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Mathematical Model for Biological
Reactor Design Incorporating Characteristics
of Cell Aggregates

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INTRODUCTION

Quantitative concepts of microbial growth have been developed from highly specialized kinds of growth studies, e.g. pure culture growth of bacteria in batch or single continuous fermenters. Such concepts may not apply to more general types of growth, characteristic of biological waste water treatment processes.

Attempts to improve biological reactor technology will require reassessment of concepts and methodologies used in research, design and control problems. In particular, quantitative techniques such as mathematical modeling of processes will probably become more important. Such techniques are already common in chemical technology and have proven very useful.

"UNSTRUCTURED" VS "STRUCTURED" MODELS OF MICROBIAL GROWTH AND METABOLISM*

Experimental data on the growth of pure microbial populations in submerged culture generally include some measure or measures of the amount of living material present in a unit volume of the reactor called concentration of biomaterial, X . There is no unique measure that is suitable, although dry mass per unit volume of reactor is commonly used in wastewater treatment. This measure often includes significant quantities of suspended material.

*After Frederickson, et al.

Additional data includes concentrations of various substances present in the non-living phase of the culture (e.g. substrate, nutrients, etc.). These concentrations will be denoted by $S_1, S_2, \dots$

Most models of microbial growth that have been and are being used start from the basic assumption that there must be a generally valid relation... and a unique relation...between the variables X and $S_1, S_2, \dots, S_n$. These are unstructured models.

This assumption confuses what is measured with what should be measured. There is no apparent reason for assuming a generally valid and unique relation between the variables S and $S_1, S_2, \dots, S_n$. In fact, other variables, measureable but not ordinarily measured, may be very important in determining dynamic behavior of a microbial population.

The variable X is a measure of the quantity of the biomaterial present but indicates nothing about the form of the biological material. However, the state of the biomaterial must be important in determining the rate processes of the population. Thus X grams of dispersed cells in stationary phase culture do not carry on metabolism at the rates as X grams of dispersed cells from an exponential phase. Nor will X grams of dispersed cells metabolize at

the same rate as X grams of flocculated cells.

Unstructured models that are commonly applied to microbial growth processes must of necessity fail in situations that are not balanced growth because there is no way of accounting for changes in cellular rate processes caused by changes in the state of the living material.

The inadequacies are not evident in growth situations commonly observed in a single chemostat or a batch fermenter. Mixed culture reactors, cascades of biological reactors, and recycle systems are examples of growth systems that do not approach balanced growth. Consequently, unbalanced growth is prevalent in biological waste water treatment.

These considerations suggest that "unstructured" models of microbial growth cannot be of general utility. Consequently, the model that is presented incorporates a somewhat more "structured" model of microbial metabolism into biological wastewater treatment design by indicating how cell aggregates or floc characteristics influences process performance.

LITERATURE REVIEW

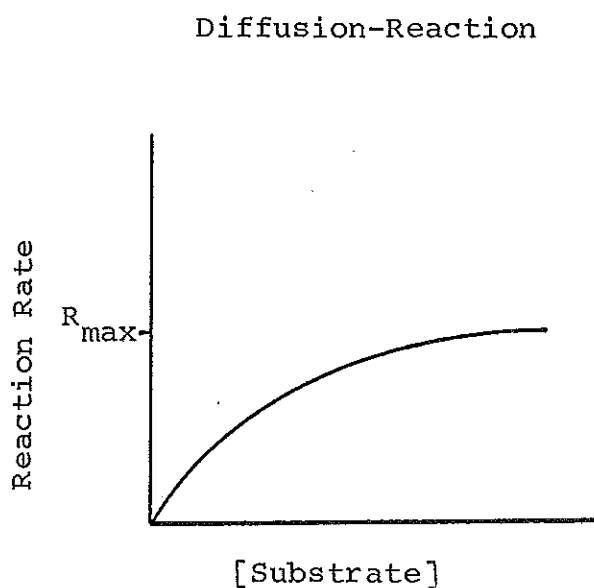
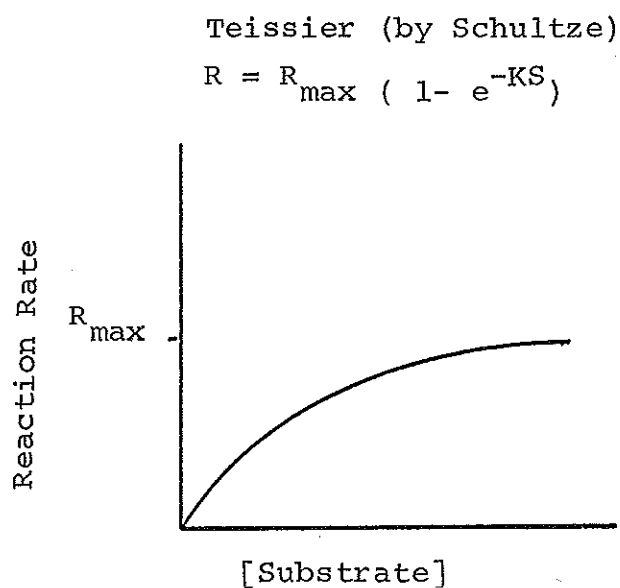
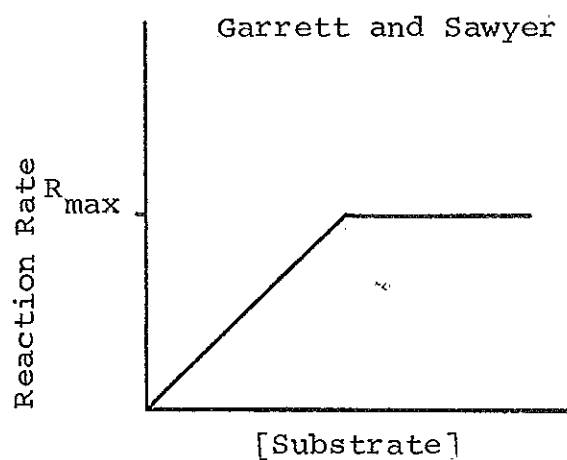
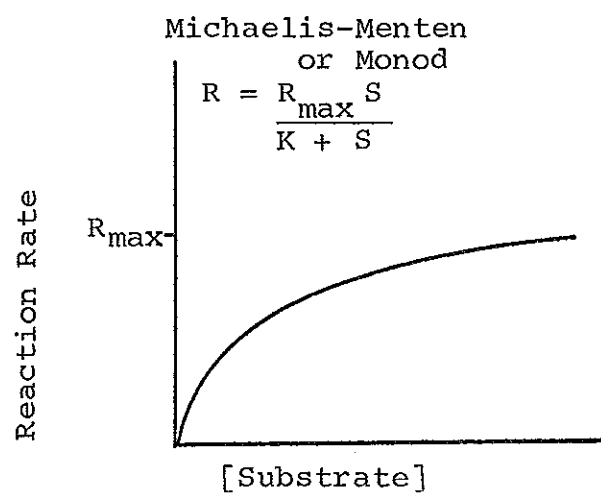
Progress in biological wastewater reactor design concepts shows a pattern of sequential developments. Initially, design was based on quantitative relationships between waste loadings and reactor volume. For example, 33 lbs BOD/1000 ft³/day was the design loading criteria for sewage treatment plants for many years.

The involvement of rate process concepts into design also required consideration of the composition and concentration of waste components. Engineers developed empirical relationships between reaction rates and waste concentrations. Experimental observations frequently were fitted to an equation developed by Michaelis and Menten (1913) for enzyme rate processes.

The Michaelis-Menten equation indicates that reaction rate is proportional to substrate concentration at low substrate levels, and is independent of substrate concentration at high substrate levels (See Fig. 1). Monod (1949) developed a similar expression for relating the growth rate of microorganisms in pure dispersed cultures to substrate levels. Monod pointed out that the equation was just a convenient curve fit of the data and had no theoretical significance.

Garrett and Sawyer (1952) questioned the applicability

Figure 1. Various Rate Process Models



of these expressions to biological growth and substrate removal in multisubstrate, mixed culture systems such as activated sludge. Garrett found that the relationship between growth rate and substrate level was best represented by two functions. The first function relates growth rate as directly proportional to substrate level up to some maximum substrate value. The second function shows growth rate to be independent of substrate concentrations above that level (Fig. 1).

Schultze (1964) advocated still another expression to represent his data, the Teissier equation which, relates reaction rate to an exponential function of substrate concentration (Fig. 1). The shape of the curve is similar to others previously proposed. The reaction rate approaches a maximum value with increasing substrate levels. Schultze concluded that the application of the Teissier equation to his data produced the best fit.

Although experimental data is frequently fitted to various equations, agreement does not necessarily reflect the assumed mechanisms that formed the basis for the equations. Zollinger (1964) and others have pointed out that very different theories have been able to produce agreement with data obtained from quite different systems. The apparent relationships between substrate level and reaction

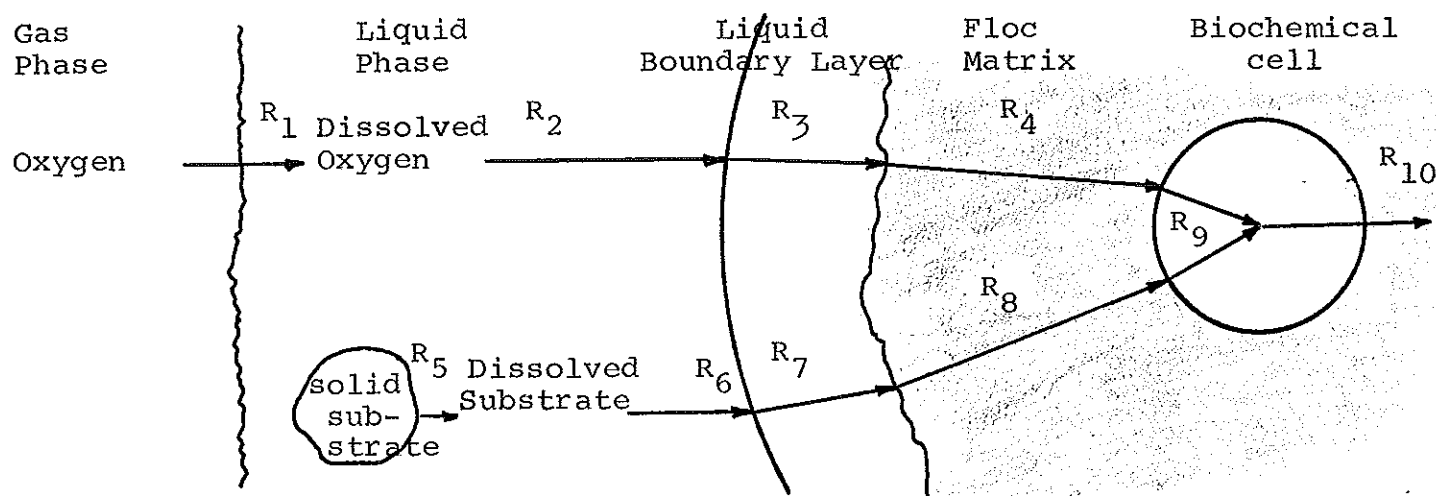
rate can be attributed in many cases to the non-specificity of the analytical method (e.g. BOD, COD, etc.), and by differences in removal rates of various substrates. Superposition of concurrent and sequential zero order removal rates can lead to a non-linear curve, frequently of exponential shape. Zollinger suggests that for the development of a generally applicable model to predict the performance of biochemical reaction, a much more detailed knowledge of the system is required.

Several recent studies have discussed the complexity of rate processes in the biological reactor. Swilley et al (1964) listed eight rate processes relevant to an aerobic dispersed culture reactor. If the biomass is aggregated in flocs, two more rate processes can be added as indicated in Figure 2.

The overall (or apparent) reaction rate for a given reaction environment can describe the interaction among the various rate processes, the rate limiting step usually involving one of the mass transfer steps. In this paper, only the rate processes involved with transport through the floc matrix to the microorganisms are considered. These processes are assumed to be the rate limiting steps in the overall reaction rate.

Mass transfer control of reaction rate by the floc

Figure 2. Rate Processes in a Biochemical Reactor - Reaction and Diffusion Model



Rate Processes:

- R_1 - Interphase transport of gaseous oxygen into solution
- R_2 - Convective transport of dissolved oxygen
- R_3 - Diffusion of dissolved oxygen through mass transfer boundary layer
- R_4 - Diffusion of dissolved oxygen through floc matrix
- R_5 - Hydrolysis of solid substrate
- R_6 - Convective transport of dissolved substrate
- R_7 - Diffusion of substrate through liquid boundary layer
- R_8 - Diffusion of substrate through the floc matrix
- R_9 - Biochemical reaction
- R_{10} - Diffusion of reaction products

1. The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is shown that the function $f(x)$ is increasing and concave down on the interval $(-\infty, \infty)$.

2. In the second part of the paper, we consider the function $g(x)$ defined by the equation

$$g(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is shown that the function $g(x)$ is increasing and concave down on the interval $(-\infty, \infty)$.

3. In the third part of the paper, we consider the function $h(x)$ defined by the equation

$$h(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is shown that the function $h(x)$ is increasing and concave down on the interval $(-\infty, \infty)$.

4. In the fourth part of the paper, we consider the function $k(x)$ defined by the equation

$$k(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is shown that the function $k(x)$ is increasing and concave down on the interval $(-\infty, \infty)$.

5. In the fifth part of the paper, we consider the function $l(x)$ defined by the equation

$$l(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is shown that the function $l(x)$ is increasing and concave down on the interval $(-\infty, \infty)$.

7

matrix was demonstrated by Ballooid and Boyle (1970). Activated sludge floc was subjected to variable shear forces to obtain different particle sizes. Results indicated that mass transfer of glucose through the floc matrix controlled the overall reaction rate.

Floc matrix resistance to oxygen transfer was recognized as a potentially controlling factor by Pasveer (1954) and Wuhrman (1962). Their calculations indicated that the resistance to mass transfer of oxygen through the floc could be significant under reactor conditions that promoted large floc particles and/or low dissolved oxygen levels.

Claims have been made that pure oxygen enhances performance in the biochemical reactor. Poon and Wang (1972) compared pure oxygen to air in laboratory scale reactors and found the overall reaction rates a function of oxygen tension. Ball and Humenick (1972) reviewed a full scale test of pure oxygen conducted by Union Carbide at Batavia, New York, and concluded that there was not significant difference between reaction rates in air and pure oxygen activated sludge systems. However, mass transfer effects were not considered in either study.

In an effort to understand the apparent contradictions, Matson et al (1972) derived a diffusion-reaction model of a biochemical reactor. The model differentiated between

oxygen and substrate mass transfer effects within the floc matrix. It also included the effects of turbulence on floc size and interphase transfer of substrate.

The model in this study relates the effects of floc size, biomass concentration, and oxygen tension to the overall reaction rate in mass transfer controlled reactors. The model also quantifies the capacity of the floc to absorb transient or shock loadings. McLellan and Busch (1968) found that the existing kinetic models did not explain the changes in the rate of substrate removal when significant step changes in substrate concentration were applied to their systems (i.e., a sizeable increase in influent substrate concentration did not cause a proportional increase in effluent substrate concentration). They concluded that the floc particles had a reserve potential which they termed "available reaction potential (ARP)". The proposed diffusion-reaction model considers the ARP as the unreactive core of a floc particle, i.e. where substrate has not diffused because of mass transfer limitations. When a shock loading is applied, the substrate concentration gradient increases and substrate diffuses into the core and is reacted. Such factors as oxygen tension, endogenous respiration rate, and floc size affect the ARP of the reactor, as will be shown.

KINETIC MODEL

Steady state diffusion and reaction of a given substrate through a spherical floc matrix containing a homogeneous concentration of microorganisms can be described by the following equation:

$$\begin{array}{c} \text{steady} \\ \text{state} \end{array} \begin{array}{c} \nearrow \\ \frac{\partial C_s}{\partial t} \end{array} + \begin{array}{c} \text{no} \\ \text{convection} \end{array} \begin{array}{c} \nearrow \\ V \nabla C_s \end{array} = \begin{array}{c} \text{Diffusion} \\ D_{sf} \nabla^2 C_s \end{array} - \begin{array}{c} \text{Reaction} \\ R \end{array} \quad (1)$$

In spherical coordinates:

$$D_{sf} \left(\frac{\partial^2 C_s}{\partial r^2} + \frac{\partial C_s}{\partial r} \right) = R = \rho_f k_s$$

The reaction between the microorganisms and substrate in the matrix is assumed to be zero order (Matson, 1972).

The boundary conditions are: 1. C_s finite for all r

$$2. C_s = C_f \text{ at } r = r_f$$

$$\text{The integration yields: } C_f - C_s = \frac{\rho_f k_s}{6 D_{sf}} (r_f^2 - r^2) \quad (2)$$

Solving this equation for r at the point $C_s = 0$ yields:

$$r = \left[r_f^2 - \frac{6 C_f D_{sf}}{\rho_f k_s} \right]^{1/2} \quad (3)$$

The distance substrate penetrates into the floc particle

is $(r_f - r)$, (See Fig. 3). The penetration of oxygen or any substrate into a floc particle can be calculated given the reaction rate constant, the diffusivity (D_{sf}) of the substrate through the floc, and the size and density of the floc particle.

The utility of equation 3 can be illustrated by the following examples. If the substrate penetrates deeper into a floc particle than oxygen, two reactions will occur simultaneously; an aerobic in the outer shell where oxygen is present, an anaerobic reaction in an inner shell where oxygen is absent. Each reaction is governed by its individual rate constant. The overall substrate reaction rate is a weighted sum of these two rates.

$$k_s = k_{sa} \left[1 - \left(\frac{r_o}{r_f} \right)^3 \right] + k_{san} \left(\frac{r_o^3 - r_{san}^3}{r_f^3} \right) \quad (4)$$

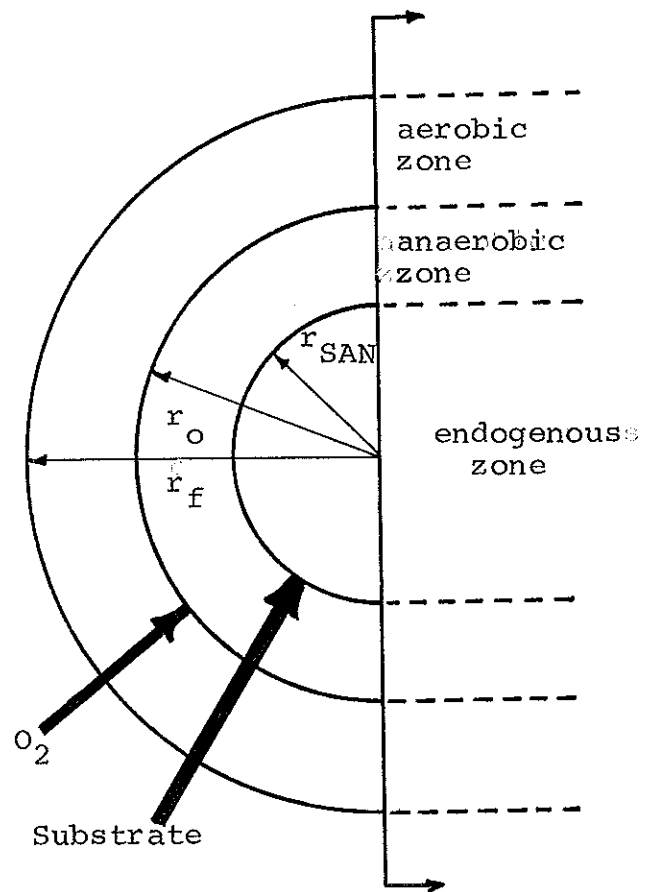
Overall substrate reaction rate constant = Aerobic fraction + Anaerobic fraction

The radii r_o and r_{san} , which are distances to which oxygen and substrate penetrate can be calculated from forms of equation 3 listed in Table I.

The oxygen reaction rate is simply its rate of consumption in the aerobic zone:



Figure 3. Cutaway View of Floc Particle



$$k_o = k_{oa} \left[1 - \left(\frac{r_o}{r_f} \right)^3 \right] \quad (5)$$

where k_{oa} is the oxygen reaction rate constant.

In this example, substrate does not penetrate completely into the floc particle which leaves an inner core containing no substrate, called the endogenous zone. Biological solids are produced in the aerobic and anaerobic zones and are degraded in the endogenous zone. The net rate of solids production is a function of the relative sizes of these zones:

$$k_x = Y_{xa} k_{sa} \left[1 - \left(\frac{r_o}{r_f} \right)^3 \right] + Y_{xan} k_{san} \times \left(\frac{r_o^3 - r_{san}^3}{r_f^3} \right) - k_{xane} \left(\frac{r_{san}}{r_f} \right)^3 \quad (6)$$

Net solids production = aerobic production + anaerobic production - endogenous consumption

The terms Y_{xa} and Y_{xan} are the aerobic and anaerobic yield coefficients respectively. k_{xane} is the rate constant for endogenous solids consumption.

The available reaction potential (ARP) is the ratio of the unused reaction volume (the endogenous zone) to the total reaction volume. For this example it is calculated

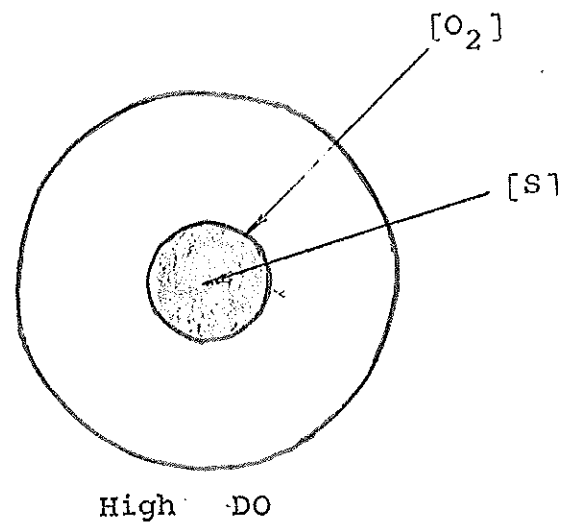
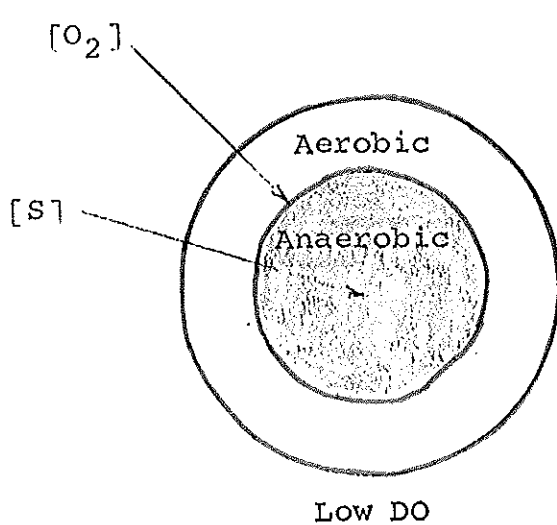
as:

$$\text{ARP} = \frac{k_{\text{san}} \left(\frac{r_{\text{san}}}{r_f} \right)^3}{k_{\text{sa}} \left[1 - \left(\frac{r_o}{r_f} \right)^3 \right] + k_{\text{san}} \left(\frac{r_o}{r_f} \right)^3} \times 100\%$$

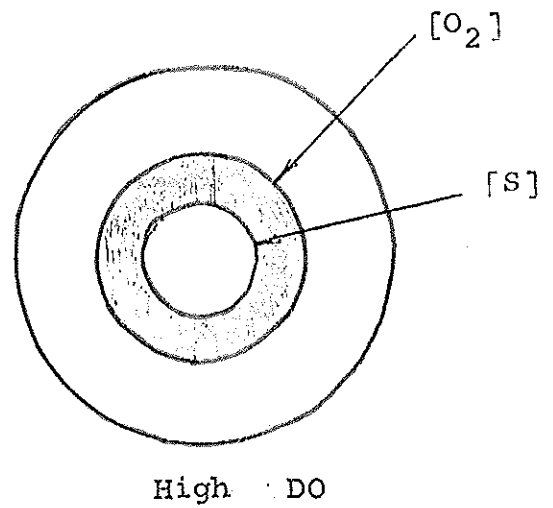
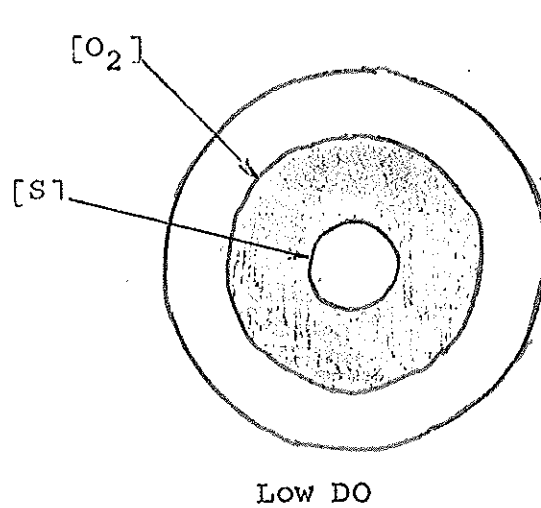
The example illustrates one of several conditions in a biochemical reactor which depend on oxygen tension and substrate concentration in the reactor. See Figure IV for illustration of other conditions that can occur in the floc.

Figure 4. Representation of Aerobic and Anaerobic Zones
In a Spherical Floc Particle

Case A. Floc Saturated with Substrate



Case B. Floc Not Saturated with Substrate



TREATMENT PROCESS MODEL

The model simulates a conventional completely stirred tank reactor (CSTR) with solids recycle. Liquid waste flows into the reactor at a constant rate and is instantaneously mixed with the reactor contents. Influent and effluent flow rates are equal. Solids are produced and wasted at the same rate, the reactor solids concentration remaining constant.

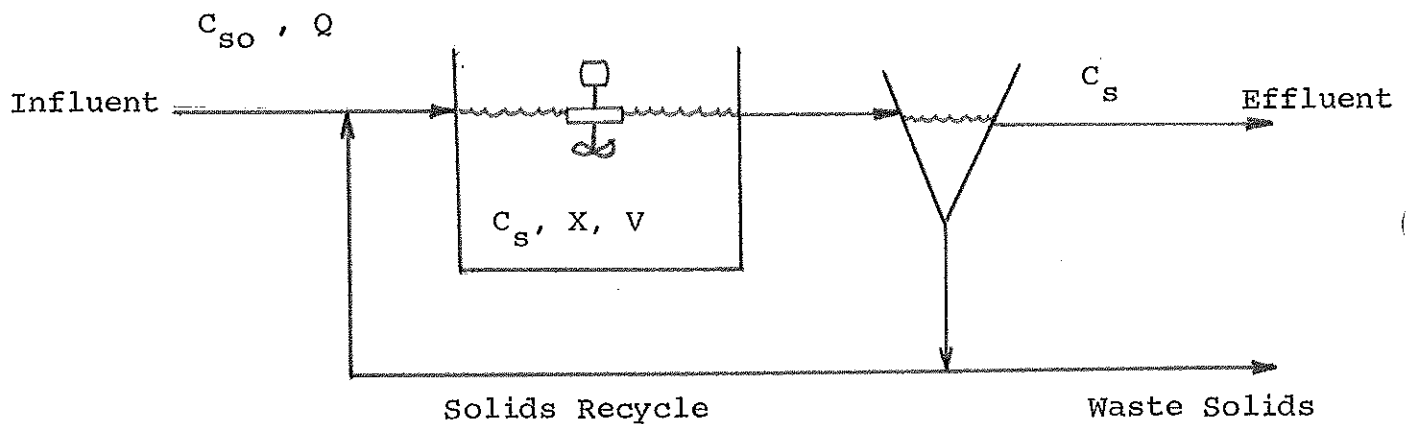


Figure 5. Schematic Representation of Continuously Stirred Tank Reactor with Recycle Waste Treatment Process

The materials balance for the net rate of change of substrate in the reactor is:

$$\begin{array}{ccccc}
 Q C_{s0} & - & Q C_s & = & V_f k_o \\
 \text{mass substrate} & & \text{mass substrate} & & \text{mass substrate} \\
 \text{in} & & \text{out} & & \text{reacted}
 \end{array}$$

$$\text{Rearranging: } C_s = \frac{V_f}{Q} k_o - C_{s0}$$

The substrate reaction rate, k_o , is given in equation 14 or 15 (Table I), depending on bulk concentrations of substrate and oxygen and the related reaction zones in the floc particles. In any case, k_o will be expressed in terms of C_{s0} , (solution obtained by numerical methods). A computer program was developed to illustrate a variety of reactor conditions commonly encountered in practice, based on this model.

Rate and physical constants for the model are listed in Table II. The constants were not determined from any one experimental test, but represent results obtained from previous work at Rice University. The constants are consistent from stoichiometric considerations using glucose as a substrate in an activated sludge reactor.

RESULTS

Kinetics

A comparison between the various kinetic expressions and the diffusion-reaction model is shown in Fig. 1. For the diffusion reaction model, an increase in substrate concentration in the reactor raises the concentration gradient driving force across the floc exposing more reactive material. The result is a nearly proportional increase in overall reaction rate per unit increase in reactor substrate concentration, i.e. apparent first order kinetics even though the biochemical reaction is zero order. This response continues with increasing substrate concentration until the floc is saturated. Further increases in substrate concentration have no effect on overall reaction rate, i.e. apparent zero order kinetics.

Oxygen Concentration

The model indicates that the oxygen tension can affect the overall reaction rate only, when oxygen partially penetrates floc which is saturated with substrate. Within the floc particle an outer aerobic reaction zone and an inner reaction zone will exist. An increase in oxygen tension will increase oxygen penetration through the floc, expanding the more reactive aerobic zone at the expense

of the anaerobic zone. This will continue until the oxygen saturates the floc. In most practical situations, substrate does not saturate the floc. In such cases, increases in oxygen tension will not affect reaction rate, but can increase the floc ARP. For example, when substrate penetrates the floc more deeply than oxygen, aerobic, anaerobic, and anaerobic endogenous zones are formed. Increasing the bulk oxygen tension will expand the aerobic and endogenous zones and decrease the anaerobic volume. The endogenous zone volume is a measure of the floc ARP, which is increased. As a result of increasing oxygen tension, the ability of the reactor to respond to transient loadings is enhanced. Figure 6 indicates the possible effects of oxygen tension on ARP. In laboratory systems, with high shear mixing, floc sizes are small and O_2 concentration is high. No changes in reaction rate with increasing oxygen concentration are observed.

Transient Loadings

Fluctuations in substrate concentration can markedly alter the diffusion-reaction regime in floc. If the floc has a sufficient endogenous zone (ARP), it may be able to adsorb and react the excess substrate. If the ARP is low or the transient peak high, the additional substrate may be more than sufficient to saturate the floc as reflected by higher effluent substrate concentration.

Thus, it is important to be able to quantify the size and capacity of the endogenous zone. The endogenous zone may be either aerobic or anaerobic depending on the relative penetrations of oxygen and substrate. An aerobic endogenous zone has a higher ARP than an anaerobic one. It would be desirable to have an oxygen tension in the reactor sufficient to saturate the floc at all times because an aerobic endogenous zone has a higher ARP than an anaerobic endogenous zone. Figure 7 illustrates the effect of a transient load on an aerobic endogenous zone.

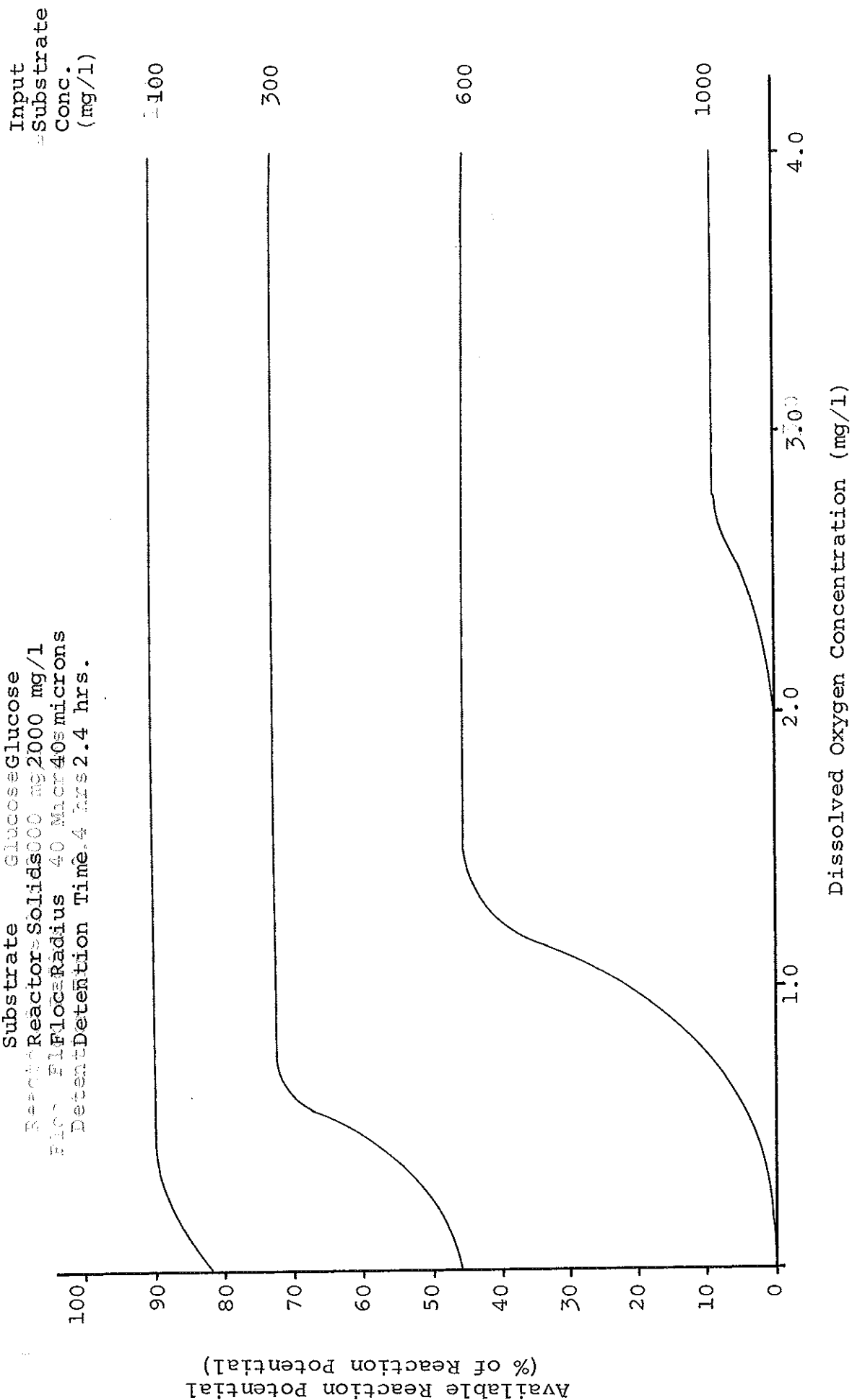
Floc Characteristics

Floc size is a measure of surface area, and is primarily a function of the turbulence level in the reactor. Figure 8 indicates that floc size can have significant effect on effluent concentration and reactor efficiency. Floc surface area per unit mass increases with decreasing floc radius, thereby increasing reaction rate which lowers effluent substrate concentration. Total biological solids concentration influences effluent substrate concentration by providing reactive capacity. An increase in solids level will stimulate a higher overall reaction rate (Fig. 9). A compromise can be developed between floc size and solids concentration in the reactor. A low concentration of small floc could provide the same surface area as a high concentration of large floc, and at low

Figure 6. ARP as a Function of Reactor Dissolved Oxygen Concentration

Reactor Conditions

Substrate Glucose
 Reactor Solids 2000 mg/l
 Flocc Radius 40 Microns
 Detention Time 2.4 hrs.



1. The first part of the document is a letter from the President of the United States to the Congress, dated January 1, 1861. It is a very important document, as it sets out the President's policy for the new year. The President states that he is pleased to see the Congress assembled, and that he is confident that the country is in a good position to meet the challenges of the future. He also mentions the recent election of Abraham Lincoln as President, and expresses his confidence in the new administration.

Figure 7. Response to Step Changes in Substrate Loadings
at Constant Solids Concentration

Reactor Conditions

Substrate	Glucose
Dissolved Oxygen	30mg/l
Floc Radius	40 microns
Detention Time	2.4 hrs.

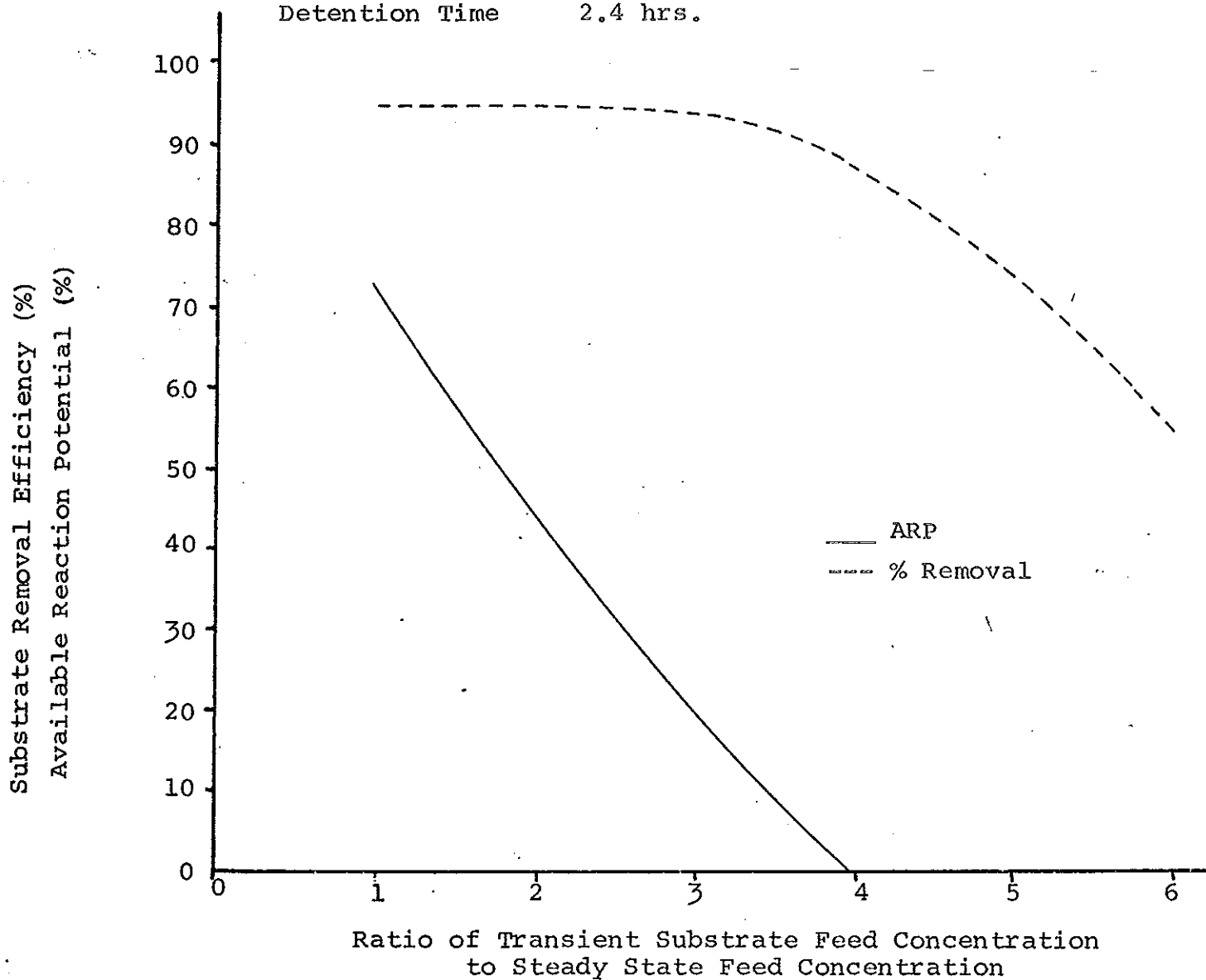
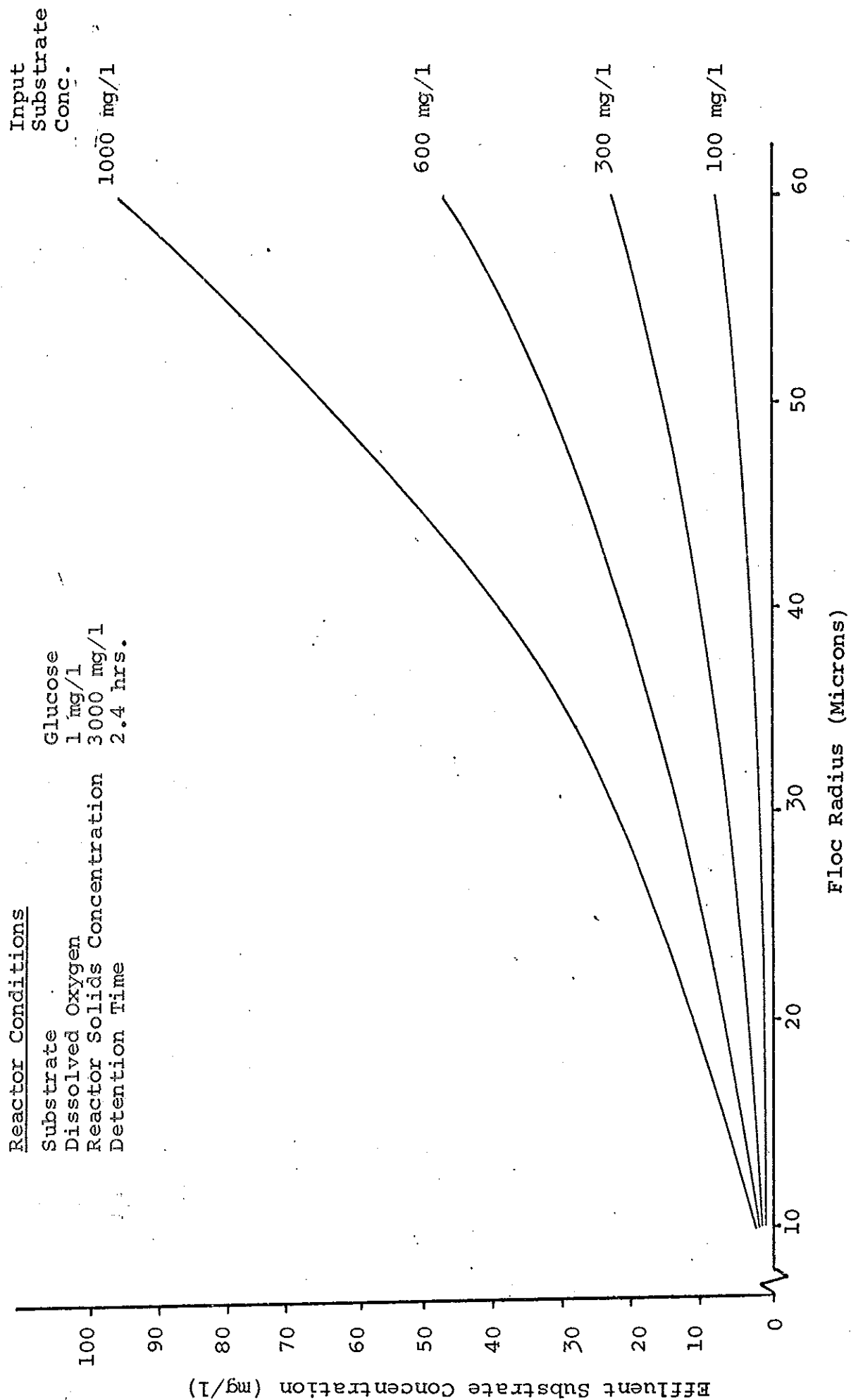


Figure 8. Effluent Substrate Concentrations as a Function of Biological Floc Size at Constant Solids Level



Figure 9. Effluent Substrate Concentration as a Function of Biological Solids Concentration



substrate concentrations the total participating reaction volume would be the same for either case. However, the large floc would have a high ARP in comparison to the small floc and could respond better to shock loads.

Internal characteristics such as structural makeup of the floc are also important in the diffusion-reaction process. For example, the variety and concentration of microorganisms in the floc matrix is an important parameter that enters into the model through the evaluation of the substrate and oxygen reaction rate constants. The macromolecular constituents of the floc matrix play a major role in the determination of the various diffusivity coefficients in the model.

The diffusion-reaction model attempts to rationalize the traditional concepts of reactor design in terms that have a sound theoretical basis. For instance the Food/Microorganism (F/M) ratio is an indirect measure of the ARP of a system. A low F/M reactor will generally have a high ARP, and vice versa. With a low substrate level in the reactor, it is likely that substrate does not diffuse or penetrate far into the floc particles; thus the ARP is high. Sludge age can be related to the various reaction rate constants. A longer sludge age usually means a lower viable cell concentration, thus the reaction rates will be lower. Sludge age will affect the diffusivities of the various

substrates through the floc matrix. Older sludges can excrete larger amounts of extracellular material, thus altering the diffusivity relationships.

Sludge or biomass concentration has traditionally been measured in terms of mixed liquor volatile suspended solids (MLVSS). The quality of the sludge, or active biomass, is not considered. However MLVSS indirectly indicates the reactive capacity of the system. Determination of the rate constants and physical parameters of the solids further quantifies these complex relationships in the reactor.

SUMMARY

A mathematical model has been developed to further understand the physical and biochemical complexities of biological reactor design. The model describes the internal structure of cellular flocs and how it affects diffusion and reaction of soluble substrates. Varying rates of diffusion and reaction rates within the floc account for available reaction potential observed in actual systems subjected to transient loadings.

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Table II. Various Physical Constants Used in Examples.

ρ_f	=	.2 gm/ml
k_{san}	=	6 mg glu/gm floc-sec
k_{oa}	=	6 gm O_2 /gm floc-sec
k_{sa}	=	12 gm glu/gm floc-sec
D_{of}	=	5×10^{-6} cm ² /sec
D_{sf}	=	4×10^{-7} cm ² /sec
k_{oe}	=	12 mg O_2 /gm floc-sec
k_{xae}	=	.7 day ⁻¹
k_{xane}	=	.35 day ⁻¹
Y_{xa}	=	0.4 gm X/gm glu
Y_{xane}	=	0.2 gm X/gm glu

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Table I. Summary of Formulae for Spherical Floc Particle

A. Floc Particle with aerobic and anaerobic reaction zones, anaerobic endogenous zone

$$r_o = \left[r_f^2 - \frac{6 C_o D_{of}}{\rho_f k_{oa}} \right]^{\frac{1}{2}} ; \quad k_o = k_{oa} \left[1 - \left(\frac{r_o}{r_f} \right)^3 \right] ; \quad r_{san} = \left[r_o^2 - \frac{6 C_{sa} D_{sf}}{\rho_f k_{san}} \right] ;$$

$$k_s = k_{sa} \left[1 - \left(\frac{r_o}{r_f} \right)^3 \right] + k_{san} \left(\frac{r_o^3 - r_{san}^3}{r_f^3} \right) ;$$

$$C_{sa} = C_s - \frac{\rho_f k_{sa}}{6 D_{sf}} (r_f^2 - r_o^2) ;$$

$$k_x = Y_{xa} k_{sa} \left[1 - \left(\frac{r_o}{r_f} \right)^3 \right] + Y_{xan} k_{san} \left(\frac{r_o^3 - r_{san}^3}{r_f^3} \right) - k_{xane} \left(\frac{r_{san}}{r_f} \right)^3 ; \quad ARP = k_{san} \left(\frac{r_{san}}{r_f} \right)^3 \times 100\% ;$$

$$k_{sa} \left[1 - \left(\frac{r_o}{r_f} \right)^3 \right] + k_{san} \left(\frac{r_o}{r_f} \right)^3$$

B. Floc particle with aerobic reaction zone; aerobic endogenous zone.

$$r_s = \left[r_f^2 - \frac{6 C_s D_{sf}}{\rho_f k_{sa}} \right]^{\frac{1}{2}} ; \quad k_o = k_{oa} \left[1 - \left(\frac{r_s}{r_f} \right)^3 \right] + k_{oe} \left(\frac{r_s}{r_f} \right)^3 ; \quad ARP = \left(\frac{r_s}{r_f} \right)^3 = 100\% ;$$

$$k_s = k_{sa} \left[1 - \left(\frac{r_s}{r_f} \right)^3 \right] ; \quad k_x = Y_{xa} k_{sa} \left[1 - \left(\frac{r_s}{r_f} \right)^3 \right] + k_{xae} \left(\frac{r_s}{r_f} \right)^3 ;$$

C. Floc particle with aerobic reaction zone, partially aerobic and partially anaerobic endogenous zone.

$$r_{oe} = \left[r_s^2 - \frac{6 C_{oa} D_{sf}}{\rho_f k_{oe}} \right]^{\frac{1}{2}} ; \quad k_o = k_{oa} \left[1 - \left(\frac{r_s}{r_f} \right)^3 \right] + k_{oe} \left[\frac{r_s^3 - r_{oe}^3}{r_f^3} \right] ;$$

$$C_{oa} = C_o - \frac{\rho_f k_{oe}}{6 D_{of}} (r_f^2 - r_s^2) ; \quad k_x = Y_{xa} k_{sa} \left[1 - \left(\frac{r_s}{r_f} \right)^3 \right] - k_{xae} \left(\frac{r_s^3 - r_{oe}^3}{r_f^3} \right) - k_{xane} \left(\frac{r_{oe}}{r_f} \right)^3 ;$$

$$ARP = k_{sa} \left[\frac{r_o^3 - r_{oe}^3}{r_f^3} \right] + k_{san} \left(\frac{r_{oe}}{r_f} \right)^3 \times 100\%$$

$$\frac{k_{sa} \left[1 - \left(\frac{r_{oe}}{r_f} \right)^3 \right] + k_{san} \left(\frac{r_{oe}}{r_f} \right)^3}{k_{sa} \left[1 - \left(\frac{r_{oe}}{r_f} \right)^3 \right] + k_{san} \left(\frac{r_{oe}}{r_f} \right)^3}$$

LIST OF SYMBOLS

C_{oa}	oxygen concentration at some r in floc
C_o	bulk oxygen concentration
C_s	bulk substrate concentration in reactor
C_{sa}	substrate concentration at r_o
C_{so}	influent substrate concentration
D_{of}	oxygen diffusivity in floc
D_{sf}	substrate diffusivity in floc
k_{oa}	oxygen reaction rate constant in substrate reaction zone
k_{sa}	aerobic substrate reaction rate constant
k_{san}	anaerobic substrate reaction rate constant
k_s	overall substrate reaction rate constant
k_o	overall oxygen reaction rate constant
k_{oe}	oxygen reaction rate constant for endogenous zone
k_x	overall sludge generation rate constant
k_{xae}	aerobic solids endogenous decay rate constant
k_{xane}	anaerobic solids endogenous decay rate constant
K	arbitrary constant
r_f	floc radius
r_o	distance to which oxygen penetrates the floc
r_s	distance to which substrate penetrates the floc under aerobic conditions

r_{san}	distance to which substrate penetrates the floc under anaerobic conditions
r_{oe}	distance to which oxygen penetrates endogenous zone
R	overall reaction rate
R_{max}	maximum reaction rate
Y_{xa}	aerobic sludge yield coefficient
Y_{xan}	anaerobic sludge yield coefficient
X	biological solids concentration
θ_x	solids detention time
V	reactor volume
τ	reactor detention time
ρ_f	solids density
V_f	floc volume

