



An intrusive slotted cylinder antenna array for subsurface moisture profiling
by David Leo Herrick

A thesis submitted in partial fulfillment of the requirements for the degree of DOCTOR OF
PHILOSOPHY in ELECTRICAL ENGINEERING

Montana State University

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Abstract:

Theoretical analysis and measured performance of an intrusive microwave probe for sensing the distribution of soil moisture from near surface to root zone depths (1-2 m) are presented. The probe consists of a linear array of slot antennas mounted along the axis of a metal cylinder and is intended for borehole installation.

Measurements of self and mutual slot antenna admittances are used to derive a discrete depth profile of the soil's complex permittivity. The moisture distribution is then deduced from this permittivity profile by empirical relations.

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A thesis submitted in partial fulfillment
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ABSTRACT

Theoretical analysis and measured performance of an intrusive microwave probe for sensing the distribution of soil moisture from near surface to root zone depths (1-2 m) are presented. The probe consists of a linear array of slot antennas mounted along the axis of a metal cylinder and is intended for borehole installation. Measurements of self and mutual slot antenna admittances are used to derive a discrete depth profile of the soil's complex permittivity. The moisture distribution is then deduced from this permittivity profile by empirical relations.

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

Knowledge of soil moisture conditions at root zone depth is important for a variety of reasons. In some instances the particular strain of crop to be planted is chosen based on soil moisture estimates. For example, winter wheat is normally planted if moisture appears plentiful in the fall. Otherwise winter snows are allowed to replenish soil water and then spring wheat is planted. Soil moisture conditions are also an important factor in the design and efficient utilization of irrigation systems.

Although soil moisture is an important parameter in crop management decisions, it is not an easy quantity to measure. The distribution of soil moisture varies as a function of time and position due to such factors as percolation, evaporation, irrigation, rainfall, and plant use. Due to the nonhomogeneous nature of soil, there may be great differences in the distribution of soil moisture in adjacent areas. Traditional field

measurements of soil water have depended upon sampling given locations at specified times and depths to determine profiles of the moisture distribution. More recently, remote sensing satellites have made it possible to monitor large tracts of land but the attainable profile depth has been limited to a few centimeters [11].

There are two basic approaches to measuring soil moisture conditions:

- 1) those which determine the potential energy of the moisture;
- 2) those which measure moisture content per unit mass or per unit volume of soil.

The potential energy measurement determines the ease with which a plant may remove a given quantity of water from the soil. This is a fundamental factor governing a plant's response to moisture and may be measured by tensiometers, vapor pressure determination, freezing point depression, etc. However, the potential energy of soil water is, in principle, related to the soil's gravimetric (weight basis) and volumetric (volume basis) moisture content. Thus an indirect estimate of crop performance can be made from measurements of soil

moisture content. Since moisture content is more easily determined than potential energy, the latter approach is commonly taken.

Some Techniques for Measuring Soil Moisture Content

Moisture content can be measured in a variety of ways. A standard laboratory technique is the oven drying of soil samples. The moisture content is expressed as the difference in the sample's wet and dry weights normalized to either the sample volume or the dry weight. In equation form,

$$\%W_g = \frac{\text{Wet Weight} - \text{Dry Weight}}{\text{Dry Weight}} \times 100\% \quad (1-1)$$

or

$$\%W_v = \frac{\text{Wet Weight} - \text{Dry Weight}}{\text{Volume of Soil Sample}} \times 100\% \quad (1-2)$$

where $\%W_g$ is the gravimetric percentage and $\%W_v$ is the volumetric percentage.

Oven drying is probably the most accurate way to measure $\%W$, but this method has several disadvantages:

- 1) it is difficult to automate;
- 2) it is very slow (sample drying often takes 48 hours);

3) continuous real time measurements are impossible.

A newer method which overcomes these limitations is the neutron probe [2]. This instrument consists primarily of a fast neutron source and a detector/counter unit. When inserted in a bore hole, fast neutrons collide with hydrogen atoms in the soil, lose energy, and are scattered back to the detector as slow neutrons. The particle count per second as observed by the detector is thus proportional to the number of hydrogen atoms in the surrounding soil. Since there is more hydrogen associated with soil water than with clay, organic matter, or other soil particles, the slow neutron flux is also proportional to the amount of water in a bulk volume of soil. The relationship between particle count and %W is empirical and the operator must rely on calibration curves. Although less accurate than oven drying, the neutron probe is especially useful when repeated sampling must be performed in a limited area where numerous sampling holes are undesirable. One disadvantage of the neutron probe is that the sensing volume is essentially a sphere 18 inches across. This makes it difficult to resolve fine layers in the vertical

soil profile. Also, deployment of the probe is strictly regulated since radioactive materials are involved.

As an alternative to the neutron probe, the possibility of a microwave based measurement system arises. Considerable attention, both theoretical and experimental, has been given to electromagnetic sensing of soil properties. It is convenient to organize the existing literature into two major categories:

- 1) efforts to relate a soil's electromagnetic properties (e.g. complex permittivity) to its other physical properties (e.g. soil moisture);
- 2) efforts to find practical field techniques for measuring soil electromagnetic properties.

Literature Review of Electromagnetic Properties of Soil

The electromagnetic behavior of soil may be generally described in terms of complex permittivity and permeability tensors. In practice, however, permittivity and permeability are generally treated as scalars. This assumption implies that soil is essentially isotropic. Furthermore, soil permeability is usually assigned a free space value of $\mu_0 = 4\pi \times 10^{-7}$ H/m. This

approximation is reasonable provided the soil has a negligible iron content. Thus, most electromagnetic sensing of soil consists of measuring its complex permittivity $\hat{\epsilon}$.

The permittivity of soils is influenced by many factors such as soil type, moisture content, temperature, and density. Many authors have attempted to derive dielectric mixing formulas for predicting $\hat{\epsilon}_m$, the permittivity of a mixture, in terms of the permittivity of its constituents and the nature of their dispersion. Bruggeman [3] derived expressions which set upper and lower bounds on $\hat{\epsilon}_m$ for a porphyritic two phase mixture. His expressions were experimentally verified by Pierce [4]. De Loor [5] presents a mixing formula for solutions having ellipsoidal, spherical, or disc shaped inclusions. He also discusses various dielectric loss mechanisms and the frequencies at which they become important. Tinga and Voss [6] review a host of dielectric mixture formulas from the literature and then proceed to treat multiphase inclusions of confocal ellipsoids in a uniform host medium. These theoretical results are then used

to predict the complex permittivity of Douglas Fir versus moisture content.

One problem in applying dielectric mixing formulas is that the permittivities of some constituents are highly variable. Thus it is not clear which values to use in the mixing formulas. In particular, the variable properties of water are well known. Windle and Shaw [7] studied the dielectric properties of wool/water systems and found that the permittivity of localized or "bound" water was substantially less than that of bulk water. The so-called bound water consists of a thin layer of water molecules intimately bonded to polar sites in the host medium. This bonding reduces the rotational polarizability of the water molecules and thus lowers their relaxation frequency. This effect has also been reported by Schwan [8] on measurements of water/hemoglobin mixtures from .01 to 1 Ghz. De Loor [9] states that the relaxation frequency of bound water is typically between .3 and 1 Ghz, compared to about 22 Ghz for bulk water. The contribution of bound water to the dielectric properties of a mixture is significant at the lower microwave frequencies and can lead to

errors when applying dielectric mixture formulas. This fact was noted by Tinga and Voss [6].

It is not necessary to rely on dielectric mixture formulas to relate soil permittivity to moisture content. Instead one may choose to generate a data base from experimental measurements on actual soils and then use curve fitting techniques to establish empirical relations. This approach has been used successfully with the neutron probe [2]. Hoekstra and Delaney [10] measured the permittivity of several clay, silt and sandy soils as a function of moisture content and temperature at frequencies ranging from 0.1 Ghz to 26 Ghz. They found that soil type had a strong effect on dielectric properties below about 100 Mhz and that the relaxation frequency of bound soil water was lower than that of bulk water. At frequencies above 3 Ghz they observed that the relation between volumetric water content and complex permittivity is relatively independent of soil type. Unfortunately penetration of more than a few centimeters at these frequencies is difficult because of the high attenuation offered by moist soil.

Literature Review of Field Techniques for Measuring Soil Permittivity

Most field measurement techniques may be classified as either intrusive or nonintrusive. Intrusive probes include buried transmission lines [11] and antennas [12]. Nonintrusive probes include various radar [13-16] and radiometry [17-20] techniques. The Journal of Microwave Power [21] contains a bibliography of 186 publications regarding nonintrusive microwave sensing. Lytle [22] gives an excellent survey of intrusive techniques.

It would be convenient to use nonintrusive (remote sensing) techniques for soil moisture profiling. Large surface areas could be conveniently scanned and no disturbance of the soil would be necessary. Unfortunately such methods appear incapable of sensing to root zone depth. To show this, consider the penetration of plane waves into a lossy medium. Assuming normal incidence and an $e^{j\omega t}$ time dependence, the magnitude of the electric field as a function of depth (z) can be written [23]

$$|E_x| = |E_o|e^{-\alpha z} \quad \text{volts/m} \quad (1-3)$$

where

E_o = magnitude of electric field at surface ($z=0$)

$\alpha = \text{Im}\{k\} = \text{Im}\{w \sqrt{u_o e_o} \sqrt{\hat{\epsilon}_r}\} = \text{attenuation factor}$
(nepers/m).

The attenuation in the lossy medium may be expressed in decibels as

$$A = 20 \log_{10} e^{-\alpha z} \text{ decibels.} \quad (1-4)$$

Typical $\hat{\epsilon}_r$ for a dry soil (5% $\text{gH}_2\text{O}/\text{cm}^3$) at 4 Ghz is $2.6 - j0.3$ [10]. This is equivalent to an attenuation of 68 dB/m, or a round trip attenuation of 136 dB/m. Even with efficient coupling of the plane wave into the soil, this excessive attenuation precludes remotely sensing moisture at significant depths.

This penetration problem is acknowledged in the literature. Ulaby and Batalivala [13] using frequencies between 2 Ghz and 8 Ghz in a radar sensing system obtained useful correlation against measured moisture profiles to a depth of only 15 cm. Deeper penetration may be achieved by using lower frequencies at the cost of reduced resolution and increased dependence on soil type [10].

Another difficulty encountered with surface based measurements is uniquely resolving the nonuniform vertical moisture profile of soils. This problem has been treated theoretically for the case of sensing natural microwave emissions (i.e. radiometry) by Stogryn [20].

If depth of penetration is a prime consideration, some form of intrusive probe is necessary. Many of the intrusive systems described by Lytle [22] involve two or more boreholes. These would be adequate for low frequency applications where attenuation between boreholes is small, but are not very useful at microwave frequencies.

Examples of single hole probes include buried transmission lines and monopole antennas. A transmission line system described by Chudobiak et al [24] employed time domain reflectometry (TDR) techniques. A balanced open wire transmission line was inserted vertically into the soil to a depth of several meters. A fast rise time pulse was then propagated down the line and all reflections were monitored at the surface. The amplitudes of the reflections are proportional to the change in permittivity between soil layers, and the

arrival times are proportional to the depth of the layers. To calibrate the system in terms of depth, marker reflections were established by designing step impedance changes into the transmission line at preset intervals. Unfortunately the lossiness of the soil (discussed above) causes reflections to die out as rapidly as for a nonintrusive probe. Scott and King [25] used a buried monopole to measure the complex permittivity of lake water at VHF frequencies. This procedure worked well for homogeneous media. However, resolution in a layered medium such as soil would be poor because the permittivity is averaged over the length of the monopole.

The Slotted Cylinder Geometry for Soil Moisture Sensing

As was pointed out in the literature review, effects of soil type on permittivity can be minimized by increasing the operating frequency of the sensor. However, increasing the operating frequency seriously restricts attainable depth of penetration due to propagation losses in the soil. One solution to these combined problems is to employ a slotted cylinder geometry of the form shown

in Figure 1-1. This probe consists of a conducting cylinder with an array of slot antennas (also called aperture radiators) situated along its axis. The array is installed intrusively so that each slot antenna senses a different soil depth. The antennas are individually fed from the surface by coaxial transmission lines running through the center of the cylinder. Measurements of slot antenna properties (e.g. VSWR and mutual coupling) provide a discrete (histogram) profile of the soil's electromagnetic characteristics along the length of the cylinder. Experimentally derived curves relating soil moisture and complex permittivity, such as those of Hoekstra and Delaney [10], could then be used to convert the permittivity profile to a moisture profile.

Root zone moisture measurement at arbitrarily high frequencies are possible with this geometry because the sensing antenna is located at the desired depth. In fact the total depth of the profile is governed by the length of the array, and the resolution of the profile is determined by the spacing and beam pattern of the

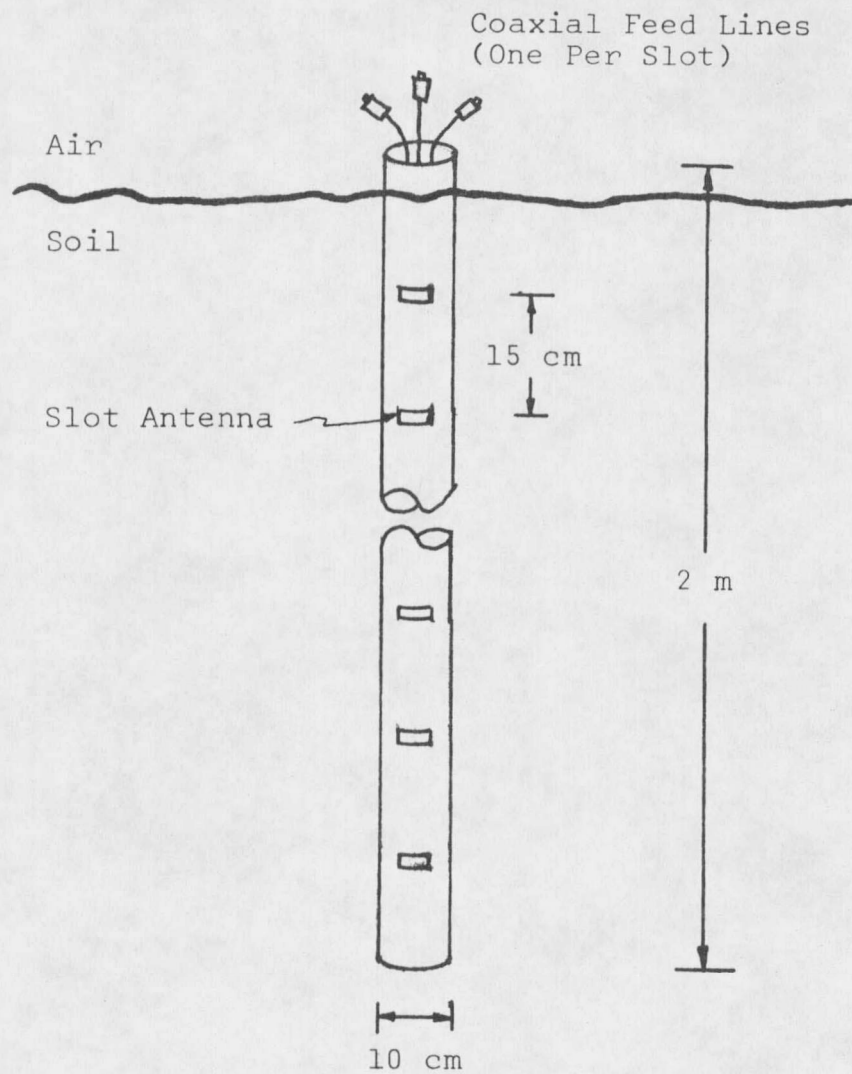


Figure 1-1. A Slotted Cylinder Array for Soil Moisture Sensing.

individual radiators. These two very important parameters are thus under designer control.

It should be pointed out that a circular cylinder geometry was chosen for the probe so that it could be easily installed in a borehole. There is nothing especially convenient about the electrical properties of a slotted circular cylinder. A square cylinder would suffice just as well. Aperture radiators were chosen for array elements so that the array could be flush with the cylinder's surface. The array elements are fed separately from the surface to maximize the possible array configurations. This allows the user to design redundancy into his measurement strategy since the array can be reconfigured at will from the surface.

In effect the slotted cylinder array comprises a multipoint microwave network which can be characterized by a scattering matrix. As will be shown in Chapter 2, it is possible to relate the scattering matrix of the array elements to the soil's effective complex permittivity. These parameter calculations prove costly for a slotted cylinder geometry but it is shown that less costly slotted plane results may often be

substituted with little error. Also since accurate phase measurements are difficult to obtain in the field and require relatively expensive instrumentation, the usefulness of s-parameter magnitudes only is discussed. Chapter 3 presents comparisons between theory and experiment for both slotted cylinder and slotted plane arrays. Chapter 4 summarizes the potential usefulness of the slotted cylinder as a moisture probe.

CHAPTER 2

ANALYSIS OF THE SLOTTED CYLINDER AND SLOTTED GROUND PLANE ARRAYS

Introduction

The primary purpose of this chapter is to develop theoretical expressions for the self admittance, mutual admittance, and scattering parameters of a slotted cylinder array in terms of the complex permittivity of the medium in which it is immersed. These expressions are useful for two reasons:

- 1) if the expected variation of permittivity with soil moisture is known, then the sensitivity of the array to small changes in moisture may be predicted;
- 2) unknown soil permittivity may be measured by solving the inverse expression, i.e. a value of permittivity is selected which fits the measured admittance data over a range of frequencies.

The admittance versus permittivity expressions are also presented for a slotted ground plane geometry because they are considerably easier to evaluate and, under certain conditions, may be substituted for the slotted cylinder equations with negligible error. A

numerical comparison between the self admittance and mutual coupling of a slotted plane and a 6 inch (15.2 cm) diameter slotted cylinder is included.

Previous Work

The analysis of a slotted cylinder array radiating into a lossy environment has been treated previously [26-32]. Much of this work concerned the study of spacecraft aperture antennas subjected to thin lossy plasmas during reentry.

Silver and Saunders [30] gave a general method of analysis for the fields due to a single slot with an arbitrary two dimensional aperture distribution. Then expressions for the far field of a uniformly excited circumferential slot and an infinite axial slot were derived. Knop [29] presents integral expressions for the admittance of a finite axial slot on a dielectric clad cylinder. He assumes a TE_{10} aperture source distribution. Golden and Stewart [27,28] compute the self and mutual admittance of TE_{10} excited axial slots radiating into lossy plasmas. Tetenbaum [32] compares the properties of slotted cylinders with those of slotted ground planes.

Assumptions

The theoretical analysis of the slotted cylinder array as a moisture probe is based on the following major assumptions:

- 1) soil can be represented for moisture sensing purposes as a homogeneous isotropic linear medium;
- 2) the slot antennas are excited by a dominant mode TE_{10} aperture distribution.

The basis for the first assumption is that soil should appear locally homogeneous to the side-looking slot antennas because of the shallow outward penetration of the microwaves (i.e. small skin depth) and the natural horizontal layering of the soil. Also the soil should be isotropic if ferrous compounds are not abundant and should behave linearly for small field intensities. A TE_{10} aperture distribution is assumed because it is the dominant mode of the waveguide feed which excites the aperture. Although some distortion of this distribution is likely, Compton [33] has shown that a first order error in assumed aperture distribution results in only a second order error in the calculated slot admittance.

External Self Admittance of Rectangular Slots on an Infinitely Long Conducting Cylinder

The steps required to formulate the external self admittance of a slot antenna may be summarized as follows:

- 1) determine the Fourier expansion for the tangential components of electric field over the cylinder's surface;
- 2) invoke Poynting's theorem and Parseval's theorem to calculate the complex power flow through the aperture;
- 3) express the self admittance as the ratio of the complex power leaving the aperture to the square of the effective aperture voltage.

Fourier Expansions for the External Fields of the Slotted Cylinder

Consider the single slot geometry shown in Figure 2-1. Let the tangential components of electric field at the cylinder's surface ($r = r_0$) be

$$\vec{E}_\phi = E_\phi(\phi, z, r_0) \vec{a}_\phi$$

and

$$\vec{E}_z = E_z(\phi, z, r_0) \vec{a}_z$$

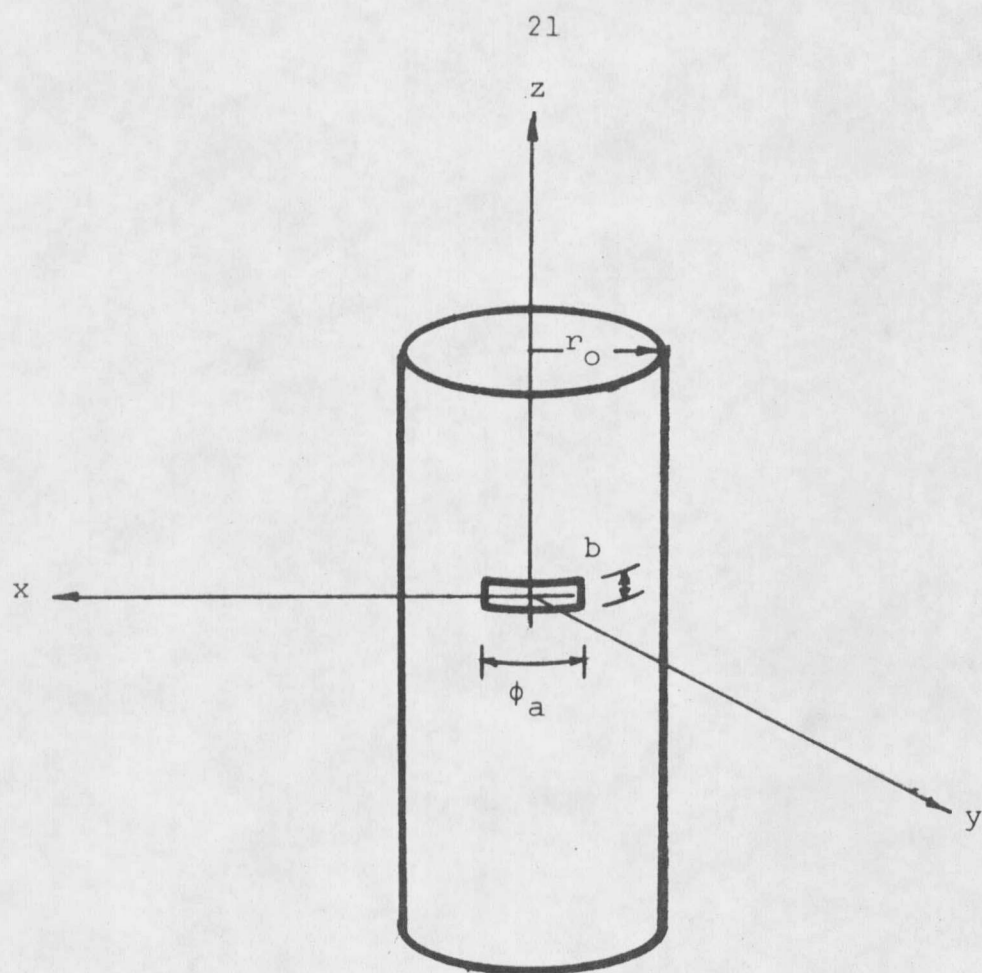


Figure 2-1. Rectangular Aperture in an Infinitely Long Conducting Cylinder.

or simply

$$\vec{E}_{\phi,z} = E_{\phi,z}(\phi,z,r_o) \vec{a}_{\phi,z}$$

where $\vec{a}_{\phi}$ and $\vec{a}_z$ are unit vectors in the ϕ and z directions.

Because of the cylindrical geometry, the fields must be periodic in ϕ and aperiodic in z . Thus $E_{\phi,z}$ may be written as a Fourier series in terms of ϕ and a Fourier integral in terms of z , i.e.

$$E_{\phi,z}(\phi,z,r_o) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \bar{E}_{\phi,z}(n,k_z) e^{-jn\phi} e^{-jk_z z} dk_z \quad (2-1)$$

where $\bar{E}_{\phi,z}(n,k_z)$ is the two dimensional Fourier transform of $E_{\phi,z}(\phi,z,r_o)$ found from

$$\bar{E}_{\phi,z}(n,k_z) = \frac{1}{4\pi^2} \int_{-b/2}^{b/2} \int_{-\phi a/2}^{\phi a/2} E_{\phi,z}(\phi,z,r_o) e^{jn\phi} e^{jk_z z} d\phi dz. \quad (2-2)$$

Equation (2-2) may be expressed more compactly as

$$\bar{E}_{\phi,z}(n,k_z) = I[E_{\phi,z}(\phi,z,r_o)]$$

where the operator $I[]$ is defined as

$$\Gamma[\] = \frac{1}{4\pi^2} \iint_{\text{aperture}} [\] e^{jn\phi} e^{jk_z z} d\phi dz. \quad (2-4)$$

The integration specified by (2-2) is only evaluated over the aperture because the conducting cylinder geometry forces the tangential electric fields to be zero elsewhere.

Following Wait's notation [35], let the operator Γ be defined as

$$\Gamma[\] = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} [\] e^{-jn\phi} e^{-jk_z z} dk_z. \quad (2-5)$$

Using this notation, (2-1) may be written more succinctly as

$$E_{\phi,z}(\phi,z,r_o) = \Gamma[\bar{E}_{\phi,z}(n,k_z)]. \quad (2-6)$$

As shown by Stratton [36] solutions to fields from slotted cylinders may be expressed as a superposition of cylindrical wavefunctions. Assuming an $e^{j\omega t}$ time dependence, the possible components of electric and magnetic fields are [30]

$$E_z = \Gamma[k_c^2 a_n H_n^{(2)}(k_c r)]; \quad (2-7)$$

$$E_{\phi} = \Gamma[jw\mu b_n \frac{\partial H_n^{(2)}(k_c r)}{\partial r} - \frac{nk_z}{r} a_n H_n^{(2)}(k_c r)]; \quad (2-8)$$

$$E_r = \Gamma[\frac{-nw\mu}{r} b_n H_n^{(2)}(k_c r) - jk_z a_n \frac{\partial H_n^{(2)}(k_c r)}{\partial r}]; \quad (2-9)$$

$$H_z = \Gamma[k_c^2 b_n H_n^{(2)}(k_c r)]; \quad (2-10)$$

$$H_{\phi} = \Gamma[\frac{-nk_z}{r} b_n H_n^{(2)}(k_c r) - \frac{jk^2}{w\mu} a_n \frac{\partial H_n^{(2)}(k_c r)}{\partial r}]; \quad (2-11)$$

$$H_r = \Gamma[-jk_z b_n \frac{\partial H_n^{(2)}(k_c r)}{\partial r} + \frac{nk^2}{w\mu r} a_n H_n^{(2)}(k_c r)]; \quad (2-12)$$

where

ϵ_0 = permittivity of free space, 8.854×10^{-12} fd/m

μ = permeability of medium (assumed equal to free space permeability $\mu_0 = 4\pi \times 10^{-7}$ h/m;

$\epsilon_r = \epsilon_r' - j\epsilon_r''$ is the complex relative permittivity of the medium;

$k = w\sqrt{\mu_0 \epsilon_0 \epsilon_r}$ is the plane wave propagation constant of the medium;

$$k_c = \pm \sqrt{k^2 - k_z^2};$$

$H_n^{(2)}(z) = J_n(z) - jY_n(z)$ is the n^{th} order Hankel function of the second kind;

a_n = coefficients of TM cylindrical waves;

b_n = coefficients of TE cylindrical waves.

The expression for k_c contains a sign ambiguity. It is resolved by applying the physical constraint that the desired solution represents outward traveling waves which are bounded at infinity. To show this, consider the large argument asymptotic approximation for $H_n^{(2)}(k_c r)$ [37]

$$H_n^{(2)}(k_c r) \approx \sqrt{\frac{2}{\pi k_c r}} e^{-jk_c r} e^{j(2n+1)\pi/4}.$$

The $e^{-jk_c r}$ term requires

$$\text{Re}\{k_c\} \geq 0$$

to represent outward traveling waves and

$$\text{Im}\{k_c\} \leq 0$$

for the fields to be zero at infinite radius. In other words, k_c is restricted to the fourth quadrant.

Evaluation of the Coefficients a_n and b_n

Equations (2-7) to (2-12) show that all six field components can be expressed in terms of just two sets of Fourier coefficients, a_n and b_n . Alternately, a_n and b_n may be evaluated if any two of the six field components are known. The solution for a_n and b_n is performed most simply in terms of the tangential electric fields at the cylinder's surface for these fields vanish everywhere except on the slot. From (2-2) and (2-6)

$$E_{\phi,z}(\phi,z,r_o) = \Gamma \left[\frac{1}{4\pi^2} \int_{-b/2}^{b/2} \int_{-\phi_a/2}^{\phi_a/2} E_{\phi,z}(\phi,z,r_o) e^{jn\phi} e^{jk_z z} d\phi dz \right]. \quad (2-13)$$

Since (2-7) is a general expression for E_z , it must satisfy the $r = r_o$ boundary condition prescribed by (2-13). Equating (2-7) and (2-13) gives

$$\frac{1}{4\pi^2} \int_{-b/2}^{b/2} \int_{-\phi_a/2}^{\phi_a/2} E_z(\phi,z,r_o) e^{jn\phi} e^{jk_z z} d\phi dz = k_c^2 a_n H_n^{(2)}(k_c r_o)$$

or

$$a_n = \frac{I[E_z(\phi,z,r_o)]}{k_c^2 H_n^{(2)}(k_c r_o)} \quad (2-14)$$

where $I[]$ is the Fourier operator defined by (2-4).

The coefficient b_n is similarly determined. Equating (2-13) with (2-8) gives

$$\frac{1}{4\pi^2} \iint E_\phi(\phi, z, r_0) e^{jn\phi} e^{jk_z z} d\phi dz =$$

$$j\omega\mu b_n \left. \frac{\partial H_n^{(2)}(k_c r)}{\partial r} \right|_{r=r_0} - \frac{nk_z}{r_0} a_n H_n^{(2)}(k_c r_0)$$

or

$$b_n = \frac{-jnk_z I[E_z(\phi, z, r_0)]}{\omega\mu r_0 k_c^3 H_n^{(2)}(k_c r_0)} -$$

$$j \frac{I[E_\phi(\phi, z, r_0)]}{\omega\mu k_c H_n^{(2)}(k_c r_0)} \quad (2-15)$$

where

$$H_n^{(2)}(k_c r_0) \triangleq \frac{1}{k_c} \left. \frac{\partial H_n^{(2)}(k_c r)}{\partial r} \right|_{r=r_0} \quad (2-16)$$

Once a_n and b_n are evaluated, (2-7) through (2-12) may be used to find any of the six field components.

Self Admittance of a Slot Antenna

The external self admittance of a slot in a perfectly conducting cylinder may be expressed as [29]

$$Y_{11} = \frac{2P}{|V_o|^2} \quad (2-17)$$

where P is the complex power flowing through the slot and V_o is some reference voltage, e.g. the dominant mode excitation voltage. The power flow through the slot may be determined by integrating the Poynting vector over the surface of the slot, i.e.

$$P = \frac{1}{2} \iint_{\text{slot}} [\vec{E}_T \times \vec{H}_T^*] \cdot d\vec{s} \quad (2-18)$$

where $\vec{E}_T$ and $\vec{H}_T$ represent the tangential electric and magnetic aperture fields.

It is also possible to evaluate P in terms of the Fourier components of $\vec{E}_T$ and $\vec{H}_T$. This may be shown by applying Parseval's theorem to (2-18) [36]. The resulting expression is

$$P = \frac{r_o}{2} 4\pi^2 \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{E}_T \bar{H}_T^* dk_z \quad (2-19)$$

where r_o is the radius of the cylinder. This latter form is more convenient to apply when $\vec{E}_T$ and $\vec{H}_T$ are expressed in Fourier transform space as in (2-7) through (2-12).

Substituting (2-19) into (2-17) gives the following relation between slot self admittance and the Fourier transforms of the tangential aperture fields:

$$Y_{11} = \frac{4\pi^2 r_o}{|V_o|^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{E}_T \bar{H}_T^* dk_z. \quad (2-20)$$

Before Y_{11} can be evaluated it is necessary to determine $\bar{E}_T$ and $\bar{H}_T$ for a particular slot excitation. Several possibilities are considered in the following sections.

Self Admittance of an Axial Rectangular Slot in a Conducting Cylinder of Infinite Length

The geometry of the axially slotted cylinder is shown in Figure 2-1. By definition, the long dimension of the slot parallels the axis of the cylinder. If a

TE₁₀ excitation is assumed then the tangential electric fields at the cylinder's surface may be written as

$$E_{\phi}(r=r_0, \phi, z) = \begin{cases} V_0 \sqrt{\frac{2}{ab}} \cos \frac{\pi}{b} z; & -\frac{b}{2} \leq z \leq \frac{b}{2} \\ & -\frac{\phi_a}{2} \leq \phi \leq \frac{\phi_a}{2} \\ 0 & ; \text{ elsewhere} \end{cases} \quad (2-21)$$

$$E_z(r=r_0, \phi, z) = 0$$

where

$$a = r_0 \phi_a$$

is the width of the slot and V_0 is the peak dominant mode voltage. The constant $\sqrt{\frac{2}{ab}}$ arises from the requirement that mode functions must be normalized [33]. The slot excitation of (2-21) is then substituted into (2-14) and (2-15) to solve for the a_n and b_n coefficients. Inspection of (2-14) shows that $a_n = 0$ for this case because $E_z = 0$ (there is no E_z field component). The expression for b_n reduces to

$$b_n = \frac{-jV_o \sqrt{\frac{2}{ab}}}{k_c^4 \pi^2 w \mu_o H_n^{(2)'}(k_c r_o)} \int_{-b/2}^{b/2} \int_{-\phi_a/2}^{\phi_a/2} \cos \frac{\pi}{b} z e^{jn\phi_e} e^{jk_z z} d\phi dz \quad (2-22)$$

$$\text{where } H_n^{(2)'}(k_c r_o) \triangleq \frac{1}{k_c} \left. \frac{\partial H_n^{(2)}(k_c r)}{\partial r} \right|_{r=r_o}$$

or

$$b_n = -j \sqrt{\frac{2}{ab}} \frac{V_o b \phi_a \sin(n \phi_a/2) \cos(k_z b/2)}{2k_c \pi w \mu_o (n \phi_a/2) H_n^{(2)'}(k_c r_o) (\pi^2 - b^2 k_z^2)} \quad (2-23)$$

The power flowing through the slot is given by (2-19) where $\bar{E}_t = \bar{E}_\phi$ and $\bar{H}_t^* = \bar{H}_z^*$ for the geometry being considered. Substituting (2-8) and (2-10) into (2-20) and noting that $a_n = 0$ gives the slot admittance as

$$Y_{11} = \frac{r_o^4 \pi^2}{|V_o|^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} j w \mu_o |b_n|^2 H_n^{(2)'}(k_c r_o) k_c H_n^{(2)*}(k_c r_o) k_c^2 dk_z \quad (2-24)$$

or

$$Y_{11} = j \frac{2b\phi_a}{w \mu_o} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{k_c^* \sin^2(n \frac{\phi_a}{2}) \cos^2(k_z \frac{b}{2}) H_n^{(2)*}(k_c r_o) dk_z}{(n \frac{\phi_a}{2})^2 (\pi^2 - b^2 k_z^2)^2 H_n^{(2)'}(k_c r_o)} \quad (2-25)$$

Since the integral of (2-25) is an even function of n and k_z , the expression for axial slot admittance may also be written in the following more computationally efficient form:

$$Y_{11} = +j \frac{8b\phi_a}{w\mu_o} \sum_{n=0}^{\infty} \int_0^{\infty} \frac{k_c^* \sin^2(n \frac{\phi_a}{2}) \cos^2(k_z \frac{b}{2}) H_n^{(2)*}(k_c r_o) dk_z}{(1+\delta_o^n) (n \frac{\phi_a}{2})^2 (\pi^2 - b^2 k_z^2)^2 H_n^{*'}(2)(k_c r_o)} \quad (2-26)$$

where

$$\delta_o^n = \begin{cases} 1; & n = 0 \\ 0; & \text{otherwise.} \end{cases}$$

A FORTRAN computer program (AXIAL) was written to evaluate (2-26) for various frequencies, cylinder diameters, slot dimensions, and complex permittivities. A listing of this program is contained in Appendix A.

Self Admittance of a Transverse Rectangular Slot in a Conducting Cylinder of Infinite Length

The geometry of the transversely slotted cylinder is shown in Figure 2-1. In this configuration, the long dimension of the slot is perpendicular to the cylinder's

axis. The aperture excitation, assumed to be the dominant TE_{10} mode, is written

$$E_z(r=r_0, \phi, z) = \begin{cases} V_0 \sqrt{\frac{2}{ab}} \cos \frac{\pi \phi}{\phi_a}; & -\phi_a \leq \phi \leq \phi_a, \quad -\frac{b}{2} \leq z \leq \frac{b}{2} \\ 0 & ; \text{elsewhere.} \end{cases}$$

$$E_\phi(r=r_0, \phi, z) = 0.$$

Given that the aperture electric field is aligned with the z axis, the complex power leaving the slot (and, hence, the slot's complex admittance) can be expressed in terms of the H_ϕ magnetic field component. This follows from Poynting vector considerations, i.e.

$$P_r = -\frac{1}{2} \iint_{\text{slot}} \vec{E}_z \times \vec{H}_\phi^* dz d\phi$$

where P_r represents radial power flow.

The Fourier transforms of E_z and H_ϕ evaluated at $r = r_0$ are given by (2-7) and (2-11):

$$\bar{E}_z = k_c^2 a_n H_n^{(2)}(k_c r_0) \quad (2-28)$$

$$\bar{H}_\phi = \frac{-nk_z b_n}{r_0} H_n^{(2)}(k_c r_0) - \frac{jk^2 a_n}{w\mu} \frac{\partial H_n^{(2)}(k_c r)}{\partial r} \bigg|_{r=r_0} \quad (2-29)$$

Inspection of (2-14) and (2-15) show that both a_n and b_n are nonzero for the transverse slot. Specifically,

$$a_n = \frac{I[E_z]}{k_c^2 H_n^{(2)}(k_c r_o)} \quad (2-30)$$

and

$$b_n = \frac{-jn k_z I[E_z]}{w \mu_o r_o k_c^3 H_n^{(2)*}(k_c r_o)} \quad (2-31)$$

where

$$I[E_z] = \frac{1}{4\pi^2} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{\phi_a}{2}}^{\frac{\phi_a}{2}} E_z(\phi, z) e^{jn\phi} e^{jk_z z} d\phi dz, \quad (2-32)$$

The slot self admittance is obtained by substituting (2-30) and (2-31) into (2-20):

$$Y_{11} = j \frac{r_o}{|V_o|^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{|I|^2}{16\pi^4 w \mu_o k_c} \left[\frac{n^2 k_z^2 |H_n^{(2)}(k_c r_o)|^2}{r_o^2 k_c^2 H_n^{(2)*}(k_c r_o) H_n^{(2)}(k_c r_o)} - \frac{k_c^2 H_n^{(2)*}(k_c r_o) H_n^{(2)}(k_c r_o)}{|H_n^{(2)}(k_c r_o)|^2} \right] dk_z. \quad (2-33)$$

In the above expression, I is determined by applying the aperture excitation of (2-27) to equation (2-32). This gives

$$I = V_o \sqrt{\frac{2}{ab}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \cos\left(\frac{\pi}{\phi_a} \phi\right) e^{jn\phi} e^{jk_z z} d\phi dz$$

or

$$I = \frac{4V_o \phi_a \sqrt{\frac{2}{ab}} \cos\left(n\frac{\phi_a}{2}\right) \sin\left(k_z \frac{b}{2}\right)}{k_z [\pi^2 - n^2 \phi_a^2]} \quad (2-34)$$

Substituting (2-34) into (2-33) and noting that the integrand is an even function of k_z and n results in the desired expression for transverse slot admittance:

$$Y_{11} = \frac{j8r_o^3 b}{2\mu_o a^3} \sum_{n=0}^{\infty} \int_0^{\infty} \frac{\cos^2\left(n\frac{\phi_a}{2}\right) \sin^2\left(k_z \frac{b}{2}\right)}{(1+\delta_o^n) (k_z \frac{b}{2})^2 \left[\left(\frac{\pi}{\phi_a}\right)^2 - n^2\right]^2} dk_z$$

$$\left[\frac{n^2 k_z^2 H_n^{(2)}(k_c r_o)}{r_o k_c^2 H_n^{(2)}(k_c r_o)} - k^2 \frac{H_n^{(2)'}(k_c r_o)}{H_n^{(2)}(k_c r_o)} \right] dk_z \quad (2-35)$$

A Fortran computer program (TRANS) was written to evaluate (2-35) for various slot dimensions, cylinder diameters, permittivities, and frequencies. A listing of this program is contained in Appendix B.

Mutual Admittance of Slotted Cylinder Array

The analysis which leads to an expression for the self admittance of a single slot on a cylinder can be extended to determine the mutual admittance of an array of two or more slots [28,29]. For the case of two slots, the method depends on evaluating the change in one slot's self admittance as both slots are excited in phase and then 180° out of phase. Assuming the two slot array is a reciprocal network, e.g. $Y_{12} = Y_{21}$, this procedure may be justified from consideration of the pi equivalent circuit for a two port network.

Referring to Figure 2-2,

$$I_1 = Y_{11}E_1 + Y_{12}E_2 \quad (2-36)$$

$$I_2 = Y_{12}E_1 + Y_{22}E_2 \quad (2-37)$$

For in phase (even) excitation, $E_1 = E_2$ and the driving point admittance measured at port one is

$$Y_e = \left. \frac{I_1}{E_1} \right|_{E_1=E_2} = Y_{11} + Y_{12} \quad (2-38)$$

where Y_e denotes the "even" admittance. For out of phase (odd) excitation, $E_1 = -E_2$ and

$$Y_o = \left. \frac{I_1}{E_1} \right|_{E_1=-E_2} = Y_{11} - Y_{12}. \quad (2-39)$$

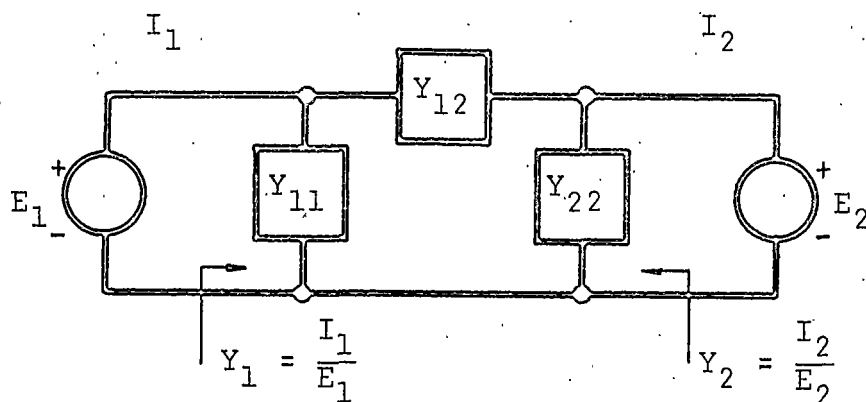


Figure 2-2. Equivalent Circuit for a Reciprocal Array of Two Slot Antennas.

Solving (2-38) and (2-39) for Y_{12} gives

$$Y_{12} = \frac{Y_e - Y_o}{2}. \quad (2-40)$$

Thus the mutual admittance of a reciprocal two port network can be calculated from its response to even and odd excitations. A similar procedure can be derived for a reciprocal n port network.

Next it is necessary to determine Y_e and Y_o for a pair of slot antennas. It will be shown that only a slight modification to the self admittance expressions makes this calculation possible. The geometry of the slotted cylinder array is shown in Figure 2-3. Axial slots are assumed for convenience ($a_n = 0$), however, this analysis holds for all configurations in which the slots remain parallel. Equation (2-24) shows that the slot driving point admittance is a linear function of the parameter $b_n b_n^* = |b_n|^2$, i.e.

$$Y_1 = f(b_n b_n^*) = f(|b_n|^2). \quad (2-41)$$

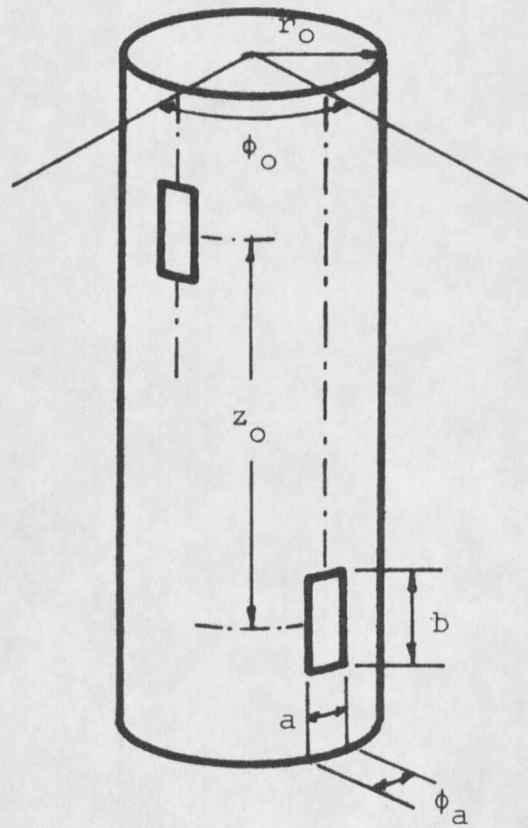


Figure 2-3. Array of Two Parallel Rectangular Slots on an Infinitely Long Conducting Cylinder.

Equation (2-15) in turn shows that b_n is a linear function of the aperture excitation field, i.e.

$$b_n = g(E_\phi) \quad (2-42)$$

where E_ϕ is the transverse electric field.

For even excitation of the slots, the combined aperture field is

$$E_{\phi_e} = E_\phi(z, \phi) + E_\phi(z - z_o, \phi - \phi_o). \quad (2-43)$$

Similarly for odd excitation

$$E_{\phi_o} = E_\phi(z, \phi) - E_\phi(z - z_o, \phi - \phi_o). \quad (2-44)$$

Applying superposition and the Fourier delay/linear phase shift equivalence property [40] to (2-42), (2-43), and (2-44) gives

$$b_{n_e} = g\{E_\phi(z, \phi) + E_\phi(z - z_o, \phi - \phi_o)\} = g\{E_\phi(z, \phi)\} + g\{E_\phi(z, \phi)\}e^{j(n\phi_o + k_z z_o)} \quad (2-45)$$

and

$$b_{n_o} = g\{E_\phi(z, \phi) - E_\phi(z - z_o, \phi - \phi_o)\} = \\ g\{E_\phi(z, \phi)\} - g\{E_\phi(z, \phi)\} e^{j(n\phi_o + k_z z_o)} \quad (2-46)$$

where the subscripts on b_n refer to even and odd excitation.

Substituting (2-45) and (2-46) into (2-41) gives

$$Y_e = f(b_{n_e} b_{n_e}^*) = f\{|g(E_\phi)|^2 (2 + e^{j(n\phi_o + k_z z_o)} + \\ e^{-j(n\phi_o + k_z z_o)})\} \quad (2-47)$$

and

$$Y_o = f(b_{n_o} b_{n_o}^*) = f\{|g(E_\phi)|^2 (2 + e^{j(n\phi_o + k_z z_o)} + \\ e^{-j(n\phi_o + k_z z_o)})\}. \quad (2-48)$$

Since f is linear in $|b_n|^2$, the superposition property may be applied to (2-40):

$$Y_{12} = \frac{Y_e - Y_o}{2} = \frac{f(b_{n_e} b_{n_e}^*) - f(b_{n_o} b_{n_o}^*)}{2} \quad (2-49)$$

or

$$Y_{12} = f\{|g(E_\phi)|^2 e^{-j(n\phi_o + k_z z_o)}\}. \quad (2-50)$$

By definition, $g(E_\phi) = b_n$ so finally

$$Y_{12} = f\{|b_n|^2 e^{-j(n\phi_o + k_z z_o)}\}. \quad (2-51)$$

Thus an expression for the mutual admittance of a pair of axial slots is obtained by replacing $|b_n|^2$ with $|b_n|^2 e^{-j(n\phi_o + k_z z_o)}$ in the expression for self admittance, i.e.

$$Y_{12} = \frac{j2b\phi_a}{w\mu_o}.$$

$$\sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{k_c^* \sin^2(n\frac{\phi_a}{2}) \cos^2(k_z \frac{b}{2}) H_n^{*(2)}(k_c r_o) e^{-j(n\phi_o + k_z z_o)} dk_z}{(n\frac{\phi_a}{2})^2 (\pi^2 - b^2 k_z^2)^2 H_n^{*'}(2)(k_c r_o)}.$$

(2-52)

A similar derivation shows the mutual admittance of a transverse slot pair is

$$Y_{12} = \frac{j2r_o^3 b}{w\mu_o a^3} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\cos^2(n\frac{\phi_a}{2}) \sin^2(k\frac{b}{2}) e^{-j(n\phi_o + k_z z_o)}}{(k\frac{b}{2})^2 [(\frac{\pi}{\phi_a})^2 - n^2]^2} \cdot$$

$$\left[\frac{n^2 k_z^2 H_n^{(2)}(k_c r_o)}{r_o k_c^2 H_n^{(2)}(k_c r_o)} - k^2 \frac{H_n^{(2)'}(k_c r_o)}{H_n^{(2)}(k_c r_o)} \right] dk_z. \quad (2-53)$$

The addition of the complex exponential linear phase shift term to the self admittance expression results in an integrand that is neither odd nor even. For this reason the two-sided integration limits $(-\infty$ to $+\infty)$ are retained.

Frequently it is more meaningful to work with the mutual isolation of array elements rather than their mutual admittance. This mutual isolation parameter is simply the magnitude of s_{ij} (the mutual scattering coefficient) expressed in decibels, i.e.

$$I_{ij} = -20 \log_{10} |s_{ij}|; \quad i \neq j.$$

Isolation is a useful quantity primarily because it is simple to measure in the field. One merely injects a signal of known level at the i^{th} slot and measures the magnitude received at the j^{th} slot. No phase determination is required.

Isolation may be expressed in terms of the self and mutual admittance of the array elements [26]:

$$I = -20 \log_{10} \left| \frac{\frac{2Y_{12}}{Y_g}}{\left(1 + \frac{Y_{11}}{Y_g}\right)^2 - \left(\frac{Y_{12}}{Y_g}\right)^2} \right|$$

where Y_g is the characteristic admittance of the waveguide feed.

Values of X-band isolation for a 6 inch (15.2 cm) diameter, transversely slotted cylinder are given in Figure 2-8. A soil permittivity of $\hat{\epsilon}_r = 2.5 - j0.3$ is assumed. This value is typical for a soil volumetric moisture content of 5% grams H_2O/cc . The distance between slots was chosen to be 2.4 inches (6.1 cm). The isolation values were calculated by program TRANS, a listing of which may be found in Appendix B. These values are so high that practical measurements would be

restricted to nearest neighbor isolation only. Even higher isolations would be obtained for an axially slotted cylinder [25], thus a transverse slot geometry is preferred.

Sensing Radius of the Slotted Cylinder

One important property of a soil moisture probe is its effective coverage or "sensing volume." The sensing volume is defined as the region of soil which contributes substantially to a moisture measurement. For example, a neutron probe typically has a spherical sensing volume with a radius of approximately 23 cm (9 inches) [2]. The suitability of a large or small sensing volume depends on the particular application. A large sensing volume tends to provide consistent readings in the presence of inhomogeneities because the measurement represents an average moisture content for a large region. However, when the spatial distribution of soil moisture is of interest, a large sensing volume may average out the very differences being sought. Clearly the coverage of a given moisture probe should be consistent with the desired spatial resolution.

One approach to evaluating the effective coverage of the slotted cylinder probe is to compute the ratio of time-average power flux through a hypothetical cylindrical surface of radius $r_s > r_o$ to the power leaving the slotted cylinder at radius $r = r_o$, i.e.

$$\ell(r_s) = \frac{p(r_s)}{p(r_o)} = \frac{\frac{1}{2}R_e \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} \vec{E}_t \times \vec{H}_t^* \cdot \vec{r}_s d\phi d_z \vec{r}}{\frac{1}{2}R_e \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} \vec{E}_t \times \vec{H}_t^* \cdot \vec{r}_o d\phi d_z \vec{r}} \quad (2-54)$$

where $p(r)$ is the real time-average power flux through a cylindrical surface of radius r , and $\ell(r)$ is the power attenuation ratio. Only the real part of $\vec{E} \times \vec{H}^*$ is considered here because this quantity is related to the amount of power dissipated within the region bounded by $r_o < r < r_s$. The imaginary component of $\vec{E} \times \vec{H}$ is related to energy stored within the region. This power attenuation ratio is analogous to the ratio used to define the skin depth for a plane wave impinging on a lossy medium [23]. When ℓ becomes small, most of the power radiated by the slotted cylinder is being dissipated within the region bounded by $r_o < r < r_s$.

Since soil outside this region can have but little effect on the measured properties of the slot antennas, r_s may be regarded as the effective sensing radius of the slotted cylinder probe. To be consistent with the skin depth concept, r_s will be defined as the radius for which the radiated power has decayed to e^{-2} of its initial value, i.e.

$$10 \log_{10} \ell(r_s) = 20 \log_{10} e^{-1} = -8.686 \text{ dB.} \quad (2-55)$$

In order to solve for the value of r_s which satisfies (2-55) an expression for $p(r)$ must be developed. This is easily accomplished since (2-7) through (2-12) express all six field components for $r \geq r_0$ in terms of the Fourier coefficients a_n and b_n , and these coefficients have previously been derived for slot admittance calculations. From Poynting's theorem, the real time-average power flow through a cylindrical surface of radius r is given by

$$p(r) = \frac{1}{2} R_e \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} (E_{\phi} H_z^* - E_z H_{\phi}^*) r d\phi dz. \quad (2-56)$$

Parseval's theorem may be applied to (2-56) giving $p(r)$ in terms of the Fourier transforms of the transverse fields:

$$p(r) = (2\pi)^{\frac{2r}{2}} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} (\bar{E}_{\phi} \bar{H}_z^* - \bar{E}_z \bar{H}_{\phi}^*) dk_z. \quad (2-57)$$

this latter form is convenient to use since (2-7) through (2-12) give the transformed fields for all values of $r \geq r_0$.

Power Attenuation Ratio for Axial Slots

It was shown in the section on axial slot self admittance that $a_n = 0$ for a TE_{10} excitation. Equation (2-7) shows that $E_z = 0$ for $r \geq r_0$. Thus (2-57) reduces to

$$P(r) = R_e \left\{ (2\pi)^{\frac{2r}{2}} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{E}_{\phi} \bar{H}_z^* dk_z \right\}$$

or

$$p(r) = R_e \left\{ (2\pi)^{\frac{2r}{2}} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} -jk_c^2 \omega \mu_0 |b_n|^2 k_c^{*H_n^{(2)}}(k_c r) H_n^{(2)}(k_c r) dk_z \right\}. \quad (2-58)$$

The expression for b_n for axial slots is given by (2-23).

Substituting this into (2-58) gives

$$p(r) = \text{Re} \left\{ \frac{-j r V_0^2 b_n^2}{w \mu_0 a} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\sin^2(n \frac{\phi_a}{2}) \cos^2(k_z \frac{b}{2}) k_c^* H_n^{(2)}(k_c r) H_n(k_c r) dk_z}{(n \frac{\phi_a}{2})^2 (\pi^2 - b^2 k_z^2)^2 |H_n^{(2)}(k_c r_0)|} \right\}. \quad (2-59)$$

The power attenuation ratio may be computed by substituting the above expression for $p(r)$ into (2-54). The sensing radius r_s is then determined numerically by finding the value of r for which $10 \log_{10} \ell(r) = -8.686$ dB.

Power Attenuation Ratio for Transverse Slots

The expression for $p(r)$ for transverse slots has more terms than the axial slot configuration because both a_n and b_n are nonzero for transverse excitation. Substituting (2-7), (2-8), (2-10), and (2-11) into (2-57) gives

$$p(r) = (2\pi)^2 \frac{r}{2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{-2nk_z a_n^* b_n |H_n^{(2)}(k_c r)|^2 \text{Im}(k_c^2)}{r} +$$

$$+ jk_c |k_c|^2 \left[\frac{k^2 |a_n|^2 H_n^{*(2)}(k_c r) H_n'^{(2)}(k_c r)}{w\mu_o} - \right. \\ \left. w\mu_o |b_n|^2 H_n^{(2)}(k_c r) H_n'^{(2)*}(k_c r) \right] dk_z \quad (2-60)$$

where a_n and b_n are given by (2-30); (2-31), and (2-34),
i.e.

$$a_n = \frac{I[E_z]}{k_c^2 H_n^{(2)}(k_c r_o)} = \frac{4V_o \phi_a \sqrt{\frac{2}{ab}} \cos(n\frac{\phi_a}{2}) \sin(k_z \frac{b}{2})}{k_z k_c^2 [\pi^2 - n^2 \phi_a^2] H_n^{(2)}(k_c r_o)}; \quad (2-61)$$

$$b_n = \frac{-jnk_z I[E_z]}{w\mu_o r_o k_c^3 H_n^{(2)*}(k_c r_o)} =$$

$$\frac{-j4V_o \phi_a \frac{2}{ab} nk_z \cos(n\frac{\phi_a}{2}) \sin(k_z \frac{b}{2})}{w\mu_o r_o k_c^3 k_z [\pi^2 - n^2 \phi_a^2] H_n^{(2)*}(k_c r_o)}.$$

Self and Mutual Admittance of Slotted Plane Antennas

Introduction

In some circumstances the admittance of a slotted cylinder antenna closely approximates that of a slotted

ground plane [32, 28, 26]. The agreement is quite good when the circumference of the cylinder is large compared to a free space wavelength. This is equivalent to requiring that

$$k_0 r_0 > 1$$

where r_0 is the cylinder's radius and

$$k_0 = \frac{2\pi}{\lambda_0} = \omega_0 / c$$

is the free space propagation constant. Generally the agreement improves as $k_0 r_0$ increases (which is reasonable since a plane may be regarded as a cylinder of infinite radius), although Fante [26] has reported less than 20% deviation from slotted plane admittance for $k_0 r_0$ values as small as 1.5.

This similarity between cylinder and plane admittances is a useful property to exploit since the analysis, construction, and operation of a slotted plane array are comparatively easy. For example, it will be shown that the admittance expression for a plane aperture can be cast as a finite integral of sinusoidal functions

rather than the infinite integral of Hankel functions required for slotted cylinder admittance. The former is, by far, more amenable to numerical evaluation. The construction of a slotted plane antenna is relatively simple because standard waveguide flanges can mate directly to a planar surface. In contrast the slotted cylinder requires specially machined transition elements between flange and cylinder. These transitions must be carefully aligned with the waveguide flange and the aperture to minimize spurious responses. The alignment procedure is time consuming and must be repeated several times during s parameter calibration as various terminations are hung on the aperture. It should be stressed that a slotted cylinder geometry was chosen for the moisture probe for convenient borehole installation in the field. This advantage outweighs the problems described above.

Self Admittance of a Rectangular Slot in an Infinite, Perfectly Conducting Plane

Compton [33] has derived an expression for the self admittance of a rectangular aperture radiating into a semi-infinite lossy medium. The geometry he considers

is shown in Figure 2-4. For a TE_{10} aperture electric field, i.e.

$$E_x(x,y,z=0) = \begin{cases} \sqrt{\frac{2}{ab}} \cos \frac{\pi y}{b}: & (x,y) \in \text{aperture} \\ 0 & : (x,y) \notin \text{aperture} \end{cases}$$

where $\sqrt{\frac{2}{ab}}$ is a normalizing constant, Compton shows that the self admittance is given by

$$Y_{11} = \frac{2b}{(2\pi)^2 a w \mu_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(4\pi)^2 (k^2 - k_y^2) \sin^2\left(\frac{k_x a}{2}\right) \cos^2\left(\frac{k_y b}{2}\right) dk_x dk_y}{k_x^2 \sqrt{k^2 - k_x^2 - k_y^2} (\pi^2 - k_y^2 b^2)^2} \quad (2-63)$$

where

$$k = w \sqrt{\mu_0 \epsilon_0 \epsilon_r}.$$

However, he goes on to cast this infinite integral in the wave number domain as a finite integral in the spatial domain. The details of this transformation are repeated here because they are useful in extending Compton's analysis to cover mutual admittances. The first step is to rewrite (2-63) as

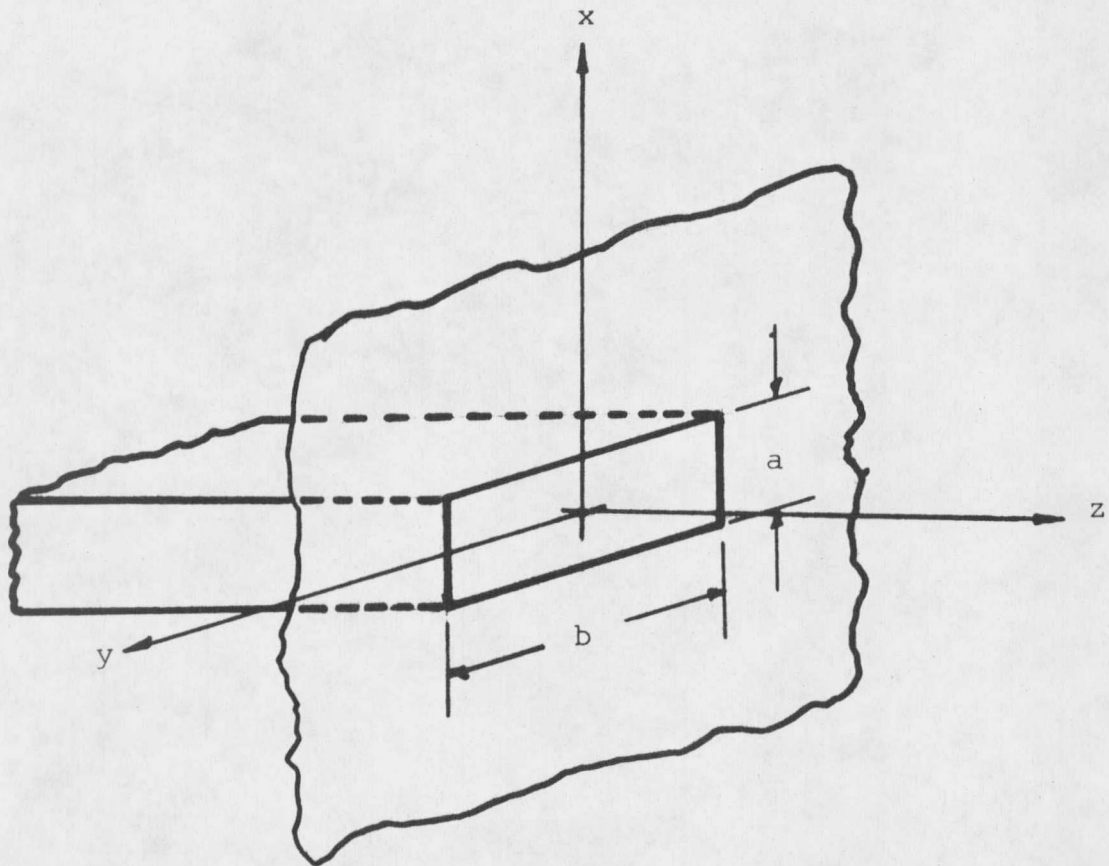


Figure 2-4. Rectangular Aperture in an Infinite Conducting Ground Plane.

$$Y_{11} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_1(k_x, k_y) F_2(k_x, k_y) dk_x dk_y \quad (2-64)$$

where

$$F_1(k_x, k_y) = \frac{8b(k_x^2 - k_y^2) \sin^2\left(\frac{k_x a}{2}\right) \cos^2\left(\frac{k_y b}{2}\right)}{a \omega_o k_x^2 (\pi^2 - k_y^2 b^2)^2} \quad (2-65)$$

and

$$F_2(k_x, k_y) = \frac{4\pi^2}{\sqrt{k^2 - k_x^2 - k_y^2}} \quad (2-66)$$

It can be shown by means of the convolution integral that

$$Y_{11} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_1(k_x, k_y) F_2(k_x, k_y) dk_x dk_y =$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(x, y) f_2(x, y) dx dy \quad (2-67)$$

where $f_1(x, y)$ and $f_2(x, y)$ are the inverse transforms of F_1 and F_2 , i.e.

$$f_i = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_i(k_x, k_y) e^{-jk_x x} e^{-jk_y y} dk_x dk_y; \quad i=1,2 \quad (2-68)$$

Substituting (2-65) into (2-68) gives

$$\begin{aligned} f_1(x,y) &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{8b(k^2 - k_y^2)}{a\omega\mu_0 k_x^2} \frac{\sin^2(\frac{k_x a}{2}) \cos^2(\frac{k_y b}{2})}{(\pi^2 - k_y^2 b^2)^2} \\ &\quad e^{-jk_x x} e^{-jk_y y} dk_x dk_y \\ &= \frac{2b}{a\omega\mu_0 \pi^2} \int_{-\infty}^{\infty} \frac{\sin^2(\frac{k_x a}{2})}{k_x^2} e^{-jk_x x} dk_x \cdot \\ &\quad \int_{-\infty}^{\infty} \frac{(k^2 - k_y^2) \cos^2(\frac{k_y b}{2})}{(\pi^2 - k_y^2 b^2)^2} e^{-jk_y y} dk_y. \end{aligned} \quad (2-69)$$

This expression integrates to

$$f_1(x,y) = \frac{2b}{a\omega\mu_0 \pi^2} g(x)h(y) \quad (2-70)$$

where

$$g(x) = \begin{cases} \frac{\pi}{2}(a - |x|); & |x| \leq a \\ 0 & ; |x| > a \end{cases} \quad (2-71)$$

$$h(y) = \begin{cases} D_1(b - |y|)\cos\frac{\pi y}{b} + D_2\sin\frac{\pi|y|}{b}; & |y| \leq b \\ 0 & ; |y| > b \end{cases} \quad (2-72)$$

$$D_1 = \frac{1}{b^2} \left[\frac{k^2}{4\pi} - \frac{\pi}{4b^2} \right] \quad (2-73)$$

$$D_2 = \frac{1}{\pi b} \left[\frac{k^2}{4\pi} + \frac{\pi}{4b^2} \right]. \quad (2-74)$$

Next $f_2(x,y)$ is found by substituting (2-66) into (2-68) and integrating:

$$f_2(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-jk_x x} e^{-jk_y y}}{\sqrt{k^2 - k_y^2 - k_x^2}} dk_x dk_y \quad (2-75)$$

or

$$f_2(x,y) = 2\pi j \frac{e^{-jk\sqrt{x^2 + y^2}}}{\sqrt{x^2 + y^2}}. \quad (2-76)$$

An expression for the self admittance is then obtained by substituting (2-76) and (2-70) into (2-67):

$$Y_{11} = j \frac{4b}{\omega \mu_0 a \pi} \int_{y=-b}^b \int_{x=-a}^a (a - |x|) [D_1(b - |y|) \cos \frac{\pi y}{b} + D_2 \sin \frac{\pi y}{b}] \frac{e^{-jk\sqrt{x^2 + y^2}}}{\sqrt{x^2 + y^2}} dx dy. \quad (2-77)$$

Because of its finite integration limits, this last expression is much easier to evaluate than (2-63).

Mutual Admittance of Rectangular Slots in an Infinite, Perfectly Conducting Ground Plane

Compton's dissertation [33] did not consider the mutual admittance of slotted plane antennas. However, his approach to deriving a finite integral expression

for self admittance can also be applied to the mutual admittance problem. The geometry to be considered is two identical rectangular apertures in an infinite, perfectly conducting ground plane. The apertures may be offset in both x and y dimensions but are parallel to each other. This configuration is illustrated in Figure 2-5.

Equation (2-51) showed that the mutual admittance of two parallel slots on a conducting cylinder could be derived from the self admittance expression by including a linear phase shift term whose slope is proportional to the spatial separation of the apertures. This result also applies to ground plane apertures, i.e.

$$Y_{12} = \frac{2b}{(2\pi)^2 a w \mu_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(4\pi)^2 (k^2 - k_y^2) \sin^2\left(\frac{k_x a}{2}\right) \cos^2\left(\frac{k_y b}{2}\right) e^{-jk_x x_0} e^{-jk_y y_0}}{k_x^2 \sqrt{k^2 - k_x^2 - k_y^2} (\pi^2 - k_y^2 b^2)^2} dk_x dk_y \quad (2-78)$$

where $e^{-jk_x x_0}$ and $e^{-jk_y y_0}$ are the linear phase shifts.

If the slots completely overlap ($x_0 = y_0 = 0$), then

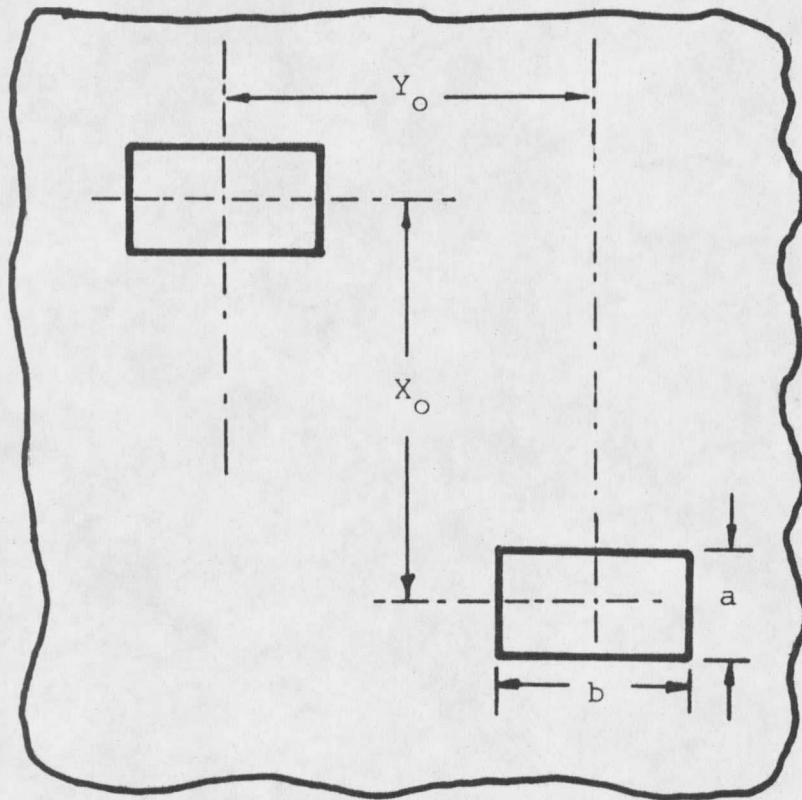


Figure 2-5. Array of Two Parallel Rectangular Slots in an Infinite Conducting Ground Plane.

(2-78) reduces to the self admittance expression. Following Compton's procedure, (2-78) is rewritten in terms of the functions $F_{2,3}(k_x, k_y)$ and $f_{2,3}(x, y)$:

$$\begin{aligned}
 Y_{12} &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_3(k_x, k_y) F_2(k_x, k_y) dk_x dk_y \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_3(x, y) f_2(x, y) dx dy
 \end{aligned} \tag{2-79}$$

where

$$F_3(k_x, k_y) = F_1(k_x, k_y) e^{-j(k_x x_0 + k_y y_0)} \tag{2-80}$$

$$f_3(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_1(k_x, k_y) e^{-j(k_x x_0 + k_y y_0)} dk_x dk_y. \tag{2-81}$$

From Fourier transform theory it is known that the introduction of a linear phase term in one domain corresponds to an offset of the function in the transform domain [40]. For example, if the functions $h(t)$ and $H(w)$ comprise a Fourier transform pair, i.e.

$$h(t) \Leftrightarrow H(w) \quad (2-82)$$

then it is readily shown that

$$h(t \pm t_0) \Leftrightarrow H(w) e^{\pm j w t_0} \quad (2-83)$$

and

$$h(t) e^{\pm j t w_0} \Leftrightarrow H(w \pm w_0) \quad (2-84)$$

are also Fourier transform pairs. This result applied to (2-81) gives an expression for $f_3(x,y)$ in terms of $f_1(x,y)$:

$$f_3(x,y) = \frac{2b}{a w_0 \pi^2} g(x - x_0) h(y - y_0) \quad (2-85)$$

where

$$g(x - x_0) = \begin{cases} \frac{\pi}{2}(a - |x - x_0|); & |x - x_0| \leq a \\ 0 & ; |x - x_0| \geq a \end{cases} \quad (2-86)$$

$$h(y - y_0) = \begin{cases} D_1(b - |y - y_0|) \cos\left(\frac{\pi |y - y_0|}{b}\right) + D_2 \sin\left(\frac{\pi |y - y_0|}{b}\right); & |y - y_0| \leq b \\ 0 & ; |y - y_0| > b \end{cases} \quad (2-87)$$

Substituting these results into (2-79) gives a finite integral expression for the mutual admittance of two rectangular apertures:

$$Y_{12} = j \frac{4b}{2\mu_0 a \pi} \int_{y=y_0-b}^{y_0+b} \int_{x=x_0-a}^{x_0+a} (a - |x-x_0|) \cdot$$

$$[D_1(b - |y-y_0|) \cos(\frac{\pi}{b}|y-y_0|) +$$

$$D_2 \sin(\frac{\pi}{b}|y-y_0|)] \frac{e^{-jk\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} dx dy. \quad (2-88)$$

Numerical Comparison of Slotted Plane and Slotted Cylinder Properties

As stated earlier in this chapter, FORTRAN computer programs were written to numerically evaluate the expressions derived for aperture self and mutual admittance. These programs are summarized in Table 2-1. The slotted cylinder programs must evaluate an infinite summation

Table 2-1. Summary of FORTRAN Computer Programs for Slotted Cylinder and Slotted Plane Computations.

Program Name	Array Geometry	Equations Evaluated
AXIAL	Axial Rectangular Slots on a Conducting Cylinder	(2-26):Self Admittance (2-52):Mutual Admittance
TRANS	Transverse Rectangular Slots on a Conducting Cylinder	(2-35):Self Admittance (2-52):Mutual Admittance
PLANE	Rectangular Slots in a Ground Plane	(2-77):Self Admittance (2-88):Mutual Admittance

over an infinite integral of Hankel functions, while the slotted plane calculations involve a finite double integral of sinusoidal functions.

A graphical display capability was written for the slotted cylinder programs to help establish the points at which the infinite series and integral could be truncated without unduly effecting accuracy. In this diagnostic mode, contours of the integral's magnitude are plotted as a function of summation index n and integration variable k_z . An example is shown in Figure 2-6. The characters "0" through "9" are proportional to the number of decibels down from the peak value of the integrand magnitude. In this example, "0" denote 0 dB down, "1" denotes 3 dB down, "2" denotes 6 dB down, etc. In other words, the decibel values are obtained by multiplying the characters in the plot by 3 dB. Blanks denote values below -30 dB. Figure 2-6 shows that the integrand exhibits a two dimensional $\sin^2 x/x^2$ variation as would be expected from examination of the integrand. The spacing of the lobes is largely determined by the dimensions of the aperture and the k_z increment must be chosen small enough for the numerical integration routine

Increasing n $\longrightarrow$

Increasing k_z $\downarrow$

.00	00000000000000011111222233445679	8765554
25.00	10000000000000011111222233445679	8766555
50.00	10000000000000011111222233445679	8766555
75.00	10000000000000011111222233445679	8766555
100.00	10000000000000011111222233445679	8766555
125.00	10000000000111111222233445679	9776655
150.00	10000011111111122222334456789	9876665
175.00	21111111111112222233445678	9877666
200.00	21111111112222223333444556678	887766
225.00	3222222222222233334445566789	987776
250.00	322222222233333344455667789	998877
275.00	43333333333334444556667789	99888
300.00	43344444444444555566677889	9998
325.00	5444455555555566667778899	9
350.00	766666666666677777888999	
375.00	8777888888888899999	
400.00		
425.00		
450.00	999999999999999	
475.00	9888888888888999999	
500.00	8777777788888889999	
525.00	87777777888888899999	
550.00	8777778888888899999	
575.00	98888888888899999	
600.00	988889999999999	
625.00	99999	
650.00		
675.00		
700.00		

dB Increment is 3.000
 Max Value is 2.6404781E 03

Figure 2-6. Example of Contour Map of Admittance Expression Integrand Generated by Program TRANS. The Geometry is a Transverse 2.286 by 1.016 cm Aperture on a 5.057 cm Radius Cylinder. A Permittivity of $\epsilon_r = 10.0 - j75$ is assumed.

to follow the peaks and valleys of the function. The rate at which the integrand decreases with increasing n and k_z is largely determined by the effective circumference of the cylinder, i.e. by $k_0 r_0$. As $k_0 r_0$ increases, the summation order must be extended before truncation occurs. The contour plots are very helpful in establishing efficient k_z increments and suitable truncation limits. The slotted plane program does not produce diagnostic plots since its calculations involve finite integration limits only.

As discussed earlier, slotted plane properties closely approximate slotted cylinder properties for $k_0 r_0$ greater than 1.5. This is shown by Figures 2-7 and 2-8 which present comparisons for the case where $k_0 r_0 = 13.6$, i.e. a 6 inch (15.2 cm) cylinder at 8.5 Ghz. A permittivity of $\epsilon_r = 2.5 - j 0.3$ is used for this example. This value is typical of a dry soil (3% grams H_2O/cc). The aperture dimensions are 2.286 by 1.016 cm (standard X-band waveguide dimensions). Spacing between apertures is 6.1 cm. Excellent agreement between slotted cylinder and slotted plane admittance and isolation was obtained, however, the computational effort was much greater for the cylindrical geometry (6375

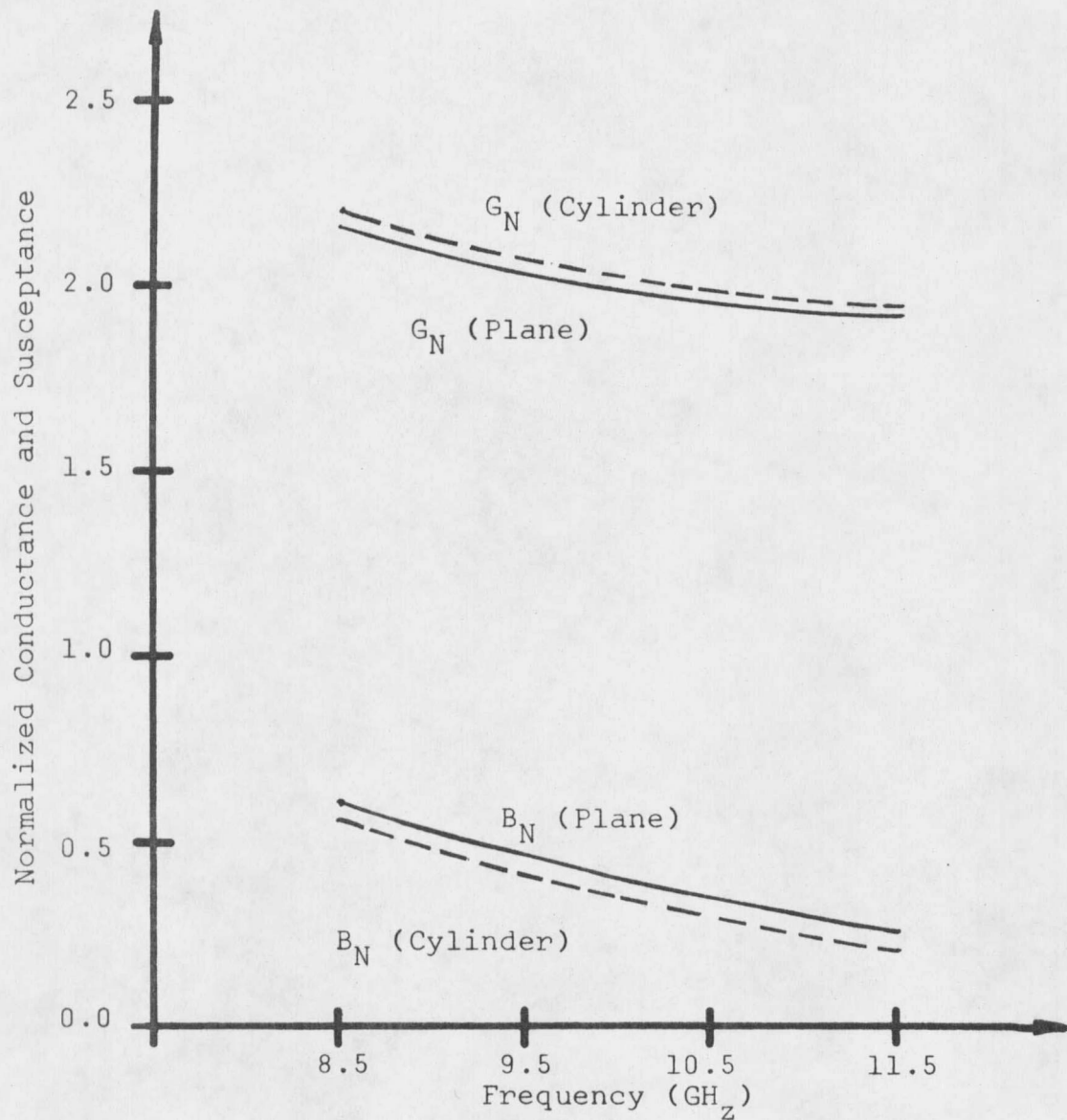


Figure 2-7. Comparison of Slotted Plane and Slotted Cylinder Self Admittance. Cylinder Diameter is 6.0 inches (15 cm), $\hat{\epsilon}_r = 2.5 - j0.3$. Slot is 2.286 cm by 1.016 cm.

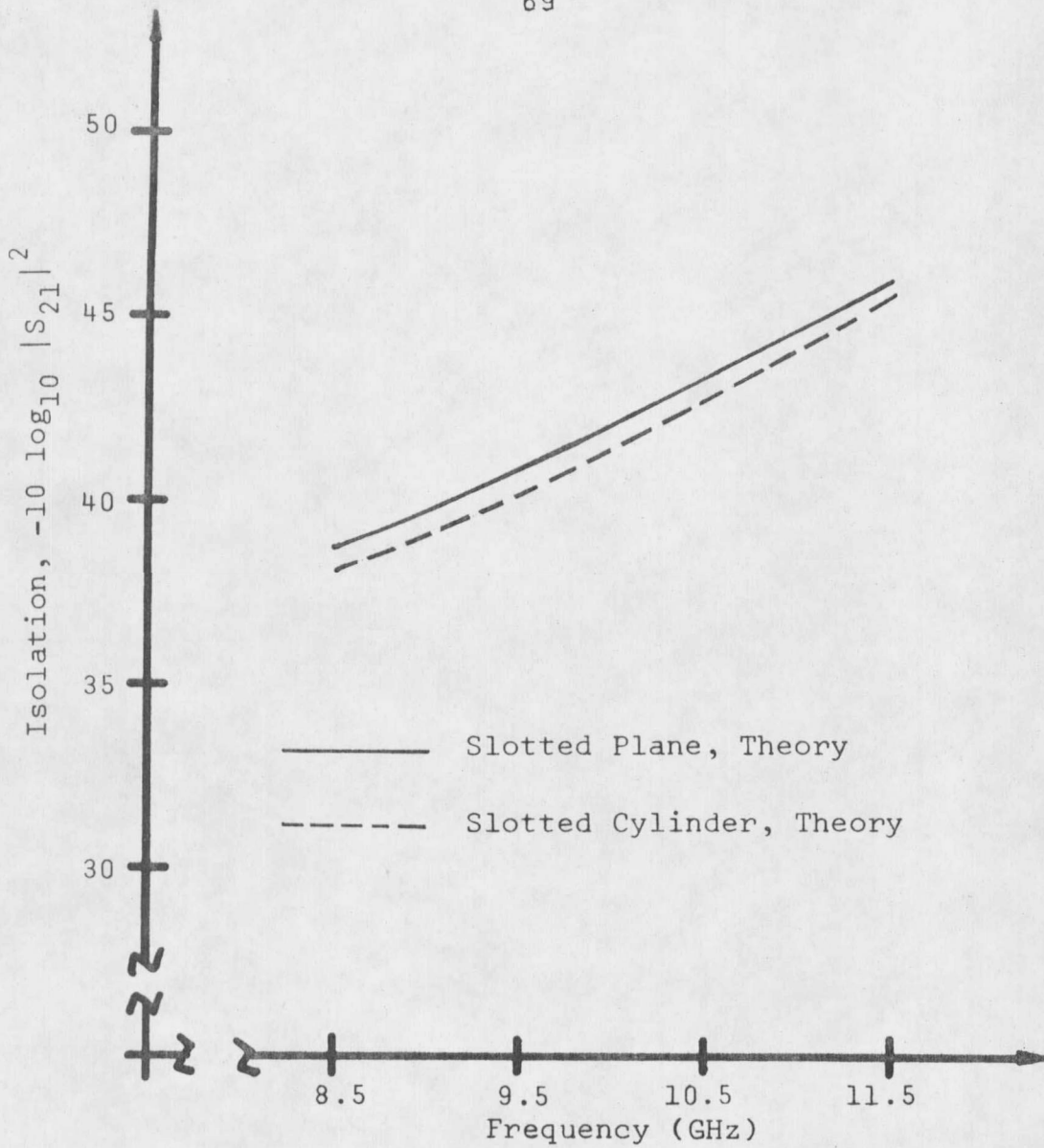


Figure 2-8. Comparison of Slotted Plane and Slotted Cylinder Isolation. Cylinder Diameter is 6 inches (15 cm), $\hat{\epsilon}_r = 2.5 - j0.3$. Slot is 2.286 cm by 1.016 cm.

integrand evaluations for the cylinder versus 253 evaluations for the plane). Thus it is highly advantageous to approximate the slotted cylinder results by slotted plane results whenever possible. A comparison of experimental measurements using both slotted cylinder and slotted plane arrays is contained in Chapter 3.

Estimation of Complex Permittivity from Scattering Parameter Magnitudes

In previous sections it was implied that both real and imaginary admittance components could be used to determine soil permittivity. However, measurement of complex admittance under field operating conditions is difficult since both phase and magnitude measurements must be made. Magnitudes are rather simple to measure but phase measurement requires considerably more sophistication. For example, diurnal temperature variations cause slight extensions or contractions of transmission lines connecting the measurement apparatus to the slotted array. This introduces phase errors which are not easily compensated for. Another consideration is that phase determination requires greater equipment complexity

(hence greater cost) than magnitude measurement. Both VSWR and isolation can be measured with just a modulated Gunn oscillator, bidirectional coupler, diode detector and microammeter.

Fortunately much information can be obtained from magnitude observations alone. For example, Table 2-2 shows how small changes in complex permittivity affect the reflection coefficient and isolation of an X-band aperture array. The aperture dimensions are 2.286 by 1.016 cm and their spacing is 6.1 cm. Calculations were performed by program PLANE. It is clear that a small change in real part permittivity ϵ_r' significantly alters $|s_{11}|$ but $|s_{21}|$ remains essentially unchanged. Similarly, a small change in imaginary part permittivity ϵ_r'' primarily affects $|s_{21}|$ but not $|s_{11}|$. These observations suggest that $|s_{11}|$ and $|s_{21}|$ measurements may be used to estimate ϵ_r' and ϵ_r'' , respectively. This method was, in fact, used successfully and is further discussed in Chapter 3.

Table 2-2. Effects of Small Permittivity Changes on Aperture Reflection Coefficient and Isolation. Aperture Dimensions are 2.286 by 1.016 cm and are Spaced 6.10 cm. Frequency is 8.5 Ghz.

Permittivity $\hat{\epsilon}_r = \epsilon'_r - j\epsilon''_r$	Reflection Coefficient $ S_{11} \angle \theta_{11}$	Isolation $-10 \log_{10} S_{21} ^2 \angle \theta_{21}$
2.8-j0.5	0.428*-169°	44.6*103°
3.0-j0.5	0.448*-170°	44.8*68°
3.0-j0.7	0.442*-172°	50.0*69°

CHAPTER 3

EXPERIMENTAL RESULTS

Introduction

In previous chapters expressions for the driving point admittance and mutual coupling of both slotted cylinder and planar arrays were derived. This chapter describes the experimental procedures followed to verify the theoretical results. Specifically, data were collected to

- 1) check accuracy of computer programs by measuring array properties for a known permittivity (free space);
- 2) compare behavior of slotted cylinder and slotted ground plane arrays; and
- 3) measure soil permittivity as a function of moisture content and compare with published results.

Experimental Apparatus

Figure 3-1 shows a block diagram of the system used to study array properties. A standard Hewlett Packard HP8410A network analyzer was used in conjunction with a HP8690B X-band swept frequency generator to measure two

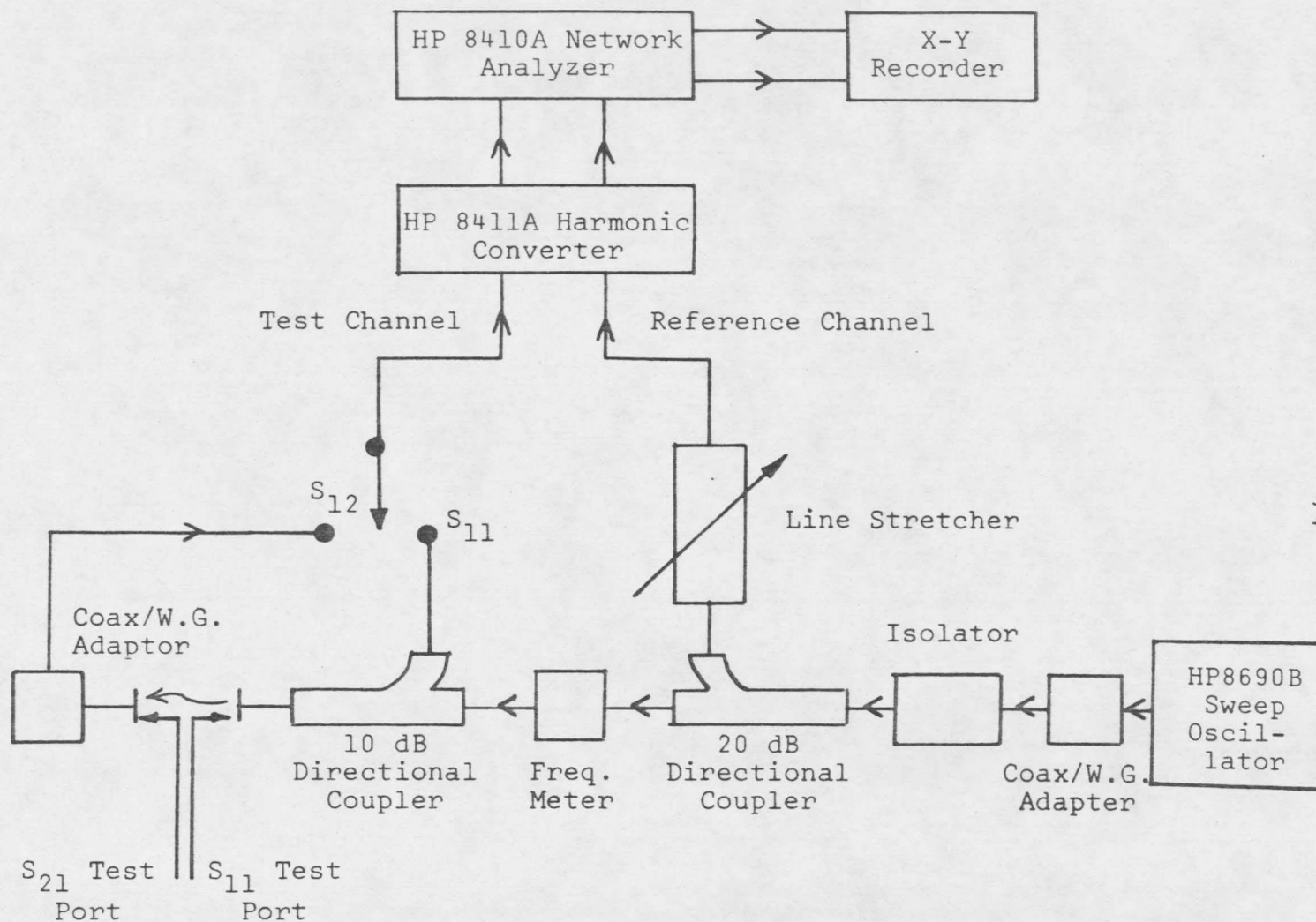


Figure 3-1. Network Analyzer Configuration for S_{11} and S_{12} Measurements.

port scattering parameters. Graphical output of phase and magnitude versus frequency was obtained via an HP7034A X-Y recorder. All data was taken at X-band (8.0-12.4 GHz) primarily because: 1) at these frequencies effects of soil type on permittivity are slight, yet permittivity is still a strong function of moisture content [10]; 2) precision X-band terminations are available which allow admittance measurements to be referred to the plane of the slot antenna; and 3) a relatively small volume of soil is needed to simulate an infinite medium.

Construction of the Slotted Plane and Slotted Cylinder

Arrays

Both planar and cylindrical arrays were constructed so that their properties could be compared. The planar array consists of two E-plane coupled rectangular 0.900" x 0.400" (2.286 x 1.016 cm) slots spaced 2.40" (6.096 cm) and milled from a 9" x 9" x 0.25" (23 x 23 x 0.64 cm) aluminum plate. E-plane coupling has the long slot dimensions juxtaposed and was chosen to maximize the mutual coupling between slots [27]. A front view of the array is shown in Figure 3-2 and a rear view is shown

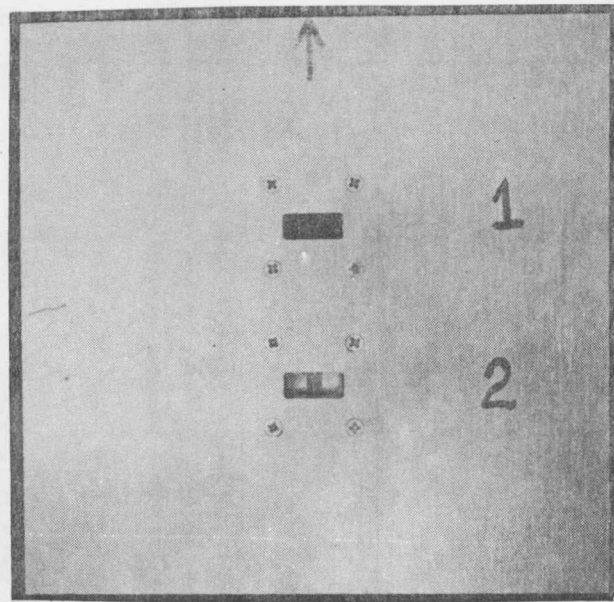


Figure 3-2. Front View of X-Band Slotted Plane Array.

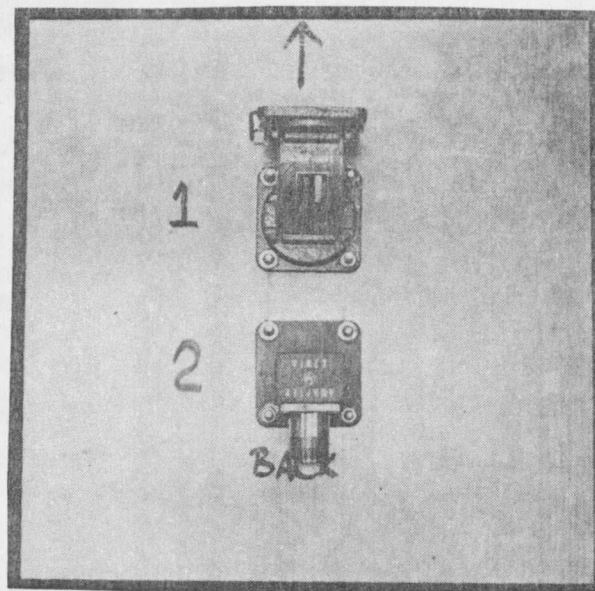


Figure 3-3. Rear View of X-Band Slotted Plane Array.

in Figure 3-3. The slots are fed from behind the ground plane by either waveguide or coax (the latter requires a coax to waveguide adaptor). Countersunk mounting holes were used to keep bolts from protruding above the ground plane and possibly disturbing the fields. To improve alignment of the slots with their waveguide feeds, special sleeves were constructed which fit over the mounting bolts thus taking up slack.

The theoretical calculations for a planar array were based on an infinite ground plane. In practice, however, induced surface currents diminish rapidly with distance from the radiating apertures so fairly small ground planes may be used. For example, Croswell, et al [34] found that the standard flange of an open ended waveguide is sufficiently large for self admittance measurements. When mutual coupling between slots is of interest, however, the ground plane dimensions should be several times the slot-to-slot spacing.

The slotted cylinder array was constructed from a 38 cm (15 inch) length of 15.24 cm (6 inch) diameter by 0.318 cm (1/8 inch) thick aluminum irrigation pipe. The 15.24 cm (6 inch) diameter is probably too large for

a practical moisture probe because of the large bore-hole required for field installation. This diameter was a convenient size for laboratory measurements, however, because there was ample room inside the cylinder for installing waveguide feeds and flange adaptors. As for the planar array, rectangular E-plane coupled slots 2.286×1.016 cm ($0.900'' \times 0.400''$) spaced 6.096 cm ($2.40''$) were employed. Special flange adaptors were machined to mate the curved surface of the cylinder to standard planar waveguide flanges. Convex adaptors were used for the waveguide feeds within the cylinder, while concave adaptors were used outside to attach standard terminations for system calibration. A photograph of the slotted cylinder array is shown in Figure 3-4 and the waveguide flange adaptors are shown in Figure 3-5.

System Calibration

Scattering parameter measurements of a distributed network such as the slotted array may be referred to an arbitrary plane. Since the driving point admittance of the slot antennas is of interest (this is what the computer programs calculate), it seems natural to

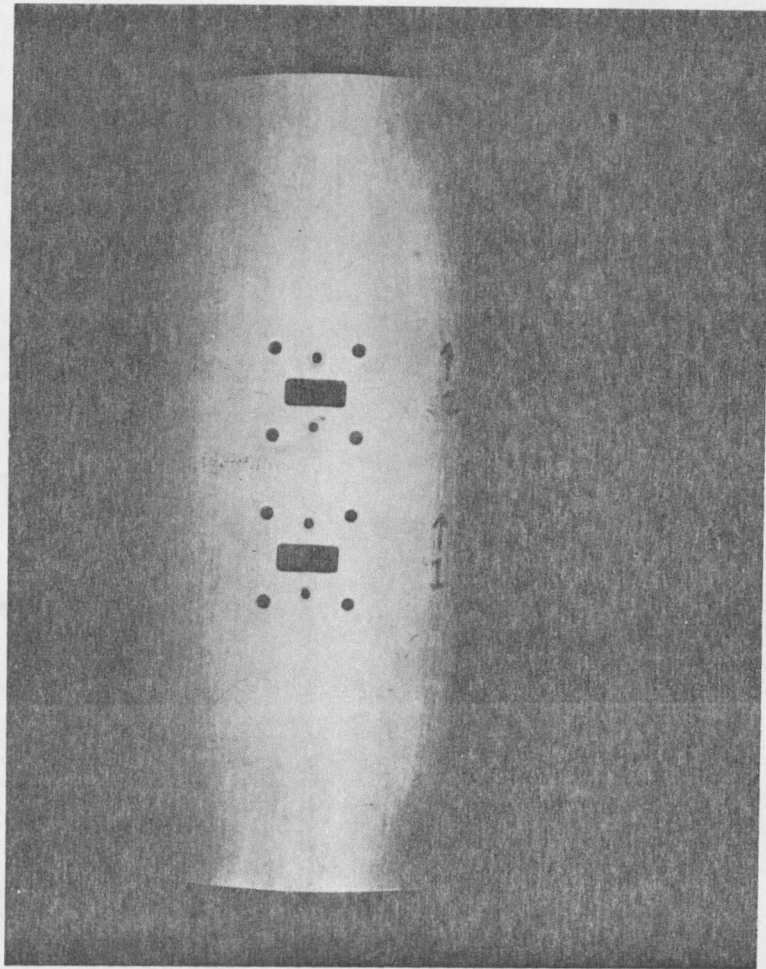


Figure 3-4. Front View of 15.24 cm (6 inch)
Diameter X-Band Slotted Cylinder
Array.

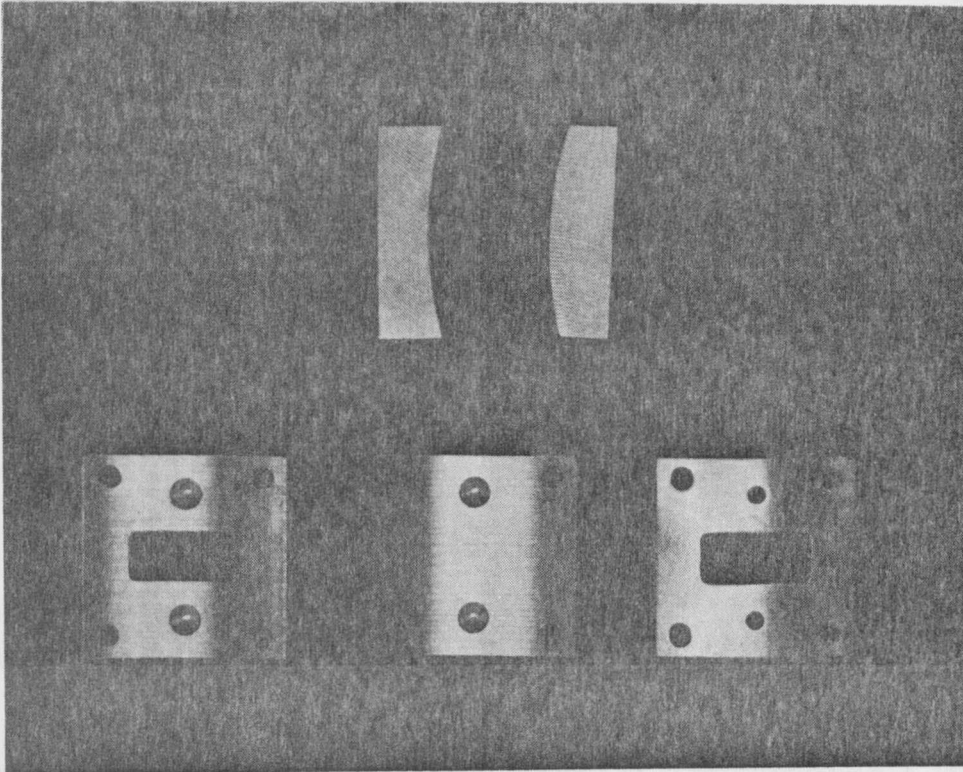


Figure 3-5, View of Waveguide Flange Adaptors.

establish a reference at the plane of the slot. In practice this may be done by measuring the self scattering from a slot terminated successively in three known loads (for example a short, an offset short, and a matched load). It can be shown [41] that these measurements are sufficient to characterize the distributed network (consisting of transmission lines, directional couplers, coax to waveguide adaptors, etc.) between the network analyzer and the desired reference plane. When this network is characterized, one can compensate for imperfections in the measurement system (such as variations in magnitude and phase with frequency). A computer program (SCAL) was written to derive s parameter corrections based on measurements with three known terminations. These corrections allow the self admittance to be referred to the plane of the slot. A program listing of SCAL is given in Appendix D.

Although it is common to speak of reference planes when describing properties of distributed circuits, this concept does not directly apply to the slotted cylinder geometry. To be rigorous, the driving point admittance should in this case be described in terms of a reference

cylinder. In practice the distinction between reference plane and reference cylinder becomes more important as the radius of curvature decreases compared to a free space wavelength. Also the effect of curvature is more pronounced for transverse slots than for axial slots.

To determine the effects of curvature on reference plane location, one of the slotted cylinder apertures was terminated in a short circuit having the same radius of curvature as the cylinder. A slotted waveguide probe was then used to measure the locations of voltage nodes as a function of frequency. Next the curved short was replaced by an offset planar short mated to the cylinder with a concave flange adaptor. Again node locations were noted as a function of frequency. It was found that the node shift between the curved short and the offset planar short was constant, independent of frequency. Thus a definite reference plane can be established for the 15.24 cm (6 inch) diameter cylinder. For cylinders with small radius of curvature, however, the apparent short circuit location may be frequency dependent.

Free Space Measurements

Measuring the admittance and isolation of slot antennas radiating into a medium with known dielectric constant provides a basic check on the experimental procedures and theoretical results. Perhaps the most convenient medium with known properties is free space, for which $\hat{\epsilon}_r$ is very nearly $1 - j0$. Ideally, free space aperture characteristics should be measured in a microwave anechoic chamber to minimize spurious reflections. Although an anechoic chamber was unavailable, good agreement with theory was obtained when the slotted arrays were directed into a 61 cm (2 foot) square sheet of Emerson and Cummings' Echosorb[®] AN-73 material from a distance of about 1 m (30 free space wavelengths). A comparison between the calculated and measured values of normalized admittance for the planar array are shown in Figure 3-6. Slot admittances are normalized with respect to the admittance of the waveguide feeds. Excellent agreement between theory and experiment was obtained. These values also agree with results reported by Croswell, et al [34].

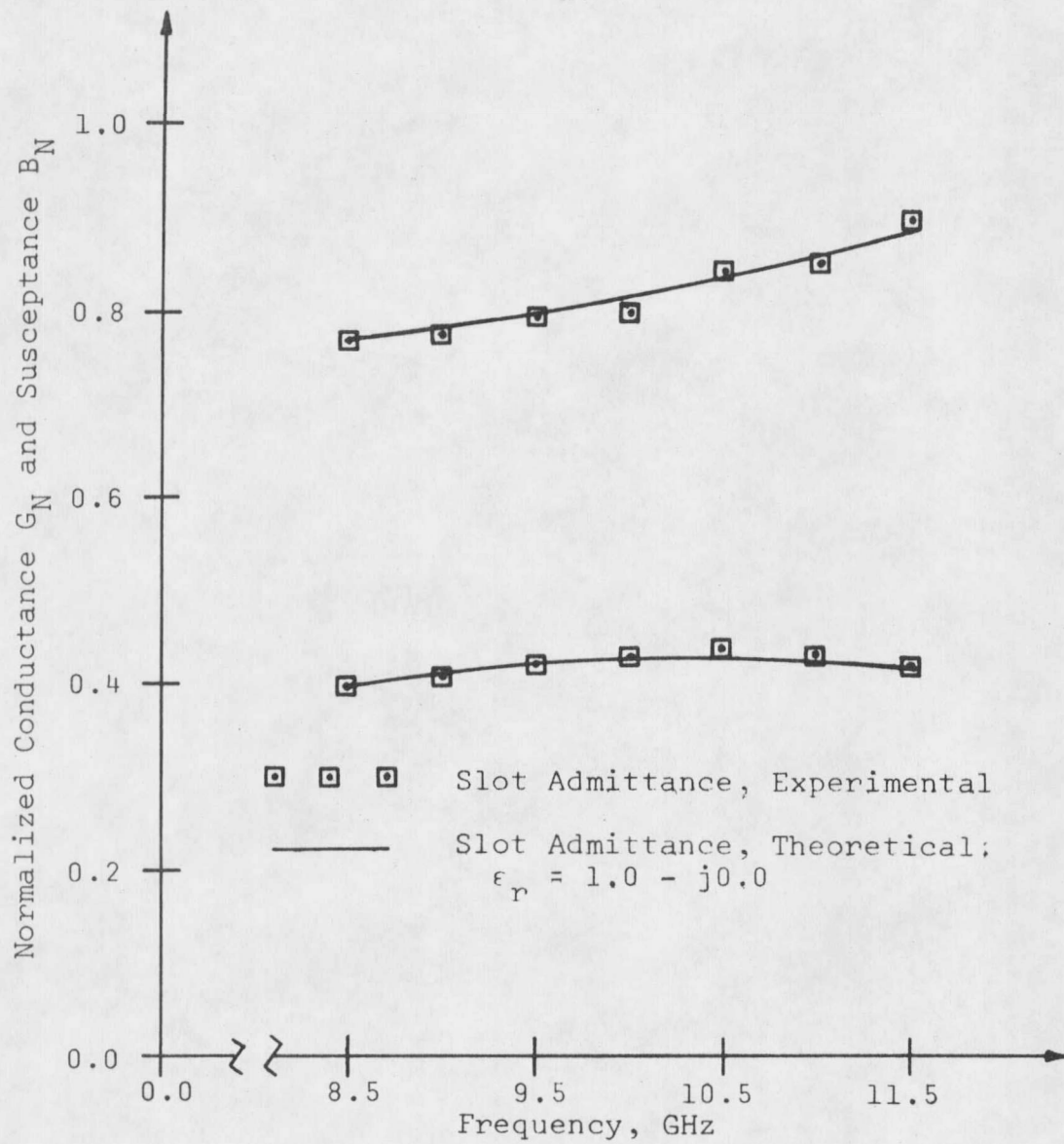


Figure 3-6. Normalized Free Space Admittance $Y_N = G_N + jB_N$ of 2.286 cm x 1.016 cm Slot in a 23 cm Square Ground Plane.

Figure 3-7 shows normalized admittances for the 15.24 cm (6 inch) diameter slotted cylinder. Again good agreement between measured and theoretical values was obtained. It is interesting to compare the measured self admittances of the slotted cylinder and slotted plane arrays. The conductance values for the cylinder are approximately 6% to 10% greater than for the planar array. The closest agreement occurs at the highest frequencies. This behavior is expected since the cylinder behaves more like a ground plane at the higher frequencies where its effective circumference ($k_0 r_0$) is greatest.

Figure 3-8 shows the theoretical and experimental free space isolation (isolation = $-20 \log_{10} |s_{21}|$) for the two arrays. The arrays had identical slot spacing (6.096 cm or 2.40 in). The measured values fall within ± 1 dB of calculated free space values. This is consistent with the accuracy achieved by Fante [26] for isolation of axially slotted cylinders.

The preceding admittance and isolation measurements confirmed that the slotted plane and slotted cylinder arrays behaved as expected in a medium of known electromagnetic properties. Next the arrays were used to measure the properties of an unknown medium.

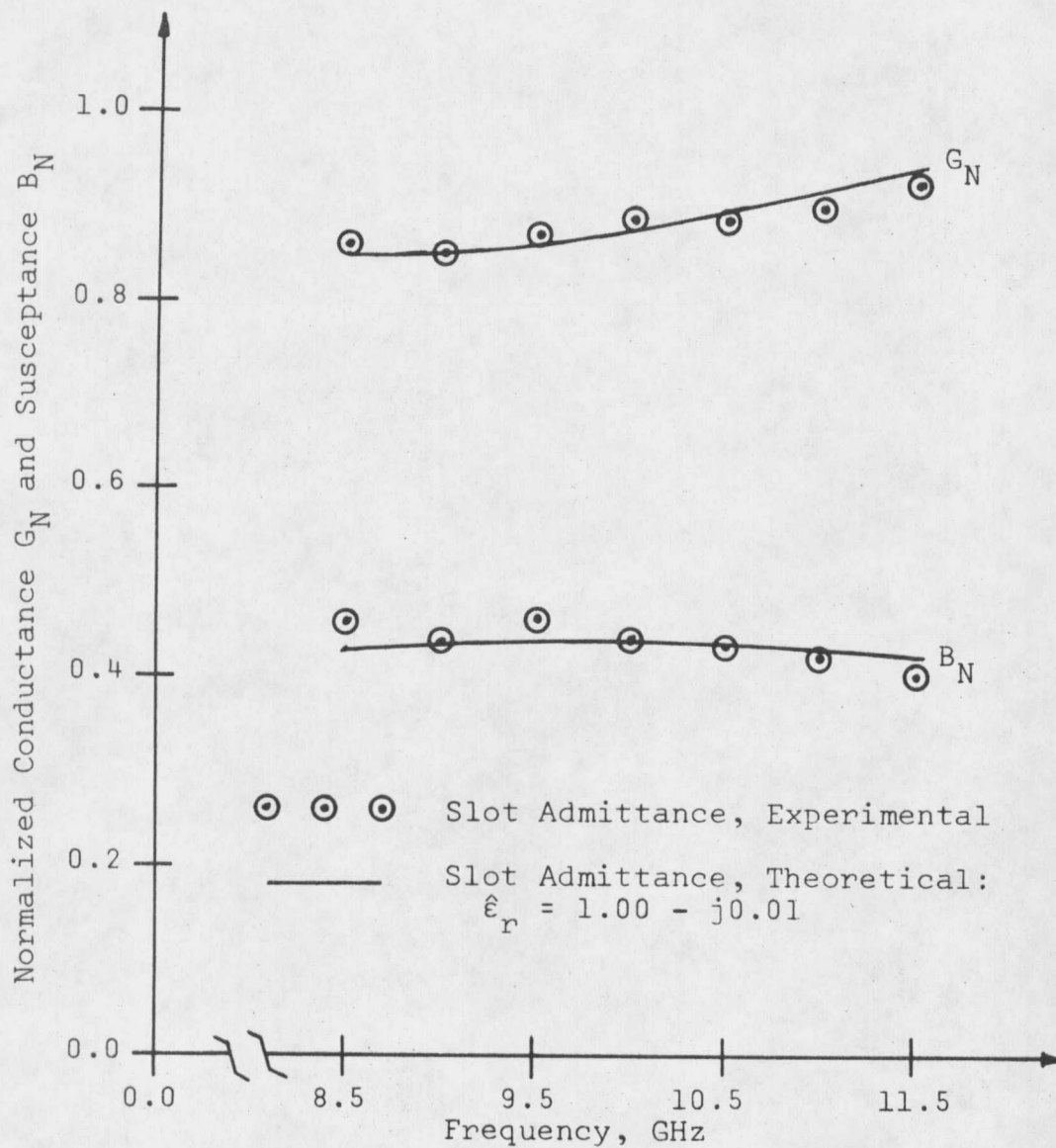


Figure 3-7. Normalized Free Space Admittance $Y_N = G_N + jB_N$ of 2.286 cm x 1.016 cm Transverse Slot on 15.24 cm Diameter Cylinder.

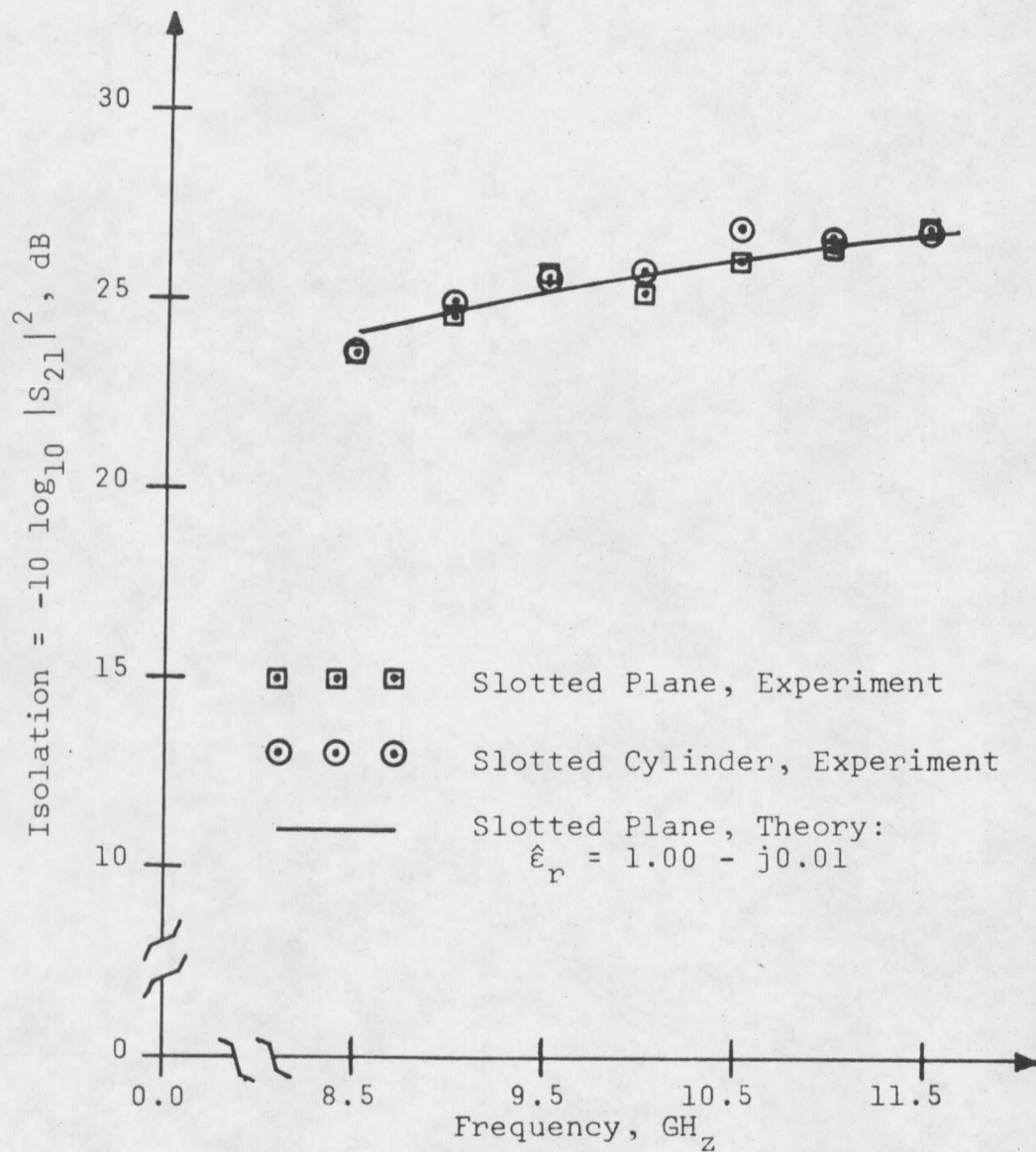


Figure 3-8. Free Space E-Plane Slot Isolation.
Slot Spacing = 6.10 cm.

Soil Measurements

The soils used in this project were supplied by the Montana State University Department of Plant and Soil Science. Initially "greenhouse" soil was used. Essentially a mixture of sand and organic material, greenhouse soil is capable of absorbing large amounts of water. Unfortunately, its bulk density (≈ 0.8) is much lower than that of normal crop soils (≈ 1.3). This lower density results in fewer dipole moments per unit volume. Consequently, permittivities measured for greenhouse soil are small compared to those of denser soils with the same volumetric moisture content. This relationship between soil density and permittivity was not discussed in detail by Hoekstra and Delaney [10], possibly because their soil samples had nearly equal densities. For the 57 soil samples they measured, the mean density was 1.49 and the standard deviation was 0.14. The mean density of their soils was about 85% higher than the density of greenhouse soil. Although soil type may have little effect on permittivity at X band, soil density cannot be neglected. Fortunately, the density of in situ crop soil is fairly constant from just below the surface to

root zone depth [2] so a nominal density value should suffice for establishing experimental relationships between permittivity and moisture content.

Because of its atypical density, measurements on greenhouse soil were suspended and a denser soil was obtained. The new soil had a density of approximately 1.30 and was very homogeneous, having been thoroughly ground and passed through a sieve with 5 mm (2×10^{-3} inch) openings.

Sample Preparation

Although the new soil was quite homogeneous when dry, care had to be taken when moistening it and packing it into the sample holder. To moisten the soil a thin layer (approximately 2 cm thick) was loosely packed into the bottom of a large plastic bucket. A mist of water was then directed at the center of the sample as the moisture was thoroughly worked into the soil by hand. When the desired amount of water had been absorbed, another layer of dry soil was added and the process repeated. Since soil density and sample volume were known, the total amount of water required for a given percent moisture could be calculated. After this amount of water

had been thoroughly worked into the soil, the plastic container was covered and set aside for 12 to 24 hours. This allowed the mixture to reach equilibrium.

Prior to the slotted array measurements the soil was transferred from the storage container to an appropriate sample holder. Figure 3-9 shows a photograph of the slotted cylinder and slotted plane sample holders. To help achieve a uniform packing density the loose soil was added to the sample holder in layers, each successive layer being leveled and tamped with an aluminum plate. The entire sample was then weighed to determine its density. It was very important to ensure that the soil surface was level to eliminate air gaps between ground plane and soil. Such discontinuities effect both self admittance and isolation measurements. When the slotted cylinder was being used, soil was packed around it to follow the curvature as closely as possible. In field measurements air gap effects can be minimized by driving the cylinder into a snug fitting bore hole.

After the microwave measurements were taken the soil moisture was determined by oven drying. The oven was operated at a temperature of 90°C. Higher temperatures were avoided to prevent burning off any organic

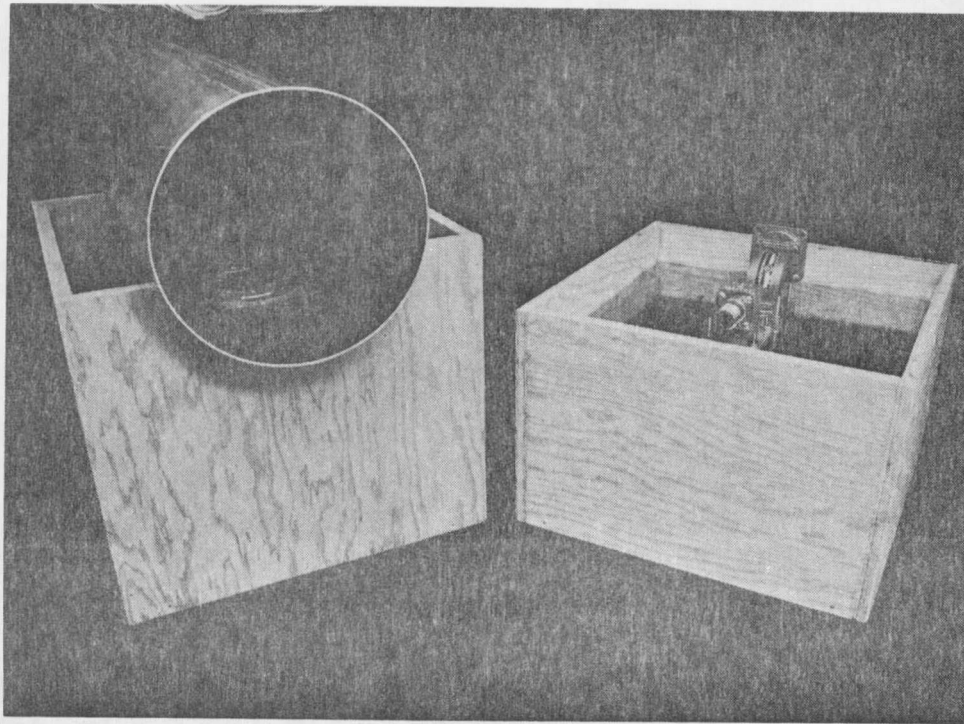


Figure 3-9. View of Soil Sample Holders for the Slotted Cylinder and Slotted Plane Arrays.

material in the soil. A tray of dessicant was also placed in the oven to reduce vapor pressure, thus stimulating evaporation. A total drying time of about 36 hours was found to be adequate for moisture contents up to 25% H_2O/cc . Due to limitations on oven size, it was not feasible to dry the entire 32 cm x 23 cm x 15 cm sample. Instead soil from the sample holder was packed into a small metal cup (137 cc) which would conveniently fit in the drying oven. When volumetric water content (grams H_2O/cc) is being measured, the cup must be packed to the same density as was the large sample holder. Overpacking would increase the total amount of soil and water without affecting the sample's volume. This would give an inflated volumetric moisture reading. Gravimetric water content (grams $H_2O/grams$ soil) is less sensitive to packing density.

The procedure for proper packing of each cup is as follows. First, the entire soil sample is weighed to determine its average density. The weight of the metal cup when packed to this same density is given by

$$W_T = W_E + (P_{AVG})(V_C) \quad (5-1)$$

where

W_T = total packed weight of cup

W_E = empty weight of cup

P_{AVG} = average soil density

V_C = volume of cup.

Thus soil from the sample holder is packed into the cup until the desired weight W_T is obtained. This packing problem is a source of error for volumetric water contents but not for gravimetric water contents. However, volumetric measurements are preferred for electromagnetic based methods because this quantity is more nearly proportional to the number of dipole moments/volume of soil [10].

Soil Measurements with the Slotted Cylinder and Slotted Plane Arrays

Results of Chapter 2 predict that the X-band admittance and isolation of the 15 cm (6 inch) diameter cylindrical array should closely match that of a planar array having identical slot dimensions, spacing, and orientation. If true this result is quite useful for two reasons. First, the fast and very efficient planar array algorithm could be used to compute theoretical cylindrical

array performance. This would result in substantial savings in computer time compared to running the slotted cylinder program (6000 integrand evaluations versus 200). Second, laboratory soil moisture measurements made with a planar array could be used to establish calibration curves for the slotted cylinder array. This would save time in the laboratory because it is difficult to pack soil in a contour which fits around the cylinder without gaps while maintaining uniform density. This should not be a problem in actual field measurements, however, because the soil will be properly contoured from the bore hole left by the core sampler. Another factor which makes laboratory slotted cylinder measurements time consuming is the manner in which the convex and concave flange adaptors are attached to the cylinder. As Figure 3-10 shows, the waveguide feed, flange, adaptors, and termination are held together by four long bolts passing through the standard flange mounting holes. These bolts must be removed each time the termination is changed (three different terminations must be used for system calibration). Even though special sleeves were fitted over the bolts to take up slack, the possibility of random misalignments between waveguide and apertures

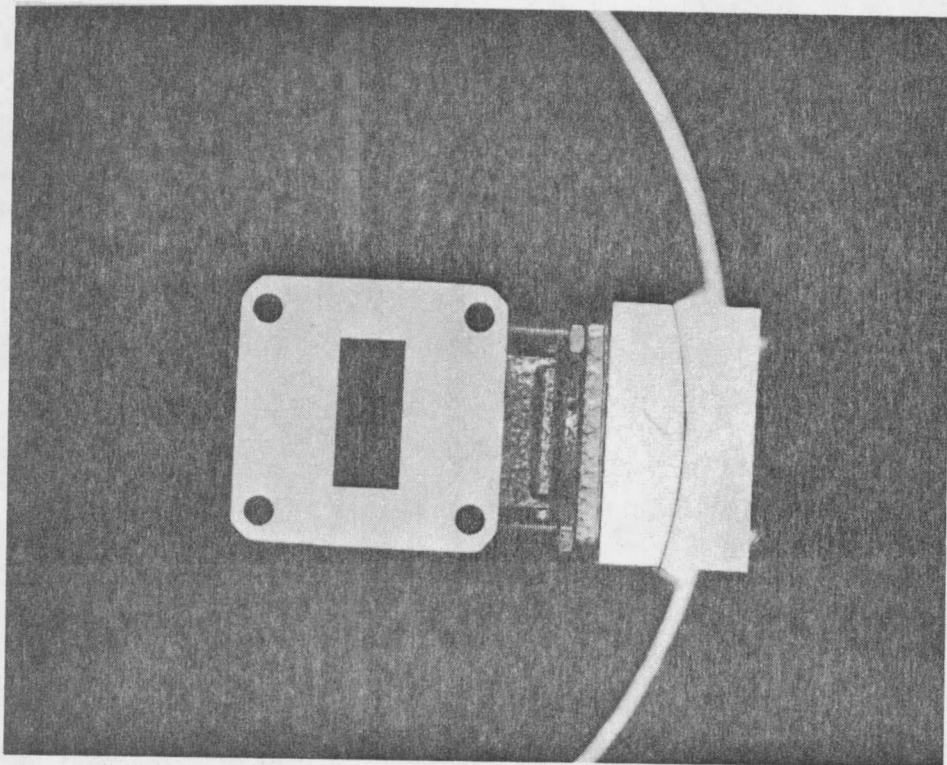


Figure 3-10. View of Slotted Cylinder Waveguide Feeds and Flange Adaptors.

is much greater for the slotted cylinder than for the slotted plane. This problem can be eliminated in future designs by permanently bonding the convex flange adaptor to the inside of the cylinder. Self-tapping screws could be used on the outside to attach terminations and on the inside to attach the waveguide feeds.

Figure 3-11 shows the conductance G_n (normalized to the admittance of the waveguide feed) versus frequency for the slotted plane and cylinder radiating into 11% $\text{gH}_2\text{O/cc}$ soil. Susceptance values are not plotted because they have little significance (S_{11} angles are very near 180° , so slight phase errors lead to large susceptance errors but have little effect on conductance). Figure 3-11 also shows theoretical conductance curves corresponding to different assumed soil permittivities. Two sets of planar array data are shown which represent different orientations of the soil sample. The conductances for these two runs agree within 5%. This corresponds to about 0.5 dB difference in measured return loss. This range of repeatability is typical for different antenna placements on the same soil sample. The principal source of disagreement is probably local density fluctuations

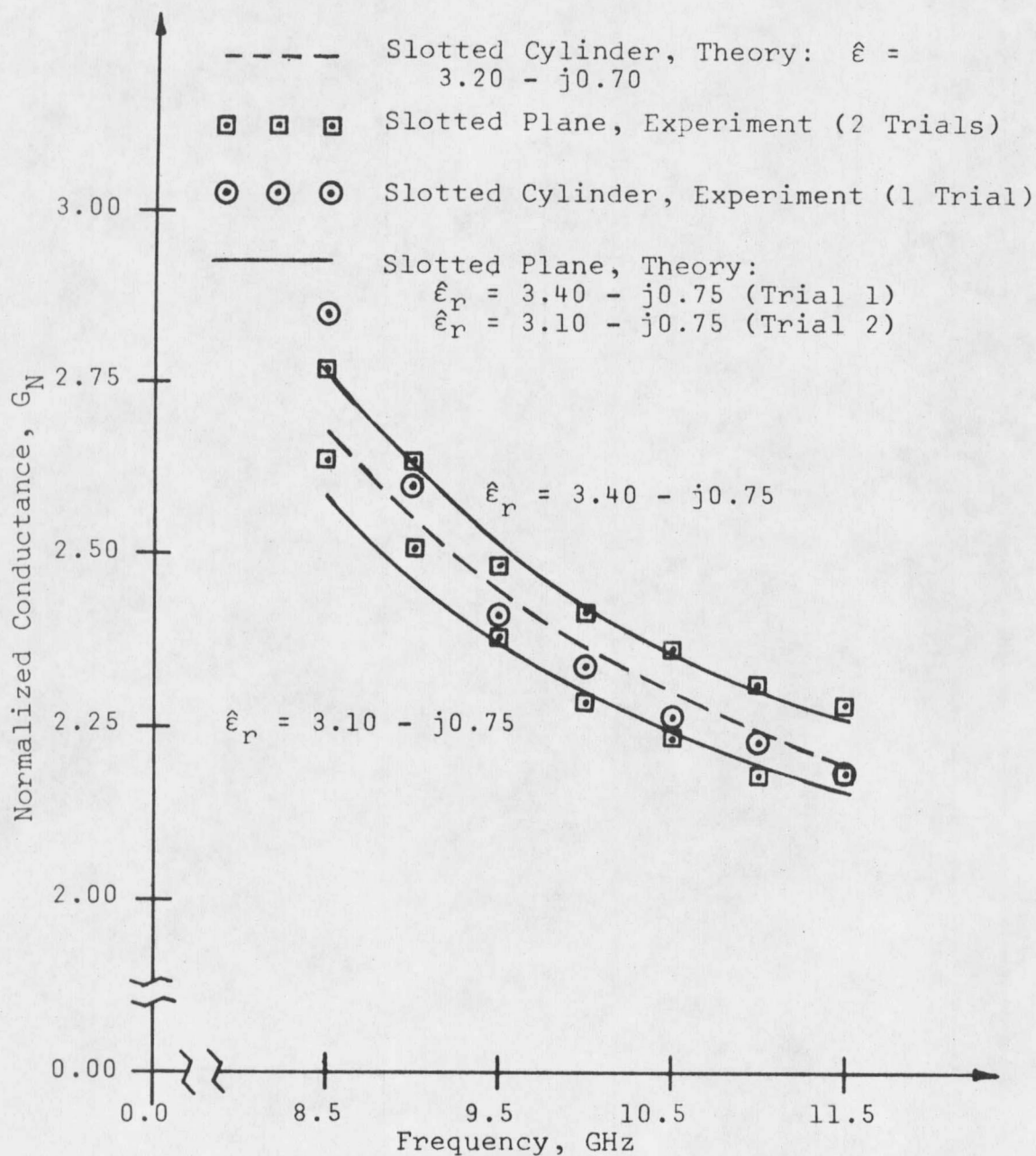


Figure 3-11. Comparison of Slotted Plane and Slotted Cylinder Normalized Conductance G_N for 11% $\text{gH}_2\text{O/cc}$ Soil.

which arise as the ground plane is pressed against a somewhat unlevel soil surface.

The theoretical curve fits for the two sets of planar conductances indicated real permittivities (ϵ_r') of 3.4 and 3.1, respectively. This is a 10% difference. As Figure 3-11 shows, the theoretical curves based on these values fit the measured data over the entire 3 Ghz bandwidth.

Conductance measurements were also taken with the slotted cylinder. These values fall between the two sets of ground plane data. Using the slotted cylinder program (TRANS) to fit measured conductances, the real part permittivity was found to be approximately 3.2. This agrees within 6% of the slotted plane values.

The experimental conductance values shown in Figure 3-11 are based on both phase and magnitude measurements of the reflection coefficient. Unfortunately, there are problems associated with measuring phase under field conditions. This was discussed in Chapter 2.

Fortunately, much information can be obtained from just scattering parameter magnitudes. Table 3-1 shows the relationship between G_N , VSWR, and slot reflection coefficients (at a frequency of 8.5 Ghz) for several

Table 3-1. Comparison of VSWR and G_N as a Function of Soil Volumetric Water Content.

$gH_2O/cc \times 100\%$	$\hat{\epsilon}_r$	ρ	G_N	VSWR	$\frac{VSWR - G_N}{VSWR} \times 100\%$
5%	2.7 - j0.4	.422 $\angle -167^\circ$	2.301	2.459	6.9%
10%	3.1 - j0.7	.451 $\angle -173^\circ$	2.579	2.644	2.5%
15%	3.5 - j1.0	.479 $\angle -176^\circ$	2.819	2.840	0.7%
20%	4.0 - j1.4	.510 $\angle -179^\circ$	3.081	3.082	0.03%
25%	4.4 - j1.7	.531 $\angle +179^\circ$	3.267	3.269	0.06%

typical soil permittivities. These values were calculated with the slotted ground plane program (PLANE). It can be seen that the magnitude of the reflection coefficient is a much stronger function of permittivity than is the angle. Also the reflection coefficient angles are approximately 180° . These observations indicate that the reflection coefficient ρ may be approximated as

$$\rho = |\rho| \angle -|\rho|. \quad (3-1)$$

The relationship between reflection coefficient and normalized permittivity Y_n is

$$Y_n = \frac{1 - \rho}{1 + \rho}. \quad (3-2)$$

Using the approximation of (3-1) in (3-2) gives

$$Y_n \approx G_n \frac{1 - |\rho| \angle 180^\circ}{1 + |\rho| \angle 180^\circ} = \frac{1 + |\rho|}{1 - |\rho|}. \quad (3-3)$$

Recalling that the righthand term of (3-3) is also the equation for voltage standing wave ratio VSWR, it is clear that

$$G_n \approx \text{VSWR}. \quad (3-4)$$

The three right hand columns of Table 3-1 show that this approximation holds very well for soil moisture contents of 10% to 25% grams $\text{H}_2\text{O}/\text{cc}$. Thus it is possible to use VSWR to estimate slot conductance. This result is important because field measurement of VSWR requires minimal equipment, e.g., a modulated Gunn oscillator and directional coupler. Since transmission line loss may affect measured VSWR, an s parameter equivalent circuit of the slot antenna feed system should be derived before the array is installed. Then transmission line effects may be compensated for.

Another quantity which may be measured rather simply in the field is isolation. Isolation is defined as the magnitude of the transmission scattering parameters (s_{ij} 's) expressed in decibels, i.e.

$$I_{ij} = -10 \log_{10} |s_{ij}|^2 \quad (3-5)$$

where $|s_{ij}|^2$ is the power reaching the i^{th} slot antenna due to excitation of the j^{th} slot antenna. The i^{th} antenna is terminated in a matched load.

Figure 3-12 shows isolation measurements made on the same soil sample whose admittance measurements are shown in Figure 3-11 (11% $\text{gH}_2\text{O/cc}$). The slotted plane and slotted cylinder data agree to within approximately 2 dB. Curve fitting of the slotted plane isolation indicates an imaginary permittivity components of $\epsilon_r'' = -0.75$ while the slotted cylinder isolation gave $\epsilon_r'' = -0.70$. This is a difference of 7%. As Figure 3-12 shows, the theoretical curves fit measured data quite well over the entire 3 GHz bandwidth.

The preceding conductance and isolation measurements indicate that slotted plane data typically agrees within about 10% with slotted cylinder data (15 cm diameter). Theory predicts much closer agreement. Table 3-2 compares theoretical admittance and isolation values for the two arrays. A permittivity of $\epsilon_r = 3.2 - j0.7$ is assumed.

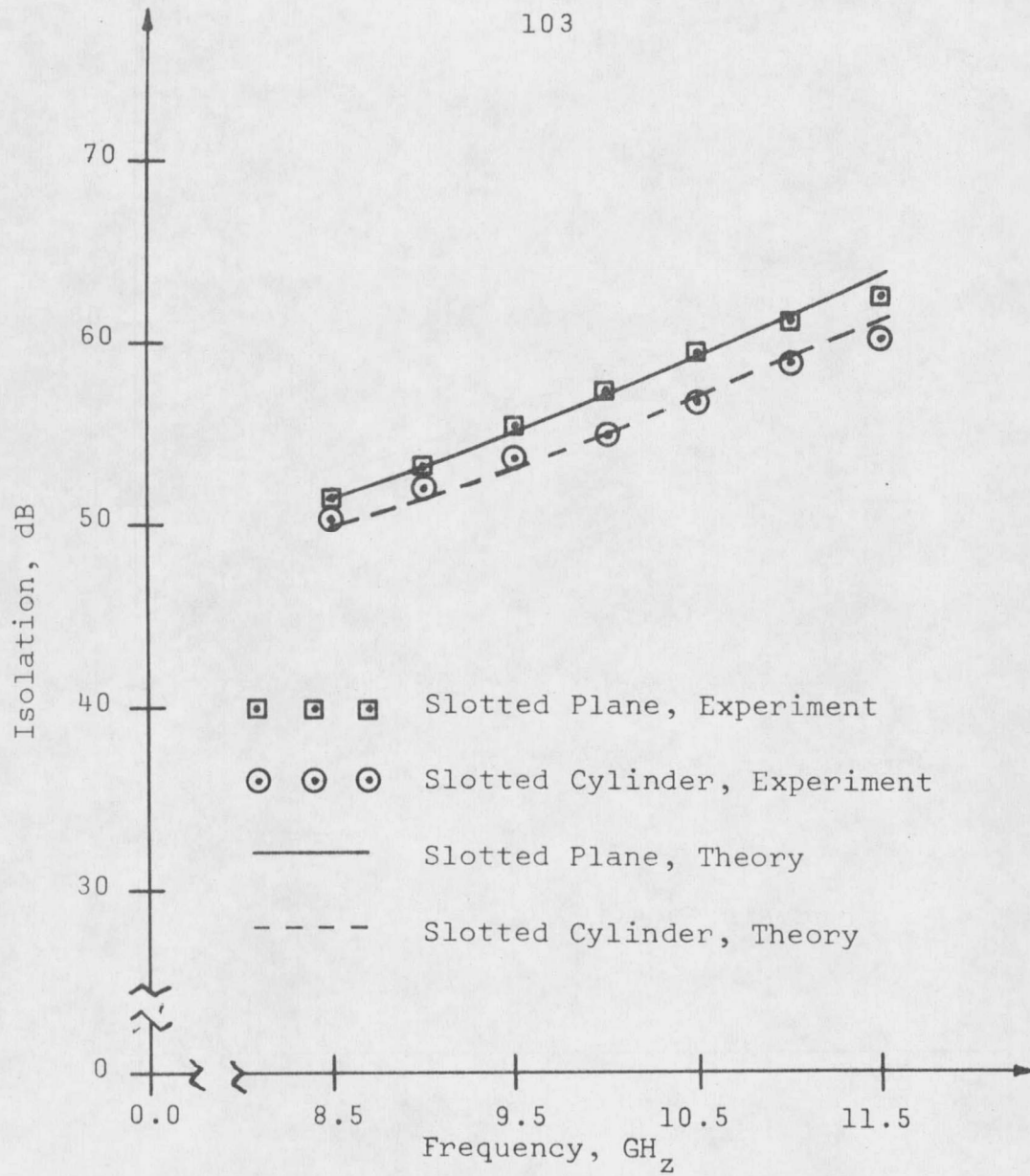


Figure 3-12. Comparison of Slotted Plane and Slotted Cylinder Isolation for 11% $\text{gH}_2\text{O/cc}$ Soil, Slot Spacing = 6.10 cm.

Table 3-2. Theoretical Comparison of Slotted Plane and Slotted Cylinder Properties for $\epsilon_r = 3.2 - j0.7$.

Frequency	Normalized Admittance		Isolation, dB	
	Plane	Cylinder	Plane	Cylinder
8.5	2.64+j.38	2.68+j.33	50.0 $\angle$ 35°	49.3 $\angle$ 31°
9.5	2.42+j.24	2.44+j.19	53.5 $\angle$ -88°	52.8 $\angle$ -92°
10.5	2.29+j.13	2.30+j.09	57.4 $\angle$ 150°	56.9 $\angle$ 145°
11.5	2.19+j.05	2.19+j.01	61.8 $\angle$ 29°	61.1 $\angle$ 28°

Admittances agree within 1.5% and isolations agree within about 0.7 dB. The fact that greater differences were observed experimentally is probably due to variations in packing the sample holder and possibly from slight drying of the soil between observations. The slotted cylinder data was taken 11 days after the slotted plane data. Even though the soil was sealed in a plastic bag slight drying may have occurred. This could explain why the slotted cylinder isolation was less than the slotted plane isolation.

Effective Sensing Radius of the Slotted Cylinder

Sensing radius, like bandwidth or rise time, may be defined in many different ways. A theoretical definition was given in Chapter 2. A convenient operational definition for moisture measurement purposes is the minimum distance between a slot antenna and a reflecting ground plane for which the ground plane has no measurable effect on return loss. This definition of sensing radius indicates the range at which significant soil inhomogeneities (such as cracks or rocks) have negligible effect on moisture readings.

Laboratory measurement of sensing radius involved burying a 14 cm (5.5 inch) square aluminum plate in the soil sample holder. The slot return loss was measured as a function of the plate's depth. Table 3-3 shows results obtained for 21% gH₂O/cc soil. Return losses for plate depths of 3.8 cm and 8.6 cm were found to agree within 0.3 dB of the value measured with the plate absent. Since the soil had to be repacked each time plate depth was changed, differences on the order of 0.3 dB may be attributed to packing

Table 3-3. Effect of Buried Metal Plate on Slot Antenna Return Loss

Plate Depth (cm)	Return Loss (dB)		
	With Plate	Without Plate	Difference
0.0	0.0	7.2	7.2
0.8	4.1	7.2	3.1
2.3	8.3	7.2	1.1
3.8	6.9	7.2	0.3
8.6	7.5	7.2	0.3

variations. Thus, within experimental error, the effective sensing radius is roughly 3.8 cm.

If the soil's complex permittivity is known, the sensing radius may be expressed in terms of skin depth as follows. The complex plane wave propagation factor $\hat{k}$ is given by

$$\hat{k} = \beta - j\alpha = w \sqrt{\epsilon_o \epsilon_r} \quad (3-6)$$

where

β = real propagation constant (radians/meter)

α = attenuation constant (nepers/meter)

ω = frequency (radians/second).

Skin depth is defined as

$$\sigma = \frac{1}{\alpha} = -\text{Im}\left\{\frac{1}{\omega \sqrt{\epsilon_0 \epsilon_r}}\right\}. \quad (3.7)$$

The measured permittivity of 21% $\text{gH}_2\text{O/cc}$ soil was $\epsilon_r = 4.0 - j1.4$. Thus the skin depth at a frequency of 8.5 GHz is

$$\sigma = -\text{Im}\left\{\frac{1}{1.780 \sqrt{4.8 - j1.4}}\right\} = 1.63 \text{ cm}. \quad (3-8)$$

The sensing radius in skin depths is thus

$$R \approx 3.8 \text{ cm} / 1.63 \text{ cm} = 2.4 \text{ skin depths}. \quad (3-9)$$

The sensing radius given in (3-9) is approximate but is a useful rule of thumb for estimating the volume of soil sensed by one of the slot antennas.

Soil Permittivity as a Function of Volumetric Moisture Content

Having previously established theoretically and experimentally that the slotted cylinder and slotted plane arrays have essentially similar properties, the remaining soil measurements were made with the slotted plane.

Figure 3-13 shows measured permittivity as a function of volumetric moisture content. The solidlines are linear regressions of the experimental data. The average density of the soil samples used in these measurements was 1.35 and the standard deviation was 0.08. For comparison purposes both slotted cylinder and slotted plane data are shown for a moisture content of 11% gH₂O/cc.

The linear regression equations for ϵ'_r and ϵ''_r in terms of %gH₂O/cc are

$$\epsilon'_r = 2.314 + (0.0845)(\%gH_2O/cc) \quad (3-10)$$

and

$$\epsilon''_r = 0.056 + (0.0657)(\%gH_2O/cc). \quad (3-11)$$

To determine soil moisture from measured permittivity, (3-10) and (3-11) may be rewritten as

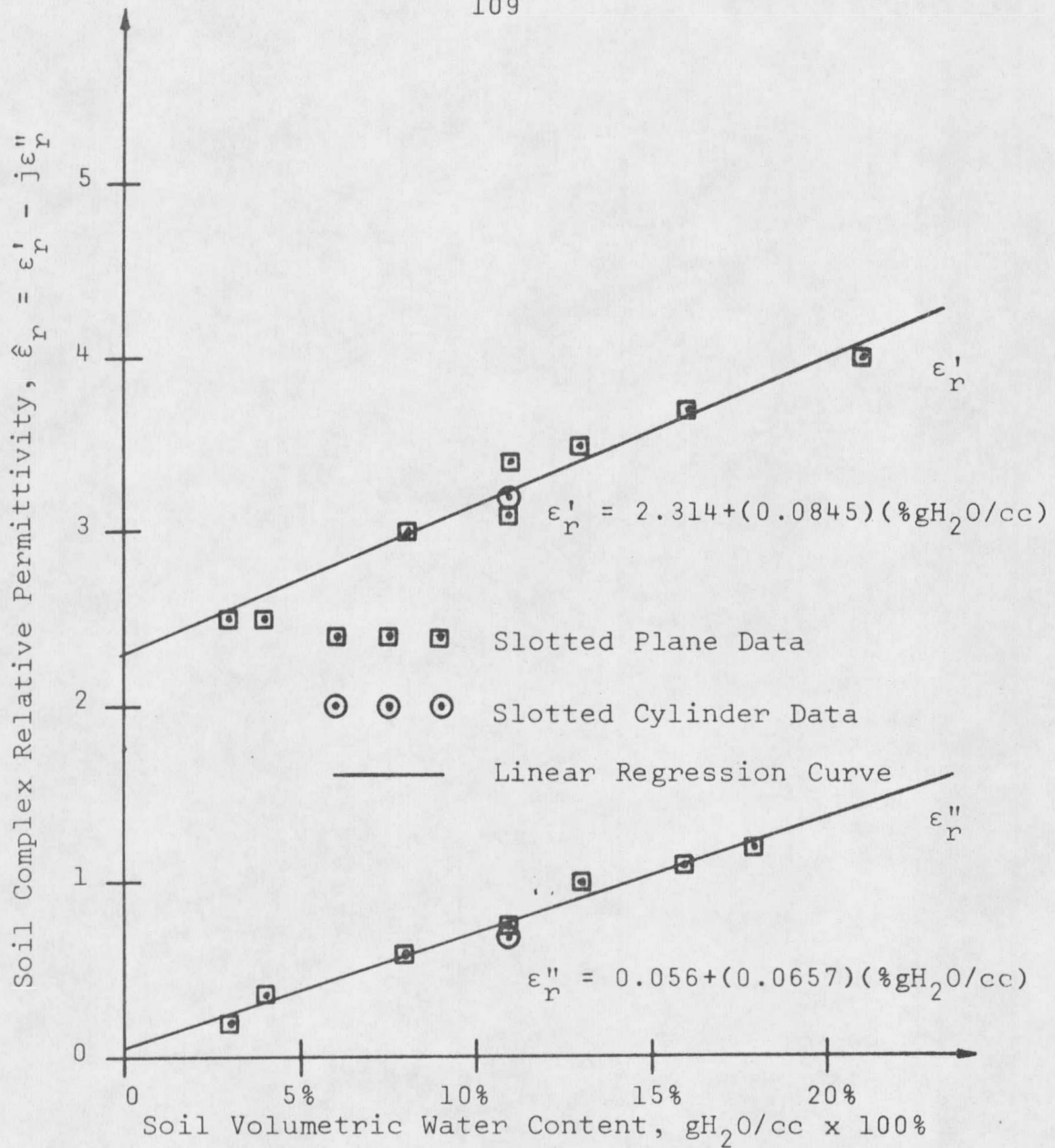


Figure 3-13. X Band Plot Antenna Measurements of Soil Permittivity Versus Volumetric Water Content ($\text{gH}_2\text{O/cc}$). Mean Soil Density = 1.35 g/cc . Standard Deviation = 0.08 .

$$\% \text{gH}_2\text{O/cc} = -27.38 + (11.83)(\epsilon_r') \quad (3-12)$$

and

$$\% \text{gH}_2\text{O/cc} = -0.852 + (15.22)(\epsilon_r''). \quad (3-13)$$

The above equations are based on X-band measurements of soils with bulk densities near 1.35 g/cc. Although Hoekstra and Delaney showed soil type has minimal effect on permittivity at X-band [10], additional work is required to determine the effect of soil density. Fortunately in situ soil density does not vary greatly. Typical values range from 1.1 to 1.5 g/cc.

Equations (3-10) to (3-13) are useful in estimating error sensitivity. For example, $\epsilon_r' = 3.16$ corresponds to 10% gH₂O/cc. If ϵ_r' was erroneously measured as 3.32 (5% high), the estimated moisture content given by (3-12) would be 11.9%. This is an error of 2% gH₂O/cc. Similarly, a 5% low measurement of ϵ_r' results in a 2% low moisture estimate. A $\pm 5\%$ error in determining ϵ_r' is not unreasonable so absolute accuracy of the slotted cylinder moisture probe is on the order of $\pm 2\%$. This estimated accuracy does not include effects of soil type or density.

It also assumes negligible error in the oven drying determination of true soil moisture.

Frequently relative changes in soil moisture are more important than absolute values. This would be true in studying dynamics of the downward percolation of rain water or in comparing the relative moisture contents of adjacent experimental plots. The sensitivity of the slotted cylinder to changes in soil moisture may be estimated as follows. The computer programs for calculating slot admittance show that a change in ϵ_r' from 3.16 to 3.26 produces a 0.15 dB change in return loss. A change of this magnitude is easily measured in the field. From (3-12) this same change in permittivity corresponds to soil moisture increasing from 10.0% to 11.2% $\text{gH}_2\text{O/cc}$. Thus the slotted cylinder can resolve 1% changes in volumetric moisture content.

Comparison of Permittivities Measured by Slot Antenna and Shorted Waveguide Methods

Figure 3-14 compares soil permittivities derived from slot antenna measurements with values given by Hoekstra and Delaney [10]. Hoekstra and Delaney's data was based on soil measurements in a short circuited

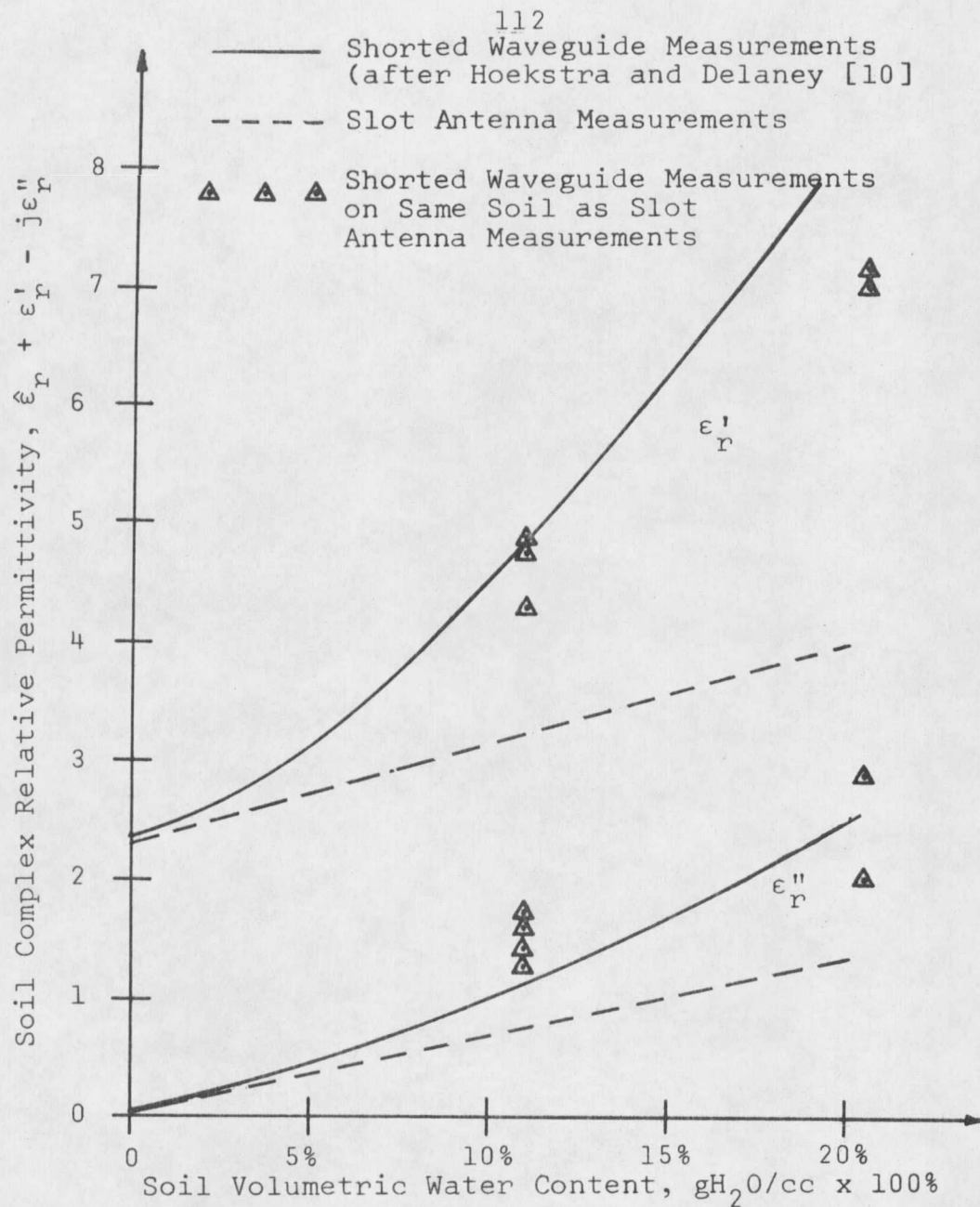


Figure 3-14. Comparison of Soil Permittivity as Measured by Slot Antenna and Shorted Waveguide Methods.

waveguide sample holder. Poor agreement was obtained. The slot antenna measurements are consistently low and the disparity increases with moisture content. When the same soil samples used for slotted array measurements were tested in a waveguide sample holder, results similar to Hoekstra and Delaney's were obtained. These measurements are also shown in Figure 3-14.

Croswell et al [34] observed similar behavior for a rectangular slot radiating into a dielectric slab ($\epsilon'_r = 3.76$). Their measured admittances were also consistently lower than theory predicted. Suspecting that higher order modes induced at the aperture-dielectric boundary were the problem, they attempted to measure the aperture field distribution. For free space radiation (no dielectric) the aperture distribution was approximately TE_{10} as expected. When the dielectric slab was introduced, a more peaked distribution resulted. Croswell suggested that the presence of a TE_{30} mode would explain the observed aperture field.

The effect of multimoding at the soil-aperture boundary may be qualitatively treated as follows. Figure 3-15 shows a power conservation diagram for a

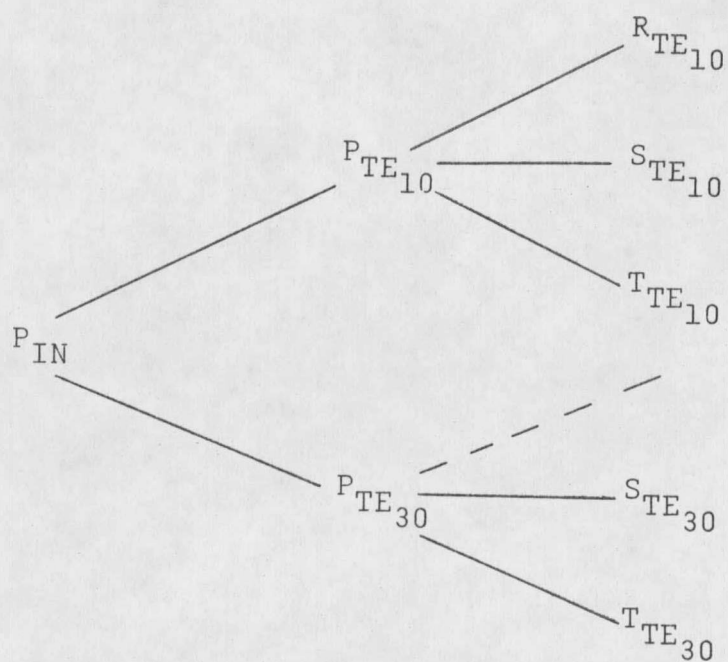


Figure 3-15. Power Conservation Diagram for a Multimoding Aperture.

multimoding aperture. Assuming power orthogonality (i.e. each possible mode carries energy independently of the other modes)[39], the total incident power P_{IN} is the sum of the powers contained in the TE_{10} and TE_{30} modes, i.e.

$$P_{IN} = P_{TE_{10}} + P_{TE_{30}} \quad (3-15)$$

The power contained in $P_{TE_{10}}$ and $P_{TE_{30}}$ is in turn reflected ($R_{TE_{10}}$), stored ($S_{TE_{10}}$, $S_{TE_{30}}$), or transmitted ($T_{TE_{10}}$, $T_{TE_{30}}$) from the soil-aperture boundary. The TE_{30} mode has no reflected component because it is evanescent in the waveguide which illuminates the aperture. Since the network analyzer compares incident to reflected power at a point many wavelengths from the aperture, evanescent modes are not detected. Thus the measured scattering from the aperture is

$$|S_{11}|^2 = \frac{R_{TE_{10}}}{P_{IN}} = \frac{R_{TE_{10}}}{P_{TE_{10}} + P_{TE_{30}}} \quad (3-16)$$

If multimoding does occur, $P_{TE_{30}}$ is nonzero and the

apparent reflection coefficient magnitude is reduced.

Since admittance magnitude is given by

$$|Y_N| \approx \frac{1 + |S_{11}|}{1 - |S_{11}|} \quad (5-17)$$

experimental admittance values would also be low.

When the soil is contained in a waveguide sample holder multimoding is less likely because the boundary conditions at the interface can then be satisfied by a pure TE_{10} mode (assuming the interface plane is normal to the waveguide axis). Thus reflection coefficients measured with the shorted waveguide sample holder are of proper size because the incident power is not diluted among several modes.

Since distortion in the aperture field (and hence multimoding) can be expected to increase with soil permittivity, the disparity between theoretical and experimental admittance should be greater for wet soil than for dry. This assumption would explain the difference between slotted plane data and shorted waveguide data shown in Figure 3-14.

By following the procedure developed in Chapter 2, the admittance of a slot antenna can be evaluated for any desired aperture field distribution. However, since the correct distribution to assume for a given soil permittivity is not known in advance, it does not seem worthwhile to pursue such derivations.

Fortunately multimoding effects do not limit the usefulness of the slotted cylinder as a moisture probe. The only restriction is that moisture versus permittivity data measured with coaxial or waveguide sample holders may not be used to calibrate the slotted cylinder.

Recommendations for Field Trial of Slotted Cylinder Array

Although initial laboratory tests of the slotted cylinder array were performed at X-band (8.2 - 12.4 GHz), this frequency range may not be suitable for field tests. The high attenuation offered by moist soil at X-band not only restricts sensing radius but also severely reduces mutual coupling of the slot antennas. These conditions may be improved by using lower frequencies. Table 3-4 compares isolation ($-20 \log_{10} |S_{21}|$) and sensing radius (2.5 skin depths) for slot arrays operating at X-band and S-band (2.60 - 3.95 GHz). For the worst case

Table 3-4. Comparison of Isolation and Sensing Radius of Slotted Arrays Operating at 8.5 GHz and 3.0 GHz. Slot Separation = 10.0 cm.

$\text{gH}_2\text{O/cc}$ $\times 100$	$\hat{\epsilon}_r$	Isolation		Sensing Radius	
		8.5 GHz	3.0 GHz	8.5 GHz	3.0 GHz
5%	2.7-j0.5	53.4 dB	33.4 dB	11.6 cm	32.8 cm
15%	3.6-j1.0	77.3 dB	43.8 dB	5.4 cm	15.2 cm
25%	4.4-j1.7	100.2 dB	53.4 dB	3.5 cm	10.0 cm

shown (25% $\text{gH}_2\text{O/cc}$), slot isolation is 100.2 dB at 8.5 GHz and 53.4 dB at 3.0 GHz. A slot spacing of 10.0 cm (4 inches) is assumed in both cases. A coupling loss of 53 dB is well within the dynamic range of a network analyzer but 100 dB is not. Of course, coupling could be increased by decreasing slot separation but then many more array elements are needed to cover a given soil profile. The worst case sensing radius shown is 3.5 cm for 8.5 GHz and 10.0 cm for 3.0 GHz. Although this parameter is not as important to effective array operation as isolation, a significant sensing radius permits the moisture measurement to be averaged over a larger volume of soil. Thus less emphasis is placed on effects at the soil-cylinder boundary. It should be noted that the same permittivity values were assumed for 3.0 GHz and 8.5 GHz. In reality the 3.0 GHz values would be somewhat higher [10].

Frequencies lower than S-band are not recommended for field testing because the apertures of the slot antennas become excessively large. In fact, a transverse S-band slot (7.214 cm x 3.404 cm) subtends an angle of 81° on a 10.16 cm (4.0 inch) diameter cylinder. As the

angle subtended by the slot increases, fringing effects become significant and the aperture field may deviate substantially from the assumed TE_{10} distribution even for free space radiation. Another argument against lower frequencies is the increasing dependence of permittivity on soil type noted by Hoekstra and Delaney [10]. S-band is proposed as a reasonable compromise between adverse high frequency and low frequency effects.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

This dissertation has demonstrated the feasibility of measuring the distribution of soil moisture from near surface to root zone depths (1-2 m) by means of an intrusive slotted cylinder probe. The probe consists of a linear array of slot antennas mounted along the axis of a conducting cylinder and is intended for borehole installation. It was shown that magnitude measurements of self and mutual slot antenna scattering parameters are sufficient to derive a discrete depth profile of soil permittivity. The moisture distribution is then determined from this permittivity profile by empirical relations. The total depth of the profile is determined by the overall length of the array, while the vertical resolution is largely dependent on the slot antenna spacing.

To aid in designing the slotted cylinder probe, theoretical expressions were derived which predict array performance for arbitrary soil complex permittivity. Computer programs were written to evaluate these expressions for both axial and transverse slot geometries. Similar expressions were also derived and evaluated for

a slotted ground plane array since they are much simpler to evaluate and closely approximate the slotted cylinder results in many cases.

The validity of the theoretical expressions was confirmed from laboratory measurements at X-band (8.2-12.4 GHz) of slotted plane and slotted cylinder arrays. Excellent agreement was obtained between measurement and theory for a medium of known permittivity (free space, $\hat{\epsilon}_r = 1 - j0$). Also, the slotted plane and slotted cylinder results were quite close, as theory predicted. A direct comparison between theory and experiment was not possible with soil measurements because the true soil permittivity was not known. Instead the measured slot admittances were compared to families of theoretical curves generated for a number of assumed permittivities. The permittivity providing the best fit to the experimental data was chosen as the estimated soil permittivity. As before, good agreement was obtained between slotted cylinder and slotted plane data.

Using the curve fitting method described above, soil permittivity measurements were made for soil moisture contents between 0.05 and 0.20 grams H_2O/cc (i.e. volu-

metric basis). It was not possible to measure slot-to-slot coupling at higher moisture contents with the antenna spacings used in the laboratory arrays. However by decreasing slot spacing or lowering the operating frequency, this measurement range could be extended.

The moisture content of the soil samples were determined by oven drying. Based on a linear regression analysis of the available data, the apparent accuracy of the slotted cylinder probe was on the order of ± 0.02 grams H_2O/cc . However, this estimated accuracy does not include errors induced by soil type variations because only one soil type was measured. The regression analysis was also used to show that the slotted cylinder should be able to resolve changes in soil moisture as small as 0.01 grams H_2O/cc . This sensitivity should prove very useful in studying the dynamics of percolation. In fact the potential resolution and sensitivity of the slotted cylinder appear superior to that of the neutron probe. Deployment of the slotted cylinder would not be regulated, as is the slotted probe, since it uses no radioactive materials.

It is recommended that the slotted cylinder probe

be tested under field conditions. This would reveal any difficulties associated with borehole installation, such as air gaps occurring between the soil and the slot antennas. Fortunately such air gaps can be detected from the surface by the presence of periodic VSWR variations with frequency. The slotted cylinder used for field tests should be designed for S-band (2.60 - 3.95 GHz) operation to improve slot-to-slot coupling and to extend the sensing radius over that of the X-band laboratory version.

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APPENDIX A

FORTRAN LISTING OF AXIAL, A COMPUTER PROGRAM
FOR CALCULATING SELF AND MUTUAL ADMITTANCES
OF PARALLEL AXIAL RECTANGULAR SLOTS
IN AN INFINITE CONDUCTING CYLINDER

PROGRAM AXIAL

THIS PROGRAM COMPUTES THE SELF AND MUTUAL ADMITTANCE
OF AXIAL RECTANGULAR TE¹⁰ EXCITED SLOTS ON AN

INFINITELY LONG CONDUCTING CYLINDER.

```

INTEGER RDR,PRT
INTEGER DISC,PLOT(11),BLANK,LINE(120)
COMPLEX DIV,HANK(51),GAMMA,ZN
COMPLEX YNM1,YNM2,YN,ADMIT(201),Y0,Z0
COMPLEX ER,ARG,H0,H1,HNM1,HNM2,HN,HNP,IN(120)
COMPLEX JN,ARG2,JNM1,ARGRAD,KE
COMPLEX Y12,Y12SUM,MUTUAL(201),S12,Y12N
REAL MU,KOL,KOLY,MAG(120),K0,JMAG,FACT(51),A(51)
REAL KZ02,B(51)
COMMON /FACTOR/FACT
DATA PLOT/'0','1','2','3','4','5','6','7','8','9',' ' /
DATA BLANK/' ','/','LINE/120*','/','5','6','7','8','9',' ' /
DISC=7
RDR=105
PRT=108

```

COMPUTE FACTORIALS FROM 0 TO 50

```

FACT(1)=1.
FACT(2)=1.
FACT(3)=2.
DO 60 I=3,50
J=I+1
FACT(J)=FACT(I)*FLOAT(I)
DBINC=3.
YDELTA=.12
IMAX=51
PI=3.14159265358979324
PI2=PI/2.
ETA=376.7300543
EO=8.8542E-12
MU=PI*4E-7
WRITE(PRT,91)

```

```

91  FORMAT(' ENTER RADIUS, SLOT LENGTH, AND ',
1'  SLOT WIDTH (CM) ...')
C
C  NOTE: FOR AXIAL SLOTS THE LENGTH IS GREATER THAN
C  THE WIDTH. OPPOSITE IS TRUE FOR TRANSVERSE SLOTS.
C
  READ(RDR,92,END=997) RADIUS,SLOTL,SLOTW
92  FORMAT(3F.0)
  IF(RADIUS.EQ.0.) GO TO 997
  WRITE(PRT,201)
201  FORMAT('///' ENTER SLOT AXIAL AND AZIMUTHAL',
1'  SEPARATION (CM AND DEGREES)...')
  READ(RDR,202) AX,AZ
202  FORMAT(2F.0)
  RAZ=AZ*PI/180.
  AZ=AZ/100.
  AX=AX/100.
  RADIUS=RADIUS/100.
  SLOTL=SLOTL/100.
  SLOTW=SLOTW/100.
  WCUT=9.418251357E8/SLOTL
  PHI=SLOTW/RADIUS/2.
  B(1)=1.
  A(1)=.5
  DO 103 KK=2,51
  THETA=PHI*FLOAT(KK-1)
  THETA=SIN(THETA)/THETA
  THETA=THETA*THETA
  B(KK)=COS((KK-1)*RAZ)
  A(KK)=THETA
103  WRITE(PRT,93) RADIUS,SLOTL,SLOTW,AX,AZ
108  FORMAT('///' RADIUS=,2PF10.5, LENGTH=,2PF10.5,
93  1'  WIDTH=,2PF10.5/ SPACING=,2PF10.5,5X, ANGLE=,
22PF10.5///' ENTER COMPLEX PERMITTIVITY...')
  READ(RDR,94,END=997) ER
94  FORMAT(2F.0)
  IF(ER.EQ.(0.,0.)) GO TO 997
  WRITE(PRT,95)ER
95  FORMAT(' PERMITTIVITY=,2F10.5///' ENTER INITIAL',
1'  FREQ, DELTA FREQ (GHZ), # FREQ'S, AND MAP...')
C
C  NOTE: UPPER FREQUENCY LIMIT SHOULD BE RESTRICTED
C  TO VALUES FOR WHICH ONLY TE MODE CAN EXIST.

```

MAP OPTION IS SELECTED BY SETTING RMAP.NE.0. AND
WILL PROVIDE A TWO DIMENSIONAL CONTOUR MAP OF
INTEGRAND MAGNITUDE (IN DECIBELS). THIS MAP IS
VERY USEFUL FOR DECIDING WHERE TO CUTOFF THE
INFINITE SUMMATION AND INFINITE INTEGRAL.

```

96 READ(RDR,96,END=997) FREQ,FQINC,RFMAX,RMAP
   FORMAT(4F.0)
   LFMAX=IFIX(RFMAX)
97 WRITE(PRT,97) FREQ,FQINC,LFMAX,RMAP
   FORMAT(' F0=',F10.5,' DELTAF=',F10.5,
1' NO. OF FREQ'S=',I3,' MAP=',F3.0///
1' ENTER SUMMATION LIMIT (MAX=51),',
2' DKZ, AND NO. OF KZ POINTS...')
   READ(RDR,98) RNMAX,YDELTA,RMAX
   THE STEP SIZE USED FOR NUMERICAL INTEGRATION DESERVES
   CAREFUL THOUGHT. MUCH FINER SIZES ARE NECESSARY FOR
   ACCURATE S21 CALCULATIONS THAN FOR S11 CALCULATIONS.
   THE FOLLOWING DEFAULT VALUES ARE ADEQUATE FOR RADII
   OF 3 INCHES OR LES AND FOR REAL PERMITTIVITIES OF
   6 OR LESS. IF LOSS TANGENTS ARE VERY SMALL (AS FOR
   FREE SPACE) SPECIAL ATTENTION MUST BE GIVEN TO STEP
   SIZE NEAR THE POLE OF THE INTEGRAND. AS LOSS TANGENT
   INCREASES, THIS POLE MOVES AWAY FROM THE REAL AXIS
   AND THE INTEGRAND BECOMES SMOOTHER-HENCE STEP SIZE
   IS LESS CRITICAL. WHEN TO TRUNCATE THE INFINITE
   INTEGRAL IS ALSO OF IMPORTANCE. AS THE SLOT SIZE
   BECOMES SMALLER COMPARED TO THE CIRCUMFERENCE IN
   WAVELENGTHS OF THE CYLINDER (E.G. AS FREQUENCY AND/OR
   REAL PERMITTIVITY INCREASE) THE UPPER LIMIT OF
   INTEGRATION INCREASES. THE MAP OPTION IS VERY USEFUL
   FOR DETERMINING CUTOFF POINTS FOR BOTH THE INTEGRATION
   AND THE INFINITE SERIES.
98 FORMAT(3F.0)
   IF(YDELTA.EQ.0.) YDELTA=.07
   IMAX=IFIX(RMAX)
   IF(IMAX.EQ.0) IMAX=129
   IF(MOD(IMAX,2).EQ.0) IMAX=IMAX+1
   IF(IMAX.GT.201) IMAX=201
   NMAX=IFIX(RNMAX)
   IF(NMAX.EQ.0) NMAX=51

```

```

IF(NMAX.GT.51) NMAX=51
RMAX=YDELTA*(RMAX-1.)
WRITE(PRT,101) NMAX,YDELTA,RMAX
101  FORMAT( ' SUMMATION LIMIT = ',I3,
1° DKZ=°,F8.3, ' KZ MAX=°,F12.5)
C
UNITS ARE CM. AND GHZ.
FREQ=FREQ-FQINC
DO 90 L=1,LFMAX
FREQ=FREQ+FQINC
REWIND DISC
W =2.E9*PI*FREQ
K0= W*SQRT(E0*MU)
KE=K0*CSQRT(ER)
DY=YDELTA*K0
NMAXM1=NMAX-1
Y=1.E-6-DY
VALMAX=0.
DO 100 I=1,IMAX
Y=Y+DY
Y2=Y*Y
C
ARG=CSQRT(KE*KE-Y2)
IF(AIMAG(ARG).GT.0.) ARG=-ARG
ARGRAD=ARG*RADIUS
KZB2=Y*SLOTL*.5
PROD=SIN(KZB2+PI2)/(KZB2+PI2)
PROD=PROD+SIN(KZB2-PI2)/(KZB2-PI2)
PROD=PROD*PROD
IF(AIMAG(ARG).GT.0.) ARG=-ARG
CALL HANKEL(ARGRAD,HANK,NMAX)
H0=HANK(1)
H1=HANK(2)
N=0
IN(1)=-A(1)*PROD*ARG*H0/H1
105  FORMAT(I3,F10.5,2E12.5,F10.5/15X,8E12.5//)
MAG(1)=CABS(IN(1))
IF(MAG(1).GT.VALMAX) VALMAX=MAG(1)
IN(2)=A(2)*PROD*ARG*H1/(H0-(H1/ARGRAD))
MAG(2)=CABS(IN(2))
IF(MAG(2).GT.VALMAX) VALMAX=MAG(2)
DO 99 N=3,NMAX
NM1=N-1
RN=FLOAT(N-1)
67  HN=HANK(N)

```

```

C      HNP=HANK(NM1)-(RN*HN/ARGRAD)
C      IF HN OR HNP IS VERY LARGE, MUST SCALE THEM OR COMPLEX
      DIVIDE WILL OVERFLOW
      RHN=AIMAG(HN)
      RHN=ABS(RHN)
      HNPR=ABS(REAL(HNP))
      HNPI=ABS(AIMAG(HNP))
65     FMULT=1.
      IF(RHN.GE.1.E38.OR.HNPR.GE.1.E38) FMULT=1.E-43
      IF(HNPR.GE.1.E38.OR.HNPI.GE.1.E38) FMULT=1.E-42
      HN=FMULT*HN
      HNP=FMULT*HNP
      DIV=HN/HNP
      IN(N)=A(N)*PROD*ARG*DIV
104    FORMAT(I3,10E12.5)
66     MAG(N)=CABS(IN(N))
      IF(MAG(N).GT.VALMAX) VALMAX=MAG(N)
99     CONTINUE
C
C      PERFORM SUMMATION PRIOR TO INTEGRATION
C
      Y12SUM=(0.,0.)
      JN=(0.,0.)
      DO 70 N=1,NMAX
      Y12SUM=Y12SUM+IN(N)*B(N)
70     JN=JN+IN(N)
      MUTUAL(I)=Y12SUM
      ADMIT(I)=JN
      IF(L.EQ.1.OR.L.EQ.LFMAX.AND.RMAP.NE.0)
1WRITE(DISC,2) (MAG(II),II=1,NMAX)
2     FORMAT(E15.7)
100    CONTINUE
      IF(L.NE.1.AND.L.NE.LFMAX.OR.RMAP.EQ.0.) GO TO 998
      REWIND DISC
      WRITE(PRT,996)
996    FORMAT(1, 'CONTOUR MAP OF INTEGRAND"/' HORIZONTAL',
1' AXIS IS SUMMATION INDEX, VERTICAL AXIS IS HANKEL',
2' FUNCTION ARGUMENT..."/')
      Y=-DY
      DO 30 I=1,IMAX
      Y=Y+DY
      ARGAD=CSQRT(KE*KE-Y*Y)*RADIUS
      READ(DISC,2) (MAG(II),II=1,NMAX)
      DO 22 N=1,NMAX

```

```

IF(MAG(N).LT.1.E-10) MAG(N)=1.E-10
MAG(N)=10.*ALOG10(VALMAX/MAG(N))
22 CONTINUE
DO 23 N=1,NMAX
23 LINE(N)=BLANK
DO 24 N=1,NMAX
DB=0.
DO 25 K=1,10
DB=DB+DBINC
IF(MAG(N).GE.DB) GO TO 25
LINE(N)=PLOT(K)
GO TO 24
25 CONTINUE
24 CONTINUE
WRITE(PRT,27) ARGRAD,(LINE(KKK),KKK=1,NMAX)
27 FORMAT(2F7.2,4X,51A1)
30 CONTINUE
WRITE(PRT,51) DBINC
51 FORMAT(// DB INCREMENT IS ',F8.3)
WRITE(PRT,52) VALMAX
52 FORMAT(' MAX VALUE IS ',1PE15.7)
C
C
C
C
998 PERFORM SIMPSON'S RULE INTEGRATION. NOTE MUST HAVE EVEN
NUMBER OF STRIPS, IMPLYING IMAX MUST BE ODD.
IMAXM1=IMAX-1
YMAX=DY*IMAXM1
Y12=MUTUAL(1)+MUTUAL(IMAX)*COS(YMAX*AX)
Y=0.
Y0=ADMIT(1)+ADMIT(IMAX)
DO 80 I=2,IMAXM1
RIM1=FLOAT(I-1)
Y=Y+DY
Y12=Y12+MUTUAL(I)*COS(Y*AX)*2.*(1.+AMOD(RIM1,2.))
80 Y0=Y0+ADMIT(I)*2.*(1.+AMOD(RIM1,2.))
Y0=Y0*DY/3.
Y0=Y0*(0.,-1.)*SLOTL*SLOTW*.5/PI/PI/RADIUS/W/MU
Y12=Y12*DY/3.
Y12=Y12*(0.,-1.)*SLOTL*SLOTW*.5/PI/PI/RADIUS/W/MU
Z0=(1.,0.)/Y0
ZGUIDE=ETA/SQRT(1.-(WCUT/W)**2)
YN=ZGUIDE/Z0
WRITE(PRT,81) FREQ,YN
81 FORMAT(///',F10.5, 'GHZ.....('F10.5,', 'F10.5,') ')

```

```

      GAMMA=(Z0-ZGUIDE)/(Z0+ZGUIDE)
      GAMMAG=CABS(GAMMA)
      GAMANG=ATAN(AIMAG(GAMMA),REAL(GAMMA))*57.29577951
      VSWR=(1.+GAMMAG)/(1.-GAMMAG)
      RETLOS=20.*ALOG10(GAMMAG)
      ZN=Z0/ZGUIDE
111  WRITE(PRT,111) ZGUIDE,GAMMAG,GAMANG,YN,VSWR,RETLOS,ZN
      FORMAT(/F15.5,5X,F10.6,2X,F7.2,5X,2F10.5/F10.3,5X,F15.5,2F10.5)
      Y12N=Y12*ZGUIDE
      S12=2.*Y12N
      S12=S12/((1.+YN)**2-(Y12N*Y12N))
      GAIN=-20.*ALOG10(CABS(S12))
      S12ANG=ATAN(AIMAG(S12),REAL(S12))*57.29577951
213  WRITE(PRT,213) Y12N,GAIN,S12ANG
90   FORMAT(/Y12,S12 ARE ,2F8.4,3X,2F7.2)
999  CONTINUE
997  GO TO 108
      STOP
      END
      SUBROUTINE HANKEL(Z,HN,NMAX)

```

C
C
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C
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C
C
C

THIS SUBROUTINE COMPUTES HANKEL FUNCTIONS OF THE SECOND KIND FOR COMPLEX ARGUMENT Z FROM ORDER ZERO UP TO POSITIVE ORDER (NMAX-1). THE RESULTS ARE STORED IN ARRAY HN. MAXIMUM ORDER IS 50. THIS ROUTINE CALLS SUBROUTINE JEESL WHICH CALCULATES BESSEL FUNCTIONS OF THE FIRST KIND. OVERFLOW OR UNDERFLOW MAY OCCUR FOR HIGH ORDER HANKEL FUNCTIONS OF SMALL ARGUMENT (CABS(Z).LT.2).

137

```

31  COMPLEX Y0,Y1,J0,J1,YN(S1),HN(S1),SUM,J,Z
      COMPLEX JNC(102),CHI,Z8,P,Q
      IF(CABS(Z).LT.2.) WRITE(108,31)
      FORMAT(/' ARG.LT.2,HN(N>40) MAY OVERFLOW'/)
      J=(0.,1.)
      GAMMA=.577215664901533
      PI=3.14159265358979324
      IF(CABS(Z).LE.14) GO TO 23
      CHI=Z-.25*PI
      Z8=8.*Z
      P=(1.,0.)-4.5/(Z8*Z8)+459.375/(Z8**4)
      Q=37.5/(Z8**3)-1./Z8

```

```

      Y0=CSQRT(2./PI/Z)*(P*CSIN(CHI)+Q*CCOS(CHI))
      HN(1)=CSQRT(2./PI/Z)*CEXP(-J*CHI)*(P-J*Q)
      CHI=Z-.75*PI
      P=(1.,0.)+7.5/(Z8*Z8)-590.625/(Z8**4)
      Q=3./Z8-52.5/(Z8**3)
      HN(2)=CSQRT(2./PI/Z)*CEXP(-J*CHI)*(P-J*Q)
      DO 15 I=3,51
      RI=FLOAT(I-2)
15      HN(I)=2.*RI/Z*HN(I-1)-HN(I-2)
      RETURN
23      NMAXP1=NMAX+1
      CALL JBESL(Z,NMAXP1,JN)
      J0=JN(1)
      J1=JN(2)
28      Y0=2./PI*(CLOG(.5*Z)+GAMMA)*J0
      SUMB4=1.
      SUM=(0.,0.)
      DO 10 I=1,20
      I2=I*2+1
      SUM=SUM+(-1.)**I*JN(I2)/FLOAT(I)
      SMAG=CABS(SUM)
      IF(ABS(SMAG-SUMB4)/SUMB4.LE.1.E-12) GO TO 4
      SUMB4=SMAG
10      CONTINUE
      Y0=Y0-4.*SUM/PI
22      Y1=(Y0+J1-2./PI/Z)/J0
26      YN(1)=Y0
      YN(2)=Y1
      DO 13 I=3,NMAX
      RI=FLOAT(I-2)
13      YN(I)=2.*RI/Z*YN(I-1)-YN(I-2)
      DO 14 I=1,NMAX
14      HN(I)=JN(I)-J*YN(I)
      RETURN
      END
      SUBROUTINE JBESL(Z,NMAX,JN)

```

C
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C
C
C

THIS SUBROUTINE COMPUTES BESSEL FUNCTIONS OF THE FIRST KIND FOR COMPLEX ARGUMENT Z. JBESL REQUIRES AN ARRAY OF FACTORIAL VALUES FROM ORDERS 0 TO 50 SUPPLIED AS A COMMON BLOCK. THIS SAVES RECALCULATING THEM EACH TIME THE ROUTINE IS CALLED.

```

COMPLEX JN(102),Z,SUM,Z2
COMMON /FACTOR/FACT(51)
ZMAG=CABS(Z)+20.
IF((ZMAG-20.)-.2) 4,4,9
9 RN=FLOAT(NMAX)+10.
K=IFIX(ZMAG)
IF(RN.GT.ZMAG) K=IFIX(RN)
IF(MOD(K,2).EQ.0) K=K+1
IF(K.GT.77) K=77
IF(CABS(Z).LE.2.) K=39
C IF Z IS SMALL, SUM GETS TOO LARGE.
JN(K+2)=(0.,0.)
JN(K+1)=(1.E-37,0.)
IF(CABS(Z).LE.2.) JN(K+1)=(1.E-60,0.)
DO 1 I=1,K
L=K-I+1
RL=FLOAT(L)
1 JN(L)=2.*RL/Z*JN(L+1)-JN(L+2)
SUM=JN(1)
DO 2 I=3,K,2
2 SUM=SUM+2.*JN(I)
DO 3 I=1,K
3 JN(I)=JN(I)/SUM
RETURN
4 Z2=-.25*Z*Z
DO 7 I=1,NMAX
SUM=1./FACT(I)
SB4=.9
DO 5 K=1,20
SUM=SUM+Z2**K/FACT(K+1)/FACT(K+I)
SMAG=CABS(SUM)
IF(CABS(SMAG-SB4)/SB4.LT.1.E-7) GO TO 6
5 SB4=SMAG
CONTINUE
6 JN(I)=(.5*Z)**(I-1)*SUM
7 CONTINUE
RETURN
END

```

APPENDIX B

FORTRAN LISTING OF TRANS, A COMPUTER PROGRAM
FOR CALCULATING SELF AND MUTUAL ADMITTANCES
OF PARALLEL TRANSVERSE RECTANGULAR SLOTS
ON AN INFINITE CONDUCTING CYLINDER

CCCCCCCC

PROGRAM TRANS

THIS PROGRAM COMPUTES THE SELF AND MUTUAL
ADMITTANCE OF TRANSVERSE RECTANGULAR TE¹⁰ EXCITED
SLOTS ON AN INFINITELY LONG CONDUCTING CYLINDER.

```

INTEGER RDR,PRT
INTEGER DISC, PLOT(11), BLANK, LINE(120), NUM(120)
COMPLEX TERM, KE2, PROD
COMPLEX HNHNP, HANK(51), GAMMA, ZN
COMPLEX YN, ADMTT(201), YO, ZO
COMPLEX ER, ARG, HO, H1, HN, HNP, IN(120)
COMPLEX JN, ARG2, ARGRAD, KE
COMPLEX Y12, Y12SUM, MUTUAL(201), S12, Y12N
REAL MU, MAG(120), KO, FACT(51), A(120)
REAL KZB2, B(120)
COMMON /FACTOR/ FACT
DATA PLOT/'0','1','2','3','4','5','6','7','8','9',' ' /
DATA BLANK/' ' /, LINE/120*' ' /
DISC=7
RDR=105
PRT=108

```

CCC

COMPUTE FACTORIALS FROM 0 TO 50

```

FACT(1)=1.
FACT(2)=1.
FACT(3)=2.
DO 60 I=3,50
J=I+1
FACT(J)=FACT(I)*FLOAT(I)
DBINC=3.
YDELTA=.12
IMAX=51
PI=3.14159265358979324
PI2=PI/2.
ETA=376.7300543
EO=8.8542E-12
MU=PI*4E-7
WRITE(PRT,91)

```

60

147

```

91  FORMAT(' ENTER RADIUS, SLOT LENGTH, AND ',
C    ' 1" SLOT WIDTH (CM) ...')
C
C    NOTE: FOR TRANSVERSE SLOTS THE WIDTH IS GREATER
C    THAN THE LENGTH. OPPOSITE IS TRUE FOR AXIAL SLOTS.
92  READ(RDR,92,END=997) RADIUS,SLOTL,SLOTW
    FORMAT(3F,0)
    IF(RADIUS.EQ.0.) GO TO 997
    WRITE(PRT,201)
201  FORMAT('/// ENTER SLOT AXIAL AND AZIMUTHAL ',
C    ' 1" SEPARATION (CM AND DEGREES)...')
    READ(RDR,202) AX,AZ
202  FORMAT(2F,0)
    RAZ=AZ*PI/180.
    AZ=AZ/100.
    AX=AX/100.
    RADIUS=RADIUS/100.
    SLOTL=SLOTL/100.
    SLOTW=SLOTW/100.
    WCUT=9.418251357E8/SLOTW
    PHI=SLOTW/RADIUS/2.
    PIPHI2=PI*.5/PHI
    PIPHI2=PIPHI2*PIPHI2
    B(1)=1.
    A(1)=.5
    DO 103 KK=2,120
    THETA=PHI*FLOAT(KK-1)
    THETA=COS(THETA)
    THETA=THETA*THETA
    B(KK)=COS((KK-1)*RAZ)
103  A(KK)=THETA
108  WRITE(PRT,93) RADIUS,SLOTL,SLOTW,AX,AZ
93  FORMAT('/// RADIUS=',2PF10.5, ' LENGTH=',2PF10.5,
C    ' 1" WIDTH=',2PF10.5, ' SPACING=',2PF10.5, ' 5X', ' ANGLE=',
C    ' 22PF10.5/// ENTER COMPLEX PERMITTIVITY...')
    READ(RDR,94,END=997) ER
94  FORMAT(2F,0)
    IF(ER.EQ.(0.,0.)) GO TO 997
    WRITE(PRT,95) ER
95  FORMAT(' PERMITTIVITY=',2F10.5, '/// ENTER INITIAL ',
C    ' 1" FREQ, DELTA FREQ (GHZ), # FREQ"S, AND MAP...')
C

```

NOTE: UPPER FREQUENCY LIMIT SHOULD BE RESTRICTED
TO VALUES FOR WHICH ONLY TE MODE CAN EXIST.

10

MAP OPTION IS SELECTED BY SETTING RMAP.NE.0. AND
WILL PROVIDE A TWO DIMENSIONAL CONTOUR MAP OF
INTEGRAND MAGNITUDE (IN DECIBELS). THIS MAP IS
VERY USEFUL FOR DECIDING WHERE TO CUTOFF THE
INFINITE SUMMATION AND INFINITE INTEGRAL.

```

96 READ(RDR,96,END=997) FREQ,FQINC,RMAX,RMAP
   FORMAT(4F.0)
   LFMAX=IFIX(RMAX)
97 WRITE(PRT,97) FREQ,FQINC,LFMAX,RMAP
   FORMAT( ' F0= ',F10.5, ' DELTAF= ',F10.5,
1 ' NO. OF FREQ. S= ',I3, ' MAP= ',F3.0///
1 ' ENTER SUMMATION LIMIT (MAX=51), '
2 ' OKZ, AND NO. OF KZ POINTS...' )
98 READ(RDR,98) RNMAX,YDELTA,RMAX
   FORMAT(3F.0)
   THE STEPSIZE USED FOR NUMERICAL INTEGRATION DESERVES
   CAREFUL THOUGHT. MUCH FINER SIZES ARE NECESSARY FOR
   ACCURATE S21 CALCULATIONS THAN FOR S11 CALCULATIONS.
   THE FOLLOWING DEFAULT VALUES ARE ADEQUATE FOR RADII
   OF 3 INCHES OR LES AND FOR REAL PERMITTIVITIES OF
   6 OR LESS. IF LOSS TANGENTS ARE VERY SMALL (AS FOR
   FREE SPACE) SPECIAL ATTENTION MUST BE GIVEN TO STEP
   SIZE NEAR THE POLE OF THE INTEGRAND. AS LOSS TANGENT
   INCREASES, THIS POLE MOVES AWAY FROM THE REAL AXIS
   AND THE INTEGRAND BECOMES SMOOTHER-HENCE STEP SIZE
   IS LESS CRITICAL. WHEN TO TRUNCATE THE INFINITE
   INTEGRAL IS ALSO OF IMPORTANCE. AS THE SLOT SIZE
   BECOMES SMALLER COMPARED TO THE CIRCUMFERENCE IN
   WAVELENGTHS OF THE CYLINDER (E.G. AS FREQUENCY AND/OR
   REAL PERMITTIVITY INCREASE) THE UPPER LIMIT OF
   INTEGRATION INCREASES. THE MAP OPTION IS VERY USEFUL
   FOR DETERMINING CUTOFF POINTS FOR BOTH THE INTEGRATION
   AND THE INFINITE SERIES.
   IFC(YDELTA.EQ.0.) YDELTA=.07
   IMAX=IFIX(RMAX)
   IFC(IMAX.EQ.0) IMAX=129
   IF(MOD(IMAX,2).EQ.0) IMAX=IMAX+1
   IFC(IMAX.GT.201) IMAX=201

```

```

NMAX=IFIX(RNMAX)
IF(NMAX.EQ.0) NMAX=51
IF(NMAX.GT.51) NMAX=51
NMAX2=NMAX
RMAX=YDELTA*(RMAX-1.)
WRITE(PRT,101) NMAX,YDELTA,RMAX
101 FORMAT(' SUMMATION LIMIT =',I3,
1' DKZ=',F8.3,' KZ MAX=',F8.3)
C
C
UNITS ARE CM AND GHZ.
FREQ=FREQ-FQINC
DO 90 L=1,LFMAX
FREQ=FREQ+FQINC
REWIND DISC
W =2.E9*PI*FREQ
K0= W*SQRT(E0*MU)
KE=K0*CSQRT(ER)
DY=YDELTA*K0
KE2=KE*KE
NMAXM1=NMAX-1
Y=-DY+1.E-6
VALMAX=0.
DO 100 I=1,IMAX
Y=Y+DY
Y2=Y*Y
ARG=CSQRT(KE*KE-Y2)
IF(AIMAG(ARG).GT.0.) ARG=-ARG
ARGRAD=ARG*RADIUS
NLIMIT=NMAX
IF(CABS(ARGRAD).LE.15.) NLIMIT=NMAX2
IF(AIMAG(ARGRAD).LT.-45.) NLIMIT=25
DONT NEED HIGH ORDERS FOR LARGE KZ VALUES
ARG2=ARGRAD*ARGRAD
KZB2=Y*SLOTL*.5
PROD=SIN(KZB2)/KZB2
PRGD=PROD*PROD/ARG
IF(AIMAG(ARG).GT.0.) ARG=-ARG
CALL HANKEL(ARGRAD,HANK,NMAX)
H0=HANK(1)
H1=HANK(2)
HNR=ABS(REAL(H0))
HNI=ABS(AIMAG(H0))
FMULT=1.
IF(HNR.LT.1.E-38.AND.HNI.LT.1.E-38) FMULT=1.E42

```

```

H0=FMULT*H0
H1=FMULT*H1
N=0
105 IN(1)=A(1)*PROD*KE2*H1/(PIPHI2**2*H0)
      FORMAT(I3,F10.5,2E12.5,F10.5/15X,8E12.5//)
      MAG(1)=CABS(IN(1))
      IF(MAG(1).GT.VALMAX) VALMAX=MAG(1)
      HNP=H0-(H1/ARGRAD)
      TERM=Y2*H1/(ARG2*HNP)-KE2*HNP/H1
      IN(2)=A(2)*TERM*PROD/(PIPHI2-1.0)**2
      MAG(2)=CABS(IN(2))
      IF(MAG(2).GT.VALMAX) VALMAX=MAG(2)
      DO 99 N=3,NLIMIT
      NM1=N-1
      RN*12=FLOAT(NM1)**2
      RN=FLOAT(N-1)
      IF(N-NMAX) 67,67,116
116  HNHNP=-ARGRAD/RN
      GO TO 114
67  HN=HANK(N)
      HNP=HANK(NM1)-(RN*HN/ARGRAD)
C   IF HN OR HNP IS VERY LARGE,MUST SCALE THEM OR COMPLEX
C   DIVIDE WILL OVERFLOW
      HNI=ABS(AIMAG(HN))
      HNR=ABS(REAL(HN))
      HNPR=ABS(REAL(HNP))
      HNPI=ABS(AIMAG(HNP))
65  FMULT=1.
      IF(HNR.GE.1.E38.OR.HNI.GE.1.E38) FMULT=1.E-43
      IF(HNPR.GE.1.E38.OR.HNPI.GE.1.E38) FMULT=1.E-42
      IF(HNR.LT.1.E-38.AND.HNI.LT.1.E-38) FMULT=1.E-42
      IF(HNPR.LT.1.E-38.AND.HNPI.LT.1.E-38) FMULT=1.E-42
      HN=FMULT*HN
      HNP=FMULT*HNP
      HNHNP=HN/HNP
114  TERM=RN*12*Y2*HNHNP/ARG2-KE2/HNHNP
      IN(N)=A(N)*PROD*TERM/(PIPHI2-RN*12)**2
104  FORMAT(I3,10E12.5)
66  MAG(N)=CABS(IN(N))
      IF(MAG(N).GT.VALMAX) VALMAX=MAG(N)
99  CONTINUE
C
C
C   PERFORM SUMMATION PRIOR TO INTEGRATION

```

```

Y12SUM=(0.,0.)
JN=(0.,0.)
DO 70 N=1,NLIMIT
Y12SUM=Y12SUM+IN(N)*B(N)
70 JN=JN+IN(N)
MUTUAL(I)=Y12SUM
ADMIT(I)=JN
IF(L.EQ.1.OR.L.EQ.LFMAX.AND.RMAP.NE.0)
1 WRITE(DISC,2) (MAG(II),II=1,NLIMIT)
NUM(I)=NLIMIT
2 FORMAT(E15.7)
100 CONTINUE
IF(L.NE.1.AND.L.NE.LFMAX.OR.RMAP.EQ.0.) GO TO 998
REIND DISC
Y=-YDELTA*K0
DO 30 I=1,IMAX
Y=Y+YDELTA*K0
ARGRAD=CSQRT(KE*KE-Y*Y)*RADIUS
NNA=NUM(I)
READ(DISC,2) (MAG(II),II=1,NNN)
DO 22 N=1,NNN
IF(MAG(N).LT.1.E-10) MAG(N)=1.E-10
MAG(N)=10.*ALOG10(VALMAX/MAG(N))
22 CONTINUE
DO 23 N=1,NNN
23 LINE(N)=BLANK
DO 24 N=1,NNN
DB=0.
DO 25 K=1,10
DB=DB+DBINC
IF(MAG(N).GE.DB) GO TO 25
LINE(N)=PLOT(K)
GO TO 24
25 CONTINUE
24 CONTINUE
WRITE(PRT,27) ARGRAID, (LINE(KKK),KKK=1,NNN)
27 FORMAT(2F7.2,4X,112A1)
30 CONTINUE
WRITE(PRT,51) DBINC
51 FORMAT(//,2 DB INCREMENT IS ",F8.3)
WRITE(PRT,52) VALMAX
52 FORMAT(" MAX VALUE IS ",1PE15.7)
C
C PERFORM SIMPSON'S RULE INTEGRATION. NOTE MUST HAVE EVEN

```

```

C      NUMBER OF STRIPS, IMPLYING IMAX MUST BE ODD.
C
998    IMAXM1=IMAX-1
      YMAX=DY*IMAXM1
      Y12=MUTUAL(1)+MUTUAL(IMAX)*COS(YMAX*AX)
      Y=C.
      Y0=ADMIT(1)+ADMIT(IMAX)
      DO 80 I=2,IMAXM1
      RIM1=FLOAT(I-1)
      Y=Y+DY
80     Y12=Y12+MUTUAL(I)*COS(Y*AX)*2.*(1.+AMOD(RIM1,2.))
      Y0=Y0+ADMIT(I)*2.*(1.+AMOD(RIM1,2.))
      Y0=Y0+DY/3.
      Y0=Y0*(0.,-8.)*SLOTL*(RADIUS/SLOTH)**3/W/MU
      Y12=Y12+DY/3.
      Y12=Y12*(0.,-8.)*SLOTL*(RADIUS/SLOTH)**3/W/MU
      Z0=(1.,0.)/Y0
      ZGUIDE=ETA/SQRT(1.-(WCUT/W)**2)
      YN=Y0*ZGUIDE
      WRITE(PRT,81) FREQ,YN
81     FORMAT(///,F10.5, 'GHZ.....(,F15.5,', ' ',F15.5,') ')
      GAMMA=(Z0-ZGUIDE)/(Z0+ZGUIDE)
      GAMMAG=CABS(GAMMA)
      GAMMANG=ATAN(AIMAG(GAMMA),REAL(GAMMA))*57.29577951
      VSWR=(1.+GAMMAG)/(1.-GAMMAG)
      RETLOS=20.*ALOG10(GAMMAG)
      ZN=Z0/ZGUIDE
111    WRITE(PRT,111) ZGUIDE,GAMMAG,GAMMANG,YN,VSWR,RETLOS,ZN
      FORMAT(/F15.5,5X,F10.6,2X,F7.2,5X,2E12.5/F10.3,5X,F15.5,2F10.5)
      Y12N=Y12*ZGUIDE
      S12=2.*Y12N
      S12=S12/((1.+YN)**2-(Y12N*Y12N))
      GAIN=-20.*ALOG10(CABS(S12))
      S12ANG=ATAN(AIMAG(S12),REAL(S12))*57.29577951
      WRITE(PRT,213) Y12N,GAIN,S12ANG
213    FORMAT(//,Y12,S12 ARE ',2F8.4,3X,2F7.2)
90     CONTINUE
999    GO TO 108
997    STOP
      END
      SUBROUTINE HANKEL(Z,HN,NMAX)

```

```

C
C
C      THIS SUBROUTINE COMPUTES HANKEL FUNCTIONS OF THE

```

C
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C
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C
C
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C
C

SECOND KIND FOR COMPLEX ARGUMENT Z FROM ORDER ZERO
UP TO POSITIVE ORDER (NMAX-1). THE RESULTS ARE
STORED IN ARRAY HN. MAXIMUM ORDER IS 50. THE
ROUTINE REQUIRES AN ARRAY OF FACTORIAL VALUES
FROM 0! TO 50! SUPPLIED AS A COMMON BLOCK. THIS
SAVES RECALCULATING THEM EACH TIME HANKEL IS
CALLED. THIS ROUTINE CALLS SUBROUTINE JBESL WHICH
CALCULATES BESSEL FUNCTIONS OF THE FIRST KIND.
OVERFLOW OR UNDERFLOW MAY OCCUR FOR HIGH ORDER
HANKEL FUNCTIONS OF SMALL ARGUMENT (CABS(Z).LT.2).

31

```

COMPLEX Y0,Y1,J0,J1,YN(51),HN(51),SUM,J,Z
COMPLEX JN(102),CHI,Z8,P,Q
IF(CABS(Z).LT.2.) WRITE(108,31)
FORMAT(/' ARG.LT.2,HN(N>40) MAY OVERFLOW'/)
J=(0.,1.)
GAMMA=.577215664901533
PI=3.14159265358979324
IF(CABS(Z).LE.14) GO TO 23
CHI=Z-.25*PI
Z8=8.*Z
P=(1.,0.)-4.5/(Z8*Z8)+459.375/(Z8**4)
Q=37.5/(Z8**3)-1./Z8
Y0=CSQRT(2./PI/Z)*(P*CSIN(CHI)+Q*CCOS(CHI))
HN(1)=CSQRT(2./PI/Z)*CEXP(-J*CHI)*(P-J*Q)
CHI=Z-.75*PI
P=(1.,0.)+7.5/(Z8*Z8)-590.625/(Z8**4)
Q=3./Z8-52.5/(Z8**3)
HN(2)=CSQRT(2./PI/Z)*CEXP(-J*CHI)*(P-J*Q)
DO 15 I=3,51
RI=FLOAT(I-2)
HN(I)=2.*RI/Z*HN(I-1)-HN(I-2)
RETURN
23 NMAXP1=NMAX+1
CALL JBESL(Z,NMAXP1,JN)
J0=JN(1)
J1=JN(2)
28 Y0=2./PI*(CLOG(.5*Z)+GAMMA)*J0
SUMB4=1.
SUM=(0.,0.)
DO 10 I=1,20
I2=I*2+1
SUM=SUM+(-1.)**I*JN(I2)/FLOAT(I)

```

15

23

28

148

```

      SMAG=CABS(SUM)
      IF(CABS(SMAG-SUMB4)/SUMB4.LE.1.E-12) GO TO 4
      SUMB4=SMAG
10     CONTINUE
4      Y0=Y0-4.*SUM/PI
22     Y1=(YC*J1-2./PI/Z)/J0
26     YN(1)=Y0
      YN(2)=Y1
      DO 13 I=3,NMAX
      RI=FLOAT(I-2)
13     YN(I)=2.*RI/Z*YN(I-1)-YN(I-2)
      DO 14 I=1,NMAX
14     HN(I)=JN(I)-J*YN(I)
      RETURN
      END
      SUBROUTINE JBESL(Z,NMAX,JN)

```

```

C
C
C
C
      THIS SUBROUTINE COMPUTES BESSEL FUNCTIONS OF
      THE FIRST KIND FOR COMPLEX ARGUMENT Z.

```

```

      COMPLEX JN(102),Z,SUM,Z2
      COMMON /FACTOR/FACT(51)
      ZMAG=CABS(Z)+20.
      IF((ZMAG-20.)-.2) 4,4,9
9      RN=FLOAT(NMAX)+10.
      K=IFIX(ZMAG)
      IF(RN.GT.ZMAG) K=IFIX(RN)
      IF(MOD(K,2).EQ.0) K=K+1
      IF(K.GT.77) K=77
      IF(CABS(Z).LE.2.) K=39
C IF Z IS SMALL, SUM GETS TOO LARGE.
      JN(K+2)=(0.,0.)
      JN(K+1)=(1.E-37,0.)
      IF(CABS(Z).LE.2.) JN(K+1)=(1.E-60,0.)
      DO 1 I=1,K
      L=K-I+1
      RL=FLOAT(L)
1      JN(L)=2.*RL/Z*JN(L+1)-JN(L+2)
      SUM=JN(1)
      DO 2 I=3,K,2
2      SUM=SUM+2.*JN(I)
      DO 3 I=1,K
3      JN(I)=JN(I)/SUM

```

```

4      RETURN
      ZZ=-.25*Z*Z
      DO 7 I=1,NMAX
      SUM=1./FACT(I)
      SB4=.9
      DO 5 K=1,20
      SUM=SUM+ZZ**K/FACT(K+1)/FACT(K+1)
      SMAG=CABS(SUM)
      IF(CABS(SMAG-SB4)/SB4.LT.1.E-7) GO TO 6
      SB4=SMAG
5      CONTINUE
6      JN(I)=(.5*Z)**(I-1)*SUM
7      CONTINUE
      RETURN
      END

```

APPENDIX C

FORTRAN LISTING OF PLANAR, A COMPUTER PROGRAM
WHICH CALCULATES SELF AND MUTUAL ADMITTANCES
OF PARALLEL RECTANGULAR APERTURES IN
AN INFINITE CONDUCTING GROUND PLANE

PROGRAM PLANAR

THIS PROGRAM COMPUTES THE SELF AND MUTUAL
ADMITTANCES OF RECTANGULAR TE¹⁰ EXCITED
SLOTS IN AN INFINITE GROUND PLANE.

PROGRAM USE

TO CALCULATE SLOT SELF ADMITTANCE, ENTER A SLOT
SPACING OF ZERO. PROGRAM WILL THEN PRINT SELF
ADMITTANCE. MUTUAL ADMITTANCE REQUIRES A SEPARATE
RUNNING OF THE PROGRAM USING THE DESIRED SLOT
SPACING (IN CM). THE USER MUST SUPPLY THE
PROGRAM WITH THE PROPER VALUE OF SELF ADMITTANCE
OBTAINED FROM RUNNING THE PROGRAM WITH ZERO
SLOT SPACING. THIS PROCEDURE REQUIRES TWO
SEPARATE RUNS OF THE PROGRAM BUT RESULTS IN
VERY EFFICIENT EXECUTION.

INTEGER RDR,PRT
COMPLEX GAMMA,ZN,YN,YO,ZO,ER,ARG, INC(49),PARSUM(49)
COMPLEX SUM,EVAL,S12,Y11N,YN,N,YNN2
REAL MU, MAG(120),KO, FACT(40),A(40)
RDR=105
PRT=108
DBINC=3.
PI=3.14159265358979324
PI2=PI*PI
ETA =376.7300543
EO=8.8542E-12
MU=PI*4E-7
WRITE(PRT,91)
FORMAT(' ENTER SLOT LENGTH, ',
1 'SLOT WIDTH AND SPACING (CM)...')
1

SLOT LENGTH MUST BE GREATER THAN SLOT WIDTH.
OTHERWISE, THE WRONG TE¹⁰ CUTOFF FREQUENCY WILL
BE CALCULATED.

C
C
C
C
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C

NOTE: SLOT SPACING IS DEFINED ALONG AN AXIS
PERPENDICULAR TO THE LONG DIMENSION OF THE
SLOT ANTENNAS. THIS SLOT GEOMETRY MAXIMIZES
MUTUAL COUPLING BETWEEN THE ANTENNAS.

92

READ(RDR,92,END=997) SLOTL,SLOTW,SEP
FORMAT(3F,0)
IF(SLOTL.EQ.0.) GO TO 997
RADIUS=RADIUS/100.
SLOTL=SLOTL/100.
SLOTW=SLOTW/100.
SLOTL2=SLOTL*SLOTL
SLOTW2=SLOTW*SLOTW
SEP=SEP/100.

108

WCUT=9.416251357E8/SLOTL
WRITE(PRT,93) SLOTL,SLOTW,SEP

93

FORMAT(' LENGTH=',2PF10.5,
1' WIDTH=',2PF10.5,' SPACING=',2PF10.5////' ENTER COMPLEX ',
2' PERMITTIVITY...')

READ(RDR,94,END=997) ER
IF(ER.EQ.(0.,0.)) GO TO 997
IF(AIMAG(ER).GT.0.) WRITE(PRT,121)

121

FORMAT(///' WARNING, UNREALIZEABLE PERMITTIVITY...')
94

94

FORMAT(2F,0)
WRITE(PRT,95)ER

95

FORMAT(' PERMITTIVITY=',2F10.5////' ENTER INITIAL',
1' FREQ, DELTA FREQ (GHZ), # FREQ'S')

READ(RDR,96,END=997) FREQ,FQINC,RFMAX

96

FORMAT(3F,0)
LFMAX=IFIX(RFMAX)

WRITE(PRT,97) FREQ,FQINC,LFMAX

97

FORMAT(' F0=',F10.5,' DELTAF=',F10.5,
1' NO. OF FREQ'S=',I3,///
1' ENTER # DRHO POINTS, # OTHETA POINTS...')

READ(RDR,98) NRHO,NTHETA

98

FORMAT(2F,0)
IF(NRHO.EQ.0) NRHO=11

IF(MOD(NRHO,2).EQ.0) NRHO=NRHO+1
IF(NRHO.GT.49) NRHO=49

IF(NTHETA.EQ.0) NTHETA=23

C
C

THE ABOVE DEFAULT VALUES FOR INTEGRATION STEP

C
C
C

SIZE ARE ADEQUATE FOR SELF ADMITTANCE COMPUTATION.
FOR MUTUAL ADMITTANCE RUNS USE NRHO=NTHETA=15

```

101 IF(MOD(NTHETA,2).EQ.0) NTHETA=NTHETA+1
    WRITE(PRT,101) NRHO,NTHETA
    DTHETA=PI*.5/(NTHETA-1)
    FORMAT(' # DRHO POINTS=',I3,' # DTHETA POINTS=',I3)
    NTM2=NTHETA-2
    FREQ=FREQ-FQINC
    DO 90 L=1,LF4AX
    SUM=(0.,0.)
    FREQ=FREQ+FQINC
    W =2.*PI*FREQ
    KO= W*SQRT(E0*NU)
    ARG=KO*CSQRT(ER)
    AWIDE=KO*SLOTH
    BLONG=KO*SLOTL
    DELTA=KO*SEP
    NTM1=NTHETA-1
    NTM2=NTHETA-2
    NRH2=NRHO-2
    BEGHT=0.
    C IF DELTA>AWIDE, MUST DO RECT INTEGRATION
    C IF(DELTA-AWIDE) 152,153,153
    C LET THETA = X AND RHO = Y
153 DTHETA=2.*AWIDE/(NTHETA-1)
    BEGHT=ABS(DELTA-AWIDE)
    DRHO=BLONG/(NRHO-1)
    GO TO 154
152 DTHETA=PI*.5/(NTHETA-1)
    THETA1=ATAN(BLONG/ABS(DELTA+AWIDE))
    THETA2=PI-ATAN(BLONG/ABS(DELTA-AWIDE))
154 THETA=-DTHETA+BEGHT
    DO 100 I=1,NTHETA
    THETA=THETA+DTHETA
    IF(DELTA-AWIDE) 155,156,156
155 RHOMAX=ABS(AWIDE+DELTA)/COS(THETA)
    IF(THETA.GE.THETA1) RHOMAX=BLONG/SIN(THETA)
    IF(THETA.GE.THETA2) RHOMAX=-ABS(DELTA-AWIDE)/COS(THETA)
    DRHO=RHOMAX/(NRHO-1)
156 RHO=-DRHO
    DO 99 N=1,NRHO
    RHO=RHO+DRHO
    IN(N)=EVAL(AWIDE,BLONG,RHO,THETA,ARG,KO,DELTA)

```

```

99  CONTINUE
C   INTEGRATE IN RHO DIMENSION USING DRHO AS STEPSIZE
CALL INTR8(IN,PARSUM(I),1.,4.,1.,NRM2,DRHO)
102  FORMAT(2E15.7)
100  CONTINUE
C   NOW INTEGRATE IN THE THETA DIMENSION
998  CALL INTR8(PARSUM,SUM,1.,4.,1.,NTM2,DTHETA)
C   CALCULATE ADMITTANCE NORMALIZED TO FREE SPACE ADMIT.
Y0=(0.,8.)*BLONG/AWIDE*SUM
Z0=ETA/Y0
ZGUIDE=ETA/SQRT(1.-(WCUT/W)**2)
YN=ZGUIDE/Z0
ZN=Z0/ZGUIDE
WRITE(PRT,81) FREQ,YN
81  FORMAT(///"  ",F10.5," GHZ.....(",F10.5," ",F10.5," ")")
IFC(W.LE.WCUT) GO TO 90
GAMMA=(Z0-ZGUIDE)/(Z0+ZGUIDE)
GAMMAG=CABS(GAMMA)
GAMANG=ATAN(AIMAG(GAMMA),REAL(GAMMA))*57.29577951
VSWR=(1.+GAMMAG)/(1.-GAMMAG)
RETLOS=20.*ALOG10(GAMMAG)
IF(DELTA.NE.0.) GO TO 596
YNN=YN/ZGUIDE*ETA
C   YNN IS SELF ADMITTANCE NORMALIZED TO FREE
C   SPACE ADMITTANCE.
C   YNN IS APPROX. PROPORTIONAL TO SQUARE ROOT OF
C   COMPLEX RELATIVE PERMITTIVITY.
YNN2=YNN*YNN
WRITE(PRT,111) ZGUIDE,GAMMAG,GAMANG,YN,VSWR,RETLOS,ZN
111  FORMAT(/F15.5,5X,F10.6,2X,F7.2,5X,2F10.5/F10.3,5X,F15.5,2F10.5)
WRITE(PRT,601) YNN2
601  FORMAT(" (Y11/Y0)**2 = ",2F10.3)
IF(DELTA.EQ.0.) GO TO 90
596  WRITE(PRT,594)
594  FORMAT(///" ENTER Y11/YG...")
READ(RDR,595) Y11N
595  FORMAT(2F.0)
S12=2.*YN
S12=S12/((1.+Y11N)**2-YN**2)
GAIN=-20.*ALOG10(CABS(S12))
S12ANG=ATAN(AIMAG(S12),REAL(S12))*57.29577951
WRITE(PRT,213) YN,GAIN,S12ANG
213  FORMAT("//  Y12, S12 ARE ",2E12.5,7X,F7.2,3X,F7.2//)
90  CONTINUE

```

```

999 GO TO 108
997 STOP
END
SUBROUTINE INTGR8(IN,SUM,W1,W2,W3,NMAXM2,DX)
COMPLEX IN(49),SUM
SUM=(0.,0.)
DO 1 I=1,NMAXM2,2
1 SUM=SUM+W1*IN(I)+W2*IN(I+1)+W3*IN(I+2)
SUM=SUM*DX/3.
RETURN
END
COMPLEX FUNCTION EVAL(A,B,R,THETA,K1,K0,DELTA)
COMPLEX EVAL,K1,C1,C2,SUM,J
REAL K0
J=(0.,1.)
PI=3.14159265358979324
C1=(K1/K0)**2-(PI/8)**2
C1=C1/(4.*PI*B*B)
C2=(K1/K0)**2+(PI/8)**2
C2=C2/(4.*PI*PI*B)
IF(DELTA-A) 1,2,2
2 FAC=SQRT(THETA*THETA+R*R)
SUM=C1*(B-R)*COS(PI/8*R)
SUM=SUM+C2*SIN(PI/8*R)
SUM=.5*SUM*(A-ABS(THETA-DELTA))*CEXP(-J*K1/K0*FAC)/FAC
GO TO 3
1 SUM=C1*(B-R*SIN(THETA))*COS(PI/8*R*SIN(THETA))
SUM=SUM+(C2*SIN(PI/8*R*SIN(THETA)))
3 SUM=SUM*(A-R*COS(THETA))*CEXP(-J*K1/K0*R)
EVAL=SUM
RETURN
END

```

APPENDIX D

FORTRAN LISTING OF SKIN, A COMPUTER PROGRAM
FOR CALCULATING THE RATE AT WHICH POWER
IS DISSIPATED WITH INCREASING RADIAL
DISTANCE FROM A SLOTTED CYLINDER

PROGRAM SKIN

THIS PROGRAM CALCULATES THE RATIO OF REAL POWER LEAVING A CYLINDRICAL SURFACE SURROUNDING THE SLOTTED CYLINDER ARRAY TO THE REAL POWER LEAVING A SLOT ANTENNA ON THE ARRAY'S SURFACE. TRANSVERSE SLOTS ARE ASSUMED. THIS RATIO IS USEFUL IN STUDYING THE RATE AT WHICH POWER IS DISSIPATED WITH DISTANCE IN A LOSSY MEDIUM SURROUNDING THE SLOTTED CYLINDER.

THE USER MUST ENTER THE RADIUS OF THE SURFACE AT WHICH THE POWER FLUX IS TO BE EVALUATED. INITIALLY THE SURFACE SHOULD COINCIDE WITH THE SLOTTED CYLINDER (I.E. THE FIRST TRIAL SKIN DEPTH SHOULD BE ZERO). THIS GIVES A REFERENCE POWER FLUX WHICH MUST BE ENTERED FOR EACH SUBSEQUENT TRIAL SKIN DEPTH.

```

INTEGER RDR,PRT
INTEGER DISC,PLOT(11),BLANK,LINE(120),NUM(120)
COMPLEX TERM,KE2,PROD
COMPLEX HHNP,HANK(51),GAMMA,ZN
COMPLEX YN,ADMIT(201),YO,ZO,HANK2(51),HNP2,ARGSKN
COMPLEX ER,ARG,H0,H1,HNM1,HNM2,HN,HNP,IN(120)
COMPLEX JN,ARG2,ARGRAD,KE,H02,H12,YREF,HN2
COMPLEX Y12,Y12SUM,MUTUAL(201),S12,Y12N
REAL MU,MAG(120),K0,JMAG,FACT(51),AC(120)
REAL KZB2
COMMON /FACTOR/FACT
DATA PLOT/'0','1','2','3','4','5','6','7','8','9','/'
DATA BLANK/' ','/ ',LINE/120*2 '/'
DISC=7
RDR=105
PRT=108

```

COMPUTE FACTORIALS FROM 0 TO 50

```

FACT(1)=1.
FACT(2)=1.
FACT(3)=2.

```

```

DO 60 I=3,50
J=I+1
60 FACT(J)=FACT(I)*FLOAT(I)
DBINC=3.
YDELTA=.12
IMAX=51
PI=3.14159265358979324
PI2=PI/2.
ETA=376.7300543
EQ=8.8542E-12
MU=PI*4E-7
WRITE(PRT,91)
91 FORMAT(' ENTER RADIUS, SLOT LENGTH, AND ',
1 ' SLOT WIDTH (CM) ')
READ(RDR,92,END=997) RADIUS,SLOTL,SLOTW
92 FORMAT(3F.0)
IF(RADIUS.EQ.0.) GO TO 997
RADIUS=RADIUS/100.
SLOTL=SLOTL/100.
SLOTW=SLOTW/100.
WCUT=9.418251357E8/SLOTW
PHI=SLOTW/RADIUS/2.
PIPHI2=PI*.5/PHI
PIPHI2=PIPHI2*PIPHI2
A(1)=.5
DO 103 KK=2,120
THETA=PHI*FLOAT(KK-1)
THETA=COS(THETA)
THETA=THETA*THETA
103 A(KK)=THETA
108 WRITE(PRT,93) RADIUS,SLOTL,SLOTW
93 FORMAT('/// RADIUS=',2PF10.5, ' LENGTH=',2PF10.5,
1 ' WIDTH=',2PF10.5, ' ENTER COMPLEX PERMITTIVITY...')
READ(RDR,94,END=997) ER
94 FORMAT(2F.0)
IF(ER.EQ.(0.,0.)) GO TO 997
WRITE(PRT,201)
201 FORMAT('/// ENTER TRIAL SKIN DEPTH (CM)...')
READ(RDR,202) SKIN2
202 FORMAT(F.0)
SKIN2=SKIN2/100.
WRITE(PRT,117) SKIN2
117 FORMAT(' SKIN DEPTH = ',2PF10.5)
SKIN=SKIN2+RADIUS

```

```

95  WRITE(PRT,95)ER
    FORMAT( " PERMITTIVITY=",2F10.5///" ENTER INITIAL",
1    " FREQ, DELTA FREQ (GHZ), # FREQ'S, AND MAP..." )
96  READ(CRDR,96,END=997) FREQ,FQINC,RFMAX,RMAP
    FORMAT(4F.0)
    LFMAX=IFIX(RFMAX)
    WRITE(PRT,97) FREQ,FQINC,LFMAX,RMAP
97  FORMAT( " F0=",F10.5, " DELTAF=",F10.5,
1    " NO. OF FREQ'S=",I3, " MAP=",F3.0///
1    " ENTER SUMMATION LIMIT (MAX=51),",
2    " OKZ, AND NO. OF KZ POINTS..." )
98  READ(CRDR,98) RNMAX,YDELTA,RMAX,RNMAX2
    FORMAT(4F.0)
    IF(YDELTA.EQ.0.) YDELTA=.22
    IMAX=IFIX(RMAX)
    IF(IMAX.EQ.0) IMAX=51
    IF(MOD(IMAX,2).EQ.0) IMAX=IMAX+1
    IF(IMAX.GT.201) IMAX=201
    NMAX=IFIX(RNMAX)
    IF(NMAX.EQ.0) NMAX=20
    IF(NMAX.GT.51) NMAX=51
    NMAX2=IFIX(RNMAX2)
    IF(NMAX2.EQ.0) NMAX2=NMAX
    RMAX=YDELTA*(RMAX-1.)
    WRITE(PRT,101) NMAX,YDELTA,RMAX,NMAX2
101. FORMAT( " SUMMATION LIMIT = ",I3,
1    " OKZ=",F8.3, " KZ MAX=",F12.5, " NMAX2=",I3)

```

C
C

```

    UNITS ARE CM AND GHZ.
    FREQ=FREQ-FQINC
    DO 90 L=1,LFMAX
    FREQ=FREQ+FQINC
    REWIND DISC
    W =2.E9*PI*FREQ
    K0= W*SQRT(E0*MU)
    KE=K0*CSQRT(ER)
    KE2=KE*KE
    NMAXM1=NMAX-1
    Y=-YDELTA+1.E-6
    VALMAX=0.
    DO 100 I=1,IMAX
    Y=Y+YDELTA
    Y2=Y*Y
    ARG=CSQRT(KE*KE-Y2)

```

```

IF(AIMAG(ARG).GT.0.) ARG=-ARG
ARGRAD=ARG*RADIUS
ARGSKN=ARG*SKIN
NLIMIT=NMAX
IF(CABS(ARGRAD).LE.15.) NLIMIT=NMAX2
IF(AIMAG(ARGRAD).LT.-45.) NLIMIT=25
C DONT NEED HIGH ORDERS FOR LARGE KZ VALUES
ARG2=ARG*ARG*RADIUS*SKIN
KZB2=Y*SLOTL*.5
PROD=SIN(KZB2)/KZB2
PROD=PROD*PROD/ARG
C IF(AIMAG(ARG).GT.0.) ARG=-ARG
CALL HANKEL(ARGRAD,HANK,NMAX)
CALL HANKEL(ARGSKN,HANK2,NMAX)
H02=HANK2(1)
H12=HANK2(2)
H0=HANK(1)
H1=HANK(2)
N=C
IN(1)=A(1)*PROD*KE2/(PIPHI2**2)*H12*CONJG(H02)
105 1/(H0*CONJG(H0))
FORMAT(I3,F10.5,2E12.5,F10.5/15X,8E12.5//)
MAG(1)=CABS(IN(1))
IF(MAG(1).GT.VALMAX) VALMAX=MAG(1)
HNP=H0-(H1/ARGRAD)
HNP2=H02-(H12/ARGSKN)
TERM=Y2*H12*CONJG(H12)/(ARG2*CONJG(H1)*HNP)
TERM=TERM-KE2*CONJG(H12)*HNP2/(H1*CONJG(H1))
IN(2)=A(2)*TERM*PROD/(PIPHI2-1.0)**2
MAG(2)=CABS(IN(2))
IF(MAG(2).GT.VALMAX) VALMAX=MAG(2)
DO 99 N=3,NLIMIT
NM1=N-1
RN#12=FLOAT(NM1)**2
RN=FLOAT(N-1)
67 HN=HANK(N)
HN2=HANK2(N)
HNP2=HANK2(NM1)-(RN*HN2/ARGSKN)
HNP=HANK(NM1)-(RN*HN/ARGRAD)
C IF HN OR HNP IS VERY LARGE,MUST SCALE THEM OR COMPLEX
C DIVIDE WILL OVERFLOW
HNI=ABS(AIMAG(HN))
HNR=ABS(REAL(HN))
HNPR=ABS(REAL(HNP))

```

```

65      HNPI=ABS(AIMAG(HNP))
      FMULT=1.
      IF(HNR.GE.1.E38.OR.HNI.GE.1.E38) FMULT=1.E-43
      IF(HNPR.GE.1.E38.OR.HNPI.GE.1.E38) FMULT=1.E-42
      IF(HNR.LT.1.E-38.AND.HNI.LT.1.E-38) FMULT=1.E42
      IF(HNPR.LT.1.E-38.AND.HNPI.LT.1.E-38) FMULT=1.E42
      HN=FMULT*HN
      HNP=FMULT*HNP
      HN2=FMULT*HN2
      HNP2=FMULT*HNP2
114     TERM=PNM12*Y2*HN2*CONJG(HN2)/(ARG2*CONJG(HN)*HNP)
      TERM=TERM-KE2*CONJG(HN2)*HNP2/(HN*CONJG(HN))
      IN(N)=A(N)*PROD*TERM/(PIPHI2-RNM12)**2
104     FORMAT(I3,10E12.5)
66     MAG(N)=ABS(IN(N))
      IF(MAG(N).GT.VALMAX) VALMAX=MAG(N)
99     CONTINUE
C
C     PERFORM SUMMATION PRIOR TO INTEGRATION
C
      JN=(0.,0.)
      DO 70 N=1,NLIMIT
170     JN=JN+IN(N)
      ADMIT(I)=JN
      IF(L.EQ.1.OR.L.EQ.LFMAX.AND.RMAP.NE.0)
1WRITE(DISC,2) (MAG(II),II=1,NLIMIT)
      NUM(I)=NLIMIT
      FORMAT(E15.7)
2     CONTINUE
100    IF(L.NE.1.AND.L.NE.LFMAX.OR.RMAP.EQ.0.) GO TO 998
      REWIND DISC
      Y=-YDELTA
      DO 30 I=1,IMAX
      Y=Y+YDELTA
      ARGRAD=CSQRT(KE*KE-Y*Y)*RADIUS
      NNN=NUM(I)
      READ(DISC,2) (MAG(II),II=1,NNN)
      DO 22 N=1,NNN
      IF(MAG(N).LT.1.E-10) MAG(N)=1.E-10
      MAG(N)=10.*ALOG10(VALMAX/MAG(N))
22     CONTINUE
      DO 23 N=1,NNN
23     LINE(N)=BLANK
      DO 24 N=1,NNN

```

```

DB=0.
DO 25 K=1,10
DB=DB+DBINC
IF(MAG(N).GE.DB) GO TO 25
LINE(N)=PLOT(K)
GO TO 24
25 CONTINUE
24 CONTINUE
WRITE(PRT,27) ARGRAD,(LINE(KKK),KKK=1,NNN)
27 FORMAT(2F7.2,4X,112A1)
30 CONTINUE
WRITE(PRT,51) DBINC
51 FORMAT(//,DB INCREMENT IS ',F8.3)
WRITE(PRT,52) VALMAX
52 FORMAT(' MAX VALUE IS ',1PE15.7)
C
C PERFORM SIMPSON'S RULE INTEGRATION. NOTE MUST HAVE EVEN
C NUMBER OF STRIPS, IMPLYING IMAX MUST BE ODD.
998 IMAXM1=IMAX-1
Y0=ADMIT(1)+ADMIT(IMAX)
DO 80 I=2,IMAXM1
RIM1=FLOAT(I-1)
80 Y0=Y0+ADMIT(I)*2.*(1.+AMOD(RIM1,2.))
Y0=Y0*YDELTA/3.
Y0=Y0*(0.-8.)*SLOTL*SKIN*RADIUS**2/(SLOTW**3*W*MU)
ZGUIDE=ETA/SQRT(1.-(WCUT/W)**2)
YN=Y0*ZGUIDE
WRITE(PRT,81) FREQ,YN
81 FORMAT(///,F10.5, 'GHZ.....('',E15.7,', ' ',E15.7,',') MHOS')
WRITE(PRT,118)
118 FORMAT(//,ENTER REFERENCE ADMITTANCE...')
READ(CRD,119) YREF
119 FORMAT(2F.0)
RATIO=REAL(YN)/REAL(YREF)
POWER=10.*ALOG10(RATIO)
WRITE(PRT,120) RATIO,POWER
120 FORMAT(//,POWER RATIO =',E12.5,5X,'DB =',F10.3)
90 CONTINUE
999 GO TO 108
997 STOP
END
SUBROUTINE HANKEL(Z,HN,NMAX)
COMPLEX Y0,Y1,J0,J1,YN(51),HN(51),SUM,J,Z

```

```

31 COMPLEX JN(102),CHI,Z8,P,Q
   IF(CABS(Z).LT.2.) WRITE(108,31)
   FORMAT(' ARG.LT.2,HN(N>40) MAY OVERFLOW'/)
   J=(0.,1.)
   GAMMA=.577215664901533
   PI=3.14159265358979324
   IF(CABS(Z).LE.14) GO TO 23
   CHI=Z-.25*PI
   Z8=8.*Z
   P=(1.,0.)-4.5/(Z8*Z8)+459.375/(Z8**4)
   Q=37.5/(Z8**3)-1./Z8
   Y0=CSQRT(2./PI/Z)*(P*CSIN(CHI)+Q*CCOS(CHI))
   HN(1)=CSQRT(2./PI/Z)*CEXP(-J*CHI)*(P-J*Q)
   CHI=Z-.75*PI
   P=(1.,0.)+7.5/(Z8*Z8)-590.625/(Z8**4)
   Q=3./Z8-52.5/(Z8**3)
   HN(2)=CSQRT(2./PI/Z)*CEXP(-J*CHI)*(P-J*Q)
   DO 15 I=3,51
   RI=FLOAT(I-2)
15  HN(I)=2.*RI/Z*HN(I-1)-HN(I-2)
   RETURN
23  NMAXP1=NMAX+1
   CALL JBESL(Z,NMAXP1,JN)
   J0=JN(1)
   J1=JN(2)
28  Y0=2./PI*(CLOG(.5*Z)+GAMMA)*J0
   SUMB4=1.
   SUM=(0.,0.)
   DO 10 I=1,20
   I2=I+2+1
   SUM=SUM+(-1.)**I*JN(I2)/FLOAT(I)
   SMAG=CABS(SUM)
   IF(CABS(SMAG-SUMB4)/SUMB4.LE.1.E-12) GO TO 4
   SUMB4=SMAG
10  CONTINUE
   Y0=Y0-4.*SUM/PI
22  Y1=(Y0*J1-2./PI/Z)/J0
26  YN(1)=Y0
   YN(2)=Y1
   DO 13 I=3,NMAX
   RI=FLOAT(I-2)
13  YN(I)=2.*RI/Z*YN(I-1)-YN(I-2)
   DO 14 I=1,NMAX
14  HN(I)=JN(I)-J*YN(I)

```

```

RETURN
END
SUBROUTINE JBESL(Z,NMAX,JN)
COMPLEX JN(102),Z,SUM,Z2
COMMON /FACTOR/FACT(51)
ZMAG=CABS(Z)+20.
IF((ZMAG-20.)-.2) 4,4,9
9 RN=FLOAT(NMAX)+10.
K=IFIX(ZMAG)
IF(RN.GT.ZMAG) K=IFIX(RN)
IF(MOD(K,2).EQ.0) K=K+1
IF(K.GT.77) K=77
IF(CABS(Z).LE.2.) K=39
C IF Z IS SMALL, SUM GETS TOO LARGE.
JN(K+2)=(0.,0.)
JN(K+1)=(1.E-37,0.)
IF(CABS(Z).LE.2.) JN(K+1)=(1.E-60,0.)
DO 1 I=1,K
L=K-I+1
1 RL=FLOAT(L)
JN(L)=2.*RL/Z*JN(L+1)-JN(L+2)
SUM=JN(1)
DO 2 I=3,K,2
2 SUM=SUM+2.*JN(I)
DO 3 I=1,K
3 JN(I)=JN(I)/SUM
RETURN
4 Z2=-.25*Z*Z
DO 7 I=1,NMAX
SUM=1./FACT(I)
SB4=.9
DO 5 K=1,20
5 SUM=SUM+Z2**K/FACT(K+1)/FACT(K+1)
SMAG=CABS(SUM)
IF(ABS(SMAG-SB4)/SB4.LT.1.E-7) GO TO 6
6 SB4=SMAG
CONTINUE
7 JN(I)=(.5*Z)**(I-1)*SUM
CONTINUE
RETURN
END

```

APPENDIX E

FORTRAN LISTINGS OF DERIVE AND SCAL, A PROGRAM
PACKAGE WHICH FACILITATES THE REDUCTION
OF LABORATORY SCATTERING PARAMETER
MEASUREMENTS

PROGRAM DERIVE

THIS PROGRAM DERIVES THE SCATTERING PARAMETERS OF AN INTERMEDIATE TWO PORT NETWORK BETWEEN THE NETWORK ANALYZER PORT AND THE DESIRED REFERENCE PLANE.

THE USER MUST INPUT S₁₁ AS MEASURED WITH THE NETWORK ANALYZER PORT TERMINATED IN THREE KNOWN LOADS (MATCH, SHORT, AND OFFSET SHORT) AND ALSO WITH THE INTERMEDIATE NETWORK TERMINATED IN THREE KNOWN LOADS.

THE RESULTING S PARAMETERS MAY BE USED IN PROGRAM SCAL TO COMPENSATE FOR THE INTERVENING NETWORK.

```

COMPLEX S11,S12,S22,J,RL,RM,RSC,ROC,R,A1,A2
COMPLEX AINV,BINV,CINV,DINV,DELTA,A,B,C,D
COMPLEX AN,BN,CN,DN,AT,BT,CT,DT,RMT,RSC,ROCT
COMPLEX S11T,S12T,S22T,S11N,S12N,S22N
COMPLEX YN,S11I,S112,S122
DIMENSION FREQ(25)
PI=3.141592654
J=CMPLX(0.,1.)
20 I=0
WRITE(108,14)
14 FORMAT(//,ENTER OFFSET SHORT LENGTH AND GUIDE WIDTH (CM)...')
READ(105,1) SHORTL,WIDE
1 FORMAT(2F,0)
IF(SHORTL.EQ.0.) GO TO 6
WRITE(108,28)
28 FORMAT(//,ENTER NETWORK OFFSET SHORT (CM)...')
READ(105,29) SHORT2
29 FORMAT(F,0)
IF(SHORT2.EQ.0.) SHORT2=SHORTL
5 I=I+1
WRITE(108,15)
15 FORMAT(//,ENTER FREQ, MATCH, S.C., OFFSET ',
1',FOR DIRECT. COUPLER PORT (DB,DEG)...')

```

```

2   READ(105,2) FREQ(I),RM,RSC,ROC,RL
   FORMAT(9F,0)
   IF(FREQ(I).EQ.0.) GO TO 20
   IF(FREQ(I).EQ.999.) GO TO 6
12  WRITE(108,12) RM,RSC,ROC,RL
   FORMAT(8F7.2)
   CALL PTOR(RM)
   CALL PTOR(RSC)
   CALL PTOR(ROC)
   CALL PTOR(RL)
   Y0=30./FREQ(I)
   YCUT=2.*WIDE
   YG=Y0/SQRT(1.-(Y0/YCUT)**2)
   TANBL=TAN(2.*PI*SHORTL/YG)
   A1=(J*TANBL-1.)/(J*TANBL+1.)
   TANBL=TAN(2.*PI*SHORT2/YG)
   A2=(J*TANBL-1.)/(J*TANBL+1.)
   S11=RM
   S22=(RM-RSC+(RM-ROC)/A1)/(RSC-ROC)
   S12=(RM-RSC)*(1.+S22)
   S12=CSQRT(S12)
   WRITE(108,22)
22  FORMAT(/,' ENTER 3 VALUES FOR TOTAL NETWORK(DB,DEG)...')
23  READ(105,23) RMT,RSCT,ROCT
   FORMAT(6F,0)
   CALL PTOR(RMT)
   CALL PTOR(RSCT)
   CALL PTOR(ROCT)
   S11T=RMT
   S22T=(RMT-RSCT+(RMT-ROCT)/A2)/(RSCT-ROCT)
   S12T=(RMT-RSCT)*(1.+S22T)
   S12T=CSQRT(S12T)
C   FIND ABCD PARAMETERS
   CALL S2ABCD(S11,S12,S22,A,B,C,D)
   CALL S2ABCD(S11T,S12T,S22T,AT,BT,CT,DT)
C   PREMULTIPLY BY INVERSE ABCD MATRIX
   DELTA=D*A-B*C
   AINV=D/DELTA
   BINV=-B/DELTA
   CINV=-C/DELTA
   DINV=A/DELTA
   AN=AINV*AT+BINV*BT
   BN=AINV*BT+BINV*DT
   CN=CINV*AT+DINV*DT

```

```

C      DN=CINV*BT+DINV*DT
      FIND EQUIVALENT S PARAMETERS
      CALL ABCD2S(AN,BN,CN,DN,S11N,S12N,S22N)
      CALL RTOP(S11N,S111)
      CALL RTOP(S12N,S112)
      CALL RTOP(S22N,S122)
      WRITE(108,3) FREQ(1),S111,S112,S112,S122
3      FORMAT(//F8.3,' GHZ'//2(F15.7,2X,F6.1,5X)/2(F15.7,2X,F6.1,5X))
      GO TO 5
19     R=(RM-RL)/(S22*(RM-RL)-S12**2)
      YN=(1.-R)/(1.+R)
      CALL RTOP(R,R)
21     WRITE(108,4) R
4      FORMAT(// 'RET. LOSS = ',F7.3,5X,F6.1)
      WRITE(108,16) YN
16     FORMAT(// 'ADMITTANCE = ',2F12.4)
13     WRITE(108,18)
18     FORMAT(// 'ENTER NEW UNKNOWN (DB,DEG)...')
      READ(105,17) RL
17     FORMAT(2F.0)
      IF(RL.EQ.(0.,0.)) GO TO 5
      IF(RL.EQ.(999.,0.)) GO TO 6
      CALL PTOR(RL)
      GO TO 19
6      STOP
      END
      SUBROUTINE PTOR(ARG)
      COMPLEX ARG
      REAL MAG
      MAG=REAL(ARG)
      ANG=AIMAG(ARG)
      MAG=10.**(MAG/20.)
      ANG=ANG/57.29577951
      R=MAG*COS(ANG)
      AI=MAG*SIN(ANG)
      ARG=CMPLX(R,AI)
      RETURN
      END
      SUBROUTINE RTOP(ARG,ARG2)
      COMPLEX ARG,ARG2
      REAL MAG
      R=REAL(ARG)
      AI=AIMAG(ARG)
      MAG=SQRT(R*R+AI*AI)

```

```

ANG=57.29577951*ATAN(AI,R)
ARG2=CMPLX(MAG,ANG)
RETURN
END
SUBROUTINE S2ABCD(S11,S12,S22,A,B,C,D)
COMPLEX S11,S12,S22,A,B,C,D
A=(S12*S12+(1.+S11)*(1.-S22))/(2.*S12)
B=((1.+S11)*(1.+S22)-S12*S12)/2./S12
C=((1.-S11)*(1.-S22)-S12*S12)/2./S12
D=(S12*S12+(1.-S11)*(1.+S22))/2./S12
RETURN
END
SUBROUTINE ABCD2S(A,B,C,D,S11,S12,S22)
COMPLEX S11,S12,S22,A,B,C,D,Y11,Y12,Y22,DELTA
Y11=D/B
Y12=-1./B
Y22=A/B
DELTA=(1.+Y11)*(1.+Y22)-Y12*Y12
S11=((1.-Y11)*(1.+Y22)+Y12*Y12)/DELTA
S12=-2.*Y12/DELTA
S22=((1.+Y11)*(1.-Y22)+Y12*Y12)/DELTA
RETURN
END

```

PROGRAM SCAL

THIS PROGRAM CALCULATES THE EQUIVALENT
S PARAMETERS OF THE NETWORK ANALYZER SYSTEM.
THIS INCLUDES DIRECTIONAL COUPLERS AND THE
TRANSMISSION LINE USED TO CONNECT THE ANALYZER
TO THE SLOT ANTENNA.

THE USER MUST ENTER S¹¹ AS MEASURED WITH THE
DESIRED PORT TERMINATED IN THREE KNOWN LOADS
(MATCH, SHORT, AND OFFSET SHORT). IF DESIRED,
THE S PARAMETERS OF ANY INTERVENING TWO PORT MAY
ALSO BE ENTERED. THIS NETWORK WILL THEN BE
INCLUDED IN THE OVERALL SYSTEM PARAMETERS. THE
S PARAMETERS OF THE INTERVENING TWO PORT MAY BE
DETERMINED WITH PROGRAM DERIVE.

AFTER SYSTEM PARAMETERS HAVE BEEN CALCULATED, USER
MAY INPUT S¹¹ FOR THE UNKNOWN LOAD (I.E. THE SLOT
ANTENNA). THEN THIS MEASUREMENT WILL BE CORRECTED
TO THE PROPER REFERENCE PLANE.

COMPLEX S11 ,S12 ,S22 ,J,RL,RM,RSC,ROC,R,A
COMPLEX S11N,S12N,S22N,AN,BN,CN,DN
COMPLEX A,B,C,D,AT,BT,CT,DT
COMPLEX YN,S111,S112,S122
DIMENSION FREQ(100)
PI=3.141592654
J=CMPLX(0.,1.)
I=0

20 WRITE(108,14)
14 FORMAT(//, 'ENTER OFFSET SHORT LENGTH AND GUIDE WIDTH (CM)...')
READ(105,1) SHORTL,WIDE
IF(SHORTL.EQ.0.) GO TO 6
1 FORMAT(2F.0)
WRITE(108,22)
22 FORMAT(//, 'HIT RETURN TO INCLUDE SECOND NETWORK...')
READ(105,23) QUERY
23 FORMAT(F.0)

```

5      I=I+1
      WRITE(108,15)
15     FORMAT(//,15) ENTER FREQ, MATCH, S.C., OFFSET,"
1     AND UNKNOWN (OB,DEG)...."
      READ(105,2) FREQ(I),RM,RSC,ROC,RL
2     FORMAT(9F.0)
      IF(FREQ(I).EQ.0.) GO TO 20
      IF(FREQ(I).EQ.999.) GO TO 6
12     WRITE(108,12) RM,RSC,ROC,RL
      FORMAT(8F7.2)
      CALL PTOR(RM)
      CALL PTOR(RSC)
      CALL PTOR(ROC)
      CALL PTOR(RL)
      Y0=30./FREQ(I)
      YCUT=2.*WIDE
      YG=Y0/SQRT(1.-(Y0/YCUT)**2)
      TANBL=TAN(2.*PI*SHORTL/YG)
      A=(J*TANBL-1.)/(J*TANBL+1.)
      S11=RM
      S22=(RM-RSC+(RM-ROC)/A)/(RSC-ROC)
      S12=(RM-RSC)*(1.+S22)
      S12=CSQRT(S12)
      IF(QUERY.NE.0.) GO TO 24
25     WRITE(108,25)
      FORMAT(//,25) ENTER 3 NETWORK PARAMETERS (MAG,ANG)...."
26     READ(105,26) S11N,S12N,S22N
      FORMAT(6F.0)
      CALL PLTOR(S11N)
      CALL PLTOR(S12N)
      CALL PLTOR(S22N)
C     FIND ABCD PARAMETERS
      CALL S2ABCD(S11N,S12N,S22N,AN,BN,CN,DN)
C     CALL S2ABCD(S11,S12,S22,A,B,C,D)
      MULTIPLY ABCD PARAMETERS
      AT=A*AN+B*CN
      BT=A*BN+B*DN
      CT=C*AN+D*CN
      DT=C*BN+D*DN
C     FIND EQUIVALENT S MATRIX
      CALL ABCD2S(AT,BT,CT,DT,S11,S12,S22)
24     CALL RTOP(S11,S111)
      CALL RTOP(S12,S112)
      CALL RTOP(S22,S122)

```

```

3      WRITE(108,3) FREQ(I),S111,S112,S112,S122
19     FORMAT(//F8.3," GHZ",//2(F7.3,2X,F6.1,5X)/2(F7.3,2X,F6.1,5X))
      R=(S11-RL)/(S22*(S11-RL)-S12**2)
      YN=(1.-R)/(1.+R)
      CALL RTOP(R,R)
21     WRITE(108,4) R
4      FORMAT(//," RET. LOSS = ",F7.3,5X,F6.1)
      WRITE(108,16) YN
16     FORMAT(//," ADMITTANCE = ",2F12.4)
13     WRITE(108,18)
18     FORMAT(//," ENTER NEW UNKNOWN (DB,DEG)...")
      READ(105,17) RL
17     FORMAT(2F,0)
      IF(RL.EQ.(0.,0.)) GO TO 5
      IF(RL.EQ.(999.,0.)) GO TO 6
      CALL PTOR(RL)
      GO TO 19
6      STOP
      END
      SUBROUTINE PTOR(ARG)
      COMPLEX ARG
      REAL MAG
      MAG=REAL(ARG)
      ANG=AIMAG(ARG)
      MAG=10.**(MAG/20.)
      ANG=ANG/57.29577951
      R=MAG*COS(ANG)
      AI=MAG*SIN(ANG)
      ARG=CMPLX(R,AI)
      RETURN
      END
      SUBROUTINE RTOP(ARG,ARG2)
      COMPLEX ARG,ARG2
      REAL MAG
      R=REAL(ARG)
      AI=AIMAG(ARG)
      MAG=SQRT(R*R+AI*AI)
      ANG=57.29577951*ATAN(AI,R)
      ARG2=CMPLX(MAG,ANG)
      RETURN
      END
      SUBROUTINE S2ABCD(S11,S12,S22,A,B,C,D)
      COMPLEX S11,S12,S22,A,B,C,D
      A=(S12*S12+(1.+S11)*(1.-S22))/(2.*S12)

```

```

B=((1.+S11)*(1.+S22)-S12*S12)/2./S12
C=((1.-S11)*(1.-S22)-S12*S12)/2./S12
D=(S12*S12+(1.-S11)*(1.+S22))/2./S12
RETURN
END
SUBROUTINE ABCD2S(A,B,C,D,S11,S12,S22)
COMPLEX S11,S12,S22,A,B,C,D,Y11,Y12,Y22,DELTA
Y11=D/B
Y12=-1./B
Y22=A/B
DELTA=(1.+Y11)*(1.+Y22)-Y12*Y12
S11=((1.-Y11)*(1.+Y22)+Y12*Y12)/DELTA
S12=-2.*Y12/DELTA
S22=((1.+Y11)*(1.-Y22)+Y12*Y12)/DELTA
RETURN
END
SUBROUTINE PLTOR(ARG)
COMPLEX ARG
REAL MAG
MAG=REAL(ARG)
ANG=AIMAG(ARG)/57.29577951
R=MAG*COS(ANG)
AI=MAG*SIN(ANG)
ARG=CMPLX(R,AI)
RETURN
END

```

[illegible]

3 1762 10005643 9

D378 Herrick, David L
H435 An intrusive slotted
cop.2 cylinder antenna array...

[illegible]

D378
H435
cop. 2