



Replacement heifer development alternatives: food costs and expected reproductive performance
by Vernon Francis Fogle

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
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Abstract:

The major capital assets of a commercial cow-calf ranch, the cows in the breeding herd, must periodically be replaced. Optimal replacement heifer development policy is jointly determined by the expected productivity of cows presently in the herd, the expected productivity of potential replacements, and the cost of providing replacements.

If the replacements are to be heifer calves produced in the herd, the rancher must decide at weaning time which heifer calves should be kept as replacements and how they should be fed from weaning until the coming breeding season.

It was hypothesized that the occurrence of puberty and thus of pregnancy in replacement heifers can be affected by postweaning feeding practices. Probability of puberty was estimated as a function of heifer age and weight. The estimated equation supports the hypothesis that heifers, if fed for high rates of gain during the wintering period, will be younger and heavier at puberty than if fed for low rates of gain.

Using linear programming techniques, least cost rations were determined for heifer calves varying in weight and fed for four alternative rates of gain. Utilizing these least cost rations, total winter feed costs were estimated for various heifer wintering alternatives, each alternative consisted of specific heifer weight at weaning and a specific rate of gain to be fed for during an 180 day wintering period.

Various replacement heifer development alternatives were then analyzed in terms of costs and expected reproductive performance.

Each alternative consisted of a state of nature, a heifer of specific age and weight at weaning, and an action, the rate of gain to be fed for during the wintering period. The results indicate that for a heifer of any state probability of pregnancy increases and expected time of pregnancy becomes earlier as rate of gain is increased.

This indicates that within limits young light heifers, if fed for higher rates of gain, can achieve the same level of expected reproductive performance as old heavy heifers. However, the benefits of higher probability of pregnancy and earlier expected pregnancy must be weighed against increased feed costs. The inclusion of an opportunity cost of sale of heifer calves at weaning changed the ranking of the alternatives, indicating that costs other than feed costs should be considered when planning replacement policy. Altering the length of the breeding season also changed the ranking, indicating that individual management practices should be considered.

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REPLACEMENT HEIFER DEVELOPMENT ALTERNATIVES:
FEED COSTS AND EXPECTED REPRODUCTIVE PERFORMANCE

by

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ABSTRACT

The major capital assets of a commercial cow-calf ranch, the cows in the breeding herd, must periodically be replaced. Optimal replacement heifer development policy is jointly determined by the expected productivity of cows presently in the herd, the expected productivity of potential replacements, and the cost of providing replacements. If the replacements are to be heifer calves produced in the herd, the rancher must decide at weaning time which heifer calves should be kept as replacements and how they should be fed from weaning until the coming breeding season.

It was hypothesized that the occurrence of puberty and thus of pregnancy in replacement heifers can be affected by postweaning feeding practices. Probability of puberty was estimated as a function of heifer age and weight. The estimated equation supports the hypothesis that heifers, if fed for high rates of gain during the wintering period, will be younger and heavier at puberty than if fed for low rates of gain.

Using linear programming techniques, least cost rations were determined for heifer calves varying in weight and fed for four alternative rates of gain. Utilizing these least cost rations, total winter feed costs were estimated for various heifer wintering alternatives, each alternative consisted of specific heifer weight at weaning and a specific rate of gain to be fed for during an 180 day wintering period.

Various replacement heifer development alternatives were then analyzed in terms of costs and expected reproductive performance. Each alternative consisted of a state of nature, a heifer of specific age and weight at weaning, and an action, the rate of gain to be fed for during the wintering period. The results indicate that for a heifer of any state probability of pregnancy increases and expected time of pregnancy becomes earlier as rate of gain is increased. This indicates that within limits young light heifers, if fed for higher rates of gain, can achieve the same level of expected reproductive performance as old heavy heifers. However, the benefits of higher probability of pregnancy and earlier expected pregnancy must be weighed against increased feed costs. The inclusion of an opportunity cost of sale of heifer calves at weaning changed the ranking of the alternatives, indicating that costs other than feed costs should be considered when planning replacement policy. Altering the length of the breeding season also changed the ranking, indicating that individual management practices should be considered.

Chapter 1

INTRODUCTION

Included in Montana's agricultural sector is a large number of beef cattle operations, the most prevalent of which are commercial cow-calf ranches. The chief income-producing activity of a commercial cow-calf ranch is raising feeder cattle for sale, generally steer calves and those heifer calves which are not kept for replacement of cows to be culled from the breeding herd.

The major capital asset used in producing cattle for beef is the beef cow herd. As is the case with most productive assets, beef cows are periodically replaced. Cows in the herd may be replaced by females of any age. However, it is common practice to replace cows with heifers which are bred as yearlings to produce their first calves as two-year-olds. Optimal replacement policy is jointly determined by the expected productivity of cows presently in the herd, the expected productivity of potential replacements, and the cost of providing replacements.

Replacement heifers can be purchased or raised. There is risk associated with both methods of providing replacements. Heifers that are available for purchase have in most cases been culled from other herds. This would indicate that they are either relatively poor performers or too young to be economically bred as yearlings.

In addition, there is some degree of uncertainty concerning the possible introduction of disease into the herd. Consequently, the common practice is for ranchers to raise their own replacements.

Raising replacements is also subject to conditions of risk and uncertainty. Among the decisions a rancher must make at weaning time are two interrelated decisions concerning replacement policy: Which heifer calves should be kept as replacements? How should the heifers be fed from weaning through breeding? In order to make these decisions the rancher must be able to predict the effects of various feeding alternatives on the future reproductive performance of the replacements and to estimate the costs associated with the feeding alternatives.

Future reproductive performance, as it is used in this research, is defined as weight of calf weaned each year during the heifer's productive life. Future reproductive performance is affected to a large extent by first calf performance. Heifers which calve early in their first calving season tend to calve early in subsequent years, and thus tend to wean older and therefore heavier calves throughout their lifetimes (Lesmeister, et.al. (1973), pp. 3-4). In order to calve early as a two-year-old a heifer must reach puberty and become pregnant by the early part of the breeding season when she is a yearling.

It is hypothesized that the occurrence of puberty and thus of first pregnancy in replacement heifers can be hastened by feeding for high rates of gain during the post weaning period. There is a trade-off involved, however; high rates of gain result in high feed costs.

Purpose of the Study

Replacement policy decisions could more easily be made if there existed a decision model which would enable ranchers to relate the effects of various post weaning feeding alternatives on the occurrence of puberty and pregnancy to the costs associated with the feeding alternatives. The purpose of this research is to construct such a model.

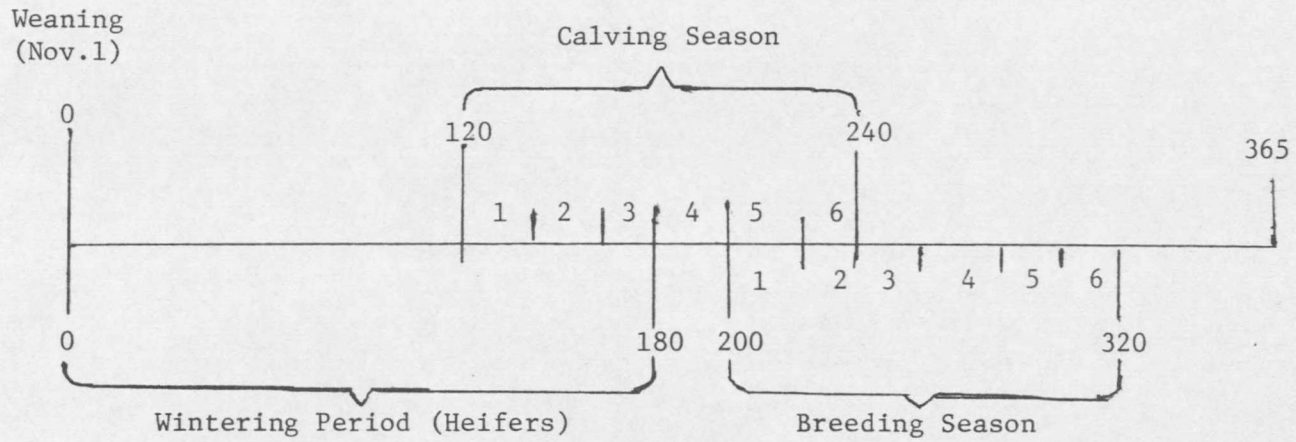
The objectives of this research are:

1. To estimate a probability distribution for the occurrence of puberty in Hereford heifers;
2. To estimate winter feed costs for heifer calves varying in weight at weaning and fed for alternative rates of gain;
3. To estimate probability distributions of pregnancy during the breeding season for heifers of various ages and weights at weaning which are fed for alternative rates of gain; and
4. To evaluate the resulting replacement heifer policy alternatives in terms of cost per bred heifer and expected time of breeding.

Assumptions and Delimitations

Although it would be desirable to include other breeds in this study, the data required for their inclusion are not available. As a result, this study is restricted to Hereford cattle.

The major activities in the production of feeder cattle are winter feeding, calving, breeding, summer grazing, and weaning. The specific timing and duration of the activities varies from ranch to ranch. For purposes of this research, the timing and duration of the activities are assumed to be as presented in Figure 1.1. All calves are weaned at one time in the fall. The first calendar period is a wintering period which for the mature cow herd is assumed to be 120 days in length and 180 days for heifer calves kept as replacements. Cows and heifers are bred during a 120 day breeding season beginning on day 200. The breeding season is divided into six 20-day periods. The 20 day period corresponds to the average length of the estrous cycle in cattle. Calves are born during a 120 day calving season beginning on day 120 of the next year, 285 days after the beginning of the breeding season, which corresponds to the average length of the gestation period for beef cattle. The calving season is divided into six 20-day periods corresponding to the six 20-day periods of the breeding season. If a cow (or heifer) is bred during period 3 of the breeding season, her calf is born during period 3 of the following calving season.



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Figure 1.1 Cow-Calf Calendar

Outline of Procedures

In Chapter 2 a probability distribution will be estimated for the occurrence of puberty by age and weight in Hereford heifers. In Chapter 3, winter feed costs will be estimated for heifers of various weights at weaning and fed for alternative rates of gain. In Chapter 4, the estimated probability distribution of puberty (Chapter 2) will be combined with the assumptions of the Cow-Calf Calendar (Figure 1.1) to estimate probability distributions of pregnancy during the breeding season for heifers of various ages and weights at weaning which are fed for alternative rates of gain. The winter feed cost estimations (Chapter 3) will then be applied to the estimated pregnancy distributions and the replacement heifer policy alternatives will be evaluated in terms of cost per bred heifer and expected time of pregnancy.

Chapter 2

ESTIMATION OF A PROBABILITY DISTRIBUTION FOR THE OCCURRENCE OF PUBERTY IN HEIFERS

In Chapter 1 it was hypothesized that the onset of puberty in heifers, a precondition for pregnancy, can be affected by postweaning feeding practices. In this chapter literature concerning the factors affecting the onset of puberty will be reviewed and a method will be developed for estimating a probability distribution for the occurrence of puberty.

Prediction of Puberty

The onset of puberty in heifers is affected by many factors, both genetic and environmental.

Laster et.al. (1972) showed that average age and weight at puberty varied significantly among breeds and breed crosses (Table 2, page 1033).

A heifer must reach a certain minimum age and a certain minimum weight before puberty can occur (Wiltbank (1973) page 3, and Table 4, page 4).

This relationship between age and weight and the onset of puberty is affected by the growth pathway¹ by which a heifer grows to

¹A growth pathway is the relationship of heifer weight to age from birth to maturity.

maturity. One important component of the growth pathway is the postweaning rate of gain achieved by a heifer during the winter feeding period which is determined by postweaning feeding practices and level of nutrition provided. Wiltbank et.al. (1969), in a study of 74 heifers (16 Hereford, 24 Angus, 15 Angus-Hereford crossbreds, and 19 Hereford-Angus crossbreds) found that, for heifers within all breed groups, heifers fed at high nutritional levels from weaning until puberty were significantly younger and heavier at puberty than heifers fed at low nutritional levels during the same period (Table 4, page 604). Short and Bellows (1971) confirmed these findings (Table 2, page 128). In their study 89 heifers (50 Angus-Hereford cross breeds and 39 Hereford-Angus crossbreds) were assigned at random within breed to three feed groups at weaning time. The groups were fed to gain 0.23, 0.45, and 0.68 kg./da., respectively, through the wintering period, and all heifers were put on pasture in the spring of the year.

In addition to confirming the findings of the Wiltbank study, Short and Bellows observed the phenomenon of compensatory growth. That is, heifers fed at low nutritional levels during the wintering period achieved higher rates of gain on pasture than heifers fed at high nutritional levels during the wintering period. The compensatory gains achieved by the heifers fed at low nutritional levels were insufficient, however, to allow them to reach the same weight by the

following fall as the heifers fed at high nutritional levels (Table 1, page 128). The phenomenon of compensatory growth demonstrates that winter feeding practices affect heifer growth pathways later in life as well as during the winter feeding period.

The results of the Wiltbank et.al. and Short and Bellows studies lead one to hypothesize that there is some degree of tradeoff between age and weight in influencing the onset of puberty. This tradeoff, if it exists, can be exploited through the choice of winter feeding practices. Within limits, a heifer will reach puberty earlier and at a heavier weight if she is fed for a high rate of gain during the wintering period than if she is fed for a low rate of gain during the wintering period.

Varner et.al. (1977) demonstrated that benefits can be obtained by wintering replacement heifers in groups that are similar in certain characteristics, in this case weaning weight. In this study 59 crossbred heifers (one-half Charolais one-fourth Angus, one-fourth Hereford) were divided into three groups at weaning time: heavies (heifers heavier than the group average at weaning), lights (heifers lighter than the group average at weaning), and a control group composed of both heavies and lights. All groups were fed over the winter to achieve the same target weight at the end of the winter feeding period, and all were put on pasture in the spring. The

results show that both heavy and light heifers tended to reach puberty sooner when fed in separate groups than when fed in one group (Table 6, page 170). In addition, the phenomenon of compensatory gain was observed in this study (Table 4, page 169), confirming the findings of Short and Bellows.

In a study of 510 Hereford heifers Arije and Wiltbank (1971) used simple regression analysis and correlation analysis to determine what factors are significantly related to age at puberty and weight at puberty. The results of the regression analysis indicated that age at puberty is affected by day of birth ($p < 0.005$) and postweaning rate of gain ($P < 0.005$), and weight at puberty is affected by day of birth ($P < 0.005$), 205 day adjusted weaning weight ($P < 0.005$), actual weaning weight ($P < 0.005$), and postweaning rate of gain ($P < 0.005$). The correlation analysis indicated that age at puberty is related to weight at puberty ($P < 0.01$), day of birth ($P < 0.01$), rate of gain from birth to weaning ($P < 0.01$), postweaning rate of gain ($P < 0.01$), and weaning weight ($P < 0.01$), and that weight at puberty is related (in addition to age at puberty) to rate of gain from birth to weaning ($P < 0.01$), postweaning rate of gain ($P < 0.01$), day of birth ($P < 0.01$), and weight at weaning ($P < 0.01$). Generally, these relationships are consistent with and are what would be expected from the tradeoff between age and weight in determining the onset of puberty. The exceptions are the relationships between day of birth and age and

weight at puberty. According to the authors, these seeming relationships were due to abnormal local conditions, and thus are not indicative of any true functional relationship.

Arije and Wiltbank (1974) used multiple regression analysis to develop predictive equations for age and weight at puberty for heifers of various breeds. For the 52 Hereford heifers included in this study the following regression equations were estimated:

$$\text{Age-P} = 585 - 0.21B - 0.38W - 200.83A \quad R^2 = 0.414$$

(0.37) (0.35) (110.01)

$$\text{Wt-P} = 67 + 0.71B + 0.83W + 31.58A \quad R^2 = 0.386$$

(0.37) (0.28) (85.51)

Where Age-P = age of puberty

Wt-P = weight at puberty

B = birthdate (day of year, Jan 1 = 1, Feb. 1 = 32, etc.)

W = weight at weaning

A = average daily gain during the winter feeding period.

Numbers in parentheses are the standard errors of the respective parameters.

The statistical tests ($P < 0.05$) indicated that none of the estimated parameters in the age equation were significant, and that in the weight equation only the estimated coefficient for weight at weaning was significant. R^2 measures the proportion of the variation in the dependent variable which is explained by the variables in the

regression equation. The R^2 values, 0.414 and 0.386 for the age and weight equations, respectively, indicate that these equations have relatively weak predictive ability. Part of the problem with this approach is that age at puberty and weight at puberty are interdependent as shown by Arije and Wiltbanks' earlier correlation analysis. This difficulty could possibly be overcome by including both age and weight considerations, complete with interaction terms describing the tradeoff between age and weight at puberty in a single equation, or by using simultaneous equation techniques.

In summary, this review of past research suggests that to predict the onset of puberty in heifers, breed, age, weight, and growth pathway from birth through puberty should be considered.

Past research has concentrated on predicting point values for heifer age and weight at the onset of puberty. In contrast, this study will estimate a probability distribution for the occurrence of puberty at various ages and weights for Hereford heifers. The effect of growth pathway on the onset of puberty will not explicitly be considered in the estimation. The inclusion of age and weight in the estimation will to some degree capture the effect of growth pathway on the onset of puberty. The growth pathway is the relationship of weight to age from birth through maturity, and different growth pathways result in different combinations of age and weight through time. Two representative growth pathways are illustrated in Figure 2.1.

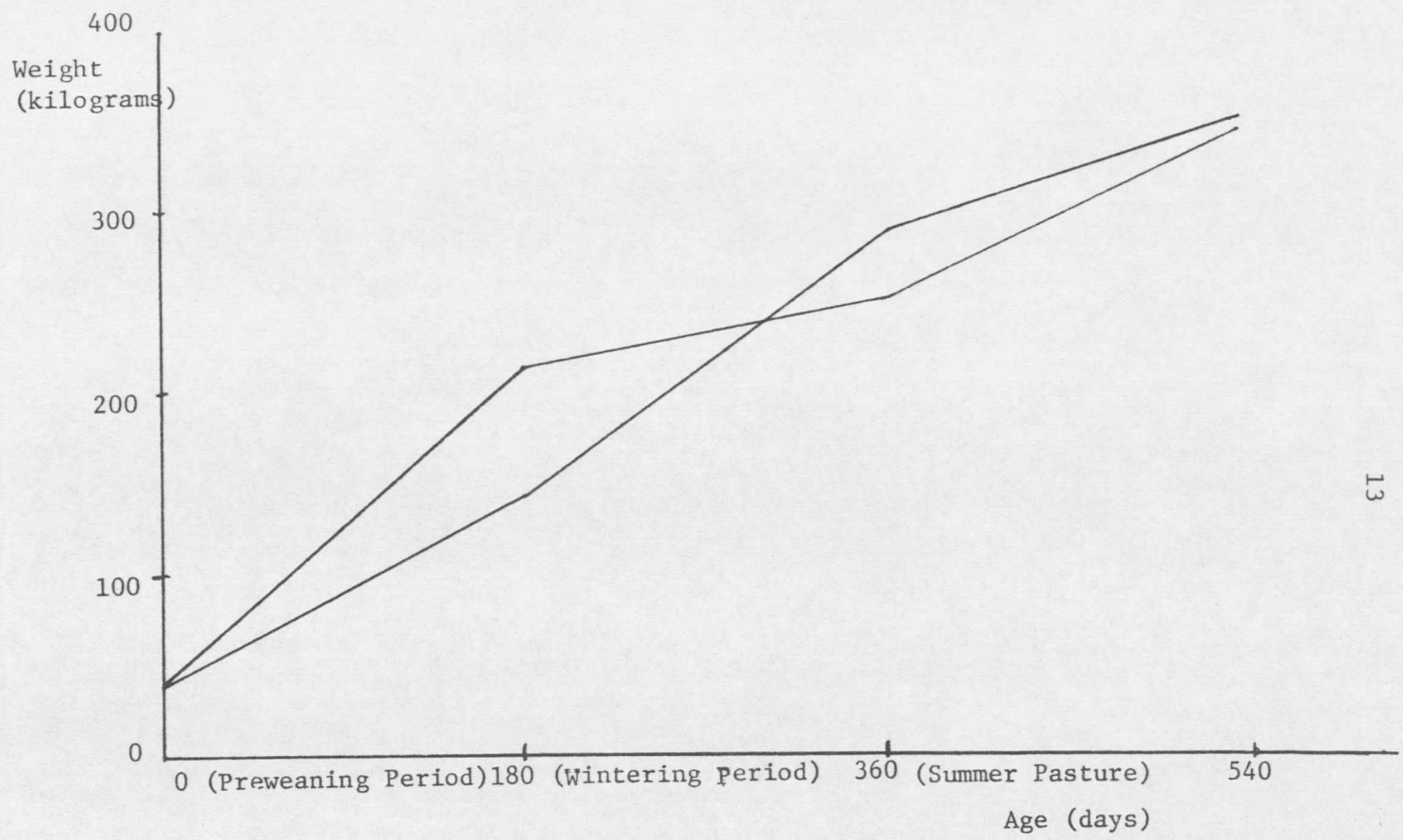


Figure 2.1 Representative Heifer Growth Pathways.

The data used in this study are age and weight at puberty for a group of Hereford heifers. The source of the data to be used is the performance records of 190 Hereford heifers born at the Northern Agricultural Research Center, Havre, Montana, in the years 1968, 1969, and 1970. The data set and a description of pertinent characteristics are presented in Appendix C.

Stepwise, the estimation technique proceeds as follows:

1. Groupings are developed for age and weight at puberty. Age at puberty is divided into 20 day increments ranging from 301 days to 480 days. Weight at puberty is divided into 50 lb. (22.7 kg.) increments ranging from 451 lb. (204.6 kg.) to 850 lb. (385.6 kg.). The size and range of the groupings for age and weight are dictated by the size and distribution characteristics of the sample.
2. The sample is crosstabulated by age and weight to develop a contingency table. Each cell in the table corresponds to an age group-weight group category. The value in each cell is the number of heifers in the sample that reach puberty in the age-weight category. The table thus represents the frequency distribution of the occurrence of puberty. From this table, a second contingency table showing the relative frequency distribution is calculated and is a discrete estimate of the

probability distribution of the occurrence of puberty at the various ages and weights.

3. The cell relative frequencies are then summed over age and weight through each age-weight category. The resultant contingency table represents the cumulative relative frequency distribution of the occurrence of puberty in the sample. That is, the number in each cell is the proportion of the sample that has reached puberty by the given age-weight category. This table is a discrete estimate of the cumulative probability distribution of the occurrence of puberty.
4. Multiple regression techniques are used to convert this discrete estimator into a smooth continuous function by essentially a curve-fitting operation. The dependent variable in the regression is the cumulative probability of the occurrence of puberty. The explanatory variables are age, weight and various age-weight interaction terms. The data set for this regression comes from the contingency table of cumulative probabilities. Each cell is a data point consisting of the cumulative probability and the midpoint values of the age group and weight group.

Estimation of the Probability of Puberty in One Set of Hereford Heifers

The contingency table crosstabulating the sample by age and weight at puberty is presented in Table 2.1. Because of the hypothesized

Table 2.1
 Frequency Distribution of the Occurrence of Puberty in 190 Hereford Heifers

Weight at Puberty (kg.)	Age at Puberty (da.)								
	301- 320	321- 340	341- 360	361- 380	381- 400	401- 420	421- 440	441- 460	461- 480
204.6-226.8				1					
226.9-249.5	2	1	2	1	1	1			
249.6-272.2		4	9	7	9	4	2	1	1
272.3-294.8		5	15	9	7	6	5	3	2
294.9-317.5		3	12	14	6	5	3	4	2
317.6-340.2	1		5	8	7	4	3	4	1
340.3-362.9				5		1		2	
363.0-385.6						1		1	

tradeoff between age and weight at puberty one test statistic of interest is Kendall's tau c.² The statistical test considers the relationships between the classificatory variables (age and weight at puberty). If, in a rectangular table with these dimensions, with variables measured ordinally or cardinally, there is a perfectly positive relationship (all observations lie on the major diagonal) between classificatory variables the tau c value would be +1.14. If there is a perfectly negative relationship (all observations lie on the minor diagonal) between classificatory variables the tau c value would be -1.14. A finding indicating a relationship between the classificatory variables, age and weight at puberty, would suggest inclusion of interaction terms in the regression equation.

The tau c value calculated by the CROSSTABS subroutine³ for this data set is 0.14708 (significance level, $p = 0.00283$), which indicates that there is a small but highly significant positive relationship between age and weight at puberty. This result is consistent with the findings of Arije and Wiltbank (1971). Arije and Wiltbank interpreted their result to mean that heifers that are older at puberty tend to be heavier at puberty.

¹ Nie et.al. (1975), pp. 223,228.

² Ibid., pp. 218-248.

The relative frequency distribution of age and weight at puberty is presented in Table 2.2. The cumulative relative frequency distribution for the occurrence of puberty in the sample is presented in Table 2.3. Since both tables are computed directly from Table 2.1, independent statistical analyses are not appropriate.

One of the objectives of this study was to estimate the probability of puberty in beef heifers at various ages and weights. Table 2.3 provides discrete estimates. The information contained in Table 2.3 may be exploited by multiple regression analysis. Regression analysis will provide a continuous function which may be used to estimate the probability of puberty at any age-weight combination and may be solved for the various age-weight combinations that yield equal probabilities of puberty (iso-probability curves). While this information may be estimated by interpolation from Table 2.3, multiple regression analysis makes full use of the information regarding the interaction between explanatory variables contained in the data set.

The following equation was selected from the multiple regression curve fitting operation.

$$P(\text{Pub}) = 4.591 - 0.01187W - 0.02453A - 0.7657\sqrt{W} - 0.03267\sqrt{A} + \\ (0.00416) \quad (0.005596) \quad (0.1647) \quad (0.2317) \\ + \\ 0.06693\sqrt{W}\sqrt{A} \\ (0.004238)$$

Table 2.2
Relative Frequency Distribution of the Occurrence of Puberty
in 190 Hereford Heifers

Weight at Puberty (kg.)	Age at Puberty (da.)								
	301- 320	321- 340	341- 360	361- 380	381- 400	401- 420	421- 440	441- 460	461- 480
	Proportion								
204.6-226.8				0.0053					
226.9-249.5	0.0105	0.0053	0.0105	0.0053	0.0053	0.0053			
249.6-272.2		0.0211	0.0474	0.0368	0.0474	0.0211	0.0105	0.0053	0.0053
272.3-294.8		0.0263	0.0789	0.0474	0.0368	0.0316	0.0263	0.0158	0.0105
294.9-317.5		0.0158	0.0632	0.0737	0.0316	0.0263	0.0158	0.0211	0.0105
317.6-340.2	0.0053		0.0263	0.0421	0.0368	0.0211	0.0158	0.0211	0.0053
340.3-362.9				0.0263		0.0053		0.0105	
363.0-385.6						0.0053		0.0053	

Table 2.3
 Cumulative Relative Frequency Distribution of the Occurrence
 of Puberty in 190 Hereford Heifers

Weight at Puberty (kg.)	Age at Puberty (da.)								
	301- 320	321- 340	341- 360	361- 380	381- 400	401- 420	421- 440	441- 460	461- 480
Proportion									
204.6-226.8	0.000	0.000	0.000	0.005	0.005	0.005	0.005	0.005	0.005
226.9-249.5	0.011	0.016	0.026	0.037	0.042	0.047	0.047	0.047	0.047
249.6-272.2	0.011	0.037	0.095	0.142	0.195	0.221	0.232	0.237	0.242
272.3-294.8	0.011	0.063	0.200	0.295	0.384	0.422	0.479	0.500	0.516
294.9-317.5	0.011	0.079	0.279	0.447	0.568	0.653	0.705	0.747	0.774
317.6-340.2	0.016	0.084	0.311	0.521	0.679	0.784	0.853	0.916	0.947
340.3-362.9	0.016	0.084	0.311	0.547	0.705	0.816	0.884	0.958	0.989
363.0-385.6	0.016	0.084	0.311	0.547	0.705	0.821	0.889	0.968	1.000

$$R^2 = 0.96$$

$$\text{S.E.E.} = 0.072$$

$$F = 303.7^*$$

Where P(Pub) = cumulative probability for the occurrence of puberty

W = weight (kg.)

A = age (da.)

* = significant at 1 percent level

Numbers in parentheses are the standard errors of the estimated regression coefficients.

The R^2 of 0.96 signifies that 96 percent of the variation in the dependent variable (P(Pub)) in the sample is explained by the variables in the regression equation. The group F test⁴ indicates that the explanatory variables as a group have a highly significant effect on the dependent variable.

Since this is a curve fitting operation, it is not necessary that the explanatory variables, taken individually significantly affect the dependent variable. Because of the high degree of

⁴Ho: $B_1 = B_2 = \dots = B_6 = 0$ vs. Ha: $B_1 \neq B_2 \neq \dots \neq B_6 \neq 0$

The test statistic is $F_{k-1, n-k} = (\text{SSR}/k-1) / (\text{SSE}/N-k)$. The critical $F(0.01)$ is 3.34.

multicollinearity among the explanatory variables (see Table 2.4, Correlation Matrix) it would be expected that statistical tests would indicate that none of the explanatory variables individually have a significant effect on the dependent variable. However, individual t tests⁵ show that all the explanatory variables except \sqrt{A} significantly affect P(Pub) at the 1 percent significance level.

Figure 2.2, showing iso probability curves for 0.25, 0.50, 0.75, and 1.0 probability of puberty, was developed from the regression equation. An iso probability curve is the set of all age-weight combinations for which the cumulative probability for the occurrence of puberty is equal.

Figure 2.2 demonstrates that the estimated probability distribution of puberty in heifers supports the hypothesized tradeoff between age and weight in influencing the occurrence of puberty. Weight can be substituted for age to attain a given level of probability of puberty. The rate of substitution of weight for age at a point (an age and weight) is the negative of the slope of the iso probability curve at that point.

⁵Ho: $B = 0$ vs. $H_A: B \neq 0$)

The test statistic is $t = \hat{B}/S_{\hat{B}}$. The critical value for t (0.01) is 2.58.

Table 2.4
Correlation Matrix for Variables in the Estimated
Regression Equation

	P(Pub)	W	A	\sqrt{W}	\sqrt{A}	$\sqrt{W} \sqrt{A}$
P(Pub)	1.0					
W	0.6696	1.0				
A	0.5716	0.0	1.0			
\sqrt{W}	0.6730	0.9992	0.0	1.0		
\sqrt{A}	0.5750	0.0	0.9996	0.0	1.0	
$\sqrt{W} \sqrt{A}$	0.9032	0.7993	0.5975	0.7999	0.5978	1.0

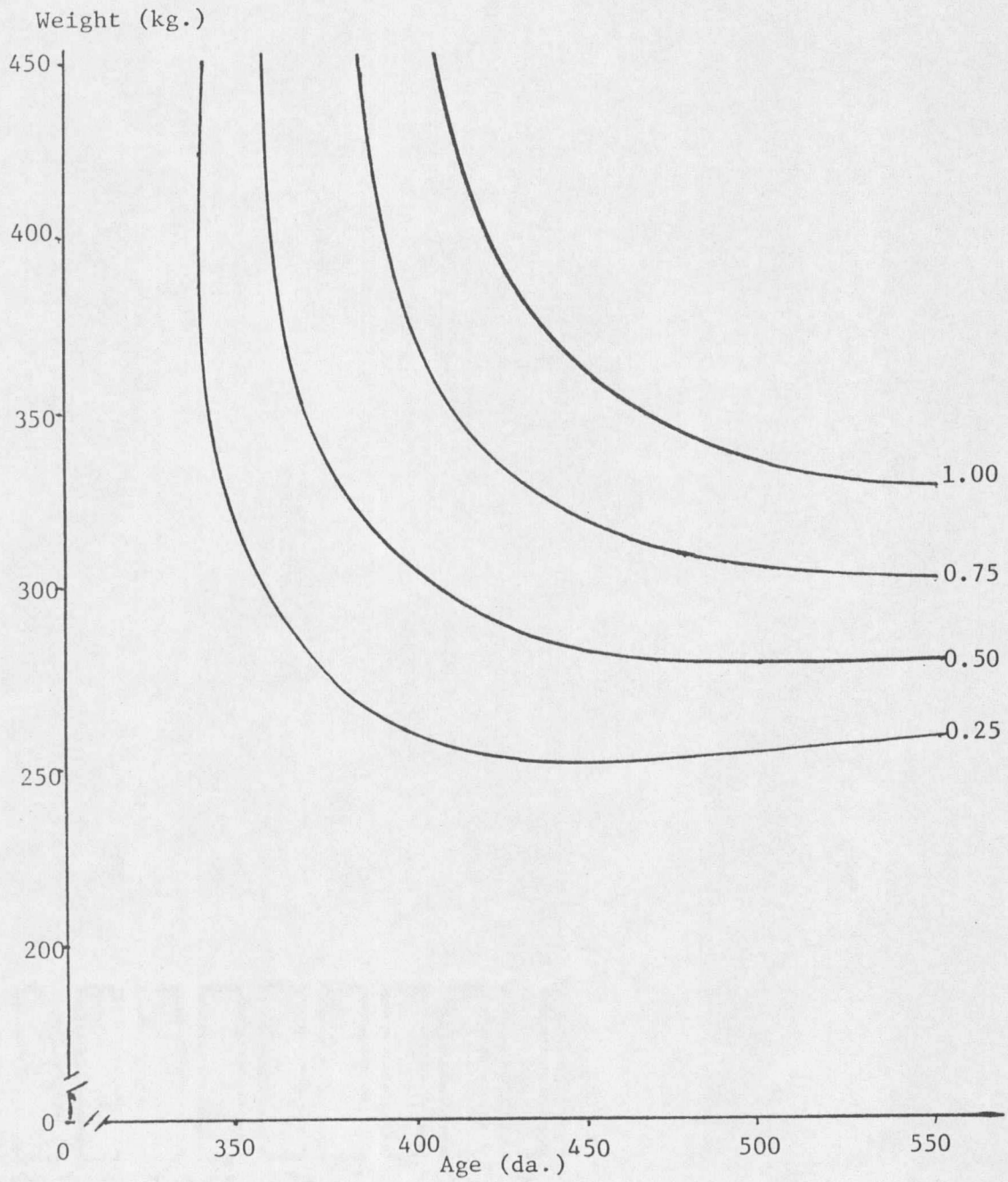


Figure 2.2
Isoprobability Curves for the Occurrence of Puberty by Age and Weight
for Hereford Heifers

Seven functional forms for the regression equation were tried before one was chosen. The choice criteria were goodness of fit (R^2) and the slope of the iso-probability curves generated by the equation. In order to compare goodness of fit between the various equations the R^2 's were adjusted to take into account the different numbers of explanatory variables (Kmenta (1971), page 365).

The adjusted R^2 's for two of the equations were 0.89, and these equations were rejected. The adjusted R^2 for each of the remaining five equations was 0.96.

The iso-probability curves generated by the five remaining regression equations were virtually identical within the range of the data set. At the edges of the range of the data set, however, the curves generated by several of the equations turned rather abruptly and exhibited large positive slopes. An iso-probability curve with a large positive slope would imply that as a heifer grows older and heavier, the probability of her having reached puberty is decreasing. The regression equation which was chosen generated iso-probability contours with areas of slight positive slopes, and only at the lower probability levels, and thus does not lead to this illogical implication.

Summary

In this chapter, the relevant literature concerning puberty in

heifers was reviewed and an estimate for the probability distribution of puberty, based on one set of Hereford heifers, was developed. In Chapter 3 the feed costs associated with several replacement heifer feeding alternatives will be estimated.

Chapter 3

ESTIMATION OF FEED COSTS FOR WINTERING

REPLACEMENT HEIFERS

Heifer calves can be wintered either for sale as yearlings or for replacements. Several costs, including opportunity cost of capital, labor, facilities, and feed costs are incurred when heifers are wintered. Feed cost, which varies with heifer weight, desired rate of gain, and cold stress, is a major cost. Thus least cost rations become an important input in heifer wintering decisions.

LINEAR PROGRAMMING

Linear programming (LP) is a mathematical technique for solving constrained optimization problems; i.e., problems for which the goal is the maximization or minimization of some objective value subject to certain constraints, which can be in the form of equalities or inequalities (Baumol (1972), pp. 70-102). LP is commonly applied to problems of constrained profit maximization and constrained cost maximization. The goal in the formulation of least cost rations for farm animals is to minimize ration costs subject to the constraint that all nutrient requirements are fulfilled.

NUTRIENT REQUIREMENTS

Nutrient requirements for wintering heifers depend on the

desired rate of gain, heifer weight, and cold stress.

Rate of Gain

Since replacement heifer development is the primary consideration of this research, the highest rate of gain considered (for the wintering period) will be 0.8 kg./da. It is felt that higher rates of gain result in heifers becoming fat with little if any increase in real growth. While increased fleshiness is desirable in some situations, previous research has shown that increased fleshiness in replacement heifers may increase breeding and/or calving difficulties. (USDA (1965)). A minimum rate of gain of 0.2 kg./da. will be assumed to assure "normal" growth. Two intermediate rates of gain, 0.4 and 0.6 kg./da. will also be considered.

Heifer Weight

In this study it is assumed that all heifers weigh 36 kg at birth and that the rate of gain from birth to weaning ranges from 0.6 to 1.0 kg./da. Based on the Cow-Calf Calendar (Figure 1.1) and these assumptions, heifers will range in weight from 120 to 270 kg. at weaning. This range in weaning weight is divided into 10 kg. increments, resulting in 16 heifer weaning weights considered.

The winter feeding period for replacement heifers is assumed to last 180 days, from November through April (cow-calf calendar days 1

through 180). Mature cows are commonly fed for about 120 days. The reason for the difference in the length of the winter feeding periods is that, generally, mature cows are being fed to maintain body weight and condition while replacement heifers are being fed to grow. Consequently, mature cows are generally placed on pasture as soon as spring pasture growth is sufficient to meet maintenance requirements, while heifers are kept on feed until pasture grasses are sufficient to sustain growth.

Projected heifer weight when placed on pasture at the end of the winter feeding period is calculated as follows:

$$W_p = W_w + (L \times ADG_w)$$

Where W_p = projected weight on pasture (kg.)

W_w = weight at weaning (kg.)

L = length of the wintering period (days)

ADG_w = rate of gain during the wintering period (kg./da.)

Projected weight on pasture varies from 156 kg. (heifers weighing 120 kg. at weaning and gaining 0.2 kg./da.) to 414 kg. (heifers weighing 270 kg. at weaning and gaining 0.8 kg./da.).

Cold Stress

During the winter months cattle may, and in Montana usually do, require additional energy intake to compensate for energy losses due

to cold stress. The amount of additional energy required depends on the ambient temperature, the body thermal insulation of the animal, and the weight of the animal (Young (1975), Brownson (1976)). Body thermal insulation depends on animal type (beef vs. dairy), age, body condition, length of time exposed to cold weather, amount of hair coat, and wind chill (Brownson, 1975).

Change in Nutrient Requirements During the Winter

Heifer nutrient requirements change over the course of the wintering period because the heifers are increasing in weight and the average ambient temperature varies between months, changing cold stress energy requirements.

Nutrient requirements during the wintering period for heifers of the specified weights at weaning and the specified rates of gain are approximated by estimating nutrient requirements for seven heifer weight groups for each rate of gain during each month of the wintering period, i.e., N_{ijk} where i = weight group 1, 2, ..., 7, (75 - 125, 125 - 175, ... 375 - 425 kg. respectively), j = rate of gain 1, 2, 3, 4 (0.2, 0.4, 0.6, 0.8 kg./da., respectively), and k = month 1, 2, ... 6, (November, December, ... April, respectively). N_{224} , for example, signifies the nutrient requirements of a heifer weighing from 125 to 175 kg., gaining 0.4 kg./da. during February. The use of 50 kilogram heifer weight intervals rather than specific heifer weights for

estimating heifer nutrient requirements greatly reduces the number of least cost rations to be calculated. The estimation of total winter feed costs for the heifer wintering alternatives, which is the objective of this chapter, will not be significantly affected.

Minimum daily nutrient requirements and maximum daily feed (dry matter) intake for heifers of the specified weights and rate of gain are presented in Table 3.1. Minimum nutrient requirements are taken from National Academy of Sciences (1976). Maximum daily feed intakes are estimated from Church and Pond (1974). Church and Pond estimate that maximum daily feed (dry matter) intake for young beef animals varies from 2.5 to 2.75 percent of body weight when going on feed to 2.2 percent of body weight when coming off feed.

The additional energy required by heifers of each weight during each month of the wintering period to compensate for cold stress (Table 3.2) is estimated using a nomogram relating body weight, daily metabolizable energy intake, body thermal insulation, and air temperature (Young, 1975, page 7). Average monthly temperatures for Montana are calculated from U.S. Weather Bureau records for the period 1931 to 1960 (U.S. Weather Bureau, 1963). Thermal insulation value for heifers is estimated to be 24 (Brownson, 1975). The lower critical temperature (the air temperature below which additional energy is required to compensate for cold stress) is assumed to vary

Table 3.1

Daily Nutrient Requirements for Beef Heifers

Body Wt. Kg	Rate of Gain Kg./da.	Minimum Requirements								Max. DM kg.
		NE _m Mcal	NE _{Gl} Mcal	ME Mcal	DP kg	TP kg	Ca g	P g	VitA 1,000 IU	
100	0.2	2.43	0.37	5.67	0.17	0.28	8.0	7.0	6.2	3.06
	0.4	2.43	0.77	6.87	0.23	0.36	12.0	10.0	6.6	3.06
	0.6	2.43	1.21	7.28	0.27	0.40	17.0	13.0	6.6	3.06
	0.8	2.43	1.67	8.03	0.32	0.45	22.0	16.0	6.6	3.06
150	0.2	3.30	0.50	7.69	0.21	0.36	9.0	8.0	8.8	4.40
	0.4	3.30	1.05	9.34	0.28	0.44	12.0	11.0	9.0	4.40
	0.6	3.30	1.64	9.88	0.31	0.48	16.0	13.0	9.0	4.40
	0.8	3.30	2.28	10.94	0.35	0.53	21.0	16.0	9.0	4.40
200	0.2	4.10	0.62	9.55	0.25	0.43	9.0	9.0	10.6	5.62
	0.4	4.10	1.30	11.76	0.32	0.54	12.0	12.0	12.5	5.62
	0.6	4.10	2.03	13.18	0.37	0.60	16.0	15.0	13.0	5.62
	0.8	4.10	2.81	14.21	0.40	0.64	20.0	17.0	13.0	5.62
250	0.2	4.84	0.74	11.30	0.29	0.50	10.0	10.0	12.5	6.75
	0.4	4.84	1.54	13.91	0.36	0.61	13.0	13.0	14.1	6.75
	0.6	4.84	2.40	14.47	0.38	0.63	15.0	14.0	14.1	6.75
	0.8	4.84	3.33	16.22	0.43	0.70	19.0	16.0	14.1	6.75
300	0.2	5.55	0.84	12.93	0.32	0.56	12.0	12.0	14.3	7.77
	0.4	5.55	1.76	15.93	0.39	0.68	14.0	14.0	16.5	7.77
	0.6	5.55	2.75	16.59	0.41	0.68	15.0	15.0	16.5	7.77
	0.8	5.55	3.82	18.60	0.46	0.76	18.0	16.0	16.5	7.77
350	0.2	6.24	0.95	14.56	0.35	0.60	13.0	13.0	15.4	8.72
	0.4	6.24	1.98	17.91	0.42	0.73	15.0	15.0	18.9	8.72
	0.6	6.24	3.09	18.65	0.44	0.74	15.0	15.0	18.9	8.72
	0.8	6.24	4.28	20.88	0.48	0.83	16.0	16.0	18.9	8.72
400	0.2	6.89	1.05	16.08	0.38	0.61	14.0	14.0	17.0	9.56
	0.4	6.89	2.18	19.46	0.44	0.73	16.0	16.0	19.0	9.56
	0.6	6.89	3.41	20.54	0.46	0.75	16.0	17.0	19.0	9.56
	0.8	6.89	4.73	22.76	0.48	0.83	17.0	17.0	19.0	9.56

Table 3.2

Additional ME Required for Cold Stress
(Mcal/da)

Heifer Weight (kg.)	Ave. Temp. (°F)	Month					
		Nov.	Dec.	Jan.	Feb.	Mar.	Apr.
		31.6	23.9	18.3	21.9	29.8	42.7
100		0.73	1.40	1.89	1.58	0.89	0
150		0.90	1.83	2.51	2.07	1.11	0
200		0.99	2.18	3.05	2.49	1.27	0
250		0.98	2.38	3.40	2.74	1.31	0
300		0.92	2.52	3.69	2.94	1.29	0
350		0.79	2.56	3.85	3.02	1.20	0
400		0.61	2.55	3.97	3.06	1.06	0

linearly with heifer weight, from 40°F at 100 kg. to 34°F at 400 kg.

Cold stress tends to stimulate appetite in cattle (Church and Pond, 1974, p. 211, and National Academy of Sciences, 1976, p. 2). Consequently, during periods of cold stress maximum daily feed intake is increased. It is assumed in this research that heifers will increase their feed intake by the amount necessary to provide the increment in energy required to compensate for cold stress. Since the concentration of metabolizable energy in feedstuffs ranges from about 2 to 3 Mcal./kg., it is assumed that maximum diet dry matter intake is increased by 1.0 kg. for every 2.5 Mcal. of metabolizable energy required for cold stress. These increases in allowable feed intake are presented in Table 3.3.

FEEDSTUFFS

Straw, corn silage, soybean meal, urea, dicalcium phosphate and monosodium phosphate, as well as feedstuffs more commonly used in Montana, are considered for inclusion in heifer wintering rations. Straw and corn silage are considered because they are potential low cost sources of some nutrients. Soybean meal and urea are included to ensure that protein requirements are satisfied. Dicalcium phosphate and monosodium phosphate are included to ensure that calcium and phosphorous requirements are satisfied.

Table 3.3.

Additional Allowable Feed (Dry Matter)
Intake (kg./da.) during Cold Stress Months

Heifer Weight (kg.)	Month					
	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.
100	0.29	0.56	0.76	0.63	0.36	0
150	0.36	0.73	1.00	0.83	0.44	0
200	0.40	0.87	1.22	1.00	0.51	0
250	0.39	0.95	1.36	1.10	0.52	0
300	0.37	1.01	1.48	1.18	0.52	0
350	0.32	1.02	1.54	1.21	0.48	0
400	0.24	1.02	1.59	1.22	0.42	0

The respective nutrient compositions and per unit prices of the feedstuffs are presented in Table 3.4. The nutrient composition of range cake was estimated from information on the feed tag. The nutrient compositions of the other feedstuffs were taken from National Academy of Sciences (1976). The per unit prices are those that existed in mid-1978. No active market existed for alfalfa hay, corn silage, and straw. The prices for these feedstuffs were estimated from prices during the preceding winter.

The inclusion of certain of these feedstuffs in heifer rations requires that additional constraints be imposed on the rations. To prevent urea toxicity, urea must be limited to no more than 1 percent of the ration, and no more than one-third of the nitrogen in the ration may come from urea (National Academy of Sciences, 1976, p. 6). The maximum amount of straw (dry matter) which cattle will consume is approximately 1 percent of body weight per day (Church and Pond, 1974, p. 216). Wheat grain is limited to no more than 20 percent of a ration to prevent protein digestion difficulties. Corn silage is limited to no more than 20 percent of a ration to ensure that the ration is at least 80 percent dry matter.

LEAST COST RATIONS

Given heifer nutrient requirements (N_{ijk}), the additional ration

Table 3.4
Prices and Nutrient Compositions (As Fed Basis)
of Feedstuffs Considered

Number	Feedstuff	Reference Number (NRC)	Price (\$/T.)
1	Alfalfa Hay (S-C,e-b)	1-00-059	65
2	Corn Silage (40% DM)	3-08-153	21
3	Barley Grain	4-00-549	105
4	Oat Grain	4-03-309	103
5	Wheat Grain (HRW)	4-05-268	115
6	Barley Straw	1-00-498	15
7	Oat Straw	1-03-283	15
8	Wheat Straw	1-05-175	15
9	Beet Pulp w/molasses	4-00-672	135
10	Soybean Meal (solv.ext.)	5-04-604	280
11	Molasses Cake (20% prot.)		150
12	Dicalcium Phosphate	6-01-080	400
13	Monosodium Phosphate	6-04-288	640
14	Urea		260

Symbol	Nutrient	Units
NEM	Net Energy for Maintenance	MCal./kg.
NEG	Net Energy for Gain	MCal./kg.
ME	Metabolizable Energy	MCal./kg.
DP	Digestible Protein	kg./kg.
TP	Total Protein	kg./kg.
CA	Calcium	g./kg.
P	Phosphorous	g./kg.
DM	Dry Matter Concentration	kg./kg.
VIT A	Vitamin A	1,000 IU/kg.

Table 3.4 Prices and Nutrient Compositions of Feedstuffs Considered.

FEEDSTUFF	NUTRIENT									
	NEM	NEG	ME	DP	TP	CA	P	DM	VIT A	
1	1.10	.50	1.85	.114	.166	11.3	2.1	.900	45.8	
2	.62	.40	1.01	.019	.032	1.1	.8	.400	.0	
3	1.90	1.25	2.67	.087	.116	.8	4.2	.890	.0	
4	1.54	1.01	2.45	.088	.117	1.0	3.5	.890	.0	
5	1.92	1.27	2.83	.102	.130	.5	5.1	.891	.0	
6	.89	.12	1.31	.004	.036	3.0	.8	.882	.0	
7	1.00	.32	1.69	.013	.040	3.0	.9	.901	.0	
8	.93	.17	1.57	.004	.032	1.5	.7	.901	.0	
9	1.87	1.23	2.47	.060	.091	5.6	1.0	.920	.0	
10	1.72	1.15	2.61	.390	.458	3.2	6.7	.890	.0	
11	1.90	1.23	2.69	.160	.200	7.0	7.0	.900	66.1	
12	.00	.00	.00	.000	.000	222.2	179.2	1.000	.0	
13	.00	.00	.00	.000	.000	.0	217.4	1.000	.0	
14	.00	.00	.00	2.810	2.810	.0	.0	1.000	.0	

constraints, feedstuff nutrient compositions, and feedstuff prices, linear programming techniques are used to determine least cost rations for each heifer weight range and rate of gain during each month of the wintering period. The least cost rations are designated R_{ijk} where, as for nutrient requirements (N_{ijk}), i = weight group 1, 2, ... 7 (75 - 125, 125 - 175, ... 375 - 425 kg. respectively), j = rate of gain 1, 2, 3, 4 (0.2, 0.4, 0.6, 0.8 kg./da., respectively), and k = month 1, 2, ... 6 (November, December, ... April, respectively).

NHILP, a mixed integer linear programming computer algorithm developed by Verner G. Hurt was used to determine the least cost rations. These least cost rations and the cost per day for each ration are presented in Appendix A.

The major constituents of the rations are roughages (alfalfa hay, corn silage, and oat straw) and concentrates (barley and wheat). In addition to these major constituents some rations contain a small amount (less than 1 percent) of dicalcium phosphate, and one ration contains a very small amount of urea. All rations contain the maximum allowable amount of corn silage (20 percent of the ration), and all but one (R_{141}) contain the maximum allowable amount of straw (1 percent of body weight per day).

The rations for the lowest rate of gain (0.2 kg./da.) are composed entirely of roughages, being 20 percent corn silage, 25 to 40 percent oat straw, and 40 to 55 percent alfalfa hay. For each successively

higher rate of gain, the proportion of alfalfa hay in the rations decreases, being replaced by concentrates, usually barley but sometimes barley and wheat. At the highest rate of gain (0.8 kg./da.) the rations consist of 10 to 20 percent alfalfa hay, 30 to 45 percent barley (or barley and wheat), 25 to 40 percent oat straw, and 20 percent corn silage.

TOTAL WINTER FEED COSTS

To calculate the total winter feed cost associated with a heifer wintering alternative, it is necessary to determine which least cost rations are required and the number of days each is required. Consider for example, a heifer weighing 200 kg. at weaning and gaining 0.6 kg./da. This heifer will be classified in weight group 3 (175 - 225 kg.) for the first 42 days of the wintering period, then in weight group 4 (225 - 275 kg.) until day 125 of the wintering period, and in weight group 5 (275 - 325 kg.) for the remainder of the wintering period. Total winter feed cost for this heifer is calculated as follows:

Month	Ration	Cost/Day	Days	Cost
November	R331	0.444	29	12.786
December	R332	0.490	13	6.370
	R432	0.502	18	9.036
January	R433	0.543	31	16.833
February	R434	0.515	28	14.420
March	R435	0.459	6	2.754
	R535	0.511	25	12.775
April	R536	0.459	30	13.770
Total Winter Feed Cost				\$88.83

Total winter feed costs for all heifer wintering alternatives are calculated in this manner and are presented in Table 3.5.

CHANGES IN FEEDSTUFF PRICES

The prices of feedstuffs in Montana vary considerably over time, both in absolute terms and relative to one another. The relationship of roughage prices to concentrate prices is important if alternative rates of gain are being considered. As rate of gain is increased, the proportion of concentrates in a ration must be increased to provide the increase in energy required for growth. Consequently, high rates of gain become more expensive relative to low rates of gain if concentrate prices increase relative to roughage prices.

Table 3.5

Feed Costs for Wintering Heifers

Weaning Weight	Rate of Gain During Winter Feeding (kg./da.)			
	0.2	0.4	0.6	0.8
(kg.)	(dollars)			
120	44.37	60.20	73.21	89.02
130	45.65	62.79	75.96	91.28
140	45.85	64.74	78.19	93.23
150	48.02	66.84	80.16	97.10
160	50.88	69.50	82.30	100.03
170	53.64	72.38	84.61	103.04
180	54.12	74.48	86.65	105.21
190	54.93	75.92	87.76	107.32
200	56.83	77.47	88.83	109.53
210	59.18	79.70	89.97	111.57
220	61.44	82.17	91.24	113.62
230	62.36	84.04	93.21	115.47
240	62.53	85.39	95.05	117.70
250	64.18	86.79	96.78	120.01
260	66.18	88.94	98.63	122.17
270	68.13	91.27	100.63	124.32

The effects on heifer wintering rations of changing the price relationship between concentrates and roughages can be investigated by lowering the price of alfalfa hay from \$65/T. to \$40/T. and computing representative least cost rations for light, medium, and heavy heifers at the four rates of gain. Since, at the given prices for corn silage and straw, the least cost rations contain the maximum allowable amounts of these roughages, lowering these prices would not affect the composition of the rations. Examples of light, medium, and heavy heifers are heifers weighing 120, 200, and 270 kg. at weaning, respectively.

The representative least cost rations calculated are those that would be required at mid-winter given heifer weaning weight and rate of gain. The composition and cost per day of the representative rations, with alfalfa hay priced at \$65/T. and \$40/T., are presented in Table 3.6. Lowering the price of alfalfa hay from \$65/T. to \$40/T. does not change the composition of the representative least cost rations for light, medium, or heavy heifers gaining 0.2 kg./da., but ration costs are reduced approximately 30 percent. At higher rates of gain alfalfa hay tends to replace part of the concentrates in the rations, and wheat replaces barley in the concentrate portion of the ration. The reduction in ration costs decreases as rate of gain increases, and ration costs for heavy heifers are decreased proportionately more than ration costs for light heifers.

Table 3.6

Effect on Representative Least Cost Rations
of Changing the Price of Alfalfa Hay

Ration Number	Alfalfa Hay Price \$/T.	Alfalfa Hay kg.	Corn Silage kg.	Barley kg.	Wheat kg.	Oat Straw kg.	Cost \$/da.
Light Heifers							
213	65	3.31	1.24			1.66	0.294
213	40	3.31	1.24			1.66	0.202
223	65	2.83	1.35	0.91		1.66	0.367
223	40	2.97	1.35		0.77	1.66	0.287
233	65	2.20	1.35	1.55		1.66	0.396
233	40	2.44	1.35		1.31	1.66	0.332
343	65	0.75	1.71	3.39	0.47	2.22	0.582
343	40	0.98	1.71	1.92	1.71	2.22	0.558
Medium Heifers							
313	65	3.94	1.54			2.22	0.355
313	40	3.94	1.54			2.22	0.246
423	65	4.46	2.03	0.87		2.77	0.513
423	40	4.60	2.03		0.73	2.77	0.388
433	65	3.80	2.03	1.53		2.77	0.543
433	40	4.04	2.03		1.29	2.77	0.435
443	65	1.74	2.03	3.61		2.77	0.636
443	40	2.12	2.03	1.20	2.03	2.77	0.583
Heavy Heifers							
513	65	4.83	2.04			3.33	0.448
513	40	4.83	2.04			3.33	0.315
523	65	5.25	2.33	0.73		3.33	0.569
523	40	5.37	2.33		0.61	3.33	0.422
533	65	4.48	2.33	1.51		3.33	0.604
533	40	4.71	2.33		1.27	3.33	0.477
643	65	2.05	2.59	4.40		3.88	0.780
643	40	2.53	2.58	1.39	2.58	3.88	0.717

SUMMARY

In this chapter winter feed costs were estimated for heifers varying in weight at weaning and fed for alternative rates of gain. In Chapter 4 probability distributions of pregnancy during the breeding season will be estimated for heifers varying in age and weight at weaning and fed for alternative rates of gain. Heifer wintering alternatives will then be evaluated in terms of cost per bred heifer and expected time of breeding.

Chapter 4

EVALUATION OF REPLACEMENT HEIFER DEVELOPMENT ALTERNATIVES- COSTS AND EXPECTED REPRODUCTIVE PERFORMANCE

In this chapter probability distributions for the occurrence of pregnancy during the coming breeding season will be estimated for various replacement heifer development alternatives. The alternatives will then be evaluated in terms of cost per bred heifer and expected time of pregnancy.

Replacement Heifer Development Alternatives

Each replacement heifer development alternative is defined by a state of nature (a heifer of specified age and weight at weaning time) and an action (the rate of gain to be fed for during the wintering period). For each state of nature there are four actions considered, rates of gain of 0.2, 0.4, 0.6, and 0.8 kg./da.

From the Cow-Calf Calender (Figure 1.1, p. 5) it can be seen that age at weaning for heifer calves varies from 135 days (mean age of heifers born during period 6 of the calving season) to 235 days (mean age of heifers born during period 1 of the calving season). Six specific ages at weaning are considered, each corresponding to the mean age of heifers born during one of the six periods of the calving season (see Table 4.1). Assuming that heifers weigh 36 kg. at birth and gain 0.6 to 1.0 kg./da. from birth until weaning, the

Table 4.1

Age and Weight at Weaning of Heifer Calves

<u>Period of Birth</u>	<u>Age at Weaning</u> (da.)	<u>Range in Weight at Weaning</u> (kg.)
1	235	180 - 270
2	215	160 - 250
3	195	150 - 230
4	175	140 - 210
5	155	130 - 190
6	135	120 - 170

range in weight at weaning for heifers of each age at weaning is as shown in Table 4.1. The ranges in weight at weaning are divided into 10 kg. increments so that for each age at weaning six to ten specific weights at weaning are considered.

PROBABILITY DISTRIBUTION OF PREGNANCY

The estimated probability distribution of pregnancy depends on the probability distribution of puberty during the breeding season and on conception rate, i.e., the probability of a heifer becoming pregnant given that she has reached puberty and is inseminated. The probability distribution of puberty is a function of heifer age and weight during the breeding season (Page 24, Chapter 2). Consequently, in order to estimate the probability distributions for puberty during the breeding season, heifer age and weight during each period of the breeding season must be estimated for each replacement heifer development alternative. Given the age of a heifer at weaning, age during each of the periods of the breeding season can be determined directly from the Cow-Calf Calendar (Figure 1.1, page 5). Since heifers are put on pasture twenty days prior to the beginning of the breeding season, the estimation of heifer weights during each of the periods of the breeding season requires that rates of gain on pasture be estimated.

Rate of Gain on Pasture

The rate of gain that a heifer will achieve on pasture depends on quality of pasture, stocking rate, and the size and condition of the heifer. Quality of pasture depends on soil and vegetation types and climatic conditions, and is highly variable. Stocking rate is an optimization problem in itself and cannot be treated specifically in this study. Size and condition of the heifer, for purposes of this study, will be described by the heifer's weight when placed on pasture and the rate of gain achieved during the wintering period, respectively.

Short and Bellows (1970), in a study of 89 Angus-Hereford and Hereford-Angus crossbred heifers at the Livestock and Range Research Station (LAARS), Miles City, Montana, observed the phenomenon of compensatory growth. Heifers fed to achieve low rates of gain during the wintering period achieved higher rates of gain on pasture than did heifers fed to achieve high rates of gain during the wintering period. The results (from Short and Bellows, Table 2, page 128) are presented in Table 4.2.

It can be seen from Table 4.2 that rate of gain on pasture (G_p) is inversely related to both weight when placed on pasture (W_p) and rate of gain during the wintering period (G_w). The information in Table 4.2 can be used to obtain an estimator for rate of gain on pasture. The methodology consists of regressing G_p on G_w to obtain one estimator ($\hat{G}_{p1} = f(G_w)$), regressing G_p on W_p to obtain another

Table 4.2

Rate of Gain of Pasture

<u>Winter Feed Level</u>	<u>Rate of Gain Winter</u> (kg./da.)	<u>Weight on Pasture</u> (kg.)	<u>Rate of Gain on Pasture</u> (kg./da.)
Low	0.283	189	0.599
Medium	0.447	218	0.525
High	0.678	254	0.420

Source: Short and Bellows (1971)

estimator ($G_{p2} = g(W_p)$), and averaging these two estimators to obtain an estimator for G_p which is a function of both G_w and W_p .

$$(G_p = 1/2(G_{p1} + G_{p2})) = 1/2(f(G_w) + g(W_p)).$$

The resultant estimators are shown below:

$$G_{pi} = 0.727 - 0.4533 G_w$$

$$R_2 = 0.99 + 0.0.F = 1$$

$$G_{p2} = 1.123 - 0.00276 W_p$$

$$R_2 = 0.99 + D.O.F. = 1$$

$$G_p = 0.925 - 0.2266 G_w - 0.00138 W_p$$

Pahnish et. al. (1971), in a study of heifers born at LARRS in 1962 through 1965, found that the mean values for rate of gain during the wintering period, weight when placed on pasture, and rate of gain on pasture did not vary significantly between Herefords and Hereford-Angus and Angus-Hereford crossbreds. Therefore, although the estimator for rate of gain on pasture was developed using information on crossbreds, it is directly applied to Herefords.

The estimator for rate of gain on pasture was not rigorously derived in a statistical sense. However, when applied to data in the Pahnish et. al. study, the estimator predicted rates of gain on pasture similar to the reported rates of gain on pasture.

Management of the heifers in the Short and Bellows' experiment was similar to that of many cow-calf ranches, and pasture conditions

at LARRS are similar to ranches in eastern Montana. It is assumed that rates of gain on pasture predicted by the estimator pertain to range conditions existing in a normal year. Climatic aberrations such as less than normal rainfall, cold spring, or dry summer could inhibit the growth of pasture grasses to the extent that these rates of gain would not be attained.

Probability Distribution of Puberty

Heifer age and weight during each of the periods of the breeding season for each of the replacement heifer development alternatives can now be estimated utilizing knowledge of age and weight at weaning, rates of gain during the winter and on pasture, and the time relationship of activities dictated by the Cow-Calf Calendar (Figure 1.1, page 5). The age and weight values are presented in Table B.1, Appendix B. The probability of puberty occurring in each of the periods of the breeding season for each of the replacement heifer development alternatives is estimated by applying the puberty estimation equation (page 11, Chapter 2) to the appropriate age and weight values. The resultant probability distributions for puberty are also presented in Table B.1, Appendix B.

Conception Rate

There is no evidence that conception rate varies from estrus to estrus. Therefore, conception rate is assumed to be the same for

all periods of the breeding season. Laster et. al. (1972), in a study of 538 heifers estimated conception rate to be 0.60 (Table 6, page 1036). Cundiff et. al. (1974) reported a range of 0.42 to 0.72 for conception rate in beef heifers. In this study conception rate is assumed to be 0.60.

Probability Distribution of Pregnancy

The probability distribution of pregnancy during the coming breeding season for each of the replacement heifer development alternatives can be estimated using the following equation:

$$PB(n) = C \left[Pp(n) - \sum_{i=1}^{n-1} PB(i) \right]$$

Where n = period of the breeding season, 1, 2, ... 6

$PB(n)$ = probability of the heifer becoming pregnant during period n

C = conception rate, 0.60

$Pp(n)$ = probability that the heifer has reached puberty by period n (from Table B.1, Appendix B).

The probability distributions of pregnancy during the breeding season for all the replacement heifer wintering alternatives are presented in Table 4.3.

COSTS AND REPRODUCTIVE PERFORMANCE

Analysis of replacement heifer development alternatives will focus on the costs associated with each alternative, expected reproductive performance, and the trade-off between costs and reproductive performance.

The analysis first considers only feed costs (estimated in Chapter 3) and a 120 day breeding season. These rather restrictive assumptions will later be relaxed in a discussion of other costs and shorter breeding seasons.

Measures of Reproductive Performance

One measure of reproductive performance is the probability of a heifer becoming pregnant during the breeding season. Probability of pregnancy during the breeding season is calculated as the sum of the probabilities of becoming pregnant during each of the periods of the breeding season (Table 4.3); i.e. $P(\text{Pregnancy}) = \sum_{n=1}^6 PB(n)$. Ceteris paribus, higher probabilities of pregnancy are preferred to lower probabilities. Another measure of reproductive performance is expected breeding period. Ceteris paribus, it is preferred that

Table 4.3 Probability Distribution of Pregnancy During the Breeding Season for Replacement Heifer Development Alternatives.

AGE AT WEANING (DA)	WEIGHT AT WEANING (KG)	RATE OF GAIN (KG/DA)	*	PERIOD OF THE BREEDING SEASON					
				1	2	3	4	5	6
135.	120.	.2	*	.000	.000	.000	.000	.041	.077
135.	120.	.4	*	.000	.009	.050	.071	.084	.093
135.	120.	.6	*	.047	.071	.083	.089	.094	.098
135.	120.	.8	*	.106	.107	.107	.106	.105	.105
135.	130.	.2	*	.000	.000	.000	.027	.067	.089
135.	130.	.4	*	.000	.038	.065	.080	.089	.097
135.	130.	.6	*	.066	.083	.091	.096	.099	.102
135.	130.	.8	*	.120	.117	.115	.113	.111	.109
135.	140.	.2	*	.000	.000	.012	.058	.081	.095
135.	140.	.4	*	.016	.056	.075	.086	.094	.100
135.	140.	.6	*	.083	.094	.099	.102	.104	.106
135.	140.	.8	*	.132	.127	.122	.119	.116	.114
135.	150.	.2	*	.000	.000	.047	.074	.089	.100
135.	150.	.4	*	.037	.068	.083	.092	.099	.104
135.	150.	.6	*	.099	.105	.107	.108	.109	.110
135.	150.	.8	*	.143	.136	.130	.126	.122	.119
135.	160.	.2	*	.000	.028	.063	.082	.094	.104
135.	160.	.4	*	.057	.080	.091	.098	.104	.108
135.	160.	.6	*	.114	.115	.115	.115	.115	.115
135.	160.	.8	*	.153	.144	.137	.132	.128	.124
135.	170.	.2	*	.005	.052	.076	.090	.099	.107
135.	170.	.4	*	.076	.091	.099	.104	.108	.112
135.	170.	.6	*	.127	.124	.122	.121	.120	.119
135.	170.	.8	*	.162	.152	.145	.138	.133	.129
155.	130.	.2	*	.000	.000	.000	.018	.065	.089
155.	130.	.4	*	.003	.046	.068	.080	.089	.096
155.	130.	.6	*	.099	.093	.093	.094	.096	.099
155.	130.	.8	*	.175	.134	.117	.109	.106	.104
155.	140.	.2	*	.000	.000	.008	.057	.082	.097
155.	140.	.4	*	.033	.061	.076	.086	.093	.099

Table 4.3 Continued.

AGE AT WEANING (DA)	WEIGHT AT WEANING (KG)	RATE OF GAIN (KG/DA)	*	PERIOD OF THE BREEDING SEASON					
				1	2	3	4	5	6
155.	140.	.6	*	.123	.107	.101	.100	.101	.102
155.	140.	.8	*	.193	.146	.125	.116	.111	.108
155.	150.	.2	*	.000	.000	.049	.075	.090	.101
155.	150.	.4	*	.061	.076	.085	.092	.097	.102
155.	150.	.6	*	.145	.119	.109	.106	.106	.106
155.	150.	.8	*	.210	.157	.133	.122	.116	.113
155.	160.	.2	*	.000	.035	.067	.084	.095	.104
155.	160.	.4	*	.088	.090	.094	.098	.102	.106
155.	160.	.6	*	.166	.131	.117	.112	.110	.110
155.	160.	.8	*	.226	.167	.141	.128	.121	.117
155.	170.	.2	*	.019	.058	.078	.090	.100	.107
155.	170.	.4	*	.113	.103	.102	.103	.106	.109
155.	170.	.6	*	.185	.143	.125	.118	.115	.114
155.	170.	.8	*	.240	.177	.149	.135	.127	.104
155.	180.	.2	*	.049	.073	.086	.096	.103	.110
155.	180.	.4	*	.135	.116	.110	.109	.111	.113
155.	180.	.6	*	.203	.154	.133	.124	.120	.118
155.	180.	.8	*	.254	.186	.156	.141	.132	.079
155.	190.	.2	*	.076	.087	.095	.101	.107	.113
155.	190.	.4	*	.157	.128	.118	.115	.115	.117
155.	190.	.6	*	.219	.164	.141	.131	.125	.123
155.	190.	.8	*	.266	.195	.163	.147	.137	.055
175.	140.	.2	*	.000	.000	.000	.053	.082	.098
175.	140.	.4	*	.042	.064	.077	.086	.093	.099
175.	140.	.6	*	.154	.116	.102	.098	.098	.099
175.	140.	.8	*	.245	.161	.127	.112	.106	.103
175.	150.	.2	*	.000	.000	.044	.074	.091	.102
175.	150.	.4	*	.077	.081	.086	.091	.096	.101
175.	150.	.6	*	.182	.131	.111	.104	.102	.103
175.	150.	.8	*	.268	.174	.135	.119	.111	.107

Table 4.3 Continued.

AGE AT WEANING (DA)	WEIGHT AT WEANING (KG)	RATE OF GAIN (KG/DA)	*	PERIOD OF THE BREEDING SEASON					
				1	2	3	4	5	6
175.	160.	.2	*	.000	.035	.067	.085	.096	.105
175.	160.	.4	*	.110	.097	.095	.097	.100	.104
175.	160.	.6	*	.209	.145	.120	.110	.107	.106
175.	160.	.8	*	.289	.187	.144	.125	.116	.084
175.	170.	.2	*	.025	.060	.079	.091	.100	.107
175.	170.	.4	*	.141	.113	.104	.102	.104	.107
175.	170.	.6	*	.234	.158	.128	.116	.111	.110
175.	170.	.8	*	.309	.198	.152	.131	.121	.054
175.	180.	.2	*	.061	.077	.088	.096	.103	.110
175.	180.	.4	*	.170	.127	.112	.108	.108	.110
175.	180.	.6	*	.257	.171	.136	.122	.116	.113
175.	180.	.8	*	.327	.210	.160	.137	.100	.040
175.	190.	.2	*	.095	.093	.097	.102	.107	.112
175.	190.	.4	*	.197	.141	.121	.114	.113	.113
175.	190.	.6	*	.279	.184	.144	.128	.121	.087
175.	190.	.8	*	.345	.221	.167	.143	.074	.030
175.	200.	.2	*	.127	.109	.105	.107	.111	.115
175.	200.	.4	*	.223	.155	.129	.120	.117	.117
175.	200.	.6	*	.300	.195	.152	.134	.125	.056
175.	200.	.8	*	.361	.231	.175	.140	.056	.022
175.	210.	.2	*	.157	.124	.114	.112	.114	.118
175.	210.	.4	*	.247	.168	.137	.125	.121	.120
175.	210.	.6	*	.319	.207	.160	.140	.105	.042
175.	210.	.8	*	.376	.241	.183	.120	.048	.019
195.	150.	.2	*	.000	.000	.032	.071	.091	.103
195.	150.	.4	*	.084	.083	.086	.091	.096	.100
195.	150.	.6	*	.211	.139	.112	.102	.099	.099
195.	150.	.8	*	.316	.189	.137	.115	.106	.082
195.	160.	.2	*	.000	.028	.065	.085	.097	.106
195.	160.	.4	*	.123	.101	.096	.096	.099	.103

Table 4.3 Continued.

AGE AT WEANING (DA)	WEIGHT AT WEANING (KG)	RATE OF GAIN (KG/DA)	*	PERIOD OF THE BREEDING SEASON					
				1	2	3	4	5	6
195.	160.	.6	*	.243	.155	.121	.108	.104	.103
195.	160.	.8	*	.343	.203	.146	.121	.111	.046
195.	170.	.2	*	.023	.060	.080	.092	.101	.108
195.	170.	.4	*	.160	.119	.105	.102	.103	.105
195.	170.	.6	*	.274	.171	.130	.114	.108	.106
195.	170.	.8	*	.368	.217	.154	.128	.080	.032
195.	180.	.2	*	.066	.079	.089	.097	.104	.110
195.	180.	.4	*	.195	.136	.114	.107	.106	.108
195.	180.	.6	*	.303	.186	.138	.120	.112	.085
195.	180.	.8	*	.392	.230	.163	.129	.052	.021
195.	190.	.2	*	.106	.097	.098	.102	.107	.112
195.	190.	.4	*	.229	.152	.123	.113	.110	.111
195.	190.	.6	*	.330	.200	.147	.125	.116	.049
195.	190.	.8	*	.415	.243	.171	.103	.041	.017
195.	200.	.2	*	.144	.115	.107	.107	.110	.114
195.	200.	.4	*	.260	.167	.131	.118	.114	.114
195.	200.	.6	*	.356	.214	.155	.131	.086	.034
195.	200.	.8	*	.436	.255	.179	.078	.031	.013
195.	210.	.2	*	.180	.132	.116	.112	.113	.116
195.	210.	.4	*	.290	.182	.140	.124	.118	.088
195.	210.	.6	*	.381	.227	.164	.137	.055	.022
195.	210.	.8	*	.456	.266	.166	.067	.027	.011
195.	220.	.2	*	.214	.148	.124	.117	.117	.119
195.	220.	.4	*	.318	.196	.148	.129	.122	.051
195.	220.	.6	*	.404	.239	.172	.111	.044	.018
195.	220.	.8	*	.475	.278	.148	.059	.024	.009
195.	230.	.2	*	.246	.164	.133	.123	.121	.122
195.	230.	.4	*	.345	.210	.156	.135	.092	.037
195.	230.	.6	*	.426	.252	.180	.085	.034	.014
195.	230.	.8	*	.493	.288	.131	.052	.021	.008

