Effect of permanent deformation on elastic properties of metals
by Grace McVicker

A THESIS SUBMITTED TO THE PHYSICS DEPARTMENT FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED SCIENCE
Montana State University

Abstract:
no abstract found in this volume

EFFECT OF PERMANENT DEFORMATION ON ELASTIC PROPERTIES OF METALS

A THESIS

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1928

In the October 1927 issue of "Engineering News-Record, Earle B. Norris, Dean of Engineering of Montana State College presented his empirical equation for plastic flow of steel which had been cold worked. It is

$$S = T + 0.2\sqrt{T}\,x - T^{(1-.0018Tx)}$$

in which T is the tensil strength, x is the percentage decrease in diameter and S is true stress. As shown in Figure 3 which accompanies his article, this curve, when plotted for various steels, follows very closely the stress-strain curve plotted from experimental observations during the cold workings of the same steels. Undoubtedly this is very close to a general one which would fit all cases, but quoting Dean Norris, "From my empirical equation it would appear that resistance to plastic flow is combination of a logarthmic or expotential function and a linear function of deformation the former predominating up to the ultimate tensil strength, as ordinarily determined, while the linear function takes control at higher true stresses.

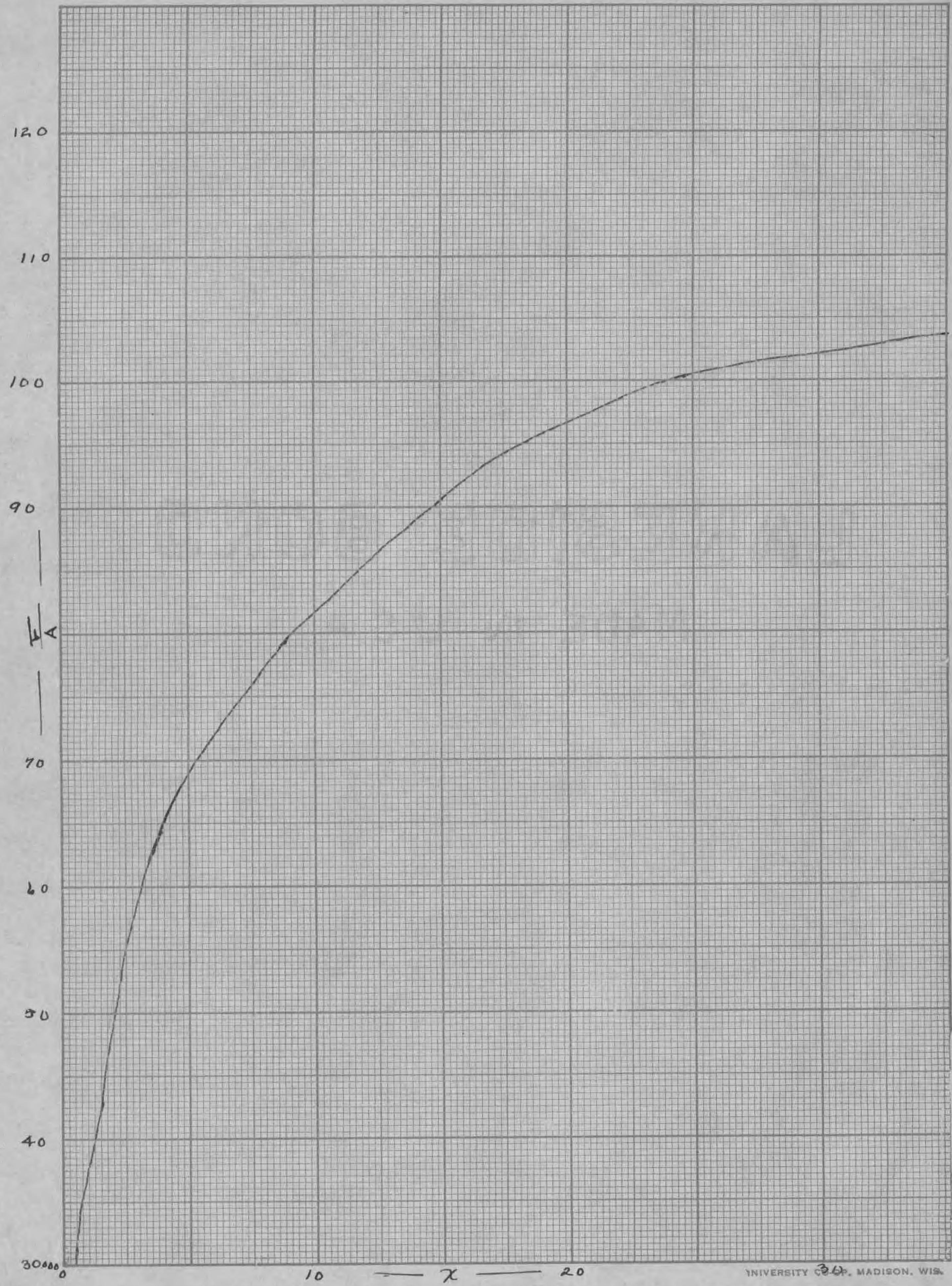
In this work, an attempt was made to derive an equation from assumptions as to the nature of the relationship between stresses and strains, in the hopes that a comparatively simple function might be found which would fit the experimental data. Steel T - 9.1 was the specific specimen from which data, as tabulated, was taken; Figure 1 is a curve of true stress from this experimental data.

Assuming that Hooke's law hold, the elongation per unit length is proportional to the force on unit area. Letting A_1, l_1, and d_1 be the original area, length, and diameter respectively and A, l, and d the new area, length, and diameter respectively then

$$\frac{A_1}{A} = \frac{d^2}{d_1^2} = \frac{l_1 + e_1}{l_1} \qquad (1)$$

STEEL T - 9.1

P	A	P/A	dD%		e	E
8600	.200	43000	0	Y.P.		
7400	.196	37740	1.07		.0206	1,850,000
9600	.192	50110	2.1		.0417	2,400,000
11400	.188	60648	3.13		.0639	2,850,000
12400	.183	67704	4.42		.094	2,480,000
12900	.179	72240	5.48		.1123	3,225,000
13100	.175	74670	6.55		.1429	3,275,000
13200	.170	77616	7.93		.1765	2,640,000
13300	.164	81130	9.50	)	.2195	2,216,660
				)Ult.		
13300	.156	85386	11.7	)	.282	1,662,500
				)		
13100	.143	91700	15.3		.3986	1,007,000
12500	.128	97500	20.0		.5645	830,000
11600	.108	107416	26.5		.852	580,000
10100	.080	126250	36.8		1.500	360,070


120
110
100
90
F/A
70
60
50
40
30000
0
10
x
20
30

where e_1 is the total elongation. When e is the elongation per unit length and x is the percentage decrease in diameter

$$x = \frac{d_1 - d}{d_1} = 1 - \sqrt{\frac{1}{1+e}} \qquad (2)$$

$$e = \frac{1}{(1-x)^2} - 1 \qquad (3)$$

By Hooke's law

$$e = KF$$

or

$$\frac{1}{(1-x)^2} - 1 = KF \qquad (4)$$

Figure 2, the curve plotted from equation (4) indicates a parabola revolving about the y axis and not conforming to conditions as they actually existed as were plotted in Figure I.

The defining equation of Young's modulus E, is

$$E = \frac{Fl}{Ae_1} \qquad (5)$$

and now it is assumed that E is a constant. From previous notations,

$$e_1 = \frac{F(l_1 + e_1)^2}{E A_1 l_1} \qquad (6)$$

or

$$e = \frac{F(1+e)^2}{EA_1} \qquad (7)$$

Then

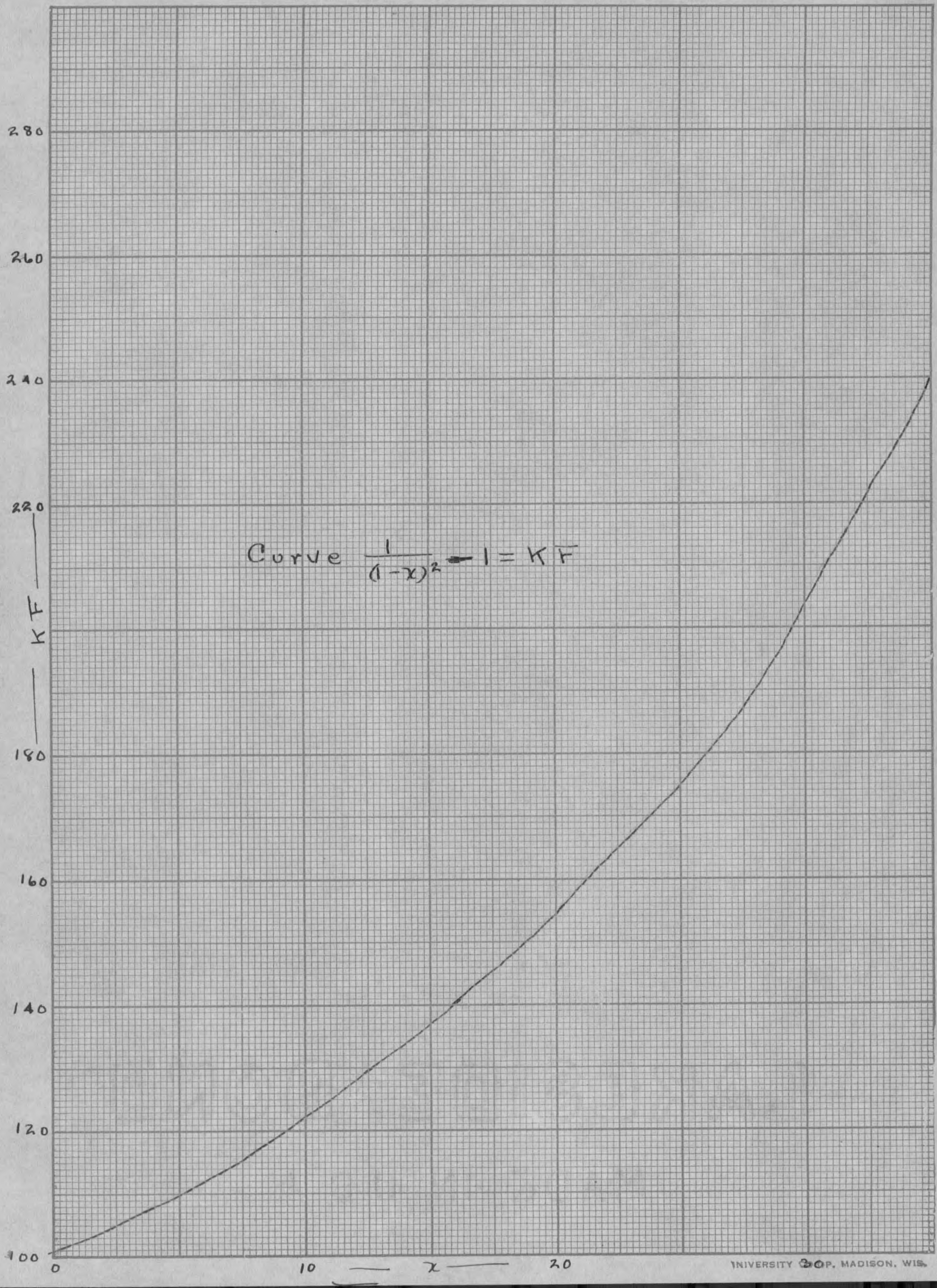
$$de = \frac{dF(1+e)^2}{EA_1} \qquad (8)$$

From equation (2)

$$dx = \frac{1}{2}\sqrt{\frac{1}{(1+e)^3}}\, de$$

and

$$(1+e) = \frac{1}{(1-x)^2}$$


280
260
240
220
KF
180
160
140
120
100
Curve $\frac{1}{(1-x)^2} - 1 = KF$
0
10
x
20
30

Then

$$de = \frac{2}{(1-x)^3}\, dx \qquad (10)$$

Substituting the values of equations (9) and (10) in equation (8), it is

$$\frac{2}{1-x}\, dx = \frac{dF}{EA_1} \qquad (10_1)$$

Since A is the original area, EA, may be expressed as K, a constant. Integrating (10) it is

$$(1-x)^2 + C = \frac{F}{k}$$

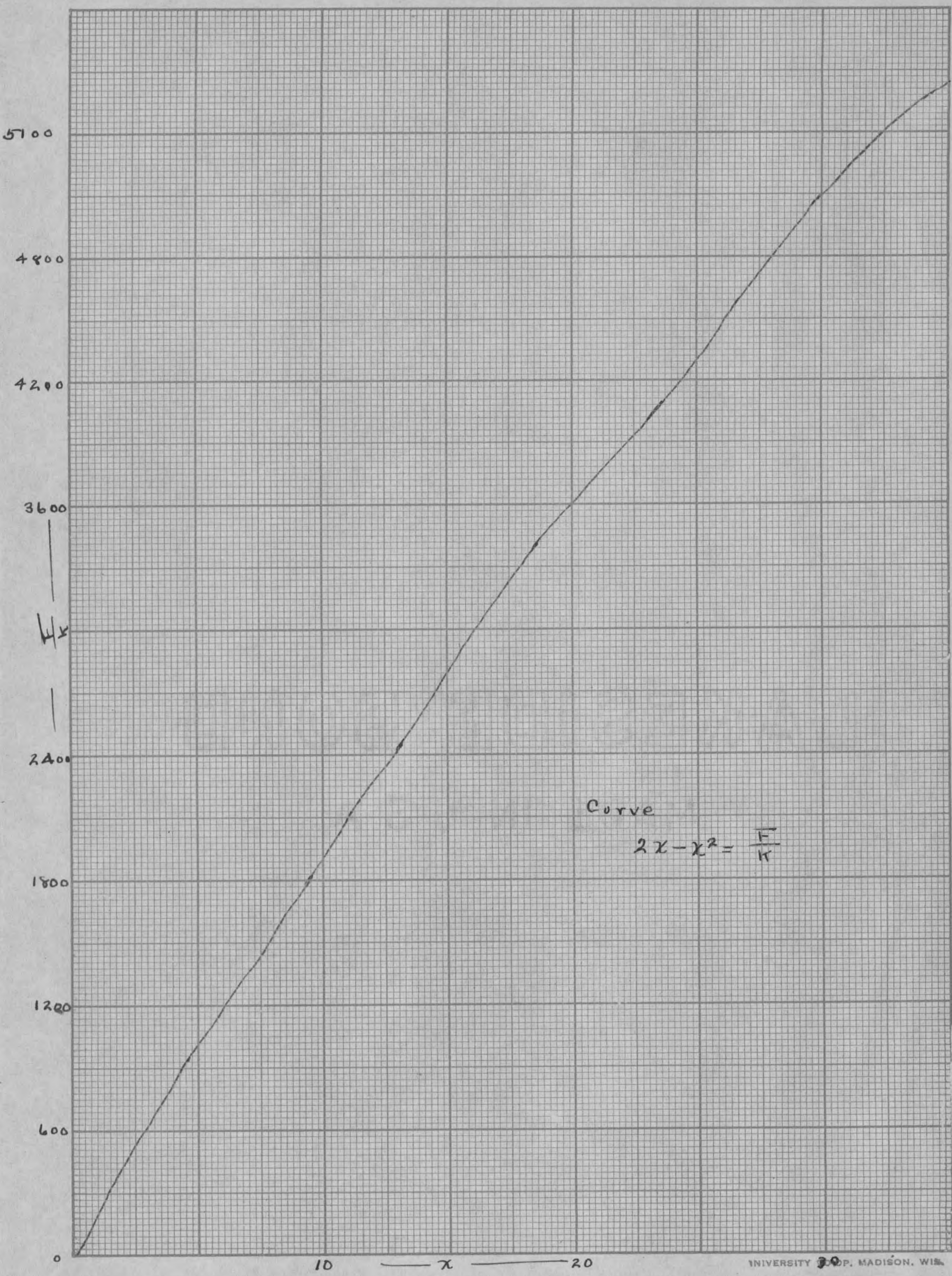
or

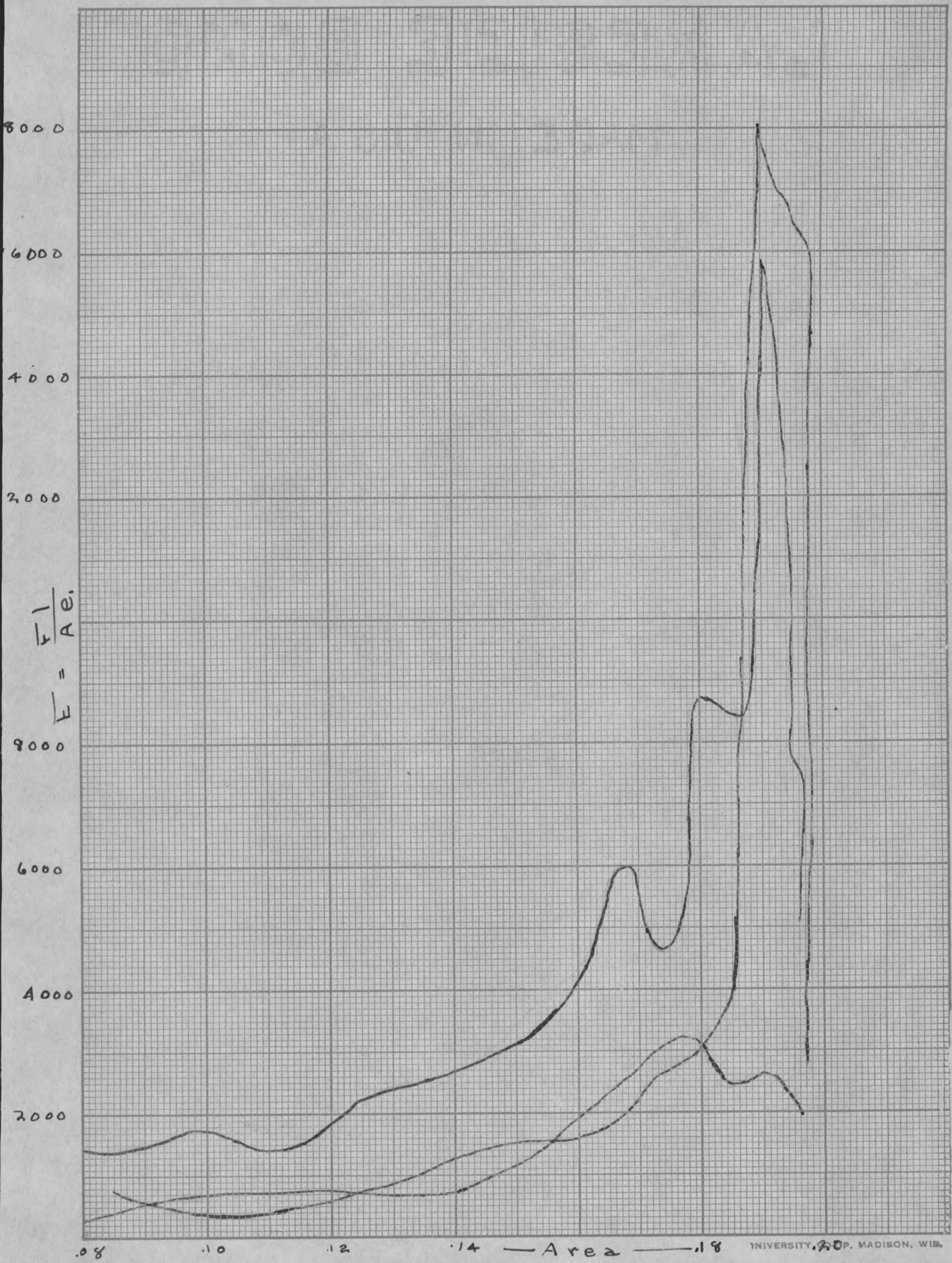
$$2x - x^2 = \frac{F}{k} \qquad (10_2)$$

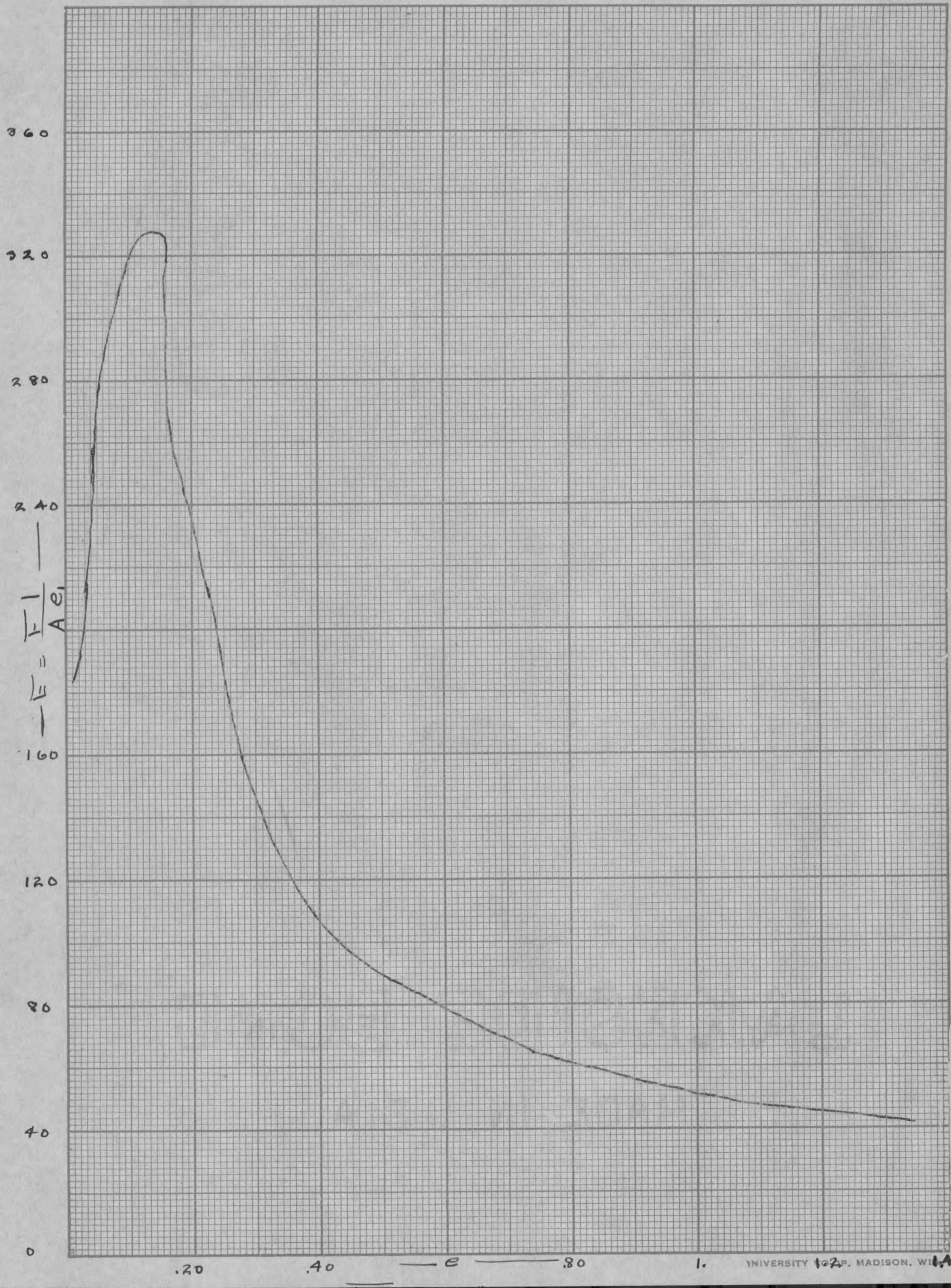
Figure 3 is the curve plotted from equation (10$_2$) but again this does not conform to Figure I.

Using equation (5), the modulus was calculated at each successive stage of elongation for several specimens of steel. Figure 4 shows the curves plotted, using the modulus as ordinate and area as abscissa; in each case it was noted that the modulus of maximum value was attained before the specimen reached the ultimate tensil strength.

Assuming E to be a function of e, a curve was plotted for the T - 9.1 steel (Figure 5) where E was again the ordinate, and the elongation e, the abscissa. Selecting three widely separated points ($e = .02$; $e = .14$; $e = .56$) an attempt was made to derive a cubic equation which would satisfy all points on the curve and thus account for the change which occurred in E according to equation (5); this was not accomplished. A quartic was the considered using the same three points and the slopes at $e = .02$ and $e = .14$, from equation (5)


5100
4800
4200
3600
Fx
2400
1800
1200
600
0
10
x
20
30
Curve
2x − x² = F/K


8000
6000
4000
2000
8000
6000
4000
2000
E = Fl / Ae
.08
.10
.12
.14
— Area —
.18
.20


360
320
280
240
160
120
80
40
0
$\frac{F}{A} = \frac{F}{A e_1}$
.20
.40
— e —
.80
1.
1.2
1.4

$$E e_1 = \frac{F(l_1 + e_1)^2}{A_1 l_1} \qquad (11)$$

$$E e_1 \, de_1 = \frac{dF(l_1 + e_1)^2}{A_1 l_1} \qquad (12)$$

Assume that

$$f(e_1) = E e_1 \qquad (13)$$

and

$$E = a_0 + a_1 e_1 + a_2 e_1^2 + a_3 e_1^3 + a_4 e_1^4 \qquad (14)$$

Then

$$f(e_1) = [a_0 e_1 + a_1 e_1^2 + a_2 e_1^3 + a_3 e_1^4 + a_4 e_1^5] \, de = \frac{dF(l_1 + e_1)^2}{A_1 l_1} \qquad (15)$$

Equation (15) was solved and the values of the constants calculated for the T - 9.1 steel, but the quartic did not satisfy the points of the curve which showed the variation in E.

Considering the experimental data in hand it was not found possible to derive a simple equation, based on the above assumptions, which would satisfy the actual observations as well as did the equation contributed by Dean Norris.