

THE EFFECT OF RANKING TASKS AND PEER INSTRUCTION
IN A MATHEMATICS CLASSROOM

by

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DEDICATION

This project is dedicated to my wife, Megan, and my children Leo and Amelia. Without your support, understanding, and love, completion of the MSSE program and this project would not have been possible.

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ABSTRACT

Conceptual understanding had regularly been an area of difficulty for students in my mathematics classes. In an attempt to improve conceptual understanding, this study examined the effectiveness of two pedagogical tools that I have used previously when teaching physics: ranking tasks and peer instruction. The use of ranking tasks has been shown to be successful in helping students understand concepts in the high-school physics classroom. In addition, peer instruction has been determined to be a pedagogical method that enhances student success. The main focus of this study was to determine if the use ranking tasks in a peer instruction environment increased conceptual understanding of mathematics.

The treatment was comprised of several peer instruction ranking task activities throughout a unit on linear relations. The activities involved an individual completion of a paper-and-pencil ranking task followed by a small group discussion and re-completion of the task with group input. After both the individual and group phases, students were prompted to record the confidence they had in their answer.

Data collection for this study included a pre- and post-concept test for the treatment unit and for comparison, a non-treatment unit as well. To triangulate the data, a questionnaire aimed at revealing student perceptions on ranking tasks, peer instruction, and mathematics in general was given post-treatment. Furthermore, several students were randomly selected to participate in an interview after the treatment was completed. Finally, each ranking task also yielded insight to the effectiveness of the treatment through the rate of successful completion and student confidence.

The results indicated that ranking tasks were successful in improving student conceptual understanding. When combined with peer instruction, the effectiveness of ranking tasks was even more prominent. Due to these positive outcomes, peer instruction ranking tasks will be an activity that I will regularly implement in my future mathematics classes. On a more general level, peer instruction is a model that I plan to use more often to enable students to learn from themselves and succeed with each other.

INTRODUCTION AND BACKGROUND

Okanagan Mission Secondary School (OMSS) is located in Kelowna, British Columbia, Canada. OMSS is one of five high schools in the Central Okanagan School District, which serves Kelowna and the surrounding area. There are 1314 students enrolled at OMSS from grades 7-12 for the 2015-2016 school year. The student body is reflective of an affluent area of Kelowna with above average mean household incomes and highly educated parents. The Grade 12 Graduation Rate was 99.5% for the 2014-2015 school year. Furthermore, math and science provincial exam achievements have typically been above the provincial average (L. Zorn, personal communication, November 20, 2015).

This is my first year at OMSS having previously taught physics and mathematics in another district in British Columbia. My course load this year focuses entirely on mathematics from grades 9-11. These courses represent a diverse mix of students with varying academic backgrounds and aspirations. A large subset of my students can be classified as honors students, who will likely go on to university. While another subset of students are enrolled in order to satisfy their graduation requirements or to transition into careers in technology or trades. This is in part because of a recent impetus placed on developing the skills of the labor force by the province of British Columbia in order to overcome forecasted shortages in technological and trades careers.

Regardless of background or career aspirations, I have noticed one ubiquitous problem amongst mathematics students: low conceptual understanding. The honors students can easily solve straightforward problems, while the technology and trades

students can do the same given scaffolding support. However, across the board markedly fewer students can give a coherent explanation of the concept that they have applied or explain fundamental mathematical properties. Conceptual questions have continually been the ones students struggle with the most on assessments in my classes. In order for learners to build expertise in mathematics, an understanding of the key principles is necessary for useful application. Therefore, improving the conceptual understanding of my students has become a high priority within all of my classes.

To address this problem I have piloted numerous pedagogical strategies aimed at bringing about conceptual change and increasing conceptual understanding. However, two specific strategies that I have used previously in my experience as a physics teacher, ranking tasks and peer instruction, could prove to help my students significantly improve conceptual understanding. A ranking task is typically a paper-and-pencil assessment where students are asked to make a comparative judgment on different variations of a physical situation (O’Kuma, Maloney, & Hieggelke, 2000). Peer instruction can be described as a pedagogical model where the instructor is seen as a facilitator and students are given an activity requiring them to apply a concept and then explain it to their classmates (Crouch & Mazur, 2001; Gokhale, 1995)). The focus of this action research is to examine how using these two pedagogical strategies in concert can enhance conceptual understanding in mathematics.

My focus question is as follows: *Will the use of mathematics-based ranking tasks in a peer instruction environment increase conceptual understanding in high-school students?* In addition, I was also interested in whether a peer instruction environment

enhances the effectiveness of ranking tasks and provides a benefit to student understanding and confidence.

CONCEPTUAL FRAMEWORK

Due to current environmental, technological, health and economic conditions, it has never been more important for an individual, or a society for that matter, to have a better understanding of science and mathematics (Bybee & Fuchs, 2006; Wieman & Perkins, 2005). Therefore, from the grassroots to the international level an increasing impetus has been put on science, technology, engineering, and mathematics (STEM) education (Sanders, 2009). Education in these fields can no longer be restricted to the small percentage of individuals who are able to educate themselves but must extend to a larger population base (Wieman & Perkins, 2005). Unfortunately, many students enter STEM courses with a genuine interest only to leave with lower than expected levels of understanding and a disinterest in STEM (Adams et al., 2006; Hudgins, Prather, Grayson, & Smits, 2007; Redish, Saul, & Steinberg, 1998). Wieman and Perkins (2005) contend that STEM educators need to seize this opportunity to usher in a new era of effective instruction by re-examining traditional instructional practices. Traditional STEM instructional practices of lecture followed by quantitative textbook homework problems and similar summative tests are ineffective in promoting expertise regardless of the instructor (Hake, 1998; Redish et al., 1998). Furthermore, traditional instruction may reinforce the ability of learners to solve quantitative, abstract, and specific problems but do little to encourage expert competence. To gain expert competence, students not only need to hold some basic knowledge but also need to be able to construct a retrieval

system where that knowledge can be organized and applied to new situations (Wieman & Perkins, 2005).

In order for learners to organize and apply their knowledge in a useful manner, they must first overcome any preconceptions (Laws, Sokoloff, & Thornton, 1999; Posner, Strike, Hewson, & Gertzog 1982; Stepan, 1996). As traditional instruction is shown to only cause positive conceptual shift in a small percentage of learners, a model for conceptual change is needed (Laws et al., 1999; Stepan, 1996). In a broad sense, a conceptual change model can be described as addressing the nature of preconceptions followed by a prescription for evolving these preconceptions into concepts (Rowlands, Graham, Berry, & McWilliam, 2007).

Preconceptions of the physical world are deeply ingrained and often formed during elementary school (Dykstra & Sweet, 2009). There is an ongoing debate as to whether preconceptions have their own system of concepts or whether they are the result of spontaneous reasoning (Rowlands et al., 2007). Nevertheless, preconceptions must be first exposed and confronted, through an activity like a discrepant event, before the learner can accommodate a new concept (Eryilmaz, 2002). Discrepant events are demonstrations where counter-intuitive outcomes occur that create cognitive dissonance in learners (Liem, 1987). Consequently, observing the outcome forces learners to relate their preconception to the true concept. To test that the true concept is satisfactory for them to adopt, learners must then apply it to new situations and make new connections (Posner et al., 1982; Stepan, 1996). Although instruction that takes preconceptions and applies some form of conceptual change to them is effective, it is a long way from being

universally adapted and perfected (Hestenes, Well, & Swackhamer, 1992; Treagust & Duit, 2008). For example, certain STEM disciplines, like mathematics, have not been studied with great depth with respect to a model of conceptual change. However, it is suggested that the tenets of conceptual change theory can apply to mathematics allowing for a framework to understand difficulties in mathematics education and how to provide optimal environments for their resolution (Tirosh & Tsamir, 2005; Vosniadou & Verschaffel, 2004).

In order to expose student preconceptions or to determine whether any conceptual change intervention has worked, instructors need quality formative assessments that can quickly expose student understanding (Black & William, 1998; Keeley, 2010). Not only can formative assessments allow for teachers to better understand students' preconceptions, but they are also shown to improve achievement for all students through increased ownership of learning (Black & William, 1998). Several formative assessments that can accurately test student conceptions have been developed and researched with respect to STEM, including ConcepTests, formative assessment (misconception) probes, and Just-in-Time Teaching questions (Crouch & Mazur, 2001; Keeley 2010; Novak, 2011).

One formative assessment that has been found successful is the rule-assessment technique. The rule-assessment technique involves a student's repeated use of an algorithm over the course of many different problems that are connected to a broad concept (Siegler, 1976). A shortened form of the rule-assessment technique, called ranking task exercises, specifically target student conceptions within the physical

sciences. Ranking tasks are structured with a brief description including constraints, the basis for making an arrangement, and a pictorial representation of the options to rank. In addition, they also contain a form to rank the options, a space for an explanation, and a confidence level form (O’Kuma, Maloney, & Hieggelke, 2000). Ranking tasks can be a pencil and paper exercise or potentially enhanced with Java-based Physlet and other software applications (Cox, Belloni, & Christian, 2005). Ranking tasks were initially designed with physics in mind. However, other disciplines, such as biology, are exploring the use of similar activities to ranking tasks (Hoskinson, Caballero, & Knight, 2013). Furthermore, recent implementations in astronomy suggest ranking tasks could be an asset for all disciplines within STEM (Hudgins et al., 2007).

Ranking tasks have several documented benefits for both instructors and learners. First, the ranking activity serves as a more authentic assessment than a multiple-choice question as it minimizes the ability of the student to develop coping strategies for eliminating answers and discourages just plugging numbers into a formula. Second, research has determined that ranking tasks increase conceptual understanding and overall student outcomes (Hudgins et al., 2007; O’Kuma et al., 2000). To that end, as students must create a model in order to rank their options, they must exhibit expert competence (Cox et al., 2005; Malone, 2008).

Ranking tasks have shown to uncover student preconceptions and have the ability to be keyed specifically to indicate a multitude of specific preconceptions (Desbiens, 1997; Keeley, 2014). As students need to explain the model they used to rank the options when performing a ranking task, further insight is also given to student conceptions (Cox

et al., 2005). Lastly, as students must indicate their confidence when responding to a ranking task, it is possible to discern the degree to which preconceptions are ingrained (O’Kuma et al., 2000).

An individual student can complete a ranking task for conceptual growth; however, the impact of a ranking task could be compounded if it was used in a peer instruction model where it was performed collaboratively, used in a think-pair-share format, or used to initiate a classroom debate (Crouch & Mazur, 2001; Keeley, 2014; O’Kuma et al., 2000). Students who learn collaboratively under a peer instruction model have been determined to have higher achievement gain scores through increased conceptual understanding, critical thinking, and problem solving skills (Crouch & Mazur, 2001; Enderle, Southerland, & Grooms, 2013; Gokhale, 1995). Another asset of using ranking tasks in a peer instruction model is the sharing of answers and explanations. By exposing students to alternate conceptions and explanations, open-mindedness can be fostered (O’Kuma et al., 2000). Finally, the use of collaborative ranking tasks has also been shown to improve student attitudes towards STEM and students have also indicated that they believe ranking tasks help them learn (Gok, 2011; Hudgins et al., 2006).

METHODOLOGY

To foster conceptual understanding in mathematics, a Math 9 Honors class was chosen for a treatment involving the use of ranking tasks in a peer instruction environment ($N=28$). The section of the course that represented the treatment was a unit on linear relations. This particular unit was chosen for its composition of both new and varied concepts that provided a challenge to students. Furthermore, success in this

particular unit is contingent on applying new and old concepts together, making it appropriate for a study seeking to increase mathematical conceptual understanding. The linear relations unit took place over the course of 3 weeks, which represented 15 80-minute periods. The research methodology for this project received an exemption by Montana State University's Institutional Review Board and compliance for working with human subjects was maintained (Appendix A).

The treatment for this study was designed to augment the normal pedagogy that students in this class had been receiving in previous units. The normal pedagogy consisted of teacher-led lessons or guided inquiry investigations as a means to introduce students to new topics and skills in order to meet the course learning objectives. The remaining class time was generally used for students to individually work on practicing these skills. To begin the following class, students were administered a brief ticket-in-the-door quiz on the previously covered learning objectives. These quizzes were either assessed by the teacher or by the student themselves formatively. The regular and prompt feedback students receive from these quizzes was an essential component of this course; therefore, the quizzes were administered during the treatment unit as well. As this class is an honors section, extra time was also devoted to working on problem solving skills through weekly group or individual problem solving activities.

The unit that preceded the treatment was a unit on linear equations and inequalities. For the purposes of comparison, progress through the linear equations and inequalities unit was monitored using the Equations and Inequalities Content Test (EICT), which I developed using a McGraw-Hill test bank (Appendix B). The EICT was

administered before commencing the unit and again once the unit was completed. The EICT consisted of 20 multiple-choice questions and was analyzed for change and normalized gains. The mean normalized gain scores were compared using Hake's (1998) classification of low (<0.3), medium ($0.3-0.7$), and high (>0.7) gains.

Students were made familiar with some of the content in both the treatment and non-treatment units through prior instruction. However, concepts in both units were developed further and new ones were introduced. From my experience, both units are central to the Math 9 curriculum and both are equally challenging.

The treatment unit was structured around nine Ranking Task Activities (RTAs) that were completed over the course of the linear relations unit (Appendix C). I developed the RTAs to focus on specific concepts introduced throughout the unit, such as the y-intercepts and extrapolation. Depending on the timing of learning objectives, one of the RTAs was typically undertaken each class period. To make time for the RTAs, lessons and investigations were streamlined and the amount of time students received to work on practice assignments in class was decreased. In addition, weekly problem solving activities were cancelled for this unit to provide additional time for the RTAs. The rationale for this shift in time allotment was that the RTAs offered students a chance to reason and discuss the concepts while allowing them time to practice and apply their skills collaboratively. Table 1 summarizes the similarities and differences between the non-treatment units and the treatment unit.

Table 1
Comparison of Treatment and Non-Treatment Units

	Non-Treatment Units	Treatment Unit
Lesson or Investigation	yes	yes, streamlined
Individual Skill Practice	yes	yes, streamlined
Ticket-in-the-door Quizzes	yes	yes
Problem Solving Activities	yes	no
Ranking Task Activities	no	yes

To begin RTAs, generic instructions were presented and the situation was described orally and in written form on the ranking task itself. After completing the first two RTAs, the generic written instructions section were removed for brevity, as it was assumed that students knew how to complete a RTA. Therefore, these subsequent RTAs began directly with the presentation of the situation.

Each RTA had three distinct phases: individual response, peer discussion and response, and whole class debriefing. The individual response section of each RTA typically took students up to ten minutes to complete. In the individual response section students performed the task by ranking six to eight options from least to greatest or vice versa based on the outlined criteria. To perform the ranking, students constructed a way to compare the options based on the criteria using unit concepts. Students also were required to explain their reasoning and indicate their confidence on a ten-point scale in their written answer.

After completing the individual response section, groups were randomly selected so that three or four students could discuss and compare their answers. The intention of this phase was to have students verbally explain their thought process, listen and engage with others, and provide the opportunity for students to use argumentation to solve the problem. By the end of the peer discussion phase, students re-completed the ranking. They either reported the same ranking or, based on their group discussion, they changed their ranking in the after discussion response section of the RTA. If they changed their answer they were asked to provide a reason as to why they made the change. Finally, students were prompted to record their confidence level post-discussion. The last phase of each RTA included a whole class debriefing on possible rankings, the correct ranking, and the possible ways to arrive at the correct ranking. Each phase normally lasted around five minutes; however, for more challenging RTAs, students required extra time in the individual and peer phases.

To establish a baseline for linear relations content understanding I created the Linear Relations Concept Test (LRCT), which was first administered prior to the treatment unit (Appendix D). The LRCT, which contained 20 multiple-choice questions, was representative of the learning objectives for the linear relations unit and was scored as a percentage. After the treatment, the same LRCT was given to the students to determine the change from pre-treatment to post-treatment by comparing means and calculating normalized gains.

Students completed the Linear Relations Questionnaire (LRQ) after the treatment unit (Appendix E). The LRQ was aimed at gaining insight to student attitudes and

confidence levels related to the treatment unit. The questions were focused on mathematics in general, the linear relations unit, ranking tasks, and learning in a peer instruction environment. The questions on the survey were Likert-type and to analyze were scored as follows: 1 = *agree*, 2 = *somewhat agree*, 3 = *undecided or not applicable*, 4 = *somewhat disagree*, and 5 = *disagree*. Questions 6-10 on the LRQ examined the theme of peer instruction and a Likert-scale was created. Questions 7-10 were worded such that agreeing with the statement could be correlated with positive opinions on the merits of discussion and instruction. However, question six was the worded in the opposite fashion; therefore, the scale was inversed for that question. A score for each student was calculated along with the mean for the sample.

After the treatment unit, seven students were randomly selected to complete the Linear Relations Student Interview (LRSI) (Appendix F). The interview was designed to add additional qualitative student input to the LRQ and was analyzed for common themes.

The treatment itself provided further data collection opportunities. As students completed each RTA both individually and after discussion with their peers, a comparison of the individual and peer ranking results was performed. In addition, before and after confidence level comparisons were also made from each ranking task. Finally, to gain further information about student confidence, a comparison of student confidence levels and ranking correctness was examined for correlation.

The data sources detailed above can be summarized in the following triangulation matrix (Table 2). Triangulation of data was made for the focus question and for the related sub-question.

Table 2
Data Triangulation Matrix

	Focus Question: Will the use of mathematics-based ranking tasks in a peer instruction environment increase conceptual understanding in high-school students?	Sub-Question: Does a peer instruction environment enhance the use of ranking tasks by increasing student understanding, attitudes, and confidence?
Instrument		
Pre- and Post Linear Relations Concept Test	X	
Pre- and Post-Equations and Inequalities Concept Test	X	
Linear Relations Questionnaire	X	X
Linear Relations Student Interviews	X	X
Ranking Task Activities	X	X

DATA AND ANALYSIS

The mean score for the pre-unit administration of the Equations and Inequalities Concept Test (EICT) was 61% ($N=28$). The standard deviation was determined to be 11%. In comparison, the post-unit EICT mean was 78% with a standard deviation of 12%. The normalized gain for the EICT was calculated to be 0.46, indicating medium gain (Figure 1).

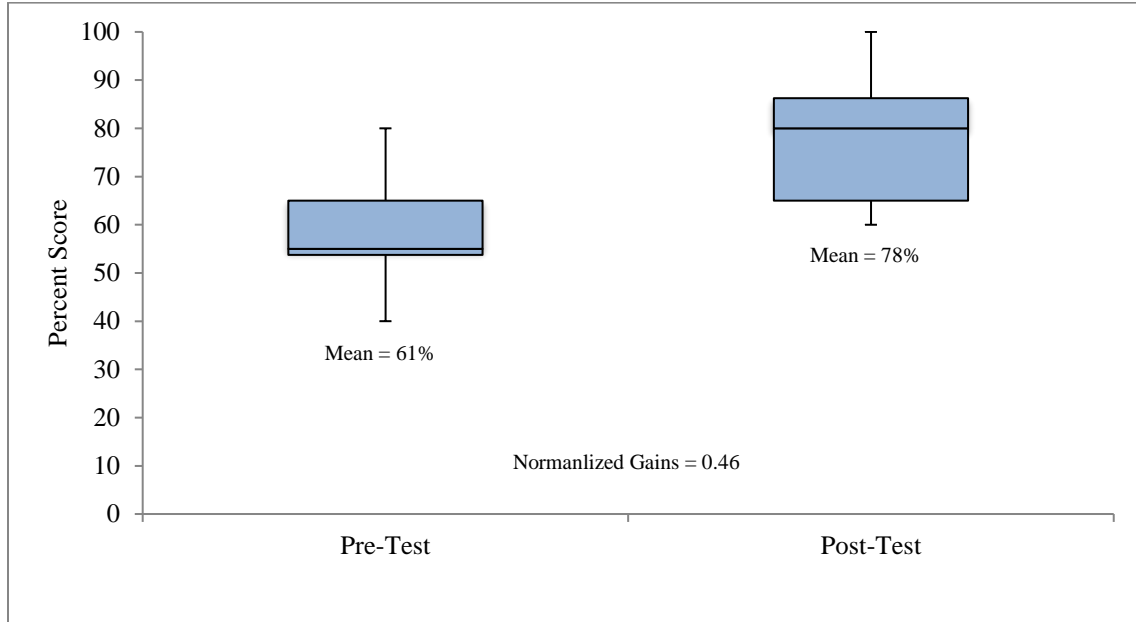


Figure 1. Box plot of pre-test and post-test Equations and Inequalities Concept Test, ($N=28$).

Figure 2 illustrates the box plot distribution of the pre- and post-treatment Linear Relations Concept Test (LRCT) ($N=28$). The mean score for the pre-treatment LRCT was 42% with a standard deviation of 12%. The sample's post-treatment LRCT mean saw an increase to 72% with a 13% standard deviation. In addition, the normalized gain for the LRCT from pre-treatment to post-treatment was calculated to be 0.52, which is classified as a medium gain. The gains students made mirrored student thinking from the Linear Relations Student Interviews (LRSI) where students thought they did much better on the post-test ($N=7$). One interviewed student said they achieved improvement because they, "Completed the ranking tasks...and learnt a bunch of strategies."

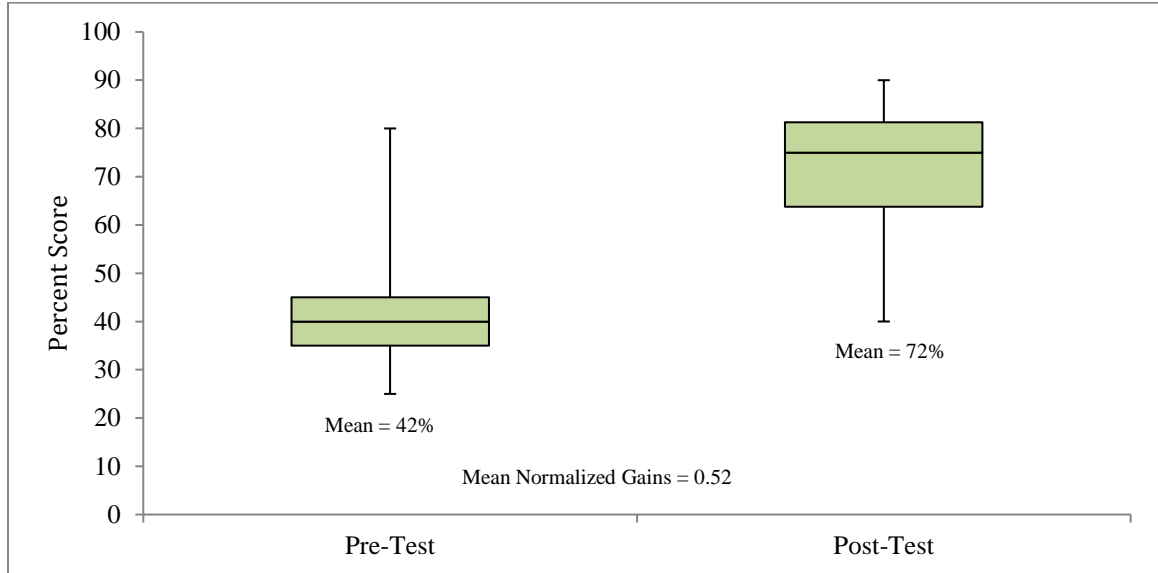


Figure 2. Box plot of pre-test and post-test Linear Relations Concept Test, ($N=28$).

The Linear Relations Questionnaire (LRQ) provided insight into the confidence and attitudes towards mathematics that characterized this class ($N=28$). Eighty-nine percent of students indicated a positive response of *agree* or *somewhat agree* when asked if they are confident when it comes to mathematics. Furthermore, 75% indicated they enjoyed solving mathematics problems. Despite having general confidence and positive attitudes towards mathematics, when responding to the statement *I feel nervous about the upcoming test for this unit (linear relations)*, 39% of students indicated that they *somewhat agree* with this statement. One student said, “I thought that this chapter was actually kind of hard.” In preparation for the test, the same student also said, “I had to study all weekend.” To that end, students identified understanding concepts along with communicating their reasoning and solutions as the most difficult aspects of mathematics (Figure 3). Both of these aspects were prominent throughout this unit.

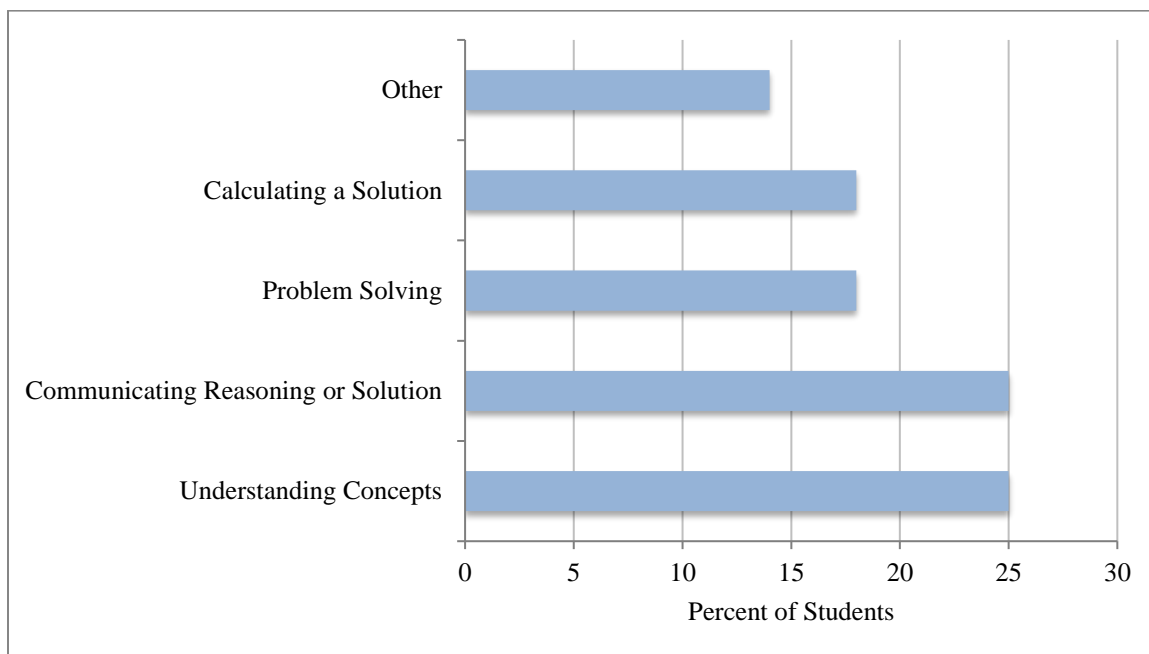


Figure 3. Student responses to the question *The most challenging part of math is...* from the Linear Relations Questionnaire, ($N=28$).

The LRQ also provided insight to student reflections of the effectiveness of Ranking Task Activities (RTAs) (Figure 4). Sixty-five percent of students had a positive perception on the effectiveness of RTAs in improving their understanding of linear relations concepts, versus 25% who shared a negative view. Students who were chosen for the LRSI mostly spoke to the benefit of RTAs, as one student said, “If there was like something you didn’t understand about the concept, then practicing it with the [ranking task] really helped.” Completing the ranking task required students to create a model, which tested their ability to understand and apply the pertinent mathematics concepts. When asked as to how students performed ranking tasks in the LRSI some students communicated that they attempted to apply one concept to all situations. For example, the thought process was explained by one student who said, “I would go through each one at a time, the same way, then try to rank them accordingly.” Other students applied

several concepts at once, like one who said, “I just use whatever one of the strategies you taught us that applies to each one, then ranked them afterwards.”

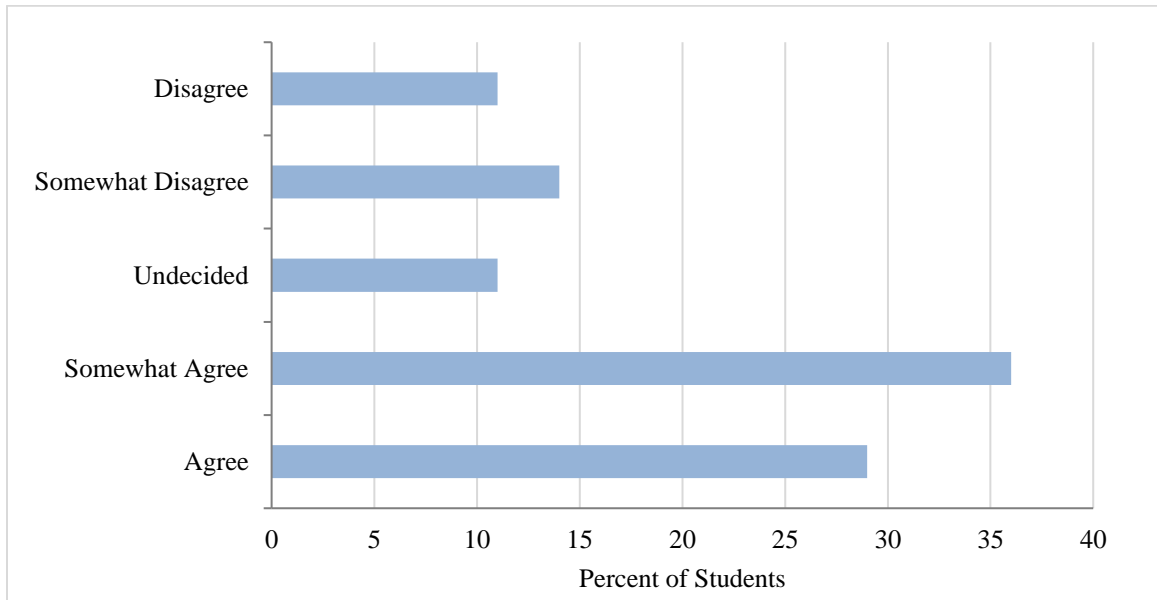


Figure 4. Student responses to the question *Completing Ranking Task Activities help my understanding of math concepts* from the Linear Relations Questionnaire, ($N=28$).

For the nine RTAs that were completed as part of the treatment unit, all but one showed an increase in correct ranking and explanation after students had time to discuss with their peers (Figure 5) ($N=28$). The mean percentage increase from before discussion to after was 16%; however, the mean normalized gain was 48%. The confidence level students indicated showed a similar pattern of increase from before discussion to after with a normalized gain of 62%.

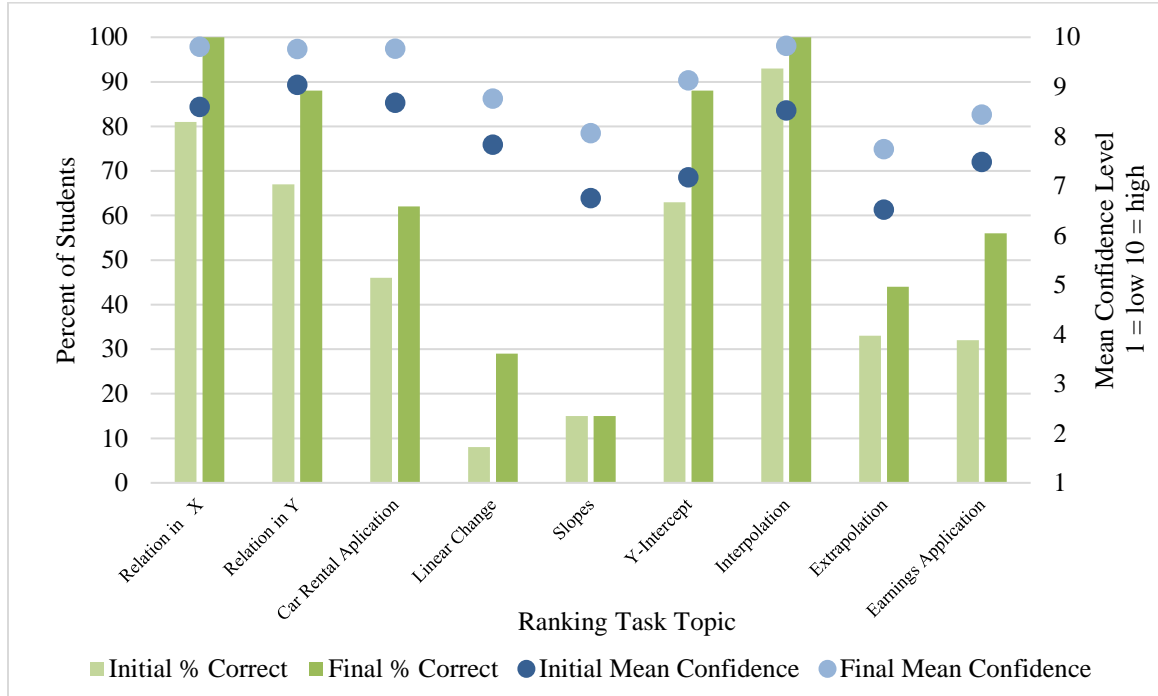


Figure 5. Initial and final correct student responses and confidence levels for nine Ranking Task Activities, ($N=28$).

The results from the LRQ also showed student thinking on discussion in the mathematics classroom (Figure 6). Seventy-five percent indicated a positive response of *agree* or *somewhat agree* to whole class discussions being beneficial. The positive responses increased when considering small group discussion, as 89% were favorable including 54% who gave the strongest response of *agree*. One student said, “[Discussion of solutions] helps you understand another student’s thought process. So if you get it wrong, you won’t continue to get it wrong which will help you.”

Students also perceived peer discussion to increase their confidence as 64% *agreed* and 21% *somewhat agreed* they were more confident with an answer to a problem after discussing it. One student said, “If everyone had the same answer as you, you would feel, like, more confident.”

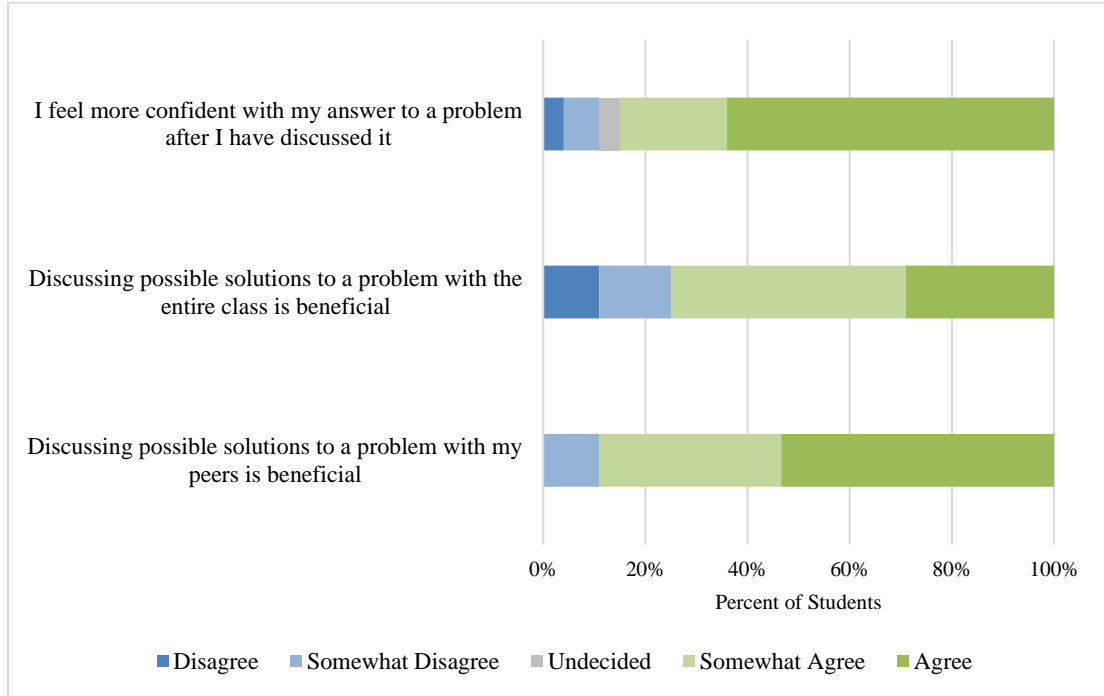


Figure 6. Student responses to peer discussion related questions from the Linear Relations Questionnaire, ($N=28$).

The results of the peer instruction themed LRQ questions that were considered as a Likert-scale further indicated that students observed a benefit from peer instruction (Figure 7). Eighty-two percent of students had a mean Likert-scale score for these questions of less than three on the five-point scale. The overall mean was determined to be 2.2. The only question that was part of the peer instruction theme that indicated a negative view of peer instruction was a question that asked students if they preferred to work by themselves (question 6). Question 6 showed 54% of students indicated that they prefer to work on mathematics problems on their own.

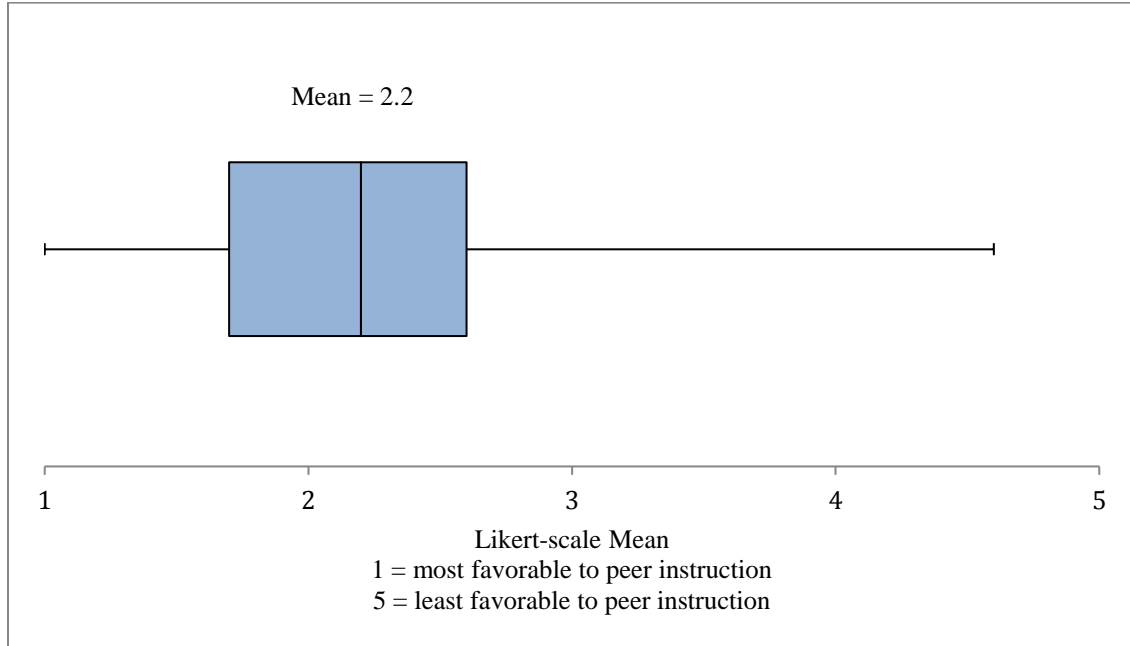


Figure 7. Student Likert-scale means on peer instruction questions from Linear Relations Questionnaire, ($N=28$).

INTERPRETATION AND CONCLUSION

The results of this study provided evidence that mathematics-based ranking tasks can benefit student understanding of mathematics concepts. When administered in a peer instruction environment, results suggested that ranking tasks could be even more powerful by improving understanding and confidence.

The mean normalized gain for the treatment unit on the Linear Relations Concept Test could be classified as a medium gain (Hake, 1998). However, it represented a gain of over 50% of the available marks from pre-test to post-test. Given that the sample class was an honors class, and students in previous studies have covered some of the material, I believe this size of a normalized gain supports the effectiveness of Ranking Task Activities (RTAs). Furthermore, the mean normalized gains for the treatment unit do compare favorably to the non-treatment unit, suggesting improved achievement. Upon

reviewing the tests used for the treatment and non-treatment units, there were some questions on the former that were difficult and were not done well on either the pre-test or post-test. One possible explanation for the greater difficulty could be that the treatment unit test contained more word problems due to the nature of the learning objectives for linear relations. Consequently, poor performance on the more difficult questions could have skewed the normalized gains achieved by the class.

The data from each RTA shows that more students were able to provide and explain the correct ranking after discussing with their peers. In addition, student confidence increased across the board. I've seen confidence play a large role in student success in my teaching career. Conversely, I've also seen a lack of confidence limit the ability of students to take risks or even attempt to solve mathematical problems. Therefore, any activity that can increase confidence is beneficial in the learning process.

The results from the Linear Relations Questionnaire (LRQ) and Linear Relations Student Interviews further support the data from the concept tests and individual ranking tasks, especially when it comes to peer instruction. The peer instruction LRQ Likert-scale data suggest that students perceive that a peer instruction environment is helpful as the overall class mean is firmly below the mid-point of the scale. The strongest opposition to peer instruction came when students were asked if they preferred to work on mathematics problems by themselves (question 6); where slightly over half of the students *agreed* or *somewhat agreed*. The preference to work by themselves was overshadowed by stronger student perception of group activities such as whole- and small-group discussion as well as instruction from peers. I believe the difference in these

items may point to a disconnect between what students prefer and what is of greatest benefit to them.

Another interesting result was the comparison of student perceptions of small-group discussion to whole-group discussion. Although the students saw both discussion types as beneficial, small-group discussion was clearly the favorite. Small group discussion is a trademark of a peer instruction model and this can be seen to validate its role in amplifying ranking tasks.

The students also had their say as to whether ranking tasks were effective or not. Although it wasn't an overwhelming majority, 65% of the students did believe RTAs were effective. I contend that these results were supportive of RTAs and I can offer two explanations as to why a larger number of students didn't agree. First, one student told me in the interview that she often wasn't able to see where she went wrong if she didn't have the same ranking as her classmates. I tried to encourage frank discussion and not just a comparison of rankings, but I could see in some cases this was primarily what certain groups were doing. If some students were not able to see where they made any errors then I can see how they would potentially view RTAs as less effective. As I will discuss in the following section, this problem will be addressed in any future usage of RTAs.

Second, I believe social dynamics and student predisposition to the individual work routine of mathematics classes may explain other opposing views for RTAs. Slightly more than half of this class said that they preferred to work on math problems by themselves. It could be concluded that some members of the class would prefer to stick

to a traditional model of individual work, and thus have a negative view of any activity which forces them out of their comfort zone and to work with their classmates on problems. However, what students prefer is not always be what is best for their learning process.

By all indications from this study, mathematics-based ranking tasks appear to positively affect student conceptual understanding of mathematics. Furthermore, based on the results of this action research, the application of a peer instruction approach to how ranking tasks are performed is shown to have a compounding effect when it comes to understanding and confidence.

VALUE

The most important outcomes from this action research were the benefits it served to my students and to my teaching practice moving forward. As the results have indicated, student understanding and confidence were positively affected by the use of ranking tasks in a peer instruction environment. However, there were some additional benefits this action research provided to my students.

Multiple students in the interviews suggested that Ranking Task Activities (RTAs) exposed students not only to the correct understanding of the concept, but alternative methods. This promotion of open-mindedness is an interesting benefit of RTAs that goes beyond just correct understanding. Especially in a discipline like mathematics where the plurality of solutions or approaches can sometimes be accidentally discouraged, I believe these activities have only helped students develop an open-minded disposition. In addition, as evidenced by how students completed their ranking tasks, I

believe critical thinking and problem solving skills were fostered throughout these activities. Students had to use criteria to evaluate similar situations with concepts acquired in class. In addition, they had to work around complicating factors in order to arrive at the correct solution.

Another stated benefit of peer instruction arises from having students teach students. Students who participated in the Linear Relations Student Interviews spoke of the benefit of learning from someone else as one student said, “One time I calculated it wrong and [another student] helped me. She explained her process and I understood how she did it and I was able to redo it and get the answer.” There was also an obvious sense of empowerment from the students I observed teaching a concept to another student. One student said, “I was able to explain it to them and tell them what they did wrong by trying to not [say] ‘this is wrong’ but [explaining] it nicely and they ended up changing their answer.”

There are also several key outcomes that I have taken from this study. One is related to how I will begin administering RTAs in the future. In subsequent uses I hope to begin using RTAs by piloting a non-mathematics RTA that would focus on, and encourage, varied solutions. If done this way, I could really encourage open-minded behavior and meaningful discussions with all students in mind. This would also help model what a good RTA would look like and make sure everyone is getting the most out of the tasks and everyone is participating appropriately and not just comparing rankings. For future uses of RTAs, I also plan to hold students more accountable to the discussion phase to curb this rush to complete and compare rankings. One possible alteration would

be to have a pencil-down stipulation to the first part of the discussion until everyone has a chance to explain their method and their ranking. Hopefully these alterations would ensure every student gains a positive experience from RTAs.

A larger scale perspective I gained was the value my students put on small-group discussion. Their indications echoed my observations that, for the most part, the peer instruction aspect of the RTAs we performed was the most influential part. Even though students thought whole-class debriefings were effective, they were even more in favor of small-group discussions. The significance of this finding for my teaching practice will be to include as much small-group discussion or formative assessments as possible. Despite a small amount of push back from some students on peer instruction methods, I believe it is a model that works in all classrooms and one that I will continue to use increasingly in the future.

Completing the action research process throughout the course of this project also provided me with a greater insight to my daily practice. I've seen the benefit that assessing my practice through data collection instruments can provide. In addition, I've also seen what can be gleaned from thorough analysis of the data. Obviously, given that a high-school teacher has to prepare and execute lessons and give feedback to students in their limited preparation time, there is not enough time to perform thorough action research on every aspect of their practice. However, I have been inspired by this experience to continue the action research process on a regular basis within my classroom practice.

REFERENCES CITED

- Adams, W. K., Perkins, K. K., Podolefsky, N. S., Dubson, M., Finkelstein, N. D., & Wieman, C. E. (2006). New instrument for measuring student beliefs about physics and learning physics: The Colorado Learning Attitudes about Science Survey. *Physical Review Special Topics-Physics Education Research*, 2(1), 010101.
- Black, P., & William, D. (1998). Inside the Black Box: Raising Standards Through Classroom Assessment. *Phi Delta Kappan*, 80, 139-144, 146-148.
- Bybee, R. W., & Fuchs, B. (2006). Preparing the 21st century workforce: A new reform in science and technology education. *Journal of Research in Science Teaching*, 43(4), 349-352.
- Cahyadi, V. (2004). The effect of interactive engagement teaching on student understanding of introductory physics at the faculty of engineering, University of Surabaya, Indonesia. *Higher Education Research & Development*, 23(4), 455-464.
- Crouch, C.H., & Mazur, E. (2001). Peer Instruction: Ten years of experience and results. *American Journal of Physics*, 69(9), 970-977.
- Cox, A.J., Belloni, M., & Christian, W. (2005). Teaching Physics with Physlet Based Ranking Task Exercises. *The Physics Teacher*, 43(10), 587-592.
- Desbien, D. (1997). Ranking Task Uses and Results. *Curriculum and Faculty Development Newsletter for Two-Year College Physics Teachers*. Summer 1997, 1, 8.
- Duit, R., & Treagust, D. F. (2003). Conceptual change: a powerful framework for improving science teaching and learning. *International Journal of Science Education*, 25(6), 671-688.
- Dykstra Jr, D. I., & Sweet, D. R. (2009). Conceptual development about motion and force in elementary and middle school students. *American Journal of Physics*, 77(5), 468-476.
- Enderle, P. J., Southerland, S. A., & Grooms, J. A. (2013). Exploring the context of change: Understanding the kinetics of a studio physics implementation effort. *Physical Review Special Topics-Physics Education Research*, 9(1), 020107.
- Eryilmaz, A. (2002). Effects of conceptual assignments and conceptual change discussions on students' misconceptions and achievement regarding force and motion. *Journal of Research in Science Teaching*, 39(10), 1001-1015.

- Gok, T. (2012). The impact of peer assessment on college students' beliefs about physics and conceptual understanding of electricity and magnetism. *International Journal of Science and Mathematics Education*, 10(2), 417-436.
- Gokhale, A. A. (1995). Collaborative learning enhances critical thinking. *Journal of Technology Education*, 7(1), from <http://scholar.lib.vt.edu/ejournals/JTE/v7n1/gokhale.jtev7n1.html?ref=Sawos.Org>
- Hake, R. R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American journal of Physics*, 66(1), 64-74.
- Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *The Physics Teacher*, 30(3), 141-158.
- Hudgins, D.W., Prather, E.E., Grayson, D.J., & Smits, D.P. (2007). Effectiveness of Collaborative Ranking Tasks on Student Understanding of Key Astronomy Concepts. *Astronomy Education Review*, 5, 1-22.
- Keeley, P.D. (2014). *Science Formative Assessment, Volume 2: 50 More Strategies for Linking Assessment, Instruction, and Learning*. Thousand Oaks, CA: Corwin Press.
- Laws, P., Sokoloff, D., & Thornton, R. (1999). Promoting active learning using the results of physics education research. *UniServe Science News*, 13, 14-19.
- Liem, T. L. (1987). *Invitations to science inquiry*. S17 Science, 333 Clark Ave West, Apt 216, Thornhill, ON, L4J 7K4, Canada (www.s17science.com).
- Lietz, M., (2010). *Using Ranking Tasks in the AP Physics Classroom*. Retrieved February 25th, 2015 from http://apcentral.collegeboard.com/apc/members/courses/teachers_corner/189936.html
- Malone, K. L. (2008). Correlations among knowledge structures, force concept inventory, and problem-solving behaviors. *Physical Review Special Topics-Physics Education Research*, 4(2), 010114.
- Novak, G. M. (2011). Just- in- time teaching. *New Directions for Teaching and Learning*, 2011(128), 63-73.
- O'Kuma, T.L., Maloney, D.P., & Hieggelke, C.J. (2000). *Ranking Task Exercises in Physics*. Upper Saddle River, NJ: Prentice Hall.

- Posner, G.J., Strike, K.A., Hewson, P.W., & Gertzog, W.A. (1982). Accommodation of a Scientific Conception: Toward a Theory of Conceptual Change. *Science Education*, 66(2), 211-227.
- Redish, E. F., Saul, J. M., & Steinberg, R. N. (1998). Student expectations in introductory physics. *American Journal of Physics*, 66(3), 212-224.
- Rowlands, S., Graham, T., Berry, J., & McWilliam, P. (2007). Conceptual change through the lens of Newtonian mechanics. *Science & Education*, 16(1), 21-42.
- Sanders, M. (2009). Stem, stem education, stemmania. *The Technology Teacher*, 68(4), 20-26.
- Savinainen, A., & Viiri, J. (2008). The force concept inventory as a measure of students conceptual coherence. *International Journal of Science and Mathematics Education*, 6(4), 719-740.
- Siegler, R. S. (1976). Three aspects of cognitive development. *Cognitive psychology*, 8(4), 481-520.
- Stepans, J. (1996). *Targeting students' science misconceptions: Physical science concepts using the conceptual change model*. Idea Factory.
- Tirosh, D., & Tsamir, P. (2004). What can mathematics education gain from the conceptual change approach? And what can the conceptual change approach gain from its application to mathematics education?. *Learning and Instruction*, 14(5), 535-540.
- Vosniadou, S., & Verschaffel, L. (2004). Extending the conceptual change approach to mathematics learning and teaching. *Learning and instruction*, 14(5), 445-451.
- Wieman, C., & Perkins, K. (2005). Transforming physics education. *Physics Today*, 58(11), 36.

APPENDICES

APPENDIX A
INSTITUTIONAL REVIEW BOARD EXEMPTION



INSTITUTIONAL REVIEW BOARD
For the Protection of Human Subjects
FWA 00000165

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 Telephone: 406-994-6783
 FAX: 406-994-4303
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Chair: Mark Quinn
 406-994-5721
 mquinn@montana.edu
Administrator:
 Cheryl Johnson
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MEMORANDUM

TO: Ryan Harvey and John Graves

FROM: Mark Quinn *Mark Quinn*

DATE: November 3, 2015

RE: "The Effect of Ranking Tasks and Peer Instruction in a Mathematics Classroom" [RH110315-EX]

The above research, described in your submission of **November 2, 2015**, is exempt from the requirement of review by the Institutional Review Board in accordance with the Code of Federal regulations, Part 46, section 101. The specific paragraph which applies to your research is:

- ☒ (b) (1) Research conducted in established or commonly accepted educational settings, involving normal educational practices such as (i) research on regular and special education instructional strategies, or (ii) research on the effectiveness of or the comparison among instructional techniques, curricula, or classroom management methods.
- ☒ (b) (2) Research involving the use of educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures or observation of public behavior, unless: (i) information obtained is recorded in such a manner that human subjects can be identified, directly or through identifiers linked to the subjects; and (ii) any disclosure of the human subjects' responses outside the research could reasonably place the subjects at risk of criminal or civil liability, or be damaging to the subjects' financial standing, employability, or reputation.
- ☐ (b) (3) Research involving the use of educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures, or observation of public behavior that is not exempt under paragraph (b)(2) of this section, if: (i) the human subjects are elected or appointed public officials or candidates for public office; or (ii) federal statute(s) without exception that the confidentiality of the personally identifiable information will be maintained throughout the research and thereafter.
- ☐ (b) (4) Research involving the collection or study of existing data, documents, records, pathological specimens, or diagnostic specimens, if these sources are publicly available, or if the information is recorded by the investigator in such a manner that the subjects cannot be identified, directly or through identifiers linked to the subjects.
- ☐ (b) (5) Research and demonstration projects, which are conducted by or subject to the approval of department or agency heads, and which are designed to study, evaluate, or otherwise examine: (i) public benefit or service programs; (ii) procedures for obtaining benefits or services under those programs; (iii) possible changes in or alternatives to those programs or procedures; or (iv) possible changes in methods or levels of payment for benefits or services under those programs.
- ☐ (b) (6) Taste and food quality evaluation and consumer acceptance studies, (i) if wholesome foods without additives are consumed, or (ii) if a food is consumed that contains a food ingredient at or below the level and for a use found to be safe, or agricultural chemical or environmental contaminant at or below the level found to be safe, by the FDA, or approved by the EPA, or the Food Safety and Inspection Service of the USDA.

Although review by the Institutional Review Board is not required for the above research, the Committee will be glad to review it. If you wish a review and committee approval, please submit 3 copies of the usual application form and it will be processed by expedited review.

APPENDIX B

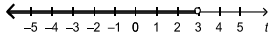
EQUATIONS AND INEQUALITIES CONCEPT TEST

Linear Equations and Inequalities Test

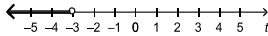
- _____ 1. Solve: $9x - 15 = 3$
 a. $\frac{46}{3}$ b. 9 c. -2 d. 2
- _____ 2. Solve: $5 = -3x + 14$
 a. $-\frac{19}{3}$ b. 3 c. -3 d. $\frac{19}{3}$
- _____ 3. Solve: $4x + 2.8 = 6.4$
 a. -1.2 b. -0.4 c. 5.7 d. 0.9
- _____ 4. Solve: $\frac{x}{7} - 4 = 5$
 a. 39 b. 2 c. 63 d. 33
- _____ 5. Write an equation for this statement: A number divided by 2, plus 5, is 8.
 a. $\frac{x+5}{2} = 8$ b. $\frac{x}{2} = 5 + 8$ c. $\frac{2}{x} + 5 = 8$ d. $\frac{x}{2} + 5 = 8$
- _____ 6. Solve: $3(x + 5) = 12$
 a. $\frac{7}{3}$ b. -6 c. -1 d. 4
- _____ 7. A number times 5, minus 6, is 8. Write an equation to determine the number.
 a. $6 - 5x = 8$ b. $5x - 6 = 8$ c. $5 - 6x = 8$ d. $6x - 5 = 8$
- _____ 8. Solve: $13 - 4x = 3x - 8$
 a. $x = -3$ b. $x = \frac{1}{3}$ c. $x = -\frac{1}{3}$ d. $x = 3$
- _____ 9. Solve: $3(5q - 4) = 2(4q + 6)$
 a. $q = -3\frac{3}{7}$ b. $q = \frac{7}{24}$ c. $q = -\frac{7}{24}$ d. $q = 3\frac{3}{7}$

____ 10. Which of these graphs is a solution of $t \leq 3$?

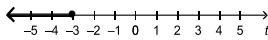
i) .



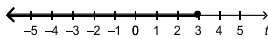
ii) .



iii) .



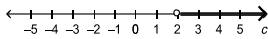
iv) .



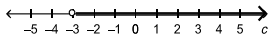
- a. Graph ii b. Graph iii c. Graph iv d. Graph i

____ 11. Which of these graphs is a solution of $c > -3$?

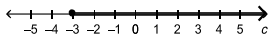
i) .



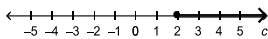
ii) .



iii) .



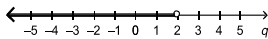
iv) .



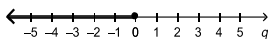
- a. Graph i b. Graph ii c. Graph iv d. Graph iii

____ 12. Which of these graphs represent the solution of the inequality $q - 2 \leq 0$?

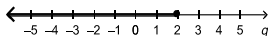
i) .



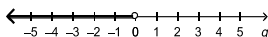
ii) .



iii) .



iv) .



- a. Graph ii b. Graph iv c. Graph iii d. Graph i

- _____ 13. Solve: $12t - 3 < 16 + 13t$
a. $t > -19$ b. $t < 13$ c. $t < -19$ d. $t > 13$
- _____ 14. Solve: $20 - 3t > 5$
a. $t > -5$ b. $t < -5$ c. $t < 5$ d. $t > 5$
- _____ 15. An equipment rental company charges a flat rate of \$25, plus \$13 per day for insurance. Kyle has \$121. Write an inequality to represent the number of days, d , for which he can rent the equipment.
a. $25 + 13d > 121$ b. $25 + 13d \geq 121$
c. $25 + 13d < 121$ d. $25 + 13d \leq 121$
- _____ 16. A plumber charges \$75 for a house call plus \$45 per hour. How many hours did the plumber work if he charged \$210?
a. 2 b. 3 c. 4 d. 6
- _____ 17. Terrance bought 3 markers. His sister bought 5 markers. Terrance and his sister spent a total of \$16 on the markers. What was the price of each marker?
a. \$16 b. \$8 c. \$4 d. \$2
- _____ 18. Which of the following correctly shows $A = \frac{1}{2}bh$ solved for h .
a. $h = \frac{1}{2}Ab$ b. $h = \frac{A}{2b}$ c. $h = 2Ab$ d. $h = \frac{2A}{b}$
- _____ 19. Solve $4k - 7 + 3 + 5k = 59$
a. $k = 6$ b. $k = 6,6$ c. $k = 7$ d. $k = 11,8$
- _____ 20. Jacob's father is 4 years more than 3 times Jacob's age. Jacob's father is 37 years old. How old is Jacob?
a. 10 b. 11 c. 13 d. 17

APPENDIX C
RANKING TASK ACTIVITIES

Your task is to rank these variations on a specified basis. After ranking the items, you will be asked to explain how you determined your ranking sequence and the reasoning behind the way you used the values of the variables to reach your answer.

Below are eight linear relations. Given that $x = 4$, rank the functions from greatest to least based on their produced y -values.

A	B	C	D
$y = 2x - 5$	$y = -2x + 5$	$y = 0.5x + 5$	$y = 2x + 2$
E	F	G	H
$y = -0.5x + 2$	$y = -2x - 2$	$y = 0.5x$	$y = 5$

Explain your reasoning.

10

After Discussion Response

Situation: Below are eight linear functions. Given that $x = 4$, rank the functions from greatest to least based on their produced y -values.

A $y = 2x - 5$	B $y = -2x + 5$	C $y = 0.5x + 5$	D $y = 2x + 2$
E $y = -0.5x + 2$	F $y = -2x - 2$	G $y = 0.5x$	H $y = 5$

Final Ranking:

Greatest 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, 7 _____, 8 _____ Least

Or, all functions give the same y -value _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

Ranking Task 2 – Linear Relation in Y

Instructions:

Each ranking task will have a number of situations, or variations of a situation, that have varying values for two or three variables.

Your task is to rank these variations on a specified basis. After ranking the items, you will be asked to explain how you determined your ranking sequence and the reasoning behind the way you used the values of the variables to reach your answer.

Situation:

Below are eight linear functions. Given that $y = -2$, rank the functions from greatest to least based on their associated x -values.

A $y = 2x - 5$	B $y = -2x + 5$	C $y = 0.5x + 5$	D $y = 2x + 2$
E $y = -0.5x + 2$	F $y = -2x - 2$	G $y = 0.5x$	H $y = x - 5$

Individual Response

Greatest 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, 7 _____, 8 _____ Least

Or, all functions give the same x -value _____

Explain your reasoning.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

After Discussion Response

Situation:

Below are eight linear functions. Given that $y = -2$, rank the functions from greatest to least based on their associated x-values.

A $y = 2x - 5$	B $y = -2x + 5$	C $y = 0.5x + 5$	D $y = 2x + 2$
E $y = -0.5x + 2$	F $y = -2x - 2$	G $y = 0.5x$	H $y = x - 5$

Final Ranking:

Greatest 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, 7 _____, 8 _____ Least

Or, all functions give the same x-value _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

Chris has decided to rent a car for his upcoming seven-day vacation. He searched online for car rental options and found eight websites offering the same car, but at different prices. Chris estimates that he will drive 200 km on his trip. Rank these websites in order from least to greatest cost over the seven-day vacation.

A \$140 + \$0.75 per km	B \$150 + \$0.50 per km	C \$250 per day + \$0.25 per km	D \$100 + \$1 per km
E \$2 per km	F \$150 + \$0.75 per km	G \$300	H \$200 + \$0.25 per km

Least 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, 7 _____, 8 _____ Greatest

Or, all websites had the same price _____

Explain your reasoning.

How confident are you in your answer?

Not very confident 1 2 3 4 5 6 7 8 9 10 Very Confident

After Discussion Response

Situation:

Chris has decided to rent a car for his upcoming seven-day vacation. He searched online for car rental options and found eight websites offering the same car, but at different prices. Chris estimates that he will drive 200 km on his trip. Rank these websites in order from least to greatest cost over the seven-day vacation.

A \$140 + \$0.75 per km	B \$150 + \$0.50 per km	C \$250 per day + \$0.25 per km	D \$100 + \$1 per km
E \$2 per km	F \$150 + \$0.75 per km	G \$300	H \$200 + \$0.25 per km

Final Ranking:

Least 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, 7 _____, 8 _____ Greatest

Or, all websites had the same price _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

Ranking Task 4 – Linear Change

Instructions:

Each ranking task will have a number of situations, or variations of a situation, that have varying values for two or three variables.

Your task is to rank these variations on a specified basis. After ranking the items, you will be asked to explain how you determined your ranking sequence and the reasoning behind the way you used the values of the variables to reach your answer.

Situation:

Below are eight linear relations. Rank the relations from greatest to least based on their constant rate of change. If the rate of change is not constant assign a value of 0.

<div>A</div> <div>$y = 2x - 5$</div>	<div>B</div> <table><tr><td>x</td><td>y</td></tr><tr><td>1</td><td>-2</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>6</td></tr></table>		x	y	1	-2	2	2	3	6	<div>C</div> <table><tr><td>x</td><td>y</td></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>12</td></tr></table>		x	y	1	3	2	8	3	12	<div>D</div> <div>$y = 2x + 2$</div>
x	y																				
1	-2																				
2	2																				
3	6																				
x	y																				
1	3																				
2	8																				
3	12																				
<div>E</div> <table><tr><td>x</td><td>y</td></tr><tr><td>1</td><td>1</td></tr><tr><td>4</td><td>4</td></tr><tr><td>7</td><td>7</td></tr></table>	x	y	1	1	4	4	7	7	<div>F</div> <table><tr><td>x</td><td>y</td></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>9</td></tr><tr><td>3</td><td>8</td></tr></table>		x	y	1	10	2	9	3	8	<div>G</div> <div>$y = x^2$</div>	<div>H</div> <div>$y = -3x + 10$</div>	
x	y																				
1	1																				
4	4																				
7	7																				
x	y																				
1	10																				
2	9																				
3	8																				

Individual Response

Greatest 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, 7 _____, 8 _____ Least

Or, all functions give the same constant linear change _____

Explain your reasoning.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

After Discussion Response

Situation: Below are eight linear relations. Rank the relations from greatest to least based on their constant rate of change. If the rate of change is not constant assign a value of 0.

A $y = 2x - 5$	B		C		D $y = 2x + 2$
	x	y	x	y	
	1	-2	1	3	
	2	2	2	8	
	3	6	3	12	
E		F		G $y = x^2$	H $y = -3x + 10$
x	y	x	y		
1	1	1	10		
4	4	2	9		
7	7	3	8		

Final Ranking:

Greatest 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, 7 ____, 8 ____ Least

Or, all functions give the same constant linear change ____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

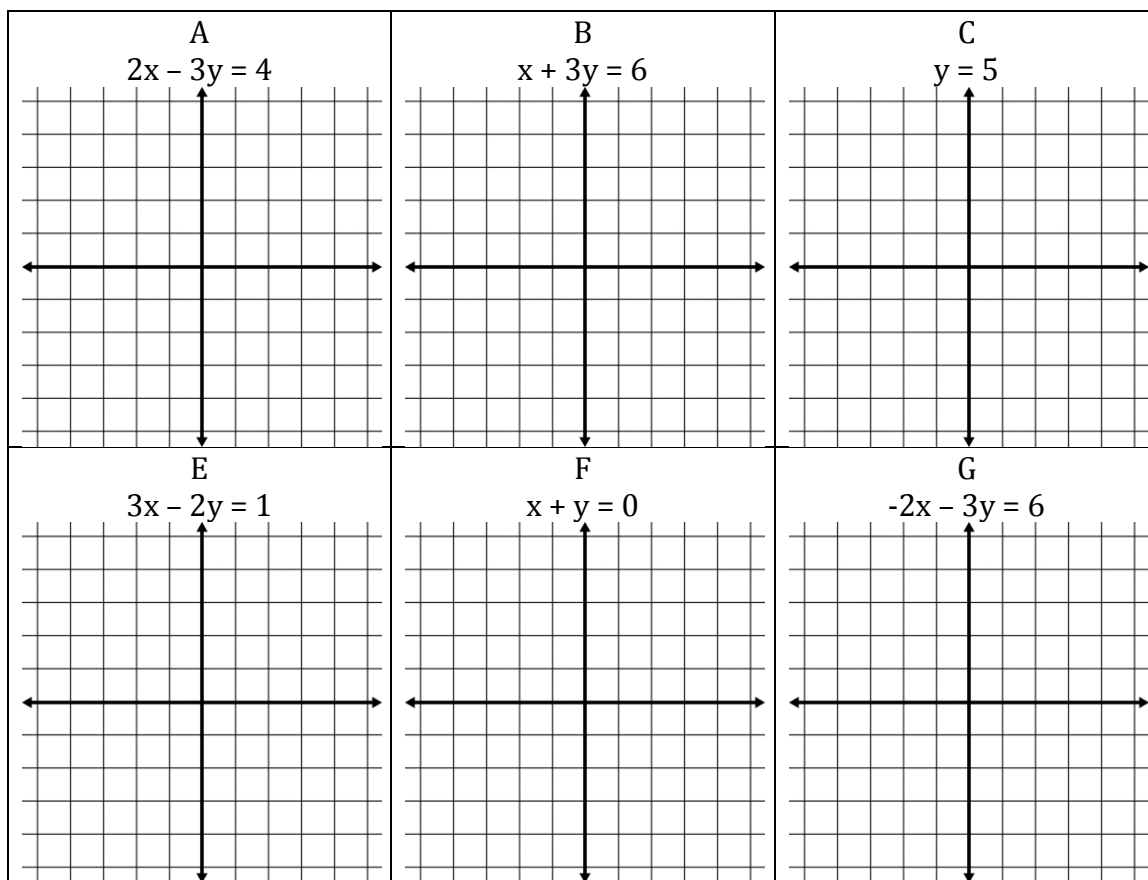
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Ranking Task 5 – Y-Axis

Situation:

Below are six linear equations. Rank them in order from least to greatest based on where they cross the y-axis.



Individual Response

Least 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, Greatest

Or, they all cross at the same spot _____

Explain your reasoning.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

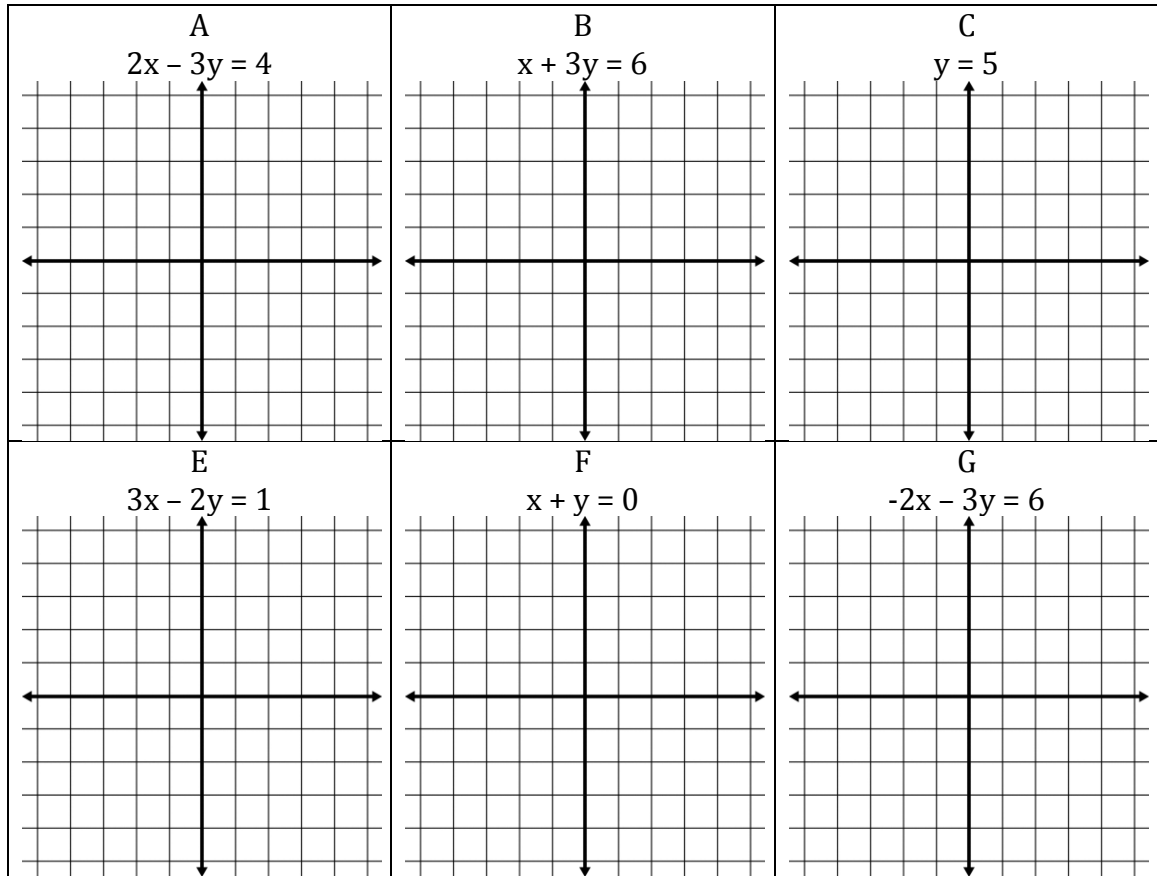
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10

After Discussion Response

Situation:

Below are six linear equations. Rank them in order from least to greatest based on where they cross the y-axis.



Final Ranking:

Least 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, Greatest

Or, they all cross at the same spot _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

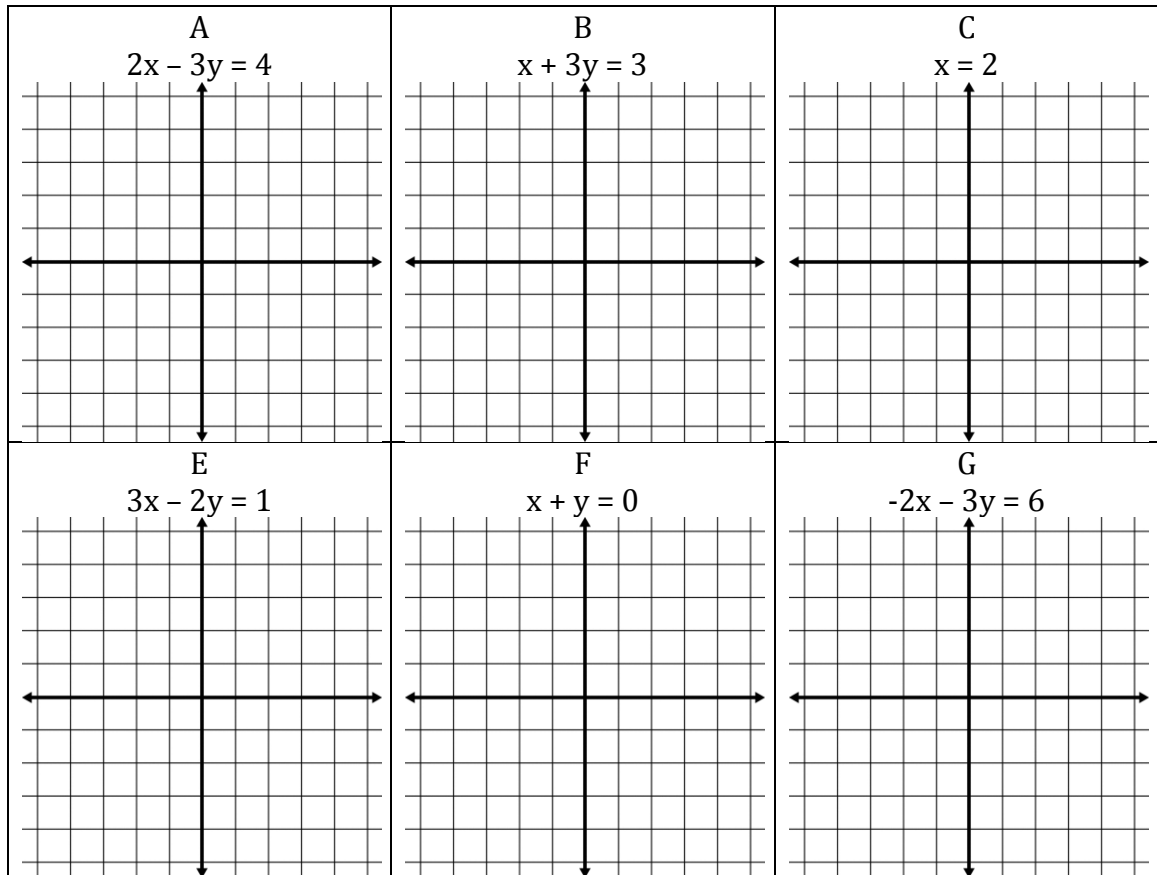
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10

Ranking Task 6 – X-Axis

Situation:

Below are six linear equations. Rank them in order from least to greatest based on where they cross the x-axis.



Individual Response

Least 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, Greatest

Or, they all cross at the same spot _____

Explain your reasoning.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

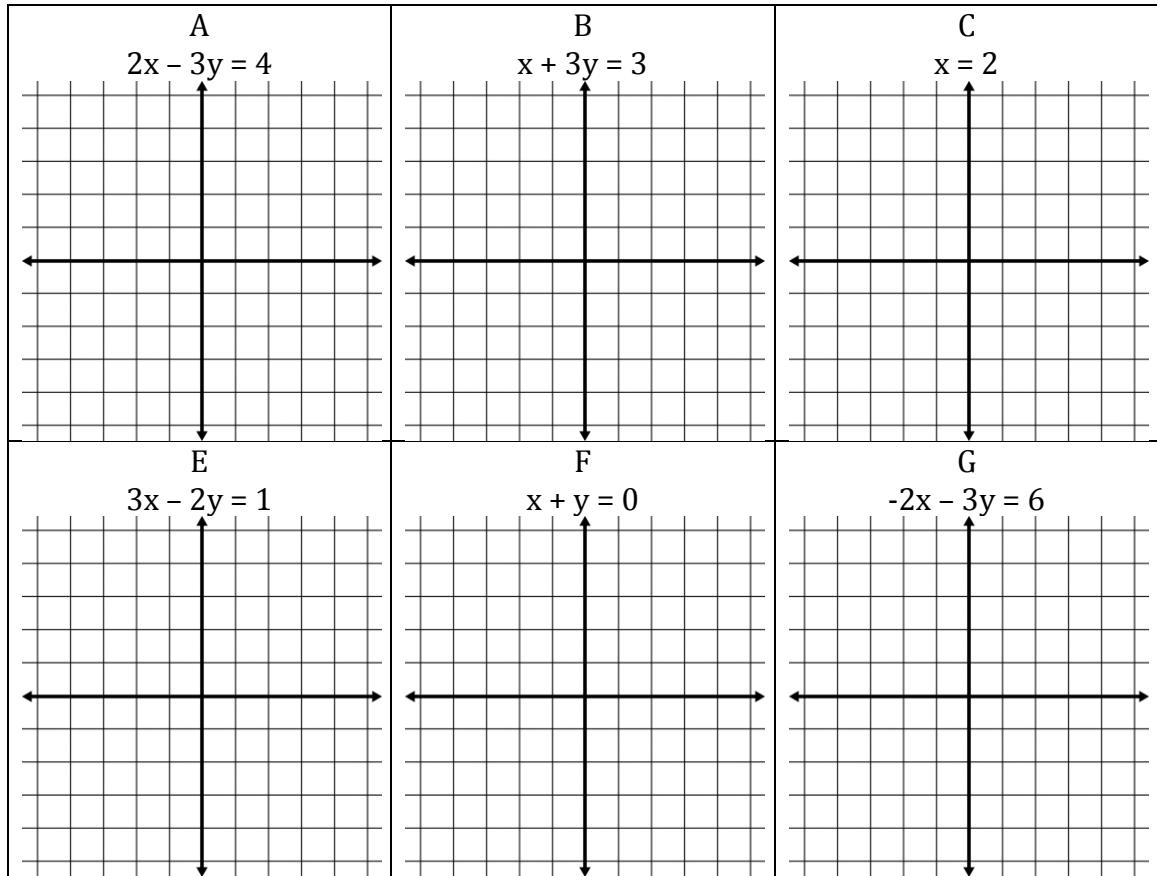
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10

After Discussion Response

Situation:

Below are six linear equations. Rank them in order from least to greatest based on where they cross the x-axis.



Final Ranking:

Least 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, Greatest

Or, they all cross at the same spot _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

Ranking Task 7 – Slopes

Situation:

Below are eight linear equations. Rank them in order from greatest to least based on where their slopes.

A $2x - y = 4$	B $y = 3x + 1$	C $y = -2x + 7$	D $y = 0.5x - 2$
E $y = 6 - 5x$	F $4x + 2y = 8$	G $y = 7x$	H $-2x - 3y = 6$

Individual Response

Greatest 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, 7 ____, 8 ____, Least

Or, they all have the same slope _____

Explain your reasoning.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

After Discussion Response

Situation:

Below are eight linear equations. Rank them in order from greatest to least based on where their slopes.

A $2x - y = 4$	B $y = 3x + 1$	C $y = -2x + 7$	D $y = 0.5x - 2$
E $y = 6 - 5x$	F $4x + 2y = 8$	G $y = 7x$	H $-2x - 3y = 6$

Final Ranking:

Greatest 1 _____, 2 _____, 3 _____, 4 _____, 5 _____, 6 _____, 7 _____, 8 _____, Least

Or, they all have the same slope _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

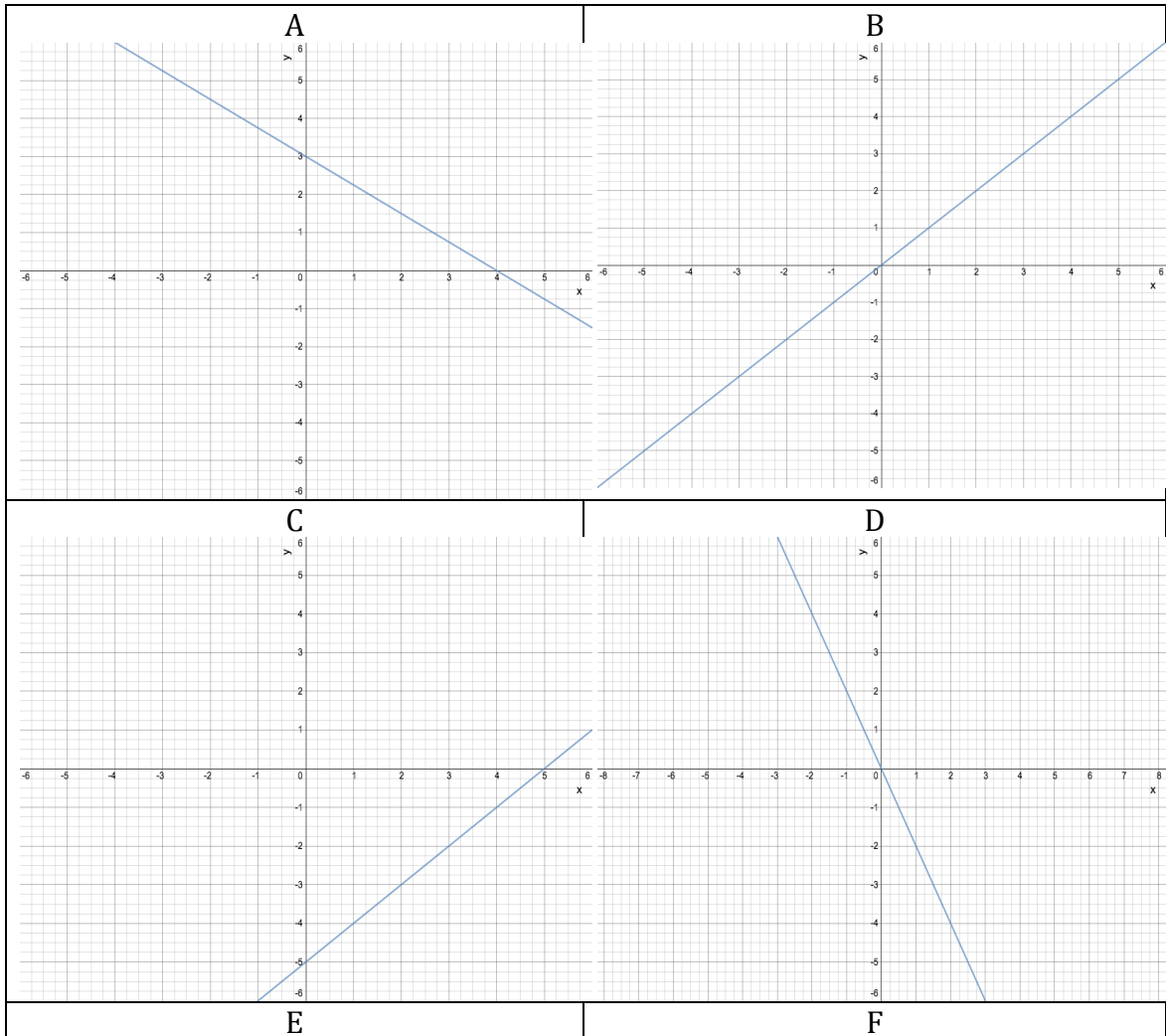
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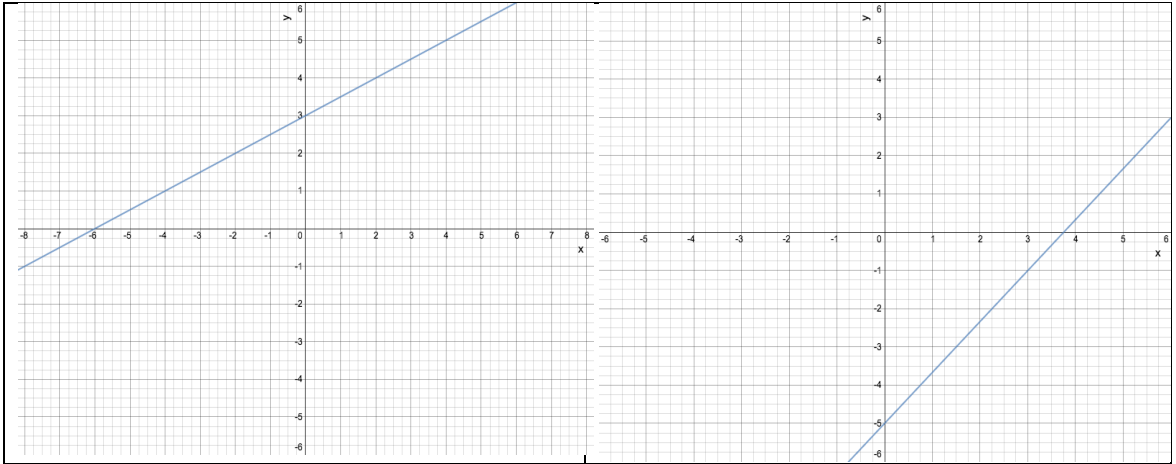
10

Ranking Task 8 – Interpolation

Situation:

Below are six graphs of linear equations. Rank them in order from least to greatest based their values when $x = 3$.





Individual Response

Least 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, Greatest

Or, they have the same value at $x = 3$ _____

Explain your reasoning.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

After Discussion Response

Situation:

On the previous page there are six graphs of linear equations. Rank them in order from least to greatest based their values when $x = 3$.

Final Ranking:

Least 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, Greatest

Or, they have the same value at $x = 3$ _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

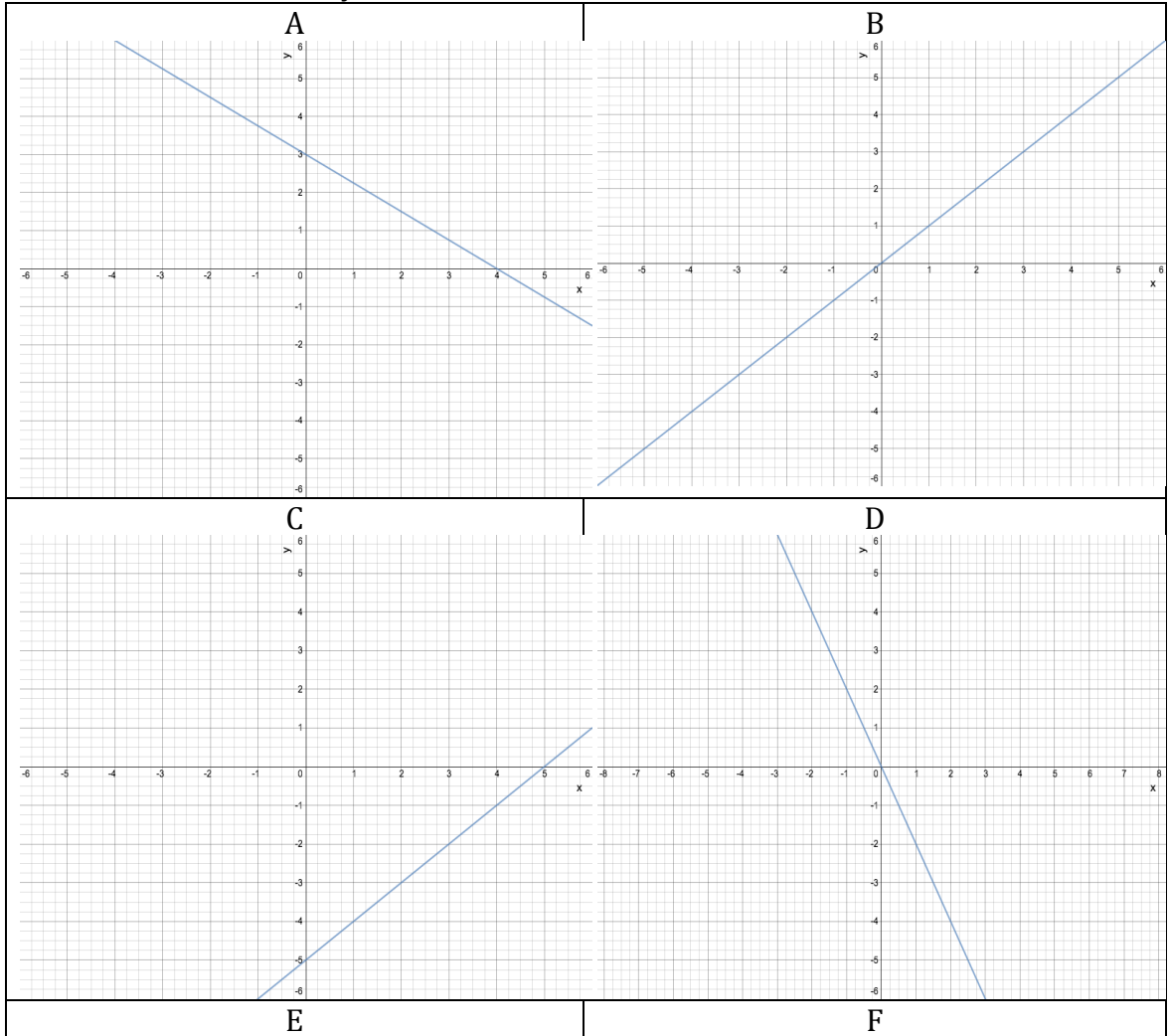
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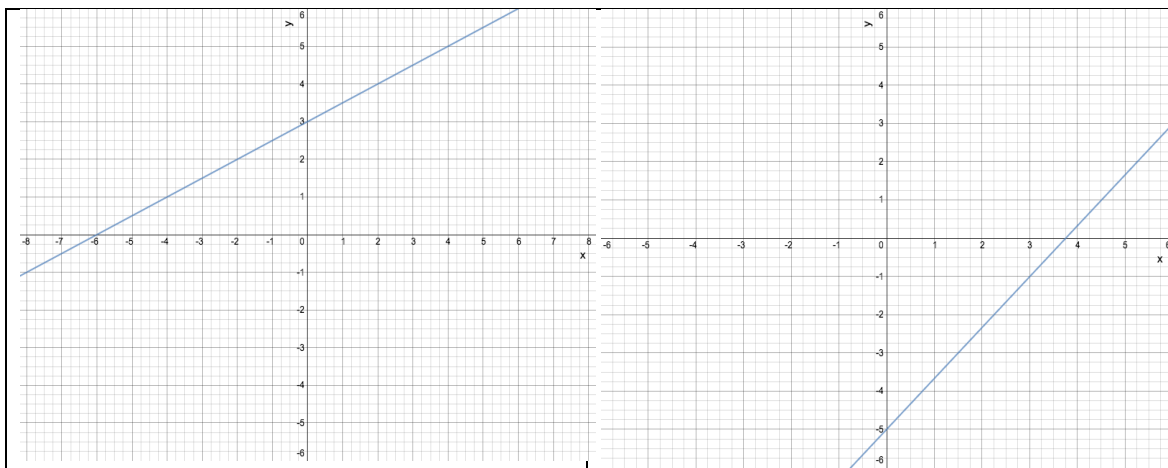
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Ranking Task 9 – Extrapolation

Situation:

Below are six graphs of linear equations. Rank them in order from greatest to least based their values when $y = 10$.





Individual Response

Greatest 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, Least

Or, they have the same value at $y = 10$ _____

Explain your reasoning.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

9

10

After Discussion Response

Situation:

On the previous page there are six graphs of linear equations. Rank them in order from least to greatest based their values when $y = 10$.

Final Ranking:

Greatest 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, Least

Or, they have the same value at $y = 10$ _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident

1

2

3

4

5

6

7

8

Very Confident

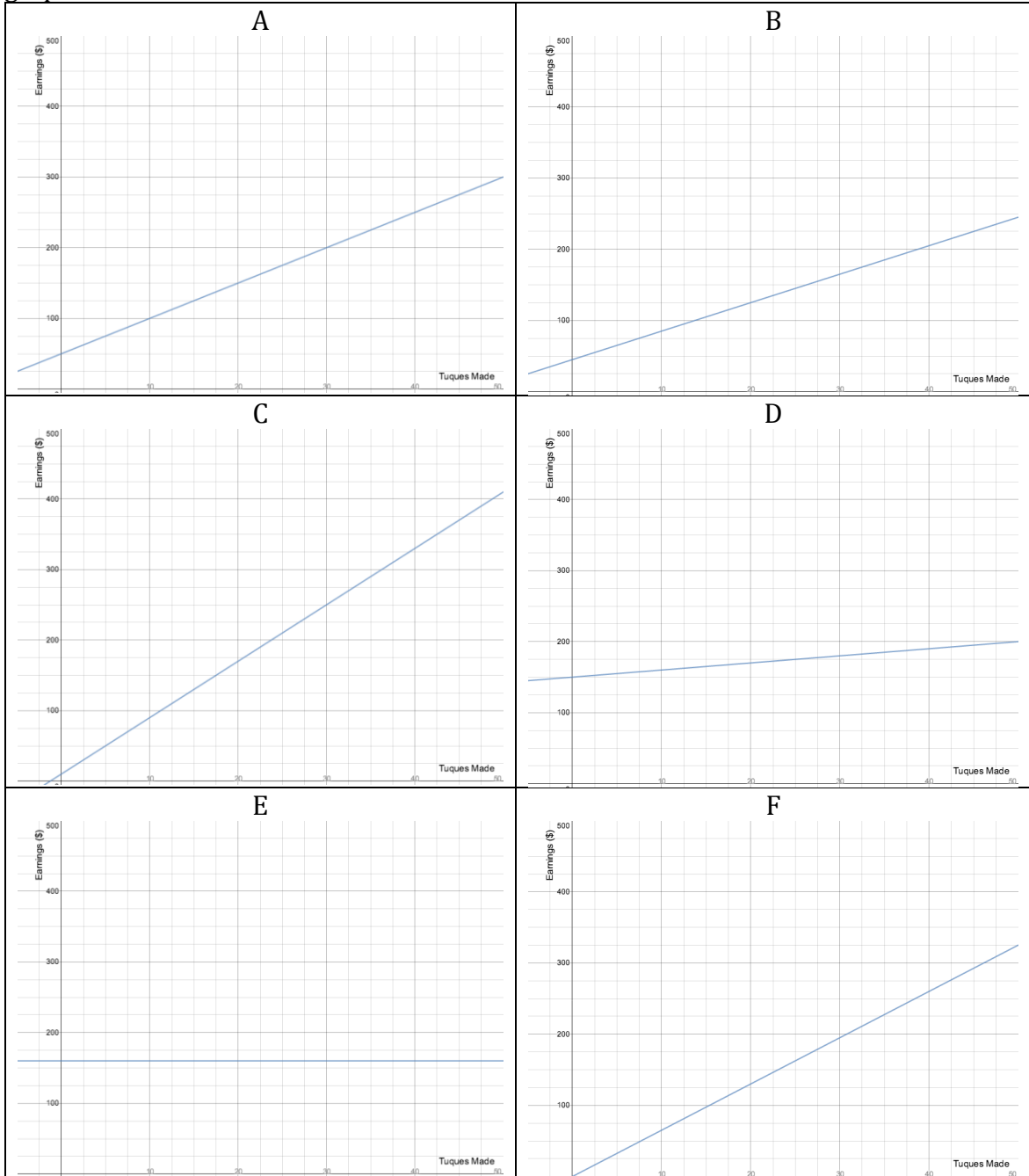
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10

Ranking Task 10 – Earnings Application

Situation:

Laura is offered a job making tuques. She has been offered six different payment structures. The payment structures see her earn a combination of a daily wage and an additional amount per tuque. She figures she can make anywhere between 25-30 tuques per day. Rank these payment structures from greatest to least based on the graphs below.



Individual Response

Greatest 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, Least

Or, all options pay the same _____

Explain your reasoning.

How confident are you in your answer?

Not very confident										Very Confident
1	2	3	4	5	6	7	8	9	10	

After Discussion Response

Situation:

Laura is offered a job making tuques. She has been offered six different payment structures. The payment structures see her earn a combination of a daily wage and an additional amount per tuque. She figures she can make anywhere between 25-30 tuques per day. Rank these payment structures from greatest to least based on the graphs on the previous page.

Final Ranking:

Greatest 1 ____, 2 ____, 3 ____, 4 ____, 5 ____, 6 ____, Least

Or, all options pay the same _____

If you made any changes to your ranking, explain why.

How confident are you in your answer?

Not very confident										Very Confident
1	2	3	4	5	6	7	8	9	10	

APPENDIX D

LINEAR RELATIONS CONCEPT TEST

1. Which of the following equations describes the relationship below:

x	y
2	13
3	16
4	19

- $y = x + 3$
- $y = 3x + 7$
- $y = 3x + 3$
- $y = x + 7$
- None of the above

2. Determine a linear equation for the pattern below where n is the figure number:

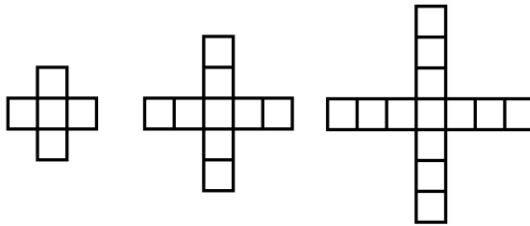


Figure 1

Figure 2

Figure 3

Source: <http://musingmathematically.blogspot.ca/2013/02/relation-stations.html>

- $4n + 5$
 - $3n + 2$
 - $5n + 4$
 - $2n + 3$
 - None of the above
3. A polar bear is positioned 7 km away from a research station. For 3 straight days the polar bear walks 2 km each day towards the research station. Which linear equation can model the relationship between the day number (n) and the distance (d) in km from the research station?
- $d = 2 - 7n$
 - $d = 2n + 7$
 - $d = 7n + 2$
 - $d = 7 - 2n$
 - None of the above

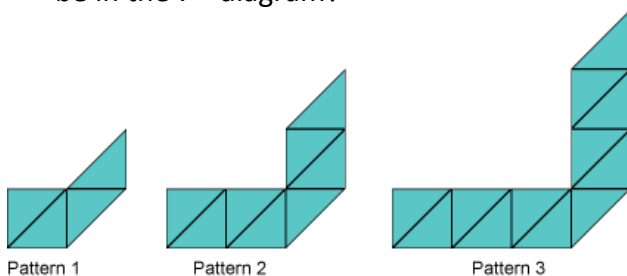
4. A commercial airplane is currently cruising at an altitude of 30 000 feet. However, the airplane is required to increase its altitude to 35 000 feet. The pilot raises the airplane's altitude to 35 000 feet in 5 minutes. Which linear equation can model the relationship between the time in minutes (t) and the altitude (A) in feet?
- $A = 1000t + 30\,000$
 - $A = 5t + 35\,000$
 - $A = 1000t + 30\,000$
 - $A = 5t + 1000$
 - None of the above
5. A bowler achieved the following scores in her last three games: 205, 217, 229. Which of the following represents the linear relationship of her scores?
- $y = 12x + 205$
 - $y = 205 - 12x$
 - $y = 229 - 12x$
 - $y = 12x + 229$
 - None of the above
6. Which of the following situations describes the relationship $y = 4x + 3$
- There were four duck eggs and three duck eggs hatch every hour
 - A student had three cookies and ate four more every hour
 - The temperature outside was 4°C and dropped three degrees every hour
 - A baker made three French loaves every four minutes
 - None of the above

7. Determine the y value when $x = 4$.

x	y
1	12
2	5
3	-2

- 4
- 7
- 7
- 9
- None of the above

8. A hiker spots 3 falcons on a hike of a nearby mountain. The next time he hikes the same mountain he spots 5 falcons. On the third time he hikes the mountain he spots 7 falcons. In this linear pattern continues, how many falcons would he spot the 11th time he hikes the mountain?
- 25
 - 9
 - 23
 - 21
 - None of the above
9. The linear equation $d = 5t + 50$ represents the relationship between distance (d) of a biker in metres and time (t) in seconds. What is the distance of the biker after 30 seconds?
- 55 m
 - 1505 m
 - 200 m
 - 85 m
 - None of the above
10. The linear equation $d = 4t + 60$ represents the relationship between distance (d) of a biker in metres and time (t) in seconds. How much time has passed when the distance is 160 m?
- 15 s
 - 55 s
 - 100 s
 - 25 s
 - None of the above
11. Observe the pattern below. If this pattern continued, how many triangles would be in the 7th diagram?

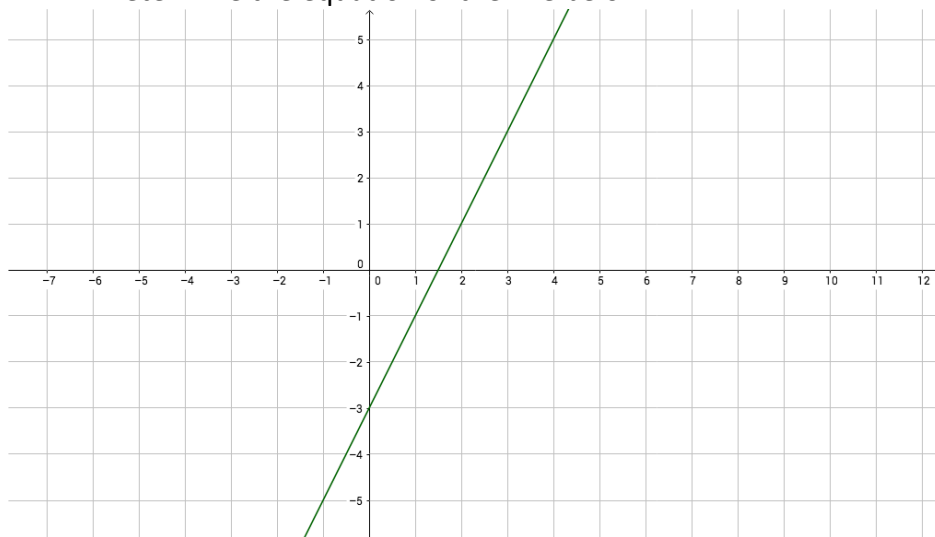


Source: www.bbc.co.uk/bitesize/ks3/maths/images/tb_linearseqs.gif

- 32

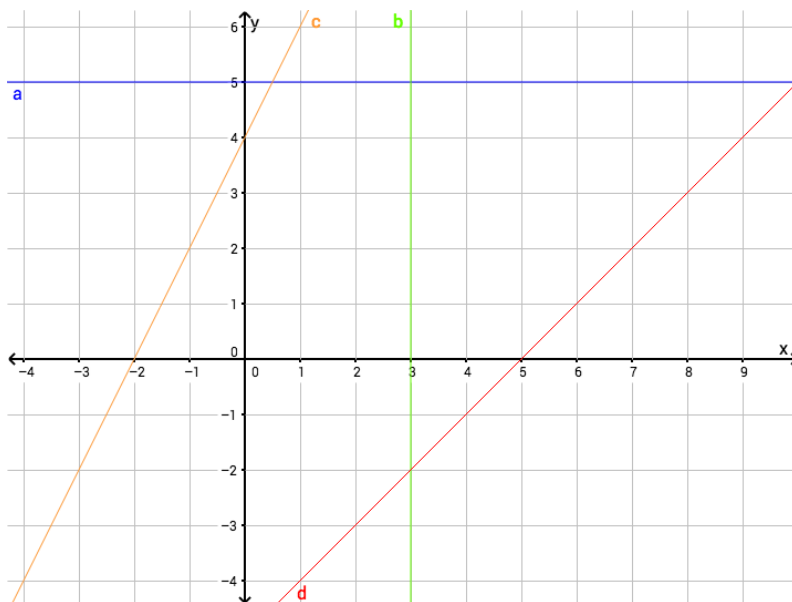
- b. 28
- c. 30
- d. 36
- e. None of the above

12. Determine the equation of the line below



- a. $y = 2x - 3$
- b. $y = x - 3$
- c. $y = 3x + 2$
- d. $y = x + 2$
- e. None of the above

Consider the graph below for questions 13 and 14.



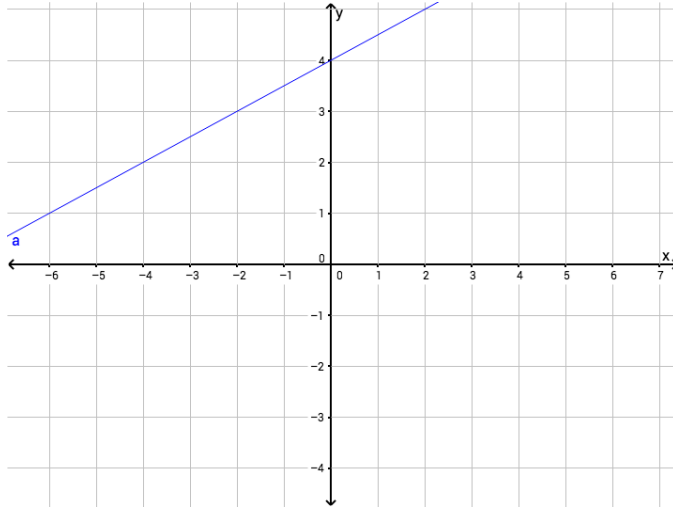
13. Which of the line represents $y = x - 5$

- a. a
- b. b
- c. c
- d. d
- e. None of the above

14. Which line represents $x = 3$

- a. a
- b. b
- c. c
- d. d
- e. None of the above

Consider the graph below for questions 15, 16 and 17



15. What is the equation of this line?

- a. $y = x + 4$
- b. $y = 2x + 4$
- c. $y = 0.5x + 4$
- d. $y = 0.25x + 4$
- e. None of the above

16. Determine the y-value when $x = -3$.

- a. -2
- b. 5.5
- c. -2.5
- d. -3
- e. None of the above

17. If this pattern continues, what would be the x-value when $y = 10$?

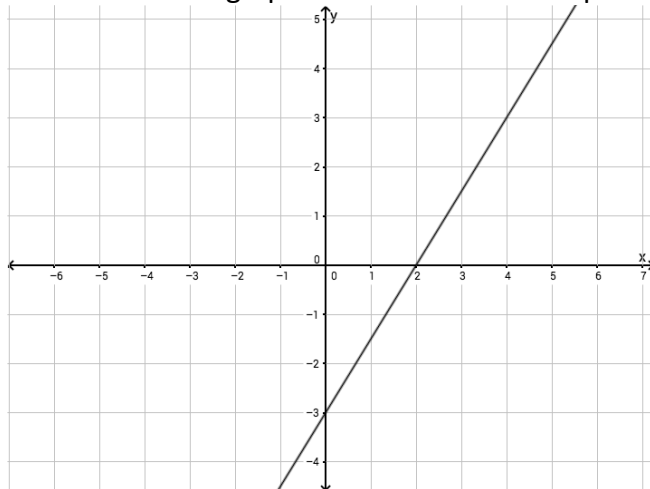
- a. 14
- b. 10
- c. -2
- d. 12
- e. None of the above

18. A long distance runner completed a 5 km race in 25 minutes last weekend. Next weekend, the same runner will attempt her first 10 km race. Which of the following best describes the most likely scenario for how long you think it will take the runner to run 10 km?

- a. 50 minutes
- b. Less than 50 minutes
- c. More than 50 minutes
- d. Not enough information

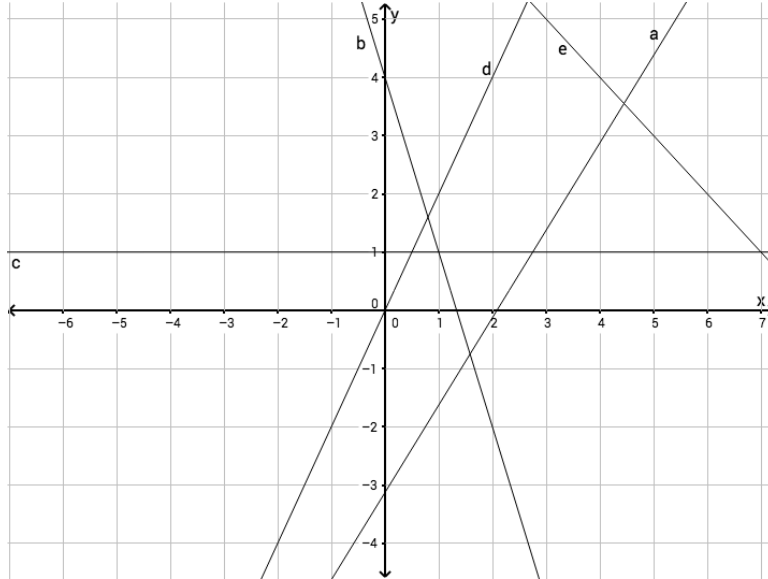
e. None of the above

19. Describe the graph below as far as its slope and y-intercept:



- a. slope = 2; y-intercept = -3
- b. slope = -3; y-intercept = 2
- c. slope = -1.5; y-intercept = -3
- d. slope = -1.5; y-intercept = 2
- e. None of the above

20. Rank the following graphs in order of increasing slopes:



- a. $b > d > a > e > c$
- b. $b > e > c > a > d$
- c. $d > a > c > e > b$
- d. $a > d > e > b > c$
- e. None of the above

APPENDIX E
LINEAR RELATIONS QUESITONNAIRE

Participation in this research is voluntary and participation or non-participation will not affect a student's grades or class standing in any way

1. I feel confident when dealing with math concepts, like linear relations

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Undecided

2. I enjoy using math concepts to solve problems

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Undecided

3. I feel nervous about the upcoming test for this unit (linear relations)

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Undecided

4. Completing Ranking Task Activities help my understanding of math concepts

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Undecided

5. The most challenging part of math is...

☐ Understanding Concepts ☐ Communicating Reasoning or Solution
☐ Problem Solving ☐ Calculating a Solution ☐ Other

6. I prefer to work on math problems by myself

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Undecided

7. Discussing possible solutions to a problem with my peers is beneficial

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Undecided

8. Discussing possible solutions to a problem with the entire class is beneficial

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Undecided

9. I feel more confident with my answer to a problem after I have discussed it

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Undecided

10. Teaching a concept to a friend furthers my understanding of a concept

☐ Agree ☐ Somewhat Agree ☐ Somewhat Disagree ☐ Disagree ☐ Not Applicable

APPENDIX F

LINEAR RELATIONS STUDENT INTERVIEW

Participation in this research is voluntary and participation or non-participation will not affect a student's grades or class standing in any way

1. Do you feel more confident in your understanding of math concepts after completing this unit? Please explain why or why not.
2. Explain your thought process for completing a ranking task.
3. Did you find the use of ranking tasks in this unit helpful? Please explain why or why not.
4. Do you feel you improved your score on the Linear Relations Concept Test after completing the unit? If so, what were some factors that caused this improvement?
5. Did you find it helpful to discuss your answers to the ranking tasks before submitting your answer? Please explain why or why not.
6. Can you think of any times you changed your answer to a ranking task or you caused a group member to change their answer after discussion? If yes, please explain an example.
7. Is there anything else you would like me to know about?