WELFARE STUDY OF LIFELINE ELECTRICITY RATES

by

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A thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Applied Economics

MONTANA STATE UNIVERSITY
Bozeman, Montana

May 1989

APPROVAL

of a thesis submitted by

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ABSTRACT

Increases in electricity prices have caused many states to consider adopting an increasing block rate "lifeline" structure. Proponents argue that a lifeline rate structure provides low income households with a basic amount of electricity at a reasonable price, and encourages all consumers to conserve electricity. This thesis developed a framework to analyze the welfare effects of adopting a lifeline rate structure for the state of Montana. The properties of a linear demand system, expenditure function, and compensated demand function are developed in detail. A method for calculating compensating variation from this demand system is applied to a utility model with a kinked budget constraint. Using this method it is determined that the proposed rate structure would not be beneficial to low income households of the state of Montana.

CHAPTER 1

INTRODUCTION

Increases in electricity costs and prices in recent years have led many states to reform rate structures for residential consumers. The most common type of reform has been the movement from a decreasing block rate structure to an increasing block rate structure (lifeline rates). The appeal of the lifeline rate structure is twofold: It provides low income consumers with basic amounts of electricity at a "reasonable" price, and it encourages efficient use of electricity by all consumers due to the higher price for discretionary use (Power, 1985).

Proponents of a lifeline rate structure in Montana argue that the lifeline rate structure for the state would be a more accurate representation of the marginal cost of electricity production. Montana Power Company has a large, cheap hydroelectric base, and the cost of additional kilowatt hours generated from thermal plants is greater than the average cost of all kilowatt hours (Power, 1985). In other words, marginal cost is above average cost; where the first units are produced by hydroelectric generation and the last units are produced by thermal plants. Proponents of lifeline rates also argue that if a lower amount was charged for the first units of

electricity, then the high profits Montana Power earns on the inexpensive hydroelectric power could be reduced.

Unfortunately, a block pricing structure will inhibit the attainment of an efficient outcome, so long as any consumer at the margin faces a price below marginal cost. A lifeline rate structure (or any block pricing structure) violates the principle of optimal pricing, that consumption be carried out to the point where the marginal value to each consumer is equal to the marginal cost to all consumers. Therefore, a lifeline rate structure can never yield an efficient solution if different consumers face different prices at different levels of consumption.

A lifeline rate structure could in principle be constructed that is more efficient than the current rate structure. Under the current rate structure electricity is priced at average cost, which results in over consumption. However, if a lifeline rate was constructed such that the price at the margin was greater than average cost, then efficiency would increase.

A second argument for adopting a lifeline rate structure is that the inverted rates will encourage conservation. The underlying logic is to set price below marginal cost where demand is least elastic (the first 300 to 500 kilowatt hours), and then raise the price above marginal cost for larger amounts of electricity consumed. The lifeline rate structure would encourage conservation by charging a higher rate for the discretionary use of electricity.

The main argument in favor of a lifeline rate structure is that it allegedly favors low income consumers. Unfortunately, a change to a lifeline rate structure would not automatically benefit low income consumers, because the demand for electricity is not solely dependent on the level of income. An individual's demand for electricity is a function of other variables such as: the propensity to use electricity for heating and cooling, the size of the home, the amount of insulation in the home, and the stock of electric appliances, as well as income. Therefore, the effectiveness of a lifeline rate to transfer wealth to low income consumers cannot be judged solely on the income elasticity of demand. There are many other effects that must be taken into account. For example, high income consumers tend to have larger homes, but the homes are newer and more energy efficient. It could, for example, require more electricity to heat a smaller, less well insulated home. Under a lifeline rate structure it is conceivable that low income consumers could be negatively impacted. The purpose of this study is to estimate a linear demand equation for residential electricity consumption, and to examine the income transfer effects of a lifeline rate structure. Specifically, the objectives of this study are:

1) To estimate a linear demand equation for residential electricity consumption.

2) To determine if lower income consumers would benefit from a lifeline rate structure.

3) To estimate the effect of a change to a lifeline rate on the average household in the state of Montana.

Outline of Thesis

Chapter 2 presents a survey of literature on electricity demand and consumer's surplus. An example of compensating variation is presented for a simple two good case with a single price change. The chapter concludes with a brief discussion of the permanent income hypothesis and its relevance to empirical studies. Chapter 3 derives the structural properties of a linear demand function. Compensating variation is derived for the simple two good case and is generalized for a kinked budget constraint. Chapter 4 provides a description of the data used in the empirical study. Residential electricity demand is estimated and compensating variation is calculated for the proposed lifeline rate structure. Chapter 5 summarizes the results and implications of the study.

CHAPTER 2

LITERATURE REVIEW

This chapter surveys the literature on the welfare effects of a lifeline rate structure, methods of estimating the demand for electricity, and methods of measuring consumer surplus. Included is a table of recent income and price elasticity estimates, and a discussion of the data used in the study.

Lifeline Rate Welfare Studies

Moffitt (1986) surveys maximum likelihood methods for estimating consumer demand functions, where the budget constraints are piecewise linear. Consumer demand functions are then derived under these constraints. The kinked budget constraints cause a variety of problems in estimation. With a nonlinear budget constraint, demand is often sticky at the kink points or will make nonmarginal leaps to a different area of the constraint. Therefore, these kink points and nonlinear leaps must be estimated along with the demand parameters. Demand can be formally derived using the indirect utility function for both convex and concave budget sets. The maximum likelihood estimation techniques can then be applied to estimate the demand function. This type of approach is not needed in this study because

demand will be estimated using a linear model and constant marginal electricity prices.

Recent research on the income distributional effects of a lifeline rate structure has concentrated on demand estimation. The income elasticity is then examined to determine if income has a strong effect on consumption. Burgess and Paglin (1981) used linear and log-linear demand functions in which the exogenous variables included the use of electricity for cooking and heating, annual income, family size, and propensity to occupy a single family dwelling. In both the linear and log-linear models income elasticity was found to be positive but small. The authors concluded that due to the small income elasticities, income did not have a strong effect on consumption. They also attempted to estimate the indirect effects income had on electricity use holding family size constant. Even with the indirect effects of income taken into account, income still did not have a strong influence on consumption. Thus Burgess and Paglin concluded that due to the small income elasticities, income did not have a strong influence on the demand for electricity. Therefore, a lifeline rate structure would not be an effective income transfer device.

Kahn (1986), in his testimony before the Public Utility Commissioner of the State of Oregon, argued that there is only a slight tendency for electricity consumption to vary with income, so that on the average, higher income families would tend to pay a higher average price than lower income families under a lifeline rate

structure. However, because the relationship between income and consumption of electricity is weak, the lifeline rate structure would tend to favor families who heat with fuels other than electricity, while injuring those who heat with electricity and those who have inadequately insulated houses. Therefore, a lifeline rate structure is an inefficient method of transferring income to the poor, because in the first block the price of electricity for all customers (regardless of income) is below the costs of providing that electricity (Kahn, 1986).

Survey of Other Electricity Demand Studies

This section reviews some of the studies on electricity demand. A table is included that lists the income and price elasticities of the various studies.

Taylor (1975) surveyed both the theoretical and the empirical problems in studying electricity demand. Although there is generally good data available for empirical electricity demand studies, there are some features of the market that make modeling difficult. The first problem is that until recently most utilities have used a multistep block pricing structure. This problem has most often been avoided by using average price in the demand equation. Although using average price in the demand equation does simplify the mechanics of the estimation process, it is an ex post approximation to the true price schedule the consumer faces, and results in biased parameter estimates. There are also problems in using marginal price

for parameter estimation. This is because the individual does not face a single marginal price. A single marginal price is only relevant to consumer decisions within a given block and does not explain why an individual chooses one block over another.

Taylor (1975) also pointed out the differences in estimating long-run versus short-run electricity demand. The demand for electricity is derived from the use of electrical appliances. Because most of these appliances are durable goods, there will be a difference in short versus long-run demand. The difference is that in the short run the appliance stock is fixed, whereas in the long run consumers will adjust to relative energy prices and their appliance stock will adjust. Taylor concluded that in the past many empirical demand studies have failed to consider block pricing rate schedules and long-run versus short-run effects in the estimation process, causing inaccurate parameter estimates.

Wilder and Willenborg (1975) estimated residential demand for electricity from a cross-sectional data set for a single city. Average price was used to estimate the parameters on the grounds that consumers respond to their total average bill and are unaware of the marginal cost. In addition, almost all of the residences are served by the same utility and are regulated under the same rate structure. The model consisted of four equations: a size of residence equation, an appliance stock equation, a demand equation, and a price equation. The first two equations were estimated using ordinary least squares. The price and demand equations were estimated simultaneously, using

two-stage least squares. Two-stage least squares is used to avoid the correlation problem between average price and the error term.

Income, family size, and race are important determinants of the appliance stock. Of the estimated income elasticity of .34, half was determined to be a direct income effect while the other half was attributed to income's effect on the size of the appliance stock. The price and income elasticities are reported in Table 1.

Table 1. Survey of Price and Income Elasticities.

Study	Income Elasticity	Price Elasticity	Type of Data
Burgess, Paglin	1.95	not estimated	cross-sectional (1970)
Wilder, Willenborg	.34	-1.31	cross-sectional (1973)
McFadden, Puig, Kirshner	.21	-.28 to -.15	cross-sectional (1975)
Dubin, McFadden	.01 to .08	-.29 to -.02	cross-sectional (1975).
Barnes, Gillingham, Hagemann	.20	-.55	cross-sectional (1972 - 1973)

McFadden et al. (1977) estimated demand using a structural model in which one equation describes intensity of electricity use given appliance holdings, and a second describes the choice of appliance portfolio. The model is fitted using individual consumer data. However, due to the lack of information in the data set about differing rate structures within the sample, typical electric bills

were constructed for five levels of consumption: 100, 250, 500, 750, and 1000 kilowatt hours per month. The typical electric bills are a weighted average of individuals' total bills in the five consumption brackets.

The demand estimates were obtained using an instrumental variable approach to avoid the correlation between average price and the error term. National survey of households was used in the empirical estimation. A predicted consumption level was found by regressing the log of monthly consumption on all typical electric bills and the demographic variables. This predicted consumption level was used to interpolate the typical electric bills for a predicted average price. This predicted average price was then used as an instrument for the measured average price, and the parameters were estimated. The income and price elasticities are reported in Table 1.

Dubin and McFadden (1984) used a unified model of electricity demand and discrete appliance choice. The optimization problem was that the consumer must weigh the alternatives of each appliance against expectations of future use, future energy prices and current financing costs. In the analysis, the authors ignored the block rate structures and used a fixed marginal cost which was equivalent to the average cost.

A discrete choice model was constructed for the choice of space and water heating fuel type. The demand for electric heating was modeled as a function of price of electricity, price of alternative

energy, costs of alternative heating, analyzed total life cycle cost of the alternative, and a vector of household demographics.

Demand elasticity was estimated using ordinary least squares. The demand for electricity is a function of the average cost of electricity, the cost of natural gas, a vector of demographic variables, and income minus the lifetime cost of a given appliance. An instrumental variable method was used to estimate demand; where an instrument variable was constructed for appliance stock of the household. A reduced form method was used where ordinary least squares was applied to income minus annual operating cost times the probability that the appliance was owned minus capital cost times its probability.

The fourth method used was a conditional expectation with a correction to permit consistent estimates. Dubin and McFadden (1984) concluded that observable factors of portfolio choices are not independent of unobservable factors influencing intensity of appliance use. Therefore, individual residential demand for electricity cannot be estimated by decomposition of total demand. The other methods of estimation are more appropriate to estimate individual demand. Price and income elasticities are reported in Table 1.

Barns et al. (1981) estimated short-run residential demand from data that covered a large geographic area. In addition, the distribution of consumption over specific area end-use categories was analyzed. Consumers faced declining block rates, so a measure of

marginal cost was constructed which the authors termed the rate structure premium. The rate structure premium for a declining block rate structure with two prices is the difference between the first price and the second price times the quantity where the price changes.

The authors modeled demand as a function of income minus the rate structure premium of the relative price of electricity, and demographic variables. Their model was estimated using ordinary least squares and an instrumental variable to avoid the correlation problem, since price and quantity are determined simultaneously. Then an instrumental variable for price was obtained by estimating the demand equation using price at the mean level of consumption. The marginal price at the predicted level of consumption was used as an instrumental variable to obtain the final demand estimates. The price and income elasticities are reported in Table 1.

Consumer's Surplus

The basic concept underlying consumer's surplus is to attach a monetary value to the change in welfare resulting from a change in relative prices. The most commonly used measures are Marshallian consumer's surplus and compensating variation. There are two distinct approaches in the literature on consumer's surplus. The first seeks to determine the restrictions that must be placed on consumer preferences in order to yield exact measures of welfare change. The second approach is concerned with finding approximations

to these welfare measures without having the restrictions placed on consumer preferences.

The Marshallian measure of consumer's surplus is measured as the area under the ordinary demand curve and above the horizontal price line. The problem with this measure is that it cannot be defined in terms of the utility of the individual. Compensating variation is defined as the amount the individual is willing to pay to consume at the new price (Freeman, 1979). It should be noted that each of these concepts measure something different and each has a different meaning. However, for small price changes the measures will not differ substantially.

For discussion purposes, a special case is developed where there are only two goods and one price change. The first measure discussed is Marshallian consumer's surplus which is measured by the area under a Marshallian demand curve and between the horizontal price lines. The problem with this measure is that it cannot be defined in terms of the individual's preference map. Figure 1 illustrates an example of Marshallian consumer's surplus.

Compensating variation is often defined as the maximum amount that an individual is willing to pay to consume at the new price. Panel A in Figure 2 shows an individual's preference map for a simple two good case. Suppose that the price of x increases from p^0 to p^1. The individual's equilibrium shifts from point A to point B, on the new budget line. Points A and B are points on the Marshallian demand if the price of y is held constant. Suppose that income is added to

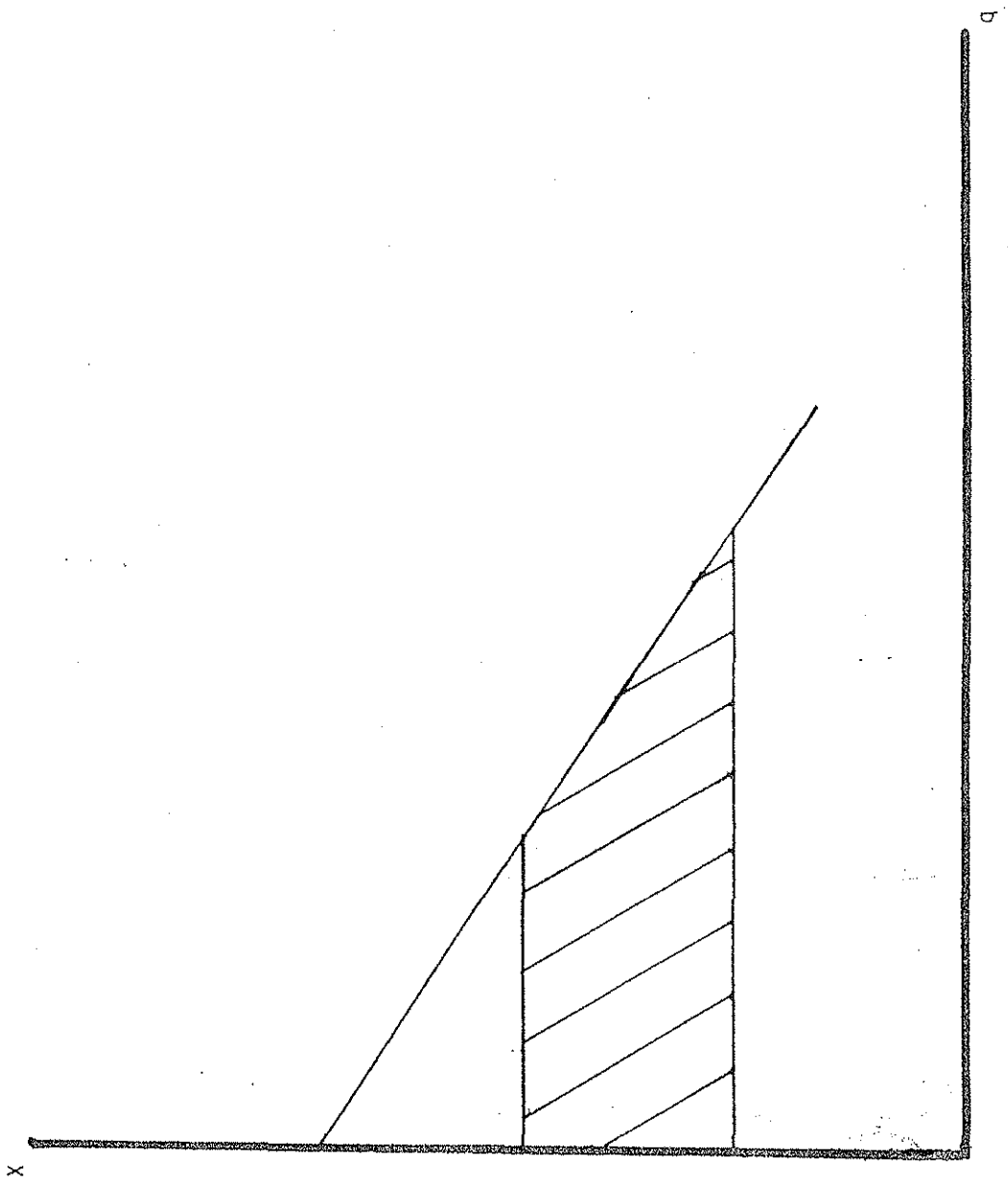


Figure 1. Marchallian Consumer's Surplus.

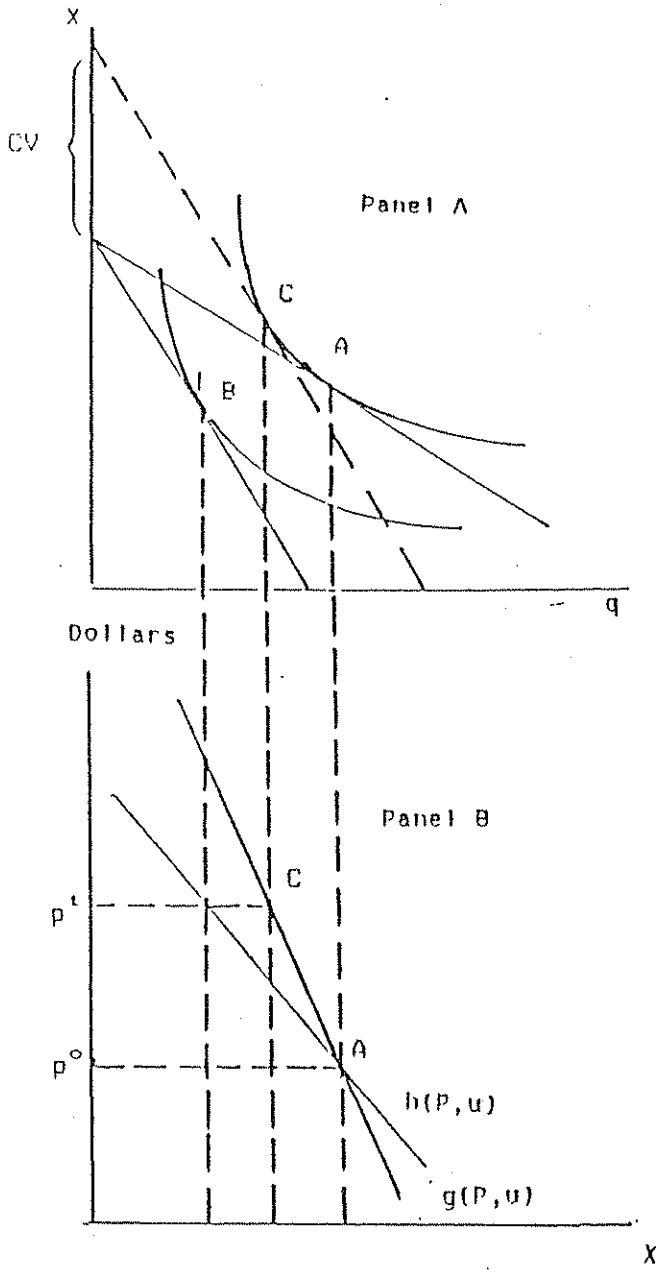


Figure 2. The Compensating Variation and the Hicks-Compensated demand curve.

the individual at the new price p^1 so that he remains at the initial utility level U^o, with equilibrium consumption at point C. Note that point C is the tangent between the original utility level and a budget line that is parallel to the higher price set. The amount of income that must be given to the individual to move the equilibrium at point C is the compensating variation.

Compensating variation can also be calculated from the expenditure function. Points A and C are plotted in panel B. These points are on the Marshallian demand curve. Note that the Hicksian demand is less elastic than the Marshallian demand if x is a normal good, because the income effect is eliminated from the Hicksian demand. In this two good case, compensating variation is the area to the left of the Hicksian demand curve between the two price lines. In panel B, area PP^oAC is compensating variation.

Willig (1976) argued that consumer surplus can be used to estimate compensating variation. He derived upper and lower bounds and the percentage errors from approximating compensating variation with Marshallian consumer surplus. The bounds can easily be calculated using observable demand data. Willig argued that in most cases the error in the approximation of compensating variation is often smaller than the error involved in estimating the original demand system. The approximation is most accurate when there is a small range between the high and low income elasticities. Willig's results indicate, in cases where proportional changes in income due to a price change are small, the percentage errors in estimating

compensation and equivalent variation with Marshallian consumer surplus are relatively small.

Hausman (1981) confirmed that Willig's approximations are relatively accurate for consumer's surplus estimation; however, Marshallian dead weight loss can be a poor approximation for the correct measure of Hicksian dead weight loss. The general argument for using the approximation is that observable market data can be used. However, compensated demand functions can be derived for many commonly used demand functions. The exact measure of consumer's surplus and dead weight loss can then be easily calculated, eliminating the need to use Willig's approximation.

Hanemann (1980) extended Hausman's (1981) results to four other commonly used demand functions. He assumed that the demand functions were homogeneous of degree zero in prices and income, and satisfy the adding up condition (individuals spend all their income). Also the utility function was assumed quasi-concave, thus the Slutsky matrix was negative semidefinite. If these conditions are met, then the expenditure function can be derived from the demand function, and an exact measure of compensating variation can be calculated. Compensating variation will equal the expenditure at the new price level and original utility level minus the original income. The basic purpose was to show that exact measures of compensating variation can be calculated for most commonly used demand functions, thus enabling direct calculation of compensating variation rather than using an approximation.

Marshallian consumer's surplus and compensating variation are two different measures and have different meanings. This study will use compensating variation as the welfare measure, because the study is concerned with welfare changes after the adoption of the lifeline rate structure.

Permanent Income

Friedman (1957) proposed that annual income (Y) and annual consumption (C) are the sum of two components: a transitory component and a permanent component. That is,

(1) $$Y = YP + YT$$

$$C = CP + CT$$

The permanent component of income (YP) is interpreted as the income of the individual over a long period of time, and the income to which an individual adjusts consumption. Factors that determine the permanent component are education, training, age, and personality. The transitory component (YP) is interpreted as accidental or chance factors that affect the individual's income for a fixed period of time. This component also contains the measurement error of statistical data. An example of the transitory component would be temporary loss of income due to injury. This temporary income loss is reflected in a worker's annual income for that given year. However, the worker is not likely to adjust his/her lifestyle significantly because of the temporary loss of income.

In empirical studies it is beneficial to construct a measure of

permanent income to avoid the bias caused by the transitory component. In this study a measure of permanent income is constructed using age, education, and the consumption of major durable goods of the household. This measure of permanent income is then used to estimate residential electricity demand.

in utility restores the original level of utility after the change in prices. Equivalent variation measures could be calculated in a manner similar to the methods shown for the compensating variation. The combination of the random utility component, ϵ, and the kink in the budget line due to the increasing block rate structure of lifeline electricity pricing introduces two complications into the estimation of the compensating variation for such a change in electricity rates. The first complication is due to the qualitative/quantitative choice problem faced by the household with respect to which segment of the budget line is preferred and where on the chosen segment to consume electricity. The second complication is due to the probabilistic nature of the qualitative/quantitative choice problem from the perspective of the analyst. Both of these aspects of the problem are dealt with in this study in a coherent manner, and the details of the necessary calculations are presented in this chapter.

The balance of this chapter is organized as follows. First, the dual structure of the linear demand model is presented and analyzed. Then the dual structure is applied to the lifeline electricity rate structure to illustrate the formal choice problem faced by the electricity consuming household with increasing block rates. The stochastic nature of calculating compensating variations is presented in detail.

The Structural Properties of Linear Demand Systems

Recall that the demand function (2) arises from the utility function,

$$(3)\quad U(X,q) = \left[(\beta + \gamma X)/\gamma^2\right]\exp\left\{\gamma(\alpha + \gamma q + \delta z + \epsilon - X)/(\beta + \gamma X)\right\}.$$

The expenditure function is derived by minimizing expenditure subject to a given utility level. Using Lagrange's method, total expenditure $PX + q$ is minimized subject to $u \leq u(q,X)$. The Lagrangian is

$$(4)\quad L = PX + q + \lambda\left[u - \left\{(\beta + \gamma X)/\gamma^2\right\}\cdot \exp\left\{\gamma(\alpha + \gamma q + \delta z + \epsilon - X)/(\beta + \gamma X)\right\}\right].$$

Differentiating (4) with respect to X, q, and λ and equating to zero gives the first-order conditions:

$$(5)\quad \partial L/\partial X = P - \lambda\left[(X - \alpha - \gamma q - \delta z - \epsilon)/(\beta + \gamma X)\cdot \exp\left\{\gamma(\alpha + \gamma q + \delta z + \epsilon - X)/(\beta + \gamma X)\right\}\right] = 0$$

$$(6)\quad \partial L/\partial q = 1 - \lambda\left[\exp\left\{\gamma(\alpha + \gamma q + \delta z + \epsilon - X)/(\beta + \gamma X)\right\}\right] = 0$$

$$(7)\quad \partial L/\partial \lambda = u - \left[\left\{(\beta + \gamma X)/\gamma^2\right\}\cdot \exp\left\{\gamma(\alpha + \gamma q + \delta z + \epsilon - X)/(\beta + \gamma X)\right\}\right] = 0.$$

The slope of the indifference curve is found by solving for the marginal rate of substitution between X and q, which is the ratio of $\partial u/\partial X$ to $\partial u/\partial q$. That is,

$$(8)\quad \frac{\partial u/\partial X}{\partial u/\partial q} = (X - \alpha - \gamma q - \delta z - \epsilon)/(\beta + \gamma X) = P.$$

Equation (8) is the price dependent demand function. Note that the original Marshallian demand function can be recovered from the price dependent demand by substituting Y for $PX + q$ from the budget constraint and solving for X.

The compensated demand function for X is obtained by substituting P into $\partial L/\partial \lambda = 0$ and solving for X:

(9) $X^* = \gamma u \exp(\gamma P) - \beta/\gamma$

The compensated demand for q is obtained by substituting X^* into equation (8) and solving for q^*.

(10) $q^* = u\exp(\gamma P) - P\gamma u\exp(\gamma P) - \left[(\beta/\gamma + \alpha + \delta z + \epsilon)/\gamma\right]$

These compensating demand functions trace the demand for X and q for various prices at a constant utility level. They are a locus of demands for X and q as traced out by tangent budget lines moving along an indifference curve.

The minimum cost of achieving a given level of utility is given by the expenditure function. The expenditure function is defined as

(11) $E(P,u) = PX^* + q^*$.

The expenditure function is obtained by substituting the compensated demand functions (9) and (10) into the budget constraint to yield

(12) $E(P,u) = u(\exp\gamma P) - (\alpha + \beta P + \delta z + \epsilon + \beta/\gamma)/\gamma$.

The maximum level of utility attainable at the given prices and income is the indirect utility function. The indirect utility function is obtained by setting $E(P,u) = Y$ and solving the expenditure function (12) for u, and is defined as

(13) $v(P,Y) = \left[Y + (\alpha + \beta P + \delta z + \epsilon + \beta/\gamma)/\gamma\right]\exp(-\gamma P)$.

Assuming expenditures are minimized, the following identity holds:

(14) $u(X,q) = v\left[P(X,q),\ P(X,q)X + q\right]$

Therefore, the direct utility function can be recovered by

substituting the price dependent demand function (8) and the budget constraint $Y = PX + q$ into the indirect utility function (13):

$$(3)\quad u(X,Y) = \left[(\beta + \gamma X)/\gamma^2\right]\exp\left\{\gamma(\alpha + \gamma q + \delta z + \epsilon - X)/(\beta + \gamma X)\right\}.$$

A comparative statics result of this utility function is that

$$(15)\quad \partial u(X,q)/\partial X = \left[(X - \alpha - \gamma q - \delta z - \epsilon)/(\beta + \gamma X)\right]\cdot \exp\left\{\gamma(\alpha + \gamma q + \delta z - \epsilon - X)/(\beta + \gamma X)\right\} > 0$$

where $\beta + \gamma X \neq 0$ and sign $(X - \alpha - \gamma q - \delta Z - \epsilon) = \text{sign}(\beta + \gamma X)$. These two side conditions must be met to satisfy the postulate of nonsatiation.

If $\partial u(X,q)/\partial X > 0$, then the marginal utility of q is also positive, since

$$(16)\quad \partial u(X,q)/\partial q = \exp\left\{\gamma(\alpha + \gamma q + \delta Z + \epsilon - X)/(\beta + \gamma X)\right\} > 0$$

where $\beta + \gamma X \neq 0$. Therefore, the monotonicity and quasi-concavity conditions for X are

$$(17)\quad \partial q/\partial X|_u = (\alpha + \gamma q + \delta Z + \epsilon - X)/(\beta + \gamma X) < 0,$$

i.e., the slope of the indifference curve must be negative indicating that the consumer is willing to make trade-offs between X and q, and

$$(18)\quad \partial^2 q/\partial X^2|_u = -1/(\beta + \gamma X) > 0 \text{ if and only if } \beta + \gamma X < 0.$$

The positive inequality in (18) is consistent with a diminishing marginal rate of substitution between X and q. Thus the marginal value of a good decreases as more of that good is consumed. The indifference curve is then convex to the origin. The restrictions implied by equations (15), (16), (17), and (18) will ensure that the utility function is continuously differentable, increasing and strictly quasi-concave, in (X,q). For the utility function u(X,q) to

have these properties, it follows that

$$\beta + \gamma X < 0 \text{ and } X < \alpha + \gamma q + \delta Z + \epsilon.$$

For the case where X is a normal good ($\gamma > 0$, $\beta < 0$), then the region of regularity for u(X,q) is defined by the monotonicity and quasi-concavity constraints as depicted in Figure 3.

Comparison of Marshallian and Hicksian Demands

The Marshallian demand is defined as

(2) $X = h(P,Y) = \alpha + \beta P + \gamma Y + \delta Z + \epsilon.$

The expenditure function for the demand function was derived as

(12) $e(P,u) = u\exp(\gamma P) - (\alpha + \beta P + \delta z + \epsilon + \beta/\gamma)/\gamma.$

The Hicksian demand is obtained by taking the first derivative of the expenditure function (12) with respect to price. The Hicksian demand for electricity is

(18) $\partial e(P,u)/\partial P = g(P,u) = \gamma u\exp(\gamma P) - \beta/\gamma.$

Hypothetical Marshallian and Hicksian demand functions are presented in Figure 4. Note that where the two functions intersect direct and indirect utility are equivalent. Assume that

P_1 is where h(P,Y) intersects the vertical axis,

P_2 is where g(P,u) intersects the vertical axis,

P^o is a constant where h(P,Y) intersects g(P,u),

X^o is a constant where h(P,Y) intersects g(P,u),

u^o is the given level of utility $u(X^o,Y^o)$.

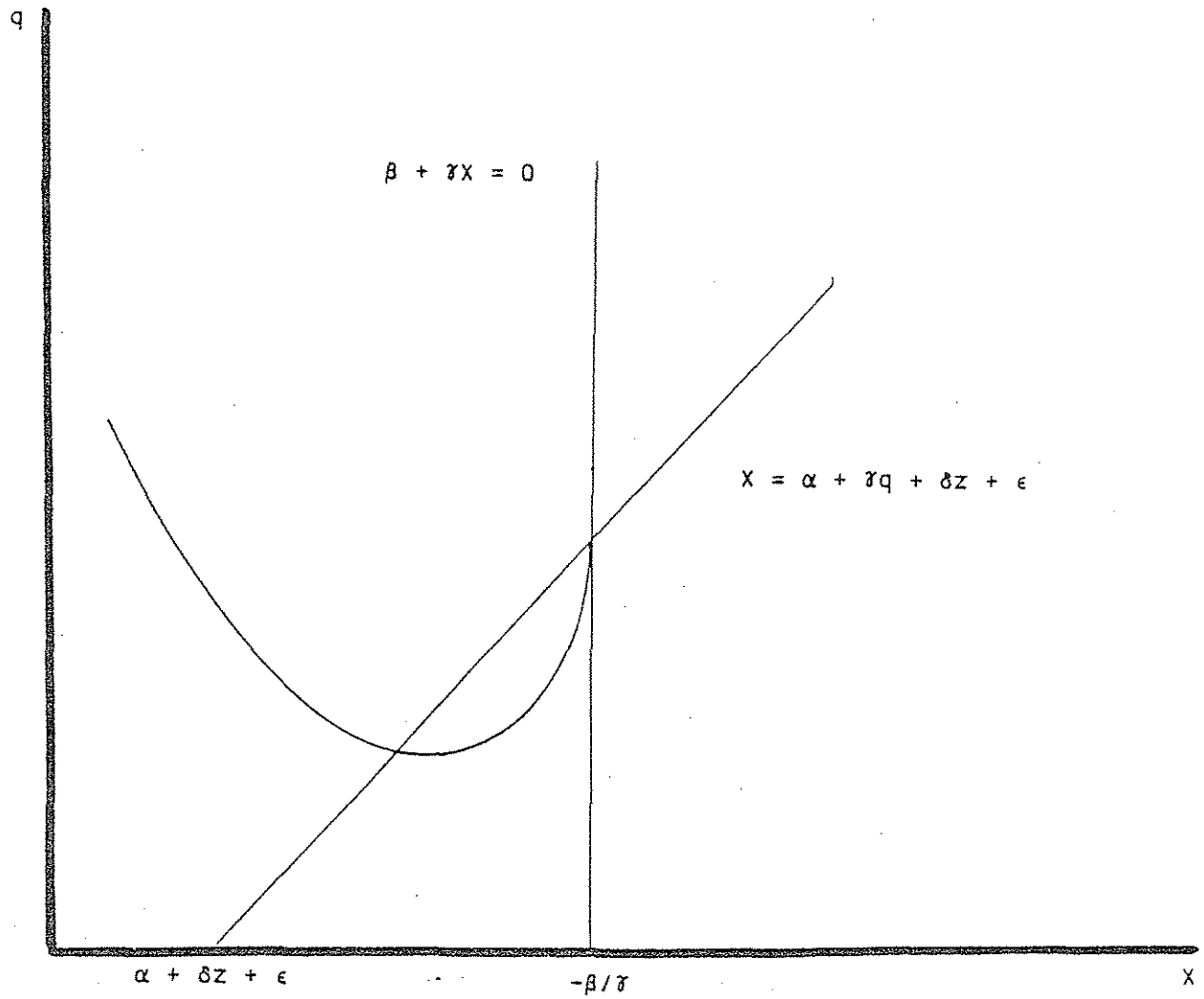


Figure 3. Region of Regularity of the Indifference Map.

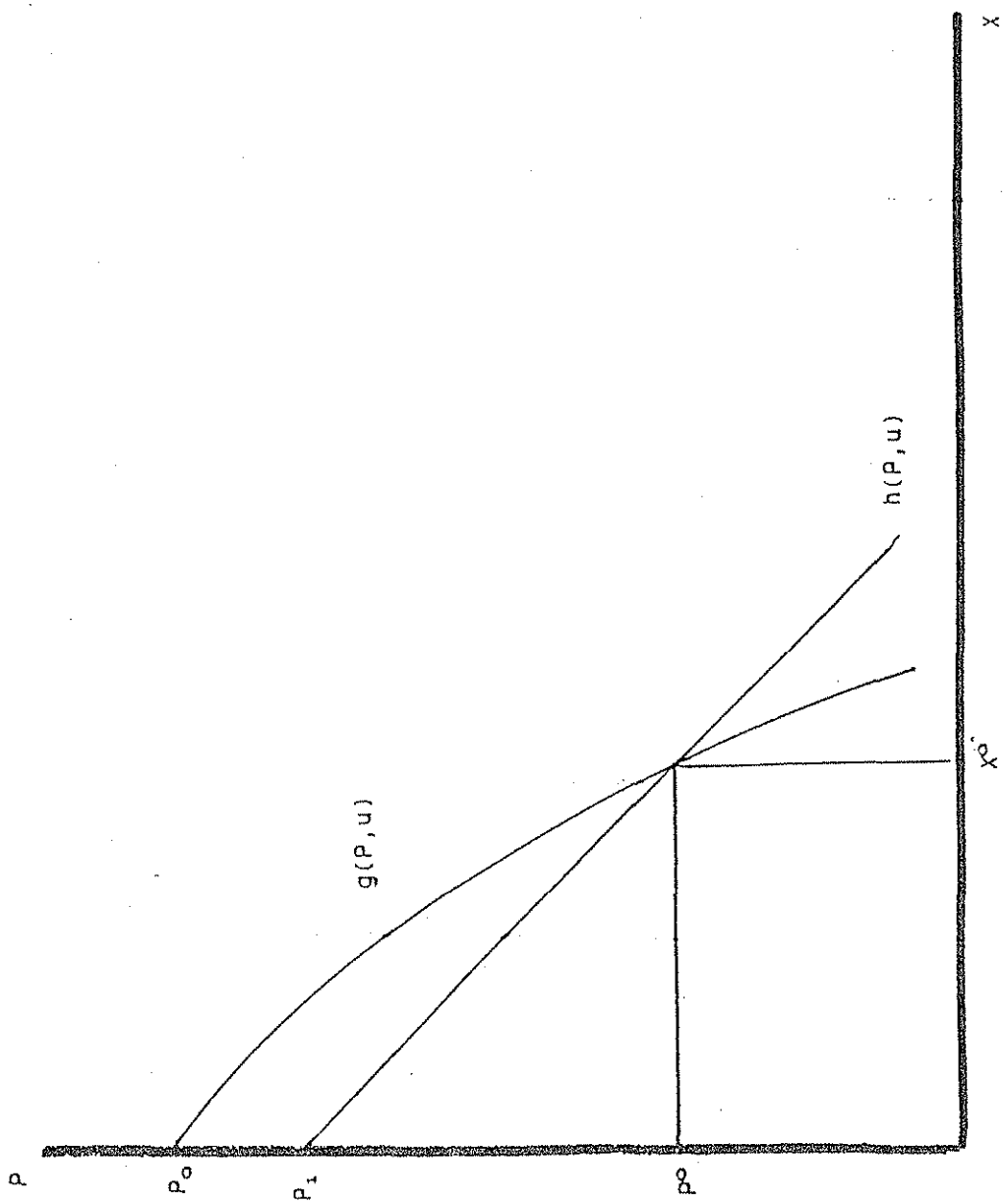


Figure 4. Marshallian and Hicksian Demand Functions.

It then follows that

1) $v(P^o,Y^o) = \left[Y + (\alpha + \beta P^o + \delta Z + \epsilon + \beta/\gamma)/\gamma\right]\exp(-\gamma P^o)$

$= (1/\gamma^2)(\beta + \gamma X^o)\exp(-\gamma P^o)$

Note $\exp(-\gamma P) = \exp\left[\gamma(\alpha + \gamma q^o + \delta Z + \epsilon - X^o)/(\beta + \gamma X^o)\right]$

$v(P^o,Y^o) = u(X^o,q^o)$

since $\beta + \gamma X < 0$ for regularity of $u(X,q)$

2) $h(P,Y) = 0 \iff P_1 = -(\alpha + \gamma Y + \delta Z + \epsilon)/\beta = P^o - X^o/\beta$

3) $g(P,u) = 0 \iff P_2 = (1/\gamma)\ln(\beta/\gamma^2 u^o)$

$= P^o - (1/\gamma)\ln(\beta + \gamma X^o/\beta)$

Slope Comparisons of Marshallian and Hicksian Demands

The first and second derivatives of the two demand functions are used to compare the slopes at X^o and P^o, the point where the Marshallian and Hicksian demand functions intersect in Figure 4. That is,

(19) $\partial h(P,Y)/\partial P = \beta < 0$,

(20) $\partial^2 h(P,Y)/\partial P^2 = 0$,

(21) $\partial g(P,u)/\partial P = \gamma^2 u\exp\gamma P = (\beta + \gamma X) < 0$,

(22) $\partial^2 g(P,u)/\partial P^2 = \gamma(\beta + \gamma X) < 0$.

Given that $\gamma > 0$, $X^o > 0$ and $\beta < 0$, the Hicksian demand is less elastic than the Marshallian demand at the point X^o and P^o for $u = u^o$. Therefore, $\partial g(P,u)/\partial P > \partial h(P,Y)/\partial P$ at X^o,P^o.

Intercept Comparisons of Marshallian and Hicksian Demands

The Marshallian demand intercept (see Figure 4) is $P_1 = P^o - X^o/\beta$. The Hicksian demand intercept equals $P_2 = P^o - (1/\gamma)\ln\left[(\beta + \gamma X^o)/\beta\right]$. Where do each of these demand functions intersect the vertical axis assuming that X is a normal good where $\gamma > 0$, $\beta < 0$? Comparing the demand functions at $u = u^o$,

$$P_1 = P^o - X^o/\beta \lessgtr P_2 = P^o - (1/\gamma)\ln\left[(\beta + \gamma X^o)/\beta\right],$$

$$\text{iff } -X^o/\beta \lessgtr (-1/\gamma)\ln(\beta + \gamma X^o/\beta),$$

$$\text{iff } X^o/\beta \lessgtr 1/\gamma\ln\left[(\beta + \gamma X^o)/\beta\right],$$

$$\text{iff } \exp(\gamma X^o/\beta) \lessgtr \beta + \gamma X^o/\beta.$$

Recall the first and second derivatives from the previous section. If

(19) $\partial h(P,Y)/\partial P = \beta < 0$

(20) $\partial^2 h(P,Y)/\partial P^2 = 0$

(21) $\partial g(P,u)/\partial P = \gamma^2 u\exp(\gamma P) = \beta + \gamma X > \beta$ for $X > 0$

(22) $\partial^2 g(P,u)/\partial P^2 = \gamma^3 u\exp(\gamma P) = \gamma(\beta + \gamma X) < 0$ and since $\partial P/\partial X = 1/(\partial X/\partial P)$.

then $\lim_{x\to 0} \beta\exp(\gamma X/\beta) = \beta = \lim_{x\to 0}(\beta + \gamma X)$. Therefore,

$$\beta\exp(\gamma X^o/\beta) = \beta + \gamma X^o = \beta \text{ at } X^o = 0,$$

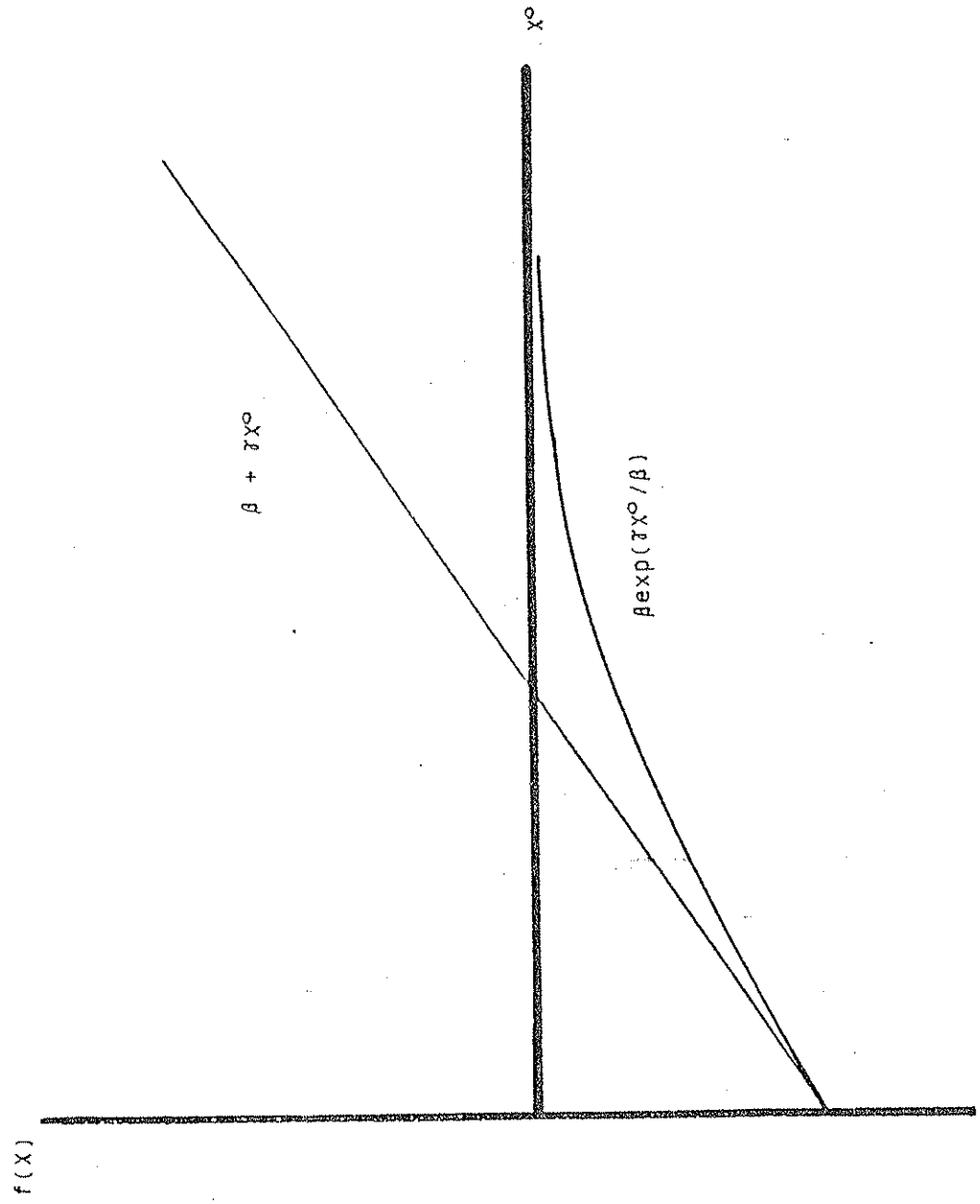


Figure 5. Slope Comparison of Marshallian and Hicksian Demand Functions.

$$X = \begin{cases} \alpha + \beta P_1 + \gamma Y + \delta Z + \epsilon & \text{if } L_1 \leq \epsilon \leq L_2 \\ X^* & \text{if } L_2 \leq \epsilon \leq L_3 \\ \alpha + \beta P_2 + \gamma Y^* + \delta Z + \epsilon & \text{if } L_3 \leq \epsilon \leq L_4 \end{cases}$$

where

$$L_1 = -(\alpha + \beta P_1 + \gamma Y + \delta Z + \epsilon),$$

$$L_2 = X^* - (\alpha + \beta P_1 + \gamma Y + \delta Z + \epsilon),$$

$$L_3 = q^* - (\alpha + \beta P_2 + \gamma Y^* + \delta Z + \epsilon),$$

$$L_4 = \min\left[Y^*/P_2,\ -\beta/\gamma\right] - (\alpha + \beta P_2 + \gamma Y^* + \delta Z + \epsilon).$$

Note X^* is the quantity of electricity where the consumer faces the price change due to the lifeline rate structure. Y^* is the perceived income of an individual that consumes in the second block due to the lower price in the first block. Refer to Figure 6.

Compensating Variation for Segment 1

For a given household on segment 1, $0 < X < X^*$. Therefore, the residual for the household on segment 1 is located between L_1 and L_2, as defined by the inequalities

$$-(\alpha + \beta P_1 + \gamma Y + \delta Z) < \epsilon < X^* - (\alpha + \beta P_2 + \gamma Y + \delta Z).$$

Assume that ϵ is normally distributed and has the probability density function

(24) $f(\epsilon) = 1/(2\pi\sigma)^{1/2}\left[\exp(-\epsilon^2/2\sigma^2)\right]$.

This study rules out the possibility that a household will not consume electricity ($X = 0$), and the case where the household spends all of its income on electricity. Therefore, the tails of $f(\epsilon)$ are removed from the distribution. The remainder of the distribution must be transformed to have probability density equal to 1.

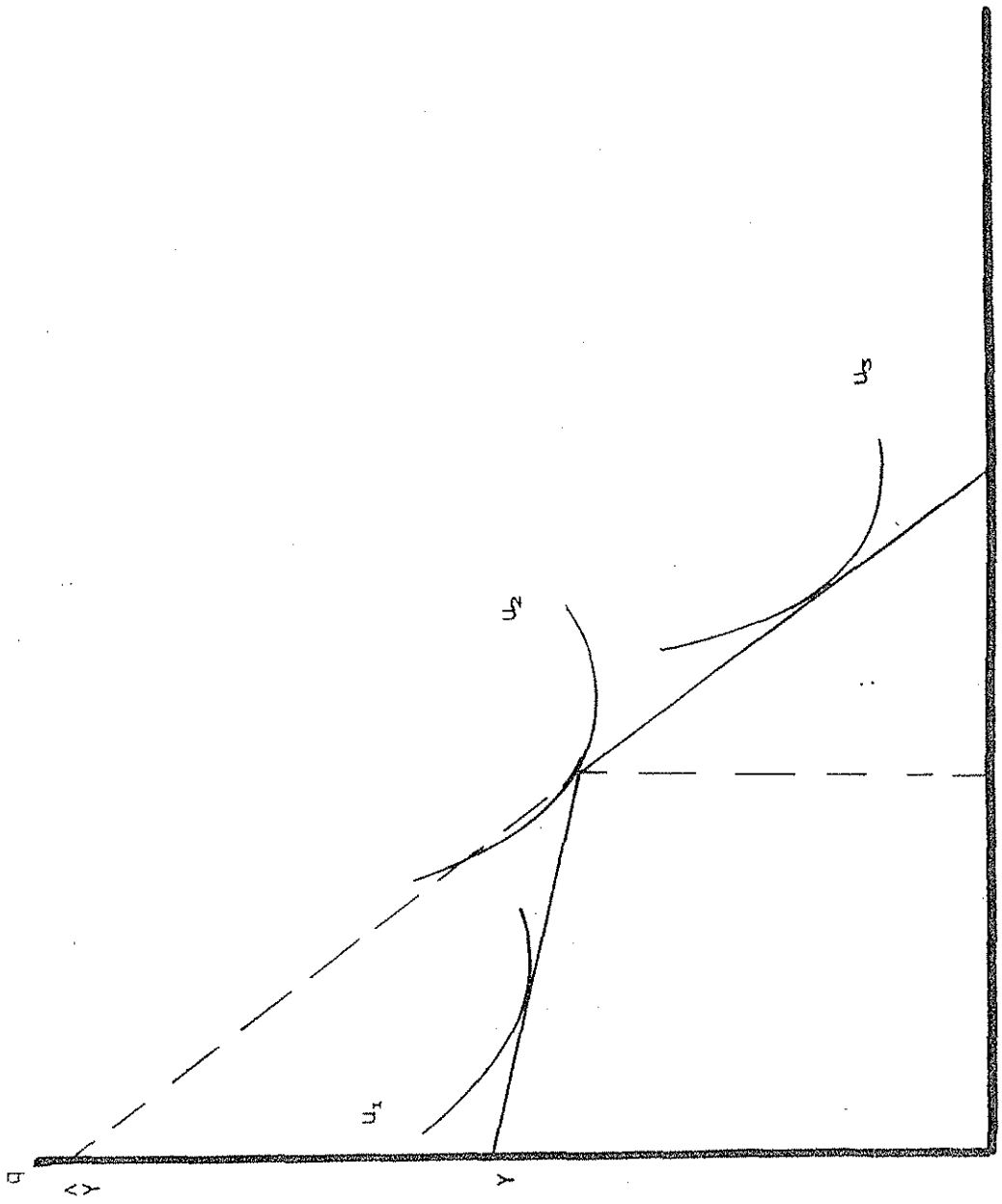


Figure 6. Lifeline Rate Structure with Linear Demand.

Define $\varphi(Z) = 1/(1\pi)^{2}\exp(-Z^{2}/2)$, a normal (0,1) probability density function,

and $\Phi(Z) \equiv \int_{-\infty}^{Z} \varphi(x)dx = \Phi(Z) = \int_{-\infty}^{Z} 1/(2\pi)^{1/2}(\exp-X^{2}/2)dx,$

$\Phi(Z) \equiv$ normal (0,1) cumulative density function.

Then the probability that $L_1 < \epsilon < L_4$, denoted $Pr(L_1 < \epsilon < L_4)$ is given by $\Phi(L_4/\sigma) - \Phi(L_1/\sigma)$. Similarly, $Pr(L_1 < \epsilon < L_2) = \Phi(L_2/\sigma) - \Phi(L_1/\sigma)$. The conditional probability density for ϵ, given that the household is on segment 1, is

$f(\epsilon \mid L_1 < \epsilon < L_2) = \left[1/(2\pi\sigma)^{1/2}\exp\left\{-\epsilon^{2}/2\sigma^{2}\right\}\right]/\left[\Phi(L_2/\sigma) - \Phi(L_1/\sigma)\right].$

The conditional mean of the residual ϵ, given $L_1 < \epsilon < L_2$, can be thought of as the center of gravity between L_1 and L_2. Thus ϵ_1 is defined as

$$(25)\quad \bar{\epsilon}_1 = E(\epsilon \mid L_1 < \epsilon < L_2) = \int_{L_1}^{L_2} \epsilon\left[f(\epsilon \mid L_1 < \epsilon \leq L_2)\right]\partial\epsilon$$

$$= \int_{L_1}^{L_2} \epsilon/(2\pi\sigma)\exp\left\{-\epsilon^{2}/2\sigma^{2}\right\}d\epsilon/\left[\Phi(L_2/\sigma) - \Phi(L_1/\sigma)\right]$$

$$= \sigma/\left[\Phi(L_2/\sigma) - \Phi(L_1/\sigma)\right]\int_{L1/\sigma}^{L2/\sigma} 1/(2\pi)^{1/2}\left[Z\left\{\exp(-Z^{2}/2)\right\}\right]dz$$

$$= \sigma\left[\varphi(L_1/\sigma) - \varphi(L_2/\sigma)\right]/\left[\Phi(L_2/\sigma) - \Phi(L_1/\sigma)\right].$$

Equation (25) adjusts the probability density function so that the conditional probability density function accounts for the loss of area due to the truncation at L_1 and L_2 when $L_1 < \epsilon < L_2$.

Note that $P_1 < P_o < P_2$ where P_o is the original flat rate price of electricity, P_1 = price of electricity for the first block, and P_2 = price of electricity for the second block. The compensating

variation for segment 1 is defined using the indirect utility function and is defined as $v(P_o, Y) = v(P_1, Y - CV_1)$.

Solving for CV_1 gives

$$(26)\quad CV_1 = (1/\gamma)\left[\beta\left(P_1 - P_o\exp\left\{\gamma(P_1 - P_o)\right\}\right) + (\alpha + \beta Y + \delta z + \epsilon + \beta/\gamma)\cdot \left[1-\exp\left\{\gamma(P_1-P_o)\right\}\right]\right].$$

In this study the conditional mean compensating variation for a household on segment 1 is calculated by using the conditional expected value $\bar{\epsilon}_1$ for the mean of ϵ, given that the household is on segment 1, which is found by substituting (25) into (26).

Compensating Variation for the Corner Solution

At the corner solution, $X = X^*$. The slope of the indifference curve at the kink in the budget constraint $(X^*, Y - P_1X^*)$ is given by:

$$(27)\quad \hat{P} = -\left[\partial u(X^*, Y - P_1X^*)/\partial q\right]/\left[\partial u(X^*, Y - P_1X^*)/\partial X\right]$$
$$= \left[X^* - (\alpha + \gamma(Y - P_1X^*) + \delta z + \epsilon)\right]/(\beta + \gamma X).$$

Note that $\hat{P} \geq P_1$. Thus at the corner solution, utility is maximized subject to the artificial budget constraint

$$(28)\quad \hat{P}X + q \leq Y + (\hat{P} - P_1)X^* = \hat{Y}.$$

Therefore, $u(q^*, Y - P_1X^*) = v(\hat{P}, \hat{Y})$

$$(29)\quad v(\hat{P}, \hat{Y}) = (1/\gamma)(\alpha + \delta z + \beta\hat{P} + \gamma\hat{Y} + \epsilon + \beta/\gamma)\exp(-\gamma\hat{P}).$$

Define CV_2 for the corner solution by $v(P_o, Y) = v(\hat{P}, \hat{Y} - CV_2)$. Solving for CV_2 gives

$$(30)\quad CV_2 = (\hat{P} - P_1)X^* + (1/\gamma)(\alpha + \delta z + \gamma Y + \epsilon + \beta/\gamma)\cdot$$

$$\left(1 - \exp\left[\delta(\hat{P} - P_1)\right]\right) + (\beta/\gamma)\left(\hat{P} - P_0\exp\left[\delta(\hat{P} - P_0)\right]\right).$$

For the corner solution, $L_2 \leq \epsilon \leq L_3$, so that the conditional probability density function for the corner solution is

$$(31)\quad f(\epsilon \mid L_2 \leq \epsilon \leq L_3) = \frac{\left[1/(2\pi\sigma)^{1/2}\exp(-\epsilon^2/2\sigma^2)\right]}{\left[\Phi(L_3/\sigma) - \Phi(L_2/\sigma)\right]}$$

and the conditional mean for ϵ, given that $L_2 \leq \epsilon \leq L_3$, is

$$(32)\quad \bar{\epsilon}_2 = \sigma\left[\varphi(L_3/\sigma) - \varphi(L_2/\sigma)\right]/\left[\Phi(L_3/\sigma) - \Phi(L_2/\sigma)\right].$$

Thus the conditional expected compensating variation for the corner solution is given by

$$(33)\quad E(CV_2 \mid L_2 \leq \epsilon \leq L_3) = \left[E(\hat{P} \mid L_2 \leq \epsilon \leq L_3) - P_1\right]X^* +$$

$$(1/\gamma)\left[\alpha + \gamma Y + \delta z + E(\epsilon \mid L_2 \leq \epsilon \leq L_3) + \beta/\gamma\right] -$$

$$(1/\gamma)(\alpha + \gamma Y + \delta z + \beta/\gamma)E\left[\exp\left\{\gamma(P^* - P_0)\right\} \mid L_2 \leq \epsilon \leq L_3\right]$$

$$-(1/\gamma)E\left(\epsilon\exp\hat{\left[\gamma(P - P_0)\right]} \mid L_2 \leq \epsilon \leq L_3\right) + (\beta/\gamma)\hat{\left[E(P \mid L_2 \leq \epsilon \leq L_3)\right.}$$

$$\left. - P_0E(\exp\left\{\gamma(\hat{P} - P_0)\right\} \mid L_2 \leq \epsilon \leq L_3)\right].$$

The required conditional expectations to evaluate $E(CV_2 \mid L_2 \leq \epsilon \leq L_3)$ are

$$(34)\quad E(\hat{P} \mid L_2 \leq \epsilon \leq L_3) = \left[X^* - \left[(\alpha + \gamma q^* + \delta z + \bar{\epsilon}_2)\right]/(\beta + \gamma X^*)\right.$$

$$E\left[\exp(\gamma\hat{P}) \mid L_2 \leq \epsilon \leq L_3\right] = \exp\left\{\gamma[X^* - \alpha - \gamma q^* - \delta z]/(\beta + \gamma X^*) + \right.$$

$$\left.0.5\left[\gamma\sigma/(\beta + \gamma X^*)\right]^2\right\} \cdot \frac{\left[\Phi(K_3/\sigma) - \Phi(K_2/\sigma)\right]}{\left[\Phi(L_3/\sigma) - \Phi(L_2/\sigma)\right]},$$

$$(35)\quad E(\epsilon \exp \gamma \hat{P} \mid L_2 \leq \epsilon \leq L_3) = \exp\left\{\gamma(X^* - \alpha - \gamma q^* - \delta z)/(\beta + \gamma X^*) + .5\left[\gamma\sigma/(\beta + \gamma X^*)\right]^2\right\} \cdot \left[\Phi(K_3/\sigma) - \Phi(K_2/\sigma)\right]/\left[\Phi(L_3/\sigma) - \Phi(L_2/\sigma)\right] \cdot \sigma\left\{\frac{\varphi(K_2/\sigma) - \varphi(K_3/\sigma) + (\gamma\sigma/\beta + \gamma X^*)[\Phi(K_2/\sigma) - \Phi(K_3/\sigma)]}{\Phi(L_3/\sigma) - \Phi(L_2/\sigma)}\right\},$$

where

$$(36)\quad K_2 = L_2 + (\gamma\sigma^2)/(\beta + \gamma X^*),$$

$$(37)\quad K_3 = L_3 + (\gamma\sigma^2)/(\beta + \gamma X^*).$$

Compensating Variation for Segment 2

For a given household on segment 2, $X^* < X < \min\left[Y^*/P_2, -\beta/\gamma\right]$. Therefore, the residual for the household on segment 2 is located between L_3 and L_4 as defined by the inequalities,

$$(38)\quad q^* - (\alpha + \beta P_2 + \gamma Y^* + \delta z) < \epsilon < \min\left[Y^*/P_2, -\beta/\gamma\right] - (\alpha + \beta P_2 + \gamma Y^* + \delta z).$$

Assume that ϵ is normally distributed and has the probability density function (24) $f(\epsilon) = 1/(2\pi\sigma)^{1/2}\left[\exp(-\epsilon^2/2\sigma^2)\right]$. The distribution is truncated; therefore, the probability $L_3 < \epsilon < L_3$ denoted $Pr(L_3 < \epsilon < L_4)$ is given by $\Phi(L_4/\sigma) - \Phi(L_3/\sigma)$.

The conditional probability density for ϵ, given that the household is on segment 2, is

$$(39)\quad f(\epsilon \mid L_3 < \epsilon < L_4) = \frac{\left[1/(2\pi\sigma)^{1/2}\exp\left\{-\epsilon^2/2\sigma^2\right\}\right]}{\left[\Phi(L_4/\sigma) - \Phi(L_3/\sigma)\right]}.$$

Thus the conditional mean $\bar{\epsilon}_3$ is

$$(40)\quad \bar{\epsilon}_3 = \sigma\left[\varphi(L_3/\sigma) - \varphi(L_4/\sigma)\right]/\left[\Phi(L_3/\sigma) - \Phi(L_4/\sigma)\right] = \sigma\left[\varphi(L_3/\sigma) - \varphi(L_4/\sigma)\right]/\left[\Phi(L_4/\sigma) - \Phi(L_3/\sigma)\right].$$

Equation (40) adjusts the probability density so that the conditional probability density function accounts for the loss of area due to the truncation at L_3 and L_4 when $L_3 < \epsilon < L_4$.

Compensating variation for segment 2 is obtained by solving $v(P_0,Y) = v(P_2,Y + (P_2 - P_1)X^* - CV_3)$ for CV_3.

$$(41)\quad CV_3 = 1/\gamma\left[\gamma\left(P_2 - P_0\exp\left\{\gamma(P_2 - P_1\right\}\right) + (\alpha + \gamma Y + \delta z + \epsilon + \beta/\gamma)\cdot \left(1-\exp\left\{\gamma(P_2-P_0)\right\}\right)\right] + (P_2 + P_1)X^*$$

To obtain the expected value of CV_3, substitute $\bar{\epsilon}_3$ into equation (40).

Note that in this problem CV_1 is necessarily positive. CV_2 can be either positive or negative and CV_3 is always negative. In general, CV_1 is always positive; CV_2 and CV_3 can be positive or negative depending on (z,Y,ϵ). For any given income class there will be households on each of the two segments and the corner.

CHAPTER 4

DATA, EMPIRICAL MODEL, COMPENSATING VARIATION ESTIMATES

This chapter provides a description of the method used to generate the data set used in the study. The empirical demand model is presented, and compensating variation is calculated from the demand parameters that were estimated.

Data

The data in this study was generated from the Pacific Northwest Residential Energy Consumption Survey (USDOE, Bonneville Power Administration, 1983). The survey questioned households about demographic aspects of the residents, along with household appliance stock, and the energy efficiency of the home. The original data contained billing information on each household from August 1981 to April 1983. However, billing information for all 20 months for each of the households was incomplete.

Description of Variables

The first step was to generate a subset of the Montana Power customers. This sample contained 553 households. At the time of the survey, Montana Power Company was regulated by a flat rate structure. During the 20 month sampling period there was a price change from

2.9053 to 3.1611 cents per kilowatt hour. In an attempt to obtain a greater variation in the price variable, each monthly price was deflated by the corresponding monthly Consumer Price Index for all items less energy. An average real monthly price per kilowatt hour was calculated for each household.

The dependent variable employed was average electricity consumption per month, where the average was taken over the months reported for each household. The number of months of billing information was used as a weight in the regression analysis to correct for heteroskedasticity.

A vector of binary variables which described the appliance stock was obtained from the original data set. These variables included electric heat, electric water heat, an electric secondary heat source, separate freezer, washer, dryer, and dishwasher. Binary variables were created for the level of education. There were five categories for the level of education: elementary school, high school, trade school or some college, college, and post-graduate. A binary variable was also used to indicate winter months which were considered to be November through March. The winter month dummy variable was averaged over the months reported for each household to obtain an index of the proportion of winter months for the household. There were also variables for the number of heated rooms, the number of refrigerators, the number of residents, and the age of the head of household.

Variables for percentage of the outside walls that contained insulation and the percentage of the ceiling that contained insulation were constructed for the six categories in the survey. The categories included: 0%, 10-39%, 40-59%, 60-89%, and 90-100%. The final variables were created by using the midpoint of the category the individual was in. A zero was assigned to the first category. The income variable was constructed from 16 categories: $0-4,999, $5,000-7,999, $8,000-9,999 $10,000-11,999, $12,000-13,999, $14,000-15,999, $16,000-17,999, $18,000-19,666, $20,000-24,999, $25,000-29,999, $30,000-34,999, $35,000-39,999, $40,000-49,999, $50,000-59,999, $60,000-75,000, and over $75,000. The midpoint of each category below $75,000 was used with $75,000 for the final category. Income was deflated to 1987 dollars by multiplying by (1982 CPI)/(1987 CPI). Annual income was then divided by 12 to obtain monthly income.

Empirical Model

The first model was constructed to estimate an instrumental variable for permanent income of each household. This permanent income estimate was then used in the demand models. Ordinary least squares was used to estimate permanent income. Permanent income was estimated by regressing annual income on age, age squared, binary variables for level of education, and binary variables for big ticket durable goods.

Age and age squared were used in the model to simulate the earning cycle of an individual over his lifetime. Level of education is generally accepted as being strongly positively correlated with income. The ownership of various big ticket disposable goods by the household was used as a proxy for information on the permanent consumption level of the household. Permanent income was estimated using the following model:

$$(42)\quad YP = \beta_0 + \beta_1 AGE + \beta_2 AGE2 + \beta_3 EG + \beta_4 TS + \beta_5 CG + \beta_6 PG + \beta_7 FRZ + \beta_8 HR + \beta_9 IR + \beta_{10} IW + \beta_{11} W + \beta_{12} D + \beta_{13} DW + \beta_{14} F2.$$

The variables are defined in Table 2.

To estimate equation (42), another data set was generated. The second data set was required because only 379 of the 553 households reported the age of the head of the household. The instrument variable for permanent income was estimated from this smaller data set. The parameters and T-Ratios are reported in Table 3.

All of the demand equations were estimated using weighted least squares where the regressions were weighted by the number of months of billing information available for the household. The first demand model used the current income variable generated from the survey categories. The demand model also contained interaction terms between income and the presence of electric heat, and income and the number of residents in the household. These variables were generated in the SHAZAM software program by multiplying YT and EH together, and YT and RR together. Demand model 1 was estimated using

Table 2. Definition of Variables.

Variable	Variable Definition
AGE	Age of the Head of the Household
AGE2	AGE*AGE of the Head of the Household
EG	Elementary School Graduate
TS	Trade School Graduate or Some College
CG	College Graduate
PG	Post-Graduate Study
FRZ	Freezer
HR	Number of Heated Rooms
IW	Percentage of Insulation in the Walls
IR	Percentage of Insulation in the Roof
W	Washer
D	Dryer
DW	Dishwasher
F2	Number of Refrigerators
EH	Electric Heat
EW	Electric Water Heat
SH	Electricity Used for a Secondary Heat Source
RR	Number of Residents
WM	Winter Month
AP	Average Price
YT	Temporal Income
PI	Permanent Income
YTEH	YT*EH
YTRR	YT*RR
YPEH	YP*EH
YPRR	YP*RR

$$(43)\quad KWH = \beta_0 + \beta_1 EH + \beta_2 EW + \beta_3 SH + \beta_4 RR + \beta_5 WM + \beta_6 AP + \beta_7 YT + \beta_8 YTEH + \beta_9 YTRR.$$

See Table 2 for a description of the variable names. Parameters and T-ratios are reported in Table 4.

Demand models 2, 3, 4, and 5 were estimated using the instrument variable for permanent income. Recall that permanent income was

Table 3. Estimation of Permanent Income.

Variable	Parameter (T-Ratio)
AGE	40.735
	(2.198)
AGE2	-.506
	(-2.819)
EG	-123.380
	(-.807)
TS	286.380
	(2.114)
CG	486.110
	(2.54)
PG	767.210
	(2.945)
FRZ	107.910
	(.950)
HR	134.910
	(4.400)
IW	2.899
	(1.762)
IR	2.008
	(1.062)
W	-36.893
	(-.154)
D	5.812
	(.025)
DW	363.220
	(2.929)
F2	269.390
	(2.007)
CONSTANT	-742.530
	(-1.670)
R^2	.324

estimated from the smaller data set. To use the permanent income variable to estimate demand with the large data set, age and age squared were taken at their means. Permanent income was generated from the following equation in the SHAZAM software program:

Table 4. Estimation of Electricity Demand for the Five Models.

Variable	Parameter (T-Ratio) Model 1	Model 2	Model 3	Model 4	Model 5
EH	663.53	458.72	458.35	458.57	458.78
	(4.81)	(1.85)	(1.85)	(1.85)	(1.85)
EW	482.86	480.04	491.37	484.79	478.21
	(6.14)	(6.01)	(6.38)	(6.30)	(6.21)
SH	313.79	278.81	280.09	279.35	278.6
	(4.24)	(3.69)	(3.71)	(3.70)	(3.69)
RR	-37.687	-113.21	-111.73	-112.59	-113.45
	(-1.14)	(-1.96)	(-1.94)	(-1.96)	(-1.97)
WM	1131.1	1295.0	1581.9	1415.2	1248.6
	(1.75)	(1.96)	(4.02)	(3.60)	(3.17)
AP	-10463	-212.49	-62543	-38552	14560
	(-.14)	(-.28)	(fixed)	(fixed)	(fixed)
YT	-.1064				
	(-2.07)				
YTEH	.1311				
	(2.41)				
YTRR	.0935				
	(6.09)				
YP		.0393	.043	.041	.039
		(.35)	(.383)	(.365)	(.347)
YPEH		.265	.266	.265	.265
		(2.04)	(2.05)	(2.04)	(2.04)
YPRR		.1243	.123	.124	.124
		(4.00)	(3.98)	(4.00)	(4.01)
CONSTANT	133.78	271.06	1583.6	821.04	58.452
	(.056)	(.11)	(5.91)	(3.06)	(2.18)
R^2	.481	.465	.465	.465	.465

$$(44)\ YP = \hat{\beta}_0 + \hat{\beta}_1 * \overline{AGE} + \hat{\beta}_2 * \overline{AGE2} + \hat{\beta}_3 * EG + \hat{\beta}_4 * TS + \hat{\beta}_5 * CG$$
$$+ \hat{\beta}_6 * CG + \hat{\beta}_7 * PG + \hat{\beta}_8 * HR + \hat{\beta}_9 * IR + \hat{\beta}_{10} * IW + \hat{\beta}_{11} * W + \hat{\beta}_{12} * D$$
$$+ \hat{\beta}_{13} * DW + \hat{\beta}_{14} * F2.$$

$\overline{AGE}$ and $\overline{AGE2}$ are mean values from the large data set and the $\hat{\beta}_{0-14}$ are the parameter estimates from equation (42).

The remaining four demand models were the same as the first demand model, except that permanent income was substituted for current income. The interaction terms were generated in the same manner as in demand model 1 except the permanent income was employed. The equation used to estimate demand models 2, 3, 4, and 5 was

$$(45)\ KWH = \beta_0 + \beta_1 EH + \beta_2 EW + \beta_3 SH + \beta_4 RR + \beta_5 WM + \beta_6 AP + \beta_7 YP$$
$$+ \beta_8 YPEH + \beta_9 YPRR.$$

The variables are defined in Table 2. These four models were estimated using ordinary least squares and were weighted by the number of months. Model 2 was estimated with no restrictions, while models 3, 4, and 5 were estimated where the income elasticities were restricted to -1.87, -1.15, and -.44. These values were the high, middle, and low values obtained from the literature (Bohi, 1981). The parameters and their T-ratios are reported in Table 4.

Compensating Variation Estimates

Compensating variations were calculated from these five demand models using FORTRAN program 1 shown in the appendix. Note that FORTRAN program 1 calculates compensating variation for the first

demand model (model 1). There are a few variables in the program that must be changed in order to calculate compensating variation for demand models 2, 3, 4, and 5. The variables that must be changed are: G the coefficient on AP, D the appropriate coefficients for YP must be substituted for YT, and the appropriate coefficients must be substituted for YPEH and YPRR. Sig must also be changed to the correct standard error of the estimate for each demand model.

The lifeline rate structure that is proposed has different rates for summer and winter months. The winter rate is 4.35 cents per kilowatt hour for the first 400 kilowatt hours; the rate then increases to 5.8 cents per kilowatt hour. The summer rate is 3.36 cents per kilowatt hour for the first 300 kilowatt hours, then increases to 4.48 cents per kilowatt hour.

Compensating variation was calculated for the 16 income categories under the summer and winter rate structures, and for households both with and without electric heat. Compensating variations are reported for each of the 16 income categories for each of the five demand models in Tables 5, 6, 7, 8, and 9.

Summer and winter values were added together and multiplied by 6 to give a yearly compensating variation. These compensating variations were calculated separately for households with electric heat and those without electric heat. These values were then multiplied by the probability that the household in that particular income category has electric heat for the former; and the later was multiplied by the probability that the household in that particular

Table 5. Calculation of Compensating Variation for Demand Model 1.

	Electric Heat		No Electric Heat	
Monthly Income	Summer Month	Winter Month	Summer Month	Winter Month
$245	-2.32	-8.89	-1.39	-2.91
$368	-2.68	-9.79	-1.77	-3.42
$883	-3.00	-10.38	-2.10	-3.78
$1079	-3.29	-10.86	-2.07	-4.06
$1276	-3.36	-11.36	-2.27	-4.35
$1472	-3.97	-11.87	-2.32	-4.66
$1668	-4.34	-12.39	-2.43	-4.96
$1864	-4.71	-12.93	-2.51	-5.26
$2208	-5.35	-13.87	-2.50	-5.78
$2698	-6.23	-15.24	-2.37	-6.49
$3189	-7.08	-16.61	-2.24	-7.14
$3679	-7.93	-17.98	-2.13	-7.75
$4415	-9.32	-20.03	-2.00	-8.60
$5397	-11.37	-22.87	-2.29	-9.71
$6378	-13.52	-25.53	-3.11	-10.93
$7359	-15.67	-28.27	-4.23	-12.29

Table 6. Calculation of Compensating Variation for Demand Model 2.

	Electric Heat		No Electric Heat	
Monthly Income	Summer Month	Winter Month	Summer Month	Winter Month
$245	-1.27	-3.95	.83	-1.70
$368	-1.78	-6.02	.45	-1.77
$883	-1.91	-7.31	-.02	-2.01
$1079	-2.06	-8.33	-.44	-2.34
$1276	-2.37	-9.37	-.86	-2.80
$1472	-2.88	-10.46	-1.28	-3.34
$1668	-3.56	-11.60	-1.62	-3.99
$1864	-4.35	-12.80	-1.87	-4.67
$2208	-5.78	-14.96	-2.04	-5.88
$2698	-7.76	-18.09	-1.98	-7.47
$3189	-9.93	-21.22	-2.02	-8.94
$3679	-12.33	-24.36	-2.67	-10.46
$4415	-16.01	-29.06	-4.53	-13.04
$5397	-20.39	-35.33	-7.01	-16.76
$6378	-25.86	-41.61	-9.38	-20.47
$7359	-30.78	-47.88	-12.20	-24.18

Table 7. Calculation of Compensating Variation for Demand Model 3.

	Electric Heat		No Electric Heat	
Monthly Income	Summer Month	Winter Month	Summer Month	Winter Month
$245	.20	-.93	.76	-.51
$368	-.87	-1.59	1.48	-.81
$883	-1.30	-2.39	1.51	-.96
$1079	-1.40	-3.27	1.36	-.93
$1276	-1.38	-4.31	1.03	-.97
$1472	-1.38	-5.43	.71	-.91
$1668	-1.50	-6.55	.23	-.92
$1864	-1.85	-7.66	-.36	-.98
$2208	-3.06	-9.55	-1.15	-1.24
$2698	-5.35	-12.38	-1.80	-2.13
$3189	-7.43	-15.47	-1.70	-3.66
$3679	-9.54	-18.64	-1.32	-5.47
$4415	-13.18	-32.35	-1.60	-8.07
$5397	-18.12	-29.64	-4.40	-11.15
$6378	-23.06	-35.35	-7.11	-14.68
$7359	-28.00	-42.22	-9.41	-18.45

Table 8. Calculation of Compensating Variation for Demand Model 4.

	Electric Heat		No Electric Heat	
Monthly Income	Summer Month	Winter Month	Summer Month	Winter Month
$245	-.65	-2.05	1.21	-1.61
$368	-1.55	-3.81	1.22	-1.44
$883	-1.69	-5.20	1.03	-1.31
$1079	-1.67	-6.30	.64	-1.25
$1276	-1.70	-7.37	.09	-1.29
$1472	-1.93	-8.40	-.45	-1.47
$1668	-2.42	-9.44	-.95	-1.81
$1864	-3.15	-10.52	-1.46	-2.29
$2208	-4.70	-12.56	-2.01	-3.41
$2698	-6.79	-15.68	-2.00	-5.28
$3189	-8.80	-18.83	-1.62	-7.02
$3679	-11.13	-21.96	-1.61	-8.53
$4415	-14.83	-26.67	-3.14	-10.77
$5397	-19.75	-32.95	-6.09	-14.32
$6378	-24.68	-39.23	-8.33	-18.02
$7359	-29.61	-45.51	-10.97	-21.78

Table 9. Calculation of Compensating Variation for Demand Model 5.

	Electric Heat		No Electric Heat	
Monthly Income	Summer Month	Winter Month	Summer Month	Winter Month
$245	-1.40	-4.79	.41	-1.92
$368	-1.86	-6.80	0.00	-2.28
$883	-2.08	-8.07	-.44	-2.71
$1079	-2.34	-9.11	-.79	-3.16
$1276	-2.76	-10.19	-1.14	-3.70
$1472	-3.33	-11.32	-1.47	-4.30
$1668	-4.02	-12.50	-1.69	-4.93
$1864	-4.79	-13.72	-1.88	-5.57
$2208	-6.15	-15.90	-2.01	-6.68
$2698	-8.15	-19.02	-2.08	-8.81
$3189	-10.39	-22.15	-2.39	-9.69
$3679	-12.80	-25.28	-3.19	-11.31
$4415	-16.47	-29.98	-4.99	-13.98
$5397	-21.39	-36.25	-7.34	-17.69
$6378	-26.31	-42.45	-9.82	-21.39
$7359	-31.23	-48.79	-12.67	-25.10

income category does not have electric heat. These two values were then added and multiplied by the probability that the household is in that particular income category. This yields the expected compensating variation for each income group. Refer to Table 10 for the probability that a household was in a given income category, and the probability that a household in a given income category had electric heat.

Due to the few number of households in the top three income categories, these categories were combined into households earning more than $65,000 dollars per year (in 1987 dollars). An average compensating variation was also calculated for each of the five demand models. The expected compensating variations for each demand model and income are shown in Table 11. The average compensating variation for each demand model is also found in Table 11. The income levels in the table are the midpoints from the survey deflated to 1987 dollars.

Table 10. Percent of Households in the Income Category and the Percentage of Households with Electric Heat.

Monthly Income	Percent Income	Percent Electric Heat
$245	.0847	.2174
$368	.0866	.1489
$883	.0700	.1579
$1079	.0626	.2059
$1276	.0810	.1136
$1472	.0681	.1351
$1668	.0350	.2632
$1864	.0442	.1667
$2208	.1363	.2432
$2698	.0847	.2474
$3189	.1031	.1964
$3679	.0405	.4091
$4415	.0571	.3226
$5397	.0110	.5000
$6378	.0147	.6250
$7359	.0203	.1818

Table 11. Expected Compensating Variation for a Year.

Yearly Income	Model 1	2	3	4	5
$245	-2.95	-.92	.01	-.46	-1.28
$638	-3.26	-1.19	.06	-.52	-1.69
$883	-2.97	-1.33	-.04	-.55	-1.79
$1079	-2.93	-1.63	-.26	-.80	-2.06
$1276	-3.68	-2.22	-.26	-1.02	-2.80
$1472	-3.34	-2.38	-.47	-1.25	-2.84
$1668	-2.07	-1.71	-.56	-1.08	-1.93
$1864	-2.50	-2.20	-.71	-1.43	-2.46
$2208	-8.95	-9.03	-4.00	-6.79	-9.77
$2698	-5.90	-6.62	-3.55	-5.38	-7.09
$3189	-7.75	-9.23	-5.44	-7.65	-9.96
$3679	-3.99	-5.53	-3.78	-4.75	-5.87
$4415	-5.70	-9.06	-6.29	-7.82	-9.54
$5397	-6.76	-12.72	-10.39	-11.47	-13.09

CHAPTER 5

SUMMARY AND IMPLICATIONS

The purpose of this study was to estimate residential electricity demand for the state of Montana and to study the welfare effects of a lifeline rate structure. The primary objective was to determine if a particular lifeline rate structure proposed by Thomas Power (1985) would benefit low income households in the state of Montana.

Summary

This study develops the properties of a linear demand function using the expenditure function, where the budget constraint is minimized subject to a given level of utility. The comparative statics are deduced and the Marshallian and Hicksian demand equations are compared.

A method for calculating compensating variation from this linear demand system is developed and applied to a random utility model with a kinked budget constraint. The conditional expected value of the residual $\bar{\epsilon}$ is employed for the unobservable mean of the residual ϵ. The value of the residual is conditional upon which segment of the budget constraint the indifference curve is tangent. An expected compensating variation is calculated for the various income

categories for households with and without electric heat, for both summer and winter months. Average expected compensating variations was also calculated for each of the five demand models.

An important result of this thesis is the development of the framework used to solve for compensating variation. The study employs a linear demand function that results from a utility function that satisfies the postulates of utility theory. The expenditure function, compensating demand function, and compensating variation are derived and their properties are analyzed in detail. This thesis provides the tools necessary to analyze welfare effects for a variety of block rate pricing problems.

Implications

A limitation of this study was the small amount of variation in the price of electricity during the sample period, which led to very imprecise estimates of the effect of price on consumption. Long-run and heteroscedastic demand models were estimated; however, they yielded positive coefficients on the price variables. Nonlinear models were also constructed to estimate demand; however, they also failed to yield negatively sloped demand curves.

For demand models 3, 4, and 5, price elasticities were restricted to values from the literature because of the limited information in the data set. For the unrestricted demand models (1 and 2) price elasticity estimates of -.31 and -.64 were obtained. Note that the permanent income model's elasticity is within the range

of the literature. However, the current model elasticity is less than the lowest estimate from the literature. Parameters from the restricted demand models were used to estimate the welfare effects of the lifeline rate structure. These models were used to provide additional calculations of compensating variation in case the other demand models were erroneous.

The results of this study refute Thomas Power's (1985) claim that a lifeline rate structure would benefit low income households. In all cases except where the price elasticity is restricted to the most elastic estimate (-1.87), all households were made worse off under the proposed lifeline rate structure. Using the most elastic demand model, only households in the first income groups were better off, and only by a few cents per year. All other models yielded results where all income classes were made worse off under the proposed rate structure. The average household's expected compensating variation ranged from -2.55 to -5.16 dollars per year depending on the demand model used.

Results of the study suggest that the proposed rate structure would be inefficient as a means to transfer income to low income households. However, a lifeline rate structure could be constructed such that, on average, low income households would benefit. If the number of kilowatt hours in the first block were increased, the lifeline rate would benefit the average low income household. However, a lifeline rate structure is a poor method to transfer income to low income households because the benefactors of the policy

are not necessarily low income households but rather households that consume small quantities of electricity.

Recall that any block pricing structure is less efficient than marginal cost pricing, if any consumer faces a price at the margin that is different from marginal cost. However, a lifeline rate could be designed to increase efficiency over the current rate structure. This could be accomplished by pricing electricity in the second block near marginal cost. This type of rate structure would greatly increase revenue to Montana Power Company because marginal cost is above average cost. The lower price in the first block could serve to reduce these monopoly profits to a level acceptable to the Public Service Commission. Assuming that consumption decisions are adjusted according to marginal price, this rate structure could increase economic efficiency.

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APPENDIX

Figure 7. FORTRAN PROGRAM 1

```
C       This program computes the compensating variation for a
C       Montana household electricity demand for lifeline rate
C       structure with a linear demand model.


      DIMENSION B(6), Y(16)
      DOUBLE PRECISION A1, A2, A3, A4, B1, B2
      DOUBLE PRECISION Z1, Z2, Z3, Z4
      EH2O = 0.70534
      SHT = 0.33517
      RR = 2.7864
      G = -10463
      DATA B / 663.53, 482.86, 313.79, -37.687, 1131.1, 133.78 /
      DATA Y / 245.30,637.78,883.09,1079.32,1275.56,1471.81,
     &1668.04,1864.29,2207.71,2698.31,3188.92,3679.52,4415.43,
     &5396.63,6377.83,7359.05 /
      OPEN (UNIT = 6, FILE = 'COMP.OUT', STATUS = 'UNKNOWN')
      DO 50 I = 0,1
      EH = I
      D=-.10640+.13109*EH+.093498*RR
      DO 100 J = 0,1
      WM = J
      SM = 1.0 - WM
      IF (I .EQ. 0 .AND. J .EQ. 0) WRITE (6,21)
   21 FORMAT (1X,'No Electric Heat  -  Summer Month',/)
      IF (I .EQ. 0 .AND. J .EQ. 1) WRITE (6,22)
   22 FORMAT ('1',1X,'No Electric Heat  -  Winter Month',/)
      IF (I .EQ. 1 .AND. J .EQ. 0) WRITE (6,23)
   23 FORMAT (1X,'Electric Heat  -  Summer Month',/)
      IF (I .EQ. 1 .AND. J .EQ. 1) WRITE (6,24)
   24 FORMAT (1X,'Electric Heat  -  Winter Month',/)
      A = B(1)*EH + B(2)*EH2O + B(3)*SHT + B(4)*RR + B(5)*WM
     &+B(6)
C
C       SUMMER MONTH RATE STRUCTURE
C
      QST = 300.0
      P0 = 0.0371
      P1 = 0.0336
      P2 = 0.0448
C
```

Figure 7 Cont.

```
C       WINTER MONTH RATE STRUCTURE
C
      IF (WM .EQ. 1.0) THEN
         QST = 400.0
         P0 = 0.0482
         P1 = 0.0435
         P2 = 0.0580
      END IF
C
C       INCOME LEVELS AND STANDARD ERROR OF DEMAND
C
      DO 150 K = 1,16
      YST = Y(K)+ (P2 - P1)*QST
      SIG = 707.28
C
C       SEGMENT BOUNDARIES AND SCALING FACTOR FOR ZERO CONSUMPTION
C
      A1 = - (A + G*P1 + D*Y(K))
      A2 = QST + A1
      A3 = QST - (A + G*P2 + D*YST)
      A4 = AMIN1((YST/P2),(-G/D)) - (A + G*P2 + D*YST)
      Z1 = A1/SIG
      Z2 = A2/SIG
      Z3 = A3/SIG
      Z4 = A4/SIG
      SCL = PHI(Z4) - PHI(Z1)
C
C       SEGMENT ONE
C
      PR1 = PHI(Z2) - PHI(Z1)
      E1 = SIG*(F(Z1) - F(Z2))/PR1
      B1 = 1.0 - EXP(D*(P1 - P0))
      B2 = P1 - P0*EXP(D*(P1 - P0))
      Q1 = A + G*P1 + D*Y(K) + E1
      CV1 = ((A + D*Y(K) + E1 + G/D)*B1 + G*B2)/D
      PR1 = PR1/SCL
C
C       CORNER SOLUTION
C
      PR2 = PHI(Z3) - PHI(Z2)
      E2 = SIG*(F(Z2) - F(Z3))/PR2
      Q2 = QST
      XST = Y(K) - P1*QST
      PHT = (QST - (A + D*XST + E2))/(G + D*QST)
      YHT = Y(K) + (PHT - P1)*QST
      B1 = D*(QST - (A + D*XST))/(G + D*QST)
      B2 = D*SIG/(G + D*QST)
      B3 = DEXP(B1 + 0.5*B2*B2)
      EX1 = B3*(PHI(Z3 + B2) - PHI(Z2 + B2))/PR2
```

Figure 7 Cont.

```
      EX2 = B3*SIG*(F(Z2 + B2) - F(Z3 + B2)
     &    + B2*(PHI(Z2 + B2) - PHI(Z3 + B2)))/PR2
      CV2 = (PHT - P1)*QST
     &    + (A + D*Y(K) + E2 + G/D)/D
     &    - (A + D*Y(K) + G/D)*EXP(-D*PO)*EX1/D
     &    - EXP(-D*PO)*EX2/D
     &    + G*(PHT - PO*EXP(-D*PO)*EX1)/D
      PR2 = PR2/SCL
C
C       SEGMENT 2
C
      PR3 = PHI(Z4) - PHI(Z3)
      E3 = SIG*(F(Z3) - F(Z4))/PR3
      Q3 = A + G*P2 + D*YST + E3
      B1 = 1.0 - EXP(D*(P2 - PO))
      B2 = P2 - PO*EXP(D*(P2 - PO))
      CV3 = ((A + D*Y(K) + E3 + G/D)*B1 + G*B2)/D + (P2 - P1)*QST
      PR3 = PR3/SCL
C
C       EXPECTED COMPENSATING VARIATION, GIVEN INCOME
C
      ECV = PR1*CV1 + PR2*CV2 + PR3*CV3
C
C       WRITE OUT THE INCOME LEVEL
C
      WRITE (6,400) Y(K)
 400  FORMAT (2X,'Monthly Income = $',F8.2)
C
C       WRITE OUT THE PROBABILITIES AND COMPENSATING VARIATIONS
C
      WRITE (6,500) PR1, Q1, CV1
 500  FORMAT (2X,'PR1 = ',F5.3,2X,'Q1 = ',F6.1,2X,'CV1 = $',F6.2)
      WRITE (6,501) PR2, Q2, CV2
 501  FORMAT (2X,'PR2 = ',F5.3,2X,'Q2 = ',F6.1,2X,'CV2 = $',F6.2)
      WRITE (6,502) PR3, Q3, CV3
 502  FORMAT (2X,'PR3 = ',F5.3,2X,'Q3 = ',F6.1,2X,'CV3 = $',F6.2)
      WRITE (6,503) ECV
 503  FORMAT (2X,'Expected CV = $',F6.2,/)
 150  CONTINUE
 100  CONTINUE
 50   CONTINUE
      CLOSE (UNIT = 6)
      STOP
      END
C
      FUNCTION F(Z)
      DOUBLE PRECISION Z
      IF (Z .LT. -20.0 .OR.  Z .GT. 20.0) F = 0.0
      IF (Z .GE. -20.0 .AND. Z .LE. 20.0) F = 0.39894228*
```

Figure 7 Cont.

```
      &  DEXP(-0.5*Z*Z)
      RETURN
      END
C
      FUNCTION PHI(Z)
      DOUBLE PRECISION P, Z
      P = 1.5947*Z + (0.74075/10.0)*(Z**3)
     &  - (0.58536/(10.0**3))*(Z**5) - (0.27625/(10.0**4))*(Z**7)
     &  + (0.14439/(10.0**5))*(Z**9) - (0.20498/(10.0**7))*(Z**11)
      IF (Z .LT. -5.0) PHI = 0.0
      IF (Z .GT.  5.0) PHI = 1.0
      IF (Z .GE. -5.0 .AND. Z .LE. 5.0) PHI = DEXP(P)/(1.0 + DEXP(P))
      RETURN
      END
```