of observation increases, the response of our displacement spectrum decreases.

intuitive explanation of what this line represents. This derivation discussed two aspects of measurement: the detection of scattered photons and the momentum transfer caused by those scattered photons. The nuance of this interaction has been overlooked in the SQL PSD, but with some effort, one can recover it and achieve a complete picture of the SQL.

Let's begin with the more complicated of the two interactions: radiation pressure shot noise (RPSN). Initially, a single photon at a time was used in our thought experiment. In practice, however, single photons are inadequate to accurately describe an object's position.

A short burst of photons from a laser can involve millions to billions of photons scattering during a position measurement, depending on the pulse’s duration and the laser’s power. Our detection devices are not instantaneous; they rely on specific exposure times. For example, if your camera collects data for 30 ms, the pixels on the detector capture photons during that period before writing the data to a computer. The collective data from these photons forms the basis for individual position measurements.

At this stage, let’s set some parameters, starting with some numbers that are *conveniently* selected to match the eventual real-world experiment discussed in this dissertation. Consider an approximately 1  $\mu\text{m}$  in diameter spherical object illuminated by 785 nm photons, putting us firmly in the Mie scattering regime. This particle size and illumination wavelength will be used throughout this dissertation. Begin by examining a simplified model of scattering light off a small particle. In Chapter 3 of Bohren and Huffman [85] titled “Absorption and Scattering of Light by Small Particles”, four elements of an amplitude scattering matrix are introduced for a particle lacking particular symmetry, denoted as  $S_j$  (where  $j = 1, 2, 3, 4$ ). This general form can be simplified by making certain assumptions and definitions.

First, we use the same coordinate system defined in Bohren and Huffman for convenience while calculating the scattering probability distribution. Once this calculation is complete, we will rotate our coordinate system to align with that of the eventual experimental setup for further calculations. For this analysis, assume that the scattering particle is spherically symmetric. Additionally, the illuminating beam, propagating along the z-axis, is linearly polarized along the x-axis, as illustrated in Figure 4.

The previously mentioned scattering light parameters result in  $S_3$  and  $S_4$  being zero when considering spherically symmetric cases, shown in Chapter 4 of Bohren and Huffman. For a spherically symmetric object, the electric field of the scattered wave is given by

$$\vec{E}(r, \theta, \phi) = \frac{P}{r} \left( \cos \phi S_2(\theta) \hat{\theta}(\theta, \phi) - \sin \phi S_1(\theta) \hat{\phi}(\phi) \right) \quad (18)$$

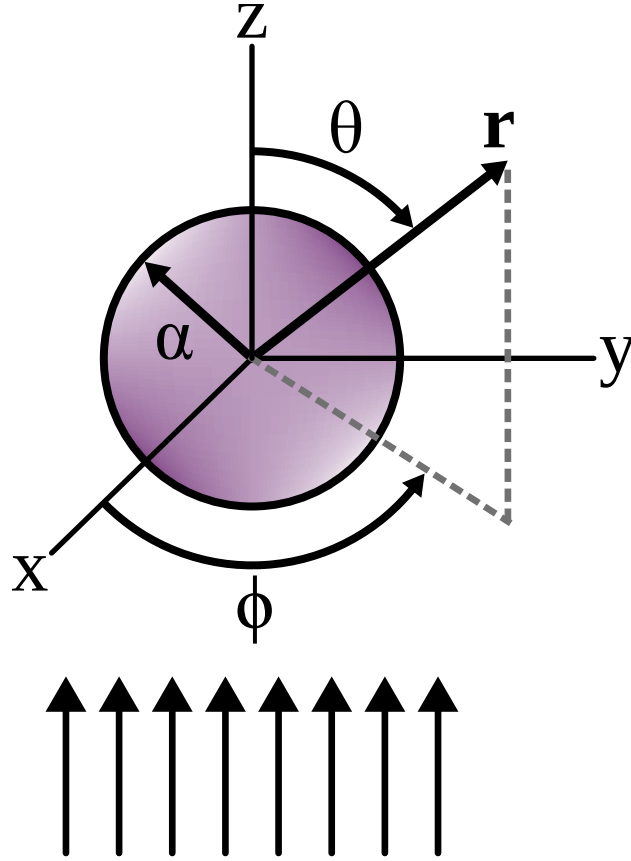


Figure 4: A spherical polar coordinate system with a particle of radius  $\alpha$  positioned at the center. Light propagates along the z-axis towards the particle. Polar coordinates are defined using  $\phi$  from the x-axis, and  $\theta$  from the z-axis, with distance  $r$  from the origin. Incoming light is polarized along the y-axis.

where  $P$  is a proportionality constant that depends on the electric-field strength from the illumination wave, its frequency, and the particle's optical properties, and  $r$  is the observed radial distance from the particle. The vectors that define our spherical coordinates can be given in the form of Cartesian unit vectors  $(\hat{x}, \hat{y}, \hat{z})$  by using

$$\hat{\theta}(\theta, \phi) = [\cos \theta \cos \phi] \hat{x} + [\cos \theta \sin \phi] \hat{y} + [-\sin \theta] \hat{z} \quad (19)$$

$$\hat{\phi}(\phi) = [-\sin \phi] \hat{x} + [\cos \phi] \hat{y} \quad (20)$$

Our main concern is not the electric field but instead the intensity and angular distribution. The intensity, once normalized out, leaves us with a normalized probability distribution for photon angular scattering,  $P(\theta, \phi)$ , which is required for the determination of RPSN. Get there by writing our intensity as

$$I(\theta, \phi) = I_0 (\sin^2 \phi |S_1(\theta)|^2 + \cos^2(\phi) |S_2(\theta)|^2) \quad (21)$$

where  $I_0$  is the intensity of scattered light. Normalizing the integral leads to the probability distribution for photon angular scattering, given by

$$P(\theta, \phi) = P_0 (\sin^2 \phi |S_1(\theta)|^2 + \cos^2 \phi |S_2(\theta)|^2) \quad (22)$$

with

$$P_0^{-1} = \pi \int_0^\pi (|S_1(\theta)|^2 + |S_2(\theta)|^2) \sin \theta d\theta \quad (23)$$

The azimuthal symmetry for the spherical scattering allows us to evaluate our  $\phi$  dependence now, which helps simplify Equation 22. Considering  $\int_0^{2\pi} \cos^2 \phi d\phi = \int_0^{2\pi} \sin^2 \phi d\phi = \pi$ , Equation 22 becomes

$$P(\theta, \phi) = \pi P_0 (|S_1(\theta)|^2 + |S_2(\theta)|^2) \quad (24)$$

Finally, the two unknown elements of our scattering integral,  $S_1$  and  $S_2$ , can be solved computationally. Values were recovered for these scattering coefficients, assuming a homogeneous sphere of  $d = 1.18 \mu\text{m}$  illuminated by linearly polarized 785 nm light and plotted in Figure 5. The numerical calculations for  $S_1$  and  $S_2$  were tested in the Rayleigh regime to ensure consistent results. The range of angles used for the ‘collection window’

comes from experimental design considerations that will be discussed later.

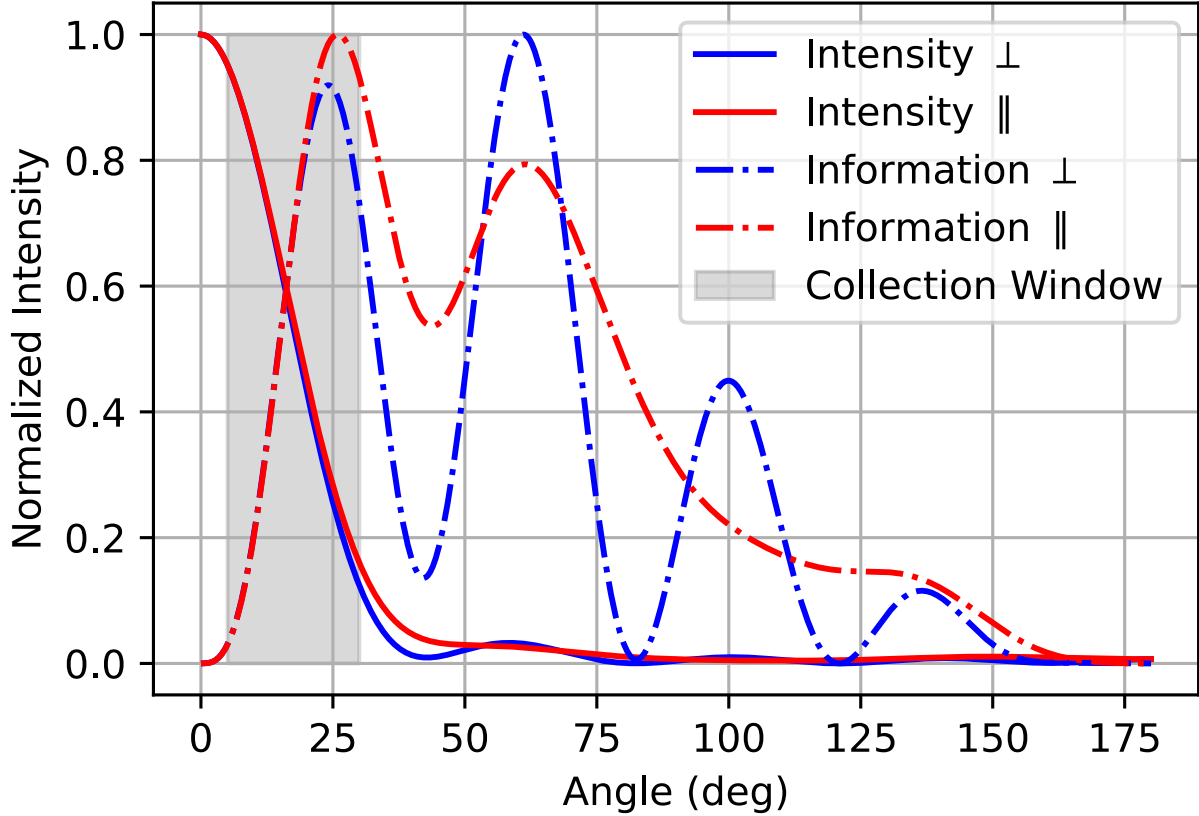


Figure 5: Normalized intensity and information profile for parameters that match our experimental setup, with a particle size of  $1.18\ \mu\text{m}$  in diameter, and illumination done with 785 nm photons. Solved computationally by adapting methods provided by Bohren and Huffman. Collection optics for our experimental setup will limit our window of interest due to limitations in our objective NA and retro-reflector (to be discussed later), and this is shown by the grey section between 5 deg and 30 deg.

This scattering profile lets us recover a useful parameter for the fractional momentum kick provided to the particle from photon scattering. This will be useful later when determining the number of photons scattered from the particle during a measurement. Given the angular profile of photon scattering, only a fraction of the energy transferred to particle momentum will occur in the direction of wavefront propagation. Solving for this leads to

our fractional kick parameter given by

$$\zeta_x = \pi P_0 \int_0^\pi (|S_1(\theta)|^2 + |S_2(\theta)|^2) (1 - \cos \theta) \sin \theta d\theta \quad (25)$$

$$\zeta_x = 0.2264 \quad (26)$$

The scattering of photons off a particle imparts different magnitudes of momentum to each spatial axis. Thus, the energy transfer from an incoming stream of photons can be broken down into Cartesian components. Starting with equations (5a, 5b, and 5c) from [70], and utilizing our derived photon scattering distribution from Equation 22, the expected energy transfer into each translational degree of freedom can be calculated. These equations have been adjusted to fit our standard coordinate system used on our optical table, where light propagates along the x-axis with linear polarization aligned to the vertical y-axis.

$$\langle \Delta E_x \rangle = \frac{\hbar^2 k^2}{2m} \int_{\Omega} P(\theta, \phi) (1 - \cos \theta^2) d\Omega \quad (27a)$$

$$\langle \Delta E_y \rangle = \frac{\hbar^2 k^2}{2m} \int_{\Omega} P(\theta, \phi) (\sin \theta \cos \phi)^2 d\Omega \quad (27b)$$

$$\langle \Delta E_z \rangle = \frac{\hbar^2 k^2}{2m} \int_{\Omega} P(\theta, \phi) (\sin \theta \sin \phi)^2 d\Omega \quad (27c)$$

where  $k = \frac{2\pi}{\lambda}$  with  $\lambda$  being the photon wavelength,  $m$  is the particle mass, and  $\hbar$  is reduced Planck's constant.

The expected change in energy occurs over the timescale of the measurement,  $\Delta t$ . As a result, the expected force produced by such an interaction can be calculated. Since force can be defined as the change in momentum over time, this can be expressed as  $\Delta p_i = F_i \Delta t = \sqrt{2m\Delta E_i}$ .

Our goal is to arrive at a PSD for the interaction that characterizes RPSN. To achieve this, focus not on  $\Delta F_i$  but on the mean square value of the radiation pressure force. This quantity will then be converted into a PSD. Based on the information given so far, the mean square value of the radiation pressure force is given by

$$\langle \Delta F_x^2 \rangle = \left( \frac{h}{\lambda \Delta t} \right)^2 \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi P(\theta, \phi) (1 - \cos \theta)^2 = \epsilon_x \left( \frac{h}{\lambda \Delta t} \right)^2 \quad (28a)$$

$$\langle \Delta F_y^2 \rangle = \left( \frac{h}{\lambda \Delta t} \right)^2 \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi P(\theta, \phi) (\sin \theta \cos \phi)^2 = \epsilon_y \left( \frac{h}{\lambda \Delta t} \right)^2 \quad (28b)$$

$$\langle \Delta F_z^2 \rangle = \left( \frac{h}{\lambda \Delta t} \right)^2 \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi P(\theta, \phi) (\sin \theta \sin \phi)^2 = \epsilon_z \left( \frac{h}{\lambda \Delta t} \right)^2 \quad (28c)$$

where  $h$  is the Planck constant and  $\langle \Delta F_i^2 \rangle$  represents the mean square of the radiation pressure force per scattered photon over time in the  $i$ -th direction. The coefficients  $\epsilon_i$  can be determined from the integrals in Equation 28. These coefficients scale how photon scattering transfers momentum to each axis, depending on the direction of  $\vec{k}$  and the initial polarization. Numerical solutions to the scattering integrals, performed using experimental parameters and adjusted to match our optical table coordinate system, produce the following results:

$$\epsilon_x = 0.2100 \quad (29a)$$

$$\epsilon_y = 0.1285 \quad (29b)$$

$$\epsilon_z = 0.1143. \quad (29c)$$

From this point, let's consider three things. First, equation 28 will scale proportionally with the discrete photon impacts from the illumination beam. Second, our end goal is to

recover RPSN as a PSD. The scattering of light is comprised of discrete photons that produce a seemingly random momentum transfer onto the orthogonal axes of light propagation. The collection of these interactions produces a constant quantity for  $\langle \Delta F_i^2 \rangle$ , meaning the fluctuations in force are steady over time. So Equation 28 becomes

$$\langle F_i^2 \rangle = \epsilon_i \left( \frac{N_\lambda}{\Delta t^2} \right) \left( \frac{h}{\lambda} \right)^2 \quad (30)$$

where  $\epsilon_i$  is our previously solved coefficient,  $N_\lambda$  is the number of photons scattered off of the particle (to be experimentally determined later),  $\Delta t$  is our measurement time,  $h$  is the Planck constant, and  $\lambda$  is the photon wavelength.

The autocorrelation of this kind of force will yield a frequency-independent result, which simplifies the force PSD for such a system to that of a white noise source. These types of forces are known as stochastic forces. Given the discrete measurements, simply convert to a double-sided force PSD by multiplying by  $2\Delta t$ , leading to our radiation pressure force PSD given by

$$S_F^{RPSN} = 2\epsilon_i R_\lambda \left( \frac{h}{\lambda} \right)^2 \quad (31)$$

where a new parameter,  $R_\lambda = (N_\lambda/\Delta t)$ , is introduced. This is the photon scattering rate. In the free particle limit, convert to the displacement spectrum via  $F = ma$ , which yields

$$S_D^{RPSN} = \left( \frac{2\epsilon_i R_\lambda}{m^2 \omega^4} \right) \left( \frac{h}{\lambda} \right)^2 \quad (32)$$

Adding this information to our SQL plot from Figure 3 to create Figure 6, we find that the RPSN actually falls off at a faster rate compared to the SQL. At some frequency cutoff (dependent on system parameters), the RPSN becomes less than the SQL. This is interesting

since the SQL is supposed to represent the lower noise limit for measurement schemes that make no attempt to evade measurement back-action. Still, it is also unsurprising since only considering a single component of our quantum noise is considered.

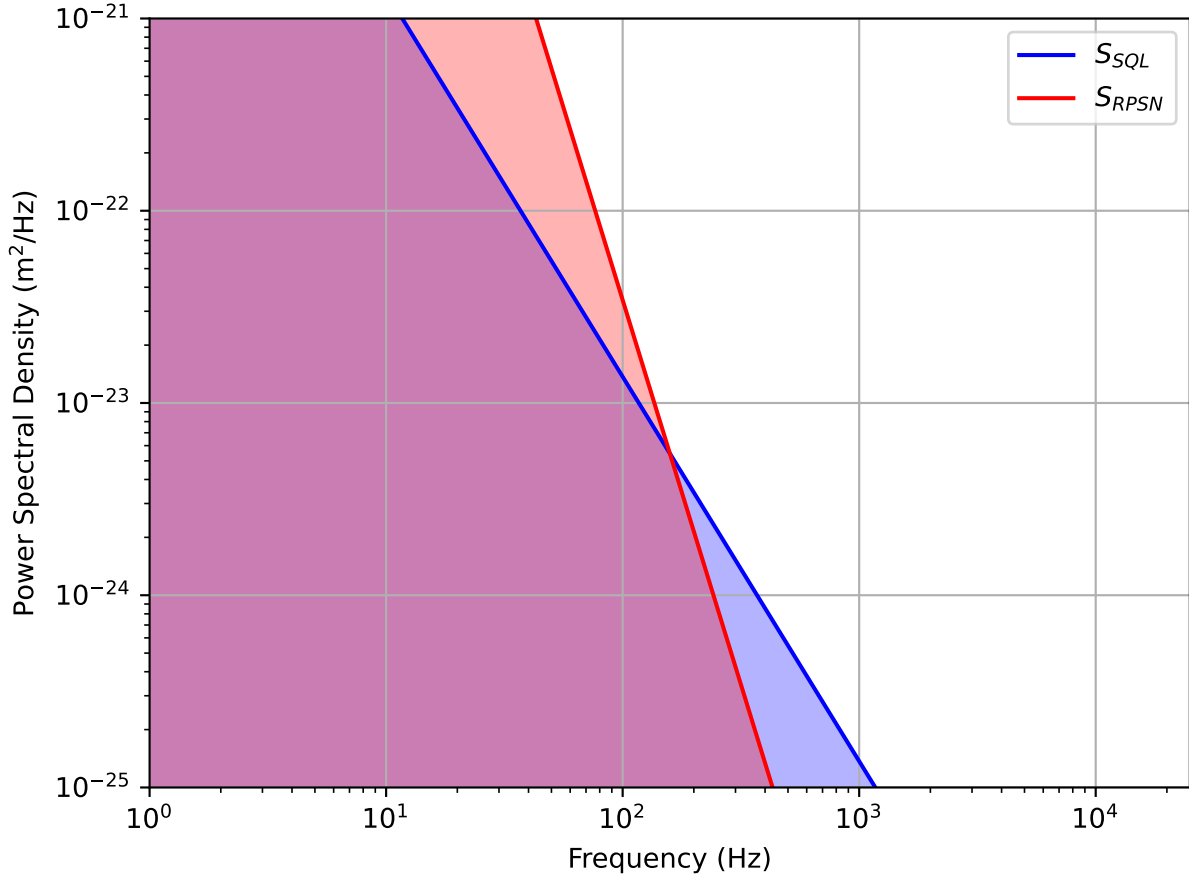


Figure 6: A plot of the displacement PSD for the SQL, along with the RPSN. The RPSN has a steeper slope compared to the SQL, and has a crossover point depending on various system parameters. This plot was made for  $\langle \Delta F_z^2 \rangle$  with,  $m = 1.54 \times 10^{-15}$  kg,  $R_\lambda = 5 \times 10^{10} \frac{\text{photons}}{\text{s}}$ , and photon wavelength  $\lambda = 785$  nm.

Next, we will discuss the other form of quantum noise in our model, detector shot noise (DSN). This noise, caused by the discrete arrival of photons at our detector, can be recovered by considering Heisenberg's uncertainty principle and what has been previously discussed when looking at RPSN. Let's begin with a reminder that a perfect microscope such that  $\Delta x \Delta p = \hbar/2$  is being used. In the discussion on RPSN we noted that  $\Delta p_i = F_i \Delta t$ , and

proceeded to find the mean square value for the radiation pressure force. This information can now be plugged into the Heisenberg relation to find the mean square value for our position variance,  $\langle \Delta r_i^2 \rangle$ , given by

$$\sqrt{\langle \Delta r_i^2 \rangle} = \frac{\hbar}{2\sqrt{\langle \Delta p_i^2 \rangle}} = \frac{\lambda}{4\pi\sqrt{\varepsilon_i N_D}} \quad (33)$$

which shows that the position uncertainty term is proportional to the wavelength of the illumination light  $\lambda$  divided by the square root of the number of photons detected  $N_D$ . This is a convenient place to introduce the effect of finite detection efficiency by introducing  $N_D$ , which lets us determine our efficiency of collection where  $N_D = \eta_D \cdot N_\lambda$ . Our efficiency factor,  $\eta_D$ , accounts for the fact that not every photon scattered is detected. This leads to the position variance for our measurement given by

$$\langle \Delta r_i^2 \rangle = \frac{\lambda^2}{16\pi^2 \varepsilon_i \eta_D N_\lambda} \quad (34)$$

which can be converted to a double-sided PSD similar to our RPSN force variance by multiplying by  $2\Delta t$ , resulting in

$$S_D^{DSN} = \frac{2\Delta t}{\varepsilon_i \eta_D N_\lambda} \left( \frac{\lambda}{4\pi} \right)^2 = \frac{\lambda^2}{8\pi^2 \varepsilon_i \eta_D R_\lambda} \quad (35)$$

This expression characterizes the DSN of our system. In particular, it does not incorporate a frequency component, indicating that the displacement spectrum remains constant across all frequencies. Higher input powers of our beam result in an increased number of photons scattering per unit time, which leads to an overall reduction in the DSN.

With this final piece of the puzzle, include the detector shot noise in our spectral density plot and finally answer what the SQL *is* in a way that should foster understanding instead

of just parroting a definition.

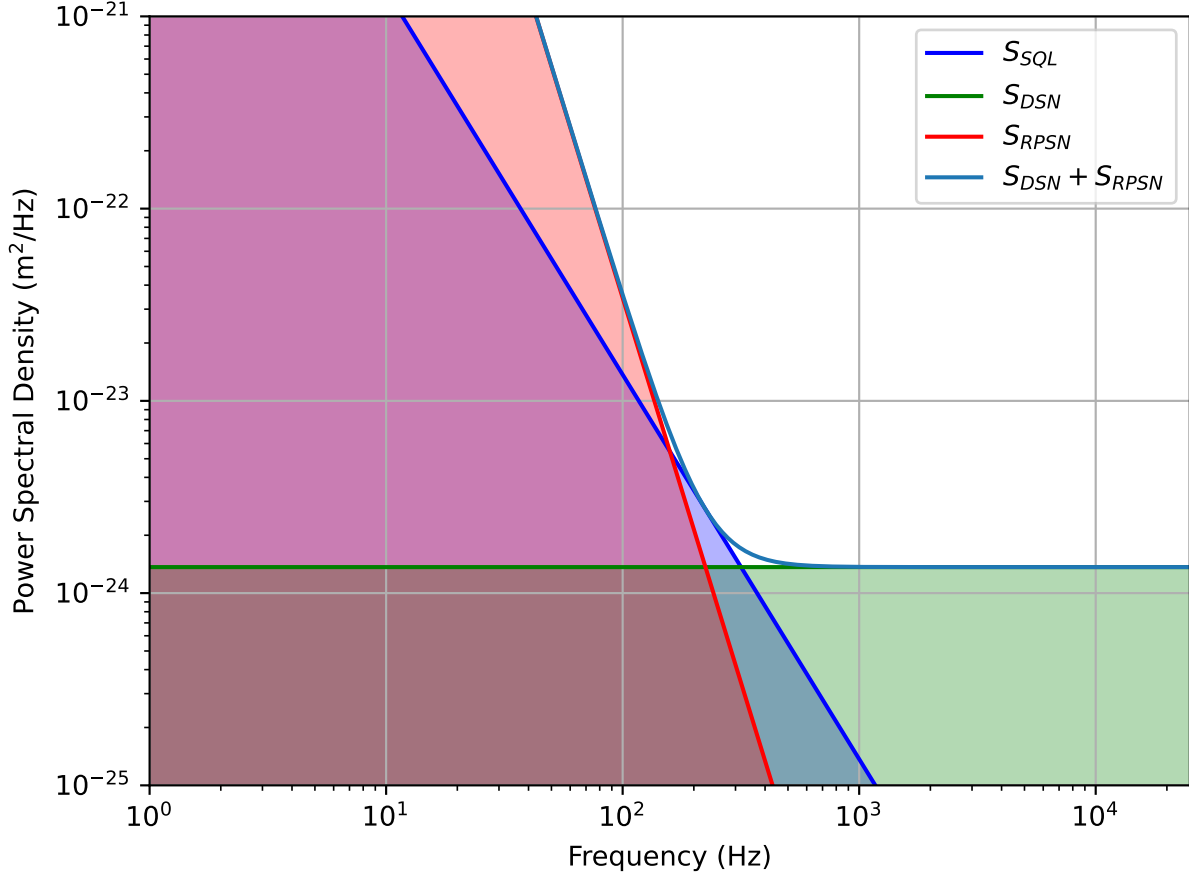


Figure 7: A plot of the displacement PSD which includes the SQL, RPSN, and DSN. This plot highlights the combined effects of quantum noise that come about from the discrete nature of photons. It is shown clearly that the combination of these noise sources produces a profile with the closest approach to the SQL occurring at the elbow, or intersection, of both sources. This is shown as the curve that comes from the sum of our quantum noise PSDs. This point is the SQL at a particular frequency (given  $\eta_D = 1$ , which is the case for this plot). Our noise sources can scale as a function of the number of photons per time, which shifts the position of this elbow. The line the elbow traces out is the SQL derived earlier in this document. This line represents the lowest noise possible without using more advanced measurement techniques (such as squeezed states, or back action evading measurement schemes). The values used to generate this plot are  $m = 1.54 \times 10^{-15}$  kg,  $\lambda = 785$  nm and  $R_\lambda = 5 \times 10^{10} \frac{\text{photons}}{\text{s}}$ .

Look at Figure 7, which was generated using the parameters listed in the figure's description. For the given values, there is an elbow that sits at 224 Hz, which also runs

tangent to the line traced by our derived SQL from before. Imagine increasing the photon scattering rate in an attempt to lower the noise observed at 224 Hz. This adjustment would cause two things to happen. First, the DSN would get lower, since it is inversely proportional to the photon scattering rate. This is good, since our goal is to reduce noise at 224 Hz, and this adjustment removes DSN as a limiting factor, but it comes at a cost. When looking at the behavior of the RPSN, notice that its curve will *increase* with a larger photon scattering rate. The net result is that our attempt to reduce noise by changing the input power caused things to worsen in our region of interest. Likewise, if input power is reduced, the same issue persists, but instead it is DSN that climbs and causes the noise located at 224 Hz to increase. Note too that these adjustments cause the location of the elbow to shift away from the 224 Hz mark. With that, we can finally define and explain SQL in a brief set of sentences.

The SQL is the noise limit produced by the limitations imposed by quantum mechanics on a measurement that involves non-commuting observables. As a function of frequency, this limit will produce a barrier that can only be touched at one particular frequency during the measurement process unless back-action evasion methods are used.

### Thermal Noise

The previous section discussed how quantum noise can be described when observing the motion of a free particle. However, our analysis must become a bit more complex, as real-world systems are influenced by additional noise sources. One significant source of noise is thermal noise. Objects that possess a non-zero temperature contain heat energy, which arises from the mechanical motion of their surrounding thermal environment. In the case of a free particle, the presence of residual air will affect the particle by imparting momentum during collisions. It is more effective to view this as an average collective behavior of the constituent parts rather than tracking each momentum transfer caused by discrete air particles.

The impacts of these interactions create a stochastic force that can either increase or decrease the average kinetic energy of a free particle. This change in energy depends on the temperature of the particle's center of mass relative to the thermal bath in which it exists. If the particle starts at rest, the impacts will increase its kinetic energy until it reaches equilibrium with the thermal bath. Conversely, suppose the particle's kinetic energy exceeds that of the thermal bath. In that case, the interactions with residual air will lead to a loss of energy from the free particle until it reaches equilibrium. This balance reflects the principles of fluctuation and dissipation.

For a random fluctuating force,  $F_i$ , the thermal force PSD can be expressed as

$$S_F^{THERM} = 4k_B T m \Gamma_i \quad (36)$$

where  $k_B$  is Boltzmann's constant,  $T$  is temperature,  $m$  is particle mass, and  $\Gamma_i$  is the damping coefficient. This equation adheres to the fluctuation-dissipation theorem, which satisfies the relation  $\langle F(t)F(t') \rangle = 2m\Gamma_0 k_B T \delta(t-t')$  [78, 79]. It's important to note that this is a situation similar to RPSN, where the ensemble of individual interactions generates a force that results in random motion. Consequently, this leads to another frequency-independent force PSD. Using the relationship  $F = ma$ , convert this force PSD into a displacement PSD. This gives us our final expression for the thermal noise displacement PSD of a free particle given by

$$S_D^{THERM} = \frac{1}{\omega^4} \frac{4k_B T \Gamma_i}{m} \quad (37)$$

### Harmonic Oscillator Systems

A new dilemma arises with all relevant noise sources identified: how to perform free particle measurements in a laboratory setting? One possibility is temporarily deactivating traps, allowing free particle behavior [42, 43]. However, this is not an option for a stable unpowered trap, such as a magneto-gravitational trap. Instead, a free particle regime must be created by leveraging the low oscillation frequencies provided by the magneto-gravitational trap. By imaging a particle at a rate significantly faster than the trap's oscillation frequency, behavior on a time scale small enough to approximate free particle conditions can be observed.

Taking inspiration from LIGO, the goal of this section is to describe the motion of an object by observing a displacement PSD in the free particle limit. The frequency domain is a valuable tool for breaking down our noise sources into easily identifiable components, as illustrated in Figure 7.

Assume a spherical particle is placed in a trap, where it behaves as a damped harmonic oscillator subjected to stochastic forces. This leads to an equation of motion (let's just say along the z-axis) given by

$$\frac{d^2 r_z}{dt^2} = -\omega_z^2 r_z - \Gamma_z \frac{dr_z}{dt} + \frac{F_z(t)}{m}, \quad (38)$$

where  $\omega_z$  is the natural oscillation frequency of the trap,  $\Gamma_z$  is the damping coefficient,  $r_z$  is the particle's position,  $m$  is the mass of the particle, and  $F_z(t)$  represents the stochastic noise sources. This differential equation has some convenient results when analyzed in the frequency domain. To facilitate this transition, take the Fourier transform of each term

$$\mathcal{F}[f(t)] = \tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt, \quad (39)$$

where  $\tilde{f}(\omega)$  is the Fourier transform of the function  $f(t)$ , and  $\omega$  is the angular frequency. Applying this transform to each term results in

$$\mathcal{F}\left[\frac{d^2 r_z}{dt^2}\right] = -\omega^2 \tilde{r}_z(\omega), \quad (40)$$

$$\mathcal{F}\left[\frac{dr_z}{dt}\right] = i\omega \tilde{r}_z(\omega), \quad (41)$$

$$\mathcal{F}[r_z(t)] = \tilde{r}_z(\omega), \quad (42)$$

$$\mathcal{F}\left[\frac{F_z(t)}{m}\right] = \frac{\tilde{F}_z(\omega)}{m}. \quad (43)$$

where  $\tilde{r}_z(\omega)$  is our Fourier transform of position  $r_z(t)$ , and  $\tilde{F}_z(\omega)$  is the Fourier transform of our noise sources given by  $F_z(t)$ . Plugging all of these transforms into our equation of motion yields

$$-\omega^2 \tilde{r}_z(\omega) = -\omega_z^2 \tilde{r}_z(\omega) - i\Gamma_z \omega \tilde{r}_z(\omega) + \frac{\tilde{F}_z(\omega)}{m}. \quad (44)$$

Recall that the spectrum produced by stochastic forces is frequency independent, meaning  $\tilde{F}_z(\omega) = \tilde{F}_z$  and can be treated as constant for both thermal noise and RPSN. Rearranging equation 45, our displacement as a function of frequency becomes

$$\tilde{r}_z(\omega) = \frac{\tilde{F}_z}{m(\omega_z^2 - \omega^2 + i\Gamma_z \omega)}. \quad (45)$$

Our next step is to convert our frequency-dependent position expression,  $\tilde{r}_z(\omega)$ , into a displacement PSD. This is achieved by taking the Fourier transform of the autocorrelation function, since the PSD and the autocorrelation function are related as a Fourier transform

pair. This leads to a PSD given by

$$S_z(\omega) = \langle |\tilde{r}_z(\omega)|^2 \rangle \quad (46)$$

$$S_z(\omega) = \frac{S_F}{m^2} \frac{1}{[(\omega_z^2 - \omega^2)^2 + (\Gamma_z \omega)^2]}. \quad (47)$$

where  $S_F$  is the force PSD from all stochastic noise sources, which is constant with frequency. The rest of this expression after  $\frac{S_F}{m}$  comes from the oscillator's response function, which here has been squared, and can be labeled as  $|\chi(\omega)|^2$ . The general shape of this spectrum has a peak whose position is determined by the natural oscillation frequency  $\omega_z$ , with a height and width determined by the damping coefficient  $\Gamma_z$ . From here, the free particle regime is given by the following conditions. First, observations are done on time scales much smaller than the oscillation frequency of the trap ( $\omega \gg \omega_z$ ). Second, our observed frequency range is much larger than the damping time of the system ( $\omega \gg \Gamma_z$ ). These conditions simplify the response function into that of a free particle, meaning  $|\chi(\omega)|^2 \approx \frac{1}{\omega^4}$ . Now plug in our previously established stochastic forces (Equation 36 and Equation 31) for  $S_F$ . The last piece of the puzzle is the noise floor, which is determined by DSN defined in Equation 35. This is already a displacement PSD, not a force PSD, and is not influenced by the response function. Our final displacement spectrum for a free particle is then given by

$$S_D = \frac{1}{m^2 \omega^4} \left( 4k_B T m \Gamma_z + 2\epsilon_z R_\lambda \left( \frac{h}{\lambda} \right)^2 \right) + \frac{\lambda^2}{8\pi^2 \epsilon_z \eta_D R_\lambda} \quad (48)$$

Moving forward, it will be common to see final results reported as an amplitude spectral density (ASD) rather than PSD. This choice is based on two primary reasons, both stemming from personal preference. First, the units used in ASD,  $\text{m}/\sqrt{\text{Hz}}$ , provide a clearer interpretation of the results, as the noise floor indicates a scale of measurement accuracy in

terms of distance. Second, the ASD produces shallower slopes, which I find preferable.

A simulation of this system is conducted to create the spectrum shown in Figure 8. This simulation is based on experimental parameters that are achievable in a laboratory setting and serve as a target for our measurement outcomes. Different colors and line styles in the plots represent contributions from each noise source. Additionally, a simplified diagram for each noise source is included for clarification.

This simulated data set has the closest approach to the SQL occurring around 300 Hz where the thermal noise curve meets the DSN curve. Given the low oscillation frequency used (7 Hz), the behavior of this regime can be described as the free particle limit. A result such as this would be an ideal outcome of our experimental setup. A real-world outcome resembling Figure 8 is the primary goal of this project, with a secondary goal to achieve high force sensitivity measurements within this limit.

### Force Sensitivity and the Weibull Distribution

With the outline of our harmonic oscillator system complete, we will demonstrate how it can be adjusted to evaluate force sensitivity. As mentioned in the literature review, force sensitivity measurements have been extensively applied to harmonic oscillator systems. Many of these systems face limitations on their measurement bandwidth due to oscillator stability. The evaluation of the force minimum is typically expressed by the formula  $F_{\min} = \sqrt{S_F \cdot b} = \sqrt{4k_B m T_N \Gamma_N b}$  [49], where  $S_F$  represents the measured force PSD,  $k_B$  is Boltzmann's constant,  $m$  is mass,  $T_N$  is temperature,  $\Gamma_N$  is the damping coefficient, and  $b$  is the bandwidth determined by the total duration of the measurement. In the case of active feedback cooling  $T_N$  and  $\Gamma_N$  can be replaced with  $T_{eff}$  and  $\Gamma_{eff}$ . For harmonic oscillator systems, a fit can be done with the Lorentzian function to determine  $F_{min}$ .

The Allen deviation of the signal at resonance presents challenges for long measurement timescales [50], which are necessary for enhancing force sensitivity. This issue affects

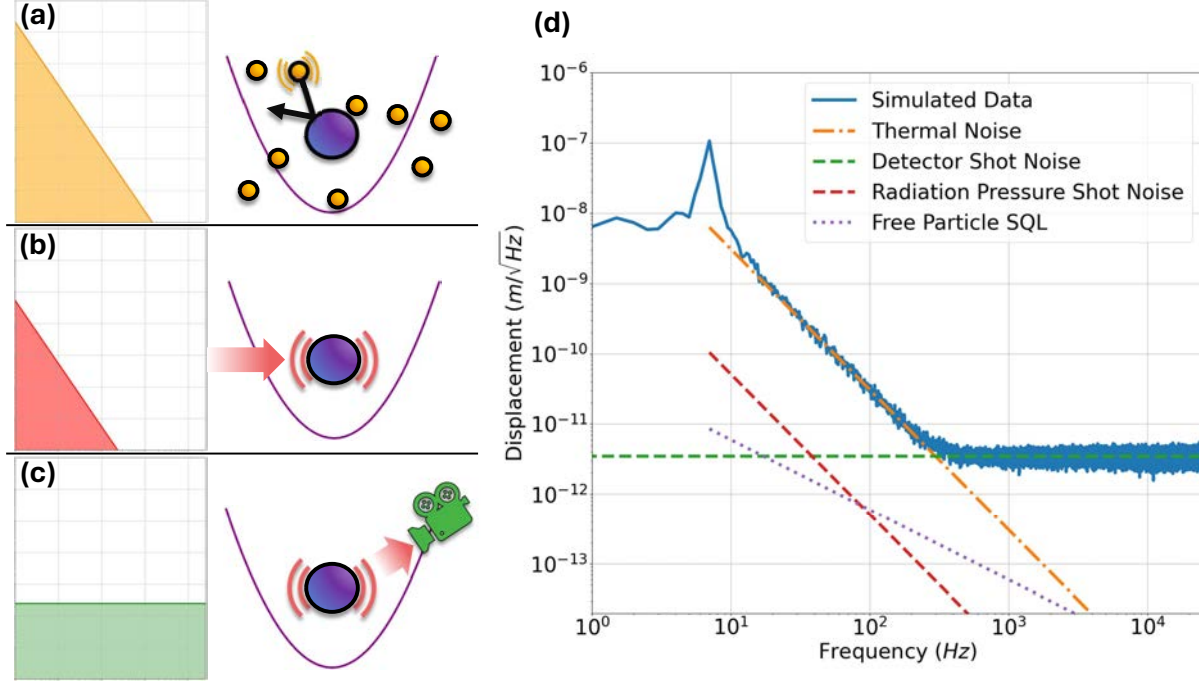


Figure 8: (a) A diagram illustrating the effects of residual air in an ultra-high vacuum environment. The interactions between the particles and this environment lead to Brownian motion, where resulting interactions are stochastic in nature. In our simulation, this is the dominant noise source at low frequencies. (b) A diagram depicting the interaction that causes RPSN. Individual photons interact with the particle, causing it to recoil. The net result of these interactions generates a white noise force. (c) A diagram highlighting the collection of scattered photons. Unlike the previously mentioned interactions, this noise exhibits a frequency-independent displacement spectrum. While it is suppressed at low frequencies, it dominates both RPSN and thermal noise at high frequencies. (d) A result from a simulated dataset generated using Equation 38 is displayed. This dataset produces time domain data, which is then transformed and plotted as an ASD. This simulation was produced to match achievable parameters on our optical bench with  $\eta_D \neq 1$ .

all measurements near the oscillator frequency, as the signal can become incoherent over extended periods. However, in the free particle regime, there is no such limitation.

Continuing from Figure 8, the equation of motion from Equation 38 allows us to simulate a time series that lets us produce our previously shown displacement ASD. Evaluating force

sensitivity requires us to alter our methods slightly. Let's start with our time series,  $x(t)$ , and proceed to recover a similar plot to Figure 8. Once complete, we will adjust our methods to highlight the differences in our force sensitivity measurements. First, perform a fast Fourier transform (FFT) on our time series data, which is broken into chunks of time smaller than its total duration. This allows the use of Welch's method [81], which improves observations of frequency spectra at the expense of frequency resolution. Performing the FFT leads to our new frequency dependent term given by

$$X(\omega) = FFT[w(t) \cdot x(t)] \quad (49)$$

where  $w(t)$  is our window function, and  $x(t)$  is a segment time domain data. To ensure proper scaling the function is normalized, leading to

$$X_{norm}(\omega) = \sqrt{\frac{2 \cdot \Delta t}{\sum w(t)^2}} X(\omega) \quad (50)$$

where  $\Delta t$  is the measurement time. Finally, convert our data into an ASD using  $\sqrt{S_D} = \sqrt{|X_{norm}(\omega)|^2}$ .

At this point, our signal from the time domain, in this case particle motion, has been converted it into a displacement ASD with units of  $\text{m}/\sqrt{\text{Hz}}$ . This process was done to generate the plot in Figure 8, with the additional step of Welch's method to produce a smooth profile for the simulated data in the frequency domain. In the case of measurements approaching the SQL, this is fine, but for force sensitivity measurements, the loss of frequency resolution becomes a limitation. To alleviate this issue, we will not use Welch's method when evaluating the spectrum of a time series for force sensitivity; instead, we will opt to use the entire time series for a single Fourier transform. From here, consider Newton's 2nd law,  $F = ma$ . For our displacement ASD, we can convert to a force ASD via  $\sqrt{S_F} = m\omega^2\sqrt{S_D}$ , as shown in Figure 9. Regions dominated by stochastic forces will appear frequency independent

when plotting a force ASD. At this point, take our reported force ASD,  $\sqrt{S_F}$  for this region, and our measurement bandwidth  $b$  and calculate the minimum force [10, 50].

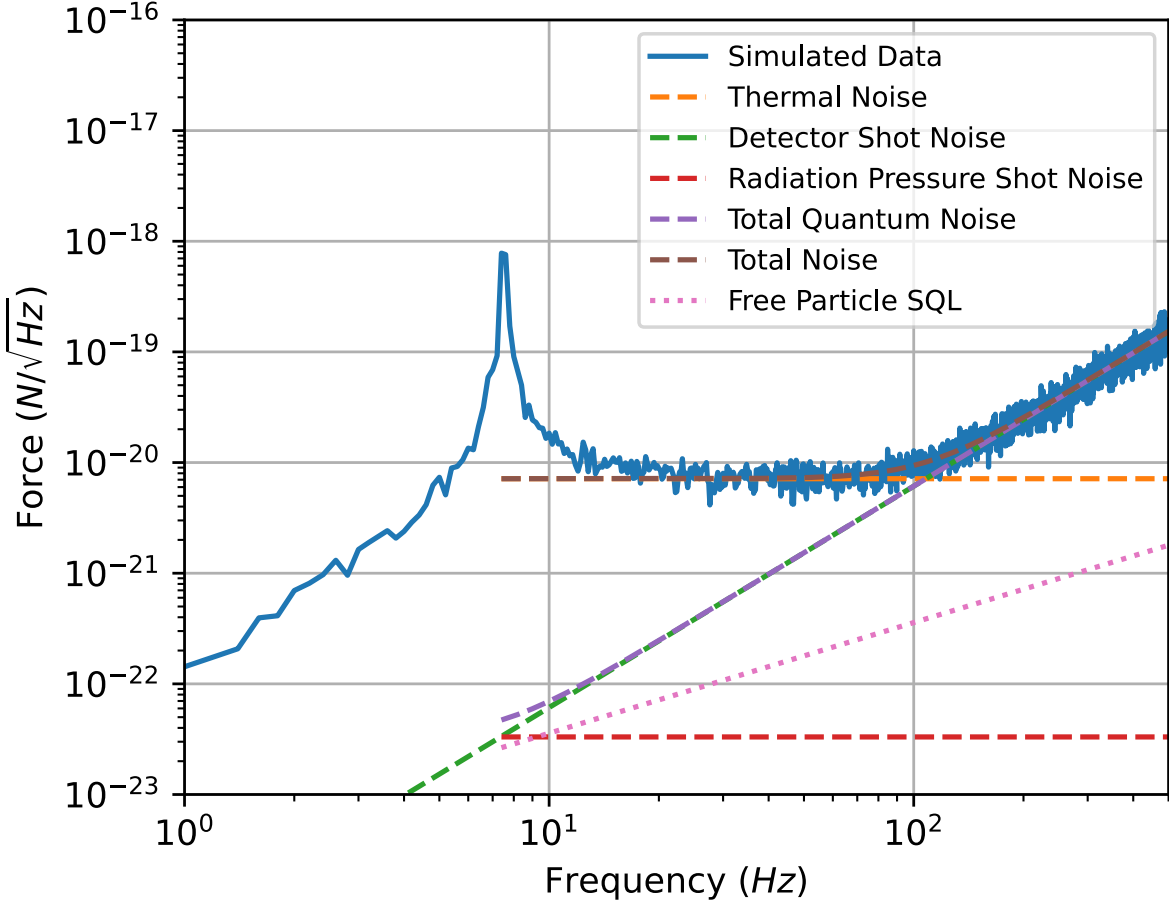


Figure 9: Another simulation of a time series plotted as a force ASD. Dominated by thermal noise, a frequency-independent region between 10 Hz and 100 Hz is shown. This region gives us  $\sqrt{S_F}$ , and combined with the measurement bandwidth allows the reporting of minimum force detection over a frequency range proportional to the bin size. Welch's method was used here to produce a more 'clean' plot, but otherwise, it will not be used during force sensitivity analysis.

In the free particle limit, Allan deviation is not a concern. As such, our only limitation to the measurement time is how long the particle can stay in the imaging plane of our optics, and how much data can physically be stored on a PC. A longer duration for the time

series will reduce the bin size, which means finer force sensitivity outcomes, but this begs the question “How do we recover  $F_{min}$  in the free particle limit?”. Unlike the previously mentioned Lorentzian fit method, our system is not evaluated near resonance, so we need a new way to characterize  $F_{min}$  for a free particle. Here, we propose a new analysis method for force sensitivity at the free particle limit.

Performing our FFT starts with a time series of real numbers  $x(t)$  that holds our particle positions. The output of our FFT yields a complex-valued array giving us a discrete analog to  $X(\omega)$  in the form  $X_k$ , which provides us with a complex number for each associated frequency bin of our transform. This complex number has a magnitude given by  $|X_k| = \sqrt{A_k^2 + B_k^2}$ , where  $A_k$  and  $B_k$  are the real and imaginary components of the complex number  $X_k$ , and  $|X_k|$  is the resulting amplitude with units  $N/\sqrt{\text{Hz}}$ . Since a real-valued time series is broken up into sinusoidal functions via the FFT, the mean of both  $A_k$  and  $B_k$  is centered at zero over a full period.

Assuming a random noise signature, such as thermal noise, the contribution of frequency components for our FFT’s cosine (real) and sine (imaginary) results yields the same variance. Finally, given the orthogonal nature of the sine and cosine functions, these two sections of our FFT are also uncorrelated.

To recap:

1. A time series,  $x(t)$ , is converted into a force ASD similar to the one shown in Figure 9.
2. When done using a FFT without Welch’s method, the result has three critical properties:
  - (a) ZERO MEAN: The real and imaginary parts of our FFT correspond to cosine and sine functions, resulting in zero mean.
  - (b) EQUAL VARIANCE: Our FFT from a real-valued signal distributes power evenly between our cosine and sine components, leading to equal variance between the

real and imaginary parts of the complex-numbered result.

- (c) **UNCORRELATED:** The cosine and sine functions are orthogonal, thus uncorrelated.

These criteria match those of a Rayleigh distribution, and given a force ASD with a region dominated by white noise, this should be observable. For a Rayleigh distribution, there are 2D vectors whose components have zero mean and equal variance and are uncorrelated with each other. The FFT of our time series produces such vectors in the form of  $\vec{X}_k$ . This parameter is shown as the amplitude in our spectrum when observing our ASD. The probability density function for a Rayleigh distribution is given by

$$P(|X_k|) = \frac{|X_k|}{\sigma^2} \exp\left(-\frac{|X_k|}{2\sigma^2}\right) \quad (51)$$

where  $\sigma$  is the scale parameter of our distribution, and  $|X_k|$  is our ASD amplitude at a particular frequency. This means that if many independent measurements centered around a single frequency bin are performed, the resulting distribution would be a Rayleigh distribution. In our case, a stochastic noise source independent of frequency is observed. Since each bin is independent, a Rayleigh distribution is obtained by throwing each bin over a range of frequencies onto a histogram. This method allows us freedom to evaluate the force sensitivity of a stochastic noise across a wide range of frequencies, so long as the behavior in this range is truly frequency independent.

At this point, there are two additional observations to make before analyzing a simulated data set. First, our ASD has units of  $\text{N}/\sqrt{\text{Hz}}$ , so to achieve a measurement of our minimum force sensitivity, an intermediate step is required, integrating for each bin, allowing us to evaluate a distribution of force measurements. Second, a Rayleigh distribution is a special case of a more general form called a Weibull distribution, given by

$$P(X_k; \lambda, k) = \frac{k}{\lambda} \left( \frac{X_k}{\lambda} \right)^{k-1} \exp \left[ - \left( \frac{X_k}{\lambda} \right)^k \right] \quad (52)$$

where  $\lambda$  is the scale parameter of the Weibull distribution, and  $k$  is the shape parameter. This form follows a Rayleigh distribution under the conditions  $k = 2$  and  $\lambda = \sqrt{2}\sigma$ . At this junction, either option to evaluate our force measurements can be used, but I opt for the form presented by the Weibull distribution. To understand why, let us continue by observing simulated data fitted to these distributions.

I used a one-minute simulated data set to generate a histogram and fit seen in Figure 10. This simulation is similar to the one done in Figure 8 with two critical changes. The first change is in parameter  $\Delta t$ , where it goes from  $20 \mu\text{s}$  to  $1000 \mu\text{s}$ . Regions dominated by DSN in the displacement spectrum are irrelevant to force sensitivity measurements. The second change is that this method performs no Welch averaging, since using Welch's method breaks the required criteria that give rise to a Rayleigh distribution. A fit to a Rayleigh distribution and its residuals are shown for a simulated data set in Figure 10.

This analysis method allows us to observe the force sensitivity for several different time scales over a wide band of frequencies, increasing the sample size for the data set and reducing uncertainty. This also eliminates the ambiguity of selecting exceedingly narrow bandwidths to describe our sensitivity, as the white noise spectrum allows analysis over the whole range of noise dominated by stochastic forces. With our demonstrated fit, there is an interesting observation where  $F_{min}$  seems fairly arbitrary when considering only the Rayleigh distribution form in Equation 51. Relating  $F_{min}$  from a Lorentzian fit method is done to recover our expected scale parameter for stochastic thermal noise, and we find for the Rayleigh distribution this is given by  $F_{min} = \sqrt{2}\sigma$ . This value doesn't relate to any other outcome for a Rayleigh distribution, such as the distribution mean, median, or mode. However, when considering the form of the Weibull distribution, then our scaling parameter,

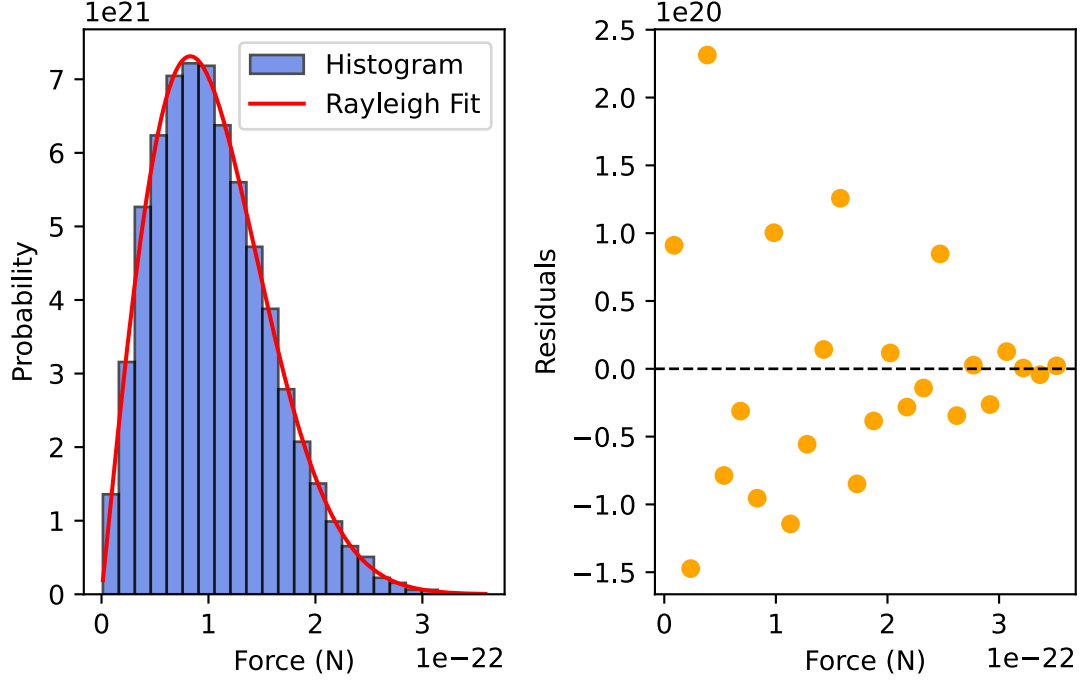


Figure 10: The force ASD bins are observed from a frequency range dominated by a stochastic noise force. Each frequency bin is stored and put into a histogram, which reports its corresponding force sensitivity. The collection of these bins results in a Rayleigh distribution with a reportable force minimum of  $F_{min} = \sqrt{2}\sigma$ .

$\lambda$ , becomes a direct reading of  $F_{min}$ , where  $\lambda$  can be found with an estimation for maximum likelihood given by

$$\hat{\lambda} = \sqrt{\frac{1}{N} \sum_{i=1}^N F_i^2} \quad (53)$$

With all of this laid out, we finally arrive at a method to evaluate the force sensitivity for a free particle dominated by a stochastic thermal noise given by

$$P(F) = \frac{2F}{\lambda^2} \exp \left[ - \left( \frac{F}{\lambda} \right)^2 \right] \quad (54)$$

where  $\lambda = F_{min}$ . Given that both this form of the Weibull distribution and the previously described Rayleigh distribution are essentially the same thing, but with a slightly different form, the choice on which one to use is entirely subjective. I prefer the form that uses a Weibull distribution, but either method will yield the same results.

### Conclusion

In this chapter, I listed most of the mathematical tools necessary for the experiments outlined in this dissertation (some math has been moved to Chapter 4 for reading convenience). Following a simple set of rules and assumptions, a mathematical model was developed to describe a real-world physical system in the free particle limit. Creating such a system would allow tests of physics near the SQL for a free mass. Given the novelty of the free particle limit, record-setting force sensitivity measurements could be achieved. These tasks are the main objectives of my research, and everything discussed after this section is done to achieve results that resemble these simulations.

## EXPERIMENT SETUP

System Overview

In the previous section, a mathematical description of a low-frequency harmonic oscillator was developed. Now, we will explore this system experimentally.

To begin, I will provide a comprehensive overview of the subsystems involved in our experimental setup before delving deeper into how everything integrates. First, let's gain an understanding of the overall experiment by examining a diagram of our system, as shown in Figure 11.

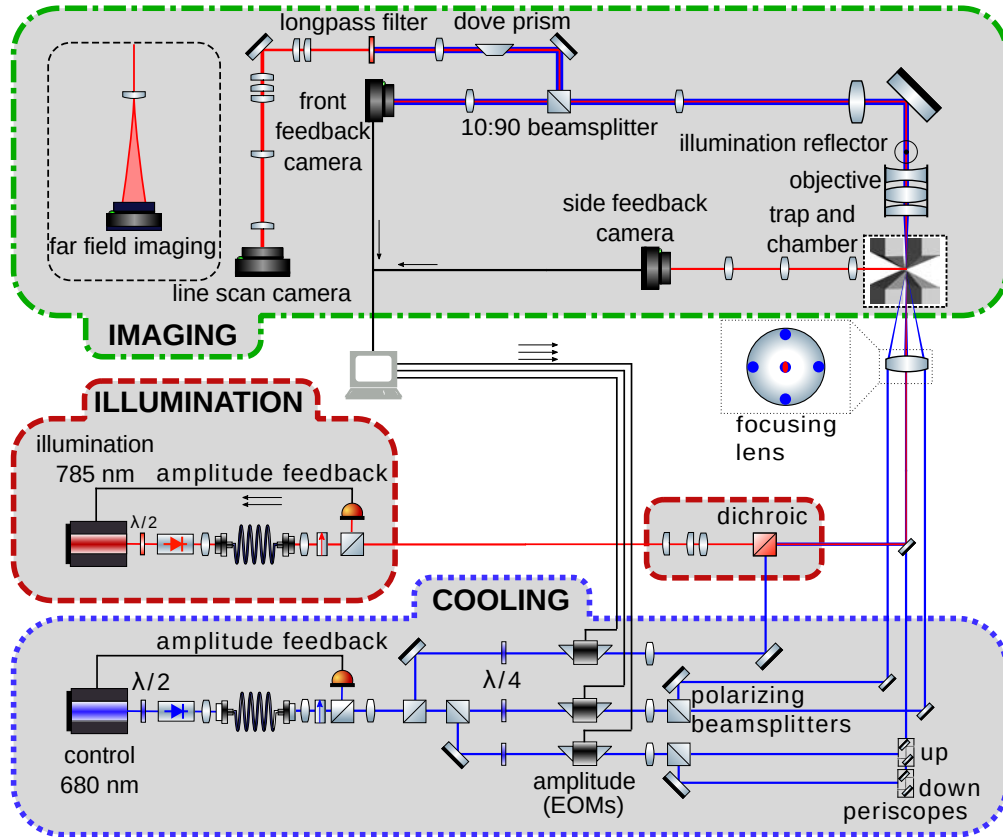


Figure 11: A diagram of the optical bench used to perform our precision measurements. This diagram highlights several core elements of our experimental setup and is labeled to identify different subsystems within the bench.

We will start with what I consider the foundation of our system: the magneto-gravitational trap. This trap is essential for achieving a trapping behavior that can be treated as a free particle. Equally important is the vacuum chamber where the trap is placed, which will be covered after our discussion of the trap. This section will address some design considerations for the vacuum chamber, as well as the pumps required to operate in an ultra-high vacuum (UHV) environment.

Next, the optical configuration of this experiment will be discussed, starting with the illumination and feedback cooling beams. It is necessary to collect light from scattering, which is accomplished with an adjustable objective that merits its own section. For feedback cooling to function, the particle location must be tracked. This is achieved using two area-scan cameras that dictate when to modulate the feedback cooling beams.

After this, I will introduce the line-scan camera. This device, consisting of a single row of pixels operating at 50 000 Hz, serves as the primary detector for our high-speed measurements. To use this device effectively, a special configuration of cylindrical lenses is required to optimize photon collection.

I conclude this chapter with a discussion on timing control for our system. To obtain accurate measurements, precisely controlling the timing of when all our various components are triggered is a must. This is achieved using a digital pulse generator, which allows us to maintain accurate control over the timing of our system.

### The Magneto-Gravitational Trap

A magneto-gravitational trap allows isolation of diamagnetic materials, enabling us to create an unconditionally stable trapping environment. This trap operates passively, requiring no power supply or active corrections, and the fields that produce the trap are static. The concept of magnetically levitating diamagnetic materials is not new; it has previously been used to levitate large objects, such as frogs [86]! The creation of this trap

is not the primary focus of this dissertation; more detailed and comprehensive studies are available on this trapping method [24, 29]. Some of these aspects will be discussed in this section.

Consider a sphere made of a homogeneous magnetic material, such as  $\text{SiO}_2$ , which has its magnetic moment induced by a magnetic field  $\vec{B}$  located near the Earth's surface. The potential energy for this system is expressed as follows [72]:

$$U = -\frac{\chi B^2 V}{2\mu_0} + mgy \quad (55)$$

where  $\chi$  is the volume magnetic susceptibility of the trapped material,  $V$  is the volume of the sphere,  $m$  is the mass of the sphere,  $\mu_0$  is the vacuum permeability,  $g$  is the acceleration due to gravity near Earth's surface, and  $y$  refers to the vertical displacement. For diamagnetic materials (where  $\chi < 0$ ), a trapping potential is achieved when the magnetic contribution balances the gravitational force. Given the low susceptibility of most diamagnetic materials (approximately  $\chi \approx 10^{-5}$ ), it is necessary to utilize a strong magnetic field or field gradient to allow trapping. Another option is to reduce gravitational acceleration  $g$ , which is fairly impractical and costly. To provide a more quantitative description of our system, begin with Laplace's equation:

$$\nabla^2 \Phi_M = 0 \quad (56)$$

In the presence of a magnetic field, the magnetic field vector is given by  $\vec{B} = -\nabla \Phi_M$ . A reasonable approach to characterize the magnetic potential is to use an expansion in spherical harmonics. This is a general solution to Equation 56 given by

$$\Phi_M(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l [A_{lm} r^l] Y_l^m(\theta, \phi) \quad (57)$$

It is important to note that the  $B_{lm}$  term is absent as it must be zero to ensure a finite magnetic field at  $r = 0$ . Symmetry considerations [24] point to a specific set of spherical harmonics that provide approximate solutions for our expansion in a magneto-gravitational trap that can be written as

$$Y_{2,-2}(\theta, \phi) = i\sqrt{\frac{1}{2}} (Y_2^{-2}(\theta, \phi) - Y_2^2(\theta, \phi)) \quad (58)$$

$$Y_{3,1}(\theta, \phi) = \sqrt{\frac{1}{2}} (Y_3^{-1}(\theta, \phi) - Y_3^1(\theta, \phi)) \quad (59)$$

$$Y_{4,-4}(\theta, \phi) = i\sqrt{\frac{1}{2}} (Y_4^{-4}(\theta, \phi) - Y_4^4(\theta, \phi)) \quad (60)$$

The trapping strength along each axis of the trap can be determined experimentally, allowing us to map the trapping potential [24]. When constructing a magneto-gravitational trap, several factors must be considered. First, the trap will be located within an UHV environment, which is achieved by using a series of pumps along with a baking procedure that raises the vacuum chamber temperature to 130 °C. Some components of our chamber, such as the ion pump, can withstand baking temperatures well over 200 °C, while others, such as the windows and gaskets, have stricter temperature limitations. The heat generated during this process can also pose challenges depending on the magnets used in our system.

The trap used in this dissertation functions with a pair of samarium cobalt (SmCo) permanent magnets because they have a remarkably high Curie temperature of 800 °C, which is well above our baking temperature. This characteristic ensures that the magnetic strength of our trap remains unaffected by thermal effects, since the magnetic fields produced by the

magnets begin to weaken at a much lower temperature than the Curie temperature. The Curie temperature is the limit where the magnetic properties of the material are lost.

While SmCo magnets are a suitable option for this kind of trap, neodymium-iron-boron (NdFeB) magnets are also available. They can produce a stronger magnetic field and have a volume independent maximum energy product, commonly referred to as  $BH_{max}$ , that can be nearly double that of SmCo magnets, reaching up to around  $440 \frac{\text{kJ}}{\text{m}^3}$  [71]. However, the Curie temperature for NdFeB magnets is only  $350^\circ\text{C}$ , making them less ideal for our applications due to the added risk of weakening the magnets.

A permanent magnet is positioned between two Hiperco 50A pole pieces to shape the magnetic field and create a horseshoe configuration. Hiperco 50A was chosen due to its high saturation magnetization compared to other magnetic alloys. By combining two of these configurations, a magnetic quadrupole that has a field minimum at its geometric center is created. An example of this type of quadrupole configuration is illustrated in Figure 12. Additionally, Figure 13 displays the same system when viewed along the xy and yz planes.

This configuration creates a complete trapping environment in the xy-plane of the trap. However, without any adjustments, the field minimum would extend flat along the z-axis, allowing diamagnetic particles to escape. To break the symmetry along this plane, the top pole pieces are shaved to a point, which raises the field minimum when moving away from the origin along the z-axis. When accounting for gravitational potential combined with this upward curve of the magnetic field minimum, a net restoring force in the yz-plane is achieved, resulting in a complete 3D trap. The trap used throughout this work typically operates with the following oscillation frequencies: Axial (z-axis) 6.5 Hz, Transverse (x-axis) 71 Hz, and Vertical (y-axis) 84 Hz.

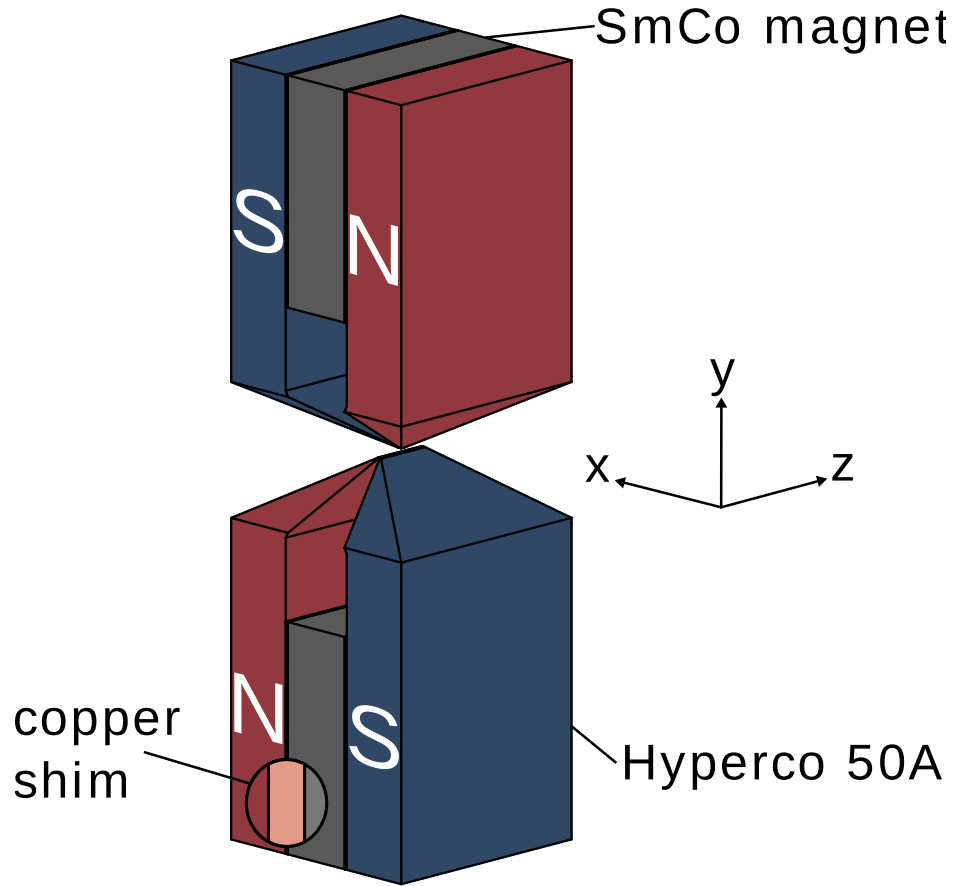


Figure 12: SmCo permanent magnets are sandwiched between Hyperco 50A pole pieces. This north-south configuration between the magnets creates a magnetic quadrupole with a minimum field around the pole pieces' geometric center. Thin copper shims are used to fine-tune the gap and conveniently maintain the electrical connection of the pole pieces to the magnets for the top and bottom pole pieces, respectively.

#### Electrical connection for magnetic trap

Electrical charge accumulation within the chamber can affect particle motion in undesirable ways. When a particle is pulled to a new stable position within the trap due to the presence of excess charge, the oscillation frequencies along each axis change. This perturbation can result in particle instability, making precision measurements more difficult. The system must be discharged to counter this, which requires evaluating its electrical response.

The pole pieces are electrically connected to their respective magnets via copper shims.

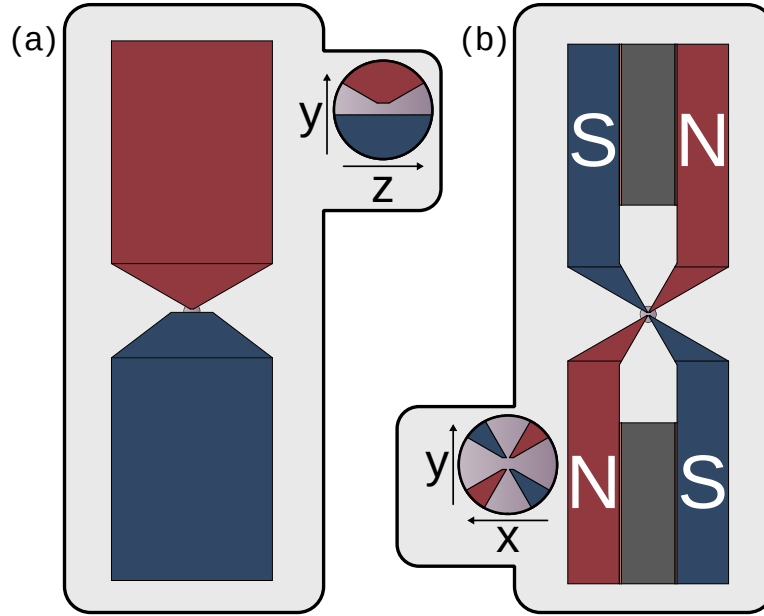


Figure 13: (a) A side profile of the yz-plane. This configuration requires the top pole pieces to be shaved down, which causes the field minimum to drift upwards when moving away from the geometric center along the z-axis. Combined with gravity, this creates a complete trapping environment along all three Cartesian axes. (b) A side profile of the xy-plane. This configuration allows a complete trapping environment for diamagnetic materials along the x and y axes, independent of the shaved top pole pieces.

The bottom pole pieces are grounded by connecting them to the trap mounting plate. In contrast, the top pole pieces remain electrically isolated by placing a polyether ether ketone (PEEK) shim between them and the mounting plate. These connections are extended through wires that exit the vacuum chamber, allowing for an electrical connection from outside the chamber to create a potential difference between the pole pieces. Figure 14 shows the pole pieces mounted and connected to our external nodes. Measuring the resistance between our output connections shows OL on a multimeter, indicating good electrical insulation since there is no good path between the wires.

The electrical connections on the exterior of the chamber are linked to a BNC cable that connects to a Stanford Research Systems Precision DC Voltage Source (Stanford Research

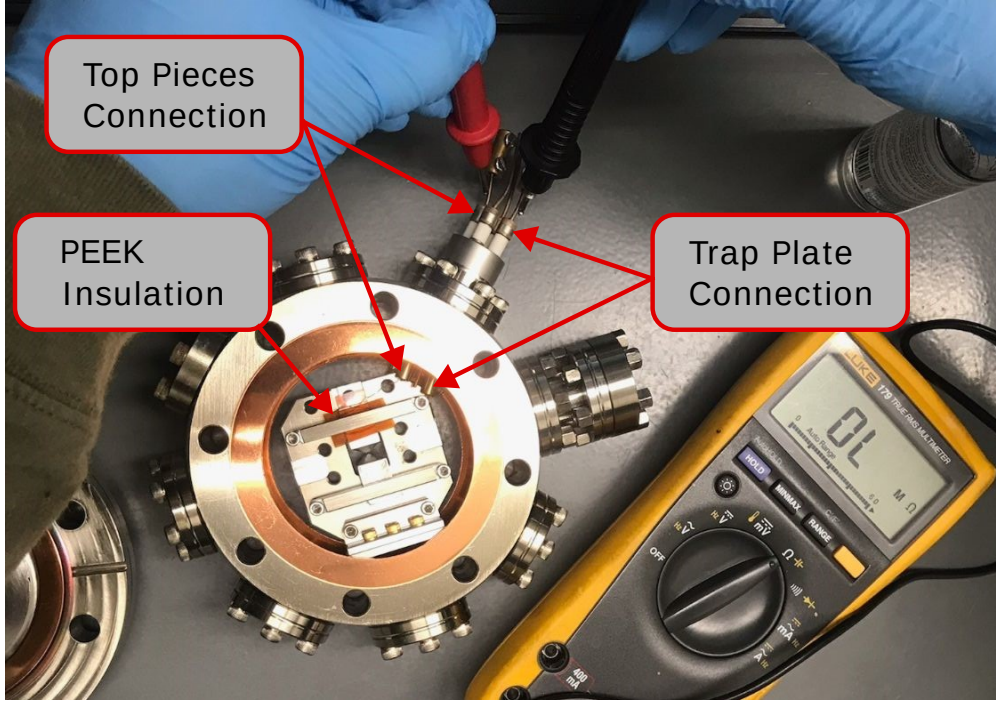


Figure 14: The trap inside the chamber mount is being checked for electrical isolation. The two wires on the outer part of the chamber mount will be used to create a potential difference via a DC voltage source. The top pole pieces are floating, while the bottom pole pieces are grounded to the mounting plate.

Systems DC205). A low-pass filter is situated between the voltage source and the magnetic trap. This filter prevents voltage surges that could dislodge the particle from the trap or cause unwanted motion. Since the DC voltage supply will remain connected to the system while a particle is trapped, the use of this filter is crucial. A diagram illustrating this configuration can be found in Figure 15.

Adjustments to the voltage supplied are used to evaluate particle charge by watching the particle react to the new electrical potential. The particle loading method utilizes an ultrasonic horn [68] to spray several  $\text{SiO}_2$  spheres (Cospheric-SiO2MS-2.0  $1.18\mu\text{m}$ ) into the trapping region. Figure 16 shows how a charged particle shifts its position in a magneto-gravitational trap when a potential difference is created between the top and bottom pole pieces.

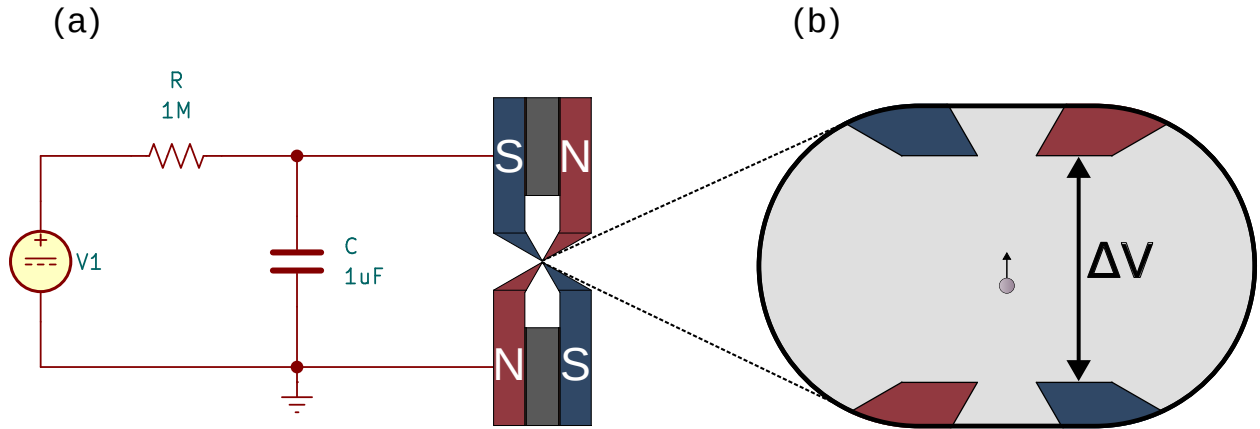


Figure 15: (a) A diagram of the electrical configuration for the pole pieces. The power supply passes through a low-pass filter to prevent surges before reaching the pole pieces. The bottom pole pieces are connected to ground, while the top pole pieces are linked to the output of the low-pass filter. (b) Trapped particles will respond to the potential difference created in the pole pieces according to their electrical charge. This figure shows a portion of the pole piece tips with a labeled potential difference generated by our voltage source.

A radiation source (Americium-241) is used during the loading procedure to discharge trapped particles. This radiation source is positioned next to the trap via a rod. When done in atmospheric pressure, this radiation source allows the ionization of nearby gas, resulting in both positive and negative charges being neutralized in the trapped particle. Particle motion is tracked by a camera to determine its response to changes in the supplied voltage. When there is no visible reaction by the particle when changing voltages and only a single particle remains, the radiation source is ready to be removed. Complete neutralization can be achieved by using ultraviolet (UV) light, but the additional step of complete neutralization was never utilized for this work.

### Vacuum Chamber

An exceptionally well-isolated environment is required to achieve measurements that approach the SQL. To this end, a vacuum chamber and a series of pumps were constructed to create a UHV environment. Much of this work was done by Brad Slezak during his time in

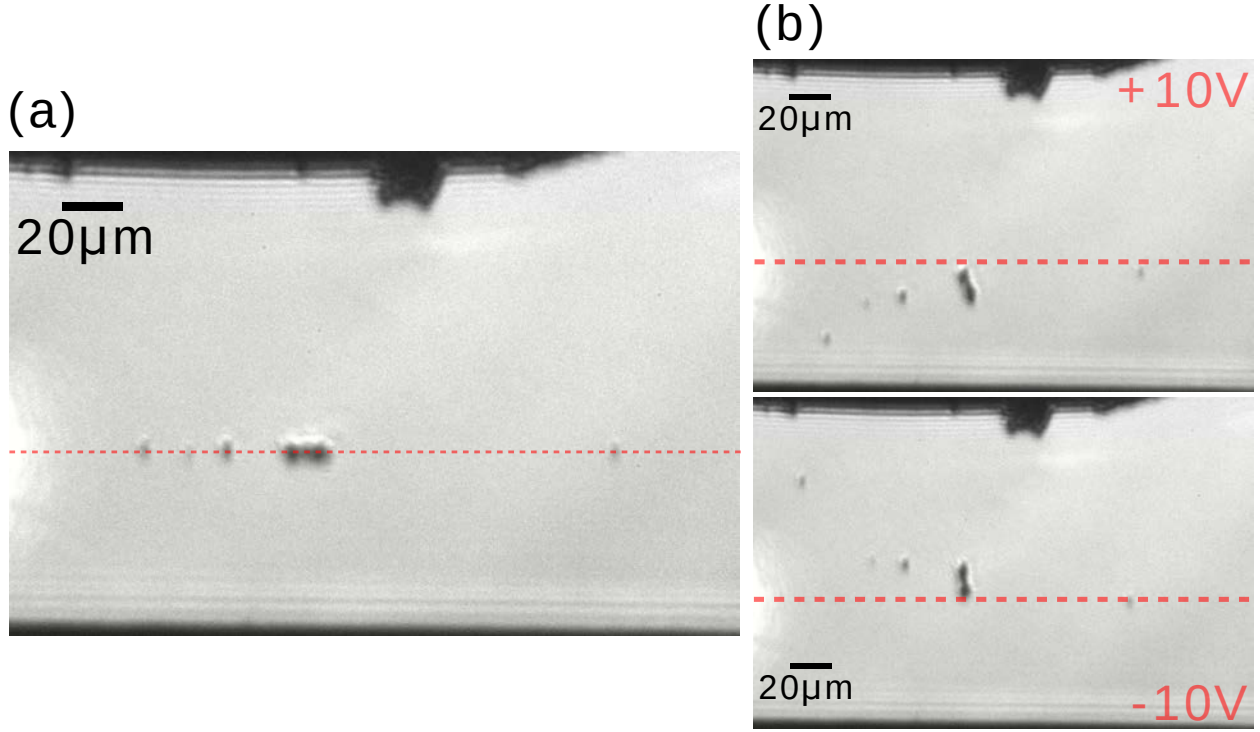


Figure 16: (a) Several particles suspended in a magneto-gravitational trap. The pole pieces are shorted together, meaning no potential difference. A red dashed line is used as a reference to indicate the starting positions of these particles. (b) Setting our voltage source to  $\pm 10\text{ V}$  causes the particles to shift their positions. The red dashed line marks the particle positions while the pole pieces are shorted. The particle on the far left has the largest response, while the particle on the right has almost no reaction.

the D’Urso lab, with some minor additions made by me. For a more comprehensive overview of the vacuum system’s construction, please refer to Brad’s dissertation [73]. Below, we will outline a brief overview of our vacuum system.

Several considerations were taken into account when constructing our vacuum chamber. All optical elements used on this bench are located outside of the chamber. This decision was made due to the stringent requirements for vacuum-compatible components that must be placed inside the chamber. To achieve a UHV environment, the chamber needs to be baked at approximately  $130^\circ\text{C}$  for several days. The main optical objective used on this bench is not compatible with such an environment. Moreover, any warping or repositioning of the optical objective under these conditions would render any distance calibration of our

system ineffective.

The particle trap and its mounting plate were designed with temperature considerations in mind. Our particle trap is connected to a compatible spherical octagon (Kimball Physics MCF450-SphOct-E2A8). To achieve faster pumping speeds, larger apertures are necessary for the pumps. Therefore, the trap-mounted octagon is also connected to a larger port (Kimball Physics MCF450-SphSq-E2C4), which features two 4 in and four 2 in flanges that facilitate quicker pumping. These components are seen in Figure 17.

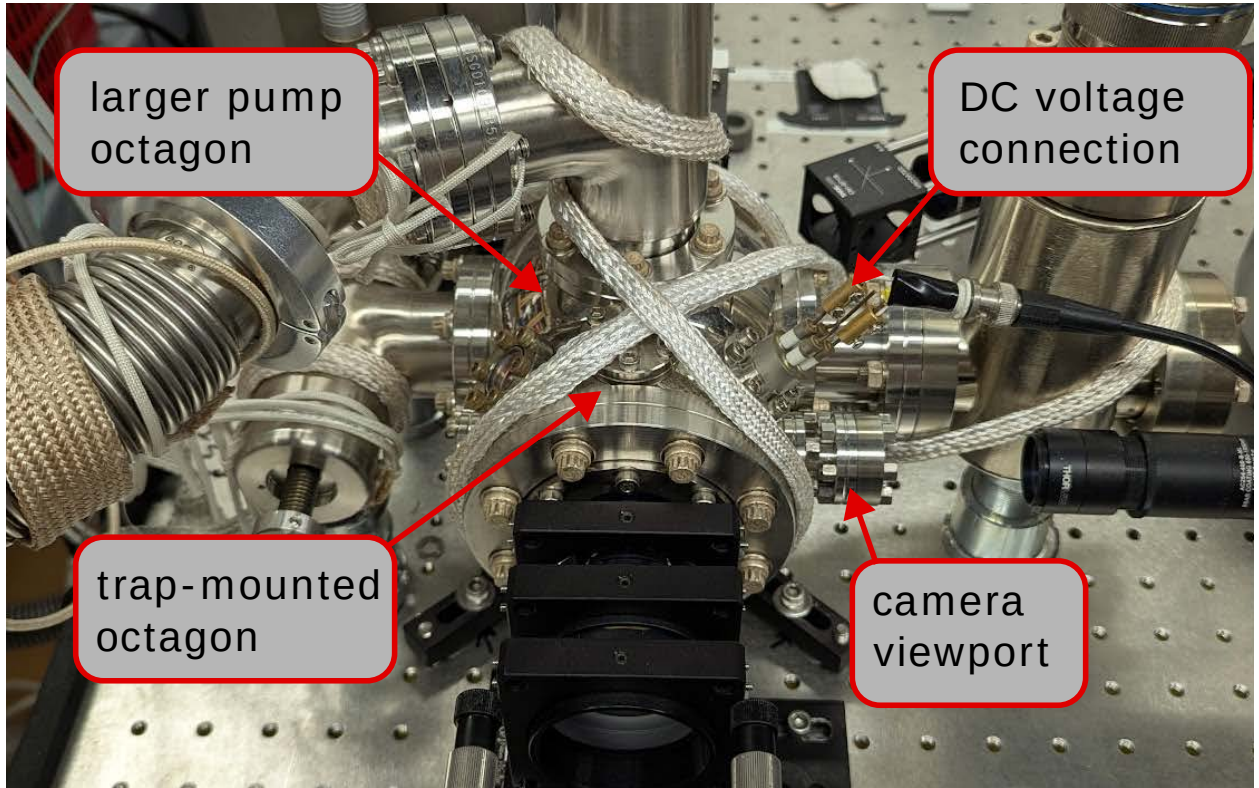


Figure 17: An image of the vacuum chamber used on our optical bench. Closest to us is the trap-mounted octagon, with a viewport on the right to observe the trap. On top of this octagon, the external wires used to connect our voltage supply to the pole pieces are visible, also shown in Figure 14. The second octagon port with larger flanges is connected behind this section to accommodate our various vacuum pumps.

This spherical octagon with a trap has a depth of  $\approx 1.5$  in, providing a wide viewing angle of the particle trap. This design allows for the use of a 2-inch optical objective outside of

the chamber with minimal separation from the trap, giving an improved numerical aperture (NA).

The main body of the chamber is situated on a top hat mount designed to minimize vibrations. This mount is a fixed stainless steel platform that supports the chamber. The height of this platform was chosen to allow a trapped particle to be conveniently positioned at the same height as our optical elements.

### Vacuum Pumps

Several pumps are used to achieve a UHV environment, using the configuration shown in Figure 18.

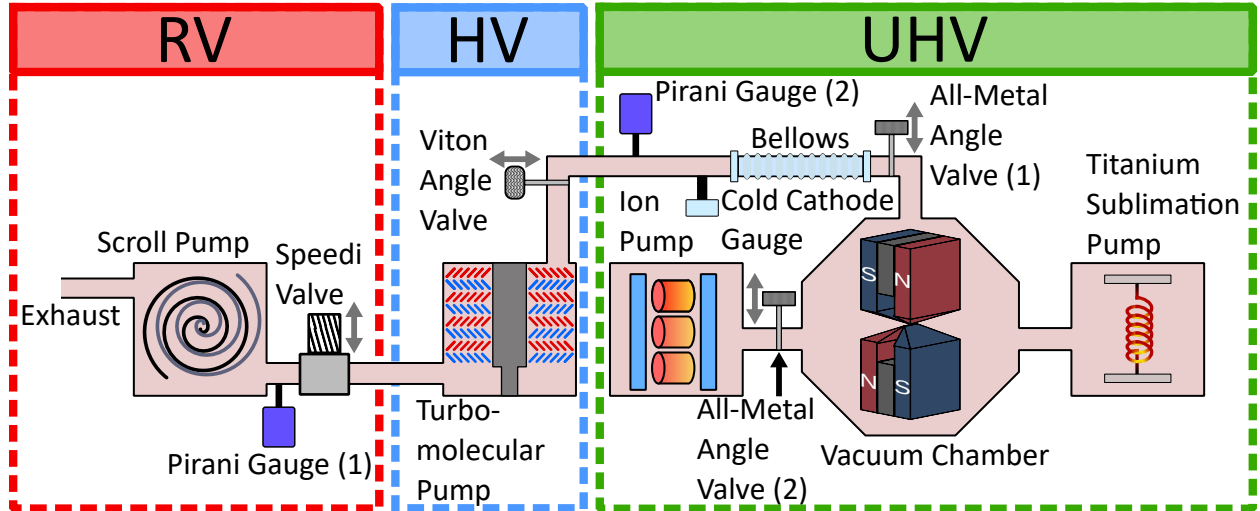


Figure 18: An overview of the various pumps and valves used to achieve UHV for our magneto-gravitational trap. Each section is labeled based on the expected vacuum quality achieved by its contribution. Rough vacuum (RV) around  $10^{-3}$  Torr, high vacuum (HV) around  $10^{-6}$  Torr, and UHV at and beyond  $10^{-9}$  Torr.

The assembly begins with a rough scroll pump (Edwards nXDS-10i Scroll Pump) that is connected to the chamber through an Edwards Speedivalve. This pump and valve combination allows for the gradual decrease in chamber pressure that lets us reach around  $10^{-3}$  Torr. The speedivalve, despite its name, must be opened very slowly since the slight decrease in pressure is critical for keeping a particle in the magneto-gravitational trap during

the pumping process. If the air leaves the chamber too quickly, it will pull the trapped particle along with it. A Pirani gauge is positioned between the rough pump and speedivalve to allow pressure readings during this process. The pump used along with the valve configuration can be seen in Figure 19.

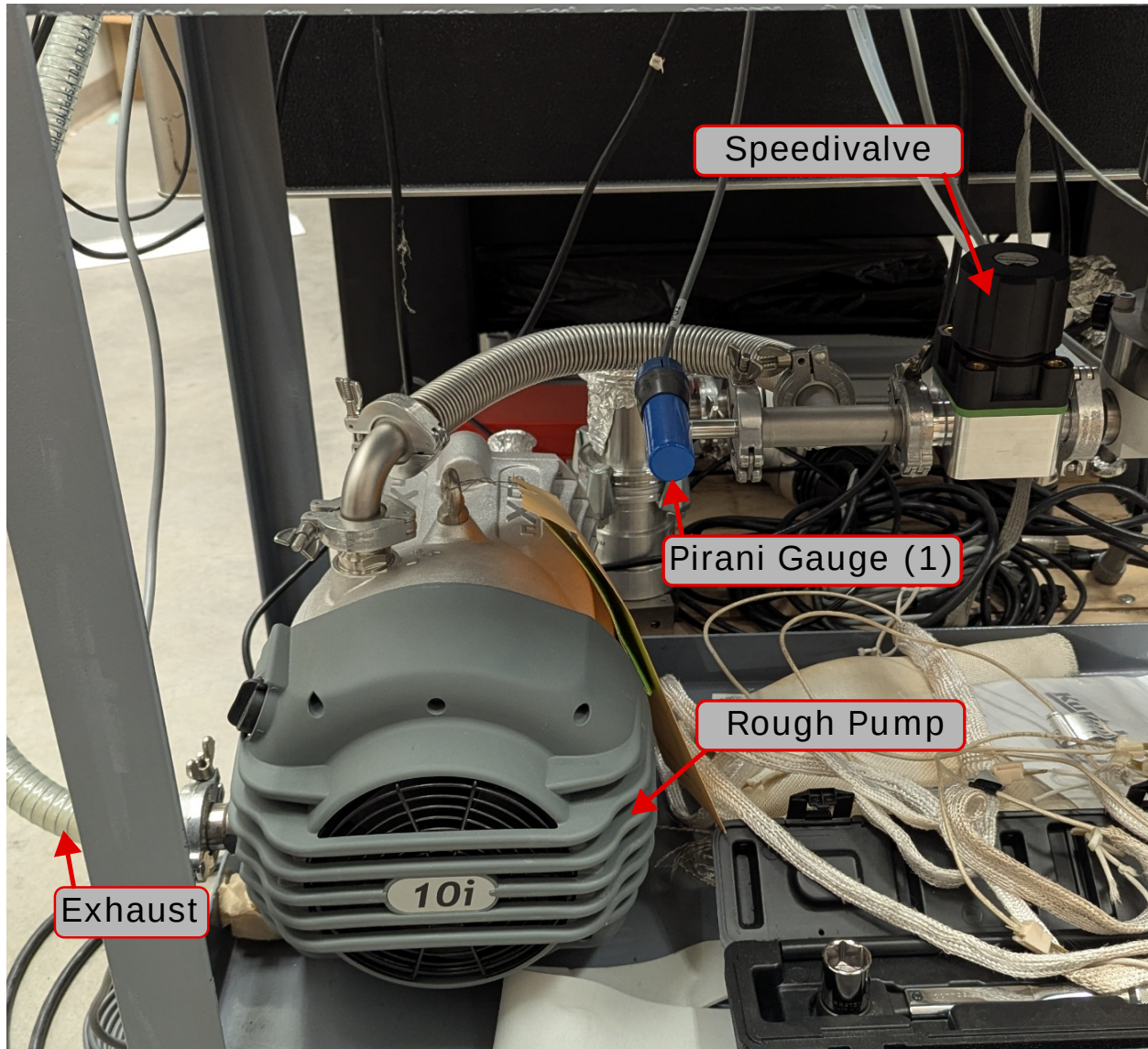


Figure 19: A scroll pump connected to a speedivalve next to the main optical bench. The speedivalve leads to the chamber through a turbo pump and bellows assembly. This pump stays near the chamber without issue since it will be turned off by the time measurements are conducted.

Reaching a higher vacuum requires an additional pump after  $10^{-3}$  Torr. A turbo molecular pump (Pfeiffer Vacuum TMU 521P PMP02855 Turbo Pump) is used to further decrease chamber pressure to around  $10^{-6}$  Torr. This pump has an exhaust that is connected to the rough pump via the speedivalve. Its input port (on top) connects to another valve (MDC valve) that allows isolation between the turbo pump and the main vacuum chamber. The chamber is connected to the turbo pump via a bellows, with another Pirani gauge positioned between the bellows connection and MDC valve. When the turbo pump is turned on, the chamber pressure will already be low enough that there is minimal risk of losing the particle due to air currents. At this stage, the Pirani gauges bottom out, and pressure monitoring is done via a Cold Cathode gauge (Varian 525 series cold cathode gauge). The turbo pump and its configuration are shown in Figure 20.

The two Pirani gauges are configured to allow real-time analysis of pumping quality at this stage. Readings that differ from expected results can have many causes, such as a contaminated chamber after maintenance on the pole pieces. Typically, this deviation would be minimal. Leaks between the chamber or pump gaskets can cause a large increase in pressure on the cold cathode gauge, which would be a clear sign to inspect the chamber and identify leaks. The most common source of a leak identified in this system is between the bellows and chamber connection. Typically, a finger-tight adjustment to the fitting between these two parts would be sufficient, but sometimes, chamber pressure would plateau at a sub-par vacuum quality. Most of the time, giving this part of the assembly an extra nudge beyond finger-tight was sufficient to stop the leak.

Contaminants from the atmosphere that lead to excess outgassing prevent us from going beyond our  $10^{-6}$  Torr vacuum on short time scales. When reaching this stage, the chamber is baked up to  $130^{\circ}\text{C}$ . This uses silica heater tape powered by a variac voltage supply (TDGC2-2KM Metered Variac). Baking temperatures are limited to around  $\approx 130^{\circ}\text{C}$  to prevent damage to gaskets and excess thermal expansion of the magneto-gravitational trap

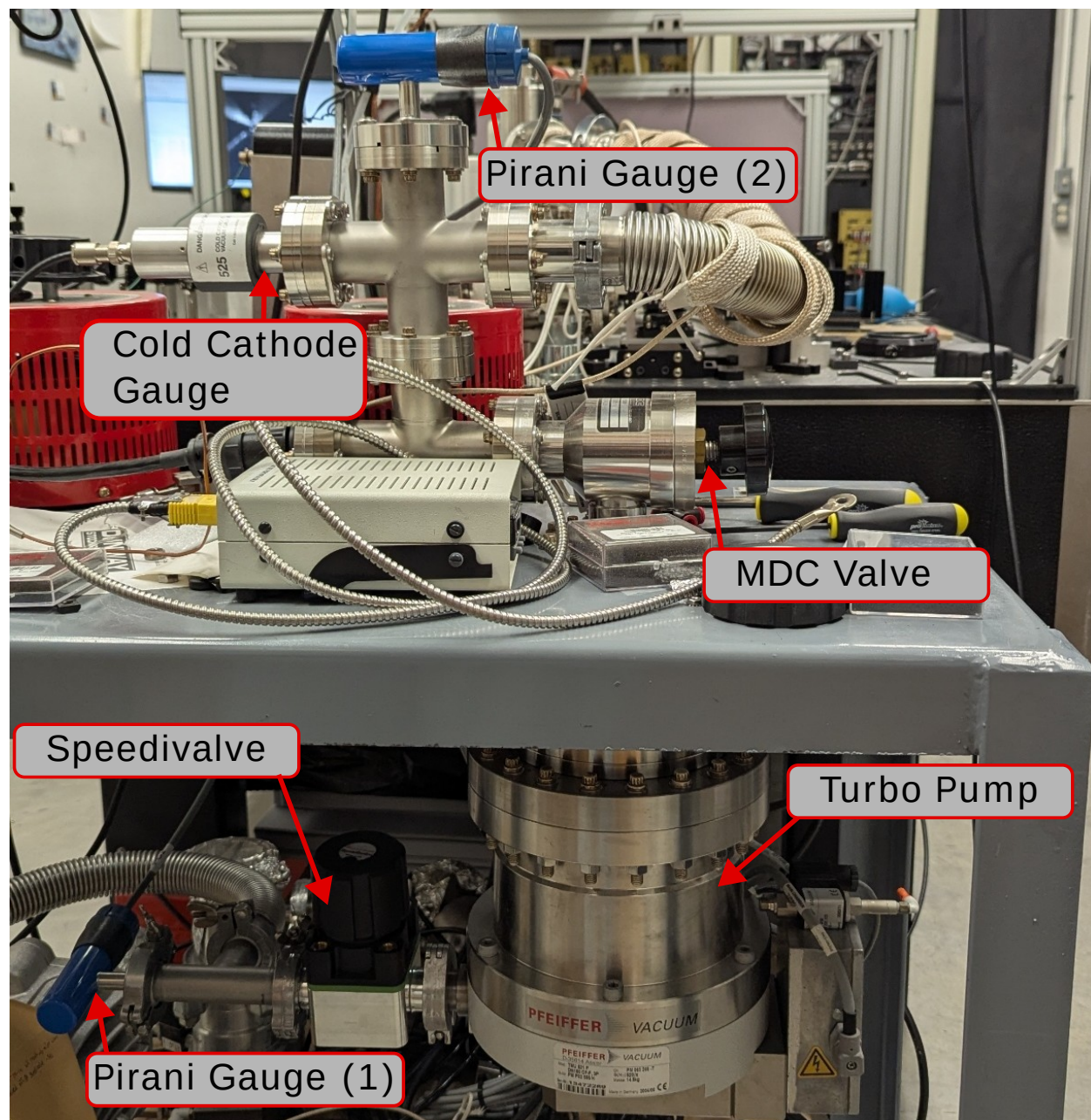


Figure 20: The turbo molecular pump used to achieve  $10^{-6}$  Torr. Positioned after the scroll pump and speedivalve, this pump sits on the same assembly. The Pirani gauges between the scroll pump and turbo pump and the turbo pump and chamber allow for real-time evaluation of pressure during the pumping process.

as outlined in Brad Slezak's dissertation [73]. Extra precautions to protect the chamber windows during baking are taken by wrapping these vulnerable parts in aluminum foil. This

prevents dangerous temperature gradients from forming on the glass, potentially causing leaks or complete part failure. Baking is done for 72h and brings the chamber pressure around  $10^{-9}$  Torr. A 72h baking time has resulted in a long-term improvement in vacuum quality. A picture of the chamber and its heating tape configuration is shown in Figure 21.

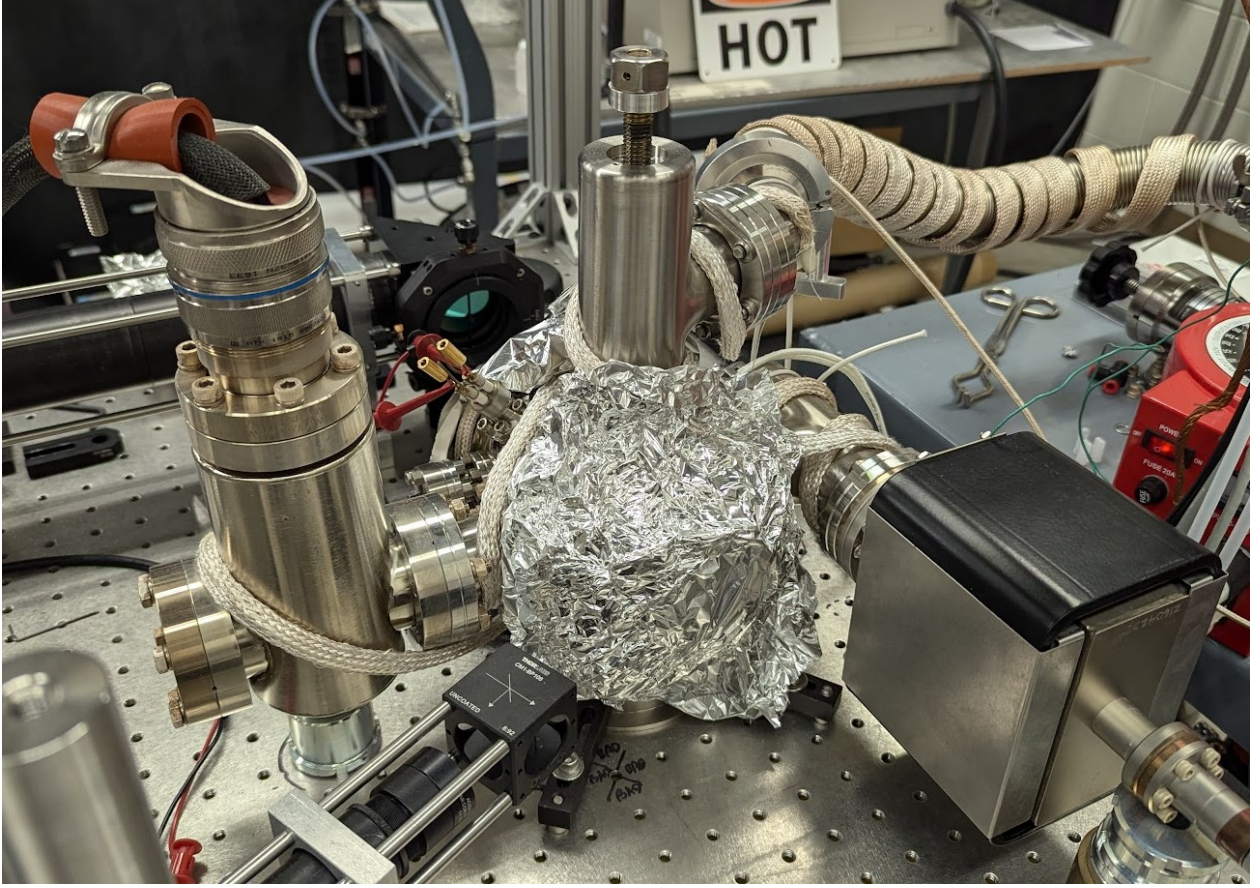


Figure 21: The chamber is wrapped in heater tape that bakes the chamber up to a temperature of  $130^{\circ}\text{C}$ . Aluminum foil is wrapped around the viewports of the chamber to prevent thermal gradients from damaging the windows. The objective used for particle loading is removed during the baking procedure. A thermocouple is periodically used to monitor chamber temperature.

After 72 h has passed, the variacs are adjusted to bring the chamber temperature below  $100^{\circ}\text{C}$ . This protects the copper gaskets that separate the ion pump (Duniway DGD-030-5125-M) and bellows tube from the chamber. This reduction in temperature also allows us to remove the aluminum foil and reposition the optical objective to view the trapped particle.

Active cooling can be turned on at this stage if illumination and feedback cooling are aligned before the pumping procedure. This adds an extra layer of protection against losing the particle during the rest of this process. This is not a required step, as the excess damping from the heated chamber is typically sufficient to keep the particle within the trap while turning the valves, but the added protection is always welcome.

While the scroll and turbo pumps are still going, the titanium sublimation pump is fired to decontaminate and prepare it for sublimation. Typically, the pressure will spike during this stage as heavy outgassing from the coil of the titanium pump fills the chamber. The turbo and rough pumps will quickly take care of this, and our titanium pump will be ready to use as needed while the chamber remains at UHV.

With the titanium pump out-gassed, the ion pump is exposed to the main chamber, which causes another drop in pressure on the cold cathode gauge and an increase in our ion pump gauge. After a short moment, these readings should equalize. At this stage, the cold cathode gauge sometimes shuts itself off due to the low vacuum, which indicates that the pumping procedure has gone well. Following this, the metal valves at the top of the chamber is closed, which isolates the chamber from the rough and turbo pumps by blocking the bellows tube. Once shut with a torque wrench to ensure a good seal, the isolated chamber operates only on the ion pump (and the titanium pump as needed). The turbo and rough pumps can be turned off at this stage, and the bellows tube can be removed. The chamber will still be hot, and its temperature will stabilize over the next 24 h. Once stable, the chamber pressure should read in the low  $10^{-9}$  Torr range and sometimes even read zero (since our ion pump gauge bottoms out at this reading, indicating a chamber pressure on the order of  $10^{-10}$  Torr). Sublimate titanium at this point with our newly isolated and cooled chamber to further reduce chamber pressure as needed.

## Optics and Control

With most of the chamber setup complete, we can focus on the rest of our optical bench. Once a particle is trapped and sealed in an UHV environment, a method to observe it must be realized. Specifically, an observation method that fits the system model as discussed in Chapter 2 is required. The next several sub-sections of this chapter cover the various components of the optics needed for imaging our trapped particle.

### Illumination Beam

To observe the particle motion, an illumination beam that can be pulsed quickly and provide stability in our measurements is used. The illumination beam used consists of both a 770 nm superluminescent diode (SLD) (Thorlabs SLD770S), and a 785 nm booster optical amplifier (BOA) (Thorlabs BOA785S). This beam operates in a range from 0.2 mW all the way up to 20 mW, with an estimated peak wavelength of 785 nm for the combined system in operation. For polarization control, a set of paddles is used to induce strain on the fiber, adjusting the output polarization. These paddles are used to maximize the power from the fiber output during the operation of the optical amplifier. These illumination beam components rest on a plate shown in Figure 22.

Following single-mode fiber coupling, the output of this beam (now Gaussian) is collimated and passed through a series of optics to ensure stable vertical polarization. This optical configuration starts with a quarter-wave plate (Thorlabs WPQ05M-780) followed by a free space optical isolator (Thorlabs IO-5-780-VLP). The isolator is a crucial component for this setup. Without it, the beam was observed to couple back into the fiber output, causing a reduction in output power. After the isolator, the beam passes through a half-wave plate (Thorlabs WPH05M-780) before passing through a final crystal polarizer to ensure clean vertical polarization (normal to the surface of the optical bench). The wave plates are set in

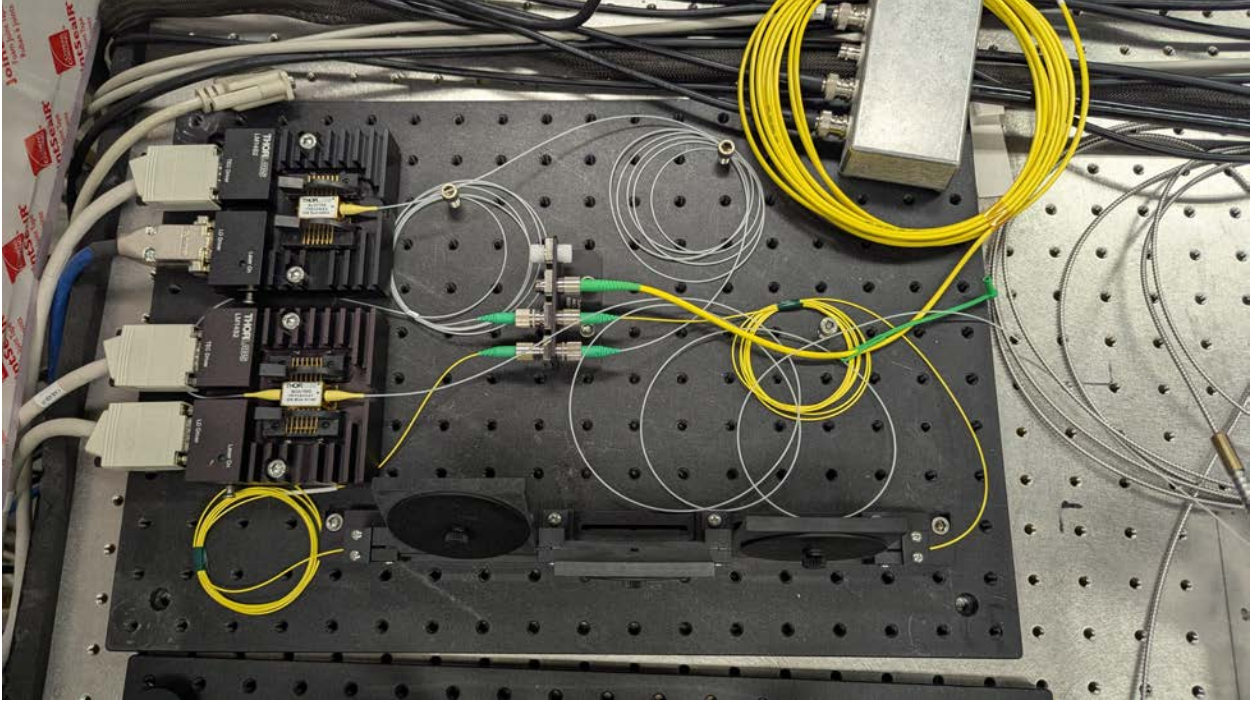


Figure 22: The illumination beam configuration used on our optical bench. Two butterfly mounts with non-polarization maintaining fiber deliver a low coherence length 785 nm beam through the single-mode fiber. The control paddles can be seen with the amplification fiber routed through them, allowing for control of output polarization.

adjustable mounts that, with the aid of the paddles, are used to maximize the beam output measured past the crystal polarizer when measured with a power meter.

A beam splitter and photodiode setup are kept next to these components for troubleshooting purposes, which will prove helpful come Chapter 5. The photodiode output can be measured using an oscilloscope to diagnose problems or maximize the power output of the beam. The next set of optics adjusts the beam size along two orthogonal axes to ensure smooth illumination of the trapped particle in the magneto-gravitational trap. This is done through the use of cylindrical lenses that adjust the beam diameters of each axis independently of one another. The final output of the beam before entering the chamber is that of a Gaussian with two different diameters along its vertical and horizontal orientations. This is done so that the horizontal beam size at the imaging plane of our optics is wider

than the vertical beam size. Since the axial motion of the particle is much more prominent in amplitude than the vertical motion, this configuration of the illumination beam allows a uniform planar wavefront to hit the particle. Figure 23 shows this optical train used on our illumination beam up to the crystal polarizer, while Figure 24 shows the cylindrical lens configuration to adjust our beam diameter.

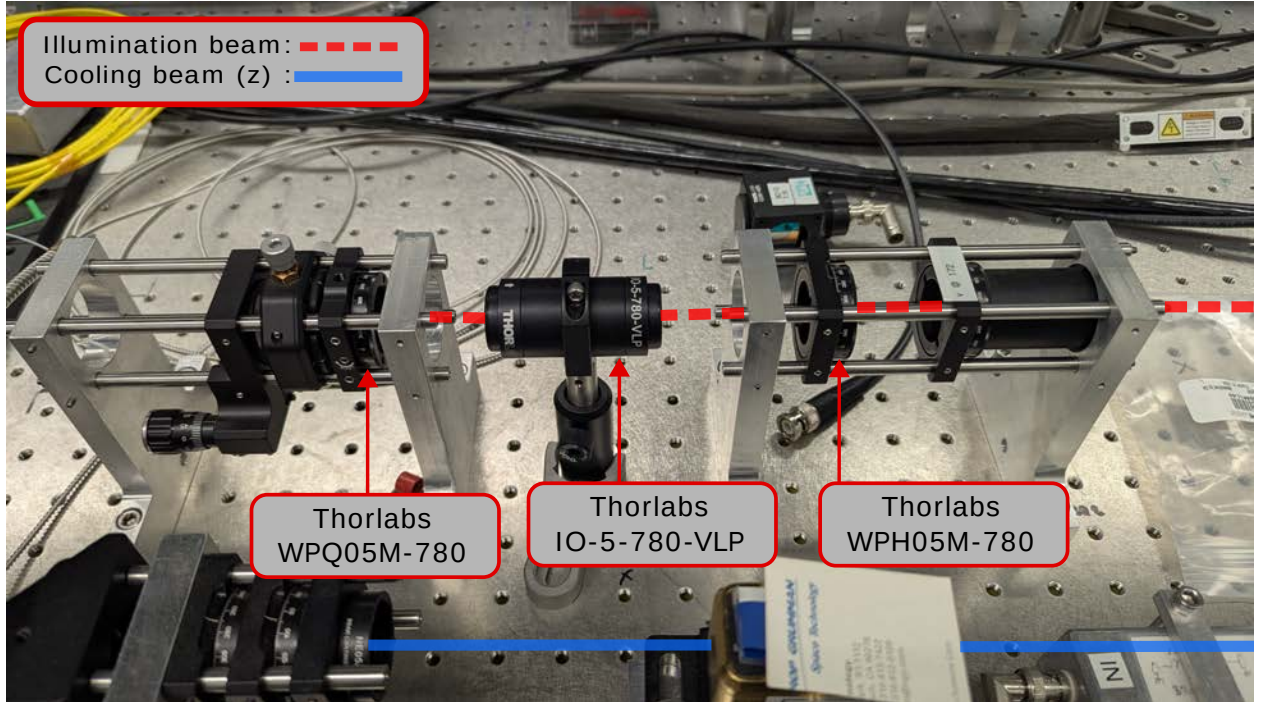


Figure 23: Here the optical train used to shape and polarize the illumination beam following the output of the single-mode fiber is shown. The beam exits the fiber on the left and is collimated before passing through the quarter-wave plate, optical isolator, and half-wave plate. Finally, it passes through a crystal polarizer to produce a vertically polarized beam.

Given a uniform distribution of light around the trapped particle, optical trapping effects are avoided since the input optics to the chamber are configured to make the wavefront of the illumination beam planar at the particle's location. Additionally, the illumination beam is adjusted to be orthogonal to the axial motion of the particle that will be tracked (the beam travels down the transverse axis). This results in less impact on the particle response caused by the illumination beam outside of the stochastic noise, as mentioned

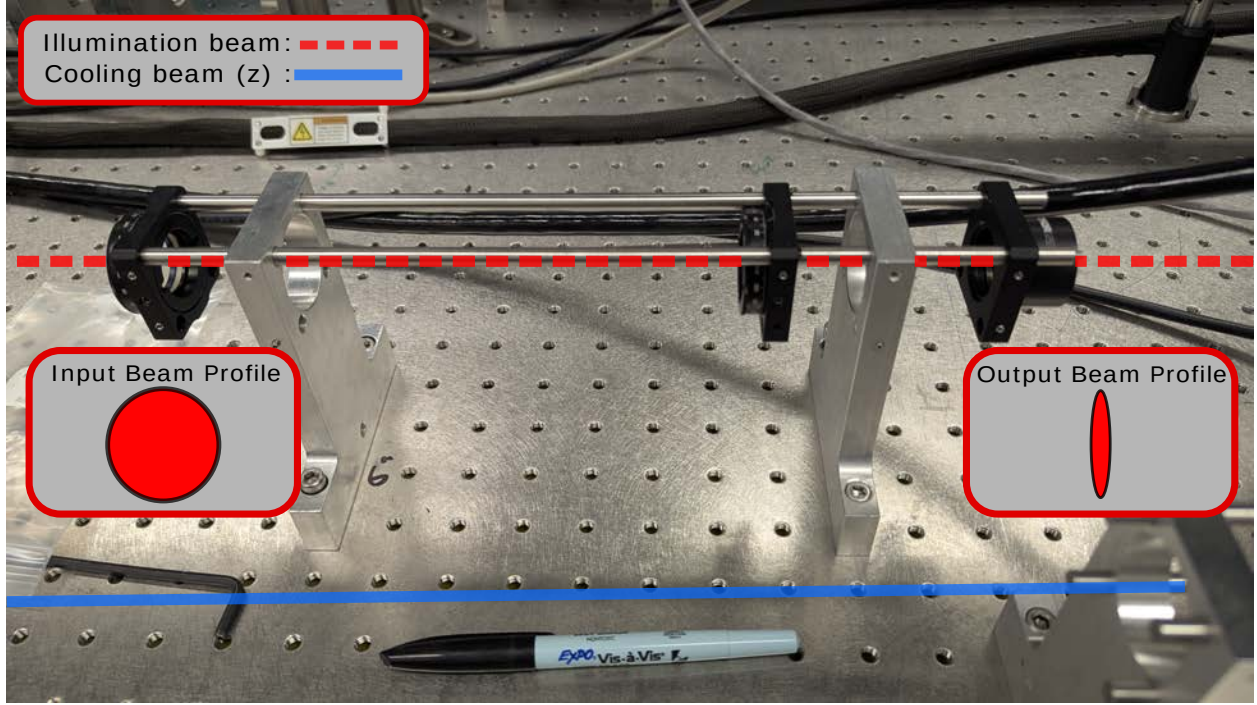


Figure 24: Two cylindrical lenses (250 mm focal length followed by 50 mm focal length) are configured to focus light on two orthogonal axes (parallel to the table surface and normal to the table surface) allowing an output of two independent beam diameters. The output of this configuration is an oval-shaped Gaussian beam roughly 0.5 mm wide and 4 mm tall. In this image, the beam propagates from left to right. An image of this beam profile when it arrives at the particle is shown in Figure 37.

in the theory section of this paper. If the beam were not orthogonal to the axial motion ( $z$ -axis), this would result in a driving force that would negatively impact measurement outcomes. Further discussion on this alignment is discussed in Chapter 4 under the section ‘kick calibration.’

### Feedback Cooling Beams

Feedback cooling is achieved by tracking particle motion with two area-scan cameras that perform real-time image analysis using a graphics processing unit (GPU) [69]. While particle tracking is a crucial part of this process, an effective way to guide the particles to their desired locations gently is also required (gently is optional). This section will provide

an overview of the cooling beams used in this setup, which facilitate this guidance.

Feedback cooling is done with a 680 nm SLD (QPhotonics QSDM-680-9) mounted in a temperature controller (Newport M700C). Unlike the illumination beam, this SLD is a free-standing diode that is not operated in a butterfly mount and must be manually coupled into a single-mode fiber. First, the beam passes through a half-wave plate (Thorlabs WPH05M-670) for polarization control; following this, the beam enters an optical isolator (Thorlabs IO-5-670-VLP) before coupling the beam into a single-mode fiber as shown in Figure 25.

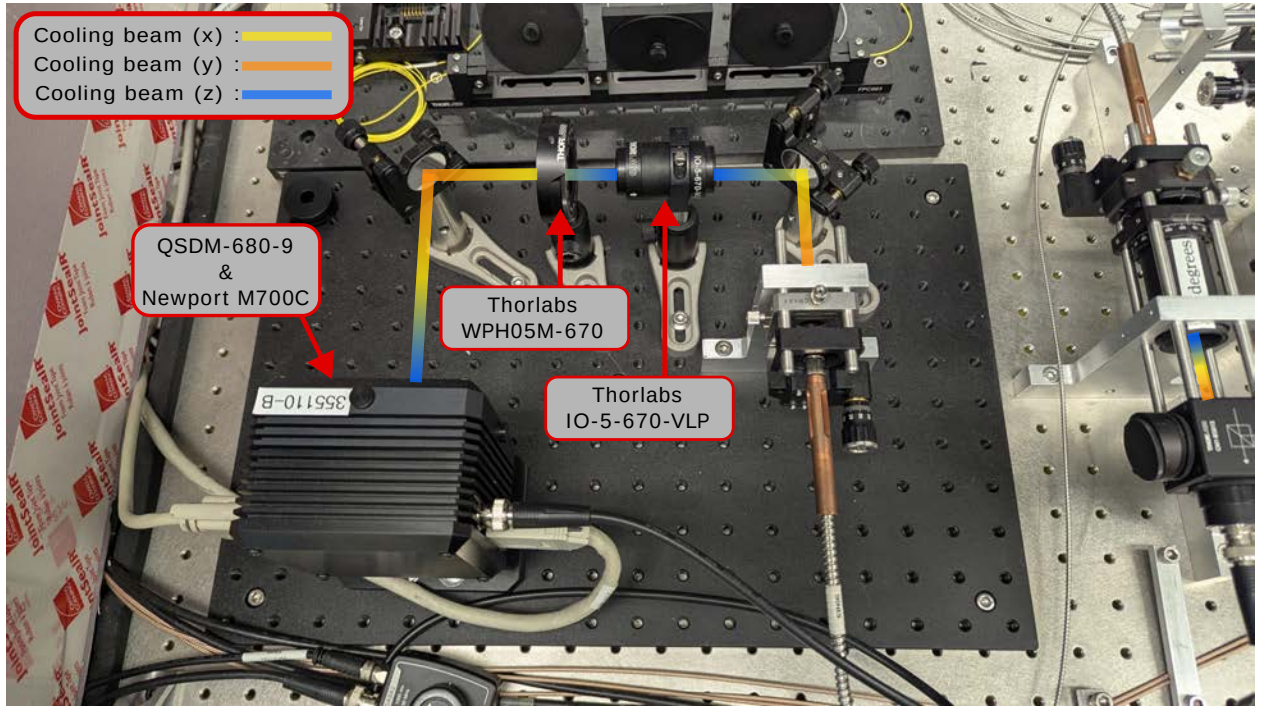


Figure 25: The cooling beam configuration used on our optical bench. This setup polarizes and isolates the beam before coupling into a single-mode fiber. The beam will eventually be split into separate paths to handle the cooling of each Cartesian axis of our trap. For now, the color gradient of the beam path represents all of these (eventual) separate beams.

The beam exits the single-mode fiber and passes through a vertically oriented crystal polarizer before reaching the first of several beam splitters. This beam splitter (Thorlabs CM1-BS013) directs a portion of the beam to a photodiode, which is used for amplitude stabilization by providing feedback to the Newport M700C temperature controller.

Meanwhile, the remaining beam portion travels through a 500 mm lens before encountering a complex arrangement of beam splitters and mirrors that separates the beam into three distinct paths. This setup is illustrated in Figure 26.

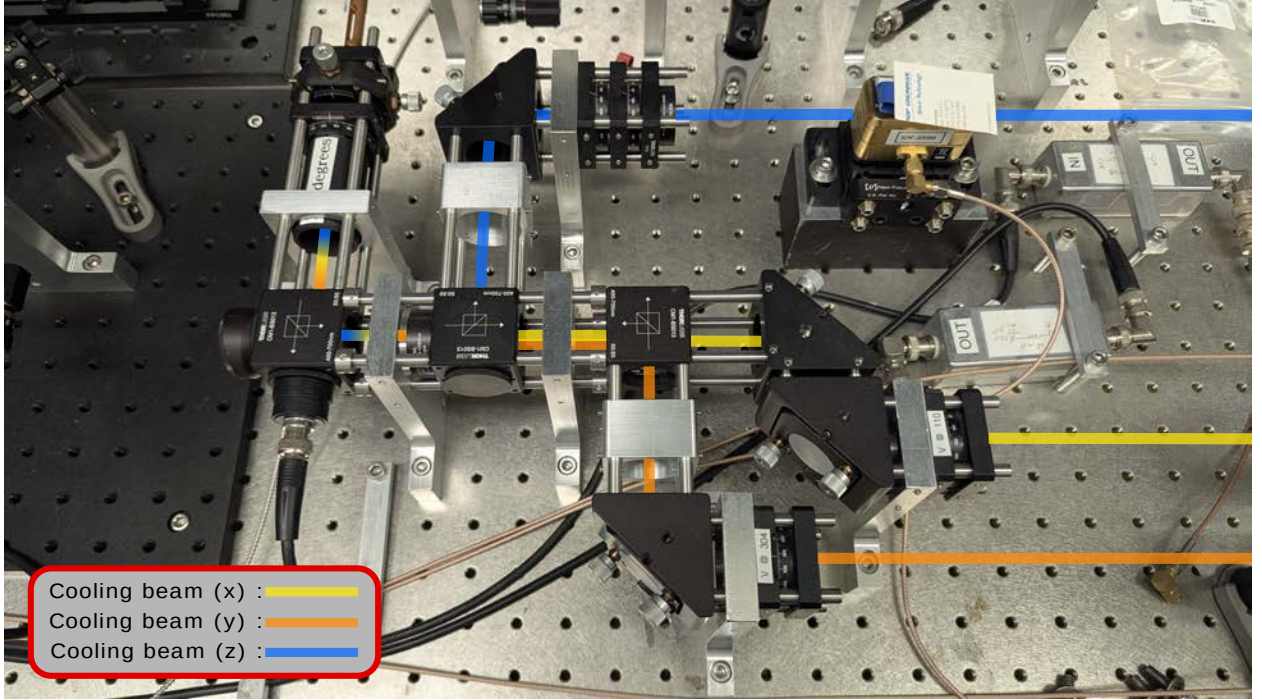


Figure 26: After exiting the single-mode fiber, our cooling beam is stabilized using a photodiode setup, as shown in the bottom left of this figure. The beam travels through a series of beam splitters and mirrors that split it into three separate paths.

All cooling beam paths, after going through their respective beam splitters, are passed through quarter-wave plates (Thorlabs WPQ05M-670). Since the topmost beam from Figure 26 is not split again like the other two beams, an attenuator is used to reduce the output power (Thorlabs NE05-A-B) along this beam path. At this point, each beam must pass through an electro-optic modulator (EOM). These EOMs (New Focus 4102 series electro-optic amplitude modulator) can modulate the amplitude of our beam along each respective path. Connected to our feedback control with a SubMiniature version A (SMA) connector, these EOMs are given a signal in response to our particle motion, allowing adjustments to provide additional damping to the particle to cool its center of mass motion.

Previous iterations of this design have used lasers with long enough coherence lengths to require phase EOMs on two of the beam paths to eliminate any phase relationship between the beams that would produce unwanted effects when attempting to cool the particle. After switching to a SLD, the low coherence length of our beam is sufficient to ensure no phase relationship, and the phase EOMs were removed.

The cooling beam paths will meet again at a final optic positioned just before the vacuum chamber. This 2 in diameter optic (Thorlabs AC508-150-B-ML) redirects all cooling beam paths to the particle.

The beam path on the bottom of Figure 27 handles vertical feedback cooling and goes through a 50:50 polarizing beam splitter (Thorlabs PBS252) before hitting a set of periscopes. These adjust the beam height relative to the optical table, allowing each respective path to approach the particle from above and below, giving more control over the vertical motion of the particle.

The middle path from Figure 27 handles cooling on the transverse axis. This axis is orthogonal to the final optic and does not require splitting. This path merges with the illumination beam path via a dichroic beam splitter (Edmund Dichroic Filter #87-065).

The final beam path from Figure 27 covers the cooling of axial motion. This beam is split once more into two separate paths that remain level with the table, unlike the vertical cooling beams. Using two separate mirrors positioned to the left and right of the optical axis leading into the chamber, the beams are directed to enter our final lens similarly to our vertical cooling beams. These beams act to cool the axial motion of our particle. A representation of these beam paths is shown in Figure 27.

The power from all of our cooling beams is several orders of magnitude lower than our illumination beam power. As a result, the RPSN caused by the cooling beams is negligible. Ideally, feedback cooling should introduce no additional noise to the system; however, in practice, there is an observed increase in noise. One indicator of this is seen in the product

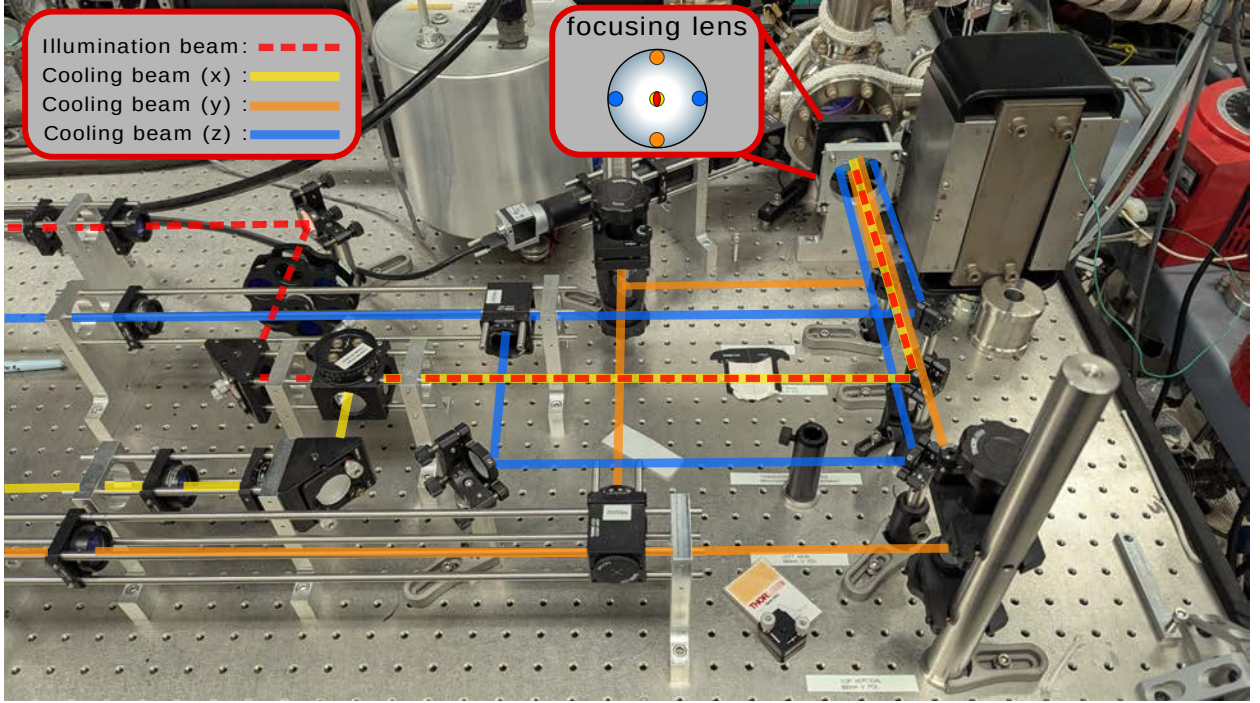


Figure 27: The optical configuration that guides all five cooling beams to the particle. Using a set of beam splitters and periscopes, the end result is two vertical beams, two axial beams, and one transverse beam. The entry point of each beam in our final optic is also shown in the focusing lens diagram, where the colors correspond to their respective beam paths.

of damping and temperature when a peak fit is done to our axial motion in the frequency domain. The product of these two parameters describes our thermal noise. This system, when operating in UHV, achieves an incredibly low natural damping rate,  $\Gamma_N \approx 10^{-6} \text{ s}^{-1}$ , while the thermal bath is at room temperature (295 K). Active cooling reduces the center of mass temperature of the particle at the expense of increased damping. Our feedback cooling can not reduce our thermal noise, and the increased noise introduced to the system from the feedback beams only makes the product of these two variables more significant. This results in a relationship between the natural temperature and damping compared to the feedback temperature and damping given by

$$T_N \cdot \Gamma_N \lesssim T_{FB} \cdot \Gamma_{FB} \quad (61)$$

where  $T_N$  and  $\Gamma_N$  represent the natural temperature and damping of the system, while  $T_{FB}$  and  $\Gamma_{FB}$  denote the particle's center of mass temperature and damping rate under feedback cooling. Fortunately, the primary interests are in the system's behavior in the free particle limit, away from the natural oscillation frequency of the trap. In Chapter 5, we will see how the increased noise occurs around resonance with our oscillator and subsequently decreases to our natural thermal noise when approaching the free particle limit.

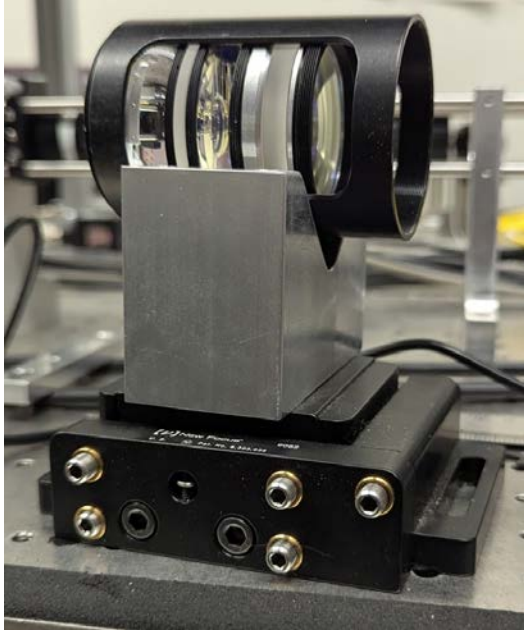
At this point, our overview of the beams used in our optical bench is complete. Referring to Figure 11, we have finished discussing the 'illumination' and 'cooling' components of our system, along with the vacuum chamber and pumps. The remainder of this chapter will focus on the opposite side of our optical bench, which contains nearly all of our imaging components and detectors, followed by a brief overview of system timing and control.

### Optical Objective

Optical aberrations are incredibly problematic during our measurement procedures. Any imperfections in the wavefront after scattering cause increases in DSN that are undesirable. Under ideal circumstances, the image detected on the camera would be that of a nice dot with tiny airy disk rings coming from it due to the diffraction limit of our optics. Figure 28 shows the optical objective used when I first took over this project. All of the brightness seen away from the central bright spot comes from scattered photons that do not contribute to reducing the DSN floor. Reducing these aberrations is a sure way to improve our DSN conditions. A process of trial and error is used to reduce the aberrations considerably from non-ideal circumstances. An example image taken using the old objective is also shown in Figure 28.

A new objective was built with aberration correction in mind. This objective, first modeled in Zemax by Cody Jessup, was constructed in the lab to be used on the SQL bench. It consists of three central optics, in order of position after the chamber (Edmund Optics

(a)



(b)

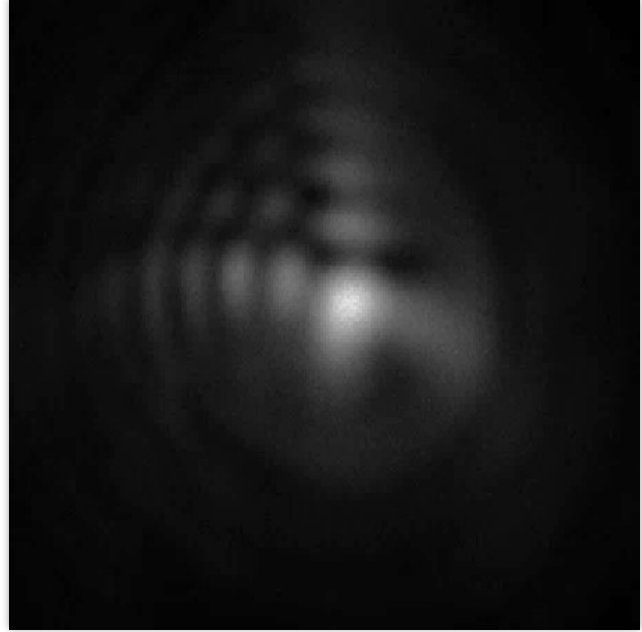


Figure 28: (a) This is the original objective used when I first took over this experiment. It gets positioned just after the vacuum chamber to collect light scattered off the particle. The setup consists of three optics housed in a non-adjustable lens tube. This tube is mounted on an adjustable stage (Newport 9082 Wide Five-Axis Aligner) that offers five degrees of freedom, enabling precise adjustments to minimize lower-order aberrations. The lens tube is also supported by a rigid aluminum mount, designed to keep the objective level with our other optical components. (b) An image captured using this old objective. Notice the quality of the image; it appears messy, with several non-uniform fringes visible next to the particle spot at the center. These fringes represent photons that do not contribute to accurately determining the particle's location, resulting in an increase to DSN.

#22-909 Asphere, Thorlabs LA1050-B, and Ross Optical L-BCC025), giving us a working distance of around 44 mm. A schematic of this objective is shown in Figure 29.

Unlike the old objective, our new one consists of additional mounts that allow more position adjustments between each lens. This new configuration benefits from the improved optics used to build the objective and the possibility of more degrees of freedom with our new configuration. Instead of a fixed lens tube, one lens (the front asphere closest to the vacuum chamber) is fixed on the top of a Newport 9082 mount. The other two lenses on this

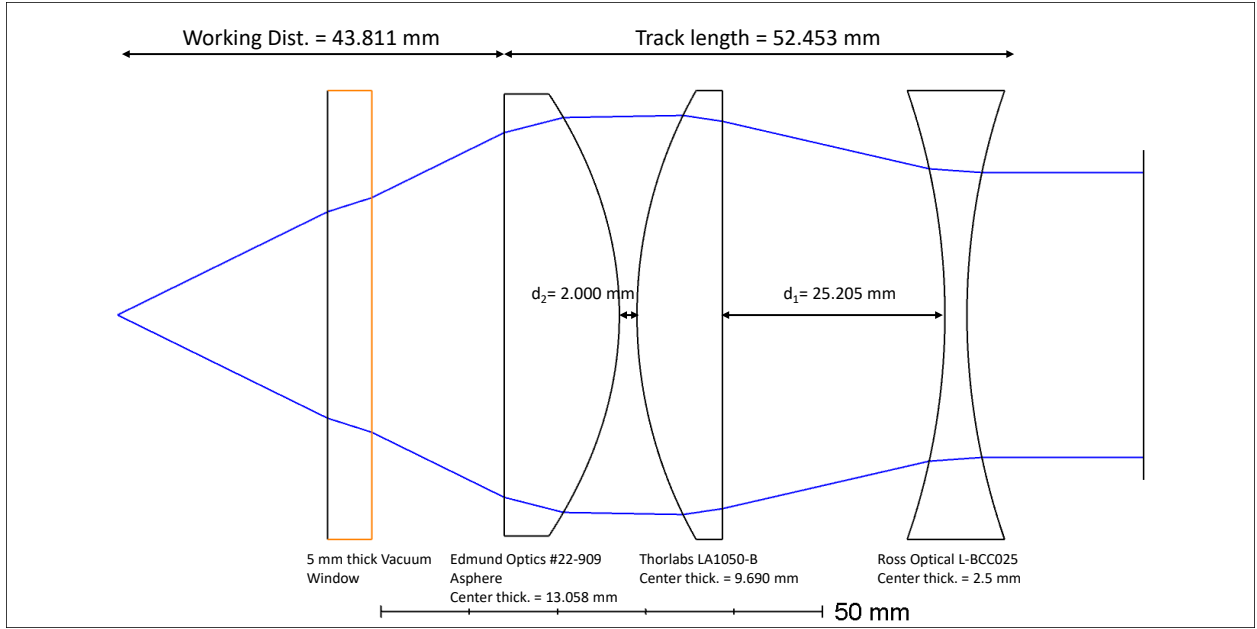
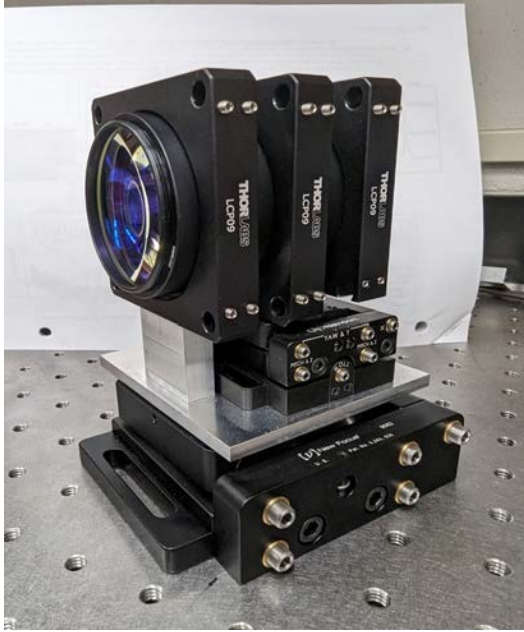


Figure 29: A schematic for the new adjustable objective.

stage are each independently adjustable on their own stages (Newport 9031 six-axis aligner) mounted on top of the Newport 9082 mount, giving us a whopping 15 degrees of freedom to make minor adjustments. While this setup took a lot of time and trial/error to configure, this eventually led to improvements in DSN. Figure 30 shows how the new objective has dramatically diminished the fringes of our particle compared to the old objective.

Once the aberrations have been minimized as much as possible using visual inspection, the two independent stages of the optics are no longer adjusted. Instead, the original five adjustable screws on the Newport 9082 are used, similar to those on the previous objective, to further reduce aberrations and maximize the peak intensity captured by our camera. By refraining from adjusting the two smaller stages that move individual optics, the need to recalibrate the distance for the objective is avoided since all lenses will stay fixed relative to each other. However, if these optics are adjusted, it would be necessary to reproduce a distance calibration for the newly adjusted objective due to changes in magnification. This

(a)



(b)

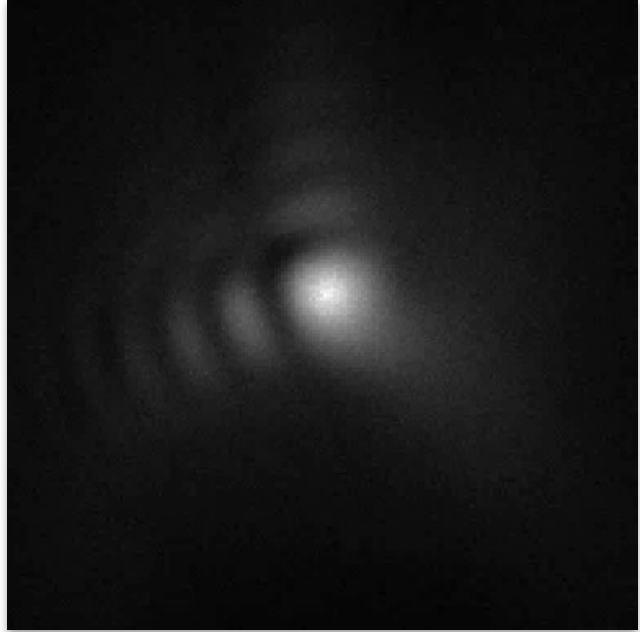


Figure 30: (a) The new objective with the asphere optic visible. Unlike the old objective, this one has each lens in an independently adjustable mount. The optical train is configured so that adjustment controls are in the same position as the old objective. (b) An image was taken with the new objective. A large reduction in aberrations results in an improved DSN floor during measurements. This objective also increases magnification compared to the old one, which is useful for avoiding camera saturation limits while taking high-power data.

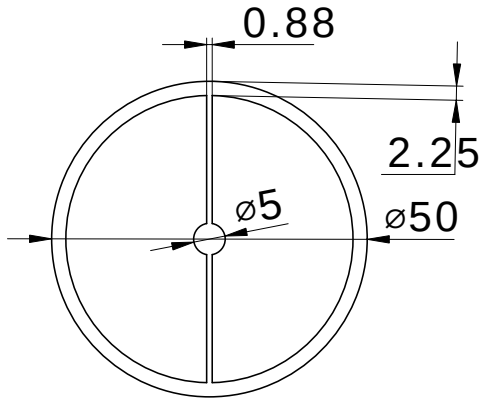
recalibration would need to be performed each time the sub-stages are used, so adjusting them as infrequently as possible is advisable.

Some attempts were made to aid in reducing aberrations by tracking how each screw adjustment affects the image from our camera. One idea was to monitor a quarter turn of each screw and analyze the image using components of Zernike polynomials. The goal was to create a library of images and corresponding screw positions that could be updated and used as a guide for future improvements in image quality. The advantage of this method would be its utility in correcting higher-order aberrations that require multi-screw adjustments. Unfortunately, this effort was unsuccessful due to the substantial time investment needed

for minimal improvements.

One aspect of the objective not covered so far is collecting light from our particle while avoiding light from the illumination beam itself. In previous project iterations, a beam blocker positioned after our collection objective would block the illumination beam. This blocker was produced from a 3D printer to fit in a 2 in optical mount to hold it centered in the beam path. This part can be seen in Figure 31. The scattered light from the particle can pass around this blocker and continue down the collection optics. New iterations of this project include a retro-reflector instead of a beam blocker, which will be discussed later in this chapter. The NA of our objective combined with the blocked region creates the ‘collection window’ shown in Figure 5.

(a)



(b)

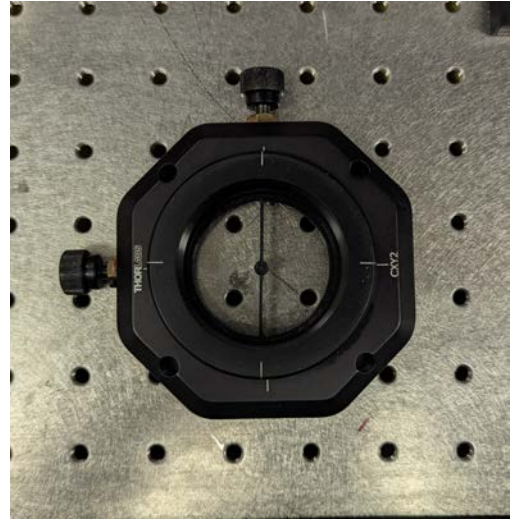


Figure 31: (a) A schematic of a simple beam blocker used on the SQL bench, measurements reported in units of (mm). (b) This device was printed to fit inside a 2 in optical mount after the collection objective. The central diameter was picked to be large enough to block the illumination beam while still allowing plenty of scattered light to pass by. Early configurations of this blocker were larger in central diameter and included four arms to keep the centerpiece stable. This final version allowed the most light to pass while still having good stability. When used on the SQL bench, it is configured so the two arms form a vertical line normal to the optical bench surface.

## Reference Cameras

This section covers the reference cameras used for particle tracking. Two area scan cameras observe particle motion along all three Cartesian axes. Figure 32 shows the optical configuration for the reference cameras.

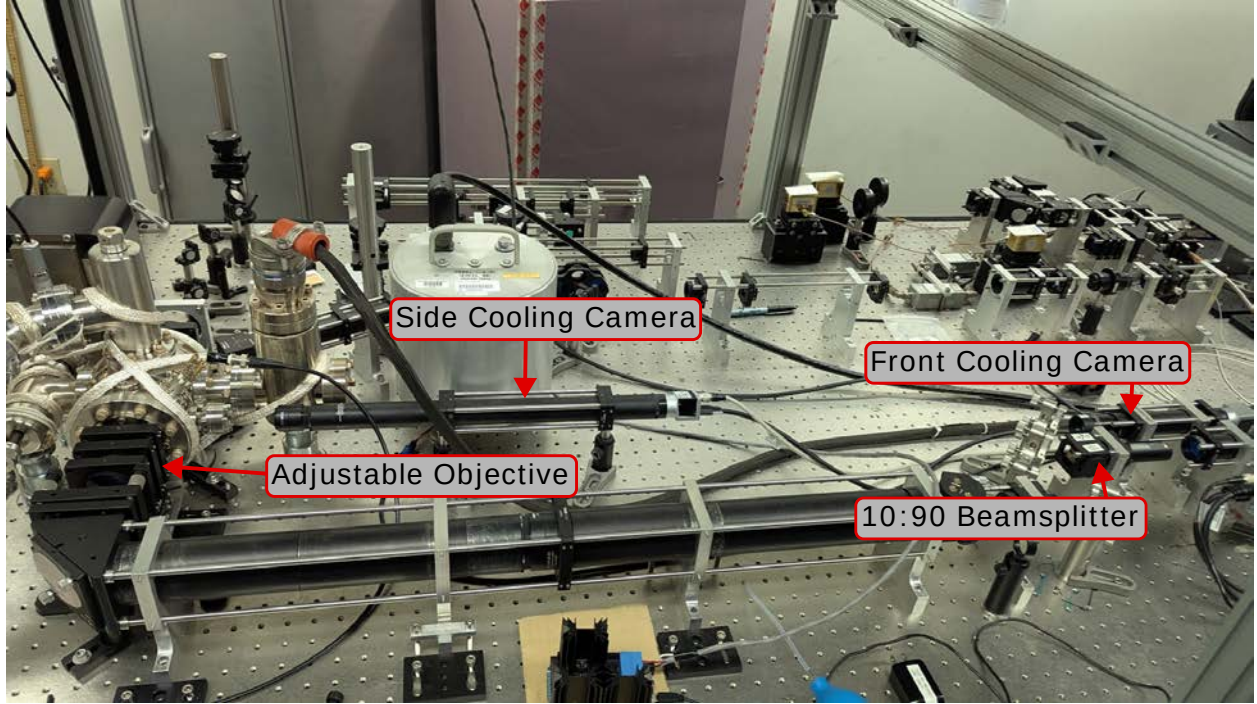


Figure 32: Scattered light exits the viewport along the z-axis and is collected by our ‘side camera’ (BASLER acA2440-75 $\mu$ m). This camera tracks particle motion along the xy plane. Light scattered along the transverse axis goes through an adjustable objective outlined in the previous section, a series of telescopes, and a 10:90 beamsplitter before hitting our other reference camera labeled as the ‘front camera.’ This camera (another BASLER acA2440-75 $\mu$ m) has much more magnification and observes motion in the yz plane.

The cameras used in our setup operate at a rate ranging from 325 Hz or 450 Hz, ensuring that the Nyquist frequency for observing particle motion exceeds the highest frequency of oscillation produced by the particle trap, which is 84 Hz. The front camera is fixed in position, with its rotation aligned so that the horizontal motion captured corresponds to the axial motion of the particle. Focusing is managed by our adjustable objective located just outside the vacuum chamber; therefore, the front-facing camera does not need to be

translated to focus the image. In contrast, the side camera does not have this limitation and can be adjusted accordingly.

The cameras are externally triggered to ensure synchronous operation. Both cameras have a limited field of view to achieve the specified frame rates, observing only a fraction of their available pixels. Each camera has a resolution of 2448 pixels in width and 2048 pixels in height; however, these full resolutions are unsuitable for high frame rate operation. Reducing the viewing window also decreases the file size during data collection if these cameras save images. Given the large file sizes generated during data collection, storage becomes a concern, so eliminating unnecessary parts of the camera's field of view during operation is advisable. The side camera is typically limited to a 100 by 100 pixel window, while the front-facing camera operates within a 500 by 200 pixel window. These camera configurations are illustrated in Figure 33.

For particle loading and chamber baking, the side camera must be removed. This camera also has no lens that significantly contributes to magnification external to the lens tube connected to the camera. This camera is focused by translating it towards or away from the chamber once its vertical height is fixed.

This camera is also rotated as needed to track transverse and vertical motion. As charge accumulation occurs, the transverse and vertical oscillation directions shift. The exact explanation of this change is not fully understood, but some suspects are the trap geometry and image charges. Figure 34 shows an example of this rotation.

Significant charge build-up can cause vertical and transverse axes to rotate excessively, limiting our cooling capabilities. In extreme cases, this leads to particle instability when attempting to run feedback cooling on the particle. In such cases, the UV-capable viewport above the trap can be used to discharge the particle. Once complete, the oscillation axes will return to their original orientation, and particle behavior will return to normal.

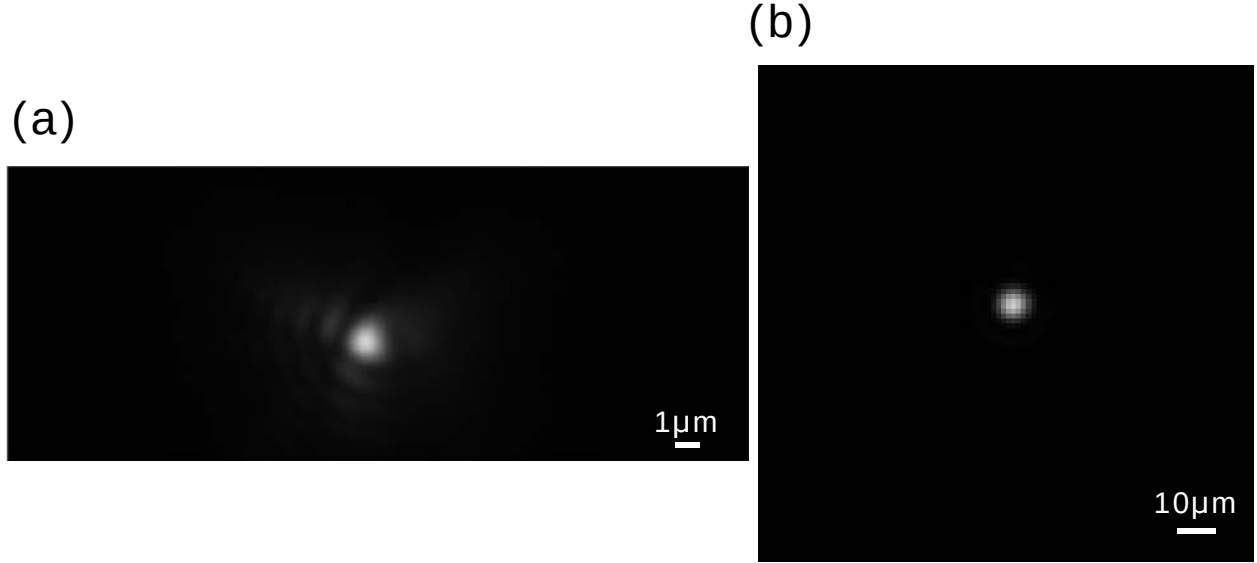


Figure 33: (a) This image was captured using the front-facing feedback camera. The viewing window has been reduced to 500 pixels wide and 200 pixels tall to enable higher refresh rates. A reference scale bar is displayed in the bottom right corner. The calibration of this camera is discussed in Chapter 4. (b) This image was taken with the side feedback camera at the same time as (a), with timing synchronized using a trigger cable. This camera has a significantly lower magnification, and our particle size is much smaller than its diffraction limit for this camera. A reference scale bar is included in this image, and its calibration will also be covered later.

### Retro Reflector

The illumination beam section discussed how the beam waist was deliberately made much larger than the trapped particle to minimize optical trapping effects. The resulting shape of the beam after this adjustment is illustrated in Figure 24. One issue briefly covered in the ‘optical objective’ section was the method used to block the illumination beam, allowing us to capture images of light scattering solely from the particle. This setup was standard for quite some time; however, it needed to be revised when attempting high-power continuous position measurements. Until that point, the blocker had sufficed for our purposes.

Introducing a continuous measurement scheme created a new challenge: the impact on momentum transfer along the transverse axis. Although the illumination beam does not

(a)



(b)

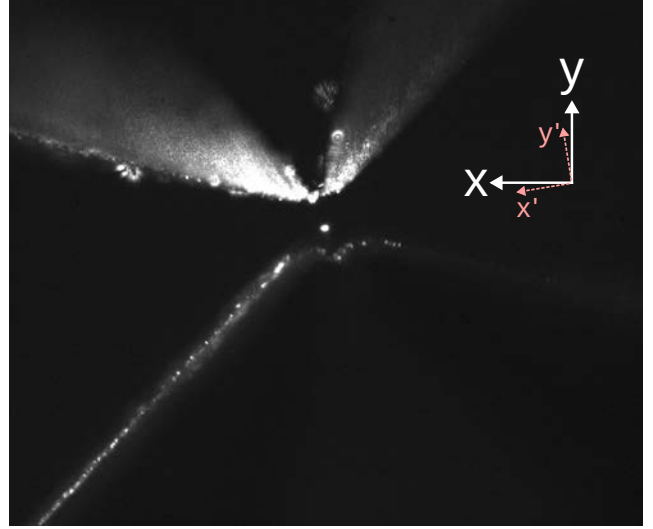


Figure 34: (a) The side camera is tilted to align the ‘transverse’ motion of the particle to the horizontal axis of the camera. (b) An image was taken by the tilted camera. The tilted configuration is aligned to the oscillation axes of the particle in the  $xy$  plane when the particle has accumulated excess charge. The dashed coordinate system represents the typical oscillation axes for a discharged particle; as charge accumulates, the axes begin to rotate, represented by the unprimed coordinate system in this image.

generate a restoring force in the  $yz$  plane due to its large diameter relative to the particle, it still pushes the particle along the transverse axis, which is orthogonal to the imaging plane of our objective. As a result, this can cause the front camera to become de-focused. In extreme cases, when the input power is sufficiently high, this momentum transfer may push the particle out of the trap.

This is rectified by the introduction of a retro-reflector. The idea here is to redirect the illumination beam back into the chamber to hit the particle from the opposite side. The net effect of the forward and backward propagating illumination beam is an equal push along the transverse axis in both directions. As a result, during high-power measurements, the particle remains around the object plane due to experiencing an equal push both forward and backward.

Early versions of this part consisted of a four-arm configuration of the beam blocker,

with additional extrusions to house a mirror. This configuration was problematic since the size of our reflector was much bigger than the beam. This resulted in significant losses of light collected from our cameras, leading to increased DSN. Eventually, a mirror (Edmund optics mirror #87-376) was found that could fit the diameter of our smaller beam blocker shown in Figure 31. Double-sided tape was used to attach this mirror without the need for additional support. This new configuration is what is used today, and a comparison of these two prints is shown in Figure 35.

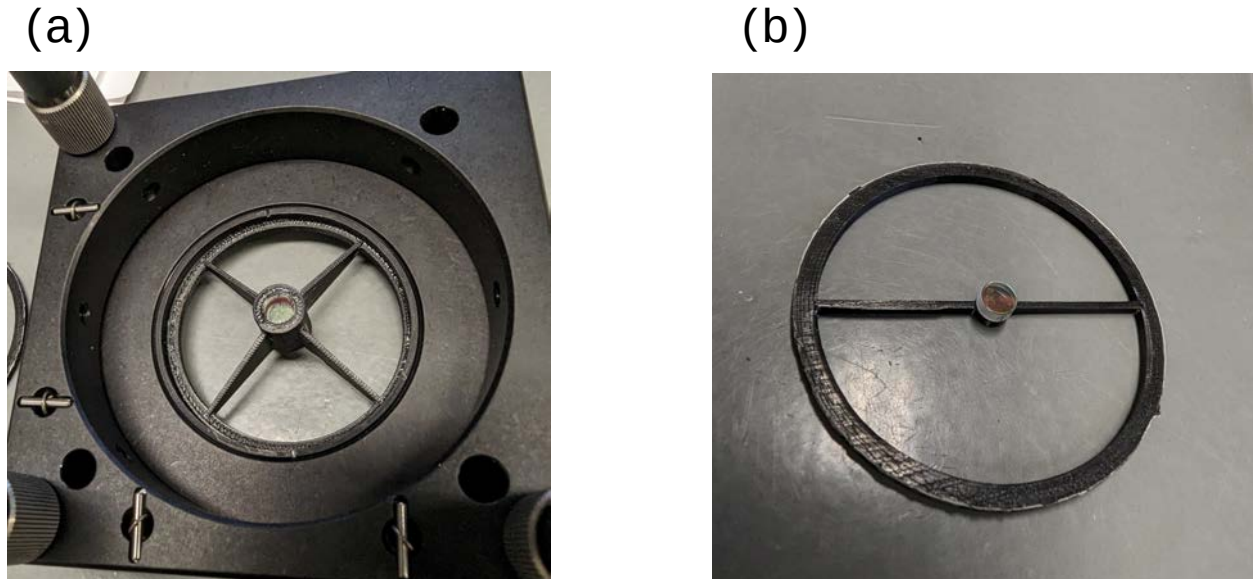


Figure 35: (a) The old retro reflector consisting of four arms instead of two and with a central spot much larger than the beam blocker, this reflector had a lot of drawbacks. Although the reflection of the illumination beam was realized, the downsides caused by the increased center diameter and extra arms were problematic. (b) The new retro-reflector mirror is mounted with double-sided tape. This configuration provides all the benefits of the old reflector with none of the downsides!

When positioned after our optical objective, the illumination SLD is redirected back into the chamber. An additional camera (BASLER acA2440-75 $\mu$ m) is used for retro-reflector alignment. When positioned just before the final 150 mm optic that directs both the cooling and illumination beam, this camera can observe the retroreflected light using a beam splitter. This setup can be seen in Figure 36 and the resulting image can be seen in Figure 37

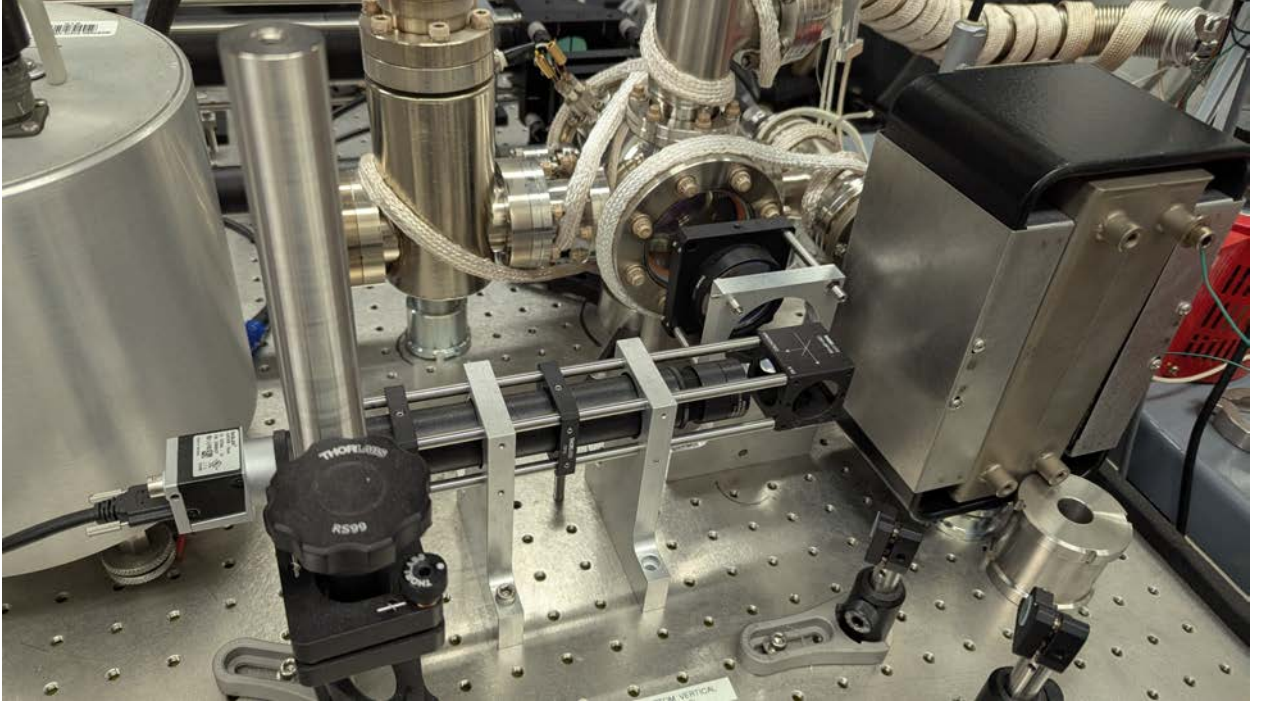


Figure 36: A camera with a beam splitter mounted to the end. In this configuration, it can observe the path of the retro-reflected beam. This camera tracks changes to the beam path when adjusting the stage housing the reflector. This camera also allows us to observe the beam profile of our illumination beam as shown in Figure 37.

The retro-reflector adds a factor of 2 to the observed RPSN, but these photons will not be collected, resulting in a loss of collection efficiency. These losses are not ideal, but this configuration is necessary for higher power measurements due to the kick the particle would otherwise receive.

Another downside of the retro-reflector seems to be an increase in feedback cooling noise. As previously mentioned, the product of  $\Gamma_{FB} \cdot T_{FB}$  determines the thermal noise level for our system, and feedback cooling increases this product above the natural  $\Gamma_N \cdot T_N$ . The increased beam power allowed by the retro-reflector causes more spontaneous particle movement, requiring greater amplitudes for the feedback cooling beams.

The optical isolator in the illumination beam path is a critical part of the optical chain that allows the retro-reflector to work correctly. The beam coupling back into the optical



Figure 37: An image of the retro-reflector beam taken using the camera from Figure 36. An outline of the pole piece structure in this image is added for extra reference, but is not seen in the raw image file due to lack of exposure. In the center, the illumination beam returns through the trapping region after hitting our retro-reflector. This camera is needed for initial retro-reflection alignment.

fiber has been observed without this piece. This coupling decreases the beam's power output as the retro-reflector was aligned, which will be discussed in Chapter 5.

### Line Scan Camera

A line scan camera (BASLER raL2048-48gm) is utilized for high-speed measurements. starting from the 10:90 beam splitter shown in Figure 11, which directs some light to a front-facing area scan camera, while the rest of the scattered photons from the particle continue along a different path, passing first through a Dove prism. This prism is strategically positioned to adjust the image of the trap's oscillation axis. The Dove prism rotates the image that passes through it, which helps correct any misalignment of the magnetic trap's z-axis without adjusting the trap or chamber.

This optical system has three critical axes that need to be aligned in order to collect accurate displacement data. The first axis is the trap itself. The trap’s geometry determines the axial motion of the particle. Due to the design of the vacuum chamber, it is not practical to shift the entire trap to align it with our other components. Instead, the Dove prism allows us to effectively “rotate” the trap axis.

Next, a high-pass filter is positioned after the Dove prism, since this allows for the collection of scattered light from the particle without interference from the cooling beams. The light continues through a series of cylindrical lenses that focus it along one axis at a time. These cylindrical lenses represent the next critical axis that requires alignment.

The imaging method used requires seven cylindrical lenses between the chamber and the line scan camera. Any adjustments to one of these lenses necessitate equivalent adjustments to the other six. I prefer to keep the lenses in a fixed orientation to avoid the hassle of constant adjustments.

The first step in this alignment process is to mount all the cylindrical lenses on fine adjustment rotation stages (Thorlabs CRM1PT). Each cylindrical lens is positioned on cage rods, and a separate test bench is used to observe a Thorlabs t-shirt with a camera. Depending on the lens’s orientation, the resulting image is one of three possible outcomes.

First, the t-shirt viewed through the cylindrical lens may appear warped vertically, creating an effect in which the entire t-shirt looks compressed into an oval shape with the longer axis running horizontally. Second, the t-shirt might look compressed along the horizontal axis, resulting in an oval with the longer side oriented vertically. Lastly, if the cylindrical lens is misaligned, it can cause warping in both the horizontal and vertical directions. In this case, the symmetry of our t-shirt is disrupted, leading to the vertical line printed on the target being tilted, breaking symmetry. All three scenarios are illustrated in Figure 38, where a single 50 mm cylindrical lens demonstrates what to look for during alignment.

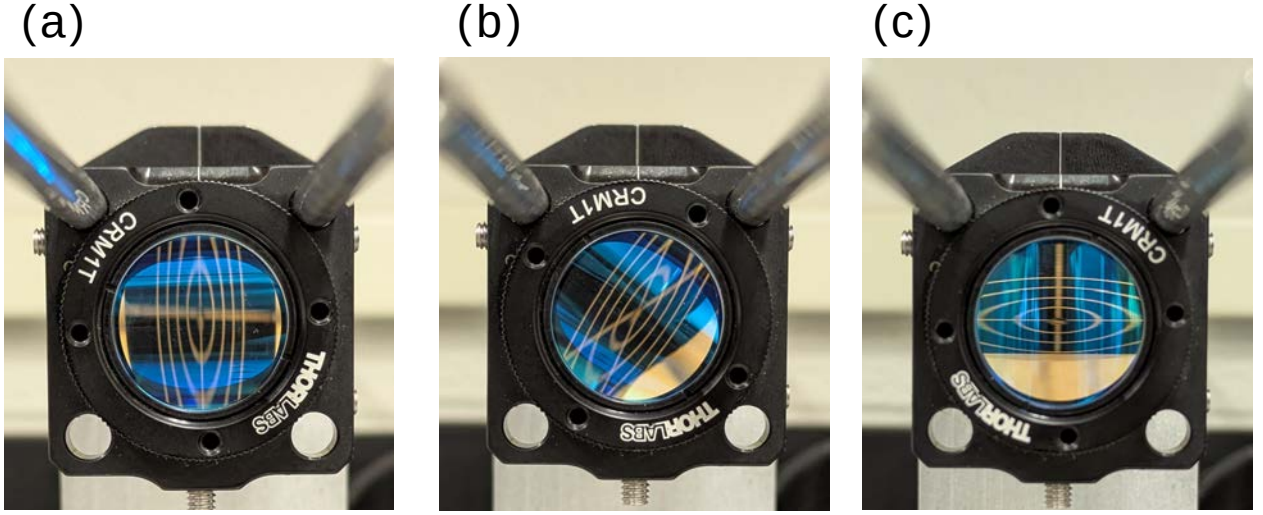


Figure 38: (a) A picture taken of a Thorlabs t-shirt observed through a cylindrical lens mounted in a rotation stage. With proper alignment, the camera observes a vertical line along the center of the t-shirt. In this orientation, the cylindrical lens redirects light along the vertical axis, causing the t-shirt to produce a horizontal oval shape. (b) When orientated at an angle that causes warping, it is clear that the center line does not appear vertical. This is an example of what to avoid when configuring cylindrical lenses. (c) In this example, the cylindrical lens is orthogonal to the position in image (a). The cylindrical lens redirects light horizontally, producing a vertical oval shape.

In this setup, each lens is oriented to redirect light along a specific axis. The cylindrical lenses are arranged in our optical train to redirect light into orthogonal components. This configuration enables variable magnification levels both vertically and horizontally, similar to the arrangement used for the illumination beam with cylindrical lenses. Doublet cylindrical lenses are used to minimize aberrations and achieve a tighter spot size, which helps reduce DSN. The exact configuration of these lenses is detailed in Table 2, and the optics mentioned in the table are also illustrated in Figure 39.

Our lenses are configured to work across various collimated spaces, and our final horizontal cylindrical lens is positioned to produce a collimated beam as its output. This beam can then be allowed to diffract as it travels toward our line scan camera, offering two key benefits.

First, with this configuration, the magnification of our imaging is not limited by the

| Lens | focal length | orientation | part number             |
|------|--------------|-------------|-------------------------|
| 1    | 50 mm        | Horizontal  | (Thorlabs ACY254-050-B) |
| 2    | 250 mm       | Vertical    | (Thorlabs ACY254-250-B) |
| 3    | 250 mm       | Horizontal  | (Thorlabs ACY254-250-B) |
| 4    | 50 mm        | Vertical    | (Thorlabs ACY254-050-B) |
| 5    | 50 mm        | Horizontal  | (Thorlabs ACY254-050-B) |
| 6    | 250 mm       | Vertical    | (Thorlabs ACY254-250-B) |
| 7    | 50 mm        | Vertical    | (Thorlabs ACY254-050-B) |

Table 2: A list of cylindrical lenses used between the Dove prism and the line scan camera.

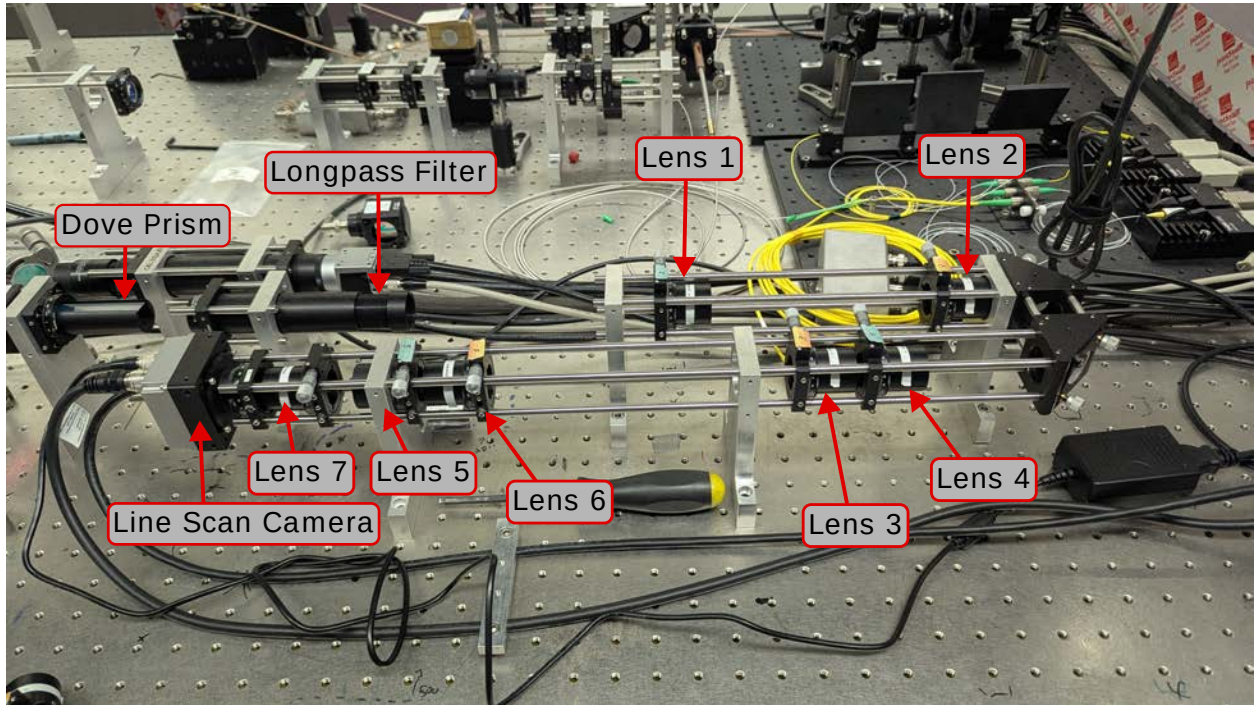


Figure 39: The optical configuration that allows observations of axial motion using the line scan camera. A Dove prism is used to align the desired motion of the trap with the cylindrical lenses. The line scan camera is also positioned on a rotation stage to align with the axes defined by the cylindrical lenses. Cylindrical lenses are fixed once placed in the optical train. The cylindrical lenses shown in this image are labeled in conjunction with Table 2.

Rayleigh range of our beam. Past optical setups in this experiment required an additional spherical lens beyond the final cylindrical lens. However, to effectively image the particles in that case, the lens would need to be positioned outside the Rayleigh range of the beam, resulting in a significant loss of efficiency. This configuration is shown in Figure 60, and

discussed in Chapter 5.

Second, by allowing the beam to spread after this lens, the linescan can be anywhere beyond this lens so long as there is space for the final vertical lens in front of the camera. This lets us control magnification by translating the line scan camera along our optical axis. Although this setup requires new distance calibration each time the camera is moved, it will be shown in Chapter 4 how this calibration is quick and painless.

Images captured with a line scan camera form a 2D array, but differ from those taken with an area scan camera. In a line scan image, one axis represents a spatial dimension, while the other (the vertical axis from the image) represents time. Each row of the spatial axis corresponds to a single ‘image’ of a particle in time. Figure 40 illustrates both an image from the line scan camera and a single slice from that image.

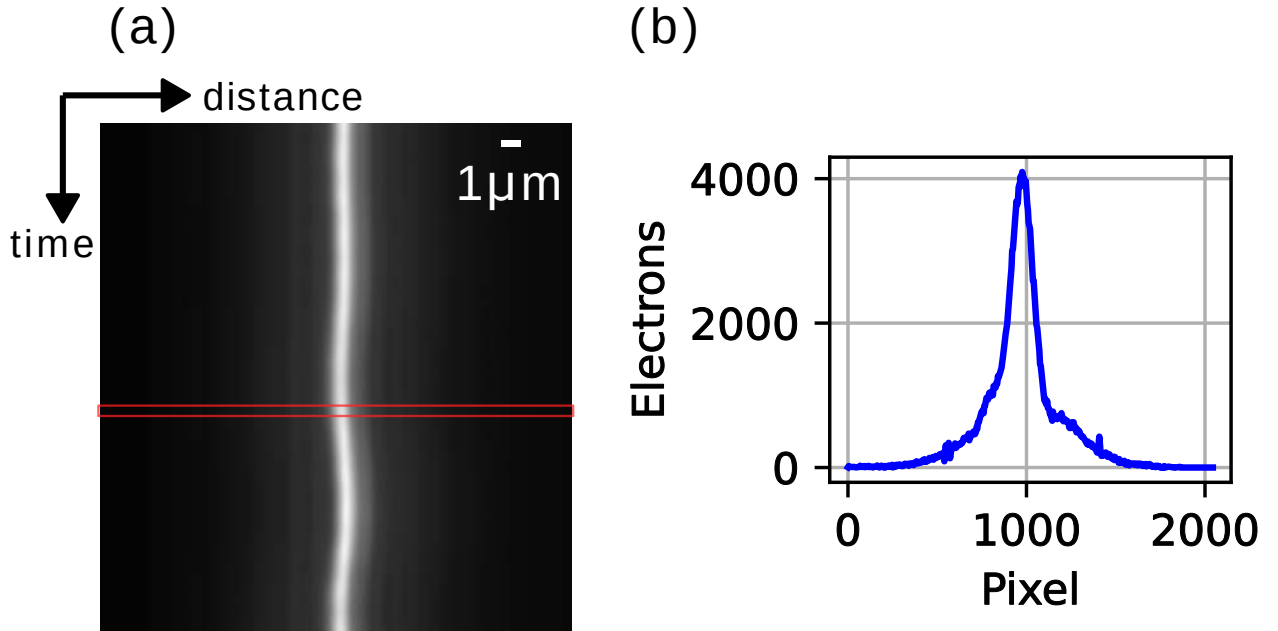


Figure 40: (a) An image captured with the line scan camera showing a cooled particle in a magneto-gravitational trap. The vertical axis of this image represents time. Each row from this camera can be considered an image of the particle. (b) A slice from the image can be displayed as a plot showing each pixel’s response. This camera has 2048 pixels, and the y-axis of this plot represents the number of electrons recorded from the pixel bins, which is determined by various camera specifications and settings.

Through post-processing, the particle positions can be recovered. Several methods of this were attempted and will be outlined in Chapter 5. The impact of particle motion along each axis of our trap can be observed by driving motion along a single axis and taking an image with the line scan camera, as shown in Figure 41. If the particle moves too much along any of these axes, particle tracking methods will fail to find the particle on the line scan camera.

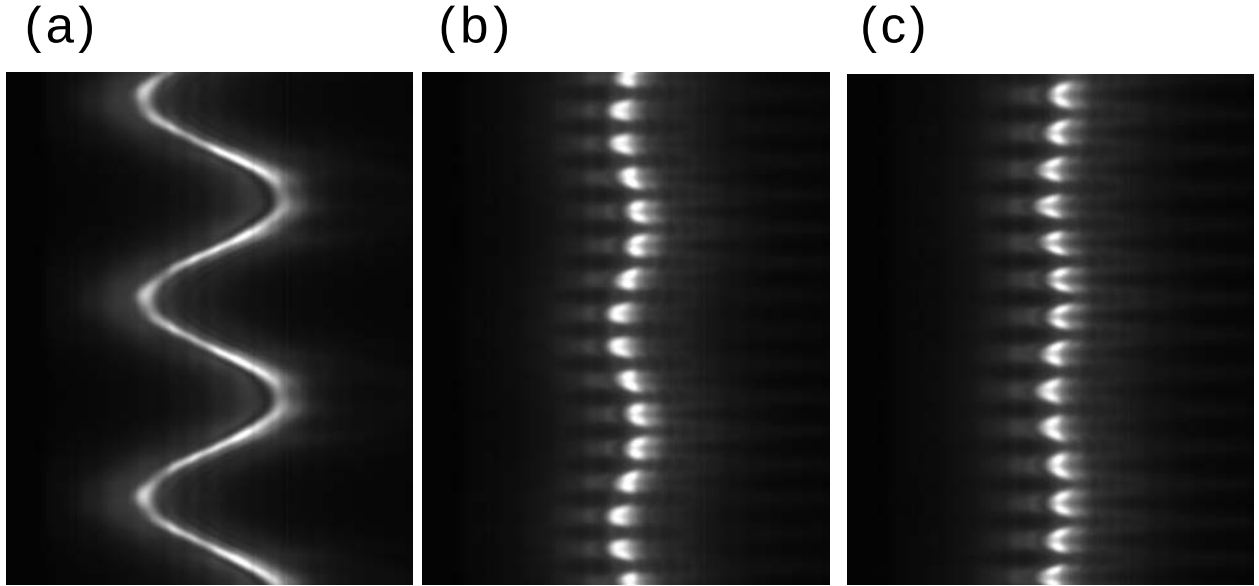


Figure 41: (a) An image was taken with the line scan camera in which axial motion is driven. (b) An image was taken with the line scan camera in which vertical motion is driven. (c) An image was taken with the line scan camera in which transverse motion is driven.

This camera is also controlled via an external trigger, similar to the side and front cooling cameras. The single row of pixels allows for an incredibly fast refresh rate during data taking, with the capacity to operate at 50 000 Hz.

### System Timing and Control

Various devices are used to control the equipment on our optical bench, and all of these systems must be linked together and triggered precisely to allow for our high-speed

measurements. A simplified diagram of these control systems is shown in Figure 42

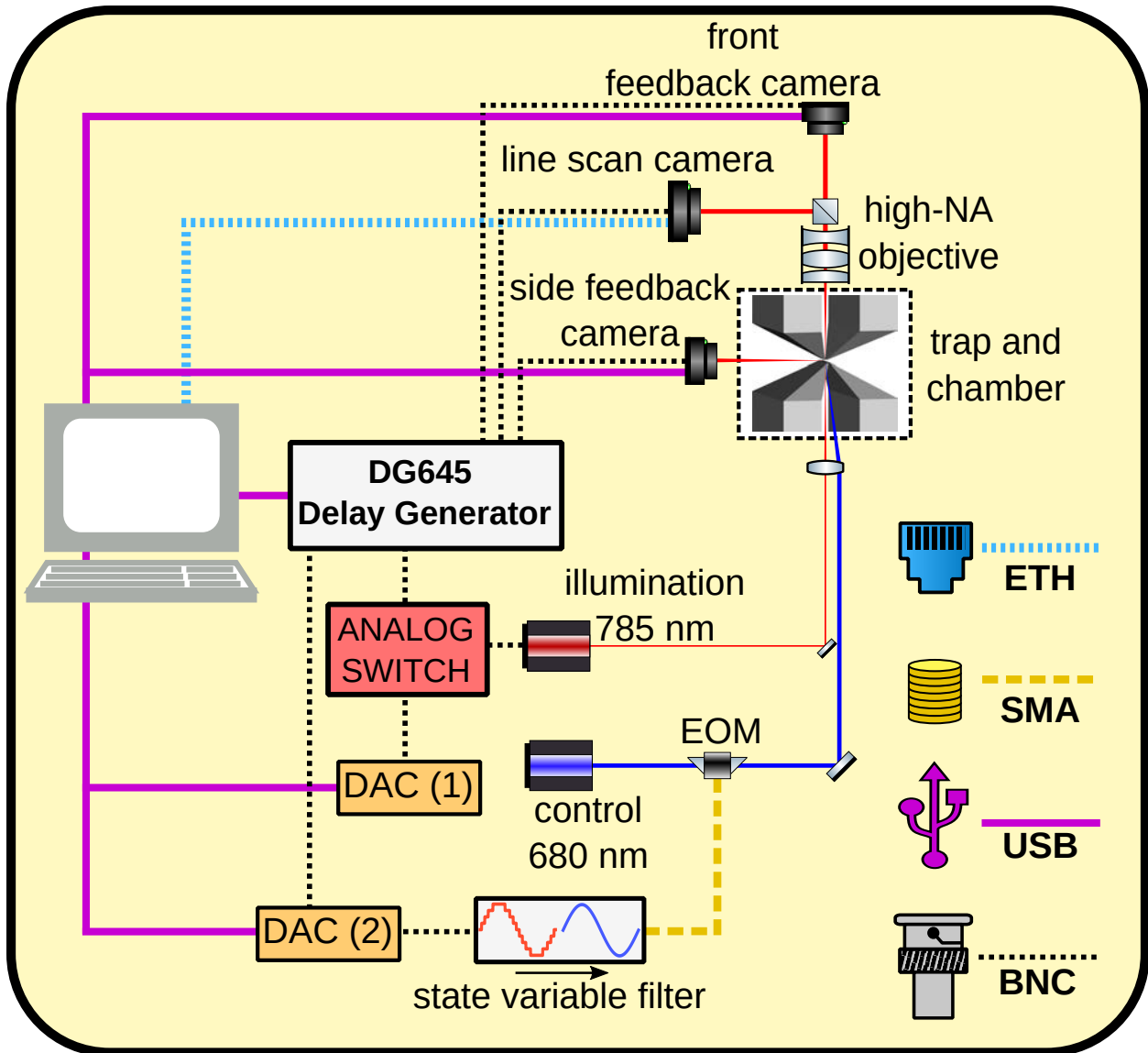


Figure 42: A diagram for the various devices used to control our measurements on the optical bench. The lab PC is connected to various 20-bit digital-to-analog converters (DAC), an analog switch for the SLD, our three primary cameras (feedback and line scan), and a DG645 delay generator. Connection type used between each component is labeled from the legend in the bottom right.

Fast and precise timing is required between all components to achieve high-speed measurements, which is accomplished using a DG645 Delay/Pulse Generator. The configuration for measurements done on this optical bench uses two DG645 boxes, with

specific port connections outlined in Table 3.

| SRS DG645 1   |                                |
|---------------|--------------------------------|
| Port          | Connection                     |
| Trigger Input | Internal Trigger               |
| 1-T0          | SRS DG645 2 Trigger Input      |
| 1-AB          | Cooling Camera 1 Trigger Input |
| 1-CD          | Cooling Camera 2 Trigger Input |
| 1-EF          | Feedback DAC Trigger Input     |
| 1-GH          | Empty                          |

| SRS DG645 2   |                                      |
|---------------|--------------------------------------|
| Port          | Connection                           |
| Trigger Input | SRS DG645 1 Output: 1-T0             |
| 2-T0          | Empty                                |
| 2-AB          | Illumination Current Controller      |
| 2-CD          | High-Speed Line Camera Trigger Input |
| 2-EF          | Auxiliary ADC Trigger Input          |
| 2-GH          | Empty                                |

Table 3: Connections of the two SRS DG645 digital delay generators used for hardware timing with camera cooling and line-scan camera measurements.

The three main cameras in our system are connected via BNC cable to our DG645 boxes, with additional connections to the PC using USB for the area scan cameras, and Ethernet for the line scan camera. This arrangement allows triggering to be handled by the pulse generation, with exposure and other camera settings handled by the PC.

Also configured to the DG645 is an analog switch that handles laser pulses. This system is able to produce controlled square wave pulses of our illumination beam with  $10\text{ }\mu\text{s}$  duration, followed by another  $10\text{ }\mu\text{s}$  rest time to allow the line scan camera time to write data to the PC. To control the output power from our 785 nm BOA, a digital-to-analog converter (DAC) is used. Another DAC is configured to receive data from the PC to allow for live feedback cooling during measurements. This DAC output is passed through a state variable filter to smooth out the signal that gets sent to the feedback EOMs. Larger discrete jumps in feedback cooling output are produced without this filter, which may produce unwanted

noise.

The measurement timing used for our ‘approaching the SQL’ measurements is identical to the force sensitivity measurements, with the main difference being an artificial reduction in line scan framerate by binning in time handled by software when writing data to storage. Data collection is done in a continuous format with active feedback cooling running.

### Conclusion

This chapter provided a thorough overview of the subsystems that constitute our optical bench, each playing a crucial role in achieving the precise measurements discussed in Chapter 2. The vacuum chamber was explored, creating a controlled environment free from atmospheric interference, along with the magneto-gravitational trap, which is essential for observing free particle behavior. The illumination and cooling beams are vital for observing and controlling a particle suspended in our trap. The retro-reflector enhances our measurement capabilities by allowing for higher optical power during continuous measurements, while the imaging cameras provide crucial data for analysis and feedback cooling. All of these subsystems are controlled by our lab PC and timing boxes that ensure proper synchronization between devices during measurements. These core components form a cohesive system designed to meet our research objectives.

## CALIBRATION AND SYSTEM PREPERATION

### Chapter Overview

This chapter will discuss several key topics that complement the information presented in Chapter 3. There are four primary considerations to remember when transitioning from the optical bench described previously to preparing a particle for measurement under UHV.

First, a reliable distance calibration must be established for the three leading cameras: two for feedback cooling and one for high-speed data collection. Given our system's geometry, this presents a significant challenge, but a novel distance calibration method was devised to address this issue.

Second, it is crucial to confirm that a single particle is confined within the trap. Our optical setup has a diffraction limit that matches the size of our trapped particles, making visual confirmation insufficient. To tackle this issue, a mass calibration technique is used to verify that the measured mass,  $m$ , corresponds to that of a single particle.

Finally, after completing the previous steps, 'kick calibration' is performed, where laser pulses are used to track particle displacement without the retro-reflector. This last calibration step allows us to adjust the illumination beam be orthogonal to the imaging axis and provides us with a readout of  $R_\lambda$ .

### Distance Calibration

Our cameras require precise distance calibration to detect and track a trapped particle. Accurate measurements are essential for mass calibration, determining the photon scattering rate, and establishing the vertical extent of spectral density plots. Quantifying how far the particle has moved between images is critical for these processes.

However, traditional calibration techniques, like a calibration slide, are inadequate for

our optical setup. One major issue is that the geometry of our system does not permit the placement of a removable calibration tool at the imaging plane of our objective. The removal of the viewing window and trap from the optical bench is required to do so. This removal could affect magnification, and since our optical objective is used to correct for aberrations, some of which are caused by the chamber window, calibration can not be performed independently of the vacuum chamber. Additionally, once the system reaches UHV, removing the viewing window without compromising the vacuum is not feasible without the aid of Maxwell's demon, which is unlikely.

Our primary measurement device is a line scan camera, which captures images along a single spatial axis. Due to our system's configuration involving cylindrical lenses, observing a calibration slide with this device is challenging. Additionally, this camera experiences the same issue as our front-facing area scan camera: the difficulty presented by removing the trap and chamber window.

Another consideration is any adjustments to the imaging objective while a particle is under UHV. Such adjustments can change the magnification of our system, necessitating a new distance-per-pixel calibration. This limitation applies only to the cameras that use the adjustable objective for focusing. One of our three cameras does not rely on the objective and can, therefore, be calibrated independently of the other two.

The side camera is connected to a single lens tube that houses all the optics used for imaging, with the exception of a viewport window. It does not encounter the same issues as other devices, making it suitable for calibration using more traditional methods. To quickly calibrate the displacement per pixel, two images of ruler markings oriented vertically and horizontally were captured. This process resulted in a calibration of  $1.29 \times 10^{-6}$  m per pixel. The calibration images are shown in Figure 43. Technically, the viewport window will affect the distance calibration, and this calibration could be refined further if necessary. However, it is sufficient for our current purposes, as the side camera's primary function is feedback

control. Final reported data on particle position will be obtained using the line scan camera, which will use a more precise method for distance calibration.

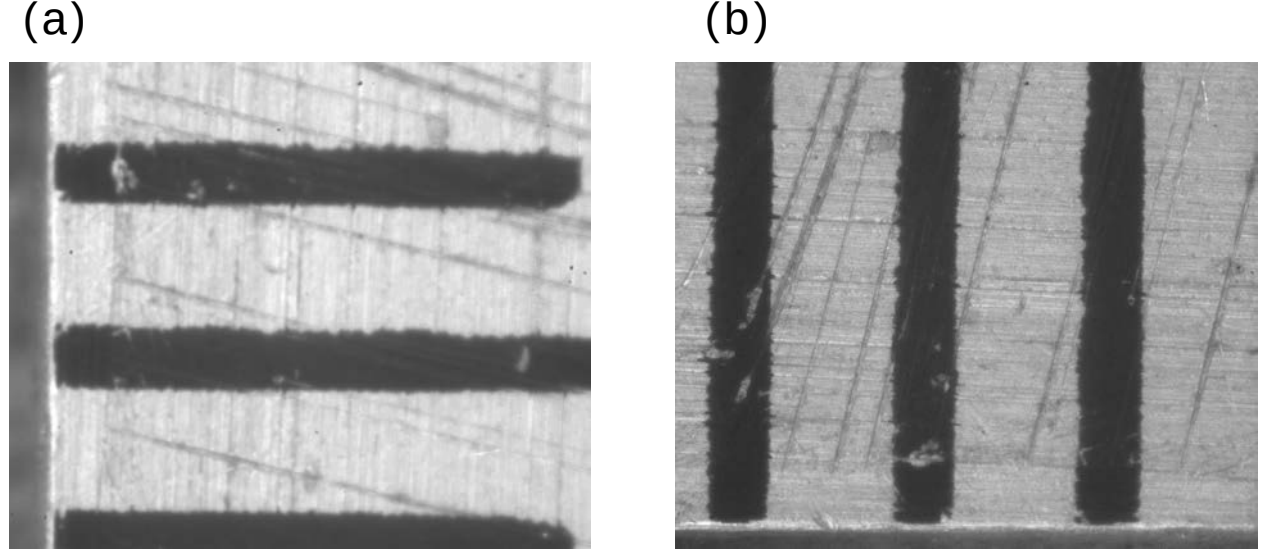


Figure 43: Images taken using the side camera for distance calibration. This was accomplished by removing the post-mounted camera and holding a ruler at the object plane of the camera. The markings in each image indicate a change of 1 mm with each step. (a) shows the ruler oriented vertically relative to the camera, while (b) shows the ruler oriented horizontally.

Now, a key concern needs to be addressed: how does one calibrate both the front-facing cooling camera and the line scan camera, given the constraints of our trapping setup? To overcome the challenges associated with the front area scan camera, a new measurement tool that our lab calls an optical ruler has been developed. This device utilizes an axicon to create a Bessel beam, which allows us to employ interferometry for distance measurement. This method is advantageous because it requires only a single axicon and a Gaussian beam to pass through, without necessitating a complex setup.

A Bessel beam is characterized by its extended range of propagation, where the rings are evenly spaced. The spacing of these rings depends solely on two parameters: the axicon angle, denoted as  $\alpha$ , and the wavelength of light  $\lambda$ . Using a reference bench, a fellow graduate student, Tahereh Naderishahab, constructed a Bessel beam, imaged it, and tested it against

a NIST-traceable calibration tool. A fit is performed to the resulting intensity profile, which is approximately described by a zeroth-order Bessel beam, defined by the equation:

$$I(r) = A_0^2 J_0^2(k \cos \beta) \quad (62)$$

where  $A_0^2$  represents the peak intensity at  $r = 0$ ,  $J_0$  is the zeroth-order Bessel function of the first kind,  $k$  is the wave vector, and  $\beta$  is the angle between the redirected beam (after passing through the axicon) and the optical axis. A diagram illustrating this setup can be found in Figure 44.

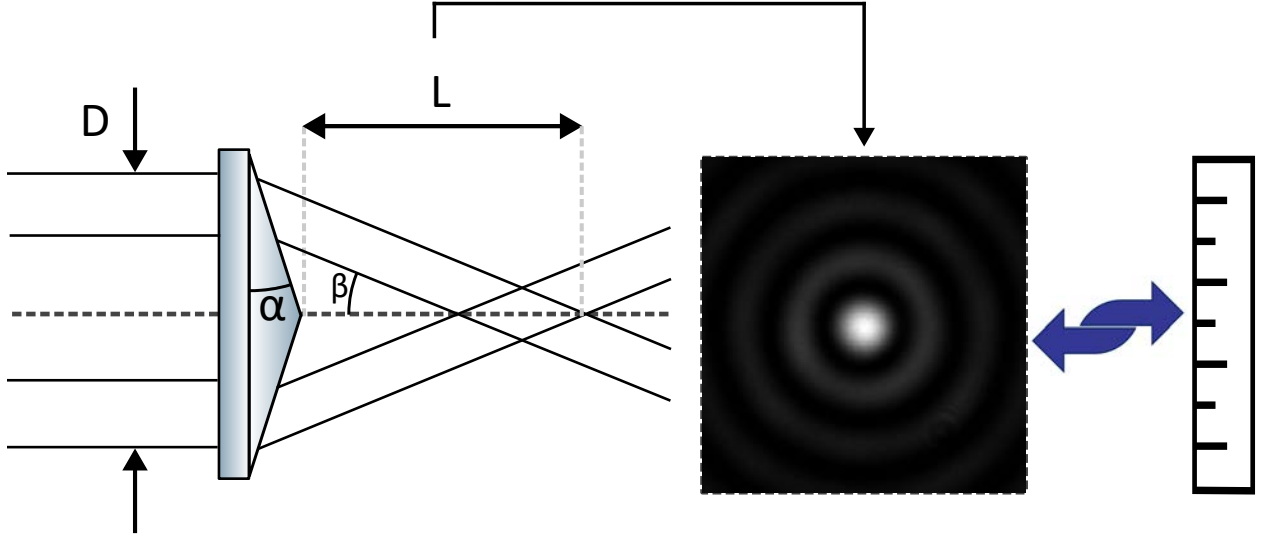


Figure 44: A collimated Gaussian beam with diameter  $D$  enters an axicon to generate a Bessel beam. The spacing of the rings for this beam is constant over the distance  $L$  as it propagates. This spacing is a function of  $\lambda$ , the beam's wavelength, and  $\beta$ , the angle of the deflected beam relative to the optical axis. On a test setup, the region  $L$  is overlapped with the focus of an objective that leads to a camera. Calibration from the Bessel beam can then be compared to a NIST traceable calibration tool. Once complete, the axicon stage can be lifted and placed on the main bench to calibrate our front-facing area scan camera.

Bessel beams have several characteristics that make them ideal for our purposes. First, Bessel beams have been shown to propagate up to 200 m with limited diffraction effects [74]. This means that the Bessel profile can be maintained over a long distance once created. In our lab, this propagation is much less than 200 m, but for our use case, this is just fine.

This long propagation distance means getting the Bessel profile in focus is trivial since this distance dramatically exceeds the depth of field for our objective.

Second, Bessel beams have self-healing properties [75–77]. Bessel beams can reconstruct their structure around some obstructions, meaning the Bessel profile can be recovered despite potential interference from the trap geometry. Even with a particle in the trap, the intensity profile of the Bessel beam can be retrieved.

Once complete, the axicon assembly can be moved to the SQL optical bench and projected towards the magneto-gravitational trap. An image of this optical ruler in use is shown in Figure 45.

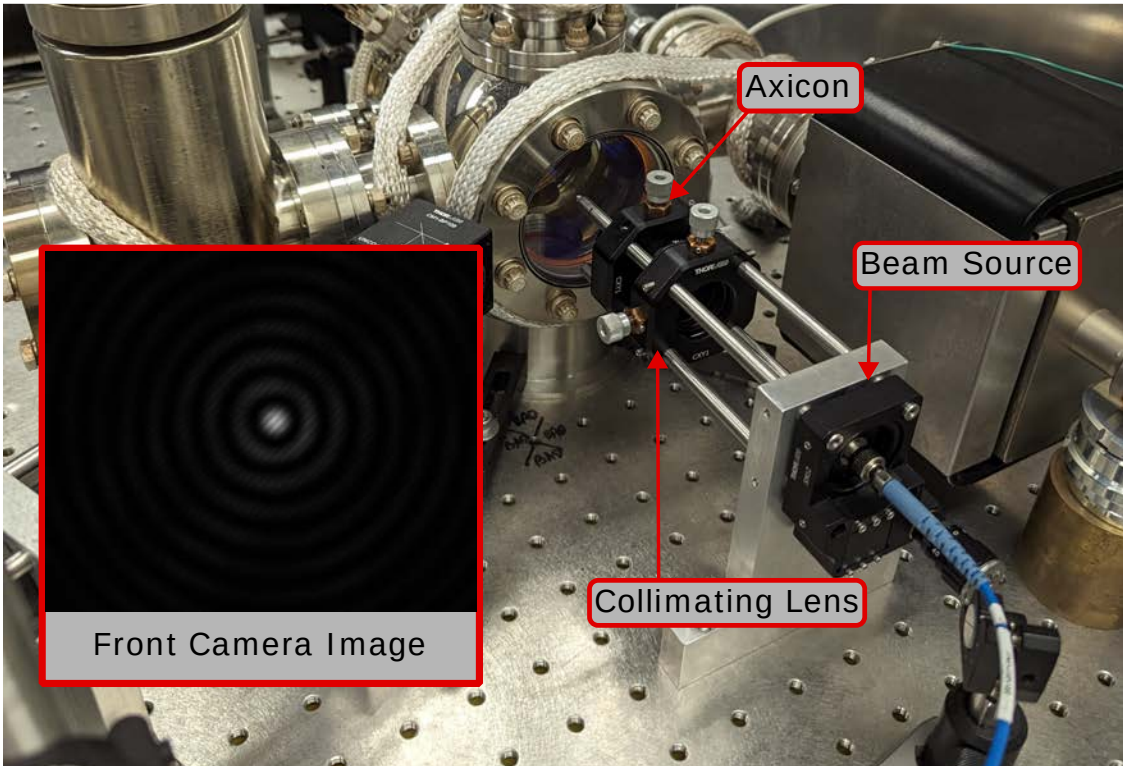


Figure 45: The Bessel beam optical ruler in use on the main bench. An image of the Bessel beam was taken with our front-facing cooling camera. A fit is performed to a Bessel function yielding a distance per pixel calibration of  $6.05 \times 10^{-8}$  m per pixel.

For reference, the spatial region that is viewed by our front-facing camera is about the size of Lincoln’s head on the ‘tails’ side of a US penny, as shown in Figure 46.

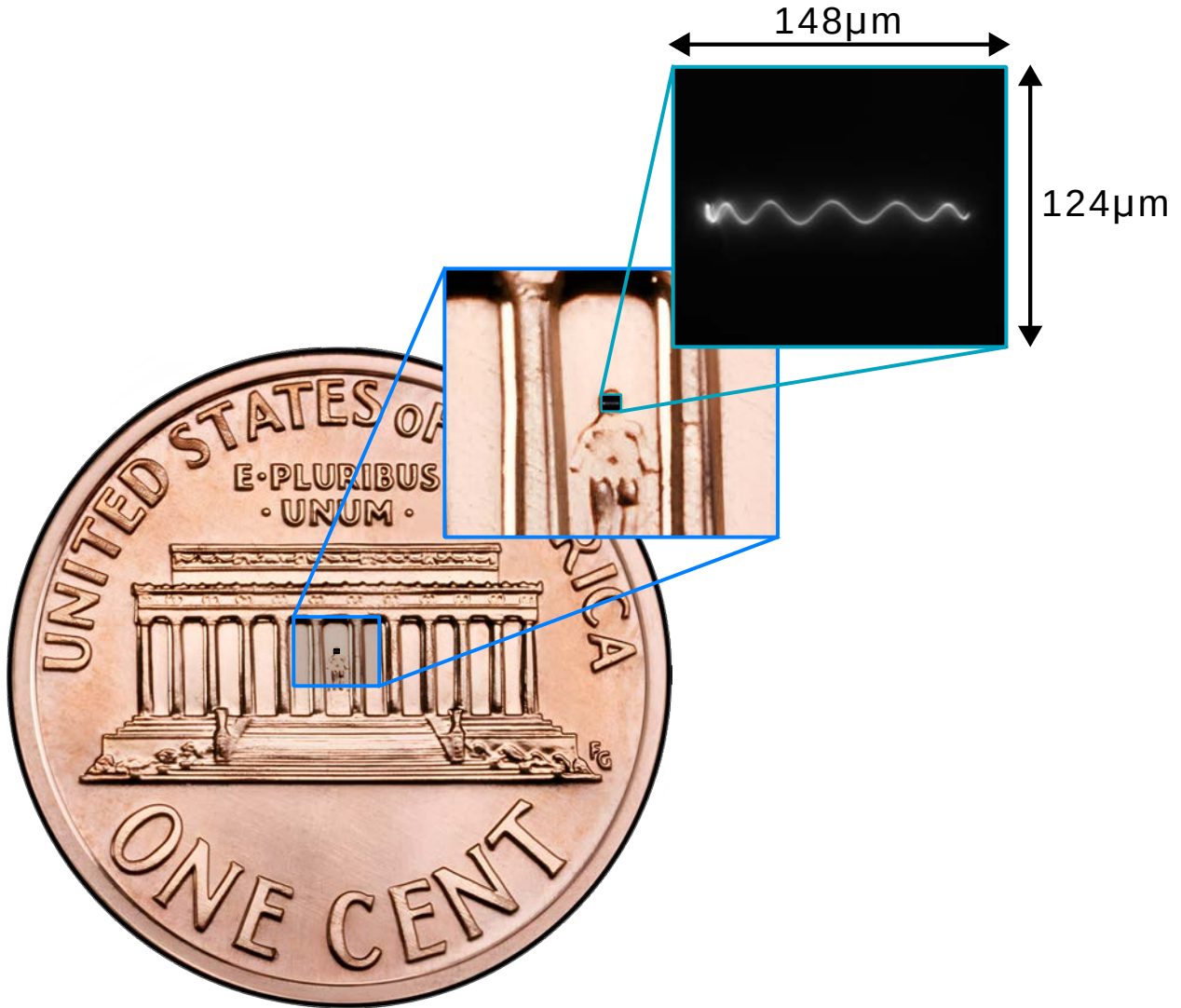


Figure 46: Comparing the extent of the front camera's field of view using its max resolution to a real-world object. The small black box around Lincoln's head represents the de-magnified spatial extent of the front camera's view. An image of the particle (without active cooling) is also displayed. Based on the Bessel beam calibration and the camera resolution, it was determined that the camera's spatial extent for an image in focus is 148  $\mu\text{m}$  wide and 124  $\mu\text{m}$  tall.

The last distance calibration needed is for the line scan camera. This camera requires a different approach to distance calibration given its cylindrical lens configuration. The line scan camera consists of a single row of pixels, and to produce useful images with it, an optical configuration of cylindrical lenses is required. If a Bessel beam is imaged on this camera,

only a single cross-section of the Bessel pattern will be shown, which could be sufficient for distance calibration. Still, given the cylindrical lenses, this cross-section will be distorted and no longer resemble a Bessel profile. This is not a problem when imaging a particle much smaller than the Bessel profile, but this issue does prevent the direct use of Bessel beam calibration on the line scan camera. Luckily, a trapped particle is a sufficient tool for distance calibration of the line scan camera.

By tracking particle motion between two cameras, one calibrated and the other not calibrated, the particle motion can be used as a bridge to transfer one distance calibration to another camera. Since the front-facing area scan camera is calibrated using a Bessel beam optical ruler, and both the area scan camera and line scan camera can track the same particle, a match to the particle trajectories between cameras can be made by scaling the line scan camera's amplitude to equal that of the area scans reported motion. The net result is a calibration transfer between the two cameras, as shown in Figure 47.

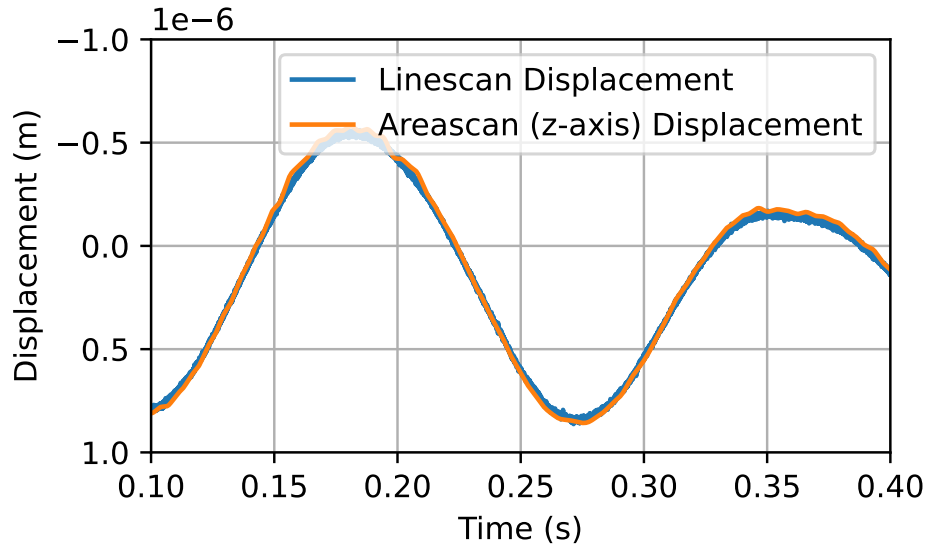


Figure 47: Particle motion tracked by two cameras. The area scan camera data was calibrated using the Bessel beam. The motion of the particle on the line scan camera is scaled to match that of the area scan camera, and from this, a distance per pixel calibration of the line scan camera is obtained. In this example, the recovered calibration was  $7.5 \times 10^{-9}$  m per pixel.

### Mass Calibration

To ensure that the mass of the particle is in line with manufacturer specifications, analysis is done by fitting the particle motion to a Boltzmann distribution. This step is typically done after the particle is loaded and before pumping down. This saves us a lot of hassle from lost particles during the baking procedure, which used to be a common occurrence. To begin, start with the classical Hamiltonian for some axis of our trap, given by

$$H = \frac{1}{2}mv_i^2 + \frac{1}{2}m\omega_i^2x_i^2 \quad (63)$$

where  $m$  is the mass of our particle,  $\omega_i$  is the oscillation frequency of the trap,  $v_i$  is particle velocity, and  $x_i$  is particle position. This Hamiltonian serves as a substitution in a Boltzmann distribution as a function of energy given by  $P(E) \propto \exp\left(-\frac{E}{k_B T}\right)$ . It is rather inconvenient that this function has a velocity dependence, but this can be integrated out, resulting in some constant value. The exact result of this integration is not relevant to mass calibration, leaving us with a probability function that is now only proportional to a position measurement given by

$$P(x_i) \propto \exp\left(-\frac{m\omega_i^2x_i^2}{2k_B T}\right). \quad (64)$$

where  $k_B$  is Boltzmann's constant, and  $T$  is temperature,  $\omega_i$  is the angular oscillation frequency of the trap, and  $x_i$  is particle displacement from equilibrium.

Our next steps require both the oscillation frequency of our trap and an average of the squared displacement for our particle. The particle's motion can be observed, and the time series produced can then be Fourier transformed to find the trap oscillation frequencies. Given our knowledge of trap geometry, three peaks are expected around the oscillation

frequencies given by each axis of our trap. In the past, this whole calibration procedure was done during the pump-down process, but we have recently discovered it works fine when done under atmospheric pressure, so long as the vertical motion of the particle is used. This is the highest oscillation frequency between all three of our axes, and it seems that this faster oscillation is sufficient to result in a Boltzmann distribution. The lower oscillation frequency of the axial direction causes the particle motion to be heavily impacted by other noise sources (such as vibration), which results in poor fits. The transverse axis may work fine, but the side camera must be used to measure displacements for this axis. Instead, the Bessel-calibrated camera is the ideal tool for this job to track vertical displacements. The ASD from particle motion to determine oscillation frequencies is shown in Figure 48.

With the vertical oscillation frequency known, analysis of collected position data can take place. To fit a Boltzmann distribution, several instances of particle position are measured over a time scale that exceeds the thermalization time of our oscillator. This is achieved by recording the particle motion for several minutes at atmospheric pressure. The mean position of the particle across the data set is subtracted so that the new mean position is our origin point. Assuming the particle is in thermal equilibrium with its surroundings, a histogram of particle positions would follow a distribution given by Equation 64. A mass calibration fit can be seen in Figure 49.

### Kick Calibration

A critical component of our measurement process is the beam path of our SLD. Ideally, the beam should be adjusted to be perfectly orthogonal to the axial motion of our trap. However, achieving this alignment by eye is challenging. To address this, our lab has developed a method that utilizes controlled beam pulses to displace our particle, and this displacement is used to analyze our beam path.

This approach assists in aligning the beam by observing the displacement caused by

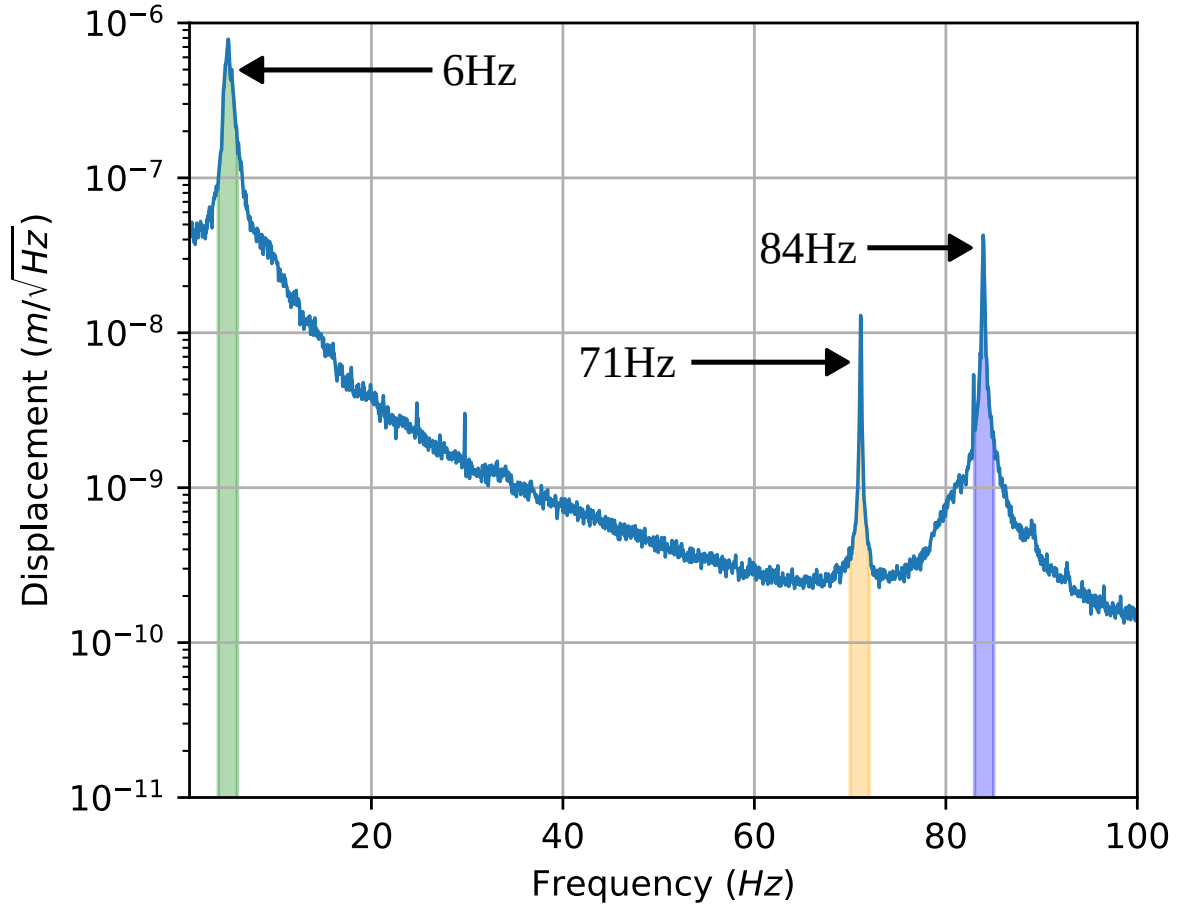


Figure 48: Three oscillation axes of the trap are shown from a displacement ASD. Each peak represents the oscillation frequency of our trap, which is needed to determine the particle mass. This plot comes from data collected on April 10th, 2024. Shown are the Axial (6 Hz), Transverse (71 Hz), and Vertical (84 Hz) oscillation frequencies.

a beam pulse and how it affects each axis. This pulse method can also determine the photon scattering rate, an essential parameter for evaluating our final data sets. Finally, this calibration tool can be employed to assess the alignment of the retro-reflector.

Let's begin with the first step: beam alignment. During this phase, the retro-reflector is adjusted to miss the particle so each beam pulse impacts the particle only once. The area scan cameras on our bench capture reference images just before the beam is pulsed to a high power state. After this, the SLD is switched to a high power output for 500  $\mu$ s. Following

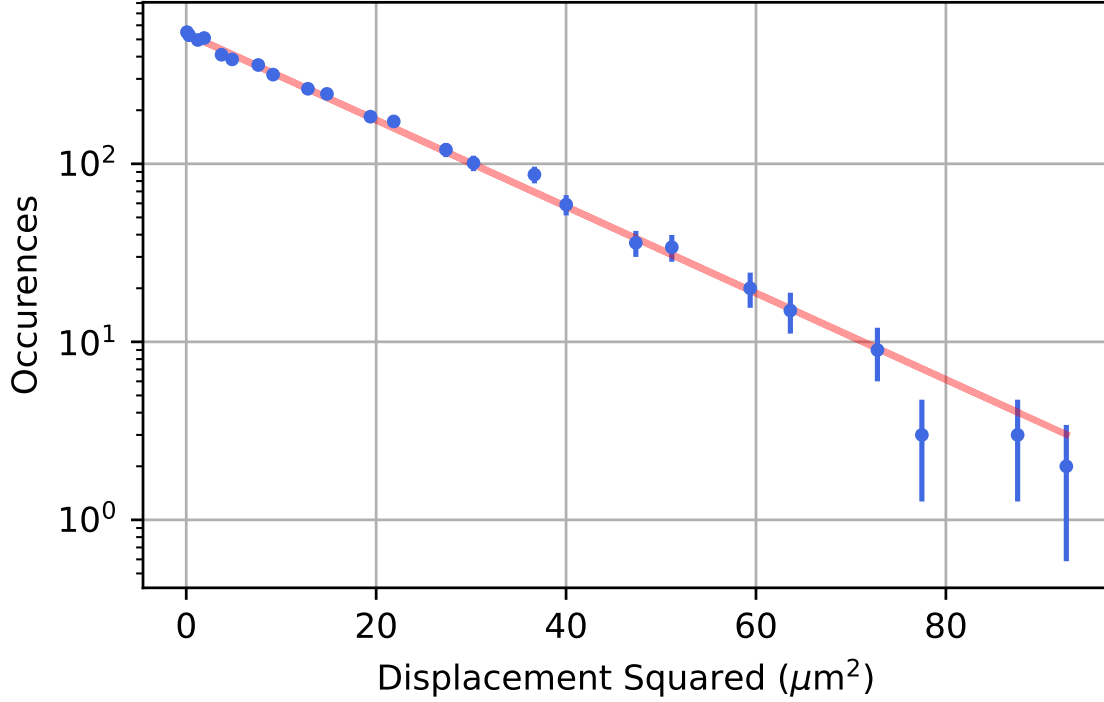


Figure 49: Position data is recorded by the front-facing reference camera over several minutes at 30 Hz operation. The resulting position measurements are plotted as a histogram of displacements squared. It is then fit to the expression for our Boltzmann distribution after subtracting off the origin point (mean location of the particle). The results of this input yield a fit that appears linear when viewed on a semi-log plot, assuming the particle is in thermal equilibrium with its surroundings. This is used to determine if our trapped object is consistent with that of a single particle using mass as our fit parameter, with an expected mass of  $m = 1.54 \times 10^{-15}$  kg. In Chapter 5, all presented data have an associated mass calibration included.

a specified time (which I will define later in this section), another image of the particle is taken, which is now displaced from its original position. By comparing the initial position data with the post-pulse data, a measure of displacement caused by the pulse is attained.

Several kicks are performed to observe the average displacement caused by the beam pulses. At this stage, the mirrors that redirect the illumination beam are adjusted to change their path. More kicks are then conducted after these adjustments, and the results can be

compared with those obtained before the adjustments.

The goal of these adjustments is to ensure that the beam path is orthogonal to the axial axis. This involves maximizing the transverse displacement caused by the pulses while minimizing axial and vertical displacements. A good guideline for this process is to balance the resulting kicks, producing an equal amount of positive and negative displacement over multiple kicks. This balance indicates that the beam is not driving motion along any specific axis, which also aligns with achieving maximum displacement along the transverse axis. Figure 50 shows how an adequately aligned illumination beam will produce an average of no displacement along the axial path of the particle.

The photon scattering rate can be measured once this alignment is complete. To effectively predict the levels of quantum noise in the system, it is necessary to know how many photons are scattered off the trapped particle per unit time. This process begins with the differential equation that describes the motion of our trapped particle when a radiation pressure force,  $F_R$ , is applied to the particle from an equilibrium position. During this process, active feedback cooling is turned off. Assuming  $x(0) = 0$  and  $\dot{x}(0) = 0$ , begin with equation 65.

$$\ddot{x} = -\frac{k}{m}x + \frac{F_R}{m} \quad (65)$$

where  $k$  is the spring constant of our trap,  $x$  is particle position, and  $m$  is particle mass. The above differential equation is satisfied by the solution shown in equation 66.

$$x(t) = \frac{F_R}{m\omega_x^2}(1 - \cos(\omega_x t)) \quad (66)$$

where  $\omega_x$  is the oscillation frequency of the transverse axis. The source of radiation pressure in this case is our 785 nm SLD that is pulsed for 500  $\mu$ s. The displacement and velocity of the particle at the end of this pulse ( $t = t_R$ ) are defined by equations 67 and 68.

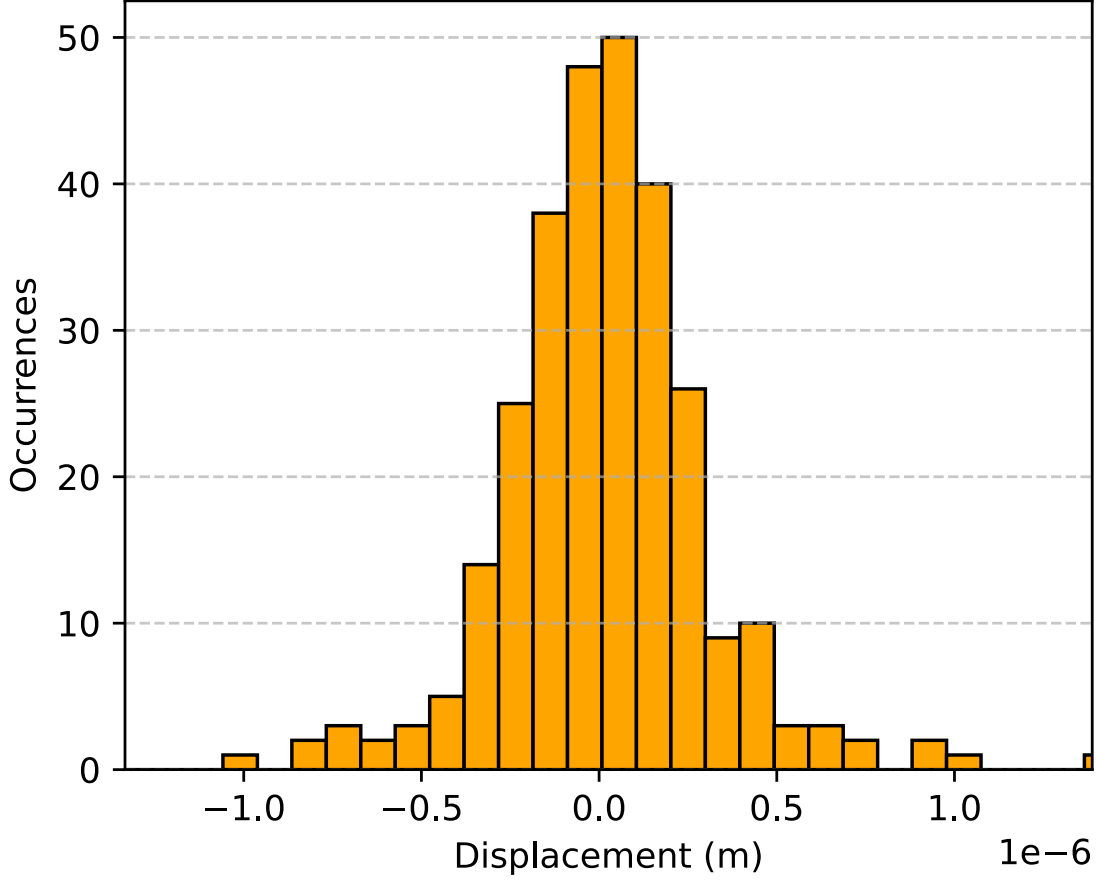


Figure 50: Measured axial displacement during kick calibration. With the illumination beam orthogonal to the z-axis, the resulting kicks, on average, produce no preference for pushing the particle to the left or right.

$$x(t_R) = \frac{F_R}{m\omega_x^2}(1 - \cos(\omega_x t_R)) \quad (67)$$

$$\dot{x}(t_R) = \frac{F_R}{m\omega_x} \sin(\omega_x t_R) \quad (68)$$

From then on, the particle behaves like a typical harmonic oscillator (ignoring any damping effects). This freely oscillating particle travels in a path specified by equation 69.

$$x(t) = X_R \sin(\omega_x(t - t_R) + \phi) \quad (69)$$

where  $X_R$ , the peak displacement amplitude, and  $\phi$  must ensure that  $x(t)$  and  $\dot{x}(t)$  are continuous at time  $t = t_R$ . Putting this condition in equations 67 and 68 results in two equations and two unknowns,  $X_R$  and  $\phi$ . Solving for both of these leads to

$$X_R = \frac{2F_R}{m\omega_x^2} \sin\left(\frac{\omega_x t_R}{2}\right) \quad (70)$$

$$\phi = \frac{\omega_x t_R}{2} \quad (71)$$

With this information, it can be shown that the maximum displacement occurs in a quarter period from the center of the driving pulse,  $t_{max} = \frac{t_R}{2} + \frac{\pi}{2\omega_x}$ . This is the time I was referencing earlier; since the beam is now traveling along the transverse axis following kick calibration alignment, the oscillation frequency for the transverse axis, 71 Hz, can be used to determine the photon scattering rate. The force from radiation pressure is a function of the number of scattered photons  $N_\lambda$ , their wavelength  $\lambda$ , and the driving pulse time  $t_R$ . This gives us equation 72 for the radiation pressure force.

$$F_R = \frac{N_\lambda \zeta_x h}{t_R \lambda} \quad (72)$$

where  $h$  is Plank's constant, and  $\zeta_x$  is a parameter that gives the fraction of momentum transferred from our photons to the particle's motion in the transverse direction. Given the angular profile of photon scattering, only a fraction of the energy transferred to particle

momentum will occur in the direction of wavefront propagation. This parameter was found with the scattering integrals discussed in Chapter 2. Plugging Equation 72 into Equation 70 leads to Equation 73, which solves for  $N_\lambda$ , the number of photons scattered during a laser pulse of duration  $t_R$ .

$$N_\lambda = \left( \frac{m\omega_x^2 t_R \lambda}{2\zeta_x h \sin\left(\frac{\omega_x t_R}{2}\right)} \right) X_R \quad (73)$$

As a procedure, the particle is cooled to an equilibrium position and imaged to give a reference  $x(0)$  before the driving pulse is initiated. After the initial reference image is taken, a square wave pulse lasting 500  $\mu\text{s}$  is triggered for the illumination beam. At  $t_{max}$ , another image is taken to determine  $X_R$ . An additional pulse is triggered roughly  $2t_{max}$  after the first pulse to stop the particle motion induced by the first kick. Repeated measurements of this prepared state are performed to generate a distribution of kick displacements, which can be used to determine  $N_\lambda$ . An example result of this process is shown in Figure 51.

### Conclusion

In this chapter, the various methods used to calibrate system parameters were discussed. Distance calibration was an essential first step to achieving accurate measurement results for particle displacement tracking. Mass calibration, only achievable following distance calibration, both confirms a single particle within the trap, and that the particle is an expected 1.18  $\mu\text{m}$  diameter  $\text{SiO}_2$  sphere. Finally, kick calibration is performed to confirm our illumination is orthogonal to the imaging axis, and it is used to measure  $N_\lambda$  experimentally. Knowing this variable is key in evaluating our quantum noise sources; without it, no estimates of photon collection efficiency, DSN, or RPSN can be given. All necessary preparation is now complete for high-speed measurements to take place.

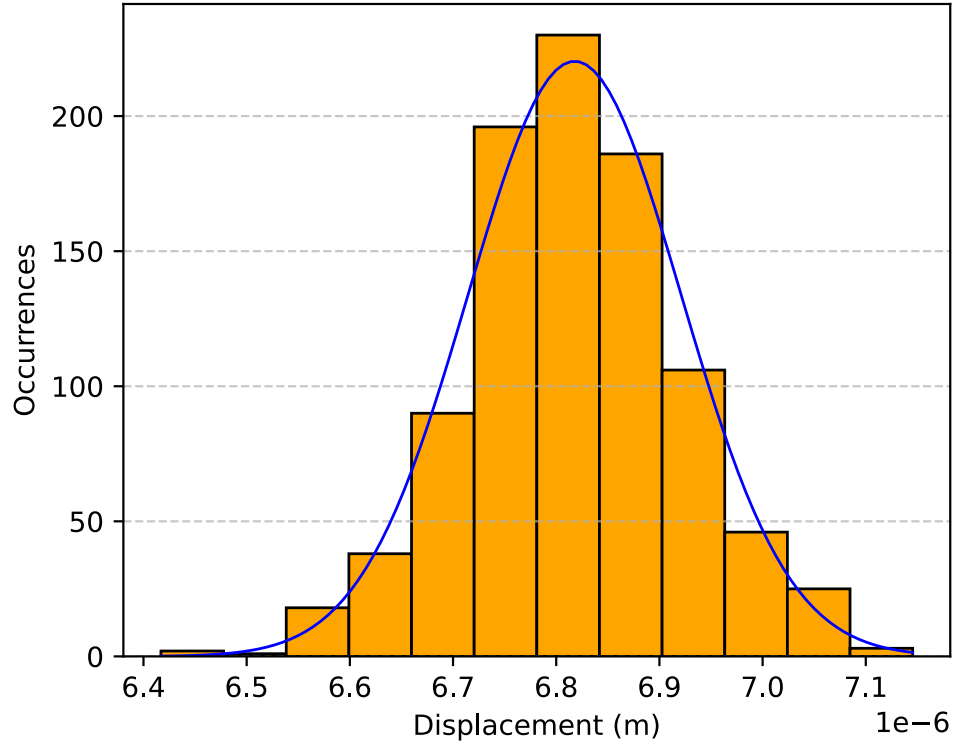


Figure 51: Pulsed laser kick calibration produces displacements that are imaged at the peak of the oscillation. This figure shows a histogram of several displacement measurements created by a 785 nm SLD operating with a pulsed power output of 20 mW with a square wave pulse that lasts for 500  $\mu$ s. Once  $X_R$  is measured,  $N_\lambda$  can be calculated.

## RESULTS AND DISCUSSION

### Chapter Overview

This section will look at the results of our experiment design and data-taking procedures. Data taken over the course of a year has yielded interesting results, and it has given us an idea of where to go in the future. Additionally, results are shown from optical bench configurations taken before the final form outlined in Chapter 3.

There are various other topics to go over in this section, such as particle charge accumulation, interference subtraction, and issues with particle stability. All of this will be addressed while analyzing collected data. Finally, a comparison of the collected data with the model outlined in Chapter 2 is used to evaluate how close the experimental results match the free particle limit. The chapter concludes with the data yielding the closest approach to the SQL, the results of force sensitivity measurements, and some additional analysis and comparisons between data sets.

### Particle Fitting

This section will detail the methods used to process raw image data from the linescan camera and convert it into a list of particle positions. However, the issue of hot pixels that can interfere with measurement outcomes needs to be addressed before proceeding. The linescan camera utilized in this dissertation has various pixel defects that affect the accuracy of the data. To mitigate the impact of these defects, dark images are taken (images captured with all illumination turned off), which are later used during various fitting processes to subtract their effects. This issue seems to impact data collected with maximum gain on the camera primarily, and is less noticeable on data collected with high illumination powers and no gain. Figure 52 illustrates an example of particle motion captured with the linescan camera,

highlighting the presence of these defective pixels in certain areas.

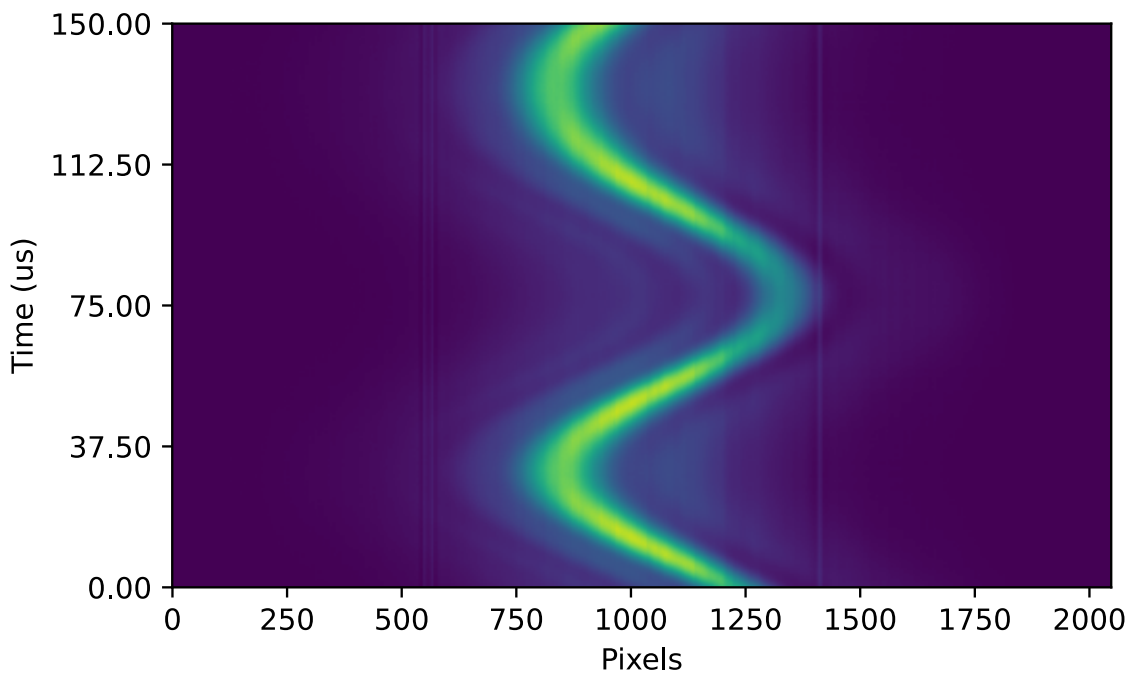


Figure 52: An example image taken without pixel correction. Errors from hot pixels can be observed around pixels 547, 566, and 1409.

Once the dark images have been collected and saved, they can be subtracted from the raw data sets to minimize the effect of these defects during particle tracking. A few fitting methods will be explored, discussing their advantages and disadvantages, ultimately focusing on the primary method employed in this work.

### Center of Brightness

Several methods were attempted to track particles from raw image data during the data analysis. The first method presented here for particle tracking from linescan data involves a Center of Brightness (CoB) approach, which considers all the electrons in each pixel bin. This method produced amplitude spectral densities (ASDs) that aligned with expectations based on our model outlined in Chapter 2. However, it had significant drawbacks. When

tracking the CoB, optical aberrations led to increased interference from transverse and vertical oscillations. Figure 53 presents three examples taken from individual rows of a line scan image.

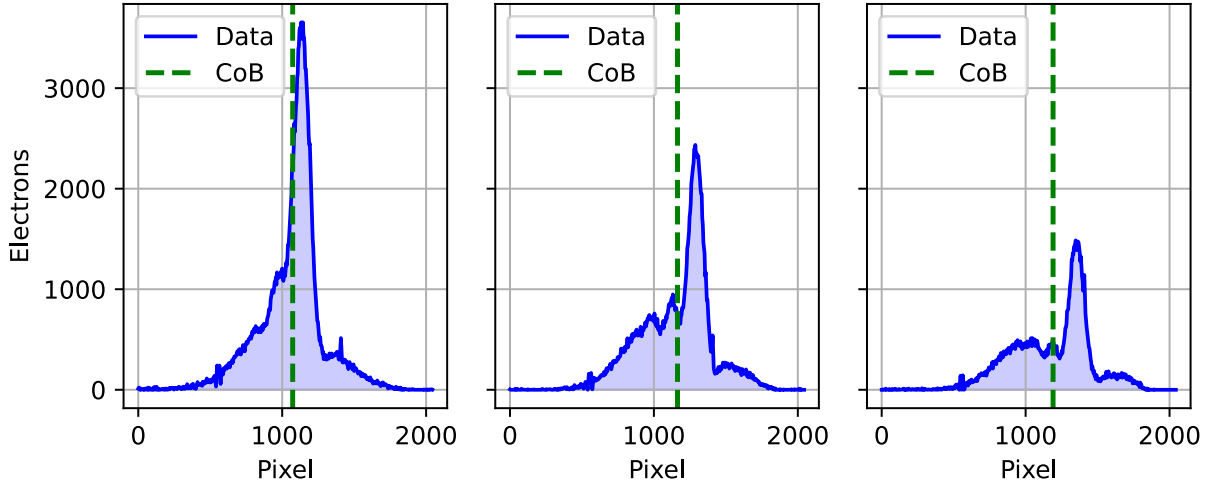


Figure 53: Three plots display various slices from an image captured using a linescan camera. The CoB method has been utilized to determine the particle positions, indicated by the vertical dashed lines. The narrow Gaussian profile to the right of these CoB positions is thought to represent the actual location of the particles in a line scan image.

When the particle was well-aligned with the linescan camera, the CoB still struggled to identify the particle location accurately. Even slight instabilities in the particle's position caused the fit to deteriorate, and the position deviations reported by CoB between each row showed excessive variance. In this context, instability refers to vertical and transverse particle motion, which gets coupled into tracking methods via aberrations and slight optical misalignment. Consequently, the time series of particle motion did not accurately reflect the particle's actual trajectory in these cases. This limitation significantly reduced the value of our collected data, leading to the quick abandonment of the CoB method.

### Single Gaussian

The next method used to track the particle involves fitting the data to a single Gaussian given by

$$f(params) = A_0 \exp \left( - \left( \frac{H - h_0}{w_0} \right)^2 \right) + B_0 \quad (74)$$

where  $H$  is position,  $A_0$  is the Gaussian amplitude,  $h_0$  is the position of the Gaussian peak,  $w_0$  give the width of the Gaussian, and  $B_0$  is a vertical offset. A comparison of the single Gaussian fits using the same data set depicted in Figure 53 is shown in Figure 54.

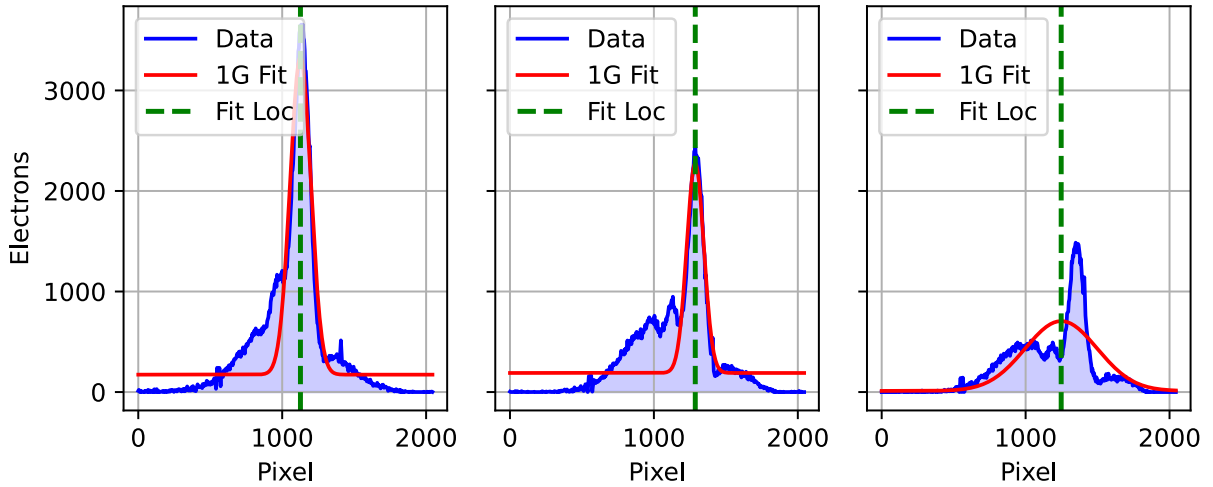


Figure 54: The three plots display various sections of an image captured with a line scan camera. The estimated particle location, indicated by the vertical dashed line, is determined by the center of a Gaussian fit. When the particle experiences significant instability, the resulting aberrations can hinder accurate tracking using this fitting method.

Although this approach is more computationally intensive than the CoB method, it was chosen for its ability to provide a more refined analysis. One significant drawback of the CoB method is a larger-than-expected variance, which can impact tracking over short time scales. This issue primarily stems from the inherent limitations of the CoB fitting technique in accounting for optical aberrations. In contrast, the single Gaussian fitting

approach effectively reduces this noise floor compared to the CoB method, resulting in a significantly tighter variance in the particle's position at fast time scales. This improvement enhances tracking precision and allows for more reliable measurements. Given the same set of raw data, a single Gaussian method recovers more information from the pixels, resulting in a lower DSN.

However, despite these advancements, the single Gaussian fitting method still faces challenges, especially in situations of high particle instability. During these moments, the method may struggle to track the particle accurately. This is particularly true for high-power data sets, where the data is often mostly unusable due to the single Gaussian fit method failing to track excessive particle motion.

#### Skew Normal

One approach for improving particle tracking was to use a skew normal distribution given by

$$f(params) = A_0 \cdot \frac{2}{w_0} \phi\left(\frac{H - h_0}{w_0}\right) \Phi\left(\alpha \frac{H - h_0}{w_0}\right) + B_0 \quad (75)$$

where  $A_0$  is the amplitude,  $h_0$  is the distribution mean,  $w_0$  is the width parameter,  $\alpha$  is the skewness parameter that controls the distributions asymmetry,  $B_0$  is the vertical offset,  $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$  is the standard normal probability density function, and  $\Phi(x) = \int_{-\infty}^x \phi(t) dt$  is our cumulative distribution function.

The objective of this method was to reduce the variance between position measurements, similar to the single Gaussian model, while also addressing the particle instability often observed during high-power measurements. A comparison of the fits with this method is illustrated in Figure 55.

Unfortunately, this method quickly turned out to be unproductive. We found that it failed to account for particle instability and, in fact, the resulting variance was slightly worse

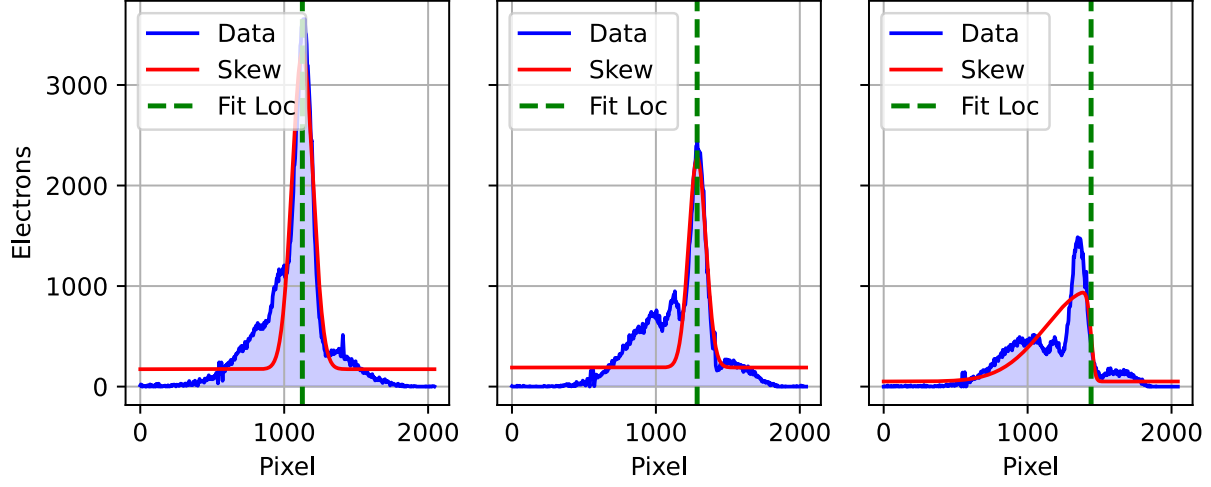


Figure 55: Three plots displaying different slices from an image captured with the linescan camera. The vertical dashed line indicates the particle position reported by the skew normal distribution. The results from this method are comparable to those from the single Gaussian approach; however, in some instances, they lead to significant discontinuous jumps in the reported particle position.

than that of a single Gaussian fit, possibly due to the influence of the skew parameter. This method showed high sensitivity to start conditions, sometimes returning differing results from the same raw data following adjustments to initial parameters. Additionally, this method frequently produced random discontinuous jumps during the fitting process, as demonstrated in Figure 58. Ultimately, this method did not resolve any new issues and was less effective at handling previously resolved problems.

### Double Gaussian

The double Gaussian fit method is the most effective technique used in this dissertation for tracking a particle on a line scan camera. It successfully tracks the particle during significant instability swings, a capability that no other tested fit method has achieved. This method begins with a double Gaussian function given by

$$f(params) = (G_0^M + G_1^M)^{\frac{1}{M}} + B_0 \quad (76)$$

where the two Gaussians,  $G_0$  and  $G_1$ , are summed,  $M$  serves as a smoothing parameter, and  $B_0$  acts as a vertical scaling factor. The use of two Gaussians in this manner accounts for optical aberrations. The primary challenge in accurately tracking the particle stems from the added Gaussian shape caused by these aberrations. When the particle image deviates significantly from the linescan camera pixels, the broad Gaussian produced by the aberrations becomes a major issue in previous fit methods. This interference is effectively addressed by characterizing the aberration contributions with their own Gaussian. The two Gaussians are given by

$$G_0(H, A_0, h_0, w_0) = A_0 \exp \left( - \left( \frac{H - h_0}{w_0} \right)^2 \right) \quad (77)$$

$$G_1(H, A_1, A_0, h_1, h_0, w_2) = A_1 A_0 \exp \left( - \left( \frac{H - h_0 - h_1}{w_2} \right)^2 \right) \quad (78)$$

with the following definitions:

| Parameter | Description                                                      |
|-----------|------------------------------------------------------------------|
| $H$       | Independent variable (horizontal axis/position)                  |
| $A_0$     | Amplitude of the first Gaussian peak ( $G_0$ )                   |
| $h_0$     | Center position of the first Gaussian peak ( $G_0$ )             |
| $B_0$     | Vertical offset (baseline)                                       |
| $w_0$     | Standard deviation (width) of the first Gaussian peak ( $G_0$ )  |
| $h_1$     | Horizontal shift between the two Gaussian peaks                  |
| $A_1$     | Amplitude scaling factor of the second Gaussian peak ( $G_1$ )   |
| $w_2$     | Standard deviation (width) of the second Gaussian peak ( $G_1$ ) |

Table 4: Parameters and their descriptions for the double Gaussian fitting function.

The final definition required is for  $w_2$ , which is defined as

$$w_2 = \sqrt{2w_0^2 + w_1^2} \quad (79)$$

Here, a relationship is established between the two Gaussian distributions where the width,  $w_2$ , of the second Gaussian must be greater than that of the first. This condition is crucial to ensure that the reported fit parameters during data processing do not swap from one Gaussian to the other. The fitting process saves the center position of the first (narrow) Gaussian as the particle location. If the two Gaussians were to switch roles, the outcome would be reporting the aberration position as the particle position, which is a key problem that the other methods failed to address. Figure 56 illustrates how this fitting approach consistently tracks the particle compared to previous methods.

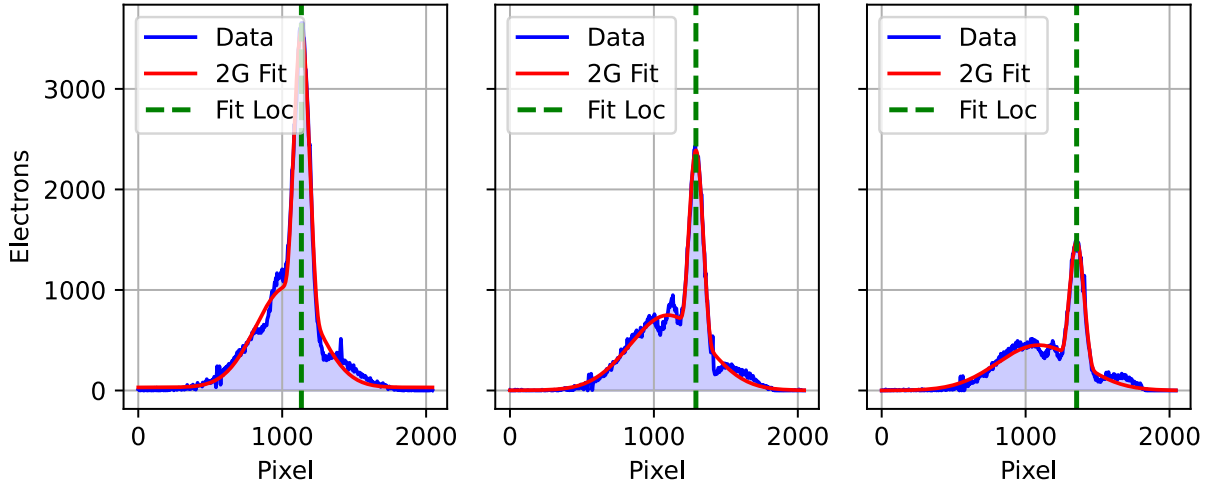


Figure 56: Three plots displaying different slices from an image captured by the line scan camera. The double Gaussian fit method has proven to be the most effective for our data to date, accurately determining the particle position in the rightmost plot where our previous fits have failed.

To further showcase the effectiveness of the various fitting methods, we will compare their performance using the same data set. Figure 57 shows an observed time series segment demonstrating how the various fit methods stack up given the same raw data set. To highlight the different outcomes in particle variance between these measurements, this same data set can be observed in the frequency domain by converting to an ASD. Figure 58 highlights how different fitting methods can drastically alter the outcome of our ASD noise floor.

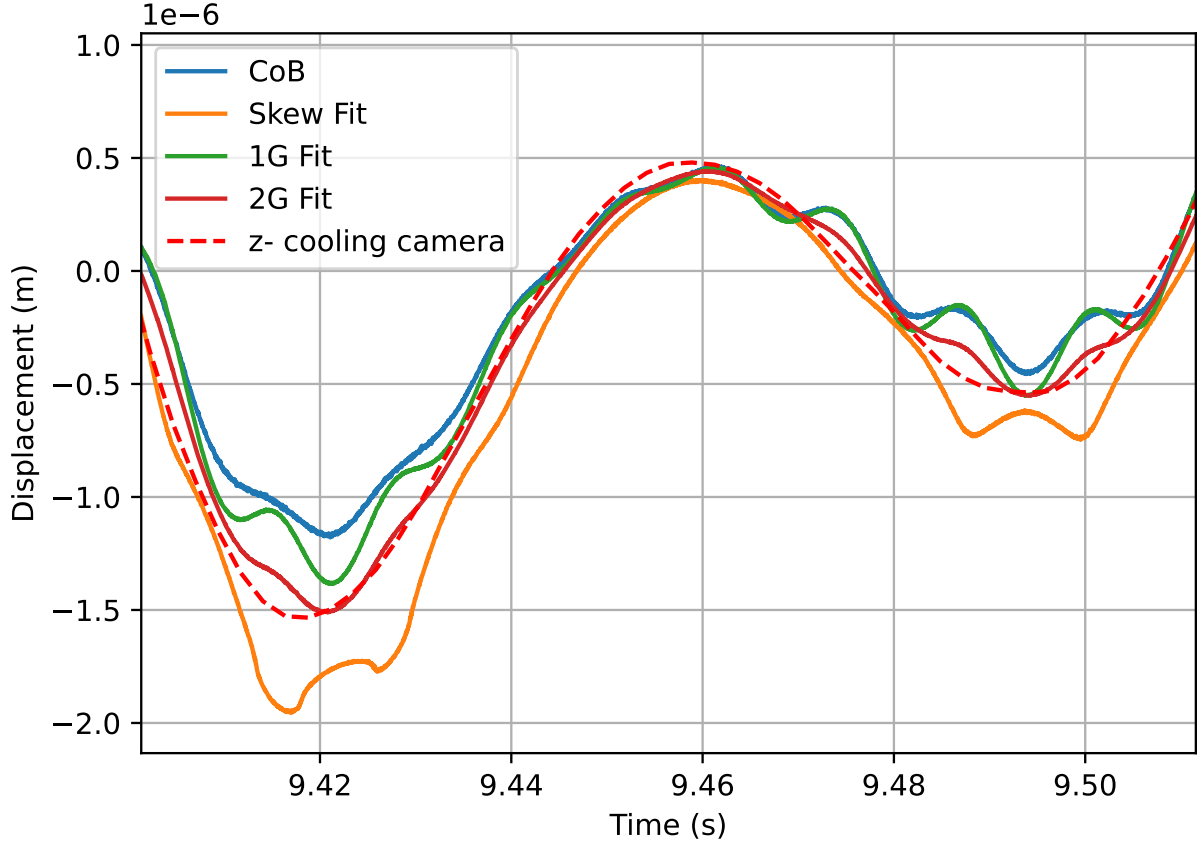


Figure 57: A position versus time plot generated by different fit methods applied to the same raw line scan data. The dotted line represents the particle's true path according to our front-facing cooling camera, which provides a much larger field of view than the linescan camera, albeit with lower time resolution. The effectiveness of each fitting method is indicated by how closely the fit follows the dotted line.

### Approaching the SQL

The following section discusses the results from particle images collected by the linescan camera and the subsequent data processing. The presented data comes from particles in a magneto-gravitational trap between 2024 and 2025. Several improvements were made to the final experimental setup during this period, with the most up-to-date configuration outlined in Chapter 3. This section will chronologically detail the events that led to our most recent dataset, which focused on approaching the SQL in the free particle limit.

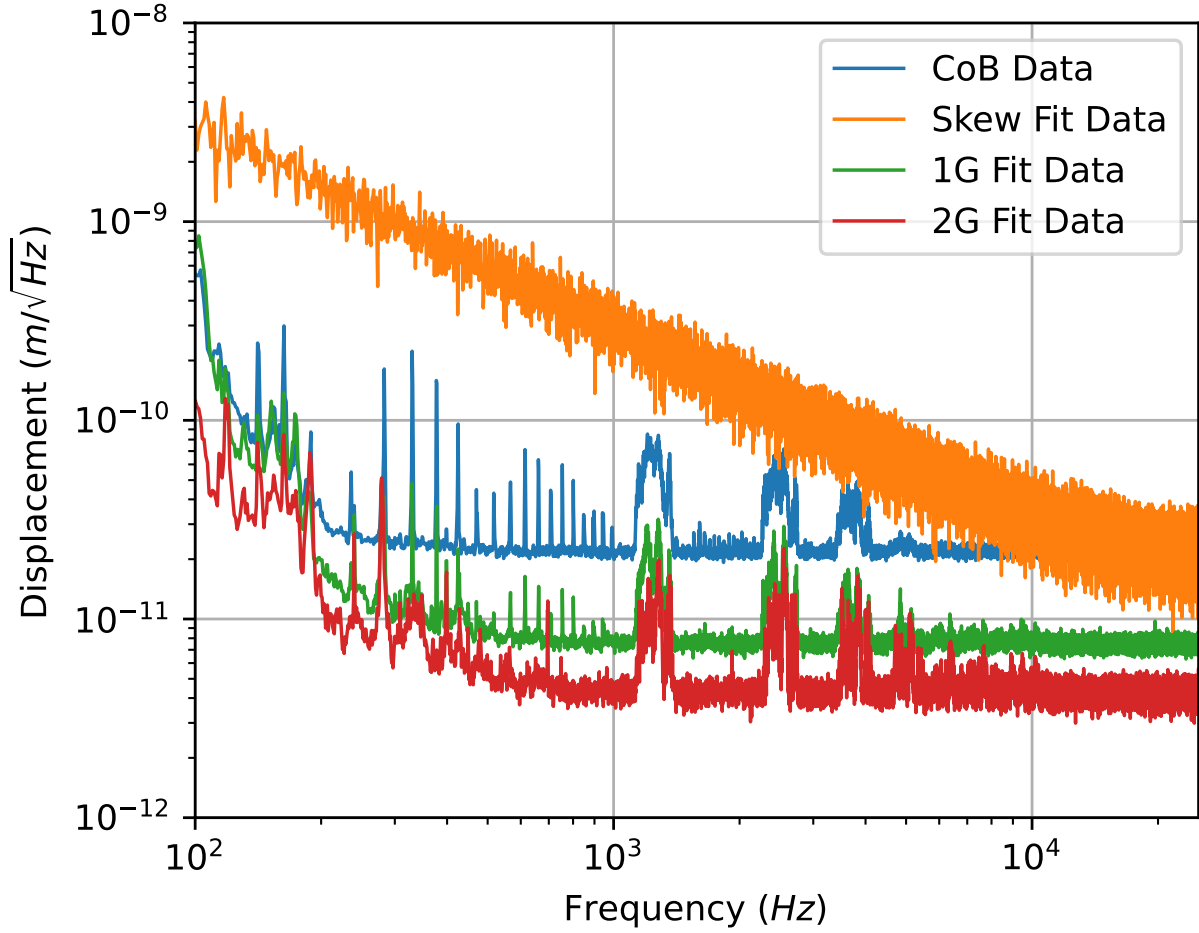


Figure 58: ASD plot generated by different fit methods applied to the same raw line scan data. Notice how each fit method impacts the noise floor of the ASD, with the double Gaussian method having the lowest floor. The odd behavior of the skew normal method stems from discontinuities in particle position reporting by the fit method.

Each of the subsections begins with the results of mass calibration. Data collected throughout 2024 and 2025 involved three distinct particles, which we refer to as ‘Bob III’, ‘Bob V’, and ‘Bob VII’. This naming convention indicates the history of the trapped particles; for example, Bob III is the third particle we trapped. The name Bob pays homage to the professional oscillator Bob Burnquist.

### Early Line Scan Results

In this section, we will discuss results obtained in April 2024 from an earlier configuration of line scan optics. This setup used different cylindrical lenses compared to the version outlined in Chapter 3, and a brief outline of that system is provided here. Figure 59 illustrates the mass calibration that confirmed the confinement of a single particle in the magneto-gravitational trap used throughout this section.

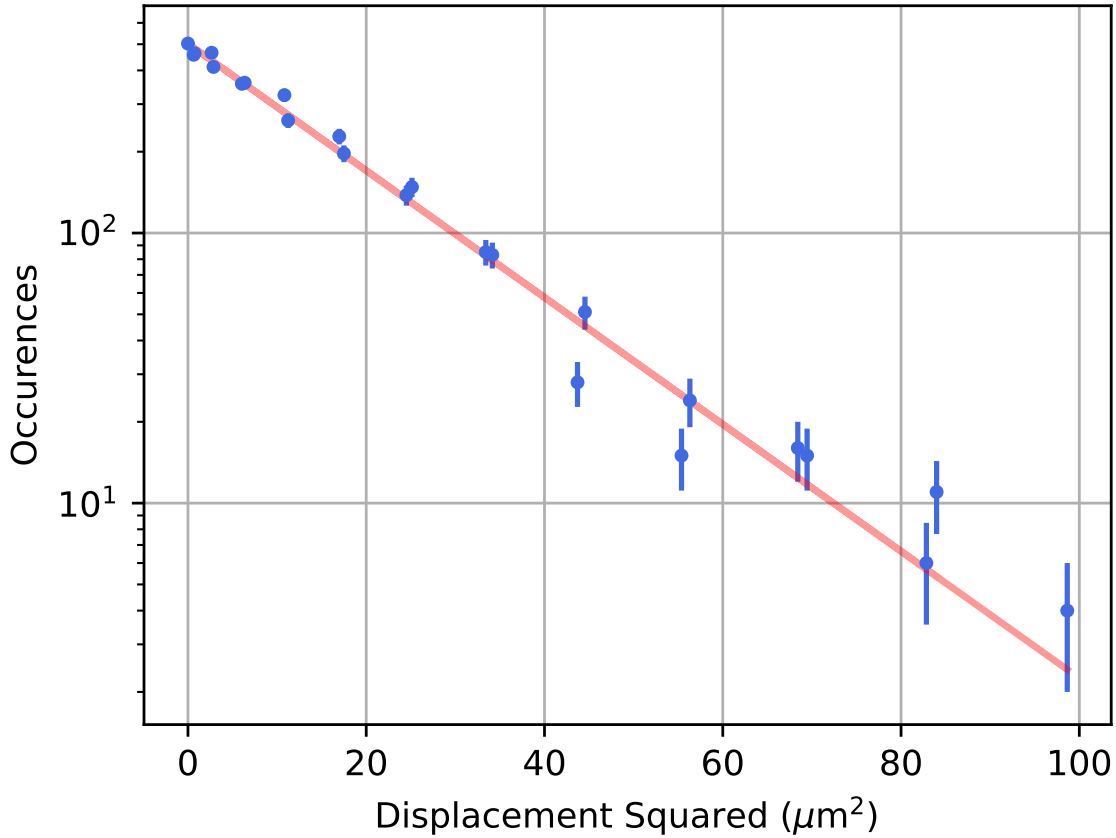


Figure 59: Mass calibration performed on ‘Bob III’ in March 2024. Fitting a histogram of the squared displacements yields a mass fit of  $1.59 \times 10^{-15}$  kg, which is consistent with the presence of a single particle.

During this period, the cooling beam was managed by an external cavity diode laser

operating at 660 nm. This required some additional phase modulation to prevent unwanted effects between the cooling beams. However, aside from these two differences compared to the most up-to-date picture, the functionality of feedback cooling remained consistent with the descriptions provided in Chapter 3.

The main changes involve the optics of the line scan camera. After the Dove prism, a different set of optics was configured to capture images for the linescan camera. The optical system consisted of pairs of singlet cylindrical lenses that created collimated spaces before reaching a final configuration of a spherical lens and a beam splitter. This splitter (Thorlabs BS020) directed a portion of the light to a reference area scan camera, which assisted in aligning the line scan camera. The specific cylindrical lenses used are detailed in Table 5, and a reference image of this optical configuration is presented in Figure 60.

| Lens | focal length | orientation | part number           |
|------|--------------|-------------|-----------------------|
| 1    | 50 mm        | Horizontal  | (Thorlabs LJ1695RM-B) |
| 2    | 100 mm       | Vertical    | (Thorlabs LJ1567RM-B) |
| 3    | 100 mm       | Horizontal  | (Thorlabs LJ1567RM-B) |
| 4    | 50 mm        | Vertical    | (Thorlabs LJ1695RM-B) |
| 5    | 50 mm        | Horizontal  | (Thorlabs LJ1695RM-B) |
| 6    | 500 mm       | Vertical    | (Thorlabs LJ1144RM-B) |
| 7    | 500 mm       | Horizontal  | (Thorlabs LJ1144RM-B) |
| 8    | 50 mm        | Vertical    | (Thorlabs LJ1695RM-B) |

Table 5: A list of cylindrical lenses used between the Dove prism and linescan camera in an earlier stage of this experiment. This configuration is no longer in use, but all data taken before ‘Bob V’ was done with this configuration.

This series of lenses produced a higher magnification level compared to later configurations of cylindrical lenses, with a distance calibration around 7.5 nm per pixel. A collection efficiency issue was identified only after collecting and analyzing various data sets. The primary causes of this issue can be broken down into two parts.

First, this configuration incorporates extra mirrors and lenses, which can lead to losses before the photons reach the linescan camera. Second, I did not account for the Rayleigh

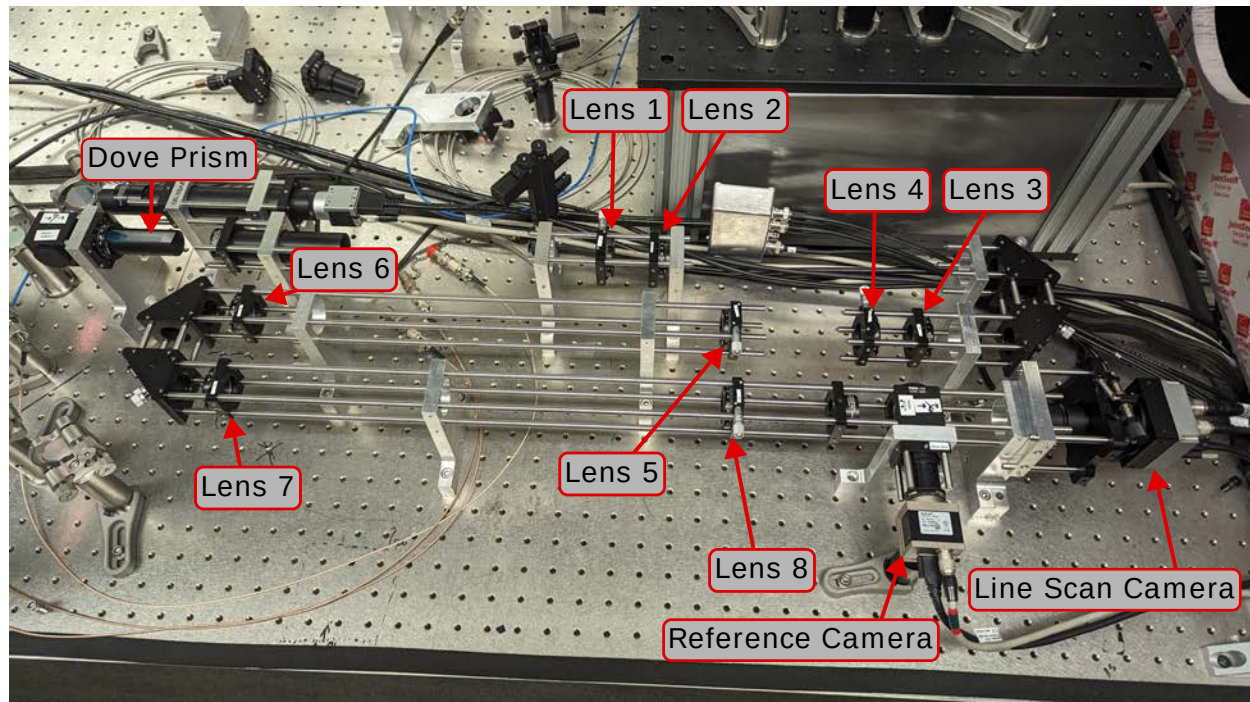


Figure 60: The old configuration of the linescan camera. Singlet cylindrical lenses (8 in total) were used to magnify the particle image before directing the light into a linescan camera. This configuration was done following geometric optic principles. It was eventually scrapped in favor of the current ‘far field’ configuration due to efficiency losses.

range of the beam as it propagated down the optical path to the linescan camera. When the beam was collimated after the final cylindrical lens, the horizontal beam diameter was so narrow that the Rayleigh range extended only a couple of millimeters beyond the previous optic. The distance to the next optic greatly exceeded the Rayleigh range of this beam. With this background established, the first significant set of data collected aimed to approach the SQL in the free particle limit can be presented. This data set was taken on April 13th, 2024, and can be seen in Figure 61.

Although kick calibration data was collected at the time, it was done before an optical isolator was included in the illumination beam path. As a result, when operating with the retro-reflector properly aligned, reduced output was experienced due to the coupling of the beam back into the optical fiber. Therefore, accurately determining the input power of

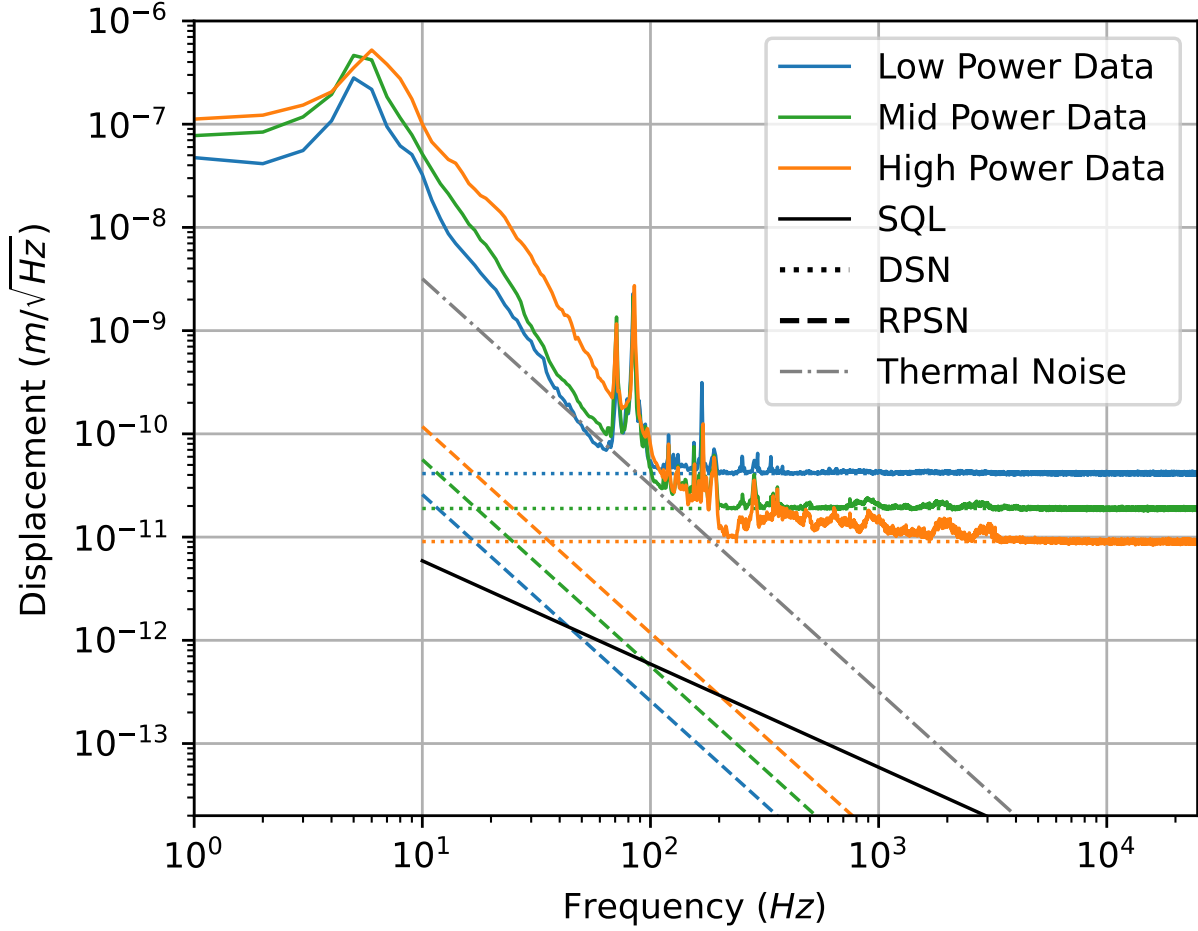


Figure 61: ASD of data collected with Bob III. This data was taken with an old optic configuration that produced far less efficient light collection than our current setup. Despite this limitation, this collected data was still able to reach a closest approach roughly  $\approx 36$  times above this SQL. This approach occurs around 219 Hz on this ASD. DSN and RPSN curves were determined qualitatively since this set was taken with no optical isolator in the illumination beam path.

the illumination beam entering the chamber during measurements is not possible. For this reason, there is no direct method of transferring kick calibration for data sets without the isolator, but this does not mean there are no options.

Low power measurements are taken without additional amplification from the 785 nm BOA, and seem less affected by this coupling issue. By calculating the ratio of the kick

calibration photons to the measured beam power entering the chamber, an estimate for the number of photons hitting the particle at low power can be obtained. This qualitative method provides a rough idea of the DSN and RPSN profiles in data collected without the isolator.

### Cylindrical Lens Upgrade

After identifying subpar collection efficiency, the cylindrical lenses were upgraded to doublet lenses and reconfigured to mitigate the negative impact caused by Rayleigh range limitations. This adjustment resulted in a significantly improved noise floor. Figure 62 shows the mass calibration for the particle Bob V, used throughout this section.

During the transition from atmosphere to UHV, errors occurred, producing poor vacuum quality for the duration of time that this particle remained trapped. Typically, after successful baking of the chamber, pressures can remain around  $10^{-10}$  Torr with the assistance of the titanium pump. However, in this case, there appeared to be a minor leak that caused the chamber pressures to stabilize around  $4 \times 10^{-8}$  Torr. The chamber pressure during Bob V's time in the trap remained at this level, nearly two orders of magnitude worse than usual. Although this situation was not ideal at the time, it did provide us with some valuable data.

First, the increased thermal noise resulted in an ASD that aligns closely with the model of our system outlined in Chapter 2. This data clearly indicates that even within an UHV environment, the particle is still significantly affected by both thermal noise and external noise sources. This finding is valuable when considering future plans for this system, which will be discussed later in Chapter 6.

Second, the data collected using Bob V was the first set obtained with the far-field optical configuration and cylindrical doublet lenses. The noise floor for this new dataset greatly exceeded the previous noise floors established with data from Bob III. While equal

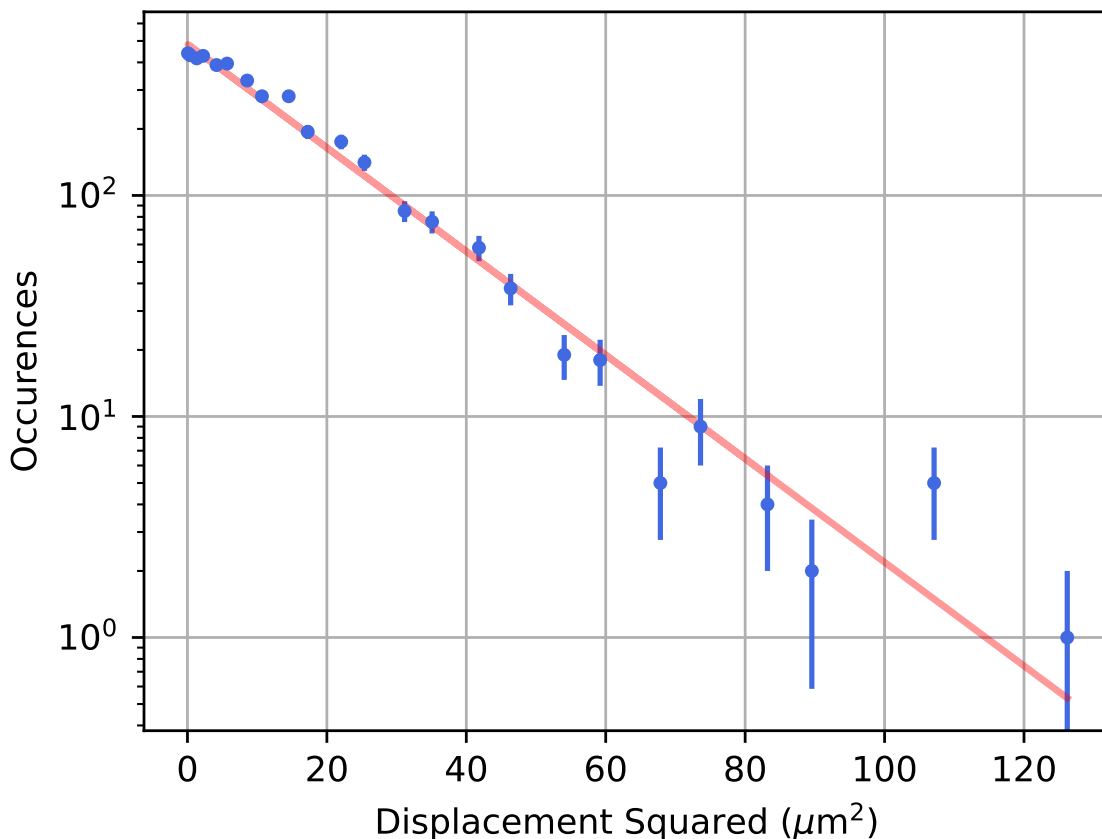


Figure 62: Mass calibration performed on ‘Bob V’ in September 2024. Fitting a histogram of the squared displacements yields a mass fit of  $1.59 \times 10^{-15}$  kg, which is consistent with the presence of a single particle.

photon scattering rates between this set and the set shown in Figure 61 can not be confirmed, the conditions and settings of the illumination beam were similar between both runs. The improvement in our noise floor was significant enough that, despite the considerable increase in thermal noise, the closest approach to the SQL was still approximately 35 times above it. An ASD generated from tracking this particle is shown in Figure 63.

Finally, having this particle in a suboptimal vacuum environment allowed us to observe a rapid increase in its response to charge buildup occurring within the chamber. It was

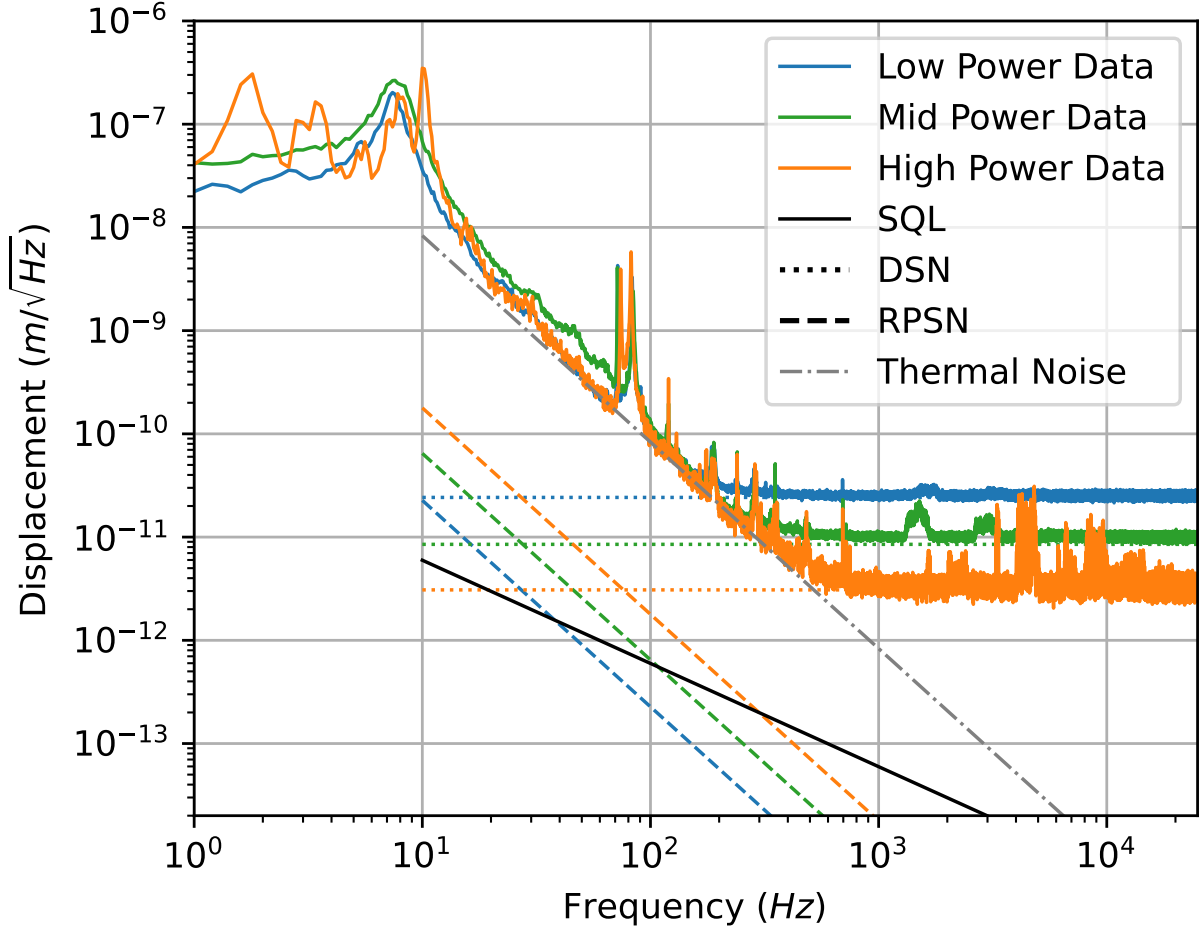


Figure 63: ASD of data collected with Bob V. This data was taken with the newer far field optical configuration that produces more efficient light collection compared to our old geometric setup. Unlike other data sets presented in this chapter, this set was taken outside of UHV, at only  $4 \times 10^{-8}$  Torr. Given  $T$  is room temperature, this pressure results in  $\Gamma_z \approx 8 \times 10^{-5} \text{ s}^{-1}$ . This set achieves the closest approach to the SQL at roughly 550 Hz reaching  $\approx 35$  times the SQL.

previously hypothesized that there is a relationship between the quality of the vacuum achieved and the rate of charge accumulation that influences particle behavior. Over time, following the successful trapping of a particle, the oscillation frequencies of our trap tend to drift at a rate that appears to be significantly affected by vacuum quality, especially when compared to our previously trapped particles. In the case of this particle, there was an

observed near doubling of the axial oscillation frequency in less than a month! Evidence of charge accumulation related to vacuum pressure can be seen in Figure 64.

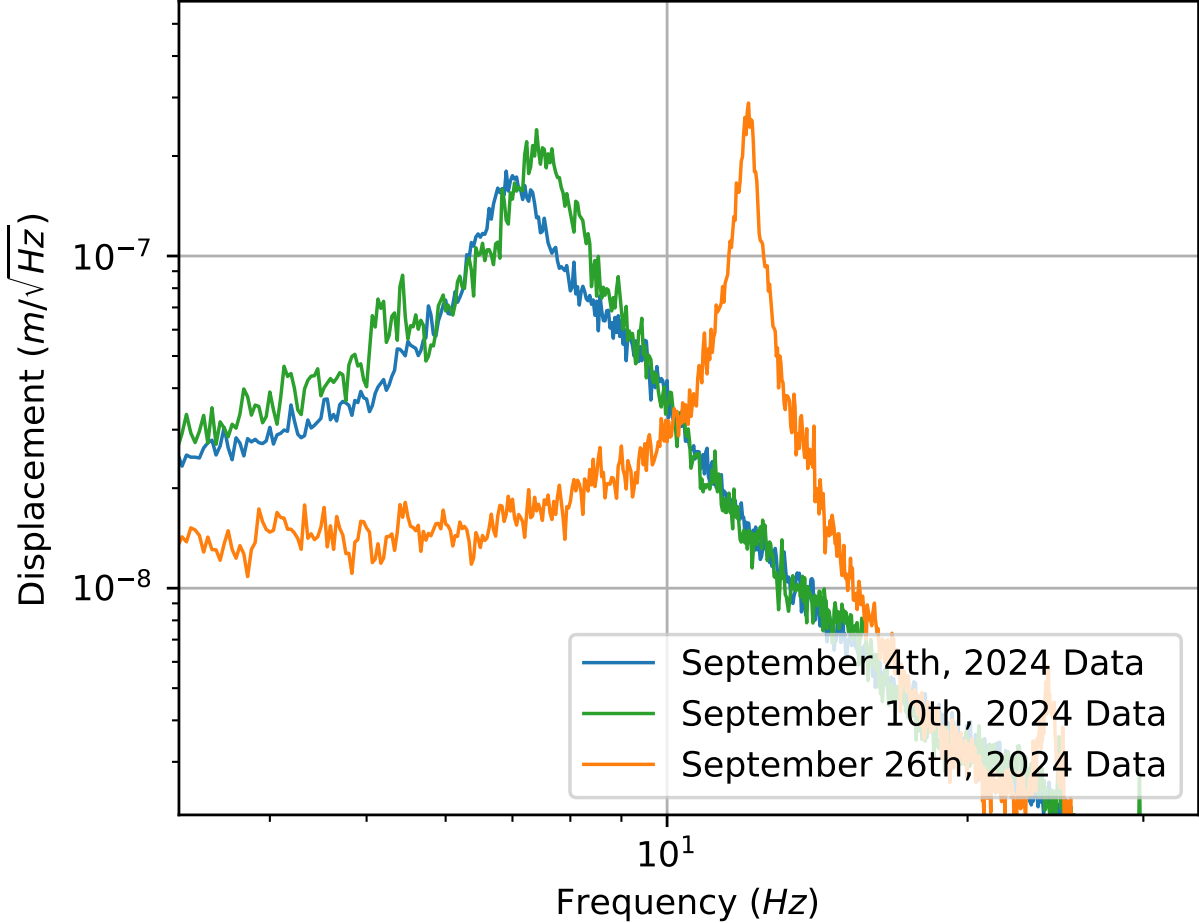


Figure 64: This figure shows various peaks from the ASDs of Bob V recorded on different dates. Notably, there is a frequency shift from 7 Hz to 12 Hz between September 4th and September 26th. Over 22 days, the oscillation frequency nearly doubled. Bob V would go missing in action in the following days.

### Final Improvements and Results

Bob VII is the final particle examined in this dissertation, with its mass calibration illustrated in Figure 65. The trapping of this particle proceeded smoothly, allowing us to maintain long-term UHV conditions. The optical bench was also upgraded with the inclusion

of an optical isolator for the illumination beam. Up to this point, DSN and RPSN in our plots were obtained via a qualitative method since kick calibrations could not be used directly due to the lack of an optical isolator affecting the beam power during measurements. This prevented the use of any hard numbers reported for various parameters such as  $\eta_D$  and  $N_\lambda$ .

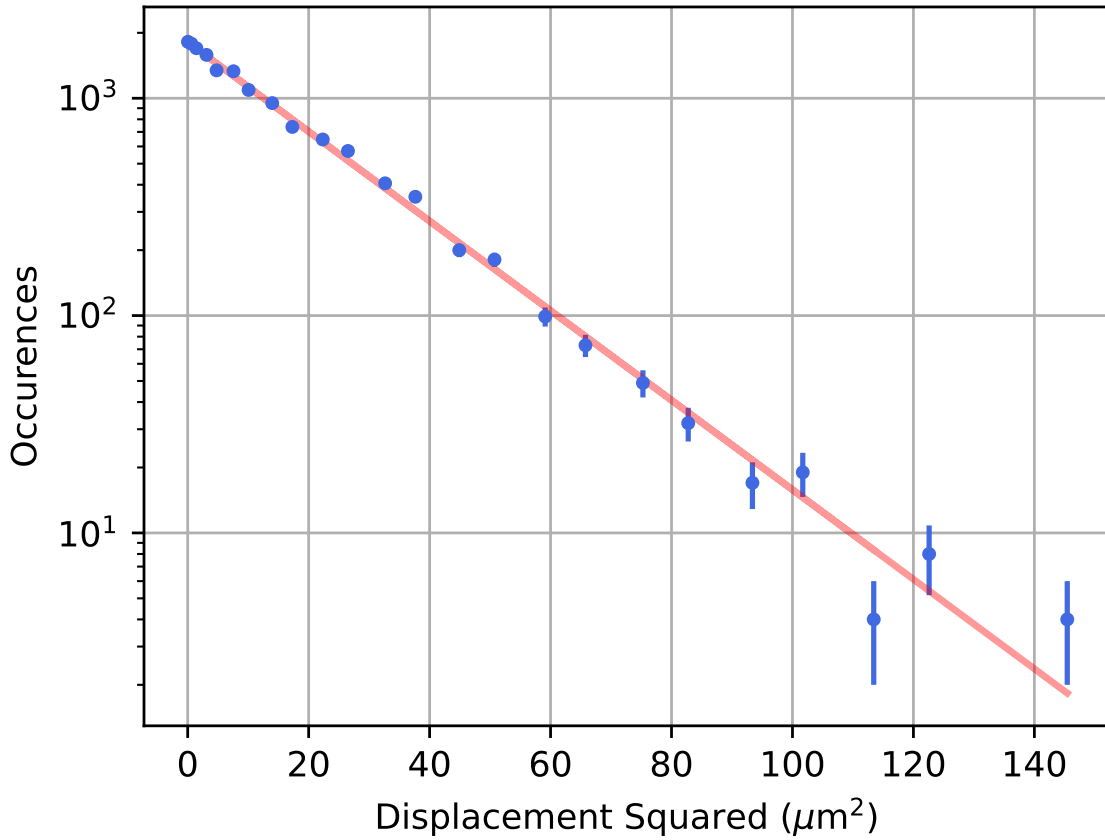


Figure 65: Mass calibration performed on ‘Bob VII’ in November 2024. Fitting a histogram of the squared displacements yields a mass fit of  $1.40 \times 10^{-15}$  kg, which is consistent with the presence of a single particle.

Following successful trapping of this particle, the strange behavior attributed to coupling along the illumination beam path was identified. It was noticed that once the retro-reflector beam passed the particle, it returned across the table and became coupled to

the amplifier. The net result of this coupling was a reduction in the output power of the beam. Consequently, the measurements taken during kick calibration without the oscillator require additional analysis to use, unlike measurements taken with the isolator. Even with this additional analysis, only a qualitative estimate of DSN and RPSN curves could be produced with pre-isolator data.

The behavior was observed by measuring the output of the illumination beam with a photodiode and a beam splitter. When the retro-reflector was adjusted to avoid striking the particle, the absence of coupling into the fiber resulted in a noticeable increase in the beam output. Conversely, when the reflector was moved to hit the particle, the beam output decreased. At this point, an optical isolator was introduced, which is positioned after the fiber output of the illumination beam, as outlined in Chapter 3. Once the isolator was in place, the power fluctuations ceased when adjusting the retro-reflector.

While trapped in UHV, the chamber has shown a high resilience to charge buildup compared to the case with Bob V. Figure 66 shows how this particle went a much longer duration with very minor oscillation frequency shift.

While operating under UHV, additional tests were conducted to evaluate various aspects of both the imaging optics and cooling performance. Initially, the dove prism was subjected to minor adjustments to assess how the particle appears on the linescan camera. This process involved collecting brief data sets as the Dove prism was rotated in 2-degree increments between measurements. The objective was to demonstrate how improper alignment of the dove prism along the trap axis can result in increased noise affecting the transverse and vertical motion of the particle. Figure 67 illustrates how these adjustments impacted particle tracking by the linescan camera. As another confirmation that this position aligned well with the cylindrical lenses, the particle position can be shifted by adjusting the first mirror in the optical train following the collection objective. When using the front cooling camera as a reference, if vertical motion on one camera does not translate to similar displacement on the

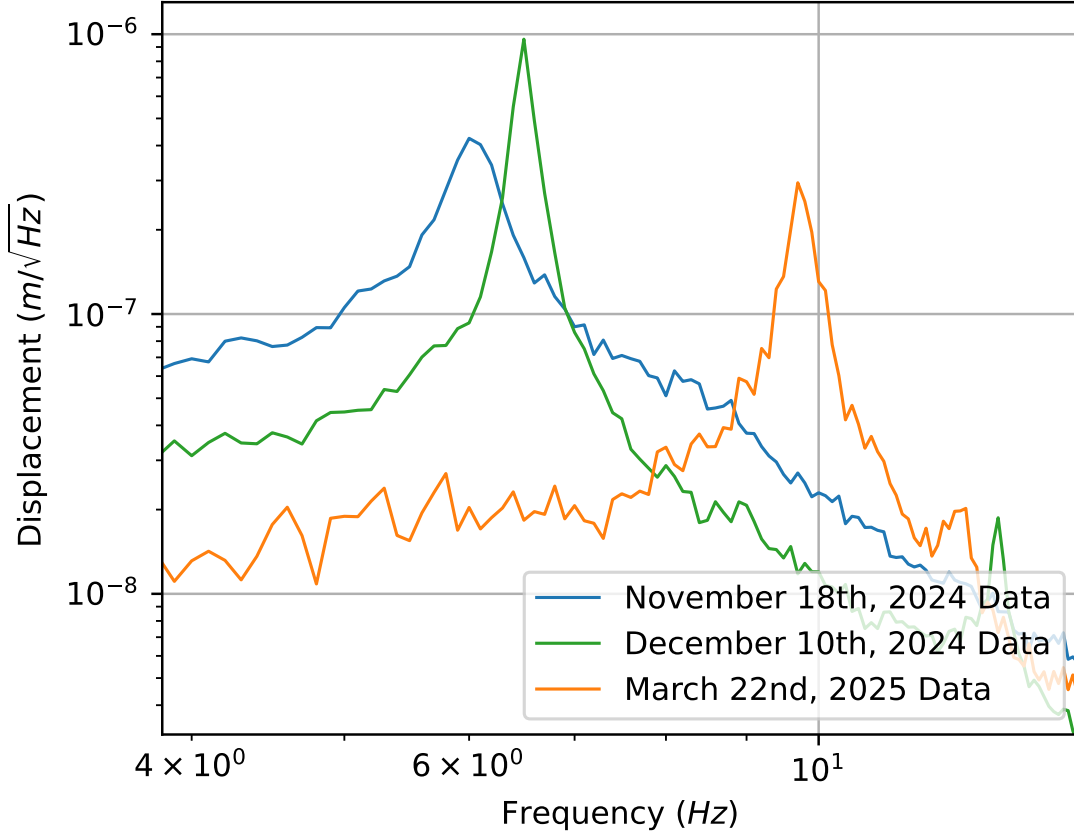


Figure 66: A plot with various peaks from ASDs of Bob VII data taken from different dates. There is an observed shift from 5.8 Hz to 9.5 Hz from November 18th, 2024, to March 22nd, 2025. In 4 months of being trapped under UHV the axial oscillation frequency remains lower than the build-up shown in Figure 64, which took place in 22 days.

linescan camera, then it is clear the dove prism is improperly aligned.

Another interesting test conducted with this particle examined the effects of cooling beam gain on particle motion. When collecting data to approach the SQL, active cooling keeps the particle within the bounds of the line scan camera's field of view. The cooling has a gain setting that can be adjusted based on the desired strength of the feedback. The particle may not be adequately cooled for data collection if the gain is too low. Conversely, if the gain is set too high, it can lead to overdamped behavior. A comparison of different gain settings (where gain is an arbitrary value input in our cooling feedback software that

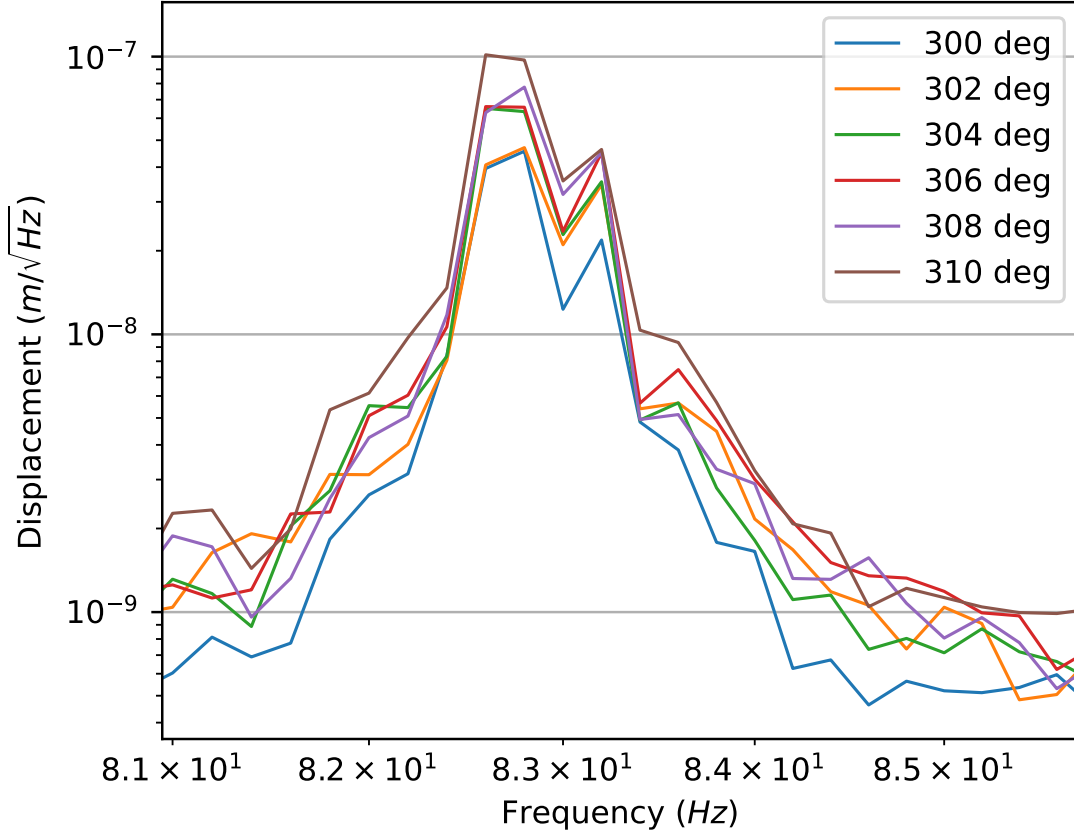


Figure 67: The Dove prism was adjusted to various positions in its rotation stage to highlight its effects on vertical motion coupled into the line scan signal.

adjusts the peak power allowed by the amplitude EOMs) is illustrated in Figure 68.

With the inclusion of our optical isolator, kick calibration can now be properly utilized. Previously, all kick calibration results were not directly included because the data collected was influenced by the retro-reflector beam coupling without the optical isolator. This limitation prevented us from accurately transferring the expected number of photons scattered per measurement to the final data sets, rendering the kick calibration conducted before the installation of the isolator not directly applicable to previously collected data sets.

In the new configuration with the optical isolator, kick calibration was performed using an input power of 15.8 mW, measured after the dichroic that combines the transverse cooling

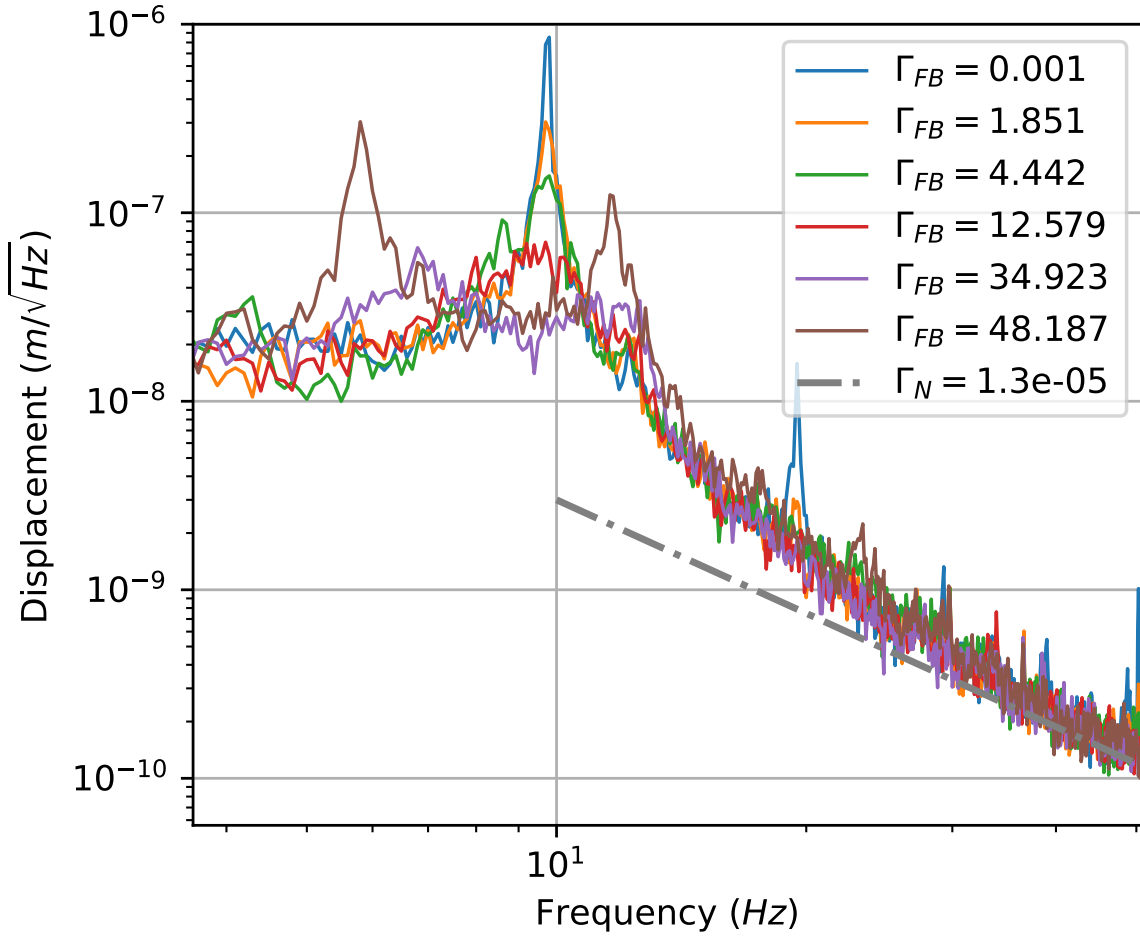


Figure 68: The effect of various cooling gain settings on linescan camera data collection, with peak fits reporting the damping rate for each data set. Each increase in  $\Gamma_{FB}$  corresponds to an increase in feedback cooling beam power, leading to overdamped behavior. This data shows that even in cases of extreme damping, all spectra converge at frequencies beyond the oscillation frequency of the trap. This is a positive sign, suggesting that the noise levels in regions away from the oscillation frequency are independent of the feedback cooling settings. Achieving this independence is essential to observe levitated optomechanics in the free particle limit. The thermal noise line for  $\Gamma_N$  assumes room temperature  $T = 295$  K.

and illumination beams. The results of this calibration are illustrated in Figure 69.

With all the background information provided and the optical isolator in place, the complete picture of our system's calculated DSN and RPSN can be included. Below are two datasets with the closest approach to the SQL in a magneto-gravitational trap to date.

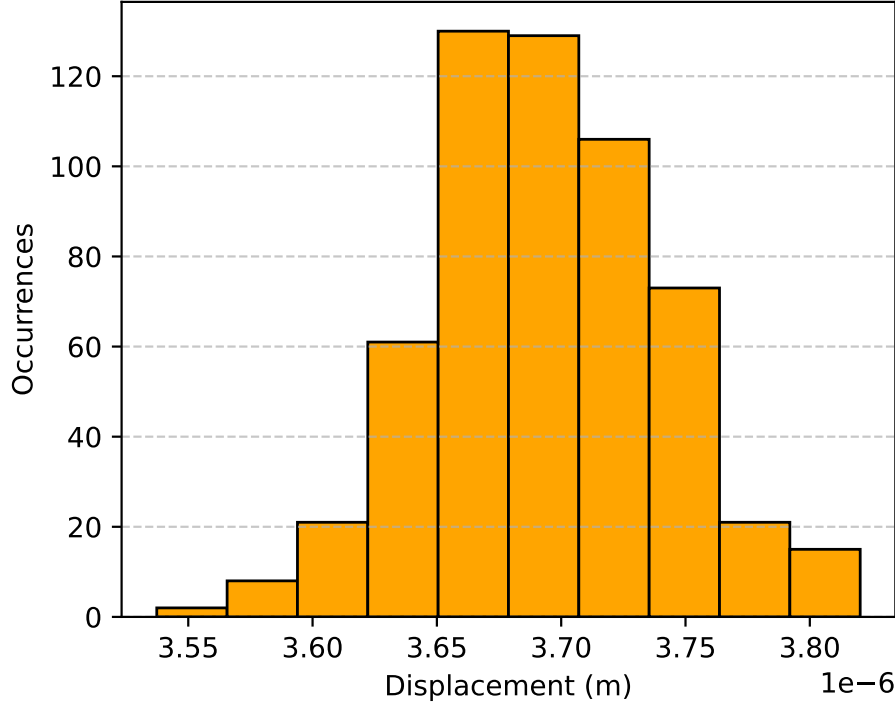


Figure 69: Kick calibration conducted using  $500\text{ }\mu\text{s}$  pulses with an input power of  $15.8\text{ mW}$  entering the chamber. This process measured particle displacement. The number of photons scattered during a typical high-speed measurement can be calculated using the conversion shown in Equation 73.

The first dataset, shown in Figure 70, was collected using a high gain on the feedback cooling beams, resulting in an overdamped system. Interestingly, there was unusual behavior at low frequencies not observed in any other set I’ve collected, the cause of which remains undetermined. However, beyond  $100\text{ Hz}$ , this behavior seems to have no impact, and the ASD profile became more consistent with what is typically observed in our other datasets.

From the kick calibration data, collection efficiency was determined, as defined in Chapter 2, to be approximately  $\eta_D \approx 0.001$  for the data shown in Figure 70. This value is significantly lower than the expected  $\eta_D \approx 0.1$ . Likely causes for this discrepancy include optical aberrations and detector quantum efficiency, which prevent some of the captured photons from being useful for particle tracking. The line scan camera is also near saturation

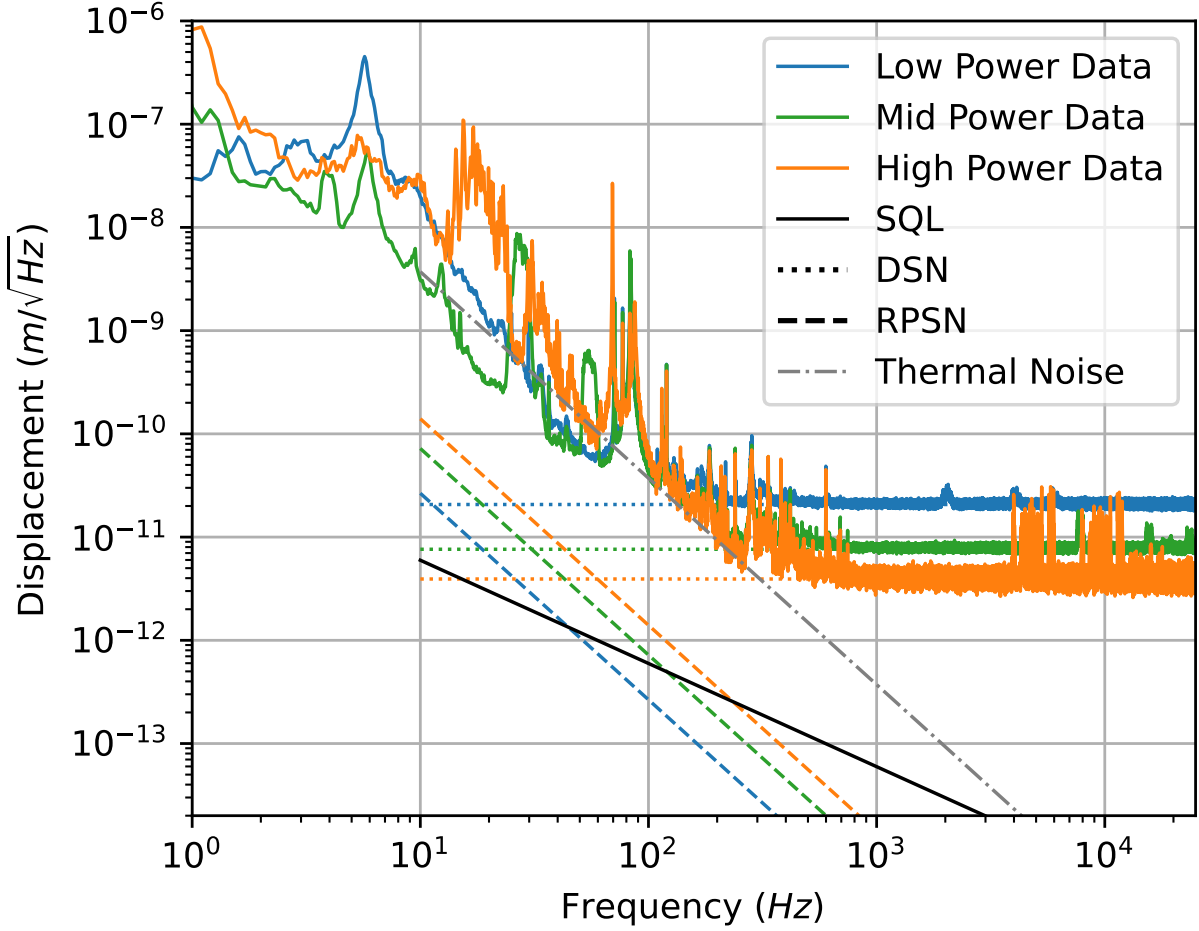


Figure 70: ASD of Bob VII data taken with high feedback cooling gain. This dataset was significantly overdamped and had unusual behavior at low frequencies. Beyond 100 Hz, the behavior became more typical. This run was conducted in UHV on the order of  $10^{-9}$  torr. The closest approach to the SQL occurs with the high power data around 256 Hz, reaching  $\approx 27$  times the SQL. The thermal noise line was generated with  $T$  at room temperature (295 K and  $\Gamma_z = 2 \times 10^{-5} \text{ s}^{-1}$ ). Using data from kick calibration, the DSN and RPSN curves are reported for the given data sets as  $N_{low} = 3.12 \times 10^6$  photons,  $N_{mid} = 2.24 \times 10^7$  photons,  $N_{high} = 8.24 \times 10^7$  photons and  $\eta_D \approx 0.001$ .

capacity during high power data collection, meaning any improvements to collection efficiency would likely result in a lower illumination power being required.

Additionally, as shown in Figure 58, our fitting method significantly influences the noise floor, suggesting that employing more robust tracking methods could further enhance  $\eta_D$ .

It's important to note that any  $\eta_D$  values reported in this dissertation take into account the effects of the retro-reflector beam. By doubling the number of scattered photons from the particle without collecting them, there is effectively a reduction in our detection efficiency by half while increasing RPSN by a factor of  $\sqrt{2}$ . Nevertheless, this efficiency and the kick calibration enable us to include plots of calculated DSN and RPSN with datasets that include the isolator without the need to infer their values using additional steps, which was done for pre-isolator data sets.

At high frequencies, the observed noise floor of our ASD scales inversely with the square root of the scattered photons, consistent with the expectations from Equation 35. The lowest noise floor value achieved so far is around  $3\text{ pm}/\sqrt{\text{Hz}}$ , which is nearly  $10^6$  times smaller than the particle diameter! This indicates that during high-power measurements, the particle displacements are tracked with an accuracy finer than the ground state of our axial harmonic oscillator (on fast timescales).

In the overdamped dataset (as well as in the previously shown ASDs), various spikes within the range of the DSN noise floor are shown. These spikes differ between datasets and are believed to be caused by particle rotation, which could be detected due to slight asphericity in the particles.

### Interference Subtraction

The data set presented in Figure 74 demonstrates the closest approach to the SQL achieved to date in a magneto-gravitational trap, approximately 25 times the SQL in the free particle limit. This data collection utilized a moderate gain setting on feedback cooling to prevent overdamping. This is the first data set presented in this dissertation that has undergone interference subtraction. Figure 71 shows the results of this process for a single data set, showing a large reduction in excess noise caused by transverse and vertical oscillations. This process utilizes reference position data collected by the feedback

cameras, allowing us to remove unwanted interference correlated between the cameras from our spectrum. A brief overview of this process is provided below.

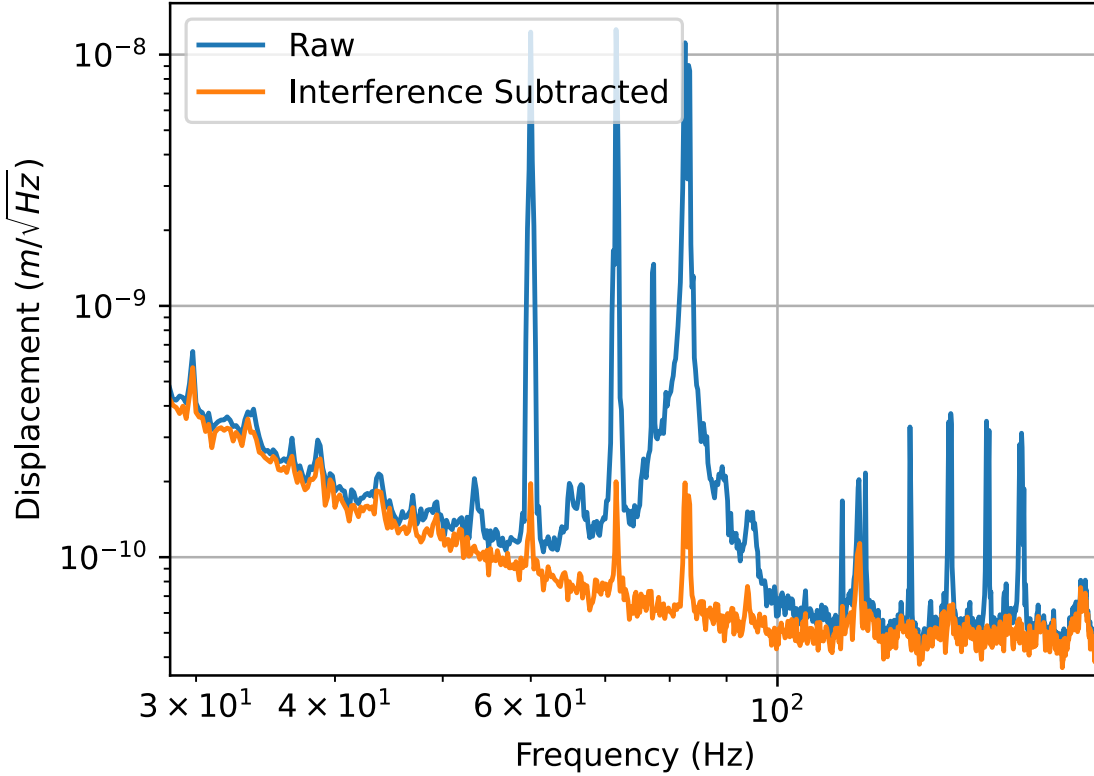


Figure 71: An ASD of particle data before and after interference subtraction. This plot shows that the excess noise caused by the other oscillation axes is reduced by using correlations between the feedback cooling cameras and the linescan camera.

Our data collection yields position data that, when broken up and converted into an average of ASDs, should resemble the spectrum shown in Figure 8. However, the particle trapping method results in three independent oscillation axes, two of which are present in all collected data. Interference subtraction, performed through a method of least-squares, identifies signal correlations and facilitates the removal of correlated noise from the linescan and feedback cooling cameras. Given the ability to track the particle along a 2D plane, the front feedback camera can isolate axial motion independent of vertical motion. Figure 57

highlights how the particle path from the cooling camera appears free of any interference from vertical particle motion. In contrast, every example taken from the linescan camera has this interference.

The cooling camera operates at a much lower sampling rate than the linescan camera and does not capture the higher frequency harmonics. To address this issue, zero-padding is applied to the feedback camera data to artificially generate these harmonics. Using an integer multiple of the feedback camera's framerate ensures that the frequency bins align during zero-padding, extending the interference subtraction capabilities beyond the cooling camera's sampling rate. Figure 72 shows the results of this process.

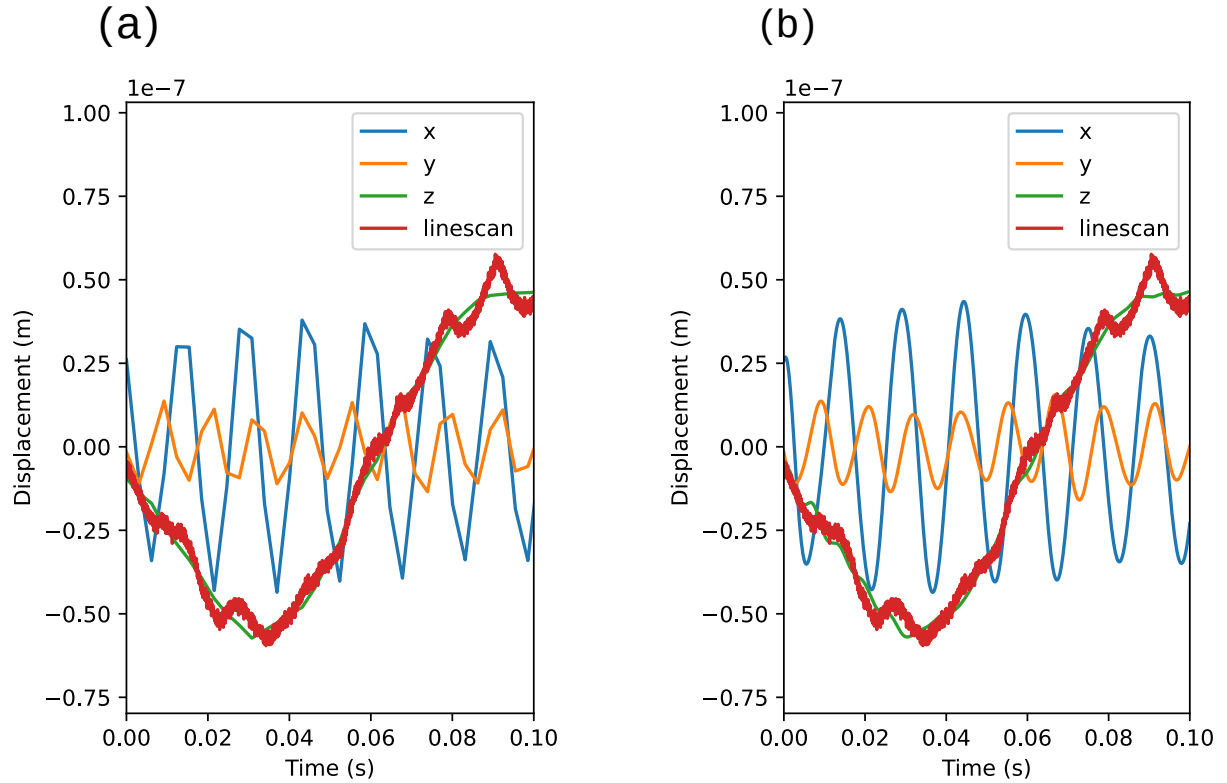


Figure 72: A set of simulated data to replicate behavior of the area scan cameras and line scan camera. (a) Data taken before additional padding. (b) Data taken following padding. Additional resolution in the time domain can be seen clearly in the 'x' and 'y' time series traces.

Data is prepared for least squares analysis by dividing the time-domain linescan data

into chunks, each of some specified length in time, and then performing FFTs on these segments. The analysis is performed on a single frequency bin,  $f_i$ , at a time. The cooling camera data inserts known peaks ( $x$  and  $y$ ), sidebands ( $y \cdot z$ ,  $y \cdot z^2$ ), and harmonics ( $y^2$ ,  $y^3$ ,  $y^4$ , and  $y^5$ ), forming 8 columns. The segmented line scan camera data at  $f_i$  is used in conjunction with the cooling camera data to define a matrix,  $A$ , at frequency  $f_i$ , along with a vector  $\vec{b}_{in}$ .

When using complex Fourier coefficients instead of just the ASD magnitudes, both the matrix  $A$  and the vector  $\vec{b}_{in}$  will contain complex values. The number of rows in the matrix  $A$  is determined by the number of segments created from a single time series. For instance, splitting a time series that lasts for 60s into 1s non-overlapping segments, the resulting matrix will have 60 rows and 8 columns. The line scan data is similarly divided into segments, producing a vector with an identical number of rows.

To solve for the vector  $\vec{x}$  in the equation  $A\vec{x} = \vec{b}_{in}$ , we use least squares fitting, where  $\vec{b}_{in}$  represents the complex-valued line scan data at frequency  $f_i$ . The coefficients obtained from this solution are then used to update  $\vec{b}_{in}$ . This allows us to subtract the computed output from the original signal with a result given by

$$\vec{b}_{out} = \vec{b}_{in} - A\vec{x} \quad (80)$$

where  $\vec{b}_{out}$  is our subtracted line scan data,  $\vec{b}_{in}$  is the non-subtracted line scan data, and  $A\vec{x}$  is the product of our least squares fit.

Low correlations between the two cameras lead to a fit where the output vector  $\vec{b}_{out}$  is approximately equal to the input vector  $\vec{b}_{in}$ . In contrast, large correlations, particularly those observed near the oscillator resonance, result in a significantly greater contribution from the term  $A\vec{x}$ . An example of these outcomes can be seen in Figure 73, which displays the complex vectors that constitute both  $\vec{b}_{in}$  and  $A\vec{x}$  plotted together.

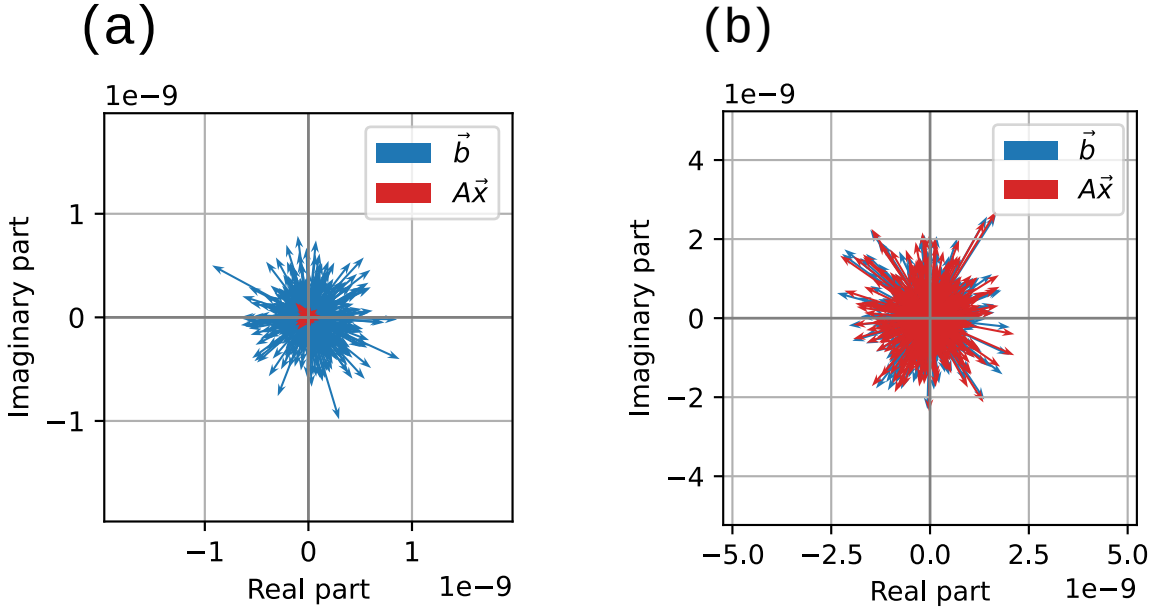


Figure 73: (a) A set of complex vectors are plotted from data at  $f_i$  equal to 50 Hz. In this range the signals between both the area scan cameras and the line scan camera are not correlated. The resulting least squares fit yields a result in which  $A\vec{x}$  and  $\vec{b}_{in}$  have minimal correlation. (b) A similar plot, but this time taken around 84 Hz. This vertical oscillation frequency gets tracked between all cameras simultaneously, yielding a very identical result between  $A\vec{x}$  and  $\vec{b}_{in}$ . This high correlation leads to interference subtraction taking place around oscillator resonance, as shown in Figure 71.

Due to the timing configuration of our measurement scheme, both the camera cooling data and linescan data must be initiated simultaneously. This is necessary to allow for interference subtraction between the two measurements. However, this setup requires a brief period during which the cooling is turned off, enabling both measurements to start simultaneously. In the past, we avoided doing this during high-power collection because turning off the cooling can create instability, potentially leading to the loss of the particle.

For the next set of high-power data, we decided to start recording with a lower power output and a longer overall measurement duration. Once both the cooling and linescan cameras began data collection, we increased the illumination beam to its high power state while continuing the cooling and data collection. As a result, we were able to gather data relevant for interference subtraction across the various power settings. While the

previously presented ASDs may benefit from interference subtraction, this process would not significantly affect the conclusions drawn from those sets.

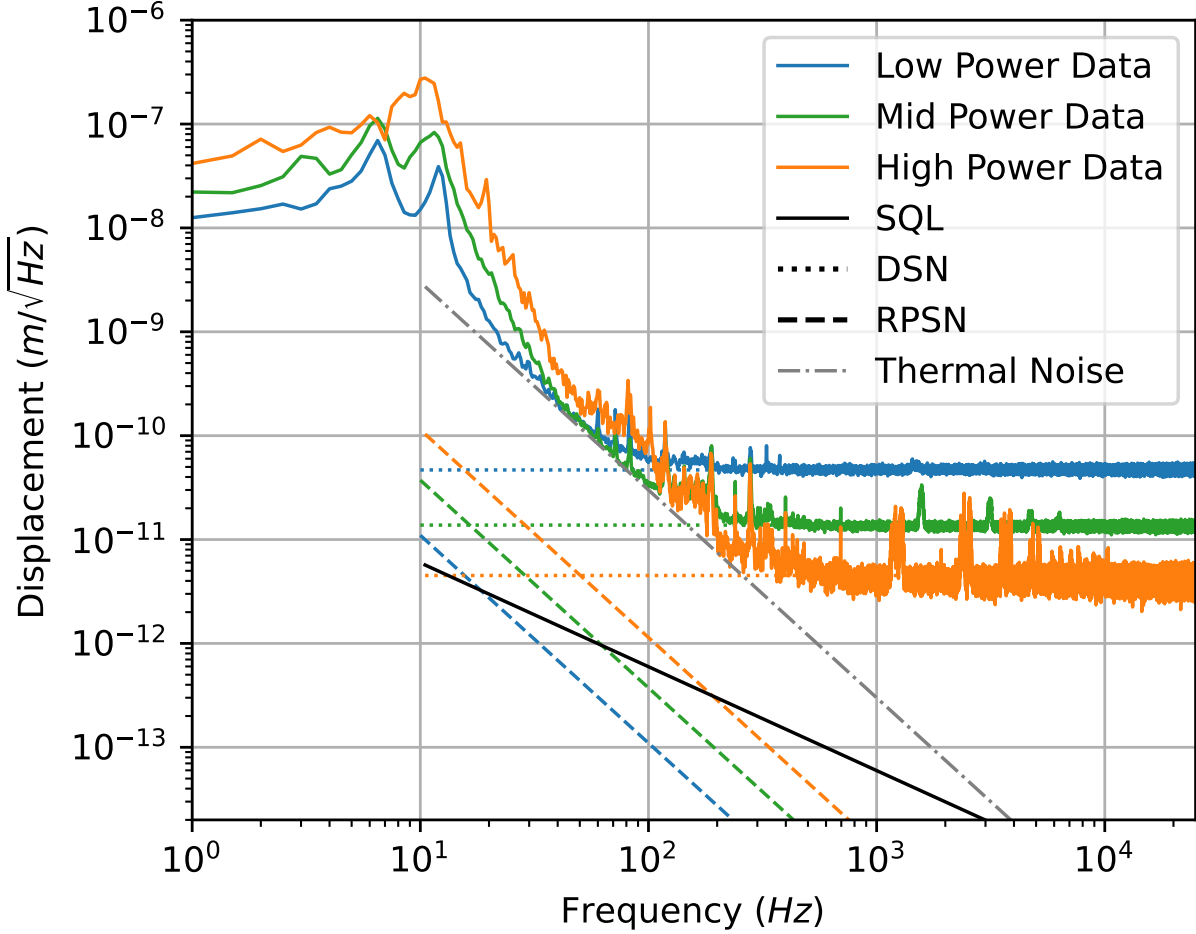


Figure 74: ASD of Bob VII data taken with moderate feedback cooling gain. This run was conducted in UHV on the order of  $10^{-9}$  torr. The closest approach to the SQL occurs with the high power data around 224 Hz, reaching  $\approx 25$  times the SQL. The thermal noise line was generated with  $T$  at room temperature (295 K and  $\Gamma_z = 1.3 \times 10^{-5} \text{ s}^{-1}$ ). Using data from kick calibration, the DSN and RPSN curves are reported for the given data sets as  $N_{low} = 4.0 \times 10^5$  photons,  $N_{mid} = 4.8 \times 10^6$  photons,  $N_{high} = 5.20 \times 10^7$  photons and  $\eta_D \approx 0.0016$ .

### Yoctonewton Force Sensitivity

While the primary goal of this dissertation was to approach the SQL in the free particle limit, a detour was made to evaluate the force sensitivity of our system, drawing inspiration from the literature reviewed in Chapter 1. Several research groups working on levitated optomechanics reported varying force sensitivities associated with force ASDs.

After completing the procedure to trap Bob VII, the system achieved a remarkably low natural damping rate of  $\Gamma_z \approx 3 \times 10^{-6} \text{ s}^{-1}$  with assistance from the titanium sublimation pump. This opportunity allowed the collection of a long-duration measurement, which involved considerations different from our previous high-speed measurements. Our objective was to analyze a force ASD, where the stochastic force-dominated regions of the spectrum are frequency-independent. A continuous set of position data was recorded for 48 h, and its ASD is shown in Figure 75

Our analysis differs from some examples outlined in the introduction in that observations are done in the free particle limit. In contrast, other systems analyze data near the resonance of their oscillator. In those cases, they must consider the Allan variance of their oscillator, as slight perturbations in oscillation frequency can lead to a loss of coherence over long measurement times. However, our free particle behavior does not have this limitation.

One significant challenge for our measurements using a magneto-gravitational trap is the storage capacity for raw data. High-speed imaging with the linescan camera does not provide additional useful information when evaluating force sensitivity since our stochastic-dominated region primarily exists in a small fraction of the total spectrum when operating at 50 000 Hz. To minimize storage requirements, the linescan camera's operation allows for binning images along the time axis before saving them to a file. This adjustment reduces our operating refresh rate to only 1000 Hz, and thus allows longer duration measurements without the data storage limitations. Additionally, this reduces the post-processing time of

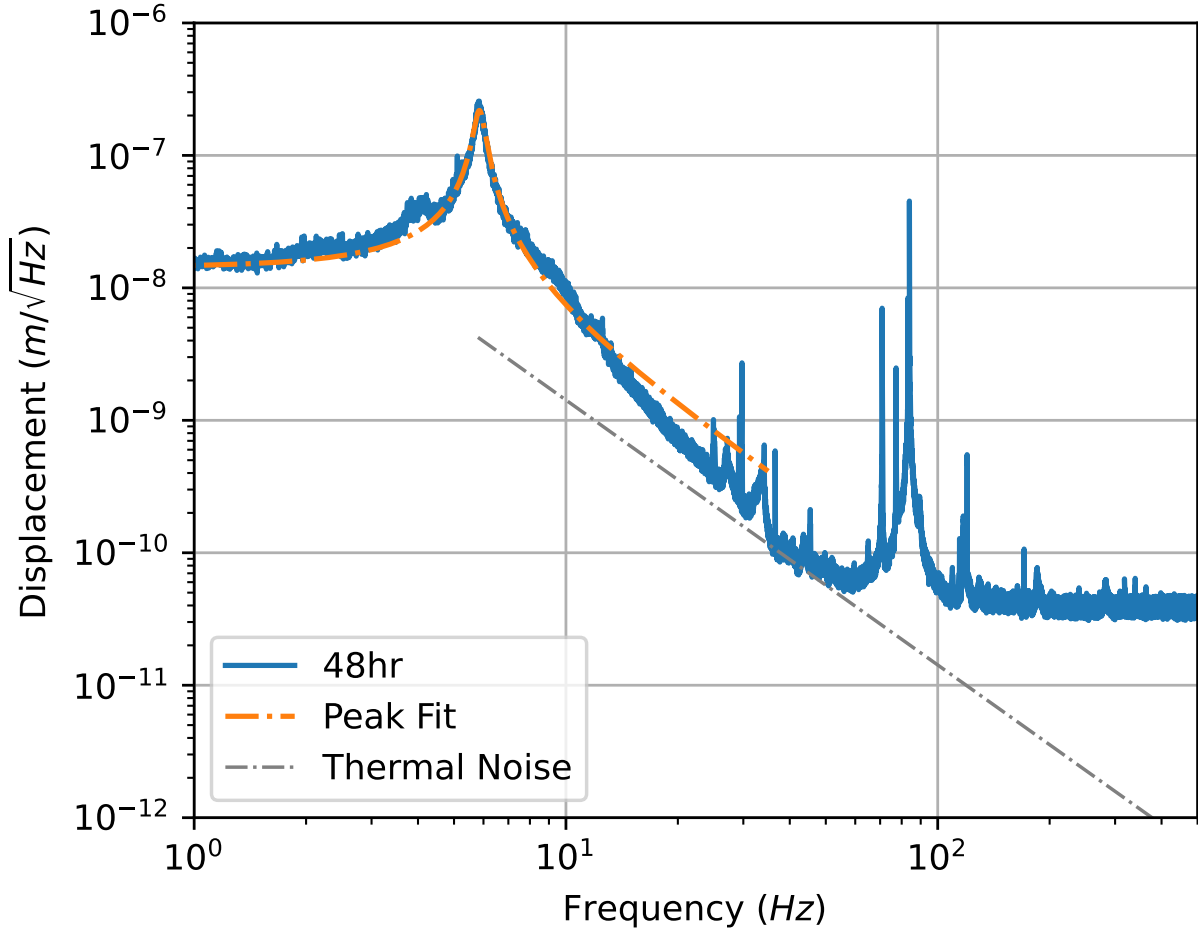


Figure 75: Position ASD collected from Bob VII with the intent of a force sensitivity measurement. Data collection lasted 48 h. This ASD was averaged with a time per segment,  $t_{seg}$ , equal to 600 s. Excess noise from illumination and feedback cooling occurs near axial resonance, and drops off as observed frequency increases towards the free particle limit.

the 48 h data when performing the double Gaussian fits from over 80 days to just 3 days!

Unlike measurements that seek to approach the SQL, force sensitivity measurements do not gain advantages from increased illumination power. Excessive illumination only leads to increased feedback cooling during the measurement process, increasing noise near the oscillator's resonance frequency. Since lowering DSN does not negatively contribute to this measurement's primary goal, the illumination beam's input power was reduced to 0.1 mW.

This lower power level enabled consistent particle tracking by the linescan camera with minimal particle instability.

Addressing particle instability is crucial for this measurement. To achieve low force sensitivity, it is essential to have a continuous time series; any breaks or discontinuities effectively mark the beginning of a new measurement, hindering the ability to reduce the frequency bin size.

Retro-reflection was not used during these measurements because the additional RPSN would only increase particle instability, increasing the risk of failed tracking throughout the measurement duration. Operating the illumination beam at low powers doesn't provide enough of a transverse kick to cause issues with camera focusing.

In November 2024, two force sensitivity measurements took place, lasting 24 h and 48 h respectively, with an input illumination set at 0.1 mW. The 24 h set was recorded with a chamber pressure reading  $0.3 \times 10^{-9}$  Torr on the ion pump. Once complete, the titanium pump further reduced chamber pressure into the  $10^{-10}$  Torr range. During the 48 h set chamber pressure remained low enough to bottom out the ion pump gauge, and this reduction in chamber pressure seems to have reduced thermal noise within the free particle regime, indicating residual gas damping still contributes to reduced sensitivity even in UHV conditions. Various interferences from vibrations and other noise sources are present in the region of interest. Still, there are multiple pockets of smooth noise profiles between 20 Hz and 70 Hz. The force ASDs obtained from these measurements are shown in Figure 76, zoomed into a region of interest for our measurement.

Given the noise floor of the spectra and the measurement time, an estimate of the force minimum expected using  $F_{min} = \sqrt{S_F \cdot b}$  can be made. Using this form as a guess of the minimum force,  $F_{min} \approx 36$  yN for the 24 h set and  $F_{min} \approx 22$  yN for the 48 h set.

These values are checked against the Weibull distribution expected from the analysis described in Chapter 2. Given the measurement duration, a region of our spectrum with a

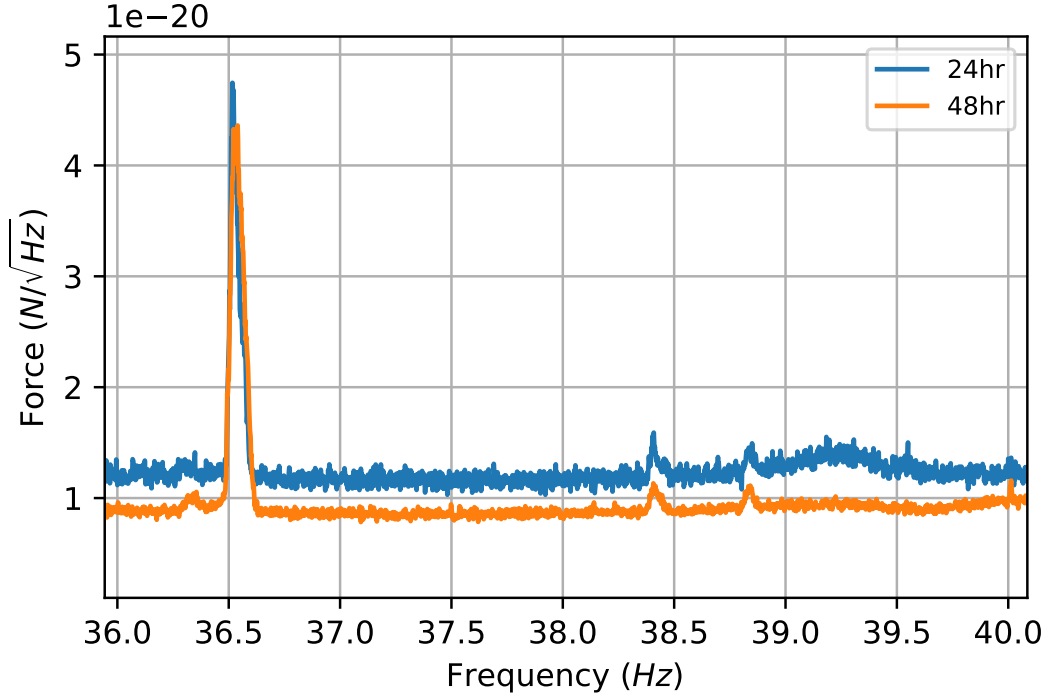


Figure 76: Force ASDs from our 24 h and 48 h data sets within the frequency range of 36 Hz to 40 Hz. The difference in noise floors is believed to result from reduced chamber pressure between runs. Small peaks and spikes from vibrations and other noise sources can be seen, with an ideal stochastic region visible between 37 Hz and 38 Hz. This plot was generated by averaging the data sets into 600 s segments for better visualization. No averaging will take place during analysis with the Weibull distribution.

width of 1 Hz, when not averaged, produces a total number of bins equal to the duration of each measurement in seconds. This results in 86 400 s for the 24 h dataset and 172 800 s for the 48 h dataset. Taking these two datasets and creating a histogram of these bins leads to the results shown in Figure 77. These results demonstrate a new record for force sensitivity achieved by a magneto-gravitational trap.

For the 24 h set the force sensitivity  $F_{min}$  was found to be  $(36.09 \pm 0.05)$  yN, and  $F_{min}$  for the 48 h set is  $(22.04 \pm 0.02)$  yN. The reported uncertainty comes from the covariance matrix recovered by the Weibull distribution fit. For the best force sensitivity this uncertainty is around 23 rN, or  $23 \times 10^{-27}$  N!

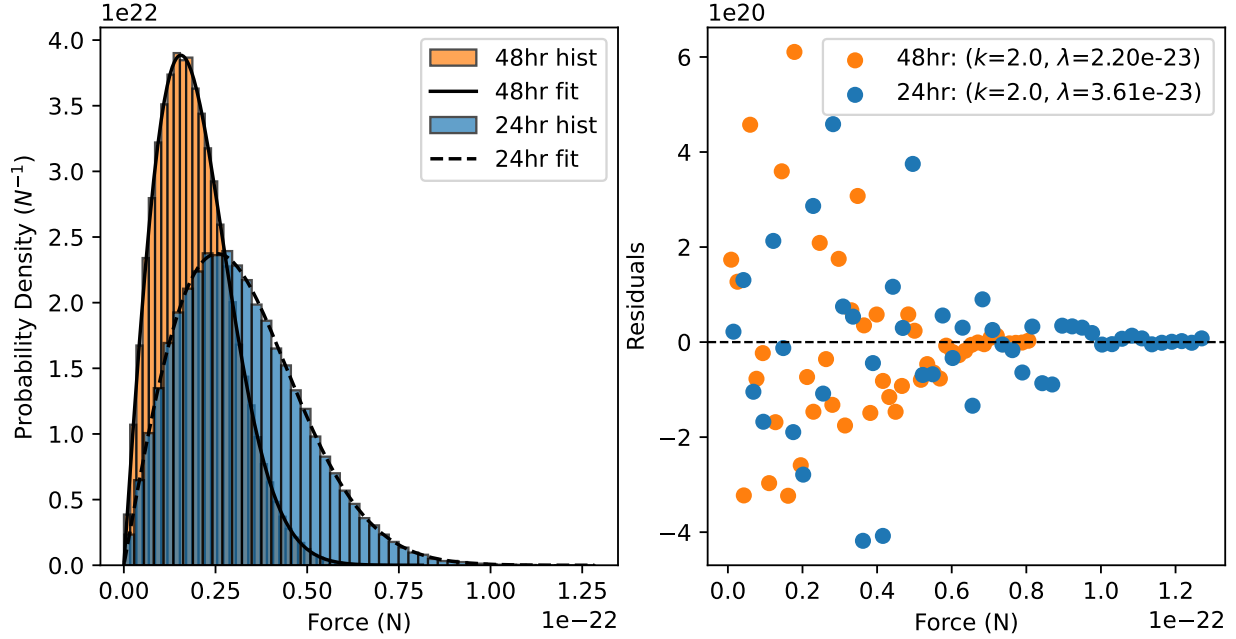


Figure 77: A histogram of forces taken from the 24 h and 48 h sets within frequency range 37 Hz to 38 Hz following the procedure outlined in Chapter 2. The resulting distribution from our continuous time series is then fitted to a general Weibull distribution to evaluate force sensitivity. Fits return  $k = 2.0$ , in line with our expected Rayleigh distribution. Between these two datasets, the 24 h set achieved a force minimum of  $F_{min} = 36.1$  yN, while the 48 h set achieved  $F_{min} = 22.0$  yN.

### Additional analysis

This section contains additional observations about the collected data. To start, let's consider the title of this dissertation, "Levitated Optics in the Free Particle Limit". As outlined in the introduction, there appears to be a gap in the literature concerning levitated systems modeled as oscillators and tethered systems that can be treated as free masses, such as those used in LIGO. An early goal from the start of this dissertation was to achieve a free particle system using a magneto-gravitational trap, and various data sets that resemble a free particle at higher frequencies have been shown. Now let's focus on a specific data set collected with Bob V, under a vacuum of  $4 \times 10^{-8}$  Torr. This data set, illustrated in Figure 78, serves as an excellent example that aligns with the model described in Chapter 2,

clearly demonstrating levitated optomechanics in the free particle limit.

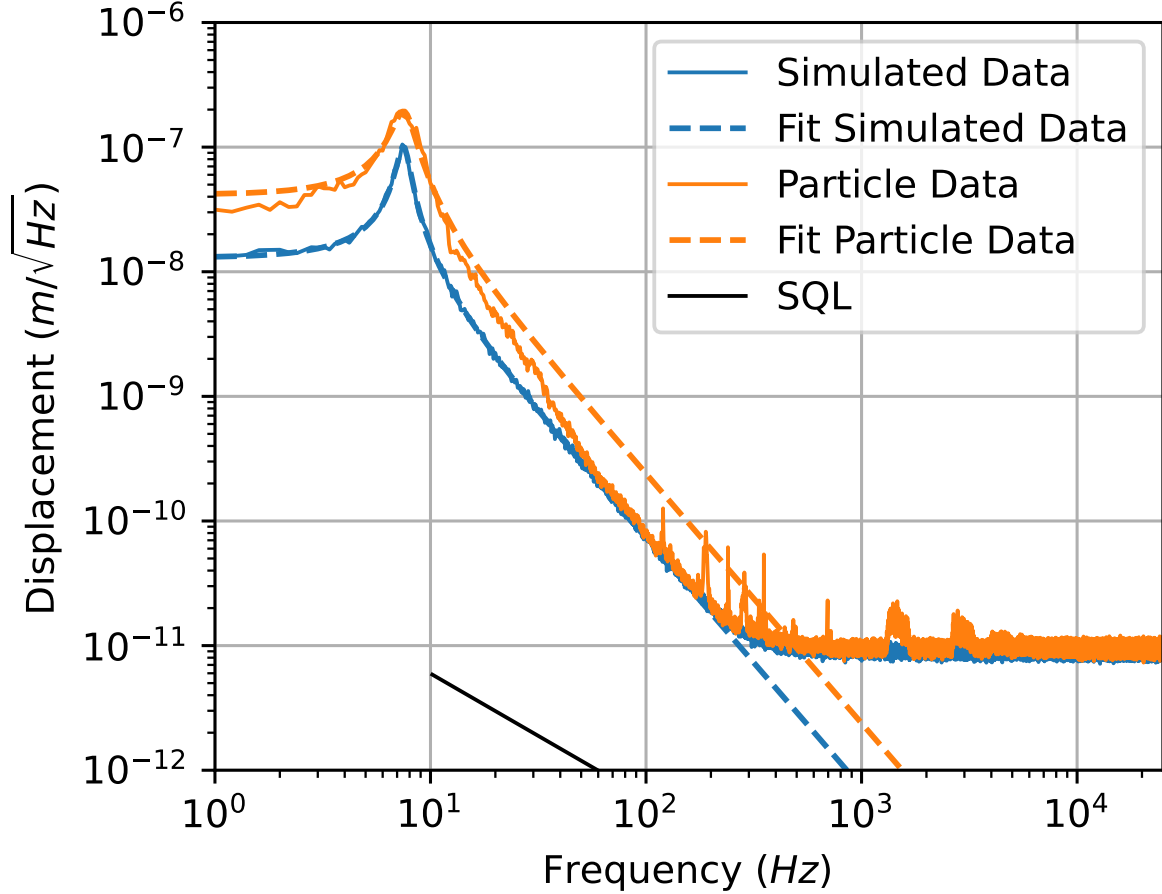


Figure 78: After subtracting the interference from the ‘Mid Power Data’ in Figure 63, this new ASD is plotted alongside simulated data with similar parameters. At low frequencies, near the axial oscillation frequency, there is an observed increase in noise relative to the simulation. By fitting these curves near oscillator resonance to our harmonic oscillator model, which is dominated by thermal noise, the result in the value of  $\Gamma \cdot T$  in the collected data is approximately ten times larger than in the simulated set. Beyond this region, the collected data ASD decreases until it converges with a line defined by the system’s natural damping rate and temperature. Afterward, aside from a few small spikes due to other noise sources, the data and simulation remain consistent.

This data set shows a successfully realized levitated optomechanical system that exhibits free particle behavior, particularly evident when subjected to significant thermal damping. In comparison, other data sets collected in UHV, such as those shown in Figure 74, are

subjectively less appealing than the model. This suggests that external noise sources can introduce interference into our ASD when the influence of stochastic forces is sufficiently minimized.

We believe a higher imaging rate and increased illumination beam power will be necessary to achieve a quantum-limited system functioning in the free particle limit. These adjustments should enable us to observe behavior at higher frequencies, reducing the impact of low-frequency vibrations and other forms of interference. Collecting additional vibration data from the environment via seismometers would also provide us with a means to subtract the interference caused by these external noise sources. This would be done alongside the reference camera subtraction by adding more rows to our previously mentioned interference subtraction matrix  $A$ .

As an example of the sensitivity of this setup, I played an 880 Hz audio tone from my phone speaker while holding it in my hand. This was done during a data collection run while I was approximately 3 m away from the vacuum chamber. The magnitude of the tone itself was not measured, and these results offer no quantitative analysis on the system's response to external audio sources. Still, this example highlights sensitivity to external noise sources that may be considered in future iterations of magneto-gravitational trapping systems. The results of this test are illustrated in Figure 79.

A further test was conducted to evaluate the response of the particle following charge accumulation within the chamber. At the time of this writing, the most recent particle, Bob VII, has an axial oscillation frequency of nearly 11 Hz. Earlier in this dissertation, we noted a method for the evaluation of our particle's response to a potential difference, as illustrated in Figure 15. This procedure was performed with Bob VII under UHV conditions to observe displacements in response to an established potential difference. The DC power supply was adjusted in increments of 0.1 volts while the particle's movements were monitored using the front-facing cooling camera. Figure 80 displays the position information for vertical

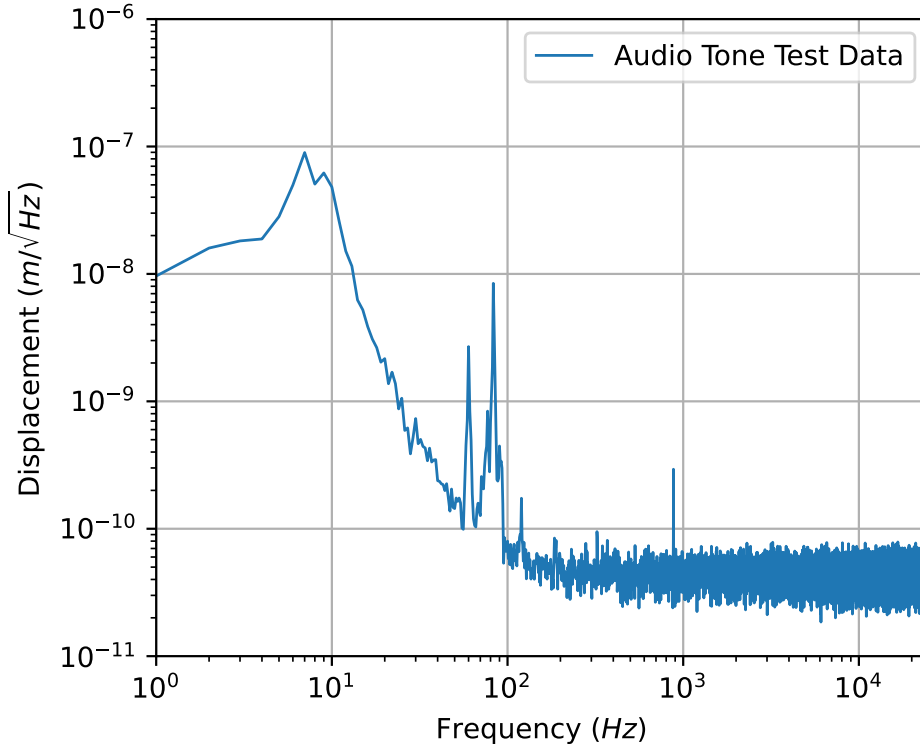


Figure 79: This data set was collected while an audio tone played from my phone speaker, positioned roughly 3m away from the vacuum chamber. The collected spectrum reveals that our system is sensitive enough to detect this audio tone, despite the chamber being mounted on a floating optical table. The spike appears precisely at 880 Hz.

displacements.

The slow transition of the particle between changing equilibrium positions is due to the low-pass filter between the pole pieces and the voltage source. In an ideal scenario, assuming the voltage difference and the distance between the pole pieces are sufficient to determine the electric field, the expected charge buildup on the particle is equivalent to only four electrons. This gradual shifting of oscillation frequencies over time suggests that charge buildup is significantly influenced by the trap and chamber itself, rather than solely by the particle.

Throughout this study, several sets of ASDs have been presented, with data collected under various vacuum conditions. There has been ongoing discussion regarding the sources

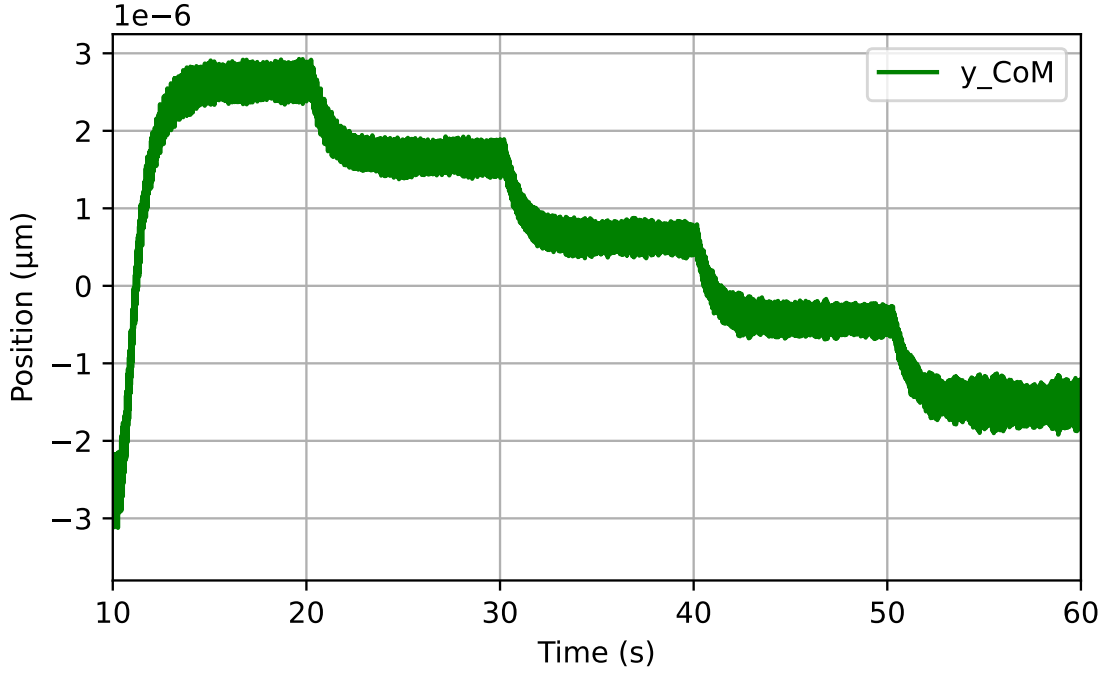


Figure 80: A trapped particle in UHV responds to changes in the supplied voltage between the pole pieces. Every 10 s, the voltage is increased in increments of 0.1 V, with an initial shift of  $-0.5$  V. The particle is displaced to a new equilibrium position within the trap with each voltage adjustment. These displacements occur in increments of approximately  $1\text{ }\mu\text{m}$ .

of particle damping in the system as chamber pressure decreases. These measurements demonstrate that even under UHV conditions, the lower limit for  $\Gamma \cdot T$  still depends on chamber pressure, although it appears to have diminishing returns. Figure 81 displays data collected at different chamber pressures, revealing a clear trend of reduced thermal noise, even when the chamber pressure falls below the detection limit of our ion pump gauge.

Given room temperature operation ( $T = 295\text{ K}$ ), only an upper bound for  $\Gamma_N$  can be determined based on an intersection between the collected data and a line produced by calculated thermal noise. For this specific trap and particle size, the 48 h force sensitivity data set shows the lowest thermal noise produced to date, with  $\Gamma_N = 2.8 \times 10^{-6}\text{ s}^{-1}$ .

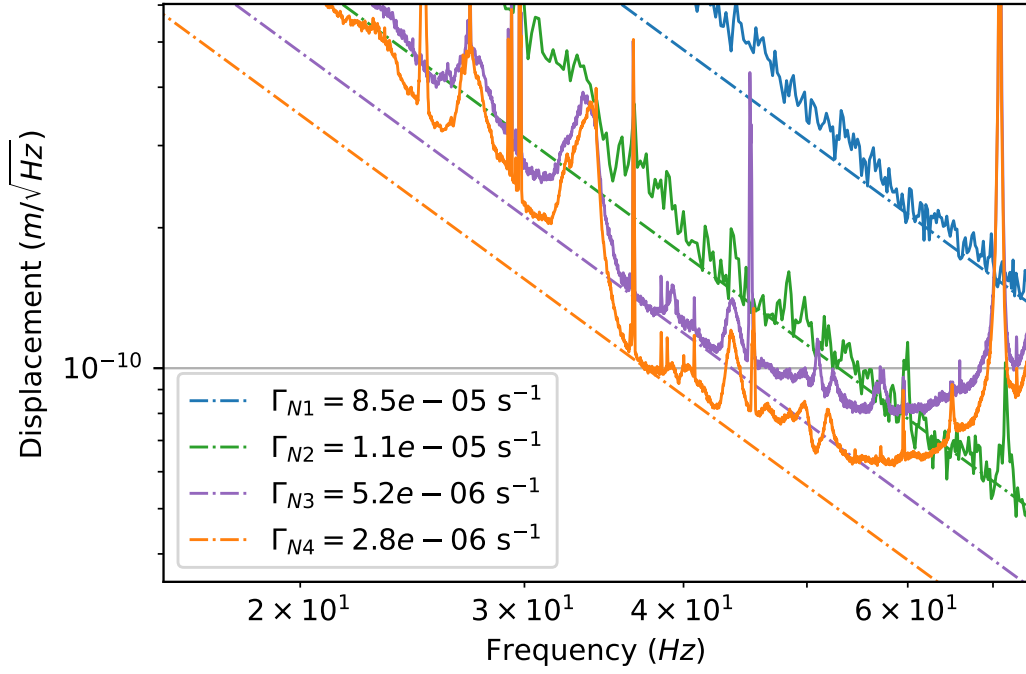


Figure 81: A comparison of different data collected at various chamber pressures.  $\Gamma_{N1}$  data was collected at  $4 \times 10^{-8}$  Torr.  $\Gamma_{N2}$  data was collected at  $3 \times 10^{-9}$  Torr.  $\Gamma_{N3}$  data was collected at  $0.3 \times 10^{-9}$  Torr.  $\Gamma_{N4}$  was collected over a period when the ion pump reading was  $0.0 \times 10^{-9}$  Torr, with no other reportable pressure; only this upper bound is known.

## CONCLUSION

This dissertation began with a discussion on SQL and the landscape of precision measurements. Drawing from LIGO, we sought to translate the free particle behavior from a system such as LIGO to a levitated optomechanical system by employing a magneto-gravitational trap. This system was outlined in detail with a mathematical framework throughout Chapter 2, and a complete overview of the experimental setup used in Chapter 3. Several advancements to this experiment were introduced, including a linescan camera for high-speed imaging, a retro-reflection system to allow for high-power illumination during a continuous measurement scheme, and a fully adjustable objective to allow for high-order aberration correction. Image analysis allowing for object tracking in the presence of optical aberrations was also produced.

The various parameter calibrations described in Chapter 4 offer wide applications. Tools such as the Bessel beam optical ruler and methods for counting scattered photons via kick calibration can be easily translated to other systems with similar limitations to our own. Much of this work has been dedicated to approaching the SQL in the free particle regime, with experimental efforts showing promising results in reducing noise and characterizing the system's performance. These results also provide much-needed insight for future improvements and routes for this project to build on, with the ultimate goal of reaching and surpassing the SQL.

Furthermore, this dissertation has demonstrated the potential of the developed system for achieving Yoctonewton force sensitivity, opening avenues for exploring new physics and pushing the boundaries of weak force detection. The challenges encountered, such as residual gas effects and the complexities of particle fitting, have been discussed, providing valuable insights for future investigations. The groundwork laid out in this dissertation offers a strong foundation for future experiments aimed at pushing the limits of quantum measurement and

force sensitivity with levitated optomechanical systems.

In conclusion, this dissertation has successfully established an experimental platform for studying levitated optomechanics in the free particle regime, which I consider to be the most significant contribution to the scientific community provided by this work. The technological advancements and the detailed characterization of the system presented here contribute to furthering our understanding of quantum measurements and hold promise for future breakthroughs in precision measurements and the exploration of fundamental physics.

### Future Work

The system outlined in this dissertation has several avenues for improvement. The most obvious of these is our current limitations imposed by thermal noise. There are several different options available to reach a fully quantum-limited system.

First, an increase in optical power is advisable. Using a MOGLabs tapered semiconductor amplifier and a Teledyne Dalsa camera, one may gain a 25 times increase in scattered light from our trapped particle when compared to our current maximum output of 20 mW. If done, this would result in an improved RPSN and a reduction in DSN by a factor of 5.

Another option currently being explored in the lab is damping reduction using insulating composite pole pieces, which would eliminate eddy currents within the system caused by particle motion. To further reduce thermal noise, the trap can be configured to operate in a cryogenic environment. Bringing the trap down to 4 K corresponds to a decrease in system temperature by a factor of  $\approx 100$ , leading to a roughly  $10\times$  reduction in thermal noise on our displacement ASDs. This system may produce improved vacuum quality, further reducing thermal noise.

Improvements in photon collection efficiency are essential to reach the SQL in the free particle limit. Current limitations to collection efficiency due to aberrations and camera pixel capabilities will need to be addressed to achieve the goal of measurements at the SQL. New

methods of cylindrical lens alignment may prove useful in improving collection efficiency, with one proposed option using a Bessel beam optical ruler to observe the orientation of each cylindrical lens used in the optical bench.

In conjunction, these improvements would bring the gap lower than ever. I estimate that a reduction in temperature to 4 K, with  $\Gamma_N \approx 1 \times 10^{-5} \text{ s}^{-1}$ ,  $\eta_D \approx 0.1$ , and an increased in  $N_\lambda$  by 25 times would bring us just 3 times above the SQL at closest approach. These improvements would put us in a quantum limited system, with our primary limitation becoming collection efficiency. At this level, it may be possible to surpass the SQL in this free particle limit using special measurement schemes (outlined in appendix D), highlighting the versatility of levitated optomechanical systems.

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## APPENDICES

APPENDIX A

BAKING PROCEDURES

Baking procedures used on the vacuum chamber in this experiment. This document has been updated numerous times. As of writing this dissertation this outline is the most up to date form.

### **Setup:**

1. The white, fibrous silica heaters work best due to their high heat capacity. The tan (ultra-high temperature) silica heaters also work well.
2. Silicone heaters are rated to 250 °C, but issues can arise around 130 °C (e.g., smoking). They must be well heat-sinked to the chamber.
3. Wrap as much of the chamber and all metal valves as possible in heater tape, including the Titanium Sublimation Pump (TSP) if attached.
4. (Optional) Attach a thermocouple to a 1.33" flange port (preferably top or side, not bottom).
5. (Optional) Connect the thermocouple and heater tape to a temperature controller.
6. Wrap bellows with separate heater tape and control with a dedicated variac.
7. Cover any coated viewports (e.g., 4") with aluminum foil according to manufacturer instructions.

### **Procedure:**

1. Load a particle and begin pumping with the rough pump.
2. Once speedivalve is fully open take additional mass calibration data (optional). After a few minutes (or when pressure on the Pirani gauge stabilizes), turn on the turbo pump.
3. Heat chamber to 130 °C using variac with heater tape.
4. Start bellows heater via variac (max 80V). Bellows should exceed 100 °C.
5. **(NEW)** Bake for at least 72 hours at 130 °C. Previous baking procedures involve much shorter baking times, but it seems we benefit from better vacuum quality by baking for longer.
6. Lower variac to 60V and monitor chamber temperatures. The chamber temp should settle close to 87 °C (Sidney Crosby superstition).
7. Wait 1 hour at this temp to protect all-metal valves.
8. Make note of the pressure at this point. If things are going well, it should be in the  $10^{-8}$  Torr range.

9. **(NEW)** (Well.. Kind of new. This procedure already existed, but for a while went unused.) Degas the TSP before sputtering:
  - (a) Use CURRENT-LIMITED mode: voltage set to max ( 5.XV), increase current slowly (max 10A per 30s step) up to 35A. Alternatively, you can do the 5A steps, which I opted for out of caution.
  - (b) Watch for pressure rise then fall. Pump down for a bit.
  - (c) Reverse ramp-down. Let the TSP nipple cool to chamber temperature.
10. Turn off variacs.
11. Carefully open the ion pump all-metal valve.
12. Close turbo pump viton valve, compare cold-cathode gauge with ion pump gauge, should be in the high  $10^{-9}$  Torr range.
13. Close bellows all-metal valve carefully.
14. Turn off turbo. Manually vent when turbo slows to 415 Hz. (As of this writing the cable throws an error that prevents this readout, instead just wait a few minutes before SLOWLY venting).
15. Disconnect bellows from chamber side while chamber is still hot. (This step is optional. The heat provides additional damping, reducing risk of particle loss. I find that it works just as well to keep it attached until active cooling can be turned on, at which point the bellows can be removed.)
16. Monitor pressure as the chamber cools. Wait at least 24 hours before any data collection.
17. Once near room temp, fire the TSP.

#### Outgassing the TSP

1. Use CURRENT-LIMITED mode (voltage 5.XV).
2. Ramp current to 35A in 10A/30s steps (or 5A/30s steps). Observe pressure behavior.
3. Reverse ramp-down. Let TSP nipple cool to match chamber.

#### Firing the TSP

1. Always use CURRENT-LIMITED mode.
2. Make note of current pressure before turning on.
3. Ramp to 35A slowly (same ramp strategy).
4. Pressure may rise to  $10^{-7}$  Torr, then fall.

5. For new filaments, ramp up to 42–45A to sublime Ti.
6. Dwell for 30–45s.
7. Reverse the ramp-down.
8. Expect pressure to drop post-sublimation. Should improve from baseline reading taken before firing TSP.
9. Target post-TSP pressure is typically on the order of  $\approx 10^{-10}$  Torr.

#### PEEK Tightening

1. **(NEW)**: I never had to bother with this section, but I will leave it in case it is useful for someone else.
2. With regular PEEK gaskets:
  - (a) Re-tighten during cooldown at 110 °C and 80 °C using torque screwdriver.
3. With PEEK HT gaskets:
  - (a) Only initial post-bake tightening needed.
  - (b) Torque setting: 15–16 ft-in (gross units. I know...).
  - (c) These gaskets are robust and leak-free.

#### Ion Pump Baking

1. Indicators of needed bake-out:
  - (a) Non-zero pressure with closed ion pump valve.
  - (b) Pressure spikes rapidly and frequently.
2. Turn off ion pump and expose to chamber.
3. Use the rough and turbo pumps to bring the chamber to rough vacuum. With no particle trapped, this should be easy to accomplish.
4. Bake chamber and bellows to 100 °C while pumping.
5. Remove ion pump magnets. Wrap the pump in silica heater tape.
6. Bake pump to 200 °C using variac for at least 6-8 hours.
7. Reinstall the magnet and turn the pump on once temperatures drop to safe levels for the magnets. Duniway documentation claims their Ion pumps can be baked up to 150 °C with the magnets on. Out of caution I wait for temperatures to drop to at least 100 °C before putting the magnets back on to avoid any unwanted reduction in magnetic field strength due to excessive heat.

8. Pressure may rise briefly to  $10^{-5}$  Torr but should stabilize quickly.
9. Close the pump valve and let the pressure settle (possibly overnight). Once fully cool, the ion gauge should read  $0.0 \times 10^{-9}$  Torr.

APPENDIX B

LOADING PROCEDURE AND SETUP

Detailed description of the particle loading procedure and associated experimental setup. This document reflects the most up-to-date practices as of writing. **(NEW)**

**Horn Setup:**

1. Connect red wire to red wire on terminal block.
2. Connect black wire to white wire on terminal block.
3. Connect a third wire to the terminal block position with the yellow wire (attached to horn handle body).
4. Ground the horn to the handle using a regular (non-nylon tipped) set screw.
5. Apply 30 –40 V to HV+ and HV- pins; current draw is 1.5 A briefly.
6. Use Acopian supply with all knobs at lowest settings before power on.
7. Increase current first, then voltage slowly; nominal current should be zero at target voltage.

**Particle Preparation:**

1. Line a small plastic container with aluminum foil to reduce static. Using new aluminum foil also prevents unwanted items from being trapped, such as dust or other-sized particles.
2. Pour a very small amount of 1.18  $\mu\text{m}$  spheres into the container.
3. Remake the container when it runs out to prevent dust/moisture accumulation.

**Chamber and Vacuum Preparation:**

1. Ensure all pole pieces are at a constant voltage by setting DC power supply to 0 V
2. Bottom poles are grounded, top poles left floating. Setting on DC power supply should be ‘float’.
3. Equilibrate all vacuum lines to room pressure before loading. Once done be sure to close the speedivalve, since turning on the rough pump later will cause particle loss if you forget this step.
4. Close bellows all-metal valve before particle loading to prevent contamination of the copper gasket.

**Optics Preparation:**

1. Remove 150 mm optic going into chamber and replace with LED to illuminate the trap.
2. Run LED in current-limited mode; power supply must be off when connecting.
3. This change in illumination allows viewing multiple particles within the trap.

Loading Procedure

1. Display trap region on camera with large FOV by replacing high NA objective with 150 mm lens. At this stage you should be observing the trap from the front cooling camera with an image similar to that shown in Figure 16.
2. Set 0 V for pole pieces potential difference via DC power supply.
3. Remove viewport and side camera shown in Figure 17 for particle loading. At this stage DO NOT remove the nuts closest to the vacuum chamber. They are very difficult to put back on. Instead just remove the assembly facing the side camera and pull off the viewport window.
4. Load particles onto the horn by gently pressing into the aluminum-lined container.
5. Insert horn into the viewport. Practice this dry before real attempts. It helps to temporarily remove the 150 mm optic during this step so you can view the trap through the chamber window. This allows you to see where the horn is positioned. Additionally having another member observe from the opposite side of the chamber helps.
6. Press Arduino button to attempt loading. This is also best done with assistance so whoever is holding the horn can keep their hands steady.
7. Place the 150 mm optic back in the optical path and focus. If trapping was successful there should now be several particles visible from the front cooling camera.
8. Use a mix of tapping on bellows and chamber, plus adjusting the DC power supply between 10 V and  $-10$  V to remove excess particles until only one remains (visually, pre mass calibration).

**Neutralization:**

1. Use DC voltage to adjust a single particle within the trap. Alternating voltages can ‘walk’ the particle towards the trap’s center.
2. Insert radiation source close to the particle and monitor motion. BE VERY CAREFUL. Do not do this without first removing the 150 mm optic. This allows you to get the radiation source close without knocking it into the trap and potentially dropping it.

3. I find it easiest to get the radiation source (and rod its taped on) to be level with the viewport first, and then loosen the key holding the rod in place to allow the rod to translate into the chamber while remaining level with the trap.
4. Remove source once the particle shows a minimal response to the potential difference in the pole pieces.
5. Set DC voltage supply to 0 V to ‘short’ the pole pieces together.

### **Verifying Single Particle:**

1. Remove LED and put the ‘focusing lens’ 150 mm optic back once a single particle is discharged. Turn on the illumination beam and observe the particle on the front camera by putting the high NA objective back on the bench.
2. Multiple particles stuck together should produce scattering asymmetries that are obvious when viewed with the illumination beam and high NA objective. If scattering appears symmetric, proceed with mass calibration.
3. Take images of the particle on the front camera while at ATM pressure. Vertical motion should follow the Boltzmann distribution with mass as a fit parameter, confirming single particle trapping. I like to use the full resolution allowed by the camera while operating at 30 Hz. This has worked best for ATM conditions.

### **Final Steps:**

1. Open closed valves (NOT the speedivalve).
2. Seal the viewport window and put the side camera back on. Make sure you can view the particle from the side camera in Pylon viewer using only the illumination beam. I recommend adjusting the beam to hit slightly above the current particle position, and operate with excessive saturation (so long as the frame rate is still above 30 Hz and you can see the particle).
3. Begin pumping down and SLOWLY open speedivalve. There is sometimes a dead spot following the first turn, so be careful and take your time. When pressure suddenly jumps, keep an eye on the particle to ensure it doesn’t get pulled out of the trap. You want this jump to be as tiny as you can muster. This is perhaps the hardest part of the whole process.
4. Monitor pressure gauge connected near the rough pump; adjust valve for 2 Torr per update.
5. Once in the single Torr range, open speedivalve fully.
6. Rough pump to 100 mTorr, then turn on turbo pump.
7. When turbo pump is fully spun up proceed with baking procedure.

APPENDIX C

BEAM ALIGNMENT

### A Word of Caution

1. This document corresponds to the optical table design as of 04-27-2025.
2. The setup changes frequently — verify all information before relying on it.

### 5 Control Beams

#### Setup

1. High voltage amplifiers are powered by Acopian supplies labeled +/-150V FOR EOM AMPS.
2. Synthesizers drive passive RLC circuits for phase modulation — verify outputs and frequencies.
3. Adjust relative beam intensities using ND filters.

#### Alignment

1. Assumes a well-centered trap: 4" off table, symmetric between optical table holes.
2. Ensure beams remain balanced after EOMs when no voltage is applied.
3. Beams should hit focusing lens in expected positions — vertical beams top/bottom, axial left/right, as shown in Figure 27.
4. Use a movable alignment target (t-shirt) to verify beam paths stay consistent along propagation. This prevents beam clipping from misalignment during propagation.
5. Position large t-shirt in front of final lens going into chamber to observe final beam location for all 5 cooling beam paths. Adjust optics to make paths resemble the 'focusing lens' from Figure 27.
6. Once trap contains a particle, verify each beam hits the particle using the side feedback camera. This should be done one at a time for each beam, blocking the other 4 as each is checked.

### Illumination Beam

1. Measure beam output following cylindrical lens setup from Figure 23. Adjust wave plates and paddles to maximize output before proceeding.
2. Adjust mirror just before rotation ND filter stage along with the mirror following this stage to get beam path going into the center of the final focusing lens.
3. Checking beam alignment for this beam requires a IR viewing card.

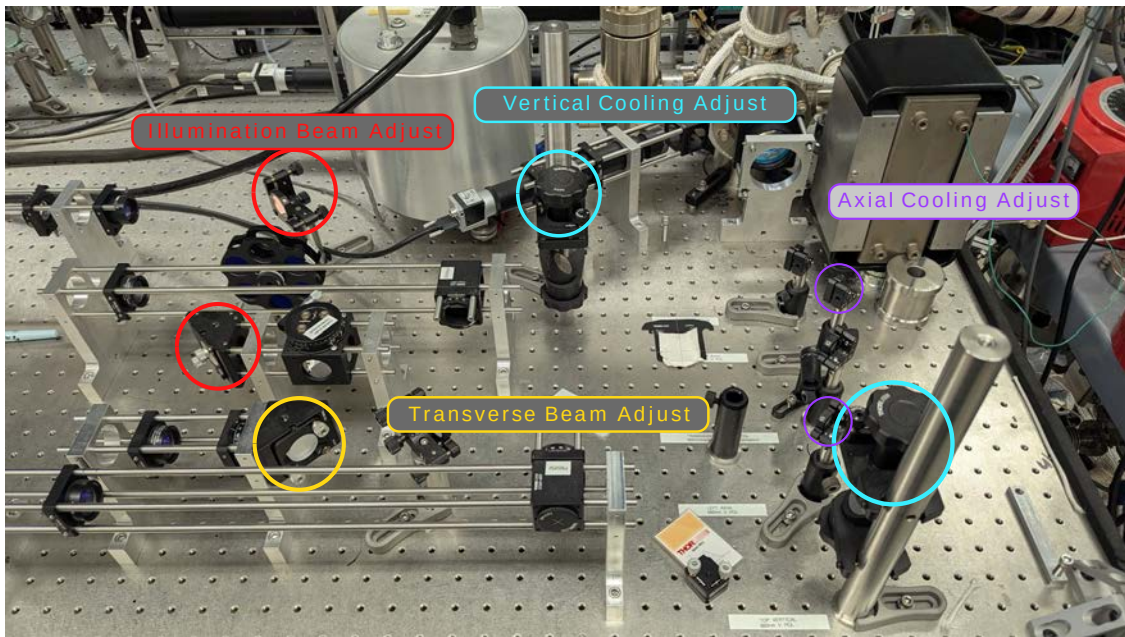


Figure 82: A diagram of which optics should be adjusted when attempting beam alignment.

For additional clarity here is an image of the optical bench with the adjustment points labeled. When altering beam paths for cooling (as of the writing of this document) the following optics are used, shown in Figure 82.

APPENDIX D

SUB-SQL MEASUREMENT SCHEME

A member of my committee, Hang Yu, drafted a document outlining a proposed measurement scheme to achieve measurements below the SQL. With Hang's permission, we have decided to include this write-up in the appendix as it highlights a future goal for our project once we have established a quantum-limited system.

### Quantum limitation

We study the quantum limitation by studying the quantized electric field, which can be written as [39]

$$\begin{aligned} \hat{E}(x, y, z, t) &= u(x, y, z) \\ &\times \int_0^{+\infty} \frac{d\omega}{2\pi} \sqrt{\frac{\hbar\omega}{2}} [\hat{a}_\omega e^{ikx-i\omega t} + \hat{a}_\omega^\dagger e^{-ikx+i\omega t}], \end{aligned} \quad (81)$$

where  $x$  is the direction of light propagation,  $\hat{a}_\omega^\dagger$  and  $\hat{a}_\omega$  are creation and annihilation operators satisfying the commutation relation

$$[\hat{a}_\omega, \hat{a}_{\omega'}] = [\hat{a}_\omega^\dagger, \hat{a}_{\omega'}^\dagger] = 0, \quad [\hat{a}_\omega, \hat{a}_{\omega'}^\dagger] = 2\pi\delta(\omega - \omega'). \quad (82)$$

The function  $u(x, y, z)$  describes the spatial mode and we normalize it such that  $\int |u(x, y, z)|^2 dydz = 1$ . Note that we have chosen the normalization such that the intensity is

$$\hat{I}(x, t) = c \int dydz \hat{E}^2. \quad (83)$$

We first ignore the  $u(x, y, z)$  part and the propagation phase  $e^{ikx}$  and focus on the temporal variation of the field. When the operator in Eq. (81) acts on a coherent state  $|\alpha_{\omega_0}\rangle = \hat{D}|0\rangle$  produced by a laser with a central frequency  $\omega_0$ , we can use a Heisenberg picture to transfer the operator  $\hat{E}$  while keeping the vacuum state  $|0\rangle$ . In this case, we have (dropping the spatial parts for now)

$$\begin{aligned} \hat{E}_{\text{coh}}(t) &= \hat{D}^\dagger \hat{E} \hat{D} \\ &= A_c \cos(\omega_0 t + \phi) + \sqrt{\hbar\omega_0} [\hat{a}_a \cos(\omega_0 t) + \hat{a}_p \sin(\omega_0 t)], \\ &\equiv E_c(t) + \hat{E}_q(t), \end{aligned} \quad (84)$$

where  $A_c$  is the amplitude of the classical light field set by the intensity via Eq. (83), and  $\hat{a}_a$  and  $\hat{a}_p$  are the amplitude and phase quadrature fields produced by quantum fluctuation defined as

$$\hat{a}_i = \int_0^\Lambda \frac{d\omega}{2\pi} [\hat{a}_{i,\omega} e^{-i\omega t} + \hat{a}_{i,\omega}^\dagger e^{i\omega t}], \quad i = a, p, \quad (85)$$

and

$$\hat{a}_{a,\omega} = \frac{a_{\omega_0+\omega} + a_{\omega_0-\omega}^\dagger}{\sqrt{2}}, \hat{a}_{p,\omega} = \frac{a_{\omega_0+\omega} - a_{\omega_0-\omega}^\dagger}{\sqrt{2}i}. \quad (86)$$

The one-sided PSDs of  $a_{a,\omega}$  and  $a_{p,\omega}$  are

$$S_{a_{i,\omega}, a_{j,\omega'}} = \left\langle 0 | \hat{a}_{i,\omega} \hat{a}_{j,\omega'}^\dagger + \hat{a}_{j,\omega'}^\dagger \hat{a}_{i,\omega} | 0 \right\rangle = \delta_{ij} \delta_{\omega,\omega'}, \quad (87)$$

for  $i, j = a, p$ . In the equations above, we have factored out the fast oscillations of the light field  $\omega_0 \sim 10^{14}$  Hz and focused on audio sidebands around it with  $\omega 10^3$  Hz. Hereafter, when we talk about  $\hat{E}$  we will refer to Eq. (84) and drop the “coh” subscript.

We can now write the light intensity

$$\begin{aligned} \frac{1}{c} \frac{d\hat{I}(t)}{dydz} &= \hat{E}^2(t) = \left[ E_c^2(t) + 2E_c(t) \hat{E}_q(t) \right] \\ &= \frac{A_c^2}{2} + A_c \sqrt{\hbar\omega_0} [\hat{a}_a \cos \phi - \hat{a}_p \sin \phi], \\ &= \frac{1}{c} \frac{d}{dydz} \left[ I_0 + \hat{I}_q(t) \right]. \end{aligned} \quad (88)$$

We see that  $E_c^2$  produces the DC intensity  $I_0$ , while  $2E_c \hat{E}_q$  produces a fluctuating  $\hat{I}_q(t)$  that is the source of quantum noise. It is also convenient to consider the Fourier transform of  $\hat{I}(t)$ ,

$$\hat{I}_q(\omega) = \sqrt{2I_0 \hbar \omega_0} (\hat{a}_{a,\omega} \cos \phi - \hat{a}_{p,\omega} \sin \phi). \quad (89)$$

The one-sided PSD of  $\hat{I}(\omega)$  is

$$\begin{aligned} S_{I_q I_q}(\omega) &= \left\langle 0 | \hat{I}_q(\omega) \hat{I}_q^\dagger(\omega) + \hat{I}_q^\dagger(\omega) \hat{I}_q(\omega) | 0 \right\rangle \\ &= 2I_0 \hbar \omega_0, \end{aligned} \quad (90)$$

where we have used Eq. (87).

As suggested by Eq. (84), an electric field can be viewed as the superposition of a classical field  $E_c$  and a quantum one  $\hat{E}_q$ . Furthermore, the quantum operators propagate in the way classical fields do. Therefore, we can effectively model the quantum fluctuations as classical fields and drop the operator (i.e., hat) notation hereafter as long as we set the PSD of intensity fluctuations produced by  $2E_c(t)E_q(t)$  to Eq. (90).

For future convenience, we promote  $E_q(t)$  to be a complex number as

$$E_q(t) = E_q^{(\text{amp})}(t) + iE_q^{(\text{ph})}(t), \quad (91)$$

to represent the amplitude (in-phase with the carrier) and the phase (90° out of phase with the carrier) quadrature. Without squeezing, they are uncorrelated with

$$\text{PSD} [2E_c E_q^{(\text{amp})}] = \text{PSD} [2iE_c E_q^{(\text{ph})}] = 2I_0 \hbar \omega_0. \quad (92)$$

The factor of  $i$  can appear from the spatial overlap as described below.

We describe the spatial dependence  $u(x, y, z)$  by decomposing it into Hermite-Gaussian modes [83],

$$u(x, y, z) = \sum_{mn} u_m(x, y) u_n(x, z) \quad (93)$$

with

$$u_m(x, y) = \left(\frac{2}{\pi}\right)^{1/4} \left(\frac{1}{2^m m! w(x)}\right)^{1/2} H_m\left(\frac{\sqrt{2}y}{w(x)}\right) \\ \times \exp\left[-y^2\left(\frac{1}{w^2(x)} + \frac{ik}{2R(x)}\right) + i(m + 1/2)\psi_g(x)\right], \quad (94)$$

where  $H_m$  is the Hermite polynomial, and  $w$ ,  $R$ , and  $\psi_g$  are respectively the spot size, radius of curvature, and gouy phase. They satisfy

$$\int u_m^*(x, y) u_n(x, y) dy = \delta_{mn} \quad (95)$$

so the light fields are decomposed as

$$E(x, y, z, t) = \sum_{mn} u_m(x, y) u_n(x, z) E_{mn}(x, t). \quad (96)$$

For future convenience, we will make the change

$$(u_m, E_{mn}) \rightarrow (u_m e^{-im\psi_g(x)}, E_{m0} e^{+im\psi_g(x)}). \quad (97)$$

In other words, we let  $E_{mn}$  absorb the gouy phase so that  $E_{mn}$  carries all the phase shifts due to beam propagation. The spatial part  $u$  carries only the beam size and radius of curvature. Other integrals will be useful include

$$\int y u_0^*(x, y) u_1(x, y) dy = \frac{w_y(x)}{2}. \quad (98)$$

### Observable and optical gain

We use unprimed fields to indicate the incoming light, and primed ones to indicate the outgoing fields scattered off the particle. For this work, we focus mainly on particles moving in the  $y$ - $z$  plane (orthogonal to the incoming light). We further simplify the analysis that the particle moves only in the  $y$  direction as the analysis along  $z$  is essentially identical. The input-output relation of the TEM modes' amplitudes at the particle's location  $x = 0$  is thus

given by [84]

$$\begin{bmatrix} E'_{00} \\ E'_{10} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{y_0+y_q}{w_y} \\ \frac{y_0+y_q}{w_y} & 1 \end{bmatrix} \begin{bmatrix} E_{00,c} \\ 0 \end{bmatrix} + \begin{bmatrix} E_{00,q} \\ E_{10,q} \end{bmatrix}, \quad (99)$$

where  $y_0$  is the classical displacement of the particle (also the shift of the centroid location of the scattered beam) that we want to measure,  $y_q$  is displacement caused by quantum radiation pressure noise, and  $w_y$  is the spot size. All quantities are evaluated at  $x = 0$  which corresponds to the nominal location of the particle. Note that the classical field is assumed in only the 00 mode; in fact, it is used to define the basis of our modal decomposition. Quantum noise, on the other hand, exists in both 00 and 10 modes. Note that within each mode, the quantum field contains two uncorrelated quadratures.

We first consider the case without any quantum fluctuations (i.e., dropping terms with the “q” subscript). This is used for us to define the observable that encodes  $y_0$ . For this, we consider intensity-weighted spot position on the detector plane at  $x_d$ ,

$$\begin{aligned} O &\equiv c \int dy y \\ &\times [u_0 E'_{00,c}(t) + u_1 E'_{10,c}(t)]^* [u_0 E'_{00,c}(t) + u_1 E'_{10,c}(t)] \\ &= c [E'_{00,c}(x_d, t) E'_{10,c}(x_d, t)] \int dy y (u_1^* u_0 + u_1 u_0^*) \\ &= y_0 \frac{w_y(x_d)}{w_y(0)} \eta_D I_0 \cos \psi_g, \end{aligned} \quad (100)$$

where in the second line we have used Eq. (98) to select out non-vanishing terms. Because of finite collecting area and finite quantum efficiency, not all the input intensity is collected, and we have let  $E_{00,c}^2(t) \propto \eta_D I_0$  instead of  $I_0$ , with  $\eta_D$  the detection efficiency of the experiment ( $\eta_D = 1$  in the ideal case). Because the  $E'_{10,c}$  field acquires an additional Gouy phase relative to the  $E'_{00,c}$  field, which in an interaction picture where the carrier’s phase fixed at 0 can be described by

$$\begin{pmatrix} E_c^{(\text{amp})} \\ E_c^{(\text{ph})} \end{pmatrix}_{x_d} = \begin{pmatrix} \cos \psi_g & -\sin \psi_g \\ \sin \psi_g & \cos \psi_g \end{pmatrix} \begin{pmatrix} E_c^{(\text{amp})} \\ E_c^{(\text{ph})} \end{pmatrix}_0 \quad (101)$$

Consequently,

$$\begin{aligned} E'_{00,c}(x_d, t) E'_{10,c}(x_d, t) &= E'_{00,c}(x_d, t) E'_{10,c}^{(\text{amp})}(x_d, t) \\ &= E'_{00,c}(0, t) E'_{10,c}^{(\text{amp})}(0, t) \cos \psi_g. \end{aligned} \quad (102)$$

This leads to the extra  $\cos \psi_g$  factor in Eq. (100). We define

$$G \equiv \frac{dO}{dy_0} = \eta_D I_0 \frac{w_y(x_d)}{w_y(0)} \cos \psi_g, \quad (103)$$

which is the optical gain converting a physical displacement to a signal in the observable.

Now adding in quantum noise. It is evident that two effects contribute to the noise. The first is due to quantum fields  $[E_{00,q}, E_{10,q}]$  reaching the detector, which cause a sensing shot noise. The second is due to the term  $y_q$ , which is quantum radiation pressure noise causing a physical displacement of the particle.

#### Sensing shot noise

We first consider the quantum sensing noise, where the field at the detector can be written as

$$\begin{aligned} E'(x_d, y, z, t) &= u_0(x_d, z) \\ &\{u_0(x_d, y) [E_{00,c}(x_d, t) + E_{00,q}(x_d, t)] \\ &+ u_1(x_d, y) E_{10,q}(x_d, t)\}. \end{aligned} \quad (104)$$

The signal created by such a field is (dropping the  $z$  integral)

$$\begin{aligned} O_{\text{shot}}(t) &= 2cE_{00,c}(x_d, t)E_{10,q}(x_d, t) \\ &\times \frac{1}{2} \int dy y [u_0^* u_1 + u_0 u_1^*] \\ &= \sqrt{\eta_D} I_q(x_d, t) \frac{w_y(x_d)}{2}, \end{aligned} \quad (105)$$

with

$$\begin{aligned} I_q(x_d, t) &= 2cE_{00,c}(x_d, t)E_{10,q}^{(\text{amp})}(x_d, t), \\ &= 2cE_{00,c}(0, t) \left[ E_{10,q}^{(\text{amp})}(0, t) \cos \psi_g - E_{10,q}^{(\text{ph})}(0, t) \sin \psi_g \right]. \end{aligned} \quad (106)$$

Note the intensity fluctuation is produced by the local amplitude quadrature and the PSD is given by Eq. (90) except for we replace  $I_0$  by  $\eta_D I_0$  to account for imperfect detection, which leads to

$$S_{OO}^{(\text{shot})} = \frac{\eta_D I_0 \hbar \omega_0 w_y^2(x_d)}{2}. \quad (107)$$

The equivalent displacement noise of the particle is then

$$\Delta y^{(\text{shot})} = \frac{O_{\text{shot}}}{G} = \frac{I_q(x_d)}{\sqrt{\eta_D} I_0} \frac{w_y(0)}{2 \cos \psi_g}. \quad (108)$$

and its PSD is

$$\begin{aligned} S_{yy}^{(\text{shot})} &= \frac{S_{OO}^{(\text{shot})}}{|G|^2} \\ &= \frac{\hbar \omega_0 w_y^2(0)}{2\eta_D I_0 \cos^2 \psi_g} \geq \frac{\hbar \omega_0 w_{y0}^2}{2\eta_D I_0}, \end{aligned} \quad (109)$$

where in the last equality, we have replaced the spot size at the particle's location  $w_y(0)$  with the waist size of the beam  $w_{y0}$  and used the fact that  $w_y(0) \geq w_{y0}$ .

### Radiation pressure noise

To compute the quantum radiation pressure noise, we need to compute the outgoing light intensity in the  $y$  direction at the far field,

$$\begin{aligned} I_y(+\infty, t) &= c \int dy dz (\hat{\mathbf{k}} \cdot \hat{\mathbf{y}}) E'^2(t), \\ &= 2c \left[ E_{00,c} E_{10,q}^{(\text{amp})} \right] (+\infty, t) \\ &\times \int (\hat{\mathbf{k}} \cdot \hat{\mathbf{y}}) u_0(+\infty) u_1(+\infty) dy, \end{aligned} \quad (110)$$

where in the second line we have dropped the  $z$  integral and kept only non-vanishing terms. The above integral is most conveniently evaluated at the far field  $x = +\infty$ . We have  $(\hat{\mathbf{k}} \cdot \hat{\mathbf{y}}) \simeq y/x$ . Together with Eq. (98), we have

$$I_y(t) \simeq I_q(+\infty, t) \frac{\beta_y}{2} = \frac{M^2 \lambda I_q(+\infty, t)}{2\pi w_{y0}} = \frac{c M^2 I_q(+\infty, t)}{\omega_0 w_{y0}}, \quad (111)$$

where  $\beta_y = M^2 \lambda / (\pi w_{y0})$  is the divergence angle and  $M^2 \geq 1$  the beam quality factor. The intensity is

$$\begin{aligned} I_q(+\infty, t) &= 2c E_{00,c}(+\infty, t) E_{10,q}^{(\text{amp})}(+\infty, t) \\ &= -2c E_{00,c}(0, t) E_{10,q}^{(\text{ph})}(0, t), \end{aligned} \quad (112)$$

Modeling the particle as a free oscillator,

$$-m\omega^2 y_q = -\frac{I_y}{c}, \quad (113)$$

we have

$$y_q = \frac{I_q(0)}{m\omega^2} \frac{M^2}{\omega_0 w_{y0}}. \quad (114)$$

The corresponding PSD is

$$S_{yy}^{(\text{RP})} = \frac{2\hbar I_0 M^4}{m^2 \omega^4 \omega_0 w_{y0}^2}. \quad (115)$$

### Standard quantum limit

The total quantum noise is the sum of the sensing shot noise and radiation pressure noise. Consider first an ideal case where  $\eta_D = M^2 = 1$  and  $w_y(0) = w_{y0}$ . Further assume

that the two noises are uncorrelated. In this case we have

$$\begin{aligned}\bar{S}_{yy}^{(\text{Quan})} &\equiv S_{yy}^{(\text{shot})} + S_{(\text{RP})} \\ &= \frac{S_{yy}^{(\text{SQL})}}{2} \left( \kappa_y + \frac{1}{\kappa_y} \right)\end{aligned}\quad (116)$$

with

$$S_{yy}^{(\text{SQL})} = \frac{2\hbar}{m\omega^2}, \quad (117)$$

the standard quantum limit of free mass, and

$$\kappa_y = \frac{2I_0}{m\omega^2\omega_0 w_{y0}^2} = \frac{I_0}{cm\omega^2 w_{y0}} \frac{\lambda}{\pi w_{y0}}. \quad (118)$$

In reality, one needs to account for the fact that the shot noise and radiation pressure noise are correlated because they are caused by the same fields. We have

$$\langle I_q(x_d, t) I_q(0, t) \rangle = \langle I_q(0, t)^2 \rangle \sin \psi_g. \quad (119)$$

The actual PSD should be computed by summing first the amplitudes, or  $\Delta y$  from Eq. (108) and  $y_q$  from Eq. (114) first and then take the square. With the help of Eq. (119), we have

$$\begin{aligned}S_{yy}^{(\text{Quan})} &= \frac{S_{yy}^{(\text{SQL})}}{2} \\ &\times \left( M^4 \kappa_y + \frac{1}{\eta_D \cos^2 \psi_g} \frac{w_y^2(0)}{w_{y0}^2} \frac{1}{\kappa_y} + 2 \frac{M^2}{\sqrt{\eta_D}} \frac{w_y(0)}{w_{y0}} \tan \psi_g \right), \\ &= \frac{S_{yy}^{(\text{SQL})}}{2} (A \tan^2 \psi_g + B \tan \psi_g + C),\end{aligned}\quad (120)$$

where in the second equality we have used the trig identity  $1/\cos^2 \psi_g = 1 + \tan^2 \psi_g$  so that the noise PSD over the SQL forms a quadratic function of  $\tan \psi_g$ . The coefficients are

$$\begin{aligned}A &= \frac{w_y^2(0)}{w_{y0}^2} \frac{1}{\eta_D \kappa_y} > 0, \\ B &= 2 \frac{M^2}{\sqrt{\eta_D}} \frac{w_y(0)}{w_{y0}}, \\ C &= \frac{w_y^2(0)}{w_{y0}^2} \frac{1}{\eta_D \kappa_y} + M^4 \kappa_y.\end{aligned}$$

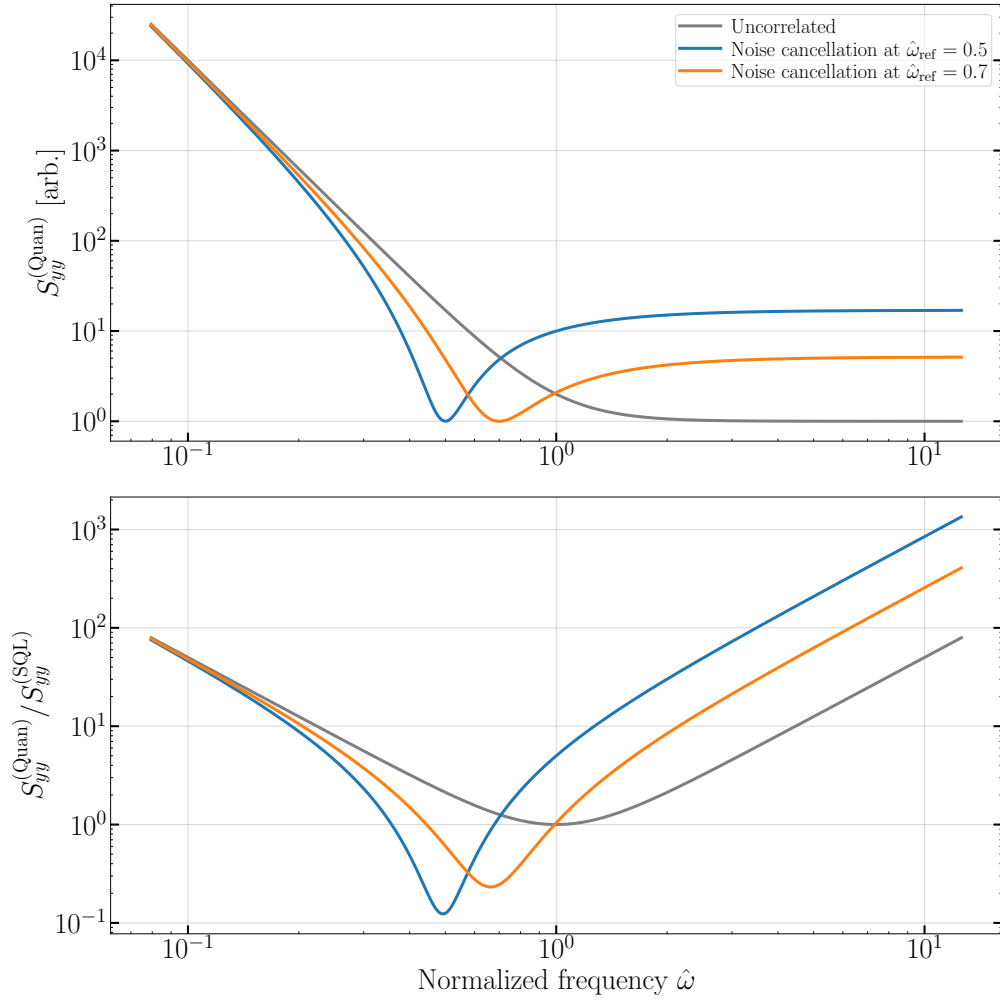


Figure 83: One-sided displacement noise PSD due to quantum fluctuations. The top panel shows normalized  $S_{yy}^{(\text{Quan})}$  and the bottom the PSD relative to the SQL. The gray line is for the case ignoring correlation [ $\tan \psi_g = 0$ ; Eq. (116)]. The blue and orange curves show the PSD when the radiation pressure is locally eliminated at  $\omega_{\text{ref}}$  with  $\kappa_{y,\text{ref}} = \kappa_y(\omega_{\text{ref}})$  [Eq. (122)].

The minimum of  $S_{yy}^{(\text{Quan})}/S_{yy}^{(\text{SQL})}$  is achieved when

$$\tan \psi_{g,\min} = -\frac{B}{2A} = -\frac{M^2}{\sqrt{\eta_D}} \frac{w_{y0}}{w_y(0)} \eta_D \kappa_{y,\text{ref}}. \quad (121)$$

Note we can only arrange  $\psi_g$  to minimize the noise at a specific frequency  $\omega_{\text{ref}}$  with  $\kappa_{y,\text{ref}} = \kappa_y(\omega_{\text{ref}})$ . The minimized PSD relative to the SQL is

$$\frac{S_{yy,\min}^{(\text{Quan})}}{S_{yy}^{(\text{SQL})}} = \frac{1}{2} \left[ M^4 (\kappa_y - 2\kappa_{y,\text{ref}} + \frac{\kappa_{y,\text{ref}}^2}{\kappa_y}) + \frac{w_y^2(0)}{w_{y0}^2} \frac{1}{\eta_D \kappa_y} \right]. \quad (122)$$

In other words, when  $\psi_g$  is arranged according to Eq. (121), the radiation pressure noise is exactly canceled at  $\omega_{\text{ref}}$  when  $\kappa_y = \kappa_{y,\text{ref}}$ , while leaving the sensing shot noise intact.

Fig. 83 shows the quantum noise PSDs in the ideal situation with  $M^2 = \eta_D = 1$  and  $w_y(0) = w_{y0}$ . The gray curve shows the case where the radiation pressure and the sensing shot noise are treated as uncorrelated [Eq. (116)]. The blue and orange curves utilize the correlation to eliminate the radiation pressure noise locally at two different values of  $\omega_{\text{ref}}$ . We have normalized the frequency such that  $\hat{\omega} = 1$  when  $\kappa_y = 1$ .

### Heisenberg Uncertainty

As a sanity check, we show that the radiation pressure noise [Eq. (115)] and the sensing shot noise [Eq. (109)] respects the Heisenberg uncertainty.

We will focus in the  $y$  direction. The momentum uncertainty caused by the radiation pressure noise is

$$\begin{aligned} \langle \Delta p_y^2 \rangle &= \int_{1/T}^{1/(2\Delta t)} m^2 \omega^2 S_{yy}^{(\text{RP})} df \\ &\simeq \frac{\hbar \omega_0 \beta_y^2 I_i T}{8\pi^2 c^2}, \end{aligned} \quad (123)$$

where  $T$  is the measurement duration,  $\delta t$  is the sampling rate, and  $f = \omega/2\pi$ .

The displacement uncertainty due to the sensing shot noise is

$$\begin{aligned} \langle \Delta y^2 \rangle &\simeq \frac{S_{yy}^{(\text{shot})}}{2\Delta t} \\ &\simeq \frac{\hbar \omega_0 w_y^2(0)}{4\eta_D I_0 \Delta t}. \end{aligned} \quad (124)$$

The product of the two thus satisfies

$$\langle \Delta p_y^2 \Delta y^2 \rangle \simeq \frac{\hbar^2}{4} \frac{1}{\eta_D} \left( \frac{w_y(0) \beta_y}{2c/\omega_0} \right)^2 \frac{T}{2\Delta t} \geq \frac{\hbar^2}{4}. \quad (125)$$

