



Slag run-off in an MHD air preheater
by Wade Richard Clowes

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
in Mechanical Engineering
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Abstract:

A model was developed for slag layer growth due to slag deposition and motion on a tube wall in a cored brick, regenerative air preheater as proposed for use in open magnetohydrodynamic (MHD) cycles. The 19 mm x 6.10 m circular tube had wall temperatures which varied with axial position and time, and ranged between 1850 K and 1000 K. The slag flux to the wall was calculated using three slag deposition functions based on an initial slag weight concentration of one percent. Two moderate slag deposition flux functions were calculated using the model developed by Sande while a third deposition function represented an upper limit on the amount of slag deposited. The three deposition flux functions were used in conjunction with three coal slag compositions resulting in seven test cases. The slag compositions with the lowest viscosity resulted in a larger maximum slag thickness than did that of a more viscous slag. Using a moderate deposition flux, a maximum cross-sectional area reduction of 9.3 percent occurs for the least viscous slag composition after ten hours of simulated operation.

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NOMENCLATURE

Symbol	Description
C	Preheater tube circumference
c	Slag specific heat
dA	Differential area
g	Local acceleration due to gravity
k	Slag thermal conductivity
M	Slag mass contained in an element
m	Slag mass entering or leaving element
$\dot{m}_d$	Slag mass flux due to deposition
P	Absolute pressure
R	Tube radius
r	Radial position
T	Slag absolute temperature
t	Time
u	Slag axial velocity component
$\dot{V}$	Volumetric flow rate
$\dot{V}$	Elemental volume
$\bar{V}$	Radially averaged axial slag velocity
v	Slag radial velocity component
y	Coordinate starting at the tube wall and progressing radially inward
z	Axial position originating at the tube entrance

NOMENCLATURE (continued)

Symbol	Description
Δr	Radial step
Δt	Time step
Δz	Axial step
δ	Slag layer thickness
ρ	Slag density
σ	Normal stress
μ	Dynamic (or absolute) viscosity
τ	Shear stress
Superscript n	Denotes time step
Subscript i	Denotes radial step
Subscript j	Denotes axial step
Subscript g	Denotes properties of, or effects of, gas flow
Subscript r	Denotes radial direction
Subscript z	Denotes axial direction
Subscript rz	Used with a differential element to denote a force acting on the radial face in the axial direction.

ABSTRACT

A model was developed for slag layer growth due to slag deposition and motion on a tube wall in a cored brick, regenerative air preheater as proposed for use in open magnetohydrodynamic (MHD) cycles. The 19 mm x 6.10 m circular tube had wall temperatures which varied with axial position and time, and ranged between 1850 K and 1000 K. The slag flux to the wall was calculated using three slag deposition functions based on an initial slag weight concentration of one percent. Two moderate slag deposition flux functions were calculated using the model developed by Sande while a third deposition function represented an upper limit on the amount of slag deposited. The three deposition flux functions were used in conjunction with three coal slag compositions resulting in seven test cases. The slag compositions with the lowest viscosity resulted in a larger maximum slag thickness than did that of a more viscous slag. Using a moderate deposition flux, a maximum cross-sectional area reduction of 9.3 percent occurs for the least viscous slag composition after ten hours of simulated operation.

INTRODUCTION

Currently proposed open-cycle magnetohydrodynamic (MHD) cycles require combustion gases to reach a temperature of 3000 K. This temperature cannot be achieved by burning coal in ambient temperature air and it therefore becomes necessary to preheat air prior to combustion. One method of preheating air is through use of a regenerative heat exchanger, using cored ceramic bricks as the heat storage and transfer medium. Hot combustion gas from the MHD channel is used to heat the cored bricks which are in turn used to preheat pre-combustion air. Since it is impossible to remove all particulate matter from the combustion products, the deposition and resulting flow of molten coal slag and MHD seed material along the heat exchanger wall becomes a factor of significance in the performance and life of the air preheater. The solidification point of the coal slag mixture is approximately 1420 K, which lies between the entrance and exit preheater wall temperatures, indicating that the slag layer will build up along the preheater wall and cause a flow restriction. Determination of the slag layer thickness as a function of position on the wall and time is of primary importance in an estimation of the life of the preheater. The objective of this work is to mathematically model the slag layer behavior with time, and to determine the region of maximum slag layer thickness.

II. LITERATURE REVIEW

To acceptably model slag layer growth, it is necessary to represent slag particle deposition as accurately as possible. Numerous experimental and analytical studies have been made in an attempt to predict slag deposition. Sande [1] developed a model for particulate deposition in turbulent flow, as have Davies [2], and Friedlander and Johnstone [3]. Ondo [4] presented a review of small particle deposition theory which compares the deposition models of Davies, Beal, Stickler and De Saro, and others. Ondo also investigated the correlation of the deposition models with the experimental results of Friedlander and Johnstone, Liu and Azarwall, and others, and showed two orders of magnitude variation in the experimental results.

Development of equations similar to those governing the flow of coal slag can be found in Schlichting [5] and in studies of condensation run off and transient freezing. Denny, Mills, and Jusionis [6] analytically investigate the film condensation on a vertical surface and the associated heat transfer. A finite difference technique predicting the thickness of a solid phase deposited by a flowing fluid is presented by Beauboef and Chapman [7].

Slag layer build up was experimentally determined for magnetohydrodynamics (MHD) generator duct conditions by Koester, Rodgers, and Eutis [8] who observed the formation of transverse ripples.

Air preheater design for open MHD cycles was investigated by Upshaw [9] who determined thermal characteristics of cored brick regenerative heat exchangers for various geometries and flow conditions.

A review of the literature, especially that presented by Ondo, shows a two-order-of-magnitude range in experimental data for particle deposition in turbulent flow while analytical models exhibit a four-order-of-magnitude variation. The large disparity in both experimental and analytical results for particle deposition indicates that there is a large degree of uncertainty associated with the use of any of the deposition models.

III. THEORY

The current MHD air preheater design consists of cored hexagonal ceramic bricks, stacked vertically such that the array of circular flow passages are aligned to form a system of tubular flow channels. The model developed herein describes slag layer growth in one tube of the system of tubular flow channels which compose the preheater.

In the model development, coal slag flow is taken as axisymmetric. Also, the effect of wall roughness on slag flow and the effect of changes in wall roughness on the slag deposition rate are not considered. The model developed is a first effort at estimating the slag layer behavior.

The slag layer thickness and change with time will ultimately be obtained from a form of the mass continuity equation. However, in the continuity equation there are slag velocity terms which are temperature dependent. This interdependence indicates a need for the general development of the combined momentum, energy, and continuity equations in order to obtain the slag growth rate.

Application in the axial direction of the principle of conservation of momentum for constant density slag flow yields the following equation (see Appendix 1 for details)

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} \right) = - \frac{\partial p}{\partial z} + \rho g + \mu \left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) + \frac{\partial \mu}{\partial r} \cdot \left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r} \right) + 2 \frac{\partial \mu}{\partial z} \frac{\partial u}{\partial z} \quad (1)$$

where u = slag velocity component in axial direction
 v = slag velocity component in radial direction
 ρ = slag density (constant)
 μ = dynamic (or absolute) viscosity
 g = local acceleration due to gravity
 r = radial position
 z = axial position

For constant viscosity, equation (1) reduces to the Navier Stokes equation in the z -direction for axisymmetric flow. However, the viscosity of coal slag is a strong function of temperature; therefore, the principle of the conservation of energy must be applied to determine the temperature variation:

An energy balance for constant density, axisymmetric slag flow in a tube, with negligible shear work yields the following equation (see Appendix 2 for details)

$$\begin{aligned} \frac{\partial}{\partial z} (u\rho cT) + \frac{1}{r} \frac{\partial}{\partial r} (rv\rho cT) - \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) \\ - \frac{1}{r} \frac{\partial}{\partial r} \left(r k \frac{\partial T}{\partial r} \right) + \rho c \frac{\partial T}{\partial t} = 0 \end{aligned} \quad (2)$$

where T = slag temperature
 c = slag specific heat
 k = slag thermal conductivity

Equation (2) can be further reduced by inspection of the order of magnitude of the radial and axial velocities, v and u , for laminar flow in a tube (see Appendix 2 for details).

$$\begin{aligned} \frac{\partial T}{\partial z} \left(\rho c u - \frac{\partial k}{\partial z} \right) - \frac{\partial T}{\partial r} \left(\frac{k}{r} + \frac{\partial k}{\partial r} \right) - k \frac{\partial^2 T}{\partial r^2} - k \frac{\partial^2 T}{\partial z^2} \\ + T \rho c \frac{\partial c}{\partial z} + \rho c \frac{\partial T}{\partial z} = 0 \end{aligned} \quad (3)$$

Equations (1) and (3) are coupled nonlinear partial differential equations. Application of the principle of mass continuity to a flow system of constant density results in

$$\rho \frac{\partial \dot{V}}{\partial t} = \rho \dot{V}_{in} - \rho \dot{V}_{out}$$

where $\dot{V}$ = elemental volume

$\dot{V}$ = volumetric flow rate

An expression for slag continuity in terms of slag thickness can easily be found by applying the principle of mass continuity to an element which has been integrated radially. For small radial velocity, the mass continuity equation becomes

$$\frac{\partial \delta}{\partial t} = \frac{-1}{2(R-\delta)} \left[(2R-\delta) \frac{\partial (\delta \bar{u})}{\partial z} - \bar{u} \delta \frac{\partial \delta}{\partial z} \right] + \frac{\dot{m}_d}{\rho} \quad (4)$$

where R = tube radius
 δ = slag layer thickness
 $\dot{m}_d$ = mass flux due to deposition
 $\bar{u}$ = radially averaged slag velocity

Equations (1), (3), and (4) form a set of three coupled equations relating the three variables u , T , and δ . Since equation (4) can easily be derived in finite difference form (see Appendix 4 for a similar derivation), the primary concern at this point is reducing equations (1) and (3) to an appropriate numerical form and investigating the resulting stability criteria.

Equations (1) and (3) can be formulated using forward and central differences. However, when any one scheme, forward or central finite difference, is used individually the result is a set of equations each of which having at least one negative coefficient. This usually means that the algorithm obtained is unconditionally unstable. The stability criterion used here is one generally accepted for finite difference schemes, and is based on an argument utilizing the second law of thermodynamics. There are, however, several methods which can have negative coefficients and remain stable while satisfying the second law of thermodynamics. These methods will be discussed later.

A combination of forward and central differences will yield a relation in which coefficients can be made positive and, hence, can be

used to determine step sizes necessary for stability (see Appendix 3 for development). Using this combination of finite difference methods, and assuming a stable slag layer thickness, the energy equation (equation 3) yields a maximum axial step size of 50 microns while the momentum equation (equation 1) limits the time step to 10 microseconds. When results from the model were used, a time step limit of .003 seconds and an axial step limit of 1 cm were obtained. When lengths of meters and times of hundreds of hours are to be used, it is obvious that the stability restrictions imposed are too severe.

In view of the difficulties encountered in the forward, central, and combination central-forward finite difference schemes, the unconditionally stable explicit methods appeared to hold the most promise for workable reformulations of equations (1) and (3). Stability for these methods is not limited by time or spatial step sizes.

The first unconditionally stable explicit method considered was the alternating direction explicit procedure (ADEP) which was first conceived by Saul'yev [10] and later expanded by Larkin [11], and Allada and Quon [12]. The ADEP method is much faster than implicit methods while being at least as accurate. Unfortunately, the ADEP method requires all boundary conditions to be fixed. The problem considered in this work has fixed boundary conditions at the inlet only. This would necessitate repeated applications of the reversed direction portion of the ADEP scheme in order to arrive at the known inlet conditions. The

difficulty associated with the unfixed boundary condition removed the ADEP method from further consideration.

Dufort and Frankel [13] present an explicit method which is unconditionally stable for the linear differential equations they considered. However, the method developed by DuFort and Frankel has no uniform definition or proof of stability for general parabolic differential equations. The method also has no general expression limiting the step sizes which will guarantee acceptable accuracy. Since neither stability nor accuracy criteria exist in general, extension of the DuFort-Frankel method to a system of coupled nonlinear partial differential equations, such as equations (1), (3), and (4), would produce questionable results. Therefore, the DuFort-Frankel method was not considered further.

When considering implicit methods for solution of partial differential equations, it is generally known that step sizes must be small relative to the total length or time span to achieve acceptable accuracy. This implies that the coupled equations (1, 3, and 4), when subjected to an implicit solution scheme, would result in very large coefficient matrices. The difficulties associated with computer storage limitations, computing time, and round off errors were considered prohibitive for the implicit methods.

Since there appeared to be no workable solution to the problem as it was formulated, another limitation was imposed which allowed completion of the slag layer growth model.

If the slag layer thickness is considered small relative to the tube diameter, several simplifications in the formulation occur. For a thin slag layer, the radial temperature variation is small. With a radial heat transfer rate of 8000 w/m^2 , as calculated by Upshaw [9], a one millimeter thickness of coal slag experiences a radial temperature drop of approximately five degrees Kelvin. Therefore, the temperature may be approximated as radially constant and equal to the tube wall surface temperature. The ceramic wall temperature as a function of axial position and time can be calculated using the model developed by Upshaw [9].

With the axial and radial temperatures known, the energy equation is not needed. Viscosity, since it is a function of temperature only, can be taken as constant radially and as a function of temperature axially. Since viscosity can be taken as constant at any one axial point, the momentum equation, when applied to one axial point, simplifies to an expression for the radial velocity profile. The thin slag layer limitation also allows use of rectangular coordinates in place of cylindrical coordinates, with no significant loss of accuracy.

Using these simplifications, the momentum equation for a thin slag layer at any axial point, j , becomes (see Appendix 4 for details)

$$u(y) = \frac{-\rho g}{2\mu_j} y^2 + \frac{(\tau_{g,j} + \rho g \delta_j)}{\mu_j} y \quad (5)$$

where τ_g = surface shear due to gas flow
 δ = slag layer thickness
 y = dimension starting at tube wall and progressing radially inward

The conservation of mass equation, equation (4), may be rewritten in finite difference form for the thin slag layer as (see Appendix 4 for development)

$$\delta_{j+1}^{n+1} = \delta_{j+1}^n + \frac{\Delta t}{\rho} (\dot{m}_d)_{j+1}^n + \frac{\Delta t}{\Delta z} (\bar{v}_j^n \delta_j^n - \bar{v}_{j+1}^n \delta_{j+1}^n) \quad (6)$$

where
$$\bar{v}_j^n = \frac{1}{\delta_j^n} \int_0^{\delta_j^n} u(y)_j^n dy$$

The thin slag layer limitation restricts the extent to which the slag growth model may be applied. When the slag layer reaches a thickness at which it begins to influence gas flow characteristics, or when an appreciable radial temperature gradient exists, the model is no longer applicable. The slag deposition model of Sande [1] and the gas and ceramic temperature model of Upshaw [9] are both based on no change in flow conditions due to slag build up. This indicates that these models are also not applicable when the slag layer is no longer thin. The model will give acceptable results for a thin slag layer which should indicate areas of maximum slag growth rate, and allow an approximation of the total slag layer thickness.

IV. RESULTS

The model developed for slag thickness and growth rate was applied to seven test cases. In all cases, the tube radius was 19.0 mm and the tube length (or bed length) was 6.10 m. A digital computer was used for all calculations and for plotting results.

Values for tube wall temperature, surface shear due to gas flow, and mass deposition flux needed in the slag thickness model were calculated using the models developed by Upshaw [9] and Sande [1]; the least squares method of curve fitting was used to obtain functional forms representing the calculated values. Liquid slag viscosity data for Montana Rosebud coal was obtained from Webster Capps [14] and was fitted to Rollands viscosity equation [15] using the method of least squares (see Figure 1).

In this work, the phrase "reheat cycle" is used to denote the heat up phase in the air preheater, while a "blowdown cycle" is the phase in which precombustion air is heated. Combined, a reheat cycle and blowdown cycle form one total cycle (or simply one cycle). Therefore, when the terms cycle or cyclic are used without being preceded by reheat or blowdown, total cycles are referred to.

Six of the seven test cases result from combinations of three coal compositions and two slag deposition flux functions. The seventh case results from a slag deposition flux function derived using the highest deposition rate measured by Liu and Azarwall [16], and represents an upper limit on the amount of slag deposited.

The three slag compositions are given in Table 1 with the effects of composition variation on viscosity shown in Figure 1. A composition number was assigned to each slag composition for simplicity. The composition which yielded the most viscous coal slag (NBS melt K-875) was assigned composition number 1, while the least viscous (NBS melt K-795) was assigned composition number 3. The remaining slag composition (NBS melt K-603) was assigned a composition number of 2.

The coal slag deposition flux functions were obtained from Sande's model by variation of the surface roughness. To avoid excessive thermal stress, it is desirable to have a maximum entrance ceramic temperature variation between reheat and blowdown of less than 55 K. To meet this criterion while varying surface roughness, it was necessary to vary the reheat and blowdown cycle times (hence the total cycle time)...

For simplicity, case numbers were assigned to the deposition flux functions. Case number 1 corresponds to the deposition flux function derived from Sande's model using a surface roughness of 254 μm and reheat and blowdown cycle times of 330 seconds. Case number 2 corresponds to the deposition flux function derived from Sande's model using a surface roughness of 508 μm and reheat and blowdown cycle times of 350 seconds. Case number 3 was assigned to the deposition flux function based on the measurements of Liu and Azarwall [16], Table 2 lists the complete flow conditions and parameters for the three cases.

The two deposition flux functions, derived from Sande's model, varied in the total amount of slag deposited on the tube wall, as well as the rate at which the deposition flux changed with axial position. The slag deposition flux function corresponding to case number 2 delivered 11.5 percent more slag to the tube than did the deposition function represented by case number 1. The slag deposition function corresponding to case number 2 also deposited slag at a much higher rate near the tube entrance than did the first slag deposition function. The difference in the rate at which slag was deposited decreased as the distance from the entrance increased.

The slag deposition flux function corresponding to case number 3 represents an upper limit to slag deposition. The deposition flux function was derived using the largest of a group of deposition rate measurements which varied by two orders of magnitude. The total slag mass deposited on the tube walls using the third deposition flux function was larger than that corresponding to the second deposition function by a factor of 4.3. The large deposition flux of the third deposition function produced such a rapid decrease in the slag particle concentration of the gas that only the first 30 percent of the tube length received significant deposition. The net effect of the third deposition flux function was that essentially all of the slag deposited was deposited in the high temperature region of the preheater tube. The results using case number 3 should be considered as an absolute upper limit for

deposition rate and probably even unreasonably high.

Results of the seven test cases are shown in Figures 2 through 12. All plots were constructed from data taken at a fixed time during a cycle, namely, the end of blowdown. The plots of slag thickness (Figure 2 through 8) therefore, represent one instant of the total cycle. The smooth curves of Figures 9 through 12 are not exact plots of growth rate and thickness variation with time, but are intended, instead, to aid visualization of the general slag flow tendencies. Since the process is a cyclic one, plots of the actual local thickness and local growth rates versus time would have, in place of the smooth lines of Figures 9 through 12, lines with small sinusoidal fluctuations.

Figures 2 through 8 show that the point of maximum slag thickness, as predicted by the model, first occurred near the top of the tube where slag viscosity is lowest and where slag deposition flux functions produce the highest deposition rates. As time progressed, the axial position of the maximum slag thickness moved down the tube.

The effect of the moving point of maximum thickness is readily visible in the local growth rate and thickness plots (Figures 9 through 12). In the local slag thickness plots (Figures 9 and 10) the motion of the point of maximum thickness through the axial position of the plot is visible as the near step change undergone by the thickness. In the local growth rate plots, the motion of the point of maximum thickness through the axial position of the plot is visible as a spike.

After the point of maximum thickness passed, an axial location is shown by the thickness plots (Figures 2 through 10) to be in a state of cyclic equilibrium; that is, the thickness at any one cyclic time did not vary. The cyclic equilibrium thickness can also be seen as equivalent to the maximum thickness experienced by that particular axial point for the specific cycle time involved. Since the thickness is cyclically stable, the growth rate, when integrated over one total cycle, must equal zero. The plots of thickness growth rate (Figures 11 and 12) are for one specific cycle time and do not represent an average value. Consequently the growth rate after the spike is not zero on the growth rate plots, but instead takes the form of a small constant; this shows that the growth rate also reached cyclic equilibrium, which it must have done to insure a cyclically stable thickness.

The slag deposition flux functions from Sande's model contained a provision which terminated slag deposition when the wall temperature dropped below 1090 K. The effect of the termination of slag deposition is visible as a tapering of slag thickness which occurs near the tube's exit in the slag thickness profile plots (Figures 2 through 8).

Inspection of the slag thickness profile and local slag thickness plots (Figures 2 through 10) for varying coal composition number shows the effect of slag composition variations. As might be expected, the more viscous slag found its point of maximum slag thickness at an axial position nearer the tube entrance than did the less viscous coal slags.

Another result of viscosity variation predicted by the thickness model was that the lower viscosity coal slag developed a larger maximum slag thickness than did the more viscous coal slags.

The effect of slag viscosity variations on the slag growth rate for one axial position is observable in Figures 11 and 12. Again as expected, the maximum growth rate for any one axial position occurred earlier in time for the low viscosity coal slag. The magnitude of the local slag growth rate was the largest for the intermediate viscosity coal slag (composition number 2): The smallest maximum growth rate occurred with the most viscous coal slag for case number 1 and occurred with the least viscous slag for case number 2.

As Figures 2 through 10 show for any specific coal composition, when the deposition function corresponding to case number 2 was used in the thickness model, the result was a larger maximum slag thickness than that which was produced using the first deposition flux function. The second deposition function also resulted in the location of the point of maximum thickness being further down the tube. No consistent change is discernable for variation of local slag thickness growth rate for change in the slag deposition flux functions.

The results for slag thickness and local growth rate predicted by the model can be explained by inspection of the effects of axial viscosity variations and the cyclic nature of slag deposition. Near the tube entrance the wall temperature is close to its maximum value which

would result in relatively low slag viscosities. The low slag viscosity would then result in the relatively rapid establishment of a cyclically stable slag thickness in the region near the tube entrance. Cyclic equilibrium for a region would imply that the amount of slag deposited on the region during reheat is equal to the slag transported out of the region during one total cycle (there is no slag deposition during blow-down). Since slag is deposited along the entire tube length, an expansion in the region of cyclic equilibrium would imply that the slag volume which must be transported out of the region increases. Slag viscosity, however, increases nearly exponentially down the tube length due to temperature variation. The exponential viscosity growth would imply that as the region of cyclically stable slag thickness expands, the slag thickness must grow in order to transport the required volume of slag.

The local slag thickness of a point not at the tube entrance would first grow nearly linearly due to the slag deposition flux at that axial position and to the low slag velocities. A large difference in thickness exists between an axial point's cyclically stable and unstable thickness. Consequently, when the region of cyclic equilibrium extends through an axial point, the slag layer would have to experience a very large local growth rate in order to change the local slag thickness to its stable value. Once cyclic stability is reached at an axial point, the large growth rate would have to drop to zero when integrated over a cycle. The rapid changes in slag thickness are visible in Figures 9 and

10 while the corresponding local growth rate spikes are shown in Figures 11 and 12.

A discussion using the cyclically stable slag thickness can be used to explain variations in the results predicted by the model due to slag viscosity variation. A less viscous slag would develop higher velocities for the same axial position than would a more viscous slag. The higher velocities would establish an initially thinner slag layer which would reach equilibrium sooner than those of a more viscous slag. A low viscosity would then result in a larger region of cyclic equilibrium compared to that of a higher viscosity for the same elapsed time. Since the region of cyclic equilibrium is larger for low viscosity slag relative to a more viscous slag, more slag is deposited and, therefore, must be transported out of the region. The increased volume of slag to be transported would result in the lower viscosity slag developing a maximum stable thickness which would eventually become larger than that of a more viscous slag. The results from the thickness model show the maximum thickness for lower viscosity slag surpassing the maximum thickness for more viscous coal slags in approximately five to eight-hours of simulated time.

A change in the slag deposition flux function from case 1 to case 2 results in 11.5 percent more slag being deposited, with the greatest increase in the deposition rate near the tube entrance. With more slag being deposited, a thicker cyclically stable slag layer would form near

the entrance. The higher slag flux would cause a more rapid extension of the cyclically stable region since more slag is transported. An extended region has more slag to transport which would result in a larger maximum slag thickness. The results from the model exhibit an extended region of cyclic stability and a larger maximum thickness for case number 2 when compared to the results from the lower deposition flux of case number 1.

Figure 14 shows the maximum slag thickness as calculated by the model for six specific times with connecting lines to aid visualization. The high viscosity slag exhibits an initially larger maximum thickness, which agrees with previously discussed results. The extended region of cyclic stability which occurred for the less viscous slags eventually resulted in a larger growth rate than that for a more viscous slag. Figure 14 shows that the largest change in slope, hence, the greatest slag layer growth rate change, occurs for the least viscous slag. If the growth rate trends were to continue as shown in Figure 14, a much larger maximum slag thickness would result for a low viscosity slag relative to that of a more viscous slag.

Figure 13 shows a slag thickness profile calculated by the model using the second deposition function for nearly twenty hours of simulated preheater time. As Figure 13 shows, the maximum slag thickness for the twenty hour run approximately tripled the ten hour maximum. The twenty hour maximum thickness represents a cross-sectional area reduction

of 24 percent compared with an area reduction of 9.3 percent for ten hours of simulated time. Since the model does not account for changes in gas flow conditions due to deposition, results from the model were accepted as reasonably accurate until slag thickness caused a cross-sectional area reduction of 10 percent. Clearly the 24 percent reduction in area caused by the twenty hour maximum slag thickness will cause major changes in gas flow and heat transfer. Figure 13 represents an extrapolation of the model results to a point of near failure in terms of restricting the flow passage although it should be considered as an approximation to the actual case.

The results from the ten hour run using the slag deposition flux function corresponding to case number 3 are shown in Figure 8. Use of the third deposition function in the model indicates that Figure 8 sets an upper bound on the slag layer thickness. Such a thickness in an actual preheater tube is much less probable than a thickness corresponding to the more moderate deposition fluxes of case numbers 1 and 2.

The magnitude of the maximum slag thickness shown in Figure 8 is such that a cross-sectional area reduction of 78 percent results. Severe changes in the gas flow and heat transfer would result from such a gas flow restriction, indicating that the model's limitations were exceeded. The thicknesses corresponding to times greater than two hours shown in Figure 8 must therefore be considered as gross extrapolations of the model's results.

V. CONCLUSIONS

From the results of the test cases, it may be concluded that:

1. An increase in the amount of slag deposited in a preheater tube results in a larger nonlinear increase in the maximum slag thickness on the tube wall.
2. Low viscosity coal slag eventually results in larger growth rates and a larger maximum thickness than those of a more viscous slag.
3. A ten percent tube cross-sectional area reduction occurs in approximately ten hours for the moderate deposition rates considered.

TABLES

TABLE 1: Montana Rosebud Coal Ash Composition

Composition number 1:

NBS melt K-875 synthetic Rosebud ash from averaged analyses done by ERDA on Rosebud ash.

Composition number 2:

NBS melt K-603 analysis on Rosebud ash done by Schwartzkopf labs.

Composition number 3:

NBS melt K-795 synthetic Rosebud ash from BiGas Corporation analysis of Rosebud ash.

Weight Percent

	<u>Composition Number 1</u>	<u>Composition Number 2</u>	<u>Composition Number 3</u>
SiO ₂	47.5	43.90	42.10
Al ₂ O ₃	21.1	15.98	19.50
Fe ₂ O ₃	7.8	6.80	7.10
CaO	14.5	18.30	24.48
MgO	4.6	3.59	5.50
K ₂ O	0.7	0.76	0.10
TiO ₂	0.8	---	0.90
P ₂ O ₅	0.4	---	---
Na ₂ O	0.4	0.23	0.40
C	---	0.69	---
SO ₃	---	0.42	---

TABLE 2: Test Case Conditions

All Case Numbers:

Weight concentration of slag in gas = 1%

Inlet gas velocity = 50 m/sec

Inlet gas temperature = 1894 K

Tube length = 6.096 m

Tube diameter = 19.05 mm

Pressure = 92,000 Pa

Slag density = 2590 kg/m³

Case Number 1:

Surface roughness = 254 μ m

Reheat and blowdown cycle times = 330 sec

Total slag mass deposited in 1 hour = .0401 kg

Deposition flux function determined using Sande's model [1]

Case Number 2:

Surface roughness = 508 μ m

Reheat and blowdown cycle times = 350 sec

Total slag mass deposited in 1 hour = .0445 kg

Deposition flux function determined using Sande's model [1]

Case Number 3:

Reheat and blowdown cycle times = 350 sec

Total slag mass deposited in 1 hour = .1947 kg

Deposition flux determined using maximum deposition rate as measured by Liu and Azarwall [16]

FIGURES

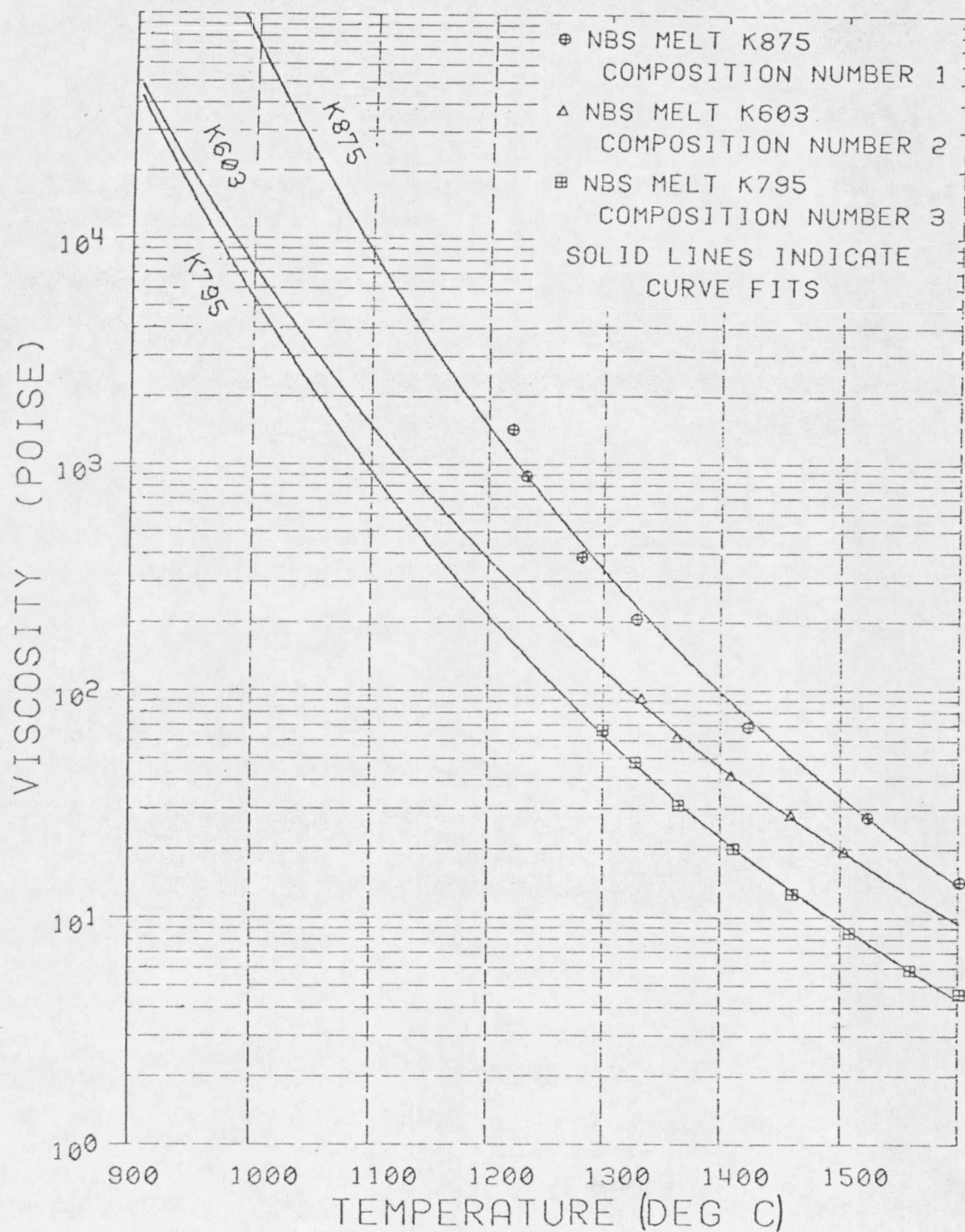


FIGURE 1. COAL SLAG VISCOSITY

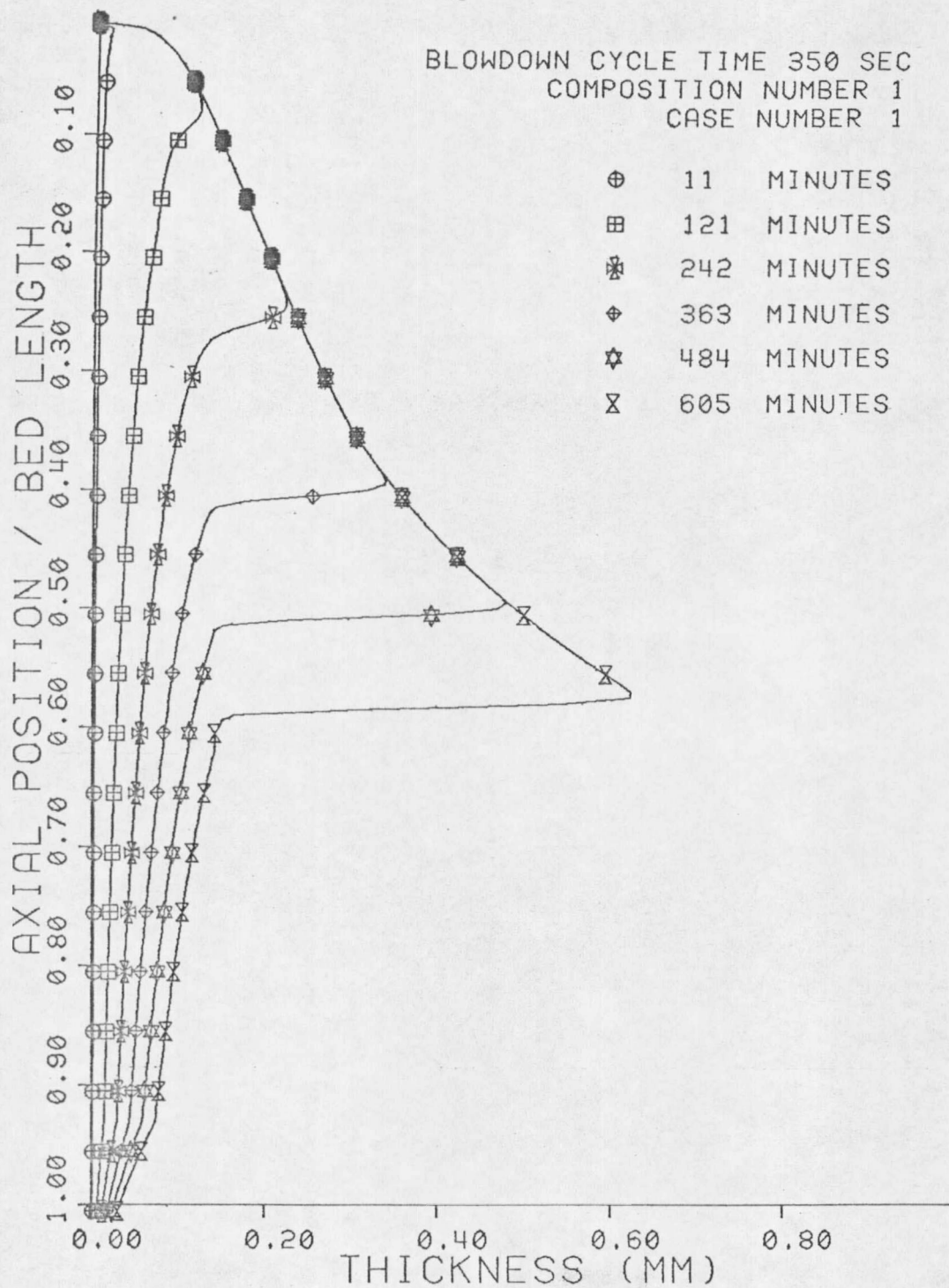


FIGURE 2. SLAG THICKNESS PROFILE

BLOWDOWN CYCLE TIME 350 SEC
COMPOSITION NUMBER 2
CASE NUMBER 1

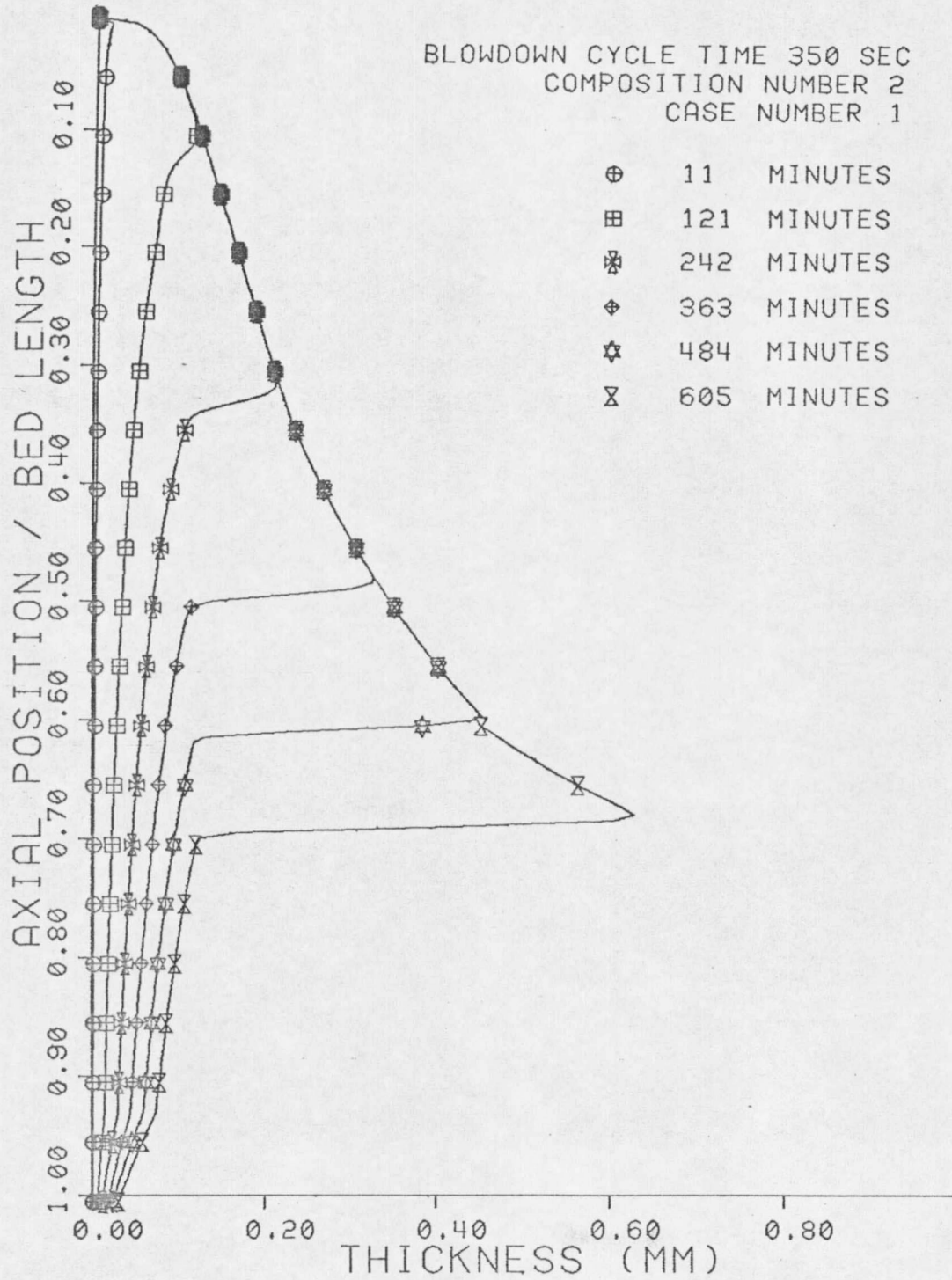


FIGURE 3. SLAG THICKNESS PROFILE

BLOWDOWN CYCLE TIME 350 SEC
COMPOSITION NUMBER 3
CASE NUMBER 1

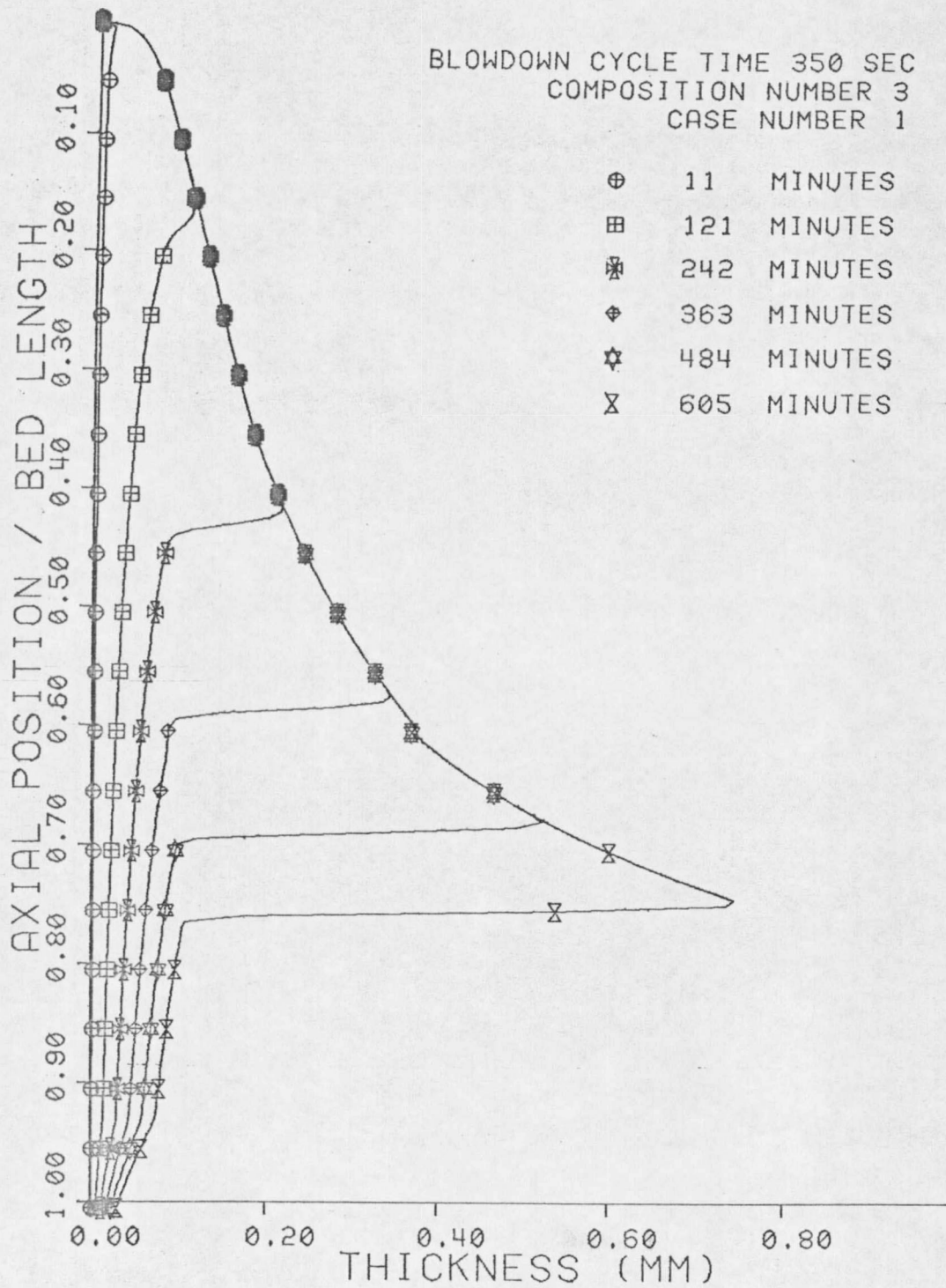


FIGURE 4. SLAG THICKNESS PROFILE

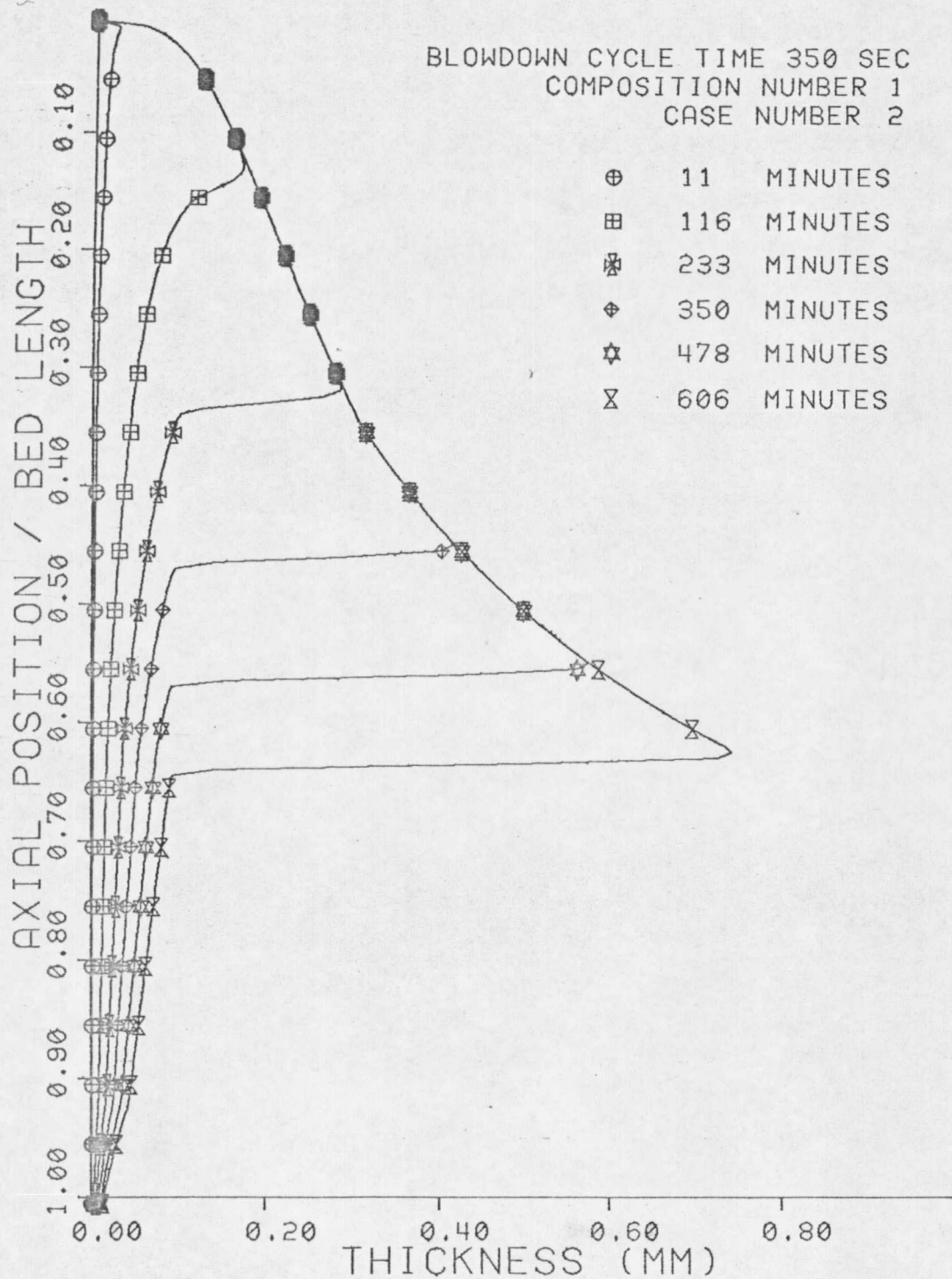


FIGURE 5. SLAG THICKNESS PROFILE

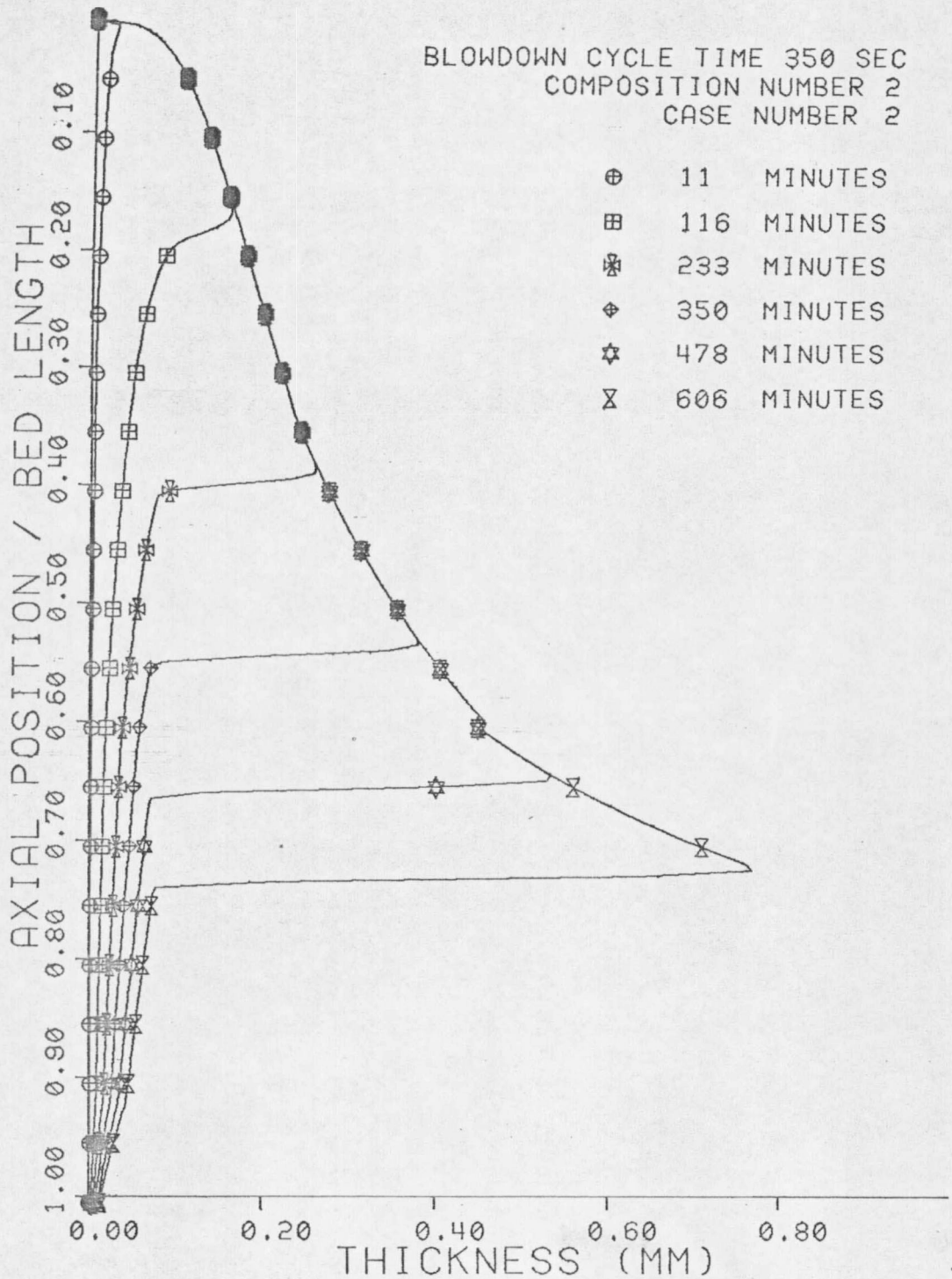


FIGURE 6. SLAG THICKNESS PROFILE

BLOWDOWN CYCLE TIME 350 SEC
COMPOSITION NUMBER 3
CASE NUMBER 2

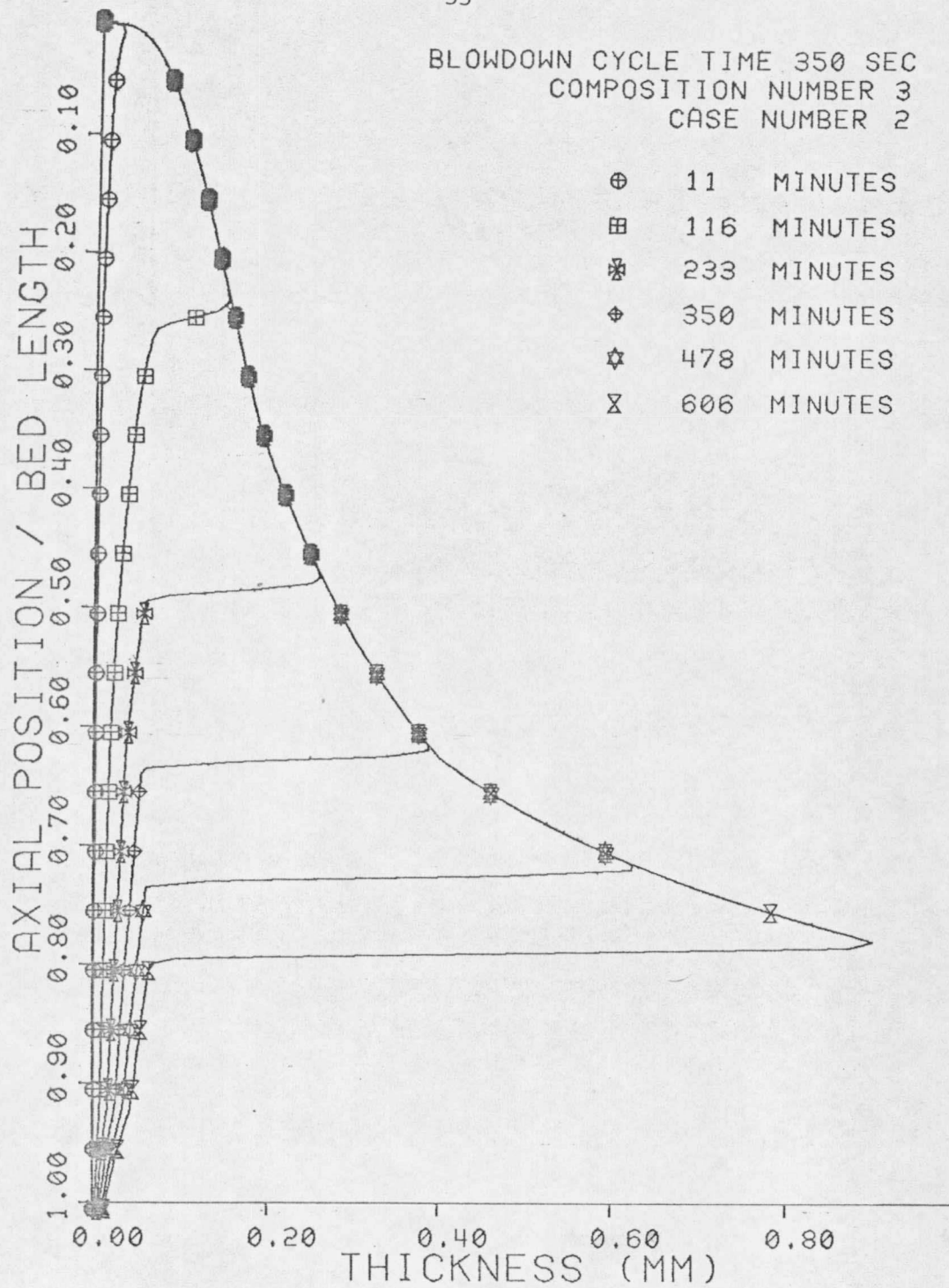


FIGURE 7. SLAG THICKNESS PROFILE

BLOWDOWN CYCLE TIME 350 SEC
COMPOSITION NUMBER 1
CASE NUMBER 3

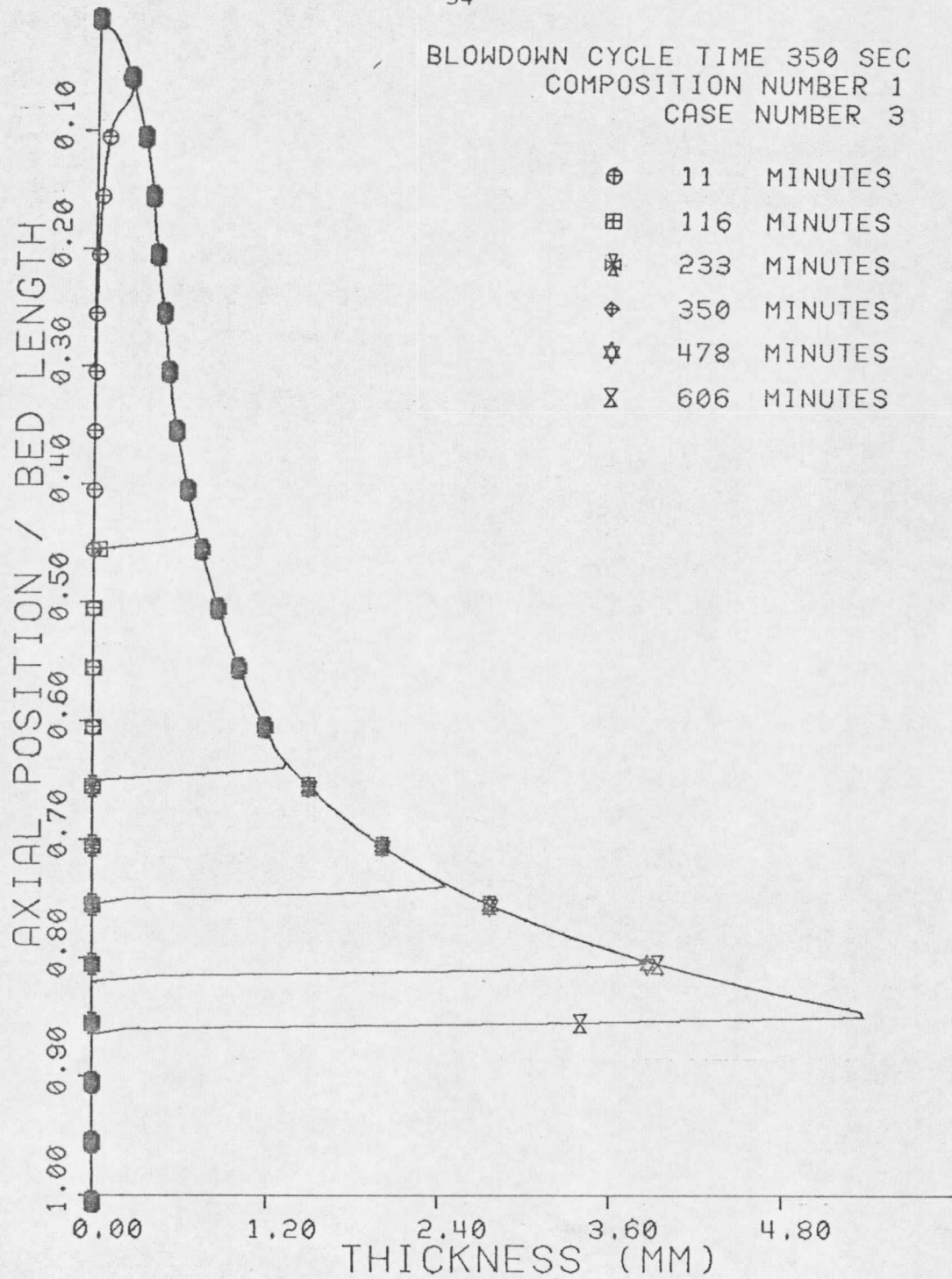


FIGURE 8. SLAG THICKNESS PROFILE

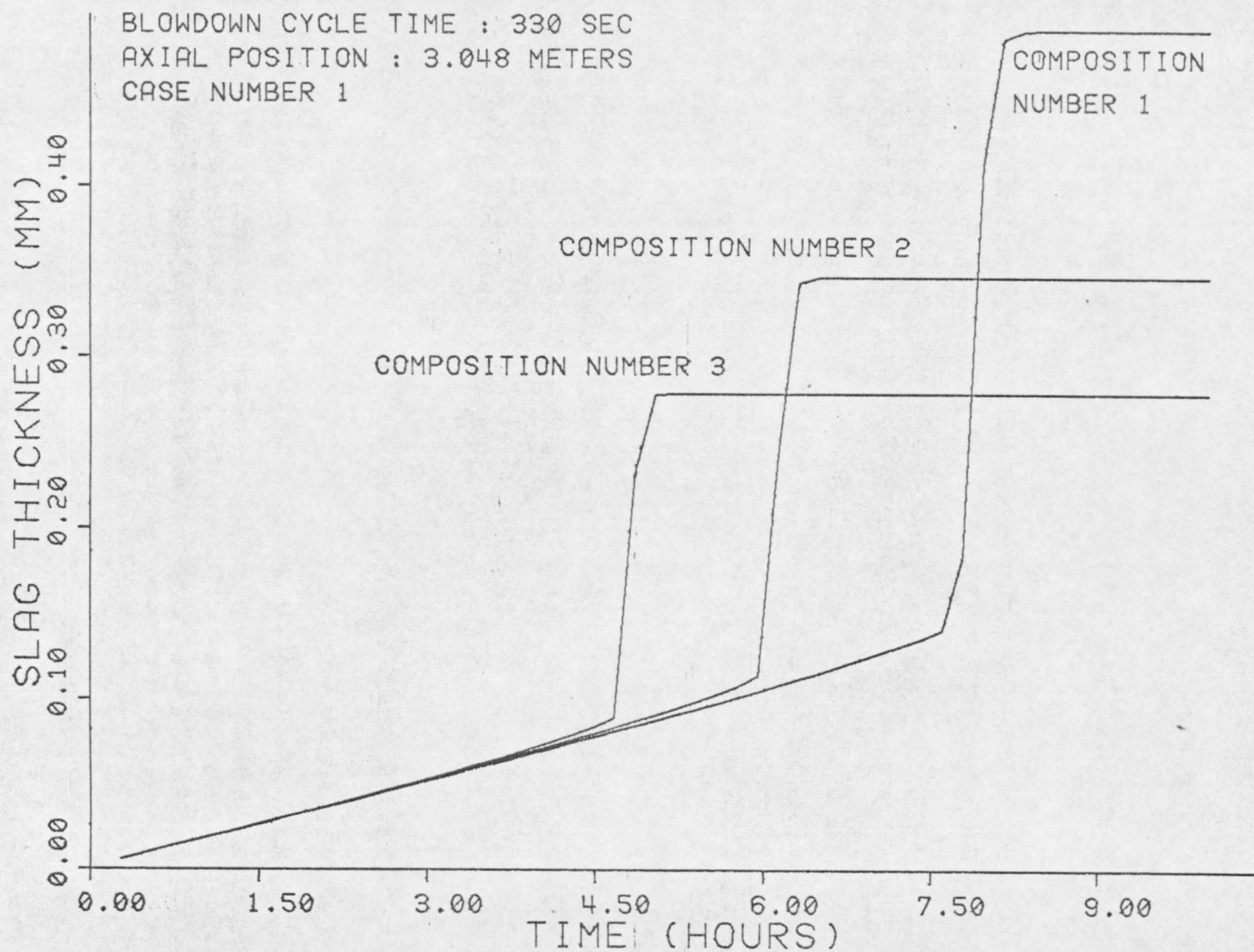


FIGURE 9. LOCAL SLAG THICKNESS

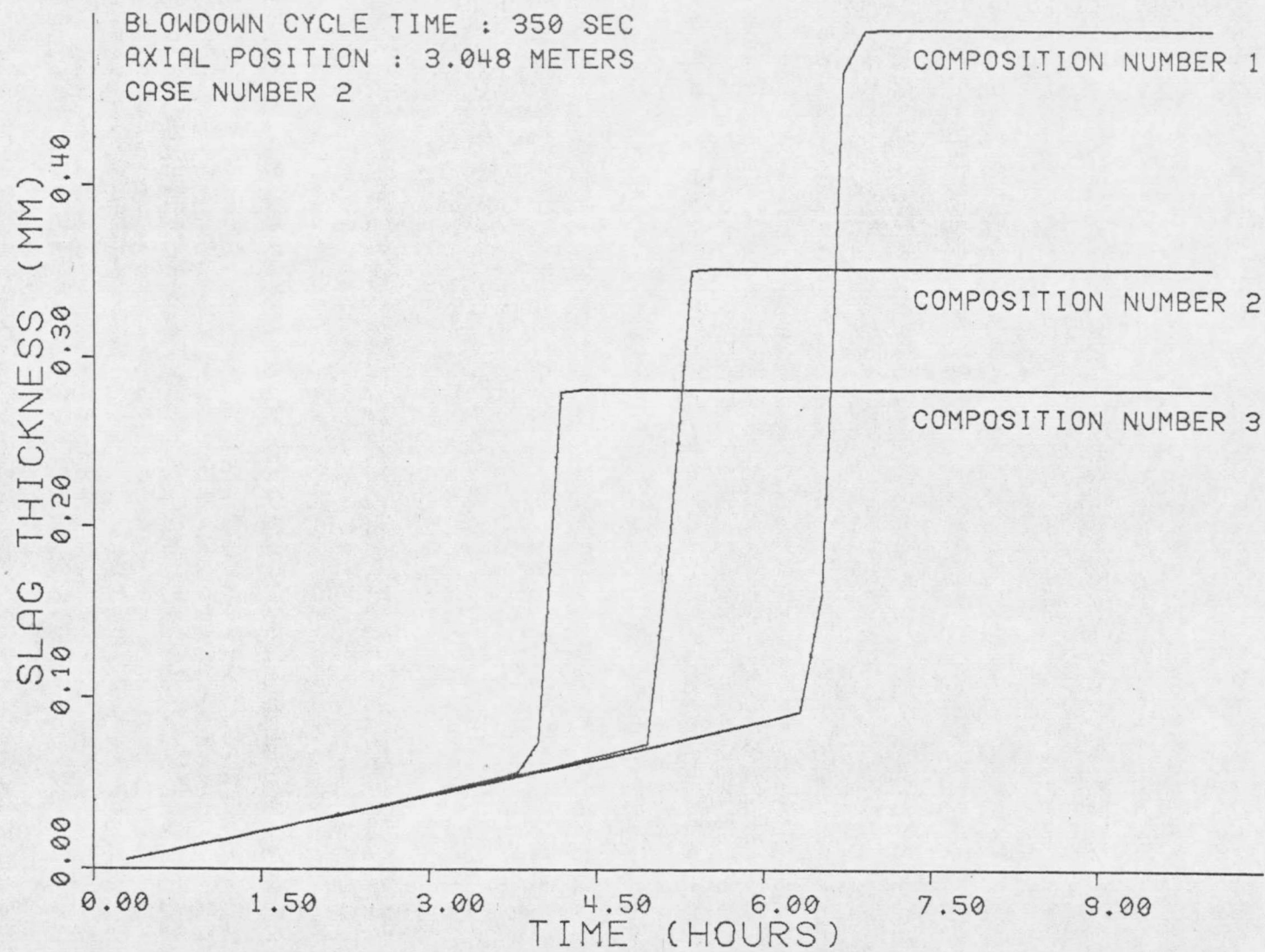


FIGURE 10. LOCAL SLAG THICKNESS

BLOWDOWN CYCLE TIME : 330 SEC
AXIAL POSITION : 3.048 METERS
CASE NUMBER 1

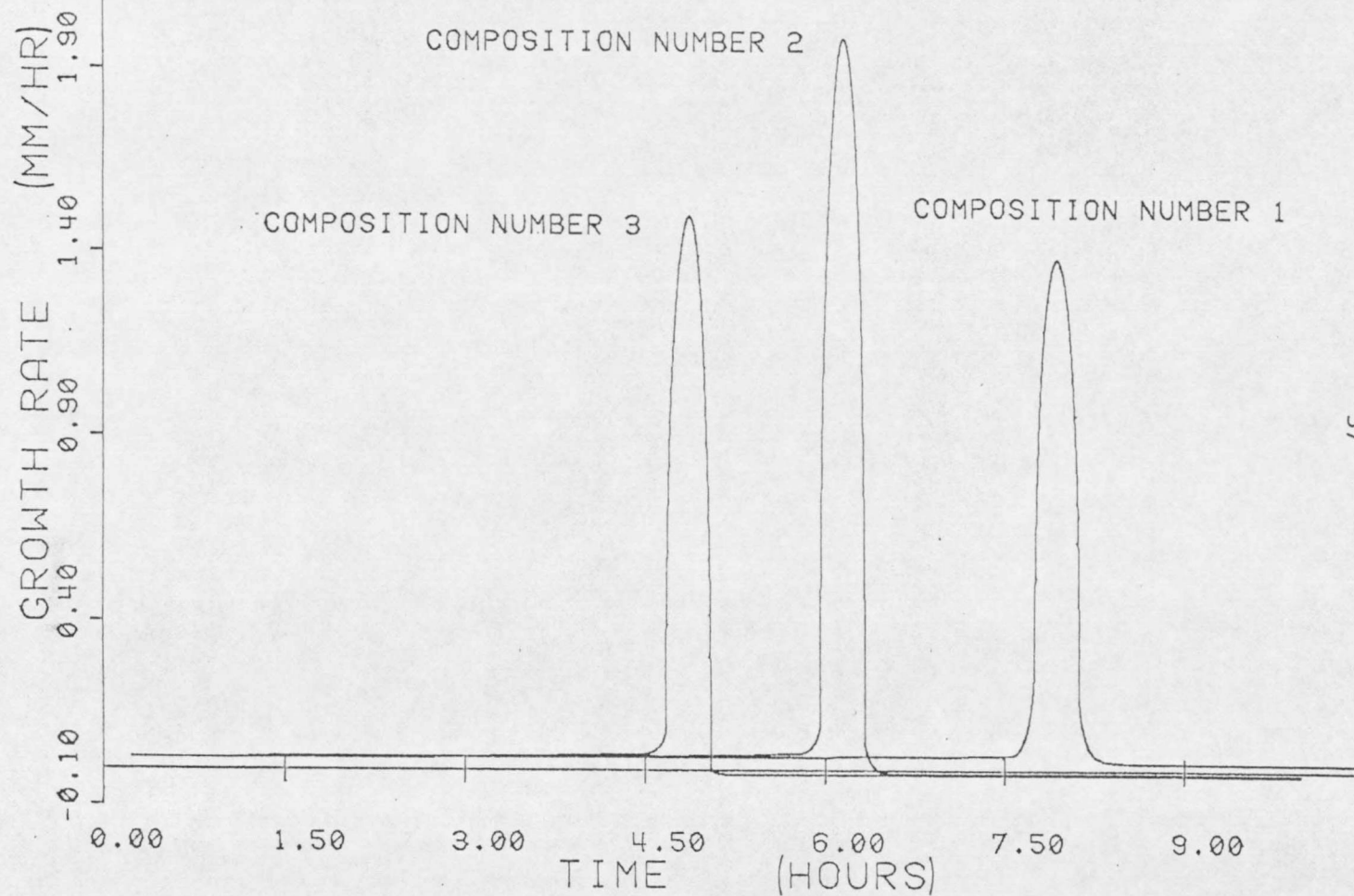


FIGURE 11. LOCAL SLAG GROWTH RATE

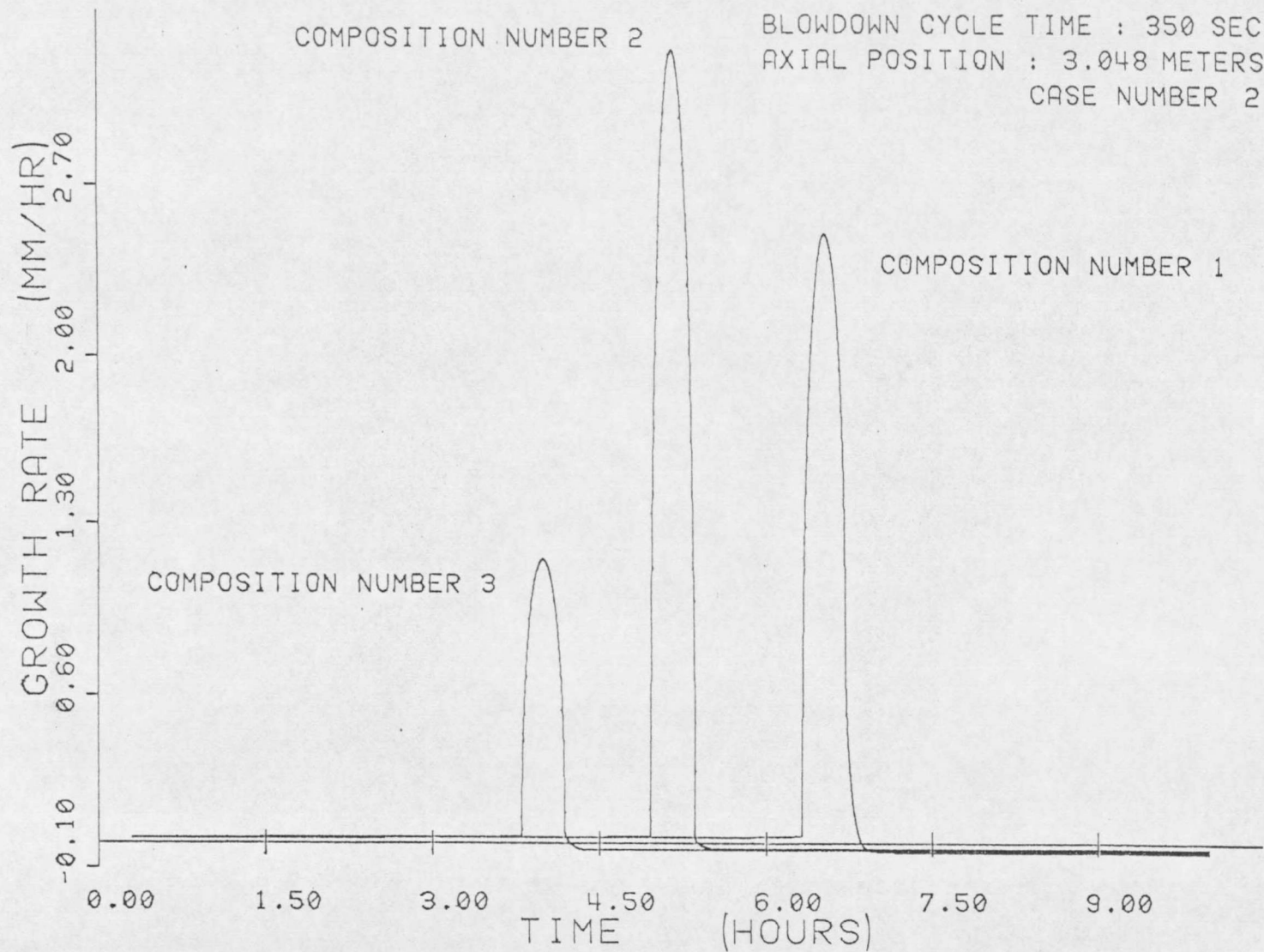


FIGURE 12. LOCAL SLAG GROWTH RATE

BLOWDOWN CYCLE TIME 350 SEC
COMPOSITION NUMBER 3
CASE NUMBER 2

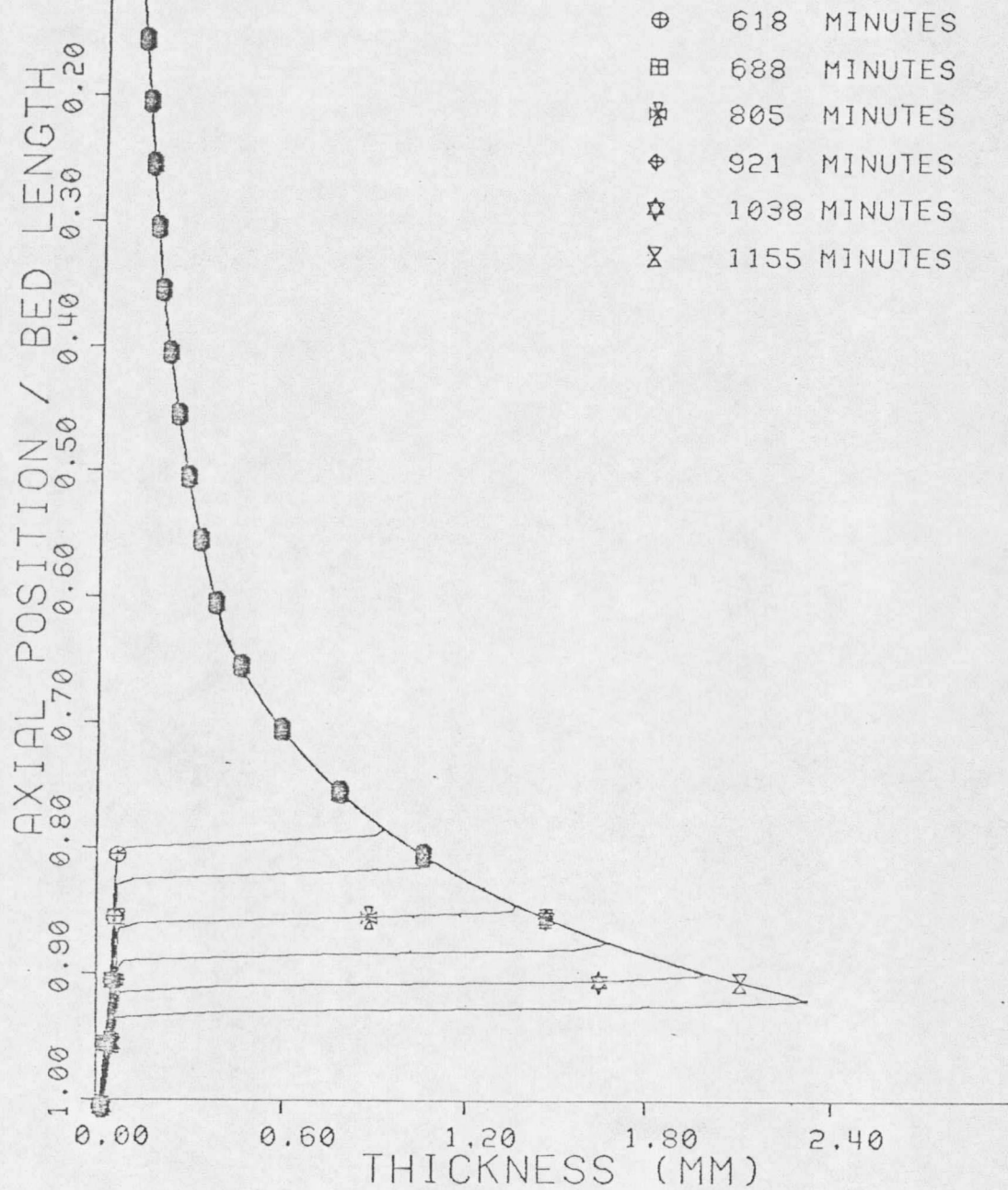


FIGURE 13. SLAG THICKNESS PROFILE

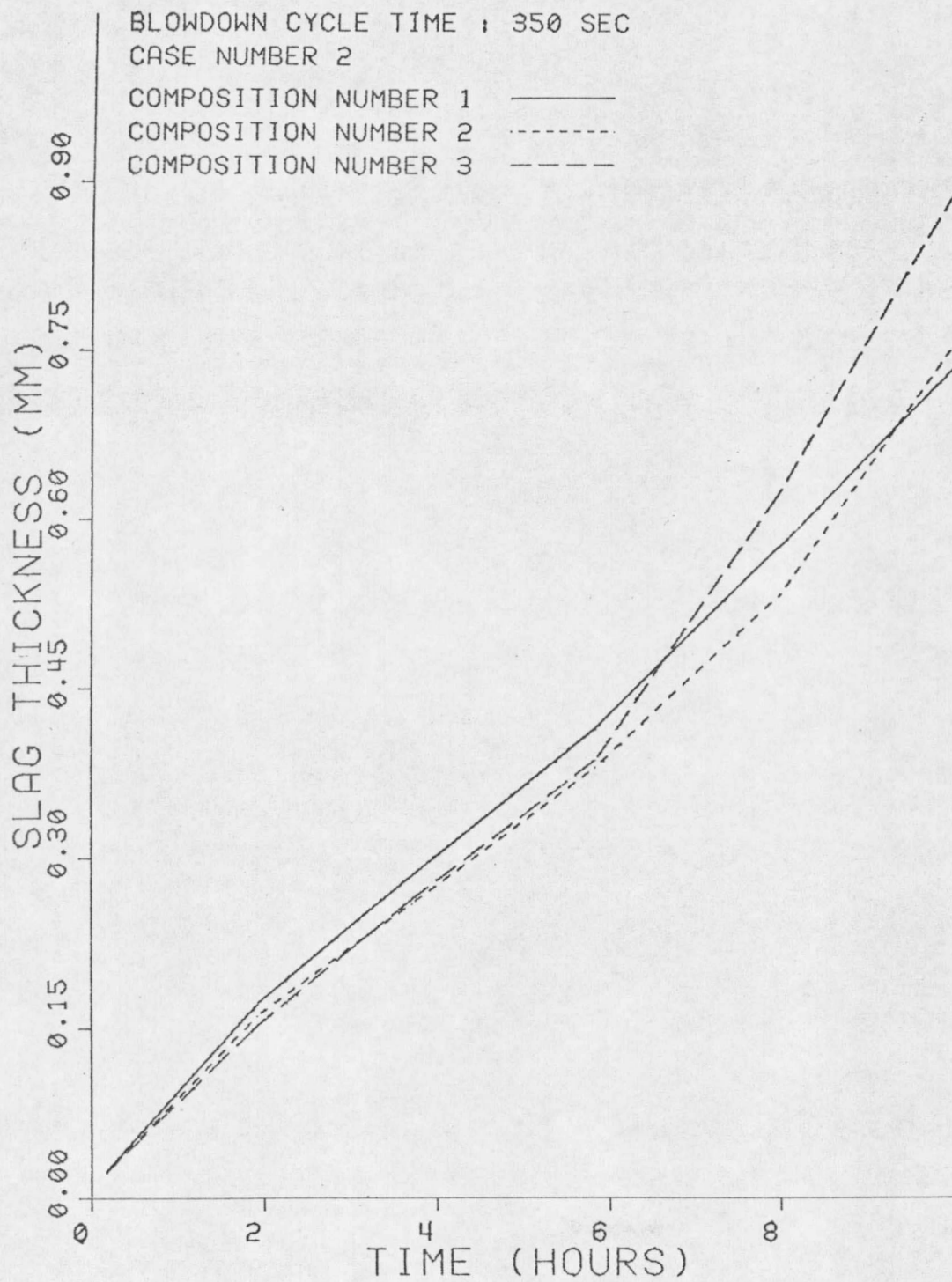


FIGURE 14, MAXIMUM SLAG THICKNESS

APPENDIX 1

Development of the Momentum

Equation for Flow in Circular Tubes

For axisymmetric flow with constant density a convenient infinitesimal element for use in development of the axial momentum equation is the differential ring shown in Figure 1.1:

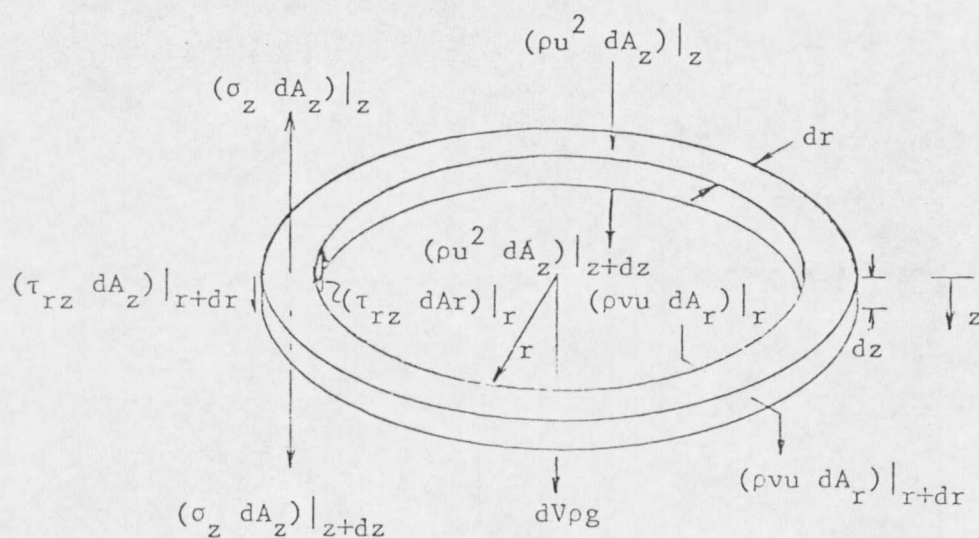


Figure 1.1 Fluid forces on a differential element

where

- σ_z = stress
- τ_{rz} = shear stress
- ρ = slag density
- g = gravitational acceleration
- u = axial velocity component
- v = radial velocity component

The differential surface area dA_r is the area perpendicular to radial direction. It is therefore a function of radius and must be evaluated at the desired radius; it may be written as

$$(dA_r)|_r = 2\pi r \, dz$$

The differential surface area dA_z is the area of the differential ring perpendicular to the axial direction. The area may be expressed as the difference of the areas of two circles,

$$\begin{aligned} dA_z &= \pi(r + dr)^2 - \pi r^2 \\ &= 2\pi r \, dr + \pi dr^2 \end{aligned}$$

If the differentials of higher order are taken as small, then $dr^2 \rightarrow 0$ and

$$dA_z = 2\pi r \, dr$$

The differential volume of the differential ring can then be written as

$$\begin{aligned} dV &= dA_z \cdot dz \\ &= 2\pi r \, dr \, dz \end{aligned}$$

The rate of axial momentum out of the differential ring is

$$(\rho v u \, dA_r) \big|_{r+dr} + (\rho u^2 \, dA_z) \big|_{z+dz}$$

The rate of axial momentum into the differential ring is

$$(\rho v u \, dA_r) \big|_r + (\rho u^2 \, dA_z) \big|_z$$

The rate of axial momentum change in the differential ring is

$$(2\pi r \, dr \, dz) \rho \frac{\partial u}{\partial t}$$

Summation of external forces gives

$$\begin{aligned} & (\sigma_z \, dA_z) \big|_{z+dz} - (\sigma_z \, dA_z) \big|_z + (dA_r \, \tau_{rz}) \big|_{r+dr} \\ & - (dA_r \, \tau_{rz}) \big|_r + (2\pi r \, dr \, dz) \rho g \end{aligned}$$

The conservation of momentum principle may be written as

Rate of momentum out - rate of momentum in + rate
of change of momentum = summation of forces.

Therefore, the conservation of axial momentum may be expressed as follows:

$$\begin{aligned} & (\rho v u \, dA_r) \big|_{r+dr} - (\rho v u \, dA_r) \big|_r + (\rho u^2 \, dA_z) \big|_{z+dz} - \\ & (\rho u^2 \, dA_z) \big|_z + (2\pi r \, dr \, dz) \rho \frac{\partial u}{\partial t} = (\sigma_z \, dA_z) \big|_{z+dz} - \\ & (\sigma_z \, dA_z) \big|_z + (\tau_{rz} \, dA_r) \big|_{r+dr} - (\tau_{rz} \, dA_r) \big|_r + (2\pi r \, dr \, dz) \rho g \end{aligned}$$

If the appropriate relations are substituted for dA_r and dA_z and the equation is divided by the differential volume the result is

$$\frac{(\rho v u r)|_{r+dr} - (\rho v u r)|_r}{r dr} + \frac{(\rho u^2)|_{z+dz} - (\rho u^2)|_z}{dz} + \rho \frac{\partial u}{\partial t} = \frac{\sigma_z|_{z+dz} - \sigma_z|_z}{dz} + \frac{(\tau_{rz} r)|_{r+dr} - (\tau_{rz} r)|_r}{r dr} + \rho g$$

If the limit as dz and dr tend to zero is taken and the definition of partial derivations is applied, the momentum equation becomes

$$\rho \left[\frac{1}{r} \frac{\partial(rvu)}{\partial r} + \frac{\partial(u^2)}{\partial z} + \frac{\partial u}{\partial t} \right] = \frac{\partial \sigma_z}{\partial z} + \frac{1}{r} \frac{\partial(r\tau_{rz})}{\partial r} + \rho g$$

application of the chain rule for the differentiation gives

$$\rho \left[u \left(\frac{1}{r} \frac{\partial(vr)}{\partial r} + \frac{\partial u}{\partial z} \right) + v \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \right] = \frac{\partial \sigma_z}{\partial z} + \frac{1}{r} \frac{\partial(r\tau_{rz})}{\partial r} + \rho g \quad (1.1)$$

Schlichting [5] gives the continuity equation for axisymmetric flow in cylindrical coordinates, with density constant, as

$$\frac{1}{r} \frac{\partial(vr)}{\partial r} + \frac{\partial u}{\partial z} = 0$$

Schlichting [5] also defines shear stress and normal stress as

$$\tau_{rz} = \mu \left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r} \right) ; \quad \sigma_z = -p + 2\mu \frac{\partial u}{\partial z}$$

Equation (1.1) becomes, upon application of the continuity equation, differentiation of the shear and stress terms and some rearrangement, the following equation:

$$\begin{aligned} \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} \right) = & - \frac{\partial P}{\partial z} + \rho g + \mu \left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) \\ & + \frac{\partial \mu}{\partial r} \left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r} \right) + 2 \frac{\partial \mu}{\partial z} + \frac{\partial u}{\partial z} \end{aligned} \quad (1.2)$$

Equation (1.2) corresponds to equation (1) in the main body of the thesis.

APPENDIX 2

Development of the Energy Equations for Axisymmetric Flow in Circular Tubes

For axisymmetric flow, a differential ring may be used in the development of a general energy equation. Figure 2.1 shows the differential ring and the energy fluxes.

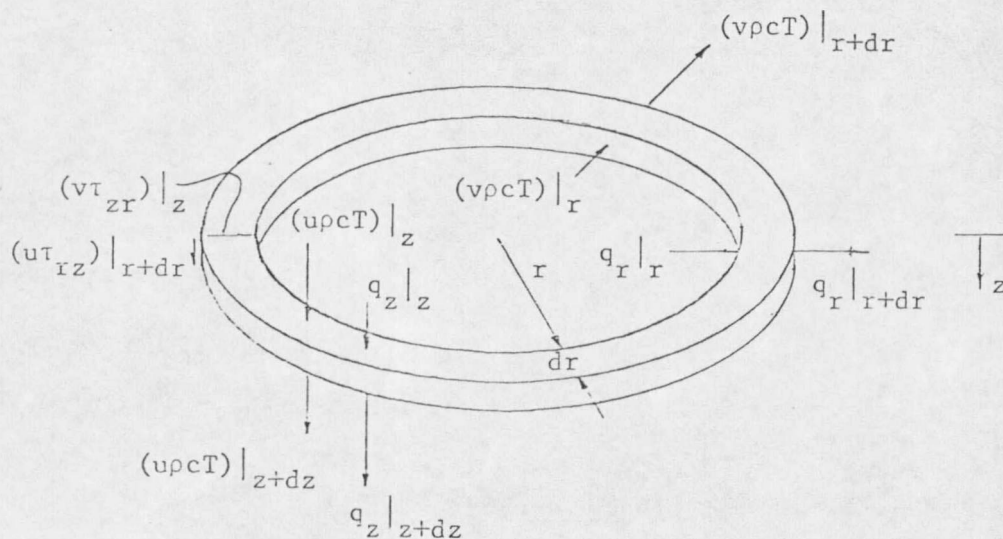


Figure 2.1 Energy fluxes for differential ring

where the terms used in Figure 2.1 are

- q_r = conductive heat transfer in axial direction
- q_z = conductive heat transfer in radial direction
- τ_{rz} = surface shear stress on r surface in axial direction
- u = axial velocity component

v = radial velocity component

ρ = density

c = specific heat

T = temperature

Net energy transfer rates may be expressed by subtracting the rate into the differential ring from the rate out of the differential ring. The different net energy transfer rates may be expressed as follows:

convection axially:

$$dA_z (upcT) \big|_{z+dz} - dA_z (upcT) \big|_z$$

convection radially:

$$[dA_r (vpcT)] \big|_{r+dr} - [dA_r (vpcT)] \big|_r$$

conduction axially:

$$dA_z q_z \big|_{z+dz} - dA_z q_z \big|_z$$

and conduction radially:

$$(dA_r q_r) \big|_{r+dr} - (dA_r q_r) \big|_r$$

The shear work is

$$dA_z (v\tau_{zr}) \big|_{z+dz} - dA_z (v\tau_{zr}) \big|_z + (dA_r u\tau_{rz}) \big|_{r+dr} - (dA_r u\tau_{rz}) \big|_r$$

and the rate of change of stored energy is

$$(dV)\rho c \frac{\partial T}{\partial t}$$

As derived in Appendix 1, the differential areas and differential volume can be written as

$$(dA_r)|_r = 2\pi r \, dz$$

$$dA_z = 2\pi r \, dr$$

$$dV = 2\pi r \, dr \, dz$$

The principle of conservation of energy for the case of no internal generation may be expressed as

Energy rate out - Energy rate in + rate of change in
stored energy = 0

The principle of energy conservation gives

$$\begin{aligned} & dA_z [(u\rho cT)|_{z+dz} - (u\rho cT)|_z] + [(dA_r v\rho cT)|_{r+dr} - (dA_r v\rho cT)|_r] \\ & + dA_z (q_z|_{z+dz} - q_z|_z) + [(dA_r q_r)|_{r+dr} - (dA_r q_r)|_r] + \\ & dA_z [(v\tau_{zr})|_{z+dz} - (v\tau_{zr})|_z] + [(dA_r u\tau_{rz})|_{r+dr} - (dA_r u\tau_{rz})|_r] \\ & + (2\pi r \, dr \, dz)\rho c \frac{\partial T}{\partial t} = 0 \end{aligned} \quad (2.1)$$

Using a method similar to that used in the development of the momentum equation in Appendix 1, equation (2.1) can be transformed to a differential equation. Following division of the equation by the

differential volume term, the limit as dr and dz tend to zero is taken. The definition of a partial derivative may then be applied whereupon equation (2.1) takes the following form:

$$\begin{aligned} \frac{\partial}{\partial z} (u\rho cT) + \frac{1}{r} \frac{\partial}{\partial r} (rv\rho cT) + \frac{\partial(q_z)}{\partial z} + \frac{1}{r} \frac{\partial(rq_r)}{\partial r} \\ + \rho c \frac{\partial T}{\partial z} = \frac{\partial}{\partial z} (v\tau_{zr}) + \frac{1}{r} \frac{\partial}{\partial r} (r\tau_{rz}) \end{aligned} \quad (2.2)$$

The two terms to the right of the equality in equation (2.2) represent shear work. For the problem being considered, heat transfer is large compared to shear work, and, therefore, the latter may be considered to have no significant effect on the energy fluxes.

Neglecting shear work and applying Fourier's conduction law to the energy equation gives

$$\begin{aligned} \frac{\partial}{\partial z} (u\rho cT) + \frac{1}{r} \frac{\partial}{\partial r} (rv\rho cT) - \frac{\partial}{\partial z} (k \frac{\partial T}{\partial z}) - \\ \frac{1}{r} \frac{\partial}{\partial r} (rk \frac{\partial T}{\partial r}) + \rho c \frac{\partial T}{\partial t} = 0 \end{aligned} \quad (2.3)$$

where k = thermal conductivity

Equation (2.3) corresponds to equation (2) in the main body.

Work done by Reihman, Townes, and Mozer [17] indicates that the thermal gradients in the axial and radial directions are of the same order of magnitude. Since $v \ll u$ for laminar flow down the tube wall, it may be written that

$$v \frac{\partial T}{\partial r} \ll u \frac{\partial T}{\partial z}$$

The specific heat, c , is a function of temperature only which gives

$$v \frac{\partial c}{\partial r} \ll u \frac{\partial T}{\partial z}$$

If the small terms are eliminated, equation (2.3) becomes, after manipulation, the following:

$$\begin{aligned} \frac{\partial T}{\partial z} \left(\rho c u - \frac{\partial k}{\partial z} \right) - \frac{\partial T}{\partial r} \left(\frac{k}{r} + \frac{\partial k}{\partial r} \right) - k \frac{\partial^2 T}{\partial z^2} - k \frac{\partial^2 T}{\partial r^2} \\ + T \left[\rho c \left(\frac{\partial u}{\partial z} + \frac{\partial (vr)}{\partial r} \right) + \rho u \frac{\partial c}{\partial z} \right] + \rho c \frac{\partial T}{\partial t} = 0 \end{aligned} \quad (2.4)$$

The continuity equation may be written as

$$\frac{\partial u}{\partial z} + \frac{\partial (vr)}{\partial r} = 0$$

Applying the continuity equation to equation (2.4) results in

$$\begin{aligned} \frac{\partial T}{\partial z} \left(\rho c u - \frac{\partial k}{\partial z} \right) - \frac{\partial T}{\partial r} \left(\frac{k}{r} + \frac{\partial k}{\partial r} \right) - k \frac{\partial^2 T}{\partial z^2} - k \frac{\partial^2 T}{\partial r^2} \\ + T \rho u \frac{\partial c}{\partial z} + \rho c \frac{\partial T}{\partial t} = 0 \end{aligned} \quad (2.5)$$

APPENDIX 3

Finite Difference Approximation and Stability

Terms for the Energy and Momentum Equations

The finite difference scheme used in the transformation of equations (1) and (3) into finite difference forms is one in which second derivations are approximated by the central difference method while first derivatives are represented by a forward difference approximation. This finite difference scheme is necessary to avoid negative coefficients which appear in the forward, central, and backward finite difference methods when applied individually to equations (1) and (3).

The momentum equation, equation (1), with superscripts denoting time and subscripts denoting radial and axial position, gives the following finite difference expression:

$$\begin{aligned}
 u_{i,j}^{n+1} = & u_{i,j}^n \left\{ 1 + \frac{\Delta t}{\Delta z} u_{i,j}^n + \frac{\Delta t}{\Delta r} v_{i,j}^n - \frac{2\mu_{i,j}^n}{\rho(\Delta z)^2} \left[\frac{\mu_{i,j+1}^n}{\mu_{i,j}^n} + \right. \right. \\
 & \left. \left. \frac{v_{i,j}^n}{2u_{i,j}^n} \left(\frac{\mu_{i,j+1}^n}{\mu_{i,j}^n} - 1 \right) \right] - \frac{\mu_{i,j}^n}{\rho(\Delta r)^2} \left(\frac{\mu_{i+1,j}^n}{\mu_{i,j}^n} + \frac{\Delta r}{r_i} + 1 \right) \right\} + u_{i-1,j}^n \\
 & \left[\frac{\Delta t}{\rho(\Delta r)^2} \mu_{i,j}^n \right] + u_{i+1,j}^n \left[\frac{\Delta t}{\rho(\Delta r)^2} \left(\frac{\mu_{i+1,j}^n}{\mu_{i,j}^n} + \frac{\Delta r}{r_i} \right) - \frac{\Delta t}{\Delta r} v_{i,j}^n \right] \\
 & + u_{i,j-1}^n \left[\frac{\Delta t}{\rho(\Delta z)^2} \mu_{i,j}^n \right] + u_{i,j+1}^n \left\{ \frac{2\mu_{i,j}^n}{\rho(\Delta z)^2} \left[\frac{\mu_{i,j+1}^n}{\mu_{i,j}^n} - \frac{1}{2} + \frac{v_{i,j+1}^n}{2u_{i,j+1}^n} \right. \right. \\
 & \left. \left. \left(\frac{\mu_{i,j+1}^n}{\mu_{i,j+1}^n} - 1 \right) \right] - \frac{\Delta t}{\Delta z} u_{i,j}^n \right\} + \frac{\Delta t}{\rho \Delta z} (P_j^n - P_{j+1}^n) + g \Delta t
 \end{aligned} \tag{3.1}$$

An investigation of the order of magnitude of the terms in equation (3.1) shows that terms containing the radial velocity component v are small relative to the other finite difference terms. As a result the terms containing v will not be considered further in the stability investigation. The energy equation, equation (4) (or 2.4), when expressed in the same finite difference scheme gives

$$\begin{aligned}
 T_{i,j}^{n+1} = & T_{i,j}^n \left\{ 1 - \left[\frac{u_{i,j}^n \Delta t}{\Delta z} \left(\frac{c_{i,j+1}^n}{c_{i,j}^n} - 2 \right) \right. \right. \\
 & + \frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n (\Delta r)^2} \left(1 + \frac{k_{i+1,j}^n}{k_{i,j}^n} + \frac{\Delta r}{r_i} \right) + \frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n (\Delta z)^2} \\
 & \left. \left. \left(1 + \frac{k_{i,j+1}^n}{k_{i,j}^n} \right) \right] \right\} + T_{i-1,j}^n \left[\frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n (\Delta r)^2} \right] \\
 & + T_{i+1,j}^n \left[\frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n r_i \Delta r} + \frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n (\Delta r)^2} \left(\frac{k_{i+1,j}^n}{k_{i,j}^n} \right) \right] \\
 & + T_{i,j-1}^n \left[\frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n (\Delta z)^2} \right] + T_{i,j+1}^n \left[\frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n (\Delta z)^2} \left(\frac{k_{i,j+1}^n}{k_{i,j}^n} \right) \right. \\
 & \left. \left. - \frac{\Delta t u_{i,j}^n}{z} \right] \right. \quad (3.2)
 \end{aligned}$$

When considering finite difference schemes, the standard stability criterion is one that requires coefficients to be positive; this criterion is a result of the second law of thermodynamics and represents a sufficiency condition for stability.

There exists, as a result of applying the stability criteria, four coefficients which can be made positive and which can subsequently be used to limit step sizes. The two from equation (3.1) are

$$1 + \frac{\Delta t}{\Delta z} u_{i,j}^n - \frac{2\mu_{i,j}^n \Delta t}{\rho(\Delta z)^2} \left(\frac{\mu_{i,j+1}^n}{\mu_{i,j}^n} \right) - \frac{\mu_{i,j+1}^n (\Delta t)}{\rho(\Delta r)^2} \left(\frac{\mu_{i+1,j}^n}{\mu_{i,j}^n} + \frac{\Delta r}{r_i} + 1 \right) \geq 0 \quad (3.3)$$

and

$$\frac{2\mu_{i,j}^n \Delta t}{\rho(\Delta z)^2} \left(\frac{\mu_{i,j+1}^n}{\mu_{i,j}^n} - \frac{1}{2} \right) - \frac{\Delta t}{\Delta z} u_{i,j}^n \geq 0 \quad (3.4)$$

Two coefficients from equation (3.2) are

$$1 - \left[\frac{\mu_{i,j}^n \Delta t}{\Delta z} \left(\frac{c_{i,j+1}^n}{c_{i,j}^n} - 2 \right) + \frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n (\Delta r)^2} \left(1 + \frac{k_{i+1,j}^n}{k_{i,j}^n} + \frac{\Delta r}{r_i} \right) + \frac{\Delta t k_{i,j}^n}{\rho c_{i,j}^n (\Delta z)} \left(1 + \frac{k_{i,j+1}^n}{k_{i,j}^n} \right) \right] \geq 0 \quad (3.5)$$

and

$$\frac{\Delta t k_{i,j}^n}{\rho(\Delta z)^2 c_{i,j}^n} \left(\frac{k_{i,j+1}^n}{k_{i,j}^n} \right) - \frac{\Delta t u_{i,j}^n}{\Delta z} \geq 0 \quad (3.6)$$

Equations (3.3) through (3.6) may be reduced to expressions explicit in either, or both, time and axial step sizes.

Two expressions which limit the time step can be derived from equations (3.3) and (3.5) and can be written as:

$$\Delta t \leq \frac{1}{\frac{\mu_{i,j}^n}{\rho} \left(\frac{2}{\Delta z^2} + \frac{2}{\Delta r^2} + \frac{1}{\Delta r \cdot r_i} \right) - \frac{u_{i,j}^n}{\Delta z}} \quad (3.7)$$

and

$$\Delta t \leq \frac{1}{\frac{k_{i,j}^n}{\rho c_{i,j}^n} \left(\frac{2}{\Delta r^2} + \frac{2}{\Delta z^2} + \frac{1}{\Delta r \cdot r_i} \right) - \frac{u_{i,j}^n}{\Delta z}} \quad (3.8)$$

Equations (3.4), (3.5), and (3.6) are found to limit the axial step size. The relations derived from the three equations are respectively

$$\Delta z \leq \frac{\mu_{i,j}^n}{\rho u_{i,j}^n} \quad , \quad (3.9)$$

$$\frac{1}{\Delta z} \leq \frac{k_{i,j}^n}{u_{i,j}^n \rho c_{i,j}^n} \left(\frac{2}{\Delta r^2} + \frac{2}{\Delta z^2} + \frac{1}{\Delta r \cdot r_i^2} \right) \quad , \quad (3.10)$$

and

$$\Delta z \leq \frac{k_{i,j}^n}{\rho c_{i,j}^n u_{i,j}^n} \quad (3.11)$$

An approximation of the step size limitations from equations (3.7) through (3.11) may be found by substitution of representative numbers

for the properties and velocity in the equations. Representative numbers are those properties which are available for the coal slag at approximately 1900 K. Those properties unavailable are approximated by the properties of silica at 1900 K. With an assumed slag thickness of .5 mm, a simplified analysis determined the maximum velocity to be of the order of magnitude of 0.2 m/s.

The most severe restriction on the axial step size is that imposed by equation (3.11) which limits the axial step to a length of approximately 50 μm .

If an axial step size of 50 μm is used to determine the time step size, it is found that equation (3.7) limits the maximum step size to approximately 10 microseconds.

Since equation (3.7) is derived from equation (3.1), it can be said that the most severe time step restriction is made by the momentum equation. Similarly, the most severe axial step size limitation is made by the energy equation.

In the stability study of the coupled differential equations (3), (4), and (5) performed above, a stable slag thickness near the tube entrance was assumed to be .5 mm. However, when a slag thickness calculated by the simplified model described in the main body was used, the axial step size limit became approximately one centimeter. When, a radial step size of one millimeter (10 percent of the tube radius) and an axial step of one centimeter was used in equation (3.7) the resulting

time step was only three milliseconds. The small time step of three milliseconds still warrants use of the limitations needed to develop the simplified model developed in the main body.

APPENDIX 4

Development of a Finite Difference Equation for Slag Layer Thickness

In development of the finite difference equation representing slag thickness, the limitation of a thin slag layer is applied; this allows development of the equation in rectangular coordinates with no significant loss of accuracy.

The slag layer may be represented as a series of axial elements as shown in Figure 4.1.

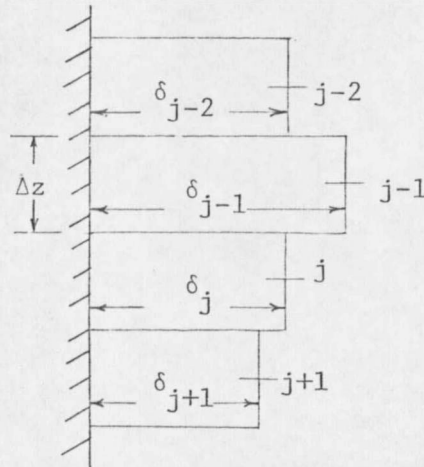


Figure 4.1 Slag layer thickness as approximated by a series of elements.

Each element has dimensions of $\Delta z \times C \times \delta$, where Δz is the axial step length, C is the circumference of the heat exchanger tube, and δ is the local slag thickness.

The principle of conservation of mass can be applied to an element resulting in an equation for the slag layer thickness. Conservation of mass can be written as:

$$\begin{aligned} &\text{mass storage change with respect to time} = \\ &\text{rate of mass in} - \text{rate of mass out} \end{aligned}$$

Since the mass storage change is the partial derivative of mass with respect to time, it can be approximated by the forward difference method. Therefore, with superscripts denoting time and subscripts denoting axial positions, the mass conservation principle may be written as

$$\frac{M_{j+1}^{n+1} - M_{j+1}^n}{\Delta t} = (\dot{m}_{in} - \dot{m}_{out})_{j+1}^n$$

where M = mass of element

$\dot{m}$ = mass flow

Δt = time step

multiplication by the time step gives:

$$M_{j+1}^{n+1} - M_{j+1}^n = m_{in}^n - m_{out}^n \quad (4.1)$$

The mass entering the $j+1$ element may be expressed as

$$m_{in} = \text{mass deposited from gas} + \text{mass flow from element } j$$

$$= (\dot{m}_d)_{j+1} \Delta t (C \Delta z) + \bar{V}_j \rho (C \delta_j) \Delta t \quad (4.2)$$

where $\dot{m}_d$ = mass deposition flux
 $\bar{V}$ = average axial velocity of element
 δ = slag thickness
 C = circumference

Similarly, the mass leaving the $j+1$ element can be written as

$$m_{out} = \bar{V}_{j+1} \rho (C \delta_{j+1}) \Delta t \quad (4.3)$$

The mass stored in element $j+1$ can be written as

$$\begin{aligned} M_{j+1} &= \text{Volume}_{j+1} \cdot \rho \\ &= (\Delta z C \delta_{j+1}) \rho \end{aligned} \quad (4.4)$$

Substitution of equations (4.2), (4.3), and (4.4) into equation (4.1) gives, after manipulation, the following equation:

$$\delta_{j+1}^{n+1} = \delta_{j+1}^n + \frac{\Delta t}{\rho} \dot{m}_{d,j+1} + \frac{\Delta t}{\Delta z} (\bar{V}_j^n \delta_j^n - \bar{V}_{j+1}^n \delta_{j+1}^n) \quad (4.5)$$

Equation (4.5) will give the results desired once an expression for the averaged velocity can be found.

Since the thin slag layer limitation has been applied, the temperature variation in the radial direction is negligible. This implies that viscosity can be taken as a function of axial position only.

The continuous change of viscosity and velocity due to their axial dependence can be approximated by a series of step changes axially, which in turn were made to occur at the beginning of the axial elements portrayed in Figure 4.1. Both velocity and viscosity are constrained by the step changes approximation to be constant for any one axial element. As a result of the approximation, viscosity can be considered constant in any one element, while axial velocity, u , varies only radially in that element.

The axial velocity can be calculated as a function of radius and viscosity by considering an infinitesimal element, as in Figure 4.2.

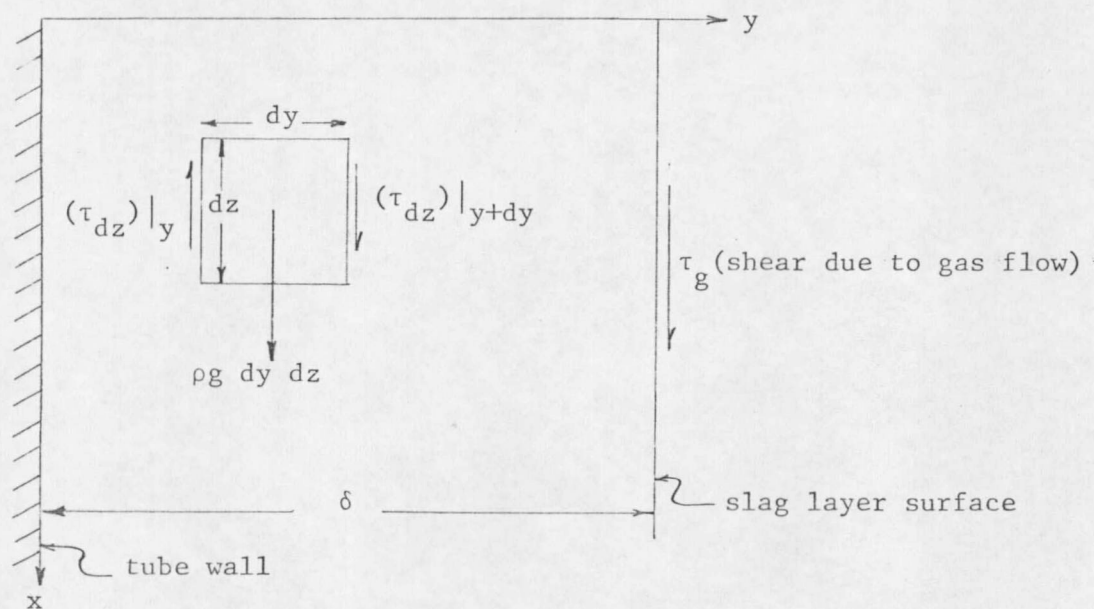


Figure 4.2 Infinitesimal element of slag

Assuming a unit depth, summation of the forces on the infinitesimal element gives

$$(\tau \, dz) \Big|_{y+dy} - (\tau \, dz) \Big|_y + \rho g \, dy \, dz = \rho \, dy \, dz \, \frac{du}{dt} \quad (4.6)$$

Since the slag flow will be slow, any change with respect to time will be small. Therefore, the slag motion can be approximated by steady flow for any one element. Equation (4.6) then becomes

$$(\tau \, dz) \Big|_{y+dy} - (\tau \, dz) \Big|_y + \rho g \, dy \, dz = 0$$

Division by the volume, $dy \cdot dz \cdot 1$, gives

$$\frac{\tau \Big|_{y+dy} - \tau \Big|_y}{dy} + \rho g = 0 \quad (4.7)$$

If the limit is taken as dy tends to zero and the definition of a derivative is applied, equation (4.7) becomes

$$\frac{\partial \tau}{\partial y} = -\rho g \quad (4.8)$$

The boundary conditions for the differential equations are

$$\tau \Big|_{y=\delta} = \tau_g \quad (4.9)$$

$$u \Big|_{y=0} = 0 \quad (4.10)$$

Integration of equation (4.8) with respect to y , and applications

of equation (4.9) gives

$$\tau = \mu \frac{du}{dy} = -\rho g y + \tau_g + \rho g \delta \quad (4.11)$$

Integrating equation (4.11) again with respect to y and applying equation (4.10) gives

$$u = \frac{-\rho g}{2\mu} y^2 + \frac{\tau_g + \rho g \delta}{\mu} y \quad (4.12)$$

Since an averaged velocity is needed to determine the slag mass flow rate, the velocity profile equation (4.12) must be integrated from the tube wall to the free surface, and divided by that thickness. The averaged velocity may be written as

$$\begin{aligned} \bar{v}_j &= \frac{1}{\delta_j} \int_0^{\delta_j} u_j dy \\ \Rightarrow \bar{v}_j &= \frac{\tau_g \delta_j}{2\mu_j} + \frac{\rho g \delta_j^2}{3\mu_j} \end{aligned} \quad (4.13)$$

Equations (4.5) and (4.13) complete the relations for determination of slag layer thickness. The values of viscosity used in the relations are determined using data provided by Webster Capps [14], while values for wall temperature and surface shear, τ_g , come from calculations made by Upshaw [9].

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