EMPIERICAL VELOCITY PREDICTIONS

AT CULVERT INLETS

by

Jesse Earl Patton

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of the requirements for the degree

of

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in

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ABSTRACT

The velocity distribution at the entrance cross section of a culvert is typically diverse, reflecting the nuances of the bed material, debris and other hydraulic factors just upstream of the culvert. These diverse inlet velocity fields have been observed to perpetuate some distance into the culvert, impacting the ability of fish to travel upstream in the culvert barrel. It is important to be able to quantitatively describe the inlet velocity field, especially as this serves as a necessary boundary condition for three-dimensional modeling of fluid flow in culverts. While there are various theory-based models of velocity distributions in open channels, velocity distributions at culvert inlets tend to be chaotic and are not well represented by analytic methods. **The goal of this project was to use field data collected at existing culverts to estimate the density at which velocity observations should be collected to adequately describe the nature of the velocity at the culvert inlet.** Two methods of data analysis were utilized to determine the required density of velocity observations. The first approach randomly selected velocities to be used as predictors and did not stress the location of the predictors, but instead emphasized the number of velocity observations needed to describe the nature of the velocity at the culvert inlet. The second method employed the idea that the location of the predictors was more important than quantity of predictors used. Results indicate that the pattern of velocity measurements is important - that is, velocities should not be measured at randomly selected positions in the cross section, but should follow a geometric pattern where the measurement density increases in zones having larger velocities. Also, it appears that if one follows the rigorous implementation of the USGS method for measuring stream flow (often referred to as the velocity-area method in texts), velocity predictions can be extrapolated using the inverse-distance-squared technique to adequately describe the inlet velocity field. The implication of this research is that there are steps that can be followed to adequately describe the nature of the velocity at culvert inlet even through the velocity distributions are chaotic in these regions.

INTRODUCTION

There were two primary incentives for conducting this study of velocity fields at culvert inlets. First, it has been observed that hydraulic features just upstream of culvert inlets can cause very diverse velocity fields at the inlet, and this diversity perpetuates into the culvert barrel (Day 1997). This influence is thought to dramatically affect the fish passage potential of the culvert. Secondly, contemporary computational models that predict the three-dimensional velocity distribution throughout a culvert require an upstream boundary condition. The upstream boundary condition is of utmost importance in the performance of the model, as culverts are typically not long enough to assume fully developed flow. That is, the *answer* that we seek from these computational models throughout most of the culvert depends heavily on an accurate upstream boundary description.

Once it is decided that adequately describing the velocity field at the inlet to a culvert is important, a reasonable next step is to determine how many velocity measurements must be made at the cross section to adequately describe the field as a whole. Probably having only one or two velocity measurements would be inadequate. Perhaps measuring the velocity at every possible location would be overkill. It is possible that velocities could be observed at some intermediate number of positions, with the remainder of the field extrapolated from these measured values. The goal of this project is to use field data collected at existing culverts to estimate the density at which velocity observations should be collected to adequately describe the nature of the velocity

at the culvert inlet**.** The hypothesis is that water velocity need not be measured at all possible locations in a cross section because an extrapolation scheme based on a predictable, but limited, number of velocity measurements will suffice.

Literature Review

Estimating the velocity distribution at a stream cross section (the culvert inlet is simply a stream cross section near a man-made structure) has been the subject of many investigations, but often the discussion centers on theory-based models or measurements taken in very controlled laboratory settings. Mathematical or theory-based models for the velocity distribution in an open channel are typically either one-dimensional models used to predict velocity as a function of flow depth, or two-dimensional models used to predict the velocity throughout a cross section (Barber 1996). One-dimensional models are frequent in the literature, but are not applicable to the setting of interest here. Two examples of one-dimensional models for estimating the velocity distribution in turbulent conditions were presented by Roberson and Crowe (1990). The first example, typical of a host of logarithmic models, is:

$$\frac{u}{u^*} = 5.75\log\left(\frac{yu^*}{\nu}\right) + 5.56 \tag{1}$$

where u is the velocity (L/T), u^* is the shear velocity (L/T), y is the depth (L), and ν is the kinematic viscosity (L^2/T). The second example is typical of power-law models of velocity distribution, and is:

$$\frac{u}{u_0} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \tag{2}$$

where u_o is the mean velocity (L/T) and δ is the boundary layer thickness (L). Both of these methods require knowledge of the boundary layer thickness in order for them to be applied (Barber 1996).

As an example of two-dimensional approaches, a recent attempt to describe velocities in open channels in two-dimensions (Chiu 1989) was based on concepts from probability and entropy. Chiu presented three models for predicting the velocity distribution. The most simplistic of the three models only requires that the mean and maximum velocities be known. The intermediate model assumes that the maximum velocity occurs on the water surface. Classical laboratory studies have shown that the maximum velocity occurs just below the water surface making both the simple and intermediate Chiu models easy to scrutinize. Chiu's most complex model does position the maximum velocity just below the water surface, but has the complexity that the second and third moments of the (time-averaged) point velocity in the longitudinal direction as well as the (cross sectional) mean and maximum values of the point velocity must be known. This approach, while more theoretically complete, can become cumbersome and may require more work than actually collecting the velocities manually. Also, personnel associated with this project have not found high coincidence between the results of the complete Chiu model and field observations in anecdotal comparisons.

An example of a different theory-based approach to estimating two-dimensional velocity fields is a model based on the Navier-Stokes equations (Goring, 1997). Goring used a finite element model that is deterministic, not empirical, and the results are two-dimensional. The major drawback to the application of Goring's model at the setting of the culvert inlet is the appropriateness of the three assumptions made about the flow in

the cross section. These assumptions are that steady-uniform flow exists, there are no velocity components in the plane of the cross section, and the water level is horizontal across the section. All three of these assumptions are often violated at a culvert inlet.

There does not seem to be much literature devoted to studies of the velocity distribution specifically at the entrance to a culvert in field settings. Barber (1996) related culvert hydraulics to juvenile fish passage, and discussed the benefits and drawbacks of the use of theory-based models of velocity distribution such as those mentioned above. Barber found that velocity distributions measured in experimental culverts were often nonsymmetrical about the centerline, a feature that even two-dimensional theory-based models fail to adequately describe.

A recent study focused on culvert design with fish passage in mind, where velocity diversity was indirectly included (House, 2005). The model House proposed considered the percentage of a cross section having velocities that a juvenile fish might be able to swim against. Using regression techniques along with physical and hydraulic parameters the model was tested directly against velocity distributions estimated from field measurements. The tests produced favorable results, but House (2005) found that the model was conservative because it underestimated the percent of the channel that exhibit velocities that a juvenile fish could swim against, particularly at low flow depths. Even though the model has the ability to predict the percent of a cross section possessing velocities within juvenile fish swimming capabilities, the discrepancy between the predicted and actual velocities show that it does not provide the capability to accurately describe a velocity distribution. Furthermore, this experiment included only culverts with continuous gravel beds, and may not be applicable to other types of culverts.

Background

Open channel flow can be classified temporally and spatially. The temporal classifications are steady and unsteady flow. Steady flow occurs when the flow rate, velocity and depth do not change with time. In unsteady flow, the flow rate, velocity and depth do change with time. On the other hand, spatially varied flow describes how the flow changes with location. Uniform flow occurs when the velocity and depth do not change as a function of location, a condition that seldom occurs in nature. Non-uniform flow in an open channel occurs when the velocity and depth change with position along the channel length. The culverts studied in this project have unsteady, non-uniform flow. The non-uniformity of the flow is not an issue, as this project uses only velocities at a cross section, not over a stream reach. The unsteady nature of the flow at the field sites, however, must be addressed.

The flow rate at the field sites is subject to typical snow-melt hydrology (flow is reduced in the cold of the night and increased in the heat of the day with a time lag) as well as the influence of precipitation events in the upstream basin. To overcome the temporal influence on the velocity fields at the culvert entrance, each independent data set was recorded over a relatively short period of time (a few hours). As such, the observed velocity fields can be thought of as snapshots in time, the equivalent of longer duration observations under steady flow. Also, perturbations (turbulence) were averaged over time regardless of the instrument used to measure velocity.

One of the methods used herein to estimate velocity fields at a cross section mimics some components of the protocol used by the USGS (United States Geologic

Survey) to estimate flow rate (the velocity-area method). As such, a brief review of the method (Rantz 1982) is presented here. The velocity is recorded at 20% and 80% of the water depth at a number of vertical transects across the cross section as shown in Figure 1.1. The four velocity values observed on the bounding transects for each polygon are then averaged and multiplied by the trapezoidal area of the polygon to estimate the flow rate in the polygon. These are summed to arrive at the total flow.

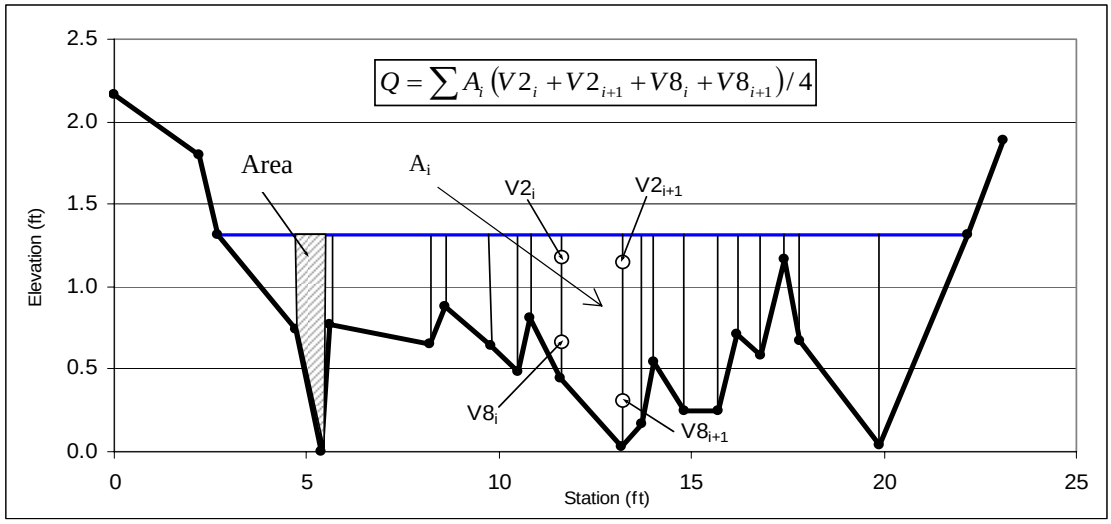


Figure 1.1 – An example of a typical cross section and the USGS method.

The USGS method recommends that for coarse estimates of flow, no more than 20% of the total flow should occur in any one polygon, and for fine estimates no more than 10% of the flow should occur in any one polygon. Thus, the method can result in a trial-and-error scheme during field observations. If any one polygon is found to have more than its share of flow, it is split in half by measuring the velocities on a transect at the horizontal center of the offending polygon. This continues until the flow criteria is met for either the coarse or fine result as desired. The end effect is that zones having higher velocities tend to be sliced into smaller areas (more dense velocity observations).

Because this method has an established protocol and is easily communicated by hydraulic engineers, it was chosen as a possible technique for measuring velocity distributions in this project (it is important to remember that the USGS method has been validated in the field for measuring *flow rates*, but not for adequately describing complete *velocity fields*).

The objective of this project was to use field data collected at existing culverts to estimate the density at which velocity observations should be collected to adequately describe the nature of the velocity at the culvert inlet. While there are various theory-based models of velocity distributions in open channels, velocity distributions at culvert inlets tend to be chaotic and are not well represented by analytic methods. Thus, two methods of data analysis were utilized to determine the required density of velocity observations. The first approach randomly selected velocities to be used as predictors and did not stress the location of the predictors, but instead emphasized the amount of velocity observations needed to describe the nature of the velocity at the culvert inlet. On the other hand, the second method employed the idea that the location of the predictors was more important than the quantity of predictors. It is important to be able to quantitatively describe the inlet velocity field, especially as this serves as a necessary boundary condition for three-dimensional modeling of fluid flow in culverts.

FIELD SITE AND OBSERVATIONS

Field Site

Water velocities were measured at the inlet face of three different culverts, each at two different flow rates for a total of six data sets. The three culverts used in this study are on Mulherin Creek (Figure 2.1), located north of Gardiner, Montana. Mulherin Creek has a fairly high gradient (between 2 and 6%) throughout the study reach. The stream contains large substrate, primarily cobble and boulder, and is a tributary to the Yellowstone River. The stream has a base flow of approximately 15 cfs with a peak measured flow of 96.6 cfs measured in June 1983 (USGS, 1986).

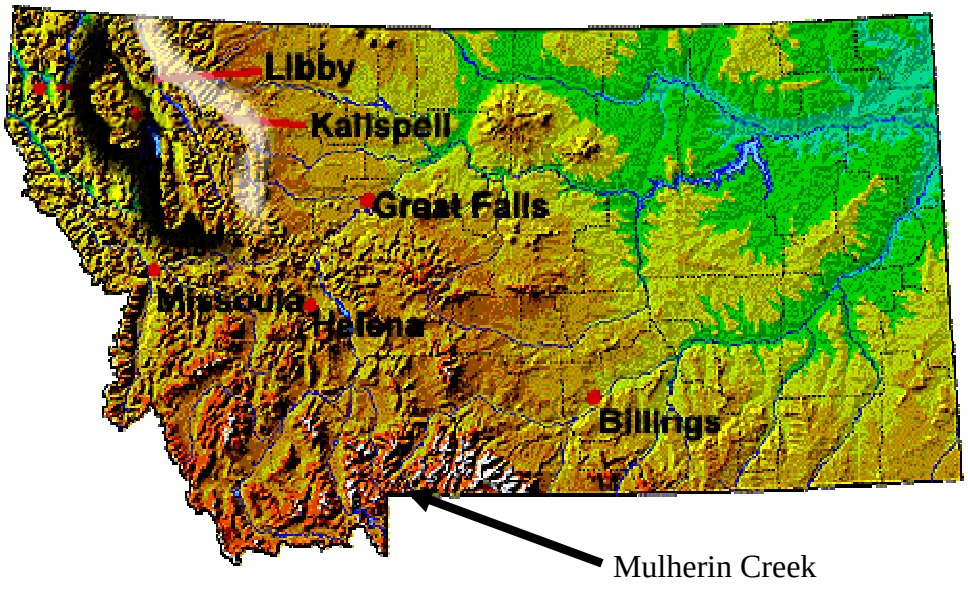


Figure 2.1 – The location of Mulherin creek by Gardiner, Montana.

Mulherin Creek has five main culverts in the six miles of stream nearest the confluence with the Yellowstone River. Figure 2.2 shows the location of the three

culverts used in this study. The first culvert studied (hereafter named Culvert 1) is the most downstream of the three and is a concrete box culvert with chamfered corners. Culvert 1 is shown in Figure 2.3, and spans 12 feet, is 6 feet tall, and is 36.7 feet long. The second culvert in the study (hereafter named Culvert 2) is also a concrete box culvert with chamfered corners. Culvert 2 (Figure 2.4) is 12 feet tall, 6 feet wide, and 30.5 feet long. This culvert has five concrete baffles perpendicular to flow along the culvert floor designed to slow the velocity by increasing the flow depth and thus the cross sectional flow area. The baffles are 7 feet long, 0.8 feet wide and 0.8 feet tall. The baffles alternately originate on each side of the culvert and are approximately 5 to 5.5 feet apart. The chamfered corners on both Culvert 1 and Culvert 2 are at 45° angles. The final culvert in the study (hereafter named Culvert 4) is a steel cylinder with a diameter of 7.5 feet. Culvert 4 (Figure 2.5) is located above the confluence of Mulherin and Cinnabar creeks and tends to run at approximately two-thirds of the main stream flow.

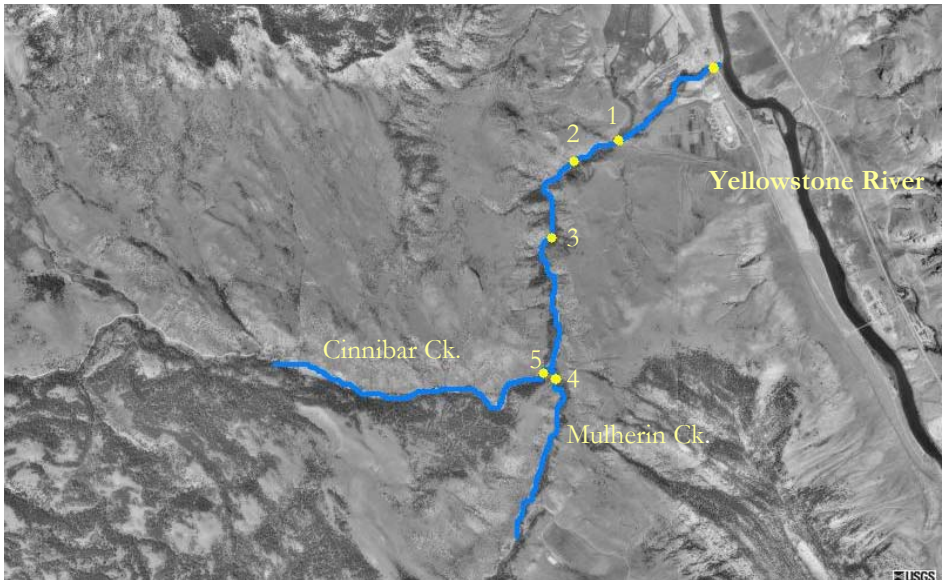


Figure 2.2 - Aerial photograph of the Mulherin Creek area showing the location of Culverts 1, 2 and 4.



Figure 2.3 – The entrance to Culvert 1 looking upstream.



Figure 2.4 – The entrance to Culvert 2 looking upstream.



Figure 2.5 – The entrance to Culvert 4 looking downstream.

Equipment and Protocol Used to Observe Velocities

Because the culvert geometry and configuration was different at Culvert 4 from that at Culverts 1 and 2, a different velocimeter and protocol was used there. The equipment and methods at Culverts 1 and 2 will be discussed first, and then those used at Culvert 4 will be covered.

Equipment Used at Culverts 1 and 2

A SonTek ADVField 10-Mhz Acoustic Doppler Velocimeter (ADV) model number SP-AV10M01 (Figure 2.6) was used to collect velocity data at Culvert 1 and Culvert 2. The acoustic sensor consists of three receivers and one transmitter that allow three dimensional velocity readings to be recorded on an attached laptop computer. The

velocities measured and recorded are the vector components of the velocity in the cardinal directions, “x” (parallel to the culvert floor and walls), “y” parallel to the culvert floor but perpendicular to the walls, and “z” vertically. The vector components could be resolved geometrically to arrive at the resultant velocity, but for the analysis here only the x component was considered. The velocimeter was set to sample at 50 Hz (50 velocities per second on each axis) and internal algorithms supplied by the manufacturer were used to partition “noise” from turbulence. The algorithm rejects velocity observations if the average correlation is 30% or less or if the average signal-to-noise ratio is 15% or less. The velocities observed over the one-minute sample were averaged in time to eliminate the turbulent component. This protocol resulted in anywhere from 1 to 1400 velocities retained in the one-minute sample. Figures 2.3 and 2.4 show how the velocimeter was suspended above the water surface in the culvert on a trolley that was used to accurately position the velocimeter in the y-z plane at many locations in the culvert cross section. The figures also show the laptop computer that served as the data logger for the trials.



Figure 2.6 – The acoustic Doppler velocimeter (ADV) used to collect velocity measurements at Culvert 1 and Culvert 2.

The ADV will accommodate independent velocity observations representing very small areas as shown in Figure 2.7. However, the flow in these streams is unsteady, so only a certain number of velocities could be observed without the flow changing

appreciably. In order to sample the cross section in a relatively short time period, velocity measurements were obtained every 6 inches horizontally and no closer than 2.4 inches vertically.

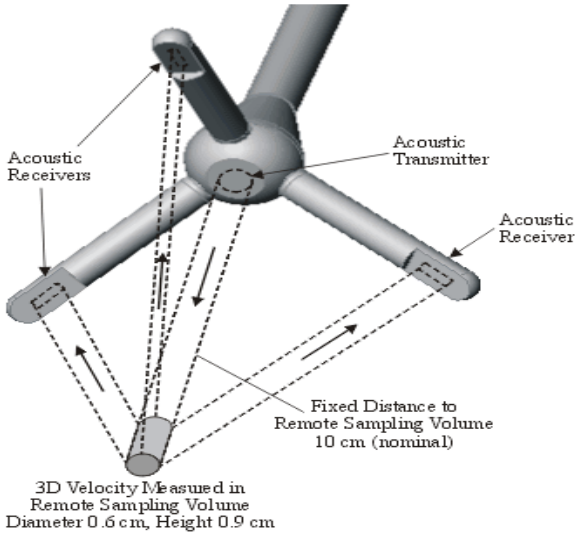


Figure 2.7 – A diagram showing the location of the volume sampled by the ADV.

When using the ADV, the first velocity at each station (station refers to the horizontal position in the cross section) was recorded near the bottom of the culvert. The last velocity at the top of each station has a larger area because each of the three prongs of the ADV must remain under the water to achieve an accurate reading. Figure 2.8 shows the proper location of the ADV in the first station, but an improper location of the ADV in the second station.

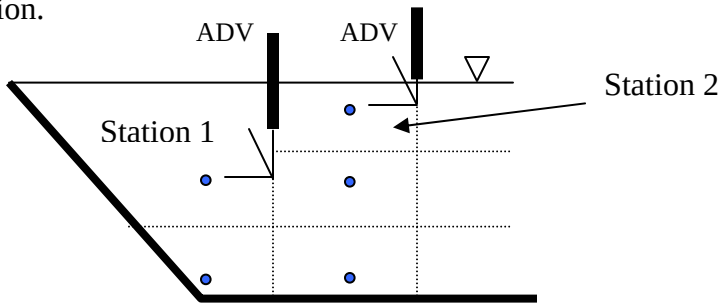


Figure 2.8 – The proper (station 1) and improper (station 2) location of the ADV arm.

In all three culverts, the horizontal and vertical distance from one velocity reading to the next was divided in half to construct the polygon for which the velocity is representative, as shown in Figure 2.9. That is, regardless of the horizontal or vertical spacing between two velocity measurements, the distance is always halved to arrive at the polygon boundary between the two observations. AutoCAD was used to calculate the area of each polygon for all six cross sections.

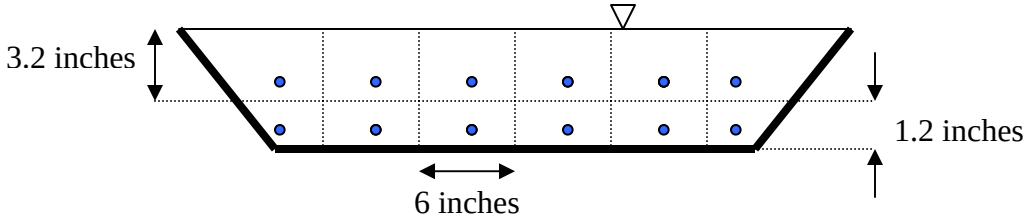


Figure 2.9 – The dots represent the velocity measurement locations and the lines delineate the polygons.

Velocity Measurements at Culvert 1

The first event observed in Culvert 1 had a bulk flow rate of 26.19 cfs, as determined by velocity observations at 51 points. Figure 2.10 shows Culvert 1 as separated into polygons, where each polygon had a velocity observation recorded in it.

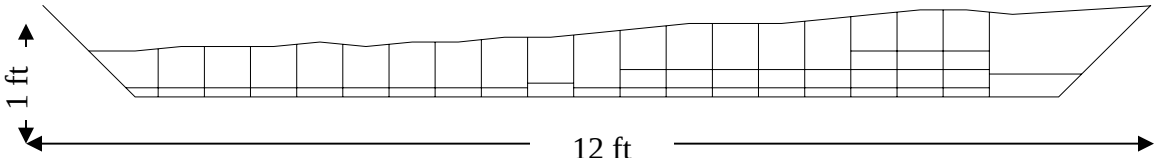


Figure 2.10 – Culvert 1 at a flow rate of 26.19 cfs.

The second event observed in Culvert 1 had a flow rate of 18.91 cfs as determined by velocity observations at 46 points as shown in Figure 2.11.

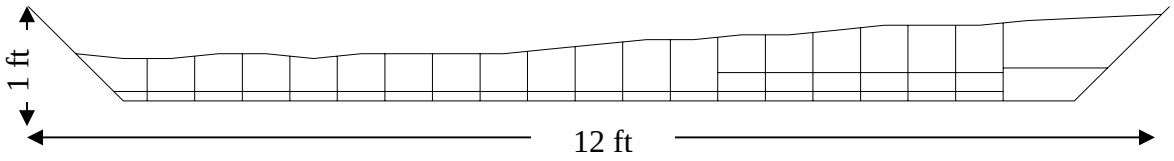


Figure 2.11 – Culvert 1 at a flow rate of 18.91 cfs.

Velocity Measurements at Culvert 2

The first event observed in Culvert 2 had a flow rate of 20.89 cfs when represented by 77 velocity observations as shown in Figure 2.12.

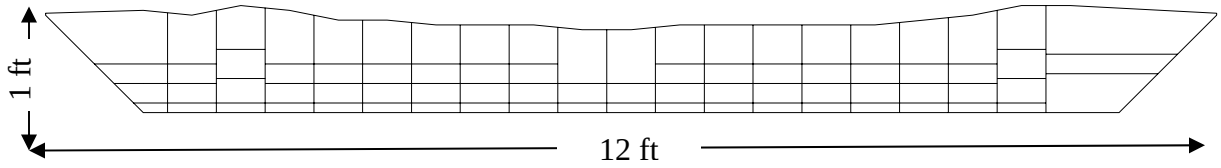


Figure 2.12 – Culvert 2 at a flow rate of 20.89 cfs.

The second event observed in Culvert 2 had a flow rate of 19.54 cfs when represented by 78 velocities as shown in Figure 2.13.

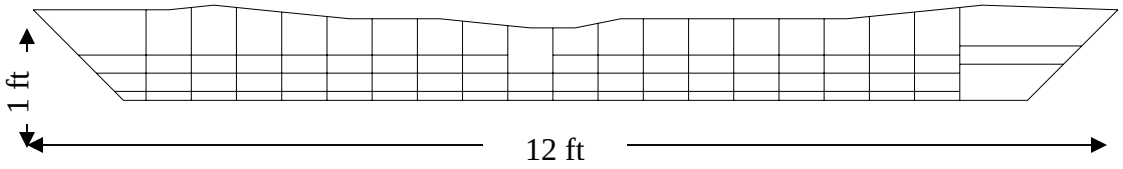


Figure 2.13 – Culvert 2 at a flow rate of 19.54 cfs.

Equipment Used at Culvert 4

Culvert 4 had a shallow water depth and circular geometry that limited the applicability of the ADV. Instead, a pigmy-type Gurley velocimeter (model number

625D) was used to collect velocity data at Culvert 4 only. The sensor unit is shown in Figure 2.14. The sensor attaches to a rod that can be placed at a location in the cross section. This type of velocimeter essentially measures the horizontal resultant of the x and y velocity vectors, as it is omni-directional in that plane. The data logger supplied with the unit time-averages automatically, and is not sensitive enough for turbulence measurements.



Figure 2.14 – The sensor of the Gurley velocimeter used to collect velocities at Culvert 4.

Velocity Measurements at Culvert 4

The first event observed in Culvert 4 had a flow rate of 15.04 cfs as represented by velocities measured at 82 points as shown in Figure 2.15. The Gurley velocimeter has the capability to collect velocities at a faster rate then the ADV because of its general design, which allowed for a larger volume of measurements to be recorded in a shorter amount of time.

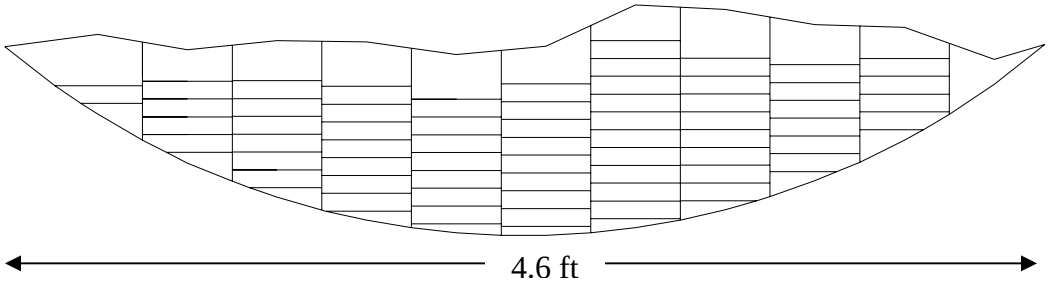


Figure 2.15 – Culvert 4 at a flow rate of 15.04 cfs.

The second event observed in Culvert 4 had a flow rate of 13.72 cfs as represented by velocity measurements at 71 points as shown in Figure 2.16.

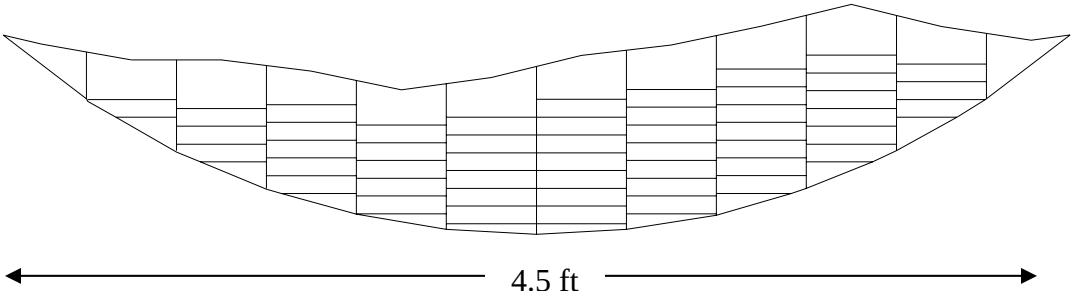


Figure 2.16 – Culvert 4 at a flow rate of 13.71 cfs.

Table 2.1 shows the associated flow rate, the instrument used to collect the velocity readings, and the number of velocities recorded for each field observation in all three culverts.

Table 2.1 - A summary of the field observations.

Field Observation	Culvert	Culvert Characteristics	Flow Rate (cfs)	Instrument	Number of Velocities Recorded
1	1	Chamfered Box	26.19	ADV	51
2	1	Chamfered Box	18.19	ADV	46
3	2	Chamfered Box w/ Baffles	20.89	ADV	77
4	2	Chamfered Box w/ Baffles	19.54	ADV	78
5	4	Circular	15.04	Gurley	82
6	4	Circular	13.72	Gurley	71

DATA ANALYSIS

General Approach

The following steps were undertaken for two different sampling schemes, the random sampling scheme and the USGS sampling scheme. In the most general terms, the data were analyzed by completing the following steps for each trial:

1. One *predictor* velocity for the random sampling scheme or two *predictor* velocities for the USGS sampling scheme were selected from the observed velocities at one culvert during one field observation.
2. The predictor velocities were used to extrapolate/interpolate the velocities at the locations of each of the observed velocities that were not selected as predictors.
3. A statistic was used to indicate the average error – the collective deviation between the velocities predicted in step 2 and those measured in the field observation at the same locations in the cross section.
4. The process was repeated (a new *sample run*) with one more predictor velocity for the random sampling scheme or two more predictor velocities for the USGS sampling scheme. This continued until the observed velocities were exhausted.

So, a *trial* consists of successive sample runs through the velocities measured at a culvert during each field observation using either one of two sampling schemes. This resulted in 72 trials (each of the 6 field operations had 12 trials) for the random sampling scheme and 6 trials (each of the 6 field operations had one trial) for the USGS sampling scheme.

The Extrapolation/Interpolation Model

In each sample run of each trial (step 2 above) the predictor velocities are used to extrapolate/interpolate estimates of the velocities at the points not selected as predictors. The interpolation/extrapolation model chosen for this analysis was the inverse distance squared:

$$V_{pred} = \frac{\sum_{i=1}^{n} \frac{V_i}{d_i^2}}{\sum_{i=1}^{n} \frac{1}{d_i^2}} \quad (3)$$

where V_{pred} is the interpolated/extrapolated velocity (L/T), V_i is an individual predictor velocity (L/T), d_i is the distance from the individual predictor velocity to the point at which the velocity is to be interpolated/extrapolated (L), and n is the number of predictor velocities in a sample run. While the variable V_{pred} is not indexed in equation 3, it is clear that equation 3 is used (m - n) times to predict all velocities not used as predictors in each sample run where m is the total number of observed velocities in the cross section. The nature of the inverse distance squared method is that estimates located in close proximity to predictor velocities are more weighted by those velocities than by predictor velocities farther away.

Maintenance of Total Flow

The total flow through the culvert should remain constant for each sample run. That is, after velocities are interpolated/extrapolated in each sample run, they should be globally adjusted such that if the total flow rate were calculated using the combination of

predictor and globally adjusted predicted velocities, the total flow would be the same as observed in the field observations. After each sample run, the flow in the culvert was calculated using:

$$Q_{pred} = \sum_{j=1}^{n} V_j \Delta A_j \tag{4}$$

where $V_j \Delta A$ is the product of each velocity (predicted or predictor) and corresponding area, with the summation providing the flow rate for the cross section. Each predicted velocity was then adjusted by equation 5 such that the resulting flow was equal to the observed flow, Q_{Total}, for that field trial.

$$V_{pred} \frac{Q_{pred}}{Q_{Total}} = V_{adj} \tag{5}$$

The Error Statistic

After each sample run for each trial, the mean absolute error of the predictions can be calculated using:

$$error = \frac{\sum_{i=1}^{n} abs\left(\frac{V_k - V_{adj}}{V_k}\right)}{n} \tag{6}$$

where V_k equals the value of the velocity measured in the field, V_{adj} equals the value of the adjusted predicted velocity at the same point in the cross section, and n equals the number of velocities being predicted in the cross section i.e., the total number of velocities minus the velocities used as predictors.

Random Sampling Scheme

As mentioned previously, two methods were used to sample predictors for each sample run through a set of field observations. The first sampling method does not have any geometric pattern; the predictors are simply randomly selected. There is no replacement – if a velocity is a predictor in one sample run, it is a predictor in the next sample run with one more predictor selected. The visual basic code to assist in the analysis is given in Appendix B. A progression through the first three sample runs is shown in Figures 3.1 through 3.3. Each location is randomly selected.

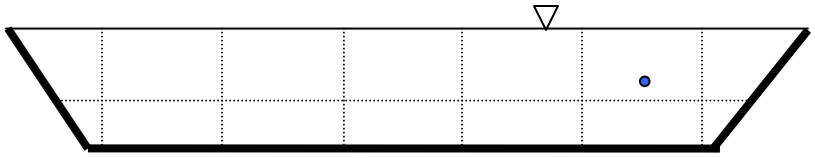


Figure 3.1 – An example of the predictors' location during the first calculation.

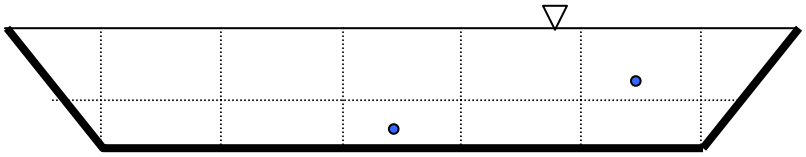


Figure 3.2 – An example of the predictors' location during the second calculation.

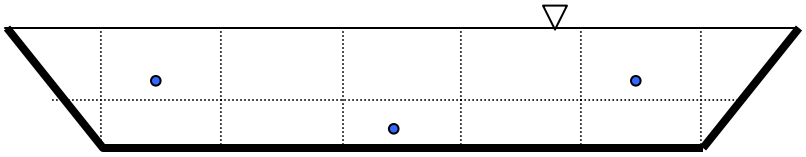


Figure 3.3 – An example of the predictors' location during the third calculation.

USGS Sampling Scheme

The second sampling scheme somewhat mimics the USGS method of measuring velocities to calculate flow. In the first sample run, two velocities are selected to

represent the entire cross section. The two velocities are located approximately at the cross section center (width-wise) and at the 20% and 80% depths. A sample run is completed using these two predictors and the error statistic is calculated. Then, the culvert is divided into two equal areas (polygons) and the process is repeated. This continues, with each sample run having two more predictors then the previous. In each sample run, the flow rate that occurs in the polygon having the largest flow rate, $(Q_{max})_{polygon}$, is noted. Figures 3.4 through 3.6 show the progress through the first three sample runs.

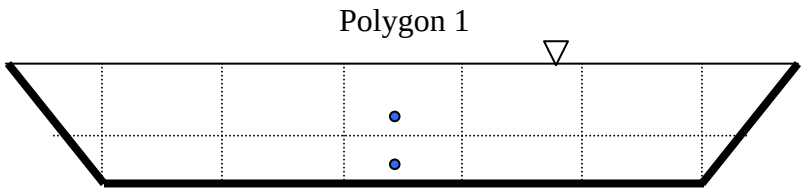


Figure 3.4 – An example of the predictors' location during the first sample run.

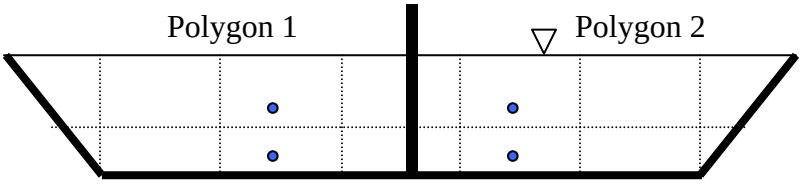


Figure 3.5 – An example of the predictors' location during the second sample run.

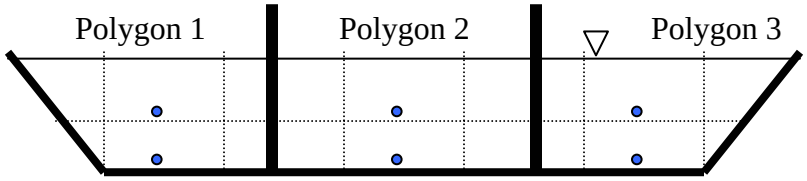


Figure 3.6 – An example of the predictors' location during the third sample run.

RESULTS AND DISCUSSION

Random Sampling Scheme

For each of the 72 trials where the random sampling scheme was followed, the results are examined in two ways. First, the progression of the error statistic with each successive sample run is examined and graphed. Then, contour plots of the sample run having an intermediate error statistic, around 50%, for 6 of the 72 trials are prepared for comparison with the observed velocity contours and the contour plots of the sample run having a low error statistic, around 20%, for the same 6 trials. Each one of the 6 trials chosen to be plotted represented the data from one of the 6 different field observation, remembering that a field observation represents the data set from one of the three culverts at one of the flows observed. The contour plots were prepared, mainly to "ground truth" the results from the numerical analyses.

Error Analysis

For each trial, randomly selected predictor velocities were used to interpolate/extrapolate the remaining velocities in the cross section. Figure 4.1 is a graph of the error statistic (Equation 6) versus the percent of velocities (Equation 7) used to

$$V_{\%} = \frac{V_{\mathrm{Pr}ed}}{V_{Total}} \tag{7}$$

predict the remaining velocities in Culvert 1 at a flow rate of 26.19 cfs. Where $V_{\%}$ is the abscissa, V_{Pred} is the number of predictors and V_{Total} is the total number of velocities in the field observation being analyzed.

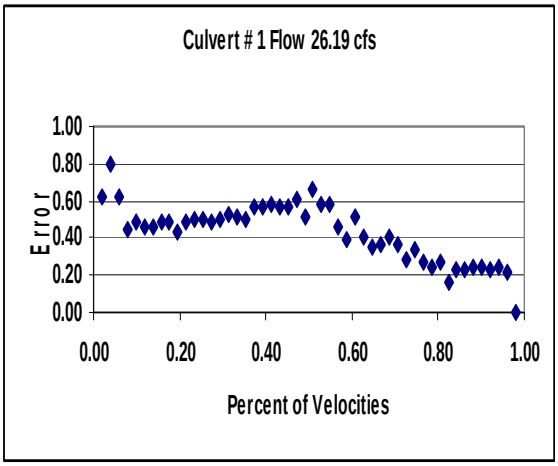


Figure 4.1 – The error analysis for Culvert 1 at 26.19 cfs.

Because a random number generator is used to select the predictors, each culvert and flow condition could have multiple "trials" if the visual basic code is used repetitively. Each time the code is run, different velocities are randomly chosen (without replacement) to be used as predictors. Figure 4.1 represents one set of sample runs, or one "trial". Multiple runs of the code better represent the capability to predict the velocity distribution using random sampling. Figure 4.2 shows the errors for each prediction at each culvert at both flow rates, with 12 trials at each culvert/flow condition (field observation).

It is shown in Figure 4.2 that the results are highly dependent on which velocities are used as predictors. Some of the trials produced results that had a distinct pattern, when more predictors were used the error statistic lessened, but the majority of the trials produced results that did not possess a pattern. The source of the scatter in these results may be caused by utilizing a majority of predictors that are non-representative of the actual velocity distribution during a trial.

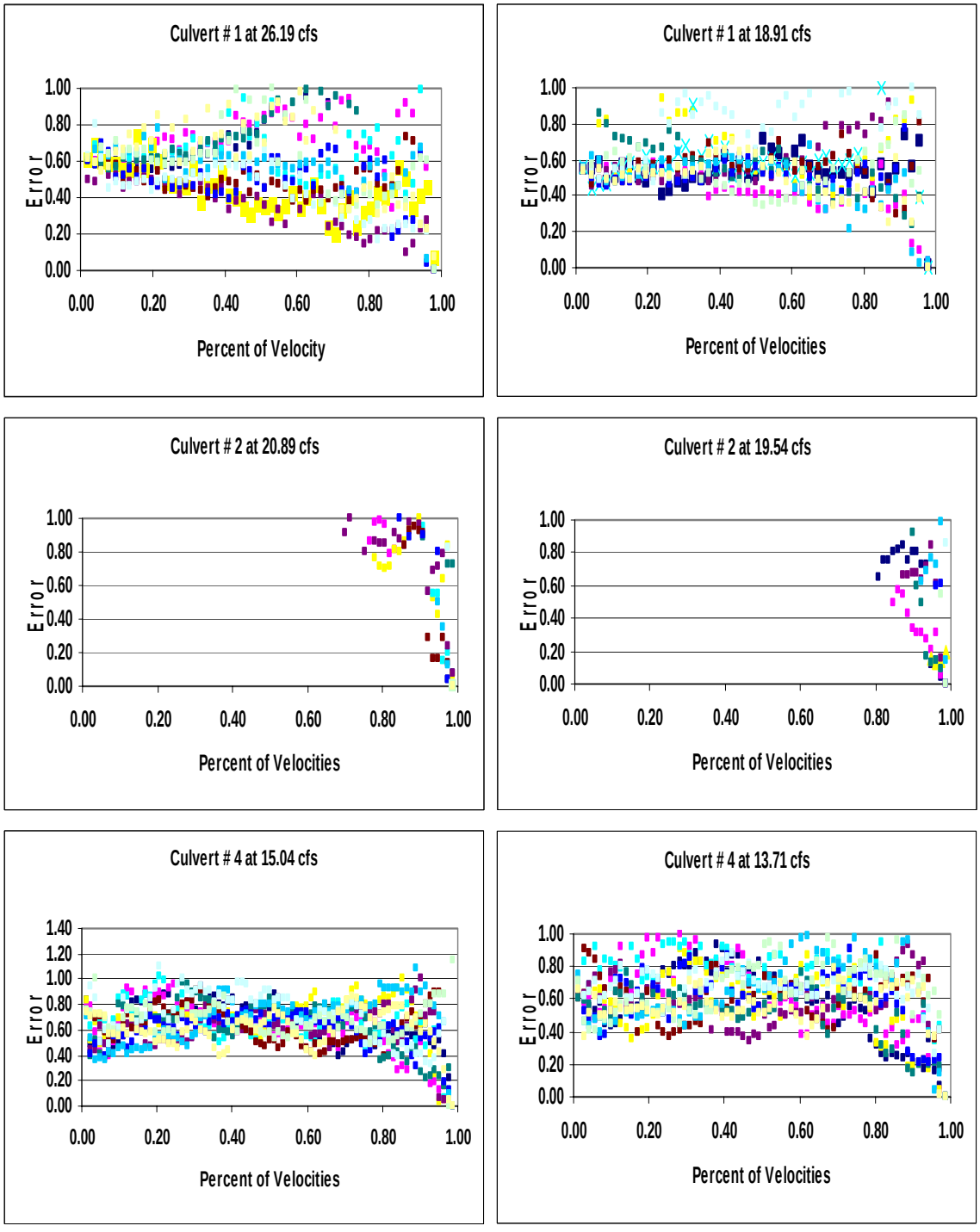


Figure 4.2 – The error statistic for dithering sample intensities using random sampling.

The scatter in the trends shown in Figure 4.2 demonstrate that randomly sampling locations at which to measure velocity in a two dimensional field is not the best choice, and in many cases is very inadequate. An analogy occurs in land surveying. To adequately describe the topography of a piece of land, surveyors visually select *break points* – locations that, when connected linearly, describe the undulation in the terrain. If stream modelers could see velocity breakpoints in a cross section, this same approach could be adopted. Velocity break points are not something that can be seen; however, it should be intuitive to land surveyors and velocity modelers alike that randomly choosing locations to measure velocities is likely to result in poor predictions of the cross sectional field as a whole.

Contour Plots

To help provide visual corroboration of the numerical analysis, contour plots of velocity in the cross section were prepared for several sample runs of each trial. These plots are archived in Appendix C, and an example sequence is shown in Figure 4.3. The graphs of Figure 4.3 are for Culvert 1 (trial 1), and show the velocity distribution based on observations alone and at two prediction intensities. It should be pointed out that some of the predicted values are larger than any of the recorded velocities because of procedure where the observed flow rate is maintained in all cases.

All of the trials resulted in the pattern of Figure 4.3 – as more predictors are included, the contour plot of velocities becomes more visually similar to the observed case.



Figure 4.3 – The velocity distributions in feet per second for Culvert 1 at 26.19 cfs using the random sampling scheme.

USGS Sampling Scheme

The procedure for examining the results for the USGS sampling scheme is similar to that of the random sampling scheme. For each of the 6 trials where the USGS sampling scheme was followed, the results are examined in two ways. First, the progression of the error statistic with each successive sample run is examined and graphed. Then, like in the random sampling scheme contour plots of sample runs having an intermediate and low error statistic were prepared, for comparison with the observed velocity contour plot.

Error Analysis

The mean absolute error (Equation 6) for each sample run of each trial is shown in Figure 4.4. The abscissa, Q_r, is the ratio of the flow in the polygon having the greatest flow to the total flow.

$$Q_r = \frac{\left(Q_{\max}\right)_{polygon}}{Q_{Total}} \tag{8}$$

This is to remain consistent with the USGS method. Recall that for coarse flow rate measurements the USGS recommends a Q_r of no more than 20% (0.2) and for fine flow measurements the maximum Q_r is 10% (0.1). Only sample runs having an error statistic less than 100% are shown in Figure 4.4 – sample runs where the interpolated velocities have large error are of little interest here.

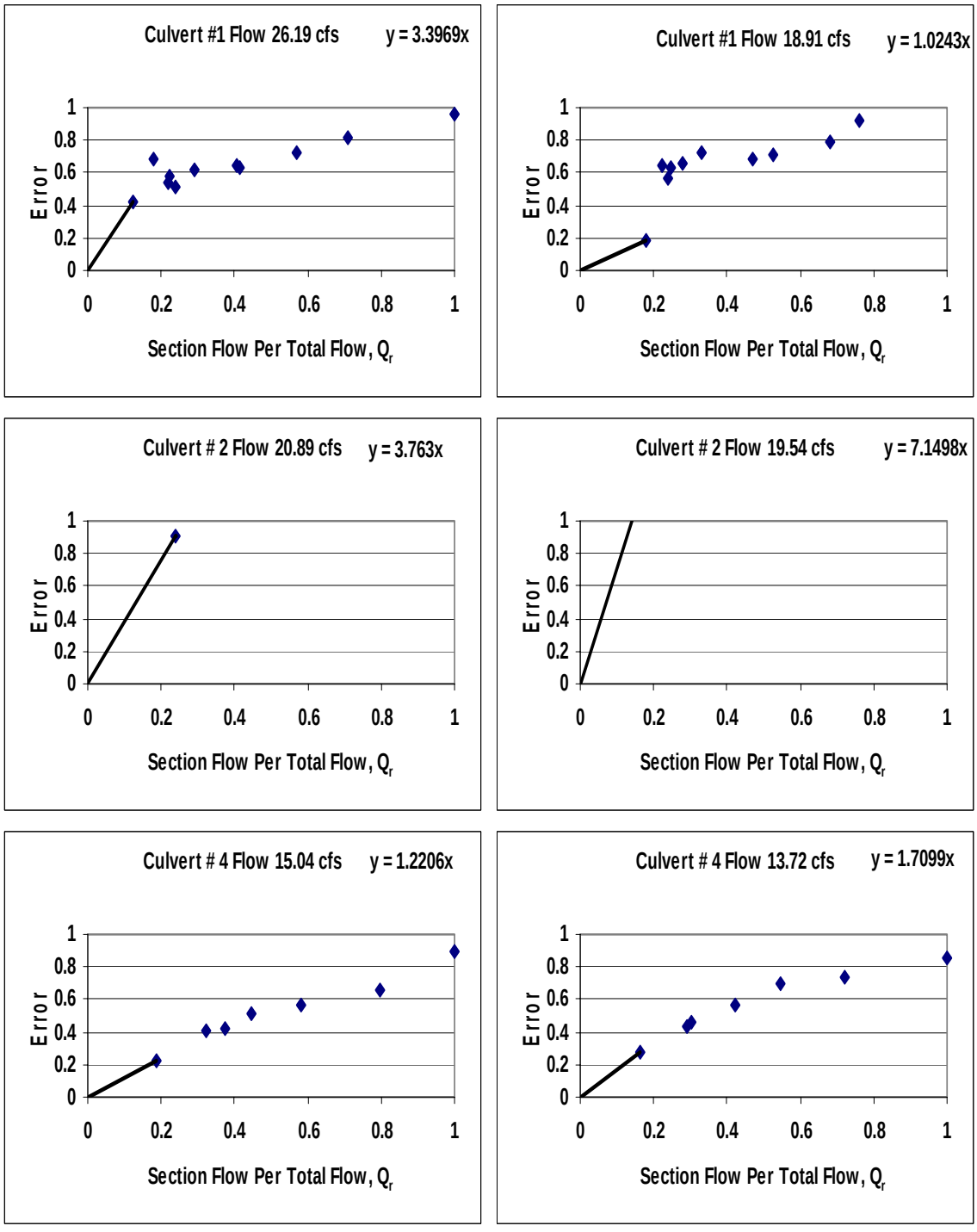


Figure 4.4 – The error statistic as a function of the USGS criteria.

In Figure 4.4 it is clear that Culverts 1 and 4 have an intuitive trend - the more velocities that are used to predict the remaining velocities, the better the prediction. Culvert 2 is an exception. Culvert 2 had a significant eddy in the cross section, making some of the x-direction velocities negative. When there are negative velocities in the cross section, many more observations of velocity are required to interpolate/extrapolate the remaining velocities. Furthermore, unless a good balance of positive and negative velocities are included as predictors, a sample run can have a very large error statistic.

Another feature of the graphs of Figure 4.4 is that all the trials share a common mathematical certainty – the trend must pass through the origin. So, for each trial the result of the sample run that produced the lowest error statistic was linearly connected to the origin. If it is assumed that the trend is linear from the origin to the lowest error statistic, then the value of the error statistic corresponding to any particular Q_r may be interpolated. The values of Q_r corresponding to error statistic of interest are shown in Table 4.1.

Table 4.1 - Linearly interpolated Q_r values for certain values of the error statistic.

Location	Flow rate (cfs)	Q_r Values When Error = 20%	Q_r Values When Error = 10%
Culvert 1	26.19	5.89	2.94
Culvert 1	18.91	19.53	9.76
Culvert 2	20.89	5.31	2.66
Culvert 2	19.54	2.80	1.40
Culvert 4	15.04	16.39	8.19
Culvert 4	13.72	11.70	5.85
-------	Average	10.27	5.13
-------	Std. Deviation	6.71	3.35

Table 4.1 provides the most enlightening result of this project. On the average, error statistics of 10% were attained by having Q_r = 5.13% plus or minus 3.35%, which is the standard deviation for the calculation. This would require slightly more detailed sampling than the "fine" USGS protocol (Q_r = 10%). The "fine" USGS protocol, on average would have achieved an error statistic of approximately 20%.

When the data points on the graphs shown in Figure 4.4 are displayed on one graph the result is shown in Figure 4.5. A linear model was used to describe all of the combined data.

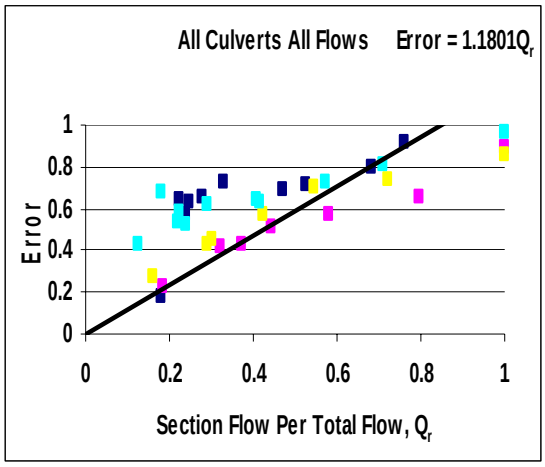


Figure 4.5 – Error statistic for all culverts using the USGS sampling scheme.

The Q_r from Figure 4.5 corresponding to an error statistic of 10% equals 8.5%, which is within one standard deviation of the average Q_r calculated in Table 4.1. The Q_r from Figure 4.5 corresponding to 20% error is 16.9%, which again is within one standard deviation of the average Q_r calculated in Table 4.1. The Q_r values from the linear model in Figure 4.5 were calculated using Equation 9.

$$Error = 1.1801(Q_r) \quad (8)$$

The discrepancy between the values is because the Q_r values from Figure 4.5 are based on a linear model that includes all of the data points, while the average Q_r values in Table 4.1 are based on linear models of six individual culverts that only included the smallest error statistic value in the regression. Although, these results again suggests that sampling using the "fine" protocol of the USGS method will arrive at velocity distribution prediction with 10 to 20 percent error.

Contour Plots

As before, contour plots were created to ground-truth the progression of the error analysis for certain trials. These too are archived in Appendix C, and an example sequence is shown in Figure 4.6. The graphs of Figure 4.6 are for Culvert 1 (trial 1), and show the velocity distribution based on observations alone and at two prediction intensities. It should be pointed out that some of the predicted values are larger than any of the recorded velocities because of procedure where the observed flow rate is maintained in all cases.

All of the trials resulted in the pattern of Figure 4.6 – as more predictors are included, the contour plot of velocities becomes more visually similar to the observed case.

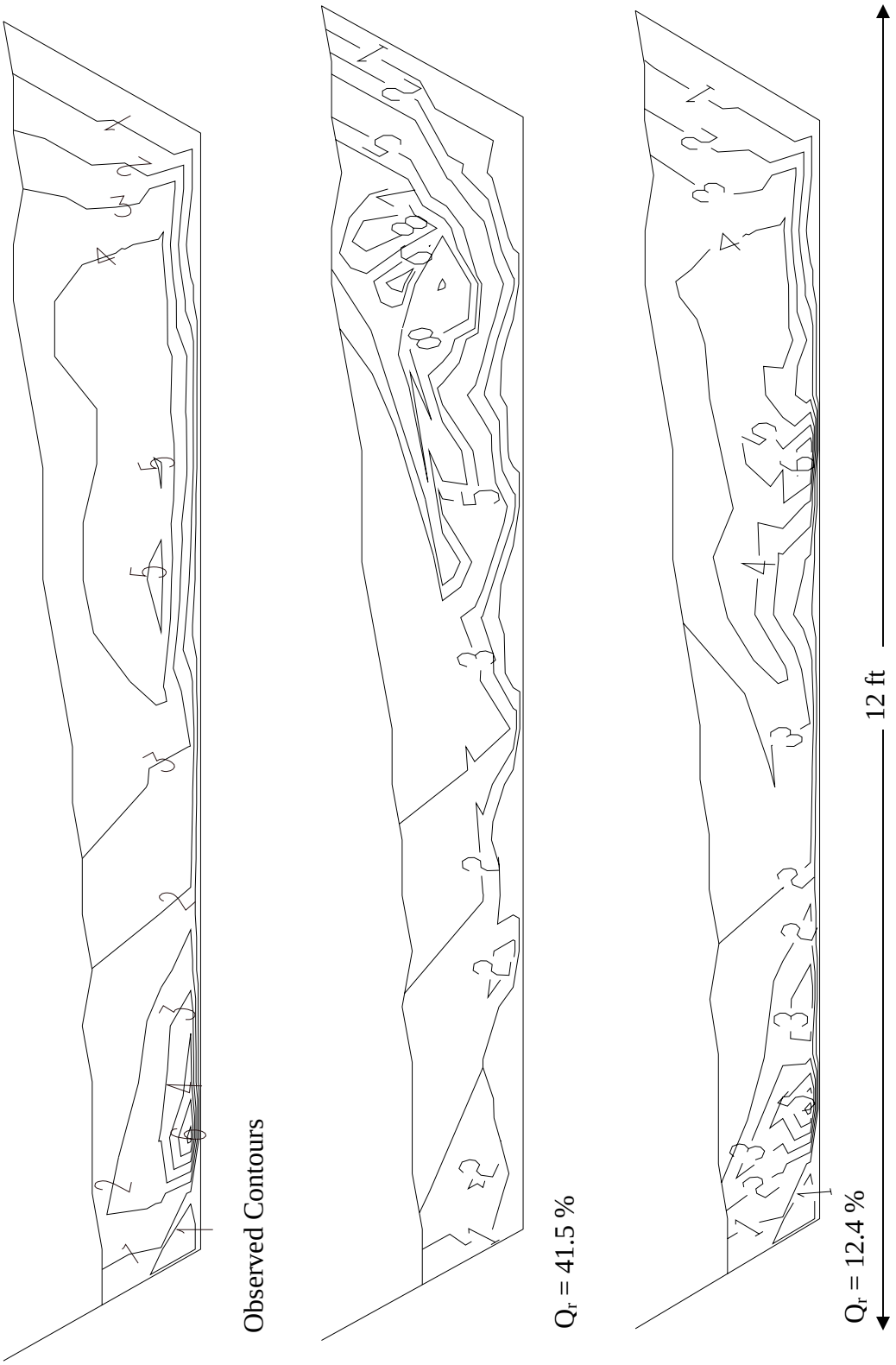


Figure 4.6 – Velocity distributions in feet per second for Culvert 1 at 26.19 cfs using the USGS sampling scheme.

CONCLUSIONS

Important conclusions can be drawn from this work. Velocities should be sampled in the cross section according to a geometric pattern that differentiates zones based on the magnitude of the velocity, rather than by random selection. The USGS method for measuring flow rate has this feature. Furthermore, if the USGS method is used with the objective of "fine" flow measurement, the velocity distribution of the cross section can be interpolated/extrapolated using the inverse distance squared method to within 20% on the average. To achieve an average velocity prediction error of 10% using the USGS protocol, the polygon having the largest percent of the total flow rate should have no more than approximately 5% of the total flow. If a polygon has more than the desired flow, it is split in half by measuring the two velocities at 80 and 20% of the water depth on a transect at the horizontal center of the offending polygon. This continues until the flow criteria is met for either the coarse or fine result as desired.

The practical implications are significant. This work indicates that the criteria for measuring flow using the USGS method can be applied to velocity prediction if the field observations are taken with rigorous adoption of the USGS method. An important warning needs to be pointed out that this research does not suggest that the USGS velocity-area method for calculating flow can be applied at the entrance of a culvert to calculate the flow. The USGS Geological Survey Water-Supply Paper 2175 defines strict criteria when locating a site to use for calculating flow. This work also shows that a casual approach to velocity measurement does not provide adequate data for the generation of detailed velocity information that has emerged as important components of fish passage research and three-dimensional flow modeling.

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APPENDICES

APPENDIX A:

FIELD DATA FOR EACH CULVERT

Culvert 1 at 26.19 cfs

Point	Station (ft)	Depth (ft)	Velocity (ft/sec)	Area (ft^2)	Flow (cfs)
1	1	0.05	1.83	0.03	0.05
2	1	0.2	0.96	0.2231	0.21
3	1.5	0.05	0.81	0.05	0.04
4	1.5	0.2	2.20	0.2219	0.49
5	2	0.05	6.41	0.05	0.32
6	2	0.2	3.05	0.225	0.69
7	2.5	0.05	4.33	0.05	0.22
8	2.5	0.2	2.74	0.2281	0.63
9	3	0.05	3.95	0.05	0.20
10	3	0.2	2.58	0.2438	0.63
11	3.5	0.05	2.45	0.05	0.12
12	3.5	0.2	1.74	0.2312	0.40
13	4	0.05	1.83	0.05	0.09
14	4	0.2	1.99	0.2469	0.49
15	4.5	0.05	2.18	0.05	0.11
16	4.5	0.2	2.34	0.2531	0.59
17	5	0.05	2.48	0.05	0.12
18	5	0.2	2.49	0.2719	0.68
19	5.5	0.05	2.98	0.075	0.22
20	5.5	0.3	3.32	0.2531	0.84
21	6	0.05	2.42	0.05	0.12
22	6	0.2	4.23	0.3	1.27
23	6.5	0.05	1.22	0.05	0.06
24	6.5	0.2	4.99	0.1	0.50
25	6.5	0.4	4.48	0.225	1.01
26	7	0.05	1.82	0.05	0.09
27	7	0.2	5.17	0.1	0.52
28	7	0.4	4.69	0.2469	1.16
29	7.5	0.05	2.14	0.05	0.11
30	7.5	0.2	4.93	0.1	0.49
31	7.5	0.4	4.59	0.25	1.15
32	8	0.05	2.70	0.05	0.13
33	8	0.2	5.04	0.1	0.50
34	8	0.4	4.48	0.2531	1.13
35	8.5	0.05	2.65	0.05	0.13
36	8.5	0.2	4.64	0.1	0.46
37	8.5	0.4	4.82	0.275	1.33
38	9	0.05	2.74	0.05	0.14
39	9	0.2	4.73	0.1	0.47
40	9	0.4	4.65	0.1	0.47
41	9	0.6	4.77	0.2	0.95
42	9.5	0.05	1.38	0.05	0.07
43	9.5	0.2	4.69	0.1	0.47
44	9.5	0.4	4.83	0.1	0.48
45	9.5	0.6	4.34	0.2219	0.96

46	10	0.05	0.46	0.05	0.02
47	10	0.2	4.15	0.1	0.41
48	10	0.4	3.94	0.1	0.39
49	10	0.6	3.81	0.2219	0.85
50	10.5	0.15	3.48	0.21875	0.76
51	10.5	0.35	2.60	0.9285	2.42

Culvert 1 at 18.91 cfs

Point	Station (ft)	Depth (ft)	Velocity (ft/sec)	Area (ft^2)	Flow (cfs)
1	1	0.05	0.79	0.03	0.02
2	1	0.2	2.25	0.195	0.44
3	1.5	0.05	-2.25	0.05	-0.11
4	1.5	0.2	1.47	0.1781	0.26
5	2	0.05	4.13	0.05	0.21
6	2	0.2	2.81	0.1969	0.55
7	2.5	0.05	3.28	0.05	0.16
8	2.5	0.2	2.85	0.1969	0.56
9	3	0.05	2.34	0.05	0.12
10	3	0.2	2.14	0.1813	0.39
11	3.5	0.05	1.76	0.05	0.09
12	3.5	0.2	1.82	0.1969	0.36
13	4	0.05	1.51	0.05	0.08
14	4	0.2	1.65	0.2	0.33
15	4.5	0.05	1.88	0.05	0.09
16	4.5	0.2	1.84	0.2	0.37
17	5	0.05	2.36	0.05	0.12
18	5	0.2	2.30	0.2031	0.47
19	5.5	0.05	2.54	0.05	0.13
20	5.5	0.2	-1.14	0.225	-0.26
21	6	0.05	2.38	0.05	0.12
22	6	0.2	-1.52	0.25	-0.38
23	6.5	0.05	1.56	0.05	0.08
24	6.5	0.2	4.26	0.2719	1.16
25	7	0.05	2.39	0.05	0.12
26	7	0.2	4.72	0.2781	1.31
27	7.5	0.05	2.52	0.05	0.13
28	7.5	0.2	4.64	0.1	0.46
29	7.5	0.4	4.25	0.1969	0.84
30	8	0.05	2.17	0.05	0.11
31	8	0.2	4.28	0.1	0.43
32	8	0.4	4.01	0.2031	0.82
33	8.5	0.05	2.79	0.05	0.14
34	8.5	0.2	4.10	0.1	0.41
35	8.5	0.4	4.03	0.225	0.91
36	9	0.05	2.87	0.05	0.14
37	9	0.2	4.14	0.1	0.41
38	9	0.4	4.04	0.2469	1.00
39	9.5	0.05	2.19	0.05	0.11
40	9.5	0.2	4.17	0.1	0.42
41	9.5	0.4	4.24	0.25	1.06
42	10	0.05	1.42	0.05	0.07
43	10	0.2	3.06	0.1	0.31
44	10	0.4	3.41	0.2531	0.86
45	10.5	0.25	3.31	0.3237	1.07
46	10.5	0.45	3.44	0.7075	2.44

Culvert 2 at 20.89 cfs

Point	Station (ft)	Depth (ft)	Velocity (ft/sec)	Area (ft^2)	Flow (cfs)
1	1	0.05	0.12	0.03	0.00
2	1	0.2	1.33	0.09	0.12
3	1	0.4	2.16	0.13	0.28
4	1	0.6	2.59	0.54	1.41
5	1.5	0.05	0.32	0.05	0.02
6	1.5	0.2	0.67	0.10	0.07
7	1.5	0.4	1.70	0.10	0.17
8	1.5	0.6	3.98	0.26	1.03
9	2	0.05	3.30	0.05	0.16
10	2	0.2	3.33	0.13	0.42
11	2	0.5	4.10	0.15	0.62
12	2	0.8	4.25	0.22	0.92
13	2.5	0.05	2.89	0.05	0.14
14	2.5	0.2	3.34	0.10	0.33
15	2.5	0.4	4.23	0.10	0.42
16	2.5	0.6	4.65	0.27	1.26
17	3	0.05	2.48	0.05	0.12
18	3	0.2	4.07	0.10	0.41
19	3	0.4	4.35	0.10	0.43
20	3	0.6	4.19	0.23	0.97
21	3.5	0.05	3.55	0.05	0.18
22	3.5	0.2	2.63	0.10	0.26
23	3.5	0.4	1.71	0.10	0.17
24	3.5	0.6	2.01	0.22	0.45
25	4	0.05	1.24	0.05	0.06
26	4	0.2	1.40	0.10	0.14
27	4	0.4	0.96	0.10	0.10
28	4	0.6	0.10	0.20	0.02
29	4.5	0.05	2.43	0.05	0.12
30	4.5	0.2	-0.08	0.10	-0.01
31	4.5	0.4	-0.25	0.10	-0.03
32	4.5	0.6	-0.10	0.20	-0.02
33	5	0.05	0.27	0.05	0.01
34	5	0.2	-0.34	0.10	-0.03
35	5	0.4	-0.42	0.10	-0.04
36	5	0.6	-0.06	0.20	-0.01
37	5.5	0.05	-0.21	0.05	-0.01
38	5.5	0.2	-0.40	0.10	-0.04
39	5.5	0.4	-0.29	0.28	-0.08
40	6	0.05	1.33	0.05	0.07
41	6	0.2	-0.20	0.10	-0.02
42	6	0.4	-0.16	0.28	-0.04
43	6.5	0.05	0.18	0.05	0.01
44	6.5	0.2	0.04	0.10	0.00
45	6.5	0.4	0.14	0.10	0.01

46	6.5	0.6	0.16	0.20	0.03
47	7	0.05	0.62	0.05	0.03
48	7	0.2	0.52	0.10	0.05
49	7	0.4	0.79	0.10	0.08
50	7	0.6	0.80	0.20	0.16
51	7.5	0.05	0.37	0.05	0.02
52	7.5	0.2	-0.08	0.10	-0.01
53	7.5	0.4	-0.33	0.10	-0.03
54	7.5	0.6	0.12	0.20	0.02
55	8	0.05	1.40	0.05	0.07
56	8	0.2	-0.17	0.10	-0.02
57	8	0.4	-0.57	0.10	-0.06
58	8	0.6	-0.18	0.20	-0.04
59	8.5	0.05	2.76	0.05	0.14
60	8.5	0.2	1.50	0.10	0.15
61	8.5	0.4	0.12	0.10	0.01
62	8.5	0.6	-0.32	0.20	-0.06
63	9	0.05	1.43	0.05	0.07
64	9	0.2	3.73	0.10	0.37
65	9	0.4	4.14	0.10	0.41
66	9	0.6	2.49	0.23	0.56
67	9.5	0.05	1.68	0.05	0.08
68	9.5	0.2	2.39	0.10	0.24
69	9.5	0.4	4.45	0.10	0.44
70	9.5	0.6	5.03	0.25	1.27
71	10	0.05	0.73	0.05	0.04
72	10	0.2	1.74	0.13	0.22
73	10	0.5	3.86	0.15	0.58
74	10	0.8	4.28	0.22	0.94
75	10.5	0.3	2.52	0.38	0.96
76	10.5	0.5	3.48	0.25	0.87
77	10.5	0.7	3.67	0.74	2.70

Culvert 2 at 19.54 cfs

Point	Station (ft)	Depth (ft)	Velocity (ft/sec)	Area (ft^2)	Flow (cfs)
1	1.00	0.05	0.46	0.03	0.01
2	1.00	0.20	0.66	0.09	0.06
3	1.00	0.40	2.09	0.13	0.27
4	1.00	0.60	3.26	0.50	1.63
5	1.50	0.05	0.28	0.05	0.01
6	1.50	0.20	0.81	0.10	0.08
7	1.50	0.40	1.61	0.10	0.16
8	1.50	0.60	4.02	0.25	1.02
9	2.00	0.05	3.14	0.05	0.16
10	2.00	0.20	3.50	0.10	0.35
11	2.00	0.40	4.03	0.10	0.40
12	2.00	0.60	4.36	0.27	1.17
13	2.50	0.05	2.67	0.05	0.13
14	2.50	0.20	3.30	0.10	0.33
15	2.50	0.40	4.27	0.10	0.43
16	2.50	0.60	4.71	0.25	1.18
17	3.00	0.05	2.50	0.05	0.13
18	3.00	0.20	4.15	0.10	0.41
19	3.00	0.40	4.25	0.10	0.42
20	3.00	0.60	4.12	0.23	0.93
21	3.50	0.05	3.73	0.05	0.19
22	3.50	0.20	2.31	0.10	0.23
23	3.50	0.40	1.11	0.10	0.11
24	3.50	0.60	1.20	0.20	0.24
25	4.00	0.05	1.22	0.05	0.06
26	4.00	0.20	1.15	0.10	0.12
27	4.00	0.40	0.77	0.10	0.08
28	4.00	0.60	-0.01	0.20	0.00
29	4.50	0.05	2.21	0.05	0.11
30	4.50	0.20	-0.03	0.10	0.00
31	4.50	0.40	-0.14	0.10	-0.01
32	4.50	0.60	0.06	0.20	0.01
33	5.00	0.05	0.22	0.05	0.01
34	5.00	0.20	-0.40	0.10	-0.04
35	5.00	0.40	-0.38	0.10	-0.04
36	5.00	0.60	0.04	0.18	0.01
37	5.50	0.05	-0.20	0.05	-0.01
38	5.50	0.20	-0.37	0.10	-0.04
39	5.50	0.40	-0.35	0.25	-0.09
40	6.00	0.05	0.10	0.05	0.00
41	6.00	0.20	-0.22	0.10	-0.02
42	6.00	0.40	-0.19	0.10	-0.02
43	6.00	0.60	0.02	0.16	0.00
44	6.50	0.05	0.00	0.05	0.00
45	6.50	0.20	-0.04	0.10	0.00
46	6.50	0.40	0.08	0.10	0.01
47	6.50	0.60	0.13	0.19	0.03
48	7.00	0.05	0.28	0.05	0.01
49	7.00	0.20	0.18	0.10	0.02
50	7.00	0.40	0.54	0.10	0.05

51	7.00	0.60	0.64	0.20	0.13
52	7.50	0.05	0.39	0.05	0.02
53	7.50	0.20	-0.02	0.10	0.00
54	7.50	0.40	-0.08	0.10	-0.01
55	7.50	0.60	0.23	0.20	0.05
56	8.00	0.05	1.33	0.05	0.07
57	8.00	0.20	-0.03	0.10	0.00
58	8.00	0.40	-0.48	0.10	-0.05
59	8.00	0.60	-0.36	0.20	-0.07
60	8.50	0.05	2.45	0.05	0.12
61	8.50	0.20	1.60	0.10	0.16
62	8.50	0.40	0.04	0.10	0.00
63	8.50	0.60	-0.29	0.20	-0.06
64	9.00	0.05	0.88	0.05	0.04
65	9.00	0.20	2.52	0.10	0.25
66	9.00	0.40	4.16	0.10	0.42
67	9.00	0.60	2.20	0.20	0.45
68	9.50	0.05	2.12	0.05	0.11
69	9.50	0.20	3.81	0.10	0.38
70	9.50	0.40	5.00	0.10	0.50
71	9.50	0.60	5.11	0.23	1.15
72	10.00	0.05	1.79	0.05	0.09
73	10.00	0.20	2.35	0.10	0.24
74	10.00	0.40	3.57	0.10	0.36
75	10.00	0.60	4.08	0.25	1.02
76	10.50	0.30	2.42	0.38	0.92
77	10.50	0.50	3.51	0.25	0.88
78	10.50	0.70	3.13	0.67	2.08

Culvert 4 at 15.04 cfs

Point	Station (ft)	Depth (ft)	Velocity (ft/sec)	Area (ft^2)	Flow (cfs)
1	1.70	0.59	1.66	0.02	0.04
2	1.70	0.62	1.65	0.03	0.04
3	1.70	0.70	0.97	0.11	0.10
4	2.10	0.38	0.71	0.02	0.01
5	2.10	0.41	1.38	0.03	0.04
6	2.10	0.49	1.32	0.03	0.04
7	2.10	0.57	1.24	0.03	0.04
8	2.10	0.65	1.34	0.03	0.04
9	2.10	0.73	1.06	0.06	0.07
10	2.50	0.22	3.07	0.02	0.05
11	2.50	0.25	3.39	0.03	0.10
12	2.50	0.33	2.98	0.03	0.10
13	2.50	0.40	4.55	0.03	0.15
14	2.50	0.48	3.97	0.03	0.13
15	2.50	0.56	3.99	0.03	0.13
16	2.50	0.64	2.79	0.03	0.09
17	2.50	0.72	2.10	0.07	0.15
18	2.90	0.12	1.39	0.02	0.02
19	2.90	0.15	1.39	0.03	0.04
20	2.90	0.23	1.68	0.03	0.05
21	2.90	0.31	1.93	0.03	0.06
22	2.90	0.39	3.01	0.03	0.10
23	2.90	0.47	4.64	0.03	0.15
24	2.90	0.55	5.49	0.03	0.18
25	2.90	0.63	5.60	0.03	0.18
26	2.90	0.71	3.87	0.08	0.30
27	3.30	0.06	3.07	0.01	0.05
28	3.30	0.09	3.78	0.03	0.12
29	3.30	0.17	5.22	0.03	0.17
30	3.30	0.25	5.36	0.03	0.17
31	3.30	0.33	5.12	0.03	0.16
32	3.30	0.41	5.70	0.03	0.18
33	3.30	0.49	5.83	0.03	0.19
34	3.30	0.57	7.35	0.03	0.24
35	3.30	0.65	8.00	0.08	0.68
36	3.70	0.05	2.98	0.01	0.04
37	3.70	0.08	3.93	0.03	0.13
38	3.70	0.16	5.31	0.03	0.17
39	3.70	0.24	6.31	0.03	0.20
40	3.70	0.32	6.37	0.03	0.20
41	3.70	0.40	6.47	0.03	0.21
42	3.70	0.48	6.95	0.03	0.22
43	3.70	0.56	6.78	0.03	0.22
44	3.70	0.64	9.25	0.03	0.30
45	3.70	0.72	9.25	0.08	0.70

46	4.10	0.08	3.17	0.01	0.05
47	4.10	0.11	4.27	0.03	0.14
48	4.10	0.19	5.07	0.03	0.16
49	4.10	0.27	4.27	0.03	0.14
50	4.10	0.35	4.74	0.03	0.15
51	4.10	0.43	5.60	0.03	0.18
52	4.10	0.51	6.40	0.03	0.20
53	4.10	0.59	6.37	0.03	0.20
54	4.10	0.67	8.58	0.03	0.27
55	4.10	0.75	8.68	0.03	0.28
56	4.10	0.83	8.29	0.03	0.27
57	4.10	0.91	7.91	0.05	0.43
58	4.50	0.16	4.26	0.02	0.07
59	4.50	0.19	4.79	0.03	0.15
60	4.50	0.27	5.89	0.03	0.19
61	4.50	0.35	5.36	0.03	0.17
62	4.50	0.43	6.27	0.03	0.20
63	4.50	0.51	6.95	0.03	0.22
64	4.50	0.59	7.16	0.03	0.23
65	4.50	0.67	9.35	0.03	0.30
66	4.50	0.75	9.73	0.03	0.31
67	4.50	0.83	9.83	0.09	0.84
68	4.90	0.29	5.49	0.02	0.09
69	4.90	0.32	6.85	0.03	0.20
70	4.90	0.40	7.14	0.03	0.23
71	4.90	0.47	6.95	0.03	0.22
72	4.90	0.55	7.26	0.03	0.23
73	4.90	0.63	8.00	0.03	0.26
74	4.90	0.71	8.21	0.03	0.26
75	4.90	0.79	5.89	0.07	0.44
76	5.30	0.48	5.03	0.02	0.11
77	5.30	0.51	5.22	0.03	0.15
78	5.30	0.59	5.60	0.03	0.18
79	5.30	0.67	5.70	0.03	0.18
80	5.30	0.75	6.08	0.03	0.19
81	5.30	0.83	5.49	0.05	0.27
82	5.70	0.73	2.53	0.06	0.14

Culvert 4 at 13.72 cfs

Point	Station (ft)	Depth (ft)	Velocity (ft/sec)	Area (ft^2)	Flow (cfs)
1	1.00	0.86	1.17	0.04	0.04
2	1.40	0.55	1.12	0.02	0.02
3	1.40	0.58	1.17	0.03	0.03
4	1.40	0.66	1.52	0.08	0.12
5	1.80	0.39	2.72	0.02	0.05
6	1.80	0.42	2.34	0.03	0.07
7	1.80	0.50	1.55	0.03	0.05
8	1.80	0.58	1.26	0.03	0.04
9	1.80	0.66	0.99	0.08	0.08
10	2.20	0.22	3.20	0.02	0.05
11	2.20	0.25	4.83	0.03	0.15
12	2.20	0.33	5.55	0.03	0.18
13	2.20	0.41	6.08	0.03	0.19
14	2.20	0.49	5.12	0.03	0.16
15	2.20	0.57	4.74	0.03	0.15
16	2.20	0.65	2.57	0.06	0.15
17	2.60	0.11	3.39	0.01	0.05
18	2.60	0.14	2.48	0.03	0.08
19	2.60	0.22	2.82	0.03	0.09
20	2.60	0.30	4.64	0.03	0.15
21	2.60	0.38	8.29	0.03	0.27
22	2.60	0.46	10.69	0.03	0.34
23	2.60	0.54	11.17	0.07	0.79
24	3.00	0.07	3.23	0.01	0.05
25	3.00	0.10	4.64	0.03	0.15
26	3.00	0.18	5.59	0.03	0.18
27	3.00	0.36	6.59	0.03	0.21
28	3.00	0.44	7.33	0.03	0.23
29	3.00	0.52	7.81	0.03	0.25
30	3.00	0.60	10.40	0.07	0.77
31	3.40	0.07	3.20	0.01	0.05
32	3.40	0.10	3.65	0.03	0.12
33	3.40	0.18	3.93	0.03	0.13
34	3.40	0.26	4.27	0.03	0.14
35	3.40	0.34	3.57	0.03	0.11
36	3.40	0.42	4.88	0.03	0.16
37	3.40	0.50	4.60	0.03	0.15
38	3.40	0.58	6.08	0.03	0.19
39	3.40	0.66	8.68	0.08	0.67
40	3.80	0.11	3.02	0.02	0.05
41	3.80	0.14	3.55	0.03	0.11
42	3.80	0.22	4.12	0.03	0.13
43	3.80	0.30	4.45	0.03	0.14
44	3.80	0.38	4.60	0.03	0.15
45	3.80	0.46	5.55	0.03	0.18

46	3.80	0.54	6.56	0.03	0.21
47	3.80	0.62	7.33	0.03	0.23
48	3.80	0.70	7.81	0.08	0.64
49	4.20	0.19	3.84	0.02	0.06
50	4.20	0.22	4.83	0.03	0.15
51	4.20	0.30	5.07	0.03	0.16
52	4.20	0.38	6.27	0.03	0.20
53	4.20	0.46	7.04	0.03	0.23
54	4.20	0.54	7.04	0.03	0.23
55	4.20	0.62	7.14	0.03	0.23
56	4.20	0.70	9.44	0.03	0.30
57	4.20	0.78	9.16	0.08	0.71
58	4.60	0.34	5.12	0.02	0.09
59	4.60	0.37	6.27	0.03	0.18
60	4.60	0.45	6.78	0.03	0.22
61	4.60	0.53	7.14	0.03	0.23
62	4.60	0.61	6.85	0.03	0.22
63	4.60	0.69	6.56	0.03	0.21
64	4.60	0.77	6.08	0.03	0.19
65	4.60	0.85	4.98	0.08	0.41
66	5.00	0.51	5.02	0.02	0.10
67	5.00	0.54	4.35	0.03	0.12
68	5.00	0.62	4.74	0.03	0.15
69	5.00	0.70	5.12	0.03	0.16
70	5.00	0.78	4.74	0.07	0.33
71	5.40	0.76	3.10	0.05	0.15

APPENDIX B:

VISUAL BASIC CODE

```
Option Explicit
Option Base 1
Sub RandomVelocity()

Dim boxTitle As String, boxInst As String
Dim rangeE As Range, output As Range
Dim vRowsCount As Integer, Random() As Integer
Dim n As Integer, i As Integer, A As Integer, z As Integer, j As Integer
Dim St() As Double, Elv() As Double, V() As Double, Vs() As Double, Vss() As Double
Dim Distance() As Double, Area() As Double, DisTerm As Double, Dim Velocity() As
Double, VelTerm As Double, VelTermTotal As Double
Dim VelWeighted As Double, VMeasured As Double, DMeasured As Double
Dim Q() As Double, QTotal As Double, Qs() As Double, DisTermTotal As Double
Dim QMeasured As Double, Flow As Double, FlowTotal As Double
Dim E As Double, C As Double, Error As Double, TrueError As Double
Dim ErrorTotal As Double, ErrorTotalTotal As Double, QGood As Double
Dim Bound() As Double, QTotalTotal As Double, Qsubtract As Double
Dim Large As Double, Small As Double, Qsub As Double

'This is the input info
boxTitle = "Mean Velocity"
boxInst = "Locate the columns of Station, Elevation, Velocity, Area, and Boundary"
Set rangeE = Application.InputBox(boxInst, boxTitle, , , , Type:=8)
boxInst = "Locate the cell you would like the out put to go"
Set output = Application.InputBox(boxInst, boxTitle, , , , Type:=8)

'This well count the number of Stations, Elevations, and Velocities
vRowsCount = rangeE.Rows.Count
n = vRowsCount

'Re-dimension arrays based on n
ReDim St(n), Elv(n), V(n), Area(n), Bound(n)
ReDim Distance(n), Velocity(n), Random(n)
ReDim Q(n), Qs(n), Vs(n), Vss(n)

'Read Station values from worksheet into St() array
For i = 1 To n
   St(i) = rangeE.Cells(i, 1).Value
   Elv(i) = rangeE.Cells(i, 2).Value
   V(i) = rangeE.Cells(i, 3).Value
   Area(i) = rangeE.Cells(i, 4).Value
   Bound(i) = rangeE.Cells(i, 5).Value
Next i
```

```
'This calculates the flow for use later when adjusting each velocity
For i = 1 To n - 1
   Q(i) = V(i) * Area(i)
   QTotal = QTotal + Q(i)
Next i
output.Cells(-7, 0).Value = QTotal
QGood = QTotal

'This calculates the first Random Number
Random(1) = Int((n - 1 - 1 + 1) * Rnd + 1)
output.Cells(1, 0).Value = Random(1)

'This calculates the rest of the Random Numbers
'The code needs a first random number to compare the rest to that is why i starts at 2
For i = 2 To n - 1
   Random(i) = Int((n - 1 - 1 + 1) * Rnd + 1)
      For A = 1 To i - 1                    'This checks the new random number against the
                                                              established numbers
            If Random(i) = Random(A) Then          'If the new random number equals an
                                                        old random number it calculates a new number
           Random(i) = Int((n - 1 - 1 + 1) * Rnd + 1)
            A = 0                          'This will start the check over again with the new
                                                                  random number
         Else
            End If
      Next A
output.Cells(i, 0).Value = Random(i)
Next i

'This calculates the Distance and the velocity Terms
'When A=n - 1 all of the velocities are the same as the measured velocity
 For A = 1 To n - 2                              'The value of A tells how many measured
                                                    velocities are being used to calculate the others
'This step organizes the Data so that the known velocities can be filtered out when
adjusting the data for flow
   For i = 2 To A
      If Random(i - 1) > Random(i) Then
         Large = Random(i - 1)
         Small = Random(i)
         Random(i - 1) = Small
         Random(i) = Large
         i = 1
      Else
         End If
   Next i
```

```
    For z = 1 To n - 1                          'The Value of z tells which velocity is
                                                being calculated
   For i = 1 To A                               'The Value of i tells which measured velocity
                                                is being used to calculate the other velocities
 If Random(i) < z Or Random(i) > z Then         'This step makes it so that the values
                                                being used to calculate the velocities are skipped
     Distance(i) = ((St(Random(i)) - St(z)) ^ 2 + (Elv(Random(i)) - Elv(z)) ^ 2) ^ (0.5)
       DisTerm = 1 / ((Distance(i) ^ 2) + (1 / Bound(i)))
       DisTermTotal = DisTerm + DisTermTotal
     Else
        DMeasured = 1                           'This value is used later to separate the
                                                measured data from the calculated data
       i = A
     End If
   Next i

For i = 1 To A
 If Random(i) < z Or Random(i) > z Then         'This step makes it so that the values being
                                                used to calculate the velocities are skipped
       Velocity(i) = V(Random(i)) / (Distance(i) ^ 2)
       VelTerm = Velocity(i)
       VelTermTotal = VelTerm + VelTermTotal
     Else
        Vs(z) = V(Random(i))                    'This step records the measured velocity to the
                                                appropriate cell
  Qsubtract = V(Random(i)) * Area(Random(i))    'This step pulls out the known flow
                                                from the flow being adjusted

       output.Cells(z, A).Value = Vs(z)
        QTotalTotal = QTotal - Qsubtract
       QTotal = QTotalTotal
       i = A
     End If
   Next i

     If DMeasured = 0 Then                      'When DisTermTotal=0 the velocity is
                                                VMeasured and has already been recorded to its cell
      VelWeighted = VelTermTotal / DisTermTotal 'This calculates the Weighted
                                                Velocity

     Vs(z) = VelWeighted
   Else
     End If

'This step rests the terms for the next calculation
   DisTermTotal = 0
   VelTermTotal = 0
```

```
   DMeasured = 0
   QMeasured = 0
   VelWeighted = 0
Next z

z = 1
E = 15
C = 1
Do Until E < 0.05
   For i = 1 To n - 1
      If Random(z) < i Or Random(z) > i Then
         Vss(i) = Vs(i) * C
         output.Cells(i, A).Value = Vss(i)
         Flow = Vss(i) * Area(i)
         FlowTotal = Flow + FlowTotal
      Else
         Vss(i) = V(i)
         If z = A Then
           z = z
         Else
            z = z + 1
         End If
      End If
   Next i

   C = (QTotalTotal / FlowTotal)
   E = Abs(FlowTotal - QTotalTotal)
   z = 1
   FlowTotal = 0
Loop

For i = 1 To n - 1
 Error = Abs((V(i) - Vss(i)) / V(i))
   output.Cells(n + i + 3, A).Value = Error
   ErrorTotal = Error
   ErrorTotalTotal = ErrorTotalTotal + ErrorTotal
Next i

TrueError = ErrorTotalTotal / (n - 1)
output.Cells(n + n + 5, A).Value = TrueError

ErrorTotalTotal = 0
Qsub = 0
QTotal = QGood
Next A
End Sub
```

APPENDIX C:

CONTOUR PLOTS



Figure C.1 – The velocity distributions in feet per second for Culvert 1 at 26.19 cfs using the random sampling scheme.



Figure C.2 – The velocity distributions in feet per second for Culvert 1 at 18.91 cfs using the random sampling scheme.



Figure C.3 – The velocity distributions in feet per second for Culvert 2 at 20.89 cfs using the random sampling scheme.

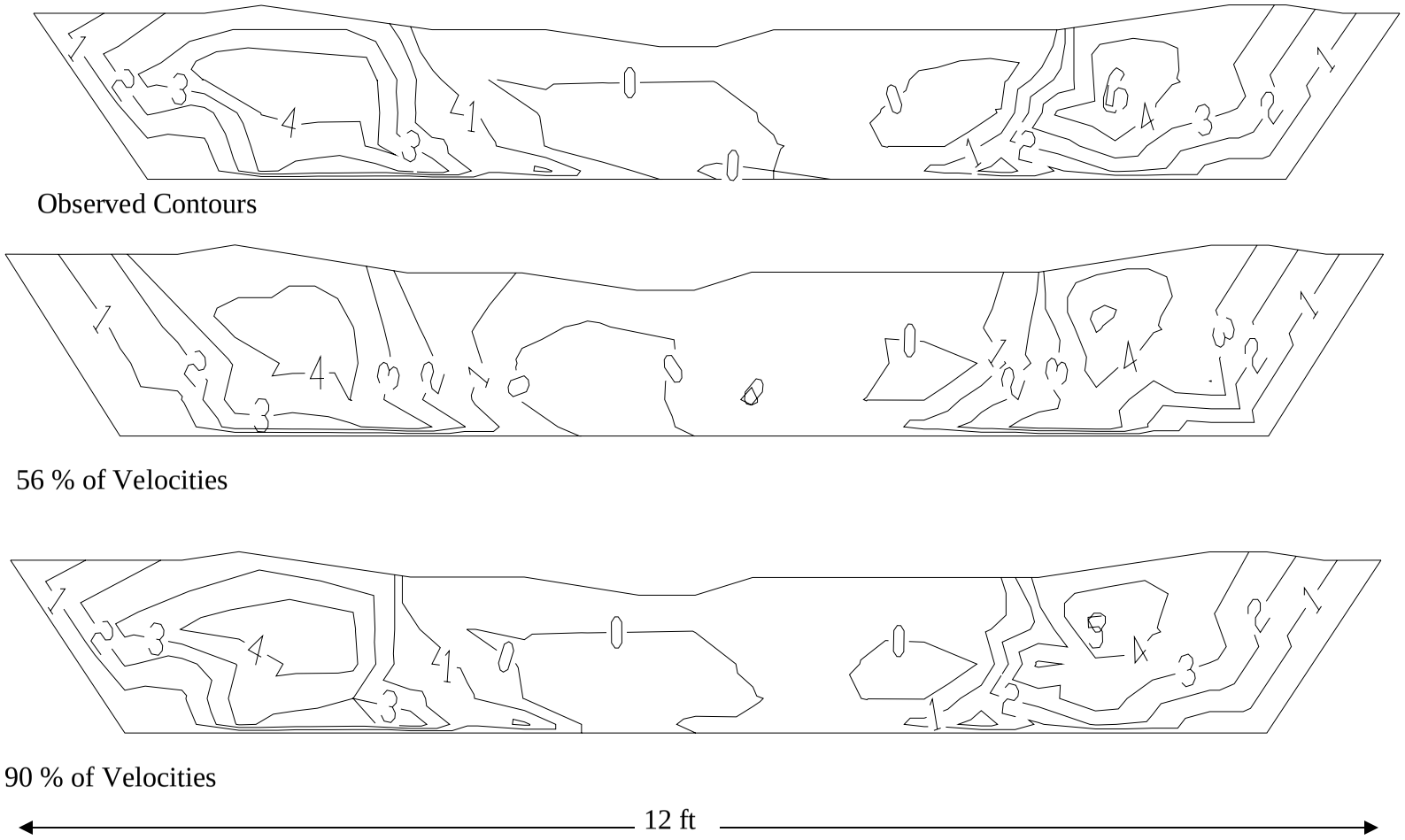


Figure C.4 – The velocity distributions in feet per second for Culvert 2 at 19.54 cfs using the random sampling scheme.

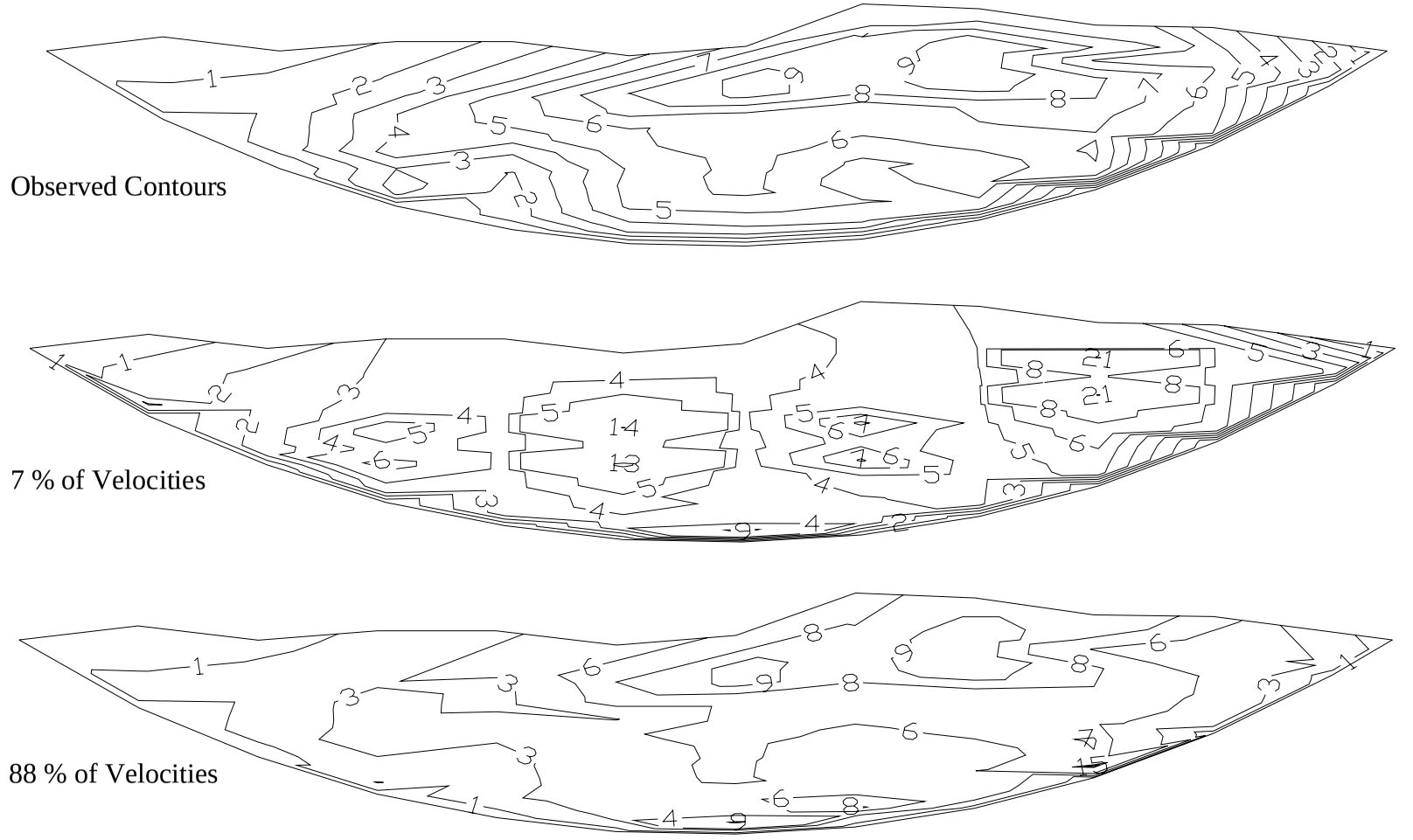


Figure C.5 – The velocity distributions in feet per second for Culvert 4 at 15.04 cfs using the random sampling scheme.

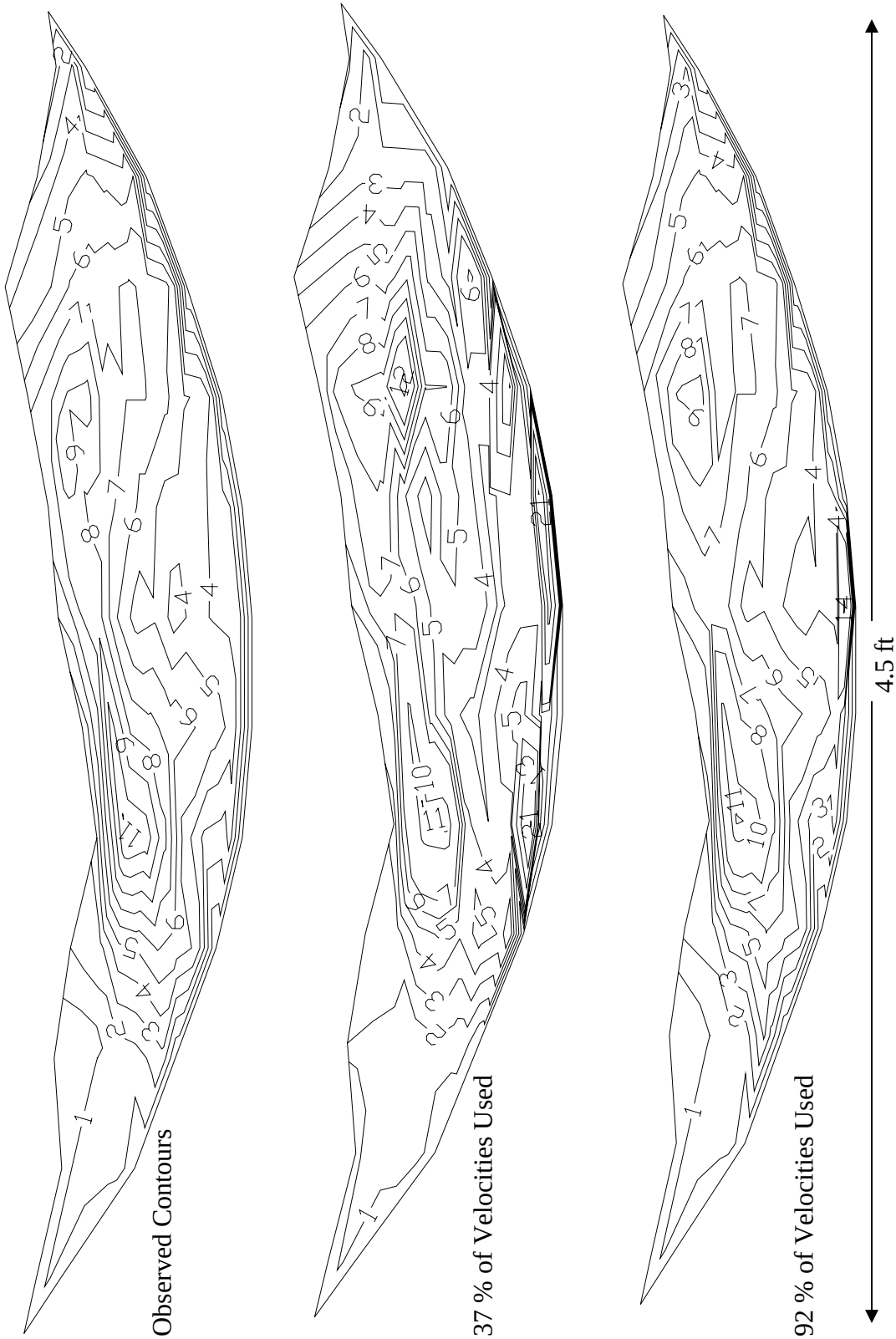


Figure C.6 – The velocity distributions in feet per second for Culvert 4 at 13.72 cfs using the random sampling scheme.



Figure C.7 – Velocity distributions in feet per second for Culvert 1 at 26.19 cfs using the USGS sampling scheme.

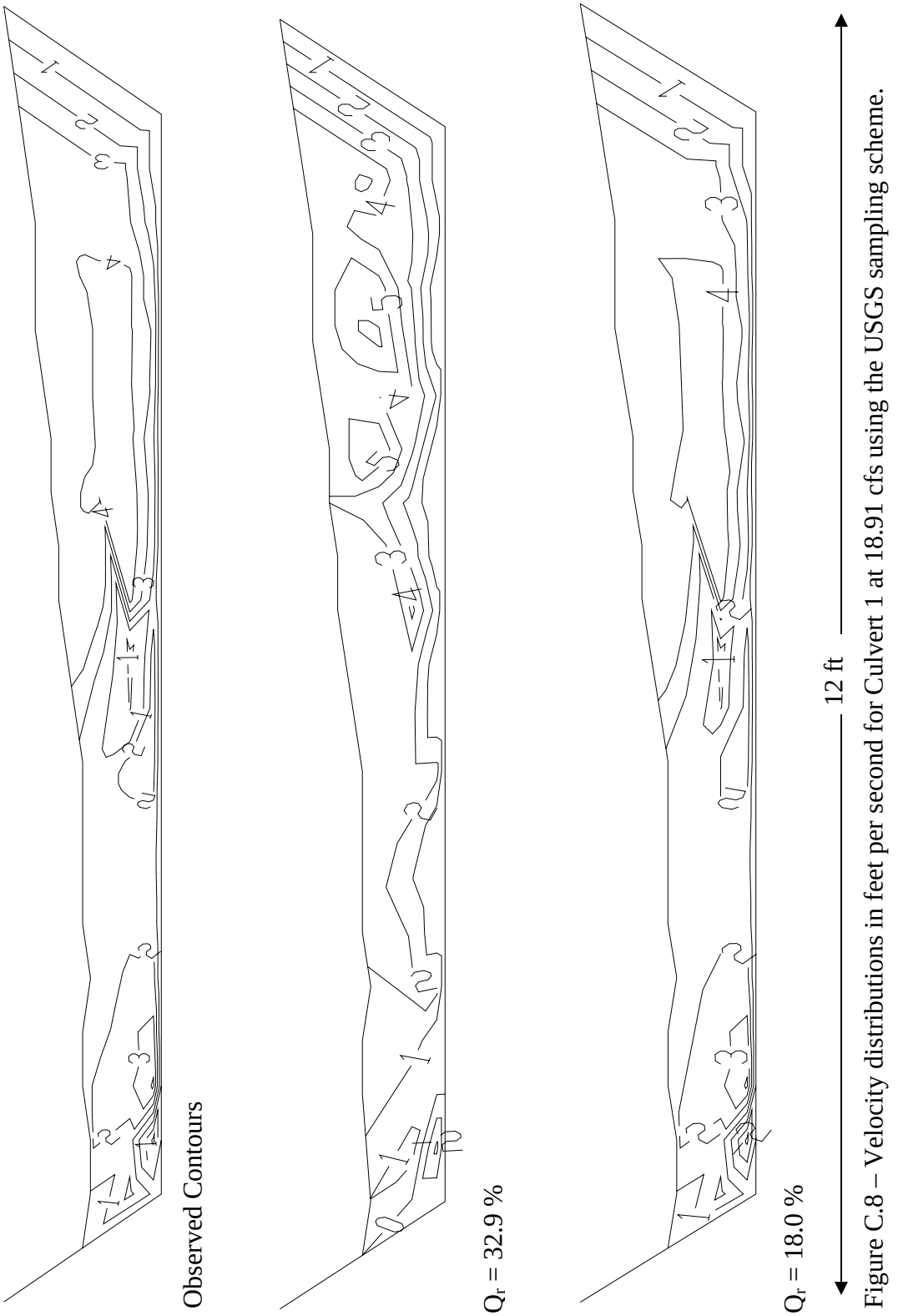
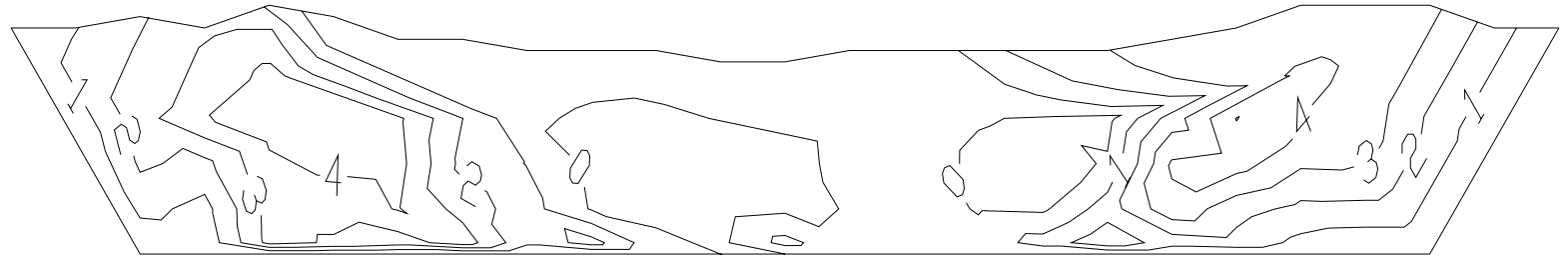
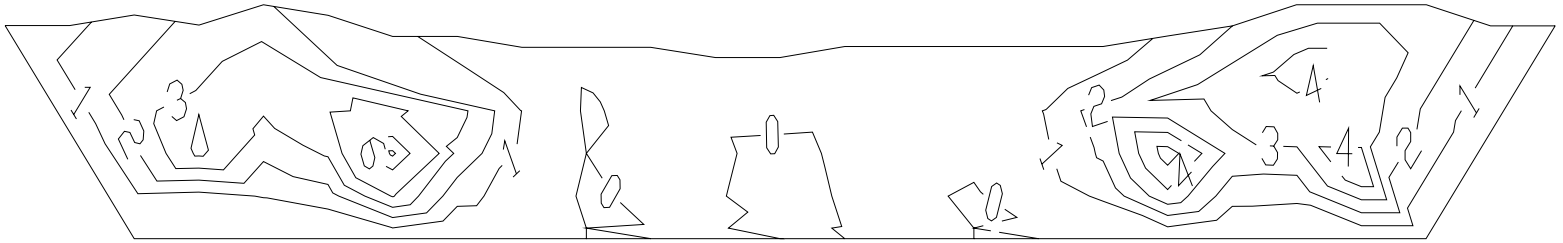


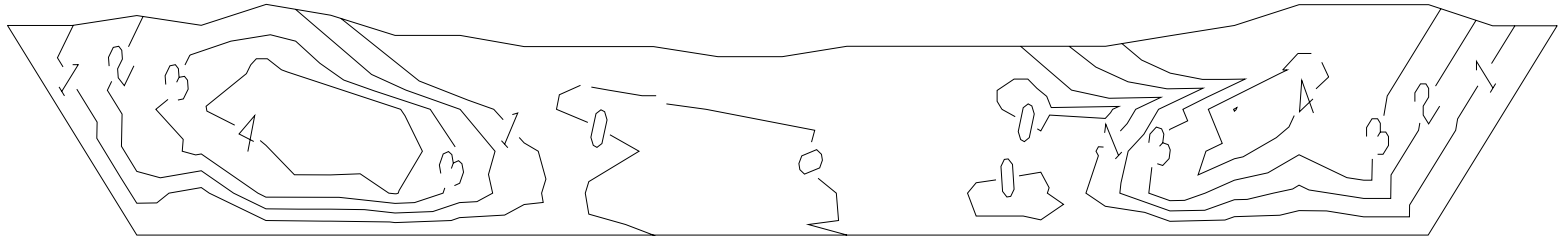
Figure C.8 – Velocity distributions in feet per second for Culvert 1 at 18.91 cfs using the USGS sampling scheme.



Observed Contours



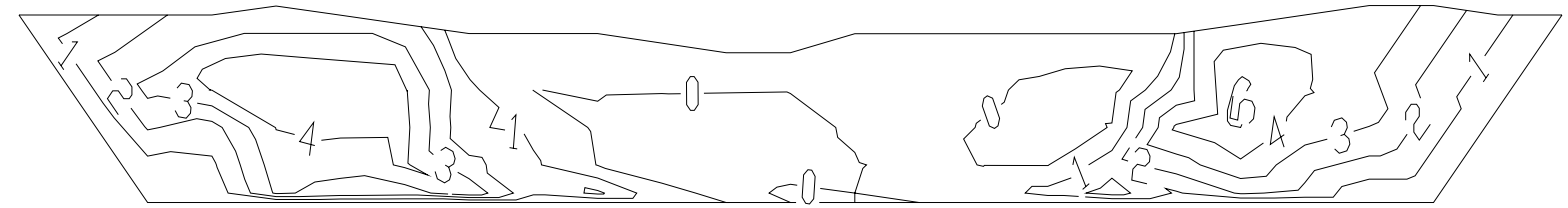
Q_r = 34.8 %



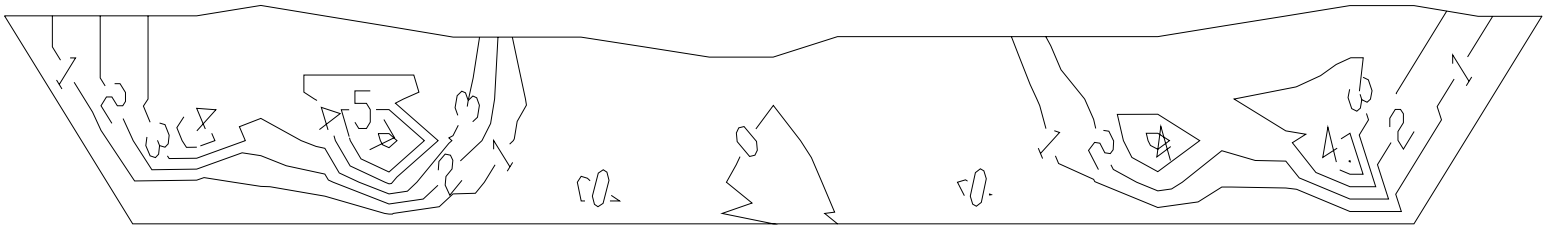
Q_r = 24.0 %

12 ft

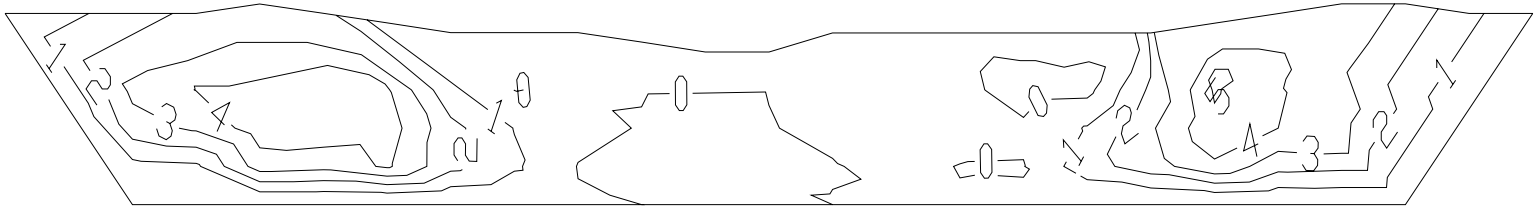
Figure C.9 – Velocity distributions in feet per second for Culvert 2 at 20.89 cfs using the USGS sampling scheme.



Observed Contours



Q_r = 30.7 %



Q_r = 22.6 %

12 ft

Figure C.10 – Velocity distributions in feet per second for Culvert 2 at 19.54 cfs using the USGS sampling scheme.

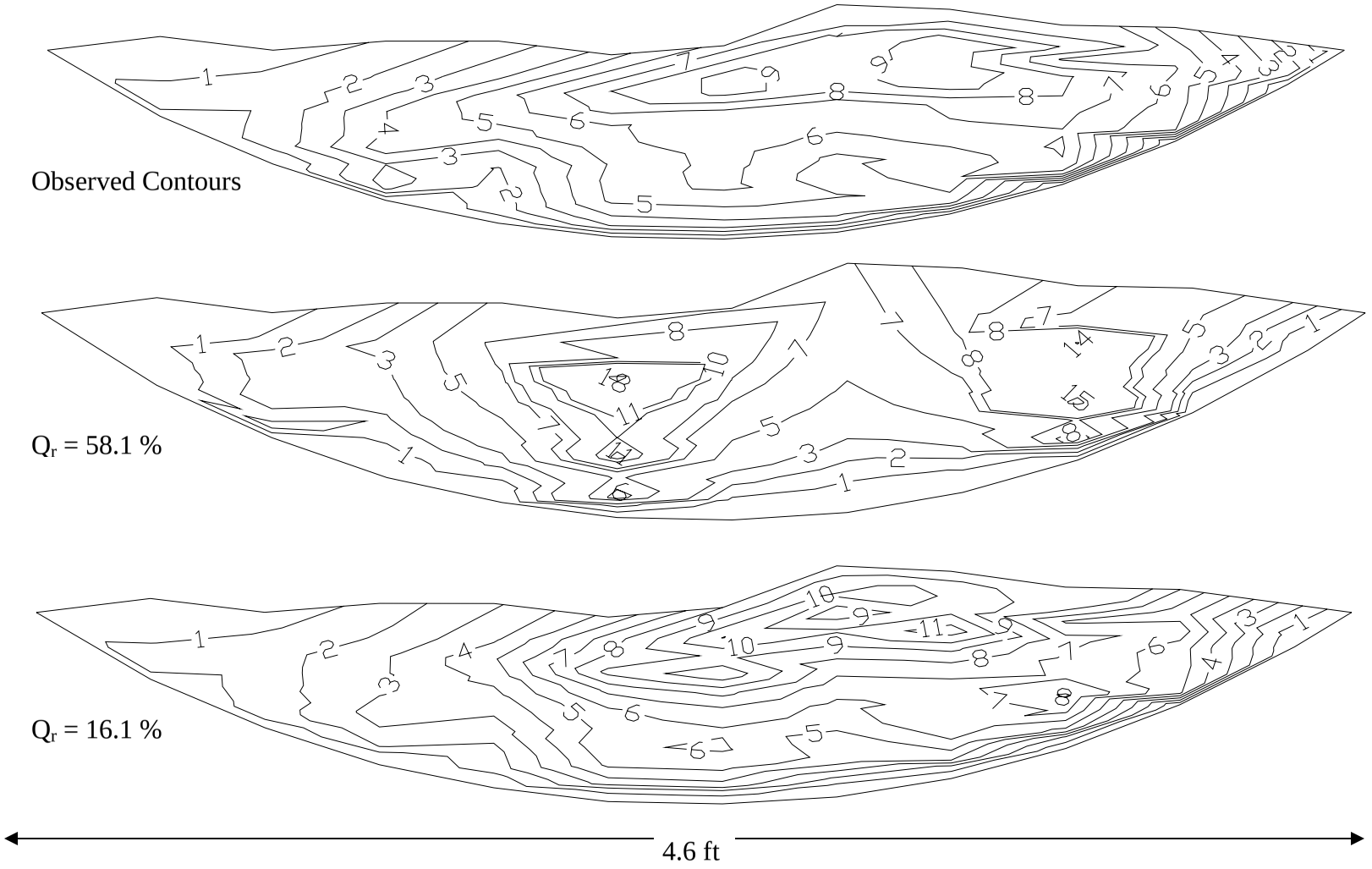


Figure C.11 – Velocity distributions in feet per second for Culvert 4 at 15.04 cfs using the USGS sampling scheme.



Figure C.12 – Velocity distributions in feet per second for Culvert 4 at 13.72 cfs using the USGS sampling scheme.