

Non-magnetic slug-tuned inductors
by Charles L A Gies

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering
Montana State University
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Abstract:
The determination of the effects of a nonmagnetic slug for tuning inductors at radio frequencies, to facilitate the design of coils used in tuned r. f. circuits having a band-pass characteristic was the primary purpose of this thesis.

The accomplishment of this purpose necessitated the ascertaining of the manner in which the reactance, the resistance, and the Q of the coil varied with coil size, slug size,and slug position. Equipment limitations prevented a complete analytical derivation of formulae for the design of non-magnetic slug-tuned coils. However, sufficient information was obtained to serve as a basis for the design of non-magnetic slug-tuned coils by use of graphs.

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Master of Science in Electrical Engineering

at

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Approved:

E. W. Schilling
Head, Major Department

R. C. Seibel
Chairman, Examining Committee

L. A. Nelson
Dean, Graduate Division

Bozeman, Montana
June, 1952

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Charles L. A. Gies

ABSTRACT

The determination of the effects of a non-magnetic slug for tuning inductors at radio frequencies, to facilitate the design of coils used in tuned r. f. circuits having a band-pass characteristic was the primary purpose of this thesis.

The accomplishment of this purpose necessitated the ascertaining of the manner in which the reactance, the resistance, and the Q of the coil varied with coil size, slug size, and slug position. Equipment limitations prevented a complete analytical derivation of formulae for the design of non-magnetic slug-tuned coils. However, sufficient information was obtained to serve as a basis for the design of non-magnetic slug-tuned coils by use of graphs.

INTRODUCTION

The usual radio transmitter consists of several cascaded stages - oscillator, buffer, multipliers, driver, and final. Normally the plate circuit and occasionally both the plate and grid circuit of each stage, are tuned. This tuning is customarily accomplished by varying the capacitance which is in parallel with an inductance.

In recent years the manufacturers of transmitters have tended to use a variable inductance and a constant capacitance in the low-powered stages. This has been accomplished by inserting a metallic slug into the inductor and varying the slug position to get the variation of inductance. At first, the type of slug most commonly used for this purpose was either iron or powdered iron. These magnetic slugs increased the inductance and the Q of the inductor, which resulted in a sharp resonance curve and, in turn, required precision tuning.

When a non-magnetic slug, such as brass, was used the inductance and the Q of the inductor was decreased. Thus, the resonance curve was broadened and tuning was no longer so difficult. The tank circuit was no longer resonant at one frequency but resonated over a short range of frequencies. It follows then, that if a transmitter were constructed using non-magnetic slug-tuned inductors

in all tuned circuits except the frequency determining circuit of the oscillator and the final stage, only the final stage would require re-tuning when the frequency of the transmitter was varied over the short range at which these coils were resonant.

When this thesis was undertaken, the commercial practice in designing non-magnetic tuned inductors was by the use of trial and error methods for the physical size of the inductors and slugs. This thesis research was undertaken to determine the relationship between coil and slug sizes for a given inductance change and band-pass width.

LIMITS OF THE INVESTIGATION

To find the relationship between coil and slug sizes for a given inductance change and band-pass width the parameters which had to be found were: inductive reactance, effective resistance of the inductor or coil, power, voltage, and current of the coil. A radio frequency signal generator, radio frequency bridge, and a suitable receiver was used to measure the reactance and effective resistance of the coils.

The currents involved were of the order of tenths of a milliampere, and the voltages were of the order of 1 volt at a frequency of 2 megacycles. This presented a difficult instrumentation problem which was not solved because of equipment limitations.

According to Prof. F. E. Terman[1], the best coil for use in a radio tank circuit is one with a length to diameter ratio of approximately one. Using this ratio and constructing coils of the size that are normally found in low-powered transmitter stages, the coils used were a one-inch, a three-quarter-inch, and a one-half-inch in diameter. The inductance of the coils was small because of the dimension limits from the ℓ/d ratio and diameters.

1. Terman, F.E., 1937 - RADIO ENGINEERING pp 38-41 McGraw Hill Book Company, Inc., New York and London

The small inductance resulted in a high self-resonant frequency, well above the frequency at which measurements were made.

Considering the foregoing, therefore, there were certain requirements to be met in proceeding with the investigation:

1. Use of a coil whose ℓ/d ratio was approximately 1.
2. A small coil similar to those used in commercial transmitters.
3. A coil whose reactance was within the range of the bridge available.

With the coils used, it was necessary to use a frequency of 2 megacycles to keep the reactance within the range of the bridge used.

Thus, because of instrumentation difficulties, this investigation was limited to the change of inductances, change of effective resistances, and Q of the coils selected with non-magnetic slug tuning.

METHOD OF INVESTIGATION

The analysis of the non-magnetic slug-tuned coil by means of coupled-circuit theory was undertaken first. The laboratory investigation followed.

The selection of coils was made so that one set of coils had the same wire size and ℓ/d ratio; another set had the same inductance, but different wire sizes and lengths; and the third set had the same wire size and length. The specifications for the coils are:

	Coil Length inches	Wire Size gauge
Constant length all coils	1.35	24
Constant ℓ/d		
1" diameter coil	1	24
3/4" diameter coil	3/4	24
1/2" diameter coil	1/2	24
Constant Inductance		
1" diameter coil	1-1/8	22
3/4" diameter coil	1-3/16	24
1/2" diameter coil	1-1/8	28

A preliminary investigation was made as to the effects of the chemical composition of the slug on the reactance and resistance variations of the slug-tuned coil. The bridge readings of the reactance and resistance were identical within the accuracy of the bridge, for the three types of brass tested, 62 per cent, 65 per cent and 72 per cent copper.

The voltage applied to the reactance bridge was constant at 0.5 volts and two megacycles for all experimentation.

MAGNETIC FIELD THEORY

The inductance of an air-core coil is difficult to compute and adding a non-magnetic slug increases the difficulties. The inductance is given by the equation

$$L = N \frac{d\phi}{di}$$

where

L is inductance of coil

N is number of turns in the coil

ϕ is the magnetic flux of the coil

The difficulty is in computing ϕ. The type of coil in which the magnetic flux is easiest to compute is a long single-layer coil. In this type of coil only half of the flux reaches the end of the coil; the rest leaks out through the coil itself. This causes complications in computing the magnetic flux produced by the flow of current in the coil. An assumption made by most authors in the treatment of long coils is that there is a "current sheet" in the coil. This is not true with an actual coil as round wires with insulation are used. The use of round wire also results in a winding pitch which will give a current component parallel to the axis. For these reasons the generally adopted treatment gives an approximation, but the results are well within engineering accuracy if the diameter of the wire is very small compared to the

diameter of the coil.

The magnetic field intensity of an air-core coil (H_o) along the axis of the coil is

$$H_o = \frac{2\pi N I_o}{\ell}\left[\frac{\frac{\ell}{2}+b}{\sqrt{r^2+\left(\frac{\ell}{2}+b\right)^2}}+\frac{\frac{\ell}{2}-b}{\sqrt{r^2+\left(\frac{\ell}{2}-b\right)^2}}\right]$$

where

ℓ is length of coil

b is distance from any point on the axis to center of coil

r is radius of coil.

The magnetic flux on the coil axis is given by the equation

$$\phi_o = \mu H_o A$$

where

A is the area normal to the direction of H_o

μ is permeability.

Assuming the coil is an infinitely long coil, the flux density across the mid-section is uniform and

$$\phi = \frac{4\pi\mu NIA}{\ell}$$

When the brass slug is inserted inside the coil, a voltage is induced in the slug. This voltage is

$$-e = \frac{N d\phi}{dt}$$

The self-induced voltage in the slug is approximately zero because the inductance of the slug is practically zero. The current in the slug is determined solely by the induced voltage and effective resistance of the slug. The current, (eddy current) is

$$I_e = \frac{e}{R_2}$$

and is proportional to the field of the coil

$$-I_e = \frac{N\frac{d\phi}{dt}}{R_2}$$

The eddy current (I_e) in turn creates a field around the slug in opposition to the original field and causes a decrease in the original coil field. The smaller ϕ_o results in a decrease in ϕ_s (slug flux) which causes the coil flux to increase slightly and ultimately a point of equilibrium is reached giving a constant resultant flux.

With no slug in the coil, the flux density is uniform and parallel to the axis at the center section of the coil and decreases in magnitude and changes direction as the point in question moves from the center towards either end of the coil. The addition of the slug changes this distribution. When current flows in the slug it flows only in the outer periphery because of skin effect. With current only in the outer portion of the

slug, the flux lines will distort due to the eddy current acting as an inside boundary of the magnetic field. This was proven to be so by the reactance curves of a solid slug and a hollow slug (Fig. 4).

Based on the foregoing findings, the assumption can be made that the total H varies as the H along the coil axis because all the flux goes through the portion of the slug which is carrying the current. With this assumption, the flux around the brass slug due to eddy currents should be proportional to the H of the coil, ($\phi_e \sim \phi_o$). Then as

$$L_o = \frac{N d\phi_o}{di}$$

$$L_o' = L_{(o+s)} = \frac{N d\phi_o'}{di}$$

and

$$\phi_o' = \phi_o - \phi_e$$

$$L_o' - L_o = N \frac{d(\phi_o - \phi_e)}{di} - N \frac{d\phi_o}{di}$$

then

$$\Delta L = N \frac{d\phi_e}{di}$$

but

$$\phi_e \sim \phi_o = \mu HA \text{ or } \phi_e \sim \mu HA$$

therefore

$$\Delta L = N \frac{d(KH)}{di}$$

where K is a proportionality constant. This is substantiated by the graphs (Figs. 1, 5 and 6) of the computed H and the per cent ΔL. These curves have approximately the same slope in the linear portion of the curves.

COUPLED-CIRCUIT THEORY

Using coupled-circuit theory, with the brass slug being thought of as a short-circuited coil inside of the actual coil, the brass slug has an impedance of

$$\dot{z}_2 = R_2 + j\omega L_2$$

and the coil has an impedance of

$$\dot{z}_1 = R_1 + j\omega L_1 \ .$$

Then, using Tang's[2] coupled-circuit solution, equating reals and imaginaries

$$R' = R_1 + \frac{|z_M|^2}{R_2^2 + X_2^2} R_2$$

and

$$X' = X_{L_1} - \frac{|z_M|^2 X_2}{R_2^2 + X_2^2}$$

The unknowns are z_M, R_2, and X_2 and none of these can be measured directly with present equipment. z_M varies with slug position because the mutual inductance between two coils varies with the flux linkage between them, and in this instance, it is impossible to determine the flux linkage with the equipment available. The d.c. value of R_2 can be found and an approximate value of a.c. resistance can be computed by finding the depth of penetration at 2 megacycles. Using the equation $R = \rho A/\ell$ and

2. Tang, K. Y., 1949, ALTERNATING-CURRENT CIRCUITS, pp. 208-211, The International Textbook Company, Scranton, Pennsylvania.

knowing the depth of penetration, R_2 can be computed approximately. An approximation for L_2 can also be made knowing the depth of penetration, using only that portion of the slug which carries current as a single-turn coil and computing the inductance from

$$L = \frac{0.0395\ r^2N^2}{\ell}\ K$$

where K is found from Tables developed by Nagaoka[3]. If R_2 and X_2 could be assumed constant, the Z_M could be solved for different slug positions. This calculation was made, and the results showed that X_2 and R_2 did not remain constant. The value of X_2 will change with the flux of the coil which links the slug. Therefore, X_2 will be greatest when the slug center is at the coil center. R_2 will vary only slightly with a variation in slug position.

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3. U.S.Department of Commerce National Bureau of Standards, 1937, RADIO INSTRUMENTS AND MEASUREMENTS CIRCULAR C74, Table 10, page 283. United States Government Printing Office, Washington, D. C.

SHORT COIL DISCUSSION

A. Reactance Changes

The physical dimensions of the coils used in this research were determined upon the basis of the following factors: the ℓ/d ratios, equipment available, and the desire to use commercial type coils. The choice of slug was determined by experimental data. The inductance change with length of slug, Fig. 2, was found to be approximately linear when the length of the slug was between five-sixteenths of an inch and fourteen-sixteenths of an inch. To simplify the problem, slug lengths were chosen in the linear portion of the curve.

The manner in which the diameter of the slug varied the coil inductance, Fig. 3, was found by experimental data to be approximately $\frac{1}{d^3}$. The slug diameters were chosen so that an appreciable change in inductance would be obtained. The three slug diameters chosen were: seven-eighths of an inch, five eighths of an inch, and three-eighths of an inch.

It was previously shown, page 14, that the change in inductance or reactance was proportional to the magnetic field intensity (H_o) of the coil. As H is dependent upon the length and diameter of the coils, the inductance change will also be dependent upon the ℓ/d

ratio of the coils. This is indicated by Fig. 5 and Fig. 6. Both sets of curves have the same slope in the linear portion of the curve. Figure 5 is for a coil one inch in diameter and one inch in length. Figure 6 also has the same ℓ/d ratio, but the diameter of the coil is three-quarters of an inch. Figure 7 also shows that with the same ℓ/d ratio, the slope of the reactance curves is the same when the same diameter of slug is used.

Figures 7 through 11 show how the inductance varies with length of slug, diameter of slug and ℓ/d ratios of the coils in the proportions stated earlier.

The inductance of a coil is determined by the flux linkage. Treating the slug as a one-turn coil, as the slug moved toward the center of the coil, the flux linkage increased and the eddy currents increased. This produced an increase in slug inductance and slug reactance. The mutual inductance also increased as the flux linkage between the coil and slug increased. In the equation

$$X' = X_1 - \frac{|Z_M|^2 \quad X_2}{R_2^2 + X_2^2}$$

the change in reactance is due to the term

$$\frac{|Z_M|^2 \ X_2}{R_2^2 + X_2^2} \ .$$

With Z_M, X_2 and R_2 varying, it is impossible to solve for any of them with the data obtained. If power measurements could have been made, the equation could have been solved. The only means of doing this known to the author is by calorimetry.

B. Resistance and Q Changes

The equivalent resistance of the coil is composed of two parts: R_1 which is the a.c. resistance of the coil without the slug and the reflected resistance which is given by $\frac{|Z_M|^2 R_2}{R_2^2 + X_2}$. As stated in the reactance discussion, Z_M, R_2, and X_2 vary with slug position and, as of the time of writing, no method of determining the variables has been found using the equipment available.

The experimental data curves of Resistance versus Slug Position, Figures 12 through 17, indicate that the point of maximum resistance varies with coil diameter, slug diameter and slug length. With the largest diameter coils and slugs, the point of maximum resistance occurs where the slug center is approximately at the end of the coil. As the diameters decrease, the point of maximum resistance moves toward the coil center.

As was to be expected, if the coil dimensions remained constant, the magnitude of the resistance maximum decreased with the diameter of the slug.

As the evidence is insufficient to form any reliable conclusions as to the variations of the resistance, the manner in which Q varies is also left in doubt as Q is dependent on X and R, $Q = X_2/R$.

The variation of Q is not linear with slug position, nor does the general shape of these curves vary consistently with various ℓ/d ratios.

SUMMARY OF RESULTS

Preliminary investigation revealed that brass slugs with a percentage of copper ranging from 62 to 72 per cent had, within the accuracy of the reactance bridge used, almost identical effects on the reactance and resistance variations of the non-magnetic slug coil. After this was discovered, all further experimentation was done with brass having 65 per cent copper content.

With the depth of penetration of the eddy currents acting as flux boundaries, the percentage of inductance or reactance change was directly proportional to the magnetic intensity distribution along the axis of the coil. The equation for this is

$$\Delta L = N \frac{d(KH)}{di}$$ where K is a proportionality constant.

From coupled-circuit theory, two equations express the relationships of the reactance and resistance within the slug-tuned coil

$$R_1' = R_1 + \frac{|Z_M|^2}{R_2^2 + X_2^2} R_2$$

and

$$X_1' = X_1' - \frac{|Z_M|^2}{R_2^2 + X_2^2} X_2$$

The term

$$\frac{|Z_M|^2}{R_2^2 + X_2^2} X_2$$

is the change in reactance due to the slug and its position. It is this term which is proportional to

$$N \frac{d(KH)}{di} .$$

The solution of the above two equations required the finding of one additional variable which, at present, could not be determined due to equipment limitations.

Sufficient data were procured to show, by means of curves, the trend of the variations of reactance and resistance of the coil.

CONCLUSIONS AND RECOMMENDATIONS

The non-magnetic slug tuning gives a linear decrease in inductance over most of the variable slug position range. The Q of the coil is reduced sufficiently so as to allow the coil to be resonant over a small range of frequencies without re-tuning. Thus, the non-magnetic slug-tuned coil can be used to advantage in low-powered r.f. amplifiers, especially with pentode or beam-power tubes which require small excitation power.

The results of this thesis indicate that, in the opinion of the author, a nomograph could be constructed to facilitate the design of the non-magnetic slug-tuned coil. To achieve the construction of the nomograph and to prove it mathematically, more information on the slug variables must be procured.

To obtain more information about slug variables, the equipment limitations must be overcome. Special instruments must be constructed, such as an r.f. voltmeter, capable of measuring 0.01 millivolts; a means of measuring power of the order of milliwatts at radio frequency; a means of determining the actual field within the coil; and some means of determining X_2, R_2 or Z_M.

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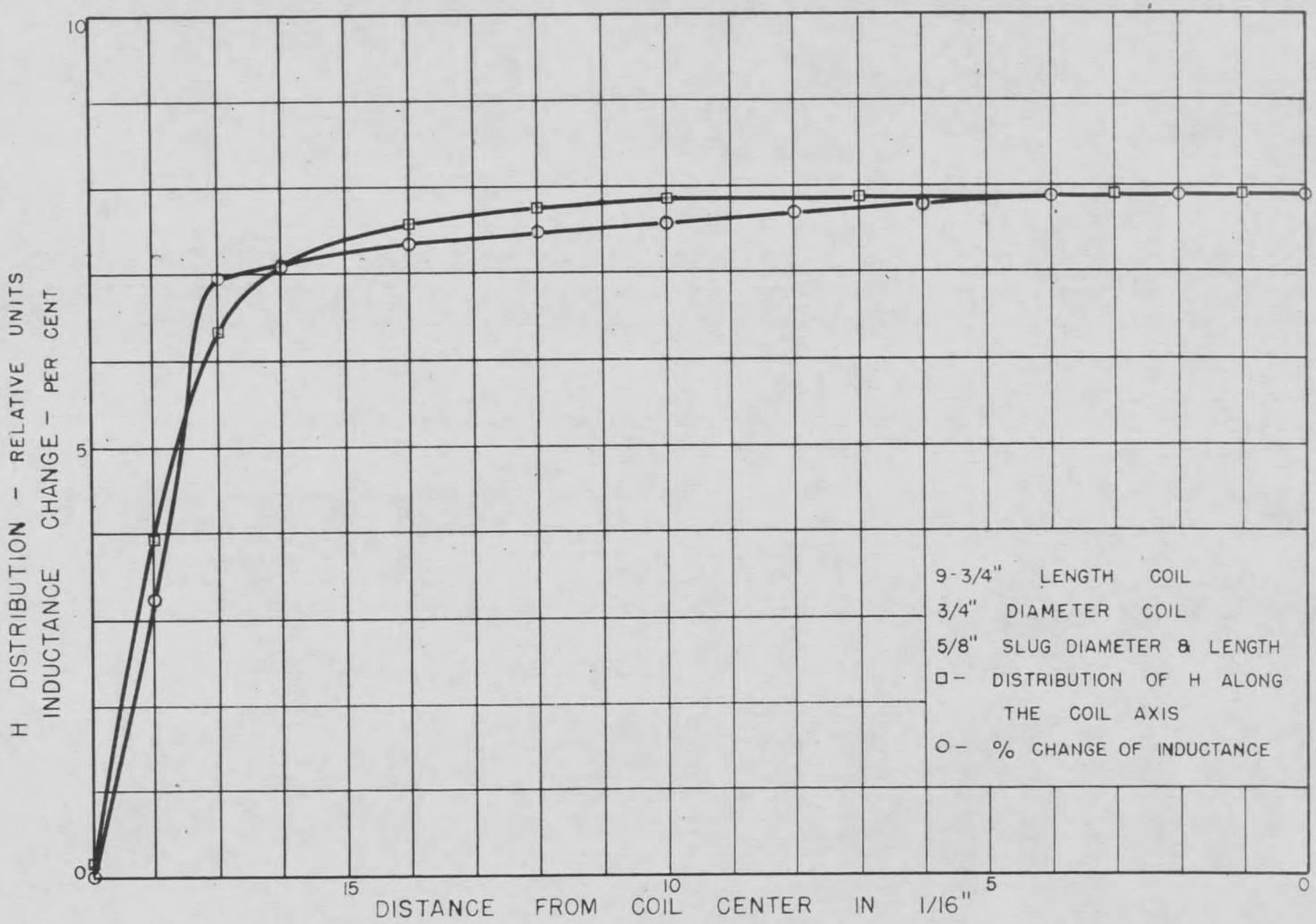
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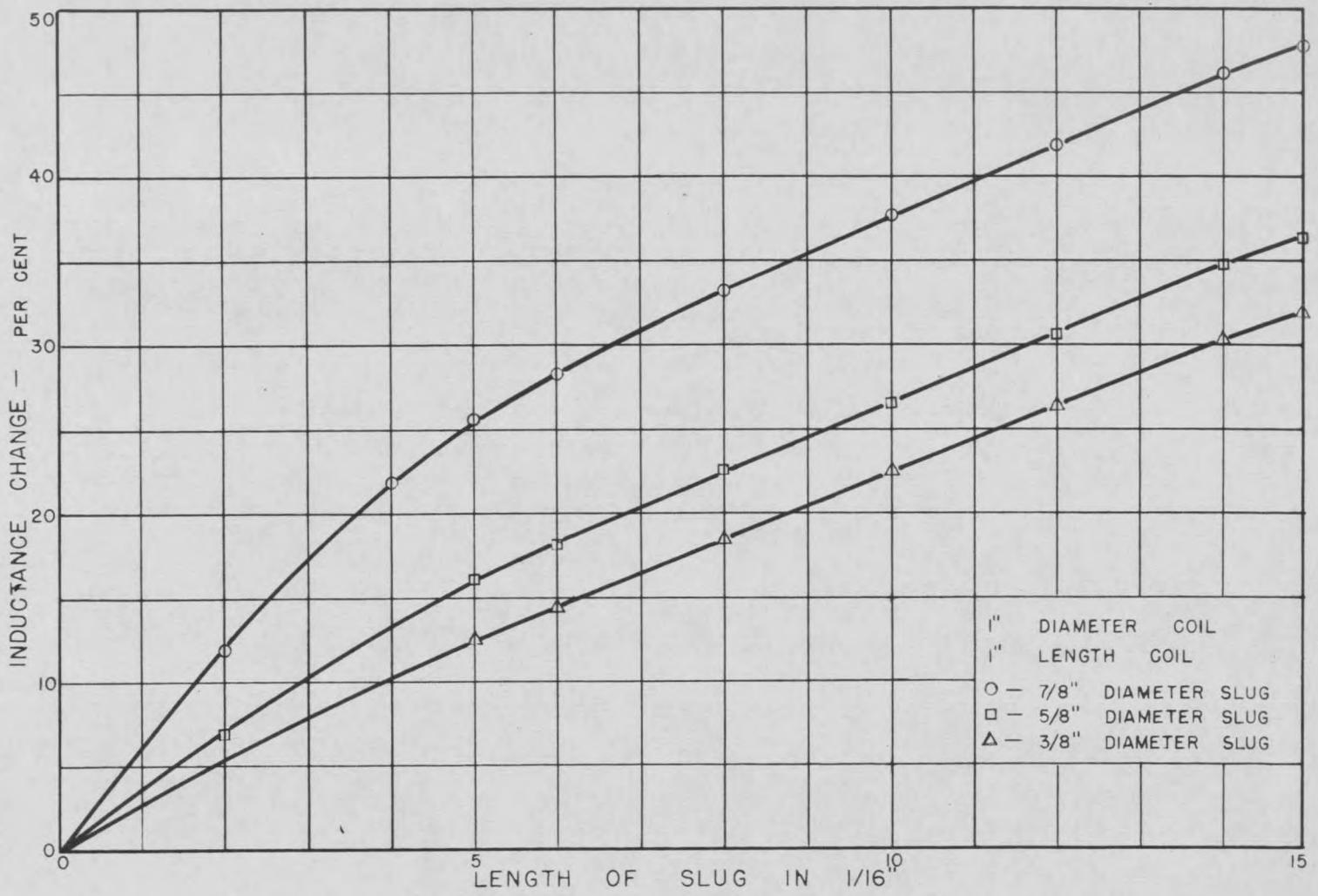
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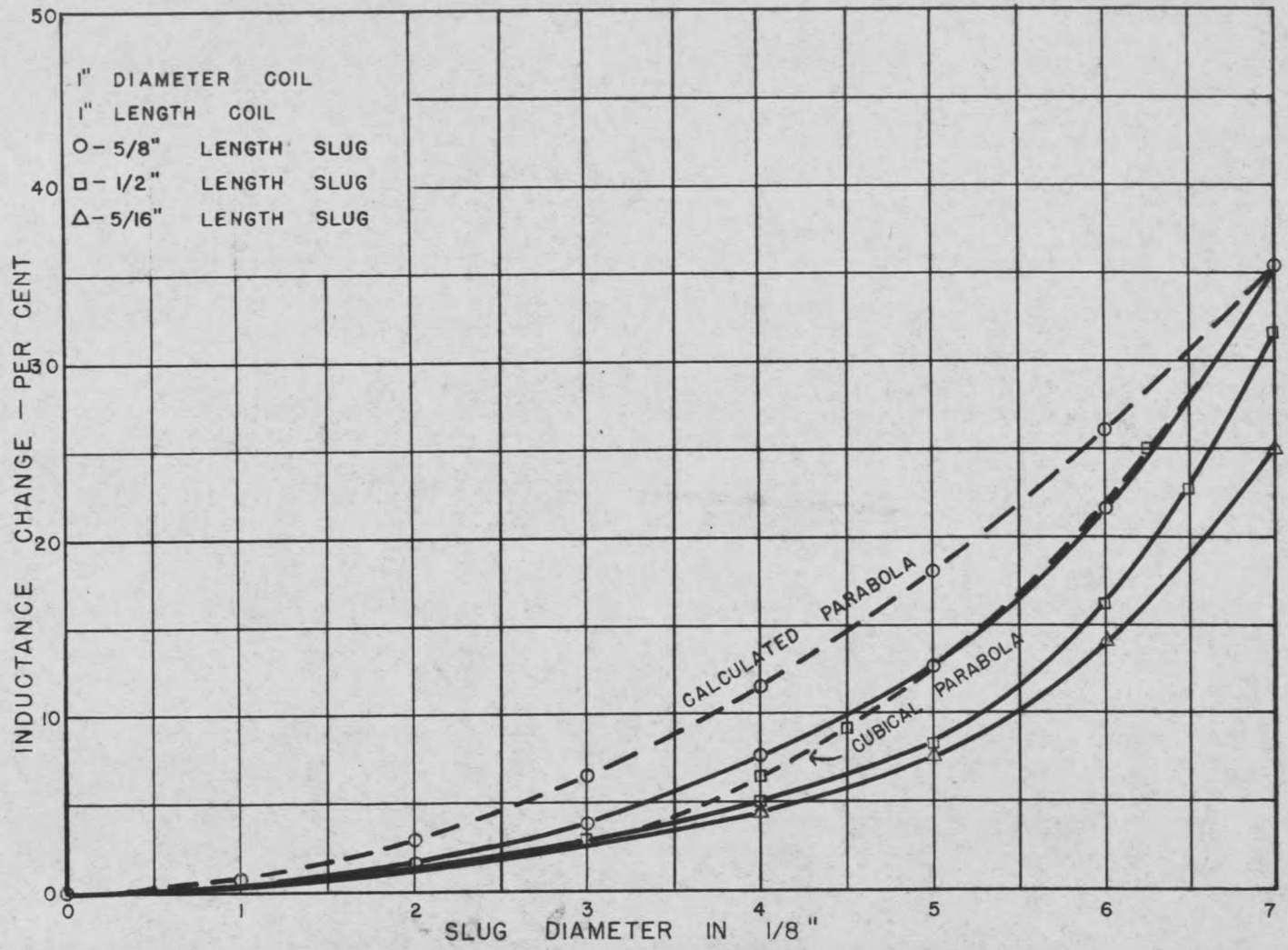
INDUCTANCE CHANGE AND H DISTRIBUTION VERSUS SLUG POSITION FOR A LONG COIL

FIG. 1



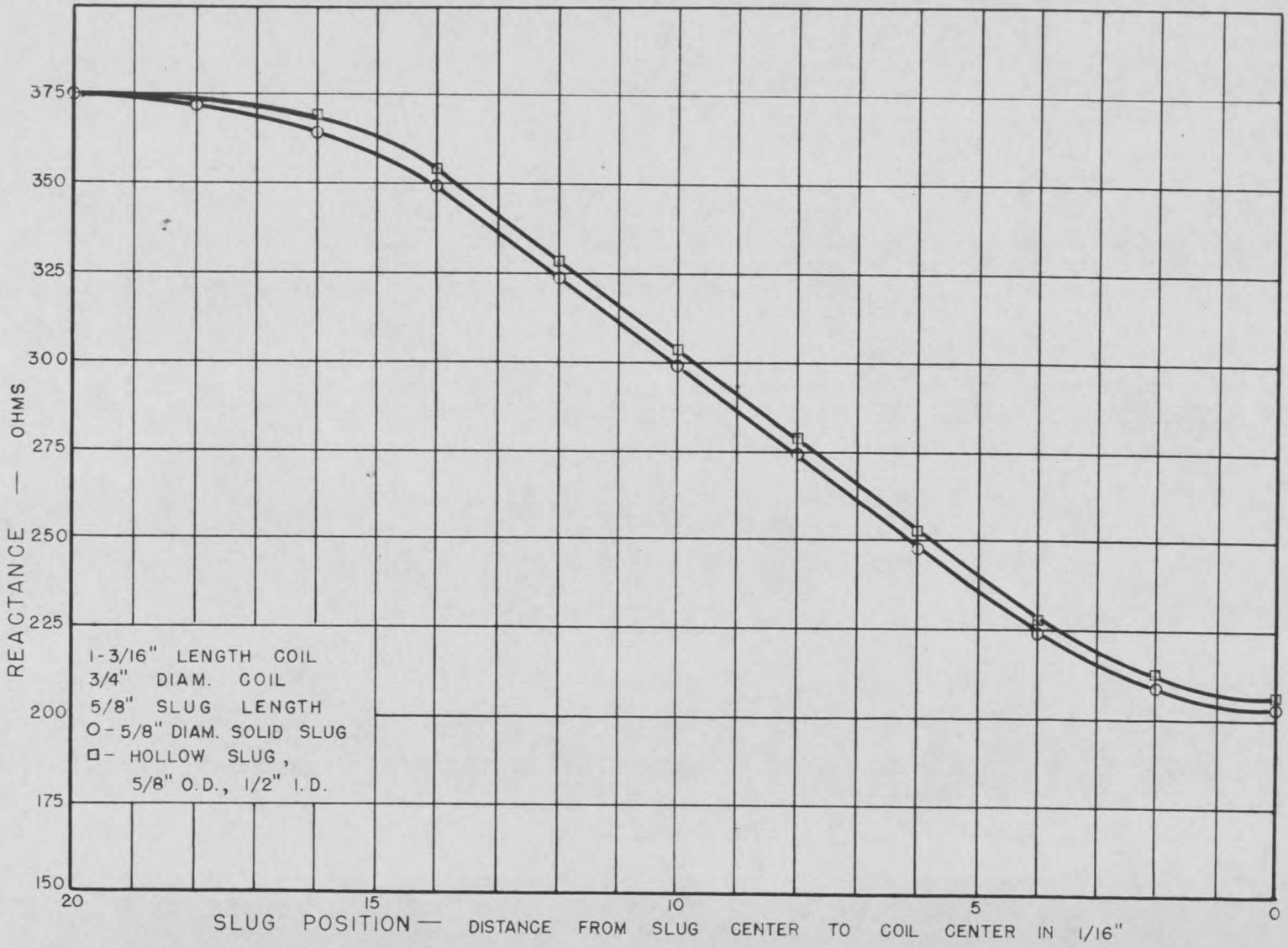
INDUCTANCE CHANGE VERSUS LENGTH OF SLUG

FIG. 2



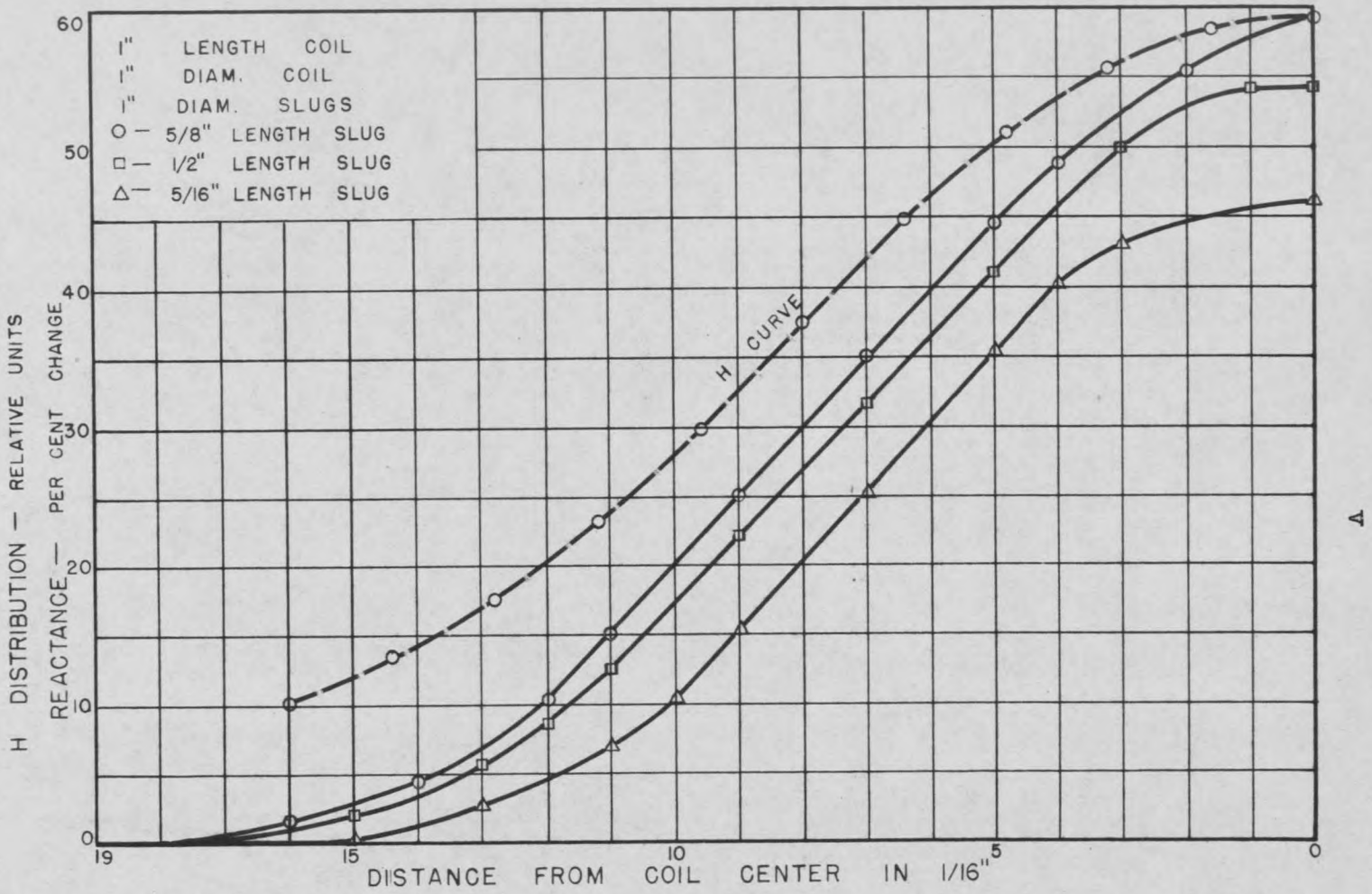
INDUCTANCE CHANGE VERSUS SLUG DIAMETER

FIG. 3



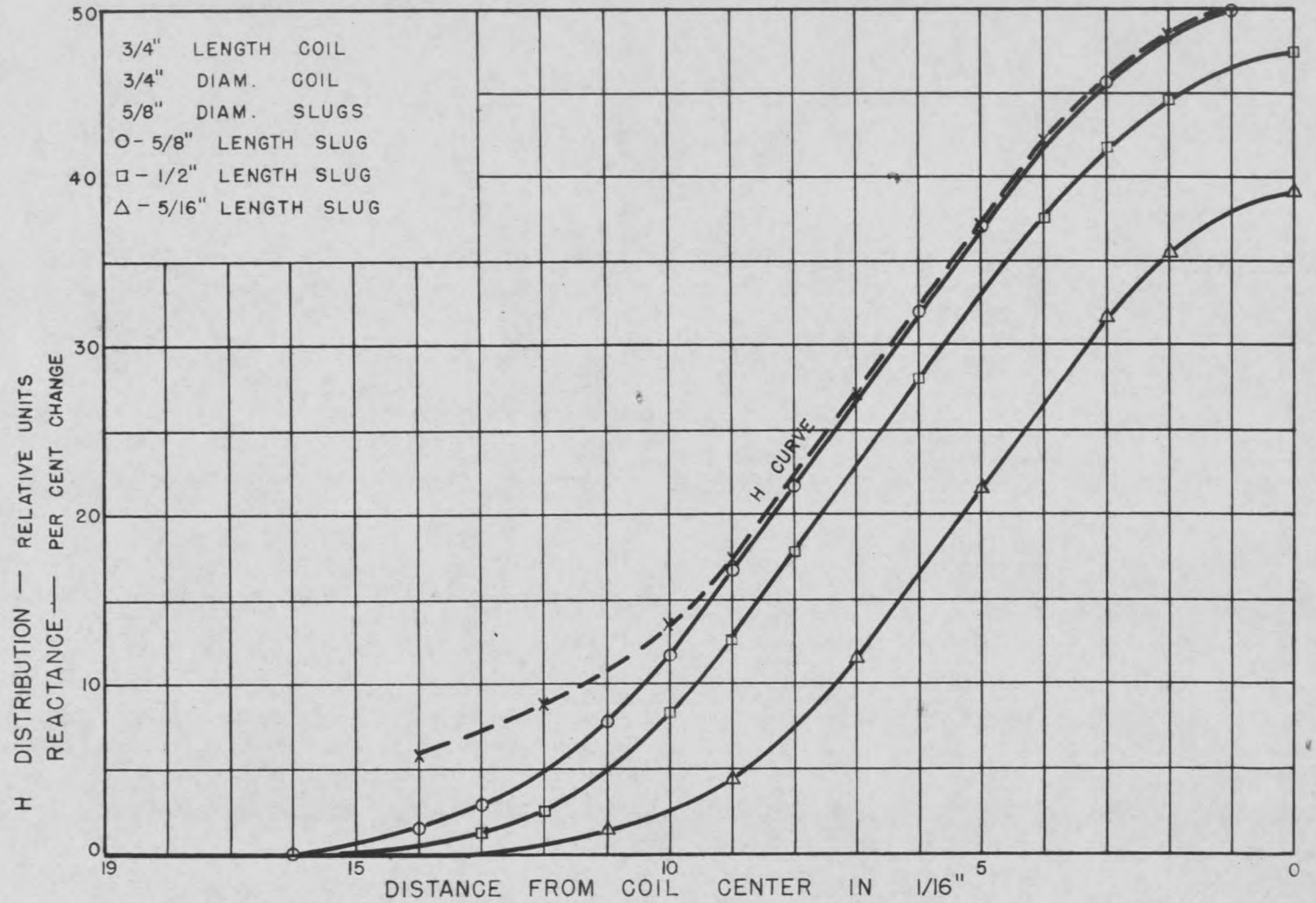
INDUCTIVE REACTANCE VERSUS SLUG POSITION FOR SOLID AND HOLLOW SLUGS

FIG. 4



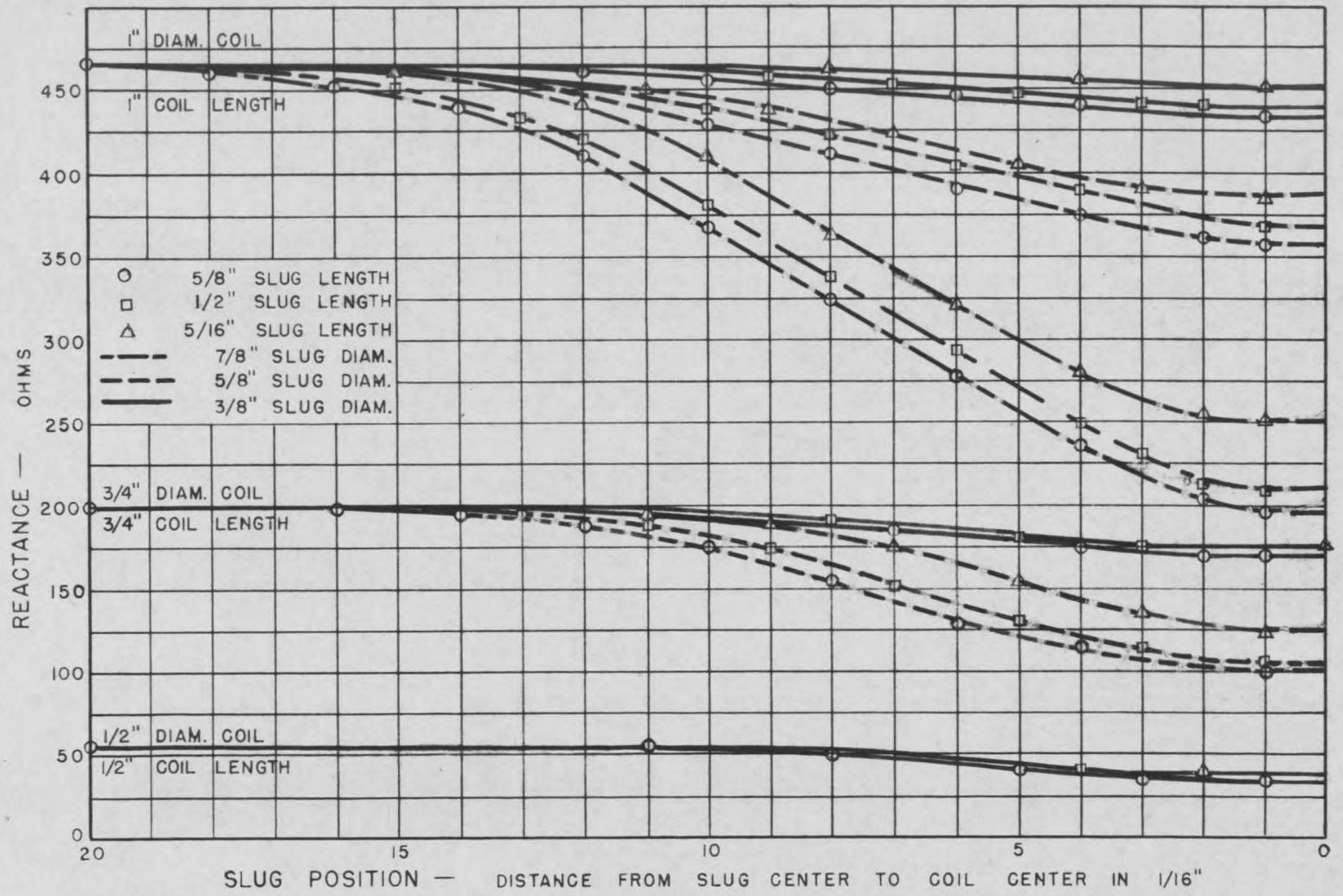
REACTANCE CHANGE AND H DISTRIBUTION VERSUS SLUG POSITION FOR SHORT COIL

FIG. 5

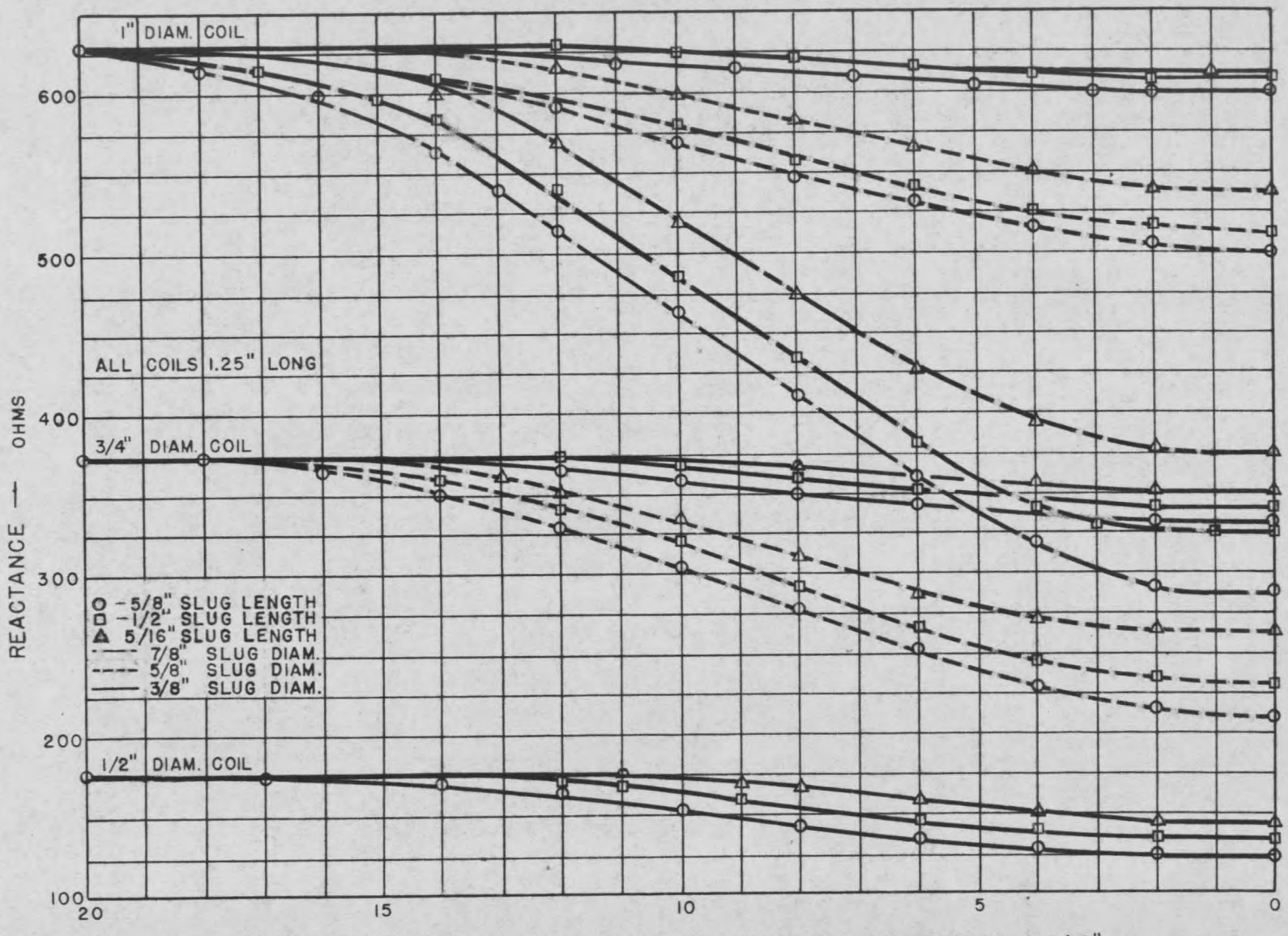


REACTANCE CHANGE AND H DISTRIBUTION VERSUS SLUG POSITION FOR SHORT COIL

FIG. 6

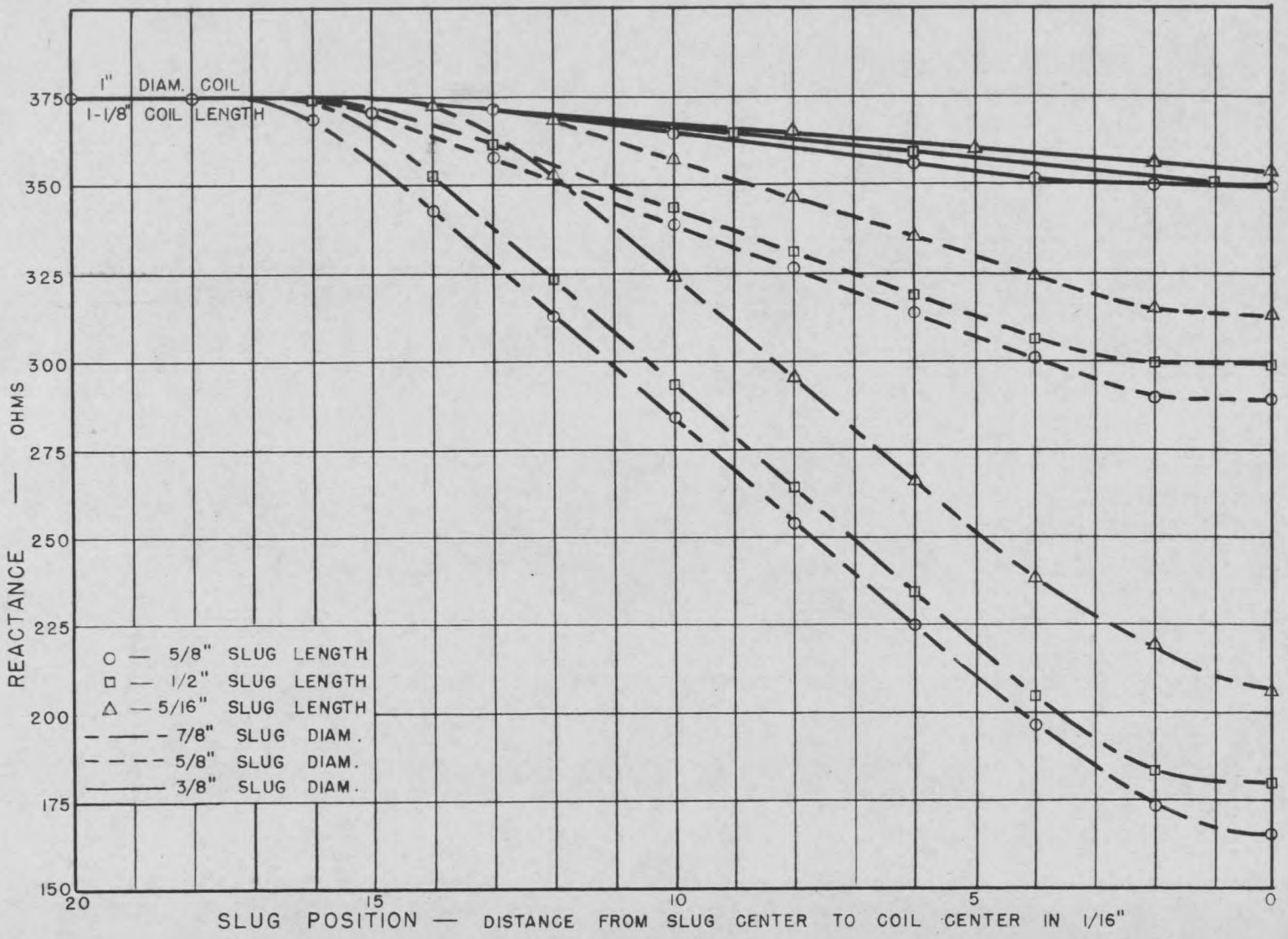


REACTANCE VERSUS SLUG POSITION
FOR
COILS WITH L/D = 1
FIG. 7



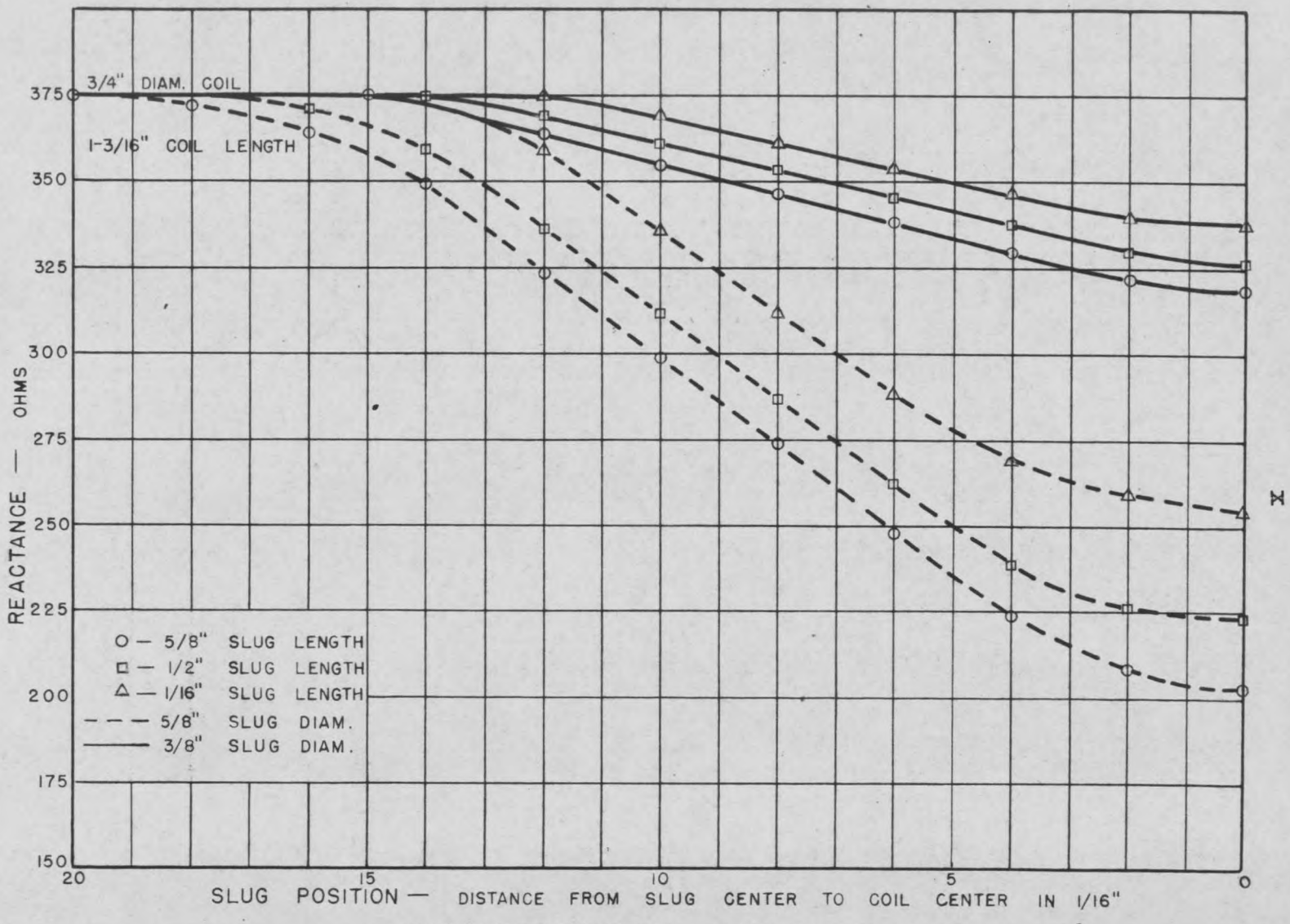
REACTANCE VERSUS SLUG POSITION

FIG. 8



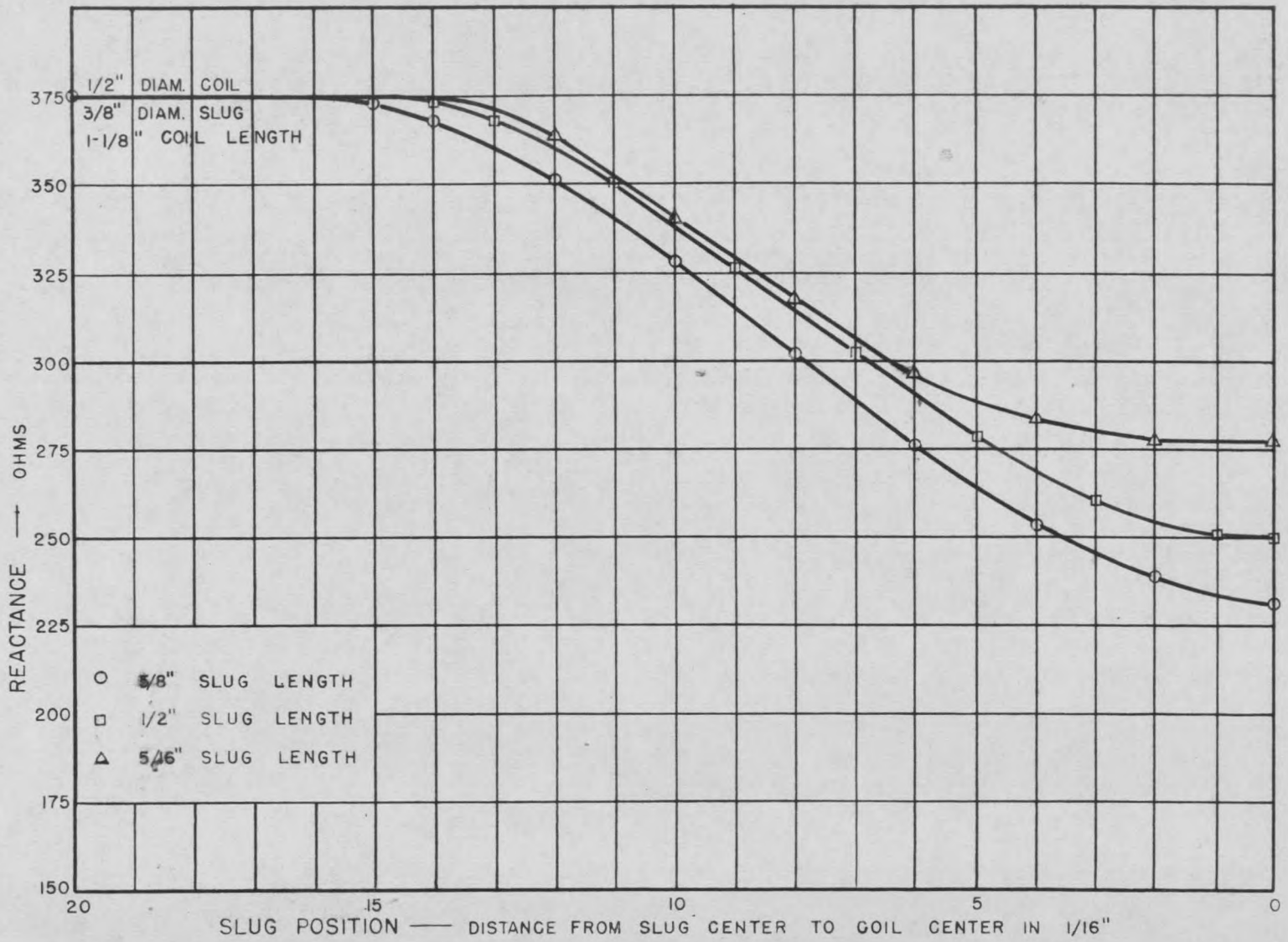
REACTANCE VERSUS SLUG POSITION

FIG. 9



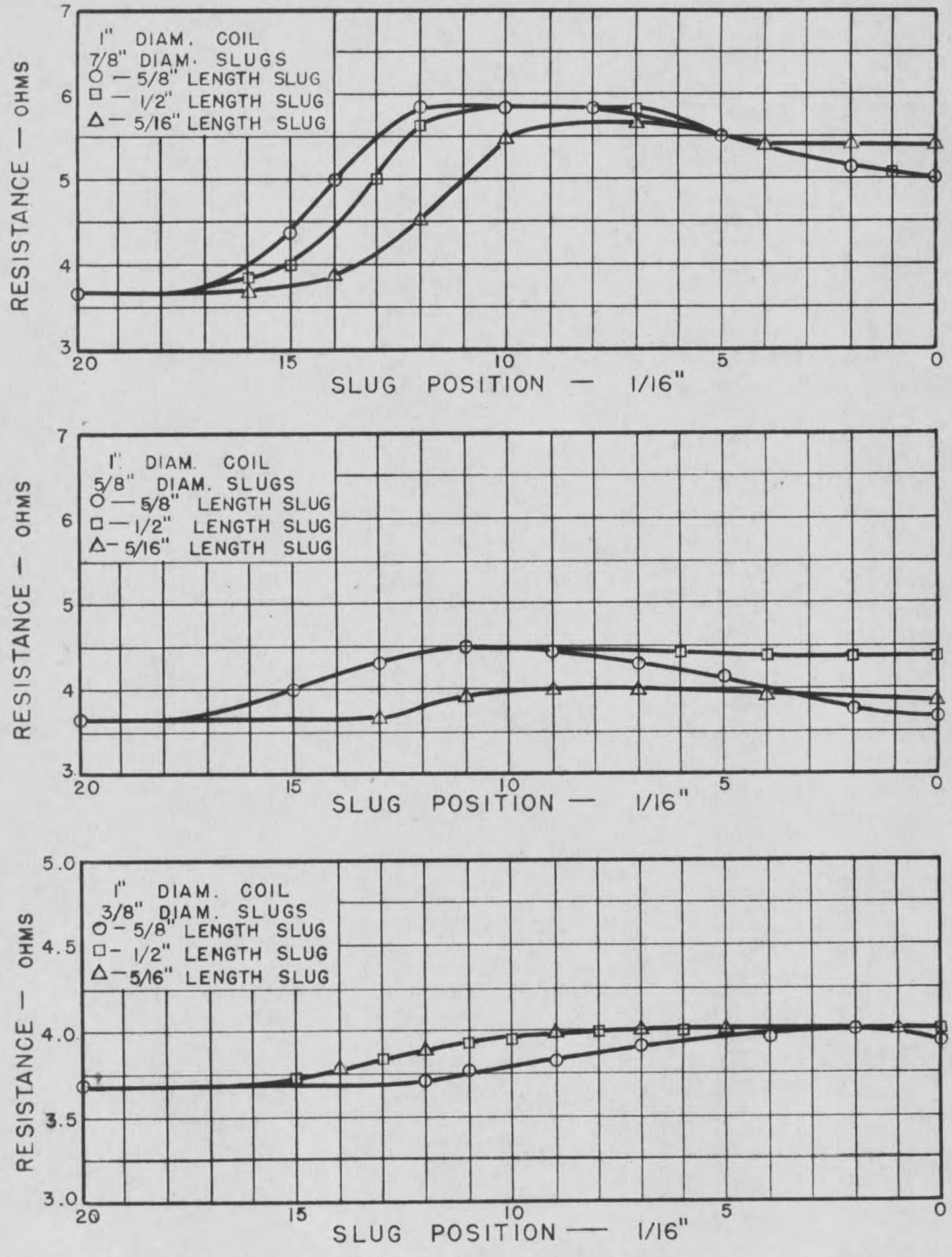
REACTANCE VERSUS SLUG POSITION

FIG. 10

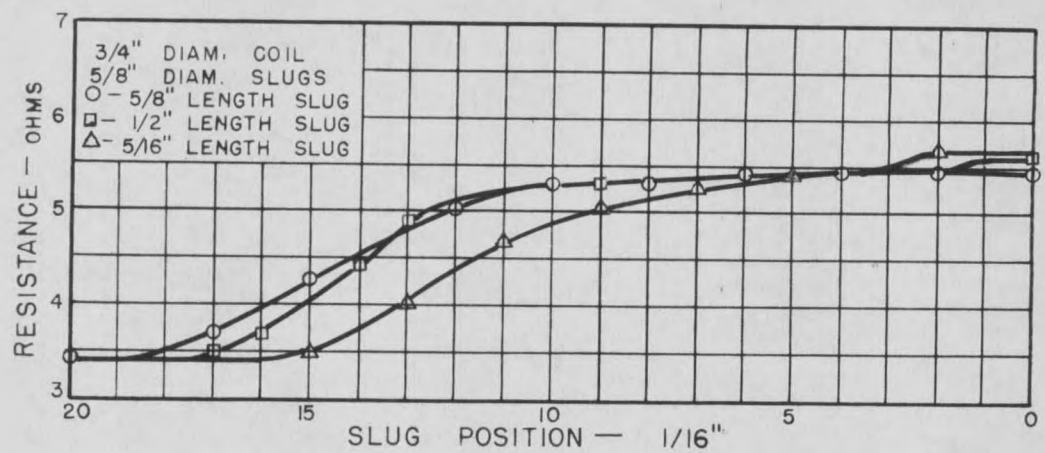


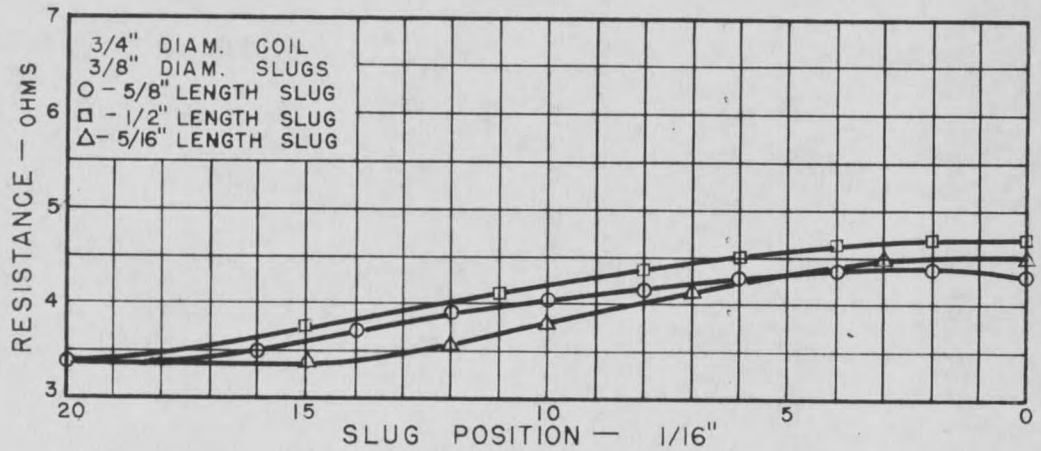
REACTANCE VERSUS SLUG POSITION

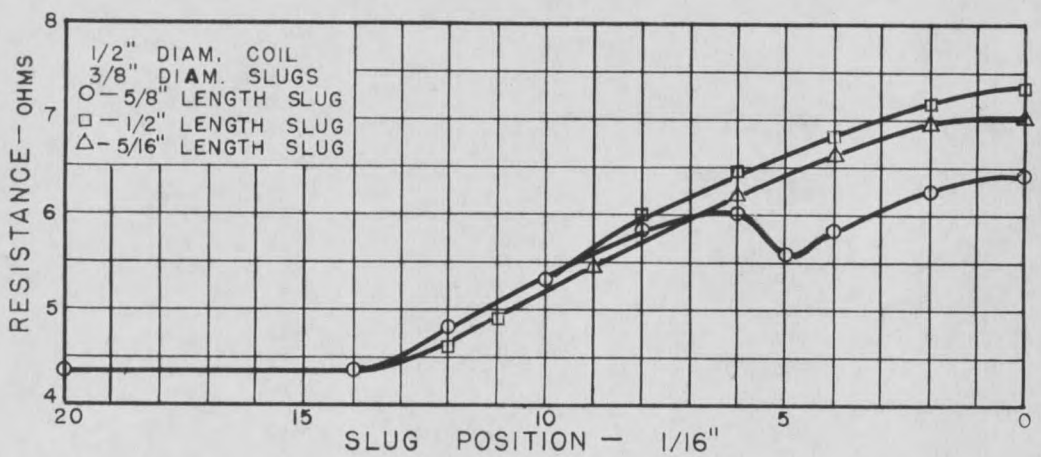
FIG. 11



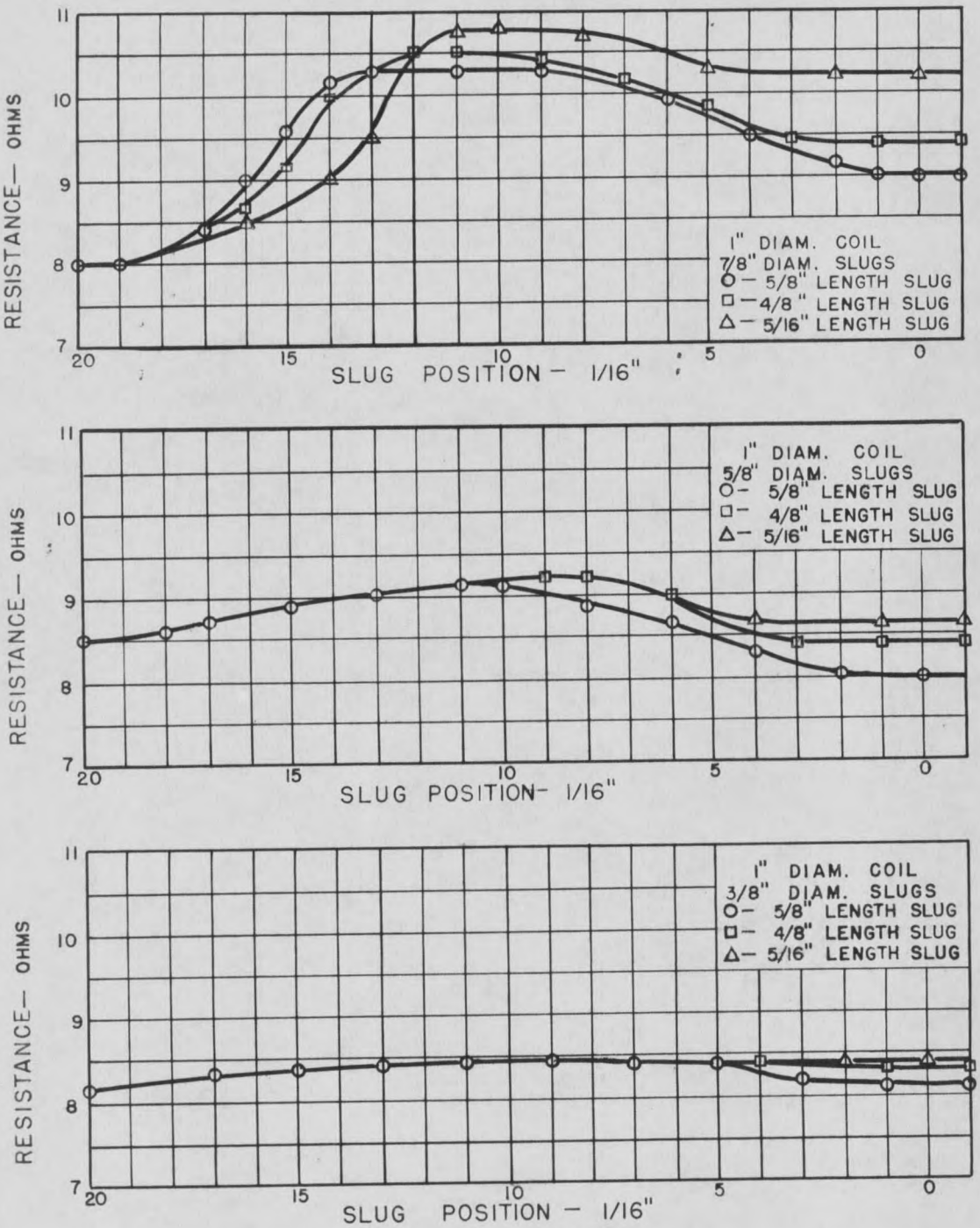
EFFECTIVE RESISTANCE VERSUS SLUG POSITION
FOR THE COILS HAVING 750 OHMS REACTANCE
FIG. 12





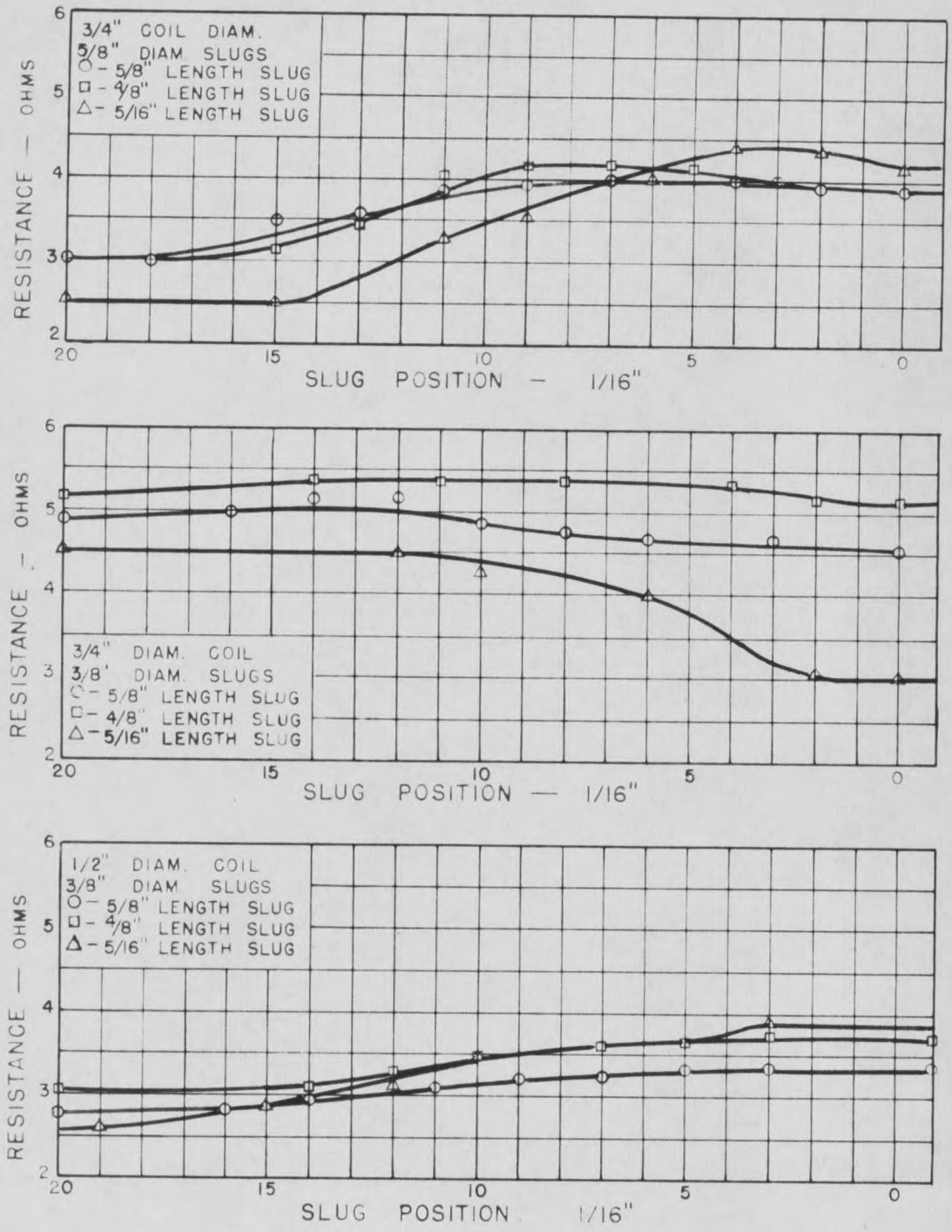


EFFECTIVE RESISTANCE VERSUS SLUG POSITION
FOR THE COILS HAVING 750 OHMS REACTANCE
FIG. 13

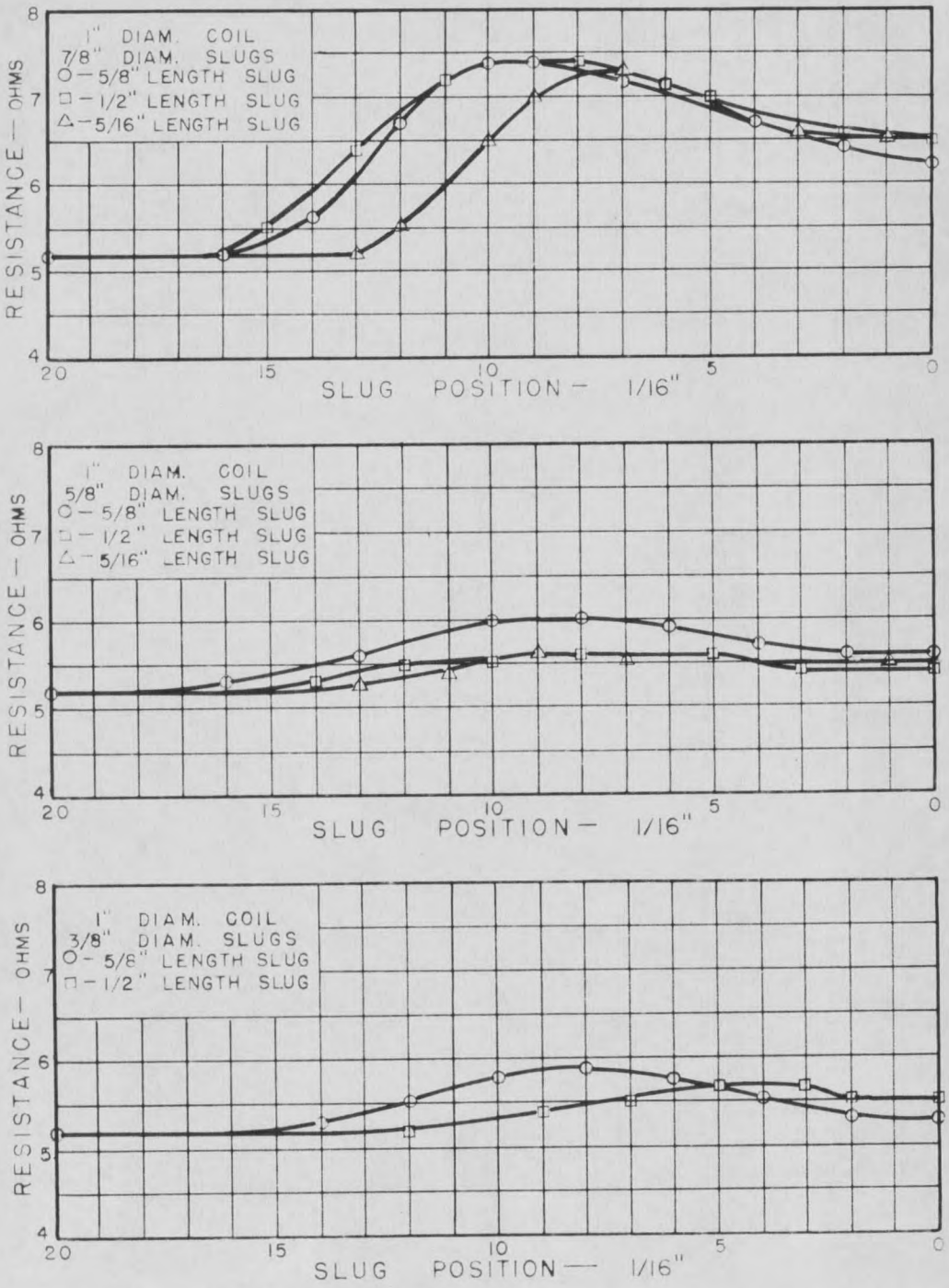


EFFECTIVE RESISTANCE VERSUS SLUG POSITION
FOR THE COILS 1.25" IN LENGTH

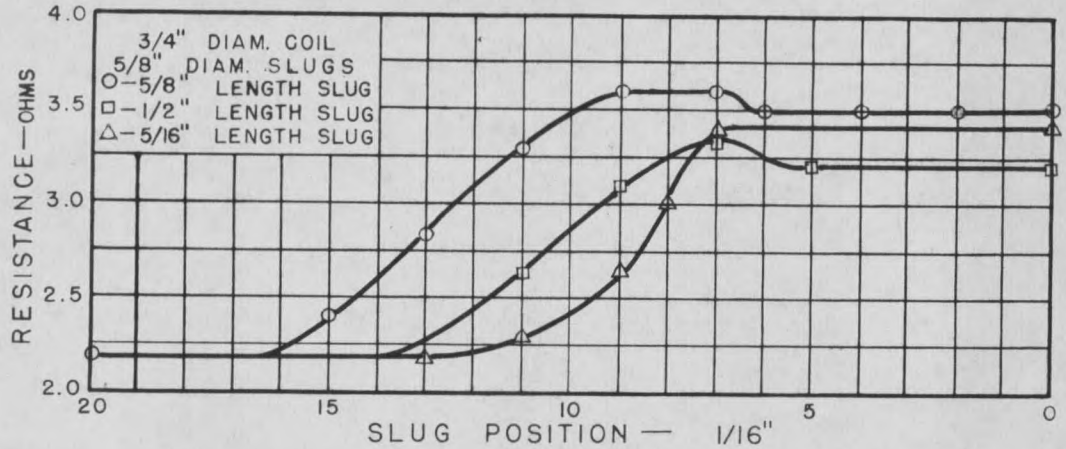
FIG. 14

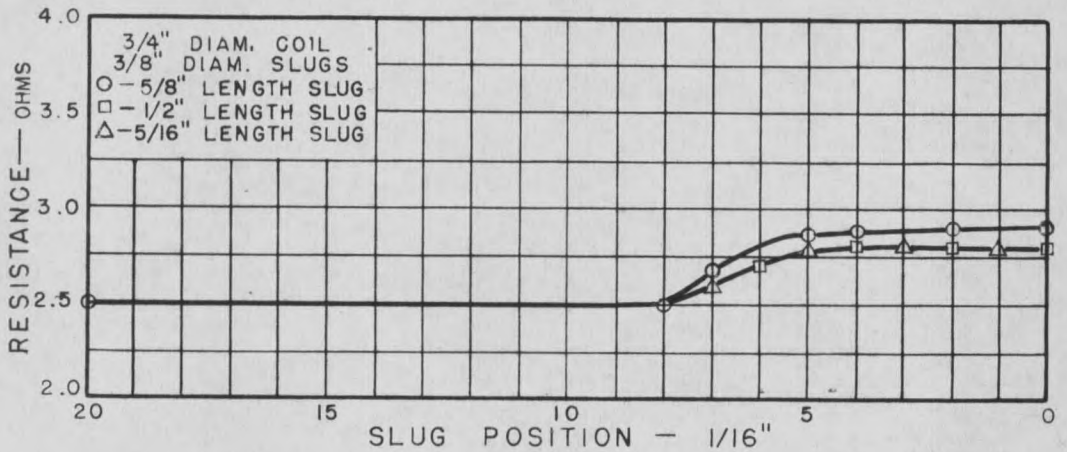


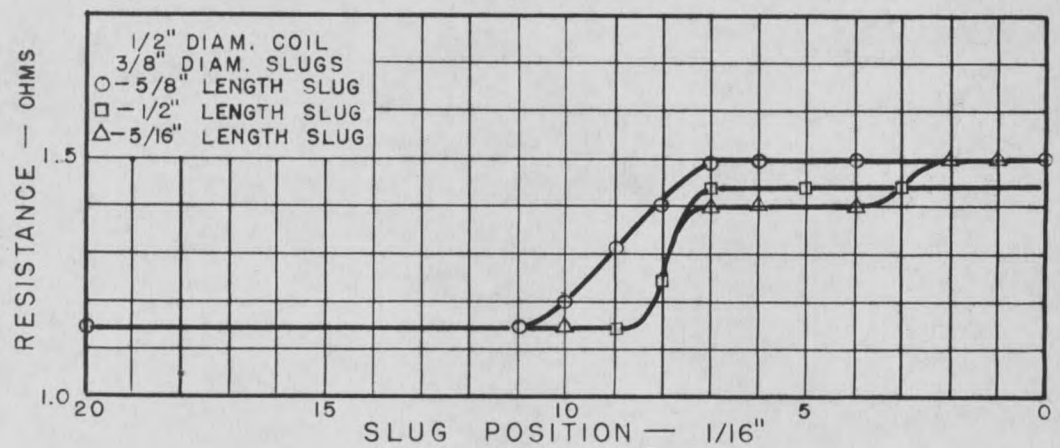
EFFECTIVE RESISTANCE VERSUS SLUG POSITION
FOR THE COILS 1.25" IN LENGTH
FIG. 15



EFFECTIVE RESISTANCE VERSUS SLUG POSITION
FOR COILS WITH L/D = 1
FIG. 16

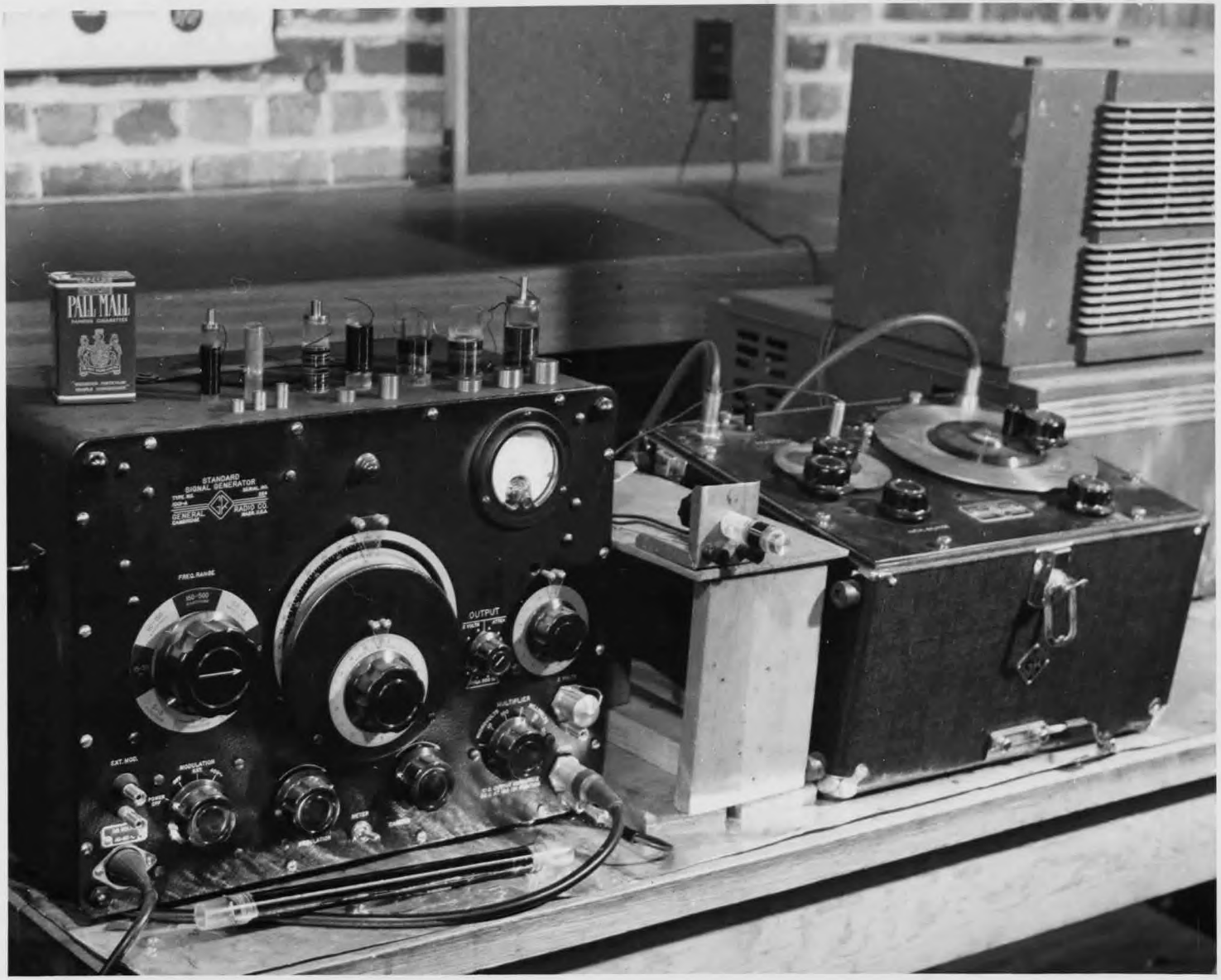






EFFECTIVE RESISTANCE VERSUS SLUG POSITION
FOR COILS WITH L/D = 1
FIG. 17

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