A MONTE CARLO STUDY OF SEVERAL REGRESSION ESTIMATORS IN THE CONTEXT OF SECOND-ORDER AUTOCORRELATION

by

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APPROVAL

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TABLE OF CONTENTS

	Page
APPROVAL	ii
STATEMENT OF PERMISSION TO USE	iii
ACKNOWLEDGMENTS	iv
TABLE OF CONTENTS	v
LIST OF TABLES	vii
LIST OF FIGURES	xiii
ABSTRACT	xiv
CHAPTER	
1. LINEAR REGRESSION MODELS WITH SECOND-ORDER AUTOCORRELATED ERRORS	1
Introduction	1
Purpose	5
The Model	6
Estimation Techniques	8
The OLS Method	8
The Two-Stage and Iterative CO Method	8
The Two-Stage and Iterative PW Method	10
The Full-Maximum-Likelihood (FML) Method	11
The Initially Nonstationary (INS) Method	12
Plan of the Study	13
Methodology	14
A Preview	15
2. THEORY OF ESTIMATION	16
Introduction	16
Theoretical Development of the Estimators	16
The GLS Estimator	16
The Maximum Likelihood Estimator	20
Computational Forms of the Estimators	22
The OLS Estimator	25
The TSCO and ICO Estimators	26

TABLE OF CONTENTS--Continued

	Page
The TSPW and IPW Estimators	28
The FML Estimator	30
The INS Method	32
3. THE MONTE CARLO EXPERIMENT	36
Introduction	36
Data Generation	37
Numerical Subtleties	45
Results	49
4. CONCLUSIONS	61
Introduction	61
Summary	61
Conclusions and Recommendations	63
Further Research	65
REFERENCES CITED	67
APPENDICES	
A: FORTRAN Program	71
B: Tables 2-51	103

LIST OF TABLES

Table		Page
1.	Empirical Moments of a Sample of Size 500,000, Generated by a Pseudo-Normal (0,1) Random Number Generator and Their, Respective, Theoretical Values	39
2.	BEST Estimators in Terms of RMSE for the Experiment with Stationary Errors and White Noise on the Independent Variable	104
3.	BEST Estimators in Terms of RMSE for the Experiment with Nonstationary Errors and White Noise on the Independent Variable	105
4.	BEST Estimators in Terms of RMSE for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable	106
5.	BEST Estimators in Terms of RMSE for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable	107
6.	BEST Estimators in Terms of RMSE for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable	108
7.	BEST Estimators in Terms of RMSE for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable	109
8.	BEST Estimators in Terms of RMSE for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable	110
9.	BEST Estimators in Terms of RMSE for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable	111
10.	Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Stationary Errors and White Noise on the Independent Variable	112

LIST OF TABLES--Continued

Table		Page
11.	Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and White Noise on the Independent Variable.....	114
12.	Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.....	116
13.	Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.....	118
14.	Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.....	120
15.	Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.....	122
16.	Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.....	124
17.	Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.....	126
18.	Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and White Noise on the Independent Variable.....	128

LIST OF TABLES--Continued

Table		Page
19.	Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and White Noise on the Independent Variable.....	130
20.	Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.....	132
21.	Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.....	134
22.	Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.....	136
23.	Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.....	138
24.	Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.....	140
25.	Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.....	142
26.	Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and White Noise on the Independent Variable.....	144

LIST OF TABLES--Continued

Table		Page
27.	Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and White Noise on the Independent Variable	146
28.	Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable	148
29.	Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable	150
30.	Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable	152
31.	Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable	154
32.	Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable	156
33.	Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable	158
34.	Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Stationary Errors and White Noise on the Independent Variable	160

LIST OF TABLES--Continued

Table		Page
35.	Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and White Noise on the Independent Variable.....	162
36.	Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.....	164
37.	Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.....	166
38.	Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.....	168
39.	Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.....	170
40.	Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.....	172
41.	Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.....	174
42.	Iteration Statistics for the Four Iterative Estimators in the Experiment with Stationary Errors and White Noise on the Independent Variable.....	176

LIST OF TABLES--Continued

Table		Page
43.	Iteration Statistics for the Four Iterative Estimators in the Experiment with Nonstationary Errors and White Noise on the Independent Variable	177
44.	Iteration Statistics for the Four Iterative Estimators in the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable	178
45.	Iteration Statistics for the Four Iterative Estimators in the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable	179
46.	Iteration Statistics for the Four Iterative Estimators in the Experiment with Stationary Errors and a Growth Trend on the Independent Variable	180
47.	Iteration Statistics for the Four Iterative Estimators in the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable	181
48.	Iteration Statistics for the Four Iterative Estimators in the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable	182
49.	Iteration Statistics for the Four Iterative Estimators in the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable	183
50.	Relative Bias of $\hat{\sigma}_{\beta_1}$	184
51.	Relative Bias of $\hat{\sigma}_{\beta_2}$	188

LIST OF FIGURES

Figure		Page
1.	Stationary Region for AR(2) Process.....	7
2.	Stationary Region Subdivision Scheme.....	44
3.	Stationary Region Boundary Violation.....	49
4.	Regions of Superior Estimator Performance Within the Stability Triangle for the Subexperiments with Nonstationary Errors.....	56
5.	FORTRAN Program.....	72

ABSTRACT

The purpose of this study is to compare the small sample properties of several estimators in the context of second-order autocorrelation by using Monte-Carlo methods. Each estimator is applied to data sets generated under several experimental design assumptions. The design assumptions consist of four structures on the independent variable, iterated over two types of error structures, with each error structure being iterated over twenty-five combinations of the autoregressive parameters evenly distributed within the second-order autoregressive stability triangle. The four independent variable structures included: (1) white noise; (2) dampened trend; (3) growth trend; and (4) dampened cyclical trend. The error structures are generated first under the assumption that the process is stationary, and second under the assumption that the process is initially nonstationary. Each permutation of the experimental design consists of a sample size of T=25, with each permutation being replicated 2500 times. Several statistics relevant to the selection of an appropriate estimator based on efficiency, computational complexities, and inference validity of each estimator are offered.

From an overall perspective of choosing an estimator that uniformly demonstrates robustness across the various independent variable types, error structure assumptions, and autoregressive parameter values, a general ranking of the estimators is apparent. The ranking of these estimators is: (1) full maximum likelihood; (2) initially nonstationary; (3) Prais-Winsten estimators; and (4) Cochrane-Orcutt estimators. Ordinary least squares, however, is the superior estimator when the distance of the point defined by the value of the autoregressive parameters is close to the center of the second-order autoregressive stability triangle. The performance of the initially nonstationary estimator is not substantially below that of the full maximum likelihood estimator when viewed from the perspective of overall performance across the various subexperimental designs. This result coupled with the ease in implementing the initially nonstationary estimator for any order autoregressive process suggests strong potential for this estimator to efficiently estimate linear regression models with autoregressive errors.

CHAPTER 1

LINEAR REGRESSION MODELS WITH SECOND-ORDER AUTOCORRELATED ERRORS

Introduction

Several arguments, both theoretical and empirical, have been offered for the existence of autocorrelated error structures in many economic data sets (Cochrane and Orcutt, 1949; Johnston, 1984; Judge et al., 1985). Generally, three sources are most frequently cited (Johnston, 1984). First, if the variables omitted from the empirical model are autocorrelated, then the autocorrelated disturbances represent the net influence of these omitted explanatory variables. The omission of variables is due to the pragmatic fact that no model, given the constraints on available data, can include all relevant explanatory variables. A second source is misspecification of the model. The classic case is a linear relationship imposed on an independent variable that inherently has a nonlinear relationship with the dependent variable. In multiple regression models this misspecification may be difficult to detect and/or accurately take into account. Finally, statistical measurement of the dependent variable can introduce autocorrelation. Because the same sorts of measurements, with their associated errors, tend to be used in data

collection, it is possible to have the mistakes manifest themselves as autocorrelated error structures.

Regardless of the source of autocorrelated error structures, once we acknowledge their presence, the ramifications of not accounting for an error structure of this type are far reaching in the efficiency of estimation, prediction and hypothesis testing of econometric models. Specifically, as noted by Johnston (1984), the consequences of not accounting for autocorrelated disturbances are unbiased but inefficient estimates, and invalid inference procedures if ordinary least squares (OLS) is applied.

Since the publication of Cochrane and Orcutt's now classic article in 1949, there has been a substantial body of research performed on various methods of estimating models that are autocorrelated. The general goal has been to increase the efficiency of the estimator over OLS, thus making hypothesis tests more reliable. In current applied econometric estimation there are five commonly used methods of transforming autocorrelated models into ones that contain spherical errors. They are: (1) two-stage Prais-Winsten (TSPW); (2) iterative Prais-Winston (IPW) (Prais and Winsten, 1954); (3) full maximum likelihood (FML) (Beach and MacKinnon, 1978); (4) two-stage Cochrane-Orcutt (TSCO); and (5) iterative Cochrane-Orcutt (ICO) (Cochrane and Orcutt, 1949). In addition to these standard estimators another estimator alluded to by Judge et al. (1985) has recently been studied (Siebrasse, 1987). Referred to as the initially nonstationary estimator (INS), this method assumes that the first k residuals, u_t,

where k equals the order of the autoregressive error process, were generated by an autoregressive error process with a starting point at time t=0, or that the k estimates of the presample residuals, u_t, t=0,-1,...,-k+1, are fixed at their expected values of zero.

Asymptotically, all of the above estimators are equivalent (Judge et al., 1985). In small samples, however, the properties of these estimators are not apparent and in some cases may actually be more inefficient than OLS. For example, in the case of first-order autoregressive (AR(1)) errors with trended independent variables, it has been demonstrated that the Cochrane-Orcutt estimator can become extremely inefficient when compared to OLS in small samples (Maeshiro, 1976). As the current body of research illustrates, evaluating the small sample properties of these various estimators analytically is a formidable task. Thus, many researchers have analyzed the small sample properties of these estimators with Monte Carlo simulations (Rao and Griliches, 1969; Spitzer, 1979; Beach and MacKinnon, 1978).

A comprehensive body of literature has been developed on the small-sample properties of the estimators noted in the context of AR(1) error structures (Siebrasse, 1987). Unfortunately, the dynamics of first-order autoregressive processes, as described in Box and Jenkins (1976), are very restrictive since they do not accommodate cyclical error structures. To compensate for this deficiency one must introduce second-order autoregressive (AR(2)) structures which facilitate the possibility of complex roots, thus cyclical dynamics. Fortunately, this gain in flexibility is not at the expense of losing

the dynamics found in the AR(1) process. Instead, the dynamics of the AR(1) process are a subset of the possible processes generated by the AR(2) process (Box and Jenkins, 1976).

There has been very little research on the small-sample properties of the estimators discussed above in the context of second-order autocorrelation (Judge et al., 1985). The Monte Carlo results of Gallant and Goebel (1976) found that the generalized nonlinear least squares estimator (GNLS), using the autoregressive assumption to obtain the approximating variance-covariance matrix, performed better than the ordinary nonlinear least squares (ONLS). The authors noted that the benefits of iterating the estimation process are ambiguous. Iterating improved performance when the model was correctly specified and the errors were autoregressive, performance deteriorated slightly when they were not. Thus, it may be argued that if the marginal cost of computing is negligible and the potential gain in performance greatly outweighs the possibility of loss, then iterative estimation should be used in applications (Gallant and Goebel, 1976).

In a 1978 paper, Beach and MacKinnon presented a full maximum likelihood (FML) procedure that estimated a general linear regression model in the context of a stationary AR(2) error process. Incorporating the absolute value of the Jacobian of the transformation matrix into the likelihood function, the authors derived an iterative estimation procedure that forces the estimates of the autoregressive parameters to lie within the stability triangle of a stationary AR(2)

process. The resulting variances of the regression parameter estimates asymptotically reach their Cramer-Rao Lower Bounds.

Using a consumption function from Klein's Model I (Klein, 1950) Beach and MacKinnon applied OLS, FML assuming AR(1) and AR(2), and ICO assuming AR(1) and AR(2) to annual U.S. data for the period 1921 to 1941. The results suggest, based on theoretical values of the parameters, that the FML estimates are more plausible than the CO estimates. For the other two structural equations of Klein's Model I, coefficient estimates were essentially the same between the two estimators.

Although it is obvious from the Beach and MacKinnon study that the CO and FML procedures perform differently in a statistical sense, it is also obvious that the results are inconclusive with respect to the question of efficiency and inference validity, due to the fact that the actual functional form and parameter values of the data being analyzed are unknown.

Purpose

Results from the Beach and MacKinnon (1978), and Gallant and Goebel (1976) articles, while informational, do not comprehensively study the estimators under AR(2) error process considerations. Acknowledging this deficiency, the purpose of this thesis is to examine the small-sample properties of the seven estimators noted above in the context of AR(2) errors.

Generally, applied econometricians are concerned with parameter

estimates that are efficient, have estimated standard errors that lead to valid inferences, and minimize computational complexities. Bearing these concerns in mind, this research attempts to compute statistics that will make it possible for the applied econometrician to choose the estimator that most satisfactorily estimates the linear regression model with AR(2) errors.

In the present chapter the specific model is defined, as are the estimators used, the currently known properties are discussed, and the methodology of the study is presented.

The Model

The general form of the simple linear regression model with AR(2) errors is

$$y_t = \beta_1 + \beta_2 x_t + u_t, \qquad (1.1)$$

with

$$u_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + \epsilon_t, \; t = 1,\ldots,T, \qquad (1.2)$$

where at time t, the value of the dependent variable is y_t, the nonstochastic independent variable is x_t, and the error is u_t. The ϵ_t's are independently and identically distributed random variables with mean zero, $E(\epsilon_t)=0$, and constant variance, σ_ϵ^2, for all t. β_1 and β_2 are the regression coefficients and θ_1 and θ_2 are the autoregressive coefficients. It is further assumed that the autoregressive coefficients lie in the stability triangle (Figure 1), i.e. $\theta_1 + \theta_2 < 1$, $\theta_2 - \theta_1 < 1$ and $|\theta_2| < 1$, implied by a stationary AR(2) process (Box and Jenkins, 1976).

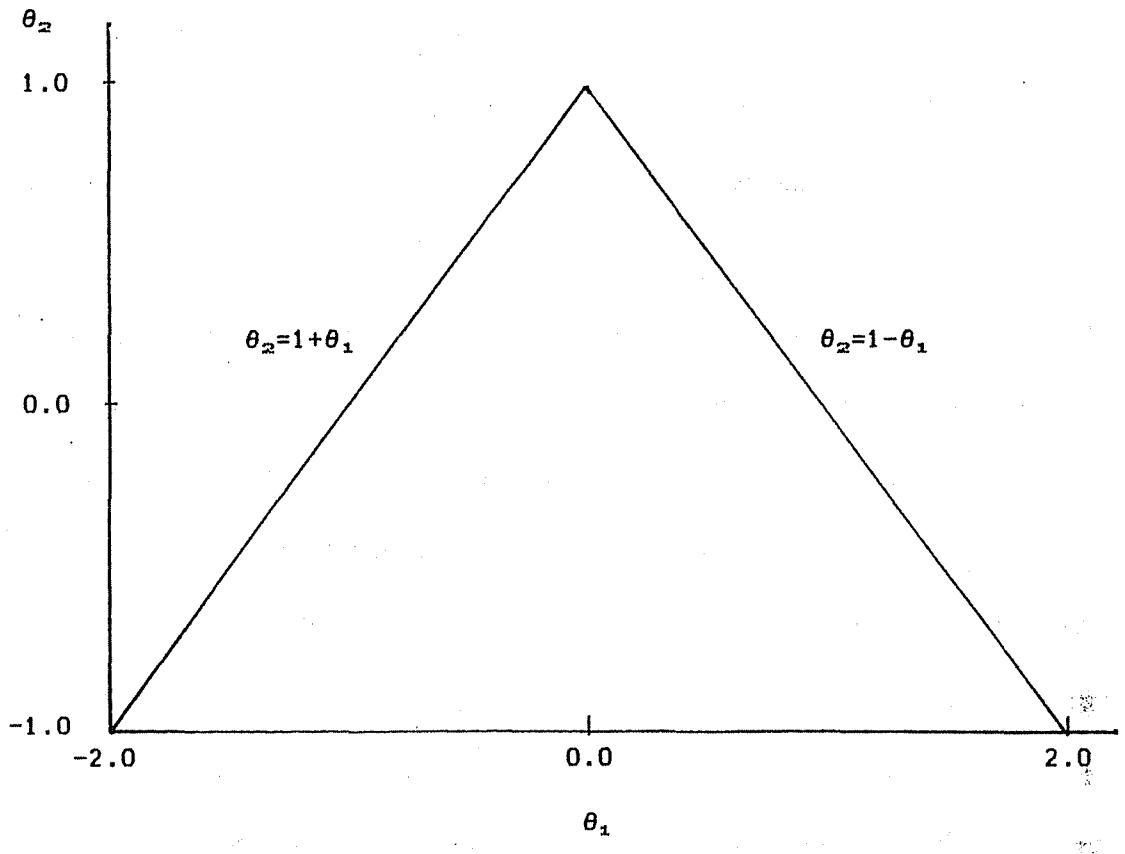

Figure 1. Stationary Region for the AR(2) Process.

In matrix form this model is represented as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{U}, \qquad (1.3)$$

where

$$\mathbf{Y} = (y_1\ y_2\ \ldots\ y_T)'$$

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ x_1 & x_2 & \ldots & x_T \end{bmatrix}'$$

$$\mathbf{U} = (u_1\ u_2\ \ldots\ u_T)'$$

$$\boldsymbol{\beta} = (\beta_1\ \beta_2)',$$

with

u_t = relation (1.2).

Estimation Techniques

In this section the estimators of interest are briefly reviewed. A more formal development of the estimators and their methods of implementation is elaborated upon in the next chapter.

The OLS Method

Let $\hat{\boldsymbol{\beta}}_{ols}$ be the estimator of $\boldsymbol{\beta}$; it is well known that

$$\hat{\boldsymbol{\beta}}_{ols} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}. \tag{1.4}$$

The OLS estimators of θ_1 and θ_2, derived from solving the Yule-Walker equations (Judge et al., 1985), are

$$\hat{\theta}_{1,ols} = \hat{\rho}_1(1-\hat{\rho}_2)/(1-\hat{\rho}_1^2) \tag{1.5}$$

and

$$\hat{\theta}_{2,ols} = \left(\hat{\rho}_2-\hat{\rho}_1^2\right)/(1-\hat{\rho}_1^2), \tag{1.6}$$

respectively, where

$\hat{\rho}_j$, j=1,2 are the estimated autocorrelation coefficients at lag j, defined by

$$\hat{\rho}_j = \left(\Sigma_{t=j+1}^{T}\hat{u}_t\hat{u}_{t-j}\right)/\left(\Sigma_{t=1}^{T}\hat{u}_t^2\right), \text{ j=1,2 and T=sample size.} \tag{1.7}$$

The estimated errors, $\hat{u}_t$, t=1,...,T are defined by

$$\hat{u}_t = y_t - (\hat{\beta}_1+\hat{\beta}_2x_t). \tag{1.8}$$

The Two-Stage and Iterative CO Method

In 1949 Cochrane and Orcutt published an article that brought to light the importance of recognizing autocorrelated error structures in the least squares regression problem. This article laid the

groundwork for a great deal of research on testing for, and methods of treating, regression models with autocorrelated error structures.

Satisfied, via theoretical and empirical evidence, with the assumption of positive autocorrelation in many of the economic data sets analyzed, especially time series, Cochrane and Orcutt proposed a method to regain most of the OLS efficiency losses. This method effectively minimizes the sum of squared residuals given by

$$SSR_{co} = \Sigma_{t=3}^{t}[y_t - \theta_{1,co}y_{t-1} - \theta_{2,co}y_{t-2} - (X_t - \theta_{1,co}X_{t-1} - \theta_{2,co}X_{t-2})\beta_{co}]^2, \quad (1.9)$$

where

$X_{t-j} = (1\ x_{t-j})$, $t=3,\ldots,T$ and $j=0,1,2$, and

$\beta_{co} = (\beta_1\ \beta_2)'$

When the true error parameters are known, applying OLS to the transformed variables implied by the above equation results in an approximate generalized least squares (GLS) estimator of β in (1.3). If, however, the error parameters are not known, Cochrane and Orcutt suggested a two-stage procedure that takes advantage of the stochastic properties of certain second-order derivatives of (1.9). In the first stage $\theta_{1,co}$ and $\theta_{2,co}$ are fixed, initially to zero, and SSR_{co} is minimized with respect to β_{co}. β_{co} is then fixed to its estimated value and SSR_{co} is minimized with respect to $\theta_{1,co}$ and $\theta_{2,co}$. Two iterations of this procedure is known as two-stage CO (TSCO) and iteration till convergence of the parameters is known as iterative CO (ICO).

Two important points are to be noted in this method. First, the transformation matrix reduces, by the order of autocorrelation assumed (in this case two) the number of observations from the beginning of the data series. Asymptotically this will not make a difference. In small samples, however, especially if they are trended (Rao and Griliches, 1969; and Maeshiro, 1976), the reduction of the sample size can result in a considerable loss of efficiency. Secondly, as Cochrane and Orcutt (1949) express, the procedure may not do enough to randomize the errors because of the strong bias they already have towards randomness. Thus, all potential efficiency gains may not be obtained from this procedure.

The Two-Stage and Iterative PW Method

Recognizing the possible inefficiencies associated with using the TSCO and ICO estimators in small samples when the data contains a trend, Prais and Winston (1954) developed an estimator that minimizes the sum of squared residuals given by

$$SSR_{pw} = u_1^{*2} + u_2^{*2} + SSR_{co}, \tag{1.10}$$

where

$$u_1^* = \left\{[(1+\theta_2)[(1-\theta_2)^2-\theta_1^2]]/(1-\theta_2)\right\}^{1/2}(y_1-\mathbf{X}_1\boldsymbol{\beta})$$

$$u_2^* = (1-\theta_2^2)^{1/2}[y_2-\theta_1/(1-\theta_2)y_1]$$

$$- (1-\theta_2^2)^{1/2}[\mathbf{X}_2 - \theta_1/(1-\theta_2)\mathbf{X}_1]\boldsymbol{\beta}$$

SSR_{co} = Sum of squared residuals equation (1.9)

$\boldsymbol{\beta} = (\beta_1\ \beta_2)'$

$\mathbf{X}_t = (1\ x_t)$, $t=1,\ldots,T$.

When the true error parameters are known, applying OLS to the transformed variables implied by (1.10) results in the GLS estimator of $\boldsymbol{\beta}$ in (1.3). If θ_1 and θ_2 are not known, the estimated parameters $\hat{\beta}_{1,PW}$, $\hat{\beta}_{2,PW}$, $\hat{\theta}_{1,PW}$ and $\hat{\theta}_{2,PW}$ are found by application of the same process undergone in the CO estimation procedure with the exception that (1.10) is now the objective function to be minimized. Two iterations of this procedure gives us the two-stage PW (TSPW) estimator, while iteration till convergence is known as the iterative PW (IPW) estimator.

The Full-Maximum-Likelihood (FML) Method

If the autoregressive parameters are unknown, none of the estimators under consideration are FML, with the exception of the method proposed by Beach and MacKinnon (1978).

The joint likelihood function for the spherical normal error vector ϵ, with variance σ_ϵ^2, up to a factor of proportionality is

$$L(\epsilon) \propto \sigma_\epsilon^{-T}\exp[-\epsilon'\epsilon/(2\sigma_\epsilon^2)]. \qquad (1.11)$$

Letting $\mathbf{U}$ be the error vector whose elements u_t, t=1,2,...,T, are generated by the AR(2) process (1.2), we can represent the relation between ϵ and $\mathbf{U}$ by

$$\epsilon = \mathbf{PU}, \qquad (1.12)$$

where $\mathbf{P}$ is a linear transformation matrix that is derived based on the assumption that $\mathbf{U}$ is a stationary error process. The likelihood function can thus be written in terms of $\mathbf{U}$ as

$$L(\mathbf{U}) \propto \sigma_\epsilon^{-T}\exp[-\mathbf{U}'\mathbf{P}'\mathbf{PU}/(2\sigma_\epsilon^2)]|\mathbf{P}|, \qquad (1.13)$$

where $|P|$ is the absolute value of the Jacobian of the transformation from ϵ to U.

Concentrating out σ_ϵ^2 from the likelihood function (1.13), and taking the logarithm yields the concentrated log-likelihood function

$$\mathcal{L} \propto (-T/2)\log(U'P'PU) + \log|P|. \qquad (1.14)$$

Taking advantage of the block diagonal nature of the expected value of the Hessian matrix of the log-likelihood function with respect to the parameters, known as Fisher's information matrix, Beach and MacKinnon suggest an iterative procedure, similar to CO and PW, that finds a solution for β given θ_1 and θ_2, then for θ_1 and θ_2 given β. The iteration process terminates when the successive values of the parameters are sufficiently close.

Because the concentrated log-likelihood function (1.14) explicitly constrains the parameters θ_1 and θ_2 to lie within the stability triangle and assuming **a priori** stationarity conditions exist, Beach and MacKinnon maintain this estimation method to be superior.

The Initially Nonstationary (INS) Estimator

As noted earlier, this method sets the presample residuals, u_t, $t=0,-1,\ldots,-k+1$, equal to zero, thus assuming that either the autoregressive process has a starting point at $t=0$, or the presample residuals are set to their expected values. Setting the presample residuals equal to zero is not unrealistic if we are willing to replace the assumption that the errors were generated from a process

that reaches back infinitely into the past by, say, an assumption that the error process started with a major war or political shock.

Accepting the rationale of setting the presample residuals equal to zero, it is a straightforward matter to show that the designation "initially nonstationary" is due to the second moments of the first and second transformed errors; the variances of the first two errors are time dependent, therefore nonstationary.

The sum of squared residuals to be minimized in this case is

$$SSR_{inm} = u_1^{**} + u_2^{**} + SSR_{co}, \qquad (1.15)$$

where

$u_1^{**} = (y_1 - X_1\beta)^2$,

$u_2^{**} = [y_2 - X_2\beta - \theta_1(y_1 - X_1\beta)]^2$,

SSR_{co} = Sum of squared residuals equation (1.9),

$X_t = (1\ x_t)$, $t=1,\ldots,T$.

Although SSR_{inm} results in a nonlinear least squares estimation process, we can once again take advantage of the block diagonal nature of the expected value of the Hessian matrix to mitigate the difficulty in estimation. In other words, it is possible to use the same iterative process employed in the other nonlinear least squares estimators, with SSR_{inm} being the objective function minimized.

Plan of the Study

In this section, statement of the basic methodology employed and the statistics compiled by the Monte Carlo study are presented. A preview of the the remainder of the study is also discussed.

Methodology

The Monte Carlo experiment for each of the seven estimators consists of estimating the simple linear regression model (1.1) with errors (1.2) under several experimental designs for the underlying structure of the independent variable x_t and the errors u_t. The x_t's, of sample size T=25, are generated under each of four assumptions: (1) white noise; (2) dampened trend; (3) growth trend; and (4) dampened cyclical. The x_t values are common to, and fixed for, the 2500 replications of each error structure. The error structures considered consist of twenty-five combinations of θ_1 and θ_2 within the stability triangle under stationary and initially nonstationary assumptions. In other words, for each combination of θ_1 and θ_2, two sets of errors are generated, one stationary, the other, initially nonstationary.

Several statistics pertinent to the small-sample properties of the estimators under consideration are computed; they are: (1) the mean estimate of β_1, β_2, θ_1, and θ_2; (2) the root mean square error (RMSE) of the estimators for β_1, β_2, θ_1, and θ_2; and (3) the RMSE and percent bias of the standard errors $\hat{\sigma}_{\beta_1}$ and $\hat{\sigma}_{\beta_2}$. The first two sets of statistics address the efficiency of each estimator and have been the statistics most emphasized in many of the past Monte Carlo studies. The final set of statistics is concerned with the validity of the inferences made by each of the estimators. Because it is common practice to use estimated asymptotic distributions to make inferences in small samples, it is apparent that an estimate of the

variance that more closely approximates the true variance would be better from a hypothesis testing perspective. In addition, the average number of iterations and the number of times the maximum iteration count (100) is encountered are tabulated for the iterative estimators.

A Preview

Chapter 2 explains in detail the theoretical motivation behind the estimation of linear regression models in the context of AR(2) processes. With this foundation, the actual computational form used for each estimator is derived.

Armed with the computational formulas from Chapter 2, Chapter 3 details the actual Monte Carlo implementation -- random number generation, parameter specification, etc. -- as well as the computational subtleties of the FORTRAN 77 program employed. This chapter also discusses the results of the research.

Conclusions, summarization of main points, recommendations, and suggestions for further research are discussed in Chapter 4.

CHAPTER 2

THEORY OF ESTIMATION

Introduction

The theory supporting the use of the GLS and the FML estimators when the disturbance vector is nonspherical has been well developed (Judge et al., 1985). In what follows, the theoretical GLS and equivalent ML estimators, under the assumption that the disturbance vector is normally distributed, for linear models with AR(2) errors are developed. Using these theoretical constructs in conjunction with statements concerning underlying assumptions, the actual computational form of each estimator used in the Monte Carlo experiment is derived.

Theoretical Development of the Estimators

The GLS Estimator

Consider the model described by (1.3) where $\mathbf{U}$ is an error vector whose elements are described by the AR(2) error process (1.2). The mean and variance of $\mathbf{U}$ are

$$E(\mathbf{U}) = \mathbf{0} \tag{2.1}$$

and

$$E(\mathbf{U}\mathbf{U}') = \sigma_{\epsilon}^{2}\Psi=\Omega, \tag{2.2}$$

respectively, where Ψ is a (T×T) known, real, symmetric and positive

definite matrix. Given these assumptions about Ψ, there exists a (TxT) matrix P such that $P\Psi P'=I_T$, $\Psi=P^{-1}(P')^{-1}$, and $\Psi^{-1}=P'P$ (Judge et al., 1985). Transforming the model (1.3) as

$$PY = PX\beta + PU, \tag{2.3}$$

results in a model where the random vector $PU=\epsilon$ has a mean of **0** and a scalar covariance matrix, $\sigma_\epsilon^2 P\Psi P'=\sigma_\epsilon^2 I_T$. Adding the assumption of normality, the elements of the random vector ϵ are independently and identically distributed. Furthermore, if the assumption of a nonstochastic independent variable matrix, X, is included, we have a model that satisfies the assumptions of the classical regression model (Judge et al., 1985).

By minimizing the sum of squared errors

$$\begin{aligned} S(\beta,\sigma_\epsilon^2\Psi) &= U'P'PU \\ &= (Y-X\beta)'\Psi^{-1}(Y-X\beta) \\ &= (PY-PX\beta)'(PY-PX\beta) \end{aligned} \tag{2.4}$$

implied by the transformed model (2.3), with respect to β and solving, we obtain the GLS estimator

$$\hat{\beta} = (X'\Psi^{-1}X)^{-1}X'\Psi^{-1}Y, \tag{2.5}$$

where Ψ is the (TxT) known covariance matrix for U. Similarly,

$$\begin{aligned} \tilde{\sigma}_\epsilon^2 &= (PY-PX\hat{\beta})'(PY-PX\hat{\beta})/(T-K) \\ &= (Y-X\hat{\beta})'\Psi^{-1}(Y-X\hat{\beta})/(T-K) \end{aligned} \tag{2.6}$$

is an unbiased estimator of σ_ϵ^2.

$\hat{\beta}$ is a Best Linear Unbiased Estimator (BLUE). If we further assume the ϵ_t's in (1.2) to be independently identically distributed $N(0,\sigma_\epsilon^2)$, then $\hat{\beta}\sim N(\beta,\sigma_\epsilon^2(X'\Psi^{-1}X)^{-1})$ (Johnston, 1984).

In the case of AR(q) error processes, the transformation matrix **P** can be derived by constraining **P'P** to be proportional to the inverse of the variance-covariance matrix of the stationary Yule error process (Gallant and Goebel, 1976). This will be done for the case of q=2.

Let the autocovariance function be $\gamma(h) = E(u_t u_{t+h})$, $h=0,\pm1,\pm2,\ldots$, where h is the lag. At zero lag we have the constant variance of the series

$$\begin{aligned}\gamma(0) &= E(u_t^2) \\ &= \theta_1\gamma(1) + \theta_2\gamma(2) + \sigma_\epsilon^2 \\ &= \sigma_u^2. \end{aligned} \tag{2.7}$$

The autocorrelation coefficient at lag h is defined by

$$\rho_h = \gamma(h)/\gamma(0), \quad h=\pm1,\pm2,\ldots. \tag{2.8}$$

Let

$$\begin{aligned}\Psi_{q=2} &= \begin{bmatrix}\gamma(0) & \gamma(1)\\ \gamma(1) & \gamma(0)\end{bmatrix} \\ &= \sigma_u^2 \begin{bmatrix}1 & \rho_1\\ \rho_1 & 1\end{bmatrix}, \end{aligned} \tag{2.9}$$

and

$$\begin{aligned}\boldsymbol{\gamma}_{q=2} &= [\gamma(1)\ \gamma(2)]' \\ &= \sigma_u^2[\rho_1\ \rho_2]'. \end{aligned} \tag{2.10}$$

Hence, the Yule-Walker equations can be written as

$$\boldsymbol{\theta} = \Psi_q^{-1}\boldsymbol{\gamma},$$

with

$$\begin{aligned}\sigma_\epsilon^2 &= \gamma(0) - \boldsymbol{\theta}'\boldsymbol{\gamma}_q \\ &= \sigma_u^2 - \boldsymbol{\theta}'\boldsymbol{\gamma}_q, \end{aligned} \tag{2.11}$$

where $\boldsymbol{\theta}$ is the (2x1) vector of autoregressive parameters stated as a

function of the autocorrelation coefficients ρ_1 and ρ_2, specifically

$$\theta_1 = \rho_1(1-\rho_2)/(1-\rho_1^2) \qquad (2.12)$$

and

$$\theta_2 = (\rho_2-\rho_1^2)/(1-\rho_1^2). \qquad (2.13)$$

Solving (2.12) and (2.13) for ρ_1 and ρ_2 implies

$$\rho_1 = \theta_1/(1-\theta_2) \qquad (2.14)$$

and

$$\rho_2 = \theta_2 + \theta_1^2/(1-\theta_2). \qquad (2.15)$$

If the assumption of stationarity holds, then Ψ_q^{-1} is a real, symmetric and positive definite matrix that can be factored by, say, Choleskey's decomposition into the relation $\Psi_q^{-1} = \mathbf{P}_q'\mathbf{P}_q$, which results in the transformation matrix

$$\mathbf{P} = \begin{bmatrix} \sigma_\epsilon \mathbf{P}_q & \vdots & \mathbf{0} \\ \hline -\theta_2 & -\theta_1 & 1 & 0 & \cdots & 0 \\ 0 & -\theta_2 & -\theta_1 & 1 & 0 & \cdots & 0 \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & -\theta_2 & -\theta_1 & 1 & \end{bmatrix} \begin{matrix} q=2 \text{ rows} \\ \\ T-q \text{ rows} \\ \\ \\ \end{matrix} \qquad (2.16)$$

(Gallant and Goebel, 1976). Substitution of expressions (2.14) and (2.15) into (2.16) results in the (TxT) AR(2) transformation matrix expressed in terms of the autoregressive coefficients θ_1 and θ_2

$$P = \begin{bmatrix} \sigma_\epsilon/\sigma_u & 0 & 0 & \cdots & 0 & 0 \\ -\theta_1(1-\theta_2^2)^{0.5}/(1-\theta_2) & (1-\theta_2^2)^{0.5} & 0 & \cdots & 0 & 0 \\ -\theta_2 & -\theta_1 & 1 & 0 \cdots & 0 & 0 \\ 0 & -\theta_2 & -\theta_1 & 1 \; 0 \cdots & 0 & 0 \\ & \cdot & \cdot & \cdot \; \cdot & & \\ & & & \cdot \; \cdot \; \cdot & & \\ 0 & \cdots & \cdots & 0 & -\theta_2 & -\theta_1 \; 1 \end{bmatrix}, \qquad (2.17)$$

where

$$\sigma_\epsilon/\sigma_u = \left\{(1+\theta_2)[(1-\theta_2)^2 - \theta_1^2]/(1-\theta_2)\right\}^{0.5}, \qquad (2.18)$$

with $P'P = \Psi^{-1}$. Thus, the sum of squared residuals relation (2.4) can be reexpressed as

$$SSR(\theta_1,\theta_2,\beta) = u_1^{*2} + u_2^{*2} + \Sigma_{t=3}^{T} e_t^2, \qquad (2.19)$$

where

$$u_1^* = (\sigma_\epsilon/\sigma_u)(y_1 - X_1\beta),$$

$$u_2^* = (1-\theta_2^2)^{0.5}\left\{y_2 - X_2\beta - [\theta_1/(1-\theta_2)](y_1 - X_1\beta)\right\},$$

and

$$e_t = y_t - \theta_1 y_{t-1} - \theta_2 y_{t-2} - (X_t - \theta_1 X_{t-1} - \theta_2 X_{t-2})\beta,$$

with

$$X_t = [1 \; x_t], \; t=3,4,5,\ldots,T.$$

The Maximum Likelihood Estimator

Previously we assumed the spherical error vector ϵ to be independently identically distributed as normal $N(0,\sigma_\epsilon^2 I)$. Since the stationary error vector U is a linear transformation of ϵ, we can write the likelihood function for U, up to a factor of proportionality, as

$$L(U) \propto \sigma_\epsilon^{-T}\exp[-U'P'PU/2\sigma_\epsilon^2]|P|, \qquad (2.20)$$

where $|\mathbf{P}|$ is the absolute value of the Jacobian of the transformation matrix $\mathbf{P}$ from $\boldsymbol{\epsilon}$ to $\mathbf{U}$. In this case

$$|\mathbf{P}| = (1+\theta_2)(1-\theta_1-\theta_2)^{1/2}(1+\theta_1-\theta_2)^{1/2}. \tag{2.21}$$

Concentrating out σ_ϵ^2 from the likelihood function (2.20), and taking the natural logarithm, yields the concentrated log-likelihood function

$$\mathfrak{J} \propto (-T/2)\log(SSR) + \log(1+\theta_2) + \tfrac{1}{2}\log(1-\theta_1-\theta_2) + \tfrac{1}{2}\log(1+\theta_1-\theta_2), \tag{2.22}$$

where

SSR = the sum of squared residuals (2.19).

The three terms at the end of (2.22) explicitly restrict the ML estimates of θ_1 and θ_2 to lie within the stability region because as any one of the three boundaries is approached the value of the term associated with that boundary approaches minus infinity.

The maximum likelihood estimator of $\boldsymbol{\beta}$, given that $\mathbf{P}$ is known, is seen to be the GLS estimator

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{P}'\mathbf{P}\mathbf{X})^{-1}\mathbf{X}'\mathbf{P}'\mathbf{P}\mathbf{Y}. \tag{2.23}$$

Moreover, if $\mathbf{P}$ is not known, the ML estimators of θ_1, θ_2 and $\boldsymbol{\beta}$ will differ from the LS estimators found by minimizing (2.19) because of the last terms in (2.22). Finding the exact ML estimates of θ_1 and θ_2, given $\boldsymbol{\beta}$, is a difficult problem that involves the use of either nonlinear optimization techniques, or simplical search algorithms (Beach and MacKinnon, 1978).

As in the case of the GLS estimator, if θ_1 and θ_2 are known, then $\hat{\boldsymbol{\beta}} \sim N(\boldsymbol{\beta}, \sigma_\epsilon^2(\mathbf{X}'\boldsymbol{\Psi}^{-1}\mathbf{X})^{-1})$ with (2.6) being the unbiased estimator of σ_ϵ^2 (Judge et al., 1985).

Estimation when θ_1 and θ_2 are unknown, using either the theoretical GLS or FML methods, results in estimators for $\boldsymbol{\beta}$ that are asymptotically normally distributed with mean $\boldsymbol{\beta}$ and covariance matrix $\sigma_\epsilon^2(\mathbf{X}'\mathbf{P}'\mathbf{P}\mathbf{X})^{-1}$. Replacing $\mathbf{P}$ by $\hat{\mathbf{P}}$ and σ_ϵ^2 by $\hat{\sigma}_\epsilon^2=(\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}})'\hat{\mathbf{P}}'\hat{\mathbf{P}}(\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}})/(T-k)$ we can proceed to test hypotheses relative to $\boldsymbol{\beta}$. It is crucial to note that each of the estimators finds $\hat{\mathbf{P}}$ and $\hat{\sigma}_\epsilon^2$ differently, hence, potentially different conclusions upon testing $H_o:\beta_1=\beta_1^o$ and $H_o:\beta_2=\beta_2^o$.

Computational Forms of the Estimators

This section undertakes the derivation of the actual computational formulas implemented in the Monte Carlo program, given each estimator's associated assumptions.

It is important to note that none of the estimation techniques, with the exception of the FML, TSPW and IPW procedures, constrain the estimates of θ_1 and θ_2 to lie within the stability region. It was decided that an **ad hoc** constraint placed on the estimators to limit θ_1 and θ_2 to values inside the stability triangle would bias the true properties of the unconstrained methods. Therefore, the TSCO, ICO, OLS, and INS estimators do not explicitly place constraints on the estimation of θ_1 and θ_2, such that the estimates of these parameters are stationary.

In the case of the FML procedure, the likelihood function explicitly restricts θ_1 and θ_2 to lie in the stability triangle. This explicit constraint is a result of the absolute value of the Jacobian of the transformation being included in the likelihood function. Effectively, the terms introduced force the value of the likelihood function to minus infinity when one of the boundaries is approached, thus guaranteeing a maximum within the stability region.

For the two Prais-Winston procedures the constraint is not so evident. Nevertheless, if we consider that the matrix $\mathbf{P}_{pw}$ was derived by constraining $\mathbf{P'P}$ to be proportional to the variance-covariance matrix of a stationary AR(2) error process, it is obvious that the values of θ_1 and θ_2 should lie in the stability region. Thus, explicit constraints were incorporated into the TSPW and IPW procedures. If one does not constrain θ_1 and θ_2, the matrix $\mathbf{X'P'PX}$ is non-positive definite, therefore not statistically meaningful.

Another computational subtlety common to all the estimators in this study results from taking advantage of the assumption that the regression parameters $\boldsymbol{\beta}$ are stochastically independent of the AR(2) error process parameters $\boldsymbol{\theta}=[\theta_1\ \theta_2]'$. If this assumption is true, it is easily shown that the expected value of cross partial derivatives of the objective function that consist of any combination of regression parameters and error parameters is zero. Specifically, the expectations of these cross partials are

$$E[\partial^2\Phi(\boldsymbol{\beta},\boldsymbol{\theta})/\partial\boldsymbol{\beta}\partial\boldsymbol{\theta}'] = \mathbf{0}, \qquad (2.24)$$

where the function Φ can be any of the objective functions relevant to

the estimators in the study. By Young's Theorem the expected value of the appropriate transposes of (2.24) are also zero. Incorporating these results into the Hessian matrix $\mathbf{H} = \partial^2\Phi(\boldsymbol{\beta},\boldsymbol{\theta})/\partial\boldsymbol{\varphi}\partial\boldsymbol{\varphi}'$, where $\boldsymbol{\varphi} = [\boldsymbol{\beta}' \ \boldsymbol{\theta}']'$, gives us the modified block diagonal Hessian matrix

$$\mathbf{H}^* = \begin{bmatrix} \partial^2\Phi(\boldsymbol{\beta},\boldsymbol{\theta})/\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}' & \mathbf{0} \\ \mathbf{0} & \partial^2\Phi(\boldsymbol{\beta},\boldsymbol{\theta})/\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}' \end{bmatrix} . \qquad (2.25)$$

The important implication of (2.25) is that estimation of $\boldsymbol{\beta}$ can be made conditional on $\boldsymbol{\theta}$, and estimation of $\boldsymbol{\theta}$ can be made conditional on $\boldsymbol{\beta}$. More expressly, the procedures optimized their respective objective function $\Phi(\boldsymbol{\beta},\boldsymbol{\theta})$ by alternatively minimizing $\Phi(\boldsymbol{\beta},\boldsymbol{\theta})$ with respect to $\boldsymbol{\beta}$ given $\boldsymbol{\theta}$, then minimizing $\Phi(\boldsymbol{\beta},\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ given $\boldsymbol{\beta}$. Two iterations of this alternating procedure is defined as two-stage estimation, while iteration till convergence of the parameters is known as iterative estimation.

For all of the estimation methods, with the exception of FML, utilization of this alternating process resulted in closed form solutions to the first order conditions of the conditional objective functions, thus greatly reducing the computational complexities necessary to estimate the parameters $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$.

In the FML procedure, Beach and MacKinnon (1978) suggest an iterative procedure that also takes advantage of (2.25); however, a closed form solution was found only for the conditional estimate of $\boldsymbol{\beta}$. The conditional estimation of $\boldsymbol{\theta}$ involves the solution of mixed third and fourth order polynomials by either nonlinear techniques, or, as Beach and MacKinnon (1978) suggest, a simplical search algorithm. The nonlinear method was chosen for this study.

The OLS Estimator

Minimizing the sum of squared errors of the untransformed residuals $\mathbf{U}$ in (1.3) with respect to $\boldsymbol{\beta}$ results in the OLS estimator

$$\hat{\boldsymbol{\beta}}_{ols} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}. \tag{2.26}$$

From the residuals

$$\hat{\mathbf{U}} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}, \tag{2.27}$$

the sample autocorrelation coefficients of ρ_1 and ρ_2 are given by

$$\hat{\rho}_j = (\Sigma_{t=j+1}^{T}\hat{u}_t\hat{u}_{t-j})/(\Sigma_{t=1}^{T}\hat{u}_t^2), \quad j=1,2. \tag{2.28}$$

Substituting the sample counterparts into equations (2.12) and (2.13) gives us the sample autoregressive coefficients

$$\hat{\theta}_1 = [\hat{\rho}_1(1-\hat{\rho}_2)]/(1-\hat{\rho}_1^2), \tag{2.29}$$

and

$$\hat{\theta}_2 = (\hat{\rho}_2-\hat{\rho}_1^2)/(1-\hat{\rho}_1^2). \tag{2.30}$$

Assuming (2.1) and (2.2) to be true, the mean and variance of $\hat{\boldsymbol{\beta}}_{ols}$ are

$$E(\hat{\boldsymbol{\beta}}_{ols}) = \boldsymbol{\beta} \tag{2.31}$$

and

$$Var^*(\hat{\boldsymbol{\beta}}_{ols}) = \sigma_\epsilon^2(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\Psi\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}, \tag{2.32}$$

respectively, with

$$\hat{\sigma}_\epsilon^2 = (\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}}_{ols})'(\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}}_{ols})/(T-2). \tag{2.33}$$

Thus, the conventional formula

$$Var(\hat{\boldsymbol{\beta}}_{ols}) = \hat{\sigma}_\epsilon^2(\mathbf{X}'\mathbf{X})^{-1}, \tag{2.34}$$

no longer measures the sample variance-covariance of $\hat{\boldsymbol{\beta}}_{ols}$. Furthermore, although (2.32) is the true sampling variance, it is

invalid to use this estimate in t-tests and confidence intervals because the underlying assumptions used to derive these inference procedures no longer apply (Johnston, 1984).

The consequences of using the conventional variance-covariance formula (2.34) in small-samples, given an AR(2) process, on hypothesis testing have not been explored. Considering that there are probably many cases when OLS and its respective inference methods are used on data that are potentially autocorrelated, a measure of the consequences of using the standard variance-covariance estimator (2.34) is pertinent to this study. Hence, in the computation of RMSE and percent bias of $\hat{\sigma}_{\beta_1}$ and $\hat{\sigma}_{\beta_2}$ for the OLS estimator, the standard estimator (2.34) of the parameter variance-covariance matrix will be utilized.

The TSCO and ICO Estimators

Ignoring the first two observations implies an ML estimator conditional on y_1 and y_2 that has a [(T-2)xT] transformation matrix P_{co} that is given by P in (2.17) with the first two rows deleted. The resulting objective function to be minimized is defined by (2.19) without the terms u_1^{*2} and u_2^{*2}. This can be rewritten as

$$SSR_{co} = \Sigma_{t=3}^{T}(u_t-\theta_1 u_{t-1}-\theta_2 u_{t-2})^2, \qquad (2.35)$$

where u_t are elements of the vector defined by (2.27). Assuming the stochastic independence of $\boldsymbol{\beta}$ from $\boldsymbol{\theta}$, as discussed above, the CO estimators of $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$ can be derived. Fixing θ_1 and θ_2 initially to zero, and minimizing (2.35) with respect to $\boldsymbol{\beta}$ gives us the estimator

$$\hat{\boldsymbol{\beta}}_{co} = (\mathbf{X}'\hat{\mathbf{P}}_{co}'\hat{\mathbf{P}}_{co}\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{P}}_{co}'\hat{\mathbf{P}}_{co}\mathbf{Y}, \qquad (2.36)$$

where $\hat{\mathbf{P}}_{co}$ is $\mathbf{P}_{co}$ with $\boldsymbol{\theta}$ replaced by $\hat{\boldsymbol{\theta}}$. Alternatively, holding $\boldsymbol{\beta}$ fixed, initially to the OLS estimates, and minimizing (2.35) with respect to θ_1 and θ_2 implies an estimator of the autoregressive terms. The closed forms for $\hat{\theta}_{1,co}$ and $\hat{\theta}_{2,co}$ are found by setting the first order conditions

$$\partial SSR_{co}/\partial\theta_1 = -2\Sigma_{t=3}^{T} u_{t-1}(u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2}) \quad (2.37)$$

$$= -2\Sigma_{t=3}^{T} u_{t-1}u_t + 2\theta_1\Sigma_{t=3}^{T} u_{t-1}^2 + 2\theta_2\Sigma_{t=3}^{T} u_{t-1}u_{t-2},$$

and

$$\partial SSR_{co}/\partial\theta_2 = -2\Sigma_{t=3}^{T} u_{t-2}(u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2}) \quad (2.38)$$

$$= -2\Sigma_{t=3}^{T} u_{t-2}u_t + 2\theta_1\Sigma_{t=3}^{T} u_{t-2}u_{t-1} + 2\theta_2\Sigma_{t=3}^{T} u_{t-2}^2,$$

both equal to zero and solving the two resulting simultaneous equations for $\hat{\theta}_1$ and $\hat{\theta}_2$. The solution is found to be

$$\begin{bmatrix} \hat{\theta}_{1,co} \\ \hat{\theta}_{2,co} \end{bmatrix} = \begin{bmatrix} \Sigma_{t=2}^{T-1}\hat{u}_t^2 & \Sigma_{t=2}^{T-1}\hat{u}_t\hat{u}_{t-1} \\ \Sigma_{t=2}^{T-1}\hat{u}_t\hat{u}_{t-1} & \Sigma_{t=1}^{T-2}\hat{u}_t^2 \end{bmatrix}^{-1} \begin{bmatrix} \Sigma_{t=3}^{T}\hat{u}_{t-1}\hat{u}_t \\ \Sigma_{t=3}^{T}\hat{u}_{t-2}\hat{u}_t \end{bmatrix}, \quad (2.39)$$

where $\hat{u}_t = y_t - \mathbf{x}_t\hat{\boldsymbol{\beta}}_{co}$, t=1,...,T, are the residuals. It is important to note that $\mathbf{P}_{co}$ no longer contains the implicit stability constraints due to the fact that the first two rows of $\mathbf{P}_{pw}$ in (2.17) have been deleted; therefore it is possible to obtain nonstable estimates of θ_1 and θ_2.

Two iterations of the procedure gives us the TSCO estimates. Iteration until the subsequent changes in the parameters are less than some arbitrary amount, or an iteration count limit is encountered, results in the ICO estimates.

The variance estimator used to compute RMSE and percent bias of $\hat{\sigma}_{\beta_1}$ and $\hat{\sigma}_{\beta_2}$ was

$$\text{Var}(\hat{\boldsymbol{\beta}}_{co}) = \hat{\sigma}^2_{\epsilon,co}(\mathbf{X}'\hat{\mathbf{P}}'_{co}\hat{\mathbf{P}}_{co}\mathbf{X})^{-1}, \tag{2.40}$$

with

$$\hat{\sigma}^2_{\epsilon,co} = (\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}}_{co})'\hat{\mathbf{P}}'_{co}\hat{\mathbf{P}}_{co}(\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}}_{co})/[(T-2)-2]. \tag{2.41}$$

The TSPW and IPW Estimators

Employing the full data set, the PW estimator uses the (TxT) transformation matrix $\mathbf{P}_{pw}$ defined by $\mathbf{P}$ in (2.17), implying the sum of squared residuals equation (2.19). Rewriting (2.19) as

$$SSR_{pw} = (1-\theta_2^2)u_1^2 - 2\theta_1(1+\theta_2)u_1u_2 + (1-\theta_2)u_2^2 + SSR_{co}, \tag{2.42}$$

where u_t is the t^{th} element of the vector (2.27) and SSR_{co} is the Cochrane-Orcutt sum of squared errors defined by (2.35). Equation (2.42) defines the PW objective function to be minimized.

The estimators of the stochastically independent parameters $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$ can again be found by optimizing the conditional sum of squared residuals implied by (2.42). Fixing $\boldsymbol{\theta}$, initially to zero, and minimizing (2.42) with respect to $\boldsymbol{\beta}$ leads to the estimator

$$\hat{\boldsymbol{\beta}}_{pw} = (\mathbf{X}'\hat{\mathbf{P}}'_{pw}\hat{\mathbf{P}}_{pw}\mathbf{Y})^{-1}\mathbf{X}'\hat{\mathbf{P}}'_{pw}\hat{\mathbf{P}}_{pw}\mathbf{Y}, \tag{2.43}$$

where $\hat{\mathbf{P}}_{pw}$ is $\mathbf{P}_{pw}$ with $\boldsymbol{\theta}$ replaced by $\hat{\boldsymbol{\theta}}$. Holding $\boldsymbol{\beta}$ fixed, initially to the OLS estimates, and minimizing (2.42) with respect to $\boldsymbol{\theta}$ gives the estimates of the autoregressive coefficients. Setting the first order partial derivatives

$$\partial SSR_{pw}/\partial\theta_1 = -2(1+\theta_2)u_1u_2 - 2\Sigma_{t=3}^{T}u_{t-1}(u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2})$$

$$= 2(-\Sigma_{t=2}^{T}u_t u_{t-1} + \theta_1\Sigma_{t=2}^{T-1}u_t^2 + \theta_2\Sigma_{t=3}^{T-1}u_t u_{t-1}), \tag{2.44}$$

and

$$\partial SSR_{pw}/\partial\theta_2 = -2[\theta_2(u_1^2+u_2^2) + \theta_1 u_1 u_2$$

$$+ \Sigma_{t=3}^{T}u_{t-2}(u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2})]$$

$$= 2(-\Sigma_{t=3}^{T}u_t u_{t-2} + \theta_1\Sigma_{t=3}^{T-1}u_t u_{t-1} + \theta_2\Sigma_{t=3}^{T-2}u_t^2), \tag{2.45}$$

equal to zero and solving the simultaneous system gives the estimators of θ_1 and θ_2 expressed as

$$\begin{bmatrix} \hat{\theta}_{1,pw} \\ \hat{\theta}_{2,pw} \end{bmatrix} = \begin{bmatrix} \Sigma_{t=2}^{T-1}\hat{u}_t^2 & \Sigma_{t=3}^{T-1}\hat{u}_t\hat{u}_{t-1} \\ \Sigma_{t=3}^{T-1}\hat{u}_t\hat{u}_{t-1} & \Sigma_{t=3}^{T-2}\hat{u}_t^2 \end{bmatrix}^{-1} \begin{bmatrix} \Sigma_{t=2}^{T}\hat{u}_{t-1}\hat{u}_t \\ \Sigma_{t=3}^{T}\hat{u}_{t-2}\hat{u}_t \end{bmatrix}, \tag{2.46}$$

where $\hat{u}_t = y_t - \mathbf{x}_t\hat{\boldsymbol{\beta}}_{pw}$, t=1,...,T, are the residuals. The estimators of θ_1 and θ_2 were constrained to the stability region by the following algorithm:

1) if $\theta_2 > .99999$, then $\theta_2 = .99999$

2) if $\theta_2 < -.99999$, then $\theta_2 = -.99999$

3) if $1-\theta_2 < -\theta_1$, then $\theta_1 = \theta_2-1+0.5\times10^{-5}$

4) if $1-\theta_2 < \theta_1$, then $\theta_1 = 1-\theta_2-0.5\times10^{-5}$.

Since it is possible to violate two boundaries at one time in a triangle, order of execution is important. If the boundaries associated with steps 3 and 4 were checked first prior to executing

steps 1 and 2, it would be possible to have ambiguous situations where $\hat{\theta}_{1,PW}$ and $\hat{\theta}_{2,PW}$ would not be constrained to the stability region.

Similar to the CO estimation, two iterations of the procedure defines TSPW and iteration until convergence of the parameters results in IPW.

The variance estimator used to compute RMSE and percent bias of $\hat{\sigma}_{\beta_1}$ and $\hat{\sigma}_{\beta_2}$ was

$$\text{Var}(\hat{\beta}_{PW}) = \hat{\sigma}^2_{\epsilon,PW}(\mathbf{X}'\hat{\mathbf{P}}'_{PW}\hat{\mathbf{P}}_{PW}\mathbf{X})^{-1}, \tag{2.47}$$

with

$$\hat{\sigma}^2_{\epsilon,PW} = (\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}}_{PW})'\hat{\mathbf{P}}'_{PW}\hat{\mathbf{P}}_{PW}(\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}}_{PW})/(T-2). \tag{2.48}$$

The FML Estimator

Following Beach and MacKinnon (1978), the log-likelihood function of the full maximum likelihood estimator in the context of the linear regression model (1.3) with stationary AR(2) errors is, up to a factor of proportionality,

$$\mathcal{J}(\mathbf{U}) \propto (-T/2)\log(\mathbf{U}'\mathbf{P}'\mathbf{P}\mathbf{U}) + \log|\mathbf{P}|, \tag{2.49}$$

where the last term is the log of the absolute value of the Jacobian of the transformation $\mathbf{P}$ (defined by (2.17)) from $\boldsymbol{\epsilon}$ to $\mathbf{U}$. Explicitly, the last term of (2.49) can be written as

$$\log|\mathbf{P}| = \log(1+\theta_2) + \tfrac{1}{2}\log(1-\theta_1-\theta_2) + \tfrac{1}{2}\log(1+\theta_1-\theta_2) \tag{2.50}$$

(Beach and MacKinnon, 1978). Substituting (2.50) into (2.49) and reexpressing yields the objective function

$$\mathcal{J} \propto (-T/2)\log(SSR_{fml}) + \log(1+\theta_2) + \tfrac{1}{2}\log(1-\theta_1-\theta_2) + \tfrac{1}{2}\log(1+\theta_1-\theta_2), \tag{2.51}$$

where

$$SSR_{fml} = (\mathbf{U'P'PU})$$

$$= (\mathbf{Y}-\mathbf{X}\boldsymbol{\beta})'\mathbf{P}'\mathbf{P}(\mathbf{Y}-\mathbf{X}\boldsymbol{\beta})$$

$$= (1-\theta_2^2)u_1^2 - 2\theta_1(1+\theta_2)u_1u_2 + (1-\theta_2^2)u_2^2$$

$$+ \Sigma_{t=3}^{T}(u_t-\theta_1u_{t-1}-\theta_2u_{t-2})^2, \qquad (2.52)$$

and

$$u_t = y_t-\mathbf{x}_t\boldsymbol{\beta},\ t=1,2,\ldots,T.$$

Upon examination of (2.50), it is obvious that as the parameters θ_1 and θ_2 approach any one of the three stability triangle boundaries that the value of the likelihood function will approach minus infinity, thus implying the existence of stationary parameter estimates. Beach and MacKinnon (1978) suggest, on theoretical grounds, that the explicit inclusion of the stationarity constraints should provide for more efficient parameter estimates.

The maximum likelihood estimator of $\boldsymbol{\beta}$, given particular values of θ_1 and θ_2, is seen to be the GLS estimator

$$\hat{\boldsymbol{\beta}}_{fml} = (\mathbf{X}'\hat{\mathbf{P}}_{fml}'\hat{\mathbf{P}}_{fml}\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{P}}_{fml}'\hat{\mathbf{P}}_{fml}\mathbf{Y}, \qquad (2.53)$$

where $\hat{\mathbf{P}}_{fml}$ is the transformation matrix $\mathbf{P}$ with θ_1 and θ_2 replaced by their respective estimates.

As mentioned earlier, obtaining FML estimates of θ_1 and θ_2, given $\boldsymbol{\beta}$, presents a more difficult problem. The FML estimates of θ_1 and θ_2 are found by setting the first order derivatives of (2.49)

$$\partial\mathcal{J}/\partial\theta_1 = -(T/2)(\partial SSR_{fml}/\partial\theta_1)/SSR_{fml} - \tfrac{1}{2}(1-\theta_1-\theta_2)^{-1}$$

$$+ \tfrac{1}{2}(1+\theta_1-\theta_2)^{-1}, \qquad (2.54)$$

and

$$\partial\mathcal{J}/\partial\theta_2 = -(T/2)(\partial SSR_{fm1}/\partial\theta_2)/SSR_{fm1} - \tfrac{1}{2}(1-\theta_1-\theta_2)^{-1}$$
$$- \tfrac{1}{2}(1+\theta_1-\theta_2)^{-1} + (1+\theta_2)^{-1}, \qquad (2.55)$$

equal to zero and solving the resulting simultaneous equations. Unfortunately, (2.54) and (2.55) are mixed third and fourth order polynomials and therefore must be solved through the use of numerical techniques. Beach and MacKinnon (1978) chose to use a simplical search algorithm known as the Vector Sandwich Method (MacKinnon, 1975). The Marquardt Method (Bard, 1974) was chosen for this study.

Iterating the process of estimating $\boldsymbol{\beta}$, given θ_1 and θ_2, then estimating θ_1 and θ_2, given $\boldsymbol{\beta}$, resulted in the algorithm used to maximize the objective function (2.49). The algorithm starts with the initial values of θ_1 and θ_2 set equal to the TSPW estimates of these parameters, and the initial value of $\boldsymbol{\beta}$ set to the OLS estimate. Termination of the process occurs when successive values of the parameters are arbitrarily close or the iteration limit is reached.

The variance estimator used to compute RMSE and percent bias of $\hat{\sigma}_{\beta_1}$ and $\hat{\sigma}_{\beta_2}$ was

$$\text{Var}(\hat{\beta}_{fm1}) = \hat{\sigma}^2_{\epsilon^*,fm1}(\mathbf{X}'\hat{\mathbf{P}}'_{fm1}\hat{\mathbf{P}}_{fm1}\mathbf{X})^{-1}, \qquad (2.56)$$

with

$$\hat{\sigma}^2_{\epsilon^*,fm1} = (\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}}_{fm1})'\hat{\mathbf{P}}'_{fm1}\hat{\mathbf{P}}_{fm1}(\mathbf{Y}-\mathbf{X}\hat{\boldsymbol{\beta}}_{fm1})/(T-2). \qquad (2.57)$$

The INS Method

Setting the value of the presample residuals to zero implies a transformation matrix that differs from the transformation matrices used by the other estimators. Specifically, the initially nonstationary approximation results in the (TxT) transformation matrix

$$\mathbf{P}_{ins} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & 0 \\ -\theta_1 & 1 & 0 & \cdots & \cdots & 0 \\ -\theta_2 & -\theta_1 & 1 & 0 & \cdots & 0 \\ 0 & -\theta_2 & -\theta_1 & 1 & 0 & 0 \\ & \cdot & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot & \cdot \\ & & & & & \\ 0 & \cdots & \cdots & 0 & -\theta_2 & -\theta_1 & 1 \end{bmatrix}. \quad (2.58)$$

Cognizant that $\mathbf{P}_{ins}$ is (T×T), it is obvious that this method employs the full data set, much like PW and FML, but with different weights on the first and second observations. Define $\mathbf{e} = \mathbf{P}_{ins}\mathbf{U}$, where $\mathbf{e}$ is a T×1 vector of transformed errors, $\mathbf{P}_{ins}$ is the transformation matrix (2.58) and $\mathbf{U}$ is the AR(2) error vector defined by (1.2). The elements of $\mathbf{e}$ are

$$\begin{aligned} e_1 &= u_1 = \epsilon\left\{(1-\theta_2)/[(1+\theta_2)((1-\theta_2)^2-\theta_1^2)]\right\}^{1/2} \\ e_2 &= u_2 - \theta_1 u_1 = \epsilon_2/(1-\theta_2^2)^{1/2} \\ e_3 &= u_3 - \theta_1 u_1 - \theta_2 u_2 = \epsilon_3 \\ &\cdot \\ &\cdot \\ e_T &= u_T - \theta_1 u_{T-1} - \theta_2 u_{T-2} = \epsilon_T \end{aligned} \quad (2.59)$$

Examination of the variance of $\mathbf{e}$ reveals that $\mathrm{Var}(e_1) = \sigma_\epsilon^2(1-\theta_2)/[(1+\theta_2)((1-\theta_2)^2-\theta_1^2)]$ and $\mathrm{Var}(e_2) = \sigma_\epsilon^2/(1-\theta_2^2)$, while $\mathrm{Var}(e_t) = \sigma_\epsilon^2$ for t=3,...,T. Observing the time dependency of the variances of e_1 and e_2, the reason for the designation "initially nonstationary" is readily apparent.

Asymptotically, applying the transformation matrix $\mathbf{P}_{ins}$ in the estimated GLS procedure is equivalent to the other methods of transformation. In addition, like the CO transformation matrix, $\mathbf{P}_{ins}$

is easily extended to higher order error processes, while, unlike the CO method, using the full data set.

Transformation of **U** by $\mathbf{P}_{ins}$ results in the sum of squares objective function

$$SSR_{ins} = u_1^2 + (u_2-\theta_1 u_1)^2 + \Sigma_{t=3}^{T}(u_t-\theta_1 u_{t-1}-\theta_2 u_{t-2})^2. \tag{2.60}$$

Again noting the stochastic independence of $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$, estimators of the parameters can be derived by conditionally minimizing (2.60) with respect to both sets of parameters. Minimizing (2.60) with respect to $\boldsymbol{\beta}$, while holding θ_1 and θ_2 fixed, initially at zero, implies the estimator

$$\hat{\boldsymbol{\beta}}_{ins} = (\mathbf{X}\hat{\mathbf{P}}_{ins}'\hat{\mathbf{P}}_{ins}'\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{P}}_{ins}'\hat{\mathbf{P}}_{ins}\mathbf{Y}, \tag{2.61}$$

where $\hat{\mathbf{P}}_{ins}$ is $\mathbf{P}_{ins}$ with θ_1 and θ_2 replaced by their estimates. Subsequent estimation of the autoregressive parameters is accomplished by holding $\boldsymbol{\beta}$ fixed, initially to the OLS estimates, and minimizing (2.60) with respect to θ_1 and θ_2. Setting the first order conditions

$$\begin{aligned}\partial SSR_{ins}/\partial\theta_1 &= -2u_1(u_2-\theta_1 u_1) - 2\Sigma_{t=3}^{T}u_{t-1}(u_t-\theta_1 u_{t-1}-\theta_2 u_{t-2}) \\ &= 2(-\Sigma_{t=2}^{T}u_t u_{t-1} + \theta_1\Sigma_{t=1}^{T-1}u_t^2 + \theta_2\Sigma_{t=2}^{T-1}u_t u_{t-1}),\end{aligned} \tag{2.62}$$

and

$$\begin{aligned}\partial SSR_{ins}/\partial\theta_2 &= -2\Sigma_{t=3}^{T}u_{t-2}(u_t-\theta_1 u_{t-1}-\theta_2 u_{t-2}) \\ &= 2(-\Sigma_{t=3}^{T}u_t u_{t-2} + \theta_1\Sigma_{t=2}^{T-1}u_t u_{t-1} + \theta_2\Sigma_{t=1}^{T-2}u_t^2),\end{aligned} \tag{2.63}$$

equal to zero and solving the simultaneous equations for θ_1 and θ_2 implies the closed form estimators

$$\begin{bmatrix} \hat{\theta}_{1,ins} \\ \hat{\theta}_{2,ins} \end{bmatrix} = \begin{bmatrix} \Sigma_{t=1}^{T-1}\hat{u}_t^2 & \Sigma_{t=2}^{T-1}\hat{u}_t\hat{u}_{t-1} \\ \Sigma_{t=2}^{T-1}\hat{u}_t\hat{u}_{t-1} & \Sigma_{t=1}^{T-2}\hat{u}_t^2 \end{bmatrix}^{-1} \begin{bmatrix} \Sigma_{t=2}^{T}\hat{u}_{t-1}\hat{u}_t \\ \Sigma_{t=3}^{T}\hat{u}_{t-2}\hat{u}_t \end{bmatrix}. \tag{2.64}$$

Iteration of this alternating procedure results in the estimates for the initially nonstationary estimator.

The variance estimator used to compute RMSE and percent bias of $\hat{\sigma}_{\beta_1}$ and $\hat{\sigma}_{\beta_2}$ was

$$\text{Var}(\hat{\beta}_{ins}) = \hat{\sigma}^2_{\epsilon,ins}(X'\hat{P}'_{ins}\hat{P}_{ins}X)^{-1}, \tag{2.65}$$

with

$$\hat{\sigma}^2_{\epsilon,ins} = (Y-X\hat{\beta}_{ins})'\hat{P}'_{ins}\hat{P}_{ins}(Y-X\hat{\beta}_{ins})/(T-2). \tag{2.66}$$

CHAPTER 3

THE MONTE CARLO EXPERIMENT

Introduction

Methodological procedures of the Monte Carlo experiment were briefly discussed in Chapter 1. Succinctly stated, the study consisted of computing statistics relevant to the efficiency, computational complexity, and inference validity of seven estimators (OLS, TSPW, IPW, FML, TSCO, ICO, and INS) in the context of the simple linear regression model (1.3) with AR(2) errors (1.2). The experimental design consisted of four types of independent variables, iterated over two types of error structures (stationary and initially nonstationary), which was further iterated over twenty-five combinations of the autoregressive parameters θ_1 and θ_2, for a total of 200 unique subexperiments. Each subexperiment consisted of twenty-five observations, with each subexperiment being replicated 2500 times. The previous discussion on the design of the experiment, however, did not elaborate on several important points such as random number generation, parameter specifications, independent variable specifications, and other methodological fine points. In addition to these data generation concerns, there were several computational subtleties encountered in the computer implementation of the estimators that warrant discussion. Bearing these unanswered concerns

in mind, this chapter attempts to address these issues in a detailed manner.

Having fully discussed data generation methods, parameter values, computational subtleties, etc., the final section of this chapter details the results of the Monte Carlo experiment. Generally, emphasis of the results will be placed on presenting information most useful to the applied econometrician in choosing an estimator most suited to estimation of linear regression models with AR(2) errors.

Appendix A contains the FORTRAN 77 program used in the research. The program was executed on a VAX 8550 with a VMS operating system at Montana State University. Numerical precision necessary to successfully execute the program was obtained by compiling the program with the G_FLOATING option.

Data Generation

Throughout the execution of the experiment, the parameters β_1 and β_2 of the regression model (1.3) were fixed equal to one. Pseudorandom numbers q_t, uniformly distributed in the interval (0,1), were generated by the compound congruential random number generator number one, written by Bruce Collings (1986).

Essentially, this algorithm has a congruential generator G, known as the mixing generator, that produces a sequence of integers uniformly distributed over the set of integers (0,1,...,n). Letting n=15, the mixing generator G could randomly produce sixteen unique integer values, denoted as n_j, j=0,1,...,15. Component congruential

generators G_0, G_1, ..., G_n, all producing sequences of random values within the uniform (0,1) distribution, were then selected to produce the next value in the series based on the values of n_j generated by G. Intermixing the values output from the sixteen component generators by the random series of integers generated from the mixing generator, the resulting compound generator passes an extensive series of tests for randomness (Collings, 1986).

Pairs of pseudorandom uniform (0,1) numbers, q_t and q_{t+1}, were then transformed into stochastically independent N(0,1) deviates by the Box-Muller transformation (Hogg and Craig, 1978)

$$v_t = (-2\log(q_t))^{1/2}\cos(2\pi q_{t+1})$$

and

$$v_{t+1} = (-2\log(q_t))^{1/2}\sin(2\pi q_{t+1}). \qquad (3.1)$$

Comparison of the first four empirical moments, i.e. the mean, variance, skewness, and kurtosis, from a sample of size 500,000 generated by the above method, with the true moments of an N(0,1) distribution resulted in very adequate results. Table 1 presents the results of the comparison between the empirical and theoretical values of the mean, variance, skewness and kurtosis.

Let $w_t=v_t\sigma_w$, where $\sigma_w^2=0.01$, and v_t, t=1,...,T are values generated by the pseudo-normal (0,1) random number generator described above. Thus, the w_t, t=1,..,T, are independent identically distributed random values with mean equal to zero and variance equal to 0.01. The sample size for all variations of the experiment was set

to T=25. Once generated, the vector **w** remained fixed throughout the duration of the experiment.

Table 1. Empirical Moments of a Sample of Size 500,000, Generated by a Pseudo-Normal (0,1) Random Number Generator and Their, Respective, Theoretical Values.

Moment	Empirical Value	Theoretical Value
Mean	-0.00061	0.0
Variance	0.99844	1.0
Skewness	-0.00048	0.0
Kurtosis	2.99082	3.0

Given the vector **w** of pseudo-normal $(0,\sigma_w^2)$ random numbers, subsequent generation of the four types of independent variables was accomplished as follows. For the white noise independent variables, the vector **X** was generated by setting x_t equal to w_t, t=1,..,25, where x_t is the t^{th} element of **X**. This implies that $x_t \sim N(0,\sigma_w^2)$, t=1,..,25.

The independent variable following a dampened trend was generated by the relationships

$$x_1 = \varphi + w_1/(1-\lambda^2)^{1/2}, \qquad (3.2)$$

and

$$x_t = \lambda x_{t-1} + w_t\sigma_w, \quad t=2,\ldots,25, \qquad (3.3)$$

where λ=0.9 and φ=25. Specifying the first term x_1 in this manner is important in that it facilitates a constant variance across all observations. The assumption of a dampened trend structure implies that $x_t \sim N(\lambda^t\varphi,\ \sigma_w^2/(1-\lambda^2))$, t=1,..,25. It must be noted that the series has a mean that is dependent on time, hence, not strictly

stationary (Box and Jenkins, 1976). However, given that $|\lambda|<1$, it is obvious that the series converges to a mean of zero.

The third structure on the independent variable assumes the presence of a growth trend as described by Beach and MacKinnon in their 1978 Monte Carlo study of first-order autocorrelation. Specifically, this assumption was realized by the relationship

$$x_t = \exp(0.04t) + w_t, \quad t=1,\ldots,T. \tag{3.4}$$

Constructed in this manner, x_t is distributed N(exp(0.04t), .01), t=1,..,25. Once again it is noted that the variance is constant, while the mean is dependent on time; thus the series is not strictly stationary.

The fourth, and final structure assumes that the x_t are generated from a stationary second-order lagged independent variable process with complex roots. Imposing complex roots on a stationary process implies that the time path of the lag effect will be a dampened cycle. Somewhat arbitrarily, the process is assumed to have a period of ten years and a modulus (absolute value) of 0.9. The phrase "somewhat arbitrarily" must be emphasized in that it does seem that many economic relationships display behavior approximately consistent with these assumed values, for example, cattle production, small grain production, etc.

Reversing the process of solving the complimentary equation

$$x_t - \lambda_1 x_{t-1} - \lambda_2 x_{t-2} = 0 \tag{3.5}$$

implies the solution for the autoregressive parameters, λ_1 and λ_2, from the given period and modulus (Chaing, 1984). Specifically, the

values of the autoregressive parameters λ_1 and λ_2 are 1.45622 and -0.81, respectively. Since the transformation matrix **P** in relation (2.17) is derived from a stationary AR(2) process, the relation can be stated

$$\mathbf{X} = \mathbf{P}_x^{-1}\mathbf{w}, \tag{3.6}$$

where

$\mathbf{P}_x$ = **P** in expression (2.17) with σ_u, σ_ϵ, θ_1, and θ_2 replaced by σ_x, σ_w, λ_1, and λ_2, respectively,

and

w = vector of size T=25, whose elements w_t are pseudorandom $N(0,\sigma_w^2)$ deviates.

The vector **X** is then the desired independent variable generated from a stationary AR(2) process whose period is ten years and has a modulus of 0.9. Elements of the **X** are

$$x_1 = w_t\left\{(1-\lambda_2)/[(1+\lambda_2)((1-\lambda_2)^2-\lambda_1^2)]\right\}^{1/2},$$

$$x_2 = w_t/(1-\lambda_2^2)^{1/2},$$

and

$$x_t = \lambda_1 x_{t-1} + \lambda_2 x_{t-2} + w_t, \quad t=3,\ldots,25. \tag{3.7}$$

Design of the independent variable in this manner implies that $x_t \sim N(0, (1-\lambda_2)\sigma_w^2/[(1+\lambda_2)(1-\lambda_2)^2-\lambda_1^2)])$.

The first three types of independent variables are consistent with earlier Monte Carlo studies with AR(1) processes (Siebrasse, 1987). Inclusion of the final independent variable following an AR(2) process was added to test for an influence on estimator performance when the error process was AR(2).

Once generated, the independent variable values were fixed for all permutations of the error process and the 2500 replications of each permutation. Statistically, fixing the vector **X** over all replications of the experiment implies that the independent variable is nonstochastic. This is critical in the sense that the study is concerned only with estimation of the simple linear regression model when only one of the assumptions of the classical linear regression model (Kmenta, 1986) is violated, namely nonautocorrelated errors.

As stated earlier, the AR(2) error process

$$u_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + \epsilon_t, \quad t=1,..,25, \tag{3.8}$$

was generated under each of two assumptions, stationarity and nonstationarity. Setting $\epsilon_t = \sigma_\epsilon v_t$, t=1,..,T, where σ_ϵ^2=0.01 and $v_t \sim N(0,1)$, t=1,..,25, are the pseudo-random deviates described by relation (3.8), it is obvious that ϵ is a vector of spherical errors whose elements are distributed $N(0,\sigma_\epsilon^2)$. Assuming (3.8) to be a stationary AR(2) error process, the vector **U** of generated errors is then given by

$$\mathbf{U} = \mathbf{P}^{-1}\epsilon, \tag{3.9}$$

where **P** is the stationary transformation matrix (2.17), with the autoregressive parameters θ_1 and θ_2 prespecified as described below. **U** is then a stationary error vector whose elements are

$$u_1 = \epsilon_1 \left\{ (1-\theta_2) / \left((1+\theta_2)((1-\theta_2)^2 - \theta_1^2) \right) \right\}^{1/2},$$

$$u_2 = \epsilon_2 / (1-\theta_2^2)^{1/2},$$

and

$$u_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + \epsilon_t, \quad t=3,\ldots,T,$$

with

$$u_t \sim N\left(0, (1-\theta_2)\sigma_\epsilon^2 / \{(1+\theta_2)[(1-\theta_2)^2-\theta_1^2]\}\right). \qquad (3.10)$$

As discussed in Chapter 1, the initially nonstationary assumption is equivalent to setting the first two presample residuals equal to zero. Accepting the argument for the plausibility of this assumption given in Chapter 1, it was shown in Chapter 2 that the use of the resulting transformation matrix $\mathbf{P}_{inn}$, given by expression (2.57), resulted in variances of the first two transformed errors that are time dependent when the true error process was in fact stationary. If the true errors were generated from a stationary AR(2) error process, one would expect the initially nonstationary transformation to result in estimates that are not as "good" in a statistical sense as those found by using the true transformation matrix given by expression (2.17). On the other hand, if the true error process did in fact start at a fixed point in time, then one would be led to believe that the initially nonstationary estimator would perform the "best" in a statistical sense because $\mathbf{P}_{inn}$ would be the true transformation matrix derived from constraining $\mathbf{P}'_{inn}\mathbf{P}_{inn}$ to be proportional to the variance-covariance matrix of the initially nonstationary AR(2) error process. Specifically, the error vector $\mathbf{U}$ in this case was generated by the relation

$$\mathbf{U} = \mathbf{P}_{inn}^{-1}\boldsymbol{\epsilon}, \qquad (3.11)$$

where

$\mathbf{P}_{inn}$ = initially nonstationary transformation matrix (2.57), and $\boldsymbol{\epsilon}$ is a vector of pseudo-normal $(0,\sigma_\epsilon^2)$ random deviates.

The final data generation point to be addressed is concerned with the selection of the twenty-five combinations of the autoregressive parameters θ_1 and θ_2. Because the research is concerned with examining the properties of estimators over the range of stable parameter values, the logical choice is to select combinations of θ_1 and θ_2 that optimally cover the stability triangle. Subdividing the stability triangle defined by $\theta_1+\theta_2<1$, $\theta_2-\theta_1<1$ and $|\theta_2|<1$ into twenty-five subtriangles, subsequently selecting the center of each subtriangle results in the desired optimal parameter selection. Graphically, this subdivision scheme is represented in Figure 2.

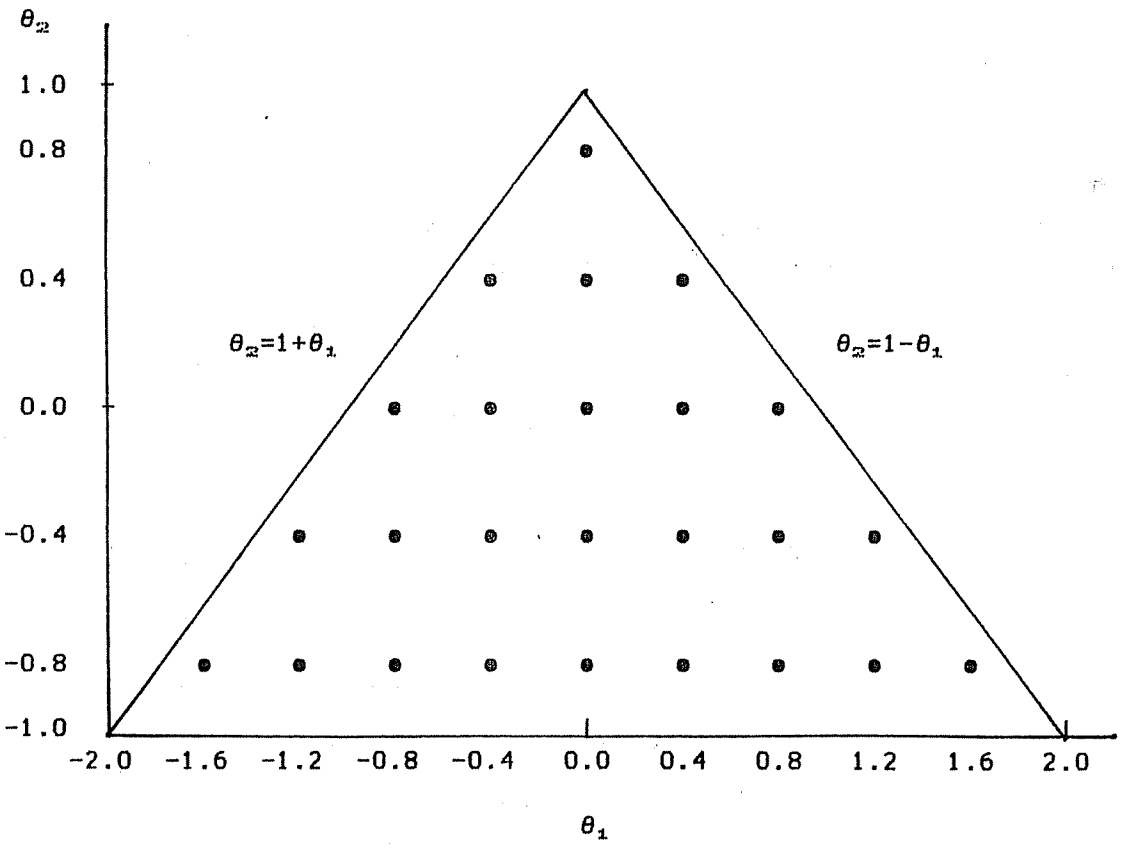

Figure 2. Stationary Region Subdivision Scheme.

Numerical Subtleties

The purpose of this section is to explicitly describe fine points of the program execution such as convergence criteria of the iterative estimators, Marquardt Algorithm implementation, etc.

Let $\varphi^j = [\hat{\beta}_1^j \ \hat{\beta}_2^j \ \hat{\theta}_1^j \ \hat{\theta}_2^j]'$ be the vector whose n^{th} element φ_n^j contains the value of the n^{th} parameter at the j^{th} iteration. Convergence for the iterative estimators IPW, ICO, INS and FML was achieved when the condition

$$\max_{n=1,\ldots,4}\left\{|\varphi_n^j - \varphi_n^{j-1}|/(\max(|\varphi_n^{j-1}|,1))\right\} < 10^{-6}$$

was satisfied. If the estimation did not converge in 100 iterations for a particular estimator, the estimation process was terminated, with the values of the parameters at the 100^{th} iteration being accepted for that replication of that estimator.

OLS estimates of β_1 and β_2 were used as starting values in the TSPW, IPW, FML, TSCO, ICO, and INS estimators. With the exception of the FML estimator, all of the estimators used starting values of the autoregressive parameter estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ set equal to zero. The FML estimator used the TSPW estimates of θ_1 and θ_2 as starting values. Using the TSPW estimates as starting values in the FML procedure effectively reduced the number of iterations needed for convergence, as compared to using starting values of zero. The results of a small test suggested that the use of different starting values did not change the estimates of θ_1 and θ_2 for the FML procedure. From the pragmatic perspective of the applied econometrician, the use of good

starting values that are easily acquired and do not appear to result in different final estimates is justifiable.

Without question, the computer implementation of the FML estimation procedure resulted in more complexities than any other procedure. One may argue that once the computer code is written for an estimator, this cost is sunk and the relevant cost is the marginal cost of having the computer calculate the estimates. From this marginal cost perspective, the FML procedure may be only slightly more costly in terms of additional computation than the other estimators, given the memory capacity and relatively low cost of program execution on most modern computers.

Basically, two factors contributed to the increased difficulty encountered in the FML procedure. The first factor resulted from the fact that the first-order derivatives of the likelihood function, conditional on $\hat{\beta}_1$ and $\hat{\beta}_2$, with respect to θ_1 and θ_2 were mixed third and fourth order polynomials, therefore not having an analytical solution. Solution of the two equations for $\hat{\theta}_1$ and $\hat{\theta}_2$ was found numerically through the use of the Marquardt Method (Bard, 1974). This method forced the Hessian matrix of second-order derivatives with respect to θ_1 and θ_2 of the likelihood function, conditional on $\hat{\beta}_1$ and $\hat{\beta}_2$, to be positive definite. The fact that the likelihood function, conditional on $\hat{\beta}_1$ and $\hat{\beta}_2$, was very flat when the point defined by $\hat{\theta}_1$ and $\hat{\theta}_2$ was not close to the boundary of the stability triangle (the conditional likelihood function logarithmically approaching negative infinity as a boundary was approached) made this modification to the

Hessian matrix very important in computing an acceptable step-size and direction. Specifically, the modified Hessian matrix was computed by adding increasing multiples of the absolute value of the original diagonal elements, with the original diagonal elements less than one being replaced by one, to the original Hessian matrix until the step vector was in the same quarter space as the gradient vector. Initially, a factor of 10^{-8} was used as a multiplicative value on the absolute value of the original Hessian diagonal elements. This value was increased by a factor of ten until the cosine of the angle between the step vector and the gradient vector was greater than $0.5^{1/2}$. This guaranteed that the step vector and the gradient vector were in the same quarter space, thus an acceptable direction leading to improvement of the likelihood function.

Given an acceptable direction, the second factor contributing to the computational problems associated with the FML estimator was related to the computation of an acceptable step-size, i.e. a step-size that leads to an optimal increase in the likelihood function. Inclusion of the terms associated with the absolute value of the Jacobian guaranteed the existence of a stationary point inside the stability triangle because as any boundary is approached the value of the likelihood function approaches minus infinity. However, observing the likelihood function outside of the stability triangle, it becomes apparent that the maximum within the stability triangle may not be a global maximum of the likelihood function. Therefore, although the step-direction may be acceptable, a step-size that is too large in

that direction may cross the asymptote at the boundary of the stability triangle and find a value of the likelihood function that is larger than the value inside the stability triangle. The Marquardt Algorithm will then search for a set of parameter estimates that maximize the likelihood function, but, if found, will violate the stationarity assumptions explicitly contained in the FML method. Prevention of boundary violations was achieved by incorporating the following subalgorithm into the Marquardt subroutine:

1) If the new parameter vector $\hat{\boldsymbol{\theta}}^{j+1} = [\hat{\theta}_1^{j+1} \ \hat{\theta}_2^{j+1}]'$ at the $(j+1)^{th}$ iteration violates one of the boundaries given by the conditions $|\theta_2| < 1$, $\theta_2 + \theta_1 < 1$, and $\theta_2 - \theta_1 < 1$ go to 2; else go to 3.
2) Solve the system $\hat{\boldsymbol{\theta}} = w\hat{\boldsymbol{\theta}}^{j+1} + (1-w)\hat{\boldsymbol{\theta}}^{j}$, where $\hat{\boldsymbol{\theta}}$ is the point at the intersection of the violated boundary and the vector between the coordinates defined by the new estimates $\hat{\boldsymbol{\theta}}^{j+1}$ and the old estimates $\hat{\boldsymbol{\theta}}^{j}$ (Figure 3), for w. Go to 3.
3) Set the step-size equal to one if a boundary was not violated; else equal to 0.99w if a boundary was violated. Go to 4.
4) Reduce the step-size until the maximum of the conditional likelihood function is found given that particular direction vector. Go to 5.
5) Check for convergence. If convergence is attained, then return to the FML procedure with the estimates of the autoregressive parameters; else continue the Marquardt procedure.

Computer implementation of the other estimators was relatively straightforward once a library of matrix routines had been developed.

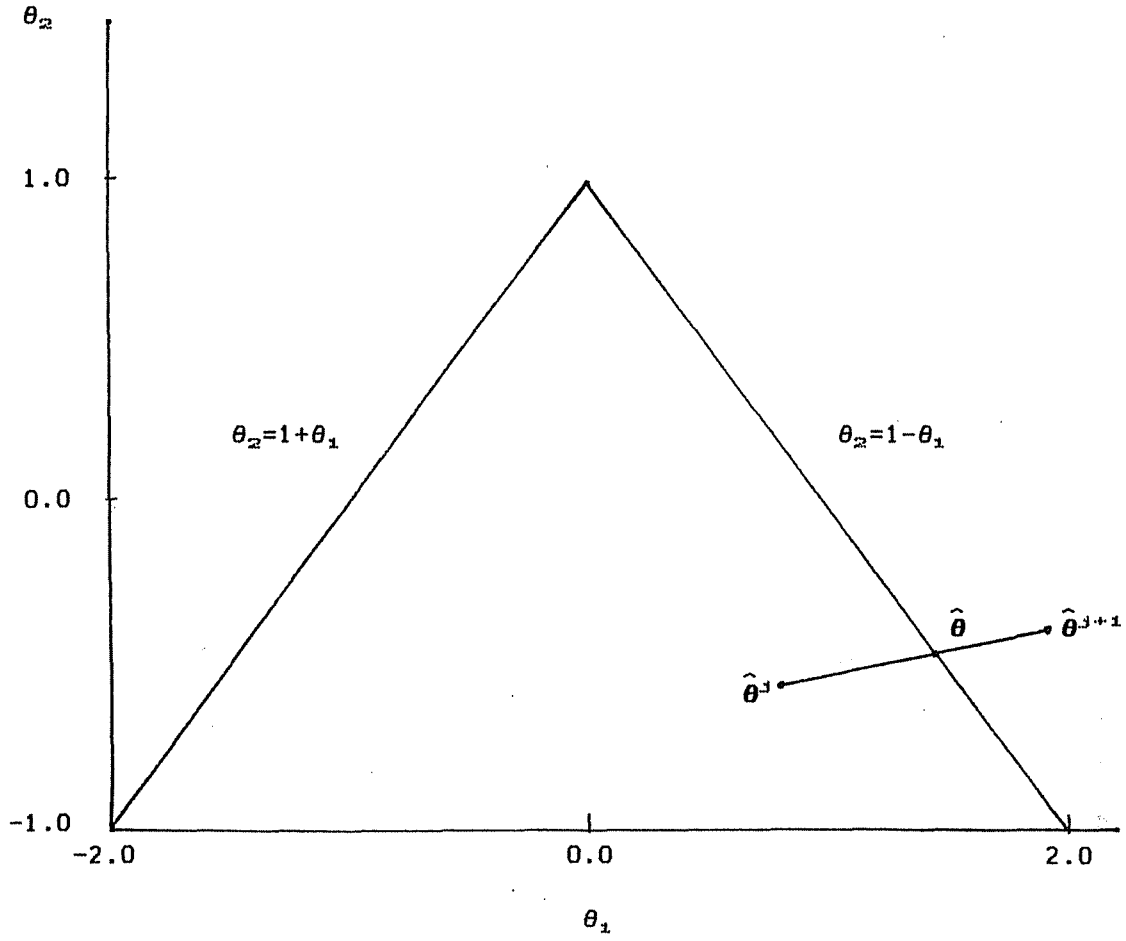

Figure 3. Stationary Region Boundary Violation.

Results

As is the case with most Monte Carlo studies, the volume of raw information generated is considerable. Due to this almost overwhelming body of information, it is very easy to generate such a ponderous volume of tables that the research loses track of "the forest for the trees." Once again noting that this research is primarily concerned with presenting information to the applied

econometrician, an effort will be made to report summary statistics useful in choosing the most appropriate estimator in the context of linear regression models with AR(2) errors. For the reader interested in a more detailed survey of the results, a complete listing can be obtained by either contacting Dr. Jeffrey T. LaFrance, Associate Professor of Agricultural Economics, Montana State University, or the author.

Econometric theory suggests that all seven estimators should produce estimates of the regression parameters β_1 and β_2 that are unbiased in the presence of autocorrelation (Judge et al., 1985). The results of the study definitely show this to be true for all seven estimators. Generally, the mean estimates were within $\pm 5 \times 10^{-3}$ of the prescribed value of one for both parameters, regardless of the subexperimental design.

Unbiasedness of the estimates of the autoregressive parameters θ_1 and θ_2 was a different matter. Delineating the stability triangle into two sections, where the boundary formed by setting θ_2 equal to zero divides the sections, a pattern in the bias of the estimates of θ_1 and θ_2 is made clear. For values of θ_1 and θ_2, such that $\theta_2 \leq 0$, the estimators TSCO, ICO, FML, TSPW, IPW, and INS, hereafter known as the approximate generalized least squares (AGLS) estimators, all had small biases (approximately ± 0.15) that were not consistently in any one direction. Estimates of θ_1 and θ_2, however, obtained from the Yule-Walker equations and OLS residuals, in the same portion of the stability triangle were substatially biased. The bias of the

Yule-Walker estimates of θ_1 and θ_2 were as large as ±0.6 for values near the bottom of the stability triangle, and would decrease as θ_2 increased. The direction of the bias for the Yule-Walker estimates of θ_1 and θ_2 was consistently toward the center of the triangle, i.e. $\theta_1=0$ and $\theta_2=0$.

Analyzing the upper portion of the stability triangle, i.e. combinations of θ_1 and θ_2 such that θ_2 is greater than zero, it was observed that all of the estimators tended to produce mean values of $\hat{\theta}_1$ and $\hat{\theta}_2$ that were biased towards $\hat{\theta}_1=0$ and $\hat{\theta}_2=0$. This bias tended to increase as θ_1 and θ_2 moved away from the center of the triangle, and was more pronounced, in general, for the Yule-Walker estimator than for the AGLS estimators.

In a similar manner, this pattern of bias was characteristic of the means of $\hat{\theta}_1$ and $\hat{\theta}_2$, regardless of the type of experimental design. In other words, estimator performance in the estimation of $\hat{\theta}_1$ and $\hat{\theta}_2$ was very consistent across subexperimental designs that permuted the four types of independent variables over the two types of error structures.

One would expect these results concerning the bias of $\hat{\theta}_1$ and $\hat{\theta}_2$ to carry over into the RMSEs of $\hat{\theta}_1$ and $\hat{\theta}_2$. The patterns did in fact carry over, and in an interesting manner. In general, it was observed that at the bottom of the stability triangle the AGLS estimators had RMSEs of $\hat{\theta}_1$ and $\hat{\theta}_2$ that were smaller than those of the Yule-Walker estimates. Moving up the triangle, i.e. increasing θ_2, the RMSE values of the AGLS estimators tended to increase, while the RMSE

values of the Yule-Walker estimates tended to decrease.

Because most applied econometricians are generally concerned with the estimates of the regression parameters β_1 and β_2, not with the dynamics of the error process, the bias of $\hat{\theta}_1$ and $\hat{\theta}_2$ is most often of little concern as long as $\hat{\beta}_1$ and $\hat{\beta}_2$ have good statistical properties. Since $\hat{\theta}_1$ and $\hat{\theta}_2$ are used in the transformation matrices of the various estimators, however, it is obviously important that the estimates of θ_1 and θ_2 have statistical qualities that are consistent with efficiently transforming the error process from one that is nonspherical to one that is spherical. Thus, it would be interesting to know if the Yule-Walker estimates of θ_1 and θ_2, used in place of the appropriate AGLS estimates, would greatly influence the efficiency of the respective AGLS estimators. Unfortunately, this research does not address this issue, which may be important in the sense that some regression packages use the Yule-Walker estimates of θ_1 and θ_2 instead of the appropriate AGLS estimator. Relevance of this concern was demonstrated by Rao and Griliches (1969) in the case of estimators that used different estimation procedures to compute the autoregressive parameter for the model with AR(1) errors. Rao and Griliches noted substantial gains in the efficiency of the regression parameter estimates through the proper choice of an autoregressive parameter estimator.

Tables 2 through 9 in Appendix B tabulate the estimators that had the lowest RMSE values for each given set of assumptions about the error process, type of independent variable, and error process

parameter values. For example, Table 2 contains the estimators that had the lowest RMSE values for the subexperiment with stationary errors and white noise on the independent variables. Integer values 1 through 7 are assigned to each estimator as follows: 1=OLS; 2=TSPW; 3=IPW; 4=FML; 5=TSCO; 6=ICO; and 7=INS. Each table is broken into four triangles, with the relevant RMSE statistic at the top of the respective triangle. The location of each integer code within a triangle is associated with the center of each of the twenty-five subtriangles described in Figure 2. Thus, for example, the 1 at the top of the upper left-hand triangle indicates that OLS had the lowest RMSE for $\hat{\beta}_1$, given θ_1=0 and θ_2=0.8. The 4 at the top of the upper right-hand triangle indicates that FML had the lowest RMSE for the standard error of $\hat{\beta}_1$, given θ_1=0 and θ_2=0.8, and so on.

Taking various composite views of Tables 2 through 9 three general characteristics are identified. The first characteristic common to all subexperiments (each table being a subexperiment) is the fact that OLS performed the best when θ_1 and θ_2 are zero, as theory would predict. The superior performance of OLS to the other estimators extended consistently to combinations of θ_1 and θ_2 that lie immediately above, to the right, and to the lower right of θ_1=0 and θ_2=0. Interestingly, the OLS estimates did not perform as well to the left of θ_1=0 and θ_2=0, although the Euclidean distance is exactly the same as combinations of θ_1 and θ_2 that lie to the right. Thus, it appears that the point made by Spitzer (1979), asserting the superior efficiency of OLS for values of $|\theta_1|$<0.2 in the case of linear

regression models with AR(1) errors, can be modified to extend to models with AR(2) errors. The patterns observed in the AR(2) case suggest that as the point defined by θ_1 and θ_2 moves farther away from the origin, the efficiency of OLS relative to the other estimators decreases. However, the change in efficiency may not be symmetric in all directions from the origin, i.e. the change appears to be less for movements to the northeast than for movements to the southwest.

Secondly, for the subexperiments with stationary errors (Tables 2, 4, 6, and 8) the FML, TSPW and IPW estimators recorded the lowest RMSE values for the most part, with FML showing up more often than TSPW and IPW, especially around the borders of the stability triangle. Only two divergences from this general pattern were noted.

The first divergence from this general pattern occurred in the subexperiment with white noise on the independent variable and the other with a dampened cyclical trend (Tables 2 and 8). For values of θ_1 and θ_2 lying in the lower right-hand corner of the stability triangle it was observed that the TSCO and ICO estimators dominate in the estimation of $\hat{\beta}_2$. This observation is very interesting when one considers that the two Cochrane-Orcutt estimators rarely displayed a dominant element, let alone a pattern in any of the other subexperimental designs, for either regression parameter.

Another interesting divergence of the subexperiments with stationary errors was the pattern of superior performance displayed by the INS estimator. For the INS estimator a diagonal band of superior performance starting at the left of $\theta_1=0$ and $\theta_2=0$, then running down

and to the right, appears, to a varying degree, throughout these subexperiments. It is very interesting that the estimator based on initially nonstationary errors would perform well on errors that are stationary in a pattern that would suggest more than a chance outcome. Unfortunately, a reason is not immediately obvious for this observation.

Analyzing the results of the subexperiments with nonstationary errors (Tables 3, 5, 7, and 9 in Appendix B) a final general characteristic was observed. The pervading pattern in this set of tables suggests that the stability triangle could be broken into three general regions of estimator performance (Figure 4). The first region is the area at the center of the triangle which is dominated by the OLS estimator, as discussed above. Defining the second region as the combination of θ_1 and θ_2 such that θ_2 is less than or equal to zero, and excluding the portion dominated by the OLS estimator, it is observed that the INS estimator is the most dominant in terms of efficiency. In the final area at the top of the stability triangle it is observed that the FML, TSPW and IPW estimators generally outperform the INS estimator. This observation is contrary to what one would expect **a priori**, given that the error process was in fact initially nonstationary.

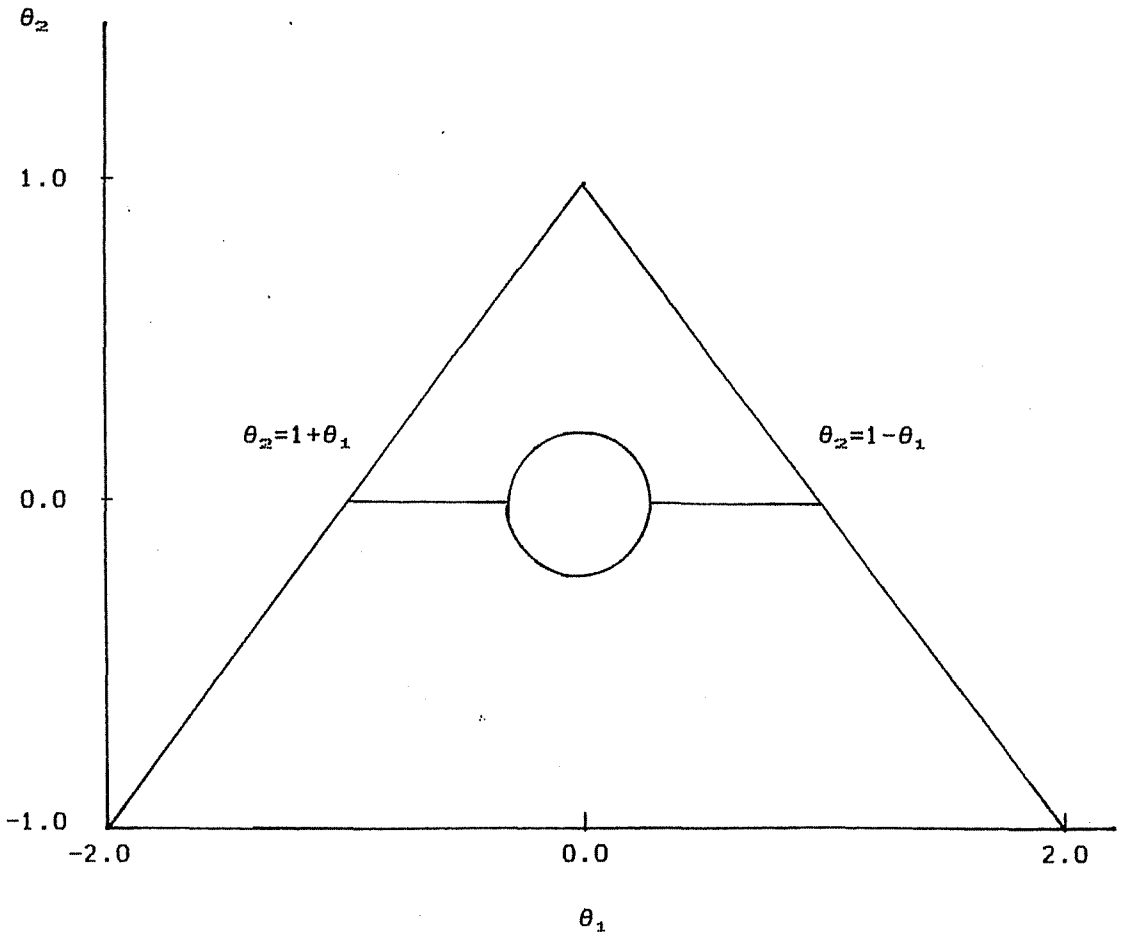

Figure 4. Regions of Superior Estimator Performance Within the Stability Triangle for the Subexperiments with Nonstationary Errors.

In absolute terms, the difference between the lowest RMSE value and the RMSE value for other estimators was not very large for many combinations of θ_1 and θ_2 within a given subexperiment. Because robustness of an estimator across various subexperimental designs is generally considered to be of more importance to the applied econometrician than choosing the estimator that performs the best

given a specific model design and set of autoregressive parameter values, it is important to look at the loss in efficiency associated with the general application of a particular estimator. In other words, if the cost associated with the use of one particular estimator were low relative to the benefit of being able to apply that particular estimator in a general manner, the particular estimator would be accepted as the optimal estimator in the context of most applied econometric analysis. Of course, it is obvious that the cost of lost efficiency may be higher for one application than for another; thus, a presentation of statistics that would allow for flexibility in estimator selection given the various model assumptions would be deemed appropriate. Flexibility of estimator selection was attained in this research through the generation of several tables stating relative efficiencies, iteration statistics, and relative bias of the standard errors of $\hat{\beta}_1$ and $\hat{\beta}_2$.

Tables 10 through 17, 18 through 25, 26 through 33, and 34 through 41 present the results of computing the relative efficiency of the INS, IPW, TSPW, and FML estimators, respectively, to the value of the estimator that performed the best at each combination of θ_1 and θ_2, for each RMSE statistic, for a given subexperiment. For example, the value of 1.27 located at the top of Table 10 was computed from the ratio RMSE $\hat{\beta}_{1,INS}$ / RMSE $\hat{\beta}_{1,OLS}$, where RMSE $\hat{\beta}_{1,OLS}$ is the value in the denominator because OLS had the lowest RMSE for $\hat{\beta}_1$ for that particular subexperimental design at θ_1=0.0 and θ_2=0.8 (see Table 2). Tabulation of the relative efficiencies for the two Cochrane-Orcutt procedures

was not performed as a result of the fact that, excluding the anomalies discussed earlier, these two procedures behaved poorly in terms of relative RMSE statistics.

Several patterns emerged from a comparison of relative efficiencies computed for one estimator with those of another estimator. In the case of the subexperiments with stationary errors, it was noted that the FML estimator uniformly performed the best across all values of the autoregressive parameters. Interestingly enough, the INS estimator proved to be the runner-up over the TSPW and IPW estimators, although the errors were stationary. Even in the growth trend and dampened cyclical subexperiments where TSPW and IPW were observed to perform better, the difference was not very large. The most significant difference between the performance of INS, relative to the two Prais-Winston estimators, can be observed by noting the incidence of very high relative efficiency ratios -- the largest was 438.58 for the RMSE $\hat{\sigma}_{\beta_1}$ of the IPW estimator -- for combinations of θ_1 and θ_2 lying close to the right boundary of the stability triangle. It appeared that this problem was worse for the IPW estimator than it was for the TSPW estimator. This problem associated with the Prais-Winston estimators also appeared to be of more significance in the estimation of β_1 than it was for β_2. As a rule, if one ignored the autoregressive parameter values close to the right-hand border of the stability triangle, one would find that the INS, TSPW, and IPW estimators performed essentially the same when

judged on their respective robustness across all subexperimental designs with stationary errors.

Examining the tables of relative efficiencies for the experiments with nonstationary errors, it was readily apparent that the INS estimator was uniformly the best estimator, thus bearing out what estimation theory would predict. However, upon studying the relative efficiencies associated with the FML estimator, it was very clear that the FML estimator demonstrated performance just slightly below that of the INS estimator. In most cases, the FML estimator came closer to matching the performance of the INS estimator in the subexperiments with initially nonstationary errors, than comparison of INS to FML when the errors were stationary.

Once again, the TSPW and IPW estimators did not perform quite as well due to the poor performance of these two estimators when θ_1 and θ_2 were close to the right-hand border of the stability triangle. Interestingly, the problem of very large relative efficiencies associated with these two estimators was mitigated by the initially nonstationary error structure.

Iteration statistics for the iterative estimators IPW, FML, ICO, and INS are found in Tables 42 through 49. Each table represents a particular subexperiment. For example, Table 42 tabulates the iteration statistics for the subexperiment with stationary errors and white noise on the independent variable. In general, one observes that each iterative estimator exhibits difficulty in converging as the values of θ_1 and θ_2 approached any of the combinations associated

with the right-hand side of the stability triangle. Dividing the subexperiments into those with stationary errors and those with initially nonstationary errors, one observes that the subexperiments with stationary errors posed more convergence difficulties for the iterative estimators at the borders than the subexperiments with initially nonstationary errors did. This observation was especially apparent for the IPW estimator.

The final set of statistics generated by this experiment presents the relative biases of the standard errors of β_1 (Table 50) and that of β_2 (Table 51), where

$$\text{relative bias of } \hat{\sigma}_{\beta_i} = (\hat{\sigma}_{\beta_i} - \sigma_{\beta_i}) \,/\, \sigma_{\beta_i},\ i=1,2.$$

Identification of some sort of ranking based on preferential estimator behavior was not immediately obvious for this set of statistics. However, one quite general observation should be noted; that is the fact that all of the estimators generally bias the standard error down. Thus we would expect our t-statistics to be inflated.

CHAPTER 4

CONCLUSIONS

Introduction

Applied analysis of time series econometric models is quite common in many fields of economics. Often, the data sets used to estimate the parameters of the model are small, with autoregressive error structures implicit to either the data set, the model, or both. Due to the existence of these nonspherical errors, it is important to implement an estimator that effectively transforms the model into one with a spherical error structure. Specifically, this research was concerned with the efficient estimation of linear regression models with AR(2) errors.

The discussion that follows is separated into three sections. The first section briefly reiterates the objectives and methodology employed by the study. The second section presents the conclusions of the study. The final section offers suggestions for further research.

Summary

Six conventional estimators (OLS, TSPW, IPW, FML, TSCO, and ICO) and one nonconventional estimator (INS) were suggested as potential estimators of the linear regression model with AR(2) errors. The primary objective of this research was to analyze the small-sample

properties of each estimator in the context of linear regression models with AR(2) errors. Because analytically tractable small-sample properties of these estimators are very difficult, if not impossible, to derive, this research opted to use Monte Carlo methods to arrive at these small-sample properties.

Several statistics relevant to the selection of an appropriate estimator based on efficiency, computational complexities, and inference validity of each of the seven estimators were offered. The statistics included: (1) the mean of $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\theta}_1$, and $\hat{\theta}_2$; (2) the RMSE of $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\theta}_1$, and $\hat{\theta}_2$; (3) the RMSE and relative bias of the standard errors $\hat{\sigma}_{\beta_1}$ and $\hat{\sigma}_{\beta_2}$; and (4) iteration statistics for the iterative estimators IPW, FML, ICO, and INS.

The study design consisted of four structures on the independent variable, iterated over two types of error structures, with each error structure being iterated over twenty-five combinations of θ_1 and θ_2 evenly distributed within the AR(2) stability triangle. The four independent variable structures included: (1) white noise; (2) dampened trend; (3) growth trend; and (4) dampened cyclical trend. The error structures were generated first under the assumption that the process was stationary, and second under the assumption that the process was initially nonstationary. Each of the 200 permutations consisted of a sample size of T=25, with each permutation being replicated 2500 times.

Conclusions and Recommendations

From an overall perspective of choosing an estimator that uniformly demonstrates robustness across the various independent variable types, error structure assumptions, and autoregressive parameter values, a general ranking of the estimators was apparent. The ranking of these estimators is: (1) FML; (2) INS; (3) Prais-Winsten estimators; and (4) Cochrane-Orcutt estimators. OLS, however, was the superior estimator when the distance of the point defined by θ_1 and θ_2 to the center of the stability triangle was not very large, approximately $(\theta_1^2 + \theta_2^2)^{1/2} < 0.2$. Thus, in most applied econometric estimation situations where the econometrician is interested in uniformly robust estimation, a suggested estimation procedure would be to use OLS if $(\theta_1^2 + \theta_2^2)^{1/2} < 0.2$; otherwise use FML. For AR(2) error structures, FML imposes a slightly higher cost in terms of programming difficulty and additional computation, but given that a program exists and the cost of modern computing time is minimal, these costs are negligible.

Interestingly, the performance of the INS estimator was not substantially below that of the FML estimator when viewed from the perspective of overall performance across the various subexperimental designs. In the subexperiments where the errors were initially nonstationary, the INS estimator uniformly out-performed all other estimators except for values of the autoregressive parameter θ_2, such that $\theta_2 \geq 0.0$, i.e. the upper portion of the stability triangle. Furthermore, in contrast to the FML estimator, the INS estimator is

very easy to implement, and very easily extended to higher order autoregressive processes.

One very interesting result of this research was the performance of the two Prais-Winston estimators (TSPW and IPW). Both estimators performed very close to that noted for the FML and INS estimators with one very costly exception. The cost, in terms of lost efficiency and invalid inference testing, associated with these two estimators was incurred when the values of θ_1 and θ_2 were close to the boundary of the stability triangle defined by $\theta_2=1-\theta_1$. If the econometrician was assured of not having values close to this boundary, then the Prais-Winston estimators would result in essentially the same small-sample properties as the FML and INS estimators. In addition, it was also obvious that iteration of the Prais-Winston estimator amplified the undesirable performance for values of θ_1 and θ_2 close to the boundary of the stability triangle defined by $\theta_2=1-\theta_1$.

A final conclusion resulted from the general tendency of all six estimators that transform the data to bias the standard errors of $\hat{\beta}_1$ and $\hat{\beta}_2$ downward. Given this tendency, the t-statistics would be expected to be slightly over-inflated. Therefore, a pragmatic strategy that could be employed by the econometrician would be to use smaller critical values when testing the significance of the regression parameters.

Further Research

A variety of potential research projects are suggested by the results of this study. Essentially, the potential projects can be categorized into three main topics.

It was apparent that the bias associated with the estimation of the autoregressive parameters θ_1 and θ_2 could be large when estimated by the Yule-Walker equations, as compared to the bias associated with the other estimators of these parameters. Some statistical packages, for example SAS, exist that use the Yule-Walker estimates of θ_1 and θ_2 in their respective transformation procedures. Bearing this in mind, it would be interesting to study the effects of using the Yule-Walker estimates of θ_1 and θ_2 in place of the respective least squares or full maximum likelihood estimates for the various estimators included in this study.

A second prospective project results from the anomalies observed in analyzing the performance of the estimators. Among the anomalies observed were: (1) the poor performance of the Prais-Winston estimators in estimating the regression parameters when the values of θ_1 and θ_2 were close to the boundary of the stability triangle defined by $\theta_2=1-\theta_1$; (2) the pattern of strong performance exhibited by the Cochrane-Orcutt estimators in the estimation of β_2 for the white noise and dampened cyclical subexperiments with stationary errors for values of θ_1 and θ_2 in the lower-right portion of the stability triangle; (3) the tendency for the INS estimator to fail to perform, by a slight margin, as well as the "BEST" estimator in the subexperiments with

initially nonstationary errors for values of the autoregressive parameters such that $\theta_2 \geq 0.0$; and (4) the superior performance of the INS estimator in the subexperiments with stationary errors for values of θ_1 and θ_2 in the region of the stability triangle where $\theta_1<0$ and $\theta_2<0$. Explanation of these observations may lead to insights into the estimation of time series econometric models.

Contingent upon evidence for the existence of higher order autocorrelated error processes in economic data, it would be interesting to look at the performance of the estimators in this study when estimating linear regression models with higher order autoregressive error processes. If there was evidence for higher order autoregressive error processes, the easy extension of the INS estimator may prove to be of considerable value. Conversely, the analytical and numerical problems associated with the FML estimator may prove to be prohibitively costly when compared to potential gains in efficiency over the INS estimator.

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APPENDICES

APPENDIX A:

FORTRAN PROGRAM

Figure 5. FORTRAN Program

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C                              MONTE
C                          BY WALTER ZIDACK
C                       LAST REVISION 2/25/88
C
C  This is a Monte Carlo simulation routine for estimating the
C  small sample properties of common estimators in the context of
C  second order autoregressive errors.
C  Seven estimators are tested.
C  1) Ordinary Least Squares (subroutine ABROLS)
C  2) Beach-McKinnon Full Maximum likelihood (subroutines FML & MARQ)
C  3) Two-stage Praise-Winston (subroutine PW)
C  4)  "    "    Cochrane-Orcutt (subroutine CO)
C  5) Iterative Praise-Winston (subroutine PW)
C  6)     "     Cochrane-Orcutt (subroutine CO)
C  7)     "     Initially Nonstationary (subroutine INS)
C  The sample x's generated will be of four types:
C  1) White noise
C  2) Dampened trend
C  3) Growth trend
C  4) Dampened cyclical
C  Stationary and initially nonstationary error processes will be
C  generated.  Rho1 and rho2 will be iterated over 25 combinations
C  that lie within the stability triangle.
C  The sample size and replication values are read in by the variables
C  NR and IT, respectively.
C
C  Variable Declarations
C
      INTEGER SIZE, IE, NR, IT, IX, ITR1, ITR2, L, M, I, J, J1, J2,
     &        MARK
      DOUBLE PRECISION PI
      PARAMETER (SIZE=25, PI = 3.14159265358979323846)
      DOUBLE PRECISION X(SIZE,2), Y(SIZE), W(SIZE), E(SIZE),
     &      XBAR, XVAR, ALPHA, BETA, RHO1, RHO2, EPS, WVAR, EVAR,
     &      U(SIZE), DATA, DM, DML, SEED, Q(SIZE,SIZE), QIN(SIZE,SIZE),
     &      ESTIM1(14), ESTIM2(14), EPSSQT4, RESULT(7,14), AOLS, BOLS,
     &      TOTNS, TOTPW, TOTCO, TOTML, PHI1, PHI2, H, X1(SIZE), VARA,
     &      VARB
      COMMON /SEEDBK/  DM(0:15), DML, SEED(0:16)
      COMMON /DATABK/  DATA(100)
      COMMON /PARAM/   EPS
      COMMON /COEFF/   ALPHA, BETA, RHO1, RHO2
C
C  Program Code
C
      OPEN (5, FILE = 'MONTE.CTL', STATUS = 'OLD')
      OPEN (6, FILE = 'MONTE.OUT', RECL=300, STATUS = 'UNKNOWN')
```

```
      THETA = 25.0
C
C  Get parameter values from file # 5
C
      READ (5, 10) ALPHA, BETA, EVAR, WVAR, EPS, NR, IT
  10  FORMAT (4F10.5,F10.8,2I5)
      WRITE (6, 20) ALPHA, BETA, EPS, EVAR, WVAR, NR, IT
  20  FORMAT (1X,'Original parameter values:',9X,'Alpha',12X,
     & F10.5,/36X,'Beta',13X,F10.5,/36X,'Epsilon',10X,F10.8,
     & /36X,'Variance of E',4X,F10.5,/36X,'Variance of W',4X,
     & F10.5,/36X,'Sample size',13X,I3,/36X,'Replications',11X,I4)
C*****
C*   Generate the sample of predetermined x's for the white noise,
C*   dampened trend, growth trend and dampened cyclical experimental
C*   designs.
C*****
      EVAR = SQRT(EVAR)
      WVAR = SQRT(WVAR)
      CALL GTSEED
      CALL NORMAL (NR, W)
      DO 1600 IX = 1,4
C***  White noise X
      IF (IX .EQ. 1) THEN
         DO 21 I = 1,NR
          X(I,1) = 1.0
          X(I,2) = W(I)
  21     CONTINUE
C***  Dampened trend X
      ELSE IF (IX .EQ. 2) THEN
         X(1,2) = THETA + W(1)*WVAR/(1.0-0.81)**0.5
         X(1,1) = 1.0
         DO 22 I = 2,NR
          X(I,1) = 1.0
          X(I,2) = 0.9*X(I-1,2) + W(I)*WVAR
  22     CONTINUE
C***  Growth trend on X
      ELSE IF (IX .EQ. 3) THEN
         XGR = 0.04
         X(1,1) = 1.0
         X(1,2) = EXP(XGR) + W(1)*WVAR
         DO 23 I = 2,NR
          XGR = 0.04*I
          X(I,1) = 1.0
          X(I,2) = EXP(XGR) + W(I)*WVAR
  23     CONTINUE
C***  Dampened cyclical X
      ELSE
         PHI2 = -(0.9D0**2)
         H = -(DSQRT(-PHI2 / (1.0D0 + (DTAN(2.0D0 * PI / 10.D0))**2)))
         PHI1 = -2.0D0 * H
```

```
          DO 24 I = 1,NR
           W(I) = W(I) * WVAR
   24     CONTINUE
          CALL QINIT (NR, Q, PHI1, PHI2, 1)
          CALL INVCH (Q, QIN, NR)
          CALL ATIMB (X1, QIN, W, NR, NR, 1)
          DO 25 I=1,NR
           X(I,1) = 1.0D0
           X(I,2) = X1(I)
   25     CONTINUE
        END IF
C******
C* Loop over stationary (IE=1) and nonstationary error processes (IE=2).
C******
      DO 1500 IE=1,2
       IF (IX .EQ. 1 .AND. IE .EQ. 1) WRITE(6,31)
   31  FORMAT ('1',1X,'Stationary errors, white noise on x')
       IF (IX .EQ. 1 .AND. IE .EQ. 2) WRITE(6,32)
   32  FORMAT ('1',1X,'Nonstationary errors, white noise on x')
       IF (IX .EQ. 2 .AND. IE .EQ. 1) WRITE(6,33)
   33  FORMAT ('1',1X,'Stationary errors, dampened trend on x')
       IF (IX .EQ. 2 .AND. IE .EQ. 2) WRITE(6,34)
   34  FORMAT ('1',1X,'Nonstationary errors, dampened trend on x')
       IF (IX .EQ. 3 .AND. IE .EQ. 1) WRITE(6,35)
   35  FORMAT ('1',1X,'Stationary errors, growth trend on x')
       IF (IX .EQ. 3 .AND. IE .EQ. 2) WRITE(6,36)
   36  FORMAT ('1',1X,'Nonstationary errors, growth trend on x')
       IF (IX .EQ. 4 .AND. IE .EQ. 1) WRITE(6,37)
   37  FORMAT ('1',1X,'Stationary errors, dampened cyclical x')
       IF (IX .EQ. 4 .AND. IE .EQ. 2) WRITE(6,38)
   38  FORMAT ('1',1X,'Nonstationary errors, dampened cyclical x')
C*****
C*    Initialize total average iterations for iterative estimators.
C*****
      TOTML = 0.0
      TOTNS = 0.0
      TOTPW = 0.0
      TOTCO = 0.0
C*****
C*    Iterate over the 25 combinations of RHO1 and RHO2.
C*****
      DO 1400 ITR2=1,5
       RHO2 = -0.8D0 + 0.4D0 * (ITR2 - 1.0D0)
       DO 1300 ITR1=1,2*(5-ITR2) + 1
        RHO1 = -1.6D0 + 0.4D0 * (ITR2 - 1.0D0) + 0.4D0 * (ITR1 - 1.0D0)
C******
C*    Initialize the array RESULT which will store the accumulated
C*    statistics for all seven estimators.
C*    RESULT(1,*) = OLS
C*      "   (2,*) = Two-step P-W
```

```
C*       "      .    = Iter. P-W
C*       "      .    = FML
C*       "      .    = Two-step C-O
C*       "      .    = Iter. C-O
C*       "   (7,*) = INS
C*     RESULT(*,1) = Alpha
C*       "   (*,2) = Beta
C*       "      .    = Rho1
C*       "      .    = Rho2
C*       "      .    = MSE alpha
C*       "      .    = MSE beta
C*       "      .    = MSE rho1
C*       "      .    = MSE rho2
C*       "      .    = Average # of iterations
C*       "      .    = Sum of variances for alpha
C*       "      .    = Sum of standard deviations for alpha
C*       "      .    = Sum of variances for beta
C*       "      .    = Sum of standard deviations for beta
C*       "   (*,14)= # of times max. iteration count is encountered
C******
      DO 30 L=1,7
       DO 40 M=1,14
        RESULT(L,M) = 0.0D0
  40   CONTINUE
  30  CONTINUE
C****
C  Run IT replications
C****
      DO 310 J = 1,IT
C****
C*   Generate the vector E of NR, N(0,1) elements
C****
      CALL NORMAL (NR, E)
C****
C*  Generate the error vector U given RHO1 and RHO2.  If IE=1 then
C*  stationary errors are generated, else, if IE=2 then nonstationary
C*  errors are generated.  Compute Y given ALPHA, BETA, X, and U.
C****
      CALL QINIT (NR, Q, RHO1, RHO2, IE)
      CALL GENXY (Q, QIN, E, X, Y, EVAR, ALPHA, BETA, U, NR)
C****
C*  Find the OLS estimates of the parameters
C****
      CALL ABROLS(X, Y, ESTIM1)
      DO 12 L=1,14
       RESULT(1,L) = RESULT(1,L) + ESTIM1(L)
  12  CONTINUE
C****
C*  Save the OLS estimates of ALPHA and BETA to be used as starting
C*  values for the other estimators.
```

```
C****
      AOLS = ESTIM1(1)
      BOLS = ESTIM1(2)
      ESTIM2(1) = AOLS
      ESTIM2(2) = BOLS
C****
C*  Find the two-step P-W and Iter. P-W estimates
C****
      CALL PW(X, Y, ESTIM1, ESTIM2)
      DO 13 I=1,14
       RESULT(2,I) = RESULT(2,I) + ESTIM1(I)
       RESULT(3,I) = RESULT(3,I) + ESTIM2(I)
   13 CONTINUE
      ESTIM1(1) = AOLS
      ESTIM1(2) = BOLS
C*****
C*   Note FML uses Two-step P-W estimates of RHO1 and RHO2, in addition
C*   to the OLS estimates of ALPHA and BETA, as starting values.
C*****
      CALL FML (X, Y, ESTIM1)
      DO 14 I=1,14
       RESULT(4,I) = RESULT(4,I) + ESTIM1(I)
   14 CONTINUE
      ESTIM2(1) = AOLS
      ESTIM2(2) = BOLS
C*****
C*   Find the Two-step C-O and Iter. C-O parameter estimates.
C*****
      CALL CO(X, Y, ESTIM1, ESTIM2)
      DO 15 I=1,14
       RESULT(5,I) = RESULT(5,I) + ESTIM1(I)
       RESULT(6,I) = RESULT(6,I) + ESTIM2(I)
   15 CONTINUE
      ESTIM1(1) = AOLS
      ESTIM1(2) = BOLS
C*****
C*   Find the INS parameter estimates.
C*****
      CALL INS(X, Y, ESTIM1)
      DO 16 I=1,14
       RESULT(7,I) = RESULT(7,I) + ESTIM1(I)
   16 CONTINUE
  310 CONTINUE
C*****
C*   End of replication loop
C*****
C*****
C*   Update the means
C*****
      DO 17 I=1,7
```

```
      DO 18 L=1,13
       RESULT(I,L) = RESULT(I,L) / IT
   18  CONTINUE
   17 CONTINUE
C*****
C*   Compute the RMSE and bias of the standard errors of alpha
C*   beta.
C*****
      DO 51 I=1,7
       VARA = RESULT(I,5) - (RESULT(I,1) - ALPHA)**2
       RESULT(I,10) = (RESULT(I,10) - 2.0 * DSQRT(VARA) * RESULT(I,11)
     &              + VARA)**0.5
       RESULT(I,11) = RESULT(I,11) - DSQRT(VARA)
       VARB = RESULT(I,6) - (RESULT(I,2) - BETA)**2
       RESULT(I,12) = (RESULT(I,12) - 2.0 * DSQRT(VARB) * RESULT(I,13)
     &              + VARB)**0.5
       RESULT(I,13) = RESULT(I,13) - DSQRT(VARB)
       DO 52 L=5,8
        RESULT(I,L) = DSQRT(RESULT(I,L))
  52   CONTINUE
  51  CONTINUE
C*****
C*   Accumulate the sums for total average iteration statistics.
C*****
      TOTPW = TOTPW + RESULT(3,9)
      TOTML = TOTML + RESULT(4,9)
      TOTCO = TOTCO + RESULT(6,9)
      TOTNS = TOTNS + RESULT(7,9)
C*****
C* Output the results
C*****
      WRITE (6,401) RHO1,RHO2
      WRITE (6,500)
      WRITE (6,600)
      DO 19 I=1,4
       GO TO (102, 202, 302, 402) I
  102   WRITE(6,100)
       GO TO 502
  202   WRITE(6,200)
       GO TO 502
  302   WRITE(6,300)
       GO TO 502
  402   WRITE(6,400)
  502  CONTINUE
C******
C*    End case statement
C******
       IF (I .LT. 3) THEN
        IF (I .EQ. 1) THEN
         MARK = 10
```

```
        ELSE
         MARK = 12
        END IF
         WRITE(6,1001)((RESULT(L,M),M=I,I+4,4),(RESULT(L,M),
     &          M=MARK,MARK+1),L=1,7)
       ELSE
        WRITE(6,1000)((RESULT(L,M),M=I,I+4,4),L=1,7)
       END IF
   19 CONTINUE
      WRITE (6,1310) RESULT(4,9)
      WRITE (6,1350) RESULT(6,9), RESULT(3,9), RESULT(7,9)
      WRITE(6,1375) INT(RESULT(6,14))
      WRITE(6,1380) INT(RESULT(3,14))
      WRITE(6,1385) INT(RESULT(7,14))
      WRITE(6,1390) INT(RESULT(4,14))
1300  CONTINUE
1400  CONTINUE
      TOTML = TOTML / 25.0
      TOTNS = TOTNS / 25.0
      TOTPW = TOTPW / 25.0
      TOTCO = TOTCO / 25.0
      WRITE(6,1550) TOTML, TOTNS, TOTPW, TOTCO
1500  CONTINUE
1600  CONTINUE
C*****
C*   Format statements
C*****
 401  FORMAT ( '1',1X,'For rho1 = ',F5.2,' and rho2 = ',F5.2)
 100  FORMAT (/,' ALPHA')
 200  FORMAT (/,' BETA')
 300  FORMAT (/,' RHO1')
 400  FORMAT (/,' RHO2')
 500  FORMAT (/38X,'root mean',9X,'root mean',
     &      /18X,'estimated value',3X,'squared error 1',3X,
     &      'squared error 2',3X,'bias of std. error')
 600  FORMAT (' ___________________________________________',
     &        '_____________________________')
 1000 FORMAT (' OLS               ',E13.7,5X,E13.7,
     &      /,' Prais-Winston     ',E13.7,5X,E13.7,
     &      /,' Iterative PW      ',E13.7,5X,E13.7,
     &      /,' Iterative ML      ',E13.7,5X,E13.7,
     &      /,' Cochrane-Orcutt   ',E13.7,5X,E13.7,
     &      /,' Iterative CO      ',E13.7,5X,E13.7,
     &      /,' Iterative NS      ',E13.7,5X,E13.7)
 1001 FORMAT (' OLS               ',E13.7,5X,E13.7,5X,E13.7,5X,E13.7,
     &      /,' Prais-Winston     ',E13.7,5X,E13.7,5X,E13.7,5X,E13.7,
     &      /,' Iterative PW      ',E13.7,5X,E13.7,5X,E13.7,5X,E13.7,
     &      /,' Iterative ML      ',E13.7,5X,E13.7,5X,E13.7,5X,E13.7,
     &      /,' Cochrane-Orcutt   ',E13.7,5X,E13.7,5X,E13.7,5X,E13.7,
     &      /,' Iterative CO      ',E13.7,5X,E13.7,5X,E13.7,5X,E13.7,
```

```
     &      /,' Iterative NS      ',E13.7,5X,E13.7,5X,E13.7,5X,E13.7)
 1310 FORMAT (/,' Average Iterations',/5X,'ML = ',F7.2)
 1350 FORMAT (5X,'CO = ',F7.2,/5X,'PW = ',F7.2,/5X,'NS = ',F7.2,/)
 1375 FORMAT (' CO HIT THE ITERATIVE LIMIT (100)',I3,' TIMES.')
 1380 FORMAT (' PW HIT THE ITERATIVE LIMIT (100)',I3,' TIMES.')
 1385 FORMAT (' NS HIT THE ITERATIVE LIMIT (100)',I3,' TIMES.')
 1390 FORMAT (' ML HIT THE ITERATIVE LIMIT (100)',I3,' TIMES.')
 1550 FORMAT (/,' Total average iterations',/5X,'ML = ',F7.2,
     & /5X,'NS = ',F7.2,/5X,'PW = ',F7.2,/5X,'CO = ',F7.2)
C*****
C*   Save new seeds and close files.
C*****
      CALL SVSEED
      CLOSE(5)
      CLOSE(6)
      STOP
      END
C********************** END MAIN ***************************************



C ***************************************************************************
      SUBROUTINE NORMAL (N, Z)
C***************************************************************************
C     This subroutine generates normal (0,1) pseudo-random numbers     *
C     by the method of Box and Muller.  N is the dimension of the      *
C     vector of i.i.d. N(0,1) observations, Z.  The uniform (0,1)      *
C     pseudo-random numbers are generated with the compound con-       *
C     gruential generator # 1 of Collings (1986).                      *
C     The following subroutines are used to generate the random #'s:   *
C       1) UNIF                                                        *
C       2) GTSEED.                                                     *
C     The BLOCK DATA statement contains the prime #'s used by the      *
C     generators.                                                      *
C***************************************************************************


       INTEGER     M, N, L, LT2P2, I, L2
       DOUBLE PRECISION   Z(100)
       DOUBLE PRECISION   DATA, DM, DML, SEED, PI
       COMMON /SEEDBK/    DM(0:15), DML, SEED(0:16)
       COMMON /DATABK/    DATA(100)
       PI = 3.14159265358979323846
       M = 1
       L = N/2
       LT2P2 = L*2 + 2
       CALL UNIF(LT2P2)
       IF(N .EQ. 1) GO TO 200
       DO 100 I = 1,L
```

```
      Z(M)   = (DSQRT(-2.0D0*DLOG(DATA(M))))*DCOS(2.0D0*PI*DATA(M+1))
      Z(M+1) = (DSQRT(-2.0D0*DLOG(DATA(M))))*DSIN(2.0D0*PI*DATA(M+1))
      M = M + 2
  100 CONTINUE
  200 L2 = 2*L
      IF (N .NE. L2)
     &    Z(N) = (DSQRT(-2.0D0*DLOG(DATA(M))))*DCOS(2.0D0*PI*DATA(M+1))
      RETURN
      END
C ***************** END NORMAL ****************************************


C ********************************************************************
      BLOCK DATA
      DOUBLE PRECISION  DATA, DM, DML, SEED
      COMMON /SEEDBK/   DM(0:15), DML, SEED(0:16)
      COMMON /DATABK/   DATA(100)
      DATA   DM/45915.0D0,32887.0D0,33314.0D0,36857.0D0,58487.0D0,
     &       40945.0D0,48691.0D0,31167.0D0,34791.0D0,61948.0D0,
     &   60839.0D0,43288.0D0,54757.0D0,52744.0D0,59704.0D0,
     &   44923.0D0/
      DATA   DML/62406.0D0/
      END
C ********************** END BLOCK DATA ***********************************


C ********************************************************************
      SUBROUTINE UNIF(M)
      DOUBLE PRECISION  DATA, DM, DML, SEED
      COMMON /SEEDBK/ DM(0:15), DML, SEED(0:16)
      COMMON /DATABK/ DATA(100)
      DO 1 J = 1,M
      SEED(16) = DMOD(SEED(16)*DML,2147483647.0D0)
      I = DMOD(SEED(16),16.0D0)
      SEED(I) = DMOD(SEED(I)*DM(I),2147483647.0D0)
      DATA(J) = SEED(I)/2147483647.0D0
    1 CONTINUE
      RETURN
      END
C ********************** END UNIF *************************************


C ********************************************************************
      SUBROUTINE GTSEED
      DOUBLE PRECISION  DM, DML, SEED
      COMMON /SEEDBK/ DM(0:15), DML, SEED(0:16)
      OPEN (1, FILE = 'SEED.DAT', STATUS = 'OLD')
      READ (1,1) (SEED(I),I = 0,16)
    1 FORMAT (6F13.0)
```

```
      CLOSE (1)
      RETURN
      END
C ************************** END GTSEED *********************************



C*********************************************************************

      SUBROUTINE SVSEED
C *********************************************************************
C    This subroutine saves the randomly permuted seeds generated by    *
C    the call to RESEED.  This is done upon completion of the MAIN      *
C    program.                                                           *
C*********************************************************************

      DOUBLE PRECISION  DM, DML, SEED
      COMMON /SEEDBK/ DM(0:15), DML, SEED(0:16)
      CALL RESEED
      OPEN (1, FILE = 'SEED.DAT', STATUS = 'OLD')
      WRITE (1,1) (SEED(I),I = 0,16)
   1  FORMAT (6F13.0)
      CLOSE (1)
      RETURN
      END
C ************************** END SVSEED *********************************



C *********************************************************************
      SUBROUTINE RESEED
C*****
C*  Randomly permutes the seeds for the multiplicative generators
C*****
      DOUBLE PRECISION  DATA, DM, DML, SEED, SV
      COMMON /SEEDBK/ DM(0:15), DML, SEED(0:16)
      COMMON /DATABK/ DATA(100)
      CALL UNIF(16)
      DO 1 I = 1,16
      M = 17-I
      N = M*DATA(I)
      SV = SEED(M)
      SEED(M) = SEED(N)
      SEED(N) = SV
   1  CONTINUE
      RETURN
      END
C ********************** END RESEED *********************************
```

```
C ***************************************************************************
      SUBROUTINE OLS(X, Y, A, B, XXINV)



      INTEGER I, ORIGIN, SIZE
      PARAMETER(SIZE=25)
      DOUBLE PRECISION X(SIZE,2), Y(SIZE), A, B, XTRAN(2,SIZE), XX(2,2),
     &       XXINV(2,2), XXINVX(2,SIZE), XXINVXY(2)

C***************************************************************************
C THIS PROGRAM COMPUTES THE O.L.S. ESTIMATES FOR THE EQUATION            *
C              Y = A + B * X                                             *
C                                                                        *
C X     : N*2 INDEPENDENT VECTOR                                         *
C Y     : N*1 DEPENDENT VECTOR                                           *
C A     : INTERCEPT TERM                                                 *
C B     : SLOPE PARAMETER                                                *
C***************************************************************************

      CALL TRANR(X, XTRAN, SIZE, 2)
      CALL ATIMB(XX, XTRAN, X, 2, SIZE, 2)
      CALL INVS2(XXINV, XX)
      CALL ATIMB(XXINVX, XXINV, XTRAN, 2, 2, SIZE)
      CALL ATIMB(XXINVXY, XXINVX, Y, 2, SIZE, 1)
      A= XXINVXY(1)
      B= XXINVXY(2)
      RETURN
      END
C ********************** END OLS ***************************************



C ***************************************************************************
      SUBROUTINE GENXY(Q, QIN, E, X, Y, EVAR, ALPHA, BETA, U, NR)

      INTEGER I, NR
      DOUBLE PRECISION Q(NR,NR), E(NR), X(NR,2), Y(NR), EVAR, ALPHA,
     &                 BETA, QIN(NR,NR), U(NR)

C***************************************************************************
C THIS SUBROUTINE GENERATES THE DEPENDENT Y VECTOR FROM THE TRANSFOR-    *
C MATION MATRIX "Q" AND THE PARAMETERS ALPHA AND BETA.                   *
C                                                                        *
C NR : THE NUMBER OF OBSERVATIONS IN THE SAMPLE                          *
C X  : A VECTOR OF INDEPENDENT VALUES                                    *
C***************************************************************************
```

```
C*****
C*   INVERT Q AND MULT. RESULT TIMES E TO GET TRANSFORMED ERRORS
C*****
      CALL INVCH(Q, QIN, NR)
      CALL ATIMB(U, QIN, E, NR, NR, 1)
C*****
C*   Compute the Y vector.
C*****
      DO 10 I=1,NR
       Y(I)=ALPHA + BETA * X(I,2) + U(I) * EVAR
  10  CONTINUE
      RETURN
      END
C ***************************** END GENXY ********************************



C ************************************************************************
      SUBROUTINE QINIT(NR, Q, RHO1, RHO2, FLAG)

C************************************************************************
C*  This subroutine initializes the transformation matrix Q.  It is    *
C*  both in the generation of the sample and in the estimation of the  *
C*  parameters.  The type of matrix is dependent on the value of FLAG. *
C*  If FLAG :                                                          *
C*   =1, then the stationary values are used                           *
C*   =2, then the nonstationary values are used                        *
C*   =3, then the first two observations are omitted (C-O).            *
C************************************************************************

      INTEGER I, J, NR, FLAG
      DOUBLE PRECISION Q(NR,NR), Q11, Q21, Q22, RHO1, RHO2

      IF (FLAG .EQ. 1) THEN
       Q11=DSQRT(1.0 - RHO2**2 - RHO1**2 * (1.0 + RHO2) / (1.0 - RHO2))
       Q21=DSQRT(RHO1**2 * (1.0 + RHO2) / (1.0 - RHO2))
       Q22=DSQRT(1.0 - RHO2**2)
      ELSE IF (FLAG .EQ. 2) THEN
       Q11=1.0
       Q21=-RHO1
       Q22=1.0
      ELSE
       Q11=0.0
       Q21=0.0
       Q22=0.0
      END IF
      DO 10 I=1,NR
       DO 10 J=1,NR
        Q(I,J)=0.0
```

```
        IF (I .EQ. 2 .AND. J .EQ. NR) THEN
         Q(1,1)=Q11
         Q(2,1)=Q21
         Q(2,2)=Q22
        ELSE IF (I .GT. 2 .AND. I .EQ. J) THEN
         Q(I,I)=1.0
         Q(I,I-1)=-RHO1
         Q(I,I-2)=-RHO2
        END IF
  10  CONTINUE
      RETURN
      END
C ******************************** END QINIT ****************************


C ***************************************************************************
      SUBROUTINE INVCH(S, A, N)

C***************************************************************************
C THIS PROGRAM COMPUTES THE INVERSE OF A LOWER TRIANGULAR MATRIX        *
C STORED IN "S", PLACING THE RESULTS IN "A".                            *
C                                                                       *
C N : ACTUAL ORDER OF A AND S                                           *
C ***************************************************************************

      INTEGER I, K, J, L, N
      DOUBLE PRECISION A(N,N), D, S(N,N)
C*****
C*   COMPUTE DIAGONAL TERMS OF A
C*****
      DO 10 I=1,N
       A(I,I)=1.0 / S(I,I)
  10  CONTINUE
C*****
C*   COMPUTE THE TERMS OF THE KTH DIAGONAL OF A
C*****
      DO 20 K=1,N-1
       DO 20 I=1,N-K
        J=I + K
        D=0.0
        DO 30 L=I,J-1
         D=D + S(J,L) * A(L,I)
  30    CONTINUE
        A(J,I)= -D / S(J,J)
  20  CONTINUE
      RETURN
      END
C ******************** END INVCH ****************************************
```

```
C ***************************************************************************
      SUBROUTINE ATIMB(C, A, B, N, M, L)

C***************************************************************************
C  THIS PROGRAM COMPUTES THE MATRIX OPERATION C = A * B                      *
C                                                                            *
C  N : ACTUAL NUMBER OF ROWS OF A AND C                                      *
C  M : ACTUAL NUMBER OF COLUMNS OF A AND ROWS OF B                           *
C  L : ACTUAL NUMBER OF COLUMNS OF B AND C                                   *
C***************************************************************************

      DOUBLE PRECISION A(N,M), B(M,L), C(N,L)
      INTEGER I, J, K

      DO 10 I=1,N
       DO 10 J=1,L
        C(I,J)=0.0
        DO 10 K=1,M
         C(I,J)=C(I,J) + A(I,K) * B(K,J)
  10  CONTINUE
      RETURN
      END
C ******************** END ATIMB ***************************************


C ***************************************************************************
      SUBROUTINE ADSUB(C, A, B, N, M, L)

C***************************************************************************
C THIS SUBROUTINE COMPUTES THE MATRIX OPERATION                             *
C C = A + B, WHEN L = 1 AND                                                 *
C C = A - B, WHEN L = 2.                                                    *
C                                                                           *
C N : ACTUAL NUMBER OF ROWS OF A, B, AND C                                  *
C M : ACTUAL NUMBER OF COLUMNS OF A, B, AND C                               *
C***************************************************************************

      INTEGER I, J, N, M, L
      DOUBLE PRECISION C(N,M), A(N,M), B(N,M)

      DO 10 I=1,N
       DO 10 J=1,M
        GO TO (2,4), L
 2      C(I,J)=A(I,J) + B(I,J)
        GO TO 10
 4      C(I,J)=A(I,J) - B(I,J)
```

```
 10   CONTINUE
      RETURN
      END
C ****************************** END ADSUB ******************************


C ***************************************************************************
      SUBROUTINE TRANR (A, B, N, M)

C***************************************************************************
C THIS SUBROUTINE COMPUTES THE TRANSPOSE OF THE NxM MATRIX "A" AND      *
C RETURNS THE TRANPOSE AS THE MATRIX "B".                               *
C                                                                       *
C N : ACTUAL NUMBER OF ROWS OF "A"                                      *
C M : ACTUAL NUMBER OF COLUMNS OF "A"                                   *
C B : THE MxN TRANSPOSE OF "A"                                          *
C***************************************************************************

      INTEGER N, M
      DOUBLE PRECISION B(M,N), A(N,M)

      DO 10 I=1,N
       DO 10 J=1,M
        B(J,I)=A(I,J)
 10   CONTINUE
      RETURN
      END
C ***************************** END TRANR *******************************


C ***************************************************************************
      SUBROUTINE INVS2(A, B)

C***************************************************************************
C THIS SUBROUTINE COMPUTES THE INVERSE OF THE 2 x 2 MATRIX "B".         *
C THE INVERTED MATRIX IS STORED IN "A". THE DETERMINANT OF "B" IS       *
C STORED IN "D".                                                        *
C***************************************************************************

      DOUBLE PRECISION A(2,2), B(2,2), D
      D=B(1,1) * B(2,2) - B(1,2) * B(2,1)
      A(1,1)=B(2,2) / D
      A(1,2)=-B(1,2) / D
      A(2,1)=-B(2,1) / D
      A(2,2)=B(1,1) / D
      RETURN
      END
C ************************** END INVS2 ******************************
```

```
C ***************************************************************************
      SUBROUTINE FML (X, Y, ESTIM1)

C***************************************************************************
C*    This subroutine finds the Beach-McKinnon Full Maximum Likelihood
C*    estimates of the parameters.  Alpha and beta are estimated using
C*    OLS conditional on rho1 and rho2.  Rho1 and rho2 are estimated
C*    using a Marquardt algorithm conditional on alpha and beta.  The
C*    process iterates until convergence of all four parameters is
C*    attained.  The results are returned in the vector ESTIM1.
C*    Two-step P-W estimates of RHO1 and RHO2 and OLS estimates of ALPHA
C*    and BETA are used as starting values.
C***************************************************************************

      INTEGER COUNT, I, SIZE, ITER
      PARAMETER (SIZE=25)
      DOUBLE PRECISION Q(SIZE,SIZE), AM1, BM1, RHO1M1, RHO2M1,
     &     Y(SIZE), UT1SQ, UT2SQ, UTSQ, UT1UT, UT1UT2, U1SQ, U1U2, U2SQ,
     &     UHAT(SIZE), UT2UT, XSTAR(SIZE,2), YSTAR(SIZE), ESTIM1(14),
     &     EPS, X(SIZE,2), T1, T2, T3, T4, ALPHA, BETA, RHO1, RHO2,
     &     SSQU, USTAR(SIZE), XXINV(2,2)
      COMMON /PARAM/ EPS
      COMMON /COEFF/ ALPHA, BETA, RHO1, RHO2

      ITER = 0
      COUNT=0
C******
C*    Make sure that the P-W estimates are inside the stability tri-
C*    angle.  If they are not then use initial parameter values that
C*    just lie inside the triangle that are near the P-W estimates.
C******
      IF (.NOT. (ESTIM1(4) .LT. 0.85D0)) ESTIM1(4) = 0.8D0
      IF (.NOT. (ESTIM1(4) .GT. -0.85D0)) ESTIM1(4) = -0.8D0
      IF (.NOT. (1.0D0 - ESTIM1(4) + ESTIM1(3) .GT. 0.2D0))
     &    ESTIM1(3) = ESTIM1(4) - 1.0D0 + 0.2D0
      IF (.NOT. (1.0D0 - ESTIM1(4) - ESTIM1(3) .GT. 0.2D0))
     &    ESTIM1(3) = 1.0D0 - ESTIM1(4) - 0.2D0
 1000 AM1=ESTIM1(1)
      BM1=ESTIM1(2)
      RHO1M1=ESTIM1(3)
      RHO2M1=ESTIM1(4)
C*****
C*   Transform X and Y and find the conditional OLS estimates.
C*****
      CALL QINIT(SIZE, Q, ESTIM1(3), ESTIM1(4), 1)
```

```
      CALL ATIMB(XSTAR, Q, X, SIZE, SIZE, 2)
      CALL ATIMB(YSTAR, Q, Y, SIZE, SIZE, 1)
      CALL OLS(XSTAR, YSTAR, ESTIM1(1), ESTIM1(2), XXINV)
C*****
C*   Compute the error vector.
C*****
      DO 10 I=1,SIZE
       UHAT(I)=Y(I) - ESTIM1(1) - ESTIM1(2) * X(I,2)
  10  CONTINUE
      UT2UT=0.0
      UT1SQ=0.0
      UT2SQ=0.0
      UTSQ=0.0
      UT1UT=0.0
      UT1UT2=0.0
      U1SQ=UHAT(1)**2
      U1U2=UHAT(1) * UHAT(2)
      U2SQ=UHAT(2)**2
      DO 20 I=3,SIZE
       UT1SQ=UT1SQ + UHAT(I-1)**2
       UT2SQ=UT2SQ + UHAT(I-2)**2
       UTSQ=UTSQ + UHAT(I)**2
       UT1UT=UT1UT + UHAT(I) * UHAT(I-1)
       UT1UT2=UT1UT2 + UHAT(I-1) * UHAT(I-2)
       UT2UT=UT2UT + UHAT(I) * UHAT(I-2)
  20  CONTINUE
C*****
C*   Use the Marquardt algorthm to find new estimates of RHO1 and RHO2.
C*****
      CALL MARQ(ESTIM1(3), ESTIM1(4), U1SQ, U1U2, U2SQ, UT1SQ,
     &     UT2SQ, UTSQ, UT1UT, UT1UT2, UT2UT)
      COUNT=COUNT + 1
      IF (COUNT .GE. 100) ITER = 1
C*****
C*   Check for convergence.
C*****
      T1 = DABS(ESTIM1(1) - AM1) / (EPS * MAX(DABS(AM1), 1.0D0))
      T2 = DABS(ESTIM1(2) - BM1) / (EPS * MAX(DABS(BM1), 1.0D0))
      T3 = DABS(ESTIM1(3) - RHO1M1) / (EPS * MAX(DABS(RHO1M1), 1.0D0))
      T4 = DABS(ESTIM1(4) - RHO2M1) / (EPS * MAX(DABS(RHO2M1), 1.0D0))
      IF (.NOT. (MAX(T1, T2, T3, T4) .LT. 1.0D0) .AND. ITER .EQ. 0)
     &   GO TO 1000
C*****
C*   Compute MSE, iter. count, and max. iter. flag values.
C*****
       CALL ATIMB(USTAR, Q, UHAT, SIZE, SIZE, 1)
       CALL ATIMB(SSQU, USTAR, USTAR, 1, SIZE, 1)
       ESTIM1(5) = (ESTIM1(1) - ALPHA)**2
       ESTIM1(6) = (ESTIM1(2) - BETA)**2
       ESTIM1(7) = (ESTIM1(3) - RHO1)**2
```

```
       ESTIM1(8) = (ESTIM1(4) - RHO2)**2
       ESTIM1(9) = COUNT
       ESTIM1(10) = SSQU / 23.0 * XXINV(1,1)
       ESTIM1(11) = DSQRT(ESTIM1(10))
       ESTIM1(12) = SSQU / 23.0 * XXINV(2,2)
       ESTIM1(13) = DSQRT(ESTIM1(12))
       ESTIM1(14) = ITER
      RETURN
      END
C ********************** END FML ***************************************



C ************************************************************************
      SUBROUTINE MARQ (RHO1, RHO2, U1SQ, U1U2, U2SQ, UT1SQ, UT2SQ,
     &          UTSQ, UT1UT, UT1UT2, UT2UT)

C ************************************************************************
C *     Marquardt algorithm for the maximization of the conditional     *
C *     log-liklehood function with respect to rho1 and rho2.           *
C ************************************************************************

      INTEGER SIZE, MAXIT, ICNV, METHOUT, I, ITER, NR, FLAG
      DOUBLE PRECISION  D(2), ZETA, SUMD, SUMDG, SUMG, DELTA, GAMMA,
     &       RATIO, STEP, STEP1, STEP2, TAU, B(2), B1(2), BCHG, C, EPS,
     &       G(2), R(2,2), NU, U1SQ, U1U2, U2SQ, UT1SQ, UT2SQ, UTSQ,
     &       UT1UT, UT1UT2, UT2UT, TST, IDEN(2,2), L, L1, RINV(2,2), T1,
     &       T2, T3, SSR, SSRR1, SSRR2, NRD2, RHO1, RHO2, RINV1(2,2),
     &       SSRR1R1, SSRR1R2, SSRR2R2, THETA, W, TST1, TST2
      PARAMETER (SIZE=25, ZETA=0.707106781D0, TAU=1.0D-15, NU=1.0D-7,
     &       GAMMA=1.0D0, DELTA=1.0D-60)
      COMMON /PARAM/ EPS


      ICNV = 0
      ITER = 0
      METHOUT = 0
      NRD2 = SIZE / 2.0D0
      B(1) = RHO1
      B(2) = RHO2
      MAXIT = 0
C*****
C*       B is the vector of parameter estimates.
C*       G is the gradient vector.
C*       L and L1 are the likelihood function values at B and B1.
C*       R is the value of the Hessian matrix for B.
C*****
  1   T1= 1.0 - B(1) - B(2)
      T2= 1.0 + B(1) - B(2)
```

```
      T3= 1.0 + B(2)
      SSR=(1.0 - B(2)**2) * U1SQ - 2.0 * B(1) * T3 * U1U2
     &   + (1.0 - B(2)**2) * U2SQ + UTSQ - 2.0 * B(1) * UT1UT - 2.0 *
     &   B(2) * UT2UT + 2.0 * B(1) * B(2) * UT1UT2 + B(1)**2 * UT1SQ
     &   + B(2)**2 * UT2SQ
      SSRR1=2.0 * (-T3 * U1U2 - UT1UT + B(1) * UT1SQ + B(2)
     &     * UT1UT2)
      SSRR2=2.0 * (-B(2) * (U1SQ + U2SQ) - B(1) * U1U2 - UT2UT + B(1)
     &     * UT1UT2 + B(2) * UT2SQ)
      G(1)=-NRD2 * SSRR1 / SSR - 0.5 / T1 + 0.5 / T2
      G(2)=-NRD2 * SSRR2 / SSR + 1.0 / T3 - 0.5 / T1 - 0.5 / T2
      L=-NRD2 * LOG(SSR) + LOG(T3) + 0.5 * LOG(T1) +
     &     0.5 * LOG(T2)
      SSRR1R2=2.0 * (-U1U2 + UT1UT2)
      SSRR1R1=2.0 * UT1SQ
      SSRR2R2=-2.0 * (U1SQ + U2SQ - UT2SQ)
      R(1,1)=-NRD2 * (SSRR1R1 / SSR - SSRR1 * SSRR1 / (SSR * SSR)) -
     &    0.5 / (T1 * T1) - 0.5 / (T2 * T2)
      R(1,2)=-NRD2 * (SSRR1R2 / SSR - SSRR1 * SSRR2 / (SSR * SSR)) -
     &    0.5 / (T1 * T1) + 0.5 / (T2 * T2)
      R(2,1)=R(1,2)
      R(2,2)=-NRD2 * (SSRR2R2 / SSR - SSRR2 * SSRR2 / (SSR * SSR)) -
     &    1.0 / (T3 * T3) - 0.5 / (T1 * T1) - 0.5 / (T2 * T2)
C***
C*       Choose the direction vector D such that
C*              D = Inverse(R + THETA*IDEN)*G.
C*       Where IDEN is a matrix composed of the absolute value of
C*       R(i,i) on the main diagonal and zero elsewhere. If abs(R(i,i))
C*       is less than 1 then IDEN(i,i) = 1.0.
C*       The factor THETA follows the sequence:
C*         THETA = 0.0000001, 0.000001, 0.00001, ...
C*       until the angle PHI between D and G satisfies
C*          COS(PHI) = D'G/SQRT((D'D)*(G'G)) > SQRT(0.5).
C*       This guarantees that D and G are always in the
C*       same quarter space.
C*****
      THETA = NU
  6   IF (DABS(R(1,1)) .LT. 1.0D0) THEN
       IDEN(1,1)=1.0D0 * THETA
      ELSE
       IDEN(1,1)=DABS(R(1,1)) * THETA
      END IF
      IDEN(1,2)=0.0D0
      IDEN(2,1)=0.0D0
      IF (DABS(R(2,2)) .LT. 1.0D0) THEN
       IDEN(2,2)= 1.0D0 * THETA
      ELSE
       IDEN(2,2)=DABS(R(2,2)) * THETA
      END IF
      CALL ADSUB(RINV, R, IDEN, 2, 2, 1)
```

```
C*****
C*   Make sure RINV is positive definite
C*****
      IF ((RINV(1,1) .LT. TAU) .OR.
     & (RINV(1,1) * RINV(2,2) - RINV(1,2) * RINV(2,1)) .LT. TAU) THEN
       THETA=10.0D0 * THETA
       GO TO 6
      END IF
      CALL INVS2(RINV1, RINV)
      CALL ATIMB(D, RINV1, G, 2, 2, 1)
      CALL ATIMB(SUMG, G, G, 1, 2, 1)
      CALL ATIMB(SUMD, D, D, 1, 2, 1)
      CALL ATIMB(SUMDG, D, G, 1, 2, 1)
C*****
C*   Check for convergence of either the gradient or the distance of
C*   the new estimate from the old.
C*****
      TST=MIN(SUMG, SUMD)
      IF (TST .LT. DELTA) THEN
       ICNV=1
       METHOUT=1
       GO TO 17
      END IF
C*****
C*   Are the vectors in the same quarter space?  If not make RINV
C*   more positive definite.
C*****
      RATIO=SUMDG / (DSQRT(SUMD * SUMG))
      IF (RATIO .GT. ZETA) THEN
       GO TO 10
      ELSE
       THETA=10.0D0 * THETA
       GO TO 6
      END IF
  10  FLAG=0
C*****
C*   Find new parameter estimates, then test to see if there is
C*   improvement.
C*****
      B1(1)= B(1) + D(1)
      B1(2)= B(2) + D(2)
C*****
C*    Check if stability triangle boundary has been violated by
C*    the new step.  If a boundary has been violated decrease
C*    the step by an amount that just brings the new parameter
C*    estimates back into the stability triangle. This is
C*    important because of the discontinuity that occurs at the
C*    boundary of the stability triangle.
C*    W is the proportion of the distance between the old and new
C*    parameter values and the boundary of the stability triangle.
```

```
C*****
      IF (B1(2) .GE. 1.0D0) THEN
       B1(2)= 1.0D0
       D(2)= B1(2) - B(2)
      END IF
      IF (1.0D0 + B1(2) .LE. 0.0) THEN
       W= (-1.0D0 - B(2)) / (B1(2) - B(2))
       FLAG= 1
      END IF
      IF (FLAG .EQ. 1) THEN
       IF (1.0D0 - B1(1) - B1(2) .LE. 0.0) THEN
        B1(1)= W * B1(1) + (1.0D0 - W) * B(1)
        B1(2)= W * B1(2) + (1.0D0 - W) * B(2)
        D(1)= B1(1) - B(1)
        D(2)= B1(2) - B(2)
        W= (1.0D0 - B(1) - B(2)) / (B1(1) + B1(2) - B(1) - B(2))
       ELSE IF (1.0D0 + B1(1) - B1(2) .LE. 0.0) THEN
        B1(1)= W * B1(1) + (1.0D0 - W) * B(1)
        B1(2)= W * B1(2) + (1.0D0 - W) * B(2)
        D(1)= B1(1) - B(1)
        D(2)= B1(2) - B(2)
        W= (B(2) - B(1) - 1.0D0) / (B1(1) - B1(2) - B(1) + B(2))
       END IF
      ELSE IF (1.0D0 - B1(1) - B1(2) .LE. 0.0) THEN
       W= (1.0D0 - B(1) - B(2)) / (B1(1) + B1(2) - B(1) - B(2))
       FLAG= 1
      ELSE IF (1.0D0 + B1(1) - B1(2) .LE. 0.0) THEN
       W= (B(2) - B(1) - 1.0D0) / (B1(1) - B1(2) - B(1) + B(2))
       FLAG= 1
      END IF
C*****
C*   Update the step size to bring estimates inside triangle.
C*   If the new estimate is already in the triangle let the
C*   step size equal 1.
C*****
      IF (FLAG .EQ. 1) THEN
       STEP= .99 * W
       STEP1= STEP
      ELSE
       STEP = 1.0D0
       STEP1 = 1.0D0
      END IF
      STEP2 = 0.0D0
C*****
C*      Choose the stepsize, STEP, that improves L.
C*                      B1 = B + STEP*D.
C*      If (L1 - L)/(D'G) > 0, then STEP = 1.  If not, the
C*      interval (0,1) is successively halved to obtain a
C*      stepsize such that
C*        1.0D-7 < (L - L1)/(STEP*D'G) < 1.0D-7 - 0.1.
```

```
C*      The parameter estimates are updated if such a stepsize
C*      is found or if
C*            ABS(STEP1 - STEP2) < 1.0D-7 and L1 > L.
C*****
C*      Note that D is the actual direction of the parameter
C*      change (this is convenient for the tests).
C*****
   13 B1(1) = B(1) + STEP*D(1)
      B1(2) = B(2) + STEP*D(2)
      T1= 1.0 - B1(1) - B1(2)
      T2= 1.0 + B1(1) - B1(2)
      T3= 1.0 + B1(2)
      SSR=(1.0 - B1(2)**2) * U1SQ - 2.0 * B1(1) * T3 * U1U2
     &   + (1.0 - B1(2)**2) * U2SQ + UTSQ - 2.0 * B1(1) * UT1UT -
     &   2.0 * B1(2) * UT2UT + 2.0 * B1(1) * B1(2) * UT1UT2 +
     &   B1(1)**2 * UT1SQ + B1(2)**2 * UT2SQ
      SSRR1=2.0 * (-T3 * U1U2 - UT1UT + B1(1) * UT1SQ + B1(2)
     &     * UT1UT2)
      SSRR2=2.0 * (-B1(2) * (U1SQ + U2SQ) - B1(1) * U1U2 - UT2UT +
     &      B1(1) * UT1UT2 + B1(2) * UT2SQ)
      G(1)=-NRD2 * SSRR1 / SSR - 0.5 / T1 + 0.5 / T2
      G(2)=-NRD2 * SSRR2 / SSR + 1.0 / T3 - 0.5 / T1 - 0.5 / T2
C*****
C*   Compute the value of the likelihood function given the new para-
C*   meter estimates.
C*****
      L1=-NRD2 * LOG(SSR) + LOG(T3) + 0.5 * LOG(T1) +
     &     0.5 * LOG(T2)
C*****
C*      We stop if:
C*         STEP > 10.0**(-15) and ABS(L1 - L) < 10.0**(-15);
C*         STEP < or = 10.0**(-15) and L > or = L1;
C*         ABS(STEP*SUMDG) < 10.0**(-15).
C*****
      IF (STEP .GT. TAU .AND. DABS(L - L1) .LT. TAU) THEN
      ICNV = 1
      METHOUT = 2
      GO TO 17
      END IF
      IF (STEP .LE. TAU .AND. L1 .LE. L) THEN
      ICNV = 1
      METHOUT = 3
      GO TO 17
      END IF
      IF (DABS(STEP*SUMDG) .LT. TAU) THEN
      ICNV = 1
      METHOUT = 4
      GO TO 17
      END IF
      RATIO = (L1 - L)/(STEP*SUMDG)
```

```
      IF (STEP .GE. GAMMA .AND. L1 .GT. L) GO TO 15
      IF (RATIO .GE. NU .AND. (RATIO + NU) .LE. GAMMA) GO TO 15
      IF (DABS(STEP1 - STEP2) .LT. NU .AND. L1 .GT. L) GO TO 15
      IF (RATIO .LT. NU) STEP1 = STEP
      IF ((RATIO + NU) .GT. GAMMA) STEP2 = STEP
      STEP = (STEP1 + STEP2)/2.0D0
      GO TO 13
C*****
C*      Check for convergence.
C*****
   15 TST1 = 0.1D0 * EPS * MAX(B(1), 1.0D0)
      TST2 = 0.1D0 * EPS * MAX(B(2), 1.0D0)
      IF (DABS(B(1) - B1(1)) .LE. TST1 .AND. DABS(B(2) - B1(2))
     &   .LE. TST2) THEN
        ICNV = 1
        METHOUT = 5
      END IF
C*****
C*       Update B.  Iterate again if not converged.
C*****
      B(1) = B1(1)
      B(2) = B1(2)
   17 ITER = ITER + 1
C*****
C*       Check the iteration count and the convergence criterion.
C*****
      IF (ITER .GE. 50 .OR. ICNV .EQ. 1) GO TO 18
      GO TO 1
   18 RHO1= B(1)
      RHO2= B(2)
      RETURN
      END


C***********************************************************************


      SUBROUTINE ABROLS (X, Y, ESTIM1)

C***********************************************************************
C This subroutine computes the OLS estimates of ALPHA, BETA, RHO1, and *
C RHO2.  The estimates of the rhos are found from the residual error   *
C terms using the Yule-Walker equations.                               *
C***********************************************************************

      INTEGER I, SIZE
      PARAMETER (SIZE=25)
      DOUBLE PRECISION UTUT1, UTUT2, UTSQ, UHAT(SIZE), X(SIZE,2),
     &      Y(SIZE), R(2,2), RINV(2,2), G(2), ESTIM1(14), RHAT(2),
     &      ALPHA, BETA, RHO1, RHO2, USTAR(SIZE), XXINV(2,2), SSQU
```

```
      COMMON /COEFF/ ALPHA, BETA, RHO1, RHO2

C***
C* Find the estimates of alpha and beta.
C***
      CALL OLS (X, Y, ESTIM1(1), ESTIM1(2), XXINV)

C***
C* Compute residuals.
C***
      UTUT1= 0.0D0
      UTUT2= 0.0D0
      UTSQ= 0.0D0
      DO 10 I=1,SIZE
       UHAT(I)= Y(I) - ESTIM1(1) - ESTIM1(2) * X(I,2)
       UTSQ= UTSQ + UHAT(I) * UHAT(I)
       IF (I .GT. 1) UTUT1= UTUT1 + UHAT(I) * UHAT(I-1)
       IF (I .GT. 2) UTUT2= UTUT2 + UHAT(I) * UHAT(I-2)
  10  CONTINUE

C***
C* Estimate RHO1 and RHO2 using the Yule-Walker equations.
C***
      R(1,1)= 1.0D0
      R(2,2)= 1.0D0
      R(1,2)= UTUT1 / UTSQ
      R(2,1)= R(1,2)
      G(1)= R(1,2)
      G(2)= UTUT2 / UTSQ
      CALL INVS2 (RINV, R)
      CALL ATIMB (RHAT, RINV, G, 2, 2, 1)
C*****
C*   Compute the variance
C*****
      CALL ATIMB(SSQU, UHAT, UHAT, 1, SIZE, 1)
C***
C* The results are placed in the vector ESTIM1.
C***
      ESTIM1(3)= RHAT(1)
      ESTIM1(4)= RHAT(2)
      ESTIM1(5)= (ESTIM1(1) - ALPHA)**2
      ESTIM1(6)= (ESTIM1(2) - BETA)**2
      ESTIM1(7)= (ESTIM1(3) - RHO1)**2
      ESTIM1(8)= (ESTIM1(4) - RHO2)**2
      ESTIM1(9)= 0.0D0
      ESTIM1(10)= SSQU / 23.0 * XXINV(1,1)
      ESTIM1(11)= DSQRT(ESTIM1(10))
      ESTIM1(12)= SSQU / 23.0 * XXINV(2,2)
      ESTIM1(13)= DSQRT(ESTIM1(12))
      ESTIM1(14)= 0.0D0
```

```
      RETURN
      END

C********************** END ABROLS ************************************



C ***************************************************************************
      SUBROUTINE PW (X, Y, ESTIM1, ESTIM2)

C***************************************************************************
C*   This subroutine computes the two-step P-W and Iter. P-W estimates.*
C*   The OLS estimates of ALPHA and BETA are used as starting values.  *
C***************************************************************************

      INTEGER I, SIZE, COUNT, MAXFLAG
      PARAMETER (SIZE=25)
      DOUBLE PRECISION X(SIZE,2), Y(SIZE), ESTIM1(14), ESTIM2(14),
     &    RHAT(2), RHO1M1, RHO2M1, AM1, BM1, UTSQ, UTUT1, UTUT2,
     &    UHAT(SIZE), R(2,2), RINV(2,2), G(2), EPSSQT4, ALPHA, BETA,
     &    RHO1, RHO2, T1, T2, T3, T4, EPS, SSQU, USTAR(SIZE),
     &    XXINV(2,2), Q(SIZE,SIZE), XSTAR(SIZE,2), YSTAR(SIZE)
      COMMON /PARAM/ EPS
      COMMON /COEFF/ ALPHA, BETA, RHO1, RHO2

      RHAT(1)= 0.0D0
      RHAT(2)= 0.0D0
      COUNT= 0
      MAXFLAG= 0
 1000 RHO1M1= RHAT(1)
      RHO2M1= RHAT(2)
      AM1= ESTIM2(1)
      BM1= ESTIM2(2)
C*****
C*   Find the residual values.
C*****
      UTSQ= 0.0D0
      UTUT1= 0.0D0
      UTUT2= 0.0D0
      DO 20 I=1,SIZE
       UHAT(I)= Y(I) - ESTIM2(1) - ESTIM2(2) * X(I,2)
       IF (I .GT. 1) THEN
        UTSQ= UTSQ + UHAT(I) * UHAT(I)
        UTUT1= UTUT1 + UHAT(I) * UHAT(I-1)
       END IF
       IF (I .GT. 2) UTUT2= UTUT2 + UHAT(I) * UHAT(I-2)
  20  CONTINUE
C*****
```

```
C*    The matrices R and the vector G are the values of the coefficients
C*    from the first order conditions of the sum of squared errors with
C*    respect to RHO1 and RHO2.  The conditional estimates of RHO1 and
C*    RHO2 are then
C*           RHAT=INV(R) * G
C*****
       R(1,1)= UTSQ - UHAT(SIZE) * UHAT(SIZE)
       R(1,2)= UTUT1 - UHAT(SIZE) * UHAT(SIZE-1) - UHAT(2) * UHAT(1)
       R(2,1)= R(1,2)
       R(2,2)= R(1,1) - UHAT(SIZE-1) * UHAT(SIZE-1) - UHAT(2) *
      &         UHAT(2)
       G(1)= UTUT1
       G(2)= UTUT2
       CALL INVS2 (RINV, R)
       CALL ATIMB(RHAT, RINV, G, 2, 2, 1)
       IF (.NOT. (RHAT(2) .LT. .99999)) RHAT(2) = .99999D0
       IF (.NOT. (RHAT(2) .GT. -.99999)) RHAT(2) = -.99999D0
       IF (.NOT. (1.0D0 - RHAT(2) .GT. -RHAT(1)))
      &     RHAT(1) = RHAT(2) - 1.0D0 + 0.5D-5
       IF (.NOT. (1.0D0 - RHAT(2) .GT. RHAT(1)))
      &     RHAT(1) = 1.0D0 - RHAT(2) - 0.5D-5
C*****
C*    Store the Two-step P-W estimates.
C*****
       IF (COUNT .EQ. 1) THEN
C*****
C*     Compute the variance
C*****
        CALL ATIMB(USTAR, Q, UHAT, SIZE, SIZE, 1)
        CALL ATIMB(SSQU, USTAR, USTAR, 1, SIZE, 1)
        ESTIM1(1) = ESTIM2(1)
        ESTIM1(2) = ESTIM2(2)
        ESTIM1(3) = RHAT(1)
        ESTIM1(4) = RHAT(2)
        ESTIM1(5) = (ESTIM1(1) - ALPHA)**2
        ESTIM1(6) = (ESTIM1(2) - BETA)**2
        ESTIM1(7) = (ESTIM1(3) - RHO1)**2
        ESTIM1(8) = (ESTIM1(4) - RHO2)**2
        ESTIM1(9) = 0.0
        ESTIM1(10) = SSQU / 23.0 * XXINV(1,1)
        ESTIM1(11) = DSQRT(ESTIM1(10))
        ESTIM1(12) = SSQU / 23.0 * XXINV(2,2)
        ESTIM1(13) = DSQRT(ESTIM1(12))
        ESTIM1(14) = 0.0
       END IF
C*****
C*    Use a Generalized Least Squares algorithm to find the conditional
C*    estimates of ALPHA and BETA.
C*****
       CALL QINIT (SIZE, Q, RHAT(1), RHAT(2), 1)
```

```
      CALL ATIMB (XSTAR, Q, X, SIZE, SIZE, 2)
      CALL ATIMB (YSTAR, Q, Y, SIZE, SIZE, 1)
      CALL OLS (XSTAR, YSTAR, ESTIM2(1), ESTIM2(2), XXINV)
      COUNT= COUNT + 1
C*****
C*   Check for convergence
C*****
      T1 = DABS(ESTIM2(1) - AM1) / (EPS * MAX(DABS(AM1), 1.0D0))
      T2 = DABS(ESTIM2(2) - BM1) / (EPS * MAX(DABS(BM1), 1.0D0))
      T3 = DABS(RHAT(1) - RH01M1) / (EPS * MAX(DABS(RH01M1), 1.0D0))
      T4 = DABS(RHAT(2) - RH02M1) / (EPS * MAX(DABS(RH02M1), 1.0D0))
      IF (.NOT. (MAX(T1, T2, T3, T4) .LT. 1.0D0) .AND. COUNT .LE. 100)
     &   GO TO 1000
      IF (COUNT .GE. 100) MAXFLAG= 1
C*****
C*   Compute the variance
C*****
      CALL ATIMB(USTAR, Q, UHAT, SIZE, SIZE, 1)
      CALL ATIMB(SSQU, USTAR, USTAR, 1, SIZE, 1)
C*****
C*   Save the iterative P-W estimates.
C*****
      ESTIM2(3) = RHAT(1)
      ESTIM2(4) = RHAT(2)
      ESTIM2(5) = (ESTIM2(1) - ALPHA)**2
      ESTIM2(6) = (ESTIM2(2) - BETA)**2
      ESTIM2(7) = (ESTIM2(3) - RH01)**2
      ESTIM2(8) = (ESTIM2(4) - RH02)**2
      ESTIM2(9) = COUNT
      ESTIM2(10) = SSQU / 23.0 * XXINV(1,1)
      ESTIM2(11) = DSQRT(ESTIM2(10))
      ESTIM2(12) = SSQU / 23.0 * XXINV(2,2)
      ESTIM2(13) = DSQRT(ESTIM2(12))
      ESTIM2(14) = MAXFLAG
      RETURN
      END
C ************************** END PW **********************************



C ***********************************************************************
      SUBROUTINE CO (X, Y, ESTIM1, ESTIM2)

C***********************************************************************
C*   This subroutine computes the two-step C-O and Iter. C-O estimates.*
C*   The OLS estimates of ALPHA and BETA are used as starting values.  *
C***********************************************************************

      INTEGER I, SIZE, COUNT, MAXFLAG
```

```
      PARAMETER (SIZE=25)
      DOUBLE PRECISION X(SIZE,2), Y(SIZE), ESTIM1(14), ESTIM2(14),
     &    RHAT(2), RHO1M1, RHO2M1, AM1, BM1, UTSQ, UTUT1, UTUT2,
     &    UHAT(SIZE), R(2,2), RINV(2,2), G(2), Q(SIZE,SIZE),
     &    XSTAR(SIZE,2), YSTAR(SIZE), EPSSQT4, ALPHA, BETA, RHO1,
     &    RHO2, T1, T2, T3, T4, EPS, USTAR(SIZE), SSQU, XXINV(2,2)
      COMMON /PARAM/ EPS
      COMMON /COEFF/ ALPHA, BETA, RHO1, RHO2

      RHAT(1)= 0.0D0
      RHAT(2)= 0.0D0
      COUNT= 0
      MAXFLAG= 0
 1000 RHO1M1= RHAT(1)
      RHO2M1= RHAT(2)
      AM1= ESTIM2(1)
      BM1= ESTIM2(2)
C*****
C*   Find the residual values.
C*****
      UTSQ= 0.0D0
      UTUT1= 0.0D0
      UTUT2= 0.0D0
      DO 20 I=1,SIZE
       UHAT(I)= Y(I) - ESTIM2(1) - ESTIM2(2) * X(I,2)
       UTSQ= UTSQ + UHAT(I) * UHAT(I)
       IF (I .GT. 1) UTUT1= UTUT1 + UHAT(I) * UHAT(I-1)
       IF (I .GT. 2) UTUT2= UTUT2 + UHAT(I) * UHAT(I-2)
  20  CONTINUE
C*****
C*   The matrices R and the vector G are the values of the coefficients
C*   from the first order conditions of the sum of squared errors with
C*   respect to RHO1 and RHO2.  The conditional estimates of RHO1 and
C*   RHO2 are then
C*          RHAT=INV(R) * G
C*****
      R(1,1)= UTSQ - UHAT(SIZE) * UHAT(SIZE) - UHAT(1) * UHAT(1)
      R(1,2)= UTUT1 - UHAT(SIZE) * UHAT(SIZE-1)
      R(2,1)= R(1,2)
      R(2,2)= UTSQ - UHAT(SIZE) * UHAT(SIZE) - UHAT(SIZE-1) *
     &        UHAT(SIZE-1)
      G(1)= UTUT1 - UHAT(2) * UHAT(1)
      G(2)= UTUT2
      CALL INVS2 (RINV, R)
      CALL ATIMB(RHAT, RINV, G, 2, 2, 1)
C*****
C*   Store the Two-step C-O estimates.
C*****
      IF (COUNT .EQ. 1) THEN
C*****
```

```
C*    Compute the variance
C*****
       CALL ATIMB(USTAR, Q, UHAT, SIZE, SIZE, 1)
       CALL ATIMB(SSQU, USTAR, USTAR, 1, SIZE, 1)
       ESTIM1(1) = ESTIM2(1)
       ESTIM1(2) = ESTIM2(2)
       ESTIM1(3) = RHAT(1)
       ESTIM1(4) = RHAT(2)
       ESTIM1(5) = (ESTIM1(1) - ALPHA)**2
       ESTIM1(6) = (ESTIM1(2) - BETA)**2
       ESTIM1(7) = (ESTIM1(3) - RHO1)**2
       ESTIM1(8) = (ESTIM1(4) - RHO2)**2
       ESTIM1(9) = 0.0
       ESTIM1(10) = SSQU / 21.0 * XXINV(1,1)
       ESTIM1(11) = DSQRT(ESTIM1(10))
       ESTIM1(12) = SSQU / 21.0 * XXINV(2,2)
       ESTIM1(13) = DSQRT(ESTIM1(12))
       ESTIM1(14) = 0.0
      END IF
      CALL QINIT (SIZE, Q, RHAT(1), RHAT(2), 3)
      CALL ATIMB (XSTAR, Q, X, SIZE, SIZE, 2)
      CALL ATIMB (YSTAR, Q, Y, SIZE, SIZE, 1)
      CALL OLS (XSTAR, YSTAR, ESTIM2(1), ESTIM2(2), XXINV)
      COUNT= COUNT + 1
C*****
C*   Check for convergence
C*****
      T1 = DABS(ESTIM2(1) - AM1) / (EPS * MAX(DABS(AM1), 1.0D0))
      T2 = DABS(ESTIM2(2) - BM1) / (EPS * MAX(DABS(BM1), 1.0D0))
      T3 = DABS(RHAT(1) - RHO1M1) / (EPS * MAX(DABS(RHO1M1), 1.0D0))
      T4 = DABS(RHAT(2) - RHO2M1) / (EPS * MAX(DABS(RHO2M1), 1.0D0))
      IF (.NOT. (MAX(T1, T2, T3, T4) .LT. 1.0D0) .AND. COUNT .LE. 100)
     &   GO TO 1000
      IF (COUNT .GE. 100) MAXFLAG= 1
C*****
C*   Compute the variance
C*****
      CALL ATIMB(USTAR, Q, UHAT, SIZE, SIZE, 1)
      CALL ATIMB(SSQU, USTAR, USTAR, 1, SIZE, 1)
C*****
C*   Save the iterative C-O estimates.
C*****
      ESTIM2(3) = RHAT(1)
      ESTIM2(4) = RHAT(2)
      ESTIM2(5) = (ESTIM2(1) - ALPHA)**2
      ESTIM2(6) = (ESTIM2(2) - BETA)**2
      ESTIM2(7) = (ESTIM2(3) - RHO1)**2
      ESTIM2(8) = (ESTIM2(4) - RHO2)**2
      ESTIM2(9) = COUNT
      ESTIM2(10) = SSQU / 21.0 * XXINV(1,1)
```

```
      ESTIM2(11) = DSQRT(ESTIM2(10))
      ESTIM2(12) = SSQU / 21.0 * XXINV(2,2)
      ESTIM2(13) = DSQRT(ESTIM2(12))
      ESTIM2(14) = MAXFLAG
      RETURN
      END
C ************************** END CO **********************************


C ***************************************************************************
      SUBROUTINE INS (X, Y, ESTIM1)

C***************************************************************************
C*   This subroutine computes the Iter. INS estimates. The OLS          *
C*   estimates of ALPHA and BETA are used as starting values.           *
C***************************************************************************
      INTEGER I, SIZE, COUNT, MAXFLAG
      PARAMETER (SIZE=25)
      DOUBLE PRECISION X(SIZE,2), Y(SIZE), ESTIM1(10),
     &    RHAT(2), RHO1M1, RHO2M1, AM1, BM1, UTSQ, UTUT1, UTUT2,
     &    UHAT(SIZE), R(2,2), RINV(2,2), G(2), Q(SIZE,SIZE),
     &    XSTAR(SIZE,2), YSTAR(SIZE), EPSSQT4, ALPHA, BETA, RHO1,
     &    RHO2, T1, T2, T3, T4, EPS, USTAR(SIZE), SSQU, XXINV(2,2)
      COMMON /PARAM/ EPS
      COMMON /COEFF/ ALPHA, BETA, RHO1, RHO2

      RHAT(1)= 0.0D0
      RHAT(2)= 0.0D0
      COUNT= 0
      MAXFLAG= 0
 1000 RHO1M1= RHAT(1)
      RHO2M1= RHAT(2)
      AM1= ESTIM1(1)
      BM1= ESTIM1(2)
C*****
C*   Find the residual values.
C*****
      UTSQ= 0.0D0
      UTUT1= 0.0D0
      UTUT2= 0.0D0
      DO 20 I=1,SIZE
       UHAT(I)= Y(I) - ESTIM1(1) - ESTIM1(2) * X(I,2)
       UTSQ= UTSQ + UHAT(I) * UHAT(I)
       IF (I .GT. 1) UTUT1= UTUT1 + UHAT(I) * UHAT(I-1)
       IF (I .GT. 2) UTUT2= UTUT2 + UHAT(I) * UHAT(I-2)
  20  CONTINUE
C*****
C*   The matrices R and the vector G are the values of the coefficients
C*   from the first order conditions of the sum of squared errors with
```

```
C*    respect to RHO1 and RHO2.  The conditional estimates of RHO1 and
C*    RHO2 are then
C*           RHAT=INV(R) * G
C*****
      R(1,1)= UTSQ - UHAT(SIZE) * UHAT(SIZE)
      R(1,2)= UTUT1 - UHAT(SIZE) * UHAT(SIZE-1)
      R(2,1)= R(1,2)
      R(2,2)= R(1,1) - UHAT(SIZE-1) * UHAT(SIZE-1)
      G(1)= UTUT1
      G(2)= UTUT2
      CALL INVS2 (RINV, R)
      CALL ATIMB(RHAT, RINV, G, 2, 2, 1)
      CALL QINIT (SIZE, Q, RHAT(1), RHAT(2), 2)
      CALL ATIMB (XSTAR, Q, X, SIZE, SIZE, 2)
      CALL ATIMB (YSTAR, Q, Y, SIZE, SIZE, 1)
      CALL OLS (XSTAR, YSTAR, ESTIM1(1), ESTIM1(2), XXINV)
      COUNT= COUNT + 1
C*****
C*    Check for convergence
C*****
      T1 = DABS(ESTIM1(1) - AM1) / (EPS * MAX(DABS(AM1), 1.0D0))
      T2 = DABS(ESTIM1(2) - BM1) / (EPS * MAX(DABS(BM1), 1.0D0))
      T3 = DABS(RHAT(1) - RHO1M1) / (EPS * MAX(DABS(RHO1M1), 1.0D0))
      T4 = DABS(RHAT(2) - RHO2M1) / (EPS * MAX(DABS(RHO2M1), 1.0D0))
      IF (.NOT. (MAX(T1, T2, T3, T4) .LT. 1.0D0) .AND. COUNT .LE. 100)
     &   GO TO 1000
      IF (COUNT .GE. 100) MAXFLAG= 1
C*****
C*    Compute the variance
C*****
      CALL ATIMB(USTAR, Q, UHAT, SIZE, SIZE, 1)
      CALL ATIMB(SSQU, USTAR, USTAR, 1, SIZE, 1)
C*****
C*    Save the iterative INS estimates.
C*****
      ESTIM1(3) = RHAT(1)
      ESTIM1(4) = RHAT(2)
      ESTIM1(5) = (ESTIM1(1) - ALPHA)**2
      ESTIM1(6) = (ESTIM1(2) - BETA)**2
      ESTIM1(7) = (ESTIM1(3) - RHO1)**2
      ESTIM1(8) = (ESTIM1(4) - RHO2)**2
      ESTIM1(9) = COUNT
      ESTIM1(10) = SSQU / 23.0 * XXINV(1,1)
      ESTIM1(11) = DSQRT(ESTIM1(10))
      ESTIM1(12) = SSQU / 23.0 * XXINV(2,2)
      ESTIM1(13) = DSQRT(ESTIM1(12))
      ESTIM1(14) = MAXFLAG
      RETURN
      END
C ************************** END INS **********************************
```

APPENDIX B:

TABLES 2-51

Table 2. BEST Estimators in Terms of RMSE for the Experiment with Stationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

1

4 1 4

4 7 1 2 3

4 3 2 7 2 2 4

4 4 4 2 4 7 2 2 4

RMSE $\hat{\sigma}_{\beta_1}$

4

5 1 4

7 7 1 2 3

4 7 7 7 1 1 4

4 4 4 4 4 4 2 4 4

RMSE $\hat{\beta}_2$

3

2 2 3

4 2 1 2 6

4 3 4 2 2 4 4

4 4 4 3 4 4 4 4 6

RMSE $\hat{\sigma}_{\beta_2}$

4

2 1 5

7 1 1 1 6

4 4 7 1 5 4 6

4 4 4 4 5 4 4 4 6

*Estimator 1=OLS, 2=TSPW, 3=IPW, 4=FML, 5=TSCO, 6=ICO, and 7=INS. The location of the values in each triangle corresponds to the respective combinations of θ_1 and θ_2 represented by the center of the subtriangles in Figure 2.

Table 3. BEST Estimators in Terms of RMSE for the Experiment with Nonstationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

4

2 1 2

7 7 1 1 2

7 7 2 7 7 1 7

7 7 7 7 7 7 7 7 7

RMSE $\hat{\sigma}_{\beta_1}$

4

2 1 2

7 2 1 1 7

7 7 7 2 1 1 7

7 7 7 7 7 7 7 7 7

RMSE $\hat{\beta}_2$

3

2 2 7

7 2 1 2 7

7 7 7 2 2 7 7

7 7 7 7 7 7 7 7 7

RMSE $\hat{\sigma}_{\beta_2}$

4

2 1 5

2 1 1 1 7

7 7 7 5 1 7 4

7 7 7 7 7 7 7 7 7

*Estimator 1=OLS, 2=TSPW, 3=IPW, 4=FML, 5=TSCO, 6=ICO, and 7=INS. The location of the values in each triangle corresponds to the respective combinations of θ_1 and θ_2 represented by the center of the subtriangles in Figure 2.

Table 4. BEST Estimators in Terms of RMSE for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

4

4 4 4

4 7 1 4 4

4 2 4 7 1 4 4

4 3 4 2 4 4 4 4 4

RMSE $\hat{\sigma}_{\beta_1}$

4

4 1 4

7 7 1 4 4

4 4 7 7 1 1 4

4 4 4 4 4 4 4 4 4

RMSE $\hat{\beta}_2$

3

2 2 4

4 7 1 4 4

4 4 4 7 1 4 4

3 4 4 2 4 4 4 4 4

RMSE $\hat{\sigma}_{\beta_2}$

3

2 1 4

4 4 1 4 4

4 4 4 4 4 4 4

4 4 4 4 4 4 4 4 4

*Estimator 1=OLS, 2=TSPW, 3=IPW, 4=FML, 5=TSCO, 6=ICO, and 7=INS. The location of the values in each triangle corresponds to the respective combinations of θ_1 and θ_2 represented by the center of the subtriangles in Figure 2.

Table 5. BEST Estimators in Terms of RMSE for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

3

1 2 4

7 7 1 1 4

7 7 7 7 7 7 7

7 7 7 7 7 7 7 7 7

RMSE $\hat{\sigma}_{\beta_1}$

4

4 1 3

7 4 1 4 3

7 7 7 7 1 1 7

7 7 7 7 7 7 7 7 4

RMSE $\hat{\beta}_2$

3

1 2 4

7 7 1 1 4

7 7 7 7 7 7 7

7 7 7 7 7 7 7 7 7

RMSE $\hat{\sigma}_{\beta_2}$

3

4 1 7

7 4 1 7 7

7 7 4 4 1 7 7

7 7 7 7 7 4 7 7 7

*Estimator 1=OLS, 2=TSPW, 3=IPW, 4=FML, 5=TSCO, 6=ICO, and 7=INS. The location of the values in each triangle corresponds to the respective combinations of θ_1 and θ_2 represented by the center of the subtriangles in Figure 2.

Table 6. BEST Estimators in Terms of RMSE for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

3
4 1 3
4 2 1 4 4
4 4 2 7 2 4 4
4 4 4 4 4 7 2 4 4

RMSE $\hat{\sigma}_{\beta_1}$

1
7 1 4
2 7 1 4 4
4 4 7 7 1 4 4
4 4 4 4 4 4 4 4 4

RMSE $\hat{\beta}_2$

2
2 1 4
4 2 1 4 4
4 4 2 7 2 4 4
4 4 4 3 4 7 7 4 4

RMSE $\hat{\sigma}_{\beta_2}$

3
7 1 4
7 7 1 4 4
4 4 7 7 1 4 4
4 4 4 4 4 4 4 4 4

*Estimator 1=OLS, 2=TSPW, 3=IPW, 4=FML, 5=TSCO, 6=ICO, and 7=INS. The location of the values in each triangle corresponds to the respective combinations of θ_1 and θ_2 represented by the center of the subtriangles in Figure 2.

Table 7. BEST Estimators in Terms of RMSE for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

7

2 2 4

7 7 1 1 3

7 7 7 7 1 7 7

7 7 7 7 7 7 7 7 7

RMSE $\hat{\sigma}_{\beta_1}$

7

2 1 3

7 7 1 7 7

7 7 7 7 1 7 7

7 7 7 7 7 7 7 7 7

RMSE $\hat{\beta}_2$

4

2 2 2

7 7 1 1 4

7 7 7 7 1 7 7

7 7 7 7 7 7 7 7 7

RMSE $\hat{\sigma}_{\beta_2}$

3

2 1 4

7 7 1 1 4

7 7 7 7 1 7 7

7 7 7 7 7 7 7 7 7

*Estimator 1=OLS, 2=TSPW, 3=IPW, 4=FML, 5=TSCO, 6=ICO, and 7=INS. The location of the values in each triangle corresponds to the respective combinations of θ_1 and θ_2 represented by the center of the subtriangles in Figure 2.

Table 8. BEST Estimators in Terms of RMSE for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

			1					
			2	1	4			
		4	7	1	2	4		
	2	2	7	7	7	2	4	
4	3	2	4	4	7	7	2	4

RMSE $\hat{\sigma}_{\beta_1}$

			4					
			7	1	4			
		7	7	1	2	4		
	4	7	7	7	1	1	4	
4	4	4	4	4	4	4	2	4

RMSE $\hat{\beta}_2$

			3					
			2	3	2			
		4	4	1	4	5		
	2	4	4	7	2	2	5	
5	5	4	4	3	4	2	5	5

RMSE $\hat{\sigma}_{\beta_2}$

			4					
			7	1	4			
		7	7	1	4	5		
	5	4	7	7	1	5	5	
4	4	4	4	4	4	5	5	5

*Estimator 1=OLS, 2=TSPW, 3=IPW, 4=FML, 5=TSCO, 6=ICO, and 7=INS. The location of the values in each triangle corresponds to the respective combinations of θ_1 and θ_2 represented by the center of the subtriangles in Figure 2.

Table 9. BEST Estimators in Terms of RMSE for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

			2					
			1	1	2			
		7	7	1	1	2		
	7	7	7	7	1	1	7	
7	7	7	7	7	7	7	7	7

RMSE $\hat{\sigma}_{\beta_1}$

			4					
			4	1	4			
		7	7	1	1	7		
	7	7	7	7	1	1	7	
7	7	7	7	7	7	7	7	7

RMSE $\hat{\beta}_2$

			7					
			1	7	7			
		4	7	1	7	7		
	7	7	7	7	7	7	7	
7	7	7	3	7	7	7	7	7

RMSE $\hat{\sigma}_{\beta_2}$

			7					
			7	1	7			
		7	7	1	7	7		
	7	7	7	7	1	7	7	
7	7	7	7	7	7	7	7	7

*Estimator 1=OLS, 2=TSPW, 3=IPW, 4=FML, 5=TSCO, 6=ICO, and 7=INS. The location of the values in each triangle corresponds to the respective combinations of θ_1 and θ_2 represented by the center of the subtriangles in Figure 2.

Table 10. Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Stationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.27				
			1.02	1.18	1.10			
		1.00	1.00	1.01	1.04	1.50		
	1.01	1.00	1.00	1.00	1.01	1.06	4.73	
1.08	1.02	1.01	1.01	1.01	1.00	1.02	1.49	8.17

RMSE $\hat{\sigma}_{\beta_1}$

				1.34				
			1.04	1.51	1.14			
		1.00	1.00	2.23	1.17	1.84		
	1.24	1.00	1.00	1.00	1.24	1.20	10.71	
3.17	1.58	1.22	1.11	1.06	1.09	1.33	2.34	22.73

Table 10--Continued.

RMSE $\hat{\beta}_2$

				1.04				
			1.02	1.02	1.01			
		1.02	1.01	1.08	1.02	1.06		
	1.05	1.01	1.00	1.02	1.01	1.05	1.10	
1.28	1.12	1.07	1.04	1.01	1.02	1.04	1.21	1.14

RMSE $\hat{\sigma}_{\beta_2}$

				1.06				
			1.08	1.24	1.00			
		1.00	1.15	1.87	1.15	1.34		
	1.18	1.01	1.00	1.04	1.02	1.11	2.16	
2.82	1.45	1.19	1.06	1.00	1.15	1.65	2.96	3.93

*Efficiency = INS / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 11. Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.03				
			1.02	1.10	1.13			
		1.00	1.00	1.01	1.11	1.09		
	1.00	1.00	1.00	1.00	1.00	1.03	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_1}$

				1.03				
			1.08	1.32	1.09			
		1.00	1.01	2.31	1.23	1.00		
	1.00	1.00	1.00	1.02	1.25	1.16	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

able 11.--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.02	1.02	1.00			
		1.00	1.01	1.09	1.01	1.00		
	1.00	1.00	1.00	1.02	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_2}$

				1.02				
			1.14	1.15	1.06			
		1.01	1.18	1.79	1.05	1.00		
	1.00	1.00	1.00	1.06	1.08	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

'ficiency = INS / BEST, where BEST is the estimator that had the)west RMSE for the particular combination of θ_1 and θ_2. The)cation of the ratio corresponds to the subtriangles defined in gure 2.

Table 12. Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.05				
			1.00	1.00	1.07			
		1.01	1.00	1.01	1.02	1.11		
	1.06	1.01	1.00	1.00	1.01	1.12	1.62	
1.30	1.11	1.05	1.03	1.02	1.01	1.05	2.19	2.64

RMSE $\hat{\sigma}_{\beta_1}$

				1.06				
			1.01	1.22	1.10			
		1.00	1.00	2.14	1.05	1.16		
	1.19	1.00	1.00	1.00	1.17	1.60	1.94	
2.57	1.41	1.13	1.06	1.04	1.05	1.31	4.52	4.49

Table 12.--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.01	1.00	1.08			
		1.02	1.00	1.02	1.07	1.50		
	1.16	1.03	1.00	1.00	1.03	1.44	3.29	
1.69	1.25	1.14	1.08	1.05	1.01	1.12	3.39	4.79

RMSE $\hat{\sigma}_{\beta_2}$

				1.07				
			1.01	1.20	1.08			
		1.04	1.01	2.06	1.14	1.75		
	1.40	1.08	1.02	1.01	1.09	2.08	6.34	
3.64	1.86	1.45	1.16	1.12	1.06	1.37	9.69	10.36

*Efficiency = INS / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 13. Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.03				
			1.00	1.00	1.01			
		1.00	1.00	1.01	1.00	1.02		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_1}$

				1.02				
			1.02	1.18	1.02			
		1.00	1.00	2.05	1.01	1.02		
	1.00	1.00	1.00	1.00	1.36	1.06	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 13--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.00	1.02			
		1.00	1.00	1.01	1.00	1.02		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_2}$

				1.04				
			1.00	1.17	1.00			
		1.00	1.02	1.90	1.00	1.00		
	1.00	1.00	1.02	1.03	1.37	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.02	1.00	1.00	1.00

*Efficiency = INS / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 14. Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.01				
			1.00	1.02	1.05			
		1.02	1.00	1.01	1.06	1.35		
	1.07	1.01	1.00	1.00	1.01	1.14	2.41	
1.37	1.12	1.07	1.04	1.03	1.00	1.00	1.67	4.12

RMSE $\hat{\sigma}_{\beta_1}$

				1.04				
			1.00	1.26	1.04			
		1.00	1.00	2.02	1.10	1.55		
	1.21	1.02	1.00	1.00	1.15	1.23	4.47	
2.70	1.42	1.19	1.06	1.04	1.03	1.22	2.90	11.70

Table 14--Continued.

RMSE $\hat{\beta}_2$

				1.04				
			1.00	1.02	1.03			
		1.01	1.00	1.01	1.04	1.17		
	1.06	1.01	1.00	1.00	1.01	1.09	1.37	
1.29	1.09	1.05	1.03	1.02	1.00	1.00	1.43	1.50

RMSE $\hat{\sigma}_{\beta_2}$

				1.09				
			1.00	1.28	1.03			
		1.00	1.00	2.01	1.08	1.24		
	1.21	1.01	1.00	1.00	1.20	1.15	1.75	
2.70	1.42	1.18	1.05	1.02	1.01	1.20	2.28	3.44

*Efficiency = INS / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 15. Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.01	1.04			
		1.00	1.00	1.02	1.01	1.03		
	1.00	1.00	1.00	1.00	1.01	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_1}$

				1.00				
			1.00	1.21	1.02			
		1.00	1.00	2.08	1.00	1.00		
	1.00	1.00	1.00	1.00	1.44	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 15--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.00	1.01	1.06			
		1.00	1.00	1.02	1.05	1.05		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_2}$

				1.04				
			1.00	1.22	1.09			
		1.00	1.00	2.05	1.08	1.01		
	1.00	1.00	1.00	1.00	1.43	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Efficiency = INS / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

able 16. Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.28				
			1.00	1.13	1.08			
		1.01	1.00	1.02	1.04	1.40		
	1.02	1.00	1.00	1.00	1.00	1.08	2.81	
1.19	1.06	1.03	1.02	1.01	1.00	1.00	1.17	3.56

RMSE $\hat{\sigma}_{\beta_1}$

				1.32				
			1.00	1.39	1.12			
		1.00	1.00	2.26	1.12	1.59		
	1.21	1.00	1.00	1.00	1.19	1.28	4.55	
3.00	1.47	1.24	1.10	1.01	1.07	1.29	2.03	7.45

Table 16--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.00	1.00	1.04			
		1.01	1.00	1.01	1.03	1.26		
	1.02	1.00	1.00	1.00	1.02	1.26	2.03	
1.07	1.02	1.01	1.00	1.01	1.05	1.33	2.97	2.94

RMSE $\hat{\sigma}_{\beta_2}$

				1.02				
			1.00	1.44	1.06			
		1.00	1.00	1.78	1.06	1.70		
	1.26	1.00	1.00	1.00	1.47	1.74	4.12	
3.21	1.60	1.28	1.12	1.05	1.06	1.86	7.32	9.31

*Efficiency = INS / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 17. Relative Efficiency of the INS Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.04				
			1.04	1.10	1.07			
		1.00	1.00	1.01	1.08	1.02		
	1.00	1.00	1.00	1.00	1.02	1.03	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_1}$

				1.03				
			1.07	1.37	1.00			
		1.00	1.00	2.27	1.16	1.00		
	1.00	1.00	1.00	1.00	1.53	1.19	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 17--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.00	1.00			
		1.00	1.00	1.01	1.00	1.00		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_2}$

				1.00				
			1.00	1.44	1.00			
		1.00	1.00	1.85	1.00	1.00		
	1.00	1.00	1.00	1.00	1.43	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Efficiency = INS / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 18. Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.15				
			1.01	1.34	1.01			
		1.02	1.01	1.16	1.15	1.00		
	1.00	1.00	1.00	1.01	1.08	1.05	2.97	
1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.03	5.81

RMSE $\hat{\sigma}_{\beta_1}$

				43.63				
			1.26	2.03	34.76			
		76.22	1.10	3.02	1.34	1.00		
	1.05	1.04	1.08	1.10	1.48	1.19	218.49	
1.07	1.06	1.07	1.06	1.06	1.06	1.07	35.84	438.58

Table 18--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.02	1.03	1.00			
		1.01	1.02	1.09	1.02	1.04		
	1.00	1.00	1.00	1.02	1.02	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02

RMSE $\hat{\sigma}_{\beta_2}$

				1.02				
			1.15	1.31	1.04			
		1.08	1.25	2.02	1.26	1.22		
	1.06	1.04	1.11	1.16	1.17	1.07	1.93	
1.05	1.04	1.05	1.09	1.13	1.10	1.08	1.03	18.30

*Efficiency = IPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 19. Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.02				
			1.01	1.23	1.12			
		1.00	1.02	1.15	1.67	1.19		
	1.00	1.00	1.00	1.03	1.22	1.63	1.27	
1.00	1.00	1.00	1.00	1.00	1.01	1.05	1.18	1.88

RMSE $\hat{\sigma}_{\beta_1}$

				27.48				
			1.17	14.61	1.12			
		1.09	1.12	2.99	2.48	1.34		
	1.07	1.08	1.06	1.14	2.03	2.99	1.90	
1.07	1.09	1.08	1.06	1.08	1.06	1.17	1.48	6.17

Table 19--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.03	1.02	1.01			
		1.01	1.02	1.11	1.03	1.02		
	1.01	1.01	1.01	1.03	1.02	1.03	1.01	
1.02	1.01	1.02	1.01	1.01	1.01	1.01	1.01	1.01

RMSE $\hat{\sigma}_{\beta_2}$

				1.03				
			1.20	1.21	1.14			
		1.08	1.28	1.96	1.19	1.10		
	1.07	1.09	1.09	1.19	1.22	1.15	1.06	
1.08	1.09	1.11	1.13	1.14	1.11	1.12	1.08	2.09

*Efficiency = IPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 20. Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.01	1.01			
		1.00	1.00	1.01	1.00	1.67		
	1.00	1.00	1.00	1.00	1.00	2.53	20.01	
1.00	1.00	1.00	1.00	1.00	1.01	1.01	14.86	49.00

RMSE $\hat{\sigma}_{\beta_1}$

				15.51				
			24.81	1.25	17.30			
		1.05	1.05	2.24	1.04	12.46		
	1.05	1.04	1.05	1.08	1.20	39.00	76.40	
1.07	1.06	1.10	1.06	1.07	1.08	1.07	119.57	191.56

Table 20--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.01	1.00			
		1.00	1.01	1.03	1.00	1.57		
	1.00	1.00	1.00	1.02	1.02	1.70	17.88	
1.00	1.00	1.00	1.00	1.00	1.01	1.03	7.70	37.78

RMSE $\hat{\sigma}_{\beta_2}$

				1.00				
			1.06	1.23	1.01			
		1.05	1.04	2.12	1.02	3.66		
	1.05	1.05	1.04	1.06	1.06	7.65	52.32	
1.08	1.08	1.11	1.06	1.08	1.09	1.11	35.73	85.57

*Efficiency = IPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 21. Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.00	1.00			
		1.01	1.00	1.01	1.01	1.00		
	1.02	1.01	1.01	1.00	1.01	1.02	1.01	
1.03	1.02	1.02	1.01	1.02	1.01	1.02	1.06	1.86

RMSE $\hat{\sigma}_{\beta_1}$

				10.61				
			21.46	1.20	1.00			
		1.07	1.05	2.15	1.04	1.00		
	1.06	1.08	1.05	1.07	1.52	1.15	1.05	
1.08	1.09	1.11	1.09	1.09	1.08	1.09	1.17	14.46

Table 21--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.01	1.00	1.00			
		1.03	1.01	1.03	1.02	1.00		
	1.04	1.03	1.01	1.01	1.05	1.08	1.14	
1.08	1.05	1.05	1.04	1.04	1.03	1.05	1.18	1.96

RMSE $\hat{\sigma}_{\beta_2}$

				1.00				
			1.06	1.19	1.04			
		1.07	1.05	1.95	1.14	1.15		
	1.07	1.06	1.05	1.05	1.60	1.44	1.58	
1.10	1.12	1.13	1.08	1.09	1.09	1.12	1.58	5.82

:fficiency = IPW / BEST, where BEST is the estimator that had the owest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 22. Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.04	1.00			
		1.00	1.00	1.03	1.01	1.02		
	1.00	1.00	1.00	1.01	1.01	1.02	1.63	
1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.06	4.99

RMSE $\hat{\sigma}_{\beta_1}$

				8.31				
			10.62	1.31	16.87			
		1.05	1.04	2.18	1.03	1.02		
	1.04	1.06	1.06	1.09	1.18	1.09	96.38	
1.08	1.03	1.05	1.09	1.08	1.08	1.08	1.22	192.38

Table 22--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.00	1.03	1.02			
		1.00	1.00	1.03	1.01	1.04		
	1.00	1.00	1.00	1.01	1.00	1.02	1.11	
1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.05	1.15

RMSE $\hat{\sigma}_{\beta_2}$

				1.00				
			1.06	1.30	1.03			
		1.06	1.05	2.18	1.05	1.07		
	1.04	1.06	1.06	1.09	1.25	1.08	1.69	
1.07	1.04	1.07	1.09	1.09	1.09	1.09	1.18	2.75

*Efficiency = IPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 23. Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.02				
			1.01	1.01	1.01			
		1.01	1.01	1.05	1.08	1.00		
	1.02	1.01	1.00	1.02	1.05	1.11	1.09	
1.03	1.02	1.01	1.02	1.02	1.01	1.03	1.09	1.28

RMSE $\hat{\sigma}_{\beta_1}$

				8.44				
			1.04	1.23	1.00			
		1.06	1.05	2.27	1.20	1.10		
	1.08	1.04	1.05	1.08	1.67	1.44	1.50	
1.10	1.11	1.09	1.09	1.08	1.08	1.12	1.33	6.35

Table 23--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.01	1.02	1.04			
		1.01	1.01	1.04	1.09	1.04		
	1.01	1.01	1.00	1.02	1.04	1.08	1.05	
1.03	1.01	1.01	1.01	1.01	1.01	1.02	1.06	1.08

RMSE $\tilde{\sigma}_{\beta_2}$

				1.00				
			1.04	1.26	1.04			
		1.06	1.06	2.24	1.23	1.04		
	1.09	1.04	1.06	1.11	1.65	1.29	1.19	
1.08	1.11	1.09	1.10	1.08	1.08	1.12	1.22	1.30

*Efficiency = IPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 24. Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.17				
			1.04	1.25	1.01			
		1.00	1.01	1.07	1.11	1.01		
	1.00	1.00	1.00	1.11	1.06	1.04	1.53	
1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.05	6.46

RMSE $\hat{\sigma}_{\beta_1}$

				37.05				
			39.13	1.66	34.34			
		1.07	1.10	2.53	1.25	34.23		
	1.05	1.06	1.06	1.37	1.42	1.17	76.10	
1.04	1.07	1.04	1.08	1.09	1.06	1.06	1.18	191.56

Table 24--Continued.

RMSE $\hat{\beta}_{2}$

				1.00				
			1.00	1.00	1.00			
		1.00	1.00	1.01	1.00	1.11		
	1.00	1.00	1.00	1.00	1.00	1.04	1.28	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.03	1.47

RMSE $\hat{\sigma}_{\beta_{2}}$

				1.01				
			1.05	1.47	1.03			
		1.09	1.06	1.88	1.02	1.51		
	1.04	1.04	1.06	1.10	1.51	1.14	1.64	
1.07	1.06	1.05	1.07	1.06	1.05	1.05	1.12	2.91

*Efficiency = IPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 25. Relative Efficiency of the IPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.03				
			1.01	1.21	1.08			
		1.00	1.03	1.17	1.59	1.12		
	1.00	1.00	1.00	1.13	1.13	1.70	1.22	
1.01	1.02	1.01	1.01	1.00	1.01	1.05	1.21	1.14

RMSE $\hat{\sigma}_{\beta_1}$

				36.00				
			1.13	1.60	7.67			
		1.07	1.11	3.23	2.24	1.34		
	1.07	1.07	1.05	1.44	2.06	3.11	1.81	
1.07	1.14	1.06	1.10	1.10	1.05	1.21	1.55	12.09

Table 25--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.01	1.00	1.01			
		1.00	1.00	1.01	1.02	1.09		
	1.00	1.00	1.00	1.00	1.01	1.10	1.47	
1.00	1.00	1.00	1.00	1.00	1.01	1.05	1.33	1.56

RMSE $\hat{\sigma}_{\beta_2}$

				1.02				
			1.04	1.48	1.07			
		1.06	1.06	1.96	1.07	1.26		
	1.09	1.08	1.07	1.09	1.57	1.22	2.13	
1.07	1.11	1.11	1.09	1.08	1.09	1.20	1.72	2.04

*Efficiency = IPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 26. Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.01				
			1.00	1.06	1.05			
		1.01	1.00	1.02	1.00	1.10		
	1.02	1.01	1.00	1.00	1.00	1.00	1.52	
1.19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.09

RMSE $\hat{\sigma}_{\beta_1}$

				22.64				
			26.89	1.20	19.54			
		1.29	1.00	2.12	1.00	1.33		
	2.48	1.15	1.05	1.00	1.18	1.05	34.90	
9.21	1.58	1.12	1.05	1.04	1.06	1.00	1.01	18.45

Table 26--Continued.

RMSE $\hat{\beta}_2$

				1.06				
			1.00	1.00	1.03			
		1.04	1.00	1.05	1.00	1.13		
	1.17	1.03	1.00	1.00	1.00	1.05	1.21	
1.79	1.03	1.00	1.01	1.00	1.01	1.02	1.03	1.41

RMSE $\hat{\sigma}_{\beta_2}$

				1.16				
			1.00	1.14	1.05			
		1.00	1.09	1.69	1.11	1.31		
	1.69	1.09	1.02	1.03	1.03	1.14	1.68	
6.55	1.51	1.11	1.06	1.06	1.09	1.09	1.27	5.52

*Efficiency = TSPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 27. Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.01				
			1.00	1.02	1.00			
		1.00	1.00	1.02	1.10	1.00		
	1.03	1.00	1.00	1.00	1.02	1.09	1.04	
1.18	1.00	1.00	1.00	1.00	1.01	1.01	1.05	1.31

RMSE $\hat{\sigma}_{\beta_1}$

				16.81				
			1.00	1.10	1.00			
		1.16	1.00	2.14	1.15	1.06		
	2.41	1.16	1.05	1.00	1.25	1.31	1.42	
8.91	1.74	1.15	1.07	1.06	1.05	1.00	1.08	2.15

Table 27--Continued.

RMSE $\hat{\beta}_{22}$

				1.05				
			1.00	1.00	1.02			
		1.03	1.00	1.05	1.00	1.14		
	1.18	1.04	1.01	1.00	1.00	1.07	1.27	
1.83	1.06	1.03	1.01	1.01	1.01	1.04	1.06	1.78

RMSE $\hat{\sigma}_{\beta_{22}}$

				1.15				
			1.00	1.08	1.09			
		1.00	1.10	1.58	1.03	1.33		
	1.75	1.10	1.03	1.02	1.08	1.18	1.76	
6.25	1.59	1.13	1.09	1.09	1.07	1.14	1.31	4.69

*Efficiency = TSPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_{22}. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 28. Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.00	1.00			
		1.00	1.00	1.01	1.00	1.00		
	1.00	1.00	1.00	1.00	1.00	1.00	8.46	
1.01	1.00	1.00	1.00	1.00	1.00	1.01	1.03	21.50

RMSE $\hat{\sigma}_{\beta_1}$

				9.05				
			1.01	1.18	11.25			
		1.02	1.01	2.12	1.02	4.59		
	1.09	1.01	1.03	1.03	1.15	1.21	38.76	
1.29	1.04	1.05	1.03	1.04	1.04	1.03	1.04	103.97

Table 28--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.00	1.01			
		1.00	1.00	1.02	1.01	1.03		
	1.00	1.00	1.00	1.01	1.01	1.02	8.35	
1.01	1.00	1.00	1.00	1.00	1.00	1.01	1.07	21.58

RMSE $\hat{\sigma}_{\beta_2}$

				1.02				
			1.00	1.20	1.04			
		1.00	1.00	2.04	1.03	1.10		
	1.08	1.01	1.02	1.01	1.02	1.09	23.80	
1.24	1.04	1.05	1.03	1.04	1.05	1.05	1.15	53.44

*Efficiency = TSPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 29. Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.00	1.00			
		1.01	1.00	1.01	1.00	1.01		
	1.01	1.01	1.00	1.00	1.01	1.01	1.01	
1.03	1.02	1.01	1.01	1.01	1.01	1.01	1.05	1.05

RMSE $\hat{\sigma}_{\beta_1}$

				7.32				
			1.03	1.16	1.04			
		1.03	1.01	2.04	1.01	1.07		
	1.04	1.05	1.02	1.02	1.38	1.09	1.10	
1.07	1.07	1.08	1.06	1.06	1.05	1.04	1.12	1.09

Table 29--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.01	1.00	1.01			
		1.02	1.01	1.02	1.01	1.02		
	1.04	1.02	1.01	1.01	1.02	1.05	1.15	
1.07	1.05	1.04	1.04	1.03	1.02	1.04	1.15	1.46

RMSE $\hat{\sigma}_{\beta_2}$

				1.02				
			1.01	1.17	1.09			
		1.02	1.01	1.87	1.12	1.23		
	1.05	1.03	1.02	1.01	1.47	1.37	1.68	
1.10	1.09	1.09	1.06	1.05	1.04	1.07	1.50	2.09

*Efficiency = TSPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 30. Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.02	1.02			
		1.00	1.00	1.02	1.00	1.08		
	1.00	1.00	1.00	1.00	1.00	1.01	1.55	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	3.86

RMSE $\hat{\sigma}_{\beta_1}$

				4.31				
			10.61	1.24	10.90			
		1.00	1.01	2.05	1.01	1.22		
	1.03	1.01	1.02	1.03	1.15	1.06	38.20	
1.15	1.03	1.01	1.05	1.05	1.05	1.03	1.07	48.12

Table 30--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.01	1.00			
		1.00	1.00	1.02	1.00	1.03		
	1.00	1.00	1.00	1.00	1.00	1.01	1.12	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.15

RMSE $\hat{\sigma}_{\beta_2}$

				1.01				
			1.01	1.23	1.01			
		1.00	1.01	2.05	1.01	1.11		
	1.03	1.01	1.03	1.04	1.21	1.05	1.40	
1.16	1.03	1.01	1.05	1.06	1.05	1.03	1.06	1.89

*Efficiency = TSPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 31. Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.03				
			1.00	1.00	1.01			
		1.01	1.01	1.02	1.03	1.02		
	1.01	1.01	1.00	1.01	1.02	1.05	1.14	
1.03	1.02	1.01	1.01	1.01	1.01	1.01	1.07	1.36

RMSE $\hat{\sigma}_{\beta_1}$

				1.05				
			1.00	1.18	1.04			
		1.02	1.02	2.10	1.09	1.21		
	1.05	1.02	1.02	1.02	1.51	1.28	1.74	
1.14	1.07	1.05	1.06	1.05	1.06	1.06	1.24	2.52

Table 31--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.00	1.00			
		1.01	1.00	1.02	1.02	1.01		
	1.01	1.00	1.00	1.01	1.01	1.03	1.05	
1.03	1.01	1.01	1.01	1.01	1.01	1.01	1.04	1.18

RMSE $\hat{\sigma}_{\beta_2}$

				1.01				
			1.00	1.18	1.01			
		1.02	1.02	2.07	1.04	1.07		
	1.06	1.02	1.03	1.02	1.50	1.16	1.32	
1.14	1.07	1.04	1.07	1.05	1.06	1.05	1.16	1.76

*Efficiency = TSPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 32. Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.11				
			1.00	1.07	1.01			
		1.00	1.00	1.02	1.00	1.04		
	1.00	1.00	1.00	1.03	1.01	1.00	4.60	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	8.54

RMSE$\hat{\sigma}_{\beta_1}$

				33.88				
			1.09	1.26	30.28			
		1.05	1.05	2.25	1.00	25.76		
	1.02	1.04	1.05	1.11	1.23	1.06	78.21	
1.05	1.05	1.03	1.07	1.07	1.05	1.02	1.00	128.99

able 32--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.00	1.00			
		1.00	1.00	1.01	1.00	1.04		
	1.00	1.00	1.00	1.00	1.00	1.00	1.16	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.41

RMSE $\hat{\sigma}_{\beta_2}$

				1.01				
			1.02	1.47	1.01			
		1.05	1.04	1.85	1.01	1.14		
	1.02	1.02	1.05	1.08	1.49	1.03	1.39	
1.04	1.04	1.04	1.06	1.05	1.04	1.03	1.02	2.42

Efficiency = TSPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 33. Relative Efficiency of the TSPW Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.05	1.00			
		1.00	1.00	1.02	1.13	1.00		
	1.00	1.00	1.00	1.01	1.03	1.14	1.03	
1.00	1.01	1.01	1.01	1.00	1.00	1.02	1.04	1.04

RMSE $\hat{\sigma}_{\beta_1}$

				34.92				
			1.08	1.25	5.64			
		1.05	1.03	2.35	1.21	1.16		
	1.06	1.06	1.04	1.05	1.59	1.42	1.41	
1.05	1.12	1.05	1.08	1.09	1.05	1.06	1.04	7.11

Table 33--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.01	1.00	1.02			
		1.00	1.00	1.01	1.01	1.04		
	1.00	1.00	1.00	1.00	1.01	1.04	1.24	
1.00	1.00	1.00	1.00	1.00	1.01	1.05	1.25	1.49

RMSE $\hat{\sigma}_{\beta_2}$

				1.04				
			1.03	1.47	1.09			
		1.04	1.04	1.92	1.05	1.15		
	1.07	1.06	1.06	1.07	1.54	1.09	1.41	
1.05	1.09	1.10	1.08	1.07	1.08	1.17	1.50	1.67

*Efficiency = TSPW / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 34. Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Stationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.05				
			1.00	1.19	1.00			
		1.00	1.01	1.10	1.10	1.00		
	1.00	1.00	1.00	1.00	1.06	1.02	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.02	1.00

RMSE $\hat{\sigma}_{\beta_1}$

				1.00				
			1.02	1.50	1.00			
		1.04	1.03	2.57	1.20	1.03		
	1.00	1.00	1.03	1.02	1.36	1.08	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.00	1.00

Table 34--Continued

RMSE $\hat{\beta}_2$

				1.01				
			1.01	1.02	1.00			
		1.00	1.01	1.08	1.01	1.04		
	1.00	1.00	1.00	1.02	1.01	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_2}$

				1.00				
			1.10	1.25	1.01			
		1.01	1.18	1.90	1.19	1.16		
	1.00	1.00	1.03	1.07	1.07	1.00	1.00	
1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.03

*Efficiency = FML / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 35. Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and White Noise on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.10	1.06			
		1.00	1.01	1.10	1.49	1.14		
	1.00	1.00	1.00	1.01	1.15	1.58	1.23	
1.00	1.00	1.00	1.00	1.00	1.01	1.04	1.18	1.40

RMSE $\hat{\sigma}_{\beta_1}$

				1.00				
			1.05	1.28	1.06			
		1.02	1.04	2.59	2.04	1.26		
	1.02	1.03	1.02	1.03	1.67	2.78	1.85	
1.02	1.03	1.03	1.02	1.02	1.01	1.06	1.43	2.51

Table 35--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.03	1.02	1.00			
		1.00	1.01	1.10	1.02	1.01		
	1.01	1.01	1.01	1.02	1.01	1.03	1.01	
1.02	1.01	1.01	1.01	1.01	1.00	1.01	1.01	1.01

RMSE $\hat{\sigma}_{\beta_2}$

				1.00				
			1.16	1.17	1.09			
		1.02	1.20	1.83	1.11	1.05		
	1.03	1.03	1.03	1.09	1.13	1.08	1.00	
1.03	1.03	1.04	1.03	1.04	1.02	1.03	1.03	1.03

[k]Efficiency = FML / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 36. Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.00	1.00			
		1.00	1.00	1.01	1.00	1.00		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_1}$

				1.00				
			1.00	1.18	1.00			
		1.01	1.01	2.13	1.00	1.00		
	1.00	1.00	1.01	1.02	1.14	1.16	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 36--Continued.

RMSE $\hat{\beta}_{22}$

				1.00				
			1.00	1.00	1.00			
		1.00	1.00	1.02	1.00	1.00		
	1.00	1.00	1.00	1.01	1.02	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_{22}}$

				1.04				
			1.01	1.21	1.00			
		1.00	1.00	2.04	1.00	1.00		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

'Efficiency = FML / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 37. Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.00	1.00			
		1.01	1.00	1.01	1.00	1.00		
	1.01	1.01	1.00	1.00	1.01	1.02	1.01	
1.03	1.02	1.01	1.01	1.01	1.00	1.02	1.05	1.03

RMSE $\hat{\sigma}_{\beta_1}$

				1.00				
			1.00	1.15	1.01			
		1.02	1.00	2.04	1.00	1.00		
	1.03	1.03	1.01	1.01	1.43	1.10	1.02	
1.04	1.04	1.04	1.03	1.03	1.01	1.03	1.11	1.00

able 37--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.01	1.00	1.00			
		1.02	1.01	1.02	1.02	1.00		
	1.04	1.02	1.01	1.01	1.05	1.08	1.14	
1.07	1.05	1.04	1.03	1.03	1.02	1.05	1.17	1.36

RMSE $\hat{\sigma}_{\beta_2}$

				1.03				
			1.00	1.18	1.06			
		1.01	1.00	1.87	1.12	1.17		
	1.04	1.01	1.00	1.00	1.54	1.44	1.61	
1.07	1.06	1.06	1.04	1.02	1.00	1.06	1.62	1.82

fficiency = FML / BEST, where BEST is the estimator that had the owest RMSE for the particular combination of θ_1 and θ_2. The ocation of the ratio corresponds to the subtriangles defined in igure 2.

Table 38. Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.03	1.00			
		1.00	1.00	1.03	1.00	1.00		
	1.00	1.00	1.00	1.01	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_1}$

				1.01				
			1.01	1.28	1.00			
		1.01	1.00	2.09	1.00	1.00		
	1.00	1.00	1.00	1.02	1.12	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 38--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.01	1.00			
		1.00	1.00	1.02	1.00	1.00		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

RMSE $\hat{\sigma}_{\beta_2}$

				1.02				
			1.01	1.26	1.00			
		1.01	1.00	2.07	1.00	1.00		
	1.00	1.00	1.01	1.02	1.19	1.00	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Efficiency = FML / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 39. Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.03				
			1.00	1.00	1.00			
		1.01	1.01	1.04	1.07	1.00		
	1.01	1.01	1.00	1.02	1.05	1.11	1.10	
1.03	1.02	1.01	1.01	1.01	1.01	1.03	1.11	1.27

RMSE $\hat{\sigma}_{\beta_1}$

				1.06				
			1.00	1.20	1.02			
		1.01	1.01	2.17	1.16	1.13		
	1.03	1.01	1.01	1.02	1.58	1.43	1.58	
1.05	1.04	1.03	1.04	1.02	1.01	1.05	1.37	2.20

Table 39--Continued.

RMSE $\widehat{\beta}_2$

				1.00				
			1.00	1.00	1.00			
		1.01	1.01	1.03	1.05	1.00		
	1.01	1.00	1.00	1.01	1.03	1.07	1.02	
1.03	1.01	1.01	1.01	1.01	1.01	1.01	1.07	1.10

RMSE $\widehat{\sigma}_{\beta_2}$

				1.02				
			1.01	1.20	1.00			
		1.01	1.01	2.12	1.10	1.00		
	1.03	1.01	1.01	1.03	1.54	1.24	1.17	
1.05	1.04	1.03	1.04	1.02	1.01	1.03	1.21	1.37

'Efficiency = FML / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 40. Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.07				
			1.01	1.13	1.00			
		1.00	1.00	1.05	1.06	1.00		
	1.00	1.00	1.00	1.10	1.04	1.01	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.03	1.00

RMSE $\hat{\sigma}_{\beta_1}$

				1.00				
			1.05	1.37	1.00			
		1.02	1.02	2.33	1.12	1.00		
	1.00	1.01	1.02	1.27	1.29	1.08	1.00	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.00

Table 40--Continued.

RMSE $\hat{\beta}_2$

				1.00				
			1.00	1.00	1.00			
		1.00	1.00	1.01	1.00	1.10		
	1.00	1.00	1.00	1.00	1.00	1.01	1.32	
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.14

RMSE $\hat{\sigma}_{\beta_2}$

				1.00				
			1.03	1.45	1.00			
		1.04	1.02	1.82	1.00	1.31		
	1.00	1.00	1.02	1.05	1.45	1.05	1.74	
1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.04	1.37

*Efficiency = FML / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 41. Relative Efficiency of the FML Estimator to the BEST Estimator for the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.*

RMSE $\hat{\beta}_1$

				1.00				
			1.00	1.12	1.02			
		1.00	1.02	1.10	1.45	1.07		
	1.00	1.00	1.00	1.11	1.10	1.61	1.18	
1.00	1.01	1.01	1.01	1.00	1.00	1.07	1.24	1.14

RMSE $\hat{\sigma}_{\beta_1}$

				1.00				
			1.00	1.36	1.00			
		1.02	1.05	2.72	1.91	1.28		
	1.03	1.02	1.02	1.31	1.82	2.79	1.76	
1.02	1.05	1.03	1.02	1.03	1.01	1.16	1.58	1.78

Table 41--Continued.

RMSE $\hat{\beta}_2$

				1.01				
			1.01	1.00	1.01			
		1.00	1.00	1.01	1.01	1.07		
	1.00	1.00	1.00	1.00	1.01	1.06	1.38	
1.00	1.00	1.00	1.00	1.00	1.01	1.05	1.31	1.60

RMSE $\hat{\sigma}_{\beta_2}$

				1.01				
			1.02	1.46	1.04			
		1.02	1.02	1.89	1.04	1.15		
	1.04	1.03	1.03	1.04	1.51	1.12	1.88	
1.02	1.04	1.04	1.02	1.03	1.04	1.14	1.62	2.19

*Efficiency = FML / BEST, where BEST is the estimator that had the lowest RMSE for the particular combination of θ_1 and θ_2. The location of the ratio corresponds to the subtriangles defined in Figure 2.

Table 42. Iteration Statistics for the Four Iterative Estimators in the Experiment with Stationary Errors and White Noise on the Independent Variable.

		FML		ICO		IPW		INS	
θ_1	θ_2	AVE.[a]	LIMIT[b]	AVE.	LIMIT	AVE.	LIMIT	AVE.	LIMIT
-1.60	-0.80	10.30	0	4.53	0	4.41	0	4.34	0
-1.20	-0.80	4.69	0	4.27	0	4.24	0	4.15	0
-0.80	-0.80	3.80	0	4.40	0	4.38	0	4.30	0
-0.40	-0.80	3.58	0	4.54	0	4.57	0	4.48	0
0.00	-0.80	3.72	0	4.84	0	4.90	0	4.80	0
0.40	-0.80	3.95	0	4.89	0	4.97	0	4.89	0
0.80	-0.80	4.23	0	4.82	0	4.96	0	5.05	0
1.20	-0.80	5.01	1	4.67	0	5.15	2	6.63	0
1.60	-0.80	9.37	0	5.67	1	27.60	559	10.24	0
-1.20	-0.40	10.90	0	4.77	0	4.73	0	4.57	0
-0.80	-0.40	5.03	0	4.93	0	4.95	0	4.80	0
-0.40	-0.40	4.25	0	5.27	0	5.31	0	5.18	0
0.00	-0.40	4.48	0	5.87	0	5.97	0	5.76	0
0.40	-0.40	5.03	0	6.18	0	6.47	0	6.09	0
0.80	-0.40	5.77	0	5.83	0	6.53	0	6.58	0
1.20	-0.40	10.38	0	7.07	11	10.57	87	13.71	5
-0.80	0.00	10.69	3	5.48	0	5.51	0	5.20	0
-0.40	0.00	5.08	0	6.11	0	6.15	1	5.90	1
0.00	0.00	5.09	0	6.73	0	6.94	0	6.50	0
0.40	0.00	6.08	0	7.20	1	7.90	1	7.16	0
0.80	0.00	9.68	0	8.77	28	8.93	1	10.18	4
-0.40	0.40	9.74	2	6.41	0	6.45	0	6.20	0
0.00	0.40	6.06	0	7.26	3	7.79	0	7.50	0
0.40	0.40	7.91	1	10.00	41	9.03	1	8.90	1
0.00	0.80	8.27	1	11.23	65	7.48	8	9.13	4

[a]AVE. is the average number of iterations to reach convergence for that particular estimator over 2500 replications of the experiment.

[b]LIMIT is the number of times the iteration limit of 100 was encountered over 2500 replications of the experiment.

Table 43. Iteration Statistics for the Four Iterative Estimators in the Experiment with Nonstationary Errors and White Noise on the Independent Variable.

		FML		ICO		IPW		INS	
θ_1	θ_2	AVE.[a]	LIMIT[b]	AVE.	LIMIT	AVE.	LIMIT	AVE.	LIMIT
-1.60	-0.80	10.57	2	4.51	0	4.41	0	4.36	0
-1.20	-0.80	4.93	0	4.27	0	4.20	0	4.17	0
-0.80	-0.80	3.85	0	4.38	0	4.36	0	4.33	0
-0.40	-0.80	3.55	0	4.59	0	4.58	0	4.54	0
0.00	-0.80	3.64	0	4.83	0	4.87	0	4.80	0
0.40	-0.80	3.98	0	5.04	0	5.12	0	5.03	0
0.80	-0.80	4.41	0	4.97	0	5.19	0	5.01	0
1.20	-0.80	5.75	1	4.75	0	5.54	0	4.98	0
1.60	-0.80	11.86	1	5.79	6	6.99	5	6.52	0
-1.20	-0.40	10.31	0	4.73	0	4.66	0	4.56	0
-0.80	-0.40	5.03	0	4.92	0	4.92	0	4.83	0
-0.40	-0.40	4.27	0	5.39	0	5.38	0	5.27	0
0.00	-0.40	4.47	0	5.85	0	5.90	0	5.73	0
0.40	-0.40	5.24	0	6.19	0	6.76	3	6.09	0
0.80	-0.40	6.89	1	5.90	0	7.79	3	6.17	0
1.20	-0.40	12.03	2	6.85	15	7.85	0	8.02	0
-0.80	0.00	9.32	0	5.44	0	5.44	0	5.25	0
-0.40	0.00	5.08	0	6.06	0	6.08	0	5.86	0
0.00	0.00	5.18	0	6.86	1	7.01	0	6.55	0
0.40	0.00	7.13	0	7.15	1	9.05	5	7.29	1
0.80	0.00	11.51	2	8.56	25	9.21	2	9.01	0
-0.40	0.40	8.62	0	6.42	0	6.39	0	6.33	1
0.00	0.40	5.98	0	7.21	0	7.85	2	7.48	1
0.40	0.40	9.47	1	9.94	30	9.43	1	9.20	3
0.00	0.80	7.58	0	10.37	50	7.49	6	8.96	2

[a]AVE. is the average number of iterations to reach convergence for that particular estimator over 2500 replications of the experiment.

[b]LIMIT is the number of times the iteration limit of 100 was encountered over 2500 replications of the experiment.

Table 44. Iteration Statistics for the Four Iterative Estimators in the Experiment with Stationary Errors and a Dampened Trend on the Independent Variable.

		FML		ICO		IPW		INS	
θ_1	θ_2	AVE.[a]	LIMIT[b]	AVE.	LIMIT	AVE.	LIMIT	AVE.	LIMIT
-1.60	-0.80	10.49	0	4.21	0	4.22	0	3.65	0
-1.20	-0.80	4.71	0	4.06	0	4.21	0	3.64	0
-0.80	-0.80	3.77	0	4.06	0	4.23	0	3.70	0
-0.40	-0.80	3.43	0	4.04	0	4.25	0	3.76	0
0.00	-0.80	3.47	0	4.06	0	4.30	0	3.88	0
0.40	-0.80	3.85	0	4.17	0	4.46	0	4.26	0
0.80	-0.80	4.51	0	4.37	0	4.78	0	5.97	0
1.20	-0.80	5.46	0	5.00	0	6.23	13	11.22	0
1.60	-0.80	9.37	0	8.57	1	24.51	461	6.97	0
-1.20	-0.40	10.50	2	4.27	0	4.29	0	3.71	0
-0.80	-0.40	4.85	0	4.12	0	4.27	0	3.69	0
-0.40	-0.40	3.76	0	4.08	0	4.27	0	3.76	0
0.00	-0.40	3.62	0	4.15	0	4.41	0	3.91	0
0.40	-0.40	4.08	0	4.33	0	4.69	0	4.54	0
0.80	-0.40	5.24	0	5.00	0	5.58	1	8.30	2
1.20	-0.40	8.83	0	10.26	22	13.35	176	10.73	2
-0.80	0.00	9.70	0	4.45	0	4.40	0	3.85	0
-0.40	0.00	4.12	0	4.34	0	4.36	0	3.90	0
0.00	0.00	3.69	0	4.51	0	4.53	0	4.17	0
0.40	0.00	4.30	0	5.39	2	5.31	0	5.34	0
0.80	0.00	6.64	0	11.87	36	6.53	1	11.17	5
-0.40	0.40	8.38	0	5.01	0	4.58	0	4.29	0
0.00	0.40	4.25	0	6.19	6	5.03	0	5.06	0
0.40	0.40	5.14	0	13.20	62	6.06	0	6.93	0
0.00	0.80	7.81	0	14.87	83	5.88	4	6.13	1

[a]AVE. is the average number of iterations to reach convergence for that particular estimator over 2500 replications of the experiment.

[b]LIMIT is the number of times the iteration limit of 100 was encountered over 2500 replications of the experiment.

Table 45. Iteration Statistics for the Four Iterative Estimators in the Experiment with Nonstationary Errors and a Dampened Trend on the Independent Variable.

		FML		ICO		IPW		INS	
θ_1	θ_2	AVE.[a]	LIMIT[b]	AVE.	LIMIT	AVE.	LIMIT	AVE.	LIMIT
-1.60	-0.80	10.71	0	3.91	0	3.88	0	3.61	0
-1.20	-0.80	4.90	0	3.86	0	3.90	0	3.62	0
-0.80	-0.80	3.77	0	3.84	0	3.92	0	3.64	0
-0.40	-0.80	3.29	0	3.86	0	3.99	0	3.70	0
0.00	-0.80	3.27	0	3.88	0	4.04	0	3.76	0
0.40	-0.80	3.68	0	3.96	0	4.29	0	3.93	0
0.80	-0.80	4.39	0	4.08	0	4.64	0	4.29	0
1.20	-0.80	5.62	0	4.41	0	5.20	0	5.05	0
1.60	-0.80	9.79	0	6.79	6	6.12	11	5.72	0
-1.20	-0.40	9.91	0	4.00	0	4.03	0	3.62	0
-0.80	-0.40	4.83	0	3.99	0	4.09	0	3.63	0
-0.40	-0.40	3.73	0	4.03	0	4.19	0	3.75	0
0.00	-0.40	3.57	0	4.08	0	4.33	0	3.88	0
0.40	-0.40	4.24	0	4.32	0	4.90	0	4.30	0
0.80	-0.40	5.26	0	4.77	0	5.60	0	5.43	0
1.20	-0.40	8.01	0	9.42	25	5.76	0	6.98	0
-0.80	0.00	8.73	0	4.23	0	4.21	0	3.76	0
-0.40	0.00	4.06	0	4.27	0	4.25	0	3.85	0
0.00	0.00	3.69	0	4.58	0	4.58	0	4.15	0
0.40	0.00	4.34	0	5.37	2	5.40	0	5.14	0
0.80	0.00	5.97	0	11.79	52	5.89	0	7.05	0
-0.40	0.40	7.14	0	4.84	1	4.45	0	4.30	0
0.00	0.40	4.23	0	6.16	8	4.98	0	5.06	0
0.40	0.40	5.03	0	13.15	64	5.93	0	6.53	0
0.00	0.80	6.82	0	13.95	71	5.72	4	6.08	0

[a]AVE. is the average number of iterations to reach convergence for that particular estimator over 2500 replications of the experiment.

[b]LIMIT is the number of times the iteration limit of 100 was encountered over 2500 replications of the experiment.

Table 46. Iteration Statistics for the Four Iterative Estimators in the Experiment with Stationary Errors and a Growth Trend on the Independent Variable.

		FML		ICO		IPW		INS	
θ_1	θ_2	AVE.[a]	LIMIT[b]	AVE.	LIMIT	AVE.	LIMIT	AVE.	LIMIT
-1.60	-0.80	10.41	0	4.14	0	4.16	0	3.88	0
-1.20	-0.80	4.71	0	4.12	0	4.16	0	3.93	0
-0.80	-0.80	3.83	0	4.12	0	4.18	0	3.95	0
-0.40	-0.80	3.56	0	4.11	0	4.18	0	3.97	0
0.00	-0.80	3.59	0	4.11	0	4.19	0	3.97	0
0.40	-0.80	3.98	0	4.17	0	4.33	0	4.14	0
0.80	-0.80	4.64	0	4.43	0	4.72	0	4.94	0
1.20	-0.80	5.80	0	5.04	0	5.51	0	9.86	4
1.60	-0.80	10.11	1	6.58	2	26.57	513	12.06	1
-1.20	-0.40	10.29	1	4.17	0	4.20	0	3.88	0
-0.80	-0.40	4.81	1	4.14	2	4.192	0	3.92	0
-1.20	-0.40	10.29	1	4.17	0	4.20	0	3.88	0
-0.80	-0.40	4.81	0	4.14	0	4.19	0	3.92	0
-0.40	-0.40	3.77	0	4.10	0	4.18	0	3.92	0
0.00	-0.40	3.67	0	4.15	0	4.29	0	3.96	0
0.40	-0.40	4.15	0	4.38	0	4.65	0	4.30	0
0.80	-0.40	5.52	0	5.37	1	5.87	0	6.21	1
1.20	-0.40	10.27	1	9.29	13	11.69	81	15.11	10
-0.80	0.00	9.57	1	4.27	0	4.26	0	3.95	0
-0.40	0.00	4.13	0	4.26	0	4.24	0	4.01	0
0.00	0.00	3.75	0	4.51	1	4.45	0	4.12	0
0.40	0.00	4.64	0	5.59	8	5.76	2	5.08	0
0.80	0.00	8.28	2	9.71	39	8.79	2	10.42	3
-0.40	0.40	8.41	0	4.65	1	4.48	0	4.29	0
0.00	0.40	4.46	0	5.95	14	5.19	0	5.17	1
0.40	0.40	6.30	3	8.78	39	7.56	2	7.57	1
0.00	0.80	8.23	1	9.61	58	6.71	3	7.84	3

[a]AVE. is the average number of iterations to reach convergence for that particular estimator over 2500 replications of the experiment.

[b]LIMIT is the number of times the iteration limit of 100 was encountered over 2500 replications of the experiment.

Table 47. Iteration Statistics for the Four Iterative Estimators in the Experiment with Nonstationary Errors and a Growth Trend on the Independent Variable.

		FML		ICO		IPW		INS	
θ_1	θ_{22}	AVE.[a]	LIMIT[b]	AVE.	LIMIT	AVE.	LIMIT	AVE.	LIMIT
-1.60	-0.80	10.69	0	4.03	0	4.03	0	3.91	0
-1.20	-0.80	4.97	0	4.06	0	4.04	0	3.97	0
-0.80	-0.80	3.86	0	4.09	0	4.11	0	3.99	0
-0.40	-0.80	3.53	0	4.08	0	4.10	0	4.01	0
0.00	-0.80	3.50	0	4.06	0	4.11	0	3.99	0
0.40	-0.80	3.85	0	4.14	0	4.26	0	4.09	0
0.80	-0.80	4.58	0	4.37	0	4.66	0	4.44	0
1.20	-0.80	6.27	0	5.00	0	5.76	2	5.26	0
1.60	-0.80	11.73	2	7.26	11	8.14	7	7.71	1
-1.20	-0.40	9.86	1	4.10	0	4.10	0	3.92	0
-0.80	-0.40	4.83	0	4.09	0	4.11	0	3.92	0
-0.40	-0.40	3.77	0	4.08	0	4.15	0	3.92	0
0.00	-0.40	3.65	0	4.11	0	4.23	0	3.95	0
0.40	-0.40	4.31	0	4.36	0	4.83	0	4.24	0
0.80	-0.40	6.05	0	5.15	0	6.65	0	5.26	0
1.20	-0.40	11.29	7	9.22	23	9.37	2	9.56	1
-0.80	0.00	8.63	0	4.19	0	4.19	0	3.98	0
-0.40	0.00	4.11	0	4.22	0	4.20	0	3.98	0
0.00	0.00	3.76	0	4.47	0	4.49	0	4.12	0
0.40	0.00	5.12	2	5.45	6	6.26	0	5.20	2
0.80	0.00	8.69	8	9.20	31	8.93	1	8.89	3
-0.40	0.40	7.27	0	4.73	2	4.41	0	4.32	0
0.00	0.40	4.41	0	5.73	8	5.20	0	5.11	0
0.40	0.40	6.62	4	8.77	38	8.12	2	7.94	1
0.00	0.80	7.40	0	8.45	37	6.56	3	7.67	1

[a]AVE. is the average number of iterations to reach convergence for that particular estimator over 2500 replications of the experiment.

[b]LIMIT is the number of times the iteration limit of 100 was encountered over 2500 replications of the experiment.

Table 48. Iteration Statistics for the Four Iterative Estimators in the Experiment with Stationary Errors and a Dampened Cyclical Trend on the Independent Variable.

		FML		ICO		IPW		INS	
θ_1	θ_2	AVE.[a]	LIMIT[b]	AVE.	LIMIT	AVE.	LIMIT	AVE.	LIMIT
-1.60	-0.80	10.21	0	3.70	0	3.75	0	3.56	0
-1.20	-0.80	4.64	0	3.54	0	3.62	0	3.43	0
-0.80	-0.80	3.59	0	3.52	0	3.62	0	3.44	0
-0.40	-0.80	3.11	0	3.56	0	3.64	0	3.51	0
0.00	-0.80	3.09	0	3.63	0	3.72	0	3.67	0
0.40	-0.80	3.43	0	3.71	0	3.83	0	4.03	0
0.80	-0.80	4.10	0	4.07	0	4.13	1	5.56	0
1.20	-0.80	6.27	0	5.65	0	5.87	0	10.58	4
1.60	-0.80	12.00	2	9.41	3	21.54	366	11.52	1
-1.20	-0.40	10.30	2	3.70	0	3.79	0	3.61	0
-0.80	-0.40	4.80	0	3.58	0	3.73	0	3.54	0
-0.40	-0.40	3.56	0	3.63	0	3.79	0	3.64	0
0.00	-0.40	3.36	0	3.75	0	3.93	0	3.79	0
0.40	-0.40	3.89	0	4.07	0	4.32	0	4.33	0
0.80	-0.40	5.62	0	5.29	2	5.96	1	6.92	0
1.20	-0.40	12.53	3	9.11	16	14.24	100	13.44	6
-0.80	0.00	9.43	0	3.86	0	3.94	0	3.75	0
-0.40	0.00	4.05	0	3.87	0	3.99	0	3.79	0
0.00	0.00	3.62	0	4.08	0	4.31	0	4.04	0
0.40	0.00	4.60	0	4.70	1	5.50	1	4.95	0
0.80	0.00	9.14	2	7.95	25	8.46	9	9.46	4
-0.40	0.40	8.39	1	4.30	0	4.34	0	4.20	0
0.00	0.40	4.53	0	4.71	2	5.28	0	5.13	0
0.40	0.40	6.64	1	7.94	42	6.70	0	7.13	1
0.00	0.80	7.63	0	9.05	55	5.92	8	7.65	2

[a]AVE. is the average number of iterations to reach convergence for that particular estimator over 2500 replications of the experiment.

[b]LIMIT is the number of times the iteration limit of 100 was encountered over 2500 replications of the experiment.

Table 49. Iteration Statistics for the Four Iterative Estimators in the Experiment with Nonstationary Errors and a Dampened Cyclical Trend on the Independent Variable.

		FML		ICO		IPW		INS	
θ_1	θ_2	AVE.[a]	LIMIT[b]	AVE.	LIMIT	AVE.	LIMIT	AVE.	LIMIT
-1.60	-0.80	10.58	0	3.53	0	3.52	0	3.43	0
-1.20	-0.80	4.89	0	3.43	0	3.46	0	3.38	0
-0.80	-0.80	3.68	0	3.42	0	3.48	0	3.40	0
-0.40	-0.80	3.11	0	3.45	0	3.51	0	3.45	0
0.00	-0.80	3.06	0	3.52	0	3.60	0	3.57	0
0.40	-0.80	3.38	0	3.66	0	3.77	0	3.79	0
0.80	-0.80	4.31	0	4.02	0	4.31	0	4.35	0
1.20	-0.80	7.23	0	5.64	0	6.51	2	6.02	1
1.60	-0.80	14.59	5	8.63	6	9.00	6	7.86	0
-1.20	-0.40	9.88	1	3.58	0	3.62	0	3.48	0
-0.80	-0.40	4.78	0	3.54	0	3.63	0	3.49	0
-0.40	-0.40	3.53	0	3.57	0	3.72	0	3.57	0
0.00	-0.40	3.35	0	3.73	0	3.93	0	3.78	0
0.40	-0.40	4.10	0	4.07	0	4.59	0	4.22	0
0.80	-0.40	7.08	1	5.07	0	7.47	2	5.69	0
1.20	-0.40	14.07	4	9.03	28	9.43	3	8.92	1
-0.80	0.00	8.59	1	3.73	0	3.80	0	3.64	0
-0.40	0.00	4.02	0	3.80	0	3.93	0	3.74	0
0.00	0.00	3.80	0	4.11	0	4.45	1	4.07	0
0.40	0.00	5.55	1	4.73	0	6.66	1	5.08	0
0.80	0.00	10.93	4	8.40	37	7.91	1	7.93	1
-0.40	0.40	7.31	1	4.19	0	4.22	0	4.21	0
0.00	0.40	4.52	0	4.78	2	5.19	0	5.07	0
0.40	0.40	7.88	3	8.02	35	7.39	1	7.43	1
0.00	0.80	6.74	2	8.50	39	5.71	8	7.31	2

[a]AVE. is the average number of iterations to reach convergence for that particular estimator over 2500 replications of the experiment.

[b]LIMIT is the number of times the iteration limit of 100 was encountered over 2500 replications of the experiment.

Table 50. Relative Bias of σ_{β_1}.

θ_1	θ_2	EST.	WNS[a]	WNN[b]	DTS[c]	DTN[d]	GTS[e]	GTN[f]	DCS[g]	DCN[h]
-1.60	-0.80	OLS	0.197E+01	0.195E+01	0.337E+01	0.406E+01	0.283E+01	0.381E+01	0.481E+01	0.592E+01
		TSPW	0.682E+00	0.666E+00	0.546E-02	-0.289E-01	-0.163E-01	-0.186E-01	-0.204E-01	-0.529E-01
		IPW	-0.944E-01	-0.802E-01	-0.912E-01	-0.628E-01	-0.966E-01	-0.715E-01	-0.745E-01	-0.730E-01
		FML	-0.496E-01	-0.383E-01	-0.469E-01	-0.201E-01	-0.520E-01	-0.303E-01	-0.288E-01	-0.305E-01
		TSCO	0.880E+00	0.870E+00	0.533E-01	0.786E-02	0.196E-01	0.176E-01	0.219E-01	-0.767E-02
		ICO	-0.489E-01	-0.382E-01	-0.465E-01	-0.256E-01	-0.541E-01	-0.325E-01	-0.301E-01	-0.287E-01
		INS	0.251E+00	-0.343E-01	-0.146E-01	-0.101E-01	-0.547E-01	-0.197E-01	0.149E+00	-0.314E-01
-1.20	-0.80	OLS	0.281E+01	0.279E+01	0.262E+01	0.308E+01	0.198E+01	0.227E+01	0.362E+01	0.389E+01
		TSPW	0.469E-01	0.559E-01	-0.425E-01	-0.535E-01	-0.192E-01	-0.548E-01	-0.751E-01	-0.102E+00
		IPW	-0.792E-01	-0.876E-01	-0.804E-01	-0.704E-01	-0.631E-01	-0.863E-01	-0.903E-01	-0.110E+00
		FML	-0.318E-01	-0.455E-01	-0.346E-01	-0.276E-01	-0.157E-01	-0.456E-01	-0.445E-01	-0.684E-01
		TSCO	0.103E+00	0.108E+00	-0.327E-02	-0.157E-01	0.207E-01	-0.143E-01	-0.315E-01	-0.616E-01
		ICO	-0.322E-01	-0.445E-01	-0.335E-01	-0.307E-01	-0.181E-01	-0.452E-01	-0.454E-01	-0.696E-01
		INS	0.106E+00	-0.408E-01	-0.184E-01	-0.249E-01	-0.130E-01	-0.456E-01	0.340E-01	-0.578E-01
-0.80	-0.80	OLS	0.247E+01	0.258E+01	0.207E+01	0.237E+01	0.143E+01	0.148E+01	0.282E+01	0.289E+01
		TSPW	-0.300E-01	-0.127E-01	-0.877E-01	-0.714E-01	-0.378E-01	-0.550E-01	-0.620E-01	-0.572E-01
		IPW	-0.925E-01	-0.785E-01	-0.115E+00	-0.862E-01	-0.746E-01	-0.836E-01	-0.728E-01	-0.641E-01
		FML	-0.446E-01	-0.361E-01	-0.699E-01	-0.445E-01	-0.268E-01	-0.436E-01	-0.257E-01	-0.215E-01
		TSCO	0.200E-01	0.377E-01	-0.582E-01	-0.343E-01	0.173E-02	-0.184E-01	-0.186E-01	-0.138E-01
		ICO	-0.463E-01	-0.335E-01	-0.767E-01	-0.459E-01	-0.303E-01	-0.465E-01	-0.274E-01	-0.201E-01
		INS	0.534E-01	-0.308E-01	-0.462E-01	-0.416E-01	-0.182E-01	-0.470E-01	0.374E-01	-0.141E-01
-0.40	-0.80	OLS	0.203E+01	0.190E+01	0.167E+01	0.174E+01	0.133E+01	0.144E+01	0.211E+01	0.209E+01
		TSPW	-0.412E-01	-0.302E-01	-0.650E-01	-0.706E-01	-0.972E-01	-0.542E-01	-0.940E-01	-0.931E-01
		IPW	-0.817E-01	-0.688E-01	-0.894E-01	-0.855E-01	-0.118E+00	-0.764E-01	-0.103E+00	-0.996E-01
		FML	-0.344E-01	-0.265E-01	-0.441E-01	-0.441E-01	-0.760E-01	-0.347E-01	-0.571E-01	-0.581E-01
		TSCO	0.226E-02	0.149E-01	-0.326E-01	-0.369E-01	-0.575E-01	-0.181E-01	-0.486E-01	-0.537E-01
		ICO	-0.360E-01	-0.243E-01	-0.472E-01	-0.481E-01	-0.763E-01	-0.388E-01	-0.553E-01	-0.594E-01
		INS	0.334E-01	-0.175E-01	-0.129E-01	-0.438E-01	-0.595E-01	-0.321E-01	-0.410E-02	-0.525E-01
0.00	-0.80	OLS	0.138E+01	0.119E+01	0.129E+01	0.135E+01	0.128E+01	0.131E+01	0.153E+01	0.150E+01
		TSPW	-0.538E-01	-0.591E-01	-0.828E-01	-0.718E-01	-0.951E-01	-0.694E-01	-0.108E+00	-0.101E+00
		IPW	-0.925E-01	-0.884E-01	-0.103E+00	-0.865E-01	-0.109E+00	-0.835E-01	-0.117E+00	-0.108E+00
		FML	-0.465E-01	-0.496E-01	-0.589E-01	-0.451E-01	-0.672E-01	-0.435E-01	-0.727E-01	-0.681E-01
		TSCO	-0.164E-01	-0.202E-01	-0.517E-01	-0.447E-01	-0.559E-01	-0.365E-01	-0.647E-01	-0.623E-01
		ICO	-0.490E-01	-0.472E-01	-0.641E-01	-0.554E-01	-0.681E-01	-0.489E-01	-0.717E-01	-0.680E-01
		INS	0.170E-01	-0.449E-01	-0.199E-01	-0.410E-01	-0.439E-01	-0.401E-01	-0.211E-01	-0.613E-01
0.40	-0.80	OLS	0.958E+00	0.848E+00	0.971E+00	0.850E+00	0.974E+00	0.839E+00	0.118E+01	0.105E+01
		TSPW	-0.717E-01	-0.554E-01	-0.101E+00	-0.997E-01	-0.113E+00	-0.107E+00	-0.968E-01	-0.725E-01
		IPW	-0.968E-01	-0.819E-01	-0.114E+00	-0.113E+00	-0.123E+00	-0.116E+00	-0.105E+00	-0.804E-01
		FML	-0.561E-01	-0.434E-01	-0.745E-01	-0.746E-01	-0.886E-01	-0.820E-01	-0.653E-01	-0.415E-01
		TSCO	-0.309E-01	-0.174E-01	-0.662E-01	-0.682E-01	-0.647E-01	-0.735E-01	-0.517E-01	-0.329E-01
		ICO	-0.507E-01	-0.397E-01	-0.755E-01	-0.778E-01	-0.729E-01	-0.814E-01	-0.579E-01	-0.397E-01
		INS	0.229E-01	-0.347E-01	-0.170E-01	-0.735E-01	-0.147E-01	-0.742E-01	0.148E-01	-0.341E-01

Table 50--Continued.

0.80 -0.80	OLS	0.558E+00	0.495E+00	0.648E+00	0.467E+00	0.432E+00	0.420E+00	0.738E+00	0.588E+00
	TSPW	-0.765E-01	-0.639E-01	-0.121E+00	-0.112E+00	-0.135E+00	-0.123E+00	-0.116E+00	-0.120E+00
	IPW	-0.108E+00	-0.126E+00	-0.125E+00	-0.120E+00	-0.143E+00	-0.141E+00	-0.126E+00	-0.156E+00
	FML	-0.806E-01	-0.805E-01	-0.959E-01	-0.920E-01	-0.121E+00	-0.115E+00	-0.978E-01	-0.134E+00
	TSCO	-0.198E-01	-0.233E-01	-0.904E-01	-0.740E-01	-0.973E-01	-0.693E-01	-0.728E-01	-0.704E-01
	ICO	-0.530E-01	-0.505E-01	-0.968E-01	-0.821E-01	-0.103E+00	-0.772E-01	-0.785E-01	-0.791E-01
	INS	0.485E-01	-0.441E-01	-0.152E-01	-0.809E-01	0.153E-01	-0.828E-01	0.525E-01	-0.722E-01
1.20 -0.80	OLS	0.199E+00	0.141E+00	0.242E+00	0.436E-01	-0.486E-01	0.838E-03	0.220E+00	0.133E+00
	TSPW	-0.850E-01	-0.155E+00	-0.122E+00	-0.173E+00	-0.147E+00	-0.185E+00	-0.148E+00	-0.157E+00
	IPW	0.808E-01	-0.237E+00	-0.816E+00	-0.167E+00	-0.151E+00	-0.199E+00	-0.172E+00	-0.269E+00
	FML	-0.911E-01	-0.224E+00	-0.102E+00	-0.150E+00	-0.145E+00	-0.207E+00	-0.176E+00	-0.273E+00
	TSCO	-0.247E-01	-0.563E-01	-0.110E+00	-0.137E+00	-0.131E+00	-0.112E+00	-0.104E+00	-0.702E-01
	ICO	-0.454E-01	-0.566E-01	-0.109E+00	-0.144E+00	-0.135E+00	-0.119E+00	-0.117E+00	-0.803E-01
	INS	-0.168E+00	-0.738E-01	-0.374E+00	-0.117E+00	-0.299E+00	-0.958E-01	0.349E-01	-0.672E-01
1.60 -0.80	OLS	-0.274E+00	-0.396E+00	-0.297E+00	-0.445E+00	-0.513E+00	-0.497E+00	-0.372E+00	-0.447E+00
	TSPW	0.124E-01	-0.379E+00	-0.589E+00	-0.262E+00	-0.385E+00	-0.388E+00	-0.849E-02	-0.351E+00
	IPW	0.845E+01	-0.527E+00	-0.253E+00	-0.381E+00	0.271E+01	-0.299E+00	0.136E+01	-0.324E+00
	FML	-0.900E-01	-0.439E+00	-0.126E+00	-0.226E+00	-0.400E-01	-0.358E+00	-0.904E-01	-0.466E+00
	TSCO	-0.307E+00	-0.622E+00	-0.295E+00	-0.271E+00	-0.241E+00	-0.284E+00	-0.245E+00	-0.321E+00
	ICO	-0.822E+00	-0.854E+00	-0.279E+00	-0.334E+00	-0.490E+00	-0.568E+00	-0.430E+00	-0.543E+00
	INS	-0.823E+00	-0.168E+00	-0.408E+00	-0.176E+00	-0.669E+00	-0.154E+00	-0.601E+00	-0.235E+00
-1.20 -0.40	OLS	0.231E+01	0.239E+01	0.260E+01	0.301E+01	0.243E+01	0.269E+01	0.339E+01	0.358E+01
	TSPW	0.205E+00	0.174E+00	-0.180E-01	-0.391E-01	-0.284E-01	-0.529E-01	-0.711E-01	-0.709E-01
	IPW	-0.931E-01	-0.927E-01	-0.782E-01	-0.650E-01	-0.764E-01	-0.904E-01	-0.103E+00	-0.858E-01
	FML	-0.618E-01	-0.613E-01	-0.473E-01	-0.332E-01	-0.456E-01	-0.598E-01	-0.720E-01	-0.546E-01
	TSCO	0.246E+00	0.221E+00	0.105E-01	-0.649E-02	-0.503E-02	-0.251E-01	-0.423E-01	-0.358E-01
	ICO	-0.595E-01	-0.615E-01	-0.453E-01	-0.295E-01	-0.489E-01	-0.598E-01	-0.713E-01	-0.505E-01
	INS	0.450E-01	-0.538E-01	-0.124E-01	-0.298E-01	-0.269E-01	-0.593E-01	0.214E-01	-0.512E-01
-0.80 -0.40	OLS	0.145E+01	0.150E+01	0.153E+01	0.158E+01	0.129E+01	0.141E+01	0.171E+01	0.173E+01
	TSPW	0.155E-02	-0.241E-01	-0.630E-01	-0.960E-01	-0.884E-01	-0.430E-01	-0.912E-01	-0.989E-01
	IPW	-0.809E-01	-0.113E+00	-0.884E-01	-0.113E+00	-0.112E+00	-0.650E-01	-0.104E+00	-0.108E+00
	FML	-0.498E-01	-0.823E-01	-0.571E-01	-0.831E-01	-0.824E-01	-0.344E-01	-0.730E-01	-0.773E-01
	TSCO	0.238E-01	0.933E-02	-0.354E-01	-0.660E-01	-0.584E-01	-0.182E-01	-0.655E-01	-0.645E-01
	ICO	-0.559E-01	-0.794E-01	-0.559E-01	-0.791E-01	-0.803E-01	-0.380E-01	-0.762E-01	-0.722E-01
	INS	-0.190E-01	-0.727E-01	-0.490E-01	-0.791E-01	-0.763E-01	-0.340E-01	-0.480E-01	-0.709E-01
-0.40 -0.40	OLS	0.876E+00	0.925E+00	0.874E+00	0.913E+00	0.766E+00	0.798E+00	0.976E+00	0.101E+01
	TSPW	-0.604E-01	-0.441E-01	-0.808E-01	-0.764E-01	-0.958E-01	-0.812E-01	-0.925E-01	-0.824E-01
	IPW	-0.104E+00	-0.902E-01	-0.981E-01	-0.926E-01	-0.113E+00	-0.966E-01	-0.102E+00	-0.905E-01
	FML	-0.755E-01	-0.614E-01	-0.698E-01	-0.644E-01	-0.856E-01	-0.696E-01	-0.735E-01	-0.619E-01
	TSCO	-0.316E-01	-0.101E-01	-0.515E-01	-0.521E-01	-0.648E-01	-0.558E-01	-0.665E-01	-0.547E-01
	ICO	-0.726E-01	-0.537E-01	-0.648E-01	-0.648E-01	-0.801E-01	-0.699E-01	-0.742E-01	-0.612E-01
	INS	-0.588E-01	-0.543E-01	-0.650E-01	-0.629E-01	-0.844E-01	-0.710E-01	-0.573E-01	-0.528E-01
0.00 -0.40	OLS	0.426E+00	0.485E+00	0.404E+00	0.442E+00	0.401E+00	0.418E+00	0.501E+00	0.463E+00
	TSPW	-0.964E-01	-0.571E-01	-0.124E+00	-0.105E+00	-0.134E+00	-0.118E+00	-0.129E+00	-0.126E+00
	IPW	-0.128E+00	-0.106E+00	-0.138E+00	-0.117E+00	-0.149E+00	-0.137E+00	-0.202E+00	-0.227E+00
	FML	-0.972E-01	-0.616E-01	-0.113E+00	-0.931E-01	-0.124E+00	-0.114E+00	-0.172E+00	-0.190E+00
	TSCO	-0.688E-01	-0.264E-01	-0.102E+00	-0.818E-01	-0.117E+00	-0.903E-01	-0.753E-01	-0.947E-01
	ICO	-0.882E-01	-0.498E-01	-0.114E+00	-0.933E-01	-0.128E+00	-0.101E+00	-0.819E-01	-0.101E+00
	INS	-0.825E-01	-0.497E-01	-0.107E+00	-0.905E-01	-0.110E+00	-0.958E-01	-0.789E-01	-0.988E-01

Table 50--Continued.

0.40	-0.40	OLS	0.102E+00	0.853E-01	0.114E+00	0.670E-01	0.103E+00	0.459E-01	0.114E+00	0.567E-01
		TSPW	-0.130E+00	-0.126E+00	-0.143E+00	-0.157E+00	-0.157E+00	-0.202E+00	-0.163E+00	-0.199E+00
		IPW	-0.192E+00	-0.265E+00	-0.144E+00	-0.159E+00	-0.162E+00	-0.223E+00	-0.205E+00	-0.271E+00
		FML	-0.165E+00	-0.207E+00	-0.126E+00	-0.144E+00	-0.144E+00	-0.208E+00	-0.175E+00	-0.231E+00
		TSCO	-0.941E-01	-0.855E-01	-0.116E+00	-0.122E+00	-0.130E+00	-0.140E+00	-0.119E+00	-0.137E+00
		ICO	-0.933E-01	-0.860E-01	-0.125E+00	-0.133E+00	-0.137E+00	-0.145E+00	-0.123E+00	-0.143E+00
		INS	-0.953E-01	-0.849E-01	-0.107E+00	-0.127E+00	-0.108E+00	-0.155E+00	-0.114E+00	-0.163E+00
0.80	-0.40	OLS	-0.219E+00	-0.233E+00	-0.216E+00	-0.241E+00	-0.230E+00	-0.274E+00	-0.250E+00	-0.261E+00
		TSPW	-0.212E+00	-0.272E+00	-0.227E+00	-0.210E+00	-0.230E+00	-0.330E+00	-0.237E+00	-0.315E+00
		IPW	-0.224E+00	-0.481E+00	-0.600E+00	-0.180E+00	-0.226E+00	-0.351E+00	-0.257E+00	-0.518E+00
		FML	-0.203E+00	-0.462E+00	-0.199E+00	-0.181E+00	-0.211E+00	-0.352E+00	-0.238E+00	-0.490E+00
		TSCO	-0.155E+00	-0.142E+00	-0.189E+00	-0.186E+00	-0.209E+00	-0.198E+00	-0.187E+00	-0.168E+00
		ICO	-0.123E+00	-0.107E+00	-0.201E+00	-0.195E+00	-0.242E+00	-0.227E+00	-0.191E+00	-0.170E+00
		INS	-0.119E+00	-0.151E+00	-0.178E+00	-0.153E+00	-0.183E+00	-0.210E+00	-0.168E+00	-0.179E+00
1.20	-0.40	OLS	-0.595E+00	-0.622E+00	-0.569E+00	-0.625E+00	-0.625E+00	-0.622E+00	-0.633E+00	-0.625E+00
		TSPW	-0.270E+00	-0.513E+00	-0.704E+00	-0.389E+00	0.170E+00	-0.489E+00	-0.190E+00	-0.537E+00
		IPW	0.419E+01	-0.573E+00	-0.562E+00	-0.316E+00	0.329E+01	-0.434E+00	0.132E+01	-0.590E+00
		FML	-0.242E+00	-0.572E+00	-0.332E+00	-0.351E+00	-0.203E+00	-0.455E+00	-0.295E+00	-0.589E+00
		TSCO	-0.370E+00	-0.575E+00	-0.692E+00	-0.537E+00	-0.420E+00	-0.404E+00	-0.399E+00	-0.554E+00
		ICO	-0.437E+00	-0.714E+00	-0.747E+00	-0.530E+00	-0.534E+00	-0.596E+00	-0.585E+00	-0.680E+00
		INS	-0.715E+00	-0.307E+00	-0.264E+00	-0.289E+00	-0.513E+00	-0.273E+00	-0.544E+00	-0.305E+00
-0.80	0.00	OLS	0.152E+01	0.149E+01	0.160E+01	0.165E+01	0.154E+01	0.155E+01	0.175E+01	0.165E+01
		TSPW	0.223E-01	0.156E-01	-0.656E-01	-0.109E+00	-0.962E-01	-0.855E-01	-0.100E+00	-0.137E+00
		IPW	0.273E+00	-0.107E+00	-0.105E+00	-0.129E+00	-0.125E+00	-0.109E+00	-0.120E+00	-0.148E+00
		FML	-0.922E-01	-0.912E-01	-0.885E-01	-0.112E+00	-0.109E+00	-0.914E-01	-0.105E+00	-0.132E+00
		TSCO	0.427E-01	0.362E-01	-0.561E-01	-0.909E-01	-0.698E-01	-0.655E-01	-0.863E-01	-0.115E+00
		ICO	-0.842E-01	-0.849E-01	-0.932E-01	-0.109E+00	-0.996E-01	-0.889E-01	-0.105E+00	-0.125E+00
		INS	-0.567E-01	-0.885E-01	-0.669E-01	-0.106E+00	-0.966E-01	-0.887E-01	-0.740E-01	-0.126E+00
-0.40	0.00	OLS	0.477E+00	0.505E+00	0.468E+00	0.459E+00	0.440E+00	0.491E+00	0.521E+00	0.501E+00
		TSPW	-0.649E-01	-0.680E-01	-0.146E+00	-0.148E+00	-0.149E+00	-0.132E+00	-0.126E+00	-0.137E+00
		IPW	-0.119E+00	-0.126E+00	-0.159E+00	-0.159E+00	-0.160E+00	-0.144E+00	-0.140E+00	-0.162E+00
		FML	-0.104E+00	-0.107E+00	-0.145E+00	-0.146E+00	-0.147E+00	-0.130E+00	-0.121E+00	-0.143E+00
		TSCO	-0.506E-01	-0.525E-01	-0.130E+00	-0.124E+00	-0.122E+00	-0.122E+00	-0.113E+00	-0.116E+00
		ICO	-0.921E-01	-0.941E-01	-0.145E+00	-0.136E+00	-0.135E+00	-0.134E+00	-0.120E+00	-0.122E+00
		INS	-0.902E-01	-0.950E-01	-0.142E+00	-0.147E+00	-0.145E+00	-0.125E+00	-0.114E+00	-0.124E+00
0.00	0.00	OLS	-0.205E-01	-0.193E-01	-0.186E-01	-0.141E-01	-0.203E-01	-0.133E-01	-0.433E-02	-0.479E-02
		TSPW	-0.149E+00	-0.146E+00	-0.188E+00	-0.170E+00	-0.195E+00	-0.201E+00	-0.168E+00	-0.167E+00
		IPW	-0.245E+00	-0.235E+00	-0.188E+00	-0.171E+00	-0.209E+00	-0.220E+00	-0.201E+00	-0.268E+00
		FML	-0.205E+00	-0.200E+00	-0.183E+00	-0.167E+00	-0.200E+00	-0.210E+00	-0.183E+00	-0.218E+00
		TSCO	-0.127E+00	-0.122E+00	-0.168E+00	-0.158E+00	-0.173E+00	-0.173E+00	-0.145E+00	-0.149E+00
		ICO	-0.125E+00	-0.191E+00	-0.178E+00	-0.169E+00	-0.221E+00	-0.189E+00	-0.148E+00	-0.151E+00
		INS	-0.132E+00	-0.129E+00	-0.177E+00	-0.160E+00	-0.177E+00	-0.184E+00	-0.159E+00	-0.147E+00
0.40	0.00	OLS	-0.339E+00	-0.338E+00	-0.352E+00	-0.365E+00	-0.320E+00	-0.351E+00	-0.365E+00	-0.369E+00
		TSPW	-0.229E+00	-0.285E+00	-0.270E+00	-0.272E+00	-0.267E+00	-0.343E+00	-0.265E+00	-0.330E+00
		IPW	-0.290E+00	-0.482E+00	-0.246E+00	-0.242E+00	-0.260E+00	-0.357E+00	-0.320E+00	-0.494E+00
		FML	-0.270E+00	-0.437E+00	-0.256E+00	-0.256E+00	-0.259E+00	-0.353E+00	-0.299E+00	-0.458E+00
		TSCO	-0.196E+00	-0.201E+00	-0.266E+00	-0.249E+00	-0.276E+00	-0.245E+00	-0.249E+00	-0.211E+00
		ICO	-0.172E+00	-0.293E+00	-0.273E+00	-0.259E+00	-0.576E+00	-0.715E+00	-0.447E+00	-0.208E+00
		INS	-0.176E+00	-0.219E+00	-0.236E+00	-0.238E+00	-0.235E+00	-0.268E+00	-0.235E+00	-0.250E+00

Table 50--Continued.

0.80	0.00	OLS	-0.685E+00	-0.694E+00	-0.670E+00	-0.703E+00	-0.607E+00	-0.641E+00	-0.681E+00	-0.700E+00
		TSPW	-0.460E+00	-0.522E+00	-0.437E+00	-0.476E+00	-0.377E+00	-0.509E+00	-0.122E+00	-0.562E+00
		IPW	-0.353E+00	-0.558E+00	-0.529E+00	-0.424E+00	-0.306E+00	-0.463E+00	0.205E+00	-0.583E+00
		FML	-0.375E+00	-0.550E+00	-0.470E+00	-0.469E+00	-0.314E+00	-0.481E+00	-0.380E+00	-0.577E+00
		TSCD	-0.838E+00	-0.494E+00	-0.499E+00	-0.859E+00	-0.452E+00	-0.444E+00	-0.672E+00	-0.866E+00
		ICD	-0.779E+00	-0.724E+00	-0.598E+00	-0.859E+00	-0.732E+00	-0.680E+00	-0.727E+00	-0.861E+00
		INS	-0.399E+00	-0.383E+00	-0.355E+00	-0.430E+00	-0.331E+00	-0.381E+00	-0.352E+00	-0.396E+00
-0.40	0.40	OLS	0.394E+00	0.355E+00	0.490E+00	0.402E+00	0.471E+00	0.379E+00	0.477E+00	0.396E+00
		TSPW	0.715E-01	-0.116E+00	-0.155E+00	-0.197E+00	-0.133E+00	-0.227E+00	-0.137E+00	-0.168E+00
		IPW	-0.133E+00	-0.134E+00	-0.633E-02	-0.547E-01	-0.143E+00	-0.235E+00	0.893E-01	-0.172E+00
		FML	-0.165E+00	-0.158E+00	-0.179E+00	-0.210E+00	-0.216E+00	-0.237E+00	-0.167E+00	-0.184E+00
		TSCD	-0.135E+00	-0.128E+00	-0.169E+00	-0.208E+00	-0.200E+00	-0.232E+00	-0.168E+00	-0.179E+00
		ICD	-0.155E+00	-0.148E+00	-0.189E+00	-0.221E+00	-0.261E+00	-0.461E+00	-0.172E+00	-0.180E+00
		INS	-0.183E+00	-0.168E+00	-0.174E+00	-0.205E+00	-0.214E+00	-0.240E+00	-0.156E+00	-0.208E+00
0.00	0.40	OLS	-0.334E+00	-0.354E+00	-0.328E+00	-0.333E+00	-0.303E+00	-0.281E+00	-0.348E+00	-0.332E+00
		TSPW	-0.266E+00	-0.264E+00	-0.298E+00	-0.297E+00	-0.329E+00	-0.277E+00	-0.275E+00	-0.243E+00
		IPW	-0.364E+00	-0.247E+00	-0.283E+00	-0.282E+00	-0.334E+00	-0.276E+00	-0.351E+00	-0.317E+00
		FML	-0.335E+00	-0.308E+00	-0.304E+00	-0.304E+00	-0.340E+00	-0.285E+00	-0.327E+00	-0.301E+00
		TSCD	-0.254E+00	-0.266E+00	-0.317E+00	-0.353E+00	-0.298E+00	-0.286E+00	-0.272E+00	-0.266E+00
		ICD	-0.581E+00	-0.234E+00	-0.325E+00	-0.397E+00	-0.731E+00	-0.567E+00	-0.410E+00	-0.334E+00
		INS	-0.316E+00	-0.289E+00	-0.286E+00	-0.287E+00	-0.329E+00	-0.275E+00	-0.311E+00	-0.278E+00
0.40	0.40	OLS	-0.703E+00	-0.717E+00	-0.691E+00	-0.708E+00	-0.570E+00	-0.621E+00	-0.694E+00	-0.713E+00
		TSPW	-0.111E+00	-0.490E+00	-0.257E+00	-0.509E+00	-0.204E+00	-0.463E+00	0.388E+00	-0.435E+00
		IPW	0.689E+00	-0.479E+00	0.161E+00	-0.471E+00	0.137E+00	-0.435E+00	0.684E+00	-0.391E+00
		FML	-0.399E+00	-0.487E+00	-0.523E+00	-0.518E+00	-0.373E+00	-0.453E+00	-0.396E+00	-0.506E+00
		TSCD	-0.843E+00	-0.497E+00	-0.808E+00	-0.565E+00	-0.460E+00	-0.482E+00	-0.800E+00	-0.927E+00
		ICD	-0.820E+00	-0.671E+00	-0.823E+00	-0.620E+00	-0.710E+00	-0.680E+00	-0.778E+00	-0.912E+00
		INS	-0.364E+00	-0.420E+00	-0.507E+00	-0.481E+00	-0.335E+00	-0.411E+00	-0.342E+00	-0.431E+00
0.00	0.80	OLS	-0.642E+00	-0.637E+00	-0.584E+00	-0.609E+00	-0.515E+00	-0.447E+00	-0.643E+00	-0.622E+00
		TSPW	0.775E+00	0.150E-01	-0.242E+00	-0.357E+00	-0.428E+00	-0.406E+00	0.239E+01	0.151E+01
		IPW	0.349E+01	0.837E+00	0.267E+00	-0.130E+00	-0.111E+00	-0.209E+00	0.264E+01	0.160E+01
		FML	-0.494E+00	-0.436E+00	-0.571E+00	-0.568E+00	-0.539E+00	-0.422E+00	-0.509E+00	-0.404E+00
		TSCD	-0.944E+00	-0.853E+00	-0.764E+00	-0.624E+00	-0.533E+00	-0.501E+00	-0.884E+00	-0.900E+00
		ICD	-0.925E+00	-0.844E+00	-0.785E+00	-0.692E+00	-0.744E+00	-0.695E+00	-0.871E+00	-0.883E+00
		INS	-0.587E+00	-0.441E+00	-0.556E+00	-0.539E+00	-0.550E+00	-0.407E+00	-0.596E+00	-0.417E+00

[a]White noise independent variable, stationary errors.
[b]White noise independent variable, nonstationary errors.
[c]Dampened trend independent variable, stationary errors.
[d]Dampened trend independent variable, nonstationary errors.
[e]Growth trend independent variable, stationary errors.
[f]Growth trend independent variable, nonstationary errors.
[g]Dampened cyclical trend independent variable, stationary errors.
[h]Dampened cyclical trend independent variable, nonstationary errors.

Table 51. Relative Bias of σ_{β_2}.

θ_1	θ_2	EST.	WNS[a]	WNN[b]	DTS[c]	DTN[d]	GTS[e]	GTN[f]	DCS[g]	DCN[h]
-1.60	-0.80	OLS	-0.166E+00	-0.199E+00	0.242E+01	0.505E+01	0.281E+01	0.341E+01	0.384E+01	0.586E+01
		TSPW	0.459E-01	0.597E-02	-0.125E-01	-0.229E-01	-0.130E-01	-0.156E-01	-0.385E-01	-0.536E-01
		IPW	-0.766E-01	-0.795E-01	-0.101E+00	-0.580E-01	-0.927E-01	-0.680E-01	-0.915E-01	-0.736E-01
		FML	-0.314E-01	-0.385E-01	-0.594E-01	-0.142E-01	-0.484E-01	-0.271E-01	-0.464E-01	-0.314E-01
		TSCO	-0.248E-01	-0.221E-01	0.407E-01	0.582E-02	0.233E-01	0.215E-01	0.602E-02	-0.859E-02
		ICO	-0.349E-01	-0.393E-01	-0.563E-01	-0.274E-01	-0.501E-01	-0.285E-01	-0.464E-01	-0.295E-01
		INS	0.484E-01	-0.371E-01	-0.279E+00	-0.500E-02	0.942E-02	-0.154E-01	0.267E+00	-0.260E-01
-1.20	-0.80	OLS	0.293E+00	0.189E+00	0.196E+01	0.341E+01	0.186E+01	0.198E+01	0.325E+01	0.384E+01
		TSPW	0.311E-01	0.916E-02	-0.564E-01	-0.562E-01	-0.217E-01	-0.605E-01	-0.655E-01	-0.861E-01
		IPW	-0.674E-01	-0.804E-01	-0.926E-01	-0.748E-01	-0.652E-01	-0.912E-01	-0.819E-01	-0.941E-01
		FML	-0.209E-01	-0.393E-01	-0.487E-01	-0.305E-01	-0.183E-01	-0.516E-01	-0.346E-01	-0.526E-01
		TSCO	0.681E-01	0.399E-01	-0.100E-01	-0.273E-01	0.166E-01	-0.202E-01	-0.192E-01	-0.426E-01
		ICO	-0.197E-01	-0.391E-01	-0.412E-01	-0.422E-01	-0.217E-01	-0.506E-01	-0.340E-01	-0.505E-01
		INS	0.190E-02	-0.404E-01	-0.179E+00	-0.358E-01	0.184E-01	-0.499E-01	0.101E+00	-0.451E-01
-0.80	-0.80	OLS	0.265E+00	0.170E+00	0.155E+01	0.251E+01	0.134E+01	0.129E+01	0.246E+01	0.256E+01
		TSPW	-0.216E-01	-0.415E-01	-0.951E-01	-0.688E-01	-0.465E-01	-0.548E-01	-0.640E-01	-0.997E-01
		IPW	-0.765E-01	-0.908E-01	-0.121E+00	-0.852E-01	-0.832E-01	-0.829E-01	-0.751E-01	-0.106E+00
		FML	-0.308E-01	-0.508E-01	-0.779E-01	-0.418E-01	-0.359E-01	-0.437E-01	-0.275E-01	-0.658E-01
		TSCO	0.296E-01	-0.558E-02	-0.634E-01	-0.403E-01	-0.767E-02	-0.170E-01	-0.193E-01	-0.565E-01
		ICO	-0.287E-01	-0.523E-01	-0.815E-01	-0.521E-01	-0.395E-01	-0.448E-01	-0.282E-01	-0.627E-01
		INS	-0.100E-01	-0.440E-01	-0.160E+00	-0.455E-01	-0.279E-02	-0.449E-01	0.666E-01	-0.572E-01
-0.40	-0.80	OLS	0.248E+00	0.265E+00	0.130E+01	0.182E+01	0.124E+01	0.127E+01	0.175E+01	0.169E+01
		TSPW	-0.748E-01	-0.800E-01	-0.588E-01	-0.521E-01	-0.930E-01	-0.647E-01	-0.884E-01	-0.965E-01
		IPW	-0.104E+00	-0.109E+00	-0.836E-01	-0.688E-01	-0.113E+00	-0.862E-01	-0.974E-01	-0.103E+00
		FML	-0.617E-01	-0.701E-01	-0.388E-01	-0.248E-01	-0.721E-01	-0.460E-01	-0.516E-01	-0.633E-01
		TSCO	-0.332E-01	-0.449E-01	-0.276E-01	-0.311E-01	-0.535E-01	-0.283E-01	-0.471E-01	-0.540E-01
		ICO	-0.610E-01	-0.732E-01	-0.430E-01	-0.424E-01	-0.720E-01	-0.486E-01	-0.541E-01	-0.597E-01
		INS	-0.326E-01	-0.633E-01	-0.877E-01	-0.317E-01	-0.437E-01	-0.408E-01	0.269E-01	-0.577E-01
0.00	-0.80	OLS	0.685E-01	0.135E+00	0.100E+01	0.136E+01	0.120E+01	0.119E+01	0.111E+01	0.113E+01
		TSPW	-0.961E-01	-0.856E-01	-0.858E-01	-0.673E-01	-0.104E+00	-0.696E-01	-0.844E-01	-0.730E-01
		IPW	-0.128E+00	-0.110E+00	-0.106E+00	-0.836E-01	-0.118E+00	-0.835E-01	-0.936E-01	-0.800E-01
		FML	-0.861E-01	-0.734E-01	-0.642E-01	-0.403E-01	-0.765E-01	-0.444E-01	-0.500E-01	-0.410E-01
		TSCO	-0.570E-01	-0.503E-01	-0.537E-01	-0.525E-01	-0.648E-01	-0.360E-01	-0.424E-01	-0.324E-01
		ICO	-0.870E-01	-0.747E-01	-0.672E-01	-0.635E-01	-0.764E-01	-0.484E-01	-0.499E-01	-0.385E-01
		INS	-0.353E-01	-0.647E-01	-0.854E-01	-0.442E-01	-0.414E-01	-0.397E-01	0.780E-02	-0.330E-01
0.40	-0.80	OLS	0.119E+00	0.741E-01	0.862E+00	0.849E+00	0.903E+00	0.769E+00	0.688E+00	0.575E+00
		TSPW	-0.795E-01	-0.737E-01	-0.110E+00	-0.977E-01	-0.119E+00	-0.104E+00	-0.901E-01	-0.815E-01
		IPW	-0.104E+00	-0.104E+00	-0.124E+00	-0.113E+00	-0.129E+00	-0.113E+00	-0.979E-01	-0.895E-01
		FML	-0.634E-01	-0.667E-01	-0.897E-01	-0.760E-01	-0.943E-01	-0.796E-01	-0.593E-01	-0.533E-01
		TSCO	-0.337E-01	-0.371E-01	-0.712E-01	-0.718E-01	-0.757E-01	-0.716E-01	-0.522E-01	-0.475E-01
		ICO	-0.614E-01	-0.699E-01	-0.816E-01	-0.819E-01	-0.838E-01	-0.794E-01	-0.588E-01	-0.543E-01
		INS	0.469E-02	-0.624E-01	-0.517E-01	-0.746E-01	-0.210E-01	-0.718E-01	-0.440E-01	-0.533E-01

Table 51--Continued.

0.80 -0.80	OLS	-0.817E-01	-0.524E-01	0.593E+00	0.475E+00	0.399E+00	0.366E+00	0.147E+00	0.783E-01
	TSPW	-0.647E-01	-0.830E-01	-0.135E+00	-0.126E+00	-0.137E+00	-0.121E+00	-0.124E+00	-0.114E+00
	IPW	-0.105E+00	-0.110E+00	-0.146E+00	-0.138E+00	-0.145E+00	-0.136E+00	-0.130E+00	-0.127E+00
	FML	-0.671E-01	-0.714E-01	-0.124E+00	-0.123E+00	-0.120E+00	-0.107E+00	-0.109E+00	-0.968E-01
	TSCO	-0.146E-01	-0.468E-01	-0.102E+00	-0.752E-01	-0.101E+00	-0.743E-01	-0.896E-01	-0.988E-01
	ICO	-0.613E-01	-0.780E-01	-0.111E+00	-0.838E-01	-0.107E+00	-0.824E-01	-0.955E-01	-0.108E+00
	INS	0.760E-01	-0.715E-01	-0.106E+00	-0.777E-01	0.163E-01	-0.890E-01	-0.270E+00	-0.965E-01
1.20 -0.80	OLS	-0.208E-01	0.687E-01	0.839E-01	0.573E-01	-0.428E-01	-0.411E-01	-0.456E+00	-0.405E+00
	TSPW	-0.154E-01	-0.267E-01	-0.126E+00	-0.230E+00	-0.145E+00	-0.168E+00	-0.172E+00	-0.195E+00
	IPW	-0.631E-01	-0.637E-01	-0.814E+00	-0.239E+00	-0.145E+00	-0.172E+00	-0.177E+00	-0.237E+00
	FML	-0.359E-01	-0.200E-01	-0.121E+00	-0.257E+00	-0.138E+00	-0.170E+00	-0.192E+00	-0.219E+00
	TSCO	0.309E-01	-0.350E-02	-0.109E+00	-0.134E+00	-0.133E+00	-0.117E+00	-0.168E+00	-0.207E+00
	ICO	-0.324E-01	-0.344E-01	-0.111E+00	-0.144E+00	-0.136E+00	-0.123E+00	-0.186E+00	-0.238E+00
	INS	0.114E+00	-0.356E-01	-0.654E+00	-0.122E+00	-0.170E+00	-0.999E-01	-0.720E+00	-0.185E+00
1.60 -0.80	OLS	-0.322E+00	-0.373E+00	-0.535E+00	-0.401E+00	-0.469E+00	-0.515E+00	-0.420E+00	-0.571E+00
	TSPW	0.174E+00	-0.110E+00	-0.798E+00	-0.316E+00	-0.682E-01	-0.344E+00	-0.380E+00	-0.193E+00
	IPW	0.393E+00	-0.477E-01	-0.703E+00	-0.427E+00	0.867E-01	-0.251E+00	-0.408E+00	-0.265E+00
	FML	-0.191E-01	-0.128E-01	-0.186E-01	-0.329E+00	-0.715E-01	-0.267E+00	-0.226E+00	-0.285E+00
	TSCO	0.141E+00	-0.107E+00	-0.365E+00	-0.326E+00	-0.234E+00	-0.302E+00	-0.161E+00	-0.271E+00
	ICO	-0.433E-01	-0.255E-01	-0.402E+00	-0.547E+00	-0.158E+00	-0.231E+00	-0.233E+00	-0.281E+00
	INS	0.311E+00	-0.296E-01	-0.743E+00	-0.132E+00	0.456E-01	-0.181E+00	-0.737E+00	-0.195E+00
-1.20 -0.40	OLS	0.547E-01	0.606E-01	0.198E+01	0.322E+01	0.235E+01	0.245E+01	0.275E+01	0.338E+01
	TSPW	-0.208E-01	-0.126E-01	-0.254E-01	-0.445E-01	-0.239E-01	-0.576E-01	-0.540E-01	-0.824E-01
	IPW	-0.106E+00	-0.865E-01	-0.808E-01	-0.707E-01	-0.718E-01	-0.942E-01	-0.853E-01	-0.969E-01
	FML	-0.762E-01	-0.571E-01	-0.507E-01	-0.388E-01	-0.410E-01	-0.640E-01	-0.536E-01	-0.662E-01
	TSCO	-0.115E-01	-0.871E-02	0.988E-02	-0.188E-01	0.149E-02	-0.279E-01	-0.226E-01	-0.505E-01
	ICO	-0.731E-01	-0.588E-01	-0.450E-01	-0.412E-01	-0.422E-01	-0.620E-01	-0.513E-01	-0.641E-01
	INS	-0.291E-01	-0.506E-01	-0.133E+00	-0.412E-01	-0.287E-02	-0.603E-01	0.453E-01	-0.582E-01
-0.80 -0.40	OLS	0.152E+00	0.113E+00	0.124E+01	0.155E+01	0.125E+01	0.133E+01	0.150E+01	0.153E+01
	TSPW	-0.404E-01	-0.722E-01	-0.788E-01	-0.818E-01	-0.868E-01	-0.452E-01	-0.666E-01	-0.962E-01
	IPW	-0.802E-01	-0.118E+00	-0.103E+00	-0.100E+00	-0.111E+00	-0.670E-01	-0.792E-01	-0.105E+00
	FML	-0.530E-01	-0.901E-01	-0.732E-01	-0.695E-01	-0.809E-01	-0.368E-01	-0.481E-01	-0.753E-01
	TSCO	-0.161E-01	-0.535E-01	-0.524E-01	-0.604E-01	-0.577E-01	-0.173E-01	-0.369E-01	-0.627E-01
	ICO	-0.535E-01	-0.927E-01	-0.721E-01	-0.735E-01	-0.794E-01	-0.371E-01	-0.474E-01	-0.702E-01
	INS	-0.441E-01	-0.846E-01	-0.106E+00	-0.777E-01	-0.677E-01	-0.351E-01	-0.139E-01	-0.644E-01
-0.40 -0.40	OLS	0.134E+00	0.140E+00	0.787E+00	0.838E+00	0.744E+00	0.755E+00	0.775E+00	0.768E+00
	TSPW	-0.117E+00	-0.100E+00	-0.688E-01	-0.788E-01	-0.929E-01	-0.842E-01	-0.822E-01	-0.102E+00
	IPW	-0.150E+00	-0.131E+00	-0.866E-01	-0.952E-01	-0.110E+00	-0.993E-01	-0.916E-01	-0.109E+00
	FML	-0.126E+00	-0.108E+00	-0.585E-01	-0.675E-01	-0.826E-01	-0.729E-01	-0.639E-01	-0.830E-01
	TSCO	-0.984E-01	-0.834E-01	-0.423E-01	-0.635E-01	-0.639E-01	-0.592E-01	-0.570E-01	-0.742E-01
	ICO	-0.131E+00	-0.115E+00	-0.558E-01	-0.763E-01	-0.789E-01	-0.729E-01	-0.645E-01	-0.802E-01
	INS	-0.117E+00	-0.101E+00	-0.729E-01	-0.777E-01	-0.784E-01	-0.721E-01	-0.445E-01	-0.712E-01
0.00 -0.40	OLS	0.650E-01	0.100E+00	0.354E+00	0.406E+00	0.395E+00	0.401E+00	0.284E+00	0.239E+00
	TSPW	-0.138E+00	-0.130E+00	-0.130E+00	-0.104E+00	-0.127E+00	-0.117E+00	-0.114E+00	-0.145E+00
	IPW	-0.175E+00	-0.174E+00	-0.147E+00	-0.119E+00	-0.140E+00	-0.133E+00	-0.122E+00	-0.153E+00
	FML	-0.152E+00	-0.149E+00	-0.124E+00	-0.957E-01	-0.116E+00	-0.110E+00	-0.100E+00	-0.133E+00
	TSCO	-0.121E+00	-0.108E+00	-0.101E+00	-0.795E-01	-0.111E+00	-0.902E-01	-0.980E-01	-0.127E+00
	ICO	-0.161E+00	-0.154E+00	-0.114E+00	-0.912E-01	-0.121E+00	-0.100E+00	-0.104E+00	-0.132E+00
	INS	-0.144E+00	-0.142E+00	-0.115E+00	-0.922E-01	-0.103E+00	-0.960E-01	-0.897E-01	-0.126E+00

Table 51--Continued.

0.40 -0.40	OLS	0.332E-01	0.259E-02	0.154E+00	0.459E-01	0.864E-01	0.400E-01	-0.104E+00	-0.818E-01
	TSPW	-0.150E+00	-0.156E+00	-0.143E+00	-0.211E+00	-0.156E+00	-0.188E+00	-0.178E+00	-0.147E+00
	IPW	-0.186E+00	-0.191E+00	-0.151E+00	-0.229E+00	-0.159E+00	-0.201E+00	-0.182E+00	-0.152E+00
	FML	-0.161E+00	-0.168E+00	-0.134E+00	-0.218E+00	-0.142E+00	-0.186E+00	-0.171E+00	-0.140E+00
	TSCO	-0.128E+00	-0.130E+00	-0.114E+00	-0.121E+00	-0.133E+00	-0.138E+00	-0.156E+00	-0.146E+00
	ICO	-0.167E+00	-0.165E+00	-0.125E+00	-0.132E+00	-0.140E+00	-0.144E+00	-0.160E+00	-0.153E+00
	INS	-0.137E+00	-0.156E+00	-0.916E-01	-0.141E+00	-0.114E+00	-0.146E+00	-0.187E+00	-0.148E+00
0.80 -0.40	OLS	-0.756E-01	-0.688E-01	-0.138E+00	-0.240E+00	-0.236E+00	-0.277E+00	-0.384E+00	-0.362E+00
	TSPW	-0.146E+00	-0.135E+00	-0.194E+00	-0.334E+00	-0.227E+00	-0.291E+00	-0.214E+00	-0.167E+00
	IPW	-0.147E+00	-0.144E+00	-0.489E+00	-0.340E+00	-0.218E+00	-0.295E+00	-0.237E+00	-0.202E+00
	FML	-0.123E+00	-0.122E+00	-0.172E+00	-0.343E+00	-0.206E+00	-0.293E+00	-0.229E+00	-0.186E+00
	TSCO	-0.118E+00	-0.114E+00	-0.353E+00	-0.194E+00	-0.204E+00	-0.201E+00	-0.197E+00	-0.202E+00
	ICO	-0.120E+00	-0.115E+00	-0.690E+00	-0.206E+00	-0.225E+00	-0.221E+00	-0.217E+00	-0.222E+00
	INS	-0.647E-01	-0.105E+00	-0.282E+00	-0.185E+00	-0.171E+00	-0.208E+00	-0.397E+00	-0.215E+00
1.20 -0.40	OLS	-0.313E+00	-0.364E+00	-0.557E+00	-0.569E+00	-0.608E+00	-0.625E+00	-0.444E+00	-0.442E+00
	TSPW	-0.854E-01	-0.123E+00	-0.818E+00	-0.447E+00	-0.338E+00	-0.438E+00	-0.159E+00	-0.166E+00
	IPW	-0.759E-01	-0.102E+00	-0.781E+00	-0.405E+00	-0.233E+00	-0.367E+00	-0.200E+00	-0.320E+00
	FML	-0.744E-01	-0.753E-01	-0.204E+00	-0.432E+00	-0.264E+00	-0.379E+00	-0.273E+00	-0.300E+00
	TSCO	-0.656E-01	-0.906E-01	-0.453E+00	-0.442E+00	-0.410E+00	-0.404E+00	-0.220E+00	-0.205E+00
	ICO	-0.735E-01	-0.731E-01	-0.722E+00	-0.730E+00	-0.368E+00	-0.387E+00	-0.305E+00	-0.302E+00
	INS	0.738E-01	-0.796E-01	-0.619E+00	-0.239E+00	-0.199E+00	-0.305E+00	-0.622E+00	-0.197E+00
-0.80 0.00	OLS	0.181E+00	0.198E+00	0.138E+01	0.166E+01	0.150E+01	0.148E+01	0.148E+01	0.169E+01
	TSPW	-0.104E+00	-0.769E-01	-0.740E-01	-0.108E+00	-0.997E-01	-0.890E-01	-0.127E+00	-0.941E-01
	IPW	-0.156E+00	-0.136E+00	-0.112E+00	-0.129E+00	-0.128E+00	-0.112E+00	-0.143E+00	-0.104E+00
	FML	-0.133E+00	-0.113E+00	-0.947E-01	-0.110E+00	-0.112E+00	-0.949E-01	-0.126E+00	-0.857E-01
	TSCO	-0.895E-01	-0.606E-01	-0.684E-01	-0.890E-01	-0.753E-01	-0.715E-01	-0.109E+00	-0.732E-01
	ICO	-0.142E+00	-0.117E+00	-0.103E+00	-0.106E+00	-0.105E+00	-0.944E-01	-0.124E+00	-0.827E-01
	INS	-0.118E+00	-0.112E+00	-0.103E+00	-0.110E+00	-0.949E-01	-0.916E-01	-0.951E-01	-0.824E-01
-0.40 0.00	OLS	0.552E-01	0.628E-01	0.451E+00	0.459E+00	0.433E+00	0.495E+00	0.462E+00	0.471E+00
	TSPW	-0.145E+00	-0.151E+00	-0.133E+00	-0.136E+00	-0.148E+00	-0.126E+00	-0.129E+00	-0.125E+00
	IPW	-0.187E+00	-0.197E+00	-0.147E+00	-0.148E+00	-0.159E+00	-0.139E+00	-0.135E+00	-0.131E+00
	FML	-0.171E+00	-0.180E+00	-0.133E+00	-0.134E+00	-0.146E+00	-0.124E+00	-0.121E+00	-0.116E+00
	TSCO	-0.138E+00	-0.138E+00	-0.125E+00	-0.115E+00	-0.124E+00	-0.116E+00	-0.115E+00	-0.111E+00
	ICO	-0.185E+00	-0.189E+00	-0.139E+00	-0.127E+00	-0.137E+00	-0.129E+00	-0.122E+00	-0.117E+00
	INS	-0.166E+00	-0.175E+00	-0.137E+00	-0.139E+00	-0.143E+00	-0.119E+00	-0.113E+00	-0.110E+00
0.00 0.00	OLS	-0.327E-01	0.230E-02	-0.261E-01	0.381E-02	-0.259E-01	-0.106E-01	-0.141E-02	-0.118E-01
	TSPW	-0.179E+00	-0.152E+00	-0.208E+00	-0.171E+00	-0.194E+00	-0.193E+00	-0.144E+00	-0.155E+00
	IPW	-0.223E+00	-0.209E+00	-0.215E+00	-0.180E+00	-0.203E+00	-0.206E+00	-0.148E+00	-0.160E+00
	FML	-0.208E+00	-0.191E+00	-0.207E+00	-0.172E+00	-0.195E+00	-0.197E+00	-0.141E+00	-0.152E+00
	TSCO	-0.166E+00	-0.145E+00	-0.172E+00	-0.146E+00	-0.178E+00	-0.170E+00	-0.139E+00	-0.152E+00
	ICO	-0.216E+00	-0.206E+00	-0.180E+00	-0.167E+00	-0.203E+00	-0.184E+00	-0.144E+00	-0.158E+00
	INS	-0.204E+00	-0.185E+00	-0.191E+00	-0.151E+00	-0.181E+00	-0.180E+00	-0.139E+00	-0.151E+00
0.40 0.00	OLS	-0.123E+00	-0.118E+00	-0.259E+00	-0.333E+00	-0.320E+00	-0.347E+00	-0.285E+00	-0.284E+00
	TSPW	-0.176E+00	-0.152E+00	-0.229E+00	-0.340E+00	-0.263E+00	-0.313E+00	-0.194E+00	-0.175E+00
	IPW	-0.210E+00	-0.193E+00	-0.216E+00	-0.338E+00	-0.253E+00	-0.329E+00	-0.195E+00	-0.182E+00
	FML	-0.195E+00	-0.176E+00	-0.217E+00	-0.339E+00	-0.252E+00	-0.313E+00	-0.196E+00	-0.180E+00
	TSCO	-0.165E+00	-0.133E+00	-0.281E+00	-0.281E+00	-0.271E+00	-0.247E+00	-0.185E+00	-0.201E+00
	ICO	-0.202E+00	-0.176E+00	-0.562E+00	-0.548E+00	-0.354E+00	-0.323E+00	-0.188E+00	-0.206E+00
	INS	-0.180E+00	-0.160E+00	-0.171E+00	-0.248E+00	-0.238E+00	-0.282E+00	-0.225E+00	-0.191E+00

Table 51--Continued.

0.80	0.00	OLS	-0.310E+00	-0.379E+00	-0.551E+00	-0.619E+00	-0.606E+00	-0.644E+00	-0.335E+00	-0.322E+00
		TSPW	-0.160E+00	-0.195E+00	-0.355E+00	-0.482E+00	-0.411E+00	-0.481E+00	-0.162E+00	-0.103E+00
		IPW	-0.163E+00	-0.173E+00	-0.544E+00	-0.449E+00	-0.356E+00	-0.424E+00	-0.214E+00	-0.173E+00
		FML	-0.143E+00	-0.158E+00	-0.341E+00	-0.475E+00	-0.362E+00	-0.439E+00	-0.224E+00	-0.157E+00
		TSCO	-0.135E+00	-0.149E+00	-0.553E+00	-0.549E+00	-0.440E+00	-0.442E+00	-0.165E+00	-0.142E+00
		ICO	-0.137E+00	-0.144E+00	-0.786E+00	-0.792E+00	-0.468E+00	-0.466E+00	-0.193E+00	-0.167E+00
		INS	-0.493E-01	-0.143E+00	-0.374E+00	-0.360E+00	-0.331E+00	-0.404E+00	-0.346E+00	-0.161E+00
-0.40	0.40	OLS	0.168E+00	0.144E+00	0.467E+00	0.447E+00	0.480E+00	0.375E+00	0.677E+00	0.618E+00
		TSPW	-0.153E+00	-0.131E+00	-0.159E+00	-0.186E+00	-0.194E+00	-0.228E+00	-0.142E+00	-0.173E+00
		IPW	-0.189E+00	-0.179E+00	-0.172E+00	-0.193E+00	-0.205E+00	-0.236E+00	-0.148E+00	-0.176E+00
		FML	-0.174E+00	-0.166E+00	-0.179E+00	-0.196E+00	-0.210E+00	-0.238E+00	-0.141E+00	-0.169E+00
		TSCO	-0.144E+00	-0.119E+00	-0.193E+00	-0.217E+00	-0.197E+00	-0.232E+00	-0.141E+00	-0.167E+00
		ICO	-0.186E+00	-0.178E+00	-0.207E+00	-0.224E+00	-0.220E+00	-0.270E+00	-0.148E+00	-0.172E+00
		INS	-0.165E+00	-0.164E+00	-0.188E+00	-0.201E+00	-0.206E+00	-0.238E+00	-0.128E+00	-0.164E+00
0.00	0.40	OLS	-0.112E+00	-0.113E+00	-0.299E+00	-0.282E+00	-0.298E+00	-0.281E+00	-0.478E-01	-0.865E-01
		TSPW	-0.174E+00	-0.159E+00	-0.319E+00	-0.288E+00	-0.319E+00	-0.275E+00	-0.144E+00	-0.174E+00
		IPW	-0.209E+00	-0.187E+00	-0.316E+00	-0.283E+00	-0.317E+00	-0.274E+00	-0.145E+00	-0.176E+00
		FML	-0.196E+00	-0.177E+00	-0.325E+00	-0.294E+00	-0.325E+00	-0.281E+00	-0.141E+00	-0.172E+00
		TSCO	-0.169E+00	-0.145E+00	-0.367E+00	-0.361E+00	-0.301E+00	-0.285E+00	-0.152E+00	-0.180E+00
		ICO	-0.210E+00	-0.181E+00	-0.643E+00	-0.734E+00	-0.387E+00	-0.355E+00	-0.154E+00	-0.182E+00
		INS	-0.193E+00	-0.174E+00	-0.313E+00	-0.281E+00	-0.323E+00	-0.274E+00	-0.141E+00	-0.171E+00
0.40	0.40	OLS	-0.286E+00	-0.318E+00	-0.546E+00	-0.591E+00	-0.587E+00	-0.625E+00	-0.156E+00	-0.187E+00
		TSPW	-0.165E+00	-0.183E+00	-0.367E+00	-0.436E+00	-0.424E+00	-0.462E+00	-0.111E+00	-0.943E-01
		IPW	-0.171E+00	-0.203E+00	-0.341E+00	-0.409E+00	-0.392E+00	-0.431E+00	-0.129E+00	-0.111E+00
		FML	-0.159E+00	-0.189E+00	-0.375E+00	-0.437E+00	-0.418E+00	-0.450E+00	-0.115E+00	-0.981E-01
		TSCO	-0.160E+00	-0.152E+00	-0.631E+00	-0.612E+00	-0.450E+00	-0.474E+00	-0.104E+00	-0.115E+00
		ICO	-0.176E+00	-0.177E+00	-0.811E+00	-0.791E+00	-0.483E+00	-0.513E+00	-0.113E+00	-0.122E+00
		INS	-0.135E+00	-0.181E+00	-0.342E+00	-0.368E+00	-0.388E+00	-0.442E+00	-0.133E+00	-0.107E+00
0.00	0.80	OLS	-0.146E+00	-0.159E+00	-0.470E+00	-0.430E+00	-0.483E+00	-0.461E+00	0.328E+00	0.239E+00
		TSPW	-0.125E+00	-0.155E+00	-0.481E+00	-0.386E+00	-0.488E+00	-0.444E+00	-0.874E-01	-0.949E-01
		IPW	-0.101E+00	-0.133E+00	-0.468E+00	-0.368E+00	-0.468E+00	-0.421E+00	-0.908E-01	-0.966E-01
		FML	-0.863E-01	-0.122E+00	-0.505E+00	-0.408E+00	-0.504E+00	-0.455E+00	-0.712E-01	-0.802E-01
		TSCO	-0.103E+00	-0.131E+00	-0.618E+00	-0.653E+00	-0.505E+00	-0.492E+00	-0.106E+00	-0.999E-01
		ICO	-0.959E-01	-0.128E+00	-0.807E+00	-0.801E+00	-0.534E+00	-0.515E+00	-0.108E+00	-0.101E+00
		INS	-0.895E-01	-0.129E+00	-0.520E+00	-0.414E+00	-0.504E+00	-0.442E+00	-0.665E-01	-0.855E-01

[a]White noise independent variable, stationary errors.
[b]White noise independent variable, nonstationary errors.
[c]Dampened trend independent variable, stationary errors.
[d]Dampened trend independent variable, nonstationary errors.
[e]Growth trend independent variable, stationary errors.
[f]Growth trend independent variable, nonstationary errors.
[g]Dampened cyclical trend independent variable, stationary errors.
[h]Dampened cyclical trend independent variable, nonstationary errors.