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ABSTRACT

Data envelopment analysis (DEA) is an extensively used method to estimate capacity and technical and economic efficiency of fishing vessels. However, DEA is often criticized because of the influence that outliers, or noisy data, can have on the DEA estimates. Recently, quantile data envelopment analysis (QDEA) was introduced to identify and address issues caused by influential data outliers. QDEA endogenously identifies potential outliers and eliminates them from a given observation's DEA reference set. In this study, we utilize QDEA to estimate fishing fleet capacity for vessels operating in the northwest Atlantic Ocean during 2019. We present methods for implementing the QDEA model that we think are practical and can be adapted for fishing fleets worldwide. Results show lower capacity estimates using the QDEA model than what the standard capacity model would yield. Our results are quite encouraging in terms of utilizing the QDEA model in future work.

Key words: Data envelopment analysis, fishing capacity, quantile DEA.
JEL codes: C61, D24, Q22.

INTRODUCTION

Fishing capacity is the ability of fishing vessels and fleets to harvest fish, and has been of interest to fisheries researchers and policy makers since the late 1990s. Efforts undertaken by the United Nations Food and Agriculture Organization (FAO) led to seminal publications on both the management and the measurement of fishing capacity (FAO 1998, 2000). This started a more than decade-long effort by various researchers to measure fishing capacity worldwide (Kirkley et al. 2004; Felthoven and Morrison Paul 2004; Pascoe and Greboval 2003; Pascoe et al. 2012; Walden, Kirkley, and Färe 2010; Walden, Kirkley, and Kitts 2003; Terry, Walden, and Kirkley 2008; Färe, Kirkley, and Walden 2011; Kirkley et al. 2001). Most of the capacity studies cited above adopted a technical (primal) concept of capacity proposed by Johansen (1968, 50, 57), which is the "maximum amount that can be produced per unit time with existing plant and equipment provided the availability of variable factors of production are not limited." This definition formed the basis of the US government definition of "harvesting capacity," which was the "maximum amount of fish that the fishing fleets could have reasonably expected to catch or land during the year

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under the normal and realistic operating conditions of each vessel, fully utilizing the machinery and equipment in place, and given the technology, the availability and skill of skippers and crew, the abundance of the stocks of fish, some or all fishery regulations, and other relevant constraints" (Terry, Walden, and Kirkley 2008, 15). Fishing vessel capacity based on this physical output notion has primarily been estimated with data envelopment analysis (DEA), a method developed in the operations research literature using linear programming (LP) techniques, which were originally operationalized by Färe, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989). Work utilizing DEA to measure fishing capacity has expanded beyond the simple measurement and tackled issues such as buyouts and fleet restructuring (Walden, Kirkley, and Kitts 2003; Kirkley, Walden, and Waters 2004, 2007; Thunberg, Kitts, and Walden 2007; Kerstens, Squires, and Vestergaard 2005; Kerstens, Vestergaard, and Squires 2006). Clearly, when answering policy questions such as how to best buy out fishing vessels or restructure a fleet, DEA estimates of fishing capacity must be as accurate as possible and be able to properly rank vessels by their estimated capacity.

DEA models that estimate fishing capacity based on the Johansen (1968) capacity definition seek to expand a vessel's outputs given their fixed factors of production, such as length, horsepower, gross tonnage, or other relevant vessel attributes.[1] Thus, it is a measure of technical efficiency based on only fixed factors of production (i.e., subvector efficiency). The most efficient observations based on output and fixed inputs determine the frontier and have a DEA value of 1, and observations that are interior to the frontier have a DEA value greater than 1. For those observations with a value greater than 1, the DEA score is showing by how much output would need to be scaled up proportionately for the observation to be on the efficient production frontier (Färe, Grosskopf, and Lovell 1994). For example, a value of 1.10 means all outputs would need to be scaled up by 10%. In a multispecies fishery, the 10% increase applies to all species. Some types of DEA models relax the requirement that all outputs be expanded proportionately and allow some outputs to be expanded while others are not allowed to expand using a model based on directional distance functions (Scheld and Walden 2018). Finally, we note that this technical definition based on Johansen (1968) is not the same as an economic definition of capacity based on the cost function (Kerstens, Sadeghi, and Van de Woestyne 2019; Ray 2015; Ray, Walden, and Chen 2021). However, it is attractive to use because it is easily calculated in situations where input price data are missing, or when data are limited (Kirkley et al. 2001).

Vessel catches in a fishery can be highly variable and influenced by external factors such as weather, oceanographic conditions, and fish behavior. Random events can lead to a particularly good trip, or tow, which then influences the estimation of the position of the production frontier and the resulting capacity estimates. Simply put, when a rare event serves as a benchmark for the other observations, the resulting DEA capacity estimate will be higher than what would occur in the absence of the rare event. The sensitivity of DEA scores to outliers and extreme values is widely recognized in the broader DEA literature, leading to the development of methods to detect them prior to estimating DEA models (Simar 2003; Wilson 1993, 1995). However, even if all outliers were perfectly identified and removed from the analysis, DEA scores are still biased by construction. Output-oriented models are biased downward and input-oriented models are biased upward (Simar and Wilson 2008). This problem can be easily corrected by bias-correcting

1. Generally, DEA models can be constructed to contract inputs (input orientation), expand outputs (output orientation), or both expand outputs and contract inputs (i.e., graph efficiency, directional distance function).

the DEA values using bootstrap methods (Simar and Wilson 1998, 2000a, 2000b; Walden 2006). The problem of outliers and extreme values in fishing vessel capacity and efficiency studies is the more difficult problem to overcome and has been recognized as an issue dating back to the work of Holland and Lee (2002).

In the broader DEA literature, the outlier question has been tackled by several different authors and is still the subject of ongoing research (Khezrimotlagh, Cook, and Zhu 2020; Chen and Johnson 2010). Alternative frontier estimators have been developed that reduce the influence of outliers based on the free disposal hull, specifically the order-m and order-α estimators (Cazals, Florens, and Simar 2002; Daraio and Simar 2005, 2006). However, in fishing capacity studies, there have been few examples using these alternate estimators, and many of the methods used to address the outlier issue have been ad hoc in nature (Walden and Tomberlin 2010; Terry, Walden, and Kirkley 2008). For example, in a US government report on commercial fishing fleet capacity, trips were often stratified into calendar year quarters to help "reduce the impact of unobserved factors that explain differences in catches among trips" (Terry, Walden, and Kirkley 2008, 21). Additionally, a lower "capacity" estimate was generated by adding the difference between capacity output and technically efficient output to the observed catch. Both of these were done in order to lower the influence of outlier levels of catch on the capacity estimate.

Recently, Atwood and Shaik (2020) proposed quantile DEA (QDEA) as a method to account for outliers in a DEA model. Their methods allow up to a certain predetermined percentage of endogenously identified observations to lie external to an individual observation's DEA frontier, thus eliminating the "external" observations' influence on the efficiency score. In an output-oriented DEA model, such as one used to estimate fishing vessel capacity, this will result in a lower estimate of potential output expansion for an observation that now lies interior to a new set of reference observations. The work by Atwood and Shaik (2020) initially used a small eight-firm example, with one output and one input assuming constant returns to scale, to demonstrate the QDEA approach. In this manuscript, we seek to demonstrate how QDEA can be used to estimate capacity for a multiple-input, multiple-output fishing fleet. We compare the QDEA capacity estimates with those derived from a standard directional distance function model to determine whether the QDEA model changes our capacity assessment regarding overcapacity. QDEA requires the analyst to make two key decisions: how many observations are allowed to lie external to the frontier, and what capacity value to assign to those observations. We demonstrate an approach for answering these two questions that is easy to implement. This is the first use of QDEA we are aware of for estimating capacity in any setting, and perhaps more importantly, for a problem that is real and has practical policy ramifications.

Our findings after using QDEA show that QDEA capacity levels for a fishing fleet operating in 2019 are less than what a standard DEA capacity model would yield. Our approach uses the R software language and a recently developed R package.[2] We outline our methods for applying the QDEA model in the next section, and then show results. Based on our experience, we believe that the QDEA methods we utilize can be applied to most DEA problems. More importantly, QDEA gives economists another method to estimate fishing capacity. We conclude that when the production environment is characterized by high levels of noisy data, as in most fisheries, QDEA limits the influence of outliers and yields a more conservative estimate of fishing capacity.

2. An R package implementing QDEA is available on request from the authors.

METHODS

ESTIMATING VESSEL CAPACITY USING DEA

Färe, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989) first operationalized the Johansen capacity measure by utilizing an output-oriented DEA model where only the fixed factors constrain outputs. Given j observations producing outputs y ($y = 1, \ldots, M$) with inputs x ($x = 1, \ldots, N$) where the x vector is partitioned into fixed (F_x) and variable inputs (V_x), the LP model used to estimate Johansen definition of capacity with variable returns to scale is

$$Max_{\theta,z,}\theta, \tag{1a}$$

s.t.

$$\sum_{j=1}^{J} z_j y_{jm} \geq \theta y'_{jm}, m = 1, 2, \ldots, M; \tag{1b}$$

$$\sum_{j=1}^{J} z_j x_{jn} \leq x'_{jn}, n \in F_x; \tag{1c}$$

$$\sum_{j=1}^{J} z_j = 1; \tag{1d}$$

$$z_j \geq 0. \tag{1e}$$

Note that the only input constraint is for the fixed inputs (equation 1c). The value of theta returned by the model is a radial efficiency measure yielding the proportional expansion of each output needed in order to reach the production frontier. A value of 1 means that the observation in question is operating on the frontier and no output expansion is possible. A value greater than 1 means that all outputs will need to be expanded by the objective function value in order for the observation to reach the frontier. The optimal level of each variable input needed to produce at capacity is calculated as $\Sigma_{j=1}^{J} z_j^{*} x_{jv}$, where $z_j^{*}$is the optimal value of peer j determined by the model. One or more of the fixed input constraints (equation 1c) will bind the system. Equation 1d is a convexity constraint that imposes variable returns to scale. Dropping the constraint imposes constant returns to scale. Setting the sign on constraint 1d to $\leq$ 1imposes nonincreasing returns to scale on the model.

The DEA model shown above has been widely applied to measure capacity in a variety of industries (Kerstens, Sadeghi, and Van de Woestyne 2019). However, a more general framework using a directional distance function (DDF), rather than the radial model above, can be constructed. Doing so grants additional possibilities for capacity measurement. In order to show this possibility, we begin with the directional technology (DDF) distance function defined as in Färe and Grosskopf (2006):

$$\overrightarrow{D_T}\left(x, y; g_x, g_y\right) = sup\left\{\beta : \left(x - \beta g_x, y + \beta g_y\right) \in T\right\}, \tag{2}$$

where g_x and g_y are nonnegative "movement directions", β is a scalar, and T is the feasible technology set. The Johansen capacity model shown in equation 1 is based on the radial output distance function, while the DDF version we seek uses the directional output distance function. In order to derive the directional output distance function from the directional technology distance function, the directional vector for the inputs is set equal to 0 (Färe and Grosskopf 2006):

$$\vec{D}_O(x, y; g_y) = sup\{\beta : (y + \beta g_y) \in P(x)\}, \tag{3}$$

where $P(x)$ is the efficient set of output vectors. The Johansen capacity model based on the directional output distance function (DDF) with variable returns to scale is shown in equations 4a–4e below.

$$Max_{Z,\beta}\beta, \tag{4a}$$

s.t.

$$\sum_{j=1}^{J} z_j y_{jm} \geq y'_{jm} + \beta g_y, m = 1, 2, \ldots, M; \tag{4b}$$

$$\sum_{j=1}^{J} z_j x_{jn} \leq x'_{jn}, n \in F_x; \tag{4c}$$

$$\sum_{j=1}^{J} z_j = 1; \tag{4d}$$

$$z_j \geq 0. \tag{4e}$$

A value of 0 for beta ($\beta^* = 0$, where β^* is the optimal value of β returned by the LP model) means the observation is considered to be "efficient" and no output expansion is possible, while $\beta^* > 0$ indicates the observation is "inefficient" and $\beta^* < 0$ indicates the observation is "super-efficient" (a concept discussed in more detail below). If the directional vector g_y is set equal to the observed output, the value $(1 + \beta^*)$ is equivalent to the radial measure in the Johansen model shown above, and the results can be used in a multiplicative expansion of the outputs (Färe and Grosskopf 2006). Alternatively, if we set the directional vector equal to 1, the DDF model has an additive structure in that a value of β^* greater than 0 indicates that β^* can be directly added to the observed output to yield the frontier level of output (i.e., capacity). As in model 1, the optimal levels of the variable inputs can be calculated by $\Sigma_{j=1}^{J} z_j^* x_{jv}$.

DEA models, including the capacity models discussed above, are known to produce efficiency estimates that are biased by their makeup (Simar and Wilson 1998, 2008). One method to bias-correct the efficiency scores is through the use of a bootstrap (Simar 1992; Simar and Wilson 1998, 2000a, 2000b, 2011). In addition to bias-correcting the efficiency scores, bootstrapping also allows for construction of confidence intervals and hypothesis testing. Ray (2004) points out that it may be more meaningful to present results in terms of confidence intervals rather than bias-corrected point estimates. It is clear from numerous studies that the ability to bootstrap DEA models yields better information for managers than a simple point estimate. For this study, we utilized subsample bootstrapping procedures where the subsample size is determined utilizing algorithm 6.1 from Politis, Romano, and Wolf (2001). Their algorithm requires using a range of possible subsample sizes m_i, computing bootstrapped parameter confidence intervals for each subsample size m_i, and selecting the subsample size $\hat{m}_i$ that minimizes the volatility of the confidence interval estimates. Simar and Wilson (2011) present simulation-based evidence that shows using this procedure to endogenously select the subsample size $\hat{m}_i$ provided good nominal coverage levels when bootstrapping DEA-based statistics.

QUANTILE DEA

Quantile DEA is a method to identify sets of one or more influential observations and reestimate DEA efficiency scores, while excluding the identified influential observations. It is well known that DEA is sensitive to outliers in the data, and numerous individuals have tried to deal with this issue in a variety of ways (Bogetoft and Otto 2010; Cazals, Florens, and Simar 2002; Holland and Lee 2002; Simar and Wilson 1998; Walden and Tomberlin 2010). We note here that the influence of outliers introduces a different type of bias than that discussed above. If there are no outliers in the data, the DEA scores are still biased, but that bias can be corrected. Although the sensitivity of results to outliers in not unique to DEA, the number of studies that propose methods to account for the outlier issue in DEA suggests that it is a problem that needs serious consideration (Atwood and Shaik 2020). Other suggested methods include the order-m and order-α frontiers, which are partial frontier approaches (Cazals, Florens, and Simar 2002; Daraio and Simar 2005, 2006). Quantile DEA is similar in concept to the order-α approach and allows one to directly identify observations that are most influencing a given observation's DEA estimate. QDEA is a two-stage method, where a set of influential observations or "outliers" are identified in the first stage, and then the DEA model is reestimated with the influential observations removed from the reference set.

QDEA's deletion of points from an observation's reference set introduces the potential for several complications, including the possibility of "super-efficiency" and unbounded dual infeasible primal linear programs when estimated with a variable returns to scale (VRS) technology. In order to first explain the QDEA method, we present a small example using Cooper, Seiford, and Tone's (2007, 3) one-input, one-output example. Figure 1 plots CST's input-output data, the VRS DEA frontier, and potential VRS QDEA frontiers when one data observation is allowed to lie external to a given observation's DEA frontier.

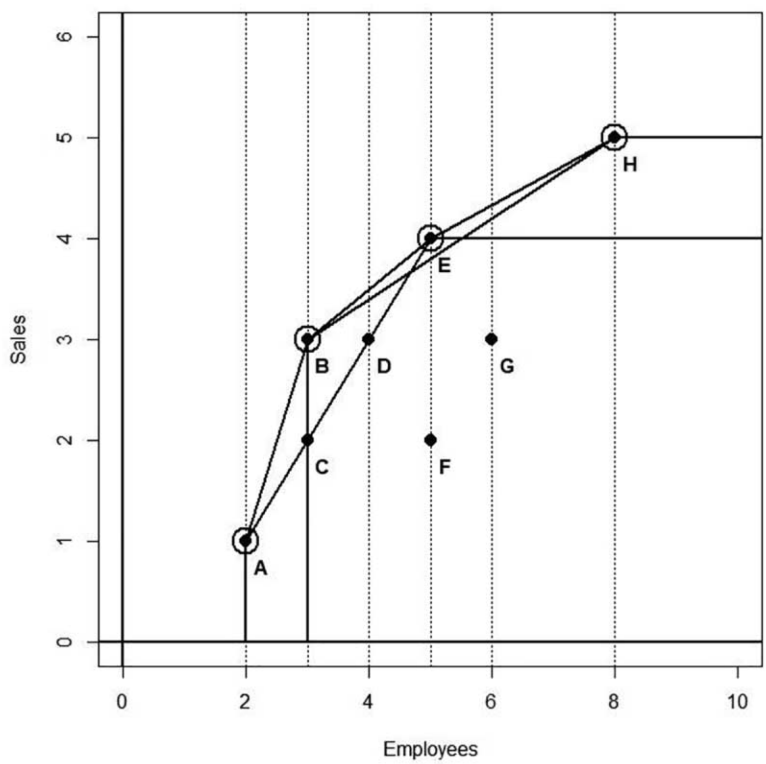

Figure 1. Cooper, Seiford, and Tone Example Data with Variable Returns to Scale

In this example, the number of employees is the input, and sales the output. The vertical dotted lines in figure 1 indicate potential output movements from each observation's initial input-output position while restricting their input movement to 0. The original output-oriented DEA model assumes that all observations are in the reference set used to estimate the technology and output is limited by the observations defining the frontier, which are A, B, E, and H. The DEA problem can be visualized as attempting to move as far as possible upward from a given observation's initial point (x_0, y_0) while requiring that the resulting "projected point" $(x_0, \hat{y}_0) = (x_0, y_0 + \beta g_y)$ to lie on or below the hyperplanes defining the A-B-E-H frontier. The circled points in figure 1 indicate that points A, B, E, and H are "DEA efficient" in that no further upward movement is feasible for those points. The remaining points C, D, F, and G are "inefficient," as they lie below the frontier traced out by the points A, B, E, and H. We adopt a directional distance function (DDF) model to determine projected output for each observation (i.e., the projected point for the output $\hat{y}_0$ is determined by $(x_0, \hat{y}_0) = (x_0, y_0 + \beta g_y)$). Setting the directional vector (g_y) equal to 1 and solving the DDF model results in an objective function value (beta) of 1.0 for point C, and an efficient output (sales) value of 3.0 (table 1). For point D, the objective function value returned is 0.5, yielding an efficient output of 3.5. Efficient output for point F is 4.0 (objective function value is 2.0), and for point G efficient output is 4.33 (objective function value is 1.33). As can be seen in figure 1, point C is being projected upward to point B, point D is projected to a point lying between points B and E, point F is projected to point E, and point G to a point lying between points E and H.[3] Points A, B, E, and H will all have an objective function value of 0 returned, meaning they lie on the efficient frontier.

We now allow up to one point to lie external to a given observation's DEA frontier. Using the DDF model's dual linear program, QDEA can be visually interpreted as attempting to find a point that, if deleted from the observation's reference set, results in the observation's new projected point lying as "far below" its original DEA projected point as possible while also requiring that the new projected point lie on or "above and to left" of a separating hyperplane enveloping all remaining reference points on or "below and to the right" of the hyperplane.[4] QDEA's associated primal problem is a traditional distance-maximizing primal linear program but with points eliminated from the given observation's original reference set. Since each QDEA problem is observation specific, we find that with VRS the omitted data points will commonly differ by observation and some QDEA β_j distances will be negative or negative unbounded.[5]

We first consider the QDEA results for the "inefficient" points C, D, F, and G. For points C and D, the QDEA model endogenously allows point B to be excluded, resulting in a revised DEA bounding hyperplane defined by any two of the points A, C, D, or E. Points C and D become "QDEA efficient" and the QDEA distance metrics reduce to $\beta_C = \beta_D = 0$ (table 1). For point F, the QDEA model chooses to delete point E, its revised bounding hyperplane is defined by points B

3. More accurately, points can be projected upward to points on hyperplanes containing one or more of the "DEA efficient" points. For example, point G can be projected upward until its projected point lies on the hyperplane containing points E and H.

4. A more formal and rigorous statement and proof of this statement is readily obtained using duality theory and Farkas' lemma.

5. For a complete discussion of the QDEA algorithm see Atwood and Shaik (2020). QDEA uses a variant of the stochastically constrained LP models discussed by Atwood et al. (1988) and Rockafellar and Uryasev (2000) (hereafter, AWHH and RU, respectively). These models allow up to a user-specified proportion of the LP's constraints to be endogenously identified and violated within the LP model. Since the dual DEA model has a constraint for each observation in the LP problem, the AWHH and RU procedures can endogenously identify one or more influential constraints that can be deleted from the given observation's reference set.

Table 1. DDF and QDEA Results Using Cooper, Seiford, and Tone Example Data

Point	Sales, Y	Employees, X	DDF Change (units)	Projected Y Value	QDEA Point Out	QDEA Change (units)	Projected QDEA Y Value
A	1	2	0.0	1.0	A	$-\infty$	unbounded
B	3	3	0.0	3.0	B	–1.0	2.0
C	2	3	1.0	3.0	B	0.0	2.0
D	3	4	0.5	3.5	B	0.0	3.0
E	4	5	0.0	4.0	E	–0.2	3.8
F	2	5	2.0	4.0	E	1.8	3.8
G	3	6	1.33	4.33	H	1.0	4.0
H	5	8	0.0	5.0	H	–1.0	4.0

Note: DDF, directional distance function. QDEA, quantile data envelopment analysis.

and H, and its revised β_F distance metric is 1.8. The projected value for point F is 3.8, which lies on the line segment BH, and is below point E. For point G, the QDEA model chooses to delete point H, the bounding hyperplane becomes the horizontal segment beginning at point E, and its distance metric β_G is 1.0 (table 1), resulting in a projected output of 4.0.

For each of the initially efficient points A, B, E, and H, the QDEA model decides to remove their own data from their respective QDEA reference sets (table 1). Each of these points now becomes "quantile super-efficient" or "external" to its QDEA reference set with $\beta_j < 0$. Point B is projected downward to point C (3, 2.0), point E is projected downward to a point on the hyperplane line defined by points B and H (5, 3.8), and point H is projected downward to a point (8, 4.0) on the horizontal line through point E (figure 1). The QDEA dual solution for point A becomes dual unbounded primal infeasible ($\beta_A^Q \rightarrow -\infty$) since, if point A is dropped from its reference set, point A lies above all convex combinations of the points in the remaining QDEA reference set. We conclude the discussion of this example by noting two results that bear directly upon our following discussions. We first note that originally 50% (4 of 8) of the observations were efficient. If we allow only one external observation for any given QDEA problem, six out of eight observations (or 75%) are classified as "efficient" or "super-efficient."[6] We also note that while only one external observation was allowed for a given observation's QDEA problem (table1), with variable returns to scale, four of the eight points (A, B, E, and H) were chosen as external points in at least one QDEA problem.[7]

DATA

We utilize a detailed trip-level dataset of 33 steel-hulled fishing vessels that made 456 trips targeting *Illex* squid in the northwest Atlantic during 2019. By using a trip-level dataset, we are estimating capacity per fishing trip. This is a natural unit of time, as a fishing vessel leaves port, steams to a fishing ground, and fishes for a certain number of hours, or days. The fish is stored in the hold, and when the hold approaches being full, or the vessel feels that they need to maintain a certain level of freshness, the vessel returns to port. Our data contain trip information on three different types of vessels based on their processing capability. The first type are freezer-trawlers (FT), where the squid are frozen after they are harvested; the second are ice vessels (ICE), where the squid are stored on ice until the vessel returns to port; and the third are refrigerated seawater

6. In the VRS QDEA model, observations A, B, C, D, E, and H were deemed "efficient" or "super-efficient."
7. With constant returns to scale, point B is chosen as the "external" point for each observation's QDEA problem.

vessels (RSW), where the squid are stored in chilled seawater. Each vessel type is considered a different technology and fishes differently. Consequently, we estimate three different DDF and QDEA models where the vessels are stratified by refrigeration type.

Outputs for our model include *Illex* squid, *Loligo* squid, and all "other" species caught on *Illex* trips. Trips landing more than 10,000 pounds of *Illex* squid, and where the percentage of *Illex* by weight was more than 50% of the trip total weight, were chosen for the analysis. These trips are considered to be targeting *Illex* squid, and their inclusion is consistent with the data used in *Illex* squid stock assessment models. Fixed inputs were vessel length, engine horsepower, and gross tonnage. Variable inputs were days at sea and crew size. Although the variable inputs will not constrain the model, we use them to calculate the optimal variable inputs needed on each trip.

In 2019, there were seven FT vessels, which made 65 trips; 12 ICE vessels (202 trips); and 14 RSW vessels (189 trips, table 2). In terms of *Illex* production, the FT fleet on average landed the most per trip (261,300 pounds), followed by RSW vessels (151,542 pounds per trip) and ICE vessels (67,077 pounds per trip). FT vessels also were bigger, on average, then either ICE vessels or RSW vessels (table 2). The FT fleet landed 16,984,500 pounds of *Illex*, the ICE fleet landed 13,549,603 pounds, and the RSW fleet landed 28,641,434 pounds (table 3), for a total of 59,175,537 pounds. There was very little *Loligo* squid or other species landed on trips where *Illex* was targeted. These are considered "bycatch" species, as they were not the main target of the trip. FT vessels caught more of these species groups compared with the other two vessel types. However, on a percentage basis, the bycatch averaged only 1% of the average trip. For ICE vessels, the average was far below 1%, and RSW vessels did not catch any *Loligo* squid.

Illex squid is managed as part of the Mackerel, Squid, and Butterfish Fishery Management Plan by the Mid-Atlantic Fishery Management Council (MAFMC). Since *Illex* squid are a short-lived species, they are managed through a single coast-wide quota. The majority of landings occur between June 1 and October 31. There are additional effort-control measures in place to minimize bycatch, such as restricting fishing beyond the 50-fathom depth line. During 2019,

Table 2. Number of Vessels, Trips Taken, Vessel Characteristics, and Output by *Illex* Squid Vessels in 2019

	Vessel Type		
	FT	ICE	RSW
Number of vessels	7	12	14
Number of trips	65	202	189
	Outputs (mean pounds per trip)		
Illex squid	261,300	67,077	151,542
Loligo squid	2,597	5.3	0
Other species	268	111.8	277.4
	Inputs (mean)		
Length (feet)	104.9	76.8	100
Gross tonnage	251.7	145.8	161.8
Engine horsepower	1,460	702.5	1,173
Crew per trip	7.4	4.3	4.3
Days at sea per trip	5.9	2.7	2.7

Note: FT, freezer-trawlers. ICE, ice vessels. RSW, refrigerated seawater vessels.

Table 3. Capacity by Vessel Type Based on the DDF Model

	Vessel Type		
	FT	ICE	RSW
Illex (total pounds landed)	16,984,500	13,549,603	28,641,434
Capacity (total pounds)	24,970,806	20,604,723	46,885,479
Capacity (pounds) per vessel	3,567,258	1,717,060	3,348,963
Capacity (pounds) per trip	384,166	102,003	248,071
Observed as percentage of trip capacity	0.68	0.66	0.61
Days per trip	8.0	2.6	2.5
Crew per trip	8.5	4.4	4.2
	Bootstrap DDF Results		
Bias-corrected capacity (total pounds)	26,374,080	21,836,125	51,302,494
Bias-corrected capacity (pounds) per vessel	3,767,726	1,819,677	3,664,464
Bias-corrected capacity (pounds) per trip	405,755	108,100	271,442
Observed as percentage of bias-corrected capacity	0.64	0.62	0.56
Days per trip	8.1	2.7	2.6
Crew per trip	8.6	4.4	4.3

Note: DDF, directional distance function. FT, freezer-trawlers. ICE, ice vessels. RSW, refrigerated seawater vessels.

landings of *Illex* totaled a little over 58 million pounds with a value of $27.3 million.[8] This exceeded the approximately 55 million pounds of quota for the year.

APPLYING THE QDEA MODEL

In the following discussion, we demonstrate how we implemented quantile DEA to estimate capacity for our squid fleet. We also discuss several additional issues concerning the selection of the appropriate quantile, or number of possibly external points, as well as how we utilize the DDF and QDEA results when estimating capacity for our fishing fleet. Admittedly, this procedure is narrowly applied to a specific fleet of vessels in one fishery. For other fleets and fisheries, the procedure will likely need to be modified. However, the process of identifying observations that define the frontier as a first step in determining how many observations should lie external to the QDEA frontier is a step that can be applied to any QDEA model. The set of efficient observations can then be examined further to determine whether any members of the set could be considered an outlier. Once the ideal number of points is chosen, further sensitivity analysis should be conducted to see how changes in the number of external points influences the results. If no points lie external to the QDEA frontier, the QDEA and DDF frontier will coincide with each other. As the number of points that are external to the QDEA frontier increases, our two frontiers will diverge and our estimate of total output will change.[9]

Before discussing how to determine the number of external points, we first explain how we set capacity for observations that are external to the QDEA frontier. Since the trip occurred, and unless there is evidence of a data error, we assigned a value of 0 for their distance from the frontier, meaning their capacity output is equal to their observed output. Since these observations are external to the QDEA frontier, no expansion is possible, so a distance value of 0 seems appropriate.

8. https://www.fisheries.noaa.gov/species/shortfin-squid; accessed August 30, 2022.
9. The solved LP problem is the dual version of equation 4.

Using our small eight-point problem shown in figure 1 and table 1, the points A, B, E, and H, which all had negative distance values returned by the QDEA model, would have their capacity output set equal to their observed values (i.e., A = 1, B = 3, E = 4, H = 5).

Returning to the question of how many observations are allowed to lie external to the frontier, we adopt an approach that is similar to that of Khezrimotlagh, Cook, and Zhu (2020), and utilize an upper bound for the *Illex* squid landings. We first run the initial DDF model and identify observations that are found to be DDF efficient (i.e., $\beta = 0$). Separately, we set an upper bound for squid landings equal to the median squid landings plus 1.5 times the interquartile range (i.e., median + 1.5 × IQR). From the set of efficient observations, we determine the subset of these observations where the output of *Illex* squid is greater than our upper bound. The number of total observations that lies in the set where $\beta = 0$ and the *Illex* squid quantity is above the upper bound determines the number of observations held out in the QDEA model. We consider these observations to be the most influential. Initial results from the DDF model applied to our fleet of vessels showed 11 efficient observations for the FT and RSW fleets, and 12 observations for the ICE fleet. The upper bound squid landings were 570,250, 118,602, and 338,423 pounds for the FT, ICE, and RSW fleets, respectively. Combining these upper bounds with the number of efficient observations resulted in three influential observations (4.6%) for the FT fleet, two influential observations (1%) for the ICE fleet, and three influential observations (1.6%) for the RSW fleet.

Graphically, the impact of withholding different amounts of observations on capacity estimates for each fleet is shown in figures 2–4. We use this graphical approach as an initial check on the number of points withheld that was calculated above. As the graphs show, moving from zero external observations to one external observation in each observation's LP problem decreases the sum of the capacity estimates over one million pounds for the FT and ICE fleets (figures 2 and 3), and over two million pounds for the RSW fleet (figure 4). For the FT fleet, moving beyond one observation yields very little difference in projected capacity until seven external observations are withheld (figure 2). For the RSW fleet, moving from one to two external observations held out decreases the sum of the distance values by an additional two million pounds, while moving to three observations drops about one million more pounds. Examination of the graphical results for the ICE fleet shows that moving from two to four observations would reduce capacity by roughly another million pounds, after which the rate of capacity reduction was much slower. Because two observations for the ICE fleet were slightly less than 1%, we followed the graphical information and increased our withholding to four observations.

We then use a Kruskal-Wallis test to compare the distributions of capacity from the DDF model with that from the QDEA model using different numbers of points out. If the two distributions are significantly different at a desired level of significance, no further observations are withheld. We adopted a 10% significance level. If they are not significantly different, the percentage of observations held out is increased slightly until there is a significant difference. This is the initial stopping rule we used for this study, and could be easily altered. For example, once the number of points leading to a significant change in the distributions is determined, this number could be reduced until no significant difference is seen. This would give a different total for capacity with the same distribution.

The null hypothesis is that the distribution of projected outputs from the QDEA model is equivalent to the DDF model, and the Kruskal-Wallis test yields a chi-squared value with one degree of freedom for each fleet. For all three fleets, withholding one observation resulted in a significant difference between the two distributions. The chi-squared value for the FT fleet was

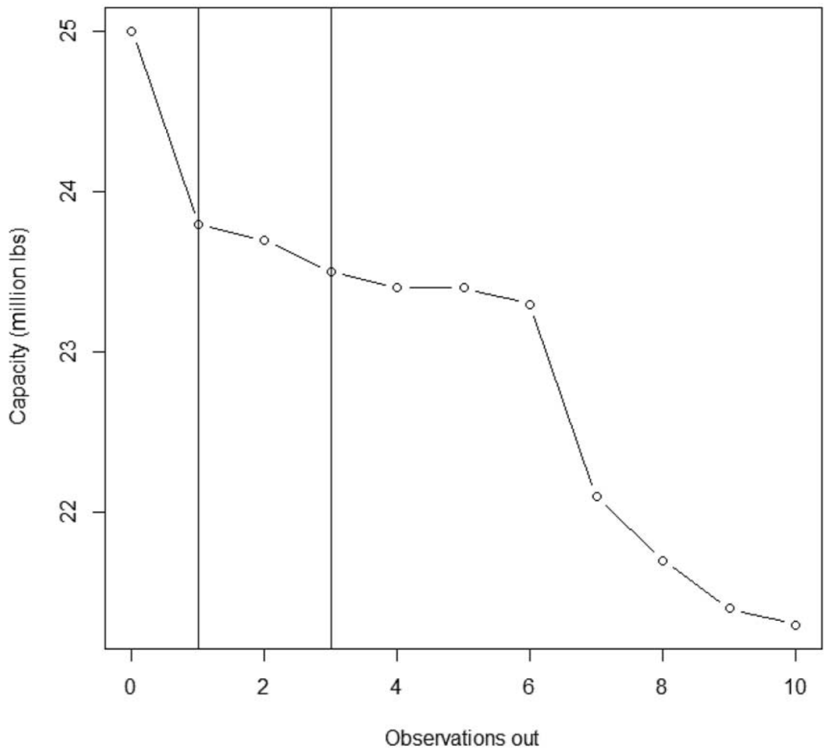

Figure 2. Capacity Estimates for the FT Fleet by the Number of Observations Allowed to Lie External the QDEA Frontier. Vertical lines represent values with one or three external observations.

significant at the 10% level (3.26), while the ICE fleet (10.01) was significant at the 1% level, as was the value for the RSW fleet (7.77). In order to obtain a 5% level of significance for the FT fleet, there would need to be seven observations held out (4.37 chi-squared value). This is consistent with the large drop in capacity when moving to seven observations shown in figure 2.

In summary, there are several steps that we undertook to implement QDEA for our fleet, which can be used if QDEA is applied in a different fishery. First, we set capacity for observations external to the QDEA frontier equal to their observed value. Second, we chose an initial number of observations to withhold based on the number of observations that were on the DDF boundary (i.e., distance $=0$) and an upper bound level of landings. As a visual check, the analysts could plot QDEA results showing estimated capacity given changes in the number of observations withheld, and look for inflection points in the graph or large drops in capacity. For our fishery, we held out additional points for one of our fleets based on the graphical results and wanting at least a 1% minimum for points withheld. Finally, use of a nonparametric statistical test, such as the Kruskal-Wallis test, to see whether there was a significant difference in capacity distributions between the DDF model results and the QDEA results given different numbers of withheld points can help determine whether additional points need to be withheld.

RESULTS

Before presenting the results for the quantile DEA model, we first present the results for the DDF model, including the bootstrap (250 replications) results. Because the landings of both *Loligo*

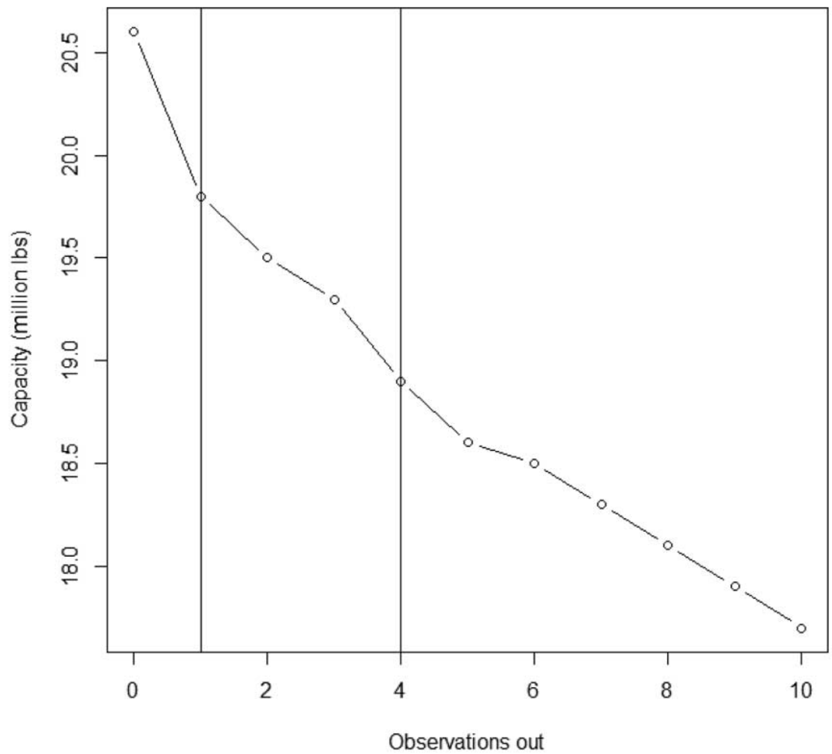

Figure 3. Capacity Estimates for the ICE Fleet by the Number of Observations Allowed to Lie External the QDEA Frontier. Vertical lines represent values with one or four external observations.

squid and "other" species is so low on targeted *Illex* trips, the directional vector used for expansion was [1,0,0]. This allowed *Illex* to be expanded, while the other two species were not allowed to expand. This is a slightly different measure than that used in most capacity studies where all outputs are radially expanded. Ultimately, we are interested in the possible expansion of *Illex* squid to arrive at a measure of *Illex* capacity for the fleet. This measure yields capacity given a fixed level of bycatch. Based on the DDF results, the fleets are on average operating at between 61% and 68% of their trip capacity (table 3). FT vessels average 261,300 pounds of *Illex* per trip, which was 68% of their projected trip capacity of 384,166 pounds. ICE vessels landed on average 67,077 pounds per trip, or 66% of their projected trip capacity of 102,003 pounds. RSW vessels landed 151,542 pounds per trip, which was 61% of their projected capacity of 248,071 pounds per trip. FT vessels would need to fish a little over two more days per trip (8.0 vs. 5.9) to reach this level of capacity, and increase their crew size by one (8.5 vs. 7.5), while the other two vessel types would need slightly fewer days, and in the case of RSW vessels, slightly fewer crew members. Crew size for ICE vessels would increase slightly, on average (4.4 vs. 4.3). Overall, if vessels fished the same number of trips, their DDF estimated potential capacity was 92.5 million pounds of *Illex* squid, which is 56% greater than the actual landings of 59.2 million pounds.

Results from the bootstrap DDF model showed a slightly higher level of estimated capacity, which was expected (table 3). FT vessels were operating at 64% of their bias-corrected trip-level capacity, ICE vessels were at 62% of their bias-corrected capacity, and RSW vessels were at 56%

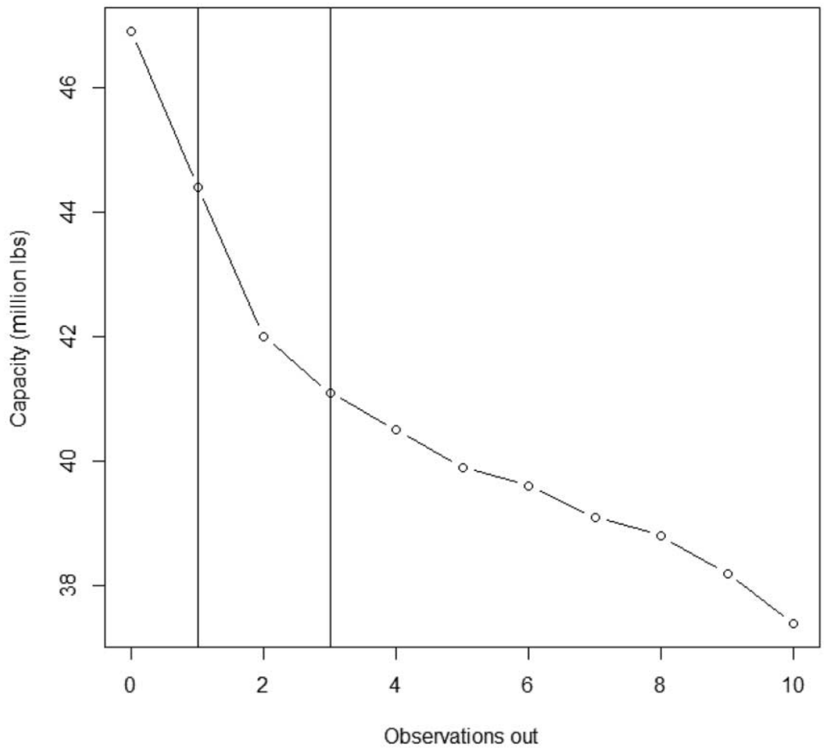

Figure 4. Capacity Estimates for the RSW Fleet by the Number of Observations Allowed to Lie External the QDEA Frontier. Vertical lines represent values with one or three external observations.

of their bias-corrected capacity. The average days at sea and crew size needed to reach capacity were only slightly different than what the DDF model returned.[10] Total bootstrapped DDF estimated potential *Illex* output was 99.5 million pounds, which was 68% higher than the observed output of 59.2 million pounds.

QUANTILE DEA RESULTS

Results from the QDEA model with three, four, and three external points per QDEA problem for the FT, ICE, and RSW fleets, respectively, showed an estimated capacity of 83.4 million pounds (table 4), which was 9.8% lower than the DDF capacity estimate of 92.5 million pounds. The FT fleet had a slightly lower capacity estimate of 23.5 million pounds, compared with the DDF estimate of 25.0 million pounds. The current output for the FT fleet was 72% of the QDEA capacity output, an increase of 4% over the DDF capacity estimate. The ICE fleet QDEA capacity was 18.9 million pounds, compared with the DDF estimate of 20.6 million pounds, an 8.3% reduction. Current output for the ICE fleet was 72% of the QDEA output, while it was 66% for the DDF capacity estimate. The RSW fleet had a QDEA estimate of 41.1 million pounds, compared with 46.9 million pounds from the DDF model, a 14.1% reduction. Current output was 70% of the QDEA capacity output, compared with 61% for the DDF model. Using QDEA for the RSW fleet

10. We did not attempt to bias-correct days at sea or crew size, and present just the mean values based on all bootstrap replications

Table 4. QDEA Results by Vessel Type

	Vessel Type		
	FT	ICE	RSW
Capacity (total pounds)	23,481,179	18,853,418	41,106,502
Capacity pounds per vessel	3,354,454	1,571,118	2,936,179
Capacity pounds per trip	361,249	93,333.75	217,494.72
Observed as percentage of QDEA capacity	0.72	0.72	0.70
Days per trip	8.0	2.7	2.5
Crew per trip	8.6	4.2	4.4
	Bootstrap Results		
Bias-corrected capacity (total pounds)	24,087,766	19,168,077	42,107,068
Bias-corrected capacity (pounds) per vessel	3,441,109	1,597,340	3,007,648
Bias-corrected capacity (pounds) per trip	370,581	94,891	222,789
Observed as percentage of bias-corrected capacity	0.71	0.71	0.68
Days per trip	7.4	2.7	2.6
Crew per trip	8.2	4.3	4.3

Note: QDEA, quantile data envelopment analysis. FT, freezer-trawlers. ICE, ice vessels. RSW, refrigerated seawater vessels.

yielded the largest change in outcomes in terms of both total capacity reduction and increase in capacity utilization.

Bias-corrected results from the bootstrapped QDEA model for all three vessel types were slightly higher than the initial QDEA results (table 5), which was the expected result. The biggest absolute difference was found in the RSW fleet (1.0 million pounds), which was 2.4% greater than the QDEA total (table 5). For the ICE fleet, the difference between the bias-corrected QDEA model and the QDEA results was 314,659 pounds (1.7%), and for the FT fleet, it was 606,587 pounds (2.6%). The QDEA model removes a number of observations from the QDEA reference set for each observation, and this same procedure occurs for each bootstrap replication. Since these are being endogenously identified, they are likely to be the same observations being removed by the non-bootstrap QDEA model. Because the influence of the outliers is reduced through the QDEA procedure, the bootstrapped QDEA results are lower than the bootstrapped DDF results. However, they will still be higher than the non-bootstrapped QDEA model because

Table 5. Landings and Capacity Totals by Vessel Type, 2019

	Vessel Type			
	FT (pounds)	ICE (pounds)	RSW (pounds)	Totals (pounds)
DDF results	24,970,806	20,604,723	46,885,479	92,461,008
Bootstrap DEA results	26,374,080	21,836,125	51,302,494	99,512,699
Quantile DEA results	23,481,179	18,853,418	41,106,502	83,441,099
Bootstrap QDEA results	24,087,766	19,168,077	42,107,068	85,362,911

Note: DDF, directional distance function. DEA, data envelopment analysis. QDEA, quantile data envelopment analysis. FT, freezer-trawlers. ICE, ice vessels. RSW, refrigerated seawater vessels.

the structural bias in the DEA model is being corrected with the bootstrapped model. In summary, the bias-corrected DDF model yielded the highest overall capacity for all three vessel types (table 5). This was followed by the DDF capacity model, the bias-corrected QDEA model, and finally, the QDEA model.

CONCLUSIONS

Quantile DEA is an innovative approach for handling noisy data in a DEA setting. The ability to endogenously identify and drop sets of observations from an observation's reference set, rather than examining the influence of removing single observations, is a key strength of QDEA. Our results showed about a 10% difference in capacity estimated using QDEA after allowing a small number of observations to lie external to the DEA frontier. This was a difference of a little more than nine million pounds. On a per trip basis, the DDF results show our fleets harvesting between 61% and 68% of their potential output, while the QDEA results show the fleets at between 70% and 72% of their potential output. Although the QDEA model results indicate a lower level of potential capacity than the DDF model, they still suggest there is a capacity issue in this fishery. One caveat that we note in our results is that total estimated capacity is calculated using a static number of trips. Any expected increase in trips would increase our estimate of overall capacity.

After obtaining our initial results, we applied a subsample bootstrap routine to both the DDF and the QDEA models. In terms of bias-corrected total capacity, the bootstrapped DDF results increased our capacity estimate by 7.6%, while the QDEA bias-corrected total capacity increased by 2.3%. Bootstrapping the QDEA model had much less of an impact in terms of increasing total capacity because the influence of outliers is being reduced at each iteration of the bootstrap model. For our three fleets, the bias-corrected QDEA estimate was about 2.6% more for the FT fleet, 1.7% for the ICE fleet, and 2.4% for the RSW fleet than the uncorrected QDEA estimates.

In order to use the QDEA method for this fishery, the analyst must make two key decisions. The first is how to estimate capacity for observations that are deemed external to the QDEA frontier. For our three fleets, we set capacity for these observations equal to their observed output. This is consistent with saying that these observations cannot expand their output further, but the observed output did actually occur. Depending on what information is desired, this step may not be necessary. For example, if just mean inefficiency is desired, these observations could be excluded from the calculation of mean inefficiency, or given a value of 0. There are numerous ways to handle this question, and the answer depends on the data and the question posed. Second, and perhaps more importantly, the analyst must decide on the number of points allowed to lie external to the QDEA frontier. In order to set this number, we used graphical results, a more formal Kruskal-Wallis test, results from the initial DDF model, and the establishment of an upper bound for *Illex* landings. Again, we stress that there is no single answer to this question. This step requires data exploration by the analyst.

This was the first attempt that we are aware of that sought to apply QDEA to a relevant and timely fisheries policy question, which was the estimation of fishing capacity in the *Illex* squid fishery. We found that the QDEA model performed well in terms of ease of use, speed, and returning credible results. Additionally, we find that because QDEA endogenously chooses the observations to lie external to the QDEA frontier, the analyst is free to focus on answering relevant-policy questions, rather than trying to determine which observations are truly outliers. In a problem with multiple inputs and outputs, choosing the right dimension to determine whether an observation is an outlier can be difficult. Future work with QDEA using simulated data may reveal additional insights into how much bias reduction is occurring due to outliers the QDEA model is removing. Although our results are specific to the *Illex* fishery, we are optimistic that QDEA will prove to be a useful addition to the DEA toolbox for estimating fishing capacity.

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