COST: A POSSIBLE EXPLANATION FOR RISK PREMIUM?

by

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This thesis has been read by each member of the thesis committee and has been ɔund to be satisfactory regarding content, English usage, format, citations, bibliographic ɣle, and consistency, and is ready for submission to the College of Graduate Studies.

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ABSTRACT

Transaction costs, information costs and defaults costs are suspected to partially explain differences in returns which were previously attributed to risk premiums in the financial markets. Two portfolios with identical costs are constructed from a Put, a Call, and underlying S&P 500 stocks, with the first Portfolio being hedged (Put-Call Parity) and the second Portfolio being unhedged (with systematic beta close to 2). The expected stock prices of S&P 500 were calculated based on 52-year historical data using several methods, and returns of the two portfolios were obtained and compared, using 935 observations of S&P 500 from January 2, 1992 through June 30, 1992. A linear regression model adjusted for cross-sectional heteroskedasticity and auto-correlation was used to estimate the expected risk premium rate.

Transaction, information and default costs were statistically significant and estimated at 0.6% of value annually. These costs reduce the risk premium estimated by the Capital Asset Pricing Model.

CHAPTER 1

INTRODUCTION

Statement of the Problem

Since the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT) were developed by Sharpe, Ross, and other economists in the 1960s, the subject of risk premium has been widely studied in finance. Standard financial theory states that more risky assets generate higher average rates of return and that there exists a market price for risk, identified as "risk premium" (defined as systematic risk or beta in this paper).

However factors other than risk premium may explain differences in returns across risky assets. One possible explanation is the costs involved in buying and selling stocks. Several costs are incurred by investing in financial markets: transaction costs, information costs, and default costs. Transaction costs include brokerage fees, bid-ask spreads, the seat price in the New York Stock Exchange (NYSE) or the American Stock Exchange (AMEX), and taxes. Information costs include time, money spent in searching for the stock and price, data collecting costs, analyzing fees, and human capital costs, some of which are nonmonetary. Finally, every company has a risk of going bankrupt. When default occurs, the stockholders of that company may lose their investments. Those losses are defined as default costs.

For example, banks usually charge higher interest rates to those borrowers with higher possibility of default (Benjamin, 1978). While this can be explained by the risk premium theory, it also can be explained by the costs associated with default. Default usually results in banks sustaining losses. Thus banks must charge a higher contractual interest rate to recoup those losses (Watts, etc., 1992).

Some of those costs are explicit (e.g., taxes), while others are implicit, such as time consumed in searching for the best price. Some costs are proportional percentages, or directly related with each transaction, such as commission fee, while some are fixed, such as the seat price of the NYSE. The complexity of costs makes estimation difficult.

As discussed in the literature, as well as in this thesis (see Chapter 6), the returns from the Put-Call Parity (PCP), which is free from risk, are consistently and significantly greater than the most obvious opportunity cost--riskless interest rate--implying that costs such as transaction costs, information costs, and default costs exist in the financial markets. In most of the previous research, however, the yields from an asset are calculated by summing the market price appreciation and the accumulated dividends. Such an approach ignores differences in the costs earlier discussed, thereby influencing the comparison of returns across different stocks.

For example, a broker may charge a higher commission for trading rare, low-volume stocks, and a lower commission for trading prominent and frequently-traded stocks. It is much easier to collect information regarding performances and prospects of those companies that are large and well known than for the small, local, or marginal companies. The information costs associated with stocks are therefore not identical from one company

to another. Furthermore, default costs which are closely associated with characteristics of production technology of a firm, which also vary from one company to another. Because these costs differ from one stock to another, it is questionable that the differences in returns of various stocks are caused solely by risk premium based on expected utility theory rather than by varied costs.

Modigliani and Pogue (1974) found a linear relationship between the returns and the systematic risks (beta), predicted by the CAPM model from 1945-1970 historical data of some companies (see Chapter 2, Literature Review). However those companies examined are long-run survivors. It is possible that other, nonsurviving companies had the same beta but yielded lower or even negative returns during the period they were in existence. Thus, the "observed" returns over-represent the true returns. Incorrect estimation of costs can significantly affect the conclusion reached. To the degree that costs are underestimated, it is likely that the differences in "observed" rates of returns are not caused solely by a risk premium.

Model Development

To correctly analyze the role of risk aversion in financial markets, the transaction costs, information costs, and default costs must be separated from the risk premium. Ignoring those costs may result in over estimation of the risk premium and failure to fully explain differences in gross returns.

A way to avoid this problem is to compare portfolios that have identical costs but that have different risks. The portfolios used in this thesis are constructed from call and put

options of the same underlying stock, S&P 500. S&P 500 is an index stock (composite stock) that reflects the market value of the 500 most actively-traded U.S. domestic stocks, and is a proxy for overall market performance.

Portfolio 1 in this thesis is the well-known Put-Call Parity (PCP). A call option is the right to purchase a specific stock at a given price (called the exercise price, or striking price) at a given date (called maturity date). A put option is the right to sell a specific stock at the exercise price (or striking price) at maturity date. The holder of the put has the right to sell a stock at an exercise price on an expiration date. If a call or put option can be exercised only at the date of maturity, it is called a European option. If it can be exercised at any day before maturity, it is called an American option. All options terminate on the third Friday of each month. Almost all of the options traded on the Chicago Board Option Exchange (CBOE) are the American type, except a few such as the S&P 500 option.

As an example, suppose on January 9, 1992, one S&P 500 call option is bought with an exercise price of $410 and a maturity date of March 20, 1992. The call price is $4 and the call is kept through the expiration date. If the market price of S&P 500 goes up to $420 on the expiration day, the investor will exercise the call, buy S&P 500 at $410 (exercise price), and sell it at the market price of $420. The investor will net $6 ($420 - $410 - $4). If the market value goes below $410 on the expiration day, the investor will not exercise the call option and simply will lose $4 of the call price originally paid.

Now suppose the investor simultaneously buys a stock, buys one put option and sells (writes) one call option. Both of the options are written on the same stock, maturity date,

and exercise price. On the maturity day, whatever the new market condition is, the value of the portfolio will end with the same fixed value--the exercise price.

Let: S = current stock price,

P = current put price,

C = current call price,

X = current exercise price, and

S' = stock price at expiration day.

The total investment at time 0 (current time) is S + P - C. On the expiration day, all possible outcomes of S' can be divided into two states: S' < = X, and S' > X. Omitting dividends and other costs:

If S' < = X,	Return
(a) The investor holds the stock	S'
(b) The call option is worthless	0
(c) The put option has worth (makes money)	X - S'
(d) Therefore, the net value is	X
If S' > X	**Return**
(a) The investor holds the stock	S'
(b) The call option has worth (loses money)	-(S' - X)
(c) The put option is worthless	0
(d) Therefore, the net value is	X

This put-call parity reveals an inherent relationship among put price, call price, exercise price and stock price at any time. The cash outflow at time 0 is (S + P - C) and inflow at time t is X. Since X is generated regardless of the market movement, Portfolio 1 is defined as a riskless asset.

Portfolio 2 in this thesis is a reversed, unhedged trading strategy in which a stock is bought, a call is bought, and a put is sold (i.e., S - P + C). On the maturity date:

If S' <= X	Return
(a) The investor holds the stock	S'
(b) The call option is worthless	0
(c) The put option has worth (loses money)	-(X - S')
(d) Therefore, the net value is	2S' - X
If S' > X	Return
(a) The investor holds the stock	S'
(b) The call option has worth (makes money)	S' - X
(c) The put option is worthless	0
(d) Therefore, the net value is	2S' - X

Portfolio 1 is a hedged asset equivalent to a "riskless bond," X. Portfolio 2 is an unhedged asset equivalent to a "risky stock," 2*S' - X (where X is predetermined).

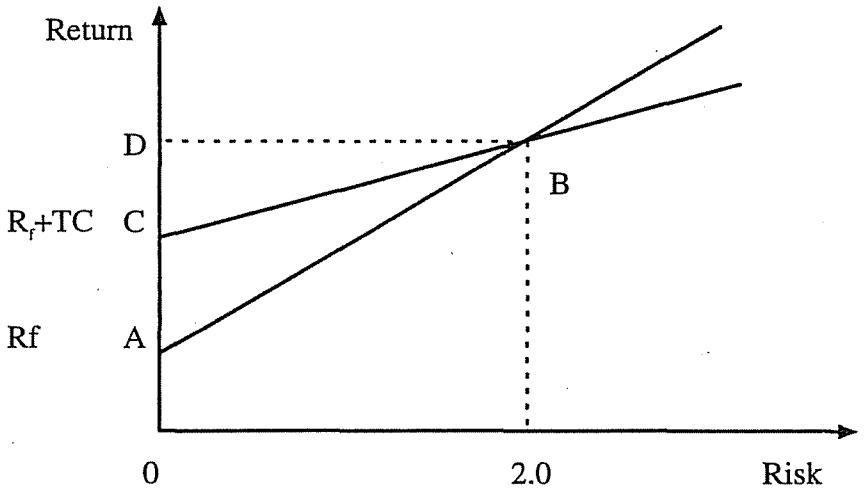
The two portfolios in this study are constructed upon the same underlying stock--S&P 500. Since it is reasonable to expect that the commission fees for the call or put option of the same type (i.e., exercise price, maturity date, and the underlying stock) are the same, the

commission fees implied in the realized returns are the same. Also, the side holding either Portfolio 1 or Portfolio 2 equally value the information of stock price going up or down. This symmetry guarantees that the transaction costs and information costs associated with the portfolio are theoretically identical.

Therefore, the problem of correctly estimating transaction, information, and default costs to compare the two portfolios is avoided since both portfolios contain the same underlying stock but with different systematic risks. The difference between the two portfolio returns is caused solely by the risk premium, since the costs are the same.

Figure 1 illustrates this concept. Suppose a bond (with risk-free rate R_f) is at point A, and a risky asset is at point B. A corresponding risk-free asset with the same costs as asset B is at point C. The estimation of risk premium using the CAPM model is the slope of AB because it estimates risk premium from various kinds of assets. The estimation using the method offered in this paper however is the slope of CB. If these costs exist, the slope of CB should be smaller than that of AB, and the difference between the two should be AC, which is just the costs suspected (TC).

Figure 1. Risk Premium Estimation in this Thesis and in the CAPM Model



Objective

The objectives of this thesis are:

To examine whether a significant difference still exists between the rate of return of riskless and risky portfolios after adjusting for cost difference (i.e., identical underlying stock), as predicted by the CAPM model, and to examine whether the risk premium estimated by this model is smaller than or equal to the risk premium estimated by CAPM model.

To examine whether the put-call parity (PCP) holds and if the costs (transaction costs, information costs, and default costs) significantly exist in real financial markets.

Outline of Thesis

The introduction is followed by a review of the literature related to this thesis, which includes the market efficiency theory, option theory and PCP, the CAPM model, and the random walk theory. Chapter 3 focuses on the theoretical model and the statistical specification. In Chapter 4, four methods of forming expected returns for Portfolio 2, based on random walk are discussed. The data collecting and processing is summarized in Chapter 5. Statistical results and findings are discussed in Chapter 6. And finally Chapter 7 provides conclusions and remarks.

CHAPTER 2

LITERATURE REVIEW

In this chapter, literature relevant to this thesis is reviewed. In the first section, a discussion of market efficiency and the role of transaction cost in financial markets is presented. In the following section, option pricing theory is introduced, upon which the two portfolios detailed in this thesis are based. Next, an introduction of the concept of risk premium is presented, followed by a summary of the development of the CAPM model as well as its empirical testing. In the final section, random walk theory is discussed, which is the theoretical basis upon which the expected returns are formed as in Chapter 4.

Market Efficiency Theory

Capital markets are efficient when security prices fully reflect all available information. In such a market, security prices adjust rapidly to new information, leaving no opportunity for arbitrage. For example, the foreign exchange market is efficient if the current exchange rate of Japanese Yen quickly goes down in response to the news of strict regulations on Japanese imports into the U.S. Also, the future exchange rate of Japanese Yen must go down to maintain the interest-rate parity (assuming that interest rates in both nations do not change).

An efficient market is not necessarily a perfect market. A perfect market means:

A. Transaction costs do not exist in the markets.

B. There is perfect competition in the markets. All participants are price takers and are free to enter or leave.

C. Information is costless and available to all individuals.

D. Individual behavior is rational, maximizing either profit or utility.

A market may be efficient without being perfect, since an efficient market may include transaction costs. Fama (1970, 1976) defined the following three degrees of efficiency:

A. Weak degree: An unanticipated return in financial markets is not related to any historical data or information.

B. Semi-strong degree: An unanticipated return is not related to any publicly available information. No investors can make extra returns from obvious public information (e.g., announcement of stock splits, annual reports, new security issues, etc.).

C. Strong degree: An unanticipated return is not related to any information, whether publicly available or from an insider. It is concerned with whether some investors can profit from "monopolistic information" relevant to the stock price.

Rubinstein (1975) and Latham (1985) used a stronger definition of efficiency by arguing that it is possible that people might disagree with the implication of a piece of information and thereby some investors buy an asset and others sell it in such a way that the market prices are unaffected.

Grossman and Stiglitz (1980) formed an interesting paradox that implies market prices will not reflect all available costly information. They argue that if prices reflected all available information, the trader would have no incentive to collect information because information is costly to obtain. Therefore, in equilibrium, prices will reveal only part (not all) of the information available to traders. A market will not reflect all information because information is costly.

Transaction costs play a significant role in markets. Coase (1937) showed that markets will expand to a point where marginal benefit from using the market is equal to the transaction costs. Phillips and Smith (1980) differentiate types of costs in financial markets. The first kind of costs are explicit: commission, brokerage fee, and bid-ask spread, which are usually proportional to the size of the transaction. The second kind of costs are implicit, such risk-free interest rate and seat prices in the NYSE. Market efficiency means zero economic profit, which should be net of all costs. Including the ownership, costs of seats is $125,000 in the NYSE and is $115,000 in the option market CBOE.

J.P. Gould and D. Galai (1974) wrote:

> "A belief once held by many economists has been that while transaction costs exist in the real world, they are of relatively negligible importance as an empirical matter, especially in the financial markets. In contrast to this belief, the finding of this paper is that transaction costs appear to play a nontrivial role in the explanation of observed premium on puts and calls."

They concluded that the market seemed so inefficient as to raise the question whether the transaction costs were estimated correctly or not. The transaction costs they used included broker fee and taxes, but failed to include seat price in the NYSE and other implicit costs.

Option Pricing and Put-Call Parity

Black and Scholes (1973) independently derived a continuous-time option pricing formula for a European option with no dividends from the underlying asset. The formula is:

$$C = SN(d_1) - Xe^{-R_f T}N(d_2), \tag{1}$$

where

$$d_1 = \frac{\ln(S/N) + R_f T}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T};$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

where e is Euler's number, S is the underlying stock price, X is striking price of option, T is time to expiration of the option, R_f is annual risk-free rate, and σ is instantaneous variance rate of the stock. The terms $N(d_1)$ and $N(d_2)$ are the cumulative probabilities for the standard normal distribution.

The Black-Scholes model was generalized for dividend payment by Merton (1973a). In Merton's model, dividends are assumed to be paid continuously such that the yield is constant. Merton also used the Dominance Theorem (e.g., First-order stochastic dominance, which means that one asset would be preferred by all investors if it offers the highest possible expectation of return at any given level of risk) to establish a lower bound for call pricing:

$$C \geq S - e^{-rt} * X - \sum e^{-rt} * D_t. \tag{2}$$

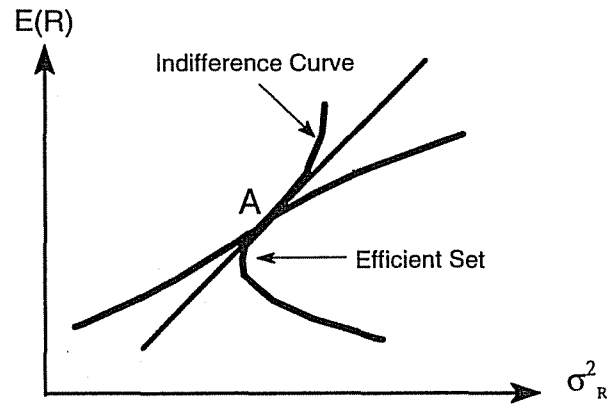
That is, the call price must be greater than the stock price minus the present value of exercise and cumulative dividends (using riskless interest rate).

Stoll (1969) showed that European options exhibit a fixed relationship between the price of a put and call with the same maturity date, which is denoted as put-call parity (PCP). Once we know a European call on a stock, we can easily determine the price of the put on the same stock, regardless of how options are valued. Merton (1973c) showed that the PCP does not necessarily hold for American options because the possibility of early exercise cannot be ruled out when the portfolio is established. Klemkosky and Resnick (1979) tested the put-call parity by using market data from July 1977 through June 1978. Their results were consistent with the put-call parity.

Risk Premium and the CAPM Model

Risk premium begins with an assumption that all individuals are risk averse. From the concave utility function of wealth, a convex, positively-sloped indifference function of expected return and risk is developed, implying a positive substitution between expected returns and risk. Given an efficient set of portfolios (a set of mean-variance choices from different combinations of securities), an optimal choice is reached at point A where the trade-off between the expected return E(R) and risk σ^2_R is in equilibrium (Figure 2).

Figure 2. Optimal Portfolio Choice for a Risk Averse Investor and an Efficient Set.



The CAPM model, used to show the existence and level of risk premiums, was first introduced simultaneously by Treynor (1961) and Sharpe (1963, 1964). Trenyor's contribution to the theory of finance was to devise a method for predicting the risk premium and to demonstrate its importance in the behavior of capital markets as well as in portfolio selection. Both authors reached the conclusion that the only thing investors should worry about is how much any asset contributes to the risk of the portfolio as a whole (beta). The model was further developed by Mossin (1966) and Lintner (1969).

The CAPM model is based on the following assumptions:

A, All investors are risk averse and maximize the expected utility of end-of-period wealth.

B, Investors are price takers and have homogeneous expectations about asset returns.

C, There exists a risk-free interest rate (R_f).

D, Total quantity of the assets is given.

E, Assets are frictionless and information is costless.

F, There are no transaction costs, such as taxes and restrictions.

Based on these assumption, an efficient market would reach an equilibrium where riskier assets would have higher expected returns than less risky assets.

Mathematically, CAPM is

$$E(R_i) = R_f + [E(R_m) - R_f]\beta_i, \quad (3)$$

where $E(R_i)$ is expected rate of return of an individual asset: R_f is risk-free interest rate, $E(R_m)$ is expected rate of return of market portfolio, and β_i is the covariance of returns between the market portfolio and the risky asset, or beta. The higher the risk (beta), the higher the expected return $[E(R_i)]$.

Roll (1977) argued that the efficiency of the market portfolio and the capital asset pricing model are inseparable, joint hypotheses, i.e., it is not possible to test the validity of one without the other.

There are two kinds of risks: systematic and unsystematic risk. Investors can eliminate all the unsystematic risks by appropriate diversification among risky assets. Systematic risk is risk which cannot be avoided by diversification. Beta measures systematic risk and reflects the covariance with the entire market. A portfolio of the entire market, having a beta of one, generates a return equal to risk-free rate plus the risk premium.

Modigliani and Pogue (1974) found a positive relationship between annual rate of return and beta (Table 1).

Table 1. Rates of Return and Betas for Selected Companies (1945-1970).

Company	Average Annual Returns (%)	Standard Deviation (%)	Beta
City Investing Co.	17.4	11.09	1.67
Radio Corp. of American	11.4	8.30	1.35
Chrysler Corp.	7.0	7.73	1.21
Continental Steel Co.	11.9	7.50	1.21
100-Stock Portfolio	10.9	4.45	1.11
NYSE Index	8.3	3.73	1.00
Swift and Co.	5.7	5.89	0.81
Bayside Cigar	5.4	7.26	0.71
American Snuff	6.5	4.77	0.54
Homestake Mining Co.	4.0	6.55	0.24

Since the 1970s, many economists such as Friend and Blume (1970), Miller and Scholes (1972), Fama and MacBeth (1973), Litzenberger and Ramaswamy (1979), and Banz (1981) have empirically tested the CAPM by transforming the ex ante form (expected return) into ex post form (actual return), under the assumption of a fair game (i.e., average realized rate of return is equal to expected rate of return):

$$E(R_i) = R_f + [E(R_m) - R_f]\beta_i \tag{4}$$

$$\Rightarrow \quad R_i - R_f = (R_m - R_f)\beta_i + \varepsilon_i.$$

They found that although beta is a good explanatory variable, low-beta securities earn more than predicted by the CAPM and high-beta securities earn less. That implies that the actual differences in returns across assets are less than those predicted by the CAPM.

Gibbons (1982), Stambaugh (1982), and Shanken (1985) examined an alternative test of the CAPM using the following transformation from a two-factor CAPM developed by Black (1972), in which riskless rate (R_f) is replaced by the return of zero-beta, minimum variance portfolio (R_z).

$$E(R_i) = E(R_z) + [E(R_m) - E(R_z)]\beta_i \tag{5}$$

$$\Rightarrow \quad E(R_i) = R_f(1 - \beta_z) + E(R_m)\beta_i.$$

Gibbons (1982) noticed that there exists a restriction on the intercept of the model:

$$\alpha_i = E(R_z)^*(1 - \beta_i) \tag{6}$$

Gibbons tested this restriction and found that it is violated, implying the CAPM is rejected.

Arbitrage Pricing Theory (APT), a more general model developed by Ross (1976), shows that many other factors can explain asset returns. Those factors include such macroeconomic variables as index of industrial production (or national income), default risk premium, and unanticipated inflation (Chen etc., 1983). The CAPM can be viewed as a special case of APT when the market rate of return is assumed to be the single relevant factor.

Random Walk Theory

As discussed in the introduction of Chapter 4, investors base their decision on the expected future stock prices [E(S')] rather than the actual future stock prices. Therefore, a forecast of stock prices is necessary to calculate the expected return from the Portfolio 2.

Extensive work has been done to explain and predict the movement of stock prices. Firm-foundation theory (fundamental analysis) stresses that the value of a stock (or investment instrument) ought to be based on the stream of earnings a firm will be able to distribute in the future (intrinsic value). Castle-in-the-air theory (technical analysis), on the other hand, focuses on psychic values (beauty contest, i.e., a thing has worth only when someone else will pay for it) (Malkiel 22). However random walk process appears to be the most commonly accepted model. In this thesis, the random walk model is used to form the expected stock prices.

The notion of a "random walk" originated in 1959 from "Brownian Motion in the Stock Market", a paper published by Osborne, which observed that the movement of stock prices is no more predictable than the movement of an "ensemble" of molecules. Random walk theory states that the expected return of a security is stationary throughout time, or the successive price changes are independent and identically distributed.

If S_t is security price at time t,

$$S_t = S_{t-1} + \varepsilon_t, \tag{7}$$

$$E(\varepsilon_t) = 0,$$

$$\text{Cov}\ (\varepsilon_t, \varepsilon_{t-1}) = 0,$$

and $\varepsilon \sim \text{NID}\ (0, \sigma_2)$.

Random Walk Theory asserts that in an efficient market, a security price is random, not because it is insensitive to any new information, but because it adjusts so quickly to new information that future stock price changes seem unpredictable.

The weak form of efficiency in Fama's (1970) definition implies that price change follows a random walk. If unanticipated return is not related to any past information, the distribution of return conditional on a given past information structure is equal to the unconditional distribution of return.

Samuelson (1965) showed that prices could follow a deterministic trend while still fluctuating randomly (random walk with drift). The form is:

$$S_t = w + S_{t-1} + \varepsilon_t \tag{8}$$

where w is a drift, and $\varepsilon \sim NID\ (0, \sigma_2)$.

CHAPTER 3

THE MODEL

As discussed in Chapter 1, to avoid the problem of adjusting for costs (which vary from one stock to another) and to correctly value the role of risk aversion in financial markets, Portfolio 1 and 2 based on one put, one call, and the underlying stocks (S&P 500) were constructed. The two portfolios have identical costs but have different risks. Risk premium, defined as market premium per unit of beta in this thesis, should be the difference between the expected returns from the two portfolios divided by the difference in betas.

Let:

$E(S')$ = Expected S&P 500 price on maturity day.

D = Future value of accumulated dividend payments (Calculated by using risk-free interest rate).

TC = Transaction costs and information costs summarized as percentage of annual rate of return.

R_f = Annual real riskless interest rate.

R_p = Annual risk premium rate (per unit of beta).

N = Total number of days before expiration (from the current date to the maturity date, including weekends and holidays).

R1 = Ratio of the maturity-day value to the current-day value of Portfolio 1.

R2 = Ratio of the maturity-day value to the current-day value of Portfolio 2.

The two portfolios involve the following transactions (Table 2):

Table 2. Transaction Summaries For Portfolio 1 and 2.

Asset	Current Date	Expiration Date
Portfolio 1:	S + P - C	X + D
Portfolio 2:	S - P + C	2S' - X + D

The underlying S&P 500 stock is a representative of the well-diversified market portfolio. D is assumed to be certain and constant. As shown in Figure 3 and 4, the volatility of accumulated dividends is very small compared to that of the stock price. Figlewski (1984) argued that dividend uncertainty for S&P 500 is insignificant. This assumption implies that the change of rate of returns is solely from the change in stock price (market appreciation).

Portfolios 1 and 2 have different risks. Portfolio 1 has a beta of 0 and Portfolio 2 has a beta close to 2 (see Appendix A, Note 1).

According to the CAPM model (Chapter 2, equation (3)),

$$E(R_i) = R_f + [E(R_m) - R_f]\beta_i \qquad (3)$$

Figure 3. Decompositions of Returns

Dividends & Appreciation Jan-Jun 1992

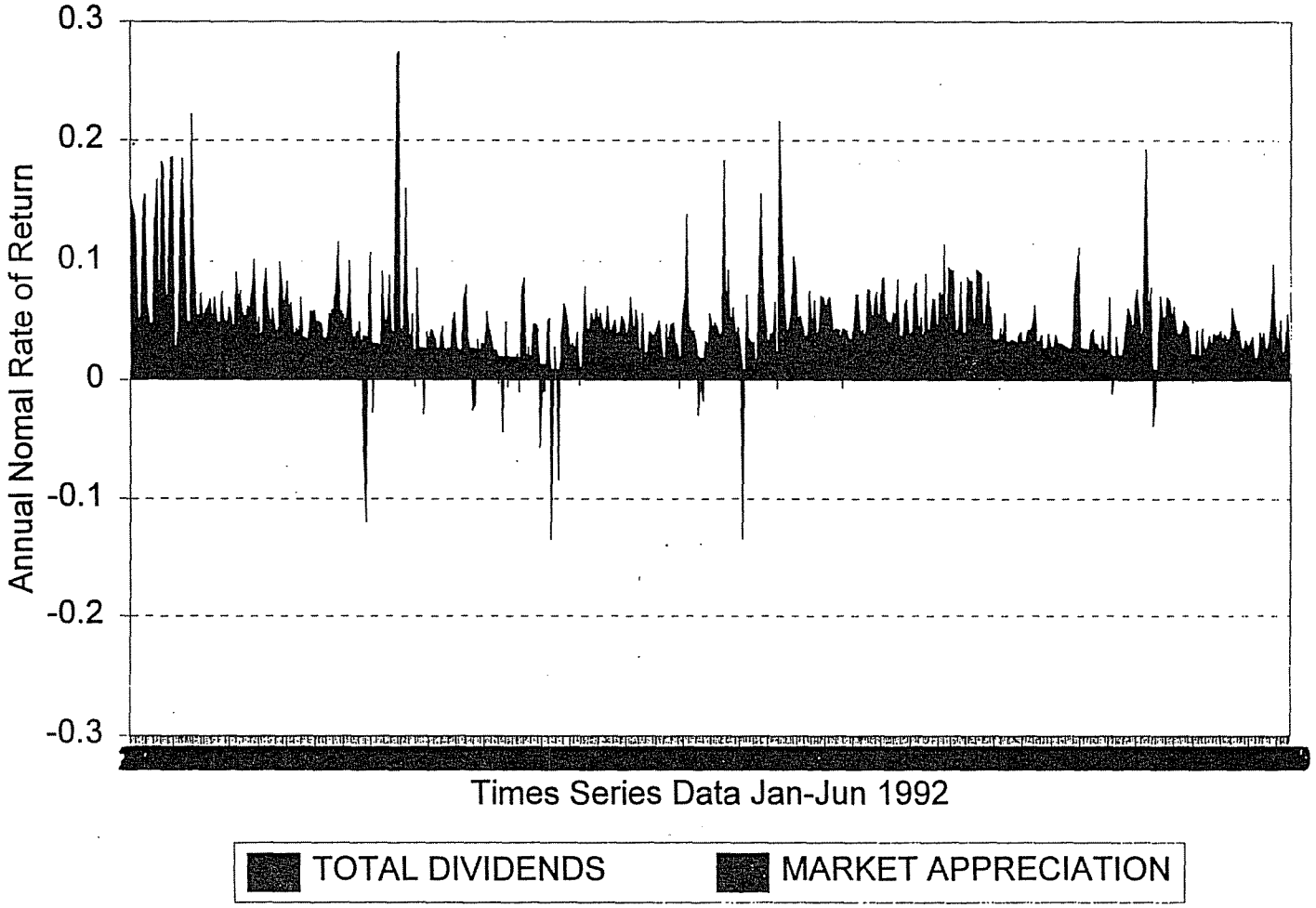
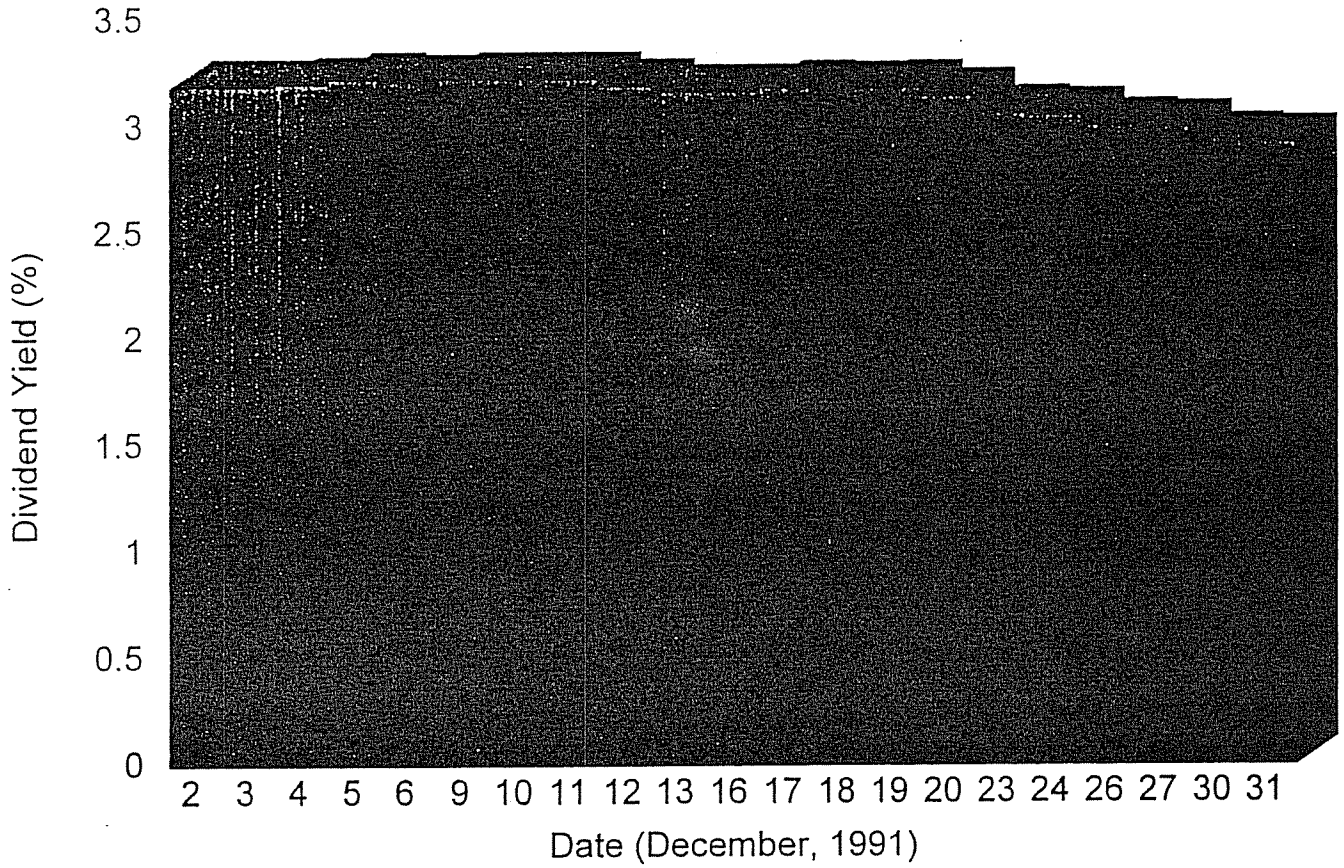


Figure 4. Dividend of S&P 500 Stock
Annual Quarterly Dividend, Dec 1991



That is, the expected rate of return of a risky asset is the linear combination of risk-free rate and risk premium times its beta (systematic risk).

In this study, since

$$R_1 = \frac{X + D}{S + P - C} \quad \text{and} \quad R_2 = \frac{2E(S) - X + D}{S - P + C},$$

and so:

$$R = 1 + (R_f + TC)*N/365 + \beta*R_P*N/365 + \varepsilon. \tag{9}$$

where 365 is the number of days in 1992 and when R = R1, $\beta = 0$, and when R = R2, $\beta = 2$.

Based on the previous analysis, R_f (real risk-free rate or opportunity costs) and TC (transaction costs and information costs) are constant in both portfolios. The real Treasury Bill rate is used as real riskless rate in this thesis.

The error term (ε) reflects market noise and degree of market inefficiency. Black points out that noise is the primary motivation of market activity. Market noises also reflect the degree of understanding of the real world as well as degree of market efficiency. In this thesis, the error terms in both portfolios are assumed to have the same variance (i.e., the same degree of market efficiency).

The statistical form is as follows:

$$R_i = a + b*N_i/365 + c*D_i*N_i/365 + \varepsilon_i, \tag{9*}$$

where D is a dummy variable of beta, D = 0 when R1 is used, D = 2 when R2 is used, and $\varepsilon_i \sim NID(O, \sigma^2)$

It is expected that

1. $a = 1$,
2. $b = R_f + TC > R_f$, and
3. c is a test of the existence and level of the risk premium.

To remove the effect of inflation from the returns, R1 and R2 were deflated by the actual inflation rate. Various inflation rates have been suggested. Since the primary concern is the difference between R1 and R2 as measured by c, the analysis is unlikely to be sensitive to minor differences among alternative choices of the inflation rates.

CHAPTER 4

EXPECTED STOCK PRICE

All of the variables used to calculate R1 and R2 are directly observable except E(S'), the expected future stock prices. Many economists avoid this problem by simply using the actual stock prices at the expiration date (S') as an unbiased instrument of E(S'), justified by the efficient-market theory.

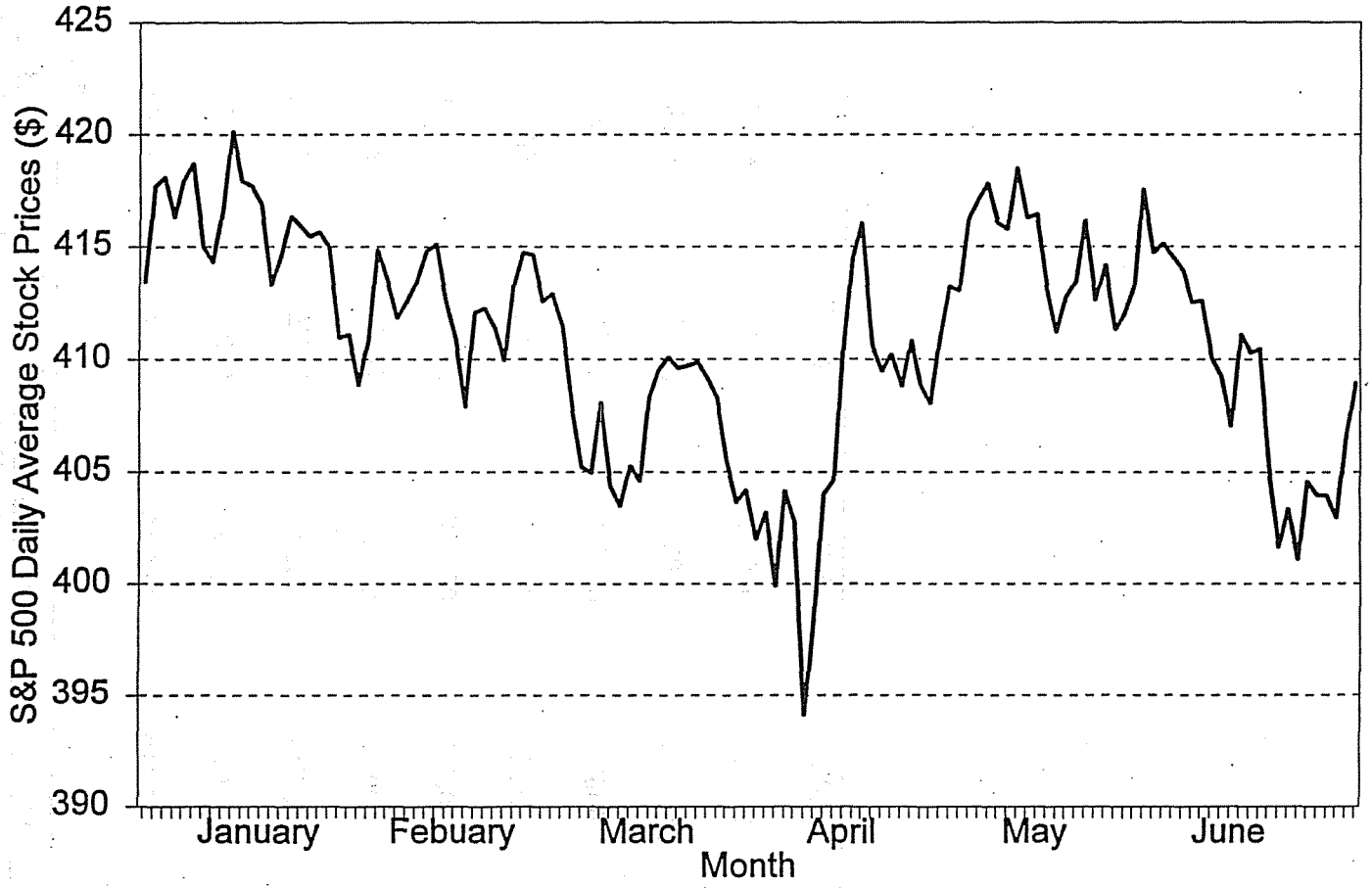
The data set in this thesis (from January 2, 1992 through June 30, 1992) is relatively short and the short-term variations may adversely affect the results. The S&P 500 during that period is generally a bear market (Figure 5). Replacing E(S') by S' would therefore result in a negative risk premium. Nor can we use S for E(S') because under zero drift R2 is always smaller than R1 (see the mathematical proof in Appendix A, Note 2).

Various approaches to the formation of expected stock prices E(S') based on the random walk theory were investigated.

Testing Random Walk

The most commonly accepted stock market movement pattern is the random walk. In a random walk, price changes do not follow a pattern and past information is useless in

Figure 5. S&P 500 Market Prices
January - June 1992, In Norminal Terms



predicting the current price movements (Figure 6).

Suppose S_t is the real stock price at time t and r_t is the change rate of market price in real terms at time t. Then

$$E(S_t \mid S_{t-1}, S_{t-2},...) = E(S_t \mid S_{t-1}) = S_{t-1} \quad (10)$$

(Martingale Process)

and

$$E(r_t \mid r_{t-1}, r_{t-2},...) = E(r_t), \quad (11)$$

(Random Walk).

The empirical forms to test the random walk are:

$$S_t = a + b*S_{t-1} + \varepsilon_t \quad (10^*)$$

$$\varepsilon_t \sim \text{NID}(0, \sigma^2),$$

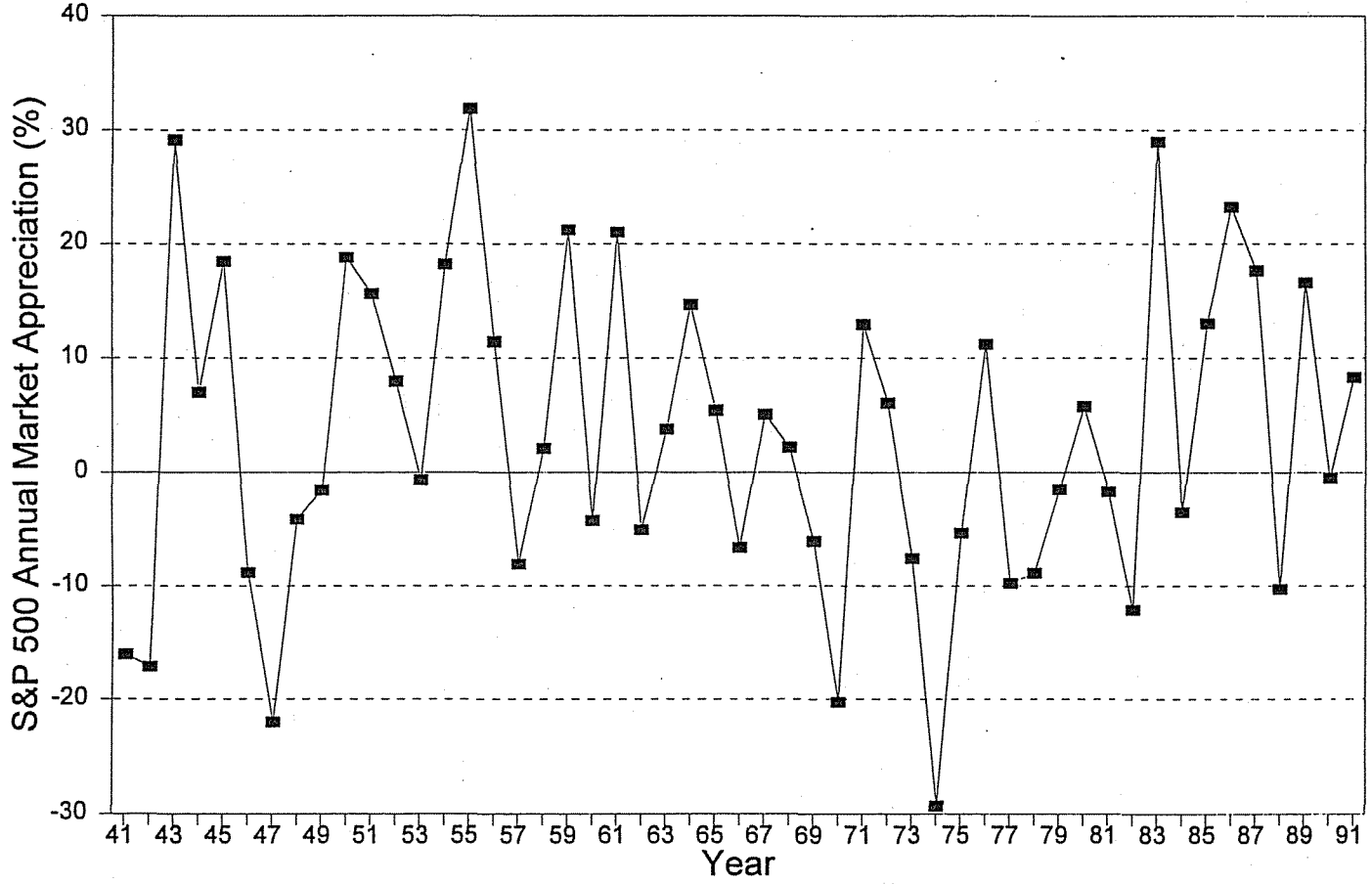
and

$$r_t = a + b\, r_{t-1} + \varepsilon_t \quad (11^*)$$

$$\varepsilon_t \sim \text{NID}(0, \sigma^2)$$

It is expected that in (10*) the coefficient on S_{t-1} will be 1 (that is, the past price is an unbiased estimate of the current price), and the intercept will be 0 (which means no drift). In the (11*), the coefficient on r_{t-1} is expected to be 0 (that is, the past price changes have no power in predicting the current price change), and R^2 is expected to be low (poor fit).

Figure 6. Random Walk - S&P 500 Returns
in Real Terms, 1941-1991



The Ordinary Least Square (OLS) results of equations (10*) and (11*), using yearly average S&P 500 stock prices during 1940-1991 (with t-values in parentheses) are as follows:

$$S_t = 10.132 + 0.960S_{t-1} \qquad (10^{**})$$
$$(1.232) \quad (20.192)$$

$N = 51, \qquad R^2 = 0.8919, \qquad F = 404.1, \qquad Dh = 0.441.$

Zero autocorrelation is accepted at the 1% significance level and the coefficient on S_{t-1} is not significantly different from 1.

and $$r_t = 0.034 + 0.075\, r_{t-1} \qquad (11^{**})$$
$$(1.699) \quad (0.529)$$

$N = 50, \qquad R^2 = 0.0058, \qquad F = 0.279.$

Zero auto-correlation is accepted at 1% significance level. The results conform to what was expected. Neither estimated equations provides evidence upon which to reject a random walk. (Details about the data are provided in Chapter 5.)

Four Methods of Estimating Stock Price

Based on the random walk theory, the following four methods of estimating E(S') were employed using data of yearly average S&P stock price from 1940 through 1991 in either real or nominal terms (plot of price movements are in Figure 7 and 8).

Figure 7. S&P 500 Prices 1940-1991
in Nominal Terms

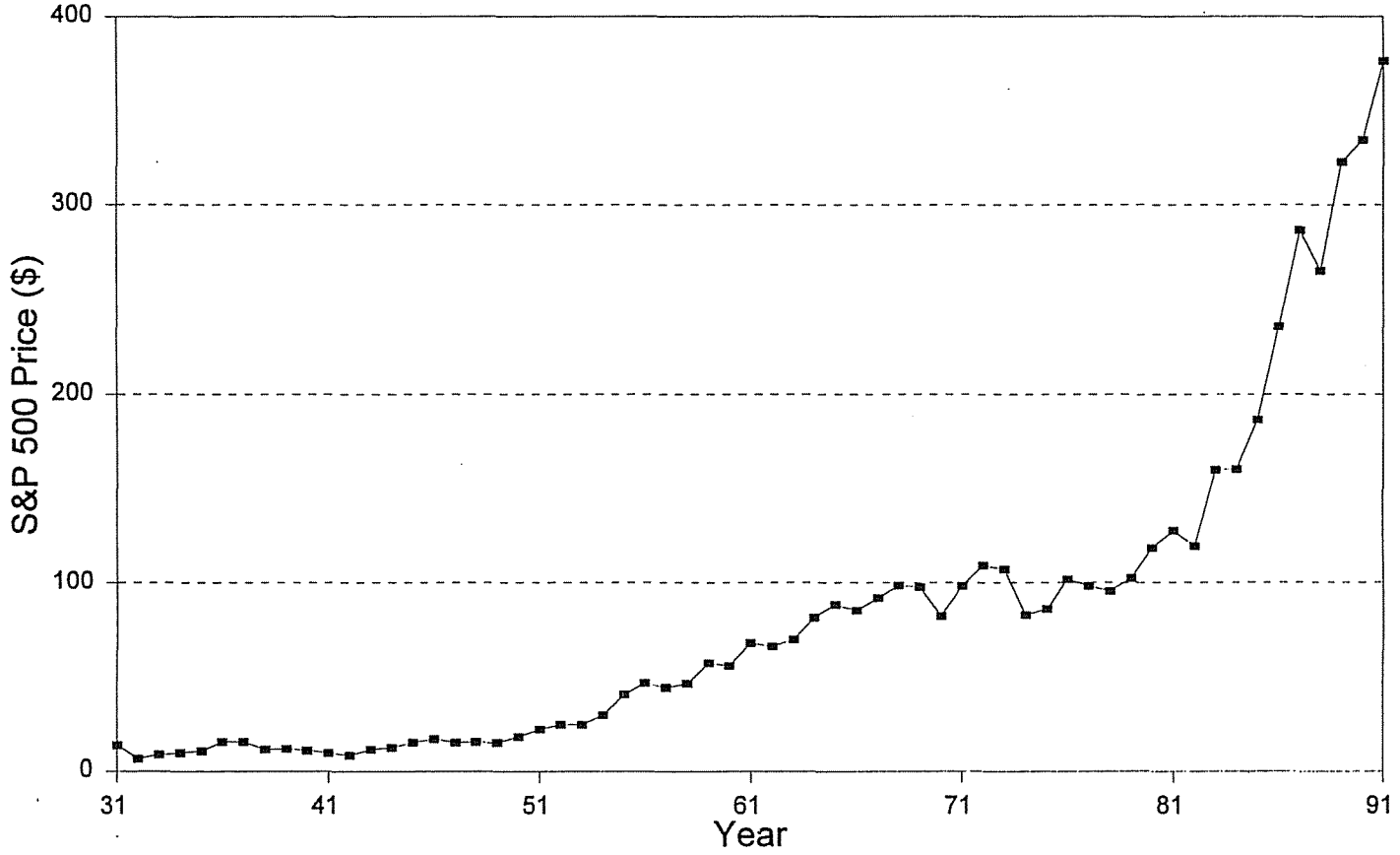
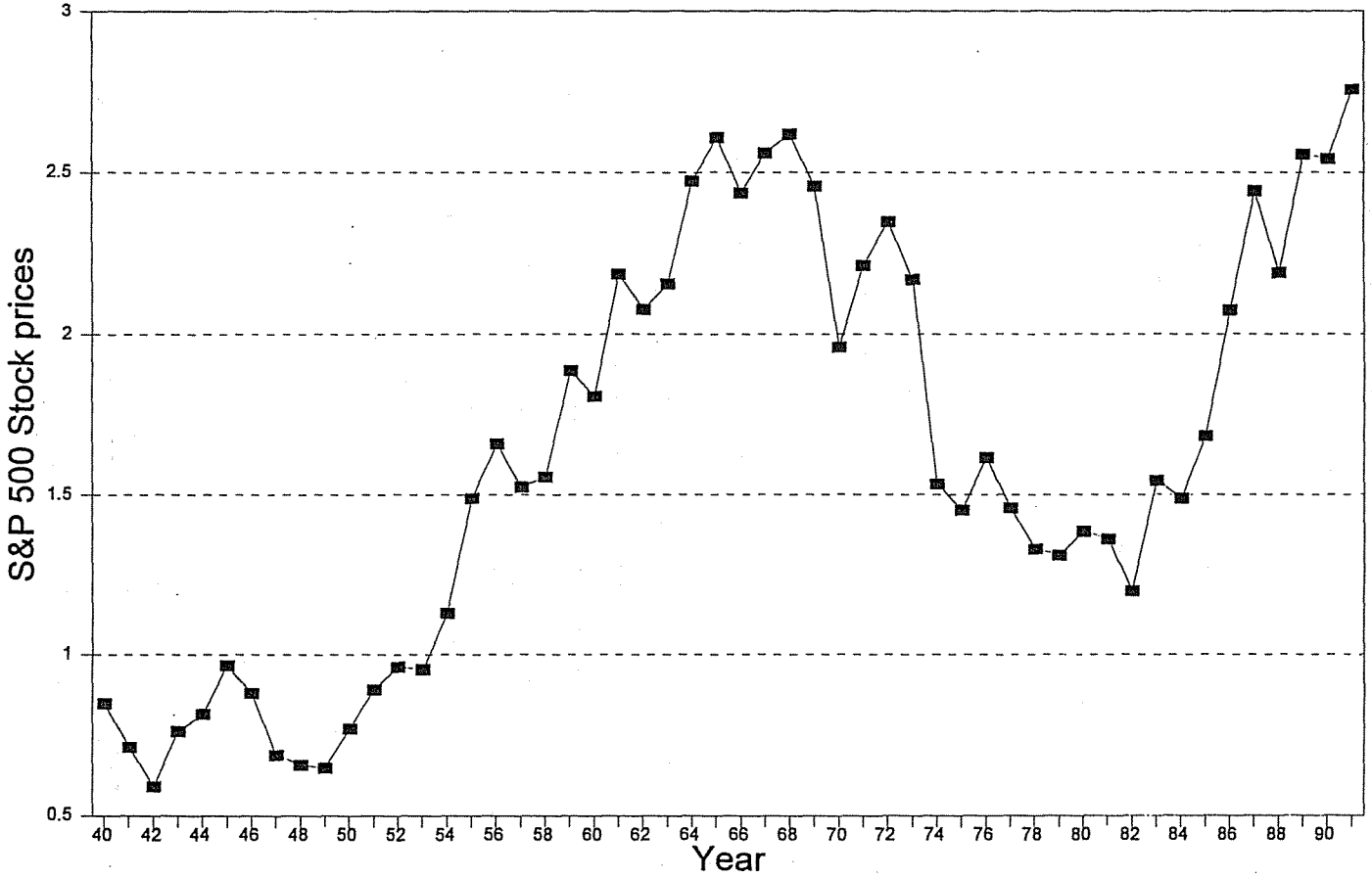


Figure 8. S&P 500 Prices 1940-1991
Inflation Adjusted (1982=100)



Method 1

Based on the earlier discussion in this Chapter, the movement of stock price in real terms is assumed to be a random walk, and the expected stock price in nominal terms is therefore the past stock price plus the expected inflation rate. In this model, it is assumed that the expected inflation rate is formed in such a way that people adjust their expectation according to the past inflation.

Suppose:

S'_t = Nominal stock price at time t,

π_t = Inflation rate at given time t,

$E(S'_t)$ = Expected nominal stock price at time t,

$E(\pi_t)$ = Expected inflation at time t,

that:

$$E(S'_t) = S'_{t-1}*(1 + E(\pi_t)) \tag{12}$$

and

$$E(\pi_t) = a + b\pi_{t-1} \tag{13}$$

Taking the natural logarithm of both sides:

$$\Rightarrow \quad \text{Ln } E(S'_t) = \text{Ln } S'_{t-1} + \text{Ln}(1 + E(\pi_t)) \tag{12*}$$

$$\text{Ln } S'_t = \text{Ln } S'_{t-1} + b*\text{Ln}(1 + E(\pi_t)) + \varepsilon_t$$

$$\varepsilon \sim \text{NID}(0, \sigma^2)$$

and $\Rightarrow$

$$\pi_t = a + b*\pi_{t-1} + \mu_t \tag{13*}$$

$$\mu \sim \text{NID}(0, \sigma^2)$$

Equations (12*) and (13*) were estimated by a two-stage process. The predicted value (π_t^*) was used for $E(\pi)$ in estimating (12*), with a restriction of 1 on the coefficient of Ln S'_{t-1}. The OLS regression results are:

$$\pi_t^* = \underset{(3.033)}{0.023} + \underset{(4.061)}{0.505}\pi_{t-1} \qquad (13^{**})$$

$N = 50, \quad R^2 = 0.2557, \quad F = 16.5, \quad Dh = 1.56.$

and $$\text{Ln } S'_t = \text{Ln } S'_{t-1} + \underset{(2.812)}{1.06} \text{ Ln } (1 + \pi_t^*) \qquad (12^{**})$$

$N = 50, \quad R^2 = 0.9813, \quad F = 52152, \quad DW = 1.521.$

Second-order serial correlation is not significant at the 1% level in either of the regressions. The actual and predicted inflation rates, (π_t and π_t^*), are shown in Figure 9. The predicted inflation rate for 1992 is 0.0429. The estimated equation (12**) will be used in calculating the expected value of S&P 500 on the expiration date.

$$E(S'_i) = \exp(\text{Ln } S_i + 1.061 \text{ Ln}(1 + 0.0429*N_i/365)),$$

where$E(S'_i)$ is in nominal terms.

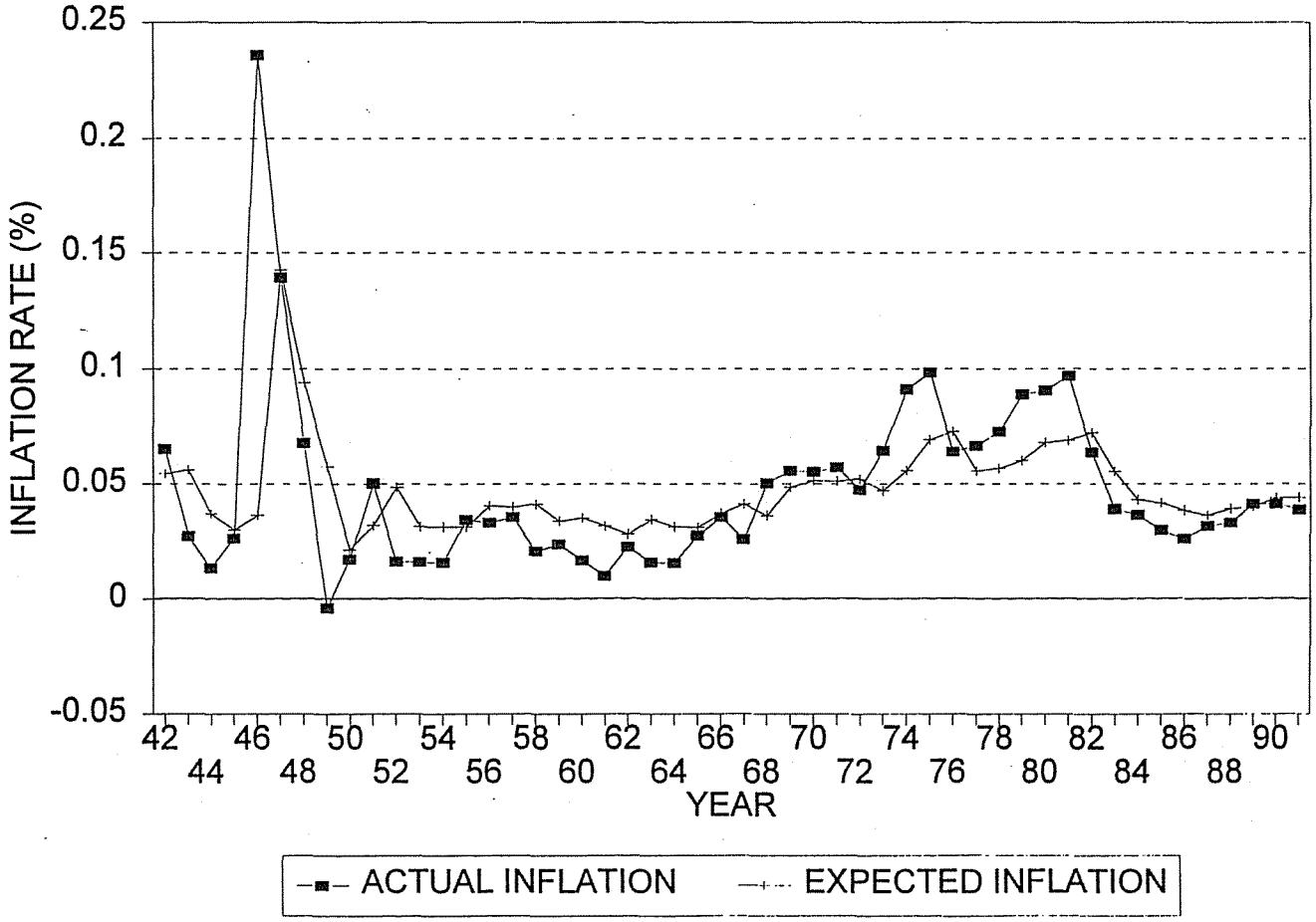
Method 2

Since $E(S_t \mid S_{t-1}, S_{t-2},...) = E(S_t \mid S_{t-1})$, the expected real price S_t is formed upon S_{t-1} only. Then:

$$S_t = a + b*S_{t-1} + \varepsilon_t \qquad (14)$$

$$\varepsilon_t \sim \text{NID } (0, \sigma^2),$$

FIGURE 9. ACTUAL VS EXPECTED INFLATION
1942-1991



where a is expected to equal 0 and b is expected to equal 1. The OLS results (with t-values in parentheses) are:

$$S_t = \underset{(1.232)}{10.132} + \underset{(20.1)}{0.960}S_{t-1} \qquad (14^*)$$

$N = 51, \qquad R^2 = 0.8919, \qquad F = 404.1, \qquad Dh = 0.441.$

Second-order autocorrelation is not significant. Although a minimum drift can't be significantly rejected from this regression, the empirical form (14*) was used to compute the expected stock price.

Method 3 and 4, Estimating a Specific Drift in Real Terms

If the investor expects stock prices to drift, then the drift will affect the stock price and thereby the expected rate of return in Portfolio 2 as:

$$R2 = \frac{2E(S) - X + D}{S - P + C} = \frac{S(1 + w*N/365) - X + D}{S - P + C}, \qquad (15)$$

where w is the real, annual drift, and S is S&P 500 stock price at current time.

Following are two methods of estimating the drift, w.

Method 3 (Arithmetic Market Price Appreciation Rate). From the historical data of the S&P 500 (1940-1991), the average annual nominal market appreciation is 7.98% and

the average annual inflation rate is 4.78%, and so the average real price appreciation rate is 3.28%.

Method 4 (Geometric Market Price Appreciation Rate). Suppose a long-run price trend is an unbiased estimate of the expected drift. From historical data, during the 50-year-period from 1941 through 1991, the S&P 500 stock price appreciated in nominal terms at a compounding annual rate of 7.6%, while the overall price index (using the GNP implicit deflator) increased at a compounding annual rate of 4.7%. The real capital gain, which is net of inflation, is therefore 2.9%.

CHAPTER 5

DATA

The two sets of data in this thesis are discussed below.

The First Data Set

The first set is the data used in testing for the random walk and estimating the expected appreciation in S&P 500 stock price. The S&P 500 yearly average stock prices from 1940 through 1991 were collected from the 1994 Price Index Security Record published by Standard and Poor's Statistical Reporting Service. The annual price index, which is used to deflate nominal stock price into real terms, was obtained from the Gross National Product (GNP) implicit deflator in the Economic Report of the President, 1992. The price index for 1992 was constructed from a Gross Domestic Product (GDP) index because the GNP index was not found

The Second Data Set

The second data set is used to estimate the risk premium by comparing the two portfolios constructed from S&P put and call options. The S&P 500 option was used

because it is a European type option, and thus the put-call parity (PCP) strictly holds (Merton 1973b).

The data are from January 2, 1992 through June 30, 1992. There are 148,192 observations and 44,704 observations respectively, in the original cash price and option price data. The data were purchased from Future Industry Institute, located in Washington, DC. Because the large data set was cumbersome, it was compressed by using the daily average value as an observation for each variable. This reduced the number of observations to 935.

The Treasury Bill (T-Bill) rate was used as the risk-free interest rate. Daily T-Bill rates were collected from the Wall Street Journal. For each option type, the Treasury Bill rate whose maturity date was closest to the option's expiration date was used.

The dividend payments for each business day were also collected from the Wall Street Journal. From the daily data, the future value (at maturity day) of the accumulative dividends was calculated using the T-Bill rate as the discount rate.

The data set analyzed contained 935 observations on the following variables: current date, type of contract, expiration date, exercise price (X), future accumulated dividends (D), call price (C), put price (P), current stock price (S), actual stock price at maturity (S'), number of days before expiration (N), and T-Bill rate (TBILL). Those data were sorted first by date, then by the type of contract (first by maturity day, then by strike price). The first 935 observations are Portfolio 2 and the second 935 observations are Portfolio 1.

A sample of the data (first 50 observations) is included in appendix B . The actual inflation rate in 1992 used to deflate returns was 2.78%, which was calculated from the GDP implicit price index (U.S. Congress, p272).

CHAPTER 6

EMPIRICAL FINDINGS

OLS Estimation

E(S′) were calculated by the four methods discussed in chapter 5. Table 3 summarizes the four estimation methods. R1 and R2 in real terms were obtained using the following formula (16). π is inflation adjustment factor.

Table 3. Summary of Four Estimation Methods Used to Estimate E(S′).

Method No.	Brief Description	Estimated E(S′)
1	Expected nominal stock price is based upon past stock price plus expected inflation	equation (12**)
2	Real stock price is formed only on past real stock price	equation (14*)
3	Real, long-run arithmetic market price appreciation	drift (w = 3.28%)
4	Real, long-run geometric market price appreciation	drift (w = 2.9%)

$$\pi = 1 + 0.0278 * \frac{n}{365} \qquad (16)$$

$$R1 = \frac{(X + P) / \pi}{S + P - C}$$

$$R2 = \frac{2E(S) + (D - X) / \pi}{S - P + C}, \quad E(S) = S(1 + w * N / 365),$$

or using method 1 to calculate the expected return:

$$R2 = \frac{(2E(S) + D - X) / \pi}{S - P + C}$$

The plots of R1 or expected R2 using different methods of estimating E(S'), against N (maturity period) are shown in Figures 10-14. Both R1 and R2 are in real terms. All of the five plots show a consistent and strong relationship between returns and the time period.

The OLS results of the statistical model [equation (9) in chapter 3], using 1,870 observations are summarized in Table 4.

All of the coefficients are significant at the 1% level. Durbin-Watson statistics (DW) show significant serial correlation in each method. As expected (Chapter 3), the intercept coefficient is not significantly different from 1 (with the exception of method 2). The

Figure 10. Plot of R1 Against N
In Real Terms, 935 Observations

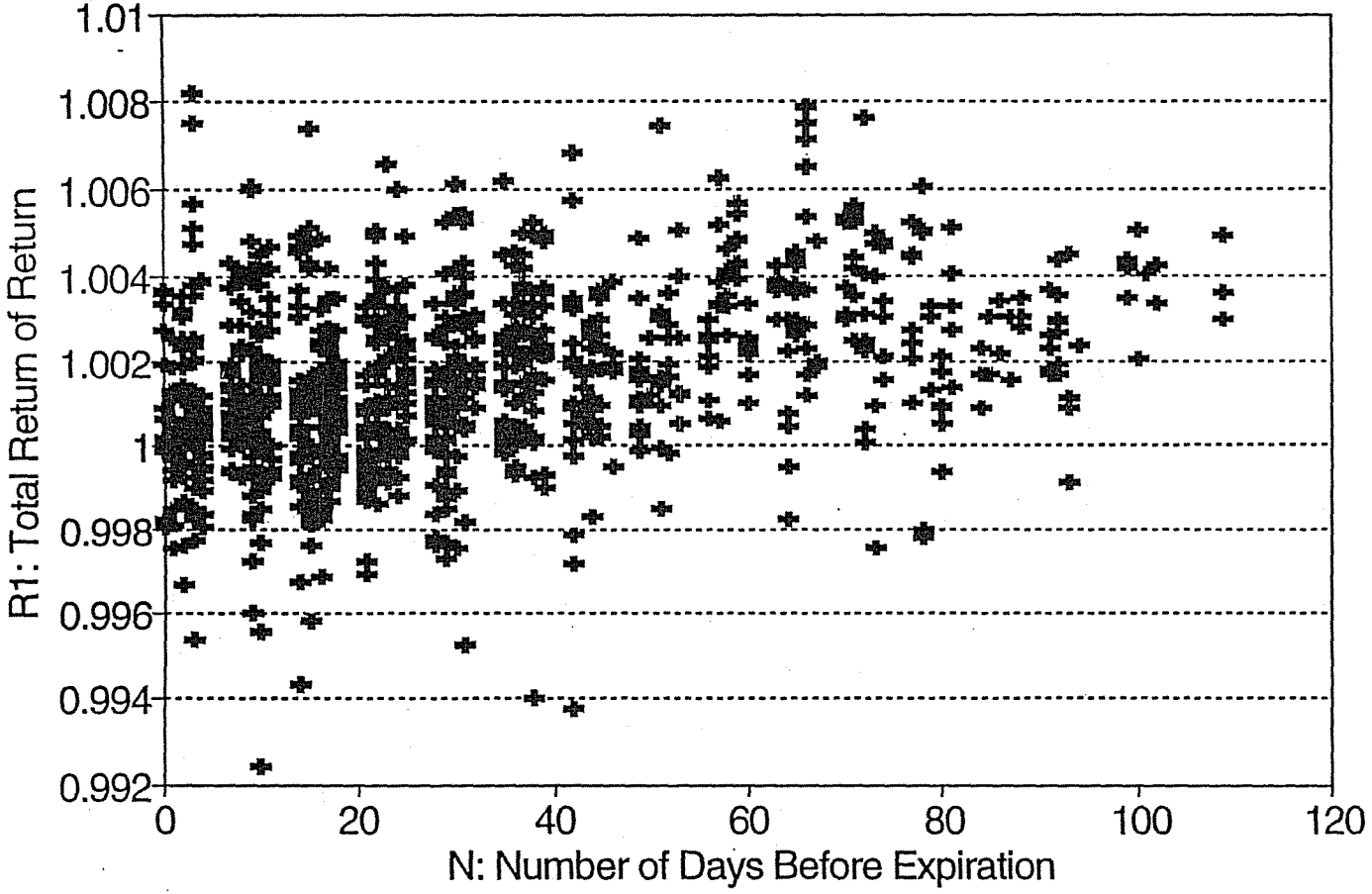


Figure 11. Plot of R2 Against N
Method 1, In Real Terms, 935 Obs

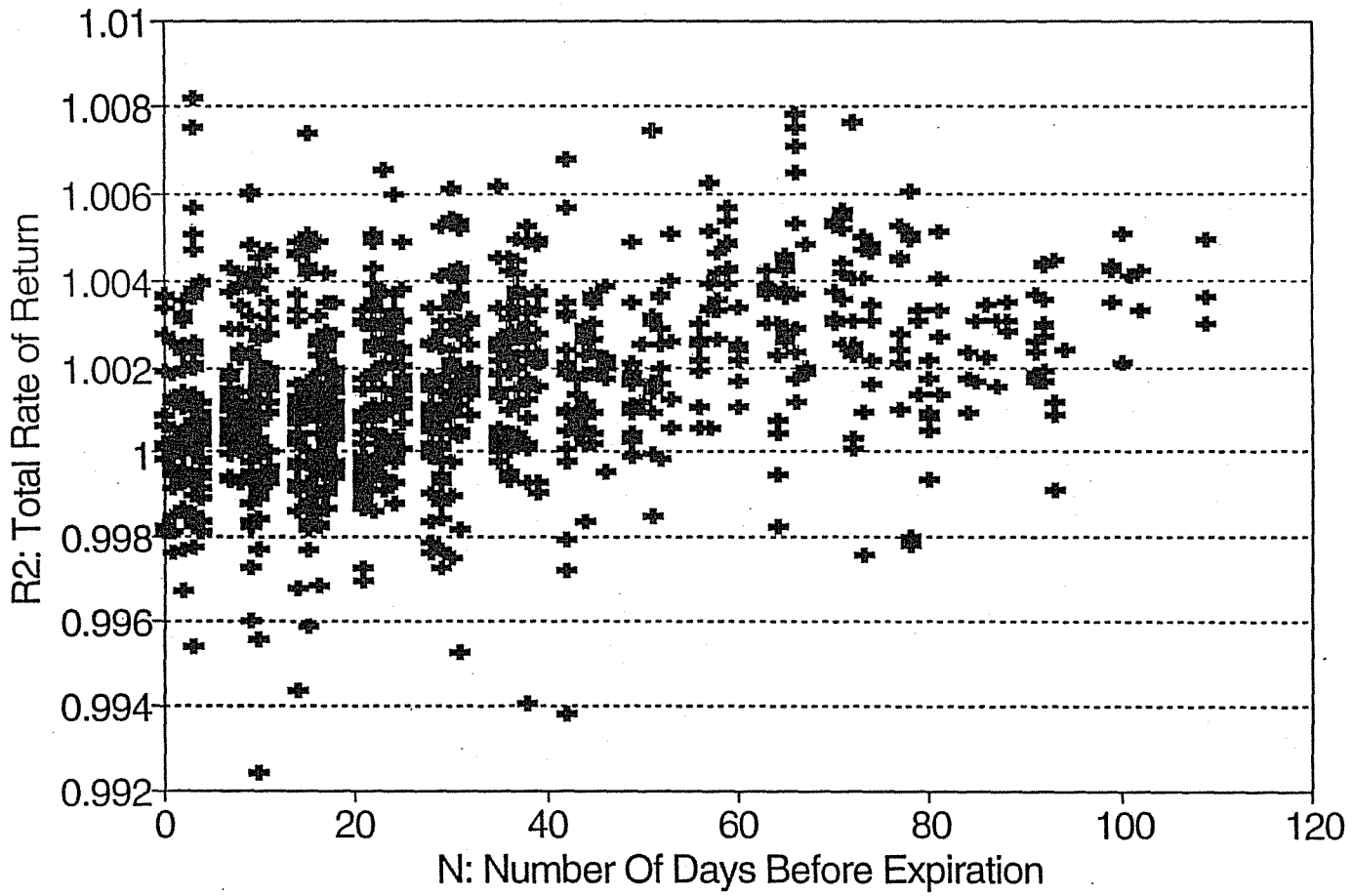


Figure 12. Plot of R2 Against N
Method 2, In Real Terms, 935 Obs

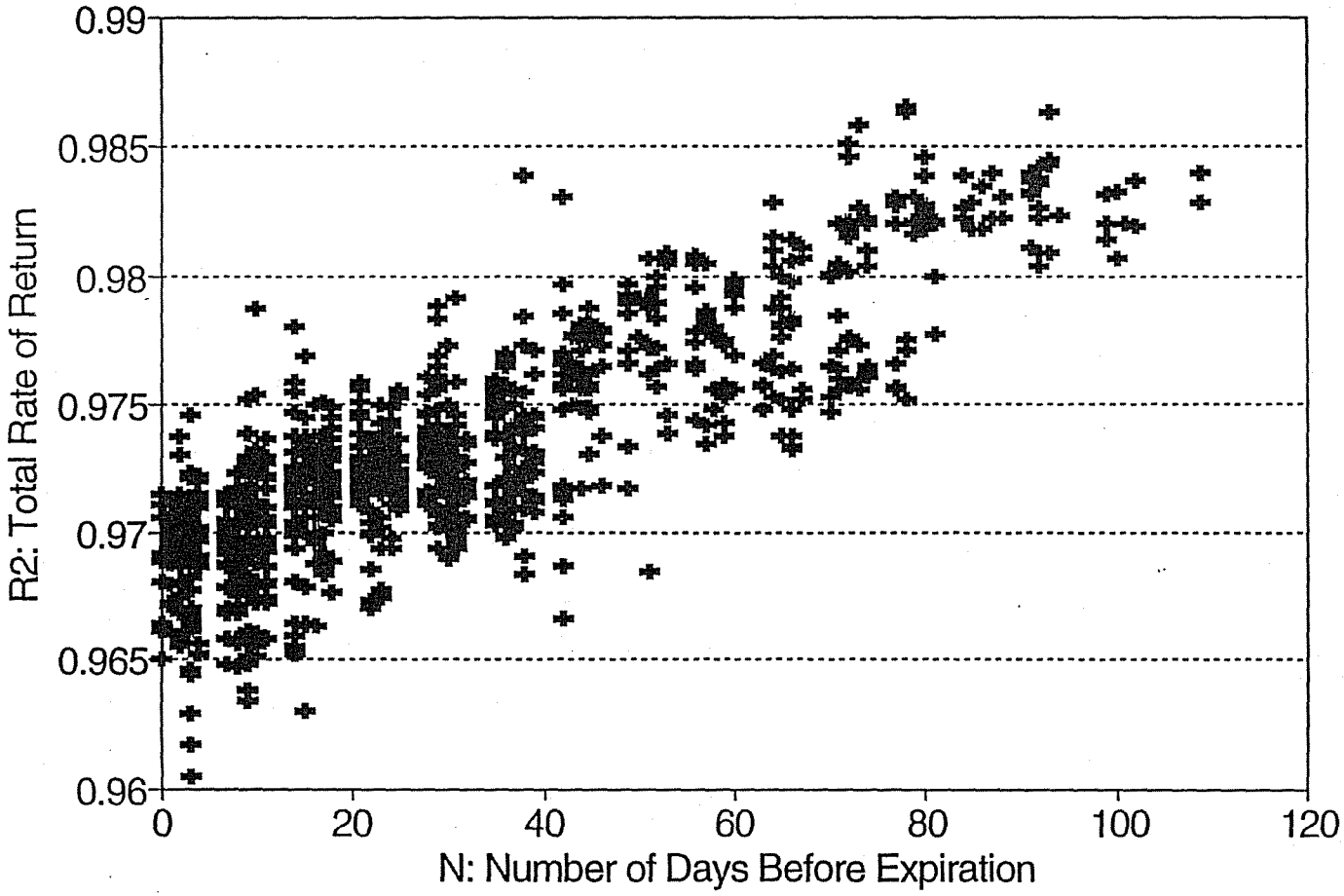


Figure 13. Plot of R2 Against N
Method 3, (3.28% Drift), In Real Terms

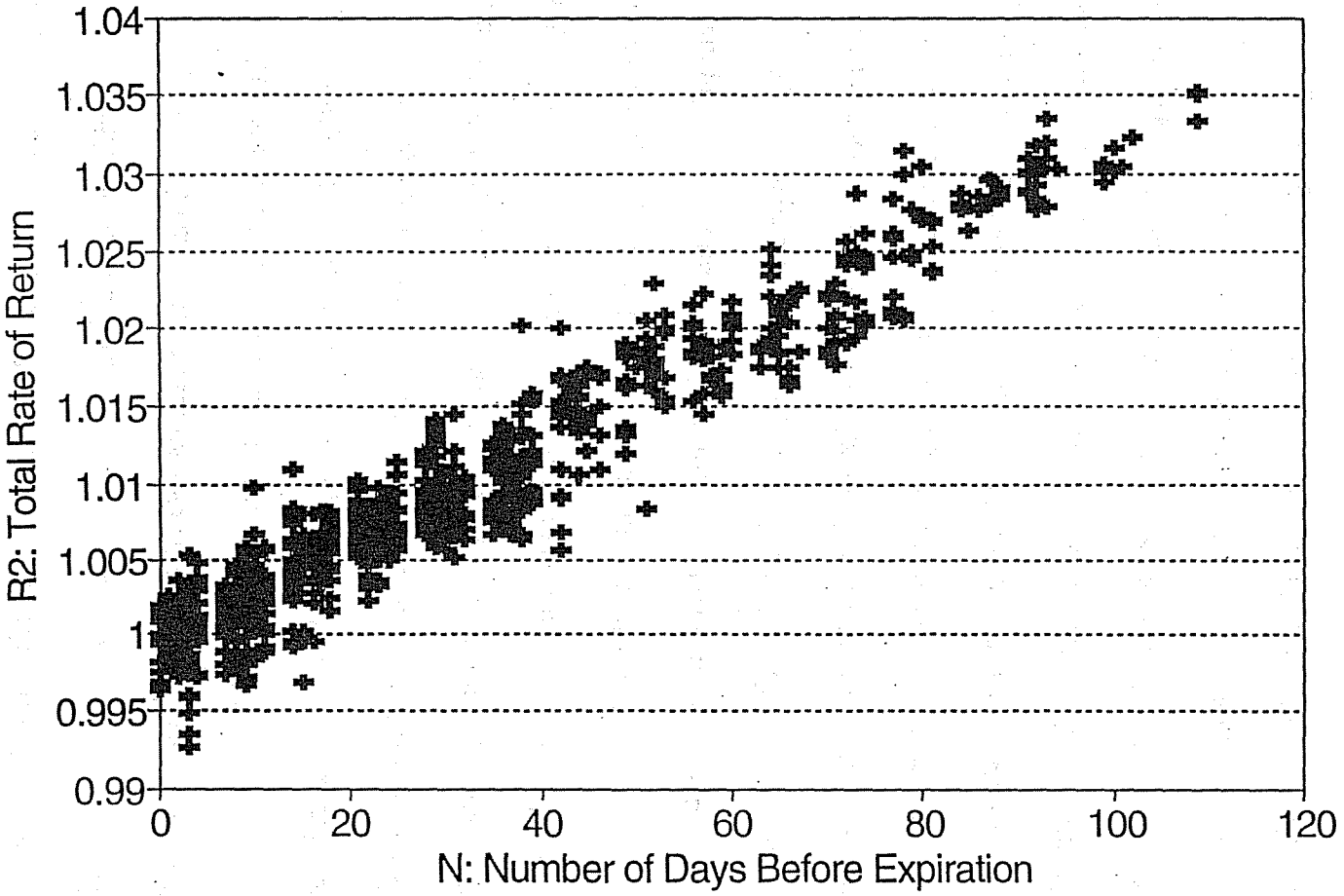
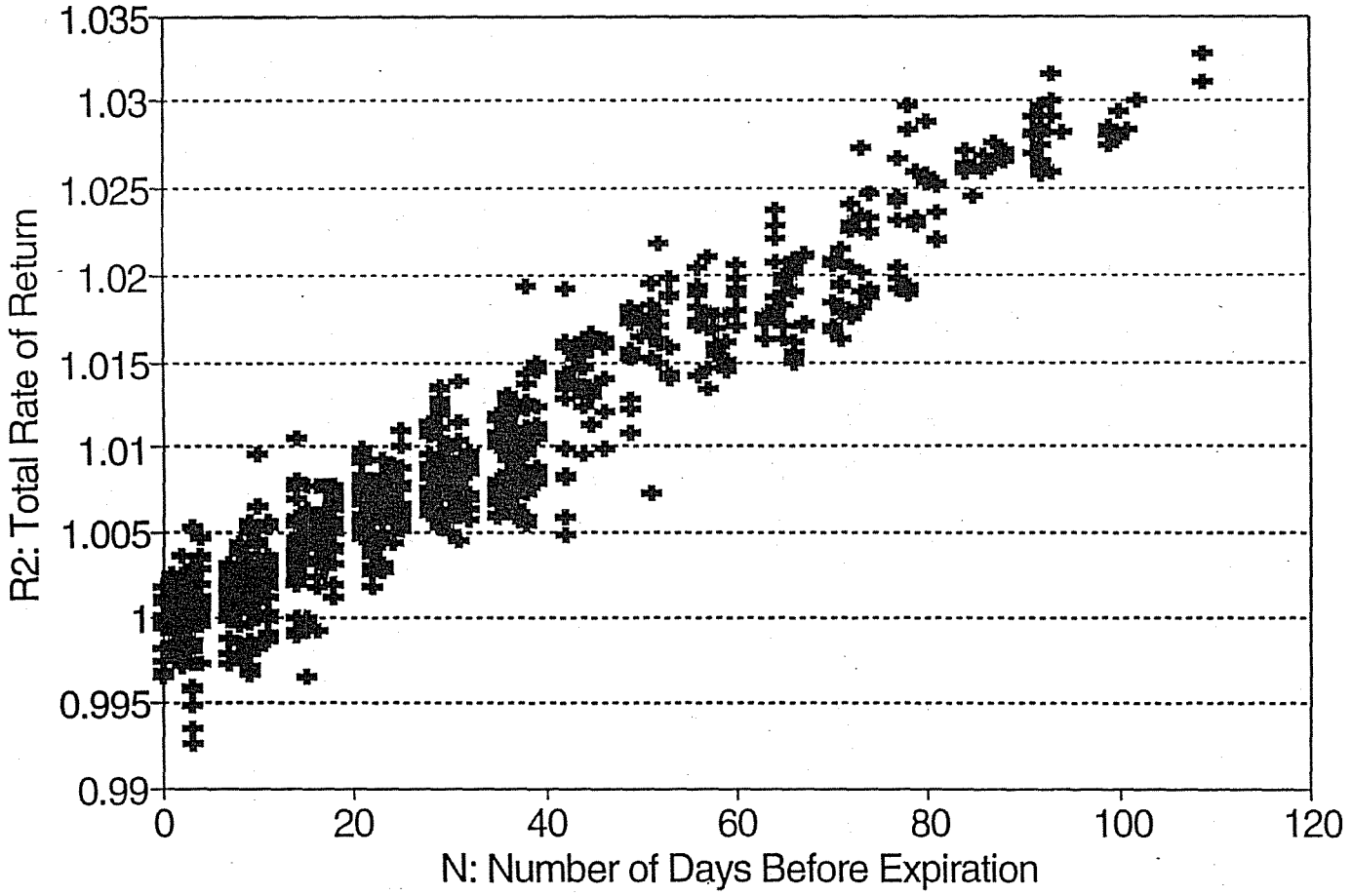


Figure 14. Plot of R2 Against N
METHOD 4 (2.9% Drift), In Real Terms



coefficient on N/365, R_f + TC, is around 1.6%. The estimated risk premium is significant with a range from 3% to 5% (method 2 produces a negative premium).

Method 2 produces a negative premium and an intercept less than 1 because the model [equation (14)] used to predict the expected price is simple and open to over- or under-forecasting from year to year. It generally undervalues the expected price in 1992. The rate of return and risk premium (Rp) using method 2 is therefore underestimated.

Table 4. OLS Estimation of Coefficients in Equation (9), Using 935 Observations.

Method	Intercept	R_f+TC	R_p	R^2	F	RHO	DW
1	0.99997 (0.00008)	0.0159 (0.0009)	0.034 (0.0005)	0.8474	5198.7	0.424	1.15
2	0.98465 (0.00004)	0.127 (0.0041)	-0.094 (0.0021)	0.5184	1004.7	0.694	0.611
3	0.99997 (0.00008)	0.0159 (0.0009)	0.049 (0.0005)	0.9140	9918.4	0.611	0.775
4	0.99997 (0.00008)	0.0159 (0.0009)	0.046 (0.0005)	0.9020	8591.8	0.611	0.774

(DW is Durbin-Watson Statistics. RHO is estimated first-order auto-correlation coefficient in error terms. Standard deviations are in parenthesis.)

Table 5. Statistics Summary of The Treasury Bill Rate, January - June, 1992.

Mean	Standard Deviation	Maximum	Minimum
3.68%	0.22%	3.07%	4.17%

The T-Bill rate was not employed as an explanatory variable in this thesis because the testing period is short (from January to June, 1992) and the Treasury Bill rate is relatively stable in this period (see Table 5).

Autocorrelation and Adjusted Estimation

The strong RHO and DW found in Table 4 suggest a first-degree serial correlation in the residual of regressions.

One possibility might be that the return of one type of option is strongly, positively related with the return of another type of option on the same day. This is not serial correlation because there are several types of contracts (observations) within a day whose profits are all likely to be related to each other.

Another possibility might be a positive relationship between the average return on one day (R_t) and the average return on the day before (R_{t-1}).To more accurately test for autocorrelation, the 1,870 observations were sorted first by the date, then by the type of option (first by maturity date, second by striking price). The OLS results are:

$$R_t = 0.002 + 0.545\, R_{t-1} \qquad (17)$$
$$(5.318)\ \ (7.331)$$

$$N = 125,\ \ R^2 = 0.3041,\ \ F = 53.8,\ \ Dh = 1.403.$$

where 125 is total business day in the first half of 1992. There is no second-degree autocorrelation in the error term. The coefficient on R_{t-1} is different from 0 at the 1% significance level.

Since autocorrelation exists in the returns of consecutive days, OLS estimation is, though unbiased, not efficient. To correct for the serial correlation, a uniform time-series cross-sectional pooled data set was constructed in which each cross-section contained a time series of 23 observations. Each observation within a cross-section has the same strike price and maturity date. From the original 935 observations, 19 cross-sections, each containing 23 observations (a total of 437 observations) were used as a new data set to examine the influence of the serial correlated errors. The total number of observations available for the regression is 874. Details of the data processing are provided in Appendix A, Note 3.

Table 6 summarizes OLS results of the statistical model [equation (9)], using the second set of data (874 observations). The results are very close to those estimated for the entire sample in Table 4, indicating little effect from truncating the data set. The RHO in Table 6 is generally lower than that in Table 4 because the RHO in Table 6 only accounts for correlation between the days.

Assuming a constant autocorrelation coefficient across types of contract (cross-section), the statistical form of the model (Kmenta, 1986, 616) is:

$$R_{it} = a + b*N_{it}/365 + C*D*N_{it}/365 + \varepsilon_{it}, \qquad (18)$$

where

$R_{it} = R1$, if $D = 0$

$R_{it} = R2$, if $D = 2$

$E(\varepsilon_{it}^2) = \sigma_i^2, \qquad E(\varepsilon_{it}, \varepsilon_{is}) = \rho^{t-s}\sigma_i^2, \qquad (t > s).$

$E(\varepsilon_{it}, \varepsilon_{jt}) = 0, \qquad E(\varepsilon_{it}, \varepsilon_{js}) = 0, \qquad (i \neq j).$

Table 6. OLS Estimation of Coefficients in Equation (9), Using Cross-sectional Time-Series Data (874 Observations).

Method	Intercept	R_f+TC	R_p	R^2	F	RHO	DW
1	1.0000 (0.0001)	0.0162 (0.0013)	0.035 (0.0006)	0.8682	2869.3	0.469	1.058
2	0.98471 (0.0006)	0.130 (0.005)	-0.096 (0.0025)	0.6418	780.3	0.900	0.203
3	1.0000 (0.0001)	0.0163 (0.0013)	0.050 (0.0006)	0.9269	5523.6	0.467	1.063
4	1.0000 (0.0001)	0.0163 (0.0013)	0.046 (0.0006)	0.9164	4775.9	0.467	1.063

(Standard deviations are in parentheses.)

The statistical results after adjusting for cross-sectional heteroskedasticity and time-series autoregression are presented in Table 7. The estimated RHO for each method is significant and similar to those reported in Tables 4 and 5. R_f + TC is estimated to be approximately 1.5%. Since the real riskless treasury rate during the first half of 1992 averages 0.9% (the average nominal T-Bill rate in 1992 is 3.68% as shown in Table 5 and the expected inflation rate in 1992 is 2.78%), the estimated transaction costs plus information costs equal 0.6% (1.5% - 0.9%). Since the T-Bill rate of 3.68% does not include any implied costs such as commissions, and so the real opportunity cost must be smaller than 3.68%, the estimated transaction costs plus information costs actually exceed 0.6%. This number coincides with the fee charged by most financial companies. For

example, T. Rowe Price, a mutual fund which does not charge commission fees, spends about 0.35% of its total $34.7 billion assets on operating costs, which is an approximation of the minimized underlying costs. The commission rate, if charged by a stock broker, ranges from less than 1% to several points, depending on total investment, type of services and stocks. Kennedy, Cabots & Company, for instance, charges $30 for buying or selling 600 shares of stock and $100 for 5,000 shares of stock, which equals commissions of 0.2% and 0.5%, respectively, when the stock price is $10/share.

Table 7. Adjusted Estimation of Coefficients in Equation (18), Using Cross-sectional Time-Series Data (874 Observations).

Method	Intercept	R_f+TC	R_p	R^2	F	RHO
1	1.0001 (0.0002)	0.0150 (0.0018)	0.035 (0.002)	0.8680	2863.7	0.469
2	0.98425 (0.002)	0.0202 (0.012)	0.012 (0.008)	0.1140	56.1	0.992
3	1.0001 (0.0002)	0.0150 (0.0018)	0.051 (0.0008)	0.9268	5514.0	0.468
4	1.0001 (0.0002)	0.0150 (0.0018)	0.047 (0.0008)	0.9163	4767.6	0.468

(Standard deviations are in parentheses. R^2 and F were calculated by the untransformed results.)

In this thesis, the risk premium is estimated to be from 3.5% to 5.1% [depending on the method used to calculate E(S')], and is even as low as 1.6% (method 2). This result is

305) estimated that the risk premium during the 1980s was about 4 to 6 percentage points. Van Horne (1992, p. 74) concluded that the expected market risk premium has ranged from 3 to 7 % in recent years.

Heteroskedasiticity

Heteroskedasticity was previously suspected in this model because the variance of returns from the two portfolios could possibly increase with N, the time period before maturity. However, as shown in Figures 10-14, the variance of returns seems to have no significant relationship with the number of days to maturity. Using White's test, in which N is treated as an explanatory variable, heteroskedasticity was found to be insignificant (details of testing is in Appendix A, Note 4).

Summary of Empirical Results

The empirical results of this study are summarized below.

(1) The high R^2 and F value indicate that the proposed model, based on the put-call parity, fits well overall. The estimated opportunity costs plus transaction costs and information costs (R_f + TC), is significant and at least 1.5% (autocorrelation-adjusted estimation). Transaction costs and information costs are estimated to be at least 0.6% in the financial markets.

(2) The estimated risk premium varies from 3.5% to 5.1%, except in method 2 which estimates the risk premium at 1.6%. These results are similar to the risk premium estimated in most of the literature.

(3) The estimated correlation of today's average returns with yesterday's average returns is 0.5 (Equation 17 and Table 7). About half of the autocorrelation can be explained by the data averaging process (see Working 1960), leaving the other half unexplainable. If the market is efficient and provides no arbitrage opportunities, especially over a longer period, this finding implies a possible misspecification in this model. However, if the possible misspecification affects both portfolios similarly, the estimated risk premium may still be reliable.

(4) White's test does not show strong heteroskedasticity when N, the time until maturity, is used to explain the variance.

(5) Historical yearly average S&P 500 price (1940-1991) shows that random walk holds in real terms (refer to Chapter 4).

CHAPTER 7

CONCLUSIONS AND REMARKS

The CAPM model begins with the premise that all investors are risk averse and that risk is not escapable. Previous estimates of risk premium have not been separated from the transaction costs, information costs, and default costs. After adjusting for the transaction and information costs, risk premium (R_p) ranges from 3.6% to 5.1%.

The empirical estimation of R_p is based on how stock price expectation was formed. However, the choice of method used does not strongly affect the conclusion reached. Table 8 summarizes the results of risk premium per unit of beta estimated by Proposed model and by the CAPM model, under method 1,3 and 4.

Table 8. Comparison of R_p by Proposed Model and R_p Estimated by the CAPM Model.

Method No.	This model R_p (%)	Std. Dev. (%)	CAPM R_p (%)	Difference (%)
1	3.4	0.05	4.00	-0.60
3	4.9	0.08	5.52	-0.62
4	4.6	0.08	5.14	-0.54

(Those estimates are the means of the total 935 observations adjusted to an annual rate, without considering any autocorrelation.)

The difference between the risk premium estimated by this model and that estimated by the CAPM model is significant and stable, though small, regardless of which method is used to estimate the expected return (see mathematics discussion in appendix A, Note 5). Theoretically speaking, the difference in risk premium between the two models should be equal to the summation of transaction and information costs. The transaction and information costs estimated in this study are about 0.6% (Chapter 6), which is very close to the differences shown in Table 8.

The costs associated with Portfolios 1 and 2 are not strictly identical because Portfolio 1 does not contain any default risks and default costs. The estimated risk premium is therefore larger than the theoretical or pure risk premium because it picks up the default costs existing in R2 only. Since dividends account for a large portion of total returns (about 2/3 in put-call parity) and are not strictly constant, the beta of Portfolio 2 is not strictly equal to 2 (Refer to Appendix A, Note 1). Its variance, as well as correlation with the market appreciation, needs further research.

Another limitation of this thesis is availability of the data. Although long-term data do not necessarily eliminate the need for a stock price expectation model, the actual returns also could be compared because both realized returns and risk premium are fairly easy to calculate.

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APPENDICES

APPENDIX A

NOTES

(1) Quantity of risk is defined as systematic risk (beta), β_i:

$$\beta_i = \frac{COV(R_i, R_m)}{VAR(R_m)}. \qquad (19)$$

It is the covariance between returns on the risky assets (I) and the market portfolio (M), divided by the variance of the market portfolio returns.

Portfolio 1 has a beta of zero because its covariance with the market portfolio is zero. The beta of S&P 500 would be one because the covariance of the market portfolio with itself is identical to the variance of the market portfolio, using the assumption in Chapter 3.

$$\beta_m = \frac{COV(R_m, R_m)}{VAR(R_m)} = \frac{VAR(R_m)}{VAR(R_m)} = 1. \qquad (20)$$

Since Portfolio 2 ends up with twice the market value of stock minus a constant (X), the change of return, when stock price S' varies, will be near twice the change of return of the market portfolio. Let:

$$R_2 = \frac{2S' - X + D}{S - P + C}$$

$$R_m = \frac{S' + D}{S}$$

$$\beta = \frac{COV(R_2,\ R_m)}{VAR(R_m)} = \frac{COV(\frac{2S' - X + D}{S - P + C},\ \frac{S' + D}{S})}{VAR(R_m)}$$

$$= \frac{COV(\frac{2S'}{S - P + C} + \frac{D - X}{S - P + C},\ \frac{S'}{S} + \frac{D}{S})}{COV(\frac{S' + D}{S},\ \frac{S' + D}{S})} = \frac{COV(\frac{2S'}{S - P + C},\ \frac{S'}{S})}{COV(\frac{S'}{S},\ \frac{S'}{S})}$$

$$= \frac{COV(\frac{2S'}{S - P + C} + \frac{D - X}{S - P + C},\ \frac{S'}{S} + \frac{D}{S})}{COV(\frac{S' + D}{S},\ \frac{S' + D}{S})} = \frac{COV(\frac{2S'}{S - P + C},\ \frac{S'}{S})}{COV(\frac{S'}{S},\ \frac{S'}{S})}.$$

(2) The mathematical proof of R1 > R2 with zero drift is shown below:

$$R1 = \frac{X + D}{S + P - C},$$

$$R2 = \frac{2E(S) - X + D}{S - P + C} = \frac{2S - X + D}{S - P + C}.$$

If R1 > R2,

$$-\frac{X + D}{S + P - C} > \frac{2S - X + D}{S - P + C}$$

$$\Leftrightarrow XS - XP + XC + DS - DP + DC > 2S^2 + 2SP - 2SC - XS - XP + XC + DS + DP - DC$$

$$\Leftrightarrow 2XS + 2DC - 2DP > 2S^2 + 2SP - 2SC$$

$$\Leftrightarrow S(X - S - P + C) + D(C - P) > 0$$

It is known that S + P - C (at time 0) X + D (at time t), and return from PCP is more than dividends. (As shown in Table 9, dividend yield accounts for about 2/3 of total return, and market appreciation accounts for 1/3 of total return.)

Table 9. Composition of Annual Return of PCP Based on Thesis Data

Total Yield	Market Appreciation Yield	Dividends Yield
4.97% (100%)	1.83% (37%)	3.14% (63%)

(Shares of the total yield are in parentheses.)

Since, on average, $X > S + P - C$, and on average, $X = S$ (random walk), then $P - C < 0$. Therefore, on average:

$$X - S - P + C > 0 \text{ and } C - P > 0$$

$$\Leftrightarrow S(X - S - P + C) + D(C - P) > 0.$$

So R1 > R2 with zero drift (on the average).

(3) The processing of the second set of data (a final total of 437 observations) was done as follows:

First, the data were sorted by types of option contract (first by maturity, then by exercise price). Second, those observations discontinuous in time series were thrown away, reducing the total number of observations to 873.

Next, to keep cross-section unit (type of contract) having 23 time-series data was found to retain the maximum number of observations. Those cross-sections with less than 23 time-series observations were deleted and those with more than 23 observations were cut to exactly 23. (The observations with the nearest expiration days were thrown out because their returns were suspected to be more volatile.) The final data then consisted of 19 cross-sections with 23 time-series observations in each (a total of 437 observations).

(4) Testing Heteroskedasticity:

$$R = a + b*N/365 + c*D*N/365 + \varepsilon_t, \qquad (21)$$

$$E(\varepsilon_t^2) = \sigma_i^2.$$

$$H_0: \ \sigma_i^2 = \sigma^2$$

$$H_1: \ \sigma_i^2 \neq \sigma^2$$

The first step was to run this regression (OLS) and to calculate the residual (ε). Then ε^2 (residual square) was regressed on a constant, N and N^2, from which R^2 was obtained. The last step was to use White's test:

$$n*R^2 = 1{,}870*0.0001 = 0.187 < \lambda^2(0.95) = 5.99.$$

Therefore, H_0 failed to be rejected.

(5) Suppose the stock price is expected to increase at w%. Theoretically, the risk premium estimated by this model should be the difference between R1 and R2 with zero drift (a) plus w. The risk premium estimated by the CAPM model should be annual dividend yield (DY) plus w, minus nominal risk-free rate (T-Bill) (assume underlying stock is a proxy for market portfolio).

Comparing:

$$a + w >=< DY + w - \text{T-Bill},$$

w on both sides can be canceled:

$$a >=< DY - \text{T-Bill}.$$

Therefore, the difference between the risk premium estimated by this model and by the CAPM model is constant regardless of which method is used to estimate the expected return.

APPENDIX B

SAMPLE DATA

Variables: OBS, DATE, MATURITY, EXPIDATE, X, D, C, P, S, S', NDAYS, TBILL.

1 01/02/92 F92 01/17/92 390 0.32728 26.3000 0.7553 413.449 417.695 15 3.78
2 01/02/92 F92 01/17/92 400 0.32728 17.0000 1.7569 413.449 417.695 15 3.78
3 01/02/92 F92 01/17/92 410 0.32728 9.4136 3.8939 413.449 417.695 15 3.78
4 01/02/92 F92 01/17/92 415 0.32728 6.2406 5.8444 413.449 417.695 15 3.78
5 01/02/92 F92 01/17/92 420 0.32728 4.4556 7.7800 413.449 417.695 15 3.78
6 01/02/92 H92 03/20/92 400 2.57427 22.1000 6.8571 413.449 409.609 78 3.93
7 01/02/92 H92 03/20/92 405 2.57427 18.8500 8.5250 413.449 409.609 78 3.93
8 01/02/92 H92 03/20/92 420 2.57427 10.1714 14.2667 413.449 409.609 78 3.93
9 01/03/92 F92 01/17/92 410 0.31826 12.1667 2.4646 417.706 417.695 14 3.78
10 01/03/92 F92 01/17/92 415 0.31826 8.4438 3.6800 417.706 417.695 14 3.78
11 01/03/92 F92 01/17/92 420 0.31826 5.5804 5.6912 417.706 417.695 14 3.78
12 01/03/92 F92 01/17/92 430 0.31826 1.8545 12.4250 417.706 417.695 14 3.78
13 01/03/92 G92 02/21/92 420 1.66951 9.4500 9.8000 417.706 412.298 49 3.85
14 01/03/92 H92 03/20/92 415 2.56514 14.3500 9.9000 417.706 409.609 77 3.91
15 01/03/92 H92 03/20/92 420 2.56514 11.7200 11.9182 417.706 409.609 77 3.91
16 01/03/92 H92 03/20/92 430 2.56514 6.8375 17.2667 417.706 409.609 77 3.91
17 01/06/92 F92 01/17/92 410 0.31026 11.7714 2.2289 418.080 417.695 11 3.86
18 01/06/92 F92 01/17/92 415 0.31026 7.9750 3.5543 418.080 417.695 11 3.86
19 01/06/92 F92 01/17/92 420 0.31026 5.1682 5.2867 418.080 417.695 11 3.86
20 01/06/92 H92 03/20/92 410 2.55707 17.8000 7.9833 418.080 409.609 74 3.91
21 01/06/92 H92 03/20/92 415 2.55707 14.5000 9.7083 418.080 409.609 74 3.91
22 01/06/92 H92 03/20/92 420 2.55707 11.6000 11.7571 418.080 409.609 74 3.91
23 01/07/92 F92 01/17/92 410 0.13806 10.5500 2.2900 416.363 417.695 10 3.96
24 01/07/92 F92 01/17/92 415 0.13806 6.8091 3.5743 416.363 417.695 10 3.96
25 01/07/92 F92 01/17/92 420 0.13806 3.9174 5.7885 416.363 417.695 10 3.96
26 01/07/92 H92 03/20/92 415 2.38366 13.4500 10.2000 416.363 409.609 73 3.89
27 01/07/92 H92 03/20/92 420 2.38366 10.6250 12.6267 416.363 409.609 73 3.89
28 01/08/92 F92 01/17/92 410 0.12104 10.8000 1.7352 417.955 417.695 9 3.82
29 01/08/92 F92 01/17/92 415 0.12104 7.6455 2.9230 417.955 417.695 9 3.82
30 01/08/92 F92 01/17/92 420 0.12104 4.4455 4.6037 417.955 417.695 9 3.82
31 01/08/92 F92 01/17/92 425 0.12104 2.2868 7.5100 417.955 417.695 9 3.82
32 01/08/92 F92 01/17/92 430 0.12104 1.1441 10.9500 417.955 417.695 9 3.82
33 01/08/92 G92 02/21/92 420 1.47141 8.3000 11.1000 417.955 412.298 44 3.64
34 01/08/92 H92 03/20/92 390 2.36635 33.4667 4.1500 417.955 409.609 72 3.81
35 01/08/92 H92 03/20/92 420 2.36635 10.4500 11.6100 417.955 409.609 72 3.81
36 01/09/92 F92 01/17/92 415 0.12104 7.4500 2.6694 418.665 417.695 8 3.77
37 01/09/92 F92 01/17/92 420 0.12104 4.5708 4.3311 418.665 417.695 8 3.77
38 01/09/92 F92 01/17/92 425 0.12104 2.3138 6.8400 418.665 417.695 8 3.77

39 01/09/92 F92 01/17/92 430 0.12104 0.9929 11.1750 418.665 417.695 8 3.77
40 01/09/92 H92 03/20/92 390 2.36626 34.6500 4.1167 418.665 409.609 71 3.77
41 01/09/92 H92 03/20/92 410 2.36626 18.7000 8.3789 418.665 409.609 71 3.77
42 01/09/92 H92 03/20/92 420 2.36626 12.1750 11.8571 418.665 409.609 71 3.77
43 01/10/92 F92 01/17/92 410 0.09502 8.3417 2.0421 414.997 417.695 7 3.76
44 01/10/92 F92 01/17/92 415 0.09502 5.2324 3.5680 414.997 417.695 7 3.76
45 01/10/92 F92 01/17/92 420 0.09502 2.6940 5.7667 414.997 417.695 7 3.76
46 01/10/92 F92 01/17/92 425 0.09502 1.1619 10.3000 414.997 417.695 7 3.76
47 01/10/92 H92 03/20/92 410 2.34016 16.3800 9.3429 414.997 409.609 70 3.81
48 01/10/92 H92 03/20/92 415 2.34016 12.8167 11.4000 414.997 409.609 70 3.81
49 01/10/92 H92 03/20/92 420 2.34016 10.5833 13.4857 414.997 409.609 70 3.81
50 01/13/92 F92 01/17/92 415 0.09402 3.8625 2.8851 414.309 417.695 4 3.79