

THE EFFECTS OF THE EQUATION OF STATE AND THE COMPOSITION  
ON THE THERMAL EVOLUTION OF NEUTRON STARS

by

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A dissertation submitted in partial fulfillment  
of the requirements for the degree

of

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DEDICATION

To my family.  
Without your extraordinary love and continuous support  
this work would not have been possible.

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## GLOSSARY

EOS – equation of state, the equation describing the state of matter under a given set of physical conditions, namely, pressure, density, temperature, and composition.

Urca process – a set of nuclear reactions that take place in a neutron star, and contribute to its cooling via neutrino emissions. The name “Urca” was chosen by the discovering physicists George Gamow and Mario Schoenberg after the “Casino da Urca”, where they thought they were losing money as fast as a neutron star loses energy through these processes.

TNI – three body nucleon interaction, an interaction of nucleons with each other and/or with exotic particles such as hyperons and pions.

LMXB – low mass X-ray binary, a binary system that contains a neutron star and a low mass stellar companion that accretes mass onto neutron star surface.

SXT – soft X-ray transient, a neutron star that has low energy X-ray burst events followed by extended quiescent periods.

$M_{\odot}$  – solar mass, a stellar mass unit that is equivalent to the mass of the sun.

## ABSTRACT

Even though the neutron stars have been studied for over six decades there is little consensus over the mechanisms involved in their thermal evolution. Similarly, despite decades of study, the behavior of exotic particles, especially under highly degenerate situations, garner little consensus in academia. We combined these two fields in the study of observational data in an attempt to identify involvement of specific exotic particles in neutron star thermal evolution. We generated various equations of state (EOS) that lie within the theoretically established limits for the pion condensates and hyperons, and attempted to find thermal evolution regimens that are consistent with observations that cannot be explained by standard neutron star cooling models. Our chosen observations are; the Cassiopeia-A (Cas-A) supernova remnant for our pion model, and various low mass X-ray binaries (LMXB) for our hyperon and pion models. Our results strongly suggest the involvement of these aforementioned exotic particles in thermal evolution and related problems of certain neutron stars that lead to their unusual thermal behavior.

## CHAPTER 1

## INTRODUCTION

1.1 Theoretical and Observational Discovery of Neutron Stars

In the early 20<sup>th</sup> century, astrophysicists hypothesized that the mechanism that allows the energy production inside a star was a newly postulated nuclear mechanism called fusion which allowed colliding nuclei to fuse into a larger nucleus, producing a sizable amount of energy in the process. While in theory this process could produce energy from any two nuclei, it was clear that in the dominant process in a star that was primarily made of hydrogen, the energy production must have been through the fusing process of hydrogen into heavier helium. While a revolutionary discovery, this invited an even bigger question regarding the long term evolution of a star since the hydrogen material that fueled the energy production inside the star was finite, and once depleted the star could no longer produce the energy that was necessary to maintain the internal pressure that balanced the gravitational collapsing force of the star's mass. This meant that any star would eventually have to collapse into some other much smaller stellar body. The collapse to a smaller body was confirmed in 1914 by the discovery of one of the hypothetical outcomes of the process, a white dwarf (Adams W. S., 1914), which was a dim star estimated to have a mass comparable to the sun, but with a radius four orders of magnitude smaller.

In the years following the discovery of the first white dwarf, the field of stellar evolution advanced significantly, and many supernova processes that result in various

remnants were theorized for the progenitor stars of different mass. One such theoretical remnant was a neutron star. Following the discovery of neutrons by Sir J. Chadwick (1932), Baade and Zwicky (1934) proposed that a progenitor star would experience a supernova event, through which the bulk of its mass would be lost to the core material which collapsed into a neutron star, and the binding energy of the lost mass would be released fueling the supernova event which would eject the remaining outer layers into space.

In the several decades following this initial proposal, considerable theoretical work on neutron stars was carried out. By studying the structure of neutron stars, the central density of the cores of these stellar bodies was shown to be as high as  $10^{15} \text{ g/cm}^3$  when the mass of the neutron star was only 0.7 solar mass by Oppenheimer & Volkoff (1939). Subsequent work between 1959 and 1964 by a group of scientists, including Cameron (1959), Hayakawa (1964), Chiu and Salpeter (1964) showed that the broader structure as well as the evolution of the neutron star would be heavily dependent on the properties of the highly degenerate material it is made of, amongst the most important is the equation of state. Finally in 1964, Tsuruta carried out the first detailed theoretical work that described the evolution of neutron stars through neutrino and photon emission processes based on specific equations of state (Tsuruta S., 1964).

The theoretical works on these stars were heavily challenged until the first discovery of a neutron star in 1968 (Hewish et al., 1968). A faint radio source was later identified as a stellar source which was either a white dwarf or a neutron star. The most important feature of this faint signal was its extreme pulsation regularity at a period of

1.34s, which suggested that the pulsation was a quality of the source rather than a result of random atmospheric obstructions. This discovery was considered to be definitive evidence of the existence of neutron stars, since in earlier work by Meltzer & Thorne (1966), the period of a remnant star with core density of  $10^7 \text{ g/cm}^3$ , which corresponds to a white dwarf, was shown to be no shorter than 8 s. Since the period of pulsations is inversely related to the density of the remnant, this particular source had to be an object of much greater density, perhaps as high as  $10^{13} \text{ g/cm}^3$  which would be consistent with a neutron star.

Initial possible observational support for the existence of neutron stars came in the form of X-ray observations before the radio observations. These are: two previously unknown X-ray sources, Sco X-1 which was discovered in the northern part of Scorpius constellation by Giacconi et al. (1962), and another source which was discovered in the Crab supernova remnant by Bowyer et al. (1964). Some of the contemporary researchers (Gianconni et al. 1962, Tsuruta & Cameron 1965 and Hayakawa 1964) initially suggested that if the X-ray emissions from these sources were shown to follow a blackbody spectrum, then the emitting region would have to as small as  $\sim 10 \text{ km}$ , which would be consistent with the size of a neutron star. However, further studies showed that the spectrum of Sco X-1 was consistent with bremsstrahlung radiation, and the Crab source was shown to be an “extended” object by the observation of the light curve as it passed behind the moon. Since these initial observations, however, many such sources have been observed by X-ray satellites, and identified as neutron stars. In addition, Sco X-1 X-ray emissions were later identified as being produced by the accretion disk around the central

neutron star, and the Crab source was found to have X-ray pulsations consistent with a neutron star.

The observational data collected by X-ray satellites in the five decades that followed the discovery of first X-ray sources suggested to be neutron stars have allowed detailed theoretical studies of these stars to be carried out. Through these studies, not only a very comprehensive understanding of the neutron star structure and evolution was developed, but further advancements in other fields such as nuclear physics were made possible. Some of these discoveries in broader fields, and specific ones that came out of this particular study will be discussed in detail in the upcoming chapters.

## 1.2 Synopsis

The primary goal of this thesis is to discuss the new and existing exotic particle core models that were used to study some recently observed phenomena, such as the rapid cooling of Cassiopeia-A neutron star, and the accretion driven steady state heating of neutron stars. Chapter 2 focuses on the primary characteristics of neutron star thermal evolution, the effects of various parameters on the neutron star cooling, how these parameters and characteristic were used to create a new neutron star core model, TNI7, and the results that were obtained from the study of Cas-A neutron star. Chapter 3 focuses on the study of accretion driven steady state heating of low mass X-ray binaries using the new TNI7 pion core model as well as older TNI3u and TNI6u hyperon mixed core models. In both chapters 2 and 3, the exotic core models are shown to be very promising methods for studying the aforementioned neutron star phenomena, as well as

isolated neutron stars, and provide conclusive evidence for the effect of superfluid suppression on non-standard cooling processes.

## CHAPTER 2

THERMAL EVOLUTION OF ISOLATED NEUTRON  
STARS INCLUDING NEW CASSIOPEIA-A DATA2.1 Basic Equations

Neutron stars are governed by the same principles and laws as ordinary stars, but with the effects of weak field general relativity which are studied in detail by Tsuruta (1998). The first condition that must be fulfilled in order for a stable neutron star to exist is the *hydrodynamic equilibrium* condition between gravitational force and the hydrostatic pressure. The pressure gradient equation that describes this condition is as follows (Oppenheimer & Volkoff, 1979):

$$\frac{dP}{dr} = \frac{-G(m + 4\pi r^2 P/c^2)(P/c^2 + \rho)}{r^2(1 - 2Gm/rc^2)} \quad (2.1)$$

where  $P$ ,  $r$ ,  $G$ ,  $m$ ,  $c$  and  $\rho$  are the pressure, radial distance from the center, universal gravitation constant, stellar mass, speed of light and density respectively.

The second condition that must be fulfilled is the linear relationship between the mass and the density of the neutron star, also known as the density function:

$$\frac{dm}{dr} = 4\pi r^2 \rho \quad (2.2)$$

Finally, the general relativistic gravitational potential is given by:

$$\frac{d\phi}{dr} = \frac{dP/dr}{\rho c^2 + P} \quad , \quad (2.3)$$

where  $\phi$  is the gravitational potential.

The mechanical properties of the neutron star then can be determined by simultaneous solutions of these three differential equations. In this study, these equations, as well as “governing equations” which allow the determination of stellar mass  $M$ , radius  $R$ , density distribution  $\rho$  and pressure  $P$  as a function of radius, are used.

The thermal state and evolution of a neutron star is described by two general relativistic equations: The energy balance equation describes the evolution of the total luminosity and the temperature with time (Thorne, 1977)

$$\frac{dLe^{2\phi}}{dr} = -\frac{4\pi r^2 n e^\phi C_v dT/dt}{(1 - 2Gm/rc^2)^{1/2}} \quad (2.4)$$

where

$$L = L_v + L_\gamma \quad (2.5)$$

and the radiative energy transport equation:

$$\frac{d(Te^\phi)}{dr} = \frac{-3\kappa\rho L_\gamma e^\phi}{16\sigma T^3 4\pi r^2 (1 - 2Gm/rc^2)} \quad (2.6)$$

where  $L$  is the total luminosity ( $L_\gamma + L_v$ ),  $L_\gamma$  is the photon luminosity,  $L_v$  is the neutrino luminosity,  $T$  is the temperature,  $n$  is the total number density,  $C_v$  is the total specific heat,  $\kappa$  is the total opacity,  $\rho$  is the total density, and  $e^\phi$  is the general relativistic correction factor which makes the necessary correction to account for the redshift and time dilation effects, given by:

$$e^\phi = 1 - \left[ \frac{2Gm}{rc^2} \right]^{\frac{1}{2}} \quad (2.7)$$

While the equation (2.4) is complex in its relativistic form, a proper neutron star cooling function can be better understood by evaluating this equation at the Newtonian limit where the equation (2.4) reduces to:

$$\frac{dE}{dt} = -C_v \frac{dT}{dt} = L_\gamma + L_\nu - H \quad (2.8)$$

where  $E$  is the total internal energy, and  $H$  is the heating rate due to any possible heating processes. This equation describes the total change in the internal energy of the neutron star as a function of energy loss due to photon and neutrino emissions, and energy gain due to various heating processes, and consequently it can be used to evaluate the thermal evolution of the neutron star over time.

## 2.2 Simulation of Thermal Evolution of Neutron Stars

The isothermal approximation method that is used by many groups in the neutron star research field employs a stellar model where the interior of the star is assumed to be isothermal and any temperature differential is assumed to be confined to a very thin layer near the surface of the star (Tsuruta, 1998). These assumptions allow decoupling of mechanical processes from thermodynamic ones, which subsequently permits separate solutions for the equations (2.1) – (2.3) from the equations (2.4) and (2.5). By solving the first three differential equations, the mechanical properties of the neutron star, such as the total stellar mass, radius versus central density and internal distribution of density, pressure and mass of the star, are obtained, which then can be used to solve the remaining two differential equations to obtain the thermodynamic properties, such as the core and surface temperatures, and neutrino and photon luminosities, of the neutron star as a

function of time. While this method often yields reasonable results, it is somewhat imprecise. The Nomoto-Tsuruta (1987) exact code that is employed for this study on the other hand, solves all differential equations simultaneously; and the two solutions - one starting from the center of the core and moving outward, and another starting from the surface and moving inward - are done at the same time and the results are matched at the boundary at which the two meet. The results of this method are significantly more precise than the isothermal method as shown by previous work by Tsuruta (1998).

In the process of evaluating each mechanical and thermodynamic equations self consistently, the exact code employs various microphysical input parameters. A discussion of these parameters is therefore relevant to understanding the results.

## 2.3 Input Microphysics

### 2.3.1 The Composition and Structure

The first property that is to be obtained, as the result of solving the basic equations 2.1 to 2.7, is the physical structure of the neutron star interior. The general structure of a neutron star interior involves three layers: The outer crust, the inner crust and the core. Some of the details of this structure is related to the equation of state for the neutron star matter, and will be discussed in the following chapter. However, regardless of the equation of state there are certain conditions for various regions/layers of the neutron star that are preset in the Nomoto – Tsuruta “exact evolutionary code” (Nomoto & Tsuruta, 1987). The preset conditions for these layers are as follows:

The density of the material that makes up the outer crust is less than  $4.3 \times 10^{11} \text{ g/cm}^3$ . Near the surface of the neutron star, where the density is below  $10^4 \text{ g/cm}^3$ , the crust is primarily composed of main sequence fusion elements, dominantly iron. As the density increases above  $10^4 \text{ g/cm}^3$  further into the outer crust, the heavy elements are ionized and the region becomes saturated with heavy ions and free electrons. However, as the density increases to  $10^9 \text{ g/cm}^3$  towards the inner regions of the layer, these free electrons are captured by protons bound inside nuclei, creating neutron rich heavy ions that make up the inner portion of the outer crust of the neutron star.

The next layer, the inner crust, starts at a density of  $4.3 \times 10^{11} \text{ g/cm}^3$ , and continues up to a density of  $2.8 \times 10^{14} \text{ g/cm}^3$ . As the density of the stellar matter passes  $4.3 \times 10^{11} \text{ g/cm}^3$ , the potential energy of neutrons that are normally bound inside the nucleus grows beyond the binding energy, which allows these neutrons to “drip out” of the nucleus. As a result this layer of the neutron star contains a combination of free neutrons, free electrons and neutron rich nuclei.

The core is defined as the region of density above the nuclear density,  $2.8 \times 10^{14} \text{ g/cm}^3$ . Due to the high density, the core is predominantly made of free neutrons. The maximum density is related to the mass of the neutron star, with more massive stars having greater core densities. When the core conditions allow densities that are greater than the transition densities for exotic particles, such as pions and hyperons, these particles also appear in the core. Appearance of such particles is highly consequential to the neutron star evolution and discussed in some detail later in this chapter.

### 2.3.2 Neutrino Processes

While the thermal evolution of a neutron star is determined by emissions of both neutrinos and photons, neutrino emissivity has distinct importance in cooling of neutron stars especially in the earlier stages of the evolution before the cooling process is dominated by photon emissions (Tsuruta, 1979). This is due to the fact that the neutron stars are transparent to neutrinos almost immediately after their formation, and the neutrinos have mean free paths that are many orders of magnitude greater than the radius of a neutron star (Qin, 1995). As a result, properly defining the contributions of neutrino emissivity to the cooling of neutron stars is especially important. The impact of various neutrino processes is shown qualitatively in figure 2.1.

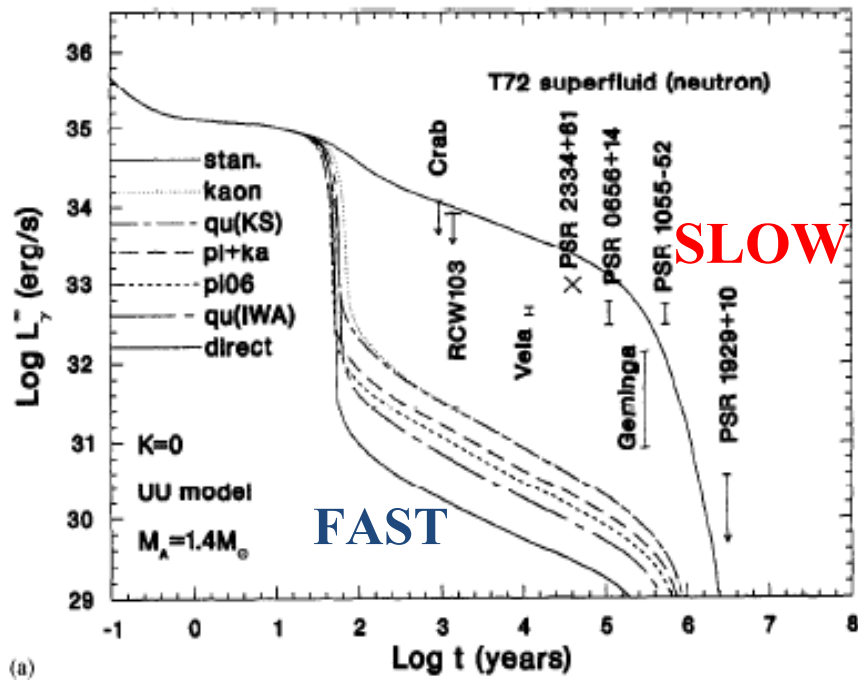


Figure 2.1. Thermal evolution of neutron stars is shown under various neutrino processes that dominate their early evolution. Additional details about the detections shown can be found in the publication this figure is taken from, Tsuruta, 1998.

As it can be seen from the figure 2.1, different processes that dominate the earlier stages of cooling can make significant difference in thermal evolution. The slow cooling processes, also referred to as “standard cooling”, are common for all neutron stars, while the fast cooling processes, also referred to as “non-standard cooling”, are only seen under specific conditions. The conditions that allow the fast processes to occur are studied in detail by Tsuruta (1998).

The common neutrino processes that give rise to standard and non-standard cooling are listed in table 2.1. Additional details of the core neutrino processes can be found in the Appendix, while the details of non-standard neutrino processes that are central to this study are discussed later in this chapter and in Chapter 3.

| Standard                          | Non-standard           |
|-----------------------------------|------------------------|
| Modified Urca                     | Direct Urca (nucleons) |
| n-n bremsstrahlung                | Pion Urca              |
| n-p bremsstrahlung                | Kaon Urca              |
| plasmon neutrino                  | Hyperon Urca           |
| Nuclear Urca                      | Quark Urca             |
| Electron heavy ion bremsstrahlung |                        |
| Electron-positron pair neutrino   |                        |
| Photo-neutrino                    |                        |

Table 2.1. List of standard and non-standard neutrino emission processes.

All standard cooling processes listed in table 2.1 are included in the Nomoto-Tsuruta exact evolutionary code for all studies, while individual non-standard cooling processes are selected for different parts of this study based on the involvement of specific constituent particles.

### 2.3.3 Superfluidity

In the study of some more recent isolated neutron star observations, Tsuruta (1998) reported that non-standard cooling processes, while necessary, did produce cooling rates that were too high to be consistent with some of the observations. The clear conclusion that was drawn from this inconsistency was the presence of a suppression mechanism that influenced the thermal evolution process, and reduced the cooling rates. The evident source of this suppression was the superfluidity of the constituent particles that formed the neutron star.

Superfluidity in neutron stars works quite differently than the ordinary material that is studied in terrestrial experiments, which requires very low temperatures in order for ordinary matter to exhibit superfluid behavior, due to the extreme density of neutron stars which allows superfluid transition of the material to take place at very high temperatures. Also different layers of the neutron star interior are expected to exhibit different superfluid behaviors since each layer has a significantly different composition than the others. For example, the outer crust does not have the sufficient density that is necessary to generate superfluid behavior. The inner core, on the other hand, is dense enough for free neutrons to be present. These free neutrons interact via short range attractive forces and form Cooper pairs which exhibit  $^1S_0$  state superfluidity, when the temperature is below the critical value  $T_{cr}$ , similar to what is seen in hydrogen in ordinary matter. Once the density reaches the extreme levels of the core, however, short range interactions between the constituent particles, which are attractive under these conditions, also come into play, and the neutron superfluidity switches to  $^3P_2$  state. Inside the core,

the remaining protons will also experience short range attraction, and will also become superfluid/superconductor in the  $^1S_0$  state. The critical temperature at which these transitions would take place is model dependent value that can vary significantly.

In addition to neutrons and protons, exotic particles that appear in the core under proper conditions may also be affected by superfluidity. The effect of the appearance of superfluid nucleons is enhanced cooling for some standard neutrino emission processes by the formation of Cooper pairs (see the Appendix for the details of this process). On the other hand, the superfluid state formation of the exotic particles has the opposite effect, which can reduce, or even completely cancel the fast cooling processes by exotic particles due to the reduces reaction rates of these states.

The critical temperature below which the superfluidity of neutron star cores appears is dependent on the superfluid energy gap which represents the potency of the attraction force between the constituent particles. The size of the superfluid energy gap is directly related to the critical temperature, where larger energy gap means superfluid transitions can take place at higher temperatures, and is heavily model dependent. Various critical temperature – core density profiles that correspond to various superfluid energy gap models as studied by Tsuruta (1998) are shown in figure 2.2. As can be seen from this graph, both the critical temperature for superfluid transition, as well as the core density range over which the superfluidity is effective vary significantly between models.

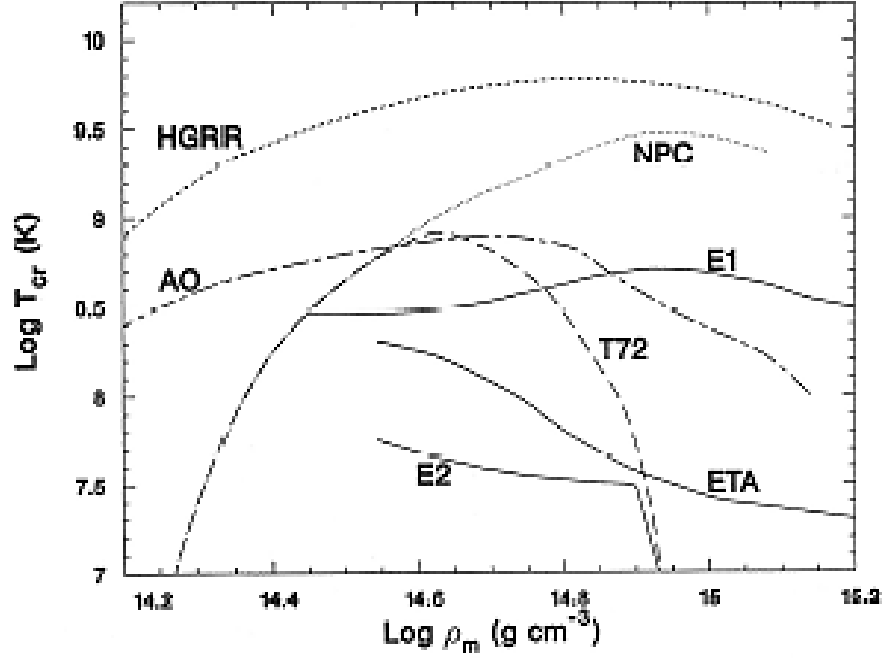


Figure 2.2 Critical temperature versus core density plots of various superfluid models are shown. E1, E2 and ETA correspond to pion superfluid models while all others are 3P2 neutron superfluid models. HGRIR model is presented by Hoffberg et al. 1970, AO model is presented by Amundsen & Ostgaard 1985, T72, NPC, E1, E2 and ETA model are presented by Takatsuka & Tamagaki 1972, 1980-1982.

The primary outcome of the introduction of superfluidity to the neutron star cooling is the suppression of the cooling process when the core temperature of the neutron star falls below the critical temperature. The approximate reduction in neutrino luminosity due to superfluid suppression is defined as:

$$L_v^S = L_v^N R(T/T_{cr}) \quad (2.8)$$

where  $L_v^S$  and  $L_v^N$  are the neutrino luminosities with and without the effects of the superfluid suppression, and  $R(T/T_{cr})$  is the reduction factor (Takatsuka and Tamagaki, 1972). Since cooling rate due to neutrino emissions in the presence of superfluidity is lower, neutron stars with a core structure with larger superfluid energy gaps, and

consequently with stronger superfluid effects, are expected to have higher surface temperatures during the neutrino dominated cooling era. In the cases of very strong superfluid suppression the enhancement to the cooling process due to non-standard neutrino processes can be weakened to the point that the cooling regime cannot be distinguished from the standard cooling regime.

#### 2.3.4 Internal Heating

While many internal heating mechanisms were proposed for neutron stars, for isolated neutron stars with moderate magnetic fields ( $<10^{13}$  Gauss), the only mechanism that has a noticeable impact on the cooling of the star is frictional heating. As explained earlier, the neutrons in the inner crust of the neutron stars may be in a superfluid state through the Cooper pair formation by these free neutrons, while the rigid heavy ions in the crust are not. As the neutron star rotates, the magnetic field of the neutron star exerts a torque on the ions, causing it to spin down which in-turn causes the rotational kinetic energy to be lost through radiation. The spin down process only affects the heavy ion crust, but not the superfluid neutrons. Consequently, a spin difference between the crustal ions and the superfluid neutrons is created, which gives rise to a friction force that creates an internal heating source for the neutron star (Tsuruta, 1998). The effectiveness of this heating mechanism is determined by the strength of the superfluid vortex pinning in the inner crust where the efficiency of the frictional interaction between the crustal ions and superfluid neutrons is directly related to how strongly magnetic vortexes are pinned to the crust. The relationship between vortex pinning, and heating rates was studied by Umeda

et al. (1993) and Tsuruta (1998). The resulting heating term that is used in the evolutionary code is expressed as:

$$H = 1.27 \times 10^4 (t + \tau_0(\text{yr}))^{-\frac{3}{2}} K (\text{ergs } m^{-3/2} s^2) \text{ ergs } s^{-1} \quad (2.9)$$

where  $t$  is the age of the neutron star,  $\tau_0$  is the spin-down time at  $t = 0$ ,  $K$  is the parameter that represents the strength of the vortex pinning.

An example of the effect of the inclusion of frictional heating and the resulting alteration to the cooling of the neutron star as studied by Tsuruta (1998) is shown in figure 2.3. In this figure the effects of internal heating are demonstrated for various heating scenarios. The solid curve represents the thermal evolution of a neutron star with no internal heating. The dashed curves represent various cases with the intensity of heating increasing along the ascending order of these curves, and the uppermost long-dashed curve representing the case where the neutron star experiences the maximum heating due to internal friction. The downward arrows represent the upper limit for the temperature of some detected neutron stars, and the vertical error bars represent actual ranges of the surface temperatures from some neutron star detections. The upper limit information in this graph indicates that the actual value of temperature may be at any point below that point, not just in the parameter space that is covered by the length of the arrow. Therefore many of these data points may be consistent with all cooling curves shown in the figure. Only the right-most five data points favor cases with moderate to high heating conditions exclusively.

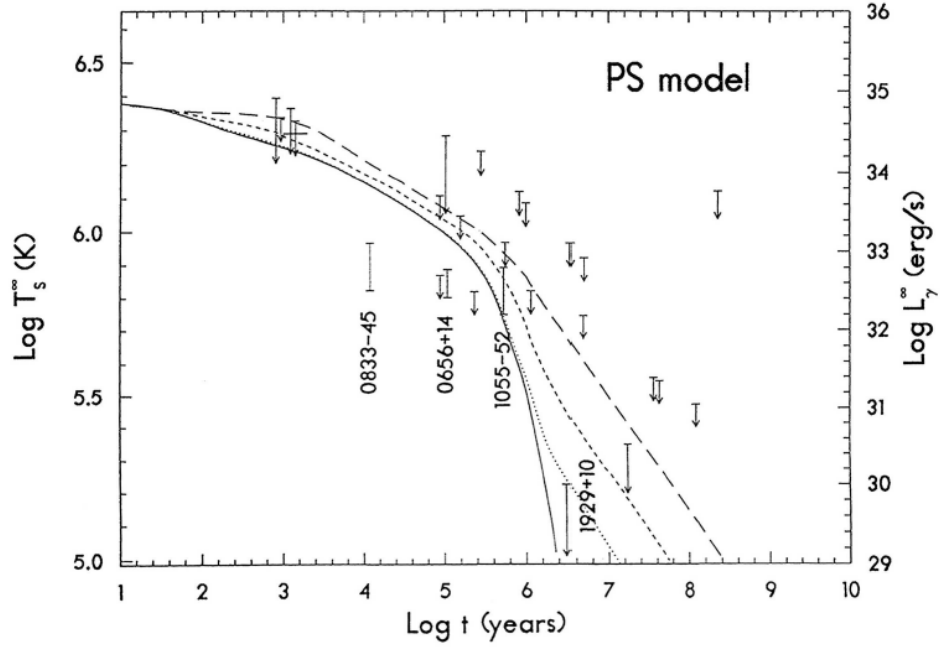


Figure 2.3. The effects of frictional heating regimes of various strengths are shown in the figure. The surface temperature as detected by an observer at infinite distance is shown as a function of the age of the neutron star that has a core which follows the extremely stiff PS model. The long-dashed, short-dashed and dotted lines represent the maximum, intermediate and low heating regimes respectively. The vertical arrows and error bars correspond to upper limits and actual detections of the surface temperatures of various observed isolated neutron stars.

## 2.4 Equation of State

### 2.4.1 Overview of Core Equation of State

The equation of state for a neutron star core basically represents the compressibility of the material that constitutes the neutron star core, and arguably is one of the most important parameters that determine the structure, characteristics, and thermal evolution. While compressibility of normal material is well studied and understood, the highly degenerate material inside a neutron star does not conform to the physics of terrestrial material. For example, the pressure of completely degenerate particles is

independent of the temperature. An equation of state in a neutron star core depends on nuclear and strong forces, which are not well understood. That gives rise to many competing theories, and a relatively large parameter space to study. The study of neutron stars has been thought of as among the best methods to place constraints to these nuclear models and parameter space (Takatsuka et al. 1972).

The equation of state for the neutron star matter inside a stellar core is primarily determined by the competing attractive and repulsive interaction forces between the hadrons (neutrons and/or protons, and exotic particles such as hyperons and pions). A simplified representation of these interactions is illustrated by Tsuruta (2010) as shown in figure 2.4. In this figure, the negative potential represents a net attractive interaction, while the positive potential represents a net repulsive interaction between the particles. Regardless of the model, as the separation between nucleons gets smaller from long

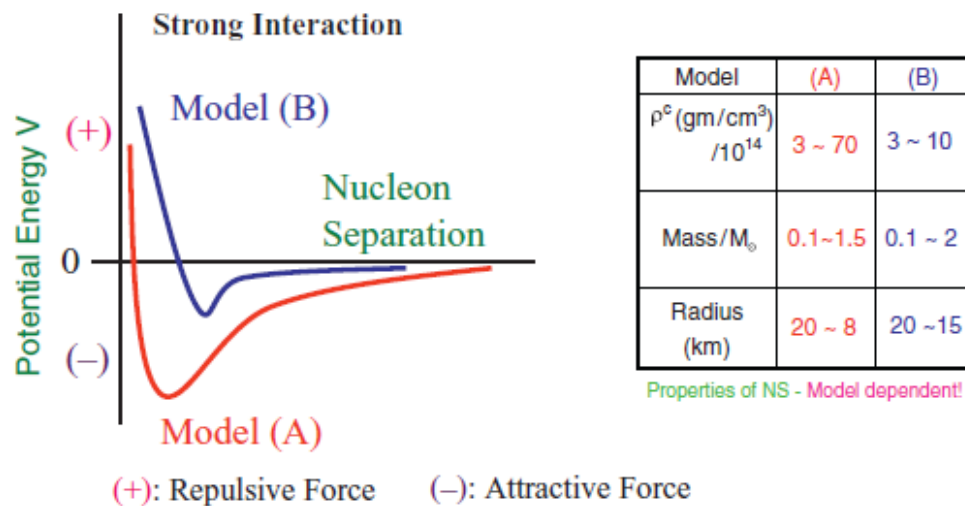


Figure 2.4 A simplified version of the nuclear potential energy as a function of separation between nucleons is shown for two different models.

distance, the interactions are net attractive, and turn repulsive only when the particle separation is sufficiently small. The separation distance where this change from attraction to repulsion takes place and the depth of the attractive potential are the primary determining factors for the nature of the EOS of the neutron star core.

Model A in figure 2.4 represents a scenario where the attractive forces are very strong, and consequently the interactions do not turn repulsive until the nucleon separation is very small. Therefore a neutron star that follows this EOS would be very dense, very compact and less extended. This kind of EOS model is typically referred to as “soft”. Model B, on the other hand, represents a case where the attractive forces between the nucleons are not as strong, and consequently the interactions become repulsive at much greater separations. As a result a neutron star that follows this type of EOS would be less dense, more extended and larger. This kind of EOS model is typically referred to as “stiff”. One of the most important consequences of the EOS stiffness is the maximum allowed stellar mass; stiffer EOSs allow larger mass neutron stars to form.

It is important to note that while the overview in this section is focused on the EOS as a result of interactions of nucleons, the inclusion of other particles, specifically exotic particles such as hyperons and pions has a significant impact on the EOS, as well as the neutron star cooling as a whole (Takatsuka et al, 1997, Umeda et al. 1994, Tsuruta 1998). If such particles exist in a stable state in the neutron star core, then they will not only alter the EOS through additional inter-particle interactions, and therefore can alter the neutron star characteristics, but also may contribute to the superfluidity regime due to

these interactions. Consequences of the presence of these particles are the main subject of study of this thesis and will be discussed in some detail.

#### 2.4.2 Previous EOS Studies of Neutron Stars

The involvement of exotic particles in neutron star cores was hypothesized nearly half a century ago, and has been studied since. While there are many different models that involve various exotic particles, the theoretical work for pion and hyperon involvement in EOS have the most advanced and well developed theoretical work (Tsuruta 1998). As a result, models involving these particles were included in this study.

2.4.2.1 Pandirapande – Smith (PS) Neutron Star Core Model. The PS model was first introduced to the neutron star research arena by Pandharipande and Smith (1974). In their publication, the authors stated that when the neutron star core is sufficiently dense neutron-neutron two body interactions could create sufficiently high potentials for a stable  $\pi^0$  condensate to form. While  $\pi^0$  particles are already involved in the short range interaction of the neutrons, once the pion condensate is present and reaches a certain density ( $\sim 2 \rho_N$ ), it can excite the neutrons to a virtual  $\Delta^0$  baryon state. Once the excited states are formed, additional two body interactions of the coupled states such as  $N\Delta - N\Delta$  and  $\Delta\Delta - \Delta\Delta$  would not only play a role, but also dominate the nuclear potential due to interaction forces that are as strong as 4 times that of ordinary  $N - N$  interactions. Pandharipande and Smith (1974) considered various solid and liquid phases of the pion condensate potential along with lower and upper limits for neutron effective mass, and generated three possible EOSs for the neutron stars. The stellar mass – energy density relationship of these EOS is shown in figure 2.5 ( Pandirapande and Smith, 1974).

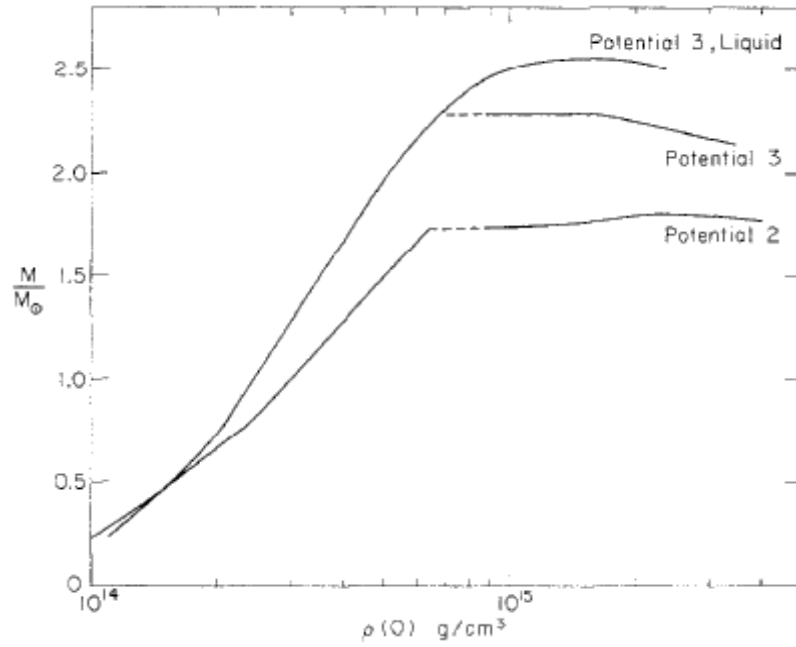


Figure 2.5. Mass versus mass energy density for non-rotating neutron stars is shown.

The top two curves in the figure 2.5 correspond to the same potential with the top curve representing a core in liquid state, while lower one is a solid core; and the bottom curve represents a solid core with a weaker repulsive potential.

The most important outcome of these EOSs is the size of the neutron stars they can support. The highest potential liquid core model could allow a neutron star with  $2.5 M_{\odot}$  (solar mass) and a radius of roughly 18 km, while the two solid cores could support neutrons stars with  $1.8 M_{\odot}$  and  $2.28 M_{\odot}$ , and radii of 12 km and 16 km respectively. The central densities of the solid core models were estimated to be  $2.7 \times 10^{15} \text{ g/cm}^3$  and  $1.1 \times 10^{15} \text{ g/cm}^3$  for the lower and higher potential models respectively, while the central density of the liquid model was estimates as  $1.0 \times 10^{15} \text{ g/cm}^3$ .

While the PS model is somewhat controversial due to its possibly unrealistic stiffness, the high maximum mass liquid model is still considered to be an upper limit for

the EOS models and continues to be used accordingly by many others, including some earlier work by Umeda et al. (1994). This liquid model is also adopted as the upper limit of the EOS stiffness parameter space that is examined in this thesis.

The most important outcome of the study by Pandhirapande and Smith was the introduction of a real possibility of a really massive and extended neutron star to the research arena since the mass and radius play a pivotal role in the thermal evolution of the neutron star.

2.4.2.2 Friedman – Pandirapande (FP) Neutron Star Core Model. In 1981, Friedman and Pandharipande (1981) discussed the properties of neutron matter in reference to recent the nucleon-nucleon scattering data. In this work, in addition to a different model for two-nucleon interactions (referred to as  $v_{14}$  interactions by the authors), which were weaker than what were used for the PS model, they also included three-nucleon interactions (TNIs).

In the FP model study, authors used a phenomenological fit of the experimental nuclear data to determine the three-nucleon interaction (TNI) contributions to the nuclear potential which they assumed were solely caused by neutron excitation to  $\Delta$  particles. The authors realized that, while the TNIs have an attractive and repulsive component, the repulsive component rapidly dominated the contributions as the density increased, though they also realized the  $v_{14}$  interactions had a greater contribution to the overall potential. The resulting nuclear potential due to the net contributions of the TNIs and the  $v_{14}$  interactions were such that the overall equation of state had to be significantly softer than the PS model. Consequently, the neutron stars that contained a pion core that followed

the FP model would be much smaller and far less extended compared with the PS model. This relationship is shown in figure 2.6.

In figure 2.6, the lowest curve represents a neutron star whose core experiences only two body interactions, the middle dashed curve represents a neutron star core that experiences only select two & three body interactions, and the topmost curve represents a neutron star core that experiences all plausible two and three body interactions. The top curve shows that the largest neutron star that can be supported under this model would have an approximate gravitational mass of  $1.8 M_{\odot}$  which has a corresponding radius of 8.7 km (Friedman and Pandirapande, 1981).

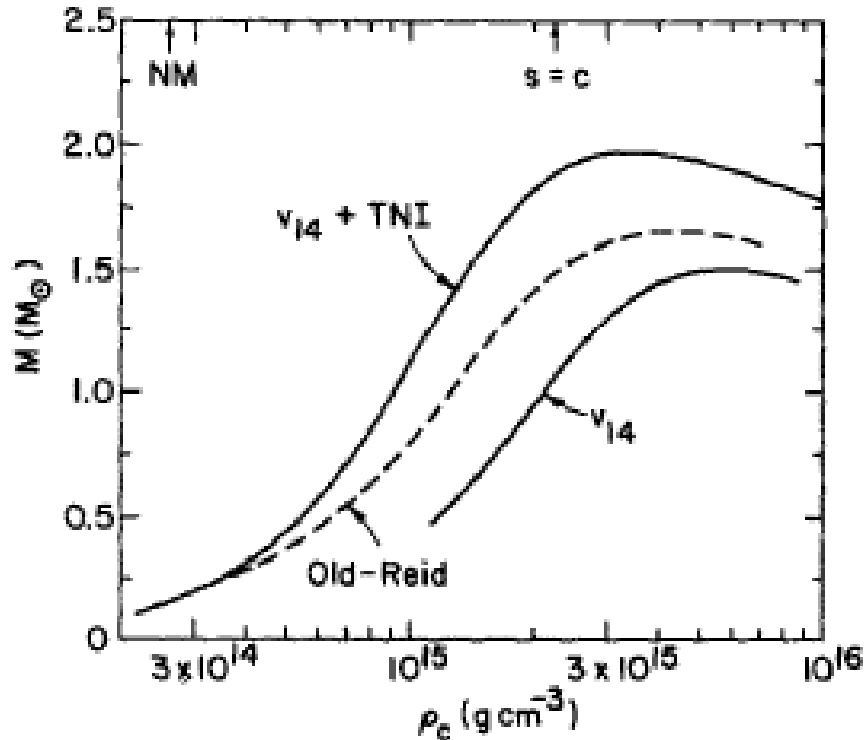


Figure 2.6. Neutron star mass as a function of central density for various models are shown. The masses shown correspond to the baryon mass of the neutron stars, consequently the actual gravitational mass of the neutron stars are less than the values shown in the figure.

2.4.2.3 FP and PS Models with Pion Cores. The effects of a pion-condensate core on the thermal evolution of neutron stars were studied by Umeda et al. (1994). The authors included the pressure correction due to the presence of pion-condensate and the superfluid behavior of these particles in their calculation. Their simulations suggested that the neutron stars following the FP and PS EOSs would have noticeably different cooling regimes, one of the most visible differences being the timeline for thermal relaxation which lasted for 100 and 1000 years for the FP and PS models respectively.

Over the past two decades both models have been studied by many researchers. While the medium FP model was considered to be a much better model than the stiff PS model at the time of its introduction, and continues to be used by many researchers to this day, it fails to explain the full range of new observational data due to its low maximum allowed mass. The PS model, on the other hand, is thought to be unrealistically stiff. Consequently it would be safe to conclude that the actual strength of repulsive three-body forces involved in a neutron star core with pions would have to be greater than what is used for the FP model, but perhaps not as strong as the PS model.

2.4.2.4 Neutron Star Core Models with Hyperons. After the introduction of the possibility of the presence of exotic particles in the core of a neutron star by the various pion models, additional ideas were explored for other exotic particle models. One such model involves hyperons. While the hyperons (Y particles) under normal circumstances are unstable particles that quickly decay into nucleons, under the extreme conditions of a neutron star core, these particles may exist in a stable state. If these particles are indeed present in a stable state and in large enough populations, their interactions could

potentially alter the EOS in a very minor way, but may provide additional fast cooling neutrino processes which in turn may influence the overall cooling of the neutron star significantly.

The involvement of hyperons in the neutron star EOS was studied and discussed extensively in the publications by Takatsuka et al. (2001-2006). In these publications, the authors indicated that for a neutron star with sufficiently high core density stable hyperon components will indeed be present, intermix, and interact with the nucleonic matter. They pointed out that it is most likely that only the lower mass components,  $\Lambda$  and  $\Sigma^-$  hyperons, will appear; and consequently they included only these two flavors in their modeling. Additionally, they stated that their experimental hypernuclear data were compatible with only YN and NN interactions, therefore those were chosen for sole consideration in the EOS construction.

In their study, Takatsuka et al. (2006) considered both two-nucleon and three-nucleon interactions' contributions to the nuclear potential. While they assumed two-body interactions were universal for nucleonic and hyperon material they tested various scenarios where the hyperons may or may not participate in the three-body interactions. They tested the models for three possible repulsive force strength regimes which are denoted by three separate values for the nuclear incompressibility constant  $\kappa$ . These three models are the soft EOS model TNI2, where  $\kappa = 250$  MeV, the stiff EOS model TNI3, where  $\kappa = 300$  MeV, and the intermediate EOS model TNI6, where  $\kappa = 280$  MeV.

When the authors ignored the hyperon-mixing, and considered only the effect of two-nucleon interactions and TNIs, they determined that the maximum mass permissible

for the TNI2, TNI3 and TNI6 models were  $1.62 M_{\odot}$ ,  $1.88 M_{\odot}$  and  $1.78 M_{\odot}$ . While these values were consistent with the existing observations at the time, the inclusion of hyperons in their model that interacted only through two-body interactions created a major problem. When the hyperon-mixed material was considered, the EOS softened considerably; because while the neutron-part of the EOS actually became stiffer, this allowed Y-mixing to take place at much lower densities, which in turn rendered the enhanced NN repulsion ineffective. The resulting soft EOS allowed much smaller maximum masses of  $1.08 M_{\odot}$ ,  $1.1 M_{\odot}$  and  $1.09 M_{\odot}$  for the three models, rendering all three models incompatible with the observations, since a number of neutron stars with much greater mass are known to exist, most notably a recently discovered 2 solar mass neutron star (Demorest et al. 2010).

The clear resolution of the overly softened EOS by the introduction of Y-mixing to the neutron star core would be the presence of additional repulsive forces in the hypernucleus systems. According to Takatsuka et al. (2007), this additional repulsive force can be obtained from extending it to the hyperon matter. When the YN three body interactions were included, the new EOS, while still slightly softened, was in significantly better agreement with the observational data for neutron stars. According to the authors, this is a clear indication that the well-recognized three body interactions of the nuclear systems cannot be restricted to just NN interactions, but should be extended to YN interactions.

The energy per particle versus density of the hyperon mixed neutron star matter, and the corresponding mass versus central density relationship for the TNI2u, TNI3u,

TNI6u (the addition of hyperon-mixing is denoted by the addition of the letter “u” to the model’s name) models are shown in the figures 2.7 and 2.8 (Reproduced from Takatsuka et al. 2007). These represent the EOSs that include the two-nucleon as well as YN TNI potentials of the Y-mixed stars. In figure 2.7, an additional curve that does not include hyperon mixing effects is shown in order to signify the small softening of the EOS when the hyperons intermix with the nucleonic matter in the stellar core.

In figure 2.8, the small crosses denote the maximum permissible mass by the three hyperon core models. They correspond to  $1.52 M_{\odot}$ ,  $1.71 M_{\odot}$  and  $1.83 M_{\odot}$  respectively from the bottom-most curve to the top-most curve. The threshold density for Y-mixing to take place is also shown in the figure: According to Takatsuka et al. (2007), for densities ranging from  $2.5$  to  $3 \rho_0$ , there would be no extra repulsion and the additional effect of the YN contribution to extended TNI will be present only for densities larger than  $4 \rho_0$ .

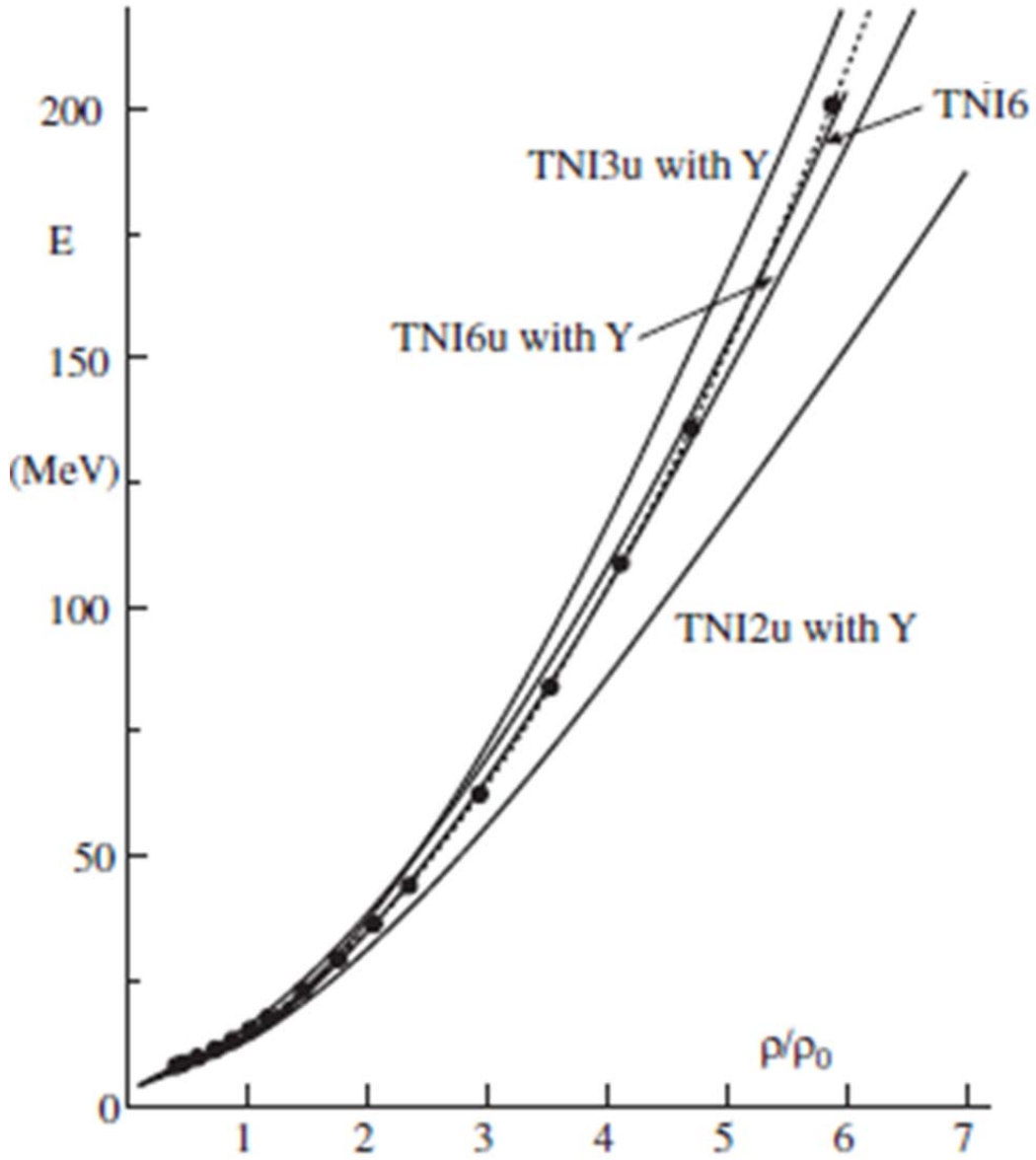


Figure 2.7. The energy per baryon  $E$  vs. matter density  $\rho$  (in units of nuclear density  $\rho_0$ , where  $\rho_0 = 0.17$  nucleons/fm<sup>3</sup> or  $2.8 \times 10^{14}$  g/cm<sup>3</sup>) relation for various hyperon mixed neutron star EOSs are shown. TNI2u, TNI3u and TNI6u represent soft, moderately stiff and stiff EOSs respectively for neutron stars with hyperon mixed cores. TNI6 is a moderately stiff neutron star EOS model that assumes that the neutron star core is only composed of neutrons, electrons, protons and muons.

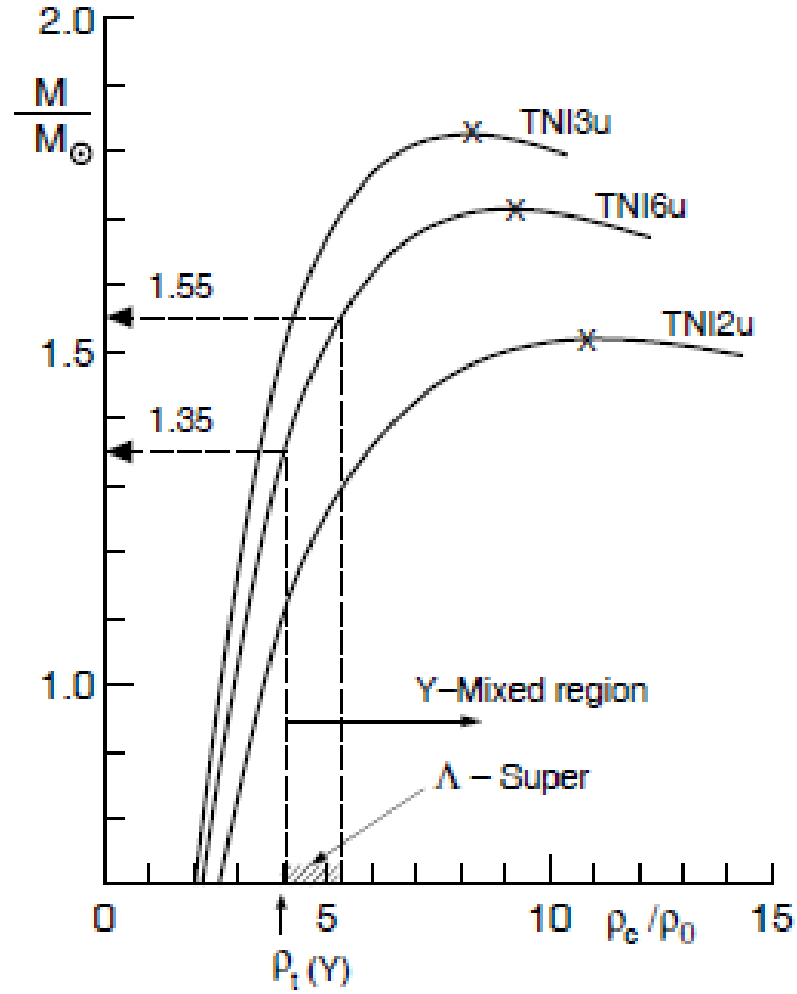


Figure 2.8. Mass (in units of solar mass) versus the central density (in units of nuclear density) of the neutron stars corresponding to the EOS of hyperon mixed neutron star matter is shown.

While the three Y-mixed models that are presented by Takatsuka et al. (2006) are used for several following studies, including Tsuruta et al. (2009) as well as parts of the chapter 3 of this study, some questions were raised when they were applied to neutron star cooling problems due to the results of the NAGARA event. The NAGARA event was first observed in a hybrid-emulsion experiment, and describes a hyperon hypernucleus formation through interactions between Y-particles. Hyperon hypernuclei

would have much weaker attractive strong force interactions compared to the individually interacting hyperons (Takahashi et al., 2001). Since the superfluid gap energy is determined by the strength of the attractive strong force, the formation of hypernuclei would lead to much smaller superfluid gaps. Then the superfluid suppression of fast hyperon cooling would not be effective, and that in turn would lead to an extremely rapid cooling of the neutron star that is inconsistent with some of the observational data, especially those for the lower temperature neutron stars such as the Vela pulsar (0833-45), that are shown in figure 2.3. At this time, however, there is no conclusive proof of this effect, therefore the TNI Y-mixed models are still considered valid.

## 2.5 Thermal Evolution of Isolated Neutron Stars

### 2.5.1 Earlier Studies of Neutron Star Cooling via Standard Processes

The cooling of neutron stars is typically observed in two stages: The earlier ‘neutrino era’ where the cooling takes place primarily due to neutrinos escaping from the interior; and the later ‘photon era’ where the cooling takes place primarily due to photon radiation from the surface of the star (Tsuruta 1979). The neutrino cooling era of neutron stars is dominated by standard processes such as neutrino bremsstrahlung, plasmon and modified Urca processes, and, under certain conditions (see section 2.6), by enhanced neutrino processes involving exotic particles. The neutrino era should last  $10^4$  to  $10^6$  years (Nomoto and Tsuruta, 1987). During the early neutrino era, the core cools due to the dominant modified and enhanced Urca processes that are taking place in the central core, but the finite thermal conductivity of the crustal layers does not allow the cooling

information of the core to reach the surface for 10 to 1000 years (Nomoto and Tsuruta, 1987). The process through which the core cooling information reaches the surface is often referred to as “the thermal relaxation”.

While the standard cooling processes are well understood and almost universally considered to be part of the neutron star cooling, their ability to fully explain the behavior of neutron stars was brought into question when theoretical cooling results were compared with the X-ray satellite observations of actual neutron stars. Examples of this discrepancy are shown in the thermal evolution curves in figure 2.9 (Umeda et al., 1994). The cooling curves shown in figure 2.9 are calculated using the Nomoto – Tsuruta exact

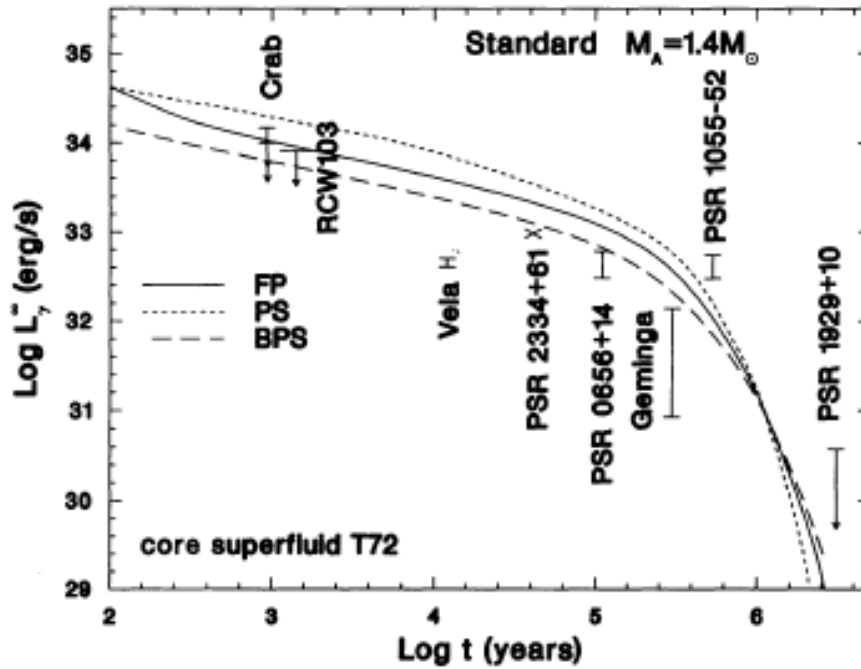


Figure 2.9. Standard Cooling is shown for identical 1.4 solar mass neutron stars where the stellar core constituents follow the FP, PS and BPS (this is an earlier equation of state developed by Baym, Pethick and Surther, and considered to be too soft to be realistic by contemporary researchers, therefore is not discussed in this study) equations of state, and the superfluid model of Takatsuka (1972). Photon luminosity detected by an observer at infinite distance is shown as a function of the age of the neutron star in logarithmic scale.

method (Nomoto-Tsuruta 1987). In the figure, while some of the hotter neutron stars, such as the Crab, display thermal evolution consistent with the observations, some other stars, such as Vela, were significantly colder than predicted by the standard cooling. While several ideas for enhancing the standard cooling processes were proposed in order to allow an explanation for the behavior of colder neutron stars (Page et al., 1990; Pines et al. 1992) that are shown as the observational results in figure 2.9, the theory of the involvement of exotic particles and the associated non-standard faster cooling processes that were proposed over three decades ago (Migdal, 1979) also gained more credence. Consequently, many studies of the involvement of these exotic particles in neutron star cooling (e.g. Tsuruta 1998 for review) have been carried out with very promising results.

### 2.5.2 Non-standard Cooling of Neutron Stars

Since the discovery of pulsars that appear to have cooled more quickly than what is predicted by the standard cooling scenario, several authors have studied various non-standard faster cooling mechanisms. Two of the most promising studies, one involving the participation of pions, and the other involving participation of hyperons in the neutron star cores, will be discussed in this section.

2.5.2.1 Cooling of Neutron Stars with Pion Condensate Cores. Although the possibility of the involvement of pions in neutron star cooling was proposed over forty years ago (Bachall and Wolf, 1965), it was not thought to be a serious contender due to some serious problems with the theory of free pions. Nearly a decade later, a new theory involving pion condensate (Migdal, 1979), has been shown to be very promising.

According to this theory, an additional Urca cycle with very high emissivity can take place in neutron stars with cores containing pion condensates:

$$n + \langle \pi^- \rangle \rightarrow n + e^- + \bar{\nu}_e \quad (2.10)$$

This new theory stated that the additional Urca cycle could provide much more efficient neutrino emissions that are several orders of magnitude stronger than any of the standard processes, and could account for the faster than expected cooling of some of the observed pulsars.

Over the decades following the initial proposal, the pion cooling theories underwent significant improvements, primarily due to the developing understanding of the behavior of pion condensates. One of the most detailed studies of the contribution of the non-standard cooling in neutron stars due to the presence of pion condensates in the core was carried out by Umeda et al. (1994). In their work, the authors included the latest estimates of enhanced cooling due to additional neutrino processes in addition to the existing standard processes (Muto et al., 1993; Takatsuka et al., 1993). In the work by Muto et al. (1993), the efficiency of the non-standard cooling was linked to the fractional size of the exotic core, and that was constrained by the stellar mass. The thermal evolution of neutron stars with pion cores were calculated for the medium stiffness FP and stiff PS models, and the results are shown in figures 2.10 and 2.11.

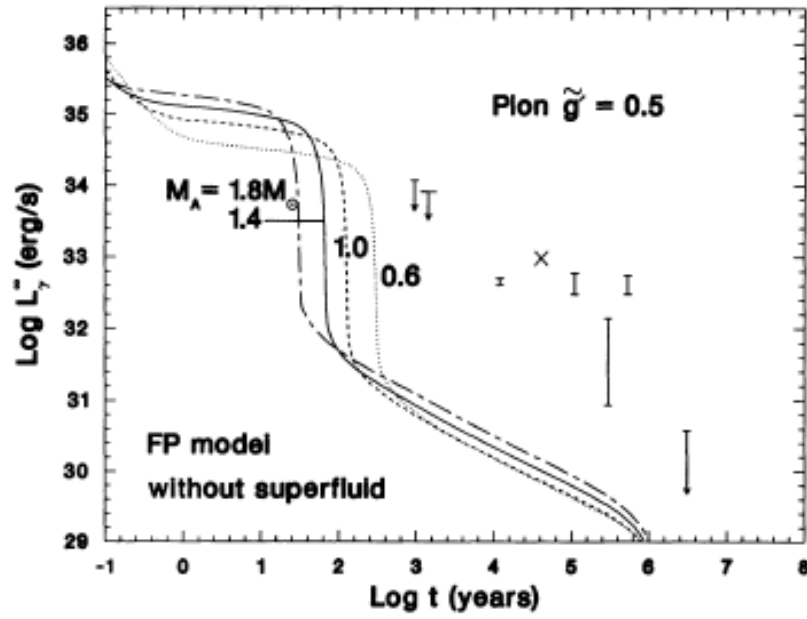


Figure 2.10. The pion cooling curves of neutrons stars of varying mass are shown for the FP Equation of state. Photon luminosity as detected by an observer at infinite distance is shown as a function of the age of the neutron star in logarithmic scale. The pulsar data shown are identical to those in figure 2.9.

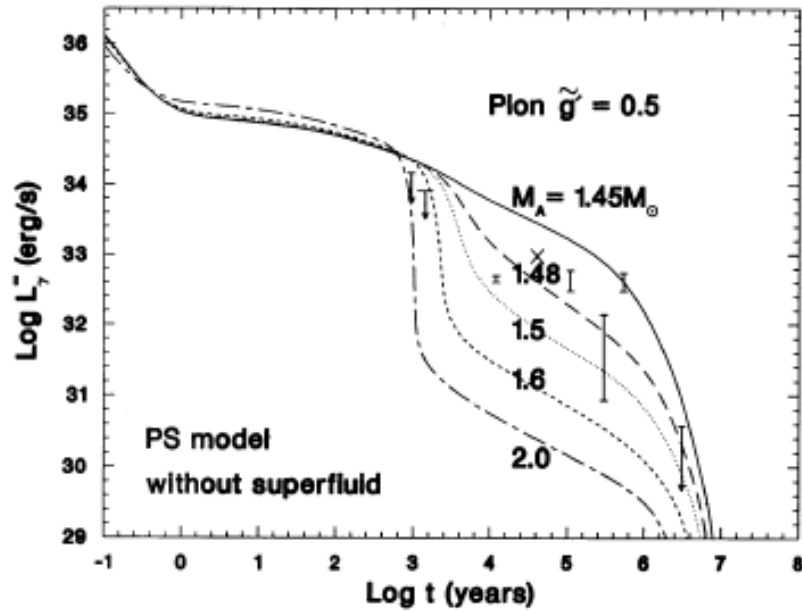


Figure 2.11. The pion cooling curves of neutrons stars of varying mass are shown for the PS Equation of state. Photon luminosity as detected by an observer at infinite distance is shown as a function of the age of the neutron star in logarithmic scale. The pulsar data shown are identical to those in figure 2.9.

In both figures 2.10 and 2.11, the drastic effects of the enhanced cooling due to the involvement of pion condensates in the neutron star are visible. One of the most pronounced differences between the standard cooling and the non-standard cooling cases is the dramatic difference in the temperature drop during the thermal relaxation period. With the standard cooling shown in the figure 2.9, the stellar core cools relatively slowly, and while there is small sudden drop in temperature as the core cooling information reaches the surface, as seen as a minor drop at the beginning of the figure, the cooling is generally slow without major sudden changes. In figures 2.10 and 2.11, the exact cooling method is used with finite thermal conductivity and non-standard cooling processes are included. Subsequently, the cooling information reaches the surface after  $10^{1.5} - 10^{2.5}$  years in the FP model, and after  $10^{2.5} - 10^3$  years in the PS model. Once the core neutrino cooling information reaches the surface at the end of these periods the surface temperature of the star drops rapidly. This is indicative of the much more efficient core cooling processes involved in the neutron stars when pion condensates participate.

It is also important to note some of the differences between the results of the FP and PS models. When figures 2.10 and 2.11 are compared, the thermal relaxation period takes place much sooner for the FP model than the PS model. This is due to the fact that the neutron stars that follow the very stiff PS model will be far more extended and larger in size with larger crust, and consequently the core cooling information will take longer to reach the surface of the star. Also the minimum mass benchmark in order for the star to possess a pion core is much higher for the PS model due to the reduced density of the

stiffer model. Finally, the mass dependence of the cooling is far more pronounced in the PS case.

While the enhanced cooling of the non-standard processes may provide an explanation for the colder neutron stars, they may not be consistent with the hotter stars. If the FP model represents the actual stiffness of a neutron star, then the cooling of the star takes place too rapidly to be consistent with the detections. However, this issue is resolved if the efficiency of the non-standard processes were reduced. Umeda et al. (1994) pointed out that the natural source of such a reduction would be the superfluid suppression of the neutrino emissions from the core.

As in ordinary nuclear matter, pions can also become superfluid under certain conditions. According to the superfluid theory employed by various authors (e.g. Takatsuka & Tamagaki, 1993; Umeda et al. 1994), in the superfluid state both the specific heat and the pion cooling decrease as a function of  $T_{cr}/T$ , where  $T_{cr}$  is the critical temperature for superfluid transition and  $T$  is the current temperature of the neutron star. Subsequently, the neutrino emissivity due to the non-standard processes involving pions is reduced as  $\exp(-\Delta/T)$ , where  $\Delta = a T_{cr}$  is the superfluid energy gap. The values for the constant  $a$  are given by Takatsuka (1972), and  $a = 1.76$  for the  $^1S_0$  state where  $a = 8.4$  for the  $^3P_2$  state superfluidity. While the reduction in neutrino emissivity due to the superfluidity of ordinary nuclear material is also an exponential function, the suppression in the non-standard processes of the pions would be significantly more manifest, because the neutrino luminosity of these processes is significantly stronger. Since the size of the superfluid gap and the efficiency of the superfluid suppression of neutrino emissions

from non-standard processes are directly related, models with smaller gaps can be expected to demonstrate relatively minor suppression on the cooling of the neutron stars, while the models with larger gaps will show very noticeable suppression of cooling. The superfluid suppression effects of various superfluid models on the neutron star evolution for the FP EOS models is demonstrated in figure 2.12 (Umeda et al., 1994).

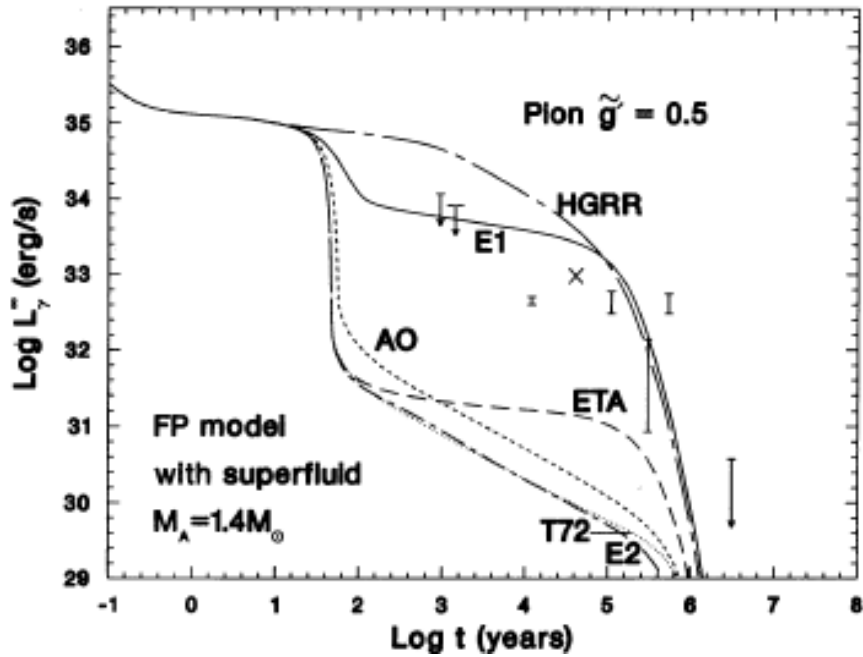


Figure 2.12. Effects of the superfluid suppression of various superfluidity models on the cooling of a  $1.4 M_{\odot}$  neutron star are shown for the FP model.

As can be seen in the figure 2.12, the models with very small gaps such as E2 and T72 have almost no noticeable effect on the cooling of the  $1.4 M_{\odot}$  FP model neutron star. The cases with a large superfluid gap such as the HGRR model, on the other hand, leads to extremely strong suppression of the non-standard processes, almost to the point that

the thermal evolution of the star closely resembles the result of standard cooling as seen in figure 2.9.

Finally, Umeda et al. (1994) also explored the effects of stellar mass on the superfluid suppression in their publication. Since the superfluid effect is inversely related to the density of the core material as shown in figure 2.2, and since the density and the stellar mass are directly related, the increasing stellar mass should weaken the superfluid suppression of the non-standard processes. The result from Umeda et al. (1994) for the superfluid effect and mass relationship for the FP and PS stars are shown in figure 2.13 and 2.14. For the FP model star E1 superfluid model adopted, while the PS model star uses ETA superfluid model. Both stars demonstrate the diminishing effect of the superfluid suppression on the cooling with increasing mass, but the PS model star experiences this effect more noticeably than the FP model star.

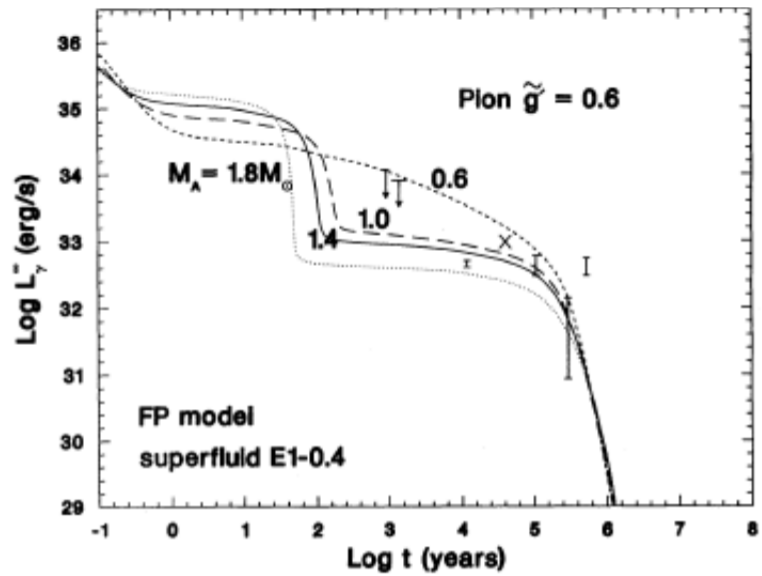


Figure 2.13. Effects of the stellar mass on the superfluid suppression of pion cooling for the FP EOS stars with E1 superfluid models is shown for several different mass values.

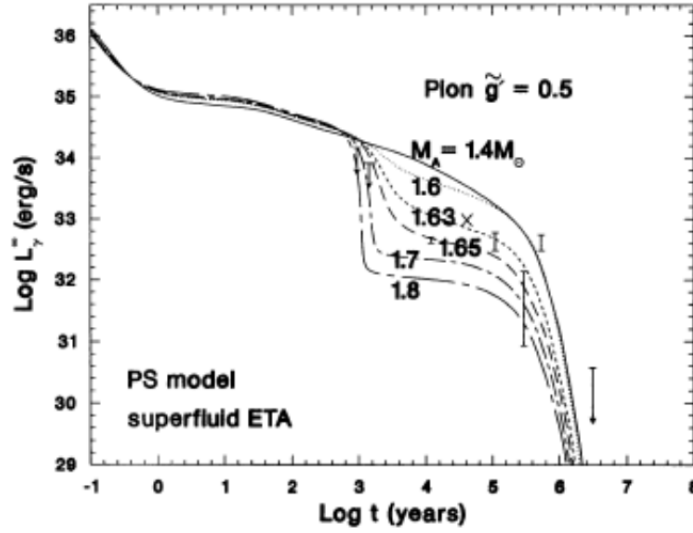


Figure 2.14. Effects of the stellar mass on the superfluid suppression of pion cooling for the PS EOS stars with ETA superfluid models is shown for several different mass values.

2.5.2.2 Cooling of Neutron Stars with Hyperon-Mixed Cores. The possibility of the involvement of hyperon particles in the neutron star cooling processes was first proposed by Prakash et al. (1992), followed by works by Haensel and Gnedin (1994), Schaab et al. (1998), and Page (1998). All of these works were based on parameterized emissivity models that only carried out qualitative studies of the thermal evolution of neutron stars with hyperon-mixed cores. The far more in-depth and complete study of hyperon mixed neutron star models was carried out by Tsuruta et al. (2009), who adopted three different equations of state, TNI3u, TNI6u and TNI2u, that were based on the then most recent development in nuclear and particle theories (Takatsuka et al. 2007).

While ordinary neutron star cores are primarily composed of neutrons along with very small fraction of protons, electrons, and muons (Tsuruta, 1964), Takatsuka et al. (1995) had shown that exotic particles may also be present in the core if the

density/pressure is high enough. The critical density for transformation to hyperon-mixed material is given as four times the nuclear density where nuclear density is  $\rho_0 = 2.8 \times 10^{14} \text{ gm cm}^{-3}$  by the results of Takatsuka et al (2002 & 2006). Once the temperature of the stellar core after the supernova explosion drops below the critical level for the formation of superfluid hyperon mixture, the cooling of the star becomes heavily influenced by the presence of these particles.

The appearance of the hyperon matter in the stellar core has several effects on the cooling process of the neutron star. Firstly, as explained in the previous chapter, the EOS for the core becomes somewhat softened. Secondly, some highly efficient non-standard neutrino processes involving these particles become possible. The examples of these processes are found in Takatsuka et al. (2006):

$$\Lambda \rightarrow p + l + \bar{\nu}_l \quad (2.11)$$

$$\Sigma^- \rightarrow \Lambda + l + \bar{\nu}_l \quad (2.12)$$

where  $l$  is a lepton, either an electron or a muon. Thirdly, the hyperon material has its own superfluid properties, which leads to the suppression of the non-standard processes when the core conditions are suitable (Takatsuka & Tamagaki, 2004). The surface temperature at the early stages of the cooling is high because the fast cooling by neutrinos in the core has not been transmitted to the surface. However after the thermal relaxation the fast cooling information of the core reaches the surface, and about that time the core temperature drops below the transition temperature for hyperon superfluidity, and the hyperon direct Urca process (Eqn. 2.11) is greatly suppressed. The effects of the hyperon-mixing and the hyperon superfluidity on the neutron star cooling are evident in the

cooling curves that are shown in figure 2.15 and 2.16 which correspond to the medium stiffness TNI6u and soft TNI2u models respectively (Tsuruta et al. 2009). Similar to pion-condensate core neutron stars, these cooling curves show a rapid drop in the photon luminosity of the neutron star which is indicative of the rapid cooling due to enhanced neutrino processes of the hyperon-mixed matter in the core of the neutron star, and subsequent sudden drop in neutron star surface temperature as the core cooling information reaches the surface of the star. They also show the effects of superfluidity, and the EOS dependence of the superfluid energy gap. In the figure 2.15 the top most curve, which corresponds to a 1.26 solar mass neutron star, does not show any cooling effect by the fast neutrino processes, suggesting that the density is not high enough for the hyperon transitions to occur. The lower curves that correspond to 1.44, 1.47, 1.52 and 1.53 solar mass neutron stars in descending order, on the other hand, show the diminishing effect of the superfluid suppression on the fast neutrino processes involving hyperons with increasing mass. These authors identify the hyperon transitions as occurring for masses greater than 1.4 solar mass. In the figure 2.16, the results are somewhat different. The top two solid curves which correspond to a 1.2 solar mass neutron star are with and without internal heating, neither of which indicating any effects from non-standard cooling processes. On the other hand, the lower curves that correspond to 1.28, 1.3, 1.32, 1.33 and 1.5 solar mass neutron stars in descending order, once again display the effect of superfluid suppression on the non-standard processes which diminishes with the increasing mass. The hyperon transition mass for the neutron star is identified as 1.25 solar mass by the authors for the softer EOS model as expected.

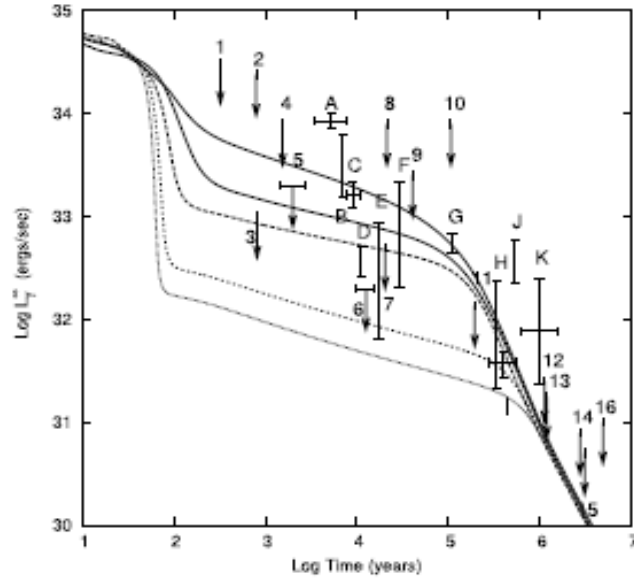


Figure 2.15. Thermal evolution of hyperon stars with TNI6u EOS and vortex creep heating with moderate strength  $K = 10^{36} \text{ ergs}^{-3/2} \text{ s}^2$ . The curves refer to stars with gravitational mass of 1.26, 1.44, 1.47, 1.52, and 1.53  $M_\odot$ , respectively, in the decreasing order in photon luminosity.

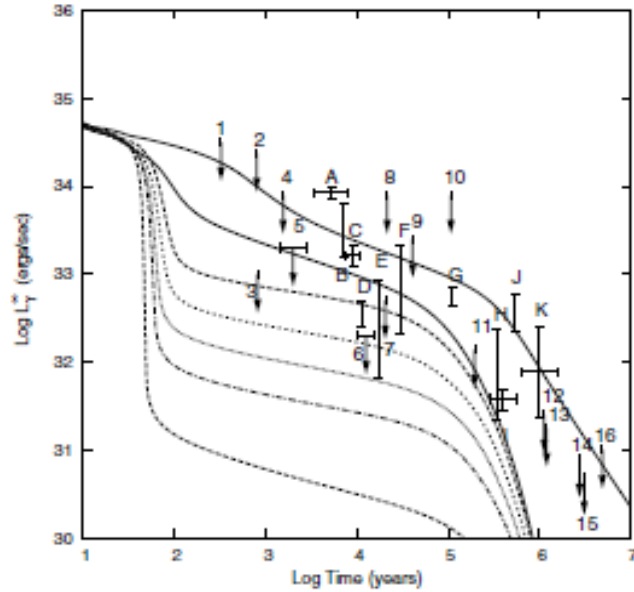


Figure 2.16 Thermal evolution of hyperon stars with TNI2u EOS. The hottest two solid curves refer to stars with gravitational mass of 1.2  $M_\odot$  with and without maximum heating for the higher and lower curves, respectively. The rest (the cooler five curves) are for hyperon stars with gravitational mass of 1.28, 1.3, 1.32, 1.33, and 1.5  $M_\odot$ , respectively, in the order of decreasing luminosity.

## 2.6 Construction of the New TNI7 EOS Model for Neutron Stars with Exotic Particle Cores

While previously created exotic core neutron star equations of state have shown great promise in explaining the cooling behavior of existing observations, a major setback to these theories came in the form of an additional observation in 2010. A neutron star was measured to have a mass very close to  $2 M_{\odot}$  using the Shapiro effect (Demorest et al. 2010), which links the time delay of radiation as it passes near a neutron star to the mass of the neutron star. Since most existing exotic core neutron star models have a maximum mass less than  $2 M_{\odot}$ , if correct this observation would rule out all these models. On the other hand, this shortfall in mass does not invalidate the idea of the involvement of exotic particles in the cores of the neutrons stars. However, it is an indication that the EOS for the neutron star core matter needs to be the noticeably stiffer than the ones previously studied.

As part of this doctoral study such a model was created that is consistent with most up-to-date nuclear and strong force information such as the net three body interaction (TNI) strength described in Takatsuka et al. 2009. The new model that is created for this study is dubbed TNI7. Since the stiffest model that was previously created by Takatsuka et al. (2007), the TNI3u model, was shown to support a maximum of  $1.85 M_{\odot}$  for neutron stars, the new TNI7 model would have to be stiffer than the TNI3u model. The latest results from nuclear theories and experiments (Takatsuka et al. 2009) supported that the universal three body model with hyperons can be strong enough to create a significantly stiffer model than the TNI3u model. According to Takatsuka et

al. (2009), the suggestion of a more massive neutron star than what is supported by their earlier work led them to believe that “extra repulsion” is needed to allow a stiffer EOS that can support a larger maximum neutron star mass. The authors concluded that a promising source for the extra repulsion was the extension of three-body interaction to include possible phenomenological hyperon – hyperon (YY) interactions and therefore making the TNI consideration in the EOS model truly “universal”. Their results showed that with the inclusion of this “extra repulsion”, the new universal three-body interaction based EOS would be capable of supporting a neutron star mass greater than 2 solar masses.

Based on the work by Takatsuka et al. (2009), it was determined that an EOS could be built that is stiffer than the TNI3u model, that is not only capable of supporting the  $2 M_{\odot}$  neutron star, but also explain a variety of neutron star properties such as temperature of other isolated neutron stars, and neutron stars in binaries, and neutron stars that display unique cooling properties such as Cassiopeia-A. Therefore the initial goal was set to gradually increase the stiffness of the TNI3u model, until the EOS could support a  $2 M_{\odot}$  neutron star. Also, an additional consideration had to be given to the results of the NAGARA effect, which, even though still controversial with no consensus for its validity, suggested that the hyperon models may lack the necessary superfluid suppression that is required to explain some of the observations for cooler pulsars such as the Vela pulsar. On the other hand, in a private communique (2009), Tamagaki and Takatsuka also suggested that the interactions involving pion condensates would create nearly identical results to the hyperon case, but would not be affected by possible

consequences of the NAGARA event. Since the previously studied FP and PS models can be considered as the lower and upper limits for a pion condensate core neutron star EOS, even though they are respectively considered to be not stiff enough and too stiff to be realistic, the idea of gradually increasing the stiffness of TNI3u model until it can support the  $2 M_{\odot}$  neutron star is still valid, since it falls into the parameter space between these two models. However, an additional difficulty may be present when considering the PS model as the ceiling for the EOS, since the parameter space in between the PS and TNI3u is fairly substantial which would allow many different EOSs that can support a  $2 M_{\odot}$  neutron star. Fortunately, just around the same time an additional piece of data, detections of neutron star temperatures of the Cassiopeia-A neutron star, became available and allowed a firm constraint to be placed on the EOS (Heinke and Ho, 2010). While further details of the role of these detections will be given later in this chapter, it is important to note that these data were sufficient to limit the parameter space significantly to the point that a statistically separable EOS could be created. In this PhD study, such an EOS was created. This EOS model is what will be called the TNI7 model hereafter.

In the Nomoto – Tsuruta (1987) exact code, the thermal evolution of a neutron star is affected primarily by four properties of the neutron star as described earlier in this chapter: equation of state, composition, superfluid behavior and internal heating. While the equation of state is independent from the other properties, all four properties had to be taken into consideration in the creation of TNI7 model, therefore it is important to discuss the details of the involvement of each property in the creation of TNI7 model.

The equation of state is defined by data tables instead of a singular equation in the Nomoto – Tsuruta (1987) exact code. Three data tables corresponding to internal pressure, mass density and energy density distributions of neutron star are used to construct an equation of state. The equation of state is defined in two separate parts, one representing the region inside the core and the other representing the crust. Since the composition of the crust is identical for all neutron stars, the equation of state of the crust will also be identical for various models. The core EOS, on the other hand, is model dependent, since the compressibility of the core material is strongly dependent on the strength of the repulsive strong forces between the constituent particles which is determined by the model. The increasing strength of the repulsive force is established by increasing the pressure of the core material for a given core density. By repeating this procedure for all data points in the core EOS data tables a new EOS is obtained. While in theory any set of data could be used to create a new equation of state, there are two constraints in this case that limits how the data tables are generated. First constraint is the existing models. Since there are already existing data tables for the core EOSs of the FP and PS models, a weighted average of the two has to be used to create the new EOS so that the new EOS lies somewhere between the two. How close the EOS would be to either FP or PS model EOS can be controlled by the weight given to the individual EOSs in the calculation of the average. Second constraint is the mathematical limitations. Since the Nomoto – Tsuruta (1987) exact code evaluates equations 2.1 – 2.6 starting from the center of the core and moving outward, and checking the continuity of the results at each step, the core EOS data table it will refer to while evaluating these equations must also

correspond to a very smooth and continuous equation. It is impossible to determine whether a newly created core EOS is “smooth enough” prior to running the code. Instead, the code fails to complete the evaluation at a given step if the corresponding pressure – density part of the EOS function is not smooth enough, indicating the step at which it failed. Using this information, the EOS function can be smoothed at the necessary location and the code can be rerun. It is important to note that the mathematical difficulty is more likely to appear at stiffer EOSs since the core distribution varies faster and therefore is more prone to discontinuities. Therefore it is advisable to start with softer EOSs and increase the stiffness incrementally until reaching the desired stiffness. By following this procedure, first an EOS with the required condition of  $2 M_{\odot}$  maximum was established. This EOS was found to be slightly softer (approximately 4%) than the linear average of FP and PS model EOSs. Then the EOS was further stiffened to yield the results that are consistent with Cas-A detections data. Further details of this process will be given later in this chapter.

The second parameter that was studied during the creation of TNI7 model was the composition. While the core composition, specifically the presence of exotic particles, may have a minor softening effect on the EOS, this effect is not included in the Nomoto – Tsuruta (1987) exact code. This does not exclude the interdependence of EOS and composition completely. Although the only possible adjustment to the composition is whether a specific constituent particle would be present or not, the presence of specific exotic particles is also determined by the EOS since the exact code allows the involvement of exotic particles only if the necessary core density conditions are met. For

the study of Cas-A, pion condensate was chosen as the exotic core model to study and therefore the code was set to assume the existence of pion condensate. However, because the pion condensate appears only for large enough core density, pions would not be involved for all neutron star simulations. For this study the transition density determined by the latest work by Takatsuka et al. (2009) was used. This transition density is three times the nuclear density for pions (four times the nuclear density for hyperons as show in figure 2.8). It is important to note that the core density of a neutron star of certain mass is inversely related to the stiffness of the EOS of its core material. Therefore, even though the transition density is fixed, as the EOS becomes stiffer, the appearance of exotic particles will require increasingly larger minimum stellar mass. This effect is also visible in figure 2.8 where the hyperons appear at  $1.35 M_{\odot}$  for the medium stiff TNI6u model, while the transition does not take place until approximately  $1.45 M_{\odot}$  for the stiff TNI3u model. Furthermore, the appearance of exotic particles does not immediately bring about the maximum cooling due to the enhanced neutrino processes, since only a small portion of the core has these exotic particles for lower mass stars, and the full enhancement is only seen in higher mass neutron stars. The TNI7 model is determined to have its exotic particle transition limit at  $1.5 M_{\odot}$ , and therefore any neutron star with a smaller mass cools with identical standard processes under this model. Moreover, the fraction of the exotic core does not reach large enough size until about  $1.6 M_{\odot}$ , and therefore only neutron stars with  $1.6 M_{\odot}$  and greater can demonstrate the full enhancement of cooling by non-standard processes. Further details of the mass – cooling relation will be shown later in this chapter during the discussion of the study of Cas-A.

Another crucial property of the neutron star thermal evolution is the superfluid suppression of the enhanced neutrino processes. Once again this is not a property of the EOS, but a quality of the neutron star core constituents. However, because the superfluidity of the constituent particles affects the cooling regime of neutron stars, and because the superfluid behavior itself is affected by the EOS, it must be taken into account when constructing an EOS for a specific purpose such as fitting the Cas-A data. As explained earlier in section 2.2.3, the previous study of neutron star revealed that the maximum cooling enhancement by non-standard cooling processes causes the temperature of neutron stars to drop too rapidly to be consistent with the hotter neutron star observations. This issue, however, can be resolved if the enhanced processes are partially suppressed by the superfluidity of the constituent particles (Tsuruta, 1998). Similarly, a superfluid regime must be part of the consideration of a new, stiff EOS in order for the new model to be consistent with the full spectrum of neutron star temperature observations. While there are many superfluid models that can be employed for this purpose, the most recent pion condensate superfluid model that is developed by Takatsuka et al. (2009), which is based on most up-to-date particle and nuclear physics data, is adopted for this study. In the case of hyperon superfluid model, the superfluid suppression of enhanced neutrino processes is effective only within a certain density range. This range can be seen in figure 2.8, and is approximately between 4 and 5.3 times the nuclear density for hyperon cores. In the case of pions, superfluidity becomes effective at three time nuclear density. While superfluid density region is not affected by the changes in the stiffness of EOS, the neutron star masses for which the superfluid

suppression is effective is dependent on the stiffness of the EOS. As it can be seen in figure 2.8, as the stiffness of the EOS is increased, the range of stellar masses that are affected by the superfluid suppression shifts toward the more massive side. In addition, the efficiency of the superfluid suppression is lowered with increasing stellar mass since the superfluid suppression is dependent on the superfluid gap size which gets smaller as the core density gets larger with increasing stellar mass. The TNI7 model also displays a similar superfluid behavior. Neutron stars with TNI7 core model that have a mass of  $1.5 M_{\odot}$  or less do not contain exotic particle core, therefore are unaffected by the superfluid suppression of these particle. Instead, all such neutron stars cool with identical slow, standard neutrino processes. Neutron stars with TNI7 core model that have masses greater than  $1.5 M_{\odot}$ , on the other hand, contain pion condensates in their cores, and are affected by the superfluid suppression of the non-standard neutrino processes. For the lower mass neutron stars in this range, this effect is very pronounced, therefore the enhancement to the cooling is not very strong, and the neutron stars cool only slightly faster than the standard case. As the stellar mass increases, however, the effect of superfluid suppression diminishes, and the cooling becomes increasingly more efficient. In the TNI7 model this increase in the efficiency of enhanced neutrino processes continues until neutron star mass is around  $1.9 M_{\odot}$  where the superfluid suppression completely disappears, and all neutron stars of  $1.9 M_{\odot}$  or greater cool with identical maximum efficiency enhanced neutrino processes. Consequently, all variations in the thermal evolution of neutron stars that follow TNI7 model will be for neutron stars that fall in the  $1.5$  to  $1.9 M_{\odot}$  range.

The final neutron star property that needs to be considered is the involvement of internal heating in the neutron stars. Once again the studies by Tsuruta (1998) had shown that even the neutron stars that are cooling via standard processes may be cooling too fast to be consistent with the hottest neutron star detections. The obvious solution for this discrepancy was determined to be an internal heating source. In the creation of TNI7 model, it was assumed that the discrepancy would be present for the new model as well. Consequently an internal heating regime was also included for some simulations in order to make the results consistent with the full range of observational temperature data. While there are many possible sources for internal heating, frictional heating given by equation 2.9 was deemed to be best understood and studied, and therefore was chosen for this study. While this heating regime has three versions; -weak, moderate and strong heating, - the moderate heating was deemed sufficient to explain the full range of observations. Internal heating was also shown to be necessary only for the hottest neutron star detections.

While by taking the aforementioned four neutron star properties into account any number of EOS can be created that will also fulfill the prerequisite of supporting a  $2 M_{\odot}$  stable neutron star, these properties alone do not provide a way of distinguishing between these EOSs. Also the behavior of exotic particles as well as the nucleonic matter at the extremely high densities of the cores of these neutron stars is not very well understood, and as a result no strong constraints can be placed on these EOSs based on particle and nuclear physics theory either. Therefore creation of a specific EOS requires additional external information that can be matched to the result of neutron star simulations based

on this specific EOS. In the case of TNI7 model this external information came from the observations of the neutron star in Cas-A supernova remnant. The unique nature of this observation is the availability of the neutron star age and short term cooling rate information. This information is particularly useful because the neutron stars typically cool in four stages regardless of EOS and composition, and each of these stages takes place during a unique period of thermal evolution and has their unique cooling profiles. The stages of neutron star cooling are shown in figure 2.17. The initial stage of the neutron star cooling is dominated by the neutrino emissions from the crust with relatively low emissivity. During this stage, the previously explained core neutrino processes are also in effect, but due to the finite conductivity of the neutron star material, the core cooling information does not reach the surface immediately. However, once the heat wave from the rapidly cooling core reaches the surface, the surface temperature of the neutron star drops significantly in a relatively short period of time, in an event commonly referred to as “thermal relaxation”. This is marked as stage II in figure 2.17. It is also evident in the figure that the thermal relaxation surface temperature drop is far more dramatic in the cases where enhanced neutrino processes are in effect in the core of the neutron star. At the end of thermal relaxation, the neutron star becomes isothermal, and the surface temperature continues to drop at a lower rate primarily due to the neutrino emissions from the core during the stage III of the cooling. Finally, as the temperature of the neutron star drops below a certain level, the neutron star enters its final stage of cooling where the neutrino emissions are weakened to the point that they do not contribute to the overall cooling in any significant way and the neutron star cooling is

dominated by the photon emissions from the crust. This final stage of cooling is the stage IV shown in figure 2.17.

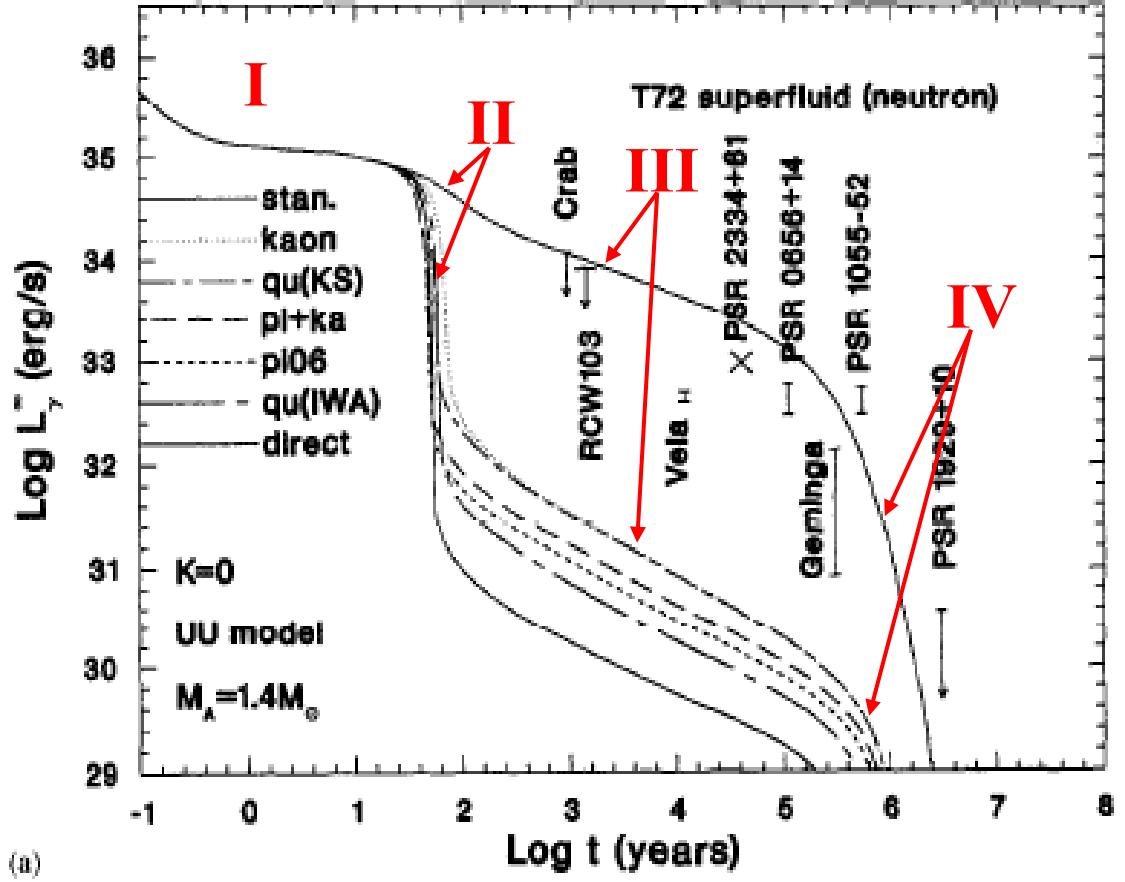


Figure 2.17. Stages of thermal evolution of neutron stars is shown under various neutrino processes that dominate their early evolution (Tsuruta, 1998). Regions I, II, III and IV refer to the four stages of cooling as described in the text.

The cooling stage that was primarily of interest during the construction of the TNI7 model was the thermal relaxation period. The short term observations of Cas-A suggested that this neutron star was cooling at an unusually rapid rate (4% over ten years) for a neutron star (Heinke et al. 2010). The only period of cooling during the overall thermal evolution of a neutron star that is capable of displaying such a high cooling rate

is the thermal relaxation phase, and only when the core cooling is influenced by strong enhancement of non-standard neutrino processes. Furthermore, the timing of thermal relaxation is dependent on the stiffness of the equation of state. Since the thermal relaxation takes place as the core cooling information reaches the surface of the neutron star, it is dependent on the radius of the neutron star. The radius of the neutron star, in turn, is dependent on the matter compressibility, therefore the stiffness of the EOS of the neutron star where a stiffer EOS neutron star would be larger and more extended. This fact, combined with the information about the age of Cas-A neutron star, made it possible for the EOS to be stiffened until the neutron star's thermal relaxation was delayed to the point that is consistent with the observational data. Some further discussion of this process will be given in the following section during the discussion of Cas-A study. While this allowed the creation of a unique EOS TNI7, in principal, same process can be used to generate EOS models if and when similar data becomes available for other neutron stars.

Ultimately, the TNI7 was constructed using the ideas that are discussed earlier in this section. The energy per particle versus density information of the newly created TNI7 model in comparison to various other models is shown in figure 2.18. As it can be seen from this figure, the new model is significantly stiffer than the FP model, and moderately stiffer than the TNI3u model, but it is significantly softer than the PS model. The new model is also very close to the average of the stiffness of the FP and PS models, but it is slightly stiffer than this average.

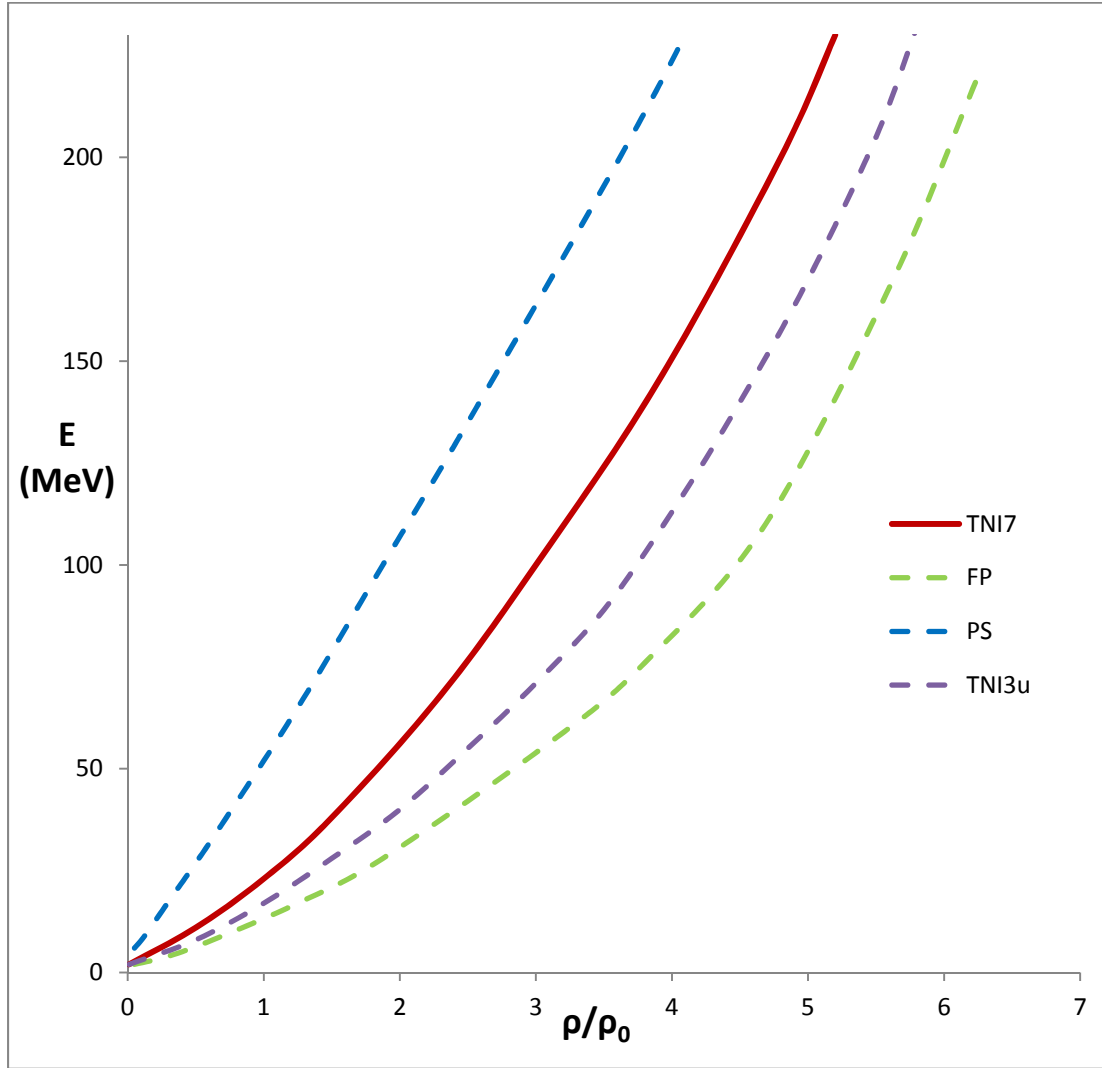


Figure 2.18. The energy per baryon  $E$  vs. matter density  $\rho$  (in units of nuclear density  $\rho_0$ , where  $\rho_0 = 0.17$  nucleons/fm<sup>3</sup> or  $2.8 \times 10^{14}$  g/cm<sup>3</sup>) relations for various neutron star EOSs are shown. Dashed blue, purple and green lines are the PS, TNI3u and FP models, while the solid red line is the newly created TNI7 model.

This new EOS allows neutron stars that are as nearly as massive as  $2.1 M_\odot$ , and therefore it is compatible with the observation of a  $2 M_\odot$  neutron star (Demorest et al. 2010). Due to its very stiff nature, the neutron stars that are based on the TNI7 EOS would give very extended and larger neutron stars. In this model, the  $2 M_\odot$  star

corresponds to a radius of 13.2 km, and a core density of  $1.8 \times 10^{15} \text{ g/cm}^3$ . These values are essentially identical to the parameters that are given by Takatsuka et al. (2009). This similarity between the TNI7 model and the theoretical work by Takatsuka et al. (2009) is particularly important. The TNI7 model is originally created in a purely phenomenological fashion in order to create neutron star thermal evolution simulation results that are consistent with Cas-A observations. While the idea of stiff model such as TNI7 is justified based on the earlier suggestions by Takatsuka et al. (2007) that the inclusion of universal TNI forces would lead to stiffer EOSs than their previously created models, the fact that the latest theoretical particle and nuclear physics adopted by Takatsuka et al. (2009) led them to an EOS that is essentially identical to TNI7 model created for this study, gives further credence to the efficacy of the TNI7 EOS.

The gravitational mass versus central density relation for neutron stars with pion cores based on the new TNI7 EOS model is shown in figure 2.19. The neutron star whose cooling curves are consistent with Cas-A detections is marked with a red dot on the figure, and corresponds to a neutron star with an approximate mass of  $1.7 M_{\odot}$ . Further details and discussion of Cas-A will be given in following section.

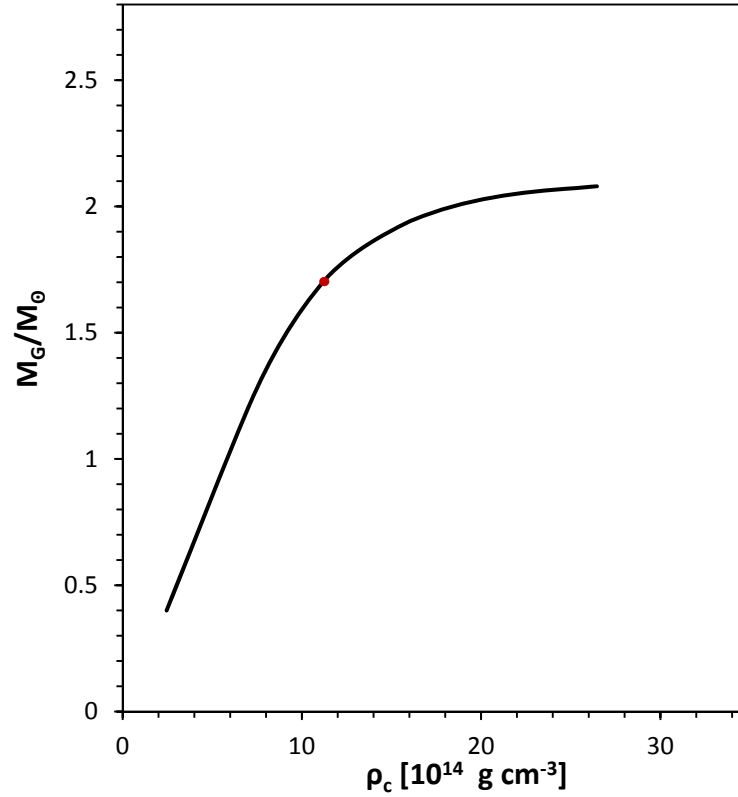


Figure 2.19. Gravitational mass vs. central density relation for TNI7 Model is shown. The red point refers to Cas-A neutron star.

One final question that is important to answer about the TNI7 model is whether its high stiffness is justifiable based on previous work in this field. As previously discussed, at least one significantly stiffer model, the PS model, is already available. However, many other additional EOSs were constructed and studied in the past. A collection of the most well-known EOSs can be found in Yakovlev et al. (2007), and is shown in figure 2.20. In this figure, the TNI7 model would occupy the parameter space between the FP model (line 3) and the PS model (essentially identical to the BGN2 model of line 8) as stated earlier. While the details of the additional EOS model that are shown in this figure

are irrelevant to this study, this figure is still useful for establishing the soundness of the TNI7 model, since it falls within the mid-range of the parameter space, near model 7 (APR), that was previously studied by neutron star researchers, and therefore is by no mean an extreme case.

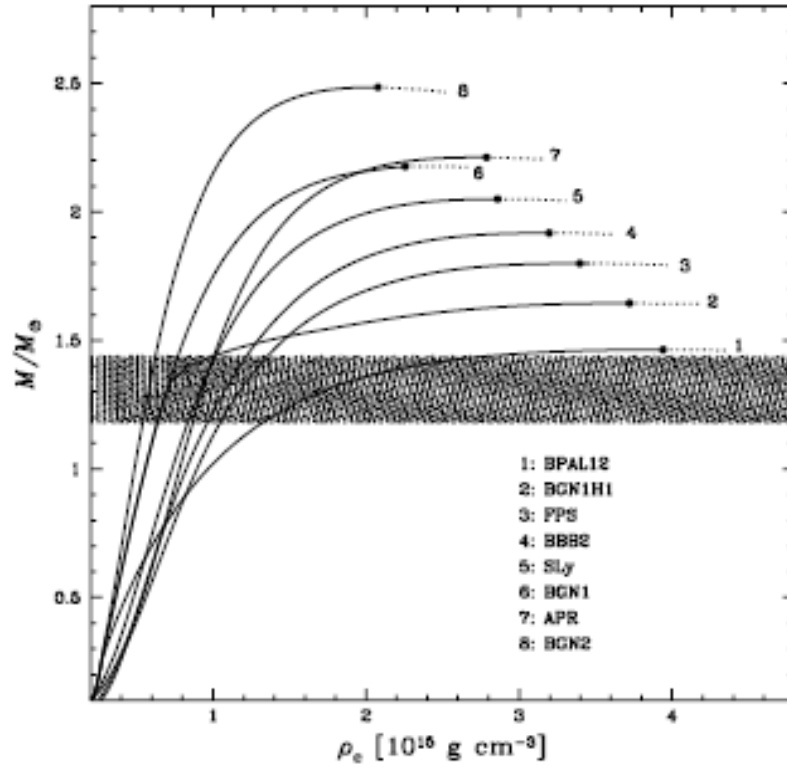


Figure 2.20. Gravitational mass versus core mass density plots for various neutron star core models are shown. The shadowed are represent the mass range of observed observed pulsar, though this information is very outdated since there are current pulsar observations with masses as high as  $2 M_\odot$ .

## 2.7 Background on Cassiopeia-A Neutron Star

One of the biggest challenges for neutron star studies is the observed data for the characteristics of a stellar objects, one of the most important characteristics being the age of the neutron star. While great many strides have been made in neutron star research, due to the lack of accurate age information, it has been difficult to distinguish between various thermal evolution models. The recent study of the Cassiopeia-A neutron star has brought new rigor to such studies, because relatively higher quality data had the promise that it may allow the formulation of better constraints for the number of reasonable neutron star models.

Cassiopeia-A is the strongest radio source in the sky beyond the solar system that has been identified to this day (Baars et al. 1977). While there is no confirmed observation of the supernova that led to the creation of this neutron star, a historical observation in 1630 by John Flamsteed is highly suspected as being of that supernova (Hughes, 1980). Moreover, the age of the supernova and the neutron star was estimated by the studies of the propagation of the nebular expansion of the supernova ejecta through Hubble Space Telescope observations, and this age is generally accepted to be  $330 \pm 20$  years (Fesen et al., 2006). The neutron star in Cas-A supernova remnant was first discovered by the Chandra X-ray Observatory in 1999, but until recently only the upper limit for its temperature was known (Tananbaum, 1999; Hughes et al. 2000). Additional observations and studies of the thermal properties of the Cas-A neutron star were carried out over a 10 year period from 1999 to 2009 (Pavlov et al., 2000; Umeda et al. 2000; Chakrabarty et al. 2001; Ho & Heinke, 2009), and the neutron star was shown to

have an unexpectedly high cooling rate during this period (Heinke & Ho, 2010; Shternin et al. 2011). The age and short-term cooling profile combined present a unique and unparalleled opportunity for the study of the neutron stars that may allow very strong constraints for realistic neutron star models.

### 2.8 Exotic Particle Neutron Star Core Model for Cassiopeia-A

The primary goal of the neutron star modeling that will be applied to Cas-A is to provide the thermal evolution timeline and the cooling rate that are consistent with the observed extremely rapid 4% drop in temperature over the 10 year period (Heinke & Ho, 2010),  $330 \pm 20$  years after the formation of the neutron star. Such a rapid drop in temperature for a neutron star this young is inconsistent with most of the existing models, which has led several groups to formulate new neutron star models that include a specific version of neutron superfluidity (Page et al. 2011, Shternin et al. 2011). While these models do indeed yield the desired cooling rates at the expected age of the neutron star that are consistent with Cas-A, the neutron superfluid model that was used for these models was highly phenomenological and highly contrived theoretically when compared with the previous work on the subject (Tsuruta, 2002; Tsuruta, 2009). An alternate explanation for the behavior of Cas-A is possible if the involvement of exotic particles in the core of the neutron star is taken into account instead of this phenomenological neutron superfluidity.

As discussed in the previous section, a period of very rapid cooling is present naturally during the thermal relaxation period of neutron stars when the core cools

rapidly due to enhanced neutrino processes that involve exotic particles such as pions and hyperons (Umeda et al. 1994; Tsuruta et al. 2009). While most traditional models place the thermal relaxation period at a time no later than 100 years after the formation of a neutron star, it was also shown that thermal relaxation can be delayed to as late as 1000 years after the formation if a sufficiently stiff equation of state, such as the PS model, that allowed the formation of a much larger and extended neutron star, were to be adopted (Umeda et al., 1994). This idea, combined with the goal of creating an equation of state that can support a neutron star mass of  $2 M_{\odot}$ , provided the basis for the search for a stiff enough equation of state that will create the thermal evolution profile that is consistent with the Cas-A observation.

Through a parameter search, accomplished by creating various intermediate EOS models between FP (or TNI3) and PS models, and testing their cooling profiles against Cas-A observational data, TNI7 model was created (see section 2.6 for details). The TNI7 model is consistent with the latest universal three-nucleon interaction based equation of state that is outlined by Tamagaki and Takatsuka (2009). Characteristic properties of the neutron stars with a pion condensate core based on the new TNI7 equation of state are shown in comparison with the FP and PS models in Table 2.2.

**CHARACTERISTIC PROPERTIES OF TNI7, FP AND PS  
MODELS WITH A PION CONDENSED CORE**

| $M_A$<br>( $M_\odot$ ) | $M_g$<br>( $M_\odot$ ) | $\rho_c^m$<br>( $10^{14} \text{g} \cdot \text{cm}^{-3}$ ) | R<br>(km) | $R_e$<br>(km) |
|------------------------|------------------------|-----------------------------------------------------------|-----------|---------------|
| <b>TNI7 Model</b>      |                        |                                                           |           |               |
| 1.2                    | 1.14                   | 6.98                                                      | 14.43     | 16.53         |
| 1.4                    | 1.30                   | 8.34                                                      | 14.14     | 16.74         |
| 1.6                    | 1.47                   | 10.12                                                     | 13.84     | 16.99         |
| 1.8                    | 1.59                   | 12.81                                                     | 13.53     | 17.28         |
| 1.9                    | 1.71                   | 14.88                                                     | 13.37     | 17.46         |
| 2.0                    | 1.77                   | 18.42                                                     | 13.21     | 17.65         |
| 2.1                    | 1.83                   | 25.75                                                     | 13.08     | 17.80         |
| <b>FP Model</b>        |                        |                                                           |           |               |
| 0.6                    | 0.59                   | 7.99                                                      | 10.85     | 11.84         |
| 0.8                    | 0.77                   | 9.92                                                      | 10.46     | 11.82         |
| 1.0                    | 0.95                   | 11.91                                                     | 9.98      | 11.74         |
| 1.2                    | 1.12                   | 13.13                                                     | 9.63      | 11.85         |
| 1.4                    | 1.28                   | 14.17                                                     | 9.40      | 12.13         |
| 1.6                    | 1.44                   | 15.58                                                     | 9.02      | 12.43         |
| 1.8                    | 1.60                   | 17.97                                                     | 8.65      | 12.86         |
| <b>PS Model</b>        |                        |                                                           |           |               |
| 1.4                    | 1.31                   | 4.04                                                      | 16.15     | 18.52         |
| 1.6                    | 1.48                   | 4.86                                                      | 15.89     | 18.67         |
| 1.8                    | 1.65                   | 5.93                                                      | 15.48     | 18.70         |
| 2.0                    | 1.81                   | 6.63                                                      | 15.00     | 18.70         |
| 2.1                    | 1.89                   | 6.88                                                      | 14.84     | 18.75         |
| 2.2                    | 1.97                   | 7.12                                                      | 14.62     | 18.83         |
| 2.3                    | 2.04                   | 7.35                                                      | 14.47     | 18.94         |

Table 2.2. Characteristic properties of various neutron stars that follow TNI7, FP and PS models are shown. The columns from the left are; the proper mass in units of solar masses, gravitational mass in units of solar masses, central mass density of the core in units of grams per cubic centimeter, stellar radius as detected by an observer at infinite distance in units of kilometers and the stellar radius detected by a local observer (an observer near the vicinity of the neutron star) in units of kilometers.

The new EOS is adopted into the fully general relativistic “exact evolutionary code” (Nomoto & Tsuruta, 1987). While the most important quality of the new cooling simulations based on the TNI7 EOS is the addition of enhanced cooling through non-standard neutrino processes involving exotic particles, the standard cooling mechanisms such as neutrino bremsstrahlung and plasmon neutrinos as described in the earlier work by Nomoto & Tsuruta (1987) are still included. These processes are expected to be the dominant cooling processes for the lower mass neutron stars without pion-condensate cores, and therefore they will play a significant role in the overall study. The simulations in this study also adopt the superfluid model for the core neutrons as described by Nishizaki et al. (2002) and Takatsuka et al. (2004). The superfluid effects for the pion-condensate are given by Takatsuka et al. (2009). For the Cooper pair enhancement to neutrino emissions due to superfluid neutrons and protons, the results of Flowers et al. (1976) and Yakovlev & Pethick (2004) are utilized. The  $^1S_0$  superfluidity for the crust neutrons and  $^1S_0$  superfluidity-superconductivity of the core protons, and their respective suppression effects on the neutrino emission and the specific heat are treated in accordance with the earlier work by Tsuruta (1998).

In order to better understand the behavior of a neutron star based on the new TNI7 EOS, it is useful to compare its results to those of the previously studied FP and PS models that were used to define the parameter space. While the three EOSs significantly differ, the neutron stars that follow them are expected to share similar pressure-density functions outside the core, but have different characteristics inside the core. This relationship is shown in figure 2.21. The PS model has the greatest stiffness, and

therefore displays the greatest pressure inside the core for a given mass density. The FP model, on the other hand, is the softest equation of state amongst the three, therefore has the lowest pressure inside the core for the same given mass density. The new TNI7 model, being the intermediate EOS, is close to the average of the other two models, but slightly closer to the PS model, and displays intermediate pressure inside the core for the same given mass density. The three models share identical pressure-density properties inside their crusts.

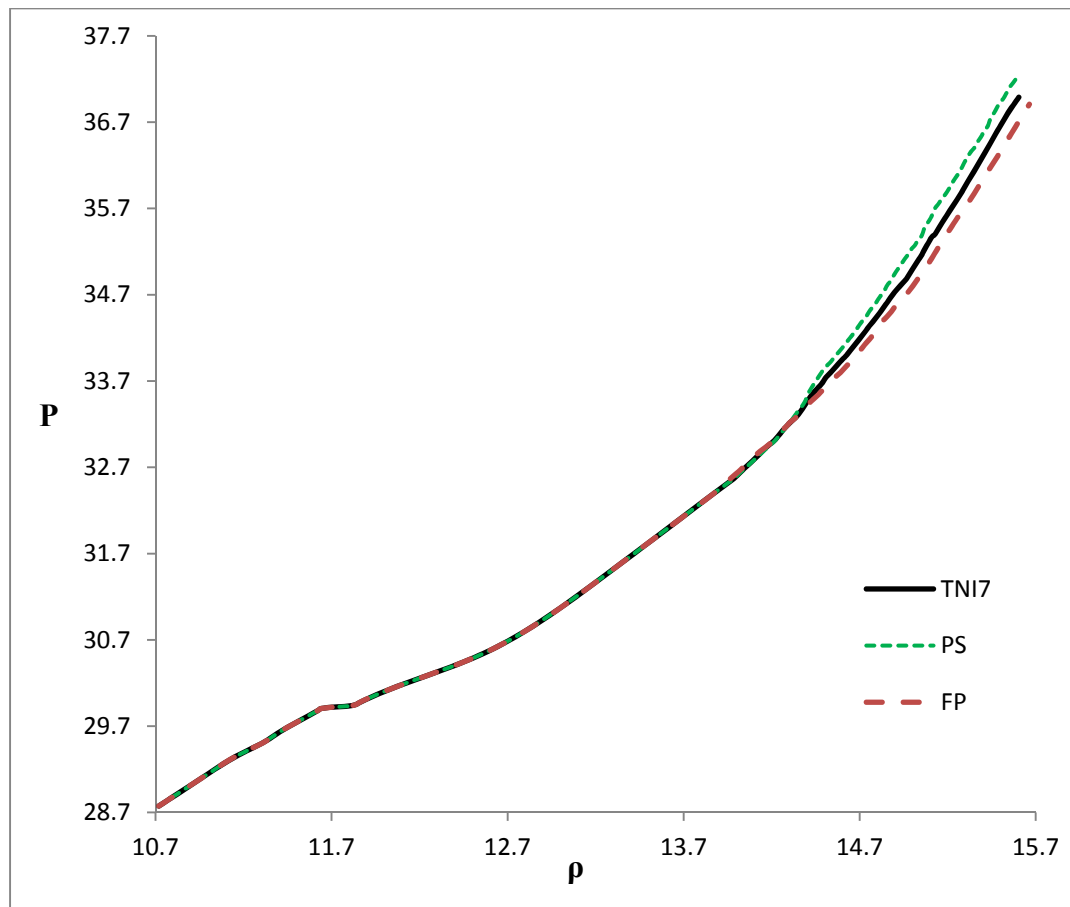


Figure 2.21. The pressure-mass density profiles of the PS, TNI7 and FP models are shown. Pressure is given in the units of dynes/cm<sup>2</sup>, while the logarithmic density is in the units of nuclear density.

While the cores of neutrons stars based on the three pion-condensate models would have nearly identical compositions, the equation of state determines the actual central density of a neutron star. A comparison of the identical mass neutron stars in table 2.21 shows this dependence where the central density decreases with the increasing EOS stiffness. This, once again, is the result of stronger repulsive forces between constituent particles in stiffer equations of state. While the central densities of the neutron stars differ noticeably for different EOSs, the size of the stellar core should not be significantly impacted by the stiffness of the EOS. Since a stiff EOS neutron star has significantly lower central density with a core volume that is similar to the core volume of a soft EOS neutron star, the stiff EOS neutron star will have a smaller portion of its overall mass contained within its core compared to the softer EOS star with the same stellar mass, and the remainder of its mass will have to be outside the core, namely in its crust. The density distribution inside the crust should not be affected by the core EOS either. Therefore a neutron star that has a greater portion of its mass in the crust with the same boundary conditions and density distribution will have to have a thicker crust. Consequently, stiffer EOS neutron stars would be larger, with thicker crusts.

Figure 2.22 displays the density distribution of the crusts of the neutron stars with identical mass of  $1.7 M_{\odot}$  based on the three models as a function of the radial distance from the inner edge of the crust / the outer edge of the core to the stellar surface. Figure 2.22 shows the identical distributions of density, but over increasingly thick crusts for increasing EOS stiffness.

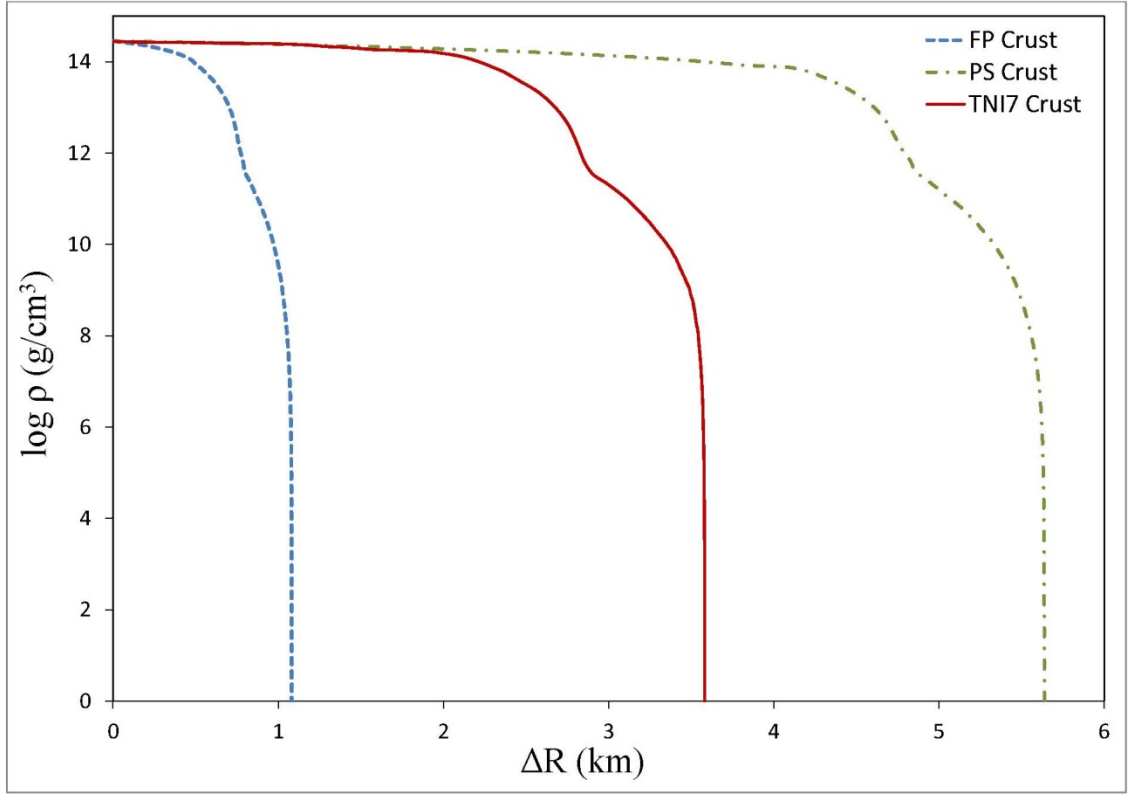


Figure 2.22. Crust Property of Various Models. Density distribution is shown as a function of  $\Delta R$ , the distance from the outer edge of the core (where the crust begins) outward towards the surface. The blue dashed line represents the crustal properties of a neutron star that follows the moderately stiff FP model, the green dash-dotted line represents the crust properties of a neutron star that follows the very stiff PS model, and the red solid line represents the crustal properties of a neutron star that follows the newly created intermediate, stiff TNI7 model. All three curves are created for stars of identical gravitational mass of  $1.7 M_{\odot}$ .

From figure 2.22, it can easily be seen that while the densities at the boundary between the cores and the crusts of the three models are identical, the central densities differ significantly, which in turn leads the crust thicknesses to differ rather significantly with the stiffer models yielding thicker crusts. The details of the radial parameters of the three models are displayed in table 2.3. It should also be noted that the core sizes of the three models are influenced to a relatively minor degree compared to the crusts as

predicted. The core size changes only by about 5%, while the crust size changes by more than 500% between the softest FP and stiffest PS models.

**CRUST PROPERTIES FOR NEUTRON STAR MODELS  
WITH PION CONDENSATE CORES**

|                   | <b><math>R_{\text{inner}}</math><br/>(km)</b> | <b><math>R_{\text{outer}}</math><br/>(km)</b> | <b><math>R_{\text{ns}}</math><br/>(km)</b> | <b><math>\Delta R_{\text{crust}}</math><br/>(km)</b> |
|-------------------|-----------------------------------------------|-----------------------------------------------|--------------------------------------------|------------------------------------------------------|
| <b>FP Model</b>   | <b>9.54 km</b>                                | <b>10.34 km</b>                               | <b>10.62 km</b>                            | <b>1.08 km</b>                                       |
| <b>TNI7 Model</b> | <b>9.82 km</b>                                | <b>12.24 km</b>                               | <b>13.58 km</b>                            | <b>3.76 km</b>                                       |
| <b>PS Model</b>   | <b>10.04 km</b>                               | <b>14.5 km</b>                                | <b>15.68 km</b>                            | <b>5.64 km</b>                                       |

Table 2.3. The stellar radius and crust thickness properties for FP, TNI7 and PS models, all with  $M_G = 1.7 M_\odot$  are shown.  $R_{\text{inner}}$ ,  $R_{\text{outer}}$ ,  $R_{\text{ns}}$ ,  $\Delta R_{\text{crust}}$  are the boundary where the inner crust starts and therefore the core radius, the boundary where the outer crust starts at the neutron dripping density, stellar radius, and the whole thickness of the crust, respectively.

The location of the starting point of the inner crust is determined by the density function as the density drops below the nuclear density, while the location of the starting point of the outer crust is the point where the density drops below the neutron dripping density. As density crosses the neutron dripping point inward, there is a significant change in the composition, with the appearance of free neutrons, and this is seen as a jump in the density function. This density change at the boundary manifests itself as a kink in the plots of the density functions in figure 2.22 (e.g. at  $\Delta R = 3$  km for TNI7). As discussed in section 2.6, the overall effect of the thicker crust due to a stiffer EOS based model is a delay in the start of the thermal relaxation period of the neutron star, since a thicker crust translates to a longer period for the core temperature information to be transmitted to the surface, therefore delaying the beginning of the isothermal cooling

stage. This delay allows the rapid surface temperature drop during the thermal relaxation period to take place at a delayed time consistent with the Cas-A observations for the TNI7 model.

### 2.9 Results of the Study of Neutron Star Thermal Evolution Simulations Including New Cas-A Data Based on TNI7 EOS

The exact thermal evolution code developed by Nomoto & Tsuruta (1987) was utilized for the study of the cooling regimen of neutron stars with pion condensate cores that are based on the new TNI7 EOS. Since the Cas-A observational data are taken over 10 years, which is a period that is significantly shorter than the timescale for the complete thermal evolution of observed neutron stars of a million years or so, and since the data from these detections indicate a specific rapid cooling rate that can only be seen for a specific and short period during the thermal evolution, it was clear that the data generated by the exact code that can fit the observational data for Cas-A neutron star could only be the result of a specific set of neutron star parameters. While the actual parameter space that allows the accomplishment of the earlier goal of finding an EOS that can support a  $2 M_{\odot}$  neutron star is relatively large, this range would shrink to a substantially smaller size when the Cas-A consideration is added to the study. While numerous different EOSs were created that can support a  $2 M_{\odot}$  neutron star that is within the parameter space defined by the FP and PS models as seen in the figure 2.21, those which match all known properties of Cas-A neutron star including detected mass, temperature, age and cooling rate are strongly constrained. It was determined that the TNI7 model that is constructed

for this study is a promising model to satisfy all these constraints that are defined by Cas-A observations, as well as the temperature data of all other isolated neutron stars.

Figure 2.23 demonstrates the thermal evolution curves that are generated by the Nomoto & Tsuruta exact code and based on the new TNI7 EOS. A number of different neutron star parameters, including various masses, and presence of heating, were utilized along with a full range of observational data for known isolated neutron stars to generate this graph. The error bars correspond to the most recent observational data corresponding to isolated neutron stars, while the upper limits are shown with vertical arrows. The sources are: (A)CXO J232327.8 (Cas A), (B) RX J0822-4247 (in Puppis A), (C) 1E 1207.45-5209,(D) RXJ0002+6246, (E) PSR 0833-45 (Vela pulsar), (F) PSR 1706-44, (G) PSR 0538+2817, (H)PSR 0656+14, (I) PSR 0630+1748 (Geminga), (J) RX J1856.5-3754, (K) PSR 1055-52, (L)RX J0720.4-3125 for detections, and (1)PSR 0531+21 (Crab pulsar), (2) PSR J0205+6449(in 3C 58), (3) PSR 1124-5916 (in G292.0+1.8), (4) PSR 1509-58 (in MSH-15-52), (5) RXJ0007.0+7302 (in CTA 1), (6) PSR 1046-58 (Vela twin), (7) PSR 1823-13(Vela-like), (8) PSR B2334+61, (9) PSR B1951+32 (CTB 80), (10) PSRJ0154+61, (11) PSR 2224+61, (12) PSR 2043+2740,(13) PSR 0628-28, (14) PSR 1929+10, and (15) PSR B0823+26, for upper limits.

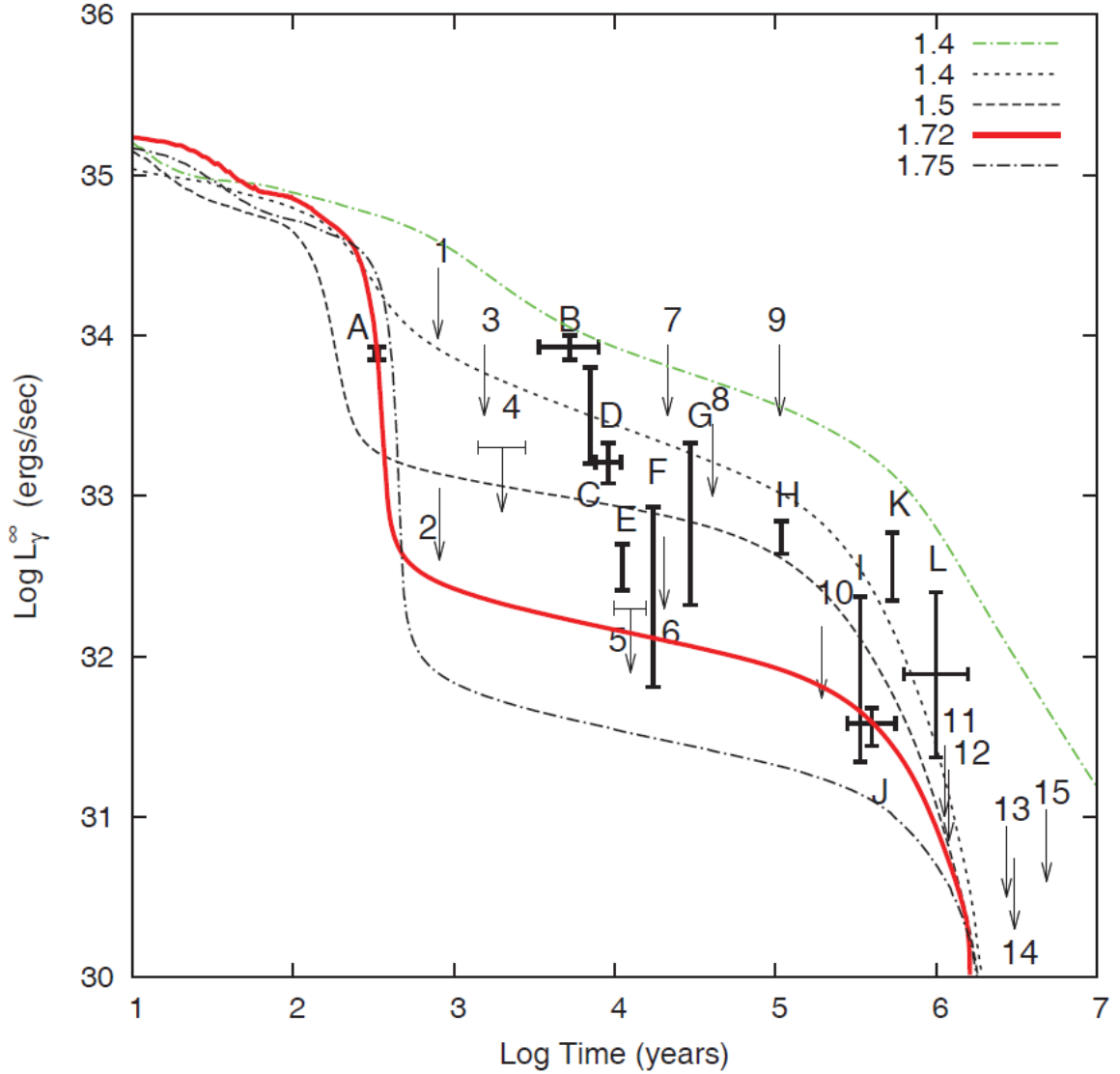


Figure 2.23 Thermal evolution of NSs with stiff TNI7 equation of state is shown by the photon luminosity versus neutron star age curves. The uppermost dot-dashed curve (green) refers to stars with gravitational mass  $M_G = 1.4 M_\odot$  with standard cooling and undergoing frictional heating with the heating parameter  $K = 10^{37} \text{ ergs cm}^{-3/2} \text{ s}^2$ . The next hot curve (black short dashed) is also for  $1.4 M_\odot$  stars but without heating. In all of the rest of the cooler curves the stars cool by pion cooling with no heating. The two cooler curves (black dashed and dot-dashed) refer to  $1.5 M_\odot$  and  $1.75 M_\odot$  stars, respectively. The red thick solid curve refers to Cas-A NS with  $M_G = 1.72 M_\odot$ , carbon atmosphere, carbon-contaminated envelope and no heating. The  $1.72 M_\odot$  curve is identical to the  $1.63 M_\odot$  curve obtained for a neutron star with iron atmosphere, therefore only one is shown in the figure. The observational data for various sources are shown as bars for detections and downward arrows for upper limits with the numbers and letters referring to specific observations as described in the text.

The following figure 2.24 is the close-up of the thermal evolution curve that focuses on the ten year long period over which the Cas-A temperature data were collected. In this figure, the surface temperature of the neutron star as observed at infinite distance as a function of time is shown. The error bars represent actual Cas-A detections by the Chandra X-ray observatory and are taken directly from Shternin et al. (2011). Figures 2.23 and 2.24 have different units, with figure 2.23 using the luminosity information and figure 2.24 using the temperature information. They are essentially identical graphs with different timescales as the temperature and luminosity information are related to one another through the blackbody emissivity equation. Therefore it is valid to say that the quality of the fit in figure 2.24, is also indicative of the quality of the fit in figure 2.23.

In figure 2.23, five separate cooling curves for various mass neutron stars with pion condensate cores based on TNI7 EOS are shown indicating the evolution of stellar luminosity as observed at an infinite distance as a function of time. The uppermost curve drawn by the dot-dashed line corresponds to a neutron star with  $1.4 M_{\odot}$  that is influenced by frictional heating, as described in section 2.3.4, with the heating parameter  $K=10^{37}$  ergs.cm<sup>-3/2</sup>s<sup>2</sup> (Tsuruta, 1998). The details of the K value can be found earlier in this chapter. This particular regimen that includes a heating element had to be included in order to explain the observations of hotter pulsars such as RX 0833-45 (Vela Pulsar), shown as the detection B in figure 2.23. As it can easily be seen from the graph, the cooling regimen of such a neutron star will be significantly slower which would not be consistent with the Cas-A observations even during the relaxation phase. Also as a result

of the slower cooling, the complete thermal evolution of a neutron star with an internal frictional heating mechanism lasts more than an order of magnitude longer compared to the unheated case to complete.

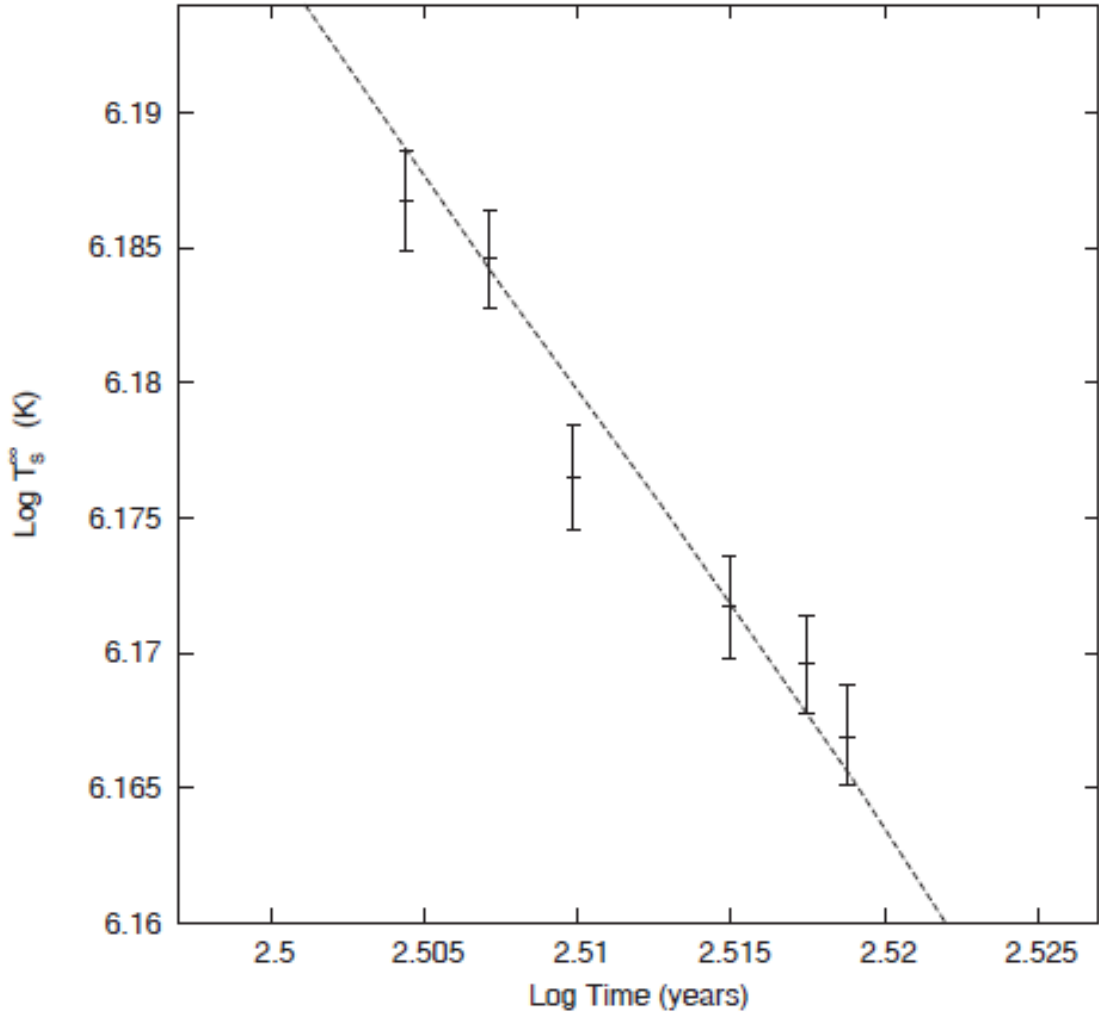


Figure 2.24. Surface temperature as detected by an observer at infinite distance as a function of logarithmic time is shown. The curve corresponding to Cas-A neutron star, the thick solid red curve in figure 2.23, is magnified in the vicinity of Cas-A neutron star data, to show the fast cooling in this NS with the detection data points with error bars.

The second uppermost curve (black dotted line) once again corresponds to a  $1.4 M_{\odot}$  neutron star, but does not include any heating element. While cooling at a much faster pace than the case with heating, a neutron star of this nature also does not have the extremely rapid rate of temperature drop that is seen in Cas-A observations during its relaxation period. This is due to the fact that the core of a neutron star of this mass would not reach a high enough density that surpasses the transition density for pion condensates to form, and consequently there is no contribution to the cooling by the non-standard fast cooling mechanisms involving these particles, and the neutron star cools with the standard, relatively slow scenario. However, cooling is still somewhat accelerated by the presence of superfluid neutron cooper pairs.

Two remaining black curves corresponding to 1.5 and 1.75 solar mass neutron stars, respectively drawn by dashed and dot-dashed lines, have cores that are dense enough for the pion condensate to form, and subsequently display the much higher cooling rates due to the enhanced neutrino emission via the non-standard processes involving these particles. It is also important to note that the superfluid suppression effect in these stars impacts their cooling significantly. Since the core density and the superfluid suppression of the enhanced neutrino processes are inversely related, as explained in section 2.6, the more massive neutron stars are affected by this suppression to a lesser degree. As a result, the cores of the more massive stars cool faster through the enhanced neutrino emissions. This results in larger drops in surface temperature, and subsequently the luminosity, during the thermal relaxation period. This effect can be seen in the cooling curves of the 1.5 and 1.75 solar mass neutron stars. The 1.5 solar mass neutron

star experiences a far stronger superfluid effect due to its lower core density. It also contains a smaller pion-condensate portion in its core due to the relatively low density. As a result of the cumulative effects of the superfluid suppression and relatively small pion involvement, its core does not experience as great a temperature drop as the  $1.7 M_{\odot}$  neutron star's core due to the reduced rate of neutrino emissions through enhanced processes. This is manifest in the smaller drop during the thermal relaxation era as seen in the cooling curve. However, despite its slower rate of cooling, the  $1.5 M_{\odot}$  neutron star reaches its isothermal cooling stage somewhat more quickly due to the greater contribution from the enhanced cooper pair cooling of the neutron matter shell. For simulations of all neutron stars except  $1.72 M_{\odot}$  star, they are assumed to have iron atmospheres and the crustal heavy element composition which becomes increasingly neutron-rich with increasing densities inward from the surface as studied by Tsuruta et al (1998).

The final curve drawn with a solid red line corresponds to a  $1.72 M_{\odot}$  neutron star with a carbon atmosphere. The carbon atmosphere is chosen specifically to generate a curve that would be consistent with Cas-A detections. The observations of the Cassiopeia-A supernova compact remnant indicated that this neutron star lacks pulsations that are typical for pulsars. This absence of pulsations was shown to be consistent with a weak magnetic field by Heinke & Ho (2010). The presence of a carbon atmosphere creates higher conductivity which results in higher surface temperature for any given core temperature as well as a somewhat shortened relaxation period due to the faster rate of surface radiation. It is important to note that while the carbon atmosphere effects were

included in modeling this curve in order to be consistent with aforementioned Cas-A neutron star observations, a nearly identical and equally well fitting curve was obtained for a  $1.63 M_{\odot}$  neutron star with an iron atmosphere. The resulting curve is observed to pass through the observational data point for Cas-A neutron star which is labeled as point A in the figure 2.23. No cooling curves were found for a neutron star with internal heating that is compatible with the rapid cooling that is observed for Cas-A.

In figure 2.24 it can further be seen, upon closer inspection, that the red curve from figure 2.23 is an excellent fit, representing a fast cooling regimen that is consistent with what was observed for the Cas-A neutron star. It is important to note that for several of the years in which these detections were taken, more than one data point corresponding to several observations throughout the year are available. While the inclusion of those additional data would broaden the error range, which may in turn lead to a different set of parameters to also generate well-fitting cooling curves, these specific data points were chosen and used by all the authors who previously studied Cas-A due to the fact that they represent the longest exposure times of all observations of their respective years. Also, despite the wider error margins, such data still represent a very small portion of the actual cooling timeline, and consequently no statistically significant enough parameter set that results in an equally well fitting cooling can be found even with such broader error bars. As a result, assuming that the detection of fast cooling of Cas-A neutron star are perfectly accurate, it is safe to state that if Cas-A neutron star contains a core that is contaminated by superfluid pion condensates and follows the stiff TNI7 EOS, then it is most likely a star with 1.72 solar mass and radius of 13.6 km and

core density of  $1.14 \times 10^{15} \text{ g.cm}^{-3}$ , which is enveloped by an atmosphere that is 30% carbon contaminated and currently in the crustal relaxation phase of its thermal evolution.

### 2.10 Discussion

As proposed earlier, if there is pion condensate matter present inside the core of a neutron star, the resulting superfluid effect and non-standard neutrino processes can lead to both suppression and enhancement of cooling of neutron stars during different stages. Consequently, the neutron star's thermal evolution will be significantly different than what is predicted by standard cooling models. The newly constructed TNI7 EOS, clearly shows the influences of these effects. Since TNI7 is a stiff equation of state with relatively lower core densities, it does not create the necessary conditions for the formation of stable pion condensate for stellar masses until about  $1.4 M_{\odot}$ , and as a result less massive neutron stars continue to cool following the standard scenario. Neutron stars with masses moderately higher than  $1.4 M_{\odot}$ , on the other hand, cool at a slower rate than unsuppressed pion cooling due to superfluid suppression of the enhanced processes in the pion core, and the most massive stars ( $1.7 M_{\odot}$  for example) cool at a rapid rate due to the fast direct Urca processes with diminishing superfluid suppression as the mass increases.

The thermal evolution simulations based on the TNI7 pion condensate core model with superfluid suppression show results that are consistent with rapid cooling as implied by the Cas-A data. While it may be possible to explain the fast pace of cooling in Cas-A neutron star with the projections of all standard models under certain conditions, any such model must also be consistent with all of the observational temperature data for isolated

neutron stars as well. Based on this criterion, the TNI7 model can be seen as a very good candidate for explaining the unusual behavior of the Cas-A supernova compact remnant, as well as the full range of isolated neutron star temperature data. Despite the consistency of the results, the TNI7 model does not come without some challenges (as described below), and must compete with several other models including the phenomenological neutron superfluid models by Heinke & Ho (2010) and Yakovlev et al. (2010) that have serious shortcomings of their own.

The core idea behind this study of Cas-A neutron star was that the proper conditions inside a sufficiently large neutron star should be present to allow the formation of exotic particles, in this case pions, in a stable condensate form. Once these particles are present, they would not only allow additional heat loss via non-standard pion Urca processes, but also display the unique superfluid suppression property associated with those processes. Combined with the alteration to the neutron star size due to the stiffer EOS that is a consequence of the universal three-body interactions and resulting strong repulsion, the thermal evolution profile of the new model would be noticeably different from the standard models. The resulting thermal evolution would have two unique characteristics that cannot be explained by any standard cooling regimen: A delay in the thermal relaxation of the crust, in this case as late as 300 years, and a high rate of surface cooling for a hot, young star that is consistent with Cas-A observations during this period. While pion condensate material based core model appears to be naturally consistent with Cas-A, providing proper surface cooling rates for a neutron star with Cas-A's composition, its behavior may not be unique to pions and may also be seen in the

presence of other exotic particles. In fact, the original idea for the stiffer EOS was a result of the study of a hyperon-mixed neutron star core model by Tsuruta et al. (2009) which was based on the EOS models of Takatsuka et al. (2007). The study by Tsuruta et al. (2009) also showed the presence of rapid cooling during the thermal relaxation period, albeit without the delay in this period that is necessary for the results to be consistent with Cas-A observations. However, if the new TNI7 EOS is adapted for the thermal evolution simulations of neutron stars with hyperon contaminated cores, similar results as the pion case are obtained. The only reason the pion condensate core model was selected instead of the hyperon-mixed core model was the certain doubts that were cast by the “NAGARA event”. The so-called “NAGARA event” suggested that the superfluid suppression in this material may not be as strong as previously thought (Takatsuka et al., 2001) due to the smaller than expected superfluid energy gap. Without a strong superfluid suppression of the fast neutrino processes, it would be impossible to explain the full range of isolated neutron star temperature data with hyperon-mixed core models, since the stars would cool too rapidly to be consistent with some detection data, such as Vela pulsar data. This effect, however, is by no means proven, and therefore hyperon-mixed core models are still considered as valid possibilities for neutron star core EOS at this time. Consequently, a moderately stiff hyperon-mixed core model was also tested as part of this study for the Cas-A observations. The results are shown in figure 2.25.

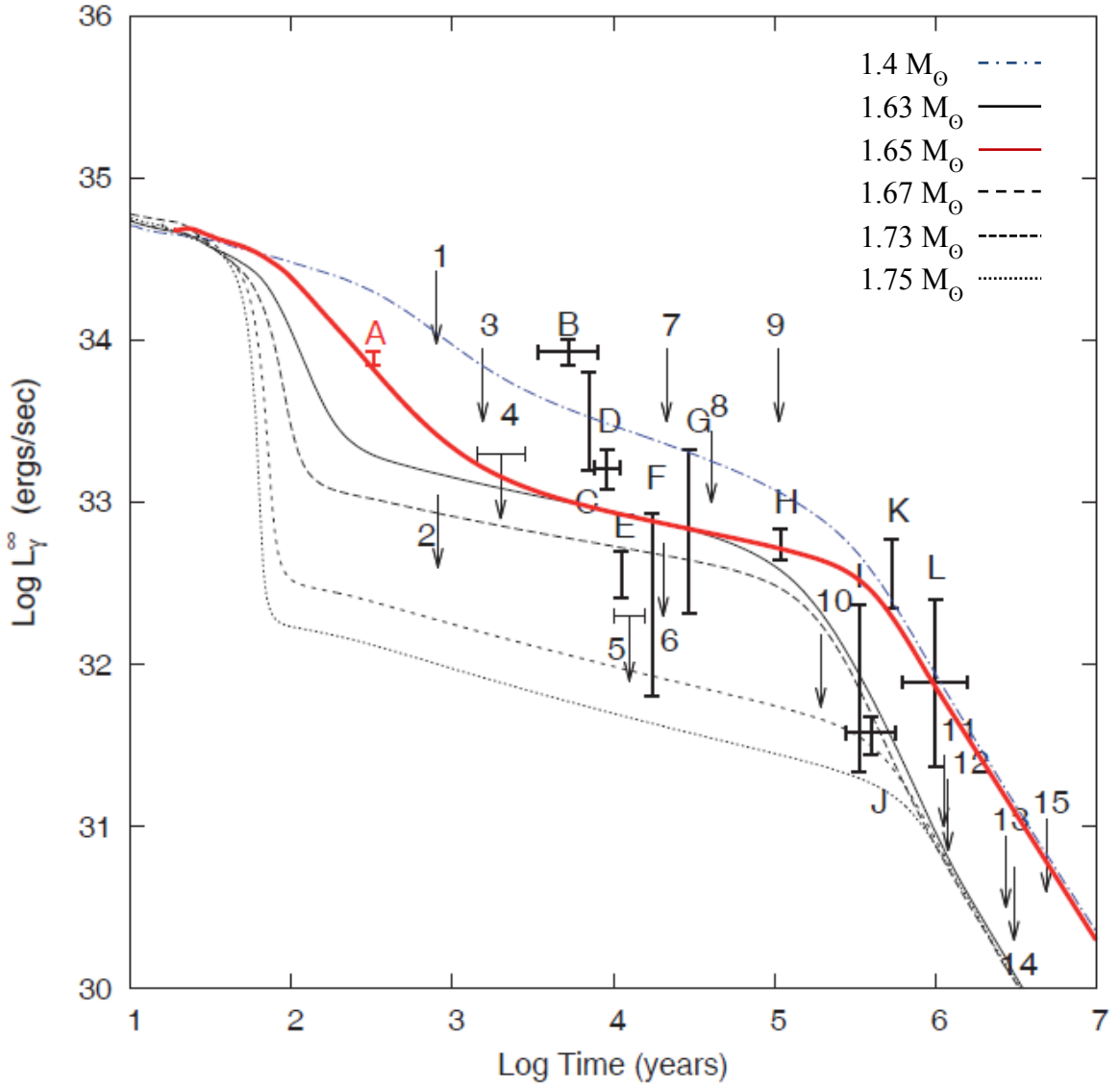


Figure 2.25 Thermal evolution of NSs with a moderately stiff hyperon-mixed equation of state TNI6u is shown. The uppermost dot-dashed curve (blue) refers to stars with gravitational mass  $M_G = 1.4 M_\odot$  with standard cooling and undergoing frictional heating with the heating parameter  $K = 10^{37} \text{ ergs cm}^{-3/2} \text{ s}^2$ . The next solid curve (red) is for  $1.65 M_\odot$  stars with enhanced cooling and undergoing frictional heating with the heating parameter  $K = 10^{37} \text{ ergs cm}^{-3/2} \text{ s}^2$ . In all of the rest of the cooler curves the stars cool by hyperon-mixed enhanced cooling with no heating. The four cooler curves (black solid, black dashed, black short-dashed and black dotted) refer to  $1.63 M_\odot$ ,  $1.67 M_\odot$ ,  $1.73 M_\odot$  and  $1.75 M_\odot$  stars, respectively.

Figure 2.25 shows the thermal evolution simulations obtained by the exact evolutionary code for a neutron star with  $\Lambda$  hyperon-mixed core. The figure is based on the moderately stiff equation of state TNI6u, and displays similar characteristics to the pion condensate based model with rapid cooling during the thermal relaxation period with rapid cooling rates for neutron stars that are sufficiently large masses in order to have hyperon matter to appear in their cores.

As can be seen in figure 2.25, while a hyperon-mixed neutron star core with a medium stiffness equation of state would also allow a small delay in the crustal relaxation period, this delay would not be great enough to be consistent with the Cas-A detections. Therefore, some additional effect, either a very low stellar mass or a moderate stellar mass with internal heating mechanism, would be necessary in order to explain the Cas-A detection. Very low mass neutron stars had no chance of showing cooling rates that are as high as needed for Cas-A, since the core of the neutron star will not be dense enough to allow hyperon matter formation, and therefore would not exhibit the rapid neutrino cooling of non-standard processes. Therefore, only the second scenario (with internal heating) is tested, and it is shown as the red curve in the figure. Once again this particular regime, while allowing the delay of the thermal relaxation to be consistent with Cas-A detections data, does not allow a high enough cooling rate. Therefore it is safe to conclude that a relatively soft EOS is unlikely, if not completely unable, to explain the behavior of Cas-A neutron star even when internal heating is taken into consideration.

The results of the TNI6u model, however, do not exclude the possibility of a hyperon model that is suitable for Cas-A observations. In theory, it is possible to generate

a significantly stiffer model, possibly identical to the TNI7 model, that may allow the relaxation period to be delayed to a point that will be consistent with the Cas-A neutron star data, yet still have the necessary rapid cooling rate. However, if the suggestions of the NAGARA event, where the  $\Lambda\Lambda$  hyperon pair interaction is reported to be too weak, leading to a small superfluid energy gap, this model would lack any significant suppression of its enhanced cooling processes, and subsequently would be unable to explain the behavior of the intermediate temperature neutron stars such as Vela pulsar. Consequently, this avenue of research was deemed unwarranted until more information on the hyperon-mixed core superfluidity becomes available.

Additional alternative models based on exotic particles are still available for future exploration, such as quark cooling. In their recent work, Noda et al. (2013) reported that they were able to explain the rapid cooling of Cas-A with a model that involves a quark core. The most important characteristic of this model is that the hotter neutron star detection data correspond to the cooling of more massive neutron stars rather than the less massive ones as predicted by the models used in this study. If confirmed, this would also be a novel and interesting way of studying neutron stars. However, these models are still at their infancy; especially the superfluid properties of such particles, as well as the transition conditions, remain largely unexplored (Takatsuka et al. 2007). Since the theoretical work on these models is not advanced enough to place reasonable constraints to the model, their testing as part of this study was also deemed unwarranted at the time, but remains as possible avenues of exploration of neutron stars in general and Cas-A specifically in the future.

One additional possible challenge to the results of the TNI7 model comes from some recent reports of some models that neutron star radius may be confined to  $\sim 11$ - $12$  km, which is in contradiction with the central idea of the TNI7 core model for Cas-A. The stiff TNI7 model creates a neutron star that has a moderately thicker crust compared to many other models, which in turn leads to the delay in thermal relaxation since the time for the cooling information of the core to reach the surface is directly related to the thickness of the crust. In comparison, in some models the relaxation event takes place within the first 100 years, due to the smaller radius. In a recent work by Lattimer and Prakash (2010), the authors concluded that the radius of a neutron star will be less than 12 km, which is somewhat smaller than the predictions of the TNI7 model and what is necessary to explain the Cas-A detections data based on this model since the explanation requires the delayed thermal relaxation which is caused by the thicker crust of the larger neutron star. It is important to point out, however, that this conclusion is far from certain with too many assumptions involved, and many groups around the world continue to use models that result in larger neutron stars radii, including the PS model which can yield neutron star radii in excess of 15 km (Yagi & Yunes, 2013). The only conclusive determination for an upper limit for the neutron star radius will be made possible by continued experiments and observation with more advanced techniques that may become available in the near future (e.g. Shibata, 2013). This may likely take another decade or longer. Therefore, at this time the TNI7 model continues to be a very valid explanation of Cas-A detections.

Another challenge to the TNI7 model and all other models that are currently being used, is based on how the matter in neutron stars behaves in general. As explained in the introduction, neutron stars form when the gravitational force that is trying to collapse the star cannot be countered by the mere electron degeneracy pressure, and subsequently the star collapses further to an extremely dense state where degenerate pressure of neutrons can support the star. The nucleons at this density are highly degenerate, and do not behave the same way as ordinary matter behaves. Specifically, the mass-radius relationship is quite different from ordinary matter. The total energy at this state is given by the equation when the effect of nuclear force is not included:

$$E = K + U \approx \frac{\hbar^2 M^{5/3}}{m_e m_n^{5/3} R^2} - \frac{GM^2}{R} \quad (2.13)$$

where  $M$  and  $R$  are the stellar mass and radius,  $m_e$  and  $m_n$  are the masses of electrons and nucleons,  $G$  and  $\hbar$  are the gravitational and Plank constants (Clayton, 1983). The mass radius relationship for such a star can be found by minimizing energy:

$$\frac{dE}{dR} \approx -\frac{2\hbar^2 M^{5/3}}{m_e m_n^{5/3} R^3} + \frac{GM^2}{R^2} = 0 \quad (2.14)$$

Solution of equation 2.14 gives:

$$R \approx -\frac{2\hbar^2 M^{-1/3}}{G m_e m_n^{5/3}} \quad (2.15)$$

The resulting radius function predicts an inverse relationship between the radius and the mass of a neutron star, where a more massive star would have a smaller radius. This is a relationship that is quite unlike other stellar bodies that are made of non-degenerate material, but is the typical and widely accepted behavior for neutron stars by many

models that are used in the contemporary studies. The TNI7 EOS model also adopts this particular inverse relationship between the mass and the radius of a neutron star. Figure 2.26 demonstrates the gravitational mass versus radius relation for a neutron star that follows the TNI7 EOS core model. The red dot on the graph corresponds to a specific neutron star that has the cooling properties that are consistent with the Cas-A detections. As can be seen in the figure, the radii of the neutron stars range from approximately 13.1 km to 14.4 km, which correspond to the radii of 2.1 and 1.2 solar mass neutron stars respectively. This inverse relationship is in direct contradiction with the report of Lattimer et al. (2010) which predicts that the actual radius of the neutron stars should be affected by increasing mass in a relatively minor way, leading to a range of neutron star masses that have nearly identical radii.

The inverse relationship between the stellar mass and radius based is not unique to TNI7 model, and is adopted by many other researchers in this field including by Umeda et al. (1994). Reversing this mass – radius relationship would require a specific nuclear model. Such nuclear models are not well-studies at this time. However, while the results of Lattimer et al. (2010) are far from conclusive at this time, this contradiction is still worth some exploration. The most likely explanation for this inconsistency may come from the incompleteness of the current understanding of the behavior of neutron star matter at ultra-high densities. As explained earlier in this chapter, the basis for the EOS models is the strength of two and three body strong forces. However, these forces are not well understood. The EOSs used in this study are constructed based on the idea of

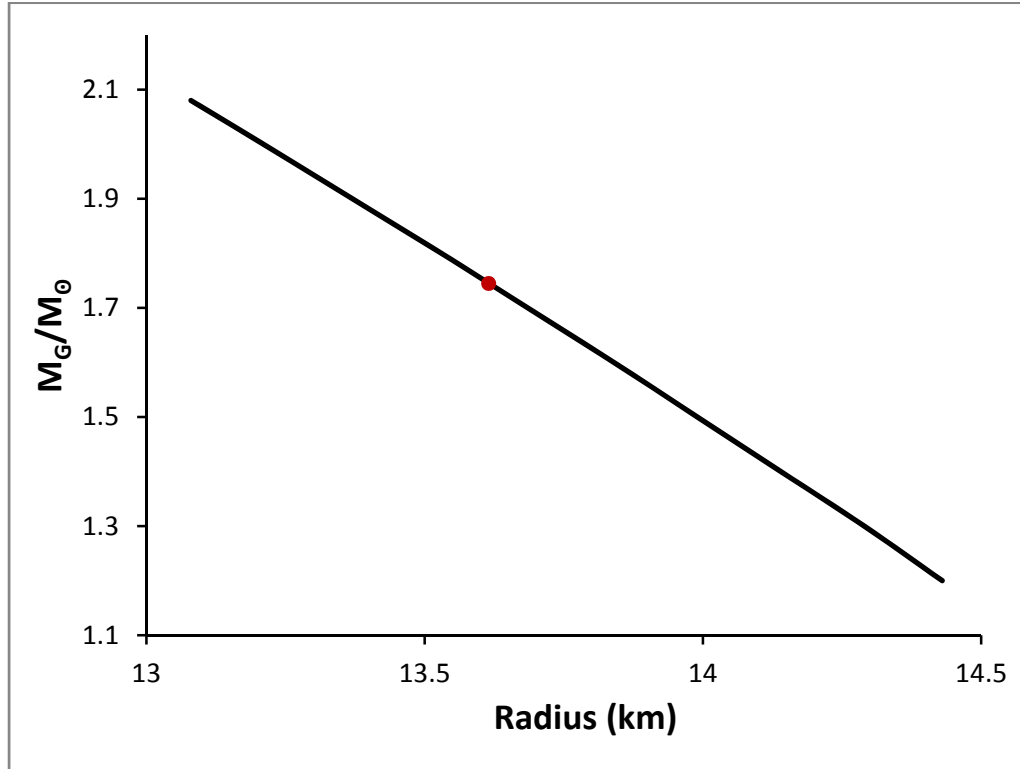


Figure 2.26. The gravitational mass in the units of solar mass versus radius in the units of km for the TNI7 EOS neutron stars is shown. The red dot represents Cas-A neutron star.

uniform repulsive forces that vary in strength from one model to another with stiffer models corresponding to cases with stronger repulsive potentials. While the relationship between the stiffness and the strength of repulsive force cannot be disputed, the assertion that these forces are uniform is a speculative statement. It is entirely possible that the repulsive forces between the constituent particles of the neutron star are dependent on the density and subsequently the mass of the neutron star itself, where the constituent particles in the core of a more massive neutron star would experience stronger repulsive forces. If this is the case, then the stiffness of the equation of state will not be manifested as uniform for all densities. Instead the equation of state will be such that it will appear as

if it is getting stiffer with increasing density in order to reflect the increasing strength of the repulsive forces between the constituent particles in the neutron star. Consequently, then the neutron star mass and radius relationship can be significantly altered to the point that the radius is either nearly constant or ever increasing with the increasing stellar mass. This new relationship, while speculative, is relatively easy to demonstrate. In figure 2.27, such a relationship is qualitatively demonstrated. In this figure, the blue, green and red solid lines correspond to mass-radius relationship of the existing FP, TNI7 and PS models, while the black dashed lines represent qualitatively created intermediate EOSs. Finally, the purple solid curve is drawn to represent the variable repulsion based EOS. The points where this new EOS curve crosses the others would indicate a neutron star that is identical to the ones that would be the result of the existing EOSs, such as the red dot which represent the Cas-A neutron star of the current TNI7 model. For instance, if the radius of a less massive and therefore less dense neutron star (e.g.  $1.3 M_{\odot}$ ) is experimentally and observationally shown to be  $\sim 12$  km, an EOS with increasing radii for more massive stars will be favored over the current model for Cas-A neutron star data where a more massive star with  $1.7 M_{\odot}$  will have a radius of  $\sim 13.5$  km (see figure 2.27).

This idea of a variable repulsion EOS model appears to have a very good potential for consolidating the theoretical work when the full spectrum of high quality observations and experiments becomes available. It presents a prospective study of the theory that can be carried out over the next few decades as additional and more precise data for neutron star masses and radii, as well as cooling profiles, become available by the detections from more modern observatories.

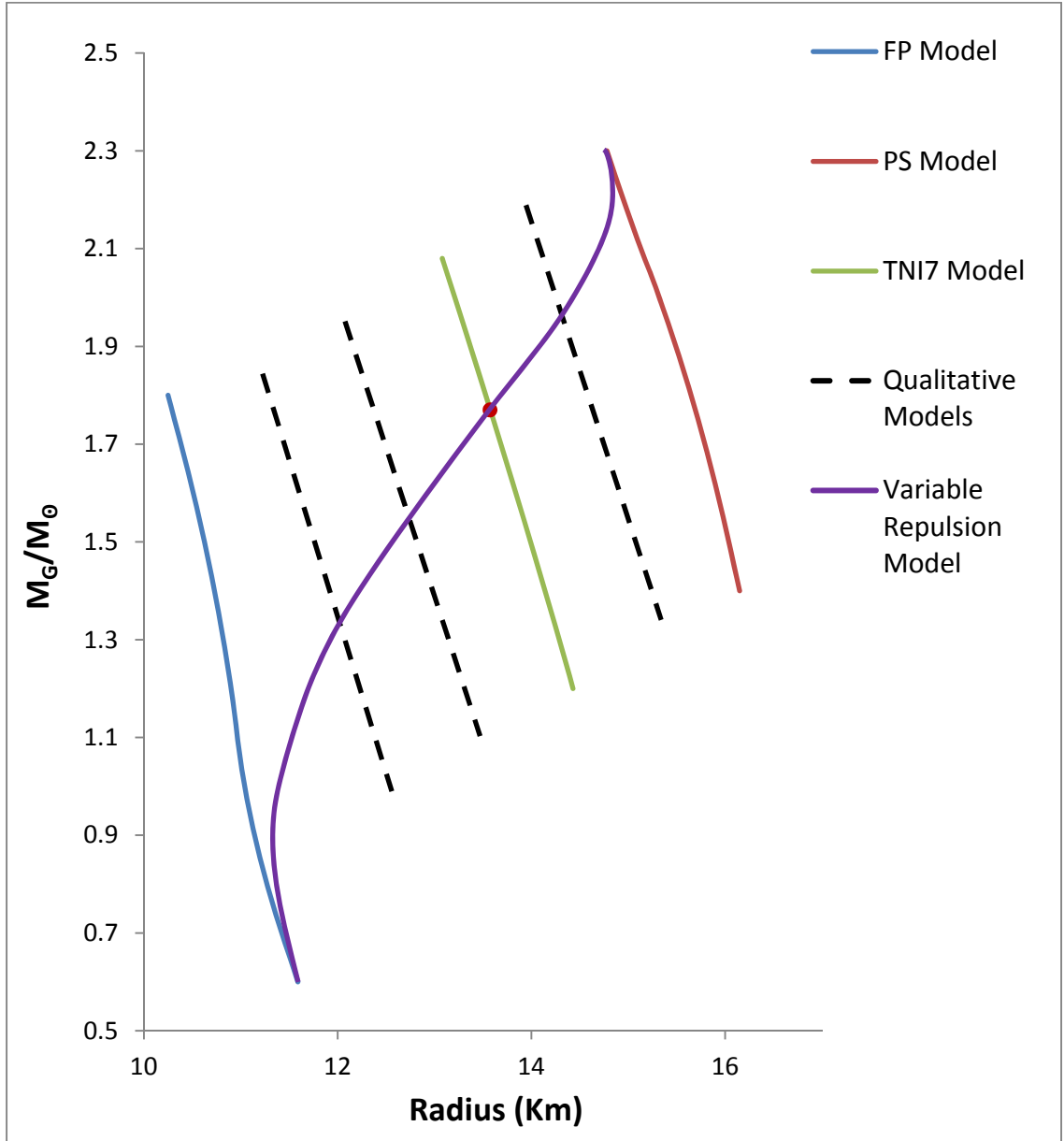


Figure 2.27. The mass-radius relationships of various existing EOSs, several qualitatively created intermediate EOSs, and an EOS that is based on variable strong repulsive force are shown.

### 2.11 Summary and Conclusions

A new EOS, TNI7, is created to explain the rapid cooling indicated by Cas-A neutron star detections. The TNI7 EOS is shown to produce cooling results consistent with Cas-A neutron star data by delaying the thermal relaxation phase of a  $1.72 M_{\odot}$  neutron star to the observed age of Cas-A neutron star of 330 years by creating a larger radius neutron star. This  $1.72 M_{\odot}$  star is shown to have a cooling rate that is consistent the rapid cooling rate (4% temperature drop over 10 years) observed for Cas-A neutron star due to the inclusion of enhanced neutrino processes involving pion condensates. The new EOS is also shown to be capable of producing results consistent with the full range of isolated neutron star temperature data. Moreover, the intermediate temperature neutron stars are shown to be consistent with the results only if the superfluid suppression of the enhanced neutrino processes for these stars is effective, confirming earlier findings of Tsuruta (2010).

Additionally several possible challenges to the TNI7 model are discussed. The radial limit suggested by Lattimer and Prakash (2010) that is somewhat lower than what is predicted by TNI7 model is discussed, and it is concluded that additional observational data would be necessary in order to verify the validity of this challenge. The challenge to the mass – radius relationship that is introduced by Lattimer et al. (2010) is discussed. A modified EOS based on variable repulsion force is introduced for comparison, which offers a possible resolution for this challenge.

## CHAPTER 3

EXOTIC PARTICLE CORE NEUTRON STAR MODELS FOR  
ACCRETING LOW MASS X-RAY BINARIES (LMXB)3.1 Introduction

While many X-ray sources were detected and studied since the beginning of X-ray astronomy era, this particular subset of neutron stars, low mass X-ray binaries, was not discovered until 1975 by Grindlay et al. (1975), and was only identified indirectly after studies of corresponding X-ray bursts. In this particular subset of neutron stars, a binary system is believed to be present that contains an ordinary albeit older and compact neutron star alongside a donor object that is often less massive than the neutron star itself. The donor object is typically a low mass post main sequence star, up to a red giant. The resulting system allows accretion from the donor object onto the neutron star through Roche lobe overflow processes.

The accreted matter is typically hydrogen rich and forms a burning shell surrounding the neutron star releasing energy at a rate of approximately 5 MeV/nucleon when the fusion process is stable (Chamel & Haensel, 2008). If the accretion rate is greater than what is permissible for stable hydrogen burning, then the helium that formed from these nuclear reactions and seeped into the neutron star crust may also be triggered into a nuclear flash. These flashes lead to sudden and short lived energy releases in the form of X-ray bursts with luminosity of  $10^{38}$  erg s<sup>-1</sup>, lasting anywhere from a few seconds to a few hours, followed by quiescent periods that last anywhere from months to years.

During the accretion, a single hydrogen atom falling on a neutron star from an infinite distance would release approximately 200 MeV of energy, most of which is radiated away in the form of X-rays (Chamel & Haensel, 2008). This allows the calculation of X-ray luminosity for an accreting neutron star as:

$$L_x \approx (\dot{M}/10^{-10}M_\odot/y). 10^{36} \text{erg.s}^{-1} \quad (3.1)$$

in terms of the accretion rate  $\dot{M}$  in units of solar mass per  $10^{10}$  years. The most important consequence of the non-equilibrium nuclear processes for the thermal evolution problem of a neutron star is the introduction of a new heating element due to deep crustal heating, caused by the nuclear reactions of accreted material that sinks into the outer crust.

In this chapter, the steady state accretion rate – luminosity relationship for LMXBs will be studied using exotic neutron core models. The specific models chosen for this study are the medium stiff TNI6u, stiff TNI3u hyperon models and the stiff TNI7 pion model.

### 3.2 Basic Equations

The detailed calculations of the contributions of an accretion driven heating mechanism were carried out by Haensel and Zdunik (2003 & 2009), and showed that depending on the choice of nuclear model adopted, such reactions would produce a heating rate of  $Q_{\text{tot}} = 1.45$  or  $1.12$  MeV/accreted nucleon. This difference in heat generation rate is primarily due to the assumptions about which nuclear reaction chains will take place for the accreted matter. It is important to note that this heating contribution is mainly due to the accretion driven mechanisms in the crust. In contrast,

the energy generated during burst events near the surface and by hydrogen fusion is expected to be almost completely radiated away, and consequently has little effect on the heating of the neutron star itself.

The nuclear reaction of the accreted matter inside the crust is thus the only significant contributor to heating of a steady state neutron star. When the accretion driven heating and the heat loss due to radiation are equal to each other, the neutron star would be at thermal equilibrium. Therefore, the relationship between the steady state accretion rate and neutron star luminosity can be determined by setting these two equal to each other. A study of such a neutron star at the equilibrium accreting state was carried out by Gnedin et al. (2001), and determined the stationary state thermal balance equations:

$$L_{dh}^{\infty}(\dot{M}) = L_{\nu}^{\infty}(T_{in}) + L_{\gamma}^{\infty}(T_{eff}) \quad (3.2)$$

where  $L_{dh}^{\infty}$  is the deep heating power as observed at an infinite distance from the neutron star, and  $L_{\nu}^{\infty}$  and  $L_{\gamma}^{\infty}$  are the neutrino and photon luminosities of the neutron star as detected by the same observer. The deep crustal heating is a function of the accretion rate and heating per nucleon, and is given by:

$$L_{dh} = \frac{Q\dot{M}}{m_N} \approx 6.03 \times 10^{33} \left( \frac{\dot{M}}{10^{-10} M_{\odot} \text{ yr}^{-1}} \right) \left( \frac{Q}{\text{MeV}} \right) \text{ erg s}^{-1} , \quad (3.3)$$

where  $Q$  is the heat released per accreted nucleon given by Haensel and Zdunik (2003 & 2009),  $\dot{M}$  is the accretion rate, and  $m_N$  is nucleon mass. By equating the two definitions of the deep crustal heating power in equations 3.2 and 3.3, the equilibrium accretion rate for a thermally steady state neutron star can be determined in terms of its photon and neutrino luminosities, and is given by:

$$\dot{M} = \frac{10^{-10} M_{\odot} \text{ yr}^{-1}}{6.03 \times 10^{33}} [L_{\nu}^{\infty}(T_{in}) + L_{\gamma}^{\infty}(T_{eff})] \left( \frac{\text{MeV}}{Q} \right) \text{ erg}^{-1} \text{ s} . \quad (3.4)$$

Since the steady state accretion rate given by equation 3.4 depends only on the photon and neutrino luminosities, the results of neutron star thermal evolution simulations can easily be checked against existing LMXB detection data. The steady state accretion rates can be determined from this equation for a specific neutron star model along with luminosity information, which is amongst the direct results of neutron star thermal evolution simulations. Consequently theoretical luminosity and accretion rate information can be compared with the observational data, which will help to constrain the specific neutron star model.

### 3.3 Hyperon Core Model for Low Mass X-ray Binaries

#### 3.3.1 Introduction

Hyperon-mixed neutron star core models were studied in the past by Tsuruta et al. (2009). Their goal was to show that the consequences of a neutron star having an exotic core would be a natural explanation for observational temperature data of an intermediate temperature neutron star such as the Vela pulsar. As discussed earlier in section 2.6, neutron stars do not possess exotic cores unless a special condition is met, specifically the core density of the star must exceed the transition density. Therefore, the high temperature observations are naturally explained by low mass neutron stars that did not meet this requirement, and subsequently evolve through standard (i.e., slow) thermal processes. Similarly, the low temperature observations are naturally explained by high mass neutron stars that possess an exotic core and therefore cool rapidly through fast non-

standard cooling processes. On the other hand, the detections for intermediate temperature neutron stars can only be explained if these stars do possess an exotic core, and therefore experience fast cooling through non-standard cooling processes, but these processes were less efficient than expected. The non-standard processes can indeed be partially suppressed by superfluidity of exotic particles as explained in chapter 2. While this suppression may also explain the behavior of high temperature detections by the complete superfluid suppression of enhanced processes, the only possible way to conclusively prove the existence of superfluid suppression phenomenon is through the study of intermediate temperature neutron stars. Tsuruta et al. (2009) showed that the explanation of intermediate temperature isolated neutron stars did indeed require the superfluid suppression of the enhanced cooling processes of hyperons. Their results motivated the study described herein, where the requirement of enhanced processes and their superfluid suppression can be further verified by the study of the behavior of neutron stars in low-mass X-ray binary (LMXB) systems.

For this study, the methodology of Tsuruta et al. (2009) was adopted. While Tsuruta et al. (2009) tested soft TNI2u model and medium TNI6u model, stiff TNI3u and the medium TNI6u models were adopted in this study, and the soft TNI2u model was not included, because only the stiffer models were deemed to have the proper neutron star mass range for a realistic study of LMXBs: The maximum mass supported by the soft TNI2u model was shown to be as low as  $1.5 M_{\odot}$ , while the maximum mass of a neutron star was observationally reported to be as large as  $2 M_{\odot}$  (Demorest et al. 2010). Though none of the EOS models tested by Tsuruta et al. (2009) is capable of supporting a neutron

star with that mass, the TNI3u and TNI6u models have the closest with 1.95 and 1.75 solar mass maxima, therefore they were chosen for this study.

Thermal evolution of neutron stars with hyperon-mixed cores was simulated with the use of the two aforementioned EOSs for a variety of neutron star masses using the same “exact evolutionary code” by Nomoto & Tsuruta (1987) that was discussed earlier. This code is designed not only to consider hyperon based direct Urca processes but also contributions from all neutrino emission processes of standard models. Consequently the resulting cooling curves are expected to display varying degrees of superfluidity effects from the hyperon mixture where very low and very high masses are less affected by the superfluidity of these exotic particles, since for low masses the core density is not expected to be high enough for the hyperon-mixture to appear, and for the high masses the superfluid gap may be too small to display a strong superfluid effect. The exact contribution of the hyperons is also dependent on the EOS itself, since a stiffer EOS is affected to a lesser degree by the introduction of hyperon cooling. Once neutrino and photon luminosity profiles of the neutron star over time as a function of temperature are calculated, the steady state accretion rate and corresponding deep crustal heating can also be calculated and the results can be compared with the existing LMXB X-ray detection data.

### 3.3.2 Results and Comparison with Observational Data

Figures 3.1 – 3.6 in the following pages demonstrate the steady state accretion rate versus photon luminosity relation where figures 3.2, 3.4, 3.6 are for the medium TNI6u model for 1.4, 1.5 and 1.75 solar mass stars respectively, while figures 3.1, 3.3,

3.5 are for stiff TNI3u model neutron star in the same order of mass. The two curves in each figure are for the same neutron star but they are based on different nuclear heating models with the black curve following the heating regimen of HZ90 model (Haensel & Zdunik, 1990), and the red curve following the heating regimen of HZ03 model (Haensel & Zdunik, 2003). In all these figures the vertical axis is the log of photon luminosity as observed at an infinite distance in the units of ergs per second, and the horizontal axis is the steady state accretion rate in the units of solar masses per year.

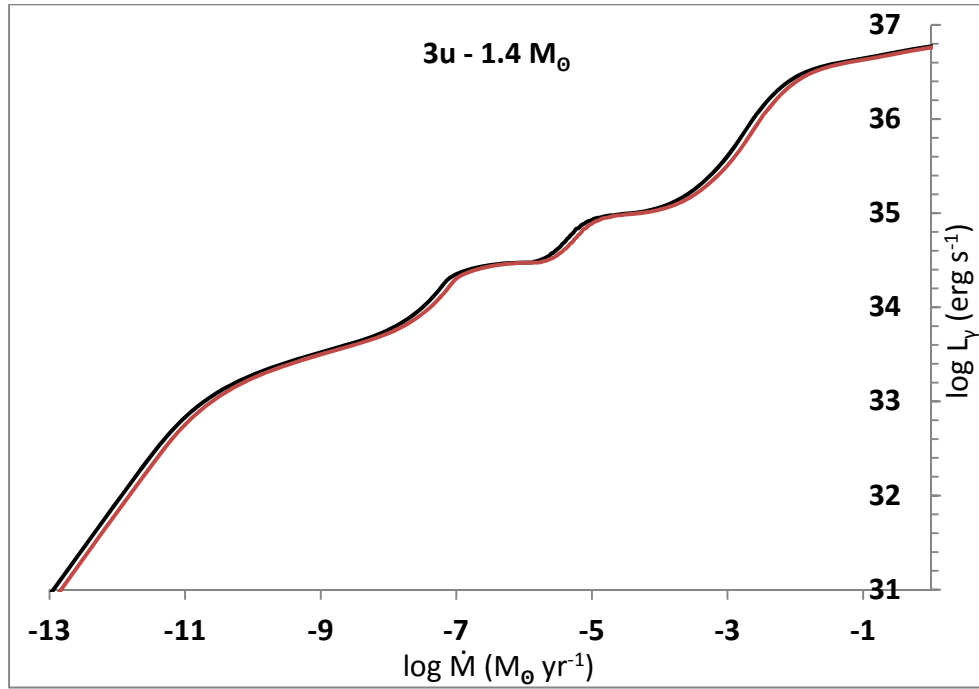


Figure 3.1 Steady state accretion rate vs. thermal luminosity for a  $1.4 M_{\odot}$  neutron star that follows the stiff TNI3u hyperon core model

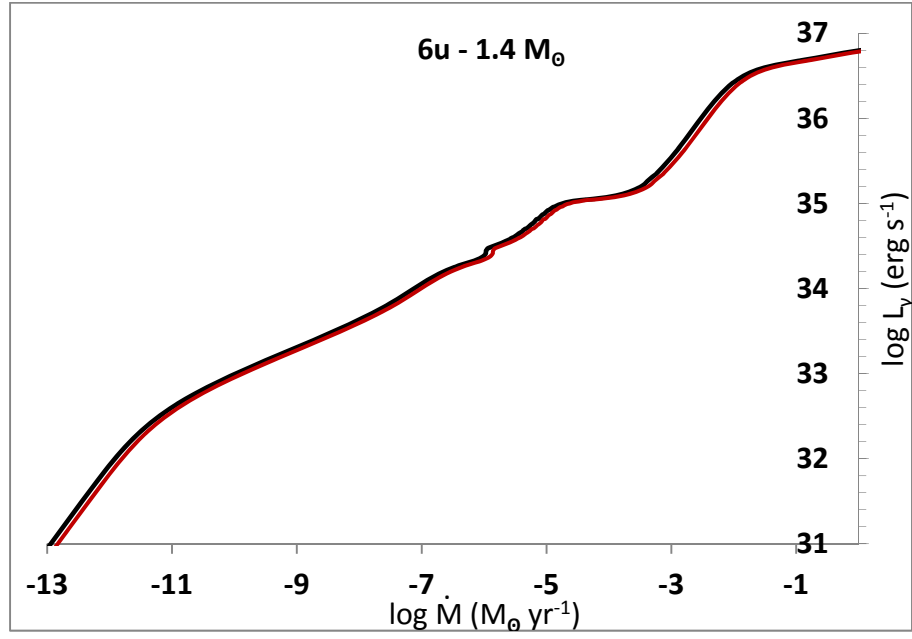


Figure 3.2 Steady state accretion rate vs. thermal luminosity for a  $1.4 M_\odot$  neutron star that follows the medium TNI6u hyperon core model.

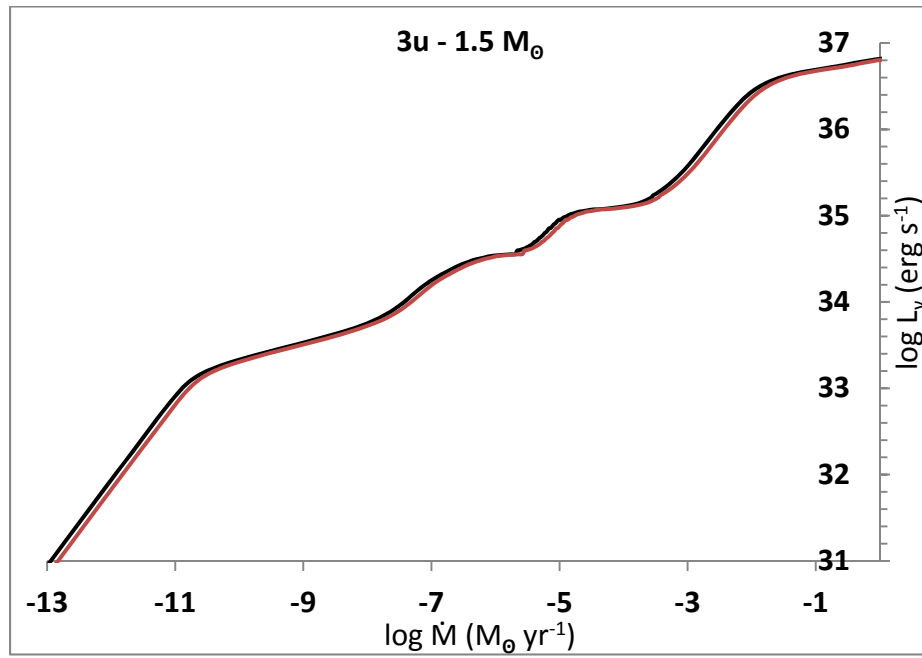


Figure 3.3. Steady state accretion rate vs. thermal luminosity for a  $1.5 M_\odot$  neutron star that follows the stiff TNI3u hyperon core model.

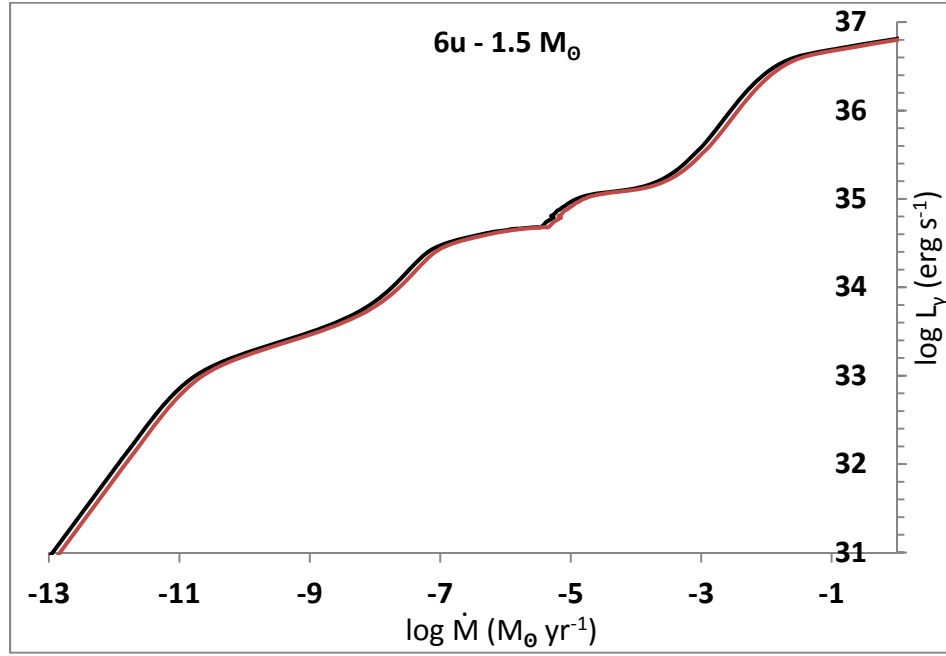


Figure 3.4. Steady state accretion rate vs. thermal luminosity for a  $1.5 M_\odot$  neutron star that follows the medium TNI6u hyperon core model.

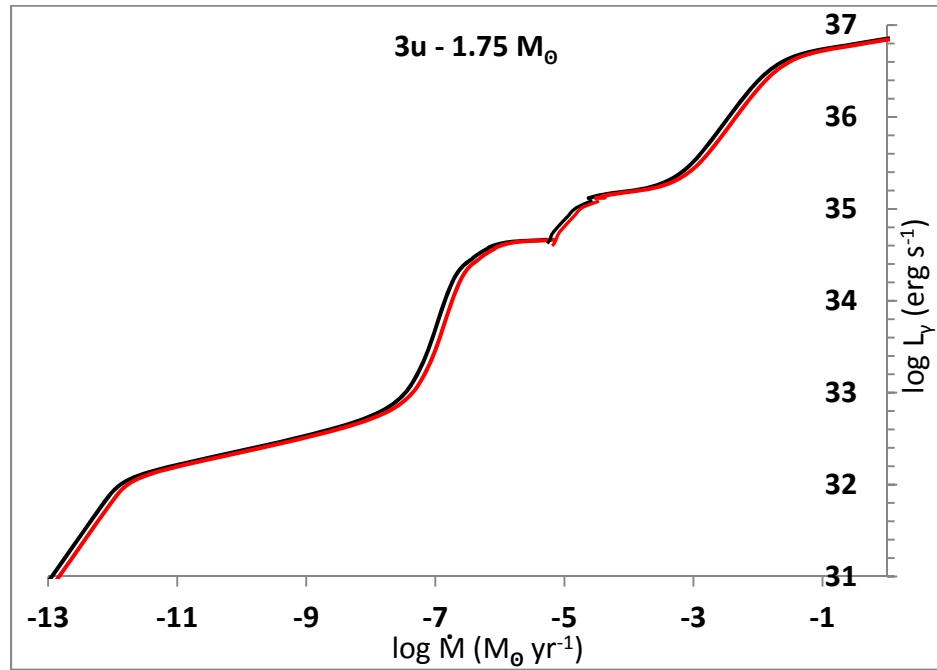


Figure 3.5. Steady state accretion rate vs. thermal luminosity for a  $1.75 M_\odot$  neutron star that follows the stiff TNI3u hyperon core model

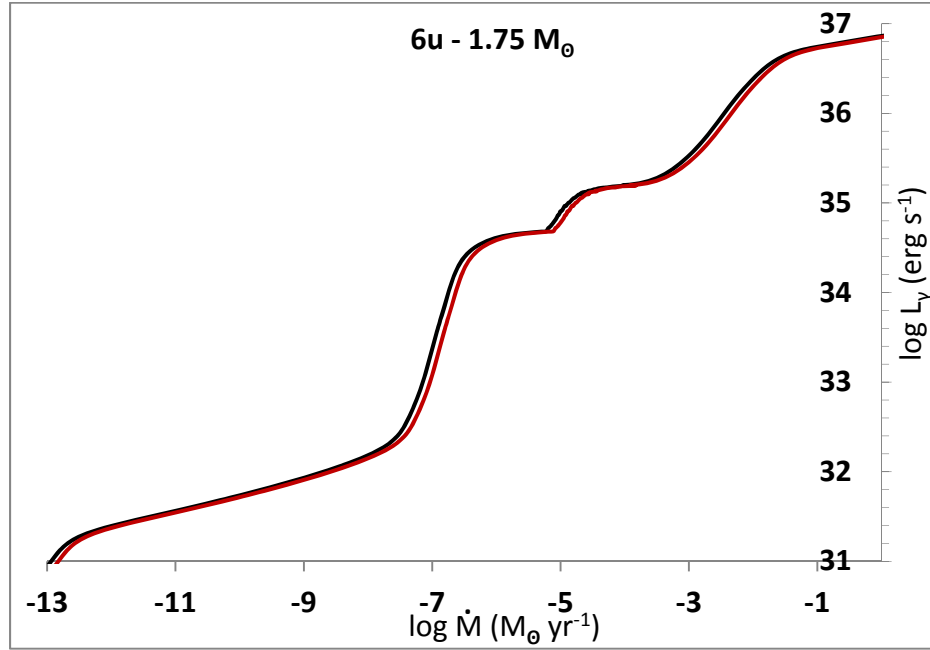


Figure 3.6. Steady state accretion rate vs. thermal luminosity for a  $1.75 M_{\odot}$  neutron star that follows the medium TNI6u hyperon core model.

It is important to note that the luminosity information on these graphs only reflects the quiescent period of the neutron star, and does not include any information about fusion fueled bursts. This does not have any impact on the results since the energy created during the burst events should almost entirely be radiated away, and therefore have no significant contribution to the luminosity of the star overall, but create some crustal heating.

Figures 3.1 – 3.6 display the characteristics of the four stages of neutron star cooling as described in section 2.6. However, only the late stages of the neutron star thermal evolution are relevant to the LMXB data since the neutron star will be at steady state only after it becomes isothermal, past the thermal relaxation period. In figures 3.1 – 3.6 the region that corresponds to possible steady state neutron stars is the leftmost region

of the overall graphs where the accretion rate is less than  $10^{-8} M_{\odot}$  per year. By focusing on the relevant portions of these figures, and combining the curves for each model in a single figure along with LMXB detections, the composite figures 3.7 and 3.8 are created. It is important to note that since in figures 3.1 – 3.6 the HZ03 (Haensel & Zdunik, 2003) and HZ90 (Haensel & Zdunik, 1990) models yielded very similar curves, it was deemed unnecessary to show both on the composite figures, and only the results of HZ03 are shown in figures 3.7 and 3.8. The error bars in these figures correspond to actual LMXB detections, while the arrows represent the upper limits. The masses of neutron stars are attached to each curve. The luminosity information is given in ergs/s while the accretion rate has the units of solar mass per year.

### 3.3.3 Discussion

A qualitative comparison of figures 3.7 and 3.8 does not reveal a natural preference for the stiffness of the equation of state. Both figures show steady-state accretion rate versus photon luminosity curves that span the entire parameter space that contains the LMXB data. The non-preferential nature of the equation of state is further revealed by examining the limiting results from the two figures. At the high end of the luminosity range, the data range for Aql X-1 is fitted by 1.3 and 1.4 solar mass curves for TNI6u and TNI3u models respectively in figures 3.7 and 3.8. However, it is important to note that both curves represent a neutron star that is below the hyperon transition mass for its respective EOS.

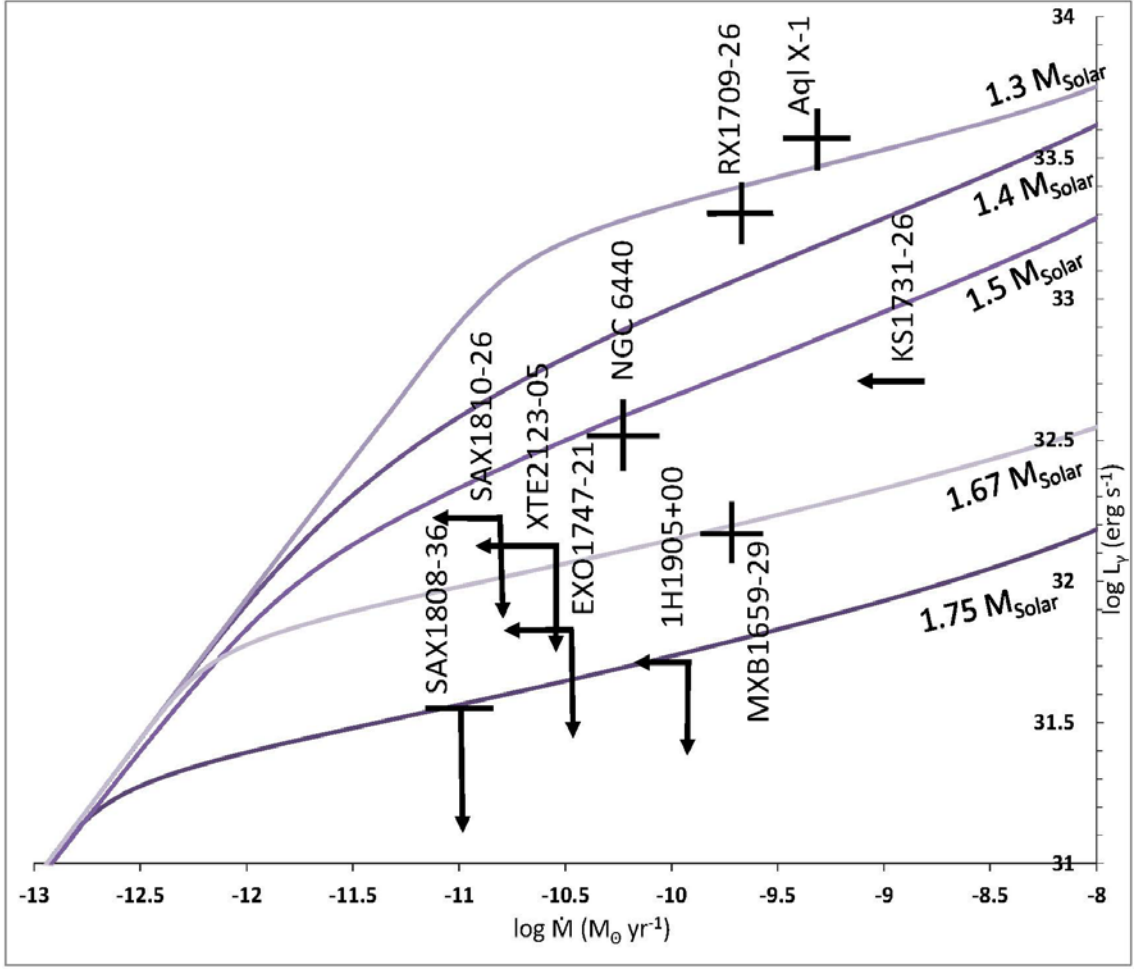


Figure 3.7. The logarithmic mass accretion rate versus logarithmic luminosity curves for TNI6u hyperon-mixed model neutron stars of various mass along with observational data for several LMXBs are shown.

Also a nearly identical curve could be obtained for a lower mass neutron star that follows the same EOS. Similarly, at the low end of the luminosity range, the data range for SAX 1808-36 is fitted by 1.75 and 1.9 solar mass curves for TNI6u and TNI3u respectively. However, these masses are greater than the superfluid suppression limit, and therefore should experience maximum cooling. Consequently, curves for 1.75 and 1.9 solar mass neutron stars for two EOSs appear nearly identical. Consequently, the existing

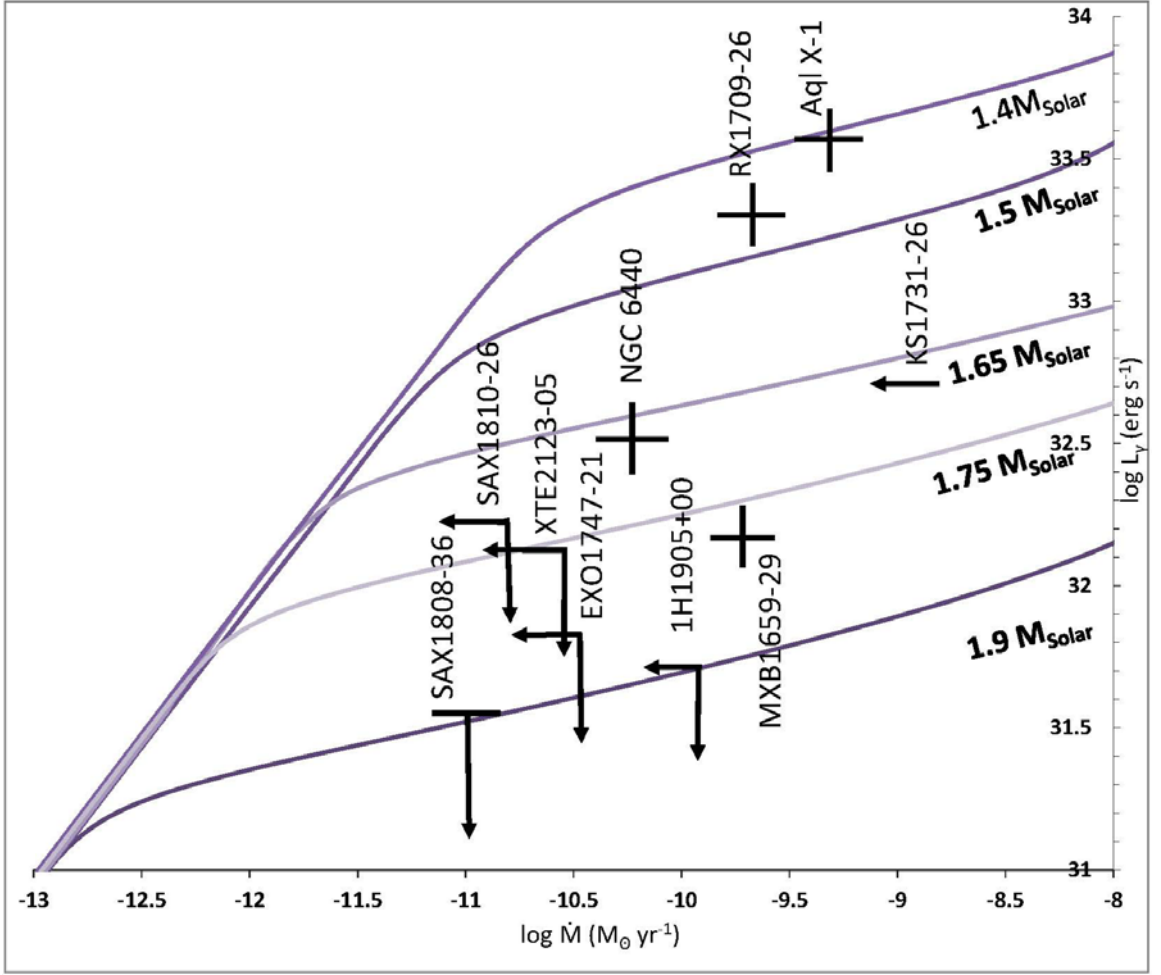


Figure 3.8. The logarithmic mass accretion rate versus logarithmic luminosity curves for TNI3u hyperon-mixed model neutron stars of various mass along with observational data for several LMXBs are shown.

observations do not allow a natural preference for either of the EOSs. This may change with further observational studies of the low-end detection of SAX 1808-36. If this particular soft X-ray transient is shown to have a mass greater than  $1.75 M_{\odot}$ , which is the mass limit of TNI6u model, but less than  $1.95 M_{\odot}$ , which is the mass limit of TNI3u model, it may indicate a preference for the stiffer EOS for the high mass soft X-ray Transients (SXTs). On the other hand, if the actual luminosity for this neutron star is

shown to be significantly lower than the currently established upper limit, then neither hyperon model will be able to explain that detection, and may indicate either the emissivity of the hyperon neutrino processes to be stronger than what is currently accepted, or the presence of faster cooling processes, such as direct Urca involving nucleons (as shown in the Appendix), for this neutron star.

While both TNI3u and TNI6u models appear to be very promising for explaining the behavior of LMXBs, it is difficult to verify the validity of their results since no firm mass information for these LMXBs exists, primarily due to the difficulty of finding the angle of inclination of the binary system, and separating the donor object from the neutron star itself observationally. However, it is informative to make comparisons with other existing theoretical work on LMXBs and observationally determined work that produced mass ranges for some of the binary neutron stars that are shown in the figures in order to confirm the agreement between the results of this study and others.

Previous studies by Yakovlev et al. (2003) follow a similar methodology and formalism as this study, therefore are appropriate for a comparison. While the authors considered the involvement of exotic particles in the neutron star cores, and consequent modifications to the EOS, their superfluid model was highly phenomenological, and consequently could have led them to somewhat different results than the results of the study herein. While the authors also suggested that the mass of the individual neutron stars could not be determined by their study alone, at least until better observational data becomes available, they were able to put some limits to the masses of two of the neutron stars. The two stars that were chosen specifically for this task are Aql X-1 and SAX

J1808.4-3658 (SAX 1808-36 in figures 3.7 and 3.8). The reason for this choice is not only that the observations for these two stars are of reasonably high quality, but also they represent the upper and lower ends of the luminosity range for the detections. Their results indicated that the mass of Aql X-1 was in the range of  $1.1 - 1.3$  solar mass, while the mass of SAX J1808.4-3658 ranged from  $1.74 - 2$  solar mass. When compared to the curves in figures 3.7 and 3.8, these values are indeed in agreement with the results of both hyperon models even though the curves cross the detections near the edges of the error bars. For the Aql X-1 results, the virtual inconsistency between the results in figure 3.8, where  $1.4 M_{\odot}$  curve fits the data, and the findings of Yakovlev et al. (2003) is completely superficial since as previously stated a lower mass star would yield a nearly identical curve for the TNI3u model due to the fact that any neutron star mass of  $1.4 M_{\odot}$  and under does not have the participation of hyperons in its core, therefore cools through identical standard processes. For the SAX 1808-36 data point, the result of the TNI3u model has a slightly greater overlap in the mass range with their results than the TNI6u model, but this does not provide a preference for either EOS either based on the currently available observational data.

Despite the fact that the mass of a neutron star in a binary system is notoriously difficult to identify, there are several papers published that attempt to directly identify or at least constrain the mass of some of the stars shown in figures 3.7 and 3.8 using various observational and data analysis methods. The detection corresponding to NGC 6440 is one of the focuses of these studies. NGC 6440 is one of the detections that has been studied by Freire et al. (2008), and the neutron star is estimated to be within the mass

range of  $1.26 - 1.65$  solar mass. This is clearly a very wide range. Consequently, the agreement between the results of the TNI3u and the TNI6u models, and the findings of Freire et al. (2008) provides no significant preference for either model. However, the resulting constraint that this neutron star is a medium mass one is a more important conclusion. Both the TNI3u and TNI6u models predict that luminosity and mass of neutron stars are inversely related where higher mass neutron stars have lower steady-state luminosity for the same accretion rate compared with the lower mass neutron stars. A medium mass neutron star such as NGC 6440 being found to be in the middle of the LMXB observation data will be indicative of the validity of this prediction. Another observationally studied neutron star is SAX1808-36. A recent publication by Heinke et al. (2013) reported that they were able to constrain the lower mass limit for this neutron star to a value greater than  $1.6 M_{\odot}$  using a multicolor light curve. This would indicate that the SAX 1808-36 neutron star is a relatively massive neutron star which is once again consistent with the findings of both the TNI3u and TNI6u models. It is important to note that the results presented by Freire et al. (2008) and Heinke et al. (2013) are still controversial, and therefore the agreement with the results of the present study cannot be taken as a proof of the validity of this study, but still gives some level of confidence about applicability of hyperon-mixed neutron star core models to the study of LMXBs.

Based on the comparison between the results of other theoretical accretion-driven heating studies, independent mass identification attempts and the results of this study, it is safe to state that the hyperon-mixed neutron star core models are excellent candidates for the study of steady state accretion rate – deep crustal heating of low mass X-ray

binaries. Both the medium TNI6u and the stiff TNI3u model are shown to yield results that are consistent with existing observational data. While there is no indication of which of these models may be preferable, one quality of the results may give some clues. If the neutron star of the SAX 1808-36 is shown to be more massive than the  $1.75 M_{\odot}$  limit of the TNI6u model, or another observation for a different SXT with low luminosity and very high mass is identified, it may necessitate a stiffer EOS for the high mass observations. However, neither of these outcomes could invalidate either hyperon model, and could be an indication of an EOS that has density dependent repulsion as discussed in the previous chapter. At this time, however, these are speculative and both hyperon models are considered valid methods for the study of accretion driven deep crustal heating mechanisms in LMXBs.

Consequently, the most important conclusion of this study is that there is indeed evidence and need for superfluid suppression of enhanced cooling processes in order to explain the full range of observations. Three SXT data, NGC 6440, MXB 1659-29 and KS 1731-26, fall into the mid-range of observations. The low mass neutron stars yield curves that are too high to be consistent with these data due to the lack of enhanced processes for such low mass stars. The high mass neutron stars, on the other hand, yield curves too low to be consistent with these observations due to the unsuppressed fast neutrino processes of hyperon matter. Therefore it is safe to say that the only possible explanation for the mid-range temperature data of these three neutron stars is the presence of superfluid suppression of enhanced neutrino processes if these stars indeed contain hyperon-mixed cores. While it is difficult, if not impossible, to distinguish

between the results of various EOSs, this conclusion will be valid regardless of which EOS prevails. This provides further evidence of superfluid suppression of non-standard cooling in neutron stars with hyperon-mixed cores, which was also shown to be necessary by earlier isolated neutron star studies (Tsuruta, 2009). Further confirmation of this statement will be possible once additional LMXB observations, as well as better mass constraints for observed SXTs become available.

### 3.4 TNI7 Model for Low Mass X-ray Binaries

#### 3.4.1 Introduction

The hyperon-mixed model results were consistent with the observational results if effective superfluid suppression of the enhanced neutrino processes is present. However, as earlier discussed, the superfluid model for neutron stars with hyperon-mixed cores may not be able to yield such results if the suggestions of possibly weaker attractive forces due to the “NAGARA event” are valid which would lead to small superfluid energy gaps, and consequently would not allow superfluid suppression of the fast neutrino processes (Takatsuka et al. 2001). Even though this effect is highly debated, it would be prudent to test alternative models for the LMXB case. In addition, even the relatively stiff TNI3u hyperon-mixed core model falls slightly short of matching the mass requirements of the newly discovered 2 solar mass neutron star, which is at this point almost universally accepted as an accurate detection (Demorest et al., 2010). The TNI7 model can successfully address the high maximum mass requirement (see Chapter 2 for details). Since Tsuruta et al. (2010) states that the cooling of hyperon and pion models behave nearly identically, this model could also be extended to pion cores which would

avoid the superfluid gap issues that may be present for hyperon model should the “NAGARA event” results be validated (Takahashi, 2001). Since the TNI7 model does not refer to a specific neutron star composition, but rather indicates the universal density – pressure relation of all neutron star cores, it is also applicable to hyperon-mixed neutron stars. Therefore the TNI7 model was adopted for an additional study of LMXBs.

### 3.4.2 Results and Comparison with Observational Data

Similar to the study of hyperon-mixed core models, cooling profiles for TNI7 pion condensate model neutron stars of various masses were generated using the Nomoto-Tsuruta exact code (Nomoto & Tsuruta, 1984), and the steady state accretion rates of these stars were calculated using their luminosity profiles. The calculated steady state accretion rates were plotted against the thermal luminosity of the neutron stars, and the fit of the resulting curves were checked against the existing LMXB detections.

Figures 3.9 – 3.11 in the following pages demonstrate the steady state accretion rate versus photon luminosity for TNI7 model neutron stars of 1.3, 1.4 and 1.8 solar mass respectively. The two curves in each figure are for the same neutron star but they are based on the different nuclear heating models with the black curve following the HZ90 heating regimen (Haensel & Zdunik, 1990), and the red curve following the HZ03 heating regimen of (Haensel & Zdunik, 2003). In all these figures the vertical axis is the log of photon luminosity as observed at an infinite distance in the units of ergs per second and the horizontal axis is the steady state accretion rate in the units of solar mass per year.

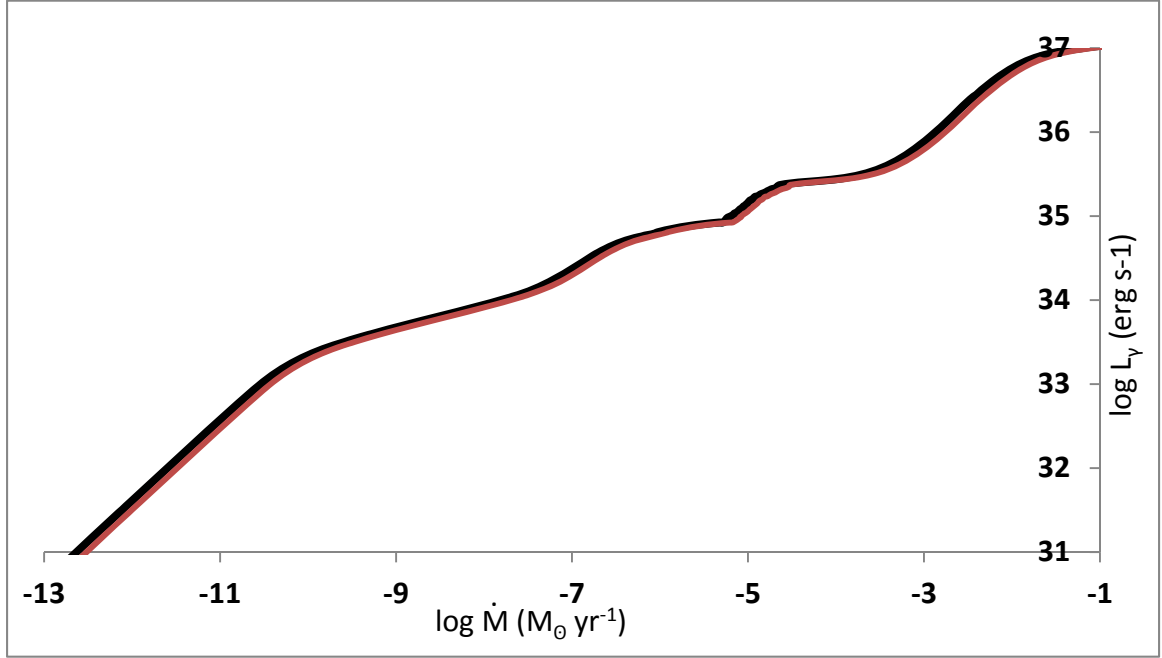


Figure 3.9 Steady state accretion rate vs. thermal luminosity for a 1.3  $M_{\odot}$  neutron star that follows the very stiff TNI7 core model.

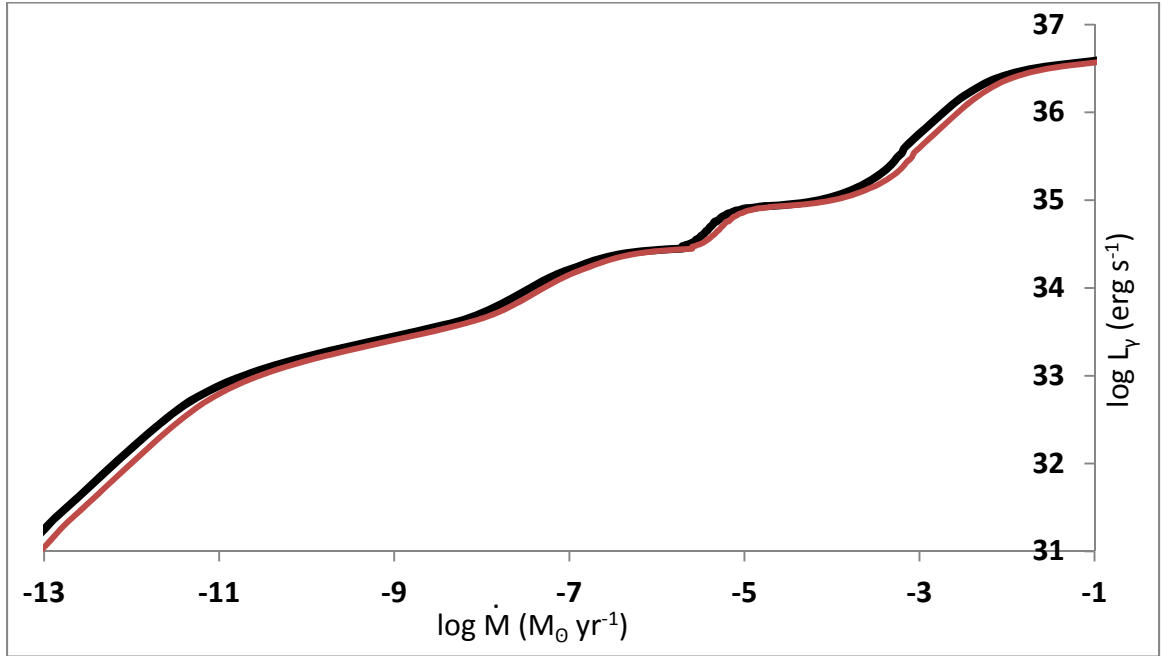


Figure 3.10 Steady state accretion rate vs. thermal luminosity for a 1.5  $M_{\odot}$  neutron star that follows the very stiff TNI7 core model.

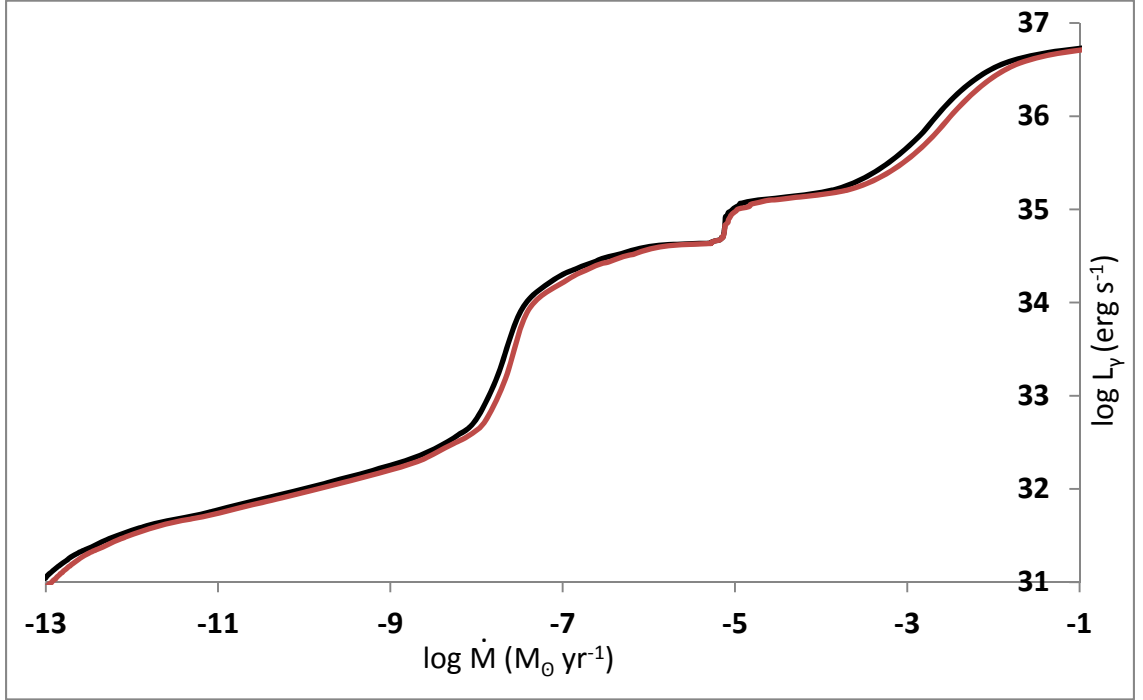


Figure 3.11. Steady state accretion rate vs. thermal luminosity for a  $1.8 M_{\odot}$  neutron star that follows the very stiff TNI7 core model.

These curves, once again, show the four stages of the thermal evolution of the neutron star evolution, but only the leftmost parts of the graphs representing the isothermal stage of the neutron stars are relevant to the study of the steady state accretion rate versus luminosity relationship. The steady state accretion – photon luminosity curves obtained in this section resemble the results of hyperon models that were studied earlier in Section 3.3.1. As expected the more massive stars once again appear to be colder, corresponding to relatively smaller steady state accretion rates. While a qualitative examination of these graphs reveals that the TNI7 model results cover the full range of the LMXB observation, a closer examination to determine whether the results of the TNI7 model are in agreement with the results of previous LMXB work is warranted.

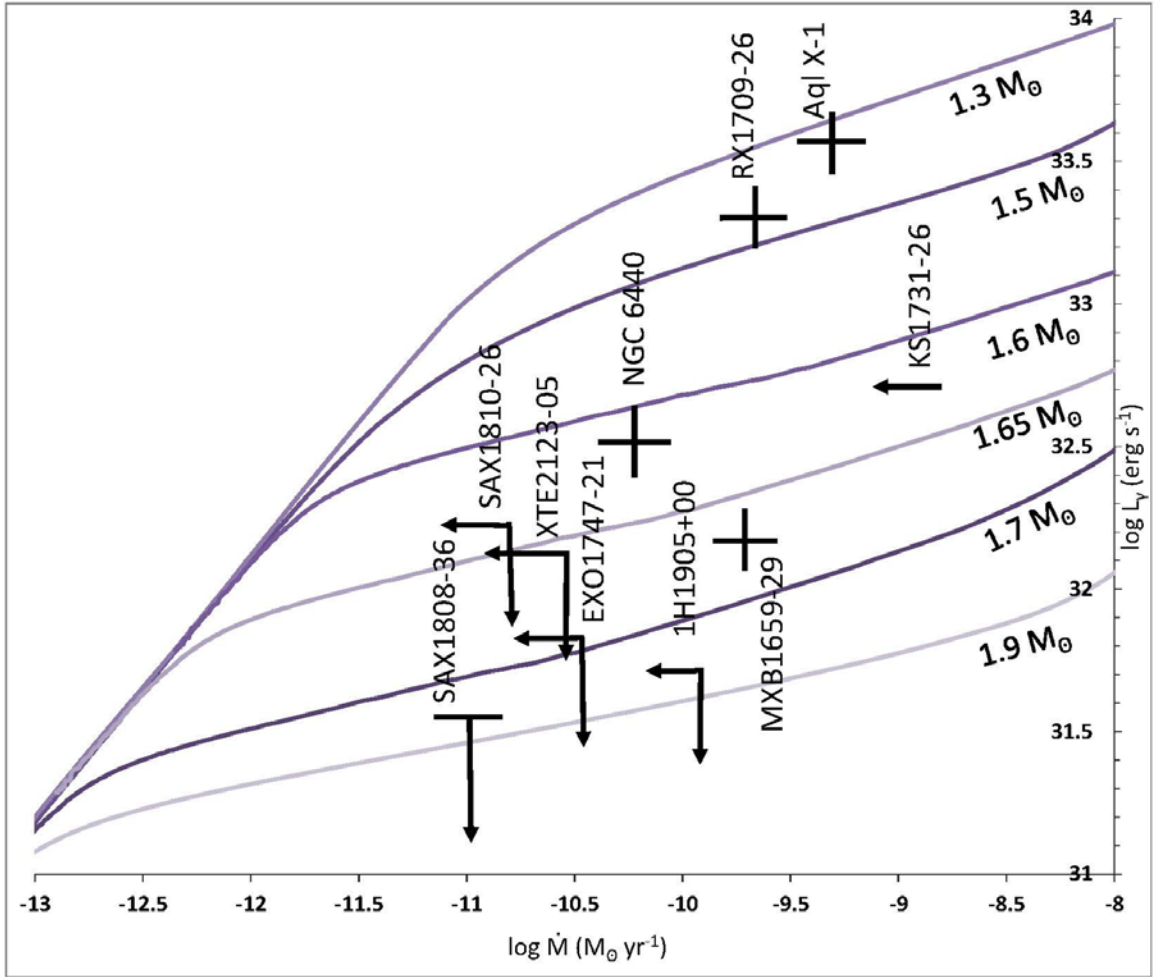


Figure 3.12. The logarithmic mass accretion rate versus logarithmic photon luminosity curves for the TNI7 pion condensate model neutron stars of various mass along with observational data for several LMXBs are shown.

Figure 3.12 above is another steady state mass accretion rate versus photon luminosity graph that shows the curves that belong to several different mass neutron stars for TNI7 pion condensate core model, but it magnifies the range for the LMXB data for closer investigation. Similar to the results of the hyperon model, the HZ03 (Haensel and Zdunik, 2003) and HZ90 (Haensel and Zdunik, 1990) heating regimes created results that are essentially indistinguishable against the available observational data; therefore only

HZ03 was chosen for further study. The detections shown in this graph are identical to those used in figure 3.7.

### 3.4.3 Discussion

As can be seen from figure 3.12, the results of the TNI7 core model cover the full range of the observations. The results are also in agreement with previous work on the subject. Once again this can be seen by focusing on the data for Aql X-1 and SAX 1808-36 since these are the upper and lower limits for the luminosity of the current detection range. The results indicate Aql X-1 has a low mass which may be as low as 1.3 solar mass. This is consistent with the empirical upper limit for the mass/luminosity of Aql X-1 determined by Yakovlev et al. (2003). The figure 3.12 also shows that SAX1808-36 is a massive star, close to 1.9 solar mass. This is also consistent with the findings of the Yakovlev et al. (2003) and Heinke et al. (2013), though it is near the higher end of the result of the former. On the other hand, an additional advantage of the TNI7 model for the SAX 1808-36 detection can be inferred from figure 3.12. Since there is no established mass for this neutron star, but is expected to be relatively large, it may be in excess of the maximum mass allowances of the softer models. Since the maximum mass limit for the TNI7 model is greater than  $2 M_{\odot}$ , if the masses of lowest luminosity SXTs are eventually shown to be close to or in excess of  $2 M_{\odot}$ , it will have an advantage over most other models that do not extend that far. Also similar to the findings of the study of hyperon models, TNI7 model results indicate that the data associated with intermediate luminosity neutron stars such as NGC 6440, MXB 1659-29 and KS 1731-26 can only be explained by the presence of superfluid suppression of the enhanced neutrino processes.

### 3.5 Summary

In this chapter, the results of steady state heating studies of soft X-ray transient neutron stars that are located in low mass X-ray binary systems are discussed. Various neutron star core models with exotic particles, including the medium stiff and stiff hyperon models TNI6u and TNI3u and the stiff pion model TNI7, are used for the study. The results indicate that all tested models are capable of explaining the full range of currently available LMXB data, with no discernible preference for one model over the others. An important conclusion of this study is that the lower mass neutron stars in binary systems can be expected to have higher photon luminosity and support greater steady state accretion rates, while the high mass neutron stars can be expected to have lower luminosities and support smaller steady state accretion rates. While this conclusion cannot be irrefutably verified at this time due to lack of definitive mass data for the observed LMXBs, the results are consistent with other theoretical work (Yakovlev et al. 2003) and observational work (Freire et al. 2008, Heinke et al. 2013). The most important result of this study, however, is the clear indication of the necessity of superfluid suppression of enhanced neutrino processes. While the results of all models are capable of explaining the full range of LMXB data, the intermediate data points are consistent with cases where the enhanced neutrino processes are partially suppressed due to the superfluidity of the constituent particles and therefore allow an intermediate cooling case between the standard and maximum enhanced cases. This result confirms the earlier findings of Tsuruta (2009) which also had an identical requirement of partial superfluid

suppression of enhanced neutrino processes in order to explain the intermediate temperature isolated neutron star data.

## CHAPTER 4

## CONCLUDING REMARKS

4.1 Summary

A new neutron star EOS model is created to study the behavior of Cas-A neutron star. The newly created TNI7 model is shown to produce results consistent with the rapid cooling rates indicated by Cas-A neutron star detections by delaying the thermal relaxation of a  $1.72 M_{\odot}$  pion condensate core star until the current age of Cas-A of 330 years. This model is also shown to be capable of explaining the full range of isolated neutron star temperature detections, but only if the superfluid suppression of enhanced cooling processes is also taken into account. This requirement of superfluid suppression confirms the findings by Tsuruta (2010).

The new EOS TNI7 is also applied to the study of steady state accretion rate – luminosity relation of neutron stars in low mass X-ray binary systems along with two additional older hyperon models TNI3u and TNI6u. All EOS models are shown to be capable of explaining the full spectrum of currently existing LMXB data. This explanation further strengthens the need for neutron star thermal evolution models with superfluid suppression of fast non-standard cooling processes involving exotic particle cores for the intermediate range temperature observational data.

## 4.2 Conclusions

In chapter 2, the newly constructed TNI7 stiff equation of state was used to study the thermal evolution of neutron stars with pion-condensate cores. The results showed that the thermal relaxation timescale can be as late as 300 – 380 years, due to the fact that the neutron star based on this EOS is somewhat larger than what was predicted by some other models which also place the thermal relaxation time to an earlier period, 10 – 100 years after the formation. When the results of the new model were tested with the short term observations of Cas-A neutron star as well as the detections of all other neutron stars, the model was shown to be very promising.

Due to the unusual cooling behavior of Cas-A, which was shown to drop nearly 4% in temperature over a 10 year period of observations, at the age of  $330 \pm 20$  years, this star provided a unique set of parameters that were not compatible with most existing models. The results of the new model, on the other hand, are in complete agreement with these observations. According to the results of the TNI7 model, the unusually rapid cooling of the Cas-A neutron star is due to the presence of non-standard cooling processes involving pions. This provides the high rate of cooling observed for Cas-A, and cannot be explained by standard cooling mechanisms. Furthermore, the thermal relaxation of the neutron star is delayed until its current age due to the fact that this is a relatively larger and more extended neutron star. The involvement of exotic particles in the neutron star core generates additional repulsive interactions that result in a stiffer EOS, and subsequently a larger crust for the neutron star that allows the necessary delay in thermal relaxation.

When the superfluid-suppression scenarios that are given by recent nuclear/particle models (Takatsuka et al. 2007) and the possible internal heating by vortex creep (Tsuruta, 1998) were included, the cooling regimes predicted by the new TNI7 model were shown to be also consistent with the full range of isolated neutron star temperature observational data.

The TNI7 model results were also applied to the study of accreting low-mass X-ray binaries. The results of this study gave additional credence to the new model since its luminosity results were well within the range of observational data. It was also shown to be particularly useful if some of the colder binary neutron stars are shown to have greater masses than the maximum mass allowance of softer models. The TNI7 model allows a maximum stable neutron star mass of roughly 2.1 solar masses which is consistent with new observational results. It was also determined that if the actual luminosities of the low luminosity LMXBs are shown to be noticeably lower than the currently available upper limits, however, a stronger cooling regime than the enhanced pion processes will likely be needed.

In addition to the new model, the results of older hyperon-mixed core model neutron star thermal evolution simulations were tested for the study of low-mass X-ray binaries. Both the stiff TNI3u and medium TNI6u models were shown to be able to explain the detections of all such neutron stars. While the hyperon-mixed neutron star core models were consistent with the existing LMXB observations, certain limits for the applicability for these stellar bodies were also identified. Due to the fact that the theoretical low end limits for the neutron star luminosity approximately correspond to the

upper-limit for the luminosity observations of some LMXBs, it was determined that if the actual values of luminosity for these stellar objects were to be identified as lower than the current limit, the observation would require a different model, which would allow either higher emissivity from hyperon neutrino processes, or would project different processes such as direct Urca with nucleons with higher proton concentration. Additional information about the nucleon Urca processes can be found in the Appendix.

The study of LMXBs with exotic neutron star core models also confirmed the necessity of superfluid suppression of non-standard cooling processes, since there are at least three intermediate range LMXB sources that cannot be explained otherwise. This result supports the earlier findings of Tsuruta et al. (2009) where superfluid suppression of non-standard cooling was also shown to be vital for explaining the temperature data of intermediate temperature neutron stars such as the Vela pulsar.

In conclusion, various exotic particle based core models, along with various equations of state were tested for a variety of neutron star phenomena. Both the hyperon-mixed and the pion-condensate core models were shown to be very promising especially when used along with a stiff equation of state such as the newly created universal three-body interaction model TNI7. This new EOS model also allows the neutron star mass projections to be extended to include the recently established observational maximum of 2 solar masses. Although the larger radius that is the consequence of this stiff EOS is yet to be observationally verified, the exotic-particle core models that follow this new EOS are ideal for both current and future studies of neutron stars.

### 4.3 Future Prospects

While a variety of EOS models based on the latest nuclear experiments have been tested as part of this study, much remains to be explored. The newly created TNI7 model is in full agreement with the latest observational and nuclear/particle experimental data. It is based on the current observations of Cas-A neutron star, and the neutron star models based on this EOS display thermal evolution results that are in perfect agreement with the observed data. However, the true challenge to the model remains in the future. The TNI7 EOS based neutron star model predicts that Cas-A is still in its early stages of thermal relaxation, and the currently observed fast cooling rate should continue for another 60 – 80 years. This is verifiable through continued observations of Cas-A. The accuracy of the model can easily be tested with such observation, though it is a relatively long term project.

Additionally, while TNI7 model cooling curves were shown to be able to fit the full range of neutron star observational data, the quality of the fit is unknown at this time. There are two possible future avenues of exploration that can determine the accuracy of TNI7. Firstly, as additional neutron star mass data become available by the use of the Shapiro effect on additional sources, these data can be compared with the mass predictions of the TNI7 based cooling curves. Secondly, additional sources with well constrained evolutionary ages can be observed over a period, perhaps over 10 years like the Cas-A observational data used for this study, and their behavior over that period can be compared with the predictions of TNI7 model neutron stars. The second method would be especially useful if additional sources that are in their thermal relaxation period

can be found, since this particular period in the neutron star evolution has the most distinguishing properties such as the cooling rate and the relaxation time of the overall timeline.

The most important point of contention that needs to be resolved for the future of hyperon based core models is the result of the “NAGARA event”. This result suggests that the superfluid suppression of the fast processes involving hyperons may not be possible due to much smaller than expected superfluid energy gap. This small energy gap would in turn make it impossible to explain the detections of intermediate temperature neutron stars since the superfluid suppression of the enhanced neutrino processes that allow the explanation of the temperature data of these neutron stars would vanish with the diminishing energy gap. However the results of the “NAGARA event” are highly controversial at this time. Further developments in nuclear physics in the next decade should be able to determine whether the hyperon-mixed core models are indeed valid or not. If they are proven to be valid, then the TNI7 EOS can be extended to the hyperon-mixed core models, though the results are not expected to be significantly different from the pion-condensate core model.

The research can also be expanded further to include the involvements of additional exotic particles, such as quarks, in neutron star cores. However, the particle physics research for these exotic particles is still in its infancy, and will likely require at least another decade to mature to the point where they can be applied to neutron star research for reasonably precise results.

Finally, the results of this study suggest that a variable repulsive force based EOS warrants further exploration. On the other hand, there is no good way of constraining the parameters of such a model at this time. Additional neutron star data, specifically precise information for the radii and masses of individual neutron stars, will constitute the basis for such a study in the future as they become available.

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APPENDIX A:

NEUTRINO EMISSION PROCESSES OF NEUTRON STARS

MODIFIED URCA PROCESSES:

First developed by Chiu and Salpeter (1968), these processes were thought to be the primary mechanism of neutrino generation in the standard neutron star cooling regime:

$$n + n \rightarrow n + p + e^- + \bar{\nu}_e \quad (\text{A.1})$$

$$n + p + e^- \rightarrow n + n + \nu_e \quad (\text{A.2})$$

$$p + n \rightarrow n + p + e^- + \bar{\nu}_e \quad (\text{A.3})$$

$$p + p + e^- \rightarrow p + n + \nu_e \quad (\text{A.4})$$

The emissivity of the neutrino mechanisms in (A.1) and (A.2) are  $\sim 2 \times 10^{21} RT_9^8 \text{ erg s}^{-1}$ , while the emissivity of (A.3) and (A.4) are  $\sim 10^{21} RT_9^8 \text{ erg s}^{-1}$  respectively where R is the radius of the neutron star in km and  $T_9$  is the temperature in units of  $10^9 \text{ K}$ .

NUCLEON-NUCLEON SCATTERING BREMSSTRAHLUNG PROCESSES:

The n-n and n-p bremsstrahlung processes are:

$$n + n \rightarrow n + n + \nu + \bar{\nu} \quad (\text{A.5})$$

$$n + p \rightarrow n + p + \nu + \bar{\nu} \quad (\text{A.6})$$

$$p + p \rightarrow p + p + \nu + \bar{\nu} \quad (\text{A.7})$$

The emissivity of the neutrino mechanisms in Urca cycles (A.5), (A.6) and (A.7) are  $\sim 10^{19} RT_9^8 \text{ erg s}^{-1}$  where R is the radius of the neutron star in km and  $T_9$  is the temperature in units of  $10^9 \text{ K}$ .

NUCLEON COOPER PAIRING PROCESSES:

The nucleon cooper pairing processes are:

$$n + n \rightarrow [nn] + \nu + \bar{\nu} \quad (\text{A.8})$$

$$p + p \rightarrow [pp] + \nu + \bar{\nu} \quad (\text{A.9})$$

The emissivity of the neutrino mechanisms in (A.8) and (A.9) are  $\sim 5 \times 10^{21} RT_9^8 \text{ erg s}^{-1}$  and  $\sim 5 \times 10^{19} RT_9^8 \text{ erg s}^{-1}$  respectively where R is the radius of the neutron star in km and  $T_9$  is the temperature in units of  $10^9 \text{ K}$ .

NUCLEON DIRECT URCA PROCESS:

The nucleon nucleon direct Urca process is:

$$n \rightarrow p + e^- + \bar{\nu}_e \quad (\text{A.10})$$

$$p + e^- \rightarrow n + \nu_e \quad (\text{A.11})$$

The emissivity of the neutrino mechanisms in (A.8) and (A.9) are both  $\sim 10^{27} RT_9^8 \text{ erg s}^{-1}$  respectively where R is the radius of the neutron star in km and  $T_9$  is the temperature in units of  $10^9 \text{ K}$ .