Application of the minimum rate of energy dissipation theory to gradually varied flow in gravel-bed streams

by Paul Arthur Janke

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering
Montana State University

Abstract:
Predictability of the magnitude of stream changes resulting from man-made works can be improved. This study develops the MINRED method. The MINRED method utilizes (a) the minimum rate of energy dissipation theory, (b) a combination of Shield's diagram and Raudkivi's graphical relation, and (c) the direct step method of determining the gradually varied flow profile. This method calculates equilibrium bankfull values of the dependent variables (stream depth, width, and sediment size) in reaches of gravel-bed alluvial streams having gradually varied flow and low sediment discharge. The MINRED method's accuracy is evaluated through application of the method to two Alaskan streams, Copper River tributary and Brooks Creek tributary. Results of this application indicate poor reliability when compared with measured values and trends in values predicted in the literature. This study supplies a rationale for the low reliability and offers suggestions for future work.

APPLICATION OF THE MINIMUM RATE OF ENERGY DISSIPATION THEORY TO GRADUALLY VARIED FLOW IN GRAVEL-BED STREAMS

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APPROVAL

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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TABLE OF CONTENTS

TABLE OF CONTENTS--Continued

TABLE OF CONTENTS--Continued

TABLE OF CONTENTS -- Continued

LIST OF TABLES

LIST OF TABLES -- Continued

LIST OF FIGURES

TABLE OF CONTENTS -- Continued

LIST OF TABLES

LIST OF TABLES -- Continued

LIST OF FIGURES

ABSTRACT

Predictability of the magnitude of stream changes resulting from man-made works can be improved. This study develops the MINRED method. The MINRED method utilizes (a) the minimum rate of energy dissipation theory, (b) a combination of Shield's diagram and Raudkivi's graphical relation, and (c) the direct step method of determining the gradually varied flow profile. This method calculates equilibrium bankfull values of the dependent variables (stream depth, width, and sediment size) in reaches of gravel-bed alluvial streams having gradually varied flow and low sediment discharge. The MINRED method's accuracy is evaluated through application of the method to two Alaskan streams, Copper River tributary and Brooks Creek tributary. Results of this application indicate poor reliability when compared with measured values and trends in values predicted in the literature. This study supplies a rationale for the low reliability and offers suggestions for future work.

CHAPTER 1

INTRODUCTION

Alluvial streams are self-adjusting. Through the processes of scour and deposition, the physical configuration and flow characteristics in a reach of an alluvial stream can be modified by changes in conditions imposed at a particular location. These imposed conditions are the independent variables. A stream's physical configuration and flow characteristics are its dependent variables. When values of the independent variables change, it is desirable to estimate values of the dependent variables in equilibrium with the modified values of the independent variables.

Establishment of the Problem

Hey (1975b) considers it important to know what impacts imposed human activity will have on alluvial channels. With this information, appropriate measures can be taken to sustain channel stability, to prevent destruction of riparian land, to maintain channel flood flow capacity, and to retain recreational appeal. Hyra (1978) expounds that instream recreational activities have specific water velocity and depth requirements. Further, Bovee (1978) emphasizes that some fish species have specific instream habitat requirements for water velocity, depth, and mean sediment size.

Regardless of the degree of equilibrium attained by a stream, localized human activity may produce major changes in values of a stream's dependent variables both near the activity and throughout an entire reach (Simons, 1979). Conversely, alterations in values of a stream's dependent variables may impact human activities. Knowing that a specific human activity will be imposed on a stream, it is desirable to estimate values of the dependent variables that will be in equilibrium with the modified values of the independent variables. These estimated values can be compared to the existing values to predict impacts from the human activity on (a) channel flood flow capacity; (b) riparian land use; and (c) various instream activities such as aquatic habitat, recreation, and commerce. With this information, appropriate measures could be taken to minimize negative impacts. For example, estimates could indicate that a specific human activity would modify the stream mean sediment size so as to make it unusable to an important fish species during spawning. This information could be used to modify the human activity to minimize this negative impact.

Trends in values of a stream's dependent variables due to a specific human activity can be predicted with a high degree of reliability. However, methods currently available to water resources engineers are inadequate to predict the magnitude of these changes.

Minimum Rate of Energy Dissipation Theory

By determining the values of a stream's dependent variables before and after a human activity is imposed, the magnitude of changes in these values can be predicted. Values of a stream's dependent variables can be calculated by the simultaneous solution of relationships which link the independent and dependent variables. This solution method requires that one such relationship is used for each dependent variable. However, alluvial streams have more dependent variables than relationships available to link them to independent variables. Also, many human activities imposed on streams create a considerable length of gradually varied flow. The availability of another relationship usable in alluvial streams with gradually varied flow increases the ability of engineers to predict the magnitude of stream changes resulting from human activity.

The minimum rate of energy dissipation theory has recently been proposed for use with alluvial streams. This theory states that, provided certain conditions are met, equilibrium values of the dependent variables are adjusted so that alluvial streams flow with the minimum rate of energy dissipation that is compatible with the independent variable values.

Alluvial streams with no imposed human activity experience conditions approaching uniform flow. Although not correct, this study considers such streams to have uniform flow conditions. The terms gradually varied flow and rapidly varied flow are used in the traditional sense.

Recently, investigators have used the minimum rate of energy dissipation theory as a relationship to link the independent and dependent variables of alluvial streams with no imposed human activity. Results of the simultaneous solution of this with other relationships indicate values of the calculated dependent variables agree with published data measured in flumes and streams which have low sediment discharges and uniform flow.

Objectives

The first objective is to develop a method that uses the minimum rate of energy dissipation theory to calculate equilibrium bankfull values of the dependent variables in reaches of gravel-bed alluvial streams having gradually varied flow. The second objective is to answer the question: Given a gravel-bed alluvial stream which satisfies certain conditions, with what reliability can the above method be used to calculate equilibrium bankfull values of the dependent variables in reaches having gradually varied flow imposed by a highway culvert?

The first objective will be addressed by developing a method which solves three relationships to calculate equilibrium bankfull values of the dependent variables in reaches of gravel-bed alluvial streams having gradually varied flow. The three relationships used by this method are the direct step method of determining the gradually varied flow profile, a combination of Shields diagram and Raudkivi's graphical relation, and the minimum rate of energy dissipation theory. The dependent variables of gravel-bed alluvial streams are the stream width, stream depth, and mean size of the sediment that is stationary on the bed. Because this method uses the minimum rate of energy dissipation theory, it is named the MINRED method.

Two gravel-bed streams are studied. Each has rapidly and gradually varied flow imposed by a highway culvert at the bankfull water discharge. The second objective will be addressed by evaluating the reliability with which the MINRED method can be used to calculate equilibrium bankfull values of the dependent variables in the gradually varied flow reaches of the two streams studied.

CHAPTER 2

LITERATURE REVIEW

This literature review provides the background for developing the MINRED method and for evaluating the reliability with which it can be applied to reaches of gravel-bed alluvial streams having gradually varied flow. The literature review is divided into five sections discussing: the time period required for equilibrium to be developed between the independent and dependent variables, the independent variables of gravel-bed streams for the required time period, the dependent variables of gravel-bed alluvial streams for the required time period, the trends in values of the dependent variables expected after values of the independent variables are changed by the installation of a culvert, and the minimum rate of energy dissipation theory as applied to alluvial streams, including its use to estimate equilibrium values of the dependent variables.

Adjustment Time Periods

Independent and dependent variables of a stream are considered to be in equilibrium if values of the dependent variables are adjusted in response to values of the independent variables. Such a stream may be defined as "graded" (Mackin, 1948), "in regime" (Blench, 1969), or "poised" (Simons, 1979). In this study, the term equilibrium is used exclusively.

The time periods which can be considered in a study of stream response to independent variables are short term, long term, and very long term (Vanoni, 1975). Short term time periods are days, weeks, or months. Streams seldom if ever achieve equilibrium in the short term. This is because after values of the independent variables change, the time required for the dependent variables to change to the new equilibrium values is greater than the length of the short term time period. Therefore, the short term is too brief for studying stream equilibrium. The long term is required for an equilibrium condition to be created between the independent and dependent variables (Vanoni, 1975). Alluvial streams with 10 to 100 years to react to a change in an imposed condition can be considered to be adjusting toward equilibrium (Santos-Cayade and Simons, 1973). The very long term is a geologic time period (Vanoni, 1975). The long term time period is considered in this study.

Independent Variables

For the long term time period, the independent variables of alluvial streams are sediment density, fluid molecular viscosity and density, acceleration due to gravity, water discharge, and sediment discharge (Vanoni, 1975). "Man-made works" are independent variables in the very long term (Vanoni, 1975). In this study, however, man-made works are considered to be independent variables in the long term time period. This is because man-made works are known to cause major impacts on streams in the short term and long term time periods and are believed to have only a negligible influence on stream morphology in the very long term.

The number of independent variables to be considered herein is reduced below that given by Vanoni (1975). Simons and Senturk (1976) and Stelczer (1981) indicate that when low density particles of volcanic origin can be neglected, the sediment specific gravity can be considered equal to a constant 2.65. The measured data indicate that this is true for the study streams. Therefore, the sediment density is constant. Daily and Harleman (1966) and Olson (1966) indicate the water molecular viscosity and density depend on the temperature. In this study, a constant water temperature is approximated to represent the temperature fluctuations in streams. Therefore, the water molecular viscosity and density are constant.

Water Discharge

According to Prins and de Vries (1971) and Bray (1982b), river studies frequently assume that dependent variables are adjusted in response to a constant discharge rather than to the discharge fluctuations. This constant, channel forming flow rate is called the dominant discharge. Wolman and Leopold (1957), Wolman and Miller (1960), and Hey (1975a) studied natural channels and laboratory flumes. These studies conclude that compared to any other single discharge, the bankfull flow rate transports the most sediment in the long term and hence is the dominant discharge. Hey (1975b) concludes that for alluvial channels in equilibrium, the average bankfull dimensions that are formed in response to a constant bankfull discharge are similar to those formed in response to discharge fluctuations. Ackers (1982) studied meander wavelength in laboratory flumes at a constant discharge concluding that the bankfull flow rate provided the best correlation with the results than any other discharge. Therefore, the dominant discharge is the bankfull flow rate.

Consider two streams in equilibrium differing only in the imposed water discharge. The first stream has a constant water discharge and the second stream does not. The average bankfull dimensions of these two streams are similar if the constant discharge imposed on the first stream equals the bankfull discharge of the second stream.

Wolman and Leopold (1957), Wolman and Miller (1960), Hey (1975a), Williams (1978), and others have studied streams in the United States, United Kingdom, and India. These studies show the recurrence interval of the bankfull discharge to be one to two years for many streams flowing in diverse climatic and physiographic environments. These studies also show that a recurrence interval of 1.5 years on the annual maximum series is a good average, although some localities diverge greatly from this value. Dunne and Leopold (1978) indicate that for streams with limited discharge information, the best approximation of the bankfull flow rate is the discharge with a recurrence interval of 1.5 years on the annual maximum series. This recurrence interval of the bankfull discharge is widely accepted in the literature.

Sediment Discharge

The sediment discharge can be divided into the wash load and the bed material load. The wash load is that part of the sediment discharge which remains in suspension at any water flow rate and is composed of particle sizes finer than those found on the stream bed. Therefore, the wash load is determined by the upslope supply rate and can not be predicted by the transport capacity of the stream (Shen, 1971). Wash load can be imposed on streams by glacial runoff, human activity, or erosion from the drainage basin. The study streams presented in Chapter 3 are not fed by glacial runoff. There is no human activity nor is there any exposed soil in the drainage basins upstream from the highway culverts in question. The latter because vegetation ranging from grass to spruce and willow completely cover the ground. For these reasons, there is a limited source of wash load to the sites of interest in the study streams and hence the wash load is not considered further.

Lane (1947) indicates the bed material load is that part of the sediment discharge transported as either bed load or suspended load and is composed of particle sizes which can be found on the stream bed. Shen (1971) indicates the bed material load equals the sediment transport capability of the flow. According to Leopold, Wolman, and Miller (1964), the bed load is that part of the bed material load which moves on or very near the bed. Dunne and Leopold (1978) indicate that the suspended load is that part of the bed material load which is supported by the fluid column. They also indicate that the sediment grains of the suspended load are nearly always less than 0.5 mm in diameter. Lane (1947) indicates that gravel-bed streams have a mean grain diameter larger than 2 mm. For the gravel-bed study streams, therefore, suspended load is low; hence, most sediment transported must move as bed load.

Bray (1982a) indicates that initiation of motion of a gravel-bed occurs near bankfull stage. This infers that the sediment discharge and the bed load is low in gravel-bed streams at the bankfull water flow rate. Also, Hey (1982) indicates that tracer experiments at a variety of gravel-bed rivers in the United Kingdom found bed loads low at the bankfull flow rate.

Vanoni (1975) indicates that bed load samplers collect between roughly 45% and 70% of the material transported as bed load. He also compares various analytical sediment discharge relationships and concludes that predictions vary by roughly two orders of magnitude. Therefore, accurate sediment discharge information can not be determined by either direct measurements or calculations.

Sediment discharge is low and can not be accurately determined in gravel-bed streams at the bankfull water flow rate. For simplicity and possibly for accuracy, the sediment discharge is assumed to be constant at the bankfull discharge for the gravel-bed study streams.

Dependent Variables

Values of the dependent variables are determined in response to values of the independent variables. According to Yang and Song (1979), in general, the dependent variables describe the physical configuration and flow characteristics of streams. There is some disagreement in the literature regarding which parameters are the dependent variables of alluvial streams. Vanoni (1975) indicates much of the disagreement stems from not making adequate distinctions between the behavior of streams in the short term, long term, and very long term.

For the long term, the dependent variables of alluvial streams are:

b = stream width,

d = stream depth,

d_s = mean size of sediment stationary on the bed,

f = Darcy-Weisbach friction factor,

p_f = planform,

R = hydraulic radius,

S = slope of the energy grade line,

V = mean velocity of flow,

w_f = fall velocity of the bed material, and

σ = geometric standard deviation of the bed material size distribution (Vanoni, 1975).

Some parameters not on Vanoni's list are considered to be dependent variables by other investigators, for example: sinuosity, wetted perimeter, and bed form height. These parameters are less important than those given by Vanoni because, for gravel-bed alluvial streams at the bankfull discharge, these parameters are either constant, negligible, or functions of the dependent variables given by Vanoni (1975).

Functional Relationships

Table 1 presents functional relationships between the independent and dependent variables and the references from which these were determined. These relationships reduce the

Table 1. Functional relationships for dependent variables.

Variable	Function of	References
σ	Constant	c,h
d_s	Q	c,h,m
d	Q, d_s	b,i,k,l
b	Q, d_s	b,i,k
S	Q, d_s, d, b	b,e,f,h,l
p_f	Q, d_s, d, b	a,j
R	Q, d_s, d	d,g,h,o
V	Q, d_s, d, b	h,i,k,l
w_f	d_s	c,o
f	Q, d_s, d, b	n

Note. References are as follows: (a) Ackers (1982), (b) Ackers and Charlton (1970), (c) Bathurst (1982), (d) Bray (1982a), (e) Bray (1982b), (f) Fahnestock (1963), (g) Hey (1978), (h) Hey (1982), (i) Leopold and Maddock (1953), (j) Leopold and Wolman (1957), (k) Leopold et al. (1964), (l) Schumm and Khan (1972), (m) Simons (1971), (n) Simons and Richardson (1966), and (o) Vanoni (1975).

dependent variables of gravel-bed alluvial streams to functions of the mean sediment size that is stationary on the bed, stream depth, and stream width. Independent variableswere omitted from these relationships if their impacts on the dependent variables are constant or can not be predicted in general. Therefore, the water discharge, Q, was the only independent variable considered.

Cross Sectional Shape

Vanoni (1975) indicates that there have been many successful studies that assume channel cross sections are wide and rectangular. Presumably this means the width is at least several times the depth. Bray (1982b) adds that this generally results in an error of less than 3% in the value of the hydraulic radius, even for small channels near bankfull stages. Further, Leopold et al. (1964) show that the assumption of a rectangular cross section is accurate for a wide variety of streams in the United States and Europe. Therefore, channels in which the width is at least several times the depth can be considered rectangular, and the cross section defined by the width and depth.

Depth, Width, and Velocity

Leopold and Maddock (1953), Leopold et al. (1964), and others indicate stream depth, stream width, and mean velocity of flow are exponentially related to the water discharge. These investigators also show the discharge exponent for each of these variables is constant for flumes, canals, and streams that are in equilibrium, with no imposed human activity, and flowing in temperate climates. Therefore, the equilibrium condition of a stream satisfying these criteria can be evaluated by comparing its discharge exponents to those given in the literature.

Parameter Value Trends

The independent variables of gravel-bed streams are the bankfull water discharge and man-made works. A culvert placed in a stream is a man-made work. Values of the dependent variables respond to changes in the independent variable values. Therefore, after the installation of a culvert, bankfull values of the dependent variables change until equilibrium is established.

A culvert placed in a stream creates a gradually varied flow condition upstream when its discharge capacity is less than that of the stream. Schumm (1971) and Simons (1979) indicate that this causes an increase in the depth and a decrease in the slope of the energy grade line and sediment discharge. Because of this reduced sediment discharge, subreaches upstream of the culvert with gradually varied flow can not transport the larger particles and thus experience sediment deposition. The moving sediment particles are smaller than the stationary bed sediment. Thus, the mean sediment size of the sediment deposited in the gradually varied flow reach during a certain water discharge is smaller than the mean sediment size of the pre-culvert bed material which was stationary during the same water discharge. Consequently, the mean sediment size on the stream bed upstream of the culvert decreases as a result of the culvert.

The culvert reduces sediment discharge and the mean sediment size transported into the reach downstream of the culvert to below the pre-culvert condition. Degradation occurs downstream of the culvert increasing the sediment discharge and compensating for the reduction in the imposed sediment discharge (Simons, 1979). As a result of this degradation, stream depth increases downstream of the culvert reducing the velocity head and the rate of head loss. Therefore, the slope of the energy grade line decreases (Schumm, 1971 and Simons, 1979). This increased stream depth and reduced slope of the energy grade line decreases the sediment discharge of the stream. In time this condition will equal the imposed sediment discharge. Similar to upstream of the culvert, the mean sediment size on the stream bed downstream of the culvert decreases as a result of the reduced sediment discharge (Simons, 1979).

For the above conditions, channel width will decrease if the stream transports a significant amount of fine material (Santos-Cayade and Simons, 1973). This does not occur in gravel-bed streams, hence the width remains unchanged as a result of the culvert.

A culvert placed in a gravel-bed stream may impose gradually varied flow reducing its sediment discharge. This causes no change in the width, an increase in the depth, and a decrease in the slope of the energy grade line and the mean sediment size stationary on the bed. This occurs both upstream and downstream of the culvert.

Minimum Rate of Energy Dissipation Theory

Discussion of the minimum rate of energy dissipation theory is divided into two subsections. The first subsection states the theory and the conditions where it can be applied to alluvial streams. The second subsection discusses a method which uses the theory to estimate equilibrium values of the dependent variables for alluvial streams with uniform flow.

Statement for Alluvial Streams

A development of the minimum rate of energy dissipation theory is summarized in Appendix C. The development considers three velocity vectors. Provided certain conditions are met, the flow described by a velocity vector satisfying the equation of motion necessarily results in a lesser rate of energy dissipation than the flow described by a velocity vector not satisfying the equation of motion.

Fluid flows must satisfy the equation of motion. Thus, the flow with the least rate of energy dissipation necessarily satisfies the equation of motion and the real flow situation.

The energy grade line is a line connecting values of the total energy in the flow at various locations. The difference in the energy grade line at two locations is the difference in total energy (Chow, 1959). Assuming no external energy is added to or removed from the fluid, this energy difference is the amount of energy dissipated by the flow. The average rate with which energy is dissipated by the flow is determined by dividing the change of total energy by the distance between the two locations. This is equivalent to the slope of the energy grade line. Therefore, to flow with the least rate of energy dissipation is to necessarily flow with the least slope of the energy grade line. Thus, the minimum rate of energy dissipation theory, as it applies to alluvial streams, is as stated below. Provided certain conditions are met, equilibrium values of the dependent variables are adjusted so alluvial streams flow with the minimum slope of the energy grade line that is compatible with the imposed independent variable values (Chang, 1979). The above conditions require alluvial streams to have a constant water temperature with steady uniform or gradually varied flow and negligible sediment discharge.

Application

Chang (1979, 1980a, 1980b), presented a method which solves three relationships to calculate the equilibrium values of the stream width, stream depth, and slope of the energy grade line for alluvial streams with uniform flow. This method uses a specific water discharge, sediment discharge, mean sediment size, water temperature, and trapezoidal cross sectional shape. Chang's method uses three relationships: an empirical sediment transport equation, an empirical flow resistance equation, and the minimum rate of energy dissipation theory represented by a minimum slope of the energy grade line. The steps used by Chang's method are:

1. Input data.
2. Assign a value to the stream width.
3. Assign a value to the stream depth.
4. Calculate the slope of the energy grade line using the sediment transport equation.
5. Calculate the water discharge using the flow resistance equation.
6. If the calculated water discharge does not equal the input water discharge, modify the stream depth and go to item 4.
7. If the calculated slope of the energy grade line does not equal the global minimum slope, modify the stream width and go to item 3.

8. Output stream width, stream depth, and slope of the energy grade line.

Chang's 1979 study used a mean sediment size of medium sand and the method given. Calculated values of the minimum energy grade line slope and of the associated stream width and stream depth were found for six different sediment discharge values. With a low sediment discharge, one minimum value of the slope was found. For a higher sediment discharge two minimum values of the slope were found; each at different values of the width and depth. One minimum slope was found in the lower flow regime and the other in the upper flow regime. The slope in the lower flow regime had the lower value, the global minimum. Therefore, Chang (1979) determined a minimum value of the slope can be found. He did not explain, however, why the physical processes produce this minimum slope. These results were compared to published river and canal data; conclusions related planform to the water discharge and slope.

For low sediment discharges, both of Chang's 1980 studies yield calculations similar to his 1979 work. In one study (1980a) he used a mean sediment size of medium sand and compared the calculated values to published data describing the equilibrium values of alluvial sand- and gravel-bed canals with uniform flow. In his subsequent study, Chang (1980b) used a mean sediment size of coarse gravel to large cobbles and compared the calculated values to published data describing the equilibrium values of alluvial gravel- and cobble-bed streams with uniform flow. The values of the width and depth compared were those associated with a sediment discharge which produced a minimum slope equal to the measured slope. The calculated values of the width and depth in both studies compared favorably to the published data. Chang concludes that a method using the minimum rate of energy dissipation theory in combination with other relationships can accurately calculate equilibrium values of the dependent variables for alluvial streams with uniform flow and low sediment discharge.

The mean sediment size is a dependent variable. In the above studies, however, Chang assigns this variable a constant value for each stream studied. This practice is acceptable for stream reaches with uniform flow because values of the dependent variables are relatively constant with location. For stream reaches with gradually varied flow, however, values of the dependent variables change with location. Hence, for the gradually varied flow reaches under study herein, values of the mean sediment size can not be considered constant for each study stream.

Summary

Alluvial streams with 10 to 100 years (a "long-term period") reaction time to a change in the value of an independent variable are considered to be adjusting toward equilibrium. The independent variables of gravel-bed streams are the bankfull water discharge and man-made works. The dependent variables of gravel-bed alluvial streams are the mean size of the sediment that is stationary on the bed, stream depth, and stream width. After a culvert is placed in a gravel-bed stream, the stream width does not change, the stream depth increases, and the slope and mean sediment size decrease from the pre-culvert condition.

The minimum rate of energy dissipation theory states that provided certain conditions are met, equilibrium values of the dependent variables adjust so alluvial streams flow with the minimum slope of the energy grade line compatible with the imposed independent variable values. These conditions require alluvial streams to have a constant water temperature with steady uniform or gradually varied flow and negligible sediment discharge. A method using the minimum rate of energy dissipation theory in combination with other relationships accurately calculates equilibrium values of the dependent variables for alluvial streams with uniform flow and low sediment discharge.

Information presented in this chapter provides the background for developing the MINRED method and for evaluating the reliability with which it can be applied to reaches of gravel-bed alluvial streams having gradually varied flow. Information in Chapter 3 describes the data collection and presents measured data from two gravel-bed streams having gradually varied flow imposed by a highway culvert. The MINRED method is presented in Chapter 4. The application of the MINRED method to the study streams is discussed in Chapter 5 along with the resulting calculated data. The results of applying the MINRED method to the study streams is presented in Chapter 6.

CHAPTER 3

FIELD INVESTIGATION AND MEASURED VALUES

Preliminary study sites were chosen where long term U.S. Geological Survey (U.S.G.S.) annual maximum water discharge records are available and where gradually varied flow has been imposed by a single highway culvert. Eleven streams located in the southcentral and the interior portions of Alaska meet these criteria. Each stream was visually inspected prior to continuing the study; nine were discarded. To simplify the analysis, study of four streams was discontinued because man-made works other than the culvert appeared to have significantly altered the dependent variable values. The MINRED method utilizes the minimum rate of energy dissipation theory which is not valid for rapidly varied flow. Study of three streams was discontinued because the bed was composed of boulders equal to or greater than the bankfull depth, creating rapidly varied flow at the bankfull discharge. Study of two streams was discontinued because the mean sediment size on the bed was significantly smaller than gravel.

The two streams selected for study are located in Alaska as shown in Figure 1. The 9.5 foot diameter culvert

Figure 1. Location map.

in the Copper River tributary is located in southcentral Alaska at mile 51.8 of the Glenn Highway, or about nine miles west of Slana. U.S.G.S. water resources publications indicate this has a drainage area of 4.32 square miles. The five foot diameter culvert in the Brooks Creek tributary is located in the Alaskan interior at mile 84 of the Elliott Highway, or about 15 miles southwest of Livengood. U.S.G.S. water resources publications indicate this has a drainage area of 7.81 square miles. These culverts were installed in the mid 1950s.

Both streams flow through forests composed mainly of spruce and willow. The banks are held nearly vertical, by roots of the adjacent vegetation. This gives further justification for using the rectangular cross section discussed in Chapter 2. Most of the spruce trees near the streams are small suggesting much of the ground surrounding these trees is frozen all year. Both streams flow in soils of discontinuous permafrost (Hartman & Johnson, 1978).

Data Collection

Independent Variables

Provided the sediment density is constant, the independent variables of gravel-bed streams are the bankfull water discharge and man-made works.

Sediment Density

There is no evidence of volcanic activity near the sites studied. For such streams, the sediment specific gravity can be considered equal to 2.65. Therefore, the sediment density for the study streams is constant.

Water Discharge

Each study stream has a small drainage area at the culvert of interest and hence the time of concentration for each is small. Both study streams are remotely located. The travel time to each site from any population center is much greater than the time of concentration. Therefore, direct measurement of the bankfull water discharge is impractical and was not done.

The bankfull water discharge can be approximated as the discharge having a 1.5 year recurrence interval on the annual maximum series. The study streams have long term U.S.G.S. annual maximum water discharge records at the culverts of interest. The bankfull water discharge was obtained from these records. The method and the resulting discharges are presented in Appendix D.

Man-Made Works

See Appendix E, Table 11 for the measured data describing the culvert that is imposing gradually varied flow on each study stream.

Dependent Variables

The dependent variables of gravel-bed alluvial streams are the mean size of the sediment that is stationary on the stream bed, stream depth, and stream width. As discussed in Chapter 2, channels in which the width is at least several times the depth can be considered rectangular and the cross section defined by the width and depth. A preliminary inspection of the study streams found that the width was approximately five to ten times the depth and the cross sectional shape was approximately rectangular. Therefore, values of the width and depth are the only data that need be collected to describe the cross sections of the study streams.

The discharge of interest is the bankfull value. Therefore, the bankfull values of the dependent variables are those of interest. However, because data were collected during low flow rates, some estimation was required to obtain values representing the dependent variables at the bankfull discharge.

Method and Equipment

The study streams were divided into subreaches. Cross section lines define the subreach boundaries. The cross section located in the stream, adjacent to the upstream end of the culvert is designated cross section 0U. Moving upstream, cross sections are designated 1U, 2U, 3U, and so forth. Similarly, the cross section located in the stream, adjacent to the downstream end of the culvert is designated cross section 0D. Moving downstream, the cross sections are designated 1D, 2D, 3D, and so forth.

Measured values were determined with a Hilger and Watts SL 9060-2 automatic level, level rod, and Lietz-Eslon 165 foot tape. There were no bench marks near the streams. A temporary bench mark of elevation 100.00 feet was arbitrarily assigned to each upstream culvert invert.

Mean Sediment Size

The sediment discharge is low at the bankfull water discharge. However, some of the smaller particles stationary on the bed at low flow rates are transported at the bankfull discharge. Gravel-bed rivers usually have a marked sorting of bed materials with sections composed of the largest sizes spaced about every five to seven channel widths (Mercer, 1971). This was found in the study streams. For these two reasons, at a low water flow rate it is difficult to determine the mean sediment size that is stationary on the bed at the bankfull discharge.

The bed sediment at each cross section was visually examined. A location was selected such that the mean sediment size at that location is representative of the cross section. At this location, the finer material on the stream bed was assumed to be in transport during bankfull flow. The infrequently occurring smaller and larger sizes were excluded and the mean of the remaining sizes was measured. This is the value of the mean sediment size recorded.

Although the above method to determine the mean sediment size at the bankfull discharge is crude, other methods are prohibitively expensive and/or would not yield results of higher quality. For example, sophisticated bed load samplers are very expensive and do not collect representative samples. Also, a shovel or similar device could be used to collect samples for later analysis but the water velocity would not allow the smaller sizes to be retained. Further, unless these samples were collected at bankfull flow, the results would have to be adjusted for this anyway. Finally, to collect samples at bankfull flow would create both logistical and operational problems. Therefore, the measured values describing the mean sediment size are as representative of the bed as values which could have been determined by other, more sophisticated methods.

Lane (1947) standardizes the terms for a particle diameter as follows: gravel: 64 mm - 2 mm (0.21 ft - 0.0067 ft), sand: 2.0 mm - 0.250 mm (0.0067 ft - 0.0008 ft), and fine sand: 0.250 mm - 0.125 mm (0.0008 ft - 0.0004 ft). Nearly all of the study cross sections have a mean sediment size in the range of gravel.

Stream Depth

For streams in equilibrium, the difference between the elevations of the bank and the thalweg is equal to the stream depth at the bankfull discharge. Dunne and Leopold (1978) and others provide guidelines to help field locate the stream bank at low flow rates. Although these guidelines were used, field determination of the stream bank was difficult. For this reason, some variation in measured values of the stream depth is anticipated.

Stream Width

The stream width at the bankfull discharge was measured. Because field location of the stream bank was difficult, some variation in measured values of the stream width is expected.

Thalweg Slope

For both uniform and gradually varied flow in prismatic channels, the slopes of the energy grade line, stream bed, and hydraulic grade line are similar. For an open channel, the hydraulic grade line coincides with the water surface. For streams in equilibrium, the water surface at the bankfull discharge coincides with the stream bank. By approximating the study streams to be prismatic, the measured slope of the stream bed and/or stream bank represents the slope of the energy grade line during the bankfull discharge. Because field determination of the stream bank was difficult, the measured thalweg slope is used.

Collecting data to determine the thalweg slope was no simple task. Mercer (1971) indicates gravel-bed streams form a series of riffles and pools; this was found in the study streams. Hence, the thalweg slope fluctuates. The MINRED method does not account for pools and riffles and therefore calculates the average slope of the energy grade line. To allow for valid comparisons between the measured and calculated data, measurements were not collected near either the top of riffles or the bottom of pools. Thus, attempts were made to collect data representing the average thalweg slope. Because this was difficult to do by eye, some variance of the thalweg slope is expected.

Presentation and Discussion of Measured Values

Measured data are presented in Appendix E. These describe each culvert and study stream.

Average Measured Values

Because streams rarely flow in homogeneous soils, it is not surprising that the measured values fluctuate somewhat with location. In spite of this fluctuation, the measured data presented in Tables 12 through 15 and Figures 9 through 12 indicate, for most parameters, the values in the subreaches near the culvert are different from those in the subreaches farther away. Values of the dependent variables in the subreaches near the culvert are changed as a result of the culvert installation. Therefore, subreaches near the culvert are altered by the culvert and subreaches farther away are unaltered. Cross sections approximating the break in the measured data are given in Table 18. Subreaches between the culvert and these cross sections are considered to have been altered by the culvert. Subreaches further from the culvert than these cross sections are considered to have been unaltered by the culvert.

The minimum rate of energy dissipation theory is valid for uniform and gradually varied flow. However, rapidly varied flow occurs in the subreaches adjacent the culvert at the bankfull discharge. For valid comparisons between the measured and calculated data, values measured in the subreaches adjacent the culvert are not considered further.

The measured values fluctuate with location. For simplicity, average measured values from the subreaches altered by the culvert are used rather than using measured values from each altered subreach. Similarly, average measured values from the subreaches unaltered by the culvert are used. These average measured values are presented in Table 2.

For the portion of the Copper River tributary not altered by the culvert, the data presented in Table 2 are

Table 2. Average measured data.

Parameter		Copper River tributary Not altered	Copper River tributary Altered	Brooks Creek tributary Not altered	Brooks Creek tributary Altered
Cross sections	Upstm	5U-14U	1U-4U	6U-14U	1U-5U
	Dnstm	7D-12D	3D-6D	5D-12D	3D-4D
Width, ft	Upstm	7.10	15.88	7.80	8.48
	Dnstm	11.08	23.25	8.19	12.15
Depth, ft	Upstm	1.66	1.96	0.84	1.42
	Dnstm	1.60	3.70	1.29	1.37
MSS, ft	Upstm	0.0312	0.0048	0.0290	0.0419
	Dnstm	0.0660	0.0417	0.0451	0.0208
Thalweg slope	Upstm	0.0267	0.0115	0.0184	0.0095
	Dnstm	0.0378	0.0476	0.0234	0.0278

Note. MSS = mean sediment size.

averages from 10 cross sections upstream and six cross sections downstream. For the portion of the Copper River tributary altered by the culvert, the data presented are averages from four cross sections upstream and four cross sections downstream.

For the portion of the Brooks Creek tributary not altered by the culvert, the data presented in Table 2 are averages from nine cross sections upstream and eight cross sections downstream. For the portion of the Brooks Creek tributary altered by the culvert, the data presented are averages from five cross sections upstream and two cross sections downstream.

Subreaches Not Altered by the Culvert

The bankfull water discharge at each stream has probably been constant for many years. Also, the culvert is the only man-made work having any significant impact on the study streams. Therefore, the independent variable values imposed on the study streams were probably constant for many years prior to the culvert installation. For this reason, the study streams are presumed to have attained equilibrium before the culvert was installed. For the subreaches unaltered by the culvert, this equilibrium has not been changed; hence, the measured values describe the pre-culvert equilibrium.

Subreaches Altered by the Culvert

The culverts in the study streams were installed approximately 30 years ago. These streams should be adjusting toward equilibrium. Thus, values of the dependent variables in subreaches altered by the culvert should be adjusting toward equilibrium with post-culvert values of the independent variables. This trend is examined by comparing the measured bank elevation just upstream of the culvert to the elevations at the same location estimated for both the pre-culvert and post-culvert equilibrium conditions.

Pre-Culvert

Subreaches unaltered by the culvert have attained equilibrium unchanged by culvert installation. For these subreaches, measured values describe the pre-culvert equilibrium condition; the pre-culvert equilibrium slope is equal to the measured slope. The pre-culvert equilibrium bank elevation at cross section OU is approximated by extending this measured slope. For determination of the pre-culvert equilibrium bank elevation, see Appendix F, Table 19. The results are given in Table 3.

Post-Culvert

For determination of the post-culvert water surface elevation at the bankfull discharge and at stream cross section OU, see Appendix F, Tables 20 and 21. This

Table 3. Bankfull elevations at cross section OU.

Condition	Copper River tributary	Brooks Creek tributary
Equilibrium pre-culvert	99.27	100.25
Measured	101.18	101.32
Equilibrium post-culvert	102.17	102.65

Note. Elevations are in feet.calculated water surface elevation is the estimated post-culvert equilibrium bank elevation. These elevations for cross section OU are presented in Table 3.

calculated water surface elevation is the estimated post-culvert equilibrium bank elevation. These elevations for cross section OU are presented in Table 3.

Conclusion

The measured bank elevation at cross section OU is between the estimated pre-culvert and the estimated post-culvert elevation. For this reason, the bankfull water surface elevations are probably adjusting toward their post-culvert equilibrium values. It is assumed that all values of the dependent variables in subreaches altered by the culvert have not attained equilibrium with the post-culvert values of the independent variables, but are adjusting toward this condition.

Gradually Varied Flow

Upstream

Values in Table 2 show that upstream of each culvert, the average measured depth in the unaltered subreaches is less than the average measured depth in the altered subreaches. Also, values in Tables 2 and 21 show that the depth at cross section OU required to pass the bankfull discharge through the culvert is greater than the depth in the unaltered subreaches upstream of the culvert. At the bankfull discharge, an M1 gradually varied flow profile is created in those subreaches with subcritical flow that have been altered by the culvert. This is shown in Figure 2.

Downstream

Values in Table 2 show that downstream of each culvert, the average measured depth in the altered subreaches is greater than the average measured depth in the unaltered subreaches. Further, values in Tables 12 and 14 show that a scour hole is created downstream of both culverts. At the bankfull discharge, an A2 gradually varied flow profile is created at the downstream end of the scour hole. This is shown in Figure 3.

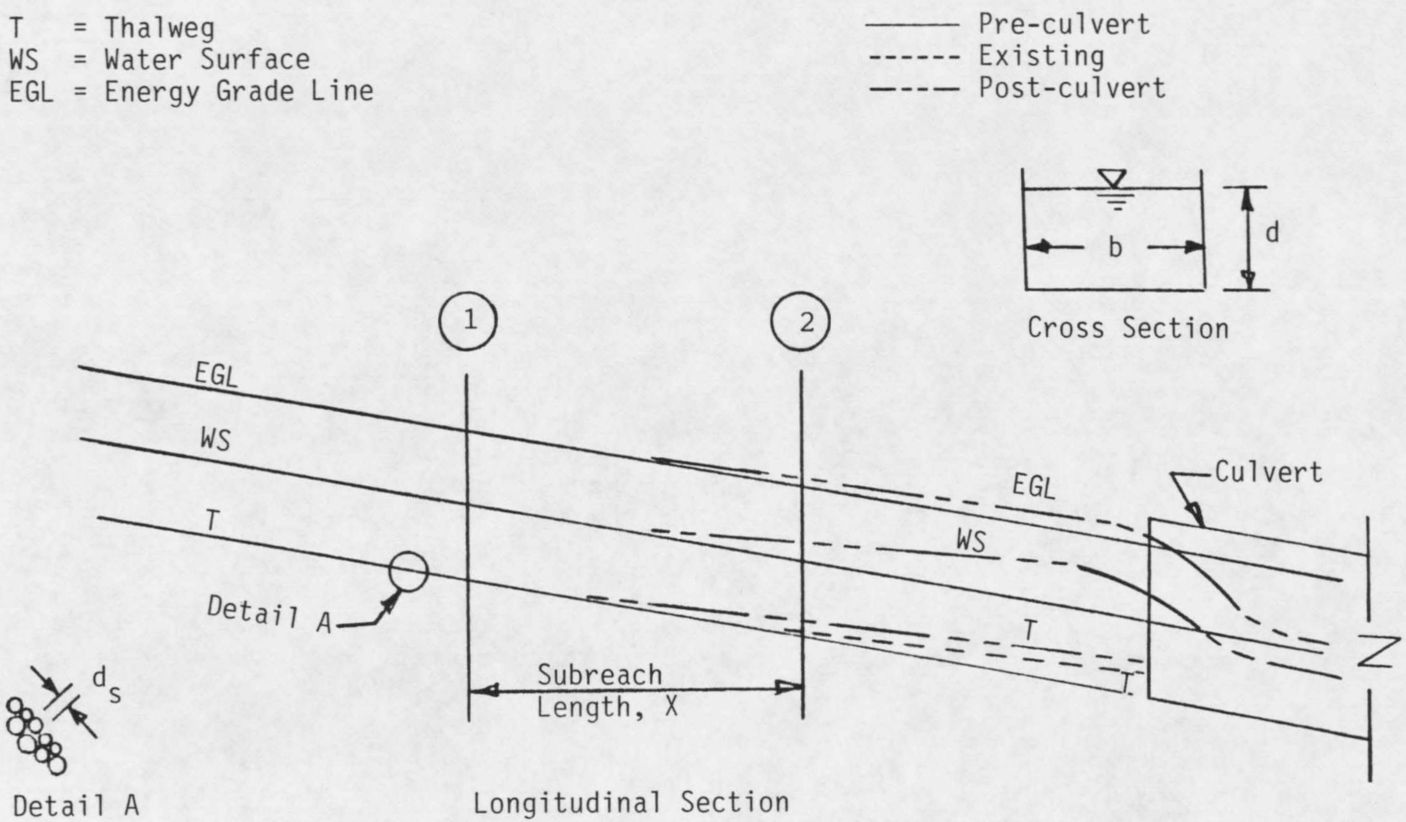

Figure 2. Typical stream with culvert creating gradually varied flow upstream.

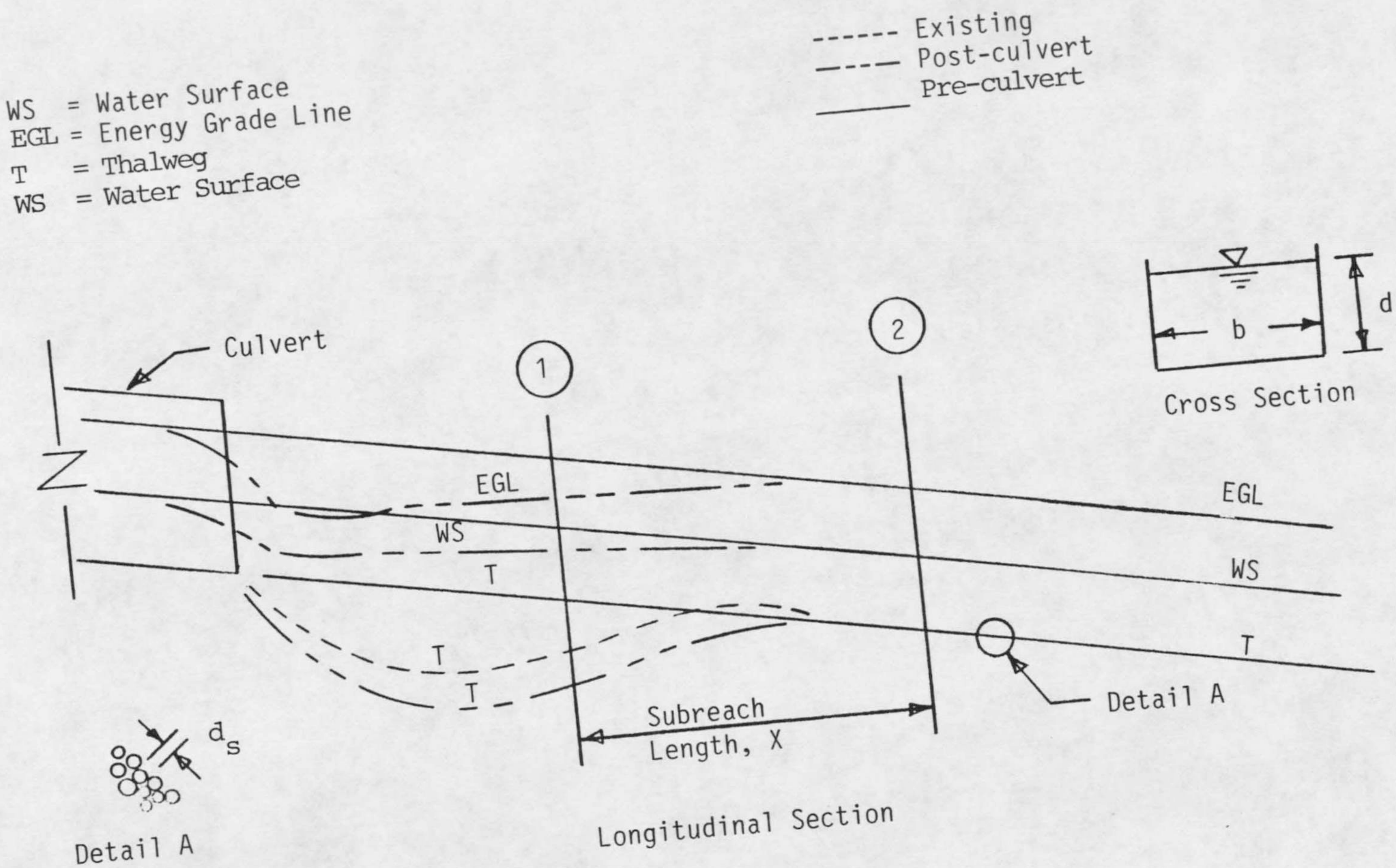

Figure 3. Typical stream with culvert creating gradually varied flow downstream.

CHAPTER 4

MINRED METHOD

The MINRED method was developed for this study. This method uses the minimum rate of energy dissipation theory and two other relationships. Results should describe the equilibrium bankfull values of the dependent variables in reaches of gravel-bed alluvial streams having gradually varied flow and low sediment discharge.

Development

The MINRED method uses measured values describing the independent and dependent variables for the subreach and the downstream cross section. The independent variables of the gravel-bed study streams are the bankfull water discharge and the culvert (man-made works). The dependent variables of the gravel-bed study streams are the mean sediment size that is stationary on the bed, stream depth, and stream width. The downstream end of the stream subreach is designated the downstream cross section. The upstream end of this subreach is designated the upstream cross section. The MINRED method uses a trial and error solution of three relationships linking the independent and dependent variables: (a) the direct step method of determining the gradually varied flow profile, (b) a combination of Shields diagram and Raudkivi's graphical relation, and (c) the minimum rate of energy dissipation theory. These relationships have not been previously coupled in the literature. Results of the MINRED method are values of the dependent variables at the upstream cross section.

The MINRED method uses measured values from the subreach and the downstream cross section to calculate values at the upstream cross section. Because stream subreaches flowing subcritical have downstream control, calculations must start downstream and work upstream. Conversely, calculations for stream subreaches flowing supercritical must work downstream. Therefore, the MINRED method can only be used for stream subreaches with subcritical flow.

Figures 2 and 3 are control volume diagrams showing a typical stream with a culvert creating gradually varied flow. Parameters shown in these figures are the stream width, stream depth, mean sediment size, and subreach length (distance between the upstream and downstream cross sections). Some parameters appearing in equations used by the MINRED method are not shown in Figures 2 and 3 because these parameters do not have a demonstrable physical significance. These figures do, however, show all the dependent variables. The parameters not shown can best be understood as mathematical combinations of, or functions of, the dependent variables and the water discharge as given in the following equations. Recall that the cross section of the study streams can be considered rectangular and thus defined by the width and depth.

General

The MINRED method assumes many values for the stream width at the upstream cross section. For each assumed width, a value is calculated for both the stream depth at the upstream cross section and the mean size of sediment that remains stationary on the stream bed at the upstream cross section. With this information, the slope of the energy grade line at the upstream cross section is also determined for each assumed width. Therefore, the MINRED method determines many sets of values for the stream width, depth, mean sediment size stationary on the bed, and energy grade line slope compatible with the independent variable values.

The minimum rate of energy dissipation theory states that of all these sets of dependent variable values, the one set that describes the stream equilibrium condition is the set associated with the smallest slope of the energy grade line. As discussed in Chapter 2, Chang (1979, 1980a, 1980b) studied streams with uniform flow and concluded that a minimum slope satisfying the independent variable values can be found. These studies, however, did not state any reason why this should occur. Chang also concludes that the dependent variable values associated with this minimum slope describe the stream's equilibrium condition.

The MINRED method determines the one set of dependent variable values (width, depth, and mean sediment size) associated with the smallest calculated slope of the energy grade line. If the MINRED method finds the minimum slope of the energy grade line that satisfies the independent variable values, the results should be the dependent variable values describing the post-culvert equilibrium condition. Details of the MINRED method are given in this chapter and in Appendix G.

Preliminary

The MINRED method uses the bankfull water discharge, the measured distance between the upstream and downstream cross sections, and the measured thalweg slope between the upstream and downstream cross sections. The MINRED method also uses measured values of the stream depth, width, and mean sediment size at the downstream cross section. The latter value is used to determine the Manning roughness coefficient.

Chow (1959) indicates the Manning roughness coefficient can be calculated by:

$$n = (n_0 + n_1 + n_2 + n_3 + n_4)\, m_5. \qquad (4\text{-}1)$$

The above symbols are as follows:

n = Manning roughness coefficient;

n_0 = the value for a straight, uniform, smooth channel in the natural materials and thus is due to the effect of grain roughness;

n_1 = correction for the effect of surface irregularities;

n_2 = correction for the effect of variations in channel cross section size and shape;

n_3 = correction for the effect of obstructions;

n_4 = correction for the effect of vegetation; and

m_5 = correction for the effect of channel meandering.

The determination of the Manning roughness coefficient for the study streams is given in Appendix G, item 6.

Direct Step Method

The MINRED method uses the direct step method to determine the depth of flow at the upstream cross section. The depth in a stream varies with location when gradually varied flow is imposed. Chow (1959) gives several methods for calculating the stream depth in gradually varied flow. One of these is the direct step method. This method divides the gradually varied flow reach into subreaches and considers each to have uniform flow.

The stream cross sectional shape is considered rectangular, see Chapter 2. Therefore:

$$A = b\,d, \tag{4-2}$$

$$R = bd/(b + 2d), \text{ and} \tag{4-3}$$

$$V = Q/A. \tag{4-4}$$

The symbols are as follows:

A = cross sectional area of flow,

b = stream width,

d = stream depth,

R = hydraulic radius,

V = mean velocity of flow, and

Q = water discharge.

The specific energy, E, and slope of the energy grade line, S, are calculated for the downstream cross section. The equations used are:

$$E = d + V^2/2g \quad \text{and} \tag{4-5}$$

$$S = [Vn/(1.49R^{0.67})]^2. \tag{4-6}$$

The symbol g represents the acceleration due to gravity. Equation 4-5 is the definition of specific energy and equation 4-6 is derived from the Manning uniform flow equation.

The MINRED method temporarily assumes values for the stream width, depth, and mean sediment size at the upstream cross section. Equations 4-1 through 4-6 are recalculated for the upstream cross section. The distance between the upstream and downstream cross sections, X, is calculated by:

$$X = (E_2 - E_1)/\{S_0 - [(S_1 + S_2)/2]\}. \qquad (4\text{-}7)$$

The subscripts 1 and 2 represent the upstream and downstream cross sections, respectively. The symbol S_0 represents the measured thalweg slope between the upstream and downstream cross sections. Equation 4-7 is given in Chow (1959) and is derived from the energy equation.

The MINRED method uses a trial and error procedure that modifies the value of the depth at the upstream cross section until the calculated distance between the two cross sections equals the measured distance. Details of how the MINRED method uses the above equations are given later in this chapter and in Appendix G.

Shields Diagram and Raudkivi's Graphical Relation

The MINRED method uses a combination of Shields diagram and Raudkivi's graphical relation to determine the mean size of the sediment that remains stationary on the stream bed at the upstream cross section. This is the mean sediment size associated with both the calculated depth and assumed width at the upstream cross section from the direct step method.

Shields Diagram

Shields diagram is an empirical curve proposed by Rouse from data obtained by Shields and other workers from experiments in flumes with fully developed turbulent flows of noncohesive sediments. This diagram is given by Vanoni (1975) and is shown in Figure 4. The left portion of the curve is the laminar flow region, the right portion of the curve is the turbulent flow region, and the central portion of the curve is the transition between the two.

Critical conditions are said to exist when the hydrodynamic force acting on a sediment grain has reached its maximum value for which no sediment movement occurs (Vanoni, 1975). The critical value of a variable is defined as that occurring at the critical conditions. These terms are used below.

Shields diagram is used by the MINRED method to determine the critical value of one parameter. Knowing the boundary Reynolds number, Shields diagram is used to determine the critical value of the dimensionless shear stress. This value is used by the MINRED method to determine the critical value of other parameters, including the mean sediment size that is stationary on the stream bed during bankfull conditions.

The original work to develop Shields diagram was

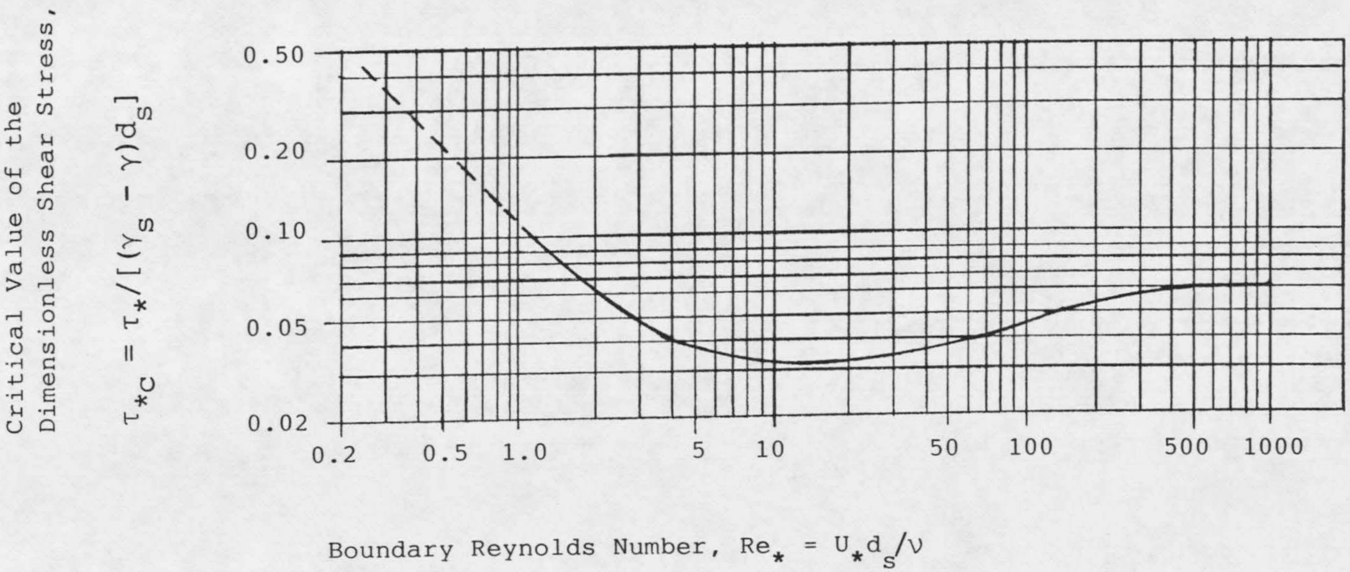

Figure 4. Shields diagram.

published in 1936. Some workers since then have reported slightly different results for portions of the curve. Shields diagram is the standard by which these new results are evaluated. Therefore, Shields diagram is the most widely accepted means for determining the critical conditions.

The results from Shields diagram are approximate. The reason for this is described below. Shields diagram uses mean parameter values to determine the critical conditions. For turbulent flow, however, there is no single mean parameter value for which sediment motion begins or ends suddenly. Rather, near critical conditions, the motion of sediment grains occurs randomly in both time and space. The incidence of such movement increases as the mean shear stress increases. For this reason, to describe the initiation of sediment movement by mean parameter values requires a range of such values corresponding to a low to high incidence of sediment movement. Vanoni (1975) indicates that one researcher found the results of Shields diagram correspond to a high incidence of sediment movement for turbulent flow.

The MINRED method assumes values for the stream width and mean sediment size at the upstream cross section. With these, the direct step method calculates values for the stream depth and energy grade line slope at the upstream cross section. The MINRED method approximates a water temperature and uses these above values to determine values for the shear velocity, U_*, and the boundary Reynolds number, Re_* at the upstream cross section. The equations used to make these calculations are given by Vanoni (1975) as follows:

$$U_* = (gRS)^{0.5} \quad \text{and} \tag{4-8}$$

$$Re_* = U_* d_s/\nu . \tag{4-9}$$

The symbol ν represents the kinematic molecular viscosity.

The sediment specific gravity for the study streams equals 2.65. Using this value, the value for the boundary Reynolds number just calculated, and equations for the curve in Shields diagram, the MINRED method calculates the critical value of the dimensionless shear stress, τ_{*c} , at the upstream cross section. The equations used to make this calculation are discussed in the next paragraph. The MINRED method uses this value and the equation given by Shields diagram to calculate the critical value of the bed shear stress, τ_c, at the upstream cross section. This is done as follows:

$$\tau_c = \tau_{*c}(\gamma_s - \gamma)d_s . \tag{4-10}$$

Previously undefined symbols are:

γ_s = sediment specific weight and

γ = fluid specific weight.

Shields diagram is empirical and the literature does not give equations for the curve. Equations were developed by using program 1415 C from the Hewlett-Packard Users Library. This program, run on a Hewlett-Packard 41CV calculator, uses data points from the curve and the method of least squares to determine second order through ninth order equations of the form:

$$Y = A_0 + A_1 x + A_2 x^2 + \ldots A_9 x^9. \quad (4\text{-}11)$$

For each equation, this program calculates the coefficient of determination, r^2. The lowest order equation with the largest coefficient of determination is used by the MINRED method. Curve equations for Shields diagram and the coefficients of determination are given in Appendix H.

Raudkivi's Graphical Relation

Two relationships given in the literature can be used to correlate the depth and mean sediment size of gravel-bed streams. These are given by Raudkivi (1967, 1971) and Chang (1980b). Both are empirical.

Empirical relationships have limited applicability; therefore, care should be exercised to ensure they are applied only to the conditions for which they were developed. Both the Raudkivi and the Chang relationship were developed from information similar to but not completely representative of all the measured data. Raudkivi's relationship is used in this study because it was developed from data more closely resembling the study streams than the data used to develop Chang's relationship. The relationship developed by Raudkivi is known in the literature as Raudkivi's graphical relation.

Raudkivi's graphical relation is a set of empirical curves that relate the mean velocity of flow to the shear velocity and mean sediment size. This relation is presented in Figure 5. Raudkivi developed this relation by using data from his own flume experiments and data reported by others from large rivers to laboratory flumes. The data cover a sediment size from 0.0003 ft to 0.016 ft (very fine sand to very fine gravel).

The left portion of the curves in Raudkivi's graphical relation corresponds to the initiation of sediment movement and the development of dunes on gravel-bed streams. Vanoni (1975) states that left of this portion of the curve corresponds to flows with velocities just above the critical value for the initiation of motion. It is this portion of the curves that is used by the MINRED method. The central portion of the curves corresponds to a moderate sediment discharge, erosion of the dunes, and the development of a flat bed. The right portion of the curves corresponds to a

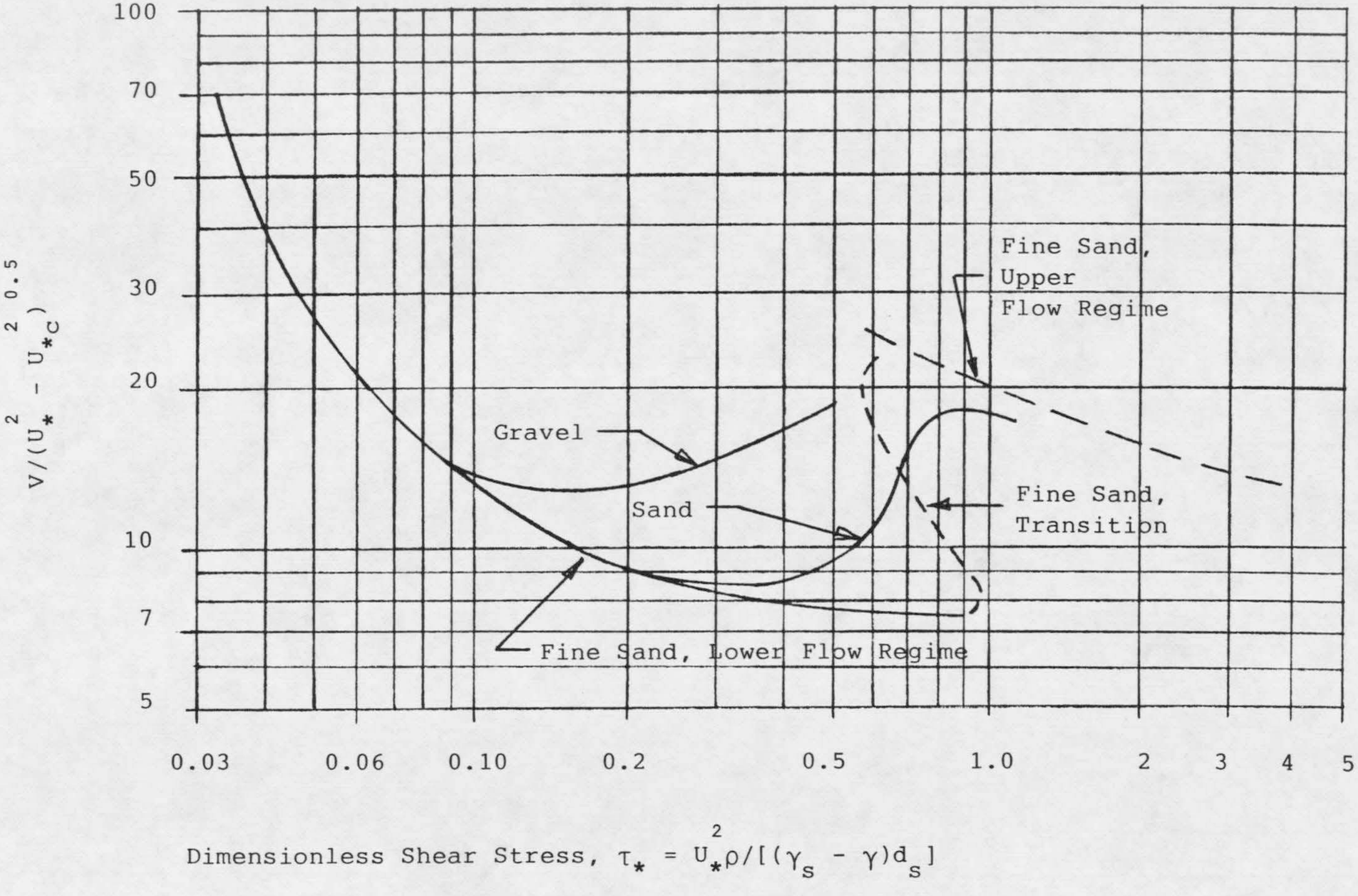

Figure 5. Raudkivi's graphical relation.

high sediment discharge and the development of antidunes on the stream bed.

The left portion of the curves in Raudkivi's graphical relation is used by the MINRED method to determine the critical value of the mean velocity. This value is used to correlate the stream depth and mean sediment size. Details of this procedure are described below.

Results from Raudkivi's graphical relation must be considered approximate for the following three reasons. First, Raudkivi (1976) indicates there is appreciable scatter of the data points from which Raudkivi's graphical relation was developed. He adds that extensive systematic investigation to check or modify the curves is necessary before results from the relation can be considered to be more than approximations. Second, the measured values of this study are not identical to the data from which this empirical relation was developed. Third, the left portion of the curve for gravel in Raudkivi's graphical relation has a moderately steep slope. When using the steep portion of the curve, the accuracy of the results depends a great deal on the accuracy of the input data. Inaccuracy in the input data can adversely effect the accuracy of the results obtained. For the above three reasons, some inaccuracy in the results of the MINRED method is anticipated from the use of Raudkivi's graphical relation.

The MINRED method determines the critical value for the shear velocity, U_{*c}, and the dimensionless shear stress, τ_*, at the upstream cross section. The equations used to make these calculations are given by Vanoni (1975) as follows:

$$U_{*c} = (\tau_c/\rho)^{0.5} \quad \text{and} \tag{4-12}$$

$$\tau_* = U_*^2\rho/[(\gamma_s - \gamma)d_s]. \tag{4-13}$$

The symbol ρ represents the fluid density. The MINRED method uses values for τ_*, d_s, U_*, and U_{*c} and the equations representing the curves in Raudkivi's graphical relation to determine the critical value for the mean velocity, V_{cr}, at the upstream cross section. This is the maximum mean velocity value for which the assumed size of the mean sediment will not move. The equations used to make this calculation are discussed in the next paragraph.

Raudkivi's graphical relation is empirical and the literature does not give equations for the curves. These equations were developed in the same manner as previously explained for Shields diagram. Curve equations and the coefficients of determination are given in Appendix H.

The MINRED method modifies the assumed value for the mean sediment size and reexecutes Shields diagram and Raudkivi's graphical relation until the critical value of the mean velocity is equal to the value of the mean velocity found by the direct step method. This is done by a trial and error procedure.

The value of the mean sediment size last used by the direct step method may not be the same as that determined by Raudkivi's graphical relation. The MINRED method simultaneously solves the direct step method and the combination of Shields diagram and Raudkivi's graphical relation until the mean velocity and the Manning roughness coefficient used by both are the same. The calculated value of the mean sediment size is therefore associated with the critical value of the mean velocity. A sediment particle smaller than this is associated with a critical value of the mean velocity smaller than that calculated; hence, will not remain stationary on the bed. For this reason, the result of this procedure is the value of the mean sediment size that will remain stationary on the stream bed at the upstream cross section. Recall that this value is associated with an assumed value of the stream width and with calculated values of the stream depth and energy grade line slope. Details of the calculation procedure used to determine these values are given later in this chapter and in Appendix G.

Minimum Rate of Energy Dissipation Theory

The MINRED method uses the minimum rate of energy dissipation theory to determine the values of the mean sediment size, depth, and width that should describe the equilibrium condition at the upstream cross section. The literature presents studies that use the minimum rate of energy dissipation theory to calculate equilibrium values of the dependent variables in stream reaches with uniform flow. However, this theory has not been used to calculate equilibrium values of the dependent variables in stream reaches with gradually varied flow.

Many values of the stream width at the upstream cross section are assumed. For each assumed value, the MINRED method determines values of the mean sediment size stationary on the bed, depth, and energy grade line slope at the upstream cross section as discussed earlier. Many sets of values are obtained. The minimum rate of energy dissipation theory is used to determine which set of values describe the post-culvert equilibrium condition. Values of the width are selected in an attempt to calculate a smaller value of the energy grade line slope than previously found. The output of the MINRED method is the smallest calculated slope of the energy grade line and the associated values of the mean sediment size stationary on the stream bed, stream depth, and stream width. Details of the calculation procedure used to determine these values are given later in this chapter and in Appendix G. The procedure of finding this minimum slope can be called "minimizing the slope of the energy grade line."

The minimum rate of energy dissipation theory states that the set of dependent variable values describing a stream's equilibrium condition is the set associated with the smallest slope of the energy grade line compatible with the independent variable values. The following discussion indicates that the values calculated by the MINRED method are compatible with the independent variable values. The bankfull water discharge and the culvert are the independent variables of the study streams. The condition imposed by the culvert is manifested in the values of the dependent variables. To calculate values of the dependent variables at the upstream cross section, the MINRED method uses the bankfull water discharge and measured values of the dependent variables at the downstream cross section. As discussed in Chapters 2 and 3, these measured values either describe the post-culvert equilibrium condition or are changing toward these values. Hence, measured values can be used to represent the condition imposed by the culvert. Therefore, by starting calculations with the bankfull water discharge and these measured values, the MINRED method uses values representing the imposed independent variables. Thus, values calculated by the MINRED method are necessarily compatible with the values of the independent variables. For this reason, if the MINRED method calculates the smallest possible slope of the energy grade line, the associated values of the dependent variables should describe the post-culvert equilibrium condition.

The calculated values depend only on the input data and the relationships linking the independent and dependent variables. The input data used in the calculations for each upstream cross section do not depend on the input data used to make the calculations for other cross sections. Therefore, the values calculated for any one cross section are independent of the values calculated for any other cross section.

A possible misinterpretation of the MINRED method and the minimum rate of energy dissipation theory could raise the following questions. Where is the slope of the energy grade line minimized? Does minimizing the slope of the energy grade line at one location necessarily require that the slope is not minimized at other locations? The answers to these questions follow from the previous discussion. The slope of the energy grade line is minimized at every location considered to be an upstream cross section. Minimizing the slope does not mean reducing it below its pre-culvert value. Also, minimizing the slope at one location does not preclude minimizing the slope at other locations because the calculations for each upstream cross section are done independently of the calculations for other cross sections.

Description

The MINRED method uses a trial and error procedure executed by a computer program written in FORTRAN and run on a Honeywell series 60 computer. The MINRED method is divided into three iteration loops. These are named D loop, DS loop, and B loop. D loop uses the direct step method and DS loop uses a combination of Shields diagram and Raudkivi's graphical relation. The MINRED method assumes a value for the stream width at the upstream cross section. This method then simultaneously solves D loop and DS loop to calculate values for the stream depth, energy grade line slope, and mean sediment size stationary on the stream bed at the upstream cross section. These values are compatible with the independent variable values. B loop assumes many values of the stream width. For each assumed value, the MINRED method simultaneously solves D loop and DS loop. Many sets of values are calculated for the dependent variables and energy grade line slope. The result of the MINRED method is the set of dependent variable values associated with the minimum calculated slope of the energy grade line. If the minimum slope is found, the minimum rate of energy dissipation theory states that these dependent variable values should describe the equilibrium condition. A summary of the MINRED method is given in Appendix G and a flow chart of this procedure is given in Figure 6. A description of the MINRED method follows.

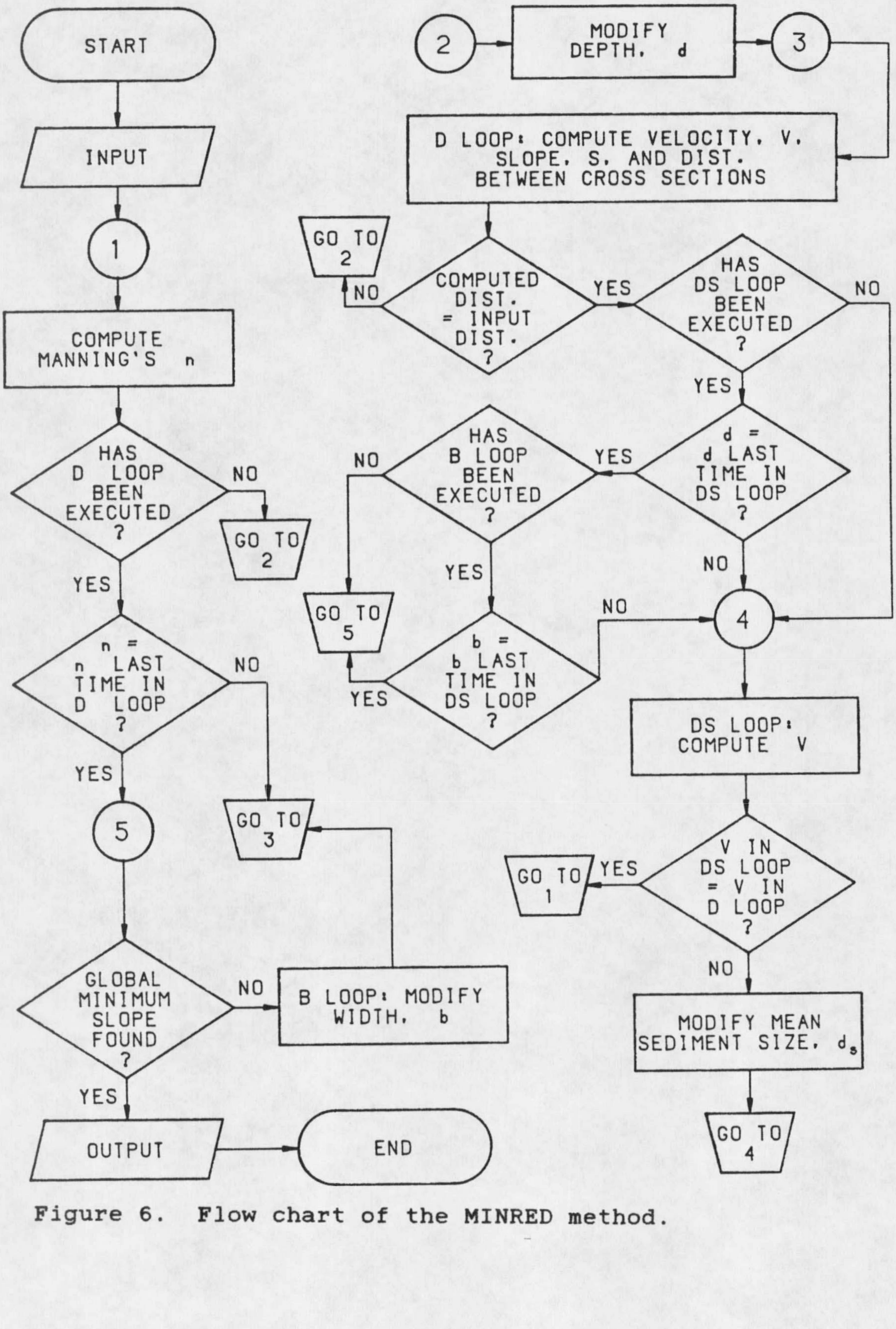

Figure 6. Flow chart of the MINRED method.

The channel cross sectional shape is considered rectangular and the sediment specific gravity is considered equal to 2.65. Also, the MINRED method approximates the water temperature to be 38 degrees F. Prior to each execution of the MINRED method, the bankfull water discharge is specified. The measured distance between the upstream and downstream cross sections and the measured thalweg slope between the upstream and downstream cross sections is given. Also input are the measured values of the stream depth, stream width, and mean sediment size at the downstream cross section. The Manning roughness coefficient at the downstream cross section is calculated using the latter value.

D Loop

The MINRED method temporarily assumes values for the stream width and mean sediment size at the upstream cross section. The first time through D loop, the MINRED method temporarily assumes these values equal the values measured at the downstream cross section. During reiterations of D loop, the MINRED method uses the values of the stream width and mean sediment size for the upstream cross section assigned at last exit from B loop and DS loop, respectively. For each of these mean sediment size values used, a value is calculated for the Manning roughness coefficient at the upstream cross section. D loop uses a trial and error modification of the stream depth. Calculations determine a value for the stream depth, slope of the energy grade line, and mean velocity at the upstream cross section. This loop also calculates the distance between the upstream and downstream cross sections. Execution of this loop ends when the calculated distance between the upstream and downstream cross sections equals the measured distance.

DS Loop

This loop begins calculations with values of the stream width, depth, and mean sediment size stationary on the stream bed assigned at last exit from D loop. DS loop uses a trial and error modification of the mean sediment size. A value is calculated for the mean sediment size stationary on the stream bed and the critical value of the mean velocity at the upstream cross section. Execution of this loop ends when the critical value of the mean velocity calculated in DS loop equals the mean velocity at last exit from D loop.

Simultaneous solution of D Loop and DS Loop

The Manning roughness coefficient at the upstream cross section is calculated again using the value of the mean sediment size assigned at last exit from DS loop. This value of the Manning roughness coefficient is compared to the value of the Manning roughness coefficient last assigned in D loop for the upstream cross section. If the two values are identical, program execution proceeds to B loop. If the two values are different, D loop is reexecuted using the new value of the Manning roughness coefficient. After reexecution of D loop, the new calculated value of the stream depth is compared to the value of the stream depth at last exit from DS loop. If the two values are identical, program execution proceeds to B loop. If the two values are different, DS loop is reexecuted, using the new value of the stream depth. Simultaneous solution of D loop and DS loop ends when two conditions occur. First, the mean velocity found in D loop equals the critical value of the mean velocity found in DS loop. Second, the value of the Manning roughness coefficient at the upstream cross section last assigned in D loop equals the Manning roughness coefficient found using the value of the mean sediment size at last exit from DS loop.

B Loop

This loop assumes many values for the stream width. For each assumed value, D loop and DS loop are solved. Many sets of values compatible with the independent variable values are obtained for the stream width, depth, mean sediment size stationary on the stream bed, and energy grade line slope. The minimum rate of energy dissipation theory states that the equilibrium values of the dependent variables are associated with the minimum slope of the energy grade line compatible with the independent variable values. The result of the MINRED method is the set of dependent variable values associated with the minimum calculated slope of the energy grade line compatible with the independent variable values. If the minimum slope is found, these dependent variable values should describe the stream equilibrium condition.

Summary

A development of the MINRED method and a description of the procedure used for its application are presented. The MINRED method solves three relationships to calculate the dependent variable values that should describe the equilibrium condition in reaches of gravel-bed alluvial streams having gradually varied flow and low sediment discharge. The dependent variables are the stream depth, stream width, and mean sediment size that remains stationary on the stream bed. This method uses the bankfull water discharge, a water temperature of 38 degrees F, a sediment specific gravity of 2.65, and a rectangular cross sectional shape. The three relationships used by the MINRED method are (a) the direct step method of determining the gradually varied flow profile, (b) a combination of Shields diagram and Raudkivi's graphical relation, and (c) the minimum rate of energy dissipation theory represented by a minimum slope

of the energy grade line.

The steps used by the MINRED method are:

1. Input data.

2. Calculate the Manning roughness coefficient.

3. Determine the stream depth, energy grade line slope, and mean velocity at the upstream cross section. This is done by using the direct step method and an iterative procedure that modifies the value of the stream depth.

4. Determine the mean sediment size associated with a critical value of the mean velocity that is similar to the mean velocity found in item 3. This is done by using Shields diagram, Raudkivi's graphical relation, and an iterative procedure that modifies the value of the mean sediment size.

5. Calculate the Manning roughness coefficient, using the value of the mean sediment size found in item 4. If this value is not similar to the value used last time by the direct step method in item 3, go to item 3.

6. If the minimum value of the energy grade line slope has not been found, modify the stream width and go to item 3.

7. Output stream width, depth, mean sediment size, and energy grade line slope at the upstream cross section.

As discussed in Chapter 2, Chang (1979, 1980a, 1980b) developed a method that calculates the equilibrium values of the dependent variables in alluvial streams. The differences between Chang's method and the MINRED method are as follows:

1. Chang's method uses a trapezoidal cross sectional shape, whereas the MINRED method uses a rectangular shape.

2. Chang's method is for streams with uniform flow. The MINRED method is for streams with gradually varied flow.

3. The sediment discharge is calculated by Chang's method, but the MINRED method assumes it is constant.

4. Both Chang's method and the MINRED method use three relationships to calculate the dependent variable values. The only relationship used by both methods is the minimum rate of energy dissipation theory.

5. Chang's method uses a constant mean sediment size whereas the MINRED method considers this parameter to be a dependent variable.

6. Chang's method uses the depth, width, and energy grade line slope as the dependent variables. However, the MINRED method considers the dependent variables to be the depth, width, and mean sediment size.

CHAPTER 5

CALCULATED VALUES

The application of the MINRED method to the study streams is discussed in this chapter along with the resulting calculated data.

Application

The reliability with which the MINRED method calculates bankfull values of the dependent variables is evaluated by applying the method to reaches of two gravel-bed streams having gradually varied flow imposed by a highway culvert. The minimum rate of energy dissipation theory is valid for uniform and gradually varied flow. However, rapidly varied flow occurs in the subreaches adjacent each culvert at the bankfull discharge. Therefore, the MINRED method is not applied to subreaches adjacent the culverts.

The MINRED method can only be used for stream subreaches with subcritical flow, see Chapter 4. Tables 16 and 17 present the Froude number at bankfull discharge calculated from measured values for each cross section under study. Evaluation indicates some cross sections flow supercritical at the bankfull discharge. The MINRED method is not used between two cross sections if the calculated

Froude number for either cross section is above unity.

Presentation and Discussion of Calculated Values

Slope of the Energy Grade Line

A review of values calculated by iterations of the MINRED method indicates that as the stream width increases with each iteration, the mean sediment size and stream depth decrease occasionally and the slope almost always decreases. Further, the slope increases with some decreases in stream depth and/or mean sediment size. However, the slope almost always decreases with subsequent iterations as stream width increases with a constant stream depth and mean sediment size.

When searching for the minimum slope, iterations of the MINRED method do not consider all possible combinations of dependent variable values. Therefore, the minimum calculated slope of the energy grade line may not be the global minimum compatible with the independent variable values. If the MINRED method calculates the global minimum slope, the calculated values of the dependent variables associated with this slope should describe the post-culvert equilibrium condition at the bankfull discharge. However, the calculated values of the dependent variables do not necessarily describe this equilibrium condition if the global minimum slope is less than the calculated minimum slope. From the iteration data discussed in the previous paragraph, it is concluded that for this latter situation the calculated values of the stream width should be smaller than the post-culvert equilibrium values. Also the stream depth and mean sediment size should be larger than the post-culvert equilibrium values.

Average Calculated Values

Results of the MINRED method are presented in Appendix I. Some executions of the MINRED method continued indefinitely without calculating a minimum slope. For these executions, no results are presented.

Measured values presented in Chapter 3 are used to estimate which subreaches in the study streams have been altered by the culvert and which subreaches have been unaltered by the culvert. The locations where the unaltered and altered subreaches converge are the same locations where no minimum slope was calculated by the MINRED method.

Table 2 presents average measured data for the subreaches altered by the culvert and the subreaches unaltered by the culvert. Table 4 presents average calculated data for these same subreach groups. These average calculated values are determined from the calculated values (see Appendix I).

For the portion of the Copper River tributary not altered by the culvert, the data presented in Table 4 are

Table 4. Average calculated data.

Parameter		Copper River tributary		Brooks Creek tributary	
		Not altered	Altered	Not altered	Altered
Width,	Upstm	50.10	50.41	48.63	61.96
ft	Dnstm	10.08	34.88	15.16	9.00
Depth,	Upstm	1.21	1.84	1.23	0.96
ft	Dnstm	0.84	2.12	1.10	1.13
MSS,	Upstm	0.0383	0.0007	0.0056	0.0063
ft	Dnstm	0.3820	0.0044	0.1702	0.1082
EGL	Upstm	0.00490	0.00005	0.00025	0.00035
slope	Dnstm	0.05363	0.00011	0.02058	0.01480

Note. MSS = mean sediment size.

averages from six cross sections upstream and four cross sections downstream. For the portion of the Copper River tributary altered by the culvert, the data presented are averages from three cross sections upstream and four cross sections downstream.

For the portion of the Brooks Creek tributary not altered by the culvert, the data presented in Table 4 are from one cross section upstream and seven cross sections downstream. For the portion of the Brooks Creek tributary altered by the culvert, the data presented are from one cross section upstream and one cross section downstream.

Summary

The application of the MINRED method to two gravel-bed streams that have had gradually varied flow imposed by a highway culvert is discussed and the resulting calculated values are presented.

CHAPTER 6

RESULTS AND DISCUSSION

The minimum rate of energy dissipation theory is valid for both uniform and gradually varied flow in alluvial streams with low sediment discharge and constant water temperature. This theory can be combined with other relationships to accurately calculate equilibrium values of the dependent variables for alluvial streams with uniform flow and low sediment discharge. The MINRED method combines this theory with two other relationships and is applied to two gravel-bed streams with gradually varied flow and low sediment discharge. Results of the MINRED method should describe the equilibrium bankfull values of the dependent variables for the study streams

This chapter compares values calculated by the MINRED method to both measured values and to changes of these values predicted in the literature. These comparisons divide the chapter into two sections. The first section compares changes in calculated values resulting from culvert installation to both changes in measured values resulting from culvert installation and to changes in these values predicted in the literature. The second section compares ratios of measured to calculated values from subreaches altered by the culvert to ratios predicted in the literature. Information from these comparisons is necessary to evaluate the reliability of the MINRED method.

The MINRED method may not calculate the global minimum slope of the energy grade line that is compatible with the imposed independent variable values. When this may alter the results, comparisons in this chapter are made twice; once assuming the global minimum slope was calculated and once assuming it was not calculated.

Parameter Changes Between Subreaches

Measured post-culvert values from subreaches unaltered by the culvert describe the equilibrium condition existing before culvert installation. The culvert is the only man-made work altering the study streams. Therefore, prior to the culvert installation, the independent variables must have been uniformly imposed on the study streams; hence, the pre-culvert equilibrium condition must have been constant with location. For these reasons, measured post-culvert parameter values from subreaches unaltered by the culvert equal the pre-culvert equilibrium values from subreaches altered by the culvert. Therefore, comparing average post-culvert parameter values from subreaches unaltered by the culvert to average post-culvert parameter values from subreaches altered by the culvert indicates the average direction changes, or trends, of these parameter values resulting from culvert installation.

Measured Value Changes

Average measured values (Table 2) from subreaches unaltered by the culvert are compared to average measured values from subreaches altered by the culvert and on the same side of the culvert. The results of these comparisons are given in Table 5. These results indicate the trends of average measured parameter values resulting from culvert installation. This is because values from subreaches unaltered by the culvert represent the pre-culvert condition and values from subreaches altered by the culvert represent the post-culvert condition.

Calculated Value Changes

Information in Table 4 is used to compare average calculated values from subreaches unaltered by the culvert to average calculated values from subreaches altered by the culvert and on the same side of the culvert. For the results of these comparisons, see Table 5. Similar to the comparisons of the average measured parameters, these results indicate the trends of average calculated parameter values resulting from culvert installation.

Table 5. Direction of average parameter value changes from unaltered to altered subreaches.

Parameter		Copper River tributary		Brooks Creek tributary		
		Measured	Calculated	Measured	Calculated	Literature prediction
Width	Upstm	+	NC	NC	+	NC
	Dnstm	+	+	+	−	NC
Depth	Upstm	+	+	+	−	+
	Dnstm	+	+	+	+	+
MSS	Upstm	−	−	+	+	−
	Dnstm	−	−	−	−	−
Slope	Upstm	−	−	−	+	−
	Dnstm	+	−	+	−	−

Note. No change in average parameter = NC. Widths are considered unchanged if the average values change less than 10%. Increase in average parameter = +. Decrease in average parameter = −. MSS = mean sediment size.

Literature Predictions

The trends of parameter values resulting from culvert installation as predicted in the literature are given in Table 5.

Results of Comparisons

The MINRED method may not calculate the global minimum slope of the energy grade line that is compatible with the imposed independent variable values. However, any error made in not calculating the global minimum slope is similar for all runs. Consequently, trends in calculated values are not dependent on whether the MINRED method calculates the global minimum slope. Therefore, comparisons of trends in calculated values to both trends in measured values and to trends predicted in the literature are done only once.

The information in Table 5 is used to compare trends in average calculated values resulting from culvert installation to trends in average measured values resulting from culvert installation . For the Copper River tributary, six of the eight comparisons agree. For the Brooks Creek tributary, three of the eight comparisons agree.

The information in Table 5 is used to compare trends in average calculated values resulting from culvert installation to trends predicted in the literature. For the Copper River tributary, seven of the eight comparisons agree. For the Brooks Creek tributary, three of the eight comparisons agree.

Parameter Changes At Subreaches

The ratios of average measured values to average calculated values for subreaches altered by the culvert are discussed in this section. These ratios are compared to the ratios predicted in the literature. Such comparisons are done twice.

Global Minimum Calculated

The MINRED method calculates values which should describe the post-culvert equilibrium condition if the global minimum slope of the energy grade line is found. The analysis in this subsection assumes the minimum slope is determined.

Literature Predictions

Culvert installations can impose gradually varied flow on streams, hence changing the equilibrium values of the dependent variables and slope. The trends in the equilibrium values of these parameters resulting from culvert installations are predicted in the literature. The predicted trends include an increase in the equilibrium values of the stream depth, a decrease in the equilibrium values of the mean sediment size and slope, and no change in the equilibrium values of the stream width.

Ideal Ratios

Values of the measured parameters from subreaches altered by the culvert are adjusting toward post-culvert equilibrium. For subreaches altered by the culvert, the value of each average measured parameter should be changing in the direction predicted in the literature. For this reason, measured values of the stream depth should be less than the post-culvert equilibrium values, hence the ratios of measured to calculated values should be somewhat less than unity. Also, measured values of the mean sediment size and slope should be greater than the post-culvert equilibrium values, hence the ratios of measured to calculated values should be somewhat greater than unity. Parameter values from subreaches unaltered by the culvert represent the stream equilibrium condition. Because the stream width is unchanged by the culvert, measured values of the stream width should represent the post-culvert equilibrium values; hence, the ratios of measured to calculated values should equal unity. The ideal ratios mentioned in this paragraph are presented in Table 6.

Actual Ratios

Table 6 presents the actual ratios of average measured values to average calculated values for subreaches altered

Table 6. Ratios of average measured values to average calculated values for subreaches altered by the culvert.

Parameter		Copper River trib.	Brooks Creek trib.	Ideal, minimum	Ideal, no minimum
Width	Upstm	0.32	0.14	=1	>1
	Dnstm	0.67	1.35	=1	>1
Depth	Upstm	1.07	1.48	<1	<1
	Dnstm	1.75	1.21	<1	<1
MSS	Upstm	6.86	6.65	>1	≃1
	Dnstm	9.48	0.19	>1	≃1
Slope	Upstm	230.	27.14	>1	≃1
	Dnstm	432.	1.88	>1	≃1

Note. MSS = mean sediment size. Ideal, minimum = ideal ratio assuming global minimum slope was calculated. Ideal, no minimum = ideal ratio assuming global minimum slope was not calculated.

by the culvert. These ratios are calculated from values in Tables 2 and 4.

Results of Comparisons

Measured values used in this study are approximations of the actual stream conditions. Therefore, ratios of average measured values to average calculated values must also be considered approximations. Actual ratios between 0.25 and 1.00 are considered to be somewhat less than unity. Also, actual ratios between 1.00 and 4.00 are considered to be somewhat greater than unity and actual ratios between 0.67 and 1.50 are considered to be equal to unity.

For the Copper River tributary, the actual ratios and the ideal ratios agree for one of the eight comparisons. This is the downstream width. For the Brooks Creek tributary, the two ratios agree for two of the eight comparisons. These are the downstream width and downstream slope.

Global Minimum Not Calculated

The following is a summary of information presented in the previous subsection. For subreaches altered by the culvert, measured values of the stream depth should be smaller and measured values of the mean sediment size and slope should be larger than the post-culvert equilibrium values. Also for these subreaches, measured values of the stream width should equal the post-culvert equilibrium values.

The MINRED method does not calculate values describing the equilibrium condition if the global minimum slope of the energy grade line is not found. The following is an extension of information presented in Chapter 5. When the minimum slope is not determined, calculated values of the stream width should be smaller and calculated values of the stream depth, mean sediment size, and slope should be larger than the post-culvert equilibrium values.

Ideal Ratios

The information in the previous two paragraphs is combined to yield conclusions valid for subreaches altered by the culvert when the minimum slope is not found. Ratios of measured to calculated values should be somewhat greater than unity for the stream width, somewhat less than unity for the stream depth, and approximately equal to unity for the mean sediment size and slope. These ideal ratios are given in Table 6.

Actual Ratios

Actual ratios of average measured values to average calculated values for subreaches altered by the culvert are given in Table 6. These ratios are calculated from values in Tables 2 and 4.

Results of Comparisons

Measured values are approximations of the actual stream conditions. For this reason, actual ratios between 1.00 and 4.00 are considered to be somewhat greater than unity and actual ratios between 0.25 and 1.00 are considered to be somewhat less than unity. Also, actual ratios between 0.50 and 2.00 are considered to be approximately equal to unity.

For the Copper River tributary, the actual ratios and the ideal ratios do not agree for any of the eight comparisons. For the Brooks Creek tributary, the two ratios agree for two of the eight comparisons. These are the downstream width and downstream slope.

Summary

This chapter compares values calculated by the MINRED method to measured values and to changes of these values predicted in the literature. A summary of these comparisons is given in Table 7. For the Copper River tributary, 14 of 32 or 44% of the comparisons agree. For the Brooks Creek tributary, 10 of 32 or 31% of the comparisons agree. Assuming the minimum slope is found, 3 of 16 or 19% of the comparisons agree. Assuming the minimum slope is not found, 2 of 16 or 13% of the comparisons agree.

Table 7. Summary of comparisons that agree.

Item	Conditions	Copper River trib.	Brooks Creek trib.
Parameter changes between subreaches	Calc to meas	6 of 8	3 of 8
	Calc to lit	7 of 8	3 of 8
Parameter changes at subreaches	Minimum	1 of 8	2 of 8
	No minimum	0 of 8	2 of 8

Note. Calc to meas = comparisons of changes in average calculated values to changes in average measured values. Calc to lit = comparisons of changes in average calculated values to changes predicted by the literature. Minimum = comparisons assuming global minimum slope was calculated. No minimum = comparisons assuming global minimum slope was not calculated.

CHAPTER 7

SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE WORK

Summary and Conclusions

1. This study develops a method (referred to herein as the MINRED method) to calculate equilibrium bankfull values of the dependent variables in reaches of gravel-bed alluvial streams which have gradually varied flow.

2. The MINRED method uses three relationships. These are (a) the direct step method of determining the gradually varied flow profile, (b) a combination of Shields diagram and Raudkivi's graphical relation, and (c) the minimum rate of energy dissipation theory.

3. Data from two gravel-bed alluvial streams which have gradually varied flow imposed by a highway culvert are used to evaluate the reliability of the MINRED method.

4. Values of the dependent variables calculated by the MINRED method are compared to measured values and to changes of these values resulting from the culvert installation, as predicted in the literature. The results of these comparisons are similar whether or not the MINRED method calculates the global minimum slope of the energy grade line. Therefore, conclusions do not depend on whether the global minimum slope is found.

5. Values calculated by the MINRED method compare poorly to both measured values and to changes of these values resulting from the culvert installation, as predicted in the literature. Thus, for the data available, the MINRED method calculates equilibrium bankfull values with a low degree of reliability.

6. The MINRED method needs further refinement and/or field testing before it can be recommended for use by practicing engineers.

Assumptions and Approximations

There is no evidence to indicate that the MINRED method is not valid for use in gradually varied flow. Therefore, the low reliability of the MINRED method must be due to the assumptions and approximations which are required to apply this method to the gravel-bed study streams.

Assumptions

Following is a summary of the assumptions made in applying the MINRED method to the gravel-bed study streams.

1. Sediment discharge is low and can not be accurately determined in gravel-bed streams at the bankfull water flow rate. For simplicity and possibly for accuracy, the sediment discharge is assumed to be constant at the bankfull water discharge for the gravel-bed study streams.

2. Streams with over ten years to react to a change in

an imposed condition can be considered to be adjusting toward equilibrium. The study streams have had approximately 30 years to react to the culvert. Just upstream of the culvert in each study stream, the bankfull water surface elevation is adjusting toward post-culvert equilibrium. For these reasons, all stream locations altered by the culvert are assumed to have not attained post-culvert equilibrium, but are assumed to be adjusting toward this condition.

Approximations

Following is a summary of the approximations made in applying the MINRED method to the gravel-bed study streams.

1. The discharge of interest is the bankfull value. Therefore, the bankfull values of the dependent variables are those of interest. For practical reasons however, data were collected during low flow rates. For this reason, measured values representing the dependent variables at the bankfull discharge are approximate. The procedures used to obtain data representative of the bankfull conditions are discussed in Chapter 3.

2. Subreaches near the culvert are altered by the culvert and subreaches farther away are unaltered. Measured data are used to determine which subreaches are altered and which are unaltered by the culvert. However, because the measured data are approximate and fluctuate somewhat with

location, this determination is approximate.

3. As described in Chapter 4, values determined by Shields diagram can only approximate the critical conditions for turbulent flow. Therefore, results obtained from Shields diagram must be considered approximate.

4. As discussed in Chapter 4, results from Raudkivi's graphical relation must be considered approximate for the following three reasons. First, there is some scatter of the data points from which this relation was developed. Second, the measured values of this study are not identical to the data from which this empirical relation was developed. Third, the portion of the curve in this relation that is used has a moderately steep slope, compounding any error in the input data.

5. There are no water temperature data for the study streams. The drainage area for both streams is in discontinuous permafrost and therefore the temperature of the groundwater influent to these streams is just above freezing. Data in Appendix D show that the annual maximum discharge for the Copper River tributary occurs primarily during June, July, and August and thus results from summer rainstorms. Data in Appendix D also show that the annual maximum discharge for the Brooks Creek tributary occurs primarily in May, June, and August. Although early May is typically snow melt season at this stream, these events are mostly a result of summer rainstorms. The temperature of

the falling rain from these storms is significantly above freezing. The bankfull water discharge has a recurrence interval of 1.5 years on the annual maximum series and thus occurs at a similar time of year as the annual maximum discharge. From above, the water temperature during a typical bankfull discharge event is not known exactly, but must be somewhat above freezing. This water temperature is approximated to be 38 degrees F.

Suggestions for Future Work

Reliability of the MINRED method could be improved by:

1. Developing a calculation method or measurement technique which can accurately determine the sediment discharge in gravel-bed streams at the bankfull discharge. This would eliminate the need to assume a constant sediment discharge at the bankfull condition. This would also ensure that the minimum rate of energy dissipation theory and the MINRED method are applied only to streams with a negligible to low sediment discharge.

2. Either waiting much longer after culvert installation to study the streams or collecting data over many years to determine changes in the variables. This would eliminate the need to assume that all the dependent variables in the study streams are adjusting toward equilibrium with the post-culvert independent variables.

3. Determining exponents of the stream width, depth,

and mean velocity that relate these variables to the discharge, for streams in equilibrium with no imposed human activity, and flowing in soils with discontinuous permafrost. Such exponents are available for the above conditions in temperate climates only. Determining these exponents for streams flowing in discontinuous permafrost would allow the altered and unaltered subreaches in the study streams to be determined, rather than approximated.

4. Improving Shields diagram or developing some other method which more accurately determines the critical conditions for turbulent flow. This would allow a better estimate of the mean sediment size.

5. Improving Raudkivi's graphical relation or some other method which more accurately applies to the gravel-bed streams under study. This would allow a better estimate of the mean sediment size.

6. Developing a method which calculates the one water temperature having the same influence on the dependent variables as the combined effects of all the water temperatures actually occurring. This would eliminate the need to approximate such a water temperature and hence result in a more accurate determination of the mean sediment size.

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APPENDICES

APPENDIX A

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APPENDIX B

DEFINITION OF SYMBOLS

A	= cross sectional area of flow;
b	= stream width;
	= exponent in equation D-2;
d	= stream depth;
d_s	= mean sediment size;
E	= total energy at a location;
E_1	= specific energy at the upstream cross section;
E_2	= specific energy at the downstream cross section;
f	= Darcy-Weisbach friction factor;
Fr	= Froude number;
g	= acceleration due to gravity;
h	= vertical direction;
h_1	= head loss at culvert entrance;
$\vec{i}$	= unit vector in the x-direction;
$\vec{j}$	= unit vector in the y-direction;
$\vec{k}$	= unit vector in the z-direction;
m_5	= correction of the Manning roughness coefficient for the effect of channel meandering;
n	= Manning roughness coefficient;
n_0	= Manning roughness coefficient due to the effect of grain roughness;
n_1	= correction of the Manning roughness coefficient for the effect of surface irregularities;
n_2	= correction of the Manning roughness coefficient for the effect of variations in the channel cross section size and shape;

n_3 = correction of the Manning roughness coefficient for the effect of obstructions;

n_4 = correction of the Manning roughness coefficient for the effect of vegetation;

p = fluid pressure;

P_f = planform;

$p(\vec{V})$ = rate of energy dissipation due to the original velocity vector;

$P(V \mp \hat{V})$ = rate of energy dissipation due to the hypothetical velocity vector;

P_e = probability of exceedance;

P_{ne} = probability of nonexceedance;

P_w = wetted perimeter;

Q = water discharge;

$\overline{Q}$ = mean water discharge;

Q_b = bankfull water discharge;

r^2 = coefficient of determination;

R = hydraulic radius;

Re_* = boundary Reynolds number;

S = slope of the energy grade line;

S_0 = measured thalweg slope between the upstream and downstream cross sections;

S_1 = slope of the energy grade line at the upstream cross section;

S_2 = slope of the energy grade line at the downstream cross section;

t = time;

T = recurrance interval;

u = component of the original velocity vector in the x-direction;

$\hat{u}$ = component of the velocity variation vector in the x-direction;

U_* = shear velocity;

U_{*c} = critical value of the shear velocity;

v = component of the original velocity vector in the y-direction;

$\hat{v}$ = component of the velocity variation vector in the y-direction;

V = magnitude of the mean velocity of flow;

V_c = velocity in the culvert at critical flow;

V_{cr} = critical value of the mean velocity;

$\vec{V}$ = original velocity vector representing the mean velocity vector at a point;

$\vec{V} + \hat{V}$ = hypothetical velocity vector;

$\vec{V}$ = velocity variation vector;

w = component of the original velocity vector in the z-direction;

$\hat{w}$ = component of the velocity variation vector in the z-direction;

w_f = fall velocity of the bed material;

x = axis in the longitudinal stream direction;

X = distance between the upstream and downstream cross sections;

y = axis in the lateral stream direction;

y_c = critical depth in the culvert;

y_n = normal depth in the culvert;

z = axis in the vertical stream direction;

$\forall$ = fluid volume;

γ = fluid specific weight;

γ_s = sediment specific weight;

ε = kinematic eddy viscosity;

η = dynamic eddy viscosity;

μ = dynamic molecular viscosity;

ν = kinematic molecular viscosity;

ρ = fluid density;

σ = geometric standard deviation of the bed material size distribution;

= standard deviation of discharges;

$\vec{\tau}$ = stress tensor;

τ = stress component;

τ_c = critical value of the bed shear stress;

τ_* = dimensionless shear stress;

τ_{*c} = critical value of the dimensionless shear stress;

$\phi(\vec{V})$ = rate of energy dissipation per unit volume of fluid due to the original velocity vector;

$\phi(\vec{V} + \hat{V})$ = rate of energy dissipation per unit volume of fluid due to the hypothetical velocity vector; and

$\phi(\vec{V})$ = rate of energy dissipation per unit volume of fluid due to the velocity variation vector.

APPENDIX C

DEVELOPMENT OF THE MINIMUM RATE OF ENERGY DISSIPATION THEORY FOR ALLUVIAL STREAMS

A development of the minimum rate of energy dissipation theory applicable to alluvial streams is given in condensed form by Yang and Song (1979) and Song and Yang (1980). This development is summarized and expanded in this appendix. The development shows that the theory is obtained from the equation of motion. Therefore, provided the conditions of the development are met, the minimum rate of energy dissipation theory is an alternate way of stating the equation of motion. The discussion in this appendix is divided into four sections. The first three sections present the equation of continuity, equations describing the stress tensor, and equation of motion. The last section develops the theory from these equations and determines the conditions for which it is applicable to alluvial streams. In order to develop the theory from these equations, three velocity vectors are considered: the original velocity vector, the hypothetical velocity vector, and the velocity variation vector. The equations to be developed by the first three sections consider the original velocity vector only. The hypothetical velocity vector and the velocity variation vector are discussed in the last section.

The minimum rate of energy dissipation theory is developed from equations written in rectangular co-ordinates. The x, y, and z axes represent the longitudinal, lateral and vertical stream directions, respectively. The symbols u, v, and w represent the components of the time averaged or mean velocity vector at a point in the x, y, and z directions, respectively. Therefore,

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k} \qquad \text{(C-1)}$$

is the mean velocity vector at a point. This is named the original velocity vector. The symbols $\vec{i}$, $\vec{j}$, and $\vec{k}$ represent unit vectors in the x, y, and z directions, respectively. An arrow superscript indicates a vector.

Equation of Continuity

Daily and Harleman (1966) indicate that water can be considered an incompressible fluid. For an incompressible fluid, Bird, Stewart, and Lightfoot (1960) indicate the equation of continuity is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \vec{\nabla} \cdot \vec{V} = 0 \ . \qquad \text{(C-2)}$$

Stress Tensor

Fox and McDonald (1978) state that water can be considered a Newtonian fluid. Equations given by Bird et al. (1960) show for laminar flow of Newtonian fluids, the constant of proportionality between the fluid stress components and the velocity gradient is the dynamic molecular viscosity. Bird et al. (1960) and Daily and Harleman (1966) indicate equations describing the fluid stress components in turbulent flow are similar to those in laminar flow except the constant of proportionality is the total fluid viscosity. They further indicate the total fluid viscosity is the sum of the dynamic molecular viscosity and the dynamic eddy viscosity.

Equation C-2 and the total fluid viscosity can be substituted into the equations describing the fluid stress components for laminar flow of Newtonian fluids. This yields the fluid stress components for laminar or turbulent flow of incompressible Newtonian fluids. These are:

$$\tau_{xx} = -2\rho(\nu + \varepsilon)\frac{\partial u}{\partial x} , \qquad \text{(C-3)}$$

$$\tau_{yy} = -2\rho(\nu + \varepsilon)\frac{\partial v}{\partial y} , \qquad \text{(C-4)}$$

$$\tau_{zz} = -2\rho(\nu + \varepsilon)\frac{\partial w}{\partial z} , \qquad \text{(C-5)}$$

$$\tau_{xy} = \tau_{yx} = -\rho(\nu + \varepsilon)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) , \qquad \text{(C-6)}$$

$$\tau_{yz} = \tau_{zy} = -\rho(\nu + \varepsilon)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) , \text{ and} \qquad \text{(C-7)}$$

$$\tau_{zx} = \tau_{xz} = -\rho(\nu + \varepsilon)\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) . \qquad \text{(C-8)}$$

The above symbols are as follows:

τ = fluid stress component,

ρ = fluid density,

ν = μ/ρ = kinematic molecular viscosity,

ε = η/ρ = kinematic eddy viscosity,

μ = dynamic molecular viscosity, and

η = dynamic eddy viscosity.

The sum of all the fluid stress components acting on a water volume equals the stress tensor, $\vec{\tau}$.

The water temperature is considered constant, hence the molecular viscosity is also constant. The eddy viscosity depends on the state of the turbulent motion, thus varies with location in the flow.

The subscript convention for the stress components in equations C-3 to C-8 can be better understood by considering the cubic fluid volume shown in Figure 7. The first subscript indicates the direction of the normal to the face of the fluid volume on which the stress acts. The second subscript indicates the direction in which the stress acts.

A normal stress has identical subscripts and a shear stress does not. A stress tensor is symmetric if the shear stress components with subscripts differing only in their order are equal in magnitude (Bird et al., 1960). The shear stress components given in equations C-6 to C-8 are symmetric.

Equation of Motion

Bird et al. (1960) and Daily and Harleman (1966) indicate the equation of motion for steady flow with

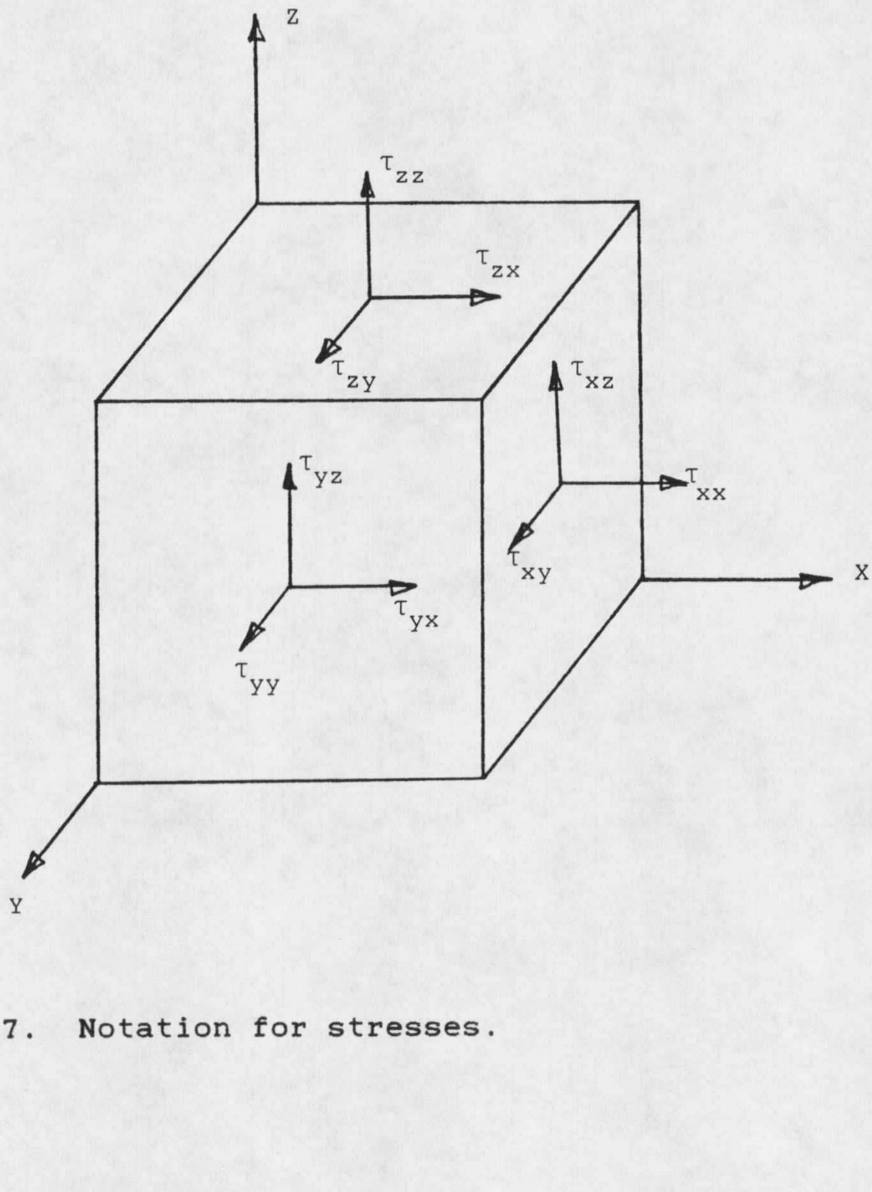

Figure 7. Notation for stresses.

gravitational body forces is:

$$\rho\left(u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z}\right) + \vec{\nabla}P + \gamma\vec{\nabla}h = -\vec{\nabla}\circ\vec{\tau} \ . \qquad \text{(C-9)}$$

Previously undefined symbols are as follows:

p = fluid pressure,

γ = fluid specific weight, and

h = vertical direction measured positive upward.

Bird et al. (1960) indicate the term on the far left of equation C-9 represents the rate of change of momentum per unit volume of fluid. They further indicate from left to right, the other terms in equation C-9 represent pressure surface force per unit volume of fluid, gravitational body force per unit volume of fluid, and viscous surface force per unit volume of fluid. For forces per unit area applied on a typical water volume between two cross sections in an alluvial stream, see Figure 8.

Rate of Energy Dissipation

This section states the minimum rate of energy dissipation theory and the conditions for which it is applicable to alluvial streams.

Velocity Vector Satisfying the Equation of Motion

The substantial time derivative of a variable describes its rate of change with respect to both time and location

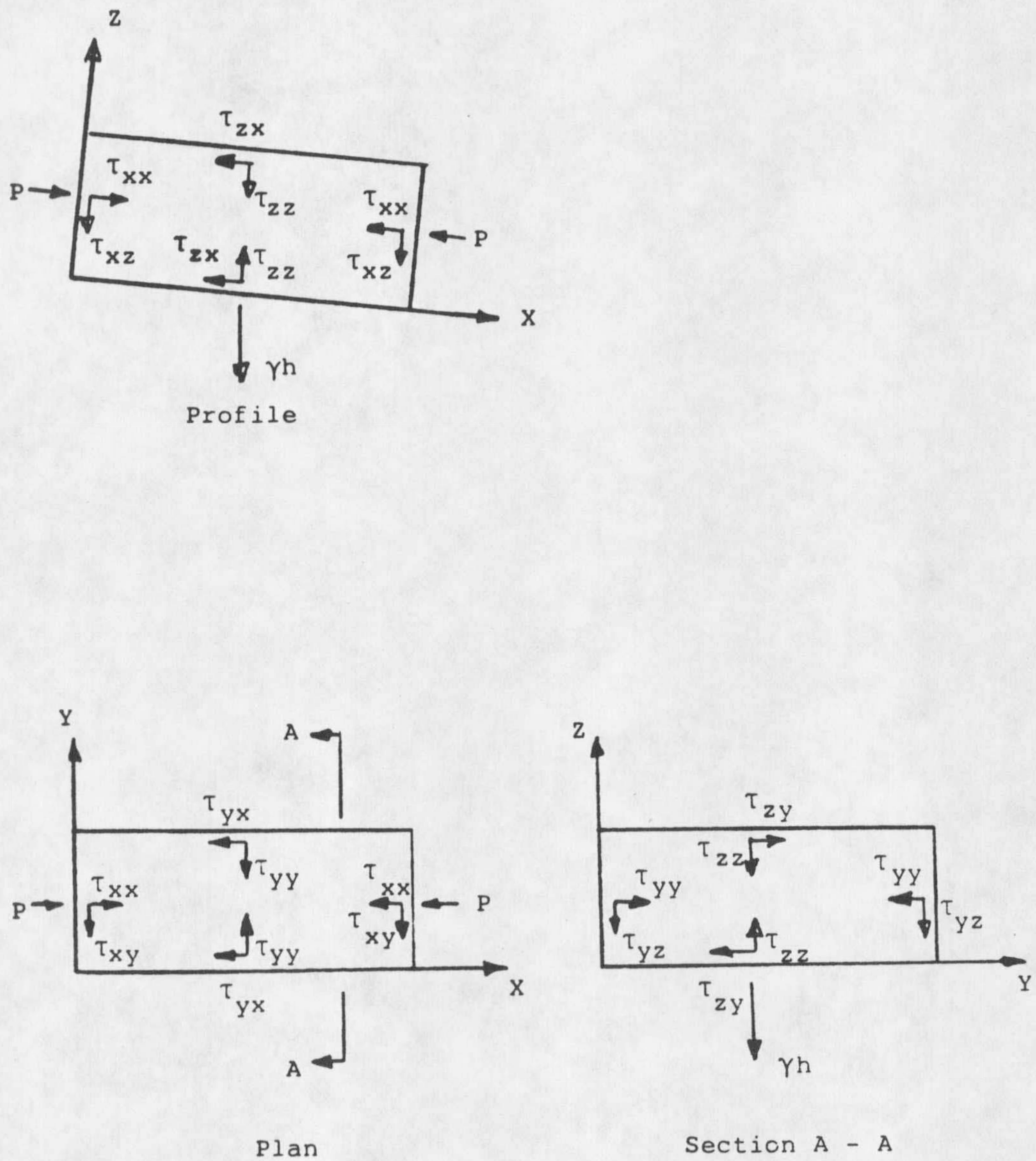

Figure 8. Forces per unit area on a typical water volume between two cross sections in an alluvial stream.

for a path following the fluid motion (Bird et al., 1960). For fluid at a constant temperature under steady flow with gravitational body forces, the substantial time derivative of the total energy per unit volume of fluid is:

$$\frac{D}{Dt}\left(\frac{E}{\forall}\right) = \vec{V}\cdot\left[\rho\left(u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z}\right) + \vec{\nabla}P + \gamma\vec{\nabla}h\right] . \qquad \text{(C-10)}$$

This is deduced from Bird et al. (1960). Previously undefined symbols are as follows:

t = time

E = total energy at a location, and

$\forall$ = fluid volume.

The right side of equation C-10 equals the dot product of the original velocity vector (given in equation C-1) and the left side of equation C-9. Combining equations C-9 and C-10 and rewriting yields:

$$\frac{D}{Dt}\left(\frac{E}{\forall}\right) = -\vec{\nabla}\cdot(\vec{\tau}\cdot\vec{V}) + \vec{\tau}:\vec{\nabla}\vec{V} . \qquad \text{(C-11)}$$

Equation C-11 describes the rate of change of energy per unit volume of fluid for a path following the fluid motion. This equation is valid for fluid at a constant temperature under steady flow with gravitational body forces.

Bird et al. (1960) indicate the first term on the right of equation C-11 describes an energy change due to viscous surface forces which is reversible. Therefore, this term does not describe the rate at which energy is lost from the fluid.

Dunne and Leopold (1978) posit most energy lost from a stream is converted into internal energy and dissipated as heat and a small part of energy lost from a stream is used to transport sediment. Bird et al. (1960) indicate the second term on the right of equation C-11 describes the rate of irreversible conversion of kinetic energy to internal energy by viscous dissipation, per unit volume of fluid. Bird et al. (1960) and Daily and Harleman (1966) show flow in open channels does not cause appreciable temperature changes in the water, hence all the energy converted into internal energy is lost. Thus, when no energy is used to transport sediment, the second term on the right of equation C-11 represents the rate at which energy is lost from a stream (Bird et al., 1960).

Energy dissipation is a loss of energy from a stream, hence a negative change of energy. Therefore, the rate of energy dissipation per unit volume of fluid equals the negative of the second term on the right of equation C-11. Bird et al. (1960) expand the negative of the second term on the right of equation C-11 for symmetrical stress tensors. Combining this with equations C-3 to C-8 yields:

$$\phi(\vec{V}) = \rho(\nu + \varepsilon)\{2[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2 + (\frac{\partial w}{\partial z})^2] \qquad \text{(C-12)}$$
$$+ (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})^2 + (\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y})^2 + (\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z})^2\} .$$

The symbol ϕ represents the rate of energy dissipation per unit volume of fluid. Equation C-12 is valid for incompressible Newtonian fluids at a constant temperature in steady, laminar or turbulent flow conditions under gravitational body forces with negligible sediment transport. The $(\vec{V})$ on the left side indicates that this equation is written considering the original velocity vector given in equation C-1. This velocity vector satisfies the equation of motion. Similar notation is also used with other velocity vectors.

Velocity Vector Not Satisfying the Equation of Motion

The original velocity vector satisfies the equation of continuity and the equation of motion. Also, this velocity vector has a certain kinematic eddy viscosity and certain boundary conditions. Yang and Song (1979) consider a hypothetical velocity vector that is assigned the following characteristics. This vector does not satisfy the equation of motion but does satisfy the equation of continuity and has both the same kinematic eddy viscosity and the same boundary conditions as the original velocity vector. The equation of motion as given in equation C-9 does not place any specific requirements on either the kinematic eddy viscosity or the boundary conditions. Therefore, it is possible for the hypothetical velocity vector to not satisfy the equation of motion but have the same kinematic eddy viscosity and boundary conditions as the original velocity vector.

A velocity vector describing flow in an alluvial stream must necessarily satisfy the equation of motion. Therefore, the hypothetical velocity vector does not describe a real flow situation. This vector is considered merely for the purposes of argument to develop the minimum rate of energy dissipation theory for alluvial streams. This development will show that the flow described by a velocity vector satisfying the equation of motion results in a lesser rate of energy dissipation than the flow described by a velocity vector not satisfying the equation of motion.

The hypothetical velocity vector is denoted by:

$$\vec{V} + \hat{v} = (u + \hat{u})\vec{i} + (v + \hat{v})\vec{j} + (w + \hat{w})\vec{k} \ . \qquad \text{(C-13)}$$

The velocity variation vector is the difference between the original velocity vector and the hypothetical velocity vector. The velocity variation vector is denoted by:

$$\hat{\vec{V}} = \hat{u}\vec{i} + \hat{v}\vec{j} + \hat{w}\vec{k} \ . \qquad \text{(C-14)}$$

Velocity Variation Vector

Albertson and Simons (1964) and Daily and Harleman (1966) indicate the kinematic eddy viscosity is proportional to the average intensity of the turbulence. Because the original and hypothetical velocity vectors given in equations C-1 and C-13 have the same kinematic eddy viscosity, they must also have the same turbulent characteristics. The sum of the original and velocity variation vectors given in equations C-1 and C-14 equals the hypothetical velocity vector given in equation C-13. Because the velocity vectors given in equations C-1 and C-13 have the same turbulent characteristics, the velocity variation vector given in equation C-14 necessarily has no turbulent characteristics, hence represents laminar flow. Therefore, the kinematic eddy viscosity of the velocity variation vector is zero. Thus, similar to equation C-12, the rate of energy dissipation per unit volume of fluid due to the velocity variation vector is:

$$\phi(\hat{\vec{V}}) = \rho\nu\{2[(\frac{\partial \hat{u}}{\partial x})^2 + (\frac{\partial \hat{v}}{\partial y})^2 + (\frac{\partial \hat{w}}{\partial z})^2] + (\frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x})^2 \qquad \text{(C-15)}$$

$$+ (\frac{\partial \hat{v}}{\partial z} + \frac{\partial \hat{w}}{\partial y})^2 + (\frac{\partial \hat{w}}{\partial x} + \frac{\partial \hat{u}}{\partial z})^2\} \ .$$

Hypothetical Velocity Vector

The rate of energy dissipation per unit volume of fluid due to the hypothetical velocity vector is developed below. Similar to equation C-12,

$$\phi(\vec{V} + \hat{V}) = \rho(\nu + \varepsilon)[2\{[\frac{\partial}{\partial x}(u + \hat{u})]^2 + [\frac{\partial}{\partial y}(v + \hat{v})]^2 + [\frac{\partial}{\partial z}(w + \hat{w})]^2\} + [\frac{\partial}{\partial y}(u + \hat{u}) + \frac{\partial}{\partial x}(v + \hat{v})]^2 + [\frac{\partial}{\partial z}(v + \hat{v}) + \frac{\partial}{\partial y}(w + \hat{w})]^2 + [\frac{\partial}{\partial x}(w + \hat{w}) + \frac{\partial}{\partial z}(u + \hat{u})]^2] \,. \tag{C-16}$$

Expanding equation C-16 yields:

$$\phi(\vec{V} + \hat{V}) = \rho(\nu + \varepsilon)\{2[(\frac{\partial u}{\partial x} + \frac{\partial \hat{u}}{\partial x})^2 + (\frac{\partial v}{\partial y} + \frac{\partial \hat{v}}{\partial y})^2 + (\frac{\partial w}{\partial z} + \frac{\partial \hat{w}}{\partial z})^2] + (\frac{\partial u}{\partial y} + \frac{\partial \hat{u}}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial \hat{v}}{\partial x})^2 + (\frac{\partial v}{\partial z} + \frac{\partial \hat{v}}{\partial z} + \frac{\partial w}{\partial y} + \frac{\partial \hat{w}}{\partial y})^2 + (\frac{\partial w}{\partial x} + \frac{\partial \hat{w}}{\partial x} + \frac{\partial u}{\partial z} + \frac{\partial \hat{u}}{\partial z})^2\} \,. \tag{C-17}$$

Expanding and rearranging equation C-17 yields:

$$
\begin{aligned}
\phi(V + \hat{V}) = {} & \rho(\nu + \varepsilon)\{2[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2 + (\frac{\partial w}{\partial z})^2] \\
& + (\frac{\partial u}{\partial y})^2 + 2(\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}) + (\frac{\partial v}{\partial x})^2 + (\frac{\partial v}{\partial z})^2 \\
& + 2(\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}) + (\frac{\partial w}{\partial y})^2 + (\frac{\partial w}{\partial x})^2 + 2(\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}) + (\frac{\partial u}{\partial z})^2\} \\
& + \rho(\nu + \varepsilon)\{2[2(\frac{\partial u}{\partial x}\frac{\partial \hat{u}}{\partial x}) + 2(\frac{\partial v}{\partial y}\frac{\partial \hat{v}}{\partial y}) + 2(\frac{\partial w}{\partial z}\frac{\partial \hat{w}}{\partial z})] \\
& + 2(\frac{\partial u}{\partial y}\frac{\partial \hat{u}}{\partial y}) + 2(\frac{\partial u}{\partial y}\frac{\partial \hat{v}}{\partial x}) + 2(\frac{\partial \hat{u}}{\partial y}\frac{\partial v}{\partial x}) + 2(\frac{\partial v}{\partial x}\frac{\partial \hat{v}}{\partial x}) \\
& + 2(\frac{\partial v}{\partial z}\frac{\partial \hat{v}}{\partial z}) + 2(\frac{\partial v}{\partial z}\frac{\partial \hat{w}}{\partial y}) + 2(\frac{\partial \hat{v}}{\partial z}\frac{\partial w}{\partial y}) + 2(\frac{\partial w}{\partial y}\frac{\partial \hat{w}}{\partial y}) \\
& + 2(\frac{\partial w}{\partial x}\frac{\partial \hat{w}}{\partial x}) + 2(\frac{\partial w}{\partial x}\frac{\partial \hat{u}}{\partial z}) + 2(\frac{\partial \hat{w}}{\partial x}\frac{\partial u}{\partial z}) + 2(\frac{\partial u}{\partial z}\frac{\partial \hat{u}}{\partial z})\} \\
& + \rho(\nu + \varepsilon)\{2[(\frac{\partial \hat{u}}{\partial x})^2 + (\frac{\partial \hat{v}}{\partial y})^2 + (\frac{\partial \hat{w}}{\partial z})^2] + (\frac{\partial \hat{u}}{\partial y})^2 \\
& + 2(\frac{\partial \hat{u}}{\partial y}\frac{\partial \hat{v}}{\partial x}) + (\frac{\partial \hat{v}}{\partial x})^2 + (\frac{\partial \hat{v}}{\partial z})^2 + 2(\frac{\partial \hat{v}}{\partial z}\frac{\partial \hat{w}}{\partial y}) \\
& + (\frac{\partial \hat{w}}{\partial y})^2 + (\frac{\partial \hat{w}}{\partial x})^2 + 2(\frac{\partial \hat{w}}{\partial x}\frac{\partial \hat{u}}{\partial z}) + (\frac{\partial \hat{u}}{\partial z})^2\} .
\end{aligned}
\qquad \text{(C-18)}
$$

Equation C-18 can be simplified to:

$$\phi(V \dot{+} \hat{V}) = \rho(\nu + \varepsilon)\{2[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2 + (\frac{\partial w}{\partial z})^2] \tag{C-19}$$

$$+ (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})^2 + (\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y})^2 + (\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z})^2\}$$

$$+ 2\rho(\nu + \varepsilon)\{2[(\frac{\partial u}{\partial x}\frac{\partial \hat{u}}{\partial x}) + (\frac{\partial v}{\partial y}\frac{\partial \hat{v}}{\partial y}) + (\frac{\partial w}{\partial z}\frac{\partial \hat{w}}{\partial z})]$$

$$+ (\frac{\partial u}{\partial y}\frac{\partial \hat{u}}{\partial y}) + (\frac{\partial u}{\partial y}\frac{\partial \hat{v}}{\partial x}) + (\frac{\partial \hat{u}}{\partial y}\frac{\partial v}{\partial x}) + (\frac{\partial v}{\partial x}\frac{\partial \hat{v}}{\partial x})$$

$$+ (\frac{\partial v}{\partial z}\frac{\partial \hat{v}}{\partial z}) + (\frac{\partial v}{\partial z}\frac{\partial \hat{w}}{\partial y}) + (\frac{\partial \hat{v}}{\partial z}\frac{\partial w}{\partial y}) + (\frac{\partial w}{\partial y}\frac{\partial \hat{w}}{\partial y})$$

$$+ (\frac{\partial w}{\partial x}\frac{\partial \hat{w}}{\partial x}) + (\frac{\partial w}{\partial x}\frac{\partial \hat{u}}{\partial z}) + (\frac{\partial \hat{w}}{\partial x}\frac{\partial u}{\partial z}) + (\frac{\partial u}{\partial z}\frac{\partial \hat{u}}{\partial z})\}$$

$$+ \rho(\nu + \varepsilon)\{2[(\frac{\partial \hat{u}}{\partial x})^2 + (\frac{\partial \hat{v}}{\partial y})^2 + (\frac{\partial \hat{w}}{\partial z})^2]$$

$$+ (\frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x})^2 + (\frac{\partial \hat{v}}{\partial z} + \frac{\partial \hat{w}}{\partial y})^2 + (\frac{\partial \hat{w}}{\partial x} + \frac{\partial \hat{u}}{\partial z})^2\} .$$

Note that:

$$\rho(\nu + \varepsilon) = \rho\nu + \rho\varepsilon = \rho\nu(1 + \frac{\varepsilon}{\nu}) . \tag{C-20}$$

Substituting equations C-12, C-15, and C-20 into equation C-19 gives:

$$\phi(\vec{V} + \hat{V}) = \phi(\vec{V}) + 2\rho(\nu + \epsilon)\{2[(\frac{\partial u}{\partial x}\frac{\partial \hat{u}}{\partial x}) \tag{C-21}$$

$$+ (\frac{\partial v}{\partial y}\frac{\partial \hat{v}}{\partial y}) + (\frac{\partial w}{\partial z}\frac{\partial \hat{w}}{\partial z})] + (\frac{\partial u}{\partial y}\frac{\partial \hat{u}}{\partial y}) + (\frac{\partial u}{\partial y}\frac{\partial \hat{v}}{\partial x})$$

$$+ (\frac{\partial \hat{u}}{\partial y}\frac{\partial v}{\partial x}) + (\frac{\partial v}{\partial x}\frac{\partial \hat{v}}{\partial x}) + (\frac{\partial v}{\partial z}\frac{\partial \hat{v}}{\partial z}) + (\frac{\partial v}{\partial z}\frac{\partial \hat{w}}{\partial y})$$

$$+ (\frac{\partial \hat{v}}{\partial z}\frac{\partial w}{\partial y}) + (\frac{\partial w}{\partial y}\frac{\partial \hat{w}}{\partial y}) + (\frac{\partial w}{\partial x}\frac{\partial \hat{w}}{\partial x}) + (\frac{\partial w}{\partial x}\frac{\partial \hat{u}}{\partial z})$$

$$+ (\frac{\partial \hat{w}}{\partial x}\frac{\partial u}{\partial z}) + (\frac{\partial u}{\partial z}\frac{\partial \hat{u}}{\partial z})\} + (1 + \frac{\epsilon}{\nu})\phi(\hat{\vec{V}}) .$$

Equation C-21 can be simplified to:

$$\phi(\vec{V} + \hat{V}) = \phi(\vec{V}) + 2\rho(\nu + \epsilon)\{2[(\frac{\partial u}{\partial x}\frac{\partial \hat{u}}{\partial x}) \tag{C-22}$$

$$+ (\frac{\partial v}{\partial y}\frac{\partial \hat{v}}{\partial y}) + (\frac{\partial w}{\partial z}\frac{\partial \hat{w}}{\partial z})] + (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})(\frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x})$$

$$+ (\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y})(\frac{\partial \hat{v}}{\partial z} + \frac{\partial \hat{w}}{\partial y}) + (\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z})(\frac{\partial \hat{w}}{\partial x} + \frac{\partial \hat{u}}{\partial z})\}$$

$$+ (1 + \frac{\epsilon}{\nu})\phi(\hat{\vec{V}}) .$$

Substituting equations C-3 to C-8 into equation C-22 yields:

$$\phi(\vec{V} + \hat{\vec{V}}) = \phi(\vec{V}) - 2[\tau_{xx}\frac{\partial\hat{u}}{\partial x} + \tau_{yy}\frac{\partial\hat{v}}{\partial y} + \tau_{zz}\frac{\partial\hat{w}}{\partial z} + \tau_{xy}(\frac{\partial\hat{u}}{\partial y} + \frac{\partial\hat{v}}{\partial x}) + \tau_{yz}(\frac{\partial\hat{v}}{\partial z} + \frac{\partial\hat{w}}{\partial y}) + \tau_{zx}(\frac{\partial\hat{w}}{\partial x} + \frac{\partial\hat{u}}{\partial z})] + (1 + \frac{\varepsilon}{\nu})\phi(\hat{\vec{V}}) \; . \quad \text{(C-23)}$$

Integration by parts is described in many calculus textbooks. This procedure is:

$$\int u dv = uv - \int v du \; . \quad \text{(C-24)}$$

The original velocity vector and the hypothetical velocity vector have the same boundary conditions. Therefore, the velocity variation vector is zero at the boundaries. Integrating the middle term on the right side of equation C-23 by parts as given above yields:

$$\phi(\vec{V} + \hat{\vec{V}}) = \phi(\vec{V}) + 2[\hat{u}(\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}) + \hat{v}(\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\tau_{yy}}{\partial y} + \frac{\partial\tau_{yz}}{\partial z}) + \hat{w}(\frac{\partial\tau_{zx}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\tau_{zz}}{\partial z})] + (1 + \frac{\varepsilon}{\nu})\phi(\hat{\vec{V}}) \; . \quad \text{(C-25)}$$

Substituting equations C-3 to C-8 into equation C-25 yields:

$$\phi(\vec{V} + \hat{V}) = \phi(\vec{V}) - 2\rho(\nu + \varepsilon)\{\hat{u}[2\frac{\partial}{\partial x}(\frac{\partial u}{\partial x}) \qquad (C\text{-}26)$$

$$+ \frac{\partial}{\partial y}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) + \frac{\partial}{\partial z}(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z})]$$

$$+ \hat{v}[\frac{\partial}{\partial x}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) + 2\frac{\partial}{\partial y}(\frac{\partial v}{\partial y}) + \frac{\partial}{\partial z}(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y})]$$

$$+ \hat{w}[\frac{\partial}{\partial x}(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}) + \frac{\partial}{\partial y}(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}) + 2\frac{\partial}{\partial z}(\frac{\partial w}{\partial z})]\}$$

$$+ (1 + \frac{\varepsilon}{\nu})\phi(\hat{\vec{V}}) .$$

Evaluating and rearranging equation C-26 gives:

$$\phi(\vec{V} + \hat{V}) = \phi(\vec{V}) - 2\rho(\nu + \varepsilon)\{\hat{u}[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad (C\text{-}27)$$

$$+ \frac{\partial^2 u}{\partial z^2} + \frac{\partial}{\partial x}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})]$$

$$+ \hat{v}[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} + \frac{\partial}{\partial y}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})]$$

$$+ \hat{w}[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} + \frac{\partial}{\partial z}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})]\}$$

$$+ (1 + \frac{\varepsilon}{\nu})\phi(\hat{\vec{V}}) .$$

Substituting equation C-2 into equation C-27 yields:

$$\phi(\vec{V} + \hat{V}) = \phi(\vec{V}) - 2\rho(\nu + \varepsilon)[\hat{u}(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) \quad (C-28)$$

$$+ \hat{v}(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2})$$

$$+ \hat{w}(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2})] + (1 + \frac{\varepsilon}{\nu})\phi(\vec{\hat{V}}) .$$

Statement of the Theory

Equations C-12 and C-15 indicate that the first and last terms on the right of equation C-28 are the sum of the squares of the first derivative of the velocity components. The first derivative of the velocity can also be called the velocity gradient or acceleration. It represents the change in the velocity with location. The middle term on the right of equation C-28 is the sum of the second derivative of the velocity components. The second derivative of the velocity can also be called the first derivative of the velocity gradient, the first derivative of the acceleration, or the acceleration gradient. It represents the change in the acceleration with location.

For rapidly varied flow, the velocity, acceleration, and acceleration gradient can all change significantly with location. Therefore, the acceleration in the first and/or the last term in equation C-28 may be comparable to the acceleration gradient in the middle term. For this reason, equation C-28 can not be reduced when considering rapidly varied flow. For uniform and gradually varied flow, however, the velocity changes little with location. For this type of flow, the acceleration is small and the acceleration gradient is very small. Therefore, the acceleration gradient in the middle term of equation C-28 is much less than the acceleration in the first and last term. For this reason, the middle term on the right of equation C-28 is negligible for uniform and gradually varied flow. Under these conditions, equation C-28 can be reduced to:

$$\phi(\vec{V} + \hat{V}) = \phi(\vec{V}) + (1 + \frac{\varepsilon}{\nu})\phi(\vec{\hat{V}}) \quad . \tag{C-29}$$

Rearranging this gives:

$$\phi(\vec{V}) = \phi(\vec{V} + \hat{V}) - (1 + \frac{\varepsilon}{\nu})\phi(\vec{\hat{V}}) \quad . \tag{C-30}$$

Equation C-15 indicates the second term on the right of equation C-30 is the sum of squared terms, hence is always positive. Equation C-30 can be rewritten:

$$\phi(\vec{V}) \leq \phi(\vec{V} + \hat{V}) \quad . \tag{C-31}$$

Integrating equation C-31 over the total volume of fluid yields:

$$P(\vec{V}) \leq P(\vec{V} + \hat{V}) \quad , \tag{C-32}$$

where P represents the rate of energy dissipation. Equation C-32 mathematically states the minimum rate of energy dissipation theory.

Equation C-32 is valid for incompressible Newtonian fluids at a constant temperature in steady, laminar or turbulent flow conditions, uniform or gradually varied flow under gravitational body forces with negligible sediment discharge. Water is an incompressible Newtonian fluid. Alluvial streams flow turbulent under gravitational body forces. Thus, for alluvial streams, these requirements are met. Therefore, equation C-32 is valid for alluvial streams having a constant water temperature with steady uniform or gradually varied flow and negligible sediment discharge.

The original velocity vector satisfies the equations of continuity and motion. The hypothetical velocity vector does not satisfy the equation of motion, but satisfies the equation of continuity and has the same kinematic eddy viscosity and boundary conditions as the original velocity vector. Therefore, the minimum rate of energy dissipation theory as represented by equation C-32 states: provided certain conditions are met, the flow described by a velocity vector satisfying the equation of motion necessarily results in a lesser rate of energy dissipation than the flow described by a velocity vector not satisfying the equation of motion.

APPENDIX D

BANKFULL DISCHARGE

Langbein (1949) and others define the annual maximum discharge as the highest instantaneous value in a given water year. Dunne and Leopold (1978) indicate a compilation of these discharges is the annual maximum series. The recurrence interval from this series is the average interval between the occurrence of annual maximum discharges that equal or exceed a given value. The bankfull discharge has a recurrence interval of approximately 1.5 years on the annual maximum series. Therefore, on the average once in 1.5 years, or two years out of three, the highest discharge for the year will be equal to or will exceed the bankfull capacity of the channel. Dunne and Leopold (1978) and Morel-Seytoux (1979) indicate:

$$P_{ne} = 1 - P_e = 1 - 1/T = 1 - 2/3 = .3333. \qquad \text{(D-1)}$$

The above symbols are as follows:

P_{ne} = probability the bankfull discharge will not be exceeded;

P_e = probability the bankfull discharge will be exceeded; and

T = recurrence interval, in years, between events that will equal or exceed the bankfull discharge.

Morel-Seytoux (1979) indicates using the Gumbel distribution, the probability of non-exceedance of any discharge is:

$$P_{ne} = e^{-e^{-b}} . \qquad (D-2)$$

He further indicates that:

$$b = \left(\frac{1}{0.78\sigma}\right)(Q - \overline{Q} + 0.45\sigma) . \qquad (D-3)$$

The above symbols are as follows:

σ = standard deviation of the discharges,

Q = any water discharge, and

$\overline{Q}$ = mean water discharge.

Combining equations D-2 and D-3 gives:

$$Q = \overline{Q} - 0.45\sigma - 0.78\sigma \ln[-\ln(P_{ne})] . \qquad (D-4)$$

Combining equations D-1 and D-4 yields:

$$Q_b = \overline{Q} - 0.52\sigma . \qquad (D-5)$$

The symbol Q_b represents the bankfull water discharge.

The annual maximum series for the streams under study, is presented in Tables 8 and 9. The bankfull discharges can be found by combining the information in these tables and equation D-5. These discharges are given in Table 10.

Table 8. Annual maximum discharges for Copper River tributary.

Water year	Month	Discharge, cfs	Water year	Month	Discharge, cfs
1963	Aug.	13	1973	June	98
1964	July	173	1974	June	57
1965	July	135	1975	July	97
1966	June	28	1976	Aug.	40
1967	N.R.	7	1977	Aug.	20
1968	June	29	1978	July	110
1969	Aug.	15	1979	June	45
1970	July	26	1980	June	205
1971	Aug.	19	1981	Aug.	62
1972	July	48	1982	Aug.	115

Note. These data are from U.S.G.S. crest stage gage station no. 15199000. N.R. = not recorded.

Table 9. Annual maximum discharges for Brooks Creek tributary.

Water year	Month	Discharge, cfs	Water year	Month	Discharge, cfs
1964	Aug.	137	1974	June	2
1965	Sept.	85	1975	May	168
1966	May	90	1976	N.R.	96
1967	Aug.	43	1977	May	43
1968	N.R.	98	1978	June	47
1969	June	2	1979	May	98
1970	June	42	1980	May	40
1971	Aug.	46	1981	Aug.	9
1972	N.R.	67	1982	May	45
1973	May	32			

Note. These data are from U.S.G.S. crest stage gage, station no. 15519200. N.R. = not recorded.

Table 10. Bankfull discharge calculation.

Stream name	Mean discharge, $\bar{Q}$, cfs	Standard deviation, σ	Bankfull discharge, Q_b, cfs
Copper River tributary	67.10	56.66	37.64
Brooks Creek tributary	62.63	44.24	39.63

APPENDIX E

MEASURED VALUES

Table 11. Culvert data.

Parameter	Copper River tributary	Brooks Creek tributary
Diameter, ft	9.5	5
Length, ft	164	86
Elevation of upstream invert, ft	100.00	100.00
Elevation of downstream invert, ft	91.67	94.90
Slope	0.0508	0.0593
Material	corrugated metal	corrugated metal

Table 12. Copper River tributary, downstream, measured data.

Cross section	Thalweg elev, ft	Bank elev, ft	Bankfull width, ft	MSS, ft	Distance, ft
0D	88.63	91.62	21.5	–	3
1D	88.43	91.93	40	–	60
2D	88.52	91.58	18	–	35
3D	88.16	93.04	41	0.0417	52
4D	85.97	89.41	20	0.0417	33
5D	84.44	87.49	12	0.0417	34
6D	82.49	85.91	20	0.0417	36
7D	82.03	83.40	20	0.0625	31
8D	80.37	81.38	7.5	0.0625	131
9D	74.81	76.16	10	0.0625	162
10D	68.77	70.32	6.5	0.0625	262
11D	59.19	61.21	13	0.0833	207
12D	52.04	54.33	9.5	0.0625	+

Note. Elev = elevation. MSS = mean sediment size. Unreported data indicated by a –. Unapplicable data indicated by a +. Distance is that to next downstream cross section.

Table 13. Copper River tributary, upstream, measured data.

Cross section	Thalweg elev, ft	Bank elev, ft	Bankfull width ft	MSS, ft	Distance, ft
14U	124.93	126.78	7.5	0.0276	+
13U	120.99	122.47	3.5	0.0276	260
12U	113.69	114.51	6	0.0208	236
11U	111.15	112.92	4	0.0833	103
10U	107.69	109.10	5.5	0.0417	111
9U	105.57	107.28	8	0.0104	75
8U	103.72	105.49	12.5	0.0276	42
7U	102.68	104.30	11	0.0417	44
6U	101.38	103.70	6.5	0.0104	13
5U	100.09	101.91	6.5	0.0208	48
4U	99.98	101.85	9	0.0130	8
3U	99.90	101.51	13.5	0.0052	32
2U	99.52	102.14	21	0.0005	8
1U	99.30	101.05	20	0.0005	19
0U	99.60	101.18	27	0.0005	28

Note. Elev = elevation. MSS = mean sediment size. Unapplicable data indicated by a +. Distance is that to next upstream cross section.

Table 14. Brooks Creek tributary, downstream, measured data.

Cross section	Thalweg elev, ft	Bank elev, ft	Bankfull width, ft	MSS, ft	Distance, ft
0D	94.71	97.16	7.5	-	10
1D	93.42	96.55	21	-	19
2D	95.38	97.40	19.4	-	23
3D	95.32	96.57	15.3	0.0208	32
4D	94.43	95.91	9	0.0208	28
5D	94.22	95.08	10.7	0.0208	40
6D	93.09	94.76	7.3	0.0272	63
7D	90.71	91.90	7	0.0208	131
8D	86.61	88.04	6	0.0208	115
9D	84.34	85.82	6.5	0.1042	170
10D	81.28	82.60	8	0.1250	172
11D	77.90	79.09	12	0.0208	213
12D	73.09	74.29	8	0.0208	+

Note. Elev = elevation. MSS = mean sediment size. Unreported data indicated by a -. Unapplicable data indicated by a +. Distance is that to next downstream cross section.

Table 15. Brooks Creek tributary, upstream, measured data.

Cross section	Thalweg elev, ft	Bank elev, ft	Bankfull width, ft	MSS, ft	Distance, ft
14U	118.30	118.99	7	0.0104	+
13U	113.25	113.95	10	0.0208	212
12U	111.06	111.46	7.5	0.0313	167
11U	108.37	109.09	7	0.0417	137
10U	105.62	106.71	8.5	0.0313	151
9U	104.17	104.88	7	0.0313	108
8U	102.00	102.95	7.5	0.0058	103
7U	101.25	102.29	8.7	0.0449	43
6U	101.04	102.31	7	0.0438	17
5U	100.74	103.21	6.7	0.0428	16
4U	100.71	102.03	6.2	0.0417	19
3U	101.05	101.69	7.5	0.0417	11
2U	100.79	102.21	10	0.0417	14
1U	100.20	101.45	12	0.0417	13
0U	100.08	101.32	7.2	0.0417	10

Note. Elev = elevation. MSS = mean sediment size. Unapplicable data indicated by a +. Distance is that to next upstream cross section.

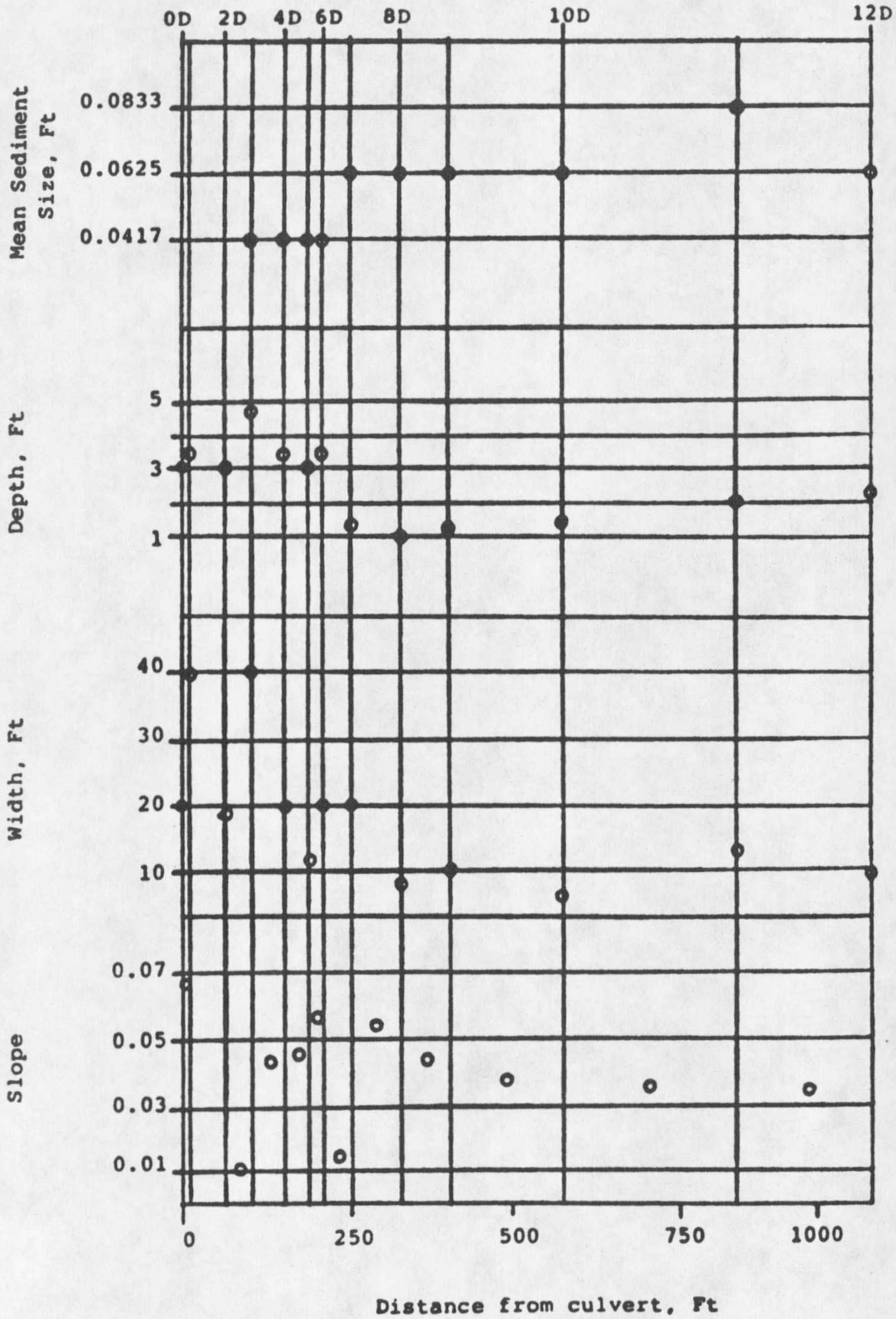

Figure 9. Copper River tributary, downstream, measured data.

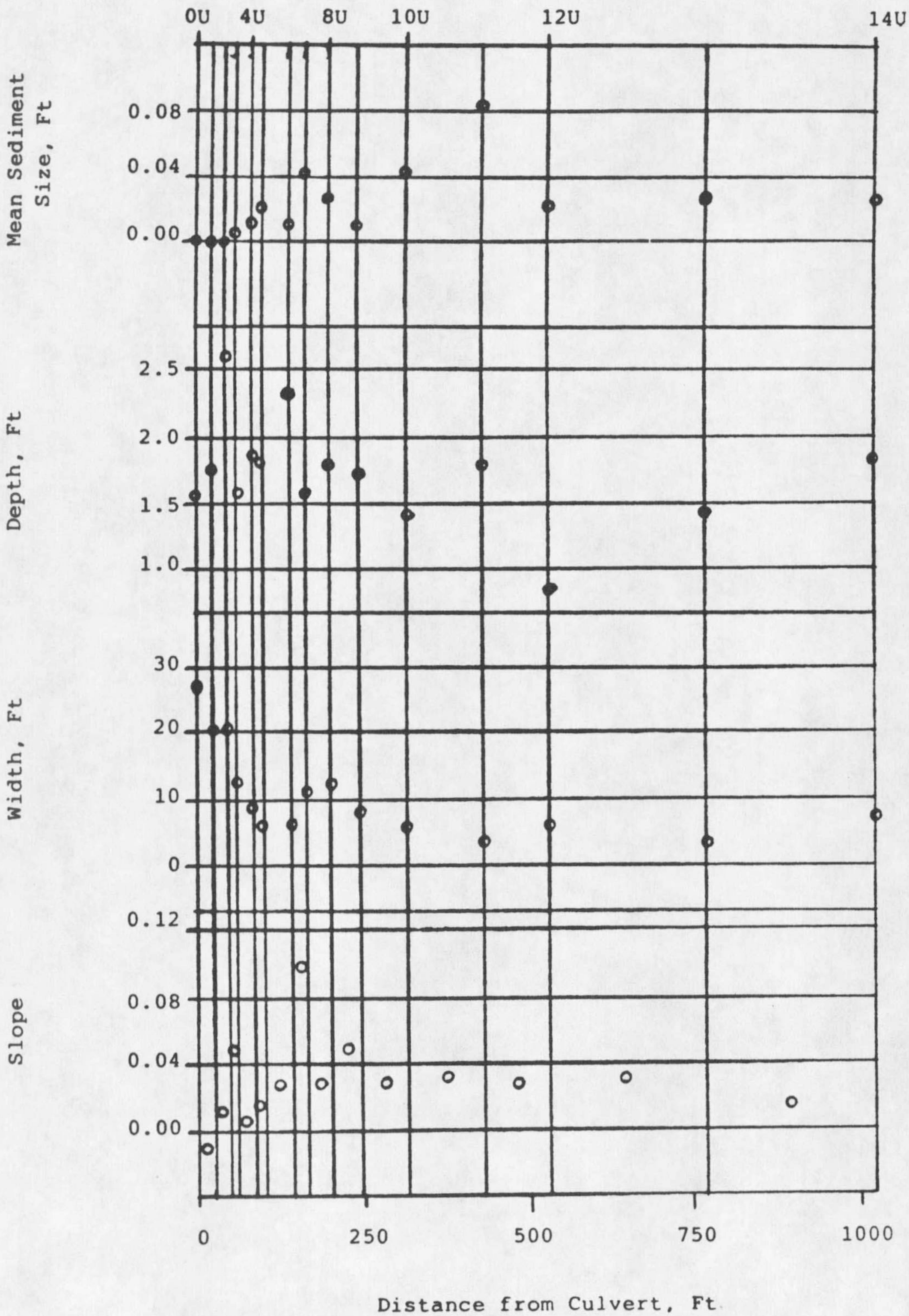

Figure 10. Copper River tributary, upstream, measured data.

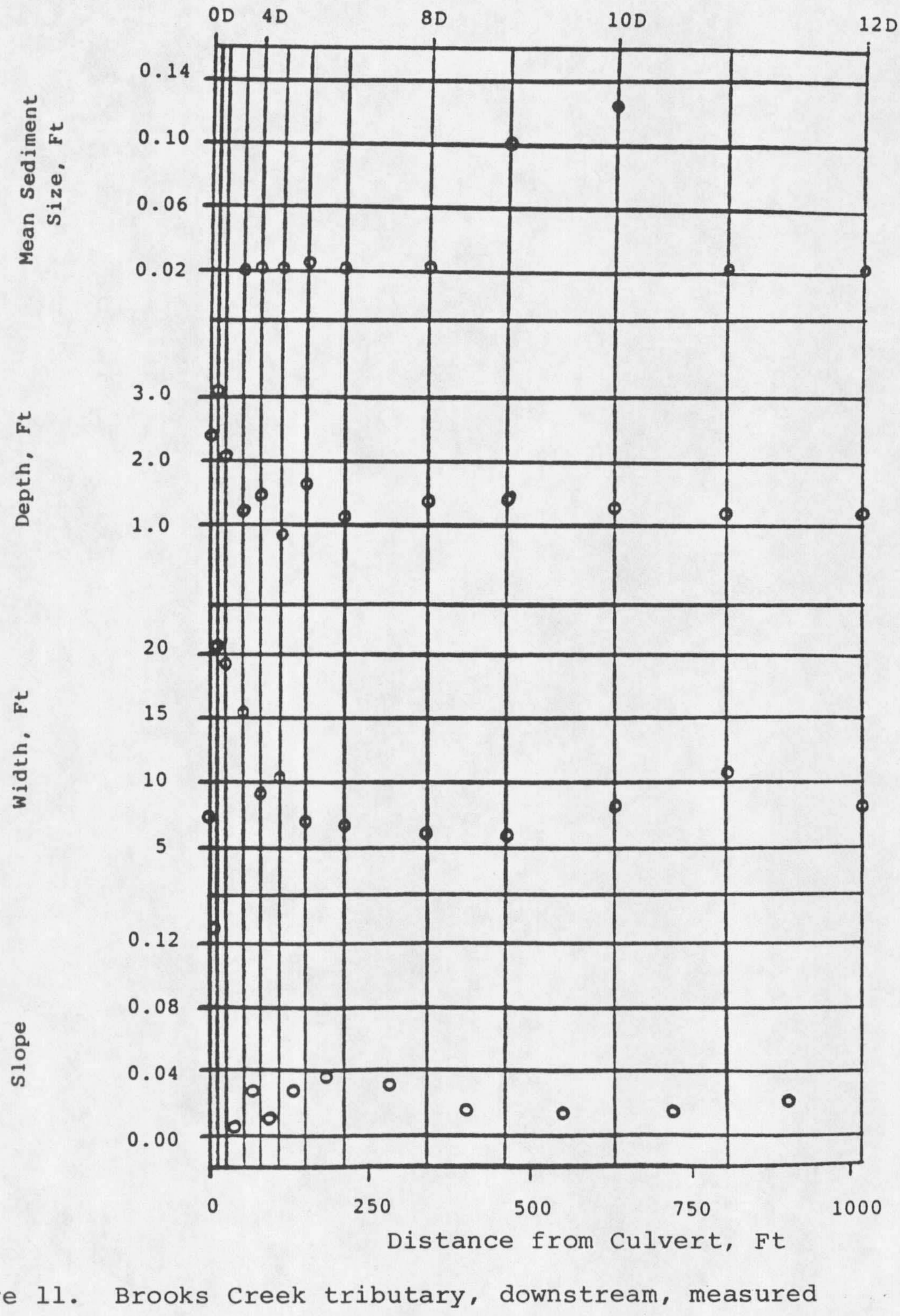

Figure 11. Brooks Creek tributary, downstream, measured data.

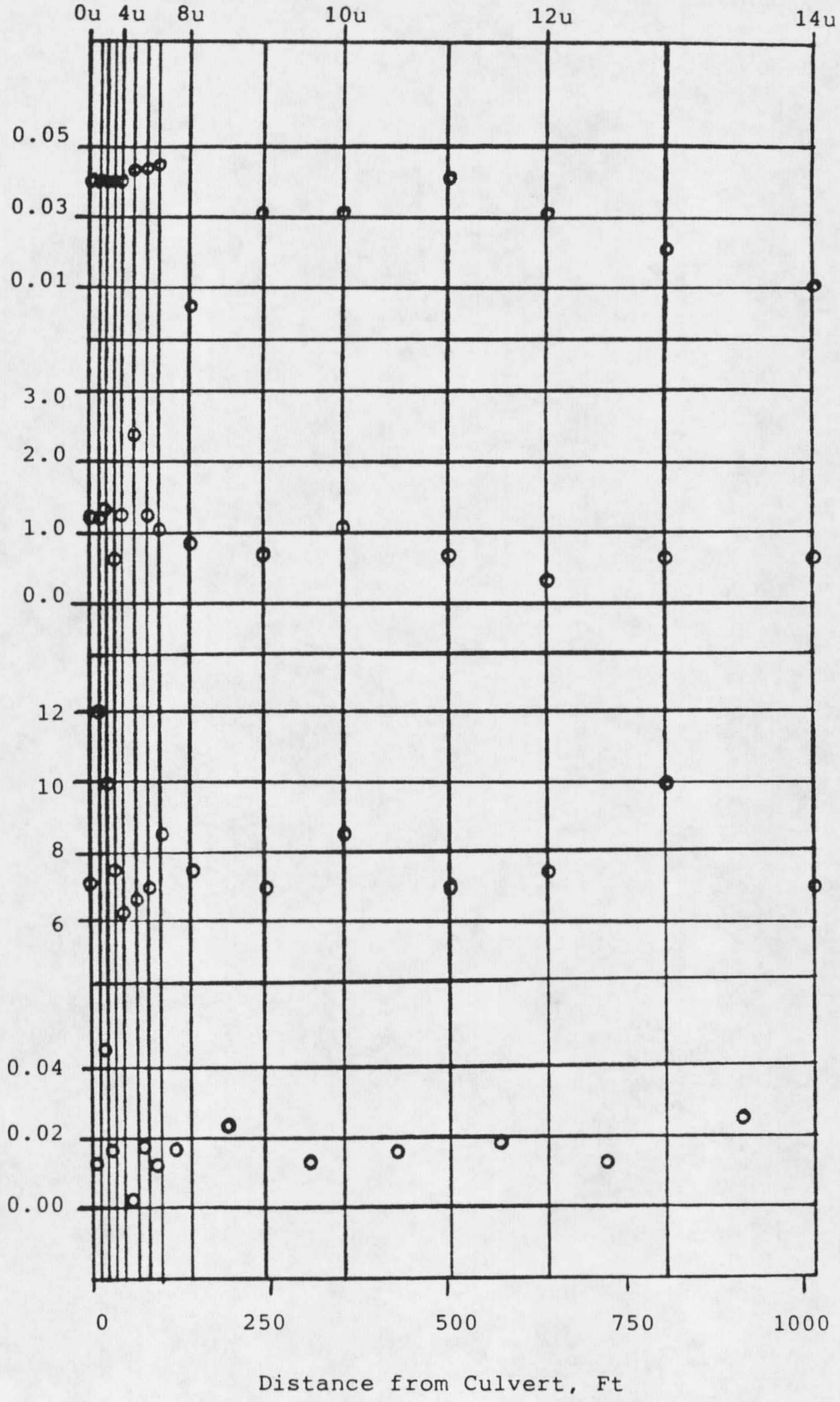

Figure 12. Brooks Creek tributary, upstream, measured data.

Table 16. Copper River tributary, bankfull Froude number.

Cross section	Froude number	Cross section	Froude number
0D	0.06	0U	0.12
1D	0.03	1U	0.14
2D	0.07	2U	0.07
3D	0.02	3U	0.24
4D	0.05	4U	0.29
5D	0.10	5U	0.42
6D	0.05	6U	0.29
7D	0.21	7U	0.29
8D	0.87	8U	0.23
9D	0.42	9U	0.37
10D	0.53	10U	0.72
11D	0.18	11U	0.70
12D	0.20	12U	1.49
		13U	1.05
		14U	0.35

Table 17. Brooks Creek tributary, bankfull Froude number.

Cross section	Froude number	Cross section	Froude number
0D	0.24	0U	0.70
1D	0.06	1U	0.42
2D	0.13	2U	0.41
3D	0.33	3U	1.82
4D	0.43	4U	0.74
5D	0.82	5U	0.27
6D	0.44	6U	0.70
7D	0.77	7U	0.76
8D	0.68	8U	1.00
9D	0.60	9U	1.67
10D	0.58	10U	0.72
11D	0.45	11U	1.63
12D	0.66	12U	3.68
		13U	1.19
		14U	1.74

Table 18. Cross sections locating break in measured data.

	Copper River tributary		Brooks Creek tributary	
Parameter	Upstream	Down-stream	Upstream	Down-stream
Width	3-4	7-8	2-3	5-6
Depth	NB	6-7	6-7	2-3
MSS	4-5	6-7	7-8	NB
Slope	4-5	NB	NB	4-5
Average	4-5	6-7	5-6	4-5

Note. MSS = mean sediment size. NB = no break in measured data.

APPENDIX F

FLOW CONDITIONS NEAR CULVERTS

This appendix estimates the pre-culvert equilibrium bank elevations and the post-culvert equilibrium bank elevations at stream cross section OU. These elevations are used in Table 3 to determine whether the dependent variables in subreaches altered by the culvert are adjusting toward equilibrium.

Table 19 estimates the pre-culvert equilibrium bank elevations at stream cross section OU. The measured bank elevation from the unaltered cross section nearest the culvert is extended to cross section OU by using the average measured thalweg slope from the subreaches unaltered by the culvert.

Table 20 shows that the normal depth in both culverts is less than critical depth at the bankfull discharge. For the Brooks Creek tributary, however, at bankfull discharge the tailwater depth above the culvert invert is greater than the critical depth. Does this force the culvert to flow subcritical? Table 20 also shows this stream flows sub-critical downstream of the culvert. Further, it shows that if at least part of the culvert flows supercritical, the sequent depth to the culvert normal depth, that is, the depth in the culvert after a hydraulic jump, is greater than the imposed tailwater depth. When the sequent depth is greater than the tailwater depth, the hydraulic jump occurs in the mild channel below the culvert (Chow, 1959). Table 20 shows for the Copper River tributary, the tailwater

Table 19. Estimated pre-culvert bank elevation, cross section OU.

		Copper River tributary		Brooks Creek tributary	
Item		Upstm	Dnstm	Upstm	Dnstm
Closest unaltered cross section	(a)	5U	7D	6U	5D
Bank elevation in (a), ft	(b)	101.91	83.40	102.31	95.08
Distance from (a) to OU, ft	(c)	95	417	83	198
Slope in un-altered subreaches	(d)	0.0267	0.0378	0.0184	0.0234
Estimated bank elev. at OU, ft	(e)	99.37	99.16	100.78	99.71
Average of (e), ft		99.27		100.25	

Table 20. Culvert control location at bankfull conditions.

Item	Copper R. tributary	Brooks Creek tributary
Critical depth, y_c ft[a]	1.43	1.74
Manning's coeff., n [a]	0.024	0.024
Normal depth, y_n, ft[a]	0.96	1.14
Stream WS, culvert exit, ft	91.62	97.16
Dnstm culvert inv. el., ft	91.67	94.90
Tailwater depth above culvert invert, ft	Below invert	2.26
Stream Fr dnstm culvert	-	0.24
Culvert Fr at y_n	2.23	2.27
Sequent depth, ft[b]	-	2.46

<u>Note</u>. Unapplicable data indicated by a -. WS = water surface elevation, inv. = invert, and Fr = Froude number. Critical depth in culvert = y_c. Normal depth in the culvert = y_n. [a] Calculated from Chow (1959). [b] Calculated from Straub (1978).

elevation is below the culvert invert at the bankfull discharge. Therefore, both of the study culverts flow supercritical at the bankfull discharge. Hence, the discharge through each culvert is controlled at the culvert entrance provided the following two conditions are met. First, flow must pass through critical at the culvert entrance. This requires subcritical flow in the stream just upstream of the culvert. Second, the culvert entrance must not be submerged. This requires the headwater depth over the top of the culvert to not exceed about half the culvert diameter.

The steady flow energy equation written between a cross section just upstream and just downstream of the culvert entrance is:

$$Z_s + d + V^2/2g = Z_c + y_c + V_c^2/2g + h_1 \qquad \text{(F-1)}$$

The above symbols are as follows:

Z_s = thalweg elevation in the stream at cross section OU,

d = depth in the stream at cross section OU,

V = velocity in the stream at cross section OU,

Z_c = culvert invert elevation,

y_c = critical depth in the culvert,

V_c = velocity in the culvert at critical flow, and

h_1 = 0.5 $V_c^2/2g$ = head loss at culvert entrance.

Using equation F-1, Table 21 presents the depth and the water surface elevation in each stream just upstream of the culvert. Table 21 shows for each depth found, the assumptions required for inlet control are true. That is, flow does pass through critical at the culvert entrance and the culvert entrance is unsubmerged.

Table 21. Estimated post-culvert bank elevation, cross section OU.

Item		Copper River tributary	Brooks Creek tributary
Discharge, cfs	(a)	37.6	39.6
Culvert dia., ft	(b)	9.5	5.0
Z_c, ft	(c)	100.00	100.00
y_c, ft	(d)	1.43	1.74
V_c, fps	(e)	5.64	6.47
Z_s, ft	(f)	99.60	100.08
V, fps	(g)	1.39/d	5.50/d
Depth, d, ft	(h)	2.57	2.57
WS at OU, ft	(f) + (h)	102.17	102.65
Elev. at top of upstm end of culvert, ft		109.50	105.00
Stream Froude no. upstm culvert		0.06	0.24

<u>Note</u>. Culvert invert elevation = Z_c. Critical depth in culvert = y_c. Velocity in culvert at critical flow = V_c. Thalweg elevation at OU = Z_s. Velocity at OU = V. Depth at OU = d. WS = water surface elevation.

APPENDIX G

SUMMARY OF THE MINRED METHOD

Preliminary

1. Input Q, the water discharge with a 1.5 year recurrence interval on the annual maximum series.

2. Input S_0, the measured thalweg slope between the upstream and downstream cross sections.

3. Input the measured distance between the upstream and downstream cross sections.

4. Input the stream width, b, measured at the downstream cross section.

5. Input the mean sediment size, d_s, measured at the downstream cross section.

6. Calculate the Manning roughness coefficient. Chow (1959) indicates that $n_0 = 0.0342 d_s^{1/6}$. For the streams under study, it is estimated from Chow (1959) that (a) $n_1 =$ 0.010 (b) $n_2 = 0.005$, (c) $n_3 = 0.000$, (d) $n_4 = 0.005$, and (e) $m_5 = 1.15$. Therefore, from equation 4-1, the Manning roughness coefficient, $n = (0.0342 d_s^{1/6} + 0.02)(1.15)$.

7. If D loop has never been executed, go to item 10.

8. If the existing value of n is similar to the value of n at last exit from D loop, go to item 40.

9. Go to item 20.

10. Input the stream depth, d, measured at the downstream cross section.

D Loop

11. Calculate the cross sectional area of flow, A=bd.

12. Calculate the wetted perimeter, P_w=b+2d.

13. Calculate the hydraulic radius, R=A/P_w.

14. Calculate the magnitude of the mean velocity of flow, V=Q/A.

15. If this is not the first time through D loop, go to item 20.

16. Calculate the specific energy at the downstream cross section, $E_2=d+V^2/(2g)$.

17. Calculate the slope of the energy grade line at the downstream cross section, $S_2 = [Vn/(1.49R^{0.67})]^2$.

18. Modify d.

19. Go to item 11.

20. Calculate the specific energy at the upstream cross section, $E_1=d+V^2/(2g)$.

21. Calculate the slope of the energy grade line at the upstream cross section, $S_1 = [Vn/(1.49R^{0.67})]^2$.

22. Calculate the distance between the upstream and downstream cross sections,

$X = (E_2 - E_1)/\{S_0 - [(S_1 + S_2)/2]\}$.

23. If the measured distance between the upstream and downstream cross sections is not similar to the calculated distance, go to item 18.

24. If DS loop has never been executed, go to item 28.

25. If the current value of d is different from the value of d at last exit from DS loop, go to item 28.

26. If B loop has never been executed, go to item 40.

27. If the current value of b is the same as the value of b at last exit from DS loop, go to item 40.

DS Loop

28. Calculate the shear velocity, $U_* = (gRS_1)^{0.5}$.

29. Calculate the boundary Reynolds number, $Re_* = U_* d_s/\nu$.

30. Enter Shields diagram with the value of Re_* to determine the critical value of the dimensionless shear stress, τ_{*c} .

31. Calculate the critical value of the bed shear stress, $\tau_* = \tau_{*c}(\gamma_s - \gamma)d_s$.

32. Calculate the critical value of the shear velocity, $U_{*c} = (\tau_c/\rho)^{0.5}$.

33. Calculate the dimensionless shear stress, $\tau_* = U_*^2\rho/[(\gamma_s - \gamma)d_s]$.

34. Enter Raudkivi's graphical relation with the values of τ_* , d_s, U_*, and U_{*c} to determine the critical value of the mean velocity of flow, V_{cr}.

35. If V calculated in D loop is not similar to V_{cr} calculated in DS loop, go to item 38.

36. If D loop has been executed after the last modification of d_s, go to item 40.

37. Go to item 6.

38. Modify d_s.

39. Go to item 29.

B Loop

40. Modify b.

41. If this is the eleventh time in this series through B loop, go to item 43.

42. Go to item 11.

43. Find the minimum calculated value of S_1 and the associated values of d, d_s, and b.

44. If (a) the last calculated value of S_1 is not the minimum found and (b) this is not the first series through B loop, go to item 46.

45. Start a new series through B loop, using the values of d, d_s, and b associated with the minimum value of S_1 and go to item 11.

46. Write the initial and final values.

47. End program execution.

APPENDIX H

EQUATIONS FOR SHIELDS DIAGRAM AND RAUDKIVI'S GRAPHICAL RELATION

Shields Diagram

1. $0.3 \leq Re_{*} < 1.0$ (H-1)

$$\tau_{*c} = 1.01000 - 3.0667\, Re_{*} + 3.8333\, Re_{*}^{2} - 1.6667\, Re_{*}^{3}$$

$r^2 = 1.0000$

2. $1.0 \leq Re_{*} < 7.0$ (H-2)

$$\tau_{*c} = 0.2527 - 0.2204\, Re_{*} + 9.7082 \times 10^{-2} Re_{*}^{2} - 2.1774 \times 10^{-2}\, Re_{*}^{3} + 2.4090 \times 10^{-3}\, Re_{*}^{4} - 1.0416 \times 10^{-4}\, Re_{*}^{5}$$

$r^2 = .9995$

3. $7.0 \leq Re_{*} \leq 10.0$ (H-3)

$\tau_{*c} = 0.032$

4. $10.0 < Re_* < 500.0$ (H-4)

$$\tau_{*c} = 2.9282 \times 10^{-2} + 2.5119 \times 10^{-4} Re_* - 1.0533 \times 10^{-6} Re_*^2 + 2.2796 \times 10^{-9} Re_*^3 - 1.8652 \times 10^{-12} Re_*^4$$

$r^2 = .9979$

5. $Re_* \geq 500.0$ (H-5)

$\tau_{*c} = 0.0600$

Raudkivi's Graphical Relation

1. Gravel:

a. $0.0325 \leq \tau_* \leq 0.081$ (H-6)

$$V_{cr} = (1.8125 \times 10^3 - 1.4854 \times 10^5 \tau_* + 4.9529 \times 10^6 \tau_*^2 - 8.2270 \times 10^7 \tau_*^3 + 6.7710 \times 10^8 \tau_*^4 + 2.2050 \times 10^9 \tau_*^5)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9948$

b. $0.081 < \tau_* \leq 0.14$ (H-7)

$$V_{cr} = (53.1301 - 9.7323 \times 10^2 \tau_* + 7.8340 \times 10^3 \tau_*^2 - 2.1152 \times 10^4 \tau_*^3)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9944$

c. $0.14 < \tau_* \leq 0.51$ (H-8)

$$V_{cr} = (13.8092 - 20.9980\tau_* + 88.5439\tau_*^2 - 58.4911\tau_*^3)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9964$

2. Sand

a. $0.0325 \leq \tau_* \leq 0.081$

Equation H-6

b. $0.081 < \tau_* \leq 0.20$ (H-9)

$$V_{cr} = (31.1591 - 3.1520 \times 10^2 \tau_* + 1.5535 \times 10^3 \tau_*^2 - 2.6422 \times 10^3 \tau_*^3)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9982$

c. $0.20 < \tau_* \leq 0.325$ (H-10)

$$V_{cr} = (15.2180 - 50.9079\tau_* + 1.1585 \times 10^2 \tau_*^2 - 70.0097\tau_*^3)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9916$

d. $0.325 < \tau_* \leq 0.70$ (H-11)

$$V_{cr} = (7.9830 + 8.0064\tau_* - 36.7524\tau_*^2 + 53.8419\tau_*^3)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9973$

e. $0.70 < \tau_* \leq 1.10$ (H-12)

$$V_{cr} = (-1.3042 \times 10^2 + 4.4408 \times 10^2 \tau_* - 4.3875 \times 10^2 \tau_*^2 + 1.4198 \times 10^2 \tau_*^3)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9775$

3. **Fine Sand, Lower Flow Regime**

a. $0.0325 \leq \tau_* \leq 0.081$

Equation H-6

b. $0.081 < \tau_* \leq 0.20$

Equation H-9

c. $0.20 < \tau_* \leq 0.95$ (H-13)

$$V_{cr} = (11.6343 - 18.8550\,\tau_* + 28.5969\,\tau_*^2 - 14.4928\,\tau_*^3)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9667$

4. Fine Sand, Upper Flow Regime (H-14)

$0.575 \leq \tau_* \leq 3.70$

$$V_{cr} = (25.0635 + 9.5466\tau_* - 28.6237\tau_*^2 + 18.2024\tau_*^3 - 4.6962\tau_*^4 + 0.4356\tau_*^5)(U_*^2 - U_{*c}^2)^{\frac{1}{2}}$$

$r^2 = .9964$

APPENDIX I

CALCULATED VALUES

Table 22. Copper River tributary, downstream, calculated data.

Cross section	Width, ft	Depth, ft	MSS, ft	EGL slope
3D	26.93	2.40	0.0048	0.0000880
4D	26.61	2.33	0.0042	0.0000999
5D	27.08	2.62	0.0041	0.0000663
6D	58.91	1.14	0.0043	0.0001880
7D	-	-	-	-
8D	13.08	0.64	0.3845	0.0606194
9D	6.18	1.12	0.4531	0.057848
10D	11.21	0.72	0.3305	0.054898
11D	9.85	0.87	0.3600	0.041153

Note. MSS = mean sediment size. EGL = energy grade line. No calculated data indicated by a -.

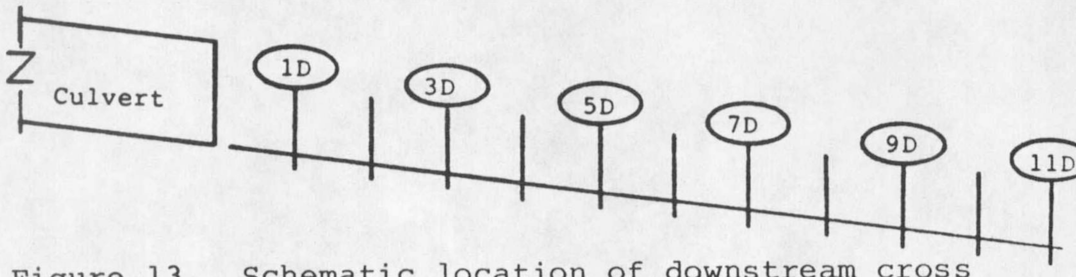

Figure 13. Schematic location of downstream cross sections.

Table 23. Copper River tributary, upstream, calculated data.

Cross section	Width, ft	Depth, ft	MSS, ft	EGL slope
11U	6.77	1.27	0.2082	0.0284891
10U	68.97	0.96	0.0047	0.0002418
9U	73.37	1.00	0.0039	0.0001855
8U	56.80	1.13	0.0048	0.0002119
7U	41.30	1.62	0.0039	0.0001237
6U	53.37	1.27	0.0041	0.0001611
5U	-	-	-	-
4U	55.88	1.59	0.0008	0.0000567
3U	33.50	2.36	0.0007	0.0000459
2U	61.85	1.58	0.0007	0.0000473

Note. MSS = mean sediment size. EGL = energy grade line. No calculated data indicated by a -.

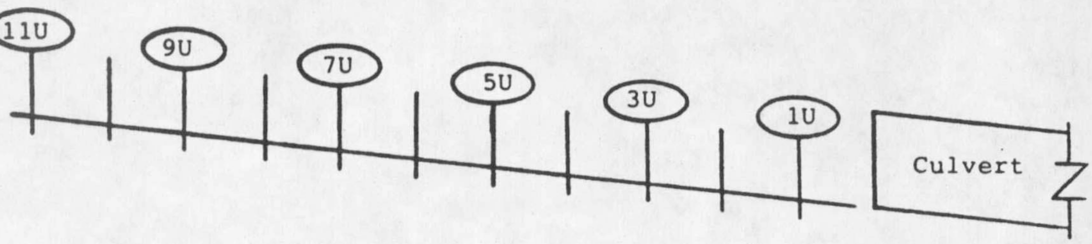

Figure 14. Schematic location of upstream cross sections.

Table 24. Brooks Creek tributary, downstream, calculated data.

Cross section	Width, ft	Depth, ft	MSS, ft	EGL slope
3D	9.00	1.13	0.1082	0.0147986
4D	–	–	–	–
5D	46.39	1.16	0.0068	0.0003389
6D	8.62	1.02	0.2895	0.0373274
7D	7.83	1.14	0.2669	0.0322400
8D	8.90	1.38	0.1099	0.0109072
9D	9.40	1.22	0.1315	0.0152865
10D	14.77	0.77	0.1765	0.0239626
11D	10.19	1.01	0.2104	0.0240062

Note. MSS = mean sediment size. EGL = energy grade line. No calculated data indicated by a –.

Table 25. Brooks Creek tributary, upstream, calculated data.

Cross section	Width, ft	Depth, ft	MSS, ft	EGL slope
8U	48.63	1.23	0.0056	0.0002491
7U	-	-	-	-
6U	-	-	-	-
5U	-	-	-	-
4U	-	-	-	-
3U	-	-	-	-
2U	61.96	0.96	0.0063	0.0003537

Note. MSS = mean sediment size. EGL = energy grade line. No calculated data indicated by a -.