



The effect of chip-shaped solids on energy losses, in axi-symmetric pipe expansions
by Robert Walter Charley

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE in Civil Engineering
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Abstract:

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THE EFFECT OF CHIP-SHAPED SOLIDS ON ENERGY LOSSES
IN AXI-SYMMETRIC PIPE EXPANSIONS

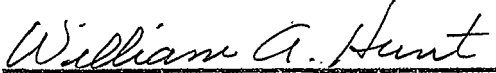
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ROBERT W. CHARLEY

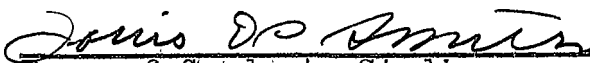
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LIST OF SYMBOLS

A	- cross sectional area.
B	- best fit slope of a straight line.
C	- volumetric concentration of solids.
D	- pipe diameter.
E	- error to test significance of two slopes.
EGL ₁	- upstream energy grade line.
EGL ₂	- downstream energy grade line.
(EGL ₁) _e	- elevation of EGL ₁ when projected to diffuser entrance.
(EGL ₂) _e	- elevation of EGL ₂ when projected to diffuser entrance.
(EGL ₂) _x	- elevation of EGL ₂ at point x.
F _x	- forces in the x direction.
ff	- fluid flowing in test section.
fps	- feet per second.
g	- gravitational acceleration.
gpm	- gallons per minute.
h _L	- head loss.
HGL	- hydraulic grade line.
K _L	- loss coefficient.
(K _L) _c	- loss coefficient for a given solids concentration.
(K _L) _o	- loss coefficient for clear water.
mf	- manometer fluid.

n	- number of observations.
NDS	- station number of pressure tap immediately upstream from diffuser.
P	- pressure.
Q	- flowrate.
Q_M	- flowrate of mix.
Q_W	- flowrate of water into mix tank.
Q_S	- flowrate of solids.
SEGL	- slope energy grade line.
SHGL	- slope hydraulic grade line.
S.G.	- specific gravity.
V	- mean velocity.
v	- variable velocity with boundary layer profile.
X	- distance from station 1 to any given pressure tap.
X_R	- distance from station 1 to diffuser entrance.
Z	- elevation head.
Δ	- differential reading or distance.
δ	- boundary layer thickness.
γ	- specific weight.
θ	- expansion angle.
ρ	- density.
μ	- viscosity.
$\partial P / \partial x$	- change in pressure with respect to distance.

ABSTRACT

The primary purpose of this study was to investigate the effect of chip-shaped solids on the energy losses in axi-symmetric pipe expansions and to compare these losses with water alone flowing through the expansion. Since the energy loss h_L is calculated using an empirically-determined loss coefficient K_L , the study was performed by comparing the value of K_L obtained at various concentrations for a given flow rate with that from clear water.

Five separate expansion angles were tested with downstream velocities of 4, 6 and 8 feet per second. The individual velocities were run with solids at volumetric concentrations of 0, 5, 10, 15 and 20%.

At the 4 feet per second velocity the energy loss remained essentially constant for a given expansion as the concentration ranged from 0 to 20%. The higher velocities showed an almost linear decrease in the energy loss as the concentration of solids was increased.

CHAPTER I

INTRODUCTION

A. Need for study

The main purposes of this paper will be to determine the energy losses (head losses) in axi-symmetric flow expansions carrying a mixture of water and relatively large rectangular chip-shaped solids and to compare these losses to expansion losses with clear water flowing. This study was initiated to gain technical knowledge necessary for estimating the head losses in pipe lines which may be used for transporting solids whose density is approximately that of water.

This study is part of a project sponsored by the U.S. Forest Service investigating the hydraulics of transporting

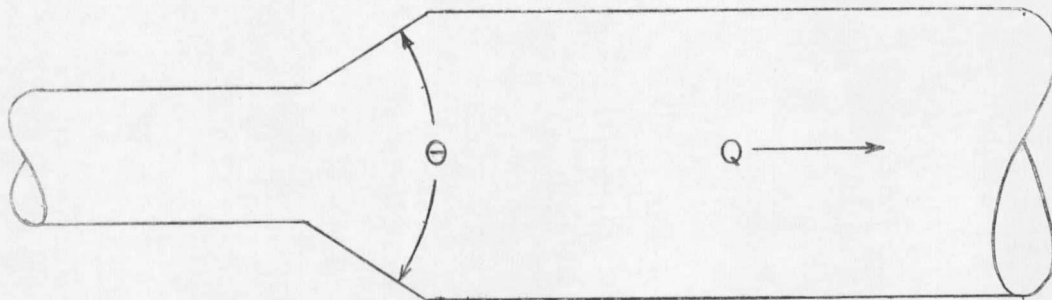


Fig. 1. Axi-symmetric pipe expansion.

wood chips in pipe lines. The potential for moving large quantities of wood chips over long distances in this matter has aroused the interest of several pulp and paper companies in the United States and Canada. Such a study will be a first step in obtaining design data for planning subsequent pipe line systems.

B. Basic head loss equation

The equation most commonly used in engineering calculations for estimating head loss, h_L , for a diffuser shown in Fig. 1, is a modified version of that developed for an abrupt enlargement ($\theta = 180^\circ$). The head loss for an abrupt expansion is:

$$h_L = \frac{(V_1 - V_2)^2}{2g} \quad (1)$$

The theoretical development of this is in Appendix A.

Theoretically, Eq. (1) is good only for abrupt expansions. Archer and Gibson both found the head loss slightly larger than that indicated by this equation which is then modified by introducing a coefficient K_L as shown by Eq. (2).

$$h_L = K_L (V_1 - V_2)^2 / 2g \quad (2)$$

K_L is generally called the loss coefficient and is determined empirically. For abrupt expansions, W. H. Archer [1] found that K_L varied from 0.754 to 1.225. Gibson [3] found

that for a given area ratio with downstream velocities V_2 greater than 5 feet per second, the loss coefficient was essentially constant. Below 5 feet per second the loss coefficient falls off somewhat rapidly with decreasing velocities. Gibson [2] plotted values of K_L vs. the expansion angle θ for various upstream and downstream area ratios (A_1/A_2).

The purpose of this study is to compare values of K_L for clear water to K_L for a known concentration of a solids-water mixture. For a given flowrate, a change in head loss h_L between clear water and water carrying solids would indicate that K_L is a function of the solids concentration. Then by experimentally determining head loss h_L for different concentrations of solids at given flowrates and solving for K_L from Eq. (2), comparisons of K_L can be obtained.

CHAPTER II

DEVELOPMENT OF HYPOTHESIS

An understanding of the flow characteristics in expansions is necessary to formulate a hypothesis which will relate head losses of clear water to losses for liquid-solid mixtures flowing for such sections and to design an experiment to test this hypothesis.

A literature review was conducted to formulate a hypothesis concerning the change in K_L when solid particles were introduced into the flow. No specific literature on the head losses due to axi-symmetric expansions carrying a liquid-solids mixture could be found. This led to a literature search which pursued two separate areas: (1) The mechanics of head loss in expansions with a liquid flowing, and (2) the transportation of liquid-solid mixtures in pipe lines. A correlation of the two studies allowed for the hypothesis to be formed.

A. Head loss in expansion

The flow of fluids through expansions is a process which converts kinetic energy to pressure head (P/γ). This process is less than one hundred per-cent efficient because of the dissipation of the turbulent energy in the fluid.

This reduced efficiency is a measure of the head loss in the expansion.

The loss in any expanding section can be divided into two parts; that due to friction on the boundaries and that due to the shape or form of the conduit, called the "form loss". The total loss is dependent on the upstream and downstream area ratio [3], the boundary geometry [7], and the velocity distribution [11]. Except for gradual expansions (θ less than 10°) the friction loss is negligible compared to the "form loss".

The "form loss", which is taken to be the total head loss will be explained by the use of Prandtl's [9] boundary layer theory of flat plates which can be modified for circular pipes.

The theoretical boundary layers for flat plates are shown in Figs. 2 and 3. Fig. 2 shows a boundary layer for a zero pressure gradient ($\partial P / \partial x = 0$). This is also the general shape of the boundary layer for a negative pressure gradient, ($\partial P / \partial x < 0$). Fig. 3 is for an adverse pressure gradient ($\partial P / \partial x > 0$).

If the pressure is decreasing ($\partial P / \partial x < 0$) in the downstream direction, the pressure forces and the inertia forces of the free stream flow are in the same direction and com-

plement each other in overcoming the viscous frictional forces within the boundary layer. This results in a reduction of the downstream boundary layer thickness, δ .

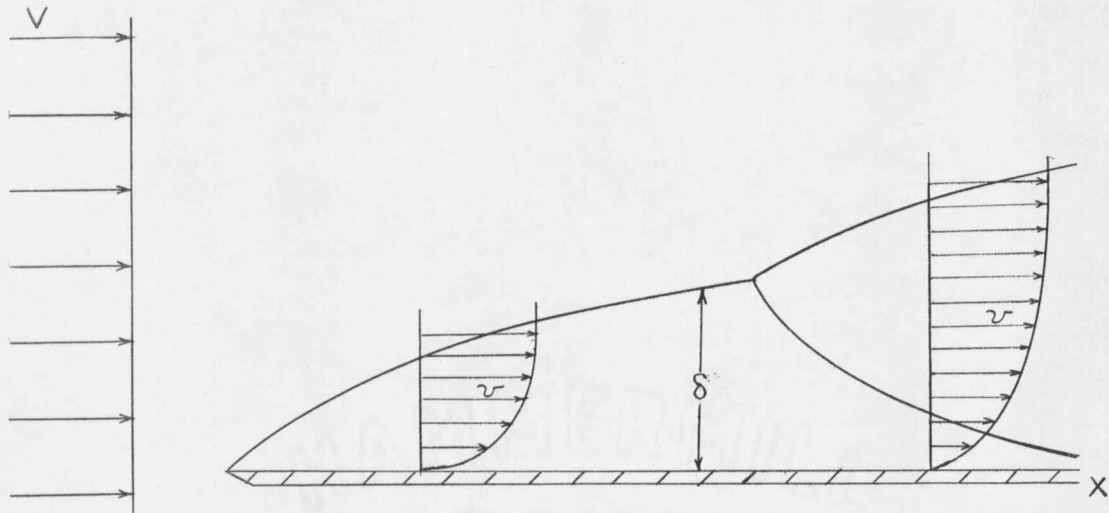


Fig. 2. Boundary layer profile for flat plates $\partial P / \partial x = 0$.

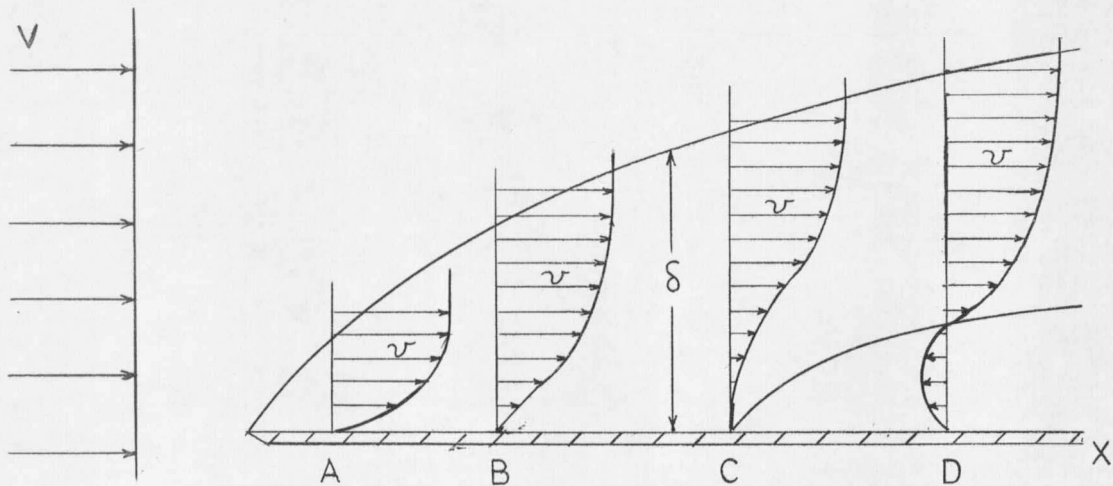


Fig. 3. Boundary layer profile for flat plates $\partial P / \partial x > 0$.

The adverse pressure gradient of Fig. 3 has its forces directed upstream and tends to augment the retarding effect of the viscous frictional forces in reducing the momentum of the boundary layer flow so that the boundary layer thickens rapidly. If these forces act over a sufficient distance a separation of flow from the plate surface will occur at point C as shown in Fig. 3. At this point the velocity can no longer move against the pressure gradient in the region adjacent to the wall. Farther downstream at D, a backflow or eddy exists. These eddies will exist for some distance downstream until they are damped out by the viscous fluid action.

Expansions in pipelines afford adverse pressure gradients necessary for the separation of flow and the formation of eddies similar to those described for flat plates.

An experiment performed by S. J. Kline [7] showed the effect of diverging boundaries on eddy formations. Kline found that he could control the separation point by adjusting a flexible lucite wall used as a diffuser. The position of the separation point has not only been found to be dependent on the geometric shape, but also on the boundary roughness and the Reynolds number, $(VD\rho/\mu)$. Separation builds up gradually, forcing the main stream of flow away from the boundary, causing the separation point to move

upstream until equilibrium is reached. Separation is generally located at the upstream end of an expansion for central angles greater than 10° .

The eddies formed due to flow separation have relatively low forward velocities and occur near the boundaries or in regions where rapidly-moving streams enter or move past stagnant or slower moving streams. As these slow moving eddies mingle with the rapidly moving central part of the stream, the kinetic energy of the central core is decreased. The resulting discontinuity in the velocity profile results in high shearing stresses occurring in the fluid and violent turbulence being generated. The efficiency of changing the kinetic energy to flow work energy (also referred to as pressure head, P/γ) depends on how much of the initial energy is used forming and dissipating eddies and how much energy is direct frictional loss. Kinetic energy is inherent in the large eddies and is dissipated in the form of heat and is called the head loss due to the expansion.

Fig. 4 is a schematic representation of the location of eddies which have been measured in and downstream from diffusers. The form loss of a diffuser results from slow moving eddies shearing on the faster moving central core of the diffuser. When separation occurs, the entire flow

pattern and pressure distribution is greatly altered from that of established uniform pipe flow.

The distance required to dissipate these eddies is called the settling length. This settling length is the distance from the diffuser entrance to the point where the hydraulic grade line becomes linear as shown on Fig. 5. Kalinske [6] found that the settling length for $\theta = 7.5^\circ$ occurred approximately 13 diameters downstream from the diffuser entrance. Both Archer and Kalinske found that the flowrate had little effect on the settling length.

Since the velocity profile within the settling length is irregular and consists of eddies, the settling length is

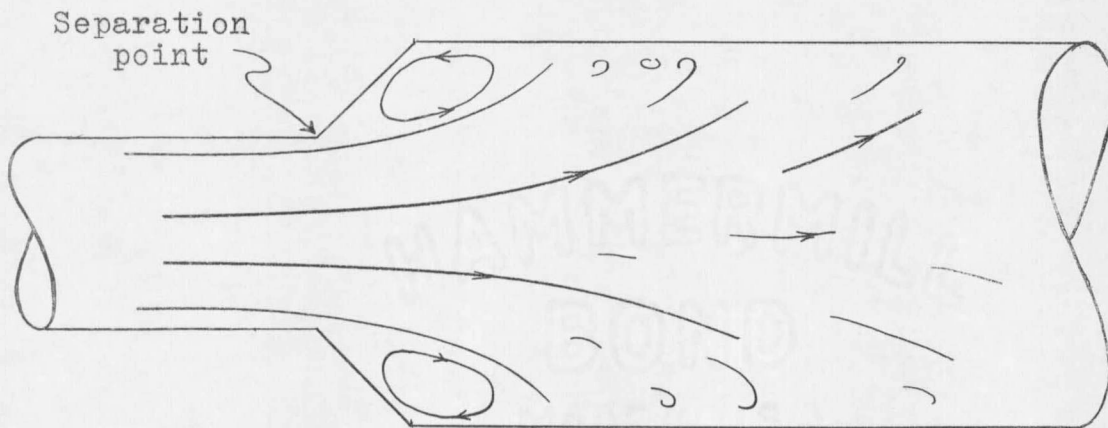


Fig. 4. Schematic representation of diffuser flow.

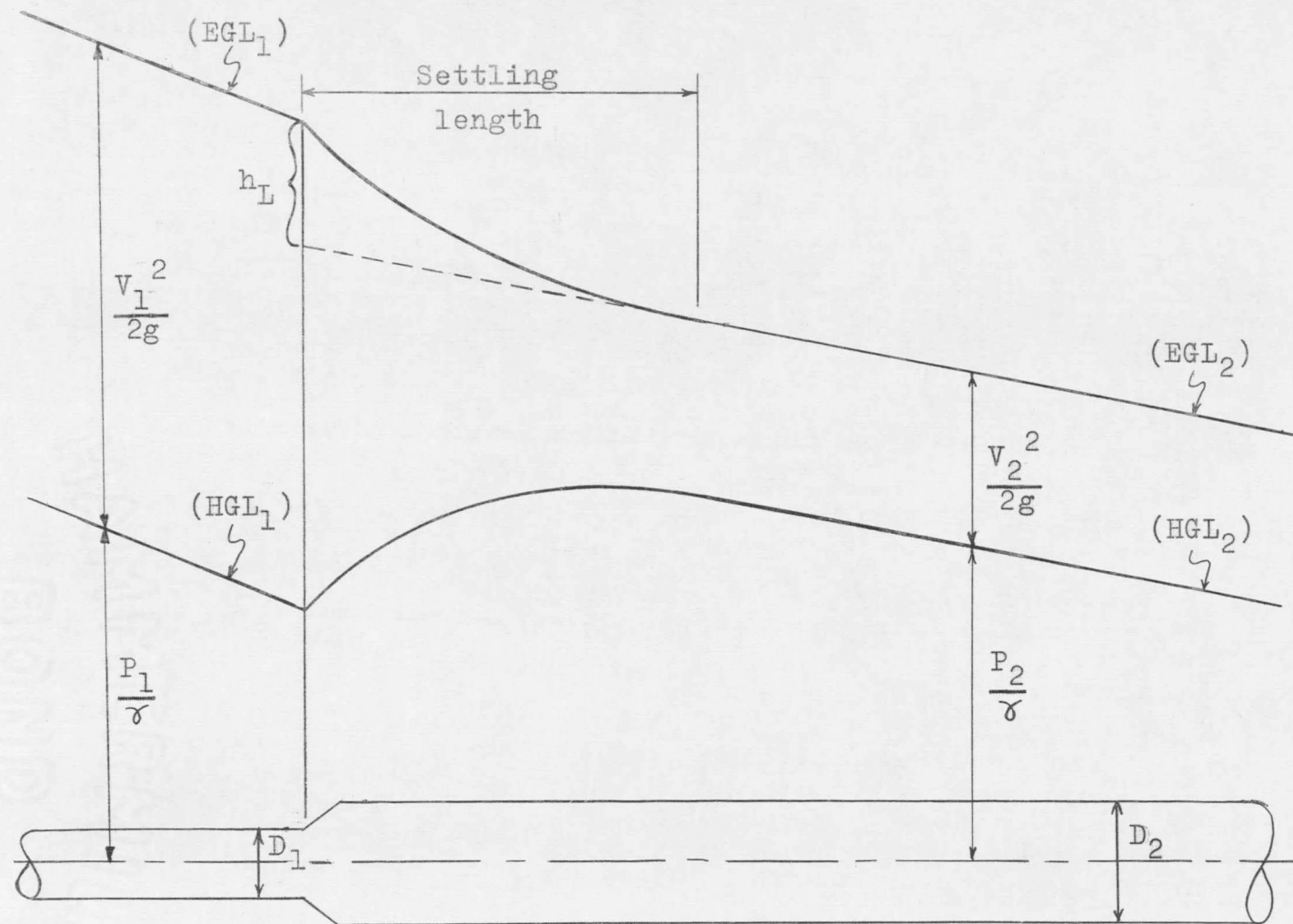


Fig. 5. Diagram of energy grade line and hydraulic grade line in the vicinity of a diffuser.

a function of the magnitude and distribution of the turbulence created. That is, the larger the eddies to be dissipated the longer the settling length. The pressure recovery of the flow is not complete until the flow has become uniform in the downstream section. A graphical indication of the head loss, h_L , due to the expansion is represented on Fig. 5. Establishing the elevations of upstream and downstream energy grade lines at the diffuser entrance will allow for the head loss, h_L , to be determined.

B. Flow of liquid-solid mixtures

Liquid-solid mixtures may be divided into two classifications for pipe flow: "settling" mixtures and "non-settling" mixtures, according to the rate of settling of the solid material which depends upon the particle size, shape, density, the concentration of particles, the liquid density and viscosity, and the velocity of the liquid. The solids used for this study may be classified as a non-settling mixture. The literature available for non-settling mixtures is quite limited.

Mikio Hino [4] developed theoretical equations relating turbulence intensity and the decay time of eddies to the concentration of solids in the flow. The non-settling solids in this case had an average size of 100 microns to 155 microns. Hino found that the decay time of eddies

decreased as the solids concentration increased. A decrease of 60% in the decay time was found for a concentration of 3.0%. He also found that an increase in concentration broke up the large scale eddies into smaller eddies, therefore increasing the turbulence intensity. Breaking one large rotating eddy into several smaller rotating eddies increases the amount of rotation in the fluid, or as stated, the turbulence intensity.

K. E. Spells has stated [12] that in turbulent flow it is the velocity fluctuations with instantaneous components perpendicular to the general direction of flow which are able to disperse solid particles through the bulk of the fluid and maintain them in suspension. The ease with which this is done must depend upon the particle size and density, and the intensity of turbulence.

Assuming that Hino's results can be applied to particles larger than 155 microns in diameter, a correlation between solid-liquid mixtures in pipe flow and flow through a diffuser can be made.

The turbulence of the fluid tends to spread the solid particles throughout the entire bulk of flow and therefore into the eddies near the diffuser boundaries. The particles dispersed throughout the flow shortens the distance down-

stream required to dissipate the energy in these eddies indicating a reduction in the head loss.

Since the form losses of a diffuser results from large slow moving eddies shearing on the faster moving central core of the diffuser, a reduction in the form loss can be expected if these same eddies are not allowed to form. Suppressing them will increase the turbulence intensity and reduce the settling length.

Assuming this is how solids will change the flow structure in a diffuser, the hypothesis can be stated as: "For a liquid-solids mixture flowing through a diffuser, the head loss will be less than that obtained for liquid alone."

To test the hypothesis, experimental tests were run to determine K_L for expansions with various solids concentrations for given flow rates and to compare these values of K_L to those occurring with no solids at the same flow rates.

CHAPTER III

EXPERIMENTAL METHODS

To test the hypothesis, that solids added to flowing fluids reduced the head loss caused by enlargements, the following experimental design included (1) head loss measurements, and (2) the determination of solids concentrations.

A. Head loss measurements

The head loss occurring in fittings is equal to the elevation of the upstream energy line $(EGL_1)_e$ at the beginning of the fitting minus that of the downstream energy grade line $(EGL_2)_e$ projected back to the fitting entrance as shown on Fig. 5. This loss includes both the form losses and friction losses on the boundaries of the diffuser. Eq. (3) gives the relationship for head loss resulting from fittings.

$$h_L = (EGL_1)_e - (EGL_2)_e \quad (3)$$

The subscripts indicate the elevation of the individual energy grade lines directly above the diffuser entrance. Combining Eq. (2) with Eq. (3) and rearranging:

$$K_L = \frac{(EGL_1)_e - (EGL_2)_e}{(V_1 - V_2)^2 / 2g} \quad (4)$$

The energy grade line (EGL) at any point in the flow is equal to the hydraulic grade line (HGL) at that point plus the velocity head ($V^2/2g$) based on the average velocity. The elevation of the hydraulic grade line (HGL) is equal to the pressure head (P/γ) plus the elevation head (Z).

Computations of head loss, h_L , require that the upstream energy grade line (EGL_1) and downstream energy grade line (EGL_2) be established at the diffuser entrance. To establish this, the slopes of the energy grade lines (SEGL) must be determined.

The HGL has a constant downward slope in the direction of flow for a uniform flow in a pipe of constant cross section. This uniform slope, or rate of head loss, is due to a constant rate of friction loss in the fluid occurring at the pipe boundaries and is called the normal friction slope. The non-linear portion of the downstream hydraulic grade line, HGL_2 , in Fig. 5 is caused by a non-uniform velocity profile of the flow resulting from a boundary layer separation and the formation of eddies in and downstream from the expansion. Average pressures measured in this region of flow result in the curved portion of the HGL_2 . The velocity distribution in the region of a pipe expansion is shown on Fig. 4.

If the horizontal datum is taken at the center line of the pipe, the elevation head is then everywhere equal to zero. Referring to Fig. 6, the EGL₂ at point 1, (EGL₂)₁, is equal to P₁/γ plus V₂²/2g. At the diffuser entrance (EGL₂)_e is represented by Eq. (5).

$$(EGL_2)_e = (EGL_2)_1 + (SEGL_2)(XR) \quad (5)$$

where XR is the distance in feet from point 1 to the diffuser entrance as shown in Fig. 6.

(EGL₁)_e is established in the same manner and is represented by Eq. (6) using NDS as the station number for the pressure tap just upstream from the diffuser entrance.

$$(EGL_1)_e = (EGL_1)_{NDS} - (SEGL_1)(X_{NDS} - XR) \quad (6)$$

where SEGL₁ is the slope of the energy grade line upstream from the diffuser.

Since the velocity head is constant for a given cross sectional area, the slope of the hydraulic grade line, SHGL, is equal to SEGL in the region of uniform flow. The following describes the procedure to determine SHGL, and consequently, SEGL.

To establish the SHGL, the h_L between every pair of pressure taps is determined by Eq. (7), which is derived in Appendix B.

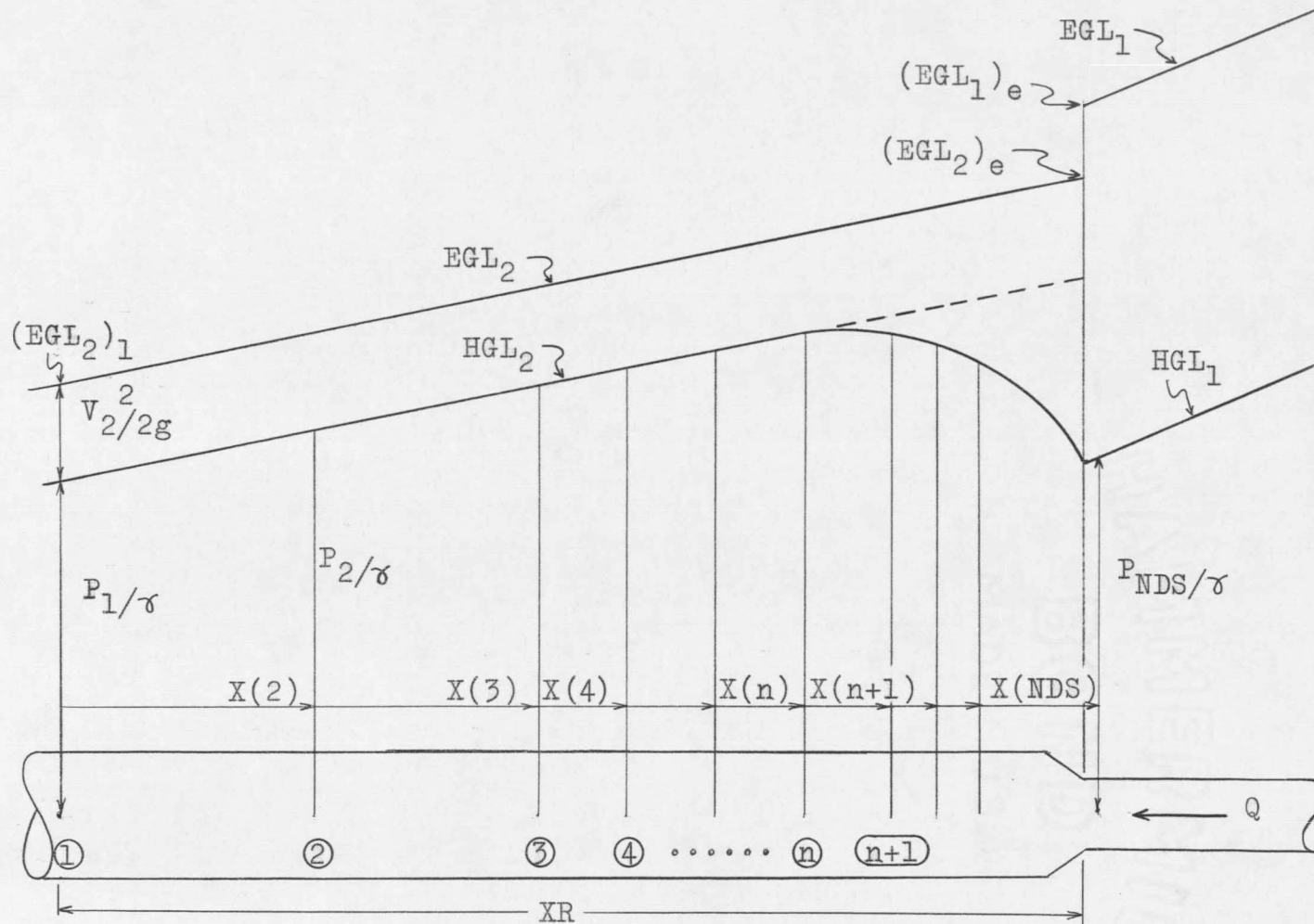


Fig. 6. Diagram showing notations used in computational analysis.

$$h_L = \left[\frac{S.G._{mf} - S.G._{ff}}{S.G._{ff}} \right] \left[\frac{\Delta Y}{12} \right] \quad (7)$$

$S.G._{mf}$ and $S.G._{ff}$ are the specific gravities of the manometer fluid and fluid flowing in the system respectively.

ΔY is a differential manometer reading in inches between two pressure taps. Knowing the h_L and distance between each pair of pressure taps, ΔX , the SHGL between each pair of pressure taps is computed by:

$$SHGL = (h_L) / \Delta X \quad (8)$$

The slopes between successive pairs of pressure taps in both the upstream and downstream regions of uniform flow varied slightly because of experimental errors and slight irregularities in the flow.

Using Eq. (8) and statistical methods as explained in Appendix C, the best fit downstream slope of the hydraulic grade line, $SHGL_2$, is established from pressure tap 1 to the high point on the HGL shown on Fig. 7, using the following iterative procedure.

Starting at the downstream end of the test section with the pair of pressure taps 1 and 2, an approximation of the downstream slope of the hydraulic grade line, $SHGL_2$, is determined from:

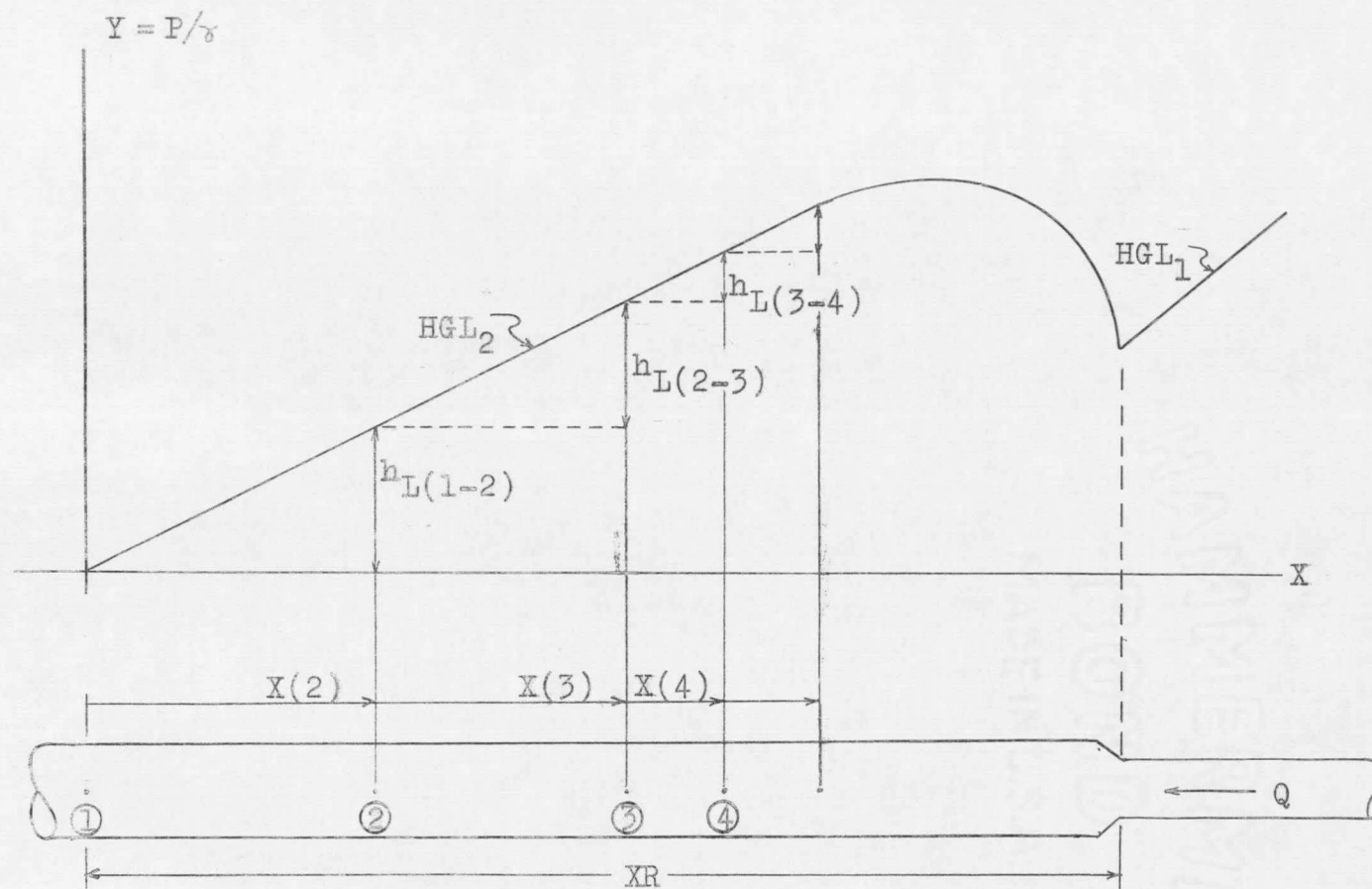


Fig. 7. Coordinate system used to determine $SHGL_2$.

$$(\text{SHGL}_2)_1 = h_{L(1-2)} / [X(2) - X(1)] \quad (9)$$

with subscript 1 denoting the section of pipe between pressure taps 1 and 2. Eq. (9) is a first approximation of the best fit slope of the downstream hydraulic grade line and is set equal to SHGL_2 .

The slope of the hydraulic grade line between the next successive pair of pressure taps $(\text{SHGL}_2)_2$ is computed:

$$(\text{SHGL}_2)_2 = h_{L(2-3)} / [X(3) - X(2)]$$

where the subscript 2 denotes the section of pipe between pressure taps 2 and 3.

A comparison of the slopes is made by the following equation:

$$|(\text{SHGL}_2) - (\text{SHGL}_2)_2| - E \leq 0 \quad (10)$$

E is the statistical test for the significance of the difference between the two slopes. The confidence of the E is dependent on the number of points used to determine the slopes being compared. E is discussed in Appendix C. If this inequality is satisfied, SHGL_2 is then corrected by a statistical method called "the method of least squares" which is also discussed in Appendix C. From linear regression the general slope equation for a line that best fits a set of points X_i and Y_i may be written as:

$$B = \frac{\sum X_i Y_i - \sum X_i \sum Y_i / n}{\sum X_i^2 - (\sum X_i)^2 / n} \quad (11)$$

where n is the number of observations. In Eq. (11) X_i is the horizontal distance from the origin of coordinates and Y_i corresponds to a vertical measurement, or pressure head (P_i/γ). B represents the slope of the best fit line and now becomes the new approximation for $SHGL_2$. The origin of coordinates was set at the final pressure tap of the experimental system as shown by Fig. 7.

If the inequality is not satisfied in any portion of the test section further downstream than is consistent with other results the data is checked and the apparatus inspected for physical conditions causing this. The data is subsequently run again.

Assuming that the inequality is satisfied, using the new approximate slope $SHGL_2$ the procedure was repeated to compare this slope with that between pressure taps 3 and 4 by:

$$|(SHGL_2) - (SHGL_2)_3| - B \leq 0 \quad (12)$$

If the inequality of Eq. (12) is satisfied the slope is once again corrected by the general slope equation, Eq. (11).

From Fig. 6 the above procedure was repeated until Eq. (12) was no longer satisfied, or until the inequality of Eq. (13) is satisfied.

$$|SHGL_2 - h_{L(n-(n+1))} / [X(n+1) - X(n)]| - E > 0 \quad (13)$$

For the section of pipe represented as $(n-(n+1))$ the absolute value in Eq. (13) is greater than E , indicating that the change in slope is significant according to the statistical t test used to determine E . The $SHGL_2$ is then equal to the corrected value of slope obtained up to section (n) by Eq. (10); this is used to determine $SEGL_2$ and $(EGL_2)_e$.

The slope of the upstream energy grade line $SEGL_1$ was established in much the same manner as $SEGL_2$. Since the entire EGL_1 is linear the use of Eq. (10) was not required to determine the significance of slopes between upstream pressure taps.

Knowing $SEGL_1$ and $SEGL_2$, Eq. (3), (5) and (6) are used to determine the h_L . Since the mix flowrate QM was measured in each case, the velocities V_1 and V_2 could be determined from the continuity equation $V = Q/A$. Eq. (2) is then used to determine K_L .

B. Concentration of solids

The concentration of the solids in the test line was determined using the following procedure. The inflow of

clear water and outflow of mix flow were measured simultaneously while holding the volume in the mix tank constant. When these conditions were met the flow of solids, Q_S , could be determined from the following equation:

$$Q_S = Q_M - Q_W$$

The volume concentration C , of solids in the system can then be expressed by:

$$C = \frac{Q_S}{Q_M} = \frac{Q_M - Q_W}{Q_M}$$

C is the volumetric concentration of solids expressed as a per centage of the total volume of the mixture.

CHAPTER IV

APPARATUS DESCRIPTION

The apparatus description will be given in three parts: (1) a description of the plastic chips used to simulate wood chips, (2) a general description of the entire apparatus, and (3) a detailed description of the 30 ft. test section and measuring equipment used for determining the head loss due to the expansion. Fig. 8 shows an overall view of the general apparatus with the test section removed.

A. Plastic chips

The solids material used for the test was plastic chips with a specific gravity close to 1.01. The chips, red in color, had dimensions of $3/32$ in. x $3/8$ in. x $1/2$ in. with a tolerance of $\pm 1/16$ in. in the width and length.

B. Complete laboratory apparatus

It was necessary to have a separate feed system for both the chips and water in order to obtain different chip concentrations in the system. Two storage bins were maintained for this purpose. The required volumetric flowrate of chips and water were then discharged into a mixing tank for injection into the test line.

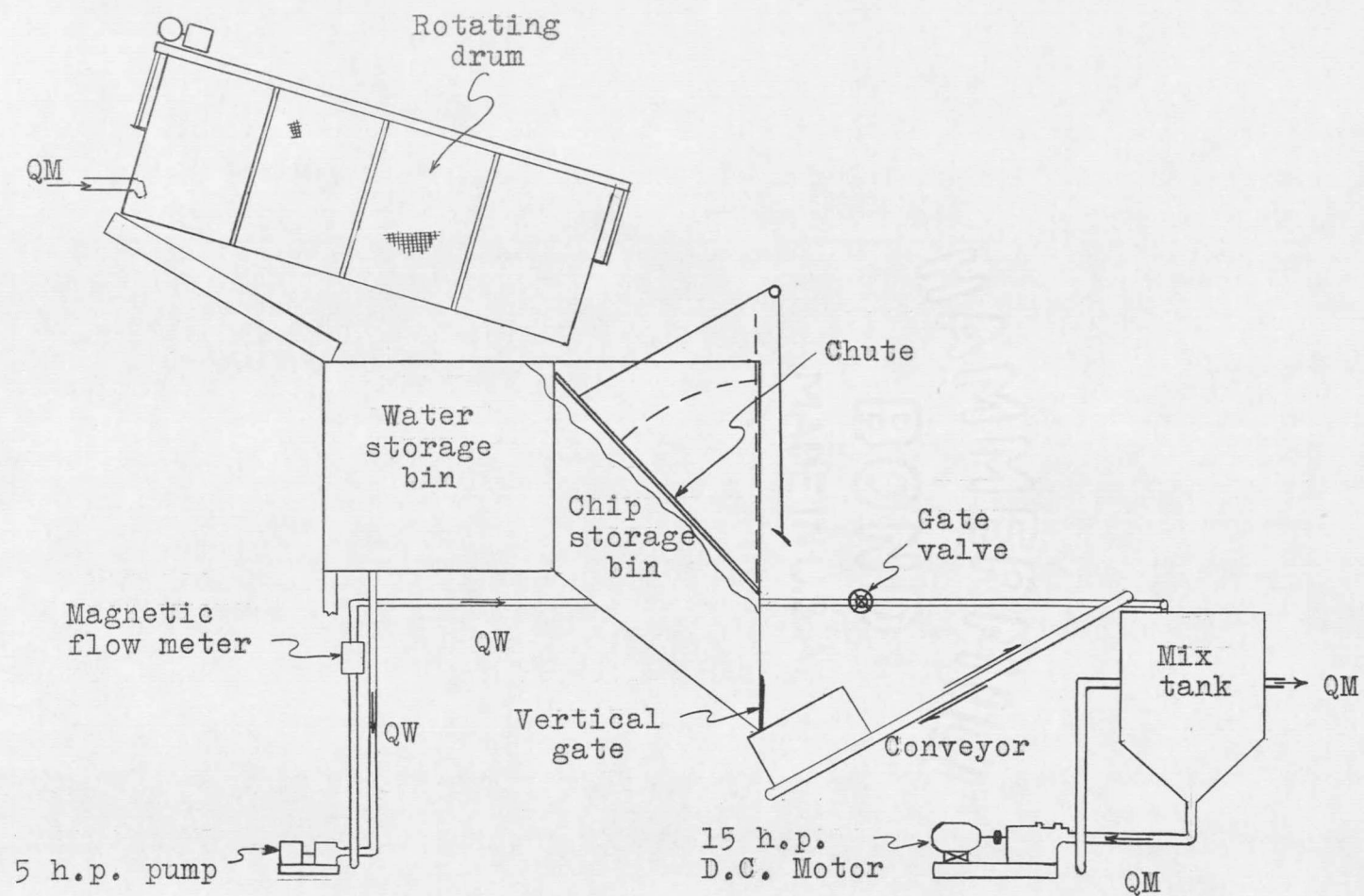


Fig. 8. General apparatus.

Water was pumped from the storage bin through a 3-inch pipe line into the mixing tank by a 1740 rpm, 5 h.p. centrifugal pump. The desired flow of water was regulated by a gate valve in the 3-inch line. Installed in this same 3-inch line was a magnetic flow meter for measuring the water flow into the mix tank.

The plastic chips were dumped from their storage bin by gravity onto a conveyor belt 18 inches wide then elevated before being dumped into the mix tank as shown on Fig. 8. The amount of chip feed was controlled by a vertical gate mounted on the chip storage bin. For recirculation of chips through the system, a chute was lowered to the position shown by a cross sectional view of the chip bin in Fig. 8 allowing the chips to pass directly from the rotating drum down to the conveyor. To remove chips from the system, the chute was raised to a vertical position.

The contents in the mixing tank were then pumped by a 400 gpm, 1090 rpm centrifugal pump into a 4-inch line leading to the test section. The rate of flow of the mixture was controlled by installing a 15 h.p. direct current motor to the pump. The motor speed and consequently the pump speed was controlled by using the rheostat on the central control panel as shown on Fig. 9.

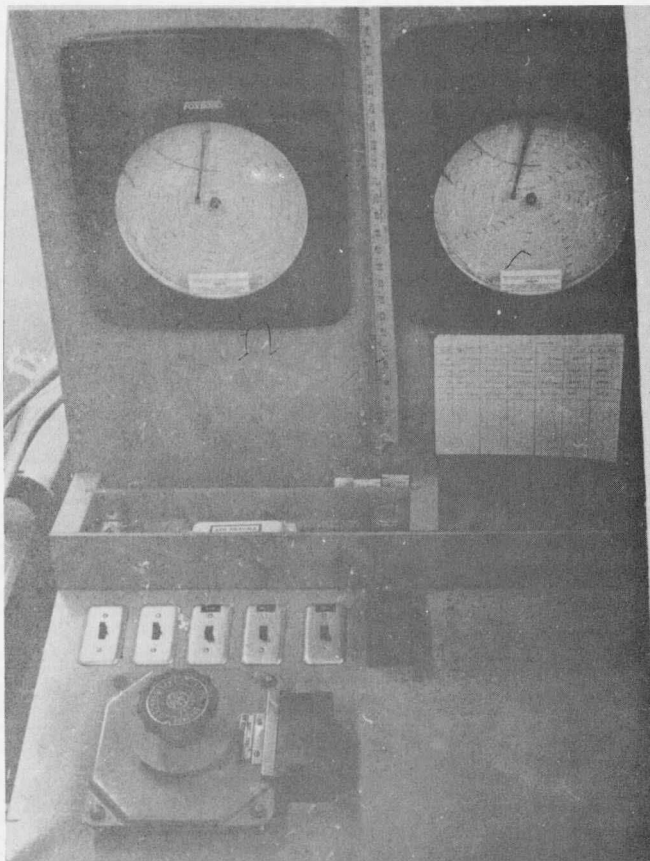


Fig. 9. Control panel.

From the 4-inch line, the mixture passed through another magnetic flow meter before being throttled down to a 3-inch plastic line 12 feet long. The flow then passed through a clear plastic diffuser into a $16\frac{1}{2}$ -foot length of 4-inch plastic line. This particular 3-inch and 4-inch line constituted the test section in which all pressure measurements were performed. Six-foot lengths of plastic pipe

were joined by tongue and groove joints to form the required pipe lengths.

After passing through the test section the flow was then elevated and dumped into a rotating drum. The outer wall of this drum was constructed of $\frac{1}{4}$ -inch wire mesh. This wire mesh allowed the water to separate from the chips. The water dropped directly downward onto a pan where it then ran into the water storage tank. The chips continued down the inclined drum and dropped either into the chip bin or onto the chute for recirculation.

Fig. 9 shows the control panel which consisted of switches, flow meter charts, a manometer showing the mix tank level and a rheostat for control of the direct current motor. Also, within easy reach of the operator was a gate valve handle for controlling the flow of clear water into the mix tank. Off-on switches were provided for the conveyor belt, rotating drum, both pumps and the two flow meter charts. Located directly under the control panel were the relay and fuse boxes for all electrical equipment used.

C. Test section and measuring devices

1. Flow and concentration measurement

The operation of the magnetic flow meter, Fig. 10, is based on the principle that the voltage induced by a con-

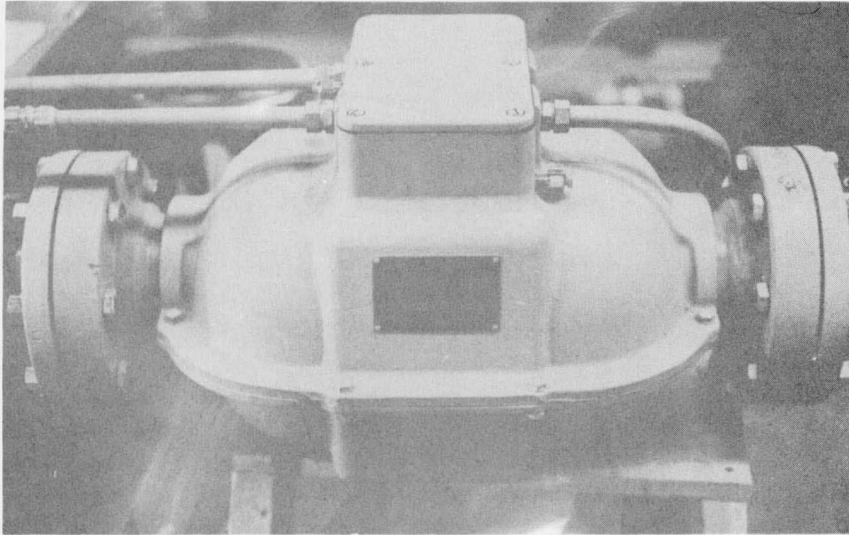


Fig. 10. Magnetic flow meter.

ductive fluid moving through a magnetic field is proportional to the velocity of the fluid. A magnetic field is produced by saddle-shaped coils wrapped around the flow. The induced voltage generated by the conductive fluid is sensed by two electrodes located between the coils and at the fluid boundaries. These two electrodes are connected to one of the dynalog flow recorders shown in Fig. 9.

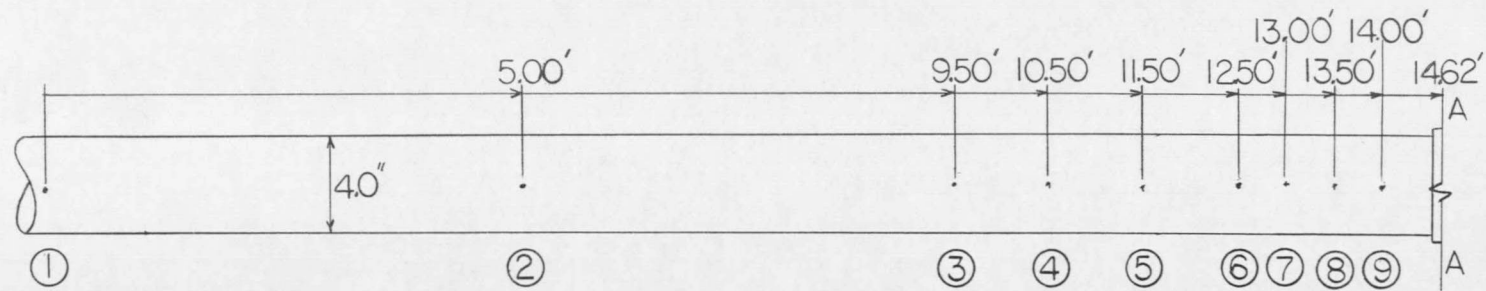
The faster the fluid moves through the magnetic field, the greater the voltage generated. A direct, linear measurement of the flowrate of the fluid is thus provided. The two flow meters were calibrated to provide direct readings in gallons per minute through the transducer system.

As the density of the plastic chips was approximately that of the water, they were assumed to have the same velocity as the water and thus did not affect the induced voltage. The chart readings therefore indicated volumetric flow rates of either clear water or the water-chip mixture.

2. Pipe and test section

The test section consisted of 3- and 4-inch inside diameter pipe made of acrylic plastic $\frac{1}{4}$ -inch thick. Fig. 11 shows the test section and measurements between manometer taps.

At each pressure reading station three different manometer taps were installed at 120° from each other as shown on Fig. 12. A $5/16$ -inch hole was first drilled $1/8$ -inch deep into the $1/4$ -inch plastic section. Using $5/16$ -inch plastic rod, nipples 1 inch in length were shaped on a lathe to fit the drilled hole in the test pipe. The nipples were then drilled completely through with a $1/8$ -inch drill. The nipples were glued into the prescribed holes and allowed to dry. A $3/32$ -inch drill was then used to finish drilling through the $1/4$ -inch wall of the test section. Tygon tubing of $1/4$ -inch I.D. was placed over these nipples for connection to the manometer board.



The spacing between pressure taps 9 through 18 varied for individual expansions.

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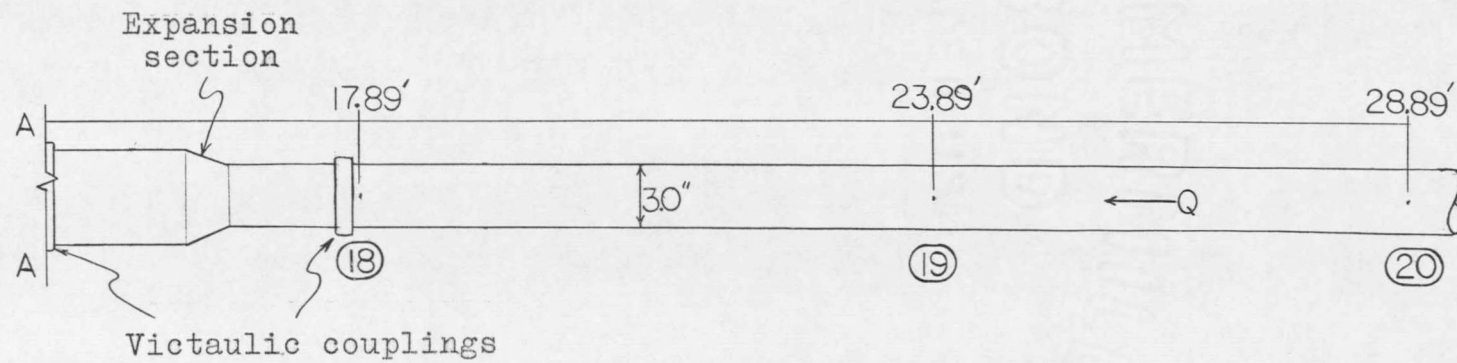


Fig. 11. Test section.

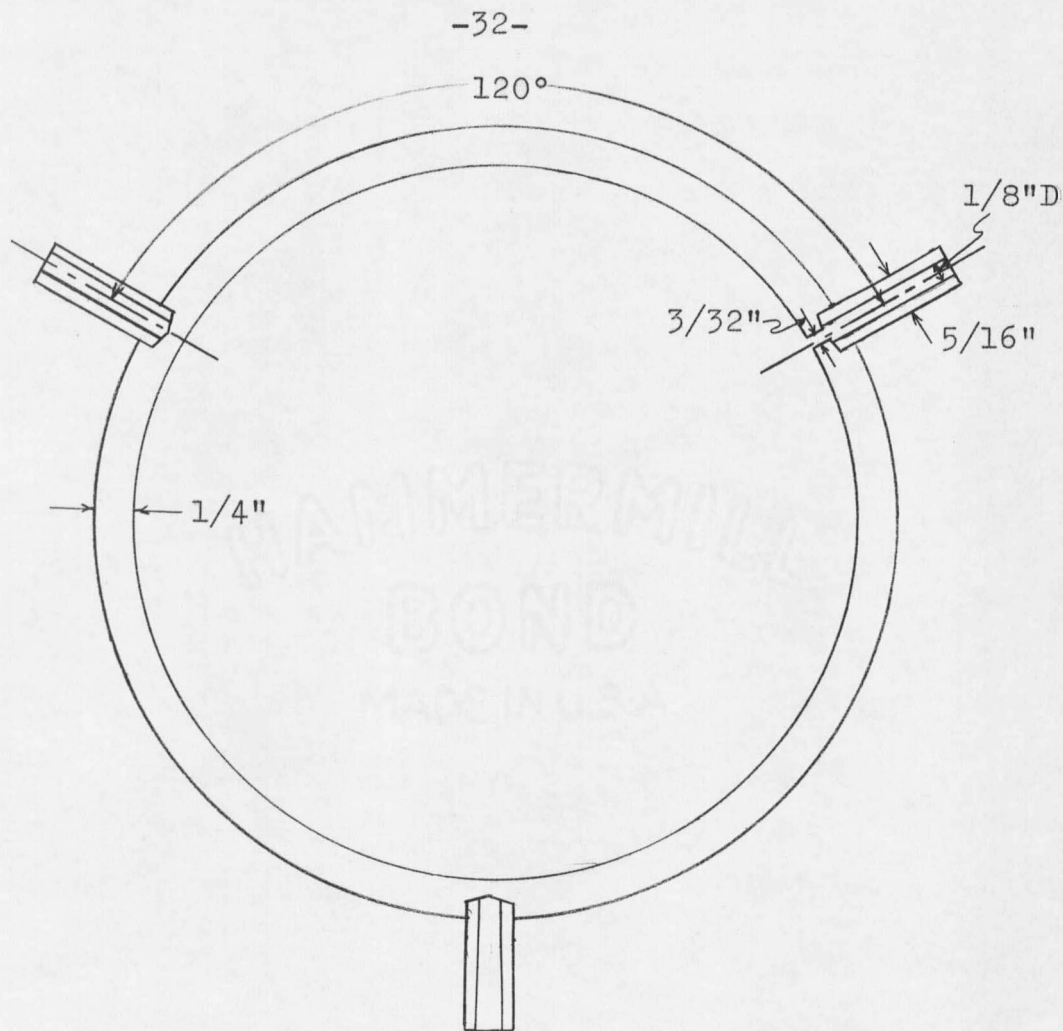


Fig. 12. Pipe cross section showing pressure tap installations.

Five separate expansion angles were used for the test. The central angles θ were 10° , 30° , 60° , 90° and 180° . Fig. 13 shows each individual expansion angle.

Each diffuser piece was so constructed that the overall length was 3 feet. This allowed for the removal and instal-

lation of individual diffusers into the test section within a matter of minutes.

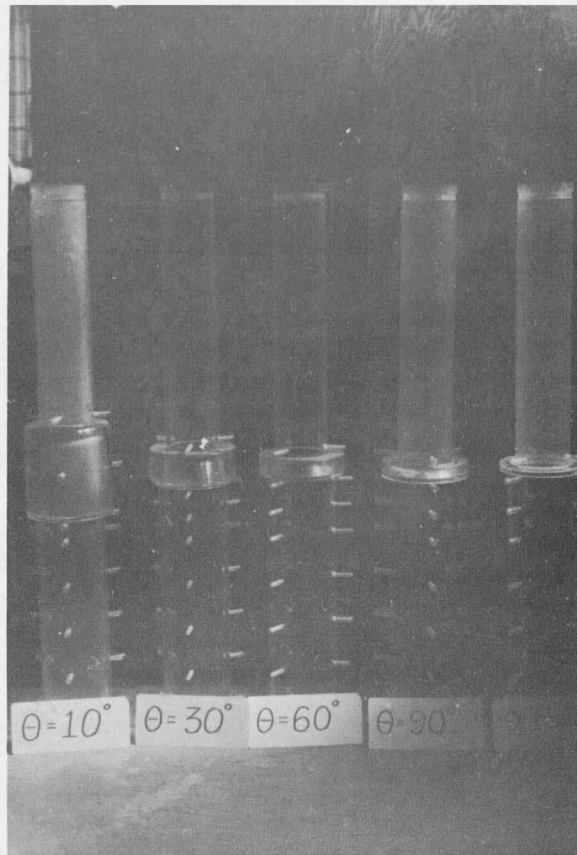


Fig. 13. Diffuser sections.

The diffuser formed butt joints between the upstream and downstream test sections. Victaulic couplings were used to hold the pipe sections in place. A victaulic coupling consists of a clamp fitted over a rubber gasket. Each end of pipe was grooved for the setting of the clamp. Fig. 14 is an illustration of a victaulic coupling.

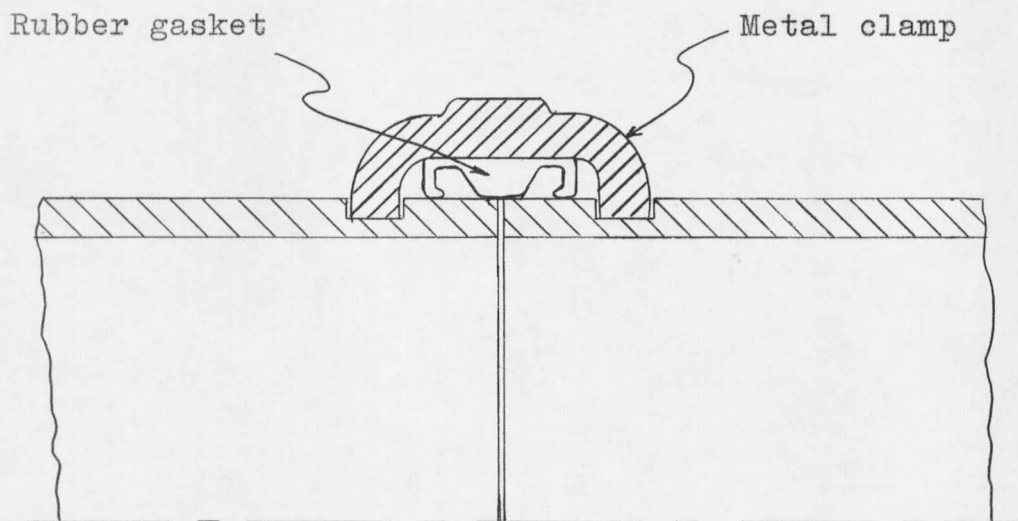


Fig. 14. Cross section of victaulic coupling.

3. Manometer board

The pressure measuring devices consisted of a manometer bank and flexible tygon tubing connecting this bank to the individual pressure taps. The manometer bank was designed to measure differential pressures, not total pressures. Fig. 15 shows the manometer board.

The manometer fluid was injected into the system through a valve on top of the reservoir tank. With valves A and B opened and C closed, the manometer fluid entered the manometer tubes. The fluid from the test pipe then filled the remainder of the manometer tube and tubing which connected the top of the manometer to the pressure tap.

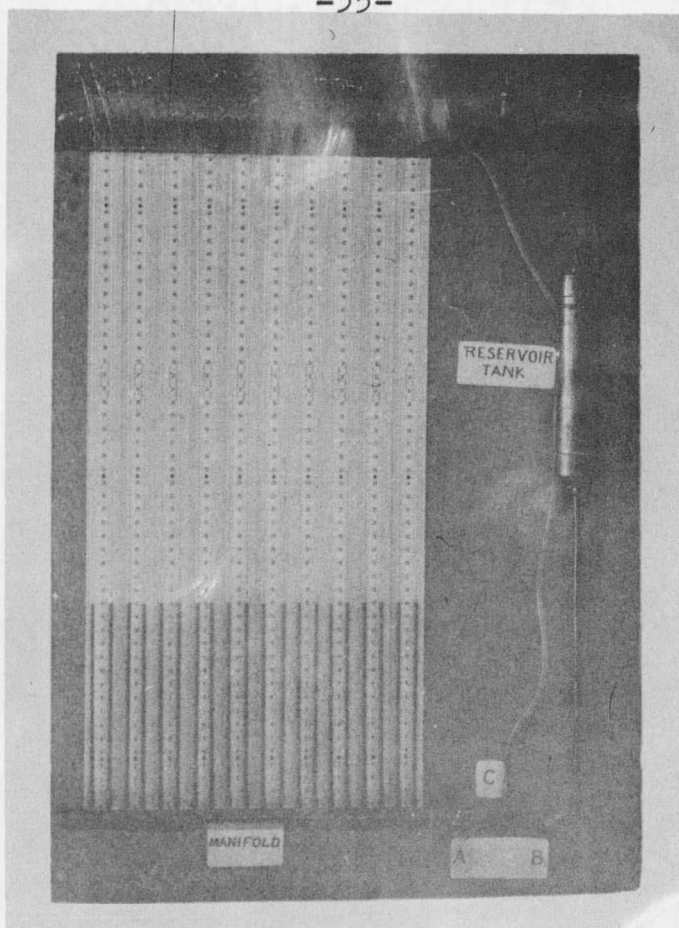


Fig. 15. Manometer board.

The top of each manometer tube was provided with a glass "T" section for bleeding air from the system. One arm of the "T" led to the manometer, one to the pressure tap on the test line and the other contained a short piece of flexible tubing which was opened and closed by means of a screw clamp. With the system in operation, opening the screw clamps allowed water and air to move through the tygon tubing and out this arm of the glass "T" section.

Valve C was used to flush clean water through the manometers and tygon tubing back into the test system. For this operation valve B was closed and valve A was open.

When the system was in operation, valve A was closed, subjecting the entire system to the pressure in the manifold as shown on Fig. 15. For maximum deflections of 48 inches at maximum velocities with water as test fluid, carbon tetrachloride was found suitable for the manometer fluid. The carbon tetrachloride was colored red with Sudan III dye.

The reservoir tank, manifold, and connecting lines to the valves were constructed of copper tubing. Every connection was fitted with solder. The manifold had 5/16-inch copper tubing protruding upward one inch for connecting the glass manometers. Tygon tubing was fitted over both the protruding copper nipples and glass tubing. The upper end of the glass manometers were held in place by extending them through 3/8-inch holes in a piece of sheet metal.

A 35 mm camera was used to obtain pictures of the manometer board for data collection. Instantaneous readings were obtained and the running time of the machinery was reduced considerably by the use of a camera. A tripod was used to give the camera rigid support. For obtaining pic-

-37-

tures without the use of lighting black and white film with an ASA number of 400 was used.

CHAPTER V

TEST PROCEDURE

A. Preparation of apparatus

For each individual diffuser, data was collected for mix flowrates of 151, 226, and 301 gallons per minute, gpm. In addition, a test was run at 330 gpm for the 10 degree expansion. The above flowrates correspond to velocities of 4, 6, 8 and 8.75 fps in the 4-inch line. At each flowrate tests were run with chips at volumetric concentrations of 0, 5, 10, 15 and 20%.

Prior to a data collection run it was necessary to clean the manometer board. The manometer fluid was removed from the board by siphoning. Valves A and C were opened while B was closed and the tygon tubing connected to valve C was then lowered to siphon the fluid into a container. By opening valves A and C and closing B, as shown on Fig. 15, clean water could be flushed through the manometer tubes. After removing the screw clamp from the "T" section on top of the board, a small brush was inserted into the manometer tube for cleaning. Clear water would then flow through the open tube. The clamp was then replaced and the brushing procedure repeated for each tube.

To remove impurities the carbon tetrachloride was strained before being placed into the reservoir tank. Valve C was then closed and B was opened to allow the carbon tetrachloride to flow into the manometer board. The system was then started and the air bled from the manometer tubes by opening the screw clamps one at a time.

Two people were required to collect the necessary data. The console operator controlled the mix flow Q_M , the clear water flow Q_W , and the chip flow while notifying the cameraman when to take each individual picture of the manometer board. This notification was necessary since Q_M would continually vary up to as much as ± 25 gpm.

The cameraman prepared an identification tag for each specific run. This tag was placed beside the manometer board and contained Q_M , Q_W , the concentration C , the expansion angle θ , and the date. This information corresponded to the headings in the data book.

B. Procedure for data collection

The data collection procedure was to obtain ten pictures for all concentrations at a given flowrate. A new flowrate was then selected and the procedure repeated. For an example of the procedure the 151 gpm flow will be used.

The data collection started with QM and QW both being set to 151 gpm. When these values were reached (zero concentration) the console operator would notify the cameraman when to take a picture. For each picture taken the console operator would enter into the data book QM, QW and the time on the flow chart corresponding to each specific picture. Ten pictures of the manometer board were taken.

With 10 pictures of the pressure data secured for zero concentration the console operator steadily increased the flow of chips in the system and at the same time slowly decreased the flow of clear water to a predetermined flow-rate required for the transportation of 5 per cent solids in 151 gpm of mixture. Prior to starting the general testing program a chart with values of QW was prepared for given values of QM and C.

When the level of the mix tank became steady, the console operator then notified the cameraman that QM was steady and close to the desired value, at which time a series of ten pictures was taken. After obtaining 10 pictures more chips were injected into the system and the procedure repeated for mix flowrate of 151 gpm and 10% concentration. When the data for all concentrations at QM = 151 gpm was secured, the system was cleared of chips and the entire

procedure was repeated for $QM = 226$ gpm and $QM = 301$ gpm. Two runs at all concentrations were obtained at $QM = 151$ and 226 gpm to check the repeatability of the data.

After processing the film, a strip film projector was used to project the pictures of the manometer board on a screen. The manometer readings from each negative were recorded along with the corresponding QM and QW from the data book. This data was recorded on a data form standardized for the IBM 1620 Model II computer and is shown on Fig. 16. From these computation sheets the data was transferred to input data cards for use in a Model II, 1620 IBM computer.

Upon comparison of results of the analyzed data, reruns were made if necessity occurred. A plot of K_L vs. C for a given mix flowrate gave an indication of which data was consistent in its trends. After the reruns were completed, the expansion angle was replaced and the entire procedure repeated.

Party _____

Date _____

Col No.	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
1	152	142	11.75	10.45	9.50	9.20	8.95	8.70	8.60	8.55	8.50	8.50			
2	150	142	11.90	10.55	9.50	9.20	9.00	8.75	8.65	8.60	8.50	8.55			
3	152	142	11.65	10.45	9.50	9.20	8.95	8.75	8.65	8.60	8.50	8.55			
4	152	142	11.90	10.60	9.55	9.20	9.00	8.75	8.65	8.60	8.50	8.55			
5	152	142	11.70	10.50	9.50	9.20	9.00	8.75	8.60	8.60	8.50	8.55			
6	151	142	11.60	10.45	9.50	9.20	9.00	8.75	8.65	8.60	8.50	8.50			
7	151	142	11.90	10.50	9.50	9.25	9.00	8.80	8.65	8.60	8.55	8.55			
8	151	142	11.50	10.40	9.45	9.20	8.95	8.70	8.65	8.55	8.50	8.50			
9	152	142	11.90	10.55	9.50	9.20	9.00	8.75	8.60	8.55	8.50	8.50			
10	150	142	11.65	10.45	9.50	9.20	8.95	8.70	8.60	8.55	8.50	8.50			

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Fig. 16. FORTRAN sheet with reduced data.

CHAPTER VI

DATA ANALYSIS

One complete set of data at a given flowrate and concentration for use in an IBM 1620 digital computer is shown in Fig. 16. Columns 1 through 3 indicate the run number. Columns 7 through 18 are for the flowrates of the mix and clear water, respectively, given in gpm. The manometer readings for one run required the use of two lines between the columns 20 and 78. For a given run, going left to right, the readings correspond to pressure taps 1 through 10 on the first line and pressure taps 11 through 20 on the second line. The manometer readings are in inches.

Using the above input data, the computer program performs the following operations in sequence for the computation of K_L :

- (a) Computes the average manometer reading for each pressure tap.
- (b) Computes the variance for each manometer reading as explained in Appendix C.
- (c) Computes the slope between each pair of pressure taps and determines the variance of this slope.

(d). Computes the downstream HGL slope as explained in Chapter 3 and Appendix C. This requires a significance test for the difference between two slopes and the computation of the approximate HGL slope using Eq. (11).

(e) Computes the upstream HGL slope.

(f) Computes K_L from Eq. (2) and the probable error in K_L .

(g). Computes the volumetric concentration of solids.

A sample of the output data tabulation is shown in Appendix D.

To establish a confidence interval of 90% for the downstream HGL slope 10 separate readings of each monometer tube were obtained for each set of data.

Table I is a summary of results for the 10 degree expansion showing values of K_L and the probable error of K_L , PEK_L , for various flowrates and concentrations. Table II through V, which are similar to Table I, for the expansion angles of 30, 60, 90 and 180 degrees are in Appendix E.

On the graphs of K_L vs. C , shown in Figs. 17, 18, 19, 20 and 21, K_L is the loss coefficient determined from Eq. (2) for a given concentration. The results of K_L for clear

TABLE I

SUMMARY OF COMPUTED RESULTS $\theta=10^\circ$

QM = 151 gpm			QM = 226 gpm			QM = 301 gpm			QM = 330 gpm		
C	K _L	PEK _L	C	K _L	PEK _L	C	K _L	PEK _L	C	K _L	PEK _L
0.000	0.02	0.01	0.000	0.09	0.01	0.000	0.15	0.01	-0.011	0.17	0.00
.000	.20	.01	.000	.16	.00	.000	.11	.00	-0.003	.11	.00
.000	.29	.00	.000	.20	.00	.000	.20	.00	.042	.12	.00
.052	.26	.01	.048	.20	.01	.052	.18	.00	.043	.17	.00
.056	.35	.01	.052	.20	.01	.101	.12	.00	.048	.13	.00
.061	.31	.01	.102	.19	.01	.102	.13	.00	.089	.04	.00
.094	.27	.01	.106	.20	.00	.149	.09	.01	.097	.09	.00
.103	.30	.02	.149	.12	.01	.150	.09	.00	.102	.08	.00
.151	.28	.01	.158	.12	.01	.194	.02	.01	.146	-0.01	.00
.153	.30	.01	.204	.08	.01	.246	-0.02	.00	.151	.02	.00
.195	.32	.03	.202	.00	.02				.160	.03	.00
.200	.19	.03							.193	.02	.00
.206	.22	.02							.196	-0.05	.00
									.197	-0.02	.00

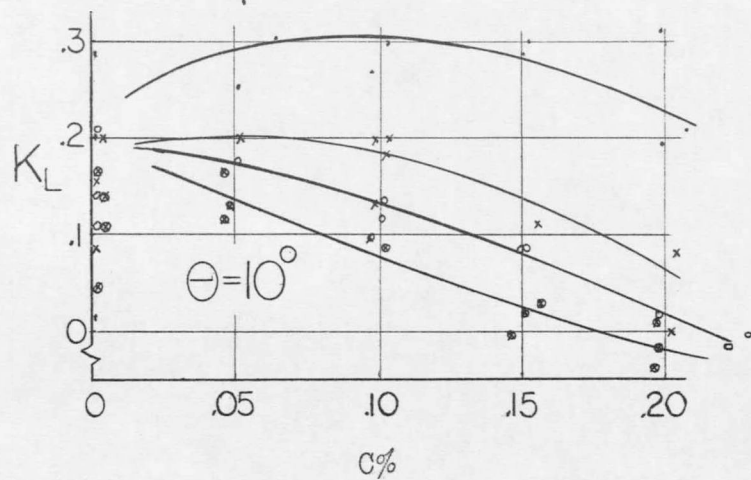


Fig. 17. Graph K_L vs. C

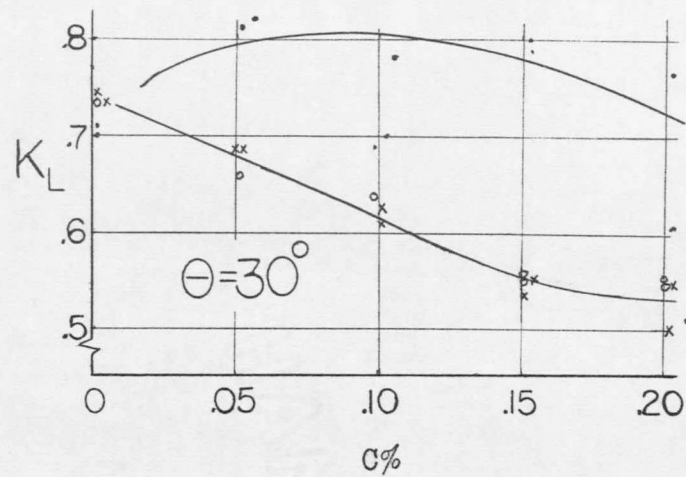


Fig. 18. Graph K_L vs. C

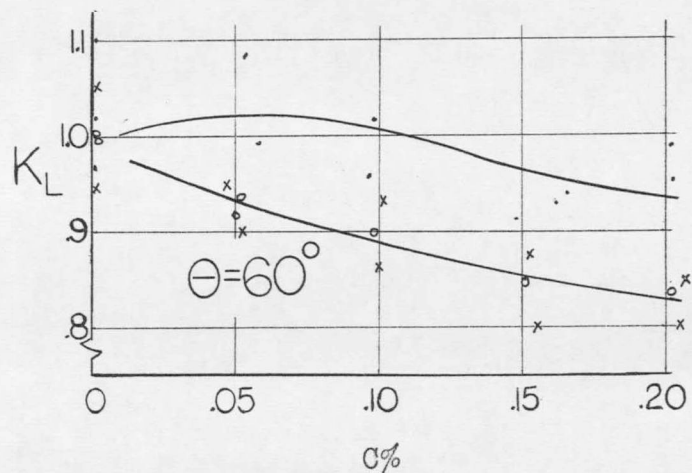


Fig. 19. Graph K_L vs. C

- QM = 151 gpm
- x = 226 gpm
- o = 301 gpm
- = 330 gpm

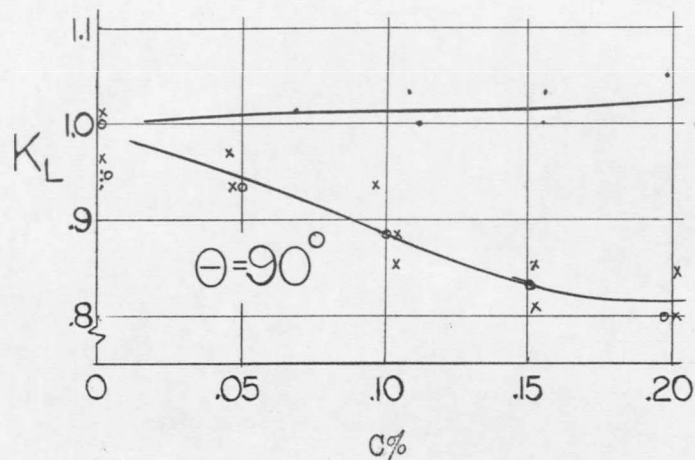


Fig. 20. Graph K_L vs. C

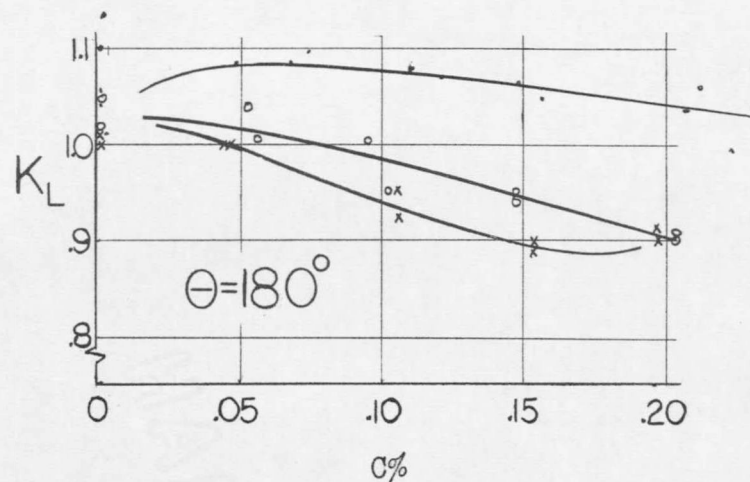


Fig. 21. Graph K_L vs. C

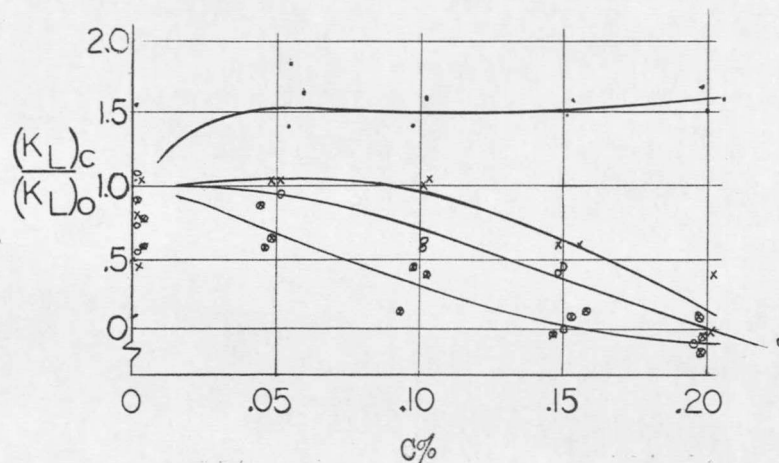


Fig. 22. $(K_L)_c / (K_L)_o$ vs. C ,
 $\theta = 10^\circ$

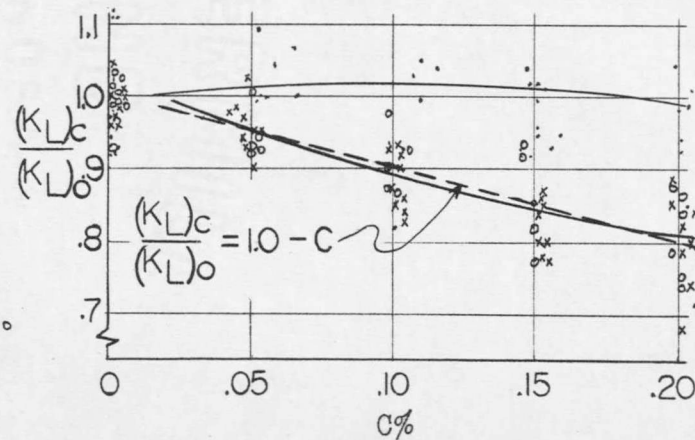


Fig. 23. $(K_L)_c / (K_L)_o$ vs. C ,
 $\theta = 30^\circ, 60^\circ, 90^\circ$ and 180°

water, $(K_L)_0$, agreed closely with the results of both Gibson [3] and Archer [1]. For flowrates of 226 gpm and larger K_L was found to be a function of the expansion angle and the chip concentration. $(K_L)_c$, the loss coefficient determined for a given concentration C , can be expressed as a function of $(K_L)_0$ and C . Using the graphs K_L vs. C , an empirical relationship was derived for any expansion angle to express $(K_L)_c$ and is expressed by:

$$(K_L)_c = (K_L)_0 - 0.9C \quad (14)$$

where C is the concentration in per cent. Eq. (14) is good for concentration in the range of 3 to 20%. The effect of chips on K_L is shown by Eq. (14) and Figs. 17, 18, 19, 20 and 21. An increase in the chip concentration will produce a decrease in K_L .

The trend for the 151 gpm flows was for $(K_L)_c$ to remain essentially constant with a maximum variation up to 0.1 for concentrations of 5 and 10% and to then decrease to $(K_L)_c$ equal to $(K_L)_0$ as the concentration approached 20%. The large variation in analysed data for the 151 gpm flow existed because of the small differentials in the manometer readings. A difference of ± 0.1 inch between the pressure taps P_{NDS} and P_1 shown on Fig. 6 results in a change of

± 0.03 for K_L at this flow. A change in the downstream hydraulic grade line slope of ± 0.0001 changes K_L by ± 0.01 .

Because of the similarity of the graphs of K_L vs. C for all expansion angles, dimensionless curves of $(K_L)_c / (K_L)_o$ vs. C were plotted in Figs. 22 and 23 for use in engineering computations. To use these graphs, a value of $(K_L)_c / (K_L)_o$ is obtained for the desired concentration. Multiplying this value by that obtained for clear water in Gibson's work will yield the desired $(K_L)_c$ value. To obtain an immediate comparison of results a composite graph of $(K_L)_c / (K_L)_o$ vs. C for the expansion angles of 30° , 60° , 90° and 180° was prepared. The 10° diffuser was not included due to the wide variation in $(K_L)_c / (K_L)_o$ created by the small loss coefficients involved. The following approximate empirical relationship was derived for 30, 60, 90 and 180 degree expansions:

$$\frac{(K_L)_c}{(K_L)_o} = 1.0 - C$$

C is in per cent.

When the difference between two slopes is non-significant as determined by the statistical method in Appendix C, the two slopes are assumed to be in the linear portion of the HGL. When the difference in slopes is significant the

HGL is now assumed to be in the curved portion and the computer no longer continues to compute a new approximate slope, which is noted as BT on the output data in Appendix D. This is the position at which the settling length ends as shown by Fig. 5. Due to insufficient data and test methods, the settling length was not determined for any of the test data.

The sample of output data in Appendix D is for a flow-rate of 151 gpm and concentrations ranging from 0 to 20%, all for the 10 degree expansion. A list describing the symbols used on the output is also present. The remainder of the data may be found on file in the Department of Civil Engineering and Engineering Mechanics, Montana State University, Bozeman, Montana.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

A decrease in K_L was observed as the chip concentration increased at flowrates of 226 gpm and larger. This is in agreement with the hypothesis formed in Chapter 2. The value of K_L shows a decrease which seems to be a linear function of the increase in concentration. In the range of data observed K_L decreased by a numerical value of 0.2 for concentrations of 20%.

For the 151 gpm flow at concentrations of 5 and 10% a small increase in $(K_L)_c$ was noted. At higher concentrations $(K_L)_c$ decreased to values of $(K_L)_o$ and slightly smaller. A possible explanation lies in the magnitude of the head loss and probable error of h_L .

For this particular study, improvements in the physical apparatus could be made by:

- (a) Testing various upstream and downstream area ratios.
- (b) Discarding the victaulic couplings which create undue turbulence.
- (c) Using a larger manometer board with a lighter manometer fluid for more accurate results. This would

be particularly helpful at low velocities and would also aid in determining the settling length.

To further increase the technical knowledge of chip transportation in pipe lines, the following studies are recommended for liquid-chip flows at various concentrations and flowrates:

- (a) A study of head losses through valves at various openings.
- (b) A study of losses for various types of pipe intersections.
- (c) A study of losses through pipe bends with various degrees of radius.
- (d) A study of head losses for horizontal and inclined pipes and the effect of diameter of these losses. These effects should then be compared with clear water losses.
- (e) The above test should be run with chips of different specific gravities. Various combinations of these chips should be tested.
- (f) The above test should at some time be performed with actual woodchips.
- (g) A study of the injections and separation of solids for the pipe line system.

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APPENDIX A
DEVELOPMENT HEAD LOSS EQUATION
FOR ABRUPT EXPANSIONS

The flowrate Q through the central volume ABCD of Fig. (A-1) is:

$$Q = A_1 V_1 = A_2 V_2 \quad (\text{A-1})$$

where a is the cross sectional area through which the flow is being measured, and V is the mean velocity through A . The rate of change of momentum $\frac{d(MV)}{dt}$ is equal to the summation of external forces and can be written for non-compressible fluids as:

$$\frac{d}{dt} (MV) = \frac{Q\gamma}{g} (V_2 - V_1) = \frac{A_2 V_2 \gamma}{g} (V_2 - V_1) = \sum F_x = P_1 A_2 - P_2 A_2$$

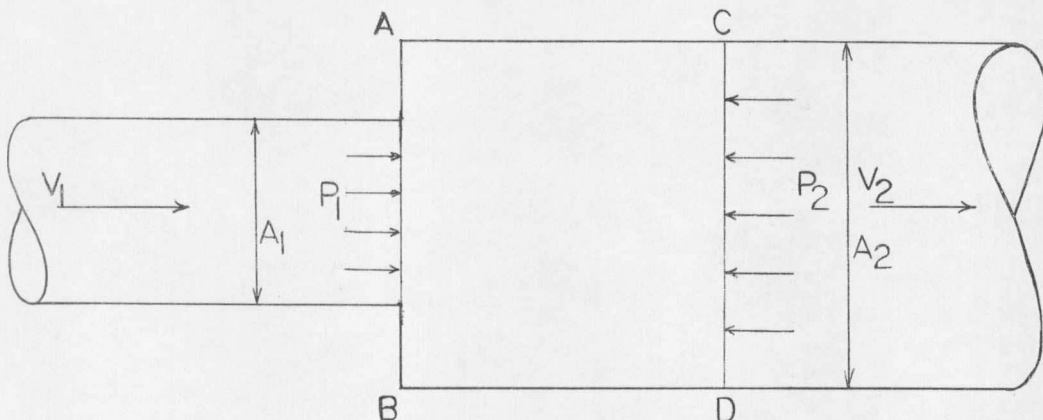


Fig. A-1. Abrupt pipe expansion.

where P_1 and P_2 are pressures in the up and downstream areas respectively and γ is the unit weight of water. Rearranging the above equation:

$$\frac{P_1 - P_2}{\gamma} = V_2 \frac{(V_2 - V_1)}{2g} \quad (A-2)$$

Applying the energy equation between section AB and CD gives:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

Rearranging the energy equation to read:

$$\frac{P_1 - P_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} + h_L \quad (A-3)$$

and equating Eq. (A-2) to Eq. (A-3) for $(P_1 - P_2)/\gamma$ gives:

$$V_2 \frac{(V_2 - V_1)}{g} = \frac{V_2^2 - V_1^2}{2g} + h_L \quad (A-4)$$

Solving Eq. (A-4) for h_L gives:

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

APPENDIX B

HEAD LOSS EQUATION FOR USE

WITH MANOMETER READINGS

The manometer bank was designed to measure differential pressures, not total pressures. With the manifold pressure constant, the larger pressures force the manometer fluid into the manifold as shown by Fig. B-1 where $P_3 > P_2 > P_1$. The head loss in a pipe with steady flow is usually expressed as a change in pressure head, thus

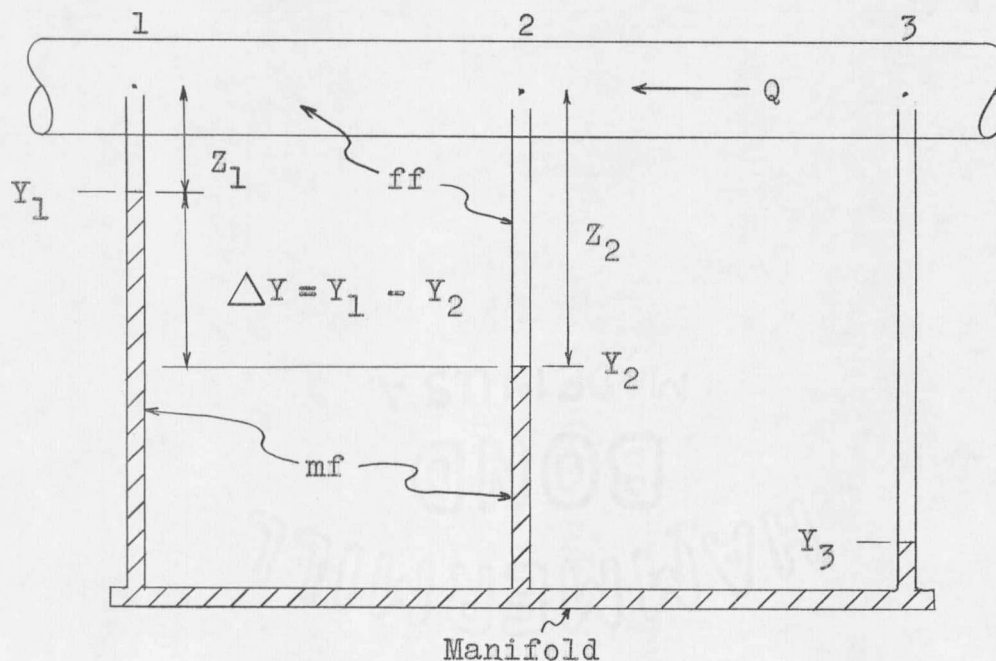


Fig. B-1. Manometer readings in relation to pipe flow.

$$h_L = \frac{\Delta P}{\gamma_{ff}}$$

γ_{ff} is the specific weight of the fluid flowing in the test section and may be written as Eq. (B-1). The dimensions of γ are lb/ft³.

$$\gamma_{ff} = (\gamma_w)(S.G._{ff}) \quad (B-1)$$

From Fig. B-1, Eq. (B-2) can be written with Y_1 in inches.

$$\begin{aligned} P_1 + S.G._{ff} (\gamma_w) Z_1 + S.G._{mf} \frac{(Y_1 - Y_2)}{12} (\gamma_w) = \\ P_2 + S.G._{ff} (\gamma_w) Z_1 + S.G._{ff} \frac{(Y_1 - Y_2)}{12} (\gamma_w) \end{aligned} \quad (B-2)$$

Eq. (B-2) can be rearranged to give the head loss as:

$$h_L = \frac{P_2 - P_1}{(\gamma_w)(S.G._{ff})} = \frac{\Delta P}{\gamma_{ff}} = \left[\frac{S.G._{mf} - S.G._{ff}}{S.G._{ff}} \right] \left[\frac{\Delta Y}{12} \right]$$

where $\Delta Y = Y_1 - Y_2$.

APPENDIX C

DEVELOPMENT STATISTICAL EQUATIONS

A statistical analysis was carried out to find: (1) the variance of the individual manometer readings, and (2) the significance of difference of slopes for finding the best fit slope of the energy grade lines. The latter analysis required the computation of the slope of the best fit line for linear regression.

A. Variance of manometer readings

The first objective after obtaining the manometer readings was to find the statistical confidence interval for each individual reading. The method to obtain this interval is described below.

The average value of all the readings $\bar{y}$ is found by:

$$\bar{y} = \sum_{i=1}^n Y_i / n \quad (C-1)$$

Y_i is each individual reading and n is the number of individual readings. The variance of n readings is:

$$s_y^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{y})^2 \quad (C-2)$$

For ease of computation Eq. (C-2) can be modified for sum of squares of variance SS as follows:

$$\begin{aligned}
 SS &= \sum_{i=1}^n (Y_i - \bar{y})^2 = \sum_{i=1}^n (Y_i^2 - 2\bar{y} Y_i + \bar{y}^2) \\
 &= \sum_{i=1}^n (Y_i)^2 - 2\bar{y} \sum_{i=1}^n Y_i + n\bar{y}^2
 \end{aligned} \tag{C-3}$$

substituting Eq. (C-1) into Eq. (C-3):

$$SS = \sum_{i=1}^n Y_i^2 - 2\bar{y}n\bar{y} + n\bar{y}^2 = \sum_{i=1}^n Y_i^2 - \left(\sum_{i=1}^n Y_i\right)^2/n$$

Eq. (C-2) can now be written as:

$$S_y^2 = \frac{1}{n-1} \left[\sum_{i=1}^n Y_i^2 - \left(\sum_{i=1}^n Y_i\right)^2/n \right] \tag{C-2b}$$

The standard error of the mean $S_{\bar{y}}$ is defined as:

$$S_{\bar{y}} = \sqrt{\frac{S_y^2}{n}} = \sqrt{\frac{\sum_{i=1}^n Y_i^2 - \left(\sum_{i=1}^n Y_i\right)^2/n}{n(n-1)}} \tag{C-4}$$

Using the standard error the confidence interval for $\bar{y}$ can be found by:

$$\bar{y} \pm (t)(S_{\bar{y}}) \tag{C-5}$$

where t is a statistical test number called Student's "t". Student's "t" is dependent on the desired confidence limits and n , the number of observations taken. Values of Student's "t" may be taken from standard statistical tables. For the above test "t" has $n-1$ degrees of freedom, df .

B. Significance between two slopes

In linear regression, Y_i values are obtained from several populations, each population being determined by a corresponding X_i value.

The equation for a straight line may be expressed as:

$$Y = a + bX$$

where a is the Y intercept and b is the slope of the line. In linear regression it is assumed that a straight line best fits a certain set of experimental data. The statistical model for this straight line is written as:

$$Y = a + bX + \epsilon_i \quad (C-6)$$

where ϵ_i is the experimental error. The procedure to make ϵ_i a minimum is explained below. Making ϵ_i a minimum produces the best fit line.

Eq. (C-6) is rearranged and then squared as shown by Eq. (C-7).

$$Q = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (Y_i - a - b X_i)^2 \quad (C-7)$$

For ϵ_i to be a minimum, the following differentials must be set equal to zero.

$$\frac{\partial Q}{\partial a} = \frac{\partial Q}{\partial b} = 0$$

where

$$\frac{\partial Q}{\partial a} = -2 \sum (Y_i - a - b X_i) = 0 \quad (C-8)$$

and

$$\frac{\partial Q}{\partial b} = -2 \sum (Y_i - a - b X_i) X_i = 0 \quad (C-9)$$

Eq. (C-8a) is obtained after rearranging Eq. (C-8) and multiplying by X_i .

$$na \sum X_i + b (\sum X_i)^2 = (\sum Y_i) (\sum X_i) \quad (C-8a)$$

where $na = \sum a$ when a is a constant.

Eq. (C-9a) is derived by rearranging Eq. (C-9) and multiplying by n .

$$b [\sum X_i^2 - (\sum X_i)^2/n] = \sum X_i Y_i - \sum X_i \sum Y_i / n \quad (C-10)$$

Solving Eq. (C-10) for b gives:

$$b = \frac{\sum X_i Y_i - \sum X_i \sum Y_i / n}{\sum X_i^2 - (\sum X_i)^2 / n} \quad (C-11)$$

where Y_i is an individual reading dependent on a particular X_i value. For this study Y_i represents a pressure head, P_i/γ .

An estimate of the variance about regression $S_{y \cdot x}^2$ is denoted by:

$$S_{y \cdot x}^2 = \frac{\sum y^2 - (\sum xy)^2 / \sum x^2}{n-2} \quad (C-12)$$

where x and y are deviations of individual readings about their average, i.e. $x_i = X_i - \bar{x}$. Eq. (C-12) can then be rearranged as:

$$S_{y \cdot x}^2 = \Sigma y^2 - b(\Sigma xy)/(n-2)$$

$$= \Sigma Y_i^2 - (\Sigma Y_i)^2/n - b(\Sigma X_i Y_i - \Sigma X_i \Sigma Y_i/n)/(n-2)$$

where Σy^2 is developed the same as it was for Eq. (C-2b), and b is calculated from Eq. (C-11). The variance of b is written as:

$$S_b^2 = \frac{S_{y \cdot x}^2}{\Sigma x^2} = \frac{S_{y \cdot x}^2}{\Sigma X_i^2 - (\Sigma X_i)^2/n}$$

Σx^2 is developed using the same method to develop Σy^2 . The confidence interval for b is:

$$b \pm t \sqrt{S_b^2} = b \pm t \sqrt{\frac{S_{y \cdot x}^2}{\Sigma X_i^2 - (\Sigma X_i)^2/n}}$$

Student's "t" now had $n-2$ df.

To test the homogeneity of two slopes the following equation applies:

$$|b_1 - b_2| \leq t \sqrt{S_p^2 (1/\Sigma x_{1j}^2 + 1/\Sigma x_{2j}^2)} \quad (C-13)$$

The right side of Eq. (C-13) is called the error E as stated in Chapter 3. If the absolute value of the difference of the slope $b_1 - b_2$ is less than $t \sqrt{S_p^2 (1/\Sigma x_{1j}^2 + 1/\Sigma x_{2j}^2)}$ or E , then the difference is non-significant and the total slope is assumed to include both b_1 and b_2 . If $b_1 - b_2$ is

greater than E the difference is significant and no longer assumed to be experimental error. The slope b_2 is assumed to be in the non-linear region of the $(HGL)_2$ as shown on Fig. 6. For this test t has $(n_1 + n_2 - 4)$ df. x_{1j} is the deviation of X_{1j} about $\bar{x}$, or $x_{1j} = X_{1j} - \bar{x}_1$. X_{2j} has similar meaning for the second set of data. S_p^2 is named "the best estimate of the variation about regression" and is expressed by Eq. (C-14).

$$S_p^2 = \left[\sum y_{1j}^2 - (\sum x_{1j} y_{1j})^2 / \sum x_{1j}^2 + \sum y_{2j}^2 - (\sum x_{2j} y_{2j})^2 / \sum x_{2j}^2 \right] / (n_1 + n_2 - 4) \quad (C-14)$$

for ease of computation, substitutions for $\sum y^2$, $\sum x^2$, $\sum xy$ and $b = \sum xy / \sum x^2$, Eq. (C-14) is written as:

$$S_p^2 = \left\{ \left[\sum_{j=1}^{n_1} y_{1j}^2 - \left(\sum_{j=1}^{n_1} y_{1j} \right)^2 / n_1 - b_1 \left(\sum_{j=1}^{n_1} x_{1j} y_{1j} - \sum_{j=1}^{n_1} x_{1j} \sum_{j=1}^{n_1} y_{1j} / n_1 \right) \right] + \left[\sum_{j=1}^{n_2} y_{2j}^2 - \left(\sum_{j=1}^{n_2} y_{2j} \right)^2 / n_2 - b_2 \left(\sum_{j=1}^{n_2} x_{2j} y_{2j} - \sum_{j=1}^{n_2} x_{2j} \sum_{j=1}^{n_2} y_{2j} / n_2 \right) \right] \right\} / (n_1 + n_2 - 4)$$

where b_1 and b_2 are obtained from Eq. (C-11) for each respective sample.

The computational program developed for the IBM 1620 included the above statistical analysis as part of the data reduction.

APPENDIX D

COMPUTER OUTPUT OF ANALYSED DATA

A sample of the computer output with $Q_M = 151$ gpm and concentrations of 0 to 20% for the 10 degree expansion follow herein. Listed below are the identification symbols for the computer output which follows:

- BI - slope between each pair of pressure taps.
- BT - approximate slope determined by the best fit of the hydraulic grade line in the upstream and downstream reach.
- CONC - concentration of solids, %.
- DD - downstream pipe diameter, inches.
- DU - upstream pipe diameter, inches.
- KL - loss coefficient.
- OB/STA - observations per station.
- PE CONC - probable error of concentration.
- PEKL - probable error of K_L .
- QM - mix flowrate, gpm.
- QW - clear water flow into mix tank, gpm.
- SBI - variance of BI.
- SBT - variance of BT.
- SGF - specific gravity of fluid in test section.

SGM - specific gravity of manometer fluid.

SHGLD - downstream hydraulic grade line slope.

SHGLU - upstream hydraulic grade line slope.

STA - station or pressure tap number.

TB - statistical test number to determine the
significance of two slopes.

X - distance from STA 1. to any given STA in feet.

Y BAR - average manometer reading in inches.

Y ER - probable error of Y BAR.

RUN IDENTIFICATION - 10-DEG ENLGMT 150-00-(07-21-65)

STA	X	YBAR	Y ER	BI	BT	SBI	SBT	TB
1	0.000	13.68	.03					
2	5.000	12.53	.01	.0113	.0113	.00022	.00022	0.000
3	9.500	11.44	.02	.0118	.0113	.00017	.00022	-1.879
4	10.500	11.14	.01	.0149	.0115	.00065	.00011	-3.686
5	11.500	10.92	.01	.0105	.0118	.00053	.00009	1.243
6	12.500	10.64	.01	.0137	.0118	.00059	.00007	-2.079
7	13.000	10.50	.01	.0142	.0119	.00113	.00006	-1.242
8	13.500	10.48	.01	.0019	.0120	.00111	.00006	5.161
9	14.000	10.35	.01	.0127		.00135		
10	14.750	10.33	.01	.0013		.00088		
11	15.000	10.40	0.00	-.0137		.00157		
12	15.250	10.57	.02	-.0344		.00273		
13	15.500	10.92	.01	-.0678		.00318		
14	15.750	11.18	.01	-.0521		.00274		
15	15.933	11.44	.02	-.0698		.00420		
16	16.204	13.92	.01	-.4499		.00275		
17	16.496	19.40	.02	-.9218		.00307		
18	17.891	17.55	.02	.0652	.0652	.00076	.00076	0.000
18	17.891	17.55	.02	.0433	.0433	.00016	.00016	0.000
19	23.891	12.26	.02	.0427	.0433	.00024	.00016	1.908
20	27.891	8.78	.02					

QM	QW	CONC	PE CON	SHGLD	SHGLU	KL	PEKL
150.	150.	0.000	0.000	.01200	.04334	.20	.10

RUN IDENTIFICATION - 10-DEG ENLGMT 151-05-(07-30-65)

STA	X	Y	ER	BI	BT	SBI	SBT	TB
1	0.000	11.74	.06					
2	5.000	10.49	.03	.0122	.0122	.00036	.00036	0.000
3	9.500	9.48	.01	.0110	.0122	.00021	.00036	2.763
4	10.500	9.24	.01	.0115	.0117	.00060	.00017	.133
5	11.500	9.00	0.00	.0118	.0116	.00051	.00012	-.113
6	12.500	8.82	.02	.0090	.0116	.00059	.00009	2.199
7	13.000	8.72	.02	.0098		.00154		
8	13.500	8.66	.01	.0054		.00154		
9	14.000	8.61	.01	.0049		.00151		
10	14.750	8.69	.02	-.0052		.00117		
11	15.000	8.79	.02	-.0196		.00390		
12	15.250	8.95	.02	-.0314		.00412		
13	15.500	9.19	.02	-.0462		.00395		
14	15.750	9.65	.03	-.0904		.00414		
15	15.933	9.85	.03	-.0550		.00658		
16	16.204	12.08	.02	-.4036		.00397		
17	16.496	17.43	.08	-.9008		.00780		
18	17.891	15.59	.07	.0646	.0646	.00209	.00209	0.000
18	17.891	15.59	.07					
19	23.891	10.20	.06	.0441	.0441	.00043	.00043	0.000
20	27.891	6.86	.07	.0410	.0441	.00067	.00043	2.879

QM	QW	CONC	PE	CON	SHGLD	SHGLU	KL	PEKL
150.	143.	.052	.002		.01164	.04416	.26	.01

RUN IDENTIFICATION - 10-DEC ENLCMT 151-10-(07-30-65)

	QM	CONC	DU	DD	SGM	SGF	CB/STA	
	151.	.09	2.970	3.940	1.590	1.000	10	
STA	X	YBAR	Y	ER	BI	BT	SBI	SBT
1	0.000	11.80	.04					
2	5.000	10.53	.02	.0124	.0124	.00025	.00025	0.000
3	9.500	9.49	.02	.0113	.0124	.00021	.00025	3.503
4	10.500	9.26	.01	.0113	.0119	.00077	.00013	.551
5	11.500	9.01	.01	.0122	.0118	.00065	.00010	-.426
6	12.500	8.82	.02	.0095	.0118	.00093	.00008	2.087
7	13.000	8.72	.02	.0093		.00200		
8	13.500	8.67	.02	.0049		.00174		
9	14.000	8.65	.01	.0024		.00133		
10	14.750	8.72	.01	-.0045		.00073		
11	15.000	8.81	.02	-.0186		.00311		
12	15.250	8.93	.03	-.0236		.00442		
13	15.500	9.19	.02	-.0511		.00427		
14	15.750	9.66	.01	-.0924		.00282		
15	15.933	9.86	.01	-.0537		.00362		
16	16.204	12.07	.02	-.4009		.00308		
17	16.496	17.40	.08	-.8966		.00784		
18	17.891	15.55	.06	.0650	.0650	.00204	.00204	0.000
18	17.891	15.55	.06	.0446	.0446	.00040	.00040	0.000
19	23.891	10.10	.06	.0413	.0446	.00066	.00040	4.284
20	27.891	0.74	.07					
QM	QW	CONC	PE	CON	SHGLD	SHGLD	KL	PEKL
151.	136.	.094	.002		.01192	.04465	.27	.01

RUN IDENTIFICATION - 10-DEG ENLGMT 151-15-(07-30-65)

	QM	CONC	DU	DD	SGM	SCF	OB/STA	
	150.	.15	2.970	3.940	1.590	1.000	10	
STA	X	YBAR	Y ER	BI	BT	SBI	SBT	TB
1	0.000	11.81	.03					
				.0128	.0128	.00022	.00022	0.000
2	5.000	10.50	.02					
				.0111	.0128	.00015	.00022	5.956
3	9.500	9.48	.01					
				.0118	.0120	.00063	.00013	.271
4	10.500	9.24	.01					
				.0110	.0119	.00062	.00010	.883
5	11.500	9.01	.01					
				.0086	.0119	.00081	.00008	3.111
6	12.500	8.84	.02					
				.0073		.00191		
7	13.000	8.76	.02					
				.0059		.00139		
8	13.500	8.70	0.00					
				.0029		.00098		
9	14.000	8.67	.01					
				-.0042		.00074		
10	14.750	8.74	.01					
				-.0196		.00200		
11	15.000	8.84	.01					
				-.0177		.00212		
12	15.250	8.93	.01					
				-.0531		.00160		
13	15.500	9.20	0.00					
				-.0944		.00168		
14	15.750	9.68	.01					
				-.0483		.00291		
15	15.933	9.86	.01					
				-.3882		.00116		
16	16.204	12.00	0.00					
				-.8991		.00286		
17	16.496	17.34	.02					
				.0671	.0671	.00114	.00114	0.000
18	17.891	15.43	.05					
18	17.891	15.43	.05					
				.0423	.0433	.00030	.00030	0.000
19	23.891	10.14	.04					
				.0415	.0433	.00053	.00030	2.908
20	27.891	6.75	.06					
QM	QW	CONC	PE CON	SHGLD	SHGLU	KL	PEKL	
150.	128.	.151	.002	.01191	.04334	.28	.01	

RUN IDENTIFICATION - 10-DEG ENLGMT 151-20-(08-09-65)

STA	X	YBAR	Y ER	BI	BT	SB1	SBT	TB
1	0.000	12.18	.11					
2	5.000	10.63	.08	.0151	.0151	.00077	.00077	0.000
3	9.500	9.56	.04	.0117	.0151	.00059	.00077	3.481
4	10.500	9.24	.02	.0154	.0135	.00144	.00038	-.625
5	11.500	9.03	.03	.0103	.0135	.00119	.00027	1.166
6	12.500	8.80	.02	.0115	.0133	.00111	.00021	.693
7	13.000	8.71	.02	.0088	.0132	.00206	.00018	.872
8	13.500	8.62	.02	.0088	.0130	.00195	.00016	.856
9	14.000	8.54	.01	.0073	.0129	.00139	.00015	1.139
10	14.750	8.49	0.00	.0032	.0127	.00067	.00014	2.878
11	15.000	8.59	.02	-.0188		.00273		
12	15.250	8.74	.01	-.0295		.00281		
13	15.500	8.93	.01	-.0373		.00216		
14	15.750	9.40	.01	-.0934		.00248		
15	15.933	9.58	.03	-.0470		.00522		
16	16.204	12.14	.02	-.4644		.00405		
17	16.496	17.64	.08	-.9260		.00789		
18	17.891	15.55	.05	.0736	.0736	.00191	.00191	0.000
18	17.891	15.55	.05					
19	23.891	9.75	.09	.0475	.0475	.00049	.00049	0.000
20	27.891	5.99	.18	.0462	.0475	.00142	.00049	.960

APPENDIX E
SUMMARY OF COMPUTED RESULTS

TABLE II
Summary of Computed Results, $\theta = 30^\circ$.

QM								
151 gpm			226 gpm			301 gpm		
C	K _L	PEK _L	C	K _L	PEK _L	C	K _L	PEK _L
0.000	0.71	0.01	0.000	0.73	0.00	0.000	0.73	0.01
.000	.70	.01	.000	.74	.00	.056	.67	.00
.052	.72	.01	.052	.69	.00	.096	.64	.01
.056	.73	.01	.052	.69	.01	.150	.56	.01
.099	.68	.01	.104	.63	.01	.201	.56	.00
.105	.68	.01	.106	.61	.01	.201	.55	.01
.105	.60	.02	.138	.44	.04			
.152	.69	.01	.151	.56	.04			
.152	.70	.02	.154	.56	.01			
.205	.61	.02	.202	.50	.01			
.210	.52	.03	.202	.55	.01			
.203	.67	.04						

TABLE III

Summary of Computed Results, $\theta = 60^\circ$

QM								
151 gpm			226 gpm			301 gpm		
C	K _L	PEK _L	C	K _L	PEK _L	C	K _L	PEK _L
0.000	1.02	0.00	0.000	1.05	0.01	0.000	0.99	0.01
.000	.97	.01	.000	.96	.01	.000	1.01	.01
.054	1.09	.01	.046	.95	.01	.050	.92	.00
.057	.99	.01	.049	1.03	.01	.051	.93	.00
.095	.96	.01	.052	.90	.01	.099	.90	.00
.099	1.02	.01	.100	.87	.01	.151	.86	.00
.149	.91	.02	.101	.94	.01	.202	.84	.00
.161	.93	.02	.154	.88	.01			
.163	.94	.01	.155	.80	.01			
.201	.98	.02	.207	.85	.01			
.203	.94	.04	.207	.80	.01			
.210	.85	.04						

TABLE IV

Summary of Computed Results, $\theta = 90^\circ$

QM								
151 gpm			226 gpm			301 gpm		
C	K_L	PEK_L	C	K_L	PEK_L	C	K_L	PEK_L
0.000	0.93	0.01	0.000	0.97	0.01	0.000	0.94	0.00
.000	.93	.01	.000	1.01	.01	.000	1.00	.00
.000	1.12	.01	.045	.98	.01	.050	.94	.01
.000	1.12	.02	.048	.94	.01	.101	.88	.01
.070	1.01	.01	.097	.94	.01	.151	.83	.00
.108	1.04	.02	.105	.88	.01	.196	.80	.01
.110	1.00	.01	.106	.86	.01			
.155	1.03	.02	.153	.81	.02			
.155	1.00	.03	.153	.85	.01			
.198	1.05	.02	.202	.80	.01			
.206	.99	.03	.202	.84	.01			

TABLE V

Summary of Computed Results, $\theta = 180^\circ$

QM								
151 gpm			226 gpm			301 gpm		
C	K _L	PEK _L	C	K _L	PEK _L	C	K _L	PEK _L
0.000	1.16	0.02	0.000	1.00	0.01	0.000	1.06	0.00
.000	1.05	.01	.000	1.01	.00	.000	1.02	.00
.000	.96	.01	.040	1.00	.00	.054	1.04	.00
.000	1.20	.01	.044	1.00	.00	.055	1.01	.00
.063	1.07	.01	.105	.96	.01	.099	1.01	.00
.072	1.10	.02	.106	.93	.01	.105	.96	.01
.110	1.08	.03	.154	.89	.00	.145	.94	.00
.120	1.07	.02	.154	.90	.01	.147	.96	.01
.148	1.07	.02	.196	.92	.01	.203	.91	.00
.157	1.04	.02	.197	.90	.01	.203	.90	.01
.209	1.04	.04						
.212	1.07	.03						
.233	1.07	.02						



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Charley, R. W.

The effect of chip-shaped
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	13-2-68

N3178

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