

STUDIES IN ALTERNATIVE THEORIES OF GRAVITY
AND ADVANCED DATA ANALYSIS

by
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DEDICATION

I dedicate this dissertation to my family for always supporting me and believing in me. I couldn't have done it without your faith, love, and encouragement.

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ABSTRACT

The field of gravitational wave astronomy is generating groundbreaking findings, yielding unique insights on some of the most extraordinary phenomena in the universe and providing invaluable information on testing the principles of general relativity. All gravitational wave signals detected so far appear to come from compact binaries - black holes and neutron stars. We use information from these sources to probe strong fields of gravity and to constrain modified theories of gravity. However, solely relying on template-based searches for known astrophysical sources biases our gravitational wave signal search towards well-modeled systems, potentially overlooking unpredicted sources with limited theoretical models, hindering the extraction of new physics. Further work in this thesis focuses on building improved signal and noise models to enhance our capability of detecting gravitational signals of all within and beyond the constraints of theoretical predictions. This includes introduction of new basis functions with added modifications to develop a signal-agnostic waveform reconstruction model using Bayesian inference. Additionally, this study discusses improvements in the speed and performance of the **BayesWave** trans-dimensional Bayesian spectral estimation algorithm, which includes implementing a low-latency analysis and various enhancements to the algorithm itself. In essence, this study is centered on developing a comprehensive understanding, both theoretical and observational, of astrophysical objects along with the spacetime that governs their dynamics.

INTRODUCTION

Gravitational waves (GW) constitute one of the most crucial observational pillars for understanding the concepts of extreme gravity. Physicists don't always observe a phenomenon and then formulate a theory to explain it. Instead, they often propose a theory that governs the motion of objects and subsequently verify if the dynamics observed in the future align with those equations. General relativity (GR) is one of the theoretical concepts aimed at explaining the geometry of space time. In this theory, the movement of celestial objects is dictated by the curvature of spacetime—an effect induced by the presence of massive entities. In 1915, Albert Einstein proposed this non-linear relationship between the mass-energy of objects and the geometry of spacetime. Einstein's equations within this theory not only describe the curvature of spacetime but also predict the generation of gravitational waves from accelerating objects due to loss of energy and angular momentum. The emitted gravitational waves carry valuable information about the intrinsic properties of the highly gravitating objects, providing a unique observational window into their characteristics.

GR is crucial for understanding the large-scale structures and dynamics of the universe. It forms the backbone of our understanding of gravity, offering insights into the nature of spacetime curvature, gravitational wave generation, and the interactions between massive entities. As a cornerstone of modern theoretical physics, general relativity continues to shape our understanding of the cosmos at both macroscopic and microscopic scales.

GR has successfully withstood a plethora of tests with a high level of accuracy. [55, 73, 226, 267, 269, 271]. In our solar system, weak field tests such as gravitational redshift, gravitational time dilation, observations of how light gets bent by gravity (photon-deflection),

Shapiro time-delay, Mercury’s perihelion advance, the Nordtvedt effect, and more [268] show no deviations from GR. In the binary pulsar tests [244, 265], GR successfully predicted and explained phenomena in binary systems, like the orbital decay due to gravitational wave emissions. On a large scale, in cosmological tests [83, 125, 164, 176, 232], GR has precisely explained the expansion of the universe, seen through the redshifts of distant galaxies. Moreover, it matches up with what we see in the cosmic microwave background radiation and the arrangement of large-scale structures in the universe. However, these tests assume quasi-stationary, quasi-linear weak field regimes of gravity. ‘Quasi-stationary’ denotes a gravitational field that is nearly stationary or changes at a considerably slower pace compared to the characteristic size of the system. ‘Quasi-linear’ refers to the region in spacetime where the gravitational field is weak relative to the mass-energy of the system, and the characteristic velocities of gravitating bodies are small relative to the speed of light [291]. Recent detections of GWs at LIGO, VIRGO and KAGRA observatories has allowed us to probe gravity in strong dynamical regimes of gravity [7, 11, 13]. GWs from over 100 binary black holes, binary neutron stars and black hole - neutron star binaries are excellent sources to probe gravity in strong field regimes.

Despite these tests and success stories, there remain some unexplained concepts that question GR’s validity, such as its incompatibility with quantum mechanics and some of the dark sectors of cosmology like the understanding of dark matter and dark energy. In process of testing GR, scientists have come up with alternative theories of gravity that hold true for weak fields but propose modifications in other realms of gravity. In this work, we discuss two such theories - dynamical Chern Simons (dCS) theory of gravity [154] and Einstein-æther theory of gravity [156]. dCS theory, which is a scalar field parity violating effective theory of gravity, is motivated by anomaly cancellation in particle physics and string theory [35, 154]. The dynamical formulation to it is because the CS scalar is treated as a dynamical field [295]. Einstein-æther is a vector-tensor theory of gravity violating Lorentz variance motivated to

understand beyond GR concepts at quantum scales by having a preferred frame, dynamical unit time like vector field.

To understand the non-GR effects in dCS theory that arise due to coupling between spacetime and matter, one would require neutron star (NS) observations from gravitational waves or binary pulsars. But the strong field modifications can be degenerate with the neutron star equation of state. This could hide the non-GR astrophysical effects. To break this degeneracy, the relation between three NS observables - moment of inertia (I), tidal love numbers (L) and quadrupole moment (Q) that do not dependent strongly on the equation of state of NSs is discussed in Chapter 2. The origin of these relations is related to the isodensity contours being approximately elliptical throughout inside the star [247]. These I-Love-Q relations play a significant role in the test of GR in strong field regimes. Having observations from binary pulsars and gravitational waves about the moment of inertia and tidal love numbers gives an area in I-Love bounded by the uncertainties in measurements. Modified theories that predict deviations away from GR can then be constrained in this I-Love plane. We show that if we are able to measure I and Love numbers using future based observations we can constrain the coupling constant of the theory by six orders of magnitude stronger than the current ones from Solar system [37] and table-top experiments [284].

Another way of constraining a modified theory of gravity discussed in Chapter 3 is using the observations of orbital period decay rate from binary pulsars and triple system [43]. We used an analogy along with observation from GW170817 to put bounds on the coupling constants of Einstein-æther theory. All recent observations from binary pulsars have verified GR. This allowed us to put stringent constraints on the Einstein-æther theory, by requiring that the non-GR effects are contained within the observational uncertainty [245], to an order of magnitude stronger than the current bounds [208, 269]. The influence on the orbital period decay rate due to the presence of dipolar radiations in this theory is reflected in a parameter called sensitivity which is related to charge in NS.

All these observations constitute tests of GR with a modified underlying theory of gravity. Since we don't understand the exact theory that describes the spacetime in strong regimes, can we still understand the source properties of the detected signal through gravitational waves? In Chapter 4, we discuss the possibility of exploring what is unexplored without any assumption on the underlying theory of gravity. The LIGO/VIRGO detectors have been in operation intermittently since September 12, 2015 [7] and have detected a number of events thus far [13]. With increasing detector sensitivity, the duration over which the signals can be detected has also increased. The detection of these signals is quite challenging since these detectors measure the differential displacement in spacetime caused by passing gravitational waves using laser beams, which is $\mathcal{O}(10^{-18})$ m in current ground-based detectors, roughly one-hundredth of the size of a proton. We require highly sensitive data analysis techniques to model the data and discern whether there is a signal embedded in it or if it is just noise. Instead of subtracting noise with limited understanding, it is essential to implement well-defined models for noise characterization to effectively detect weak signals. Lacking pre-defined templates, the burst searches rely on having multiple detectors to distinguish noise transients from gravitational wave signals. Also, for burst searches, understanding and accurately modeling the noise is crucial for all gravitational wave detections, especially considering our limited knowledge of signal characteristics. In scenarios with low signal-to-noise ratios, where signal amplitudes are on the order of noise, the detection task becomes particularly challenging. Therefore, in order to understand the signals, we need good models for both the signal and noise data. Our plan is to extend and improve both the signal and noise models in this work.

The LIGO/Virgo data exhibit two main types of non-stationary behavior. The first is adiabatic drifts in the power spectrum occurring over minutes or hours, and the second is short duration noise transients, or glitches, that are typically localized in time and frequency. Glitches are also responsible for much of the non-Gaussianity seen in the data. Additional

non-stationarity has been observed in the vicinity of spectral lines, such as those due to electromagnetic couplings to the 50/60 Hz AC power supply. The adiabatic drifts in the power spectrum can be described in terms of *locally stationary* processes [95, 241].

Chapter 4 and Chapter 5 present improved models for noise estimation and the detection of arbitrary gravitational wave signals, as compared to the current **BayesWave** model. Chapter 4 focuses on methods to model the non-stationary, non-Gaussian components of noise which are reflected in the estimation of the power spectrum. Classical methods like Welch averaging [263] and other traditional moving average techniques [94] are used to model the noise power spectral density (PSD). The current **BayesWave** model uses a combination of splines for estimating the broadband and Lorentzian features of noise [193]. The number and placement of spline points is determined by trans-dimensional reversible jump Markov Chain Monte Carlo (RJMCMC) algorithm. Indeed, **BayesWave** is a reliable technique to model PSDs but it comes with additional cost. We propose improvements to the current **BayesWave** PSD models to handle large long segments of data and observe speed-ups. We devise a new low latency spectral estimation algorithm, **FastSpec**, to deliver an independent Bayesian spectral estimate with reduced computational cost. This algorithm enhances the initial step for the **BayesWave** algorithm. The details to the low latency algorithm and improvements to **BayesWave** are discussed in Chapter 4.

Moving forward with the improvements to the signal models. **BayesWave** uses Morlet-Gabor wavelets in the wavelet domain to model short duration burst type signals or instrument transient noises (glitches) [89, 91]. But with increased detector sensitivity the demand arises for detecting weak gravitational wave signals that stay in the detectors for a long time. We build an algorithm called **BayesWaveVoices** to model weak, long duration gravitational wave signals which have a smooth amplitude and phase evolution. Since the gravitational wave signals are primarily generated by coherent flows of matter and energy, it results in the smooth evolution of amplitude and phase. Since gravitational wave data

is recorded in the time domain, and to avoid time reversibility, we choose to conduct this analysis in the time domain. The time-domain waveform $h(t)$ is commonly expressed in terms of amplitude, $A(t)$, and frequency, $f(t)$, as $h(t) = A(t)e^{i\int 2\pi f(t)dt}$. We build the signal using different basis functions (“voices”), acknowledging the behavior of Akima splines [32] which are neither as stiff as Steffan splines [246] nor as highly varying as cubic splines [3]. In the specific case of binary black hole mergers, which serves as our reference in this paper, a sharp discontinuity is observed in the signal near the merger time, where the signal amplitude rapidly drops to zero. To address these sudden discontinuities, we introduce modifications to the model detailed in Chapter 5. The new algorithm, named **BayesWaveVoices**, consistently outperforms signal reconstruction, even in scenarios with weak signals, low signal-to-noise ratios, and low-mass systems, when compared to **BayesWave**.

I-LOVE-Q RELATIONS FOR NEUTRON STARS IN DYNAMICAL CHERN SIMONS
GRAVITY

Contribution of Authors and Co-Authors

Manuscript in Chapter 2

Author: Toral Gupta

Contributions: Performed the analysis, wrote Maple codes and drafted the initial manuscript.

Author: Barun Majumder

Contributions: Developed and provided essential codes to perform the analysis.

Author: Kent Yagi

Contributions: Developed codes, offered continuous guidance throughout the analysis process, and edited the manuscript.

Author: Nicolás Yunes

Contributions: Provided valuable guidance throughout the analysis and edited the manuscript.

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Introduction

General Relativity (GR) has successfully passed all the tests performed so far in various regimes of gravity. Solar System experiments have placed very tight constraints on weak-field modifications to GR [266, 273]. Various cosmological observations have placed bounds on large scale modifications [83, 161, 164, 176, 232]. The recent discovery of gravitational waves (GWs) from binary black hole (BH) coalescences by the LIGO Scientific Collaboration (LSC) and the Virgo Collaboration [15–17, 22, 23] allowed one to probe gravity in both the strong and dynamical field regimes for the first time [18, 294]. Future GW observations of binary BH coalescences will significantly improve the ability of probing gravity in such a regime [49, 55, 76, 133, 278, 291].

Neutron star (NS) observations are also excellent tools to perform strong-field tests of gravity [226, 244, 265]. One very timely example is the detection of GWs and electromagnetic waves from a binary neutron star merger GW170817 [25]. This observation allows for bounds on the propagation speed of GWs, on gravitational Lorentz violation and on the equivalence principle [19, 141]. Unlike BH observations, one can use NS observations to probe how non-GR effects may arise in spacetimes with matter. For example, binary pulsar observations [100, 101, 132, 238] have been used to probe spontaneous scalarization in scalar-tensor theories [98, 99], which arises due to the nonlinear coupling between matter and a scalar field, and is absent in BH spacetimes. Since the density inside a NS exceeds the nuclear saturation density, one can use independent measurements of NS quantities, such as the mass and radius, to probe the equation of state (i.e. the relation between pressure and density) of supranuclear matter, which is currently unknown [185–187, 203, 214–216, 248]. Unfortunately, however, this means that there typically exists a degeneracy between equation-of-state effects and strong-field modifications in NS observations, rendering the latter unfeasible without knowledge of the former.

One way to overcome such a problem is to use approximate universal relations among NS observables, i.e. relations that do not depend strongly on the underlying equation of state (EoS), to project out uncertainties in nuclear physics and allow for tests of GR. Universal relations are known to exist for example between the oscillation frequencies of NSs and the stellar average density [40, 41], or between the binding energy of a star and its compactness [186]. Recently, two of us found another example of universal relations, one that holds among dimensionless versions of the moment of inertia (I), the tidal Love number (Love) and the quadrupole moment (Q) [275, 276]. The universality, in fact, depends sensitively on how one adimensionalizes these NS observables [198]. For example, the I-Q universality is lost for rapidly-rotating NSs if one fixes the angular velocity [111], but it is restored if one fixes the dimensionless spin parameter instead [75, 219].

What is the origin of the I-Love-Q universality? Reference [247] argued that the fact that isodensity contours inside a star are approximately elliptically self similar is the reason. In other words, the fact that the eccentricity of such isodensity contours is approximately constant throughout the star is responsible for the approximate universality. Indeed, Ref. [247] showed that the NS eccentricity varies by only $\sim 20\%$ throughout its interior for all EoSs they considered, providing evidence for this explanation. Moreover, Ref. [289] showed that the relations become significantly less universal if one artificially forces the variation of the eccentricity of isodensity contours to increase. This explanation is also consistent with the results of [77, 235], who suggested the origin of the universality is associated with all realistic EoSs being “similar” to an incompressible (constant density star) one, in which case the elliptical isodensity approximation becomes exact.

The I-Love-Q relations have applications to strong-field tests of gravity, astrophysics [209], gravitational-wave physics [275, 276] and nuclear physics [52, 227] (see [109, 282] and references therein for detailed and recent reviews), but let us concentrate on the former. Future radio observations may provide the first measurements of the moment

of inertia of the primary pulsar in J0737-3039 [177, 188], while future gravitational-wave observations may provide the first measurements of the NS tidal Love number [104, 126, 148, 149, 180, 181, 229, 230]¹. These two independent observations define a region in the I-Love plane, whose size is determined by the uncertainty in the observations. If GR holds, then the GR I-Love relation will pass through this region, thus placing constraints on any deviation. Modified theories that predict parametric deviations in the GR I-Love curve can then be constrained, as is the case in scalar tensor theories [110, 112, 217], $f(R)$ gravity [113], Einstein-dilaton Gauss-Bonnet gravity [169, 170], dynamical Chern-Simons (dCS) gravity [275, 276] and Eddington-inspired Born-Infeld gravity [234]. Whether such I-Love tests are more constraining than other existing tests, using for example Solar System observations, depends of course on how stringent the other constraints are.

In this paper, we study the I-Love-Q universal relations in dCS gravity [35, 154, 243], a parity violating effective field theory of gravity. The effective action in this theory consists of the Einstein-Hilbert term and a Pontryagin density (i.e. the contraction of the Riemann tensor and its dual) coupled to a dynamical scalar field with a standard kinetic term. Such a modification to the action arises naturally in string theory [222, 223], in loop quantum gravity upon the promotion of the Barbero-Immirzi parameter to a field [69, 251], and in effective field theories of inflation [262]. Stellar solutions in dCS gravity have of course been studied before: non-rotating solutions do not acquire any modifications, while slowly-rotating solutions to first order in rotation were found in [293] and to second order in rotation in [285], following the Hartle-Thorne formalism [144, 145].

We here extend previous results in several ways. First, we evaluate the quadrupole moment in dCS gravity following [285], and study the universality in the I-Q and Q-Love relations. Second, we investigate how the universality depends on the normalization of

¹The recent GW170817 detection already placed upper bounds on the tidal Love number with the NS mass of $1.4M_\odot$ [25], although these are quite weak.

the coupling constant in dCS gravity. Third, we look at the eccentricity profile of isodensity contours inside a NS in dCS gravity to determine whether the elliptical isodensity explanation continues to hold in this theory. Lastly, we investigate the dependence of the projected bounds on dCS gravity using the I-Love relation on the measurement accuracy of the moment of inertia and the tidal Love number.

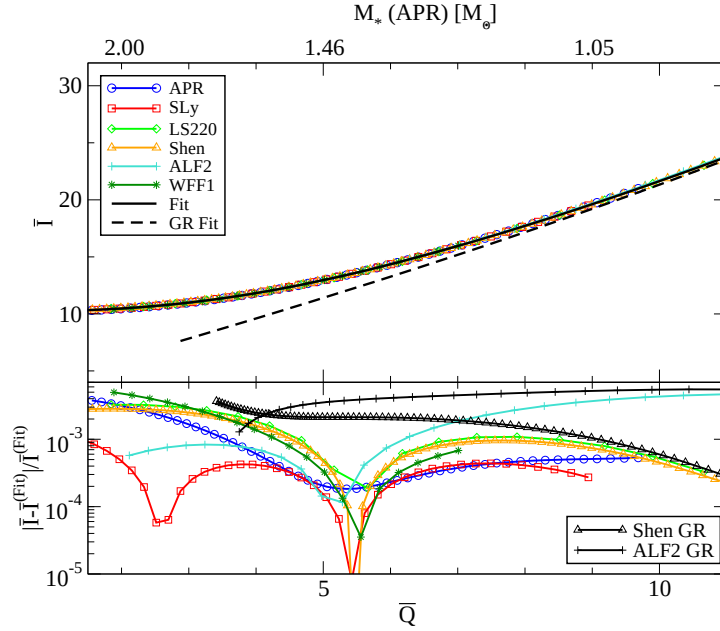


Figure 2.1: (Color Online) (Top) Universal relation between the dimensionless moment of inertia $\bar{I}$ and quadrupole moment $\bar{Q}$ for various realistic EoSs in dCS gravity, using a dimensionless coupling constant normalized by the NS mass and set to $\bar{\xi} = 10^3$. We also show a global fit to all dCS I-Q curves (black solid), together with a global fit to all I-Q curves in GR [275, 276] (black dashed). Observe how the relation deviates from the GR one as one decreases $\bar{Q}$ (as one considers more massive and compact NSs). (Bottom) Fractional difference between any one I-Q curve in dCS gravity and the global fit, together with the fractional difference between two I-Q relations in GR and a global fit in GR using the Shen and ALF2 EoSs. Observe that the amount of universality in dCS gravity is similar to that in GR for large or small $\bar{Q}$, while the dCS universality is stronger when $\bar{Q} \sim 5$.

Executive Summary

The first topic we investigate is the universality of the I-Love-Q relations. The top panel of Fig. 2.1 presents the I-Q relation in dCS gravity for various EoSs, where we adimensionalize

the dCS coupling constant by the NS mass. Observe that the relation is still approximately EoS universal, with dCS deviations from the GR I-Q relation becoming more prominent for more compact (smaller $\bar{Q}$) stars. The bottom panel shows the fractional difference between any one I-Q curve in dCS and a global fit constructed from all dCS data sets, together with the EoS variation found in GR for two EoSs for comparison. Observe that the EoS variation in dCS gravity is similar to that in the GR case, for large and small $\bar{Q}$. Observe also that when $\bar{Q} \sim 5$, the EoS variation in dCS gravity is roughly one order of magnitude smaller than that in GR; this is because when $\bar{Q}$ is large (small), the EoS variation is dominated by the GR (dCS) contribution, while the dCS and GR contributions partially cancel each other when $\bar{Q} \sim 5$ for the specific value of the dimensionless coupling constant we chose in this example.

The second topic we investigate is how the degree of universality changes when one chooses different normalizations for the dCS coupling constant. As we showed in Fig. 2.1, the degree of universality in dCS gravity is similar to (and in some cases even stronger than) the degree of universality in the GR case, when one adimensionalizes the coupling constant by the NS mass. We find, however, that if one adimensionalizes the coupling constant with either the curvature length of the system, or simply with a solar mass, the universality is lost. These results demonstrate that, even in dCS gravity, the existence of universality in the I-Love-Q relations depends sensitively on how one adimensionalizes the dimensional quantities of the problem. We expect this to remain true in other theories of gravity that depend on dimensional coupling constants, such as in Einstein-dilaton Gauss-Bonnet gravity.

The third topic we investigate is whether the self-similar elliptical isodensity explanation of the I-Love-Q universality persists in dCS gravity. We find that as one increases the dCS coupling, the eccentricity variation throughout the star becomes smaller *and* the universality becomes stronger, thus supporting the self-similar elliptical isodensity explanation [247, 289]. However, if one increases the dCS coupling too much, the EoS variation is dominated by

the dCS contribution, and then, although the eccentricity variation continues to decrease, the universality does not necessarily improve. This suggests that although the origin of the universality can be understood in terms of the elliptical isodensity explanation in GR, it may not be sufficient for non-GR theories as additional non-GR contributions come into play. The reasoning for this discrepancy is yet to be explored, and one way to approach this problem is to study other non-GR theories and see if the behavior is similar to dCS. Work along this direction is currently in progress, taking Lorentz-violating theories [63, 120, 156, 159] as examples.

With the universality of the I-Love-Q relations established and explored, we then investigate the dependence of I-Love constraints on dCS on the accuracy to which the moment of inertia and the Love number could be measured by binary pulsar and gravitational wave observations. Figure 2.2 shows how the bounds on the dCS coupling constant varies with the measurement accuracy. Observe that the bounds are $\mathcal{O}(10^2)\text{km}$, namely six orders of magnitude stronger than current Solar System [37] and table top [284] experiments. This is because NS observations allow one to probe stronger gravitational field regimes, where the dCS corrections would be naturally larger than those present in the weak field regime. Observe also that these constraints only depend on these measurements being possible, with the strength of the constraint not strongly dependent of the accuracy to which these quantities are measured.

The remainder of this paper presents the details of the results summarized above and it is organized as follows. Section 2 briefly reviews dCS gravity and explains a few approximations that we adopt throughout the paper. Section 2 describes the perturbation scheme that we use to construct slowly-rotating NS solutions in the theory by extending the Hartle-Thorne formalism. Section 2 reviews the structure of the field equations and how we numerically obtain dCS corrections to physical observables, such as the moment of inertia, the quadrupole moment, the tidal Love number and the stellar eccentricity. We also explain

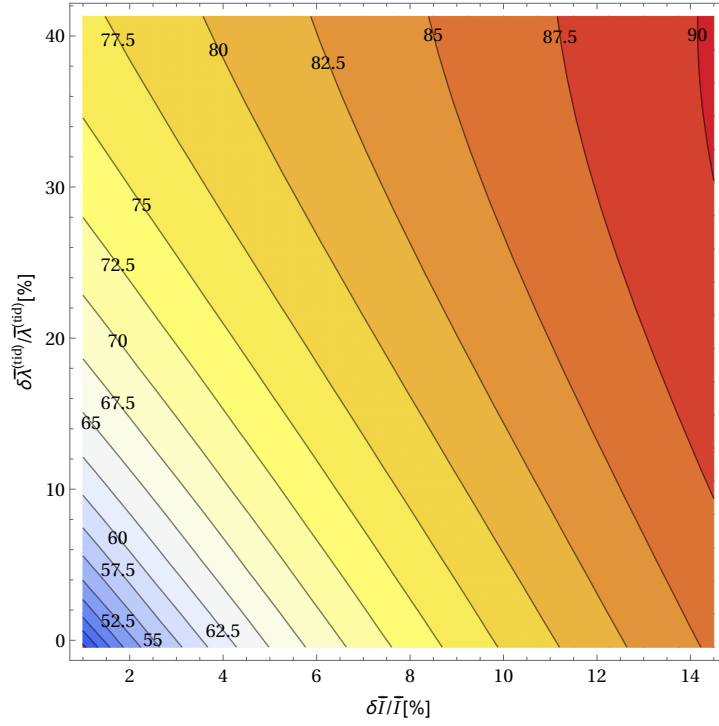


Figure 2.2: (Color Online) Contour plot of projected upper bounds on the dCS coupling constant $\sqrt{\alpha}$ in km as a function of the fractional measurement accuracy of the NS moment of inertia (horizontal axis) and tidal Love number (vertical axis). We assume that the former is measured from future radio observations of binary pulsars, while the latter is measured from future GW observations of NS binaries. Observe that the constraints are always six orders of magnitude stronger than current bounds ($\sqrt{\alpha} < \mathcal{O}(10^8)\text{km}$) from Solar System [37] and table-top [284] experiments, approximately independently of the accuracy to which these quantities can be measured.

how we adimensionalize the moment of inertia, tidal Love number and quadrupole moment and derive dCS corrections for such dimensionless quantities. We further comment on how to evaluate the stellar eccentricity. Section 2 presents all of our numerical results. Section 2 provides a final summary of our work and describes possible avenues for future work.

Review of Dynamical Chern-Simons Gravity

In this section, we review dCS gravity and explain the approximations that we adopt throughout the paper. We mostly follow [279, 285].

Action and Field Equations

We begin by presenting the action in dCS gravity. The modification to the Einstein Hilbert action is introduced by adding the Pontryagin density coupled to a dynamical scalar field ϑ to the Lagrangian, namely [35]

$$S \equiv \int d^4x \sqrt{-g} \left[\kappa_g \mathcal{R} + \frac{\alpha}{4} \vartheta \mathcal{R}_{\nu\mu\rho\sigma} {}^* \mathcal{R}^{\mu\nu\rho\sigma} - \frac{\beta}{2} \nabla_\mu \vartheta \nabla^\mu \vartheta + \mathcal{L}_{\text{mat}} \right]. \quad (2.1)$$

Here, $\mathcal{R}$ is the Ricci scalar, $\mathcal{L}_{\text{mat}}$ is the matter Lagrangian density, g is the determinant of the metric, $\kappa_g \equiv 1/(16\pi)$, and α and β are coupling constants of the theory. The dual of the Riemann tensor ${}^* \mathcal{R}^{\mu\nu\rho\sigma}$ is here defined by

$${}^* \mathcal{R}^{\mu\nu\rho\sigma} \equiv \frac{1}{2} \varepsilon^{\rho\sigma\alpha\beta} \mathcal{R}^{\mu\nu}_{\alpha\beta}, \quad (2.2)$$

where $\varepsilon^{\rho\sigma\alpha\beta}$ is the Levi-Civita tensor. For simplicity, we have neglected any potential for the scalar field. We take ϑ and β to be dimensionless, which forces α to have dimensions of (length)² [284, 290]. Solar System [37] and table-top [284] experiments place bounds on the

characteristic length scale of the theory, $\xi_{\text{CS}}^{1/4} < \mathcal{O}(10^8)$ km [37, 284] where

$$\xi_{\text{CS}} \equiv \frac{\alpha^2}{\beta \kappa_g}. \quad (2.3)$$

We next look at the field equations derived from the action in Eq. (2.1). The modified Einstein equations are given by

$$G_{\mu\nu} + \frac{\alpha}{\kappa_g} C_{\mu\nu} = \frac{1}{2\kappa_g} (T_{\mu\nu}^{\text{mat}} + T_{\mu\nu}^{\vartheta}). \quad (2.4)$$

Here, $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}^{\text{mat}}$ is the matter stress-energy tensor. The C-tensor and the scalar field stress-energy tensor in Eq. (2.4) are given by

$$C^{\mu\nu} \equiv (\nabla_\sigma \vartheta) \epsilon^{\sigma\delta\alpha(\mu} \nabla_\alpha \mathcal{R}^{\nu)}_{\delta} + (\nabla_\sigma \nabla_\delta \vartheta)^* \mathcal{R}^{\delta(\mu\nu)\sigma}, \quad (2.5)$$

$$T_{\mu\nu}^{\vartheta} \equiv \beta (\nabla_\mu \vartheta) (\nabla_\nu \vartheta) - \frac{\beta}{2} g_{\mu\nu} \nabla_\delta \vartheta \nabla^\delta \vartheta. \quad (2.6)$$

The dynamical scalar field ϑ satisfies the evolution equation

$$\square \vartheta = -\frac{\alpha}{4\beta} \mathcal{R}_{\nu\mu\rho\sigma} {}^* \mathcal{R}^{\mu\nu\rho\sigma}. \quad (2.7)$$

Taking the divergence of the field equation (2.4), and using the Bianchi identity and Eq. (2.7), we find that the matter stress-energy tensor obeys the same conservation law as in GR:

$$\nabla_\nu T_{\text{mat}}^{\mu\nu} = 0. \quad (2.8)$$

Slow-Rotation and Small-Coupling Approximation

Throughout this paper, we work within the slow-rotation approximation. We assume that the dimensionless spin parameter of a NS satisfies

$$\chi \equiv \frac{J}{M_*^2} \ll 1, \quad (2.9)$$

where M_* is the NS mass and $J \equiv |\vec{J}|$ is the magnitude of the spin angular momentum. This is an excellent approximation for old NSs, such as binary pulsars and NSs in a binary that are about to coalesce, one of the main sources of GWs for ground-based detectors. We estimate NS quantities in dCS gravity within this slow-rotation approximation, keeping terms up to $\mathcal{O}(\chi^2)$.

Throughout this paper we also work in the small-coupling approximation. For isolated NSs, this approximation requires that the dimensionless coupling

$$\zeta \equiv \frac{\xi_{\text{CS}} M_*^2}{R_*^6} \ll 1, \quad (2.10)$$

where R_* and M_* are the NS radius and mass, $\sqrt{R_*^3/M_*}$ is the curvature length scale of the star, and ξ_{CS} is defined in Eq. (2.3). This requirement ensures that the dCS modification term (second term) in the action in Eq. (2.1) is always much smaller than the Einstein-Hilbert term. This, in turn, allows us to treat dCS gravity as an effective theory, so that we can safely neglect possible higher-order terms in the action, and allowing the theory to easily reduce to GR in the low energy limit. When calculating NS quantities in dCS gravity we will work in the small-coupling approximation to $\mathcal{O}(\zeta)$.

One may wonder whether the small-coupling approximation is consistent with the current constraint on dCS gravity. Normalizing the small-coupling condition to neutron

stars, one can rewrite Eq. (2.10) as

$$\xi_{\text{CS}}^{1/4} \ll 25 \text{ km} \left(\frac{M_*}{1.4 M_\odot} \right) \left(\frac{0.18}{C} \right)^{3/2}, \quad (2.11)$$

where $C \equiv M_*/R_*$ is the stellar compactness. Observe that such a requirement is much more stringent than current bounds from Solar System [37] and table-top [284] experiments, $\xi_{\text{CS}}^{1/4} \leq \mathcal{O}(10^8) \text{ km}$, mentioned earlier [37, 284]. Therefore, requiring the small-coupling approximation is clearly not in conflict with current bounds on dCS gravity.

Given these approximations, all solutions we obtain will be *bivariate expansions*, i.e. expansions in two independent small parameters, χ and ζ . Therefore, it will be natural to first decompose any quantity A as follows

$$A = A^{\text{GR}} + \alpha'^2 A^{\text{CS}}, \quad (2.12)$$

and then to further decompose the GR and dCS pieces as a sum over χ to $\mathcal{O}(\chi^2)$, where α' is a book-keeping parameter that labels the half-order in the small-coupling approximation, i.e. $\mathcal{O}(\zeta) = \mathcal{O}(\alpha'^2)$. Composing both expansions, we can typically write any quantity A as

$$A = \sum_{k=0}^{k=2} \sum_{\ell=0}^{\ell=2} \chi'^k \alpha'^\ell A_{(k,\ell)}, \quad (2.13)$$

where χ' is another book-keeping parameter that labels the order in the slow-rotation approximation.

Ansatz for Slowly-Rotating Neutron Stars

In this section we first describe the metric ansatz and the stress-energy tensor for matter that we use to construct slowly-rotating NS solutions in dCS gravity.

Metric Ansatz and Bivariate Expansion

We begin by considering the following ansatz for the metric, which corresponds to a generic stationary and axisymmetric metric in the Hartle-Thorne coordinates [144]:

$$ds^2 = -e^{\bar{\nu}(r)} [1 + 2\bar{h}(r, \theta)] dt^2 + e^{\bar{\lambda}(r)} \left[1 + \frac{2\bar{m}(r, \theta)}{r - 2\bar{M}(r)} \right] dr^2 \\ + r^2 [1 + 2\bar{k}(r, \theta)] \{ d\theta^2 + \sin^2 \theta [d\phi - \bar{\omega}(r, \theta) dt]^2 \} . \quad (2.14)$$

Here $\bar{\nu}$ and $\bar{\lambda}$ are functions on r only that represent spin-independent contributions, while $\bar{h}$, $\bar{k}$, $\bar{m}$ and $\bar{\omega}$ are functions of (r, θ) and correspond to spin corrections. The quantity

$$\bar{M}(r) \equiv \frac{(1 - e^{-\bar{\lambda}(r)}) r}{2} \quad (2.15)$$

measures the enclosed mass within the radius r .

Since we are treating rotation perturbatively, we need to be careful with the coordinate system we work in [144]. When considering rotating stars in polar coordinates (r, θ) , the fractional change in quantities like the energy density $\bar{\rho}$ and pressure $\bar{p}$ does not remain small near the surface. Thus, we adopt new coordinates (R, Θ) , first proposed by Hartle [144], such that the energy density in the new radial coordinate $\rho(R)$ in the rotating configuration is the same as that in the non-rotating configuration:

$$\bar{\rho}[r(R, \Theta), \Theta] = \rho(R) = \rho^{(\text{non-rot})}(R), \quad \Theta = \theta . \quad (2.16)$$

Since the energy density and pressure are connected via an EoS, it follows that the pressure also does not acquire any spin corrections in the new coordinate system:

$$\rho(R) = \rho^{(\text{non-rot})}(R) \rightarrow p(R) = p^{(\text{non-rot})}(R) . \quad (2.17)$$

Moreover, since we treat rotation perturbatively and the changes in the density and pressure enter at $\mathcal{O}(\chi^2)$, we can introduce the new coordinates perturbatively via

$$r(R, \Theta) = R + \xi(R, \Theta), \quad (2.18)$$

where $\xi = \mathcal{O}(\chi^2)$.

In the new coordinates, the line element becomes

$$\begin{aligned} ds^2 = & - \left[\left(1 + 2h + \xi \frac{d\nu}{dR} \right) e^\nu - R^2 \omega^2 \sin^2 \Theta \right] dt^2 \\ & - 2R^2 \omega \sin^2 \Theta dt d\phi + [R^2(1 + 2k) + 2R\xi] \sin^2 \Theta d\phi^2 \\ & + e^\lambda \left(1 + \frac{2m}{R - 2M} + \xi \frac{d\lambda}{dR} + 2 \frac{\partial \xi}{\partial R} \right) dR^2 \\ & + 2e^\lambda \frac{\partial \xi}{\partial \Theta} dR d\Theta + [R^2(1 + 2k) + 2R\xi] d\Theta^2, \end{aligned} \quad (2.19)$$

where we have retained terms only up to $\mathcal{O}(\chi^2)$ and we have defined

$$M(R) \equiv \frac{(1 - e^{-\lambda(R)}) R}{2}, \quad (2.20)$$

with the stellar mass $M_* = M(R_*)$. Bivariately expanding the above line element, we then find that all metric functions and the coordinate function ξ can be expanded as in Eq. (2.13):

$$\nu(R) = \nu_{(0,0)}(R), \quad (2.21)$$

$$\lambda(R) = \lambda_{(0,0)}(R), \quad (2.22)$$

$$\omega(R, \Theta) = \chi' \omega_{(1,0)}(R, \Theta) + \alpha'^2 \chi' \omega_{(1,2)}(R, \Theta) + \mathcal{O}(\chi'^3), \quad (2.23)$$

$$A(R, \Theta) = \chi'^2 A_{(2,0)}(R, \theta) + \alpha'^2 \chi'^2 A_{(2,2)}(R, \Theta) + \mathcal{O}(\chi'^4), \quad (2.24)$$

with $A = (h, k, m, \xi)$.

Let us make several observations about the above decomposition. First, the metric functions ν and λ do not contain dCS corrections because non-rotating NS solutions in dCS gravity are the same as those in GR due to parity protection [35, 154, 292]. Second, also because of parity considerations, only the (t, ϕ) component of the metric contains terms odd in χ' . Third, terms of $\mathcal{O}(\alpha')$ do not appear in the metric because of the structure of the field equations, which force the scalar field ϑ to be proportional to α in the small-coupling approximation [285]. The above decomposition is therefore the minimal line element for a slowly-rotating star in dCS gravity within the small-coupling approximation.

Matter Stress-Energy Tensor and EoS

We assume we can describe the matter inside the NS as a perfect fluid, so that its stress-energy tensor is given by

$$T_{\mu\nu}^{\text{mat}} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} , \quad (2.25)$$

where u^μ is the fluid's four-velocity given by

$$u^\mu = (u^0, 0, 0, \Omega_* u^0) . \quad (2.26)$$

The quantity Ω_* is the *constant* angular velocity of the fluid, and thus of the entire NS, since we model stars in uniform rotation. The normalization condition of the four-velocity $u_\mu u^\mu = -1$ determines u^0 in terms of the metric functions [285].

To close the system of equations, one needs to specify the EoS that relates p and ρ . In this paper, we consider six different EoSs: APR [33], SLy [114], Lattimer-Swesty with nuclear incompressibility of 220 MeV (LS220) [189, 211], Shen [211, 239, 240], WFF1 [274] and ALF2 [36]. For the LS220 and Shen EoSs, we impose neutrino-less, beta-equilibrium conditions. The APR, SLy and WFF1 EoSs are relatively soft, while LS220 is intermediate

and Shen is a stiff EoS. The ALF2 EoS is a hybrid between APR nuclear matter and color-flavor-locked quark matter.

Constructing Neutron Star Solutions

In this section, we first review the structure of the field equations at each order in the dimensionless spin parameter. We then explain how we make each of the I-Love-Q quantities dimensionless following [275, 276] and derive dCS corrections to these quantities. We also derive the dCS correction to the stellar eccentricity in terms of the metric perturbation.

Structure of the Field Equations

The field equations are first expanded order by order in both χ' and α' and then solved numerically also order by order. At any given order, the equations are first solved numerically in the stellar interior, imposing regularity at the center, and then analytically in the exterior region, imposing asymptotic flatness at spatial infinity. One then matches the two solutions at the stellar surface, the radial location where the pressure vanishes, by requiring continuity and smoothness of all metric functions. From these metric functions, we then construct NS observables, such as the mass, moment of inertia and quadrupole moment. Below, we present a brief summary of the structure of the field equations at each order in spin; the full expressions can be found in [285].

At zeroth order in spin, there is no dCS correction. The field equations reduce to the Einstein equations for a non-rotating configuration. Using the (t,t) and (R,R) components of the equations, together with the conservation of matter stress-energy tensor from Eq. (2.8), one finds the Tolman-Oppenheimer-Volkoff (TOV) equation. The stellar mass M_* appears as an integration constant in the exterior solution.

At first order in spin, the only non-vanishing part of the modified Einstein equations is the (t, ϕ) component, which can be decomposed into a GR and dCS part of $\mathcal{O}(\alpha'^2)$. The dCS

part depends on the scalar field, whose evolution equation forces it to be of $\mathcal{O}(\alpha')$. One can Legendre decompose both of these equations such that the set of partial differential equations becomes a set of ordinary differential equations. Boundary conditions at the stellar center and at spatial infinity make the $\ell = 1$ mode the only non-vanishing piece of the equations. The magnitude of the angular momentum J can be read off from the asymptotic behavior of the $g_{t\phi}$ metric component, and the moment of inertia is then found by

$$I \equiv \frac{J}{\Omega_*} . \quad (2.27)$$

At second order in spin, one can again apply a Legendre decomposition to find that both the $\ell = 0$ and $\ell = 2$ modes contribute in the modified Einstein equations. Both modes have a GR and dCS contribution, with the latter again entering at $\mathcal{O}(\alpha'^2)$. The $\ell = 0$ mode gives a spin correction to the mass, which is irrelevant in this paper. We thus focus on the $\ell = 2$ mode, which allows us to calculate the quadrupole moment Q from the integration constants in the exterior solution. In particular, the gravitational potential U , which can be read off from the g_{tt} component, has the asymptotic form²

$$U^{\text{CS}} = -\frac{3}{2} \frac{Q}{R^3} \hat{S}^i \hat{S}^j n^{<ij>} + \mathcal{O}\left(\frac{M_*^4}{R^4}\right) , \quad (2.28)$$

where $\hat{S}^i$ is the unit spin angular momentum vector of a NS and n^i is the unit vector that points to a field point.

The above solutions automatically lead to the mass and radius of the star M_* and R_* , the moment of inertia I and the quadrupole moment Q , but other important quantities can also be computed, such as the electric-type tidal Love number $\lambda^{(\text{tid})}$. This quantity

²This definition of Q is different from the one used in [285] by a factor of 2. We choose to use this definition here so that the GR contribution matches the Geroch-Hansen multipole moments [136, 142] and the moments used by Hartle and Thorne [145].

characterizes the quadrupolar tidal deformability of a star and it is defined as [147]

$$Q^{(\text{tid})} = -\lambda^{(\text{tid})} \mathcal{E}^{(\text{tid})}, \quad (2.29)$$

where $Q^{(\text{tid})}$ is the *tidally-induced* quadrupole moment (not to be confused with the spin-induced quadrupole moment Q), while $\mathcal{E}^{(\text{tid})}$ is the external tidal field strength. One can treat tidal perturbations in a similar manner to perturbations due to spin. The main difference is that one must set $\omega = 0$ (i.e. no parity-odd perturbation) in Eq. (2.19) and not impose asymptotic flatness when solving the equations. Then, $Q^{(\text{tid})}$ and $\mathcal{E}^{(\text{tid})}$ can be read off from the coefficients of the $P_2(\Theta)/R^3$ and $P_2(\Theta)R^2$ parts of the metric perturbation h when asymptotically expanded away from the NS surface, with P_2 the $\ell = 2$ Legendre polynomial. Since dCS gravity modifies the odd-parity sector, $\lambda^{(\text{tid})}$ does not acquire any dCS correction [283].

Dimensionless I-Love-Q and Eccentricity Profiles

We next explain how we adimensionalize physical quantities. Let us first focus on the moment of inertia. We make this quantity dimensionless via

$$\bar{I} \equiv \frac{I}{M_*^3} = \frac{I_{\text{GR}} + \alpha'^2 I_{\text{CS}}}{M_{\text{GR}}^3}, \quad (2.30)$$

which we expand as

$$\bar{I} = \bar{I}_{\text{GR}} + \alpha'^2 \bar{I}_{\text{CS}}, \quad (2.31)$$

where clearly

$$\bar{I}_{\text{GR}} \equiv \frac{I_{\text{GR}}}{M_{\text{GR}}^3}, \quad \bar{I}_{\text{CS}} \equiv \frac{I_{\text{CS}}}{M_{\text{GR}}^3}. \quad (2.32)$$

We recall that the NS mass does not acquire dCS corrections to leading order in spin.

Next, we look at the dCS correction to the dimensionless quadrupole moment. The latter is defined by

$$\bar{Q} \equiv -\frac{Q}{M_*^3 \chi^2} = -\frac{Q_{\text{GR}} + \alpha'^2 Q_{\text{CS}}}{M_{\text{GR}}^3 (\chi_{\text{GR}} + \alpha'^2 \chi_{\text{CS}})^2}, \quad (2.33)$$

which we expand as

$$\bar{Q} = \bar{Q}_{\text{GR}} + \alpha'^2 \bar{Q}_{\text{CS}} + \mathcal{O}(\alpha'^4), \quad (2.34)$$

where we have defined

$$\bar{Q}_{\text{GR}} \equiv -\frac{Q_{\text{GR}}}{M_{\text{GR}}^3 \chi_{\text{GR}}^2}, \quad \bar{Q}_{\text{CS}} \equiv -\frac{Q_{\text{CS}}}{M_{\text{GR}}^3 \chi_{\text{GR}}^2} + 2\bar{Q}_{\text{GR}} \frac{I_{\text{CS}}}{I_{\text{GR}}}. \quad (2.35)$$

Here we used the relation $\chi_{\text{CS}}/\chi_{\text{GR}} = I_{\text{CS}}/I_{\text{GR}}$. The second term in $\bar{Q}_{\text{CS}}$ above typically dominates the first term in the stellar solutions we consider in this paper.

Let us finally focus on the Love number. We make this quantity dimensionless via

$$\bar{\lambda}^{(\text{tid})} \equiv \frac{\lambda^{(\text{tid})}}{M_*^5} = \frac{\lambda_{\text{GR}}^{(\text{tid})}}{M_{\text{GR}}^5}, \quad (2.36)$$

where the last equality follows from the fact that the electric-type tidal deformability is not modified in dCS gravity as already explained.

Before presenting numerical results, let us also discuss the eccentricity contours inside the star at a constant radius. Following [145], the eccentricity can be defined via

$$e(R) = \sqrt{-3 \left(k_2(R) + \frac{\xi_2(R)}{R} \right)}, \quad (2.37)$$

which we expand as

$$e = e_{\text{GR}} + \alpha'^2 e_{\text{CS}} + \mathcal{O}(\alpha'^4), \quad (2.38)$$

where we have defined

$$e_{\text{GR}}(R) = \sqrt{-3 \left(k_2^{\text{GR}}(R) + \frac{\xi_2^{\text{GR}}(R)}{R} \right)}, \quad (2.39)$$

$$e_{\text{CS}}(R) = -\frac{1}{2} \sqrt{\frac{-3}{k_2^{\text{GR}}(R) + \frac{\xi_2^{\text{GR}}(R)}{R}}} \left(k_2^{\text{CS}}(R) + \frac{\xi_2^{\text{CS}}(R)}{R} \right). \quad (2.40)$$

Here k_2^{GR} and ξ_2^{GR} (k_2^{CS} and ξ_2^{CS}) are the coefficients of the $\ell = 2$ modes of $k_{(2,0)}$ and $\xi_{(2,0)}$ ($k_{(2,2)}$ and $\xi_{(2,2)}$) in a Legendre decomposition. Adimensionalizing this quantity, one further finds

$$\begin{aligned} \frac{e(R)}{\chi} &= \frac{e_{\text{GR}} + \alpha'^2 e_{\text{CS}}}{\chi_{\text{GR}} + \alpha'^2 \chi_{\text{CS}}} \\ &= \frac{e_{\text{GR}}}{\chi_{\text{GR}}} + \alpha'^2 \left(\frac{e_{\text{CS}}}{\chi_{\text{GR}}} - \frac{e_{\text{GR}}}{\chi_{\text{GR}}} \frac{I_{\text{CS}}}{I_{\text{GR}}} \right) + \mathcal{O}(\alpha'^4). \end{aligned} \quad (2.41)$$

Numerical Results

In this section we present numerical results obtained by solving the field equations order by order, and calculating the dimensionless quantities defined in the previous section. We conclude this section with a discussion of projected bounds we will be able to place on the theory given future observations of the moment of inertia and the tidal deformability.

I-Love-Q Relations

We begin by studying dCS corrections to the I-Love and Q-Love relations, which we present in the top panels of Fig. 2.3. We recall that $\bar{\lambda}^{(\text{tid})}$ does not acquire any dCS corrections, and hence corrections to the I-Love-Q relations originate from changes to $\bar{I}$ and

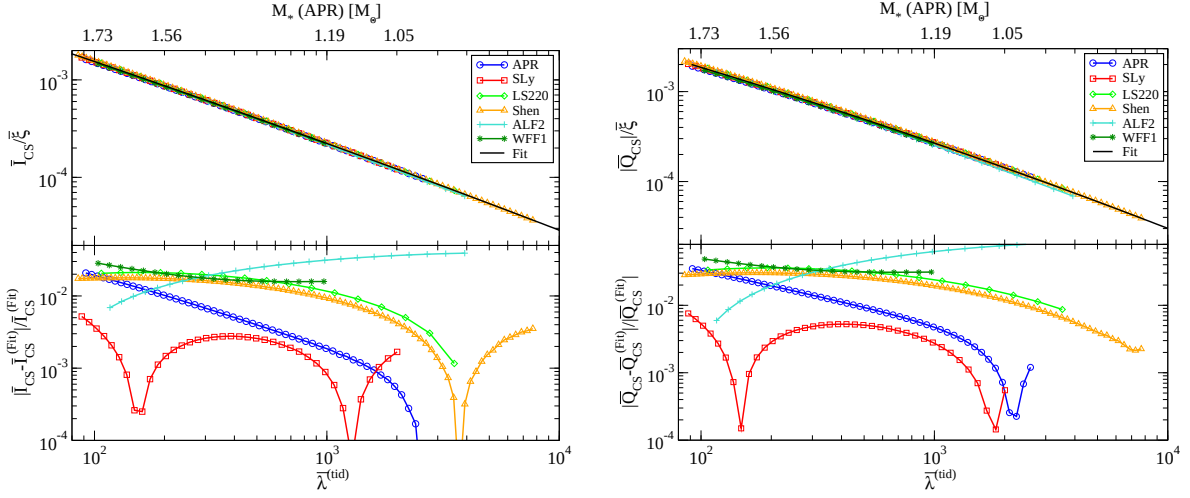


Figure 2.3: (Color Online) (Top) dCS corrections to the I-Love and Q-Love relations, normalized to $\bar{\xi}$, for various EoSs and using the global fit of Eq. (2.43). The top axis shows the corresponding NS mass for a given $\bar{\lambda}^{(tid)}$ assuming the APR EoS in GR. (Bottom) Relative fractional difference between the data and the global fit, which represents the EoS variation in the relations. Observe that the universality holds to better than 5%.

$\bar{Q}$. In Fig. 2.3, we normalize $\bar{I}_{CS}$ and $\bar{Q}_{CS}$ with

$$\bar{\xi} \equiv \frac{\xi_{CS}}{M_*^4}, \quad (2.42)$$

so that the dependence on the dCS coupling constant is factored out from corrections to the I-Love-Q relations in this figure. Observe that the dCS correction to the relations remains universal with respect to various EoSs. To estimate the degree to which these relations remain universal, we construct a global fit (shown by the black solid curves in the figure) for all the numerical data points in the form

$$y_i = \exp [a_i + b_i(\log x_i) + c_i(\log x_i)^2], \quad (2.43)$$

y_i	x_i	a_i	b_i	c_i
$\bar{I}_{\text{CS}}/\bar{\xi}$	$\bar{\lambda}^{(\text{tid})}$	-2.943	-0.718	-0.011
$\bar{Q}_{\text{CS}}/\bar{\xi}$	$\bar{\lambda}^{(\text{tid})}$	-3.443	-0.550	-0.023

Table 2.1: Estimated numerical coefficients for the fitting formula in Eq. (2.43) for dCS corrections to the I-Love and Q-Love relations.

where the coefficients are summarized in Table 2.1. The bottom panel of Fig. 2.3 shows the fractional difference between each numerical data point and the fit, which corresponds to the EoS variation in these relations. Observe that the relations are universal to within 5%.

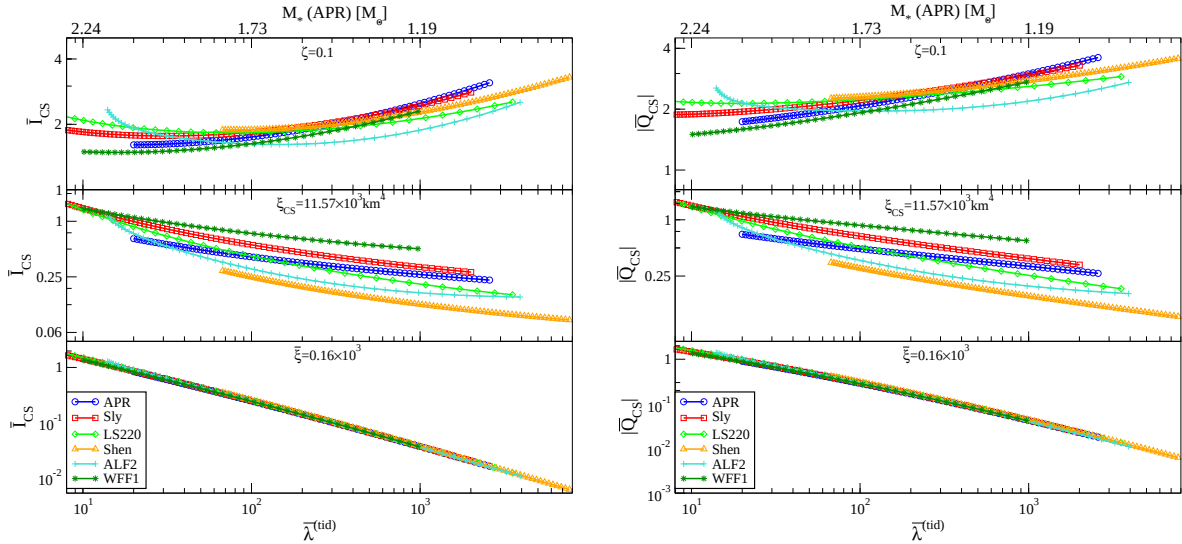


Figure 2.4: (Color Online) dCS corrections to the I-Love (left) and Q-Love (right) relations with $\zeta = 0.1$ (top), $\xi_{\text{CS}} = 11.57 \times 10^3 \text{ km}^4$ (middle) and $\bar{\xi} = 0.16 \times 10^3$ (bottom). Observe that the relations remain universal only if one fixes $\bar{\xi}$.

Figure 2.3 corresponds to fixing the value of $\bar{\xi}$ (to unity), but how does the universality change if one normalizes the I-Love-Q relations differently? To address this question, Fig. 2.4 presents the dCS corrections to the I-Love and Q-Love relations fixing ζ (top), ξ_{CS} (middle)

and $\bar{\xi}$ (bottom), where the first two quantities are given in Eqs. (2.10) and (2.3) respectively. We choose $\zeta = 0.1$ to ensure the validity of the small-coupling approximation, and thus we choose values for ξ_{CS} and $\bar{\xi}$ that correspond to $\zeta = 0.1$ for a NS with mass $2M_\odot$ and radius of 10km. Observe that the universality is lost when the dCS correction is normalized by ζ or ξ_{CS} , while it is recovered if one fixes $\bar{\xi}$. This clearly shows that whether the relations remain universal in modified theories of gravity depends crucially on how one normalizes observables with respect to the coupling constants of the theory.

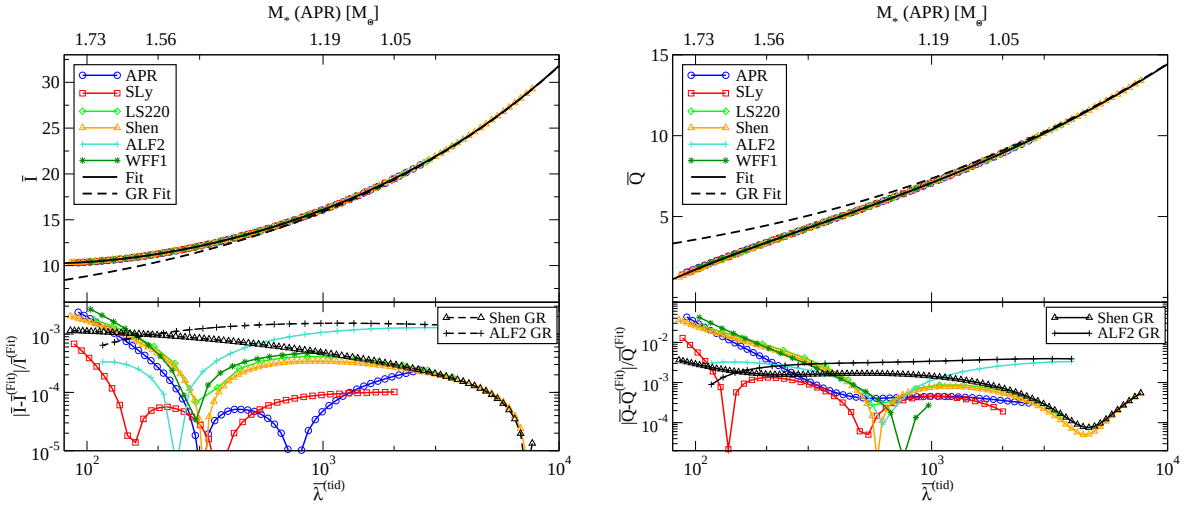


Figure 2.5: (Color Online) (Top) I-Love (left) and Q-Love (right) relations for various EoSs in dCS gravity with $\bar{\xi} = 10^3$. The black dashed curve in each panel represents the best-fit relation in GR. (Bottom) Fractional difference in each relation with respect to a global fit. For reference, we also present the fractional difference for the Shen and ALF2 EoSs in GR. Observe that the EoS variation in dCS gravity is comparable to that in GR for the I-Love relation, while the former is larger than the latter in the Q-Love relation for relativistic NSs with relatively small $\bar{\lambda}^{(\text{tid})}$.

Having studied the dCS corrections to the I-Love-Q relations and which parameter one should fix to retain universality, we now study the full I-Love-Q relations in dCS gravity, including the GR contribution. Figures 2.1 and 2.5 present such relations with $\bar{\xi} = 10^3$, together with the fractional difference from the global fit. Observe that the EoS variation is

of $\mathcal{O}(0.1\%)$ for the I-Love and I-Q relations, and of $\mathcal{O}(1\%)$ for the Q-Love relation. We can compare this EoS variability with that present in GR, represented here through the fractional difference between the Shen and ALF2 EoSs relations in GR. Observe that in general, the EoS variation in dCS gravity is comparable to that in GR for the I-Love and I-Q relations, while the former is larger than the latter for the Q-Love relation, in particular for relativistic stars with smaller $\bar{\lambda}^{(\text{tid})}$. Observe also that there is a certain parameter range for each relation in dCS gravity where the EoS variation is highly suppressed (e.g. $\bar{\lambda}^{(\text{tid})} \sim 300$ for the I-Love relation). This is because the GR (dCS) contribution in the EoS variation dominates for large (small) $\lambda^{(\text{tid})}$, while these two contributions partially cancel in the intermediate region.

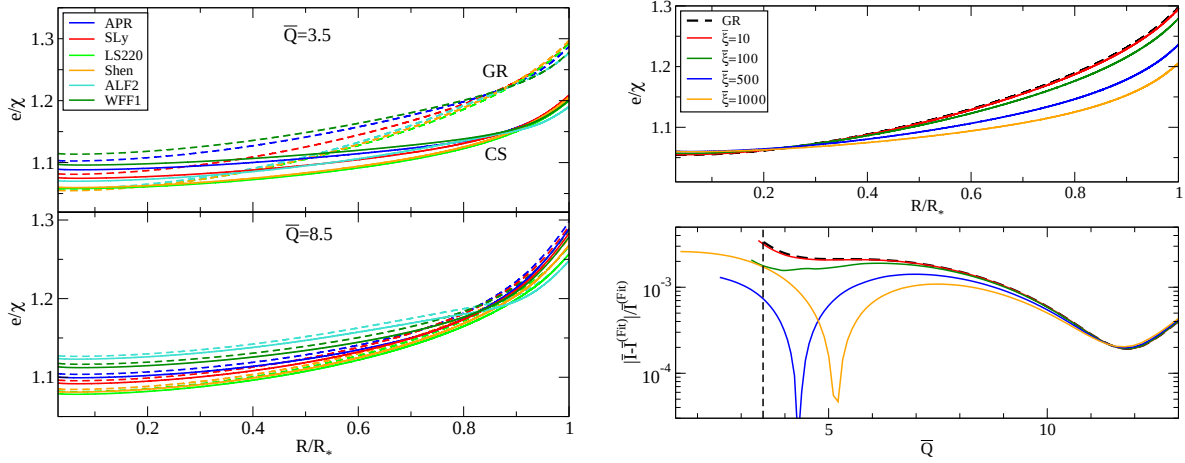


Figure 2.6: (Color Online) (Left) Stellar eccentricity profile with $\bar{Q} = 3.5$ (top) and $\bar{Q} = 8.5$ (bottom) for various EoSs in GR and dCS gravity with $\bar{\xi} = 10^3$. (Top Right) Stellar eccentricity profile for the Shen EoS with $\bar{Q} = 3.5$ and various $\bar{\xi}$. Observe that the eccentricity variation throughout the star becomes smaller as one increases $\bar{\xi}$. (Bottom Right) Fractional difference from the fit in the I-Q relation for the Shen EoS and various $\bar{\xi}$. The vertical dashed line corresponds to $\bar{Q} = 3.5$. Observe that the EoS variation tends to become smaller for larger $\bar{\xi}$ where the eccentricity variation becomes also smaller. However, the EoS variation increases again if one increase $\bar{\xi}$ too much.

Eccentricity Profile in dCS Gravity

Let us now study whether the elliptical isodensity explanation as the origin of the universality in GR [289] mentioned in Sec. 5 is still applicable in modified theories of gravity. First, we look at how the stellar eccentricity variation inside a star in dCS gravity changes from that in GR. The left panels of Fig. 2.6 present the eccentricity profile for various EoSs with $\bar{Q} = 3.5$ (top) and $\bar{Q} = 8.5$ (bottom) in GR and dCS with $\bar{\xi} = 10^3$. Observe that the eccentricity variation is smaller in dCS than in GR when $\bar{Q}$ is relatively small. On the other hand such a variation in dCS is almost indistinguishable from that in GR for relatively large $\bar{Q}$. This is because one approaches a Newtonian regime for larger $\bar{Q}$ values, in which dCS corrections are suppressed. The top right panel of Fig. 2.6 shows the eccentricity profile in dCS gravity in more detail. Here we choose the Shen EoS and $\bar{Q} = 3.5$, and study the eccentricity profile dependence on $\bar{\xi}$. One clearly sees that the variation becomes smaller for larger $\bar{\xi}$.

We now investigate how such an eccentricity variation is related to the amount of EoS variation in the I-Love-Q relations. The bottom right panel of Fig. 2.6 presents the fractional difference in the I-Q relation for the Shen EoS from the fit for various values of $\bar{\xi}$. The vertical dashed line corresponds to $\bar{Q} = 3.5$. For this value of $\bar{Q}$, observe that the fractional difference becomes smaller as one increases $\bar{\xi}$ from 0 to 500, with which the eccentricity variation becomes also smaller as explained in the previous paragraph. This finding supports the conclusion in [289] that the I-Q relation (or relations among multipole moments) becomes stronger as the eccentricity variation becomes smaller. However, if one goes beyond $\bar{\xi} = 500$ (as done also e.g. in Fig. 2.5), the EoS variation in the I-Q relation starts to increase, although the eccentricity variation keeps decreasing. This feature is opposite of what happens in GR and of what one would expect if the self-similarity of isodensity contours was solely responsible of the universality. This suggests that the origin of the universality in modified theories of gravity may be more complicated than that in GR due to the additional

degrees of freedom present.

Future Observational Bounds

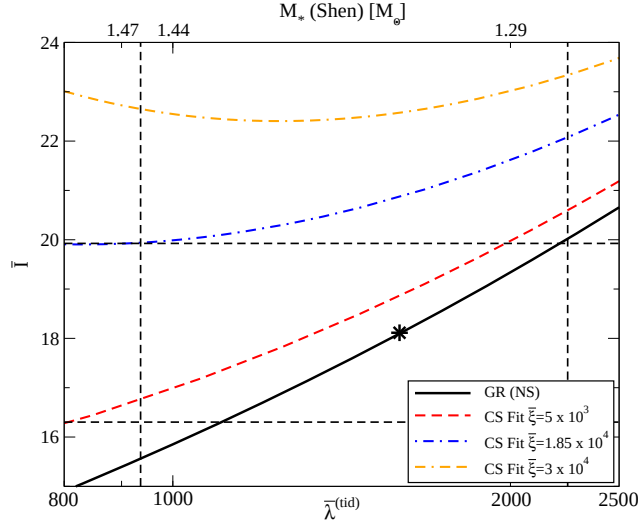


Figure 2.7: (Color Online) The best-fit I-Love relation in dCS gravity with various $\bar{\xi}$. We also show a 10% measurement accuracy of the NS moment of inertia with future double binary pulsar observations, and a 40% measurement accuracy of the NS tidal Love number with future gravitational wave observations, with a fiducial mass of $1.338M_{\odot}$ assuming the Shen EoS, which corresponds to the observed mass of the primary pulsar in J0737-3039. Observe that such observations, if realized, allow us to place the bound $\bar{\xi} < 1.85 \times 10^4$, which is six orders of magnitude more stringent than those from Solar System [37] and table-top [284] experiments in terms of the characteristic length scale $\xi_{\text{CS}}^{1/4}$.

Universal relations project out uncertainties in nuclear physics, and thus they are very useful in probing strong-field gravity with NS observations. Let us then begin by reviewing how one can use these relations to probe dCS gravity following [275, 276]. Let us imagine that we measure the moment of inertia of the primary pulsar (whose mass is estimated as $1.338M_{\odot}$ [68, 178, 196]) in the double binary pulsar J0737-3039 to 10% accuracy [177, 188]. Let us also assume that we measure the tidal Love number of a $\sim 1.338M_{\odot}$ NS³ with an

³Even if we detect GWs from a NS binary with component masses different than the mass of the primary

accuracy of 40% [275, 276]. Figure 2.7 presents these errors in the I-Love plane as dashed lines with the fiducial values taken to be a $1.338M_\odot$ NS (black star) assuming the Shen EoS (which gives the most conservative bound on dCS gravity among all the EoSs considered in this paper). The black solid curve corresponds to the I-Love relation in GR.

The relation in modified theories of gravity has to pass through the error box in order to be consistent with the above hypothetical measurements. In Fig. 2.7, we show the I-Love relation in dCS gravity with three different choices of $\bar{\xi}$. Observe that the relation with $\bar{\xi} < 1.85 \times 10^4$ is consistent with the measurement errors, thus ruling out $\bar{\xi} > 1.85 \times 10^4$. Using $M_* = 1.338M_\odot$, one can map such a bound to a constraint on the characteristic length scale $\sqrt{\alpha} \lesssim 86$ km, which is six orders of magnitude stronger [275, 276] than the bounds from Solar System [37] and table-top [284] experiments. Such a large improvement on the bound is realized because NS observations allow us to probe the strong-field regime, where dCS corrections become naturally large. One can also check that the above bound corresponds to $\zeta < 0.1$ in terms of the dimensionless coupling constant, and thus satisfies the small-coupling approximation. This finding shows the impact of using universal relations on probes of strong-field gravity.

We now go one step further and study how this putative bound on dCS changes with different measurement accuracy of the moment of inertia $\delta\bar{I}$ and tidal Love number $\delta\bar{\lambda}^{(\text{tid})}$. The contours in Fig. 2.2 present the bounds on $\sqrt{\alpha}$ as a function of the measurement accuracy $\delta\bar{I}/\bar{I}$ and $\delta\bar{\lambda}^{(\text{tid})}/\bar{\lambda}^{(\text{tid})}$. Observe that the bounds are all of $\mathcal{O}(10^2)$ km and are not very sensitive to $\delta\bar{I}$ or $\delta\bar{\lambda}^{(\text{tid})}$. This is because to map from bounds on $\bar{\xi}$ to bounds on $\sqrt{\alpha}$ one must take a fourth root, $\sqrt{\alpha} \sim \bar{\xi}^{1/4}$, which then softens the dependance of the $\sqrt{\alpha}$ constraint on $\delta\bar{I}$ and $\delta\bar{\lambda}^{(\text{tid})}$. One should therefore be able to place bounds on dCS gravity that are roughly six orders of magnitude stronger than current ones if the moment of inertia and tidal Love

pulsar in J0737-3039, one can still measure the tidal Love number of a $1.338M_\odot$ NS by Taylor expanding the tidal Love number as a function of the NS mass about $M_* = 1.338M_\odot$ and measuring the leading-order coefficient, as done in [31, 106, 201, 280, 281].

number are measured with future NS observations, irrespective of the precise details of the measurement accuracy.

Discussion

In this paper we studied the I-Love-Q relations in dCS gravity. We found that whether such relations remain universal depends on what coupling parameter one fixes. If one fixes $\bar{\xi}$, the relations are universal, while the universality is lost for fixed ζ or ξ_{cs} . By fixing $\bar{\xi}$, we found that the I-Love and I-Q relations are universal to $\mathcal{O}(0.1\%)$, which is comparable to that in GR. On the other hand, the Q-Love relation is universal to $\mathcal{O}(1\%)$, which is larger than in the GR case. We next studied whether the elliptical isodensity explanation as the origin of the universality still holds in dCS gravity. We found that the eccentricity variation inside a star in dCS is smaller than that in GR, and the universality in the I-Q relation becomes stronger as one increases $\bar{\xi}$. However, if one increases $\bar{\xi}$ too much, the universality becomes weaker while the eccentricity variation keeps decreasing. This suggests that the origin of the universality in non-GR theories may be more complicated than in GR. Finally, we studied how one can use the I-Love relation to probe strong-field gravity. We found that future radio and gravitational wave observations should be able to place bounds on dCS gravity that are roughly six orders of magnitude stronger than current bounds, irrespective of the details of the measurement accuracy of the moment of inertia and tidal Love number.

One obvious direction for future work is to study the I-Love-Q relations for rapidly-rotating NSs. This could be done by modifying the publicly-available code RNS [249]. Such an extension is important to e.g. apply the I-Q relations to rapidly-spinning NS observations using NICER. Another avenue for future work includes studying other universal relations, such as those among higher-order multipole moments [288] and various tidal Love numbers [277]. In particular, we studied parity-even tidal Love numbers in this paper but it would be interesting to consider parity-odd ones [57, 102, 107, 182–184, 218], as these

quantities would acquire non-vanishing dCS corrections. One can also study the universal relations and the origin of the universality in theories other than dCS gravity. For example, universal relations have not been studied within Lorentz-violating theories of gravity, such as Einstein-Æther [120, 156, 159] and khronometric [63] gravity. In these theories, non-rotating [121] and slowly-moving [286, 287] NS solutions have been constructed but slowly-rotating NS solutions have not been studied in the literature yet. Work along this direction is currently in progress.

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NEW BINARY PULSAR CONSTRAINTS ON EINSTEIN-ÆTHER THEORY AFTER
GW170817

Contribution of Authors and Co-Authors

Manuscript in Chapter 3

Author: Toral Gupta

Contributions: Led the development of all necessary codes and algorithms for the study, conducted the analysis, and authored the initial draft of the manuscript.

Author: Mario Herrero-Valea

Contributions: Contributed to theoretical calculations for the equations used in the analysis, and edited the manuscript.

Author: Diego Blas

Contributions: Offered guidance on the existing bounds of the theory and identified the viable parameter space.

Author: Enrico Barausse

Contributions: Detailed the theoretical understanding for the analysis, performed calculations, and edited the manuscript.

Author: Neil Cornish

Contributions: Guided the computational side of the analysis which is based on Bayesian inference and provided valuable feedback on the manuscript.

Author: Kent Yagi

Contributions: Offered innovative and alternative solutions to enhance the analysis described in the paper and edited the manuscript.

Author: Nicolás Yunes

Contributions: Initiated the project, offered comprehensive guidance across all aspects of the analysis, and edited the manuscript.

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Kent Yagi and Nicolás Yunes

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Introduction

Lorentz symmetry has been the foundation of the magnificent edifice of theoretical physics for more than a century, playing a central role in special and general relativity (GR), as well as in the quantum theory of fields. Because of its special status, Lorentz invariance has been tested to exquisite precision in the matter sector via particle physics experiments [160, 174, 175, 199]. More recently, this experimental program has been extended to the matter-gravity [173], dark matter [56, 64], and pure-gravity sectors [156, 191], where bounds on Lorentz violations (LVs) have been historically looser (because of the intrinsic weakness of the gravitational interaction).

Compelling theoretical reasons to seriously consider the possibility of LVs in the purely gravitational sector were provided by the realization that they could generate a better behavior in the ultraviolet (UV) limit. In particular, P. Hořava [151] showed that by allowing for a non-isotropic scaling between space and time, one can construct a theory that is power-counting renormalizable in the UV. Renormalizability beyond power counting (i.e. perturbative renormalizability) in special (“projectable”) versions of Hořava gravity has also been proven [50].

The low-energy limit of Hořava gravity reduces to “khronometric theory” [61, 157], which consists of GR plus an additional hypersurface-orthogonal and timelike vector field, often referred to as the “æther”. Because this vector field is hypersurface orthogonal, it selects a preferred spacetime foliation, which makes LVs manifest. A more general boost-violating low-energy gravitational theory, however, can be obtained by relaxing the assumption that the æther be hypersurface-orthogonal, in which case it selects a preferred time *threading* of the spacetime rather than a preferred foliation. The resulting theory is known as Einstein-æther theory [159].

Despite allowing for an improved UV behavior, LVs in gravity face long-standing

experimental challenges, particularly when it comes to their percolation into the matter sector, where particle physics experiments are in excellent agreement with Lorentz symmetry. While some degree of percolation is inevitable, because of the coupling between matter and gravity, mechanisms suppressing it have been put forward, including suppression by a large energy scale [224], or the effective emergence of Lorentz symmetry at low energies as a result of renormalization group flows [51, 54, 74] or accidental symmetries [139].

At the same time, purely gravitational bounds on LVs are becoming increasingly compelling. The parameters (“coupling constants”) of both Einstein-æther and khronometric theory have been historically constrained by theoretical considerations (absence of ghosts and gradient instabilities [62, 134, 155], well-posedness of the Cauchy problem [233]), by the absence of vacuum Cherenkov cascades in cosmic-ray experiments [122]), by solar-system tests [62, 65, 129, 208, 268], by observations of the primordial abundances of elements from Big-Bang nucleosynthesis [72], by other cosmological tests [46], and by precision timing of binary pulsars (where LVs generically predict violations of the strong equivalence principle) [48, 127, 128, 286, 287]. More recently, the coincident detection [20, 135] of gravitational waves (GW170817) and gamma rays (GRB170817A) emitted by the coalescence of two neutron stars and the subsequent kilonova explosion has allowed extremely strong constraints on the propagation speed of gravitational waves, which must equal that of light to within¹ 10^{-15} [19], which in turn places even more stringent bounds on the couplings of both theories [123, 212, 228, 233].

The bounds from the coincident GW170817/GRB170817A observations force us to rethink the parameter spaces of both Einstein-æther and khronometric theory, as the only currently allowed regions appear to be ones that were previously thought to be of little interest, and which were not explored extensively. In the case of khronometric theory, Refs. [48, 228] found that the couplings that remain viable after GW170817 and GRB170817

¹See also [84] for looser bounds coming from mergers of black holes.

produce exactly no deviations away from the predictions of GR, not only in the solar system, but also in binary systems of compact objects, be they black holes (BHs) or neutron stars (NSs), to leading post-Newtonian (PN) order. Reference [130] extended this result to the quasinormal modes of spherically symmetric black holes and to fully non-linear (spherical) gravitational collapse, where again no deviations from the GR predictions are found. It would therefore seem that the most promising avenue to further test khronometric theory may be provided by cosmological observables (e.g. Big-Bang nucleosynthesis abundances or CMB physics), where the viable couplings do produce non-vanishing deviations away from the GR phenomenology.

Like for khronometric theory, the parameter space where detailed predictions for isolated/binary pulsars were obtained in Einstein-æther theory [286, 287] does *not* include the region singled out by the combination of the GW170817/GRB170817A bound and existing solar-system constraints (see Ref. [233] for a discussion). The goal of this paper is therefore to extend the previous analysis of binary/isolated-pulsar data by some of us [286, 287] to this region of parameter space. This will require a significant modification of the formalism that Refs. [286, 287] utilized to calculate pulsar “sensitivities”, i.e. the parameters that quantify violations of the strong-equivalence principle in these systems. Moreover, we will extend our analysis to include additional data over that considered in Refs. [286, 287], namely the triple system PSR J0337+1715 [43]. Overall, we find that observations of the damping of the period of quasi-circular binary pulsars, and that of the triple system PSR J0337+1715, reduce the viable parameter space of Einstein-æther theory by about an order of magnitude over previous constraints.

We will also amend an error (originally pointed out in Ref. [270]) in the calculation of the strong-field preferred-frame parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ for isolated pulsars, which were presented in Refs. [286, 287]. While we have checked that this error does not impact the bounds presented in Refs. [286, 287], we present in A a detailed derivation of $\hat{\alpha}_1$ and $\hat{\alpha}_2$ for

possible future applications, also correcting a few typos present in the original calculation of Ref. [270].

This paper is organized as follows. In Sec. 3 we give a succinct introduction to Einstein-æther theory, including the modified field equations and the current observational bounds on the coupling constants. In Sec. 3 we introduce the concept of stellar sensitivities as parameters regulating violations of the strong equivalence principle. Solutions describing slowly moving stars are derived in Sec. 3, and they are used in Sec. 3 to compute the sensitivities. Section 3 uses the sensitivities to obtain the constraints on Einstein-æther theory resulting from observations of binary and triple pulsar systems. We summarize our conclusions in Sec. 3. A contains a calculation of the strong-field preferred-frame parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in Einstein-æther theory, fixing an oversight in [286], which was pointed out by [270], and correcting also a few typos present in [270] itself. We will adopt units where $c = 1$ and a signature $+ - - -$, in accordance with most of the literature on Einstein-æther theory.

Einstein Æther Theory

In order to break boost (and thus Lorentz) symmetry, Einstein-æther theory introduces a dynamical threading of the spacetime by a unit-norm, time-like vector field $\mathbf{U}$. This vector field, often referred to as the æther, physically represents a preferred “time direction” at each spacetime event. Requiring the action to also include the usual spin-2 graviton of GR, to be quadratic in the æther derivatives, and to feature no direct coupling between the matter and the æther (so as to enforce the weak equivalence principle, i.e. the universality of free fall, and the absence of matter LVs at tree level), one obtains the action [158, 159]

$$S = -\frac{1}{16\pi G} \int \left[R + \frac{1}{3}c_\theta\theta^2 + c_\sigma\sigma_{\mu\nu}\sigma^{\mu\nu} + c_\omega\omega_{\mu\nu}\omega^{\mu\nu} + c_a A_\mu A^\mu + \lambda(U^\mu U_\mu - 1) \right] \sqrt{-g} d^4x + S_{\text{mat}}(\psi, g_{\mu\nu}), \quad (3.1)$$

where R is the four-dimensional Ricci scalar, g the determinant of the metric, G the bare gravitational constant (related to the value G_N measured locally by $G_N = G/(1 - c_a/2)$ [72, 156]), ψ collectively denotes the matter degrees of freedom, λ is a Lagrange multiplier enforcing the æther's unit norm, c_θ , c_σ , c_ω and c_a are dimensionless constants², and we have decomposed the æther congruence into the expansion θ , the shear $\sigma_{\mu\nu}$, the vorticity $\omega_{\mu\nu}$ and the acceleration A_μ as follows:

$$A^\mu = U^\nu \nabla_\nu U^\mu, \quad (3.2)$$

$$\theta = \nabla_\mu U^\mu, \quad (3.3)$$

$$\sigma_{\mu\nu} = \nabla_{(\nu} U_{\mu)} + A_{(\mu} U_{\nu)} - \frac{1}{3} \theta h_{\mu\nu}, \quad (3.4)$$

$$\omega_{\mu\nu} = \nabla_{[\nu} U_{\mu]} + A_{[\mu} U_{\nu]}, \quad (3.5)$$

with $h_{\mu\nu} = g_{\mu\nu} - U_\mu U_\nu$ the projector onto the hyperspace orthogonal to $\mathbf{U}$.

By varying the action with respect to the metric, the æther and the Lagrange multiplier, and by eliminating the latter from the equations, one obtains the generalized Einstein equations

$$E_{\alpha\beta} \equiv G_{\alpha\beta} - T_{\alpha\beta}^{\mathbb{A}} - 8\pi G T_{\alpha\beta}^{\text{mat}} = 0 \quad (3.6)$$

and the æther equations

$$\mathbb{A}_\mu = \left[\nabla_\alpha J^{\alpha\nu} - \left(c_a - \frac{c_\sigma + c_\omega}{2} \right) A_\alpha \nabla^\nu U^\alpha \right] h_{\mu\nu} = 0, \quad (3.7)$$

²Note that much of the earlier literature on Einstein-æther theory uses a different set of coupling constants c_i ($i = 1, \dots, 4$), which are related to our parameters by $c_1 = (c_\omega + c_\sigma)/2$, $c_2 = (c_\theta - c_\sigma)/3$, $c_3 = (c_\sigma - c_\omega)/2$ and $c_4 = c_a - (c_\sigma + c_\omega)/2$.

where $G_{\alpha\beta}$ is the Einstein tensor, the æther stress-energy tensor is

$$\begin{aligned} T_{\alpha\beta}^{\mathcal{E}} = & \nabla_{\mu} (J_{(\alpha}{}^{\mu} U_{\beta)} - J^{\mu}{}_{(\alpha} U_{\beta)} - J_{(\alpha\beta)} U^{\mu}) + \frac{c_{\omega} + c_{\sigma}}{2} [(\nabla_{\mu} U_{\alpha})(\nabla^{\mu} U_{\beta}) - (\nabla_{\alpha} U_{\mu})(\nabla_{\beta} U^{\mu})] \\ & + U_{\nu}(\nabla_{\mu} J^{\mu\nu}) U_{\alpha} U_{\beta} - \left(c_a - \frac{c_{\sigma} + c_{\omega}}{2}\right) [A^2 U_{\alpha} U_{\beta} - A_{\alpha} A_{\beta}] + \frac{1}{2} M^{\sigma\rho}{}_{\mu\nu} \nabla_{\sigma} U^{\mu} \nabla_{\rho} U^{\nu} g_{\alpha\beta}, \end{aligned} \quad (3.8)$$

with

$$\begin{aligned} J^{\alpha}{}_{\mu} & \equiv M^{\alpha\beta}{}_{\mu\nu} \nabla_{\beta} U^{\nu}, \\ M^{\alpha\beta}{}_{\mu\nu} & = \left(\frac{c_{\sigma} + c_{\omega}}{2}\right) h^{\alpha\beta} g_{\mu\nu} + \left(\frac{c_{\theta} - c_{\sigma}}{3}\right) \delta_{\mu}^{\alpha} \delta_{\nu}^{\beta} + \left(\frac{c_{\sigma} - c_{\omega}}{2}\right) \delta_{\nu}^{\alpha} \delta_{\mu}^{\beta} + c_a U^{\alpha} U^{\beta} g_{\mu\nu}, \end{aligned}$$

and the matter stress-energy tensor is defined as usual by

$$T_{\text{mat}}^{\alpha\beta} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{mat}}}{\delta g_{\alpha\beta}}. \quad (3.9)$$

As already mentioned, a number of experimental and theoretical results constrain Einstein-æther theory and the couplings c_i . In more detail, perturbing the field equations about Minkowski space yields propagation equations for spin-0 (i.e. scalar), spin-1 (i.e. vector) and spin-2 (i.e. tensor gravitons) modes. Their propagation speeds are respectively given by [155]

$$c_T^2 = \frac{1}{1 - c_{\sigma}}, \quad (3.10)$$

$$c_V^2 = \frac{c_{\sigma} + c_{\omega} - c_{\sigma} c_{\omega}}{2c_a(1 - c_{\sigma})}, \quad (3.11)$$

$$c_S^2 = \frac{(c_{\theta} + 2c_{\sigma})(1 - c_a/2)}{3c_a(1 - c_{\sigma})(1 + c_{\theta}/2)}. \quad (3.12)$$

In order to ensure stability at the classical level (i.e. no gradient instabilities) and at the

quantum level (i.e. no ghosts) one needs to have $c_T^2 > 0$, $c_V^2 > 0$ and $c_S^2 > 0$ [134, 155]. If we also require the modes to carry positive energy, we get $c_a > 0$ and $c_\omega > 0$ [119]. Furthermore, significantly subluminal graviton propagation would cause ultrarelativistic matter to lose energy to gravitons via a Cherenkov-like process [122]. Since this effect is not observed e.g. in ultrahigh energy cosmic rays, one must have $c_I^2 \gtrsim 1 - \mathcal{O}(10^{-15})$ (with $I = T, V, S$). More recently, the coincident detection of a neutron-star merger in GW170817 (gravitational waves) and GRB170817A (gamma rays) had led to the bound $-3 \times 10^{-15} < c_T - 1 < 7 \times 10^{-16}$ [19].

Expanding the field equations through 1PN order leads to the conclusion that the 1PN dynamics is well described (like in GR) by the parametrized PN (PPN) expansion [208, 268]. However, unlike in GR, the preferred frame parameters α_1 and α_2 appearing in the PPN expansion do not vanish, but are given by [129]

$$\alpha_1 = 4 \frac{c_\omega(c_a - 2c_\sigma) + c_a c_\sigma}{c_\omega(c_\sigma - 1) - c_\sigma}, \quad (3.13)$$

$$\alpha_2 = \frac{\alpha_1}{2} + \frac{3(c_a - 2c_\sigma)(c_\theta + c_a)}{(2 - c_a)(c_\theta + 2c_\sigma)}. \quad (3.14)$$

Solar system experiments require $|\alpha_1| \lesssim 10^{-4}$ and $|\alpha_2| \lesssim 10^{-7}$ [208, 268]. By *saturating* these bounds (i.e. requiring in particular that $|\alpha_1| \lesssim 10^{-4}$ but *not* $|\alpha_1| \ll 10^{-4}$) and combining them with the constraints on the propagation speeds, one finds $c_\sigma \approx \mathcal{O}(10^{-15})$, $c_a \approx \mathcal{O}(10^{-4})$, and $c_\theta \approx 3c_a[1 + \mathcal{O}(10^{-3})]$. The resulting experimentally viable parameter space, therefore, is effectively (i.e. to within a fractional width of 10^{-4} or better in the parameters) one-dimensional: $c_\sigma, c_a, c_\theta \approx 0$, but c_ω is essentially unconstrained [233].

Another viable region of the parameter space can be obtained by *not* saturating the PPN constraints [233]. In more detail, one may require $|\alpha_1|$ be much smaller than its upper limit, so as to automatically satisfy the bound on α_2 (since $\alpha_2 \propto \alpha_1$ if $c_\sigma \approx 0$, as imposed by GW170817 and GRB170817A). This leads to $|c_a| \lesssim 10^{-7}$ and thus to an effectively

two-dimensional experimentally viable parameter space (c_θ, c_ω) , with the only additional requirement that $|c_\theta| \lesssim 0.3$ to ensure that the production of light elements during Big Bang Nucleosynthesis gives predictions in agreement with observations [72].

Both of the viable regions of parameter space identified above were *not* considered in Refs. [286, 287], where neutron-star sensitivities in Einstein-æther theory were first computed. This is because back when Refs. [286, 287] were written, the strongest constraints available were the solar system ones (since the GW170817/GRB170817A constraint was not yet available). Refs. [286, 287] solved for c_a and c_θ in terms of c_σ , c_ω , α_1 and α_2 , and fixed the latter two to their largest allowed values (respectively $\alpha_1 = 10^{-4}$ and $\alpha_2 = 10^{-7}$). Therefore, (i) this does not include the first region listed above $[(c_\theta, c_\sigma, c_a) \lesssim 10^{-4}$ with c_ω kept free], which is obtained by choosing $\alpha_1 \propto \alpha_2$; and (ii) Refs. [286, 287] did not sample accurately enough the sub-region $c_\sigma \approx 0$, $\alpha_1 \lesssim 10^{-4}$, $\alpha_2 \lesssim 10^{-7}$, since there was no reason to do so at that time and because the techniques employed broke down in that sub-region (as we will show in the following).

Strong-Equivalence Principle Violations and Sensitivities

Most theories extending/modifying GR involve additional degrees of freedom besides the massless tensor graviton of GR. These additional gravitational polarizations cannot directly couple with matter significantly, to avoid introducing unwanted fifth forces in particle physics experiments, and to prevent violations of the weak equivalence principle (and particularly violations of the universality of free fall for weakly gravitating objects). Nevertheless, effective couplings between the extra gravitons and matter may be mediated by the metric perturbations (i.e. by the tensor gravitons present also in GR), which are typically coupled non-minimally to the extra gravitational degrees of freedom. These effective couplings become important when the metric perturbations are “large”, which is the case for strongly gravitating systems such as those involving NSs and/or BHs.

A useful way to parametrize this effective coupling is provided by the sensitivity parameters. Because of the aforementioned effective couplings, the mass of strongly gravitating objects will be comprised not only of the contributions from matter and the metric (like in GR), but it will also generally depend on the additional gravitational fields. We can thus describe isolated objects, and members of a widely separated binary, by a point particle model (like in GR), but with a non-constant mass depending on the extra fields. Because the mass is a scalar quantity, it must depend on a scalar constructed from the æther field $\mathbf{U}$, the simplest of which is the Lorentz factor $\gamma \equiv \mathbf{u} \cdot \mathbf{U}$, where $\mathbf{u}$ is the particle's (i.e. the body's) four-velocity.

In many practical situations (including the long inspiral of a binary system of compact objects) one may assume that the relative speed between the æther and the object is small compared to the speed of light, and thus Taylor-expand the mass $\mu(\gamma)$ around $\gamma = 1$:

$$\mu(\gamma) = \tilde{m} \left[1 + \sigma(1 - \gamma) + \frac{1}{2}\sigma'(1 - \gamma)^2 + \dots \right] \quad (3.15)$$

where $\tilde{m}$, σ and σ' are constant parameters. In particular, the latter two are often referred to as the “sensitivities” and their derivatives:

$$\sigma \equiv - \frac{d \ln \mu(\gamma)}{d \ln \gamma} \Big|_{\gamma=1}, \quad (3.16)$$

$$\sigma' \equiv \sigma + \sigma^2 + \frac{d^2 \ln \mu(\gamma)}{d(\ln \gamma)^2} \Big|_{\gamma=1}. \quad (3.17)$$

In order to understand the effect of the sensitivities and their derivatives on the dynamics of binary systems, one can derive the equations of motion simply by varying the point particle action

$$S_{\text{pp}} = - \sum_A \int \mu_A(\gamma_A) d\tau_A, \quad (3.18)$$

where A is an index identifying the objects, and τ is the proper time. This yields the equation

of motion

$$[\mu_A(\gamma_A) - \mu'_A(\gamma_A)\gamma_A]a_\beta^A = -\mu'_A(\gamma_A)(-u_A^\mu \nabla_\beta U_\mu^A + u_A^\mu \nabla_\mu U_\beta^A) - \mu''_A(\gamma_A)\dot{\gamma}_A(U_\beta^A - \gamma_A u_\beta^A), \quad (3.19)$$

where again the index A identifies the particle under consideration (when used in μ , γ and $\mathbf{u}$) or at which position the æther field $\mathbf{U}$ and its acceleration $\mathbf{A}$ are to be computed, the prime denotes a derivative with respect to the function's argument, and the overdot represents a derivative along $\mathbf{u}$ (i.e. with respect to the proper time).

Reinstating the dependence on the speed of light c and expanding in PN orders (i.e. for $c \rightarrow \infty$), one obtains the 1PN equations of motion for a binary as

$$\begin{aligned} \frac{d\mathbf{v}_A}{dt} = & -\frac{m_B \mathbf{n}}{r^2} \left\{ \mathcal{G}_{AB} - (3\mathcal{G}_{AB}\mathcal{B}_{AB} + \mathcal{D}_{ABB}) \frac{m_B}{r} \right. \\ & -\frac{1}{2} [2\mathcal{G}_{AB}^2 + 6\mathcal{G}_{AB}\mathcal{B}_{(AB)} + 2\mathcal{D}_{BAA} + \mathcal{G}_{AB}(\mathcal{C}_{AB} + \mathcal{E}_{AB})] \frac{m_A}{r} \\ & +\frac{1}{2} [3\mathcal{B}_{AB} - \mathcal{G}_{AB}(1 + \mathcal{A}_A)] v_A^2 + \frac{1}{2} (3\mathcal{B}_{21} + \mathcal{G}_{AB} + \mathcal{E}_{AB}) v_B^2 \\ & -\frac{1}{2} (6\mathcal{B}_{(AB)} + 2\mathcal{G}_{AB} + \mathcal{C}_{AB} + \mathcal{E}_{AB}) \mathbf{v}_A \cdot \mathbf{v}_B - \frac{3}{2} (\mathcal{G}_{AB} + \mathcal{E}_{AB}) (\mathbf{n} \cdot \mathbf{v}_B)^2 \Big\} \\ & +\frac{m_B \mathbf{v}_A}{r^2} \mathbf{n} \cdot \{ [3\mathcal{B}_{AB} + \mathcal{G}_{AB}(1 + \mathcal{A}_A)] \mathbf{v}_A - 3\mathcal{B}_{AB} \mathbf{v}_B \} \\ & -\frac{1}{2} \frac{m_B \mathbf{v}_B}{r^2} \mathbf{n} \cdot [(6\mathcal{B}_{(AB)} + 2\mathcal{G}_{AB} + \mathcal{C}_{AB} + \mathcal{E}_{AB}) \mathbf{v}_A - (6\mathcal{B}_{(AB)} + \mathcal{C}_{AB} - \mathcal{E}_{AB}) \mathbf{v}_B] \\ & -\frac{1}{2} \frac{m_B \mathbf{w}}{r^2} \mathbf{n} \cdot [(\mathcal{C}_{AB} - 6\mathcal{B}_{[AB]} + \mathcal{E}_{AB} - 2\mathcal{G}_{AB}\mathcal{A}_A) \mathbf{v}_A - (\mathcal{C}_{AB} - 6\mathcal{B}_{[AB]} - \mathcal{E}_{AB}) \mathbf{v}_B] , \end{aligned} \quad (3.20)$$

where indices $A \neq B$, $r \equiv |\mathbf{x}_A - \mathbf{x}_B|$, $\mathbf{n} \equiv (\mathbf{x}_A - \mathbf{x}_B)/r$, $\mathbf{v}_A \equiv d\mathbf{x}/dt$, and

$$\begin{aligned}
\mathcal{G}_{AB} &= \frac{G_N}{(1+\sigma_A)(1+\sigma_B)} , \\
\mathcal{A}_A &= -\frac{\sigma'_A}{1+\sigma_A} , \\
\mathcal{B}_{AB} &= \mathcal{G}_{AB}(1+\sigma_A) , \\
\mathcal{D}_{ABB} &= \mathcal{G}_{AB}^2(1+\sigma_A) , \\
\mathcal{C}_{AB} &= \mathcal{G}_{AB} [\alpha_1 - \alpha_2 - 3(\sigma_A + \sigma_B) - \mathcal{Q}_{AB} - \mathcal{R}_{AB}] , \\
\mathcal{E}_{AB} &= \mathcal{G}_{AB} [\alpha_2 + \mathcal{Q}_{AB} - \mathcal{R}_{AB}] , \\
\mathcal{Q}_{AB} &= -\frac{1}{2} \left(\frac{2-c_a}{2c_\sigma - c_a} \right) (\alpha_1 - 2\alpha_2)(\sigma_A + \sigma_B) + 3 \left(\frac{2-c_a}{2c_\sigma + c_\theta} \right) \sigma_A \sigma_B , \\
\mathcal{R}_{AB} &= \frac{1}{2} \left(\frac{8+\alpha_1}{c_\omega + c_\sigma} \right) [-c_\omega(\sigma_A + \sigma_B) + (1 - c_\omega)\sigma_A \sigma_B] .
\end{aligned} \tag{3.21}$$

Note that we have defined the “active” masses $m_A \equiv \tilde{m}_A(1+\sigma_A)$, in terms of which the Newtonian acceleration matches the GR result, albeit with a rescaled gravitational constant $\mathcal{G}_{AB}$. To derive Eq. (3.20) we have also used the PN-expanded solutions for the metric and æther found in [128, 286] (dropping divergent terms due to the point particle approximation,

as usual in PN calculations):

$$g_{00} = 1 - \frac{2G_N\tilde{m}_1}{r_1c^2} + \frac{1}{c^4} \left[\frac{2G_N^2\tilde{m}_1^2}{r_1^2} + \frac{2G_N^2\tilde{m}_1\tilde{m}_2}{r_1r_2} + \frac{2G_N^2\tilde{m}_1\tilde{m}_2}{r_1r_{12}} - \frac{3G_N\tilde{m}_1}{r_1}v_1^2(1+\sigma_1) \right] \\ + 1 \leftrightarrow 2 + \mathcal{O}(1/c^6), \quad (3.22)$$

$$g_{0i} = -\frac{1}{c^3} \left[B_1^- \frac{G_N\tilde{m}_1}{r_1}v_1^i + B_1^+ \frac{G_N\tilde{m}_1}{r_1}v_1^jn_1^jn_1^i \right] + 1 \leftrightarrow 2 + \mathcal{O}(1/c^4), \quad (3.23)$$

$$g_{ij} = -\left(1 + \frac{1}{c^2} \frac{2G_N\tilde{m}_1}{r_1}\right) \delta_{ij} + 1 \leftrightarrow 2 + \mathcal{O}(1/c^4), \quad (3.24)$$

$$U^0 = 1 + \frac{1}{c^2} \frac{G_N\tilde{m}_1}{r_1} + 1 \leftrightarrow 2 + \mathcal{O}(1/c^4), \quad (3.25)$$

$$U^i = \frac{1}{c^3} \frac{G_N\tilde{m}_1}{r_1} (C_1^- v_1^i + C_1^+ v_1^jn_1^jn_1^i) + 1 \leftrightarrow 2 + \mathcal{O}(1/c^5), \quad (3.26)$$

$$B_A^\pm \equiv \pm \frac{3}{2} - 2 \pm \frac{1}{4}(\alpha_1 - 2\alpha_2) \left(1 + \frac{2-c_a}{2c_\sigma - c_a} \sigma_A \right) - \frac{2c_\omega}{c_\omega + c_\sigma} \sigma_A - \frac{1}{4} \alpha_1 \left(1 + \frac{c_\omega}{c_\omega + c_\sigma} \sigma_A \right), \quad (3.27)$$

$$C_A^\pm \equiv \frac{1}{4} \left(\frac{8 + \alpha_1}{c_\omega + c_\sigma} \right) [c_\omega - (1 - c_\omega) \sigma_A] \pm \frac{2 - c_a}{2} \left(\frac{2\alpha_2 - \alpha_1}{2(2c_\sigma - c_a)} + \frac{3\sigma_A}{2c_\sigma + c_\theta} \right), \quad (3.28)$$

where $r_A \equiv |\mathbf{x} - \mathbf{x}_A|$ and $\mathbf{n}_A \equiv (\mathbf{x} - \mathbf{x}_A)/r_A$.

Note that the æther solution (3.25)–(3.26) has space components U^i vanishing at large distances from the binary, i.e. the equations of motion are valid in a preferred reference frame in which the æther is asymptotically at rest. The dependence of the dynamics on the velocity $\mathbf{w}$ of the binary's center of mass with respect to the preferred frame can be

reinstated by performing a boost. If $w \ll c$, one then obtains [270]

$$\begin{aligned}
\frac{d\mathbf{v}_A}{dt} = & -\frac{m_B \mathbf{n}}{r^2} \left\{ \mathcal{G}_{AB} - (3\mathcal{G}_{AB}\mathcal{B}_{AB} + \mathcal{D}_{ABB}) \frac{m_B}{r} \right. \\
& -\frac{1}{2} \left[2\mathcal{G}_{AB}^2 + 6\mathcal{G}_{AB}\mathcal{B}_{(AB)} + 2\mathcal{D}_{BAA} + \mathcal{G}_{AB}(\mathcal{C}_{AB} + \mathcal{E}_{AB}) \right] \frac{m_A}{r} \\
& +\frac{1}{2} [3\mathcal{B}_{AB} - \mathcal{G}_{AB}(1 + \mathcal{A}_A)] v_A^2 + \frac{1}{2} (3\mathcal{B}_{21} + \mathcal{G}_{AB} + \mathcal{E}_{AB}) v_B^2 \\
& -\frac{1}{2} (6\mathcal{B}_{(AB)} + 2\mathcal{G}_{AB} + \mathcal{C}_{AB} + \mathcal{E}_{AB}) \mathbf{v}_A \cdot \mathbf{v}_B - \frac{3}{2} (\mathcal{G}_{AB} + \mathcal{E}_{AB}) (\mathbf{n} \cdot \mathbf{v}_B)^2 \\
& +\frac{1}{2} (\mathcal{C}_{AB} + \mathcal{G}_{AB}\mathcal{A}_A) w^2 + \frac{1}{2} (\mathcal{C}_{AB} - 6\mathcal{B}_{[AB]} + \mathcal{E}_{AB} + 2\mathcal{G}_{AB}\mathcal{A}_A) \mathbf{v}_A \cdot \mathbf{w} \\
& +\frac{1}{2} (\mathcal{C}_{AB} + 6\mathcal{B}_{[AB]} - \mathcal{E}_{AB}) \mathbf{v}_B \cdot \mathbf{w} \\
& \left. +\frac{3}{2} \mathcal{E}_{AB} [(\mathbf{w} \cdot \mathbf{n})^2 + 2(\mathbf{w} \cdot \mathbf{n})(\mathbf{v}_B \cdot \mathbf{n})] \right\} \\
& +\frac{m_B \mathbf{v}_A}{r^2} \mathbf{n} \cdot \{ [3\mathcal{B}_{AB} + \mathcal{G}_{AB}(1 + \mathcal{A}_A)] \mathbf{v}_A - 3\mathcal{B}_{AB} \mathbf{v}_B + \mathcal{G}_{AB} \mathcal{A}_A \mathbf{w} \} \\
& -\frac{1}{2} \frac{m_B \mathbf{v}_B}{r^2} \mathbf{n} \cdot \{ (6\mathcal{B}_{(AB)} + 2\mathcal{G}_{AB} + \mathcal{C}_{AB} + \mathcal{E}_{AB}) \mathbf{v}_A \\
& - (6\mathcal{B}_{(AB)} + \mathcal{C}_{AB} - \mathcal{E}_{AB}) \mathbf{v}_B + 2\mathcal{E}_{AB} \mathbf{w} \} \\
& -\frac{1}{2} \frac{m_B \mathbf{w}}{r^2} \mathbf{n} \cdot \{ (\mathcal{C}_{AB} - 6\mathcal{B}_{[AB]} + \mathcal{E}_{AB} - 2\mathcal{G}_{AB}\mathcal{A}_A) \mathbf{v}_A \\
& - (\mathcal{C}_{AB} - 6\mathcal{B}_{[AB]} - \mathcal{E}_{AB}) \mathbf{v}_B - 2(\mathcal{G}_{AB}\mathcal{A}_A - \mathcal{E}_{AB}) \mathbf{w} \} ,
\end{aligned} \tag{3.29}$$

with which Eq. (3.20) of course agrees for $\mathbf{w} = 0$.

The sensitivities and their derivatives enter into the conservative dynamics of the binary system at Newtonian and 1PN order, as can be seen explicitly in Eqs. (3.20) and (3.29). In A we will use these equations as starting point for studying in more detail the 1PN dynamics of binaries in Einstein-æther theory. In doing so, we will also amend the calculation of the strong-field preferred-frame parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ performed in [286], fixing an oversight pointed out by [270] and correcting also a few typos present in [270] itself³.

³The corrections to $\hat{\alpha}_1$ and $\hat{\alpha}_2$ do not significantly affect the final results of [286], since the strong-field preferred-frame parameters did not play a crucial role in constraining the parameter space of Lorentz-

The sensitivities and their derivatives, however, also enter the dissipative dynamics. In more detail the total energy emitted in GWs (including not only tensor but also scalar and vector æther modes) by a binary in quasi-circular orbits was derived in [128, 286] via a standard multipole expansion and reads

$$\begin{aligned} \frac{\dot{E}_b}{E_b} = 2 \left(\frac{\mathcal{G}_{12} G m_1 m_2}{r^3} \right) & \left\{ \frac{32}{5} (\Psi_1 + \mathcal{S} \Psi_2 + \mathcal{S}^2 \Psi_3) v_{21}^2 \right. \\ & \left. + (s_1 - s_2)^2 \left[\zeta_2 + 2\zeta_3 [w^2 - (\mathbf{w} \cdot \mathbf{n})^2] + \frac{18}{5} \Psi_3 w^2 + \left(\frac{6}{5} \Psi_3 + 36\zeta_1 \right) (\mathbf{w} \cdot \mathbf{n})^2 \right] \right\}, \quad (3.30) \end{aligned}$$

where $s_A \equiv \sigma_A / (1 + \sigma_A)$ is the rescaled sensitivity for the A -th body; $\mathbf{v}_{21} = \mathbf{v}_2 - \mathbf{v}_1$ is the relative velocity of the two bodies; the total (potential and kinetic) energy of the binary is

$$E_b = -\frac{\mathcal{G}_{12} m_1 m_2}{2r}; \quad (3.31)$$

w is the velocity of the binary's center of mass with respect to the preferred frame; and we have introduced the definitions

$$\Psi_1 \equiv \frac{1}{c_T} + \frac{2c_a c_\sigma^2}{(c_\sigma + c_\omega - c_\sigma c_\omega)^2 c_V} + \frac{3c_a (Z - 1)^2}{2c_S (2 - c_a)}, \quad (3.32)$$

$$\Psi_2 \equiv \frac{2(Z - 1)}{(c_a - 2)c_S^3} - \frac{2c_\sigma}{(c_\sigma + c_\omega - c_\sigma c_\omega)c_V^3}, \quad (3.33)$$

$$\Psi_3 \equiv \frac{1}{2c_V^5 c_a} + \frac{2}{3c_a (2 - c_a) c_S^5}, \quad \zeta_1 \equiv \frac{1}{9c_a c_S^5 (2 - c_a)}, \quad (3.34)$$

$$\zeta_2 \equiv \frac{4}{3c_S^3 c_a (2 - c_a)} + \frac{4}{3c_a c_V^3}, \quad \zeta_3 \equiv \frac{1}{6c_V^5 c_a}, \quad (3.35)$$

$$Z \equiv \frac{(\alpha_1 - 2\alpha_2)(1 - c_\sigma)}{3(2c_\sigma - c_a)}, \quad \mathcal{S} \equiv \frac{m_B s_A + m_A s_B}{m_A + m_B}. \quad (3.36)$$

Note that the dipole flux is proportional to ζ_2 and to $(s_1 - s_2)^2$ (just like in scalar-tensor

violating gravity in that paper. The measurements of $\hat{\alpha}_1$ and $\hat{\alpha}_2$ were used mainly to constrain c_ω , but similar bounds on that coupling constant can be obtained from the measurement of the damping of the period of PSR J0348+0432 (see Fig. 7 of [286]).

theories of the Fierz-Jordan-Brans-Dicke type [97, 116, 272]). Therefore, it may dominate over GR's quadrupole emission at low frequencies, depending on the sensitivities and the coupling parameters of the theory [296].

Solutions for Slowly Moving Stars

In order to compute the sensitivities, we start from the observation that the metric and æther solutions for a single point particle [Eqs. (3.22)–(3.26) with $\tilde{m}_2 = 0$] depend on the sensitivity σ already at linear order in the particle's velocity. Moreover, σ regulates the decay of the metric and æther components at large radii and enters already at $\mathcal{O}(1/r)$. The sensitivity is of course a free parameter in the metric and æther solutions for a point particle, but it can be determined by replacing the point particle with a body of finite size. Once a fully non-linear solution for such a body (e.g., in our case, a NS) has been obtained, one can extract its sensitivity from the asymptotic fall-off of the metric and æther fields. Obviously, since σ appears at linear order in velocity, the NS must be moving relative to the preferred foliation. Here, we follow [286] and consider a star in slow motion with respect to the æther, solve the field equations through linear order in the star's velocity, and extract the sensitivities from the asymptotic decay of the fields.

Metric Ansatz

Here, we consider the case of a non-spinning NS at rest with a background æther field moving relative to it. The system is in a stationary regime, i.e, there is no dependence of the metric and the æther field on the time coordinate. For this configuration, letting v^i be the velocity of the star relative to the æther, we consider the following ansatz for the metric

and the æther

$$ds^2 = e^{\nu(r)} dt^2 - \left(1 - \frac{2M(r)}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + 2vV(r, \theta) dt dr + 2vS(r, \theta) dt d\theta + \mathcal{O}(v^2), \quad (3.37)$$

$$U_\mu = e^{\nu(r)/2} \delta_\mu^t + vW(r, \theta) \delta_\mu^r + vQ(r, \theta) \delta_\mu^\theta + \mathcal{O}(v^2), \quad (3.38)$$

and we will set $G_N = 1$ from here on. Note that $M(r)$ has dimensions of length and $M(r) \rightarrow M_\star$ as $r \rightarrow \infty$, with $M_\star$ thus being the measured mass of the star. Here we have adopted a coordinate system that is comoving with the fluid elements of the NS, by aligning the time coordinate vector to the fluid 4-velocity u^μ [286]. In these comoving coordinates, the fluid elements are at rest while the æther is moving. The fluid 4-velocity field is

$$u^\mu = e^{-\nu/2} \delta_t^\mu. \quad (3.39)$$

The ansatz of Eqs. (3.37)–(3.39) depends on $M(r)$ and $\nu(r)$ at $\mathcal{O}(v^0)$ and on four potentials $V(r, \theta)$, $S(r, \theta)$, $W(r, \theta)$ and $Q(r, \theta)$ at $\mathcal{O}(v)$. However, one can perform a coordinate transformation of the form

$$t' = t + vH(r, \theta), \quad (3.40)$$

which allows for any one of the four potentials to be set to zero while keeping the ansatz valid at $\mathcal{O}(v)$. Here we choose to set $Q = 0$ without loss of generality.

Zeroth Order in Velocity

From Eqs. (3.6)–(3.7) let us derive the field equations at zeroth order in velocity, which we will solve to construct the background NS solution. The (t, t) , (r, r) and (θ, θ) components

of the field equations are the only non-trivial ones and give three independent equations [286]

$$16 \frac{dM}{dr} - 4c_a r(r-2M) \frac{d^2\nu}{dr^2} - c_a r(r-2M) \left(\frac{d\nu}{dr} \right)^2 + 4c_a \left(r \frac{dM}{dr} - 2r + 3M \right) \frac{d\nu}{dr} = 64\pi \rho r^2, \quad (3.41)$$

$$c_a r^2(r-2M) \left(\frac{d\nu}{dr} \right)^2 + 8r(r-2M) \frac{d\nu}{dr} - 16M = 64\pi r^3 P, \quad (3.42)$$

$$4r^2(r-2M) \frac{d^2\nu}{dr^2} - (c_a - 2)r^2(r-2M) \left(\frac{d\nu}{dr} \right)^2 - 4r \left(r \frac{dM}{dr} - r + M \right) \frac{d\nu}{dr} - 8r \frac{dM}{dr} + 8M = 64\pi r^3 P, \quad (3.43)$$

where $P(r)$ and $\rho(r)$ are rescaled NS pressure and density respectively (rescaled because $G_N = \frac{2G}{2-c_a}$). These can be expressed as

$$P \equiv \frac{2-c_a}{2} \tilde{P}, \quad \rho \equiv \frac{2-c_a}{2} \tilde{\rho}, \quad (3.44)$$

with $\tilde{P}$ and $\tilde{\rho}$ representing the pressure and energy density that enter directly in the stress-energy tensor for the matter field, which we take to be of a perfect fluid form

$$T_{\mu\nu}^{\text{mat}} = (\tilde{\rho} + \tilde{P}) u_\mu u_\nu - \tilde{P} g_{\mu\nu} + \mathcal{O}(v^2). \quad (3.45)$$

Note, that there is a bijective correspondence between the original parametrization $(c_a, c_\theta, c_\omega, c_\sigma)$ and $(\alpha_1, \alpha_2, c_\omega, c_\sigma)$ that can be derived from Eqs. (3.13) and (3.14). As discussed in Sec. 3, $c_\sigma \ll 10^{-15}$ from gravitational wave observations; we thus rewrite Eqs. (3.41)–(3.43) in terms of $(\alpha_1, \alpha_2, c_\sigma, c_\omega)$. This is justified because the bound on c_σ is much stronger than those on the other parameters, which are constrained by solar-system and stability requirements (absence of gradient instabilities and vacuum Cherenkov

radiation, and positive energy) to satisfy

$$\alpha_1 < 8\alpha_2 < 0, \quad c_\omega > -\frac{\alpha_1}{2}, \quad (3.46)$$

with $|\alpha_1| \lesssim 10^{-4}$ and $|\alpha_2| \lesssim 10^{-7}$, in the limit $c_\sigma \rightarrow 0$. Upon simplification, we get the modified Tolman-Oppenheimer-Volkoff (TOV) equations

$$\frac{dM}{dr} = \frac{1}{\alpha_1(\alpha_1 + 8)r} \left\{ -4\sqrt{r - 2M}(\alpha_1 + 8)\sqrt{(-\alpha_1 + 8)M - 4P\alpha_1\pi r^3 + 4r} \right. \\ \left. - (\alpha_1^2 + 24\alpha_1 + 128)M - 16r \left(\alpha_1\pi r^2(\alpha_1 + 2)P - 2\pi r^2\alpha_1\rho - \frac{\alpha_1}{2} - 4 \right) \right\}, \quad (3.47)$$

$$\frac{d\nu}{dr} = \frac{1}{\alpha_1(r - 2M)r} \left[-8\sqrt{r - 2M}\sqrt{(-\alpha_1 - 8)M - 4P\alpha_1\pi r^3 + 4r} + 16r - 32M \right], \quad (3.48)$$

$$\frac{dp}{dr} = \frac{1}{\alpha_1(r - 2M)r} 4(P + \rho) \left[\sqrt{r - 2M}\sqrt{(-\alpha_1 - 8)M - 4P\alpha_1\pi r^3 + 4r} - 2r + 4M \right]. \quad (3.49)$$

Modifications to the GR TOV equations can be singled out by expanding the above equations (3.41)–(3.43) in a small coupling approximation, i.e., $c_a \ll 1$ or $\alpha_1 \ll 1$ [213, 259, 286].

First Order in Velocity

We derive field equations at first order in velocity from Eqs. (3.6) and (3.7), which include the potentials as functions of r and θ , at first order in velocity. We can separate variables in r and θ using a Legendre decomposition [286] to obtain

$$V(r, \theta) = \sum_n K_n(r) P_n(\cos \theta), \quad (3.50)$$

$$S(r, \theta) = \sum_n S_n(r) \frac{P_n(\cos \theta)}{d\theta}, \quad (3.51)$$

$$W(r, \theta) = \sum_n W_n(r) P_n(\cos \theta), \quad (3.52)$$

where P_n is the Legendre polynomial of order n . More details on tensor harmonic decomposition can be found in [254]. By separation of variables, we arrive at $\mathcal{O}(v)$ equations, where only the (t, r) and (t, θ) components of the modified Einstein equations and the r and θ components of the æther field equations are non-trivial. We are only interested in $n = 1$ component of Legendre decomposition since these functions determine sensitivities and consequently the change in orbital period.

Since $c_\sigma \ll 10^{-15}$ (c.f. Sec. 3), we proceed with calculations in the limit $c_\sigma \rightarrow 0$ to obtain [286]

$$\begin{aligned}
\frac{dS_1}{dr} &= \frac{1}{\alpha_1 r(r-2M)} \left\{ -2S_1 \sqrt{r-2M} (\alpha_1 + 4) \sqrt{(-\alpha_1 - 8)M - 4P\alpha_1 \pi r^3 + 4r} - \right. \\
&\quad \left. (r-2M) (J_1 e^{\nu/2} c_\omega \alpha_1 - (3\alpha_1 + 16)S_1 + K_1 \alpha_1 (c_\omega - 1)) \right\}, \tag{3.53} \\
\frac{dK_1}{dr} &= \frac{1}{c_\omega (r-2M)(\alpha_1^2 + (2-2\alpha_2)\alpha_1 - 16\alpha_2)\alpha_1(\alpha_1 + 8)r} \left\{ 2 \left[((c_\omega + 1)\alpha_1 + 2c_\omega) J_1 e^{\nu/2} \right. \right. \\
&\quad \left. \left. - (6\alpha_1 + 32)S_1 + c_\omega K_1 \alpha_1 \right] (\alpha_1^2 + (2-2\alpha_2)\alpha_1 - 16\alpha_2)(\alpha_1 + 8) \sqrt{r-2M} \right. \\
&\quad \left. \times \sqrt{(-\alpha_1 - 8)M - 4P\alpha_1 \pi r^3 + 4r} + [-(\alpha_1 + 8)(r-2M)\alpha_1 r \right. \\
&\quad \left. ((c_\omega + 1)\alpha_1^2 - 2c_\omega(\alpha_2 - 1)\alpha_1 - 16\alpha_2 c_\omega) \left(\frac{\partial J_1}{\partial r} \right) - 16 \left(-\frac{1}{8} \left(3 \left(\left(c_\omega + \frac{5}{3} \right) \alpha_1^3 \right. \right. \right. \right. \\
&\quad \left. \left. \left. + \left(\left(-2\alpha_2 + \frac{14}{3} \right) c_\omega - \frac{8\alpha_2}{3} + \frac{8}{3} \right) \alpha_1^2 + \left(\left(-\frac{64\alpha_2}{3} + \frac{16}{3} \right) c_\omega - \frac{64\alpha_2}{3} \right) \alpha_1 \right. \right. \right. \right. \\
&\quad \left. \left. \left. - \frac{128\alpha_2 c_\omega}{3} \right) (\alpha_1 + 8)M \right) + (\pi(\alpha_1 + 2)\alpha_1 r^2 ((c_\omega + 1)\alpha_1^2 - 2c_\omega(\alpha_2 - 1)\alpha_1 \right. \\
&\quad \left. - 16\alpha_2 c_\omega)P - 2\pi\alpha_1 r^2 ((c_\omega + 1)\alpha_1^2 - 2c_\omega(\alpha_2 - 1)\alpha_1 - 16\alpha_2 c_\omega)\rho \right. \\
&\quad \left. + \frac{1}{4} \left((\alpha_1 + 8) \left(\left(c_\omega + \frac{3}{2} \right) \alpha_1^3 + ((-2\alpha_2 + 4)c_\omega - 2\alpha_2 + 2)\alpha_1^2 \right. \right. \right. \\
&\quad \left. \left. \left. + ((-20\alpha_2 + 4)c_\omega - 16\alpha_2)\alpha_1 - 32\alpha_2 c_\omega) \right) r \right] J_1 \right] e^{\nu/2} \\
&\quad + 6 \left(-\frac{2(\alpha_1 + 8)^2 S_1}{3} + c_\omega K_1 \alpha_1 \right) (\alpha_1 + 8)(\alpha_1^2 + (2-2\alpha_2)\alpha_1 - 16\alpha_2)M \\
&\quad \left. - 16 \left[\left(\pi(\alpha_1^2 + (2-2\alpha_2)\alpha_1 - 16\alpha_2)(\alpha_1 + 4)\alpha_1 r^2 P - \frac{11\alpha_2^3}{8} \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + (3\alpha_2 - 11)\alpha_1^2 + (40\alpha_2 - 16)\alpha_1 + 128\alpha_2) (\alpha_1 + 8)S_1 + c_\omega K_1(\alpha_1^2 \\
& + (2 - 2\alpha_2)\alpha_1 - 16\alpha_2)\alpha_1 (\pi r^2(\alpha_1 + 2)P - 2\pi r^2\rho + \alpha_1/4 + 2)] r\} , \tag{3.54} \\
\frac{d^2 J_1}{dr^2} = & \frac{1}{\sqrt{(-\alpha_1 - 8)M - 4P\alpha_1\pi r^3 + 4r}(r - 2M)^{3/2}\alpha_1^3 r^2(\alpha_1 + 8)}} (4 \{[(\alpha_1 + 8) \\
& \left((\alpha_1^2 - 4\alpha_1\alpha_2 - 32\alpha_2)S_1 + \left(-\frac{\alpha_1^2}{2} + c_\omega(\alpha_2 - 1)\alpha_1 + 8\alpha_2 c_\omega \right) K_1 \right) \alpha_1 r e^{-\nu/2} \\
& - 12\alpha_1^2 \left(\left(\frac{1}{24}\alpha_1^2 + \alpha_1 + \frac{16}{3} \right) M + \left(\alpha_1\pi r^2(\alpha_1 + 2)P - 2\pi r^2\alpha_1\rho + \frac{\alpha_1^2}{24} - \frac{8}{3} \right) r \right) \\
& \times r \left(\frac{\partial J_1}{\partial r} \right) + \left(8 \left(\frac{\partial \rho}{\partial r} \right) \alpha_1^3 \pi r^4 + \frac{1}{2} ((\alpha_1 + 8) (\alpha_1^3 + (-8\alpha_2 + 56)\alpha_1^2 \right. \\
& + (-192\alpha_2 + 128)\alpha_1 - 1024\alpha_2) M) + (8\pi(\alpha_1^3 + (-5\alpha_2 + 14)\alpha_1^2 \\
& + (-56\alpha_2 + 24)\alpha_1 - 128\alpha_2)\alpha_1 r^2 P + 4\pi\alpha_1^2 r^2 (\alpha_1^2 + (-2\alpha_2 + 16)\alpha_1 - 16\alpha_2 \\
& + 16)\rho + \left(\frac{\alpha_1^3}{2} + (\alpha_2 c_\omega - c_\omega - 12)\alpha_1^2 + (-32 + (8c_\omega + 32)\alpha_2)\alpha_1 + 256\alpha_2 \right) \\
& \times (\alpha_1 + 8)r) J_1] \sqrt{r - 2M} \sqrt{(-\alpha_1 - 8)M - 4P\alpha_1\pi r^3 + 4r} \\
& + 64 \left(\left(\frac{\alpha_1}{4} + 2 \right) M + P\alpha_1\pi r^3 - r \right) \\
& \times \left[\frac{1}{8} \left(-S_1\alpha_1\alpha_2 r(\alpha_1 + 8)^2 e^{-\nu/2} + \alpha_1^2 r(\alpha_1 + 8)(r - 2M) \frac{\partial J_1}{\partial r} \right) + J_1 \left(\frac{3(\alpha_1 + 8)}{4} \right. \right. \\
& \times \left(\alpha_1^2 + \left(\frac{-8\alpha_2}{3} + \frac{8}{3} \right) \alpha_1 - \frac{64\alpha_2}{3} \right) M + (r^2\pi\alpha_1^2(\alpha_1 + 2)P \\
& \left. \left. + \alpha_1^2\pi r^2(\alpha_1 + 2)\rho - \frac{3\alpha_1^3}{8} + (\alpha_2 - 4)\alpha_1^2 + (16\alpha_2 - 8)\alpha_1 + 64\alpha_2 \right) r \right] \} \} , \tag{3.55}
\end{aligned}$$

where we have defined $J_n = W_n + e^{-\nu/2}K_n$ [286]. With the above set of equations at hand, the next section describes the methods of solving these equations at each order in velocity.

The Calculation of the Sensitivities

The sensitivities are calculated by solving the coupled differential equations in Eqs. (3.47)-(3.49) and Eqs. (3.53)-(3.55), which are obtained from the modified Einstein and the æther field equations in a $v \ll 1$ expansion at $\mathcal{O}(v^0)$ and $\mathcal{O}(v)$ respectively [286]. In

Secs. 3 and 3 we describe and apply two methods to solve these equations and find the NS sensitivities. The first method, outlined in Sec. 3, was used previously in Ref. [286], but we will explain how it leads to unstable solutions in particular regions of parameter space. A second method outlined in Sec. 3 provides stable results in all regions of parameter space.

The $\mathcal{O}(v^0)$ solutions are common to both methods, as they both involve solving $\mathcal{O}(v^0)$ differential equations (3.47)–(3.49) numerically once in the interior and then in the exterior of the NS. The initial and boundary conditions to it are obtained by imposing regularity at the NS center, while imposing asymptotic flatness at spatial infinity respectively. The differential equations are solved from a core radius (i.e. some small initial radius) to the stellar surface radius, where the pressure goes to zero. These numerical solutions at the NS surface are now used as initial conditions to solve the exterior evolution equations from the stellar surface to an extraction radius r_b . Using continuity and differentiability of the solutions, the asymptotic solutions at spatial infinity are matched to the numerical solutions evaluated at r_b . This gives the observed mass of the NS and the integration constant corresponding to $\nu(0)$ (obtained by solving the $\mathcal{O}(v^0)$ differential equations [286]) which will be used in solving the $\mathcal{O}(v)$ equations discussed further.

Method 1: Direct Numerical Solutions

In this method, the aforementioned $\mathcal{O}(v)$ differential equations are solved in two regions, the interior of the star, and the exterior. The initial conditions at $\mathcal{O}(v)$ are obtained by solving the corresponding differential equations asymptotically about a core radius, while imposing regularity at the core, and asymptotically about spatial infinity, while imposing asymptotic flatness [286]. In both cases, the solutions depend on integration constants – $\tilde{C}$ and $\tilde{D}$ in the interior asymptotic solution and $\tilde{A}$ and $\tilde{B}$ in the exterior asymptotic solution – that must be chosen so as to guarantee that the numerical interior and exterior solutions are continuous and differentiable at the stellar surface, where pressure becomes significantly

smaller than their core values.

As defined above, the global solution reduces to finding the right constants $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$, which in turn is a shooting problem. In practice, Ref. [286] solved this shooting problem by first picking two sets of values for interior constants, $\vec{c}_{(1)} = (\tilde{C}_{(1)}, \tilde{D}_{(1)})$ and $\vec{c}_{(2)} = (\tilde{C}_{(2)}, \tilde{D}_{(2)})$, and then solving the interior equations twice from the core radius r_c to the NS surface $R_\star$ to find the solutions $\vec{f}_{(1)}^{\text{int}}(r) = [S_1^{(1,\text{int})}(r), K_1^{(1,\text{int})}(r), J_1^{(1,\text{int})}(r), J_1'^{(1,\text{int})}(r)]$ and $\vec{f}_{(2)}^{\text{int}}(r) = [S_1^{(2,\text{int})}(r), K_1^{(2,\text{int})}(r), J_1^{(2,\text{int})}(r), J_1'^{(2,\text{int})}(r)]$. Then, each interior numerical solution is evaluated at the stellar surface and used as initial conditions for a numerical evolution in the exterior, leading to two exterior solutions $\vec{f}_1^{\text{ext}}(r) = [S_1^{(1,\text{ext})}(r), K_1^{(1,\text{ext})}(r), J_1^{(1,\text{ext})}(r), J_1'^{(1,\text{ext})}(r)]$ and $\vec{f}_2^{\text{ext}}(r) = [S_1^{(2,\text{ext})}(r), K_1^{(2,\text{ext})}(r), J_1^{(2,\text{ext})}(r), J_1'^{(2,\text{ext})}(r)]$.

The global solutions $\vec{f}_{1,2}^{\text{glo}}(r) = \vec{f}_{1,2}^{\text{int}}(r) \cup \vec{f}_{1,2}^{\text{ext}}(r)$ are then automatically continuous and differentiable at the surface, but in general they will not satisfy the boundary conditions at spatial infinity. Because of the linear and homogeneous structure of the differential system, one can find the correct global solution through linear superposition

$$\vec{f}^{\text{glo}}(r; C', D') = C' \vec{f}_1^{\text{glo}}(r) + D' \vec{f}_2^{\text{glo}}(r), \quad (3.56)$$

where C' and D' are new constants, chosen to guarantee that $\vec{f}^{\text{glo}}$ satisfies the correct asymptotic conditions near spatial infinity, which in turn depend on $(\tilde{A}, \tilde{B})$, i.e.

$$\vec{f}^{\text{glo}}(r_b; C', D') = \vec{f}^{\text{glo},\infty}(r_b; \tilde{A}, \tilde{B}), \quad (3.57)$$

where $r_b \gg R_\star$ is the matching radius, $\vec{f}^{\text{glo}}(r_b; C', D')$ is given by Eq. (3.56) evaluated at $r = r_b$ (which depends on (C', D')) and $\vec{f}^{\text{glo},\infty}(r_b; \tilde{A}, \tilde{B})$ is the asymptotic solution to the differential equations near spatial infinity evaluated at the matching radius (which depends on $(\tilde{A}, \tilde{B})$).

With this at hand, one can calculate the NS sensitivities via [286]

$$\sigma = 2\tilde{A}\frac{\alpha_1}{\alpha_1 + 8}, \quad (3.58)$$

where $\tilde{A}$ is the coefficient of $1/r$ in the near-spatial infinity asymptotic solution of W_1^{ext} such that

$$W_1^{\text{ext}}(r) = \tilde{A}\frac{M_\star}{r} + \mathcal{O}\left(\frac{M_\star^2}{r^2}\right), \quad (3.59)$$

while we recall that $\alpha_1 \lesssim 10^{-4}$ (c.f. Sec. 3). Because of the latter constraint, it is obvious that the sensitivities are essentially controlled by $\sigma \approx \tilde{A}\alpha_1$, so the numerical stability of its calculation relies entirely on the numerical stability of the calculation of this coefficient. Unfortunately, as we show below, this calculation is not numerically stable in the region of parameter space we are interested in.

Figure 3.1 shows S_1 as a function of radius, assuming $(\alpha_1, \alpha_2, c_\omega) = (10^{-4}, 4 \times 10^{-7}, -0.1)$, and setting $(r_c, r_b) = (10^2, 2 \times 10^7)$ cm. Observe that both $S_1^{(1, \text{glo})}$ and $S_1^{(2, \text{glo})}$ diverge at spatial infinity, so in order to find an S_1^{glo} that is finite at spatial infinity, a very delicate cancellation of large numbers needs to take place. This cancellation needs to lead to $\tilde{A}\alpha_1 \approx 0$ but in general $\tilde{A}\alpha_1 \neq 0$, since $\sigma \approx \tilde{A}\alpha_1/4 \ll 1 \neq 0$, and precisely by how much $\tilde{A}\alpha_1$ deviates from 0 is what determines the value of the sensitivity. We find in practice that σ is highly sensitive to the accuracy of the numerical algorithm used to solve for $\vec{f}_{(1,2)}^{\text{glo}}$, as well as the choice of r_c, r_b and the value of $p(R_\star)$ that defines the stellar surface. Figure 3.1 is in the parameter region that is outside of interest but it indicates how sensitive the calculations are to the aforementioned cancellation, making it difficult to find numerically stable solutions.

Method 2: Post-Minkowskian Approach

Given that the first method does not allow us to robustly compute the sensitivities in the regime of interest, we developed a new post-Minkowskian method, which we describe here.

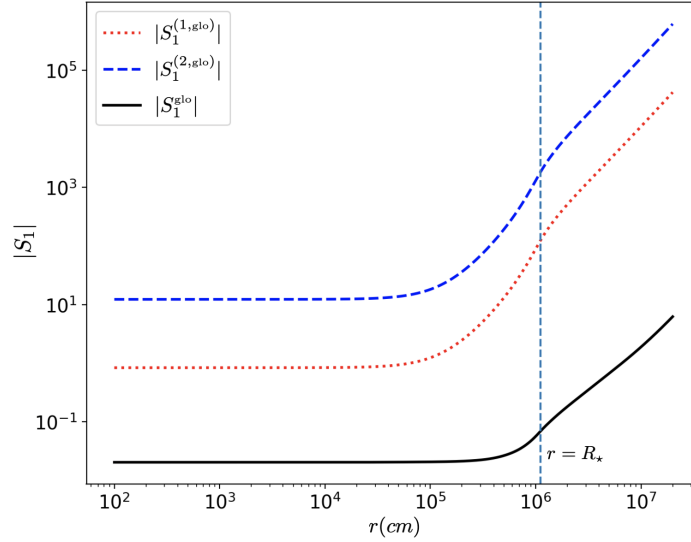


Figure 3.1: The metric function $|S_1|$ is plotted against the radius in the entire numerical domain, where the radius of the star is at 11.1 km (vertical dashed line). Observe that both of the trial solutions $S_1^{(1,glo)}$ and $S_1^{(2,glo)}$ diverge at spatial infinity. Hence, the global linearly combined solution $S_1^{(glo)}$ shows a diverging behaviour representing numerical instabilities in the calculation of sensitivities.

In this method, the background $O(v^0)$ equations are solved by direct integration, as done in method 1. The differential equations at $O(v)$, however, are expanded in compactness $\mathcal{C}$ and solved order by order. This is a post-Minkowskian approximation because the compactness always appears multiplied by G/c^2 , so in this sense it is a weak-field expansion. NSs are not weak-field objects, but their compactness is always smaller than $\sim 1/3$ (and usually between $[0.1, 0.3]$), so provided enough terms are kept in the series, this approximation has the potential to be valid. Moreover, and perhaps more importantly, we will show below that such a perturbative scheme stabilizes the numerical solution for the NS sensitivities.

The procedure presented above is not technically a standard post-Minkowskian series solution because the background equations (or their solutions) are not expanded in powers of $\mathcal{C}$. Had we expanded in powers of $\mathcal{C}$ everywhere, we would have encountered terms in the differential equations with derivatives of the equation of state (EoS). Such derivatives

would introduce numerical noise because “realistic” (tabulated) EoSs are not usually smooth functions, potentially introducing steep jumps, see, e.g., [285]. By not expanding the $\mathcal{O}(v^0)$ differential equations, we are implicitly adding higher order terms in compactness, so this procedure could be seen as a resummation technique.

Through this approach, the differential equations at $\mathcal{O}(v^1)$ turn into n sets of differential equations for an expansion carried out to $\mathcal{O}(\mathcal{C}^n)$, with n therefore labeling the compactness order. In order to derive these equations, however, one must first establish the order of the background solutions, which can be shown to satisfy

$$M(r) = \mathcal{O}(\mathcal{C}), \quad P(r) = \mathcal{O}(\mathcal{C}^2), \quad (3.60)$$

$$\rho(r) = \mathcal{O}(\mathcal{C}), \quad \text{and} \quad \nu(r) = \mathcal{O}(\mathcal{C}), \quad (3.61)$$

by looking at the differential equations these functions obey at $\mathcal{O}(v^0)$. The metric perturbation functions at $\mathcal{O}(v^1)$ are then expanded in powers of compactness through

$$Y_i(r) = \sum_{j=1}^n Y_{ij} \epsilon^j, \quad (3.62)$$

where $Y_i \equiv (S_1, K_1, W_1)$, ϵ is a bookkeeping parameter of $\mathcal{O}(\mathcal{C})$ and j indicates the order of $\mathcal{C}$ to be summed over. We work with $W_1(r)$ instead of $J_1(r)$ to avoid introducing numerical error during the conversion between these two functions.

Using these expansions in the differential equations at $\mathcal{O}(v)$, and re-expanding them in powers of compactness, one finds n sets of differential equations. At $\mathcal{O}(\mathcal{C})$, the differential system becomes

$$\begin{aligned} \frac{dS_{11}}{dr} &= \frac{2rK_{11} - 2r(S_{11} + \alpha_1 W_{11}) - (4 + \alpha_1)M}{2r^2}, \\ \frac{dK_{11}}{dr} &= \frac{-1}{2c_\omega r^2 \alpha_1} [8c_\omega \pi r^3 \alpha_1 \rho + 4c_\omega r \alpha_1 (K_{11} - S_{11}) - 4r \alpha_1^2 W_{11} \end{aligned} \quad (3.63)$$

$$\begin{aligned}
& + c_\omega(8\alpha_1 + \alpha_1^2 - 16\alpha_2 - 2\alpha_1\alpha_2)M - 4c_\omega r^2 \alpha_1 W'_{11} \\
& - 2r^2(\alpha_1^2 + c_\omega \alpha_1^2 - 16c_\omega \alpha_2 - 2c_\omega \alpha_1 \alpha_2)W'_{11} \Big] , \tag{3.64}
\end{aligned}$$

$$\frac{d^2 W_{11}}{dr^2} = \frac{2W_{11}}{r^2} , \tag{3.65}$$

where $S_{11}(r)$, $K_{11}(r)$ and $W_{11}(r)$ are metric functions at $\mathcal{O}(\mathcal{C})$, and recall that the density $\rho(r)$ is related to pressure through the EoS as $\rho(r) = \rho(P(r))$. We note that $W_{11}(r)$ is decoupled from $S_{11}(r)$ and $K_{11}(r)$, and so we can solve Eq. (3.65) separately and analytically in the regions $r \leq r_b$ and $r \geq r_b$. The solutions to them are $W_{11}(r) = \tilde{D}_1 r^2$ for $r \leq r_b$ and $W_{11}^\infty(r) = \tilde{A}_1/r$ for $r \geq r_b$, where $\tilde{A}_1$ and $\tilde{D}_1$ are integration constants. By requiring continuity and differentiability of metric functions $W_{11}(r)$ and $W'_{11}(r)$, we match the solutions at the extraction radius r_b . This fixes the values of two integration constants $\tilde{A}_1 = 0$ and $\tilde{D}_1 = 0$.

The remaining two equations, namely Eqs. (3.63) and (3.64), are solved numerically with initial conditions obtained using regularity at the NS center and asymptotic flatness at spatial infinity. At the center, we have

$$\begin{aligned}
S_{11}(r) &= \tilde{C}_1 - \frac{1}{120\alpha_1} \pi \left[(240\alpha_1 + 40\alpha_1^2 - 128\alpha_2 - 16\alpha_1\alpha_2)\rho_c \right. \\
&\quad \left. + (48\alpha_1\alpha_2 - 72\alpha_1^2 - 15\alpha_1^3 + 6\alpha_1^2\alpha_2)p_c \right] r^2 \\
&\quad + \frac{1}{1260} \pi^2 \left[(-160\alpha_1 - 35\alpha_1^2 + 80\alpha_2 + 10\alpha_1\alpha_2)\rho_c^2 \right. \\
&\quad \left. + (288\alpha_2 - 576\alpha_1 - 126\alpha_1^2 + 36\alpha_1\alpha_2)p_c^2 \right. \\
&\quad \left. + (336\alpha_2 - 672\alpha_1 - 147\alpha_1^2 + 42\alpha_1\alpha_2)\rho_c p_c \right] r^4 + \mathcal{O}(r^6) , \tag{3.66} \\
K_{11}(r) &= \tilde{C}_1 - \frac{1}{120\alpha_1} \pi \left[(400\alpha_1 + 40\alpha_1^2 - 384\alpha_2 - 48\alpha_1\alpha_2)\rho_c \right. \\
&\quad \left. + (144\alpha_1\alpha_2 - 96\alpha_1^2 - 15\alpha_1^3 + 18\alpha_1^2\alpha_2)p_c \right] r^2 \\
&\quad + \frac{1}{1260} \pi^2 \left[(400\alpha_2 - 240\alpha_1 - 35\alpha_1^2 + 50\alpha_1\alpha_2)\rho_c^2 \right.
\end{aligned}$$

$$\begin{aligned}
& + (1440\alpha_2 - 864\alpha_1 - 126\alpha_1^2 + 180\alpha_1\alpha_2)p_c^2 \\
& + (1680\alpha_2 - 1088\alpha_1 - 147\alpha_1^2 + 210\alpha_1\alpha_2)\rho_c p_c] r^4 + \mathcal{O}(r^6), \tag{3.67}
\end{aligned}$$

where $\tilde{C}_1$ is an integration constant⁴. At spatial infinity, we have

$$\begin{aligned}
S_{11}^\infty(r) = & -\frac{\tilde{B}_1}{2r^3} - \frac{1}{2c_\omega r \alpha_1} \left[\tilde{A}_1(\alpha_1^2 - 2c_\omega \alpha_1 - 2\alpha_1^2 c_\omega + 16c_\omega \alpha_2 + 2c_\omega \alpha_1 \alpha_2) \right. \\
& + c_\omega(8\alpha_2 - 6\alpha_1 - \alpha_1^2 + \alpha_1 \alpha_2)M_\star \left. \right] + (16\alpha_2 - 8\alpha_1 - \alpha_1^2 + 2\alpha_1 \alpha_2) \frac{M_\star^2}{64r^2} \\
& + (16\alpha_2 - 16\alpha_1 - 3\alpha_1^2 + 2\alpha_1 \alpha_2) \frac{M_\star^3}{192r^3} \\
& + (8\alpha_2 - 2\alpha_1 + \alpha_1 \alpha_2) \ln\left(\frac{r}{M_\star}\right) \frac{M_\star^3}{48r^3} + \mathcal{O}\left(\frac{M_\star^4}{r^4}\right), \tag{3.68}
\end{aligned}$$

$$\begin{aligned}
K_{11}^\infty(r) = & \frac{\tilde{B}_1}{r^3} + \frac{4M_\star + 2\tilde{A}_1\alpha_1 + M_\star\alpha_1}{2r} - (\alpha_1^2 + 16\alpha_2 + 2\alpha_1\alpha_2) \frac{M_\star^2}{64r^2} \\
& + (2\alpha_1 - 8\alpha_2 - \alpha_1\alpha_2) \ln\left(\frac{r}{M_\star}\right) \frac{M_\star^3}{24r^3} + \mathcal{O}\left(\frac{M_\star^4}{r^4}\right) \tag{3.69}
\end{aligned}$$

where $\tilde{B}_1$ is an integration constant and $M_\star$ is the mass of the star.

We next explain how to solve Eqs. (3.63) and (3.64) to construct the solution for S_{11} and K_{11} . First, homogeneous solutions are given by $S_{11}^{\text{hom}} = K_{11}^{\text{hom}} = \tilde{C}_1$. Next, one can construct particular solutions S_{11}^{part} and K_{11}^{part} by setting $\tilde{C}_1 = 0$ and numerically integrate the equations from r_c to $R_\star$. We then use the numerically calculated interior solutions, evaluated at $R_\star$, as the initial conditions to solve the exterior evolution equations with zero pressure and density from $R_\star$ to r_b . The true solutions are simply the sum of the homogeneous and particular solutions, namely

$$S_{11}(r) = \tilde{C}_1 + S_{11}^{\text{part}}, \tag{3.70}$$

$$K_{11}(r) = \tilde{C}_1 + K_{11}^{\text{part}}. \tag{3.71}$$

⁴Equations (3.66) and (3.67) (and also Eqs. (3.68) and (3.69)) contain terms higher than $\mathcal{O}(C)$ because the background functions are not expanded in a series of C and thus contain higher order contributions.

By requiring continuity and differentiability of all metric functions, we match the true numerical solution to the analytic asymptotic solution in Eqs. (3.68) and (3.69) at r_b . Applying this matching condition gives the values of $\tilde{B}_1$ and $\tilde{C}_1$.

Now let us focus on the solution to $\mathcal{O}(\mathcal{C}^2)$ differential equations. The equation for the metric function $W_{12}(r)$ is

$$\begin{aligned} \frac{d^2 W_{12}}{dr^2} = & \frac{2W_{12}}{r^2} + 4\pi r W_{11} \rho' - 2\pi \rho \left(-2K_{11} + 2S_{11} + (4 + \alpha_1 + \frac{2\alpha_1}{c_\omega}) W_{11} - 6r W'_{11} \right) \\ & - 2\pi \rho (6 + \alpha_1) \frac{M}{r} - W'_{11} \left(4 + \alpha_2 + \frac{8\alpha_2}{\alpha_1} \right) \frac{M}{r^2} \\ & - \left(3K_{11} - 3S_{11} - 10W_{11} - 2\alpha_1 W_{11} - \frac{2\alpha_1 W_{11}}{c_\omega} \right) \frac{M}{r^3} \\ & + \left(5 + \alpha_1 + \frac{\alpha_2}{2} + \frac{4\alpha_2}{\alpha_1} \right) \frac{M^2}{r^4}, \end{aligned} \quad (3.72)$$

where $\rho'(r)$ is the derivative of $\rho(P(r))$ obtained from the EoS. This equation is decoupled from the remaining metric functions at $\mathcal{O}(\mathcal{C}^2)$, S_{12} and K_{12} , and can be solved numerically on its own. The initial condition obtained at the center of the NS is

$$\begin{aligned} W_{12}(r) = & \tilde{D}_2 r^2 + \frac{1}{529200\alpha_1} \pi^2 r^4 \left[560\rho_c^3 \pi r^2 \alpha_1 (\alpha_1 + 8) (5\alpha_1 + 4\alpha_2) \right. \\ & - 27p_c^2 \alpha_1^2 (70(8\pi p_c r^2 - 7)\alpha_1^2 - 8\pi p_c r^2 \alpha_1 (\alpha_2 - 422) + 49\alpha_1 (\alpha_2 - 62) \\ & + 8(49 - 8\pi p_c r^2) \alpha_2) - 252\rho_c p_c \alpha_1 \{ 70p_c \pi r^2 \alpha_1^3 + \alpha_1^2 (70 - p_c \pi r^2 (\alpha_2 - 382)) \\ & + 64(7 - 4\pi p_c r^2) \alpha_2 - 8\alpha_1 (5\pi p_c r^2 (\alpha_2 + 8) - 7(\alpha_2 + 10)) \} \\ & - 12\rho_c^2 (350\pi p_c r^2 \alpha_1^4 - 5\pi p_c r^2 \alpha_1^3 (-226 + \alpha_2) - 31360\alpha_2 \\ & \left. - 784\alpha_1 (-40 + (5 + 8\pi p_c r^2) \alpha_2) - 8\alpha_1^2 (-490 + \pi p_c r^2 (980 + 103\alpha_2)) \right] + \mathcal{O}(r^6), \end{aligned} \quad (3.73)$$

where $\tilde{D}_2$ is an integration constant. The boundary condition to Eq. (3.72) at spatial infinity

is

$$\begin{aligned}
W_{12}^{\infty}(r) = & \frac{\tilde{A}_2}{r} + \frac{1}{320r^3\alpha_1c_\omega} \left[\tilde{A}_1 M_\star (40r - M_\star \alpha_1)(\alpha_1^2 + c_\omega(4\alpha_1^2 \right. \\
& + \alpha_1(34 - 4\alpha_2) - 32\alpha_2)) + c_\omega(80M_\star^2 r(\alpha_1 + 8)(\alpha_1 - \alpha_2) \\
& \left. + M_\star^3 \alpha_1(-4\alpha_1^2 + 56\alpha_2 + \alpha_1(-38 + 7\alpha_2))) \right] + \mathcal{O}\left(\frac{M_\star^4}{r^4}\right), \tag{3.74}
\end{aligned}$$

where $\tilde{A}_2$ is an integration constant. To construct the solution, we first note that the homogeneous solution is given by $W_{12}^{\text{hom}} = \tilde{D}_2 r^2$. Next, we set $\tilde{D}_2 = 0$ and find the particular solution $W_{12}^{\text{part}}(r)$ numerically in the interior of the NS by solving Eq. (3.72). This interior solution evaluated at the NS surface now serves as initial conditions to solve the differential equations in the exterior up to the boundary radius r_b . The correct solution in the entire numerical domain is then

$$W_{12}(r) = \tilde{D}_2 r^2 + W_{12}^{\text{part}}(r), \tag{3.75}$$

where the values of $\tilde{A}_2$ and $\tilde{D}_2$ are obtained using the matching condition at r_b . The equations for S_{12} and K_{12} are solved similar to the way Eqs. (3.63) and (3.64) are solved, so we omit a more detailed description here for brevity. We can use the above method to solve differential equations at higher order in $\mathcal{C}$.

Comparison Between Numerical and Analytical Approaches

Tabulated APR4 EoS The sensitivity in the æther theory σ for an isolated NS depends on the EoS chosen, and here we perform the calculations of the previous section for the APR4 tabulated EoS [34]. The results are representative of what one finds with other EoSs.

Eq. (3.58) gives the expression of sensitivity in terms of the integration constant $\tilde{A}$ [286]

where $\tilde{A}$ can be expressed as

$$\tilde{A} \equiv \frac{1}{M_\star} \sum_{j=2}^n \tilde{A}_j \epsilon^j, \quad (3.76)$$

where n is the order of the compactness expansion, with $\tilde{A}_1 = 0$. The coefficients $\tilde{A}_j$ can be calculated numerically as described in the previous subsection. Notice that the leading contribution to $\tilde{A}$ (and hence to the sensitivities) is of $\mathcal{O}(\epsilon)$, since $M_\star = \mathcal{O}(\epsilon)$.

The calculation of the sensitivity as described above requires one to choose the truncation order n of the post-Minkowskian expansion. We will choose n by the sensitivities computed from methods 1 and 2 in a regime of parameter space where method 1 yields stable results [286]. In particular, we will focus on the choice $(\alpha_1, \alpha_2, c_\omega, c_\sigma) = (10^{-4}, 4 \times 10^{-7}, 10^{-4}, 0)$. Figure 3.2 compares the sensitivities computed with the two methods with this parameter choice. Observe that as the order of post-Minkowskian approximation increases (i.e. as n increases), the curves approach the method 1 solution, but in an oscillatory manner. The bottom panel of Fig 3.2 shows the stability of post-Minkowskian method at order $n = 3$.

In the weak field limit, the sensitivity can be well-approximated as the ratio of the binding energy to the NS mass ($\Omega/M_\star$) through [128]

$$s_{\text{wf}} = \left(\alpha_1 - \frac{2}{3} \alpha_2 \right) \frac{\Omega}{M_\star}, \quad (3.77)$$

where the stellar binding energy Ω is [127]

$$\Omega = -\frac{1}{2} \int d^3x \rho(r) \int d^3x' \frac{\rho(r')}{|\mathbf{x} - \mathbf{x}'|}, \quad (3.78)$$

with $r = |\mathbf{x}|$ and $r' = |\mathbf{x}'|$. We can use a Legendre expansion of the Green's function to evaluate this integral, and to leading order in $\mathcal{C}$, we find a result that is identical to that

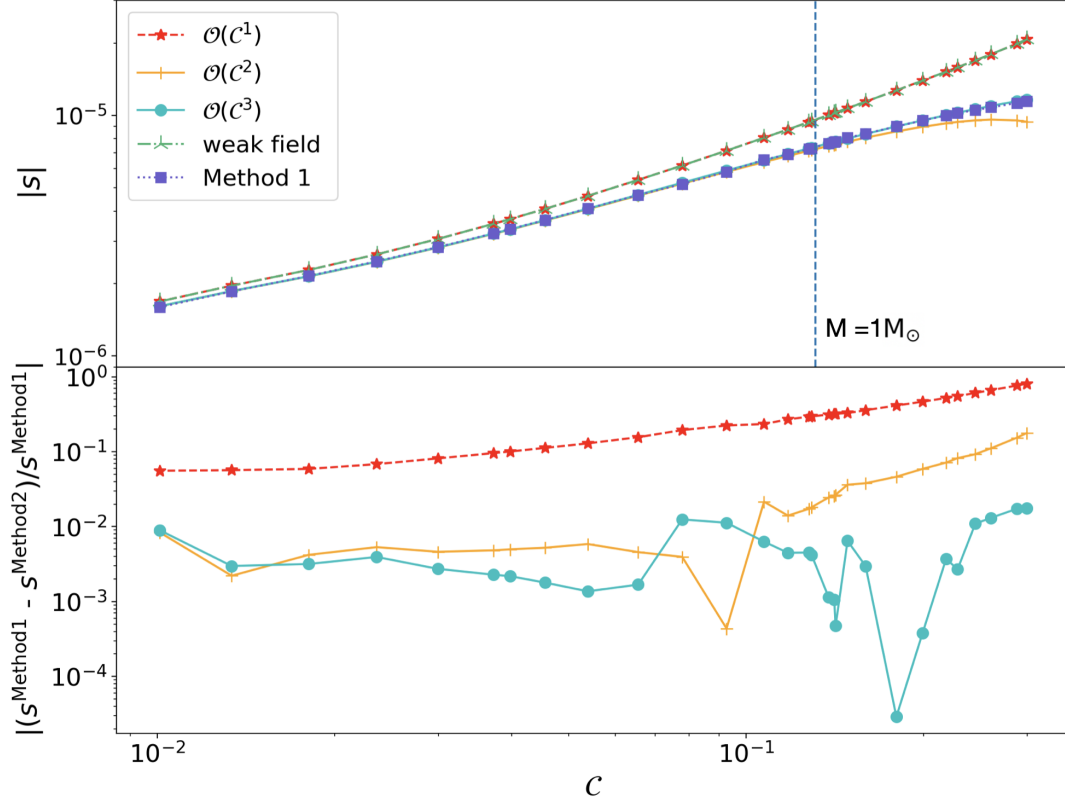


Figure 3.2: (Top) Sensitivity as a function of compactness using the APR4 EoS, for various post-Minkowskian truncation orders at $(\alpha_1, \alpha_2, c_\omega, c_\sigma) = (10^{-4}, 4 \times 10^{-7}, 10^{-4}, 0)$. At leading order in C , the sensitivity curve overlaps with that computed analytically in the weak field limit in [127] (Eq. (3.77)). As the compactness order is increased, the sensitivity curve starts to converge toward the solution found with method 1. (Bottom) Fractional difference between the sensitivities at different order of compactness and those found from method 1. Observe that when $n = 3$, the truncated post-Minkowskian series is already an excellent approximation. The vertical dashed line corresponds to the compactness of a $1M_\odot$ NS.

computed in the weak field limit by [127]. This can also be seen numerically in Fig. 3.2, where the weak field curve coincides with $\mathcal{O}(\mathcal{C}^1)$ post-Minkowskian approximation.

Tolman VII EoS We now focus on the sensitivity of a NS using the Tolman VII EoS. The latter is an analytic model that accurately describes non-rotating NSs [259] by the energy density profile

$$\rho(r) = \rho_c \left(1 - \frac{r^2}{R_\star^2} \right). \quad (3.79)$$

The advantage of using the Tolman VII EoS is that the background solution is known analytically in GR [162, 259]. We expand analytically both the $\mathcal{O}(v^0)$ and $\mathcal{O}(v)$ equations order by order in compactness. The sensitivity obtained is then

$$\begin{aligned} s = & \frac{5}{21} \mathcal{C} (-3\alpha_1 + 2\alpha_2) \\ & + 5 \left(\frac{573\alpha_1^3 + \alpha_1^2(67669 - 764\alpha_2) + 96416\alpha_2^2 + 68\alpha_1\alpha_2(-2632 + 9\alpha_2)}{252252\alpha_1} \right) \mathcal{C}^2 \\ & + \frac{1}{1801079280c_\omega\alpha_1^2} \left\{ (4\alpha_1)^2(8 + \alpha_1)(36773030\alpha_1^2 - 39543679\alpha_1\alpha_2 \right. \\ & + 11403314\alpha_2^2) + c_\omega [-1970100\alpha_1^5 + 13995878400\alpha_2^3 \\ & + 640\alpha_1\alpha_2^2(-49528371 + 345040\alpha_2) + 5\alpha_1^4(-19596941 + 788040\alpha_2) \\ & + \alpha_1^3(-2699192440 + 440184934\alpha_2 - 5974000\alpha_2^2) \\ & \left. 16\alpha_1^2\alpha_2(1294533212 - 29152855\alpha_2 + 212350\alpha_2^2)] \right\} \mathcal{C}^3 + \mathcal{O}(\mathcal{C}^4). \end{aligned} \quad (3.80)$$

Note that the above expression is not regular in the limit of $\alpha_1 \rightarrow 0$ while keeping α_2 finite or $c_\omega \rightarrow 0$ while keeping α_1 or α_2 finite. This is a known feature of Einstein-æther theory, which recovers GR only when a certain combination of coupling constants is taken to zero at a specific rate.

With this EoS, the compactness can be expressed as a function on $\Omega/M_\star$ as

$$\mathcal{C} = -\frac{7\Omega}{5M_\star} + \frac{35819\alpha_1\Omega^3}{85800M_\star^3} + \mathcal{O}\left(\frac{\Omega^4}{M_\star^4}\right). \quad (3.81)$$

Here $\mathcal{C}$ and $M_\star$ are the observed values with æther corrections included. With this at hand, we can rewrite the sensitivity as a function of Ω to find

$$\begin{aligned} s = & \frac{(3\alpha_1 + 2\alpha_2)}{3} \frac{\Omega}{M_\star} \\ & + \left(\frac{573\alpha_1^3 + \alpha_1^2(67669 - 764\alpha_2) + 96416\alpha_2^2 + 68\alpha_1\alpha_2(9\alpha_2 - 2632)}{25740\alpha_1} \right) \frac{\Omega^2}{M_\star^2} \\ & + \frac{1}{656370000c_\omega\alpha_1^2} \left\{ -4\alpha_1^2(\alpha_1 + 8) [36773030\alpha_1^2 - 39543679\alpha_1\alpha_2 \right. \\ & + 11403314\alpha_2^2] + c_\omega [1970100\alpha_1^5 - 13995878400\alpha_2^3 \\ & - 640\alpha_1\alpha_2^2(-49528371 + 345040\alpha_2) - 5\alpha_1^4(19548109 + 788040\alpha_2) \\ & - 16\alpha_1^2\alpha_2(1294533212 - 29152855\alpha_2 + 212350\alpha_2^2) \\ & \left. + \alpha_1^3(2699192440 - 309701434\alpha_2 + 5974000\alpha_2^2)] \right\} \frac{\Omega^3}{M_\star^3} + \mathcal{O}\left(\frac{\Omega^4}{M_\star^4}\right). \quad (3.82) \end{aligned}$$

Note that this expression matches identically to that of [128] when working to leading order in the binding energy.

One may wonder whether the above analytic expression is capable of approximating the sensitivity when using other EoSs. Figure 3.3 shows the absolute magnitude of the sensitivity as a function of Ω computed analytically with Eq. (3.82), as well as numerically with six other EoSs. Here we have chosen to work in a different region of parameter space, namely $(\alpha_1, \alpha_2, c_\omega) = (-10^{-4}, -4 \times 10^{-7}, 10^{-3})$, where we obtain a stable smoothly varying sensitivity curve. Observe that the sensitivities differ by less than 3%, exhibiting an approximate universality already discovered in [286] as a function of compactness. Given these results, in all future calculations we will use the analytic sensitivities computed with the Tolman VII

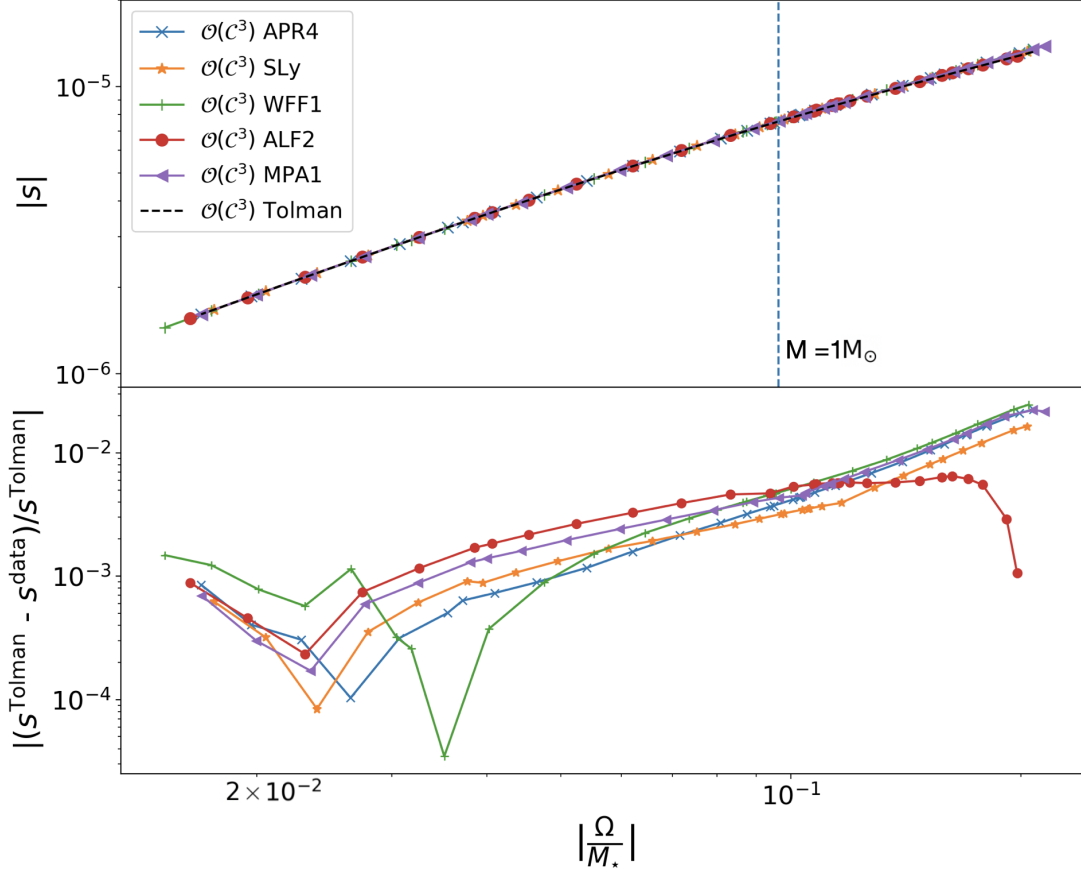


Figure 3.3: Top panel shows the plot of sensitivity as a function of binding energy for different EoS including Tolman VII (Eq. (3.82)) valid to $\mathcal{O}(\mathcal{C}^3)$. The bottom panel shows the relative fractional difference between the EoS from data and the Tolman case, which represents the EoS variation in the relations. Observe that the universality holds to better than 3%.

EoS.

Constraints from Binary Pulsars and Triple Systems

The majority of millisecond pulsars are found in binary and triple systems. The orbital dynamics of these systems modulate the time of arrival of radio waves and allow for precise measurements of the orbital parameters [103, 153, 252, 253]. In this section, we discuss the use of precise orbital parameter data to place constraints on the Lorentz-violating Einstein-æther theory. In GR energy is carried away at quadrupolar order due to propagation of

tensor modes whereas in this theory (and many of other modified theories of gravity), one usually finds radiation from extra scalar and vector modes which are responsible for energy loss at dipole order, i.e., -1PN order (c.f. the term proportional to $\mathcal{E}$ in Eq. (3.30)) as compared to GR. Hence, energy is radiated faster than what is predicted in GR. This results in a decrease in the orbital separation and orbital period (P_b) of the binary. The modified orbital period decay rate ($\dot{P}_b$) relates to the total energy of the binary, i.e., Eq. (3.30) via

$$\frac{\dot{P}_b}{P_b} = -\frac{3}{2} \frac{\dot{E}_b}{E_b}, \quad (3.83)$$

suggesting a strong dependence of $\dot{P}_b$ on the sensitivity of the NS [63]. Since the GR predictions agree with the observed value of $\dot{P}_b$ within observational uncertainty, this allows for stringent constraints to be placed on æther theory.

Observations

Following from Eqs. (3.30) (with LV terms set to zero) and (3.83), one can relate the post-Keplerian parameter $\dot{P}_b$ in GR to the Keplerian parameter P_b via [286]

$$\left(\frac{\dot{P}_b}{P_b}\right)_{\text{GR}} = -\frac{384\pi}{5} 2^{2/3} \left(\frac{\pi(m_1 + m_2)}{P_b}\right)^{5/3} \frac{m_1 m_2}{(m_1 + m_2)^2} \frac{1}{P_b}. \quad (3.84)$$

Here m_1 and m_2 are the masses in the binary system. In principle, if we can measure the masses, orbital periods and the orbital period decay rates with some uncertainty and find that they are consistent with GR predictions, then we can place constraints on Einstein-æther theory. In this section we focus on data from the measurements of Keplerian and post Keplerian parameters of four different pulsar systems PSR J1738+0333, PSR J0348+0432, PSR J1012+5307 and PSR J0737-3039 (Table 3.1) and a stellar triple system [261]. The first three are pulsar-white dwarf binaries in orbits with $\mathcal{O}(10^{-7})$ eccentricity and 8.5-hour period, 0.17 eccentricity and 4.74-hour period and $\mathcal{O}(10^{-7})$ eccentricity and 14.5-hour period

respectively. The fourth is the double pulsar binary system with 0.088 eccentricity and 2.45-hour period. Because of the small eccentricity of these systems, we will ignore it in the following, i.e. we will consider quasi-circular binaries.

Table 3.1: Orbital parameters as measured for the binary systems studied in this paper. Table shows the estimated values of the parameters and the $1\text{-}\sigma$ uncertainty in the last digits in parentheses. Here, $\dot{P}_b^{\text{obs}}$ is the observed value of $\dot{P}_b$.

Pulsar System	$m_1(M_\odot)$	$m_2(M_\odot)$	P_b (days)	$\dot{P}_b^{\text{obs}}$
PSR J1738+0333[131]	$1.46^{+0.06}_{-0.05}$	$0.181^{+0.008}_{-0.007}$	0.3547907398724(13)	$-25.9(3.2) \times 10^{-15}$
PSR J0348+0432 [42]	2.01(4)	0.172(3)	0.102424062722(7)	$-0.273(45) \times 10^{-12}$
PSR J1012+5307 [70] [190]	1.64(0.22)	0.16(0.02)	0.60467272355(3)	$-1.5(1.5) \times 10^{-14}$
PSR J0737-3039 [179]	1.3381(7)	1.2489(7)	0.10225256248(5)	$-1.252(17) \times 10^{-12}$

Parameter Estimation and Bayesian Analysis

Our goal is to constrain the theory parameters using measurements of $\dot{P}_b$. We discuss briefly the Bayesian formalism with Markov-Chain Monte-Carlo (MCMC) exploration used to calculate the posteriors on the model parameters [c.f. Sec. 3] and derive robust constraints.

For the parameter estimation, we need the expression for the orbital period decay, which depends on both the relative velocity of the binary constituents v_{21} and the center-of-mass velocity w of the binary's center of mass with respect to the æther field. A natural choice for the æther field direction is provided by the cosmic microwave background (i.e. the æther is expected to be approximately aligned with the cosmological background time direction). In this case, a typical value for the center-of-mass velocity is $w \sim 10^{-3}$ [128], which for binary pulsar observations is of the same order as v_{21} . If so, the w -dependent corrections in the rate of change of the binding energy [Eq. (3.30)] and orbital period [Eq. (3.85)] enter at the same PN order (0PN) as the quadrupole emission terms of GR, but multiplied by

either $(s_1 - s_2)$ or powers of it. As such, these w -dependent corrections are negligible for both white dwarf-pulsar systems (for which the dominant term is the -1PN dipole emission) and also for the relativistic double pulsar system (for which $s_1 - s_2 \approx 0$ as a result of the similar pulsar masses, which kills both the dipole emission and the w -dependent corrections to quadrupole emission). Therefore, our results are independent of the exact value of w as long as that is of order $w \sim 10^{-3}$ or smaller [286]

From the above assumption and using Eqs. (3.83) and (3.30), the orbital period decay rate in Einstein-æther theory is a function of the individual masses (m_1, m_2) ⁵, pulsar radii $(R_{\star,1}, R_{\star,2})$, orbital period (P_b) and coupling constants $(\alpha_1, \alpha_2, c_\omega)$ as shown below

$$\begin{aligned} \frac{\dot{P}_b}{P_b} = & \frac{1}{5(m_1 + m_2)^4 P_b \sqrt{\frac{\alpha_1}{(\alpha_1 - 8\alpha_2)}} \sqrt{\frac{-c_\omega}{\alpha_1}} \alpha_1^3 c_\omega^2} \left\{ 3 \cdot 2^{8/3} \pi \left(\frac{\pi(m_1 + m_2)}{P_b} \right)^{5/3} \right. \\ & m_1 m_2 \left[-\frac{1}{12} \left(5\alpha_1 c_\omega (m_1 + m_2)^2 (s_1 - s_2)^2 \right. \right. \\ & \left. \left(2^{5/6} \alpha_1^2 (\alpha_1 + 8) \sqrt{\frac{\alpha_1}{(\alpha_1 - 8\alpha_2)}} - 2^{13/3} \sqrt{\frac{-c_\omega}{\alpha_1}} c_\omega (\alpha_1 - 8\alpha_2) \right) \right. \\ & \left. \left(\frac{P_b}{\pi m} \right)^{2/3} \right) + \left((s_1 - 1)(s_2 - 1) \right)^{2/3} \left((\alpha_1 + 8) \alpha_1^3 \right. \\ & \left. \left(-4c_\omega^2 (m_1 + m_2)^2 \sqrt{\frac{-c_\omega}{\alpha_1}} + \sqrt{2} \alpha_1 (m_1 s_2 + m_2 s_1)^2 \right) \sqrt{\frac{\alpha_1}{(\alpha_1 - 8\alpha_2)}} + \right. \\ & \left. \left. \left. \frac{2\sqrt{\frac{-c_\omega}{\alpha_1}} (\alpha_1 - 8\alpha_2)^2 c_\omega^2 ((m_1 + m_2)\alpha_1 + 8m_1 s_2 + 8m_2 s_1)^2}{3} \right) \right] \right\}, \end{aligned} \quad (3.85)$$

where s_1 and s_2 are functions of $\mathcal{C}$ and coupling constants. One may worry that the terms inside the square roots in the above expression may be negative, leading to a complex orbital

⁵These are the active masses, whose fractional difference from the “real” masses $(\tilde{m}_1, \tilde{m}_2)$ is of the order of the sensitivities and thus negligible. In the following we will therefore typically identify (m_1, m_2) and $(\tilde{m}_1, \tilde{m}_2)$. Note that this could however introduce correlations not captured by our sufficient statistics approach, but as we show, even large correlations would have little impact on the results.

decay rate, but this is not the case because when $\alpha_1 > 0$, then $c_\omega \leq -\alpha_1/2$, while when $\alpha_1 < 0$, then $c_\omega \geq -\alpha_1/2$. As noted in Fig. 3.3, sensitivities are independent of the EoS. Here we choose to work with the Tolman VII EoS since it gives stable analytic solutions for the sensitivities.

There are some phenomenological constraints on the æther coupling constants as discussed in Sec. 3, i.e., $|\alpha_1| \lesssim 10^{-4}$, $|\alpha_2| \lesssim 10^{-7}$ (Solar system constraints), $\alpha_1 < 0$, $\alpha_1 < 8\alpha_2 < 0$ and $c_\omega > -\alpha_1/2$ (positive energy, absence of vacuum Cherenkov radiation and gradient instabilities) and $c_\sigma \lesssim 10^{-15}$ (GW constraint).⁶ Using these pre-existing constraints and by determining if the estimated value of $\dot{P}_b$ lies within the range $\dot{P}_b^{\text{obs}} \pm \delta\dot{P}_b^{\text{obs}}$ (Table 3.1), we determine the consistency of points in the parameter space with observations.

One important point is that we are not using the pulsar timing data directly [39], but instead we are using existing constraints on $\dot{P}_b$, m_1 and m_2 derived from the primary pulsar timing data as a *sufficient statistic*. Unfortunately, the published results only quote values for the individual parameters and their uncertainties, so we do not have access to the joint posterior distributions. For simplicity we assume that the parameter correlations are negligible. To check the impact of this assumption, we compared results with zero correlations with a case with 90% correlation between the parameters, and found that it only changed the results by a maximum of 17%.

Bayesian Analysis We are interested in constructing a posterior distribution on a set of model parameters $\vec{\lambda} = (m_1, m_2, P_b, R_{\star,1}, R_{\star,2}, \alpha_1, \alpha_2, c_\omega)$ and using an MCMC algorithm to explore the parameter space. According to Bayes' theorem, the probability density for

⁶Note that we cannot use existing bounds on $\hat{\alpha}_1$ [236, 268] and $\hat{\alpha}_2$ [236, 237, 268] as priors. This is because those quantities depend on the derivatives of the sensitivities (c.f. A), which are currently unknown.

parameters $\vec{\lambda}$ given data D and hypothesis H (the theory) is

$$P(\vec{\lambda}|\text{D}, \text{H}) = \frac{P(\text{D}|\vec{\lambda}, \text{H})P(\vec{\lambda}|\text{H})}{P(\text{D}|\text{H})}, \quad (3.86)$$

where $P(\vec{\lambda}|\text{H})$ is called the prior which represents the state of knowledge about the parameters before we analyze the data. $P(\text{D}|\vec{\lambda}, \text{H})$ is called the likelihood which describes the probability of measuring data D given the model H and a set of parameters $\vec{\lambda}$. $P(\text{D}|\text{H})$ is called the model evidence which represents the overall normalization factor. In practice it is better to work with log probability densities to better cover the dynamic range of the densities.

We assumed uniform priors on α_2 and c_ω such that $-4 \times 10^{-7} \leq \alpha_2 \leq 4 \times 10^{-7}$ and $-10^5 \leq c_\omega \leq 10^5$ and a Gaussian prior for α_1 , m_1 , m_2 , $\dot{P}_b$ with mean and standard deviation given by the existing bounds listed in Table 3.1. We use Gaussian priors on $R_{\star,1}$ and $R_{\star,2}$ with mean and standard deviation given $12.4 \pm 1.1\text{km}$ based on LIGO and NICER measurements [93]. While these bounds are derived assuming GR, the corrections due to LV effects are sub-dominant compared to those impacting $\dot{P}_b$ (c.f. e.g. footnote $\mathcal{P}$). Using lunar laser ranging experiments, the bounds on α_1 were obtained to be $\alpha_1 = (-0.7 \pm 0.9) \times 10^{-4}$ [208] (c.f. also Sec. 3).

The log likelihood function is

$$\ln(P(\text{D}|\vec{\lambda}, \text{H})) \propto -\frac{1}{2} \frac{\left(\left(\dot{P}_b/P_b \right)^{\text{obs}} - \left(\dot{P}_b/P_b \right)^{\text{th}} \right)^2}{\sigma_{(\dot{P}_b/P_b)}^2}, \quad (3.87)$$

where $(\dot{P}_b/P_b)^{\text{th}}$ is the theoretically predicted value of $(\dot{P}_b/P_b)$ from the model. With the likelihood and the priors in place, we can find the posterior using a MCMC algorithm.

We start the MCMC simulation near the mean values for the model parameters, calculate the posterior and iterate through these steps. Model parameters are allowed to

explore the entire range of parameter space and that gives the joint posterior distribution on all parameters $\vec{\lambda}$. For the proposal distribution we use the prior distribution for a certain set of parameters, and a relative jump from the current position for the remaining. Proposed jumps are accepted or rejected based on the Metropolis-Hastings acceptance probability

$$H = \min \left(\frac{P(\vec{\lambda}_{\text{new}}|H)P(D|\vec{\lambda}_{\text{new}}, H)Q(\vec{\lambda}_{\text{old}}|\vec{\lambda}_{\text{new}})}{P(\vec{\lambda}_{\text{old}}|H)P(D|\vec{\lambda}_{\text{old}}, H)Q(\vec{\lambda}_{\text{new}}|\vec{\lambda}_{\text{old}})}, 1 \right). \quad (3.88)$$

A random number $u \sim U[0, 1]$ is drawn, and if $H > u$ the proposed jump is accepted, otherwise it is rejected. This process is repeated multiple times to ensure convergence.

We begin by considering a single observation from the pulsar-white dwarf system PSR J1738+0333. Since the sensitivity of a white dwarf (WD) is negligible compared to the NS we can set $s_2 = s_{\text{WD}} = 0$ (thus $R_{\star,2}$ is excluded from $\vec{\lambda}$) but for a double pulsar binary we should have $s_2 \neq 0$. Figure 3.4 shows the prior and posterior distribution on the model parameters. We are recovering our priors on the masses and radii, given that these are well constrained as can be noted from Table 3.1. The distribution on α_1 and α_2 is such that $\alpha_1 < 0$ and $\alpha_2 < 0$ from existing constraints. This pulsar system further constrains the value of parameter α_1 by approximately a factor of 2 while the coupling constants α_2 and c_ω remain unconstrained. Notice that the posteriors on α_2 and c_ω are flat and very similar to the priors. Therefore, one cannot model them as Gaussian and construct confidence region, as no information is gained for the values of these parameters.

We then consider constraints on the coupling parameters by stacking all four different binary systems from Table 3.1 and computing the joint constraints (Fig. 3.5). These joint constraints also restrict the region of α_1 by a factor of 2 better than the existing constraints.

In the coming years we expect to have more observations, as the sensitivities of radio telescopes will improve as a result of larger collecting areas (e.g. the Square Kilometre Array

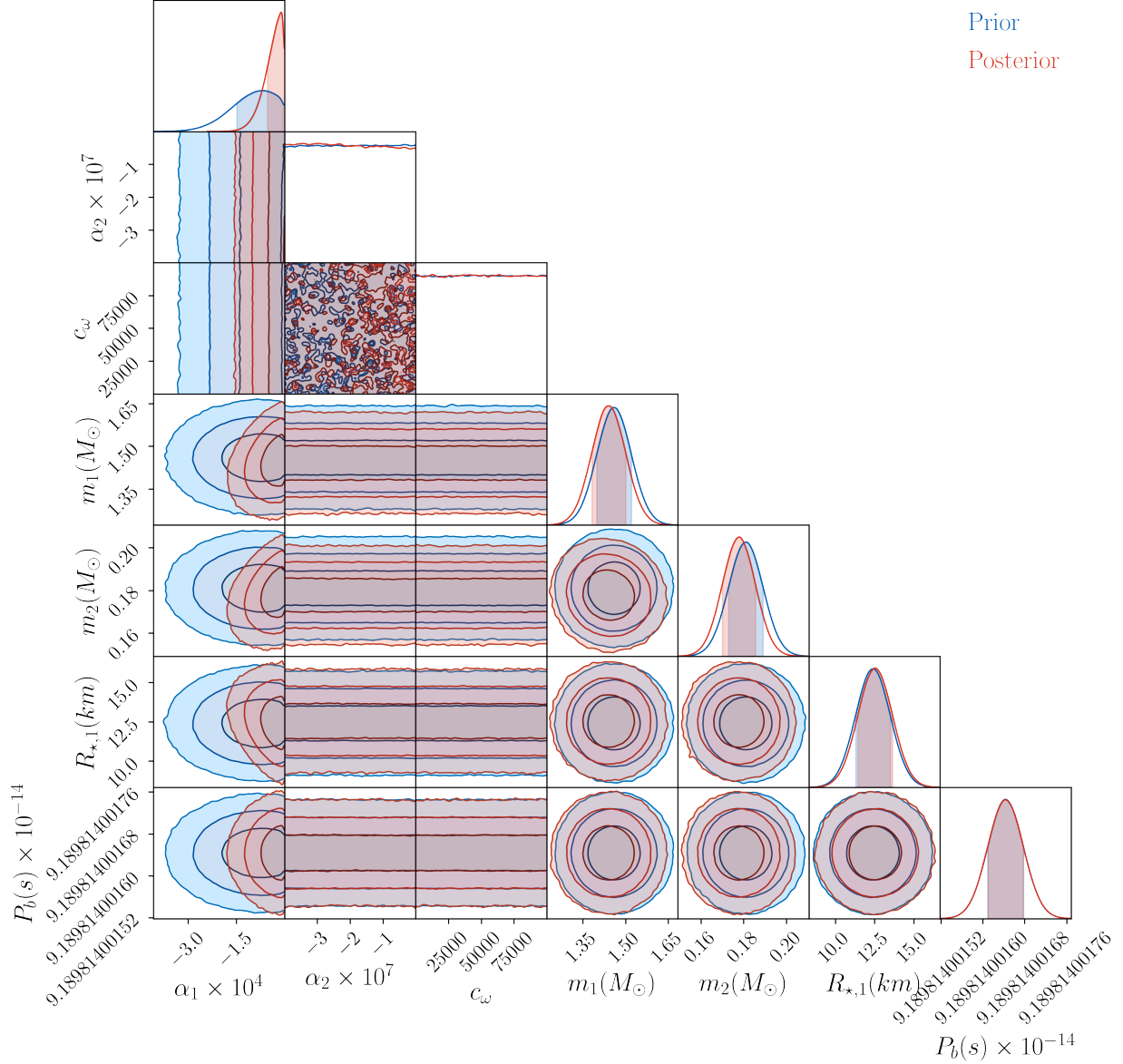


Figure 3.4: Prior and posterior distribution on the model parameters $\vec{\lambda}$ from $\dot{P}_b$ constraints for PSR J1738+0333. The pre-existing constraints from solar system, Big-Bang nucleosynthesis and stability requirements are applied to uniform priors shown in blue where, for α_1 negative it results in a negative α_2 and positive c_ω . The posterior distributions depend on the $\dot{P}_b$ constraints for PSR J1738+0333. The three shades of contours in the prior and posterior distribution in the off-diagonal cross-correlation panels represent 1- σ , 2- σ and 3- σ uncertainty on model parameters starting from the center (we only show 1- σ shaded regions for the one-dimensional marginal distributions). There is a small dip at very small magnitudes of α_1 , as that is the region where the pre-existing constraints come into play, while for larger magnitudes the constraints on α_1 are automatically satisfied. Observe that the value of α_1 is further constrained by a factor of 2 compared to existing solar system constraints (prior).

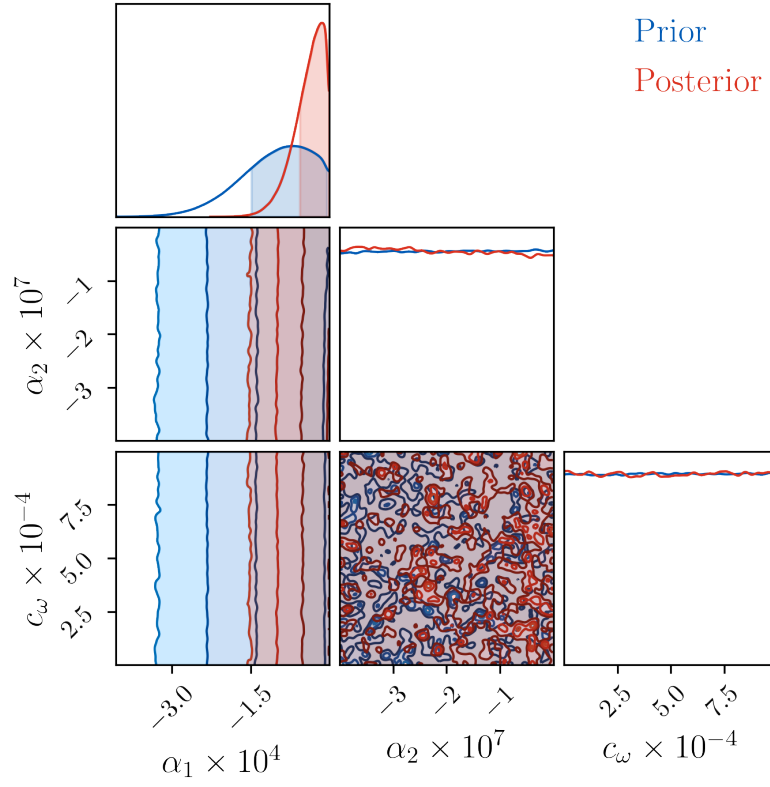


Figure 3.5: Joint prior and posterior distribution on α_1 , α_2 and c_ω from $\dot{P}_b$ constraints for all four pulsars listed in (Table 3.1) The constraint on α_1 is improved by a factor of 2.

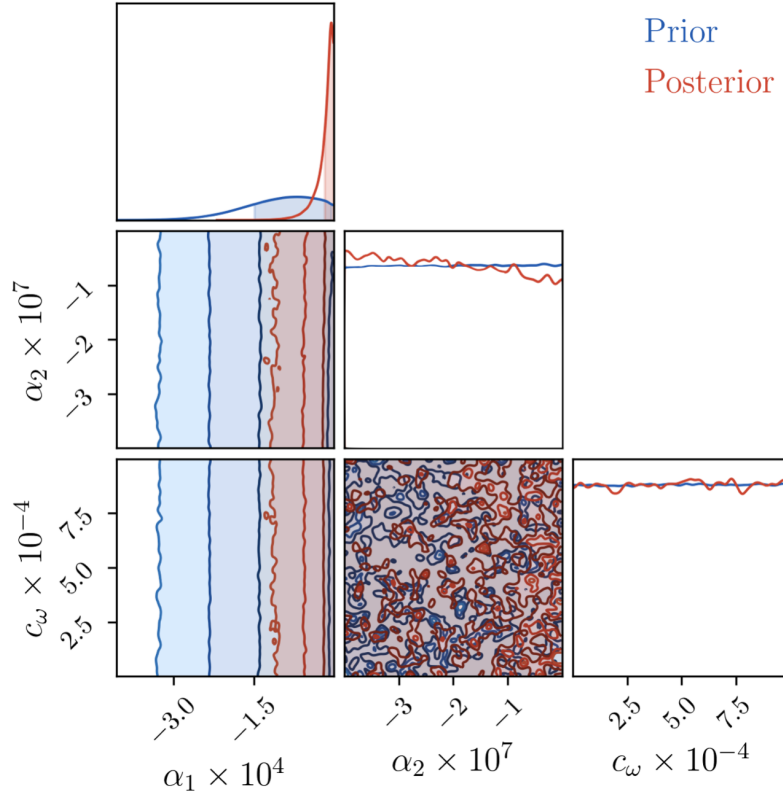


Figure 3.6: Prior and posterior distribution on the model parameters α_1 , α_2 and c_ω from $\dot{P}_b$ constraints for PSR J0348+0432, in a scenario where the observational uncertainties tighten by a factor of 10 and the value of $\dot{P}_b$ matches the GR prediction. It shows that the GR values are favoured and the value of α_1 is very closely centered around zero.

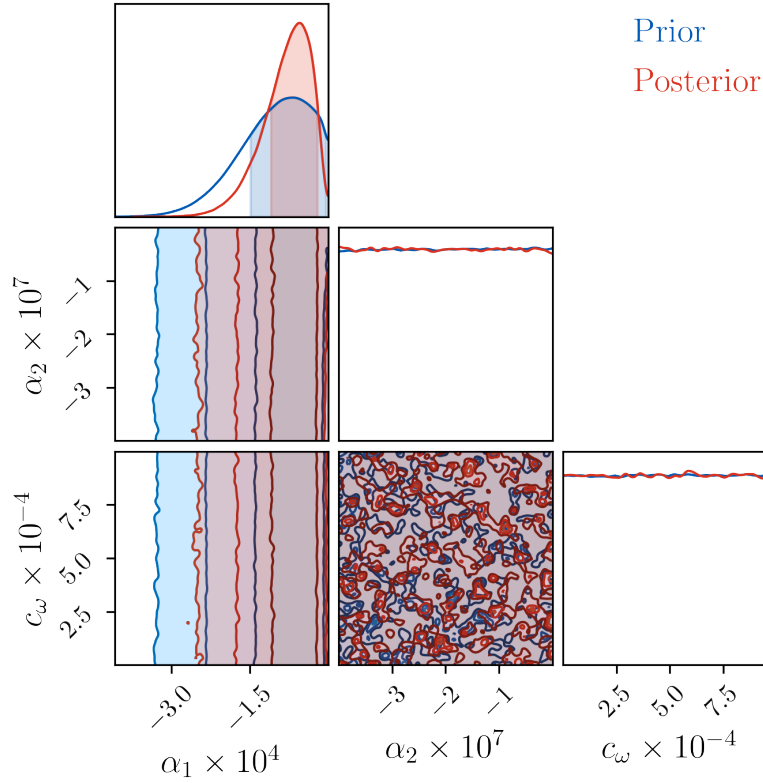


Figure 3.7: Prior and posterior distribution on the model parameters α_1 , α_2 and c_ω from $\dot{P}_b$ constraints for PSR J0348+0432, in a scenario where the uncertainties in measurements are reduced by a factor of 10, and the value of $\dot{P}_b$ stays at the currently observed value. The $1\text{-}\sigma$ uncertainty shows that $\alpha_1 = 0$, i.e., the GR value is disfavoured. It can also be noted from Table 3.2 that posterior does not include $\alpha_1 = 0$.

(SKA) project [66, 264]), which will allow for discovering more pulsars. Moreover the longer observation time (T) will reduce the error in measurements of $\dot{P}_b$ by $T^{-5/2}$ [60], allowing for more precise measurements of the orbital parameters. One may wonder, whether we can get tighter constraints from finding N similar systems or a single system measured with higher SNR (signal-to-noise ratio). The SNR^2 grows linearly with number of sources N and the observing time T , and quadratically with the effective collecting area of the radio telescope. Significant improvements in the measurement sensitivity are more likely to come from some combination of larger telescopes and additional observing time than from discovering large numbers of systems similar to those known. Figure 3.6 illustrates the kind of bounds we will get for a PSR J0348+0432 system if $\dot{P}_b^{\text{obs}}$ matches the GR prediction $\dot{P}_b^{\text{GR}}$ and the uncertainties are tightened by a factor of 10. The improved constraints on $\dot{P}_b$ translate directly into similarly improved bounds on α_1 . We also considered an alternative scenario, in which the uncertainties in $\dot{P}_b$ improved by a factor of 10, but stayed centered on the current observed value. As shown in Figure 3.7, this leads to a value for α_1 bounded away from zero. In other words, in a scenario where the observed period derivative stays at the current value while the uncertainty drops by a factor of ten, we would find that Einstein-æther theory would be favored over GR!

Constraints from the Triple System

Next we have constraints coming from a pulsar in a stellar triple system PSR J0337+1715 consisting of an inner millisecond pulsar-white dwarf binary and a second white dwarf (WD) in an outer orbit [43]. Due to the gravitational pull of the outer WD, the pulsar and the inner WD experience accelerations that differ fractionally. If the strong equivalence principle is violated (as a result of the sensitivities), the triple system constrains the fractional acceleration difference parameter δ_a to $(+0.5 \pm 1.8) \times 10^{-6}$ [261]. The relation between δ_a

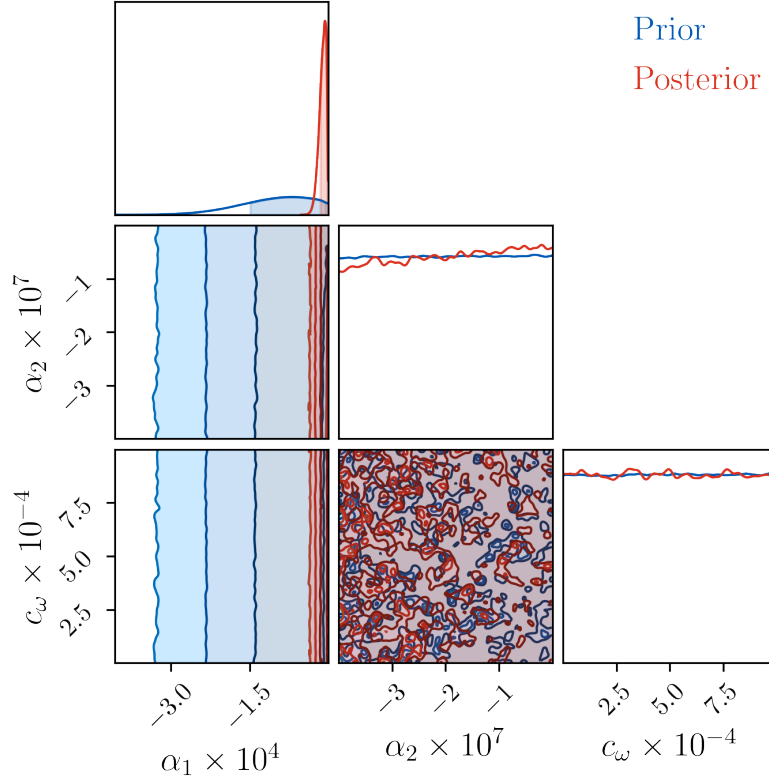


Figure 3.8: Joint prior and posterior distribution on the model parameters $\vec{\lambda}$ from $\dot{P}_b$ constraints for all four pulsars listed in (Table 3.1) and stellar triple system. Observe that inclusion of stellar triple system improves the constraints and parameter α_1 is now constrained by a factor of 10 better than lunar laser ranging experiments.

and the sensitivity parameter σ_{pulsar} (before rescaling) in Einstein-æther theory is [48, 270]

$$|\delta_a| = \left| \frac{\sigma_{\text{pulsar}}}{1 + \sigma_{\text{pulsar}}/2} \right| \approx |\sigma_{\text{pulsar}}|, \quad (3.89)$$

as can be obtained directly from Eq. (3.29) (in the Newtonian limit).

We use MCMC simulations in Bayesian analysis similar to that for the $\dot{P}_b$ constraint and with the likelihood

$$P(D|\vec{\lambda}, H) \propto \exp \left(-\frac{1}{2} \frac{(\sigma_{\text{pulsar}}^{\text{obs}} - \sigma_{\text{pulsar}}^{\text{th}})^2}{\sigma_{(\sigma_{\text{pulsar}})}^2} \right), \quad (3.90)$$

Table 3.2: Our bounds on α_1 from different pulsar systems shown in Figs. 3.4–3.8 with $1\text{-}\sigma$ uncertainty. The first half shows the bounds from existing measurements, while the second half shows projected future bounds assuming that the measurement error on $\dot{P}_b^{\text{obs}}$ reduces by a factor of 10 with the central value of $\dot{P}_b^{\text{obs}}$ at the GR predicted value (-27.3×10^{-14}) and at the current measured value (-25.3×10^{-14}).

Pulsar System	α_1
PSR J1738+0333	$(-3.975 \pm 2.968) \times 10^{-5}$
Joint binary system	$(-4.073 \pm 2.936) \times 10^{-5}$
Joint binary + triple system	$(-1.111 \pm 0.674) \times 10^{-5}$
PSR J0348+0432 ($\dot{P}_b^{\text{obs}} = -27.3 \times 10^{-14}$)	$(-8.119 \pm 4.622) \times 10^{-5}$
PSR J0348+0432 ($\dot{P}_b^{\text{obs}} = -25.3 \times 10^{-14}$)	$(-1.729 \pm 1.805) \times 10^{-5}$

where $\sigma_{\text{pulsar}}^{\text{obs}} = (+0.5 \pm 1.8) \times 10^{-6}$ from Eq. (3.89) and $\sigma_{\text{pulsar}}^{\text{th}}$ is given by Eq. (3.58), to constrain the model parameters. Figure 3.8 shows the joint pulsar and triple system constraints on the model parameters $\vec{\lambda}$ assuming uniform distribution in α_2 and c_ω . The 95% upper limit on α_1 , which was -2.4×10^{-4} (from the prior constraints) has now shifted to $\alpha_1 = -2.4 \times 10^{-5}$. It shows that the preferred frame parameter α_1 is constrained by a factor of 10 better than the lunar laser ranging experiments.

Table 3.2 shows bounds on α_1 from binary and triple systems mentioned in this paper. The data from joint binary + triple system allows us to put a stringent constraint on α_1 , which is an order of magnitude stronger than the bounds from lunar laser ranging experiments [208, 271].

Conclusions

We have investigated Einstein-æther theory in the context of binary pulsars and NSs. We have recalculated the sensitivities in the regime of coupling parameter space that still survives after the recent measurement of the speed of GWs. This required the development of a new post-Minkowskian approach that allows for stable numerical evaluation of the sensitivities, in addition to the derivation a closed form analytic solution for the Tolman VII EoS. We used these results to place a constraint on certain coupling constants of Einstein-æther theory using Bayesian analysis of binary pulsar observations, including recent observations on the triple system. We find that these data allows for constraints on a certain combination of the coupling constants, α_1 , of $\mathcal{O}(10^{-5})$, improving current Solar System constraints by one order of magnitude.

The work carried out here opens the door to several avenues for future research. One such avenue is to use gravitational wave data directly to place constraints on Einstein-æther theory, now that the sensitivities have been analytically calculated. This can be done today to leading post-Newtonian order in the inspiral, and it remains to be seen whether it is enough to lead to interesting constraints. To include the very late inspiral and merger phase, numerical simulations of coalescing NSs would have to be carried out in Einstein-æther theory. However, since the parameter space of the theory is already quite well constrained, it is not clear whether stronger bounds can be achieved with gravitational wave data.

Another avenue for future research concerns computing sensitivities for black holes. This has been done in khronometric theory [228] but not yet in Einstein-æther theory. Once the black hole sensitivities are in hand, and assuming they do not vanish, one could use the existing GW data for binary black hole mergers to constrain Einstein-æther theory, including the dipole radiation effect in the gravitational waveform.

One more avenue for future work would be along the lines of improving the analysis in

this paper by directly analyzing binary pulsar data and carrying out a parameter estimation and model selection study with a GR and a non-GR timing model. For this, it would be ideal to compute the derivative of the NS sensitivities that enter in the conservative post-Keplerian parameters, such as the periastron precession and Shapiro time delay.

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BAYESIAN POWER SPECTRAL ESTIMATION OF GRAVITATIONAL WAVE
DETECTOR NOISE REVISITED

Contribution of Authors and Co-Authors

Manuscript in Chapter 4

Author: Toral Gupta

Contributions: Developed and implemented various improvements to the original `BayesWave` algorithm and contributed to the essential material for the manuscript.

Author: Neil J. Cornish

Contributions: Led the development of the low latency algorithm for the analysis and wrote the first draft of the manuscript.

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Toral Gupta, Neil J. Cornish

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Introduction

As the sensitivity of ground based gravitational wave detectors improves, the number of events increases, as does the time span over which the signals are detectable. To be able to handle the ever increasing event rate and signal duration, fast analysis methods are needed. A first step in any analysis is an estimate of the noise properties, which for Gaussian noise is given by the noise covariance matrix. For stationary, Gaussian noise the noise properties are summarized by the power spectral density (PSD). In this work, we describe a new, fast Bayesian spectral estimation algorithm - **FastSpec** that can be used for low latency analyses, and we also introduce numerous improvements and speed-ups to the higher fidelity, but slower, **BayesWave** trans-dimensional spectral estimation algorithm.

In reality, the data from the LIGO [14, 67] and Virgo [29, 30] detectors are neither perfectly Gaussian or stationary [24], and more complex models are needed to handle noise transients (glitches) [79, 89, 91, 105, 150] and variations in the noise level with time [85, 108]. However, spectral estimates still play an important role in these analyses. For example, in applications that use a wavelet domain analysis to handle non-stationary noise [85], the usual spectral estimates describe the average noise levels as a function of frequency, and provide a good starting for estimating the dynamic power spectrum, which varies with both frequency and time.

Currently, most low latency analyses of LIGO/Virgo/Kagra (LVK) data, including online searches for signals from binary merger events [45, 81, 96, 124, 210] and sky localization for electromagnetic follow-up [242], use classical spectral estimation techniques such as Welch averaging to estimate the noise power spectrum. For example, the PyCBC Live algorithm [96, 210] uses Welch’s method [263] and median averaging to estimate the power spectrum. Typical settings for the Welch spectral estimates use 32 segments of “off-source” data, with same duration as the on-source window, from times immediately preceding the analysis

segment. Drifts in the noise levels and impulsive noise transients can bias these off-source estimates [78]. Moreover, even for perfectly stationary Gaussian noise, it is known that data whitened using Welch’s method does not follow a standard zero mean, unit variance $\mathcal{N}(0, 1)$ normal distribution [231, 250]. Welch’s method with mean averaging results in a Student’s- t distribution, while median averaging results in another more complicated distribution with even heavier tails [250]. Both of these distributions approach a $\mathcal{N}(0, 1)$ normal distribution in the limit that the number of data segments being averaged over is large, but using more segments makes the method more susceptible to non-stationarity. Using Welch’s method to estimate the power spectral has been shown to lead to biased parameter estimation for compact binary mergers [78, 250].

For these reasons, “on-source” Bayesian spectral estimates, such as those provided by the **BayesWave** algorithm [89, 91, 193], have been adopted as the preferred method for the more in-depth, longer latency parameter estimation studies that are performed on candidate signals from the search pipelines (see e.g. [26, 27]). The Bayesian spectral estimates have been shown to yield unbiased and statistically consistent results, with whitened residuals that follow a $\mathcal{N}(0, 1)$ normal distribution [78].

Ideally, the spectral estimation should be performed concurrently with the source inference. This procedure was introduced in the original **BayesWave** analysis of minimally modelled sources using a wavelet based signal [89]. The **BayesWave** analysis has now been extended to include template based models for compact binary mergers, with full marginalization over the spectral model [150]. The joint modeling of the noise spectrum and the signal was found to have a relatively small impact on parameter estimation for compact binary mergers [220], which supported the findings of an earlier study that took multiple fair draws from the **BayesWave** spectral model, and marginalized over these by alternating which one was used when computing the likelihood [58] (using off-source Welch spectral estimates rather than the on-source **BayesWave** model was shown to cause a much larger

bias in the parameter estimation [78]). While sequential and joint spectral modeling was found to produce similar parameter recovery at current sensitivity levels, the differences will become more significant as the detector sensitivities improve. It is also worth noting that joint signal and noise modeling will be essential for next generation detectors on the ground and in space, as there will no signal-free off-source data to work with, and the signals will be a significant contribution to the total power. Joint signal and noise modeling will be a key element of the *Global Fit* analysis [87, 88, 192, 195] for the future Laser Interferometer Space Antenna (LISA).

While Bayesian spectral estimates are more reliable, they come with a higher computational cost. For example, producing a spectral estimate for 128 seconds of LIGO data covering frequencies up to 4096 Hz, which are standard choices for binary Neutron star merger events, would take a day or so with the version of **BayesWave** used during the LVK third observing run (O3). In preparation for the fourth observing run (O4), a new low-latency spectral estimation algorithm, **FastSpec**, has been developed which can be used to provide standalone low latency Bayesian spectral estimates, and also provides a starting point for the full **BayesWave** analysis. In addition, numerous changes were made to the **BayesWave** algorithm that improved the quality of the spectral estimates and reduced the computational cost. The combination of the better starting solution from **FastSpec**, faster likelihood evaluations and improved sampling, result in spectral estimates that take one to two orders of magnitude less time using the O4 version of **BayesWave** than with the O3 version. The **FastSpec** analysis is faster still, with run times that are roughly equal to the duration of the data segments being analyzed.

The paper is organized as follows: Section 4 reviews the classical and Bayesian spectral estimation methods used in LVK data analysis. Section 4 describes the **FastSpec** low-latency spectral estimation algorithm. Section 4 describes the improvements made to the **BayesWave** trans-dimensional Bayesian spectral estimation algorithm. Section 4 compares

the performance of Welch’s method to the **FastSpec** and **BayesWave** algorithms using data from the LVK third observing run [28]. Section 4 provides a summary and a discussion of possible future improvements.

Spectral Estimation

A variety of classical and Bayesian techniques have been used to produce power spectral density estimates of the instrument noise for use in gravitational wave data analysis. Classical approaches include the aforementioned Welch’s method [263], and the fitting of Auto Regressive Moving Average models $\text{ARMA}(p, q)$ with p autoregressive terms and q moving-average terms [94]. A variety of Bayesian spectral estimation methods have also been tried, including both non-parametric [118, 168, 200] and parametric models [78, 193]. A more extensive review of the various approaches can be found in Ref. [80]. The focus here will be on the two main methods that are currently used in LVK data analysis, Welch’s method and Bayesian parametric modeling using **BayesWave**.

Welch’s method uses multiple, often overlapping, time segments. The data in each segment are windowed then Fourier transformed to produce a collection of periodograms. By taking either the mean or median of the periodograms at each frequency yields an estimate for the power spectrum. Taking the median is more robust against outliers, such as noise transients that might impact some of the data segments. Welch’s method makes no assumptions about the functional shape of the spectrum, but it does rely on the data being stationary. If the data are perfectly stationary, the Welch estimate improves with the number of segments used, and the distribution of the whitened residuals approaches at $\mathcal{N}(0, 1)$ distribution [231, 250]. In practice, the LVK data are non-stationary over timescales of minutes or less, so for reasons of both computational efficiency and to avoid biases from non-stationarity, a relatively small number of segments are used. The Welch estimates are referred to as “off-source” estimates since the data segments are usually taken from times

immediately before the analysis segment.

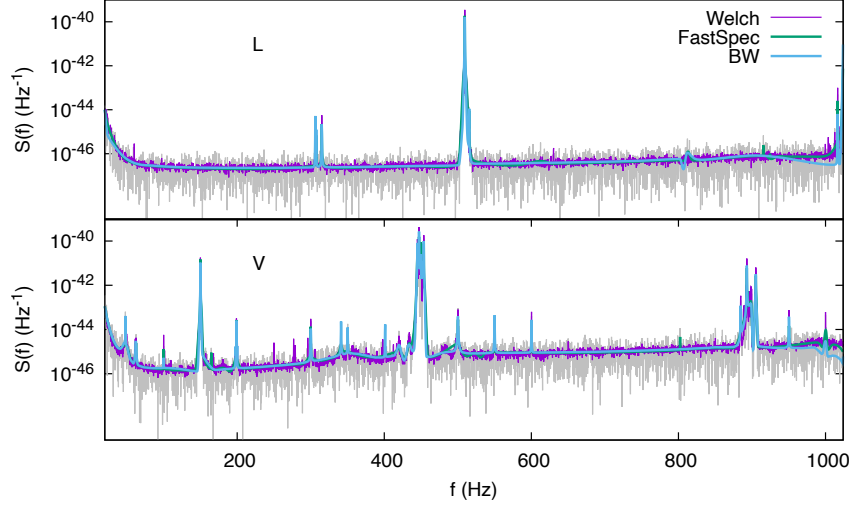


Figure 4.1: Comparison of PSD estimates for 4 seconds of O3 LIGO Livingston and Virgo data ending at UTC 0:0:0, January 1, 2020 (GPS time 1261872018). The **FastSpec** and **BayesWave** estimates are almost identical, and are in good agreement with the Welch estimate. Periodograms of the data used in the **FastSpec** analysis are shown in grey. Note that the apparent discrepancy between the **BayesWave** and **FastSpec** analyses above 950 Hz are due to differences in the bandpass filters used in each analysis.

The **BayesWave** algorithm models the power spectral density function $S(f)$ using a combination of splines for the broadband features, and Lorentzians for the spectral lines. Crucially, the number of spline knots and the number of Lorentzian line features are dynamically determined using a trans-dimensional Reversible Jump Markov Chain Monte Carlo (RJMCMC) routine. The analysis uses the Whittle likelihood, the log of which can be written as

$$\log p(\mathbf{n}|S(f)) = - \sum_j \left[\frac{|\tilde{n}(f_j)|^2}{TS(f_j)} + \log(\pi TS(f_j)) \right] \quad (4.1)$$

where T is the observation time and $f_j = j/T$ are the discrete frequency bins. A detailed description of the **BayesWave** spectral estimation method and the improvements that have been made are provided in Section 4. Another key feature of the **BayesWave** algorithm is that, concurrent with the spectral estimation, any non-Gaussian excess caused by either

noise transients or loud signals are subtracted from the data. The non-Gaussian features are modeled as a sum of Morlet-Gabor wavelets, where again, the number of wavelets is dynamically determined by a trans-dimensional RJMCMC sampler [89, 91]. The **FastSpec** algorithm is designed to mimic the main elements of the **BayesWave** algorithm while significantly reducing the computational cost by replacing fully trans-dimensional Bayesian inference with a mixture of deterministic data processing steps followed by a fixed dimension Bayesian refinement. A detailed description of the **FastSpec** algorithm can be found in Section 4.

Figure 4.1 compares PSD estimates from the Welch method, **FastSpec** and **BayesWave** applied to 4 seconds of O3 LIGO Livingston and Virgo data ending at UTC 0:0:0, January 1, 2020 (GPS time 1261872018). This data, and all the other data used in our analyses, was sourced from the LVK public data release [28], which uses the channels `H/L1:DCS-CALIB-STRAIN-CLEAN-SUB60HZ-C01` and `V1:Hrec_hoft_16384Hz`.

All three methods produced similar estimates for this example. The Welch estimate is not very smooth since just 32 4-second segments of data from times immediately preceding the on-source segment were used in the averaging. Increasing the number of segments used in the averaging yields smoother estimates, but this also increases the computational cost and the impact of any non-stationarity in the data.

Fast Bayesian Spectral Estimation

The **FastSpec** algorithm shares many of the attributes of the **BayesWave** algorithm, but with a much reduced computational cost. Both perform “on-source” PSD estimation with subtraction of non-Gaussian features (either gravitational wave signals or instrumental noise transients). The main differences are that **FastSpec** uses wavelet de-noising [260] to remove transients during an initial cleaning phase, while **BayesWave** jointly infers the structure of the transients and the PSD model. Additionally, **FastSpec** uses a fixed dimension MCMC

sampler to refine the spline and line model, while **BayesWave** uses a trans-dimensional RJMCMC sampler.

Initial Spectral Estimation and Wavelet De-noising

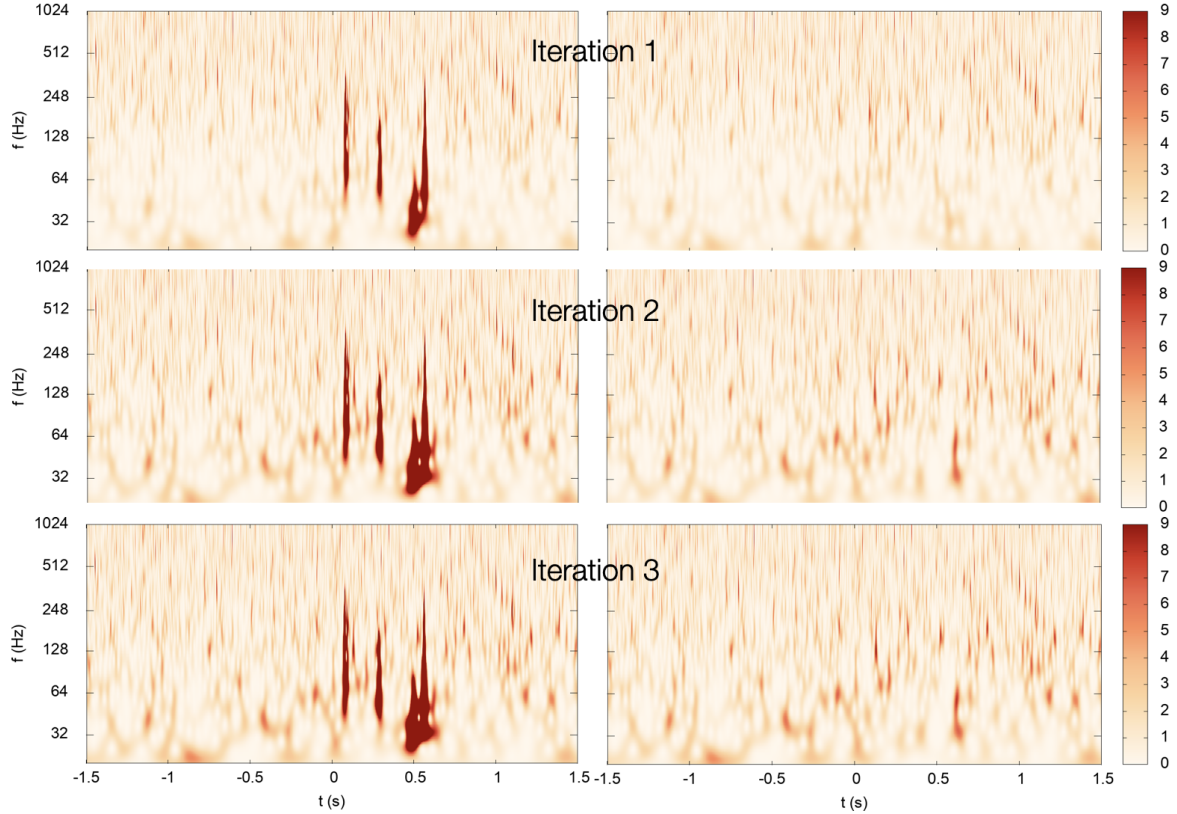


Figure 4.2: PSD estimation and Glitch reconstruction for the LIGO Hanford detector centered on GPS time 1263011016. The spectrograms on the left show successive stages in the initial data whitening, while the panels on the right show the corresponding de-noised data that is used in subsequent stages of the PSD estimation. Initially the three loud glitches distort the PSD estimation for frequencies below 300 Hz. Wavelet denoising is used to reconstruct and remove these glitches, resulting in improved PSD estimates that reveal the underlying Gaussian noise contribution. The S/N of the extracted non-Gaussian features at each iteration were 21.2, 31.2 and 32.1 respectively.

The initial cleaning phase in **FastSpec** is iterative and deterministic. The first step is to estimate the PSD using a running median of the periodogram. The width of the smoothing window is chosen to be wider than that of the line features, but smaller than scale over which

the broadband noise varies appreciably. Window widths between 4 Hz and 16 Hz wide work well. Lines are identified as regions where the periodogram is a factor of 10 higher than the running median. The PSD model is then given by the value of the running median for regions away from the lines, and set equal to the periodogram value for regions in a line. This initial PSD estimate is then used to whiten the data. The data is then transformed to the time-frequency domain using continuous Morlet-Gabor (sine-Gaussian) wavelets (known as a Q-transform in the LVK literature). The wavelet quality factor, which fixes the width of the Gaussian envelope, is usually set at $Q = 8$ since this value does a good job of capturing short duration transients. Wavelet de-noising [260] is then used to remove any features that exceed 3-sigma in the wavelet transform. If the signal-to-noise of the reconstructed non-Gaussian transient exceeds some threshold, (usually set at $S/N = 5$), the median plus line PSD estimation procedure is repeated on the cleaned data. This process is repeated until the S/N of the transient stabilizes, which is taken to be when the change in S/N between iterations is less than 5. The PSD model and the model for the non-Gaussian excess typically converge to a stable solution after a few iterations.

Figure 4.2 shows spectrograms of the data during the iterative wavelet de-noising procedure. Notice that the contrast in the Gaussian noise component for frequencies below 300 Hz grows as the fit for the noise transient is improved. Figure 4.3 shows how the running median plus line model for the PSD evolves during the iterative refinement of the glitch model.

The plots in Figures 4.2, 4.3 are for 4 seconds of LIGO Hanford data. The performance of the cleaning stage improves for longer data segments as there are more frequency samples in the window used to compute the running median, and isolated glitches or signals have a smaller impact on the initial PSD estimate, so the algorithm converges more rapidly. It is interesting to compare the glitch reconstruction from the **FastSpec** cleaning phase (which takes a fraction of a second) with the median glitch reconstruction from the full **BayesWave**

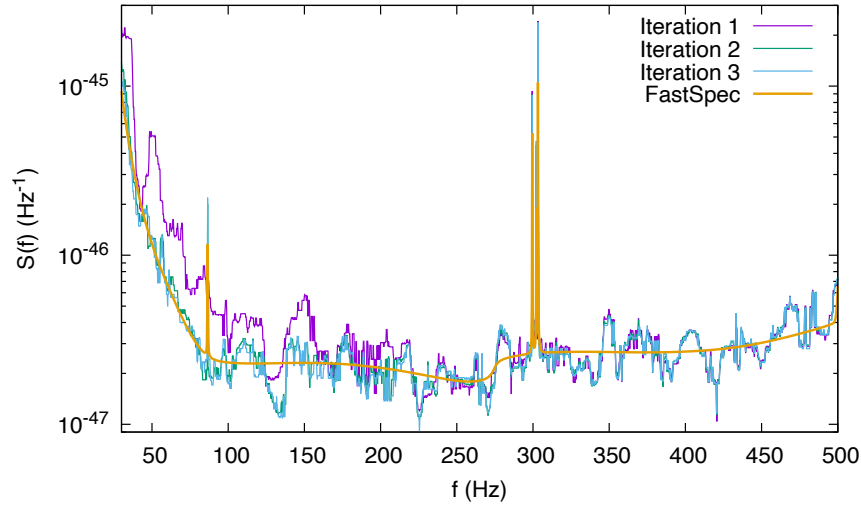


Figure 4.3: Successive stages in the refinement of PSD estimate for 4 seconds of LIGO Hanford detector centered on GPS time 1263011016. The loud glitches bias the initial PSD estimate at frequencies below 300 Hz. The PSD estimate converges to a lower value as the wavelet de-noising procedure reconstructs and removes the glitches. The coarse initial PSD estimates based on a running median are compared to the final much smoother spectrum inferred by the subsequent Bayesian spectral modeling stage of the FastSpec algorithm.

analysis (which takes minutes). The upper panel of Figure 4.4 shows the iterative refinement of the (whitened) glitch model in the time domain during the **FastSpec** cleaning phase. In this particular case, the solution converged in three iterations. For louder glitches, such as the one that impacted the LIGO Livingston detector data for the BNS event GW170817 [21], it can take four or five iterations to converge. The lower panel of Figure 4.4 compares the final **FastSpec** glitch reconstruction to the median **BayesWave** reconstruction. The agreement is remarkable, especially considering that the two algorithms are very different.

Bayesian Refinement

The initial PSD estimates from the **FastSpec** cleaning phase are not very smooth, so the next step is to refine the modeling using a fixed dimension version of the **BayesWave** spline and line model. Both models fit to the log of the PSD. The analysis is initialized by judiciously placing the spline knots and fitting the line features by Lorentzians. The spline

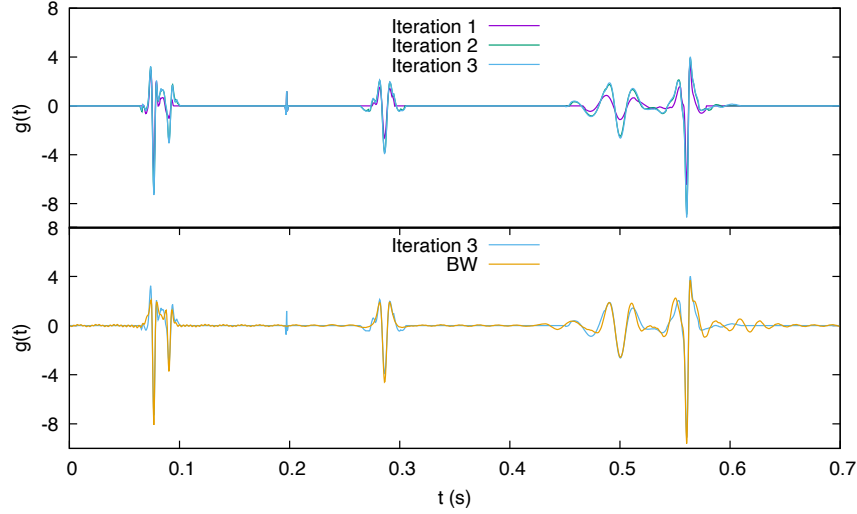


Figure 4.4: Glitch reconstruction for the LIGO Hanford detector centered on GPS time 1263011016. The top panel shows the refinement of glitch reconstruction during the iterative wavelet denoising procedure. The lower panel compares the final wavelet denoising glitch reconstruction to the BayesWave glitch reconstruction.

knots are placed by computing linear least squares fits to the running-median portion of the initial log PSD estimate. Starting at the minimum frequency used in the analysis, frequency windows of increasing size are tried until the chi-squared of the linear fit exceeds a pre-set threshold. A spline knot is then placed at the end of the window, with amplitude set equal to the the running-median of the log PSD at that location. The same procedure is then repeated until the maximum frequency used in the analysis is reached. This approach puts a higher density of spline knots in regions where the spectrum is changing most rapidly. The next step is to fit Lorentzians to each line features in the log PSD from the cleaning stage. The location in frequency, width in frequency, and the the height of lines are used to set the initial parameters for the Lorentzians. With the model initialized, a fixed dimension MCMC is used to refine the spline and line models. The parameters are the locations of the spline knots, the height of the spline knots, and the location, width and height of the Lorentzians. We found that the method used to initialize the spline and line models tended to put in

more knots and lines than the transdimensional **BayesWave** model, so we introduced a “poor man’s” version of the transdimensional step by periodically evaluating whether deleting a given spline knot or line changed the log likelihood by $\Delta\log L < -2$, and if it did not, that knot or line was removed. This pruning procedure is only applied during the burn-in phase since it violates detailed balance. After burn-in, the dimension of the spline and line models is fixed, and the analysis is fully Bayesian.

Improvements to the BayesWave Algorithm

The **BayesWave** spectral estimation algorithm [193] is used by the LIGO/Virgo/Kagra collaborations to produce point estimates for PSD that are used in parameter estimation studies of compact binary mergers. The on-source **BayesWave** PSD has been shown to allow for more accurate source parameter recovery than traditional off-source PSDs, such as those founding using Welch averaging [78]. During the break between the LVK third and fourth observing runs we took the opportunity to improve the **BayesWave** PSD estimation in several ways, resulting in an updated algorithm that is both faster and more effective at whitening the data.

The most significant change was incorporating the **FastSpec** algorithm as a rapid pre-processing step that can provide a very good starting solution for the spline and line models.. This improves the convergence of both the **BayesWave** glitch model and the PSD model. The **BayesWave** glitch model benefits from having both a good starting solution for the wavelets [91], and by starting with **FastSpec** PSDs that are robust against noise transients, making the separation between glitches and the Gaussian component of the noise more obvious. The **FastSpec** PSD is now used to define the prior bounds on the PSD model by defining an envelope around the median PSD estimate. The envelope is chosen to be wide enough so that the **BayesWave** PSD 99% credible interval is well inside the envelope, yet narrow enough that the analysis converges quickly.

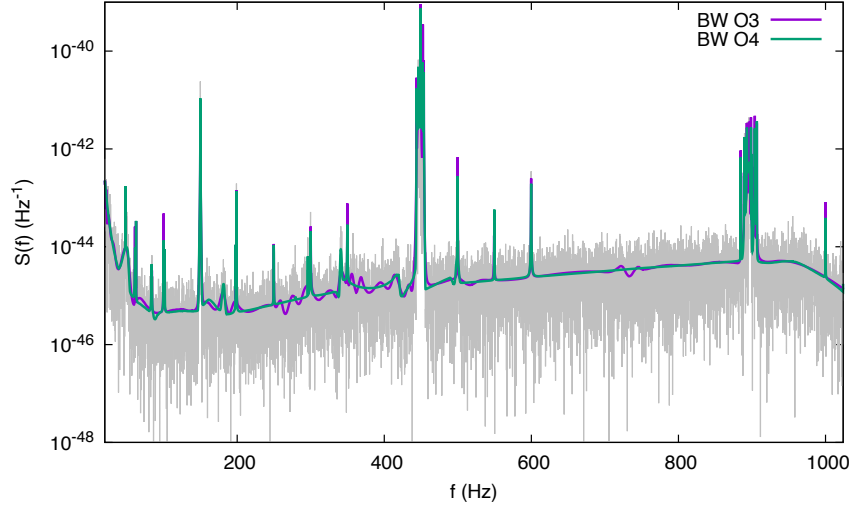


Figure 4.5: PSD estimates for 8 seconds of Virgo data centered on GPS time 1263011014. A periodogram of the data is shown in light gray, with spectral estimates from the version of BayesWave used in LVK observation run 3 (O3) and the version used in observation run 4 (O4). The O4 version uses Akima splines, and shows less variation than the O3 version, which uses Cubic splines.

Numerous changes were also made to the **BayesWave** PSD model and the sampling procedure that improved the fidelity of the model and the convergence of the sampler. The original spline model used cubic splines and placed the spline control points (also called knots) on a discrete grid in frequency. The discrete frequency grid was used since cubic splines have a tendency to produce interpolating functions that suffer from large oscillations when the knots are spaced too closely together. In the original version, the location of the knots could only be changed in the trans-dimensional moves where knots were added or deleted. In the updated version, cubic splines have been replaced by Akima splines. Akima splines produce a continuously differentiable sub-spline interpolation that is built from piecewise third order polynomials. Only data from the next neighbor points is used to determine the coefficients of the interpolation polynomial. Akima splines offer numerous advantages over cubic splines. Firstly, Akima are local, while cubic splines are computed using all the knots. Thus, when a single knot is updated the likelihood only has to be recomputed in

the immediate vicinity of that knot, which reduces the computational cost. In addition, the locality means that Akima splines are not prone to develop unphysical oscillations, so we can dispense with the frequency grid and allow the knots to be placed at any position in frequency. Lateral moves are now allowed, so both the knot location and height can be varied in the fixed dimension MCMC updates. This added flexibility allows the spline knots to be optimally placed, which is especially important in regions where there are lines or other sharp features. Fig. 4.5 compares the original O3 version **BayesWave** PSDs to the updated O4 version for 8 seconds of Virgo data centered on GPS time 1263011014. Virgo data was chosen to illustrate the difference between the O3 and O4 versions of **BayesWave** because it is more challenging to fit due to the larger number of spectral lines than the LIGO detectors. Note that the O4 version has far fewer wiggles than the O3 version thanks to the Akima splines. The O4 version allows for a prior to be set on the minimum spacing between points. When the minimum spacing is set to zero the prior on the number of spline knots is uniform. When a non-zero minimum spacing is used there are limitations on where the knots can be placed, resulting in an effective prior that exponentially decays. The results shown here use a minimum spacing of 2 Hz, though we found that setting the minimum to zero also produced good PSD models.

The LIGO and Virgo PSDs are known to have sharp line features from instrumental effects such as thermal excitation of the violin modes of the fibers that support the end mirrors. Many of the lines come from processes that can be model as damped-driven harmonic oscillators, which in the Fourier domain have power spectra that take the form of a Lorentzian. Each Lorentzian line is described by three parameters: central frequency f_i , quality factor Q_i and height A_i , with the full model given by

$$S_L(f) = \sum_{i=0}^{N_L} \frac{A_i}{\left(\frac{f}{f_i}\right)^2 + Q_i^2 \left(1 - \left(\frac{f}{f_i}\right)^2\right)^2}. \quad (4.2)$$

The number of lines, N_L , is explored by the trans-dimensional RJMCMC sampler. In the original version of **BayesWave** the spectral model was written as the sum of the spline and line components $S(f) = S_S(f) + S_L(f)$, while in the new version the default is to write the model as the product $S(f) = S_S(f) (1 + S_L(f))$. The advantage of the product model is that the line amplitudes are then always relative to the height of the spline model, making it easier to set priors on the line amplitudes. The switch from the sum to the product model has a much smaller impact on the resulting PSDs than the switch from cubic splines to Akima splines. The initial parameters for the line model come from the **FastSpec** analysis. If the number of lines identified by **FastSpec** exceeds the maximum number of lines specified for **BayesWave**, the lines are sorted by amplitude and the highest amplitude lines are used.

Many changes were made to the sampler, including some bug fixes in the line and spline proposals that resulted in the O3 version of the code failing to recover the prior distribution on quantities such as the number of spline knots and the number of lines when the likelihood was set equal to a constant. These issues had been missed since the O3 version of the code used no-constant priors on the number of model components, which masked the issues with prior recovery. For example, the O3 version used an exponential prior on the number of lines, which hid the fact that the error in the proposal densities would otherwise lead to a runaway in the number of lines. In the O4 version we now use uniform priors on the number of knots and lines, and the updated sampler recovers these distributions when the likelihood is held fixed.

The biggest changes to the sampling were related to the placement of the spline knots going from a discrete frequency grid to continuous. A new proposal was introduced for the trans-dimensional moves associated with adding or removing a spline knot. In addition to prior draws, a directed proposal was added that first randomly selects a frequency, then proposes that the amplitude of the spline knot is drawn from a Gaussian distribution centered on the current interpolated log PSD model at that frequency. This proposal

involves no-trivial proposal densities for adding and deleting a knot. When deleting a knot, the interpolated log PSD model with the knot removed has to be calculated, with the log proposal density proportional to the squared difference in the log amplitude of the knot and the amplitude of the interpolated log PSD at the frequency of the knot that is being proposed for deletion. This targeted proposal has a higher acceptance rate than the simpler prior draws on the log amplitude. The updated sampler recovers the priors on all parameters when the likelihood is held constant.

Performance Comparison

If the noise covariance matrix $\mathbf{C}$ has been correctly estimated from the data, then the data $\mathbf{d}$ can be whitened: $\mathbf{d}_w = \mathbf{W}^T \mathbf{d}$, where $\mathbf{W}$ is the Cholesky decomposition of $\mathbf{C}^{-1} = \mathbf{W}\mathbf{W}^T$. For stationary data in the Fourier domain this procedure reduces to dividing the Fourier transform of the data $\tilde{\mathbf{d}}(f)$ by the amplitude spectral density $\sqrt{S(f)}$, such that $\tilde{\mathbf{d}}_w(f) = \tilde{\mathbf{d}}(f)/\sqrt{S(f)}$. Both the real and imaginary parts of $\tilde{\mathbf{d}}_w(f)$ should follow a $\mathcal{N}(0, 1)$ normal distribution. This suggests a natural test for the PSD estimation methods - checking whether the whitened Fourier domain data is consistent with draws from a $\mathcal{N}(0, 1)$ normal distribution. We chose to use two versions of the Anderson-Darling test, the first being the version that assumes the mean and variance are known, and the second being the version which makes no assumptions about the mean or variance, instead estimating these directly from the data samples. The latter $\mathcal{N}(\mu, \sigma)$ version is a stronger test of normality, while the $\mathcal{N}(0, 1)$ test is useful in that it checks that the overall scaling is correct. We refer to the $\mathcal{N}(\mu, \sigma)$ version as the normality test.

Fig. 4.6 compares the PSD estimates and Anderson-Darling normality test p-values applied to 4 seconds of Virgo data starting at GPS time 1261967014. Rather than just applying the normality test to the entire frequency range, we instead compute p-values for successive frequency bands of width 8 Hz. This helps to identify regions where the spectral

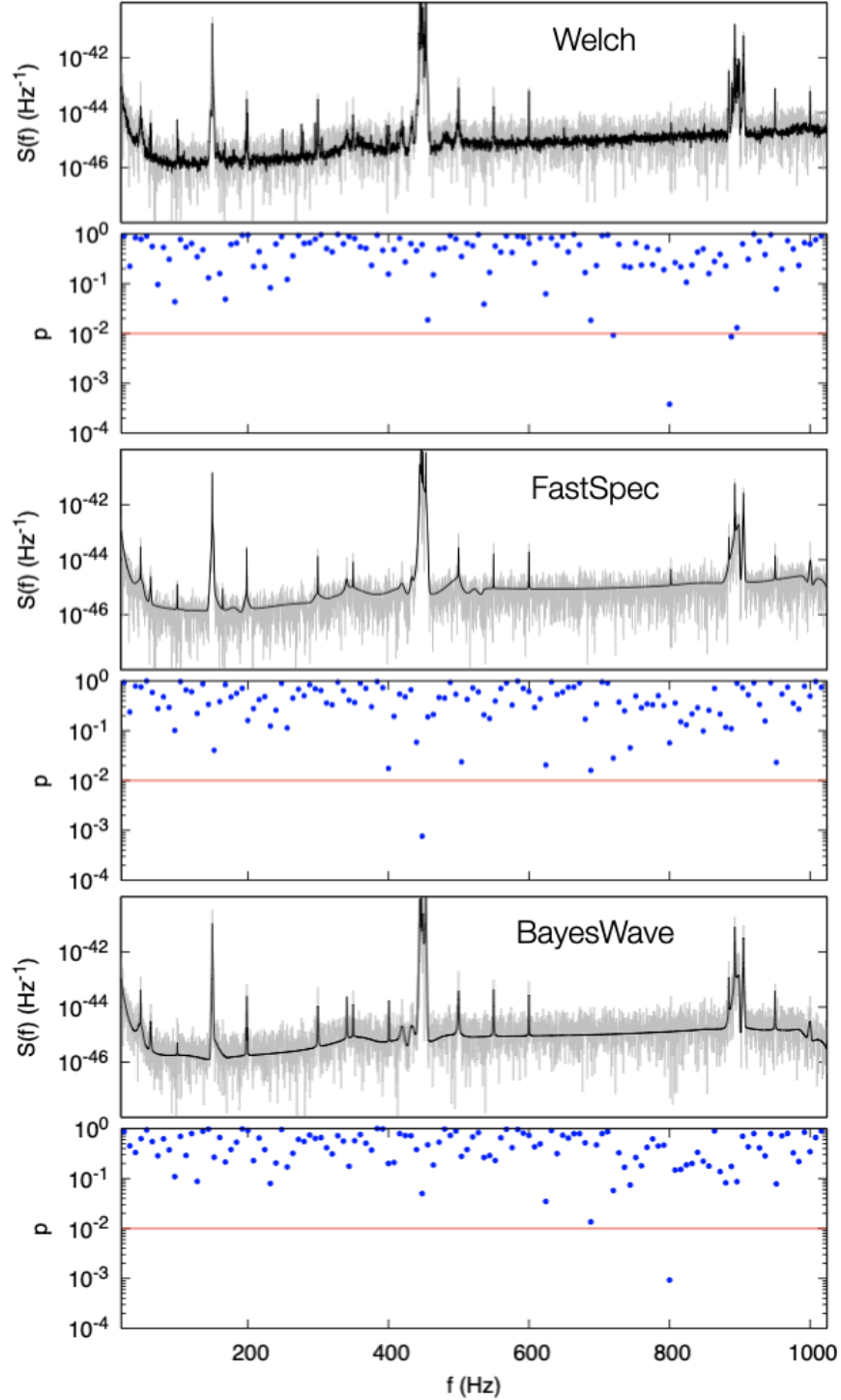


Figure 4.6: PSD estimates and Anderson-Darling normality test p-values applied to 4 seconds of Virgo data starting at GPS time 1261967014. The Anderson-Darling normality test was applied to data that was whitened with the corresponding PSD estimates. The p-values are shown for bands of width 8 Hz, each containing $8 \times 4 \times 2 = 64$ samples. Applying the Anderson-Darling normality test to the full 1000 Hz band yielded p-values of: Welch $p = 0.40$, FastSpec $p = 0.79$, BayesWave $p = 0.92$. In this example, all three spectral estimation methods produced whitened data that are consistent with Gaussian noise.

estimates might be struggling. In the example shown in Fig. 4.6, all three spectral estimation methods produced whitened data that passes both version of the Anderson-Darling for the full bandwidth. The **BayesWave** and Welch methods both fail the normality test in the 8 Hz window around 800 Hz, while the **FastSpec** analysis fails the normality test in the 8 Hz region around 440 Hz. Note that the **FastSpec** analysis picked up a small line at 800 Hz, while **BayesWave** did not find the line to significant. The 440 Hz band where the **FastSpec** analysis failed the normality test includes a very strong line. In almost all examples we looked at, very low p-values were found to be associated with frequency bands containing spectral lines.

Note that failing the normality test does not necessarily imply that the PSD estimation procedure is incorrect as the data might not be Gaussian distributed. For example, if a loud signal or a loud glitch were present in the data we would not expect it to be Gaussian. Additionally, some instrument lines are known to drift (move around in frequency over time), which the models do not account for. The **FastSpec** and **BayesWave** analyses have an advantage since they remove non-Gaussian features when estimating the PSDs, and the normality tests are applied to the residuals from the analyses. As a result, the **FastSpec** and **BayesWave** analyses typically pass the normality tests even when very loud glitches are present in the original data.

The performance of the various spectral estimation techniques was tested using data from the third observing run of the LIGO and Virgo detectors. The first set of tests used one thousand 4 second data segments starting at UTC 0:0:0, January 1, 2020. The data segments were spaced by 1000 seconds. The Welch averaging was performed using utilities from the **Bilby** [44] Python package, which uses the Welch median sub-routines from the **GWpy** [197] Python package. The median averaging was performed using 32 non-overlapping 4 second segments of data immediately prior to the analysis segment. The **FastSpec** and **BayesWave** analyses only used the 4 seconds of data in the analysis window. All three analyses used

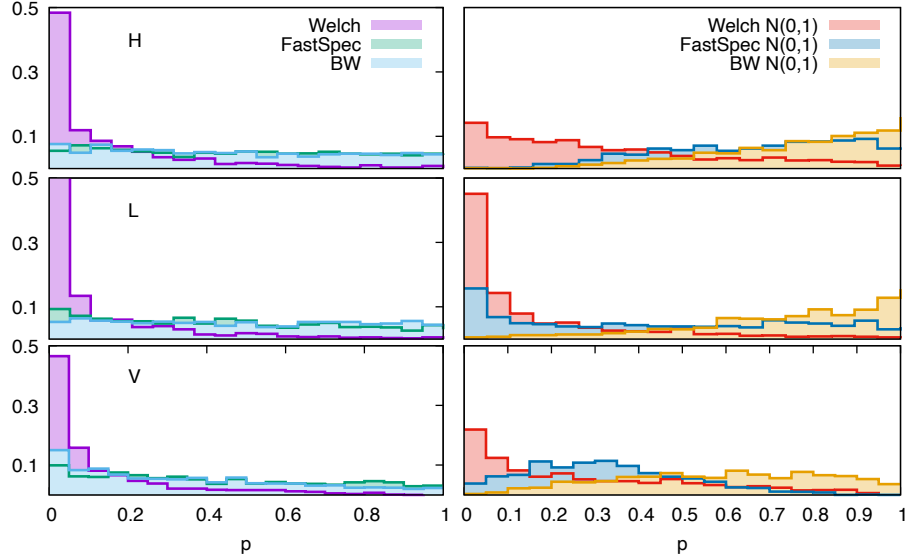


Figure 4.7: The distribution of Anderson-Darling normality test p-values and Anderson-Darling $\mathcal{N}(0,1)$ test p-values for one thousand 4 second segments of triple co-incident LIGO Hanford, Livingston and Virgo data, starting from UTC 0:0:0, January 1, 2020 (GPS time 1261872018). The data segments were separated by at least 1000 seconds. For each 4-second segment, the data were whitened using three methods, Welch, FastSpec and BayesWave, and p-values were computed for data between 20 Hz and 1020 Hz. The p-value distribution should be flat for data drawn from a Gaussian distribution. The Welch method failed both tests, whereas FastSpec and BayesWave, while still showing some bias, performed far better.

a sample rate of 2048 Hz. The Anderson-Darling $\mathcal{N}(0, 1)$ test and $\mathcal{N}(\mu, \sigma)$ normality test were applied to the real and imaginary whitened Fourier domain data between 20 Hz and 1020 Hz. Histograms of the p-values for these tests are shown in Fig. 4.7. The p-value distribution should be flat for data drawn from a Gaussian distribution. The Welch method failed both tests in all three detectors. **FastSpec** and **BayesWave** produced distributions that are consistent with an $\mathcal{N}(0, 1)$ distribution for the LIGO Hanford and Livingston data, but showed a slight excess of low p-values for the Virgo data. The Virgo instrument tends to have many more spectral lines than the LIGO instruments, and when looking at the p-values in 8 Hz frequency windows, it was found that the low p-values corresponded to regions with strong spectral lines. The **FastSpec** algorithm did not perform as well as **BayesWave** under the $\mathcal{N}(0, 1)$ test, showing a slight excess of small p-values for both the LIGO Livingston and Virgo data. In contrast, the **BayesWave** algorithm tended to produce p-values that were “too good”, in other words, distributions that are skewed towards large p-values. Further investigation revealed that the bias in the p-values was due to using the median of the **BayesWave** PSD distribution as a point estimate for the PSD. Repeating the test using fair draws from the posterior results in more uniform p-value distributions. This behavior was also noted in Ref. [78]. An in-depth investigation into the impact of using point estimates for PSD estimation can be found in the Appendix.

Fig. 4.8 focuses on the variance of the whitened data, which should be distributed around unity if the whitening is successful. The Welch method produced a distribution that is wider than expected and biased low, while the **FastSpec** and **BayesWave** algorithms produced much narrower distributions, with slight biases high or low. The narrowness of the distributions is due to the use of point estimates based on means or medians of the posterior distributions.

When testing the algorithms on real data it is difficult to say if departures from normality are due to a problem with the whitening procedure, or intrinsic to the data itself.

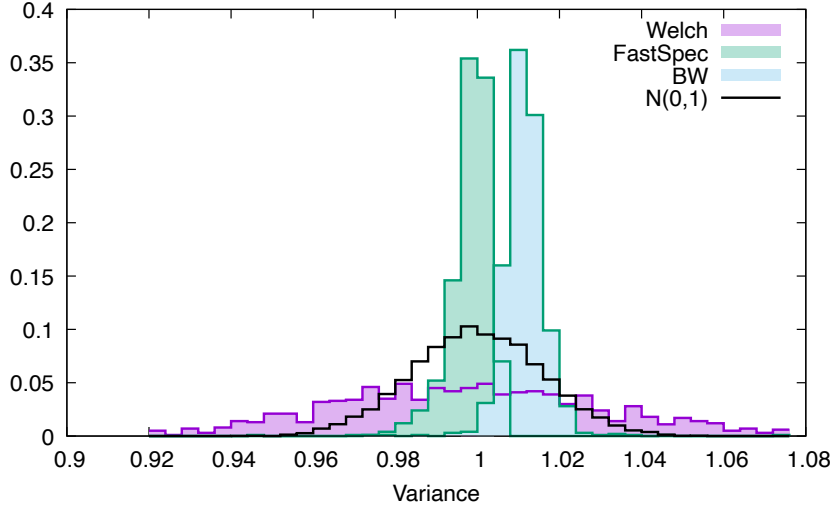


Figure 4.8: Histograms of the variances of the whiten LIGO Livingston data for the data segments shown in Figure 4.7. The variance distribution for the same number of samples drawn from a $\mathcal{N}(0, 1)$ Gaussian noise is shown for reference. The Welch method produces a spread that is wider than expected, while the FastSpec and BayesWave analyses produce variances that are narrower than expected, with slight biases either high or low.

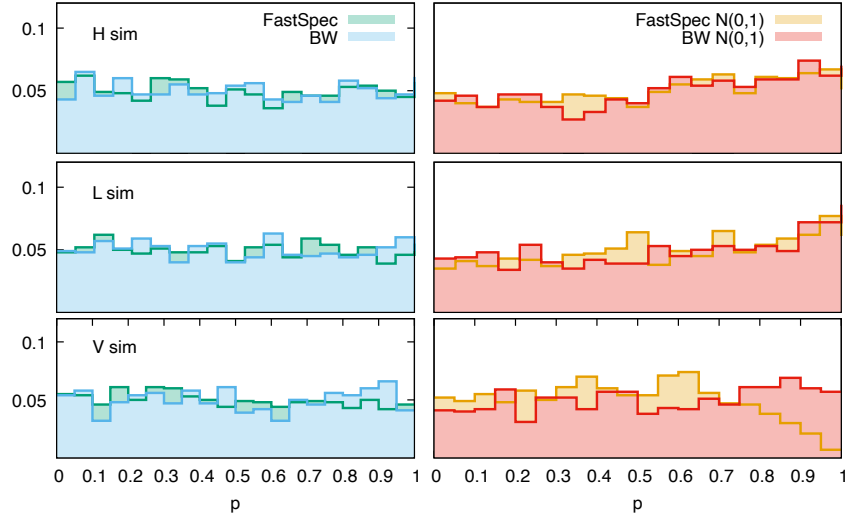


Figure 4.9: The distribution of Anderson-Darling normality test p-values and Anderson-Darling $\mathcal{N}(0, 1)$ test p-values for one thousand 4 second segments of simulated data that were generated using the PSDs derived from the real data used in Figure 4.7. Overall, both FastSpec and BayesWave passed this self-consistency check, with a slight bias toward high p-values for the $\mathcal{N}(0, 1)$ test for reasons explained in the text.

To test the internal self-consistency of the **FastSpec** and **BayesWave** algorithms we took the one-thousand PSD estimates from the analysis of the real data and generated one-thousand sets of simulated Gaussian noise drawn from those PSD estimates. The same analysis as before was then performed on the simulated data. Fig. 4.9 shows the p-value distributions for the analyses performed on these simulated data sets. The whitened data passes the normality test in all detectors for both algorithms, while most of the p-value distributions for the $\mathcal{N}(0, 1)$ test are biased towards high p-values for reasons explained in the Appendix. The exception is the **FastSpec** analysis of the simulated Virgo data, where the large number of lines still presented a challenge.

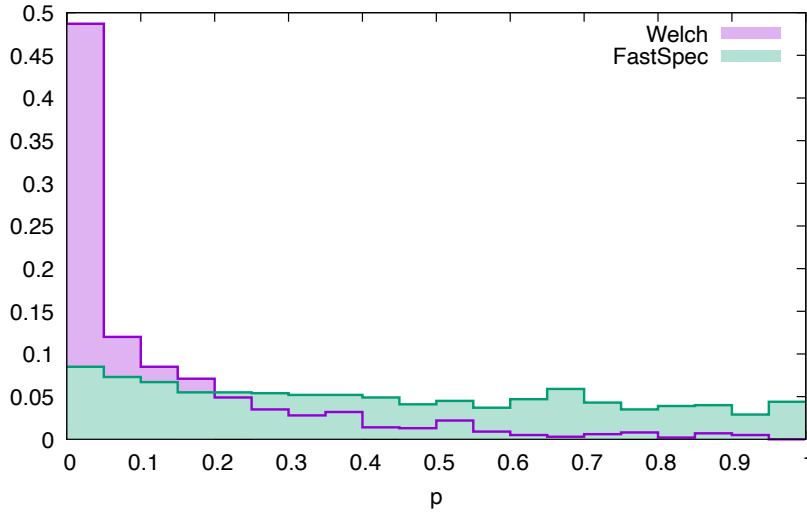


Figure 4.10: The distribution of Anderson-Darling normality test p-values for one thousand 16 second segments of LIGO Hanford data, starting from GPS time 1261872018. The **FastSpec** algorithm continues to significantly outperform the Welch method on these longer data segments. Similar results were found for longer segments of data, but non-stationarity starts to become an issue for minutes long segment lengths.

Returning to real data, a similar analysis was repeated using one thousand 16 second long segments of LIGO Hanford data, in this case focusing on the two low-latency analyses, Welch averaging and **FastSpec**. The distribution of p-values for the $\mathcal{N}(0, 1)$ Anderson-Darling test are shown in Fig. 4.10. The **FastSpec** algorithm performed very well on these

longer data stretches, delivering a nearly uniform p-value distribution. The Welch method fared considerably worse, failing the $\mathcal{N}(0,1)$ test almost half the time. Similar tests were applied to longer segments of data, with the **FastSpec** algorithm continuing to perform well, while the Welch method failed even more often. The main reason the Welch method fails is that LIGO and Virgo data are not stationary over extended periods of time. For example, the Welch method applied to 16 second segments used over 8 minutes of off-source data, and the noise levels fluctuate significantly on these timescales.

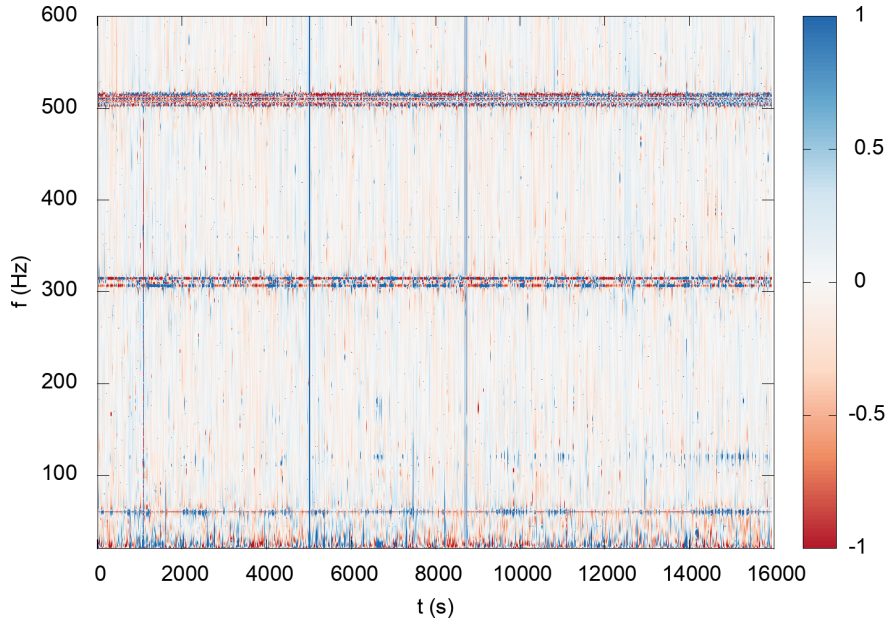


Figure 4.11: Non-stationarity in the PSD estimates for sequential 16 second stretches of LIGO Livingston data, starting at GPS time 1261872018. Here the PSD estimates for each segment were divided by the median of the PSD estimates for all 1000 segments, stacked vertically and plotted using a $\log_2$ color scale. Significant variation is seen around the dominant spectral lines (saturating the color scale), and for frequencies below 50 Hz. More moderate variation of order 10%-50% occurs across the spectrum.

The variation in the PSD over time is illustrated in Fig. 4.11. Here 1600 seconds of LIGO Livingston data, starting at GPS time 1261872018, were broken up into 16 second segments. **FastSpec** was used to estimate the PSD for each segment. The data in each segment was then whitened using the median of the 1000 **FastSpec** PSD estimates, stacked

vertically and plotted using a $\log_2$ color scale to produce a spectrogram of the whitened data. Significant variation is seen around the strong spectral lines near 300 Hz and 500 Hz. The data below 30 Hz also shows significant variation. More moderate variation of order 10%-50% occurs across the spectrum. Several time slices also stand out as being especially noisy. It is these variations in the power spectrum that degrade the Welch estimates.

Conclusions

Bayesian parametric spectral estimation, especially when coupled with noise transient removal, is a powerful method for producing robust spectral estimates. Traditional methods such as Welch averaging use additional data outside the analysis window, which can cause problems when the noise levels fluctuate over time. Here we introduced a new low latency spectral estimation algorithm, **FastSpec**, that while not quite as good as **BayesWave**, should prove useful for low latency parameter estimation. We also made many changes to the original **BayesWave** algorithm, improving the parametric model and significantly speeding up the analysis. The reduction in run time will be particularly important for long duration signals such as binary neutron star mergers, where the previous version took days to run while the new version takes hours - or just minutes if **FastSpec** is used instead.

Going forward, both **BayesWave** and **FastSpec** could be improved by implementing better models for the spectral lines, but the gains would be marginal. Much more significant will be generalizing these models to handle non-stationary noise by switching from the frequency domain to the time-frequency domain. The standard LVK parameter estimation packages, such as **Bilby** [44], use point estimates of the PSD. It would make sense to convert these analyses to being fully Bayesian, and marginalize over the PSD estimates jointly with the signal analysis, as is done in the **BayesWave** signal analyses. Importing the fixed dimension sampler from **FastSpec** should be relatively straightforward. The more adventurous could try incorporating the trans-dimensional **BayesWave** model.

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DETECTING GRAVITATIONAL WAVE SIGNALS USING A FLEXIBLE MODEL FOR
THE AMPLITUDE AND FREQUENCY EVOLUTION

Contribution of Authors and Co-Authors

Manuscript in Chapter 5

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Contributions: Conceived the study design, provided guidance throughout the analysis, and edited the manuscript.

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Introduction

Black holes and neutron stars stand out as the two most prominent sources of gravitational Waves (GW) observed to date [7, 9, 10, 12] using ground-based detectors such as the Laser Interferometer Gravitational-Wave Observatory (LIGO) in the USA and the Virgo detector in Italy [4–6, 8]. With the increasing number of detections and improved detector sensitivity, the observation of longer-duration signals has become possible. This emphasizes the necessity for robust signal models capable of capturing signatures over extended time spans. While models for compact binary mergers have steadily improved in the last two decades, we currently lack reliable models for many interesting sources, such as the post merger dynamics of binary neutron stars and core collapse supernovae. Additionally, there may be signals coming from unanticipated sources, for which no models exist. And while general relativity has successfully passed all current tests, future observation of longer duration signals, with high signal to noise, will allow for more stringent tests of the theory. These considerations motivate the development of techniques that can detect and reconstruct arbitrary gravitational wave signals.

Ground-based detectors are most sensitive to transient signals from Compact Binary Coalescence (CBC) systems, particularly those involving Neutron Stars and Black Holes, with a total mass M_T falling within the stellar mass range ($2M_\odot \leq M_T \leq 200M_\odot$). The gravitational wave emission from these systems become most pronounced in the sensitive frequency band of the LIGO-Virgo detectors (20 - 1000 Hz) [137, 143], reaching a peak amplitude milliseconds before the merger. These observations allow for the potential to uncover new physical effects in the strong gravity regime.

Unlike traditional match-filtering techniques [38] that rely on accurate template waveforms derived from theoretical models and numerical simulations [47, 71, 225], wavelet based methods have been developed to detect and characterize generic signals. Prominent

examples include coherent Wave Burst [115, 171, 172] and a Bayesian algorithm known as **BayesWave** [90, 92]. These algorithms stand apart by avoiding assumptions of a predefined template for signal detection, instead utilizing a wavelet basis, or frame, that allows the data to determine the possible morphology of the signal. **BayesWave** employs the sum of continuous Morlet-Gabor wavelets to reconstruct a signal, where both the number of wavelets and the parameters of the wavelets are explored by a trans-dimensional sampling algorithm. **BayesWave** performs very well on short duration signals, but the performance drops for long duration signals. Specialized search techniques have been developed for long duration burst signals, including seedless clustering algorithms [255–258] and variants of the coherent Wave Burst algorithm that are tuned to detect longer duration signals [172]. However, there are currently no effective Bayesian inference algorithms for reconstructing long duration burst signals.

To address these challenges, a new algorithm, **BayesWaveVoices**, is introduced in this study. The name is inspired by the literature on extreme mass ratio inspirals, where the signal can be decomposed as a collection of “voices”, each with a define amplitude and frequency evolution [152]. These “voices” do not necessarily have instantaneous frequencies that are related by integer multiples, so the term “voices” is used instead of “harmonics”. For un-expected or un-modeled sources, we have reason to expect that the gravitational waves they produce can be described by a collection of voices with time evolving amplitudes and frequencies. Gravitational waves are sourced by time varying stress-energy and current multipole moments, with bulk, coherent motion likely to produce the strongest contribution [59, 221]. This is certainly true of known sources such as compact binaries, including those beyond GR, but it will also be true for any source where the emission is dominated by a few multipoles [165, 166]. This reasoning extends to more complicated examples such as the post-merger dynamics of neutron star binaries [82]. Numerical simulations of astrophysical sources such as core collapse supernovae show evidence for a

few dominant voices, in addition to a broader spectrum of waves generated by turbulent fluid motion [207].

The traditional **BayesWave** algorithm models signals and noise transients as sums of Morlet-Gabor wavelets. There is currently no restriction on the overall amplitude and phase of the resulting waveform. In particular, neither are constrained to be smoothly varying. In contrast, **BayesWaveVoices** ensures continuity of both the amplitude and phase by modeling the amplitude, $A(t)$, and instantaneous frequency, $f(t)$, of each “voice” as smooth functions such as splines or Legendre polynomials. The idea of modeling generic waveforms using splines to describe the amplitude and frequency evolution is not new [204, 205], but our implementation of the model is very different as it uses trans-dimensional Bayesian sampling. Fixed dimension spline models have also been used to model possible departures from the predictions of general relativity [117].

We begin in Sec. 5 by motivating the new approach by comparing the waveform reconstructions of the **BayesWave** and **BayesWaveVoices** models using binary black hole mergers as an example. The **BayesWaveVoices** model, basis functions, methodology and refinements to the model in the time domain, along with corresponding priors and proposals, are described in Sec. 5. Sec. 5 discusses the results and conclusions drawn from the methodology presented in this paper and also concludes with a discussion of potential avenues for future development.

Motivation

The **BayesWave** algorithm models burst-type signals while accounting for any non-stationary and non-Gaussian detector noise features. The wavelet based **BayesWave** signal model works well for short duration signals and short duration noise transients [90, 194]. The new **BayesWaveVoices** model is designed to cover longer duration signals.

Binary black hole mergers provide a good test bed for comparing the wavelet and

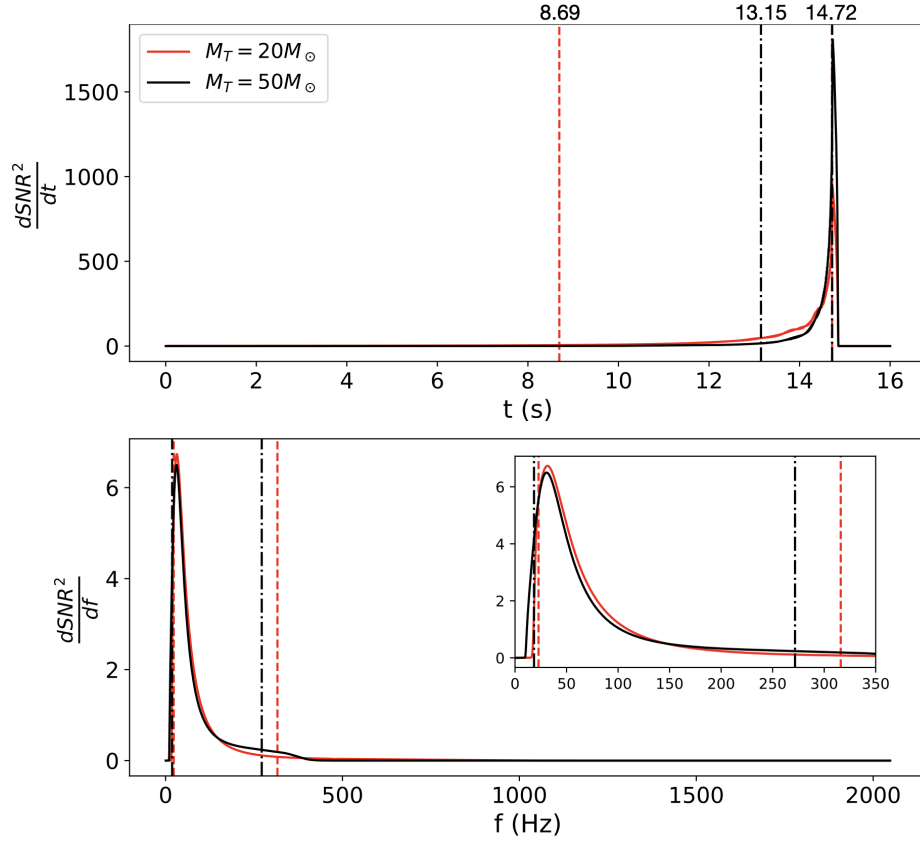


Figure 5.1: Change in accumulated SNR^2 with time (upper) and frequency (lower) for a binary black hole system with total $\text{SNR} = 20$ and $M_T = 20M_\odot$ (red line) and $M_T = 50M_\odot$ (black line). The vertical lines bound the region that contains 90% SNR^2 . We don't expect the signal reconstructions to have support outside of these regions.

voices models as the time that the signal spends in the sensitive band of the detectors can be adjusted by changing the total mass of the system.

To illustrate the performance of the wavelet and voices versions of **BayesWave**, we consider simulated signals embedded in Gaussian noise with a spectrum that follows the advanced LIGO design sensitivity curve [1]. In contrast to analyses that use theoretical waveform templates, which allow for the model to be extrapolated beyond regions with significant signal power, generic models are only expected to recover the signal in the regions with significant signal power.

To illustrate the performance of the two models, we simulated 16 seconds of advanced

LIGO data (for a single detector). In one example we added the signal from a binary black hole system with total mass $20M_{\odot}$, and in the other we added a signal with total mass $50M_{\odot}$. We used the IMRPhenomD waveform model, which describes the dominant harmonic for spin-aligned quasi-circular binaries [167]. The mass ratio and spins were $q = 1$, $\chi_1 = 0.2$, $\chi_2 = 0.1$, and the merger time was set at $t = 14.73s$. Both signals were scaled to a total SNR of 20. Figure 5.1 illustrates the accumulation of SNR^2 over time, $d\text{SNR}^2/dt$ and frequency $d\text{SNR}^2/df$ for these examples. The vertical lines bound the region that contains 90% SNR^2 . For a $20M_{\odot}$ binary, these bounds, indicated by the red dashed line, cover a time span 6.02 seconds between 8.69s and 14.72s in time (top panel), and frequency bandwidth 293.3 Hertz between 22.8 Hz and 316.1 Hz in frequency (bottom panel). Similarly, for a $50M_{\odot}$ binary the black dashed-dotted line represent the boundary of the sensitive region covers a time span of 1.57 seconds between 13.15s and 14.72s and a frequency bandwidth of 253.3 Hertz between 18.4Hz and 271.7Hz. As expected, high mass systems spend less time in band than low mass systems.

Figure 5.2 shows a comparison between the **BayesWave** (left panel) and **BayesWaveVoices** (right panel) analyses for a simulated binary black hole signal of $M_T = 20M_{\odot}$ (top) and $50M_{\odot}$ (bottom) with an injected SNR = 20. The simulated whitened data for a spin aligned black hole binary with spins $\chi_1 = 0.2$ and $\chi_2 = 0.1$ is shown in light grey. The time range shown in this Figure correspond to the 90% SNR^2 regions shown in Figure 5.1. We observe that **BayesWave** encounters difficulties in capturing the signal for low-mass binaries ($M_T = 20M_{\odot}$). In contrast, **BayesWaveVoices** effectively captures the high SNR signal region, achieving a match of 0.92, as opposed to **BayesWave** 's match of 0.56. The improvement persists for the $M_T = 50M_{\odot}$ binary, with the **BayesWaveVoices** reconstruction achieving a match of 0.95 compared to 0.82 for **BayesWave**, with the **BayesWaveVoices** reconstruction extending to earlier times. The subplots in each panel highlight the high SNR region around merger.

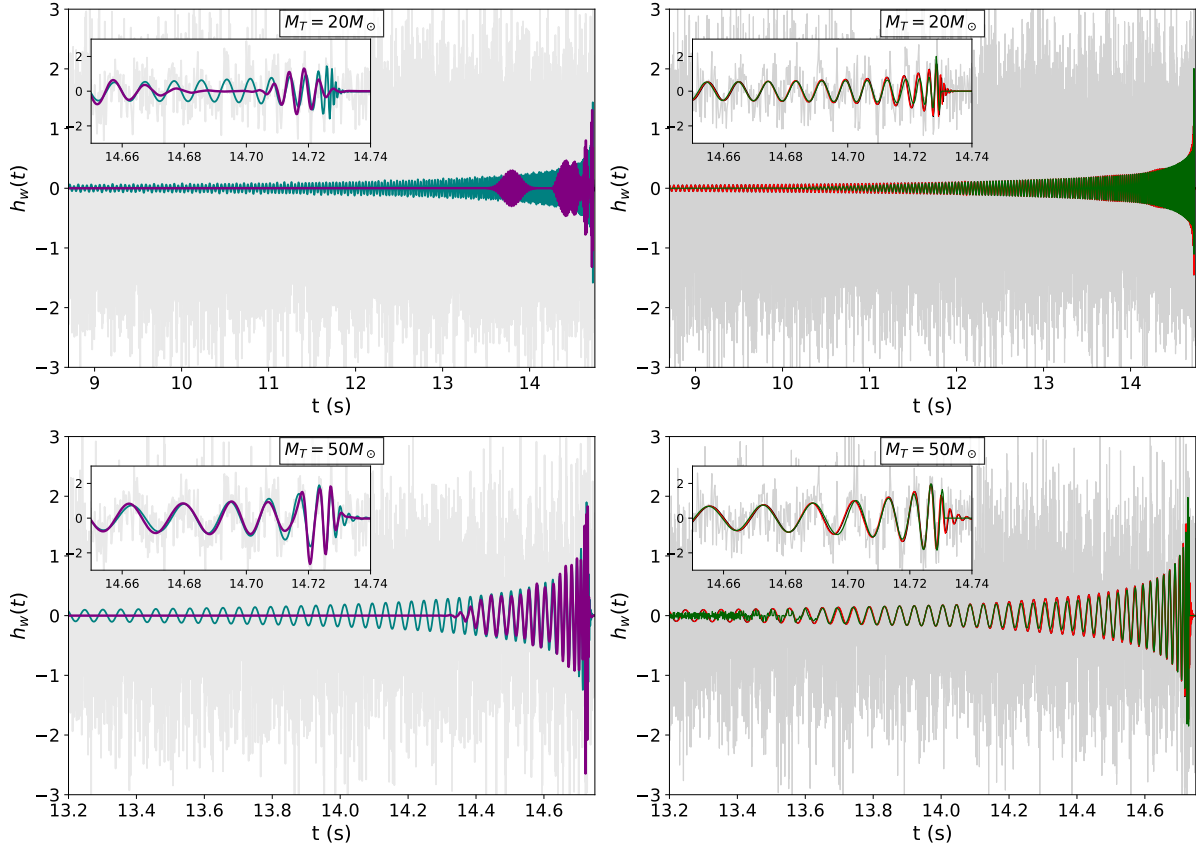


Figure 5.2: **BayesWave** [90, 92] (left panel) and **BayesWaveVoices** (right panel) reconstruction for an injected signal of $\text{SNR} = 20$. This reconstruction is for an Akima spline model that incorporates all refinements detailed in the subsequent sections of this paper. These plots are shown in the same 90% SNR^2 region as indicated in Figure 5.1. The subplots show the zoomed in regions near merger. The top and bottom panels represent an equal mass binary of total mass $20M_\odot$ and $50M_\odot$ respectively. Injected signal shown in teal and red a has spin of $\chi_1 = 0.2$ and $\chi_2 = 0.1$. The recovered median reconstruction is shown in purple for **BayesWave** with a match of 0.567 and 0.823 for a $20M_\odot$ and $50M_\odot$ respectively. Importantly, **BayesWaveVoices** enhances the overall fit for the same binary system, achieving match values of 0.92 (for $20M_\odot$) and 0.95 (for $50M_\odot$).

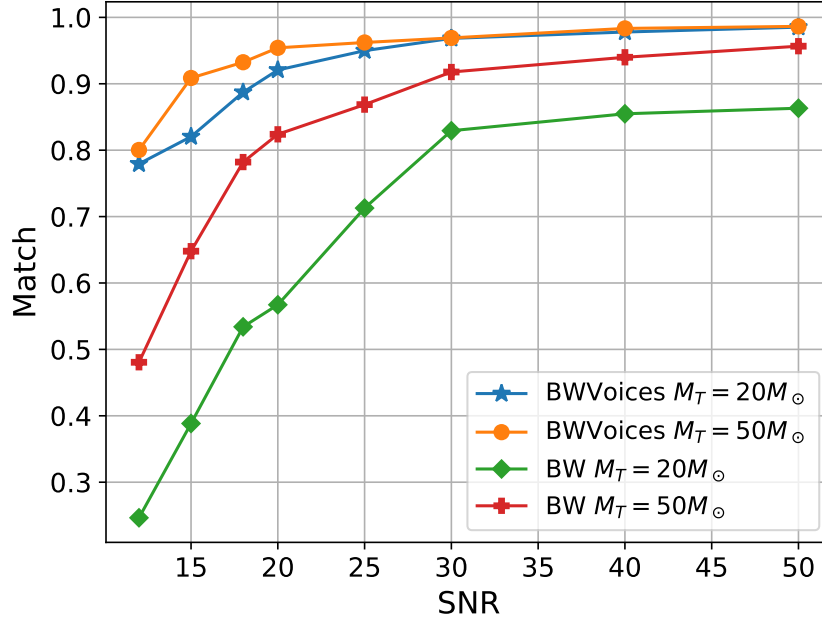


Figure 5.3: Matches between the simulated and recovered waveforms using **BayesWave** and **BayesWaveVoices** as a function of signal-to-noise ratio for simulated black hole mergers. The green and red line shows **BayesWave** match for $20M_\odot$ and $50M_\odot$ binary, while the blue and orange lines show the corresponding matches found using **BayesWaveVoices**. The voices model significantly outperforms the wavelet model in all cases.

The match is computed using a point estimate for the waveform given by the median of the posterior samples of the waveform model. Here the match between waveforms h and $\bar{h}$ is defined as

$$M = \frac{(h|\bar{h})}{(\bar{h}|\bar{h})(h|h)} . \quad (5.1)$$

The match as a function of SNR is shown in Figure 5.3 for binaries with total mass $20M_\odot$ and $50M_\odot$ for both **BayesWave** and **BayesWaveVoices**. The match increases with increasing SNR and M_T . The value approaches 1 for high mass and high SNR signals in either model. We notice that the **BayesWaveVoices** model outperforms the **BayesWave** model for the entire SNR-mass parameter space. Note that the matches for **BayesWave** found here are somewhat lower than those shown in Ref. [53], the reason being that here we are using data

at advanced LIGO design sensitivity, which results in longer time in band compared to the earlier work which used the lower sensitivities corresponding to the first observing runs.

BayesWaveVoices Algorithm

The **BayesWaveVoices** algorithm models signals as a collection of “voices”, which are each described by a smoothly changing amplitude and frequency. The current **BayesWave** algorithm computes the likelihood in the frequency domain, since the noise correlation matrix is diagonal in that representation, assuming that the detector noise is stationary over the relatively short time segments being analyzed. It might seem natural then to model the waveforms directly in the frequency domain such that for a single voice

$$h(f) = A(f)e^{-i2\pi(\int^f t(f')df')+\phi_0}, \quad (5.2)$$

One problem with this approach is that there is nothing stopping time from becoming multi-valued. This pathology is actually present in the frequency domain IMRPhenomD model [167], which models $A(f)$ and the phase $\Phi(f)$. Extracting $t(f)$ via

$$t(f) = \frac{1}{2\pi} \frac{d\Phi(f)}{df} \quad (5.3)$$

reveals that time becomes multi-valued near merger for this model.

To avoid this pathology we instead model the signals in the time domain:

$$h(t) = A(t)e^{-i2\pi(\int^t f(t')dt')+\phi_0}. \quad (5.4)$$

There are several advantages to this approach. Firstly, time is single valued, and the model is able to accommodate signals that have instantaneous frequencies that can increase or decrease in time. Secondly, spectrograms of the data, such as the Q-scans used in **BayesWave**

tfQ proposal for placing wavelets, can be used to identify tracks in the time-frequency plane that serve as proposals for placing points along the frequency track $f(t)$. Thirdly, expressing the signal in terms of $A(t)$ and $f(t)$ allows for very efficient, automatically heterodyned likelihood calculations in the discrete wavelet domain [86].

In the current implementation we chose to compute the likelihoods in the time domain to avoid having to Fourier transform the signals. The time domain analysis is performed using whitened data. The data can be whitened either by transforming to the frequency domain, dividing the data by the amplitude spectral density $\sqrt{S(f)}$ then returning to the time domain, or by staying in the time domain and transforming the data using a Cholesky decomposition of the inverse of the noise correlation matrix. The signals can be whitened by the replacement $A(t) \rightarrow A(t)/\sqrt{S(f(t))}$. Alternatively, we can model the whitened amplitude directly.

Basis Functions / Voices

Morlet-Gabor wavelets were originally chosen when constructing the **BayesWave** algorithm since they are maximally compact in time-frequency, and have simple analytic forms in both the time and frequency domains. But wavelets are not an essential part of the **BayesWave** algorithm. Far more important is the trans-dimensional modeling component, which can in principle build a signal out of any complete set of basis functions or over-complete frame functions. The performance of the algorithm can be improved if the chosen basis is able to capture signals using fewer parameters. For gravitational wave signals originating from coherent flows of matter and energy, we expect that the signal can be modelled by a few modulated voices, with distinct instantaneous frequencies. This is true for (low eccentricity) compact binary mergers [206], and appears to be a decent approximation for some core-collapse supernovae models [207] and for neutron star post-merger signals [82].

These considerations motivate a new signal model based on the decomposition in time

domain:

$$h(t) = \sum_{k=0}^{N_v} A_k(t_k) \cos \left(\int^t 2\pi f_k(t', \phi) dt' + \phi_k \right), \quad (5.5)$$

where k labels each of the N_v voices and ϕ_k is the overall phase for k^{th} voice.

There are many ways we can specify the functions describing the amplitude and frequency. For example, we could use a polynomial expansion (similar to the analytic post-Newtonian waveforms), or a sum of orthogonal basis functions (e.g. Chebyshev or Legendre polynomials), or a spline (as is used in the BW spectral model and for marginalizing over the LIGO calibration uncertainties), or some non-parametric model such as a Gaussian process. Regardless of the choice, the key idea is that the complexity of the model (e.g. the number of terms to keep in a polynomial expansion or the number of spline control points or knots to retain) should be determined from the data using trans-dimensional Bayesian inference. This allows the model to adapt and capture the inherent complexity of the gravitational wave signals present in the observed data.

We chose to use Akima splines [32] to model the smoothly evolving amplitude and frequency functions in time. Akima splines have compact support, and they are much less prone to develop unphysical oscillations than cubic splines. The location of the spline knots, their amplitude and also the number of spline knots are variable. These parameters are explored using a trans-dimensional Reversible Jump Markov Chain Monte Carlo (RJCMCMC) algorithm.

Methodology

The **BayesWaveVoices** algorithm returns samples from the posterior distribution of the waveform model. The waveform model is described by a collection of parameters $\vec{\theta}$ that specify the values and locations of the spline knots for the amplitude and instantaneous frequency of each voice. The dimension of $\vec{\theta}$ is variable. Bayes theorem states that for a

given set of data $\mathbf{d}$, the probability of measuring parameters $\vec{\theta}$ is expressed as

$$p(\vec{\theta}|\mathbf{d}) = \frac{p(\mathbf{d}|\vec{\theta})p(\vec{\theta})}{p(\mathbf{d})}, \quad (5.6)$$

where the posterior distribution function, $p(\vec{\theta}|\mathbf{d})$, quantifies the probability that the model is described by parameters $\vec{\theta}$ given the observed data $\mathbf{d}$. The likelihood function, $p(\mathbf{d}|\vec{\theta})$ or $\mathcal{L}$, evaluates the probability of observing data $\mathbf{d}$ given the parameter set $\vec{\theta}$. Assuming the noise is Gaussian distributed and the spectral model is fixed, the log likelihood can be written as

$$\log \mathcal{L} = -\frac{1}{2} \int_0^{T_{\text{obs}}} (d_w(t) - h_w(t))^2 dt + \text{const}. \quad (5.7)$$

where the subscript w indicates that the quantities are whitened. The prior probability distribution function, $p(\vec{\theta})$, allows us modify the behavior of the spline model. The simplest choice, which we refer to as the reference model, assumes a uniform distribution for each parameter over some specified range.

The transdimensional MCMC method uses the Metropolis-Hastings algorithm [202] to sample the parameter space. We start from an initial guess for $\vec{\theta}$ and then a new value is proposed based on the proposal distribution. The acceptance ratio, H ,

$$H = \frac{\mathcal{L}(\mathbf{d}|y)Q(x|y)P(x)}{\mathcal{L}(\mathbf{d}|x)Q(y|x)P(x)} \quad (5.8)$$

is determined as the product of the likelihood ($\mathcal{L}$), the proposal distribution (Q), and the prior distribution (P). Here, x represents to the current state and y denotes the proposed state. The proposed jump is accepted if, $H > U(0, 1)$, where $U(0, 1)$ is a random number between 0 and 1, and rejected otherwise. This entire process is repeated until the model converges to a true posterior distribution.

In our analysis, we utilize Akima splines as the basis functions to model the time-

varying amplitude $A_w(t)$ and frequency $f(t)$. We also explored different models, including splines [3], Steffen splines [246], Legendre polynomials, and a smooth fitting function [163]. Cubic splines, being global and C^2 differentiable, offer continuity and differentiability in the first and second derivatives. While this allows for flexibility, it can lead to undesired wiggles in the fitting functions. To address this, we introduce a separate prior on the derivatives of the cubic spline points to ensure their small values. However, this approach has the tendency to place control points too close to each other, resulting in oscillations in the fit over time. Steffen splines, on the other hand, exhibit high stiffness and demand monotonicity, presenting challenges in capturing complex features. Legendre polynomials, being a global basis functions defined between the interval 1 and -1, require additional scaling and expansion coefficients, introducing extra parameters.

A common characteristic of the aforementioned basis functions is their global nature, meaning that altering one point affects the entire curve. In contrast, Akima splines are more local in nature. They are evaluated over a five-point stencil and are C^1 differentiable, making them well-suited for our analysis. Due to their local nature and smoothness, Akima splines are more efficient in capturing localized features and are therefore chosen as the basis functions for modeling the waveform parameters throughout the rest of the paper.

In this model, the location of spline knots, their amplitudes, the coalescing phase ϕ_c and even the number of spline knots are variable. The key idea of using Akima splines voices with variable number of spline knots within the framework of trans dimensional MCMC is to avoid oscillations and facilitate a balance between model intricacy and the fidelity of fit. This approach embraces a natural parsimony, allowing the data to determine the delicate balance between model complexity and goodness of fit.

We inject simulated binary black hole signal generated from the IMRPhenomD phenomenological model [167]. This model accurately describes the $l = |m| = 2$ harmonic mode for a spin-aligned, non-precessing quasi-circular binary, incorporating information from

numerical relativity. This waveform model provides amplitude and phase information in the frequency domain. To transition from the frequency domain to the time domain, we employ a two-step process. In the early and late inspiral-merger times, we utilize a combination of SPA and inverse fast Fourier transform (iFFT) respectively [86]. SPA is effective in the inspiral phase but breaks down near the merger, prompting the switch to iFFT. Moreover, the iFFT is tapered to zero at low frequencies to prevent Gibbs oscillation in the time domain. This approach ensures a smooth and accurate representation of the gravitational wave signal across the entire evolution of the binary system.

We can extract reference values for the amplitude and phase from the simulated IMRPhenomD signal by computing the instantaneous frequency $f(t) = \dot{\phi}/2\pi$ and the whitened amplitude $A_w(t) = A(t)/\sqrt{S(t(f))}$. Up to merger-ringdown, the frequency can be extracted analytically using the stationary phase approximation. During merger-ringdown the amplitude and phase have to be extracted numerically from the inverse Fourier transform of the frequency waveform [86].

The trans dimensional Markov Chain Monte Carlo (RJCMCMC) algorithm [138, 146] is used to vary the number of spline knots in each model and to vary the values and locations of the knots.

In reality, we do not know in advance where to place the boundaries of the model in time, so we have to make the start and end times of the spline model variable (the alternative is to fix the first and last knots at the the end points of the time interval being analyzed, but this forces the model to cover regions where there is no information about the signal.)

In the current implementation we restrict the model to use a single voice ($N_v = 1$). Going forward we plan to allow N_v to vary as dictated by the data.

Priors and Proposals Priors on the number and placement of spline knots in the amplitude and frequency models can be used to enhance the performance of the algorithm.

In what we refer to as the base model, the priors on all these quantities are taken to be uniform in some range. The prior on knot point locations follows a uniform distribution $U(0, T_{obs})$, where T_{obs} represents the duration of the analysis segment, and we define $t = 0$ to be the start time of the analysis segment. The magnitude of amplitude and frequency priors are $U(0, A_{max})$ and $U(f_{min}, f_{max})$ respectively. We set A_{max} at some multiple of the noise variance $\sigma^2 = 1$, with a typical choice being $A_{max} = 6\sigma^2$. In the current study with simulated data we set A_{max} to twice the maximum amplitude of the injected signal. We set f_{max} equal to the Nyquist frequency corresponding to the given sample rate and f_{min} is set equal to 10Hz, reflecting the steep loss in sensitivity below this frequency. The prior distribution for the overall phase parameter is defined as a uniform distribution in the range $U(0, 2\pi)$ associated with amplitude and frequency is specified as $U(0, N_{max})$, where a typical choice of $N_{max} = 150$ is an effective choice.

A variety of proposals are used to update the amplitude and frequency models. These include prior draws on the location and values of the spline knots. In the within-dimensional update, the amplitude is either proposed from a uniform distribution or a Gaussian jump centered around the current state. Lateral moves facilitate the movement of spline points in time, with the randomly selected location proposed as a Gaussian jump about its current position.

The location of the end knots is allowed to vary, so that the model can expand or shrink in time to cover the region where there is significant signal power. In the default model, the splines for the amplitude and frequency share the same end knots. This adds some complications to the sampling. It is possible to allow the models to have different end knot locations, but then we either have to have some prescription for how to extend the models to cover the same total time interval, or we have to restrict the time interval to the region where the two models both have support.

Transdimensional moves involve adding a point anywhere between the current end-

points. The knot locations are drawn uniformly between the boundaries, and amplitudes are drawn from the prior distribution. The end points are treated differently since they are shared between the two models. Jumps in either endpoint location are proposed from a Gaussian distribution centered on the current location. Jumps are rejected if the proposed location is inside any of the other knot locations. Updates of the overall phase parameter ϕ_0 are drawn from a Gaussian distribution centered on the current value.

Special jumps are permitted for amplitude and frequency models. Proposals involve drawing from a density centered around the fit, with reverse moves drawing a point near the current Akima fit and using the fit at x (the previous state) as a reference. Alongside these special proposals, we include symmetric proposals, such as a uniform draw from the prior range or a Gaussian jump from the current location with a selection of different variances. We ensure detailed balance and verify prior recovery for model reversibility. This comprehensive set of proposals enhances the exploration of the parameter space throughout the Bayesian inference process.

In principle, the model should be able to lock onto a signal starting from a random draw from the prior. However, Bayesian models, especially complex trans-dimensional models can take a long time to converge from a random starting point. A better approach is to start the model off with a good guess. For example, a scalogram of the whitened data you be used to identify the region in time and frequency where a signal is present, and could also be used to extract a guess for the the amplitude and frequency tracks. In the current analysis we have focused on the sampling after a signal has been identified, and we have yet to develop a general purpose initialization algorithm. In this study we are focusing on simulated binary black hole mergers with known waveforms, so we are able to initialize the model using our knowledge of the injected waveforms.

We initialize our model by judiciously placing spline knots positioned in amplitude and frequency using the linear least squares fitting methods. The procedure involves calculating

the chi-squared of the linear fit until it surpasses a predefined tolerance, at which point a knot is placed. This process is repeated, proceeding from the established knot point to the subsequent one, until the chi-squared value once again exceeds the specified tolerance. This iterative placement of knots persists until all points are positioned up to 0.01 seconds after the merger event. Subsequently, the model places spline points at intervals of 0.5 seconds until reaching the end of the injected signal. The selection of the tolerance value is aimed to strike a balance. It is not set too high to prevent having too few points initially, nor is it set too low to avoid an excess of control points that may not be useful.

Consider two reference system with fixed masses $m_1 = m_2 = 10M_\odot$ and $m_1 = m_2 = 25M_\odot$ with spins $\chi_1 = 0.2$ and $\chi_2 = 0.1$. We set the initial model with NA and NF number of spline points, defining the time-varying $A_w(t)$ and $f(t)$ functions using the least square fitting method discussed above. Our reference model uses uniform priors and proposals for location, amplitude and number of spline knots for both amplitude and frequency model. The boundaries are variable.

Testing the Model

To test the **BayesWaveVoices** algorithm, we simulated Gaussian noise with a spectrum that follows the advanced LIGO design curve, and injected signal of $M_T = 20M_\odot$ and $\text{SNR} = 15$. For our initial test we start with our model defined in the entire time range as our boundaries, the boundaries are allowed to move based on the likelihood estimates as defined in the reference model. Figure 5.4 shows the reconstructed waveform (top panel) and its 90% confidence interval (CI), while the injected and initial signals span the entire time range. These results are from our reference model defined in Sec. 5.

Inspecting the amplitude extracted from the simulated signal, depicted in red in middle panel of Figure 5.4, reveals a sharp feature at the merger. One of the key feature of splines is to enforce smoothness, and the Akima splines used in the analysis struggle to model sharp

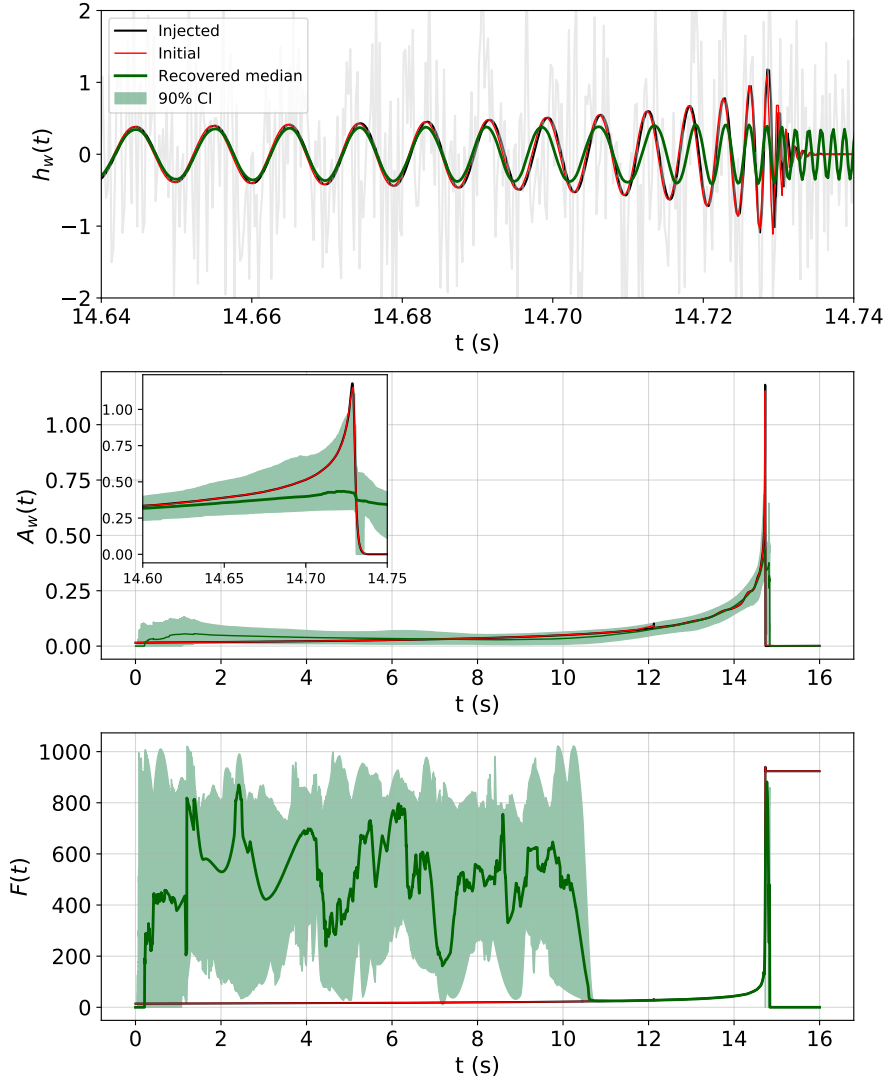


Figure 5.4: The top panel shows the median waveform reconstruction in green with the 90% credible interval using an Akima spline model with variable boundaries starting with the full time range of 16s. The injected (black line) signal is same as the initial start signal (red line) over the full length scale. The middle panel shows the injected amplitude in black which again is same as the initial start amplitude. A noteworthy observation is the abrupt decay in amplitude just after the merger, leading to suboptimal performance of the model. The lowest panel shows the reconstruction for frequency as a function of time. The model encounters challenges in accurately fitting the initial times due to a low signal-to-Noise ratio during these early stages. This resulted in a $\text{match} = 0.85$ using median waveforms.

features in the amplitude or frequency evolution.

The frequency model, shown in the lower panel of Figure 5.4, recovers the frequency track fairly well for roughly two seconds before merger, which is in keeping with our expectations that the model should recover the signal in regions with significant contribution to the SNR^2 . However, contrary to expectations, the boundaries for the amplitude and frequency models did not shrink down to focus on the high SNR^2 . The locations for the end knots did move around as the RJMCMC sampler explored the model space, but the end knots typically covered a much wider region than where the signal was concentrated. At early times, where there is little signal power, the frequency model covers most of its prior range, indicating that it was not recovering the signal in this region. Ideally the boundaries of the model would have shrunk to focus on the region where the signal power is concentrated.

The inability of the amplitude model to capture sharp features, and the fact that the boundaries did not shrink to focus on the region of high SNR^2 suggests that the base model can be improved.

While the spline model may not readily capture sharp features, given sufficient SNR, it demonstrates an ability to do so. Additionally, for lower masses, the SNR is distributed, whereas for higher masses, a more concentrated SNR is observed. Figure 5.5 illustrates the behavior of Akima splines for a system with $M_T = 20M_\odot$ and $\text{SNR} = 30$ (top panel) and another system with $M_T = 50M_\odot$ and $\text{SNR} = 15$ (bottom panel). It is evident that increasing either the SNR or the mass compels Akima splines to adeptly handle sharp discontinuities. Therefore, it is a combined effect of total mass and SNR; with sufficient power, the model exhibits proficiency in fitting sharp features, extending its capability even to $\text{SNR} = 20$.

To understand these issues better, we looked at what happens when the boundaries of the spline models are fixed to cover the time interval during which 95% of the SNR^2 is deposited. Crucially, in this example, the 95% region does not include the ringdown portion of the waveform, which we saw poses challenges for the reference `BayesWaveVoices` model.

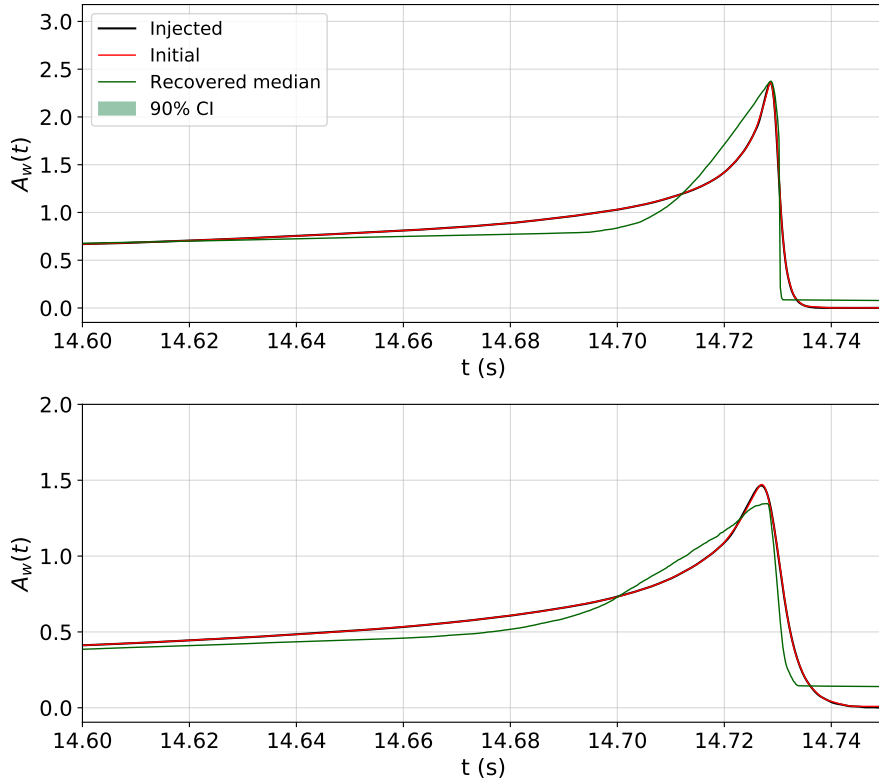


Figure 5.5: Amplitude reconstruction depicted in green using the basic Akima spline model, with the initial signal shown in red covering the entire range, alongside the injected signal in black. The top panel displays the amplitude reconstruction for a system with $\text{SNR} = 30$ and $M_T = 20M_\odot$, while the bottom panel showcases $\text{SNR} = 15$ and $M_T = 50M_\odot$. Notably, the amplitude reconstruction effectively captures sharp features in signals with high mass and high SNR.

Figure 5.6 shows the injected signal in black and the portion of the signal that accounts for 95% of the SNR^2 in red. The median of the signal model posterior is shown in dark green, along with the 90% CI in lighter green. The match for the recovered median waveform over the fixed region is 0.88, which improves on the fit we found when the boundaries were free to move. In this fixed boundary example where the signal is cut off at merger, the amplitude smoothly increases with no sharp drop, and we see in the bottom panel that the spline model is able to accurately recover the peak amplitude of the signal, in contrast to the case where the amplitude drops sharply through ringdown.

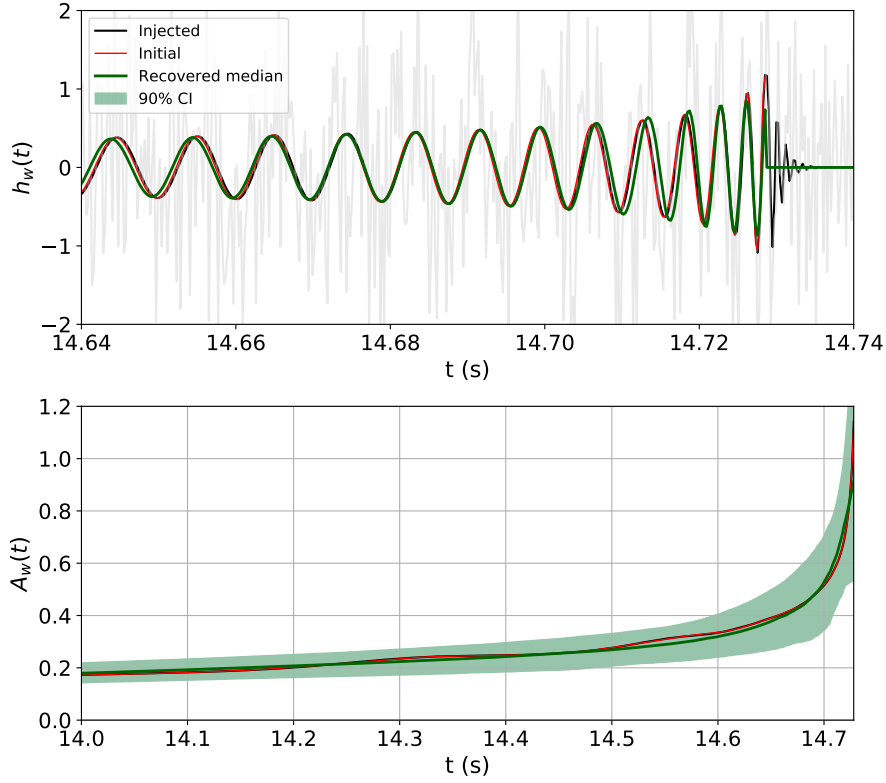


Figure 5.6: Median reconstruction is shown in green for a system with $\text{SNR} = 15$ and $M_T = 20M_\odot$. A basic Akima spline model with fixed boundaries and an injected signal in black while an initial start signal covering 95% SNR^2 region shown in red, which doesn't include the sharp drop in amplitude near merger as can be seen in the bottom panel. The model captures the peak around merger and recovers the signal with $\text{match} = 0.88$. The shaded region is the 90% credible interval for the reconstruction.

The match versus SNR curve for this case is depicted in Figure 5.7 for a binary black hole signal with a total mass of $20M_\odot$ and $50M_\odot$. The graph suggests that as the SNR increases, the match proportionally rises, eventually stabilizing at a near-constant value close to unity. Hence, the model performs effectively for signals initiated with fixed boundaries and no abrupt discontinuities.

However, it is essential to enable variable boundaries in reality, as we lack information on where to place the boundaries in time. Allowing the locations of the end knots in the amplitude and frequency models to vary adds some complications to the sampling, especially

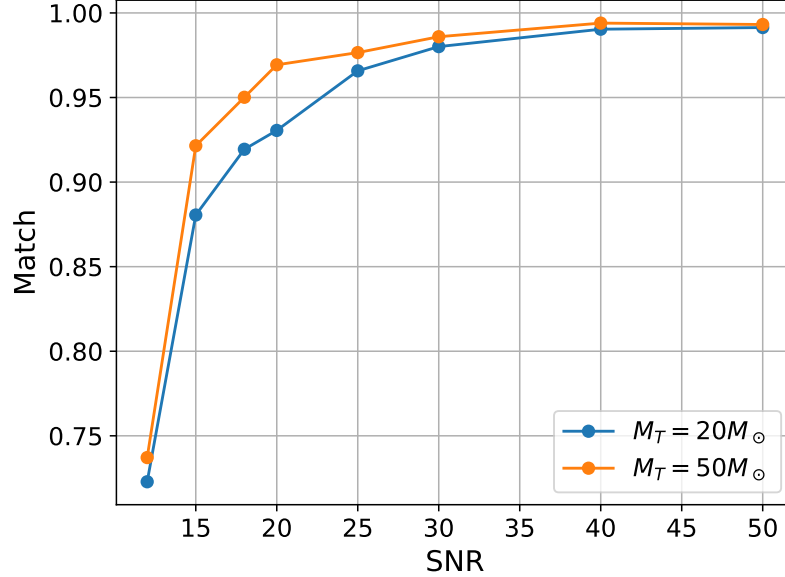


Figure 5.7: Match as a function of signal-to-noise ratio using basic Akima spline model and fixed boundaries covering 95% SNR² region for binary systems of $20M_\odot$ and $50M_\odot$, excluding the sharp drop around the merger. The match increases with increase in SNR and mass and slowly asymptotes to unity.

if both models are required to share the same end points. The advantage is that the **BayesWaveVoices** model inherits some of the advantages of the wavelet version since the model is then compact in time and frequency.

To enhance the understanding of the variable boundary, Figure 5.8 illustrates the fraction of time a specific region remains active. As the boundaries shift, the spline is modeled within that particular temporal segment. It is observed that the model exhibits reduced preference for about initial 2 s region but becomes active thereafter, extending up to the merger. Essentially, the end boundary retracts to exclude the post-merger region where SNR is very low.

The inclusion of additional information and complexities necessitates an enhanced model focused on capturing the most interesting and informative regions. If we consider

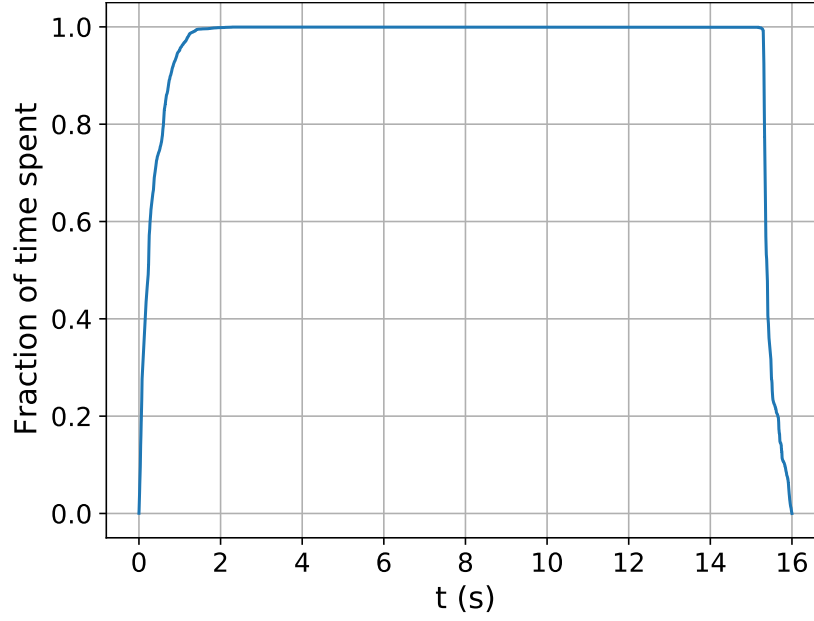


Figure 5.8: Fraction of active time for a specific temporal region in the basic model using Akima splines with variable boundaries, as generated through transdimensional MCMC iterations.

continuous $A_w(t)$ and $f(t)$ without any discontinuity and allow the boundaries to move, the defined model should extend to effectively capture the merger region. Figure 5.9 illustrates how the recovered signal not only captures the initial signal but also extends beyond to capture the injected features for the same reference case discussed above. This replicates the effects that an original full-length signal would exhibit. The end boundary, initially at $t = 14.724\text{s}$, moved to 14.728s while capturing the high SNR region and extending to encompass the merger region with an overall match of 0.88. This improvement in the model allows for a more accurate representation of the gravitational wave signal, showcasing its flexibility in adapting to diverse signal characteristics and optimizing information retrieval.

The presence of sharp features in initial model significantly impacts the overall fit. In scenarios where amplitude is the sole parameter, decoupled from frequency, Akima splines excel in capturing these sharp discontinuities. Extensive testing has confirmed the effectiveness of this approach. However, the challenge arises when frequency and

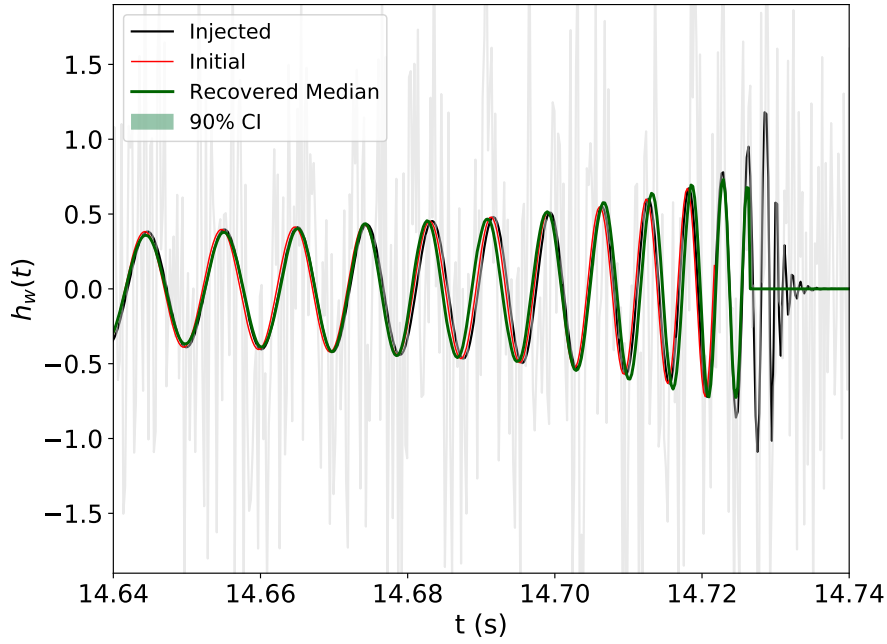


Figure 5.9: The injected signal is shown in black while the signal is initiated with the end boundary at 14.724s (red line), about 6ms before merger. The boundaries are allowed to move and model extends its end boundary location to 14.728s extrapolating the information to about 2ms before merger. This reconstruction recovers signal of $\text{SNR} = 15$ and $M_T = 20M_\odot$ with a match of 0.88.

amplitude are coupled in likelihood estimations, introducing added complexity to the modeling process. The interdependence of these parameters necessitates a more sophisticated approach, considering their joint influence on the likelihood calculations.

Further Refinements

By their very nature, spline models have a tendency to smooth over sharp features. To overcome this limitation, the model can be extended to include sharp features in addition to the smooth spline model. This is what is done in the **BayesLine** Bayesian spectral estimation algorithm [140, 194], where a combination of splines and sharp line features, modeled as Lorentzians, are used to model the LIGO/Virgo noise power spectra. An example of the **BayesLine** spectral modeling is shown in Figure 5.10, one with just the spline model and

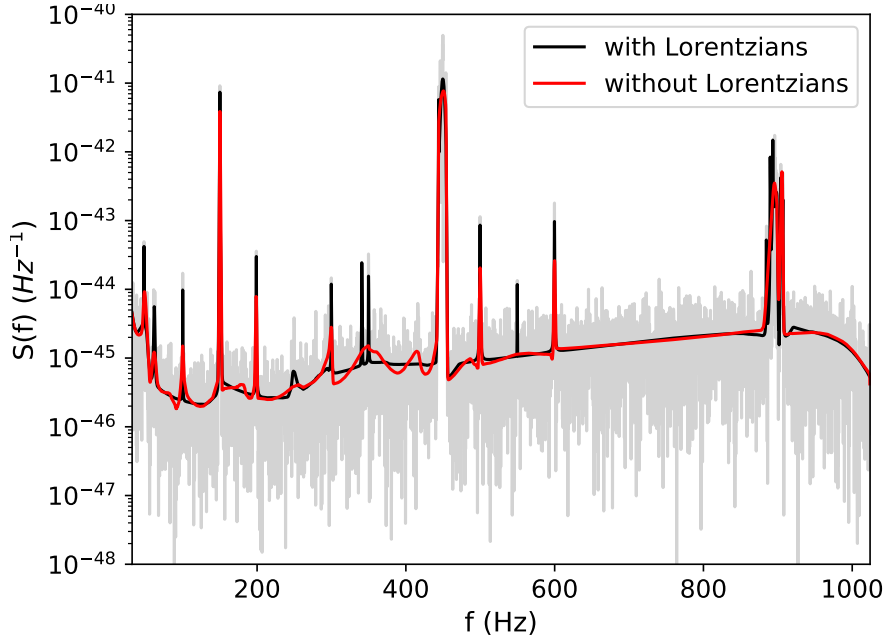


Figure 5.10: PSD estimation for a 4s Virgo data with a 2048Hz sample rate is illustrated. In light gray, a periodogram of the data is presented, while the spectral estimates using a **BayesWave** model are shown in red (utilizing only the smooth component of the PSD model) and in black (including the Lorentzian line feature modeling). The model with the Lorentzian feature more accurately captures the sharp features compared to the ones without.

the other with splines and lines. The model without Lorentzian lines misses many of the sharp, low-amplitude (low SNR) line features. The spline model is able to pick up the high amplitude (high SNR) lines, showing that with enough SNR the splines can fit any shape, despite their preference to produce smooth curves. The model that includes lines does a much better job of fitting the power spectrum (as measured by the Anderson-Darling statistic computed for the whitened data [140]).

The effectiveness of the multi-component **BayesLine** model motivated us to introduce a similar strategy by incorporating a variety of additional functions that are better suited to modeling sharp features. We included five distinct types of growing and decaying functions, all appropriately scaled to yield similar curves for a comparable rate parameter, to help ensure that proposals that seek to swap one function for another will get accepted. These

tapering functions can either be rising or falling, and are scaled to go from 0 to 1. The complete model for $A_w(t)$ is given by the product of the spline model and the tapers.

We extend this approach by incorporating a more refined strategy, employing a combination of both growing and decaying functions to construct a spike feature. Figure 5.11 illustrates five distinct functions utilized to create this spike. This method goes beyond a simplistic swap between functions, allowing for different components on the left and right sides of the peak. For instance, the spike could comprise a Gaussian fit on the left side and a power law on the right side of the peak. This flexibility enables the model to adapt to various signal features, capturing intricate details that a singular function might overlook. Specifically, for a binary black hole merger, a spike that rises gradually slowly and then decays rapidly (exponentially) should fit the final part of the amplitude fairly well. To enhance the fitting process, we started by dividing the injected amplitude by this spike function. Subsequently, by fitting the initial spline model to this modified amplitude, the product of the spline and the spike is expected to yield a robust initial fit with faster convergence. This approach not only allows for the seamless transition between different functions but also enables tailored adjustments on either side of the peak, enhancing the model's adaptability and performance. Although this additional feature moderately improves the overall fit, further refinements were required to enhance signal reconstruction.

To ensure that splines exhibit a smooth fall-off and avoid unexpected oscillations, we introduce a mechanism involving predefined knots in a specific time region. Let ΔT_s and ΔT_e represent the time region where this predefined function $\mathcal{S}(t)$ is evaluated. The locations of the fixed knots are positioned at some fraction of ΔT_s and ΔT_e , with their amplitudes gradually approaching zero on either ends. The spline function takes into account these fixed knot data points during evaluation, ensuring a smooth fall-off on both ends. Although the spline is evaluated within the specified boundary (as discussed in Sec. 5), the inclusion of these additional knots is incorporate to help mitigate oscillations, especially in regions of low

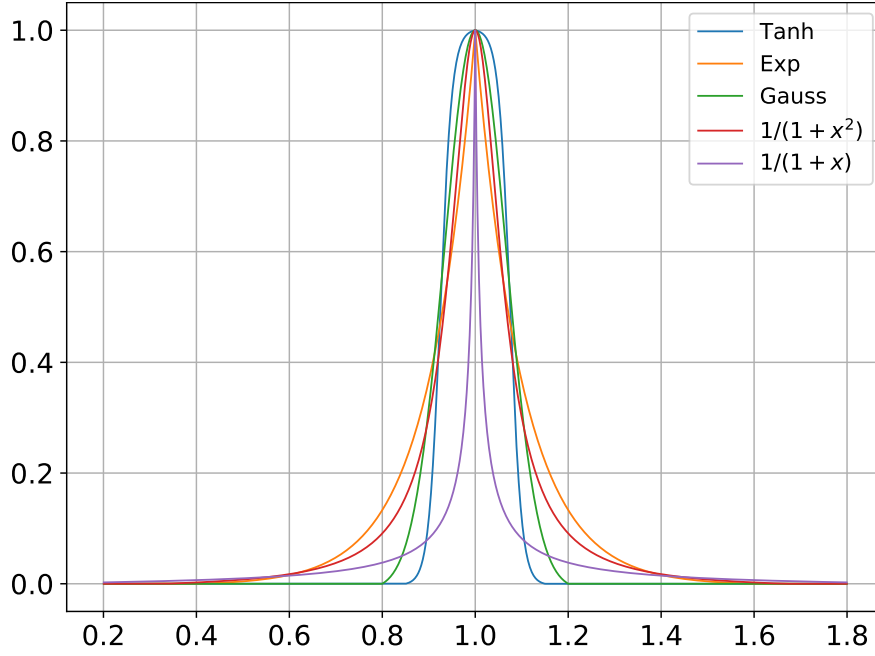


Figure 5.11: Five distinct spike functions, all appropriately scaled to yield similar curves are shown. The purple, green and blue line represents a tan hyperbolic, exponential and Gaussian function respectively. The orange and yellow curve shows inverse power law function of $1+x^2$ and $1+x$ respectively. Smooth Akima splines can be multiplied by $(1 + \text{spike})$ to potentially incorporate the sharp changes in amplitude for a given signal.

SNR, as observed in the initial times in the frequency fit (see bottom panel Figure 5.4).

It is worth noting that splines are not inherently local, meaning the positions of control points influence a broader region. By guiding the amplitudes to gracefully approach zero, we enhance the spline model's behavior, as abrupt endings are generally unfavorable for splines. This approach prioritizes a smooth descent to zero, contributing to the model's stability and reliability in capturing complex signal characteristics. Mathematically, the predefined function for the two ends can be expressed as follows:

$$\mathcal{S}_{start}(t) = A_w(t) \frac{e^{-(t+\Delta T_s - T_s)} - 1}{e^1 - 1}, \quad (5.9)$$

$$\mathcal{S}_{end}(t) = A_w(t) \frac{e^{-(t+\Delta T_e - T_e)} - 1}{e^{-1} - 1}, \quad (5.10)$$

where $\mathcal{S}_{start}(t)$ and $\mathcal{S}_{end}(t)$ indicated the two predefined functions before the start and after the end boundary knots.

To prevent frequency from exhibiting unwanted oscillations during the MCMC sampling process, a potential solution is to initiate the frequency integration from the peak of the maximum amplitude. Cumulative integration is specifically centered around the frequency point at which the maximum amplitude occurs. This approach aims to avoid the accumulation of unwanted area at low SNR, particularly given the limited SNR in the initial times.

As we allow the boundaries to move for capturing the signal around the maximum power, we have the option to either tie both the frequency and amplitude boundaries to move together or allow them to move independently. Untying these boundaries provides the model with flexibility in defining frequency and amplitude where needed, potentially reducing the total number of parameters to work with. Although untied boundaries increase the complexity of the model, but they offer greater flexibility. We chose to have the model set the amplitude to zero outside of the amplitude model end knots and to set the frequency to a constant outside of the frequency model end knots.

It's important to note that the first and last points with tied boundaries remain fixed, meaning there are no birth or death moves; they only move when there is a boundary update. Consequently, the endpoints do not have a uniform prior. To address this, we initially run the model with a constant likelihood, where the prior for the first and last points is exponentially growing and decaying, respectively. The prior is then fitted to a function, and its inverse is used as a counter-prior. This counter-prior, obtained from the inverse of the prior in the constant likelihood run on the first and last points with proper normalization, mitigates the effect of the non-uniform prior on the location of knot points.

In an ideal scenario, the amplitude would serve as a guide for the model to allocate more points in regions where the signal undergoes rapid evolution. If the amplitude is properly

normalized to the noise level, the probability of placing points is influenced by the amplitude plus a random variable drawn from a uniform distribution $U(0,1)$. Consequently, in regions with higher amplitude, the prior probability of placing points is elevated.

Building on this concept, we introduced a prior based on the second derivative of the amplitude given by

$$\left(\frac{dA_w}{dt}\right)^2 + \left|A_w \frac{d^2 A_w}{dt^2}\right|. \quad (5.11)$$

This addition is motivated by the fact that this prior scales with SNR^2 , representing the curvature of the SNR^2 . While it might seem redundant given the inherent influence of likelihood on knot placement, this prior becomes particularly valuable for low SNR signals. Physically, gravitational wave signals are not random segments of power in time and frequency but rather exhibit clustering. Hence, we incorporate a model for the cluster prior, which is particularly informative for low SNR signals.

Although the likelihood should guide the model to place knots where sharp features are present, the physical model benefits from explicit guidance regarding regions where the signal undergoes rapid variation. To achieve this, we normalize $d^2|A_w^2|$ to unity and create a probability function. This function is then utilized as a prior draw, with a 50% weightage to strike a balance and avoid overly stringent priors that would only place knots where derivatives are high. This nuanced approach aims to enhance the adaptability of the model while respecting physical constraints and data characteristics.

After exploring the introduced modifications, the signal reconstruction closely resembles the top right panel of Figure 5.2. For a binary black hole signal with $M_T = 20M_\odot$ and $\text{SNR} = 20$, the match is 0.92. Furthermore, for an even lower $\text{SNR} = 15$ injected signal with the same remaining parameters, the match is 0.87.

Considering a slightly more massive and higher SNR signal, the **BayesWaveVoices** reconstruction is presented in Figure 5.12, achieving a match of 0.97. Additionally,

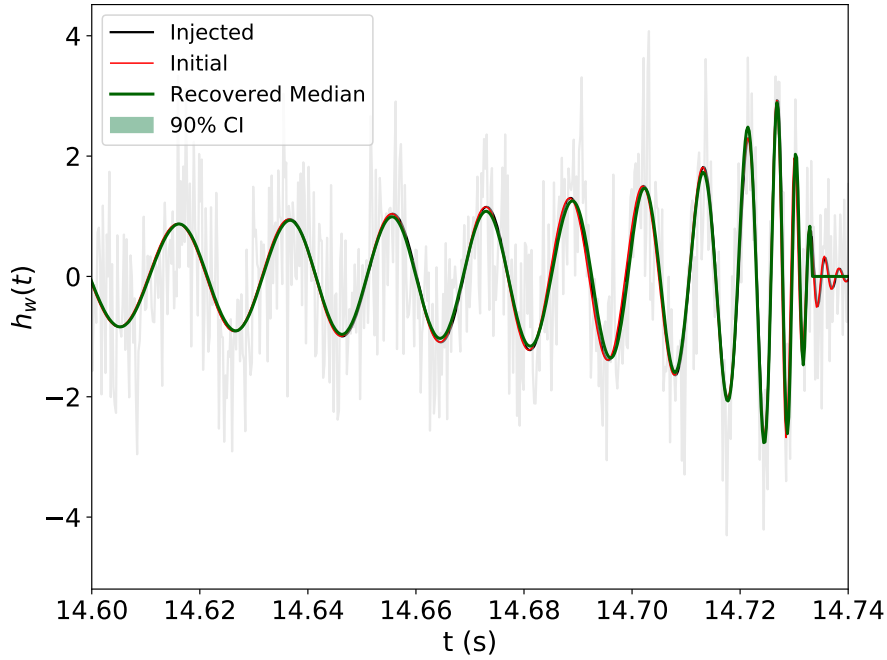


Figure 5.12: For a high mass binary of $M_T = 50M_\odot$ and $\text{SNR} = 30$, the recovered reconstruction (green) of the injected signal (black) with sharp features yield a match of 0.97 using the current `BayesWaveVoices` model. This model does not use the derivative prior on amplitude showing a more realistic case.

Figure 5.3 illustrates a general trend where massive systems tend to exhibit higher matches than lower mass systems. This phenomenon arises from the relatively higher signal amplitude compared to noise in the case of a massive binary. It can be asserted that `BayesWaveVoices` consistently outperforms `BayesWave` for signals of varying masses, demonstrating effectiveness for both weak and strong signals. Nonetheless, there remains untapped potential for further enhancement. This potential could be realized through the implementation of more focused proposals or the incorporation of a well-defined spike model. Such refinements hold promise for enabling the model to effectively capture sharp features, especially in scenarios characterized by low SNR.

Conclusions and Future Direction

We conclude that a localized spline model like Akima splines are suited to avoid oscillatory behaviors of cubic and monotonicity of Steffan splines. Although with the imposed priors and proposals, and added refinements to the model, the current **BayesWaveVoices** model as it stands now is not just an Akima fit but with multiple layers of added constraints.

Our analysis sets the stage for further exploration by incorporating multiple detectors and including information about both cross and plus polarizations. Extending the **BayesWaveVoices** model could provide invaluable insights into the most intriguing potential signals from core collapse supernovae and conducting neutron star post-merger studies. The associated gravitational radiation is poised to provide abundant information about the underlying dynamics and processes driving supernovae. Representing any signal as a product of smoothly evolving amplitude and phase over time, **BayesWaveVoices** offers a promising avenue for studying these signals all the way up to the merger, particularly in neutron star post-merger scenarios.

Furthermore, the algorithm we have created opens new avenues for testing the general theory of relativity. By introducing a correction $\Delta h(t)$ to the waveform model and constraining the parameters of these perturbations using gravitational wave and binary pulsar observations, we can explore and validate the predictions of general relativity in the context of gravitational wave phenomena.

Acknowledgment

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CONCLUSION

Black holes, neutron stars, and rapidly rotating neutron stars (i.e., millisecond pulsars) serve as ideal probes for testing GR. Utilizing gravitational waves from these astrophysical objects and binary pulsar observations, we place constraints on the coupling constants of modified theories of gravity, allowing us to test GR in strong gravity regimes.

In Chapter 2, we delve into the details of constraining the coupling constant α of the dynamical Chern-Simons gravity theory, reflecting the coupling between matter and the scalar field. Provided that moment of inertia and Love numbers can be measured in future observations, we find that the latter could place bounds on dCS gravity that are about six orders of magnitude stronger than previous bounds placed by solar system [37] and table top [284] experiments. This is possible due to the universalities found among three normalized NS observables, the moment of inertia, tidal love numbers and quadrupole moment. Whether these relations are universal depends on what coupling parameter one fixes. The universality holds if we fix $\bar{\xi}$, and the universality breaks down if we fix ζ or ξ_{CS} . These universalities are expected to hold due to the presence of isodensity contours that are elliptical throughout the star. With the dCS correction, we find that eccentricity variations inside a star are even smaller in dCS than in GR. Consequently, the universality between the moment of inertia and quadrupole moment strengthens as we increase $\bar{\xi}$, providing further support for the idea that the approximate self-similarity of isodensity contours is responsible for universality.

We also investigated the observations of binary systems involving at least one millisecond pulsar to place bounds on the parameters of Einstein-æther theory in Chapter 3. With the recent detection of GW170817 event from a binary neutron star, accompanied by the gamma ray burst, GRB170817A [20, 135], the speed of gravitational waves is constrained to one part in 10^{15} [19], which constrains a certain combination of the coupling constant in

Einstein-æther theory, $c_\sigma \ll 10^{-15}$. We recalculated the sensitivities within the viable region of parameter space that persists after the recent measurement of the speed of GWs. Using these pre-existing constraints, Bayesian analysis is applied to binary pulsar and triple system observations. By assessing whether the estimated orbital period decay rate in the presence of added dipolar radiation in the Einstein-æther theory falls within observational uncertainty, we constrain another combination of the coupling constant α_1 to be $\mathcal{O}(10^{-5})$. These constraints are an order of magnitude stronger than the current solar system constraints.

An extension to this work would be to use GW data directly to place constraints on the Einstein-Æther theory. Therefore, the precision of observations derived from LIGO-VIRGO become extremely crucial, serving either to tightly bound the theory or to extract information about the source parameters based on the GWs data. To better understand GW source parameters, we require good signal and noise models. In Chapter 4 we build upon the existing parametric spectral estimation analysis in the model-independent **BayesWave** algorithm to improve noise models. Classical methods such as Welch averaging use additional data outside the analysis window to estimate the power spectrum (PSD), which can cause problems when the noise levels fluctuate over time. We enhanced the parametric analysis using **BayesWave** by introducing a low-latency spectral estimation algorithm. While not as powerful as the complete **BayesWave**, this new algorithm provides rapid parameter estimation, taking only seconds to minutes depending on the signal length, compared to the full **BayesWave**, which requires days for longer duration signals (such as 128 seconds of data). Additional speed ups and improvements detailed in chapter 4 have been made to the PSD estimation in **BayesWave**.

Added to the noise is signal. Once we have good models for noise that can handle the non-stationarities and non-Gaussianities in the data, we propose a parametric signal model called **BayesWaveVoices** in 5. **BayesWaveVoices** algorithm utilizes new basis functions or “voices” to model the amplitude and frequency of the signal, which have a continuous

evolution. This continuity is physically motivated as GW signals are primarily generated by coherent flows of matter and energy inside a star. The current analysis singularly focuses on a single voice corresponding to a single harmonic. In general, there would be multiple voices corresponding to different harmonics. Utilizing Akima splines as the basis functions and incorporating an array of added modifications and features, **BayesWaveVoices** outperforms **BayesWave**, especially for weak and long-duration signals with low signal-to-noise ratios and low masses. These modifications and features, detailed in Chapter 5, are including variable boundaries, taper or spike features, smooth falloff near the ends, and improved priors and proposals for both fixed-dimensional and trans-dimensional moves.

There is potential to further enhance this methodology by incorporating more targeted features into the model. Nevertheless, our methodology can be used as an invaluable flexible technique not only to model binary black holes but also signals of any morphology and length, given the continuity of amplitudes and frequencies.

In this thesis, we presented several new methodologies to utilize gravitational wave and binary pulsar data to elucidate their potential in enhancing our understanding of existing astrophysical objects, detecting unprecedented GW sources, and constraining theories of gravity that describe the space and time in which we live.

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APPENDICES

APPENDIX A

MODIFIED EIH TECHNIQUE

In this Appendix, we start from the 1PN acceleration (3.29) and analyze the effect of the 1PN conservative dynamics on the orbital parameters of a binary of compact objects. We will follow the osculating-orbits technique of [270], which will lead us to amend the calculation of the preferred frame parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ presented in [286]. In doing so, we will also correct a few typos that we found in the expressions of [270].¹

The relative acceleration between the two gravitating bodies is obtained by simply letting

$$\mathbf{a} = \frac{d\mathbf{v}_1}{dt} - \frac{d\mathbf{v}_2}{dt}, \quad (\text{A.1})$$

while the position of the center of mass is not accelerated. Thus, we can set $\dot{\mathbf{X}} = \mathbf{X} = 0$ at Newtonian order without any loss of generality, getting

$$\mathbf{x}_1 = \left(\frac{m_2}{m} + \mathcal{O}(\epsilon) \right) \mathbf{x}, \quad (\text{A.2})$$

$$\mathbf{x}_2 = - \left(\frac{m_1}{m} + \mathcal{O}(\epsilon) \right) \mathbf{x}, \quad (\text{A.3})$$

where $\epsilon \sim m/r \sim v_{21}^2$ is a book-keeping parameter that counts PN order.

Here the acceleration of every individual body is given by (3.29). Hereinafter we will borrow the notation of [270] and thus we define

$$m = m_1 + m_2, \quad \eta = \frac{m_1 m_2}{m^2}, \quad \Delta = \frac{m_2 - m_1}{m}, \quad (\text{A.4})$$

and the functions of the sensitivities

$$\begin{aligned} \mathcal{G} &= \mathcal{G}_{12}, \quad \mathcal{B}_+ = \mathcal{B}_{(12)}, \quad \mathcal{B}_- = \mathcal{B}_{[12]}, \quad \mathcal{D} = \frac{m_2}{m} \mathcal{D}_{122} + \frac{m_1}{m} \mathcal{D}_{211}, \\ \mathcal{E} &= \mathcal{E}_{12}, \quad \mathcal{A}^{(n)} = \left(\frac{m_2}{m} \right)^n \mathcal{A}_1 - \left(-\frac{m_1}{m} \right)^n \mathcal{A}_2. \end{aligned} \quad (\text{A.5})$$

Using this, the relative acceleration can be written in a compact form

$$\mathbf{a} = \mathbf{a}_L + \mathbf{a}_{\text{PF}}, \quad (\text{A.6})$$

where we have separated the purely local contributions and those coming from preferred frame effects. The former reads

$$\mathbf{a}_L = \frac{m}{r^2} \left[\mathbf{n} \left(\hat{A}_1 v_{21}^2 + \hat{A}_2 \dot{r}^2 + \hat{A}_3 \frac{m}{r} \right) + \dot{r} \hat{B} \mathbf{v}_{21} \right], \quad (\text{A.7})$$

where

$$\mathbf{v}_{21} = \dot{\mathbf{x}}_2 - \dot{\mathbf{x}}_1, \quad (\text{A.8})$$

¹We have double checked the correctness of our expressions with the authors of [270].

$$\hat{A}_1 = \frac{1}{2} [\mathcal{G}(1 - 6\eta) - 3\mathcal{B}_+ - 3\Delta\mathcal{B}_- - \eta(\mathcal{C}_{12} + 2\mathcal{E}) + \mathcal{G}\mathcal{A}^{(3)}], \quad (\text{A.9})$$

$$\hat{A}_2 = \frac{3\eta}{2}(\mathcal{G} + \mathcal{E}), \quad (\text{A.10})$$

$$\hat{A}_3 = \mathcal{D} + \mathcal{G} [2\eta\mathcal{G} + 3\mathcal{B}_+ + \eta(\mathcal{C}_{12} + \mathcal{E}) + 3\Delta\mathcal{B}_-], \quad (\text{A.11})$$

$$\hat{B} = \mathcal{G}(1 - 2\eta) + 3\mathcal{B}_+ + 3\Delta\mathcal{B}_- + \eta\mathcal{G} + \mathcal{G}\mathcal{A}^{(3)}. \quad (\text{A.12})$$

These expressions agree with those of [270]. However, we find a difference in the acceleration due to preferred frame effects

$$\begin{aligned} \mathbf{a}_{\text{PF}} = \frac{m}{r^2} \left\{ -\mathbf{n} \left[\left(\frac{\hat{\alpha}_1}{2} + 2\mathcal{G}\mathcal{A}^{(2)} \right) (\boldsymbol{\omega} \cdot \mathbf{v}_{21}) + \frac{3}{2} (\hat{\alpha}_2 + \mathcal{G}\mathcal{A}^{(1)}) (\boldsymbol{\omega} \cdot \mathbf{n})^2 \right] \right. \\ \left. - \boldsymbol{\omega} \left[\frac{\hat{\alpha}_1}{2} (\mathbf{n} \cdot \mathbf{v}_{21}) + \hat{\alpha}_2 (\mathbf{n} \cdot \boldsymbol{\omega}) \right] + \mathcal{G}\mathcal{A}^{(2)} \mathbf{v}_{21} (\mathbf{n} \cdot \boldsymbol{\omega}) \right\} - \frac{m\omega^2}{2r^2} (\mathcal{C}_{12} + \mathcal{G}\mathcal{A}^{(1)}) \mathbf{n}. \end{aligned} \quad (\text{A.13})$$

where we have already specified the generic boost velocity $\boldsymbol{\omega}$ in (3.29) to match the velocity of the preferred frame $\boldsymbol{\omega}$.

This differs from the result of [270] in a sign multiplying the first whole line, as well as in the last term, which is absent in [270]. Here we have defined the following compact-body effective PPN parameters

$$\hat{\alpha}_1 = \Delta(\mathcal{C}_{12} + \mathcal{E}) - 6\mathcal{B}_- - 2\mathcal{G}\mathcal{A}^{(2)}, \quad \hat{\alpha}_2 = \mathcal{E} - \mathcal{G}\mathcal{A}^{(1)}. \quad (\text{A.14})$$

These are the strong field versions of the parameters α_1 and α_2 , which are contained inside the definition of the calligraphic objects in (A.14), so that $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are implicit functions of them. They are directly proportional to each other only in the case in which the sensitivities vanish exactly.

In the absence of PN corrections, the motion of the two-body system describes a Keplerian orbit, parametrized by $\mathbf{x} = r\mathbf{n}$ with

$$\begin{aligned} \mathbf{n} = [-\cos\Omega \cos(\omega + f) - \cos\iota \sin\Omega \sin(\omega + f)]\mathbf{e}_X + [\sin\Omega \cos(\omega + f) \\ + \cos\iota \cos\Omega \sin(\omega + f)]\mathbf{e}_Y + \sin\iota \sin(\omega + f)\mathbf{e}_Z, \end{aligned} \quad (\text{A.15})$$

where the orbital elements are: inclination ι , longitude of the ascending node Ω and pericenter angle ω . The element $f = \omega - \phi$ is the true anomaly, with ϕ the orbital phase measured from the ascending node. The reference vectors $\mathbf{e}_i$ form an orthonormal basis.

When the extra force (A.6) is included, Keplerian orbits are not solutions to the equations of motion anymore. However, provided that the force is small enough relative to the Newtonian force, we can use perturbation theory and translate the dependence on time of the motion to the orbital parameters. This is the method of *osculating orbits* described in [271], which leads to a secular variation of the orbital elements under the effect of $\mathbf{a}$. In order to parametrize this change in terms of the velocity vector of the preferred frame, we

decompose the latter by projecting it onto the orbital plane by defining

$$\omega_P = \boldsymbol{\omega} \cdot \mathbf{e}_P, \quad \omega_Q = \boldsymbol{\omega} \cdot \mathbf{e}_Q, \quad \omega_Z = \boldsymbol{\omega} \cdot \mathbf{z}, \quad (\text{A.16})$$

as well as onto the angular momentum vector

$$\omega_h = \boldsymbol{\omega} \cdot \mathbf{h} = \omega_Z \sqrt{\mathcal{G}mp}, \quad (\text{A.17})$$

Following the computation in [271], we thus find that the local terms in $\mathbf{a}_L$ induce a change only on the pericenter angle ω , which in an orbit changes by

$$\Delta_L \omega = \frac{6\pi m}{\mathcal{G}p} \left[\mathcal{G}\mathcal{B}_+ + \frac{1}{6} (\mathcal{G}^2 - \mathcal{D}) + \frac{1}{6} \mathcal{G} (6\Delta\mathcal{B}_- + \eta(2\mathcal{C}_{12} + \mathcal{E}) + \mathcal{G}\mathcal{A}^{(3)}) \right], \quad (\text{A.18})$$

and is of course independent of $\boldsymbol{\omega}$. Again, this agrees with [270] up to a typographical error in their result. The rest of secular changes vanish, either because they are identically zero or because they compensate along the orbit.

On the other hand, the force induced by preferred frame effects produces a secular change in all orbital parameters

$$\Delta_{\text{PF}a} = \frac{2\pi e\omega_P}{(1-e^2)^2} \left(\frac{mp}{\mathcal{G}} \right)^{\frac{1}{2}} (\hat{\alpha}_1 + 4\mathcal{A}^{(2)}\mathcal{G}), \quad (\text{A.19})$$

$$\Delta_{\text{PF}\iota} = \pi\hat{\alpha}_1 \left(\frac{m}{\mathcal{G}p} \right)^{\frac{1}{2}} \omega_h \sin(\omega) e F(e) - \frac{2\pi\hat{\alpha}_2 \omega_h \omega_R F(e)}{\mathcal{G}\sqrt{1-e^2}}, \quad (\text{A.20})$$

$$\Delta_{\text{PF}\Omega} = -\pi\hat{\alpha}_1 \left(\frac{m}{\mathcal{G}p} \right)^{\frac{1}{2}} \frac{\omega_h}{\sin(\iota)} \cos(\omega) e F(e) - \frac{2\pi\hat{\alpha}_2 \omega_h \omega_S F(e)}{\mathcal{G} \sin(\iota) \sqrt{1-e^2}}, \quad (\text{A.21})$$

$$\begin{aligned} \Delta_{\text{PF}\varpi} = & -\pi\hat{\alpha}_1 \left(\frac{m}{\mathcal{G}p} \right)^{\frac{1}{2}} \omega_Q \frac{\sqrt{1-e^2} F(e)}{e} - \pi\hat{\alpha}_2 (\omega_P^2 \omega_Q^2) F(e)^2 \\ & + \frac{\pi\omega_Q}{e} \left(\frac{m}{\mathcal{G}p} \right)^{\frac{1}{2}} (\hat{\alpha}_1 + 4\mathcal{A}^{(2)}\mathcal{G}), \end{aligned} \quad (\text{A.22})$$

$$\begin{aligned} \Delta_{\text{PF}e} = & -\pi\hat{\alpha}_1 \left(\frac{m}{\mathcal{G}p} \right)^{\frac{1}{2}} \omega_P (1-e^2) F(e) + 2\pi\hat{\alpha}_2 \omega_P \omega_Q e \sqrt{1-e^2} F(e)^2 \\ & + \pi\omega_P \left(\frac{m}{\mathcal{G}p} \right)^{\frac{1}{2}} (\hat{\alpha}_1 + 4\mathcal{A}^{(2)}\mathcal{G}), \end{aligned} \quad (\text{A.23})$$

where $\Delta_{\text{PF}\varpi} = \Delta_{\text{PF}\omega} + \cos(\iota)\Delta_{\text{PF}\Omega}$ and

$$F(e) = \frac{1}{1 + \sqrt{1-e^2}}, \quad (\text{A.24})$$

$$\omega_R = \omega_P \cos \omega - \omega_Q \sqrt{1-e^2} \sin \omega, \quad (\text{A.25})$$

$$\omega_S = \omega_P \sin \omega + \omega_Q \sqrt{1 - e^2} \cos \omega. \quad (\text{A.26})$$

Out of these deviations, the most relevant one is the variation of the semimajor axis, which can be related to the change in the period of the orbit by using Kepler's third law

$$\frac{\Delta T}{T} = \frac{3}{2} \frac{\Delta a}{a}. \quad (\text{A.27})$$

Note however that this change is sub-leading with respect to the change expected from emission of gravitational radiation in a binary system like the one considered throughout this paper [c.f. Eq. (3.30)], which is actually the dominant factor.

APPENDIX B

P-VALUES FOR POINT ESTIMATES

Ideally, PSD estimation and glitch fitting should be performed jointly with the parameter estimation for gravitational wave signals. This is what is done in the **BayesWave** analyses for compact binary coalescences (CBCs) [79, 150]. However, other parameter estimation pipelines such as LALInference and Bilby do not yet have the facility to marginalize over the PSD estimates, instead relying on point estimates. Thus the Bayesian PSD estimation codes needs to supply some kind of point estimate from the full posterior distribution of PSD models. Possible choices include maximum likelihood estimates, maximum a posteriori estimates, a randomly chosen fair draw, or means or medians of the PSD estimates. For FastSpec we chose to use the average of the log PSD since it is easy to compute on the fly, and for BayesWave we choose to use the median log PSD since this estimate is more robust against outlier samples. Both the average and median estimates are examples of “super resolution” fits that do a better job of whitening the data than any random draw from posterior. The super resolution behavior is sometimes referred to as “the wisdom of the crowd”. One downside of using super resolution estimates is that they do not produce uniform p-value distributions, instead they produce an over abundance of good fits, which skews the distributions towards higher p-values.

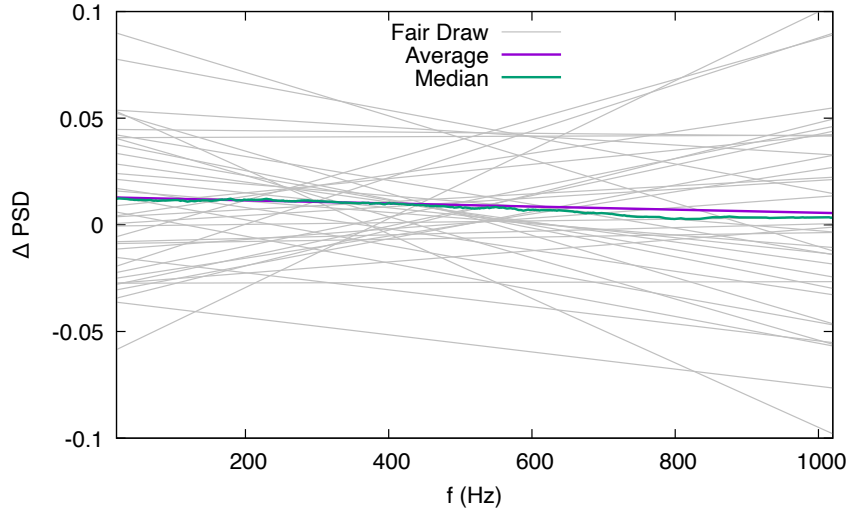


Figure B.1: Posterior draws with the mean and median post estimates for a toy model with a linearly increasing power spectrum. The true PSD used to generate the data has been subtracted to show the differences more clearly.

To illustrate this behavior using a simple model, we considered a toy version of Bayesian spectral estimation using simulated data with a linearly increasing power spectrum, and fixed dimension MCMC analysis with a linear model for the spectrum controlled by the two fixed points at the edges of the frequency range. Figure B.1 shows a collection of fair draws from the posterior samples, along with the mean and median of the posterior samples. The PSD model used to generate the data has been subtracted from these estimates to show the

differences more clearly. Notice that both the mean and median track the simulated model far more accurately than a typical draw from the posterior.

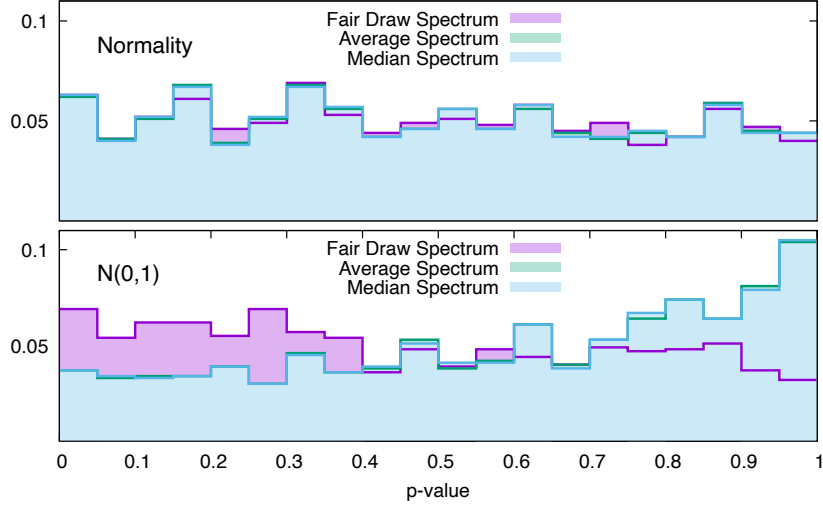


Figure B.2: Comparing the p-value distributions for the whitened toy model data using fair draw, mean and median PSD estimates.

Repeating the analysis on hundreds of simulated data sets and using the Anderson-Darling normality to produce p-values results on the distributions shown in Fig. B.2. When a fair draw from each posterior is used to whiten the data the resulting p-value distribution is uniform, while when the the mean and median of the posterior draws are used the p-value distribution is skewed towards larger values.

The over performance of the super resolution fits using the mean and median also yield whitened data with a very narrow range of variances. Fig. B.3 shows the variance of the data whitened using fair draws and the mean and median. The plot also shows the expected distribution of the variance for a reference $N(0,1)$ distribution. The data whitened using fair draws from the PSD posterior follows the expected distribution, while both the mean and median produce narrower variance distributions.

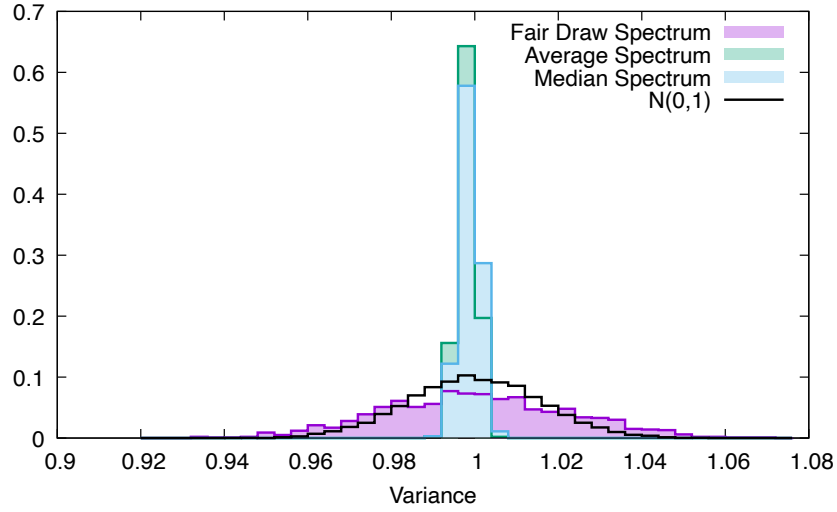


Figure B.3: Histograms of the variances of the whitened toy model data using fair draw, mean and median PSD estimates. The reference distribution for the same number of samples drawn from $\mathcal{N}(0, 1)$ Gaussian noise is also shown. The fair draw PSDs yield distributions in line with the expected distribution, while the mean and median point estimates are narrowly peaked around unit variance.