A study of arterial blood noises (cervical bruits)

by Joel Morris Bowers

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Aerospace and Mechanical Engineering
Montana State University

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Significant difference between the innocent and stenotic bruit was evident from an examination of the zero crossing frequency of the signal. The bandwidth, mean frequency, and number of peaks in the energy spectrum of the signal also showed significant difference between the innocent and stenotic bruit.

The average stenotic bruit studied was found to have 90% of its energy contained in a frequency band width of 188 Hz. with a center frequency of 131 Hz. The frequency band containing 90% of the energy of the average innocent bruit was 123 Hz. wide and centered at 82 Hz. Counting the number of spectral peaks in the energy density spectrum proved to be the most reliable test for identifying the two types of bruits. Stenosis was diagnosed correctly in 77% to 85% of the patients studied using the spectral peak count.

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Approved:

Head, Major Department

Chairman, Examining Committee

Graduate Dean

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ABSTRACT

The purpose of this investigation, a mathematical analysis of the acoustical properties of cervical bruits, was to differentiate the auscultatory signals of diseased neck arteries (stenotic bruits) from similar sounding healthy artery signals (innocent bruits).

By studying the variation and first moment of the acoustical signal distribution curve, no significant difference was found between the stenotic and innocent bruit.

Significant difference between the innocent and stenotic bruit was evident from an examination of the zero crossing frequency of the signal. The bandwidth, mean frequency, and number of peaks in the energy spectrum of the signal also showed significant difference between the innocent and stenotic bruit. The average stenotic bruit studied was found to have 90% of its energy contained in a frequency band width of 188 Hz. with a center frequency of 131 Hz. The frequency band containing 90% of the energy of the average innocent bruit was 123 Hz. wide and centered at 82 Hz. Counting the number of spectral peaks in the energy density spectrum proved to be the most reliable test for identifying the two types of bruits. Stenosis was diagnosed correctly in 77% to 85% of the patients studied using the spectral peak count.

I. INTRODUCTION

Early Warning and Prevention of Stroke

Many strokes occuring in older people are the result of an obstruction or narrowing in the major neck (carotid) artery leading to the brain. Such an obstruction, an ailment labeled stenosis by physicians, can be diagnosed and repaired by surgery because of the accessibility of the artery. Diagnosis is made by auscultation (listening to the sound). An abnormal sound heard between the first and second heart sounds may be an indication of stenosis. The medical term applied to this abnormal sound is the cervical bruit.

The Existing Problem

Unfortunately, the cervical bruit, referred to in future references as the bruit, may also occur in normal, healthy people. This type of bruit is called an innocent bruit, and its existence can make the diagnosis of stenosis frustrating and uncertain. It is possible to increase the efficiency and reliability of diagnosis of stenosis by applying modern mathematical methods to the acoustical bruit signal. The purpose of this investigation is to find a method of analysis which will allow differentiation of the stenotic bruit from the innocent bruit. Also, any identifying characteristics which help to explain the mechanism of bruit are labeled.

The General Approach

An attempt is made to solve the problem by analyzing the recorded sound from the neck artery of 13 patients, seven of whom have healthy arteries and six of whom have diseased arteries. In all cases studied, a bruit exists. In this study an attempt is made to separate the innocent bruit from the stenotic bruit by three techniques -- statistical analysis, spectral analysis, and zero crossing analysis.

II. REVIEW OF THE LITERATURE

Bruit Research

A study of over 4,000 patients made by Braun, et al, (1966) revealed that bruit occurance varies with age. Table I summarizes the type of variation which was discovered in his study.

TABLE I

INCIDENCE OF BRUIT BY AGE

Age in Years	Bruit Occurance Percentage	Number Examined
0-9	20	30
10-19	14	605
20-29	6	1082
30-39	5	680
40-49	3	685
50-59	3	566
60-69	4	387
70-79	3	232
80-89	14	28

It is apparent from this table that bruits occur most commonly in the very young and the very old. Bruits occuring in the young can usually be assumed to be of an innocent nature because the incidence of arterial disease at this age is practically nil. Braun, et al, (1966) also found that the innocent bruit occuring in 20% of all young healthy people is of a shorter duration, appearing closer to the first heart sound than the stenotic bruit. This does not necessarily hold true for the innocent bruit in older people.

A study by Rennie, Ejrup and McDowell (1964) found that, especially in young adults, the innocent bruit originates lower in the neck on the right side, while the stenotic bruit is normally found near the middle of the neck over the carotid artery. An X-ray picture of an opaque solution injected locally into the blood stream, an arteriogram, is used to verify the existance of a stenosis in this type of study, but the discomfort and trouble of this procedure make it impractical for use on healthy people whose bruits may be innocent.

The bruit has been identified with stenosis since 1954, (Fisher, 1954) and little research has been done which would help to differentiate between the stenotic and innocent bruit. Neither Braun's nor Rennie's studies were conclusive in identifying the characteristics of the innocent or stenotic bruit. Some characteristics must be found to enable simpler methods of diagnosis.

Related Research

In order to determine and understand the origin of bruits and the diagnosis of stenosis it is necessary to search related topics for pertinent information. Especially important, because of their close relation to the bruit, are four such topics: 1) the structure of the arteries, 2) the flow of blood in arteries, 3) the mechanism of heart murmurs, and 4) the diagnosis of heart murmur.

Rodbard has been particularly active in blood vessel and blood flow research. He explains and has shown by experiment (1956,1957,1959) how hydraulic forces can act upon the vascular lining to form valves, cushions, and stenosis. It is known that blood vessels, besides growing during childhood, tend to elongate and become twisted (Rodbard, 1956) losing their elasticity (Simpson and Nakagawa, 1960) with old age. Blood is a very complex media and its flow is very difficult to describe exactly, in any but qualitative terms. Blood flow is pulsatile, "... the vessel diameter changes during each surge in pressure, filtration across the vessel wall disturbs the boundary layer, and the viscosity of the blood probably changes anomalously from moment to moment." (Rodbard and Johnson, 1962) The red blood cells have a tendency to group along the axis of a vessel giving rise to a radial viscosity gradient. (McDonald, 1960) It is apparent that there is a wide latitude of variation in both the blood flow and the vessel structure.

Bruns advances a general theory of the causes of murmur

(1959) which is also applicable to bruits since they are so closely related. Based on theoretical and experimental evidence, he discounts the importance of cavitation and turbulence as noise generators in arteries and asserts that vortex shedding or eddies are the more likely cause of the noise we hear as murmurs or bruits. Anemia or other causes of high cardiac output as well as stenosis are associated with bruits and murmurs; all these conditions can cause vortex shedding under the appropriate conditions. Bruns produced murmurs artificially by introducing obstructions in the form of paper clip wire and orifices into rubber tubing. He showed that the frequency of noise produced is related to the vessel geometry and the rate of flow, and the noise can be made similar to that of murmurs.

Bruns has shown that for large diameter orifices in tubes, the frequency of sound produced will be approximated by

$$\text{FREQUENCY} \simeq \frac{\text{velocity of fluid flow}}{6 \times \text{width of orifice shoulder}},$$

where the width of the orifice shoulder is equal to one-half the difference between the tube and orifice diameters. For very small diameter orifices, however, the frequency of the tone produced by vortex shedding is approximated by

$$\text{FREQUENCY} \simeq \frac{0.6 \times \text{velocity of flow}}{\text{orifice diameter}} .$$

"Thus, as a constriction or stenosis becomes greater (orifice diameter decreases) one should find that the basic frequency, at first high, will become lower and then increase once more." (Bruns, 1959)

Murmurs have been the subject of active study in recent years. Jacobs, Horokoshi and Petrovick (1968) have devised an instrument which uses the phonocardiogram signal plus the electrocardiogram signal to separate normal hearts from grossly abnormal ones with approximately 94% certainty. This instrument uses a filter amplifier system to boost the low-level high frequency components of the phonocardiogram signal and a zero crossing detector to identify the abnormals based on the number of times per heart beat that the filter amplified phonocardiogram amplitude crosses the zero axis. The counting is started and stopped by triggering from the electrocardiogram signal. These men also ran tests on experimentally stenosed aortic valves from sheep. They found that stenosed aortic valves have characteristic frequency spectra. They also found, for a given heart and valve, that the noise intensity increases with flow rate, but a definite correlation could not be found. A concrete model of the sheep heart with a triangular brass orifice produced a similar stenosed spectrum characteristic, while the model with no obstruction produced a normal characteristic spectrum. Jacobs, _et al_, deduced from their studies that the changes which occured in the spectral analysis of valve noise were related to the degree of stenosis induced in the valve. While unable to ascertain the parameters responsible for the frequency changes, they did conclude that the noise is not determined uniquely by the stenosis but by the conditions of the system (heart and arterial conditions) as a whole. The zero crossing analysis of this study, while effective in separating grossly different signals, may not be sensitive enough to detect differences between two similar signals -- the innocent and stenotic bruit.

The Humetrics Division of the Thiokol Chemical Corporation developed a more sophisticated detector called the PhonoCardioScan (Durin, et al, 1965) for use in school heart test projects. Specialized analog digital circuitry which not only detects the presence of congenital heart defects, but also helps to identify the particular type of defect, was developed. The instrument used spectral analysis data acquired from known diseased hearts as a basis for comparison and diagnosis. The rather elaborate data acquisition system recorded simultaneously the sounds from four chest microphones, the electrocardiogram signal, the respiratory phase signal, and a voice commentary. The instrument itself only requires two inputs; an electrocardiogram and a chest microphone input. The microphone is moved to each of the four regions and 10 to 30 heart cycles are examined for each microphone placement. The whole testing process only takes three minutes per patient. A similar approach could be taken toward identifying stenotic bruits since bruit sounds are very similar to those sounds originating in heart defects. It is hoped that spectral analysis of the bruit will reveal a significant difference between the stenotic and innocent bruit which could be detected by such an instrument.

III. NATURE OF THE BRUIT WAVEFORM

Ausculation has been employed for many years by doctors to tell the condition of the heart, but variation in hearing ability and limitations imposed by the hearing threshold have led to recording the heart sounds on strip charts and magnetic tapes. This record of the heart sound is referred to as a phonocardiogram. One cycle of the normal phonocardiogram appears as in Fig. 1a. As shown, the first and second sounds are quite distinct. The remainder of the signal is fairly silent. These two sounds are transmitted through the major arteries and a similar waveform can be obtained by listening over an artery such as over the carotid artery during examination for cervical bruits.

> The first heart sound occurs with the onset of ventricular contraction. Before the ventricles contract, the mitral and tricuspid valves close by atrial contraction. The closure of these valves is the principle source of sound, although an additional component may come from vibrations of the chamber walls ... The second heart sound is generated by closure of the aortic and pulmonary valves ... The intensity of the sound is dependent on the rapidity with which the value closes and the condition of the valve. (Jacobs, et al, 1968)

It is instructive to examine simultaneous signals obtained from an electrocardiograph and from a pressure sensing device on the carotid artery such as shown in Figs. 1b. and 1c. The two heart sounds in Fig. 1a. mark the beginning and ending of systole (contraction) as seen from the carotid pulse, Fig. 1b.

> The first sound starts after the QRS wave of the electrocardiograph and before the onset of the anacrotic limb of the carotid pulse. The second sound begins just after the end of the T-wave of the electrocardiogram and just before the diacrotic notch of the carotid pulsation. (Green, 1957)

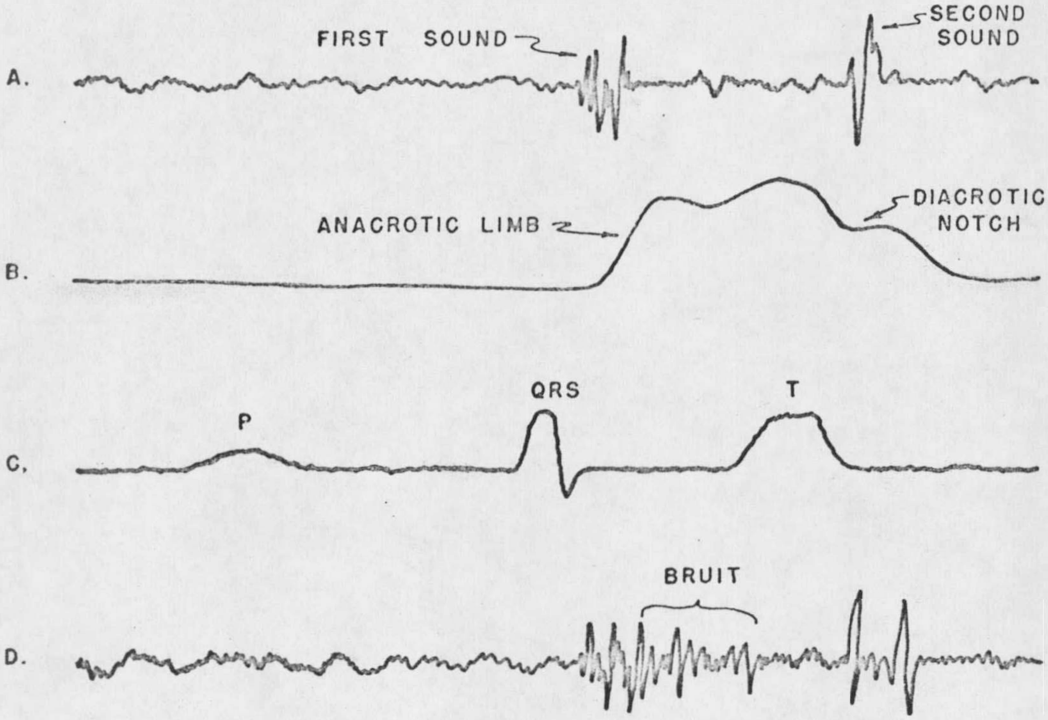

Figure 1. a) Normal phonocardiogram
b) Carotid pressure pulse
c) Normal electrocardiogram
d) Phonocardiogram with bruit.

Note: These sketches are taken from Green (1957).

The same relationships between the physiological signals shown in Fig. 1 should hold true in all persons. The diagram of the sound over the carotid artery in a normal person will appear as sketched in Fig. 1a., but where bruit is present the waveform will be that sketched in Fig. 1d., an additional noise being observed between the first and second sound.

Murmurs or other sounds may be transmitted through the artery and in some cases the sound observed at the neck could appear similar to that of Fig. 1d. but be caused by a murmur. But unlike murmur sounds the bruit appears loudest at a particular position on the artery with a diminishing intensity both up and down stream from the location of the bruit.

IV. APPROACH

Hypothesizing that the innocent bruit and the stenotic bruit belong to two different families of waveforms, this study has as its objective the identification of the characteristics which may be used to differentiate between the two waveforms. When the differences are known a reliable, more practical method of diagnosing stenosis can be devised.

Since only the characteristics of the bruit are being examined it is natural to exclude the other components of the phonocardiogram signal, such as the opening and closing sounds, from the analysis. The most useable type of record of the waveform is a digital record because it allows the utilization of the great speed and flexibility of the digital computer. A computer can be programmed to nearly duplicate any type of analysis which can be formulated, provided the sampling rate is fast enough to completely describe the signal.

Working with the sound recordings taken from a large group of stenotic and innocent bruits, the objective is to find criteria which will enable the separation of the bruits by family. Previous investigations of the waveform analysis type have been successful using one of three types of signal analysis: 1) statistical analysis, used most successfully on the random noise type signal; 2) zero crossing analysis, which has proved to be a simple, highly accurate method of speech analysis (Scarr, 1968) and murmur analysis (Jacobs, _et al_, 1968); 3) spectral analysis using Fourier transform methods as used in the development of the PhonoCardioScan. This investigation includes all three of these methods of analysis.

The statistical methods used here include the determination of the first moment and the variation of the bruit histogram.

A zero crossing analysis, which is actually a form of spectral analysis, usually includes a series of broad band filters whose outputs are all analyzed for zero crossings. But for this study, the zero crossings of the unfiltered bruit signal were counted.

A spectral analysis was made of the energy density spectrum obtained from a Fourier transform of the signal record. Recent advances in computing science have made this transform on a digital record feasible using the fast Fourier transform code, a very efficient method of obtaining the transform coefficients.

V. DATA COLLECTION

The recordings of arterial noise used in this research were made at the Western Montana Clinic in Missoula by Dr. Harold Braun using a Crown Model SS800-S tape recorder and a Sanborn surface contact microphone, Model 572-M, placed over the bruit in the neck artery of the patients. Scotch 202 silicon lubricated, one-fourth inch magnetic tape with a 1.5 millimeter polyester backing was used for the recordings. During the recording process, patients were instructed to take a breath, let it out, and remain still without breathing for a few seconds. One channel of the two-track recording was used for voice commentary; the other was used to record the arterial noise.

From the recordings, digital samples were taken using a digital controller, a Model EECO 765 multiplexer, a Model EECO 761 analog-to-digital converter, and a voltage limiter built specifically (See Appendix C) to protect the analog-to-digital circuitry from overload. The first sample records were made digitizing at a rate of 4,000 samples per second with the analog-to-digital equipment coupled directly to the IBM 1620 computer and card punch. The sampling was done at a fairly "clean" spot on the tape, where the signal wasn't obviously obliterated by skin noise made by microphone slippage or by voice or breathing interference. Fairly long sample records of two-second or three-second duration were taken and punched directly on cards. The bruit record was then hand selected by removing the unwanted first and second heart sounds. Only the portion of the signal labeled bruit in Fig. 1d. remains in the record.

The remainder of the sampling was done on the Hewlett-Packard 2116A computer utilizing an improved record selection process and a faster digitizing rate -- 10,000 samples per second. A trigger and delay system which eliminated the hand selection of records was established using a type 549 Tektronix storage oscilloscope with a four-channel type 1A4 plug-in unit. A trigger signal from the scope calibration output -- a one-kilohertz square wave -- was recorded on the voice channel near a clean portion of the tape. Triggering the oscilloscope from this signal and looking at the waveform of the arterial sound on the display screen, time delays were calculated to the beginning and end of the bruit part of the signal. The proper time delays were then set on the digital controller, the tape was positioned at the correct trigger signal, and the tape recorder was started for each sample; thus initiating the digitizing process. The Hewlett-Packard program accepted two lines of description from the teletype after each sample and punched the description and digital record on paper tape. A high-speed interface between the Hewlett-Packard and the IBM computers allowed the data to be punched on cards for later analysis on the Scientific Data Systems, Sigma 7 computer.

The removal of the IBM 1620 and its card punch cut short the data collection phase of this project. At that time 133 samples had been gathered from a total of 13 different individuals; seven of whom had innocent bruits, and six of whom had stenotic bruits. Of the total samples, about 55% are from innocent bruits. The remainder are from stenotic bruits.

VI. DATA ANALYSIS

Data Plot

Partly as a check on the analog to digital sampling process and partly as a visual check for outstanding similarities or differences between the families, a plot was made of each sampled waveform. Using the IBM 1620 computer and associated digital plotter, the magnitude of each data point in a sample was plotted against time. The time was scaled so that the time axis of the plots was of constant length. The amplitude was normalized so that the maximum amplitude in each record was 1,000 millivolts. (See Figs. 5, 6, 10 and 11 for examples of these plots.)

Statistical Analysis

If the waveforms of the innocent bruit family have a characteristic shape that is different from the waveforms of the stenotic bruit family, the difference may be more evident in the histograms than in the waveforms themselves. An histogram frequency distribution curve, was developed from the digital waveform which had any direct current bias removed and which was amplitude normalized. It was made by sorting the signal by amplitude brackets, counting the number of times that the signal falls within each bracket and plotting the frequency of occurrence versus the amplitude. For example, the waveform shown in Fig. 2 has a magnitude of +1 at three different points in time, therefore the frequency of occurrence at +1 is plotted on the histogram (Fig. 3) as three, etc. The bruit signal magnitude ranged from -1,000 to +1,000 millivolts and each data point was recorded to the nearest millivolt. Choosing a bracket width of 20 millivolts gives a 100 point histogram. This bracket width seems to preserve enough information to show any differences that exist without the confusion of a more detailed, smaller bracket, histogram.

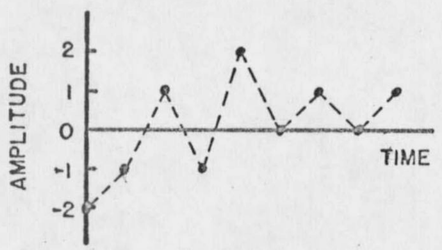

Figure 3. Digital Waveform

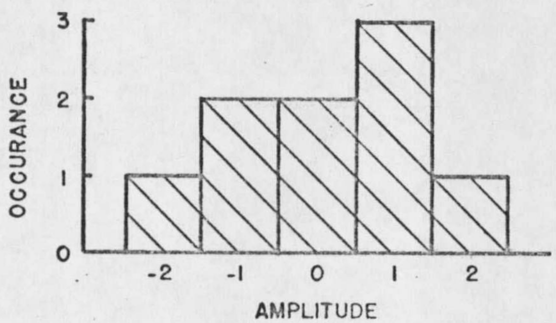

Figure 4. Associated Histogram

The common method for indicating the characteristics of a histogram, or frequency distribution curve, is to find the variation and standard deviation of the distribution. The variation is simply the second moment of the histogram about the average amplitude;

$$\text{Variation} = \sum_{i=1}^{N} f_i \, (a_i - \bar{a})^2 / N,$$

where f_i is the frequency of occurrence, a_i is the amplitude, and N is the number of points or amplitudes in the histogram. The mean amplitude, $\bar{a}$, is given by

$$\bar{a} = \sum_{i=1}^{N} a_i \, f_i / N.$$

The standard deviation is given by the square root of the variation. The variation and standard deviation are always positive numbers. By taking the first moment of the histogram,

$$\text{First moment} = \sum_{i=1}^{N} f_i \, (a_i - \bar{a}) / N,$$

any lack of symmetry of the histogram will become evident.

In addition to examining these statistical characteristics of each histogram, a plot was made of the histogram of each sample, and an attempt was made to find some differentiating characteristic visually.

Zero Crossing Analysis

The zero crossing analysis used here consisted of counting the number of times that a bruit signal passed through its average value. The average magnitude for each bruit record was adjusted to zero. The number of crossings was divided by the period of the sample to give a pseudo frequency of zero crossing. It was this pseudo frequency that was examined to see if the innocent bruit differed significantly from the stenotic. This type of zero crossing test of an unfiltered signal, in effect, disregards the low-level higher frequency components of the bruit signal.

Spectral Analysis

Using Fourier transform techniques, a waveform such as the bruit signal can be represented as a series of sine and cosine functions or as a series of complex exponentials. The energy density spectrum can then be obtained from the Fourier transform coefficients. (See Appendix A for a review of theory.) This energy spectrum provides a very complete description of the signal characteristics. Recent advances in computing science have provided the fast Fourier transform, an efficient and powerful tool in signal analysis of small digital records. The fast Fourier transform is an algorithm which computes the discrete Fourier transform coefficients with a minimum number of computational steps. It is useful in spectral analysis and in filter simulation. The fast Fourier transform changes the discrete time series representation of a waveform to a discrete frequency series. By examining the representation in the frequency domain, called spectral analysis, certain characterisitcs may appear which are not evident in the time domain. The periodogram, a form of the energy density spectrum, is a function of amplitude versus frequency constructed from the representation of the waveform in the frequency domain. Examples of periodograms constructed from two bruit samples are shown in the results (Figs. 9 and 14).

Neither the sampling rate nor the sampling period has been held constant in this study, so different sized periodograms have been generated whose sizes range from 2^8 to 2^{12}. There are several ways to obtain simplified information which is available from the periodogram. In this study four tests of the periodogram were devised and three were used in an attempt to differentiate the innocent signal from the stenotic.

Frequency Limits

In using the discrete finite Fourier transform it is necessary to assume that the bruit signal is band limited. Older sampling theory (Cochran, et al, 1967) dictates that no frequencies higher than ω, given by

$$\omega = \pi/\Delta t,$$

be present in the signal, where Δt is the time between samples.

On the other hand, a theoretical study by Lees and Dougherty (1966) using ideal examples has shown that convergence of the numerically calculated Fourier transform is obtained only when there is an upper limit of the frequency content of the signal given by

$\omega = 0.1\pi/\Delta t$ radians/second, or

$f = 0.05/\Delta t$ cycles/second.

This limit is one-tenth that given by conventional sampling theory previously used. (See Appendix A)

Data used in this investigation was digitized both at 10,000 samples per second and at 4,000 samples per second giving allowable upper frequency limits as suggested by Lees and Dougherty, of 500 cycles per second at the faster rate and 200 cycles per second at the slower rate. The computer code for the fast Fourier transform used required that the number of data points in each sample record be equal to an integral power of two. To adjust the number of total points in each record an interpolation scheme was used which lowered the effective sampling rate and with it the upper frequency limit.

The lower limit of the frequency which will be detected by discrete finite Fourier methods is given by

$\omega_o = 2\pi/T$ radians per second.

Being interested only in the bruit characteristics and having sampled over the entire bruit period, it can be assumed that any frequencies lower than ω_0 are not unique characteristics of the bruit but are associated with the longer period of the heart cycle. Any frequency present in the bruit which is lower than the ω_0 limit will show up as a zero frequency bias and is ignored in this analysis. Any frequency components appearing at 60 Hz. or less must be ignored because of the limitations in amplifier and recorder response in this range. So, in general, this study is limited to frequencies between 60 Hz. and 200 Hz. (depending on the sampling rate).

Mean Frequency and Band Width Tests

If the two bruit families are composed strongly of one frequency band and the center frequency or the band width are different for each family, then the distinction may show up by looking at the energy-mean frequency and the 90% energy band of each spectrum. The energy-mean frequency is defined here to mean that frequency which divides the signal energy in half. Half of the energy in the signal is present in frequencies above the mean and half below the mean. The 90% energy-band width is defined here to be that frequency band, centered on the mean energy frequency, encompassing 90% of the energy of the signal. A computer program was written to find this energy-mean and energy-band.

Peak Count

The fast Fourier transform generates as many complex transform coefficients as there are points in the record. When this is converted to periodogram form, there are N/2 points including the zeroth term where N is the number of points in the waveform record. This gives a spectrum with half as many points as are in the digital record from which it came, and since the digital records range in size from 256 to 4096 there are some very lengthy spectra to analyze. It is noted from examining these spectra that the magnitude of frequency components drops about two orders of magnitude in the first 50 spectral coefficients. Therefore if interest is confined to the major peaks in the spectrum, say those whose magnitude is greater than 10% of that of the greatest peak, only the first 50 spectrum components need be examined. This simplification reduces the computing time considerably.

The peak count test of the energy spectrum compares the samples by the number of different major frequency components which make up the waveform, since each major component shows up as a peak in the spectrum. For example, look at a time function,

$$f = \sin(\omega t),$$

which is a pure tone having one frequency component - namely

ω. The period of this tone is given by

$$T_t = \frac{2\pi}{\omega} .$$

If the tone is sampled for a period, T_s, the associated periodogram will have points at frequencies given by

$$\omega_k = 2\pi k/T_s, \quad k = 1,2,3,...$$

For the case where ω is equal to one of the ω_k's, the periodogram will have only one non-zero point at ω. But for the case where ω falls between two of the ω_k's, the periodogram will appear similar to that shown in Fig. 4. In either case a

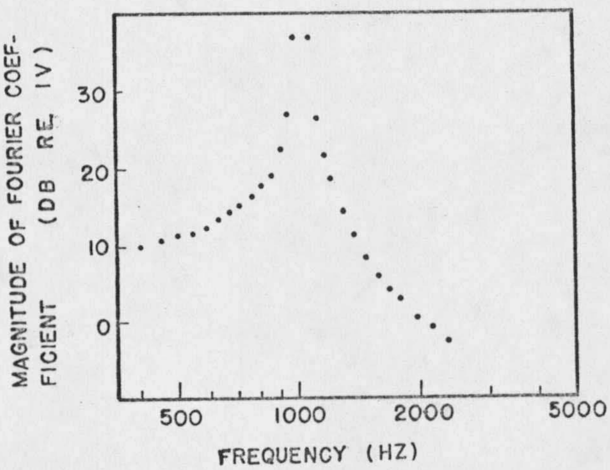

Figure 4. Typical digital computation of the spectrum of a pure tone at 1021 Hz, where the Fourier coefficients have a frequency spacing of 45.4 Hz. (Maling, <u>et al</u>, 1967)

single tone is shown to yield a single peak in the periodogram whose width is roughly proportional to the difference between ω and the nearest ω_k. Now for this analysis the peaks in our more complicated spectra can be interpreted as each originating from a tone whose frequency is that of the crest of the peak. By counting the number of major peaks in each spectrum, it is postulated that the innocent family of bruits will be found to be composed of more or less major tones than the stenotic family of bruits.

A computer code was written to count the number of peaks in the first 50 spectral coefficients which were above 10% of the maximum amplitude. See Appendix B for the listing of this program.

Filter and Zero Crossing Analysis

As stated previously some success has been realized in distinguishing normal from abnormal phonocardiograms by using broad band filter amplification together with analog-to-digital conversion and zero crossing counters (Jacobs, *et al*, 1968). In his study, Jacobs used the electrocardiograph signal as a gate to sample an entire heart beat and ran the sample through a filter amplifier series which eliminated the very low frequency components and amplified the high frequency components. The reason for the amplification of high frequency components was to boost this part of the phonocardiogram signal enough to be detected by zero crossing counters. This method of signal testing could be used on the bruit signal, but it was not pursued in this study. The advent of the fast Fourier transform has made it practical to perform this test analytically. The fast Fourier transform is especially applicable to ideal filter simulation because of its simple convolution relationship. A filtering operation can be simulated by multiplying the discrete transform coefficients by discrete filter characteristics of matching frequency and performing an inverse transform to give the system output. Then a zero crossing count can be made much the same as was done with the unfiltered bruit signal.

It should be emphasized that the mean frequency, bandwidth, peak count, and zero crossing tests in this section do not provide information in addition to that contained in the energy spectrum, but only supply a method for obtaining the same information or part of the same information in a simplified, more readable and identifiable form.

VII. DISCUSSION OF RESULTS

Plots

Figs. 5 through 9 show the type of plots that were made for each innocent bruit sample. Fig. 5 shows the noisy part of the heart beat cycle for an innocent bruit. An envelope of this curve would contain three peaks. The first envelope peak is what is called the first heart sound; the second peak contains the cervical bruit, and the third peak in the envelope is the second heart sound. This plot actually covers only about half of the normal heart beat cycle, but the other half is comparatively level or silent. Fig. 6 shows that part of the heart cycle which was chosen as a bruit sample. It is that subset of the previous plot whose position is indicated in Fig. 5. As can be seen, the attempt to exclude the first heart sound was not completely successful in this case. Figs. 7 and 8 are the histograms for Figs. 5 and 6, respectively. Fig. 9 is a plot of 50 coefficients in the energy spectrum for the innocent bruit of Fig. 6. Figs. 10 through 14 are similar to Figs. 5 through 9, but are for a stenotic bruit. Note in Fig. 10, the noisy part of a stenotic bruit beat, that the first heart sound is not easily distinguished from the bruit.

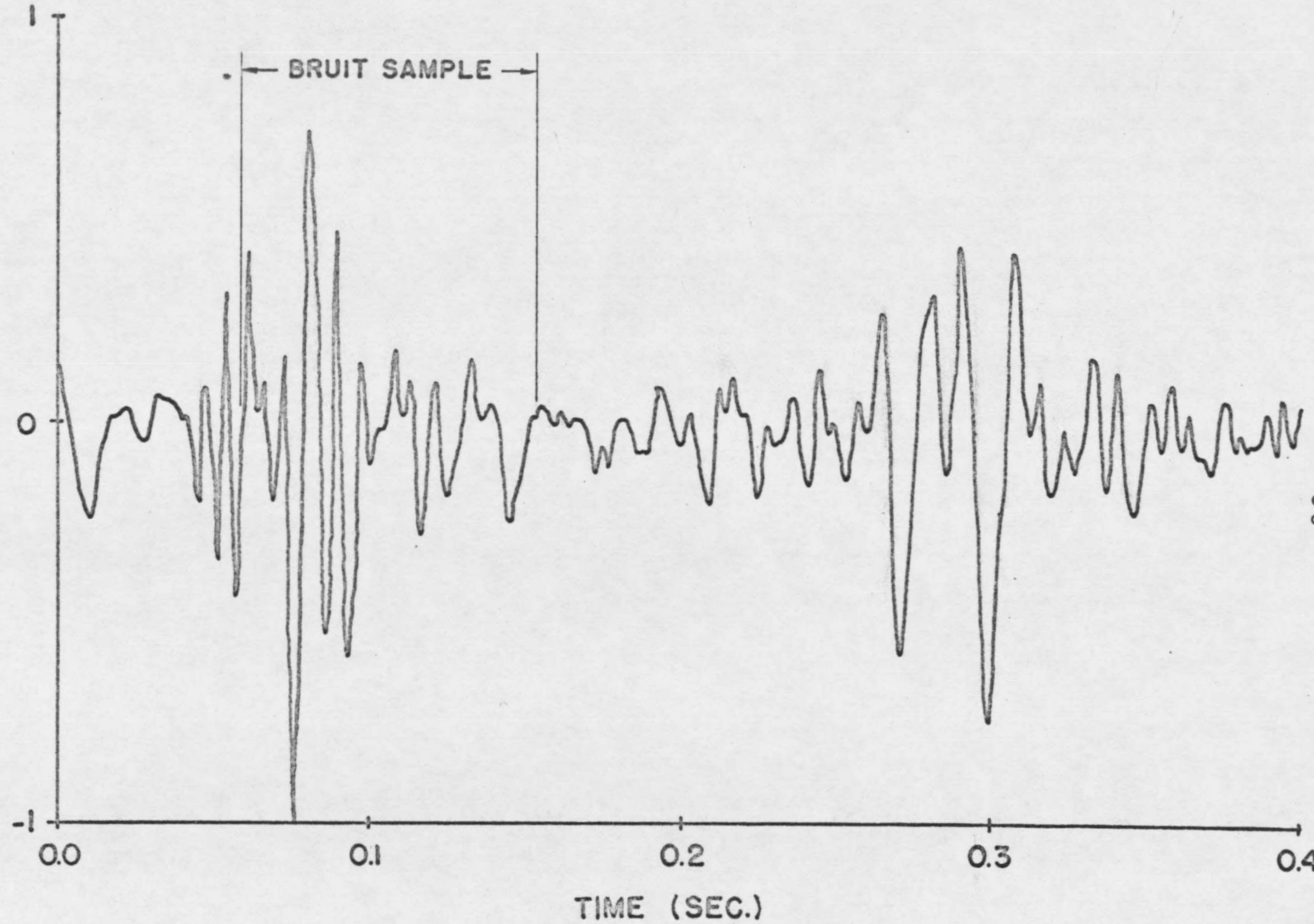

FIGURE 5. FIRST AND SECOND HEART SOUND WITH INNOCENT BRUIT
(129768-512-HE)

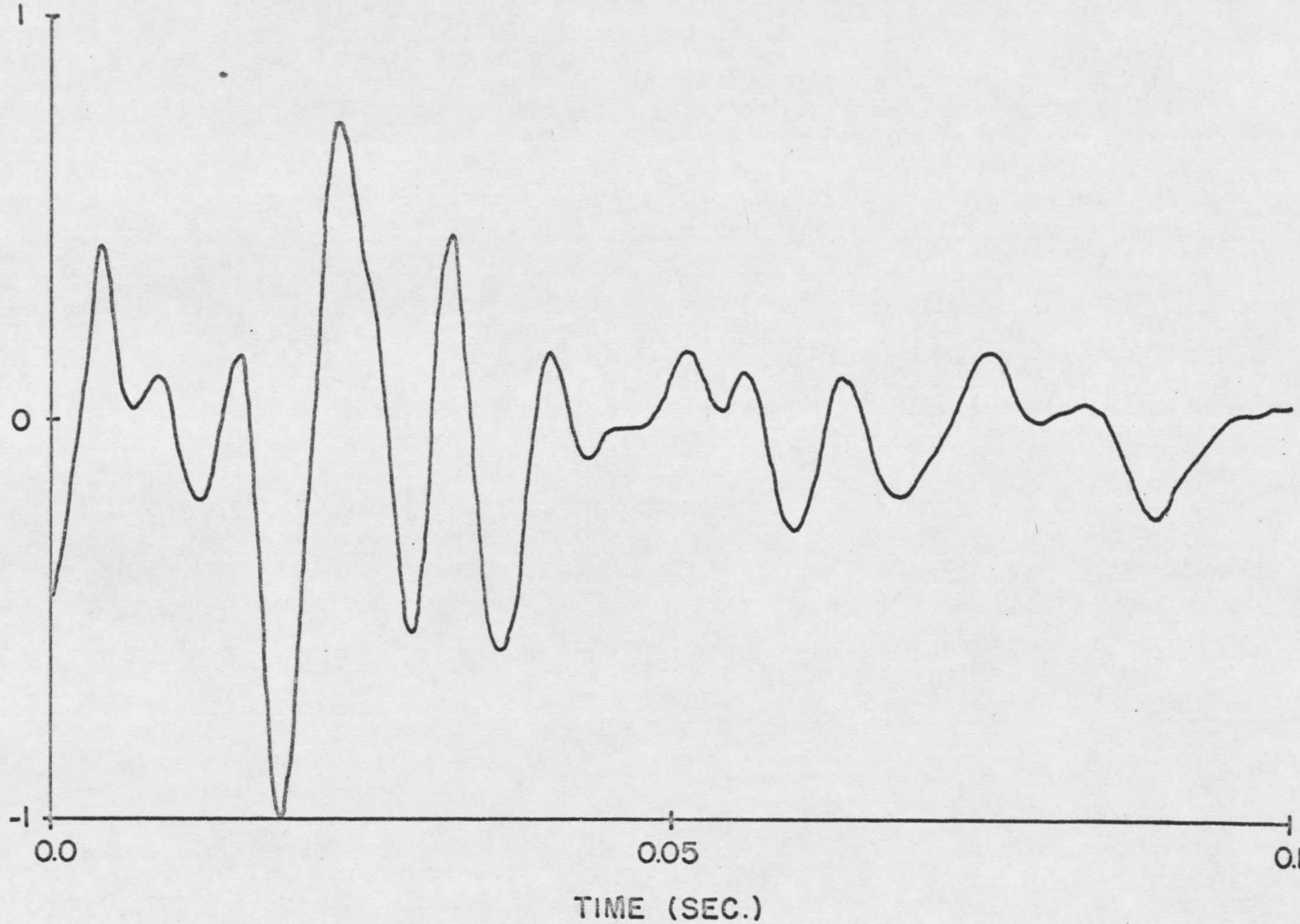

FIGURE 6. INNOCENT BRUIT SAMPLE (129768-512-H)

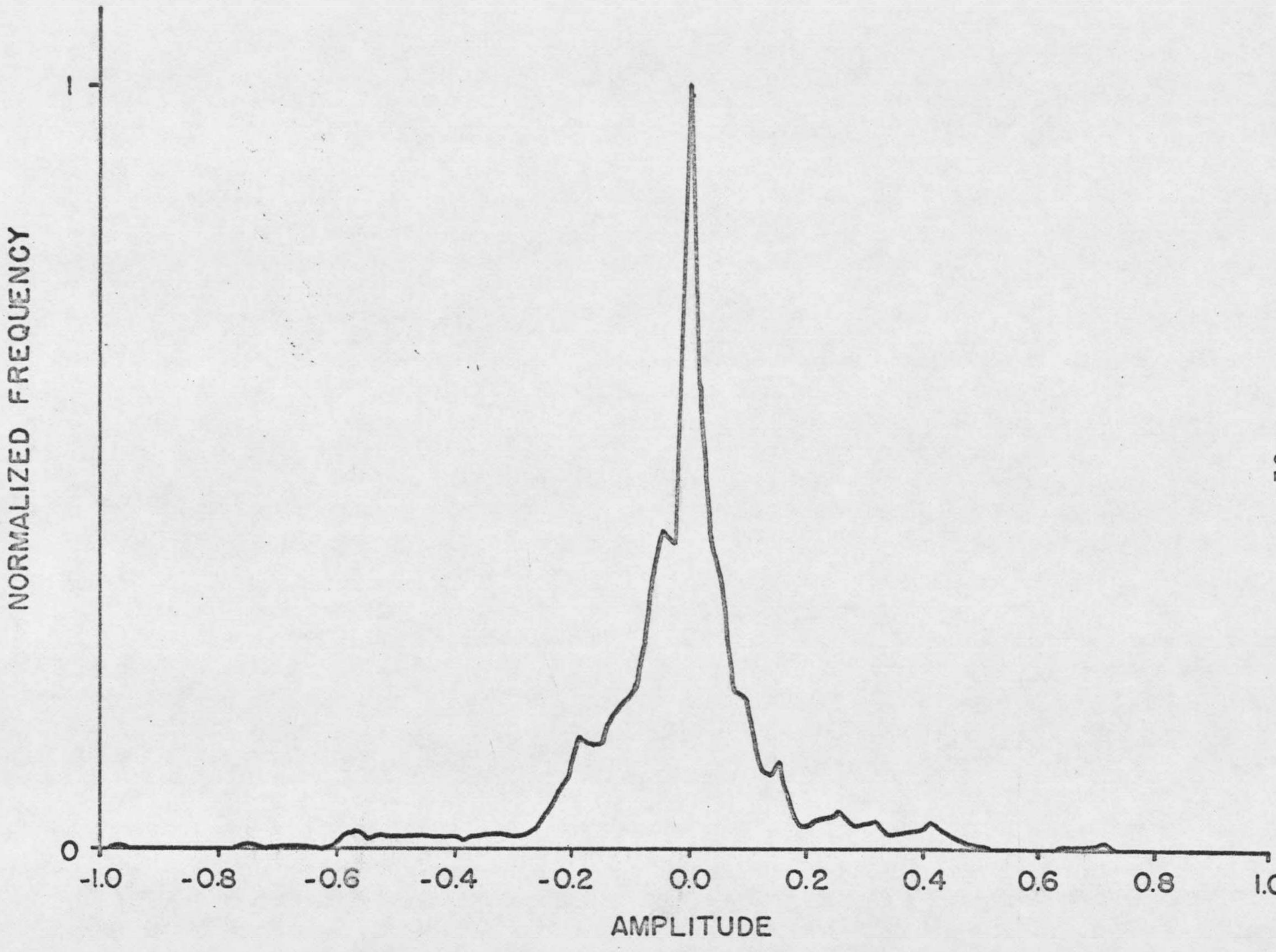

FIGURE 7. HISTOGRAM OF HEART BEAT SHOWN IN FIGURE 5 (129768-512-HE)

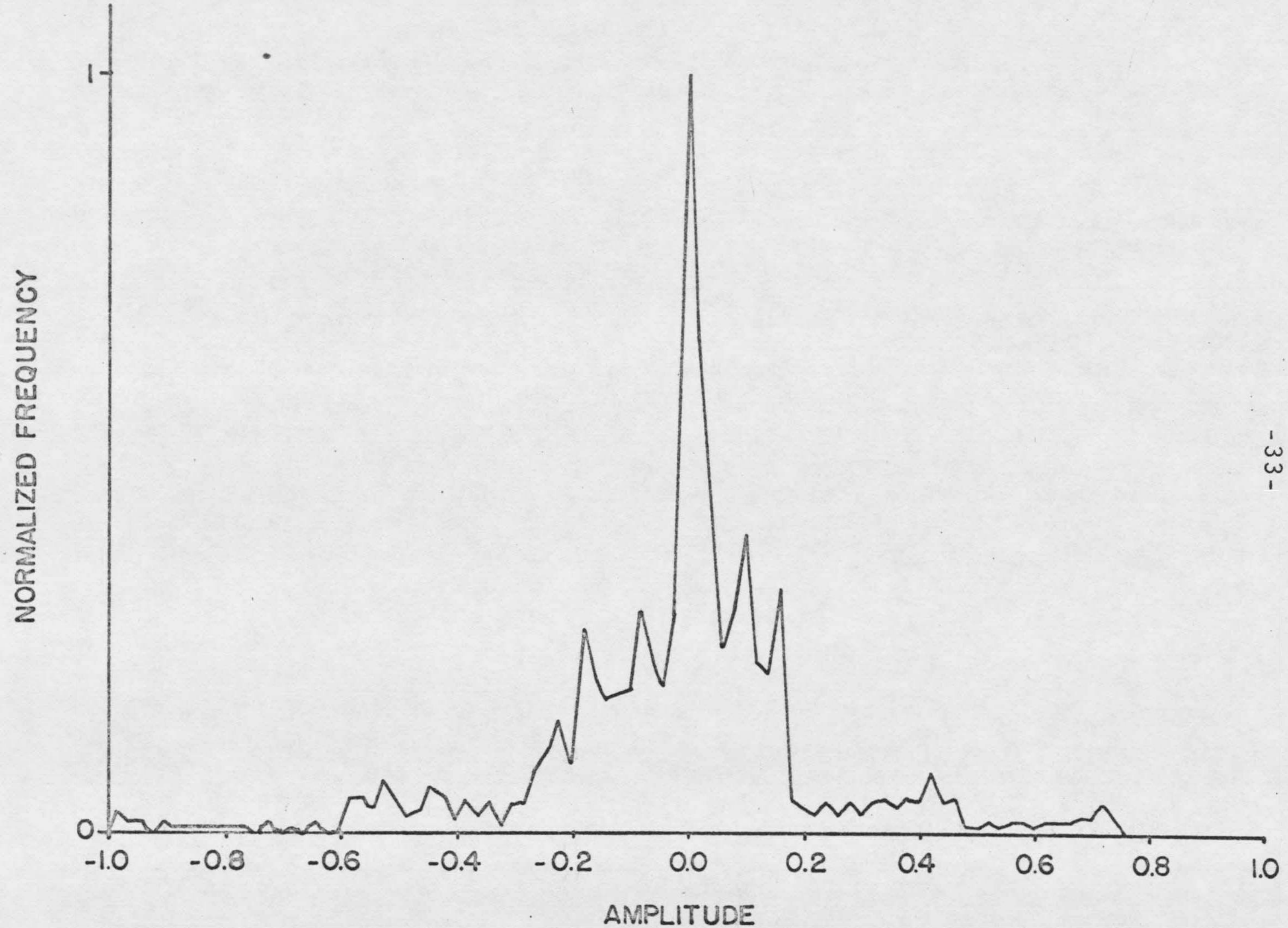

FIGURE 8. HISTOGRAM OF AN INNOCENT BRUIT SAMPLE (129768-512-H)

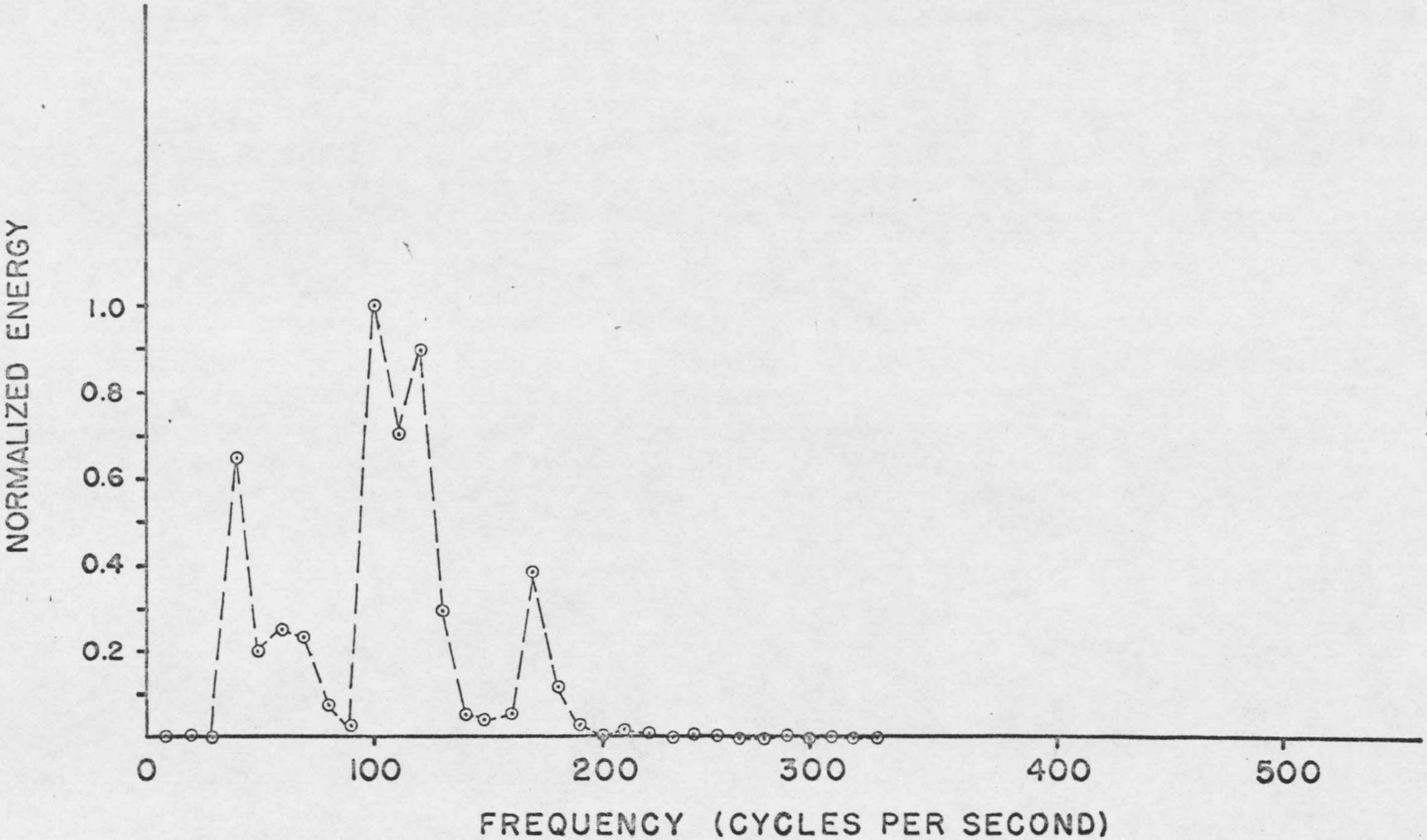

FIGURE 9. ENERGY SPECTRUM OF AN INNOCENT BRUIT (129768-512-H)

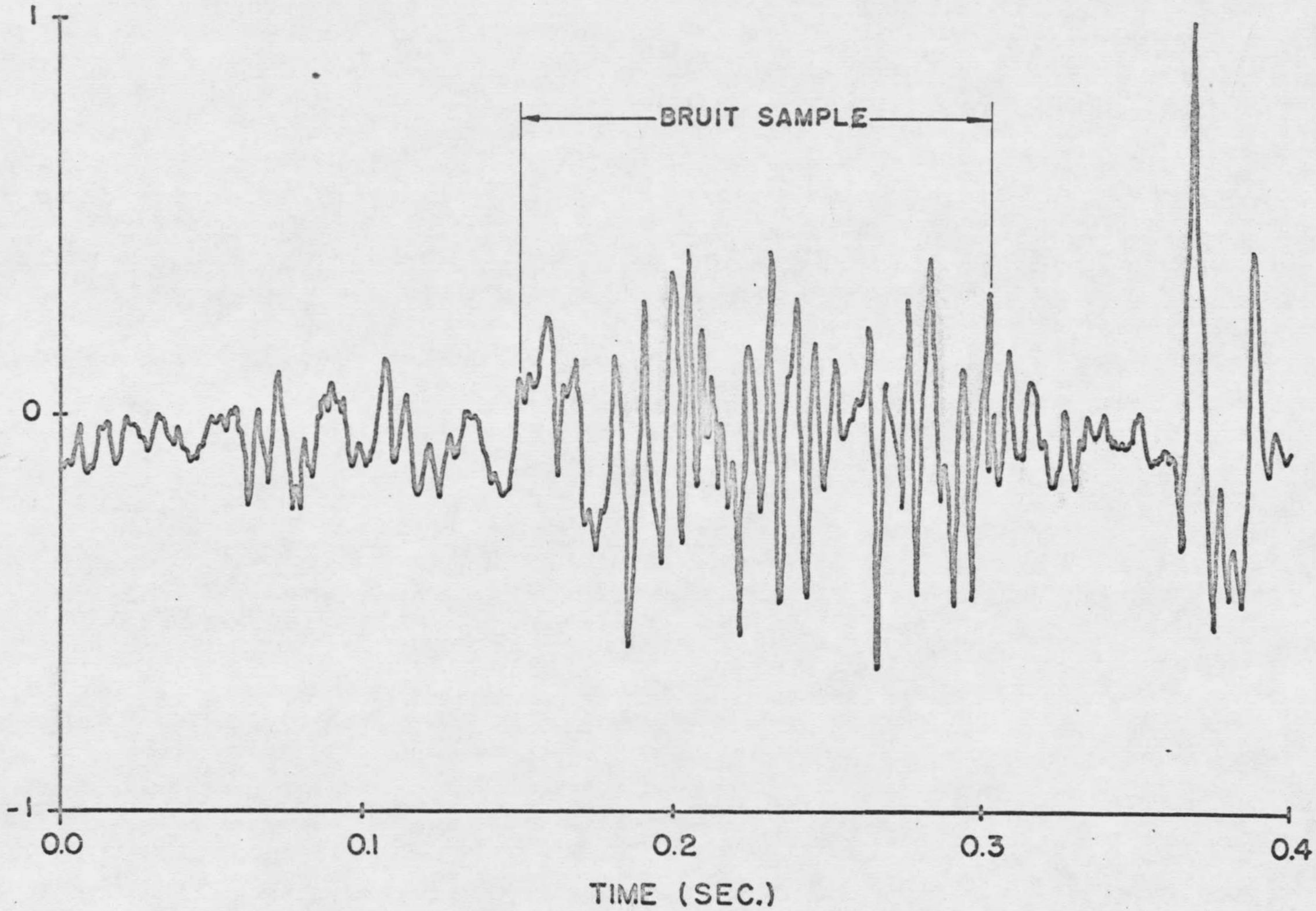

FIGURE 10. FIRST AND SECOND HEART SOUND WITH STENOTIC BRUIT

(129379-4-AA)

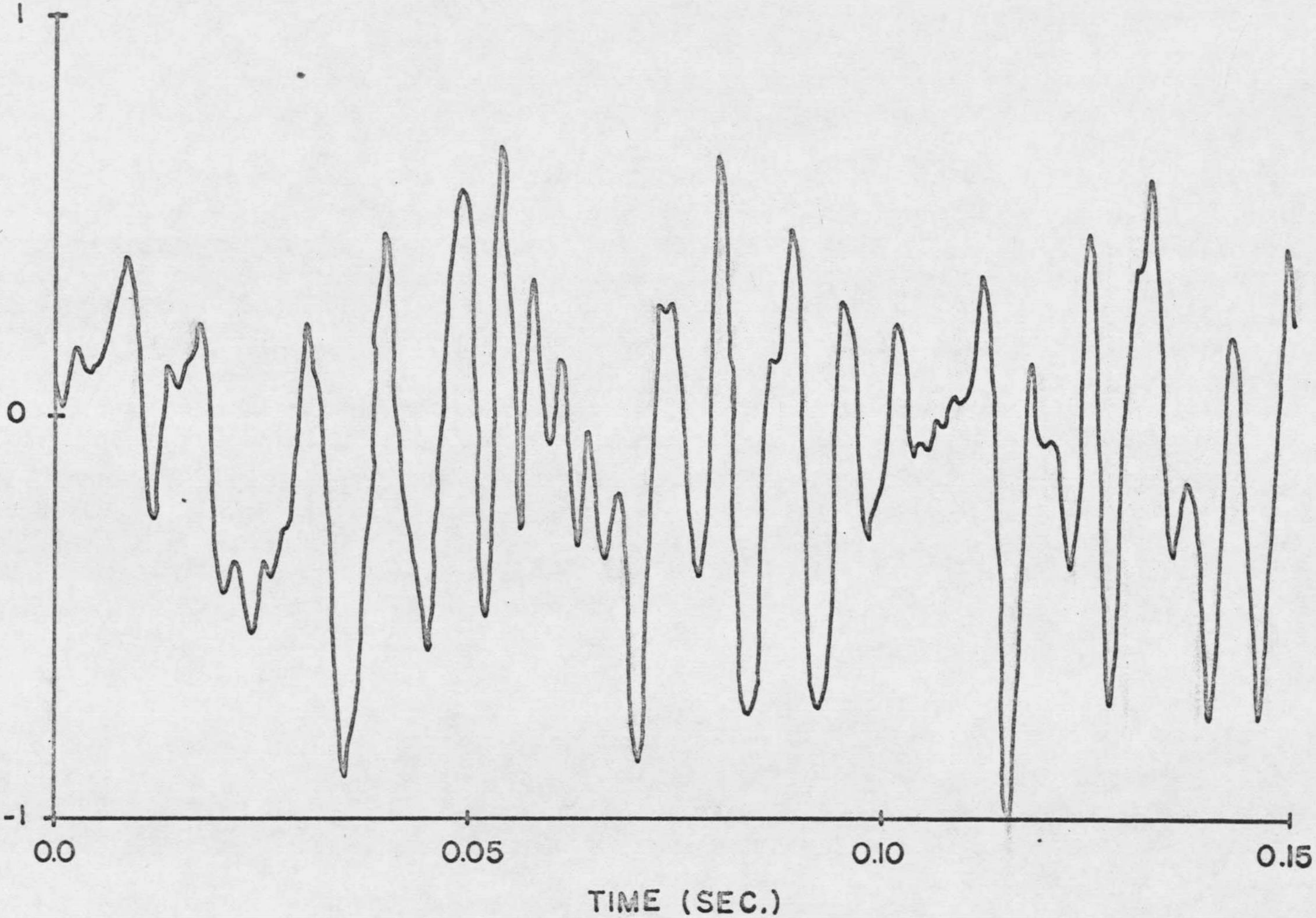

FIGURE 11. STENOTIC BRUIT SAMPLE (129379-4-A)

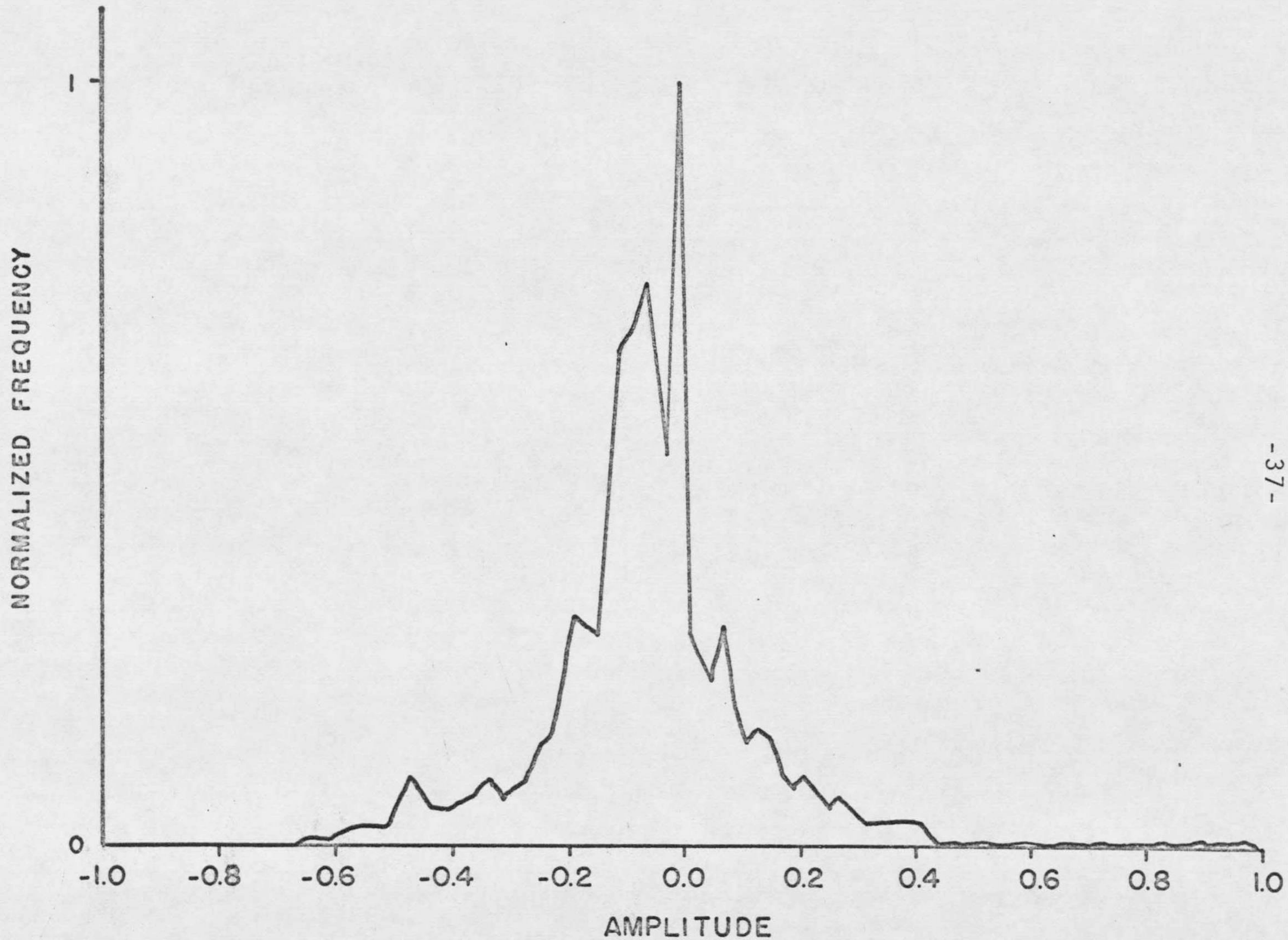

FIGURE 12 HISTOGRAM OF HEART BEAT SHOWN IN FIGURE 10 (129379-4-AA)

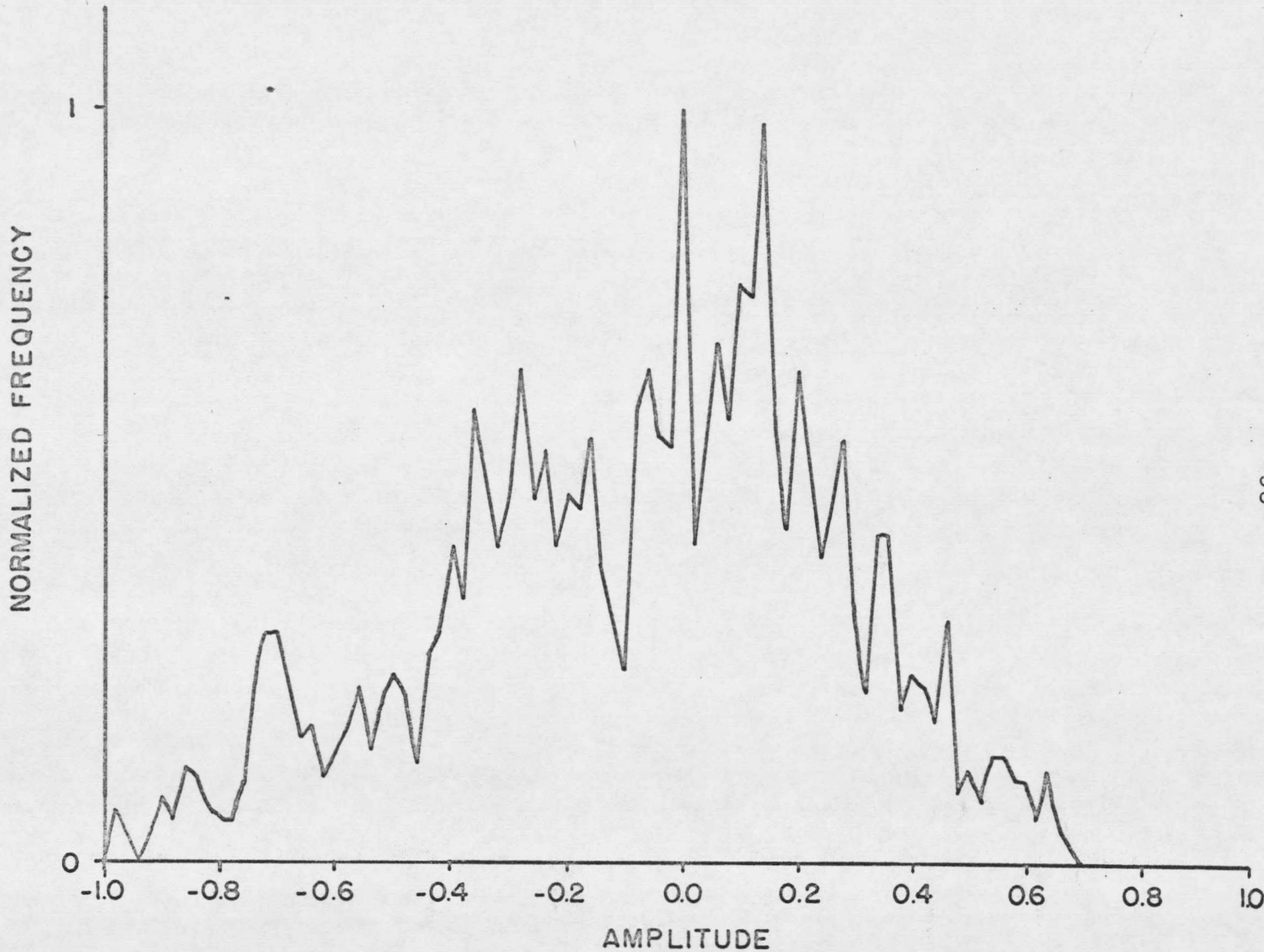

FIGURE 13. HISTOGRAM OF A STENOTIC BRUIT SAMPLE (129379-4-A)

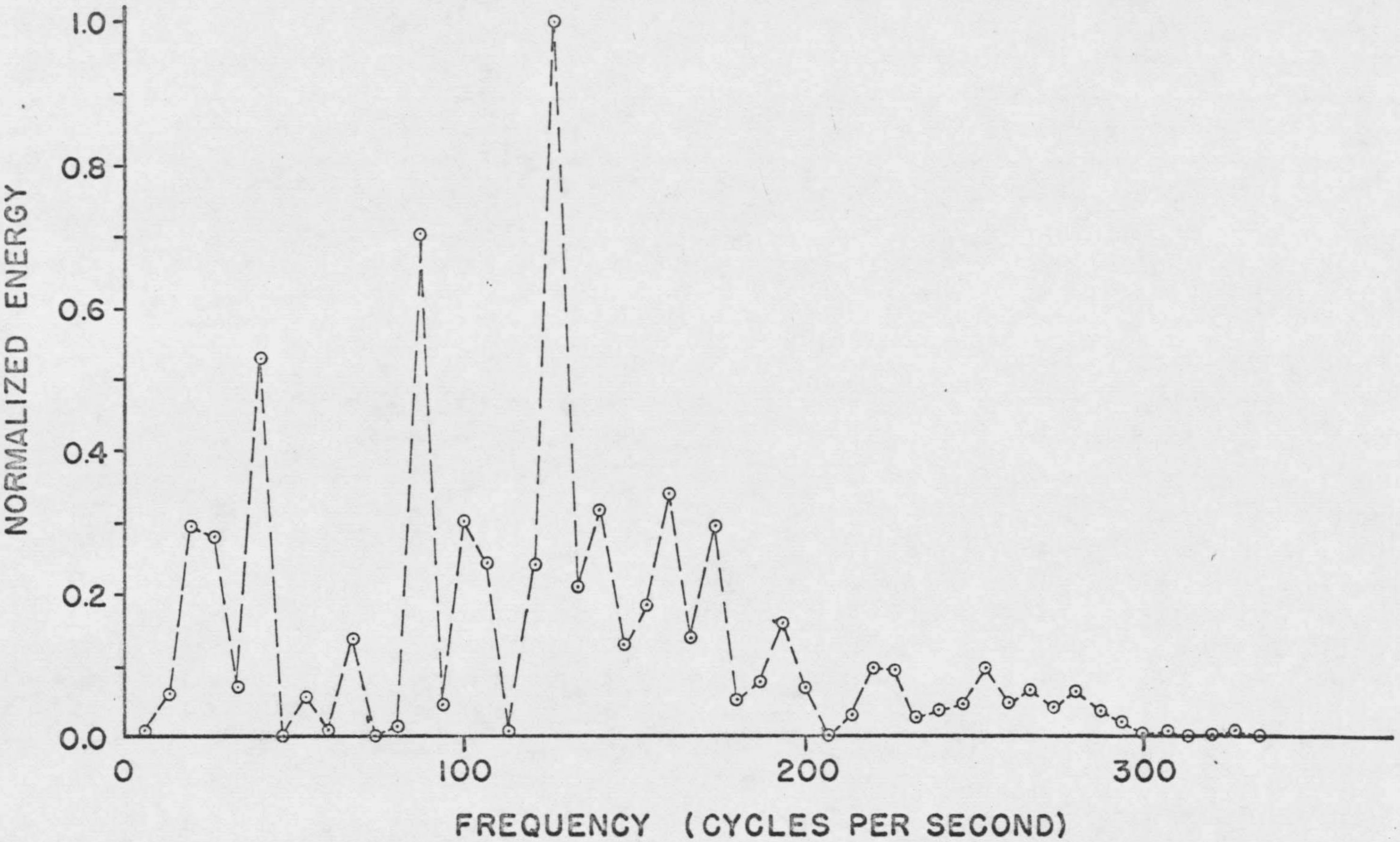

FIGURE 14. ENERGY SPECTRUM OF A STENOTIC BRUIT (129379-4-A)

A visual examination of the bruit waveform plots of the sampled populations failed to reveal any outstanding differences between the stenotic waveform and the innocent waveform. While the innocent waveforms from one patient may be similar, their variation between innocent patients is about as much as between an innocent and a stenotic patient.

The appearance of the bruit histograms is affected not only by the differences between any two heart beats of an individual but also by the differences in sample position, sample length, and by the differences between individuals. No conclusions which would help to separate the two bruit families could be drawn from a visual examination of the histogram plots.

Plots of the energy spectra, while containing as much information as the signal or histogram plots, appear in a simpler visual form. Visual examination shows the stenotic spectra to have more peaks than the innocent spectra, thus suggesting the idea of a major peak count for identification purposes.

Relating Bruit Characteristics to the Innocent or Stenotic Population

To demonstrate the success of using a bruit characteristic, such as the number of major spectral peaks, to predict disease, it must be shown that this characteristic is dependent on the population, either innocent or stenotic, from which it is taken. In order to test the independence of the variable or bruit characteristic, a t test for the significance of the difference between two sample means was used. The t test involves making the assumption that the mean of the stenotic characteristic is equal to the mean of the innocent characteristic; this assumption is called the null hypothesis. A t test of the peak count characteristic compares the mean and variation of all the innocent peak results with the mean and variation of all the stenotic peak results to show how well that characteristic can be used to separate the two populations. The t value and its associated alpha probability will allow rejection of the null hypothesis if the difference in the two family means could not be obtained by random sampling from a single population. For example, look at a t test of the stenotic population spectral peaks versus the innocent population peaks. Suppose a value of t is computed with appropriate degrees of freedom which corresponds to an alpha probability level of 0.05. This would mean that there is only one chance in 20 that the stenotic population's mean number of peaks could come from a random sampling of the innocent bruit, spectral peak population. The two populations could be said to be independent with 95% certainty.

Table II gives the results obtained using the t test to

TABLE II

T TEST OF SEVERAL BRUIT CHARACTERISTICS DETERMINING THE VALIDITY OF USING THE CHARACTERISTIC AS A METHOD OF SEPARATING THE STENOTIC AND INNOCENT BRUIT.

Characteristic	Innocent Mean	Stenotic Mean	Degrees Freedom	T Test	Alpha Prob.
Histogram					
Standard dev.	315.7	324.0	131	0.782	.5
Variation	105090.0	107580.0	131	0.315	.8
First moment	-707.5	-565.0	131	0.952	.4
Zero-Crossing					
Frequency	175.3	273.5	131	9.000	--*
Spectral Analysis					
Energy-Band width	123.3	188.4	115	7.732	--
Energy-Mean frequency	82.0	130.7	115	7.604	--
Number of major peaks	5.23	7.55	112	5.577	--

*Probability of a larger t is less than .001.

test the significance of the difference between the innocent and stenotic mean of each characteristic used in this study -- standard deviation, variation, first moment, zero crossing frequency, energy-band width, energy-mean frequency, and number of major spectral peaks. In locating a characteristic which will reliably separate the bruit families it is important to minimize the type I error, false rejection of the null hypothesis, so that time and money are not wasted working with marginal characteristic tests. Therefore, it would be reasonable to chose an alpha probability of 0.01 as the dividing line between rejection and acceptance of the null hypothesis. As seen from Table II, the null hypothesis can be rejected for all but the histogram characteristics.

The sample cross-section which has been used in this study is quite irregular. Although there are samples from several individuals with each type of bruit, the number of useable samples from each person varies considerably. One individual furnishes about 70% of the stenotic bruit samples; another individual furnishes about 50% of the innocent bruit samples; and a third individual furnishes about 25% of the innocent bruit samples. Because the two means, innocent and stenotic, in each of the tests tabulated in Table II are so greatly influenced by the values from three individuals, the mean of each individual sampled must be checked to insure that he can be considered a member of the population whose mean he helps to establish. This check is determined by a t test between the individual's mean and each family's mean.

In a t test between an individual and the innocent population one must minimize the possibility of falsely accepting the null hypothesis, a type II error, so the rejection point should be set high; say at the 0.05 level. In the t test between an individual and the stenotic population the type I error is the least desirable and the rejection level should be set low; say at the 0.01 level. In other words, care must be taken so that a diseased artery is not judged healthy even at the risk of classifying some healthy arteries as diseased.

The results of the major peak counting test are shown in Table III. Comparing the mean number of peaks for each stenotic patient with the mean of the innocent population it can be seen that the null hypothesis would be rejected at the 0.05 probability level in all cases but one.

Comparing these same patients with the stenotic population we see that at the 0.01 level the null hypothesis can in no case be rejected. All of the stenotic bruit patients have been correctly identified with this criteria. Now comparing each innocent patient's mean with the innocent population mean, we reject the null hypothesis for one patient only, identify-

TABLE III

T TESTS ON THE NUMBER OF MAJOR PEAKS OF THE BRUIT SPECTRA

Patient Tested			With Innocent Population			With Stenotic Population		
Number of Samples	Patient Identification Number	Patient's Mean Number of Peaks	Degrees of Freedom	T Test with Innocent Population	Alpha Probability	Degrees of Freedom	T Test with Stenotic Population	Alpha Probability
Innocent Bruit Patients								
31	132323	4.94	63	0.62	0.60	78	5.03	--
15	129768	5.87	63	1.07	0.30	62	2.67	0.01
77	122252	4.86	63	0.44	0.70	54	3.01	0.01
5	133441	7.60	63	2.34	0.05	52	0.04	0.90
3	50268	3.33	63	1.51	0.20	50	3.18	0.01
2	116971	4.50	63	0.48	0.70	49	1.88	0.10
2	131824	4.00	63	0.80	0.50	49	2.20	0.05
Stenotic Bruit Patients								
35	129379	7.11	98	4.20	--	47	0.89	0.40
4	111111	7.50	67	2.04	0.05	47	0.04	0.90
4	115326	8.50	67	2.88	0.01	47	0.79	0.50
3	222222	10.33	66	4.08	--	47	2.11	0.05
2	131921	7.50	65	1.48	0.20	47	0.03	0.90
1	124918	11.00	64	2.66	0.01	47	1.51	0.20

ing six out of seven correctly with this criteria. Comparing these innocents with the stenotic population there are three cases where the null hypothesis cannot be rejected, giving only 57% accuracy with this test. The accuracy of the t test identifications, summarized in Table VII show that this characteristic is very successful in separating the stenotic bruit from the innocent bruit.

The results of the individual t tests for energy-band width, energy-mean frequency and zero crossing frequency are tabulated in Tables IV, V, and VI respectively; all three showed promise in Table II. Although these three characteristics also show some trend toward correct prediction of stenosis, there are several cases in each table of results where an innocent patient is shown to be most likely classified as stenotic or the opposite mistake is made. It is felt that these discrepancies may disappear with an accurate, more uniform sampling procedure, and with a large, more balanced population.

Table VII summarizes the accuracy of the tests shown in Tables III through VI. It clearly shows that we have biased our test to correctly diagnose the stenotic bruit over the innocent bruit, and all four characteristic tests show good results in the t test between the stenotic patient mean and the stenotic population mean. The major peak counting test

TABLE IV

T TESTS ON THE ENERGY-BAND WIDTH OF THE BRUIT SPECTRA

Patient Tested			With Innocent Population			With Stenotic Population		
Number of Samples	Patient Identification Number	Patient's Mean Band Width	Degrees of Freedom	T Test with Innocent Population	Alpha Probability	Degrees of Freedom	T Test with Stenotic Population	Alpha Probability
Innocent Bruit Patients								
16	129768	130.	66	0.51	0.7	63	6.38	--
33	132323	97.	66	2.41	0.02	80	10.09	--
7	122252	163.	66	2.02	0.05	54	1.98	0.1
3	50268	89.	66	1.12	0.3	50	4.99	--
2	166971	130.	66	0.19	0.9	49	2.42	0.02
5	133441	200.	66	3.21	0.01	52	0.71	0.5
2	131824	208.	66	2.29	0.05	49	0.81	0.5
Stenotic Bruit Patients								
35	129379	191.	101	7.42	--	47	0.37	0.8
4	115326	133.	70	0.36	0.8	47	3.31	0.01
4	111111	228.	70	3.97	--	47	2.25	0.05
3	222222	231.	69	3.59	--	47	2.20	0.05
2	131921	120.	68	0.10	0.9	47	2.88	0.01
1	124918	180.	67	1.01	0.3	47	0.24	0.8

TABLE V

T TESTS ON THE ENERGY-MEAN FREQUENCY OF THE BRUIT SPECTRA

Patient Tested			With Innocent Population			With Stenotic Population		
Number of Samples	Patient Identification Number	Patient's Mean Energy Mean (cps)	Degrees of Freedom	T Test with Innocent Population	Alpha Probability	Degrees of Freedom	T Test with Stenotic Population	Alpha Probability
Innocent Bruit Patients								
16	129768	89.	66	0.73	0.5	63	5.31	--
33	132323	61.	66	2.88	0.01	80	11.03	--
7	122252	85.	66	0.20	0.9	54	4.08	--
5	133441	149.	66	3.89	--	52	1.35	0.2
3	50268	76.	66	0.29	0.01	50	3.26	0.01
2	116971	114.	66	1.19	0.3	49	0.78	0.5
2	131824	171.	66	3.32	0.01	49	1.93	0.1
Stenotic Bruit Patients								
35	129379	123.	101	6.40	--	47	1.14	0.3
4	115326	115.	70	1.78	0.1	47	1.01	0.4
4	111111	154.	70	3.78	--	47	1.55	0.3
3	222222	215.	69	6.06	--	47	4.92	--
2	131921	100.	68	0.66	0.6	47	1.50	0.3
1	124918	124.	67	1.11	0.3	47	0.23	0.9

TABLE VI

T TESTS ON THE ZERO-CROSSING FREQUENCY OF THE SAMPLES

Patient Tested			With Innocent Population			With Stenotic Population		
Number of Samples	Patient Identification Number	Patient's Mean Crossing Frequency	Degrees of Freedom	T Test with Innocent Population	Alpha Probability	Degrees of Freedom	T Test with Stenotic Population	Alpha Probability
Innocent Bruit Patients								
41	132323	146.	72	2.99	0.01	98	11.27	--
16	129768	181.	72	0.41	0.7	73	5.33	--
7	122252	212.	72	1.60	0.2	64	2.36	0.50
3	50268	167.	72	0.26	0.8	60	2.72	0.01
2	166971	249.	72	1.75	0.1	59	0.51	0.7
5	133441	322.	72	5.45	--	62	1.57	0.2
Stenotic Bruit Patients								
46	129379	257.	118	8.57	--	57	1.51	0.2
4	115326	249.	76	2.51	0.02	57	0.70	0.5
4	222222	465.	76	9.86	--	57	5.63	--
2	131921	199.	74	0.58	0.6	57	1.54	0.2
2	111111	399.	74	5.38	--	57	2.61	0.02
1	124918	263.	73	1.49	0.2	57	0.16	0.8

TABLE VII

T TEST ACCURACY*

Characteristic used for Test	Innocent Bruits Compared to		Stenotic Bruits Compared to		All Patients Compared to	
	Innocent Population $p=0.05$	Stenotic Population $p=0.01$	Innocent Population $p=0.05$	Stenotic Population $p=0.01$	Innocent Population $p=0.05$	Stenotic Population $p=0.01$
Major Peak Count	86%	57%	83%	100%	85%	77%
Zero Crossing Frequency	67%	50%	67%	83%	62%	62%
Energy-Mean Frequency	43%	57%	50%	83%	46%	69%
90% Energy Band Width	43%	43%	50%	67%	46%	54%

*Given in percentage of correct diagnosis.

is shown to be the best overall diagnostic tool, followed in accuracy by the zero crossing test, the energy-mean frequency test and the 90% energy band width test respectively.

The center or energy-mean frequency and band width of the bruits used in this study are shown for each individual in Tables IV and V. The mean values of all characteristics are shown for the two populations studied (Table VIII).

TABLE VIII

ZERO CROSSING, ENERGY-MEAN FREQUENCY, BAND WIDTH AND PEAK COUNT AVERAGES FOR THE TWO POPULATIONS STUDIED.

	Innocent	Stenotic
Energy Spectrum Mean Frequency	81.96 Hz	130.7 Hz
Energy Spectrum 90% Band Width	123.3 Hz	188.4 Hz
Zero Crossing Frequency	175.3 Hz	273.5 Hz
Major Spectral Peaks	5.23 Hz	7.55 Hz

Problem Areas

The author was responsible for choosing the period of the arterial noise to be sampled. The policy established was to exclude the first and second heart sounds which are transmitted through the blood and arise from the closing of heart valves. It has been shown (Braun, et al, 1966) that the cervical bruit can vary in length and in position relative to the first and second sounds. Braun theorized that this variation itself may be an identifying mark, since the stenotic bruit seems to be longer, extending closer to the second sound than the innocent bruit. Any identification of position is destroyed by omitting the first and second sounds, but the reasoning used here was that to find the characteristics unique to the bruit noise, all other sounds must be removed. Differences in length may show up in the band width test -- short samples having a wider energy band than long samples of the same signal. (Cochran, <u>et al</u>, 1966) But the samples taken in this study don't necessarily reflect the true bruit length since emphasis was placed on ending the sample just before onset of the second sound, rather than on trying to locate the end of the bruit noise. This could explain why the band width difference shown in Table VIII is opposite than that expected. If the stenotic bruit is longer, it should have the shorter band width. Another possible explanation is that the stenotic bruit has a more complex signal than the innocent.

For all patients contributing seven or more bruit samples to this study a sampling rate of 10,000 hertz was used, and band width studies show that the upper frequencies present are well under the suggested limit of 500 cycles per second. For patients with less than seven samples the earlier sampling method and slower sampling rate was used. Band width studies showed that the frequencies present in these bruits extended, in many cases, 50 to 100 hertz above the limit suggested by Lees and Dougherty (1966). Therefore, the spectra computed for these samples may not be correct, but with upper frequencies well under the limit suggested by sampling theory, these spectra will be an approximate representation of the actual spectra. A polynomial interpolation function has been used to convert our records to appropriate lengths. Thus, the effective sampling rate is slower and the upper signal frequency limit is lowered some. Nevertheless, all of the spectra used should still be good approximations.

There was some systematic error present in the sampling process. The sampling periods were determined as mentioned and the proper time delay settings were dialed on the digital controller. In several cases it was noticed that the digital controller was sampling earlier than was expected, but only by a fraction of a second which was enough to include the first heart sound. This could possibly be caused by a noise on the voice channel of the tape near the trigger signal.

The magnetic tape was handled a great deal in setting the proper time delays. It was necessary to stop and reverse the tape quite often in the area of the sampled bruits; the delays for as many as ten samples were measured from the same trigger signal. Since high quality tape was used for the recordings, tape handling should not have had much effect on the signal quality. The tape recorder used in this study was not extremely accurate, but it was a good quality, musical type, tape recorder which would be available to most doctors.

The actual accoustical signal originating in a stenotic lesion or other bruit production site may be quite different from the signal used in a study such as this because of several problems associated with the recording process. The effect that the skin may have as a filter on the signal is unknown and subject to change with the individual and with the location of the microphone. Some work has been done on this problem by lowering a speaker down the esophagus, but the interest in that study (Lepeschkin, et al, 1957) was centered on the chest cavity, not the neck area. According to Bruns' general theory (1959) a frequency shift can occur near sound producing orifices. The frequency heard upstream is equal to the vortex shedding frequency, but the frequency heard downstream from the orifice is lower due to vortex coalescence. This effect will also introduce a variable in the recorded signal which is a function of position, not of disease or lack of disease. The microphone used here introduced another change in the bruit waveform. It is a special type of microphone, used by the medical profession, which filters the signal so that it duplicates the sound heard through a stethoscope, but this filtering is not the most important effect of this type microphone.

> Even though we may know what the characteristics of the contact microphone itself are when placed against the chest, the various conditions or variables that exist, due to the amount of fat underlying the skin, the toneness of the skin, how hard you press, and the stiffness of the microphone itself, are all variables on which I cannot give you any data. They change from one subject to another and are just a mass of unknowns. (M. B. Rappaport, Sanborn Co., Boston, Mass. 1957)

It is felt that the problems with sample location, upper frequency limitations, tape handling and recording accuracy due to artery, tissue, microphone and recorder response were of some significance but did not invalidate the results of this study. The stated purpose of the investigation was accomplished by (1) identifying the stenotic from the innocent bruit by four methods, the most successful of which consisted of counting the major spectral peaks, and (2) discovering the important frequency components found in bruits (Table VIII).

VIII. CONCLUSIONS AND RECOMMENDATIONS

Summary of Results

The stated purpose of this study was to identify the stenotic bruit from the innocent bruit, and to label possible identifying features or characteristics of each. It was found that these two families could be reliably separated by counting the number of major peaks in a periodogram (a form of the energy density spectrum) of the bruit. From the results of a t test, rejecting the null hypothesis at the 0.01 level, significant differences were found between the following: (1) the mean number of spectral peaks in stenotic bruits (7.55) and in innocent bruits (5.23); (2) the mean zero crossing frequency for stenotic bruits (273.5 Hz) and for innocent bruits (175.3 Hz); (3) the average energy spectrum, mean frequency for stenotic bruits (130.7 Hz) and for innocent bruits (82.0 Hz); and (4) the mean of the energy spectrum 90% band width for stenotic bruits (188.4 Hz) and for innocent bruits (123.3 Hz).

Treating the accoustical signal as random stationary noise and examining the histogram for differences in variation and first moment was found to be unsatisfactory for differentiating between innocent and stenotic bruits. Although no definite conclusions about the mechanism of bruit can be drawn from these results, Appendix D suggests some possibilities.

Future Work

Some difficulty was encountered in sampling the bruit section of the auscultatory signal without including parts of the first and second heart sounds. By using the electrocardiograph signal to trigger the digital sampling process, it should be possible to pick out the bruit portion of the signal more quickly and reliably.

If the same number of discrete points could be used to completely describe each sample, a comparison of the periodograms would be more revealing by merit of having compared exactly the same frequency components in each periodogram. Eliminating the leakage in the frequency spectrum by digital filtering of the Fourier coefficients may be helpful in studying the spectrum. This technique was used by Maling (1967) to obtain a closer convergence to theory.

It is the author's belief that since zero crossing of the raw signal was so successful, a band filtered zero crossing analysis would be as reliable in diagnosis studies as it has proved to be in the case of heart murmurs. This type of analysis can be made by an electrical circuitry setup, thus eliminating the use of expensive computer time where extensive testing is involved.

APPENDIX

APPENDIX A

REVIEW OF SIGNAL THEORY AND FOURIER TECHNIQUES

Fourier Series

If a function f(t) can be expanded in the time series

$$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right) \qquad \text{(A1)}$$

then the coefficients of this series can be found by the Euler formulas for Fourier Coefficients:

$$a_n = \frac{2}{T} \int_{-T/2}^{+T/2} f(t) \cos\left[\frac{2n\pi t}{T}\right] dt, \quad n = 0,1,2,\ldots \qquad \text{(A2)}$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin\left[\frac{2n\pi t}{T}\right] dt, \quad n = 1,2,\ldots \qquad \text{(A3)}$$

For the time series in equation (1) to converge to the true value of the function, f(t), the signal f(t) must satisfy the Dirichlet conditions:[1]

> Within the finite time interval, -T/2 to +T/2, f(t) must be single valued; must have a finite number of maxima and minima; must possess a finite number of discontinuities; and must satisfy the inequality:
>
> $$\int_{-T/2}^{T/2} | f(t) | dt < \infty$$

---

[1] The actual time function f(t) corresponding to any physical signal will satisfy these conditions although some common mathematical representations do not. (Cooper and McGillem, 1967)

With a little manipulation, the Fourier series in equation (A1) can be expressed in complex notation. Using the relationship

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \text{(A4)}$$

it can be shown that

$$\cos nx = 1/2\ (e^{inx} + e^{-inx}) \qquad \text{(A5)}$$

and

$$\sin nx = 1/2\ (e^{inx} - e^{-inx}),\ \text{where } i = \sqrt{-1}. \qquad \text{(A6)}$$

Now using equations (A5) and (A6) equation (A1) becomes

$$f(x) = c_o + \sum_{n=1}^{\infty} (c_n e^{inx} + k_n e^{-inx}) \qquad \text{(A7)}$$

where $x = 2\pi t/T$, $c_o = \frac{a_o}{2}$, $c_n = (a_n - ib_n)/2$, and $k_n = (a_n + ib_n)/2$. The coefficients can be found by these relations;

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(x)\ e^{-inx}\ dx \qquad \text{(A8)}$$

$$k_n = \frac{1}{T}\int_{-T/2}^{T/2} f(x)\ e^{inx}\ dx \qquad \text{(A9)}$$

But it can be seen that $k_n = c_{-n}$, so we can write

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx} = \sum_{n=-\infty}^{\infty} c_n e^{in2\pi t/T} \qquad \text{(A10)}$$

where c_n is now expressed by

$$c_n = \frac{1}{T}\int_{-T/2}^{+T/2} f(x)\ e^{-inx}\ dx;\ n = 0, 1, 2, \ldots \qquad \text{(A11)}$$

Now let us call the fundamental component frequency ω_0, recognizing that this is also the spacing between components or harmonics ($\omega_0 = 2\pi/T$). Now the complex Fourier series can be written

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \exp(in\omega_0 t) \tag{A12}$$

and the coefficients can be written

$$c_n = \omega_0/2\pi \int_{-T/2}^{T/2} f(t) \exp(-i\omega_0 nt)\, dt \tag{A13}$$

Fourier Transform

The Fourier integral relation can be derived from the complex Fourier series by taking the limit as the period, T, approaches infinity. Letting $T \to \infty$, $\omega_0 \to d\omega$, $n \to \infty$, and $n\omega_0 \to \omega$, the summation can be written as an integral

$$f(t) = \int_{-\infty}^{+\infty} \exp(i\omega t)[d\omega/2\pi \int_{-\infty}^{+\infty} f(t) \exp(-i\omega t)dt] \tag{A14}$$

or

$$f(t) = 1/2\pi \int_{-\infty}^{+\infty} [\int_{-\infty}^{+\infty} f(t) \exp(-i\omega t)\, dt] \exp(i\omega t)\, d\omega. \tag{A15}$$

Equation (A15) is the Fourier integral relation, and the inner integral is called the Fourier Transform of f(t),

$$\mathfrak{F}[f(t)] = F(i\omega) = \int_{-\infty}^{+\infty} f(t) \exp(-i\omega t)\, dt. \tag{A16}$$

The function f(t) can be obtained from $F(i\omega)$ using equation (A15). We actually have two complete representations of the function: f(t) in the time domain and $F(i\omega)$ in the frequency domain.

In representing the bruit signal it can be assumed that the value of the function is zero outside of the sampling period; since this part of the signal doesn't interest us in this particular investigation. So with the stipulation that $f(t)=0$; $t<T/2$, $t>T/2$, the transform limits can be changed, and

$$F(i\omega) = \int_{-T/2}^{T/2} f(t) \exp(-i\omega t)\, dt. \qquad \text{(A17)}$$

Notice the similarity between equation (A13) the Fourier series coefficients, and equation (A17), the Fourier transform coefficients.

Finite Discrete Analysis

"If digital analysis techniques are to be used for analyzing a continuous waveform then it is necessary that the data be sampled (usually at equally spaced intervals of time) in order to produce a time series of discrete samples which can be fed into a digital computer. As is well known (Cochran, et al., 1966) such a time series completely represents the continuous waveform, provided this waveform is frequency band-limited ..." (Cochran, et al., 1967), and provided that the upper limit in frequency is given by ω, and

$\omega \leq 0.1\pi/\Delta t$ radians per second, (Lees and Dougherty, 1966) where Δt is the time between sampled points.

Let the continuous periodic function $x(t)$ in the range $-\frac{T}{2} \leq t \leq \frac{T}{2}$ (where $T = N\Delta t$) be represented by the finite discrete function $x(j\Delta t)$ consisting of N total samples with $j = 0,1,2,...N-1$. To represent this discrete function, $x(j\Delta t)$ we can use the discrete Fourier series,

$$x(j\Delta t) = x(j\frac{T}{N}) = \sum_{n=-\infty}^{\infty} c(n) \exp(\frac{2\pi inj}{N}) \qquad (A18)$$

where

$$c(n) = \frac{1}{N} \sum_{j=0}^{N-1} x(j\Delta t) \exp(\frac{-2\pi inj}{N}), \; n=0,\pm 1,...\pm\infty \qquad (A19)$$

(Cooley, et al, 1967)

The finite discrete Fourier transform can also be used to represent $x(j\Delta t)$ over the interval of interest in the following manner:

$$x(j\Delta t) = \frac{1}{N} \sum_{n=0}^{N-1} c_p(n) \exp(\frac{2\pi inj}{N}), \; j=0,1,2,...N-1 \qquad (A20)$$

where

$$c_p(n) = \sum_{j=0}^{N-1} x(j\Delta t) \exp(\frac{-2\pi inj}{N}), \; n=0,1,2,...N-1. \qquad (A21)$$

(Cochran, et al, 1967)

The following development of the relationship between the Fourier series and the finite Fourier transform is quoted from Cooley, et al, 1967.

RELATIONSHIP BETWEEN THE FOURIER SERIES AND THE FINITE FOURIER TRANSFORM

Suppose, we have a function $x(t)$ which is periodic of period T. Then x(t) has a Fourier series expansion

$$x(t) = \sum_{n=-\infty}^{\infty} c(n)e^{-2\pi i(nt/T)} \qquad (A22)$$

where the c(n) are given by

$$c(n) = \frac{1}{T}\int_0^T x(t)e^{-2\pi i(nt/T)}\,dt \qquad (A23)$$

Now, if we sample x(t) at N equally spaced points between 0 and T, we generate the sequence $x(j\Delta t)$ where $t = T/N$. This sequence is periodic of period N; substituting in (A22), we obtain

$$\begin{aligned} x(j\Delta t) = x(jT/N) &= \sum_{n=-\infty}^{\infty} c(n)e^{2\pi i(nj/N)} \\ &= \sum_{n=0}^{N-1} \Big[\sum_{\ell=-\infty}^{\infty} c(n+N\ell)\Big]e^{2\pi i(nj/N)} \\ &= \sum_{n=0}^{N-1} c_p(n)e^{2\pi i(nj/N)} \end{aligned} \qquad (A24)$$

Thus, we see that $x(j\Delta t)$ is the finite Fourier transform of

$$c_p(n) = \sum_{\ell=-\infty}^{\infty} c(n+N\ell)$$

This is summarized by Theorem 2.

<u>Theorem 2</u>

If the period function x(t) with period T has the Fourier series expansion c(n),

$$x(t) \leftrightarrow c(n)$$

then the periodic sequence $x(j\Delta t)$ of period N, where $t = T/N$, has the finite Fourier transform $c_p(n)$:

$$x(j\Delta t) \leftrightarrow c_p(n) = \sum_{\ell=-\infty}^{\infty} c(n+\ell N)$$

From this we see that in using the algorithm for harmonic analysis we should pick an N such that the error due to aliasing in the approximation of c(n) by $\Sigma_{\ell=-\infty}^{\infty} c(n+\ell N)$ is acceptable. Then let $\Delta t = T/N$ form $x(j\Delta t)$, and take its finite Fourier transform. Again, as with the Fourier transform, if we let

$$c_p(n) \quad \sum_{\ell=-\infty}^{\infty} c(n+\ell N)$$

then $c_p(n) \simeq c(n)$ for $n=0,1,2,\ldots,N/2$
$c_p(N-n) \simeq c(-n)$ for $n=-1,-2,\ldots,-N/2$ (Cooley, *et al*, 1967)

If we pick N large enough so that the frequency components for $|n| \geq N/2$ are negligible as would be true for a band limited function with no components above $\omega = \pi/\Delta t$, then the approximations in Theorem 2 above should become exact equalities:

$$c_p(n) = \sum_{\ell=-\infty}^{\infty} c(n+\ell N) = c(n), \quad n = 0,+1,+2,\ldots+N/2$$

and

$$c_p(N-n) = c(-n), \quad n = -1,-2,\ldots-N/2.$$

As has been shown by Lees and Dougherty (1966), the approximations in Theorem 2 can be quite accurate even for non-band limited functions provided we look only at the coefficients for frequencies lower than or equal to $0.1\pi/\Delta t$ radians per second.

Energy Spectrum

The energy spectrum, P(f), is obtained from the Fourier transform coefficients and is defined by

$$P(f) = \lim_{T\to\infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} x(t) e^{-i\omega t} \, dt \right|^2$$

This definition includes positive and negative frequencies, but the usual reference to the energy spectrum is for positive frequencies referring to 2P(f) over the range $0 \leq f$.

Since we have considered our signal to have zero value for $-T/2 < t < T/2$, our spectrum is given by

$$P(f) = (1/T)\ F(iw)^2$$

We are only interested here in the relative magnitudes of the spectral values so the constant can be dropped and the spectrum can be expressed as

$$2P(f) = A_n^{\ 2} + B_n^{\ 2}.$$

APPENDIX B

FORTRAN PROGRAMS

Contents

```
C     HISTOGRAM CHECK
C
C     THIS PROGRAM CHECKS THE HISTORAM FOR FIRST
C     MOMENT VALUE, VARIATION, AND STANDARD DEVIATION,
C     AND FINDS THE PSEUDO-FREQUENCY OR ZERO-CROSS FREQUENCY
C     OF THE BRUIT SIGNAL
      INTEGER BLANK, CLEAR,REM
      COMMON IX(9010),NPTS,DT,LERR
      DIMENSION  REM(13),DISC(6)
      LOGICAL LIT,BRUIT
      DATA BLANK/'    '/
      LIT=.TRUE.
      WRITE(108,153)
  153 FORMAT(1X,'STENOTIC'T16,'SAMPLE'T32,'MAX  MEAN  MOMENT  NCROS'T60,
     1 'CROSS FREQ' T77,'VARIATION'T91,'STANDARD DEV')
      WRITE(108,6)
   99 CONTINUE
      IREAD=1
      READ(105,150,END=999,ERR=170) CLEAR
  150 FORMAT(1X,A4)
      IF(CLEAR.NE.BLANK) GO TO 170
   98 CALL CHECK(LIT)
      IF(.NOT.LIT) GO TO 170
      LERR=0
      IREAD=2
      READ(NDSK(1),5,END=999,ERR=888)
      CALL CHECK(LIT)
      IF(.NOT.LIT) GO TO 170
      IREAD=3
      READ(NDSK(1),7,END=999,ERR=888) BRUIT,DISC
      CALL CHECK(LIT)
      IF(LIT) GO TO 170
      IREAD=4
      READ(NDSK(1),2,END=999,ERR=888) NPTS
      CALL CHECK(LIT)
      IF(LIT) GO TO 170
      IREAD=5
      READ(NDSK(1),4,ERR=888)NREC
```

```
      DT=.0001
    2 FØRMAT(14I5)
    4 FØRMAT(I5)
    5 FØRMAT(80H
     1                                        )
    6 FØRMAT(//)
    7 FØRMAT(L6,6A4)
      IF (NPTS)170,170,175
  175 TPI=6.2831852
    3 XNPTS=NPTS
      PRD=(XNPTS-1.)*DT
      FST=TPI/PRD
      FINC=FST
      N1=1
      IN14=14
      N14=IN14
      NCARD=(NPTS/IN14)
      CRD=NCARD
      CDS=XNPTS/IN14
      IF(CDS.EQ.CRD) GØ TØ 51
      NCARD=(NPTS/IN14)+1
   51 DØ 60 L=1,NCARD
      CALL CHECK(LIT)
      IF (LIT)GØ TØ 170
      IREAD=6
      READ(NDSK(1), 2,ERR=888)(IX(I),I=N1,N14)
   42 FØRMAT(15I5)
      DØ 43 IL=N1,N14
   43 CØNTINUE
      N1=N1+IN14
      N14=N14+IN14
   60 CØNTINUE
      TL=0
      DØ 41 I=1,NPTS
      TL=TL+IX(I)
   41 CØNTINUE
      MEAN=TL/NPTS
      KMAX=0
      DØ 40 I=1,NPTS
```

```
C     REMOVE THE DC COMPONENT OF THE SIGNAL
      IX(I)=IX(I)-MEAN
      IF(IABS(IX(I)).LT.KMAX)  GO TO 40
      KMAX=IABS(IX(I))
      IMAX=IX(I)
   40 CONTINUE
      CALL ZCROS(NCROS,XHZ)
      CALL      DATAID(MEAN,KMAX,MOMENT,VARIAN,STDEV)
      WRITE(106,7)BRUIT,DISC
      WRITE(106,155)BRUIT,(DISC(II),II=1,3),IMAX,MOMENT,NCROS,XHZ,VARIAN
     1 ,STDEV
  155 FORMAT(1X,L1,1X,3A4,1X,I4,1X,I5,1X,I5,3(1X,E14.5))
      WRITE(108,154)BRUIT,DISC,IMAX,MEAN,MOMENT,NCROS,XHZ,VARIAN,STDEV
  154 FORMAT(L4,1X,6A4,1X,I4,I6,2X,I6,2X,I5,3(2X,E14.5))
      GO TO 99
C     ERROR ROUTINE
  170 CONTINUE
      WRITE(108,151)
  888 WRITE(108,2)IREAD,LERR
  151 FORMAT('ERROR IN READING THIS SAMPLE')
  171 READ(105,150,END=999,ERR=888) CLEAR
      IF(CLEAR.NE.BLANK) GO TO 171
      GO TO 98
  999 CALL EXIT
      END
```

```
C      PROGRAM  PEAK
C
C      THIS PROGRAM PLOTS THE ENERGY SPECTRUM,
C      PRINTS THE FIRST 50 COEFFICIENTS,
C      AND COUNTS THE MAJOR PEAKS
       DIMENSION RM(8400),RN(4200),DATA(2,4200),TABLE(4200)
       DIMENSION IX(8400)
       DIMENSION DISC1(20)
       DIMENSION DISC(6)
       DIMENSION PDS(50)
       COMMON M,N,RM,RN
       LOGICAL LIT,BRUIT
       DATA BLANK/'    '/
       LIT=.TRUE.
       TPI=6.2831853
       IS=0
       REWIND 1
    99 CONTINUE
       READ(105,150,END=999,ERR=170) CLEAR
       IF(CLEAR.NE.BLANK) GO TO 170
    98 CALL CHECK(LIT)
       IF(.NOT.LIT) GO TO 170
       READ(NDSK(1),152,END=999,ERR=888)DISC1
   152 FORMAT(20A4)
       WRITE(108,152)DISC1
       CALL CHECK(LIT)
       IF(.NOT.LIT) GO TO 170
       READ(NDSK(1),7,END=999,ERR=888) BRUIT,DISC
       WRITE(108,7)BRUIT,DISC
       CALL CHECK(LIT)
       IF(LIT) GO TO 170
       READ(NDSK(1),3,END=999,ERR=888) M
       MDIM=8399
       IF(M.GT.MDIM) GO TO 170
       CALL CHECK(LIT)
       IF(LIT) GO TO 170
       READ(NDSK(1),1,END=999,ERR=888) DT
     1 FORMAT(1X,F14.3)
```

```
    2 FORMAT(1X,I4)
    3 FORMAT(I5)
    4 FORMAT(1X,I5)
    6 FORMAT(1X,//)
    7 FORMAT(1X,L5,6A4)
   12 FORMAT(4(I6,2E14.6))
    8 FORMAT(4(2X,I4,2(1X,E13.6)))
    5 FORMAT(80H
     1                                )
  110 FORMAT(80H
     1                                )
  115 FORMAT(1X,'     MEAN= 'I5)
  120 FORMAT(1X, 9F12.3)
  125 FORMAT('1')
  131 FORMAT(14F5.0)
  150 FORMAT(1X,A4)
C       READ IN DATA
        READ(105,42,END=999,ERR=888)(RM(I),I=1,M)
   42 FORMAT(15F5.0)
      DO 38 I=1,M
   38 IX(I)=RM(I)
C       FIND  POWER OF TWO OR NEXT SMALLER
      RDNGS=M
      NPTS=M
      PT=ALOG(RDNGS)/ALOG(2.0)
      IT=IFIX(PT)
      N=2**IT
      RMAX=ABS(RM(1))
      DO 25 J=1,M
      IF(ABS(RM(J))-RMAX) 25,25,24
   24 RMAX=ABS(RM(J))
   25 CONTINUE
      KMAX=RMAX
      DO 32 I=1,M
      RM(I)=1000.*RM(I)/RMAX
   32 CONTINUE
      CALL GNRATE
C     WE HAVE GENERATED RN(N) FROM RM(M), MUST ADD IMAGINARY PART
      DO 20 I=1,N
```

```
      DATA(1,I)=RN(I)
      DATA(2,I)=0.0
   20 CONTINUE
      CALL TAPER(DISC1,DISC,NPTS,DT,IX,BRUIT)
      CALL FRT  (DATA,TABLE,IS,N,-1)
      IS=1
      WRITE(108, 36)
   36 FORMAT( /'          HZ                PDS                AN
     1BN            XLAM   ' /)
      MEAN=IFIX(DATA(1,1)/FLOAT(N))
      WRITE(108,115)MEAN
      MF=N/2-1
      MFT=MF
      WRITE(108,37) MF
   37 FORMAT(1X,'NUMBER OF COEFFICIENTS = ' I5)
      IF (MFT.GT.50) MFT=50
      WRITE(106,7) BRUIT,DISC
      DO 111 K=1,MFT
      HZ=K/(DT*(M-1))
      XLAM=TPI*HZ
      AN=2.*DATA(1,K+1)/FLOAT(N)
      BN=2.*DATA(2,K+1)/FLOAT(N)
      PDS(K)=AN**2+BN**2
      WRITE(108,112) K,HZ,PDS(K),AN,BN,XLAM
      WRITE(106,112) K,HZ,PDS(K),AN,BN,XLAM
  112 FORMAT(1X,I3,7(2X,E13.6))
  111 CONTINUE
      WRITE(108,125)
      WRITE(108,7)BRUIT,DISC
      WRITE(108,6)
      CALL PEAK(MFT,PDS)
      CALL PLOT2(PDS,MFT,.TRUE.)
      WRITE(108,125)
      GO TO 99
C     ERROR ROUTINE
  170 CONTINUE
      WRITE(108,151)
  888 WRITE(108,6)
  151 FORMAT('ERROR IN READING THIS SAMPLE')
```

```
171 READ(105,150,END=999,ERR=888) CLEAR
    IF(CLEAR.NE.BLANK) GO TO 171
    GO TO 98
999 END FILE 1
    REWIND 1
    END
```

```
C                       BRUIT PROGRAM 10   J M BOWERS
C         CALCULATION OF THE MEAN FREQUENCY AND THE
C         WIDTH OF THE FREQUENCY BAND (ABOUT THE MEAN)
C         WHICH CONTAINS 90% OF THE SIGNAL ENERGY
          LOGICAL    BRUIT
          DIMENSION RM(8100),RN(4200),DATA(2,4200)
          DIMENSION TABLE(8400),DISC1(20)
          DIMENSION DISC(6),PDS(2100),HZ(2100)
          COMMON M,N,RM,RN
          REWIND 1
          LINE=1
       10 CALL RTAPE (DISC1,DISC,DT,BRUIT,&99)
C         FIND POWER OF 2 FOR INTERPOLATION SCHEME
          FM=M
          PT=ALOG(FM)/ALOG(2.0)
          IT=IFIX(PT)
          N= 2**IT
C         NORMALIZE SIGNAL : MAX=1000
          RMAX=ABS(RM(1))
          DO 25 J=1,M
          IF(RM(J)-RMAX) 25,25,20
       20 RMAX=ABS(RM(J))
       25 CONTINUE
          DO 30 I=1,M
       30 RM(I)=1000.*RM(I)/RMAX
C         GENERATE INTERPOLATED SAMPLE RN(N)
          CALL GNRATE
          DO 35 I=1,N
          DATA(1,I)=RN(I)
       35 DATA(2,I)=0.0
          IS=0
          IFRD=-1
          CALL FRT(DATA,TABLE,IS,N,IFRD)
          MIDF=N/2-1
          HAF=0
          DO 40 K=1,MIDF
          HZ(K)=K/(DT*(M-1))
          AN=2.*DATA(1,K+1)/FLOAT(N)
```

```
      BN=2.*DATA(2,K+1)/FLØAT(N)
      PDS(K)=AN**2+BN**2
   40 HAF=HAF+PDS(K)
      HALF=0
      DØ 45 I=1,MIDF
      PINC=PDS(I)
      HALF=HALF+PINC
      HAF  =HAF  -PINC
      IF(HALF-HAF) 45,50,50
   45 CØNTINUE
   50 FMEAN=HZ(I)
      MEEN  =I
      PMEAN =PDS(I)
      PTØT  =0.0
      DØ 55 J=1,MIDF
   55 PTØT  = PTØT+PDS(J)
      P9=0.9*PTØT
      PSUB  =PMEAN
   60 DØ 85 J=1,MIDF
      IF (MEEN - J) 70,70,65
   65 PSUB=PSUB+PDS(MEEN-J)
      ILØW =MEEN-J
   70 IF (MIDF-MEEN-J) 80,75,75
   75 PSUB  =PSUB+PDS(MEEN+J)
      IHIGH =J+MEEN
   80 IF ( PSUB-P9) 85,90,90
   85 CØNTINUE
   90 WBAND =HZ(IHIGH)-HZ(ILØW)
      WRITE(106,100) BRUIT,DISC,FMEAN,WBAND,
     1 HZ(ILØW),HZ(IHIGH)
      IF(LINE-1) 115,115,110
  110 IF(LINE-37)120,115,115
  115 WRITE(108,101)
      LINE=3
  120 WRITE(108,100) BRUIT,DISC,FMEAN,WBAND,
     1 HZ(ILØW),HZ(IHIGH)
      LINE=LINE+1
  100 FØRMAT(L6,6A4,4(2X,F10.4))
  101 FØRMAT(1H1,'STEN',T9,'DISCRIPTIØN',T33,'MEAN FREQ   BAND WIDTH'
```

```
  1 '  LØW FREQ     HIGH FREQ'/)
    GØ TØ 10
 99 REWIND 1
    END
```

```
C     T TEST OF TWO MEANS
C
C     THIS PROGRAM WILL ACCEPT TWO SETS OF SUBGROUPS
C     BELONGING TO TWO DIFFERENT POPULATIONS
C     AND PERFORM A T TEST ON THE GROUPS OR POPULATIONS
C     ALSO CHECKING EACH SUBGROUP  BY A T TEST  WITH
C     THE TWO POPULATIONS
      DIMENSION F(100,2),TR(100,2),NFS(10),NTS(10)
      DIMENSION A1(10),A2(10),T(10),TI(10),TS(10)
      LOGICAL BRUIT,IN,ST
      IN=.FALSE.
      ST=.TRUE.
      READ (105,115)
  115 FORMAT('                                                  ')
      II=1
      I=1
      NR=1
      IJ=1
      J=1
   30 CONTINUE
   35 READ(105,105,END=60) BRUIT,IDISC,XHZ
  105 FORMAT(5X,L1,I6,20X,F10.4)
      GO TO (40,65)NR
   40 IF(BRUIT) GO TO 50
   45 F(J,2)=IDISC
      F(J,1)=XHZ
      J=J+1
      GO TO 30
   50 NFS(II)=J-1
      II=II+1
      GO TO 30
   60 NR=NR+1
      IF(NR.EQ.3) GO TO 85
      GO TO 35
   65 IF(.NOT.BRUIT) GO TO 80
   70 TR(I,2)=IDISC
      TR(I,1)=XHZ
      I=I+1
```

```
      GO TO 35
   80 NTS(IJ)=I-1
      IJ=IJ+1
      GO TO 35
   85 CONTINUE
      IF1=J-1
      IF2=I-1
      PRINT 106,((F(L,M),M=1,2),L=1,IF1)
      PRINT 110
      PRINT 106,((TR(L,M),M=1,2),L=1,IF2)
  106 FORMAT (1X,F10.3,1X,F10.0)
      PRINT 110
  110 FORMAT(1X,/)
      IQ=II-1
      PRINT 112,(NFS(IP),IP=1,IQ)
      IQ=IJ-1
      PRINT 112,(NTS(IP),IP=1,IQ)
  112 FORMAT(11(1X,I7))
      PRINT 125
  125 FORMAT('1')
      I1=1
      I2=1
      CALL T TEST(F,TR,IN,I1,I2,IF1,IF2,T,A1,A2)
      PRINT 115
      PRINT 104,A1(1),NFS(II-1),A2(1),NTS(IJ-1),T(1)
  104 FORMAT(                                           //,' INNOCENT POPUL
     1ATION MEAN=',2X,E14.5,'  FOR',I5,'  SAMPLES',//,' STENOTIC POPULATI
     2ON MEAN=',2X,E14.5,'  FOR',I5,'  SAMPLES',////,' T TEST OF INNOCENT
     3 POPULATION VERSUS STENOTIC POPULATION YIELDS',F7.3,///)
      PRINT 107
  107 FORMAT(' # SAMPLES | T TEST OF |   MEAN   |   VS INNOC SAMPLE POP
     1 |   VS STEN SAMPLE POP   | ')
  103 FORMAT(4X,I4,3X,'|',3X,I6,2X,'|',1X,F8.3,1X,'|',5X,F8.3,10X,'|',5X
     1  , F8.3  )
      PRINT 109
  109 FORMAT(' -----------------------------------------------------------
     1------------------------------------------------')
      PRINT 110
      I2=1
```

```
      L=0
   94 L=L+1
C     II IS ONE MORE THAN # OF MEANS,I&J ARE ONE MORE THAN # OF SAMPLES
      IF (II-L-1.EQ.0) GO TO 95
      NUMB=NFS(II-L)-NFS(II-L-1)
      I1=NFS(II-L-1)+1
      GO TO 96
   95 I1=1
      NUMB=NFS(1)
   96 IF1=NFS(II-L)
      IDN=F(I1,2)
      IF2=J-1
      CALL TTEST(F,F,ST,I1,I2,IF1,IF2,TI,A1,A2)
      IF2=I-1
      CALL TTEST(F,TR,IN,I1,I2,IF1,IF2,TS,A1,A2)
      PRINT 103,NUMB,IDN,A1(1),TI(1),TS(1)
      PRINT 110
      IF(I1.GT.1) GO TO 94
      L=0
   97 L=L+1
      IF(IJ-L-1.EQ.0) GO TO 98
      NUMB=NTS(IJ-L)-NTS(IJ-L-1)
      I1=NTS(IJ-L-1)+1
      GO TO 99
   98 I1=1
      NUMB=NTS(1)
   99 IF1=NTS(IJ-L)
      IDN=TR(I1,2)
      IF2=I-1
      CALL TTEST(TR,TR,ST,I1,I2,IF1,IF2,TS,A1,A2)
      IF2=J-1
      CALL TTEST(TR,F,IN,I1,I2,IF1,IF2,TI,A1,A2)
      PRINT 103,NUMB,IDN,A1(1),TI(1),TS(1)
      PRINT 110
      IF(I1.GT.1) GO TO 97
      PRINT 125
      END
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C*THIS SUBROUTINE PERFORMS THE FAST FOURIER TRANSFORM ALGORITHM OF
C OF COOLEY-TUKEY
C
C
C    CALL STATEMENT:CALL FRT(A,TABLE,IS,NCPLX,ISIGN)
C
C    PARAMETERS: A=THE ARRAY OF NCPLX COMPLEX NUMBERS TO BE TRANSFORMED,(
C                      WHERE NCPLX=2**(SOME INTEGER)). DIMENSION OF A IS 2*NCPLX
C                      THE ARRAY A STORES COMPLEX NUMBERS AS ORDERED PAIRS
C                      OF REAL NUMBERS WITH THE REAL PART OF THE WORD IN ODD
C                      ADDRESS STORAGE LOCATIONS AND THE IMAGINARY PART OF THE
C                      NUMBER IN THE IMMEDIATELY ADJACENT NEXT EVEN ADDRESS
C                      STORAGE LOCATION.
C
C                NCPLX=NUMBER OF COMPLEX NUMBERS TO BE TRANSFORMED.
C                          A MUST BE DIMENSIONED AS TWICE THIS NUMBER IN THE
C                          CALLING PROGRAM.
C
C                TABLE=AN ARRAY OF TABULATED VALUES OF SINE AND COSINE.
C                          AFTER AN INITIAL CALL SETTING IS=0, THE SUBROUTINE
C                          MAY BE SUBSEQUENTLY CALLED ANY NUMBER OF TIMES
C                          WITH IS=1 WITH AN EXECUTION TIME SAVING OF APPROXIMATELY
C                          1/3. DIMENSION OF TABLE IS 2*(NCPLX-1) AND MUST
C                          BE SO DIMENSIONED IN THE CALLING PROGRAM.
C
C                ISIGN=FIXED POINT VARIABLE GIVING THE DIRECTION OF THE
C                         TRANSFORMATION. SETTING ISIGN=-1  GIVES THE FOURIER
C                         SPECTRUM*NCPLX;SETTING ISIGN=+1  GIVES THE INVERSE
C                         TRANSFORM.
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE FRT(A,TABLE,IS,NCPLX,ISIGN)
      DIMENSION A(1),TABLE(1)
      N=2*NCPLX
      J=1
      DO 11 I=1,N,2
      JP1=J+1
      IP1=I+1
```

```
      IF(I-J)2,4,4
    2 TR=A(J)
      TI=A(JP1)
      A(J)=A(I)
      A(JP1)=A(IP1)
      A(I)=TR
      A(IP1)=TI
    4 M=N/2
    5 IF(M-J)6,11,11
    6 J=J-M
      M=M/2
      IF(M-2)11,5,5
   11 J=J+M
      MMAX=2
      MCOS=-1
   13 IF(MMAX-N)14,99,99
   14 INCR=2*MMAX
      PIMI=3.14159265/FLOAT(MMAX)
      DO 21 M=1,MMAX,2
      MCOS=MCOS+2
      MSIN=MCOS+1
      IF(IS)16,16,17
   16 ANG=PIMI*FLOAT(M-1)
      TABLE(MCOS)=COS(ANG)
      TABLE(MSIN)=SIN(ANG)
   17 WI=TABLE(MSIN)
      IF(ISIGN)18,99,19
   18 WI=-WI
   19 DO 21 I=M,N,INCR
      J=I+MMAX
      JP1=J+1
      IP1=I+1
      TR=TABLE(MCOS)*A(J)-WI*A(JP1)
      TI=TABLE(MCOS)*A(JP1)+WI*A(J)
      A(J)=A(I)-TR
      A(JP1)=A(IP1)-TI
      A(I)=A(I)+TR
   21 A(IP1)=A(IP1)+TI
      MMAX=INCR
```

```
      GO TO 13
   99 RETURN
      END


C     THIS SUBROUTINE COMPUTES THE NUMBER OF ZERO-CROSSINGS IN THE
C     SAMPLE (NCROS) AND THE PSEUDOFREQUENCY OF CROSSING (XHZ)
      SUBROUTINE ZCROS(NCROS,XHZ)
      COMMON IX(9010),NPTS,DT,LERR
      NCROS=0
      NP1=NPTS-1
      DO 10 J=2,NP1
      IF(IX(J).EQ.IX(J-1)) GO TO 10
    2 IF(IX(J)) 4,10,5
    4 IF(IX(J+1))10,9,9
    5 IF(IX(J+1)) 9,9,10
    9 NCROS=NCROS+1
   10 CONTINUE
      XHZ=FLOAT(NCROS)/(DT*FLOAT(NPTS))
   20 RETURN
      END
```

```
      SUBROUTINE GNRATE
      DIMENSION RM(8400),RN(4200)
      COMMON M,N,RM,RN
C     SUBROUTINE GNRATE PRODUCES THE ARRAY RN(I) HAVING N TOTAL ELEMENTS
C     FROM THE ARRAY RM(I) HAVING M TOTAL ELEMENTS BY USE OF A FOUR TERM
C     LEGRANGIAN INTERPOLATION SCHEME (NOTE RM(1)=RN(1), RM(M)=RN(N)).
C     VARIABLES...
C           RM    GIVEN ARRAY
C           RN    ARRAY TO BE GENERATED
C           M     TOTAL NUMBER OF ELEMENTS IN RM
C           N     TOTAL NUMBER OF ELEMENTS IN RN
C           IN    ELEMENT NUMBER OF RN BEING COMPUTED
C           IM    NEAREST LOWER ELEMENT OF RM LESS ONE
C           T     EXACT FLOATING POINT LOCATION IN THE ARRAY RM OF IN
C           X     T-IM
C           DN    (M-1)/(N-1), RATIO OF THE DISTANCE BETWEEN INDICIES OF
C                              RM TO RN.
      A= M-1
      B= N-1
      DN= A/B
      RN(1)= RM(1)
      RN(N)= RM(M)
      NL1= N-1
      DO 40 IN=2,NL1
      T= IN-1
      T= T*DN+1.
      IM= T-1
C     CHECK FOR PROXIMITY OF LOWER BOUND.
      IF(IM-1)32,38,34
   32 IM=1
      GO TO 38
C     CHECK FOR PROXIMITY OF UPPER BOUND
   34 IF(IM+3-M)38,38,36
   36 IM= M-3
   38 A= IM
      X= T-A
      RN(IN)=      (X-1.)*(X-2.)*(X-3.)*RM(IM)/(-6.)
     1      +(X)*        (X-2.)*(X-3.)*RM(IM+1)/(+2.)
```

```
    2          +(X)*(X-1.)*          (X-3.)*RM(IM+2)/(-2.)
    3          +(X)*(X-1.)*(X-2.)*          RM(IM+3)/(+6.)
   40 CONTINUE
      RETURN
      END


C
C     CARD CHECKING SUBROUTINE     READS A CARD AND STORES IT ON THE
C     DISK    THE VALUE OF LIT IS SET TRUE FOR A LITERAL STRING  FALSE
C     FOR A NUMERIC STRING
      SUBROUTINE CHECK(LIT)
      COMMON IX(9010),NPTS,DT,LERR
      DIMENSION CARD(80)
      LOGICAL LIT
      INTEGER CARD,BL,NEG,DEC,L0,L9
      DATA L0,L9,DEC,BL,NEG/'0','9','.',' ','-'/
      LERR=LERR+1
      LIT=.FALSE.
      READ(105,152,END=999,ERR=11) CARD
  152 FORMAT(80A1)
      WRITE(NDSK(1),152)CARD
      DO 10 ICH=1,72
      IF(CARD(ICH).GE.L0.AND.CARD(ICH).LE.L9) GO TO 10
      IF((CARD(ICH).EQ.DEC).OR.(CARD(ICH).EQ.BL).OR.(CARD(ICH).EQ.NEG))
     1  GO TO 10
      LIT=.TRUE.
   10 CONTINUE
   60 RETURN
    4 FORMAT(I5)
   11 WRITE(108,4) LERR
      STOP 1
  999 STOP 2
      END
```

```
      SUBRØUTINE PLØT2(X,N,BAR)
C     GRØUP; BASIC
      REAL X(N),HEAD(10)
      INTEGER  LINE(100),BLANK,STAR
      LØGICAL BAR
      DATA BLANK,STAR/' ','*'/
      IF(N.LT.1)GØ TØ 25
      WRITE( 108,502)
      DØ 1 I=1,100
    1 LINE(I)=BLANK
      XMAX=-1.E70
      XMIN= 1.E70
      DØ 2 I=1,N
      IF(X(I).LT.XMIN) XMIN=X(I)
      IF(X(I).GT.XMAX) XMAX=X(I)
    2 CØNTINUE
      IF(XMAX-XMIN)25,3,4
    3 XMAX=XMIN+1.
      XMIN=XMIN-1.
    4 CØNTINUE
      DØ 5 I =1,10
      Z=I
    5 HEAD(I)=(XMAX-XMIN)*Z/10.+XMIN
      WRITE( 108,3001)
      WRITE( 108,507) XMIN,HEAD
      WRITE( 108,3002)
      WRITE( 108,504)
      DØ 6 I=1,N
      KPLØTX=((X(I)-XMIN)/(XMAX-XMIN))*99.+1.
      IF(.NØT.BAR) GØ TØ 8
      DØ 7 K=2,KPLØTX
    7 LINE(K-1)=STAR
    8 LINE(KPLØTX)=STAR
      WRITE( 108,508) I, X(I),LINE
      IF(.NØT.BAR) GØ TØ 10
      DØ 9 K =2,KPLØTX
    9 LINE(K-1)=BLANK
   10 LINE(KPLØTX)=BLANK
```

```
    6 CONTINUE
      WRITE( 108,504)
      RETURN
   25 WRITE( 108,506)
      RETURN
  502 FORMAT (1H1 )
  504 FORMAT ( 1X, 14(1H-), 1H., 20(5H----.), 1H- )
  507 FORMAT ( 1X, 5X, 11(F9.3, 1HX) )
  508 FORMAT (1X, I3, F11.4, 1HI, 99A1, A1, 1HI )
  506 FORMAT ( 1X, 12HPLOTER ERROR )
 3001 FORMAT ( 51X,12HSCALING OF X  )
 3002 FORMAT(1X,14(1H-),1HI,10(9X,1HI)/12X,1HX,2X,1HI,10(9X,1HI))
      END


      SUBROUTINE TAPER(DISC1,DISC,NPTS,DT,IX,BRUIT)
      DIMENSION IX(1),DISC(1),DISC1(1)
      LOGICAL BRUIT
      WRITE(1,4)
      WRITE(1,1)(DISC1(I),I=1,20)
      WRITE(1,2) BRUIT,(DISC(I),I=1,6)
      WRITE(1,3) NPTS
      WRITE(1,5) DT
      WRITE(1,3)(IX(I),I=1,NPTS)
    4 FORMAT(80H
     1                                  )
    1 FORMAT(20A4)
    2 FORMAT(L6,6A4)
    3 FORMAT(14I5)
    5 FORMAT(F10.6)
      RETURN
      END
```

```
C     THIS SUBROUTINE NORMALIZES THE SIGNAL (IX) SO THAT THE NAX VOLTAGE
C     IN THE SAMPLE = 1000 V.  IT OUTPUTS THE MEAN, FIRST MOMENT (MOMENT
C     SECOND MOMENT OR VARIANCE (VARIAN), AND THE STANDARD DEVIATION (
C     (STDEV).   IT REQUIRES THE NPTS AND THE KMAX PLUS THE DATA ARRAY
      SUBROUTINE DATAID(MEAN,KMAX,MOMENT,VARIAN,STDEV)
      COMMON IX(9010),NPTS,DT,LERR
      TOTAL=0
      DO 5 I=1,NPTS
C     NORMALIZE THE SIGNAL
      IX(I)=IFIX(IX(I)*1000/FLOAT(KMAX))
      TOTAL=TOTAL+IX(I)
    5 CONTINUE
      MEAN=IFIX(TOTAL/FLOAT(NPTS))
      MOMENT=0
      VARIAN=0
      DO 10 J=1,NPTS
      MOMENT=MOMENT+IX(J)-MEAN
      VARIAN=VARIAN+(IX(J)-MEAN)**2/FLOAT(NPTS-1)
   10 CONTINUE
      STDEV=ABS(VARIAN)**0.5
      RETURN
      END
```

```
      SUBROUTINE RTAPE (DISC1,DISC,DT,BRUIT,*)
      DIMENSION RM(8100),RN(4200),DISC(6),DISC1(20)
      COMMON M,N,RM,RN
      LOGICAL BRUIT
      READ (1,1,END=99) IBLANK
      READ (1,2) (DISC1(I),I=1,20)
      READ (1,3) BRUIT,(DISC(I),I=1,6)
      READ (1,4) M
      READ (1,5) DT
      READ (1,6) (RM(I),I=1,M)
    1 FORMAT(1X,A4   )
    2 FORMAT(20A4    )
    3 FORMAT(L6,6A4 )
    4 FORMAT(I5      )
    5 FORMAT(F10.6  )
    6 FORMAT(14F5.0 )
      GO TO 7
   99 RETURN 1
    7 RETURN
      END
```

```
      SUBROUTINE T TEST(Y1,Y2,LO,I1,I2,IF1,IF2,T,A1,A2)
      DIMENSION Y1(100,2),Y2(100,2),T(10),A1(10),A2(10),X1(100),
     1 X2(100)
      LOGICAL LO
      N1=IF1-I1+1
      N2=IF2-I2+1
      FN1=N1
      FN2=N2
      DO 5 I=1,10
    5 T(I)=0
      L=1
      TX1=0
      TX2=0
      DO 6 K=I1,IF1
    6 X1(K)=Y1(K,L)
      IF(.NOT.LO) GO TO 17
      DO 8 K=I2,IF2
    8 X2(K)=Y1(K,L)
      GO TO 100
   17 DO 7 K=I2,IF2
    7 X2(K)=Y2(K,L)
  100 DO 10 I=I1,IF1
   10 TX1=TX1+X1(I)
      A1(L)=TX1/FLOAT(N1)
      DO 20 I=I2,IF2
   20 TX2=TX2+X2(I)
      A2(L)=TX2/FLOAT(N2)
      SX1=0
      DO 30 I=I1,IF1
   30 SX1=SX1+(X1(I)-A1(L))**2
      SX2=0
      DO 40 I=I2,IF2
   40 SX2=SX2+(X2(I)-A2(L))**2
      SBAR=((SX1+SX2)/(N1+N2-2))**.5
      T(L)=ABS(A1(L)-A2(L))/(SBAR*SQRT(1./FN1+1./FN2))
   50 CONTINUE
      RETURN
      END
```

```
      SUBRØUTINE PEAK(M,PDS)
      DIMENSIØN PDS(1),NP(12),L(12)
      NPTS=50
      DØ 100 J=1,12
  100 NP(J)=0
      PMAX=0
      DØ 110 IJ=1,NPTS
      IF(PDS(IJ)-PMAX) 110,110,120
  120 PMAX=ABS(PDS(IJ))
  110 CØNTINUE
      LSS=0
      DØ 200 I=2,NPTS
C     CHECK SLØPE
      IF(PDS(I)-PDS(I-1)) 40,50,60
   40 LS=-1
      GØ TØ 65
   50 LS=LSS
      GØ TØ 65
   60 LS=1
   65 IF(LS-LSS+2)200,80,200
   80 PK=PDS(I-1)*10./PMAX
      KP=PK+1
      NP(KP)=NP(KP)+1
  200 LSS=LS
      NP(10)=NP(10)+NP(11)
      IJ=10
      L(11)=0
  220 L(IJ)=NP(IJ)+L(IJ+1)
      IJ=IJ-1
      IF(IJ) 250,250,220
  250 CØNTINUE
      DØ 300 K=1,11
      K1=10*(K-1)
      WRITE(108,104) L(K), K1
  104 FØRMAT(1X,I6,'  PEAKS ABØVE',I6,'  PERCENT')
  300 CØNTINUE
      RETURN
      END
```

APPENDIX C

VOLTAGE LIMITER

Back to back zeiner diodes, 1N702, having a breakdown voltage of 2.6 volts were used to insure that excessive voltage would not reach the analog to digital equipment. A schematic of the five channel voltage limiter which was built is shown in Fig. 15.

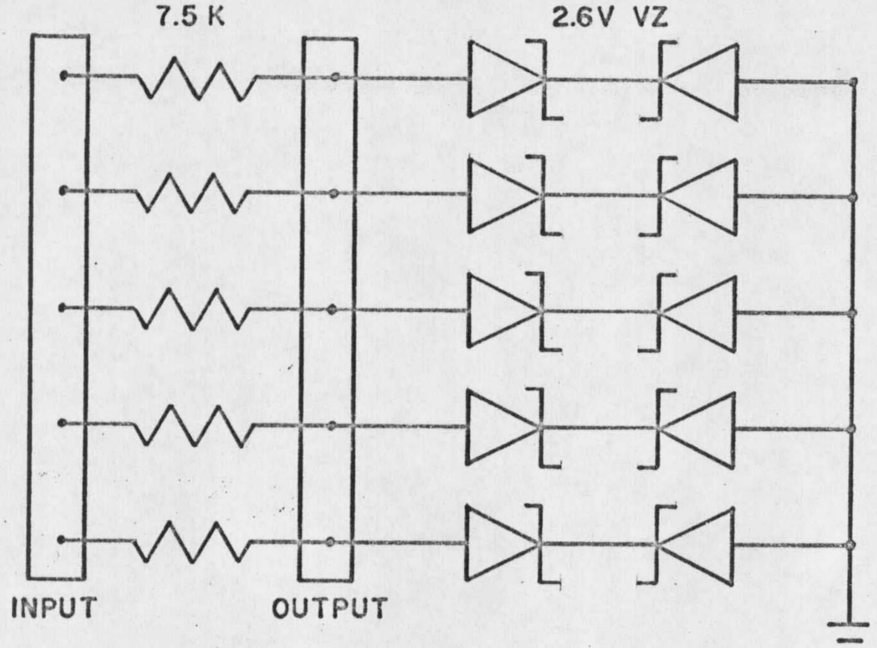

Figure 15. Schematic of the Voltage Limiter

APPENDIX D

POSSIBLE MECHANISM FOR THE BRUIT

This investigation has shown that the energy spectrum of the stenotic bruit differs from that of the innocent bruit for the 13 patients studied. The energy in the stenotic spectrum was found to be more broadly distributed and concentrated in higher frequencies than the energy in the innocent spectrum. The stenotic spectrum had an average of 7.55 major peaks while the innocent spectrum averaged only 5.23 peaks. From these results, speculation on the mechanism of bruits is possible.

Looking at the vortex shedding phenomena it can be seen that a "perfect" cylindrical rod or circular orifice in a flowing fluid of constant velocity produces a tone of discrete frequency. Should the shape be rough or of inconsistent diameter the sound produced is a noise composed of a multitude of frequencies, not a clear note. Let roughness be represented by the number of different diameters on a finely machined rod. Begin with a "slightly rough" rod half the length of which is machined to one diameter, the other half being a different diameter. From this "slightly rough" rod in a flowing stream two tones would be observed, giving two discrete peaks in the energy density spectrum. Listening to a "very rough" rod composed of a large number of perfect diameters, a large number of tones would be heard giving rise to a large number of peaks in the spectrum. Possibly this analogy could be extended to include imperfect (non-cylindrical and rough surfaced) rods and imperfect orifices. If so, it could be concluded that the stenotic bruit, having more peaks in its energy density spectrum, is produced by an obstruction which is more irregular than that producing innocent bruits. One of the problems which must be recognized with this type of analogy is that in pulsatile flow the frequency of vortex shedding from a rod is more likely to be distributed throughout a frequency band rather than a pure tone, but the same type of conclusion may be drawn.

It may be possible to make an estimate of the size of a stenotic or innocent orifice using the frequency of a few of the greater peaks in the energy density spectrum along with a value for the velocity of flow through the orifice during bruit. The flow is at its peak rate during this period, but still varying. Possibly by using the average flow rate (U_a) during this period a "typical" family of curves could be constructed relating this average to the degree of obstruction, such as the orifice diameter (d_o). Such a proposed family is shown in Fig. 16. The members of the family would differ by such factors as blood pressure, pulse rate, and/or diameter of artery (d_a). A trial and error method would then have to be employed using the points along the correct curve in the correct frequency relationship:

$$N \simeq U_a/3(d_a - d_o) \ldots \text{ innocent bruit (large orifice)}$$

$$N \simeq 0.6\ U_a/d_o \ \ldots \text{ stenotic bruit (small orifice)},$$

where N is the frequency of the peak in the spectrum which, hopefully, arises when an average pulsating velocity U_a passes through an orifice or approximate diameter d_o, in an artery of diameter d_a.

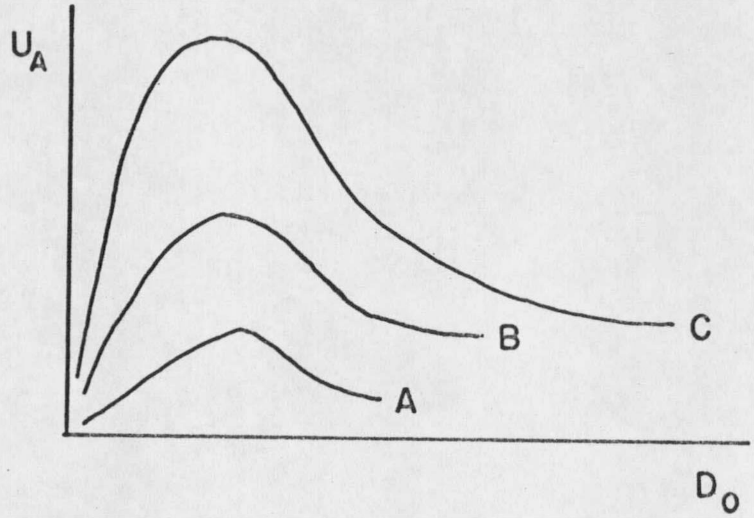

Figure 16. Average Velocity over Obstruction vs its Size.

LITERATURE CONSULTED

LITERATURE CONSULTED

Bingham, Christopher, et al. "Modern Techniques of Power Spectrum Estimation", IEEE Transactions on Audio and Electroacoustics, Au-15, No. 2 (june 1967), pp. 56-66.

Blackman, R. B. and Tukey, J. W. The Measurement of Power Spectra, New York: Dover Publications, Inc., 1958.

Braun, Harold A., et al. "Auscultation of the Neck; Incidence of Cervical Bruits in 4,296 Consecutive Patients", Rocky Mountain Medical Journal, Vol. 63, No. 5, (May 1966), pp. 51-53.

Bruns, D. L. "A General Theory of the Cause of Murmurs in the Cardiovascular System", American Journal of Medicine, Vol. 27, No. 3 (1959), pp.

Cochran, W. T., et al. "Burst Measurements in the Frequency Domain", Proceedings of the IEEE, Vol. 54, (June 1966), pp. 830-841.

Cochran, W. T., et al. "What is the Fast Fourier Transform?" IEEE Transactions on Audio and Electroacoustics, Au-15, No. 2 (Juns 1967), pp. 45-55.

Cooley, James W., et al. "Application of the Fast Fourier Transform to Computation of Fourier Integrals, Fourier Series, and Convolution Integrals", IEEE Transactions on Audio and Electroacoustics, Au-15, No. 2, (June 1967), pp. 79-83.

Cooper, George and McGillem, Clare. Methods of System and Signal Analysis, New York: Holt, Rinehart and Winston, 1967.

Durin, R. E., et al. "Heart Sound Screening in Children by Analog-Digital Circuitry", Public Health Reports, Vol. 80, No. 9, (September 1965), pp. 761-770.

Green, D. G. "Physiological Auscultation Correlations: Heart Sounds and Pressure Pulses", Institute of Radio Engineers Transaction on Medical Electronics, Vol. PGME-9, (December 1957), pp. 4-5.

Helms, Howard D. "Fast Fourier Transform Method of Computing Difference Equations and Simulating Filters", IEEE Transactions on Audio and Electroacoustics, Au-15, No. 2 (June 1967), pp. 85-90.

Jacobs, J. E., et al. "Feasibility of Automated Analysis of Phonocardiogram", (unpublished report), Biomedical Engineering Center Technological Institute, Northwestern University, Evanston, Illinois (1968).

Landowne, Milton. "A Method Using Induced Waves to Study Pressure Propagation in Human Arteries", Circulation Research, Vol. V, (November 1957), pp. 594-601.

Lees, Sidney and Dougherty, Ray C. "Numerical Fourier Transform Calculations for Pulse Testing Procedures", American Society of Mechanical Engineers Paper No. 66, Aut-A, (May 24, 1966).

Lepeschkin, E. and Lareau, D. "Quantitative Auscultation of Heart Sounds with Internal Calibration", Institute of Radio Engineers Transactions on Medical Electronics, Vol. PGME-9, (December 1957), p. 16.

Maling, George C., et al. "Digital Determination of Third-Octave and Full Octave Spectra of Acoustical Noise", IEEE Transactions on Audio and Electroacoustics, Vol. Au-15, No. 2, (June 1967), pp. 98-104.

McDonald, Donald A. Blood Flow in Arteries. London: Edward Arnold, Ltd. (1960).

Rappaport, M. B. (Discussion of Session I-A) "Symposium: Present Status of Heart Sound Production and Recording", Institute of Radio Engineers Transactions on Medical Electronics, Vol. PGME-9, (December 1957).

Rennie, Laurie, et al. "Arterial Bruits in Cerebrovascular Disease", Neurology (Minneapolis), Vol. 14, (August 1964), pp. 751-756.

Rodbard, Simon. "Vascular Modifications Induced by Flow", American Heart Journal, Vol. 51, No. 6 (June 1956), pp. 926-942.

Rodbard, Simon. "Physical Forces and the Vascular Lining", Annals of Internal Medicine, Vol. 50, No. 6, (June 1959), pp. 1339-1351.

Rodbard, Simon and Johnson, Alonza C. "Deposition of Flow-born Materials on Vessel Walls", Circulation Research, Vol. XI, (October 1962), pp. 664-668.

Scarr, R. W. A. "Zero Crossings as a Means of Obtaining Spectral Information in Speech Analysis", IEEE Transactions on Audio and Electroacoustics, Vol. Au-16, No. 2, (June 1968), pp. 247-255.

Simpson, Ernst, and Nakagawa, K. "Effect of Age on Pulse Wave Velocity and 'Aortic Ejection Time' in Healthy Men and in Men with Coronary Artery Disease", Circulation Research, Vol. XXII, (July 1960), pp. 126-129.

Smith, D. H., et al. "Possible Approaches to Multiple-Channel Tape Recording for Biomedical Purposes", Institute of Radio Engineers Transactions on Medical Electronics, Vol. ME-6, No. 3, (September 1959), pp. 171-174.

Taback, L., et al. "Digital Recording of Electrocardiographic Data for Analysis by a Digital Computer", Institute of Radio Engineers Transactions on Medical Electronics, Vol. ME-6, No. 3, (September 1959), pp. 167-171.

Volk, William. Applied Statistics for Engineers, New York, Toronto and London: McGraw-Hill Book Company, Inc. 1958.