Superfluid hydrodynamics in neutron stars
by Gregory Allen Mendell

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics
Montana State University

Abstract:
Superfluidity is predicted to exist in neutron stars. Superfluid effects on the dynamics of these stars have not been investigated in depth in the past. In this thesis, superfluid hydrodynamics in neutron stars is developed extensively. It is shown that superfluidity has important effects on the oscillation modes; dissipative properties, and stability of these stars.

Very general hydrodynamic equations are derived which describe superfluid mixtures. The fluid equations are coupled to the electromagnetic and gravitational fields. Forces due to the quantized vortices of the superfluids are also included. It is shown that new vorticity-preserving forces can be introduced into the superfluid-mixture equations. The equations are then adapted to describe neutron stars composed primarily of superfluid neutrons, superconducting protons, and degenerate electrons and muons. The set of equations is closed by constructing a model of the total energy density and using it to express the dependent variables in terms of the independent variables. The low-frequency long-wavelength limit of the equations is determined. The results can be used to study superfluid effects on the global oscillations of neutron stars.

The equations are generalized further to include dissipative effects. Most important is a form of dissipation known as mutual friction, which occurs only in superfluids. In neutron stars, mutual friction is due to electron scattering off the neutron and proton vortices. An energy functional is constructed which determines the damping time of a mode due to the various forms of dissipation, including mutual friction. Plane-wave solutions are found to the equations. Mutual friction is shown to be the dominant form of dissipation in neutron stars for sufficiently large angular velocities.

Gravitational radiation tends to make all rotating stars unstable, while internal dissipation tends to counteract this instability. Thus, gravitational radiation can limit the maximum angular velocity of neutron stars. The most important conclusion of this thesis is that mutual friction completely suppresses the gravitational-radiation instability in rotating neutron stars cooler than the superfluid-transition temperature.

SUPERFLUID HYDRODYNAMICS IN NEUTRON STARS

by

Gregory Allen Mendell

A thesis submitted in partial fulfillment
of the requirements for the degree

of

Doctor of Philosophy

in

Physics

MONTANA STATE UNIVERSITY
Bozeman, Montana

August 1991

APPROVAL

of a thesis submitted by

Gregory Allen Mendell

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

12 August 1991
Date

Lee A. Lindblom
Chairperson, Graduate Committee

Approved for the Major Department

8-12-91
Date

[signature]
Head, Major Department

Approved for the College of Graduate Studies

August 16, 1991
Date

Henry L Parsons
Graduate Dean

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a doctoral degree at Montana State University, I agree that the Library shall make it available to borrowers under rules of the Library. I further agree that copying of this thesis is allowable only for scholarly purposes, consistent with "fair use" as prescribed in the U.S. Copyright Law. Requests for extensive copying or reproduction of this thesis should be referred to University Microfilms International, 300 North Zeeb Road, Ann Arbor, Michigan 48106, to whom I have granted "the exclusive right to reproduce and distribute copies of the dissertation in and from microfilm and the right to reproduce and distribute by abstract in any format."

Signature Gregory McDell

Date August 13, 1991

ACKNOWLEDGMENTS

I wish to thank my adviser, Dr. Lee Lindblom, for many useful discussions and suggestions during the course of this work, and for providing me with financial support from his National Science Foundation and MONTS grants.

I also wish to thank my wife, Jeanette, for her support during my years in graduate school.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Superfluidity is predicted to exist in neutron stars. Superfluid effects on the dynamics of these stars have not been investigated in depth in the past. In this thesis, superfluid hydrodynamics in neutron stars is developed extensively. It is shown that superfluidity has important effects on the oscillation modes, dissipative properties, and stability of these stars.

Very general hydrodynamic equations are derived which describe superfluid mixtures. The fluid equations are coupled to the electromagnetic and gravitational fields. Forces due to the quantized vortices of the superfluids are also included. It is shown that new vorticity-preserving forces can be introduced into the superfluid-mixture equations. The equations are then adapted to describe neutron stars composed primarily of superfluid neutrons, superconducting protons, and degenerate electrons and muons. The set of equations is closed by constructing a model of the total energy density and using it to express the dependent variables in terms of the independent variables. The low-frequency long-wavelength limit of the equations is determined. The results can be used to study superfluid effects on the global oscillations of neutron stars.

The equations are generalized further to include dissipative effects. Most important is a form of dissipation known as mutual friction, which occurs only in superfluids. In neutron stars, mutual friction is due to electron scattering off the neutron and proton vortices. An energy functional is constructed which determines the damping time of a mode due to the various forms of dissipation, including mutual friction. Plane-wave solutions are found to the equations. Mutual friction is shown to be the dominant form of dissipation in neutron stars for sufficiently large angular velocities.

Gravitational radiation tends to make all rotating stars unstable, while internal dissipation tends to counteract this instability. Thus, gravitational radiation can limit the maximum angular velocity of neutron stars. The most important conclusion of this thesis is that mutual friction completely suppresses the gravitational-radiation instability in rotating neutron stars cooler than the superfluid-transition temperature.

CHAPTER 1

INTRODUCTION

Purpose and Plan of This Thesis

Neutron stars are nature's grand tour of physics. Born from the collapsing core of a dying star in a supernova explosion, a typical neutron star has the mass of the sun compressed into the size of a city. The collapse can produce a rapid rotation rate and an intense magnetic field. Outside the star, the magnetic field interacts with a surrounding plasma, sometimes producing a corotating beam of electromagnetic radiation. When the earth lies within the beam's path we observe a pulsating star in the sky, a pulsar. Inside the star, the matter can be divided into several regions of crust and core, each corresponding to a different density regime. A large fraction of the matter is at a density greater than or equal to that of atomic nuclei. With such high densities, neutron stars are among the most gravitationally compact objects in the universe. They are also cold degenerate objects. Quantum mechanics is crucial to their description too.

Indeed, one of the most striking manifestations of macroscopic quantum effects to occur in nature — superfluidity — has been predicted to exist in neutron stars (for reviews see Pines and Alpar 1985; Sauls 1989). The interior neutrons are predicted to form superfluid condensates in the inner crust and in the outer core. The small fraction of protons in the outer core are expected to form a superconducting condensate. The transition temperatures to superfluidity and superconductivity are of the order 10^9K (see Epstein 1988 for a summary of various calculations). While uncertainties exist in the transition temperatures, neutron stars are expected to cool well below 10^9K in a few months (e.g., see Tsuruta 1979; Nomoto and Tsuruta 1987). Furthermore, in rotating magnetized neutron stars the superfluid neutrons are expected to form an array of quantized vortices and the superconducting protons an array of quantized flux tubes. In general, superfluidity can be expected to have interesting effects on the dynamics of neutron stars. (Except when the distinction is important, superfluidity and superconductivity will be referred to generically as "superfluidity" and vortices and flux tubes will be referred to generically as "vortices" from here on.)

The dynamics of neutron stars are studied primarily by using hydrodynamics. Theoretical hydrodynamic models study the equilibrium configurations, oscillation modes, dissipative processes, and stability of these stars. Two significant questions these models attempt to answer are: what is the maximum mass and what is the maximum rotation rate of a neutron star? Accurate answers to these questions will involve new insights into the physics of matter at high densities. They will enhance our understanding of the universe in other ways as well. For example, the observation of a compact object with a mass exceeding the neutron star limit would probably indicate that the object is a black hole. On the other hand, the detection of an object spinning faster than the rotation limit of a neutron star would indicate that the matter has compressed far beyond nuclear density. This would probably mean that quark stars exist. An identification of one of these objects would be made more reliable by knowing more precisely the mass and rotation limits of a neutron star.

The theoretical determination of the mass and rotation limits of a neutron star requires a stability analysis of the equilibrium models of the star. In particular, the study of neutron star oscillations determines the stability of an equilibrium model. Dissipative effects either cause the amplitude of the oscillation to grow or to damp out. If the amplitude of a particular mode grows, then the star is unstable to this mode of oscillation. For example, it is known that gravitational radiation tends to make all rotating stars unstable, while internal dissipation (such as viscosity) tends to counteract this instability (for a review see Lindblom 1991). The onset of the gravitational-radiation instability, which determines the maximum rotation rate, is found by studying the nonaxisymmetric oscillations of the star. Studying neutron star oscillations may also be fruitful for other reasons. For example, these oscillations may be observable. (This possibility is discussed further in the next section of this chapter.)

One motivation for this thesis is that superfluidity can be expected to have important effects on neutron star oscillations. However, past studies in this area of research have only begun to explore the possibilities. Previously, Epstein (1988) has studied sound waves in nonrotating, nonmagnetized, superfluid neutron-star matter. However, most studies of neutron star oscillations have not used superfluid hydrodynamics (e.g., Meltzer and Thorne 1966; Thorne 1969a; Hansen and Cioffi 1980; Lindblom and Detweiler 1983; McDermott et al. 1985; Carroll et al. 1986; Cutler, Lindblom, and Splinter 1990; Ipser and Lindblom 1990; Strohmayer 1991). Also, a past study of neutron star magnetohydrodynamics by Easson and Pethick (1979) did not include dynamics for the superfluid neutrons and treated the protons as nonsuperconducting.

Another motivation for this thesis is that dissipation in superfluids is completely different from that in ordinary fluids. (For a review of dissipation in superfluids see Sonin 1987; Putterman 1974.) In a superfluid the condensate acts like a zero temperature nondissipative perfect fluid. However, the scattering of particles off the vortices can transmit forces to the superfluid condensate, leading to a form of dissipation known as mutual friction. Previous studies of neutron stars have investigated the effects of superfluidity on viscous dissipation, but have not included the effects of mutual friction (e.g., Cutler and Lindblom 1987, Cutler, Lindblom, and Splinter 1990, Ipser and Lindblom 1991).

The purpose of this thesis is to develop superfluid hydrodynamics in neutron stars. The results will be applicable to studies of oscillations of rotating and magnetized neutron stars, including dissipative effects. In particular, the focus is on neutron stars composed primarily of superfluid neutrons, superconducting protons, and degenerate electrons and muons. However, very general sets of equations are derived which can be easily adapted to other models of neutron stars (e.g., such as those containing a superfluid pion condensate), or to other situations of physical interest (e.g., such as mixtures of superfluid ^{3}He and ^{4}He). The equations are closed by relating the dependent variables to the independent variables via partial derivatives of a model of the energy density. The low-frequency–long-wavelength limit of the equations is found. The equations are generalized further to include dissipative effects. Plane wave solutions to the equations are found and compared with those for ordinary neutron-star matter. Finally, the effect of superfluidity on the gravitational-radiation instability is studied. *The most important conclusion of this thesis* is that mutual friction between the electrons and the quantized neutron vortices completely suppresses the gravitational-radiation instability in rotating neutron stars cooler than the superfluid-transition temperature.

The plan of this thesis is discussed in the remainder of this section. In general, the approach used here is phenomenological. The underlying assumption is that the macroscopic properties of a neutron star can be described by a set of continuous fields. A complete set of such fields would be the number densities, entropy densities, and velocity fields of each species of particle in the star, plus the gravitational and electromagnetic fields specified everywhere inside and outside of the star. The temperature and pressure (and several other quantities) are specified in terms of the independent variables by an equation of state. (The equation of state is one place where the phenomenological theory touches base with the microphysics of the problem.) The equilibrium and evolution of neutron stars are thus described by the equations of fluid mechanics, electromagnetism, and gravitation.

Many approximations are made in the course of this thesis. These will be described in detail and justified when they occur. However, three major simplifying assumptions are made throughout this work and deserve extra mention here. First, in principle, the equations should be fully relativistic. However, Newtonian hydrodynamics is used since the fully relativistic treatment of the nonaxisymmetric oscillations of rotating stars has yet to be accomplished (e.g., see Lindblom 1991). Further development of relativistic superfluid hydrodynamics is also needed. Relativistic corrections will probably change the Newtonian results by no more than 20% (see Cutler and Lindblom 1991). Next, interactions between vortices are ignored in this thesis. Interaction between vortices of the same species will be shown to have negligible effects on neutron star oscillations in the low-frequency–long-wavelength limit. However, in rotating magnetized neutron stars the neutron and proton vortices either will be forced to move through each other or they will become pinned onto or between each other. The details of this interaction are not yet understood (e.g., see Sauls 1989; Srinivasan 1990) but an estimate of the pinning force is given in Chapter 10. Lastly, the neutrons, which form Cooper pairs in the triplet state, are treated as if in the singlet state. This is not expected to have significant effects on the macroscopic behavior of the neutron superfluid (Sauls 1989). The equations presented here can give important results in spite of their limitations due to these simplifying approximations. An outline of what is accomplished in each chapter is given below.

In Chapter 2 superfluid hydrodynamic theory is reviewed extensively. First, the London theory is covered (e.g., see London 1950; 1954) which describes the macroscopic properties of a superfluid in terms of a macroscopic wave function. This is used to define the superfluid velocity. Next, the Landau two-fluid theory (Landau 1941) is derived phenomenologically from a set of "macroscopic" first principles. This illustrates the power of the phenomenological approach. The derivation given here is slightly different than that given elsewhere and provides a clearer starting point for the derivation of the hydrodynamics of superfluid mixtures, which comes in the next chapter. The last part of the Chapter 2 discusses how the Landau equations must be extended when vortices are present. Since the spacing between vortices is typically microscopic compared with the length scales of interest it is appropriate to perform a smooth-averaging of the equations over a volume containing many vortices. The smooth-averaging process is discussed in terms of both a coarse-graining procedure and in terms of the phenomenological approach.

In Chapter 3 the general hydrodynamic theory for a mixture of N charged superfluids is derived. In this chapter the equations from past studies of superfluid mixtures (e.g., Holm and Kupershmidt 1987) are generalized still further by including a large class of vorticity-preserving interaction terms in the dynamical equations for the superfluid velocities. A Hamiltonian formulation for the equations is presented and used to couple the charged components of the mixture to the electromagnetic field. The new vorticity-preserving forces play a nontrivial dynamical role at the locations of vortices and have a profound effect on the electromagnetic coupling terms. By properly choosing these forces, it is possible to allow each component of the charged superfluid mixture to respond to the electromagnetic field via an appropriate Lorentz force law. The electromagnetic coupling proposed here is far more natural than that proposed in the previous studies of charge superfluid mixtures.

In Chapter 4 the equations are adapted to describe the outer-core region of a neutron star and are further modified in several important ways. A specific model of the energy density is constructed and its partial derivatives are used to relate the dependent variables to the independent variables. The equilibrium state is discussed and the linearized perturbation equations are found. The energy per unit length of the vortices in calculated. The characteristic frequencies associated with each of the force terms are discussed.

In Chapter 5 the low-frequency–long-wavelength limit of the equations is determined. In this limit the equations are also greatly simplified. For an appropriate frequency range it is shown that the number of independent velocities can be reduced from three to two. The resulting equations will be applicable to studies of gravity modes (g-modes) and pressure modes (p-modes) (for a description of these modes see Chapter 4; Van Horn 1980), including electromagnetic, rotational, and vortex effects. *A new mode* is shown to exist in the superconducting-proton–degenerate-electron plasma, while Alfvén waves, found in ordinary plasmas, are shown not to exist. It is then further shown that the electromagnetic forces and the forces due to the underlying vortices can be ignored for sufficiently long length scales. The final set of equations will be applicable to studies of p-modes in rapidly rotating neutron stars. These equations generalize the work of Epstein (1988) and Easson and Pethick (1979). Furthermore, an error in Epstein's equations is corrected. Since the final results differ significantly from the normal magnetohydrodynamic (MHD) limit, the entire procedure used here will be referred to as the low-frequency–long-wavelength (LFLW) limit throughout this thesis.

In Chapter 6 the general theory of mutual friction is derived for a mixture of N charged superfluids. This generalizes the theory of mutual friction first developed by Hall and Vinen (1956), and later derived phenomenologically by Bekarevich and Khalatnikov (1961), in the context of superfluid ^{4}He. The dissipative coefficients which appear in the superfluid equations are then related to parameters which can be determined from microscopic calculations.

In Chapter 7 the first goal is to include mutual friction forces in the superfluid hydrodynamic equations which describe a neutron star. The LFLW limit of the mutual friction forces is taken. The dissipative coefficients in these forces are related to microphysical quantities by generalizing the method of Hall and Vinen (1956). Specifically, the remaining relevant coefficients are related to the velocity relaxation time between the charged fluids and the neutron vortices, as determined by Alpar, Langer, and Sauls (1984). Outside of the vortices the main dissipative process is due to electron-electron scattering. This is incorporated into the theory by including viscous and thermal conductivity terms in the equations for the electron-muon fluid.

The second goal of this chapter is to estimate the effect of mutual friction on the damping times of p-modes. An energy functional is constructed which determines the damping times due to viscosity, thermal conductivity, and mutual friction. It is argued generically that the mutual friction damping times of the large scale p-modes are much shorter than the other damping times in rapidly rotating neutron stars.

In Chapter 8 the frequencies for plane symmetric solutions are found for several interesting cases and compared with those for ordinary neutron-star matter. Two special cases are considered: axial modes where the wave vector is parallel to the rotation axis, and inplane modes where the wave vector is perpendicular to the rotation axis. The results are compared with those for ordinary neutron-star matter. Errors in the previous study by Epstein (1988), in the case of a nonrotating star, are corrected here. As in terrestrial superfluids, the resulting frequencies are modified and the number of allowed modes is doubled when compared with the results for ordinary neutron-star matter. The damping times for these modes due to electron viscosity and mutual friction are discussed and compared with the results of Chapter 7.

The purpose of Chapter 9 is to investigate how the internal dissipation mechanisms that exist within superfluid neutron-star matter influence the gravitational-radiation instability. It is argued here that mutual friction between the electrons and the quantized neutron vortices completely suppresses the gravitational-radiation instability in rotating neutron stars cooler than the superfluid-transition temperature.

In Chapter 10 the important results and conclusions of this thesis are summarized. Areas where further work is needed, and the likely outcomes of such studies, are discussed.

In the remaining sections of this chapter a background survey of neutron stars and superfluidity is given. The reader unfamiliar with either of these topics will find these sections useful and they serve as a guide to the literature. In addition: complete reviews of neutron stars can be found in Shapiro and Teukolsky (1983), Baym and Pethick (1975; 1979), Canuto (1978), Arnett and Bowers (1977), Backer and Kulkarni (1990), and Kundt (1990); superfluidity and superconductivity are covered in Khalatnikov (1989), Tilley and Tilley (1986), de Gennes (1966), Tinkham (1965, 1975), Parks (1969), and London (1950, 1954); superfluid hydrodynamics is dealt with extensively in Putterman (1974) and Sonin (1987).

Neutron Stars

Observational Properties

Soon after the discovery of the neutron by Chadwick in 1932, Zwicky and Baade (1934) proposed the idea of neutron stars. (There is a story, perhaps apocryphal, that Landau was actually the first to have this idea. See Graham-Smith 1990.) Oppenheimer and Volkoff (1939) generated the first neutron star stellar models. Neutron stars would be small degenerate bodies with roughly the mass of the sun compressed into a ball about a dozen kilometers across. Studies of neutron star cooling showed that their surfaces would emit x-rays, but that they would be too dim to be seen (e.g., Tsuruta and Cameron 1965). It seemed that neutron stars, if they existed, were unobservable. But then in 1967 Bell discovered a radio source pulsing regularly every 1.3 seconds. It passed overhead once every sidereal day, as the stars do. (Bell referred to it as a Belisha beacon in her notes. The others in her group referred to the source as the LGM or the "little green men". See Wade 1975.) More such sources were discovered and it was soon realized that a new kind of star had been discovered—pulsars (Hewish et al. 1968).

Gold (1968) quickly identified pulsars as rotating neutron stars with intense magnetic fields. It had already been realized that the collapsing core of a dying star would be composed mostly of neutrons because of electron capture by the protons (Baade and Zwicky 1934). Conservation of angular momentum and magnetic flux would give large rotation rates and magnetic fields. (See Canuto 1978 for a derivation of flux conservation.) Thus, pulsars would form in supernova explosions. The discovery of pulsars in the Crab and Vela supernova remnants supported this idea. Today almost 500 pulsars are known, with periods ranging from 4.3s to 1.56ms (e.g., see Kundt 1990).

In the simplest model a neutron star possesses a dipole magnetic field which is not aligned with the rotation axis (e.g., see Shapiro and Teukolsky 1983). In this model a neutron star emits magnetic dipole radiation. (A crude calculation of the magnetic dipole radiation emitted by the Crab pulsar gives almost exactly the energy that is needed to keep the Crab nebula lit—a significant confirmation of the essential ideas of the model. See Gold 1969). Magnetic dipole radiation, though, is not the source of the pulses (this can be shown by analysis of the shape and frequency spectrum of the pulses). The exact pulse mechanism is still not understood. However, applying the boundary conditions for the electromagnetic field at the star's surface, Goldreich and Julian (1969) show that a rotating magnetized neutron star in a vacuum would have a surface electric field strong enough to rip charges away from the surface. Their conclusion is that a rotating magnetized neutron star cannot exist in a vacuum, but must be surrounded by a plasma. The basic idea then of how the pulses are generated is that some sort of interaction between the plasma, the electromagnetic field, and/or the neutron star's surface creates a beam of electromagnetic radiation. (For example, one popular idea is that the beam is produced by a cascade of e^+e^- pairs.) This beam, however it is produced, corotates with the star. Anyone in the path of the beam detects a pulse each time the beam sweeps past them, once with each rotation of the star. (For a further discussion of pulsar models see Backer and Kulkarni 1990; Shapiro and Teukolsky 1983.)

Whatever the pulse mechanism is, the energy source of pulsars is rotation. Since they are radiating away energy they must slow down with time. In the simple dipole model the rate of rotational energy loss equals the rate that electromagnetic energy is radiated away from the star, i.e.,

$$\dot{E} = I\Omega\dot{\Omega} = -\frac{2|\ddot{\vec{m}}|}{3c^2}, \tag{1.1}$$

where $\dot{E}$ is the time derivative of the rotational energy, I the star's moment of inertia, Ω and $\dot{\Omega}$ the angular velocity of rotation and its time derivative, respectively, c the speed of light, and $\vec{m}$ the magnetic dipole moment. Using $|\vec{m}| \cong BR^3/2$, where B is the magnitude of the surface magnetic field at the pole and R is the neutron star radius, the time derivative of the pulse period can be related to the magnetic field strength at the star's surface. Using typical neutron star numbers the result is

$$B \sin\theta \approx 10^{12} \left(\frac{P\dot{P}}{10^{-8}\,\frac{\mathrm{s}^2}{\mathrm{yr}}} \right)^{1/2} \mathrm{G}; \tag{1.2}$$

where θ is the angle $\vec{m}$ makes with the rotation axis, and $P = 2\pi/\Omega$ and $\dot{P}$ are the rotation period and its time derivative, respectively. In more complicated models the relationship between B and $P\dot{P}$ can be independent of the angle θ (see Shapiro and Teukolsky 1983). Typical observed values for $P\dot{P}$ are of the order $10^{-8}\mathrm{s}^2/\mathrm{yr}$ and so typical pulsar magnetic field strengths are about 10^{12}G.

A simple estimate of the characteristic age of a pulsar is given by

$$t_{\mathrm{age}} \approx \frac{1}{2}\frac{P}{\dot{P}} \,. \tag{1.3}$$

This assumes a constant magnetic field strength and that the observed period of a pulsar is much greater than its initial period at birth. For example, this gives an age of 1243yr for the Crab pulsar which agrees well with the Crab's known age of 937yr. (The crab pulsar was formed in the supernova explosion of 1054 A.D..) A typical value for t_{age} is 10^7yr. However, this over estimates the true age. Equation (1.3) could also be interpreted as the approximate spindown time of a pulsar if $\dot{\Omega} =$ constant held true, but pulsars spin down more slowly than this linear approximation suggests. The actual time it would take a pulsar to radiate away all of its rotational energy probably exceeds the present age of the universe. However, pulsars stop pulsing before they stop spinning. This is because the e^+e^- pair cascade that is believed to be the source of the pulses turns off when the voltage between the surface of the star and the interstellar medium drops below the value necessary to produce the pairs. The voltage is generated by the star's rotation. Thus, it can be shown that e^+e^- pairs cannot be created when a pulsar's magnetic field and period lie below the curve $B/P^2 = 2 \times 10^{11} \mathrm{G} \cdot \mathrm{s}^{-2}$. All known pulsars have observed values for B and P above this curve. Typical pulsars probably evolve below this curve, and thus turn off, in approximately 10^7yr. (For a further discussion of these ideas see Backer and Kulkarni 1990).

Comparison of pulsar magnetic fields with their ages suggest that the magnetic field decays exponentially on a time scale of $10^6 - 10^7$yr (Ostriker and Gunn 1969; Gunn and Ostriker 1970; Lyne Manchester and Taylor 1985; Stollman 1987). The observations could alternatively be interpreted as decay of the alignment angle θ to zero (e.g., see Kundt 1990). However observations do not support this alternative theory (Kulkarni 1986; Srinivasan 1989). Thus, the decay is presumedly due to ohmic dissipation in the neutron star's crust (see Shapiro and Teukolsky 1983). On the other hand, Sang and Chanmugam (1987), and Sang, Chanmugam, and Tsuruta (1990) argue that the magnetic field does not decay exponentially and its decrease could be only a few orders of magnitude during the lifetime of the universe.

Even less is known about a neutron star's interior magnetic field. Its strength is unknown, but could be several orders of magnitude larger or smaller than the surface field. Early estimates suggested it would persist for the lifetime of the universe (Baym, Pethick, and Pines 1969a) and some evidence supports this suggestion (Kulkarni 1986). However, Jones (1987) calculates that the interior field could be expelled to the crust in less than 10^7yr. A more recent and completely different theory of flux expulsion from the interior is given by Srinivasan et al. (1990). (For a summary of other theories of neutron star magnetic field evolution see Sang, Chanmugam, and Tsuruta 1990; for a further list of references see

Kundt 1990.) In any case, the main point is that the interior magnetic field is poorly understood.

Neutron stars are not only associated with pulsars. Accretion of matter from a companion onto a neutron star is the accepted explanation of x-ray binaries (e.g., see Bahcall 1978; Verbunt 1990). Neutron stars are also believed to be the sources of gamma ray bursts (See Ruderman 1989). Observations of x-ray binaries give mass estimates of neutron stars in the range of $1M_\odot - 2M_\odot$ where $M_\odot$ is the mass of the sun (see Avni 1978; Rappaport and Joss 1983; also see Shapiro and Teukolsky 1983).

The most accurate determination of the mass of a neutron star is from the Hulse-Taylor binary pulsar (PSR 1913+16). This pair consists of two orbiting neutron stars, one pulsing. High precision measurements of the pulse doppler shift plus measurements of the periastron advance (predicted by general relativity) give masses of (Taylor and Weisberg 1989)

$$\begin{aligned} M_{\text{pulsar}} &= 1.442 \pm 0.003 M_\odot, \\ M_{\text{companion}} &= 1.386 \pm 0.003 M_\odot. \end{aligned} \tag{1.4}$$

The binary pulsar is, of course, even more famous for the decay of its orbital period. This provides strong observational evidence for the existence of gravitational radiation (e.g., see Taylor and Weisberg 1989).

Unlike masses, which can in principle be determined from Kepler's third law, neutron star radii are hard to measure. Observations of the flux and spectrum of x-ray bursts can be used to give an estimate of the size of the emitting area. Assuming this is roughly the surface area of a neutron star gives an estimate for the star's radius. Observations during gamma-ray bursts of the redshift of the e^+e^- pair annihilation line have also been used to determine neutron star radii.

The results are in agreement with radii of order 10km (see Van Paradijs and Lewin 1990).

Neutron star oscillations may also be observable. Quasiperiodicity in the microstructure and subpulse of pulsar pulses, in x-ray binaries and x-ray bursters and in gamma ray bursts may be due to such oscillations (see Epstein 1988 for a list of references). Explanations of these observations as neutron star oscillations are at present far from convincing, though Van Horn (1980) has made some progress along these lines.

Finally, advancements in x-ray observations may make the detection of thermal radiation from neutron star surfaces possible. Young neutron stars ($t_{\rm age} < 10^5$yr) are expected to have surface temperatures of the order 10^6K. The surfaces glow in the soft x-ray part of the spectrum. Interior temperatures will be about one to two orders of magnitude higher. (See Tsuruta 1979 for a review of early calculations.) Recently, data from the *Einstein Observatory* has set upper limits on the temperatures of several supernova remnants, including four that are known to contain neutron stars. The data has been compared with theory in several studies (e.g., see Nomoto and Tsuruta 1986, 1987; Page and Baron 1990). Most of the data points are explainable by standard cooling theories. Standard cooling means cooling without the presence of exotic matter such as pions or a quark-gluon plasma. However, the temperature of the Vela pulsar lies below the standard cooling curve. This indicates that some neutron stars may contain exotic matter.

Evidence of Superfluidity

Neutron stars are very cold objects even though their temperature seems high by terrestrial standards. The reason is that at nuclear density the Fermi energy expressed as a temperature is about 10^{12}K. Neutron star temperatures are far below this, which means neutron-star matter is very nearly in its lowest energy state. Such matter is called degenerate. It has been shown by Cooper (1956) and Bardeen, Cooper, and Schrieffer (1957) that an attractive interaction between degenerate fermions near the Fermi surface can result in the formation of Cooper pairs and a phase transition to superfluidity. (This is discussed in detail in the next section.) If the particles are charged then a superconductor is formed. The degenerate nucleons in a neutron star do attract each other via the strong nuclear force. Thus, one might expect the neutron star's neutrons to become superfluid and the protons to become superconducting at low enough temperatures. (This possibility was first suggested by Migdal 1959.) Evidence which supports this conjecture comes from the study of what are known as pulsar glitches.

A sudden increase in the pulsation rate—known as a "glitch"—has been observed on several occasions in several pulsars. For example, three glitches have been observed in the Crab pulsar with $\Delta\Omega/\Omega \sim 10^{-9} - 10^{-8}$, and seven giant glitches have been observed in the Vela pulsar with $\Delta\Omega/\Omega \sim 10^{-6}$. (Glitches are measurable because pulsars are amazingly stable clocks. Some pulse periods are known to thirteen decimal places.) The glitch rise time is less than a day and the pulsar relaxes back to its normal slow down rate on a time scale of days to years (see Sauls 1989; Pines and Alpar 1975; Shapiro and Teukolsky 1983).

The most accepted theory of glitches is that they represent a transfer of angular momentum from a portion of the superfluid interior to the remainder of the star. Rotating superfluids carry their angular momentum in an array of quantized vortices. (In equilibrium, the array itself rotates rigidly with the rest of the star.) In the inner crust these vortices tend to pin to the heavy nuclei. As the star spins down the pinned vortices are forced to slow down as well, but their density remains the same. Thus, the angular momentum of the superfluid, which is carried by the vortices, does not decrease with the star as long as the vortices remain pinned. However, an outward pressure gradient is created across the pinned vortices. When this pressure gradient reaches a critical value the vortices will violently unpin transferring angular momentum to the rest of the star, spinning it up slightly. A glitch occurs. (Alternatively, a theory in which crust breaking, rather than vortex unpinning, occurs has recently been proposed by Ruderman 1991.) In a more detailed version of the model the vortices are not absolutely pinned but can creep along the pinning sites because of thermal fluctuations. The motion of the vortices is dissipative and acts as an internal heat source. The theory of vortex motion is referred to as vortex-creep theory. This theory was first used to explain glitches by Alpar et al. 1984. (For a review see Pines and Alpar 1985; for more recent work see Epstein and Baym 1988; Bildsten and Epstein 1989.)

The measurement of the interval between glitches and the post-glitch relaxation time for one pulsar sets the parameters of vortex-creep theory. The theory applied to another pulsar then only depends on the temperature and spindown rate of that star. Using measured spindown rates and temperature estimates, vortex-creep theory gives a reasonable fit to the glitch data of various pulsars. Conversely, the theory can be used to give temperature estimates for the neutron stars (Alpar, Nandkumar, and Pines 1985). The temperature estimate for the Vela neutron star concurs with the suggestion that it contains some exotic matter, though not enough data is available to make any firm conclusions.

Another source of evidence for superfluidity in neutron stars may come in the future from studies of neutron star temperatures. The profile of cooling curves, which show the star's surface temperature as a function of time, are sensitive to the presence of superfluidity. However, measurements much more precise than the currently known upper bounds on the surface temperatures will be needed.

Equilibrium Neutron Star Models

The theoretical equilibrium mass and radius of a neutron star is found by solving the equations of hydrostatic equilibrium for a specific nuclear matter equation of state. In the nonrotating case, a one parameter family of stellar models can be generated by specifying the central density of each model and then integrating the pressure outwards from the center of the star until it drops to zero.

Figure 1 shows the mass of a neutron star as a function of central density for three equations of state of varying stiffness. (Roughly speaking, a stiffer equation of state gives a higher pressure for a given density.) In Figure 2 the density is plotted as a function of the radial coordinate from the center of the star for the same three equations of state as in Figure 1. The plots in Figure 2 are for $1.4M_{\odot}$ stars. The results shown here were obtained using the fully relativistic equations for hydrostatic equilibrium (See Arnett and Bowers 1977; Shapiro and Teukolsky 1983). It can be seen in these figures that the mass and radius of a neutron star are very dependent on the equation of state of nuclear matter. The equations of state used are given in Pandharipande (1971), Bethe and Johnson (1974), and Serot (1979). These equations of state can be described as soft, intermediate, and stiff, respectively, and are representative of the range of equations of state for nuclear matter found in the literature (e.g., see Arnett and Bowers 1977).

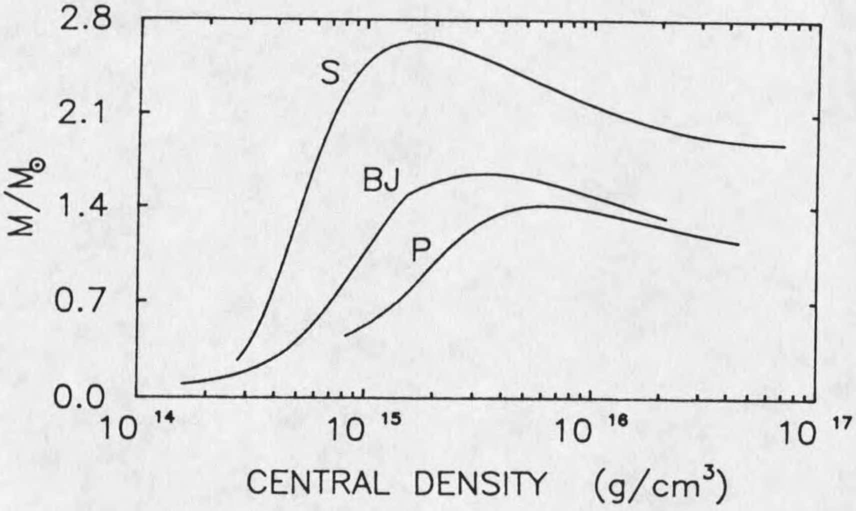

Figure 1. Mass versus central density. Result are shown for equations of state S (Serot 1979), BJ (Bethe and Johnson 1974) and P (Pandharipande 1971).

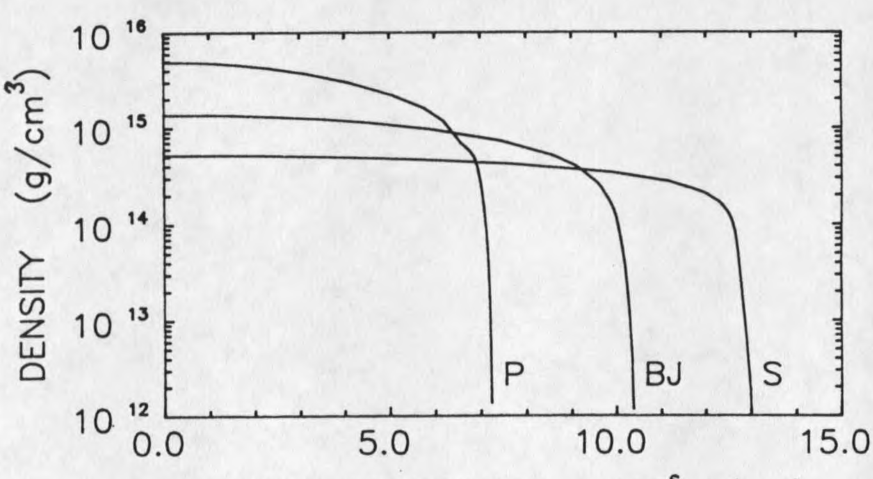

Figure 2. Density versus radial coordinate. Results are shown for equations of state S (Serot 1979), BJ (Bethe and Johnson 1974) and P (Pandharipande 1971).

In Figure 1 it can be seen that, in general, as the central density increases the mass of the star increases until a maximum mass is reached. Stars with central densities larger than that of the maximum mass star are unstable against collapse, whereas stars with central densities less than that of the maximum mass star are stable. The maximum mass increases in magnitude, but occurs at a lower central density, as the equation of state stiffens. For example the soft Pandharipande (1971) equation of state results in a maximum mass of about $1.4M_\odot$ at a central density of roughly $6 \times 10^{15}\mathrm{g/cm}^3$. The intermediate Bethe and Johnson (1974) equation of state results in a maximum mass of about $1.6M_\odot$ at a central density of $3 \times 10^{15}\mathrm{g/cm}^3$. Finally, the stiff Serot (1979) equation of state results in a maximum mass of about $2.6M_\odot$ at a central density of only $1.7 \times 10^{15}\mathrm{g/cm}^3$.

In Figure 2 the star with the softest equation of state has the smallest radius and the largest central density. The star with the stiffest equation of state has the largest radius and the smallest central density. Also, the density falls off very slowly from the central density except in the outer most part of the star. The important result is that most of the mass, about 80%, is at or above nuclear densities.

Arnett and Bowers (1977) argue that all supernova remnants, and thus all neutron stars, should have a mass of roughly $1.4M_\odot$, since above the Chandrasekhar limit (equal to $1.4M_\odot$) degenerate stellar cores become unstable against collapse. However, this does not mean that $1.4M_\odot$ is the maximum mass of a neutron star. An absolute upper limit on this mass, based on the minimal requirements of microscopic stability and causality, is about $3.6M_\odot$ (see Shapiro and Teukolsky 1983). Also, rotating stars can support more mass for a given central density. Friedman, Ipser, and Parker (1984) show that rotation can increase the maximum mass of a neutron star up to 30% above its maximum nonrotating value.

Finally, the equilibrium composition of the star as a function of the total density is determined by calculating the ground state of nuclear matter at the various densities (e.g., see Baym, Bethe and Pethick 1971, Baym and Pethick 1975). The result is that the star is divided into several regions. The crustal regions are solid, although neutrons begin to drip out of the nuclei at a density of $4.3 \times 10^{11} \mathrm{g/cm}^3$. These neutrons form a dilute gas in the inner crust. For densities above $2.8 \times 10^{14} \mathrm{g/cm}^3$ the nuclei merge into a uniform fluid outer core of neutrons, protons, electrons, and muons. The greatest uncertainty is at densities greater than two or three times nuclear density. At these high densities an inner core is expected to form as the matter undergoes a transition to some new exotic phase. Solid neutrons, a pion condensate, or a quark-gluon plasma are just three possibilities.

Typical neutron star interiors are shown in Figure 3 for the stiff Serot equation of state (on the left) and for the soft Pandharipande equation of state (on the right). The stars shown each have a mass of $1.4 M_{\odot}$. The regions where the neutrons and protons are expected to form superfluids are indicated. The important result shown in Figure 3 is that neutron stars with a stiff equation of state are primarily composed of superfluid neutrons, superconducting protons, and degenerate electrons and muons. There is some evidence in favor of a stiff equation of state as will be described in the next section.

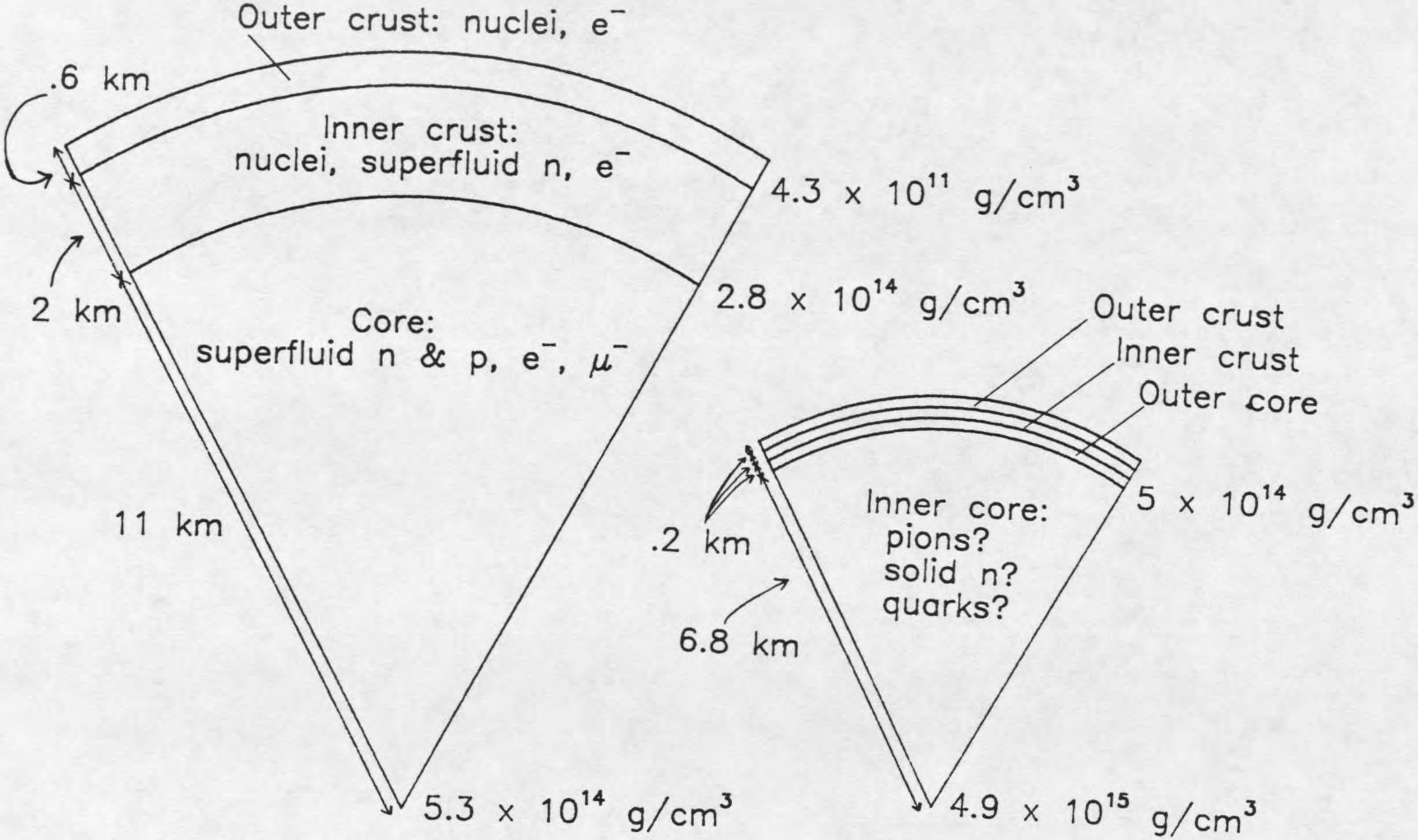

Figure 3. Cross sections of $1.4M_{\odot}$ neutron stars. Results shown are for Serot (on the left) and Pandharipande (on the right) equations of state.

How Fast Can a Neutron Star Rotate?

The discovery of millisecond pulsars (e.g., see Backer and Kulkarni 1990) naturally suggests the question: "How fast can a neutron star rotate?" The two fastest millisecond pulsars have periods of 1.56ms and 1.61ms which differ by only 3% from each other, while the next shortest period is longer by a factor of two. This suggests that $P \sim 1.6$ms represents some sort of minimum rotation period (Friedman et al. 1988).

The absolute limit on the rotation rate of a neutron star is determined by the Keplerian limit, at which point mass on the equator rotates with its orbital velocity. A star which rotates faster than this would be unstable to mass shedding at the equator. It is believed, however, that millisecond pulsars are spun up from dead pulsars by accretion (Alpar et al 1982) or are born spinning fast due to accretion induced collapse of white dwarfs (Chanmugam and Brechet 1987). Thus, it has been suggested that the limiting rotation rate is a property of the accretion process (Alpar et al. 1982; Wasserman and Cordes 1988). On the other hand, Friedman et al. (1988) argue that models which require accretion to turn off at $P \sim 1.6$ms must fine tune the values of magnetic field and the accretion rate. They argue that it is more plausible that the maximum rotation rate is determined by the equation of state and the mass of the star.

Following the arguments of Friedman et al. (1988), for a given mass star, the larger the radius the smaller the Keplerian limit on the angular velocity of rotation. Figure 2, showing the radii of $1.4M_{\odot}$ stars, then indicates that the stiffer the equation of state the smaller the maximum rotation rate. Since it is plausible that most neutron stars have a mass of at least $1.4M_{\odot}$, the equation of state is inferable from the maximum rotation rate. Furthermore, only the stiffest equations of state have a minimum rotation period of $P \sim 1.6$ms. Thus, it is reasonable to hypothesize a stiff equation of state for nuclear matter. Redshift data of the pair annihilation line also suggest a stiff equation of state (Friedman et al. 1988; Lindblom 1984). However, the existence of the Vela pulsar below the standard cooling curve suggests that it contains exotic matter and thus has a high central density. If nuclear matter is described by a stiff equation of state, then the Vela neutron star would need to have a mass significantly larger than $1.4M_{\odot}$.

The upper limit on the rotation rate as a function of equation of state may not depend only on the Keplerian limit. The studies of Chandrasekhar (1970a,b), Friedman and Schutz (1977), and Friedman (1978) show that all rotating stars are unstable to the emission of gravitational radiation. However, internal dissipation can restabilize the star. (See Lindblom and Hiscock 1983; Lindblom and Detweiler 1977; Friedman 1983; Lindblom 1986; Lindblom 1991). In ordinary neutron stars, the critical angular velocity at which the instability sets in is within 10% of the Keplerian limit. Thus, the above arguments suggesting a stiff equation of state for nuclear matter are not significantly changed by the gravitational-radiation instability. However, this instability is important to determining the precise upper limit on the rotation rate of neutron stars.

Superfluidity

Observational Properties

Superfluids and superconductors are distinguished by their ability to maintain persistent mass or charge currents without dissipation. For example Reppy and Depatie (1964) rotated a vessel packed with a porous material and containing superfluid ^{4}He. After the vessel was brought to rest the ^{4}He continued to flow, showing no reduction in its angular velocity over the twelve-hour period of the experiment. Experiments also show that superfluid ^{4}He can flow though narrow channels without viscosity. Similarly, in an experiment at MIT, a current circulated in a ring without a driving force for more than a year (see Tinkham 1965). A persistent current can be set up in a superconductor by bringing a magnet nearby. The repulsive force between the magnet and the current in the superconductor will cause the magnet to levitate above the superconductor indefinitely. (For further references see Tilley and Tilley 1986.)

Another similarity between superfluids and superconductors is the formation of vortices. Rotating superfluids do not rotate rigidly but form arrays of quantized vortices. (The array is stationary in a frame which corotates with the container.) Similarly, a type II superconductor forms an array of quantized flux-carrying vortices when placed in a magnetic field. (A type I superconductor completely expels all magnetic flux from its interior. This is known as the Meissner effect.) The array of vortices in rotating ^{4}He has been photographed by Yarmchuk and Packard (1982). The array of vortices in type II superconductors has been photographed by many investigators by spraying magnetic particles onto the superconducting surface (e.g., see Essmann and Träuble 1967.) The currents around these vortices are further examples of nondissipative flows in superfluids and superconductors.

Not all superfluid behavior corresponds to that of a perfect fluid. Experiments using piles of thinly spaced discs suspended by a torsion fiber in superfluid ^{4}He show that the oscillations of the discs are damped out (see Tilley and Tilley 1986 for references). Evidently a superfluid is capable of both viscous and nonviscous motion at the same time. This observation led Tisza (1938) to suggest a two-fluid description of superfluid ^{4}He. In the two-fluid model a superfluid is a