Variation of the resistance of copper with potential energy of deformation
by Walter T Kerttula

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Engineering Physics
Montana State University

Abstract:
The variation of the resistance of copper wire with potential energy of deformation is studied. Equations for resistivity and resistance in terms of potential energy of deforma-tion are derived oa the assumption that resistance varies linearly with tension, as found by other observers. The change In resistance is measured, and the results plotted against the potential energy of deformation. The experimental data are found to conform to the equations derived, which state that resistance varies directly with the square root of the potential energy of deformation within the elastic limit.

A determination of the unit change in resistivity for a stress of one kilogram per sgqare centimeter is made from the data, using Bridgman's equation, which remalts incidentally from the derivation of the resistance-energy equation. The value is in close agreement with the results of Bridgman and Rolnick* Data on the behavior beyond the elastic limit are includ-ed, but no definite conclusions are drawn from them.

An equation for computing the strain sensitivity factor (the ratio of unit change of resistance to unit strain) used in calibrating commercial strain gages, is included in an appendix, and the strain sensitivity factor for copper is computed from the experimental data*

VARIATION OF THE RESISTANCE OF COPPER WITH POTENTIAL ENERGY OF DEFORMATION

by

WALTER T. KERTTULA

A THESIS

Submitted to the Graduate Committee

in

partial fulfillment of the requirements

for the degree of

Master of Science in Engineering Physics

at

Montana State College

Approved:

Johnson

In Charge of Major Work

Johnson

Chairman, Examining Committee

Nelson

Chairman, Graduate Committee

Bozeman, Montana
August, 1946

TABLE OF CONTENTS

ABSTRACT

The variation of the resistance of copper wire with potential energy of deformation is studied. Equations for resistivity and resistance in terms of potential energy of deformation are derived on the assumption that resistance varies linearly with tension, as found by other observers. The change in resistance is measured, and the results plotted against the potential energy of deformation. The experimental data are found to conform to the equations derived, which state that resistance varies directly with the square root of the potential energy of deformation within the elastic limit.

A determination of the unit change in resistivity for a stress of one kilogram per square centimeter is made from the data, using Bridgman's equation, which results incidentally from the derivation of the resistance-energy equation. The value is in close agreement with the results of Bridgman and Rolnick.

Data on the behavior beyond the elastic limit are included, but no definite conclusions are drawn from them.

An equation for computing the strain sensitivity factor (the ratio of unit change of resistance to unit strain) used in calibrating commercial strain gages, is included in an appendix, and the strain sensitivity factor for copper is computed from the experimental data.

HISTORICAL BACKGROUND

The effect of tension on the resistance of metals parallel to the direction of the tensile force was studied as long ago as 1883 by Tomlinson[1], and a considerable amount of work has been done since that time, consisting principally of checking the original results and developing new techniques. Possibly the most extensive work was done by Bridgman[2], who used methods somewhat similar to those adopted for the present experiments; that is, the sample was subjected to a known tensile stress, and the resistance measured by means of a potentiometer, using a null method. The change in resistance due to geometrical changes was subtracted from the total resistance change, and the remainder was assumed to be the change due to stress.

Later determinations were made by Rolnick[3], using the method of vibrations, i.e., the sample was made to vibrate longitudinally, and the change in potential between the two points measured by means of a calibrated amplifier. The amplitude of the vibrations was measured by means of a light beam reflected from a mirror vibrating with the apparatus. Young's Modulus and Poisson's Ratio were used to compute the stress and lateral contraction. Values of these constants were taken from handbooks, or computed from values of the shear modulus.

For most metals, resistivity has been found to vary linearly with tension within the elastic limit. For most metals, the resistivity increases with strain, but in some, notably strontium and bismuth, the tension coefficient of resistivity is negative. Still others, particularly nickel, behave even more abnormally. In this metal, the resistivity decreases to a minimum, and then begins to increase again with increasing tension[4].

Copper has a normal positive tension coefficient of resistivity, which has been measured as 1.6×10^{-6}(4), 1.33×10^{-6}(2), and 1.32×10^{-6}(3) ohm-centimeters per ohm-centimeter per kilogram per square centimeter by different investigators.

Recently, this phenomenon has been put to practical use in the analysis of strain by means of resistance wire strain gages[5]. The strain sensitivity factor of these gages, defined as the ratio of unit change of resistance to unit strain, $\left(\frac{\Delta R/R_0}{\Delta L/L_0}\right)$* must, at present, be determined experimentally. Regarding this strain sensitivity factor, D. M. Nielson[5] states:

"For most metals, the strain sensitivity ... would be expected to be between 1.5 and 1.8 from a consideration of only the dimensional changes of the wire. Instead, most metals and alloys have sensitivity factors considerably higher. This has never been completely explained. The reason probably lies in an actual change of volume resistivity of the metal."

The fact that there is a change in volume resistivity (referred to herein as "resistivity") with strain has been established in the investigations mentioned above, but a method of determining this change theoretically, and thereby eliminating some of the tedious control procedures involved in the manufacture of strain gages, has not been developed. The basic

*See appendix

nature of the phenomenon is still not understood sufficiently to enable one to propound any workable theory concerning it. It is hoped that the present work, considering the resistance change from the viewpoint of potential energy of deformation may help to throw light upon the problem.

THEORY

Since resistivity varies linearly with stress, we may write for the resistivity, 'ρ', of a specimen under a tensile stress 'F',

1) $\rho = \rho_o(1+\beta F)$

where 'β' is the tension coefficient of resistivity, and 'ρ_o' the resistivity of the unstressed specimen.

The potential energy of deformation, 'W', is the work done by a tension 'T' acting through a displacement 'e', and may be written $W = \int_o^e T de$. Assuming Hooke's Law, this may be written:

2) $W = \int_o^e ke \cdot de = \frac{ke^2}{2} = \frac{Te}{2}$.

Young's Modulus is defined by the equation $E = \frac{T/A_o}{e/L_o} = \frac{F}{e/L_o}$, whence $e = \frac{L_oF}{E}$. Substituting these values in 2) we obtain

$W = \frac{A_oL_oF^2}{2E}$, or:

3) $F = \sqrt{\frac{2E}{A_oL_o}}\sqrt{W}$.

Substituting the value of 'F' from equation 3) in equation 1), we obtain for the resistivity in terms of the potential energy of deformation:

4) $\rho = \rho_o(1+\beta\sqrt{\frac{2E}{A_oL_o}}\sqrt{W})$

A curve of resistivity versus energy of deformation should, therefore, fit equation 4), while resistivity versus the square root of the energy of deformation would be a straight line, the slope being equal to $\rho_o\sqrt{\frac{2E}{A_oL_o}}\,\beta$. Since the plotting of a "theoretical" curve against which to compare an experimental curve would be of little value, because the constants 'β', 'E', 'L_o', and 'A_o' would have to be taken from the data, we have here a method of determining whether the experimental data fit a curve of the form of equation 4). If the curve of resistivity versus the square root of the energy of deformation is a straight line, we will know that the data satisfy an equation of the form of 4).

An approximate relationship between the total resistance and the energy of deformation may be derived from consideration of 'β', 'E', and Poisson's Ratio, 'μ', as follows.

The resistance of a piece of metal is given by the well-known relation

5) $R = \rho\frac{L}{A}$

where, for a sample under a tension of 'T' dynes, the length, 'L' is equal to L_o+e, and the area, 'A' equals $A_o - \Delta A$, while the resistivity 'ρ' is given by substituting $\frac{T}{A_o} = F$ in equation 1).

Poisson's Ratio is defined by $\mu = \frac{\Delta r/r_o}{e/L_o}$ whence $\Delta r = \frac{\mu r_o e}{L_o}$ $= \frac{\mu T}{\pi r_o E}$, where '$r_o$' is the original radius, (see p. 7).

Now, $A_o - \Delta A = \pi(r_o - \Delta r)^2 = \pi(r_o^2 - 2r_o\Delta r + \Delta r^2)$

Since 'μ' is less than 1/2[6], and 'T' in the present experiment never exceeded 2 x 10^7, the order of magnitude of μT is not more than 10^7. 'E' is of the order of magnitude 10^{12}, while 'r_o' was never below the order of 10^{-1}. Hence, the second term of the expression for $(A_o - \Delta A)$ is of the order of magnitude 10^{-5}, while the third term is at most 10^{-8}. We therefore neglect the term Δr^2 for the purposes of this experiment and write:

$$A_o - \Delta A = \frac{\pi E r_o^2 - 2\mu T}{E}$$

With these values for 'ρ', 'L', and 'A', the equation 5) may be written:

$$R = \rho_o(1+\beta\frac{T}{A_o}) \frac{L_o + \frac{L_o T}{\pi r_o^2 E}}{\frac{\pi r_o^2 E - 2\mu T}{E}}$$

Since $R_o = \rho_o\frac{L_o}{A_o}$, and $A_o = \pi r_o^2$, the expression for R becomes:

$$R = R_o \frac{A_o E + (1+\beta E)T + \frac{\beta T^2}{A_o}}{A_o E - 2\mu T}$$

Performing the indicated divisions:

$$6)\ R = R_o \left[1 + \frac{1+\beta E+2\mu}{A_o E}T + \frac{2\mu\beta E+6\mu+\beta E}{A_o^2 E^2}T^2 + \frac{2\mu(2\mu\beta E+6\mu+\beta E)}{A_o^3 E^3}T^3 + \ldots \right]$$

For the values of 'T' mentioned above, the series converges very rapidly, in fact, the second term is more than 10^5 times the third term, so that it is permissible to neglect all terms above the second. In this form, the equation for resistance in terms of tension becomes:

$$7)\ R = R_o \left[1 + \frac{1+\beta E+2\mu}{A_o E} T\right]$$

Substituting for 'T' its value in terms of 'W' from eq. 3):

$$8)\ R = R_o \left[1 + \frac{\sqrt{2}\,(1+\beta E+2\mu)}{\sqrt{A_o L_o E_o}} \sqrt{W}\right]$$

By comparison with equation 4), it may be seen that a curve of 'R' versus 'W' would resemble the curve of 'ρ' versus 'W', differing in this approximate form only by the terms brought in because of geometrical changes. Again, the plot of 'R' versus the square root of 'W' would be a straight line, the slope of which yields an expression for 'β'. Within limits, equation 8) is as accurate as 4), and is simpler for the purposes of this experiment, since it eliminates the necessity of computing the resistivity, 'ρ', (see pp. 15 - 16).

An interesting incidental result may be obtained from equation 7), by noting that the term $R_o \frac{1+\beta E+2\mu}{A_o E}$ is the slope of the curve of 'R' versus 'T', or

$$\frac{\Delta R}{\Delta T} = R_o \frac{1+\beta E+2\mu}{A_o E}$$

Solving for 'β', and remembering that $\beta = \frac{1}{\rho_o}\frac{\Delta\rho}{\Delta F}$, and $\Delta F = \frac{\Delta T}{A_o}$, we obtain

$$9)\ \frac{\Delta\rho}{\rho_o \Delta F} = \frac{\Delta R}{R_o \Delta F} - \frac{1+2\mu}{E}$$

This is Bridgman's equation as it is referred to in an abstract of the work of E. Lopuhkin in Techn. Phys. U. S. S. R., 4, 1, pp 25-35, 1937[7]. (The derivation of this equation could not be found in any of the available literature.) Bridgman[2] and Rolnick[3] both use the quantity $\frac{1+2\mu}{E}$ as a correction factor for the effect of dimensional changes in the computation of 'Δp', but no derivation is given by either in the literature consulted. As may be seen in the above derivation, this is but a first order approximation, holding only for low values of 'T' and fairly high values of 'r', the radius of the wire.

EXPERIMENTAL PROCEDURE

In order to obtain data relating the potential energy of deformation to resistivity, one of two variations of method might be used. One might simply measure the resistance and tension, and compute the elongation from Young's Modulus and the measured length of the specimen. Otherwise, the elongation could be measured directly. The latter method was chosen here, since data on the material used varies considerably as given in handbooks. Apparatus used and methods for preparing the data for presentation are outlined below.

The apparatus consisted of a sheet metal box, roughly 150 centimeters in length, and of square cross section, 5x5 centimeters, reenforced with one-half inch angle iron, and insulated on the sides, bottom and ends with about two inches of asbestos. A longitudinal section and a top view, showing the specimen in place, are given in figures 1A and 1B.

The clamp 'E' was a piece of iron rod, drilled longitudinally, and fitted with set screws for clamping the specimen. This was brazed securely to the angle iron reenforcements at one end of the box. The clamp 'G', constructed similarly, was used to fasten the specimen to a steel wire leading through the pulleys 'F' and 'H' to the weight hanger 'I'. The pulley 'F' was mounted on a horizontal shaft, running the width of the box. The mounting of this pulley was insulated from the box with strips of sheet mica, thus one end of the specimen could be grounded through the box to eliminate static charges without otherwise affecting the resistance measurements.

The resistance between the points 'A' and 'B' of the specimen was measured by means of a self-contained Kelvin Bridge, the current clamps of which were at 'C' and 'D', and the potential terminals were knife-edge clamps at 'A' and 'B'.

Two traveling microscopes, '1' and '2', were used to measure the elongation. These were mounted separately from the rest of the equipment on a 2" pipe running the length of the box, held in vertical supports fastened to the table with wood screws. To hold the microscopes, holes were drilled through the pipe to fit the support rods of the microscopes, the rods inserted, and clamped in place with set screws.

Transparent transformer oil was then poured into the box to submerge the specimen and clamps. During the experiment, the oil was agitated manually, it being necessary to allow the oil to come to rest before taking readings on the microscopes. Temperature was measured with ordinary mercury thermometers during the preliminary experiments, and with a Beckmann vacuum thermometer calibrated in hundredths of a degree C. on the final experiments. However, there was little difference in the data obtained in the two cases, since the temperature fluctuation was not very great as a rule.

It was necessary to start with the wire taut so that it would be and remain in focus under the microscopes. The starting tension was taken as the zero point. Microscope No. 2 measured the motion of the point 'B' of the specimen, farthest from the fixed end at 'E', i.e., the elongation of the entire wire between the point 'B' and the clamp, as well as any slipping in the clamp, and other rigid-body motions. Microscope No. 1 measured the motion of the point 'A', which consisted of the elongation of the wire between 'A' and 'E', and the other possible motions mentioned above. As each weight was added to the hanger, readings were taken on each microscope. The difference between these and the readings for the previous tension gave the motion caused by the added tension. The difference between the motion measured by No. 2 and the motion measured by No. 1 gave the elongation due to the added tension of the portion of the specimen under test (AB, figure 1), since the rigid-body motions and the elongation of EA (measured by both microscopes) was thus cancelled. To obtain the total elongation for any tension, it was necessary to sum up the elongations for each increase in tension from the zero point to the required tension.

The resistance measured as above, was recorded for each tension and elongation, together with the temperature of the oil bath at the time. The direct experimental data, then, consisted of the readings of the microscopes, the tension in kilograms, Kelvin Bridge readings, temperature, and the original length and radius of the specimen.

The method of determining the elongation 'e' has already been described. The tension was simply converted to dynes, (the weight of the connecting wire being neglected), and this tension was plotted against the elongation on ordinary graph paper, (figures 2 and 8), and the area under the curves measured with a planimeter for each point corresponding to a given elongation. The energy of deformation, 'W', was obtained from this area by multiplying by the value of 'Te' represented by each square inch.

In computing the resistivity 'ρ', the following method, which eliminates the approximations used in deriving equations 8) and 9) was adopted. The equation $\Delta r = \frac{\mu T}{\mu E r_o}$, (see p. 7) was used to compute the lateral contraction, a value of 0.33 being used for Poisson's Ratio[8]. The radius 'r_n' corresponding to the particular tension 'T_n', where the subscript refers to the number of the reading (see Tables I, II and III, and p. 20), was computed by subtracting the Δr_n, from r_o, i.e., $r_n = r_o - \Delta r_n$. In the case where the elastic limit was exceeded sufficiently to cause a measurably permanent change in radius (figures 8, 9 and 10), the average radius, 'r_a', measured after the tension had been removed, was used in place of 'r_o' for decreasing tension. The length, 'L_n' and the area 'A_n', for each tension were then computed by the equations $L_n = L_o + e_n$

and $A_n = \pi r_n^2$.

The measured resistance was corrected for temperature by the relation $R = \frac{R_t}{1 + \alpha t}$ (9), where 'R' is the resistance at a standard temperature (20° C. in this case) and 'R_t' is the resistance at a temperature 't' degrees above the standard. The value of 'α', the temperature coefficient of resistivity, was taken as 0.00382 per degree C. at 20° C.[10].

The initial-point resistivity, 'ρ_o' was computed from the initial point resistance 'R_o' and the dimensions 'L_o' and 'r_o' by equation 5).

Now, if the resistivity remained constant under tension, the anticipated resistance 'R_{an}' at a tension 'T_n' would be that computed from the resistivity 'ρ_o' and would be equal to the measured resistance 'R_n' at the tension 'T_n', that is:

$$10)\ R_{an} = \rho_o \frac{L_n}{A_n}$$

However, we find that there is a difference between the anticipated resistance, 'R_{an}', and the measured resistance 'R_n'. This difference must be due to a change in resistivity. Hence, at a tension 'T_n', we will have a resistivity 'ρ_n' different from 'ρ_o', and the resistance 'R_n' would be given by:

$$11)\ R_n = \rho_n \frac{L_n}{A_n}$$

Dividing 10) by 11) and solving for 'ρ_n', we obtain:

12) $\rho_n = \rho_o \frac{R_n}{R_{an}}$

It will be noted that in computing 'A_n' for equation 10), the equation $A_n = \pi(r_o - \Delta r)^2$ was used instead of the approximate form $A_n = \pi(r_o^2 - 2r_o\Delta r)$, (see p. 7). Equation 9) could have been used, but this would have involved the approximations used in deriving it. This method involved none of these approximations, and the error is due only to experimental error and the error in the assumed value of 'μ'.

In actual computation, 'R_{an}' was computed by equation 10), and this value combined with the measured resistance and initial-point resistivity in equation 12) to determine the resistivity 'ρ_n' for each tension 'T_n'. These values of the resistivity were then plotted against the energy of deformation to check equation 4). (figures 5 and 12)

To check equation 8) it was only necessary to plot the measured resistance against the energy of deformation. (figures 3, 9, and 11)

The samples used were copper wire, 15 and 16 B. and S. gauge, manufactured by the Anaconda Wire and Cable Company at Muskegon, Michigan. The purity of these wires is such that ordinary chemical analysis would fail to show more than a trace, at most, of impurities. The wires were used as they were, without annealing, as interest was centered mainly on the relative change in resistance with energy of deformation, and not on the measurement of any constants. This made it possible to use higher tensions, as annealing greatly reduces the tensile strength of copper. The wires were somewhat kinked as a result of having been wound on spools, but except for an increase in the measured elongation for the first, and sometimes the second readings, this seemed to have little effect.

The self contained Kelvin Bridge used was manufactured by Leeds and Northrup Company, serial no. 215536. In conjunction with this, a Leeds and Northrup Ayrton Shunt, serial no. 30226, was used to decrease the sensitivity of the galvanometer for initial bridge settings. The galvanometer, also manufactured by Leeds and Northrup, was a wall type current galvanometer having a senstivity of 105 microamperes per millimeter of deflection. The microscopes were manufactured by Wm. Gaertner and Company, and were calibrated in two-hundredths of a millimeter, thus reading to .001 millimeters by interpolation.

TABLES AND FIGURES

Typical graphs of the data taken are given on the following pages, together with tables of data from which they were plotted. Figure 1 is a drawing of the arrangement used for measuring the tension and elongation.

Figures 2 and 8 are graphs of tension versus elongation, from which the potential energy of deformation was computed. Full-size graphs, like figure 2, were used for this purpose.

Figures 3, 9, and 11 are graphs of resistance versus energy of deformation, while in figure 4 the resistance is plotted against the square root of the energy of deformation to check equation 8), (see p. 10).

Figures 5 and 12 are plots of resistivity, computed as described previously (see pp. 15 - 17), against energy of deformation, and figure 6 is a plot of resistivity versus the square root of the energy of deformation to check equation 4) (see p. 8).

Figures 7 and 10 are plots of resistance versus tension, showing the linear variation within the elastic limit, (see pp. 4 and 8).

A comparison of the value of $\frac{\Delta\rho}{\rho}$ per kilogram per square centimeter, and E in dynes per square centimeter, as determined from the data, with values from other sources is given in

Table IV. This table is included becuase it offers a comparison of the known constants obtainable from this data with the results of others, and thus another measure of the experimental accuracy.

In the discussion of the results, whenever it is necessary to refer to a specific point on a curve, that point will be referred to by the number given in the column headed "No." in the corresponding table; thus, the point 2 of figure 3 refers to the point at (2.28×10^4, 8234×10^{-6}) in figure 3. (cf. Table I)

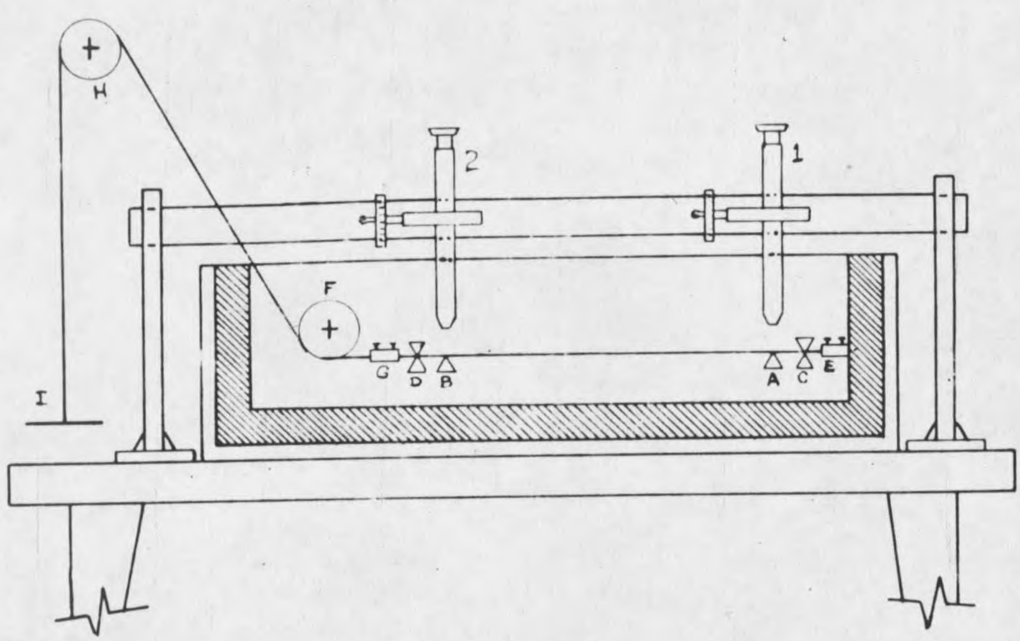

FIGURE 1A

Longitudinal Section of Apparatus

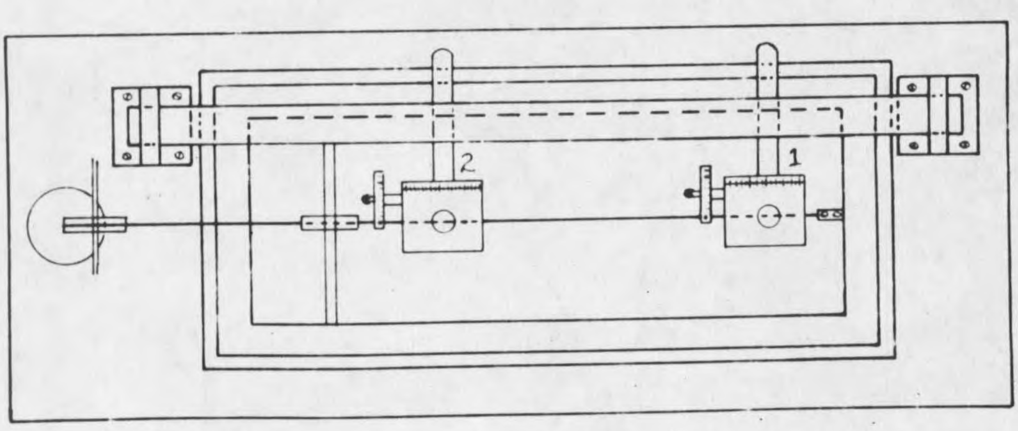

FIGURE 1B

Top View of Apparatus

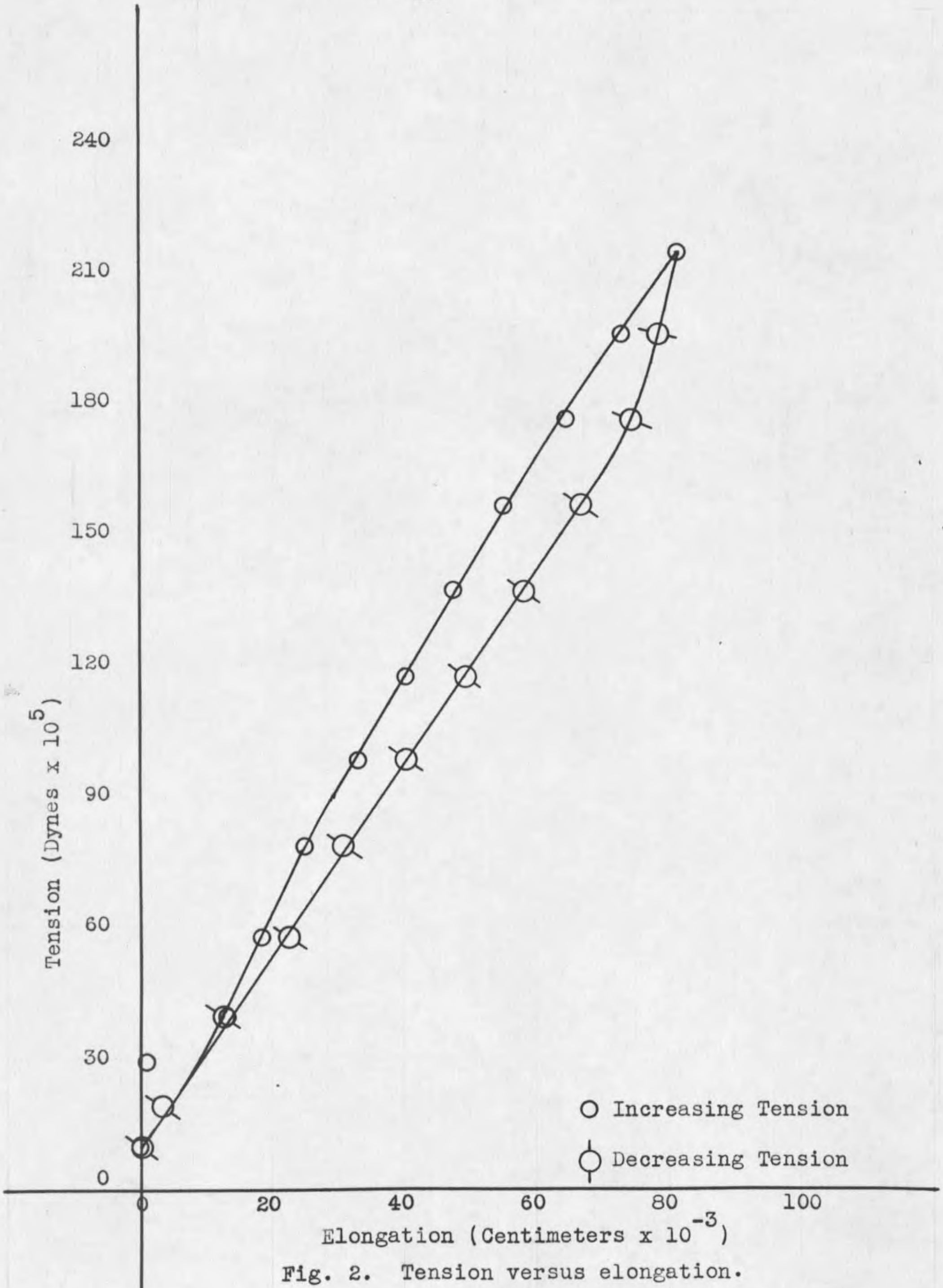

Fig. 2. Tension versus elongation.

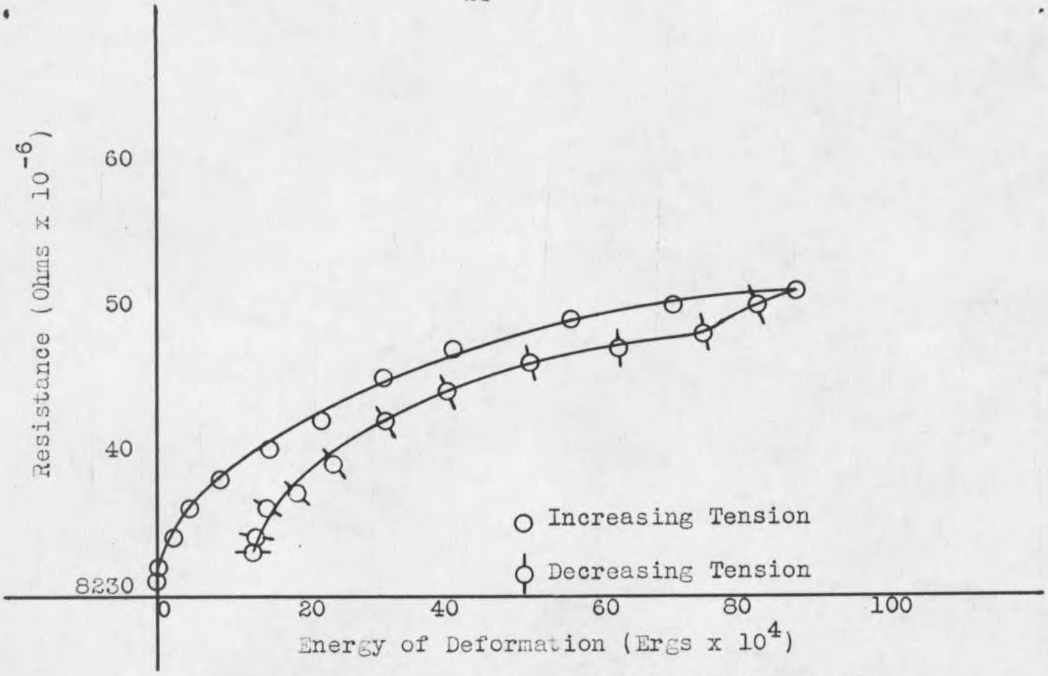

Fig. 3. Resistance versus energy of deformation.

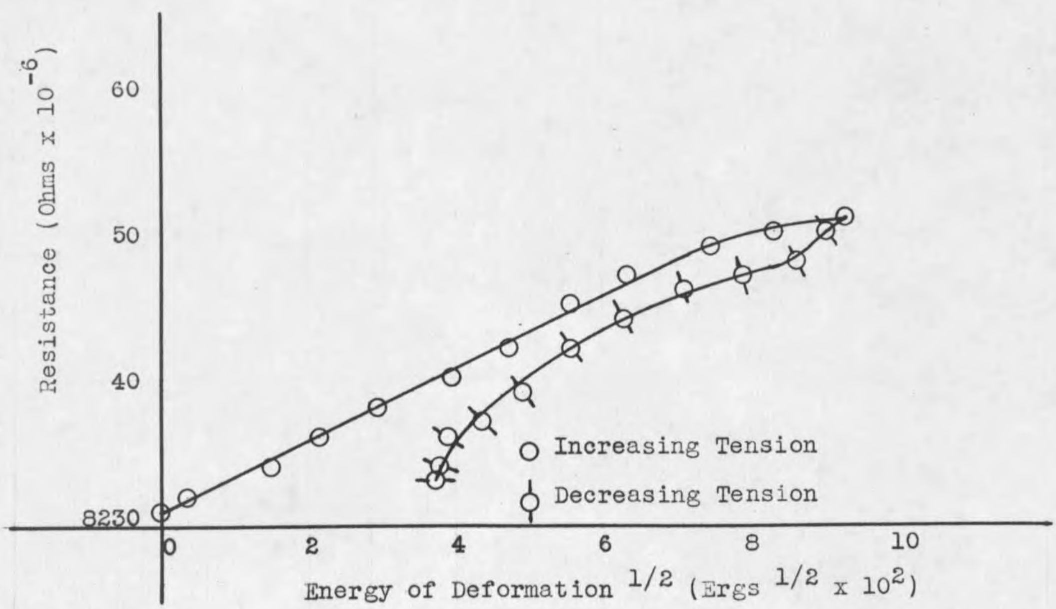

Fig. 4. Resistance versus square root of energy of deformation.

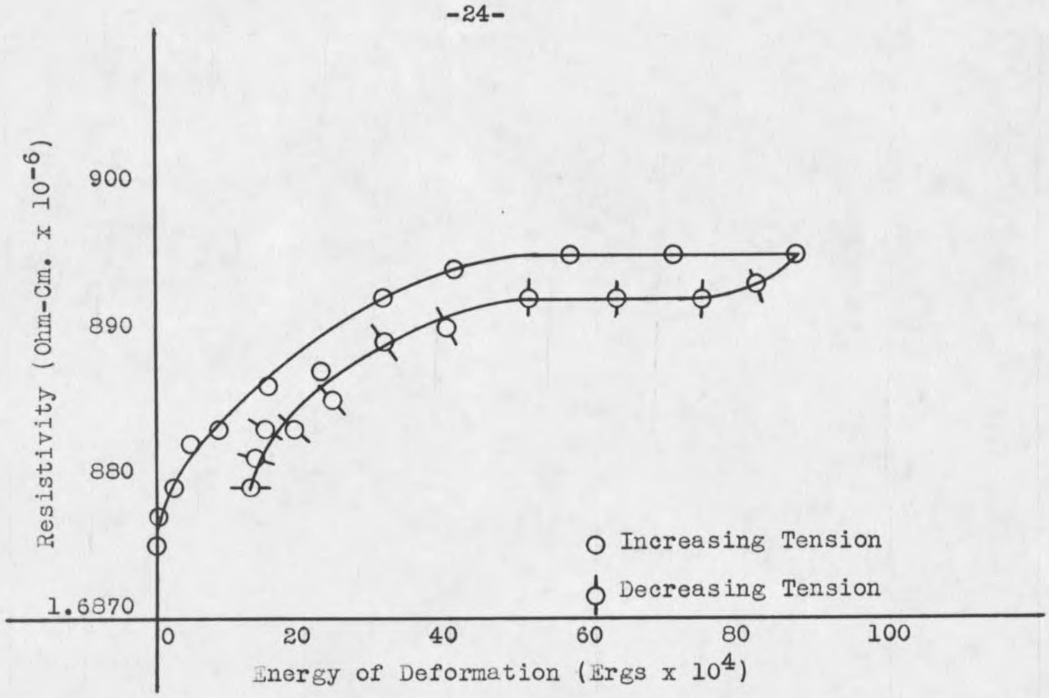

Fig. 5. Resistivity versus energy of deformation.

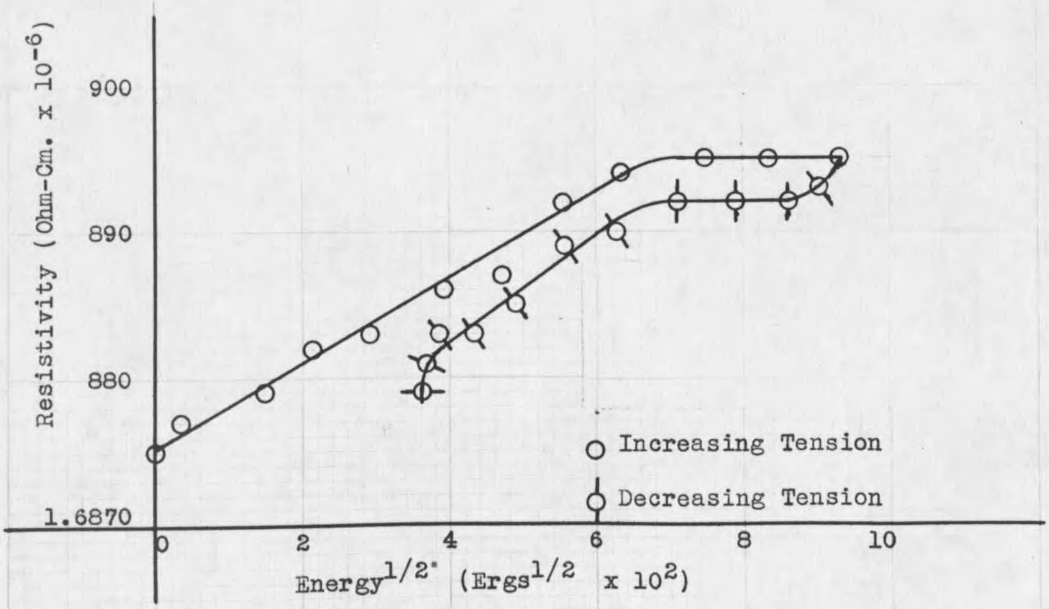

Fig. 6. Resistivity versus square root of energy of deformation.

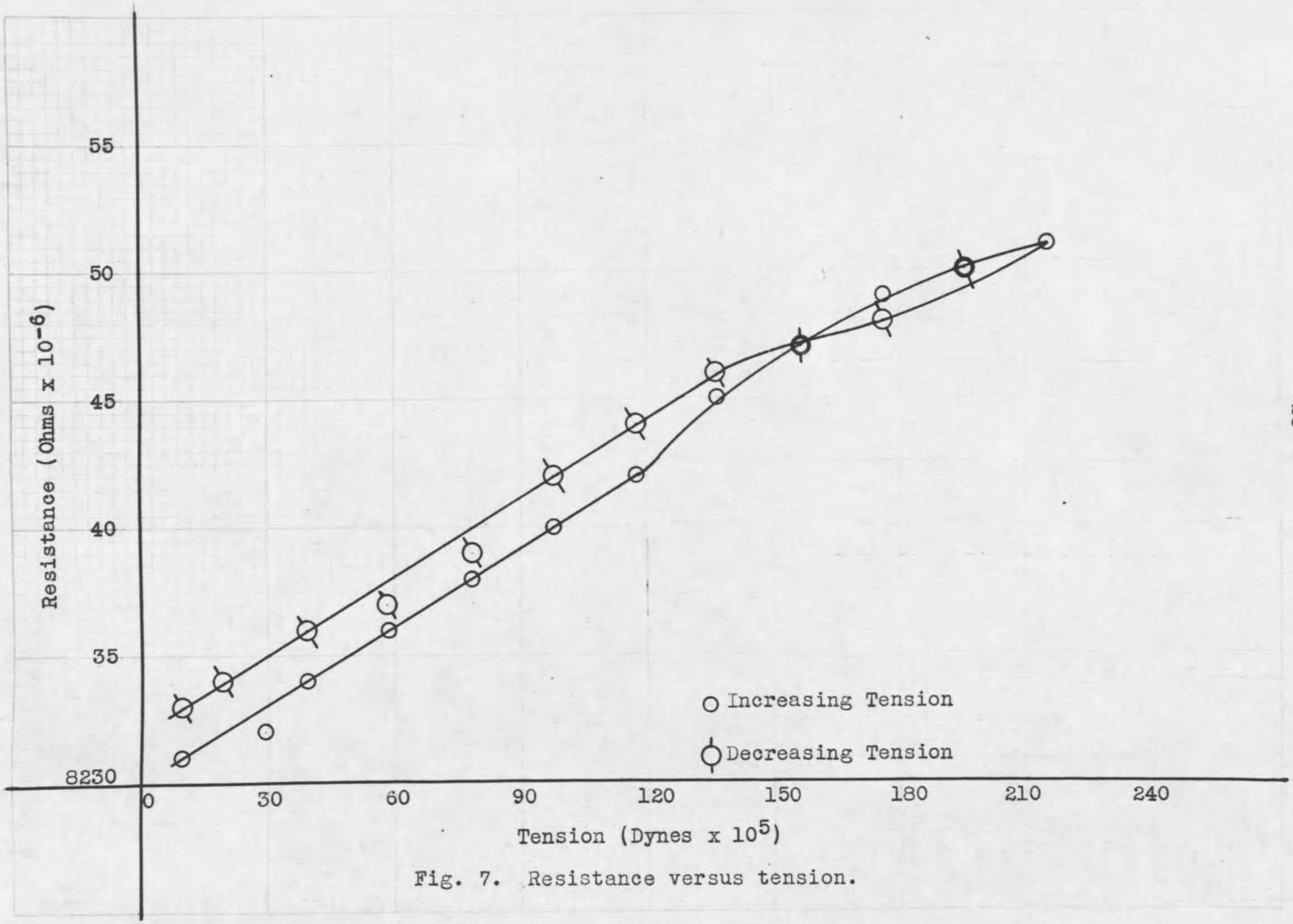

Fig. 7. Resistance versus tension.

TABLE I

Elasticity-Resistance-Energy Data of a Typical Copper Sample.

No. 14 B. and S. Gauge Bare Copper Wire June 18, 1946.

No.	Tension (Dynes)	Elongation (Cm.)	Resistance (Ohms)	Resistivity (Ohm-Cm.)	Energy (Ergs)	$(\text{Energy})^{1/2}$ $(\text{Ergs})^{1/2}$
0	$9.8\text{x}10^{5}$	0.00	$8231\text{x}10^{-6}$	$1.6875\text{x}10^{-6}$	0.00	0.00
1	29.4	$1.20\text{x}10^{-3}$	8232	1.6877	$0.12\text{x}10^{4}$	$0.35\text{x}10^{2}$
2	39.2	13.35	8234	1.6879	2.28	1.51
3	58.8	18.45	8236	1.6882	4.62	2.15
4	78.4	25.10	8238	1.6883	8.64	2.94
5	98.0	33.20	8240	1.6886	15.42	3.96
6	117.6	40.20	8242	1.6887	22.38	4.73
7	137.2	47.95	8245	1.6892	31.20	5.58
8	156.8	55.40	8247	1.6894	40.50	6.37
9	176.4	64.80	8249	1.6895	56.28	7.50
10	196.0	73.20	8250	1.6895	70.32	8.38
11	215.6	81.50	8251	1.6895	87.24	9.33
12	196.0	78.80	8250	1.6893	81.78	9.04
13	176.4	74.60	8248	1.6892	74.16	8.61
14	156.8	67.05	8247	1.6892	62.76	7.92
15	137.2	58.10	8246	1.6892	50.70	7.12
16	117.6	49.45	8244	1.6890	39.84	6.30
17	98.0	40.30	8242	1.6889	31.14	5.58
18	78.4	30.95	8239	1.6885	24.00	4.90
19	58.8	22.65	8237	1.6883	19.02	4.36
20	39.2	12.70	8236	1.6883	14.88	3.86
21	19.6	3.55	8234	1.6881	13.26	3.69
22	9.8	0.05	8233	1.6879	13.14	3.62

Length: 102.36 Cm. Radius: 0.0820 Cm.

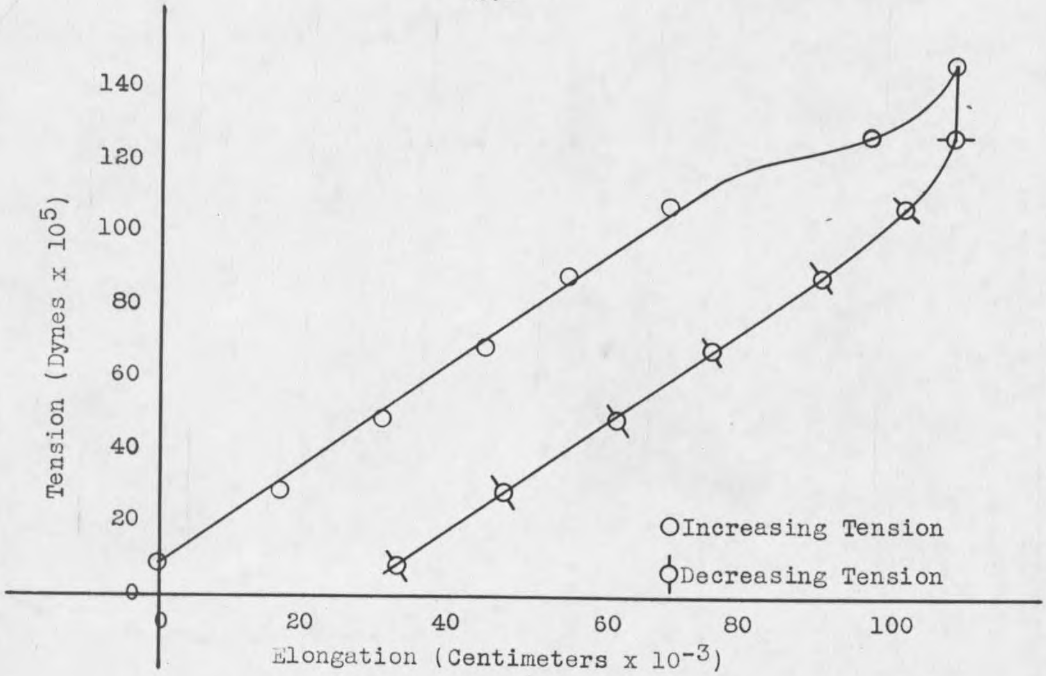

Fig. 8. Tension versus elongation

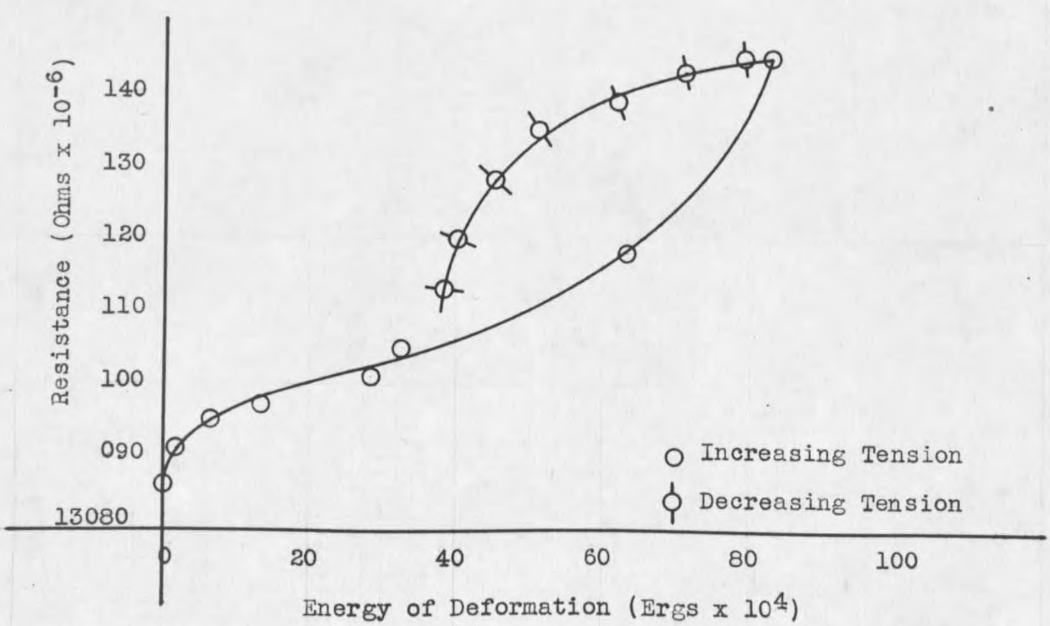

Fig. 9. Resistance versus energy of deformation.

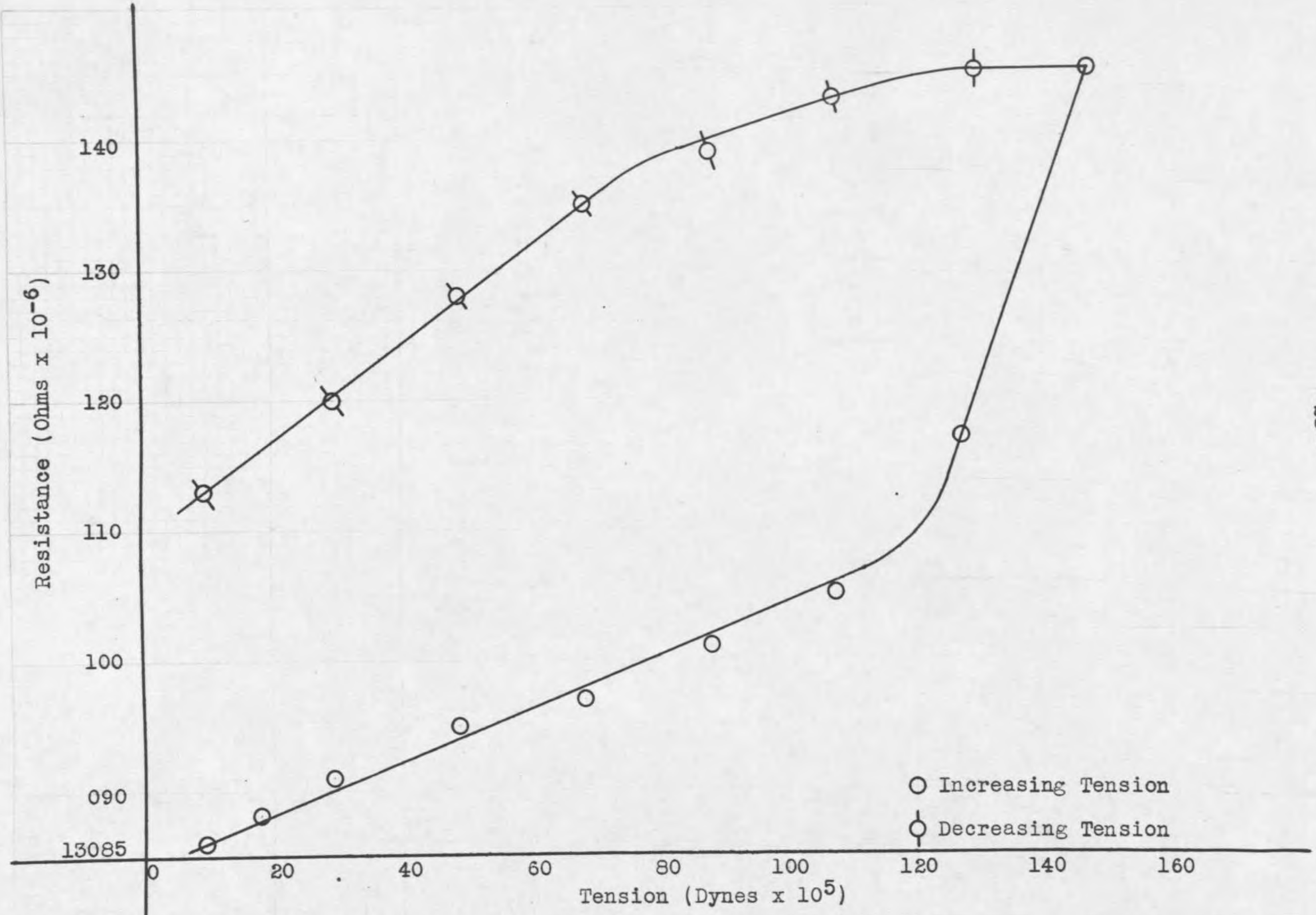

Fig. 10. Resistance versus tension.

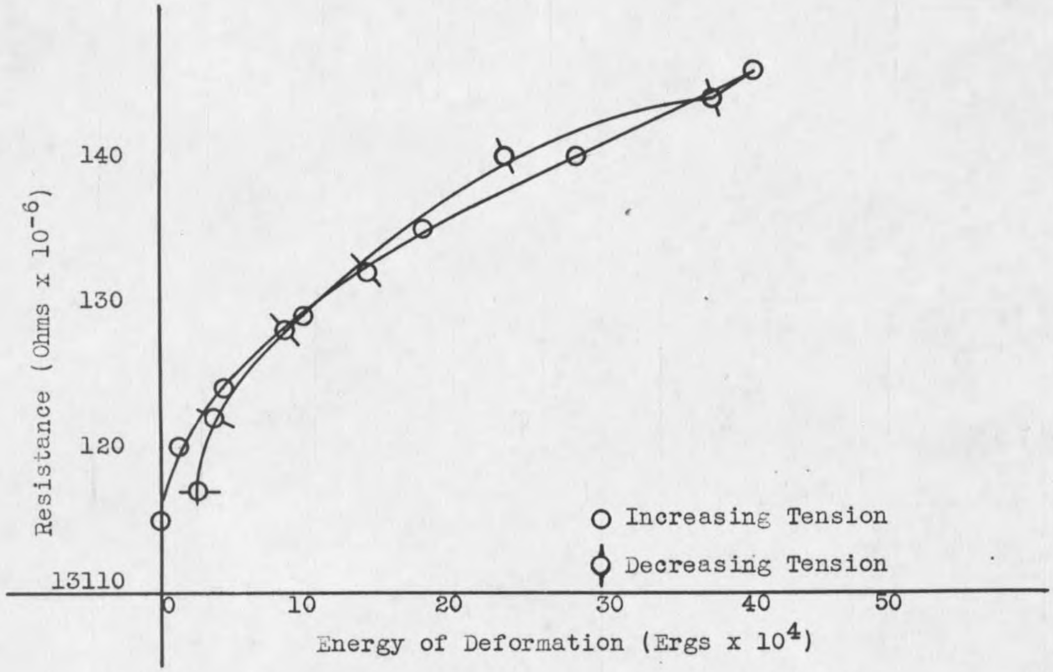

Fig. 11. Resistance versus energy of deformation.

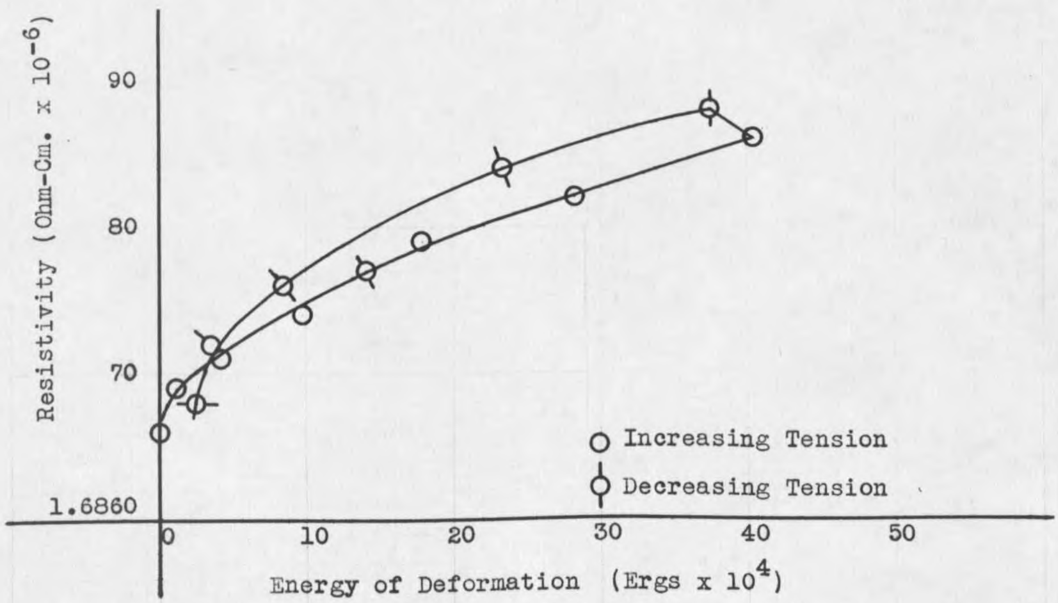

Fig. 12. Resistivity versus energy of deformation.

TABLE II

Elasticity-Resistance-Energy Data of a Typical Copper Sample.

No. 16 B. and S Gauge Bare Copper Wire June 19, 1946.

No.	Tension (Dynes)	Elongation (Cm.)	Resistance (Ohms)	Energy (Ergs)
0	9.8×10^5	0.00	13086×10^{-6}	0.00
1	29.4	16.75×10^{-3}	13091	1.88×10^4
2	49.0	30.60	13095	6.40
3	68.6	44.65	13097	13.68
4	88.2	55.95	13101	20.84
5	107.8	69.55	13105	32.76
6	127.4	96.95	13118	63.64
7	147.0	108.10	13145	83.20
8	127.4	108.10	13145	79.36
9	107.8	101.65	13143	71.31
10	88.2	90.10	13139	62.00
11	68.6	75.85	13135	51.40
12	49.0	62.45	13128	45.24
13	29.4	47.15	13120	40.28
14	9.8	32.75	13113	38.40

Length: 100.30 Cm. Radius: $r_o = 0.6410$

$r_a^o = 0.6409$

TABLE III

Resistance-Energy Data of a Typical Copper Sample.

No. 16 B. and S. Gauge Bare Copper Wire June 20, 1946.

No.	Energy (Ergs)	Resistance (Ohms)	Resistivity (Ohm-Cm.)
0	0.00	13115×10^{-6}	1.6866×10^{-6}
1	1.16×10^4	13120	1.6869
2	4.24	13124	1.6871
3	9.84	13129	1.6874
4	17.96	13135	1.6879
5	28.36	13140	1.6882
6	40.48	13146	1.6886
7	37.52	13144	1.6888
8	23.40	13140	1.6884
9	14.04	13132	1.6877
10	8.52	13128	1.6876
11	3.60	13122	1.6872
12	2.48	13117	1.6868

Length: 100.35 Cm. Radius: 0.06409 Cm.

TABLE IV

The values of Young's Modulus, E, in dynes per square centimeter, computed from elongation and tension; and $\frac{\Delta\rho}{\rho}$ per kilogram per square centimeter, computed from the data by equation 9), compared with values from other sources.

$\frac{\Delta\rho}{\rho}$ (Kg./Cm.^{2})$^{-1}$	Source	E (Dynes/Cm.^{2})	Source
1.33×10^{-6}	Bridgman	12.3×10^{11}	Kaye and Laby
1.32	Rolnick	12.4 - 12.9	Kaye and Laby
1.6	Hume-Rothery	10.19 - 12.0	Hdbk. Ch., Phy.
1.36 ($\pm$14.5%)	Kerttula	12.0 ($\pm$5.3%)	Kerttula

RESULTS AND CONCLUSIONS

Under increasing tension, resistance and resistivity vary as predicted by equations 3) and 4) respectively, so long as the elastic limit is not exceeded. (Figures 3, 4, 5, 6, 9, 11 and 12). The data conform to the approximate relation (eq. 3) as well as to the exact equation 4), in fact, there seems to be less scattering of the points when resistance is plotted against energy of deformation than in the resistivity-versus-energy curves. This is due to the fact that the elongation 'e' is used in the computation of resistivity (see p. 15, bottom of page), and hence the experimental error enters twice into the resistivity-versus-energy curves, since it is also used in the computation of the energy of deformation (see p. 15, top of page). Both equations, therefore, represent the data equally well within the limits of error.

When the yield point is reached, the basic assumptions of equations 3) and 4) break down, and the data become ambiguous.

On decreasing tension, the curves differ greatly from those predicted. Above the elastic limit, the data are ambiguous for the reason mentioned above. Below the elastic limit, the middle portions of the curves are similar to those for increasing tension (except in the extreme case of figure

9, where the elastic limit is greatly exceeded). In the lower portions, however, the resistance and resistivity both decrease more rapidly than predicted. In figures 11 and 12, where the load was kept very low, the curves almost coincide. It appears, then, that if the yield point were not exceeded, the curves for increasing and decreasing tension would be coincident, but with a metal as soft as copper, there is some "set" with even very small loads.

The shape of the curves for decreasing tension is due to the manner of computing the energy of deformation. For example, the energy in the wire at point 12 (figure 2) was taken to be the area under the top curve minus the area under that portion of the lower curve from point 11 to point 12. In actuality, this is not true, for, while the area under the curve from point 11 to point 12 represents the energy returned by the system when the tension was decreased, the energy at point 11 (i.e., the total area under the top curve) is not the energy stored in the wire, but represents the energy put into the system, part of which was dissipated as heat (see p.35), and part stored in the wire in an unavailable form. Actually, the available potential energy of deformation at point 12 is represented by the area under the lower curve from point 22 to point 12, i.e., it is the energy finally recovered from the system. The energy not regained is represented by the area between the two curves, but there is no way of determining from these experiments what portion of this energy was dissipated as heat and what portion remains stored in the wire as energy of permanent deformation. It is evident that when we proceed as above, subtracting from the energy represented by the area under the upper curve that represented by the area under the lower curve from point to point, the error is cumulative, resulting in the downward bend of the resistance versus energy curve at its lower end.

As may be seen from figures 7 and 10, resistance varies linearly with tension on both increasing and decreasing tension below the elastic limit. The steeper slope of the line for decreasing tension, particularly evident in figure 10, is due to the decrease in 'A_0', and the increase in 'R_0', caused by the permanent stretching of the wire (see equation 7).

Since this is true, if we plotted resistance against energy of deformation again, this time measuring the energy by the area under the lower curve, we would obtain a curve of the type represented by equation 8), with larger values of 'R_0' and 'A_0', and possibly some change in 'E' and 'β'. However, this would yield no more information than the method used, since we have no measure of the final energy, i.e., that energy stored in the wire by permanent deformation.

Hence, the work yields only one definite conclusion, to wit: resistance varies directly as the square root of the available potential energy of deformation within the elastic limit, as predicted by equation 8).

The data taken above the elastic limit yield little information, but the following discussion will serve to bring out what conclusions may be drawn therefrom, clarify some of the statements made above regarding the curves for decreasing tension, and will outline some of the difficulties to be overcome in a further investigation of the phenomenon.

In one experiment, (figures 3, 4, and 6) the elastic limit being exceeded only slightly, as indicated by the tension-versus-elongation curve (figure 2), the increase in resistance and resistivity was less rapid than below the yield point, while in another, (figures 11 and 12) the increase is more rapid than below the yield point, the elastic limit again being exceeded only slightly. However, when the elastic limit was greatly exceeded, the resistance increases very rapidly, as in figure 9. In this case, the load was so great that the wire continued to stretch without increase of tension. This is evident from an examination of figure 8, the tension-versus-elongation curve. Between the readings 7 and 8 (Table II), the wire evidently continued to stretch, since the elongation at point 8, at a lower tension, was the same as that at point 7, and the resistance did not decrease between these readings (figure 9). Another indication that the wire was stretching is the relative positions of points 6 and 7 (figure 8). The wire was continuing to elongate under this tension and the point 7 would have been moved farther along the elongation axis had the readings been taken just before removing the load. The resultant change in dimensions was, in this case, a great factor in causing the resistance change. Also, as the wire was continuing to elongate, the change was not isothermal, and the rise in temperature of the wire also caused the resistance to increase rapidly. These factors, then, tend to obscure the effect of tension in this case.

In addition to these factors, the energy put into the system beyond the elastic limit is not all stored as potential energy of deformation; that is, it cannot be regained from the system in the same way as if the elastic limit were not exceeded. A portion of it is used in overcoming the frictional forces within the metal, and is dissipated as heat, while a portion is retained in the deformed metal, unavailable to do work in returning the metal toward its original state. The balance is returned from the system when the weights are removed. The energy stored in the wire by stretching above the elastic limit is not the same, then, as that represented by the area under the curve.

In order to investigate the variation of resistivity beyond the elastic limit, a method of determining the amount of energy stored in the permanently deformed wire must be devised, and, since Poisson's Ratio can not be relied upon to give the change in radius at these tensions, a precision method for measuring the radius of the specimen under tension at all points of its length must also be developed. The method of these experiments permits one to measure the energy not regained from the system, but it cannot indicate what portion of this energy is retained in the wire, nor how the addition of this energy affects the resistivity, except that when a wire is permanently deformed, the resistivity is increased. Energy not regained from the system was tabulated with the corresponding change in resistance, and the resistance change for a certain increment of energy computed for each case, but the results varied as much as 200 per-cent, and were therefore deemed not worthy of consideration, except in that they established the sign of $\frac{\Delta\rho}{\rho}$ as positive above the elastic limit.

An incidental result of the experiments was a confirmation of the results of Bridgman and Rolnick in the measurement of the unit change of resistivity with tension. As may be seen from Table IV, the value of $\frac{\Delta\rho}{\rho}$ determined from this data is in close agreement with the results of Bridgman and Rolnick. The value of E also lies within the limits of the values found by others.

Bridgman's equation is also obtained incidentally in the derivation of the resistance-energy equation, the derivation being included herewith, even though it could not be determined whether the same derivation had been given elsewhere. A possible application to the problem of strain gages is drawn from this equation, and the strain sensitivity factor for the materials used is also computed. This material is included as an appendix to the thesis.

APPENDIX

DETERMINATION OF THE STRAIN SENSITIVITY FACTOR OR GAGE FACTOR

The strain sensitivity factor, or "gage factor" of a resistance wire strain gage (see p. 5) is defined as the ratio $\frac{\Delta R/R_o}{\Delta L/L_o}$. An equation for this ratio in terms of 'μ', 'β', and 'E' may be easily derived from Bridgman's Equation,

$$\text{i)}\quad \frac{\Delta\rho}{\rho_o\Delta F} = \frac{\Delta R}{R_o\Delta F} - \frac{1+2\mu}{E}$$

by remembering the definitions, $E = \frac{\Delta F}{\Delta L/L_o}$, and $\beta = \frac{\Delta\rho}{\rho_o\Delta F}$. Substituting in i), and rearranging the terms, we obtain:

$$\frac{\Delta L}{L_o}(1+\beta E+2\mu) = \frac{\Delta R}{R_o}$$

which we can solve for the gage factor:

$$\text{ii)}\quad \frac{\Delta R/R_o}{\Delta L/L_o} = 1 + 2\mu + \beta E$$

If 'β', 'μ', and 'E' can be determined to a sufficient degree of accuracy, equation ii) may be used to determine the strain sensitivity factor for the material, as is done below for the copper wire used in this experiment.

$\mu = 0.33$ (cf. p. 15)

$E = 12.0 \times 10^{11}$ dynes/cm^2. $= 1.22 \times 10^6$ Kg./cm^2. (Table IV)

$\beta = 1.36 \times 10^{-6}$ cm^2./Kg. (Table IV)

$$\frac{\Delta R/R_o}{\Delta L/L_o} = 1+2\mu+\beta E = 1+0.66+2.66 = 4.32$$

This is considerably higher than any of the values given for various types of commercial bonded strain gages[5], which is not surprising in view of the fact that the shape of the winding and the type of bonding cement used in these gages greatly affects the gage factor. The equation holds more nearly for the unbonded type of strain gage, which approximates a free wire under strain.

LITERATURE CITED AND CONSULTED

1. TOMLINSON, Trans. Roy. Soc., 1883, 174, 1 (cited in Hume-Rothery, "The Metallic State").
2. BRIDGMAN, P. W., Proc. Amer. Acad. Arts Sci., 1921, 59, 117.
3. ROLNICK, HARRY, Phys. Rev., 1930, 36, 506.
4. HUME-ROTHERY, The Metallic State, p. 25, (The Oxford University Press, London, 1931).
5. NIELSON, D. M., Electronics, 1943, 16, 106.
6. SOKOLNIKOFF, Mathematical Theory of Elasticity, (Brown University, Providence, 1941), mimeographed.
7. SCIENCE ABSTRACTS, 1937, Abstract No. 2763.
8. KAYE AND LABY, Tables of Physical and Chemical Constants, p. 29, (Longman's, Green and Company, New York, 1936).
9. KELLY, HARRY C., Textbook in Electricity and Magnetism, p. 125, (John Wiley and Sons, New York, 1941).
10. HANDBOOK OF CHEMISTRY AND PHYSICS, Twenty-Ninth Edition, (Chemical Rubber Publishing Company, Cleveland, 1945).