

NUMERICAL MODELING OF THE DEFLECTION OF AN ELECTRO-
STATICALLY ACTUATED CIRCULAR MEMBRANE MIRROR

by

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of the requirements for the degree

of

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DEDICATION

I dedicate this thesis to my loving and supportive mother, MaryAnn.

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ABSTRACT

This thesis outlines a numerical modeling method to describe the deflection behavior and investigate control schemes for an electrostatically actuated deformable membrane mirror, with application to focus control and aberration correction in microelectromechanical systems. The physics of the membrane are approximated using a finite difference approach with parameters obtained from measurements of a physical device. The model is validated by comparison of simulated and measured mirror position under static and dynamic conditions. This thesis provides simulation results for control schemes that would be difficult or potentially destructive if implemented using real devices. We suggest that the model may be useful for the development of future control strategies and in refining device design. Finally, a number of capacitive sensing circuits are presented as position feedback mechanisms and the capabilities and limitations of each are examined.

INTRODUCTION

Background

Micromachined deformable membrane mirrors are rapidly becoming critical components in many adaptive optics (AO) applications ranging from astronomical observation [1] to dynamic focus control for in-vivo medical imaging [2, 3, 4]. These Microelectromechanical systems (MEMS) consist of a thin reflective membrane supported above a cavity, at the bottom of which are situated one or more actuating electrodes. When a potential is applied between these electrodes and the membrane, an electrostatic attractive force is generated. This force deflects the membrane downward into the cavity, or gap. The resulting deflection profile is determined by the physical parameters of the device and by the pattern of the generated field. Light incident upon the membrane is reflected and altered according to this surface profile. By modulating the applied field between the electrodes, the focal properties of the beam may be altered and aberration may be corrected for. A few variations upon this basic design have been put forward, with some becoming commercially available products. Such variations have typically been specifically designed for aberration compensation wherein a relatively large number of electrodes must be patterned over the bottom of the gap. Notably, a 37 electrode device developed by OKO Technologies has gained popularity and been analyzed extensively; [5, 6, 7, 8, 9]. However, such designs are poorly suited to focus control applications in which the desire is to control the curvature of the membrane while preserving radial symmetry. Further these designs suffer from limited stroke and unnecessary complexity. This thesis will focus on devices designed for focus control applications and that have concentric actuating electrodes, such as those developed by Himmer, Lukes [10, 11].

Traditionally, AO applications for focus control involved translating lenses that respond relatively slowly and are poorly suited to miniaturization. In contrast, MEMS deformable mirrors are characterized by their rapid response, minute dimensions and by their approximately parabolic deflection. This makes micro-machined deformable membrane mirrors, in principle, an ideal AO element for focus control in miniaturized systems. There are however, a number of serious control issues that limit their performance, most importantly the electrostatic instability which limits the range of stable open-loop deflection [12]. Further, at larger deflections, the spherical aberration induced by the membrane itself and the changes in the membrane's dynamics become increasingly significant and must be addressed. Dickensheets et. al have shown that to achieve the numerical aperture required for proposed in-vivo microscopy application a deflection of at least $10\text{ }\mu\text{m}$ is required [13]. We hope to achieve deflections of between $25\text{ }\mu\text{m}$ and $30\text{ }\mu\text{m}$ which is beyond any result yet reported. Currently, this deflection range has proven difficult to achieve using open-loop (OL) actuation methods. This lead to the development of closed-loop (CL) control strategies that make possible stable deflections beyond the electrostatic instability point [11]. The process of designing a CL controller requires some knowledge of the system dynamics. However, given the nonlinear nature of the actuation source, these dynamics change considerably at large deflections. Thus, identifying the dynamic behavior beyond the electrostatic instability can present a significant challenge.

To date, efforts involving CL voltage and charge control have increased the available membrane stroke beyond the electrostatic instability point [12], but have relied on either parallel plate models or physical measurements during the development of control strategies [14, 2]. Parallel plate models have limited utility since they fail to account for the dynamics introduced by the bending motion of the membrane. Further, adequate characterization of the dynamics by physical measurement beyond

the instability point necessitates operating the membrane near the boundary of instability with a suboptimal controller, and thus involves the possibility of snap-down and device failure.

Proposed Contribution and Purpose

A numerical model of the device is developed in this thesis to aid in identification and design optimization to the end of improving CL performance. Numerical modeling by the finite difference or finite element methods is a well accepted analysis technique for predicting the behavior of deflecting membranes. However, previous efforts have primarily concentrated on modeling the steady state deflection behavior of existing membranes designed for aberration correction [5, 7, 9], or have focused on modal analysis [15], both of which have limited utility when identifying dynamic behavior. A finite difference time domain (FDTD) model is developed and analyzed in this thesis. It is proposed that the model may be used in lieu of a physical device or a parallel plate model for identifying the device dynamics in order to develop a more appropriate control strategy. This model incorporates the full dynamics introduced by the bending of the membrane and supports offline analysis without the need to construct and identify a physical device, thereby expediting the design process.

Assumptions Made

For modeling simplicity a number of assumptions about the membrane are made in this thesis, the first of which is that the device is perfectly circular and deforms in a radially symmetric fashion. This is to say that a ray extending from the center of the membrane to an arbitrary point on its perimeter completely describes it. Observed

deflection profiles obtained from interferometric imaging show that this assumption is valid.

It is also assumed that the electric field lines between the membrane and actuating electrodes are everywhere normally directed. Any effects due to fringing fields near the edges of the membrane and at the boundaries between electrodes are ignored in this thesis. These fringing fields are likely to play only a minor role in determining the shape of the membrane as the gap is quite thin, typically 30 μm , compared to the diameter of the device, a minimum of 1 mm.

The cylindrical cavity backing the membrane is open to the atmosphere through a number of ports which allow air, displaced by the deflecting membrane, to flow out of the gap. These ports may also constrict the flow of air during periods of rapid transition. When this occurs the trapped and subsequently compressed volume of gas imparts some pneumatic effect on the membrane. This pneumatic effect becomes important as the frequency of the deflection increases. However, such pneumatic effects are only modeled by a simple damping coefficient in this thesis.

Further, it is assumed that the membrane is thin enough to conform to a simple membrane model without the introduction of plate effects. These effects become more important as the localized curvature becomes increasingly strong.

Organization of Thesis

In the remainder of this thesis, a FDTD model of a MEMS deformable membrane mirror is developed. This model is then validated under static and dynamic conditions with data collected in the course of this thesis work as well as from outside sources for similar device technologies.

Next, a few examples of how this model may be of use in the design and optimization of these devices are provided. For example, the model may be used to identify the system dynamics beyond the snap-down instability. This is accomplished by first controlling the membrane in a sub-optimal manner in order to extend the range of stable deflection. Near the new CL instability point the membrane is identified by traditional system identification techniques. Given the updated information about the membrane, a more suitable sub-optimal controller may be developed, extending the range of stable deflections further to enable identification at an even more extreme deflection. The introduction of nonlinear elements in the feedback path is also considered. Each feedback strategy considered in this thesis is based upon position feedback.

Finally, in order to put the position feedback strategies considered in this thesis into application, a method of sensing the membrane position is required. By the parallel plate model the capacitance of the membrane should scale linearly with deflection. Thus, it is proposed that the capacitance of the membrane measured from the actuating terminals may serve as an accurate means to measure membrane position. A few of the circuit topologies put forward to meet this challenge in the course are presented. The advantages and limitations of each of these designs are addressed as well as the validity of directly relating capacitance to position.

NUMERICAL MODELING

Background

Numerical modeling is a widely accepted and utilized tool for the analysis and development of MEMS devices including deformable membrane mirrors. Several efforts have sought to model the deflection behavior of deformable membrane mirrors by the finite element [5] and the finite difference methods [16, 9, 8, 17]. These efforts have primarily focused on modal analysis and on modeling the deflection behavior of multi-zonally actuated membranes for aberration compensation. These studies have focused upon devices operated under open-loop conditions. For example, Vogel et al. have addressed open-loop control via pulse shaping [8].

Device Description

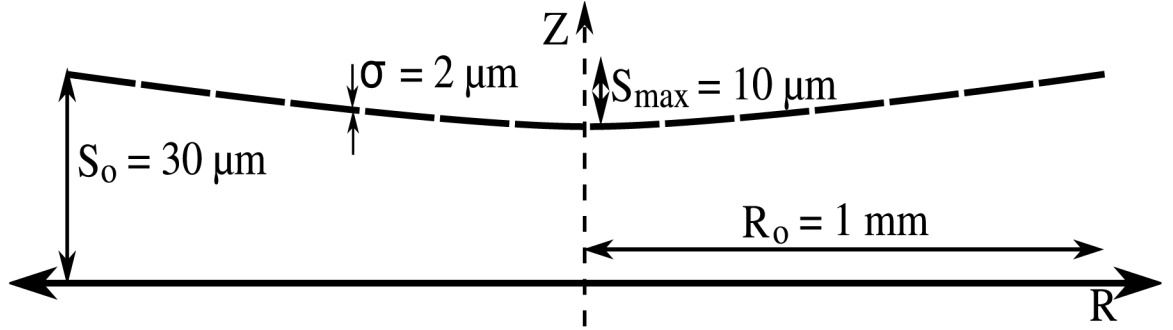


Figure 2.1: Deformable mirror with typical dimensions.

The devices modeled in this paper, Figure 2.1, are designed specifically for focus control applications and consist of a thin epoxy membrane on the order of one or two

μm thick. The epoxy membrane is situated above a cylindrical backing cavity that is bulk etched into the substrate below. It is metalized with either gold or aluminum on the top surface to act as a reflective layer. A grid of concentric electrode rings is patterned at the bottom of the gap. When a potential is applied between these electrodes and the metalized membrane, an electric field is generated within the gap. This results in an attractive electrostatic force that serves to deflect the membrane into the gap. This device is described in further detail in [18, 19]. Alternately, the concentric electrode pattern may be applied on the metalized top surface of the membrane with a continuous counterelectrode placed below, as is the case for the devices of [10], [11]. Both technologies result in the same deflection behavior and are used interchangeably within this thesis. An overview of this device, its deflection pattern and the concentric counterelectrodes is found in Figure 2.2.

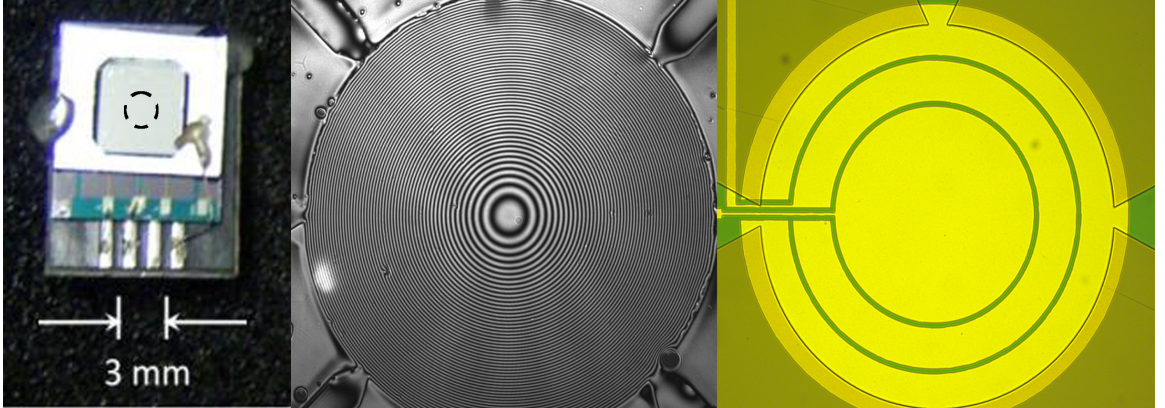


Figure 2.2: *left* : deformable membrane mirror mounted in DIP8 package, the outline of the deformable area is indicated by the dashed circle *center* : interferometric image of deflected membrane *right* : backplate electrodes exposed following removal of membrane mirror.

Electrostatic Instability

The electrostatic force is proportional to the inverse square of the distance between the membrane and the actuating electrodes. As the membrane nears the electrodes and the distance grows small, the electrostatic force tends to become unbounded. The force acting to restore the membrane to its resting state is relatively linear and is generally modeled as such [12, 20, 21, 22, 8, 23, 11]. At some point critical point the electrostatic actuating force overwhelms the linear restorative force and the membrane is pulled completely into the gap. This point is known as the electrostatic instability and is predicted to occur when deflection reaches 33 % of the gap thickness by the parallel plate model [22]. For the deformable membrane, experimental evidence has demonstrated that this instability point actually occurs when the center of the membrane has reached 43 % of the gap thickness [11], suggesting that the parallel plate model, Figure 2.3, is inadequate for predicting steady state deflection behavior beyond small excursions.

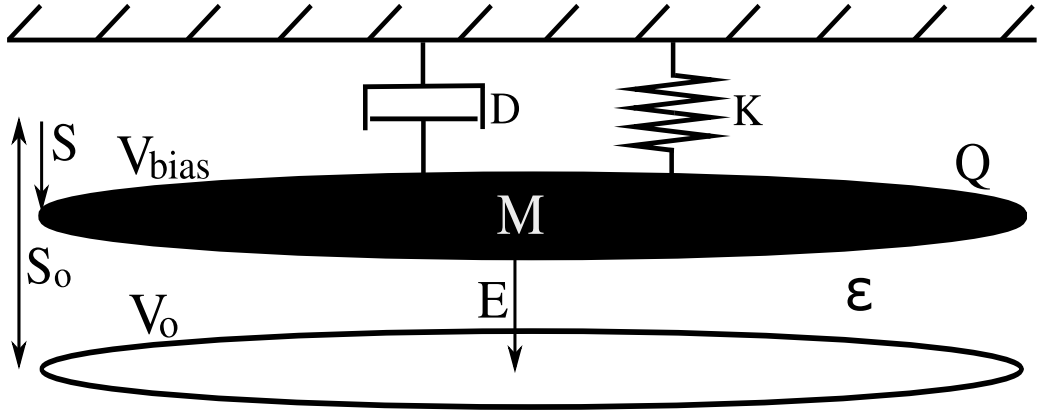


Figure 2.3: Parallel plate linear model of deformable membrane mirror.

The equation of motion describing this parallel plate model is

$$\frac{\partial^2 S}{\partial t^2} + 2\zeta \frac{\partial S}{\partial t} + S = q^2. \quad (2.1)$$

Finite Difference Method

The Finite Difference Method (FDM) is a numerical method of approximating solutions to differential equations by replacing the derivatives within the equation with difference equations. Since the first derivative of a function, $f(r)$, is defined as

$$f'(r) = \lim_{n \rightarrow 0} \frac{f(r) - f(r - n)}{n} \quad (2.2)$$

or

$$f'(r) = \lim_{n \rightarrow 0} \frac{f(r + n) - f(r)}{n} \quad (2.3)$$

then in the limit that n is small, we may make the approximation that

$$f'(r) \approx \frac{f(r) - f(r - n)}{n} \quad (2.4)$$

and

$$f'(r) \approx \frac{f(r + n) - f(r)}{n}. \quad (2.5)$$

Equations 2.4 and 2.5 are known as the backward and forward difference equations, respectively.

Similarly, the second derivative of a function, $f(r)$, is defined as

$$f''(r) = \lim_{n \rightarrow 0} \frac{\frac{f(r+n)-f(r)}{n} - \frac{f(r)-f(r-n)}{n}}{n}. \quad (2.6)$$

Again, in the limit that n is small it is reasonable to make the approximation

$$f''(r) \approx \frac{\frac{f(r+n)-f(r)}{n} - \frac{f(r)-f(r-n)}{n}}{n} \quad (2.7)$$

or after simplifying

$$f''(r) \approx \frac{f(r+n) - 2f(r) + f(r-n)}{n^2}. \quad (2.8)$$

This is known as the second difference equation.

Numerical Approximation of Membrane

To model the elastic motion of the deflecting membrane, a simple membrane equation is employed [24]

$$T\nabla^2 S(r) + p(r) = \rho \frac{\partial^2 S}{\partial t^2} \quad (2.9)$$

where ρ is the density of the membrane material, $T = h\sigma$ is the in-plane tension, h is the membrane thickness, and σ is the residual stress; all in appropriate SI units. The electrostatic pressure,

$$p(r) = \frac{1}{2} \frac{q_1^2(r)}{\epsilon_2}, \quad (2.10)$$

is a function of the localized charge on the membrane,

$$q_1(r) = \frac{V_{bias}}{\frac{S_o - S(r)}{\epsilon_2} + \frac{\delta}{\epsilon_1}}. \quad (2.11)$$

A cross-sectional illustration of this membrane is provided in Figure 2.4 along with the parameters of interest to this thesis. The charge at the bottom of the epoxy membrane, Q_2 , is presumed to be zero and the charge, Q_1 , upon the metalized top layer is a result of the potential difference applied between V_{bias} and V_o by (2.11). ϵ_1 is the relative permittivity of the membrane material. The SU-8 epoxy that the membrane is made from has a value of three [25]; this figure is used throughout. ϵ_2 represents the relative permittivity of the gap material which is assumed to be air with a value of one. E_1 is the electric field generated within the membrane material and E_2 the field within the gap. σ is the thickness of the membrane, S_k is the deflection

at some radius R_k from the center of the membrane, S_o is the gap thickness, and R_o the membrane radius.

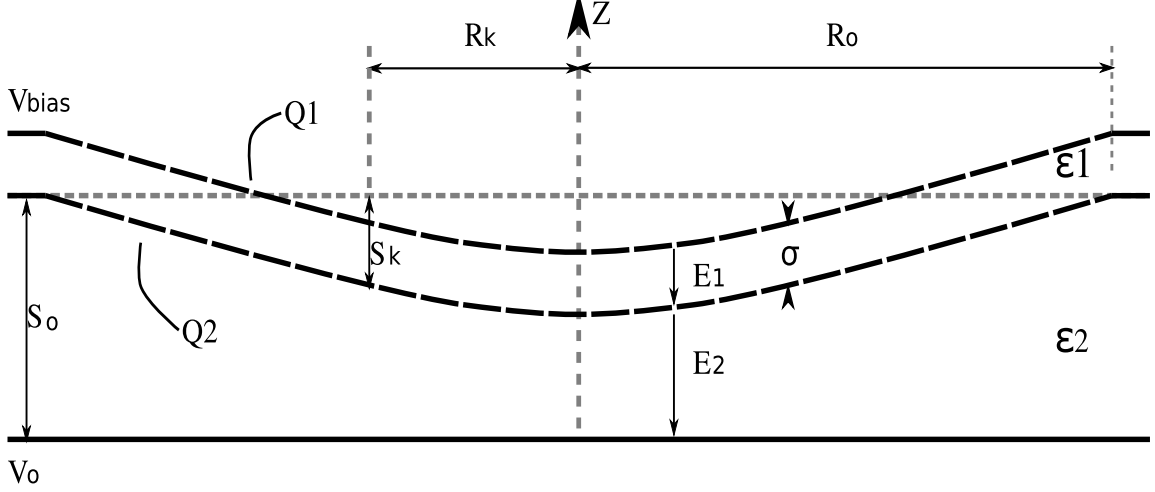


Figure 2.4: Cross sectional illustration of deforming membrane mirror with parameters of interest.

To solve the problem numerically, the continuous membrane is approximated as an array of concentric, annular parallel plate capacitors with displacements obeying a discretized form of the membrane equation. An illustration of this approximation is presented in Figure 2.5.

The first spatial partial derivative is approximated with the backwards difference method (2.4),

$$\frac{\partial S_k}{\partial r} \approx \frac{S_k - S_{k-}}{\frac{R_o}{N}}, \quad (2.12)$$

and the second spatial derivative is approximated by (2.8) as,

$$\frac{\partial^2 S_k}{\partial r^2} \approx \frac{S_{k+} - 2S_k + S_{k-}}{(\frac{R_o}{N})^2}. \quad (2.13)$$

(2.12) and (2.13) are then used to construct a discrete form of the laplacian operator, which in cylindrical coordinates with radial symmetry, $S_{k,\theta+} = S_{k,\theta} = S_{k,\theta-}$, is [26]

$$\nabla^2 S_k = \frac{\partial^2 S_k}{\partial r^2} + \frac{1}{k \frac{R_o}{N}} \frac{\partial S_k}{\partial r}. \quad (2.14)$$

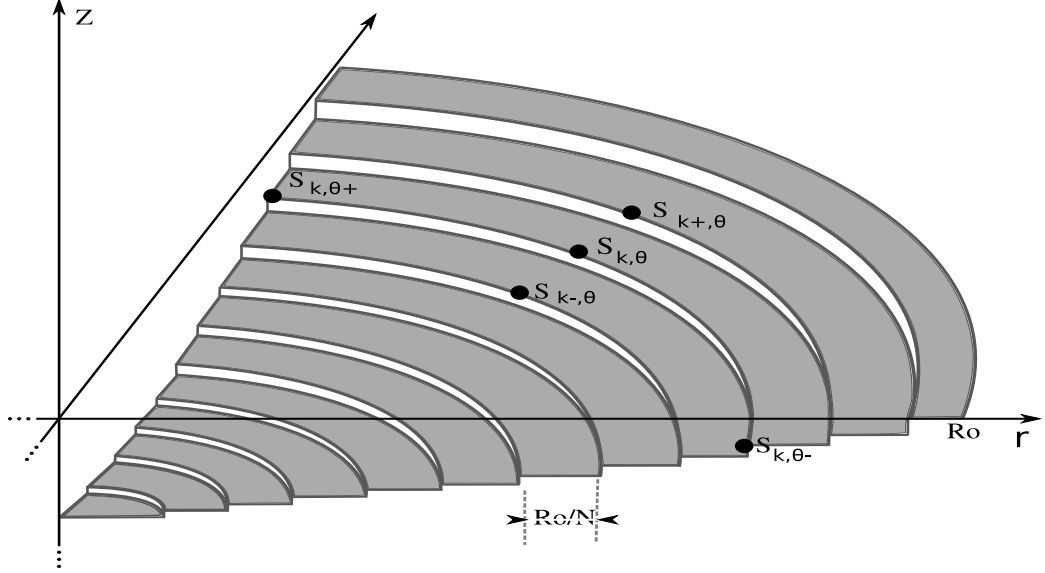


Figure 2.5: Circular deformable membrane - example discretization of continuous membrane into twelve concentric annular parallel plate capacitors with positions obeying the discrete membrane equation in polar coordinates. $S_{k,\theta+} = S_{k,\theta} = S_{k,\theta-}$

The discrete membrane equation may then be written as

$$\frac{T}{\rho} \nabla^2 S_k + \frac{1}{\rho} p_k = \frac{\partial S_k^2}{\partial t^2}. \quad (2.15)$$

To model the damping on the membrane due to the displacement of air through the constrictive ports a damping coefficient, D , is added to the model. The damping effect on each parallel plate ring is assumed to be a function of the area of that ring, A_k , where

$$A_k = \pi \frac{R_o}{N} [2r_k + \frac{R_o}{N}], \quad (2.16)$$

and also a function of the velocity of the ring, $\frac{\partial S_k}{\partial t}$. This results in an updated membrane equation given by

$$\frac{T}{\rho} \nabla^2 S_k + \frac{1}{\rho} p_k - D A_k \frac{\partial S_k}{\partial t} = \frac{\partial S_k^2}{\partial t^2}. \quad (2.17)$$

Finally, the entire model may be written in state space form as

$$\begin{bmatrix} \frac{\partial S_k}{\partial t} \\ \frac{\partial^2 S_k}{\partial t^2} \end{bmatrix} = \begin{bmatrix} 0 & I \\ \frac{T}{\rho} \nabla^2 & -DA_k * I \end{bmatrix} \begin{bmatrix} S_k \\ \frac{\partial S_k}{\partial t} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\rho} \end{bmatrix} p_k, \quad (2.18)$$

where p_k is the electrostatic pressure acting on each plate. This form represents the

Output Equation

If the position of the membrane is to be determined by measuring the capacitance from the actuating terminals, this total capacitance, C_{total} becomes the output variable. This is the summation of each ring's parallel plate capacitance which is given by:

$$C_{total} = \sum_{k=0}^N C_k = \epsilon_2 \sum_{k=0}^N \frac{A_k}{d_k} \quad (2.19)$$

where

$$C_k = \frac{\epsilon_2 A_k}{S_o - S_k}. \quad (2.20)$$

Boundary Conditions

By the radial symmetry of the model, the equation is collapsed into a one dimensional model extending from the center of the membrane to an arbitrary point at the edge. The solution to one-half of the membrane cross-section may then be reflected about the center to obtain the complete solution. At these two extreme locations, boundary conditions are provided for the numerical model. For continuity about the center of the membrane, the derivative is set to zero. The position of the membrane at this point is free to take on the value determined by the membrane equation. This is known as a Neumann or second-type boundary condition.

Conversely, at the interface between the outer boundary of the membrane and the edge of the substrate supporting it, the position is fixed and the slope is free to be determined by the solution to the membrane equation. At this point, a Dirichlet or first-type boundary condition is applied. A true Dirichlet boundary condition however, only places a constraint upon the position of the membrane, without any constraint as to its curvature. In reality, given the finite stiffness of the membrane material, the membrane may not bend with an arbitrary curvature but will instead be subjected to some plate effects that also influence the shape of the membrane near the outer ring. The overall effect of this non-ideality is likely to be small given the small deflections of the membrane when compared to its radius. This effect is further minimized if the interface is hinged, and the predicted deflection profile becomes closer to the actual. An excellent comparison of clamped membrane bending behavior near this boundary condition is provided in [10] for various residual stresses in the membrane material.

Steady state solutions for the membrane deflection may then be obtained by simply setting the time derivatives of the equation to zero. Alternately, time domain behavior may be simulated for any given actuation waveform by solving for the deflection profile without constraint on the time derivatives of the equation.

MODEL VALIDATION

The numerical model is compared with the measured performance of a physical device for agreement. Steady state validations include comparing the shape of the predicted deflection profile to measurements of a physical device and also comparing the deflection trends as a function of applied voltage. Dynamic validation is also addressed by comparing the frequency response predicted for the membrane by the model with data obtained from physical devices in the course of this thesis work as well as from outside sources.

Steady State Validation

The model is first compared to a physical device under a steady state condition. This is the state when the membrane has come to a stable resting deflection and all of its time derivatives, $\frac{\partial S_k}{\partial t}$, $\frac{\partial^2 S_k}{\partial t^2}$, approach zero.

Membrane Shape Validation

Accurately modeling the shape of the membrane is critical as the shape has a direct impact upon the capacitance measured at the actuating terminals. In this section the shape predicted by the model is compared to interferometric measurements of a physical membrane. Single zone and a number of multi zoned actuation scenarios are presented.

First, to evaluate the discretized approximation of the membrane deflection the shape predicted by the model is compared to that predicted by the integral solution to the membrane equation. A comparison of the integral solution of the membrane

equation to a solution of the discrete form at 20 percent deflection is provided in Figure 3.1.

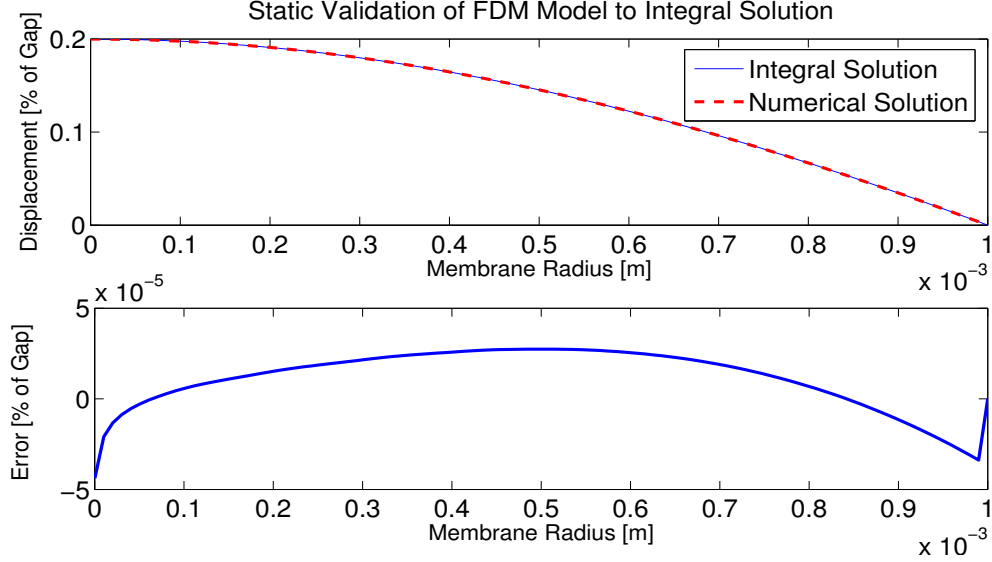


Figure 3.1: Numerical membrane models at 20 percent deflection

Figure 3.2 represents the same test at 40 percent deflection. The RMS error between the discrete deflection and the integral solution is presented in Figure 3.3 for the 20 percent deflection case. From this figure it was determined that an order of 60 provides an RMS error, below 10^{-3} , which was deemed to be sufficient for the purposes of this thesis. This order is used throughout and was found to produce good results with reasonable computation times.

Figure 3.4 is a measurement based validation of the shape of a membrane deflected by a single electrode zone to near the electrostatic instability. The data was obtained by interferometric imaging of a backside patterned membrane two mm in diameter. Agreement between measurement and model is quite good which suggests that the physical membrane is well described by the simple membrane equation.

The deflection behavior of the membrane as a function of voltage, Figure 3.5, also corresponds well to predicted and observed solution curves. The solutions shown in this figure are extended beyond the electrostatic instability by augmenting the system

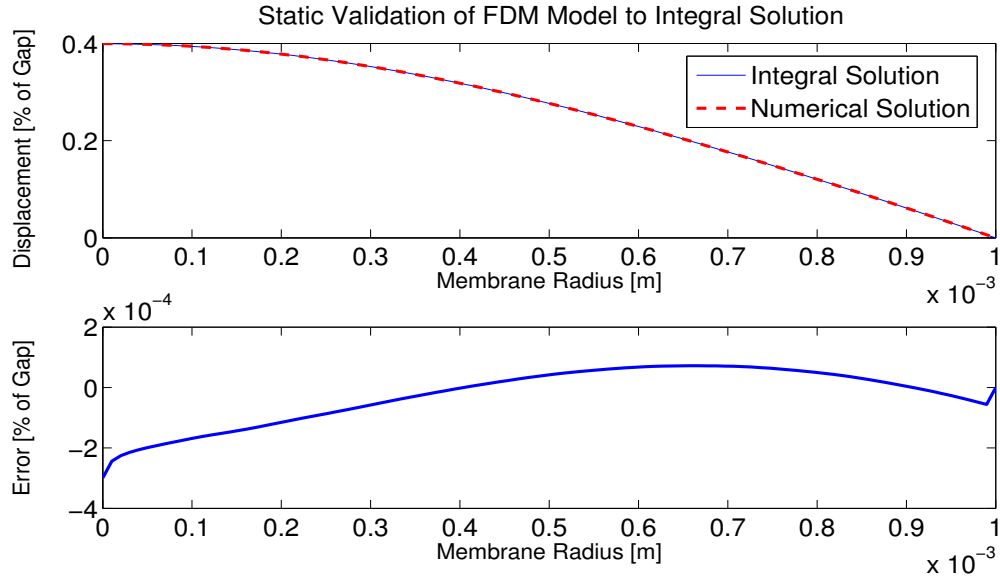


Figure 3.2: Numerical membrane models at 40 percent deflection

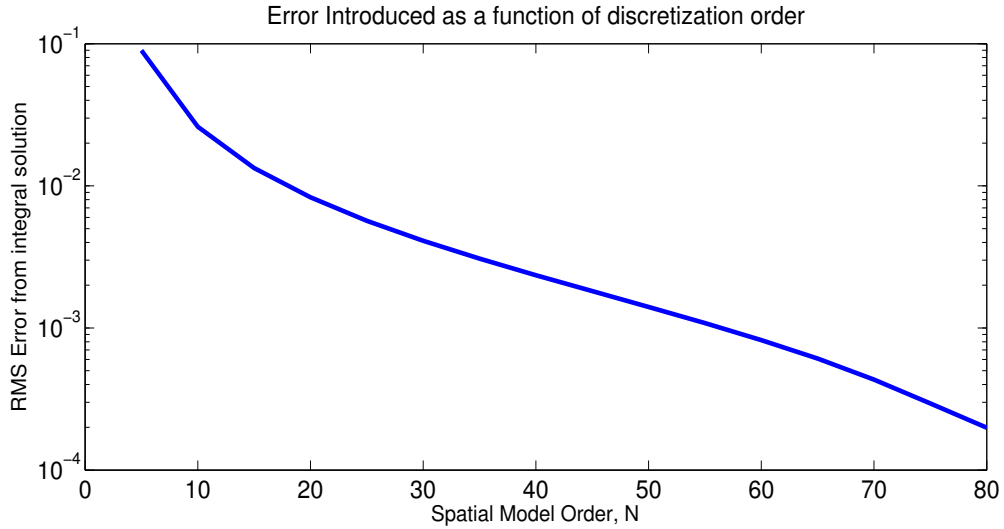


Figure 3.3: RMS error as a function of spatial model order.

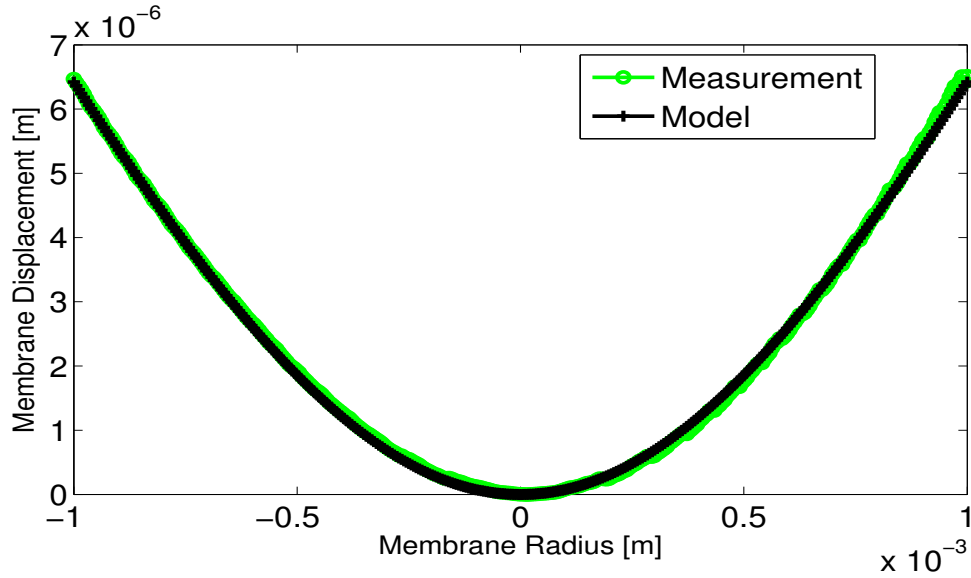


Figure 3.4: Measured and simulated profile of single zone actuated two mm membrane with a $25 \mu\text{m}$ gap thickness.

with an appropriately designed lead-lag type controller. The maximum observed deflection with this capacitive feedback stabilizing controller was 84%, which is further than the limits that have been achieved in practice using a similar control design. In [11], Lukes et. al achieved a stable deflection of 61% of the gap thickness.

Dynamic Validation

The full dynamic behavior predicted by the model may be captured by evaluating the magnitude and phase of the capacitance response resulting from a small signal actuating voltage. These dynamics are validated against those measured of a physical device in this section.

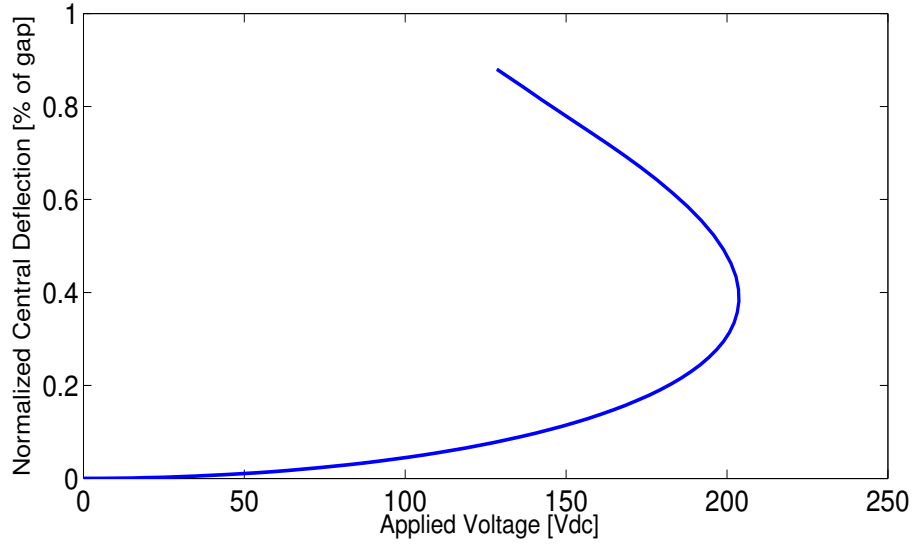


Figure 3.5: Deflection as a function of applied voltage under closed-loop control

Model Dynamics

The dynamic behavior of the model is measured by first augmenting the system with a small signal actuating voltage superimposed upon the biasing potential. The system is then solved by one of the usual methods. Typically the ODE23T algorithm, which is an implementation of the trapezoidal rule using a free interpolant, is used. This method is well suited to the application as it provides sufficient accuracy with rapid computation times and since the problem is only moderately stiff. The solution to the time evolution of the membrane shape is then converted into a time varying capacitance by (2.19). Finally, the relative magnitude and phase of the capacitance signal are compared to the actuation signal and recorded. This same test was performed under three different biasing conditions; 50, 100 and 150 V. The simulated membrane was two mm in diameter with a $30\text{ }\mu\text{m}$ gap, had a residual stress of 30 MPa and the damping coefficient was set to $1e10$. Results are presented in Figure 3.6, and seem to indicate a second-order type system as is expected by the parallel

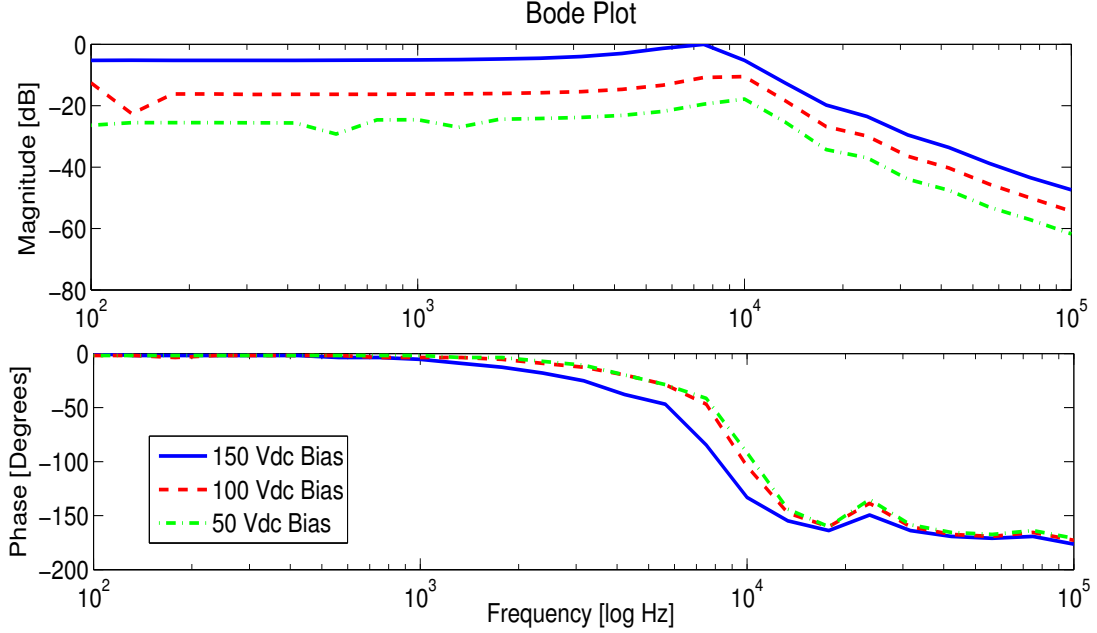


Figure 3.6: Magnitude and phase plots indicating changing dynamics with stroke.

plate model and observed experimentally in [10]. The nonlinear electrostatic force becomes increasingly strong as the membrane approaches its actuating electrode, and so the observed increase in magnitude is to be expected. Also, the bandwidth or knee position of the response seems to lower in frequency and become less “sharp” with increasing stroke. This indicates that the poles of the second-order linear approximation are separating and is suspected to be due to electrostatic spring softening.

Measured Dynamics

A laser source and a photodetector were used to measure the frequency response of a physical membrane with a 3 mm diameter, a $25\text{ }\mu\text{m}$ gap and residual stress of 20 MPa. This was achieved by shining a collimated beam onto the device at a small angle away from the normal. Some portion of the light is reflected off of the mirrored surface and is then incident on a photodetector diode through an aperture. As the

membrane deflects and changes the focus of the beam, more or less light makes it through the aperture and is incident on the photodiode resulting in a change in the measured voltage. The membrane is deflected to some operating point and a small sinusoidal perturbation is applied to the membrane. This results in small changes to the deflection and thus the observed output voltage across the diode. By measuring the magnitude and phase of this response with respect to the applied signal the magnitude and phase response of the system are determined.

The transfer function from membrane deflection to the voltage measured across the photodiode is expected to be only weakly nonlinear, since the diode is operating in the linear photoconductive mode and only small excursions are made to the nonlinear membrane. Thus, a small-signal linear approximation is assumed to be valid. Figure 3.7 illustrates the experimental setup used to perform this test.

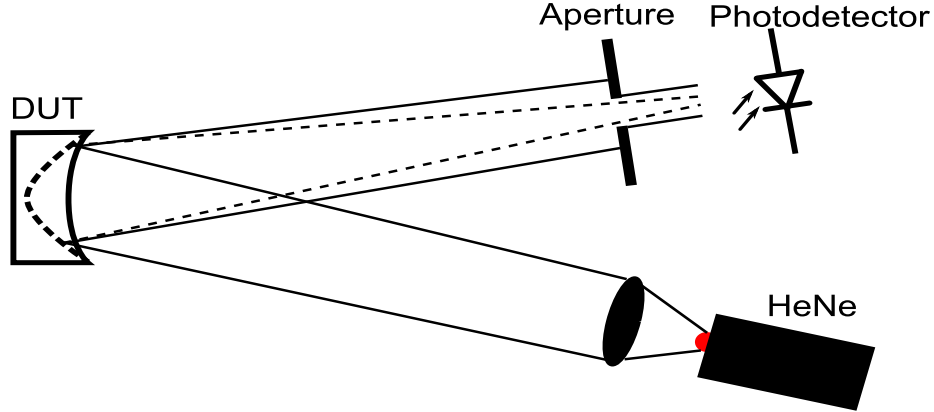


Figure 3.7: Optical frequency response measurement setup.

Figures 3.9 - 3.13 represent measured and simulated dynamic responses for three membranes that are identical except for their damping characteristics. The variable damping is achieved by using different air hole patterns perforated into the backplate of the device. This air venting technique has proven to be much less restrictive than the radial channels used in previous devices. In simulation the different damping

conditions are modeled by altering the damping coefficient, D , in (2.18). The damping coefficient is matched to the measured response by hand. The physical parameters used for these tests are $S_o = 30\mu\text{m}$, $R_o = 1\text{ mm}$, $\sigma = 2\mu\text{m}$, $\rho = 1200\text{ kg/m}^3$, and the residual stress in the membrane is 30 MPa.

The first test case is depicted in Figure 3.9 is for a three mm membrane with a lightly perforated backplate. Figure 3.8 presents the perforation pattern which consists of a ring of 8 holes patterned into the bottom of the gap at a .5 mm radius. The holes are each $50\mu\text{m}$ diameter. The response appears overdamped with a -3 dB corner frequency of approximately 5 kHz. The damping coefficient was determined to be $\approx 10^{11}$.

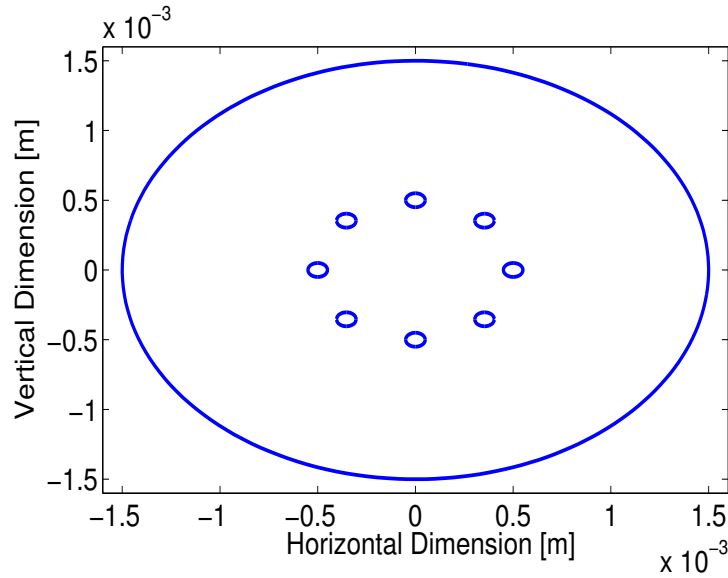


Figure 3.8: Perforated backplate exhibiting a single ring of 8 holes placed at .5 mm radius.

Figure 3.11 demonstrates the response of a moderately perforated device that has two rings of 8 $50\mu\text{m}$ holes positioned at .5 mm and 1 mm radii which is presented in Figure 3.10. The damping coefficient is decreased in the simulation and matched to the response of the measured device again by a trial and error method. This time the

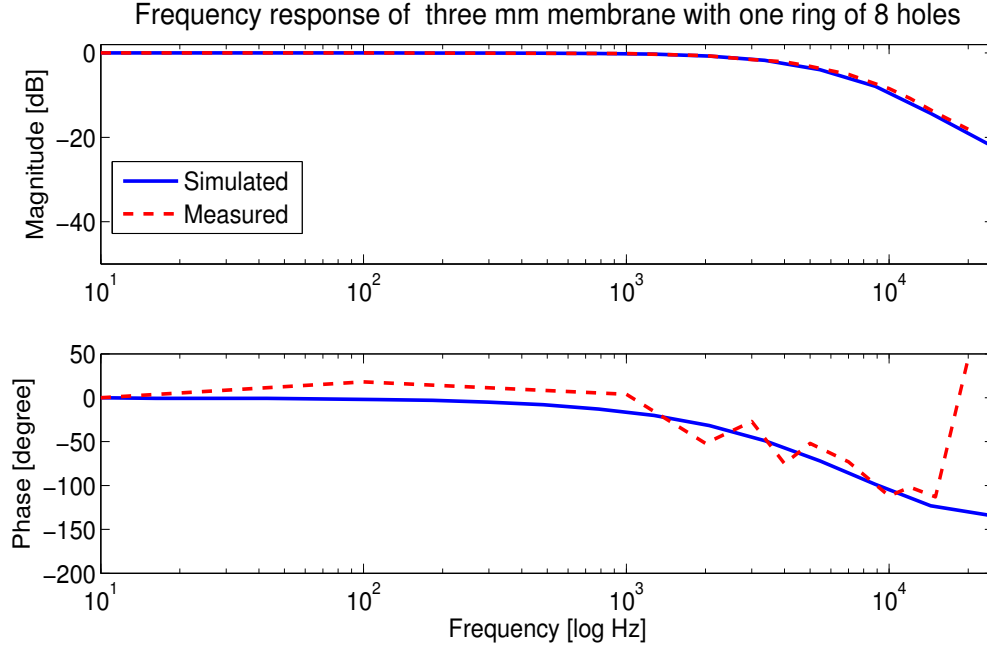


Figure 3.9: Frequency response of three mm membrane biased at 175 V with one ring of 8 holes perforated into backplate.

damping coefficient was found to be $\approx 4 * 10^{10}$. The frequency response is extended slightly and appears to be closer to a critically damped condition than the first case.

Finally, Figure 3.13 represents the measured and simulated frequency response for a device with two rings of 8 holes each placed at .5 mm and 1 mm. An approximation of a slot has also been patterned into the backplate near the perimeter that consists of a ring of 72 holes placed at a 1.4 mm radius. Again, each hole has a diameter of $50 \mu\text{m}$. An illustration of the perforated backplate is shown in Figure 3.12 The affect this slot has on the damping is much more significant than the incremental difference between the one and two ring cases. This is likely due to the fact that a much larger number of holes has been added for this test. The response has become underdamped and exhibits some peaking around 10 kHz. The damping coefficient was determined to be $\approx 1.15 * 10^{10}$.

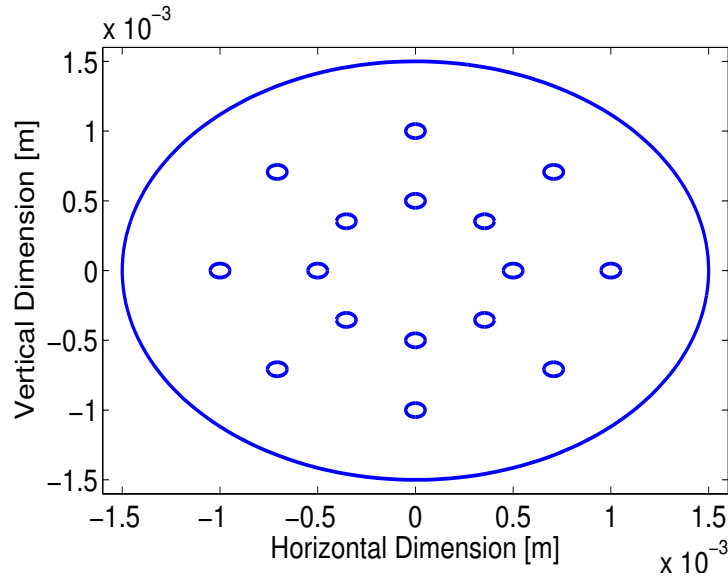


Figure 3.10: Perforated backplate exhibiting two rings of 8 holes placed at .5 mm and 1 mm radii.

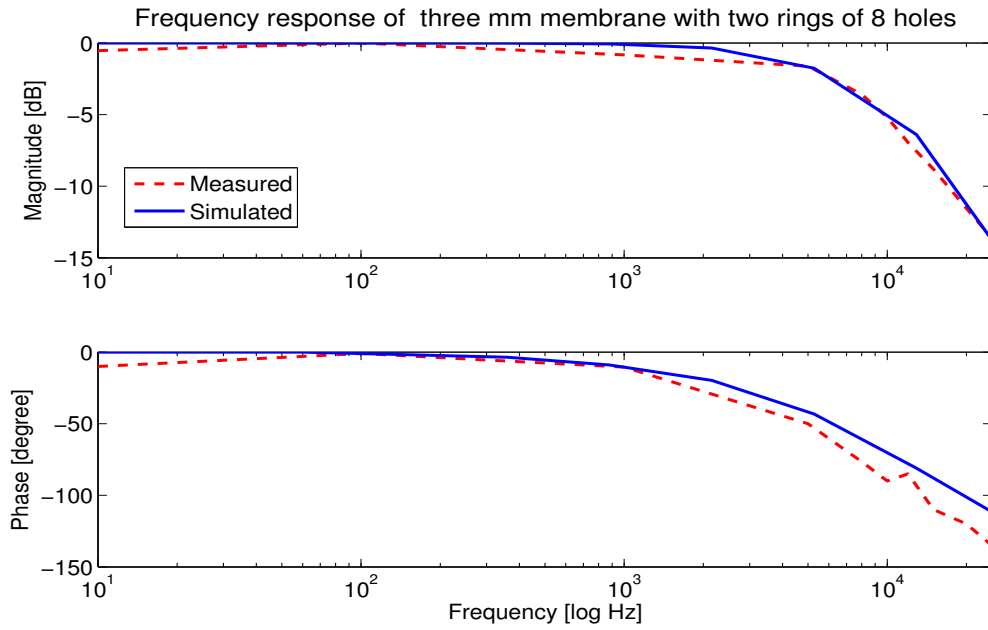


Figure 3.11: Frequency response of three mm membrane biased at 175 V with two rings of 8 holes perforated into backplate.

Overall these results indicate agreement with the expected second order linear approximation and the effect of decreasing the damping is as expected.

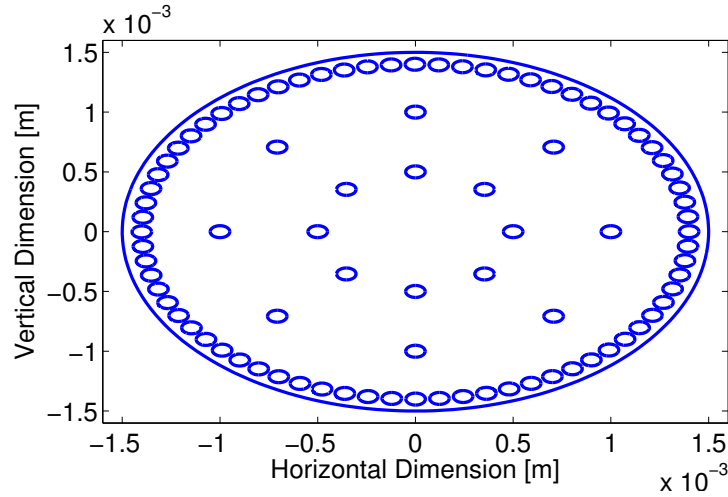


Figure 3.12: Perforated backplate exhibiting two rings of 8 holes placed at .5 mm and 1 mm radii and one ring of 72 holes placed at a 1.4 mm radius to approximate a slot port.

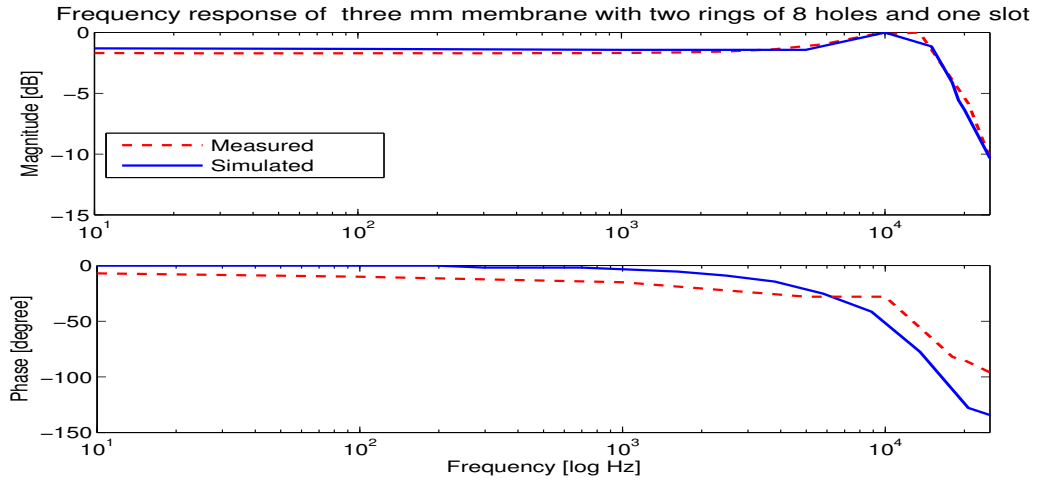


Figure 3.13: Frequency response of three mm membrane biased at 175 V with two rings of eight holes and ones slot of 72 holes perforated into backplate.

EXAMINATION OF THE MODEL

Time Domain Solutions

The time domain behavior of the membrane in a numerical environment can provide design insights that would otherwise be unavailable. As a demonstration of functionality, the step response under OL and CL conditions is simulated. Figure 4.1 represents a two mm diameter membrane subjected to a 250 volt step under open loop operation. The vertical axis represents the deflection as a function of radius (diagonal axis projected out of page) and time (horizontal axis). Figure 4.2 represents the capacitance that results from this deflection. The response indicates a time constant of about $150 \mu\text{s}$, which corresponds to a three dB bandwidth of about 6.5 kHz.

Figure 4.3 shows the time evolution of the membrane subjected to a step response under CL conditions resulting in 250 V across the membrane. The membrane has been controlled with a lead-lag type compensator with the zero placed at three kHz and a pole at 30 kHz with a gain of $1 * 10^{14} \text{ V/F}$. The closed loop step response indicates a steady state deflection of 60 percent, which is 3μ past the electrostatic instability point. It also indicates that the membrane response has been accelerated, with a closed loop time constant of about $70 \mu\text{s}$ or three dB bandwidth of 14 kHz. This demonstrates a few of the advantages of closed-loop control over open-loop actuation methods and that the numerical model can be used for control purposes.

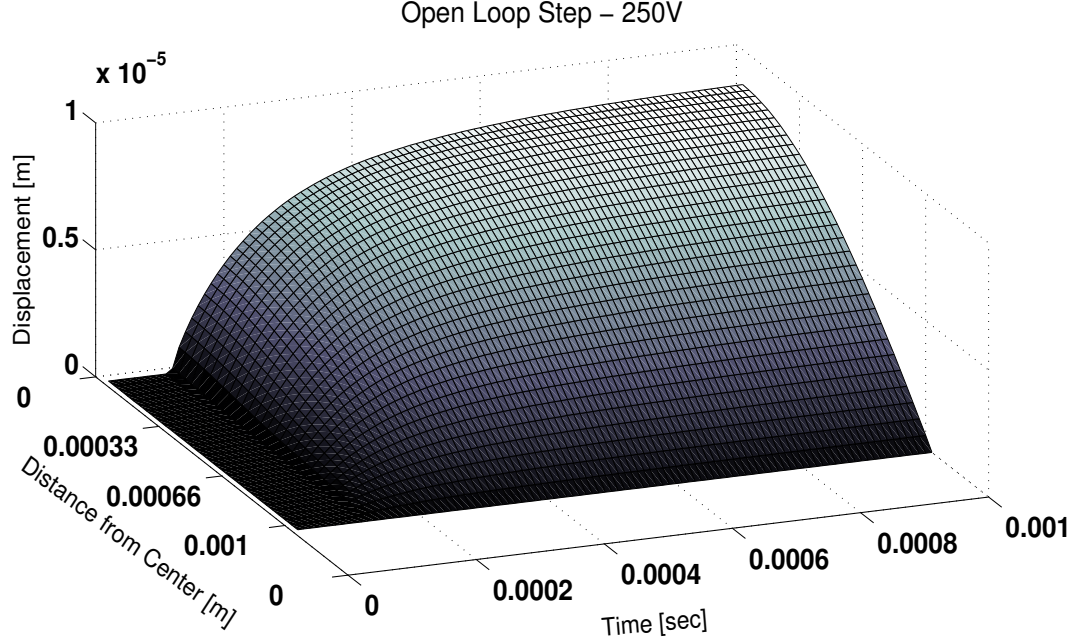


Figure 4.1: 250V step response applied to a two mm membrane under open-loop operation. Physical parameters are $S_o = 20\mu\text{m}$, $R_o = 1\text{ mm}$, $\sigma = 2\mu\text{m}$, $\rho = 1200\text{ kg/m}^3$, Stress = 30 MPa, $D = 4 * 10^{10}$

Parabolic Optimization

Under single-zoned actuation the membrane profile becomes more conical and less parabolic as the excursion increases, resulting in spherical aberration. In order to mitigate this effect, additional concentric electrodes may be added towards the perimetry of the device. As the membrane deflects and begins to lose its parabolic shape, the potential on this outer actuation zone is increased accordingly to compensate. The desired result is that the outer edge of the membrane is deflected more strongly than under single-zoned actuation and thus the overall shape becomes more parabolic and less conical, reducing the induced spherical aberration. The optimal number of electrode regions required to meet some target aberration, and the desired potentials at these electrodes is non-obvious. By modeling the deflection behavior

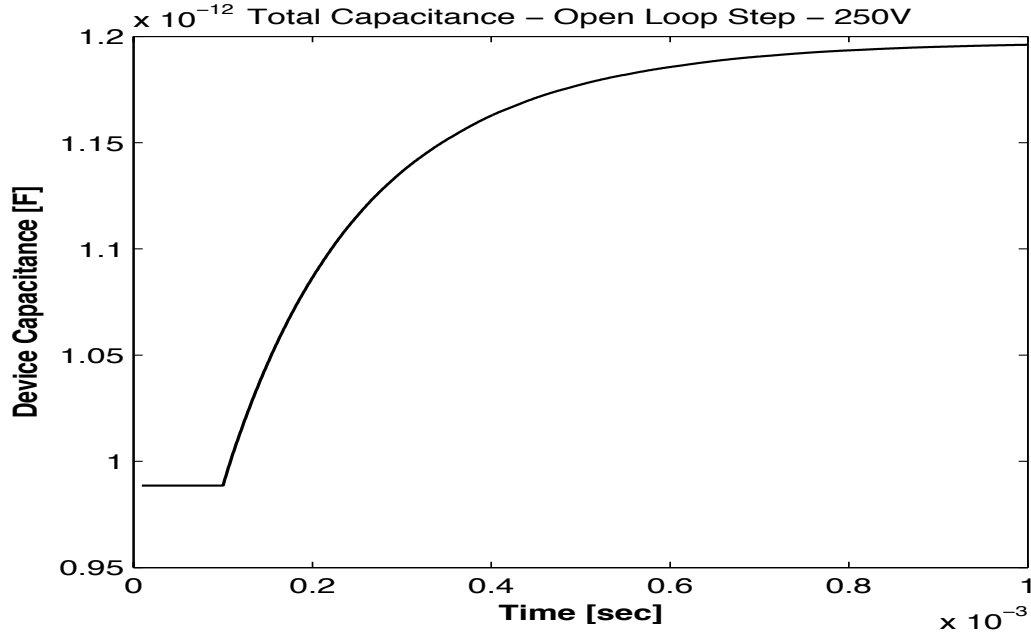


Figure 4.2: Capacitive output for 250V step response applied to a two mm membrane under open-loop operation.

numerically, optimization techniques may be easily applied to determine both the optimal potentials applied to the electrodes in order to minimize aberration, and also the number of electrode regions required to meet a target aberration figure.

A penalty equation is composed that compares the steady state deflection of the membrane to a best fit parabola with weighting by area. This penalty equation is minimized by means of the FSOLVE routine within Matlab, which solves a system of equations by minimizing an error function by the Newton method. The result is a predicted voltage trajectory for the outer zones to follow based upon the potential applied to the central zone. These voltage functions could be easily stored in a look-up table and used to drive the outermost zones while closed-loop control is implemented on the center electrode. The result is an optimally parabolic deflection given the spacing of the actuating electrodes.

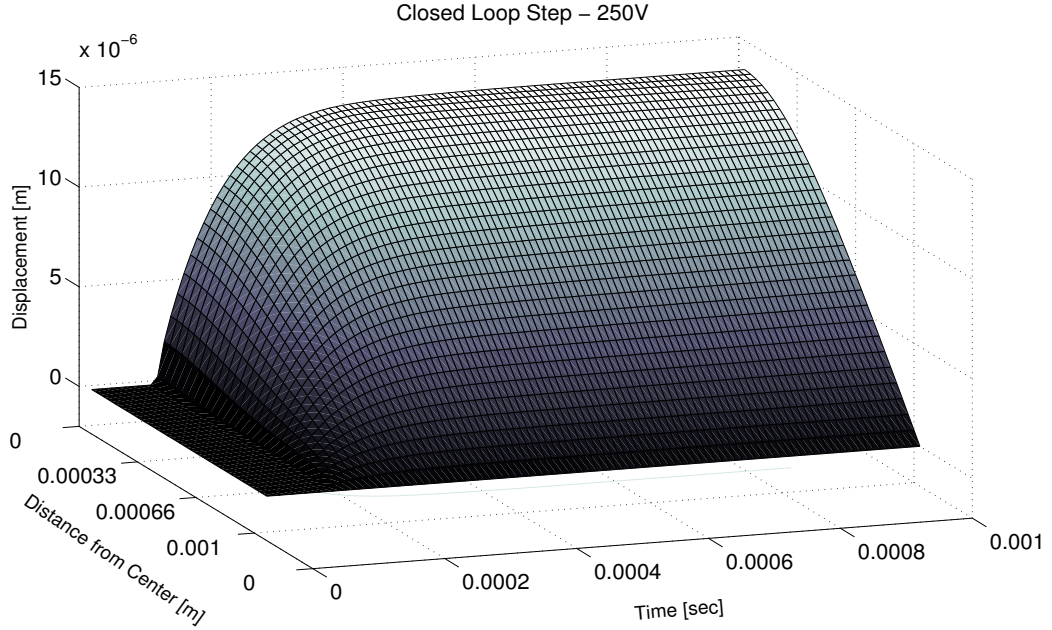


Figure 4.3: 250V step response applied to a two mm membrane under closed-loop operation. Physical parameters same as in 4.1

Figure 4.4 shows such an optimization for a three zone membrane. The electrode radii are equally spaced and a parabolic optimization routine was performed at five volt increments up to the electrostatic instability point. The membrane is two mm in diameter with a $30\text{ }\mu\text{m}$ gap thickness. The optimal outer electrode voltages begin to diverge significantly from that of the center electrode around 200 volts, which corresponds to a displacement of about five percent of the available deflection.

Figures 4.5, 4.6, 4.7, 4.8 are simulations of membrane deflection and residual error from parabolic at the optimal voltages prescribed by Figure 4.4 with center electrode potentials of 100, 200, 300, and 315 volts respectively.

Figure 4.9 represents the deflection of the same membrane near the electrostatic instability with a uniform potential of 315 volts applied to each electrode. The results

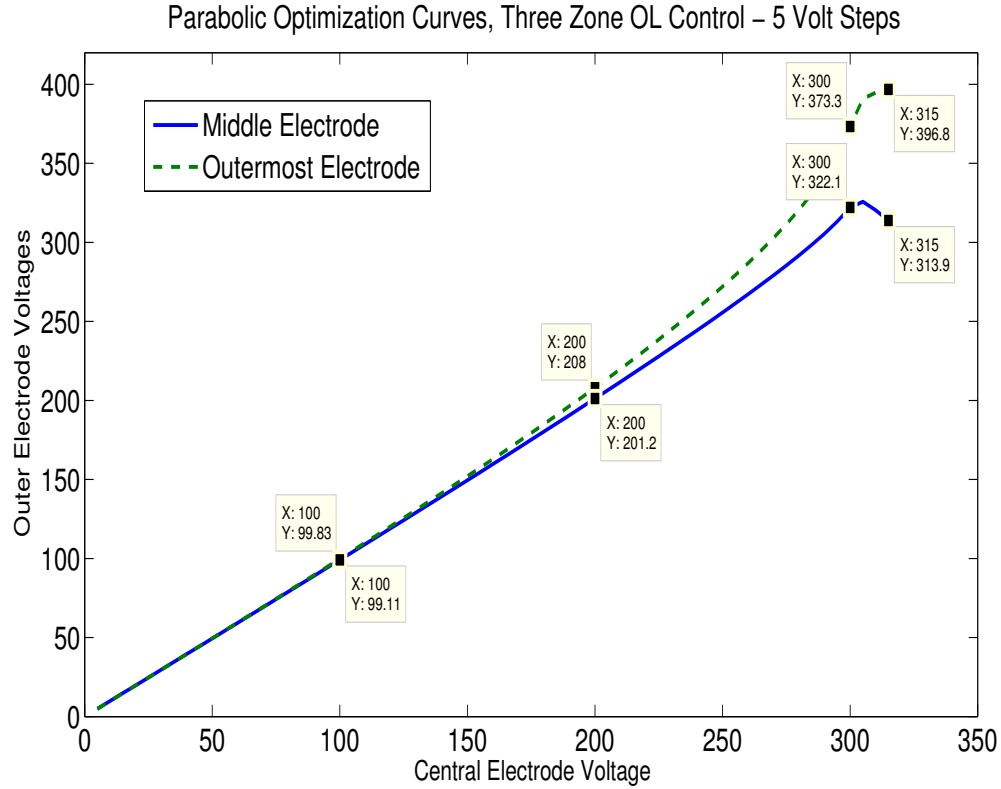


Figure 4.4: Parabolic Optimization Curves for a Membrane with a two mm Diameter and $30\ \mu\text{m}$ Gap Thickness.

indicate that the parabolic error is 100 percent greater at points along the membrane surface in this case.

Finally, Figure 4.10 represents the residual parabolic error in a membrane near snap-down with evenly spaced electrodes and with voltages upon these electrodes optimized by the process described above. Different curves correspond to different number of electrodes. As expected, the membrane may be made more parabolic by the inclusion of additional actuating electrodes.

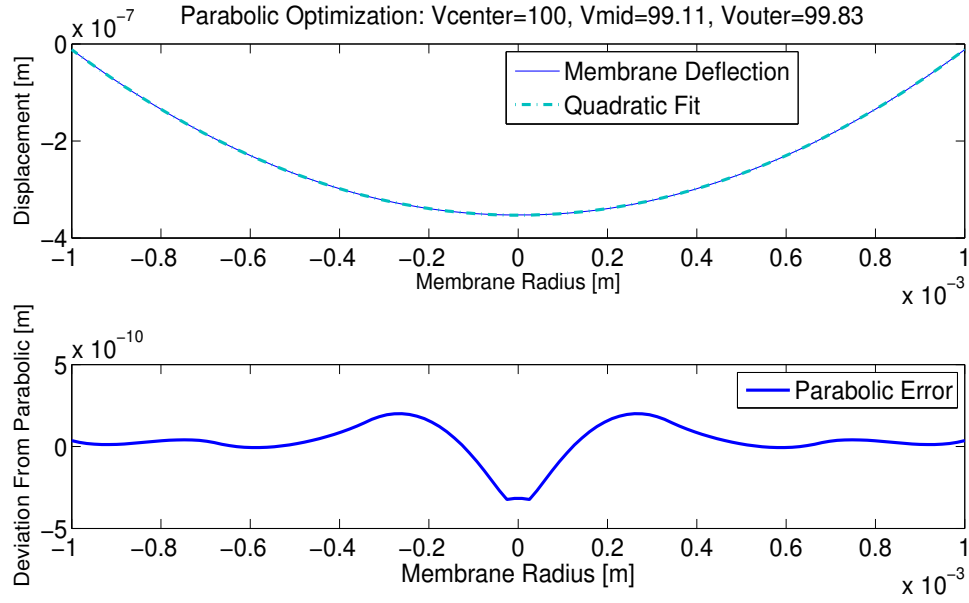


Figure 4.5: Optimally Parabolic Deflection with 100 Volts on Center Electrode.

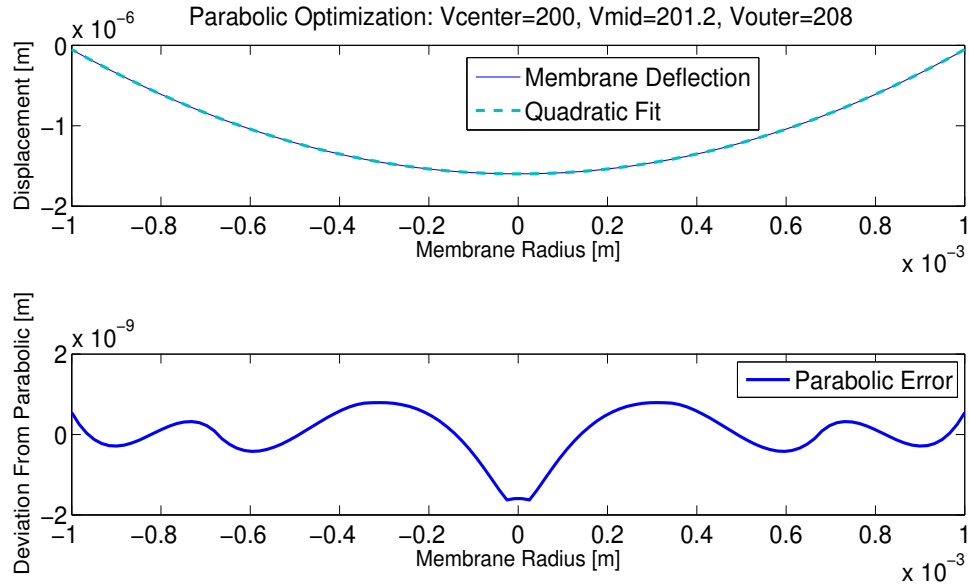


Figure 4.6: Optimally Parabolic Deflection with 200 Volts on Center Electrode.

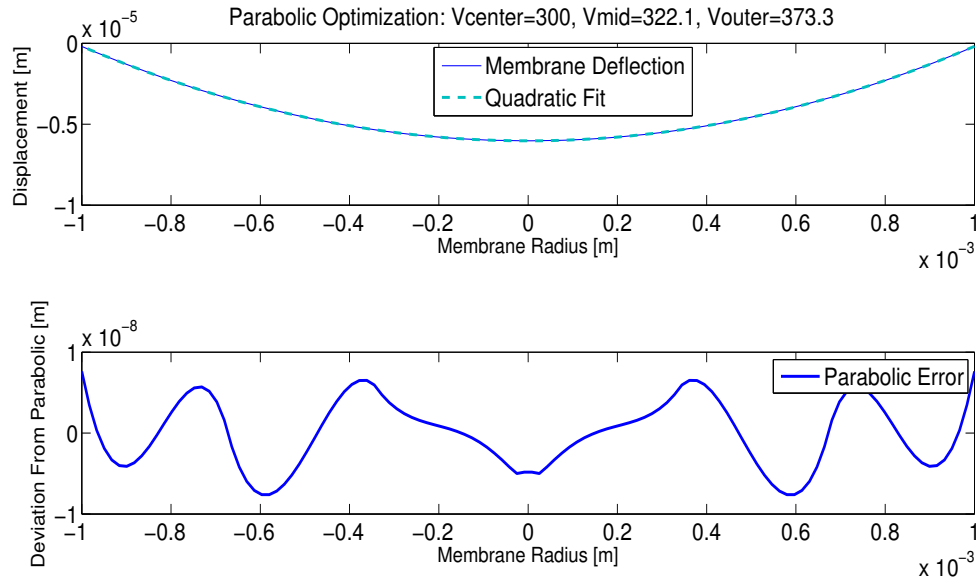


Figure 4.7: Optimally Parabolic Deflection with 300 Volts on Center Electrode.

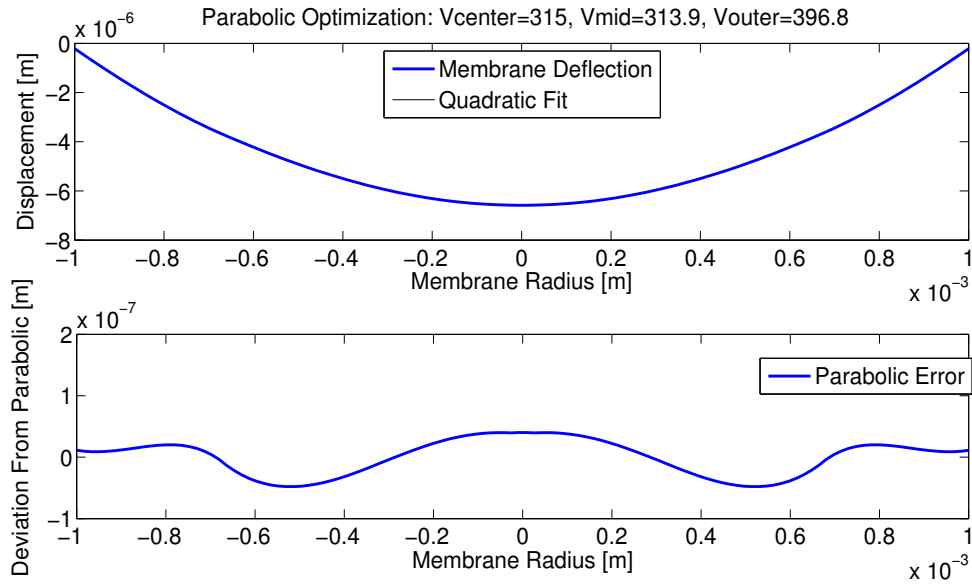


Figure 4.8: Optimally Parabolic Deflection with 315 Volts on Center Electrode.

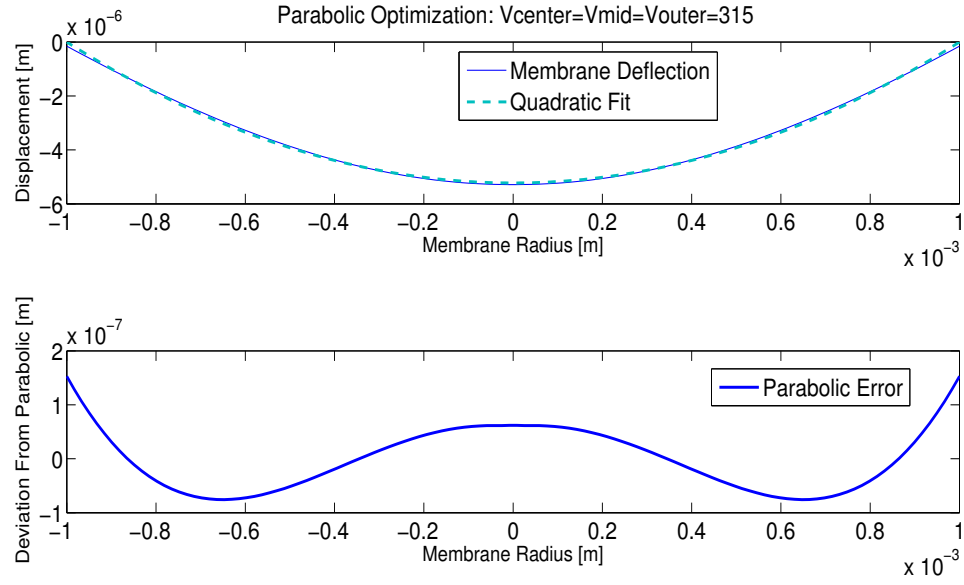


Figure 4.9: Non-parabolic Deflection with Uniform 315 Volt Actuation.

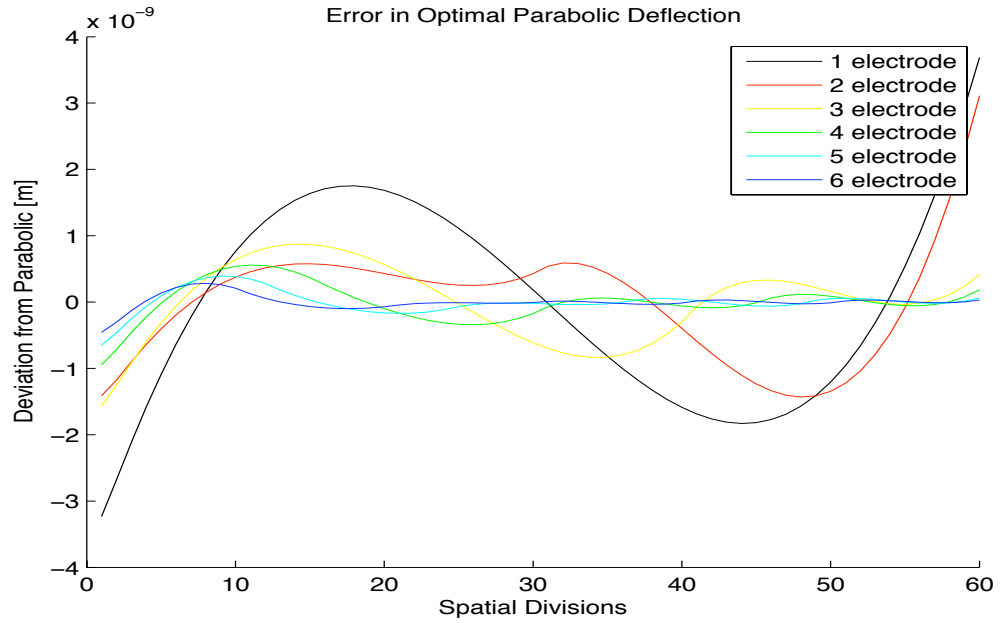


Figure 4.10: Residual Parabolic Error as a Function of the Number of Actuation Electrodes for a 2 mm Membrane Operated Near the Electrostatic Instability Point.

POSITION FEEDBACK TECHNIQUES

One critical task associated with closed-loop position control of deformable membrane mirrors is accurately measuring the position of the membrane in real time. This chapter will address optical and electronic sensing strategies. Optical sensing involves using light to gauge depth by making use of interference effects of light, and electronic sensing involves relating the changes in membrane position to observable changes at the actuating electrode terminals, namely a change in capacitance. This position measurement is the signal used for identification and feedback control purposes.

Optical Sensing

Optical position sensing is based on the principle of interferometry, which is illustrated in Figure 5.1(a). First, light from a coherent source is passed through a beam splitter. One of the resultant beams is reflected back into the beam splitter by a fixed mirror. This is known as the reference arm. The second beam is passed through a microscope objective onto the membrane mirror, which again reflects the light back through the beam splitter via the objective. As the two beams converge, they interfere with one another and form a fringe pattern composed of zones of constructive and destructive interference. Finally, this pattern is captured with a CCD. Figure 5.1(b) shows a typical interference pattern obtained from a deflected membrane mirror in this manner. The device is a 2mm membrane with a $11\mu\text{m}$ gap thickness that has been deflected under closed loop control to $7.3\mu\text{m}$ or about two thirds of the gap thickness [19]. The process of optical position sensing such the one described in Figure 3.7 relies on an additional light source to measure the deflection of the device, and outside of the small signal regime has a great deal of nonlinearity. The increased optical complexity

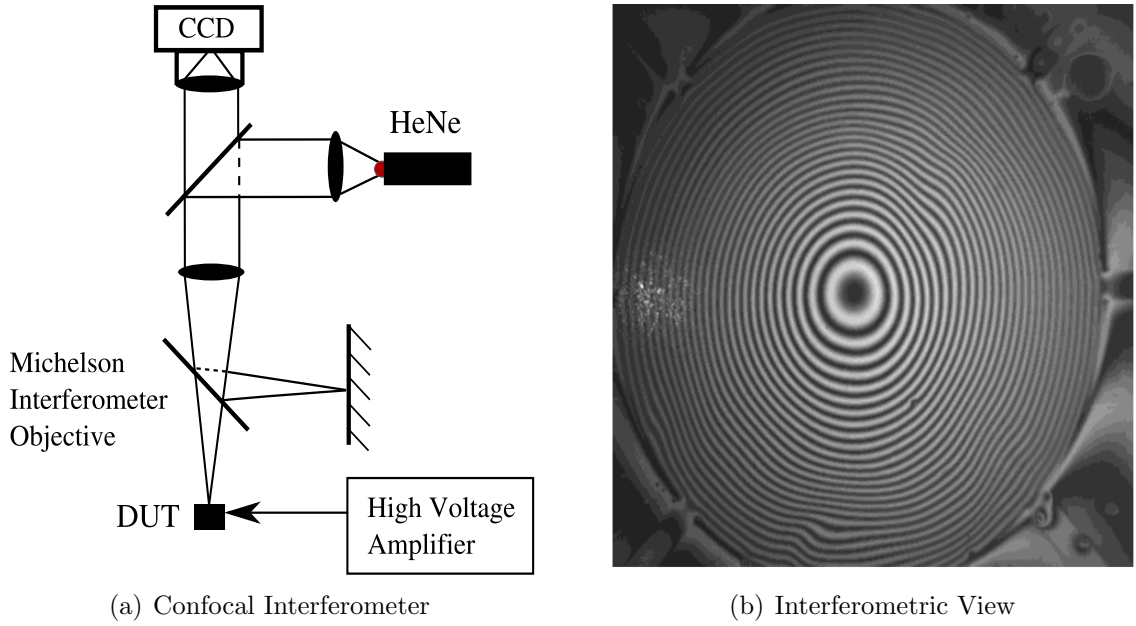


Figure 5.1: Illustration of confocal Michelson interferometer (a) a typical interference pattern for a deflected two mm diameter membrane (b).

leads to increased cost as well as difficulties with miniaturization and the nonlinearity makes this a less than ideal sensing strategy. As such, a different sensing technique was desired.

Electronic Sensing

Another method of estimating the position of the membrane within the gap is to measure the capacitance of the device from its electrical actuating terminals and then to refer this measurement back to the position of the membrane. By the parallel plate model the capacitance is linearly related to the position of the membrane. In practice the bending motion of the membrane results in some nonlinear behavior in this regard, but the effect is relatively small. Figure 5.2 represents a three mm membrane deflected over 70 % of its gap and the resulting capacitance measured at

the terminals showing some small nonlinear behavior represented by the curvature of the line. This figure also illustrates the total capacitance change that may be expected for a three mm device over its full operating range, which is about 6 pF. It is proposed in this thesis that a measurement of the device capacitance may function as an effective position measurement.

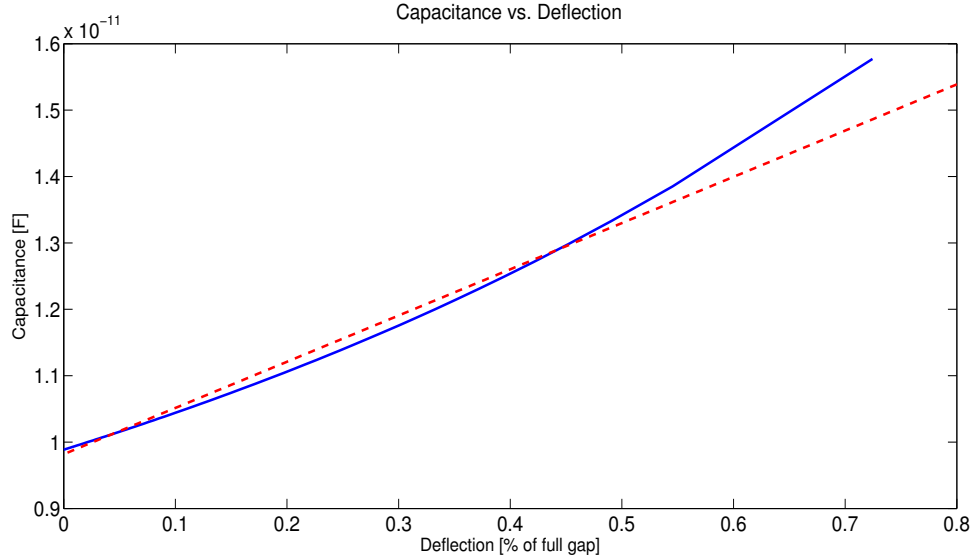


Figure 5.2: Simulated device capacitance as a function of membrane (center) deflection for a 3 mm membrane.

For this measurement to be useful for closed-loop control beyond the open-loop instability point, the measurement bandwidth must be considerably higher than the mechanical bandwidth of the device being controlled. This guarantees that should the membrane begin to become unstable that it can be actuated back into stability faster before it can snap down. The typical mechanical response of membranes such as those discussed in this thesis begin to drop off precipitously between 1-10kHz. Control theory would suggest that a measurement bandwidth of 10-100kHz would be required for closed-loop control. An analytical description of this requirement is that

if the membrane were about to become unstable, a controller must be able to adjust the actuation before the instability occurs.

Advantages of this technique are that the mirror is free to be utilized in application without the need for it to be simultaneously incorporated into an interferometer. This technique is also free from the limitations imposed by the processing required to convert an interference pattern into a position measurement.

Homodyne Sensing

The first proposed method of measuring the capacitance of the device relies on the variable detuning or "pulling" of an electronic oscillator by the changing capacitance of the device. This oscillator could be either a harmonic oscillator in which the capacitance of the device is used in place of a fixed capacitor in a tank circuit, or a relaxation oscillator detuned from its free-running-frequency (FRF) by the addition of the device capacitance to one of its nodes.

Due to the high potentials required to deflect the membrane, using the device capacitance as part of a tank circuit in a harmonic oscillator is somewhat inconvenient. For this reason we chose to detune a three inverter ring oscillator by playing the device capacitance off of the output impedance of an inverter gate. As the capacitance of the device increases, more charge movement is required to achieve the transition voltage of the next inverter in the ring. Since the output impedance of the driving inverter is unchanging, the transition time increases. This results in an overall decrease in the frequency of the oscillator which may be sensed directly. However, the frequency is determined not only by the transition time of the inverter driving the device capacitance but also by the transitions of the other inverters in the ring. This means that about two thirds of the sensitivity of the detector is lost due to these other transition times when sensing the frequency of the oscillator alone.

Figure 5.3 illustrates a capacitive sensor based on a three inverter ring oscillator. V_{bias} represents the actuating potential that is applied to the device capacitor, C_{dut} through the current-limiting protection resistance R_p . The high voltage blocking capacitor, C_b protects the low voltage sensing circuitry from the high actuating potentials on the membrane, while simultaneously AC coupling the oscillator to the device capacitance. The resistance R_s represents the output resistance of $InvC$, and additional resistance may be added in series at this point to detune the FRF of the oscillator further.

Numerous logic families were considered for this application including the LS, HC, ACT families. The use of a programmable logic controller, Atmel 22v10, was also considered and implemented. Ultimately the ACT technology was chosen for further study due to its rapid transition time and low output impedance. These characteristics allow for a faster FRF. A fast FRF means that a smaller blocking capacitor, C_b may be used. This capacitor appears to be in parallel with the device capacitance, C_{DUT} , from the actuation terminals and thus limits the bandwidth of the system.

To improve the sensitivity of this detector, a circuit is developed that is sensitive only to the transition times associated with the inverter driving the device capacitance. During these transition periods, the input and output logic states of the driving inverter ($InvC$ as shown in 5.3) are the same, so by the addition of an exclusive-or (XOR) gate between the input and output of INVC, these transition times may be isolated. The truth table for an XOR gate is provided in Table 5.1, illustrating that the output of this gate is logic low only when its inputs are equivalent.

This time period corresponds to the time spent charging and discharging the device capacitance as is illustrated by Table 5.2.

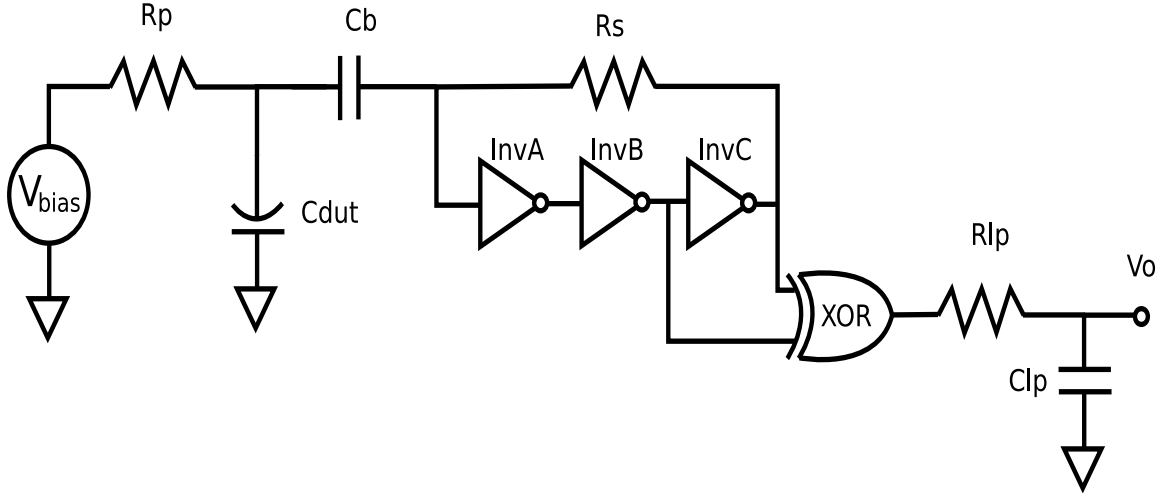


Figure 5.3: Homodyne capacitive position sensing circuit.

Table 5.1: Exclusive OR (XOR) Truth Table

$Input_1$	$Input_2$	$Output$
0	0	0
0	1	1
1	0	1
1	1	0

The waveforms for each node of the oscillator and the resultant negative-going duty width modulated output are presented in Figure 5.4. This output signal may then be easily demodulated by the simple addition of a low pass filter, represented in Figure 5.4 by R_{lp} and C_{lp} .

Though this circuit offers a relatively simple implementation of a capacitive sensor using only logic components, it is plagued by a number of issues which limit its performance. Among these issues is the fact that changing load (device) capacitance is not the only factor which affects the transition time of the inverters in the ring. Changes in the power supply for these devices as well as changes in temperature both affect the transition times of the logic gates as well. Thus, significant fluctuations in either of these parameters (and perhaps others which have yet to be identified)

Table 5.2: Ring Oscillator State Transition Table

State	$OutA$	$OutB$	$OutC$	XOR
	0	1	0	1
	1	1	0	1
charging C_{DUT--}	1	0	0	0
	1	0	1	1
	0	0	1	1
discharging C_{DUT--}	0	1	1	0

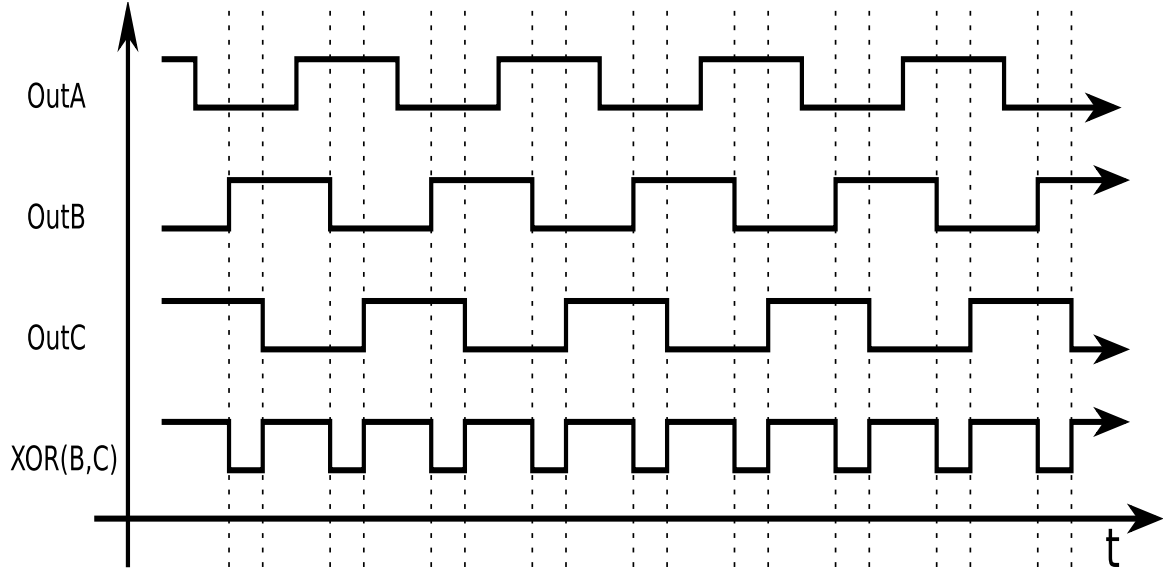


Figure 5.4: Signals of Single Oscillator Capacitive Position Sensing Circuit with Duty Width Modulation.

may erroneously be attributed to membrane motion. To minimize any measurement drift introduced by these effects, precision supply rails and temperature control are required, adding to the complexity of the system.

Heterodyne Sensing

An alternate capacitive sensing strategy makes use of the same principle of a detuned oscillator as described in the previous section and illustrated by $Inv1a - c$

in Figure 5.5, but also incorporates a second oscillator, represented by $Inv2a - c$, operating in close proximity.

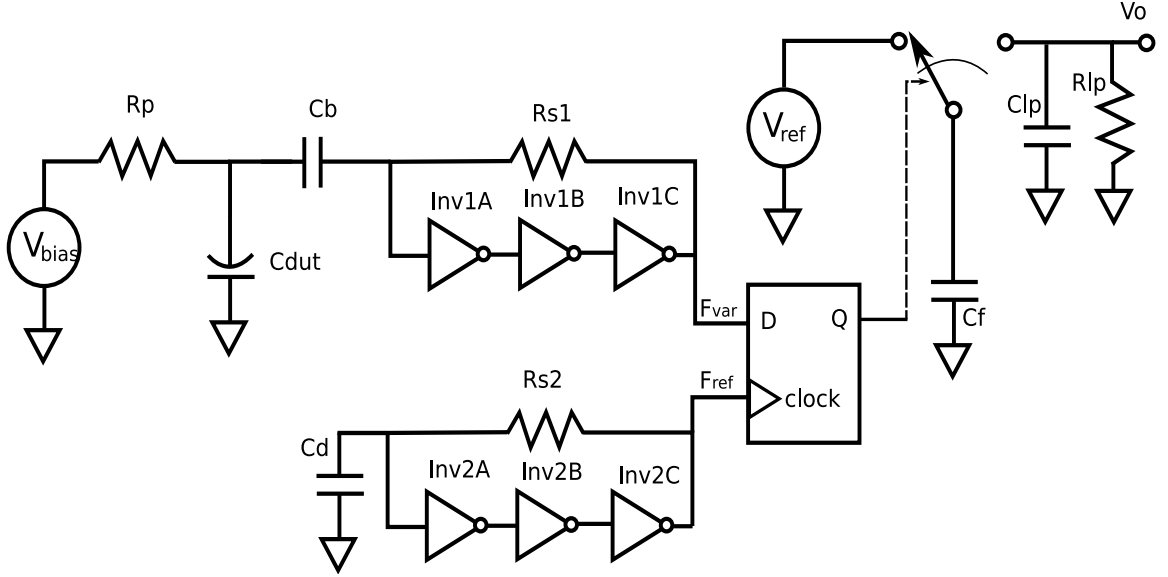


Figure 5.5: Heterodyne Capacitive Position Sensing Circuit.

The FRF of this second oscillator, f_{ref} , is determined by the detuning capacitance, C_d and any series resistance R_{s2} , but is unmodified in principle by the changing device capacitance. The reference oscillator is however equally affected by fluctuations in supply rails and by temperature, assuming any temperature gradient between the oscillators is negligible. The variable frequency, f_{var} is then mixed with f_{ref} by means of a D-latch to produce $f_{var} - f_{ref}$ by connecting one signal to the D terminal and one to the clock source. The truth table for a D-latch is provided in Table 5.3, and signals demonstrating this mixing strategy illustrated in Figure 5.6.

Table 5.3: D Latch Truth Table

D	$Clock$	Q_{next}
0	$\uparrow$	0
1	$\uparrow$	1
x	$else$	Q_{prev}

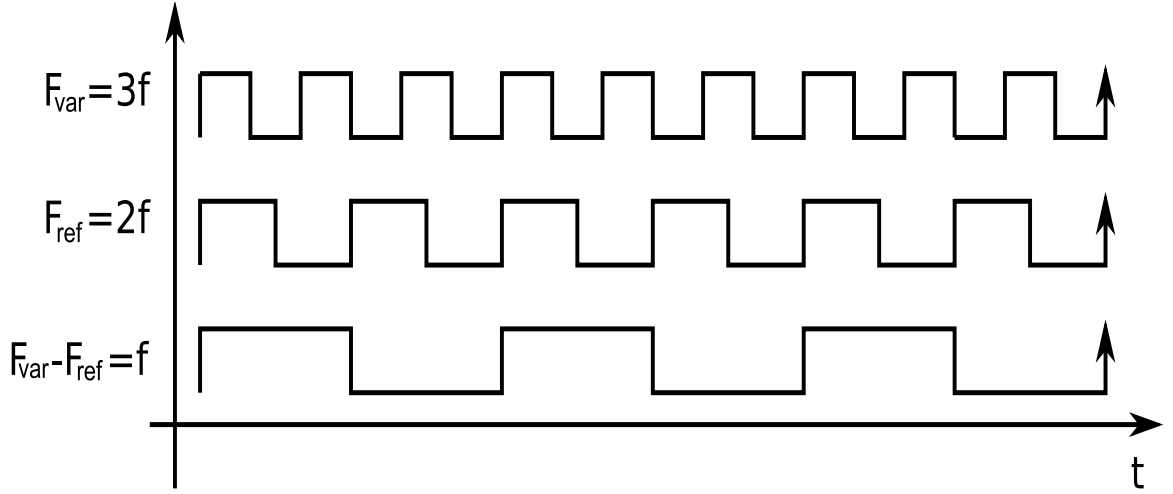


Figure 5.6: Example Heterodyne Sensing Circuit Signal: Frequency Subtraction by D-Latch.

The result is that any fluctuations imposed upon the oscillators will be common mode rejected by this process, and the resulting difference frequency then represents the capacitance measurement.

This difference frequency is demodulated into an analog voltage signal by means of a CMOS switch, a flying capacitor C_f , and a precision voltage source V_{ref} . The LTC1043C was chosen as a switch for its fast transition time and low resistance. Each time the flying capacitor is connected to the reference source, it is charged to that voltage. When the capacitor is then switched to its other state, the charge it has stored is transferred to the low pass output filter represented by C_{lp} and R_{lp} . By using the difference frequency resulting from the Q output of the D latch to toggle the switch, the resultant demodulated voltage at the output of the filter will be linearly related to this frequency.

Phase locking was identified as a significant issue associated with operating two or more oscillators in close physical proximity to one another. This is especially true if the oscillators are operated near the same frequency. Through various modes of

parasitic “communication” the oscillators will tend to lock onto precisely the same frequency, thus destroying the measurement. To mitigate this effect, the oscillators were detuned away from one another in frequency space. Further, it was found that phase locking can occur not just at a 1:1 frequency ratio, but, indeed, seems to occur to some degree at nearly every K:L ratio, where K,L are integers (rational numbers).

To illustrate this effect, two 3-inverter oscillators of the ACT logic family were constructed on separate, independently decoupled logic chips in close proximity (1 cm). One oscillator was detuned with a fixed 20 pF capacitor and the other with a 0-20 pF single-turn variable capacitor. The ratio of the two oscillators was recorded by means of an BNC Model 1105 frequency counter. As the variable capacitance was manually moved through its full range, fractional entrainment was observed at a number of rational ratios. The result of this test is represented in Figure 5.7. As expected, there is a very strong tendency for the oscillators to tend towards a one to one ratio, represented by the deep troughs at the right of the figure. This fractional entrainment tendency seems to become increasingly severe as the sum of L and K becomes smaller, for example $4/3$ (1.333) is a stronger entrainment than $13/9$ (1.444).

The primary modes by which this effect is thought to occur are through the common power supply, through parasitic capacitances between the oscillators. This phase locking tendency applies both to heterodyne sensing strategy as well as to multiple homodyne sensing circuits used to measure the various zones of a multizonal device.

Synchronous Sensing

Finally, a synchronous excitation and demodulation sensing scheme was devised for measuring the device capacitance. The excitation signal is applied to the actuating

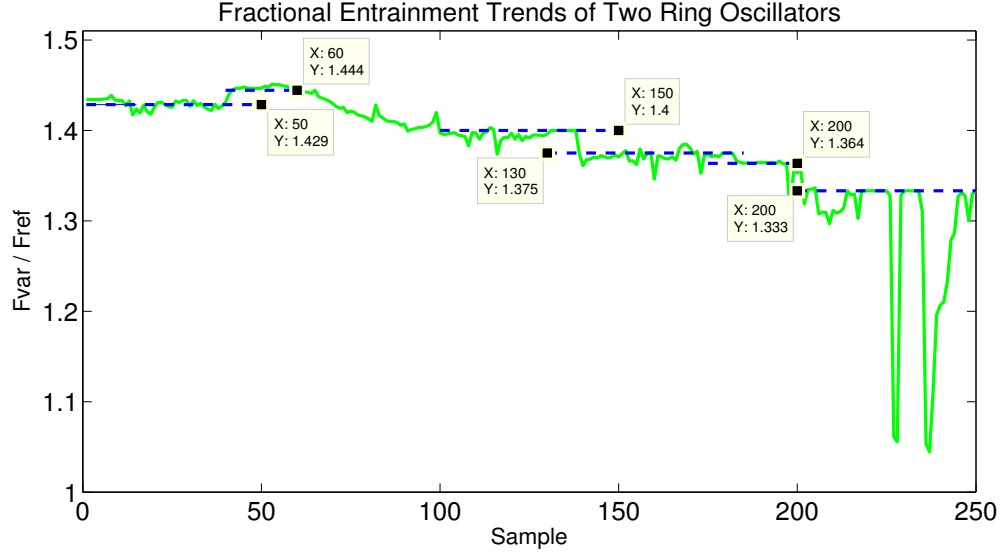


Figure 5.7: Fractional Entrainment in Two ACT Logic Ring Oscillators.

electrode of the membrane and consists of a both a slow biasing component and a fast excitation component. The biasing signal serves as a command input to the membrane, resulting in some desired excursion as determined by the controller. The excitation component is selected to be fast with respect to the mechanical bandwidth of the membrane. As a result the excitation signal will not affect the deflection of the membrane which would adversely affect optical performance.

The biasing voltage results in no steady state current through the device and the series connected sensing resistance, except through parallel leakage resistance. This leakage resistance was measured for a number of devices and was found to be >10 M Ω for each.

The current resulting from the applied excitation is synchronously measured by way of a sensing resistance. From its magnitude and phase with respect to the applied signal, the complex impedance of the membrane may be determined. This complex impedance may then be converted into a measure of the membrane capacitance and

therefore the membrane's position within the gap. Figure 5.8 illustrates the basic components of the synchronous detection scheme just described.

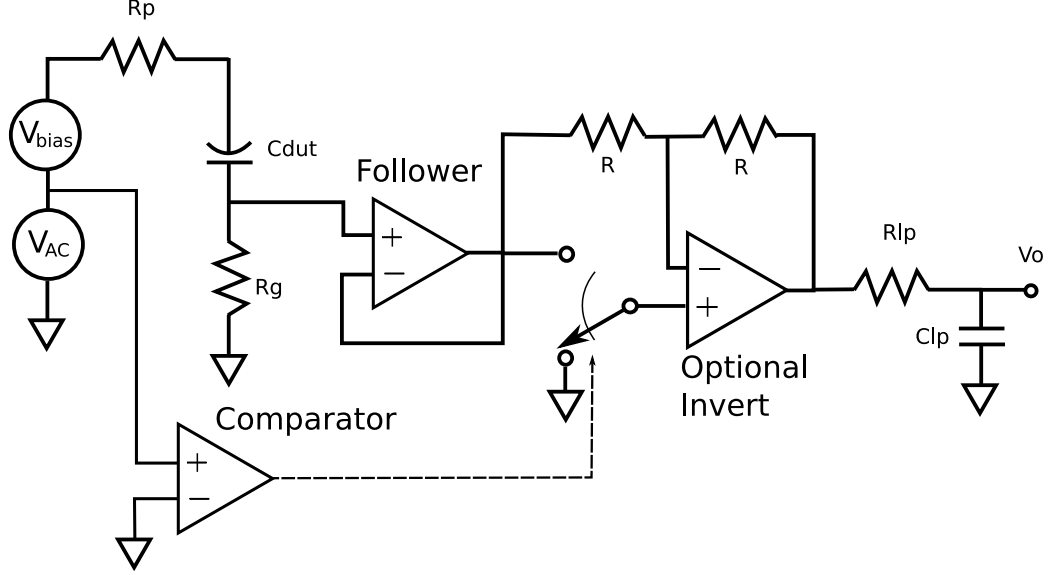


Figure 5.8: Synchronous Capacitive Sensing Circuit.

The sources V_{Bias} , and V_{AC} represent the DC and AC components of the excitation signal respectively. The sum of these sources, $V_{Total} = V_{Bias} + V_{AC}$ is applied to the terminals of the membrane, C_{DUT} , through a current-limiting protection resistance, R_p as in previous sensing techniques. The DC component is blocked by the device capacitance and produces no measurable voltage drop across the sensing resistance, R_g . However, the AC component of the signal results in a time-varying current, I_{cap} , which in turn produces a time varying potential, V_g , across R_g .

Since the current through the device capacitance is given by

$$I_{cap} = C_{DUT} \frac{dV_{total}}{dt}, \quad (5.1)$$

and the voltage measured across the sensing resistor, R_g , is

$$V_g = I_{cap} R_g, \quad (5.2)$$

then the voltage resulting from the capacitor current is

$$V_g = C_{DUT} R_g \frac{dV_{total}}{dt}. \quad (5.3)$$

The total rate of voltage change due to both sources is

$$\frac{dV_{total}}{dt} = \frac{dV_{AC}}{dt} + \frac{dV_{Bias}}{dt}, \quad (5.4)$$

where

$$\frac{dV_{AC}}{dt} = A_{AC} \omega_{AC} \cos(\omega_{AC} t), \quad (5.5)$$

and

$$\frac{dV_{Bias}}{dt} = A_{Bias} \omega_{Bias} \cos(\omega_{Bias} t); \quad (5.6)$$

and where

$$V_{AC} = A_{AC} \cos(\omega_{AC} t), \quad (5.7)$$

and

$$V_{Bias} = A_{Bias} \cos(\omega_{Bias} t). \quad (5.8)$$

And so

$$\frac{dV_{total}}{dt} = A_{AC} \omega_{AC} \cos(\omega_{AC} t) + A_{Bias} \omega_{Bias} \cos(\omega_{Bias} t). \quad (5.9)$$

Finally,

$$V_g = C_{DUT} R_g (A_{AC} \omega_{AC} \cos(\omega_{AC} t) + A_{Bias} \omega_{Bias} \cos(\omega_{Bias} t)), \quad (5.10)$$

is the equation describing the output voltage measured across R_g as a function of the AC and DC components of the excitation signal.

To accurately measure the effect of the AC excitation upon the resulting current through the device, we must suppress the terms in 5.10 that result from any changes to the biasing condition and pick out the terms due only to the AC excitation. By

properly selecting the magnitude and frequency of the excitation, the magnitude of $A_{AC}\omega_{AC}$ can be made to dominate over $A_{Bias}\omega_{Bias}$. This tradeoff can be optimized by increasing either the magnitude, A_{AC} , or frequency, ω_{AC} . In this thesis the requirements upon the magnitude and frequency of the biasing signal are assumed to be largely fixed by the physical parameters of the device under test.

The high input impedance voltage following amplifier, *Follower*, translates the voltage measured across R_g to its output and the rest of the circuit while maintaining a high impedance at the node between C_{DUT} and R_g . This amplifier is placed in close proximity to the device as a preamplifier and also used as a cable driver, sending a copy of the measured voltage to the demodulation circuitry.

A lock-in amplifier is chosen as a demodulation scheme in order to further isolate the desired excitation from any spurious signals that may be introduced by the biasing conditions, and also from noise. The lock-in amplifier consists of a zero-crossing detector and an optionally inverting amplifier. The zero-crossing amplifier detects the crossings of the zero-mean excitation signal, V_{AC} , and produces a zero-mean square wave output. This square wave is used as a control signal to toggle whether the optionally inverting amplifier has a gain of one or a gain of negative one at a 50 % duty cycle. The result is that any signal which is not synchronous with the excitation experiences an amplifier with an average gain of zero. Any synchronous signal however experiences a gain of one while positive, for example, and a gain of negative one while negative. The result is that synchronous signals are rectified and passed through to the output while non-synchronous signals are attenuated. The exceptionally narrow bandwidth of the lock in amplifier makes it possible to recover the very small signal excitation signal applied to the terminals. Initial results indicate resolution on the order of hundreds of femtofarads which corresponds to a spatial resolution of tens of nanometers.

Multiple Channel Encoding

It is possible to detect the capacitance of multiple actuation zones of the same device by exciting each zone with a separate, asynchronous source and synchronously demodulating each as described above. However, one may also make use of two sources at the same frequency but separated in phase by ninety degrees, thus reducing the number of independent sources required and simplifying the implementation. The signal presented to one channel, $\text{Ex. } \sin(\omega_{AC}t)$, is rectified by the lock-in amplifier sensitive to the zero-crossings of $\sin(\omega_{AC}t)$ while the corresponding quadrature signal, $\cos(\omega_{AC}t)$ is nulled by the amplifier. The converse is also true for the second channel, and thus good channel separation can be achieved. Further, even harmonics of the fundamental may also be used for actuation of other channels. These harmonics will be blocked by the lock-in amplifier of the fundamental and vice-versa, providing channel separation.

On Relating Capacitance to Deflection

The total capacitance of the device serves as a useful and easily instrumented metric for determining the position of the membrane within its cylindrical backing cavity, or gap. There may be, however, a certain amount of error in relating this measurement to the position of the membrane. This is due to the fact that the capacitance measured at the electrical terminals is the aggregate sum of the parallel plate capacitance over the entire membrane, and does not reveal information about the position of the membrane at any specific radius from the center. As a result, one can imagine that there exist certain membrane shapes may result in the same measured terminal capacitance. Typically when the membrane is actuated slowly its shape follows a well-defined and repeatable trajectory, but as the speed of actuation

increases and begins to excite resonant behavior in the membrane, much more complicated deflection shapes result. This leads to an increase in the possibility of error in position relation from capacitive measurement.

As an illustration, the model is subjected to a 200 Vpp, 4 kHz excitation signal, Figure 5.9 multiplied by a 150 Vdc step function. The physical parameters are $S_o = 30\mu\text{m}$, $R_o = 1\text{ mm}$, $\sigma = 2.25\mu\text{m}$, $\rho = 1200\text{ kg/m}^3$, and the residual stress in the membrane is 35 MPa. The capacitance of the membrane is represented in Figure 5.10

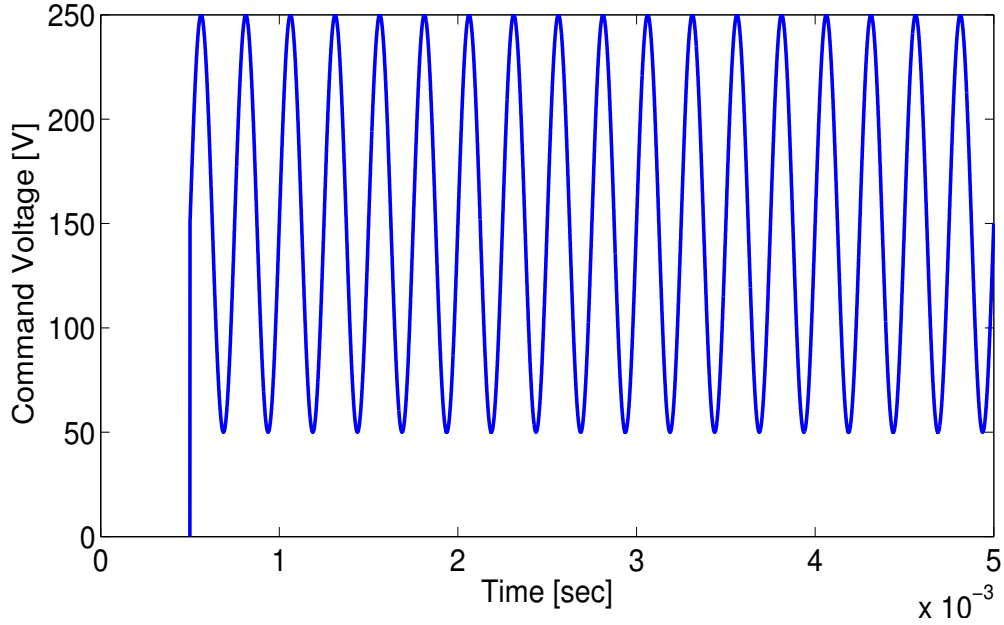


Figure 5.9: 4 kHz Sinusoidal Excitation Voltage.

by the vertical axis and the deflection at the center of the membrane placed on the horizontal axis. The repeatable trajectory of the function suggests that the shape of the membrane at this frequency is subject to little hysteresis. However, when the same test is performed at a frequency just beyond the first resonant mode of the membrane, 40 kHz, the result, as displayed in Figure 5.11, is quite different.

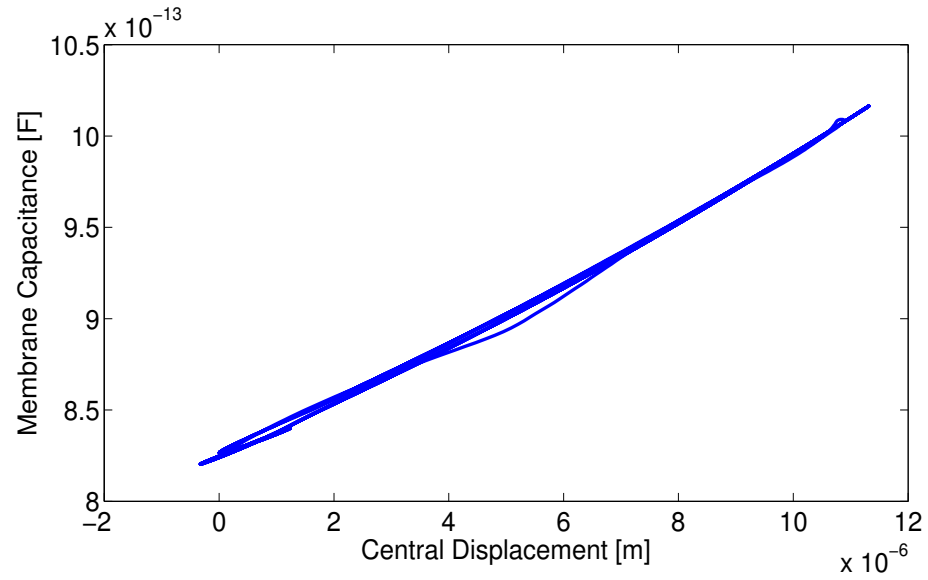


Figure 5.10: Capacitance as a Function of Central Deflection of Membrane at 4 kHz.

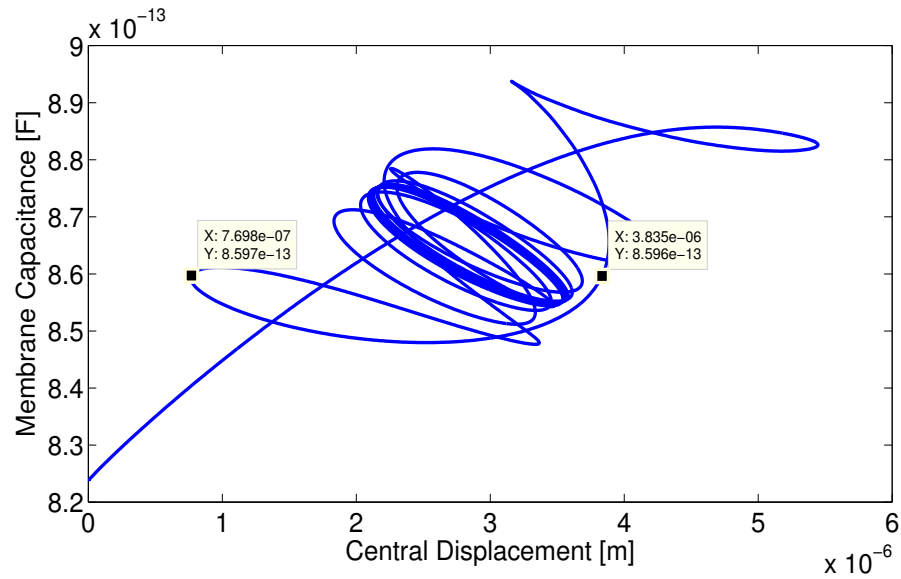


Figure 5.11: Capacitance as a Function of Central Deflection of Membrane at 40 kHz.

Clearly, as the bending motion of the membrane alters from the steady state predicted trajectory, hysteresis is induced resulting in sensor error at high frequency

excitations. The error associated with projecting the deflection from a measure of the capacitance is represented by the width of the loops in 5.11.

DISCUSSION

This thesis demonstrates a technique for numerically modeling of the deflection behavior of a MEMS deformable membrane mirror by the FDTD method. Results indicate that this technique may be useful for identifying the nonlinear behavior of the membrane under open-loop and closed-loop conditions without the error incurred by a linear parallel plate model. The numerical nature of the model also allows for identification and control simulation without the hazards associated with operating a physical device beyond the instability point with a suboptimal controller.

The steady state deflection predicted by the discrete membrane model was compared to the integral solution of the membrane equation as well as to the deflection measured of a physical device under single and multi-zonal actuation. The results show that a simple membrane model accurately predicts the deflection behavior of the thin membrane device and that the discrete formulation of the model provides a close approximation. The model has also demonstrated utility in the design of MEMS deformable mirrors by providing insight into how many zones may be required to achieve a desired level of parabolic optimality. Optimization techniques may be easily applied to the numerical model and used to determine optimal device parameters or actuation techniques. In this thesis the voltages required to achieve optimally parabolic deflection for a multi-zonally actuated device were determined by applying such optimization routines. Further, the minimum residual parabolic error was analyzed as a function of the number of radial electrode zones.

To control the membrane beyond its electrostatic instability, the dynamic behavior of the device must be known throughout the range of its stroke. Due to the increased tension in the membrane and the nonlinear nature of the actuating force, as the membrane deflects towards the actuating electrode these dynamics are subject to

change which suggests that a controller must be either robust to such changes or be able to adapt to them in real time. In this thesis the dynamic behavior of the membrane has been modeled in the time domain and used to determine the frequency response at a variety of biasing deflections. Simulated results indicate that the poles of the second order linear approximation of the membrane begin to slow significantly and separate in a non-obvious fashion as the stroke increases. These results are in close agreement with the measured open-loop frequency response of several devices with various damping conditions. Though only open-loop characterization is presented in this thesis, the membrane may be identified beyond the instability point by closing the loop and then separating the membrane dynamics from those of the controller during analysis.

Finally, to realize position feedback control in application the location of the membrane within the gap must be measured with good fidelity. This thesis has put forward a number of techniques that may be used to infer this position by measuring the device capacitance. Phase locking was observed with the homodyne and heterodyne sensing strategies put forward and ultimately led to the development of a synchronous sensing scheme similar to that used by an impedance analyzer. Although not extensively investigated in this thesis, initial results show that the sensitivity and bandwidth of this technique is sufficient to provide a good feedback signal not corrupted by the phase locking nonlinearity.

REFERENCES CITED

- [1] Bruce A. Macintosh, James R. Graham, Ben Oppenheimer, Lisa Poyneer, Anand Sivaramakrishnan, Jean Pierre Veran. Mems-based extreme adaptive optics for planet detection. *SPIE Conference 6113: MEMS/MOEMS Components and their Applications III*, 2005.
- [2] B. Jeffery Lutzenberger, Mohammad J. Moghimi, Sarah J. Lukes, Brant Kaylor, David L. Dickensheets. Mems deformable mirrors for focus control in vital microscopy. Wibool Piyawattanametha Harald Schenk, editor, *MOEMS and Miniaturized Systems IX, Proceedings of the SPIE Vol. 7594*, pages 61–68. SPIE, February 2010.
- [3] Nathan Doble, Geunyoung Yoon, Li Chen, Paul Bierden, Ben Singer, Scott Oliver, David R. Williams. Use of a microelectromechanical mirror for adaptive optics in the human eye. *Optics Letters*, 2002.
- [4] David L. Dickensheets, Marshall Overcast, Phillip Himmer, Victor X.D. Yang, I. Alex Vitkins. Focus tracking in time domain optical coherence tomography using membrane mirrors operated near snap-down. Wibool Piyawattanametha David L. Dickensheets, Harald Schenk, editor, *IEEE/LEOS International Conference on Optical MEMS and Their Applications Conference, 2006.*, pages 170–171, Big Sky, MT, August 2006. IEEE/LEOS.
- [5] Enrique Fernandez, Pablo Artal. Membrane deformable mirror for adaptive optics: performance limits in visual optics. *Optics Express*, 11(9):1056–1069, May 2003.
- [6] Lijun Zhu, Pang Chen Sun, Yeshaiahu Fainman. Aberration-free dynamic focusing with a multichannel micromachined membrane deformable mirror. *Applied Optics*, 38(25):5350–5354, September 1999.
- [7] Lijun Zhu, Pang Chen Sun, Dirk Uwe Bartsch, William R. Freeman, Yeshaiahu Fainman. Adaptive control of a micromachined continuous-membrane deformable mirror for aberration compensation. *Applied Optics*, 38(1):168–176, January 1999.
- [8] Curtis R. Vogel, Qiang Yang. Modeling, simulation, and open-loop control of a continuous facesheet mems deformable mirror. *J. of the Optics Society of America*, 23(5):1074–1081, May 2006.
- [9] Carl Paterson, Ian Munro, JC Dainty. A low cost adaptive optics system using a membrane mirror. *Applied Optics*, 6(9):175 – 185, April 2000.
- [10] Phillip A. Himmer, David L. Dickensheets. Dynamic behavior of high-speed silicon nitride deformable mirrors. Hakan Urey, editor, *MOEMS Display and Imaging Systems II*, pages 150–159, Bellingham, WA, January 2004. SPIE.

- [11] Sarah J. Lukes, Phillip A. Himmer, Eric J. Moog, Steven R. Shaw, David L. Dickensheets. Feedback stabilized deformable membrane mirrors for focus control. David L. Dickensheets, Harald Schenk, Wibool Piyawattanametha, editors, *MOEMS and Miniaturized Systems VIII, Proceedings of SPIE Vol. 7208*, pages 72080Q1–72080Q8. SPIE, 2009.
- [12] Janusz Bryzek, Eric Abbott, Anthony Flannery, David Cagle, Jacek Maitan. Control issues for mems. *Proceedings of the 42nd IEEE Conference on Decision and Control*, pages 3039–3047, Maui, Hawaii USA, December 2003. IEEE.
- [13] David L. Dickensheets. Requirements of mems membrane mirrors for focus adjustment and aberration correction in endoscopic confocal and optical coherence tomography imaging instruments. *Journal of Micro/Nanolithography MEMS MOEMS*, 7(2):8–1 – 8–9, April-Jun 2008.
- [14] Justin D. Mansell. Membrane deformable mirror frequency response. by Active Optical Systems, September 2010.
- [15] Miroslav Lukes. Comparison of vibration of two thin circular plates as moving electrodes of electrostatic transducers. *Audio Technologies and Processing*, 2002.
- [16] Thomas G. Bifano, Raji Krishnamoorthy Mali, John Kyle Dorton, Julie Perreault, Nelsimar Vandelli, Mark N. Horenstein, David A. Castañón. Continuous-membrane surface-micromachined silicon deformable mirror. *Society of Photo-Optical Instrumentation Engineers*, 36(5):1354–1360, May 1997.
- [17] Linda M. Miller, William J. Kaiser, Thomas W. Kenny, Michael L. Agronin, Robert L. Norton. Fabrication and characterization of a micromachined deformable mirror for adaptive optics applications. *SPIE*, pages 1–15, 1999.
- [18] Erwin Dunbar, Matthew Leone, Sarah Lukes, David L. Dickensheets. Polymer deformable membrane mirrors for focus control using su-8 2002. *Optical MEMs and Nanophotonics, 2008 IEEE/LEOS International Conference on*, pages 138–139, Freiburg, Germany, August 2008. IEEE/LEOS.
- [19] Brant M. Kaylor, Peter Roos, Jeffrey Lutzenberger, Jason R. Dahl, David L. Dickensheets. Novel mems deformable mirror for focus control and aberration correction. *Imaging Systems*, page IWD2. Optical Society of America, June 2010.
- [20] Guchuan Zhu, Jean Levine, Laurent Praly. Improving the performance of an electrostatically actuated mems by nonlinear control: Some advances and comparisons. *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference 2005*, pages 7534–7539, Seville, Spain, December 2005. IEEE/European Control Council.

- [21] Guchuan Zhu, Jean Lévine, Laurent Praly, Yves-Alain Peter. Flatness-based control of electrostatically actuated mems with application to adaptive optics: A simulation study. *Journal of Microelectromechanical Systems*, 15(5):1165–1174, October 2006.
- [22] Guchuan Zhu, Julien Penet, Lahcen Saydy. Robust control of an electrostatically actuated mems in the presence of parasitics and parametric uncertainties. *Proceedings of the 2006 American Control Conference*, pages 1233–1238, Minneapolis, MN, June 2006.
- [23] Robert C. Anderson, Balasaheb Kawade, Kandiah Ragulan, D. H. S. Maithripala. Integrated charge and position sensing for feedback control of electrostatic mems. *Proceedings of the SPIE 5765*, San Diego, CA, March 2005. SPIE.
- [24] Karl F. Graff. *Wave Motion in Elastic Solids*. Dover Publications, 1991.
- [25] Microchem. *SU-8 2000 Permanent Epoxy Negative Photoresist*.
- [26] Pei Bai Zhou. *Numerical Analysis of Electromagnetic Fields*. Springer-Verlag, 1993.

APPENDICES

APPENDIX A

MATLAB SOURCE CODE

```

function [C,X] = MM_SS_OL(Vin,Xinitialize)
% MM_SS_OL - Open Loop Membrane Model at Steady State
%
% syntax:
%[C, X] = MM_SS_OL(Vin)
% C = Capacitance [units=F]
% X = Profile Vector (of length 2N)[units=m]
% Vin = Input Voltage Vector (of length N) [units=V]
%
% all other parameters of the simulation performed
% in this function are entered manually in 'hard_code'
%
% This function computes the steady state deflection and
% resulting capacitance measured at the terminals

%Declare global parameters%
global N;          %number of spatial divisions [#]%
global Xo;         %Rest gap thickness [m]%
global Ro;         %Radius of Membrane [m]%
global eps1;       %Permittivity of membrane [F/m]%
global eps2;       %Permittivity of gap [F/m]%
global Kdamp;      %Damping constant [arbitrary units]%
global Sigma;      %thickness of membrane [m]%
global Tens;       %Tension in membrane [N]%
global d;          %Density of membrane [Kg/mexp2]%

```

```

global delR;    %ring width [m]%
global Co;      %rest capacitnace [F]%
global Area;

%%
%some initial business to take care of%
%populate parameters from hard_code section%
hard_code
%find spacing of divisions%
delR=RoN;
%find area of each division%
Area=area();
%inititalize shape to zero deflection%
Xinit=0*ones(N,1);
%find initial (resting) capacitance%
Co=capacitance(zeros(N,1));

%%
% Working Code goes here

V=voltage_vector_even([Vin]);
X=ShapeFinder(Xinitialize,V);
C=capacitance(X);

```



```

return;

%end working code

%%

%% %Functions Section %%

function [X] = ShapeFinder(Xinit,V)

%Setup options for shape Fsolve%

options2=optimset('Display','iter','TolFun',1e-6,'TolX',1e-6,'MaxFunEvals',1e3);

%Fsolve for steady state membrane solution%

.[X,Fval] = fsolve(@membrane_equation,Xinit,options2)

end %function "ShapeFinder"

%%

function [Area] = area()

for k = 1:N

Area(k)=pi*((k*delR)exp2-((k-1)*delR)exp2);

end %Area For

end %function "area"

%%

function [Y] = membrane_equation(X)

Y=((Tens).*laplacian(X)+pressure(X,V));

```

```
end %function "membrane.equation"
```

```
%%
```

```
function [P] = pressure(X,V)
```

```
%this function computes the electrostatic pressure
```

```
%experienced by the membrane
```

```
P=zeros(N,1);
```

```
Q=zeros(N,1);
```

```
for k=1:N
```

```
Q(k)=V(k)/(((Xo-X(k))/eps2)+(Sigma/eps1));
```

```
P(k)=((1/2)*Q(k)exp2)/eps2;
```

```
P(k)=(V(k).exp2.*eps1)./(2.*(Xo-X(k)).exp2);
```

```
end %for
```

```
end %function "pressure"
```

```
%%
```

```
function [Lap] = laplacian(X)
```

```
%this function generates the discrete laplacian
```

```
%of X in cylindrical coordinates
```

```
A(1,1)=-2;
```

```
A(1,2)=2;
```

```
b(1)=0;
```

```
for i=2:N-1
```

```
A(i, i-1:i+1) = [1-1/(2*(i-1)) -2 1+(1/(2*(i-1)))];
```

```
b(i)=0;
```

```

end %for
A(N,N) = -2;
A(N,N-1)=1;
b(N) = 0;
b = b';
const=(1/(2*(delR).exp2));
%Solve for Laplacian of X
Lap=const.*(A*X+b);
end %function "laplacian"
%%

function [D] = damping(dX)
for k=1:N
%calculate damping at R(k) for all k%
D(k)=Kdamp*Area(k)*dX(k);
end %for
D=D';
end %function "damping"
%%

function [C] = capacitance(X)
%set initial capacitance to zero
C=0;
%determine permittivity
Epsprime=1/(1/eps2+1/eps1);

```

```

%sum up parallel plate capacitances
for k = 1:N
C=C+(Area(k)*Epsprime)/(Xo-X(k));
end %for

end %function "capacitance"

%%

function [] = hard_code()

%manually Set device and simulation parameters here%
N = 180; %number of spatial divisions [#]
Xo = 21e-6; %gap thickness [m]
Ro = 1e-3; %radius of membrane [m]
Stress = 80e6;
Tens = 5.96*Stress*pi*Roexp2; %built-in membrane tension [N]
eps1 = 8*8.85e-12; %permittivity of membrane [F/m]
eps2 = 1*8.85e-12; %permittivity of gap [F/m]
Kdamp= 1e17; %damping coeff *!arb units!*
Sigma= 3e-6; %membrane thickness [m]
d = 1200*Sigma; %density of membrane [kg/mexp2]

end %function "hard_code"

%%

```

```

function [Vout] = voltage_vector_even(Velectrode)

%Create evenly spaced voltage vector from vector of voltages%
%check if N/length(Velectrode) is divisible
if rem(N,length(Velectrode))==0;
for j=1:length(Velectrode)
for k=1:N/length(Velectrode)

Vout((k-N/length(Velectrode))+j*N/length(Velectrode))=Velectrode(j);
end %for
end %for
else
disp('Error!')
disp('For even spacing the number of spatial divisions')
disp('must be evenly divisible by the number of electrodes')
end %if else
end %function even_voltage_vector

%%%

%End Functions Section

%%%

end %Shape_Finder

```

APPENDIX B

FOUR CHANNEL HOMODYNE SENSING PCB LAYOUT

In a multi-zonally actuated device, it is desired to independently measure the position of the membrane above each electrode. Prototyping techniques proved inconvenient when attempting to incorporate several devices in close proximity while maintaining channel isolation. As such, a printed circuit board consisting of four homodyne sensing circuits was developed. Instrumentation preamplifiers and cable driving amplifiers for each channel were incorporated into this board design as well.

A four layer board was chosen for this application. The sensing circuits were all placed on the top layer and the amplifiers on the bottom, such that the ground and power layers in between them act as isolation. Figures B.1, B.2, B.3, B.4 show the individual board layers in the final design of this multichannel sensor.

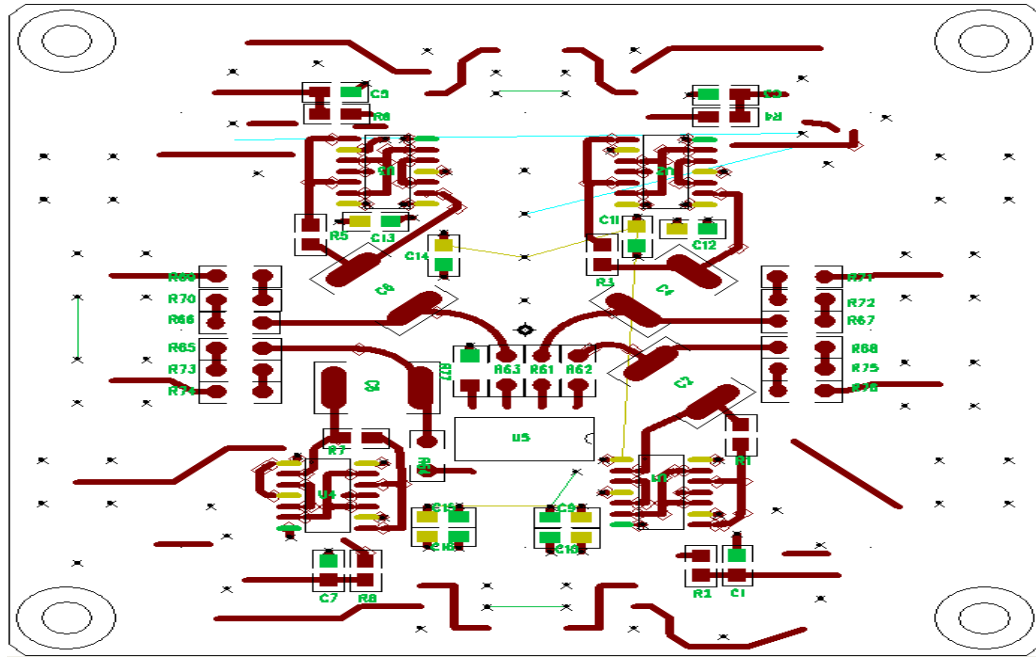


Figure B.1: Top Layer of Four Channel Sensor: Sensing Circuitry



Figure B.2: Second Layer of Four Channel Sensor: Power Plane

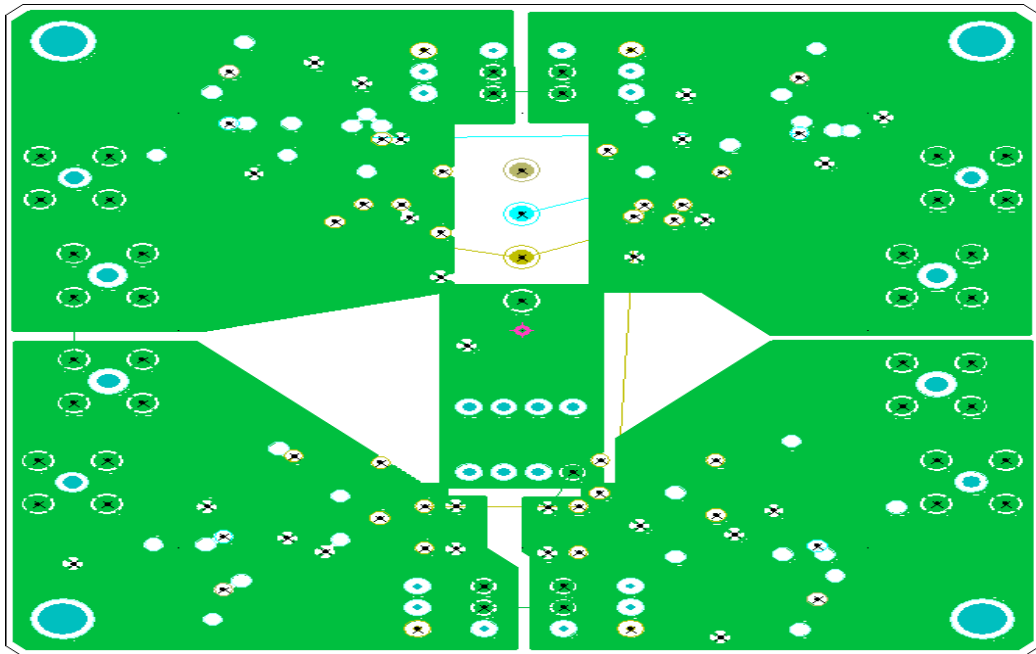


Figure B.3: Third Layer of Four Channel Sensor: Ground Plane

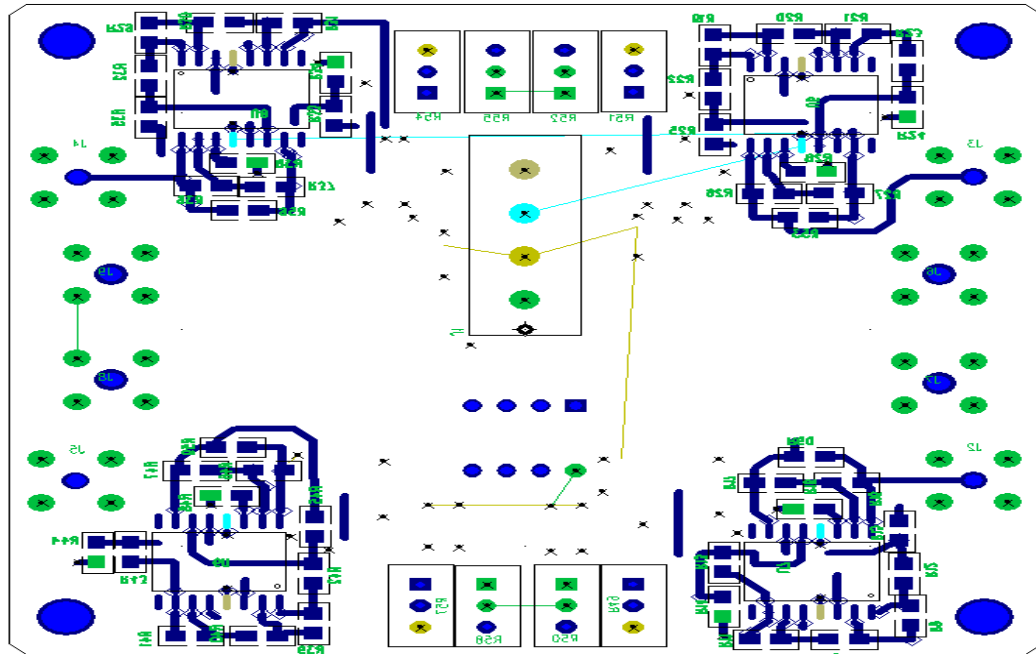
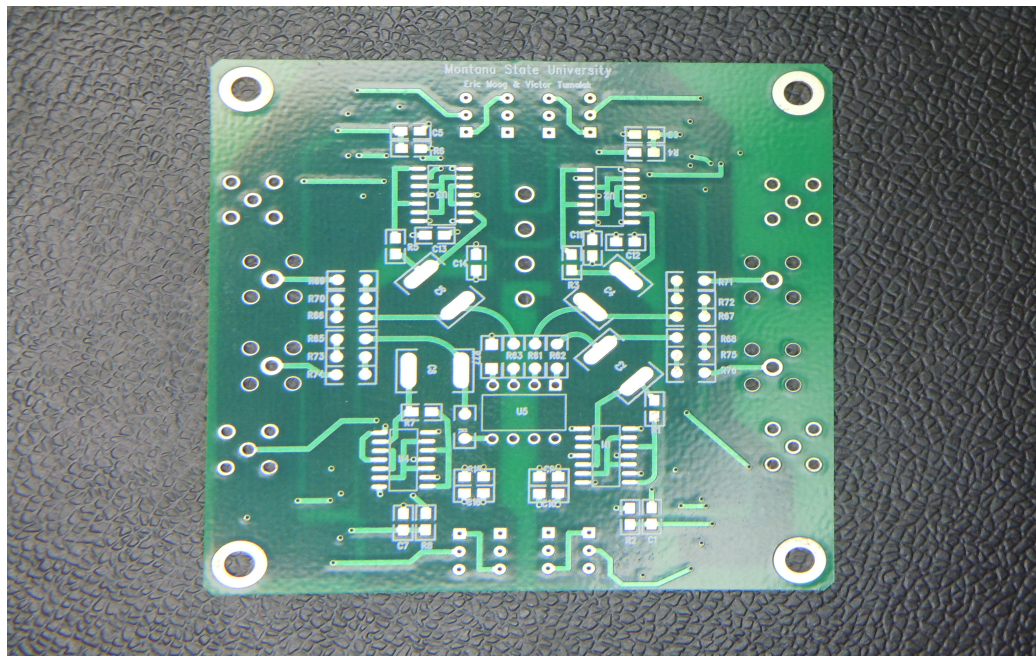


Figure B.4: Bottom Layer of Four Channel Sensor: Amplifier Circuitry



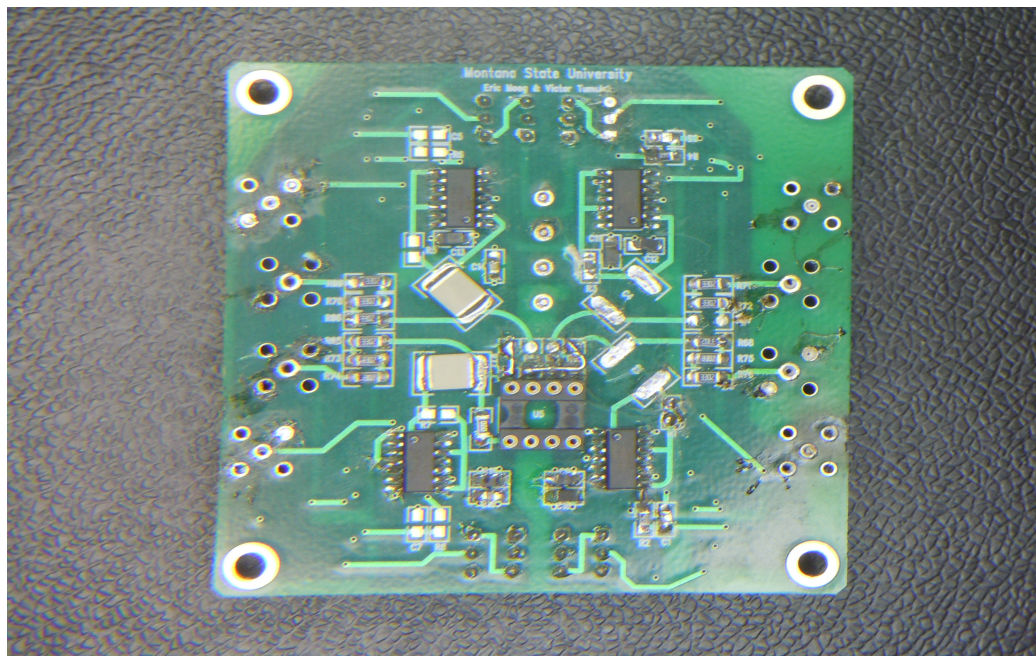


Figure B.6: Populated top layer of printed circuit board showing sensing circuitry

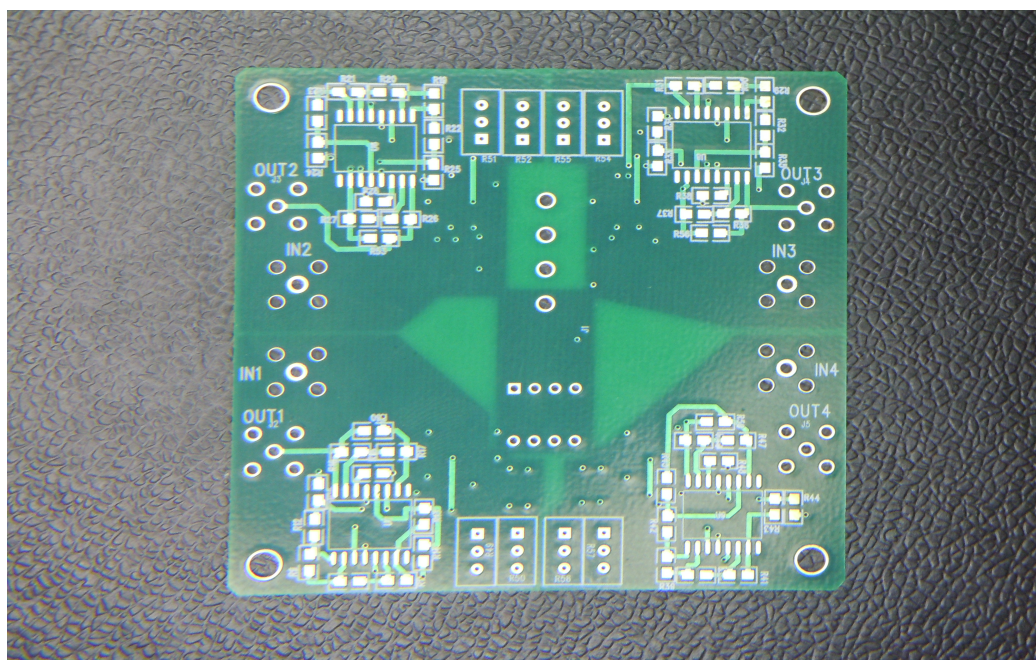


Figure B.7: Bottom layer of completed printed circuit board

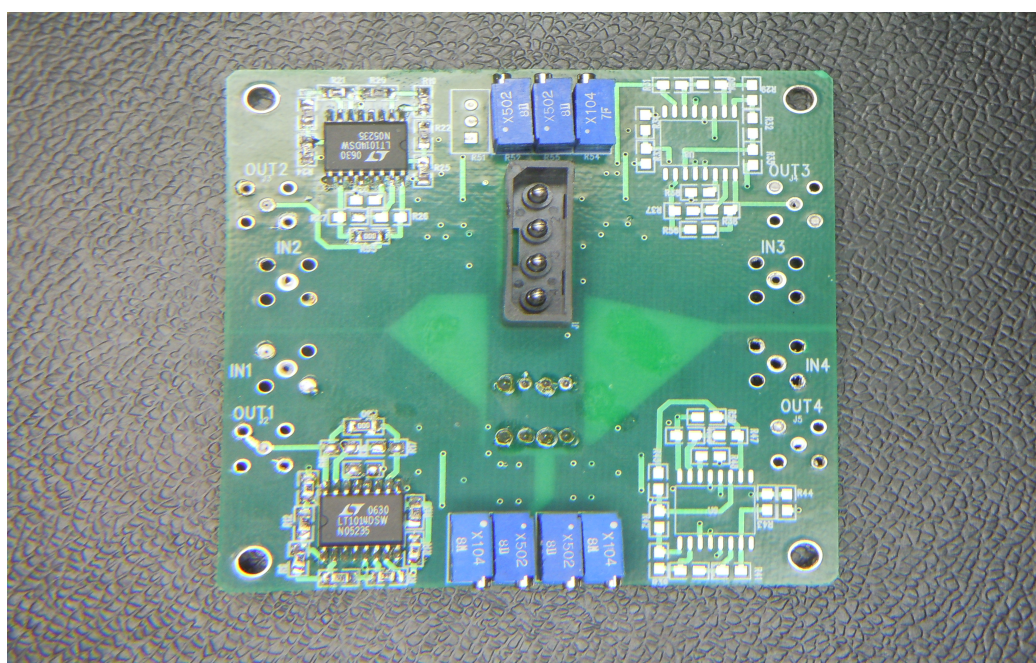


Figure B.8: Partially populated bottom layer of printed circuit board showing amplification circuitry

APPENDIX C

SPICE SCHEMATICS

The following spice schematics were generated with Orcad pSpice Capture and were used for simulation of sensor operation.

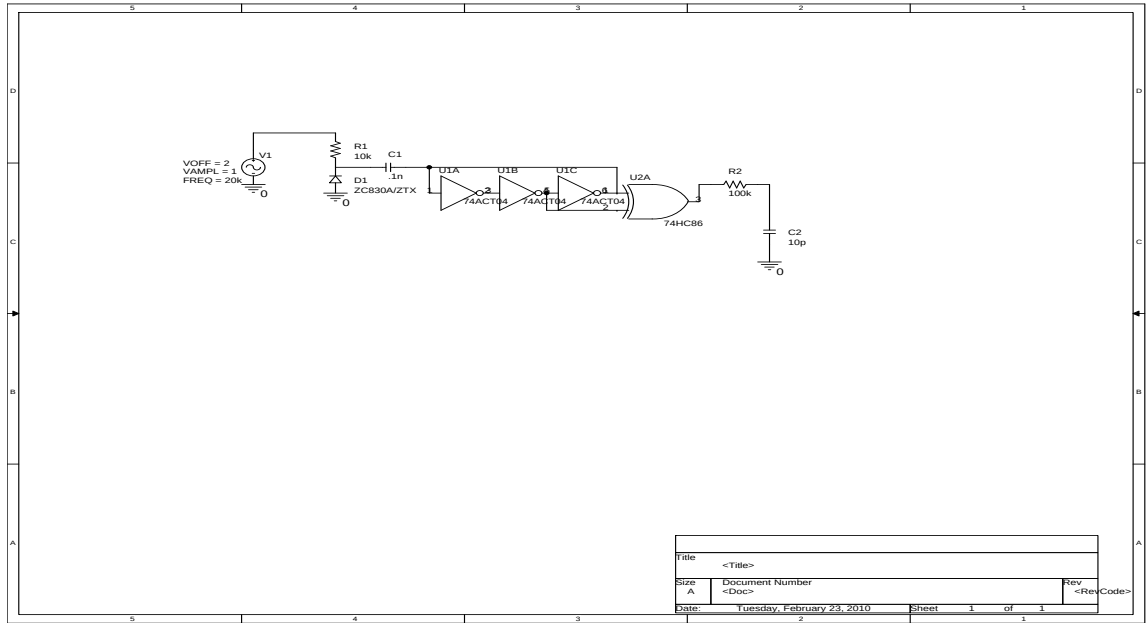


Figure C.1: Spice model of homodyne sensing circuit

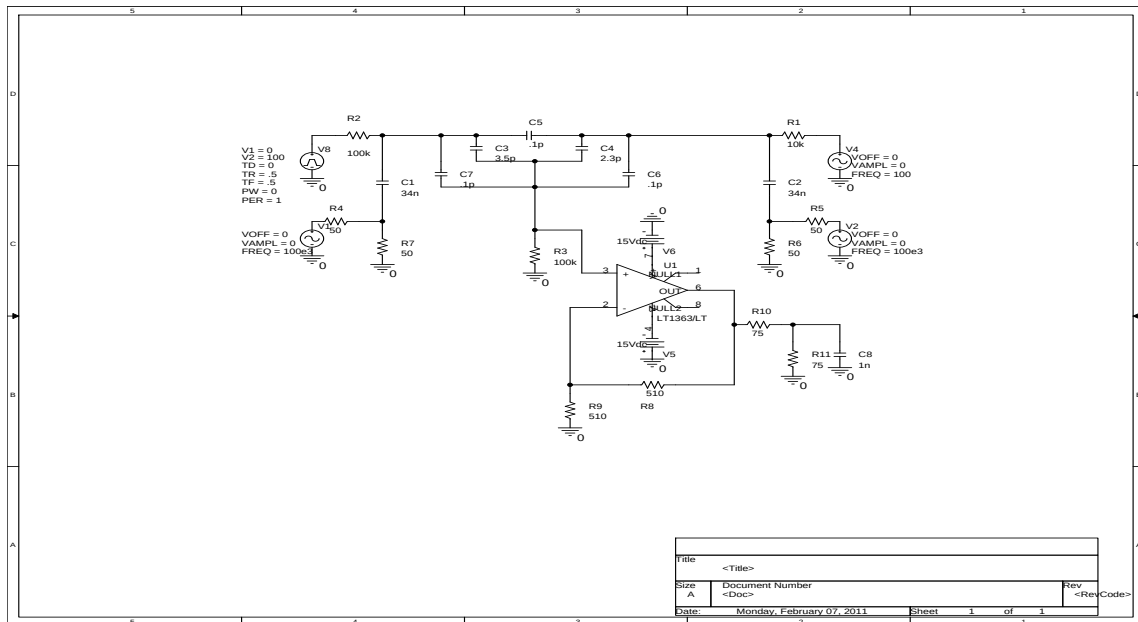


Figure C.2: Spice model of front end of synchronous detector

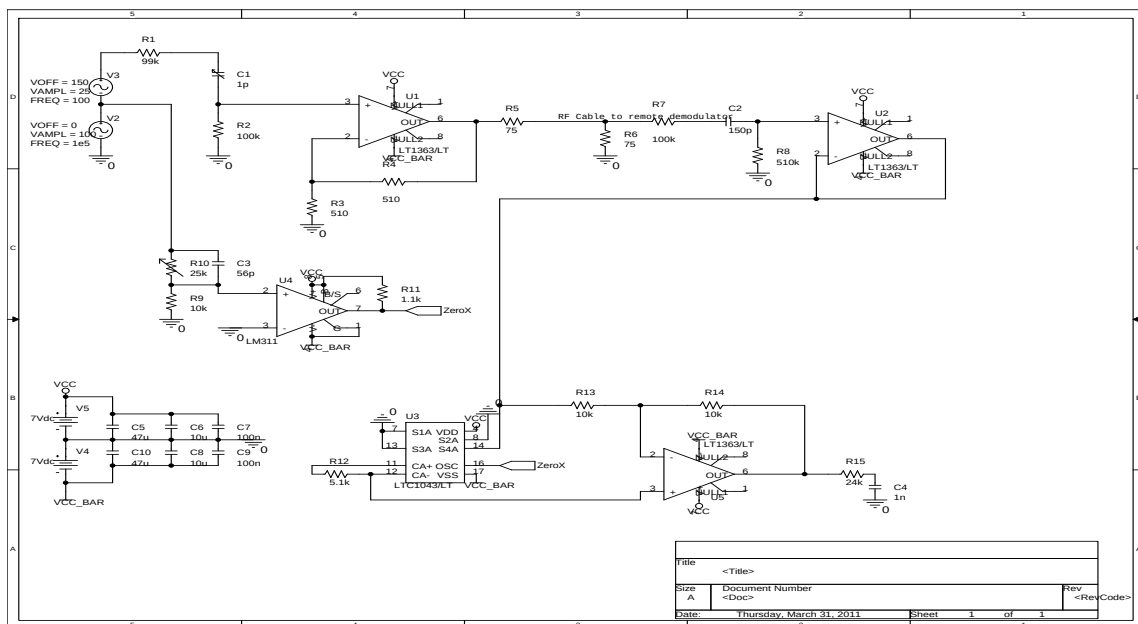


Figure C.3: Spice model of synchronous demodulator

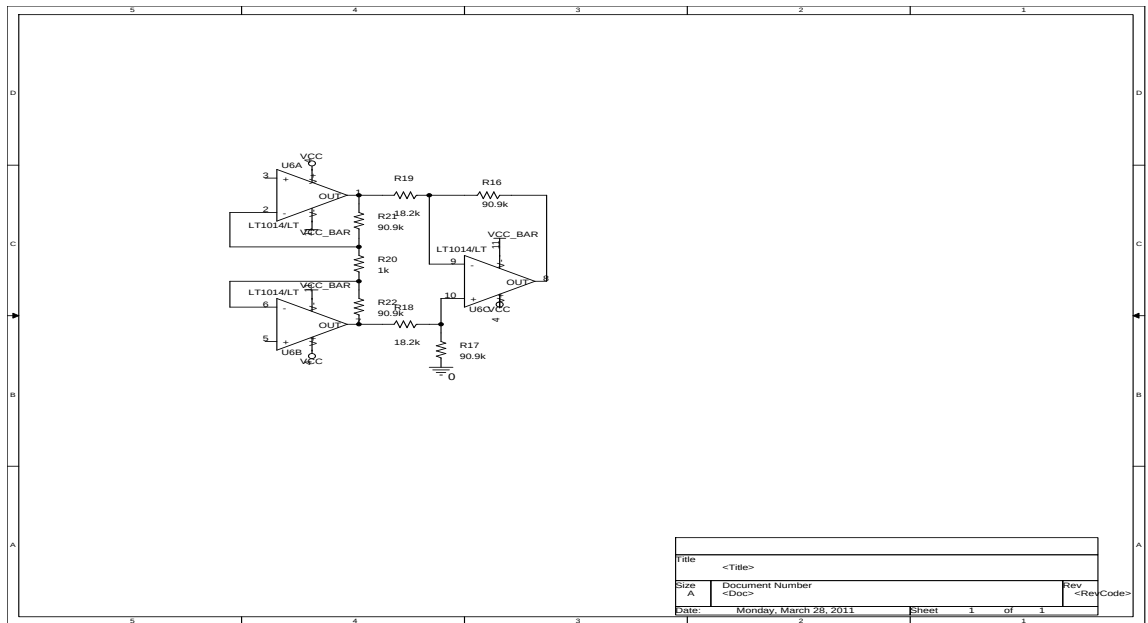


Figure C.4: Spice model of differential amplifier