



A method for the computer-aided dynamic analysis of spatial mechanisms
by Derrick Wayne Johnson

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Mechanical Engineering
Montana State University
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Abstract:

This thesis presents a method for the dynamic analysis of spatial mechanisms. A vector model of the mechanism using vector operations such as vector loops, dot products, and cross products is used to describe the kinematics of the mechanism. Newton's second law was applied to each element to describe the kinetics. The method was described such that a computer program could be written and used to generate and solve a complete set of kinematic and dynamic equations for a given mechanism. The method is limited to rigid elements. Two examples are given.

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**MONTANA STATE UNIVERSITY
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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENT	iv
CONTENTS	v
LIST OF TABLES	vi
LIST OF FIGURES	vii
LIST OF DEFINITIONS	viii
Chapter	
I. INTRODUCTION	1
II. GENERAL DESCRIPTION	4
III. KINEMATIC ANALYSIS	6
Geometry	6
Velocities	11
Accelerations	13
IV. SOLID ELEMENTS	14
V. ELEMENT DYNAMICS	21
VI. FORCE SPECIFICATION	25
VII. POINT MASS	28
VIII. REMOTE MANIPULATOR SYSTEM	32
IX. CONCLUSION	46
LITERATURE CITED	47

LIST OF TABLES

	Page
1. Point Mass Geometry	30
2. Point Mass Velocities	30
3. Point Mass Forces and Moments	31
4. RMS Element Properties	32
5. RMS Geometry	35
6. RMS Velocities	40
7. RMS Accelerations	41
8. RMS Forces and Moments	43

LIST OF FIGURES

	Page
1. System Vector	7
2. Piston-Crank and Vector Model	10
3. General Element and Element Coordinate System	16
4. Element with Element and Newtonian Coordinate Systems	18
5. Element Numbering System	20
6. Element with Connection to Center of Mass	20
7. Forces at a Connection	27
8. Point Mass Vector Representation	29
9. Sketch of the Space Shuttle RMS	33
10. Vector Representation of the RMS	34
11. RMS Element Configurations	37

LIST OF DEFINITIONS

Symbol	Page
$[C_n]$	9
d_i	8
$[E_k]$	15
$[\dot{E}_k]$	24
$E_{k,a,\beta}$	15
$e_{k,a}$	15
$\{F_{k,j}\}$	21
$F_{k,j,\gamma}$	21
$FM_{k,j,i}$	26
$\{f_{k,j}\}$	21
$\{\dot{H}_k\}$	22
$[I_k]$	22
$[IE_k]$	22
i_β	15
M_k	22
$\{P_n\}$	6
$P_{n,\beta}$	6
$\{PE_{k,m}\}$	15
$\{PS_{k,m}\}$	17
$R_n, \{R_n\}$	6
$\{\dot{R}_n\}$	11
$\{\ddot{R}_n\}$	13

[U]		21
$\{\dot{W}_k\}$		21
X_β		6
$\{\Omega_k\}$		22
$\{\Omega_n\}$		11
$\omega_{n,\beta}$		11

ABSTRACT

This thesis presents a method for the dynamic analysis of spatial mechanisms. A vector model of the mechanism using vector operations such as vector loops, dot products, and cross products is used to describe the kinematics of the mechanism. Newton's second law was applied to each element to describe the kinetics. The method was described such that a computer program could be written and used to generate and solve a complete set of kinematic and dynamic equations for a given mechanism. The method is limited to rigid elements. Two examples are given.

CHAPTER I

INTRODUCTION

There are several methods available for the dynamic analysis of mechanisms, each of which often involves the formulation and solution of a large set of equations. It is this nature of dynamic analysis that makes the utilization of computer assisted methods advantageous.

Historically, kinematic and dynamic analysis of mechanisms has been done through the use of graphical methods. Recently, however, the complexity of mechanical manipulators requires that analytical methods be used in the areas of both design and control. Hermani, Jaswa, and McGhee [3]^{*} have presented and compared several methods available for the solution of manipulator dynamics.

In the area of control, much of the present work is based on the symbolic notation and matrix transformation method developed by Denavit and Hartenberg [2]. Great advances have been made in applying both Lagrangian formulations [10,4] and Newton-Euler equations [6] to this notation. The formulations developed by Hollerbach [4] and Luh, Walker, and Paul, R. P. C. [6] were developed for the

^{*} Numbers in brackets indicate references listed in the Literature Cited section.

real-time control of mechanical manipulators. In addition, Luh and Lin [5] have developed an algorithm for the computer generation and simplification of the Newton-Euler equations used by Luh, Walker, and Paul. Walker and Orin [14] then compared several schemes for the solution of these equations.

Thomas and Tesar [11] have used a slightly different notation from that of Denavit and Hartenberg and applied principles of virtual work to develop the dynamic equations. By doing this they have increased the number of parameters and equations required to describe a mechanism but have also made it easier to generate the equations.

Another class of methods involves the use of vectors and vector loops to model the mechanism. Paul, B. and Krajcinovic [7,8] used this scheme in combination with Lagrangian formulations to develop dynamic equations for planar machinery. Paul, B. [7], later, summarized this work, compared it to graphic methods, and presented some variations to the overall approach. The use of vectors and vector loops was also used to analyze the kinematics of spatial mechanisms by Townes, Blackketter, and Lowell [12]. Since then, Townes and Blackketter [13] have enhanced this method by describing kinematic constraints with vector dot products and cross products.

In this thesis, the method of kinematic analysis by Townes and Blackketter [13] will be expanded to include the dynamic analysis of mechanisms. Emphasis will be placed on

describing the method such that a computer program can be used to generate and solve a complete set of kinematic and dynamic equations for a given mechanism. Newton's second law will be used to describe the dynamics and the method will be limited to mechanisms consisting of rigid elements.

CHAPTER II

GENERAL DESCRIPTION

The general procedure for this method consists, first, of building a kinematic vector model of the mechanism. Next, kinematic models of each element are formulated and added to the mechanism model. A kinematic analysis at this point will yield the geometry and velocities necessary to generate the kinematic acceleration equations and the kinetics equations. Elements are then defined in terms of their mass characteristics and the nodes at which forces and moments act. Finally, the kinetics equations are constructed by applying Newton's second law to each element.

A kinematic analysis is often performed on just the mechanism before considering the addition of the element models to be sure to obtain the correct mechanism model. The geometry and velocities of the mechanism are independent of the forces acting upon it. Therefore, this information is the same from the first kinematic analysis throughout the rest of the dynamic analysis.

All of the element characteristics, mass, mass moments of inertia, and geometry, are defined in terms of an element coordinate system. And since the mechanism model is defined in terms of system coordinates, the element characteristics

are transferred to system coordinates.

The description of a mechanism is accomplished by specifying a set of known parameters and calculating the unknown parameters. Often, it is difficult to determine the right set of known parameters and thus obtain a correct dynamic model of the mechanism. One procedure to simplify the analysis of a complicated mechanism is to first solve the dynamic equations in their uncoupled form. That is, the parameters can be specified such that the accelerations can be calculated independent of the forces. Once a satisfactory solution can be obtained from the uncoupled equations, the parameters can be redefined in a stepwise fashion until the desired form of the problem is reached.

CHAPTER III

KINEMATIC ANALYSIS

Geometry

In the kinematic analysis, vectors are used to represent the function of each physical link in the mechanism, locate any special points of interest, and to constrain certain elements of the mechanism. These vectors are relative to a Newtonian reference frame, or system coordinates, and are referred to as system vectors. In matrix form the system vector R_n is expressed as

$$\{R_n\} = P_{n,4}\{P_n\} \quad (1)$$

where $P_{n,4}$ is the length of the vector and $\{P_n\}$ is a unit vector in the direction of R_n . The components of the unit vector, $P_{n,\beta}$, are the directional cosines of R_n in the Newtonian reference frame X_β , $\beta = 1,2,3$. In Figure 1 a system vector is shown.

In three-dimensional space there are three parameters required to describe a vector, namely the length and two of the three directional cosines. Only two of the cosines are independent in that the sum of the squares of the cosines must equal one, or in matrix form

$$[P_n]\{P_n\} = 1 \quad (2)$$

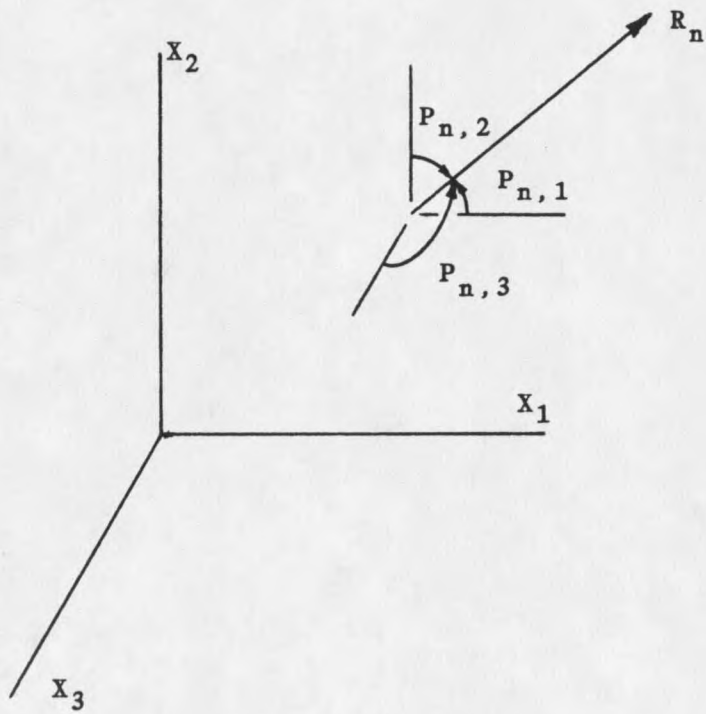


FIGURE 1. - System vector.

This is referred to as the "cosine squared constraint".

Equations can be developed to describe the geometry of the mechanism by specifying which vectors form closed loops. The sum of the vector components in each coordinate direction around the loop must equal zero. Therefore, each vector loop results in one equation for the sum in each of the three coordinate directions.

Besides vector loops and cosine squared constraints, two additional constraints available to describe the geometry of the mechanism are dot products and cross products. A dot product is used to specify the known angle between any two vectors. The dot product constraint between vectors R_n and R_m is written as

$$[P_n][P_m] = d_i \quad (3)$$

where $[P_n]$ and $[P_m]$ are unit vectors in the directions of R_n and R_m , respectively, and d_i is the cosine of the included angle. Note that the dot product constraint is independent of the vector lengths. A dot product can also be added to the set of geometry equations to find an unknown angle between two vectors.

The cross product constraint is used to define a mutually perpendicular vector to two other vectors. The cross product between unit vectors $[P_n]$ and $[P_m]$ is written as

$$\{R_h\} = [C_n][P_m] \quad (4)$$

where

$$[C_n] = \begin{bmatrix} 0 & P_{n,3} & -P_{n,2} \\ -P_{n,3} & 0 & P_{n,1} \\ P_{n,2} & -P_{n,1} & 0 \end{bmatrix} \quad (5)$$

and R_h is the resulting vector. Note that while $\{P_n\}$ and $\{P_m\}$ are unit vectors, R_h is a unit vector only if the included angle between $\{P_n\}$ and $\{P_m\}$ is 90 degrees.

In Figure 2, a piston-crank is modeled using vectors, a vector loop, two dot products, and the four cosine squared constraints. The set of geometry equations has the form

Loop Equations

$$P_{1,4}P_{1,1} + P_{2,4}P_{2,1} - P_{3,4}P_{3,1} = 0 \quad (6)$$

$$P_{1,4}P_{1,2} + P_{2,4}P_{2,2} - P_{3,4}P_{3,2} = 0$$

$$P_{1,4}P_{1,3} + P_{2,4}P_{2,3} - P_{3,4}P_{3,3} = 0$$

Dot Products

$$P_{1,1}P_{4,1} + P_{1,2}P_{4,2} + P_{1,3}P_{4,3} = d_1 = \cos 90^\circ$$

$$P_{2,1}P_{4,1} + P_{2,2}P_{4,2} + P_{2,3}P_{4,3} = d_2 = \cos 90^\circ$$

Cosine Squared

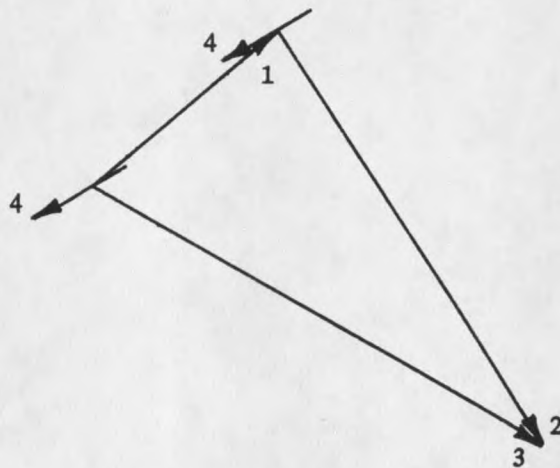
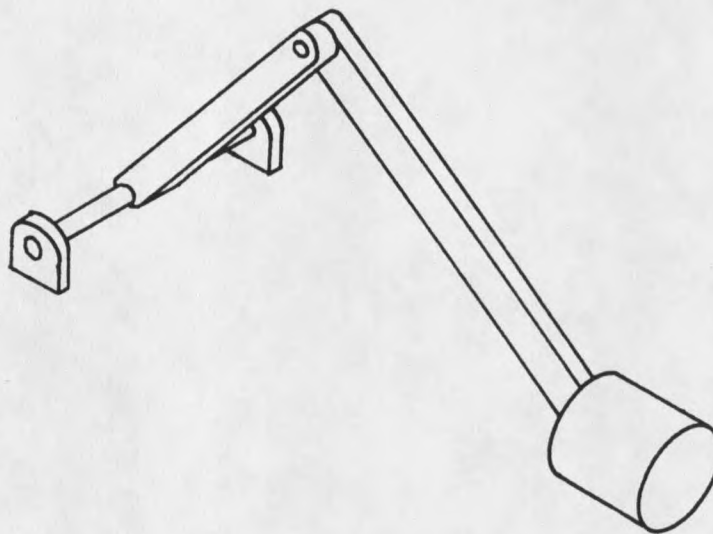
$$P_{1,1}P_{1,1} + P_{1,2}P_{1,2} + P_{1,3}P_{1,3} = 1$$

$$P_{2,1}P_{2,1} + P_{2,2}P_{2,2} + P_{2,3}P_{2,3} = 1$$

$$P_{3,1}P_{3,1} + P_{3,2}P_{3,2} + P_{3,3}P_{3,3} = 1$$

$$P_{4,1}P_{4,1} + P_{4,2}P_{4,2} + P_{4,3}P_{4,3} = 1$$

This set of equations must be reorganized before a solution can be found by placing all of the known terms on the right hand side. The resulting equations are typically nonlinear so, Newton's method is used to solve for the unknown parameters.



$$R_1 + R_2 - R_3 = 0$$

$$R_1 \cdot R_4 = \cos 90^\circ$$

$$R_2 \cdot R_4 = \cos 90^\circ$$

$$\begin{aligned} [P_1] \{P_1\} &= 1 \\ [P_2] \{P_2\} &= 1 \\ [P_3] \{P_3\} &= 1 \\ [P_4] \{P_4\} &= 1 \end{aligned}$$

FIGURE 2. - Piston-crank and vector model.

Velocities

The velocities of the mechanism are described using the first time derivative of a vector,

$$\{\dot{R}_n\} = P_{n,4} [C_n] \{\Omega_n\} + \dot{P}_{n,4} \{P_n\} \quad (7)$$

where the angular velocity vector is expressed as

$$\{\Omega_n\} = \begin{Bmatrix} \omega_{n,1} \\ \omega_{n,2} \\ \omega_{n,3} \end{Bmatrix} \quad (8)$$

Note that the first term in Equation 7 is just the cross product of the vector R_n and the angular velocity $\{\Omega_n\}$. The equations used to describe the velocities of a mechanism can be found by replacing the vectors in the loop equations with vector velocities and taking the first time derivative of the dot and cross product constraints. The first time derivative of the cosine squared constraint, however, results in a trivial equation. So, in the velocity analysis it is replaced with an equation which controls the spin of a vector about its own axes.

In this method of kinematic analysis, the spin of a vector about its own axis is arbitrary and can be specified as any value, including zero, and not affect the structure of the mechanism. Most often, this spin is set to zero and is written as

$$[P_n] \{\Omega_n\} = 0 \quad (9)$$

This expression is referred to as the "no spin constraint".

The equation set for the velocities of the piston-crank is

Loop Equations

$$\dot{R}_{1,1} + \dot{R}_{2,1} - \dot{R}_{3,1} = 0 \quad (10)$$

$$\dot{R}_{1,2} + \dot{R}_{2,2} - \dot{R}_{3,2} = 0$$

$$\dot{R}_{1,3} + \dot{R}_{2,3} - \dot{R}_{3,3} = 0$$

Dot Products

$$\begin{aligned} & (\omega_{1,2} P_{1,3} - \omega_{1,3} P_{1,2}) P_{4,1} + (\omega_{1,3} P_{1,1} - \omega_{1,1} P_{1,3}) P_{4,2} \\ & + (\omega_{1,1} P_{1,2} - \omega_{1,2} P_{1,1}) P_{4,3} + (\omega_{4,2} P_{4,3} - \omega_{4,3} P_{4,2}) P_{1,1} \\ & + (\omega_{4,3} P_{4,1} - \omega_{4,1} P_{4,3}) P_{1,2} + (\omega_{4,1} P_{4,2} - \omega_{4,2} P_{4,1}) P_{1,3} = 0 \end{aligned}$$

$$\begin{aligned} & (\omega_{2,2} P_{1,3} - \omega_{2,3} P_{1,2}) P_{4,1} + (\omega_{2,3} P_{1,1} - \omega_{2,1} P_{1,3}) P_{4,2} \\ & + (\omega_{2,1} P_{1,2} - \omega_{2,2} P_{1,1}) P_{4,3} + (\omega_{4,2} P_{4,3} - \omega_{4,3} P_{4,2}) P_{2,1} \\ & + (\omega_{4,3} P_{4,1} - \omega_{4,1} P_{4,3}) P_{2,2} + (\omega_{4,1} P_{4,2} - \omega_{4,2} P_{4,1}) P_{2,3} = 0 \end{aligned}$$

No Spin

$$\omega_{1,1} P_{1,1} + \omega_{1,2} P_{1,2} + \omega_{1,3} P_{1,3} = 0$$

$$\omega_{2,1} P_{2,1} + \omega_{2,2} P_{2,2} + \omega_{2,3} P_{2,3} = 0$$

$$\omega_{3,1} P_{3,1} + \omega_{3,2} P_{3,2} + \omega_{3,3} P_{3,3} = 0$$

$$\omega_{4,1} P_{4,1} + \omega_{4,2} P_{4,2} + \omega_{4,3} P_{4,3} = 0$$

where

$$\begin{aligned} \dot{R}_{n,1} &= P_{n,4} (P_{n,3} \omega_{n,2} - P_{n,2} \omega_{n,3}) + \dot{P}_{n,4} P_{n,1} \\ \dot{R}_{n,2} &= P_{n,4} (P_{n,1} \omega_{n,3} - P_{n,3} \omega_{n,1}) + \dot{P}_{n,4} P_{n,2} \\ \dot{R}_{n,3} &= P_{n,4} (P_{n,2} \omega_{n,1} - P_{n,1} \omega_{n,2}) + \dot{P}_{n,4} P_{n,3} \end{aligned} \quad (11)$$

Note that all of the velocity equations are linear with respect to velocities so, a routine, such as Gauss-Elimination, can be used to solve this set of equations.

Accelerations

The second time derivative of a vector, used in the acceleration analysis, is

$$\begin{aligned} \{\ddot{R}_n\} = & P_{n,4}[C_n]\{\dot{\Omega}_n\} + \ddot{P}_{n,4}\{P_n\} + P_{n,4}[\dot{C}_n]\{\Omega_n\} \\ & + 2\dot{P}_{n,4}[C_n]\{\Omega_n\} \end{aligned} \quad (12)$$

The terms in Equation 12 are, respectively, the tangential, radial, centripetal, and Coriolis accelerations. The form of the no spin constraint for accelerations is

$$[P_n]\{\dot{\Omega}_n\} = 0 \quad (13)$$

Note that in this equation the angular acceleration of the vector about its own axis is controlled and the equation is not just the time derivative of Equation 9.

The loop equations for the piston-crank can be found by replacing the velocity terms in the first three equations of Equation Set 10 with the appropriate acceleration terms. The dot products can be found by taking the time derivative of the fourth and fifth equations of Equation Set 10 and the no spin equations take the form of Equation 13.

CHAPTER IV

SOLID ELEMENTS

As stated in Chapter II, the first kinematic analysis is often performed on the mechanism before considering the effects of the solid elements which make up the mechanism. Only the function of the physical link is modeled in the first analysis. Once a correct solution is obtained from this analysis, solid element models are added to introduce the inertia effects necessary for a dynamic analysis.

To simplify the calculation of the element characteristics, such as the mass moments of inertia, a solid element is defined in terms of a local coordinate system. While the orientation of this coordinate system can be arbitrary, a scheme must be established in order to relate the element coordinate system to the Newtonian coordinate system.

The element coordinate system is defined relative to a plane formed by the intersection of two element vectors, namely the element principal vector and the element plane vector. The element principal vector corresponds to the system vector used to describe the element's function in the kinematic analysis and the element plane vector corresponds to a system vector which is most often used to locate the

center of mass of the element relative to the connection to the preceding element in the mechanism. A vector in the same direction as the principal vector defines the x_1 -axis in element coordinates and the plane formed by the intersection of the principal vector and the element plane vector is the x_1 - x_2 plane. The x_3 -axis is defined as the direction of the principal vector cross the element plane vector and, finally, the direction of the x_2 -axis is recovered by crossing the x_3 -axis with the x_1 -axis. Figure 3 demonstrates a general solid element and its local coordinate system.

Unit vectors in each element coordinate direction, for element k , are signified by $e_{k,\alpha}$ ($\alpha = 1,2,3$), whereas, the unit vectors in the Newtonian coordinate system are signified by i_β ($\beta = 1,2,3$). The relation between these two coordinate systems is found to be

$$\{e_k\} = [E_k]\{i\} \quad (14)$$

where $[E_k]$ is the set of directional cosines, $E_{k,\alpha,\beta}$, between the vectors $e_{k,\alpha}$ and the vectors i_β for element k . For example, $E_{1,2,3}$ is the directional cosine between $e_{1,2}$ and i_3 . From this relation any vector in element coordinates can be converted to system coordinates through the transformation

$$\{PS_{k,m}\} = [E_k]^T \{PE_{k,m}\} \quad (15)$$

where $\{PE_{k,m}\}$ is a unit vector in element coordinates in the

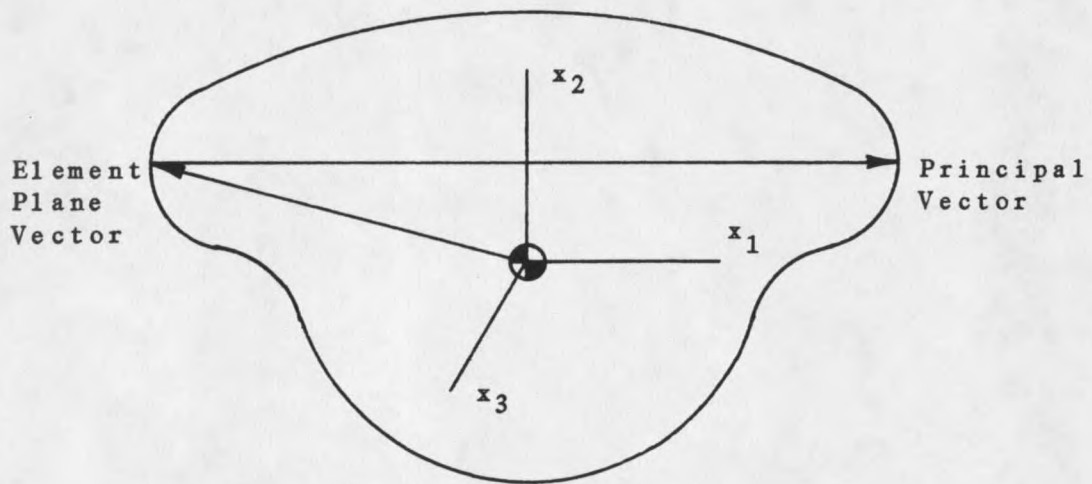


FIGURE 3. - General element and element coordinate system.

same direction as the element vector and $\{PS_{k,m}\}$ is the corresponding system unit vector. The magnitudes, $PS_{k,m,4}$ and $PE_{k,m,4}$, are the same in each coordinate system. Element k is illustrated in Figure 4 with the complete element and Newtonian coordinate systems.

Once all of the elements have been modeled, the second kinematic analysis is performed to determine the motion of each element. This time, however, the spin of an element plane vector and its corresponding principal vector should be equal to represent the motion of a solid body. To describe this constraint, the no spin constraints for each vector must be replaced with two equations which set the X_1 and X_2 angular velocities equal for the two vectors. Since there is a third possible angular velocity in the X_3 direction, another equation must also be replaced to constrain the spin of the two vectors. This other equation is the dot product constraint specifying the known angle between the element plane vector and the principal vector. Note that if all three angular velocities are known for any vector, the no spin equation is no longer a valid equation and can not be used in the system of equations. Therefore, all three angular velocities can not be specified for either the element plane vector or the principal vector.

In addition to the element plane vector and the principal vector, element vectors exist to locate every node, except the center of mass, on the element. A node is

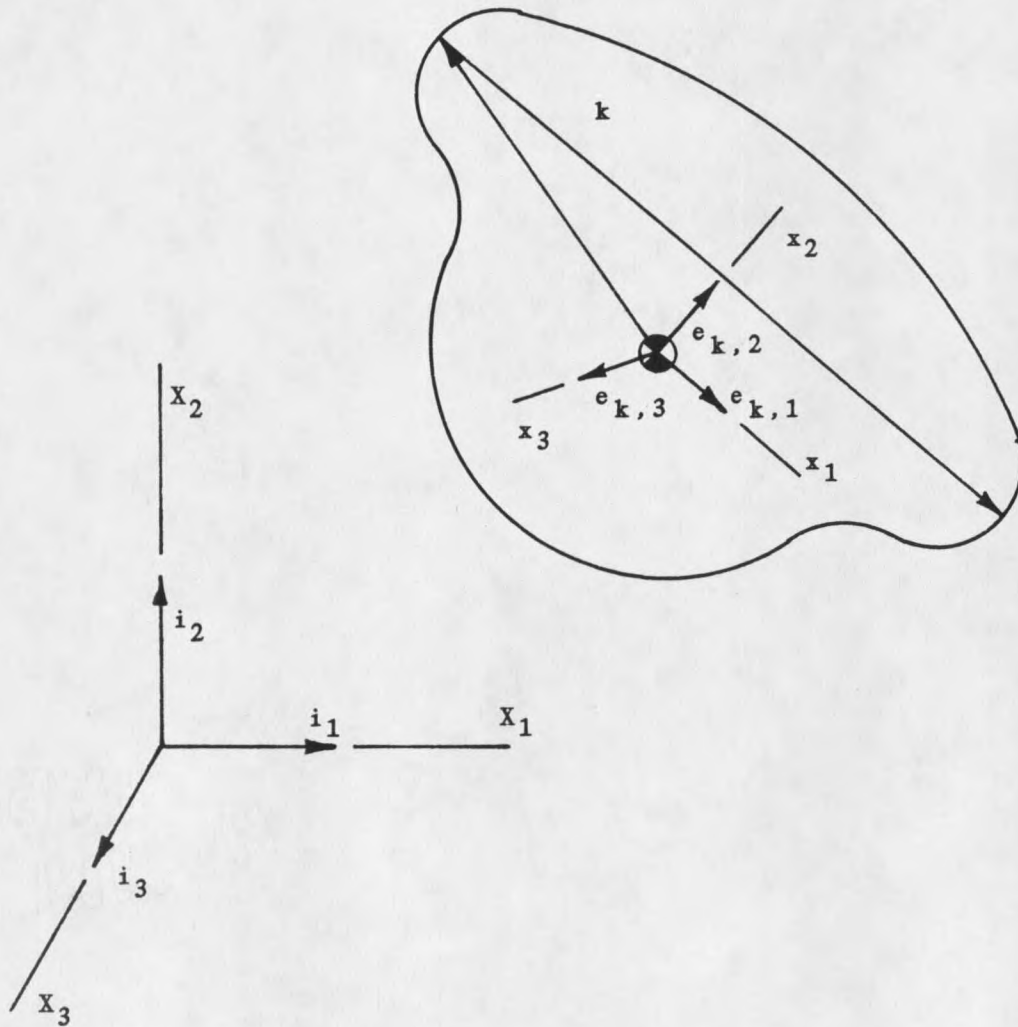


FIGURE 4. - Element with element and Newtonian coordinate systems.

any point of connection to another element, the center of mass of an element, and points of externally applied forces. Element vectors always extend from the center of mass to a node, so a set procedure can be used to calculate the moments due to the forces at that node, and are numbered the same as the node. The nodes are numbered such that the center of mass will always be the highest numbered node on the element and thus, will also be the total number of nodes on the element. A sample element is shown in Figure 5. Note that the principal vector is numbered the same as the center of mass. This will not always be true, especially in the case where connections are made to the center of mass. Figure 6 demonstrates an element to which connections were made to the center of mass. In this case, an arbitrary element vector was used as the principal vector.

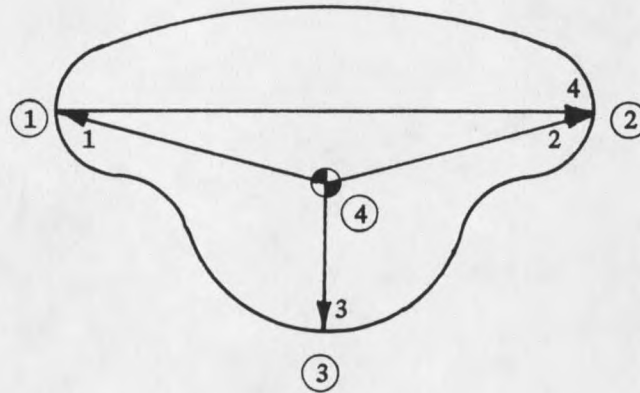


FIGURE 5. - Element numbering system (numbers inside circles indicate node numbers).

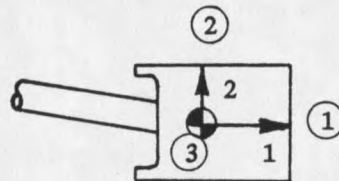


FIGURE 6. - Element with connection to center of mass.

CHAPTER V

ELEMENT DYNAMICS

Newton's second law for an element comprised of N nodes is written as

$$\sum_{j=1}^N \{f_{k,j}\} = \{\dot{W}_k\} \quad (16)$$

where $\{f_{k,j}\}$ is the set of resultant forces and moments acting on the element due to the forces and moments at node j , and $\{\dot{W}_k\}$ is the time rate of change of momentum of the element. The set of resultant forces and moments is expressed in terms of the six force and moment components at node j by

$$\{f_{k,j}\} = \begin{bmatrix} [U] & 0 \\ PS_{k,j,4}[C_{k,j}] & [U] \end{bmatrix} \{F_{k,j}\} \quad (17)$$

where $[U]$ is a 3×3 unit matrix and $\{F_{k,j}\}$ is the set of forces and moment components. For $\gamma = 1, 2, 3$, $F_{k,j,\gamma}$ represents a force component in the Newtonian coordinate system and when $\gamma = 4, 5, 6$, $F_{k,j,\gamma}$ represents a moment. The term $PS_{k,j,4}[C_{k,j}]$ is the moment due to the the element vector which locates the node, in system coordinates, crossed with the force vector at the node.

The time rate of change of momentum is expressed as

$$\{\dot{W}_k\} = \left\{ \begin{array}{c} M_k \sum_{n=1}^L \{\ddot{R}_n\} \\ \{\dot{H}_k\} \end{array} \right\} \quad (18)$$

where M_k is the mass of the element, $\sum_{n=1}^L \{\ddot{R}_n\}$ is the sum of the acceleration vectors from the origin of the Newtonian coordinate system to the center of mass of the element, and

$$\{\dot{H}_k\} = [\dot{I}_k] \{\Omega_k\} + [I_k] \{\dot{\Omega}_k\} \quad (19)$$

Here, the angular velocity of element k , $\{\Omega_k\}$, is the angular velocity of its principal vector. The inertia tensor, of Equation 19, is in system coordinates and should be expressed in terms of the inertia tensor in element coordinates which is readily calculated and constant with respect to time. Using the transformation from Equation 14 this is written as

$$[I_k] = [E_k]^T [IE_k] [E_k] \quad (20)$$

where $[IE_k]$ is the inertia tensor in element coordinates and is known as

$$[IE_k] = \begin{bmatrix} IE_{k,1,1} & -IE_{k,1,2} & -IE_{k,1,3} \\ -IE_{k,2,1} & IE_{k,2,2} & -IE_{k,2,3} \\ -IE_{k,3,1} & -IE_{k,3,2} & IE_{k,3,3} \end{bmatrix} \quad (21)$$

Substituting Equation 20 into Equation 19, the result is

$$\begin{aligned} \{\dot{H}_k\} = & [\dot{E}_k]^T [IE_k] [E_k] \{\Omega_k\} + [E_k]^T [IE_k] [\dot{E}_k] \{\Omega_k\} \\ & + [E_k]^T [IE_k] [E_k] \{\dot{\Omega}_k\} \end{aligned} \quad (22)$$

where $[\dot{E}_k]$ is given by Equation 23.

Combining the kinematic acceleration equations with the equilibrium equations results in one matrix equation used to describe the dynamics of the mechanism. This matrix equation has the form of Equation 24. Note that if the number of kinematic acceleration equations equals the number of unknown accelerations, the accelerations can be solved for independently of the forces and moments. This is referred to as the uncoupled form of the equations.

Equation 23:

$$[\dot{E}_k] = \begin{bmatrix} \omega_{k,3}E_{k,1,2} - \omega_{k,2}E_{k,1,3} & \omega_{k,1}E_{k,1,3} - \omega_{k,3}E_{k,1,1} & \omega_{k,2}E_{k,1,1} - \omega_{k,1}E_{k,1,2} \\ \omega_{k,3}E_{k,2,2} - \omega_{k,2}E_{k,2,3} & \omega_{k,1}E_{k,2,3} - \omega_{k,3}E_{k,2,1} & \omega_{k,2}E_{k,2,1} - \omega_{k,1}E_{k,2,2} \\ \omega_{k,3}E_{k,3,2} - \omega_{k,2}E_{k,3,3} & \omega_{k,1}E_{k,3,3} - \omega_{k,3}E_{k,3,1} & \omega_{k,2}E_{k,3,1} - \omega_{k,1}E_{k,3,2} \end{bmatrix}$$

Equation 24:

$$\left[\begin{array}{c|c} \text{Kinematic} & \\ \text{Acceleration} & \\ \text{Terms} & \\ \hline \text{Mass x} & \text{Force and} \\ \text{Acceleration} & \text{Moment} \\ \text{Terms} & \text{Coefficients} \end{array} \right] \left\{ \begin{array}{c} \text{Unknown} \\ \text{Accelerations} \\ \hline \text{Unknown} \\ \text{Forces and} \\ \text{Moments} \end{array} \right\} = \left\{ \begin{array}{c} \text{Known} \\ \text{Accelerations} \\ \hline \text{Known} \\ \text{Forces and} \\ \text{Moments} \end{array} \right\}$$

CHAPTER VI

FORCE SPECIFICATION

Forces, moments, and accelerations are specified as known or unknown to describe the mechanism such that the number of unknown parameters equals the number of dynamic equations. The number of dynamic equations, both kinematic and force-acceleration balance, is summarized by the following:

1. Three equations for each vector loop
2. One equation for each dot product
3. Three equations for each cross product
4. One no spin equation for each vector
5. Six force-acceleration balance equations for each element.

The choice of which forces and moments to specify as known depends on the nature of the external forces and the characteristics of the various mechanical devices used in the mechanism. For example, the external force resulting from gravity acts in one direction on the center of mass and all other components of force and moment at the center of mass are zero. Also, the moment along the axis of a pinned joint depends on the friction in the joint. If the joint is frictionless, the moment must equal zero.

Depending on the method used to couple the equilibrium equations from Newton's second law, a consistent scheme must be used to specify the forces at an element connection. One scheme is to specify corresponding forces across a connection the same and to write six additional equilibrium equations for that connection. Another scheme is to recognize that corresponding forces across a connection are equal and opposite and thus, eliminate one set of force and moment parameters. The set of forces for this last scheme is demonstrated in Figure 7. The latter scheme has the advantage of not adding additional equations to the equation set but, is restricted to only allowing two elements per connection. More than one node can be placed in the same position on an element, however, and thus permit more than one element to be connected to the same point.

When it is necessary to constrain a force or moment in a particular direction, a dot product can be used to specify the magnitude of the force vector in the direction of another vector. A force vector constrained relative to a system vector is written as

$$[P_n] \{F_{k,j}\} = FM_{k,j,i} \quad (25)$$

where $FM_{k,j,i}$ is the magnitude of the force in the direction of the vector. Likewise, a force vector constrained relative to an element vector is written as

$$[PS] \{F_{k,j}\} = FM_{k,j,i} \quad (26)$$

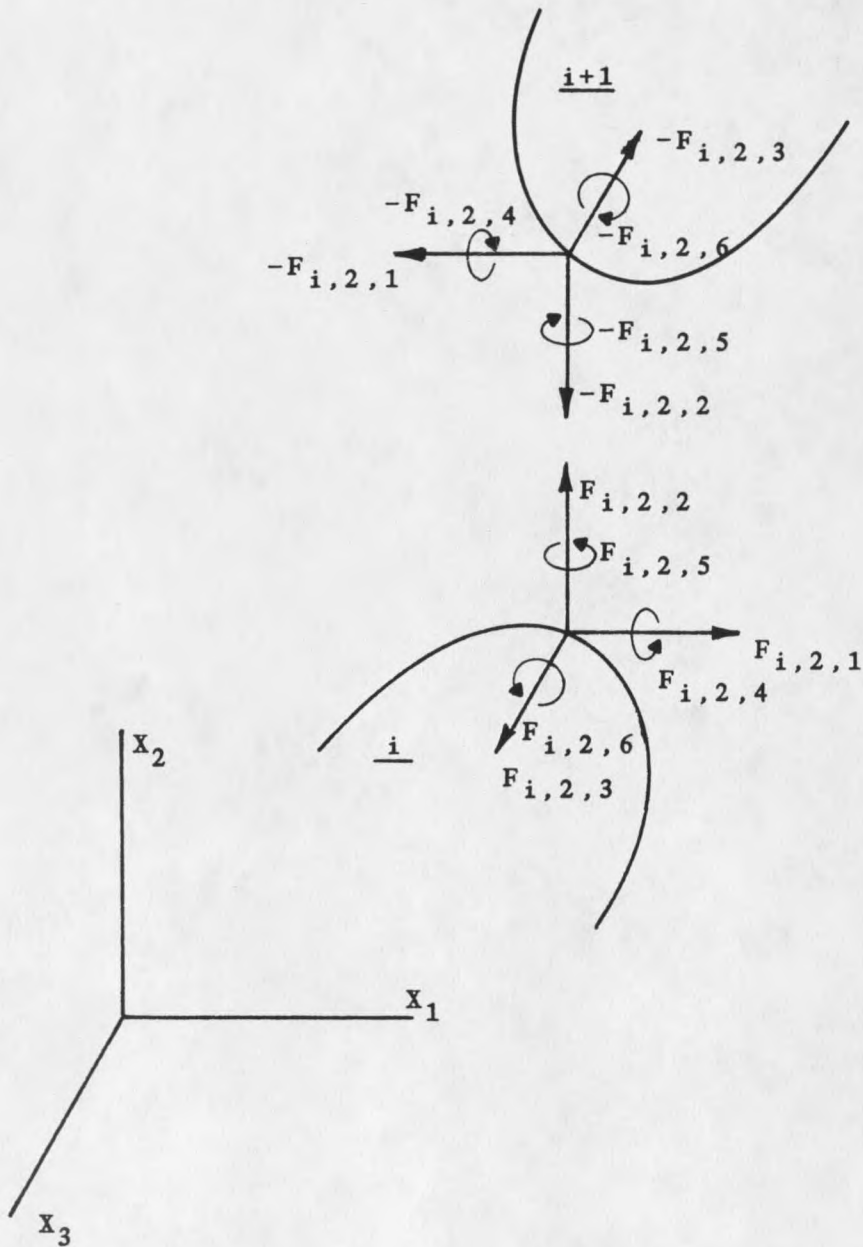


FIGURE 7. - Forces at a connection.

CHAPTER VII

POINT MASS

As an example, the circular motion of a point mass was analyzed. An element of mass 5 was considered to follow a circular path with a radius of 2 at a angular velocity of 10 rad/s.

The only vector needed to describe the kinematics of this system is the one which locates the point mass relative to the center of rotation. In giving this vector a constant length of 2 and an angular velocity of 10 rad/s about a particular axis, the motion of the point mass can be described.

To represent the element, two arbitrary unit vectors were used as the principal and element plane vectors. The total system of vectors is shown in Figure 8.

In this case, enough parameters were specified as known such that the only kinematic equations needed were the cosine squared constraints and the dot product between the principal vector (Vector 2) and the element plane vector (Vector 3). Note that the no spin constraints for the principal and element plane vectors and the dot product were preserved to so that the spins of these two vectors can be set equal to represent the motion of a rigid body.

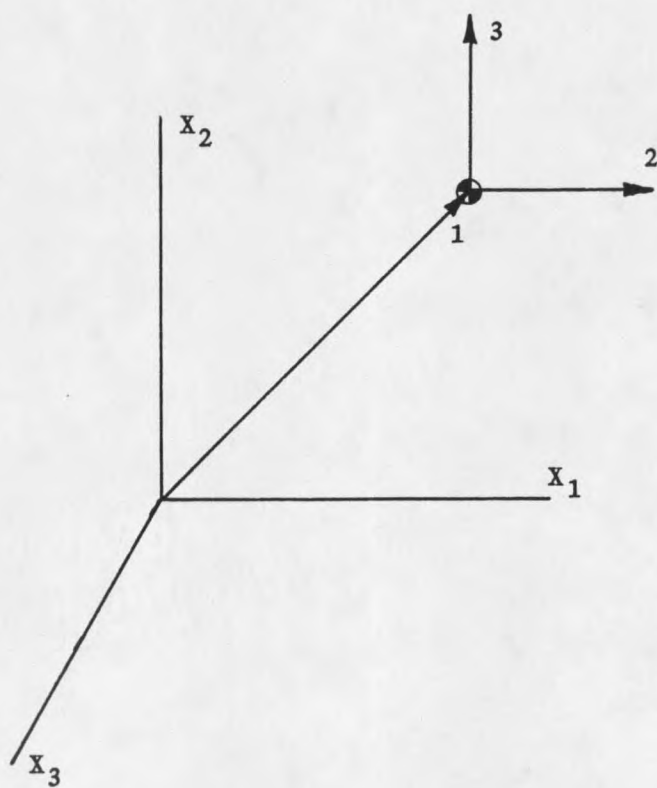


FIGURE 8. - Point mass vector representation.

The results of the kinematic geometry and velocity analyses are given in Tables 1 and 2. The asterisk (*) after a parameter indicates that the parameter was specified as a known quantity and its value was given. A question mark (?) after the the parameter indicates that the parameter was left as an unknown and its value was calculated.

TABLE 1. - Point mass geometry.

* => known				
? => unknown				
Vector Number	Length	Vector Parameters		
		X ₁ -Angle (Degrees)	X ₂ -Angle (Degrees)	X ₃ -Angle (Degrees)
1	2.00 *	45.00 *	45.00 ?	90.00 *
2	1.00 *	0.00 *	90.00 *	90.00 *
3	1.00 *	90.00 ?	0.00 ?	90.00 *
Dot Product Number	Dot Products			Included Angle (Deg)
	Vector Number	Vector Number	Vector Number	
1	2	3		90.00 *

TABLE 2. - Point mass velocities.

* => known				
? => unknown				
Vector Number	Length (Units/S)	Vector Parameters		
		X ₁ -Omega (Rad/S)	X ₂ -Omega (Rad/S)	X ₃ -Omega (Rad/S)
1	0.00 *	0.00 *	0.00 *	10.00 *
2	0.00 *	0.00 ?	0.00 *	0.00 *
3	0.00 *	0.00 *	0.00 ?	0.00 ?
Dot Product Number	Dot Products			Rate of Change of Angle (Rad/S)
	Vector Number	Vector Number	Vector Number	
1	2	3		0.00 *

The acceleration table is identical to the velocity table except that all of the values are zero. The only acceleration is the velocity induced centripetal acceleration.

The dynamic analysis results are given in Table 3. The first two nodes correspond to the ends of the principal and element plane vectors. These nodes represent points of application of external forces and since there are no external forces, all of the forces and moments at these nodes are zero. The third node corresponds to the center of mass of the element. It is at this node where forces are allowed to act to keep the mass in its circular path. Since there are six element equilibrium equations available to describe this system, all six forces and moments at Node 3 were left as unknowns. The resulting X_1 and X_2 forces at Node 3 correspond to the centripetal acceleration.

TABLE 3. - Point mass forces and moments.

Node Number	X_1 -Force	X_2 -Force	X_3 -Force
1	0.00E 0 *	0.00E 0 *	0.00E 0 *
2	0.00E 0 *	0.00E 0 *	0.00E 0 *
3	-7.07E 3 ?	-7.07E 3 ?	0.00E 0 ?
Node Number	X_1 -Moment	X_2 -Moment	X_3 -Moment
1	0.00E 0 *	0.00E 0 *	0.00E 0 *
2	0.00E 0 *	0.00E 0 *	0.00E 0 *
3	0.00E 0 ?	0.00E 0 ?	0.00E 0 ?

CHAPTER VIII

REMOTE MANIPULATOR SYSTEM

As another example, an approximation of the space shuttle remote manipulator system (RMS) was analyzed for a required performance maneuver. A design criteria for the RMS was to be able to stop a 32,000 pound payload moving at 0.2 ft/s within 2 ft [1]. A sketch of the RMS is shown in Figure 9 and its element mass properties are given in Table 4. The payload was considered to be a 30 foot long cylinder with a radius of 7.5 ft.

TABLE 4. - RMS element properties.

Element Number	Mass (Slugs)	I_{11}	I_{22}	I_{33} (Slug·ft ²)	I_{12}	I_{13}	I_{23}
1	2.20	0.4	0.8	1.0	0.1	0.0	0.0
2	9.54	2.4	735.5	736.6	-6.8	-1.4	-0.2
3	5.99	1.2	466.5	466.8	2.8	0.6	-0.1
4	0.58	0.1	0.3	0.3	0.0	0.0	0.0
5	3.15	1.7	14.9	14.2	0.3	0.7	0.3
6	994	88500	27950	88500	0.0	0.0	0.0

The vector representation of the RMS is shown in Figure 10 and the geometry parameters are listed in Table 5. Vectors 1 through 9 represent the various links of the mechanism and Vectors 10, 11, and 12 describe the cartesian coordinates of the center of mass of the payload.

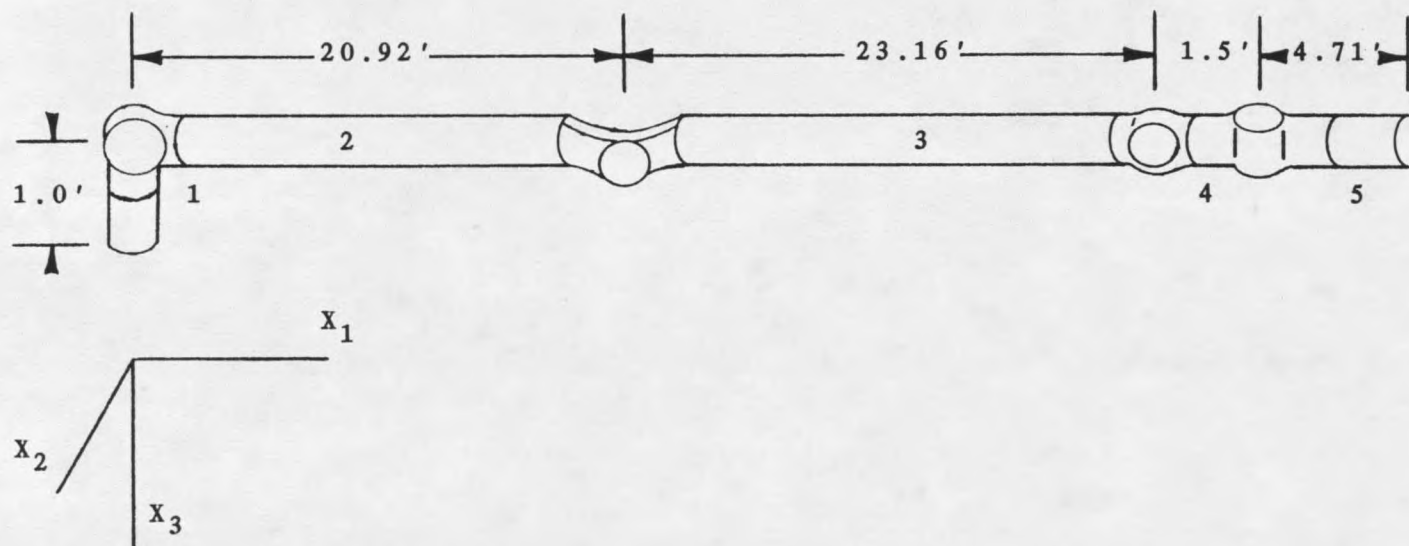


FIGURE 9. - Sketch of the space shuttle RMS.

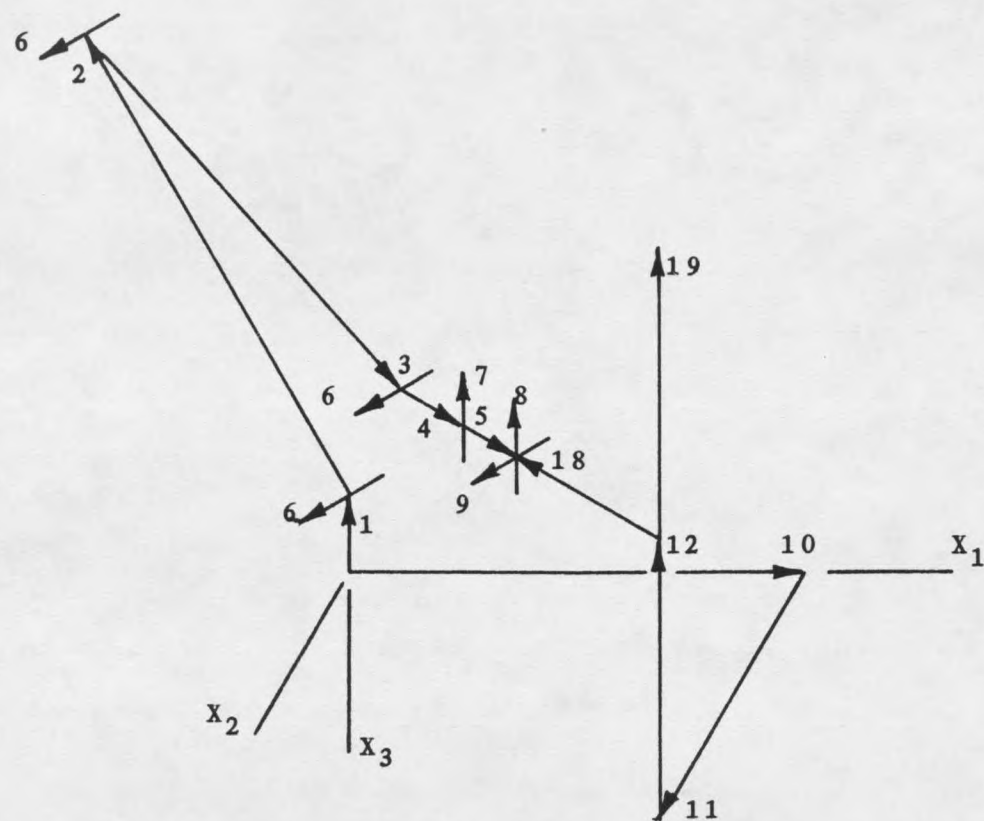


FIGURE 10. - Vector representation of the RMS.

TABLE 5. - RMS geometry.

* => known
 ? => unknown

Vectors in loop 1: (minus sign indicates the vector was traversed in the minus direction)

1 2 3 4 5 -10 -11 -12 -18

Vector Number	Length	Vector Parameters		
		X ₁ -Angle (Degrees)	X ₂ -Angle (Degrees)	X ₃ -Angle (Degrees)
1	1.00 *	90.00 ?	90.00 *	180.00 ?
2	20.92 *	100.54 ?	100.54 ?	165.00 ?
3	23.16 *	45.86 ?	45.86 ?	80.00 ?
4	1.50 *	45.00 ?	45.00 ?	90.00 ?
5	4.71 *	45.00 ?	45.00 ?	90.00 ?
6	1.00 *	135.00 *	45.00 *	90.00 *
7	1.00 *	90.00 ?	90.00 ?	180.00 ?
8	1.00 *	90.00 ?	90.00 ?	180.00 ?
9	1.00 *	135.00 ?	45.00 ?	90.00 ?
10	21.99 ?	0.00 *	90.00 *	90.00 *
11	22.00 ?	90.00 *	0.00 *	90.00 *
12	17.19 ?	90.00 *	90.00 *	0.00 *
13	0.50 *	90.71 ?	90.71 ?	0.00 ?
14	10.46 *	80.15 ?	80.15 ?	14.00 ?
15	11.58 *	133.96 ?	133.96 ?	101.00 ?
16	0.75 *	134.99 ?	134.99 ?	91.00 ?
17	2.35 *	136.00 ?	134.00 ?	90.00 ?
18	7.50 *	135.00 ?	135.00 ?	90.00 ?
19	15.00 *	90.00 *	90.00 ?	180.00 ?

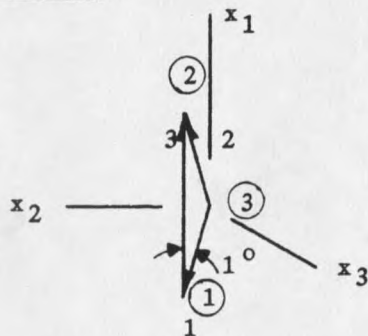
Dot Product Number	Dot Products		Included Angle (Deg)
	Vector Number	Vector Number	
1	1	6	90.00 *
2	2	6	90.00 *
3	3	6	90.00 *
4	4	6	90.00 *
5	4	7	90.00 *
6	5	7	90.00 *
7	6	7	90.00 *
8	5	8	90.00 *
9	5	9	90.00 *
10	8	9	90.00 *

TABLE 5. - Continued

Dot Product Number	Vector Number	Vector Number	Included Angle (Deg)
11	8	18	90.00 *
12	9	18	90.00 *
13	1	2	15.00 *
14	2	3	115.00 *
15	3	4	10.00 *
16	6	5	90.00 *
17	7	9	90.00 *
18	1	13	179.00 *
19	6	13	90.00 *
20	2	14	179.00 *
21	6	14	90.00 *
22	3	15	179.00 *
23	6	15	90.00 *
24	4	16	179.00 *
25	6	16	90.00 *
26	5	17	179.00 *
27	7	17	90.00 *
28	18	19	90.00 *

The first kinematic analysis was performed as described in Chapter II using only Vectors 1 through 12 with Vectors 10, 11, and 12 describing the coordinates of the end effector.

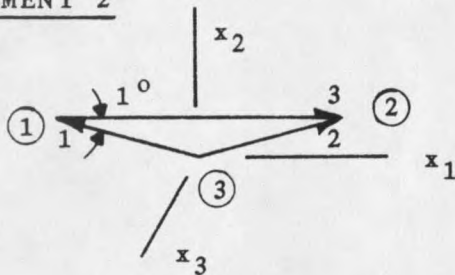
After the first analysis resulted in a satisfactory kinematic model, Vectors 13 through 19 were added to the model to include the elements of the system. The configurations of each element, the element vectors and their corresponding system vectors are given in Figure 11. The centers of mass of each element were placed such that the element plane vectors and the principal vectors were not parallel and thus, their cross product could be used to

ELEMENT 1Element
Vector

1
2
3

System
Vector

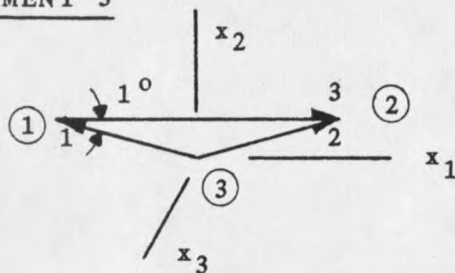
13
-
1

ELEMENT 2Element
Vector

1
2
3

System
Vector

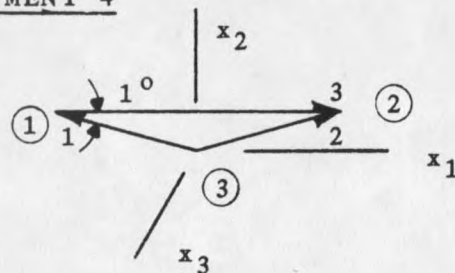
14
-
2

ELEMENT 3Element
Vector

1
2
3

System
Vector

15
-
2

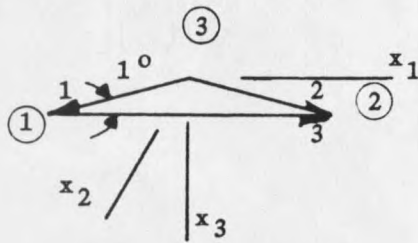
ELEMENT 4Element
Vector

1
2
3

System
Vector

16
-
2

FIGURE 11. - RMS element configurations (Node 3 is the center of mass for each element).

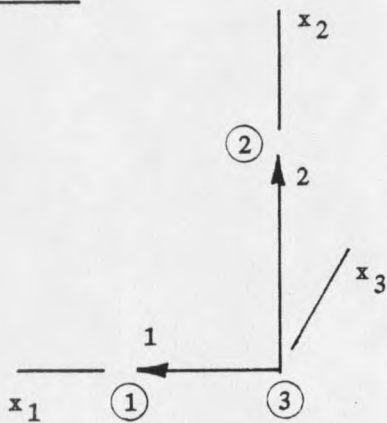
ELEMENT 5

Element Vector

1
2
3

System Vector

17
-
5

ELEMENT 6

Element Vector

1
2

System Vector

18
19

FIGURE 11. - Continued.

calculate a unique element coordinate system. A description of the dot products used in the geometry analysis is given by the following:

1. Dot products 1 through 10 represent the geometric constraints on the mechanism such as the constant angles between the joint pins and the arm booms
2. The payload is fixed to the mechanism using dot products 11 and 12
3. Dot products 13 through 17 specify the joint coordinates and thus describe the configuration of the mechanism
4. The element plane vectors are constrained to the system using dot products 18 through 28

Note that there exists a dot product between the principal and element plane vectors for each element so they can be later replaced with the equal spin equations in the velocity and acceleration analyses.

In the velocity table (Table 6), the payload was given a velocity of 0.2 ft/s in the minus X_1 -direction by specifying the length change of Vector 10. Therefore, for the given configuration, the RMS must act on the payload at an angle of 45° with the path of motion. The joint velocities, described by dot products 13, 14, and 15, and the angular velocities in the X_3 -direction indicate that the RMS is collapsing and rotating to follow the payload, as expected.

TABLE 6. - RMS velocities.

* => known
 ? => unknown

Vector Number	Length (Units/S)	Vector Parameters		
		X_1 -Omega (Rad/S)	X_2 -Omega (Rad/S)	X_3 -Omega (Rad/S)
1	0.00E 0 *	0.00E 0 ?	0.00E 0 *	4.55E-3 ?
2	0.00E 0 *	-5.19E-3 ?	5.19E-3 ?	4.55E-3 ?
3	0.00E 0 *	-1.23E-3 ?	1.23E-3 ?	4.55E-3 ?
4	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	4.55E-3 ?
5	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	4.55E-3 ?
6	0.00E 0 *	0.00E 0 ?	0.00E 0 *	4.55E-3 ?
7	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	0.00E 0 ?
8	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	0.00E 0 ?
9	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	4.55E-3 ?
10	-2.00E-1 *	0.00E 0 *	0.00E 0 *	0.00E 0 *
11	0.00E 0 *	0.00E 0 *	0.00E 0 *	0.00E 0 *
12	0.00E 0 *	0.00E 0 *	0.00E 0 *	0.00E 0 *
13	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	4.55E-3 ?
14	0.00E 0 *	-5.19E-3 ?	5.19E-3 ?	4.55E-3 ?
15	0.00E 0 *	-1.23E-3 ?	1.23E-3 ?	4.55E-3 ?
16	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	4.55E-3 ?
17	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	4.55E-3 ?
18	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	4.55E-3 ?
19	0.00E 0 *	0.00E 0 *	0.00E 0 *	4.55E-3 ?

Dot Product Number	Dot Products		Rate of Change of Angle (Rad/S)
	Vector Number	Vector Number	
1	1	6	0.00E 0 *
2	2	6	0.00E 0 *
3	3	6	0.00E 0 *
4	4	6	0.00E 0 *
5	4	7	0.00E 0 *
6	5	7	0.00E 0 *
7	6	7	0.00E 0 *
8	5	8	0.00E 0 *
9	5	9	0.00E 0 *
10	8	9	0.00E 0 *
11	8	18	0.00E 0 *
12	9	18	0.00E 0 *
13	1	2	7.35E-3 ?
14	2	3	5.60E-3 ?
15	3	4	-1.74E-3 ?
16	6	5	0.00E 0 *
17	7	9	0.00E 0 *
18	1	13	0.00E 0 *

TABLE 6. - Continued.

Dot Product Number	Dot Products		Rate of Change of Angle (Rad/S)
	Vector Number	Vector Number	
19	6	13	0.00E 0 *
20	2	14	0.00E 0 *
21	6	14	0.00E 0 *
22	3	15	0.00E 0 *
23	6	15	0.00E 0 *
24	4	16	0.00E 0 *
25	6	16	0.00E 0 *
26	5	17	0.00E 0 *
27	7	17	0.00E 0 *
28	18	19	0.00E 0 *

In this example, the number of kinematic equations equals the number of unknown accelerations thus, the dynamic equations are uncoupled. The resulting accelerations are given in Table 7. This time, Vector 10 specifies the acceleration in the X_1 -direction required to stop the payload within 2 feet.

TABLE 7. - RMS accelerations.

* => known
? => unknown

Vector Number	Vector Parameters			
	Length (Units/S ²)	X_1 -Alpha (Rad/S ²)	X_2 -Alpha (Rad/S ²)	X_3 -Alpha (Rad/S ²)
1	0.00E 0 *	0.00E 0 ?	0.00E 0 *	-1.86E-4 ?
2	0.00E 0 *	2.45E-4 ?	-2.92E-4 ?	-1.86E-4 ?
3	0.00E 0 *	2.46E-5 ?	-3.58E-5 ?	-1.86E-4 ?
4	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	-1.86E-4 ?
5	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	-1.86E-4 ?
6	0.00E 0 *	0.00E 0 ?	0.00E 0 *	-1.86E-4 ?
7	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	0.00E 0 ?
8	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	0.00E 0 ?
9	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	-1.86E-4 ?

TABLE 7. - Continued.

Vector Parameters				
Vector Number	Length (Units/S ²)	X ₁ -Alpha (Rad/S ²)	X ₂ -Alpha (Rad/S ²)	X ₃ -Alpha (Rad/S ²)
10	1.00E-2 *	0.00E 0 *	0.00E 0 *	0.00E 0 *
11	0.00E 0 *	0.00E 0 *	0.00E 0 *	0.00E 0 *
12	0.00E 0 *	0.00E 0 *	0.00E 0 *	0.00E 0 *
13	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	-1.86E-4 ?
14	0.00E 0 *	2.45E-4 ?	-2.92E-4 ?	-1.86E-4 ?
15	0.00E 0 *	2.46E-5 ?	-3.58E-5 ?	-1.86E-4 ?
16	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	-1.86E-4 ?
17	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	-1.86E-4 ?
18	0.00E 0 *	0.00E 0 ?	0.00E 0 ?	-1.86E-4 ?
19	0.00E 0 *	0.00E 0 *	0.00E 0 *	-1.86E-4 ?

Dot Products			
Dot Product Number	Vector Number	Vector Number	Rate of Change of Angle (Rad/S ²)
1	1	6	0.00E 0 *
2	2	6	0.00E 0 *
3	3	6	0.00E 0 *
4	4	6	0.00E 0 *
5	4	7	0.00E 0 *
6	5	7	0.00E 0 *
7	6	7	0.00E 0 *
8	5	8	0.00E 0 *
9	5	9	0.00E 0 *
10	8	9	0.00E 0 *
11	8	18	0.00E 0 *
12	9	18	0.00E 0 *
13	1	2	-3.79E-4 ?
14	2	3	-3.36E-4 ?
15	3	4	4.28E-5 ?
16	6	5	0.00E 0 *
17	7	9	0.00E 0 *
18	1	13	0.00E 0 *
19	6	13	0.00E 0 *
20	2	14	0.00E 0 *
21	6	14	0.00E 0 *
22	3	15	0.00E 0 *
23	6	15	0.00E 0 *
24	4	16	0.00E 0 *
25	6	16	0.00E 0 *
26	5	17	0.00E 0 *
27	7	17	0.00E 0 *
28	18	19	0.00E 0 *

The six equilibrium equations per element allowed the forces and moments at each joint to be selected as unknowns. The forces and moments at the center of mass of each element and at the external node on Element 6 are all known to be zero.

Using Gauss-Elimination with pivoting to solve the dynamics equations produced the set of resulting forces and moments given in Table 8. Back substitution into the equilibrium equations demonstrated an equation balance to thirteen significant digits. Double precision was required for the generation and solution of the equations because of the large difference in magnitude between the forces and the accelerations.

TABLE 8. - RMS forces and moments.

* => known
 ? => unknown
 # => corresponds to unknown on previous element

ELEMENT 1

Node Number	X ₁ -Force	X ₂ -Force	X ₃ -Force
1	1.00E 1 ?	5.38E-2 ?	-6.80E-3 ?
2	-1.00E 1 ?	-5.38E-2 ?	6.80E-3 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *
Node Number	X ₁ -Moment	X ₂ -Moment	X ₃ -Moment
1	1.02E 0 ?	-1.73E 2 ?	-2.24E 2 ?
2	-9.70E-1 ?	1.63E 2 ?	2.24E 2 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

TABLE 8. - Continued.

ELEMENT 2

Node Number	X ₁ -Force	X ₂ -Force	X ₃ -Force
1	1.00E 1 #	5.38E-2 #	-6.83E-3 #
2	-1.00E 1 ?	-2.80E-2 ?	2.90E-3 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

Node Number	X ₁ -Moment	X ₂ -Moment	X ₃ -Moment
1	9.70E-1 #	-1.63E 2 #	-2.24E 2 #
2	6.57E-2 ?	-4.02E 1 ?	2.63E 2 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

ELEMENT 3

Node Number	X ₁ -Force	X ₂ -Force	X ₃ -Force
1	1.00E 1 #	2.80E-2 #	-2.87E-3 #
2	-9.97E 0 ?	-5.62E-3 ?	0.00E 0 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

Node Number	X ₁ -Moment	X ₂ -Moment	X ₃ -Moment
1	-6.57E-2 #	4.02E 1 #	-2.63E 2 #
2	6.72E-4 ?	-1.26E-4 ?	1.02E 2 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

ELEMENT 4

Node Number	X ₁ -Force	X ₂ -Force	X ₃ -Force
1	9.97E 0 #	5.62E-3 #	0.00E 0 #
2	-9.97E 0 ?	-4.52E-3 ?	0.00E 0 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

Node Number	X ₁ -Moment	X ₂ -Moment	X ₃ -Moment
1	6.72E-5 #	1.26E-3 #	-1.02E 2 #
2	-6.72E-5 ?	-1.26E-3 ?	9.10E 1 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

ELEMENT 5

Node Number	X ₁ -Force	X ₂ -Force	X ₃ -Force
1	9.97E 0 #	4.52E-3 #	0.00E 0 #
2	-9.94E 0 ?	0.00E 0 ?	0.00E 0 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

Node Number	X ₁ -Moment	X ₂ -Moment	X ₃ -Moment
1	6.72E-5 #	1.26E-3 #	-9.10E 2 #
2	0.00E 0 ?	0.00E 0 ?	5.79E 2 ?
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

TABLE 8. - Continued.

ELEMENT 6

Node Number	X ₁ -Force	X ₂ -Force	X ₃ -Force
1	9.94E 0 #	0.00E 0 #	0.00E 0 #
2	0.00E 0 *	0.00E 0 *	0.00E 0 *
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

Node Number	X ₁ -Moment	X ₂ -Moment	X ₃ -Moment
1	0.00E 0 #	0.00E 0 #	-5.79E 1 #
2	0.00E 0 *	0.00E 0 *	0.00E 0 *
3	0.00E 0 *	0.00E 0 *	0.00E 0 *

The forces and moments on Node 1 of Element 1 are the resultant forces and moments at the base of the RMS. By resolving the moment components at Node 1 of Element 2 in the direction of Vector 6, it is found that the shoulder pitch joint must be able to produce a torque of 116 ft-lb_f. The same components resolved in the direction of Vector 2 indicate that the upper arm boom must withstand an axial torque of 246 ft-lb_f. The X₃-Moment of -224 ft-lb_f at Node 1 of Element 2 is the torque that the shoulder yaw joint must produce. Similar calculations at Node 1 of Element 3 indicate that the elbow joint must produce a torque of 28.5 ft-lb_f.

CHAPTER IX

CONCLUSION

In this thesis, a method for the dynamic analysis of spatial mechanisms was described. A vector model of the mechanism using vector operations such as closed vector loops, dot products, and cross products was used to describe the kinematics of the mechanism. Newton's second law was applied to each element of the mechanism to relate the accelerations and forces inherent to the mechanism.

A computer program has been written using this method and was used for the examples provided in this thesis. Several other mechanisms such as a piston-crank and a three-dimensional four-bar linkage have also been analyzed.

This method proves to be very versatile in that an inverse dynamic analysis can be as easily accomplished as a forward dynamic analysis by specifying a different set of parameters as known. The solution of inverse dynamics (calculating joint accelerations/forces given the end conditions) does not require manual reformulation of the dynamic equations. The relative simplicity of the method provides ease in programming, formulation of the dynamic equations, and description of the mechanism for use in the areas of both design and analysis.

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